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A TEXT BOOK OF PHYSICS

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TEXT BOOK OF PHYSICS

FOR THE USE OF STUDENTS OF
SCIENCE & ENGINEERING

BY

J. DUNCAN, Wh. Ex., M I.Mech E.

AND

S. G. STARLING, B.Sc., A.R C.Sc.

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A TEXT BOOK OF PHYSICS

FOR THE USE OF STUDENTS OF SCIENCE AND ENGINEERING

BY

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PREFACE

THE preparation of this volume was undertaken to meet a demand that has been growing for some years past for a text-book of Physical Science which should connect more intimately than has hitherto been usual the scientific aspects of Physics with its modern practical applications. The reader must be left to judge how far the authors have succeeded in thus combining the outlooks of the man of science and the engineer.

The contents have been selected to meet the requirements of various classes of students those preparing for Intermediate and other examinations of London and other Universities, and those entering for appointments in the Army, Navy, and Civil Service, or offering themselves for examination in Electrical Engineering (Grade I.) by the City and Guilds of London Institute

The book has been arranged in parts, in accordance with the divisions of the subject found convenient in most schools and colleges. Part I., Dynamics, comprises the sections of Mechanics and Applied Mathematics usually studied, and includes sections on motion, statics, and the properties of fluids. Part II., Heat; Part III., Light; Part IV, Sound; and Part V., Magnetism and Electricity, deal respectively with the principles of these subjects and their applications.

Complete courses of laboratory work have been provided in each Part. Many physical laboratories are equipped with apparatus differing in some respects from the instruments here described, nevertheless the guidance given will enable intelligent use to be made of other forms of apparatus designed for the same or similar purposes.

Attention is directed to the experimental treatment of dynamical principles, because its neglect, which is unfortunately common,

makes it difficult for a student to secure a thorough and systematic knowledge of physical science. The complete course of experimental work has been devised to meet both the requirements of the physicist and of the engineer, in cases where the methods of treatment adopted by these differ radically, the teacher or student may choose the experiment which best suits his special needs.

In Part V, the treatment of the Dynamo, Telegraph, and so on, is that which follows naturally and logically from the earlier theoretical principles explained; technical considerations of design and construction have been omitted as unsuitable in a text-book of Physics.

A large number of worked-out examples have been included to assist the student to understand the text and to solve the exercises at the ends of the chapters. Many of these exercises have been taken, with the permission of the authorities to whom grateful acknowledgments are made, from examination papers, the source being given in each case. Questions marked L.U. are from examination papers of the London University and those with C.G. from papers of the City and Guilds of London Institute.

Answers have been supplied in the case of numerical exercises, but it is too much to hope that these are entirely free from errors. The authors will welcome any corrections which readers may send to them.

The authors are glad of this opportunity to express their indebtedness to Prof. Sir Richard Gregory and Mr. A. T. Simmons for constant assistance and invaluable hints while the book was in preparation and passing through the Press.

J. DUNCAN.
S. G. STARLING.

1918.

In response to requests from many sources, an additional Chapter has been inserted on Wireless Telegraphy.

J. D.
S. G. S.

1919.

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PART I

DYNAMICS

CHAPTER I

INTRODUCTORY

Preliminary definitions.—**Dynamics** is that branch of physical science which investigates the behaviour of **matter** under the action of **force**.

It must suffice here to explain what is meant by matter by reference to some of its properties, of which the most obvious are, (i) it always occupies space, (ii) it always possesses weight when in the neighbourhood of the earth. A **body** is any definite portion of matter.

Force is push or pull exerted on a body, and may alter the state of motion by causing the speed of the body to increase or decrease continuously, or by producing a continuous change in the direction of motion. Our earliest appreciation of force comes usually by reason of the muscular effort which has to be exerted in sustaining the weight of a body.

Statics is that branch of the subject dealing with cases in which the forces do not produce any change in the motion of the body to which they are applied. **Kinetics** includes all problems in which change of motion occurs as a consequence of the application of force to the body. Another subdivision called **Kinematics** deals with the mere geometry of motion without reference to the applied force.

In another nomenclature in common use, the name **mechanics** is given to the entire subject, and **dynamics** to that branch in which the applied forces produce changes in the motion of the body; in this nomenclature **statics** and **kinematics** have the signification defined above.

Fundamental units.—The fundamental units—to which are referred all measurements in any scientific system—are those of **length, mass** and **time**.

The metric unit of length is the **metre**, and may be defined as the distance, under certain conditions, between the ends of a standard

bar preserved in Paris. Other practical units are the *centimetre* (0·01 metre, written one cm.), the *millimetre* (0·001 metre, written one mm.) and the *kilometre* (1000 metres).

The British unit of length is the **foot**, which is one-third of the standard *yard*. The latter may be defined as the distance between two marks on a standard bar preserved in London. The *inch* (one-twelfth of a foot) and the *mile* (5280 feet) are other practical units. One inch equals 2·539 cm., and one metre equals 39·37 inches. For convenience in showing dimensions in drawings, lengths such as 3 feet 5 inches are written 3′-5″.

Units used in measuring areas are produced by taking squares having sides equal to any of the units of length mentioned above, and are described as the square centimetre, the square inch, etc.

In measuring volumes, units are obtained by taking cubes having edges equal to any of the units of length, and are described as the cubic centimetre (written one c.c.), the cubic inch, etc. Other units of volume are the **litre** (1000 c.c., equal to 1·762 pint). the **gallon** (0·1605 cubic foot, or 8 pints, or 4·541 litres) and the **pint.**

Mass means quantity of matter. The metric unit of mass was intended to be the quantity of matter contained in a cubic centimetre of pure water at a temperature of 4 degrees Centigrade, but is actually one-thousandth of the mass of a piece of platinum preserved in Paris; this unit is called one **gram.** The *kilogram* (1000 grams) is another unit in common use. The British unit of mass is called the **pound avoirdupois,** and is the quantity of matter contained in a standard piece of platinum preserved in London. The *ton* (2240 pounds) is also used often. One gallon of fresh water has a mass of 10 pounds. One pound equals 453·6 grams.

The unit of time employed in all scientific systems is the **second,** which is derived from the mean solar day, *i.e.* the average time elapsing between two successive passages of the sun across the meridian of any one place on the surface of the earth.

It will be noted that the units of length, mass and time, on being once stated for any system of scientific measurement, remain invariable. Owing to the three metric units in common use being the centimetre, the gram, and the second, the name **C.G.S. system** is used more frequently than the term metric system.

Density.—The **density** of a given material means the mass contained in unit volume of the material. In the C.G.S. system it is

customary to measure density in grams per cubic centimetre; in the British system densities are stated usually in pounds per cubic foot or per cubic inch.

Let V = the volume of a body,
d = the density of the material,
m = the mass of the body.

Then $m = Vd$,

or $d = \frac{m}{V}$.

AVERAGE DENSITIES OF COMMON MATERIALS.*

MATERIAL	DENSITY. Grams per c c	DENSITY. Pounds per cubic ft	MATERIAL	DENSITY. Grams per c c	DENSITY. Pounds per cubic ft
Aluminium	2·65	161	Cork	0·21	15
Brass	8·6	535	Deal	0·6	37·5
Copper	8·93	555	Ebony	1·2	75
Gold	19·32	1200	Oak	0·8	50
Gunmetal	8·2	510	Pitch pine	0·65	41
Iron, Cast	7·2	450			
,, Wrought	7·8	480	Granite	2·7	168
Lead	11·37	710	Marble	2·6	162
Platinum	21·5	1340	Sandstone	2·25	140
Silver	10·5	655			
Steel	7·8	480	Glass, Flint	3·7	230
Tin	7·29	455	,, Crown	2·5	156
Fresh water	1·0	62·3	Indiarubber	0·95	59
Sea water	1·03	64	Leather	0·9	56

Dimensions of a quantity. The dimensions of any physical quantity may be stated in terms of the fundamental units. Using the symbols l, m and t to denote length, mass and time respectively, the dimensions of area, volume and density will be l^2, l^3 and m/l^3 respectively.

EXAMPLE.—Suppose that in obtaining a certain result, the final calculation takes the form

$$\frac{12\ (\text{grams}) \times 3\ (\text{cm.}) \times 3\ (\text{cm})}{6\ \text{cm.} \times 2\ (\text{sec.}) \times 2\ (\text{sec.})}.$$

*For fuller Tables of densities, see *Physical and Chemical Constants*, by Kaye and Laby. Longmans.

The numerical result is 4·5. To obtain the dimensions, cancel corresponding bracketed quantities in the numerator and denominator, giving:

$$\frac{\text{grams} \times \text{cm}}{\text{sec.} \times \text{sec}},$$

or,

$$\frac{ml}{t^2}.$$

It will be seen later that this result indicates a force.

Gravitation.—There is a universal tendency of every body to move towards every other body; every particle of matter attracts every other particle towards itself with a force in the direction of the line joining the particles. The forces of attraction between bodies of small or moderate size are very small, but, when one or both bodies is large, the forces become evident without the necessity for employing delicate means for their detection. What we call the **weight** of a body is really the attractive force which the earth exerts on the body, tending to cause the body to approach the earth's centre. The term **gravitation** is applied to this universal attraction.

Gravitational effect takes place over immense distances, thus the force of attraction which the sun exerts on the earth causes the earth to describe an orbit round the sun. The force of attraction between two small bodies is proportional to the product of their masses, and is inversely proportional to the square of the distance between them. Expressed algebraically:

$$F \propto \frac{m_1 m_2}{d^2},$$

where F is the force, m_1 and m_2 are the masses of the bodies and d is the distance between them. We may also write

$$F = k\frac{m_1 m_2}{d^2},$$

in which k is a numerical constant called the **constant of gravitation.** The value of k is about $6{\cdot}65 \times 10^{-8}$, expressed in C.G.S. units,* hence, expressed in dynes (pp. 8, 67),

$$F = 6{\cdot}65 \times 10^{-8}\frac{m_1 m_2}{d^2} \text{ dynes.}$$

Weight.—The **weight** of any given body varies somewhat, depending on the latitude of the place where the observation is made, and

*C. V. Boys, *Proc. R. Soc.*, London, 1894. The mean density of the earth has been determined and is given by Boys to be 5·527, or approximately 5½ times that of water.

on the distance of the body above or below the surface of the earth. Weight is always directed vertically downwards.

Equal masses situated at the same place possess equal weights. It follows from this fact that a common **balance** (Fig. 1) may be used for obtaining a body having a mass equal to any standard mass. A standard mass may be placed in the scale pan A, and material may be added to, or taken away from, the scale pan B until the weights acting on A and B are equal, as will be evidenced by the balance beam CD becoming horizontal, or vibrating so that it describes small equal angles above and below the horizontal. The mass in A will then be equal to that in B. The use of such a balance is facilitated by a vertical pointer fixed to the beam and vibrating over a graduated scale. Assuming that the balance is properly adjusted, the weights are equal when the pointer swings through equal angles on each side of the middle division.

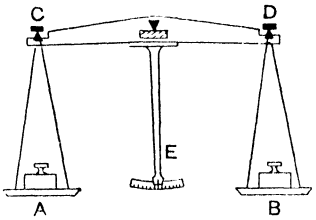
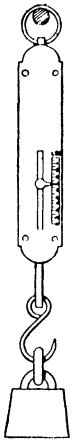


FIG. 1.—A common balance.

Standard masses ranging from 1 kilogram to 0·01 gram, and from 1 pound to 0·001 pound are provided in most laboratories. These are generally called **sets of weights**; the operation involved in using them is described as **weighing**.


FIG. 2.—Spring balance.

Units of force.—For many practical purposes the weight of the unit of mass is employed as a unit of force. As has been explained, this weight is variable, hence the unit is not strictly scientific. The name **gravitational unit of force** is given to any force unit based on weight. The C.G.S. and British gravitational force units are respectively the weight of one gram mass, written **one gram weight**, and the weight of the pound mass, written **one lb. weight**. The kilogram weight and the ton weight are other convenient gravitational units of force.

A common balance cannot be used for showing the variation in weight of a body. **Spring balances** (Fig. 2), if of sufficiently delicate construction, might be employed for this purpose. It is known that a helical spring extends by amounts proportional to the pull applied, and in spring balances

advantage is taken of this property. The body to be weighed is hung from the spring, and the extension is indicated by a pointer moving over a scale. For convenience, the scale is graduated in gram-weight or lb.-weight units, so as to enable the weight to be read direct. Such a balance will give correct readings of weight at the place where the scale was graduated, but, if used in a different latitude, will give a different reading when the same body is suspended from the balance. It may be noted that the variation in weight all over the earth is very small.

Absolute units of force are based on the fundamental units of length, mass and time, and are therefore invariable. The absolute unit of force in any system is the force which, if applied during one second to a body of unit mass, initially at rest, will give to the body a velocity of one unit of length per second. The C.G.S. absolute force unit is called the **dyne**, one dyne applied to one gram mass during one second will produce a velocity of one centimetre per second. The British absolute force unit is the **poundal**, and, if applied to a one pound mass during one second, will produce a velocity of one foot per second. These units will be referred to later and explained more fully.

Mathematical formulae. The following mathematical notes are given for reference. It is assumed that the student has studied the principles involved, or that he is doing so conjointly with his course in physics.

MENSURATION.

Determination of areas.

Square, side s; area $= s^2$.

Rectangle, adjacent sides a and b; area $= ab$.

Triangle, base b, perpendicular height h; area $= \frac{1}{2}bh$.

Triangle, sides a, b and c. $s = (a+b+c)/2$.

$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}.$$

Parallelogram, area = one side × perpendicular distance from that side to the opposite one.

Any irregular figure bounded by straight lines; split it up into triangles, find the area of each separately and take the sum.

Trapezoid; area = half the sum of the end ordinates × the base.

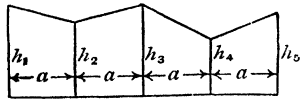


FIG. 3.—Trapezoidal figure.

A trapezoidal figure having equal intervals (Fig. 3);

$$\text{area} = a\left(\frac{h_1+h_5}{2}+h_2+h_3+h_4\right).$$

Simpson's rule for the area bounded by a curve (Fig. 4); take an odd number (say 7) of equidistant ordinates; then

$$\text{area} = \frac{a}{3}(h_1 + 4h_2 + 2h_3 + 4h_4 + 2h_5 + 4h_6 + h_7).$$

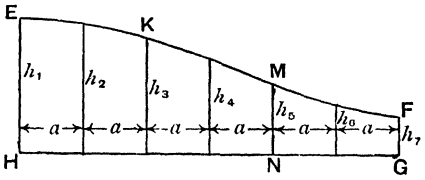


FIG 4.—Illustration of Simpson's rule

Circle, radius r, diameter d; area $= \pi r^2 = \frac{\pi d^2}{4}$.

(Circumference $= 2\pi r = \pi d$.)

Parabola, vertex at O (Fig. 5); area OBC $= \frac{2}{3}ab$.

Cylinder, diameter d, length l; area of curved surface $= \pi dl$.

Sphere, diameter d, radius r; area of curved surface $= \pi d^2 = 4\pi r^2$.

Cone; area of curved surface = circumference of base $\times \frac{1}{2}$ slant height.

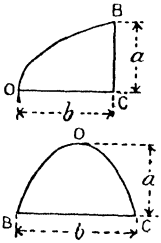


FIG 5.—Area under a parabola.

Determination of volumes.

Cube, edge s; volume $= s^3$.

Cylinder or prism, having its ends perpendicular to its axis; volume = area of one end × length of cylinder or prism.

Sphere, radius r; volume $= \frac{4}{3}\pi r^3$.

Cone or pyramid; volume = area of base $\times \frac{1}{3}$ perpendicular height.

Frustum of a cone; volume $= 0{\cdot}2618\mathrm{H}(\mathrm{D}^2 + d^2 + \mathrm{D}d)$, where D, d are the diameters of the ends and H is the perpendicular height.

TRIGONOMETRY.

A **degree** is the angle subtended at the centre of a circle by an arc of $\frac{1}{360}$th of the circumference. A **minute** is one-sixtieth of a degree, and a **second** is one-sixtieth of a minute. An angle of 42 degrees, 35 minutes, 12 seconds is written 42° 35′ 12″.

A **radian** is the angle subtended at the centre of a circle by an arc equal to the radius of the circle.

There are 2π radians in a complete circle, hence

2π radians = 360 degrees.

π ,, = 180 ,,

Let l be the length of arc subtended by an angle, and let r be the radius of the circle, both in the same units; then angle $= l/r$ radians.

Trigonometrical ratios. In Fig. 6 let OB revolve anti-clockwise about O, and let it stop successively in positions OP_1, OP_2, OP_3, OP_4; the angles described by OB are said to be as follows:

P_1OB, in the first quadrant COB.
P_2OB, in the second quadrant COA
P_3OB (greater than 180°), in the third quadrant AOD.
P_4OB (greater than 270°), in the fourth quadrant BOD.

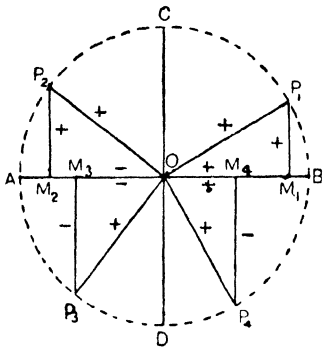


FIG 6.—Trigonometrical ratios

Drop perpendiculars such as P_1M_1 from each position of P on to AB. OP is always regarded as positive; OM is positive if on the right and negative if on the left of O; PM is positive if above and negative if below AB.

Name of ratio.	Ratio as written	Value of ratio	Algebraic sign of ratio			
			1st quad	2nd quad	3rd quad	4th quad
sine POM	sin POM	$\frac{PM}{OP}$	+	+	−	−
cosine POM	cos POM	$\frac{OM}{OP}$	+	−	−	+
tangent POM	tan POM	$\frac{PM}{OM}$	+	−	+	−
cosecant POM	cosec POM	$\frac{OP}{PM}$	+	+	−	−
secant POM	sec POM	$\frac{OP}{OM}$	+	−	−	+
cotangent POM	cot POM	$\frac{OM}{PM}$	+	−	+	−

The values of the ratios are not affected by the length of the radius OP.

The following formulae are given for reference:

$$\text{cosec } A = \frac{1}{\sin A}; \quad \sec A = \frac{1}{\cos A}; \quad \cot A = \frac{1}{\tan A}.$$

$$\tan A = \frac{\sin A}{\cos A}; \quad \cot A = \frac{\cos A}{\sin A}; \quad \cos^2 A + \sin^2 A = 1.$$

$$\sin A = \cos (90^\circ - A); \quad \sin A = \sin (180^\circ - A).$$

$$\sin (A + B) = \sin A \cos B + \cos A \sin B.$$

$$\cos (A + B) = \cos A \cos B - \sin A \sin B.$$

$$\sin (A - B) = \sin A \cos B - \cos A \sin B.$$

$$\cos (A - B) = \cos A \cos B + \sin A \sin B.$$

$$\tan (A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}.$$

$$\tan (A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}.$$

If the angles of a triangle are A, B and C, and the sides opposite these angles are a, b and c respectively, the following relations hold:

$$a = b \cos C + c \cos B.$$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}.$$

$$a^2 = b^2 + c^2 - 2bc \cos A.$$

Exercises on Chapter I.

1. Given that 1 metre = 39·37 inches, obtain a factor for converting miles to kilometres; use the factor to convert 3 miles 15 chains to kilometres. (80 chains = 1 mile.)

2. Convert 2·94 metres to feet and inches.

3. Draw a triangle having sides $4\frac{1}{2}$, $3\frac{1}{4}$, and $5\frac{3}{4}$ inches respectively. Measure its perpendicular height, and calculate the area from this and the length of the base. Check the result by use of the formula

$$\sqrt{s(s-a)(s-b)(s-c)}.$$

4. A thin circular sheet of iron has a diameter of 14 cm. Find its area, taking $\pi = \frac{22}{7}$. If the material weighs 0·1 kilogram per square metre, find the weight of the sheet.

5. Calculate the volume of a ball 9 inches in diameter. Find the mass in pounds if the material has a density of 450 pounds per cubic foot.

6. A masonry wall is trapezoidal in section, one face of the wall being vertical. Height of wall, 20 feet; thickness at top, 4 feet; thickness at base, 9 feet. The masonry weighs 150 lb. per cubic foot. Find the weight of a portion of the wall 1 foot in length.

7. A trapezoidal figure, having equal intervals of 10 cm. each, has ordinates in cm. as follows: 0, 100, 140, 120, 80, 0. Find the total area in sq. cm.

8. Draw a parabolic curve on a base $a = 60$ feet; the height y feet of the curve at any distance x from one end of the base is given by $y = 2x - \frac{x^2}{30}$. Find the area by application of Simpson's rule; check the result by use of the rule: area $= \frac{2}{3}ab$, where b is the maximum height of the curve.

9. Find the weight of a solid pyramid of lead, having a square base of 4 inches edge and a vertical height of 8 inches. Lead weighs 0·41 lb. per cubic inch.

10. A hollow conical vessel has an internal diameter of 6 inches at the top and is 9 inches deep inside. Calculate the weight of water which it can contain. Water weighs 0·036 lb. per cubic inch.

11. Calculate the diameter of a solid ball of cast iron so that the weight may be 90 lb. The material weighs 0·26 lb. per cubic inch.

12. Three small bodies, A, B and C, of masses 2, 3 and 4 grams respectively, are arranged at the corners of a triangle having sides AB = 8 cm., BC = 12 cm., CA = 10 cm. Compare the gravitational efforts which A exerts on B, A exerts on C, C exerts on B.

13. If the distance from the earth to the moon is 240,000 miles, and from the sun to the moon is ninety million miles, determine the ratio of the gravitational forces of the sun and earth upon the moon, having given that the mass of the sun is 330,000 times that of the earth.

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14. Distinguish between mass and weight. How are the mass and weight of a body affected by (*a*) variations of latitude, (*b*) variations of altitude?

If a very delicate balance is required for a laboratory near the top of a high mountain, would you advise having the weights specially adjusted for that altitude? Give careful reasons for your answer.

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15. What is meant by *weight*? Explain why a very delicate spring balance would show slight differences in the weight of a body at different places on the earth, though a common balance would give no indication of any differences. L.U.

*

CHAPTER II

SIMPLE MEASUREMENTS AND MEASURING APPLIANCES

Introductory experiments.—The experiments described in this chapter are intended to render the student familiar with the use of simple measuring appliances.

EXPT. 1.—**Scales.** Laboratory scales have generally one edge graduated in centimetres subdivided to millimetres, and the other edge graduated in inches subdivided to tenths. Reproduce a portion of one of these scales in the following manner: Take a strip of cardboard of suitable width and rule lines lengthwise on it, agreeing with those on the scale. Arrange the scale and the cardboard end to end on the bench, and fasten them to prevent slipping. Set a beam compass to a radius of about 40 cm. The compass should have a hard pencil with a sharp chisel point, or a drawing pen charged with Indian ink. Stand the needle leg of the compass successively on the marks of the scale, and mark the cardboard with corresponding lines, prolonging slightly every fifth line. Insert the numbers on the cardboard scale.

EXPT. 2.—**Use of scales and calipers.**—Several bodies of different shapes and materials are supplied. Make clear sketches of each. By means of a scale applied to the body, or by first fitting outside calipers A (Fig. 7),

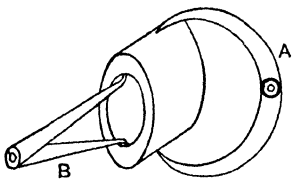


FIG. 7.—Use of calipers.

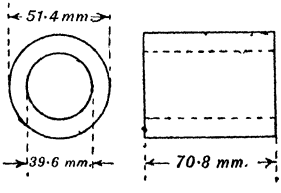


FIG. 8.—A hollow cylinder.

or inside calipers B, and then applying the calipers to the scale, measure all the dimensions of the body and insert them in the sketches. Fig. 8 shows suitable dimensioned sketches of a hollow cylinder.

Calculate the volume of each body by application of the rules of mensuration, making use of the dimensions measured.

The student should practise the estimation by eye to one-tenth of a scale division.

Verniers.—Scales do not usually have divisions smaller than half a millimetre Finer subdivision may be obtained by means of a **vernier**, an appliance which enables greater accuracy to be obtained than is possible by mere eye estimation.

In Fig. 9, A is a scale and B is a vernier, B may slide along the edge of A. The divisions on the vernier from 0 to 10 have a total

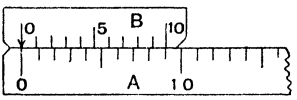


FIG 9

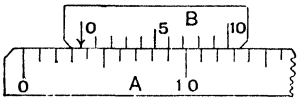


FIG 10

Forward reading verniers

length of 9 scale divisions; hence each vernier division is 0·9 of a scale division. If B is moved so that the mark 1 on the vernier is in the same straight line as the 0 1 mark on A, then the distance separating 0 on the vernier from 0 on the scale will be one-tenth of a scale division If the mark 2 on the vernier is in line with the 0·2 mark on A, the distance separating the zero marks will be two-tenths of a scale division, and so on. The vernier thus enables readings to be taken to one-tenth of a scale division by simply noting which division on the vernier is in line with any particular mark on A Fig. 10 shows a vernier and scale reading 0·36 scale divisions; 0·3 is read from the scale, and the 6 from the vernier.

The appliance described above is an example of a **forward-reading vernier**; in Fig. 11 is shown a corresponding **backward-reading vernier.** In this case the 10 vernier divisions have a total length equal to 11 scale divisions, and the graduation figures on the vernier run in the contrary direction to those on the scale. The reading of the scale is taken at the arrow on the vernier, and the second decimal is taken from the vernier as before.

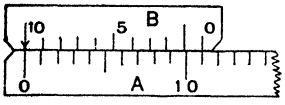


FIG. 11.—A backward reading vernier.

The following rule is useful in the construction and reading of verniers: Let the total length of a forward-reading vernier be $(N-1)$ scale divisions, or $(N+1)$ in a backward-reading vernier, and let there be N divisions on the vernier, then the vernier reads to $1/N$ scale division.

It may be verified by the student that, if the vernier has a length of $(2N \mp 1)$ scale divisions (− for forward and + for backward reading), and if there are N divisions on the vernier, then the reading may be taken to 1/N scale division.

Measurement of angles.—In Fig. 12 is shown a **protractor** by means of which angles may be measured to one minute. A semicircular piece of brass A is fitted with an arm BCD capable of rotating about a centre at D. A semicircular scale divided into half-degrees is engraved on A and the arm has a vernier. The centre of the semicircular scale lies at the intersection of two cross lines ruled on a piece of glass at D. The edge BC, on being produced, passes through the zero arrow on the vernier and also through the point of intersection of the cross lines at D.

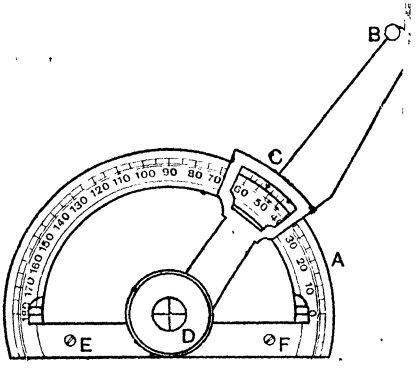


FIG 12.—Vernier protractor.

The vernier is **central reading**, and is shown enlarged on a straight scale in Fig. 13. The total length of the vernier is 29 scale divisions

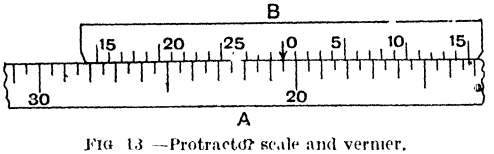


FIG 13.—Protractor scale and vernier.

and it has 30 divisions; hence it reads to one-thirtieth of a scale division, *i.e.* to one minute. The object of taking zero at the centre mark of the vernier is to remove any doubt which might arise as to which end of the vernier is to be read. Needles project at E and F on the under side of the instrument and prevent slipping.

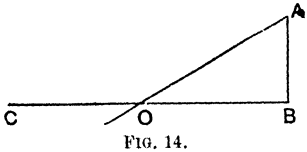


FIG. 14.

EXPT. 3.—**To measure an angle.** (*a*) Draw two lines AO and BOC intersecting at O (Fig. 14). Set the protractor so that the intersection of the cross lines coincides with O and the marks 0° and 180° fall on BOC. Set the arm so that the edge BC (Fig. 12) coincides with OA. Note the reading as the magnitude of the angle AOB. A small lens will be found useful in reading the vernier.

From any point A on OA draw AB perpendicular to OB. Measure OB, BA and OA and evaluate the trigonometrical ratios:

$$\sin AOB = \frac{AB}{OA}; \quad \cos AOB = \frac{OB}{OA}, \quad \tan AOB = \frac{AB}{OB}.$$

Consult trigonometrical tables, and write down the values of the angle AOB corresponding with the calculated values of the sine, cosine and tangent. Take the mean of these values and compare it with the value found by means of the protractor.

(*b*) Draw any triangle. Measure its three angles by means of the protractor. Verify the proposition that the sum of the three angles of any triangle is equal to 180°.

Vernier calipers.—The vernier calipers (Fig. 15) consist of a steel bar having a scale engraved on it. Another piece may slide along the bar and carries a vernier; there is a clamp and slow-motion

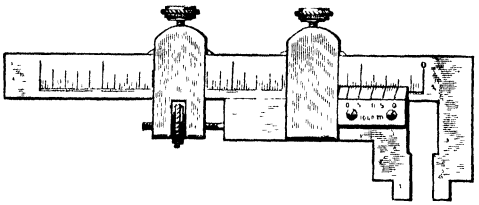


FIG 15.—Vernier calipers

screw by means of which the sliding piece may be moved slowly along the bar. The article to be measured is placed between the jaws of the calipers, and the sliding piece is brought into contact with it so as to nip it gently.

In metric calipers the scale shows centimetres, with half-millimetre subdivisions. The vernier has a length of 24 scale divisions (*i.e.* 12 mm.) and has 25 divisions; hence the instrument reads to $\frac{1}{2}\frac{1}{5} \times \frac{1}{2} = 0{\cdot}02$ mm. In British instruments the scale of inches is subdivided into fortieths of an inch. The vernier has a length of 24 scale divisions and has 25 divisions. Readings may be taken to $\frac{1}{2}\frac{1}{5} \times \frac{1}{4}\frac{1}{0} = 0{\cdot}001$ inch. In reading either scale, a small lens is desirable.

Micrometer or screw-gauge.—This instrument (Fig. 16) somewhat resembles calipers having a screw fitted to one leg. The object to be measured is inserted between the point of the screw and the fixed abutment on the other leg, and the screw is rotated until the object

is nipped gently. A scale is engraved along the barrel containing the screw, and another scale is engraved round the thimble of the screw. In Fig. 17 is shown an enlarged view of these scales. The screw has two threads per millimetre; hence one revolution will produce an axial movement of 0·5 mm. The barrel scale A shows millimetres; a supplementary scale immediately below A shows half-millimetres. The thimble scale B has 50 divisions; as one complete turn of the thimble is equivalent to 0·5 mm., one scale division movement of B past the axial line of the scale A is equivalent to $\frac{1}{50} \times 0{\cdot}5 = 0{\cdot}01$ mm. Hence readings may be taken to one hundredth of a millimetre. In Fig. 17 the scales are shown set at 7·47 mm.

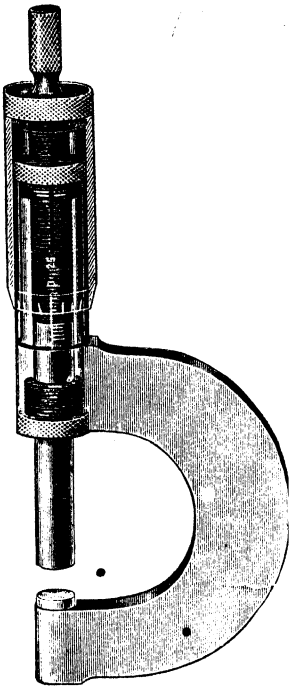


FIG. 16.—Micrometer.

In micrometers graduated in the British system the screw has usually 40 threads per inch; the barrel scale A shows inches divided into fortieths; the thimble scale has 25 divisions. Hence the instrument reads to

$$\frac{1}{25} \times \frac{1}{40} = 0{\cdot}001 \text{ inch.}$$

If the point of the screw is in contact with the abutment, the scales should read zero; if this is not so, the reading should be noted, and applied as a correction to subsequent measurements.

EXPT. 4.—**Use of vernier calipers and micrometers.** Take again the bodies used in Expt. 2. Remeasure them, using the vernier calipers and the micrometer. Calculate the volumes from these dimensions, and compare the results with those obtained by the methods employed in Expt. 2.

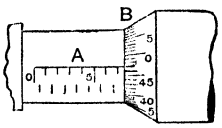


FIG. 17.—Micrometer scales.

The student is here reminded that the results of calculations should not contain a number of significant figures greater than is warranted by the accuracy of the measurements. Thus it would be absurd to state

a result of 321·46934 cubic millimetres when the instrument employed reads to 0·01 mm. only. Four significant figures are sufficient for most results; the usual plan is to state one significant figure in excess of those of which the accuracy is undoubted; for example, 321·46 may be taken to mean that 321·4 is of guaranteed accuracy, but that there is doubt regarding the last significant figure 6.

Spherometer.—An ordinary type of spherometer is shown in Fig. 18. A small stool A has three pointed legs B, C and D arranged at the corners of an equilateral triangle. A micrometer screw E is fitted at the centre of the circumscribed circle of the triangle, and is pointed at its lower end. F is a graduated circular plate fixed to the screw; there is a milled head at G for convenience in rotating the screw. A scale H is fixed to A, and has divisions cut on it at intervals equal to the pitch of the screw. The instrument rests on a glass plate K, the upper surface of which is as nearly plane as possible. L is an object the thickness of which is to be determined.

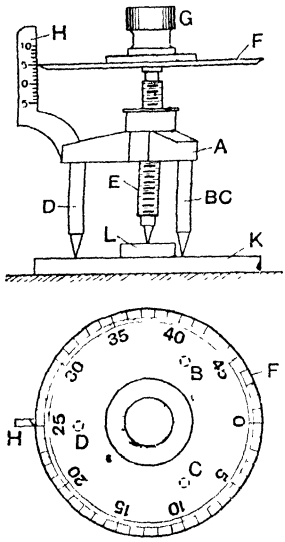


FIG 18.—Spherometer

In the instrument illustrated the screw has two threads per millimetre, and the circular scale on F has 50 divisions, each subdivided into 10; the instrument therefore reads to

$$\frac{1}{500} \times \frac{1}{2} = 0.001 \text{ mm.}$$

EXPT. 5.—**Thickness of an object by use of the spherometer.** Place the spherometer on the plane glass plate. Rotate the screw until all four points bear equally on the glass; this condition may be tested by pushing one of the legs in a direction nearly horizontal. If the instrument rotates, the screw point is bearing too strongly and must be raised. Should simple sliding occur, then the screw point is not bearing sufficiently. Note the readings of the scales. Unscrew E sufficiently to enable the object to be placed under the screw point, and make the adjustments as before. Read the scales again; the difference of the two readings will give the thickness required.

Measure the thickness of the small objects supplied at three or four spots and state the average thickness of each object.

EXPT. 6.—Use of the spherometer in determining the radius of curvature of a spherical surface. Measure the radius of curvature of the spherical surface supplied by use of the spherometer in the following manner: Place the instrument on the plane glass plate and obtain the readings of the scales; these may be denoted the zero readings. Place the spherometer on the spherical surface (Fig. 19); adjust it and again note the scale readings. The difference between these readings will be equal to AB in Fig. 19.

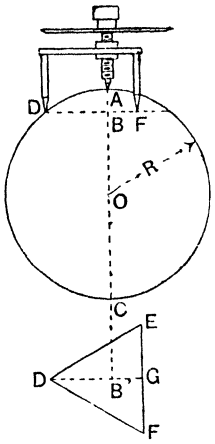


FIG. 19.—Spherometer on a spherical surface.

Let AB = h millimetres,

R = the radius of curvature in millimetres.

Then, from the geometry of Fig. 19, we have

$$CB \times BA = BD^2,$$

or,
$$(2R - h)h = BD^2;$$

$$R = \frac{BD^2}{2h} + \frac{h}{2} \quad \ldots\ldots\ldots\ldots(1)$$

To obtain BD, place the spherometer on a piece of tinfoil and press gently so as to mark the positions of the three legs D, E, F (Fig. 19). Measure DE, EF and FD, and take the mean; let this dimension be a mm. The angle EDG is 30° and BD is two-thirds of DG, hence

$$BD = \frac{2}{3}DG = \frac{2}{3}DE\cos 30°$$

$$= \frac{2}{3}a\frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{3}a;$$

$$BD^2 = \frac{3}{9}a^2 = \frac{1}{3}a^2. \quad \ldots\ldots\ldots\ldots(2)$$

Substitution in (1) gives. $$R = \frac{\frac{1}{3}a^2}{2h} + \frac{h}{2}$$

$$= \frac{a^2}{6h} + \frac{h}{2}. \quad \ldots\ldots\ldots\ldots(3)$$

In the case of a very flat spherical surface, h will be very small; the first term in (3) will then be very large when compared with the second term, and we may write:

$$R = \frac{a^2}{6h}. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(4)$$

The method of measurement and reduction is the same for both convex and concave surfaces.

Measure each of the given surfaces at two or three places; calculate the radius of curvature for each reading, and state the mean radius.

EXPT. 7.—**Micrometer microscope.** In this instrument, the object to be measured is placed opposite B and is observed through a microscope A (Fig 20) The microscope has a scale finely engraved on glass in the eyepiece at C, and is focussed so as to obtain sharp images of both scale and object when viewed through the eyepiece. The microscope may be traversed horizontally by means of a thumb-screw D, and may be raised or lowered in the supporting pillar by use of another thumb-screw E The microscope carries a scale F divided in millimetres and a vernier G reading to 0 1 mm. is attached to the pillar.*

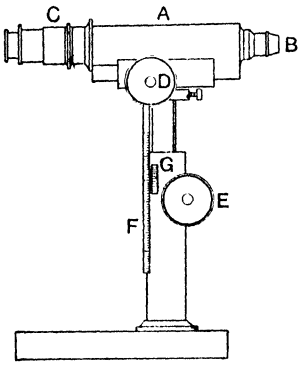


FIG 20 - -Micrometer microscope.

First obtain the value of an eyepiece scale division as follows: Focus sharply the object and scale, move the eye slightly up and down and observe whether the object and scale as seen through the eyepiece suffer any displacement relatively to one another. If so, adjust the focussing arrangements until this movement disappears. Use E to bring zero on the eyepiece scale into coincidence with one edge or other fine mark on the object; read and note the pillar scale and vernier. Use E to bring another mark on the eyepiece scale, say the fiftieth, into coincidence with the same mark on the object; again read and note the pillar scale and vernier. The difference of these readings gives the value in millimetres of 50 eyepiece scale divisions; hence calculate the value of one eyepiece scale division. Repeat the operation, using the eyepiece scale marks 20 and 70, 35 and 85, and 50 and 100. Compare the values and state the average value of an eyepiece scale division.

Measure the thickness of the objects supplied by noting the eyepiece scale marks at the top and bottom, estimating by eye to one-tenth of a scale division. Take the difference and convert into millimetres.

Measure also the bores of the glass tubes supplied.

Weighing.- The choice of a balance to be used in weighing a given body depends upon the weight of the body and also upon the accuracy required In using balances capable of dealing with heavy bodies--up to 10 kilograms say--no special precautions need be observed other than that of placing gently both body and weights in the scale pans.

Delicate balances are fitted inside glass cases, and have arrange-

* For the optical theory of this instrument, the student is referred to the Part of the volume devoted to Light.

ments by means of which the motions of the various parts may be arrested and all knife edges relieved of pressure when the balance is not in use. These arrangements are operated by a handle or lever outside the case; the handle should be moved very gently, and no weights should be placed on, or removed from either scale pan without first using the handle to arrest the motion. The sets of weights used with delicate balances are kept in partitioned boxes, and should not be fingered; forceps are provided for lifting the weights.

EXPT. 8.—**Use of a balance.** Weigh each of the bodies used in Expt. 4, thus determining its mass. Find the density of each material, making use of the equation given on p. 5, and of the volumes calculated in performing Expt. 4.

Balances are subject to errors, most of which are eliminated in the following method of weighing. Place in one of the scale pans any convenient body of weight somewhat in excess of that of the body to be weighed; add weights to the other scale pan until balance is secured; let the total weight be W_1. Remove the weights, and place in the empty scale pan the body to be weighed. Add weights (W_2 say) until balance is again restored. It is obvious that the weight of the body is equal to the difference $(W_1 - W_2)$.

EXPT. 9.—**Measurement of areas.** Draw any triangle on a piece of rectangular cardboard. Calculate the area of the triangle by use of the rules:

(i) Area = base × half the perpendicular height.

(ii) Area $= \sqrt{s(s-a)(s-b)(s-c)}$, (p. 8).

Copy the triangle on a piece of squared paper and find its area by counting the number of included squares. The copying of the figure may be obviated by use of a piece of squared tracing paper, covering the original figure.

Calculate the area of the whole card by taking the product of its length and breadth. Weigh the card, and calculate the weight per square centimetre by dividing the weight by the area. Carefully cut out the triangle and weigh it separately. Find the area of the triangle from:

Weight of triangle = area in sq. cm. × weight per sq. cm.

Compare the results of these methods.

Draw another figure by erecting equidistant ordinates of varying heights as shown in Fig. 3 (p. 8). Evaluate its area by use of the trapezoidal rule:

$$\text{Area} = a\left\{\frac{h_1+h_5}{2}+h_2+h_3+h_4\right\}, \quad \text{(p. 8)}.$$

Verify the result by use of squared paper and also by weighing.

The planimeter.—Areas may be measured by means of a planimeter (Fig. 21). This instrument consists of a bar A to which another bar B is jointed at C, so that the bars may have relative movement in a plane. B may rotate about a needle point pushed into the paper at D, and is loaded with a weight at E. A rests on a wheel F, which may roll on the paper, and has a tracing needle at G which may be carried round the boundary of the area to be measured. It may be shown that the area is proportional to the product of the distance between C and G and the distance through which the circumference of the wheel F rolls when G is carried completely round the boundary of the area.

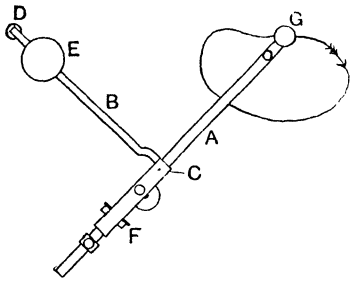


FIG 21.—Planimeter in use

The instrument is shown in greater detail in Fig 22. It will be noted that the wheel has a scale engraved round its circumference; there are 100 divisions on this scale, and a vernier enables the scale to be read to one-tenth of a scale division. A small indicator wheel H, driven from F, registers the number of complete revolutions of F.

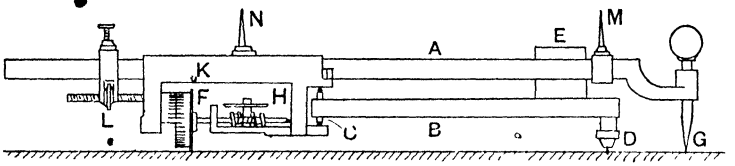


FIG 22.—Planimeter

F, H and the joint C are carried on a bracket K, which may be clamped in any position on the bar A; a slow-motion screw L enables the distance CG to be adjusted finely. Pointers M and N are fixed to the bar A and the bracket K, and are so placed as to indicate the distance CG. Marks are placed on A to facilitate the adjusting of the positions of K suitable for measuring the area in square centimetres or square inches.

The instrument should be used on a sheet of drawing paper sufficiently large to enable the whole movements of the wheel F to be completed without coming off the paper. The surface of the paper should not be highly polished, which might lead to slipping and consequent lost motion of the wheel, nor should the surface be too rough. It is best to arrange the initial position so that the arms A and B are at right angles approximately. The tracing point G should be carried clockwise round the boundary.

EXPT. 10.—**Use of the planimeter.** Draw a circle 10 cm. in diameter on the paper. Set the planimeter to the scale of square centimetres; place it on the paper with G at a mark on the circumference of the circle. Set the wheel F at zero. Carefully carry the pointer G round the boundary and stop at the mark. Read and note the scale and vernier. Carry the pointer round a second and third time, reading the scale and vernier each time the mark is reached. Take the differences, giving three results for the area; these results should be in fair agreement.

Calculate the area of the circle from:

$$\text{Area} = \pi r^2 \text{ square cm.},$$

where r is the radius of the circle in cm. Compare the calculated area with the mean area obtained by the planimeter.

Draw a figure on the drawing paper resembling Fig. 4 (p. 9). Divide it vertically by an odd number of equidistant ordinates. Estimate its area by use of Simpson's rule, viz.:

$$\text{Area} = \frac{''}{3}(h_1 + 4h_2 + 2h_3 + 4h_4 + 2h_5 + 4h_6 + h_7), \quad \text{(p. 9)}.$$

Check the result by use of the planimeter.

EXPT. 11.—**Measurement of volumes by the displacement of water.** In Fig. 23, A is a jar containing water and fitted with a hook gauge B. The hook gauge is simply a sharp pointed piece of wire bent to the proper shape and clamped to the side of the vessel; it is used for adjusting accurately the surface level of the water. C is the body the volume of which has to be determined. D is a graduated measuring jar having a scale of cubic centimetres engraved on its side. First adjust the water level so that the point of the hook gauge is just breaking the surface of the water. By means of a fine thread, lower carefully the body into the jar. Use a pipette to remove water until the level is restored as shown by the hook gauge. Discharge all the water removed by the pipette into the measuring jar. Read and note the volume of this water as shown by the scale; it is evident that this reading will give the volume of the body.

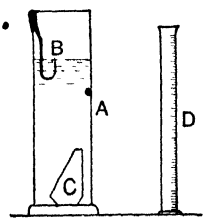


FIG. 23.—Volume by displacement.

Use this method to check the volumes of some of the larger bodies calculated in Expt. 4. The method cannot be applied with sufficient accuracy to bodies of very small dimensions, as the change in level of the water in the jar would then be inappreciable.

EXERCISES ON CHAPTER II.

1. A scale is divided into twentieths of an inch and has to be read to one twenty-fifth of a scale division by means of a vernier. Show by sketches how to construct a suitable forward-reading vernier; also a backward-reading vernier.

2. The circle of an instrument used for measuring angles is divided to show degrees, and each degree is divided into six equal parts. Show how to construct a forward-reading vernier which will enable angles to be read to the nearest third of a minute. Give sketches.

3. A micrometer, or screw gauge, has a screw having fifty threads to an inch, the barrel scale has graduations showing fiftieths of an inch. The instrument can read to the nearest thousandth of an inch. How many divisions has the thimble scale? Show, by sketches, the scales when the instrument is reading 0·437 inch.

4. A spherometer has a screw with 40 threads to an inch. How many divisions should the graduated circle have if the instrument reads to 0·0001 inch?

5. The fixed legs of a spherometer are at the corners of an equilateral triangle of 4 cm. side. When placed on a certain spherical surface the instrument reads 5·637 mm. Find the radius of curvature of the surface. The instrument has no zero error.

6. The same spherometer is used on another spherical surface and reads 0·329 mm. Find the radius of curvature of the surface.

7. In calibrating the eyepiece scale of a micrometer microscope the following readings were taken:

Eyepiece scale	0	30	50	80	100
Pillar scale, mm	35·0	33·6	32·3	30·3	29·0

What is the value in mm. of an eyepiece scale division?

8. The following dimensions of a metal frustum of a cone were measured with vernier calipers: Perpendicular height, 2·616 inches; diameter of small end, 1·876 inches; diameter of large end, 2 inches. The frustum was weighed and found to be 2·22 lb. Find the weight of the metal in lb. per cubic inch.

9. Describe how you would proceed to determine by experiment the relation between the length of the circumference of any circle and its diameter. Describe any form of screw-gauge you have used.

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10. Give sketches showing the construction of any planimeter you have used. Describe how the instrument is used in determining the area of a figure having an irregular curved boundary. State any precautions which must be observed.

11. The micrometer screw of a spherometer, instead of having two threads per millimetre, actually has 20·01 threads per centimetre. The circular scale has 500 divisions. When placed on the plane glass plate and adjusted, the scales read 0·005 mm. An object is then measured, and the reading of the scales gives 2·642 mm. What is the actual thickness of the object ?

12. A micrometer reads to 0·01 mm. When screwed home, the reading is 0·05 mm. The instrument was then applied to a steel ball, and the following diameters were obtained in three directions mutually perpendicular: 24·52 mm., 24·50 mm., 24·53 mm. State the mean diameter of the ball and calculate its volume.

CHAPTER III

DISPLACEMENT. VELOCITY. ACCELERATION

Motion of a point.—The motion of any body and its position at any instant may be specified by reference to chosen lines assumed to be fixed in space. In general, the motion of a body is complex; all points in it do not possess motions precisely alike in all respects. Hence it is convenient to commence the study of motion by the consideration of the motion of a point, or of a **particle**, *i.e.* a body so small that any differences in the motions of its parts may be disregarded

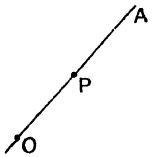


FIG. 24.—Rectilinear motion

In **rectilinear motion**, or motion in a straight line, it is sufficient to consider as fixed in space the line in which the point is moving. The position at any instant of a point P moving along the straight line OA (Fig. 24) may be specified by stating the distance OP from a fixed point O in the line: O may be called the origin.

In **uniplanar motion** the point has freedom to move in a given plane which may be taken as fixed in space. The position and motion of the point at any instant may be referred conveniently to any two fixed lines, mutually perpendicular, and lying in the plane of the motion; such lines are called **coordinate axes**. Thus in Fig. 25 a point P is describing a curve in the plane of the paper, supposed to be fixed in space. Its precise position at any instant may be defined by stating the perpendicular distances y and x from the two coordinate axes OX and OY. It will be noted that OX and OY divide the space surrounding the origin O into four compartments. Useful conventions are to describe x as positive or

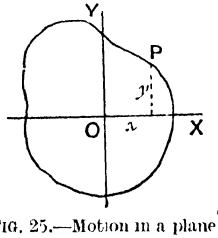


FIG. 25.—Motion in a plane

negative according as P is situated on the right or left of OY. Similarly, y is positive or negative according as P is above or below OX.

More complex states of motion arise when the moving point is not confined to one plane; for example, a person ascending a spiral staircase. Most of these cases are beyond the scope of this book.

Illustration of rectilinear and uniplanar motion.—The mechanism shown in Fig. 26 consists of a **crank** CB capable of revolving about an axis at C perpendicular to the plane of the paper. A **connecting rod** AB is jointed to the crank at B by means of a pin and also to a block D capable of sliding in a slot in the frame E. If the crank is revolving, the block D has rectilinear motion to and fro in the slot, and B has circular motion in the plane of the paper.

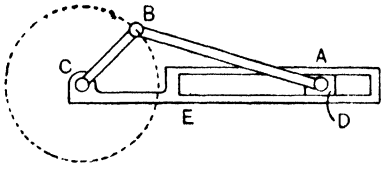


FIG. 26.—Slider crank mechanism

Locus of a moving point.—The determination of the position at any instant of a point in the above-mentioned and similar mechanisms may be made the subject of mathematical calculation. A more useful method employed in practice is to draw the **locus**, or path of the moving point; such a path will show the positions of the point throughout the whole range of possible movement of the mechanism.

An illustration of the method is given in Fig. 27, which shows the locus of a point D on the connecting rod of a mechanism similar to that given in Fig. 26. Outline drawings of the crank CB and connecting rod BA are

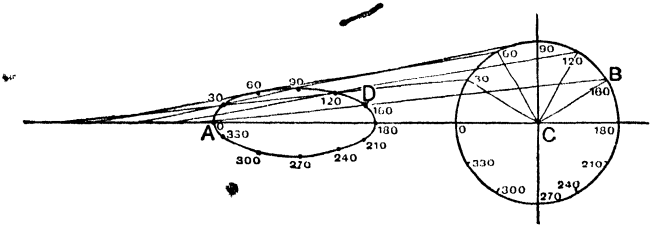


FIG. 27.—Locus of a point in a connecting rod.

constructed for successive positions of the crank, differing by 30° throughout the entire revolution. (For the sake of clearness, the positions above CA alone are shown in Fig. 27.) The position of D along AB is marked carefully on each drawing; a fair curve drawn through these points will give the required locus.

Displacement.—Suppose that a point occupies a position A at a certain instant (Fig. 28), and that at some other instant its position is B. Draw the straight line AB, AB is called the **displacement** of the point. In making this definition the precise path by which the point travelled from A to B is immaterial. For example, the point might have first a displacement from A to C, and then from C to B, with exactly the same change in position as would occur by travelling directly along the straight line AB. Hence we may say that the displacement AB is equivalent to the displacements AC and CB. AB is called the **resultant displacement,** and AC and CB are **component displacements.**

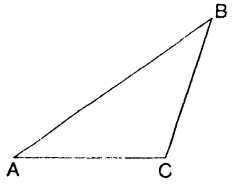


FIG. 28.—Triangle of displacements

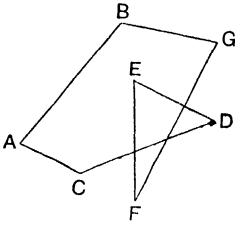


FIG. 29.—Polygon of displacements

It is evident that the number of component displacements may be unlimited. Thus, in Fig. 29, the components AC, CD, DE, EF FG and GB, successively applied to the point, are equivalent to the resultant displacement AB.

Specification of a displacement.—In stating a displacement it is necessary to specify (*a*) the initial position, (*b*) the direction of the line in which the point moves, (*c*) the **sense** of the motion, *i.e.* from A towards B or *vice-versa* (Fig. 28), (*d*) the magnitude of the displacement.

The sense may be indicated by the order of the letters defining the initial and final positions in a displacement, AB or BA, or by placing an arrow point on the line.

Vector and scalar quantities.—Any physical quantity which requires a direction to be stated in order to give a complete specification is called a **vector quantity**; other quantities are called **scalar quantities.** Displacement and force are examples of vector quantities; mass, density and volume are scalar quantities. Any vector quantity may be represented by drawing a straight line in the proper direction and sense.

The operations performed in Figs. 28 and 29 are examples of the **addition of vectors.** The operation consists in constructing a figure in which a straight line is drawn from the initial position to represent the first vector, making the line of a length to represent to scale the magnitude of the quantity, and drawing it in the proper direction and sense. From the end of this line remote from the initial position, another line is drawn in a similar manner to represent the second vector, and so on until all the components have been dealt with. The resultant vector will be represented by the line which must be drawn from the initial position in order that the completed figure may be a closed polygon.

Fig. 28, in which there are two component vectors only, may be called the **triangle of displacements**; the name **polygon of displacements** may be given to Fig. 29.

Velocity. The **velocity** of a moving point may be defined as **the rate of change of position in a given direction**; the time taken, the distance travelled, and the direction of motion are all taken into account in stating a velocity. Velocity is a vector quantity. In cases where the direction of motion does not require to be considered, the term **speed** is employed to express the rate of travelling.

Velocity may be **uniform**, in which case the point describes equal distances in equal intervals of time; the velocity is said to be **variable** if this condition be not complied with.

The velocity at any instant of a point having uniform velocity may be measured by stating the distance travelled in unit time. Thus, if a total distance s be described in t seconds, then the magnitude of the velocity v at any instant is given by

$$v = \frac{s}{t} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

This will be in cm. per sec., or feet per second, according as s is in cm. or feet. The specification of the velocity given by (1) is completed by stating also the direction of the line in which motion takes place and the sense of the motion along this line.

In the case of a variable velocity, the result given by use of equation (1) is the average value of the velocity during the interval of time t. Thus the average velocity of a train which travels a total distance of 400 miles in 8 hours (including stops) is $400 \div 8$, or 50 miles per hour.

The dimensions of velocity are l/t

If a point moves with variable velocity, the velocity at any instant may be stated as the distance which the point would travel during the succeeding second if the velocity possessed at the instant under consideration remained constant.

Acceleration.—Acceleration means **rate of change of velocity**, and involves both change of velocity and the time interval in which the change has been effected. Acceleration is measured by stating the change of velocity which takes place in unit time. Unit acceleration is possessed by a particle when unit change in velocity occurs in unit time.

EXAMPLE. At a certain instant, a particle having rectilinear motion has a velocity of 25 cm. per sec. The velocity is found to increase uniformly during the succeeding 5 seconds to 60 cm. per sec. Find the acceleration.

Increase in vel. in 5 secs. $= 60 - 25 = 35$ cm. per sec.

" " 1 sec. $= \frac{35}{5} = 7$ cm. per sec

Hence the acceleration is 7 cm. per second in every second, or, as is usually stated, 7 cm. per sec per sec, or 7 cm./sec^2.

It will be noted that time enters *twice* into the statement of a given acceleration, once in expressing the change in velocity, and again in expressing the time interval in which the change was effected.

Acceleration may be **uniform**, in which case equal changes in velocity occur in equal intervals of time. Otherwise the acceleration is **variable**. In the case of uniform acceleration, the acceleration at any instant is calculated by dividing the total change in velocity by the time in which the change takes place. A similar calculation made for the case of variable acceleration gives the average acceleration during the time interval considered.

Since acceleration involves velocity, it is a vector quantity. To specify completely a given acceleration, the magnitude, the line of direction and the sense of the acceleration along the line of direction must be stated.

The dimensions of acceleration are obtained by dividing the dimensions of velocity by time, giving $l/t \div t = l/t^2$.

Displacement, velocity and acceleration graphs.—A convenient method of studying questions involving displacement, velocity and acceleration is to construct graphs in which the magnitudes of these quantities are plotted as ordinates and the time intervals as abscissae.

EXAMPLE 1.—A point P ... a straight line OA, passes through the origin O at a certain ... has a uniform velocity of 40 cm. per sec. Plot displacement ... locity-time graphs.

Since the velocity ..., the displacement in any interval of time t seconds is giv...

Time t secs	...om O, - -	0	1	2	3	4
Displace...	...eckoned from O, -	0	40	80	120	160

These ... ...ted as shown in Fig. 30, give a straight line displacement-ti... ...n OB.

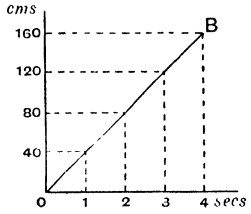


FIG 30

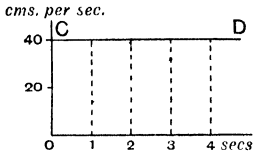


FIG 31

Graphs for uniform velocity.

The velocity is uniform, therefore the velocity-time graph CD is parallel to the time axis (Fig. 31). In this graph, since OC represents the constant velocity v, and O4 represents time t, the rectangular area C4 represents the product vt, and hence represents the displacement in the time t.

EXAMPLE 2.—A point P, travelling in a straight line OA, is at rest in the initial position O and has a constant acceleration of 20 cm. per sec. per sec. Plot velocity-time, acceleration-time and displacement-time graphs.

It is evident that the point will have a velocity of 20 cm. per sec. at the end of the first second, and that its velocity increases by 20 cm. per sec. during each second of the motion; hence the velocity v at the end of any time interval t seconds may be found from $v = 20t$ cm. per sec.

Time t secs, reckoned from O, - -	0	1	2	3	4
Vel. v cm. per sec., reckoned from O, -	0	20	40	60	80

Plotting these numbers as shown in Fig. 32, we obtain a straight-line velocity-time graph OE.

The acceleration-time graph is shown in Fig. 33. Since the acceleration is uniform, it follows that the graph FG is parallel to the time axis.

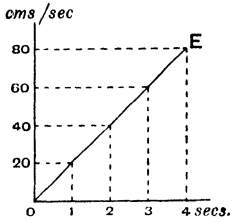


FIG 32

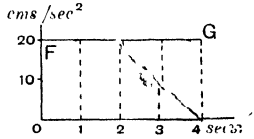


FIG 33

Graphs for constant acceleration, starting from rest

The displacement during any interval of time is given by the product of the average velocity during the interval and the interval of time. Thus, if v_a be the average velocity in cm. per sec. during a time t seconds, then the displacement s in cm. is equal to the product $v_a t$.

For example, when the point is at the origin the velocity is zero, and at the end of the first three seconds the velocity is 60 cm. per second (Fig. 32); hence the average velocity during the first three seconds is given by

$$v_a = \frac{0+60}{2} = 30 \text{ cm. per sec.}$$

And $s = 30 \times 3 = 90$ cm. in the first three seconds

Time interval in secs., reckoned from O,	0	1	2	3	4
v_a during time interval, cm. per sec. -	0	10	20	30	40
Displacement during interval, cm. -	0	10	40	90	160

Plotting displacements and time as shown in Fig. 34, we obtain the curved graph OH, which shows the relation of displacement and time.

The student will note that the average velocity during any time interval in Fig. 32 is represented by the average height of the portion of the graph inclosed by the ordinates at the beginning and end of the interval. The distance between the feet of the ordinates represents time, hence the area of the graph represents the product of average velocity and time, and therefore represents the displacement during the interval.

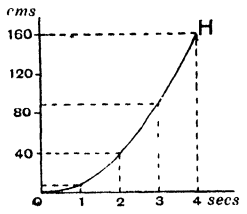


FIG. 34 — Displacement graph

Equations for rectilinear motion. The following equations are for simple cases which occur frequently, and are deduced from the velocity-time graphs.

Case 1. **Uniform velocity.**—This case has been dealt with on p 29, and the following equation was deduced .

$$s = vt \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

Case 2. **Uniform acceleration, starting from rest.**—Let the acceleration be a. The velocity at the end of the first second will be equal to a, and a velocity equal to a will be added during each subsequent second (Fig 35) Hence, at the end of t seconds, we have

$$v = at. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

The average velocity during the first t seconds is

$$v_a = \tfrac{1}{2}v = \tfrac{1}{2}at. \quad \ldots\ldots\ldots\ldots (3)$$

Therefore the displacement during the first t seconds is

$$s = v_a t = \tfrac{1}{2}vt. \quad \ldots\ldots\ldots\ldots (4)$$

$$= \tfrac{1}{2}at \times t = \tfrac{1}{2}at^2. \quad \ldots\ldots\ldots\ldots (5)$$

From (2), $t = \dfrac{v}{a}$, or $t^2 = \dfrac{v^2}{a^2}$.

Substitution in (5) gives $s = \tfrac{1}{2}a\dfrac{v^2}{a^2} = \dfrac{v^2}{2a}$,

or $$v^2 = 2as. \quad \ldots\ldots\ldots\ldots (6)$$

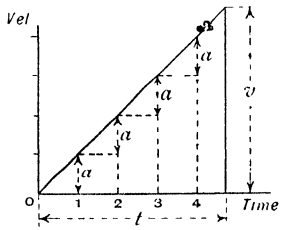


FIG. 35.—Uniform acceleration, starting from rest.

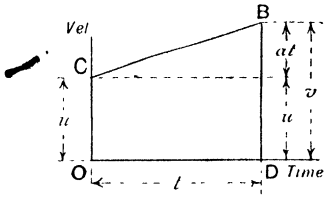


FIG 36.—Uniform acceleration, initial velocity u

Case 3. **Uniform acceleration, and starting from the initial position with a velocity u.**—The velocity-time graph is given in Fig. 36. Since the initial velocity is u, and a velocity equal to a is added during each second, the velocity at the end of t seconds is

$$v = u + at,$$

or $$v - u = at. \quad \ldots\ldots\ldots\ldots (7)$$

In Fig. 36, BD represents v and CO represents u; the average velocity during the first t seconds is

$$v_a = \frac{u+v}{2} = \frac{u+u+at}{2},$$

$$\therefore \quad \mathbf{v_a = u + \tfrac{1}{2}at} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (8)$$

The displacement during the first t seconds is given by

$$s = v_a t = (u + \tfrac{1}{2}at)t,$$

$$\mathbf{s = ut + \tfrac{1}{2}at^2}. \quad \ldots \quad (9)$$

It will be noted that the first term in (9) gives the displacement which would have occurred had the velocity u been preserved uniform throughout, the second term gives the displacement which would have taken place had the point started from rest with a uniform acceleration a

From (7), $\quad t = \dfrac{v-u}{a}$, or $t^2 = \dfrac{(v-u)^2}{a^2}$

Substitute in (9), giving

$$s = u\left(\frac{v-u}{a}\right) + \tfrac{1}{2}a\,\frac{(v-u)^2}{a^2}$$

$$= \frac{uv-u^2}{a} + \frac{v^2+u^2-2uv}{2a}$$

$$= \frac{2uv-2u^2+v^2+u^2-2uv}{2a}$$

$$= \frac{v^2-u^2}{2a},$$

or $$\mathbf{v^2 - u^2 = 2as} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (10)$$

In applying any of the above equations, either C.G.S. or British units may be employed:

v in cm. per sec., or feet per sec
a in cm. per sec. per sec., or feet per sec. per sec.
s in cm., or feet.
t in seconds.

Bodies falling freely. Experiment shows that any body falling freely under the action of gravitation has uniform acceleration. The term **freely** is used to indicate that the resistance of the atmosphere has been removed, or has been neglected. The symbol g is used to denote the acceleration of a body falling freely. All the equations obtained in Cases 2 and 3 above may be employed by

substitution of g for a, and the height h for s. Thus equations (6) and (10) will read respectively:

$$v^2 = 2gh \quad . \quad . \quad . \quad . \quad (a)$$

$$v^2 - u^2 = 2gh \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (b)$$

(a) applies to a body falling freely from rest, and may be used in calculating the velocity at the end of a fall from a height h. (b) applies to a similar case in which the body is projected downwards with an initial velocity u; the terminal velocity v may be calculated from (b).

Variations in the value of g.—The value of g varies somewhat at different parts of the earth; in Britain, 981 cm. per sec. per sec., or 32·2 feet per sec. per sec. may be used in most calculations. The value of g at any given place depends upon the distance between that place and the centre of the earth. The value of g at sea-level in latitude 45° is sometimes chosen as a standard of reference; the value at other places depends upon the height above sea-level and also upon the latitude. Latitude is a factor on account of (1) the shape of the earth, which, being flattened somewhat towards the poles, causes sea-level at the poles to be nearer to the centre of the earth than sea-level at the equator, (2) the variation of centrifugal action with distance from the equator.

Let g_0 = the value of g at sea-level in latitude 45°, cm. per sec. per sec.

g = the value of g at an elevation H metres in latitude λ, cm. per sec. per sec.

Then $g = g_0(1 - 0{\cdot}0026 \cos 2\lambda - 0{\cdot}0000002\,\mathrm{H})$.

Conventions regarding signs. In considering a point P moving in a straight line AB, it is convenient to choose one sense, say from A towards B, and to call velocities and accelerations having this sense **positive**; velocities and accelerations having the contrary sense will then be called **negative**. This convention enables graphs to be drawn in representation of such cases as that of a body projected upwards, coming gradually to rest, and then descending.

In Fig. 37 is given a velocity-time diagram illustrating this case. The body was projected upwards with a (positive) velocity u, represented by OA drawn above the time axis. Velocity is abstracted at a uniform rate g, and the body comes to rest, as indicated at B, at the end of a time interval represented by OB. Thereafter its velocity is downwards (negative) and increases numerically until it reaches

the level of the initial position, when it possesses a negative velocity v_2, represented by DC. As the graph ABC is a straight line owing to the acceleration being uniform, the time taken in ascending is equal to the time of descent, and the terminal velocity v_2 is equal to the initial velocity u. It is, of course, assumed that the resistance of the atmosphere is neglected throughout.

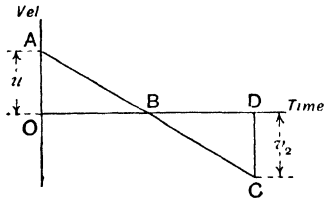


FIG. 37.—Velocities having opposite senses.

Calling downward accelerations positive, it will be noted that the acceleration in the above illustration is positive throughout the motion. During the ascent the velocity and acceleration have contrary signs, and the velocity is diminishing; during the descent the signs are similar and the velocity is increasing.

More general case of a velocity-time graph. In Fig. 38 is shown a velocity-time graph OAB. Consider two points P_1 and P_2 lying fairly close together on the graph. P_1M_1 and P_2M_2 represent velocities v_1 and v_2 respectively, and these velocities are possessed by the moving point at the end of time intervals t_1 and t_2, represented by OM_1 and OM_2 respectively. Draw P_1K parallel to the time axis. Then

$$P_2K = P_2M_2 - P_1M_1 = v_2 - v_1,$$

and is the change in velocity during a time interval

$$M_1M_2 = OM_2 - OM_1 = t_2 - t_1.$$

Hence the average acceleration during this interval is given by

$$a_a = \frac{v_2 - v_1}{t_2 - t_1} = \frac{P_2M_2 - P_1M_1}{OM_2 - OM_1}.$$

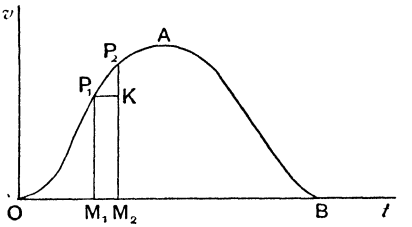


FIG. 38.—General case of a velocity-time graph.

The quantities required for making the calculation of the average acceleration may be scaled from the graph. There is no great error in assuming that the average acceleration so calculated is the actual acceleration at the middle of the interval M_1M_2. Hence an acceleration-time graph may be constructed by calculating the average accelerations as above explained for intervals throughout OB, and erecting ordinates at the middle of each interval to represent these accelerations.

It will be noted that if P_2M_2 is less than P_1M_1, the velocity is diminishing, and the acceleration has then the sign contrary to that which has been assumed for increasing velocity.

Again, the average velocity during the interval M_1M_2 is

$$\tfrac{1}{2}(P_1M_1 + P_2M_2) = \tfrac{1}{2}(v_1 + v_2).$$

Since the time interval is $M_1M_2 = t_2 - t_1$, it follows that the displacement during the interval M_1M_2 is

$$\tfrac{1}{2}(v_1 + v_2)(t_2 - t_1) = \tfrac{1}{2}(P_1M_1 + P_2M_2)M_1M_2 = \text{the area } P_1M_1M_2P_2$$

very nearly. The closeness of the approximation becomes more perfect if the interval M_1M_2 be made smaller and smaller, and is absolutely perfect if the interval be made indefinitely small. It is evident that the area of any similar strip of the graph will represent the displacement in the time interval represented by the base of the strip. Hence, for the total time OB, the total displacement is represented by the total area of the graph, and may be found by applying the rules of mensuration. Thus, find the area of the graph, using a planimeter, say. Divide this area by the length OB so as to find the average height of the graph. Multiply the average height by the scale of velocity, thus obtaining the average velocity for the whole graph. Multiply the average velocity by OB, expressed in seconds. The result gives the total displacement.

Exercises on Chapter III.

1. A flat board has two grooves cut in it, running right across the board and intersecting at right angles at the centre of the board. A rod AB 2·5 inches long moves in the plane of the board, A being constrained to move in one groove and B in the other. Draw the locus (*a*) of a point at the centre of AB, (*b*) of a point in the rod 0·75 inch from B.

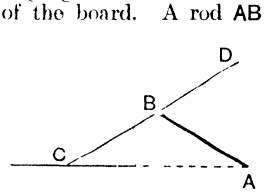


FIG. 39

2. In Fig. 39, AB is a rod 2 inches long and can revolve about a fixed centre at A. CBD is another rod, jointed to AB at B, and having the end C constrained to remain in a groove, the direction of which passes through A; CB is 2 inches long. Draw the locus of D (*a*) if BD is 2 inches long, (*b*) if BD is 3 inches long.

3. A point is given two component displacements, one of 24 cm. towards the north-east, and another of 30 cm. towards the north. Find the resultant displacement.

4. Draw a horizontal line OX as a reference axis. Starting from O, a point has the following component displacements, impressed successively: 2 inches at 30° to OX; 3 inches at 45° to OX, 5 inches at 240° to OX; 4 inches at 90° to OX. Find the resultant displacement.

5. What is the average speed in feet per second of a horse which travels a distance of 11 miles in 1·25 hours?

6. An observer notes that the peal of thunder is heard 3·5 seconds after seeing the flash of lightning. If sound travels at a speed of 1100 feet per second, find the distance in miles between the flash and the observer.

7. Two runners, A and B, start from the same place. A starts 30 seconds before B and runs at a constant speed of 8 miles per hour. B travels along the same road with a constant speed of 10 miles per hour. At what distance from the starting point will B overtake A ?

8. Two trains, A and B, travelling in opposite directions, pass through two stations 1·5 miles apart at the same instant. If A has a constant speed of 40 miles per hour, find the constant speed of B so that the trains shall pass each other at a distance of 0·9 mile from the station which A passed through.

9. A train travelling at uniform speed passes two points 480 feet apart in 10 seconds. Find the speed in miles per hour.

10. A train starts from rest and gains a speed of 10 miles per hour in 15 seconds. Find the acceleration in foot and second units. Sketch a velocity-time graph.

11. A ship travelling at 22 kilometres per hour has its speed changed to 18 kilometres per hour in 40 seconds. Find the acceleration in metre and second units. Sketch a velocity-time graph.

12. A body travelling at 800 feet per minute is brought to rest in $\frac{1}{2}$ second. Assume the acceleration to be uniform, and find it. Sketch a velocity-time graph.

13. Express an acceleration of 60 miles per hour per minute, in metres per second per second.

14. A train starts from rest with an acceleration of 1·1 feet per sec. per sec. Find its speed in miles per hour at the end of 25 seconds. What distance does it travel in this time ? Sketch a velocity-time graph.

15. A train changes speed from 60 to 50 miles per hour in 15 seconds. Find the distance travelled in this interval. Sketch a velocity-time graph.

16. A train starts from rest with an acceleration of 0·9 foot per sec. per sec. and maintains this for 30 seconds. Constant speed is then maintained until a certain instant when steam is shut off and the brakes are applied, producing a negative acceleration of 1·5 feet per sec. per sec. until the train comes to rest. If the total distance travelled is 2 miles, find the time during which the speed was uniform and the total time for the whole journey. Sketch a velocity-time graph.

17. What acceleration must be given to a train travelling at 30 miles per hour in order to bring it to rest in a distance of 200 yards ? Sketch a velocity-time graph.

18. A body falls freely from a height of 50 metres. Find the velocity just before touching the ground and the time taken. Sketch velocity-time and distance-time graphs. Take $g = 981$ cm. per sec. per sec.

19. A stone is projected vertically upwards. Find the initial velocity in order that it may reach a height of 150 feet. If the stone falls to the original level, find the total time of flight. Sketch a velocity-time graph. Take $g = 32{\cdot}2$ feet per sec. per sec.

20. A stone is dropped down a well and is observed to strike the water in 2·5 seconds. Find the depth of the well to the surface of the water.

21. Suppose in Question 20 that the sound of the splash is heard 2·6 seconds after dropping the stone; find the depth to the surface of the water. Assume that sound travels at 1100 feet per second, and that $g = 32{\cdot}2$ feet per sec. per sec.

22. A stone is thrown vertically downwards with a velocity of 20 feet per sec. Find the velocity at the end of the third second. What distance does it travel up to this instant?

23. A stone is projected vertically upwards with a velocity of 160 feet per second. Two seconds later a second stone is projected vertically from the same point. Find to what height the first will rise, and the velocity with which the second must be projected for it to strike the first as the first is just about to descend. L.U.

24. A stone is dropped from a height of 64 feet, while at the same instant a second stone is projected from the earth immediately below with sufficient velocity to enable it to ascend 64 feet. Find when and where the stones will meet. L.U.

25. Eight bodies are dropped in succession from a height at intervals of half a second. Taking $g = 32$ ft. per sec. per sec., calculate and show on a diagram the positions of the bodies at the instant at which the last is dropped. What is the relative velocity of any one of the bodies to the next succeeding one? L.U.

26. A body moves along a straight line with varying velocity, and a curve is constructed in which the ordinate represents the velocity at a time represented by the abscissa. Prove that the distance travelled by the body in any interval is measured by the area between the two corresponding ordinates. The body is observed to cover distances of 12, 30 and 63 yards in three successive intervals of 4, 5 and 7 seconds. Can it be moving with uniform acceleration? L.U.

27. From the top of a tower, 75 feet high, a stone is projected vertically upwards with a velocity of 64 feet per second. Calculate its greatest elevation, its velocity at the moment it strikes the ground, and the time it takes to reach the ground ($g = 32$.)

28. Establish the formula $s = ut + \frac{1}{2}ft^2$.

From an elevated point A a stone is projected vertically upwards. When the stone reaches a distance h below A its velocity is double what it was at a height h above A. Show that the greatest height attained by the stone above A is $\frac{1}{3}h$. Adelaide University.

29. Two trains, A and B, leave the same station on parallel lines of way. The train A starts with uniform acceleration of $\frac{1}{2}$ foot per second per second, and attains a maximum speed of 15 miles per hour, when steam is reduced so as to keep the speed constant. B leaves 40 seconds after A with uniform acceleration of 1 foot per second per second, and attains a maximum speed of 30 miles per hour. At what distance from the station will B overtake A?

30. Plot a velocity-time graph from the following particulars: Draw a horizontal line OX, 5 inches in length, and divide it into 10 equal parts; each part represents 0·2 second. Draw OY perpendicular to OX, and on

it construct a scale of velocities in which 0·5 inch represents 10 feet per second. The velocities in feet per second at the beginning of the time intervals shown on OX are as follows: 0, 16, 30, 42, 49, 49, 47, 40, 28, 14, 0.

(*a*) Find the change in velocity and the average acceleration during each interval of time; draw an acceleration-time graph by plotting the average accelerations at the centres of the time intervals. Scale for accelerations, one inch represents 20 feet per second per second.

(*b*) Find the average velocity during each time interval, and calculate the displacement during each interval; hence calculate the total displacement during the 2 seconds represented by OX.

CHAPTER IV

COMPOSITION AND RESOLUTION OF VELOCITIES AND ACCELERATIONS

Composition and resolution of velocities.—Velocity being a vector quantity may be represented by a straight line in the same manner as displacement. A given velocity may be regarded as made up of two or more component velocities, which may be compounded to obtain the resultant velocity by the methods of vector addition employed in Figs. 28 and 29 (p. 28). Thus, if a point has a velocity represented in magnitude, direction and sense by AB (Fig. 40), and if its initial position be A, then it will travel from A to B in one second. Suppose on arrival at B that the initial velocity of the point is suppressed, and that another velocity is imparted to it, represented by BC. The point will now travel from B to C in one second. Had both velocities been imparted simultaneously to the point when at A, the point would travel along the line AC and would arrive at C in one second. Hence AC represents the resultant velocity of which AB and BC represent the components. Similar reasoning may be applied to a number of component velocities. The triangle ABC in Fig. 40 may be called the **triangle of velocities.**

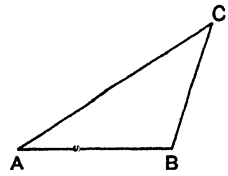


FIG. 40.—Triangle of velocities.

Composition of velocities is the process of finding the resultant velocity from given components; **resolution of velocities** is the inverse process.

Parallelogram of velocities.—Instead of a triangle of velocities ABC (Fig. 40), a construction called the **parallelogram of velocities** may be employed. In Fig. 41, a point A has component velocities v_1 and v_2 represented by AB and AC respectively. Complete the parallelogram ABDC, when the diagonal AD will represent completely the resultant

velocity v. It is evident that the triangle ABD, which is one-half of the parallelogram, is a triangle of velocities corresponding with the triangle ABC in Fig. 40. The component velocities must be arranged, prior to constructing the parallelogram of velocities, so that the

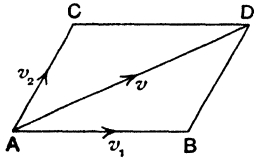


FIG. 41.—Parallelogram of velocities

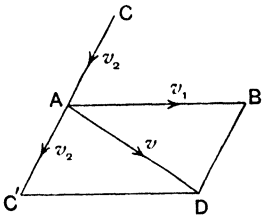


FIG. 42.

senses are either both away from A or both towards A. This process is illustrated in Fig 42, in which AB and CA represent the given velocities v_1 and v_2. AC′ is drawn to represent v_2, and v_1 and v_2 have now senses both away from A. The parallelogram is constructed as before, giving the resultant velocity v represented by AD.

Rectangular components of a velocity. - It is often convenient in the solution of problems to use components of a given velocity taken along two rectangular axes which intersect at a point on the line of the given velocity and lie in the same plane. In Fig. 43, OA represents the given velocity v; OX and OY are rectangular axes complying with the above conditions. The component velocities v_x and v_y are found by drawing AB and AC perpendicular to OX and OY respectively, when OB and OC represent v_x and v_y respectively.

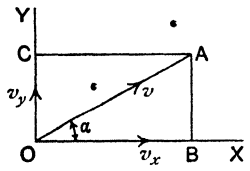


FIG. 43.—Rectangular components of v.

Let the angle XOA be α, then :

$$\frac{OB}{OA} = \cos\alpha, \quad \text{or} \quad \frac{v_x}{v} = \cos\alpha;$$

$$\therefore v_x = v\cos\alpha. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

Also,
$$\frac{OC}{OA} = \sin OAC = \sin\alpha, \quad \text{or} \quad \frac{v_y}{v} = \sin\alpha;$$

$$\therefore v_y = v\sin\alpha. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

Further,
$$OA^2 = OB^2 + BA^2 = OB^2 + OC^2,$$

or
$$v^2 = v_x^2 + v_y^2;$$

$$\therefore v = \sqrt{v_x^2 + v_y^2}. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

Relative velocity.—A person standing on the earth and watching a moving body does not perceive the absolute motion of the body; what he does observe may be described as the motion of the body **relative to the earth.** In such cases it is convenient to regard the earth, and hence the observer, as fixed in space. The velocity of one body relative to another may be defined as **that velocity which an observer, situated on and moving with the second body, would perceive in the first.**

EXAMPLE.—Suppose two trains to be moving on parallel lines of railway and to have equal velocities of like sense. A passenger in either train would perceive no velocity whatever in the other train, which would appear to him to be at rest. The velocity of either train relative to the other train is zero in this case. If one train A has a velocity of 40 miles per hour towards the north and the other B, a velocity of 30 miles per hour also towards the north, an observer in B will see A passing him at 10 miles per hour, and would describe the velocity of A relative to B as 10 miles per hour towards the north. An observer in A would see B falling behind at 10 miles per hour and would describe the velocity of B relative to A as 10 miles per hour towards the south.

These statements may be generalised by saying that **the velocity of one body A, relative to another body B, is equal and opposite to the velocity of B relative to A.**

Determination of relative velocity.—In Fig. 44 a point A has a velocity v_A relative to the paper and represented by AC. Another

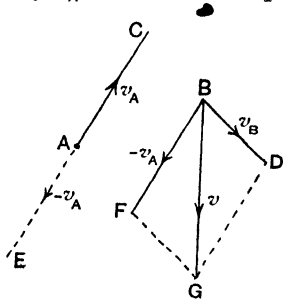


FIG. 44.—Velocity of B relative to A.

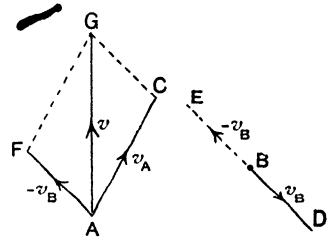


FIG. 45.—Velocity of A relative to B.

point B has a velocity v_B also relative to the paper and represented by BD. To obtain the velocity of B relative to A, stop A by imparting to it a velocity $-v_A$, represented by AE, and equal and opposite to v_A; to preserve unaltered the relative conditions, give B also a

velocity $-v_A$, represented by BF. A being now at rest relative to the paper, and B having component velocities v_B and $-v_A$, the velocity of B relative to A will be the resultant v, obtained by drawing the parallelogram of velocities BDGF.

The velocity of A relative to B may be obtained by a similar process (Fig. 45). B is stopped by imparting to it a velocity $-v_B$, and an equal and like velocity is given to A, represented by AF. A has now components v_A and $-v_B$, which when compounded give a resultant velocity v as the velocity of A relative to B. It will be noted that v in Fig. 44 is equal and opposite to v in Fig. 45.

Composition and resolution of accelerations.—Acceleration being a vector quantity, we may say at once that its representation by a straight line, the composition of two or more accelerations in order to find the resultant acceleration, and the resolution of a given acceleration into components along any pair of axes may be carried out in the same manner as for velocity. For example, if a point has an acceleration a represented in magnitude, direction and sense by OA (Fig. 46), the components along two rectangular axes OX and OY will be given by

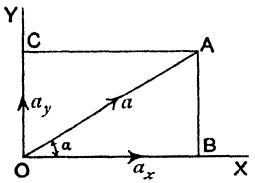


FIG. 46.—Rectangular components of an acceleration

$$a_x = a \cos \alpha. \quad \ldots\ldots (1)$$

$$a_y = a \sin \alpha. \quad \ldots\ldots (2)$$

Also,
$$a = \sqrt{a_x^2 + a_y^2}. \quad \ldots\ldots (3)$$

Velocity changed in direction.—In Fig. 47 (*a*) a point has an initial velocity v_1, represented by AB, and a final velocity v_2, repre-

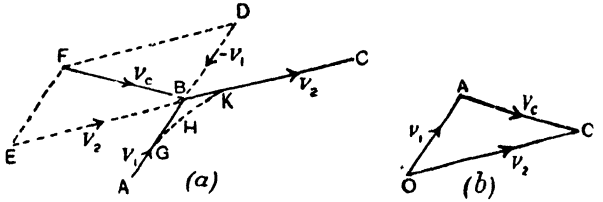


FIG. 47.—Velocity changed from v_1 to v_2.

sented by BC. To determine what has been the change in velocity during the interval of time we may proceed as follows: Stop the point when it arrives at B by impressing on it a velocity $-v_1$

represented by DB. As the point is at rest now, we may send it off in any direction with any speed. Give it the desired final velocity v_2 represented by EB. The resultant change in velocity v_c has components $-v_1$ and v_2, and may be found by constructing the parallelogram BDFE, when FB will represent v_c.

As will be understood later, it is impossible to have the change in velocity take place instantaneously at B; the direction of motion of any particle or body travelling in the line AB can be changed to another direction BC only by the body travelling along some curve such as that shown dotted at GHK. In this case the total change in velocity during the interval of time in which the particle travels from G to K, may be found by the same method and is represented by FB.

A simple alternative construction is shown in Fig. 47 (*b*). A point O is chosen and OA and OC are drawn from it to represent completely v_1 and v_2 respectively. The total change in velocity v_c is represented by AC, and has a sense indicated by AC, *i.e.* from the end of the initial velocity towards the end of the final velocity. It will be noted that the triangle OAC in Fig. 47 (*b*) is similar and equal to the triangle EFB in Fig. 47 (*a*); hence the truth of the alternative construction is established.

Since the motion along the path GHK has involved a resultant change in velocity v_c, it may be asserted that there has been a resultant acceleration having the same direction and sense as v_c. This acceleration may be calculated provided the interval of time t is known in which the particle travelled from G to K. Thus:

$$\text{Resultant acceleration} = \frac{v_c}{t}.$$

Motion in a circular path.—The case of a point travelling with uniform velocity in the circumference of a circle provides an important application of the above methods. In Fig. 48 (*a*) a point P is travelling in the circumference of a circle of radius r cm., and has a velocity of uniform magnitude v cm. per sec. When the point is at P_1, the direction of its velocity will be along the tangent at P_1, and is shown by v_1. Similarly, when the point is at P_2, the velocity has a direction as shown by v_2; both v_1 and v_2 are equal numerically to v.

v_1 and v_2 will meet, if produced, at D; the total change in velocity occurring while the point travels from P_1 to P_2 may be found by

applying the method explained on p. 45. Draw OA to represent v_1 (Fig. 48, *b*), also draw OB to represent v_2; the total change in velocity will be represented by AB, equal to v_c. Apply v_c at D, when it will be evident, from the geometry of the figure, that v_c passes through the centre C of the circle; this fact is independent of the length of the arc P_1P_2, and leads us to assert that **the resultant acceleration of the point is directed constantly towards the centre of the circle**

In applying this construction there is no limit (other than draughtsmanship) to the smallness of the arc P_1P_2. Suppose that this arc is taken very small, then the construction for obtaining the change

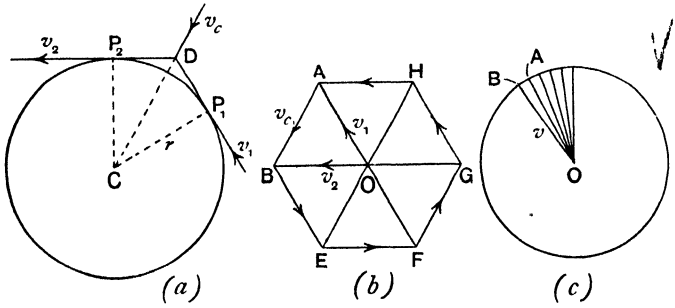


FIG 48.—Motion in a circle

in velocity becomes OAB (Fig. 48, *c*), in which OA and OB are each made equal to v and AB represents the change in velocity. Repetition of the construction for successive small arcs taken completely round the circle in Fig. 48 (*a*) will give a polygon having a very large number of sides, and this polygon becomes a circle having a radius v when the arcs are taken of indefinite smallness. Thus the total change of velocity while P describes one complete revolution in Fig. 48 (*a*) is given by the circumference of the circle in Fig. 48 (*c*), viz. $2\pi v$ cm. per sec. The interval of time in which this change in velocity takes place is equal to the time taken by P to execute one complete revolution, *i.e.* the time in which P travels a distance of $2\pi r$ along the circumference of the circle in Fig. 48 (*a*). Let t be this time in seconds, then

$$s = vt, \quad \text{(p. 33)}$$

$$2\pi r = vt,$$

or

$$t = \frac{2\pi r}{v} \text{ secs.} \quad \ldots\ldots\ldots\

$$\text{acceleration} = \frac{\text{change in velocity}}{\text{time interval}};$$

$$\therefore\ a = 2\pi v \div \frac{2\pi r}{v}$$

$$= \frac{v^2}{r} \text{ cm. per sec. per sec.} \quad \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

The conclusions are that a point travelling with uniform velocity in the circumference of a circle has a constant acceleration directed towards the centre of the circle and given numerically by the above result.

It should be noted that the appropriate British units are v in feet per sec., r in feet and a in feet per sec. per sec. The student may verify this by inserting the dimensions in equation (2).

Motion in a jet discharged horizontally.—A jet of water discharged horizontally from a small orifice at O (Fig. 49) provides an interesting example of change in direction of velocity. But for the downward acceleration g, which every particle of the water possesses, the jet would continue to travel in the horizontal line OX. Actually it travels in a curved path OPA, and the velocity v of any particle passing through a fixed point P may be taken as compounded of two velocities, viz. v_x, which may be assumed to be equal to the initial velocity u, and v_y, which follows the ordinary laws of falling bodies. These assumptions involve the neglect of effects due to the resistance of the atmospheric air.

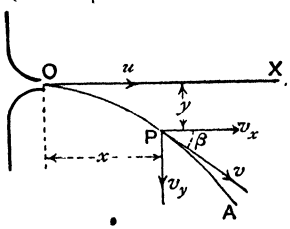


FIG 49.—Motion in a jet

Let t be the time taken by a particle in travelling from O to P, and let x and y be the coordinates of P, then

$$x = ut; \quad \therefore\ t = \frac{x}{u}.$$

$$y = \tfrac{1}{2}gt^2 = \tfrac{1}{2}g\frac{x^2}{u^2} = \frac{g}{2u^2}x^2. \quad \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

Hence, since g and u are constants, y is proportional to x^2, and the curve of the jet is a parabola.* Again,

$$v_x = u, \quad \text{and} \quad v_y = gt = g\frac{x}{u};$$

$$\therefore\ v = \sqrt{v_x^2 + v_y^2} = \sqrt{u^2 + g^2\frac{x^2}{u^2}}. \quad \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

*See *A School Geometry*, chap. xxiv., by H. S. Hall. Macmillan.

Also, $$\cot\beta = \frac{v_x}{v_y} = \frac{u}{gt} = \frac{u^2}{gx} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

For any given value of the initial velocity u, the curve of the jet may be plotted from (1); the direction of the tangent to the jet at any time t, or at any horizontal distance x from the orifice may be determined from (3), and the velocity at any point in the jet may be found from (2).

Motion of a particle projected at an angle to the horizontal.—Referring to Fig. 50, a particle is discharged at O with a velocity

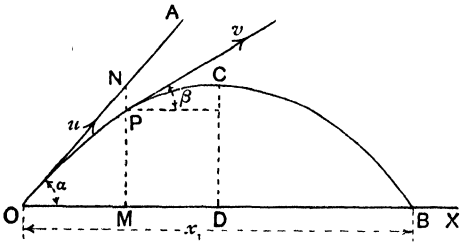


FIG 50.—Motion of a projectile

u in a line OA inclined at an angle α to the horizontal. The horizontal and vertical components of u are $u\cos\alpha$ and $u\sin\alpha$ respectively. It may be assumed, neglecting air resistance and any variations in the value of g, that $u\cos\alpha$ is the horizontal component of the velocity of the particle at any point in its flight, and that $u\sin\alpha$ is affected by the ordinary laws of falling bodies.

Let P be any point on the curved path, or **trajectory**, of the particle; let x and y be the coordinates of P, and let t be the time taken to travel from O to P. But for the downward acceleration g, the particle, after travelling for t seconds, would be found at a point N on OA, vertically over P.

Hence, $\mathrm{ON} = ut$,

$$x = \mathrm{ON}\cos\alpha = ut\cos\alpha. \quad \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

Also, $\mathrm{NP} = \frac{1}{2}gt^2$,

$$y = \mathrm{MN} - \mathrm{NP} = ut\sin\alpha - \tfrac{1}{2}gt^2. \quad \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

From (1), $$t = \frac{x}{u\cos\alpha}$$

Substitute in (2), $$y = \frac{ux\sin\alpha}{u\cos\alpha} - \tfrac{1}{2}g\frac{x^2}{u^2\cos^2\alpha}$$

$$= x\tan\alpha - \frac{g}{2u^2\cos^2\alpha}x^2. \quad \ldots\ldots\ldots\ldots\ldots\ldots (3)$$

The form of this relation of y and x indicates that the trajectory is a parabola.

The **horizontal range** is OB (Fig. 50). At B, y is zero; hence we may obtain the value of OB $= x_1$ by equating y in (3) to zero:

$$x_1 \tan\alpha - \frac{g}{2u^2\cos^2\alpha} x_1^2 = 0;$$

$$\therefore \tan\alpha - \frac{g}{2u^2\cos^2\alpha} x_1 = 0,$$

$$x_1 = \frac{2u^2 \tan\alpha \cos^2\alpha}{g} = \frac{2u^2 \sin\alpha\cos\alpha}{g}$$

$$= \frac{u^2 \sin 2\alpha}{g}. \quad \ldots\ldots (4)$$

The range will be a maximum when $\sin 2\alpha$ is a maximum, *i.e.* when $\sin 2\alpha = 1$; 2α will then be 90° and α will be 45°. Hence maximum horizontal range will be secured by projection at 45° to the horizontal.

In Fig. 50, C is the highest point in the trajectory, and evidently bisects the curve between O and B. The maximum height attained is CD. Let t_1 be the total time of flight, then the time taken to reach C from O will be $\frac{1}{2}t_1$. Now

$$x_1 = u\cos\alpha \times t_1,$$

or,
$$\frac{2u^2 \sin\alpha\cos\alpha}{g} = ut_1\cos\alpha;$$

$$\therefore t_1 = \frac{2u\sin\alpha}{g}. \quad \ldots\ldots (5)$$

And time in which C is reached $= \frac{u\sin\alpha}{g}$. $\ldots\ldots (5')$

At C, the vertical component of the initial velocity, viz. $u\sin\alpha$, has disappeared; hence, from equation (*a*), p. 35,

$$u^2\sin^2\alpha = 2g \times \text{CD},$$

$$\text{CD} = \frac{u^2\sin^2\alpha}{2g}. \quad \ldots\ldots (6)$$

At P, the velocity v of the particle is inclined at an angle β to the horizontal (Fig. 50). Writing v_x and v_y for the horizontal and vertical components of v, we have

$$v_x = v\cos\beta = u\cos\alpha,$$

$$v_y = v\sin\beta = u\sin\alpha - gt,$$

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{u^2\cos^2\alpha + (u\sin\alpha - gt)^2}$$

$$= \sqrt{u^2(\cos^2\alpha + \sin^2\alpha) - 2ugt\sin\alpha + g^2t^2}$$

$$= \sqrt{u^2 - 2ugt\sin\alpha + g^2t^2}. \quad \ldots\ldots (7)$$

Also, $\tan\beta = \frac{v_y}{v_x} = \frac{u\sin\alpha - gt}{u\cos\alpha}$. $\ldots\ldots (8)$

EXERCISES ON CHAPTER IV.

1. A point has two component velocities, each equal to 10 cm. per sec. Find, by careful drawing, the resultant velocities when the lines of direction meet at angles of 60°, 120°, 180°, 270°.

2. A boat is rowed up a straight reach on a river in a direction making 22° with the bank. If the velocity is 6 feet per second, calculate the component velocities parallel and perpendicular to the bank. In what time will the boat travel a distance of 100 yards, measured parallel to the bank ?

3. A projectile has component velocities of 1600 feet per second horizontally and 200 feet per second vertically at a certain instant. Calculate the resultant velocity.

4. A ship is sailing towards the north-east at 12 miles per hour. A person walks across the deck from port to starboard at 4 feet per second. What is his resultant velocity ?

5. A piece of coal falls vertically from rest from a height of 9 feet above the floor of a truck travelling at 2 miles per hour. Find the velocity of the coal relative to the truck just before the coal reaches the floor.

6. A train has a speed of 30 miles per hour. A drop of rain falls in a vertical plane parallel to the direction of motion of the train. Show in diagrams the direction of motion of the raindrop as seen by an observer in the train, (*a*) if the raindrop falls vertically with a velocity of 20 feet per second ; (*b*) if the raindrop has, in addition to the velocity given in (*a*), a component velocity of 5 feet per second in the direction of motion of the train ; (*c*) if the drop has a component of the same magnitude as given in (*b*) but in a direction opposite to that of the train.

7. A person runs after a tramcar travelling at 6 miles per hour. If his velocity is 8 miles per hour in a direction making 30° with the rails, find his velocity relative to the car.

8. A railway coach having ordinary cross-seats is travelling at 8 feet per second. A person about to enter a compartment runs at 10 feet per second. Show in a diagram the direction in which he must run on the platform if his velocity on entering the compartment is to be parallel to the seats ; find the magnitude of the latter velocity.

9. A person in a motor car travelling at 15 miles per hour towards the north observes a piece of paper borne by the wind and travelling towards the car apparently from the east with a velocity of 4 feet per second. Find the velocity of the wind.

10. State the parallelogram of velocities. A ship, A, is travelling from south to north with a speed of 20 miles per hour ; another ship, B, appears to an observer on A to be travelling from west to east with a velocity of 15 miles per hour. Find the magnitude and direction of B's velocity relative to the earth. L.U.

11. A steamer is travelling northward at the rate of 8 miles an hour in a current flowing westward at the rate of 3 miles an hour. Indicate in

a diagram the direction in which the steamer is heading, and find the rate at which it is steaming. If the wind is blowing at the rate of 3 miles an hour from the east, indicate in your diagram the direction in which a small flag at the masthead is pointing. L.U.

12. An aeroplane is travelling towards the north-west relative to the earth at 90 miles per hour, and the wind is blowing at 20 miles per hour towards the north. Suppose the wind were to cease suddenly, find the velocity of the aeroplane in magnitude and direction relative to the earth.

13. Two railway tracks Ox and Oy enclose an angle of 60°. A train moves along Ox with uniform velocity of 60 miles an hour, while a second train moves along Oy with equal speed, passing through O two minutes after the first. Find the velocity of the second train relatively to the first, and indicate in a diagram the shortest distance between the trains. L.U.

14. A cyclist rides at 10 miles an hour due north, and the wind (which is blowing at 6 miles an hour from a point between N. and E.) appears to the cyclist to come from a point 15° to the east of north ; find graphically or by calculation the true direction of the wind, and the direction in which the wind will appear to meet him on his return, if he rides at the same speed. Sen. Cam. Loc.

15. Two ships are steaming along straight courses with such constant velocities that they will collide unless their velocities are altered. Show that to an observer on either ship the other appears to be always moving directly towards him. L.U.

16. Explain what is meant by the velocity of one moving particle relative to another moving particle, and show how to determine it. To a ship sailing E. at 15 knots another ship whose speed is 12 knots appears to be sailing N.W. Show that there are two directions in which the latter may be moving. Find these directions, graphically or otherwise, and find the relative velocity in each case. L.U.

17. A railway coach at a certain instant has a velocity of 10 metres per second towards the north. Twenty seconds afterwards the velocity is found to be 15 metres per second towards the north-west. Find the change in velocity and the average value of the acceleration.

18. A piece of tube is bent near the middle so that the straight portions include an angle of 30°. If water flows through the tube with uniform velocity of 4 feet per second, find the total change in velocity in passing round the bend.

19. A billiard ball travelling at 3 feet per second strikes the cushion and moves thereafter in a line making 60° with the original direction of motion and with a velocity of $2\frac{1}{2}$ feet per second. Find the change in velocity.

20. A point travels in the circumference of a circle 40 cm. in diameter with a uniform velocity of 120 cm. per second. Find the acceleration towards the centre of the circle.

21. A railway coach has a speed of 60 miles per hour and travels round a curve having a radius of 1200 feet. Find the acceleration towards the centre of the curve.

22. A jet of water issues from a small hole in the vertical side of a tank with a horizontal velocity of 8 feet per second. Find the resultant velocity of a particle in the jet 2 seconds after it has issued from the orifice.

23. In Question 22, find the position of a particle in the jet at intervals of 0·1, 0 2, 0 3, 0 4 and 0 5 second after issue. From the information so obtained plot a graph showing the shape of the jet. Take $g = 32$ ft. per sec. per sec.

24. A bullet is projected with a velocity of 1200 feet per sec. horizontally from a gun which is 25 feet above the ground. Find the horizontal distance from the gun at which the bullet strikes the ground, and also the angle its direction of motion then makes with the horizontal. Sen. Cam. Loc.

25. A projectile is fired with a velocity of 2200 feet per second. Find the horizontal range, time of flight and greatest height attained when the angles of elevation are respectively 30°, 40°, 45°, 50° and 60°. Neglect atmospheric effects. Take $g = 32$ ft. per sec. per sec.

26. A gun capable of firing a projectile with a velocity of 2000 feet per second is placed at a horizontal distance of 400 feet from the foot of a vertical cliff 200 feet high. Find the angle of elevation of the gun in order that the projectile may just clear the edge of the cliff. Neglect atmospheric effects.

27. A ball is projected from a point 7 feet high with a velocity of 50 feet per second. At what angle to the horizontal must it be projected in order just to clear the top of a net 3 5 feet high at a horizontal distance of 30 feet from the point of projection ? Neglect atmospheric effects.

28. A heavy particle is projected with a velocity v in a direction making an angle θ with the horizon. Form the equations determining its position and velocity at any subsequent instant of time. Drops of water are thrown tangentially off the horizontal rim of a rotating umbrella. The rim is 3 feet in diameter, and is held 4 feet above the ground, and makes 14 revolutions in 33 seconds. Show that the drops of water will meet the ground on a circle 5 feet in diameter. Madras Univ.

CHAPTER V

ANGULAR VELOCITY AND ACCELERATION

Angular velocity.—Let one point in a straight line be fixed, and let the line revolve about this point in a fixed plane, say that of the paper. The rate of describing angles is termed the **angular velocity** of the line. Angular velocity may be measured in revolutions per minute or per second: for many purposes it is more convenient to measure angular velocity in radians per second. The symbol ω is used to denote the latter.

Since there are 2π radians in a complete revolution, the connection between ω and the revolutions per minute, N, is

$$\omega = \frac{N}{60} 2\pi = \frac{\pi N}{30} \text{ radians per second.}$$

In uniform angular velocity, equal angles are described in equal intervals of time; should this condition not be complied with the angular velocity varies, and the revolving line is said to have **angular acceleration.**

Angular velocity may be described as being **clockwise** or **anticlockwise,** according as the line appears to the observer to rotate in the same, or in the opposite direction to that of the hands of a clock. The student will note that, if there are two observers, one on each side of the plane of rotation, the angular velocity will appear to be clockwise to one observer and anticlockwise to the other.

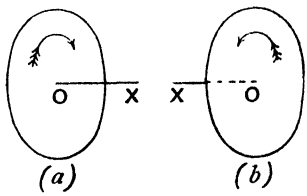


FIG. 51.—Representation of angular velocity.

A given angular velocity may be represented by drawing a line perpendicular to the plane in which the body is revolving. The length of the line represents the angular velocity to a chosen scale, and the line is drawn on one or the other side of the plane of revolution depending on the sense of rotation. Thus, in Fig. 51 (*a*), a person

situated on the right-hand side of the revolving disc observes that the angular velocity is clockwise and draws OX perpendicular to the plane of the disc and on his side of the disc In Fig. 51 (*b*), the angular velocity appears to the person to be anticlockwise, and OX is drawn on the opposite side of the disc. The student should verify by trial that two persons on opposite sides of a revolving disc will agree in placing OX on the same side of the disc.

Relation of linear and angular velocity.—Let OA (Fig. 52) revolve about O with uniform angular velocity. At any instant the point A has a linear velocity v in the direction at right angles to OA. Let r be the radius of the circle which A describes. The length of the

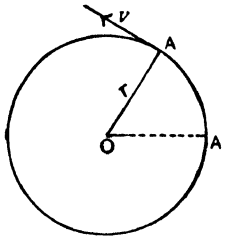


FIG. 52.—Relation of angular and linear velocities

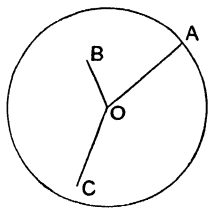


FIG. 53.—All radial lines have the same angular velocity

arc described by A in one second is v, and the angle subtended at the centre of the circle by this arc will be v/r radians, the same unit of length being used in stating both v and r Hence OA turns through v/r radians in one second, and the angular velocity is

$$\omega = \frac{v}{r} \text{ radians per second} \quad \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

In Fig. 53 a wheel rotates in the plane of the paper about an axis at O perpendicular to this plane. It is evident that the radii drawn to any fixed points, OA, OB, OC, etc., all possess the same angular velocity. Hence the angular velocity of a rotating body may be calculated by dividing the linear velocity of any point in the body by the radius drawn from that point to the axis of rotation.

Angular acceleration.—**Angular acceleration** is defined as the rate of change of angular velocity, and may be calculated by dividing the change in angular velocity by the time taken. Thus, if a revolving line changes its angular velocity from ω_1 to ω_2 radians per

second in t seconds, and if the change has been effected at a uniform rate, then

$$\text{Angular acceleration} = \phi = \frac{\omega_2 - \omega_1}{t} \text{ radians per sec. per sec.} \quad \ldots(2)$$

In Fig. 54 a line rotates about O in the plane of the paper with varying angular velocity. When passing through OA its angular velocity is ω_1, and the angular velocity increases at a uniform rate and is ω_2 when passing through OB. Let the time taken to travel from OA to OB be t seconds, then

$$\text{Angular acceleration} = \phi = \frac{\omega_2 - \omega_1}{t}.$$

Let the linear velocities of A and B be v_1 and v_2 respectively, and let r be the radius of the circle, then

$$\omega_1 = \frac{v_1}{r}, \quad \text{and} \quad \omega_2 = \frac{v_2}{r},$$

$$\therefore \phi = \frac{v_2 - v_1}{rt}.$$

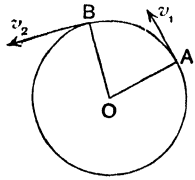


FIG 54.—Angular acceleration.

Now $(v_2 - v_1)/t$ is the tangential acceleration a of the point A in travelling from A to B, hence

$$\phi = \frac{a}{r} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3)$$

It will be noted that this rule corresponds with that for deriving angular velocity from linear velocity.

All radii of a revolving body possess the same angular acceleration, hence the angular acceleration may be calculated by dividing the tangential acceleration of any point in the body by the radius drawn to the point from the axis of rotation.

Equations of angular motion.—Equations for angular motion may be derived in the manner adopted in Chapter III. in finding equations for rectilinear motion.

Let a line revolve with uniform angular velocity ω radians per second, and let α be the angle described in t seconds. Then

$$\alpha = \omega t \text{ radians.} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

Let a line start to revolve from rest with an angular acceleration ϕ radians per second per second, the angular velocity ω at the end of t seconds is given by

$$\omega = \phi t \text{ radians per second.} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2)$$

The average angular velocity is

$$\omega_a = \frac{0+\omega}{2} = \tfrac{1}{2}\omega,$$

and $$\alpha = \omega_a t = \tfrac{1}{2}\omega t \text{ radians.} \quad \ldots\ldots\ldots\ldots (3)$$

Substituting for ω from (2) gives

$$\alpha = \tfrac{1}{2}\phi t \times t = \tfrac{1}{2}\phi t^2 \text{ radians.} \quad \ldots\ldots\ldots\ldots (4)$$

Again, from (2), $$t = \frac{\omega}{\phi}, \quad \therefore\ t^2 = \frac{\omega^2}{\phi^2}.$$

Substituting this value in (4), we have

$$\alpha = \tfrac{1}{2}\phi \frac{\omega^2}{\phi^2} = \frac{\omega^2}{2\phi},$$

$$\therefore\ \omega^2 = 2\phi\alpha. \quad \ldots\ldots\ldots\ldots (5)$$

The analogy of these equations with those for rectilinear motion is apparent. Equations for a line having an initial angular velocity ω_1 and an angular acceleration ϕ may be obtained in a similar manner. The equations are as follows :

$$\omega_2 - \omega_1 = \phi t \text{ radians per sec.} \quad \ldots\ldots\ldots\ldots (6)$$

$$\alpha = \left(\frac{\omega_2 + \omega_1}{2}\right) t \text{ radians.} \quad \ldots\ldots\ldots\ldots (7)$$

$$\alpha = \omega_1 t + \tfrac{1}{2}\phi t^2 \text{ radians.} \quad \ldots\ldots\ldots\ldots (8)$$

$$\omega_2^2 - \omega_1^2 = 2\phi\alpha. \quad \ldots\ldots\ldots\ldots (9)$$

EXAMPLE 1.—A wheel starts from rest and acquires a speed of 300 revolutions per minute in 40 seconds. Find the angular acceleration. How many revolutions did the wheel describe during the 40 seconds ?

$$\omega = \frac{300}{60} \cdot 2\pi = 10\pi = 31{\cdot}41 \text{ radians per sec.}$$

$$\phi = \frac{\omega}{t} = \frac{31{\cdot}41}{40} = 0{\cdot}785 \text{ radian per sec. per sec.}$$

$$\text{Average angular velocity} = \frac{300}{2} = 150 \text{ revs per min.}$$

$$= \frac{150}{60} = 2{\cdot}5 \text{ revs. per sec.}$$

$$\therefore \text{ Revolutions described} = 2{\cdot}5 \times 40 = 100.$$

EXAMPLE 2.—The driving wheel of a locomotive is 6 feet in diameter. Assuming that there is no slipping between the wheel and the rail, what

is the angular velocity of the wheel when the engine is running at 60 miles per hour ?

$$\text{Velocity of locomotive} = \frac{5280 \times 60}{60 \times 60} = 88 \text{ ft. per sec.}$$

As the distance travelled in one second is 88 feet, we may find the revolutions of the wheel per second by imagining 88 feet of rail to be wrapped round the circumference of the wheel.

$$\text{Number of turns of rail} = \frac{88}{\pi d} = \frac{88}{6\pi}.$$

$$\therefore \text{ Revolutions per second} = \frac{88 \times 7}{22 \times 6} = 4{\cdot}67 ;$$

$$\therefore \omega = 4{\cdot}67 \times 2\pi$$

$$= 29{\cdot}33 \text{ radians per sec.}$$

Transmission of motion of rotation.—In workshops many machines are driven by means of belts. A pulley is fixed to each shaft, and a belt is stretched round the pulleys as shown in Fig. 55. If it is intended that both shafts should rotate in the same direction, the

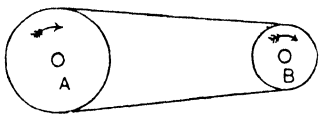


FIG. 55.—Open belt.

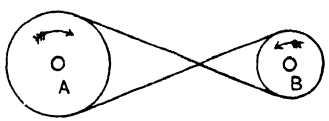


FIG. 56.—Crossed belt.

belt is open as in Fig. 55. Crossing the belt as shown in Fig. 56 enables one shaft to drive the other in the contrary direction. Neglecting any slipping between the belt and the pulleys, it is evident that the linear velocities of points on the circumferences of both pulleys are equal to the linear velocity of the belt. Let V be this velocity, and let R_A and R_B be the radii of the pulleys, then

$$\text{Angular velocity of A} = \omega_A = \frac{V}{R_A}.$$

$$\text{Angular velocity of B} = \omega_B = \frac{V}{R_B}.$$

$$\therefore \frac{\omega_A}{\omega_B} = \frac{R_B}{R_A}.$$

Thus the angular velocities of the pulleys are inversely proportional to the radii, or the diameters of the pulleys.

The arrangement shown in Fig. 57 enables a larger angular velocity ratio to be obtained. A drives B, and another pulley C, fixed

to the same shaft as B, drives D; similarly, E drives F. Taking the pulleys in pairs, we have

$$\frac{\omega_A}{\omega_B}=\frac{R_B}{R_A}\ ;\quad \frac{\omega_C}{\omega_D}=\frac{R_D}{R_C}\ ;\quad \frac{\omega_E}{\omega_F}=\frac{R_F}{R_E}.$$

Also $$\omega_B=\omega_C,\quad \text{and}\quad \omega_D=\omega_E.$$

Hence $$\frac{\omega_A\times\omega_C\times\omega_E}{\omega_B\times\omega_D\times\omega_F}=\frac{R_B\times R_D\times R_F}{R_A\times R_C\times R_E},$$

or $$\frac{\omega_A}{\omega_F}=\frac{R_B\times R_D\times R_F}{R_A\times R_C\times R_E}.$$

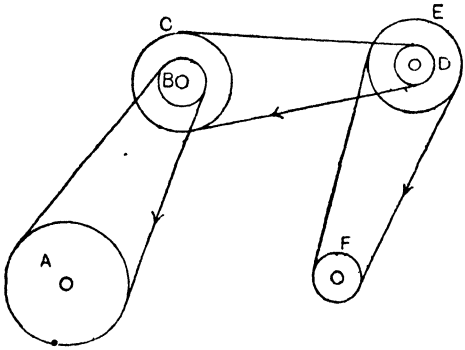


FIG. 57.—A belt pulley arrangement.

Hence the rule: the angular velocity ratio of the first and last pulleys is given by the product of the radii, or diameters, of the driven pulleys divided by the product of the radii, or diameters, of the driving pulleys.

Toothed wheels (Fig. 58) are used in cases where there must be no slipping. The teeth may be imagined to be formed on two cylinders shown dotted. It is clear that the linear velocities of points on the circumferences of the cylinders are equal, and therefore we have the same rule as for a pair of belt pulleys, viz.

$$\frac{\omega_A}{\omega_B}=\frac{R_B}{R_A}.$$

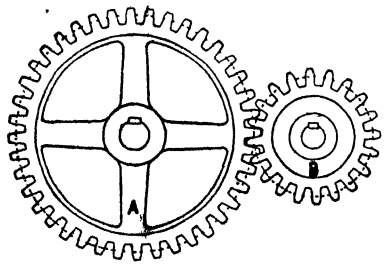


FIG. 58.—Toothed wheels in gear.

Further, the numbers of teeth, n_A and n_B, are proportional to the circumferences and therefore to the radii of the cylinders. Hence

$$\frac{\omega_A}{\omega_B} = \frac{n_B}{n_A}.$$

The wheels shown in Fig. 58 revolve in opposite directions. If angular velocities of the same sense be required, an **idle wheel** C is interposed (Fig. 59). The linear velocities of the circumferences of all three cylinders are still equal; hence, as before,

$$\frac{\omega_A}{\omega_B} = \frac{n_B}{n_A}.$$

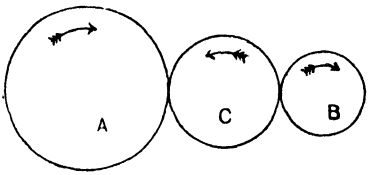


FIG. 59.—Use of an idle wheel.

A **train of wheels**, such as is used in clocks and other devices, is shown in Fig. 60. Taking the wheels in pairs, we have

$$\frac{\omega_A}{\omega_B} = \frac{n_B}{n_A}; \quad \frac{\omega_C}{\omega_D} = \frac{n_D}{n_C}; \quad \frac{\omega_E}{\omega_F} = \frac{n_F}{n_E}.$$

Also, $$\omega_B = \omega_C, \quad \text{and} \quad \omega_D = \omega_E.$$

Hence $$\frac{\omega_A \times \omega_C \times \omega_E}{\omega_B \times \omega_D \times \omega_F} = \frac{\omega_A}{\omega_F} = \frac{n_B \times n_D \times n_F}{n_A \times n_C \times n_E}.$$

It will be noticed that this result is similar to that obtained for the train of belts shown in Fig. 57.

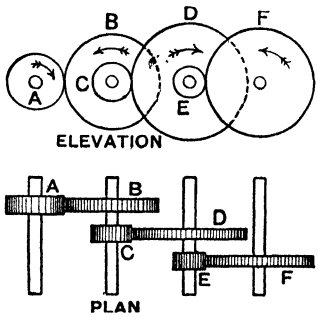


FIG. 60.—Train of wheels.

Chain drives are sometimes used instead of belts in order to avoid slipping. An ordinary bicycle provides an example. The circumferences of the toothed chain wheels, taken at the centres of the links of the chain, have the same linear velocity as the chain, hence we have the same rule as in the case of two belt pulleys, viz.

$$\frac{\omega_A}{\omega_B} = \frac{R_B}{R_A}.$$

Further, the numbers of teeth on the wheels are proportional to the circumferences, and therefore to the radii of the wheels; hence

$$\frac{\omega_A}{\omega_B} = \frac{n_B}{n_A}.$$

In early bicycles the driving was accomplished by means of cranks fixed to the axle of the front wheel; thus one revolution of the

crank gave one revolution to the wheel, and moved the bicycle *through a distance equal to the circumference of the wheel. When the statement is made that the gear of a modern bicycle is so much,* say 70, it is meant that for one revolution of the cranks the bicycle will travel a distance equal to that which would be covered by an old-fashioned machine having a driving wheel 70 inches in diameter. Let d be the diameter of the back wheel of the safety bicycle, and let n_A and n_B be the numbers of teeth on the crank chain wheel and the small chain wheel respectively, then

$$\text{Gear} = \text{D} = \frac{n_A}{n_B}\, d.$$

EXAMPLE.—**Varying angular velocity.**—In Fig. 61 a point travels with uniform velocity v in the straight line XP_1. The angular velocity of the radius vector OP_1 drawn from any fixed point O to the moving point at any instant may be determined thus: Consider two successive positions of the point, P_1 and P_2, and let these be close together. Join OP_1, OP_2, and draw P_1K perpendicular to OP_2. Let the angle P_1OX be α, and let the angle P_1OP_2 be called $\delta\alpha$. If δt is the time in which the point travels from P_1 to P_2, the radius vector describes the angle $\delta\alpha$ in the same time, and the angular velocity is given by

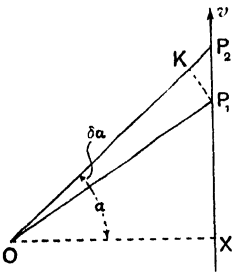


FIG 61.—Varying angular velocity.

$$\omega = \frac{\delta\alpha}{\delta t}.$$

In the similar triangles P_1OX, P_1KP_2, the angle $KP_1P_2 = P_1OX = \alpha$.

Now
$$\delta\alpha = \frac{P_1K}{OP_1} = \frac{P_1P_2 \cdot \cos\alpha}{OP_1} = \frac{P_1P_2 \cdot \cos\alpha \cdot \sin\alpha}{P_1X}.$$

Also
$$P_1P_2 = v \cdot \delta t;$$

$$\therefore\ \delta\alpha = \frac{v \cdot \delta t \cdot \cos\alpha \cdot \sin\alpha}{P_1X},$$

$$\therefore\ \omega = \frac{\delta\alpha}{\delta t} = \frac{v \cdot \sin\alpha \cdot \cos\alpha}{P_1X} \qquad \ldots\ldots(1)$$

This expression gives the angular velocity of the radius vector in terms of the distance of the point from X. If P_1X be zero, the point is passing through X, and α is zero. The expression for ω then takes the form 0/0. To determine the value, let the point be taken very close to X, when $\sin\alpha = \frac{P_1X}{OP_1}$ and $\cos\alpha = 1$. Inserting these values gives

$$\omega_0 = \frac{v \cdot P_1X}{P_1X \cdot OP_1} = \frac{v}{OP_1}$$

$$= \frac{v}{OX}. \qquad \ldots\ldots(2)$$

a result which complies with equation (1), p. 54.

Relative angular velocity.—In Fig. 62 a point A at a certain instant has a velocity v_A and another point B has a velocity v_B at the same instant. Stop A by impressing on it a velocity $-v_A$ and impress the same velocity $-v_A$ on B. Find the resultant velocity of B by means of the parallelogram B*acb*; this will be v. Instead of the given conditions of motion we now have the following equivalent conditions: A point A is at rest, and another point B is travelling along a straight line Bc with uniform velocity v and has reached B at a certain instant. Draw AX perpendicular to Bc, producing the latter if necessary. Let the angle BAX be called α, then, from equation (1) (p. 60), we have

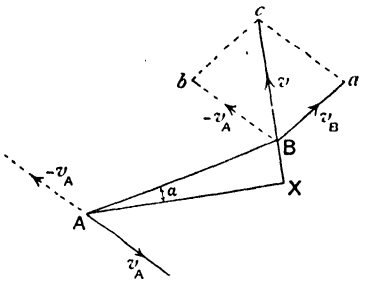


FIG. 62.—Relative angular velocity

$$\text{Relative angular velocity of B with respect to A} = \frac{v \cdot \sin\alpha \cdot \cos\alpha}{BX}.$$

With the velocities as given in Fig. 62, this relative angular velocity is counterclockwise. The angular velocity of A relative to B may be found in a similar manner, stopping B by applying $-v_B$ to B. The student should draw the diagram for this case for himself, and should verify that the angular velocity of A relative to B is equal to that of B relative to A, and has the same sense of rotation.

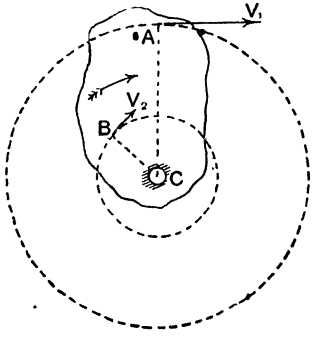


FIG. 63.—Velocities of points in a rotating body.

Velocity of any point in a rotating body.—In Fig. 63 is shown a body rotating with uniform angular velocity ω about an axis at C which is perpendicular to the plane of the paper. At any instant the direction of the velocity of any point, such as A or B, is perpendicular to the radius. Suppose that the velocity of A is given, equal to V_1 say, the velocity V_2 of B may be calculated as follows:

$$\omega = \frac{V_1}{AC} = \frac{V_2}{BC}; \qquad \therefore \frac{V_1}{V_2} = \frac{AC}{BC},$$

a result which shows that the velocity of any point is proportional to its distance from the axis of rotation.

Instantaneous centre of rotation.—Let a rod AB (Fig. 61) be moving in such a manner that at a given instant A has a velocity V_A and B has a velocity V_B in the directions shown. The direction of V_A will not be altered if we imagine that A is rotating for an instant about any centre in a line AI drawn perpendicular to V_A. Similarly, V_B will not be altered in direction if we imagine B to be rotating for an instant about any centre in BI which is perpendicular to V_B. These perpendiculars intersect at I, and we may consider that both A and B are rotating for an instant about I without thereby changing the directions of their velocities. I is called the **instantaneous centre of rotation.** It is evident that, if two points in the rod rotate for an instant about I, every point in the rod is rotating about I at the same instant

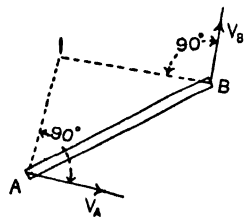


FIG. 61.—Instantaneous centre

If V_A is known, we may calculate V_B from the relation given on p. 61, viz.

$$\frac{V_A}{V_B} = \frac{IA}{IB}.$$

EXAMPLE.—In Fig. 65 is shown a slider-crank mechanism (p. 27) in which the crank BC rotates with uniform angular velocity in the plane of the paper about an axis at C. The rod AB is jointed to BC at B, and its end A is constrained to move in the line AC. Knowing the velocity V_B of B at any instant, the velocity of A may be found by application of the instantaneous centre method. Draw AI perpendicular to AC; then A may be imagined for an instant to be rotating about any centre in AI. Draw BI perpendicular to V_B, *i.e.* produce CB; B may be imagined to rotate for an instant about any centre in BI. Hence I is the instantaneous centre for the rod AB, and we have

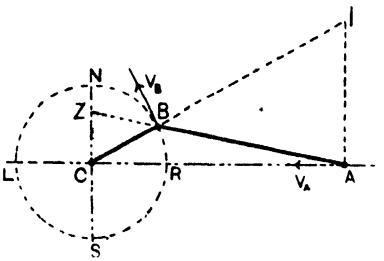


FIG. 65.—Instantaneous centre of AB

$$\frac{V_A}{V_B} = \frac{IA}{IB}.$$

In some positions of the mechanism, I will fall at a large distance from AC; when BC is at 90° to AC, I lies at infinity. A simple modification brings the whole construction required within the boundary of a piece of drawing paper of moderate size.

Draw NCS through C at 90° to AC; produce AB (if necessary) to cut ICS in Z. It is evident that the triangles IAB and CZB are similar; hence

$$\frac{IA}{IB}=\frac{CZ}{CB};$$

$$\therefore \frac{V_A}{V_B}=\frac{CZ}{CB}=\frac{1}{R}.CZ,$$

where R is the length of the rod BC. Since R and V_B are both constants, it follows that V_A is proportional to CZ.

A rolling wheel.—In Fig. 66 (*a*) is shown a wheel rolling along a road without slipping. It is evident that the velocity of the centre of the wheel, O, is equal to the velocity v of the vehicle to which

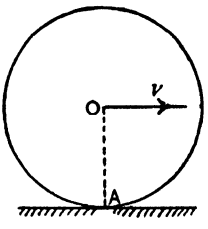


FIG. 66 (*a*).—Angular velocity of a rolling wheel

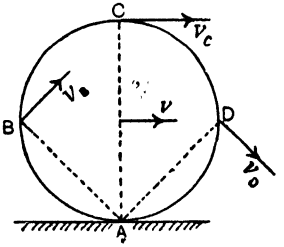


FIG 66 (*b*).—Velocities of points in a rolling wheel.

the wheel is attached. Further, if there is no slipping, then the point A in the wheel rim, being in contact with the ground for an instant, is at rest, and is therefore the instantaneous centre of the wheel. Hence the angular velocity of the wheel is v/OA, a result agreeing with that found on pp. 56 and 57 by another method.

Every point in the wheel is rotating for an instant about A; hence the velocity of any point may be found. Thus the velocity of C (Fig. 66 (*b*)) is at 90° to AC and is given by

$$\frac{v_c}{v}=\frac{AC}{AO}=2;$$

$$\therefore v_c=2v.$$

The velocities of B and D (situated on the horizontal line passing through O) are perpendicular respectively to AB and AD, and are given by

$$\frac{v_B}{v}=\frac{AB}{AO}=\sqrt{2};$$

$$\therefore v_B=v\sqrt{2}.$$

Similarly, $$v_D=v\sqrt{2}.$$

EXERCISES ON CHAPTER V.

1. A wheel revolves 90 times per minute. Find its angular velocity in radians per second.

2. What is the angular velocity in radians per second of the second hand of a watch ?

3. Find the revolutions per minute described by a wheel which has an angular velocity of 30 radians per second.

4. A revolving wheel changes speed from 50 to 49 radians per second. State the change in revolutions per minute.

5. A point on the rim of a wheel 8 feet in diameter has a linear velocity of 48 feet per second. Find the angular velocity of the wheel.

6. A wheel starts from rest and acquires a speed of 200 revolutions per minute in 24 seconds. Find the angular acceleration.

7. Find the angular acceleration of a wheel which undergoes a change in angular velocity from 50 to 48 radians per second in 0·5 second.

8. A point on the rim of a revolving wheel 8 feet in diameter has a velocity in the direction of the tangent of 80 feet per second. Five seconds afterwards the same point has a tangential velocity of 60 feet per second. Find the angular acceleration of the wheel.

9. A wheel starts from rest with an angular acceleration of 0·2 radian per second per second. In what time will it acquire a speed of 150 revolutions per minute ? How many revolutions will it make during this interval of time ?.

10. Find the angle turned through by a wheel which starts from rest and acquires an angular velocity of 30 radians per second with a uniform acceleration of 0·6 radian per second per second.

11. A wheel changes speed from 140 to 150 revolutions per minute and describes 40 revolutions while doing so. Find the angular acceleration.

12. Find the angular velocity of a bicycle wheel 28 inches diameter when the bicycle is travelling at 12 miles per hour. How many revolutions will the wheel describe in travelling a distance of one mile ?

13. A shaft A drives another shaft B by means of pulleys and a belt. If the pulley on A is 24 inches in diameter and runs at 200 revolutions per minute, find the diameter of the pulley on B in order that it may have a speed of 150 revolutions per minute.

14. A small motor has a pulley 2 inches in diameter and runs at 1200 revolutions per minute. A shaft having a pulley 12 inches in diameter is driven by a belt passing round the motor pulley. On the same shaft is another pulley 3 inches in diameter connected by a belt to a pulley 10 inches in diameter and fixed to the shaft of an experimental model. Find the speed in revolutions per minute of the model shaft.

15. The driving wheel of a bicycle is 28 inches in diameter, and has a sprocket wheel having 18 teeth. The chain wheel on the crank axle has 46 teeth. What is the "gear" of the bicycle ? How many revolutions must each crank make in travelling a distance of one mile ?

16. A wheel A having 20 teeth drives another wheel B having 54 teeth. If A runs at 110 revolutions per minute, find the speed of revolution of B. Show how A and B could be run at the same speeds as before, but both in the same direction of rotation.

17. In winding a watch 3 5 complete turns are given to the spring case; this serves to keep the watch going for 28 hours. What is the ratio of the angular velocities of the spring case and the minute hand during the ordinary working of the watch ?

18. The minute hand of a watch is connected to the hour hand by a train of wheels. A wheel A on the minute hand spindle has 12 teeth and drives a wheel having 48 teeth; on the same spindle as the latter wheel is another having 8 teeth, and this wheel drives a wheel having N teeth on the hour-hand spindle. Find N.

19. Explain how angular velocity is measured. A point P moves with uniform velocity v along a straight line. ON is drawn perpendicular to this line, O being a fixed point. Express the angular velocity of P about O in terms of the distance OP. L.U.

20. If two particles describe the circle of radius a, in the same sense and with the same speed u, show that the relative angular velocity of each with respect to the other is u/a. L.U.

21. A rod OA is pivoted to a fixed point at O, and is freely jointed at A to a second rod AB; the end B is constrained to move in a straight groove passing through O. If the rod OA rotates about O with uniform angular velocity ω, show that the velocity of B at any instant is

$$\mathrm{OA}(\sin\theta + \cos\theta \tan\phi)\omega,$$

where θ and ϕ are the acute angles made by OB with OA and AB at the instant. L.U.

22. Find the velocity at any point on the rim of a wheel rolling with uniform velocity v along a horizontal plane without sliding. Show that each point of the wheel moves as though it were revolving about the point of contact of the wheel and the ground at the instant.

Adelaide University.

CHAPTER VI

INERTIA

Newton's first law of motion.—The whole science of dynamics is based on three fundamental laws formulated by Newton. The first law is as follows.

Every body continues in its state of rest or of uniform motion in a straight line except in so far as it is compelled by forces to change that state.

The term inertia is given to the tendency of a body to preserve its state of rest, or of constant rectilinear velocity. The first law expresses the results of experience. A train at rest on a level track will not move until the locomotive applies a tractive force. If the train is travelling with constant speed, the engine exerts a pull sufficient merely to overcome the frictional resistances, and must exert a considerably greater pull while the speed is being increased. If steam be shut off, the frictional resistances gradually reduce the speed, and if the brakes be applied, the increased frictional forces bring the train to rest quickly. Thus a force having the same sense and direction as the velocity must be applied in order to obtain an increase in velocity, and a force having the opposite sense if the velocity has to be diminished.

A person standing on the top of a tramcar may experience the effects of inertia in his body: should the driver apply the brakes suddenly, the passenger will be shot forward. If the driver starts rapidly, the passenger will be left behind as it were. Should the car reach a curve on the track it will follow the track, and the passenger's body will endeavour to proceed rectilinearly, and will incline towards the outside of the curve.

To cause any body to travel in a curved path requires the application of a force in a direction transverse to that of the path.

We now proceed to discuss some principles leading to Newton's second law of motion.

Relation of force, mass and acceleration.—**All bodies at the same place fall freely with equal accelerations.** This statement may be confirmed by experiment. Two stones released simultaneously from the same height will reach the ground at the same instant. If a piece of paper be substituted for one of the stones, the paper will take a longer time to fall; this effect is owing to the resistance of the air, and may be got rid of partially by crumpling the paper into a ball, when it will be found that both stone and paper fall together.

Since the weights of two bodies are proportional to the masses, and since both bodies fall freely with equal accelerations, it follows that **the forces required are proportional to the masses if equal accelerations are to be imparted to a number of bodies.**

A laboratory experiment (p. 71) may be devised to illustrate another law, viz. **the force which must be applied to a body of given mass is proportional to the acceleration required.**

Combining these statements leads to the general law: **The force required is proportional jointly to the mass and the acceleration, and is therefore measured by the product of mass and acceleration.** The acceleration takes place in the same direction and sense as the applied force.

Let F = the force applied to the body by the external agent.
m = the mass of the body.
a = the acceleration.

Then $$F = ma.$$

Absolute units of force.—Convenient absolute units of force (p. 8) may be derived from the above result. Take m to be the unit of mass and a to be the unit of acceleration in any given system; then F becomes unity and may be accepted as the absolute unit of force for the system. The C.G.S. and British absolute units of force have been defined on p. 8, and are restated here in a slightly different form:

A force of one **dyne** applied to a gram mass produces an acceleration of one centimetre per second per second.

A force of one **poundal** applied to a pound mass produces an acceleration of one foot per second per second.

The dimensions of force may be deduced from the above equation by substitution.

Thus: $$F = ma = m\frac{l}{t^2}.$$

Relation of absolute and gravitational units of force.—Since a body of mass m falls freely under the influence of its weight W and has an acceleration g, it follows that the weight of a body, expressed in absolute units of force, is given by :

$$W = mg.$$

A force of one lb. weight, acting on a mass of one pound falling freely, produces an acceleration of g feet per second per second. A force of g poundals would produce the same acceleration ; hence one lb. weight is equivalent to g poundals. Similarly, one gram weight is equivalent to g dynes. In interpreting these statements it will be understood that g must be in feet, or centimetres, per second per second according to the system employed.

To convert from gravitational to absolute force units, multiply by g

Newton's second law of motion.—Suppose a body of mass m to be at rest in the initial position A (Fig 67). If a force F be applied, a constant acceleration a will occur, let this continue during a time interval t seconds, and let the body travel from A to B during this interval, the velocity being v on reaching B. We have :

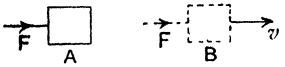


FIG. 67.—Relation of force and momentum generated.

$$F = ma.$$

Also, $v = at$ (p. 33), or $a = \frac{v}{t}$.

$$\therefore F = \frac{mv}{t} \quad \ldots\ldots\ldots\

Newton's third law of motion.—**To every action there is always an equal and contrary reaction; or, the mutual actions of any two bodies are always equally and oppositely directed.**

This law is the result of experience, of which a few instances may be noted. The hand of a person sustaining a load is subjected to a downward force—the weight of the body—and the hand applies an equal upward force to the load. Similarly, a person applying a pull to a rope experiences an equal and opposite pull which the rope exerts on his hands. Equal and opposite forces applied in the same straight line to a body balance one another; under such conditions the body, if at rest, remains at rest, or, if in motion, will experience no change of motion.

A force applied to a body by means of some external agency, such as a pull along a string attached to the body, or a push from a rod in contact with the body, produces acceleration in accordance with the law $F = ma$. In this case the body, by virtue of its inertia, supplies a reaction equal and opposite to the force applied to it by the external agency. In Fig. 68, F is the external force applied to the body; each particle of the body contributes to the equal opposite reaction by virtue of its inertia, and the total or resultant reaction is represented by the product ma. In fact, the equation $F = ma$ should be understood to represent the equality of two opposing forces, one, F, being the resultant external force applied to the body, and the other, ma, being an internal force produced by virtue of the inertia of the body.

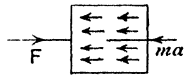


FIG 68.—Resistance due to inertia.

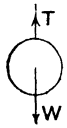


FIG 69

Should two external opposing forces T and W (Fig. 69) be applied to a body, unequal but in the same straight line, it is clear that a single external force (W − T) would produce the same effect in changing the motion. (W − T) may be called the **resultant external force**, and should be used as the value of F in the equation $F = ma$.

EXAMPLE 1.—What pull must be applied by a locomotive to give a train of 150 tons mass an acceleration of 1·5 feet per second per second if frictional resistances be neglected?

$$F = ma$$
$$= 150 \times 2240 \times 1{\cdot}5 = \underline{504{,}000} \text{ poundals.}$$
$$= \frac{504000}{32{\cdot}2} = \underline{15{,}650} \text{ lb. weight.}$$

EXAMPLE 2.—Answer the same question if there are frictional resistances opposing the motion and amounting to 10 lb. weight per ton of train.

Total frictional resistance $= Q = 150 \times 10 = 1500$ lb. weight.

Let the pull of the locomotive be P lb. weight, then the resultant force producing the acceleration will be $(P - Q)$ lb. weight. Hence

$$F = P - Q = \frac{ma}{g} = 150 \times 2240 \times 1\cdot5 \div 32\cdot2$$
$$= 15{,}650,$$
$$\therefore P = 15{,}650 + 1500$$
$$= 17{,}150 \text{ lb. weight.}$$

EXAMPLE 3.—Two bodies A and B (Fig. 70) are attached to the ends of a light cord passed over a pulley C. The cord may be assumed to be so fine that its mass may be neglected, and so flexible that the forces required in order to bend it round the pulley may be disregarded. It is assumed also that the pulley is so light that its mass may be neglected, and that its bearings are free from frictional resistance. Under these assumptions, the pulls in all parts of the cord will be equal, the pulley serving merely to change the direction of the cord. Take the masses of A and B to be m_1 and m_2 respectively, and discuss the motion.

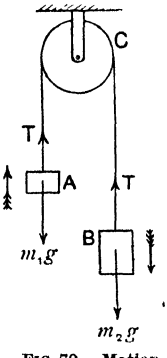


FIG. 70.—Motion under the action of gravity

Consider A; two external forces are applied to it, viz. the weight m_1g and the upward pull T exerted by the cord. If these forces are equal, no motion will occur, or if there be motion, the velocity will be uniform. Suppose T to be larger than m_1g, then an upward acceleration a will occur, and we may write:

$$T - m_1g = m_1a. \quad \ldots\ldots (1)$$

Now consider B; this body is subjected to a downward force m_2g and an upward force T, and has a downward acceleration also equal to a from the arrangement of the apparatus. Hence m_2g is greater than T, and we may write:

$$m_2g - T = m_2a. \quad \ldots\ldots (2)$$

Solving (1) and (2) in order to determine a and T, we have, by addition:

$$m_2g - m_1g = (m_1 + m_2)a,$$

or,

$$a = \left(\frac{m_2 - m_1}{m_1 + m_2}\right)g. \quad \ldots\ldots (3)$$

Dividing (1) by (2) gives:

$$\frac{T - m_1g}{m_2g - T} = \frac{m_1}{m_2},$$
$$\therefore m_2T - m_1m_2g = m_1m_2g - m_1T$$
$$T(m_1 + m_2) = 2m_1m_2g,$$
$$\therefore T = \left(\frac{2m_1m_2}{m_1 + m_2}\right)g. \quad \ldots\ldots (4)$$

It will be evident that, if m_1 and m_2 are equal, both bodies will have either no motion, or constant velocity; and the pull in the cord will be equal to the weight of one of the bodies.

This problem may be examined from another point of view. There are two bodies A and B, having a total mass $(m_1 + m_2)$ (Fig. 70); a resultant force acts, equal to the difference in their weights, viz. $(m_2 g - m_1 g)$; hence:

$$m_2 g - m_1 g = (m_1 + m_2)a,$$

or,
$$a = \left(\frac{m_2 - m_1}{m_1 + m_2}\right) g,$$

which is the same result as that given in (3) above.

Attwood's machine.—In this machine an attempt is made to realise the conditions mentioned in Example 3 above by using a very light silk cord and a light aluminium pulley mounted on ball bearings, or bearings designed to eliminate friction so far as is possible. The machine is used in the following manner:

EXPT. 12. - **Use of Attwood's machine.** Equal loads A and B are hung from the ends of the cord (Fig. 71). A small additional load B′ is added and is adjusted so as to be just sufficient to overcome friction and to cause B to have uniform downward velocity when given a slight start; A, of course, will have uniform upward velocity. Any additional weight D placed on B will produce acceleration in the whole of the moving parts. Denoting the masses of the loads by suffixes, we have, neglecting the masses of the pulleys and cord:

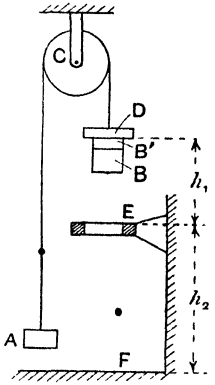


FIG. 71.—Attwood's machine.

$$\text{Total mass in motion} = m_A + m_B + m_{B'} + m_D$$
$$= 2m_A + m_{B'} + m_D.$$

Force producing acceleration $= m_D g$:

$$\therefore\ m_D g = (2m_A + m_{B'} + m_D)a,$$

or,
$$a = \frac{m_D g}{2m_A + m_{B'} + m_D}. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

To check this result we may employ the following method: A fixed ring is arranged at E, and has an internal diameter sufficiently large to permit of B and B′ passing through the ring, but will not so permit D. On arrival at E, D is arrested and the remaining moving parts will thereafter proceed with uniform velocity until they are brought to rest by B arriving at the fixed stop F. Measure h_1 and h_2; allow the motion to start unaided by any push, or otherwise, and start a stop watch simultaneously (a split-

second stop watch is useful). Note the time at which D is arrested and also that at which B reaches F. Repeat several times, and take the mean time intervals. Let the mean time interval from the start to the instant at which D was arrested be t_1 and let the mean time interval in which B travels from E to F be t_2; also let the uniform velocity of B between E and F be v, and let the acceleration from the start until D is arrested be a.

Then $h_2 = vt_2$, or $v = \frac{h_2}{t_2}$

Also, $v = at_1$, (p. 33),

$$\therefore a = \frac{v}{t_1} = \frac{h_2}{t_1 t_2} \quad \ldots\ldots (2)$$

Or we may say: $h_1 = \frac{1}{2}at_1^2$;

$$\therefore a = \frac{2h_1}{t_1^2} \quad \ldots\ldots (3)$$

Either of these expressions (2) or (3) may be used for the calculation of the acceleration, and the results should show fair agreement with that calculated from (1). It will be noted that this apparatus provides an experimental illustration of the truth of the law $F = ma$.

Impulsive forces.—Considering again the equation

$$F = \frac{mv}{t}, \quad \text{(p. 68)}, \quad \ldots\ldots (1)$$

it will be noted that the principle involved is not affected by the magnitude of the interval of time. If this interval be very small, the conception of an **impulse** is obtained, *i.e.* a force acting during a very short time. Generally it is impossible to state the magnitude of such a force at any particular instant during the action, and the calculation of F from equation (1) gives the mean value of the force, and may be called the **average force of the blow.**

The equation may be written

$$Ft = mv. \quad \ldots\ldots (2)$$

This form suggests plotting corresponding values of the force and time, should these be known, giving a diagram resembling that shown at OABC in Fig. 72. The average height of this diagram gives the average value of F. Owing to this method of deducing F from a diagram having a time base, the force F is sometimes called the **time average of the force.** This term is synonymous with the term average force of the blow.

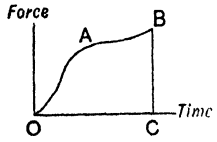


FIG. 72.—Time average of a force

Since the average force F is represented by the mean height of the diagram in Fig. 72, and the base OC represents t, it follows that the area of the diagram represents Ft. Ft may be

called the **impulse of the force**, and is equal to the total change in momentum of the body.

EXAMPLE.—A bullet has a mass of 50 grams and a velocity of 400 metres per second. If it is brought to rest in 0·01 second, find the impulse and the average force of the blow.

$$\text{Impulse} = mv = 50 \times 400 \times 100$$
$$= 2 \times 10^6 \text{ gram cm./sec. units.}$$

$$\text{Average force of the blow} = \frac{mv}{t} = \frac{2 \times 10^6}{0\cdot01} = 2 \times 10^8 \text{ dynes}$$
$$= 20\cdot4 \times 10^1 \text{ gram weight.}$$

EXERCISES ON CHAPTER VI.

1. Find the force required to give a mass of 15 pounds an acceleration of 45 feet per second per second.

2. A force of 9540 dynes acts on a mass of 2·5 kilograms. Find the acceleration

3. Find factors for converting (*a*) dynes to poundals, (*b*) poundals to dynes.

4. A cycle and rider together have a mass of 190 pounds. When travelling at 10 miles per hour on a level road the cyclist ceases to pedal and observes that he comes to rest in a distance of 200 yards. Find the average resistance to motion.

5. A train has a mass of 200 tons. Starting from rest, a distance of 400 yards is covered in the first minute. Assuming that the acceleration was uniform, find the pull required to overcome the inertia of the train.

6. The cage of a lift has a mass of 1000 pounds. Find the pull in the rope to which the cage is attached (*a*) when the lift is descending at uniform speed, (*b*) when the lift is descending with an acceleration of 2 feet per second per second, (*c*) when the lift is ascending with the same acceleration.

7. A train has a mass of 250 tons. If the engine exerts a pull of 10 tons weight in producing an acceleration of 1 foot per second per second, find the resistance due to causes other than inertia.

8. A man who weighs 160 lb. slides down a rope, that hangs freely, with a uniform speed of 4 feet per second. What pull does he exert upon the rope, and what would happen if at a given instant he should reduce his pull by one half? L.U.

9. A mass of 10 pounds is placed upon a table and is connected by a thread which passes over a smooth peg at the edge with a mass of one pound that hangs freely. Assuming the table to be smooth, determine the velocity acquired by the two masses in one second, and find the tension in the thread. What would you infer if, in actual experiment, the masses were observed to move with uniform velocity; and what would be the tension in the thread in that case? L.U.

10. A fine cord passes over a pulley and has a mass of 0·5 kilogram hanging from one end and another mass of 0·9 kilogram hanging from the other end. Neglect friction and find the acceleration and the tension in the cord.

11. In an Attwood machine, a mass of 2 pounds is attached to each end of the cord. It is then found that an additional mass of 0 2 pound on one side is sufficient to maintain steady motion. Another mass of 0 4 pound is then placed on the same side and is found to produce a velocity of 4 72 feet per second at the end of a descent from rest of 4 feet. Is this result in accordance with the theory? Compare the actual acceleration with that given by the theory. $g = 32\ 2$ feet per second per second.

12. A train moving with uniform acceleration passes three points A, B and C at 20, 30 and 45 miles an hour respectively. The distance AB is 2 miles. Find the distance BC. If steam is shut off at C and the brakes applied, find the total resistance in lb. weight per ton mass of the train in order that it may be brought to rest at a distance of one mile from C. L.U.

13. Find the momentum of a railway coach, mass 12 tons, travelling at 15 miles per hour. If the speed is changed to 12 miles per hour in 4 seconds, find the average resistance to the motion.

14. Find the impulse of a shot having a mass of 1200 pounds and travelling at 1500 feet per second. If the shot is brought to rest in 0 01 second, find the average force of the blow.

15. Define the terms "acceleration," "force," "momentum," and state their precise relation to each other.

What is incorrect in the following expression:

(i) The force with which a body moves;

(ii) An acceleration of 10 feet per second? Adelaide University.

16. A mass of 2 pounds on a smooth table is connected by a string, passing over a light frictionless pulley at the edge of the table, with a suspended mass of 1 ounce. Find (*a*) the velocities of the masses after they have moved for 1 second from rest, and (*b*) the total momentum of the system at the same time. L.U.

17. Define the impulse of a force and an impulsive force. Find the direction and magnitude of a blow that will turn the direction of motion of a cricket ball weighing 5½ oz., moving at 30 ft. per sec., through a right angle, and double its velocity. State in what units your answer is given. L.U.

18. A particle A of mass 10 oz. lies on a smooth table and is connected by a slack string which passes through a hole in the table with a particle B of mass 6 oz. lying on the ground directly beneath the hole in the table. A is projected along the table with a velocity of 8 feet per second. Find the impulsive tension when the string becomes taut and the common velocity of the particles immediately afterwards. Find also the height to which B will rise. L.U.

19. Explain what is meant by relative velocity. A ball of mass 8 ounces after falling vertically for 40 feet, is caught by a man in a motor-car travel-

ling horizontally at 30 miles an hour. Find the inclination to the vertical at which it will appear to him to be moving, and the magnitude of the impulse on the ball when it is caught. L.U.

20. Two masses of $\frac{1}{4}$ oz. and $7\frac{3}{4}$ oz., connected by an inextensible string 5 ft. long, lie on a smooth table $2\frac{1}{2}$ ft. high. The string being straight and perpendicular to the edge of the table, the lighter mass is drawn gently just over the edge and released. Find (*a*) the time that elapses before the first mass strikes the floor, and (*b*) the time that elapses before the second mass reaches the edge of the table. L.U.

21. State Newton's laws of motion, and show how from the first we obtain a definition of force, and from the second a measure of force.

A motor car, running at the rate of 15 miles per hour, can be stopped by its brakes in 10 yards. Prove that the total resistance to the car's motion when the brakes are on is approximately one-quarter of the weight of the car. L.U.

22. A particle is projected up the steepest line of a smooth inclined plane, and is observed to pass downwards through a point 18 feet distant from the place of projection 4 seconds after passing upwards through the point. Further, there is an interval of 3 seconds between its transits through a point distant 32 feet from the place of projection. Find the velocity of projection and the slope of the plane. L.U.

CHAPTER VII

STATIC FORCES ACTING AT A POINT

Specification of a force.—In specifying a force the following particulars must be stated : (*a*) the point at which the force is applied ; (*b*) the line of direction in which the force acts ; (*c*) the sense along the line of direction ; (*d*) the magnitude of the force.

Force is a vector quantity ; this statement is confirmed by the fact that mass and acceleration are involved in the measurement of a force ; mass is a scalar quantity, and acceleration is a vector quantity, hence force is also a vector quantity. It follows that two or more forces acting at a point and in the same plane may be compounded so as to give the **resultant force**, *i.e.* a force which has the same effect as the given forces, and the methods of vector addition explained in Chapter III. may be employed.

It is convenient to speak of a "force acting at a point," but this statement should not be taken literally. No material is so hard that it would not be penetrated if even a small force be applied to it at a mathematical point. What is meant is that the force may be imagined to be concentrated at the point in question without thereby affecting the condition of the body as a whole. Further, in speaking of a force applied at a point, it must not be forgotten that the mere existence of a force implies matter to which it is applied. In Fig. 73, a body A applies an action P_A to another body B, and is itself subjected to a simultaneous and equal reaction P_B applied by B.

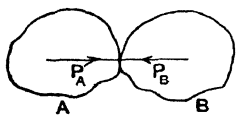


FIG. 73.—Action and reaction.

Transmission of force along the line of action.—In Fig. 74 a push P is applied to a body at a point A in a line BA. The general effect of P in producing changes of motion, or in maintaining the state of rest, will be unaltered if P be applied at any point O in the body and on the line of BA produced. There will, however, be

alterations in the mutual actions between the particles of the body; it is clear that these will not be identical whether P is applied at A or O.

The mutual actions of the particles may be ignored in considering the state of rest or motion of the body as a whole. For example a body A is subjected to pushes P_B, P_C and P_D applied by other three bodies B, C and D at points *b*, *c* and *d* (Fig. 75). The three forces intersect at O and are in the plane of the paper. Disregarding the effects of the forces in producing actions between the particles of the body A, we may say that the effect on the body as a whole would be unaltered if the three forces were applied at O instead of at the

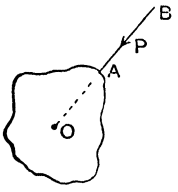


FIG. 74.—Transmission of a force

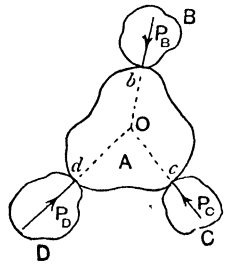


FIG. 75.—Three forces applied to a body.

given points *b*, *c* and *d*. In making this statement it is assumed that the body is **rigid**, *i.e.* its particles are assumed to adhere together so strongly as to prevent entirely any change in their relative positions. Otherwise relative motion of the parts of the body would occur independently of the motion of the body as a whole, and it is assumed that no such relative motion takes place.

Stress.—The term **stress** is given to the mutual actions which take place between one body and another, or between two parts of a body subjected to a system of forces. The term involves both of the dual aspects involved in force, and has therefore no sense. It thus becomes necessary to describe the action as **tensile stress** if the bodies, or the parts of the body, tend to separate; **compression stress** if they are forced together; and **shearing stress** if they tend to slide on one another. Stresses are discussed in more detail in Chapter XII.

Parallelogram and triangle of forces.—The **parallelogram of forces** is a construction similar to the parallelogram of velocities described on p. 41. Consider two forces, P and Q, acting at O and both in the

plane of the paper (Fig. 76); to find the resultant, choose a suitable scale, and measure OA and OB to represent the magnitudes of P and Q respectively. Complete the parallelogram OACB, when the diagonal OC will represent the resultant R. In applying this construction, care must be taken that P and Q are arranged so that they act either both towards or both away from O. It will be remembered (p. 42) that the same arrangement must be made in the parallelogram of velocities.

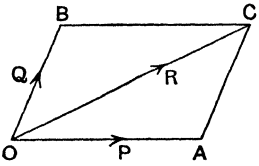


FIG. 76.—Parallelogram of forces

The **triangle of forces** may be employed, and is similar to the triangle of velocities (p. 11). Given P and Q acting at O (Fig. 77); to find the resultant, draw AB to represent P, and BC to represent Q; then the resultant is represented by AC. Note that R does not act along AC (which may be anywhere on the paper), but at O, and is so shown in Fig. 77 by a line parallel to AC.

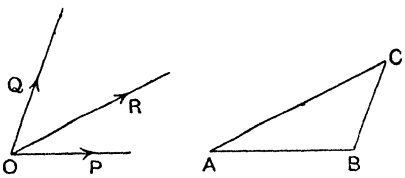


FIG. 77.—Triangle of forces

Forces acting in the same straight line.—A body is said to be in **equilibrium** if the forces applied to it balance one another, *i.e.* produce no change in the state of rest or motion. Thus, if two equal and opposite forces P, P (Fig. 78), be applied at a point O in a body, both in the same straight line, they will balance one another, and the body is in equilibrium.



FIG. 78.—Two equal opposite forces.

If several forces in the same straight line act at a point in a body, the body will be in equilibrium if the sum of the forces of one sense is equal to the sum of those of opposite sense. Calling forces of one sense positive, and those of opposite sense negative, the condition may be expressed by stating that the algebraic sum of the given forces must be zero. Thus the forces P_1, P_2, P_3, etc. (Fig. 79), will balance, provided

$$P_1 + P_2 - P_3 - P_4 - P_5 = 0,$$

or

$$\Sigma P = 0. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

The symbol Σ (sigma) means "the algebraic sum of"; P placed after the symbol is taken to mean one only of a number of forces, of which P is given as a type. Equation (1) stated in words would read: The algebraic sum of all the forces of which P is a type is equal to zero

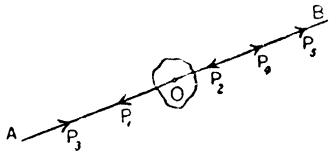


FIG. 79.—Forces in the same straight line.

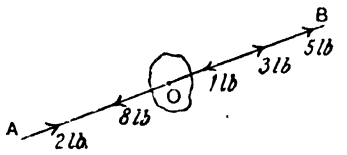


FIG. 80

Should equation (1) give a numerical result which is not zero, it may be inferred that the given forces do not balance, but have a resultant with a magnitude equal to the calculated result. Equilibrium could be obtained by applying a force equal and opposite to the resultant; this force is called the **equilibrant** of the system. Let R and E denote the resultant and equilibrant respectively, then

$$R = -E.$$

The sense of R is positive or negative, depending upon whether the sum of the given positive forces is greater or less than that of the given negative forces. Thus, in Fig. 80, forces of sense from A towards B being called positive, we have

$$2 + 3 + 5 - 8 - 1 = +1.$$

Hence the given forces may be replaced by a single force of 1 lb. weight having the sense from A towards B. This result may be expressed by the equation $\Sigma P = R$. .....(2)

Three intersecting forces.—Two forces whose lines of action intersect at a point may be balanced by first finding the resultant by means of the parallelogram, or triangle of forces; the resultant so found may be applied at the point instead of the given forces without altering the effect. The resultant so applied may then be balanced by applying an equilibrant equal and opposite to the resultant. Fig. 81 illustrates the method. Forces P and Q are given acting at O. In

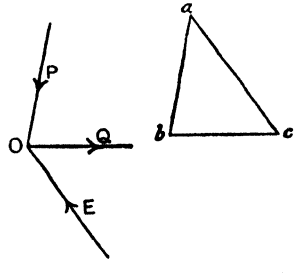


FIG. 81.—Triangle of forces applied to a push and a pull.

the triangle of forces ab represents P and bc represents Q; ac represents R, therefore ca represents the equilibrant E, which is now applied at O in a line parallel to ca and of sense given by the order of the letters ca.

The conditions of balance of three intersecting forces may be formulated now: (*a*) **They must all act in one plane**; (*b*) **they must all act at one point**; (*c*) **they must be capable of representation by the sides of a closed triangle taken in order.**

The meaning of condition (*c*) may be understood by reference to the triangle of forces abc in Fig. 81. Here P, Q and E are represented respectively by ab, bc and ca; the order of these letters indicates the sense of each force, the figure is a closed triangle, and the perimeter has been traversed from a and back to a without it being necessary to reverse the direction in order to indicate the sense of any of the forces. Should the triangle of forces for three given forces fail to close, *i.e.* if a gap occurs between a and a' in Fig. 82, in which ab, bc and ca' represent the given forces, then we infer that the given forces do not equilibrate.

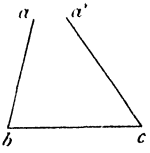


FIG. 82

EXAMPLE.—Three given forces are known to be in equilibrium (Fig. 83); draw the triangle of forces.

This example is given to illustrate a convenient method of lettering the forces called **Bow's Notation**. The method consists in giving letters to the spaces instead of to the forces. In Fig. 83 (*a*), call the space between the 4 lb. and the 2 lb. A, that between the 2 lb. and the 3 lb. B, and the remaining space C. Starting in space A and passing into space B, a line AB (Fig. 83 (*b*)) is drawn parallel and proportional to the force crossed, and the letters are so placed that their order A to B represents the sense of that force. Now pass from space B into space C, and draw BC to represent completely the force crossed. Finish the construction by crossing from space C into space A, when CA in Fig. 83 (*b*) will represent the third force completely.

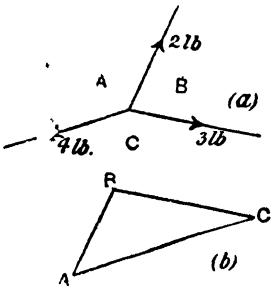


FIG. 83.—Application of Bow's Notation

Examining these diagrams, it will be observed that a complete rotation round the point of application has been performed in Fig. 83 (*a*), and that there has been no reversal of the direction of rotation. Also that, in Fig. 83 (*b*), if the same order of rotation be followed, the sides represent

correctly the senses of the various forces. Either sense of rotation may be used in proceeding round the point of application, clockwise or anti-clockwise, but once started there must be no reversal.

Relation of forces and angles.—In Fig. 84 (*a*) the three forces P, Q, S are in equilibrium, and ABC (Fig. 84 (*b*)) is the triangle of forces. We have $P : Q : S = AB : BC : CA$.

Now $$AB : BC : CA = \sin\gamma : \sin\alpha : \sin\beta,$$

or $$P : Q : S = \sin\gamma : \sin\alpha : \sin\beta. \quad \ldots\ldots(1)$$

The dotted lines in Fig. 84 (*a*) show that α, β, γ are respectively the angles between the produced directions of S and P, P and Q,

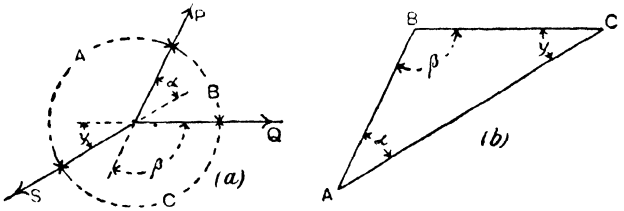


FIG. 84.—Relation of forces and angles

Q and S; also the angles or spaces denoted by A, B, C in the same figure are the supplements of these angles. Since the sine of an angle is equal to the sine of its supplement, we have, in Fig. 84 (*a*),

$$P : Q : S = \sin C : \sin A : \sin B. \quad \ldots\ldots(2)$$

Hence, if three intersecting forces are in equilibrium, **each force is proportional to the sine of the angle between the other two forces.**

Rectangular components of a force.—In the solution of problems it is often convenient to employ selected components of a force instead of the force itself. Generally these components are taken along two lines meeting at right angles on the line of the force; all three lines must be in the same plane. In Fig. 85, OC represents a given force P, and components are required along OA and OB which intersect at 90° at O. Complete the parallelogram of forces OBCA, which is a rectangle in this case,

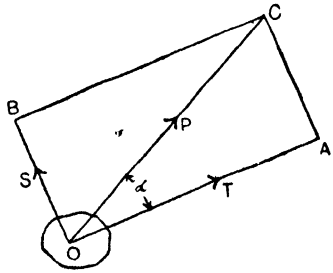


FIG. 85.—Rectangular components of force.

and let the angle COA be denoted by α, then the components S and T are obtained as follows:

$$AC = OB = OC \sin \alpha ;$$

$$\therefore S = P \sin \alpha. \quad \ldots\ldots(1)$$

$$OA = OC \cos \alpha ,$$

$$\therefore T = P \cos \alpha. \quad \ldots\ldots(2)$$

$$OC^2 = AC^2 + OA^2 = OB^2 + OA^2 ,$$

$$\therefore P^2 = S^2 + T^2. \quad \ldots\ldots(3)$$

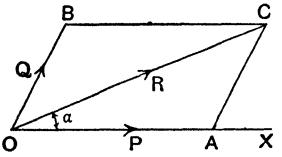


FIG. 86.—Inclined components of a force.

Relation of the resultant and inclined components.—In Fig. 86 components P and Q are given, and the resultant R has been found by means of the parallelogram of forces OACB. From trigonometry, we have $OC^2 = OA^2 + AC^2 - 2 \cdot OA \cdot AC \cdot \cos OAC.$

Also, $AC = OB$, and $\cos OAC = -\cos CAX = -\cos AOB$;

$$\therefore OC^2 = OA^2 + OB^2 + 2 \cdot OA \cdot OB \cdot \cos AOB,$$

or $$R^2 = P^2 + Q^2 + 2PQ \cos AOB. \quad \ldots\ldots(1)$$

To find the angle α which R makes with OA, we have

$$\frac{Q}{R} = \frac{\sin \alpha}{\sin OAC} = \frac{\sin \alpha}{\sin CAX} = \frac{\sin \alpha}{\sin AOB};$$

$$\therefore \sin \alpha = \frac{Q}{R} \sin AOB. \quad \ldots\ldots(2)$$

EXAMPLE 1.—A particle of weight W is kept at rest on a smooth plane inclined at an angle α to the horizontal, (*a*) by a force parallel to the plane, (*b*) by a horizontal force. Find each of these forces.

The term **smooth** is used to indicate a surface incapable of exerting any frictional forces. Such a surface, if it could be realised, would be unable to exert any action on a body in contact with it in any line other than the normal to the surface at the point of contact.

(*a*) In Fig. 87 (*a*), W is represented by *ab* ; the required force P, and the normal reaction of the plane R, are represented respectively in the triangle

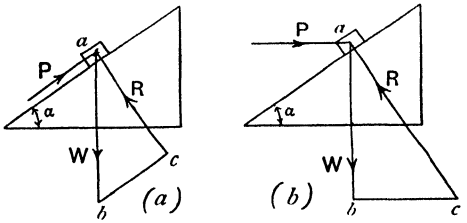


FIG. 87.—Particles on smooth inclines

of forces *abc* by *bc* and *ca*. Since these lines are respectively parallel and at right angles to the plane, *bca* is a right angle ; also the angle $bac = \alpha$.

$$\therefore \frac{P}{W} = \frac{bc}{ab} = \sin \alpha ;$$

$$\therefore P = W \sin \alpha.$$

R may be found thus :

$$\frac{R}{W} = \frac{ca}{ab} = \cos \alpha ;$$

$$\therefore R = W \cos \alpha.$$

(*b*) In this case the triangle of forces is the right-angled triangle *abc* (Fig. 87 (*b*)).

$$\frac{P}{W} = \frac{bc}{ab} = \tan \alpha ;$$

$$\therefore P = W \tan \alpha.$$

Also

$$\frac{R}{W} = \frac{ca}{ab} = \sec \alpha ;$$

$$\therefore R = W \sec \alpha.$$

EXAMPLE 2.—A particle of weight W is kept at rest on a smooth plane inclined at an angle α to the horizontal by means of a force P inclined at an angle β to the plane (Fig. 88). Find P and the reaction of the plane.

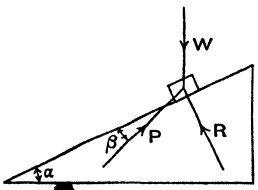


FIG. 88.

In Fig. 88, the angle between P and R is $(90° - \beta)$; also the angle between W and R is $(180° - \alpha)$. Hence (p. 81)

$$\frac{P}{W} = \frac{\sin (180 - \alpha)}{\sin (90 - \beta)} = \frac{\sin \alpha}{\cos \beta} ;$$

$$\therefore P = W \sin \alpha / \cos \beta.$$

The angle between P and W is $(90° + \alpha + \beta)$.

$$\therefore \frac{R}{W} = \frac{\sin (90 + \alpha + \beta)}{\sin (90 - \beta)} = \frac{\cos (\alpha + \beta)}{\cos \beta}$$

$$= \frac{\cos \alpha \cos \beta - \sin \alpha \sin \beta}{\cos \beta} ;$$

$$\therefore R = W (\cos \alpha - \sin \alpha \tan \beta).$$

Systems of uniplanar concurrent forces.—In Fig 89, P_1, P_2, P_3, P_4 are four typical forces, all in the same plane and intersecting at the same point O. OX and OY are two axes in the same plane as the forces and intersect at O at 90°. The angles of direction of the forces are stated with reference to OX, and are denoted by α_1, α_2, α_3, α_4. In order to take advantage of the usual conventions regarding the algebraic signs of sines and cosines, the given forces should be arranged so as to be either all pulls or all pushes. Taking components along OX and OY (Fig 90), we have .

Components along OX ; $P_1 \cos \alpha_1$, $P_2 \cos \alpha_2$, $P_3 \cos \alpha_3$, $P_4 \cos \alpha_4$

Components along OY , $P_1 \sin \alpha_1$, $P_2 \sin \alpha_2$, $P_3 \sin \alpha_3$, $P_4 \sin \alpha_4$.

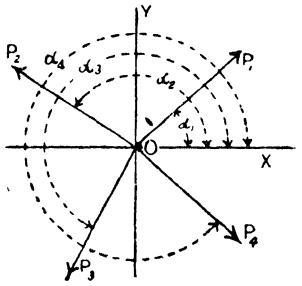


FIG. 89.—System of uniplanar forces acting at a point

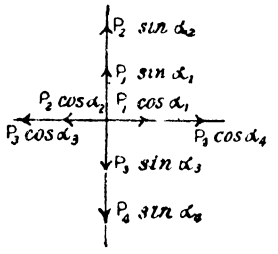


FIG 90.—First step in the reduction of the system.

Taking account of the algebraic signs, the components along OX towards the right are positive and the others are negative. Similarly, the components acting upwards along OY are positive and the others are negative. The resultants R_X and R_Y of the components in OX and OY respectively are given by :

$$P_1 \cos \alpha_1 + P_2 \cos \alpha_2 + P_3 \cos \alpha_3 + P_4 \cos \alpha_4 = R_X,$$
$$P_1 \sin \alpha_1 + P_2 \sin \alpha_2 + P_3 \sin \alpha_3 + P_4 \sin \alpha_4 = R_Y.$$

Or, using the abbreviated method of writing these,

$$\Sigma P \cos \alpha = R_X, \quad \text{.....(1)}$$
$$\Sigma P \sin \alpha = R_Y. \quad \text{.....(2)}$$

The given system has thus been reduced to two forces R_X, R_Y, as shown in Fig. 91. To find the resultant R, we have

$$R = \sqrt{R_X^2 + R_Y^2}, \quad \text{.....(3)}$$

$$\tan \alpha = \frac{CA}{OA} = \frac{OB}{OA} = \frac{R_Y}{R_X}. \quad \text{.....(4)}$$

The given system of forces will be in **equilibrium** if both R_x and R_y are zero. The algebraic conditions of equilibrium are obtained from (1) and (2) :

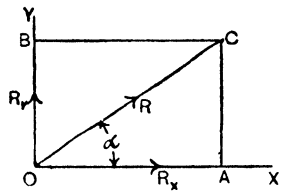


FIG. 91.—Resultant of the system shown in Fig. 89.

$$\Sigma P \cos \alpha = 0, \quad \ldots\ldots \quad \ldots\ldots (5)$$

$$\Sigma P \sin \alpha = 0. \quad \ldots\ldots \quad \ldots\ldots (6)$$

This pair of simultaneous equations may be used for the solution of any problem regarding the equilibrium of any system of uniplanar concurrent forces.

Graphical solution by the polygon of forces.—By application of the principle of vector addition, the equilibrium of a number of uniplanar forces acting at a point may be tested. Four such forces are given

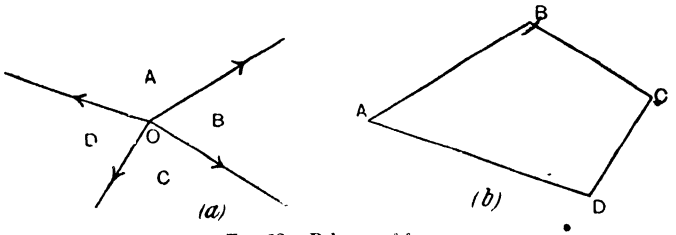


FIG. 92.—Polygon of forces.

in Fig. 92 (*a*), and are described by Bow's notation (p. 80). Starting in space A and going round O clockwise, lines are drawn in Fig. 92 (*b*) representing completely each force crossed. The production of a closed polygon ABCD is sufficient evidence that the forces are in equilibrium ; a gap would indicate that the forces have a resultant, which would be represented by the line required to close the gap, and the equilibrant of the system would be equal and opposite to the resultant. Fig. 92 (*b*) is called the **polygon of forces.**

We may therefore state that a system of uniplanar forces acting at a point will be in equilibrium, provided **a closed polygon can be drawn in which the sides taken in order represent completely the given forces.**

Concurrent forces not in the same plane.—Most of the cases of forces not in the same plane are beyond the scope of this book. The following exercise indicates the manner in which simple cases of such forces may be solved.

EXAMPLE.—*In Fig. 93 is shown the plan and elevation of a wedge. The top surface is smooth and makes an angle of 30° with the horizontal.* A particle A of weight W is kept at rest on the wedge by means of two light cords AB and AC, fastened at B and C, and making angles of 45° and 30° respectively with the line of greatest slope DE. Find the pulls T_1 and T_2 in AB and AC respectively.

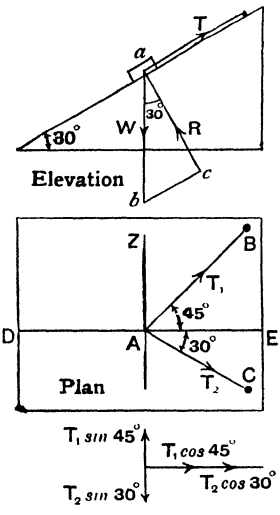


FIG 93.

(*N.B.*—The actual sizes of the angles of 45° and 30° cannot be seen in the plan in Fig. 93.)

Resolve T_1 and T_2 into components along DAE and along a horizontal axis AZ at 90° to DAE. The components along DAE are $T_1 \cos 45°$ and $T_2 \cos 30°$, both of the same sense. those along AZ are $T_1 \sin 45°$ and $T_2 \sin 30°$, and are of opposite sense (Fig. 93, plan).

For equilibrium in the direction of AZ we have: $T_1 \sin 45° - T_2 \sin 30°$,

or, $$\frac{T_1}{\sqrt{2}} = \tfrac{1}{2}T_2 \quad \ldots\ldots(1)$$

Let $T = T_1 \cos 45° + T_2 \cos 30° = \frac{T_1}{\sqrt{2}} + \frac{T_2\sqrt{3}}{2}$ (Fig. 93, elevation).

Then T, W and the reaction of the plane R are in equilibrium, and *abc* is the triangle of forces (Fig. 93, elevation). Hence

$$\frac{T}{W} = \frac{bc}{ab} = \sin 30° = \tfrac{1}{2},$$

$$\therefore\ T = \tfrac{1}{2}W,$$

or, $$\frac{T_1}{\sqrt{2}} + \frac{T_2\sqrt{3}}{2} = \tfrac{1}{2}W. \quad \ldots\ldots(2)$$

Substituting from (1), we have

$$\tfrac{1}{2}T_2 + \frac{T_2\sqrt{3}}{2} = \tfrac{1}{2}W, \quad \therefore\ T_2 = \frac{W}{1+\sqrt{3}} \quad \ldots\ldots(3)$$

And from (1), $$T_1 = \frac{\sqrt{2}}{2}T_2 = \frac{W\sqrt{2}}{2(1+\sqrt{3})} \quad \ldots\ldots(4)$$

EXPT. 13.—**Parallelogram of forces.** In Fig. 94 is shown a board attached to a wall and having three pulleys A, B and C capable of being clamped to any part of the edge of the board. These pulleys should run

easily. Pin a sheet of drawing paper to the board. Clamp the pulleys A and B in any given positions. Tie two silk cords to a small ring, pass a bradawl through the ring into the board at O, and lead the cords over the pulleys at A and B. The ends of the cords should have scale pans attached, in which weights may be placed. Thus, known forces P and Q are applied to the ring at O. Take care in noting these forces that the weight of the scale pan is added to the weight you have placed in it. Mark carefully the directions of P and Q on the paper, and find their resultant R by means of the parallelogram *Oabc*. Produce the line of R, and by means of a third cord tied to the ring, apply a force E equal to R, bringing the cord exactly into the line of R by using the pulley C clamped to the proper position on the board. Note that the proper weight to place in the scale pan is E less the weight of the scale pan, so that weight and scale pan together equal E. If the method of construction is correct, the bradawl may be withdrawn without the ring altering its position.

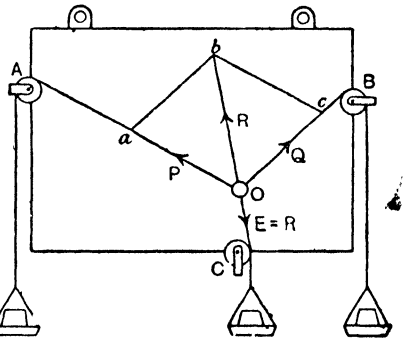


FIG 94.—Apparatus for demonstrating the parallelogram of forces.

In general it will be found that, after the bradawl is removed, the ring may be made to take up positions some little distance from O. This is due to the friction of the pulleys and to the stiffness of the cords bending round the pulleys, giving forces which cannot easily be taken into account in the above construction.

EXPT. 14.—**Pendulum.** Fig. 95 (*a*) shows a pendulum consisting of a heavy bob at A suspended by a cord attached at B and having a spring balance at F. Another cord is attached to A and is led horizontally to E, where it is fastened; a spring balance at D enables the pull to be read. Find the pulls T and P of the spring balances F and D respectively when A is at gradually increased distances x from the vertical BC. Check these by calculation as shown below, and plot P and x.

Since P, W and T are respectively horizontal, vertical and along AB, it follows that ABC is the triangle of forces for them. Hence

$$\frac{P}{W} = \frac{CA}{BC} = \frac{x}{h} \text{ (Fig. 95 (}b\text{))},$$

$$P = \frac{x}{h} W = W \tan \alpha. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

Also, $$\frac{T}{W}=\frac{AB}{BC}=\frac{l}{h}$$

$$T=\frac{l}{h}W=W\sec\alpha. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

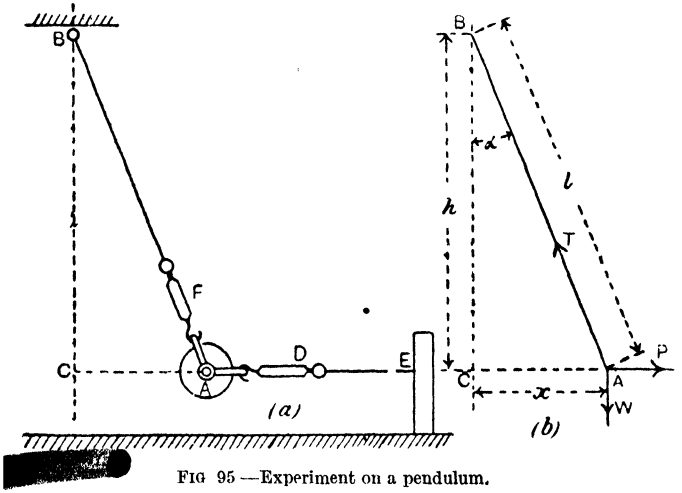


FIG 95.—Experiment on a pendulum.

Measure l, also x and h, for each position of the bob, and calculate P and T by inserting the required quantities in (1) and (2). Tabulate thus:

Weight of bob in kilograms $=W=$

Length of AB in cm. $=l=$

x cm	h cm	Calculated values, in kilograms		Observed values from spring balances, kilograms	
		$P=\frac{l}{h}W$	$T=\frac{l}{h}W$.	P	T

The curve will resemble that shown in Fig. 96. Note how nearly straight it is for comparatively small values of x.

EXPT. 15.—**Polygon of forces.** In Fig. 97 is shown the finished results of an experiment on the polygon of forces. The apparatus and method are similar to that employed in Expt. 13 (p. 86). Four forces have been assumed, and the equilibrant has been found from the closing side of

the polygon ABCDEA. On application of the equilibrant, the bradawl may be withdrawn without the ring moving.

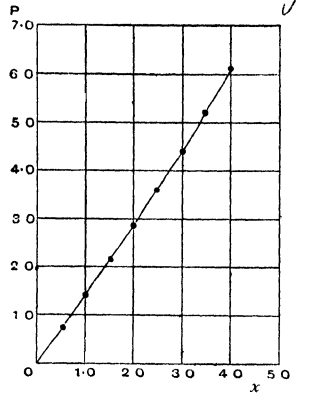


FIG. 96.—Graph of P and x for a pendulum.

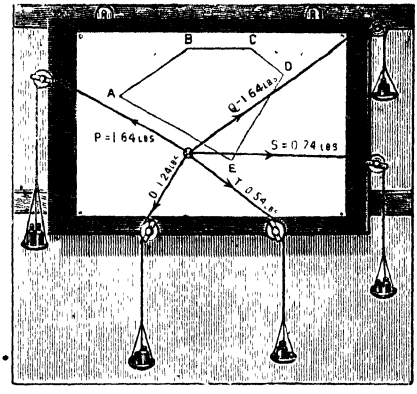


FIG. 97.—An experiment on the polygon of forces

EXPT. 16.—**Derrick crane** A derrick crane model is shown in Fig. 98, consisting of a post AB firmly fixed to a base board which is screwed to a tab.', a jib AC has a pointed end at A bearing in a cup recess, a pulley at C and a compression spring balance at D. A tie BC supports the jib and is of adjustable length; a spring balance for measuring the pull is inserted at F. The weight is supported by a cord led over the pulley at C and attached to one of the screw-eyes on the post. The inclination of the jib may be altered by adjusting the length of BC, and the inclinations of EC and BC may be changed by making use of different screw-eyes.

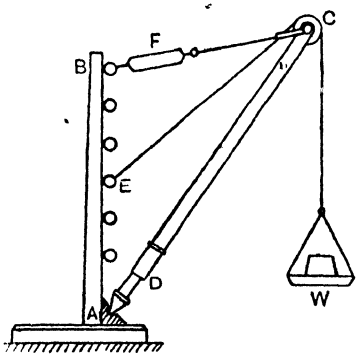


FIG. 98.—Model derrick crane

Observe the spring balances and so find the push in the jib and the pull in the tie for different values of W and different dimensions of the apparatus. Check the results by means of the polygon of forces, constructed as follows:

First measure the dimensions AB, BC, AC and AE, and construct an outline diagram of the crane (Fig. 99 (*a*)). It may be assumed that the pulley at

C merely changes the direction of the cord without altering the force in it; hence P = W (Fig. 99 (*a*)). The polygon of forces (Fig. 99 (*b*)) is drawn by making *ab* represent W, and *bc* represent P; lines are then drawn from *a* parallel to T, and from *c* parallel to Q; these intersect at *d*. Q and T may be scaled from *cd* and *da* respectively.

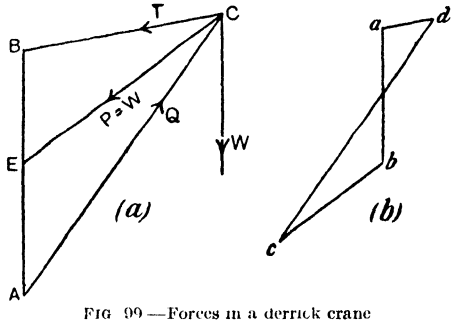


FIG. 99.—Forces in a derrick crane

The values of Q and T so found will not agree very well with those shown by the spring balances. This is owing to the weights of the parts of the apparatus not having been taken into account. Approximate corrections may be applied to the spring balance readings by removing W from the scale pan and noting the readings of the spring balances; these will give the forces in the jib and tie produced by the weights of the parts, and should be deducted from the former readings, when fair agreement will be found with the results obtained from the polygon of forces.

EXERCISES ON CHAPTER VII.

1. A nail is driven into a board and two strings are attached to it. If the angle between the strings is 60°, both strings being parallel to the board, and if one string is pulled with a force equal to 4 lb. weight and the other with a force of 8 lb. weight, find by construction the resultant force on the nail.

2. Answer Question 1 supposing a rod is substituted for the first string and is pushed with a force equal to 4 lb. weight.

3. One component of a force of 3 kilograms weight is equal to 2 kilograms weight, and the angle between this component and the force is 40°. Find the other component by construction.

4. The components of a force of 10 lb. weight are 5 lb. weight and 7 lb. weight respectively. Find by construction the lines of action of the components.

5. A pull of 6 lb. weight and another force Q of unknown magnitude act at a point, their lines of action being at 90°; they are balanced by a

force of 8 lb. weight. Calculate the magnitude of Q and the angle between Q and the force of 8 lb. weight.

6. Answer Question 5 if Q and the force of 6 lb. weight intersect at 60°.

7. Two pulls of 10 lb. weight each act at a point. Find the equilibrant by calculation in the cases when the angle between the pulls is 165°, 170°, 174°, 178°, 180°. Plot a curve showing the relation of the magnitude of the equilibrant and the angle between the pulls.

8. A particle weighing 2 lb. is kept at rest on a smooth plane inclined at 40° to the horizontal by a force P. Calculate the magnitude of P when it is (*a*) parallel to the plane, (*b*) horizontal, (*c*) pulling at 20° to the plane, (*d*) pushing at 30° to the plane. In each case find the reaction R of the plane.

9. A particle of mass m pounds slides down a smooth plane inclined at 25° to the horizontal. Find the resultant force producing acceleration; hence find the acceleration and the time taken to travel a distance of 8 feet from rest.

10. Two strings of lengths $3\frac{1}{4}$ feet and $3\frac{3}{4}$ feet are tied to a point of a body whose weight is 8 lb., and their free ends are then tied to two points in the same horizontal line $3\frac{1}{2}$ feet apart. Find the tension in each string. L.U.

11. A kite having a mass of 2 pounds is flying at a vertical height of 100 feet at the end of a string 220 feet long. If the tension of the string is equal to a weight of $1\frac{1}{2}$ lb., find graphically the magnitude and direction of the force of the wind on the kite. Tasmania Univ.

12. Three forces P, Q, E are in equilibrium. P = Q, and E = 1·25 P. Find the angle between the directions of P and Q. Answer the same question if P = Q = E.

13. A rope is fastened to two points A, B, and carries a weight of 50 lb. which can slide smoothly along the rope. The coordinates of B with respect to horizontal and vertical axes at A are 8 feet and 1·2 feet, and the length of the rope is 10 feet. Find graphically the position of equilibrium and the tension in the rope. L.U.

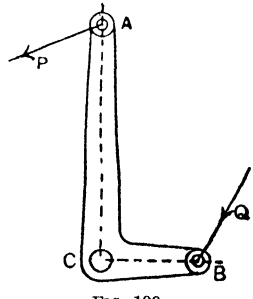


FIG. 100.

14. In Fig. 100 is shown a bent lever ABC, pivoted at C. The arms CA and CB are at 90° and are 15 inches and 6 inches respectively. A force P of 35 lb. weight is applied at A at 15° to the horizontal, and another Q is applied at B at 20° to the vertical. Find the magnitude of Q and the magnitude and direction of the reaction at C required to balance P and Q. Neglect the weight of the lever.

15. Two scaffold poles AB and AC stand on level ground in a vertical plane, their tops being lashed together at A. AB is 20 feet, AC is 15 feet and BC is 15 feet. Find the push in each pole when a load of 1 ton weight is hung from A.

16. The jib of a derrick crane measures 19 feet, the tie is $17\frac{1}{2}$ feet and the post is 9 feet long. A load of 2·5 tons weight is attached to a chain which passes over a single pulley at the top of the jib and then along the tie. Find the push in the jib and the pull in the tie. Neglect friction and the weights of the parts of the crane.

17. Answer Question 16 supposing the chain, after leaving the pulley at the top of the jib, passes along the jib.

18. Four loaded bars meet at a joint as shown in Fig. 101. P and Q are in the same horizontal line; T and W are in the same vertical; S makes 45° with P. If P = 15 tons weight, W = 12 tons weight, S = 6 tons weight, find Q and T.

19. Lines are drawn from the centre O of a hexagon to each of the corners A, B, C, D, E, F. Forces are applied in these lines as follows: From O to A, 6 lb. weight; from B to O, 2 lb. weight; from C to O, 8 lb. weight; from O to D, 12 lb. weight; from E to O, 7 lb. weight; from F to O, 3 lb. weight. Find the resultant

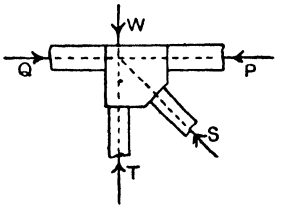


Fig 101

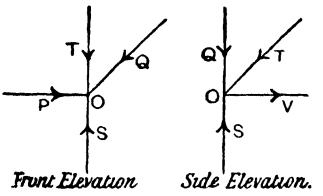


Fig 102.

20. In Fig. 102 forces in equilibrium act at O as follows: In the front elevation, P, Q and S are in the plane of the paper and T is at 45° to the plane of the paper; Q makes 135° with S. In the side elevation, T and V are in the plane of the paper; V is perpendicular to the plane containing P, Q and S, and T makes 45° with V. Given Q = 40 tons weight, T = 25 tons weight, find P, S and V.

21. State and establish the proposition known as the polygon of forces.

OA, OB, OC, OD, OE are five bars in one plane meeting at O, the angles AOB, BOC, COD, DOE being each 30°. Forces of 1, 2, 3, 4 and 5 tons respectively act outwards from O along the bars. The joint O is held in equilibrium by two other bars pulling in the opposite directions to OA and OD. Find the pull along each of these bars. Adelaide University.

22. A particle of weight W is kept in equilibrium on a smooth inclined plane (angle of inclination = θ) by a single force parallel to the plane. Find the magnitude of the force. If the particle is kept in equilibrium by three forces P, Q, R, each parallel to the plane and inclined at angles α, β, γ to the line of greatest slope, find all the relations existing between P, Q, R, W, α, β, γ, θ. Tasmania Univ.

23. Enunciate the polygon of forces and show how it may be used to find the resultant of a number of concurrent forces. Explain also the method of getting the resultant by considering the resolved parts of the forces in two directions at right angles. Bombay Univ.

24. A string 12 feet long has 11 knots at intervals of 1 foot. The ends of the string are tied to two supports A, B, 9 feet apart and in the same horizontal line. A load of 4 lb. weight is suspended from each knot in turn; find the tensions in the part of the string attached to A. Plot these tensions as ordinates, and horizontal distances of the load from A as abscissae.

25. A building measures 40 feet long and 20 feet wide. In the plan, the ridge of the roof is parallel to the long sides and bisects the short sides. The ridge is 5 feet higher than the eaves. Wind exerts a normal pressure of 40 lb. weight per square foot on one side of the roof. Find the horizontal and vertical components of the total force exerted by the wind on this side.

CHAPTER VIII

MOMENTS. PARALLEL FORCES

Moment of a force.—The **moment of a force** is the tendency of the force to rotate the body to which it is applied, and is measured by the product of the magnitude of the force and the length of a line drawn from the axis of rotation perpendicular to the line of the force. Thus, in Fig. 103, is shown a body capable of rotating about an axis passing through O and perpendicular to the plane of the paper. A force P is applied in the plane of the paper and its moment is measured by

Moment of $P = P \times OM$,

OM being drawn from O at right angles to P.

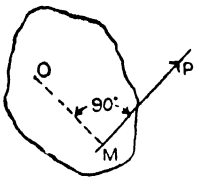


FIG. 103.—Moment of a force.

Moments involve both the units of force and of length employed in the calculation. In the C.G.S. system moments may be measured in dyne-centimetres or gram-weight-centimetres; in the British system, poundal-feet or lb.-weight-feet are customary. The dimensions of the moment of a force are obtained by taking the product of the dimensions of force and length, thus:

$$\text{Dimensions of the moment of a force} = \frac{ml}{t^2} \times l = \frac{ml^2}{t^2}.$$

The sense of the moment of a force is described as **clockwise** or **anticlockwise**, according to the direction of rotation which would result from the action of the force. In calculations it is convenient to describe moments of one sense as positive; those of contrary sense will then be negative.

It is evident that no rotation can result from the action of a force which passes through the axis of rotation; such a force has no moment.

Representation of a moment.—In Fig. 104 is shown a body free

to rotate about O and acted on by a force P, represented by the line AB. Draw OM perpendicular to AB, producing AB if necessary. Join OA and OB. Then

$$\text{Moment of } P = P \times OM = AB \times OM$$
$$= 2\triangle OAB.$$

We may therefore take twice the area of the triangle, formed by joining the extremities of the line representing the force to the point of rotation, as a measure of the moment of the force.

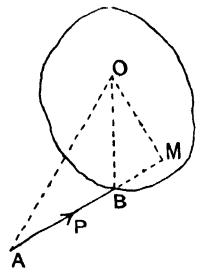


FIG 104.—Representation of a moment

The components of a force have equal opposite moments about any point on the line of the resultant.—In Fig. 105, R acts at O, and has components P and Q given by the parallelogram of forces OBDC. A is any point on the line of R, and AM and AN are perpendicular to P and Q respectively. α and β are the angles between R and P, and R and Q.

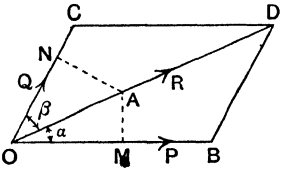


FIG 105.—Moments of P and Q

$$\frac{\text{Moment of P}}{\text{Moment of Q}} = \frac{P \times AM}{Q \times AN} = \frac{P \times OA \sin \alpha}{Q \times OA \sin \beta}$$
$$= \frac{P \sin \alpha}{Q \sin \beta}.$$

Also, $$\frac{P}{Q} = \frac{OB}{OC} = \frac{OB}{BD} = \frac{\sin \beta}{\sin \alpha};$$

$$\therefore \frac{\text{moment of P}}{\text{moment of Q}} = \frac{\sin \beta \sin \alpha}{\sin \alpha \sin \beta} = 1.$$

$$\therefore \text{moment of P} = \text{moment of Q}.$$

The moment of a force about any point is equal to the algebraic sum of the moments of its components.—There are two cases, one in which

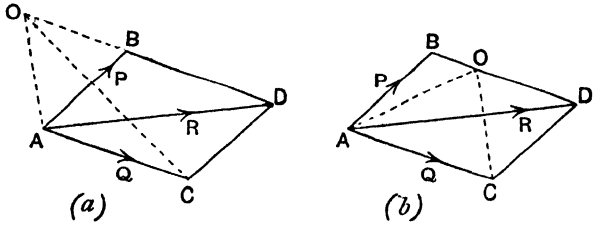


FIG 106.—Moments of P, Q and R about O.

the point is so chosen that the components have moments of the same sign (Fig. 106 (*a*)); in the other case (Fig. 106 (*b*)) the

components have moments of opposite sign. In each figure let R be the given force and let O be the point of rotation. Let the components be P and Q, and draw OBD parallel to Q, and cutting P and R in B and D respectively. Complete the parallelogram ABDC, then

$$P \,.\, Q : R = AB \,.\, AC \,.\, AD.$$

Join OA and OC. Then, in Fig. 106 (*a*),

$$\triangle OAB + \triangle ABD = \triangle OAD.$$

Also,

$$\triangle ABD = \triangle ACD = \triangle OAC.$$

$$\therefore \triangle OAB + \triangle OAC = \triangle OAD,$$

or, moment of P + moment of Q = moment of R (p. 95).

In Fig. 106 (*b*) we have

$$\triangle OAB + \triangle OAD = \triangle ABD.$$

Also,

$$\triangle ABD = \triangle ACD = \triangle OAC;$$

$$\therefore \triangle OAB + \triangle OAD = \triangle OAC,$$

or,

$$\triangle OAD = \triangle OAC - \triangle OAB,$$

or, moment of R = moment of Q − moment of P.

Hence, in taking moments, we may substitute either the components for the resultant, or the resultant for the components, without altering the effect on the body.

Principle of moments.—Let a number of forces, all in the same plane, act on a body free to rotate about a fixed axis. If no rotation occurs, then **the sum of the clockwise moments is equal to the sum of the anticlockwise moments.**

This **principle of moments** may be understood by taking any two of the forces, both having clockwise moments. The moment of the resultant of these forces is equal to the sum of the moments of the forces. Take this resultant together with another of the given forces having a clockwise moment; the moment of these will again be equal to the sum of the moments. Repeating this process gives finally a single force having a clockwise moment equal to the sum of all the given clockwise moments.

Treating the forces having anticlockwise moments in the same manner gives a single force having an anticlockwise moment equal to the sum of all the given anticlockwise moments. Hence the resultant of the two final forces has a moment equal to the algebraic sum of the given clockwise and anticlockwise moments, and if these be equal the resultant moment is zero and no rotation will occur.

Expt. 17.—Balance of two equal opposing moments. In Fig. 107, a rod AB has a hole at A through which a bradawl has been pushed into a vertical board. The rod AB hangs vertically and can turn freely about A. Fix

it in this position by pushing another bradawl through a hole near B. Attach a fine cord at C, lead it over a pulley D′ and attach a weight W_1, thus applying a pull $P = W_1$ at C. Measure the perpendicular AM drawn from A to P, and calculate the moment of $P = P \times AM$. Attach another fine cord at C′ and lead it over a pulley E′. Measure the perpendicular AN, drawn from A to the cord, and calculate Q from

$$\text{Moment of } Q = \text{Moment of } P,$$

$$Q \times AN = P \times AM,$$

$$Q = \frac{P \times AM}{AN}.$$

Apply a weight W_2 equal to the calculated value of Q, and withdraw the bradawl at B. If the rod remains vertical, the result may be taken as evidence of the principle that two equal opposing moments balance.

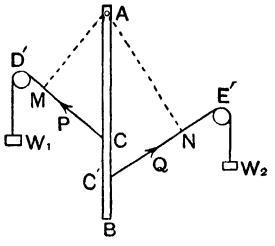


FIG 107.—Two inclined forces, having equal opposing moments.

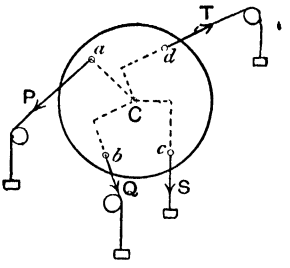


FIG. 108.—Disc in equilibrium under the action of several forces.

EXPT. 18.—**Principle of moments.** In Fig. 108 is shown a wooden disc which can turn freely about a bradawl pushed through a central hole into a vertical board. Apply forces at *a*, *b*, *c*, *d*, etc., by means of cords, pulleys and weights, and let the disc find its position of equilibrium. Calculate the moment of each force separately, and attach the proper sign, plus or minus. Take the sum of each kind, and ascertain if the sums are equal, as they should be, according to the principle of moments.

Resultant of two parallel forces.—There are two cases to be considered, viz. forces of like sense (Fig. 109 (*a*)) and forces of unlike sense (Fig. 109 (*b*)). The following method is applicable equally to both cases, and may be read in reference to both diagrams, which are lettered correspondingly.

For convenience, let the given forces P and Q act at 90° to a rod AB at the points A and B respectively. The equilibrium of the rod will not be disturbed by the application of equal opposite forces S, S, applied in the line of the rod at A and B. By means of the

parallelogram of forces A*bca*, find the resultant R_2 of P and S acting at A; in the same manner find the resultant R_1 of Q and S acting at B. Produce the lines of R_1 and R_2 until they intersect at O, and let R_1 and R_2 act at O. Resolve R_1 and R_2 into components acting at O, and respectively parallel and at right angles to AB; the components parallel to AB will be each equal to S, therefore they balance and need not be considered further. The components at right angles to AB will be equal respectively to P and Q, and are the only forces remaining. Hence R is equal to their algebraic sum, thus

In Fig. 109 (*a*) $R = P + Q$ ..... (1)

In Fig. 109 (*b*) $R = P - Q$. .....(1*a*)

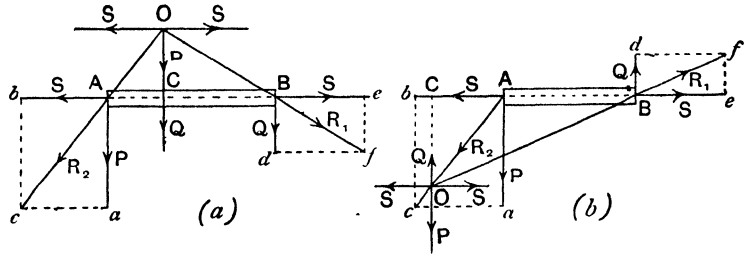


FIG 109.—Resultant of two parallel forces

Let the line of R, which passes through O and is parallel to P and Q, be produced to cut AB, or AB produced, in C. Then the triangles OAC and A*ca* are similar; hence

$$\frac{OC}{CA} = \frac{Aa}{ac} = \frac{Aa}{Ab} = \frac{P}{S}. \quad \text{.....(2)}$$

Also the triangles OBC and B*fd* are similar, therefore

$$\frac{OC}{CB} = \frac{Bd}{df} = \frac{Bd}{Be} = \frac{Q}{S}. \quad \text{.....(2}a\text{)}$$

Divide (2) by (2*a*), giving

$$\frac{CB}{CA} = \frac{P}{Q}. \quad \text{.....(3)}$$

This result indicates that the line of the resultant divides the rod into segments inversely proportional to the given forces, internally if the forces are like, and externally if the forces are unlike. It will be noted also that the resultant is always nearer to the larger force; in the case of forces of unlike sense, the resultant has the same sense as the larger force. The equilibrant of P and Q may be obtained by applying to the rod a force equal and opposite to R.

In the case of equal parallel forces of unlike sense equations (1*a*) and (3) give

$$R = P - P = 0;$$

$$\frac{CB}{CA} = \frac{P}{P} = 1; \quad \therefore CB = CA;$$

the interpretation being that the resultant is a force of zero magnitude applied at infinity—an impossibility. The name **couple** is given to two equal parallel forces of unlike sense; a couple has no resultant force, and hence cannot be balanced by a force. Some properties of couples will be discussed later.

Moments of parallel forces.—The light rods shown in Fig. 110 (*a*) and (*b*) are in equilibrium under the actions of forces P and Q, of like sense in Fig. 110 (*a*) and of unlike sense in Fig. 110 (*b*), together with the equilibrants E, supplied by the reactions of the pivots at C. From equation (3) (p. 98), we have

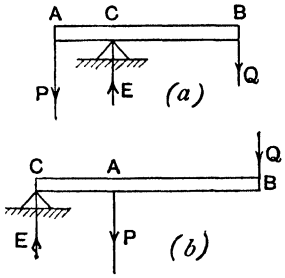


FIG. 110.—Moments of parallel forces.

$$\frac{P}{Q} = \frac{BC}{AC},$$

or,
$$P \times AC = Q \times BC. \quad \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

This result indicates that the moments of P and Q are equal and opposite, and we may infer that this condition must be fulfilled in order that the rod may not rotate.

In Fig. 111, R is the resultant of P and Q. Take any other point O in the rod, and take moments about O.

Moment of $R = R \times OC = R(OA + AC)$. $\ldots\ldots\ldots\ldots\ldots\ldots (2)$

Moment of $P = P \times OA$.

Moment of $Q = Q \times OB = Q(OA + AC + CB)$.

$\therefore$ Moment of P + moment of Q

$$= (P \times OA) + (Q \times OA) + (Q \times AC) + (Q \times CB)$$
$$= (P + Q)OA + (Q \times AC) + (P \times AC), \text{ from (1)},$$
$$= (P + Q)OA + (P + Q)AC$$
$$= R(OA + AC)$$
$$= \text{moment of } R.$$

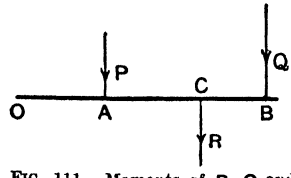


FIG. 111.—Moments of P, Q and R about O.

We may therefore assert that the algebraic sum of the moments of the parallel components of a force about any point is equal to the moment of the resultant.

Reaction of a pivot.—In Fig. 112 (*a*) a horizontal rod is in equilibrium under the action of a load W applied at A, another load P_1 applied at B_1, and a reaction E_1 exerted by the pivot at C. It is clear that E_1 is equal and opposite to the resultant of P_1 and W ; hence

$$E_1 = P_1 + W.$$

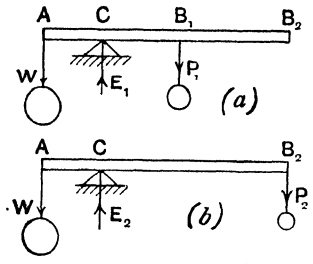


FIG 112.—Reactions of pivots

In Fig. 112 (*b*) is shown the same rod carrying the same load W at the same place, but now equilibrated by a force P_2 applied at B_2, and the equilibrant E_2 applied by the pivot. As before

$$E_2 = P_2 + W.$$

In Fig 112 (*a*) we have

$$P_1 \times B_1C = W \times AC ; \quad \therefore P_1 = \frac{AC}{B_1C} W.$$

In Fig. 112 (*b*), in the same way :

$$P_2 \times B_2C = W \times AC ; \quad \therefore P_2 = \frac{AC}{B_2C} W.$$

Since W and AC are the same in both cases, and since B_2C is greater than B_1C, P_2 is less than P_1, therefore E_2 is less than E_1. It will thus be noted that although the general effect is the same in both cases, viz. the rod is in equilibrium, the effects on the pivots are not identical, nor will be the effects of the loads in producing stresses in the material of the rod.

EXPT. 19.—**Equilibrant of two parallel forces.** Hang a rod AB from a fixed support by means of a cord attached to A (Fig. 113) ; let the rod hang in front of a vertical board and fix it in its position of equilibrium by means of bradawls at A and B. Apply parallel forces P and Q at C and D respectively, using cords, pulleys and weights W_1 and W_2. Find the resultant R and its point of application by calculation, and then apply E, equal and opposite to R by means of another cord, pulley and weight W_3. Remove the bradawls ; if the rod remains unaltered in position, the result shows that the method of calculation has been correct.

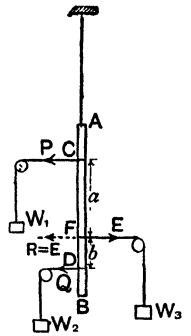


FIG 113.—Equilibrant of two parallel forces of the same sense.

Repeat the experiment using forces P and Q of unlike sense. Also verify the fact that if P and Q are equal and of opposite sense and act in parallel lines, no single force applied to the rod will preserve equilibrium.

Resultant of any number of parallel uniplanar forces.—In Fig. 114 forces P, Q, S, T are applied to a rod at A, B, C, D respectively. The resultant of these forces may be found by successive applications of the methods described on p. 97 for finding the resultant of two parallel forces. O being any convenient point of reference, first find the resultant R_1 of P and Q.

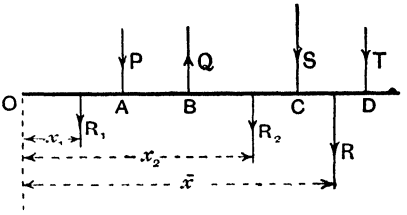


FIG 114.—Resultant of parallel forces

$$R_1 = P - Q. \quad \ldots\ldots(1)$$

$$R_1 x_1 = (P \times OA) - (Q \times OB);$$

$$\therefore\ x_1 = \frac{(P \times OA) - (Q \times OB)}{P - Q}. \quad \ldots\ldots(2)$$

Now find the resultant R_2 of R_1 and S.

$$R_2 = R_1 + S = P - Q + S. \quad \ldots\ldots(3)$$

$$R_2 x_2 = R_1 x_1 + (S \times OC) = (P \times OA) - (Q \times OB) + (S \times OC);$$

$$\therefore\ x_2 = \frac{(P \times OA) - (Q \times OB) + (S \times OC)}{P - Q + S}. \quad \ldots\ldots(4)$$

In the same manner, find the resultant R of R_2 and T; R will then be the resultant of the given forces.

$$R = R_2 + T = P - Q + S + T. \quad \ldots\ldots(5)$$

$$R\bar{x} = R_2 x_2 + (T \times OD)$$

$$= (P \times OA) - (Q \times OB) + (S \times OC) + (T \times OD);$$

$$\therefore\ \bar{x} = \frac{(P \times OA) - (Q \times OB) + (S \times OC) + (T \times OD)}{P - Q + S + T}. \quad \ldots\ldots(6)$$

In this result the numerator is the algebraic sum of the moments about O of the given forces, and may be written ΣPx. The denominator is the algebraic sum of the given forces, and may be written ΣP. Hence from (5) and (6), we have

$$R = \Sigma P. \quad \ldots\ldots(7)$$

$$\bar{x} = \frac{\Sigma Px}{\Sigma P}. \quad \ldots\ldots(8)$$

It is evident that the resultant is parallel to the forces of the given system; its sense is determined by the sign of the result calculated from (7).

Reversal of R will give the equilibrant of the given system. Should the given forces be in equilibrium, R will be zero, and the algebraic sum of the moments of the given forces will also be zero. Hence the conditions of equilibrium are

$$\Sigma P = 0. \quad \ldots\ldots (9)$$

$$\Sigma Px = 0. \quad \ldots\ldots (10)$$

These must be satisfied simultaneously.

Should ΣP be zero, and ΣPx be not zero, then the interpretation is that the system can be reduced to two equal parallel forces of opposite sense, *i.e.* a couple (p. 99). Should ΣP have a numerical value and ΣPx be zero, then the point O about which moments have been taken lies on the line of the resultant.

Reactions of a loaded beam.—The above principles may be applied in the determination of the reactions of the supports of a loaded beam. An example will render the method clear.

EXAMPLE.—A beam AB rests on supports at A and B 16 feet apart and carries loads of 2, 1, 0·75 and 0·5 tons as shown (Fig. 115). Find the reactions P and Q of the supports.

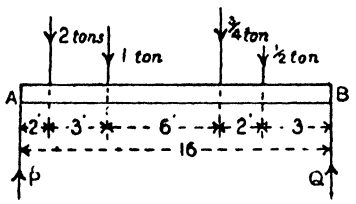


FIG. 115.—Reactions of a beam

From equation (9) above,

$$\Sigma P = 0; \quad \therefore P + Q = 2 + 1 + 0{\cdot}75 + 0{\cdot}5 = \underline{4{\cdot}25} \text{ tons}$$

P may be calculated by taking moments about B; Q has no moment about this point, and will therefore not appear in the calculation.

The sum of the clockwise moments will be equal to the sum of the anti-clockwise moments; hence

$$P \times 16 = (2 \times 14) + (1 \times 11) + (0{\cdot}75 \times 5) + (0{\cdot}5 \times 3);$$

$$\therefore P = \underline{2{\cdot}765} \text{ tons.}$$

In the same way, Q may be found by taking moments about A; thus:

$$Q \times 16 = (2 \times 2) + (1 \times 5) + (0{\cdot}75 \times 11) + (0{\cdot}5 \times 13),$$

$$\therefore Q = \underline{1{\cdot}484} \text{ tons.}$$

The sum of these calculated values gives

$$P + Q = 2{\cdot}765 + 1{\cdot}484 = \underline{4{\cdot}249} \text{ tons,}$$

a result which agrees with the sum already calculated, viz. 4·25, within the limits of accuracy adopted in the calculations.

EXPT. 20.—**Reactions of a beam.** Suspend a wooden bar from two supports, using spring balances so that the reactions of the supports may

be observed (Fig. 116). Prior to placing any loads on the beam, read the spring balances; let the readings be P_1 and Q_1 lb. weight respectively.

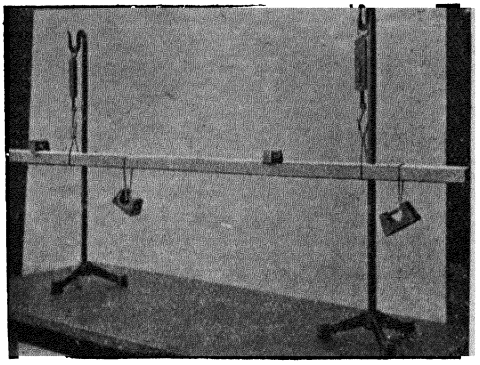


FIG. 116.—Apparatus for determining the reactions of the supports of a beam.

Place some loads on the beam and calculate the reactions, neglecting the weight of the beam. Again read the spring balances, P and Q lb. weight say. The differences $(P - P_1)$ and $(Q - Q_1)$ lb. weight should agree with the values found by calculation.

EXERCISES ON CHAPTER VIII.

1. A bicycle has cranks 7 inches long from the axis to the centre of the pedal. If the rider exerts a constant push of 20 lb. weight vertically throughout the downward stroke, find the turning moment when the crank is at the top, also when it has turned through angles of 30°, 60°, 90°, 120°, 150° and 180° from the top position.

2. A wooden disc is capable of turning freely in a vertical plane about a horizontal axis passing through its centre O. Light pegs A and B are driven into one side of the disc; OA = OB = 6 inches and OA and OB are perpendicular to one another. Fine cords are attached to A and B and hang vertically; these cords carry weights of 4 lb. and 2 lb. respectively. Find, and show in a diagram, the positions in which the disc will be in equilibrium.

3. Two parallel forces of like sense, one of 8 lb. weight and the other of 6 lb. weight, act on a body in lines 12 inches apart. Find the resultant.

4. Answer Question 3 supposing the forces to be of opposite senses.

5. A uniform horizontal rod 2 feet long has a weight of 12 lb. hanging from one end, and the rod is pivoted at its centre. Balance has to be restored by means of a weight of 18 lb. Find where it must be placed.

6. A rod AB carries bodies weighing 3 lb., 7 lb. and 10 lb. at distances of 2 inches, 9 inches and 15 inches respectively from A. Neglect the

weight of the rod and find the point at which the rod must be supported for equilibrium to be possible.

7. A beam 12 feet long is supported at its ends and carries a weight of 2·5 tons at a point 4 feet from one end. Neglect the weight of the beam and find the reactions of the supports.

8. A lever $3\frac{1}{2}$ feet long is used by a man weighing 150 lb. who can raise unaided a body weighing 300 lb. Find the load he can raise (*a*) applied at the end of the lever, the pivot being 4 inches from this end; (*b*) with the pivot at one end of the lever and the load at 4 inches from this end.

9. A light rod AB is 1 metre long and has parallel forces applied at right angles to the rod as follows: At A, 2 kilograms weight; at 15 cm. from A, 4 kilograms weight; at 55 cm. from A, 6 kilograms weight; at B, 8 kilograms weight. Find the resultant of these forces.

10. A light horizontal rod AB is 2 feet long and is supported at its ends; the reaction at A acts at 30° to the vertical. Find both reactions if a load of 3 lb. weight is placed on the rod at 8 inches from A.

11. A light horizontal rod AB, 3 feet long, is supported at its ends; the reaction at A is vertical. A force of 4 lb. weight is applied at a point C in the rod; AC is 1 foot and the angle between AC and the line of the force is 70°. Find the reactions of both supports.

12. A horizontal lever AB is 6 feet long and is pivoted at C; AC is 4 inches. If a load of 400 lb. weight is suspended at A, find the position and magnitude of the weight which must be applied to the lever in order that the reaction of the support shall be a minimum. State the minimum value of the reaction. Neglect the weight of the lever.

13. A plank AB, 10 feet long, is hinged at a point 6 feet from a vertical wall and its upper end B rests against the wall. Assume that both hinge and wall are smooth. Find the reactions of the wall and hinge if loads of 40, 60 and 100 lb. weight are hung from points in the plank at distances of 2, 6 and 8 feet respectively from A. Neglect the weight of the plank.

14. A bent lever ACB is pivoted at C; the arms AC and BC meet at 120°; AC = 18 inches, BC = 10 inches, and is horizontal. If a load of 150 lb. weight be hung from B, find, by taking moments about C, what horizontal force must be applied at A. Find also the reaction of the pivot at C

15. A beam AB, 40 feet long, is supported at A and at a point 10 feet from B. Loads of 4, 8, 6 and 10 tons weight are applied respectively at points 8, 20, 30 and 40 feet from A. Neglect the weight of the beam and find the reactions of the supports.

16. A plank 12 feet long spans an opening between two walls. A man weighing 150 lb. crosses the plank. Find the reactions of the supports of the plank when the man is at distances of 2, 4, 6. 8, 10 feet from one end. Neglect the weight of the plank. Plot a graph showing these distances as abscissae, and the reactions of the left-hand support as ordinates.

17. In Question 16, two men A, B, cross the plank from right to left, B keeping at a distance of 4 feet behind A. Each man weighs 150 lb. Find the reactions of the left-hand support when A is at the following

distances in feet from it: 12, 10, 8, 6, 4, 2, 0. Neglect the weight of the plank. Plot a graph showing the reactions of the left-hand support as ordinates, and the distances of A from this support as abscissae.

18. The seats in a rowing boat are 3 feet apart. The steersman weighs 110 lb. Starting with bow, the weights in lb. of the four oarsmen are as follows: 162, 155, 149, 166. Find the resultant weight in magnitude and position.

19. Give the conditions of equilibrium of a body under parallel forces. A thin rod of negligible weight rests horizontally on the hooks of two spring balances suspended 10 inches apart. Two bodies of weight 2 lb. and 3 lb. respectively are hung from the rod, always at a distance of 20 inches apart from each other. How will you suspend these weights so that each spring balance shows the same reading? (Calcutta.)

CHAPTER IX

CENTRE OF PARALLEL FORCES. CENTRE OF GRAVITY

Centre of parallel forces.—In Fig. 117, AB is a rod having forces P and Q applied at the ends in lines making 90° with the rod. The resultant R of P and Q divides the rods into segments given by

$$P : Q = BC : AC \text{ (p. 98)}. \quad \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

Without altering the magnitudes of P and Q, let their lines be kept parallel and rotated into new positions P′ and Q′. Through C draw DCE perpendicular to P′ and Q′. The resultant R′ of P′ and Q′ divides DE into segments inversely proportional to P′ and Q′. It is evident that the triangles ACD and BCE are similar; hence

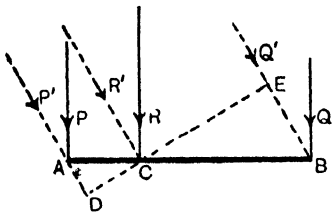


Fig. 117.—Centre of parallel forces.

$$EC : DC = BC : AC = P : Q$$

from (1). It therefore follows that R′ passes through the same point C. This point is called the **centre** of the parallel forces P and Q.

If several parallel forces act on the rod, it may be shown easily that their resultant always passes through the same point so long as the forces remain unaltered in magnitude.

Centre of gravity.—Every particle in a body possesses weight; hence the gravitational effort exerted on any body consists of a large number of forces directed towards the centre of the earth. These forces are sensibly parallel in the case of any body of moderate size. It is not possible to alter the directions of the forces of gravitation to any appreciable extent, but a similar effect may be obtained by inclining the body. The weights of all the particles still act in

vertical lines, but their directions will be altered in relation to any fixed line AB in the body (Fig. 118 (*a*) and (*b*)).

Let W be the resultant weight of the body, and let its line of action be marked at CD on the body in Fig. 118 (*a*), and to be marked again as EF in Fig. 118 (*b*). CD and EF *intersect at G, and it is clear from what has been said that* W will pass through G whatever may be the position of the body. G is the centre of the weights of the particles composing the body, and is called the **centre of gravity.**

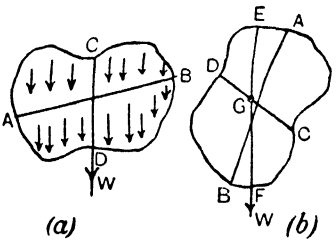


FIG. 118.—Centre of gravity

In taking moments of the forces acting on a body, the simplest way of dealing with the total weight of the body is to imagine it to be applied as a vertical force concentrated at the centre of gravity of the body. The centre of gravity of a body may be defined as **that point at which the total weight of the body may be imagined to be concentrated without thereby altering the gravitational effect on the body.**

EXAMPLE.—A uniform beam AB weighs 1·5 tons, and has its centre of gravity at the middle of its length. The beam is 16 feet long, and is supported at the end A and at a point C 4 feet from the end B (Fig. 119). Loads of 2 and 3 tons weight are applied at 3 feet from A and at B respectively. Find the reactions of the supports.

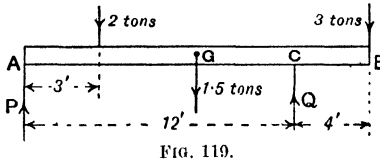


FIG. 119.

The centre of gravity G is at a distance of 8 feet from A; apply the weight of the beam, 1·5 tons weight, at G, and take moments about C in order to find P.

$$(P \times 12) + (3 \times 4) = (2 \times 9) + (1{\cdot}5 \times 4).$$

$$P = \frac{18 + 6 - 12}{12} = \frac{12}{12}$$

$$= 1 \text{ ton weight.}$$

To find Q take moments about A.

$$Q \times 12 = (3 \times 16) + (1{\cdot}5 \times 8) + (2 \times 3).$$

$$Q = \frac{48 + 12 + 6}{12} = \frac{66}{12}$$

$$= 5{\cdot}5 \text{ tons weight.}$$

Check:— $P + Q = 1 + 5{\cdot}5 = 6{\cdot}5$ tons weight

= total load on the beam.

Some simple cases of centre of gravity.—The position of the centre of gravity in certain simple cases may be located by inspection. Thus, for a slender straight rod or wire of uniform cross section, the centre of gravity G lies at the middle of the length. In a thin uniform square or rectangular plate, G lies at the intersection of the diagonals; a thin uniform circular plate has G at its geometrical centre.

A parallelogram made of a thin uniform sheet may be imagined to be built up of thin uniform rods arranged parallel to AB (Fig. 120); the centre of gravity of each rod lies at the middle of length of the rod, hence all their centres of gravity lie in HK, which bisects AB and CD. The centre of gravity of the parallelogram therefore lies in HK. Similarly, the plate may be imagined to be constructed of thin rods lying parallel to AD, and the centres of gravity of all these rods, and therefore the centre of gravity of the parallelogram, will lie in EF, which bisects AD and BC. Hence G lies at the point of intersection of HK and EF; it is evident that the diagonals AC and BD intersect at G.

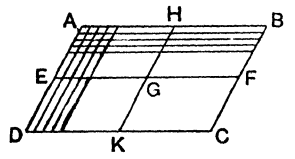


FIG. 120.—Centre of gravity of a parallelogram.

A thin triangular plate may be treated in a similar manner (Fig. 121). First take strips parallel to AB, when it is clear that the centre of gravity of the plate lies in CE, which bisects AB and also bisects all the strips parallel to AB. Then take strips parallel to BC, AD bisects BC and also all the strips parallel to BC, and therefore contains the centre of gravity. Hence G lies at the intersection of CE and AD.

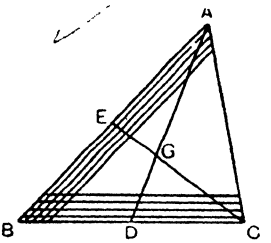


FIG. 121.—Centre of gravity of a triangle.

Let DE be joined in Fig. 121, then the triangles BED and BAC are similar, since DE is parallel to AC. Therefore $DE = \frac{1}{2}AC$. Also the triangles DEG and ACG are similar; hence

$$\frac{DG}{AG} = \frac{DE}{AC} = \frac{\frac{1}{2}AC}{AC} = \frac{1}{2};$$

$$\therefore DG = \tfrac{1}{2}AG = \tfrac{1}{3}AD.$$

We have therefore the rule that the centre of gravity of a thin triangular sheet lies on the line drawn from the centre of a side to the opposite corner, and one-third of its length from the side.

Any uniform prismatic bar has its centre of gravity in its axis at the middle of its length. The centre of gravity of a uniform sphere lies at its geometrical centre. A solid cone or pyramid has

the centre of gravity on the line joining the centre of the base to the apex, and one-quarter of its length from the base. A cone or pyramid open at the base, and made of a thin sheet bent to shape, has its centre of gravity on the line joining the centre of the base to the apex, and one-third of its length from the base.

Method of calculating the position of the centre of gravity.—The problem of finding a line which contains the centre of gravity of a body is identical with that of finding the line of action of the resultant weight of the body, having been given the weights of the separate particles of which the body is composed.

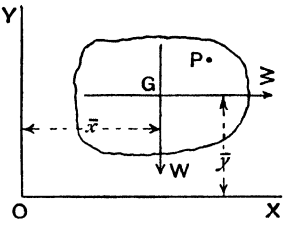


FIG 122.—Centre of gravity of a sheet.

Fig. 122 shows a thin uniform sheet in the plane of the paper; OX and OY are horizontal and vertical axes of reference. Let x_1, y_1 be the coordinates of a particle at P; let the weight of the particle be w_1, and describe similarly all the other particles of the body. Then

$$\text{Resultant weight of the body} = \mathrm{W} = w_1 + w_2 + w_3 + \text{etc.}$$
$$= \Sigma w. \quad \ldots\ldots(1)$$

Take moments about O, and let x be the horizontal distance between the line of W and OY; then

$$\mathrm{W}\bar{x} = w_1x_1 + w_2x_2 + w_3x_3 + \text{etc.}$$
$$\therefore\ \bar{x} = \frac{\Sigma wx}{\mathrm{W}}. \quad \ldots\ldots(2)$$

Now turn the figure until OX becomes vertical, and again take moments about O; let the distance between the line of W and OX be $\bar{y}$, then

$$\mathrm{W}\bar{y} = w_1y_1 + w_2y_2 + w_3y_3 + \text{etc.}$$
$$\therefore\ \bar{y} = \frac{\Sigma wy}{\mathrm{W}}. \quad \ldots\ldots(3)$$

Draw a line parallel to OX and at a distance $\bar{y}$ from it; draw another line parallel to OY and at a distance $\bar{x}$ from it; the centre of gravity G lies at the intersection of these lines.

Generally the sheet under consideration may be cut into portions for each of which the weights w_1, w_2, w_3, etc., may be calculated, and the coordinates of the centres of gravity (x_1y_1), (x_2y_2), etc., may be written down by inspection.

EXAMPLE 1.—Find the centre of gravity of the thin uniform plate shown in Fig. 123.

Take axes OX and OY as shown and let the weight of the plate per square inch of surface be w. For convenience of calculation the plate is divided into three rectangles as shown, the respective centres of gravity being G_1, G_2 and G_3. Taking moments about OY, we have

$$w\{(6\times 1)+(8\times 1)+(3\times 1)\}\bar{x}=w(6\times 1\times 3)+w(8\times 1\times \tfrac{1}{2})+w(3\times 1\times 1\tfrac{1}{2}),$$

$$\bar{x}=\frac{26{\cdot}5}{17}=\underline{1{\cdot}56} \text{ inches.}$$

Again, taking moments about OX, we have

$$17\bar{y}=(6\times 1\times 9\tfrac{1}{2})+(8\times 1\times 5)+(3\times 1\times \tfrac{1}{2}),$$

$$\bar{y}=\frac{98{\cdot}5}{17}=\underline{5{\cdot}8} \text{ inches.}$$

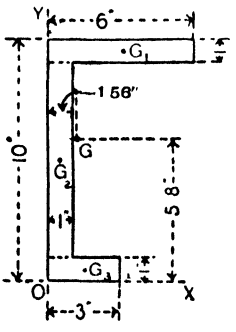


FIG. 123.

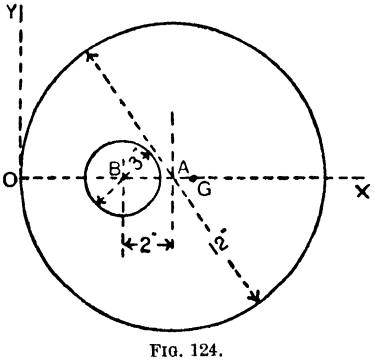


FIG. 124.

EXAMPLE 2.—A circular plate (Fig. 124) 12 inches diameter has a hole 3 inches diameter. The distance between the centre A of the plate and the centre B of the hole is 2 inches. Find the centre of gravity.

Take AB produced as OX, and take OY tangential to the circumference of the plate. It is evident that G lies in OX. Taking moments about OY, we may say that the moment of the plate as made is equal to that of the complete disc diminished by the moment of the material removed in cutting out the hole. Let w be the weight per square inch of surface, D the diameter of the plate, and d that of the hole. Then

$$\text{Weight of the complete disc} = w\frac{\pi D^2}{4}.$$

$$\text{Weight of the piece cut out} = w\frac{\pi d^2}{4}.$$

$$\text{Weight of the plate as made} = w\left(\frac{\pi D^2}{4}-\frac{\pi d^2}{4}\right)=\frac{w\pi}{4}(D^2-d^2).$$

Take moments about OY, and let OG $= \bar{x}$,

$$\frac{w\pi}{4}(D^2 - d^2)\bar{x} = \left(w\frac{\pi D^2}{4} \times 6\right) - \left(w\frac{\pi d^2}{4} \times 4\right),$$

$$(D^2 - d^2)\bar{x} = 6D^2 - 4d^2,$$

$$\bar{x} = \frac{6D^2 - 4d^2}{D^2 - d^2} = \frac{828}{135} = \underline{6{\cdot}13} \text{ inches.}$$

EXAMPLE 3.—A notice board 3 feet broad by 2 feet high, made of timber 1 inch thick, is nailed to a post 7 feet high, made of the same kind of timber, 3 inches by 3 inches (Fig. 125). Find the centre of gravity.

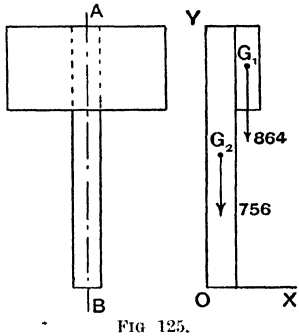


FIG 125.

The volumes of the post and board are proportional to the weights, and may be used instead of the weights.

The weight of the board is proportional to $(36 \times 24 \times 1) = 864$.

The weight of the post is proportional to $(84 \times 3 \times 3) = 756$.

The total weight is proportional to

$$(864 + 756) = 1620.$$

The vertical plane of which AB is the trace (Fig. 125) contains the centres of gravity of both board and post, and therefore contains the centre of gravity of the whole.

Take moments about O.

$$1620\bar{x} = (864 \times 3\,5) + (756 \times 1{\cdot}5)$$

$$\bar{x} = \frac{3024 + 1134}{1620} = \underline{2{\cdot}566} \text{ inches}$$

Also,

$$1620\bar{y} = (864 \times 72) + (756 \times 42).$$

$$\bar{y} = \frac{62{,}208 + 31{,}752}{1620} = \underline{58} \text{ inches.}$$

Hence the centre of gravity lies in the vertical plane AB, at a height of 58 inches and at 2·566 inches from the back of the post.

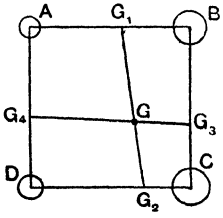


FIG. 126.

EXAMPLE 4.—Bodies having weights of w_1, w_2, w_3, w_4 are placed in order at the corners A, B, C, D of a square (Fig. 126). Find the centre of gravity.

First find the centre of gravity G_1 of w_1 and w_2; G_1 falls in AB and divides AB in the proportion

$$\frac{AG_1}{BG_1} = \frac{w_2}{w_1}.$$

In the same way, the centre of gravity of w_3 and w_4 falls in CD, and divides CD in the proportion

$$\frac{CG_2}{DG_2} = \frac{w_4}{w_3}.$$

The centre of gravity of the whole system lies in G_1G_2.

The centre of gravity G_3 of w_2 and w_3 divides BC in the proportion

$$\frac{BG_3}{CG_3} = \frac{w_3}{w_2}$$

Also the centre of gravity of w_1 and w_4 divides AD in the proportion

$$\frac{AG_4}{DG_4} = \frac{w_4}{w_1}.$$

The centre of gravity G of the whole system lies in G_3G_4. Therefore G lies at the intersection of G_1G_2 and G_3G_4. The completion of the problem may be carried out on a drawing made carefully to scale.

EXAMPLE 5.—Equal weights w_1, w_2 and w_3 are placed at the corners of any triangle ABC (Fig. 127). Find the centre of gravity.

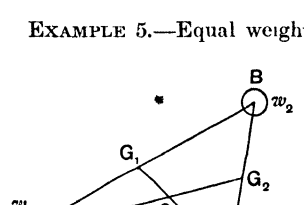


FIG. 127.

The centre of gravity G_1 of w_1 and w_2 bisects AB, and the centre of gravity of the system lies in CG_1. Also the centre of gravity G_2 of w_2 and w_3 bisects BC, and the centre of gravity G of the system lies in AG_2. Hence G is at the intersection of two lines drawn from the centres of two sides to the opposite corners of the triangle, and therefore coincides with the centre of gravity of a thin sheet having the same shape as the triangle.

EXAMPLE 6.—In Fig. 128 (*a*), ABCD is a thin sheet; AB is parallel to CD. Find the centre of gravity. (This is a case which occurs often in practice.)

Imagine the sheet to be divided into narrow strips parallel to AB. The centre of gravity of each strip will lie in EF, a straight line which bisects both AB and CD, and therefore bisects each strip. Join DE and CE, thus dividing the sheet into three triangles ADE, BCE and DEC. Since these triangles are all of the same height, their areas and hence their weights are proportional to their bases; thus

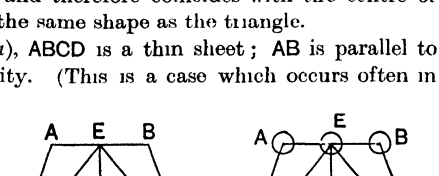


FIG 128.—A frequent case of centre of gravity

Weight of $\triangle$ ADE : weight of $\triangle$ BCE : weight of $\triangle$ DEC = $\frac{1}{2}$AB : $\frac{1}{2}$AB : DC.

Making use of the proposition discussed in Example 5 above, the weight of each triangle may be divided into three equal parts and one part placed at each corner of the triangle without altering the position of the centre of gravity of the triangle. The equivalent system of weights will be as follows (Fig. 128 (*b*))

$$\text{At A, } \tfrac{1}{6}\text{AB; at B, } \tfrac{1}{6}\text{AB; at E, } (\tfrac{2}{6}\text{AB} + \tfrac{1}{3}\text{DC}); \text{ at C, } (\tfrac{1}{6}\text{AB} + \tfrac{1}{3}\text{DC}); \text{ at D, } (\tfrac{1}{6}\text{AB} + \tfrac{1}{3}\text{DC}).$$

The centre of gravity of the weights at A, E and B lies at E, and the resultant of these weights is $(\frac{2}{6}\text{AB} + \frac{2}{6}\text{AB} + \frac{1}{3}\text{DC}) = \frac{4}{6}\text{AB} + \frac{1}{3}\text{DC}$. The centre of gravity of the weights at C and D is at F, and the resultant of these weights is $(\frac{2}{6}\text{AB} + \frac{2}{6}\text{DC})$. The centre of gravity G of the whole system divides EF in the proportion

$$\frac{\text{FG}}{\text{EG}} = \frac{\frac{4}{6}\text{AB} + \frac{1}{3}\text{DC}}{\frac{2}{6}\text{AB} + \frac{2}{3}\text{DC}} = \frac{2\text{AB} + \text{DC}}{\text{AB} + 2\text{DC}}. \quad \ldots\ldots(1)$$

The following graphical method (Fig. 129) is useful in this case: Draw EF as before; produce AB and CD to H and K respectively, making BH = CD, and DK = AB. Join HK cutting EF in O. The triangles EOH and FOK are similar, hence

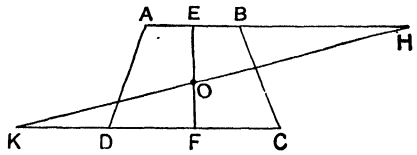


FIG 129

$$\frac{\text{FO}}{\text{EO}} = \frac{\text{FK}}{\text{EH}} = \frac{\text{DK} + \text{FD}}{\text{EB} + \text{BH}}$$

$$= \frac{\text{AB} + \frac{1}{2}\text{DC}}{\frac{1}{2}\text{AB} + \text{DC}}$$

$$= \frac{2\text{AB} + \text{DC}}{\text{AB} + 2\text{DC}}. \quad \ldots\ldots(2)$$

As this result is identical with that found in (1) for the position of G, it follows that O and G coincide. Hence Fig. 129 provides a purely graphical method of finding the centre of gravity of the sheet.

States of equilibrium of a body.—When a body is at rest under the action of a system of forces, the equilibrium is **stable** or **unstable** according as the body returns, or fails to return to its original position after being disturbed slightly. The equilibrium is **neutral** if the body remains at rest in any position. When a body is at rest under the action of gravity and given supporting forces, the state of equilibrium depends, among other conditions, upon the situation of the centre of gravity.

A cone may assume any of the three states of equilibrium. In Fig. 130 (*a*) the cone is resting with its base on a horizontal table; if disturbed slightly (Fig. 130 (*b*)), the tendency of W, acting through the centre of gravity G, and the reaction R of the table, is to restore

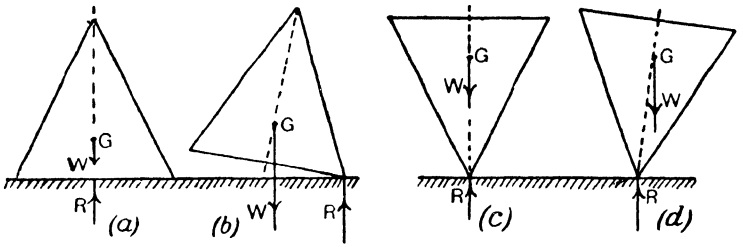


FIG 130.—Stable and unstable equilibrium

the cone to its original position. The equilibrium in Fig. 130 (*a*) is therefore stable In Fig 130 (*c*) the cone is in equilibrium when resting on its apex; the slightest disturbance (Fig. 130 (*d*)) will bring W and R into parallel lines, and they then conspire to upset the cone. The equilibrium in Fig. 130 (*c*) is therefore unstable.

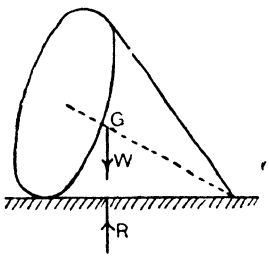


FIG. 131.—Neutral equilibrium

In Fig. 131 the cone is lying on its side on the horizontal table; in this case it is impossible for W and R to act otherwise than in the same vertical line, no matter how the cone may be turned while still lying on its side. Hence the equilibrium is neutral.

A sphere resting on a horizontal table is in neutral equilibrium, provided the centre of gravity coincides with the geometrical centre. A cylinder resting with its curved surface on a horizontal table is in neutral equilibrium, so far as disturbance by rolling is concerned, provided the centre of gravity lies in the axis of the cylinder.

In Fig. 132 (*a*) a rectangular block rests on a horizontal plank, one end of which can be raised. The vertical through G falls within the surfaces in contact *ab*, and the equilibrium is stable under the action of W and the reaction R. It is impossible that R can act outside of *ab*; hence stable equilibrium just ceases to be possible when the plank is inclined to such an angle that the vertical through G passes through *a* (Fig. 132 (*b*). It is understood that means are provided at *a* to prevent slipping of the block. If the plank be

inclined at a steeper angle (Fig. 132 (*c*)), R and W conspire to upset the block.

It will be noticed that when a body is in a position of stable equilibrium, a disturbance by tilting has the effect of raising the centre

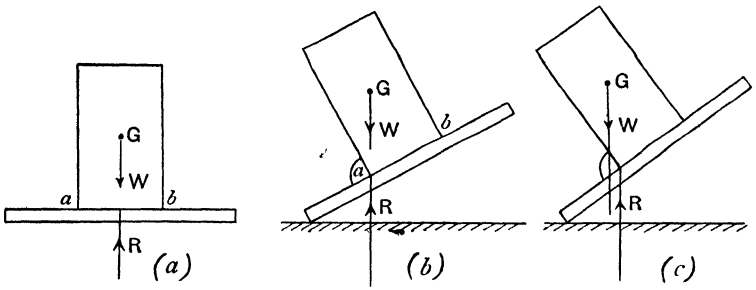


FIG. 132.—Stability of a block on an inclined plane.

of gravity. Further, if a body is capable of moving under the action of gravitational effort, it will always move in such a way as to bring the centre of gravity into a lower position. A position of stable equilibrium will be attained when (as in a pendulum) the centre of gravity has reached the lowest possible position.

In Fig. 133 (*a*) is shown a sphere having its geometrical centre at C and its centre of gravity at G. This displacement of the centre of gravity may be produced either by introducing a heavy plug into the lower hemisphere, or by cutting a slice off the top of the sphere. The sphere rests on a horizontal table, and will be in equilibrium when C and G are both in the same vertical. The reaction R and the weight W are then in the same straight line. If slightly disturbed (Fig. 133 (*b*)), R and W conspire to restore the sphere to the original position, which is therefore a position of stable equilibrium.

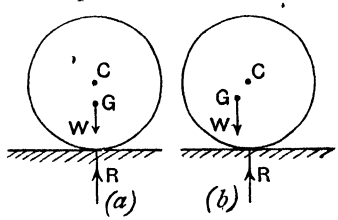


FIG 133.—Stability of a loaded sphere.

Graphical methods for finding the centre of gravity of a thin sheet.—The sheet *abcd* (Fig. 134) is quadrilateral, and is drawn carefully to scale. Divide the sheet into two triangles by joining *bd*. Bisect *bd* in *e*; join *ae* and *ce*; make $ec_1 = \frac{1}{3}ae$, and $ec_2 = \frac{1}{3}ec$; then c_1 and c_2 are the centres of gravity of the triangles *abd* and *cbd* respectively. Join c_1c_2; the centre of gravity of the sheet lies in c_1c_2. Again

divide the sheet by joining ac, and in the same way find the centres of gravity c_3 and c_4 of the triangles abc and adc. The centre of gravity G then lies at the intersection of c_1c_2 and c_3c_4.

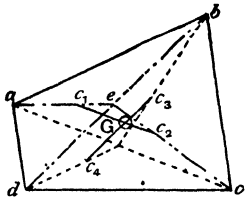


FIG 134.—Centre of gravity by construction.

In Fig. 135 (*a*) the sheet has a curved outline. Take as reference axes OX touching the outline at its lowest point, and OY at 90° to OX. Draw AB parallel to OX and touching the outline at its highest point. Let h be the perpendicular distance between OX and AB. CD is a very narrow strip parallel to OX and at a distance y from it, and has a breadth δy. The area of the strip is proportional to its weight and is equal to $CD \times \delta y$. The moment of this about OX is $CD \times \delta y \times y$. Draw CE and DF perpendicular to AB; join EO and FO cutting CD in H and K. In the similar triangles OHK and OEF, we have

$$HK : EF = y : h;$$

$$\therefore HK \times h = EF \times y = CD \times y.$$

Multiply each side by δy, giving

$$HK \times \delta y \times h = CD \times \delta y \times y.$$

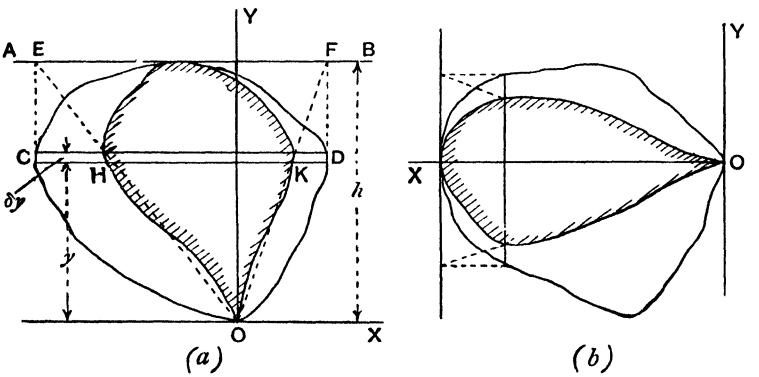


FIG 135.—Graphical method for finding the centre of gravity of a sheet

This equation indicates that the product of the area of the strip HK and h is equal to the moment of the strip CD. A similar result may be found for any other strip; hence, if a number of points, such as H and K, be found and a curve drawn through them, the area enclosed by this curve (shown shaded in Fig. 135 (*a*)), when multiplied by the constant h, will give the total moment of all the

strips resembling CD into which the sheet may be divided. Let A_1 and A_2 be the areas of the given sheet and the shaded curve respectively (these areas can be found by means of a planimeter), and let $\bar{y}$ be the distance of the centre of gravity from OX, then

$$A_2 h = A_1 \bar{y},$$

or

$$\bar{y} = \frac{A_2}{A_1} h.$$

Take another pair of axes, OX and OY (Fig. 135 (*b*)), and carry out the same process, thus determining the distance x of the centre of gravity from OY. The coordinates $\bar{x}$ and $\bar{y}$ of the centre of gravity have now been found.

EXAMPLE.—Apply the above method to find the centre of gravity of a thin semicircular sheet (Fig. 136). The diameter AB is 5 inches.

The centre of gravity of the sheet lies in OC, which is a radius drawn perpendicular to AB, hence $\bar{y}$ alone need be determined by the graphical method, which is shown in Fig. 136.

The following measurements were obtained by means of a planimeter:

The semicircular area

$A_1 = 9{\cdot}82$ sq. ins.

Also, $A_2 = 4{\cdot}12$ sq. ins.

$$\therefore \bar{y} = \frac{A_2}{A_1} h = \frac{4{\cdot}12}{9{\cdot}82} \times 2{\cdot}5$$

$$= \underline{1{\cdot}05} \text{ inches.}$$

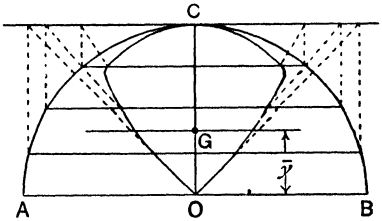


FIG. 136.—Centre of gravity of a semicircular sheet.

It is known that the centre of gravity of a semicircular sheet lies at a distance $4r/3\pi$ from AB (Fig. 136). Using this expression in order to check the above result, we obtain

$$\bar{y} = \frac{4 \times 2{\cdot}5}{3 \times \pi} = \underline{1{\cdot}06} \text{ inches.}$$

Positions of equilibrium.—If a body is suspended freely from a fixed point, the position in which it will hang in equilibrium is such that the centre of gravity falls in the vertical passing through the fixed point.

EXAMPLE 1.—A loaded rod AB (Fig. 137) is suspended by means of two cords from a fixed point C. Find its position of equilibrium.

The centre of gravity, G, of the loaded rod is first found by application of the foregoing methods. Join CG and produce it. The weight W of

the system acts in CG; hence CG is vertical. Draw DE perpendicular to CG and turn the paper round until DE is horizontal. The system will then be in its position of equilibrium.

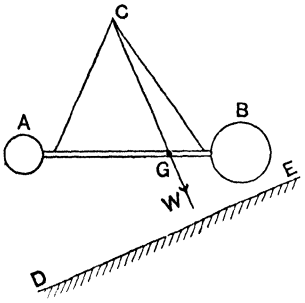


FIG. 137.—A loaded rod.

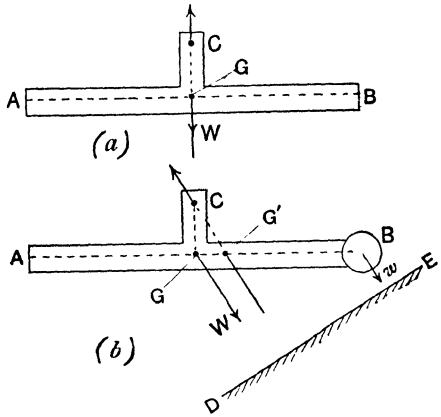


FIG 138.—Position of equilibrium of a loaded body

EXAMPLE 2.—A body ABC (Fig. 138 (*a*)), of weight W, is suspended freely from C, and AB is then horizontal. The centre of gravity G bisects AB, and CG is at 90° to AB. Find the angle which AB makes with the horizontal when a body having a weight w is attached at B (Fig. 138 (*b*)).

The centre of gravity G′ now lies in GB, and divides it in the ratio

$$\frac{GG'}{BG'} = \frac{w}{W};$$

$$\therefore \frac{GG'}{GG' + BG'} = \frac{w}{W + w};$$

$$\therefore GG' = \left(\frac{w}{W + w}\right) GB. \quad \text{.....(1)}$$

Join CG′ and produce it; draw DE at 90° to CG′, then when the paper is turned so that DE is horizontal, the system is in its position of equilibrium. The angle which AB then makes with the horizontal will be equal to the angle GCG′; let this angle be θ, then

$$\tan\theta = \frac{GG'}{CG} = \left(\frac{w}{W + w}\right)\frac{GB}{CG} \text{ (from (1))}. \quad \text{.....(2)}$$

From this result we see that if CG is diminished the angle θ becomes larger; if C and G coincide, CG is zero, $\tan\theta$ is then infinity and the system would hang in equilibrium with CB vertical.

EXPT. 21.—**Centre of gravity of sheets.** The centre of gravity of a thin sheet may be found by hanging it from a fixed support by means of a cord AB (Fig. 139); the cord extends downwards and has a small weight W, thus serving as a plumb-line. Mark the direction AC on the sheet, and then repeat the operation by hanging the sheet from D, marking the new vertical DE. G will be the point of intersection of AC and DE. Carry out this experiment for the sheets of metal or millboard supplied.

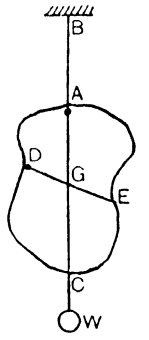


FIG. 139.—Centre of gravity of a sheet by experiment.

EXPT. 22.—**Centre of gravity of a solid body.** Arrange the body so that it is supported on knife edges placed on the pans of balances (Fig. 140). Find the weights W_1 and W_2 required to restore the balances to equilibrium; these give the reactions of the supports. Measure AB, the distance between the knife-edges. Let G be the centre of gravity, then

$$\frac{AG}{BG} = \frac{W_2}{W_1};$$

$$\therefore \frac{AG}{AG+BG} = \frac{W_2}{W_1+W_2}$$

$$\therefore AG = \frac{W_2}{W} \cdot AB,$$

where $W = W_1 + W_2$ is the weight of the body.

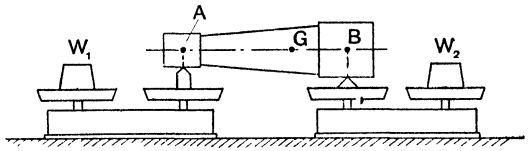


FIG. 140.—Experimental determination of the centre of gravity of a body.

The common balance.—In the outline drawing given in Fig. 141, the beam AB is capable of turning freely about a knife-edge at C, and its centre of gravity is at G. Scale-pans are hung from knife-edges at A and B. If the scale-pans be removed, the beam will remain at rest with G in the vertical passing through C. AB intersects CG at 90° at D, and is therefore horizontal. For the balance to be true, AB must

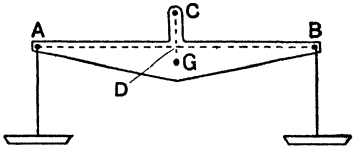


FIG. 141.—Principle of the common balance.

remain horizontal when the scale-pans are hung from the beam, and also when bodies of equal weight are placed in the pans. These conditions will be complied with if AD and BD are equal, and if the scale-pans are of equal weights.

EXAMPLE.—The beam of a balance is shown in Fig. 142. Unequal weights, W_1 and W_2, W_2 being the greater, have been placed in the pans. Determine the angle α which AB now makes with the horizontal.

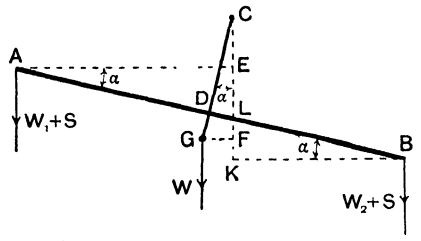


FIG 142.—Unequally loaded balance

Let S be the weight of each scale-pan, W the weight of the beam and any attachments fixed rigidly to it. Let $CD = a$, $CG = b$, and $AD = DB = c$. It is evident that CG is inclined at α to the vertical. Draw AE, BK, GF horizontally to meet the vertical CK; let this vertical cut AB in L. Take moments about C, giving

$$(W_1 + S)AE + W \cdot GF = (W_2 + S)BK,$$

$$(W_1 + S)AL \cdot \cos\alpha + W \cdot CG \cdot \sin\alpha = (W_2 + S)BL \cdot \cos\alpha,$$

$$(W_1 + S)(AD + DL)\cos\alpha + W \cdot b \cdot \sin\alpha = (W_2 + S)(BD - DL)\cos\alpha,$$

$$(W_1 + S)(c + a\tan\alpha)\cos\alpha + W \cdot b \cdot \sin\alpha = (W_2 + S)(c - a\tan\alpha)\cos\alpha,$$

$$(W_1 + S)(c + a\tan\alpha) + Wb\tan\alpha = (W_2 + S)(c - a\tan\alpha),$$

$$(aW + aS + Wb + aW_2 + aS)\tan\alpha = W_2c + Sc - W_1c - Sc,$$

$$\therefore \tan\alpha = \frac{(W_2 - W_1)c}{Wb + (W_1 + W_2 + 2S)a}$$

The magnitude of the angle α for a given difference in weights $(W_2 - W_1)$ may be taken as a measure of the **sensitiveness** of a balance. The factors influencing the magnitude of α are given in the formula found above for $\tan\alpha$. Increase in the lengths of the arms $AD = DB = c$ (Fig. 142) will increase α, and hence will increase the sensitiveness. The sensitiveness is diminished by increasing the product Wb; hence the weight W of the beam should be reduced to the minimum consistent with sufficient rigidity; greater sensitiveness can be obtained by diminishing $CG = b$ (Fig. 142). Diminishing $CD = a$ will increase the sensitiveness, and in many laboratory balances C and D coincide. If G also coincides with D, the result will be a loss of **stability**, since the beam would then be capable of resting in equilibrium at any angle to the horizontal. In a sensitive balance, G falls a little below D, and C may coincide with D. The sensitiveness is diminished by an increase in $(W_1 + W_2 + 2S)$; hence balances intended for delicate work are unsuitable for weighing

heavy bodies, and the scale-pans of delicate balances should be light. In order to understand how these principles are applied, the student should examine the parts of a delicate balance.

Truth of a balance.—A true balance having equal masses in the pans will vibrate through equal angles above and below the horizontal. The truth may be tested by placing masses in the pans until this condition is fulfilled; the masses are then interchanged, when equal angles will again be observed if the balance is true.

Referring to Fig. 141, let the arms AD and BD be unequal, and let the balance be so constructed that AB remains horizontal, or vibrates through equal angles above and below the horizontal when the scale-pans are empty. Let a body having a true weight W be placed in the left-hand pan, and let it be balanced by a weight P in the other pan. Now place W in the right-hand pan, and let Q be the weight required in order to equilibrate. Take moments in each case about C (Fig. 141).

$$W \times AD = P \times BD. \quad \ldots\ldots(1)$$

$$W \times BD = Q \times AD. \quad \ldots\ldots(2)$$

Taking products, we have

$$W^2 \times AD \times BD = P \times Q \times BD \times AD;$$

$$\therefore W = \sqrt{PQ}. \quad \ldots\ldots(3)$$

Thus the true weight is the geometrical mean of the false weights P and Q. Had the arithmetical mean $\frac{1}{2}(P+Q)$ been taken as the true weight, the result would be greater than W.

Exercises on Chapter IX.

1. A uniform beam, 12 feet long, weighs 500 lb., and carries a load of 6000 lb. distributed uniformly along its left-hand half. If the beam is supported at its ends, find the reactions of the supports.

2. The jib of a derrick crane (p. 89) is 30 feet long and weighs 800 lb.; the centre of gravity is 12 feet from the lower end. The post and tie of the crane measure 16 feet and 20 feet respectively. Find the pull in the tie necessary to support the jib.

3. A ladder AB, 20 feet long, weighs 90 lb., and its centre of gravity is 8 feet from A. The ladder is carried in a horizontal position by two men, one being at A. A bag of tools weighing 60 lb. is slung at a point 12 feet from A. Find where the second man must be situated if the men share the total load equally between them.

4. A plate of iron of uniform thickness is cut to the shape of a triangle having sides AB = 2 feet, BC = 3 feet, CA = 4 feet. If the plate weighs 50 lb. and lies on a horizontal floor, find what vertical force, applied at one corner, will just lift that corner.

5. A thin plate is cut to the shape shown in Fig. 143. Find its centre of gravity.

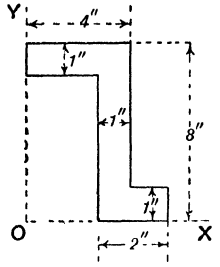


FIG. 143.

6. Draw full size a quadrilateral ABCD;

AB = 4 inches, BC = $2\frac{1}{2}$ inches,

CD = $3\frac{1}{2}$ inches, DA = $3\frac{3}{4}$ inches;

diagonal AC = $4\frac{1}{4}$ inches. The figure represents a thin plate; find its centre of gravity. If the plate weighs 2 lb. and lies on a table, what vertical force would just lift the corner D?

7. A thin plate is cut to the shape of an equilateral triangle of 18 inches side. From one corner is cut off an equilateral triangle of 6 inches side. Find the centre of gravity of the remainder of the plate.

8. A thin circular plate 20 inches diameter has two radii drawn on it meeting at 90°. A circular hole 6 inches diameter has its centre on one radius at a distance of 3 inches from the edge of the plate; another circular hole 4 inches diameter has its centre on the other radius at a distance of 2 inches from the edge of the plate. Find the centre of gravity of the plate after the holes have been cut.

9. A rectangular iron plate (Fig. 144) measures 14 inches by 8 inches by $1\frac{1}{2}$ inches thick. A hole 2 inches diameter is bored through the plate, its centre being 5 inches from one edge and 2 inches from the adjacent edge of the plate. An iron rod 2 inches diameter and 20 inches long is pushed into the hole, its end being flush with the face of the plate. Find the centre of gravity of the arrangement.

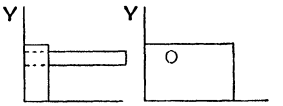


FIG 144

10. A plank of uniform cross section weighs 400 lb. and is 12 feet long. It is supported at one end and at a point 3 feet from the other end. Find the reactions of the supports. Find also the greatest load which can be placed at the end which overhangs without tilting the plank; when this load is applied, what are the reactions of the supports?

11. A rectangular block of stone stands on one end on a horizontal surface. The block measures 4 feet high, 2 feet broad and 2 feet thick. If stone weighs 150 lb. per cubic foot, find what horizontal force, applied at the top of the block at the centre of one edge, will just produce tilting. Slipping is prevented.

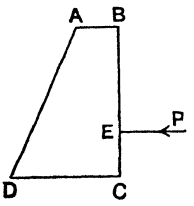


FIG. 145.

12. The block given in Question 11 rests on one end on a stiff plank, one end of which can be raised; two edges of this end are parallel to the long edges of the plank, and provision is made to prevent slipping. What angle will the plank make with the horizontal when the block is on the point of overturning?

13. ABCD is the cross section of a wall 40 feet long (Fig. 145). AB = 4 feet and is horizontal; BC = 15 feet and is vertical; CD = 9 feet and is horizontal. Find the centre of gravity of the wall. If the masonry weighs 150 lb. per cubic

foot, find what horizontal force P, applied at a height CE = 5 feet above C, will just overturn the wall.

14. A solid uniform hemisphere rests with its curved surface in contact with a horizontal table. Show that the equilibrium is stable.

15. In Fig. 146, A is a semicylindrical body resting on a horizontal table. The top face of A is rectangular, 10 inches long in the direction perpendicular to the paper, and 4 inches in the direction parallel to the plane of the paper. B is a cylindrical rod made of the same kind of material as A, 2 inches diameter, and fixed perpendicularly to the centre of the top face of A. Find the height of B so that the equilibrium of the whole shall be neutral. (The centre of gravity of A is at a distance $4r/3\pi$ below the top face.)

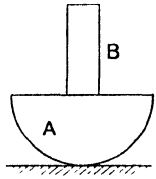


FIG. 146

16. Draw an isosceles triangle, sides AB and AC 4 inches long, base BC 3 inches long. Bodies weighing 4, 6 and 8 lb. are fastened at A, B and C respectively. The triangle is made of a thin sheet weighing 1 lb. If the arrangement is suspended by a cord attached to the centre of AB, find and show in the drawing the position it will assume.

17. Find graphically the centre of gravity of the sheet shown in Fig. 147. AB is a chord drawn at a distance of 1 inch from the centre of the circular portion, and the radius of the circular portion is 3 inches.

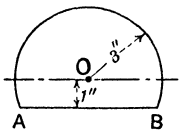


FIG. 147.

18. A body is placed first in one pan and then in the other pan of a false balance. When in the first pan, it is balanced by weights amounting to 0·562 lb. placed in the other pan; in the second operation, the weights amount to 0 557 lb. What is the true weight of the body? Assume that the balance beam swings correctly when both pans are empty. What is the error made by taking the arithmetical mean of the readings as the true weight?

19. A uniform lever weighing 85 grams rests on a knife-edge at a point 7·3 cm. from its centre, and carries upon its longer end a weight of 105 grams, distant 23 3 cm. from the support, and a weight of 113 grams 18·4 cm. from the support. What weight must be carried on the shorter end at a point 21 7 cm. from the support in order that the lever shall be in equilibrium? Adelaide University.

20. Prove that if a passenger of weight W advances a distance a along the top of a motor-bus, a weight Wa/b is transferred from the back springs to the front springs, where b is the distance between the axles. L.U.

21. A uniform bar AB, 18 inches long, has a string AC, 7 5 inches long, attached at A, and another string BC, 19 ·5 inches long, attached at B. Both strings are attached to a peg C, and the rod hangs freely. Find graphically the angle which the rod makes with the horizontal.

22. The centre of gravity of a uniform semicircular sheet is at a distance of $4r/3\pi$ from the diametrical edge, r being the radius of the semicircle. Deduce from this information the position of the centre of gravity of a

uniform sheet in the shape of a quadrant of a circle. Explain clearly the method of deduction. L.U.

23. ABC is a horizontal lever pivoted at its middle point B, and carrying a scale-pan of weight W_0 at C; AD is a light bar pivoted at A to the lever and at D, vertically above A, to a horizontal bar FDE, which is freely movable about its end F, which is fixed. The weight of this bar is W_1, and its centre of gravity is at a distance d from F and FD $= c$. Show how to graduate this bar with a movable weight w for varying weights W placed in the scale-pan at C. If inch-graduations correspond to lb. wts. and $w = \frac{1}{4}$ lb., find the value of c. In this case find the relation between W_0 and W_1, when $d = 1$ inch and the zero mark is 1 inch from F. L.U.

24. Explain the meaning of the centre of a system of parallel forces, and show how to find it.

Weights of 1, 3, 4, 10 lb. respectively are placed at the corners of a square. Find the distance of their centre of gravity from each side of the square. Tasmania University.

25. The axial distance between the wheels of a vehicle is 5 feet. The vehicle is loaded symmetrically, and the centre of gravity is at a height of 6 feet above the ground. Find the maximum angle with the vertical to which the vehicle may be tilted sideways without upsetting.

26. To determine the height of the centre of gravity of a locomotive, it is placed on rails, one of which is 5 inches above the other; and it is then found that the vertical forces on the upper and lower rails are respectively 23 and 37 tons. Calculate the height of the centre of gravity if the distance between the rails is 5 feet. Sen. Cam. Loc.

27. Prove that the sensibility of a balance is proportional to the length of the arm of the beam, and inversely proportional to the weight of the beam, and also inversely proportional to the distance between the centre of gravity of the beam and the central knife-edge.

CHAPTER X

COUPLES. SYSTEMS OF UNIPLANAR FORCES

Moment of a couple.—In Fig. 148, P_1 and P_2 are equal parallel forces of opposite sense and therefore form a couple (p. 99). By taking moments successively about points A, B, C and D, it may be shown that **the couple has the same moment about any point in its plane.** Thus :

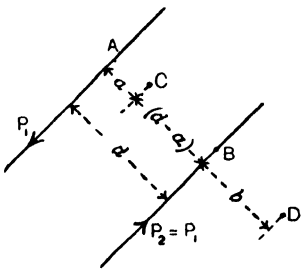


FIG. 148.—A couple has the same moment about any point in its plane.

Taking moments about A :

Moment of the couple

$$= (P_1 \times 0) - (P_2 \times d) = -P_2 d, \quad \ldots\ldots(1)$$

the negative sign indicating an anti-clockwise moment.

Taking moments about B :

Moment of the couple

$$= (P_2 \times 0) - (P_1 \times d) = -P_1 d. \quad \ldots\ldots(2)$$

Taking moments about C :

$$\text{Moment of the couple} = -(P_1 \times a) - P_2(d - a) = -P_2 d. \quad \ldots\ldots(3)$$

Taking moments about D :

$$\text{Moment of the couple} = (P_2 \times b) - P_1(d + b) = -P_1 d. \quad \ldots\ldots(4)$$

As P_1 and P_2 are equal, these four results are identical, thus proving the proposition. The perpendicular distance d between the forces is called the **arm** of the couple.

Equilibrant of a couple.—**A couple may be balanced by another couple of equal and opposite moment applied (*a*) in the same plane, or (*b*) in a parallel plane.**

(*a*) Reference is made to Fig. 149, in which are shown a clockwise couple, having equal forces P_1 and P_2 and an arm a, and an anti-clockwise couple, having equal forces Q_1 and Q_2 and an arm b. Both

couples are applied in the plane of the paper, and it is given that the moments $P_1 a$ and $Q_1 b$ are equal, or

$$Q_1 \cdot P_1 = a : b. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

Produce the lines of the four forces to intersect at A, B, C and D. From A draw AM and AN at right angles to P_1 and Q_1 respectively. Then $AM = a$ and $AN = b$. The triangles AMC and AND are similar, hence

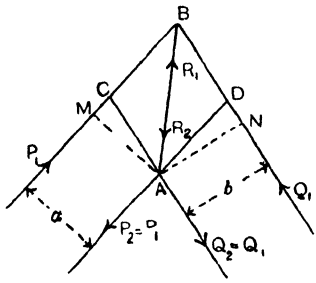


FIG. 149.—Equal opposing couples in the same plane are in equilibrium

$$AC : AD = AM : AN = a : b. \quad \ldots\ldots\ldots\ldots (2)$$

Therefore, from (1) and (2),

$$Q_1 : P_1 = AC \cdot AD. \quad \ldots\ldots\ldots\ldots\ldots (3)$$

Now AC and BD are equal, and AD and BC are also equal, being opposite sides of a parallelogram, hence we may represent P_1 by CB, P_2 by DA, Q_1 by DB, and Q_2 by CA. Let P_1 and Q_1 act at B, then their resultant R_1 is represented by the diagonal AB. Let P_2 and Q_2 act at A, and their resultant R_2 is represented by BA. As R_1 and R_2 are equal, opposite and in the same straight line, they balance, and therefore the given couples balance.

(*b*) In Fig. 150 is shown a rectangular block having equal forces P_1 and P_2 applied to AD and BC respectively, and other equal forces P_1 and P_2 applied to the back edges FG and EH. These forces being all equal, the block is subjected to two equal opposing couples in parallel planes.

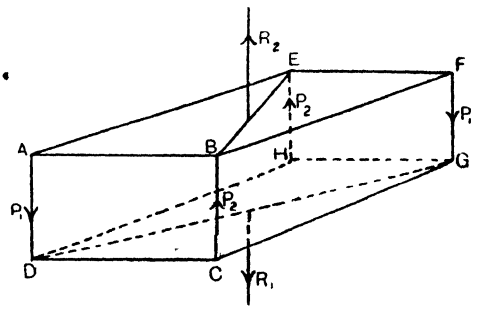


FIG. 150.—Equal opposing couples in parallel planes are in equilibrium.

The resultant R_1 of the forces $P_1 P_1$ will act perpendicularly to the bottom face and will bisect the diagonal DG; similarly, the resultant R_2 of $P_2 P_2$ will bisect the diagonal BE and will be perpendicular to the top face. It is evident that R_1 and R_2 are equal and opposite, and that they act in the same straight line; hence they balance, and therefore the given couples balance.

The effect of a couple is unaltered by shifting it to another position in the same plane or in a parallel plane.—A couple may be balanced by the application in the same plane, or in a parallel plane, of a second couple of equal opposing moment; it follows that if the second couple be reversed its effect on the body will be identical with that of the first couple. Thus the second couple, so reversed, may be substituted for the first couple; in other words, the first couple may be shifted to any new position in the same plane, or in a parallel plane, without changing its effect on the body as a whole.

Further, the second couple need not have its forces equal to those of the first couple, a fact which enables us to state that the forces of a given couple may be altered in magnitude, provided that the arm is altered to correspond, so as to leave the couple of unaltered moment. Thus, in Fig. 150, if the couple acting on the end EFGH has its forces given unequal to those of the couple acting on the end ABCD, equality of the forces may be obtained by making the arms of the couples equal.

The law that to every action there is always an equal and contrary reaction may now be extended by asserting that **to every couple there must be an equal and contrary couple, acting in the same or in a parallel plane.**

Composition of couples in the same plane or in parallel planes.—Any number of couples applied to a body and acting in the same plane, or in parallel planes, may be compounded by the substitution of a single couple having a moment equal to the algebraic sum of the moments of the given couples. This resultant couple may act in the given plane, or in any plane parallel to the given plane, without thereby altering the effect on the body as a whole.

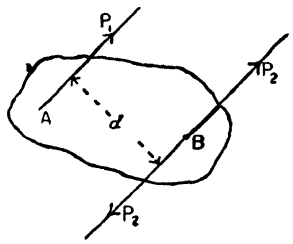


FIG. 151.—Transference of a force to a line parallel to the given line of action.

Substitution of a force and a couple for a given force.—In Fig. 151 is shown a body having a force P_1 applied at A. Suppose that it would be more convenient if P_1 were applied at another point B. To effect this change of position, let two opposing forces P_2, P_2, each equal to P_1, be applied at B in a line parallel to P_1; the forces P_2, P_2 will balance one another and therefore do not affect the given

condition of the body. Let d be the perpendicular distance between the lines of P_1 and P_2. The downward force P_2, together with P_1, forms a couple having a moment $P_1 d$; this couple may be applied at any position in the same plane, leaving a force P_2 acting at B, equal to and having the same sense and direction as the given force P_1. A given force is therefore equivalent to an equal parallel force of the same sense together with a couple having a moment equal to the product of the given force and the perpendicular distance between the lines of the parallel forces.

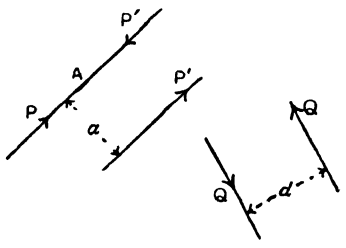


FIG 152.—Reduction of a given force and couple to a single force.

Substitution of a force for a given force and a given couple.—In Fig. 152, a force P is given acting at A, also a couple having forces Q, Q and an arm d. The system may be reduced to a single force by first altering the forces of the couple so that each is equal to P, the moment being kept unaltered by making

$$Qd = Pa,$$

where a is the new arm of the couple. Let P', P' be the new forces of the couple, and apply the couple so that one of its forces acts in the same line as the given force P, and in the sense opposite to that of P. Then P and P' acting at A balance each other, leaving a force P' of the same sense as P, and acting in a line parallel to P and at a perpendicular distance a from the line of P.

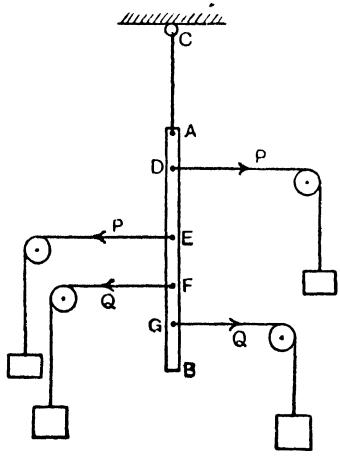


FIG 153.—An experiment on couples

EXPT. 23.—Equilibrium of two equal opposing couples. In Fig. 153 is shown a rod AB hung vertically by a string attached at A and also to a fixed support at C. By means of cords, pulleys and weights, apply two equal, opposite and parallel forces P, P, and also another pair Q, Q; all these forces are horizontal. Adjust the values so that the following equation is satisfied:

$$P \times DE = Q \times FG.$$

Note that the rod remains at rest under the action of these forces.

Repeat the experiment, inclining the parallel forces P, P, at any angle to the horizontal, and inclining the parallel forces Q, Q, to a different angle, but arranging that the moment of the P, P, couple is equal to that of the Q, Q, couple. Note whether the rod is balanced under the action of these couples.

Apply the P, P, couple only, and ascertain by actual trial whether it is possible to balance the rod in its vertical position as shown in Fig. 153 by application of a single push exerted by a finger.

Reduction of any system of uniplanar forces.—In Fig. 154 are given four typical forces P_1, P_2, P_3 and P_4, acting in the plane of the paper at A, B, C and D respectively. Take any two rectangular axes OX and OY in the plane of the paper, and let the direction angles of the forces, α_1, α_2, etc., be stated with reference to OX. Resolve each force into components parallel to OX and OY; thus P_1 will have components $P_1 \cos \alpha_1$ and $P_1 \sin \alpha_1$. Transfer into OX each component which is parallel to OX, and also transfer into OY each component which is parallel to OY. This will introduce a couple for each component so shifted; thus, the couple produced by shifting $P_1 \cos \alpha_1$ will be $(P_1 \cos \alpha_1)y_1$ and that produced by shifting $P_1 \sin \alpha_1$ will be $(P_1 \sin \alpha_1)x_1$. Some of these couples will be clockwise and others anticlockwise; to obtain the resultant moment take the algebraic sum of the set parallel to OX and also the algebraic sum of those parallel to OY, giving:

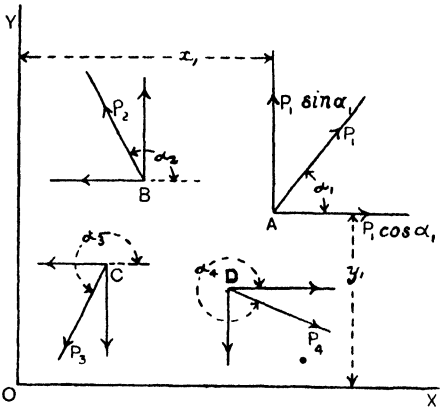


FIG 154.—A system of uniplanar forces.

Resultant moment of couples parallel to OX $= \Sigma(P \cos \alpha)y$.

Resultant moment of couples parallel to OY $= \Sigma(P \sin \alpha)x$.

The reduction of the system so far as we have proceeded is given in Fig. 155, and shows that we now have a number of forces in OX another set in OY, and two couples.

Take the resultant R_x of the forces in OX, also the resultant R_y of the forces in OY, giving:

$$R_x = \Sigma P \cos \alpha. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (1)$$

$$R_y = \Sigma P \sin \alpha. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (2)$$

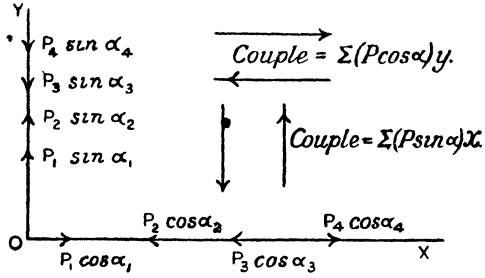


FIG 155.—A system equivalent to that in Fig. 154

The resultant R of these forces will be given by

$$R = \sqrt{R_x^2 + R_y^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (3)$$

The angle α which R makes with OX will be determined from

$$\tan \alpha = \frac{R_y}{R_x}. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (4)$$

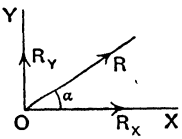


FIG 156

The system is now reduced to a force R and two couples. The resultant moment L of the two couples may be obtained by adding the couples algebraically, thus

$$L = \Sigma(P \cos \alpha)y + \Sigma(P \sin \alpha)x. \quad \ldots\ldots\ldots\ldots\ldots \quad (5)$$

Let each force of this resultant couple be made equal to R, and let the arm be a; then

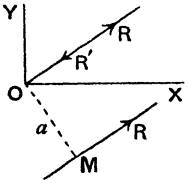


FIG. 157.—Resultant of the system.

$$Ra = L, \quad \text{or} \quad a = \frac{L}{R}. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (6)$$

Apply the couple so that one of its forces R′ is in the same straight line as R acting at O, and opposes R; the other force will be at a perpendicular distance $OM = a$ from O. It is evident that the two forces R, R′ at O balance; hence the given system has been reduced to a force R.

Special cases.—Some special solutions of equations (1), (2) and (5) above may be examined. Suppose the result given by (5) to

be zero; then the system reduces to a force acting at O. Should (2) also give zero, the system reduces to a force acting in OX, or to a force acting in OY if (1) be zero.

Should (5) have a numerical result and both (1) and (2) give zero, then the system reduces to a couple.

For **equilibrium**, there must be neither a resultant force nor a resultant couple: hence all three equations must give zero. The conditions of equilibrium may be written:

$$\Sigma P \cos \alpha = 0. \quad \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

$$\Sigma P \sin \alpha = 0. \quad \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

$$\Sigma(P \cos \alpha)y + \Sigma(P \sin \alpha)x = 0. \quad \ldots\ldots\ldots\ldots (3)$$

These equations must be satisfied simultaneously, and will serve for testing the equilibrium of any system of uniplanar forces.

The student will observe that equations (1) and (2) express the condition that a body in equilibrium does not suffer any displacement in consequence of the application of the force system, as would be the case if either or both the forces R_x, R_y had a magnitude other than zero. Equation (3) expresses the condition that no rotation of the body takes place as a consequence of the application of the forces. It will also be noted that equation (3) may be interpreted as the algebraic sum of the moments of the components of the given forces taken about an arbitrary point O.

The following typical examples should be studied thoroughly. In considering the equilibrium of a body, or of part of a body, care must be taken to show in the sketch those forces only which are applied to the body by agencies external to the body, and not those forces which the body itself exerts on other bodies.

EXAMPLE 1. AB and BC are smooth planes inclined respectively at 45° and 30° to the horizontal (Fig. 158 (*a*)). DE is a uniform rod 3 feet long and weighing 4 lb., and is maintained in a horizontal position by means of a body F, which has a weight of 2 lb. Where must F be placed?

Since the planes are smooth, the reactions, P and Q, of the planes are perpendicular respectively to AB and BC. Resolve each force into horizontal and vertical components. Thus

$$P_x = P \sin 45° = \frac{P}{\sqrt{2}}; \quad P_y = P \cos 45° = \frac{P}{\sqrt{2}}.$$

$$Q_x = Q \sin 30° = \frac{Q}{2}; \quad Q_y = Q \cos 30° = \frac{Q\sqrt{3}}{2}$$

The rod is now acted upon by forces as shown in Fig. 158 (*b*). For equilibrium, we have $P_x - Q_x = 0$; $\therefore P_x = Q_x$ .....(1)

$P_y + Q_y - 4 - 2 = 0$; $\therefore P_y + Q_y = 6$. .....(2)

Taking moments about E:

$(P_y \times 3) - (4 \times 1\frac{1}{2}) - 2x = 0$: $\therefore 3P_y = 6 + 2x$. .....(3)

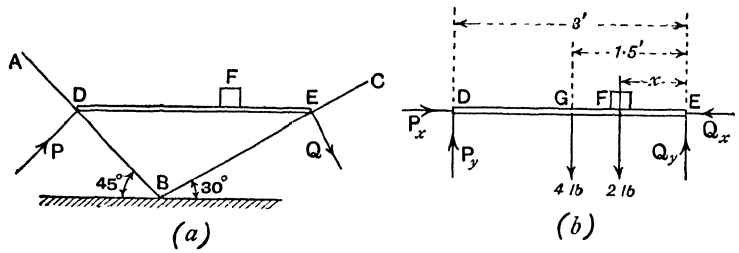


FIG 158.—A rod resting on two inclined planes

From (1), $\frac{P}{\sqrt{2}} = \frac{Q}{2}$. .....(4)

From (2), $\frac{P}{\sqrt{2}} + \frac{Q\sqrt{3}}{2} = 6$. .....(5)

Hence, $\frac{P}{\sqrt{2}} + \frac{P}{\sqrt{2}} \cdot \sqrt{3} = 6$; $\therefore P = \frac{6\sqrt{2}}{1+\sqrt{3}}$;

$\therefore P_y = \frac{P}{\sqrt{2}} = \frac{6}{1+\sqrt{3}}$. .....(6)

Inserting this value in (3), we have

$$\frac{18}{1+\sqrt{3}} = 6 + 2x,$$

whence $x = 0{\cdot}294$ foot.

EXAMPLE 2.—In Fig. 159 (*a*) AB and AC are two uniform bars each weighing 10 lb. and 6 feet long. The bars are smoothly jointed at A, and rest at B and C on a smooth horizontal surface. B and C are connected by an inextensible cord 4 feet long. A load of 40 lb.-weight is attached to D. Find, in terms of BD, the tension in the cord and the reactions communicated across the joint at A.

First consider ABC to be a rigid body, acted upon by vertical forces of 10, 40 and 10 lb.-weight, together with the reactions P and Q. Then

$P + Q - 10 - 40 - 10 = 0$; $\therefore P + Q = 60$. .....(1)

Taking moments about C gives

$$(P \times 4) - (10 \times 3) - (40 \times CE) - (10 \times 1) = 0;$$

$$\therefore\ 4P = 30 + 10 + 40 \cdot CE,$$

$$P = 10 + 10CE. \quad \ldots\ldots(2)$$

From (1), $$Q = 60 - P = 60 - 10 - 10CE$$

$$= 50 - 10CE. \quad \ldots\ldots(3)$$

Also, $$CE = BC - BE = 4 - BE.$$

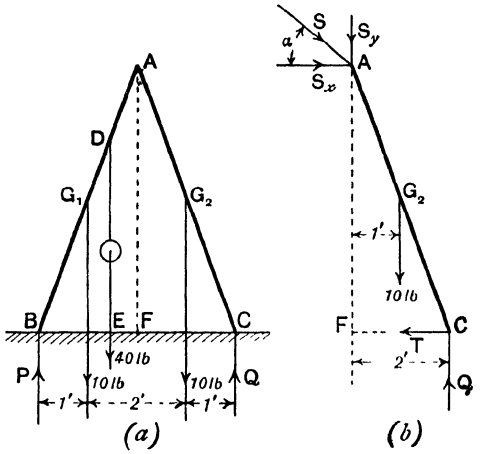


FIG 159 —Equilibrium of two rods.

Draw AF perpendicular to BC, then, in the similar triangles BED, BFA, we have

$$\frac{BE}{BD} = \frac{BF}{BA} = \frac{2}{6};$$

$$\therefore\ BE = \tfrac{1}{3}BD;$$

$$\therefore\ CE = 4 - \tfrac{1}{3}BD.$$

Hence, from (2) and (3), $$P = 10 + 10(4 - \tfrac{1}{3}BD)$$

$$= 50 - \tfrac{10}{3}BD. \quad \ldots\ldots(4)$$

And $$Q = 50 - 10(4 - \tfrac{1}{3}BD)$$

$$= 10 + \tfrac{10}{3}BD. \quad \ldots\ldots(5)$$

Now consider the bar AC separately (Fig. 159 (*b*)). The forces applied to it are its weight, acting vertically through G_2, the vertical reaction Q at C, the horizontal pull T of the cord BC, and a reaction S at A. S is exerted by the other bar AB, and its direction is guessed in Fig. 159 (*b*); the precise direction will be determined in the following calculation. Resolve

S into horizontal and vertical components S_x and S_y, and apply the conditions of equilibrium.

$$T - S_x = 0\ ; \quad \therefore\ T = S_x. \qquad \ldots\ldots (6)$$

$$Q - 10 - S_y = 0\ ; \quad \therefore\ Q = 10 + S_y. \qquad \ldots\ldots (7)$$

Take moments about A, first calculating the length of AF:

$$AF = \sqrt{AC^2 - CF^2} = \sqrt{36 - 4} = \sqrt{32}.$$

$$(Q \times 2) - (10 \times 1) - (T \times \sqrt{32}) = 0\ ;$$

$$\therefore\ 2Q = 10 + T\sqrt{32}. \qquad \ldots\ldots (8)$$

From (5) and (8), $\quad 2(10 + \frac{10}{3}BD) = 10 + T\sqrt{32}\ ;$

$$\therefore\ T = \frac{10 + \frac{20}{3}BD}{\sqrt{32}},$$

$$= \frac{30 + 20BD}{3\sqrt{32}}. \qquad \ldots\ldots (9)$$

From this result it will be seen that T increases if BD is made greater.

Again, from (6) and (9): $\quad S_x = T = \dfrac{30 + 20BD}{3\sqrt{32}}. \qquad \ldots\ldots (10)$

And from (5) and (7):

$$10 + \tfrac{10}{3}BD = 10 + S_y\ ;$$

$$\therefore\ S_y = \tfrac{10}{3}BD. \qquad \ldots\ldots (11)$$

(It will be observed from the positive sign of this result that the assumed direction of S has been chosen correctly.)

From (10) and (11):

$$S = \sqrt{S_x^2 + S_y^2}$$

$$= \sqrt{\frac{3600BD^2 + 1200BD + 900}{288}}.$$

Also, the angle α which S makes with the horizontal is given by

$$\tan\alpha = \frac{S_y}{S_x} = \frac{\frac{10}{3}BD}{\frac{30 + 20BD}{3\sqrt{32}}} = \frac{10 . BD\sqrt{32}}{30 + 20BD}$$

EXAMPLE 3.—In Fig. 160, AB is a light rod having the end A guided so as to be capable of moving freely in a horizontal line AD. At C another light bar CD is smoothly jointed to AB; CD can turn freely about D. A load W is hung from B. If AC = CD and BC = n . AC, find the horizontal and vertical reactions which must be applied at A in order to maintain the arrangement with AB at an angle θ to the horizontal.

Consider the equilibrium of the rod AB. Let Q be the reaction which DC applies to C. Take horizontal and vertical components of Q and let these be Q_x and Q_y (Fig. 161). Since Q_x and AD are parallel, the angle

between Q and Q_x is equal to the angle ADC ; also the angles ADC and CAD are equal, since AC = CD. Hence the angle between Q_x and $Q = \theta$. Therefore $Q_x = Q \cos \theta$, and $Q_y = Q \sin \theta$.

For equilibrium, $P - Q_x = 0$; $\therefore P = Q_x$ .....(1)

$Q_y - W - S = 0$; $\therefore Q_y = W + S$ .....(2)

Taking moments about C :

$P \times CE) + (S \times AE) - (W \times CF) = 0$;

$\therefore (P \times CE) + (S \times AE) = W \times CF$ .....(3)

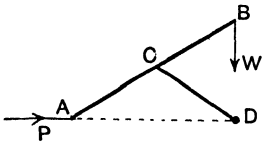


FIG. 160.

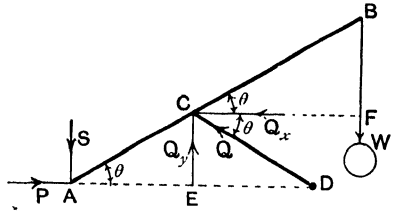


FIG. 161.—Forces acting on AB.

From (3): $P \cdot AC \sin \theta + S \cdot AC \cos \theta = W \cdot BC \cos \theta$.

Dividing this throughout by AC cos θ, we obtain

$$P \tan \theta + S = W \frac{BC}{AC} = \frac{Wn \cdot AC}{AC} = Wn. \quad \text{.....(4)}$$

From (2), $Q \sin \theta = W + S$.

From (1), $P = Q \cos \theta$, or $Q = \frac{P}{\cos \theta}$;

$$\therefore \frac{P \sin \theta}{\cos \theta} = P \tan \theta = W + S. \quad \text{.....(5)}$$

Substituting in (4), we have

$W + S + S = Wn$;

$\therefore 2S = Wn - W$,

$$S = W\left(\frac{n-1}{2}\right). \quad \text{.....(6)}$$

Hence, from (5) and (6),

$$P \tan \theta = W + W\left(\frac{n-1}{2}\right),$$

$$P = \frac{W\left\{1 + \frac{n-1}{2}\right\}}{\tan \theta}$$

$$= \left(\frac{n+1}{2 \tan \theta}\right) W. \quad \text{.....(7)}$$

It will be noted from (6) that if $n = 1$, *i.e.* AC, CD and CB are all equal (Fig. 160), then S = 0. Inspection of Fig. 161 shows that under these

conditions the line of W passes through D ; hence P, Q and W intersect in a point and can equilibrate the rod AB without the necessity for the application of a force S. If n be less than 1, the result given for S by (6) is negative, and indicates that S must act upwards.

The graphical solution of this problem depends upon the fact that the rod AB (Fig. 161) is acted upon by three forces, viz. W, Q and the resultant of P and S. The point at which these forces intersect may be found by producing W and Q ; the resultant of P and S then passes through this point, and also through A. The solution is then obtained by application of the triangle of forces, and will be found to be an interesting problem.

EXERCISES ON CHAPTER X.

1. A rectangular plate, 6 inches by 2 inches, has a force of 400 lb. weight applied along a long edge. Show how to balance the plate by means of forces acting along each of the other edges. Neglect the weight of the plate.

2. A door weighs 120 lb. and has its centre of gravity in a vertical line parallel to and at a distance of 18 inches from the axis of the hinges. The hinges are 4 feet apart and share the vertical reaction required to balance the door equally between them. Find the reaction of each hinge.

3. A vertical column has a bracket fixed to its side near the top. A load of 5 tons weight hangs from the end of the bracket at a point 8 inches from the axis of the column. Remove this load and apply an equivalent system of forces consisting partly of a force of 5 tons weight acting in the axis of the column. Show the system in a sketch.

4. Sketch a right-angled triangle in which AB = 16 feet and is vertical, and BC = 10 feet and is horizontal. The triangle represents the cross section of a wall 10 feet long and weighing 140 lb. per cubic foot. Find the reaction of the foundation of the wall, expressed as a force acting at the centre of the base together with a couple.

5. A rod AB, 4 feet long, has a pull of 20 lb. weight applied at A in a direction making 30° with AB. There is also a couple having a moment of 40 lb.-feet acting on the rod. Find the resultant force.

6. Draw any triangle ABC. Forces act in order round the sides of the triangle, and each force has a magnitude proportional to the length of the side along which it acts. Reduce the system of forces to its simplest form.

7. A square plate ABCD of 2 feet edge has forces acting along the edges as follows : From A to B, 2 lb. weight ; from B to C, 3 lb. weight ; from C to D, 4 lb. weight ; from D to A, 5 lb. weight. Find the resultant.

8. A uniform rod AB is 4 feet long and weighs 24 lb. The end A is smoothly jointed to a fixed support ; the rod is inclined at 45° and its upper end B rests against a smooth vertical wall. A load of 10 lb. weight is hung from a point in the rod 1 foot from A. Find the reactions at A and B.

9. In an isosceles triangle AC = CB ; AB is 15 feet and is horizontal ; C is 5 feet vertically above AB. The plane of the triangle is vertical, and the triangle is supported at A and B. A load of 400 lb. weight is applied at the centre of AC, another of 600 lb. weight at C, and a force of 800 lb. weight acts at the centre of BC at 90° to BC. The reaction of the support at B is vertical ; that at A is inclined. Find the reactions of both supports.

10. In the arrangement shown in Fig. 160 (p. 135), AC = CD = 4 inches, BC = 6 inches. Find P, S and Q when a load of 10 lb. weight is hung from B for values of θ of 45°, 30°, 15°, 5°.

11. Answer Question 10, (*a*) if BC = 4 inches ; (*b*) if BC = 3 inches.

12. BC is a rod 12 inches long and capable of turning in the plane of the paper about a smooth pin at C. Another rod AB, 4 feet long, is jointed smoothly to BC at B ; the end A can travel in a smooth groove, which, when produced, passes through C. The angle ACB is 30°, and a load of 200 lb. weight is hung from the centre of AB. Calculate the resultant force which must be applied at A in order to preserve the equilibrium. Check the result by solving the problem graphically.

13. A ladder, 20 feet long, is inclined at 60° to the horizontal, and rests on the ground at A and against a wall at B. The ladder weighs 80 lb., and its centre of gravity is 8 feet from A. Assuming both ground and wall to be smooth, the reactions at A and B will be vertical and horizontal respectively. The ladder is prevented from slipping by means of a rope attached at A and to a point at the foot of the wall. A man weighing 150 lb. ascends the ladder. Calculate the pull in the rope when he is 4, 8, 12, 16 and 19 feet from A. Plot a graph showing the relation of the pull and his distance from A.

14. Show how to find the resultant of two unequal parallel forces acting at different points but in opposite directions upon a rigid body. Is there a single resultant if the two forces are equal ?

A steel cylindrical bar weighing 1000 lb. is held in a vertical position by means of two thin fixed horizontal planks 5 ft. apart vertically, in which are holes through which the bar can slide. If the sides of these holes are smooth and the bar is lifted by a vertical force applied 2 inches from its axis, find the pressure on each plank. Adelaide University.

15. If a number of uniplanar forces act upon a rigid body, prove that they will be in equilibrium, provided that the algebraic sum of their resolved parts in two directions at right angles and of their moments about one given point in the plane be zero.

ABCD is a square lamina. A force of 2 lb. weight acts along AB, 4 lb. weight along BC, 6 lb. weight along CD, 8 lb. weight along DA, $2\sqrt{2}$ lb. weight along CA, and $4\sqrt{2}$ lb. weight through the point D parallel to AC. Find the resultant of the system of forces.

16. A uniform plank, 12 feet long, weighing 40 lb., hangs horizontally, and is supported by two ropes sloping outwards ; the ropes make angles of 60° and 45° respectively with the horizontal. If the plank carries a weight of 100 lb., find where this weight must be placed.

17. Four equal uniform rods, each of weight W, are jointed so as to form a square ABCD. The arrangement is hung from a cord attached at A,

and the corners B and D are connected by a light rod so that the square retains its shape. Show that the thrust along the rod BD is 2W, and find the reaction at the bottom hinge.

18. Define the expression "moment of a force about a point." Show from your definition that the sum of the moments of two equal and parallel forces acting in opposite directions is the same about every point in the plane in which they act. L.U.

19. Each half of a step-ladder is $5\frac{1}{2}$ feet long, and the two parts are connected by a cord 28 in. long, attached to points in them distant 16 in. from their free extremities. The half with the steps weighs 16 lb., and the other half weighs 4 lb. Find the tension in the cord when a man weighing 11 stone is standing on the ladder, $1\frac{1}{2}$ ft. from the top, it being assumed that the cord is fully stretched, and that the reactions between the ladder and the ground are vertical. L.U.

20. A bar AB of weight W and length $2a$ is connected by a smooth hinge at A with a vertical wall, to a point C of which, vertically above A, and such that AC = AB, B is connected by an inextensible string of length $2l$. Find in terms of these quantities the tension of the string and the action at the hinge.

If the bar is 6 ft. long and weighs 10 lb., and the string be 2 ft. long, show that its tension is $1\frac{1}{4}$ lb. wt. And if the joint at A be slightly stiff, so that in addition to the supporting force it exerts a couple G reducing the tension in the string to 1 lb. wt., then G will be approximately 3·94 lb.-ft. units. L.U.

21. ABC and ACD are two triangles in which AB = AC = AD, and the angles BAC and CAD are 60° and 30° respectively. With AD vertical, the figure represents a wall bracket of five light rods. The bracket is fixed at A, and kept just away from the wall by a small peg at D, and a mass of 5 pounds is suspended from B. Find, preferably by analytical methods, the pressure on the peg at D and the forces in the rods DC, CA and AB, stating whether they are in compression or tension. L.U.

22. Two ladders, AB and AC, each of length $2a$, are hinged at A and stand on a smooth horizontal plane. They are prevented from slipping by means of a rope of length a connecting their middle points. If the weights of the ladders are 40 and 10 lb., find the tension in the rope, and the horizontal and vertical components of the action at the hinge. L.U.

23. A rod AB can turn freely about A, and is smoothly jointed at B to a second rod BC, whose other end C is constrained to remain in a smooth groove passing through A. A force F is applied to BC along CA. Prove that the couple produced on AB is $F \times AK$, where K is the point in which BC produced cuts the perpendicular at A to AC. L.U.

24. Define a couple. What is the characteristic property of couples? Prove that a couple and a force acting in the same plane are equivalent to a single force. Three forces act along the sides of a triangular lamina and are proportional to the sides along which they act; find the magnitude of their resultant. Bombay Univ.

25 Six equal light rods are jointed together at their ends so as to form a regular hexagon ABCDEF. C and F are connected together by another light rod. The arrangement is hung by two vertical cords attached to A and B respectively, so that AB, CF and DE are horizontal. Equal weights of 20 lb each are hung from D and E. Find the forces in each rod, and state whether they are pulls or pushes.

26. Prove that the moment of two non-parallel coplanar forces about any point in the plane is equal to the moment of their resultant. Prove also that, if forces act along the sides of a plane polygon taken in order, each force being represented in magnitude and direction by the side along which it acts, they are equivalent to a torque (*i.e.* turning moment) represented by twice the area of the polygon. Madras Univ.

CHAPTER XI

GRAPHICAL METHODS OF SOLUTION OF PROBLEMS IN UNIPLANAR FORCES

Link polygon.—The following graphical method of determining the equilibrant of a system of uniplanar forces is of great practical importance. In Fig. 162 (*a*) P_1, P_2 and P_3 are any three forces acting in the plane of the paper and not meeting in a point; it is required to find their equilibrant.

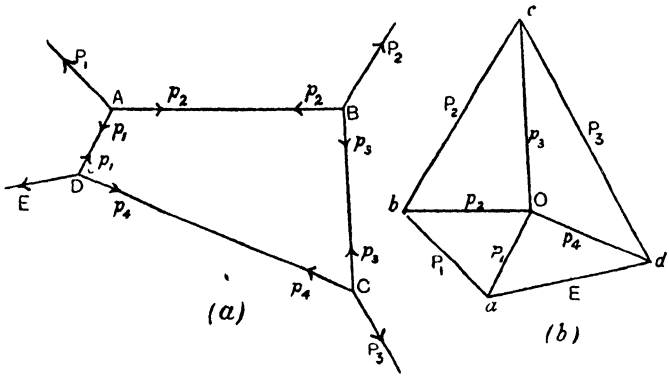


FIG. 162.—Graphical solution by the link polygon.

P_1 may be balanced by application, at any point A on its line of action, of two forces p_1 and p_2 in the plane of the paper and not in the same straight line. P_1, p_1 and p_2 must comply with the usual conditions of the triangle of forces; thus in Fig. 162 (*b*) *ab* represents P_1, *bO* and *Oa* represent p_2 and p_1 respectively. Some means must be supplied for enabling p_1 and p_2 to be applied, and it is convenient to use rods, or **links**, one of which, AB, is used for applying p_2 at A_1 and is extended to a point B on the line of action of P_2, where it applies an equal and opposite force p_2. The link AB is thus equilibrated.

Again, P_2 may be balanced by application at B of a third force p_3; P_2, p_3 and p_2 are represented respectively by the sides *bc*, *c*O and O*b* of the triangle of forces *bc*O (Fig. 162 (*b*)). Extend the line of p_3 to cut P_3 in C, and let BC be a link which is equilibrated by the forces p_3, p_3 acting at B and C. In the same manner, balance P_3 by application of the force p_4 at C; the triangle of forces for P_3, p_4 and p_3 acting at C will be *cd*O in Fig. 162 (*b*). Produce the lines of p_4 and p_1 to intersect at D, and let the links CD and AD be equilibrated by permitting them to apply forces p_4 and p_1 to D; this can only be effected provided a third force E is applied at D, the three forces acting at D being represented by the triangle of forces *da*O in Fig. 162 (*b*).

Each of the forces P_1, P_2, P_3 and E is now balanced by the forces in the links, *i.e.* the forces P_1, P_2, P_3 and E together with the forces in the links constitute an equilibrated system. Further, each link is balanced separately; hence the link system may be disregarded, and we may state that the forces P_1, P_2, P_3 and E are in equilibrium, or that E is the equilibrant of P_1, P_2 and P_3.

Inspection of Figs. 162 (*a*) and (*b*) leads to the following statement of the conditions of equilibrium: A system of uniplanar forces will be in equilibrium provided

(i) **a closed polygon of forces** *abcd* (Fig. 162 (*b*)) **may be drawn having sides which, taken in order, represent the given forces;**

(ii) **a closed link polygon,** ABCD (Fig. 162 (*a*)), **may be drawn having its sides parallel respectively to lines drawn from a common point** O **to the corners of the force polygon** (Fig. 162 (*b*)).

It will be noted that the position of the point, or **pole**, O in Fig. 162 (*b*) depends solely on the directions chosen for the first two forces p_1 and p_2. As there was liberty of choice in this respect, it follows that O may occupy any position. Further, it will be noted that any link, such as BC (Fig. 162 (*a*)), connecting the forces P_2 and P_3, is drawn parallel to O*c* (Fig. 162 (*b*)), and that O*c* falls between *bc* and *cd*, lines which represent the same pair of forces P_2 and P_3.

Should all the given forces meet at a point, the link polygon may be reduced to a point situated where the forces meet, and the polygon of forces alone suffices for the solution.

Resultant of a number of parallel forces; graphical solution.—In Fig. 163 (*a*) P, Q, S and T are given parallel forces acting on a body, and their resultant is to be determined.

First draw the polygon of forces ABCDEA (Fig. 163 (*b*)), which in this case is a straight line, as the given forces are parallel. P, Q, S

and T are represented by AB, BC, CD and DE respectively ; the closing line of the polygon is EA, which therefore represents the equilibrant. Hence the resultant R of the given forces is represented by AE, *i e.* the equilibrant reversed. Choose any pole O and join OA, OB, etc ; draw the links *ab*, *bc*, *cd* (Fig. 163 (*a*)) parallel respectively to OB, OC and OD in Fig. 163 (*b*). Draw p_1 and p_5 (Fig. 163 (*a*)) parallel respectively to OA and OE. In the triangle AEO, AE represents the

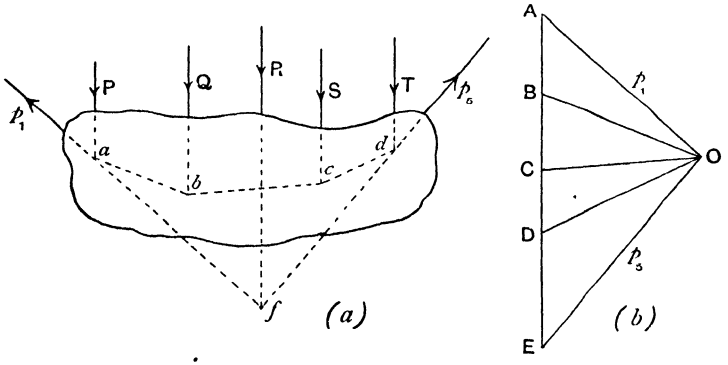


FIG 163 —Resultant of a system of parallel forces

resultant R, therefore EO and OA represent a pair of forces which would equilibrate R if applied to the body. Hence p_1 and p_5 in Fig 163 (*a*) will intersect, if produced, in a point on the line of R. This point is *f*, and R acts through *f* in a line parallel to the given forces

In practice it is customary to employ Bow's notation in using the link polygon The methods will be understood by study of the following examples.

EXAMPLE 1.—Given forces of 3, 4 and 2 tons weight respectively, find their equilibrant (Fig. 164).

The principles on which the solution depends are (*a*) the force polygon must close, (*b*) the link polygon must close. Name the spaces A, B and C, and place D provisionally near to the 2 tons force. Draw the force polygon ABCD (Fig. 164 (*b*)). The closing line DA gives the direction, sense and magnitude of the equilibrant E. To find the proper position of E, choose any pole O, and join O to the corners A, B, C and D of the polygon of forces. Construct a link polygon by choosing any point *a* on the line of the 3 tons force (Fig. 164 (*a*)), drawing in space B a line *ab* parallel to OB, in space C a line *bc* parallel to OC ; to obtain the closing sides of the link polygon, draw *ad* parallel to OA and *cd* parallel to OD, these lines intersecting in *d*.

The equilibrant E passes through *d*, and may now be shown completely in Fig. 164 (*a*).

Had the resultant of the given forces been required, proceed in the same manner to find the equilibrant, and then reverse its sense in order to obtain the resultant.

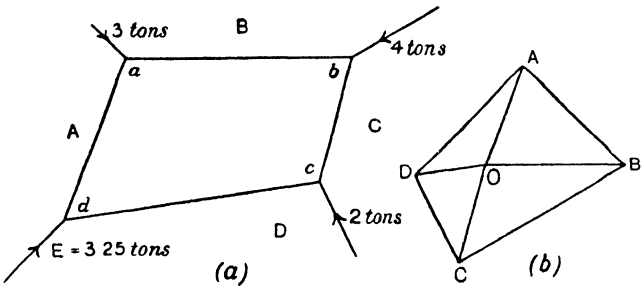


FIG. 164.—An application of the link polygon.

EXAMPLE 2.—Given a beam carrying loads as shown in Fig. 165 (*a*); find the reactions of the supports, both reactions being vertical.

In this case, as all the forces acting on the beam are parallel, the force polygon is a straight line. Begin in space B, and draw BC, CD, DE, EF and FG (Fig. 165 (*b*)) to represent the given loads. Choose any pole O,

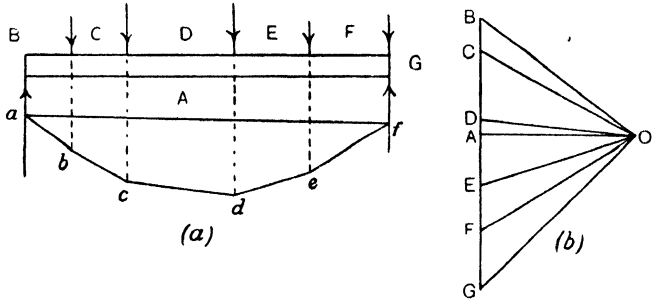


FIG 165 —Reactions of a beam by the link polygon method

and join O to B, C, D, E, F and G. Choose any point *a* on the line of the left-hand reaction, and draw in space B a line *ab* parallel to OB. Continue the construction of the link polygon by drawing in spaces C, D, E and F lines *bc*, *cd*, *de* and *ef* parallel respectively to OC, OD, OE, OF. From *f*, a point on the force FG, a side of the link polygon has to be drawn to intersect the line of the force GA; as these forces are in the same straight line, this side of the link polygon is of zero length, consequently the link

polygon has a side missing. Complete the polygon by joining *fa*, and draw OA (Fig. 165 (*b*)) parallel to *fa*. The magnitudes of the reactions may now be scaled as AB and GA.

Rigid frames.—In Fig. 166 (*a*) is shown the outline of a thin plate to which forces P_1, P_2, P_3, etc., are applied at points A, B, C, etc., respectively. The forces are all in the plane of the plate and are given in equilibrium. It will be noted that the equilibrium of the forces is independent of the shape of the plate; hence any shape may be chosen for the outline and the forces will remain in equilibrium provided no alteration is made in the given magnitudes, lines of direction, and senses of the forces. We may proceed further

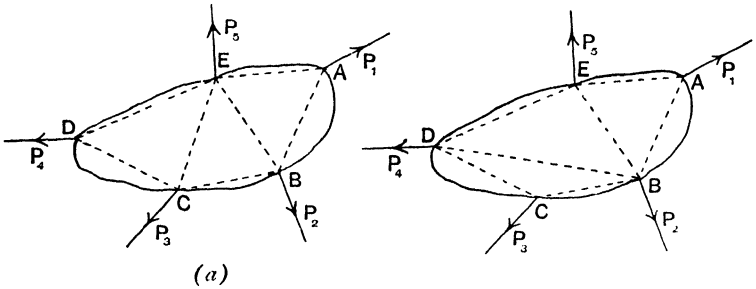


FIG. 166.—Substitution of a rigid frame for a body.

by removing the plate and substituting an arrangement of bars (shown dotted in Fig. 166 (*a*)) connected together at A, B, C, etc., by means of hinges or pins; the result will be the same, viz the given forces balance and the frame formed by the bars will be in equilibrium.

Care must be taken in devising the arrangement of bars that no motion of any bar relative to any other bar may take place. In Fig. 166 (*a*) relative motion is prevented by means of the diagonal bars EB and EC. An alternative arrangement is shown in Fig. 166 (*b*).

From these considerations it may be asserted that, if a given equilibrated system of uniplanar forces acts on a rigid frame, the equilibrium is independent of the shape of the frame or the arrangement of its parts. A **rigid frame** may be defined as an arrangement of bars jointed together and so constructed that no relative motion of the parts can take place.

Conditions of equilibrium in rigid frames.—Two points present themselves for consideration.

(*a*) The system of forces applied to the frame (called generally the **external forces**) must be in equilibrium, and must comply with the conditions already explained; *i.e.* treated analytically, the three equations of equilibrium (p. 131) must be satisfied: or, treated graphically, both the force polygon and the link polygon must close.

(*b*) Any joint in the frame is in equilibrium under the action of any external force or forces applied at the joint, together with the forces of push or pull acting along the bars meeting at the joint. As all these forces pass through the joint, the condition of equilibrium of the joint is that the force polygon for the forces acting at the joint must close. The student will observe that this fact enables the magnitude and kind of force acting in any bar of the frame to be determined.

It may be verified easily for any system of unplanar concurrent forces that it is not possible to construct a force polygon if there be more than two unknown quantities. These may be either two magnitudes, or two directions, or one magnitude and one direction. It is thus necessary to examine the number of unknowns at any joint in a given frame before attempting a solution of the forces acting at that joint.

EXAMPLE.—In Fig. 167 (*a*) is given a frame having its joints numbered 1, 2, 3, 4, 5. External forces act at each of these joints; those acting at 1, 2 and 3 are given completely. Determine the forces acting at 4 and 5 in order that the frame may be in equilibrium. Determine also the force in each bar, and indicate whether it is push or pull.

Using Bow's notation, the letters A, B, C, D and E enable the external forces to be named. The letters F, G and H placed as shown in Fig. 167 (*a*) enable the force in any bar of the frame to be named; thus the force in the bar 12 is BG, or GB.

Assuming clockwise rotation throughout, draw as much as possible of the force polygon for the external forces. Thus AB (Fig. 167 (*b*)) represents the external force acting at joint 1, BC that at joint 2, and CD that at joint 3. The forces acting at joints 4 and 5 cannot be shown meanwhile, but since the force represented by the closing side of the polygon ABCDA, viz. DA (Fig. 167 (*b*)), would equilibrate the forces acting at joints 1, 2 and 3, it follows that DA represents the resultant of the forces acting at 4 and 5, since this resultant would also equilibrate the forces acting at 1, 2 and 3.

Select any suitable pole O (Fig. 167 (*b*)), and join O to A, B, C and D. Start drawing the link polygon in Fig. 167 (*a*) by selecting a point *a* on the line of the force AB, and drawing *ab* and *bc* parallel to OB and OC respectively in Fig. 167 (*b*). From *a* draw *af* parallel to OA, and from *c* draw *cf* parallel to OD ; these lines intersect at *f*, and the resultant force represented by DA must pass through *f*. Now this force is the resultant of the forces acting at 4 and 5, and the resultant and the components must intersect in the same point, therefore the lines of direction of the forces acting at 4 and 5 will be found by joining *f*4 and *f*5.

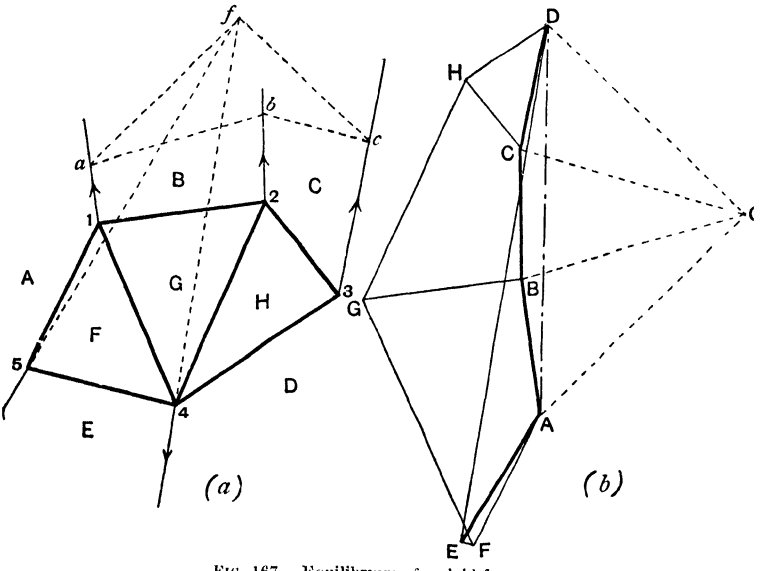


FIG 167.—Equilibrium of a rigid frame

The force polygon in Fig. 167 (*b*) may be closed now by drawing DE parallel to the line *f*4, and AE parallel to the line *f*5. DE and EA give completely the forces acting at joints 4 and 5 respectively.

The forces in the bars of the frame may now be obtained by considering each joint separately. Taking joint 3 ; there are three forces acting, and the triangle of forces is constructed by using CD (Fig 167 (*b*)) for one side, and drawing DH and CH parallel respectively to bars 34 and 23. To determine the kind of force, go round joint 3 clockwise, and find the sense of each force from the triangle of forces. The force represented by DH (Fig. 167 (*b*)) acts away from joint 3 in Fig. 167 (*a*); hence the bar 34 is under pull. The force represented by HC in Fig. 167 (*b*) acts towards joint 3 ; hence the bar 23 is under push.

At joint 4 there are five forces, and two only are known in magnitude; hence three magnitudes have to be determined, and this joint cannot be solved meanwhile. At joint 2 there are four forces, two of which are known completely, and the remaining two are known in direction. Hence this joint may be solved. The polygon of forces for joint 2 is shown in Fig. 167 (*b*) as BCHGB, and determines the forces in bars 12 and 24. Determining whether these forces are pull or push in the same manner as above, we find that 12 is under push and 24 is under pull.

Joint 1 is solved in the same manner as joint 2. The polygon of forces is ABGFA (Fig. 167 (*b*)). Inspection shows that bar 14 is under pull and bar 15 is under push.

Joint 5 is solved by the triangle of forces AFE (Fig. 167 (*b*)). The sides EA and AF already appear in the drawing, and the closing side FE must be parallel to bar 45; this fact provides a check on the accuracy of the entire drawing. Inspection shows that bar 15 is under push and that bar 45 is also under push.

There is no need to consider joint 4 specially, as it will be noted that all the forces acting there have been determined in the course of the above solutions.

Roof truss.—The frame shown in Fig. 168 (*a*) is suitable for supporting a roof, and is called a **roof truss**. Vertical loads are applied at the joints 1, 2, 4, 5 and 7, and the truss rests on supports at 1 and 7, the reactions of the supports being vertical. The external forces applied to this frame are all vertical, and the reactions may be found by applying the methods of calculation given in Chapter X., or by application of the link polygon in the same manner as for the beam on p. 143.

TABLE OF FORCES.

Name of part	Force in lb. weight		Name of part	Force in lb. weight	
	Push.	Pull.		Push.	Pull.
Reaction at 1	2000	—	13	—	4225
Reaction at 7	2000	—	36	—	2480
12	4675	—	67	—	4225
24	4250	—	23	900	—
45	4250	—	34	—	1960
57	4675	—	46	—	1960
			56	900	—

Having determined the reactions of the supports, the polygon of forces for the external forces may be completed, and is ABCDEFGA

in Fig. 168 (*b*). The joints are then solved separately in the order 1, 2, 3, 4, 5. The closing line AN in Fig 168 (*b*) should be parallel to the bar 67 in Fig. 168 (*a*), and this gives a check on the accuracy of the work. The forces in the various bars are scaled from Fig. 168 (*b*) and are shown in the table on p. 147

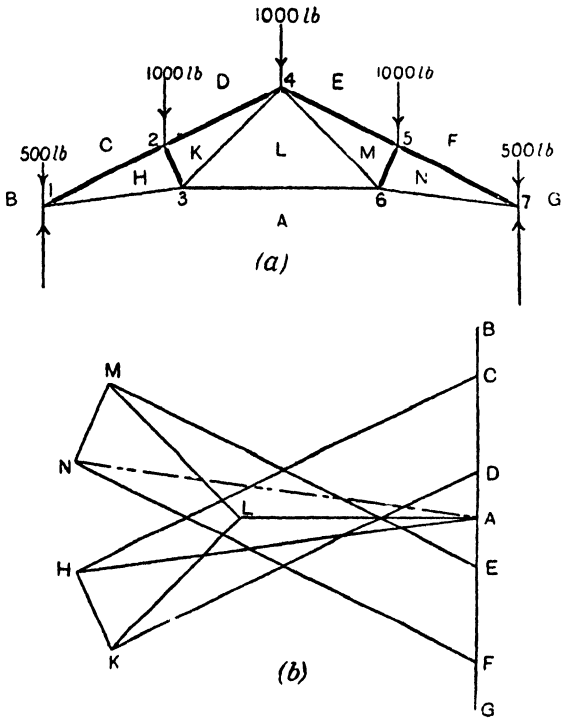


FIG 168.—Forces in a common type of roof truss

The bars which are under push have thick lines in Fig. 168 (*a*); the thin lines indicate bars under pull.

EXPT. 24.—**Link polygon.** Fig. 169 (*a*) shows a polygon ABCDEA made of light cord and having forces P, Q, S, T and V applied as shown. Let the arrangement come to rest. Show by actual drawing (*a*) that the force polygon *abcdea* closes (Fig. 169 (*b*)), its sides being drawn parallel and proportional to Q, S, T, V and P respectively; (*b*) that lines drawn from *a*, *b*, *c*, *d* and *e* parallel respectively to AB, BC, CD, DE and EA intersect in a common pole O.

EXPT. 25.—**Loaded cord.** A light cord has small rings at A, B, C and D, and may be passed over pulleys E and F attached to a wall board (Fig. 170 (*a*)). Loads W_1, W_2, W_3 and W_4 may be attached to the rings, and

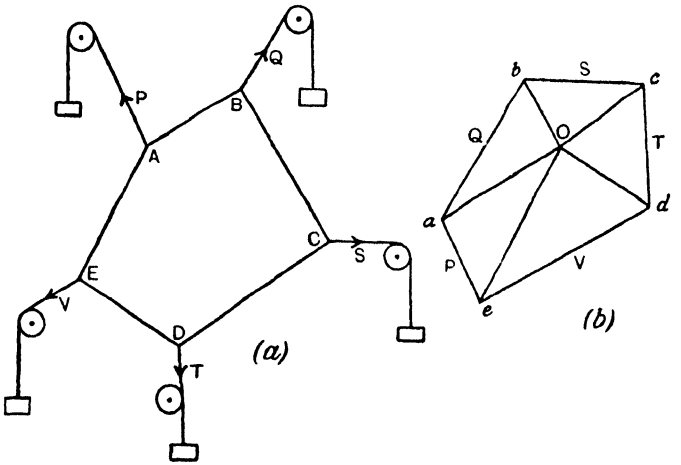


FIG. 169.— An experimental link polygon.

P and Q to the ends of the cord. Choose any values for W_1, W_2, W_3 and W_4, and draw the force polygon for them as shown at *abcde*. Choose any suitable pole O, and join O to *a*, *b*, *c*, *d* and *e*. O*a* and O*e* will give the

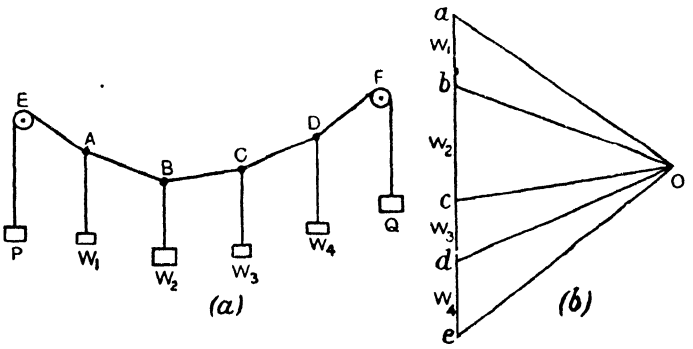


FIG. 170.—A hanging cord.

magnitudes of P and Q respectively. Fix the ring at A to the board by means of a bradawl or pin ; fix the pulley at E so that the direction of the cord AE is parallel to O*a* ; fix the ring at B by means of a pin so that the direction of the cord AB is parallel to O*b*. Fix also the other rings C and

D, and the pulley at F so that the directions of BC, CD and DF are parallel to cO, dO and eO respectively. Apply the selected weights W_1, W_2, W_3 and W_4, and also weights P and Q of magnitude given by Oa and Oe. Remove the bradawls and ascertain if the cord remains in equilibrium.

It will be noted that the shape of the cord and the values of P and Q depend upon the position of the pole O; hence a large number of solutions is possible

EXERCISES ON CHAPTER XI.

These exercises are intended to be solved graphically

1. Four forces act on a rod as shown in Fig 171. AB = BC = CD = 1 foot. The magnitudes in lb. weight are as follows: P = 4, Q = 6; S = 5; T = 7. The direction angles are:

PAB = 110°, QBC = 60°;
SCD = 45°, TDC = 120°.

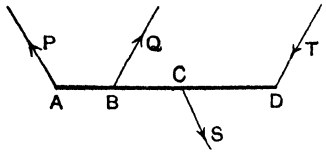


FIG. 171.

Find the equilibrant and hence the resultant of the system of forces.

2. Vertical downward forces as follows act on a body: P = 400 lb. weight; Q = 200 lb. weight, S = 600 lb. weight, T = 300 lb. weight. Horizontal distances between P and Q, 2 feet; between Q and S, 4 feet; between S and T, 3 feet. Find the resultant of the system.

3. A beam AB, 24 ft. long, is supported at its ends, and carries vertical loads of 1 5, 2, 3 and 4 5 tons weight at distances of 3, 6, 12 and 18 feet from A. Find the reactions of the supports.

4. The frame shown in Fig. 172 is made of rigid bars smoothly jointed together, and is symmetrical about the vertical line AY. AB = AC = 6 feet; BC = 9 feet; BE = CD = 4 feet; DE = 11 feet. A vertical force of 600 lb. weight is applied at A; a force of 400 lb. weight at B, making an angle of 75° with BA; a force of 800 lb. weight at C, making an angle of 90° with CD. The frame is supported by forces applied at D and E, that at D being vertical. Find the forces in all the bars of the frame, indicating push and pull, and also find the forces required at D and E in order to equilibrate the frame.

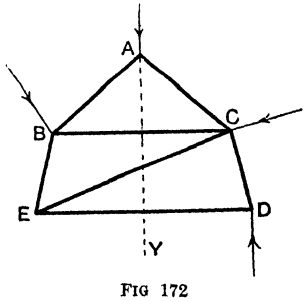


FIG 172

5. Six equal loads are hung from a cord. The ends of the cord are attached to two pegs A and B at the same level and 7 feet apart. The horizontal line AB is divided into seven equal parts by producing upwards the lines of action of the loads. The lowest part of the cord is 3 feet below AB. Make a drawing showing the cord, and find the tension in each part of it if each load weighs 2 lb.

6. Find the forces in all the bars of the frame shown in Fig. 173, indicating whether each bar is under push or pull.

7. Draw full size a rectangle ABCD having sides AB = 4 inches and AD = 3 inches. Forces of 4, 5, 8 and 3 lb. wt. act along the sides AB, CB, CD and AD respectively; a force of 7 lb. wt. acts along the diagonal AC; the senses of the forces are indicated by the order of the letters. Find the resultant.

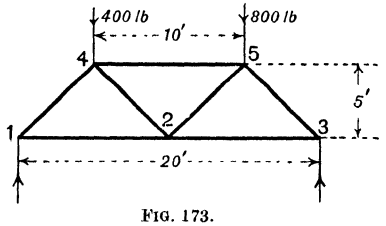


FIG. 173.

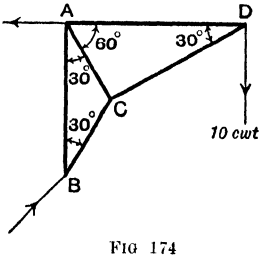


FIG 174

8. The sketch given in Fig. 174 represents a crane formed of rods smoothly jointed at A, B, C, D; the crane is kept in position by reactions at A, B, of which the former is horizontal. A load of 10 cwt. is hung from D; find, by a graphical construction or otherwise, the stresses in the rods, and determine the reactions at A and B. Sen. Cam. Loc.

9. Show how to find graphically, by means of a force polygon and a funicular (or link) polygon, the resultant of a number of forces whose lines of action lie in one plane.

Draw four parallel lines A, B, C, D, the successive distances between them being $1\frac{1}{2}$, $2\frac{1}{2}$, 2 inches. The vertices of a funicular polygon formed by a light chain are to lie on these lines supposed vertical. From the vertices A, B, C, D are to be suspended weights of 3, 5, 7, 2 lb. respectively.

Construct the figure of the polygon, so that the portion of the chain between B and C shall be horizontal, and the portion between C and D shall be inclined at 60° to the horizontal. L.U.

10. ABC is a triangle in which BC is horizontal and 32 feet long and CA = AB = 24 feet. D, E, F are the middle points of the sides BC, CA, and AB respectively, and D is joined to E, A and F. The figure represents a roof truss, supported at B and C, which is subjected to vertical loads of $\frac{1}{2}$, 1, 1, 2 and $\frac{1}{2}$ ton at B, F, A, E and C. Find graphically the stresses in each bar of the truss. L.U.

11. Seven equal light rods are freely jointed together so as to form two squares ABCD and ABEF (lying in one plane on opposite sides of AB). Two other light rods join DB and AE. The system is supported at C and carries a weight W hanging from F. Find the tension or compression of each rod, explaining the method you use. L.U.

12. Two uniform beams AB, AC of equal length and of weights respectively W, W′ are jointed at A, the ends B, C being hinged to two fixed points on the same level. The beams rest in a vertical plane. Prove by means of a force-diagram or otherwise that the vertical component of the reaction at A is $\frac{1}{4}(W - W')$, and find the horizontal component. L.U.

13. In a roof truss similar to that shown in Fig 168 (p. 148), the dimensions are as follows: Horizontal distance between the supports, 20 ft.; vertical height of joint 4 above the supports, 5 ft.; the bar 36 is 1 foot above the supports; the bars 23 and 56 bisect at right angles the rafters 14 and 47 respectively. Vertical loads are applied as follows: At joint 1, 400 lb. wt., at joint 2, 800 lb. wt.; at joint 4, 1000 lb. wt.; at joint 5, 1200 lb. wt.; at joint 7, 1000 lb. wt. The reactions of the supports are vertical. Find these reactions, and then find the forces in each bar of the frame, stating whether the forces are pushes or pulls.

CHAPTER XII

STRESS. STRAIN ELASTICITY

Stress.—The term **stress** is applied to the mutual actions which take place across any section of a body to which a system of forces is applied. Stresses are described as **tensile** or **pull**, **compressive** or **push**, and **shear**, according as the portions of the body tend to separate, to come closer together, and to slide on one another respectively.

If equal areas at every part of the section sustain equal forces, the stress is said to be **uniform**; otherwise the stress is **varying**. Stress is measured by the force per unit area, and is calculated by dividing the total force by the area over which it is distributed; the result of this calculation is called the **stress intensity**, or more usually simply the **stress**.

In the case of varying stress, the result of the above calculation gives the average stress intensity. In such cases the stress at any point is calculated by taking a very small area embracing the point, and dividing the force acting over this area by the area.

Common units of stress are the pound, or ton-weight per square inch, or per square foot. In the C G S system the dyne per square centimetre is the unit of stress; the kilogram weight per square centimetre is the practical metric unit, and is equivalent to 14·19 lb. weight per square inch. The dimensions of stress are

$$\frac{ml}{t^2} \div l^2 = \frac{m}{lt^2}.$$

Strain.—The term **strain** is applied to any change occurring in the dimensions, or shape of a body when forces are applied. A rod becomes longer or shorter during the application of pull or push, and is said to have **longitudinal strain.** This strain is calculated as follows:

Let L = the original length of the rod,

e = the alteration in length, both expressed in the same units.

Then $$\text{Longitudinal strain} = \frac{e}{L}.$$

A body subjected to uniform normal stress (hydrostatic stress) all over its surface has **volumetric** strain.

Let V = the original volume of the body;

v = the change in volume, both expressed in the same units.

Then $$\text{Volumetric strain} = \frac{v}{V}.$$

Shearing strain occurs when a body is subjected to shear stress. In this kind of stress a change of shape occurs in the body. Thus, hold one cover of a thick book firmly on the table, and apply a shearing force to the top cover (Fig. 175). The change in shape is

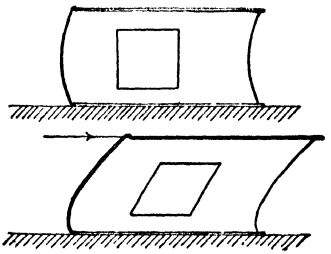


FIG. 175.—An illustration of shearing strain.

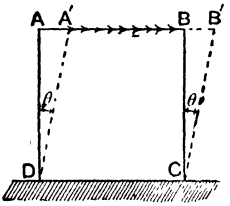


FIG. 176.—Shearing strain.

rendered evident by the square, pencilled on the end of the book, becoming a rhombus. Under similar conditions, a solid body would behave in the same manner, but in a lesser degree (Fig. 176). Shearing strain is measured by stating the angle θ in radians through which the vertical edge in Fig. 176 has rotated on application of the shearing stress. For metals θ is always very small, and it is sufficiently accurate to write

$$\text{Shearing strain} = \theta = \frac{BB'}{BC}.$$

It will be noted that strain has zero dimensions.

Elasticity.—**Elasticity** is that property of matter by virtue of which a body endeavours to return to its original shape and dimensions when strained, the recovery taking place when the disturbing forces are removed. The recovery is practically perfect in a great many kinds of material, provided that the body has not been loaded beyond a certain limit of stress which differs for different materials.

If loaded beyond this **elastic limit** of stress, the recovery of the original shape and dimensions is incomplete, and the body is said to have acquired **permanent set.**

Hooke's law.—Experiments on the pulling and pushing of rods show that the change in length is proportional very nearly to the force applied. If one end of a rod is held firmly while the other end is twisted, it is found that the angle through which this end rotates relatively to the fixed end is proportional to the twisting moment applied. Experimental evidence shows that beams are deflected, and springs are extended, by amounts proportional to the loads applied. This law was discovered by Hooke and bears his name. Since in every case the stress is proportional to the load, and the strain is proportional to the change in dimension, **Hooke's law** may be stated thus: **Strains are proportional to the stresses producing them.**

Hooke's law is obeyed by a great many materials up to a certain limit of stress, beyond which strains are produced which are larger proportionally than those for smaller stresses. In **ductile materials,** such as wrought iron, which are capable of being wire-drawn, rolled, and bent, the point of break-down of Hooke's law marks the beginning of a **plastic state** which, when fully developed, is evidenced by a large strain taking place with practically no increase in the stress. The stress at which this large increase in strain occurs is called the **yield point,** and is considerably greater than the stress at which Hooke's law breaks down

Experiments for the determination of the stress at which a given material first acquires permanent set are tedious, and when the term "elastic limit" is used, it is generally understood to mean the stress at which Hooke's law breaks down, the latter stress is determined easily by experiment.

Modulus of elasticity.—Assuming that Hooke's law is obeyed by a given material, and that s is the strain produced by a given stress p, we have

$$s \propto p, \quad \text{or} \quad as = p,$$

$$\therefore a = \frac{p}{s}, \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

where a is a constant for the material considered, and is called a **modulus of elasticity.** The value of the modulus of elasticity depends upon the kind of material and the nature of the stress applied. There are three chief moduli of elasticity.

Young's modulus applies to a pulled or pushed rod, and is obtained by dividing the stress on a cross section at 90° to the length of the rod by the longitudinal strain

Let P = the pull or push applied to the rod, in units of force.

A = the area of the cross section

L = the original length of the bar

e = the change in length of the bar.

Writing E for Young's modulus, we have

$$E = \frac{\text{stress}}{\text{strain}} = \frac{P}{A} \div \frac{e}{L} = \frac{PL}{Ae} \quad \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

The **bulk modulus** applies to the case of a body having uniform normal stress distributed over the whole of its surface

Let p = the stress intensity.

V = the original volume of the body

v = the change in volume.

Writing K for the bulk modulus, we have

$$K = \frac{\text{stress}}{\text{volumetric strain}} = p \div \frac{v}{V} = \frac{pV}{v} \quad \ldots\ldots\ldots\ldots\ldots\ldots (3)$$

Moduli of Elasticity

(Average values)

Material	Young's modulus, E		Rigidity modulus, C	
	Dynes per sq cm	Tons per sq inch	Dynes per sq cm	Tons per sq inch
Cast iron	10×10^{11}	6,000	$3{\cdot}5 \times 10^{11}$	2,200
Wrought iron	20 ,,	13,000	8·1 ,,	5,200
Mild steel	20 ,,	13,500	8·5 ,,	5,500
Copper (rolled)	9·5 ,,	6,200	3·9 ,,	2,500
Aluminium (rolled)	6·2 ,,	1,000	2·5 ,,	1,600
Brass	9·0 ,,	5,700	3·4 ,,	2,200
Gun-metal	7·8 ,,	5,000	3·1 ,,	2,000
Phosphor bronze	9·3 ,,	6,000	3·6 ,,	2,300
Timber	1·1 ,,	700	—	—
Indiarubber	0·05 ,,	32	0·0002 ,,	0·13
Glass, Crown	7·0 ,,	4,500	3·0 ,,	1,940
,, Flint	5·5 ,,	3,500	2 2 ,,	1,420
Catgut	0·3 ,,	194	—	—

The **rigidity modulus** applies to a body under shearing stress.

Let q = the shearing stress intensity.

θ = the shear strain.

Writing C for the rigidity modulus, we have

$$C = \frac{q}{\theta}.$$

The dimensions of all these moduli are the same as those of stress. The numerical values are expressed in the same units as those used in stating the stress.

EXPT. 26.—**Elastic stretching of wires.** A simple type of apparatus is shown in Fig. 177. Two wires, A and B, are hung from the same support, which should be fixed to the wall as high as possible in order that long wires may be used. One wire, B, is permanent and carries a fixed load W_1 in order to keep it taut. The other wire, A, is that under test, and may be changed readily for another of different material. The extension of A is measured by means of a vernier D, clamped to the test wire and moving over a scale E, which is clamped to the permanent wire. The arrangement of two wires prevents any drooping of the support being measured as an extension of the wire.

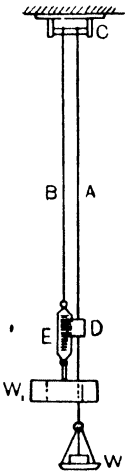


FIG 177.—Apparatus for tensile tests on wires.

See that the wires are free from kinks. Measure the length L, from C to the vernier. Measure the diameter of the wire A. State the material of the wire, and also whatever is known of its treatment before it came into your hands. Apply a series of gradually increasing loads to the wire A, and read the vernier after the application of each load. If it is not desired to reach the elastic limit, stop when a maximum safe load has been applied, and obtain confirmatory readings by removing the load step by step. In order to obtain the elastic limit, the load should be increased by small increments, and the test stopped when it becomes evident that the extensions are increasing more rapidly than the loads. Tabulate the readings thus:

TENSION TEST ON A WIRE.

Load, lb or kilograms wt.	Vernier reading		Extension, inches or mm.
	Load increasing.	Load decreasing.	

Plot loads as ordinates, and the corresponding extensions as abscissae (Fig. 178). It will be found that a straight line will pass through most of the points between O and a point A, after which the graph turns towards the right. The point A indicates the breakdown of Hooke's law.

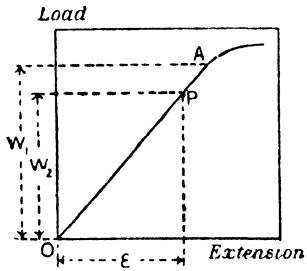


FIG. 178.—Graph of a tensile test on a wire.

Let W_1 = load at A in Fig. 178,

d = the diameter of the wire.

Then

Stress at elastic break-down $= W_1/\frac{1}{4}\pi d^2$.

Select a point P on the straight line OA (Fig. 178), and measure W_2 and e from the graph.

Let W_2 = the load at P.

e = the extension produced by P.

L = the length of the test wire.

Then Young's modulus $= E = \dfrac{\text{stress}}{\text{strain}} = \dfrac{W_2}{\frac{1}{4}\pi d^2} \dfrac{L}{e}$

Pure torsion.—In Fig. 179 is shown a rod AB having arms CD and EF fixed to it at right angles to the length of AB. Let AC = AD = BE = BF, and let equal opposite parallel forces Q, Q be applied at C and D in directions making 90° with CD. Let other equal opposite parallel forces P, P be applied in a similar manner at E and F. The rod is then under the action of two opposing couples in parallel planes. If the forces are all equal, the couples have equal moments, and the system is in equilibrium (p. 125). The rod is then said to be under **pure torsion**, *i.e.* there is no tendency to bend it, and there is no push or pull in the direction of its length. The **twisting moment** or **torque** T is given by the moment of either couple, thus

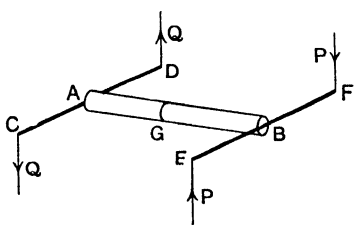


FIG. 179.—Pure torsion.

$$\text{Torque} = T = Q \times CD = P \times EF$$

The actions of the couples are transmitted from end to end of the rod AB, and produce shearing stresses on any cross section, such as G (Fig. 179).

The following experiment illustrates the twisting of a wire under pure torsion.

EXPT. 27.—**Torsion of a wire.** In Fig. 180, AB is a wire fixed firmly at A to a rigid clamp and carrying a heavy cylinder at B. The cylinder serves to keep the wire tight, and also provides means of applying a twisting couple to the wire. Two cords are wound round the cylinder and pass off in opposite parallel directions to guide pulleys. Equal weights W_1 and W_2 are attached to the ends of the cords. Pointers C and D are clamped to the wire, and move as the wire twists over fixed graduated scales E and F. The angle of twist produced in the portion CD of the wire is thus indicated.

State the material of the wire; measure its diameter d_1 and the length L between the pointers C and D. Measure the diameter d_2 of the cylinder B. Apply a series of gradually increasing loads, and read the scales E and F after each load is applied. Tabulate the readings.

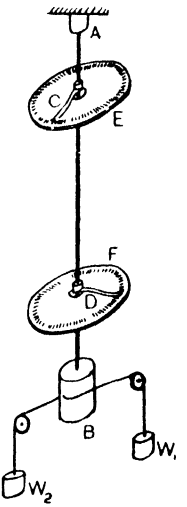


FIG. 180.—Apparatus for torsion tests on wires.

EXPERIMENT ON TORSION.

Load, $W_1=W_2$	Torque, W_1d_2	Scale readings, degrees		Angle of twist, degrees
		Scale E	Scale F	

Plot the torques as ordinates and the corresponding angles of twist as abscissae. A typical graph is shown in Fig. 181. A straight line graph indicates that the angle of twist is proportional to the torque. Select a point P on the graph, and scale the torque T and the angle α. If the graph has been plotted in degrees, convert α to radians. Calculate the modulus of rigidity from the expression:

$$C = \frac{32TL}{\pi d_1^4 \alpha}.$$

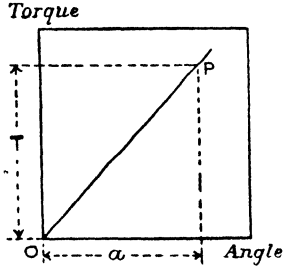


FIG. 181.—Graph of a torsion test on a wire

Bending of a beam.—The beam shown in Fig. 182 consists of a number of planks of equal lengths laid one on the top of the other and supported at the ends. Application of a load causes all the planks to bend in a similar fashion, and the planks will now be found to overlap

at the ends. Strapping the planks together (Fig. 183) prevents this action, and each end of the beam now lies in one plane; the planks now behave approximately like a solid beam. Inspection of Fig. 183 shows that planks near the top have become shorter and



FIG 182.—Bending of a loose plank beam

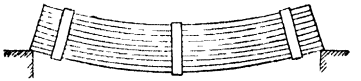


FIG. 183.—Bending of a strapped plank beam.

those near the bottom have become longer. The middle plank does not change in length. Hence we may infer that, in solid beams, there is a **neutral layer** which remains of unaltered length when the beam is bent, and that layers above the neutral layer have longitudinal strain of shortening, and must therefore be under push stress. Layers below the neutral layer have longitudinal strain of extension and are therefore under pull stress

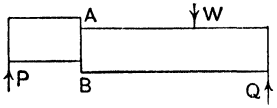


FIG. 184.—Shear at the section AB

In Fig 184 the beam carries a load W, and the reactions of the supports are P and Q Considering any cross section AB, the actions of P on the portion of the beam lying on the left-hand side of AB, and of W and Q on the other portion, produce a tendency for the material at AB to slide as shown. Hence the material at AB is under shear stress.

Beams firmly fixed in and projecting from a wall or pier are called **cantilevers.** A model cantilever is shown in Fig 185, and is arranged so as to give some conception of the stresses described above. The cantilever has been cut at AB; in order to balance the portion outside AB, a cord is required at A (indicating pull stress) and a small prop at B (indicating push stress). Further, in order to balance the tendency to shear, a cord has been arranged so as to apply a force S. In the uncut cantilever, these forces are supplied by the stresses in the material.

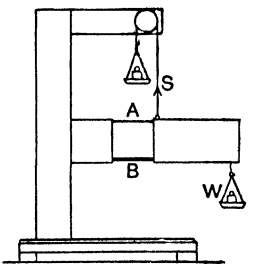


FIG 185.—Model of a cantilever, cut to show the forces at AB

Bending moment and shearing force in beams.—In Fig. 186 (*a*) is shown a beam carrying loads W_1, W_2, and supported by forces P, Q. AB is any cross section. P and W_1 have a tendency to rotate

the portion of the beam lying on the left-hand side of AB. Similarly, Q and W_2 tend to rotate the other portion of the beam in the contrary sense. These tendencies may be calculated by taking the algebraic sum of the moments of the forces about any point in AB. A little consideration will show that the resultant moment of P and W_1 must be equal to the resultant moment of Q and W_2, since both these resultant moments are balanced by the same stresses transmitted across AB. The evaluation of these stresses is beyond the scope of this book, but we may say that they give rise to equal forces X and Y (Fig. 186 (*b*)).

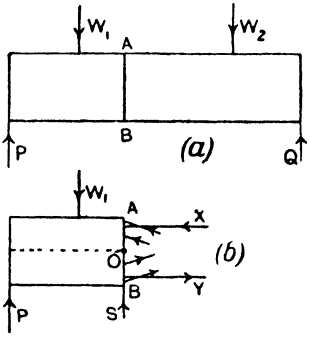


FIG. 186.—Bending moment and shearing force

The **bending moment** at any section of a beam measures the tendency to bend the beam about that section, and is calculated by taking the algebraic sum of the moments about any point on the section of all the forces applied to either one portion or the other portion of the beam.

Again, consider the left-hand portion of the beam (Fig. 186 (*b*)). If P and W_1 are equal, there is no resultant tendency to produce vertical movement of this portion. Otherwise the stresses at the section must supply an upward or downward force S according as W_1 is greater or less than P. S is called the **shearing force** at the section AB, and is calculated by taking the algebraic sum of all the forces applied to either one portion or the other portion of the beam.

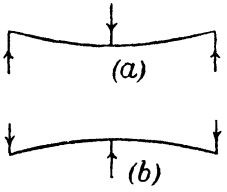


FIG. 187.—Positive and negative bending.

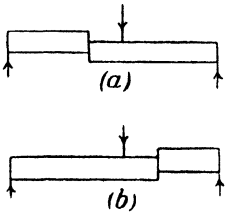


FIG. 188.—Positive and negative shear.

A common convention is to describe bending moments as positive or negative according as the beam bends as shown in Fig. 187 (*a*) or Fig. 187 (*b*). If the action is as shown in Fig. 188 (*a*), the shearing force is positive; Fig. 188 (*b*) shows the action with a negative shearing force.

EXAMPLE.—A beam of 20 feet span is supported at its ends and carries a uniformly distributed load of 1 ton weight per foot length (Fig. 189 (*a*)). Find the bending moment and shearing force at a section 6 feet from the left-hand support.

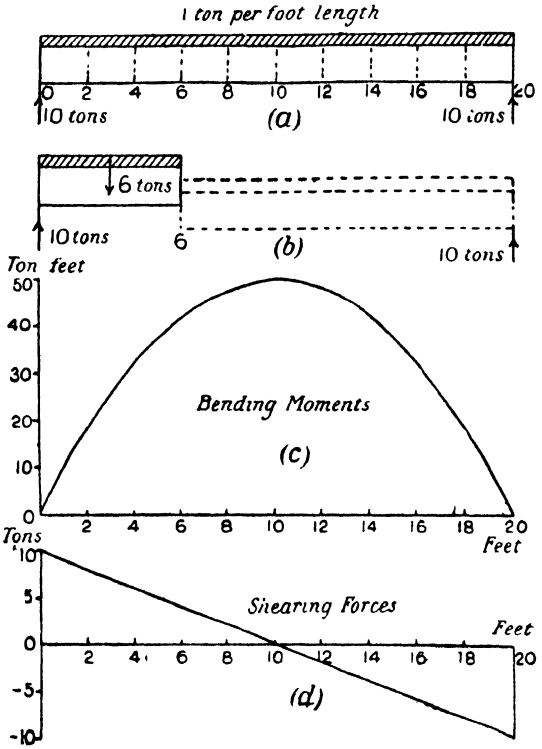


FIG. 189.—Bending moment and shearing force diagrams for a beam carrying a uniformly distributed load

The total load is 20 tons weight, and the reaction of each support is therefore 10 tons weight. Referring to Fig. 189 (*b*), it will be noted that the external forces applied to the portion of the beam lying on the left-hand side of the section are 10 tons weight acting upwards and a distributed load of 6 tons weight acting downwards. The latter may be applied at its centre of gravity, *i.e.* 3 feet from the section.

$$\text{Bending moment} = (10 \times 6) - (6 \times 3) = 60 - 18$$
$$= \underline{42} \text{ ton-feet.}$$

$$\text{Shearing force} = 10 - 6 = \underline{4} \text{ tons weight.}$$

In the same manner the bending moments and shearing forces at other sections may be calculated and the results plotted (Fig. 189 (*c*) and (*d*)). The resulting diagrams show clearly how the bending moments and shearing forces vary throughout the beam.

EXPT. 28.—**Deflection of a beam.** The apparatus employed is shown in Fig. 190, and consists of two cast iron brackets A and B, which can be

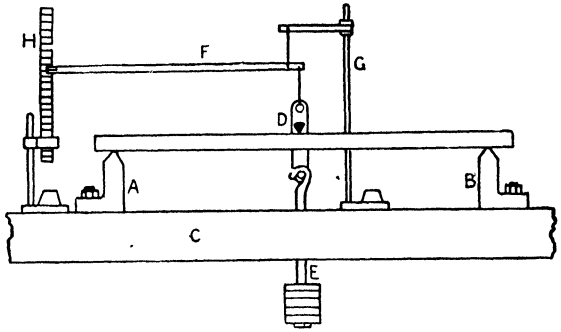


FIG. 190.—Apparatus for measuring the deflection of a beam.

clamped anywhere to a lathe bed C, or other rigid support. The brackets have knife edges at the tops, and the test beam rests on these. A wrought-iron stirrup D, with a knife-edge for resting on the beam, carries a hook E for applying the load. The deflections are measured by means of a light lever F, pivoted to a fixed support G, and attached by a fine wire at its shorter end to the stirrup; the other end moves over a fixed scale H as the beam deflects. The ratio of the lever arms may be anything

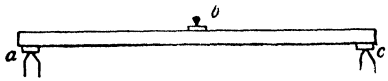


FIG. 191.

from 1 : 10 to 1 : 20. The deflection produced by any load will be obtained by dividing the difference in the scale readings before and after applying the load by the ratio of the long arm to the short arm of the lever. If the test beam is of timber, it is advisable to place small metal plates at *a*, *b* and *c* (Fig. 191) in order to prevent indentation of the soft material.

Arrange the apparatus as shown. Let the beam be of rectangular section; note the material and measure the span L, the breadth *b* and the depth *d*. Apply the load at the middle of the span, and take readings

as indicated in the table for a series of gradually increasing loads W. Take readings also when the load is removed step by step.

DEFLECTION TEST ON A BEAM.

Load, W	Scale reading		Deflection
	Load increasing.	Load decreasing	

Plot a graph showing loads as ordinates and corresponding deflections as abscissae. A straight line graph will indicate that the deflection is proportional to the load.

The deflection of a beam is chiefly due to the longitudinal strains caused by the push and pull stresses to which the fibres of the beam are subjected. Hence the deflection is related to the value of Young's modulus of the material. Select a point on the graph, and read off the values of W and the deflection Δ corresponding to this point. Calculate Young's modulus from

$$E = \frac{WL^3}{4\Delta bd^3}$$

EXERCISES ON CHAPTER XII.

1. A load of 7 tons weight is hung from a vertical bar of rectangular section 2·5 inches × 1 inch. Find the tensile stress.

2. Find the safe pull which may be applied to a bar of rectangular section 4 inches × $\frac{5}{8}$ inch, if the tensile stress allowed is 5 tons wt. per square inch.

3. A pull of 15 tons weight is applied to a bar of circular section. Find the diameter of the bar if the tensile stress permitted is 6 tons wt. per square inch.

4. Find the safe load which can be applied to a hollow cast-iron column 6 inches external and 4 5 inches internal diameter. The compressive stress allowed is 7 tons wt. per square inch.

5. A shearing force of 3 tons wt. is distributed uniformly over the cross section of a pin 1 5 inches in diameter. Find the shear stress.

6. A steel cylinder, 3 feet long and 5 inches in diameter, is subjected to hydrostatic stress, and the volume is found to change by 0·57 cubic inch. Find the volumetric strain.

7. A square steel plate 4 feet edge, plane vertical, has its lower edge fixed rigidly. Shear stress is applied and the upper edge is observed to move parallel to the lower edge through 0·02 inch. Find the shear strain.

8. A column, 20 feet high and having a cross sectional area of 12 square inches, carries a load of 36 tons weight. Find the decrease in length when the load is applied. $E = 29{,}000{,}000$ lb. wt. per sq. inch.

9. A wire, 120 inches long and having a sectional area of 0·125 square inch, hangs vertically. When a load of 450 lb. weight is applied, the wire is found to stretch 0·015 inch. Find the stress, the strain, and the value of Young's modulus.

10. Find the tensile stress in a bolt 2·5 inches diameter when a load of 30 tons weight is applied. If $E = 30 \times 10^6$ lb. wt. per square inch, find the longitudinal strain. The original length of the bolt was 102 inches; find the extension when the load is applied.

11. A cast-iron bar, diameter 0·474 inch, length 8 inches, was loaded in compression, and the contraction in length caused by a gradually increasing series of loads was measured:

Load, lb. wt.	0	100	200	300	400
Contraction in length, inches	0 000	0·00038	0 00069	0·00105	0 00137
Load, lb. wt.	500	600	700	800	
Contraction in length, inches	0 0017	0·00208	0·0024	0 0027	

Plot a graph and find the value of Young's modulus.

12. A square steel plate, 6 feet edge, has one edge rigidly fixed and shear stress of 3 704 tons wt. per square inch applied to the other edges. If the modulus of rigidity is 5,500 tons wt. per square inch, find the movement of the edge opposite the fixed edge.

13. If the bulk modulus for copper is 3,300 tons wt. per square inch, find the contraction in volume of a copper sphere 10 inches in diameter when subjected to a hydrostatic stress of 0·5 ton wt. per square inch.

14. A specimen of steel, 0 714 inch in diameter and 7·81 inches long, had an angle of twist of 0·56 degree when a torque of 400 lb.-inches was applied. Find the value of the modulus of rigidity.

15. A beam, 20 feet long, rests on supports at its ends. There is a load of 2 tons weight at the middle, and other two loads of 1 ton weight each placed at points 5 feet from each support. Find the bending moment at each load; also the shearing force at a section 6 feet from one support. Neglect the weight of the beam.

16. A cantilever projects 8 feet from a wall and carries a load of 400 lb. weight distributed uniformly over the length of the cantilever. Calculate the bending moments and shearing forces at sections 0, 2, 4, 6 and 8 feet from the wall. Draw diagrams of bending moments and shearing forces.

17. In testing a steel bar as a beam supported at the ends and loaded at the middle, it was found that a load of 10 lb. weight produced a deflection of 0 0053 inch. The beam was 1 inch broad, 1 inch deep and 40 inches span. Find the value of Young's modulus.

18. Discuss the nature of the forces acting on the fibres at any cross section of a beam fixed at one end and loaded at the other.

A uniform beam, 20 ft. long, weighing 2000 lb., is supported at its ends. The beam carries a weight of 4000 lb. at a point 5 ft. from one end. Find the bending moments at the centre of the beam and at the point where the weight is supported. Adelaide University.

19. A light horizontal beam AB, of length 7 feet, is supported at its ends, and loaded with weights 40 and 50 lb. at distances of 2 and 4 feet from A. Find the reactions at A and B, and tabulate the bending moment and shearing force at distances 1, 3, 5 and 7 feet from A. Draw a diagram from which can be found the bending moment at any point of the beam. L.U.

CHAPTER XIII

WORK. ENERGY. POWER. FRICTION

Work.—Work is said to be done by a force when the point of application undergoes a displacement along the line of action of the force. Work is measured by the product of the magnitude of the force and the displacement. Thus, if A (Fig. 192) be displaced from A to B, a distance s along the line of action of the force F, then

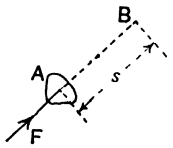


FIG. 192.—Work done by a force.

Work done by $F = Fs$ .....(1)

In Fig. 193 the point of application is displaced from A to B, and AB does not coincide with the direction of F. The displacement AB is equivalent to the component displacements AC and CB, which are respectively along and at right angles to the line of F. Let s denote the displacement AC, then

Work done by $F = Fs = F \times AB \times \cos \alpha$. .....(2)

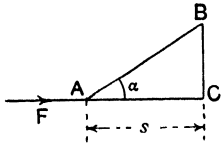


FIG. 193.

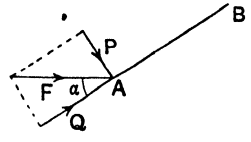


FIG 194

The work done by F may also be calculated by the following method: In Fig. 194 take components of F, (P and Q), respectively at right angles to and along AB. Q is equal to $F \cos \alpha$. P does no work during the displacement from A to B; the work is done by Q alone, and is given by

Work done $= Q \times AB = F \times \cos \alpha \times AB$, ...(3)

which is the same result as before.

No work is done against gravity when a load is carried along a level road. This follows from the consideration that the point of

application of the vertical force supporting the load moves in a horizontal plane, and therefore undergoes no displacement in the vertical line of action of the weight.

Units of work.—Unit work is performed when unit force produces unit displacement. The C.G.S. absolute unit of work is the **erg**, and is performed when a force of one dyne acts through a distance of one centimetre. The metric gravitational unit of work usually employed is the **centimetre-kilogram**, and is performed when a force of one kilogram weight acts through a distance of one centimetre; the centimetre-gram and the metre-kilogram are also used.

The British absolute unit of work is the **foot-poundal**, and is performed when a force of one poundal acts through a distance of one foot. The gravitational unit of work is the **foot-lb.**, and is performed when a force of one pound weight acts through a distance of one foot.

In practical problems in electricity the unit of work employed is the **joule**; this unit represents the work done in one second when a current of one ampere is maintained by an E M F of one volt.

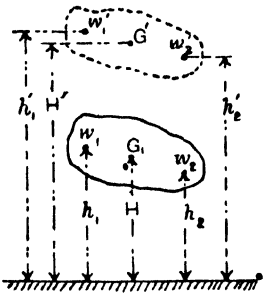


FIG. 195.—Work done in raising a body.

The dimensions of work are

$$\frac{ml}{t^2} \times l = \frac{ml^2}{t^2}.$$

Work done in elevating a body.—In Fig. 195 is shown a body having a total weight W, and having its centre of gravity G_1 at a height H above the ground. w_1, w_2, etc., are particles situated at heights h_1, h_2, etc., above the ground. Let the body be raised so that G_1 moves to G', and w_1, w_2 to w_1', w_2' at heights H', h_1' and h_2' respectively. The work done against gravity in raising w_1 and w_2 is $w_1(h_1' - h_1)$ and $w_2(h_2' - h_2)$; hence the total work done in raising the body is given by

$$\begin{aligned}\text{Work done} &= w_1(h_1' - h_1) + w_2(h_2' - h_2) + w_3(h_3' - h_3) + \text{etc.} \\ &= (w_1h_1' + w_2h_2' + w_3h_3' + \text{etc.}) - (w_1h_1 + w_2h_2 + w_3h_3 + \text{etc.}) \\ &= \text{WH}' - \text{WH}, \quad \text{(p. 109)} \\ &= \text{W}(\text{H}' - \text{H}).\end{aligned}$$

The work done in raising a body against the action of gravity may therefore be calculated by taking the product of the total weight of the body and the vertical height through which the centre of gravity is raised.

Graphic representation of work.—Since work is measured by the product of force and distance, it follows that the area of a diagram in which ordinates represent force and abscissae represent distances will represent the work done.

If the force is uniform, the diagram is a rectangle (Fig. 196). The work done by a uniform force P acting through a distance D is $P \times D$. If unit height of the diagram represents p units of force, and unit length represents d units of displacement, then one unit of area of the diagram represents pd units of work. If the diagram measures A units of area, then the total work done is given by pdA.

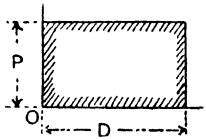


FIG. 196.—Diagram of work done by a uniform force.

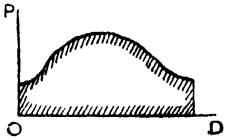


FIG. 197.—Diagram of work done by a varying force.

If the force varies (Fig. 197), the diagram of work is drawn by setting off ordinates to represent the magnitude of the force at different values of the displacement. The work done may be calculated by taking the product of the average value of the force and the displacement. Since the average height of the diagram represents the average force, and the length of the diagram represents displacement, we have, as before, the work done represented by the area of the diagram, and one unit of area of the diagram represents pd units of work. The area A of the diagram may be found by means of a planimeter, or by any convenient rule of mensuration, when pdA will give the total work done.

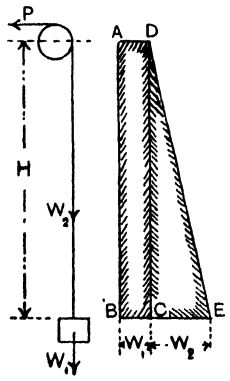


FIG. 198.—Diagram of work done in hoisting a load.

EXAMPLE.—Find the work done against gravity when a cage and load weighing W_1 are raised from a pit H deep by means of a rope having a weight W_2 (Fig. 198).

At first the pull P required at the top of the rope is $(W_1 + W_2)$, and this diminishes gradually as the cage ascends, becoming W_1 when the cage is at the top. The diagram of work for hoisting the cage and load alone is the rectangle ABCD, in which BC and AB represent W_1 and H respectively; the diagram for hoisting the

rope alone is DCE, in which W_2 is represented by CE. From the diagrams, we have

$$\text{Total work done} = W_1 H + \tfrac{1}{2} W_2 H$$
$$= (W_1 + \tfrac{1}{2} W_2) H.$$

Energy.—**Energy** means capability of doing work. A body is said to possess energy when, by reason of its position, velocity, or other conditions, work may be performed during an alteration in the conditions. Thus an elevated body is said to possess energy because work can be done by the gravitational effort if the body is permitted to descend. Energy of this kind is called **potential energy.** A flying bullet is said to possess energy because work can be done while the bullet is coming to rest Energy possessed by a body by virtue of its motion is called **kinetic energy.** There are various other forms of energy, such as heat, electric energy, etc.

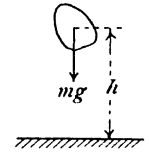


FIG 199.—Potential energy.

Energy is measured in units of work. Thus the potential energy of a body of mass m at an elevation h (Fig. 199) is mgh, since mgh absolute units of work will be done by gravitational effort whilst the body is descending.

Conservation of energy.—Experience shows that all energy at our disposal comes from natural sources. The **principle of the conservation of energy states that man is unable to create or destroy energy; he can only transform it from one kind into another** For example, a labourer carrying bricks up a ladder is not creating potential energy, but is only converting some of his internal store of energy into another form. Presently rest and food will be necessary in order that his internal store of energy may be replenished. No matter what may be the form of food, it is derived ultimately from vegetation, and vegetation depends for its growth upon the light and heat of the sun Hence the store of energy in the sun is responsible primarily for the elevation of the bricks. The student will be able to supply other examples from his own experience.

The statement that energy cannot be destroyed requires some explanation. In converting energy from one form into another, some of the energy disappears generally, so that the total energy in the new form is less than the original energy. If careful examination be made, it will be found that the missing energy has been converted into forms other than that desired, and that the total

energy in the various final forms is exactly equal to the original energy. For example, a hammer is used for driving a nail and is given kinetic energy by the operator. The hammer strikes the nail, and some of its energy is used in performing the useful work of driving the nail. The remainder is wasted in damaging the head of the nail and in the production of sound and heat. The student should accustom himself to the use of the term "wasted energy" in preference to "lost energy," which might lead to the idea that some energy had been destroyed.

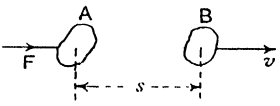


FIG 200.—Kinetic energy.

Kinetic energy.—In Fig. 200 a resultant external force F acts on a mass m, which is at rest at A. Let the body be displaced to B through a distance s, and let its velocity at B be v. Then

Work done by F = Fs. .....(1)

None of this work has been done against any external resistance, hence it must be stored in the body at B in the form of kinetic energy.

Hence Kinetic energy at B = Fs = mas.

Also, $v^2 = 2as$, or $a = \frac{v^2}{2s}$; (p. 33)

$\therefore$ Kinetic energy at B = $ms \cdot \frac{v^2}{2s}$

$= \frac{mv^2}{2}$ absolute units. .....(2)

It will be noted that the result obtained for the kinetic energy is independent of the direction of motion of the body. This follows from consideration of the fact that the velocity appears to the second power in the result, which is therefore independent of the direction or sign of the given velocity. Kinetic energy is a scalar quantity.

The dimensions of kinetic energy are ml^2/t^2, *i.e.* kinetic energy has the same dimensions as work.

Average resistance.—When a body is in motion and it is desired to bring it to rest, or to diminish its speed, a force must be applied having a sense opposite to that of the velocity. In general it is not possible to state the precise value of this resistance at any instant, but the average value may be calculated from the consideration that the change in kinetic energy must be equal to the work done against the resistance. The following example illustrates this application of the principle of the conservation of energy.

EXAMPLE.—A stone weighing 8 lb. falls from the top of a cliff 120 feet high and buries itself 4 feet deep in the sand. Find the average resistance to penetration offered by the sand, and the approximate time of penetration. L.U.

In this case it is simpler to use gravitational units of force; thus:

Total energy available = potential energy transformed
$= 8 \times (120 + 4) = 992$ foot-lb.

Let P = the average resistance in lb. weight,

then Work done against P $= P \times 4$ foot-lb.;

$\therefore 4P = 992,$

$P = \underline{248}$ lb. weight.

Again, the velocity just before reaching the sand is given by

$$v = \sqrt{2gh} = \sqrt{64\ 4 \times 120},$$

$$v = 87{\cdot}9 \text{ feet per sec.}$$

Also, Average velocity × time = distance travelled;

$$\frac{87\ 9}{2} \times t = 4,$$

$$t = \frac{8}{87\ 9},$$

$$= 0\ 091 \text{ second.}$$

Power.—**Power** means rate of doing work. The C G S unit of power is a rate of doing work of one erg per second. The British unit of power is the **horse-power**, and is a rate of working of 33,000 foot-lb. per minute; this rate of doing work is equivalent to 550 foot-lb. per second. In any given case the horse-power is calculated by dividing the work done per minute, in foot-lb, by 33,000.

The electrical power unit is the **watt**, and is 10^7 ergs per second; the watt is developed when an electric current of one ampere flows between two points of a conductor, the potential difference between the points being one volt. The product of amperes and volts gives watts. 746 watts are equivalent to one horse-power; hence

$$\text{Horse-power} = \frac{\text{amperes} \times \text{volts}}{746}.$$

The Board of Trade unit of electrical energy is one kilowatt maintained for one hour. One horse-power maintained for one hour would produce $33{,}000 \times 60 = 1{,}980{,}000$ foot-lb. The kilowatt-hour is therefore given by

$$1 \text{ kilowatt hour} = 1{,}980{,}000 \times \frac{1000}{746}$$

$$= 2{,}654{,}000 \text{ foot-lb.}$$

Friction.—In practice much energy is wasted in overcoming frictional resistances, and the general laws of friction should be understood by the student.

When two bodies are pressed together it is found that there is a resistance offered to the sliding of one upon the other. This resistance is called the **force of friction.** The force which friction applies to a body always acts in such a direction as to maintain the state of rest, or to oppose the motion of the body.

Let two bodies A and B (Fig. 201 (*a*)) be pressed together, and let the mutual force perpendicular to the surfaces in contact be R. Let B be fixed, and let a force P, parallel to the surfaces in contact, be applied (Fig. 201 (*b*)). If P is not large enough to produce sliding, or if sliding with steady speed takes place, B will apply to A a frictional force F equal and opposite to P. The force F may have any value lower than a certain maximum, which depends on the magnitude of R and on the nature and condition of the surfaces in contact. If P is less than the maximum value of F, sliding will not occur; sliding will be on the point of occurring when P is equal to the maximum possible value of F. It is found that the frictional resistance offered, after steady sliding conditions have been attained, is less than that offered when the body is on the point of sliding.

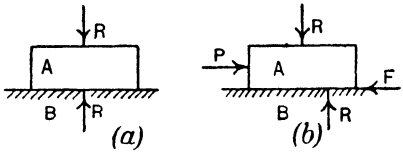


FIG. 201.—Force of friction.

Let F_s = the frictional resistance when the body is on the point of sliding.

F_k = the frictional resistance when steady sliding has been attained.

R = the perpendicular force between the surfaces in contact.

These forces should all be stated in the same units. Then

$$\mu_s = \frac{F_s}{R}; \quad \mu_k = \frac{F_k}{R}.$$

μ_s and μ_k are called respectively the **static** and **kinetic coefficients of friction.**

Friction of dry surfaces.—Owing to the great influence of apparently trifling alterations in the state of the rubbing surfaces, it is not possible to predict with any pretence at accuracy what the

frictional resistance will be in any given case. For this reason the proper place to study friction is in a laboratory having suitable apparatus For dry clean surfaces the following general laws are complied with roughly :

The force of friction is proportional to the perpendicular force between the surfaces in contact, and is independent of the extent of these surfaces and of the speed of rubbing, if moderate. It therefore follows that the **kinetic coefficient of friction for two given bodies is practically constant for moderate pressures and speeds.** Experiments on the static coefficient of friction are not performed easily, roughly, this coefficient is constant for two given bodies.

COEFFICIENTS OF FRICTION.

(Average values.)

Metal on metal, dry	0·2
Metal on wood, dry	0·6
Wood on wood, dry	0·2 to 0·5
Leather on iron	0·3 to 0·5
Leather on wood	0 3 to 0·5
Stone on stone	0 7
Wood on stone	0·6
Metal on stone	0·5

The average values given in this table should be employed only in the absence of more definite experimental values for the bodies concerned.

EXPT. 29.—**Determination of the kinetic coefficient of friction** Set up a board AB (Fig. 202) as nearly horizontal as possible, and arrange a slider C (which can be loaded to any desired amount) with a cord, pulley and scale-pan, so that the horizontal force P required to overcome the frictional resistance may be measured. Weigh the slider, and let its weight, together with the load placed on it, be called W. The perpendicular force between the surfaces in contact will be equal to W. Weigh the scale-pan, and let its weight, together with the weights placed in it in order to secure steady sliding, be called P. P and F will be equal ; hence

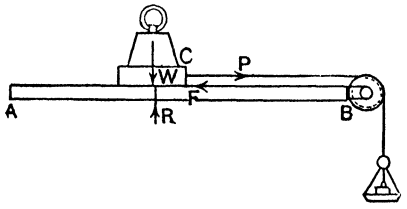


FIG 202.—Friction of a slider

$$\text{Kinetic coefficient of friction} = \frac{P}{W}.$$

It is necessary to assist the slider to start by tapping the board. The rubbing surfaces should be clean and free from dust. More consistent results can be obtained from surfaces which have been freshly planed.

Make a series of about ten experiments with gradually increasing loads. Plot P and W; the plotted points will lie approximately on a straight line. Draw the straight line which best fits the points; select one point on the graph, and read the values of P and W for it; let these values be P_1 and W_1, then

$$\text{Average kinetic coefficient of friction} = \frac{P_1}{W_1}.$$

The materials of which the slider and board are made should be stated, and, if these are timber, whether rubbing has been with the grain or across the grain of the wood.

Friction on an inclined plane.—In Fig. 203, XZ is an inclined board which has been arranged so that a block A just slides down with steady speed. Let *ca* represent the weight of the block; by means of the parallelogram of forces *cbad*, find the components Q and P of W, respectively perpendicular and parallel to XZ. The board applies a frictional force F to the block in a direction coinciding with the surface of the board and contrary to the motion of the block, *i.e.* up the plane. As there is no acceleration, P and F are equal. The plane also exerts on the block a force R, equal and opposite to Q. R is the normal or perpendicular force between the surfaces in contact. Hence, by the definition (p. 173),

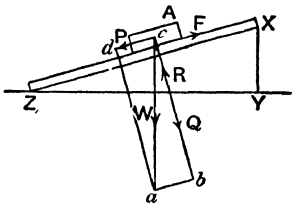


FIG. 203.—Coefficient of friction determined by inclining the board.

$$\text{Kinetic coefficient of friction} = \frac{F}{R} = \frac{P}{Q}.$$

Since *cb* and *ca* are perpendicular to XZ and ZY respectively, it follows that the angles *acb* and XZY are equal. Hence

$$Q = W \cos acb = W \cos XZY,$$
$$P = W \sin acb = W \sin XZY;$$
$$\therefore \mu_k = \frac{P}{Q} = \frac{W \sin XZY}{W \cos XZY}$$
$$= \tan XZY.$$

The angle XZY is called the **angle of sliding friction.**

EXPT. 30.—**Determination of μ_k from the angle of sliding friction.** Use the same board and slider as in Expt. 29. Raise one end of the board until, with assistance in starting, the slider travels down the inclined plane

with constant speed. Measure the angle of inclination of the plane, or measure its height and base, and so obtain the tangent of the angle of sliding friction; this will give μ_k. Place a weight on the slider, and ascertain if the block will still slide with steady speed. Compare the result with that obtained in Expt. 29.

Resultant reaction between two bodies.—In Fig. 204 is shown a block A resting on a horizontal table BC. The weight W of the block acts in a line normal to BC. Let a horizontal force P_1 be applied to the block; P_1 and W will have a resultant R_1. For equilibrium the table must exert a resultant force on the block equal and opposite to R_1 and in the same straight line. Let this force be E_1, cutting BC in D. E_1 may be resolved into two forces, Q perpendicular to BC, and F_1 along BC. Let ϕ_1 be the angle which E_1 makes with GD. Then

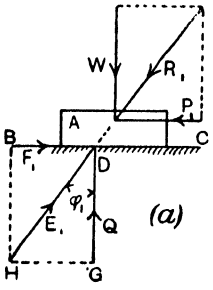


FIG. 204.—Friction angle

$$\frac{F_1}{Q} = \frac{HG}{GD} = \tan \phi_1$$

Now, when P_1 is zero, ϕ_1 and hence $\tan \phi_1$ will also be zero, and Q will act in the same line as W. ϕ_1 will increase as P_1 increases, and will reach a maximum value when the block is on the point of slipping. It is evident that Q will always be equal to W. Let ϕ be the value of the angle when the block just slips, and let F be the corresponding value of the frictional force; then

$$\text{Static coefficient of friction} = \mu_s = \frac{F}{Q} = \tan \phi.$$

ϕ is called the **friction angle** or the **limiting angle of resistance**; when steady sliding has been attained, ϕ is lower in value and is called, as noted above, the angle of sliding friction.

It is evident from Fig. 204 that P_1 and F_1 are always equal (assuming no sliding, or sliding with constant speed); W and Q are also always equal. These forces form couples having equal opposing moments, and so balance the block. It will be noted that D, the point through which Q acts, does not lie in the centre of the rubbing surface unless P_1 is zero. The effect is partially to relieve the normal pressure near the right-hand edge of the block and to increase it near the left-hand edge. With a sufficiently large value of μ_s, and by applying P at a large enough height above the table, the block can be made to overturn instead of sliding.

In Fig. 205 the resultant R of P and W may fall outside the base AB before sliding begins. Hence E, which must act on AB, cannot act in the same line as R, and the block will overturn. For overturning to be impossible, R must fall within AB.

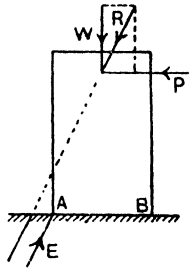


FIG. 205.—Condition that a block may overturn

EXAMPLE.—A block of weight W slides steadily on a plane inclined at an angle α to the horizontal under the action of a force P. Find the values of P in the following cases:

(a) P is horizontal and the block slides upwards.
(b) P is horizontal and the block slides downwards.
(c) P is parallel to the plane and the block slides upwards.
(d) P is parallel to the plane and the block slides downwards.

Case (a).—In Fig. 206 (a) draw AN perpendicular to the plane; the angle between W and AN is equal to α. Draw AC, making with AN an angle ϕ equal to the angle of sliding friction; the resultant reaction R of the plane acts in the line CA, and ABC is the triangle of forces for W, P and R. Let μ be the kinetic coefficient of friction, then

$$\frac{P}{W} = \frac{BC}{AB} = \tan(\alpha + \phi);$$

$$\therefore P = W\left(\frac{\tan\alpha + \tan\phi}{1 - \tan\alpha \tan\phi}\right) = W\left(\frac{\tan\alpha + \mu}{1 - \mu\tan\alpha}\right). \quad \ldots\ldots\ldots\ldots(1)$$

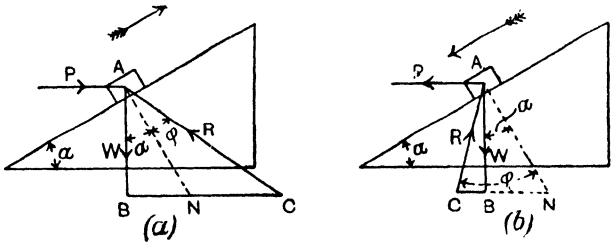


FIG 206.—Friction on an incline; P horizontal.

Case (b).—The construction is shown in Fig. 206 (b), and is made as directed under Case (a), excepting that R acts on the other side of AN. The triangle of forces is ABC.

$$\frac{P}{W} = \frac{BC}{AB} = \tan(\phi - \alpha);$$

$$\therefore P = W\left(\frac{\tan\phi - \tan\alpha}{1 + \tan\alpha \tan\phi}\right) = W\left(\frac{\mu - \tan\alpha}{1 + \mu\tan\alpha}\right). \quad \ldots\ldots\ldots\ldots(2)$$

It will be noticed in this case that, if ϕ is less than α, the block will slide down without the necessity for the application of a force P. Rest is just possible, unaided by P, if α and ϕ are equal.

Case (c).—The required construction is shown in Fig. 207 (a); the triangle of forces is ABC.

$$\frac{P}{W} = \frac{BC}{AB} = \frac{\sin BAC}{\sin ACB} = \frac{\sin(\alpha+\phi)}{\sin(90^\circ-\phi)},$$

$$= \frac{\sin\alpha\cos\phi + \cos\alpha\sin\phi}{\cos\phi};$$

$$\therefore P = W(\sin\alpha + \cos\alpha\tan\phi) = W(\sin\alpha + \mu\cos\alpha). \quad \ldots\ldots\ldots(3)$$

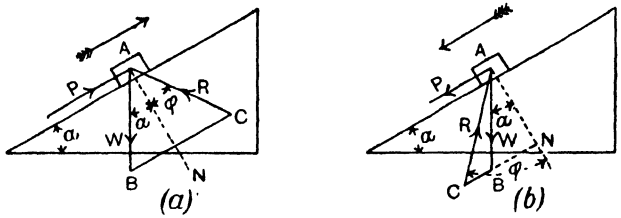


FIG. 207.—Friction on an incline; P parallel to the incline.

Case (d).—Referring to Fig. 207 (b), we have

$$\frac{P}{W} = \frac{BC}{AB} = \frac{\sin BAC}{\sin ACB} = \frac{\sin(\phi-\alpha)}{\sin(90^\circ-\phi)}$$

$$= \frac{\sin\phi\cos\alpha - \cos\phi\sin\alpha}{\cos\phi};$$

$$P = W(\tan\phi\cos\alpha - \sin\alpha) = W(\mu\cos\alpha - \sin\alpha). \quad \ldots\ldots\ldots(4)$$

Friction of a rope coiled round a post.—When a rope is coiled round a cylindrical post, slipping will not occur until the pull applied to one end is considerably greater than that applied to the other end. This is owing to the friction between the rope and the post having to be overcome before slipping can take place. As the frictional resistance is distributed throughout the surface in contact, the pull in the rope will diminish gradually from a value T_1 at one end, to T_2 at the other end.

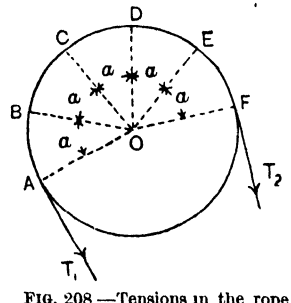


FIG. 208.—Tensions in the rope at different parts of the arc of contact.

In Fig. 208 the rope embraces the arc ACF, and this arc has been divided into equal arcs AB, BC, etc., subtending equal angles α at the centre of the post. Since these

arcs are all equal, it is reasonable to suppose that the ratios of the tensions in the rope at the beginning and end of each arc are equal, *i.e.*

$$\frac{T_1}{T_B} = \frac{T_B}{T_C} = \frac{T_C}{T_D} = \frac{T_D}{T_E} = \frac{T_E}{T_2} \quad \ldots\ldots(1)$$

This assumes that the surfaces are such that the value of the coefficient of friction is the same throughout, and the result is confirmed approximately by experiment.

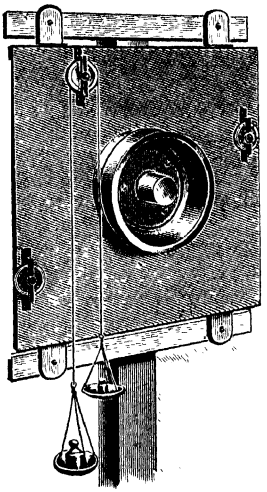


FIG 209.—Apparatus for experiments on the friction of a cord coiled on a drum

EXPT. 31.—**Friction of a cord coiled round a post.** The apparatus shown in Fig. 209 enables the tensions to be found for angles of contact differing by 90°. Weigh the scale-pans. Put equal loads in each pan, then increase one load until steady slipping occurs. Evaluate T_1 and T_2, and repeat the experiment with a different angle of contact. The following is a record of an actual experiment, using a silk cord on a pine post.

AN EXPERIMENT ON SLIPPING.

Angle of lap.	T_1 lb. wt. descending	T_2 lb. wt. ascending.	Experimental ratio $\frac{T_2}{T_1}$.	Calculated ratio $\frac{T_2}{T_1}$.
90°	0 397	0 29	0·73	0·73
180°	0 56	0·29	0 518	0·533
270°	0·79	0·29	0·367	0·39
360°	1·1	0·29	0·263	0·28

The last column is obtained as follows: Taking the first ratio of $T_2/T_1 = 0{\cdot}73$ for 90° lap. the ratio for 180° lap from (1) above would be $0{\cdot}73 \times 0{\cdot}73 = 0\ 533$; the ratio for 270° is $0{\cdot}73^3 = 0{\cdot}39$; and the ratio for 360° is $0{\cdot}73^4 = 0{\cdot}28$. These calculated and experimental values show fair agreement, remembering the assumptions that have been made regarding the constancy of the coefficient of friction.

Horse-power transmitted by a belt.—A belt drives the pulley, round which it is wrapped, by reason of the frictional resistance between the

surfaces in contact. The pulls in the straight parts of the belt differ in magnitude by an amount equal to the total frictional force round the arc of contact. Hence the pull T_1 (Fig. 210) in one straight portion is greater than T_2 in the other straight portion. Let T_1 and T_2 be stated in lb. weight, and let the speed of the belt be V feet per minute; then, since T_1 is assisting the motion and T_2 is opposing it, we have

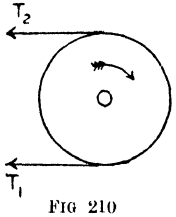


FIG 210

$$\text{Work done per minute} = (T_1 - T_2)\,V \text{ foot-lb.}$$

$$\text{Hence, Horse-power transmitted} = \frac{(T_1 - T_2)\,V}{33{,}000}.$$

EXERCISES ON CHAPTER XIII.

1. A load of 3 tons weight is raised from the bottom of a shaft 600 feet deep. Calculate the work done.

2. In Question 1 the wire rope used for raising the load weighs 12 lb. per yard. Find the total work done.

3. Calculate the work done in hauling a loaded truck, weight 12 tons, along a level track one mile long. The resistances to motion are 11 lb. weight per ton weight of truck.

4. A well is 100 feet deep and 10 feet in diameter, and is full of water (62·5 lb. weight per cubic foot). Calculate the work done in pumping the whole of the water up to ground level.

5. A pyramid of masonry has a square base of 40 feet side and is 30 feet high. If masonry weighs 150 lb. per cubic foot, how much work must be done against gravity in placing the stones into position?

6. The head of a hammer has a mass of 2 pounds and is moving at 40 feet per second. Find the kinetic energy.

7. A ship having a mass of 15,000 tons has a speed of 20 knots (1 knot = 6080 feet per hour). What is the kinetic energy in foot-tons? If the ship is brought to rest in a distance of 0·5 mile, what has been the average resistance?

8. A train having a mass of 200 tons is travelling at 30 miles per hour on a level track. Find the average pull in tons weight which must be applied in order to increase the speed to 40 miles per hour while the train travels a distance of 3000 feet. Neglect frictional resistances.

9. A bullet has a mass of 0·03 pound, and is fired with a velocity of 2400 feet per second into a sand-bank. If the bullet penetrates a distance of 3 feet, what has been the average resistance?

10. A horse walks at a steady rate of 3 miles an hour along a level road and exerts a pull of 80 lb. weight in dragging a cart. What horse-power is he developing?

11. Find the useful horse-power used in pumping 5000 gallons of water per minute from a well 40 feet deep to the surface of the water. Supposing 40 per cent. of the horse-power of the engine driving the pump is wasted, what is the horse-power of the engine?

12. A pump raises 6·2 cubic feet of water per second to a height of 7 feet; how much horse-power must be supplied if 55 per cent. is wasted? The pump is driven by an electro-motor, and current is supplied at 200 volts. How many amperes of current must be supplied to the motor assuming that the motor wastes 15 per cent. of the energy supplied to it?

13. A horizontal force of 8 lb. weight can keep a load weighing 30 lb. in steady motion along a horizontal table. What is the coefficient of friction? What is the minimum inclination of the table to the horizontal if the block is just able to slide steadily on it?

14. A block of oak rests on an oak plank 8 feet long. To what height must one end of the plank be raised before slipping will occur? The coefficient of friction is 0·45.

15. A block weighing W lb. is dragged along a level table by a force P lb. weight acting at a constant angle θ to the horizontal. The coefficient of friction is 0·25. Take successive values of $\theta = 0$, 15, 30, 45, 60 and 75 degrees, and calculate in terms of W (*a*) the values of P, (*b*) the work done in dragging the block a distance of 1 foot. Plot graphs showing the relation of P and θ, and the relation of the work done and θ.

16. A block weighs W lb. and is pushed up an incline making an angle θ with the horizontal by a force P lb. weight which acts in a direction parallel to the incline. The coefficient of friction is 0·25. Find in terms of W (*a*) the values of P, (*b*) the work done in raising the block through a vertical height of one foot, in each case taking successive values of $\theta = 0$, 15, 30, 45, 60, 75 and 90 degrees. Plot graphs of P and θ, and of the work done and θ.

17. Answer Question 16 if P is horizontal. At what value of θ does P become infinite?

18. A block slides down a plane inclined at 45° to the horizontal. If the coefficient of friction is 0·2, what will be the acceleration?

19. When a rope is coiled 180 degrees round a post, it is found that slipping occurs when one end is pulled with a force of 30 lb. weight and the other end with a force of 50 lb. weight. Supposing that the force of 30 lb. weight remains unchanged, and that three complete turns are given to the rope round the post, what force would just cause slipping?

20. A belt runs at 2000 feet per minute. The pulls in the straight portions are 200 and 440 lb. weight respectively. What horse-power is being transmitted?

21. A belt transmits 60 horse-power to a pulley. If the pulley is 16 inches in diameter and runs at 263 revolutions per minute, what is the difference of the tensions on the two straight portions?

22. A pile-driver weighing 3 cwt. falls from a height of 20 feet on a pile weighing 15 cwt.: if there is no rebound, calculate how far the pile will be driven against a constant resistance equal to the weight of 30 cwt.? Sen. Cam. Loc.

23. Define energy, kinetic energy, and potential energy; and show that when a particle of mass m is dropped from a height h, the sum of its kinetic and potential energies at any instant during motion is constant and equals mgh. Calcutta Univ.

24. A motor-car develops 20 horse-power in travelling at a speed of 40 miles per hour up a hill having a slope 1 in 50. If the frictional resistance is 80 lb. wt. per ton weight of car, find the weight of the car, and the speed it could reach on the level, supposing the horse-power developed and the resistance to be unaltered.

25. Explain the difference between the momentum and the kinetic energy of a moving body. Two bodies, A and B, weigh 10 lb. and 40 lb. respectively. Each is acted upon by a force equal to the weight of 5 lb. Compare the times the forces must act to produce in each of the bodies (*a*) the same momentum, (*b*) the same kinetic energy. L.U.

26. A cyclist always works at the rate of $\frac{1}{10}$ H.P., and rides at 12 miles an hour on level ground and 10 miles an hour up an incline of 1 in 120. If the man and his machine weigh 150 lb., and the resistance on a level road consists of two parts, one constant and the other proportional to the square of the velocity, show that, when the velocity is v miles per hour, the resistance is $\frac{5}{132}(76 + v^2)$ lb. wt. Find also the slope up which he would travel at the rate of 8 miles per hour. L.U.

27. Define work and power, and give their dimensions in terms of the fundamental units of mass, length and time. The maximum speed of a motor van weighing 3 tons is 12 miles an hour on a level road, but drops to 5 miles an hour up an incline of 1 in 10. Assuming resistances per ton to vary as the square of the velocity, find the horse-power of the engine. L.U.

28. A bicycle is geared up to 70 inches, and the length of the pedal-cranks is 6 inches. Calculate the velocity of the pedal (*a*) at its highest point, (*b*) at its lowest point, when the bicycle is travelling at 10 miles an hour. If the bicycle and rider weigh 160 lb., find the pressure on the pedals in climbing a hill of 1 in 20. L.U.

29. Explain what is meant by (1) the coefficient of friction, (2) the angle of friction.

A window curtain weighing 4 lb. hangs by 6 equidistant thin rings from a curtain rod in such a way that the weight is equally distributed between the rings. If the coefficient of friction is 0·6, and the rings are 6 inches apart, find the work done in drawing the curtain back to the position of the end ring. L.U.

30. A body having a mass of 20 pounds is placed on a rough horizontal table, and is connected by a horizontal cord passing over a pulley at the edge with a body having a mass of 10 pounds hanging vertically. If the coefficient of friction between the body and the table be 0·25, find the acceleration of the system and the pull in the cord.

31. Explain what is meant by the "angle of friction." If a body be placed on a rough horizontal plane, show that no force, however great, applied towards the plane at an angle with the normal less than the angle of friction, can push the body along the plane.

A uniform circular hoop is weighted at a point of the circumference with a mass equal to its own. Prove that the hoop can hang from a rough peg with any point of its circumference in contact with the peg, provided that the angle of friction exceeds 30°. Adelaide University.

32. A ladder of length $2a$ leans against a perfectly smooth wall, the ground being slightly rough. The weight of the ladder is w; and its centre of gravity is at its middle point. The inclination to the vertical is gradually increased till the ladder begins to slip. The inclination is then further increased, and the ladder is prevented from slipping by the smallest possible horizontal force applied at the foot. Find the magnitude of this force if μ is the coefficient of friction and θ the final inclination to the vertical. Tasmania University.

CHAPTER XIV

SIMPLE MACHINES

Machines.—A **machine** is an arrangement designed for the purpose of taking in energy in some definite form, modifying it, and delivering it in a form more suitable for the purpose in view.

There is a large class of machines designed for the purpose of raising loads, many of these machines can be used for experimental work in laboratories. The **crab** shown in Fig. 211 is an example. The rope to which the load W is attached is wound round a cylindrical barrel A. The machine is driven generally by hand by means of handles For the purposes of experiment, the handles have been removed and a wheel D substituted D is rotated by means of a cord and weights placed in a scale-pan at P, and drives the barrel by medium of the toothed wheels C and B.

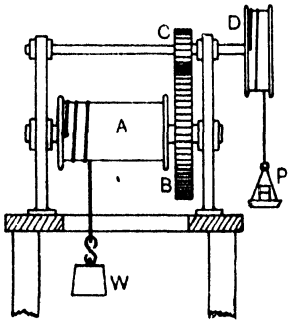


FIG 211.—A small lifting crab

Energy is supplied to this machine by means of a comparatively small force P acting through a large distance, and is delivered by the machine in the form of the work done in overcoming a large force W through a small distance.

If no energy were wasted in a machine, it would follow, from the conservation of energy, that the energy supplied must be equal to the energy delivered by the machine Thus, referring to Fig. 211,

Work done by P = Work done on W.

This statement is generally referred to as the **principle of work**, and requires modification for actual machines, in which there is always some energy wasted. Actually, the energy supplied is equal to the sum of the energy delivered by the machine and the energy

wasted. The investigation of frictional resistances in the various kinds of lubricated rubbing surfaces of machines is beyond the scope of this book. Usually, however, it is the determination of the total waste of energy in the machine which is of importance, and experiments having this object are performed easily in the case of simple machines used for raising loads.

Some definitions regarding machines.—In Fig. 212 is shown an outline diagram of the crab illustrated in Fig. 211. Let W be raised through a height h while P descends through a height H, H and h being in the same units. The **velocity ratio** of the machine is defined as the ratio of the distance moved by P to the distance moved by W in the same time, or

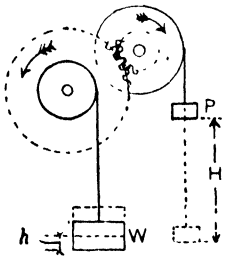


FIG. 212.—Outline diagram of an experimental crab.

$$\text{Velocity ratio} = V = \frac{H}{h}. \quad \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

H and h may be obtained by direct measurement, or they may be calculated from known dimensions of the parts of the machine.

The **mechanical advantage** of the machine is the ratio of the actual load raised to the force required to operate the machine at a constant speed.

$$\text{Mechanical advantage} = \frac{W}{P}. \quad \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

Neglecting any waste of energy in the machine, the work done by P would be equal to the work done in raising the load, and, in these circumstances, the load raised would be larger than W. Let W_1 be this hypothetical load, then

$$\text{Work done by P} = \text{work done on } W_1,$$

$$PH = W_1 h,$$

$$W_1 = P\frac{H}{h} = PV. \quad \ldots\ldots\ldots\ldots\ldots\ldots (3)$$

The effect of frictional and other sources of waste in the actual machine has been to diminish the load from W_1 to W. Hence

$$\text{Effect of friction} = F = W_1 - W$$

$$= PV - W. \quad \ldots\ldots\ldots\ldots\ldots\ldots (4)$$

The **efficiency** of any machine is defined as the ratio of the energy delivered to the energy supplied in the same time.

$$\text{Efficiency} = \frac{\text{energy delivered}}{\text{energy supplied}}$$

$$= \frac{Wh}{PH} = \frac{W}{P} \times \frac{1}{V}$$

$$= \frac{\text{mechanical advantage}}{\text{velocity ratio}}. \quad \ldots\ldots\ldots\ldots (5)$$

The efficiency thus stated will be always less than unity. Efficiency is often given as a percentage, obtained by multiplying the result given in (5) by 100. 100 per cent. efficiency could be obtained only under the condition of no energy being wasted in the machine, a condition impossible to attain in practice.

From equation (3) we have

$$W_1 = P\frac{H}{h},$$

or

$$\frac{W_1}{P} = \frac{H}{h} = V. \quad \ldots\ldots\ldots\ldots\ldots\ldots (6)$$

This result shows that the mechanical advantage of an ideal machine, having no waste of energy, is equal to the velocity ratio.

A typical experiment on a machine.—In the following experiment a complete record is given of tests on a small crab; this record will serve as a model for any other hoisting machines available.

EXPT. 32.—**Efficiency, etc., of a machine for raising loads.** The machine used was a small crab illustrated in Fig. 211 and shown in outline in Fig. 212. By direct measurement of the distances moved by P and W, the velocity ratio was found to be V = 8·78. This was confirmed by calculation:

Diameter of barrel to centre of rope sustaining W = 6·4 inches.
Diameter of wheel to centre of cord sustaining P = 7·9 inches.
Number of teeth on the barrel wheel, 128.
Number of teeth on the pinion, 18.

Let the barrel make one revolution, then

Height through which W is raised $= \pi \times 6{\cdot}4$ inches.
Number of revolutions of grooved wheel $= \frac{1}{18}$.
Height through which P descends $= \frac{128}{18} \times \pi \times 7{\cdot}9$.

Hence,

$$\text{Velocity ratio} = \frac{128 \times \pi \times 7{\cdot}9}{18 \times \pi \times 6{\cdot}4}$$

$$= 8{\cdot}78.$$

The weight of the hook from which W was suspended is 1·75 lb. The weight of the scale-pan in which were placed the weights at P is 0·665 lb.

The machine was first oiled, and a series of experiments was made, in each case finding what force P was required to produce constant speed in

the machine for each value of W. There must be no acceleration, otherwise a portion of P will be utilised in overcoming inertia in the moving parts of the machine, and also in P and W. As the test has for its object the investigation of frictional resistances only, inertia effects must be eliminated, and this is secured by arranging that the speed shall be uniform. The results obtained are given below, together with the calculated values of the loads W_1 which could be raised if there were no friction, the effect of friction F, the mechanical advantage and the efficiency.

RECORD OF EXPERIMENTS AND RESULTS.

(1) W lb wt, including weight of hook	(2) P lb wt, including weight of scale-pan	(3) Load W_1 if no frictional resistances, $W_1 = PV$ lb.	(4) Effect of friction, $F = (W_1 - W)$ lb	(5) Mechanical advantage, $\frac{W}{P}$	(6) Efficiency, per cent, $\frac{(5)}{V} \times 100$
8 75	1 785	15·7	6·95	4 9	55·8
15 75	2 665	23·4	7 65	5 9	67·2
22 75	3·565	31 3	8 55	6·38	72 6
29·75	4·405	38 7	8·95	6·74	76 6
36·75	5·335	46 8	10·05	6 89	78 5
43·75	6·215	54 6	10·85	7 04	80·0
50·75	7·115	62 5	11·75	7·14	81·2
57 75	8·065	70·8	13·05	7 16	81·6
64·75	8 915	78·4	13·65	7·26	82·7
71·75	9·815	86·2	14·45	7·30	83·2
78 75	10 705	94·1	15·35	7·36	83·7
85·75	11·59	101·8	16·05	7·40	84·3
92·75	12·515	110	17·25	7 41	84 4
99 75	13 405	118	18·25	7·43	84·6
106 75	14·285	125·4	18 65	7 47	85 0
113·75	15·205	133·8	20·05	7·48	85·2
120·75	16 065	141	20 25	7·51	85·5
127 75	16·965	149	21 25	7 53	85·7

Curves are plotted in Fig. 213 showing the relation of P and W and also that of F and W. It will be noted that these give straight lines. Curves of mechanical advantage and of efficiency in relation to W are shown in Fig. 214. It will be noted that both increase rapidly when the values of W are small and tend to become constant when the value of W is about 120 lb. The efficiency tends to attain a constant value of 86 per cent.

As both the curves showing the relation of P and of F with W are straight lines, it follows that the following equations will represent these relations:

$$P = aW + b, \quad \text{.....(1)}$$

$$F = cW + d, \quad \text{.....(2)}$$

where a, b, c and d are constants to be determined from the graphs.

Select two points on the PW graph, and read the corresponding values of P and W. P = 3·5 lb. wt. when W = 22·7 lb. wt.

P = 16·0 lb. wt. when W = 120·0 lb. wt.

Hence, from (1), $3{\cdot}5 = 22{\cdot}7a + b$,

$16 = 120a + b$.

Solving these simultaneous equations, we obtain

$a = 0{\cdot}128$, $b = 0{\cdot}64$;

$$P = 0{\cdot}128W + 0{\cdot}64. \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (3)$$

Similarly, When F = 8 lb. wt., W = 20 lb. wt.

When F = 18 lb. wt., W = 100 lb. wt.

Hence, from (2), $8 = 20c + d$,

$18 = 100c + d$.

The solution of these gives

$c = 0{\cdot}125$, $d = 5{\cdot}5$.

Hence, $$F = 0{\cdot}125W + 5{\cdot}5. \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (4)$$

If both the load and the hook sustaining the load be removed so that there is no load on the machine, the machine may be run light. The values of P and F for this case may be found from (3) and (4) by making W equal to zero, when P = 0·64 lb. wt., F = 5·5 lb. wt.

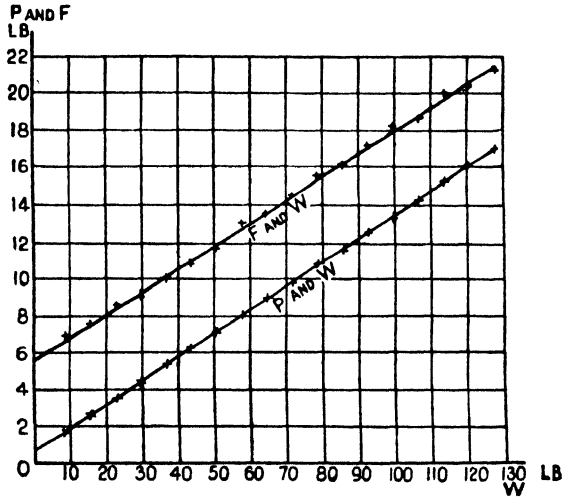


FIG. 213.—Graphs of F and W, and P and W, for a small crab.

The interpretation is that a force of 0·64 lb. wt. is required to work the machine when running light, and that, if there were no frictional waste,

a load of 5·5 lb. wt. could be raised by this force. These values are shown in Fig. 213 by the intercepts on the OY axis between O and the points

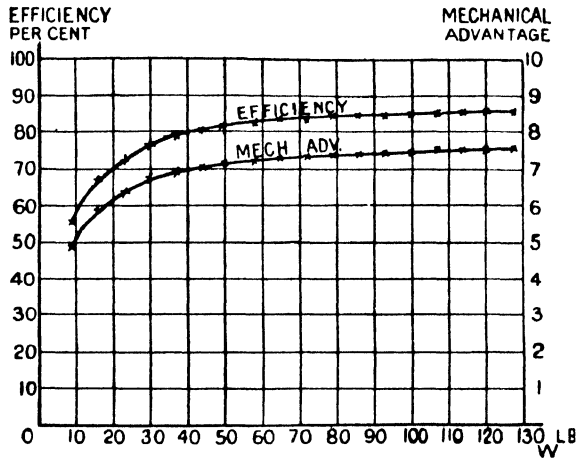


FIG. 214.—Graphs of efficiency and mechanical advantage for a small crab.

where the graphs of P and F cut the axis.

Principle of work applied to levers.—In Fig. 215 (*a*) is shown a lever AB, pivoted at C, and balanced under the action of two loads W and P. The weight of the lever is neglected. Let the lever be displaced slightly from the horizontal, taking up the position A′CB′. Work has been done by W to the amount of $W \times A'D$, and work has been done on P to the amount of $P \times B'F$. Assuming that there has been no frictional or other waste of energy, we have

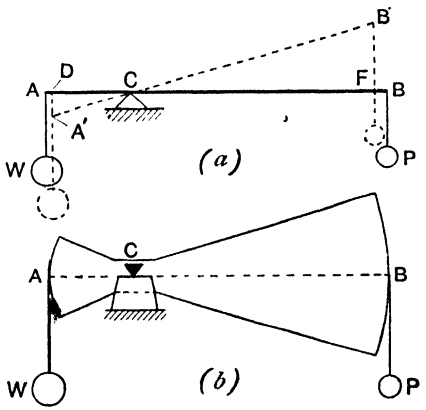


FIG. 215.—Principle of work applied to levers.

$$W \times A'D = P \times B'F.$$

The triangles A′DC and B′FC are similar; hence

$$A'D : B'F = A'C : B'C = AC : BC;$$

$$\therefore\ W \times AC = P \times BC.$$

This result agrees with that which would have been obtained by application of the principle of moments.

In Fig. 215 (*b*) is shown the same lever with the addition of circular sectors for receiving the cords. It is evident that the arms AC and BC are of constant length in this lever. If the lever is turned through a small angle α radians, W will be lowered through a height h and P will be raised through a height H, and we have

$$\frac{h}{AC} = \alpha = \frac{H}{BC},$$

$$\therefore\ h = AC \cdot \alpha, \text{ and } H = BC \cdot \alpha$$

Assuming no friction,

$$\text{Work done by W} = \text{work done on P},$$

$$Wh = PH,$$

or

$$W \times AC \times \alpha = P \times BC \times \alpha,$$

$$\therefore\ W \times AC = P \times BC,$$

a result which again agrees with the principle of moments.

In Fig. 216 the sectors are of the same radius and are extended to form a complete wheel. It is evident that P and W will be equal if there be no friction. Such wheels are called **pulleys**, and are much used for changing the direction of a rope or chain under pull, and are found often in tackle used for raising loads.

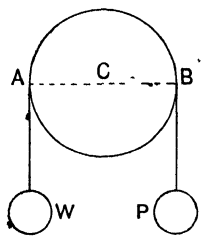


FIG. 216.—Use of a pulley

Hoisting tackle.—The fact that the mechanical advantage of a machine, neglecting friction, is equal to the velocity ratio (p 186) enables the latter to be calculated easily in the following cases of hoisting tackle.

Simple pulley arrangements.—In the pulley-block arrangement shown in Fig. 217, let n be the number of ropes leading from the lower to the upper block. Neglecting friction, each rope supports W/n; this will also be the value of P. Hence

$$V = \frac{W}{P} = \frac{Wn}{W} = n.$$

In the arrangement shown in Fig 218 (seldom used in practice) each rope A and B sustains $\frac{1}{2}W$; the pull in B is balanced by the pulls in C and D, therefore C and D have pulls each equal to $\frac{1}{4}W$;

hence E and F have pulls equal to $\frac{1}{8}$W, and the pull in G is also $\frac{1}{8}$W and is equal to P. Thus

$$V = \frac{W}{P} = \frac{W}{\frac{1}{8}W} = 8.$$

In the arrangement shown in Fig. 218 there are three inverted pulleys. Had there been n inverted pulleys, the value of P would have been

$$P = \frac{W}{2^n}, \quad \text{and} \quad V = \frac{W}{P} = 2^n.$$

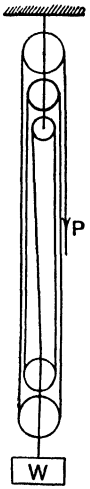


FIG. 217.—A common pulley-block arrangement.

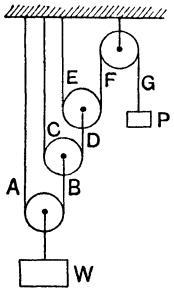


FIG 218.—Another pulley arrangement

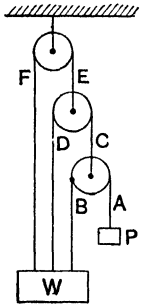


FIG 219.—Another arrangement of pulleys.

In the system shown in Fig. 219 (also seldom employed) the pulls in A and B will be each equal to P; hence the pull in C is 2P (neglecting the weight of the pulley), and equals the pull in D. The pull in E is thus 4P and equals the pull in F. Hence

$$W = \text{pull in B} + \text{pull in D} + \text{pull in F}$$
$$= P + 2P + 4P = 7P.$$
$$V = \frac{W}{P} = 7.$$

It is evident that P, 2P and 4P are terms in a geometrical progression having a common ratio 2. Hence, if there be n pulleys, we may write

$$W = P + 2P + 2^2P + 2^3P + \ldots + 2^{n-1}P$$
$$= P\left(\frac{2^n - 1}{2 - 1}\right) = P(2^n - 1);$$
$$\therefore \quad V = \frac{W}{P} = 2^n - 1.$$

The Weston's differential blocks shown in outline in Fig. 220 are much used in practice. The upper block has two pulleys of different diameters, and are fixed together, an endless chain, shown dotted, is arranged as shown. The links of the chain engage with recesses formed in the rims of the pulleys and thus cannot slip. Neglecting friction, each of the chains A and B support $\frac{1}{2}W$. Taking moments about the centre C of the upper pulleys, and calling the radii R and r respectively, we have

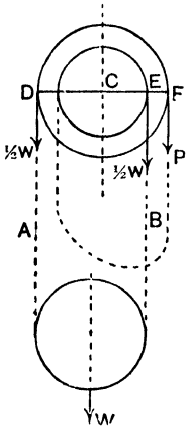


Fig. 220.—Outline diagram of Weston's differential blocks

$$\tfrac{1}{2}W \times CD = (P \times CF) + (\tfrac{1}{2}W \times CE),$$

$$\tfrac{1}{2}W(R - r) = PR,$$

$$\therefore V = \frac{W}{P} = \frac{2R}{R - r}.$$

Instead of R and r, the number of links which can be fitted round the circumferences of the upper pulleys may be used; evidently these will be numbers proportional to R and r

The **wheel and differential axle** (Fig. 221) is a similar contrivance, but has a separate pulley A for receiving the hoisting rope. Taking moments as before, we have

$$PR_A + \tfrac{1}{2}WR_B = \tfrac{1}{2}WR_C,$$

$$PR_A = \tfrac{1}{2}W(R_C - R_B),$$

$$V = \frac{W}{P} = \frac{2R_A}{R_C - R_B}.$$

A set of **helical blocks** is shown in outline in Fig. 222. The pulley A is operated by hand by means of an endless chain, and rotates a **worm** B. The worm is simply a screw cut on the spindle, and engages with the teeth on a **worm-wheel** C. Each revolution of B causes one tooth on C to advance; hence, if there be n_C teeth on the worm-wheel, B will have to rotate n_C times in order to cause C to make one revolution. Let L_A be the length of the number of links of the operating chain which will pass once round A,

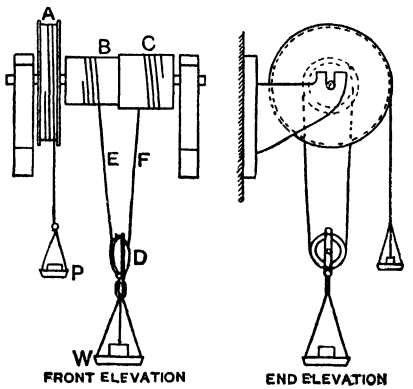


Fig. 221.—Wheel and differential axle.

then P will advance a distance $n_C L_A$ for one revolution of C. The chain sustaining the load W is fixed at E to the upper block, passes round F, and then is led round D, which has recesses fitting the links in order to prevent slipping. Let L_D be the length of the number of links which will pass once round D, then in one revolution of D, W will be raised a height equal to $\frac{1}{2}L_D$. Hence

$$V = \frac{n_C L_A}{\frac{1}{2}L_D} = \frac{2n_C L_A}{L_D}.$$

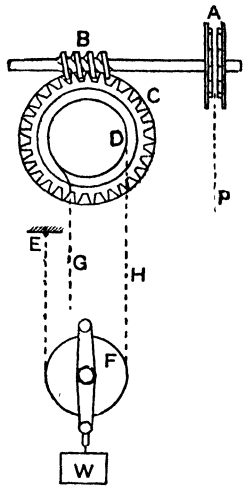


FIG. 222.—Helical blocks.

Screws.—In Fig. 223, A is a cylindrical piece having a helical groove cut in it, thus leaving a projecting screw-thread which may be of square outline as shown in Fig. 223, or V as in a common bolt. A **helix** may be defined as a curve described on the surface of a cylinder by a point which travels equal distances parallel to the axis of the cylinder for equal angles of rotation. The pitch of the screw is the distance measured parallel to the axis from a point on one thread to the corresponding point on the next thread. In Fig. 223, B is a sliding block guided so that it cannot rotate, and having a hole with threads to fit those on A. A can rotate, but the collars on it prevent axial movement. One revolution of A will therefore move

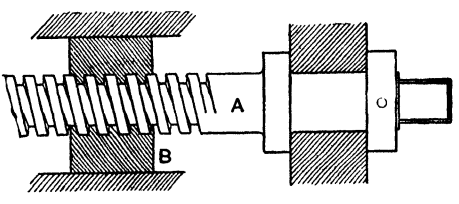
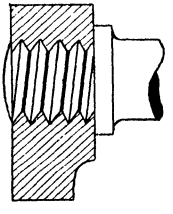


FIG. 223.—Section through a nut, B, showing screw, A.


FIG 224.—A left-handed screw.

B through a distance equal to the pitch. If there be n threads per inch, then the pitch $p = 1/n$ inch. The thread shown in Fig. 223 is right-handed; that shown in Fig. 224 is left-handed. Screws are generally made right-handed unless there is some special reason for the contrary; thus the right pedal-pin of a bicycle has generally

a left-handed screw where it is fixed to the crank; the action of pedalling then tends to fix it more firmly, whilst a right-handed screw might become unscrewed.

In Fig. 225 is shown a differential screw. A has a screw of pitch p_1 fitting a screwed hole in B. One revolution of A (the handle moving away from the observer) will advance it towards the left through a distance p_1. C has another screw of smaller pitch p_2 cut on it, and fits a screwed hole in the sliding block D. If A had no axial movement, D would move towards the right through a distance equal to p_2. The actual movement of D towards the left will therefore be $(p_1 - p_2)$ for each revolution of A. By making p_1 and p_2 very nearly equal, a very slow movement may be given to D.

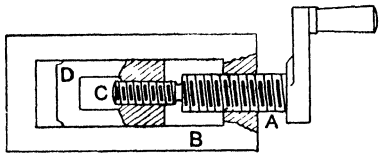


FIG. 225.—A differential screw.

EXPT. 33.—**The screw-jack.** This device for raising loads is shown in Fig. 226. A hollow case A has a hole at the top screwed to receive a square-threaded screw B. The load W lb. weight rests on the top of B; C is a loose collar interposed to prevent the load rotating with the screw. The screw is rotated by means of a bar D. Let a force P lb. weight be applied to D at a distance R inches from the axis of the screw, and let P act horizontally at right angles to the bar. Let the pitch of the screw be p inches. Then, if the screw makes one revolution,

Work done by P $= P \times 2\pi R$ inch-lb.

Work done on W $= W \times p$ inch-lb.

Assuming that there is no waste of energy, we have

$$W \times p = P \times 2\pi R,$$

or

$$\frac{W}{P} = \frac{2\pi R}{p}.$$

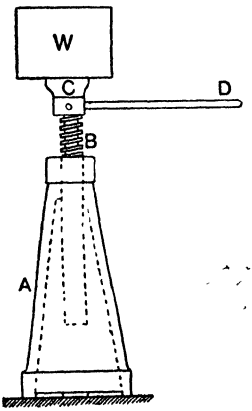


FIG. 226.—Screw-jack.

This result gives the mechanical advantage neglecting friction, and is therefore equal to the velocity ratio of the machine. Hence

$$\text{Velocity ratio} = V = \frac{2\pi R}{p}.$$

For experimental purposes the bar D is removed, and a pulley having a grooved rim takes its place. A cord is wound round the rim of the pulley, passes over another fixed pulley and has a scale-pan at its free

end. Make a series of experiments with a gradually increasing series of loads W, determining P for each. Reduce the results as directed previously (pp. 186 to 189).

Exercises on Chapter XIV.

1. In a set of pulley blocks there are two pulleys in the upper block and one in the lower block. The rope is fastened to the lower block, passes round one of the upper pulleys, then round the lower pulley, and lastly round the other upper pulley. An effort of 70 lb. weight is required to raise a load of 150 lb. weight. Find the velocity ratio and the mechanical advantage, also the effect of friction and the efficiency with this load.

2. In a system of pulleys similar to that shown in Fig. 218 there are four movable pulleys each weighing 6 lb. Neglect friction, and calculate what effort must be applied if there is no load. If the efficiency is 60 per cent.. reckoned on the work done on W and that done by P, find what effort will be required in order to raise a load of 200 lb. weight.

3. A system of pulleys resembling that shown in Fig. 219 has three movable pulleys, each of which weighs 4 lb. Neglecting friction, what effort will be required to sustain a load of 60 lb. weight ? If the efficiency is 70 per cent., reckoned as in Question 2, what effort will be required to raise a load of 60 lb. weight ?

4. The barrel of a crab is 6 inches diameter to the centre of the rope sustaining the load ; the wheel on the barrel shaft has 80 teeth, and the pinion gearing with it has 20 teeth. The machine is driven by a handle 15 inches in radius. Find the velocity ratio. If the efficiency is 70 per cent., what load can be raised by an effort of 30 lb. weight applied to the handle ? What is the mechanical advantage under these conditions ?

5. In a Weston's differential pulley block, the numbers of chain links which can be passed round the circumferences of the pulleys are 16 and 15 respectively. Find the velocity ratio of the machine. If a load of 550 lb. weight can be raised by an effort of 20 lb. weight, what are the values of the mechanical advantage, the effect of friction and the efficiency ?

6. In a wheel and differential axle the wheel is 24 inches in diameter, and the barrel has diameters of 7 and 6 inches respectively. Find the velocity ratio. What load can be raised by an effort of 30 lb. weight if the efficiency is 65 per cent. ? Under these conditions, what are the values of the mechanical advantage and the effect of friction ?

7. In a machine for testing materials under torsion, one end of the test piece is attached to the axle of a worm-wheel and the other end is fixed. The worm-wheel has 90 teeth, and is driven by a worm and hand-wheel. If the hand-wheel is rotated 785 times before the specimen breaks, how many degrees of twist have been given to the specimen ? If the average torque on the specimen was 2400 lb.-inches, and if the efficiency of the machine is 70 per cent., how much work was done on the hand-wheel ?

8. The screw of a screw-jack is 0·5 inch pitch and the handle is 19 inches long. The efficiency is 50 per cent. What effort must be applied

to the handle in raising a load of one ton weight? What is the maximum value of the efficiency of any machine?

9. A block and tackle is used to raise a load of 200 lb.; the rope passes round three pulleys in the fixed block, and round two in the movable block, to which is fastened the load and one end of the rope. Calculate the force which must be applied to the rope.

Assuming that, owing to the effect of friction, the tension on one side of a pulley is $\frac{4}{5}$ths of the tension on the other side of the pulley, prove that the force required to raise the load must be increased to over 74 lb. Sen. Cam. Loc.

10. Define the terms "velocity ratio," "mechanical advantage" and "efficiency" as applied to machines, and show that one of these quantities is equal to the product of the other two. In a lifting machine the velocity ratio is 30 to 1. An effort of 10 lb. is required to raise a load of 35 lb., and an effort of 25 lb. a load of 260 lb. Find the effort required to raise a load of 165 lb. and the efficiency under this load Assume a linear relation between effort and load. L.U.

11. How is the work done by a force measured? Define erg, foot-poundal, foot-pound. A vertical rubber cord is stretched by gradually loading a scale-pan attached to its lower end, and a graph is drawn showing the relation between the load and the extension of the cord. Explain how the work done in stretching the cord may be found from the graph L.U.

12. Find the condition of equilibrium for a system of pulleys in which each pulley hangs in the loop of a separate string, the strings being all parallel and each string attached to the beam. The weights of the pulleys are to be taken into account.

If there are 5 pulleys and each weighs 1 lb., what weight will a force equal to the weight of 6 pounds support on such a system, and what will be the total pull on the beam? L.U.

13. Find the velocity ratio, mechanical advantage and efficiency of a screw-jack, whose pitch is $\frac{1}{4}$ inch, and the length of whose arm is 15 inches, if the tangential force at the end of the arm necessary to raise one ton is 24 lb. weight. L.U.

14. Describe the construction of a differential screw, and on the assumption of the principle of work (or otherwise) calculate its velocity ratio. If the two screws have 2 threads and 3 threads to the inch respectively, and a couple of moment 20 lb.-wt.-ft. applied to the differential screw produces a thrust equal to the weight of half a ton, calculate the efficiency of the machine. L.U.

15. A body having a weight W is pushed up a rough inclined plane by a force P which acts in a line parallel to the plane. The length, height and base of the plane are L, H and B respectively. Find the work done by P, taking μ as the coefficient of friction. Show that this work is the same as the work done by a horizontal force in pushing the body along a horizontal plane of length B, and having the same value of μ, and then elevating the body through a height H. Find the mechanical advantage, *i.e.*, the ratio W/P, in the case in which $\mu = H/B$.

16. Describe the system of pulleys in which the same rope goes round all the pulleys, and find the mechanical advantage (neglecting friction). If one end of the rope is attached to the lower block, and there are five pulleys in all, find the pull which is necessary to raise a mass of one ton. Find also the power required to pull the free end at a speed of 5 ft. per second. Madras Univ.

CHAPTER XV

MOTION OF ROTATION

Centre of mass.—In Fig. 227 is shown a body travelling towards the left in such a manner that every particle has rectilinear motion only; this kind of motion is called **pure translation.** Let the body as a whole have an acceleration a, then every particle will have this acceleration If the masses of the particles be m_1, m_2, m_3, etc, the particles will offer resistances, due to their inertia, given by m_1a, m_2a, m_3a, etc. These forces are parallel; hence the resultant resistance is $R = m_1a + m_2a + m_3a + \text{etc.} = \Sigma ma = a\Sigma m.$ .....(1)

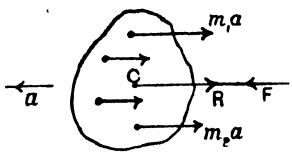


FIG. 227.—Centre of mass of a body.

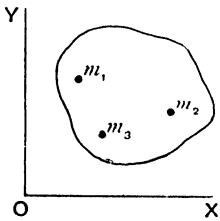


FIG. 228.—Centre of mass of a thin sheet.

The centre of these parallel forces (p. 106) is called the **centre of mass** of the body. To find the centre of mass of a thin sheet (Fig. 228), take reference axes OX and OY. Let the coordinates of m_1, m_2, m_3, etc., be (x_1y_1), (x_2y_2), (x_3y_3), etc. Let the sheet have pure translation parallel to OY, and let the acceleration be a. Take moments about O, giving

$$m_1ax_1 + m_2ax_2 + m_3ax_3 + \text{etc.} = a\Sigma mx = a(\Sigma m)\bar{x},$$

where $\bar{x}$ is the abscissa of the centre of mass. Hence

$$\bar{x} = \frac{\Sigma mx}{\Sigma m}. \quad \text{.....(2)}$$

Similarly, by assuming pure translation with acceleration a parallel to OX, we obtain

$$\bar{y} = \frac{\Sigma my}{\Sigma m}. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3)$$

The student will note that these equations are similar to those employed for finding the centre of gravity (p. 109), the only difference being the substitution of mass for weight. It may be assumed that the centre of mass coincides with the centre of gravity, and all the methods employed in Chapter IX. may be used for determining the centre of mass.

Referring again to Fig. 227, C is the centre of mass and R is the resultant resistance due to inertia and acts through C. If a force F be applied to the body, and passes through C, it is evident that F and R will act in the same straight line and the motion will be pure translation. The truth of the principle that a force passing through the centre of mass of a body produces no rotation may be tested by laying a pencil on the table and flicking it with the finger nail. An impulse applied near the end of the pencil causes the pencil to fly off, rotating as it goes; an impulse applied through the centre of mass produces no rotation.

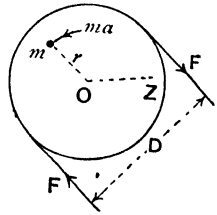


FIG 229.—Relation between the couple and the rotational inertia.

Rotational inertia.—To produce pure rotation in a body, *i.e.* the centre of mass remains at rest, requires the application of a couple. The effect of the equal opposing parallel forces is not to produce translational motion. Let a body be free to rotate about an axis OZ perpendicular to the plane of the paper (Fig. 229). Let a couple F, F, be applied, and let the couple rotate with the body so that its effect is constant. The body will have angular acceleration which we proceed to determine.

Consider a particle having a mass m and at a radius r from OZ. Let the particle have a linear acceleration a in the direction of the tangent to its circular path. The inertia of the particle causes it to offer a resistance ma. Let ϕ be the angular acceleration, then

$$a = \phi r.$$

Also, Resistance of the particle $= ma = m\phi r$.

To obtain the moment of this resistance about OZ, multiply by r, giving Moment of resistance of particle $= m\phi r^2 = \phi m r^2$.

Now ϕ is common for the whole of the particles; hence we have:

$$\text{Total moment of resistance} = \phi(m_1r_1^2 + m_2r_2^2 + m_3r_3^2 + \text{etc.})$$
$$= \phi\Sigma mr^2.$$

This moment balances the moment of the applied couple. Let the moment of the couple be $\mathsf{L} = \mathsf{FD}$, then

$$\mathsf{L} = \phi\Sigma mr^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

It will be noted that L must be stated in absolute units in using this equation

Σmr^2 is called the second moment of mass, or, more commonly, the **moment of inertia** of the body. It is a quantity which depends upon the mass and the distribution of the mass with reference to the axis of rotation. It is usual to denote it by I, and to add a suffix indicating the axis for which the moment of inertia has been calculated; thus

$$\mathsf{L} = \mathsf{I}_{oz}\phi. \quad \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

In the C.G.S. system state L in dyne-centimetres, and I in grams mass and centimetre units; in the British system state L in poundal-feet, and I in pounds mass and foot units. ϕ is in radians per second per second in both systems.

The dimensions of moment of inertia are ml^2.

Gravitational units may be employed; thus, if T is the moment of the applied couple in lb.-feet, and I is the moment of inertia in pounds-mass and foot-units, then

$$\mathsf{T} = \frac{\mathsf{I}_{oz}\phi}{g} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

EXAMPLE 1.—A wheel has a moment of inertia of 800 gram and centimetre units. Find what constant couple must be applied to it in order that the angular acceleration may be 2 radians per second per second.

$$\mathsf{L} = \mathsf{I}\phi$$
$$= 800 \times 2 = \underline{1600} \text{ dyne-centimetres.}$$

EXAMPLE 2.—A grindstone has a moment of inertia of 660 pound and foot units. A constant couple is applied and the grindstone is found to have a speed of 150 revolutions per minute 10 seconds after starting from rest. Find the couple.

$$\omega = \frac{150}{60} \times 2\pi = 5\pi \text{ radians per sec.}$$

$$\phi = \frac{\omega}{t} = \frac{5\pi}{10} = \frac{\pi}{2} \text{ radians per sec. per sec.}$$

$$\mathsf{T} = \frac{\mathsf{I}\phi}{g} = \frac{600 \times \pi}{g \times 2}$$

$$= \frac{600 \times 22}{32 \cdot 2 \times 2 \times 7} = \underline{29.3} \text{ lb. feet.}$$

Cases of moments of inertia.—A few of the simpler cases of moments of inertia are now discussed.

A thin uniform wire of mass M is arranged parallel to the axis OX (Fig. 230). Every portion of the wire is at the same distance D from the axis ; hence $I_{ox} = \Sigma mD^2 = MD^2$. .....(1)

The same wire is bent into a circle of radius R (Fig. 231) ; the axis OZ passes through the centre and is perpendicular to the plane of the circle. Every portion of the wire is at the same distance R from the axis ; hence $I_{oz} = \Sigma mR^2 = MR^2$. .....(2)

A number of such circular wires laid side by side form a tube ; hence the moment of inertia of the tube with respect to the longitudinal axis is $I_{oz} = MR^2$, ...(3)

where M is the total mass of the tube.

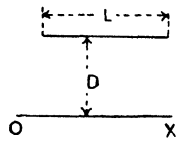


FIG 230

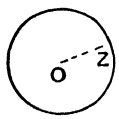


FIG 231

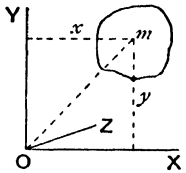


FIG. 232.—$I_{oz} = I_{ox} + I_{oy}$.

An important theorem.—In Fig. 232 is shown a thin plate in the plane of the paper. The coordinates of a small mass m, referred to the axis OX, OY are y and x. We have for the mass m,

$$I_{ox} = my^2 \; ; \quad I_{oy} = mx^2 \; ;$$

$$\therefore \; I_{ox} + I_{oy} = m(y^2 + x^2) = mr^2,$$

where r is the distance of m from the axis OZ, which passes through O and is perpendicular to the plane of the paper. Since $I_{oz} = mr^2$, we have for the particle $I_{ox} + I_{oy} = I_{oz}$.

A similar result can be obtained for any other particle in the plate ; hence, for the whole plate,

$$\begin{aligned} I_{ox} + I_{oy} &= m_1(x_1^2 + y_1^2) + m_2(x_2^2 + y_2^2) + m_3(x_3^2 + y_3^2) + \text{etc.} \\ &= m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2 + \text{etc.} \\ &= I_{oz}. \end{aligned} \quad \text{.....(4)}$$

This result enables us to calculate the moment of inertia in cases which would otherwise require mathematical work of some difficulty.

EXAMPLE.—A thin wire of mass M is bent into a circle of radius R (Fig. 233). Find the moment of inertia with respect to a diameter.

Draw the diameters AB and CD intersecting at right angles at O; let OZ be perpendicular to the plane of the circle. Then

$$I_{OZ} = MR^2.$$

Also $$I_{AB} + I_{CD} = I_{OZ}.$$

From symmetry, $$I_{AB} = I_{CD},$$

$$2I_{AB} = I_{OZ},$$

$$I_{AB} = \tfrac{1}{2}I_{OZ} = \tfrac{1}{2}MR^2.$$

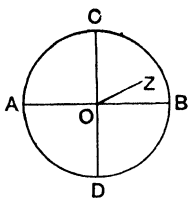


FIG. 233

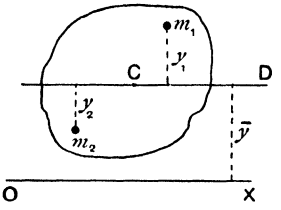


FIG. 234.—$I_{OX} = I_{CD} + M\bar{y}^2$.

Another important theorem.—In Fig. 234 is shown a thin plate in the plane of the paper. CD is in the same plane and passes through the centre of mass of the plate; OX is also in the same plane and is parallel to CD, and at a distance $\bar{y}$ from it. To find the relation of I_{CD} and I_{OX}, we proceed thus:

Considering the particle m_1 at a distance y_1 from CD, we have

$$I_{OX} = m_1(y_1 + \bar{y})^2 = m_1y_1^2 + m_1\bar{y}^2 + 2\bar{y}m_1y_1. \quad \text{.....(5)}$$

Similarly for m_2,

$$I_{OX} = m_2(y - y_2)^2 = m_2y_2^2 + m_2\bar{y}^2 - 2\bar{y}m_2y_2. \quad \text{..... .....(6)}$$

For all particles above CD the moments of inertia are given by expressions similar to (5), and for particles below CD, by expressions similar to (6); hence the total moment of inertia may be obtained by taking the sum of the equations (5) and (6) for every particle in the plate. The first and second terms in both expressions are similar; the third terms differ only in sign. When all the particles in the plate are considered, the sum of the third terms in (5) and (6) evaluates the product $2\bar{y}$ times the simple moment of mass of the plate about CD. Now CD passes through the centre of mass of the plate, and therefore the simple moment of mass with reference to CD is zero; hence we have for the whole plate

$$I_{OX} = \Sigma my^2 + \Sigma m\bar{y}^2.$$

Since $\bar{y}$ is constant, this reduces to

$$I_{OX} = \Sigma my^2 + y^2\Sigma m$$
$$= I_{CD} + My^2, \quad \text{.....(7)}$$

where M is the total mass of the plate.

EXAMPLE.—A thin wire of mass M is bent into a circle of radius R. Find the moment of inertia about a tangent.

Let AB (Fig. 235) be a diameter of the circle, and let OX be a tangent parallel to AB. Then

$$I_{AB} = \tfrac{1}{2}MR^2 \text{ (p. 202).}$$
$$I_{OX} = I_{AB} + MR^2$$
$$= \tfrac{1}{2}MR^2 + MR^2 = \tfrac{3}{2}MR^2$$

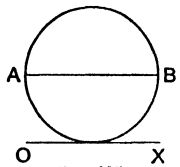


FIG. 235.

Routh's rule for calculating the moments of inertia of symmetrical solids.—If a body is symmetrical about three axes which are mutually perpendicular, the moment of inertia about one axis is equal to the mass of the body multiplied by the sum of the squares of the other two semi-axes and divided by 3, 4 or 5 according as the body is rectangular, elliptical (such as a cylinder), or ellipsoidal (such as a sphere).

EXAMPLE 1.—A rectangular plate (Fig. 236) is symmetrical about GZ and other two axes passing through G and parallel to B and T respectively. Find I_{GZ}

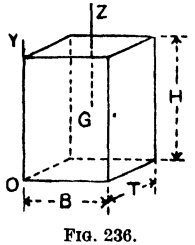


FIG. 236.

$$I_{GZ} = \frac{M\{(\frac{1}{2}B)^2 + (\frac{1}{2}T)^2\}}{3} = \frac{M(B^2+T^2)}{12}. \quad \text{.....(8)}$$

EXAMPLE 2.—A solid cylinder (special case of an elliptical body) is symmetrical about the axis OX of the cylinder, and about other two axes forming diameters at 90° and passing through the centre of mass of the cylinder. Find I_{OX}.

$$I_{OX} = \frac{M(R^2+R^2)}{4} = \tfrac{1}{2}MR^2. \quad \text{.....(9)}$$

EXAMPLE 3.—A solid sphere (ellipsoidal body) is symmetrical about any three diameters which are mutually perpendicular. Find the moment of inertia with respect to a diameter.

$$I_{OX} = \frac{M(R^2+R^2)}{5} = \tfrac{2}{5}MR^2. \quad \text{.....(10)}$$

Other cases of moments of inertia.—The following results are of service in the solution of problems.

A thin uniform wire, mass M, length L ; the axis OX passes through one end and is perpendicular to the wire.

$$I_{OX} = \tfrac{1}{3}ML^2. \quad \text{.....(11)}$$

A thin rectangular plate, mass M, breadth B, height H ; the axis OX coincides with one of the B edges

$$I_{ox} = \tfrac{1}{3}MH^2. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (12)$$

A thick rectangular plate (Fig. 236), the axis OY coincides with one edge.

$$I_{oy} = \tfrac{1}{3}M(B^2 + T^2). \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (13)$$

A thin circular plate, mass M, radius R ; the axis OZ passes normally through the centre ; OX is a diameter.

$$I_{oz} = \tfrac{1}{2}MR^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (14)$$

$$I_{ox} = \tfrac{1}{4}MR^2. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (15)$$

A thin circular plate of mass M having a concentric hole, external radius R_1, internal radius R_2 ; the axis OZ passes normally through the centre.

$$I_{oz} = \tfrac{1}{2}M(R_1^2 + R_2^2). \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (16)$$

This result also applies to a hollow cylinder having a coaxial hole.

Radius of gyration.—The radius of gyration of a body with respect to a given axis is defined as a quantity k such that, if its square be multiplied by the mass of the body, the result gives the moment of inertia of the body with reference to that axis. Thus

$$I = Mk^2.$$

$$k = \sqrt{\frac{I}{M}}.$$

For example, a solid cylinder has $I = \frac{MR^2}{2}$ with respect to the axis of the cylinder, hence

$$k = \sqrt{\frac{\frac{1}{2}MR^2}{M}} = \sqrt{\frac{R^2}{2}} = \frac{R}{\sqrt{2}}.$$

EXAMPLE.—In a laboratory experiment a flywheel of mass 100 pounds and radius of gyration 1·25 feet (Fig. 237) is mounted so that it may be rotated by a falling weight attached to a cord wrapped round the wheel axle. Neglecting friction, find what will be the accelerations if a body of 10 lb. weight is attached to the cord ; the radius of the axle is 2 inches.

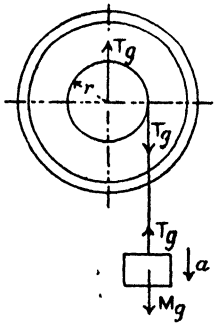


FIG. 237.—An experimental flywheel.

Let M = the mass attached to the cord, in pounds.
Mg = its weight, in poundals.
T = pull in the cord, in poundals.
r = radius of the axle, in feet.
I = moment of inertia of wheel
= 100 × 1·25 × 1·25 = 156·2 pound and foot units.
a = the linear acceleration of M, in feet per sec. per sec.
ϕ = the angular acceleration of the wheel, in radians per sec. per sec.

Then, considering M, we have

$$Mg - T = Ma. \quad \ldots\ldots(1)$$

Considering the wheel, we have

$$Tr = I\phi. \quad \ldots\ldots(2)$$

Also, $\phi = \frac{a}{r}$. (p. 55) $\ldots\ldots(3)$

These three equations enable the solution to be obtained. Thus:

From (2) and (3). $Tr = I\frac{a}{r}$;

$$T = I\frac{a}{r^2}. \quad \ldots\ldots(4)$$

Substituting in (1) gives

$$Mg - I\frac{a}{r^2} = Ma,$$

$$Mg = \left(M + \frac{I}{r^2}\right)a;$$

$$a = \frac{Mg}{M + \frac{I}{r^2}} = \frac{10 \times 32{\cdot}2}{10 + (156{\cdot}2 \times 6 \times 6)}$$

$$= \underline{0{\cdot}0572} \text{ feet per sec. per sec.}$$

From (3), $\phi = \frac{0{\cdot}0572}{r} = 0{\cdot}0572 \times 6$

$$= \underline{0{\cdot}343} \text{ radian per sec. per sec.}$$

Angular momentum.—The **angular momentum** or **moment of momentum** of a particle may be explained by reference to Fig. 238. A particle of mass m revolves in the circumference of a circle of radius r and has a linear velocity v at any instant in the direction of the tangent. Hence its linear momentum at any instant is given by mv. Now v is equal to ωr, when ω is the angular velocity; hence

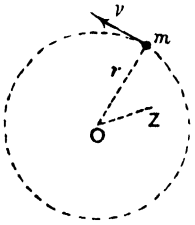


FIG. 238.—Angular momentum of a body.

Linear momentum of the particle $= \omega mr$. (1)

The moment of this momentum about OZ (Fig. 238) may be obtained by multiplying by r, the result being called the **moment of momentum**, or **angular momentum**.

Angular momentum of the particle $= \omega mr^2$. $\ldots\ldots(2)$

Each particle in a body rotating about OZ would have its angular momentum given by an expression similar to (2); hence

$$\text{Angular momentum of a body} = \omega\Sigma mr^2$$
$$= \omega I_{OZ}. \quad \ldots\ldots \quad \ldots\ldots (3)$$

Consider now a body free to rotate about a fixed axis, and, starting from rest, to be acted upon by a constant couple L. The constant angular acceleration being ϕ, we have

$$L = I_{OZ}\phi \text{ (p. 200).}$$

Let L act during a time t seconds, then the angular velocity ω at the end of this time will be

$$\omega = \phi t, \quad \text{or} \quad \phi = \frac{\omega}{t}. \quad \text{(p. 55)}$$

Hence
$$L = \frac{\omega I_{OZ}}{t}. \quad \ldots\ldots \quad \ldots\ldots (4)$$

Now ωI_{OZ} is the angular momentum acquired in the time t seconds; hence $\omega I_{OZ}/t$ will be the change in angular momentum per second. We may therefore write

$$L = \text{change in angular momentum per second.} \quad \ldots\ldots (5)$$

If the couple is expressed in gravitational units, say T, we have

$$T = \frac{\omega I_{OZ}}{gt}. \quad \ldots\ldots \quad \ldots\ldots (5')$$

If the angular velocity of a rotating body be changed from ω_1 to ω_2 in t seconds, then

$$\phi = \frac{\omega_1 - \omega_2}{t}, \quad \text{(p. 56)}$$

and the couple required is given by

$$L = \left(\frac{\omega_1 - \omega_2}{t}\right) I_{OZ}, \text{ absolute units, } \ldots\ldots (6)$$

or
$$T = \left(\frac{\omega_1 - \omega_2}{gt}\right) I_{OZ}, \text{ gravitational units, } \ldots\ldots (6')$$

Kinetic energy of a rotating body.—In Fig. 239 is shown a body rotating with uniform angular velocity ω about an axis OZ perpendicular to the plane of the paper. Consider the particle m_1, having a linear velocity v_1.

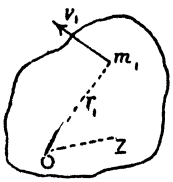


FIG. 239.—Kinetic energy of rotation.

$$\text{Kinetic energy of the particle} = \frac{m_1 v_1^2}{2}. \quad \text{(p. 171.)}$$

Now, $v_1 = \omega r_1$;

$$\therefore\ v_1^2 = \omega^2 r_1^2.$$

Hence

$$\text{Kinetic energy of particle} = \frac{m\omega^2 r_1^2}{2} = \frac{\omega^2}{2} \cdot mr_1^2 \quad \ldots\ldots(1)$$

A similar expression would result for any other particle; hence

$$\text{Total kinetic energy of the body} = \frac{\omega^2}{2} \Sigma mr^2$$

$$= \frac{\omega^2}{2} I_{oz} \text{ absolute units} \quad \ldots\ldots(2)$$

$$= \frac{\omega^2}{2g} I_{oz} \text{ gravitational units.} \quad \ldots(2')$$

EXAMPLE 1.—A wheel has a mass of 5000 pounds and a radius of gyration of 4 feet. Find its kinetic energy at 150 revolutions per minute.

$$\omega = \tfrac{150}{60} \times 2\pi = 5\pi \text{ radians per sec.}$$

$$I = Mk^2 = 5000 \times 4 \times 4 = 80{,}000 \text{ pound and foot units.}$$

$$\text{Kinetic energy} = \frac{\omega^2}{2g} I = \frac{25 \times \pi^2 \times 80{,}000}{64 \cdot 4}$$

$$= \underline{306{,}500} \text{ foot-lb.}$$

EXAMPLE 2.—The above wheel slows from 150 to 148 revolutions per minute. Find the energy which has been abstracted.

$$\text{Change in kinetic energy} = \frac{\omega_1^2}{2g} I - \frac{\omega_2^2}{2g} I$$

$$= (\omega_1^2 - \omega_2^2) \frac{I}{2g}.$$

Also,

$$\omega_1 = 5\pi,$$

$$\omega_2 = \tfrac{148}{60} \cdot 2\pi = 4 \cdot 933\pi;$$

$$\therefore \text{ Energy abstracted} = (\omega_1 - \omega_2)(\omega_1 + \omega_2) \frac{I}{2g}$$

$$= 0 \cdot 067 \times 9 \cdot 933 \times \frac{80{,}000 \times \pi^2}{64\ 4}$$

$$= \underline{8{,}160} \text{ foot-lb.}$$

Energy of a rolling wheel.—The total kinetic energy of a wheel rolling with uniform speed along a road may be separated into two parts, viz. the kinetic energy due to the motion of translation, and the kinetic energy due to the motion of rotation. The total kinetic energy will be the sum of these.

Let ω = the angular velocity.

v = the linear velocity of the carriage to which the wheel is attached (this will also be the velocity of the centre of the wheel).

M = the mass of the wheel.

k = its radius of gyration with reference to the axle.

Then Kinetic energy of rotation $= \frac{\omega^2 I}{2} = \frac{\omega^2 M k^2}{2}$.

Kinetic energy of translation $= \frac{M v^2}{2}$.

Total kinetic energy $= \frac{\omega^2 M k^2}{2} + \frac{M v^2}{2}$. ....(1)

Further, if there be no slipping between the wheel and the road, *i.e.* perfect rolling, we have

$$\omega = \frac{v}{R}, \quad \text{..... .....} \quad (2)$$

where R is the radius of the wheel.

Substituting in (1), we obtain for perfect rolling :

Total kinetic energy $= \frac{v^2 M k^2}{2R^2} + \frac{M v^2}{2}$

$= \frac{M v^2}{2}\left(\frac{k^2}{R^2} + 1\right)$ absolute units ..... (3)

$= \frac{M v^2}{2g}\left(\frac{k^2}{R^2} + 1\right)$ gravitational units .... (3′)

Energy of a wheel rolling down an inclined plane.—Fig. 240 illustrates the case of a wheel rolling from A to B down an inclined plane. A is at a height H above B. Assuming that no energy is wasted, we may apply the principle of the conservation of energy.

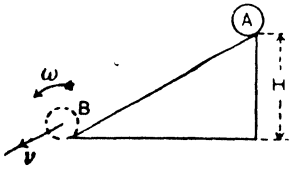


FIG 240.—Energy of a wheel rolling down an incline

Potential energy at A

$= MgH$

= total kinetic energy at B.

Let M and R be respectively the mass and radius of the wheel, and let v and ω be the linear and angular velocities at B. As there is supposed to be no waste of energy, there will be no slipping which would lead to waste in overcoming frictional resistances. Hence

$$v = \omega R.$$

Using equation (3) above for the total kinetic energy, we obtain

$$MgH = \frac{M v^2}{2}\left(\frac{k^2}{R^2} + 1\right),$$

or

$$v = \sqrt{\frac{2gH}{\frac{k^2}{R^2} + 1}}. \quad \text{..... .....} \quad (1)$$

Motion of a wheel rolling down an incline.—The above problem may be studied in the following manner. In Fig. 241 a wheel is rolling without slipping down a plane inclined at an angle α to the horizontal. Resolve the weight Mg into components $Mg \sin \alpha$ and $Mg \cos \alpha$ respectively parallel and at right angles to the incline. The normal reaction Q of the incline will be equal to $Mg \cos \alpha$, since no acceleration takes place in the direction perpendicular to the incline. The force of friction, F, acts on the wheel tangentially in the direction of the incline.

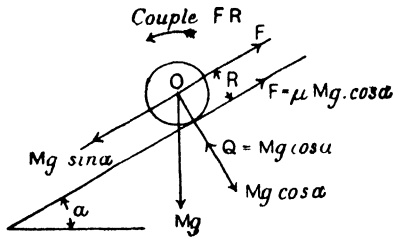


FIG. 241.—Motion of a wheel rolling down an incline

The effect of F may be ascertained by transferring it to the centre of mass O of the wheel (p. 127), introducing at the same time a couple of anticlockwise moment FR. The wheel is now under the action of opposing forces $Mg \sin \alpha$ and F, both applied at O in a direction parallel to the incline, together with a couple FR. The forces produce a linear acceleration a given by

$$Mg \sin \alpha - F = Ma. \quad \ldots\ldots(1)$$

The couple produces an angular acceleration ϕ given by

$$\phi = \frac{FR}{I_{oz}} = \frac{FR}{Mk^2}. \quad \ldots\ldots(2)$$

Also, since there is no slipping,

$$\phi = \frac{a}{R}. \quad \ldots\ldots(3)$$

From (2), $$F = \frac{\phi M k^2}{R}.$$

Substituting in (1), we obtain

$$Mg \sin \alpha - \frac{\phi M k^2}{R} = Ma.$$

Substituting for ϕ from (3), gives

$$g \sin \alpha - \frac{ak^2}{R^2} = a,$$

$$a\left(1 + \frac{k^2}{R^2}\right) = g \sin \alpha,$$

$$a = \frac{g \sin \alpha}{1 + \frac{k^2}{R^2}}. \quad \ldots\ldots(4)$$

Suppose that the body starts from rest at A (Fig. 242) and rolls to B. The linear velocity of the centre of mass when at B may be calculated thus :

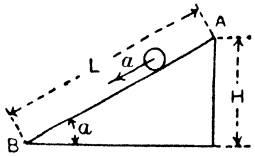


FIG 242

$v^2 = 2aL$ (p. 33).

Also, $\frac{H}{L} = \sin\alpha$; or, $L = \frac{H}{\sin\alpha}$;

$\therefore\ v^2 = 2a\frac{H}{\sin\alpha}$.

Inserting the value of a from (1), we have

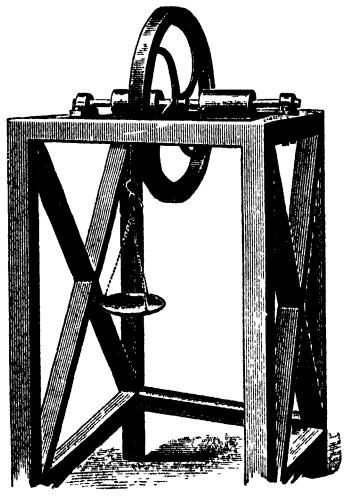
$$v^2 = \frac{2g\sin\alpha}{1+\frac{k^2}{R^2}}\ \frac{H}{\sin\alpha} = \frac{2gH}{1+\frac{k^2}{R^2}}$$

$$\therefore\ v = \sqrt{\frac{2gH}{1+\frac{k^2}{R^2}}} \quad \text{.....(5)}$$

Comparison of this with equation (1) (p. 208) indicates that the results obtained by both methods agree.

EXPT. 34.—**Kinetic energy of a flywheel.** In this experiment the wheel is driven by means of a falling weight attached to a cord which is wrapped round the wheel axle and looped to a peg on the axle so that the cord disengages when unwound (Fig. 243).

Weigh the scale-pan and let its mass together with that of the load placed in it be M. Let the scale-pan touch the floor and let the cord be taut ; turn the wheel by hand through n_1 revolutions (a chalk mark on the rim helps), and measure the height H through which the scale-pan is elevated. Allow the scale-pan to descend, being careful not to assist the wheel to start ; note the time of descent ; repeat three or four times and take the average time t seconds. Again allow the scale-pan to descend three or four times, and note the total revolutions of the wheel from starting to stopping, being careful not to


FIG. 243.—Experimental flywheel.

interfere with it in any way; let the average revolutions be n_2. Repeat the experiment, using different values of M and of H. Tabulate the results:

EXPERIMENT ON A FLYWHEEL.

No. of Expt.	Load M.	Height H	Time of descent t secs.	Revs. while M is descending n_1	Total revs. n_2.

Reduce the results for each experiment as follows:

Energy available = potential energy given up by M

$$= MgH. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

Disposal of energy:

(*a*) The energy of M at the instant it reaches the floor is given by

$$\text{Kinetic energy acquired by M} = \frac{Mv^2}{2}.$$

To find the velocity v at the instant M reaches the floor, we have

$$\text{Average velocity} \times t = H;$$

$$\therefore \text{ Average velocity} = \frac{H}{t}.$$

$$\text{Final velocity} = v = \frac{2H}{t};$$

$$\therefore \text{ Kinetic energy acquired by M} = K_M = \frac{M}{2} \cdot \frac{4H^2}{t^2} = \frac{2MH^2}{t^2}. \quad \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

Hence the energy which has been given to the wheel up to the instant that M reaches the floor is, from (1) and (2),

$$MgH - \frac{2MH^2}{t^2} = MgH - K_M. \quad \ldots\ldots\ldots\ldots\ldots\ldots (3)$$

(*b*) Some of this energy has been wasted in overcoming the friction of the bearings. Ultimately the whole of the energy given by (3) is so wasted during the entire period of motion, *i.e.* in n_2 revolutions of the wheel. Hence

$$\text{Energy wasted per revolution} = \frac{MgH - K_M}{n_2},$$

and energy wasted whilst M is descending

$$= \left(\frac{MgH - K_M}{n_2}\right) n_1. \quad \ldots\ldots\ldots\ldots\ldots\ldots (4)$$

(*c*) The kinetic energy possessed by the wheel, at the instant M reaches the floor, may be calculated by deducting the kinetic energy acquired by

M, together with the energy wasted in overcoming friction whilst M is descending, from the energy available Let this energy be K, then

$$K = MgH - K_M - \left(\frac{MgH - K_M}{n_2}\right) n_1 \quad \text{.....(5)}$$

During the descent of M, the wheel has made n_1 revolutions in t seconds. Let N be the maximum speed in revolutions per second, then

$$\text{Average speed} \times t = n_1;$$

$$\text{Average speed} = \frac{n_1}{t},$$

and

$$N = \frac{2n_1}{t}. \quad \text{.....(6)}$$

The kinetic energy is proportional to the square of the speed, hence

Kinetic energy of the wheel at 1 revolution per second

$$= \frac{K}{N^2}. \quad \text{.....(7)}$$

Obtain the value of this for each experiment; there should be fair agreement. Take the average result and call it K_1. Then

$$K_1 = \frac{\omega^2}{2} I, \quad \text{(p. 207)}$$

and

$$\omega = 2\pi \text{ radians per sec.};$$

$$K_1 = \frac{4\pi^2}{2} I = 2\pi^2 I;$$

$$I = \frac{K_1}{2\pi^2}. \quad \text{.....(8)}$$

The final result gives the moment of inertia of the wheel.

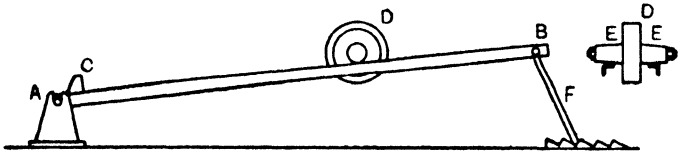


FIG. 244.—Apparatus for investigating the motion of a wheel rolling down an incline.

EXPT. 35.—**A wheel rolling down an incline.** A convenient form of apparatus is shown in Fig. 244; the incline consists of two bars AB and the axle E E of the wheel D rolls on them. The time of descent is thus increased, and it becomes possible to measure it with fair accuracy by means of a stop-watch.

Set the incline to a suitable inclination by means of the adjustable prop F. Measure the height from the horizontal table to the centre of the wheel axle: first, when the wheel is at the top, and second, when the wheel

is at the bottom of the incline. Let the difference in the heights be H. Measure the diameter of the wheel axle, and hence find its radius R. Allow the wheel to roll down, being careful not to assist it to start; note the time of descent. Repeat several times, and take the average time t seconds. Measure the length of the incline traversed by the wheel; let this be L.

Then Average velocity $\times t = L$;

$$\therefore \text{ average velocity} = \frac{L}{t},$$

and maximum velocity $= v = \frac{2L}{t}$. .....(1)

Evaluate this velocity and substitute in equation (1), p. 208, giving

$$v^2 = \frac{4L^2}{t^2} = \frac{2gH}{\frac{k^2}{R^2} + 1},$$

whence $$k^2 = \left(\frac{2gHt^2}{4L^2} - 1\right)R^2.$$ .....(2)

Weigh the wheel with its axle in order to determine its mass M. Then

Moment of inertia of the wheel $= Mk^2$

$$= \left(\frac{2gHt^2}{4L^2} - 1\right)MR^2.$$ .....(3)

Repeat the experiment, giving different slopes to the incline, and calculate the moment of inertia for each experiment; take the average value.

Exercises on Chapter XV.

1. A wheel has a moment of inertia of 10,000 in pound and foot units. If the wheel starts from rest and acquires a speed of 200 revolutions per minute in 25 seconds, what constant couple has been acting on it?

2. A wheel has a mass of 6 kilograms, and its radius of gyration is 20 centimetres. If its speed be changed from 8000 to 7000 revolutions per minute in 10 seconds, what constant couple has opposed the motion?

3. A wheel is acted upon by a constant couple of 650 poundal-inches; starting from rest it makes 6 revolutions in the first 8 seconds. What is the moment of inertia of the wheel?

4. A thin straight rod 6 feet long has a mass of 0·4 pound. Find its moment of inertia with respect to (*a*) an axis parallel to the rod and 8 inches from it; (*b*) an axis perpendicular to the rod and passing through one end; (*c*) an axis perpendicular to the rod and passing through its centre.

5. The rod given in Question 4 is bent into a complete circle. Find its moment of inertia with respect to (*a*) an axis passing through the centre of the circle and perpendicular to its plane; (*b*) a diameter of the circle; (*c*) a tangent.

6. A thin circular plate has a mass of 2 pounds and the radius is 9 inches. Find the moment of inertia with respect to the following axes: (*a*) passing through the centre and perpendicular to the plane of the plate; (*b*) a diameter; (*c*) a tangent; (*d*) a line perpendicular to the plane of the plate and passing through a point on the circumference; (*e*) a similar line to that given in (*d*), but bisecting a radius.

7. A thin rectangular plate has a mass of 1 5 pounds; the edges are 3 feet and 2 feet respectively. Find the moment of inertia with respect to (*a*) a 3 feet edge; (*b*) a 2 feet edge; (*c*) a line parallel to the 3 feet edges and bisecting the plate; (*d*) a line parallel to the 2 feet edges and bisecting the plate; (*e*) a line perpendicular to the plane of the plate and passing through the intersection of the diagonals; (*f*) a line perpendicular to the plane of the plate and passing through one corner.

8. An iron plate, 4 feet high, 2 feet wide and 2 inches thick, is hinged at a vertical edge. Find the moment of inertia with respect to the axis of the hinges. The density of iron is 480 pounds per cubic foot.

9. A hollow cylinder of iron is 60 feet long, 20 inches external and 8 inches internal diameter. The density is 480 pounds per cubic foot. Find the moment of inertia about the axis of the cylinder.

10. A solid sphere of cast iron is 12 inches in diameter. The density is 450 pounds per cubic foot. Find the moment of inertia about a diameter, and also about a tangent.

11. A wheel having a mass of 50 tons and a radius of gyration of 15 feet runs at 50 revolutions per minute. It is observed to take 4 5 minutes in coming to rest. What steady couple has been acting?

12. A wheel is mounted in bearings so that the axis of rotation is horizontal, and is driven by a cord wrapped round the axle and carrying a load. The axle is 4 inches diameter measured to the centre of the cord. A preliminary experiment shows that a load of 2 lb. weight produces steady rotation, the wheel being assisted to start by hand. The load is then increased to 4 lb. weight; starting from rest, this load descended 3 feet in 6 5 seconds. Find the moment of inertia of the wheel.

13. A solid disc, 3 feet in diameter, has a mass of 200 pounds. Calculate its angular momentum when rotating 300 times per minute. If the speed is changed to 320 revolutions per minute in 40 seconds, what constant couple has been applied?

14. A thin iron rod, 2 feet long, mass 0·6 pound, revolves about an axis perpendicular to and bisecting the rod. If the speed is 120 revolutions per minute, find the moment of momentum. If a couple of 0 3 lb.-feet be applied for 2 seconds so as to increase the speed, find the final speed of rotation.

15. Calculate the kinetic energy of a wheel having a moment of inertia of 30,000 in pound and foot units, when rotating 180 times per minute. How much energy does the wheel give up in changing speed to 179 revolutions per minute?

16. A bicycle wheel, 28 inches in diameter, has a mass of 2 pounds, and the radius of gyration is 13 inches. The bicycle is travelling at 12 miles per hour. Find (*a*) the kinetic energy of rotation of the wheel; (*b*) its kinetic energy of translation; (*c*) its total kinetic energy.

17. A solid cylinder, mass 4 pounds, diameter 6 inches, starts from rest at the top and rolls without slipping down a plane inclined at 5° to the horizontal. If the incline is 10 feet long, find the kinetic energies of translation and rotation when the cylinder reaches the bottom.

18. Find the linear and angular accelerations of the cylinder given in Question 17.

19. Two cylinders, A and B, have the same over-all dimensions and their masses are equal. The cylinder A has a lead core and the outer part is wood; the cylinder B has a wooden core and the outer part is lead. Both cylinders start simultaneously from rest at the top of an incline and roll without slipping. Which cylinder will reach the bottom first? Give reasons for your answer.

20. Write down expressions for the coordinates of the centre of mass of a number of particles of given mass, the coordinates of whose positions are given.

A uniform square plate of 1 ft. side has two circular holes punched in it, one of radius 1 inch, coordinates of centre (4, 5) inches, referred to two adjacent sides of the plate as axes, the other of radius $\frac{1}{2}$ inch, coordinates of centre (8, 1) inches; find the coordinates of the centre of mass of the remainder of the plate. L.U.

21. Write down an expression for the kinetic energy of a wheel whose moment of inertia is I, rotating n times a second.

A wheel has a cord of length 10 feet coiled round its axle; the cord is pulled with a constant force of 25 lb. wt., and when the cord leaves the axle, the wheel is rotating 5 times a second. Calculate the moment of inertia of the wheel. L.U.

22. A hollow circular cylinder, of mass M, can rotate freely about an external generator (*i.e.* a straight line drawn on the curved surface and parallel to the axis of the cylinder), which is horizontal. Its cross section consists of concentric circles of radii 3 and 5 feet. Show that its moment of inertia about the fixed generator is 42 M units, and find the least angular velocity with which the cylinder must be started when it is in equilibrium, so that it may just make a complete revolution. L.U.

23. A projectile whose radius of gyration about its axis is 5 inches is fired from a rifled gun, and on leaving the gun its total kinetic energy is 50 times as great as its kinetic energy of rotation. How far does the projectile travel on leaving the gun before making one complete turn? L.U.

24. On what does the inertia of a body, with respect to rotation about an axis, depend?

Prove that the energy of rotation of a small mass whirled in a circle is equal to half the product of its rotation-inertia (moment of inertia) about the axis of rotation into the square of its angular velocity.

Show that one-half of the kinetic energy acquired by a hoop in rolling down an inclined plane is rotational. Adelaide University.

25. What is meant by "moment of inertia" of a body? Show that the moment of inertia of a body about any axis is equal to its moment of inertia about a parallel axis through its centre of mass, plus the moment of inertia

which the body would have about the given axis if all collected at its centre of mass. Allahabad Univ.

26. A wheel runs at 240 revolutions per minute, and is required to give up 10,000 foot-lb. of energy without the speed falling below 239 revolutions per minute. Calculate the moment of inertia which the wheel must have. If the radius of gyration is 5 feet, find the mass of the wheel.

CHAPTER XVI

CENTRIFUGAL FORCE. PENDULUMS

Centrifugal force.—It has been shown (p. 45) that when a particle moves in the circumference of a circle of radius R with uniform velocity v (Fig. 245) there is a constant acceleration towards the centre of the circle given by

$$a = \frac{v^2}{R}.$$

To produce this acceleration requires the application of a uniform force F, also continually directed towards the centre of the circle and given by

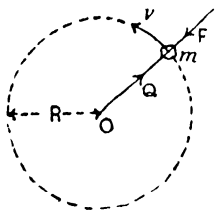


FIG. 245.—Central and centrifugal forces.

$$F = ma = \frac{mv^2}{R} \text{ absolute units, } \ldots\ldots\ldots(1)$$

or $$P = \frac{mv^2}{gR} \text{ gravitational units. } \ldots\ldots\ldots(1')$$

The force F overcomes the inertia of the particle, which would otherwise pursue a straight line path, and may be called the **central force** (sometimes called the **centripetal force**). It is resisted by an equal opposite force Q (Fig. 245) due to the inertia of the particle. Q is called the **centrifugal force.**

Expressed in terms of the angular velocity,

$$F = Q = \frac{m\omega^2 R^2}{R} = \omega^2 mR. \ldots\ldots\ldots(2)$$

Since mR is the simple moment of mass of the particle with reference to the axis of rotation, it follows that in a large body, consisting of many particles, the centrifugal force may be calculated by imagining the whole mass of the body to be concentrated at the centre of mass. Let M be the mass of the body and let

Y be the radius drawn to the centre of mass from the axis of rotation (Fig. 246), then

Centrifugal force $= \omega^2 MY$ absolute units. .....(3)

$= \frac{\omega^2}{g} MY$ gravitational units. .....(3′)

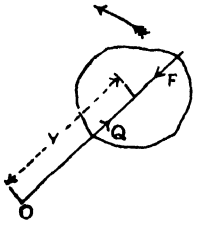


FIG. 246.—Resultant centrifugal force.

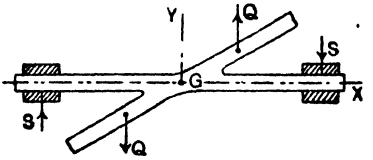


FIG. 247.—Rocking couple due to want of symmetry.

It follows from this result that if a body rotates about an axis passing through its centre of mass (in which case Y=O), there will be no resultant force on the axis due to centrifugal action. If the body is not symmetrical, a disturbing couple may act on the axis. Thus in Fig. 247 is shown a rod rotating about an axis GX, G being the centre of mass. The rod is not symmetrical about GY; hence, considering the halves separately, there will be centrifugal forces Q, Q forming a couple tending to bring the rod into the axis GY. To balance this tendency, the bearings must apply forces S, S, forming a couple equal and opposite to that produced by Q, Q. These forces will, of course, rotate with the rod and produce what is called a **rocking couple** In Fig. 248 is shown a body symmetrical about GY, and consequently having neither rocking couple nor resultant centrifugal force; in other words, this body is completely balanced.

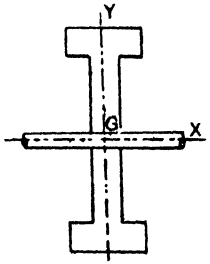


FIG. 248.—A balanced symmetrical body.

Centrifugal force on vehicles.—In Fig. 249 is shown the front view of a motor car moving in a path curved in plan. To prevent side slipping, the road is banked up to such an extent that the resultant Q of the centrifugal force and the weight falls perpendicularly to the road surface.

Let M = the mass of the car.
v = the velocity.
R = the radius of the curve, as seen in the plan.

Then Centrifugal force $= \frac{Mv^2}{R}$ absolute units.

Weight of car $= Mg$ absolute units.

The triangle of forces is ABG ; hence

$$\frac{\text{Centrifugal force}}{\text{Weight of car}} = \frac{Mv^2}{RMg} = \frac{v^2}{gR} = \frac{AB}{BG}.$$

Now $\frac{AB}{BG} = \tan \alpha$ (Fig. 249),

and α is also the angle which the section of the road surface makes with the horizontal ; hence

$$\tan \alpha = \frac{v^2}{gR}.$$

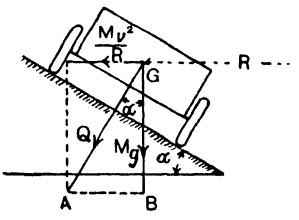


FIG. 249.—Section of a banked motor track.

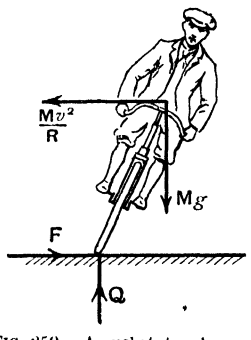


FIG. 250.—A cyclist turning a corner.

Railway tracks are banked in a similar manner ; the outer rail is laid at a greater elevation than that on the inside of the curve, and so grinding of the flanges of the outer wheels against the rail is prevented.

A cyclist turning a corner instinctively leans inwards (Fig. 250). The forces acting on machine and rider are the total weight Mg, the centrifugal force Mv^2/R—where R is the radius of the curve and v is the velocity, the vertical reaction of the ground Q—equal to Mg, and a frictional force F applied to the wheels by the ground. If all goes well, the clockwise couple formed by Mg and Q is balanced by the anticlockwise couple formed by F and Mv^2/R. It is evident that the higher the speed and the smaller the radius of the curve, the greater will be the centrifugal force, and the rider will have to lean inwards at a greater angle. Since the centrifugal force and the friction are equal, it may happen that the limiting value of the friction may be attained, when a slide slip will ensue.

Simple harmonic motion.—In Fig 251, AB is any diameter of the circle and NS is another diameter at right-angles to AB. Let a point P travel in the circumference of the circle with uniform velocity v; draw PM perpendicular to NS. It will be seen that M, the projection of P on NS, vibrates in NS as P moves round the circumference of the circle. The motion of M is called **simple harmonic motion, or vibration.**

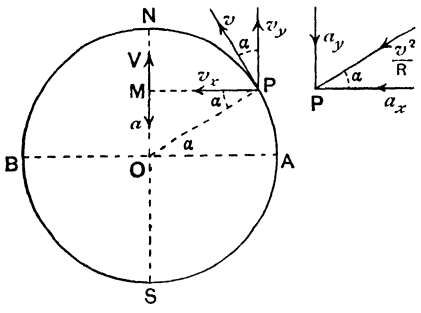


FIG. 251.—Simple harmonic motion

Let the radius of the circle be R, and let the angle α described by OP be measured from the initial position OA. The angular velocity ω of OP is v/R, and the displacement of M from the middle of the vibration at any instant is given by

$$OM = OP \sin \alpha = R \sin \alpha. \quad \ldots\ldots(1)$$

Let t seconds be the time in which OP describes the angle α, then $\alpha = \omega t$, and we may write

$$OM = R \sin \omega t. \quad \ldots\ldots(1')$$

Denoting as positive the displacements above O, and as negative those below O, the algebraic sign of $\sin \omega t$ will determine on which side of O the point M falls at the end of the time t. The maximum displacement ON or OS is called the **amplitude** of the vibration.

Velocity and acceleration in simple harmonic motion.—To obtain the velocity V of M, take components v_x and v_y of the velocity of P respectively parallel and perpendicular to AB (Fig. 251). Since v_y is perpendicular to OA and v is perpendicular to OP, it follows that the angle between v and v_y is equal to α. Hence

$$v_y = v \cos \alpha = v \cos \omega t.$$

The component v_x does not affect the velocity of M, therefore

$$V = v \cos \alpha. \quad \ldots\ldots(2)$$

$$= v \cos \omega t. \quad \ldots\ldots(2')$$

$$= \omega R \cos \omega t. \quad \ldots\ldots(2'')$$

To obtain the acceleration a of M, resolve the central acceleration of P, viz. v^2/R, into components a_x and a_y respectively parallel and perpendicular to AB, as shown separately in Fig. 251. The component a_x does not affect the motion of M; hence

$$a = a_y = \frac{v^2}{R} \sin \alpha. \quad \text{.....(3)}$$

$$= \frac{v^2}{R} \sin \omega t. \quad \text{.....(3')}$$

$$= \omega^2 R \sin \omega t. \quad \text{.....(3'')}$$

It will be noticed from (2) that the velocity of M is proportional to $\cos \alpha$. Now $\cos \alpha = PM/OP$ and is therefore proportional to PM; hence V is proportional to PM. V is zero when M is at N and also when M is at S. Maximum value of V occurs when M is passing through O and is given by

$$V_{max} = v \cos 0 = v. \quad \text{.....(4)}$$

The algebraic sign of $\cos \alpha$ indicates whether V is from S towards N (positive), or from N towards S (negative). From (1') and (3'') we may write for the acceleration of M,

$$a = \omega^2 \times \text{displacement of M from O.} \quad \text{.....(5)}$$

Hence the acceleration of M is proportional to the displacement from the middle of the vibration. The algebraic sign of $\sin \alpha$ in (3) indicates whether a is from N towards O (positive), or from S towards O (negative) (Fig. 251). It will be noted that the acceleration is directed constantly towards O. From (3), a has zero value when $\sin \alpha = 0$, *i.e.* when $\alpha = 0$ or π; M will then be passing through O. Maximum values of a occur when $\sin \alpha = \pm 1$, *i.e.* when M is at N and again at S; in these positions

$$a_{max} = \pm \frac{v^2}{R} = \pm \omega^2 R. \quad \text{.....(6)}$$

Displacement, velocity and acceleration diagrams for M have been drawn in Figs. 252 (*a*), (*b*), and (*c*) for values of α from 0 to 2π. It is evident that the displacement and acceleration graphs are curves of sines, and that the velocity graph is a curve of cosines. Further, since α is proportional to t, it follows that these diagrams are also displacement, velocity and acceleration graphs on time bases; the base line O to 2π represents the time of one revolution of P (Fig. 251), or the time of one complete vibration of M from N to S and back to N. This time is called the **period** of the vibration.

Let T = the period, then

$v T = 2\pi R$ (Fig. 251),

$$T = \frac{2\pi R}{v}, \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (7)$$

$$= \frac{2\pi R}{\omega R} = \frac{2\pi}{\omega}. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (7')$$

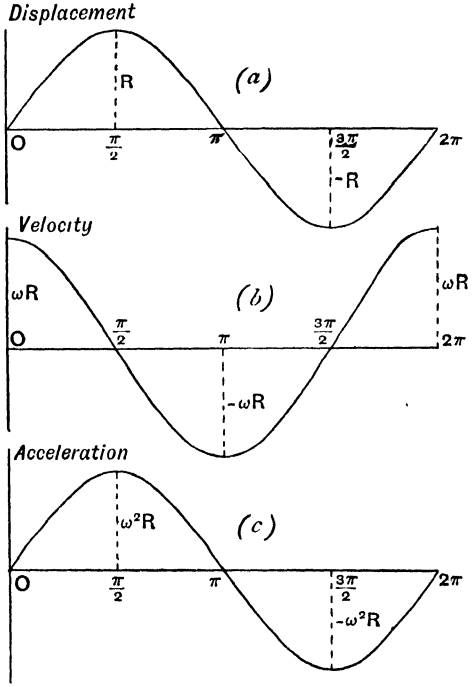


FIG. 252.—Graphs for simple harmonic motion.

The **frequency** of the vibration is the number of vibrations per second, and is obtained by taking the reciprocal of T; thus

$$\text{Frequency} = n = \frac{1}{T} \text{ vibrations per sec.} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (8)$$

EXAMPLE.—A point describes simple harmonic vibrations in a line 4 cm. long. Its velocity when passing through the centre of the line is 12 cm. per second. Find the period.

The given maximum value of V is also the velocity of P in the circumference of the circle (Fig. 251); hence

$$T = \frac{2\pi R}{v} = \frac{2 \times 22 \times 2}{12 \times 7} = \underline{1{\cdot}05} \text{ seconds.}$$

A well-known mechanism in which simple harmonic motion is realised is shown in Fig. 253. A crank revolves in the dotted circle about a fixed centre and engages a block which may slide in a slotted bar. Rods attached to the bar are guided so as to be capable of vertical motion only. The effect of the slot is to cancel the horizontal components of the velocity and acceleration of the crank pin; hence the vertical motion of the rods is simple harmonic.

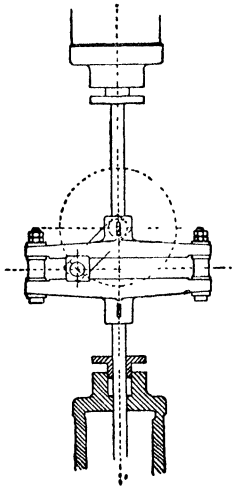


FIG. 253.—Slotted-bar mechanism.

Forces required in simple harmonic vibrations.—Referring to Fig. 254, in which a particle of mass m is executing simple harmonic vibrations in NS, the force F required to overcome inertia when the particle is at C is given by

$$F = ma.$$

Substituting the value of a obtained in (3″), p. 221, we have

$$F = m\omega^2 R \sin \omega t$$

$$= m\omega^2 \times \text{displacement OC (Fig. 254)}. \ldots\ldots\ldots (1)$$

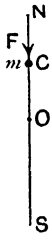


FIG. 254.—Force required in simple harmonic motion.

Hence F is proportional to the displacement OC and is directed constantly towards the middle of the vibration. The maximum values of F occur when the particle is at N and at S, and are given by

$$F_{max} = \pm m\omega^2 \times ON. \ldots\ldots\ldots (2)$$

Let μ be the value of F when the particle is at unit displacement from O, then

$$\mu = m\omega^2, \quad \text{or} \quad \omega^2 = \frac{\mu}{m}.$$

The period of the vibration is given by (7′), p. 222.

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{\mu}}. \ldots\ldots\ldots (3)$$

In using this result μ must be stated in absolute units.

EXAMPLE.—A body of mass 2 grams executes simple harmonic vibrations. When at a distance of 3 cm. from the centre of the vibration, a force of 0·4 gram weight is acting on it. Find the period.

$$\mu = \frac{0\cdot4}{3} = 0\cdot133 \text{ gram weight}$$

$$= 0\cdot133 \times 981 = 130\cdot3 \text{ dynes.}$$

$$T = 2\pi\sqrt{\frac{m}{\mu}} = \frac{2\times22}{7}\sqrt{\frac{2}{130\cdot3}}$$

$$= \underline{0\cdot777} \text{ second.}$$

The simple pendulum.—A simple pendulum may be realised by attaching a small body to a light thread and allowing it to execute small vibrations in a vertical plane under the action of gravity (Fig. 255). The forces acting on the small body at B are its weight, mg, and the pull T of the thread. The resultant of these is a force F, which may be taken as horizontal if the angle BAD is kept very small, and may be obtained from the triangle of forces ABD

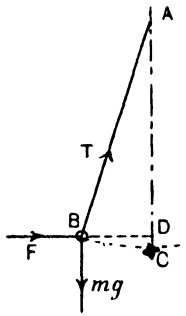


FIG. 255.—A simple pendulum.

$$\frac{F}{mg} = \frac{BD}{AD}, \quad \text{or} \quad F = mg\frac{BD}{AD}.$$

Now, if the angle BAD is very small, AC and AD will be sensibly equal. Let L be the length of the thread, then

$$F = mg\frac{BD}{AC} = \frac{mg}{L}\cdot BD. \qquad \ldots\ldots(1)$$

Hence we may say that F is proportional to BD for vibrations of small amplitude. BD and BC coincide nearly for such vibrations, and the body will execute simple harmonic vibrations under the action of a force F which varies as the displacement of B from the vertical through A. To obtain the value of μ, make BD = 1 in (1), giving

$$\mu = \frac{mg}{L}.$$

Now

$$T = 2\pi\sqrt{\frac{m}{\mu}} = 2\pi\sqrt{\frac{mL}{mg}}$$

$$= 2\pi\sqrt{\frac{L}{g}}. \qquad \ldots\ldots(2)$$

EXAMPLE.—Find the period of a simple pendulum of length 4 feet at a place when g is 32 feet per second per second. Find also the frequency.

$$T = \frac{2 \times 22}{7} \sqrt{\frac{4}{32}} = \underline{2{\cdot}222} \text{ seconds.}$$

$$n = \frac{1}{T} = \frac{1}{2{\cdot}222} = \underline{0{\cdot}449} \text{ vibration per second.}$$

Vibrations differing in phase.—In Fig. 256 (a), two points P_1 and P_2 rotate in the circumference of the circle with equal and constant angular velocities. Their projections M_1 and M_2 on AB execute simple harmonic vibrations which are said to differ in phase. The

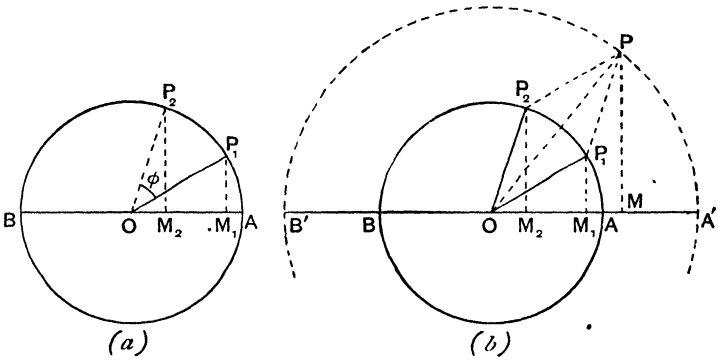


FIG. 256.—Vibrations differing in phase.

phase difference may be defined as the value of the constant angle $P_1OP_2 = \phi$, and may be stated in degrees or radians. Thus a phase difference of 90° or $\pi/2$ radians would give vibrations, such that M_1 would be at the end A of the vibration, at the instant that M_2 was passing through the point O.

The vibrations possessed separately by M_1 and M_2 may be impressed on a single particle, which will then execute simple harmonic vibrations compounded of the vibrations possessed by M_1 and M_2. In Fig. 256 (b) construct a parallelogram by making P_1P and P_2P equal and parallel respectively to OP_2 and OP_1. Join OP and draw PM perpendicular to BA produced.

OM_2 and M_1M are equal, since they are the projections on AB of equal lines equally inclined to AB. Therefore OM is equal to the sum of the component displacements OM_1 and OM_2. Hence, if the parallelogram OP_1PP_2 rotate about O with the same angular velocity possessed originally by OP_1 and OP_2, then M will execute simple harmonic vibrations in A′B′, and will have a resultant motion of which the vibrations of M_1 and M_2 are the components.

It will be evident now that if two simple harmonic vibrations in the same straight line, of equal amplitudes and periods but differing in phase by 180°, be impressed on the same particle, the particle will remain at rest

For further examples of simple harmonic vibrations, the student is referred to the Part of the volume devoted to Sound

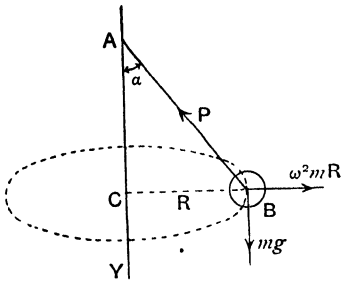


FIG 257.—A conical pendulum

Conical pendulum.—The conical pendulum consists of a small heavy particle B (Fig. 257) attached to a light cord AB which is attached at the upper end to a fixed point A The axis AY is vertical and the particle B describes a circular path, the plane of which is horizontal AB in revolving generates a conical surface

Let m = the mass of the particle,
ω = the constant angular velocity, radians/sec.,
α = the angle between AB and AY,
R = the radius of the circle described by B,
H = the height AC of the cone.

The forces acting on the particle are its weight mg, the centrifugal force $\omega^2 m$R, and the tension P of the cord. These forces balance, and the triangle of forces for them is ABC Hence

$$\frac{AC}{CB} = \frac{H}{R} = \frac{mg}{\omega^2 m R} = \frac{g}{\omega^2 R},$$

$$\therefore H = \frac{gR}{\omega^2 R} = \frac{g}{\omega^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

It will be seen from this result that the height of the cone is independent of the mass of the particle and of the length of the cord AB ; it depends solely on the angular velocity and on the value of g. For a given value of H at a stated place, for which the value of g is known, ω has a definite value, and hence the time of one revolution has a fixed value.

Let T = the time in sec. of 1 revolution.

Then $\omega T = 2\pi$, or, $T = 2\pi/\omega$.

From (1), $\omega = \sqrt{\frac{g}{H}}$,

$$\therefore T = 2\pi\sqrt{\frac{H}{g}} \quad \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

Reterring again to Fig. 257, we have from (1),

$$\cos\alpha = \frac{H}{AB} = \frac{g}{\omega^2 AB}. \quad \text{..... (3)}$$

Also,
$$\frac{P}{mg} = \frac{AB}{AC} = \frac{AB}{H} = \frac{AB \cdot \omega^2}{g};$$

$$\therefore\ P = m\omega^2 AB. \quad \text{..... .....(4)}$$

If the angular velocity be changed from ω_1 to ω_2, there will be a corresponding alteration in the height of the cone from H_1 to H_2. Thus, from (1),

$$H_1 = \frac{g}{\omega_1^2}, \quad \text{and} \quad H_2 = \frac{g}{\omega_2^2}.$$

$$\therefore\ H_1 - H_2 = g\left(\frac{1}{\omega_1^2} - \frac{1}{\omega_2^2}\right). \quad \text{.....(5)}$$

If ω_2 be greater than ω_1, $(H_1 - H_2)$ is a decrease in the height of the cone, *i.e.* the particle rises; if ω_2 be less than ω_1, the particle takes up a lower position.

This fact renders the conical pendulum useful as an engine governor, an example of which is shown in Fig. 258. The vertical spindle is driven by the engine and has two arms pivoted near the top. These arms carry masses which realise the ideal particle in the conical governor. Other arms connect the masses to a sleeve which can slide on the spindle. Movements of the sleeve as the speed changes are communicated by the bent lever and rod to a throttle valve in the steam pipe. Increase in speed causes the masses to rise; hence the sleeve also rises and the movement partially closes the throttle valve, thus reducing the quantity of steam passing to the engine and hence reducing the speed. Reduction in speed is followed by an inverse action, and more steam is supplied to the engine.

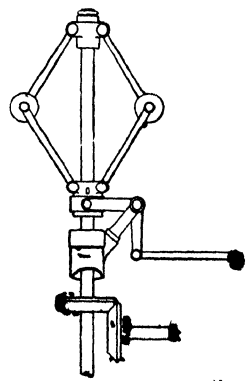


FIG. 258.—An engine governor.

Loaded governor or conical pendulum.—The speed of revolution of the simple governor shown in Fig. 258 is limited to about 70 or 80 revolutions per minute; at higher speeds H becomes too small to be suitable for fulfilling the function of governing. The speed may be increased and these defects avoided by the device of loading the sleeve (Fig. 259 (*a*)). In this governor the sleeve carries a mass of M units; all four arms are inclined at the same angle α to the vertical. Each of the pins C_1 and C_2 sustain $\frac{1}{2}Mg$; also

C_1 is balanced under the action of three forces, $\frac{1}{2}Mg$, the pull P in C_1A_1, and a horizontal force Q supplied by the sleeve (Fig. 259 (*b*)). The pull P is transmitted by the link A_1C_1, and applies a force P to the mass A_1; this force may be resolved into a vertical force $\frac{1}{2}Mg$ and a horizontal force Q. From the triangle of forces $A_1D_1C_1$, we have

$$\frac{Q}{\frac{1}{2}Mg} = \frac{A_1D_1}{D_1C_1} = \tan A_1C_1D_1,$$

or

$$Q = \tfrac{1}{2}Mg \tan \alpha. \quad \ldots\ldots \quad (1)$$

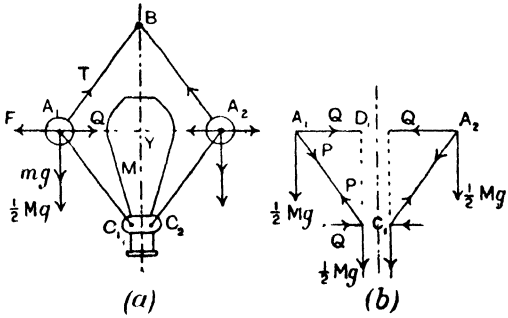


FIG. 259.—Loaded Porter governor

Referring to Fig. 259 (*a*), we see that A_1 is subjected to three forces, viz the pull T in the upper arm A_1B, the resultant $(F - Q)$ of the centrifugal force F and Q, and the resultant $(mg + \frac{1}{2}Mg)$ of the weights. A_1BY is the triangle of forces, and we have

$$\frac{F - Q}{mg + \frac{1}{2}Mg} = \frac{YA_1}{BY} = \tan \alpha. \quad \ldots\ldots \quad (2)$$

Let the angular velocity be ω, and let $A_1Y = R$ and $BY = H$, then, from (1) and (2),

$$\frac{\omega^2 mR - \frac{1}{2}Mg}{mg + \frac{1}{2}Mg} \frac{R}{H} = \frac{R}{H};$$

$$\therefore \ \omega^2 mH - \tfrac{1}{2}Mg = mg + \tfrac{1}{2}Mg;$$

$$\therefore \ \omega^2 = \left(\frac{M + m}{m}\right)\frac{g}{H}$$

$$= \left(\frac{M}{m} + 1\right)\frac{g}{H}. \quad \ldots\ldots \quad (3)$$

$$H = \left(\frac{M}{m} + 1\right)\frac{g}{\omega^2}. \quad \ldots\ldots \quad (4)$$

These results show that H is increased by an addition to the mass M; by adjusting M suitably, the governor arms can be made to run

at the most desirable angle to the vertical, whatever may be the normal speed of revolution.

Expt. 36.—Determination of the value of g by means of a simple pendulum. Arrange a simple pendulum by attaching a small heavy bob to one end of a long silk cord. Take a series of readings with varying lengths of cord, in each case taking care that the angle of vibration is small. For each length of cord L, note the time of 100 complete vibrations, and hence determine the period of vibration, T seconds.

$$T = 2\pi\sqrt{\frac{L}{g}};$$

$$\therefore T^2 \propto L.$$

Plot the values of T^2 and L obtained in the experiment; the resulting graph should be a straight line. From the graph determine the average value of the ratio

$$r = \frac{L}{T^2};$$

then

$$g = 4\pi^2 \times r.$$

Expt. 37.—Longitudinal vibrations of a helical spring. Hang a helical spring from a rigid support and attach a load to the lower end. Apply an additional smaller load and measure the extension produced by it. From the result calculate the force required to give unit extension to the spring. Remove the additional load: gently pull the load downwards and let go. Since the extension of the spring is proportional to the pull applied (p. 155), the force at any instant tending to return the load to the initial position is proportional to the displacement from this position. Hence the load will have simple harmonic vibrations. The spring also vibrates, and may be taken into account by adding one-third of its mass to that of the load.

Let m = the mass of the load + $\frac{1}{3}$ mass of the spring.

μ = the force required to produce unit extension of the spring.

Then $T = 2\pi\sqrt{\frac{m}{\mu}}$ seconds (p. 223).

Evaluate this time, and check it by finding the period of vibration experimentally. Do this by finding the time t taken to execute 100 vibrations, when

$$T = \frac{t}{100}.$$

Let m_1 be the mass of the load required to give unit extension to the spring, then $\mu = m_1 g$, and $T = 2\pi\sqrt{m/m_1 g}$, therefore $g = 4\pi^2 m/T^2 m_1$. Hence calculate the value of g from the experimental quantities.

EXERCISES ON CHAPTER XVI.

1. A body having a mass of 20 pounds revolves in a circular path of 9 inches radius with a velocity of 40 feet per second. Find the centrifugal force.

2. A small wheel revolves 24,000 times per minute. There is a body having a mass of 0 05 pound fastened to the wheel at a radius of 4 inches. Find the centrifugal force.

3. Assuming that the earth rotates once in 24 hours, and that the equatorial diameter is 8000 miles, find the centrifugal force acting on a person having a mass of 150 pounds when at the equator.

4. A cylinder has equal masses of 10 pounds each attached to its ends at radii of 9 inches. The distance between the masses, parallel to the axis of the cylinder, is 12 inches Looking at the end of the cylinder, both masses appear to be on the same diameter, on opposite sides of the centre. Calculate the rocking couple when the angular velocity is 10π radians per second.

5. A railway coach, mass 20 tons, runs round a curve of 1600 feet radius at a speed of 45 miles per hour. Calculate the centrifugal force. If both rails are on the same level, 5 feet apart centre to centre, and if the centre of mass of the coach is 6 feet above rail level, find the resultant force on each rail.

6. An oval track for motor cycles has a minimum radius of 80 yards, and has to be banked to suit a maximum speed of 65 miles per hour. Find the slope of the cross section at the places where the minimum radii occur.

7. A bicycle and rider together have a mass of 180 pounds. Find the angle which the machine must make with the horizontal in travelling round a curve of 12 feet radius at 8 miles per hour. At this speed, what frictional force must the ground exert on the wheels if no side slip occurs ? What is the minimum safe value of the coefficient of friction ?

8. A point describes simple harmonic vibrations. If the period is 0·3 second and the amplitude 1 foot, find the maximum velocity and the maximum acceleration.

9. A body having a mass of 4 grams executes simple harmonic vibrations. The force acting on the body when the displacement is 8 cm. is 24 grams weight. Find the period. If the maximum velocity is 500 cm. per second, find the amplitude and the maximum acceleration.

10. A simple pendulum beats quarter-seconds in a place where $g = 32\ 18$ feet per second per second. Find its length. If the pendulum is taken to a place where $g = 32\ 2$ feet per second per second, how many seconds per day would it gain or lose ?

11. Two simple harmonic vibrations, A and B, of equal periods and differing in phase by $\pi/2$, are impressed on the same particle. The amplitudes of A and B are 4 and 6 inches respectively. Find the amplitude of the resulting vibration and its phase difference from A.

12. In a conical pendulum find the height in feet of the cone of revolution for velocities of 20, 40, 60, 80, 100, 120 revolutions per minute. Plot a graph showing the relation of the height and the revolutions per minute.

13. The height of a conical pendulum is 8 inches and the arm is 12 inches long. Find the period. If the mass at the end of the arm is 2 pounds, find the pull in the arm. Find the revolutions per minute at which the arm will make 45° with the axis of revolution.

14. Find the change in the height of the cone of revolution of a simple unloaded governor when the speed changes from 60 to 62 revolutions per minute.

15. In a loaded governor the mass at the end of each arm is 2 pounds. The arms are each 8 inches long. The height of the cone of revolution has to be 5 inches at 180 revolutions per minute. Find the load which must be placed on the sleeve.

16. In the governor given in Question 15, the heights between which the governor works are 5·5 and 4·5 inches. Find the maximum and minimum speeds of revolution.

17. A train is travelling round a curve of 500 feet radius at a speed of 30 miles per hour. The distance between centres of rails is 3 ft. 9 in. If the resultant force on the train is to be perpendicular to the line joining the tops of the rails, find how much the outer rail must be raised above the inner. Adelaide University.

18. The roadway of a bridge over a canal is in the form of a circular arc of radius 50 ft. What is the greatest velocity (in miles per hour) with which a motor cycle can cross the bridge without leaving the ground at the highest point? L.U.

19. A train is travelling in a curve of 240 yards radius. The centre of gravity of the engine is 6 feet above the level of the rails, and the distance between the centre lines of the rails is 5 feet. Find the speed at which the engine would be just unstable, if the rails are both at the same level. L.U.

20. A motor racing track of radius a is banked at an angle α; obtain an equation which will give the speed for which the track is designed. Show that if the speed of a car is one-half this speed there will be a total transverse frictional force of $\frac{3}{4}W \sin \alpha$ between the car and the ground, W being the weight of the car. L.U.

21. The period of a simple harmonic motion is $2\pi/p$, and its amplitude is A. Prove that the displacement can be expressed in the form

$$A \cos (pt - \alpha),$$

and find the velocity.

The distance between the extreme limits of the oscillation is 6 inches, and the number of complete oscillations per minute is 100. Calculate the velocity of the point when it is 2 inches from the centre; find also the interval of time from the centre to that point. Sen. Cam. Loc.

22. A particle is performing a simple harmonic motion of period T about a centre O, and it passes through a point P with velocity v in the direction OP; prove that the time which elapses before its return to P is $(T/\pi) \tan^{-1}(vT/2\pi OP)$. L.U.

23. A particle moves with simple harmonic motion; show that its time of complete oscillation is independent of the amplitude of its motion. The amplitude of the motion is 5 feet and the complete time of oscillation

is 4 secs.; find the time occupied by the particle in passing between points which are distant 4 feet and 2 feet from the centre of force and are on the same side of it. L.U.

24. A weight of 5 lb is tied at the end of an elastic string, whose other end is fixed, and is in equilibrium when the string is of length 14 inches, its unstretched length being 12 inches. The weight is pulled gently down, through another inch, and then let go; find the time of the resulting oscillation. L.U.

25. Show that the vertical distance of the bob of a conical pendulum beneath the fixed end of the string depends only upon the number of revolutions of the pendulum per sec. If the mass of the bob is 4 pounds, and the length of the string is 2 ft., find the maximum number of revolutions per second of the pendulum when the greatest tension that can with safety be allowed in the string is 40 lb. weight. L.U.

26. Prove that the restoring force acting on a simple pendulum is proportional to the angle through which it is displaced from the equilibrium position, provided this angle be small.

Describe also a method of verifying the above result by experiment. Adelaide University.

27. A simple pendulum, 10 feet long, swings to and fro through a distance 2 inches. Find its velocity at its lowest point, its acceleration at its highest point, and the time of an oscillation, calculating each result numerically in foot and second units. L.U.

28. Investigate the time of revolution of a conical pendulum.

A ball, of mass one pound, describes a horizontal circle attached to two cords, the other ends of which are fixed to two points in the same vertical line. The cords are each of length 3 feet, and are at right angles to one another. If the ball makes 100 revolutions a minute, compute the tension of each cord in pounds weight. Adelaide University.

29. Two equal light rods, AB and BC, are freely jointed to a particle of mass m at B; the end A of the rod AB is pivoted to a fixed point A, and the end C of BC is freely jointed to a smooth ring of mass m, which can slide on a smooth vertical rod AC. Show that, when C is below A and the mass at B is describing a horizontal circle with uniform angular velocity ω, $\cos\alpha = 3g/l\omega^2$, where α is the inclination of the rods to the vertical and l is the length of either rod. L.U.

30. Show that a body moving with uniform velocity v in a circle of radius r has acceleration equal to v^2/r directed towards the centre. Hence explain why a man riding a bicycle on a curved path has always to bend his body inwards towards the centre of the path. Panjab Univ.

CHAPTER XVII

IMPACT

Direct impact.—Direct impact occurs when two bodies are both travelling in the straight line joining their centres of mass before collision, or when a moving body impinges normally on a fixed surface. It is not possible to state the precise magnitude of the stress between two bodies, A and B, at any instant during impact, but we may say that whatever action A exerts on B, at the same instant B exerts an equal opposite reaction on A. Also these actions are maintained during the same interval of time. Thus a diagram, showing the relation of the action F which A exerts on B at any instant t seconds after the commencement of impact, would be similar and equal to a diagram showing the reactions which B exerts on A. The area of such a diagram represents the change in momentum of the body (p. 72); hence, since the areas are equal, we may say that the momentum acquired by one body is equal and opposite to that lost by the other body during the impact. It follows from this that **the total momentum before impact must be equal to the total momentum after impact is completed.**

Inelastic and elastic bodies.—The motion of the bodies after collision depends greatly on the degree of elasticity possessed by them. A body having no elasticity makes no effort whatever to recover its original form and dimensions. For example, deformation of a plastic substance like putty, which is practically inelastic, progresses so long as a force is exerted on it, and the putty retains the shape it possessed at the instant of the removal of the force. When two such bodies collide with direct impact, force between them ceases at the instant when their centres of mass cease to approach each other. Hence there is no tendency for the bodies to separate, and they continue to move as one body. In other words, the relative velocity after collision is zero.

In the case of elastic, or partially elastic bodies, the force does not cease at the instant of closest approach of the centres of mass. The effort which the bodies make to recover their original dimensions causes the action and reaction to continue, with the result that there is a second period during the impact, in which the centres of mass are receding from each other. Finally, the efforts to recover the original dimensions cease, and at this instant the bodies separate, and continue to move separately. Experiment shows that, roughly, the relative velocity after collision bears a definite ratio to the relative velocity before collision, and is of opposite sense The value of this ratio differs for different materials; it is called the **coefficient of restitution**

In direct impact (Fig. 260), let

u_1 = the velocity of the body A before impact
u_2 = the velocity of the body B before impact.
v_1 = the velocity of the body A after impact
v_2 = the velocity of the body B after impact.
e = the coefficient of restitution.

Then Relative velocity of approach $= u_1 - u_2$,
Relative velocity of separation $= v_2 - v_1$,

and
$$e = \frac{v_2 - v_1}{u_1 - u_2}$$

The coefficient of restitution has values about 0·95 for glass and about 0·2 for lead. Modern experiments indicate that the value of

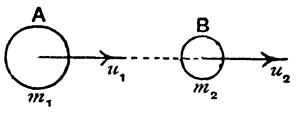


FIG. 260.—Direct impact

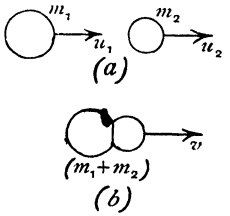


FIG. 261.—Direct impact of inelastic bodies

e may differ considerably for different parts of the surface of the same body. It is also well known that, if two metal bodies impinge twice, so that the same parts of their surfaces come into contact on both occasions, the hardness of the surfaces has been so altered during the first impact that a different value of the coefficient of restitution is apparent during the second impact.

Direct impact of inelastic bodies.—In Fig. 261 (*a*) two inelastic bodies of masses m_1 and m_2, and velocities u_1 and u_2, are about

to experience direct impact. u_1 is greater than u_2. We have (p. 233)

Total momentum before impact = total momentum after impact ;

$$\therefore\ m_1u_1 + m_2u_2 = (m_1 + m_2)v,$$

where v is the common velocity after impact (Fig. 261 (*b*)) ;

$$\therefore\ v = \frac{m_1u_1 + m_2u_2}{m_1 + m_2}. \quad \text{.....(1)}$$

If u_2 is of the sense opposite to that of u_1, then call u_2 negative ; hence

$$v = \frac{m_1u_1 - m_2u_2}{m_1 + m_2}. \quad \text{.....(2)}$$

In general,

$$v = \frac{m_1u_1 \pm m_2u_2}{m_1 + m_2}. \quad \text{.....(3)}$$

v will have the same or the opposite sense to u_1, according as the result in (3) is positive or negative.

Since work has been done in deforming the bodies, and there has been no recovery, it follows that energy has been wasted during the collision. The energy wasted may be calculated as follows :

Before impact, the total kinetic energy $= \frac{m_1u_1^2}{2} + \frac{m_2u_2^2}{2}$. .....(4)

After impact, the total kinetic energy $= \frac{(m_1 + m_2)v^2}{2}$. .....(5)

Hence, Energy wasted $= \left(\frac{m_1u_1^2}{2} + \frac{m_2u_2^2}{2}\right) - \frac{(m_1 + m_2)v^2}{2}$.

Inserting the value of v from (3), we have

$$\text{Energy wasted} = \left(\frac{m_1u_1^2 + m_2u_2^2}{2}\right) - \left(\frac{m_1 + m_2}{2}\right)\left(\frac{m_1u_1 \pm m_2u_2}{m_1 + m_2}\right)^2$$

$$= \frac{m_1u_1^2 + m_2u_2^2}{2} - \frac{(m_1u_1 \pm m_2u_2)^2}{2(m_1 + m_2)}.$$

By squaring and reducing to the simplest form, we obtain

$$\text{Energy wasted} = \frac{m_1m_2}{2(m_1 + m_2)}(u_1 \mp u_2)^2. \quad \text{.....(6)}$$

Now $(u_1 - u_2)$ is the relative velocity of approach before impact if both bodies are moving in the same sense, and $(u_1 + u_2)$ is the relative velocity if the senses of the velocities are opposite. Hence **the wasted energy is proportional to the square of the relative velocity of approach.**

Direct impact of bodies having perfect elasticity.—Perfect elasticity implies not only perfect recovery of shape and original dimensions,

but also perfect restitution of the energy expended during the deformation period. Hence no energy is wasted in the impact of perfectly elastic bodies.

To avoid complications, let the bodies be smooth spheres and let the impact be direct Let u_1 and u_2 be the velocities before collision, and let v_1 and v_2 be the velocities after collision (Fig. 262). As before, we have

Total momentum before collision = total momentum after collision ;

$$\therefore\ m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2. \quad \text{.....(7)}$$

Also

Total energy before collision = total energy after collision ,

$$\therefore\ \frac{m_1u_1^2}{2} + \frac{m_2u_2^2}{2} = \frac{m_1v_1^2}{2} + \frac{m_2v_2^2}{2} \quad \text{..... (8)}$$

From (8), $m_1(u_1^2 - v_1^2) = m_2(v_2^2 - u_2^2)$,

$$m_1(u_1 - v_1)(u_1 + v_1) = m_2(v_2 - u_2)(v_2 + u_2). \quad \text{.....(8')}$$

From (7), $m_1(u_1 - v_1) = m_2(v_2 - u_2)$.

Hence, from (8'), $u_1 + v_1 = v_2 + u_2$;

$$\therefore\ u_1 - u_2 = v_2 - v_1. \quad \text{.....(9)}$$

This result indicates that the relative velocity of approach is in this case equal to the relative velocity of separation ; in other words, the coefficient of restitution for perfectly elastic bodies is unity.

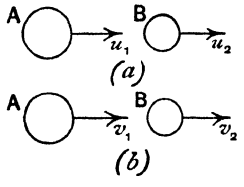


FIG 262.—Direct impact of elastic bodies

In using these and the following equations and in interpreting the results, velocities having the same sense as u_1 should be denoted positive, and those of opposite sense, negative ; negative results indicate velocities having senses opposite to u_1.

Supposing the masses to be equal, then, from (7) :

$$u_1 + u_2 = v_1 + v_2.$$

And from (9) : $u_1 - u_2 = v_2 - v_1$;

$$\therefore\ 2u_1 = 2v_2,$$

$$\therefore\ u_1 = v_2,$$

and $u_2 = v_1$.

It therefore follows that in the direct collision of perfectly elastic spheres having equal masses, the spheres interchange velocities during impact.

Direct impact of imperfectly elastic spheres.—Reference is made again to Fig. 262. As before, we have :

Total momentum before impact = total momentum after impact.

$$\therefore\ m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2. \quad \text{.....(10)}$$

Also, $$e = \frac{v_2 - v_1}{u_1 - u_2} \text{ (p. 234)};$$

$$\therefore\ eu_1 - eu_2 = v_2 - v_1. \quad \text{.....(10')}$$

Multiplying this by m_2 :

$$em_2u_1 - em_2u_2 = m_2v_2 - m_2v_1. \quad \text{.....(11)}$$

From (10) and (11),

$$(m_1 - em_2)u_1 + (1 + e)m_2u_2 = (m_1 + m_2)v_1 ;$$

$$\therefore\ v_1 = \frac{(m_1 - em_2)u_1 + (1 + e)m_2u_2}{m_1 + m_2}. \quad \text{.....(12)}$$

Multiplying (10') by m_1 gives

$$em_1u_1 - em_1u_2 = m_1v_2 - m_1v_1. \quad \text{.....} \quad (13)$$

From (10) and (13),

$$(1 + e)m_1u_1 + (m_2 - em_1)u_2 = (m_1 + m_2)v_2 ;$$

$$\therefore\ v_2 = \frac{(m_2 - em_1)u_2 + (1 + e)m_1u_1}{m_1 + m_2}. \quad \text{.....(14)}$$

Impact of a smooth sphere on a smooth fixed plane.—It is not possible to realise a plane absolutely fixed in space ; what is meant by a fixed plane is one fixed to the earth. The mass of the body against which an elastic sphere collides is then very large as compared with that of the sphere, and its velocity after impact may be taken as equal to its velocity before impact. Direct impact occurs when the line of motion of the sphere is normal to the fixed plane.

In **direct impact**, if the sphere and fixed plane are either or both inelastic, then the sphere will not rebound. If both sphere and plane are perfectly elastic, then the sphere rebounds with a velocity equal and opposite to that which it possessed before impact. If they are imperfectly elastic, and if the velocities of approach and separation are u and v respectively, then

$$v = eu. \quad \text{.....(1)}$$

The impact is **oblique** if the line of motion of the sphere prior to impact is inclined to the normal to the plane (Fig. 263). Let α be this angle, and let the sphere leave the plane in a line inclined at β to the normal. Let u and v be the initial and final velocities. Resolve these velocities parallel to and perpendicular to the plane.

Since both sphere and plane are regarded as being smooth there can be no force parallel to the plane during impact. Hence there can be no change in the component velocity parallel to the plane. Therefore

$$u \sin \alpha = v \sin \beta. \quad \ldots \quad \ldots \quad (2)$$

If the coefficient of restitution is e, then, from (1),

$$eu \cos \alpha = v \cos \beta \quad \ldots \quad \ldots \quad (3)$$

From (2) and (3),

$$u^2 \sin^2\alpha + e^2u^2 \cos^2\alpha = v^2(\sin^2\beta + \cos^2\beta) = v^2;$$

$$\therefore \ v = u\sqrt{\sin^2\alpha + e^2 \cos^2\alpha}. \quad \ldots \quad \ldots \quad (4)$$

Also, from (2) and (3)

$$\tan \beta = \frac{\tan \alpha}{e} \quad \ldots \quad \ldots \quad (5)$$

If both sphere and plane are perfectly elastic, then $e = 1$, and equations (4) and (5) become

$$v = u, \quad \ldots \quad \ldots \quad (6)$$

$$\tan \beta = \tan \alpha. \quad \ldots \quad \ldots \quad (7)$$

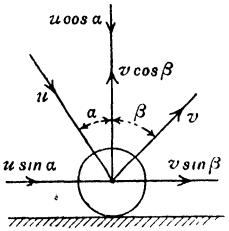


FIG 263.—Oblique impact of a sphere on a plane

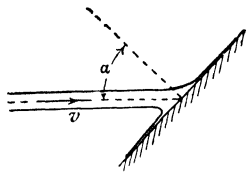


FIG 264.—Impact of a jet of water

Hence in this case the sphere leaves the plane with its initial velocity unaltered in magnitude, and the angles which the initial and final directions of motion make with the normal are equal.

If the sphere be perfectly inelastic, then the whole of the normal component $u \cos \alpha$ disappears, and the sphere will finally slide along the plane with a velocity $u \sin \alpha$

When a jet of water impinges on a fixed plate (Fig. 264), the impact practically follows the laws of inelastic bodies The jet spreads out during impact, and the water then slides along the plate.

Let v = the velocity of the jet.
α = the angle between the jet and the normal.
m = the mass of water reaching the plate per second.

Normal component of the velocity $= v \cos \alpha$.

This disappears during impact, hence .

Force acting on the plate = change in momentum per second
$= mv \cos \alpha$.

If A = the cross sectional area of the jet,
d = the density of water,

then $m = v\mathrm{A}d$;

$\therefore$ Force acting on the plate $= \mathrm{A}dv^2 \cos \alpha$.

Conservation of momentum.—This principle asserts that **the total momentum of any system of bodies which act and react on each other remains constant.** The truth of the principle will be evident when we consider the equality of the action which one body A exerts on another body B and the reaction which B exerts on A (Fig. 265). These actions continue during the same interval of time; hence whatever momentum B is losing, A is gaining an equal momentum. Hence the total momentum along AB remains constant. Similarly, the total momentum along each of the lines BC, CD, DA, AC and BD remains constant. Therefore the total momentum of the system remains constant. The forces may be caused by gravitational attraction, magnetism, or impact; their nature is immaterial; the important points are their equality, their opposing character and the equality of the times during which they act.

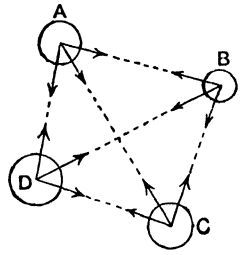


FIG 265.—Principle of the conservation of momentum.

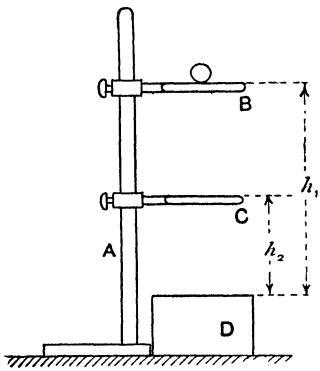


FIG. 266.—Apparatus for determining the coefficient of restitution

EXPT. 38.—**Coefficient of restitution** Arrange a tall retort stand A (Fig. 266) with two rings B and C which may be clamped at different heights. D is a massive block of cast-iron or steel. A small steel ball, $\frac{1}{4}$ inch to $\frac{1}{2}$ inch in diameter (these can be obtained from any cycle dealer), is dropped from the level of B and rebounds from D. The ring C is adjusted until it is found that the ball reaches its level in the first rebound. Measure h_1 and h_2. Then, keeping the ring C in its initial position, B is shifted to a position below

C, and the ball is dropped from the level of C. B is adjusted until it is found that the ball rebounds to its level. In this way are found the heights h_1, h_2, h_3, etc., of successive rebounds.

In the first drop, the velocity of approach $= u_1 = \sqrt{2gh_1}$
and the velocity of separation $= v_1 = \sqrt{2gh_2}$.
In the second drop, the velocity of approach $= u_2 = \sqrt{2gh_2}$,
and the velocity of separation $= v_2 = \sqrt{2gh_3}$.

The velocities for the succeeding drops may be calculated in the same way. Now

$$e = \frac{\text{velocity of separation}}{\text{velocity of approach}};$$

$$e_1 = \frac{v_1}{u_1} = \frac{\sqrt{2gh_2}}{\sqrt{2gh_1}} = \sqrt{\frac{h_2}{h_1}},$$

also

$$e_2 = \frac{v_2}{u_2} = \frac{\sqrt{2gh_3}}{\sqrt{2gh_2}} = \sqrt{\frac{h_3}{h_2}}.$$

Similarly,

$$e_3 = \sqrt{\frac{h_4}{h_3}}$$

Work out these values of e from the experimental values of h_1, h_2, etc. Are they in fair agreement? What is the mean value of e? What is the maximum error in the value of e stated as a percentage on the mean value?

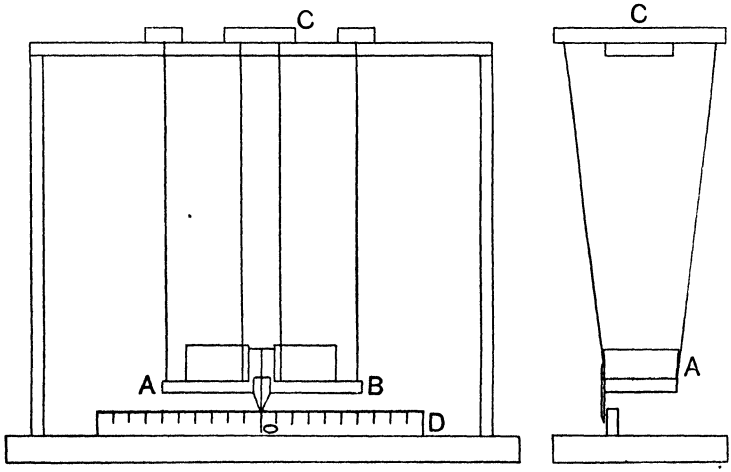


FIG 267.—Hicks's ballistic pendulum.

EXPT. 39.—**Ballistic pendulum.** In the Hicks's form of this apparatus (Fig. 267) two platforms, A and B, are each suspended from supports by means of four threads. As seen in the front elevation, the threads appear

vertical; in the end elevation the threads spread as they approach the upper support. The platforms just touch when hanging freely, and in this position the pointer which each carries is at zero on the scale D. The platforms are of equal mass and can be loaded by placing weights on them. There is a locking contrivance, by means of which the platforms become locked together automatically after impact, and thus move as one body. The suspending threads are about 3 feet in length.

Referring to Fig. 268, in which the bob of a pendulum has been displaced a distance BD from the vertical, and has been raised a height CD from the position of static equilibrium, we have for the velocity v at the instant the bob passes through C when swinging freely:

$$v = \sqrt{2g \cdot CD}.$$

Now $CD \times 2AC = BD^2$ nearly;

$$v = \sqrt{2g \frac{BD^2}{2AC}} = \text{a constant} \times BD.$$

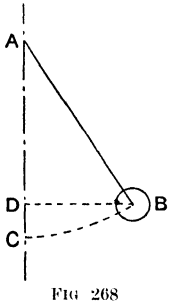


FIG 268

Hence the maximum velocity is very nearly proportional to the horizontal displacement. In the Hicks's pendulum we may therefore assume that the maximum velocity of either platform is proportional to the distance through which it has been displaced, as shown by the scale D (Fig. 267).

Place equal masses on the platforms; displace each platform to the same extent and let go. It will be found that the platforms immediately after impact are at rest. This follows from the fact that the momenta immediately before impact were equal and opposite, and hence the total momentum was zero.

Now load A until the total mass is, say, 1 pound, and load B until its total mass is, say, 2 pounds. Displace B through 2 inches, and displace A through 4 inches; again let go and observe what happens at the moment of impact. If the platforms remain at rest, the momenta before impact were equal and opposite. Repeat the experiment, varying the masses of A and B and also the displacements.

EXERCISES ON CHAPTER XVII.

1. Two inelastic bodies, A and B, moving in the same straight line come into collision. The mass of A is 4 pounds and its velocity is 10 feet per second; the mass of B is 10 pounds and its velocity is 6 feet per second. Find the common velocity after collision. How much energy has been wasted?

2. Answer Question 1, supposing the velocity of B to be -6 feet per second.

3. Direct impact occurs between two spheres A and B. The masses are 4 and 3 kilograms respectively, and the velocities are 12 and 8 metres

per second respectively. The coefficient of restitution is 0·7. Find the velocities after impact. Find also the energy wasted.

4. Answer Question 3, supposing the velocity of B to be −8 metres per second.

5. In Questions 3 and 4, what would be the velocities of A and B after collision, supposing both bodies had been perfectly elastic?

6. A sphere A, having a mass of 10 pounds, experiences direct impact with another sphere B, mass 16 pounds, velocity 12 feet per second. The coefficient of restitution is 0 5. Find the initial velocity of A if it remains at rest after impact; find also the velocity of B after impact.

7. A small steel ball is dropped vertically on to a horizontal fixed steel plane from a height of 9 feet. If the coefficient of restitution is 0 8, find the heights of the first, second, third and fourth rebounds. If the mass of the ball is 0 1 pound, how much energy is wasted during the first three impacts?

8. In Question 7 the ball is dropped vertically from the same height, and the fixed plane is at an angle of 30° to the horizontal. Find the velocity with which the ball leaves the plane. Assume both ball and plane to be smooth.

9. A jet of water having a sectional area of 0 5 square inch and a velocity of 40 feet per second, impinges on a fixed flat plate. Find the force acting on the plate when the jet makes angles of 0, 30, 45, 60 and 90 degrees with it. Plot a graph showing the relation of forces and angles.

10. If a gun of mass M fires horizontally a shot of mass m, find the ratio of energy of the recoil of the gun to the energy of the shot.

If a ½-ton gun discharges a 50-pound shot with a velocity of 1000 ft. per sec., find the uniform resistance necessary to stop the recoil of the gun in 6 inches. L.U.

11. State Newton's law of impact, and show how it can be experimentally verified. A smooth sphere of small radius moving on a horizontal table strikes an equal sphere lying at rest on the table at a distance d from a vertical cushion, the impact being along the line of centres and normal to the cushion. Show that if e be the coefficient of restitution between the spheres and between a sphere and the cushion, the next impact between the spheres will take place at a distance $\frac{e^2}{1+e^2} \cdot 2d$ from the cushion.

L.U.

12. What do you understand by Conservation of Momentum? Describe an experimental method of illustrating the conservation of momentum at the impact of two bodies. L.U.

13. Define "impulse" and energy, and give their dimensions in terms of the fundamental units of length, time and mass.

A box of sand, used as a ballistic pendulum, is suspended by four parallel ropes, and a shot is fired into its centre. In one experiment the weight of the box was 1000 lb., the weight of the shot was 10 lb, the length of the ropes was 6 feet, and the displacement of the centre of mass of the box and shot was 4½ feet. What was the velocity of the shot before hitting the box? L.U.

14. A uniform chain, 10 feet long and having a mass of 4 pounds, hangs vertically from an upper support, and its lower end touches the scale pan of a balance. The upper end is released, and the chain falls into the scale pan. Find the force acting on the pan at the instant when the last link reaches the pan. Find the energy wasted.

CHAPTER XVIII

HYDROSTATICS

Definition of a fluid.—Substances in the **fluid state** are incapable of offering permanent resistance to any forces, however small, tending to change their shape. Fluids are either **liquid** or **gaseous** Gases possess the property of indefinite expansion and liquids do not Liquids in a partially filled vessel show a distinct surface not coinciding with any of the walls of the vessel, if this surface is in contact with the air, as would be the case in an open vessel. it is called the **free surface**

Comparatively small compressive forces cause appreciable alteration in the volume of a gas, liquids show very little change in volume, even when the compressive forces are very great It may be assumed for our present purposes that liquids are incompressible. This assumption, together with the neglect of changes in volume due to changes in temperature, is equivalent to taking the density of any given liquid to be constant.

The property which differentiates a liquid from a solid is the ability of the former to flow Some liquids, such as treacle and pitch, flow with difficulty, and are said to be **viscous**; the property is called **viscosity**. **Mobile** liquids, such as alcohol and ether flow easily. No fluid is perfectly mobile.

That branch of the subject which treats of fluids at rest is called **hydrostatics**. In **hydrokinetics**, the laws of fluids in motion are discussed. **Pneumatics** deals with the pressure and flow of gases. **Hydraulics** is the branch of engineering which treats of the practical applications of the laws of the pressure and flow of liquids, especially of water.

Normal stress only can be present in a fluid at rest.—Change of shape of a body occurs always as a consequence of the application of shearing stresses (p. 154). Hence, if there be shearing stresses

present in a fluid, the fluid must be in the act of changing shape, and must therefore be in motion. Therefore **there can be none but normal stresses acting on the boundary surfaces and on any section of a fluid at rest.** Since friction is always evidenced as a force acting tangentially to the sliding surfaces, it follows that there can be no friction in a fluid at rest.

The term **pressure** is given to the normal stress which a fluid applies to any surface with which it is in contact. Pressure is stated in units of force per unit of area. The dimensions are therefore the same as those of stress, viz.

$$\frac{ml}{t^2} \times \frac{1}{l^2} = \frac{m}{lt^2}.$$

In general, the pressure of a fluid varies from place to place. The **pressure at a given point** may be defined as follows: Take a small area a embracing the point, and let P be the resultant force which the fluid exerts on a. The average value of the pressure on a is P/a. The actual value of the pressure at any part on the small area differs from the average value to a small extent only, and the difference will become smaller if a be diminished. Of course, P will then become smaller also. If a be diminished indefinitely, thus approximating to a point, the value of P/a gives the pressure at this point.

Pressures may be stated in dynes, or in grams weight, per square centimetre; for practical purposes the most convenient metric unit is the kilogram weight per square centimetre. In the British system we may use poundals per square foot, or, for practical purposes, pounds weight per square foot, or per square inch.

Pressure at a point on a horizontal area at a given depth in a liquid under the action of gravity.—In Fig. 269 is shown an open vessel containing liquid at rest. Consider the equilibrium of a vertical column of the liquid, of height y measured downwards from the free surface. Let the lower end of the column be horizontal and have an area a; this area is supposed to be small, and all horizontal sections of the column are taken to be equal and similar.

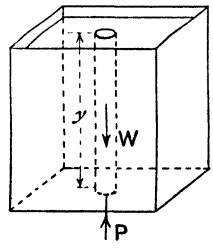


FIG. 269.—Pressure at a given depth in a liquid

Neglecting any gaseous pressure acting on the free surface, the forces acting on the column are (i) its weight W; (ii) the upward reaction P which the liquid immediately under the foot of the column exerts on the

column; (iii) the forces exerted by the liquid surrounding the column; these forces act inwards and prevent the column from spreading outwards. The forces mentioned in (iii) are applied everywhere in directions normal to the vertical sides of the column, and are therefore horizontal. Hence they cannot contribute directly to the equilibrating of the vertical force W. Therefore P and W must be equal, and must act in the same straight line.

If d is the density of the liquid, then dg, or w, is the weight of the liquid per unit of volume. Let p be the pressure at the depth y, then, since the volume of the column is ay,

$$P = W = way,$$

or

$$p = \frac{way}{a} = wy \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (1)$$

It will be noted that w has been assumed to be constant throughout the column, *i.e.* the liquid has been assumed to be incompressible. Hence the result should not be applied to a compressible fluid such as air. Note also that the pressure in a given liquid is proportional to the depth y.

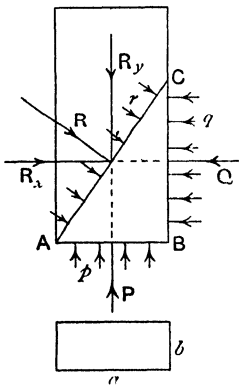


FIG. 270.—Pressure on an inclined surface.

Pressure at a point on an inclined surface.—In Fig. 270 is shown a vertical column of liquid of rectangular section and having small transverse dimensions a and b. AB is the horizontal bottom of the column, and AC is a sloping section. Consider the equilibrium of the wedge ABC, neglecting the weight of the wedge and taking account only of the pressures p, q and r acting on AB, BC and CA respectively. As the faces of the wedge are taken very small, we may assume that p, q and r are distributed uniformly; hence they give rise to resultant forces $P = p \times AB \times b$, $Q = q \times BC \times b$, and $R = r \times AC \times b$, and these forces act normally at the centres of the faces. Hence P, Q and R intersect at the centre of the circle circumscribing the triangle ABC, and thus comply with one of the conditions of equilibrium of three non-parallel forces. Taking horizontal and vertical components of R, we have

$$R_x = R \sin BAC = Q. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

$$R_y = R \cos BAC = P. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

From (1), $r \cdot AC \cdot b \cdot \sin BAC = q \cdot BC \cdot b$,

or $r \cdot \sin BAC = q \frac{BC}{AC} = q \cdot \sin BAC$;

$\therefore r = q.$ .....(3)

From (2), $r \cdot AC \cdot b \cdot \cos BAC = p \cdot AB \cdot b$,

or $r \cdot \cos BAC = p \cdot \frac{AB}{AC} = p \cdot \cos BAC$;

$\therefore r = p.$ .....(4)

$\therefore p = q = r.$ .....(5)

Strictly speaking, this result is true only when the dimensions of the wedge are reduced indefinitely, in which case the assumptions made become justifiable. In the limit, the wedge becomes a point, lying on three intersecting planes, one horizontal, one vertical and one inclined, and we may assert that the fluid pressure at the intersection of these planes is the same on each plane, *i.e.* **the pressure at a point in a fluid is the same for any plane passing through that point.** Hence equation (1) (p. 246) becomes available for calculating the pressure at a point on any immersed surface, whatever may be its inclination.

Head. - Since the pressure in a given liquid is proportional to the depth y below the free surface, pressures are often measured by stating the value of y and also the name of the liquid; y is then called the **head.** The head may be defined as the vertical height of a column of liquid reaching from the point under consideration up to the free surface level. Thus a head of 30 inches of mercury (density 13·59 grams per cubic cm.) is equivalent to a pressure of 14·7 lb. per square inch, and a head of 144 feet of water (density 62·3 pounds per cubic foot) is equivalent to a pressure of 62·3 lb. per square inch.

Pressures are also sometimes stated in **atmospheres.** One atmosphere may be defined for the present as the pressure produced at the base of a column of mercury 76 centimetres high. This is equivalent to a pressure of $76 \times 13{\cdot}59 = 1032{\cdot}8$ grams weight per square centimetre, or to 1·033 kilograms weight ($= 1{\cdot}0132 \times 10^6$ dynes) per square centimetre. In the British system one atmosphere is taken as the pressure at the base of a column of mercury 30 inches high, and is equivalent to a pressure of 14·7 lb. per square inch.

The pressure in a liquid at rest is constant at all points in a horizontal plane.—In Fig. 271 is shown a horizontal row of small liquid cubes, enlarged in the drawing for the sake of clearness. The cubes

are actually supposed to be indefinitely small, and may be thus looked upon as forming part of a horizontal line. Let the cube a be at a depth y in the liquid, then the pressure on each of its faces will be $p = wy$ (p. 246). Hence the vertical face in contact with the cube b exerts a pressure equal to p on the vertical face of b, and therefore every face of the cube b has a pressure equal to p. Similarly, all faces of the cube c will be subjected to pressures equal to p, and so on to the end of the row. Thus the pressures at all points in a horizontal immersed line are equal, and since a horizontal line can be drawn in any direction in a horizontal plane, it follows that the pressures at all points in a horizontal immersed plane are equal

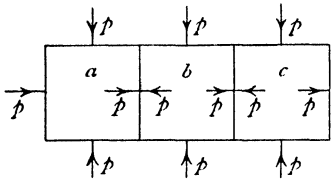


FIG 271.—Transmission of pressure through a row of cubes

It follows that the total force which a liquid exerts on a horizontal area may be calculated by taking the product of the pressure and the area.

Let d = the density of the liquid.
$w = dg$ = its weight per unit volume
A = the horizontal area.
y = the depth of the liquid.

Then Total force = $P = wyA = dg\,yA$ . . . . . (1)

The free surface of a liquid at rest is a horizontal plane.—In Fig. 272, A, B and C are points on an immersed horizontal plane, and are at depths y_1, y_2 and y_3 respectively below the free surface, which we assume not to be a horizontal plane. The pressures at A, B and C are respectively wy_1, wy_2 and wy_3, and these are equal, from what has been said above. Therefore $y_1 = y_2 = y_3$, and hence the free surface must be a plane parallel to that containing A, B and C, and must therefore be a horizontal plane This result must be modified somewhat for places near the walls of the vessel, where the effect of surface tension causes curvature in the free surface (Chap. XXII.).

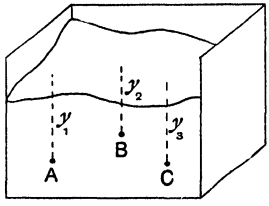


FIG 272.—Free surface of a liquid at rest

In Fig. 273 (*a*) is shown a vessel containing liquid at rest, the free surface being AB. Any portion of the liquid, such as CDE, is in equilibrium, and this state will not be disturbed by the enveloping

of this portion in a bent tube. The pressures which were supplied initially by the surrounding liquid are now supplied by the walls of

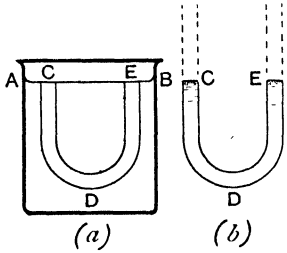


FIG. 273.—Free surfaces in a tube

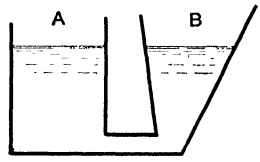


FIG. 274.—Free surfaces in communicating vessels

the tube. Further, the tube, now full of liquid, may be removed without disturbing the liquid contained in it, *i.e.* the free surfaces at C and E (Fig. 273 (*b*)) will still be in a horizontal plane. If required, both limbs of the U tube may be extended upwards without producing any effect on the state of equilibrium of the liquid. We infer that the free surface of a liquid at rest lies entirely in a horizontal plane, even when the liquid is contained in different but communicating vessels (Fig. 274). This fact leads to the popular statement that water always finds its own level.

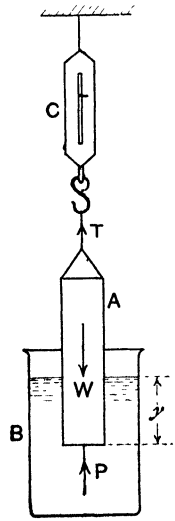


FIG 275.—Apparatus for finding the pressure at different depths.

EXPT. 40.—**Pressure on a horizontal surface at different depths.** Arrange apparatus as shown in Fig. 275. A is a brass tube suspended vertically from a spring balance C and partially immersed in a liquid contained in a vessel B. The tube is closed at its lower end, and the outside of the bottom is horizontal. The tube may be loaded internally and may thus be immersed at different depths; a scale of centimetres engraved on the outside of the tube (zero at the bottom) enables the depth y of the bottom below the free surface to be observed.

It is evident that the total upward force P which the liquid exerts on the bottom together with the upward pull T exerted by the spring balance is equal to the weight W of the tube and contents. Hence

$$P + T = W,$$

or

$$P = W - T.$$

Make a series of experiments, and evaluate P for each; note the depth y for each experiment. Since the area of the bottom of the tube is

constant. P will be proportional to the pressure at the depth y. Test if this is so by plotting P and y, a straight-line graph provides evidence of the truth of the law.

Total force acting on one side of an immersed plate. If the plate is horizontal, *e.g.* the horizontal bottom of a vessel containing a liquid, the pressure is uniform and the total force is calculated by taking the product of the pressure and the area.

Let w = the weight of the liquid per unit volume.

y = the depth of the plate below the free surface.

A = the area of one side of the plate.

Then Total force exerted on one side = $\mathsf{P} = wy\mathsf{A}$. .... (1)

The following method is applicable to both vertical and inclined plates (Fig. 276 (*a*) and (*b*)). Let a be a small area of the plate at a

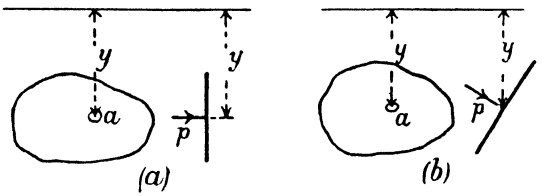


FIG. 276.—Total pressure on immersed surfaces.

depth y below the free surface. Let p be the pressure on a; then

$$p = wy,$$

and Force acting on $a = wya$.

This expression applies equally to any other small area of the plate; hence

Total force exerted on one side = $\mathsf{P} = w(a_1y_1 + a_2y_2 + a_3y_3 + \text{etc.})$

$$= w\Sigma ay. \ldots . \quad . \quad . \ldots \ldots \ldots (2)$$

Now Σay is the simple moment of area of the plate about the free surface of the liquid, and may be calculated by taking the product of the total area A of one side of the plate, and the depth y of its centre of area (a point which coincides with the centre of gravity of a thin sheet having the same shape and area as the plate). Thus:

$$\mathsf{P} = w\mathsf{A}\bar{y}. \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

$w\bar{y}$ is the pressure of the liquid at the centre of area; hence the rule: **The total force on one side of an immersed plate is given by the**

product of the area and the pressure at the centre of area. Thus the pressure at the centre of area is the average pressure on the plate.

The above proof does not depend upon the surface of the plate being plane, so that the rule applies also to curved surfaces, such as a sphere immersed in a liquid.

EXAMPLE 1.—Find the total force exerted on the wetted surface of a rectangular tank 6 feet by 4 feet by 2 feet deep when full of water.

$$\text{Total force on the bottom} = wA_1y_1$$
$$= 62{\cdot}3 \times 6 \times 4 \times 2$$
$$= \underline{2990} \text{ lb. weight.}$$

$$\text{Total force on one side} = wA_2y_2$$
$$= 62{\cdot}3 \times 6 \times 2 \times 1$$
$$= \underline{747{\cdot}6} \text{ lb. weight.}$$

$$\text{Total force on one end} = wA_3\bar{y}_3$$
$$= 62\ 3 \times 4 \times 2 \times 1$$
$$= \underline{498{\cdot}4} \text{ lb. weight.}$$

$$\text{Total force on the wetted surface} = 2990 + (747{\cdot}6 + 498{\cdot}4)2$$
$$= \underline{5482} \text{ lb. weight.}$$

EXAMPLE 2.—A cylindrical tank 7 feet in diameter has its circular bottom horizontal and contains water to a depth of 4 feet. Find the total force exerted by the water on the curved wetted surface.

The centre of area of the curved surface lies on the axis of the cylinder at a depth of 2 feet below the free surface; hence

$$\text{Total force on the curved surface} = wAy$$
$$= 62{\cdot}3 \times (\pi d \times 4) \times 2$$
$$= 62{\cdot}3 \times \tfrac{22}{7} \times 7 \times 4 \times 2$$
$$= \underline{10{,}965} \text{ lb. weight.}$$

EXAMPLE 3.—A sphere 8 cm. in diameter is sunk in an oil weighing 0·8 gram per cubic centimetre. The centre of the sphere is at a depth of 40 centimetres. Calculate the total force on the surface of the sphere.

$$\text{Total force} = wAy$$
$$= 0{\cdot}8 \times 4\pi r^2 \times 40$$
$$= 0{\cdot}8 \times 4 \times \tfrac{22}{7} \times 16 \times 40$$
$$= \underline{6437} \text{ grams weight.}$$

The student should note that the total force exerted on the horizontal bottom of a vessel containing a liquid is independent of the shape of the vessel, and consequently is independent of the weight of the contained liquid. This follows as a consequence of the pressure being constant over the whole horizontal surface. The total force

is wAy, and it is evident that this is independent of the shape of the vessel.

Resultant force exerted by a liquid.—The total force exerted by a liquid on an area with which it is in contact is the arithmetical sum of the forces which the liquid exerts on the small areas into which the given area may be divided. The resultant force is the vector sum of these forces. In Example 1, p. 251, the total force on the wetted surface of the tank was found to be 5182 lb. weight. It is evident, however, that the total force acting on one side is balanced by the equal total force acting on the opposite side of the tank. Similarly, the total forces acting on the opposite ends balance each other, and therefore the resultant force exerted on the wetted surface is equal to the total force acting on the bottom, viz. 2990 lb. weight.

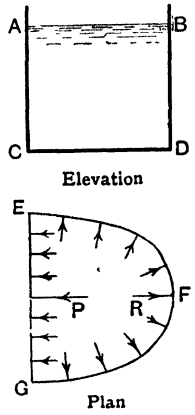


FIG 277.—Resultant force on plane and curved sides

In the case of all plane surfaces subjected to fluid pressure, the total force and the resultant force are equal. It will also be evident that the resultant force exerted by the liquid contained in a vessel of any shape is equal to the weight of the liquid. This is evident from the consideration that the resultant effect of the reactions of the walls of the vessel is to balance the weight of the contained liquid, and hence the resultant force exerted by the liquid must be equal to this weight, and must act vertically through the centre of gravity of the contained liquid.

In Fig. 277 is shown a vessel having one side plane, vertical and rectangular in shape; this side is EG in the plan and AC in the elevation. The remainder of the sides EFG is vertical, and is curved in the plan. The vessel contains liquid, the free surface of which is AB. That the vessel is equilibrated horizontally by the liquid pressures is apparent, as may be tested easily by suspending it from a long cord, when no horizontal movement will occur. Hence the resultant force P acting on the plane side EG must be equal and opposite to, and must act in the same straight line as the resultant force on the curved sides. In other words, if components of the forces which act normally on the curved sides be taken in directions perpendicular and parallel to EG, then the arithmetical sum of the components perpendicular to EG will be equal to P. Hence

the resultant force R acting on the curved sides may be found by evaluating P.

$$P = w A \bar{y} = w \times (AC \times EG) \times \tfrac{1}{2} AC ;$$

$$\therefore \quad R = \tfrac{1}{2} w \,.\, AC^2 \,.\, EG.$$

Centre of pressure.—The **centre of pressure** of an area exposed to fluid pressure is that point through which the resultant force acts. Let a vertical rectangular area ABDC (Fig. 278) be subjected to the pressure of a liquid, the free surface of which cuts the area in AB. It is evident from symmetry that the centre of pressure G lies in the vertical line HK, which divides the area into two equal and similar parts.

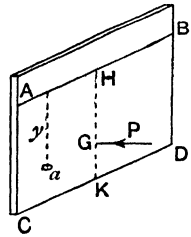


FIG 278.—Centre of pressure

To find the depth of G, consider a small area a lying in ABDC and at a depth y below AB. Then

Force acting on $a = way$.

Taking moments about AB, we have

Moment of the force acting on $a = way \times y = way^2$.

This expression serves for the moment of the force acting on any other small portion of the area ABDC; hence the total moment is given by

$$\text{Total moment about AB} = w(a_1 y_1^2 + a_2 y_2^2 + a_3 y_3^2 + \text{etc.})$$
$$= w \Sigma a y^2.$$

$\Sigma a y^2$ is called the **second moment of area**; the form of the expression is similar to that for the moment of inertia of a body, viz. $\Sigma m y^2$, and the results given on pp. 201-204 may be used by substituting the total area A for the total mass M. Writing $\Sigma a y^2 = I$, we have

$$\text{Total moment about AB} = wI. \qquad \ldots\ldots(1)$$

This moment may be expressed in another way. The resultant force P acts through G, therefore

$$\text{Total moment about AB} = P \times GH$$
$$= wA\bar{y} \times GH, \qquad \ldots\ldots(2)$$

where $\bar{y}$ is the depth of the centre of area below the free surface. Hence, from (1) and (2). $wA\bar{y} \times GH = wI$;

$$\therefore \quad GH = \frac{I}{A\bar{y}}. \qquad \ldots\ldots(3)$$

For the rectangular area ABDC (Fig. 278), and for the axis AB,

$$I = \frac{A \times HK^2}{3}, \quad \text{and} \quad \bar{y} = \tfrac{1}{2} HK ;$$

$$\therefore \quad GH = \frac{\frac{1}{3} A \times HK^2}{A \times \frac{1}{2} HK} = \tfrac{2}{3} HK.$$

It will be noted that the position of the centre of pressure is not affected by the kind of liquid, and that w disappears from the final result.

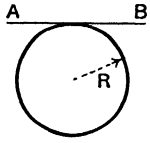


FIG 279

For a vertical circular area touching the free surface (Fig. 279) we have

$$I = \tfrac{5}{4}AR^2; \quad \bar{y} = R,$$

$$\therefore \text{ Depth of the centre of pressure} = \tfrac{5}{4}AR^2/AR = \tfrac{5}{4}R.$$

Pressure diagrams.—A pressure diagram, for an area subjected to fluid pressure, shows the pressure graphically at all points in the area. The method of construction may be understood by reference to Fig. 280, showing one side of a rectangular tank containing a liquid. Neglecting the gaseous pressure on the free surface of the liquid, the pressure at A on the side ABCD is zero, and the pressure at B is $w \times AB$. Make $BE = CF = w \times AB$ to any convenient scale of pressure, and join AE, DF and EF. The resulting figure is a wedge, and the pressure at any point in ABCD may be found by drawing a normal at that point to meet the sloping face of the wedge.

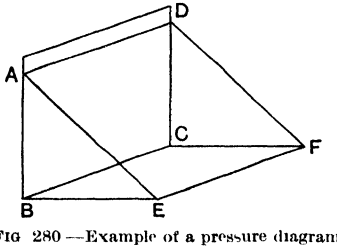


FIG 280.—Example of a pressure diagram

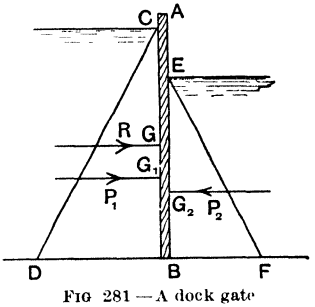


FIG 281.—A dock gate

EXAMPLE 1.—A gate closing the entrance to a dock is 40 feet wide. There is sea water on one side to a height of 30 feet, and on the other side to a height of 18 feet above the lower edge of the gate. Find the resultant force exerted by the water on the gate.

Referring to Fig. 281 (which is not drawn to scale), AB is the section of the gate, and the pressure diagrams for the high-water and low-water sides of the gate are CDB and EFB respectively. The total forces on the high-water and low-water sides are P_1 and P_2 respectively.

$$P_1 = wA_1\bar{y}_1$$
$$= 64 \times (40 \times 30) \times \tfrac{30}{2} = 1{,}152{,}000 \text{ lb. weight.}$$
$$P_2 = wA_2\bar{y}_2$$
$$= 64 \times (40 \times 18) \times \tfrac{18}{2} = 414{,}720 \text{ lb. weight.}$$

P_1 acts at the centre of pressure G_1, and BG_1 is one-third of BC (p. 253) and is therefore 10 feet. Similarly, P_2 acts at the centre of pressure G_2, and BG_2 is $18 \div 3 = 6$ feet. The resultant of P_1 and P_2 is the resultant force R required in the question.

$$R = P_1 - P_2 = 1{,}152{,}000 - 414{,}720$$
$$= \underline{737{,}280} \text{ lb. weight.}$$

Take moments about B, giving

$$R \times BG = (P_1 \times BG_1) - (P_2 \times BG_2);$$
$$\therefore BG = \frac{(1{,}152{,}000 \times 10) - (414{,}720 \times 6)}{737{,}280}$$
$$= \underline{12\ 25} \text{ feet.}$$

EXAMPLE 2.—The wall of a reservoir is rectangular in section (Fig. 282), 9 feet high and 4 feet thick. The free surface of the water is 1 foot below the top of the wall. Take moments about A, and evaluate the ratio,—overthrowing moment of the water/moment of resistance of the weight of the wall. The density of the water is 62·5 pounds per cubic foot, and the density of the material of the wall is 120 pounds per cubic foot.

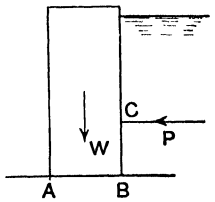


FIG 282.—A reservoir wall.

In examples of this kind it is customary to consider a portion of the wall one foot in length.

$$P = w\bar{A}\bar{y}$$
$$= 62{\cdot}5 \times (9 \times 1) \times \tfrac{9}{2}$$
$$= 2531{\cdot}25 \text{ lb. weight.}$$

And $$BG = \tfrac{9}{3} = 3 \text{ feet};$$

$$\text{Overthrowing moment} = 2531{\cdot}25 \times 3$$
$$= \underline{7593{\cdot}7} \text{ lb.-feet.}$$

$$\text{Weight of the wall} = (9 \times 4 \times 1) \times 120$$
$$= 4320 \text{ lb. weight.}$$

$$\text{Moment of resistance} = 4320 \times \tfrac{4}{2}$$
$$= \underline{8640} \text{ lb.-feet.}$$

$$\therefore \text{ Required ratio} = \frac{7593{\cdot}7}{8640} = \underline{0{\cdot}8789}.$$

EXERCISES ON CHAPTER XVIII.

1. Define a fluid. Distinguish the states solid, liquid and gaseous. Explain why normal stress only may be present in a fluid at rest.

2. What is meant by the pressure at a point in a fluid? How is pressure measured? What are the dimensions of pressure?

3. Calculate the pressure at a depth of 24 5 cm. in mercury. (Density of mercury = 13 6 grams per cubic centimetre.)

4. Find the pressure in lb. weight per square inch at a depth of 2 miles in sea water of density 64 pounds per cubic foot.

5. One side of a vessel slopes at an angle of 30° to the vertical. The vessel contains oil having a density of 52 pounds per cubic foot. Find the pressure on the sloping side at depths of 3 and 5 feet.

6. Prove that the pressure at a given depth in a liquid is the same on any plane.

7. What head of mercury corresponds to a head of 34·6 feet of water? To what pressure is the given head equal. (The density of mercury is 13·6 times that of water.)

8. Find the head of water necessary to produce a pressure of one atmosphere. State the result in feet.

9. Prove that the free surface of a liquid at rest is a horizontal plane.

10. A rectangular tank, 4 feet long, 2 feet broad and 2 feet deep, is full of water (density 62 5 pounds per cubic foot). Find the magnitudes of the total forces on the bottom, on one side and on one end.

11. Find the total force acting on the horizontal bottom of a cylindrical tank, 6 feet diameter and 3 feet deep, containing sea water (density 64 pounds per cubic foot) to a depth of 2 75 feet.

12. In Question 11 find the total force acting on the curved sides of the tank.

13. A tank 10 feet long has a rectangular horizontal bottom 4 feet wide. The ends of the tank are vertical; both sides are inclined at 45° to the horizontal. The tank contains water to a depth of 6 feet. Find the total forces acting on the bottom, on one side and on one end. (Density of water 62 5 pounds per cubic foot.)

14. In Questions 10 and 13 find the resultant forces exerted by the liquid on the tanks.

15. A hemispherical bowl, 12 cm. in diameter, is full of mercury (density 13 6 grams per cubic centimetre). Find the resultant force exerted by the liquid on the bowl.

16. A vessel has the form of an inverted cone, 6 inches diameter of base, 4 inches vertical height, and is full of oil having a density of 51 pounds per cubic foot. Find the resultant force and the total force acting on the curved inner surface of the vessel.

17. The ends of a vessel are triangular (Fig. 283). The side AB is vertical, and BC is inclined at 60° to the horizontal. AB = 3 feet, and the length of the vessel is 4 feet. Find the resultant forces acting on the vertical side, on the sloping side and on one end when the tank is full of water (density 62·5 pounds per cubic foot). Find the depth of the centre of pressure of the triangular end. (The second moment of area of a triangle about the base is $\frac{1}{6}AH^2$, where A is the area of the triangle and H is its vertical height.)

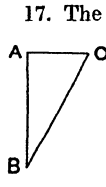


FIG. 283.

18. A rectangular opening in a reservoir wall is closed by a vertical door 4 feet high and 3 feet wide. The top edge of the door is 20 feet below the surface of the water. Find the resultant force acting on the wetted side of the door; find also the centre of pressure.

19. A hole in the vertical side of a tank containing water is 2 feet in diameter and is closed by a flap. The centre of the hole is 10 feet below the surface of the water. Find the resultant force which the water exerts on the flap, and show where it acts.

20. A cylindrical tank is 2 feet in diameter and 3 feet high, and has a vertical partition which divides the tank into two equal compartments. One compartment is full of oil of density 50 pounds per cubic foot, and the other is full of oil of density 55 pounds per cubic foot. Find the resultant forces acting on the inner curved surface of each compartment, and find also the resultant force acting on the partition.

21. A reservoir wall is rectangular in section; the wall is 20 feet long and 7 feet high. The depth of the water is 6 feet. Find the total force which the water exerts on the wall (neglect the pressure of the atmosphere). If the material of the wall weighs 120 lb. per cubic foot, what should be the thickness of the wall in order that the moment of the weight may be twice the overthrowing moment?

22. Draw a right-angled triangle ABC; AB is vertical and is 30 feet high; BC is horizontal and is 25 feet. The triangle represents the section of a reservoir wall. Take one foot length of the wall and find its weight, if the material weighs 140 lb. per cubic foot. Water pressure acts on the side AB, the free surface being 3 feet below the top of the wall. Find the resultant force which the water exerts on this portion of the wall. Find also the resultant of the force exerted by the water and the weight of the wall; mark the point in BC through which this force passes, and give its distance from B.

23. A dock gate is 12 feet wide. There is fresh water on one side of the gate to a depth of 9 feet, and on the other side to a depth of 6 feet. Find the resultant force which the water exerts on the gate and its position.

24. Obtain the dimensions of the units of force, pressure and energy in terms of the units of length, time and mass. Prove that a pressure of a million dynes per square centimetre is equivalent to a pressure of about 15 lb. wt. per square inch, having given that 1 pound = 454 grams, $g = 980$ cm./sec.^{2} and 1 inch = 2·54 cm. approximately. L.U.

25. A cubical open vessel of edge 1 ft. is filled with water; one of the vertical sides is hinged along its upper edge, and can turn freely about it. What force must be applied to the lower edge of the side so as just to keep it from opening? (The weight of a cubic foot of water is 62½ lb.) L.U.

26. A sea-wall slopes from the bottom at an angle of 30° to the horizon for 20 feet, and is then continued vertically upwards. Find the resultant horizontal and vertical forces on it, in tons weight per yard of its length, when there is a depth of 15 feet of water. (Take a cubic foot of sea-water to weigh 64 lb.) L.U.

27. A reservoir containing water to a depth of 20 feet has an opening in a vertical side 5 feet wide at the lower edge, 3 feet wide at the upper edge, and 4 feet high, and the lower edge is flush with the bottom of the reservoir. This opening is closed by a plate. If the coefficient of friction between the plate and the side of the reservoir is 0 2, find the force required to move the plate vertically. Adelaide University.

CHAPTER XIX

HYDROSTATICS (*CONTINUED*). HYDRAULIC MACHINES

Pressure of the atmosphere.—The weight of the atmosphere causes it to exert pressure on the surfaces of all bodies. This pressure may be rendered evident by the following experiment.

EXPT. 41.—**Pressure of the atmosphere.** Take a glass tube about 82 cm. in length, sealed at one end and open at the other (Fig. 284). Thoroughly clean and dry the interior of the tube. Fill it with clean mercury. Close the open end with a finger, and invert the tube two or three times so as to collect any contained air into one bubble; allow this bubble to escape and add mercury so as to fill the tube. Close the end with a finger, invert the tube and place its mouth below the surface of mercury contained in a beaker. Withdraw the finger and clamp the tube in a vertical position. It will be found that the mercury level falls to a definite height in the tube. The part of the tube above the mercury contains mercury vapour alone, at a pressure too small to be taken into account. This space is called a **Torricellian vacuum.** The pressure on the surface of the mercury in the tube may thus be taken as zero. At A the pressure of the atmosphere on the free surface of the mercury in the beaker is equal to the pressure inside the tube at the same level. The latter pressure is produced by the weight of the column of mercury in the tube. Let h be the height of the mercury column in centimetres, and let w be the weight of mercury in grams weight per cubic centimetre; then the pressure of the atmosphere at the time of the experiment is

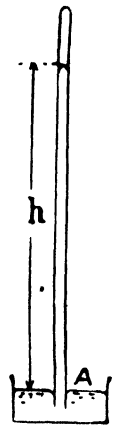


FIG. 284.—Apparatus for showing the principle of the barometer.

$$p = wh \text{ grams weight per sq. cm.}$$

Since w is constant, the height h is used in practice as a measure of the pressure of the atmosphere. The instrument described is a form of **barometer.**

From the observed height of mercury in the barometer, find the pressure of the atmosphere at the time of the experiment in grams weight per square centimetre and also in lb. weight per square inch.

Effect of gaseous pressure on the free surface of a liquid.—The pressure of the atmosphere, or other gaseous pressure, on the free surface of a liquid was neglected in Chapter XVIII.; it may be taken into account by the following artifice. In Fig. 285, AB is the free surface of a liquid contained in a vessel and is subjected to a gaseous pressure p_a. Let p_a be removed entirely, and let an equivalent pressure be obtained by the addition of another layer of the same liquid. The surface level of the liquid added is CD and is supposed to have no gaseous pressure acting on it. If the weight per unit volume of the liquid is w, the depth y_a of the layer may be found from

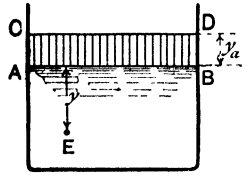


FIG. 285.—Effect of the pressure of the atmosphere.

$$p_a = wy_a,$$

or

$$y_a = \frac{p_a}{w}. \quad \ldots\ldots(1)$$

The pressure p at any point E in the liquid, situated at a depth y below the real free level AB, is given by

$$\begin{aligned} p &= w(y + y_a) \\ &= wy + wy_a \\ &= wy + p_a. \end{aligned} \quad \ldots\ldots(2)$$

It may therefore be said that the pressure at any point in the liquid is given by the sum of the pressure due to the weight of the liquid actually in the vessel, and the constant gaseous pressure applied to the free surface. This statement may be generalised by saying: **If, at a given place in a liquid, an additional pressure be applied, then that additional pressure is transmitted unaltered in magnitude to all points in the liquid.**

EXAMPLE.—The vertical side of a rectangular tank is 6 feet long and 4 feet high. If the tank is full of water, find the magnitude of the resultant force acting on the wetted side, taking into account the atmospheric pressure of 15 lb. wt. per square inch.

Due to the pressure of the atmosphere there is a uniform pressure on the wetted side of $15 \times 144 = 2160$ lb. wt. per square foot.

Total force due to the atmospheric pressure $= 2160 \times 6 \times 4$
$= 51{,}840$ lb. wt.

Due to the water alone, the total force is given by

Total force due to the water = average pressure × area
$= (62{\cdot}3 \times 2) \times (6 \times 4)$
$= 2990{\cdot}4$ lb. wt.

The magnitude of the resultant of these forces is given by their sum; hence

Resultant force $= 51{,}840 + 2990$
$= \underline{54{,}830}$ lb. wt.

In the case of open vessels and in other similar examples, the pressure of the atmosphere is neglected in practice. It is evident that both the outer and inner surfaces of the sides of the vessel are subjected to equal pressures by the atmosphere; hence the resultant forces due to these pressures balance, and the resultant effect on the sides of the vessel is the same as would be experienced by the application to the inner surfaces of the liquid pressures alone.

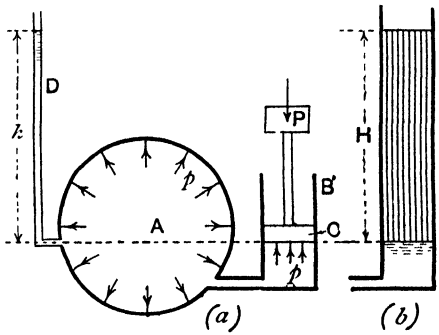


FIG. 286.—Pressure produced by a loaded piston.

Pressure produced by a piston.—In Fig. 286 (*a*), a vessel A is in communication with a cylinder B, which has a piston C capable of sliding freely in the cylinder, and nicely fitted so as to prevent leakage taking place between the piston and the walls of the cylinder. The vessel A and the portion of the cylinder below the piston are full of liquid. The piston carries a load the weight of which is P, and the area of the piston is a square units. It is evident that the downward force P is balanced by the resultant upward force which the liquid exerts on the piston. The latter force is produced by the pressure of the liquid, and if p be this pressure, we have

$$p = \frac{P}{a}.$$

It is immaterial whether this pressure is produced by means of a loaded piston, as in Fig. 286 (*a*), or by means of a column of liquid, as shown in Fig. 286 (*b*). If H is the head required to produce the pressure p, then

$$p = w\mathrm{H},$$

or

$$\mathrm{H} = \frac{p}{w},$$

where w is the weight of the liquid per unit volume.

The pressure p is transmitted uniformly throughout the liquid (p. 260), and hence the inner surfaces of the cylinder, pipes and vessel will be everywhere subjected to this pressure It will be understood in making this statement that the effects of the weight of the liquid in the vessel are disregarded, and that the effect of the loaded piston alone is being considered. The truth of the above statement may be proved by attaching a glass tube D to the vessel A in Fig. 286 (*a*) at a place on the same level as the lower side of the piston, when it will be found that the liquid rises in the tube to a height h, which will be found to be equal to the calculated value of the head H due to p. It will be noted that the actual pressure at points above the place where the tube is connected to A will be less than p, and at points below the connection greater than p, this being owing to the weight of the liquid in the vessel.

Hydraulic or Bramah press.—Very great forces may be obtained by the employment of a liquid under pressure. The principle may be understood by reference to Fig. 287, which shows an outline diagram of a **hydraulic** or **Bramah press.** A is a cylinder of small diameter fitted with a plunger rod B, which can slide in the cylinder. A load P is applied to B, thus producing pressure in the liquid which fills the lower part of the cylinder. A pipe E connects B with another cylinder C, having a diameter considerably larger than that of B. C is fitted with a ram D, which can slide in the cylinder C. The ram carries a load W. Since the pressure of the liquid is uniform throughout, we may calculate the relation of W and P as follows :

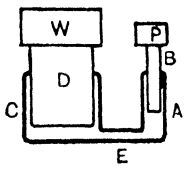


FIG. 287.—Principle of the hydraulic press.

Let d = the diameter of the plunger B.

D = ,, ,, ,, ram D.

p = the pressure of the liquid.

Then $p\frac{\pi d^2}{4}=\mathsf{P}$; and $p\frac{\pi \mathsf{D}^2}{4}=\mathsf{W}$;

$$\therefore \frac{\mathsf{W}}{\mathsf{P}}=p\frac{\pi \mathsf{D}^2}{4}\Big/p\frac{\pi d^2}{4}=\frac{\mathsf{D}^2}{d^2}. \quad \text{.....(1)}$$

It will be noted that the effect of friction in preventing free movement of the plunger and ram in the cylinders has been neglected in the above.

So far we have considered only the static balancing of W and P; the arrangement however becomes a machine if we permit the plunger B to descend. Liquid is then forced out of the cylinder A and must find accommodation in the cylinder C; therefore the ram D and load W must rise. If B descends a distance H while D rises a distance h, P does PH units of work while Wh units of work are done on W. Neglecting frictional waste, we have by the principle of the conservation of energy,

$$\mathsf{PH}=\mathsf{W}h,$$

or

$$\frac{\mathsf{H}}{h}=\frac{\mathsf{W}}{\mathsf{P}}=\frac{\mathsf{D}^2}{d^2}, \quad \text{.....(2)}$$

an expression which gives the velocity ratio of the machine.

The principle of the hydraulic press is used in many hydraulic machines. The liquid generally employed is water. The cylinder A in Fig. 287 represents a **hydraulic pump**, which in practice is so arranged as to deliver a constant stream of water under high pressure to the cylinder C.

Transmission of energy by a liquid under pressure.—In the hydraulic press discussed above it is apparent that the load P gives up potential energy while descending, and at the same time the load W is acquiring potential energy. Thus energy has been transmitted from one place to another by the medium of the flow of liquid under pressure. It is evident that the transmission of energy will continue so long as P is allowed to descend, *i.e.* so long as flow is kept up in the liquid under pressure. This principle is made use of in hydraulic power installations. Water is brought to a high pressure by means of pumps in a central station, and the water is led through pipes to various points in the district at which energy is required, and where machines capable of utilising this energy are installed.

Pressure energy of a liquid.—In Fig. 288, AB is a pipe having a piston C capable of sliding along the pipe. Liquid under a pressure p enters the pipe at A, and work is done in forcing the piston in the direction from A towards B against a resistance R. Let the area of

the piston be a square units, and let the piston move through a distance L. Then the resultant force P acting on the left-hand side of the piston is equal to pa, and the work done is given by

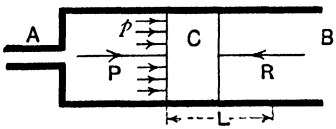


FIG. 288.—Pressure energy of a liquid

Work done by $P = PL = paL$. (1)

It is evident that the volume described by the piston is aL, and this is equal to the volume V of liquid which must be admitted at A in order to keep the pipe full of liquid while the piston is moving. Hence

Work done by $P = pV$ .....(2)

As this work has been done by supplying a volume V of liquid, we have

Work done per unit volume $= p$. .....(3)

Suppose the water to be supplied from an overhead cistern situated at a height h above A, and that the density of the liquid is d. The weight of the liquid per unit volume is dg, therefore

$$p = dgh.$$

Hence we may say

Work done by expending a mass d of liquid $= p = dgh$.

$$\text{Work done by expending unit mass of liquid} = \frac{p}{d} \quad \text{.....(4)}$$

$$= gh \quad \text{.....(5)}$$

The **pressure energy** of a liquid is defined as the energy which can be derived by expending unit mass of the liquid in the manner described; hence

$$\text{Pressure energy} = \frac{p}{d} = gh. \quad \text{.....(6)}$$

Absolute units of force have been employed in the above discussion; hence the quantities involved in (4), (5) and (6) must be expressed as follows:

	C.G.S	BRITISH.
p	dynes per sq. cm.	poundals per sq. foot.
d	grams per c.c.	pounds per cubic foot.
h	centimetres.	feet.
g	centimetres per sec. per sec.	feet per sec. per sec.
Pressure energy	ergs (*i.e.* centimetre-dynes) per gram of liquid.	foot-poundals per pound of liquid.

EXAMPLE 1.—Water is supplied by a hydraulic power company at a pressure of 700 lb. wt. per square inch. How much pressure energy in foot-lb. is available per pound of water ?

$$\text{Pressure} = p = 700 \times 144g$$
$$= 100{,}800g \text{ poundals per sq. foot.}$$
$$\text{Pressure energy} = \frac{p}{d} = \frac{100{,}800g}{62{\cdot}3} \text{ foot-poundals}$$
$$= \frac{100{,}800}{62{\cdot}3} = \underline{1618} \text{ foot-lb. per pound of water.}$$

EXAMPLE 2.—Some mercury is under a head of 30 cm. of mercury. What is the pressure energy ?

$$\text{Pressure energy} = gh = 981 \times 30$$
$$= \underline{29{,}430} \text{ ergs per gram of mercury.}$$

Hydraulic transmission of energy.—The principal apparatus required in a hydraulic installation is shown in outline in Fig. 289.

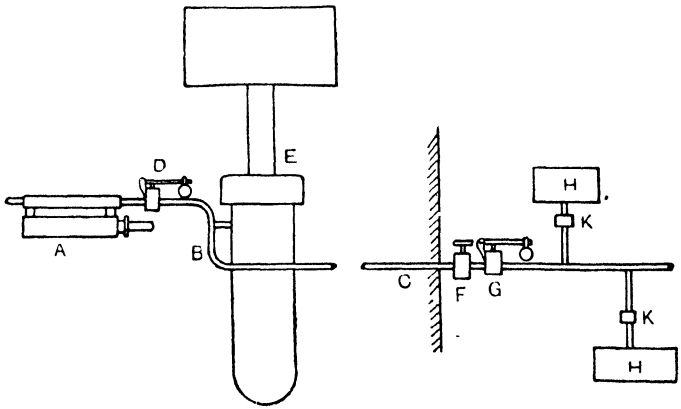


FIG. 289.—Diagram of a hydraulic installation.

A is a **hydraulic pump** driven by a steam engine, or other source of power, and delivers water under high pressure into the pipe system BC. A **safety valve** is provided at D and permits some of the water to escape should the pressure become dangerously high. Near to the pump is situated a **hydraulic accumulator** E, which is connected to the pipe system, and maintains constant pressure in the water. A branch pipe from the main pipe system is led into the consumer's premises, and a stop valve F enables him to cut the supply off when

necessary. There is also a safety valve G, which serves to protect his machinery from damage due to any excessive pressure. The machines H, H are operated by the water; each machine has a valve K, by use of which the machine may be started and stopped.

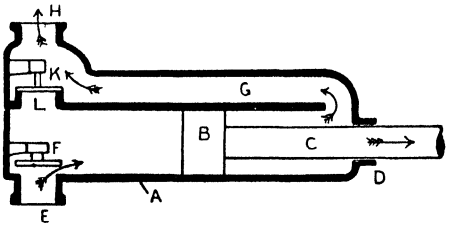


FIG. 290 — Section of a hydraulic pump

A typical **hydraulic pump** is shown in Fig. 290. A cylinder A is fitted with a piston B, which may be pushed to and fro by means of a rod C operated by an engine. Hydraulic packing at D renders water-tight the hole through which the rod passes. The valves F and K are discs which rise and fall vertically, thus opening and closing passages E and L through which the water may pass. The piston is shown moving towards the right, and water is flowing into the cylinder from E past the open valve F. At the same time, the water on the right-hand side of the piston is being expelled under high pressure through a passage G into H and so into the delivery pipe.

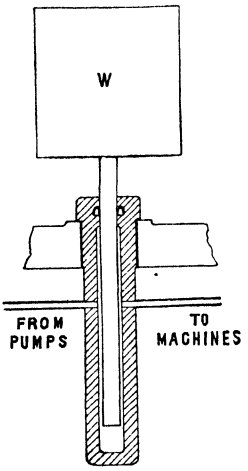


FIG. 291.—Diagram of a hydraulic accumulator

When the piston is moving towards the left, the valve F drops and closes E. The water on the left-hand side of the piston is then forced under high pressure through L, past the valve K (which has now lifted), and so partly into the delivery pipes at H, and partly through the passage G into the right-hand side of the cylinder. The pump thus delivers water during each stroke of the piston.

A **hydraulic accumulator** is illustrated in Fig. 291, and consists of a cylinder fitted with a ram which passes through a hole at the top of the cylinder and carries a load W on the top. The cylinder is connected to the pipe system of the plant (Fig. 289), and therefore the ram is subjected to the same pressure as that in the pipes.

Let p = the pressure of the water, lb. wt. per sq. in.

d = the diameter of the ram, inches.

Then Resultant upward force on the ram $= p \times \pi d^2/4 =$ W lb. wt.

Since d is constant, it is apparent that the working pressure depends on the magnitude of W, which accordingly determines the maximum pressure which may exist in the pipes.

The accumulator has another very important function. Suppose that all the machines operated by the water are cut off and that the hydraulic pumps continue to work. Owing to the incompressibility of water, either some of the pipes would be burst or the whole of the energy expended in giving pressure to the water would be wasted in the flow through the safety valve. The accumulator prevents both damage and waste. Under the conditions mentioned, the water delivered by the pumps causes the ram of the accumulator to rise. If H be the height through which the ram travels, the load W stores potential energy to the amount WH, which is available for doing useful work when the machines are started again.

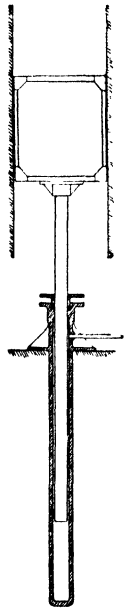


FIG. 292.—A direct-acting hydraulic lift

A system of levers, not shown in Fig. 291, is operated by W when the accumulator has been raised to the maximum safe height; the lever system is connected to the engine driving the pumps and cuts off the steam, thus stopping the pumps. Directly the machines are started again, the ram begins to descend and the lever system is operated in the reverse direction, thus restarting the pumps. The whole arrangement is automatic, and the pumps in the power house start and stop in answer to any demand for water from premises situated perhaps a considerable distance away.

Hydraulic lift.—A simple type of **hydraulic lift** is shown in Fig. 292, and consists of a hydraulic cylinder fitted with a ram which carries a cage on its top. The total weight of the ram, cage and load carried in the cage must be equal to the resultant force which the water exerts on the ram, neglecting friction.

Hydraulic engine.—Fig. 293 shows a common form of hydraulic engine whereby the pressure energy possessed by water under pressure

may be converted into useful work. The engine has three cylinders A, B and C arranged at angles of 120°, each fitted with a piston—that at A is shown in section. Each piston is connected by a rod to a crank DE, which is fixed to a shaft capable of rotating about D. The water acts on the outer sides of the pistons only, and is admitted and discharged by an arrangement of valves not shown in Fig. 293.

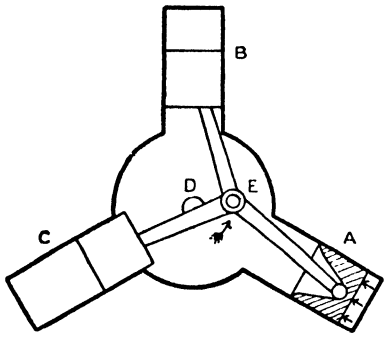


FIG 293.—Three-cylinder hydraulic engine.

The piston in A has just commenced to move towards D, and is doing work on the crank; that at C is just finishing its movement towards D, and the piston in B is moving away from D; the water in the latter cylinder is flowing out of the cylinder, and is finished with so far as the derivation of energy is concerned. Thus there is always at least one piston which is doing work on the crank, and continuous rotation of the shaft D is secured.

The horse-power may be calculated in the following manner:

Let p = the water pressure in lb wt. per square inch.
d = the diameter of each cylinder in inches.
L = the travel, or stroke of the piston towards D in feet.
N = the revolutions per minute of the shaft.

Then

Resultant force exerted by the water on one piston $= p \times \pi d^2/4$ lb.wt.

Work done on one piston per stroke $= p\pi d^2/4 \times L$ foot-lb.

As there are three pistons, there will be 3N strokes per minute, during each of which work will be done; hence

Work done per minute $= p \times \pi d^2/4 \times L \times 3N$ foot-lb.

And $$\text{Horse-power} = \frac{3p\pi d^2 LN}{4 \times 33{,}000}.$$

Pressure of a gas.—In dealing with a gas such as air, the pressure may be measured above absolute zero of pressure. Absolute zero of pressure may be defined as the state of pressure in a closed vessel containing no substance in the gaseous state, and this empty space is termed a **perfect vacuum**. Pressures measured from a perfect vacuum are called **absolute** pressures.

In practical work, the pressure of a gas is measured by an appliance called a **pressure gauge**, several types of which are described in the Part of the volume on Heat. Pressure gauges indicate the difference between the existing absolute pressure of the atmosphere and the absolute pressure inside the closed vessel containing the gas. The pressure of the atmosphere is denoted by zero on the graduated scale of the gauge, and other pressures are measured as so much above, or below the pressure of the atmosphere ; hence the term **gauge pressure.** Consider a closed vessel containing a gas under high pressure. If the absolute pressure of the gas is p, and the absolute pressure of the atmosphere is p_a, then the pressure indicated by the pressure gauge is $(p - p_a)$, and we have

Absolute pressure = gauge pressure + pressure of the atmosphere.

Boyle's law.—Experiments on the relation of the pressure and volume of gases will be described later. These show that, for gases such as air, hydrogen, oxygen and nitrogen under ordinary conditions of pressure and temperature, the absolute pressure is inversely proportional to the volume, provided the temperature is kept constant. Taking a given mass of gas, we have

$$p \propto \frac{1}{v},$$

or

$$pv = \text{a constant.}$$

If the initial conditions of pressure and volume are p_1 and v_1, and if the final conditions are p_2 and v_2, then

$$p_1 v_1 = p_2 v_2.$$

This law was discovered by Boyle and bears his name.

Lift pumps.—The **lift pump** depends for its action on the pressure exerted by the atmosphere. In Fig. 294, A is a cylinder fitted with a piston or pump bucket B ; this piston has a valve which opens upwards, thus permitting water to pass from the lower to the upper side through holes in the piston. The cylinder is connected by a pipe C, having a foot valve D at its bottom, to a cistern of water E. The pump is operated by means of a rod which is attached to the bucket and passes through a hole in the top cover of A.

During the up-stroke of the bucket, the valve B is closed and D is open ; the pressure of the air in C falls, and the pressure of the atmosphere on the surface of the water in E causes some water to

flow up the pipe. During the down-stroke, the valve D closes and B opens. No water can pass D now, and some air will be expelled through B. Repetition of these operations will bring water ultimately into the cylinder A, when it will pass B and be discharged through F. The process of starting in this manner is long, and may be hastened by first charging the cylinder and pipe C with water.

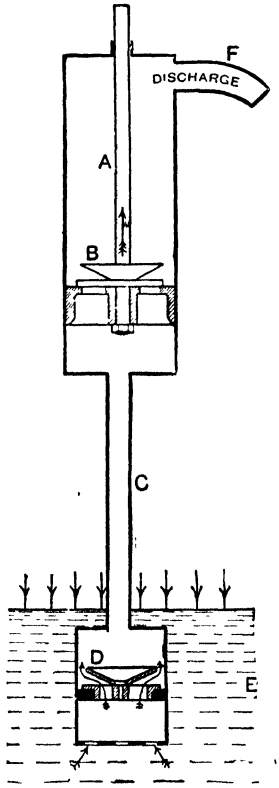


FIG. 294.—Section of a lift pump.

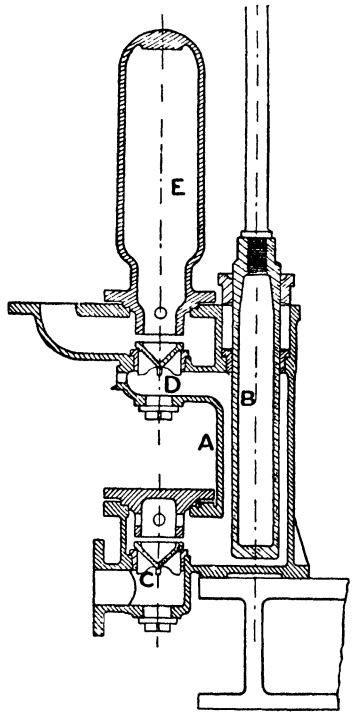


FIG. 295.—Section of a boiler feed pump.

Taking the pressure of the atmosphere to be equivalent to a head of 30 inches of mercury, or $30 \times 13{\cdot}59 = 407{\cdot}7$ inches of water, we see that the pressure of the atmosphere is incapable of forcing water to a height greater than about 34 feet. The cylinder of a lift-pump is placed usually at a height not exceeding 30 feet from the free surface of the water in the well.

Force pumps.—In force pumps the piston is employed for forcing liquids into vessels in which the pressure is higher than that of the atmosphere. For example, the pump employed to feed water into a steam boiler has to force the water to enter the boiler against the pressure of the steam in the boiler. Such a pump is shown in section in Fig. 295. A is the cylinder with a ram, or plunger, B. Water enters the cylinder, passing the valve C, during the upward stroke of the plunger, and is delivered through another valve D during the downward stroke of the plunger. The valve D opens when the pressure in the cylinder A, produced by forcing the plunger downwards, becomes greater than that exerted on the upper side of the valve.

This pump is fitted with an air-vessel E, the action of which is of interest. The vessel is in communication with the discharge pipe of the pump, and is closed entirely otherwise. Air is contained in the upper part of the vessel, and is compressed, during the early part of the downward stroke of the plunger, by some of the water discharged from A entering the vessel. Water being practically incompressible, absence of the soft cushion provided by the air in the air-vessel would lead to shocks due to the action of the plunger when it meets the water during the downward stroke, and might possibly cause the pipes to burst. Further, the pump shown in Fig. 295 is single-acting, *i.e.* water is delivered during the downward stroke only. During the upward idle stroke of the plunger, the compressed air in the air-vessel maintains some flow of water along the discharge pipe, and thus assists in producing a continuous pumping action.

Exercises on Chapter XIX.

1. If the mercury in a barometer falls from 29·8 to 29·4 inches, find the difference in the total forces which the atmosphere exerts on the outer surface of a sphere 2 feet in diameter.

2. An open rectangular tank is 6 feet long, 4 feet wide and 3 feet deep, and is full of fresh water. Find the total forces on the interior surfaces of the bottom, one side and one end, taking account of the pressure of the atmosphere of 15 lb. wt. per sq. in.

3. In a hydraulic or Bramah press the ram is 15 inches in diameter and the pump plunger is 2 inches in diameter. What is the velocity ratio of the machine? If the pressure of the water is 1000 lb. wt. per square inch, what force must be applied to the pump plunger, and what force will be exerted by the ram? Neglect friction.

4. What is the pressure energy of water when under a pressure of 1200 lb. wt. per square inch? State the result in foot-lb. per pound mass of water.

5. How many gallons of water, under the conditions given in Question 4, must be supplied per hour in order to maintain a rate of working of one horse-power ? (There are 10 pounds of water in one gallon.)

6. Water at a pressure of 700 lb. wt. per square inch acts on a piston 1 square foot in area and the piston has a stroke of 1 foot. How much work is done (*a*) by the total volume of water admitted, (*b*) by one pound of the water ? If the water company charges 20 pence per thousand gallons of water, how much energy is given for each penny ?

7. A vertical tube, 3 metres high, is full of mercury. What is the pressure energy per gram of the mercury at the bottom of the tube ?

8. The load of a hydraulic accumulator is 130 tons weight, and the ram is 20 inches in diameter. Find the pressure of the water in lb. wt. per square inch.

9. The ram of a hydraulic accumulator is 7 inches in diameter, and the stroke is 12 feet. If the pressure of the water is 700 lb. wt. per square inch, find the weight of the load. How much water is stored when the ram is at the top of the stroke ? Find also the energy then stored.

10. In the simple form of goods lift shown in Fig. 292, the ram is 3 inches in diameter and has a stroke of 12 feet. If the water is supplied under a pressure of 700 lb. wt. per square inch, what total load can be raised, neglecting friction ? How much work is done in raising this load ?

11. The hydraulic engine shown in Fig. 293 has three rams, each 3·5 inches in diameter and having a stroke of 6 inches. The pressure of the water supplied is 120 lb. per square inch, and the engine runs at 90 revolutions per minute. Neglect waste, and find the horse-power. If the efficiency is 65 per cent., find the useful horse-power.

12. The pressure in a closed vessel is known to be 150 lb. wt. per square inch above that of the atmosphere. The barometer reads 29·6 inches of mercury. Find the absolute pressure inside the vessel.

13. If the volume of a given mass of gas is 450 cubic centimetres when the absolute pressure is 2000 cm. of mercury, find the volume if the absolute pressure falls to 550 cm. of mercury without change in temperature.

14. A vertical tube has its lower end immersed in a bath of mercury, and an air pump is connected to the upper end of the tube. The barometer stands at 30 inches of mercury. By means of the pump the pressure in the interior of the tube is lowered to 10 lb. wt. per square inch absolute. Find the height at which the mercury in the tube will stand above that in the bath.

15. In a lift pump (Fig. 294) the pump bucket is 14 inches in diameter, and has a stroke of 2 feet. If the pump makes 20 double strokes (one upwards and one downwards) per minute, how many cubic feet of water will be raised per hour, neglecting waste ?

16. In Question 15 the moving parts of the pump (bucket, rod, etc.) weigh 150 lb., and the level of the water in the well is 15 feet below the top of the discharge pipe. What total upward force must be applied to the pump rod when the bucket is ascending ? Neglect friction.

17. A lift pump is used to pump oil of specific gravity 0·8 from a lower into an upper tank. What is the maximum possible height of the pump above the lower tank when the pressure of the atmosphere is 30 inches of mercury ?

18. A boiler feed pump (Fig. 295) is single-acting, and the plunger has a stroke of 12 inches. The pump makes 60 double strokes per minute, and has to force 20,000 pounds of water per hour into a boiler working at a pressure of 160 lb. wt. per square inch. Neglect waste and friction, and find (*a*) the diameter of the plunger, (*b*) the force which must be applied to the plunger during the downward stroke.

CHAPTER XX

FLOATING BODIES SPECIFIC GRAVITY

Resultant force exerted by a liquid on a floating or immersed body. —In Fig. 296 (*a*) is shown a body floating at rest in still liquid. Equilibrium is preserved by the action of two forces, viz. the weight W acting vertically through the centre of gravity of the body, and the resultant force R exerted by the liquid. It is evident

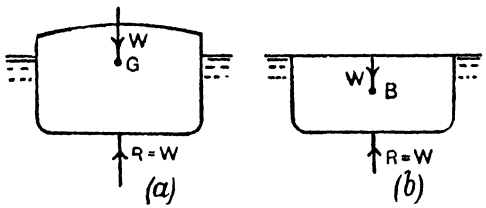


FIG 296 — Equilibrium of a floating body

that these forces must act in the same vertical line, and that they must be equal and of opposite sense. The force R is called the **buoyancy.**

Imagine for a moment that the liquid surrounding the body becomes solid, and so can preserve its shape; let the body be removed, leaving a cavity which it fits exactly (Fig. 296 (*b*)). Let this cavity be filled with some of the same liquid, and let the surrounding liquid resume its ordinary state. The pressures on the liquid now filling the cavity are identical with those which formerly acted on the body, and the effect is the same—the weight of the liquid is balanced. Hence the weight of the liquid filling the cavity and the weight of the body must be equal, since each is equal to R, the resultant force exerted by the surrounding liquid.

Further, in Fig. 296 (*b*), R must act through the centre of gravity of the liquid filling the cavity; this centre is called the **centre of**

buoyancy. It is clear that, since R acts in the same vertical line in both figures, the centre of buoyancy B, and the centre of gravity of the body G, must fall in the same vertical. Hence we have the statement: **When a body is floating at rest in still liquid, the weight of the body is equal to the weight of the liquid displaced by the body, and the centres of gravity of the body and of the displaced liquid are in the same vertical line.**

A little consideration will show that the same method of reasoning applies also to a body totally immersed in a liquid and that the same result follows. Thus, the upward resultant force, or buoyancy, which water exerts on a piece of lead lying at the bottom of a tank is equal to the weight of the water displaced by the lead.

The **principle of Archimedes** follows from the above facts, viz.: **A body wholly, or partially, immersed in a liquid experiences an apparent loss of weight which is exactly equal to the weight of the liquid displaced.**

Stability of a floating body.—The state of equilibrium of a body floating at rest in still liquid may be determined by slightly inclining the body (Fig. 297); the originally vertical line passing through G, the centre of gravity of the body, now occupies the position XY. The weight W of the body acts through G, and the resultant force R exerted on the body by the liquid acts vertically upwards through the centre of buoyancy B. It will be noted, since more liquid is now displaced on the right-hand side of XY, that the tendency has been to move B a little to the right of its first position while the body was being inclined. In Fig. 297, R and W form a couple tending to restore the body to its original position; hence the equilibrium is stable. Produce R upwards, cutting XY in M; M is called the **metacentre.** If M falls above G, as in Fig. 297, the equilibrium is stable. If M coincides with G (as in the case of a rubber ball floating in water), the lines of R and W coincide and the equilibrium is neutral. If M falls below G, the couple will have the sense of rotation opposite to that shown in Fig. 297, and has an upsetting tendency; the equilibrium was therefore unstable. The determination by calculation of the position of the metacentre is beyond the scope of this book.

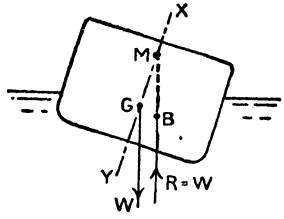


FIG. 297.—Stability of a floating body.

Force required to equilibrate an immersed body.—Should the weight of an immersed body be exactly equal to that of the liquid

displaced by the body, then the forces of weight and buoyancy balance one another, and the body is in equilibrium. Otherwise, an upward or downward force must be applied to the body, depending on whether the weight of the body is greater or smaller than that of the liquid displaced. In Fig. 298 (*a*), the weight of the body is greater than the buoyancy B, hence an upward force P is required

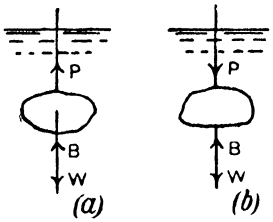


FIG. 298.—Equilibrium of immersed bodies.

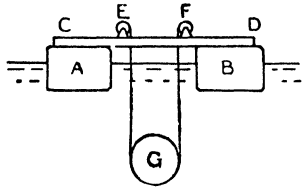


FIG. 299.—Use of pontoons.

to maintain equilibrium. In Fig. 298 (*b*), B is greater than W, and a downward force P is required in order to ensure total immersion.

In Fig. 298 (*a*), $P + B = W$.

In Fig. 298 (*b*), $P + W = B$.

A **pontoon** is a closed or partially open vessel used sometimes for raising sunken wrecks from the bottom in water of moderate depth. In Fig. 299, two pontoons, A and B, support a stage CD, having hoisting tackle at E and F. Chains are placed round the sunken body G, which may thus be raised from the bottom. The total pull in the chains is equal to the weight of the sunken body diminished by the weight of the liquid displaced by the body.

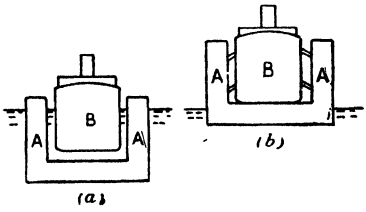


FIG. 300.—A floating dock.

In **floating docks** (Fig. 300) a large vessel A, forming the dock, may be sunk to the position shown at (*c*) by the artifice of admitting water into internal tanks. The ship B may then float into the dock. On pumping the water out of the tanks, the dock rises slowly out of the water, and the ship rests on the floor. Ultimately the position shown in Fig. 300 (*b*) is attained, in which the ship is entirely out of the water.

The immersion of **submarine boats** may also be accomplished by means of internal water tanks. When cruising, the free surface level is AB (Fig. 301), and a considerable portion of the boat is above

water. The vessel may be sunk lower in the water by admitting water into internal tanks; the free surface may then be at CD, or even higher. Pumps are provided in the interior for emptying the water tanks, and thus bringing the vessel again to its original level.

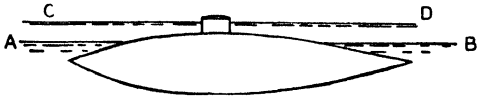


FIG 301.—A submarine boat

When the boat is in motion, diving may be accomplished by the use of horizontal rudders, which cause the longitudinal axis of the boat to become inclined.

Specific gravity.—The specific gravity of the material of a given body is defined as the ratio of the weight of the body to the weight of an equal volume of water. In Great Britain the comparison is made generally at 60° F., or 15° C.

Let W_s = the weight of the body,

W_w = the weight of an equal volume of water, both expressed in the same units.

Then
$$\text{Specific gravity} = \rho = \frac{W_s}{W_w} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

The weight of any body may be calculated from a knowledge of its volume V and specific gravity ρ. Thus, if w_0 be the weight of unit volume of water, the weight of the body, if made of water, is Vw_0, and the actual weight is

$$W = Vw_0\rho. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

Relation between the density and specific gravity of a given substance.—It will be remembered (p. 4) that the density of a substance is its mass per unit volume.

Let M = the mass of a body.

V = its volume.

d = the density of the material.

ρ = the specific gravity of the material.

w_0 = the weight of unit volume of water, in absolute units.

W = the weight of the body, in absolute units.

Then
$$W = Mg = Vdg = Vw_0\rho.$$

$$\therefore \frac{d}{\rho} = \frac{w_0}{g} = \text{a constant.} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

In the C.G.S. system, w_0 is the weight of a cubic centimetre of water and is g dynes ; hence in this system the same number expresses both the specific gravity and the density of a given substance. In the British system, w_0 is the weight of a cubic foot of water ; taking the density of water at 60° F. to be 62·3 pounds per cubic foot, w_0 is $62{\cdot}3g$ poundals ; hence in this system

$$d = 62{\cdot}3\rho.$$

It follows from these relations that any experiment having for its object the determination of the specific gravity of a substance at 60° F , gives also the density of the substance at the same temperature.

EXPT. 42.—Determination of the specific gravity of a liquid by weighing equal volumes of the liquid and of water. A specific gravity or density bottle is employed (Fig. 302), and is a small glass bottle having a fine stem. The bottle is filled with liquid by warming it slightly and dipping its mouth into the liquid ; repetition of this process will ultimately fill the bottle. The bottle and its contents are then brought to the temperature of 60° F. approximately by standing the bottle for some time in a beaker of water maintained at 60° F.

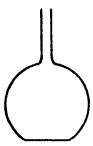


FIG. 302.—Specific gravity bottle.

Weigh the empty bottle ; let this be W_1 grams weight. Fill the bottle with distilled water, taking care to get rid of any air. Bring the contents to the temperature required, and if necessary add some more water in order to fill the bottle completely. Weigh again, and by subtracting W_1 from the result, find W_w grams weight, *i.e.* the weight of the water alone. Empty the bottle and dry it thoroughly. Fill it again with the liquid under test, adjust the temperature and again weigh. From the result deduct W_1, thus giving W_s grams weight for the weight of the liquid alone. Now W_s and W_w occupied equal volumes ; hence

$$\rho = \frac{W_s}{W_w}.$$

$d = \rho$ grams per cubic centimetre
$= 62{\cdot}3\rho$ pounds per cubic foot.

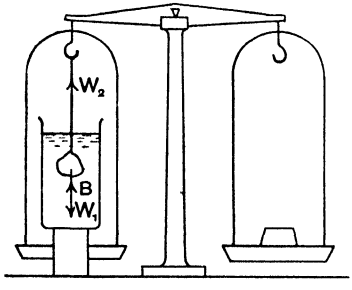


FIG. 303.—Weighing a body in water.

EXPT. 43.—Specific gravity of a solid by weighing in air and in water. First weigh the solid in air ; let the result be W_1 grams weight. Arrange a balance and a vessel of water as shown in Fig. 303, and suspend the solid by means of a fine thread attached to one arm of the balance. The solid should be completely

immersed in the water, the temperature of which should be adjusted as nearly as possible to 60° F., or 15° C. Weigh again, thus determining the pull W_2 grams weight in the thread. If the buoyancy is B grams weight, then

$$W_2 + B = W_1;$$
$$\therefore B = W_1 - W_2.$$

Now B is the weight of a quantity of water having a volume equal to that of the solid; hence

$$\rho = \frac{W_1}{B} = \frac{W_1}{W_1 - W_2} \quad \ldots\ldots(1)$$

In this way determine the specific gravities of the samples of iron, brass, lead, etc., supplied.

If some liquid other than water had been employed in the second weighing operation, let the specific gravity of this liquid be ρ'. Then

$$\text{Weight of the liquid displaced} = W_1 - W_2.$$
$$\text{Weight of an equal volume of water} = \frac{W_1 - W_2}{\rho'}.$$
$$\text{Specific gravity of the solid} = \frac{W_1\rho'}{W_1 - W_2} \quad \ldots\ldots(2)$$

EXPT. 44.—**Specific gravity of a liquid by weighing a solid in it.** Use the apparatus shown in Fig. 303. Weigh the solid (*a*) in air, (*b*) in water, (*c*) in the liquid. Let the results be W_1, W_2 and W_3 grams weight respectively. Then

$$\text{Weight of the water displaced by the solid} = W_1 - W_2.$$
$$\text{Weight of the liquid displaced by the solid} = W_1 - W_3.$$

The volumes occupied by the water and liquid displaced are equal; hence

$$\text{Specific gravity of the liquid} = \frac{W_1 - W_3}{W_1 - W_2}$$

You are supplied with a piece of brass and some turpentine. Find the specific gravity of the turpentine.

Variable immersion hydrometer.—A hydrometer is an instrument which can float in the liquid to be tested and by means of which the specific gravity of the liquid may be determined. The instrument shown in Fig. 304 consists of a glass bulb weighted with some mercury contained in an enlargement at the bottom of the bulb, for the purpose of making the instrument float in an upright position. A graduated glass stem is attached to the bulb. Since the weight of the instrument is constant, and is equal always to

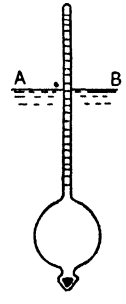


FIG. 304.—Variable immersion hydrometer.

the weight of the liquid displaced, it follows that the free surface of the liquid will cut a division on the stem depending on the specific gravity of the liquid. Deeper immersion will occur with lighter liquids. British instruments generally have the stems calibrated for a temperature of 60° F., and the liquid to be tested should be brought to this temperature. Variable immersion hydrometers can be used for a limited range only, and therefore a number of instruments is required if there is considerable difference in the specific gravities of the liquids to be tested

Expt. 45.—Use of variable immersion hydrometers Find the specific gravities of the liquids supplied, water, turpentine, petroleum, etc., employing the method described above.

Expt. 46.—Specific gravity of a solid by use of Nicholson's hydrometer. This instrument is shown in Fig. 305, and is a hydrometer of constant immersion. A hollow metal vessel C is loaded so as to float upright, and has a wire stem D, which carries a scale-pan E. Another scale-pan is attached at F. A scratch on the stem D determines the standard depth of immersion, and the instrument must be loaded so that the free surface AB cuts this mark.

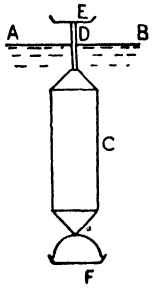


Fig 305.—Nicholson's hydrometer

Float the instrument in water, and ascertain what weight W_1 grams must be placed in E in order to bring the instrument to standard immersion. Remove W_1, and place the body under test into the scale-pan E; add weights W_2 to E so as again to produce standard immersion. Then

Weight of the body in air
$$= W = W_1 - W_2 \text{ grams weight.} \quad \ldots\ldots\ldots(1)$$

Now place the body in the scale-pan F (use a fine thread to tie it down if the body is lighter than water); place weights W_3 grams in the scale-pan E in order to secure standard immersion. Then

Weight of the body in water $= W' = W_1 - W_3$ grams weight. $\ldots\ldots\ldots(2)$

The difference of (1) and (2) gives the weight of the water displaced by the body; hence

Weight of the water displaced $= W - W'$
$$= (W_1 - W_2) - (W_1 - W_3)$$
$$= W_3 - W_2 \text{ grams weight.} \quad \ldots\ldots\ldots(3)$$
$$\rho = \frac{W_1 - W_2}{W_3 - W_2}. \quad \ldots\ldots\ldots\ldots\ldots\ldots(4)$$

In this way determine the specific gravities of the various samples supplied.

Expt. 47.—Relative specific gravities of liquids which do not mix. The U tube shown in Fig. 306 contains two liquids which do not mix; one liquid occupies the tube lying between A and B, and the other occupies the portion BC; the surface of separation is at B. Let the specific gravities of these liquids be ρ_1 and ρ_2 respectively. Let D be at the same level as B; then the pressure at D is equal to the pressure at B, *i.e.*

$$w_1 h_1 = w_2 h_2,$$

or

$$\frac{w_1}{w_2} = \frac{\rho_1}{\rho_2} = \frac{h_2}{h_1}.$$

Measure h_2 and h_1, and evaluate the ratio of the specific gravities. If the specific gravity of one of the liquids is known, find the specific gravity of the other liquid.

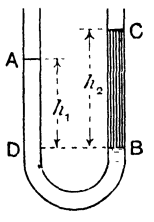


Fig. 306.—Apparatus for liquids which do not mix

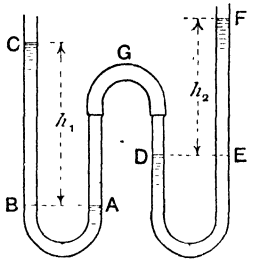


Fig. 307.—Apparatus for liquids which mix

Expt. 48.—Relative specific gravities of liquids which mix. Fig. 307 shows two U tubes connected by a short rubber tube at G. One liquid occupies the space ABC, and the other occupies the space DEF. The air trapped in AGD prevents contact of the liquids, and exerts the same pressure on the surfaces at A and D. Therefore

$$p_A = p_D.$$

A and B are at the same level, as are also D and E.

$$\therefore\ p_A = p_B = p_D = p_E;$$

$$\therefore\ w_1 h_1 = w_2 h_2;$$

$$\therefore\ \frac{w_1}{w_2} = \frac{\rho_1}{\rho_2} = \frac{h_2}{h_1}.$$

Measure h_1 and h_2, and evaluate the ratio.

Expt. 49.—Relative specific gravities of two liquids which mix, by inverted U tube. In Fig. 308 an inverted U tube is shown having a branch at the top furnished with a stop-cock, and connected to an air pump by means of which air may be withdrawn from the tube. The lower open ends of the tube are immersed in two liquids contained in separate vessels.

On operating the pump, the superior pressure of the atmosphere on the surfaces of the liquids in the vessels will cause the liquids to rise in the tubes. The pressures inside the tubes at A and C are equal to that of the atmosphere; also the air in the upper part of the tube exerts equal pressures on the surfaces at B and D. Hence

$$w_1 h_1 = w_2 h_2;$$

$$\frac{w_1}{w_2} = \frac{\rho_1}{\rho_2} = \frac{h_2}{h_1}.$$

Measure h_1 and h_2, and evaluate the ratio.

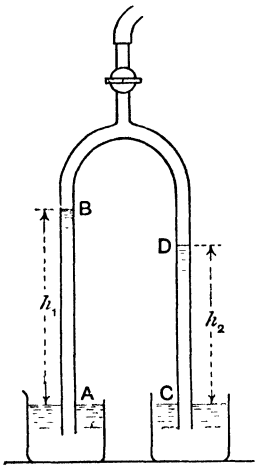


FIG 308.—Inverted U tube for liquids which mix.

Specific gravity of mixtures of liquids.—We will suppose that the volume and specific gravity of each liquid are known, that no chemical action occurs, and that there is no change in the volumes. The total volume V c.c. after mixing will be equal to the sum of the volumes V_1, V_2, V_3, etc., in c.c., of the separate liquids; further, there will be no change in weight during mixing; hence the weight W grams after mixing is equal to the sum of the initial weights W_1, W_2, W_3, etc.

$$V = V_1 + V_2 + V_3 + \text{etc. c.c.} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

$$W = W_1 + W_2 + W_3 + \text{etc. grams weight.} \quad \ldots\ldots\ldots\ldots\ldots (2)$$

From (2), $V\rho = V_1\rho_1 + V_2\rho_2 + V_3\rho_3 + \text{etc.},$

where ρ is the specific gravity of the mixture and ρ_1, ρ_2, etc., are the specific gravities of the separate liquids; hence

$$\rho = \frac{W}{V} = \frac{V_1\rho_1 + V_2\rho_2 + V_3\rho_3 + \text{etc.}}{V_1 + V_2 + V_3 + \text{etc.}}. \quad \ldots\ldots\ldots\ldots\ldots\ldots (3)$$

If the weight and specific gravity of each liquid are known, then we proceed as follows:

$$V_1 = \frac{W_1}{\rho_1}; \quad V_2 = \frac{W_2}{\rho_2}; \quad V_3 = \frac{W_3}{\rho_3}, \text{ etc.};$$

$$\therefore V = \frac{W_1}{\rho_1} + \frac{W_2}{\rho_2} + \frac{W_3}{\rho_3} + \text{etc.};$$

$$\therefore \rho = \frac{W}{V} = \frac{W_1 + W_2 + W_3 + \text{etc.}}{\frac{W_1}{\rho_1} + \frac{W_2}{\rho_2} + \frac{W_3}{\rho_3} + \text{etc.}}. \quad \ldots\ldots\ldots\ldots\ldots\ldots (4)$$

If the volume changes during mixing, becoming V′ say, then, since the weight after mixing is equal to the sum of the weights before mixing, we have

$$V'\rho = V_1\rho_1 + V_2\rho_2 + V_3\rho_3 + \text{etc.};$$

$$\therefore \rho = \frac{V_1\rho_1 + V_2\rho_2 + V_3\rho_3 + \text{etc.}}{V'} \quad \ldots\ldots\ldots\ldots\ldots\ldots (5)$$

Exercises on Chapter XX.

1. A ship displaces a volume of 400,000 cubic feet of fresh water. Find the weight of the ship. If the ship sails into sea-water (64 lb weight per cubic foot), what volume of water will it displace ?

2. A rectangular pontoon is required to carry a load of 4 tons weight, and the depression when the load is applied is not to exceed 6 inches in fresh water. Find the horizontal area of the pontoon in square feet.

3. A closed cylindrical vessel is 6 feet in diameter and 15 feet long, and weighs 5000 lb. If the vessel is floating in fresh water with the axis of the cylinder in the plane of the water surface, what load is it carrying ?

4. A body weighing 8 lb. and having a volume of 15 cubic inches lies at the bottom of a tank of fresh water. What force does it exert on the bottom of the tank ?

5. A piece of iron weighing 4 lb. is immersed in oil weighing 50 lb. per cubic foot, and is supported by means of a cord to which it is tied. If the iron weighs 0·26 lb. per cubic inch, what is the pull in the cord ?

6. Some oil is poured into a vessel containing some water. Describe what will happen if the liquids do not mix. Give reasons.

7. A rectangular body weighing 3 3 lb. in air has dimensions as follows : 4 2 inches long, 2 4 inches wide, 1·1 inches thick. What is the specific gravity of the material ?

8. A piece of lead, specific gravity 11·4, weighs 0 32 lb. in air. What will be the apparent loss of weight when the lead is immersed in water ?

9. The density of steel is 480 pounds per cubic foot. What is the specific gravity ? Explain why the density and specific gravity of a substance are represented by the same number in the c.g.s. system.

10. A plank of wood measures 6 feet by 9 inches by 3 inches, and the specific gravity is 0·6. How many cubic inches will be below the surface if the plank is floating at rest in fresh water. What vertical force must be applied in order to immerse the plank ?

11. A piece of zinc weighs 42 grams in air, and 37·8 grams when immersed in oil having a specific gravity of 0·7. Find the specific gravity of the zinc.

12. A piece of brass weighs 2 lb. in air, and the specific gravity is 8·5. Find the pull in the suspending cord when the brass is immersed in a liquid having a specific gravity of 0·82.

13. The weight of a submarine boat is 200 tons, and it lies damaged and full of water at the bottom of the sea. If the specific gravity of the material is 7·8, find the total pull which must be exerted by the lifting chains in order to raise the vessel from the bottom. Take the specific gravity of sea-water = 1 025.

14. A piece of brass was found to weigh 22 68 grams in air and 20·04 grams in water, and was then used as a sinker for determining the specific gravity of a piece of cork. The cork weighed 1·595 grams in air, and sinker and cork together weighed 14·275 grams in water. Find the specific gravities (*a*) of the brass, (*b*) of the cork.

15. To determine the length of a given tangle of copper wire the following measurements were made: Diameter (measured by means of a screw gauge), 0 0762 cm.; weight of the tangle in air, 5·43 grams; and in water, 4 81 grams. Find the specific gravity of the copper and the length of the wire.

16. The specific gravity of a piece of brass was found by use of a Nicholson's hydrometer, and the following observations were recorded: Weight required to sink the hydrometer to the standard mark, 4 48 grams; with the brass in the upper pan, 2·22 grams weight were required in the upper pan; with the brass in the lower pan, 2·48 grams weight were required in the upper pan. Find the specific gravity of the brass.

17. Some water is introduced into a U tube and fills about 12 cm. of each vertical limb. Some oil of specific gravity 0 8 is poured into one limb and fills a length of 6 cm. of the tube. Find the difference in levels of the free surfaces of the water and oil. (No mixing takes place.)

18 An inverted U tube (Fig. 308) has the open end of one limb immersed in water and the other open end is immersed in a liquid having a specific gravity 0 85. The air pump is then worked until the water stands in the tube to a height of 20 cm. Find the height to which the liquid will rise in the other tube.

19. Three liquids, A, B, C, are mixed and no chemical action takes place. The volumes and specific gravities are as follows:

Liquid - - -	A	B	C
Volume, c.c. - -	20	16	24
Specific gravity -	0 88	0·76	0·92

If no change in volume occurs, find the specific gravity of the mixture. If the volume after mixing is 59·6 c.c., find the specific gravity.

20. A closed box, whose external dimensions are 3 ft. by 2 ft. by 1 ft., is made of iron of specific gravity 7 8; show that the greatest thickness of the iron, supposed uniform, consistent with the box floating in water without sinking, is 0·42 inch nearly. L.U.

21. State the "principle of Archimedes," and explain briefly how it may be used to determine the density of a body heavier than water, and also of a body lighter than water. Adelaide University.

22. Explain how Nicholson's hydrometer may be used to find the specific gravity (*a*) of a liquid, (*b*) of a solid heavier than water. Give a sketch of the instrument.

23. A U tube, whose ends are open, whose section is one square inch, and whose vertical branches rise to a height of 33 inches, contains mercury in both branches to a height of 6·8 inches. Find the greatest amount of water that can be poured into one of the branches, assuming the specific gravity of mercury to be 13·6. L.U.

24. Explain how you would compare the specific gravities of two liquids that mix by means of a U tube. Madras Univ.

CHAPTER XXI

LIQUIDS IN MOTION

Steady and unsteady motion in fluids.—The motion of a fluid may be either **steady** or **unsteady**. In steady motion, each particle in the fluid travels in precisely the same path as the particle preceding it, thus setting up stream lines or filaments, which may be either straight

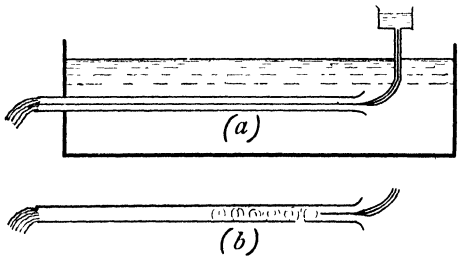


FIG 309.—Steady and unsteady motion

or curved. Thus, if a fine jet of coloured water be injected into a mass of water moving with steady motion, the coloured water will pursue the stream line which passes through the point of injection and will move unbroken, giving a coloured band which appears to remain fixed in position, and may be curved or straight, depending on the conditions under which flow takes place.

In unsteady or **turbulent motion**, eddies are formed in the fluid. If a coloured jet be injected into water moving with unsteady motion, no colour band is formed; the jet breaks up at once, and colours faintly and uniformly a considerable portion of the water.

Osborne Reynolds used the colour band method to demonstrate steady and unsteady flow of water along a glass pipe. At slow speeds of flow, a fine jet of coloured water, introduced into the body of water entering the pipe at one end, travels unbroken along the

pipe and indicates steady flow (Fig. 309 (*a*)). As the speed of flow is increased, a critical velocity is reached, above which the colour band breaks up and mingles with the whole of the water in the pipe, thus indicating unsteady flow (Fig. 309 (*b*)).

Pressure on stream lines.—Since force is required to change the direction of motion of a body, it follows that straight stream lines can exist only provided there is no resultant force acting on the boundary of the stream line in a direction perpendicular to that of the motion of the fluid (p. 217). In other words, the pressures which the adjacent stream lines exert on the filament under consideration must be uniformly distributed all over the boundary of the filament.

In a mass of fluid moving in curved stream lines, the concave side of any stream line is in contact with the convex side of an adjacent stream line (Fig. 310 (*a*)). The pressure which the concave side *ab* of the lower stream line exerts on the convex side *ab* of the upper stream line is equal and opposite to the pressure which the convex side of the upper stream line exerts on the concave side of the lower stream line. Let this pressure be p. The pressure on the concave side *cd* will be less than p by a small amount δp, and that on *ef* will be greater than p by another small amount δp. Applying the same reasoning to all stream lines in a body of fluid moving steadily in a curved path (Fig. 310 (*b*)), we see that the pressure p_1 on the convex boundary *ab* will diminish gradually across the stream, attaining a lower value p_2 at the concave boundary *cd*.

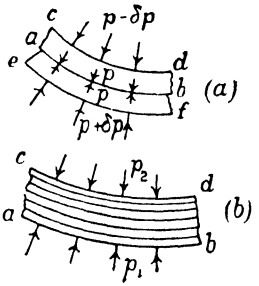


FIG 310.—Transverse pressures on curved stream lines.

Total energy of a liquid.—The total energy at any point in a liquid in motion may be separated into three different kinds of energy, and is expressed conveniently as so much energy per unit mass of liquid: (*a*) **Potential energy**, due to the elevation h above some arbitrary datum level, and given by gh absolute units of energy per unit mass. (*b*) **Pressure energy**, due to the pressure p, in absolute units, at the point under consideration, and given by p/d absolute units of energy per unit mass, d being the density of the liquid (p. 264). (*c*) **Kinetic energy**, due to the motion of the liquid, and given by $v^2/2$ absolute units of energy per unit mass, v being the speed of the liquid at the point under consideration. These energies are mutually convertible, *i.e.* any one kind may be converted into

either of the other two kinds of energy. The total energy of the liquid at the point in question is obtained by taking the sum. Thus

$$\text{Total energy} = gh + \frac{p}{d} + \frac{v^2}{2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

Bernoulli's theorem.—Suppose that a small portion of liquid flows from one point to another point, and that the change of position is effected without incurring any waste of energy, then, from the principle of the conservation of energy, we may assert that the total energy is not changed during the displacement. This statement is known as **Bernoulli's theorem**, and leads to the following equation.

Let h_1, p_1 and v_1 be respectively the elevation, pressure and velocity at a certain point in a liquid having a mass d per unit volume, and let the liquid flowing past this point arrive at another point where the elevation, pressure and velocity are respectively h_2, p_2 and v_2. Then

$$gh_1 + \frac{p_1}{d} + \frac{v_1^2}{2} = gh_2 + \frac{p_2}{d} + \frac{v_2^2}{2} \quad \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

EXPT. 50.—**An illustration of Bernoulli's theorem.** In Fig. 311, AC is a glass tube having a contraction at B; a branch D is attached at the middle of the contraction and dips into coloured water in a vessel E. The tube ABC is arranged horizontally, and is connected to a water-tap by means of rubber tubing attached to A. On opening the tap, water flows through the tube and is discharged into the atmosphere at C. Notice also that the coloured water in E ascends D, mingles with the water flowing along ABC and is also discharged at C. The arrangement constitutes a kind of lift pump, and has been used in a modified form for raising water.

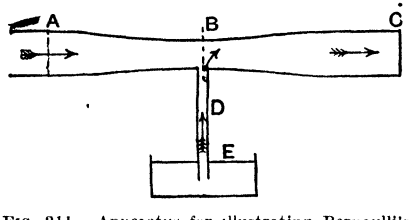


FIG 311.—Apparatus for illustrating Bernoulli's theorem

The action may be explained as follows: The tube ABC being horizontal, there will be no change in the potential energy of the water flowing along the tube. Imagine the branch D to be closed for a moment, then, neglecting any wasted energy, the sums of the pressure and kinetic energies at A, B and C will be equal. It is also evident, since the tube ABC is everywhere full of water, that the same quantity of water per second passes every cross section of the tube; therefore the velocity at B must be greater than the velocity at C. Hence the kinetic energy at B is greater than the kinetic energy at C, and therefore the pressure energy at B must be less than the pressure energy at C. Now the pressure at C is equal to that of

the atmosphere, therefore the pressure at B must be less than that of the atmosphere. Hence, if the branch D be opened, the pressure of the atmosphere on the free surface of the water in E will cause this water to ascend D, and, provided the branch is not too long, to join the water flowing along ABC.

The siphon.—The siphon consists of a bent tube, usually made with the limbs of unequal length, and is employed for emptying liquid from a vessel without the necessity for tipping the vessel.

EXPT. 51.—**Use of a siphon.** Fill a vessel with water (Fig. 312); the free surface is AB. Fill both limbs of the siphon CDEF with water, close the ends by applying the fingers, invert the siphon and place it in the position shown in Fig. 312, and remove the fingers. It will be found that water is discharged at F until the level in the vessel falls to C.

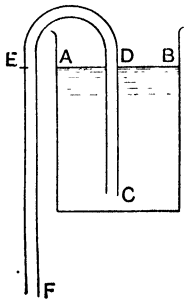


FIG 312.—Use of a siphon

The action may be explained as follows: Suppose the flow to be stopped by applying a finger at F; the pressure at D inside the tube would then be equal to the pressure of the atmosphere acting on AB. If the flow be started again, the water at D has taken up some kinetic energy, and has therefore parted with an equivalent amount of pressure energy, and its pressure is now less than that of the atmosphere. Hence there is a resultant effort tending to produce motion from AB downwards through the vessel and thence through CD towards D.

Consider now the column EF, E being on the same level as AB. On its upper surface and acting downwards there is a pressure equal to that at D; the pressure of the atmosphere acts upwards at F, and the weight of the column of liquid acts downwards. There is thus a net downward tendency which causes the liquid to be discharged at F.

Discharge from a sharp-edged orifice.—Bernoulli's theorem may be applied to the flow of water or other liquid through a small sharp-edged circular orifice. Reference is made to Fig. 313, in which *de* is the orifice; OX is an arbitrary datum level. The free surface level of the liquid is at WL, and is maintained at a constant height *h* above the centre of the orifice by allowing liquid to flow into the tank at a rate equal to that of the discharge through the orifice.

The pressure of the atmosphere, p_a in absolute units, is taken into account by removing the gaseous pressure from WL and

substituting a layer of liquid FGLW, having a depth h_a; the imaginary free surface FG has then no gaseous pressure acting on it. If the density of the liquid is d, then

$$p_a = dgh_a, \quad \therefore h_a = \frac{p_a}{dg} \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

At A the liquid has potential energy due to the elevation h_A; its pressure energy is due to the pressure of the atmosphere p_a; the velocity is too small to be taken into account, and the kinetic energy is assumed to be zero

At B the liquid has potential energy due to the elevation h_B; the pressure energy is due partly to the head h and partly to the pressure

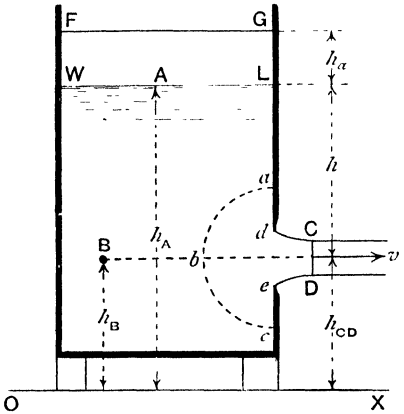


FIG 313 —Discharge through a sharp-edged orifice

p_a transmitted through the liquid; the velocity, and hence the kinetic energy, is again assumed to be zero.

As the liquid approaches the orifice, its velocity begins to be important on crossing an imaginary boundary abc (Fig. 313). Bb being taken as a horizontal line passing through the centre of the orifice, the liquid particles at B pass along the straight line Bb and are discharged; other particles, such as those at a and c, have to pass round the edges of the orifice, and can do so only by pursuing curved paths. Hence the jet contracts after passing the plane of the orifice de. The exterior surfaces of the jet are subjected to the pressure of the atmosphere p_a, but the interior of the jet has pressures in excess of p_a up to the section CD, where contraction is complete. After passing CD, the pressure throughout the interior of the jet is equal to p_a.

At CD the liquid has potential energy due to the elevation h_{CD};

its pressure energy is due to p_a; the kinetic energy is due to the velocity v (Fig. 313).

Consider unit mass of liquid initially at A, then passing slowly downwards to B, and thence along Bb until it acquires the velocity v. The energies above stated may be tabulated in absolute units as follows:

	A	B	CD
Potential energy -	gh_A	gh_B	gh_{CD}
Pressure energy - -	$\frac{p_a}{d}=gh_a$	$g(h+h_a)$	gh_a
Kinetic energy - -	0	0	$\frac{1}{2}v^2$

By Bernoulli's law, if there is no waste of energy during the passage of the liquid, the total energies at each of the three places are equal.

Hence $gh_A+gh_a+0=gh_B+g(h+h_a)+0=gh_{CD}+gh_a+\frac{1}{2}v^2$. .....(2)

The following results may now be written:

$$g(h_A-h_B)=gh. \quad \text{.....(3)}$$

This result simply verifies the fact that the gain in pressure energy in passing from A to B (Fig. 313) is equal to the potential energy transformed in descending through the height h. Also h_B and h_{CD} are equal, therefore

$$\tfrac{1}{2}v^2=gh. \quad \text{.....(4)}$$

This result indicates that the kinetic energy gained is also equal to the potential energy given up in descending through the height h.

From (4), $v^2=2gh$, ...(5)

and we may therefore state that the velocity of the jet is the same as would be acquired by a body falling freely from the free surface level to the level of the centre of the orifice.

Experiment shows that the area of the cross section of CD is about 0·64 of the area of the circular orifice de (Fig. 313). Also the actual velocity at CD is about 0·97 of the velocity given by (5).

Let A = the area of the circular orifice.

Q = the actual quantity of liquid discharged per second.

Then $Q=0\cdot64A\times0\cdot97\sqrt{2gh}$

$$=0\cdot62A\sqrt{2gh}. \quad \text{.....(6)}$$

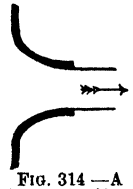


FIG. 314.—A trumpet orifice.

Contraction may be eliminated by use of a **trumpet orifice** (Fig. 314), in which case the area of the jet will be equal to that of the orifice. The velocity is about $0\cdot96\sqrt{2gh}$, and

$$Q=0\cdot96A\sqrt{2gh}. \quad \text{.....(7)}$$

Water-wheels.—There are large natural stores of energy in the water contained in lakes elevated above the level of the sea. The utilisation of this energy has provided many interesting problems for engineers. The old-fashioned method was to employ a **water-wheel.** A suitable place was selected on a river or stream where there was either a natural waterfall, or where an artificial fall could be obtained by building a dam across the stream. A difference in level being thus obtained, the water was led to the water-wheel, of which there are three types.

In the **over-shot wheel** (Fig. 315) water is brought to the top of the wheel and there enters buckets fastened all round the rim. The water remains in these buckets until the wheel, turned by the extra weight of water on one side, has brought the buckets into such a position that the water is spilled out. The wheel is thus rotated continuously and drives machinery by means of toothed wheel gearing.

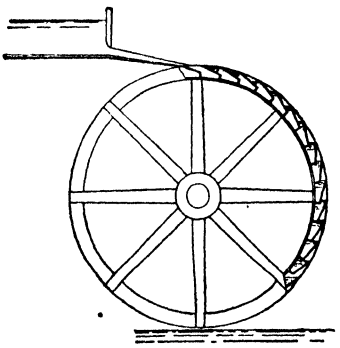


FIG. 315.—Over-shot water-wheel.

In **breast-shot wheels** the water enters the buckets about half-way up, and the action is similar to that in over-shot wheels. In both these types, the attempt is to utilise the potential energy of the water only. In **under-shot wheels** the wheel is furnished with blades, and the water is caused to impinge on these near the bottom of the wheel. The water entering the wheel must have considerable speed, and its kinetic energy is utilised.

Water-wheels are seldom constructed now; they waste a large amount of the available energy and are not suitable for developing large powers.

Water-turbines.—The modern system of utilising the energy of elevated water is by the employment of **turbines.** In these machines the water passes through a wheel furnished with blades. The action consists in causing the water to whirl before entering the wheel; in this condition it possesses angular momentum, and the function of the wheel blades is to abstract the angular momentum and to discharge the water with no whirl. A couple will thus act on the wheel (p. 206), and will cause it to rotate, thus performing mechanical work.

In **impulse turbines** arrangements are made so as to convert the whole of the available energy of the water into the kinetic form before it enters the wheel. In **reaction turbines** the energy is partly in the kinetic form and partly in the form of pressure energy.

The action in the **Girard impulse turbine** may be understood by reference to Fig. 316. Water is supplied from A and passes through a ring of guide passages B, B, having blades so shaped as to cause the water to whirl. Immediately under the guide passages is a horizontal wheel C, which is fixed to a vertical shaft DD. This wheel has a ring of blades round its rim bent contrary to the blades in the guide passages. If the wheel were prevented from rotating, the action of the wheel blades would be to direct the water backwards.

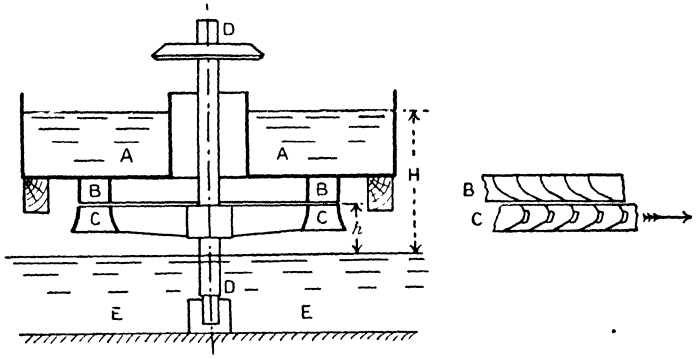


FIG 316.—Action of a Girard impulse turbine.

The wheel actually revolves in the direction shown by the arrow, and the effect is to cause the water to be discharged vertically downwards from the wheel; the whirl is thus eliminated. The water leaves the guide blades B, B in a ring of jets under atmospheric pressure; hence the potential energy—represented by $(H - h)$ units per unit mass of water—has been converted into kinetic energy in the jets. The water passes in thin layers over the wheel blades C, C, and the pressure in the wheel passages is kept equal to that of the atmosphere by means of side openings in the rim of the wheel, one at the back of each blade. It will be noted that the wheel is situated above the level of the discharged water in the tail-race E, E; the water is therefore discharged at atmospheric pressure from the wheel into the atmosphere.

In Fig. 317 is shown in outline a **Jonval reaction turbine.** The arrangement is similar to the Girard turbine. Water is supplied from A and passes through a ring of orifices B, B, having guide blades so as to whirl the water. The wheel C, C has blades so shaped as to

eliminate the whirl. The difference between the two types is that in the Jonval turbine the water passing through the wheel fills completely the passages in the wheel, and may therefore have a pressure not equal to that of the atmosphere. In the example illustrated

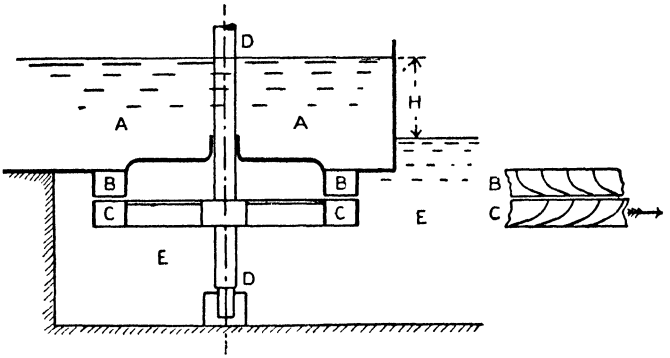


FIG 317.—Action in a Jonval reaction turbine.

the wheel is below the level of the water in the tail-race E, E, and the pressure in the wheel passages is therefore greater than that of the atmosphere.

The difference in free surface levels of the supply water in A, A and of the discharged water in E, E is H, hence H units of potential energy per unit mass of water are available for conversion into work.

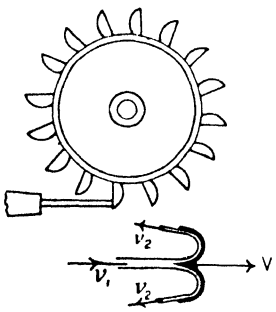


FIG 318.—Action in a Pelton wheel

Pelton wheel.—To obtain efficient conditions of working in water turbines, the wheel blades must be so formed that the water slides on to them without impact. Impact, or shock, always produces waste of energy (p. 235). Further, the water must be discharged from the wheel with as small a velocity as possible. Both of these conditions will be readily understood by reference to Fig. 318 showing a **Pelton wheel**. A jet of water is discharged into buckets which are fixed to the rim of a revolving wheel. In the plan the buckets are made double, having a sharpened dividing edge; the jet enters the buckets at this edge and divides, part flowing round one bucket and part flowing round the other. There

is thus no shock produced by the entering water, which slides tangentially into the buckets. If the wheel were at rest, the water leaving the buckets would have a velocity in the direction opposite to that of the water in the jet. Owing, however, to the velocity of the bucket, the water leaving the bucket has very little velocity relative to the earth. If the velocities of the jet and the bucket are v_1 and V respectively, and if V is equal to $\frac{1}{2}v_1$, then the velocity of the leaving water relative to the earth will be zero, and the whole of the kinetic energy of the water in the jet is available for conversion into work. If m be the mass of water per second delivered by the jet, then

$$\text{Energy supplied per sec} = \frac{mv_1^2}{2}.$$

In practice from 70 to 90 per cent. of this appears as useful work done on the wheel.

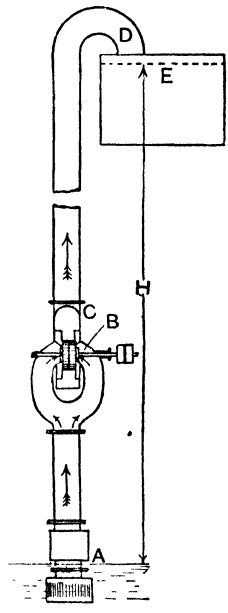


FIG. 319.—Arrangement of a centrifugal pump.

Centrifugal pumps.—Water may be raised from a lower to a higher level by means of a **centrifugal pump.** In Fig. 319 the water in A flows up a vertical pipe, and reaches a wheel B where additional kinetic energy is imparted to it. The wheel is driven by some source of power and whirls the water, giving it a higher speed. This speed is reduced gradually in the casing which surrounds the wheel, and hence the kinetic energy added by the action of the wheel is converted into pressure energy in the discharge pipe at C. The resulting pressure is sufficient to overcome the head of water in the pipe CD, hence flow is maintained upwards, and the pressure energy is converted finally into potential energy in the upper tank E. If H is the difference in free surface levels, then gH units of useful work have been done per unit mass of water.

EXERCISES ON CHAPTER XXI.

1. Describe what is meant by steady and unsteady motion in fluids. Explain what is meant by a stream line.

2. Describe briefly Osborne Reynolds's colour band experiment. What is meant by the critical velocity ?

3. Water is travelling through a bent pipe (Fig. 320), and it is found that the fluid pressure on the wall at A is greater than that at B. Explain this clearly

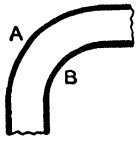


FIG 320

4. Water is flowing steadily along a pipe. Calculate the total energy possessed by one pound of the water at a point where the pressure is 30 lb. wt. per square inch, the velocity is 4 feet per second and the height is 16 feet above ground level.

5. State Bernoulli's theorem. When a liquid flows along a horizontal pipe having a gradual constriction, the pressure at the constriction is less than that at the larger parts of the pipe. Explain this, and describe briefly an experiment for demonstrating it.

6. Water flows up a vertical pipe from ground level to a point 40 feet above the ground. The speed is constant and is 6 feet per second. The top of the pipe is open to the atmosphere. Find the potential, pressure and kinetic energies of one pound of the water at points (*a*) at the top of the pipe, (*b*) at 6 feet above ground level.

7. Water is flowing steadily along a horizontal pipe of varying section. At a place where the pressure is 20 lb. wt. per square inch the speed is 4 feet per second. At another place the speed is 40 feet per second. What is the pressure at this place?

8. If a liquid flows steadily through a pipe of varying circular cross section, show that the speed is inversely proportional to the square of the diameter of the pipe provided that the liquid fills the pipe completely.

9. A horizontal pipe of circular section is 4 inches in internal diameter at a section A and contracts to 1 inch diameter at another section B. Water flows steadily along the pipe, filling it completely, and has a speed of 4 feet per second at A. If the pressure at A is 40 lb. wt. per square inch, find the pressure at B, neglecting friction.

10. Give a brief general account of the changes in energy which occur when one pound of water passes from the free surface level in a tank, through the tank and is finally discharged through an orifice in the side of the tank.

11. A tank containing water has an orifice in one vertical side. If the centre of the orifice is 9 feet below the free surface level in the tank, find the velocity of discharge, assuming that there is no wasted energy. The actual velocity is 97 per cent. of the value calculated above; find the actual velocity.

12. A circular jet of water is 0·4 inch in diameter, and has a speed of 30 feet per second. Calculate the quantity in cubic feet which passes any given section in one second.

13. A tank contains water of which the free surface level is maintained constantly at 4 feet above the centre of a sharp-edged orifice in the side of the tank. The orifice is 1 inch in diameter. Take the usual values for the various coefficients (p. 291) and calculate (*a*) the actual velocity of the jet at the section where contraction is complete, (*b*) the diameter of the jet there, (*c*) the volume discharged per second in cubic feet.

14. A stream of water supplies an overshot water-wheel which is 20 feet in diameter. The stream is 4 feet wide and 6 inches deep, and flows at 6 feet per second. Calculate the weight of water supplied per minute. If 65 per cent. of the potential energy of the water alone is converted into useful work, find the horse-power developed by the wheel.

15. Distinguish between impulse and reaction water turbines. Give clear sketches and a very brief description of the action in a turbine of each of these types.

16. A Pelton wheel is supplied by a jet of water 4 inches in diameter and having a speed of 120 feet per second. How much energy is supplied per second? If the efficiency is 80 per cent., what horse-power can the wheel develop?

17. Describe briefly, by reference to a sketch, the action of a centrifugal pump.

18. Describe the action of a siphon. Give any practical application you may have observed of the use of a siphon.

CHAPTER XXII

SURFACE TENSION DIFFUSION. OSMOSIS

Surface tension.—It is a matter of common observation that a drop of liquid, *e.g.* water, can cling to the lower side of a horizontal glass plate. This fact illustrates two properties: the liquid can adhere to the glass by reason of molecular attraction between the substances, and the liquid behaves as though it were enclosed in an elastic bag, having a constant tendency to contract, and forming a boundary between the liquid and the atmosphere which surrounds the drop. Water or other liquid in an open vessel has a horizontal free surface, and this surface shows properties similar to those of a stretched elastic film Thus a clean, dry needle may float on the surface of water, and is supported by the action of the surface film which bends under the weight of the needle

The portions of the free surface of any liquid in an open vessel lying on opposite sides of any straight line, drawn in the surface, tend to separate, showing the existence of tension in the surface. The **surface tension** is measured by the force in dynes exerted across a portion of the line one centimetre in length.

EXPT. 52.—**Surface tension of water.** Make a rectangular frame of platinum wire (Fig. 321) about 3 cm. long and 1 5 cm. high. Clean the frame by heating it in a Bunsen flame and hang it from one arm of a balance, and let the top be about 3 mm. above the surface of water in a beaker. Add weights to the balance so as to restore equilibrium. Depress the arm of the balance from which the frame hangs so as to immerse the frame. On allowing the frame to rise again, it will be found that it has taken up a film of water, and that more weights are required in order to restore equilibrium. By taking the difference in weights, obtain the total pull of the film, P grams weight, say. The

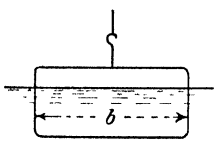


FIG. 321.—Measurement of the surface tension of water.

film of water has two surfaces, front and back; hence the surface tension T is calculated from

$$T = \frac{Pg}{2b} \text{ dynes per centimetre,}$$

where b is the breadth of the frame in centimetres.

The surface tension of water is 75 8 dynes per centimetre at 0° C., and decreases by 0 152 dyne per cm. for each degree rise of temperature. Take the temperature of the water in the beaker at the time of performing the experiment; estimate the surface tension, and compare the result with that obtained in the experiment.

Capillary elevation.—If a glass tube of fine bore, open at both ends, be dipped vertically into water it will be observed that some of the water rises in the tube to a level higher than the free surface outside the tube, and that the surface of the water in the tube which is exposed to the pressure of the atmosphere is shaped like a cup (Fig. 322). This cup is termed the **meniscus.** The elevation of the water inside the tube appears to controvert the laws of fluid pressure and is attributable to surface tension. Water wets glass and tends to spread over its surface; the tendency of the surface skin to contract is resisted by the weight of the water in the glass tube.

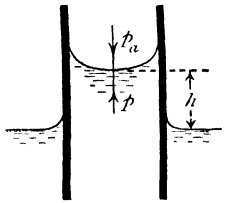


FIG. 322.—Capillary elevation.

The shape of the surface may be explained by considering that the elastic surface skin is subjected on the upper side to the pressure of the atmosphere p_a, and, on the lower side, to a pressure p which is less than p_a by an amount corresponding to the difference in head h. The superior pressure p_a therefore causes the skin to bulge downwards. If d be the density of the liquid, then the difference in the pressures on the opposite sides of the skin is hdg dynes per square centimetre. If the tube has a radius r centimetres, then the area over which the pressure is distributed is πr^2, and the resultant vertical force acting on the surface skin is given by

$$P = hdg\pi r^2. \quad \ldots\ldots (1)$$

This force is balanced by the surface tension T distributed round the inner boundary of the tube, of length $2\pi r$, and since the liquid wets the tube, the surface tensions at this boundary are upward vertical forces. Hence

$$T \times 2\pi r = P = hdg\pi r^2,$$

or

$$T = hdgr/2. \quad \ldots\ldots (2)$$

If the tube is of small diameter, the surface of the meniscus is very nearly hemispherical. The volume of water above a horizontal plane which touches the meniscus at its lowest point will be the difference between the volume of a cylinder of radius r and height r, viz. πr^3, and the volume of a hemisphere of radius r, viz. $\frac{2}{3}\pi r^3$, and is therefore $\frac{1}{3}\pi r^3$. The error in measuring h to the bottom of the meniscus may therefore be corrected in tubes of small bore by adding $\frac{1}{3}r$ to the height h.

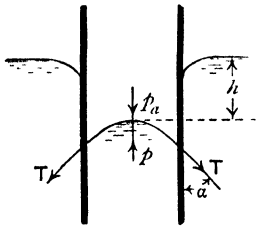


FIG 323.- Capillary depression of mercury

If a similar experiment be tried with mercury, it will be found that the surface of the mercury inside the tube is depressed below the level of the free surface outside the tube (Fig. 323) Mercury does not wet glass, and in this case the skin is bulged upwards by reason of the pressure p on the lower side of the skin being greater than the atmospheric pressure p_a on the upper side. Mercury has a definite angle of contact α with glass (about 50°), and hence it is necessary in this case to take the vertical components of T round the boundary. Thus

$$\mathrm{T}\cos\alpha \times 2\pi r = \mathrm{P} = hdg\pi r^2,$$

$$\mathrm{T} = hdgr/2\cos\alpha. \quad \ldots\ldots\ldots\ldots\ldots\ldots (3)$$

The surface tension of mercury is 547 dynes per cm. at 17·5° C., and diminishes by 0·379 dyne per cm. for each degree C. rise in temperature. The angle of contact varies considerably, depending on the freshness of the surfaces; it is 41° 5′ in a freshly formed drop on glass, and may increase to 52° 40′ for surfaces which are not fresh. Fouling of the glass in mercurial barometers accounts for the fact that the shape of the meniscus in a rising barometer differs from that when the barometer is falling.

EXPT. 53.—**Measurement of the surface tension of water by the capillary tube method.** Clean the tubes supplied by drawing through them strong sulphuric acid and then washing with distilled water. Point one end of a piece of wire, bend it twice at right angles and secure it to one of the tubes by means of rubber bands (Fig. 324). Fix the tube vertically and let the lower end dip into a beaker of water; the beaker should rest on a support so that it may be removed easily without disturbing the tube. Adjust the height of the tube until the point of the wire lies exactly in the surface of the water; the point should not be too close to the tube or the side of the beaker. Attach a piece of rubber tubing to the top of the glass tube, and draw water up the tube so as to wet the interior.

Focus a vernier microscope on the liquid in the tube, and take the reading corresponding to the bottom of the meniscus. Remove the beaker, and by means of the microscope obtain the reading corresponding to the point of the wire. The difference in these readings will give the elevation of the water in the tube above the free surface level in the beaker.

Repeat the experiment with several tubes of different diameter; in each case measure the diameter of the tube (Expt. 7, p. 20), and note the temperature of the water in the beaker.

Calculate the value of the surface tension in each experiment, using equation (2), p. 299. Apply the correction for the shape of the meniscus.

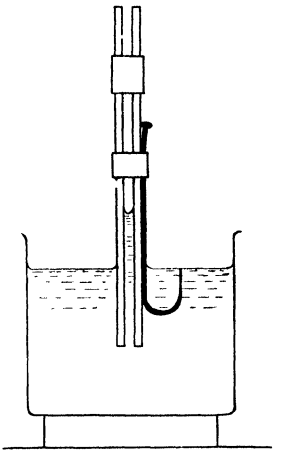


FIG. 324.—Surface tension of water by capillary tube method

Liquids which do not mix.—In Fig. 325 is shown a vessel containing two liquids which do not mix. Suppose AGKB to be the surface of separation of the liquids, and consider two points E and F in the same horizontal plane. Let w_1 and w_2 be the weights per unit volume of the upper and lower liquids respectively. The pressures at E and F must be equal; hence

$$(w_1 \times \mathrm{HG}) + (w_2 \times \mathrm{GE}) = (w_1 \times \mathrm{LK}) + (w_2 \times \mathrm{KF}) ;$$

$$\therefore\ w_1(\mathrm{HG} - \mathrm{LK}) = w_2(\mathrm{KF} - \mathrm{GE}). \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

Also,

$$\mathrm{HG} + \mathrm{GE} = \mathrm{LK} + \mathrm{KF} ;$$

$$\therefore\ \mathrm{HG} - \mathrm{LK} = \mathrm{KF} - \mathrm{GE}. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

For (1) and (2) to be true simultaneously, either w_1 and w_2 must be equal, in which case both liquids have the same specific gravity, or if w_1 and w_2 be unequal, then the result of (2) must be zero, *i.e.*

$$\mathrm{HG} = \mathrm{LK}, \quad \text{and} \quad \mathrm{KF} = \mathrm{GE}.$$

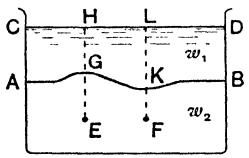


FIG. 325.—Surface of separation in non-mixing liquids.

Hence the surface of separation must be parallel to the free surface CD, and must therefore be a horizontal plane.

In Fig. 326 (*a*) the heavier liquid A is supposed to occupy the upper part of the vessel. That the equilibrium is unstable may be shown as follows: Let the surface of separation be disturbed as shown in

Fig. 326 (*b*) and consider a small area on this surface at E. The pressure p_1 on the upper side is given by

$$p_1 = w_A y. \quad \ldots\ldots (1)$$

Take another point F in the same horizontal plane as E. The pressures p_2 at E and F are equal, and are given by

$$p_2 = p_F = w_B y_1 + w_A y_2. \quad \ldots\ldots (2)$$

Also, $$y = y_1 + y_2,$$

$$\therefore\ p_1 = w_A y_1 + w_A y_2 \text{ (from (1))}. \quad \ldots\ldots (3)$$

Comparing (2) and (3), and remembering that w_A is greater than w_B, we see that p_1 is greater than p_2. Hence on the small area at E there is a resultant downward pressure $(p_1 - p_2)$. Therefore the disturbance at E will continue downwards, and the heavier liquid will occupy ultimately the lower part of the vessel. The state of equilibrium shown in Fig. 326 (*a*) is therefore unstable.

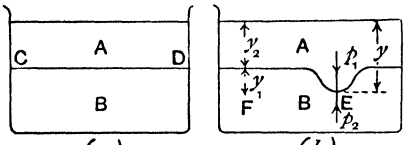


FIG. 326.—The heavier liquid must occupy the lower part.

The same principle also applies to gases. Carbon dioxide has a density greater than that of air, and therefore tends to occupy the lower part of an enclosed space. This fact has been illustrated by the death of small animals in vats containing some carbon dioxide, while men have been able to breathe the superstratum of air. Stratification of this kind is not permanent; **diffusion** takes place more or less quickly, and produces an atmosphere in which both gases are distributed uniformly.

Diffusion of liquids.—In Fig. 327 is shown a jar containing two liquids A and B, A having a greater density than B. If the liquids are incapable of mixing, no alteration will take place if the jar is left undisturbed; but if the liquids possess the capability of mixing in any proportion, it will be found that a process of self-mixing is going on, A travelling upwards in spite of its greater density, and B travelling downwards. Finally the mixture becomes uniform throughout the jar. This process is called **diffusion**.

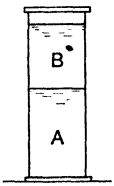


FIG. 327.—Diffusion of liquids.

Diffusion in liquids takes a long time to complete. A demonstration jar may be prepared by introducing a strong solution of copper sulphate A, (Fig. 327), the quantity being rather less than half the capacity of the jar. An equal quantity of distilled water B is then poured in carefully so as not to disturb the

copper sulphate. The jar should be covered and placed where it will not be disturbed. Periodic inspections will show that the blue colour of the copper sulphate is extending upwards, and that the tint in the lower part of the vessel is becoming fainter. At one stage the colour gradation extends throughout the whole depth of liquid. Finally, uniformity of tint is attained, showing that diffusion is complete.

Observations in experiments of this kind show that the time required to complete the diffusion process is proportional to the square of the total depth of liquid. Solutions of different substances, having the same degree of concentration and other conditions similar, have been found to possess different rates of diffusion; for example, hydrochloric acid diffuses more rapidly than potassium bromide. Solutions of the same substance, having different degrees of concentration, have been found to possess rates of diffusion proportional to the strength of the solution. Increase in temperature increases considerably the rate of diffusion.

Diffusion can be completed in a few seconds in a jar, such as is shown in Fig. 327, by using a piece of wire having a loop bent at right angles at one end and stirring the liquids vertically. The effect of such stirring is two-fold; layers of strong solution are brought into juxtaposition with layers of water, and therefore the rate of diffusion is greatly increased; further, the concentrated layers of solution have now a shorter distance to travel in completing the diffusion process.

The uniformity of distribution of the various substances dissolved in sea-water is owing to diffusion. Otherwise the ocean would consist of stratifications of salt solutions of different densities, the heaviest being at the bottom.

Diffusion of gases.—Gases possess the property of diffusion, and the process is completed much more rapidly than is the case with liquids.

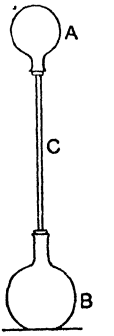


FIG. 328.—Diffusion of gases

EXPT. 54.—**Diffusion of gases.** Referring to Fig. 328, A is a flask charged with coal gas and B is another flask having a capacity about eight times that of A. The flasks are fitted with rubber stoppers, and are connected by means of a glass tube about 18 inches along and ¼ inch bore. Leave the arrangement undisturbed in a vertical position, as shown in Fig. 328, for two or three hours. It will be found that diffusion has taken place, the heavier air in B travelling upwards and the lighter gas in A downwards.

That the gases have mixed may be proved from the fact that a gaseous mixture of air and coal-gas having the stated proportions (about eight to one) is explosive. Wrap a piece of cloth round each flask; quickly remove the stoppers, and test each flask by applying a lighted taper.

Diffusion in non-uniform mixtures of gases takes place by the flow of each gas from places where its density is higher towards places where its density is lower. Ultimately uniformity of density of each gas throughout the whole space is attained The rate of diffusion of two given gases depends on the kind of gases; it is inversely proportional to the pressure of the mixed gases, and roughly is proportional to the square of the absolute temperature. The rate of diffusion also depends on the densities of adjacent layers of the two gases; hence mechanical mixing of the gases hastens the process of diffusion, as is the case also in liquids.

The property of diffusion in gases is of great importance in the prevention of accumulations of noxious gases in towns and confined spaces. Carbon dioxide does not support life, and a comparatively small percentage of this gas in the atmosphere is dangerous The exhalations of animals consist largely of carbon dioxide, which is also given off in large volumes in many industrial processes. The gas diffuses rapidly into the atmosphere, the process being assisted by the stirring produced by air currents, and thus a mixture is attained which is not dangerous. Some idea of the rate of diffusion of carbon dioxide and air may be obtained from the observed fact that in a vertical tube about 60 cm. long, and having the lower tenth of its length charged with carbon dioxide, the upper nine-tenths containing air, diffusion is completed in about two hours. The time taken is proportional to the square of the length of the tube.

Osmosis.—The term osmosis is given to the ability which some liquids have to pass through certain membranes. For example, water is able to pass through the membrane of a pig's bladder, while alcohol is unable to do so. Hence, if a pig's bladder be filled with alcohol, closed, and placed under water, it will swell and may burst. If the bladder be filled with water and placed under alcohol, shrinkage occurs. Dried currants placed under water swell and become spherical owing to the passage of water through their skins.

EXPT. 55.—**Osmosis.** Arrange apparatus as shown in Fig. 329. A is a glass vessel to which a capillary tube B is attached; the upper end of B is open. The lower end of A is closed by a piece of parchment paper (paper

treated with sulphuric acid). Fill A with a solution of sugar so that the level of the liquid is a short distance up the tube, and immerse the vessel in distilled water C, arranging that the liquid levels inside and outside the tube coincide at first. It will be found that the surface level inside B moves upwards with visible velocity, showing that osmotic flow of the water is taking place through the diaphragm into A.

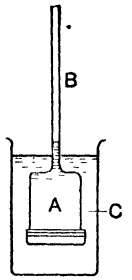


FIG 329.—Apparatus for demonstrating osmosis

Graham divided substances into two classes, **crystalloids** and **colloids**. Crystalloids include such substances as glucose, cane sugar, etc.; when dissolved in water, crystalloids can diffuse through a parchment, or animal membrane. Colloids include such substances as gum, starch and albumen; these either do not diffuse at all, or at a very slow rate.

These properties led Graham to devise a method of separating crystalloids and colloids from a mixed solution. The method is called **dialysis**. In Fig. 330, A is a tube having its lower end closed by a diaphragm of colloidal substance such as parchment paper or bladder. The mixed solution of crystalloids and colloids is poured into A, and the tube is partially immersed in a vessel of water B. The crystalloids diffuse through the membrane into the water and the colloids remain in A. If the water be changed at intervals, and sufficient time allowed, it is possible to effect nearly complete separation of the colloids from the crystalloids.

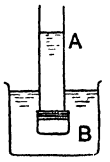


FIG. 330.—Graham's method of dialysis

The separation produced by dialysis in this way is probably due to the sizes of the constituent particles of crystalloids and of colloids. The view now held is that a colloid particle is an aggregate of molecules too small to be visible in a solution to the unaided eye and yet large enough to affect light and be seen by means of the ultra-microscope. Colloidal solutions may, therefore, be defined as uniform distributions of solids in fluids, which are transparent to ordinary light, and not separable into their constituents by the action of gravity or by filtration. Ruby glass owes its colour to the presence of gold particles in a colloidal state. In the manufacture of the glass, gold chloride is added when the glass is in a molten state. If the glass be cooled quickly, it is colourless, but if it is afterwards heated up to the point of softening it becomes suddenly ruby red. In the

coloured glass, the ultra-microscope reveals the presence of colloidal gold particles, but in the colourless glass none can be seen. Colloidal gold can be obtained red, purple, blue or green in solutions containing the same amount of metal, the difference of colour being due to the difference in the size of the particles, which may vary from $5\mu\mu$ to $20\mu\mu$ ($\mu\mu = 10^{-7}$ cm). In recent years much attention has been given to the subject of colloids both in their scientific and their industrial aspects, and Graham's original conception of them has been extended greatly.

Osmotic pressure.—In Expt. 55 if the contents of the outer vessel be examined, it will be found that some of the dissolved substance has passed through the membrane. Flow has thus taken place in both directions through the membrane. Parchment paper and bladder permit both crystalloids and water to pass, but there are certain membranes known, which will permit water to pass, and stop certain salt solutions. For experimental work the most convenient material is the gelatinous precipitate of copper ferrocyanide. This material is very weak, and Pfeffer contrived a method of precipitating it in the interior of the walls of a porous pot, thus producing a continuous film of sufficient strength for practical work.

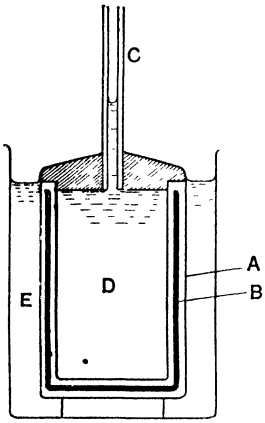


FIG. 331.—Pfeffer pot

In Fig. 331, A is a Pfeffer pot with its internal film of copper ferrocyanide B. Into the top of the pot is cemented a glass tube C, which is considerably longer than is shown in Fig. 331. The pot is filled with a dilute solution of salt D, and is then immersed in a vessel E containing distilled water. Inward flow of the water takes place through the pot and its internal film, and the increased bulk of liquid in the pot causes the level to rise in C. The process goes on until a definite pressure is attained in the pot, as indicated by a steady difference in levels in C and E. Inward flow has then ceased.

It is evident that had an artificial pressure equal to this final pressure been applied to the contents of the pot, no flow would have taken place. This pressure, which depends on the kind of solution and its strength, is called the **osmotic pressure** of the solution.

Passage of gases through porous diaphragms.—The mode by which

a gas passes through a porous obstruction depends on the size of the orifices and the thickness of the obstruction. Thus, if the obstruction is thin and the orifice is relatively large, the flow of the gas resembles the flow of a liquid through an orifice in a thin plate (p. 289) and follows the same laws. If the obstruction is thick and the passages still fairly large, the flow of the gas resembles that of a liquid through a capillary tube. If the pores are very fine, such as in plates of plaster of Paris or compressed graphite, the phenomena of flow are quite different from the other two cases. The passages in such a plate are of cross sectional dimensions comparable with the size of the gaseous molecules, and the flow through any one pore may be regarded as a stream of single molecules following one another in succession.

The laws of flow in such cases were discovered by Graham, who found that the volume, measured at standard pressure, of a given gas passing through a porous plate was directly proportional to the difference in pressure on the two sides of the plate, and inversely proportional to the square root of the molecular weight of the gas. The molecular weights of hydrogen and oxygen are in the proportion of 1 to 16 ; hence, under like conditions of pressure on the two sides of a porous plate, the rates of flow of hydrogen and oxygen will be in the proportion of 4 to 1. It therefore follows that, if there be a mixture of stated proportions of hydrogen and oxygen on one side of a porous plate, the mixture after passing through the plate will be found to contain a greater proportion of hydrogen.

EXPT. 56.—**Diffusion of a gas through a porous plug.** In Fig. 332, A is a glass tube having an enlargement near its upper end. Above the enlargement there is a thin plate B of plaster of Paris, and above this again a cork is inserted temporarily. The tube is then filled with hydrogen, and the lower end is inserted in a vessel of water D. On withdrawing the cork C, diffusion of the hydrogen outwards and of the air inwards takes place through the diaphragm. The rate of flow of the hydrogen through the porous plate is much greater than that of the air, on account of its smaller molecular weight; hence it will be observed that the level of the water rises rapidly in the tube.

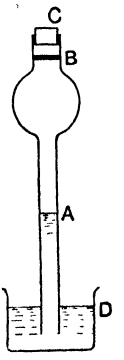


FIG. 332.—Diffusion of a gas through a porous plug.

Repeat the experiment, using coal-gas in place of the hydrogen. This gas is a mixture of gases, several of which have molecular weights more

nearly approaching those of the mixture of oxygen and nitrogen of which the atmosphere is composed. On the whole, however, the molecular weight of the coal-gas is less than that of the atmosphere, and the flow outwards is therefore greater than that inwards. Hence the level of the water rises, but the velocity is less than when hydrogen is used.

EXERCISES ON CHAPTER XXII.

1. Explain what is meant by the surface tension of a liquid. Give some instances which illustrate the existence of surface tension.

2. In an experiment for determining the surface tension of water, performed as directed on p. 298, the breadth of the platinum frame was 2·81 cm. The force required to balance the pull of the film of water was found to be 0·422 gram weight. The temperature of the water was 15° C. Find the surface tension of water at this temperature.

3. A capillary tube having an internal diameter of 0·5 mm. dips vertically into a vessel of water. At what height will the water in the tube stand above the surface level of the water in the vessel? Take the surface tension of water to be 73 dynes per cm.

4. Give a brief explanation of the shape of the meniscus in tubes containing (*a*) water, (*b*) mercury.

5. The limbs of a U tube are vertical, and have internal diameters of 5 and 1 mm. respectively. If the tube contains water, what will be the difference in the surface levels in the limbs? Take the surface tension of water to be 72 dynes per cm.

6. A glass tube, 5 mm. in internal diameter, is pushed vertically into mercury. . Take the surface tension of mercury to be 545 dynes per cm. and the angle of contact to be 50°. Calculate the difference in level of the mercury in the tube and that outside the tube.

7. A ring of glass is cut from a tube 7·4 cm. internal and 7·8 external diameter. This ring, with its lower edge horizontal, is suspended from the arm of a balance so that the lower edge is just immersed in a vessel of water. It is found that an additional weight of 3·62 grams must be placed on the other scale-pan to compensate for the pull of surface tension on the ring. Calculate in dynes per cm. the value of the surface tension.

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8. Describe briefly the phenomenon of diffusion in liquids and gases. Explain clearly why stirring hastens the process of diffusion.

9. A vertical tube 50 cm. long contains carbon dioxide in the lower 5 cm. and the remainder of the tube contains air. Diffusion is found to be completed in 1 hour 20 minutes. Supposing the proportions of the gases to be the same, in what time would diffusion be completed in a tube 10 cm. long?

10. Give a brief description of the phenomenon of osmosis. Describe an experiment for illustrating osmosis.

11. Describe Graham's method of dialysis. Explain the modern conception of a colloidal solution.

12. What is meant by the term osmotic pressure? Describe how it may be found for a given salt solution.

13. Describe briefly the methods by which a gas may flow through a porous substance, with reference to the size of the pores.

14. Describe an experiment to demonstrate that coal-gas diffuses more rapidly than air.

PART II

HEAT

CHAPTER XXIII

TEMPERATURE

Temperature.—On touching in succession two pieces of iron, one piece having been exposed for some time in the sun's rays and the other piece shaded from them, it will be noticed that the first piece is hotter than the second. Our sense of hotness is quite different from the sense of touch, which enables us to distinguish roughness and smoothness, hardness, etc. Of two bodies, the hotter is said to be at a higher **temperature**. Take two bodies, A and B, and place them in contact; **if A is at a higher temperature, heat will flow from A to B.**

The sense of hotness is not always to be relied upon in determining which of two bodies is the hotter. If pieces of wood and iron, both at the temperature of the room, be touched in succession, the iron appears to be colder than the wood. Hence the necessity for employing an instrument in the determination of temperatures; such instruments are called **thermometers.**

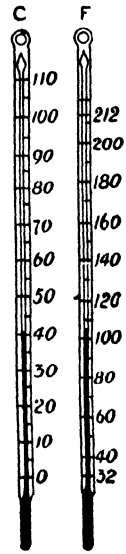


FIG. 333.—Centigrade and Fahrenheit mercury thermometers.

Mercurial thermometers.—In the commonest type of thermometer, reliance is placed upon the expansion in volume which takes place when mercury is raised in temperature. In Fig. 333 two mercurial thermometers are shown. These are made by blowing a bulb at the lower end of a fine-bore glass tube; clean dry mercury is introduced and heated to drive out any air. The top end of the tube is then sealed so that the contents consist of mercury and vapour of mercury. The quantity of mercury is adjusted so that the mercury stands some distance up the tube at ordinary temperatures. If the

bulb, which contains the greater part of the mercury, is brought into contact with a hot body, the mercury is warmed and expands, as is made evident by its level rising in the tube.

The glass walls of the bulb and tube also expand, but to a much smaller extent than the mercury. Elevation of the level of the mercury is due to the difference in expansion of mercury and glass when both are heated through the same range of temperature. Mercury has high expansive properties, and arrives quickly at the temperature of any body with which it is brought into contact; hence it is a very suitable material for the purpose.

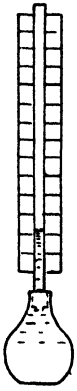


FIG. 334 — Expansion of water.

In manufacturing mercurial thermometers, it is usual to blow a small bulb at the top of the stem; this minimises the danger of the mercury expanding to an extent which would fill the whole tube, when a pressure would be exerted which would probably burst the thin-walled lower bulb.

EXPT. 57.—**Expansion of water when heated.** In Fig. 334 is shown a small glass flask fitted with a rubber stopper and a glass tube; a paper scale is attached to the tube. The flask is filled with water, preferably coloured, and the stopper is inserted and pushed in so that the water level rises a little in the tube. Place the flask in a vessel containing some hot water. Note that the water level falls slightly at first owing to the expansion of the glass (which becomes hotter first), and then rises steadily as the water in the flask rises in temperature and expands. No further rise in level takes place when the temperature of the water in the flask becomes equal to that of the water in the vessel. The action is similar to that in a mercury thermometer, the only difference being in the time taken.

EXPT. 58.—**Unequal expansion of water and alcohol.** Arrange another apparatus similar to that used in Expt. 57. The flasks should be equal in size, and the bores of the tubes should be equal. Charge the first flask as before with water and the other with alcohol, and push the stoppers in until the water and alcohol stand at the same height when at the temperature of the room. Place both flasks in the same vessel of hot water. After some time, the levels of the liquids will cease rising and it will be found that the final levels differ, showing that water and alcohol do not expand to the same extent when heated through the same range of temperature.

Fixed points and graduation of thermometers.—If the bulb and part of the stem containing mercury be immersed in a mixture of

clean ice and water, it will be found that the level of the mercury remains steady while the ice continues to melt. A mark is made on the stem at this level and is called the **freezing point.**

If the bulb and part of the stem containing mercury be immersed in steam coming from water boiling under standard barometric pressure of 760 mm., it will be found that the mercury level again remains steady. Another mark is made on the stem at this level and is called the **boiling point.**

The freezing and boiling points are called the **fixed points** of the thermometer; other temperatures are measured by reference to these points.

In the **Centigrade thermometer** (Fig. 333) the freezing point is marked 0 and the boiling point 100. The Centigrade scale of temperature is constructed by dividing the stem between the fixed points into 100 equal parts or **degrees** (written 100°). In the **Fahrenheit thermometer** the fixed points are marked 32 and 212 respectively, and the stem between these points is divided into 180 degrees. These scales may be extended above the boiling point and below the freezing point. Temperatures lower than zero on either scale are denoted by a negative sign. Thus $-15°$ C. means 15 Centigrade degrees below freezing point, $-15°$ F. means 15 Fahrenheit degrees below 0° F., or $(15+32)=47°$ F. below freezing point.

The **Réaumur scale** of temperature is not much used; in this scale the freezing and boiling points are marked 0° and 80° respectively. Other boiling points now used in thermometry as fixed points are: naphthalene, 218° C.; sulphur, 444·6° C.; zinc, 928° C.

Conversion of temperatures.—To avoid risk of error in converting from one scale of temperature to another, the method used in the following example should be employed:

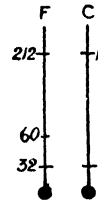


FIG. 335.—Conversion of thermometric scales.

EXAMPLE.—Find the temperature Centigrade corresponding to 60° F.

Sketch two thermometers side by side (Fig. 335); mark these F and C respectively and place the fixed points on each, putting corresponding marks opposite each other. Mark the given temperature of 60° F. on the F thermometer. Inspection shows that this temperature is $(60-32)=28$ Fahrenheit degrees above freezing point. Since 180 Fahrenheit degrees are equivalent to 100 Centigrade degrees, the number of Centigrade degrees equivalent to 28 Fahrenheit degrees is given by

$$\text{Centigrade degrees} = 28 \times \tfrac{100}{180} = 15{\cdot}5.$$

The given temperature of 60° F. therefore corresponds to 15·5° C.

EXPT. 59.—**Freezing point error of a thermometer.** Arrange a funnel and beaker on a retort stand (Fig. 336). Remove some shavings from a block of ice and put them into the funnel. Insert the thermometer to be tested and pack the ice closely round the bulb and stem up to the level of the freezing point graduation. Bring the eye to the level of the top of the mercury column, and take readings at intervals. Note the final steady reading; this may be taken as the true freezing point. The freezing point error of the thermometer is the difference between the final steady reading and 0° or 32°, according as the instrument is graduated in Centigrade or

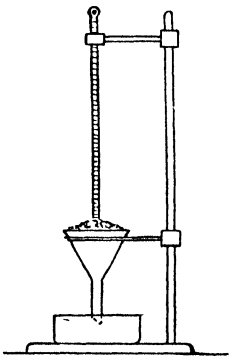


FIG 336.—Apparatus for determining the freezing point of a thermometer

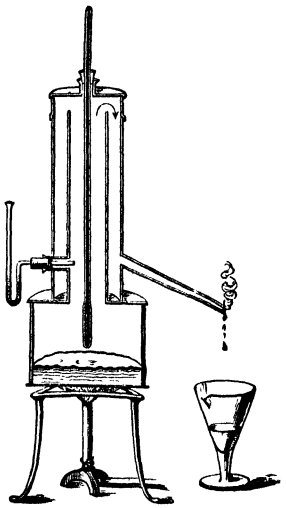


FIG 337.—Apparatus for determining the boiling point of a thermometer.

Fahrenheit degrees. The correction to be applied is equal to this difference, + or − according as the observed temperature is lower or higher than the graduation mark at the fixed point.

In carrying out this experiment, it should be noted that the temperature, as shown by the thermometer, remains steady during the whole time that the ice is melting.

EXPT. 60.—**Boiling point error of a thermometer** In Fig. 337 is shown a small copper boiler having a double copper tube attached to the cover. The thermometer used in Expt. 59 is placed in the inner tube after having been pushed through a cork which fits a hole in the top cover. The water is brought to boiling, and the steam passes up the inner tube, thus surrounding the thermometer, then down the outer tube, and is discharged

at the bottom. The object is to steam-jacket the inner tube containing the thermometer, and to ensure that this tube shall be at the same temperature as the steam. A small glass U gauge contains water, and is connected to the outer tube; when the water stands at the same level in both limbs of the gauge, evidence is provided that the pressure of the steam is equal to that of the atmosphere. The apparatus is called a **hypsometer**.

The thermometer should be arranged so that the boiling point graduation is just above the level of the cork. If the thermometer has a long stem, this condition cannot always be complied with, since the bulb of the thermometer must be situated well above the surface of the boiling water in order to prevent drops of water being thrown on to it.

After the water has been giving off steam freely during a few minutes, take readings of the thermometer. Read also the barometer. The temperature of steam coming from boiling water depends on the pressure, which in this case is equal to that of the atmosphere as shown by the barometer. By consulting the table on p. 533, the temperature of steam at this pressure will be found, and the boiling point error of the thermometer is equal to the difference between the observed boiling temperature and that shown in the table. The correction to be applied to the thermometer is equal to this difference, + or − according as the observed temperature is lower or higher than the correct temperature.

If the same thermometer be immediately retested for the freezing point error, it will be found probably that this error has altered somewhat. This is owing to the glass bulb and stem, which have expanded considerably during the boiling-point test, failing to return to the original volume. The initial volume will be recovered if sufficient time be given, some months probably. Hard glass is less liable to this effect than soft glass.

The hypsometer is sometimes used instead of the barometer for ascertaining the pressure of the atmosphere. The apparatus is set up at the required place (e.g. up a mountain), the temperature of the steam is observed, and the table (p. 533) gives the pressure of the atmosphere.

EXPT. 61.—**Graduation errors of a thermometer.** Assuming that the thermometer stem has been graduated by marking the freezing and boiling points, and then dividing the distance between these marks into the required number of equal parts, and also that the expansion of mercury is proportional to the rise in temperature throughout the range of the thermometer, it follows that the effect of any variation in the bore of the stem will cause incorrect indications of temperature to occur at different parts of the scale.

If a standard thermometer is available, for which the graduation errors are known, the thermometer under test may be examined for graduation errors by comparison. Suspend both thermometers with their bulbs immersed in a beaker containing water. Gradually raise the temperature of the water, and take simultaneous readings of the thermometers at

intervals of, say, 10°, being careful to stir the water well before taking the readings. Note these readings thus:

Standard thermometer		Thermometer under test	
Observed temp	True temp	Observed temp.	Correction

Columns 1 and 3 are filled in from the observations; column 2 from the known errors of the standard thermometer; column 4, obtained by taking the differences of columns 2 and 3, shows the corrections to be applied to the thermometer under test at various parts of the scale. Draw a correction curve by plotting columns 3 and 4.

EXPT. 62.—**Variations in the bore of a thermometer stem** Detach a thread of mercury having a length of about 10 scale divisions of the stem. This may be done by directing a small sharp-pointed flame at the place where the break in the mercury thread is required. By inverting the thermometer the detached thread may be brought to any part of the stem. If the bore is uniform, then the detached thread will have equal lengths at all parts of the stem.

Assuming that a Centigrade thermometer is being tested, shake the thermometer until the detached thread occupies approximately the space lying between 0° and 10°; read the length of the thread in stem scale divisions. Repeat the operation in the spaces lying between 10° and 20°, 20° and 30°, etc., throughout the range of the scale. Let the thread lengths be a_1, a_2, a_3, etc. The mean thread length A is given by

$$A = \frac{a_1 + a_2 + a_3 + \text{etc.}}{\text{number of readings}}.$$

Assuming meanwhile that both freezing and boiling point corrections are zero, the corrections C at other parts of the scale may be obtained in the manner indicated in the following table:

Range.	Values of a	Values of C	
		Temp	Corrections
—	—	0°	$C_0 = 0$
0° – 10°	a_1	10°	$C_{10} = A - a_1$
10° – 20°	a_2	20°	$C_{20} = A - a_2 + C_{10}$
20° – 30°	a_3	30°	$C_{30} = A - a_3 + C_{20}$
30° – 40°	a_4	40°	$C_{40} = A - a_4 + C_{30}$
....	...	...	...
90° – 100°	a_{10}	100°	$C_{100} = 0$

Plot temperatures and values of C. To obtain a complete correction curve, plot on the same diagram the known freezing and boiling point corrections, first reversing their signs, and join these points by a straight line. The intercepts between this line and the graph give the true corrections for any part of the scale.

Proportions of a thermometer.—The volume of the bulb required for a thermometer depends upon the length of stem considered to be suitable for the range of temperature intended to be measured, upon the bore of the stem, and upon the kind of fluid to be employed in the thermometer. The volume of fluid which would fill the stem between the marks placed at the extreme temperatures to be measured, represents the increase in volume of the fluid in the bulb when heated through this range in temperature. The change in volume of one cubic centimetre of the fluid when heated through this range being known, a simple calculation gives the volume of bulb necessary to provide the required expansion.

Types of thermometers.—**Alcohol** is sometimes used in thermometers instead of mercury. Mercury solidifies at $-39°$ C. and alcohol at $-130°$ C.; hence an alcohol thermometer may be used for much lower temperatures than is possible with a mercurial thermometer. Alcohol expands more than mercury for a given rise in temperature; hence the alcohol thermometer is more sensitive than the mercurial thermometer. Alcohol wets glass, and the thread of alcohol has therefore no tendency to stick in the stem. Mercury does not wet glass and is apt to move with jerky action. Alcohol boils at 78° C. approximately, and is not suitable for temperatures above 50° or 60° C. For the same reason, the upper fixed point cannot be found in the manner described on p. 315, and the stem of an alcohol thermometer must be graduated by comparison with a standard thermometer.



FIG. 338.—A clinical thermometer

Clinical thermometers are specially adapted for measuring the temperature of the human body. The body of a person in health varies only slightly from 98·4° F., and the stem of a clinical thermometer is graduated from about 95° to 110° F. The bulb of the instrument is placed in the mouth or under the armpit of the patient for a minute or two, and then withdrawn in order that the temperature may be read. There is a constriction in the stem near the bulb (Fig. 338) which prevents the mercury returning to the bulb; hence

the reading may be taken at leisure. The broken thread of mercury is then reunited by jerking the instrument so as to drive the mercury downwards.

Maximum and minimum thermometers register the highest and lowest temperatures occurring during the interval elapsing between successive settings of the instrument. In Six's thermometer (Fig. 339) a long bulb A is filled with alcohol and is connected to another bulb D by a bent tube; the bulb D also contains alcohol, which, however, does not fill the bulb completely, a space being left for expansion. The tube between B and C contains mercury, which separates the alcohol in the tube between A and B from the alcohol in the tube between D and C. The positions of the ends of the mercury thread at B and C are indicated by small steel indexes having light springs to prevent slipping due to their weights, but not strong enough to prevent the indexes being pushed along the tubes by the mercury thread. Change of volume of the alcohol in the bulb A will cause movements of the mercury thread. The minimum temperature occurring will be indicated by the position of the index controlled by B; the maximum temperature is shown by the index at C. A small magnet applied to the outside of the tube enables the indexes to be reset in contact with the mercury.

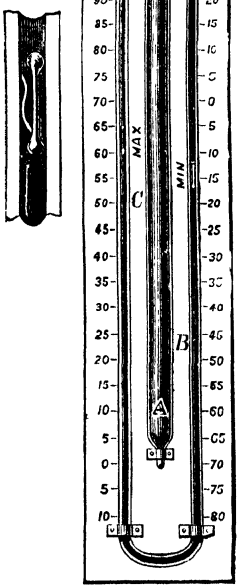


FIG. 339.—Maximum and minimum thermometer

Sensitive thermometers are used for measuring small differences in temperature in cases where the change in temperature is important, and a knowledge of the actual temperature is not required. An example is shown in Fig. 340. The short stem is graduated to show a few degrees only, and each degree is subdivided into tenths. The top of the tube is bent over so as to form a chamber into which some of the mercury may be shaken; sufficient

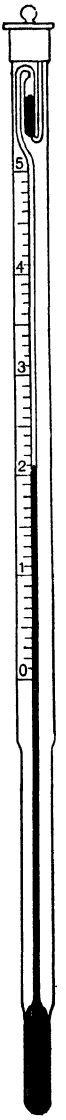


FIG. 340.—Sensitive thermometer.

mercury is left in the bulb to enable the lower temperature to be read, the level being then near the foot of the scale. The higher temperature is then read, and the difference in the readings gives the required difference in temperature. The advantage of this plan consists in greater sensitiveness being obtained without the necessity of employing a very long stem, which would be broken easily.

Precautions to be observed in using thermometers.—Do not attempt to force the thin-walled bulbs through corks. A thermometer may be injured if subjected suddenly to great changes in temperature. No thermometer should be employed where there is risk of its being exposed to temperatures higher than that to which it is graduated, otherwise the bulb may be burst by the pressure of the expanding mercury. The bulb should not be subjected to fluid pressure much in excess of that of the atmosphere; there is risk of collapse. Even if collapse does not occur, the diminution of volume of the bulb caused by the external pressure leads to false readings of temperature. In Fig. 341 is shown a means of obtaining the temperature of steam in a pipe or other closed vessel. A metal cup closed at the inner end is screwed into the pipe and is charged with oil or mercury, which comes quickly to the temperature of the steam. A thermometer inserted in the cup will indicate the required temperature

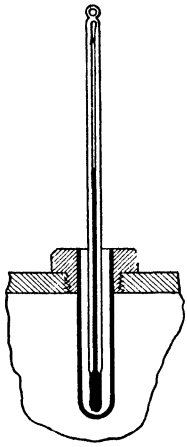


FIG 341.—Temperature in a steam pipe.

Differences in temperature at two parts of a metal pipe may be obtained by securing two thermometers with their stems lying along the pipe; flannel is then wrapped round the pipe over the bulbs. Both thermometers will then be under like conditions, and the difference in their readings may be taken as the difference in temperature of the contents of the pipe at the two places.

Measurement of high temperatures.—Under ordinary atmospheric pressure, mercury boils at 357° C.; hence ordinary mercurial thermometers can be used only for temperatures somewhat lower than this. High temperatures may be stated sometimes with sufficient accuracy by reference to the known melting temperatures of certain substances. Thus we may say that the temperature of a body is about that of melting lead (320° C.) if the temperature be such that a small piece of lead in contact with the body just melts. Naphthalene, sulphur and tin may be used in this way. The method has

been employed for rough determinations of the temperature of furnaces. Substances used in this way are called **thermoscopes.**

The temperature of a flue or furnace may be estimated by inserting a piece of platinum, copper, or other substance, allowing it to remain for some time so as to come to the temperature of the furnace, then removing and plunging it into water. The calculations involved in this method will be found in Chapter XXVI

The electrical resistance of platinum wire varies regularly with the temperature of the wire. This fact enables high temperatures to be measured by observation of the electrical resistance of a platinum wire exposed to the hot gas or liquid. Instruments used for measuring high temperatures are called **pyrometers** In **thermo-couple** pyrometers advantage is taken of the varying strength of electric current set up in a circuit consisting of two dissimilar metals, such as platinum and iridium, in contact with one another, when the junctions are at different temperatures Optical pyrometers are also used in modern metallurgical operations

Exercises on Chapter XXIII

1. Describe, with sketches, the construction of an ordinary mercurial thermometer.

2. A small spherical glass vessel having a fine stem contains cold water which stands about half-way up the stem. If the bulb is plunged into hot water, state what will happen to the water level in the stem; give a full explanation.

3. Define the terms (*a*) freezing point, (*b*) boiling point of a thermometer. What is meant by the scale of temperature of a mercury thermometer?

4. Convert the following temperatures: (*a*) 140° C. to F.; (*b*) −5° C. to F.; (*c*) −273° C. to F.

5. Convert the following temperatures: (*a*) 100° F. to C.; (*b*) 10° F. to C.; (*c*) −60° F. to C.

6. There is a certain temperature which has the same reading on both the Centigrade and Fahrenheit thermometers. Find this temperature.

7. Give sketches of the apparatus required, and explain how to determine the freezing point error of a thermometer.

8. Answer Question 7 for the boiling point error of a thermometer.

9. In testing the boiling point error of a Fahrenheit thermometer, the observed reading was 211·6 degrees. The barometer at the same time reads 76·2 cm. of mercury. Find the boiling point error. (See the table, p. 533, for any quantities required)

10. What precautions should be observed in using a mercury-in-glass thermometer?

11. What is meant by a thermoscope? Give some examples of substances which may be used as thermoscopes.

12. An accurate Centigrade thermometer registers 15·5°, while a faulty Fahrenheit thermometer, hanging beside it, registers 61·5°; what is the correction to be applied to this latter reading? Sen. Cam. Loc.

13. Explain what is meant by a scale of temperature. What properties would guide you in the selection of a liquid for use in a thermometer? What would determine the dimensions you would give to the parts of the thermometer?

14. Describe and give a sketch of a sensitive mercurial thermometer. Describe how to graduate such a thermometer with a Centigrade scale.

15. Describe, with reference to a sketch, the construction of a maximum and minimum thermometer.

16. You are supplied with a thermometer and its corrections at freezing point and boiling point. Describe how to obtain the corrections at other parts of the scale, and how to plot a complete correction graph.

17. A Fahrenheit and a Centigrade thermometer, hanging side by side, indicate 110° and 45° respectively. Describe how you would find out which thermometer was wrong, and what was wrong with it.

CHAPTER XXIV

EXPANSION OF SOLIDS

Expansion.—Most substances expand when the temperature is raised, and contract when cooled. Numerous practical examples may be cited. Wheel tyres of iron are made a little smaller in diameter than the wooden wheel; expansion occurs when the tyre is heated, and it may then be slipped on to the wheel, the wheel is then submerged in water, and the contraction of the cooling tyre binds the whole firmly together. Steam pipes become longer when steam enters them, as may be demonstrated by the following experiment:

EXPT. 63.—**Expansion of a steam pipe.** A small boiler A (Fig. 342) is connected by rubber tubing B to a copper tube C. This tube is about

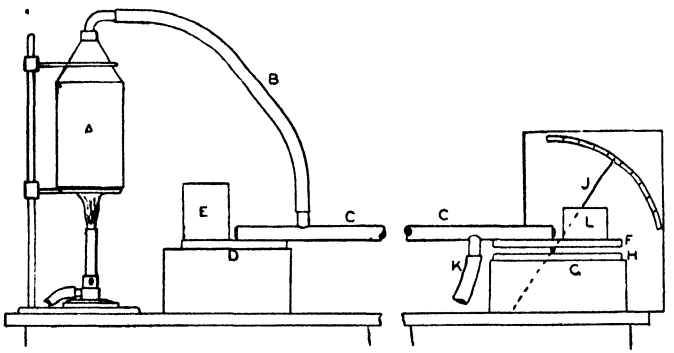


FIG 342.—Apparatus for showing the expansion of a metal tube.

3 feet long and is plugged at both ends. Branches are soldered near each end on opposite sides of the tube; the steam enters the tube through B and is discharged freely through K. Brass plates D and F are soldered to the ends of the tube. D rests on a block and is held down by a weight E; F rests on a small roller made from a portion of a steel knitting needle.

The roller is supported by a brass plate H, which is fixed to a block G; a light pointer J is attached to the roller and travels over a graduated scale. On permitting steam to flow through the pipe, expansion takes place and will be evidenced by the pointer moving over the scale.

EXPT. 64.—**Unequal expansion of metals.** Take two flat bars of equal size, say 12 inches by 0·75 inch by 0·125 inch, one of iron and the other of copper, and rivet them together, flat to flat. The composite bar so formed should be straightened at the temperature of the room. On heating the bar it will be found to have become bent in the process, the copper being on the convex side, showing that copper expands more than iron when both are heated through the same range of temperature.

Coefficient of linear expansion.—The **coefficient of linear expansion** of a substance may be defined as the increase in length which a bar of unit length undergoes when its temperature is raised through one degree.

Let α = the coefficient of linear expansion.
L = the original length of the bar.
t = the elevation of temperature.

Assuming that the expansion per degree is uniform throughout the range of temperature, we have

Increase in a bar of unit length $= \alpha t$.
Increase in a bar of length $L = L\alpha t$.
Final length of the bar $= L + L\alpha t$
$= L(1 + \alpha t)$. .....(1)

COEFFICIENTS OF LINEAR EXPANSION.*

(per degree Cent. at ordinary atmospheric temperatures).

Material	α	Material	α
Lead	$27{\cdot}6 \times 10^{-6}$	Nickel	$12{\cdot}8 \times 10^{-6}$
Zinc	26·0 ,,	Wrought iron / Mild steel	11·9 ,,
Aluminium	25·5 ,,	Cast iron	10·2 ,,
Tin	21·4 ,,	Platinum	8·9 ,,
Brass	18·9 ,,	Glass	7·8 to 9·7 ,,
Gunmetal	18·1 ,,	Masonry	4 to 7 ,,
Copper	16·7 ,,	Timber	3 to 5 ,,
Nickel steel, 10 per cent. Ni.	13·0 ,,	Nickel steel, (Invar) 36 per cent. Ni.	0·9 ,,

* See *Physical and Chemical Constants*, by Kaye and Laby (Longmans).

Coefficient of superficial expansion.—This coefficient may be defined as the increase in area which a plate of unit area undergoes when its temperature is raised through one degree.

Consider a square plate having edges of unit length. Using the same symbols as before, we have

$$\text{Final length of each edge} = 1 + \alpha t,$$

$$\text{Final area of the plate} = (1 + \alpha t)^2$$

$$= 1 + 2\alpha t + \alpha^2 t^2.$$

Now α is always very small, hence the term containing the square of α may be neglected, giving

$$\text{Final area of the plate} = 1 + 2\alpha t;$$

$$\therefore \text{ Change in area of the plate} = (1 + 2\alpha t) - 1$$

$$= 2\alpha t \quad \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

Hence we may infer that the numerical value of the coefficient of superficial expansion for a given substance is double that of the coefficient of linear expansion for the same substance.

Coefficient of cubical expansion.—The coefficient of cubical expansion of a substance is the increase in volume which unit volume undergoes when the temperature is raised through one degree

Consider a cube of unit edge, and use symbols as before.

$$\text{Final length of each edge} = 1 + \alpha t$$

$$\text{Final volume of the cube} = (1 + \alpha t)^3$$

$$= 1 + 3\alpha t + 3\alpha^2 t^2 + \alpha^3 t^3.$$

Neglect the terms containing the square and cube of α, giving

$$\text{Final volume of the cube} = 1 + 3\alpha t;$$

$$\therefore \text{ Change in volume of the cube} = (1 + 3\alpha t) - 1$$

$$= 3\alpha t. \quad \ldots\ldots\ldots\ldots\ldots\ldots (3)$$

Hence the coefficient of cubical expansion is three times the coefficient of linear expansion of the same substance

EXPT. 65.—**Coefficient of linear expansion of metal rods.** The modification of an apparatus designed originally by Weedon and illustrated in Fig. 343 has been found to give good results. A is a double copper trough having the space between the outer and inner boxes packed with asbestos. A large hole is cut in each end of the inner box, and circular discs B, B, made of thin copper slightly corrugated, are soldered over the holes. A smaller hole, coaxial with the large one, is cut in each end of the outer box. C is the rod under test, and is made of such a length as to require a little pressure to get it into position, when its ends bear against the centres of the corrugated discs B, B. Two metal stools D, D assist in supporting the rod. Any expansion in the rod will push the discs outwards. The disc at the left-hand end bears against a fixed stop E, made

of glass rod. The expansion of the rod C is measured at the right-hand end by means of a micrometer F, having a glass rod G fixed to its point and bearing against the corrugated disc when the micrometer is advanced.

The trough is supported on two rods, and arrangements are provided which enable it to slide easily. The trough is pushed towards the left by means of a spring, and thus the left-hand corrugated disc is kept bearing firmly against the fixed stop E.

The tank is filled with cold water up to the level of an overflow outlet M. The temperature of the water is taken by means of three thermometers T, T, T; steam for heating the water is supplied through a copper pipe U, which is fitted with a stop valve.

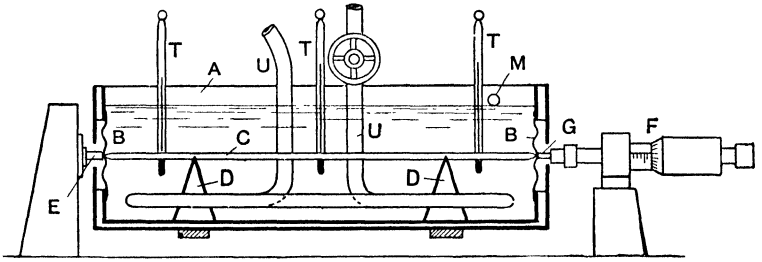


FIG. 343.—Apparatus for determining the coefficient of expansion of rods.

Rods of iron, steel, copper, brass, etc., are supplied. Select one and measure its length as accurately as possible. Place it in position in the trough, having first screwed the micrometer out of contact with the disc. Reduce the temperature of the water to 0° C. by adding ice shavings until some remain unmelted, or melt very slowly. Advance the micrometer until contact with the disc is obtained, and take the reading; read also the three thermometers, and take the mean as the temperature of the water. Screw back the micrometer (care must always be taken to do this before raising the temperature of the bath). Admit steam and raise the temperature to about 10° C.; stir the water, and take the micrometer and thermometer readings as before. Repeat this operation for every 10° up to 100° C. Tabulate the readings as follows:

EXPT. ON THE LINEAR EXPANSION OF A (name of material) ROD.

Length of rod = L mm.

Temp. Cent.	Micrometer reading. mm.	Expansion from 0° C. mm.
0°	a_1	0
t_2	a_2	$a_2 - a_1$
t_3	a_3	$a_3 - a_1$
etc	etc.	etc.

Plot columns 1 and 3. A straight-line graph indicates uniform expansion and constant value of the coefficient of expansion. Select a point on the graph, and read the temperature t and the expansion l for this point. Calculate the coefficient of expansion α from :

$$\text{Expansion} = l = L\alpha t ;$$

$$\alpha = \frac{l}{Lt}.$$

Compensated pendulums.—The time of vibration of a simple pendulum is given by $t = 2\pi\sqrt{l/g}$ (p. 224), and it is essential that l should not alter in any pendulum, otherwise the clock under its control will lose or gain time. If a simple metal rod is used for supporting the pendulum bob, variations in temperature will cause the pendulum to become longer or shorter. For this reason the pendulum rod is sometimes made of wood, which expands but little when the temperature is raised. There are several methods of compensating the pendulum rod for change in temperature, probably the best known being **Harrison's gridiron pendulum**, shown in outline in Fig. 344. The pendulum is suspended at A, and the bob B is supported by five iron rods C_1, C_2, C_3, and four brass rods D_1, D_2. These rods are attached to cross bars in such a way that the expansion of all the iron rods tends to lower the bob, and the expansion of all the brass rods tends to raise the bob.

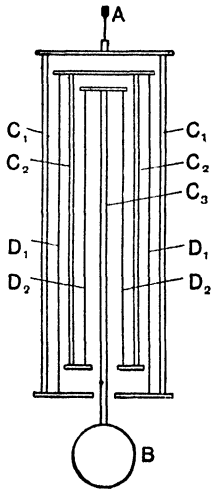


FIG. 344.—Harrison's gridiron pendulum

The coefficient of linear expansion of brass is about 1·5 times that of iron ; hence two rods, one of brass and one of iron, will expand equal amounts for the same rise in temperature if the brass rod has a length of $\frac{2}{3}$rds that of the iron rod. Referring to Fig. 344 the downward movement of the bob is equal to the expansion of three iron rods, and upward movement is equal to the expansion of two brass rods, and the total lengths of iron and brass bear an approximate ratio of 3 to 2. Hence the length of the pendulum remains practically constant.

In the **Graham compensated pendulum** (Fig. 345) an iron rod is suspended at its upper end, and has a closed cast-iron vessel

attached to its lower end and containing mercury. Expansion of the rod lowers the cup, and hence the centre of gravity of the whole pendulum. Expansion of the mercury takes place upwards in the cup, and hence raises the centre of gravity of the pendulum. By suitably adjusting the quantity of mercury, the centre of gravity of the pendulum will remain at a constant distance from the point of suspension.

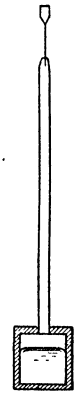


FIG 345.—Graham's mercurial pendulum.

In **chronometer escapements,** the vibrating balance wheel which controls the instrument is compensated for expansion due to changes of temperature. The arrangement is illustrated in Fig. 346. Each spoke supports a separate portion of the rim of the wheel; expansion of these spokes will cause the points A to recede from the centre of the wheel. The segments of the rim are constructed of two strips of different metal, that having the higher coefficient of linear expansion being placed on the outer circumference. Expansion will therefore cause the segments to take a smaller radius of curvature, and will diminish the radius at which the small loads B revolve. When in proper adjustment, the effect of the expansion of the spokes in increasing the radius is nullified by the expansion of the rim segments.

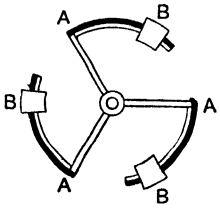


FIG 346.—Balance wheel of a chronometer

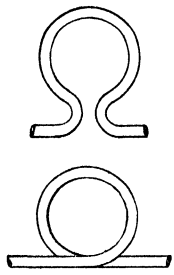


FIG 347.—Expansion loops for pipes.

Expansion of pipes and rails.—In the case of long metal pipes for conveying gas, increase in length due to alterations in atmospheric temperature may be provided for by making a loop, or circle at intervals in the pipe (Fig. 347). The elasticity of the metal permits the loop to bend easily, and thus to take up the expansion of the straight portions of the pipe.

In long steam pipes which are liable to contain water due to the condensation of some of the steam, a better plan is to cut the pipe

and introduce a stuffing box and gland. In Fig. 348 one portion of the pipe has an enlarged part in which the other portion of the pipe may slide. There is a stuffing box packed with asbestos, or other material which is squeezed in by means of a gland, and thus prevents leakage of steam

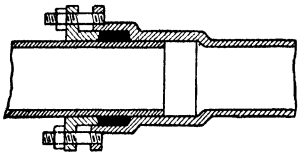


FIG. 348.—Expansion joint for a steam pipe

On railways the separate portions of rail do not butt closely end to end, but are laid with a small interval between so as to permit of expansion (Fig. 349) The joint is made by means of two fish-plates, A, A, one on each side of the rails, and four bolts. The holes in the rail are slotted as shown on the right-hand rail so that the bolts will not interfere with the rail sliding between the fish-plates. The fish-plates bear on the top and bottom of the rails, as shown in the section, thus preserving level the top surface on which the wheels run.

Lines of rail on which electric tramways run are used for electrical conductors and are generally welded end to end, thus forming a

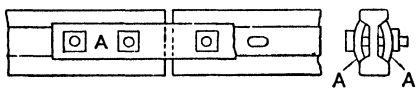


FIG 349.—Expansion joint for rails.

continuous rail. Such a procedure is rendered possible by the fact that only the top surface of the rail is exposed to atmospheric alterations in temperature. The bulk of the rail is underground, and its temperature varies to a comparatively small extent.

Stresses produced by change in temperature.—Suppose an elastic rod having a length L to be raised in temperature $t°$ C, and that free expansion is permitted. The rod will extend by an amount Lat, where a is the coefficient of linear expansion. Let the ends of the hot rod be held rigidly, and let the rod be cooled again to the initial temperature. It is evident that the forces required to hold the rod extended will have the same value as those required to produce an extension Lat at constant temperature.

Let P = the pull required.
A = the cross-sectional area of the rod.
E = Young's modulus.

Then $\text{stress} = \frac{P}{A}$, and $\text{strain} = \frac{L\alpha t}{L} = \alpha t$,

$$E = \frac{\text{stress}}{\text{strain}} = \frac{P}{A\alpha t}, \quad \text{(p. 156)},$$

or $$P = EA\alpha t. \quad \ldots\ldots\ldots\

two points on the graph, and from the readings at these points deduce the average value of the coefficient of linear expansion of the rod.

4. The pendulum rod of a clock is made of wrought iron and the pendulum swings once per second. If the range in temperature is 30 deg. Cent., find the alteration in length of the pendulum.

5. Calculate the length of a brass rod which will expand in length to the same extent as an iron rod 3 metres in length when both are heated through the same range of temperature.

6. A sheet of lead has an area of 12 square feet at 15° C. Find the area when the temperature is raised to 30° C.

7. A circular flat sheet of thin wrought iron is coated thickly with tin on one side only, and is then heated. Describe and explain any effect which may be observed.

8. A tape used for measuring distances is made of steel and is correct at the temperature of 15° C. If the tape is used for measuring a distance of 2000 feet when the temperature is 10° C., what will be the total error on the measured distance due to the expansion of the tape ?

9. A tube made of thin aluminium has a mean diameter of 40 cm. at 15° C. Find the mean diameter when the temperature is raised to 100° C.

10. Platinum wire can be fused into glass without the glass cracking, or the wire becoming loose during cooling and subsequent changes in temperature. Explain why this is possible.

11. A rod of iron, 12 feet long and 1 inch in diameter, is heated from 15° to 165° C. The rod is held forcibly at its new length and is cooled again to 15° C. Find the pull in the rod. Take $E = 30 \times 10^6$ lb. per square inch.

12. Using the same rod and temperatures as in Question 11, the rod is prevented from expanding in length during heating. Find the push exerted by the rod when the temperature of 165° C. is reached.

13. A ball of cast iron has a volume of 120 cubic inches at 20° C. Find the change in volume when the temperature is raised to 110° C.

14. Describe and give sketches of any apparatus you have used for finding the coefficient of linear expansion of a metal. Show how the results would be calculated.

CHAPTER XXV

EXPANSION OF SOLIDS AND LIQUIDS

Change of density caused by expansion.—During expansion the mass of a given body remains constant and the volume increases. Hence the density, *i.e.* the mass per unit volume, diminishes. The following applies more particularly to solids and liquids ; applications to gases will be found in Chapters XXX. and XXXI.

Let d_1 = the density of a substance at a given temperature, grams per c.c.

d_2 = the density when the temperature is raised through t degrees C.

β = the coefficient of cubical expansion.

Then, taking an initial volume of v_1 cubic centimetres,

Volume occupied at the higher temperature $= v_2 = v_1(1+\beta t)$.

Mass of the body at the higher temperature $= v_2 d_2 = v_1 d_1$;

$$\therefore (1+\beta t) d_2 = d_1,$$

$$d_2 = \frac{d_1}{1+\beta t}. \quad \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

If d_1 and β are known, the density at the higher temperature may be calculated from this equation. Equation (1) may be written thus :

$$1+\beta t = \frac{d_1}{d_2},$$

$$\beta = \left(\frac{d_1}{d_2} - 1\right)\frac{1}{t}. \quad \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

This result indicates that the value of β may be found by determining the densities of the substance at two different temperatures. The method is specially applicable to liquids.

Expansion of a vessel.—When a vessel containing a liquid is heated, both vessel and liquid expand. The observed or apparent change of volume of the liquid is the difference between the actual change in volume of the liquid and the change in volume of the vessel. If

these happened to be equal, no change in volume of the liquid would be observed.

Suppose that a vessel of thin glass is used, and that the coefficient of cubical expansion of the glass is G. Imagine the vessel to contain a piece of glass which fills it completely, so that the outer shell, which constitutes the vessel, fits closely at all places. If the volume of this piece of glass is v, and if the temperature is raised through t degrees, the volume becomes $v(1+Gt)$

As the piece of glass and the vessel form practically one piece of glass, it is clear that the shell will still fit closely at the higher temperature, *i.e* the expansion of the piece of glass contained in the vessel is the same as the expansion of the volume contained by the empty shell. Hence

Change in volume of the vessel $= vGt$. ....(1)

The **coefficient of apparent cubical expansion** of a liquid is the coefficient of the expansion of the liquid relative to the vessel. The **coefficient of absolute expansion** of the liquid refers to the expansion of the liquid which would be observed if the liquid were contained in a vessel incapable of expansion.

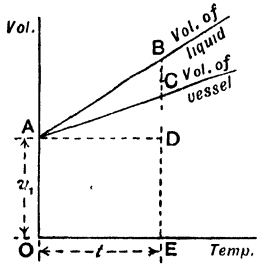


FIG. 350.—Apparent and absolute expansion

Relation of the apparent and absolute coefficients of expansion.—In Fig. 350 volumes are plotted as ordinates and temperatures as abscissae. Let a glass vessel full of liquid have a volume v_1 at 0° C. The volume of the liquid at any other temperature t is represented by the ordinate EB, and the volume of the vessel at the same temperature by the ordinate EC. The apparent change in volume of the liquid is the difference of these ordinates, viz. BC. Draw the horizontal line AD; then DB is the absolute expansion of the liquid, and DC is the absolute expansion of the vessel.

Let β_a = the coefficient of absolute expansion of the liquid.
β = the coefficient of apparent expansion of the liquid.
G = the coefficient of absolute expansion of the glass.

Then Volume of liquid = BE $= v_1(1+\beta_a t)$.
Volume of vessel = CE $= v_1(1+Gt)$

Difference in these volumes $= v_1(1+\beta_a t) - v_1(1+Gt)$
$= v_1 t(\beta_a - G)$. .. ..(1)

The same difference may be expressed by employing the coefficient of apparent expansion ; thus :

$$\text{Difference in volumes} = v_1\beta t. \quad \ldots\ldots (2)$$

$$\therefore\ v_1\beta t = v_1 t(\beta_a - \mathsf{G}),$$

$$\beta = \beta_a - \mathsf{G},$$

or

$$\mathsf{G} = \beta_a - \beta. \quad \ldots\ldots (3)$$

Hence the coefficient of absolute expansion of the vessel is the difference between the coefficients of absolute and apparent expansion of the contained liquid.

Coefficient of absolute expansion by balancing two columns of liquid.—The principle of this method is illustrated in Fig. 351. A bent tube having both limbs open to the atmosphere contains the liquid under test. Jackets round the tubes (not shown in Fig. 351) provide the means of preserving the temperature of the column AC at t_1, and for maintaining BD at a higher temperature t_2. A and B are sections at the same level ; the liquid in the tube between AB may be assumed to be at constant temperature, and therefore to have uniform density. Hence the fluid pressures at A and B are equal.

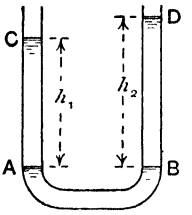


FIG. 351.—Coefficient of absolute expansion.

Let d_1 = the density of the liquid in AC.

d_2 = ,, ,, ,, ,, BD.

h_1 = the height of the column AC.

h_2 = ,, ,, ,, ,, BD.

β_a = the coefficient of absolute expansion of the liquid.

Then Pressure at A = Pressure at B,

$$h_1 d_1 g = h_2 d_2 g,$$

or

$$\frac{d_1}{d_2} = \frac{h_2}{h_1}. \quad \ldots\ldots (1)$$

Also, from equation (1), p. 333, we have

$$\frac{d_1}{d_2} = 1 + \beta_a(t_2 - t_1) ;$$

$$\therefore\ \beta_a(t_2 - t_1) = \frac{h_2}{h_1} - 1 = \frac{h_2 - h_1}{h_1},$$

or

$$\beta_a = \frac{h_2 - h_1}{h_1(t_2 - t_1)}. \quad \ldots\ldots (2)$$

The method used by Regnault in the determination of the coefficient of absolute expansion of mercury is shown in outline in Fig. 352.

Two vertical tubes AB and CD are connected near their tops by a tube AC of fine bore having a small hole at L. These tubes are connected at B and D to a bent tube BEFGHD ; a branch at K is connected to a pump, by means of which air may be forced into FG. The mercury occupies the tubes as shown in Fig. 352 , the free surfaces at A and C are subjected to the pressure of the atmosphere, and the surfaces in the tubes EF and GH are subjected to an air pressure sufficient to maintain the levels as shown. Means are provided for maintaining the mercury in CDHG and in FE at a constant temperature t_1, and that in AB at a higher temperature t_2.

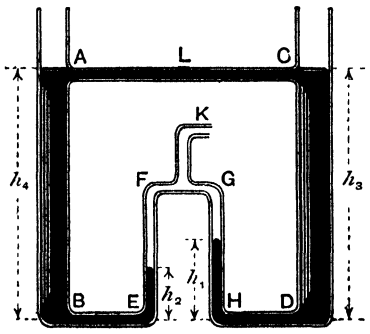


FIG. 352. Diagram of Regnault's apparatus

Let d_1 and d_2 be the densities of mercury at the temperatures t_1 and t_2 respectively. Let p be the pressure of the air in FG, and denote the pressures at other points by suffixes ; all these pressures are stated in excess of that of the atmosphere. The portions of the tube BE and HD are assumed to be coaxial, and the heads of mercury h_1, h_2, h_3, h_4 are measured from this axis Then,

$$p_D = p_H = h_3 d_1 g \; ; \qquad p_B = p_E = h_4 d_2 g.$$

Also $$p_H = p + h_1 d_1 g \; ; \qquad p_E = p + h_2 d_1 g \; ;$$

$$\therefore \; h_3 d_1 g = p + h_1 d_1 g, \; \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

and $$h_4 d_2 g = p + h_2 d_1 g. \; \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

Again, $$d_1/d_2 = 1 + \beta_a (t_2 - t_1), \text{ (p. 335)}, \; \ldots\ldots\ldots (3)$$

where β_a is the coefficient of absolute expansion of the mercury.

Solving these equations, we obtain

$$\beta_a = \frac{h_4 - h_3 + h_1 - h_2}{(h_3 - h_1 + h_2)(t_2 - t_1)},$$

or, since h_3 and h_4 are equal,

$$\beta_a = \frac{h_1 - h_2}{(h_3 - h_1 + h_2)(t_2 - t_1)}. \; \ldots\ldots\ldots\ldots\ldots (4)$$

Regnault's mean value for β_a may be taken as 0·000181.

Taking the density of mercury at 0° C. as 13·5955, the density at

any other temperature t may be calculated from equation (1), p. 333, using Regnault's value of $\beta_a = \cdot 000181$. Thus

$$d_t = \frac{d_0}{1+\beta_a t} = \frac{13{\cdot}5955}{1+0{\cdot}000181t}. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(5)$$

EXAMPLE.—Find the density of mercury at 100° C.

$$d_{100} = \frac{13\ 5955}{1+0{\cdot}0181} = \underline{13{\cdot}35}.$$

EXPT. 66.—**Determination of the coefficient of absolute expansion of a glass vessel** A small glass bottle (sometimes called a "weight thermometer") having a fine stem is employed in this experiment (Fig. 353). Determine the mass m of the bottle by weighing it when empty. Fill it with mercury by heating it slightly and dipping the mouth into the liquid, some of which will flow in as the bottle cools. Repeat the process until the bottle is quite full, taking care to get rid of air. Put the full bottle into a beaker containing some water, and let it stand for some minutes to allow the temperature to become steady. Add some more mercury if required in order to fill the bottle to the top of the stem. Take the temperature of the water, t_1 say. Remove the bottle carefully, dry its external surface, and determine the total mass by weighing; by deducting m calculate the mass m_1 of the mercury filling the bottle.



FIG. 353.

Raise the temperature of the water in the beaker and repeat the experiment, thus determining the mass m_2 of mercury which fills the bottle at the temperature t_2. Let v_1 and v_2 denote the volumes of the contained mercury at t_1 and t_2 respectively, and let G be the coefficient of absolute expansion of the glass. Then

Change in volume of the bottle $= v_2 - v_1 = \mathrm{G}v_1(t_2 - t_1)$;

$$\mathrm{G} = \frac{v_2 - v_1}{v_1(t_2 - t_1)}. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

To evaluate v_1 and v_2, we have

$$m_1 = v_1 d_1, \quad \text{and} \quad m_2 = v_2 d_2,$$

or

$$v_1 = \frac{m_1}{d_1}, \quad \text{and} \quad v_2 = \frac{m_2}{d_2},$$

where d_1 and d_2 are the densities of mercury at the temperatures t_1 and t_2 respectively, and are calculated from equation (5) above. Inserting the values in (1) above, we obtain the value of G for the material of the bottle. Also, since the volume v_1 of the bottle at t_1 is now known, together with the coefficient of absolute expansion of the glass, the volume v_2 of the bottle at any other temperature t_2 can be calculated from

$$v_2 = v_1\{1 + \mathrm{G}(t_2 - t_1)\}. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2)$$

EXPT. 67.—**Coefficients of expansion of a liquid.** The bottle used in Expt. 66 should be employed. Use it in the same manner, and thus determine the masses m_1 and m_2 of the given liquid which fill the bottle at the temperatures t_1 and t_2 respectively. Find by calculation the volumes v_1 and v_2 of the bottle at these temperatures. The densities of the liquid d_1 and d_2 at the temperatures t_1 and t_2 are then found from

$$d_1 = \frac{m_1}{v_1}, \quad \text{and} \quad d_2 = \frac{m_2}{v_2}.$$

Hence, from equation (2), p. 333, we have for the coefficient of absolute expansion of the liquid:

$$\beta_a = \left(\frac{d_1}{d_2} - 1\right) \frac{1}{(t_2 - t_1)}$$

$$= \left(\frac{m_1 v_2}{m_2 v_1} - 1\right) \frac{1}{(t_2 - t_1)}. \quad \text{.....(1)}$$

The values of G and β_a being now known, the value of β, the coefficient of apparent expansion of the liquid, may be calculated from equation (3), p. 335:

$$G = \beta_a - \beta,$$

or

$$\beta = \beta_a - G.$$

In this way, determine the values of β_a and β for the liquid for ranges from 10° to 20°, 20° to 30°, etc., up to 90° C.

If the coefficient of apparent expansion β only is required in the experiment, the change in volume of the bottle may be disregarded, when v_1 and v_2 will be equal, and equation (1) becomes

$$\beta = \left(\frac{m_1}{m_2} - 1\right) \frac{1}{(t_2 - t_1)} = \frac{m_1 - m_2}{m_2(t_2 - t_1)}. \quad \text{.....(2)}$$

Maximum density of water.—When water is cooled, it is found that a contraction occurs until a temperature of 4° C. is reached. Further cooling is accompanied by an expansion until freezing temperature is reached. During the conversion into ice considerable expansion occurs. It follows that water attains its maximum density at 4° C.

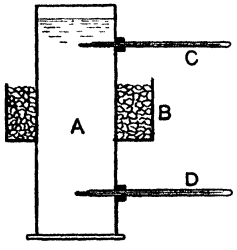


FIG. 354.—Hope's apparatus

EXPT. 68.—**Hope's experiment on the maximum density of water.** In Fig. 354 is shown a metal vessel A having a trough B surrounding the vessel near the middle of its height. C and D are thermometers.

Pour water into the vessel, and place a freezing mixture consisting of broken ice and salt in the trough. Read the thermometers simultaneously every minute.

Suppose the water to have an initial temperature of 12° to 15° C. The

water half-way up the vessel is cooled and contracts, thus acquiring greater density, which causes it to sink to the lower part of the vessel; this fact is rendered evident by the readings of the thermometer D being lower than those of C. As cooling goes on it will be found that the thermometer D shows that the water in the lower part of the vessel has attained the temperature of 4° C. From this point onwards, the cooling below 4° C. of the water near the middle of the vessel results in an increase of volume,

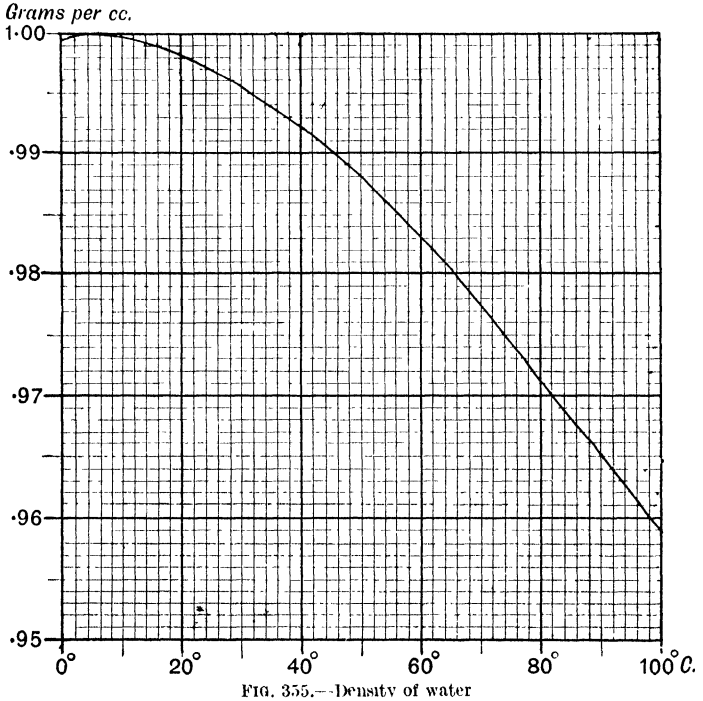


FIG. 355.—Density of water

and consequently a decrease in density, which causes the colder water to rise to the top. This is indicated by the thermometer C falling gradually to 0° C., and ultimately a layer of ice is formed on the surface of the water while the temperature shown by the thermometer D is still at or near 4° C.

These facts are of importance in the economy of nature. But for the expansion which occurs while the temperature is lowered from 4° to 0° C., the colder water would remain at the bottom of ponds and lakes, and freezing would start at the bottom and proceed

upwards. Finally the lake would be frozen into a solid mass of ice. Actually freezing occurs at the surface, and as the layer of ice conducts heat but slowly, the water underneath does not fall much below 4° C. Thus life is preserved in such waters.

Owing to the lack of uniformity in the expansion of water, it is not correct to speak of its density without also stating the temperature. The reason for taking unit mass in the C.G S. system as the quantity of matter in one cubic centimetre of water at 4° C. will be apparent. One gallon of water at 60° F. has a mass of 10 pounds. A graph showing the density of water at temperatures from 0° to 100° C. is given in Fig. 355.

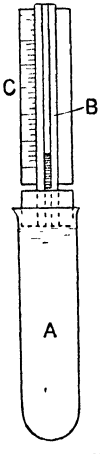


FIG. 356.—Expansion of water while freezing.

EXPT. 69.—**Expansion of water while freezing.** Fit a rubber stopper to a test tube A (Fig. 356), and bore a hole in it to receive a tube B. Determine by filling the test tube from a burette the volume v_1 of water, first well boiled and then cooled nearly to freezing, which the test tube can contain up to the stopper. Measure the bore of the tube B, and hence calculate its cross-sectional area (Expt. 7, p 20). Fit the tube to the stopper as shown in Fig. 356, and attach a scale C. Add a little water so that the level stands a short distance up the tube. Note this level on the scale.

Freeze the water from the bottom upwards by lowering the test tube very slowly into a freezing mixture. If attention be not paid to this, the upper layer will freeze first, and the tube will be burst by the expansion. When freezing is complete in A, read the level on the scale. Take the difference in level, and hence calculate the additional volume v cubic centimetres. This increase in volume has occurred in an initial volume v_1 cubic centimetres, assuming that the water in the tube B does not freeze. Evaluate the ratio $v_1/(v_1+v)$; this will give the density of the ice.

The density of ice is about 0·92 grams per cubic centimetre. Hence about 10 per cent. of the volume of a floating iceberg will be above the surface of sea-water. Melting of the submerged portion of the iceberg will proceed slowly as the iceberg travels with ocean currents into warmer waters. The melting will ultimately affect the stability of flotation of some icebergs and such have been observed occasionally in the act of capsizing.

EXERCISES ON CHAPTER XXV.

1. The density of a piece of brass is 8·456 grams per c.c. at 15° C., and the coefficient of cubical expansion is 57×10^{-6}. Find the density of the brass at 125° C.

2. The density of a piece of zinc is 7 124 grams per c.c. at 12° C. and 7·122 at 42° C. Find the coefficient of cubic expansion of the zinc.

3. Describe an experiment for determining the coefficient of cubical expansion of a solid. The density of a piece of iron is 7 81 grams per c.c. at 0° C., and its coefficient of linear expansion is $11·9 \times 10^{-6}$ per degree Centigrade. What is its density at 100° C. ?

4. A small glass vessel having a fine stem contains 4·56 c.c. of mercury which just fills the vessel at 15° C. The coefficient of linear expansion of the glass is $8·4 \times 10^{-6}$. What volume of mercury (measured at 45° C.) will flow out of the vessel if the temperature is raised to 45° C. ? The coefficient of absolute cubical expansion of mercury is 0·00018.

5. A weight thermometer contains 24 grams of mercury at 0° C. On being heated to 100° C. it is found to contain only 23·622 grams. Calculate the coefficient of linear expansion of the envelope, the coefficient of absolute expansion of mercury being 0·00018. L.U.

6. Describe how you would determine the coefficient of apparent expansion of a given sample of kerosene in a given glass envelope.

7. A glass tube of uniform bore is closed at the lower end and is arranged vertically. Mercury partly fills the tube to a height of 76 cm. at the temperature of 15° C. If the temperature is raised to 20° C., find the height of the column of mercury. The coefficient of linear expansion of the glass is 0·0000085, and the coefficient of absolute expansion of mercury is 0·00018.

8. A Centigrade thermometer has a range from − 10° to 110° C. ; the length of the scale between these marks is 25 cm. The bore of the stem is 0·5 mm. Find the volume of bulb required. Take the coefficient of linear expansion of the glass as 0·000008 and the coefficient of absolute expansion of mercury 0·00018.

9. Describe clearly how to find the coefficient expansion of a liquid by using a weight thermometer. A weight thermometer contains 100 grams of mercury at 0° C. ; when the temperature is raised to 100° C. it is found that 1·72 grams of mercury overflow. If the coefficient of absolute expansion of mercury is 0 000181, find the coefficient of cubical expansion of the material of the thermometer.

10. Describe an experiment for finding the coefficient of absolute expansion of a liquid by balancing two columns of the liquid at different temperatures in vertical tubes connected by a horizontal tube at their lower ends. The temperature of the cold column is 5° C., and that of the hot 95° C. ; the heights of the columns are 50 cm. and 50.22 cm. respectively. Find the coefficient of expansion.

11. Describe any experimental evidence you are acquainted with which proves that the maximum density of water occurs at 4° C. What bearing has this fact on the freezing of water in lakes ?

12. Describe how to determine experimentally the change in volume which occurs when water freezes.

13. If ice weighs 57·5 lb. per cubic foot, find the volume of an iceberg weighing 10,000 tons. What volume of ice will be above the surface of sea-water weighing 64 lb. per cubic foot ?

14. Give a brief description of Regnault's method of determining the coefficient of absolute expansion of mercury.

15. The coefficient of expansion of mercury is $\frac{1}{5550}$. If the bulb of a mercurial thermometer is 1 c.c., and the section of the bore of the tube 0·001 sq. cm., find the position of the mercury at 100° C., if it just fills the bulb at 0° C. Neglect the expansion of the glass. Calcutta Univ.

CHAPTER XXVI

CALORIMETRY

Quantity of heat.—When a hot and a cold body are brought into contact, a transference of some kind is evidenced by both bodies coming ultimately to the same temperature, lying between the temperatures originally possessed by the bodies.

EXPT. 70.—**Distinction between heat and temperature.** Take two vessels, one, A, containing about one litre of water at about 15° C., the other, B, containing about 0·5 litre of water at about 80° C. Place a thermometer in each vessel, stir well, and take the temperatures. Pour the water from B into A, stir again, and read the temperature. With the quantities mentioned, the final temperature will be about 36° C. The water originally in A has been warmed about 21° C., that originally in B has been cooled about 44° C.

As the temperature of the water in A has not been increased to the same extent that the temperature of the water in B has been lowered, it is evident that what has been transferred has not been temperature, but some other quantity to which the name of **heat** is given.

The quantity of heat possessed by a body depends on several factors, of which temperature is one only. For example, a quantity of water set to boil over a bunsen burner receives a great quantity of heat, as estimated by the time taken, while its final temperature is comparatively low. A wire held in the flame comes to a very high temperature almost immediately, but evidently receives only a small quantity of heat.

Heat is not a material substance which a body may absorb like a sponge taking up water. A body when hot weighs no more than the same body when cold. We shall discuss afterwards evidence which proves that **heat is a form of energy**, and exists in a body in the form of motion of the molecules.

Units of heat.—The unit of heat in any system of units is the quantity of heat required to raise the temperature of unit mass of water through 1°.

The C G S. unit of heat is the **calorie**, and is the quantity of heat required to raise the temperature of one gram of water through 1° C. When a larger heat unit is desirable, the **major-calorie** is employed; this unit is equal to 1000 calories.

In Britain two heat units besides the C.G.S. units are employed. These are:

The **Centigrade unit** of heat (lb.-deg.-Cent.), being the heat required to raise the temperature of one pound of water through 1° C.

The **Fahrenheit unit** of heat (lb.-deg.-Fah.), or **British thermal unit** (written B.Th.U.): this is the quantity of heat required to raise the temperature of one pound of water through 1° F.

Since 1·8° F. are equivalent to 1° C., it follows that 1·8 B Th.U. are required to raise the temperature of one pound of water 1° C., *i.e.*

1 Centigrade heat unit = 1·8 B Th.U.

$$1 \text{ B.Th.U.} = \tfrac{5}{9} \text{ Centigrade heat unit.}$$

EXAMPLE.—What factor must be employed to convert a given quantity of heat stated in calories into B.Th.U and into Centigrade heat units?

1 calorie can raise the temp. of 1 gram water through 1° C

453·6 calories „ „ „ 453·6 grams „ „ 1° C.

Since 1 pound = 453·6 grams, the latter statement may be written.

453·6 calories can raise the temp of 1 pound water through 1° C

$\therefore$ $(\frac{5}{9} \times 453{\cdot}6)$ „ „ „ „ „ „ „ „ 1° F.

Hence 1 B.Th.U $= \frac{5}{9} \times 453{\cdot}6 = 252$ calories.

To convert from calories to B Th U, multiply the calories by $\frac{1}{252} = 0{\cdot}003968$.

To convert from B Th U. to calories, multiply the B.Th U.'s by 252.

To convert from Centigrade heat units into calories, multiply the Centigrade heat units by 453·6.

To convert from calories to Centigrade heat units, multiply the Centigrade heat units by $\frac{1}{453{\cdot}6} = 0{\cdot}002205$.

The Centigrade heat unit has several important advantages over the B.Th.U., and its use in this country is extending rapidly.

Specific heat.—Experimental evidence shows that equal masses of different substances require unequal quantities of heat to raise their temperatures through the same range. The **specific heat** of a substance may be defined as the quantity of heat required to raise the temperature of unit mass of the substance through one degree. Thus the specific heat of iron is about $\frac{1}{9}$ calorie, *i.e.* $\frac{1}{9}$ calorie can

raise the temperature of one gram of iron through one degree Centigrade, or $\frac{1}{9}$ lb.-deg.-Cent. unit of heat can raise the temperature of one pound of iron through one degree Centigrade. The number expressing the specific heat of a substance is the same irrespectively of the system of units employed. The specific heat of most substances varies somewhat, depending upon the temperature. Thus, if the specific heat of water is taken as unity at 20° C., the values at 40°, 60° and 100° are 0·9982, 1·0000 and 1·0074 respectively. In many calculations the specific heat of water can be assumed to be unity at all temperatures.

Specific Heats.*

(The range in temperature is from ordinary atmospheric temperatures up to 100° C. unless otherwise stated.)

Material	Specific heat	Material	Specific heat
Aluminium	0·219	Tin	0·0552
Copper	0·0936	Zinc	0·093
Iron	0·119	Glass, Crown 10° to 50° C.	0·16
Lead	0·0305	Glass, Flint 10° to 50° C.	0·12
Nickel	0·109	Ice, −20° to −1° C.	0·502
Platinum	0·0324		

Heat capacity, or water equivalent of a body.—The **heat capacity, or water equivalent** of a given body is the quantity of heat which the body absorbs when its temperature is raised through one degree. The same quantity may be defined as the mass of water which requires the same quantity of heat to raise its temperature through one degree as the body itself requires. Thus the water equivalent of 9 pounds of iron is one pound, and of 9 grams of iron, one gram.

Let M = the mass of the body,

s = the specific heat of the material.

Then Heat capacity, or water equivalent of the body = Ms.

Measurements of quantities of heat are effected in vessels called **calorimeters.** The calorimeter generally contains water, and the heat undergoing measurement is passed into the water, thus causing a rise in temperature. The temperature of the calorimeter also rises, and the heat thus absorbed is taken into account by adding the

* For fuller tables of specific heats, see *Physical and Chemical Constants*, by Kaye and Laby (Longmans).

water equivalent of the calorimeter to the weight of the contained water.

Calculations regarding heat transference.—In making calculations of the ultimate temperature attained when heat is transferred from one body to another, it may be assumed in the first instance that no heat is wasted in raising the temperature of any body other than the colder one considered. Corrections may then be estimated and applied for any heat known to be wasted. Calling the two bodies A and B, we may state as an approximate solution :

Heat passing from A = heat entering B.

EXPT. 71.—**Final temperature in mixtures of water.** Put some cold water in a copper calorimeter A and heat another quantity of water in a vessel B. Obtain the masses of water M_A and M_B by weighing and deducting the weights of the empty vessels. Let t_A and t_B be the initial temperatures in A and B respectively. Pour the water from B into A, stir well, and observe the final steady temperature t. Calculate the final temperature as follows :

Heat passing from B = Heat entering A.

$$M_B(t_B - t) = M_A(t - t_A),$$

$$M_B t_B - M_B t = M_A t - M_A t_A,$$

$$t = \frac{M_A t_A + M_B t_B}{M_A + M_B}. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

Compare the result of this calculation with the experimental value of t. Setting aside errors in measuring the temperatures accurately, and the possible error due to some of the hot water being left in the vessel B, the principal source of discrepancy lies in the fact that the calorimeter A has been raised in temperature. Take account of this by adding to M_A the water equivalent of the calorimeter, which may be estimated by taking the product of the mass M of the calorimeter A and the specific heat s of its material. This now gives

$$M_B(t_B - t) = (M_A + Ms)(t - t_A),$$

from which

$$t = \frac{(M_A + Ms)t_A + M_B t_B}{M_A + Ms + M_B}. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

The result thus determined should be in close agreement with the experimental value.

The masses used in the above calculations should all be in grams, or all in pounds ; the same scale of temperature must be used throughout. If the calorimeter is made of copper, the specific heat s may be taken as 0·1.

EXPT. 72.—**Specific heat of a solid by the method of mixtures.** In this experiment, a small piece of iron, copper, brass, or other material is first heated and then lowered into a calorimeter containing water. The arrangement for heating the sample is shown in Fig. 357. The sample A has a

thread attached to it, and is contained in a copper tube B having a plug of cotton-wool at its lower end. The upper end is closed by a stopper to which a thermometer is fitted. The tube B is enclosed in another larger tube having a branch C for the introduction of steam from a boiler; D is a discharge branch. Steam is allowed to pass into the heater for a few minutes; the temperature is then read, the plug of cotton-wool is withdrawn quickly, and the sample is lowered into the calorimeter, which is placed below the heater. The arrangement enables a dry, hot sample to be obtained, and permits the minimum duration of contact with the atmosphere between the sample leaving the heater and entering the calorimeter.

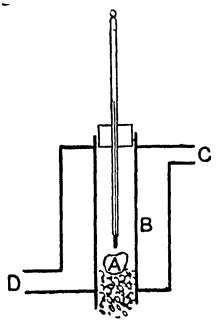


FIG. 357.—Arrangement for heating the sample

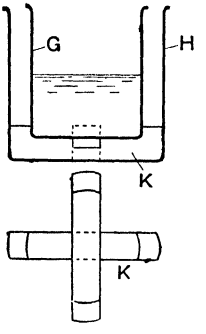


FIG. 358.—Section of a calorimeter.

The calorimeter is shown in Fig. 358, and consists of an inner vessel G and an outer vessel H, both made of copper and separated by a wooden cross K. This arrangement assists in preventing loss of heat from the calorimeter.

Weigh the sample. Determine the water equivalent of the inner vessel G. Find the mass of the water in the calorimeter by weighing. Heat the sample, and when it is ready for transference to the calorimeter, take the temperatures of the heater and of the water in the calorimeter. Lower the sample into the calorimeter, and keep it moving in the water until the temperature of the water becomes steady. Note this temperature.

Let M_s = mass of the sample.

M_w = ,, ,, water in G.

$M_G s_G$ = water equivalent of the calorimeter.

t = temperature of the hot sample.

t_1 = initial temperature of the water.

t_2 = final temperature of the water and sample.

s = specific heat of the material of the sample.

Assuming that the heat given up by the sample is equal to the heat taken up by the water and calorimeter,

$$M_s(t_s - t_2)s = (M_w + M_G s_G)(t_2 - t_1),$$

$$s = \frac{(M_w + M_G s_G)(t_2 - t_1)}{M_s(t_s - t_2)}$$

Compare the value found from this equation with that given in the Table, p. 345.

In Expt. 72, if the temperature of the room be above or below that of the calorimeter, heat will pass from the atmosphere into or out of the calorimeter during the experiment. This interference may be avoided partially by adjusting the initial temperature of the water in the calorimeter so that the temperature of the atmosphere in the room is the mean of the initial and final temperatures of the water. Thus, if a rise of 4° C. is expected and the temperature of the room is 15° C., the initial temperature of the water should be 13° C ; the heat entering the calorimeter will then be balanced approximately by the heat leaving it.

Specific heat of a liquid.—If a sufficient quantity of the given liquid is available, the specific heat may be found by the method explained in Expt. 72. The liquid is used in the calorimeter instead of water, and a hot body of known specific heat is employed.

EXPT. 73.—**Specific heat of a liquid by the method of mixtures.** Find the specific heat of the sample of lubricating oil supplied, following the method explained in Expt. 72.

Let M_B = mass of the hot body.
M = ,, ,, liquid in the calorimeter.
$M_G s_G$ = water equivalent of the calorimeter.
t_B = temperature of the hot body.
t_1 = initial temperature of the liquid.
t_2 = final temperature of the liquid and hot body.
s_B = specific heat of the hot body.
s = specific heat of the liquid.

Then

Heat entering liquid and calorimeter = heat given up by hot body ;

$$(Ms + M_G s_G)(t_2 - t_1) = M_B(t_B - t_2)s_B ;$$

$$s = \frac{M_B s_B}{M}\left(\frac{t_B - t_2}{t_2 - t_1}\right) - \frac{M_G s_G}{M}$$

EXPT. 74.—**Newton's law of cooling.** The apparatus employed for experiments on the cooling of a liquid is illustrated in Fig. 359. The liquid is contained in a test tube A, fitted with a cork and a thermometer B. A

wire stirrer C enables the liquid to be stirred prior to its temperature being observed. D and E are metal cans, one placed inside the other and having ice packed between; the test tube is suspended so as not to touch the inner can. The temperature of the air space in the inner can will remain practically constant, and may be observed by the thermometer F. The cans are closed at the top by means of a cover G. The object of the arrangement is to enable the surroundings of the liquid under test to be preserved as uniform as possible.

Put a measured volume of hot water in the test tube, and complete the arrangement of the apparatus. Observe the temperatures of the water and of the air space at one minute intervals during 20 or 30 minutes. Plot a

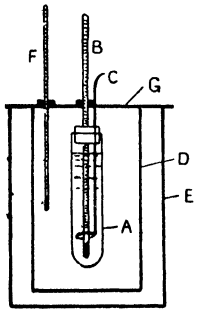


FIG. 359.—Experiment on cooling.

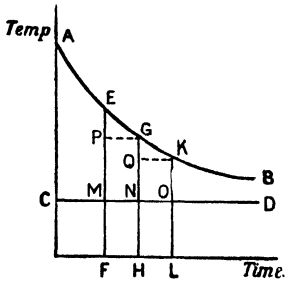


FIG 360.—Cooling curve

graph AB (Fig. 360), showing the temperatures of the water as ordinates and times as abscissae. Show the temperature of the air space on the graph as indicated by CD in Fig. 360.

Select equal intervals of time FH and HL. The fall in temperature of the water during the interval FH is EP, and during HL the fall is GQ. During the interval FH the mean difference in temperature of the water and the air space is $\frac{1}{2}$(EM + GN); during the interval HL the mean difference in temperature is $\frac{1}{2}$(GN + KO). Evaluate the ratios of fall in temperature to mean temperature difference for both intervals, viz. EP ÷ $\frac{1}{2}$(EM + GN) and GQ ÷ $\frac{1}{2}$(GN + KO). It will be found that these ratios are practically equal, showing that the rate of cooling of the water is proportional at any instant to the difference in temperature between the water and its surroundings.

Assuming that the fall in temperature of a liquid is proportional to the quantity of heat abstracted from it, we may infer from the above result that the quantity of heat passing from a cooling liquid per unit of time is proportional to the temperature difference between the liquid and its surroundings, a law which is known as **Newton's law of cooling.**

EXPT. 75.—**Specific heat of a liquid by cooling.** Repeat Expt. 74, using an equal volume of some liquid other than water. Plot a cooling graph for this liquid in the same manner as for the water Obtain from the graphs the intervals of time in which the water and the liquid cool through the same range of temperature from t_1 to t_2. Ascertain, by weighing, the masses of both liquids.

Let T_1 = the time in minutes for the water to cool through the given temperature range

T_2 = the corresponding time for the other liquid.

M_1 = the mass of the water

M_2 = the mass of an equal volume of the liquid.

s = the specific heat of the liquid.

Then, since the temperatures are identical in the two experiments,

$$\frac{\text{Heat lost by the liquid}}{\text{Heat lost by the water}} = \frac{M_2 s(t_1 - t_2)}{M_1(t_1 - t_2)} = \frac{T_2}{T_1}.$$

$$s = \frac{M_1 T_2}{M_2 T_1}$$

It will be noted that equal volumes of water and of the other liquid are used in order that the area of the wetted interior of the test tube may be the same for both. This precaution, together with the practical uniformity of the air-space temperature, ensures that the surrounding conditions are the same in both experiments.

EXERCISES ON CHAPTER XXVI.

1. Convert 1414 lb.-deg.-Centigrade units of heat into lb.-deg.-Fah. heat units and also into calories.

2. Convert 778 lb.-deg.-Fah. thermal units into lb.-deg.-Cent. units and also into calories.

3. Define "Specific heat of a substance." A copper calorimeter weighs 0·4 lb., and the specific heat of the material is 0·094. Find the quantity of heat required to raise the temperature from 15° to 55° C. What is the water equivalent of this calorimeter ?

4. A steam boiler is made of mild steel and weighs 10 tons. The specific heat of the material is 0 12. The boiler contains 8 tons of water. Find the quantity of heat required to raise the temperature of the whole from 15° to 100° C., assuming no waste.

5. Fifteen gallons of water at 10° C. are mixed in a tank with 20 gallons of water at 65° C. Find the final temperature, assuming no waste.

6. A piece of zinc weighing 65 grams is at the temperature 100° C., and is lowered into a calorimeter containing 350 grams of water at 15° C. The water equivalent of the calorimeter is 8·5 grams. The final temperature is 16·5° C. Find the specific heat of the zinc.

7. A piece of iron weighing 100 grams is allowed to remain in a current of hot gases in a flue for a few minutes, and is then lowered into a calorimeter containing 500 c.c. of water at 15° C. The water equivalent of the calorimeter is 40 grams. The final steady temperature was observed to be 22·5° C. Calculate the temperature of the gases, assuming that the average specific heat of the iron is 0·11.

8. A sample of lubricating oil weighing 240 grams is contained in a calorimeter having a water equivalent of 12 grams. The initial temperature is 14° C. A piece of copper weighing 72 grams, specific heat 0·093, is raised to 100° C., and then lowered into the calorimeter. The final steady temperature was 18·2. Find the specific heat of the oil.

9. Fifteen grams of water contained in a copper calorimeter weighing 22 grams are found to cool from 80° F. to 70° F. in 3 minutes. An equal volume of another liquid weighing 14 grams cools from 80° F. to 70° F. in the same calorimeter in 110 seconds. The specific heat of the copper is 0·09. Find the specific heat of the liquid.

10. Describe any experiment by which you would find the specific heat of a sample of metal. Give sketches of the apparatus employed.

11. Write an account of the determination of specific heats by the method of cooling, and explain the theory of the method. L.U.

12. The temperatures of equal masses of three different liquids, a, b, c, are 15°, 25°, and 35° C. respectively. On mixing a and b, the temperature of the mixture is 21° C., and on mixing b and c, the temperature of the mixture is 32° C. Supposing a and c were mixed, what would be the temperature of the mixture ?

13. If equal quantities of heat are applied to equal volumes of copper and iron, compare the rises of temperature produced. Density of copper, 8·9 ; density of iron, 7·8. Specific heat of copper, 0·094 ; specific heat, of iron, 0·12.

14. Describe how you would determine experimentally the water equivalent of a calorimeter. If 50 grams of lead shot (specific heat = 0·031) at 97° C. are poured into 75 grams of a liquid at 31° C., contained in a calorimeter of water equivalent 4·5, and the final temperature is 33° C., what is the specific heat of the liquid ? Madras Univ.

CHAPTER XXVII

NATURE OF HEAT. NATURAL SOURCES OF HEAT

Nature of heat.—At the beginning of the nineteenth century it was believed that heat was an elastic fluid, capable of being soaked in or squeezed out, as it were, by a body. The name caloric was given to this substance. This theory has been rejected on the evidence of experiments, of which the most important were made by Rumford, Davy and Joule.

Count Rumford noticed that the metal chips removed by the boring tool in boring a cannon became hot. He arranged a cannon in a bath of water, and started boring with a blunt boring tool which removed but little material. It was found that heat sufficient to boil the water could be obtained during a comparatively short run of the apparatus As there was evidently no limit to the quantity of heat which could be evolved from the bodies concerned in the experiment, he concluded that it was impossible that the heat produced could be a material substance contained in the bodies prior to starting boring.

Sir Humphry Davy experimented by rubbing together two pieces of ice, taking precautions to ensure that no heat could be communicated to them from outside sources In a short time it was found that the ice melted, showing that heat had been generated by the rubbing.

Dr. Joule, of Manchester, gave the most conclusive experimental proof that heat is not a material substance. In the experiments of Rumford and Davy, the bodies experimented on changed character during the operations. In Joule's experiments heat was evolved by the stirring of water, and the conditions at the end of the experiment were exactly the same as at the start, excepting that the temperature of the water had been raised

It is now believed that heat is energy possessed by a body by virtue of the state of motion of the molecules of which it is composed. The molecules of a solid do not alter their positions

relatively to each other, but are in a state of vibration which is increased by the addition of heat, the energy of the molecules being thus made greater. Heat imparted to a liquid may increase the motion of the molecules and at the same time may cause currents of molecules to travel from one part of the liquid to another part. In gases the molecules are in rapid motion; collisions with each other and with the walls of the vessel containing the gas are frequent. The continual bombardment produces pressure on the walls of the vessel. Heat imparted to a gas increases the speed of

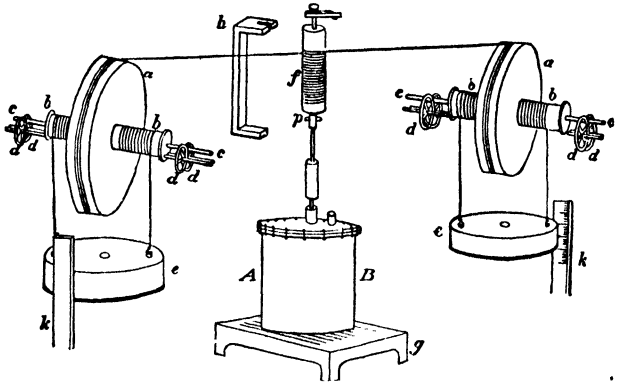


FIG. 361.— Apparatus used by Joule in his experiments on the mechanical equivalent of heat.

the molecules, thereby increasing both their kinetic energy and the pressure on the walls.

Mechanical equivalent of heat.—Heat energy is capable of being converted into other forms of energy and *vice versa*. There are many practical operations having for their object the conversion of heat into mechanical work. Since no energy can be destroyed (p. 170), it follows that a definite quantity of mechanical work is equivalent to a given quantity of heat.

Joule's experiments, already mentioned, were projected for the purpose of ascertaining the quantity of mechanical work equivalent to one thermal unit. The apparatus employed is shown in Fig. 361. Falling weights *ee* are arranged so as to drive paddles revolving in a calorimeter AB containing water. The calorimeter, shown separately in Fig. 362, was fitted with baffle plates having spaces cut so as to permit the paddles to pass, the object being thoroughly to churn the

ıter The weights were allowed to descend a measured height, volving the paddles by means of the cords wrapped round the paddle axle at f (Fig 361), f was then disconnected from the paddles by means of the pin p, and the weights were wound up again by means of the handle at the top of f. It will be noted that the arrangement of two weights and two cords wrapped round f in opposite directions applied a couple to the paddle axle, thus producing pure rotation.

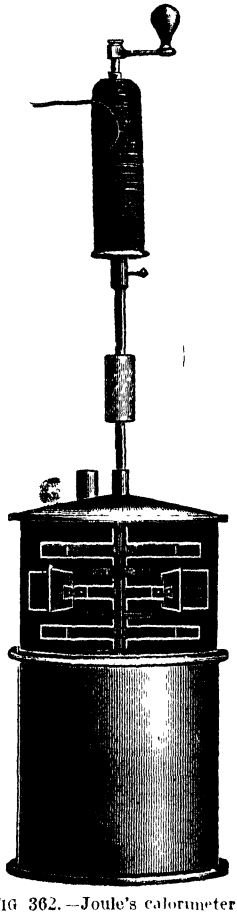


FIG 362.—Joule's calorimeter

Corrections of various kinds were applied, such as the kinetic energy of the weight on reaching the bottom, the frictional resistances of the various bearings, the heat capacity of the calorimeter, etc. The final result was that about 772 foot-lb. of mechanical work were equivalent to one British thermal unit.

Subsequent experiments by Rowland in America, and Osborne Reynolds and Griffiths in Britain, give 774 and 778 as more accurate results. The experiments of Osborne Reynolds are of interest on account of the large scale of the apparatus employed. The power developed by the experimental steam engines at Owen's College in Manchester was absorbed by the resistance provided by stirring water in a hydraulic brake. The rate of flow of the water passing through the brake and its rise in temperature were measured, as well as the horse-power absorbed by the brake, thus providing data for calculating the mechanical equivalent of heat. Carefully estimated corrections were applied.

At present the best authorities employ 778 foot.-lb as equivalent to 1 B Th U These numbers are called **Joule's mechanical equivalent** of heat, and are denoted by the symbol J In the C G S system J may be taken as $4 \cdot 18 \times 10^7$ ergs of mechanical work, equivalent to one calorie of heat Another useful value for J is 1400 foot-lb of

mechanical work equivalent to one Centigrade heat unit, *i.e.* to the heat required to raise the temperature of one pound of water through 1° C.

First law of thermodynamics. —Thermodynamics is the name given to the study of the conversion of heat into mechanical work and *vice versa*. The first law may be enunciated as follows: **Heat and mechanical work are mutually convertible, and in any operation involving such conversion $4{\cdot}18 \times 10^7$ ergs of mechanical work disappear for each calorie generated, or $4{\cdot}18 \times 10^7$ ergs of mechanical work appear for each calorie expended.** In the British system, substitute 1400 foot-lb. and one Centigrade heat unit in the latter part of the law.

The student must be prepared for very large waste in all operations involving the conversion of heat into mechanical work. It is very difficult to prevent heat being dissipated into forms which are useless for any practical purpose. The operation of converting mechanical work into heat is not accompanied by such excessive waste, and laboratory experiments on the value of Joule's equivalent usually follow this method.

EXPT. 76.—**Value of Joule's equivalent by Callendar's machine.** The Callendar apparatus provides a very convenient laboratory method of

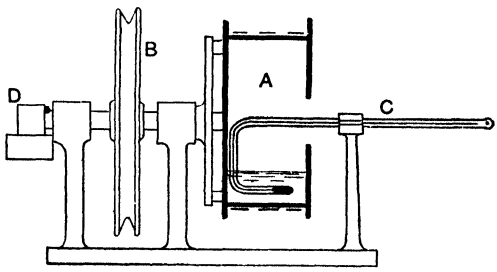


FIG. 363.—Calorimeter of Callendar's machine.

determining J, and is illustrated in Fig. 363. A brass drum A is fixed to the end of a shaft and can be rotated by means of a band passed over a pulley B; the band is driven by a small electromotor not shown in Fig. 363. A revolution counter at D enables the number of revolutions of the drum to be observed. The drum serves the purpose of a calorimeter, and contains a measured quantity of water; a bent thermometer C passes through a central hole in the end of the drum and dips into the water. A band brake, made of three strips of silk ribbon, passes round the drum and covers nearly the whole of its external cylindrical surface. The brake

carries a dead load W at one end (Fig. 364), and has a shackle E attached to the other end to which smaller weights w may be added. A light spring balance pulls the shackle E upwards and enables the brake load to be read delicately, and also helps to produce steadier running.

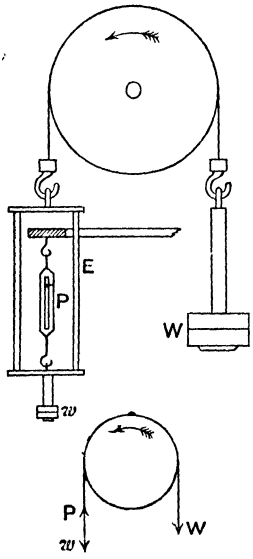


FIG 364.—Brake of Callendar's machine.

Work is done against the frictional resistances of the silk brake rubbing on the drum; this work is converted into heat, which passes with difficulty outwards through the silk, but passes very easily through the metal drum into the water. The test is made with the initial temperature of the water equal to that of the room; the drum is revolved until the thermometer indicates a rise in temperature of about 5 or 6 degrees Centigrade.

The work done against the resistance of the brake may be calculated as follows: The load W and the pull P of the spring balance both resist the motion of the drum; w assists the rotation. Hence the net resistance is $(W + P - w)$, and this is overcome through a distance equal to the circumference of the drum during each revolution. If D is the diameter of the drum, and if it makes N revolutions, then the total work done is

$$(W + P - w)\,\pi DN.$$

The following record of a test with the Callendar machine is given in order to illustrate the method of reducing the results, especially with reference to the application of cooling corrections, which often has to be done in calorimetric measurements.

DETERMINATION OF THE MECHANICAL EQUIVALENT OF HEAT BY CALLENDAR'S MACHINE.

Diameter of the drum, D	15·2 cm.
Brake load, W	4000 grams weight.
Load at spring balance end, w	300 grams weight.
Pull of the spring balance, P	30 grams weight.
Initial counter reading	62,440.
Final " "	63,335.
Total revolutions during the test, N	895.
Mass of water used, M_1	250 grams.
Water equivalent of the calorimeter, M_2	382·65 grams.

Time, min	Observed temp C	Mean temp. during interval C	Rate of cooling, from graph (Fig. 366) Deg Cent. per min	Correction to be added to the mean temp	Corrected mean temp C
0	15·1				
		15·45	0·015	0 015	15 465
1	15 8				
		16 15	0 035	0 015 + 0 035 = 0·05	16 2
2	16·5				
		16·8	0 06	0·05 + 0·06 = 0·11	16 91
3	17·1				
		17 4	0 08	0 11 + 0 08 = 0 19	17·59
4	17·7				
		18·05	0 1	0·19 + 0·1 = 0·29	18·34
5	18 4				
		18 7	0 12	0·29 + 0·12 = 0·41	19·11
6	19 0				
		19 3	0 14	0·41 + 0·14 = 0·55	19 85
7	19 6				
		19 82	0 16	0 55 + 0 16 = 0·71	20 53
8	20 04				
		20·05	0·165	0·71 + 0·165 = 0·875	20 925
9	20 06				
		19 98	0 162	0·875 + 0 162 = 1·037	21·017
10	19 9				
		19 85	0·16	1 037 + 0·16 = 1·197	21·047
11	19 8				
		19·75	0 155	1·197 + 0·155 = 1·352	21·12
12	19 7				
		19·55	0·15	1·352 + 0·15 = 1·502	21·052
13	19·4				
		19·3	0·14	1·502 + 0·14 = 1·642	21·082
14	19 2				

The machine was run for 8 minutes and then stopped. The temperature was read every minute for 14 minutes ; columns 1 and 2 show these readings. Column 3 gives the mean temperatures during each interval of one minute ; these numbers are obtained from column 2. Mean temperatures and time were plotted (Fig. 365), giving the lower graph. The effect of cooling is shown in this graph by the curve drooping after the machine was stopped. To obtain the cooling corrections throughout the test, we have from this graph :

Mean temperature at 11 minutes = 19·76° C.

Mean temperature at 13 minutes = 19·45° C.

Fall in 2 minutes = 0·31° C.

Fall per minute = 0·155° C.

$$\text{Mean temperature during this fall} = \frac{19{\cdot}76 + 19{\cdot}45}{2}$$

$$= 19\ 6^\circ \text{ C.}$$

Plot a graph (Fig. 366) in which abscissae represent mean temperatures and ordinates represent fall in temperature per minute. The fall of 0·155° C. per minute is plotted at 19·6° C., and a straight line drawn through this point and 15·1° C. (the temperature of the room) on the OX axis ; at this temperature there would be zero rate of cooling. The cooling rate at any mean temperature shown in Fig. 365 can now be obtained from Fig. 366. Thus, at the mean temperature 15·45° C., the rate of cooling is 0 015° C. per minute ; hence the corrected mean temperature for the first interval of one minute is 15·45 + 0·015 = 15·465° C. At the mean temperature of 16·15° C., the rate of cooling is 0·035° C. per minute, and as the test has now proceeded during two minutes, the correction for

the first interval must be added, giving a total correction of 0·015 + 0·035 = 0·05° C. The corrected mean temperature for the second interval of one

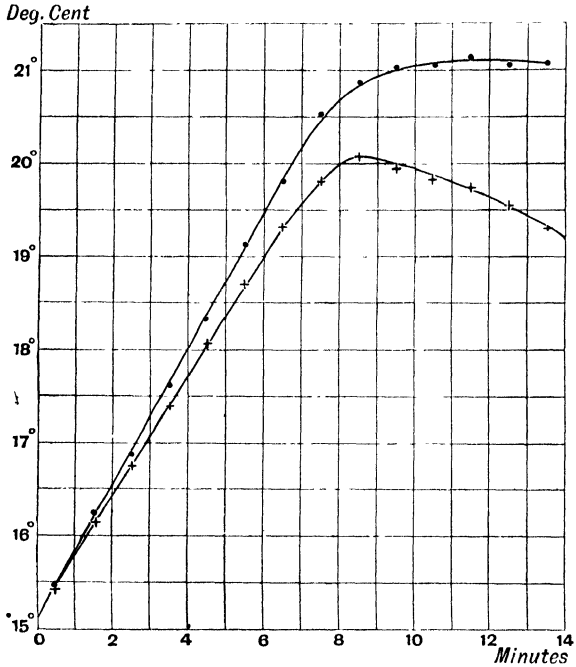


FIG. 365.—Graphs for an experiment with Callendar's machine.

minute is therefore 16·15 + 0·05 = 16·2° C. The rates of cooling obtained from Fig. 366 are shown in column 4, the corrections to be applied in

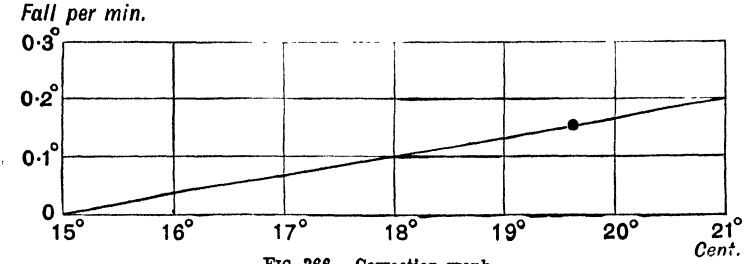


FIG 366.—Correction graph.

column 5, and the corrected mean temperatures are shown in the last column.

The upper graph (Fig. 365) shows the corrected mean temperatures, and represents what would be obtained experimentally if the conditions had been such that no heat was dissipated from the apparatus during the test. The final corrected temperature is 21·1° C. nearly; the initial temperature is 15·1° C.; the rise in temperature is therefore 6·0° C. We now have:

Work done against friction $= (W - w + P)\pi DNg$ ergs.

Heat developed $= (M_1 + M_2)t$ calories,

where t is the rise in temperature.

$$J = \frac{(W - w + P)\pi DN}{(M_1 + M_2)t} g$$

$$= \frac{3730 \times 22 \times 15 \cdot 2 \times 895 \times 981}{7 \times 632 \cdot 65 \times 6}$$

$$= 4 \cdot 12 \times 10^7 \text{ ergs.}$$

Natural sources of heat.—Heat, being a form of energy, cannot be created; all heat is obtained from natural stores, or is produced by methods which depend for their working upon natural stores of heat. The most obvious natural source of heat is the sun. Direct heat from the sun is now being used for the production of mechanical work to a small extent in Egypt and America. The plan adopted is to concentrate the sun's rays, by use of long parabolic mirrors, on a pipe containing water. The pipe serves as a steam boiler, and supplies steam to an engine. The sun's heat is also responsible indirectly for the large stores of energy available in water collected in elevated lakes in hilly country. This water comes from rain-clouds which owe their existence to water evaporated from the sea by the heat of the sun. Winds are caused by unequal heating by the sun of large masses of air, and provide energy utilised in the driving of windmills. Volcanic heat is now being utilised for power production in Central Tuscany, where powerful jets of very hot steam are discharged from cracks in the ground. This steam is employed instead of coal for heating the water in steam boilers, and the steam generated in the boilers is used for driving steam turbines, which provide motive power for electric generators. Three large installations on this system were put into operation in the year 1916.

The principal commercial sources of heat are **fuels**. A fuel is any substance capable of chemical combination with the oxygen of the atmosphere, with the evolution of heat and light (*i.e.* **combustion**), and existing in quantities sufficient to be of commercial value.

Solid fuels.—Coal is the fuel in most extensive use at present. It consists of mineralised vegetable matter, and consequently its origin may be traced to the heat and light of the sun. The vegetation of past ages being buried in the earth undergoes compression and slow mineralisation. The first product is **lignite**—a coal of very poor quality. **Anthracite** is the most perfectly mineralised coal and consists chiefly of carbon. **Bituminous coal** is intermediate in composition, and contains volatile constituents composed of compounds of hydrogen and carbon called hydrocarbons. Over 400,000,000 tons of coal are mined annually in Europe. It is estimated that the European coal supply yet remaining is about 350,000,000,000 tons. The consumption of coal is increasing rapidly every year.

The heat which may be produced by the complete combustion of one pound of good coal is about 8000 Centigrade heat units, this number is called the **heating value** of the coal.

Other solid fuels are **coke**, produced by distilling coal in closed retorts; the volatile constituents are driven off, leaving coke, which consists of carbon and ash, the latter being the incombustible material present in the coal; **timber**; **charcoal** produced from wood by driving off the moisture and volatile matter by slow heating, leaving practically pure carbon, **peat**, which is the remains of comparatively recent vegetation found in bogs.

Liquid fuels.—Mineral oils suitable for fuels are obtained as (*a*) crude petroleum, (*b*) paraffin oil; these are both mixtures of various hydrocarbons

Crude petroleum is obtained by drilling wells in the earth's crust; the bulk of the supply comes from the United States and Russia. The crude oil is refined by distillation, giving gasoline, burning oils, oils suitable for gas making, and other materials. Light gasoline oils are used for driving motor vehicles; the heavier burning oils are also used for operating engines. Crude oil is also used as fuel. The heating value ranges from about 10,800 for the lighter oils to about 12,500 for the heavier, both stated in Centigrade heat units per pound of oil. The extent of the world's store of petroleum is unknown; the consumption is enormous—about 60,000,000 tons per annum—and is increasing rapidly.

Paraffin oil is produced by the distillation of bituminous shales and boghead coal.

Gaseous fuels.—Ordinary **lighting gas** is produced by heating bituminous coal in closed retorts; coke is a bye-product of the process. The gases driven off are purified, and are then available for lighting and heating. About 50 per cent. by volume of the gas is hydrogen, the remainder consists of various hydrocarbons and carbon monoxide. A ton of coal yields about 10,000 cubic feet of gas. The average heating value is about 300 Centigrade units per cubic foot of gas.

Various gases are manufactured for power purposes on the large scale. Among these may be mentioned **Dowson gas**, manufactured by blowing a mixture of air and superheated steam through incandescent anthracite or coke. This gas has a composition by volume roughly as follows: Hydrogen, 19 per cent.; carbon monoxide, 25 per cent.; nitrogen, 49 per cent. The heating value is about 90 Centigrade units per cubic foot of gas. **Mond gas** is another power gas manufactured from cheap bituminous small coal by blowing air saturated with steam at 70° C. through the coal, which is kept burning at a dull red heat. The composition by volume is roughly as follows: Hydrogen, 28 per cent.; carbon monoxide, 12 per cent.; carbon dioxide, 15 per cent.; nitrogen, 43 per cent. The heating value is about the same as that of Dowson gas.

Natural gas is discharged from wells bored into the earth's crust in certain localities. The heating value of American (Pittsburg) natural gas is about 550 Centigrade units per cubic foot of gas. The supply is giving out rapidly.

Combustion of carbon.—In burning carbon completely to carbon dioxide, about 8040 Centigrade units are given out per pound of carbon. Theoretically about 12 pounds of air occupying a volume of about 155 cubic feet must be supplied; in practice from 18 to 24 pounds of air are required.

By limiting the supply of oxygen, carbon may be incompletely burned, producing carbon monoxide. In this process about 2470 Centigrade units of heat are given out per pound of carbon. Carbon monoxide is combustible, and when burned completely the product is carbon dioxide; it gives out about 5600 Centigrade units of heat per pound of gas.

Combustion of hydrogen.—When hydrogen is burned, the product is water vapour. About 34,500 Centigrade units of heat are given out per pound of hydrogen. About 35 pounds of air, occupying about 450 cubic feet, must be supplied per pound of hydrogen.

Hydrogen and other combustible gases and vapours may be exploded with violence when mixed with proper proportions of air. Carbon burns slowly unless it is powdered finely and mixed as a dust with oxygen, in which case an explosion may be produced.

Some of the practical methods used in the determination of the heating values of fuels may now be studied.

Heating value of coal.—In the Darling calorimeter, the heat evolved during the combustion of the sample of coal passes into a measured

quantity of water. The instrument is shown in Fig. 367. The sample of coal is first finely powdered and heated at 100° C. in an oven in order to get rid of moisture. About one gram is then weighed and introduced into a crucible C, which is held in clips at the top of a tube A. A bell-glass B encloses the crucible, and is clamped to a plate R. A gentle stream of oxygen is supplied to the bell-jar through a tube O. W, W are wires passing through the bell-glass stopper, and are connected at the lower ends by a piece of fine iron wire which dips into the coal. An electric current is used for ignition; on passing the current through the iron wire, it is heated to incandescence (some of it generally burns) and ignites the coal The vessel S contains a measured quantity of water, the temperature of which is measured by a thermometer T

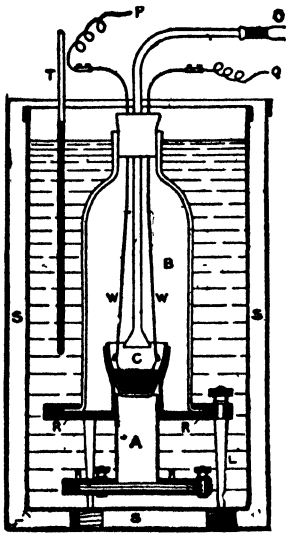


FIG 367.—Darling calorimeter for testing the heating value of coal or other solid fuel

The combustion is thus effected in an atmosphere of oxygen; the products of combustion pass downwards through A and escape into the water through a number of small holes. The resulting bubbles give up their heat to the water as they pass upwards.

Let Q = the heating value in calories per gram of coal.
M = the mass of water used, in grams.
M_c = the water equivalent of the instrument, in grams.
M_f = the mass of coal burned, in grams.
t_1 = the initial temperature of the water, C.
t_2 = the final temperature of the water, C.

Then $$Q = \frac{(M + M_c)(t_2 - t_1)}{M_f}.$$

Heating value of gaseous fuels.—In calorimeters of the Darling type, described above, a definite quantity of water is used, and the temperature of the calorimeter rises as the test proceeds. In testing gaseous fuels, calorimeters are used in which the heat evolved during the combustion is passed into water which circulates through the instrument. Arrangements are made so as to maintain, as steady as possible, both the flow of gas to the burner and the flow of water; hence the temperatures remain nearly constant throughout the

experiment. The calorimeter designed by Prof. C. V. Boys is illustrated in Fig. 368, and is typical of this kind of calorimeter.

The gas is burned at jets B; the products of combustion pass upwards into a bell H and then downwards through E, in which space is a pipe coil M made of motor-car radiator tube. Circulating water enters the pipe coil at O, where its temperature is measured, passes through the outer coil N, then through the inner coil M. Leaving M, the water enters a space K where it is thoroughly mixed before being discharged at P, where its temperature is again measured. The instrument is used in conjunction with an accurate gas meter. The quantity of water passed through the calorimeter during the test is discharged into graduated jars, and is so measured; from this quantity and the rise in temperature, the heat evolved by the combustion of the measured quantity of gas is determined.

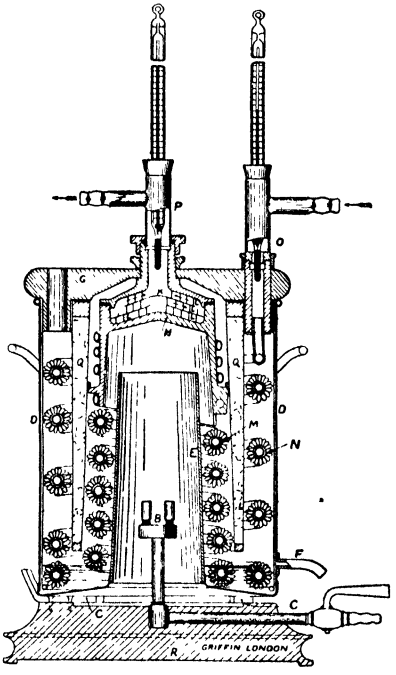


FIG. 368.—Section of Boys's calorimeter for testing the heating value of gas.

Bomb calorimeter.—The original form of the bomb calorimeter is the Berthelot-Mahler; there are now several different designs, one of which is shown in Fig. 369. Both solid and liquid fuels can be tested in this instrument, and very accurate results can be obtained on account of the completeness of the combustion. The general principles are the same as those in the Darling type, but the combustion is effected in an atmosphere of highly compressed oxygen.

A is the bomb, and consists of a strongly made metal vessel fitted with a gas-tight cover B. The measured quantity of fuel is placed in a platinum crucible C held in a stiff wire loop D. Ignition is accomplished electrically by means of a fine iron wire F, supplied with current through leads D and E. After closing the bomb, oxygen under a pressure of 20 atmospheres is passed into it through the pipe G; the valve H is then closed and the pipe G is disconnected. The

bomb is then lowered carefully into a vessel K containing a measured quantity of water and fitted with stirrers L which are rotated by hand. The temperature is observed by means of a delicate thermometer T. Another vessel M surrounds K, the space between forming

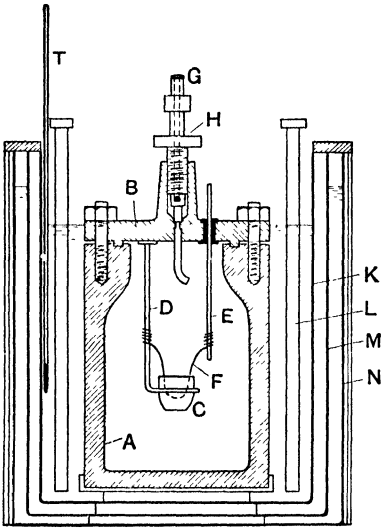


FIG 369.—Section of a bomb calorimeter.

an air jacket; the vessel N surrounds M, and the space between is charged with water; a flannel jacket is wrapped round the outside of N.

After ignition, the stirrers are operated until the temperature ceases to rise. The heating value of the fuel is then calculated in the same manner as in the Darling calorimeter (p. 362).

EXERCISES ON CHAPTER XXVII.

1. Find the mechanical energy given out when one horse-power is maintained for one hour. Find the heat equivalent of this energy. State the result in lb.-deg.-Cent., lb.-deg.-Fah. and calories.

2. Give a brief account of the evidence that we have for the statement that heat is a form of energy.

3. A tank contains 4 gallons of water, and is fitted with a stirring arrangement which takes 0·28 horse-power to drive it. Supposing that all the work done is converted into heat and retained by the water, how long will it take to raise the temperature of the water from 15° to 25° C.?

4. A train has a mass of 200 tons, and reduces speed from 40 to 30 miles per hour by application of the brakes. Assume that the whole of the work done against the frictional resistance of the brakes is converted into heat, and find this heat. State the result in Centigrade heat units.

5. Give a brief description, with sketches, of Joule's water-stirring experiment for the determination of the mechanical equivalent of heat.

6. Give sketches and describe the Callendar machine, or any other laboratory appliance you are acquainted with for determining the value of J.

7. Explain carefully how to apply cooling corrections in a calorimetric experiment.

8. One pound of coal has a heating value of 8000 lb.-deg.-Cent. By means of suitable machinery 500 gallons of water can be raised to a height of 100 feet for each pound of coal burned. What percentage of the heat contained in the coal is converted into useful work ?

9. Name the principal solid fuels used in practice, and briefly describe each.

10. Give a list and a brief description of the principal gaseous fuels.

11. One ton of coal costs 22 shillings and has a heating value of 8000 lb.-deg.-Cent. units per pound. One gallon of petrol costs 2 shillings and has a heating value of 10,800 lb.-deg.-Cent. units per pound ; a gallon of petrol weighs 7 3 lb. Lighting gas has a heating value of 300 lb.-deg.-Cent. units per cubic foot and costs 3 shillings per 1000 cubic feet. Which of these fuels gives the best heat value ? Answer this question by finding for each fuel the heat obtained for one penny

12. How much heat is available by the combustion of 100 cubic feet of hydrogen having a heating value of 34,500 lb.-deg.-Cent. units per pound ? Take the weight of one cubic foot of hydrogen as 0·0056 lb.

13. Describe any experiment for the determination of the heating value of a given sample of coal.

14. Answer Question 13 in relation to a given combustible gas.

15. Answer Question 13 in relation to a sample of liquid fuel. The calorimeter described must be different from that chosen in answer to Question 13.

16. Describe one method of determining the mechanical equivalent of heat.
A calorimeter of copper (specific heat 0·095) weighs 122 grams. It contains 1680 grams of aniline oil (specific heat 0·5). The liquid is stirred by a rotating paddle which requires a couple of moment 10^8 dyne cm. to drive it. The temperature of the liquid is raised 8° C. after 450 revolutions. Calculate the mechanical equivalent of heat. L.U.

17. One gram of coal was burned in a Darling calorimeter containing 1400 grams of water. The water equivalent of the calorimeter is 282 grams. The observed rise in temperature was 4° C. Find the heating value in lb.-deg.-Cent. units per pound of coal.

18. In a test with a bomb calorimeter, 0 742 gram of petroleum was burned. The calorimeter contained 2000 grams of water and had a water equivalent of 720 grams. The rise in temperature was 2·98° C. Find the heating value in lb.-deg.-Cent. units per pound of petroleum.

CHAPTER XXVIII

TRANSFERENCE OF HEAT

Conduction.—If heat be imparted to one part of a body, other parts in its neighbourhood have their temperatures raised owing to heat being passed on to them. Thus heat is passed through the body from layer to layer without alteration in the relative positions of the parts of the body. The process has not yet been investigated thoroughly, and is called **conduction.**

EXPT. 77.—**Conduction of heat along a wire.** Coat the surface of a copper wire with paraffin wax, and heat one end of the wire in a flame. The conduction of heat along the wire may be observed by the melting of the wax. If the wire is short, the end remote from the flame becomes in a short time too hot to be held in the hand.

Convection.—When heat is transferred from place to place by the actual motion of the hot body, the heat is said to be conveyed, and the process is called **convection.** In most cases, convection occurs automatically. Thus, in fluids the portions of the fluid adjacent to the source of heat become hot. Expansion takes place, and the density of the hotter fluid becomes smaller than that of the colder portions of the fluid. Motion is then set up under the action of gravity, and the hotter portions of the fluid move away from the source of heat, thus permitting colder portions to approach the source and to become heated in turn. Thus heat is transmitted in convection by reason of currents of fluid approaching and receding from the source of heat.

EXPT. 78.—**Convection currents in a liquid.** The apparatus shown in Fig. 370 is intended to illustrate the convection currents in an appliance sometimes fitted to steam boilers. A is a glass vessel to which is fitted a glass tube B closed at its lower end; the upper end of B is nearly flush with the bottom of A. Another tube C of smaller diameter, open at both ends, is suspended centrally in B, its lower end being at a small height

above the bottom of B, and its upper end being a little below the surface of water which is contained in A and fills both tubes. On applying a flame gently at the lower end of B, the water there is warmed and expands; the density decreases and an ascending current of warm water is established in the inner tube. At the same time a downward current of colder water passes from A through the space between the two tubes. Ultimately the whole of the water becomes heated by a source of heat applied at one part of the vessel only.

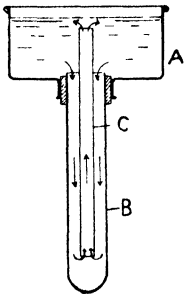


FIG 370.—Apparatus for showing the circulation of water due to convection currents

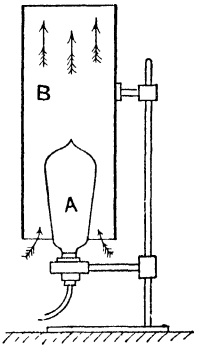


FIG 371.—Convection currents in a gas

EXPT. 79.—**Convection currents in a gas.** In Fig. 371, A is an ordinary incandescent electric lamp; B is a cardboard tube open at both ends. When the lamp is lighted, the air inside the tube near the lamp becomes heated and expands. Its density is thus diminished and an ascending current of air is established in the tube. The convection currents of air approaching the lower end of the tube and discharging from the upper end may be rendered visible by holding smouldering brown paper near the lower end of the tube. This experiment illustrates the action of an ordinary chimney, in which the upward draught is caused by the diminished density of the hotter air inside the chimney as compared with that of the colder air outside.

Radiation.—In **radiation**, heat is transferred from a source of heat to other bodies by a kind of wave motion in the ether, a medium which is assumed to fill all interstellar space as well as the spaces between the molecules of material bodies. The heat waves travel at very high speed; on arrival at a body they become absorbed, thus producing the ordinary effects of heat.

EXPT. 80.—**Radiation of heat distinguished from conduction and convection.** Hold the hand a few inches below a lighted incandescent electric lamp.

The sensation of warmth perceived cannot be due to either conduction or convection, since there are ascending currents of air in the neighbourhood of the heated lamp (Expt. 79). The hand is thus surrounded by colder air streaming *towards* the lamp, and hence cannot be affected thermally either by conduction or convection. Radiation alone can account for the sensation of warmth.

Thermal equilibrium.—A body is said to be in **thermal equilibrium** when it receives and gives out equal quantities of heat in the same interval of time. Any heat leaving the body is balanced immediately by the entrance of an equal quantity of heat, and the temperature of the body remains constant. It does not follow that a body at constant temperature is not exchanging heat with other bodies, but only that the exchanges are equal.

Temperature is the condition of matter which determines the direction in which the resultant flow of heat takes place. If a body A can exchange heat with another body B at a lower temperature, then the resultant heat flow always takes place from the body A at higher temperature to the body B. A certain quantity of heat passes per second from A to B, and a lesser quantity passes per second from B to A; ultimately both bodies arrive at the same temperature, when the exchanges of heat become equal.

The **theory of exchanges** may be explained by considering a body surrounded by an envelope. According to this theory, the body continually gives out heat at a rate which is independent of the temperature of the envelope, and depends solely upon the temperature of the body. Similarly the rate at which the envelope gives out heat depends upon its temperature and is independent of that of the body. Thermal equilibrium in both body and envelope is attained when the temperature of the body is equal to the temperature of the envelope.

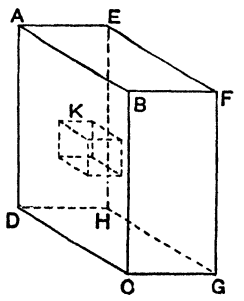


FIG. 372.—Conduction of heat through a plate.

Thermal conductivity.—Consider a large plate of a substance, having parallel faces ABCD and EFGH (Fig. 372). If the face ABCD be maintained at a temperature higher than that of EFGH, heat will flow by conduction through the plate in the sense from A towards E. The plate is supposed to be very large, so that the local effects of

heat escaping from the edges may be disregarded, and it may be assumed that sections of the plate taken parallel to the face ABCD have uniform temperature throughout.

Let all the conditions be steady, and consider a cube K (Fig. 372) of one centimetre edge, embedded in the plate and having two faces parallel to ABCD. The **coefficient of conductivity**, or the **conductivity** of the substance is defined as the quantity of heat flowing through the unit cube per second per degree difference in temperature of its opposite edges.

Let Q = the quantity of heat flowing per second through the unit cube.

t = the difference in temperature of the opposite faces of the cube.

C = the conductivity of the substance.

Then $$C = \frac{Q}{t}$$

Heat flow along an insulated bar.—In Fig. 373 is shown a metal bar AC, one end of which is heated by a bunsen burner. The portion

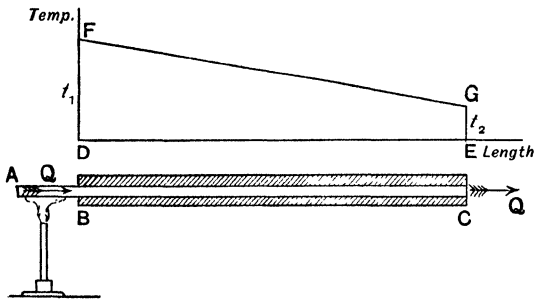


FIG. 373.—Conduction of heat along an insulated bar

BC is surrounded by non-conducting material, so that no heat entering this portion across the section at B can leave it otherwise than through the end C. Hence, if Q units of heat enter the bar at B, an equal quantity will leave it at C. Under these ideal conditions, the temperature will fall uniformly from B to C. The graph in Fig. 373 is drawn by taking DF to be the temperature of the bar at B, and EG the temperature at C. The straight line FG is drawn, and shows the temperature of the bar at any point in its length. The fall in temperature per unit length is called the **temperature gradient.**

Let Q = the heat flowing along the bar and discharged per second, in calories.
a = the sectional area of the bar, in sq. cm.
l = the length of the bar, in cm.
t_1 = the temperature at B, deg. Cent.
t_2 = ,, ,, ,, C, ,,
C = the conductivity of the material.

Then Temperature gradient = $G = \frac{t_1 - t_2}{l}$ deg. Cent. per cm. ..... (1)

Also, Heat flow per second per sq. cm. of section = $\frac{Q}{a}$.

$$\therefore C = \frac{Q}{a} \div \frac{t_1 - t_2}{l} = \frac{Q}{aG}, \quad \text{.....(2)}$$

or $$Q = \frac{Ca(t_1 - t_2)}{l} = CaG. \quad \text{.....(3)}$$

Equation (3) may be employed in calculating the heat transmitted through a plate, provided the temperatures t_1 and t_2 are known. It should be noted that there is difficulty in measuring the temperatures of the surfaces of a plate. Equation (2) indicates that experiments might be devised for determining the value of C for the material of a bar, but the difficulty of procuring a perfect heat insulator for surrounding the bar prevents the practical application of this method.

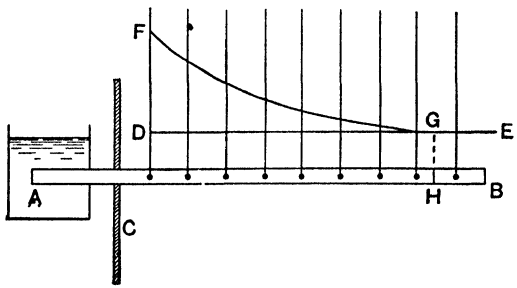


FIG. 374.—Conduction of heat along a bare bar.

Heat flow along a bare metal bar.—In Forbes's method of determining conductivity a bare metal bar AB is used (Fig. 374). The portion near the end A is maintained at constant temperature by being immersed in a bath of molten solder, and a screen C protects the remainder of the bar from being influenced by heat effects from the bath. The bar has a number of pockets, equally spaced, containing

mercury and fitted with thermometers. The temperature of the bar is raised above that of the atmosphere by the heat conducted along the bar towards the end B, and radiation takes place from the bare surfaces. The dissipation of heat which thus takes place is assisted by convection currents of air. The fall in temperature along measured lengths of the bar is thus greater than in the insulated bar discussed above. Ultimately, when conditions have become steady, and if the bar is long enough, there will be a section H at which the whole of the heat entering the bar at A has been dissipated into the surrounding atmosphere. The temperature of the bar between H and B will then be equal to that of the atmosphere.

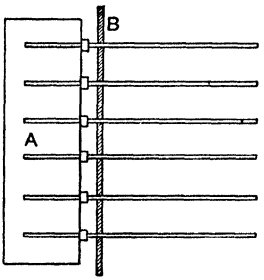


FIG 375.—Comparative conductivities

The temperature gradient may be shown by plotting the temperatures indicated by the thermometers (Fig. 374); the horizontal line DE represents the temperature of the atmosphere. The fall is more rapid than in Fig. 373, and atmospheric temperature is attained at G (Fig. 374).

A separate experiment is made on the rate of cooling of a short bar made of the same kind of material, thus determining the heat dissipated at any temperature per square centimetre of exposed surface of the longer bar. From this information and from the temperature gradient graph in Fig. 374, the conductivity of the material is determined.

Comparative conductivities.—The following experiment enables a comparison to be made between the conductivities of various metals:

EXPT. 81.—**Comparative conductivities by Ingen-Hausz's method.** Several rods of different metals are supplied. The rods are all of equal diameters and lengths, and their surfaces should be in the same state as regards polish. A trough A, shown in plan in Fig. 375, is also supplied; one end of each rod can be inserted through a hole in the side of the trough; the trough contains water which is brought to boiling, and a screen B protects the rods from the action of the bunsen flames.

Coat each rod by dipping it into a bath of melted paraffin wax; when the coating has solidified, insert the rods in the trough. Bring the water to boiling; melting of the wax will take place over lengths of the rods which will depend upon their conductivities. When the water has been boiling for 15 minutes, measure the length of each rod from which the

wax has melted; the conductivities are proportional to the squares of these lengths. Thus

$$C_1 : C_2 : C_3, \text{ etc.} = l_1^2 : l_2^2 : l_3^2, \text{ etc.}$$

Express the conductivities of the rods in terms of the conductivity of the copper rod.

The relative value of different heat insulators may be examined by means of the following experiment:

EXPT. 82.—**Comparative value of heat insulators.** A number of copper or aluminium vessels about 500 c.c. in capacity are prepared by fixing short pieces of metal tube about 2 cm. in bore to the centres of the top covers. This permits of thermometers being inserted. The vessels are completely covered, one with flannel, another with cotton-wool, another with felt, a fourth with asbestos, or any other heat insulator available. It is an advantage to have two other similar vessels, one having its bare surface brightly polished, and the other having its surface coated with lampblack. Arrange all these vessels on the bench in a place free from draughts, pour equal quantities of hot water into each through the top tube, using a funnel and being careful that no water is spilled over the insulating material. Insert thermometers, and read the temperatures at intervals of 5 minutes.

Plot temperatures and time for each vessel on a single sheet of squared paper. The resulting graphs will indicate the relative values of the various heat insulators employed, those which have the steeper curves being the poorer heat insulators. Make a list of the substances arranged in order of merit.

The vessels having polished and blackened surfaces should be specially noted; these are cases of radiation, and the results for them indicate that a polished surface provides a better heat insulator than a blackened one.

Conductivity of liquids.—The conductivity of liquids has been determined by a modification of the method of Ingen-Hausz. Experiments on liquids present some difficulty on account of the convection currents which may be set up, thus preventing conduction alone from being examined. That water is a poor conductor of heat is rendered evident by the following experiment:

EXPT. 83.—**Illustration of the poor thermal conductivity of water.** Tie a weight to a small piece of ice and sink it to the bottom of water contained in a test tube. Incline the tube, and apply a small bunsen flame near the surface of the water, thus preventing to a large extent the setting up of convection currents. It will be found possible to have the water boiling near the top of the tube whilst the ice is still unmelted at the bottom, thus showing that but little heat is conducted through the water.

Conduction of heat through a plate.—Consider the heating of water in a kettle or other metal vessel. One side of the bottom of the vessel is exposed to the flame, and the other side is in contact with the water. No part of the metal bottom, however, has a temperature anywhere approaching that of the flame. This may be shown by sticking a strip of paper on the outside of the bottom, when it will be found that the water may be boiled without charring the paper. The experiment indicates the existence of a thin film of comparatively cold gas in contact with the metal plate and practically at rest. It has been shown that the thickness of this film is approximately $\frac{1}{40}$ inch.* Its existence may be further confirmed by the experiment of boiling water in a paper bag.

We may therefore conclude that the temperature of the plate nowhere greatly exceeds the temperature of the water. A large drop in temperature occurs in the film of gas in contact with the plate. Gases are very poor thermal conductors, and this drop in temperature is necessary in order to cause the heat to be conducted through the film.

On the water side of the plate there is a similar film of water in contact with the plate and adhering thereto. The bulk of the water is heated by convection currents, but no such currents exist in this film. Heat is transmitted across it by conduction, but, as water is a better conductor of heat than gases, the drop in temperature is much less than in the gas film.

The transmission of heat from the flame into the water therefore involves a very large fall in temperature in the gas film, and comparatively trifling falls in the plate and the water film. In fact, the transmission is affected to a small degree only by the lack of perfect conductivity in the metal plate.

Methods of increasing the transmission of heat through a plate.—It will be evident—and practical experience and experiment confirm the impression—that a much larger quantity of heat may be transmitted if the mass of hot gases be projected forcibly as a strong current against the plate. The effect is partially to remove the film of gas adhering to the plate. Thus this surface of the plate becomes raised to a higher temperature, and more heat is conducted. Rapid circulation of the water by artificial means on the water side will also assist the heat transmission by partially removing the adhering film of water. Thus we infer that in steam boilers the quantity of heat

* "Heat Transmission," Prof. W. E. Dalby, *Proc. Inst. Mech. Eng.* 1909.

transmitted per square unit of plate surface will be increased very largely by scrubbing both the hot gases and the water vigorously against the surfaces of the plate.

The student will now understand the impossibility of arranging experiments on the determination of the conductivity of a metal by having a plate with, say, boiling water on one side and ice on the other. It is quite impossible to state the real temperatures of the plate surfaces, and without this information the conductivity cannot be estimated.

Effects of oil and scale on heat transmission.—If one side of a thin plate be exposed to a flame and the other side coated with a poor thermal conductor, the temperature of the plate may approach more nearly to the temperature of the flame. An illustration of this occurs in an ordinary frying-pan The oil used for frying is a very poor conductor, and the bottom of the pan is raised to a much higher temperature than would be the case if water were in the pan. The fact is evidenced by the comparatively rapid burning of the metal and the consequent formation of holes in the pan. For this reason oil must on no account enter a steam boiler.

Many waters contain solids in solution, and when the water is evaporated the solids remain in the vessel and adhere to the plates, forming scale. Such scales are often very hard and are very poor thermal conductors, leading to burning of the plates. Steam boilers require periodic cleaning in order to remove the scale.

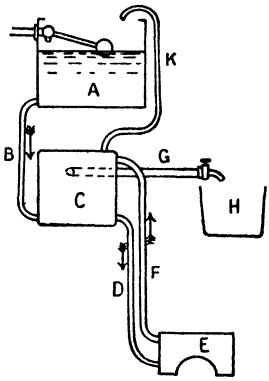


FIG. 376.—Domestic hot water supply.

Hot water supply.—In Fig. 376 is illustrated the method in general use for supplying hot water to lavatory taps. A is an open cold water tank from which the general supply of cold water is obtained, and is connected to a closed hot water storage tank C by a pipe B, which enters C near the bottom. The tank C is connected by a pipe D to a boiler E, which is usually placed at the back of the kitchen fireplace. The pipe D is connected to both C and E as low down as possible. Another pipe F is connected to the boiler near the top, and leads to the top of the storage tank C. The pipes D and F are called circulating pipes. Another pipe G leads from the upper part of C to the bath tap at H, and may have branches leading to other

taps at different parts of the house. K is a pipe for returning to A any water which may be thrown upwards from C by reason of ebullition or other causes, and also to get rid of air from the system.

In working, the water in the boiler becomes heated, and its density is lowered. Convection currents are set up and warmed water leaves the boiler, ascends the pipe F and enters C; meanwhile a further supply of cold water travels downwards from C through D, and enters the boiler to be heated in turn. After a time it will be found that the water in the upper part of C has become hot. The colder, heavier water accumulates at the bottom of C; hence the reason for the cold water supply pipe B being connected to the lower part of C, and also for the tap supply pipe being connected near the top of C If the tap at H be opened, hot water will be drawn from the upper part of C. and an equal quantity of cold water will flow from A through B into the lower part of C.

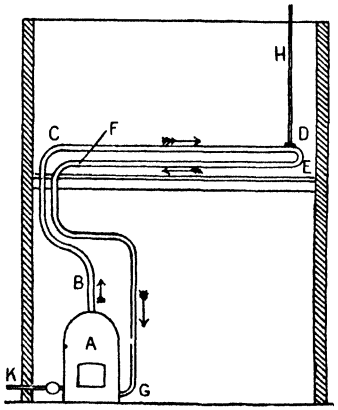


FIG. 377.—Arrangement for heating a building by hot water

Heating buildings by hot water circulation.—The arrangement is shown in outline in Fig 377. A is a boiler completely filled with water and generally situated in the basement of the building. A pipe B leads the heated water into the room or rooms to be heated, where it travels along the pipe CD, then back along EF, giving up part of its heat to the air in the room The transference of heat from the hot water to the room is effected by conduction of heat through the metal of the pipes, then principally by convection currents induced in the air in the neighbourhood of the pipes; radiation of heat from the surfaces of the pipes plays a comparatively small part. The water is returned to the lower colder part of the boiler at G. H is an air pipe leading up to the roof, and allows air to escape from the system; a small automatic air valve is often substituted for this pipe. and closes when the pipe CD becomes hot. Further supplies of cold water may be admitted as required into the boiler by means of the valve at K. Cold air may be brought into the room through openings in the walls behind the pipes CD and EF, and is carried by convection currents throughout the room.

The heating surface may be made greater by connecting to CD, at intervals, groups of short vertical pipes; these are called **radiators**, although their action is principally due to the increase in surface from which convection currents arise.

The system described above is employed also for heating greenhouses. Since it is possible to maintain the temperature of a glass house in the neighbourhood of 15° C., it may be inferred that glass is not a very good conductor of heat, nor does it permit radiant heat to pass easily. This fact is further confirmed by the possibility of holding in the hand an ordinary drinking glass containing liquid hot enough to scald the hand

Atmospheric circulation.—Circulation of the atmosphere—evidenced by winds—is caused by the air at one place being at a higher temperature, and thus having a lower density, than the air at adjacent places. Convection currents are set up by the warmed air ascending and colder air taking its place. The circulation of the atmosphere on the large scale is produced by reason of the higher temperature in equatorial regions. Lower currents of colder air flow from the temperate regions towards the equator, and upper currents of heated air travel away from the equator When very large bodies of air are in motion, the directions of the currents are modified by the rotation of the earth. A current of air which would otherwise flow from the north to the equator along a meridian will reach a point on the equator westwards of the place over which it would have passed had the earth been at rest. This is owing to the motion of the earth's surface from the west towards the east, and also to the fact that the whirling velocity of the air accompanying the earth is lower at higher latitudes than the velocity at the equator. The result is a wind coming from the north-east instead of from due north in the northern hemisphere, and from the south-east in the southern hemisphere instead of from due south. These motions of the air towards hot equatorial regions are known as the **trade winds.**

Land and **sea breezes** occur with great regularity in hot countries, and are caused by the greater specific heat of the sea compared with that of the land. During a hot day the land acquires a temperature considerably higher than that of the sea, but falls in temperature much more rapidly at night. The effect is to cause ascending currents of hot air from the land during the day, while cooler air comes from the sea to take its place, producing a **sea breeze.** The

effect is reversed at night, when the land cools quickly to a temperature lower than the sea, and a **land breeze** blows. Winds, such as the **monsoons** in the Indian Ocean, may be produced in this way, and extend in one direction or the other for definite seasons. The conditions required are a hot continent and a cooler ocean for sea breezes, produced in summer; and a cool continent and a hotter ocean for land breezes, which predominate in winter.

Owing to heating of small portions of the earth's surface, local ascending currents are often set up. It is difficult to make observations of these currents; in India the presence of such currents may be detected at certain times during the day by the circling and soaring flight of birds. The bird finds an ascending current and keeps circling in it without flapping its wings, with the result that it gradually ascends without any apparent expenditure of energy on its part.

EXERCISES ON CHAPTER XXVIII.

1. State the three ways in which heat may be transferred from one point to another, and give examples of each.

2. Give a sketch and explain the working of the ordinary household system for the supply of hot water to lavatory taps.

3. Describe briefly a method of heating a building by means of hot water. Give a sketch of the arrangement.

4. Give a brief account of the cause of winds. Explain the phenomena of trade winds, land and sea breezes and monsoons.

5. Define "thermal equilibrium"; state clearly what is meant by "the temperature of a body." One end of a long bare copper bar is maintained at a temperature 10° C. higher than that of the surrounding atmosphere. Describe clearly what is occurring at various points along the length of the bar.

6. Mention two cases in which it is advantageous to employ good thermal conductors, and other two cases in which it is advisable to employ poor thermal conductors. Give reasons for the fitting of a copper bottom to an ordinary kettle. The bottom of a steel frying-pan burns very readily; explain the reason for this.

7. Calculate the quantity of heat which will flow per hour through an iron plate 1·25 cm. thick. State the result in calories per square metre of plate. The coefficient of conductivity is 0·14, and one face of the plate is at a temperature of 10° C. higher than that of the opposite face.

8. Answer Question 7 in the case of a copper plate of the same thickness. The coefficient of conductivity is 0·91.

9. A current of gases at a temperature of 500° C. flows along one face of a wrought-iron plate 0·4 inch thick, and there is water at 100° C. in

contact with the opposite face. It is found that 5000 Centigrade heat units pass through each square foot of the plate per hour. If the coefficient of conductivity is 0 14, find the temperature of the face of the plate which is exposed to the hot gases. Account for this temperature being much lower than that of the hot gases.

10. Describe how the conductivity of a metal bar may be determined by Forbes's method.

11. Describe experiments by means of which the comparative conductivities of a number of metal rods may be found.

12. Describe briefly the passage of heat through the plates of a boiler; what must be done in order to obtain and maintain the best efficiency of transmission?

13. A room has a glass window 2 metres high, 1 metre wide and 7 mm. thick. The room is kept at a temperature of 15° C., and the temperature outside is 0° C. Assume that the temperatures of the two surfaces of the glass are 10° and 2° C. respectively, and calculate the quantity of heat which will pass through the glass per hour. The thermal conductivity of the glass is 0 0005.

14. Define thermal conductivity, and explain how its value may be determined in the case of copper. L.U.

15. Why is it difficult to make accurate measurements of the conduction of heat in liquids? Describe two experiments to illustrate the low thermal conductivity of water. Sen. Cam. Loc.

16. Heat is conducted through a slab composed of parallel layers of two different materials, of conductivities 0·32 and 0·14, and of thicknesses 3·6 cm. and 4·2 cm. respectively. The temperatures of the outer faces of the slab are 96° C. and 8° C. Find the temperature gradient in each portion. L.U.

CHAPTER XXIX

TRANSFERENCE OF HEAT—*CONTINUED*

Thermal radiations.—Radiation waves are given out by all bodies at all temperatures If a piece of metal be heated in a darkened room, the waves at first are comparatively long and the periods of vibration great; the eye is unable to detect such waves and the body is invisible As the temperature of the body is raised, the waves become shorter and the vibrations more rapid, and a point is reached at which the eye is able to detect the effects; the body is then luminous. The only difference between the phenomena of light and radiant heat is that the eye is unable to perceive any effect until the temperature of the body has been raised sufficiently; the laws of transmission are the same.

The laws of transmission of light will be found in a later section of this book. A brief statement of some of the laws of radiant heat is given here in order that reference may be made to the special methods of examining thermal radiations which may or may not be luminous.

No material substance is necessary for the transmission of thermal radiations.—This fact is evident from the consideration that heat is transmitted from the sun to the earth through space containing no ponderable matter. On a smaller scale, it has been illustrated by Sir Humphry Davy, who found that a platinum wire heated in a vessel from which the air had been extracted was capable of influencing thermometers placed outside the vessel. Another illustration is provided by the heating of the glass bulb of an incandescent electric lamp. An ordinary mercurial thermometer is not a very delicate instrument for the detection of thermal radiations, but may be made more sensitive to them by coating the bulb with dull black paint. As we shall see presently, a blackened body absorbs thermal radiations better than one having a polished surface.

Thermal radiations are transmitted with the same velocity as light.—This fact has been proved by observations taken during total eclipses of the sun. Both the light and heat radiations from the sun are cut off simultaneously; hence the velocity of propagation of heat radiations is the same as that of light, viz. about 186,000 miles per second. That the heat radiations from the sun are arrested during an eclipse also affords evidence that **thermal radiations take place in straight lines.** The heat rays are unable to bend round the moon, and a heat shadow is produced, corresponding to and coinciding with the light shadow (Chap. XLI).

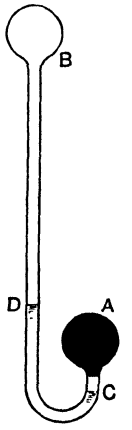


FIG 378.—Ether thermoscope.

Ether thermoscope.—The ether thermoscope (Fig. 378) affords a simple means of detecting thermal radiations. Two glass bulbs, A and B, are connected by means of a bent glass tube. Air is withdrawn entirely and some ether is introduced; the apparatus thus contains ether and ether vapour only. The bulb A is coated with dull black paint. Ether is a very volatile liquid, and any thermal radiations falling on and being absorbed by the bulb A will be rendered evident by an increase in the pressure of the vapour contained in this bulb. Consequently the level of the liquid ether at C will fall and that at D will rise.

Thermopile.—This instrument affords a sufficiently delicate means of indicating thermal radiations in many laboratory experiments. It consists of a number of bismuth and antimony bars arranged as shown in Fig. 379, in which bismuth and antimony bars are marked B and A respectively. The bars B_1 and A_1 are soldered together at their front ends, and are otherwise insulated by mica sheets. A_1 and B_2 are soldered together at their back ends, and are insulated elsewhere. The other bars are connected in similar fashion, thick lines representing insulation. A_2 and B_3 are soldered together at their back ends, as are also A_4 and B_5, also A_6 and B_7. An external electric circuit containing a galvanometer is completed by connecting wires to C and D (bars B_1 and A_8). If the front ends of the bars be

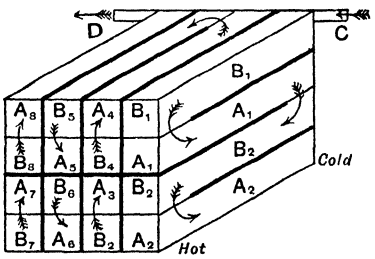


FIG. 379.—Arrangement of bars in a thermopile.

heated, an electric current will flow at the front junctions from each bismuth bar to each antimony bar, and at the colder junctions at the back end the current will flow from the antimony bars to the bismuth bars. Thus a current flows from C down the first pile of bars, then from A_2 to B_3 and up the second pile, passing from A_4 to B_5 the current passes downwards through the third pile, then upwards through the left-hand pile and makes its exit at D. The arrangement gives a cumulative effect to the electromotive force, and makes it much greater than could be secured by the use of a single pair of bismuth and antimony bars. Thus for a given difference in temperature between the two ends of the thermopile, the current is greater than could be obtained by use of a single pair of bars. The ends of the bars are blackened so as to absorb radiant heat more readily. For a fuller discussion of the theory of the thermopile, the student is referred to the Part of the volume devoted to Electricity.

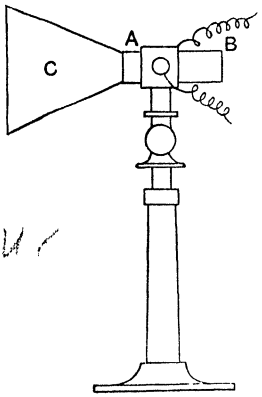


FIG 380.—Thermopile.

The pile is mounted as shown in Fig. 380. AB is the pile clamped to a stand which can be adjusted in height. Each end of the pile can be covered by a brass cap so as to prevent radiant heat reaching it. A metal cone C can be attached to one end of the pile; this cone screens the end of the pile from radiant heat coming from bodies other than that being tested, and also shields it from air currents.

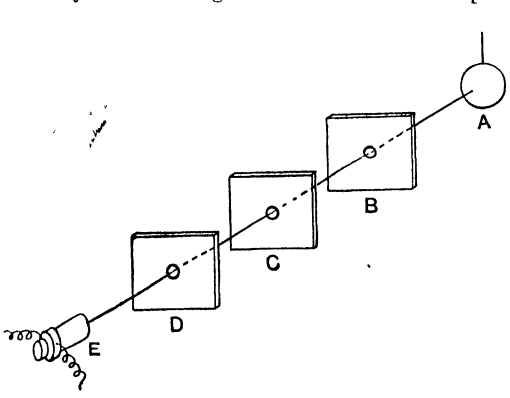


FIG 381.—Rectilinear transmission of thermal radiations.

EXPT. 84.—Thermal radiations are transmitted in straight lines. In Fig. 381, A is a source of radiant heat, such as an iron ball heated to whiteness, or an electric lamp. B, C and D are bright tin screens, each pierced with a hole. E is a thermopile. Arrange

the screens so that the holes are in a straight line. Thermal radiations can then pass from A to the thermopile, and are indicated by the galvanometer. If any one of the screens be displaced somewhat so as to put the holes out of line, it will be found that the galvanometer no longer indicates the reception of radiant heat by the thermopile.

Reflection of radiant heat.—The law of reflection of radiant heat from a plane surface is the same as the law of reflection of light, viz. the angle of incidence is equal to the angle of reflection (Chap. XLIII). This may be proved by the following experiment:

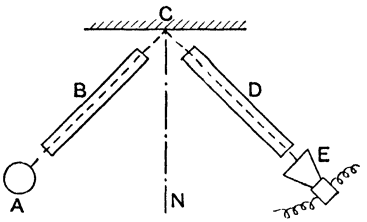


FIG. 382.—Reflection of radiant heat.

EXPT. 85.—**Reflection of radiant heat.** In Fig. 382, A is a source of heat and B and D are tin tubes arranged in plan as shown. E is a thermopile, and C is a polished tin reflector. Remove the reflector, when it will be found that no indication of thermal radiation is given by the thermopile. Replace the reflector, and adjust it so that the thermopile gives maximum indication. It will then be found that the reflector is so situated that the angle of incidence ACN (CN is normal to the reflector) is equal to the angle of reflection ECN.

Refraction of radiant heat.—Radiant heat rays passing from one medium to another, *e.g.* from air to glass, change direction unless the incident ray is normal to the surface of the medium which the ray is about to enter. The law followed is the same as that of light rays, viz. the ray after entering the medium is inclined at a smaller angle to the normal (Chap. XLV). This action is called **refraction**, and becomes evident in the case of heat rays by the well-known application of the burning glass. An ordinary lens refracts sun rays to a point called the focus, and a piece of paper placed at the focus becomes scorched, showing that heat rays have been refracted. Window curtains have sometimes been ignited by refraction of radiant heat rays passing through a globe of water placed near the window.

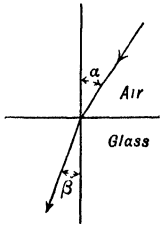


FIG. 383.—Refraction of heat rays.

If the solar spectrum, obtained by the refraction and dispersion of light through a prism, be examined by means of the thermopile,

it is found that the heating effect increases greatly as the red end of the spectrum is approached. But little heat is indicated at the violet end. Beyond the red end of the spectrum, where no light is visible, the greatest heating effect is obtained (Chap. XLIX). Since red rays have a much longer wave-length than violet rays, it may be inferred that non-luminous thermal radiations are of very long wave-length.

The inverse square law.—The quantity of heat transmitted by thermal radiations from a source of radiant energy and received by a surface of given area is inversely proportional to the square of the distance between the source and the surface

EXPT. 86.—**Proof of the inverse square law.** In Fig. 384, A is a large Leslie's cube, consisting of a copper box containing water which may be

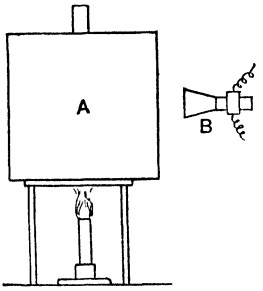


FIG 384.—Leslie's cube.

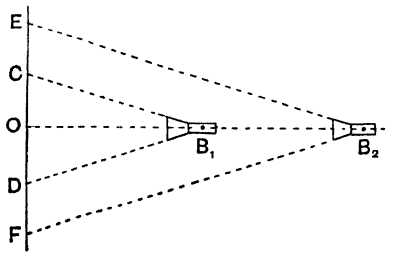


FIG 385.—The inverse square law

maintained at boiling point by means of a bunsen burner. B is a thermopile arranged to receive heat radiated from one of the sides of the cube, and fitted with a conical hood. There is thus a cone of heat rays having its apex at the thermopile, and its base falls entirely on the side of the cube provided that the distance of the thermopile from this side is not too great. On varying the distance of the thermopile from the side of the cube, it will be found that the galvanometer readings remain constant, showing that the quantity of heat received by the thermopile does not vary.

With the thermopile at B_1 (Fig. 385), the diameter of the base of the cone of rays is CD ; on moving the thermopile to B_2, the diameter is EF. The action of the conical hood is to make the cones B_1CD and B_2EF similar ; hence the areas of their bases are in the ratio of OB_1^2/OB_2^2, and therefore the heating surface emitting rays which reach the thermopile varies as the square of the distance ; hence we infer that the quantity of heat reaching the thermopile from any unit area of the side of the cube varies inversely as the square of the distance.

Radiating power of a source of heat.—It is found that different surfaces maintained at the same temperature have varying powers of radiation. The **radiating power** of a given surface depends upon the nature of the surface and also upon the temperature, and is generally expressed in terms of a surface coated with lampblack, which is taken as 100.

EXPT. 87.—**Radiating powers of different surfaces.** Use a Leslie's cube having one side lampblacked, another side polished, another side dull copper and the last side papered. Water is kept boiling gently in the cube A, and each side is presented in turn at the same distance from a thermopile B (Fig. 386). A delicate reflecting galvanometer C should be used, and a resistance box D should be inserted in the circuit.

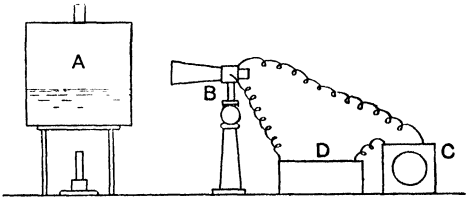


FIG. 386.—Determination of the radiating powers of different surfaces

With the cube removed or screened from the thermopile, adjust the galvanometer and scale so that the reading is zero. Then present the lampblacked surface to the thermopile, and adjust the resistance so as to obtain as large a galvanometer deflection as possible. Wait until the scale reading is steady and note it. Make similar observations with the other surfaces of the cube, taking care in each case that the base of the cone of rays falls entirely on the surface, and to preserve unaltered the resistance of the circuit. The deflections of the galvanometer so obtained are proportional to the radiating power of the surfaces.

The following experimental record indicates how the observations may be entered and reduced:

EXPERIMENT ON RADIATING POWERS.

Surface.	Galvanometer deflections.	Radiating power.
Lampblacked -	442	100
Polished - -	212	$\frac{212}{442} \times 100 = 47\cdot9$
Dull copper - -	348	$\frac{348}{442} \times 100 = 78\cdot7$
Papered - -	395	$\frac{395}{442} \times 100 = 89\cdot2$

The **emissivity** of a given surface is generally defined as the quantity of heat given out per unit area in one second per unit temperature excess between the surface and its surroundings. It will be noted

from the above experiment that polished surfaces emit much less radiant heat than blackened surfaces. Advantage is taken of this fact in the construction of the vacuum vessel devised by Dewar for storing liquid air and other very cold liquids, and in **thermos flasks**. A double-walled glass vessel has the space between the walls exhausted of air, and the walls are silvered. The vacuum minimises conduction effects, and the mirror surface reduces radiation to a minimum; hence such vessels keep practically constant the temperature of hot or cold liquids for a considerable period of time.

Diathermancy.—When thermal radiations pass into a substance, some of the heat may be absorbed, as is evidenced by the temperature of the substance rising; the remainder of the heat passes through the substance and emerges in the form of thermal radiations. The quantities of heat respectively absorbed and transmitted depend on the nature of the substance and on the wave-length of the thermal radiations. Substances which can transmit thermal radiations of a given wave-length are said to be **diathermanous** for those radiations. Substances which are unable to transmit thermal radiations of a given wave-length, but absorb them, are said to be **athermanous** for these thermal radiations.

Glass affords an example of a substance which is diathermanous to one kind of thermal radiations and athermanous to another kind. Radiations from the sun pass through glass with but little reduction, and the glass remains cool; hence the high temperature inside a green-house exposed to the sun's rays. On the other hand, thermal radiations from an ordinary fire are not well transmitted by glass, a fact which makes glass a valuable material for fire-screens. A glass fire-screen placed in front of a fire absorbs most of the thermal radiations and becomes very hot. Rock-salt is diathermanous to most non-luminous thermal radiations.

In making experiments on diathermancy, care must be taken to state the character of the thermal radiations; in particular, the wave-length should be known, otherwise it is impossible to interpret the results.

A substance which shows considerable absorbing powers for a given kind of thermal radiations also exhibits correspondingly great powers of radiating the same kind of radiations. Thus a blackened surface both emits and absorbs radiant heat freely. A china plate having a white ground and a dark pattern has this appearance to the eye because the darker portions absorb light radiations and the white portions scatter them. If the plate be heated to a high temperature, it will be found that the initially dark portions have

become bright, and the formerly white portions are now dark, showing that those portions which are good absorbers are also good radiators.

The **transmitting power** for thermal radiations of a given substance is the ratio of the thermal radiations transmitted through a unit cube of the substance to the thermal radiations incident normally on one face of the cube. Thus :

$$\text{Transmitting power} = \mathsf{T} = \frac{\text{radiation transmitted}}{\text{radiation incident}}.$$

EXPT. 88.—**Transmitting powers of different substances.** Use the Leslie's cube and a thermopile arranged as in Expt. 87 (p. 385). Observe the galvanometer deflection and let this be θ_1. Clamp one of the substances to be tested between the cube and the thermopile so as to intercept the thermal radiations. Again read the galvanometer deflection, θ_2, say. Measure the thickness t cm. of the substance. Then

$$\text{Transmitting power} = \mathsf{T} = \left(\frac{\theta_2}{\theta_1}\right)^{\frac{1}{t}}$$

In this manner determine the transmitting powers of colourless, blue and red glass.

The above equation may be obtained as follows: Suppose that the galvanometer deflection is θ when no substance is interposed between the source of thermal radiation and the thermopile, and that the deflection is θ_1 when a plate 1 cm. thick is interposed. Then

$$\mathsf{T} = \frac{\theta_1}{\theta}.$$

If the quantity of heat incident on the plate is Q, then the quantity transmitted will be

$$\mathsf{QT} = \mathsf{Q}\,\frac{\theta_1}{\theta}.$$

Suppose that the heat emerging from this plate is received by a second plate of unit thickness in contact with the first plate (Fig. 387), then the heat transmitted by the second plate will be

$$\mathsf{QT} \times \mathsf{T} = \mathsf{QT}^2.$$

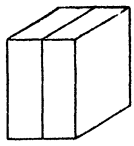


FIG 387.

If the galvanometer now gives a deflection of θ_2, the heat transmitted will be $\mathsf{Q}\,\frac{\theta_2}{\theta}$. Hence

$$\mathsf{QT}^2 = \mathsf{Q}\,\frac{\theta_2}{\theta},$$

or

$$\mathsf{T} = \sqrt[2]{\frac{\theta_2}{\theta}}.$$

In the same way, if three plates of unit thickness be employed, and if the galvanometer deflection is θ_3, then

$$T = \sqrt[3]{\frac{\theta_3}{\theta}}.$$

Hence, in general, if the thickness of the plate be t cm., and the galvanometer deflection θ_t,

$$T = \left(\frac{\theta_t}{\theta}\right)^{\frac{1}{t}}$$

Diathermancy of liquids and gases.—By substituting a cell charged with a given liquid for the plate in Expt. 88, the absorbing properties of the liquid may be examined. Experiments show that pure water is very opaque to radiant heat, and salt water almost equally so.

Tyndall, in his experiments on the diathermancy of gases and vapours, used a tube which could be charged with the gases to be examined, the ends of the tube were closed with plates of rock salt, which absorbed but little of the thermal radiations. The source of heat was placed at one end of the tube and a thermopile at the other end. The apparatus was rendered more delicate by having the thermopile fitted with a funnel at each end, one directed towards the experimental tube and the other towards a second source of heat which could be adjusted so as to nullify the thermal effects on the thermopile of the rays emerging from the experimental tube. The tube was first exhausted and the second source of heat arranged so as to bring the galvanometer needle to zero. The tube was then charged with gas and any deflection noted.

In this way it was shown that pure dry air, oxygen, nitrogen and hydrogen absorb but little radiant heat, olefiant gas and ammonia absorb a large quantity. If the heat absorbed by air is denoted by unity, that absorbed by ammonia is nearly 1200.

EXERCISES ON CHAPTER XXIX.

1. Give a brief explanation of the transmission of heat by radiation. Give any evidence you know of for the statement that no material substance is required in this method of heat transmission.

2. What evidence have we for stating that heat is radiated with the same speed as light?

3. Give sketches and describe the construction and action of a thermopile.

4. Describe an experiment showing that thermal radiations are transmitted in straight lines.

5. How would you show that the law of reflection of thermal radiations is the same as that of the reflection of light?

6. Explain how you would show that solar heat rays can be refracted.

7. Show how to prove experimentally that the radiant heat received by a given surface is inversely proportional to the square of the distance of the surface from the source of heat.

8. Describe how you would compare the thermal radiating powers of different surfaces.

9. Describe the construction and explain the principles involved in a Dewar flask.

10. Explain briefly the meaning of the term diathermancy. Give some illustrations. Describe how to compare the absorbing powers of two partially transparent substances for radiant heat.

11. What evidence have we for stating that good radiators are also good absorbers of heat ?

12. Describe how you would determine the transmitting power for thermal radiations of a given substance.

13. State the ways in which a hot body loses heat. Give some practical examples showing how these losses are made as small as possible.

14. Explain the heating of a green-house by the sun. Why are fire screens sometimes made of glass ?

15. A beam of radiant heat of a definite wave-length loses 0·12 of its energy in passing through 1 mm. of glass. What is the intensity of the beam after it has passed through 2·5 mm. of glass ?

16. Mention three facts bearing upon the similarity in character between light and radiant heat.

Describe generally the changes in character of the radiation from a body as it is raised from the ordinary temperature to white heat. L.U.

CHAPTER XXX

PROPERTIES OF GASES

Distinction between vapours and gases.—Substances in the gaseous state may exist either as **vapours**, or as **permanent gases**. The permanent gas was supposed at one time to remain in the gaseous state under all conditions of pressure and temperature, it is now known that all gases may be liquefied by means of increasing the pressure and diminishing the temperature. A vapour may be condensed by increase of pressure without reduction of temperature. The same substance, when far removed from the conditions of easy liquefaction, may be described as a permanent gas. Oxygen, hydrogen, nitrogen, air and several other gases are examples of permanent gases when under ordinary atmospheric conditions of pressure and temperature. Steam in an ordinary kettle containing some boiling water can be condensed by pressure alone, and is an example of a vapour.

Pressure of a gas.—As has been noted already (p. 353), the molecules of a substance in the gaseous state are in rapid motion and move about in all directions inside the vessel containing the gas. The continual bombardment by the molecules produces forces distributed over the walls, and the total force exerted on unit area is called the pressure of the gas. The pressure of the atmosphere is rendered evident by the barometer, an experiment on which is described on p. 259.

The height of the mercury column in a barometer at any given time depends on the atmospheric conditions then existing; 76 cm. at 0° C. is taken as a standard. The weight of mercury is about 13·59 grams per cubic centimetre; hence the standard height corresponds to a pressure of $76 \times 13{\cdot}59 = 1032{\cdot}8$ grams wt. per square centimetre, or to 1·0328 kilograms wt. per square centimetre. In reading the barometer it is customary to state the height of the mercury column only; the pressure is proportional to this height.

One atmosphere is often taken as a unit of gaseous pressure; it is the pressure produced by a mercury column 76 cm. in height at 0° C. From the above calculation it will be seen that a pressure of one atmosphere is equivalent roughly to one kilogram per square centimetre. In the British system one atmosphere of gaseous pressure is generally taken as equivalent to 30 inches of mercury (76·2 cm.), which is equivalent to a pressure of nearly 14·7 lb. per square inch.

In meteorology, the unit of pressure is the **megabar**, and is equivalent to a mercury column 750 mm. in height, at 0° C., at sea level in latitude 45°. One **bar** = 1 dyne per sq. cm.

As has been explained on p. 268, the pressure of a gas may be stated as so much above or below the pressure of the atmosphere as shown by a barometer at the time of observation; pressures so stated are called **gauge pressures**. The **absolute pressure** of a gas is measured from perfect vacuum, *i.e.* a space containing no gas and therefore having absolute zero of pressure. The absolute pressure may be calculated by adding the pressure of the atmosphere to the observed gauge pressure.

Forms of barometer.—In **Fortin's standard** barometer (Fig. 388) a screw A is fitted to the mercury cup so as to enable the level of the mercury in the cup R to be brought into coincidence with a fixed point P before taking the reading. The upper part of the case is furnished with a scale, the zero of which is at the same level as P, and a sliding vernier operated by means of a thumb-screw B. Mirrors are fitted behind the cup R and the scale S. To read the instrument, first adjust the level of the mercury in the cup R; bring the eye to the level of the top of the mercury column (the mirror aids this), and operate the screw B until the top of the vernier coincides with the top of the mercury column. Readings of the scale and vernier are then taken. The thermometer fitted in the case should also be read; a knowledge of the temperature permits of a correction being applied for the altered density of the mercury due to expansion

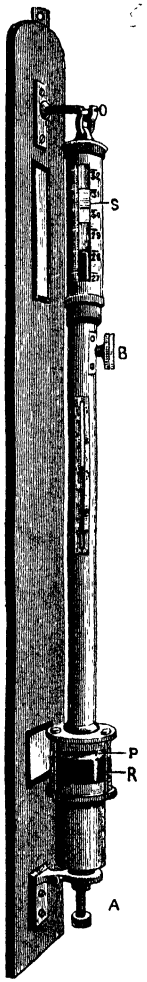


FIG. 388.—Fortin's barometer.

In the **aneroid barometer** (Fig. 389) the action depends on the movements of the flexible top and bottom of a circular closed box A under the changes of atmospheric pressure. A is fixed to the base-plate of the instrument, and is exhausted of air as thoroughly as possible; its top side is connected at B to a powerful spring C. C is held in a bracket which is secured to the base-plate at D, and has two bearing screws at E and F. A rod GH, fixed to the spring at G, communicates any rise or fall of the spring through the lever system HK, KL, LM to a fine chain MN, which is wrapped round the spindle carrying a pointer P. P moves over a graduated dial divided

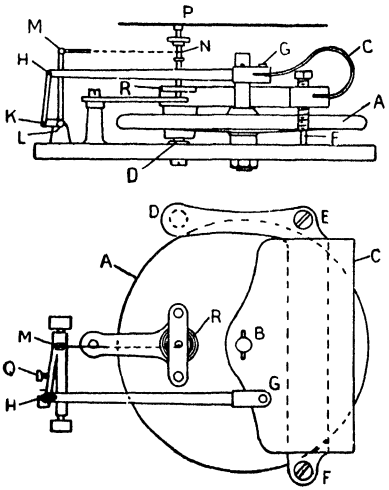


FIG 389.—Arrangement in an aneroid barometer

to show centimetres, or inches, of barometric height corresponding to the readings of a mercurial barometer. A hair-spring at R keeps the fine chain taut. Q is an adjusting screw whereby the effective length of the lever arm KL may be altered. E and F also serve as adjusting screws.

Aneroid barometers are liable to changes by reason of alterations in the elastic qualities of the metal of which the box A and the spring C are made. It is necessary to check them at frequent intervals by comparison with a mercurial barometer. A recording barometer or **barograph** may be constructed by connecting a lever carrying a pen to the rod GH; the pen draws a curve on a piece of paper wrapped round a drum which is driven by a clock. Ordinates on the resulting chart show barometric heights and the abscissae show time.

Errors in a standard mercury barometer.—The standard barometric height with which other barometric readings are compared is 76 cm. of mercury at 0° C., the instrument being at sea-level at latitude 45°. The observed barometric height is corrected as follows :

1. **Correction for the altered density of the mercury due to expansion** (Fig. 390).

Let h = the observed height in cm. at $t°$ C.

h_0 = the height in cm. which would produce the same pressure at 0° C.

d = the density of the mercury at $t°$ C.

d_0 = the density of the mercury at 0° C.

β_a = the coefficient of cubical expansion of mercury = 0·000181.

Then $hd = h_0 d_0$.

Also $d_0 = d(1 + \beta_a t)$, (p. 333) ;

$\therefore\ h_0 = h(1 - \beta_a t)$. .....(1)

The correction for the altered density of the mercury is therefore effected by multiplying the observed height by $(1 - \beta_a t)$.

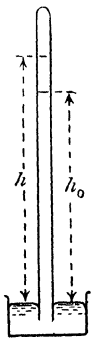


FIG 390

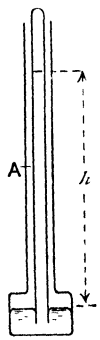


FIG 391

2. **Correction for the expansion of the scale.** In the Fortin barometer the scale is carried by a brass tube A (Fig. 391). On the temperature being raised, the length of this tube increases ; hence the height as shown by the scale will be too small at all temperatures higher than 0° C.

Let h = the observed height in cm. at $t°$ C.

h_0 = the height in cm. which would be observed if the scale were at 0° C.

α = the coefficient of linear expansion of brass = 0·000019.

The change in length of the tube A due to a fall in temperature from $t°$ to 0° C. is hat; hence

$$h_0 = h + hat = h(1 + at). \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (2)$$

Therefore the observed height must be multiplied by $(1 + at)$ in order to obtain the height free from errors due to the expansion of the scale.

The height h_0 corrected for the expansion of both mercury and scale may be obtained in one calculation Let h be the observed height, then

$$h_0 = h(1 - \beta_a t)(1 + at)$$
$$= h(1 + at - \beta_a t - \beta_a a t^2).$$

The term involving $\beta_a a$ may be neglected, as both β_a and a are very small quantities. Hence we may write

$$h_0 = h\{1 + t(a - \beta_a)\}$$
$$= h\{1 + t(0{\cdot}000019 - 0{\cdot}000181)\}$$
$$= h(1 - 0{\cdot}000162t). \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (3)$$

3. **Correction for the variations of gravitational effort on the mercury.** The value of the acceleration g due to gravity depends upon the latitude and upon the elevation above sea-level (p 35); therefore the weight of the mercury per cubic centimetre is not constant. If the barometer is situated at a height H metres above sea-level at a place in latitude λ, then the observed height may be corrected for latitude 45° by multiplying it by

$$1 - 0{\cdot}0026 \cos 2\lambda - 0{\cdot}0000002\mathrm{H}. \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (4)$$

This correction amounts to 0·13 mm. per 1000 metres above sea-level, to be deducted from the observed height.

4. **Correction for the vapour pressure of the mercury.** The pressure of the vapour of mercury in the tube above the column tends to depress the column. The correction is very small and amounts to an addition of $0{\cdot}0002t$ cm., where t is the temperature Centigrade.

5. **Correction for capillarity.** Surface tension tends to depress the mercury column; the effect is more marked in a tube of narrow bore. The method of cleaning the inside of the tube also influences the amount of depression. The correction is constant for a given instrument, and is best obtained by comparison with a standard barometer; it is usually of the order of 0·002 cm., to be added to the observed reading.

Types of pressure gauges.—A pressure gauge is an appliance for measuring the gaseous pressure inside a closed vessel. In general, pressure gauges indicate the difference between the gaseous pressure inside the vessel and the pressure of the atmosphere. If the difference is small, such as would be the case in a pipe conveying illuminating

gas, or in a boiler chimney, a simple form of **U** gauge suffices. A chimney gauge is shown in Fig. 392, and consists of a glass **U** tube A containing some water and having a scale attached by means of which differences in water level in the two limbs may be read. The tube is connected to an iron pipe BC, which passes into the interior of the chimney; the other limb of the tube is open. In action the superior pressure of the atmosphere causes a change in the surface levels of the water as shown. The difference in levels h is called the **chimney draught**, and is stated usually in inches of water. If the difference in levels is considerable, mercury may be used. The absolute pressure inside the vessel may be calculated by adding algebraically the barometer reading to the reading of the pressure gauge, first dividing the latter by 13·59 if water has been used in the gauge.

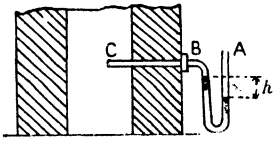


FIG. 392.—U pressure gauge.

Pressure gauges for indicating high pressures, such as that in a steam boiler, are usually of the Bourdon type. The action in these gauges depends on the tendency which a curved, partially flattened tube has to become straight when subjected to internal pressure.

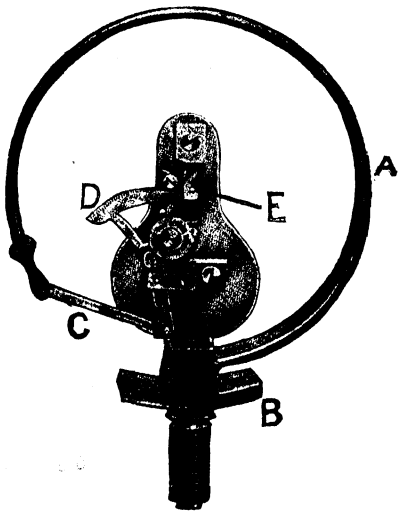


FIG. 393.—Interior parts of a steam pressure gauge.

EXPT. 89.—**Bourdon action.** Attach a piece of rubber tube about a yard long to a water tap; close the outer end by a clip, or a piece of glass rod; bend the tube into a curve lying on the table. Quickly open the water tap, when it will be found that the rubber tube, which has been slightly flattened by the bending, will distinctly show movement in the attempt to become straight.

The interior parts of a Bourdon pressure gauge are shown in Fig. 393. A is a flattened tube of hard, solid-drawn phosphor bronze

secured to a bracket B, which has passages in it forming the steam inlet to the tube. The free end of the tube is closed, and is connected by means of a short link C to a small-toothed sector D. The sector gears with a pinion on the spindle E, which carries a pointer travelling over a scale of pressures marked on the outside of the case.

Bourdon gauges show the difference between the pressure inside the vessel and the pressure of the atmosphere. If the pressure inside the vessel is lower than that of the atmosphere, the gauge is called a **vacuum gauge**, and the dial is graduated in inches or centimetres of mercury so as to facilitate the process of calculating the absolute pressure. The absolute pressure is obtained by deducting the vacuum gauge reading from the observed barometric height. For example, if a vacuum gauge reads 72 cm. at a time when the barometer stands at 75·8 cm., the absolute pressure inside the vessel is 3·8 cm of mercury.

Boyle's law.—This law has already been enunciated (p. 269), and is stated again for convenience of reference. **The absolute pressure of a given mass of any gas varies inversely as the volume, provided the temperature remains constant.** Taking a given mass of gas under conditions of absolute pressure and volume, p_1 and v_1, let these be changed, at constant temperature, to any other conditions, p_2 and v_2; then

$$p_1 : p_2 = v_2 : v_1,$$

or

$$p_1 v_1 = p_2 v_2.$$

The law may be written $pv =$ a constant.

Boyle's law is not followed by vapours, but is very closely obeyed by the permanent gases. A **perfect** gas is an ideal gas which is imagined to obey Boyle's law.

EXPT. 90.—**Verification of Boyle's law.** Reference is made to Fig. 394. A graduated burette A, of 100 c.c. capacity, is closed by a well-fitted tap at the top, and is connected to a reservoir of mercury B by means of a long piece of thick rubber tubing C. The burette scale is assumed to be so constructed as to show the volume of the gas enclosed between the tap and the surface of the mercury. The 100 c.c. scale division is near the lower end of the burette. If the scale is arranged otherwise, a preliminary experiment must be made in order to determine the volume between the highest scale division and the tap. The reservoir B may be clamped to a support at any convenient height, and has a small side branch D which travels vertically in close proximity to a scale E when B is raised or

lowered. The device enables the level of the mercury in the reservoir to be read on the scale. The level of the mercury in the burette may be read on the scale by employing a U tube containing some mercury and having both limbs open. The mercury levels F and G (Fig. 394) are in the same horizontal plane; the tube is held so that one limb is close to the burette and raised or lowered so as to obtain coincidence of mercury levels in this limb and the burette; the other limb is held close to the scale, and the scale reading at the mercury level in this limb gives the level of the mercury in the burette.

The burette is charged with dry air at atmospheric pressure in the following manner. Open the tap and raise B; the mercury level will rise in A, expelling the contents of the burette. Stop raising B when the mercury level in A is just below the tap. Now lower B slowly, when the mercury level in A will fall again, and air will be drawn through calcium chloride contained in the tube above the tap; the calcium chloride abstracts moisture from the entering air. Adjust the height of B and clamp it so that the level of the mercury in A is at the 100 c.c. mark; since the mercury is at the same level in both A and B, it follows that the air in A is at the same pressure as that of the atmosphere. Close the tap, thus isolating the contents of the burette.

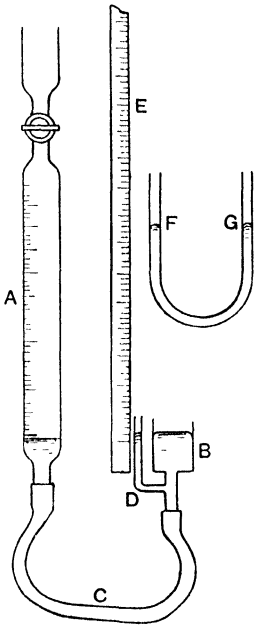


FIG. 394.—Apparatus for verifying Boyle's law.

The initial volume of the inclosed air is 100 c.c. and its absolute pressure may be obtained by reading the barometer; let this be h_1 cm. of mercury. The volume of the air in the burette may be diminished by raising B. The compression of the air may be accompanied by a rise in temperature, and the volume should not be read until two or three minutes have elapsed; the interval permits of the temperature of the air in the burette to fall again to the temperature of the room. It is advisable to have a thermometer attached to the burette; this will enable any alteration in the atmospheric temperature in the neighbourhood of the burette to be observed. Let v be the observed volume of the enclosed air in cubic centimetres.

The level of the mercury in B is now higher than that in A. Read each level on the scale and take the difference; let this be l cm. Since the pressure on the surface of the mercury in B is h_1 cm. of mercury, the pressure on the surface of that in B will be $(h_1 + l)$ cm. of mercury.

If Boyle's law is being followed by the enclosed air, the product $(h_1 + l)v$ will be equal to the product of the initial pressure and volume, viz. $h_1 \times 100$. Make eight or ten experiments, increasing the height of B progressively. Take readings also when the height of B is decreased through the same steps. Tabulate the readings and results as follows:

EXPERIMENT ON BOYLE'S LAW FOR DRY AIR.

Height of barometer = — cm. (h_1).

PRESSURE INCREASING				
Temp of room Cent	Vol of air v c c	Difference in levels, l cm	Pressure $(l+h_1)$ cm of mercury	Products $(l+h_1)v$

A similar table should be made for diminishing pressures. The products in the last columns will be found to be equal, or nearly so.

Plot a graph in which the pressures $(l + h_1)$ are plotted as ordinates, and the volumes v as abscissae.

Graphs for illustrating Boyle's law.—The graph of the equation $pv = a$ constant, represents Boyle's law, and is a rectangular hyperbola. It may be plotted from experimental data as in Expt. 90, or initial conditions of pressure and volume may be assumed, and values of p corresponding to a sufficient number of other values of v may be calculated. The following method is useful.

Let $$pv = c.$$

Take logarithms of both sides, giving

$$\log p + \log v = \log c,$$

or $$\log p = -\log v + \log c.$$

The graph for this equation is a straight line. Ascertain if this is so by plotting the logarithms of $(l + h_1)$ and of v from the experimental data obtained in carrying out Expt. 90.

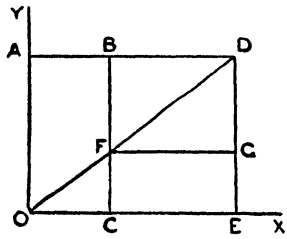


FIG. 395.

The following geometrical method of drawing a rectangular hyperbola is useful. In Fig. 395 set off OA along OY to represent p_1, and OC along OX to represent v_1 to convenient scales of pressure and volume. Make $OE = v_2$, and complete the rectangles OABC and OADE. Join OD cutting CB in F, and draw FG parallel to OX. Then EG is

equal to p_2, and G is a point on the rectangular hyperbola. The proof is as follows: Since the triangles OCF and OED are similar, we have

$$FC : OC = DE : OE,$$
$$EG : OC = BC : OE.$$

But $OC = v_1$,

$BC = p_1$,

and $OE = v_2$;

$\therefore EG : v_1 = p_1 : v_2$,

or $$EG = \frac{p_1 v_1}{v_2} = p_2.$$

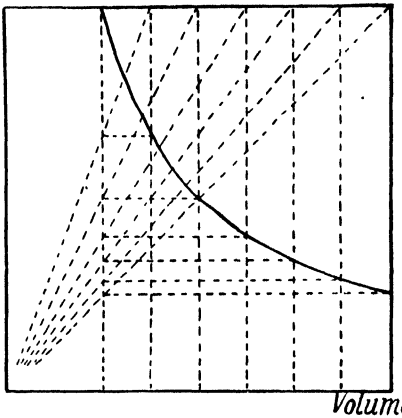


FIG. 396.—Boyle's law curve drawn by means of a geometrical construction

Other points may be found in a similar manner giving the complete curve shown in Fig. 396.

Such curves, representing operations carried out at constant temperature, are called **isothermal curves**; the operations of expansion or compression are called **isothermal operations.**

EXERCISES ON CHAPTER XXX.

1. Distinguish a vapour from a perfect gas. What is the pressure of a substance in the gaseous state due to?

2. If the barometer reads 74·32 cm. of mercury, what is the pressure of the atmosphere in grams wt. per square centimetre?

3. An oxygen cylinder is charged to a pressure of 120 atmospheres. What is the pressure in lb. wt. per square inch?

4. Give a sketch and description of a standard mercury barometer.

5. Describe, with sketches, the construction of an aneroid barometer. Criticize the accuracy of this type of barometer.

6. A standard mercury barometer reads 76·21 cm. at 15° C. Correct this reading for expansion of the mercury and scale. This barometer is situated at a height of 500 metres above sea-level in latitude 50°. Obtain the constant gravitational correction for this station.

7. A U gauge attached to a boiler chimney reads 0·8 inch of water. The barometer reads 29·45 inches of mercury. What is the absolute pressure inside the chimney?

8. Describe the construction and action of a Bourdon pressure gauge. Give a sketch.

9. State Boyle's law for gases. How would you prove it experimentally ?

10. A volume of 1 cubic foot of gas at an absolute pressure of 150 lb. wt. per square inch expands at constant temperature. Find the pressure when the volume is 2, 3, 4, 5, 6 cubic feet. Plot a graph showing the relation of pressure and volume.

11. Take the numbers obtained in answer to Question 10 and draw a graph by plotting the logarithms of the pressures and volumes.

12. Use a graphical method for drawing an isothermal curve for a gas expanding from a volume of 2000 c.c. at an absolute pressure of 15 atmospheres down to a pressure of 3 atmospheres.

13. An aneroid barometer reads 30 15 inches at ground level. When taken to the top of a tower, the barometer reads 30·06 inches. Calculate the height of the tower if the density of the air at the time was 0 00125 gram per c.c. and that of mercury 13 6.

14. Describe carefully an experiment to determine the relation between the pressure and the volume of a given mass of gas at constant temperature. Tasmania Univ.

15. The cross-sectional area of the mercury column in a barometer is 1·2 sq. cm. ; the length of the vacuum at the top is 8 cm. when the barometer reads 764 mm. Calculate the volume of external air which must be introduced into the tube in order to cause the height of the column of mercury to become 382 mm.

16. Describe the principle of the mercury barometer.

A mercury barometer is in the receiver of an ordinary air pump, and at first its height is 76 cm. After two strokes the height is 72 cm. What will it be after 10 strokes ? (Volume of barometer is insignificant compared with volume of receiver.) Tasmania Univ.

CHAPTER XXXI

PROPERTIES OF GASES—*CONTINUED*

Charles's law.—This law, first enunciated by Charles and Gay Lussac, states that **a given mass of any gas expands by a constant fraction of its volume at 0° C. when its temperature is raised one degree, provided its pressure is kept constant.** Experimental evidence shows that the value of the fraction is $\frac{1}{273}$ for a rise in temperature of 1° C. This fraction may be termed the **coefficient of expansion of a gas at constant pressure.**

Let v_0 = the volume of the given mass of gas at 0° C.

v = " " " " " " " t° C.

Then Increase in volume for a rise in temp. of 1° C. $= \frac{v_0}{273}$.

Hence
$$v = v_0 + \frac{v_0}{273} t$$
$$= v_0 \left(1 + \frac{t}{273}\right). \quad \ldots \quad (1)$$

This equation is not suitable for use in calculations, since the given initial volume would usually be at some temperature other than 0° C., and it would be necessary to evaluate v_0 prior to making any attempt to solve the problem.

Absolute scale of temperature.—Fig. 397 shows a graph (not drawn to scale) illustrating Charles's law in which volumes are plotted as ordinates and temperatures as abscissae. OA represents the volume

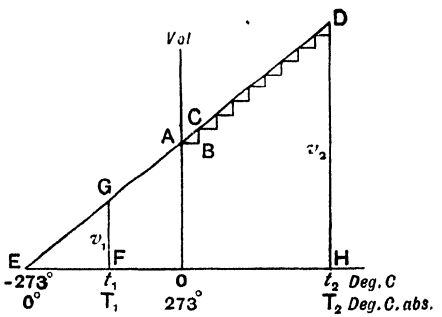


FIG. 397.—Graph illustrating Charles's law.

at 0° C.; abscissae measured to the right of O represent temperatures above freezing point, those measured to the left of O are negative and represent temperature below freezing point. AB represents a rise in temperature from 0° to 1° C, and BC represents the corresponding increase in volume. Other similar steps are shown in Fig. 397, and since the rises in temperature are all 1° C. and the increments in volume are all equal, it follows that a straight line CD may be drawn through the tops of all the steps. CD is therefore a graph illustrating the expansion of a gas obeying Charles's law.

Each of the increments, such as BC, is equal to OA/273. Produce DC until it cuts the temperature axis at E. Then, from the similar triangles CBA and AOE,

$$\frac{CB}{BA} = \frac{AO}{OE},$$

or, since BA = 1,

$$\frac{AO}{273} = \frac{AO}{OE};$$

$$\therefore OE = 273.$$

Therefore DC cuts the temperature axis at −273° C. At E the volume of the gas as shown in the diagram is zero. This would not actually be the case, but we may say that the volume and temperature of a gas at constant pressure behave in such a manner that, if the law connecting them did not change, the volume would disappear at −273° C.

A scale of temperatures having very useful properties may be illustrated in Fig. 397 by placing 0° at E and 273° at O; other temperatures may be marked along the temperature axis to correspond with the new marking of E and O. This scale is called **the gas thermometer scale of temperatures**, and is practically the same as a scale derived from considerations of energy, called the **absolute scale of temperature** (Chap. XXXVIII.). Temperatures on the absolute scale will be denoted by the letter T.

Another statement of Charles's law.—In Fig. 397 take any two points F and H on the temperature axis, and draw the ordinates FG and HD. $FG = v_1$ is the volume of the gas at an absolute temperature $EF = T_1$; $HD = v_2$ is the volume when the absolute temperature is equal to $EH = T_2$. From the similar triangles GFE and DHE, we have

$$\frac{GF}{FE} = \frac{DH}{HE},$$

or $$\frac{v_1}{T_1} = \frac{v_2}{T_2};$$

$$\therefore \frac{v_1}{v_2} = \frac{T_1}{T_2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

Hence Charles's law leads to the following statement: **The volumes of a given mass of a gas at constant pressure are proportional to the absolute temperatures.**

Temperatures stated in the Centigrade scale may be converted to the absolute scale by adding 273. Thus:

$$T = t^\circ \text{ C.} + 273.$$

Absolute zero on the Fahrenheit scale is $(273 \times \frac{9}{5}) = 491^\circ$ F. below freezing point, or $(491 - 32) = 459^\circ$ F. below zero Fahrenheit. Hence temperatures stated in the Fahrenheit scale may be converted to the appropriate absolute scale by adding 459. Thus

$$T = t^\circ \text{ F.} + 459.$$

EXAMPLE.—A room measures 50 feet × 30 feet × 25 feet. If the temperature of the air in it be raised from 10° C. to 15° C., what percentage of the initial volume of air will be expelled? The pressure is assumed to remain constant.

$$v_1 = 50 \times 30 \times 25 = 37{,}500 \text{ cubic feet.}$$

$$T_1 = 273 + 10 = 283^\circ \text{ absolute (C.).}$$

$$T_2 = 273 + 15 = 288^\circ \text{ absolute (C.).}$$

Let v_2 be the volume of the air originally in the room if heated to 15° C., then

$$v_2 = \frac{v_1 T_2}{T_1} = \frac{37500 \times 288}{283}$$

$$= 38{,}160 \text{ cubic feet.}$$

$$\text{Volume of air expelled} = 38160 - 37500$$

$$= 660 \text{ cubic feet.}$$

This volume of 660 cubic feet has been measured at 15° C., hence

$$\text{Percentage of air expelled} = \frac{660}{38160} \times 100$$

$$= \underline{1{\cdot}73}.$$

Isothermal lines of a gas.—Suppose we have a given mass of gas under initial conditions of pressure and volume, p_1 and v_1, and at a temperature of 273° absolute (C.). In Fig. 398 ordinates and abscissae represent pressure and volume respectively, and the initial conditions are plotted at A. If the gas expands following Boyle's law, the isothermal line AB results. The temperature

everywhere on AB is 273° absolute. Suppose, when the gas is under the conditions represented by the point A, that its temperature be raised to T° absolute (C.) at constant pressure. Applying Charles's law, we have for the new volume v_2,

$$\frac{v_2}{v_1} = \frac{T}{273}, \quad \therefore \ v_2 = \frac{T}{273} v_1$$

Plotting the new conditions, viz. pressure p_1 and volume v_2, we obtain the point C. Now let the gas expand at constant temperature

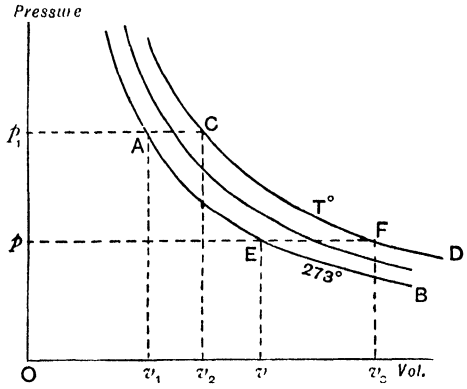


FIG. 398.—Isothermal lines of a gas.

T. Boyle's law will be followed, and the isothermal line for the temperature T will be CD As many points as may be necessary for plotting CD may be found from points such as E on AB. Thus, at E the pressure is p and the volume is v; the temperature at E is 273° and at F is T degrees, the volume at F is v_3.

$$\frac{v_3}{v} = \frac{T}{273}; \quad \therefore \ v_3 = \frac{T}{273} v.$$

The factor T/273 for converting horizontally any point on AB to the corresponding point on CD is a constant. In this way a series of isothermal lines may be drawn for 273, 274, 275, etc., degrees absolute; some of these are shown in Fig. 398.

In Fig. 399 ordinates and abscissae represent volumes and absolute temperatures respectively. The point A represents a given mass of gas at 273° absolute (C.), and having a volume of, say, 4 cubic units and a pressure of, say, 76 cm. of mercury. AOC represents changes going on in obedience to Charles's law, *i.e.* the pressure at any point in this line is 76 cm. of mercury, and the line may be called a line of constant pressure.

Let the conditions indicated at A be changed at constant temperature 273° absolute to 3 cubic units, say. The pressure p_2 will be obtained by applying Boyle's law, thus

$$p_1v_1=p_2v_2\,;\quad \therefore\ 76\times 4=p_2\times 3\,;$$
$$\therefore\ p_2=\tfrac{4}{3}\times 76=101{\cdot}33 \text{ cm. of mercury.}$$

The point E will represent these new conditions. OEH now represents changes in obedience to Charles's law under a constant pressure of 101·33 cm. of mercury. Similarly, by reducing the initial volume to 2 and 1 cubic unit, the constant pressure lines OFK and OGL are obtained.

The change in volume taking place in the given mass of gas when raised in temperature from 273° to 373° at constant pressure 76 cm. of mercury is represented by (DC – BA). The corresponding change in volume at a pressure of 304 cm. of mercury is represented by (DL – BG).

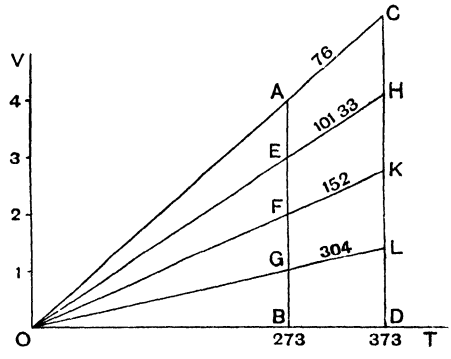


FIG. 399.—Constant pressure lines of a gas.

It is clear, by inspection of Fig. 399, that the change of volume for the given rise in temperature is much greater if the pressure is low than if the pressure is high.

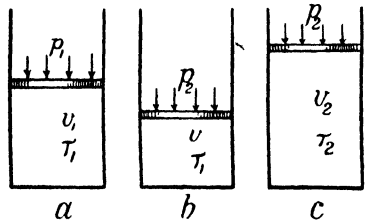


FIG. 400.—Diagram showing changes occurring in the pressure, volume and temperature of a gas.

Combination of Boyle's and Charles's laws.—If the given mass of gas is undergoing simultaneous changes in pressure, volume and temperature, the law followed may be found by the following process. In Fig. 400 (*a*) a given mass of gas is enclosed in a cylinder fitted with a piston, and is under initial conditions p_1, v_1 and T_1. Suppose, first, that p_1 and v_1 are changed at constant temperature T_1 until a pressure p_2 and a volume v are obtained. Applying Boyle's law, we have

$$p_1v_1=p_2v\,;\quad \therefore\ v=\frac{p_1v_1}{p_2}. \qquad \text{.....(1)}$$

The gas will now be as shown in Fig. 400 (*b*).

Now change the temperature from T_1 to T_2, keeping the pressure constant at p_2 The volume will change from v to v_2 as shown in Fig. 400 (*c*). Applying Charles's law,

$$\frac{v}{v_2} = \frac{T_1}{T_2}; \quad \therefore v = \frac{v_2 T}{T_2}. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

Hence, from (1) and (2), we have

$$\frac{p_1 v_1}{p_2} = \frac{v_2 T}{T_2},$$

or

$$\frac{p_1 v_1}{p_2 v_2} = \frac{T_1}{T_2}. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

We should have obtained the same result had the conditions varied simultaneously instead of by the step by step process adopted above. The result obtained in equation (3) indicates that when a given mass of a perfect gas is undergoing changes in pressure, volume and temperature, **the product of the absolute pressure and volume is proportional to the absolute temperature,** or

$$pv \propto T. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$$

The **characteristic equation for a perfect gas** is obtained from (4) by taking v as the volume occupied by unit mass of the given gas at freezing temperature ($T = 273°$ absolute) and under standard pressure of 76 cm. of mercury. By introducing a coefficient R, we have

$$pv = RT, \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5)$$

in which R has a value which depends upon the kind of gas considered.

Relation of pressure and temperature at constant volume in a perfect gas.—Take equation (3) above, and put $v_1 = v_2$, thus causing the pressure and temperature of the gas to vary at constant volume. Thus

$$\frac{p_1 v_1}{p_2 v_1} = \frac{T_1}{T_2},$$

or

$$\frac{p_1}{p_2} = \frac{T_1}{T_2}. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

Hence, if we have a closed vessel containing a given mass of perfect gas at constant volume, **the absolute pressure is proportional to the absolute temperature.**

Suppose that p_1 is the pressure of the given mass of gas at 0° C.,

or 273° absolute, and that the temperature be raised at constant volume to 274° absolute. We have

$$\frac{p_1}{p_2} = \frac{273}{274};$$

$$\therefore\ p_2 = \frac{274}{273} p_1 = p_1 + \frac{1}{273} p_1.$$

The increase in pressure per degree Centigrade is therefore $\frac{1}{273}$ of the pressure at 0° C. Similarly, if the temperature be raised to t° C. above freezing point, the pressure p is given by

$$\frac{p_1}{p} = \frac{273}{273 + t},$$

or

$$p = \left(\frac{273 + t}{273}\right) p_1 = p_1 + \frac{t}{273} p_1.$$

The fraction $\frac{1}{273}$ may be called **the coefficient of increase of pressure of a gas at constant volume.** It is evident that the value of this coefficient is the same as the coefficient of expansion of a gas at constant pressure.

Verification of Charles's law.—Direct experiments for the purpose of verifying Charles's law are difficult to carry out accurately. The relation of pressure and temperature at constant volume enables Charles's law to be proved by means of an experiment which may be carried out easily.

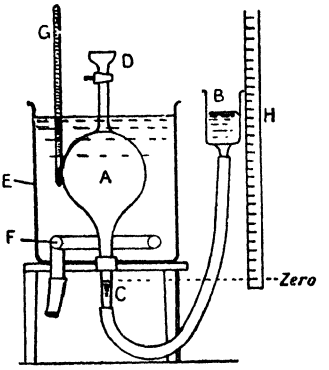


FIG. 401.—Apparatus for determining the relation of p and T.

EXPT. 91.—**Relation of pressure and temperature of air at constant volume; indirect proof of Charles's law.** Arrange apparatus as shown in Fig. 401. A is a large bulb containing dry air and connected by a long rubber tube to an open cistern of mercury at B. The bulb is immersed in water contained in a vessel E; the water may be heated by a coiled pipe F through which steam can be passed; the temperature is measured by a thermometer G placed close to the bulb. The mercury cistern may be moved vertically along a scale H and may be clamped at any height in the same way as in the Boyle's law apparatus (p. 397). The zero of the scale H is arranged so as to be at the same level as a mark C

on the neck of the bulb; this may be checked by use of the U tube mercury level described on p. 397.

Begin with cold water in E. Adjust the height of B so that the mercury level in the neck of the bulb coincides with the mark C. Wait a minute or two to make sure that the temperature of the air in the bulb is the same as that of the water; readjust the level if necessary. Note the height of the mercury level in B, say h cm., and the temperature of the water, say $t°$ C. Read the barometer; let this be h_0 cm. of mercury.

Raise the temperature of the water, say 5° C.; this will cause the mercury level at C to be depressed. Restore the level by raising B, taking the same precautions as before prior to reading the scale H and the thermometer G. Repeat the experiment several times, increasing the temperature each time by about 5° C.

If the law $p \propto T$ is being complied with, we have

$$p = cT, \quad \text{or} \quad c = \frac{p}{T},$$

where c is a constant. Try if this is so by dividing the experimental values of the absolute pressure $p = (h + h_0)$ by $T = t + 273$. Plot a graph showing the relation of p and T.

The results may be tabulated as follows:

EXPERIMENT ON THE RELATION OF p AND T FOR AIR AT CONSTANT VOLUME

Barometer reading $= (h_0)$ cm.

Temperature.		Height of mercury h cm	Pressure in bulb $p = (h_0 + h)$ cm	$c = \frac{h+h}{T}$
$t°$ C	T° abs			

Deduce from the graph the pressures at 0° and 100° C., and calculate the coefficient of increase of pressure.

Air thermometer. —In Fig. 402 is shown a modified form of the apparatus used in the above experiment. The bulb A contains dry air, and is connected by a tube of fine bore to tubes BD and EC, which may be raised or lowered along a scale. This instrument constitutes an **air thermometer**. The bulb A is introduced into the space in which the temperature has to be measured, and the sliding tubes are adjusted until the mercury level is restored to a fixed mark at B. The absolute pressure in the bulb is determined in the manner explained in Expt. 91, and the temperature is calculated from the

pressure. The instrument may be calibrated by immersing the bulb A first in melting ice and then in the steam given off by water boiling under standard pressure, taking the same precautions as in the calibration of mercurial thermometers (p. 316). The pressures corresponding to 0° and 100° C. are thus found, and intermediate graduations may be determined by permitting the bath of hot water to cool slowly and taking readings of the pressure at intervals. Used in the manner described, the instrument is a **constant volume** air thermometer.

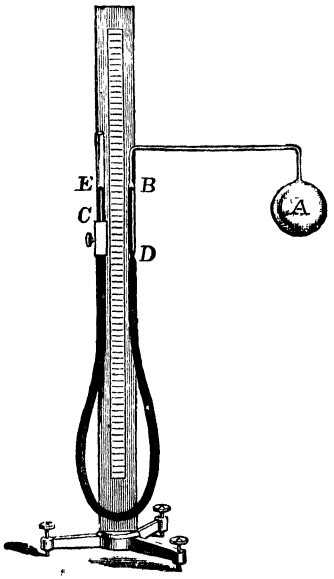


FIG. 402.—Constant volume air thermometer.

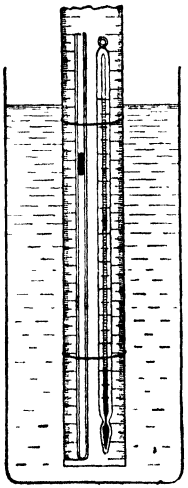


FIG. 403.—Constant pressure air thermometer.

A simple form of **constant pressure** air thermometer is shown in Fig. 403, and consists of a piece of thermometer tubing about 1 mm. in bore and 20 cm. long. The tube is carefully cleaned and dried, and one end is sealed. By slightly warming the tube, some air is expelled, and a pellet of mercury is sucked in as the tube cools. When placed vertically with the closed end at the bottom, the mercury pellet should be about 14 cm. from the lower end at ordinary atmospheric temperature. Since the upper surface of the pellet of mercury is exposed to the pressure of the atmosphere, the pressure of the enclosed air remains practically constant during an experiment. The tube is tied to a thermometer, and the scale of the thermometer

serves to indicate the position of the pellet. Assuming that the bore of the air thermometer tube is uniform, the length occupied by the enclosed air is proportional to the volume The scale readings corresponding to 0° and 100° C. may be found as before. These scale readings correspond to 273° and 373° absolute (C.) respectively, and intermediate scale readings, obtained when the instrument is at other temperatures, are converted into absolute degrees by simple proportion This type of constant pressure air thermometer is not very satisfactory on account of the sticking of the mercury piston.

Mixture of two different gases.—If a closed vessel contains a mixture of two different gases which have no chemical action on each other, then **the total pressure is the sum of the pressures which the quantity present of each gas would exert if it alone occupied the vessel.** The proof is as follows ·

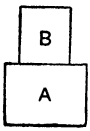


FIG 404

Suppose at first that the two gases occupy separate vessels A and B (Fig. 404). Let the pressure and volume of the gas in A be p_1 and v_1, and of that in B p_2 and v_2. The whole is supposed to be at the same temperature, which is preserved constant throughout the following operations

Let the capacity of B be changed until the pressure in this vessel is p_1, and let the volume be now v. Applying Boyle's law, we have

$$p_2v_2=p_1v, \quad \text{or} \quad v=\frac{p_2v_2}{p_1}. \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots (1)$$

Now let a hole be made in the partition separating A and B ; both gases being at the same pressure, quiet mingling will ensue, on the assumption that no chemical action occurs, and the mixed gases will exert a pressure p_1, and will occupy a total volume (v_1+v).

Let the portion B now resume its original volume, when the total volume occupied by the mixed gases will be (v_1+v_2). and the pressure will be p, say. Again applying Boyle's law, we have

$$p_1(v_1+v)=p(v_1+v_2)\,;$$

$$\therefore \; p=\left(\frac{v_1+v}{v_1+v_2}\right)p_1=\frac{p_1v_1}{v_1+v_2}+\frac{p_1v}{v_1+v_2}.$$

Substituting for v from (1), we obtain :

$$p=\left(\frac{p_1v_1}{v_1+v_2}\right)+\left(\frac{p_1}{v_1+v_2}\times\frac{p_2v_2}{p_1}\right)$$

$$=\frac{p_1v_1}{v_1+v_2}+\frac{p_2v_2}{v_1+v_2} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots (2)$$

Had the gas originally in A alone filled the volume (v_1+v_2), the pressure would have been p', say, and

$$p'(v_1+v_2)=p_1v_1,$$

or
$$p'=\frac{p_1v_1}{v_1+v_2} \quad \ldots\ldots(3)$$

Similarly, had the gas originally in B occupied the final volume (v_1+v_2), the pressure would have been p'', say, and

$$p''(v_1+v_2)=p_2v_2,$$

or
$$p''=\frac{p_2v_2}{v_1+v_2} \quad \ldots\ldots(4)$$

Hence, from (2), (3) and (4),

$$p=p'+p'', \quad \ldots\ldots(5)$$

thus proving the proposition.

Density of a gas. The density of a gas at given temperature and pressure is the mass in grams per cubic centimetre. Let the initial conditions of a given mass of gas be p_1, v_1 and T_1; let the initial density be d_1, and let the final conditions be p_2, v_2, T_2 and d_2. Let m be the constant mass of the gas, then

$$m=v_1d_1=v_2d_2;$$

$$\therefore \frac{d_1}{d_2}=\frac{v_2}{v_1}. \quad \ldots\ldots(1)$$

Also
$$\frac{p_1v_1}{T_1}=\frac{p_2v_2}{T_2};$$

$$\therefore \frac{v_2}{v_1}=\frac{p_1T_2}{p_2T_1}.$$

Hence, from (1),
$$\frac{d_1}{d_2}=\frac{p_1T_2}{p_2T_1}. \quad \ldots\ldots(2)$$

If the temperature is constant, this reduces to

$$\frac{d_1}{d_2}=\frac{p_1}{p_2}. \quad \ldots\ldots(3)$$

If the pressure is constant, the result becomes

$$\frac{d_1}{d_2}=\frac{T_2}{T_1}. \quad \ldots\ldots(4)$$

If d_0 is the density of the gas at normal pressure and temperature of 76 cm. of mercury and 0° Centigrade, the density d at any other

pressure p cm. of mercury and temperature t degrees Centigrade is obtained from (2), giving

$$\frac{d_0}{d} = \frac{76(t+273)}{273p},$$

or

$$d = \frac{273p\,d_0}{76(t+273)} \quad \ldots\ldots(5)$$

EXPT. 92.—**Density of air.** Reference is made to Fig. 405, in which A is a glass globe furnished with a tap. The globe is connected by means of a piece of thick rubber tube to a manometer BC. The manometer contains mercury, and the closed space above C is a Torricellian vacuum; hence the gaseous pressure acting on the mercury surface at B is given by the difference in levels of the mercury in the two limbs of the manometer,

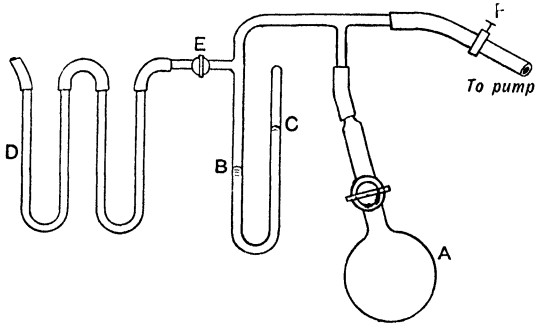


FIG 405.—Apparatus for determining the density of air

provided that the closed limb at C is not completely filled with mercury. There is another connection, having a tap at E, and leading to two drying tubes containing phosphorus pentoxide; this substance removes any moisture from the air before it enters the globe. The tube at D is open to the atmosphere. An exhausting air pump is connected to the apparatus, the connecting rubber tube being furnished with a clip at F; air may thus be withdrawn from the globe A.

Close the tap E and exhaust the globe; close the clip F and open E, thus permitting air to flow through the drying tubes and thence into the globe. Repeat the operation several times so as to ensure that the globe contains dry air only. Since the globe is in communication with the atmosphere, the pressure and temperature of the air it contains may be obtained by reading the barometer and a thermometer placed near the globe A. Close the tap on the globe; remove the globe and weigh it, thus obtaining the mass of the globe when full of dry air.

Connect the globe again to the apparatus and open its tap. Close the

tap E and exhaust the globe as thoroughly as possible. The pressure of any residual air is now shown by the manometer reading; it is best to read the levels at B and C by means of a cathetometer. Close the tap on the globe; remove the globe and weigh it, thus obtaining the mass of the globe when filled with residual air. Place the neck of the globe under water at the temperature of the room and open the tap, thus filling the globe with water; see that the water level reaches the tap, adding some water if necessary. Weigh again, thus obtaining the weight of the globe when full of water.

Let w_1 = the weight of the globe in grams when full of dry air at h_a and t.

w_2 = the weight of the globe in grams when full of residual air at h_m and t.

w_3 = the weight of the globe in grams when full of water.

h_a = the barometric pressure, cm. of mercury.

h_m = the pressure of the residual air, as shown by the manometer, cm. of mercury.

t = the constant temperature of the room, deg. Cent.

Since the temperature of the room has remained constant, the fraction h_m/h_a of the air originally in the globe remains after exhaustion. The difference in weights $(w_1 - w_2)$ therefore represents the weight of the air withdrawn, viz. $(1 - h_m/h_a)$ of the original quantity of air. Therefore the weight w of dry air originally in the globe is given by

$$\frac{w}{w_1 - w_2} = \frac{1}{1 - \frac{h_m}{h_a}} = \frac{h_a}{h_a - h_m};$$

$$w = \left(\frac{w_1 - w_2}{h_a - h_m}\right) h_a \text{ grams.} \quad \ldots\ldots\ldots\ldots(1)$$

Further, $(w_3 - w_1 + w)$ gives the weight in grams of the water which fills the globe, and the same quantity gives the volume of the globe in cubic centimetres. Hence, at the pressure h_a and temperature t,

$$\text{Density of air} = d = \frac{(w_1 - w_2)h_a}{(w_3 - w_1 + w)(h_a - h_m)}$$

$$= \frac{(w_1 - w_2)h_a}{(w_3 - w_2)h_a - (w_3 - w_1)h_m} \text{ grams per c.c. } \ldots(2)$$

The density at normal pressure and temperature may now be obtained from (5) (p. 412).

$$d_0 = \frac{76(t + 273)}{273 h_a} d$$

$$= \frac{76(t + 273)}{273} \cdot \frac{(w_1 - w_2)}{(w_3 - w_2)h_a - (w_3 - w_1)h_m} \text{ grams per c.c. } \ldots\ldots(3)$$

Influence of height on the pressure and density of the atmosphere.—It is well known that the pressure of the atmosphere decreases as

the height above sea-level is increased. If a barometer be taken up a mountain, the level of the mercury falls gradually during the journey. Water boils at a lower temperature at the top of a mountain than at sea-level, owing to the diminished pressure of the atmosphere. The following approximate method enables the law governing these changes to be understood.

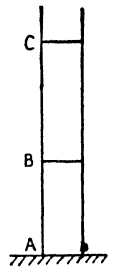


FIG 406

Consider a column of air (Fig. 406) having a cross-sectional area of one square centimetre and reaching upwards to the limit of the atmosphere. Let the temperature throughout the column be 0° C., and let the pressure at sea-level A be 76 cm. of mercury. This pressure is produced by the total weight of the column, and is equivalent to 1033 grams weight per square centimetre nearly. The density of air at this pressure and 0° C is 1·2928 grams per 1000 c.c.; assuming that the density does not change appreciably on ascending a few metres, the height of a column AB weighing one gram will be $10 \div 1{\cdot}2928 = 7{\cdot}735$ metres. The pressure at B is less than that at A by the weight of AB, and is therefore 1032 grams weight per square centimetre.

The density at this pressure and at 0° C. is given by

$$\frac{d}{1{\cdot}2928} = \frac{1032}{1033} \quad \text{(p. 411)},$$

$$d = 1{\cdot}2915 \text{ grams per 1000 c.c.}$$

Assuming that this density remains constant for another few metres above B, the height of a column BC weighing one gram is $10 \div 1{\cdot}2915 = 7{\cdot}743$ metres nearly. The pressure at C will then be 1031 grams weight per square centimetre. BC is greater than AB, and we conclude that the intervals to produce a constant pressure difference of one gram weight per square centimetre increase as we ascend.

Increase in temperature increases the heights AB and BC in Fig. 406; the effect of the change in temperature is to increase the volume, and thus to diminish the density.

Heights may be measured approximately by means of the barometer. An elevation of 900 feet corresponds nearly to one inch fall in the mercury column. The aneroid barometer (p. 392) is used generally by surveyors, and has two scales on its dial, one indicating the height of the mercury column and the other showing elevations in feet. The method is approximate only, since, for stability in the atmosphere, the temperature must fall as the altitude increases.

Flotation of balloons.—If a balloon is floating at rest in a still atmosphere, the law governing its equilibrium is the same as that for a body floating at rest in a still liquid (p. 274). Thus, in Fig. 407, W_1 is the weight of the balloon, including all material and the car; W_2 is the weight of the gas contained in the envelope; W_3 is the weight of air displaced by the balloon, and is equal to the buoyancy, *i.e.* the air surrounding the balloon exerts a resultant force equal to W_3. For equilibrium

$$W_1 + W_2 = W_3,$$

or

$$W_1 = W_3 - W_2.$$

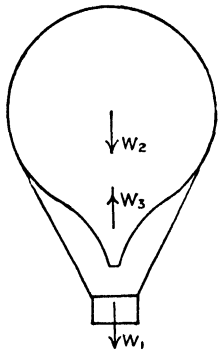


FIG. 407.—Flotation of a balloon.

It is evident that the total weight which can be raised, viz. W_1, can be increased by increasing the difference between W_3 and W_2. For a given volume of envelope, and at normal pressure and temperature, W_3 is a definite quantity, and the difference $(W_3 - W_2)$ can be increased only by choosing as light a gas as possible for filling the envelope. Hydrogen is the gas generally employed. At 0° C. and 1 atmosphere pressure, hydrogen weighs 0·00559 lb. per cubic foot, and air under like conditions weighs 0·0807 lb. Hence, at normal pressure and temperature, the weight which can be raised per cubic foot of balloon is (0·0807 – 0·00559), or 0·07511 lb. weight.

The best lifting effect would, of course, be obtained by having a vacuum inside the envelope, but this is impossible owing to the collapsing pressure of the atmosphere; an envelope strong enough to withstand this pressure would be much too heavy to be lifted. It is best to have the pressure of the internal gas equal, or nearly equal, to that of the external air, thus calling for but little strength in the material of the envelope, and enabling a very light envelope to be employed.

A balloon will ascend if $(W_1 + W_2)$ is less than W_3 (Fig. 407); since the density of the air diminishes with height of ascent, a height will be attained at which W_3 becomes equal to $(W_1 + W_2)$, and the balloon will remain at this elevation. But the pressure of the air has also decreased (p. 414); hence, if the pressure of the contained gas has not altered during the ascent, there will be a tendency to burst the envelope. This tendency may be reduced by permitting some of the gas to escape through a valve at the top of the balloon,

In modern air-ships the internal pressure can be adjusted by means of **ballonets**. These are small balloons placed inside the envelope and containing air. Air can be pumped into or withdrawn from the ballonets, which will then occupy a greater or lesser volume, and will cause the light gas in the balloon to occupy a lesser or greater volume, and thus to exert a greater or smaller pressure. The device enables the envelope to be maintained at practically constant tension without the necessity for permitting any of the light gas to escape from the balloon.

Ordinary balloons carry ballast in the form of bags of sand. If greater elevation is required, some of the sand is scattered overboard, thus diminishing the total weight to be carried. Air ships having engines and propellers for propulsion may secure greater elevation by the use of rudders pivoted on horizontal axes. The use of these introduces greater resistance to the passage of the vessel through the air, and leads to a decrease in speed if the greater elevation is maintained by use of the rudders

EXERCISES ON CHAPTER XXXI.

1. State Charles's law for perfect gases. Define absolute zero of temperature by reference to the contraction of a perfect gas. What temperature on the absolute scale (Cent.) corresponds to 20° on the ordinary Fahrenheit scale ?

2. A chimney is 120 feet high and is 3 feet in internal diameter. The average temperature of the gases inside the chimney is 280° C. What volume would the contents of the chimney occupy if the temperature were reduced to 15° C. without change in pressure ?

3. A glass bulb having a fine stem weighs 19 24 grams when empty and 74·85 grams when full of water. When full of air, the bulb was placed for a few minutes in a hot oven at atmospheric pressure and the stem was sealed. The bulb was then immersed in a bath of water at 15° C., stem downwards and the stem was unsealed. It was found, on adjusting the pressure to that of the atmosphere, that 35 68 grams of water had entered the bulb. Find the temperature of the oven.

4. Draw the isothermal line for one cubic foot of air at 15° C. and absolute pressure 10 atmospheres when it expands to 5 cubic feet. On the same drawing, show the isothermal line for the same mass of gas at 50° C.

5. Assuming the truth of the laws of Boyle and Charles for perfect gases, prove the law $pv = RT$.

6. Find the values of R in the equation $pv = RT$ for air and hydrogen, given that the mass of one cubic foot of air at 0° C. and 14 7 lb. wt. per square inch is 0·0807 pound, and one cubic foot of hydrogen under the same conditions has a mass of 0·00559 pound.

7. In the c.g.s. system, 1000 c.c. of air at 0° C. has a mass of 1·2928 grams, under a pressure of $1{\cdot}0132 \times 10^6$ dynes per sq. cm. Find the value of R in the equation $pv = RT$.

8. A cylinder fitted with a piston contains at a certain instant 6 cubic feet of gas at 15 lb. wt. per square inch absolute and 20° C. The weight of the gas in the cylinder is kept constant, and the piston is pushed in. At another instant, the pressure is found to be 150 lb. wt. per square inch absolute and the volume 2 5 cubic feet. Calculate the temperature at this instant.

9. Given that one gram of hydrogen at a temperature of 0° C. and a pressure of 76 cm. of mercury occupied a volume of 11·16 litres, what volume will be occupied by 3·85 grams at a temperature of 23° C. and a pressure of 74·6 cm. ? Adelaide Univ.

10. How would you determine the coefficient of expansion of a gas at constant pressure ? What is meant by the absolute zero of the air thermometer, and how is it calculated ? Sen. Cam. Loc.

11. A chimney is 50 metres in height ; a water pressure gauge (p. 395) connected to the base of the chimney indicates 2 cm. of water. The temperature of the atmosphere is 0° C., and the barometer reads 76 cm. (density of air under these conditions = 1·29 ; grams per litre). Find the average temperature of the gases in the chimney. The density of mercury = 13·6 grams per c.c.

12. State the two fundamental laws of change of pressure, volume, and temperature of gases, and show that they may be expressed in the form of a single equation containing one constant. What is the value of the constant in the case of hydrogen if the mass of one litre at 0° C. and 760 mm. pressure is 0·0896 gram ? (Density of mercury = 13·6.) L.U.

13. The density of oxygen at 0° C. under a pressure of 760 mm. of mercury is 1 429 grams per litre. A certain mass of the gas is enclosed in a cylinder whose volume is 2·5 litres, under a pressure of 780 mm. at a temperature of 12° C. What is the mass of gas in the cylinder ? L.U.

14. Calculate the weight of dry air in a room 30 × 18 × 15 feet in dimensions, the barometer reading 752 mm., the thermometer 30° C. (The weight of one cubic foot of air at 760 mm. pressure and 0° C. is 0·0801 lb.) Explain also the theory of your calculation. Adelaide Univ.

15. Two different gases which have no chemical action on each other are mixed in the same vessel. State and prove the law for the total pressure.

16. Describe how you would find the density of dry air. The results of an experiment are as follows : The weight of the globe when full of dry air was 35·375 grams, and when full of residual air, 35·198 grams, both at 14° C. ; the globe, when full of water, weighed 179·95 grams. The barometer read 77·19 cm., and the pressure of the residual air as shown by the manometer was 1·19 cm. of mercury. Find the density of the air in grams per c.c. at 0° C. and 76 cm. of mercury.

17. An airship 500 feet long has an average diameter of 50 feet and is charged with hydrogen. If the total weight of the structure, envelope, engines, etc., is 26 tons, find the total weight of stores (petrol, explosives, etc.) and crew which may be carried.

18. Two vessels, A and B, are connected by a pipe furnished with a tap. The tap is closed, and A and B are charged with air at pressures of 360

and 240 cm. of mercury respectively. The volume of A is 800 c.c., and that of B 600 c.c. If the tap is opened, find the final pressure in each vessel. The temperature is constant throughout.

19. A steel vessel for storing compressed air has a volume of 6 cubic feet, and has a safety valve which opens and permits air to escape when the pressure is 100 lb. wt. per square inch above that of the atmosphere, which may be taken as 15 lb. wt. per square inch absolute. The vessel contains air at a pressure of 110 lb. wt. per square inch absolute and at 15° C. If the vessel be heated, at what temperature will the safety valve open ?

CHAPTER XXXII

KINETIC THEORY OF GASES. WORK DONE BY A GAS

Pressure of gaseous molecules moving in parallel directions.—Let a hollow cube of one centimetre edge, internal dimensions, contain one molecule only. Let the mass of the molecule be m gram, and let it travel constantly in a line ab perpendicular to two opposite faces of the cube (Fig. 408). It is assumed that the velocity u is simply reversed each time the molecule strikes a face of the cube. The change in momentum during each impact is $2mu$ (p. 68), and the time taken to travel from a to b is $1/u$ second. There will therefore be u impacts per second, and the change of momentum per second is $2mu^2$. Half of the total number of impacts take place at one face, and the other half at the opposite face; hence the change of momentum per second at one face is mu^2. This is a measure of the force exerted on the face (p. 68), therefore

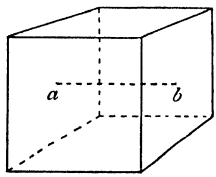


FIG 408.

$$\text{Force} = mu^2 \text{ dynes.}$$

If the centimetre cube contains n molecules, all moving in paths parallel to ab and having the same velocity u, the total force exerted on one face is nmu^2; this force is distributed over an area of one square centimetre, and is therefore the pressure on one face of the cube. $\therefore$ Pressure $= p = nmu^2$ dynes per sq. cm.

If the molecules have velocities differing in magnitude, and if $\bar{u}^2$ is the mean of the squares of all the speeds, then

$$p = nm\bar{u}^2 \text{ dynes per sq. cm.} \quad \text{.....(1)}$$

Pressure of a gas.—If the cube contains a gas, the molecules are moving in every conceivable direction. Let V (Fig. 409) be the velocity of one molecule, and let OX, OY and OZ be axes parallel to

the edges of the cube in Fig. 408. Resolve V into components along these axes, this may be done by first resolving V into components OM and OK, the latter being in the plane containing OX and OZ. OK is then resolved into velocities OL and ON along OX and OZ respectively. The components are thus u along OX, u_1 along OY and u_2 along OZ. From the geometry of the figure,

$$OA^2 = OK^2 + KA^2 = OL^2 + LK^2 + OM^2$$
$$= OL^2 + ON^2 + OM^2,$$
$$\therefore\ V^2 = u^2 + u_1^2 + u_2^2.$$

The velocities of the other molecules have different directions, but all can be resolved into components parallel to the edges of the cube. If $\bar{u}_1^2$ and $\bar{u}_2^2$ have the same relation to u_1^2 and u_2^2 respectively that $\bar{u}^2$ has to u^2, as explained above, and if $\bar{V}^2$ has the same relation to V^2, *i.e.* $\bar{V}^2$ is the mean of the squares of the actual velocities, then we may write

$$\bar{V}^2 = \bar{u}^2 + \bar{u}_1^2 + \bar{u}_2^2.$$

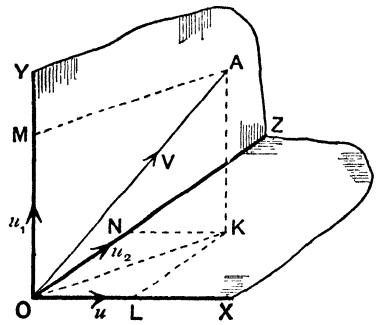


FIG 409.—Component velocities of a molecule

Since there is no tendency for molecules to accumulate in any part of the cube, it is reasonable to suppose that the velocities $\bar{u}$, $\bar{u}_1$ and $\bar{u}_2$ are equal; hence

$$\bar{u}^2 = \bar{u}_1^2 = u_2^2 = \tfrac{1}{3}\bar{V}^2. \quad \text{.....(2)}$$

Hence, in the case of a centimetre cube containing n molecules of a gas moving in all directions, we have from (1) and (2)

$$p = \tfrac{1}{3}nm\bar{V}^2 \text{ dynes per sq. cm. ... .(3)}$$

If the cube has a volume v, each cubic centimetre containing n molecules, then $pv = \tfrac{1}{3}nmv\bar{V}^2$. ....(4)

Now nm is the total mass of the molecules in one cubic centimetre, *i.e.* nm is the density d of the gas, and $d \times v$ is the total mass M in a volume v;

$$\therefore\ pv = \tfrac{1}{3}M\bar{V}^2. \text{ ....(5)}$$

If the cube contains unit mass of gas, M is unity, and

$$pv = \tfrac{1}{3}\bar{V}^2. \text{ .....(6)}$$

Some important relations.—It is known from Boyle's law that the product of the pressure and volume of a given mass of gas is constant, provided the temperature is constant, and it has just been shown that this product depends upon $\bar{V}^2$, since M in (5) above is constant. Hence it may be inferred that $\bar{V}^2$ is constant if the temperature of the gas does not alter.

Again, at constant volume, the pressure of a given mass of gas is proportional to the absolute temperature (p. 406). Writing v constant in equation (5) gives the result that p varies as $\bar{V}^2$; hence we may infer that the absolute temperature T and the mean of the squares of the velocities of the molecules are proportional, and any increase in T is accompanied by a corresponding increase in $\bar{V}^2$. Both $\bar{V}^2$ and T should become zero simultaneously, and we may define absolute zero of temperature as the temperature at which the molecules of a gas have been reduced to rest.

If p be constant in (5) above, then v varies as $\bar{V}^2$. From Charles's law we know that in a given mass of gas the volume v is proportional to the absolute temperature T, provided the pressure is constant. Hence again we infer that $\bar{V}^2$ is proportional to T.

Further, the kinetic energy of one molecule having a mass m and velocity V is $\frac{1}{2}mV^2$, and the total kinetic energy of a mass M of gas, in which the mean of the squares of the velocities is $\bar{V}^2$, is $\frac{1}{2}M\bar{V}^2$. It therefore follows, since $\bar{V}^2$ varies as T, that the total kinetic energy of the molecules is proportional to T. Suppose that a given mass of gas is maintained at constant volume, a definite quantity of heat must be added in order to produce a stated rise in temperature, and the result is a definite increase in the total kinetic energy of the molecules. We may therefore infer that the heat added has been converted into kinetic energy, and exists in the gas in the form of molecular motions. Abstraction of heat from the gas would produce a corresponding reduction in the molecular kinetic energy, and the energy would become zero at absolute zero of temperature.

Avogadro's law.—Consider two vessels of equal capacities, one containing a gas A and the other containing a different gas B. Both gases are supposed to be at the same pressure and temperature. It is assumed that single molecules of each gas have equal kinetic energies, on the average, when the temperatures are the same, *i.e.*

$$\tfrac{1}{2}m_A\bar{V}_A^2 = \tfrac{1}{2}m_B\bar{V}_B^2.$$

Since the product of pressure and volume is the same for both vessels, we have, from equation (4), p. 420,

$$\tfrac{1}{3}n_A m_A v\bar{V}_A^2 = \tfrac{1}{3}n_B m_B v\bar{V}_B^2;$$

$$\therefore\ n_A = n_B.$$

This result indicates that **in all perfect gases under the same conditions of pressure and temperature, there is the same number of molecules per cubic centimetre.** This is known as Avogadro's law.

If the gas A has a density greater than that of B, it follows that a molecule of A possesses a mass greater than that of a molecule of B. Hence, if the average kinetic energies of molecules of each gas are equal when the temperatures are the same, it follows that $\bar{v}_A^2$ must be less than $\bar{v}_B^2$. Air has a density 14 times that of hydrogen, and the mean speed of the molecules in hydrogen at normal temperature is 1800 metres per second, while the mean speed of the molecules of air at normal temperature is 450 metres per second.

Internal energy of a gas.—The **internal energy of a gas is the total heat energy stored in unit mass of the gas** by virtue of the motion and position of the molecules. Internal energy should be measured from absolute zero of temperature, but, generally speaking, there are no means available for making this estimation. It is convenient to select some arbitrary temperature—generally 0° C.—and to estimate the internal energy in excess of that possessed by the substance when at this temperature. The motion of the molecules depends upon the temperature only; hence if there is no change in the temperature of a gas undergoing changes of pressure and volume, there is no change in the internal energy of molecular motion.

Joule's experiment.—Joule made an experiment in which two vessels were connected by a pipe furnished with a tap. The tap was closed, and one of the vessels was exhausted as completely as possible, while the other vessel was charged with air. Both vessels were immersed in water and the temperature of the water was noted. On opening the tap, thus permitting the air to expand freely and fill both vessels, the temperature of the water after stirring was found to be the same as at first. It was thus inferred that no drop in temperature occurs when a gas is undergoing unresisted expansion, and hence there is no change in internal energy. This law is very nearly, but not quite true, as will be explained later.

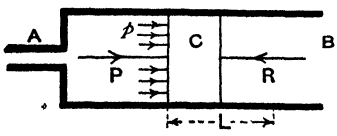


FIG. 410.—Work done by a gas

Work done by a gas at constant pressure.—In Fig. 410 is shown a cylinder fitted with a piston C. Gas under constant pressure p is admitted at A and pushes the piston through a distance L against a

resistance R. P is the total force exerted by the gas on the piston, and is equal to the product of p and the area of the piston, a say; since P is constant, the work done is given by

$$\text{Work done by P} = \text{PL} = pa\text{L}.$$

Now aL is the volume of gas which must be admitted in order to maintain constant the pressure p, and this is also the volume through which the piston sweeps in travelling a distance L. Writing v for this volume, we have

$$\text{Work done by P} = pv. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

This result will be in foot-lb. if, as is customary in engineering practice, p is in lb. wt. per square foot and v in cubic feet, and in centimetre-kilograms if p is in kilograms wt. per square centimetre and v is in cubic centimetres. In C.G.S. units, p is in dynes per square cm. and v in c.c.; the result is then in ergs.

We have obtained pv units of work by admitting v units of volume of gas; had only one unit of volume been admitted, the work done would be given by

$$\text{Work done per unit volume of gas} = \frac{pv}{v} = p. \quad \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

The diagram of work (see Chapter XIII.) done under the above conditions is shown in Fig. 411. AB represents the volume v swept by the piston; $v_1 = \text{OA}$ is the volume of gas present in the cylinder at the instant the piston begins to move, and $v_2 = \text{OB}$ is the volume of gas at the end of the movement. AC = BD represents the constant pressure p of the gas. The area ABDC represents the product pv, and therefore represents the work done by the gas.

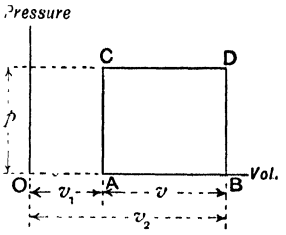


FIG. 411.—Work done by a gas at constant pressure.

Work done during the expansion of a gas.—If the supply of gas to a cylinder is cut off after the piston has moved through any given distance, and if the piston continues to move in the same direction, the pressure exerted by the gas will fall as the volume increases. The constantly diminishing pressure will continue to do work on the piston, but to a less amount in each successive centimetre of movement of the piston. In Fig. 412, $\text{OB} = v_1$ is the volume of gas in the cylinder at the instant that the supply is cut off, and $\text{BA} = p_1$ is the pressure at this instant. The diminishing pressure of the gas during the further movement of the piston is shown by the curve AC.

OD $= v_2$ is the volume to which the gas has expanded when the piston reaches D, the pressure being then p_2, represented by DC.

Consider the instant at which the pressure is EF $= p$ and the volume is OE $= v$. If the piston moves a further very small distance, the additional volume will be EG. Let this additional volume be written δv. The pressure during this small movement will remain sensibly equal to p, and the work done may be calculated from equation (1), p. 423, Work done during the small movement $= p \times \delta v$.

This work is represented by the area of the shaded strip EFHG (Fig 412) The area of similar strips will represent the work done during other small movements of the piston; hence the total work done during the expansion from the volume v_1 to the volume v_2 is represented by the area of the diagram BACD. If the gas is following Boyle's law by expanding at constant temperature, the curve AC may be plotted, and its area found by means of a planimeter, or by application of any convenient rule of mensuration. If the area is expressed in square centimetres, the result must be multiplied by the scale of pressure, say p kilograms wt. per square centimetre to a centimetre height of the diagram, and also by the scale of volume, say v cubic centimetres to a centimetre length of the diagram. The final result will then give the work done in centimetre-kilograms.

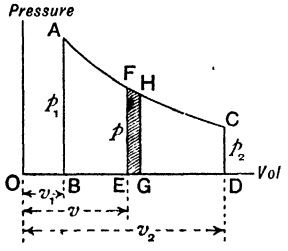
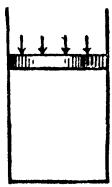


FIG. 412.—Work done by an expanding gas

Specific heat of a gas at constant volume.—If a given mass of gas be contained in a closed vessel of constant volume, the addition of heat will produce a rise in temperature, and the heat energy supplied will be stored completely by the molecules in the form of additional kinetic energy. No expansion has taken place, and therefore no work has been done against any external resistance. Under these conditions the heat which must be supplied per unit mass of gas in order to raise the temperature one degree is described as **the specific heat of the gas at constant volume**, and is written C_v.


FIG. 413

Specific heat of a gas at constant pressure.—In Fig. 413 is shown a cylinder fitted with a piston carrying a constant load and capable of travelling freely in the cylinder. The cylinder contains unit mass of gas under the piston, and the gas will be subjected to a constant pressure p_1 Let the initial temperature be

T_1 degrees absolute, and let the volume of the gas under these conditions be v_1.

Let the gas be raised in temperature through one degree by the addition of heat, when two effects will occur: (*a*) The volume will increase to v_2 in accordance with the law

$$\frac{v_1}{v_2} = \frac{T_1}{T_2} = \frac{T_1}{T_1 + 1}. \quad \text{.....(1)}$$

(*b*) The temperature of the gas has been raised, and therefore its store of internal energy has been increased.

Suppose the operation to take place in the following manner: Imagine a thin partition to be fitted to the cylinder, in contact with the lower side of the piston. This partition will prevent expansion taking place when the piston is raised. Let the piston be raised by application of an external force to such an extent that the total volume under it is v_2. Work will have to be done on the piston to an amount given by the product of p_1, and the volume swept by the piston (p. 423), *i.e.*

$$\text{External work done} = p_1(v_2 - v_1). \quad \text{.....(2)}$$

The space below the partition B (Fig. 414) now contains the gas; that between B and the piston A is a perfect vacuum. Let a hole be pierced in the partition so as to permit free expansion of the gas, which will then fill the whole space under the piston. This expansion will take place without change in temperature (p. 422), and we have now unit mass of gas at volume v_2 and temperature T_1.

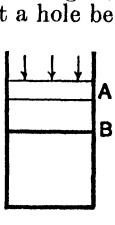


FIG. 414.

Keeping the piston fixed so as to maintain the volume constant, add sufficient heat to raise the temperature one degree. As the volume is constant, the heat required will be C_v, the specific heat at constant volume. Had the entire operation taken place without external assistance, we should have required to supply heat energy sufficient to perform the external work (given in (2) above) in addition to C_v. This additional quantity of heat may be calculated from (2) by dividing by J, the mechanical equivalent of heat. Hence

$$\text{Total heat required} = C_v + \frac{p_1(v_2 - v_1)}{J}.$$

This total heat represents the **specific heat of the gas at constant pressure**, C_p say. Hence

$$C_p = C_v + \frac{p_1(v_2 - v_1)}{J}. \quad \text{.....(3)}$$

From (1), $$v_2 = \frac{T_1+1}{T_1} v_1 ;$$

$$\therefore\ v_2 - v_1 = \left(\frac{T_1+1}{T_1}\right) v_1 - v_1 = v_1\left(\frac{T_1+1}{T_1} - 1\right) = \frac{v_1}{T_1} ;$$

$$\therefore\ C_p = C_v + \frac{p_1 v_1}{T_1} \cdot \frac{1}{J}.$$

Now, $$p_1 v_1 = RT_1 \quad \text{(p. 406)} ;$$

$$\therefore\ C_p = C_v + \frac{RT_1}{T_1} \cdot \frac{1}{J}$$

$$= C_v + \frac{R}{J}. \quad \text{..... .... (4)}$$

This equation enables the value of C_p for a given gas to be calculated, provided the other quantities are known. It is of interest to note that Mayer used known values of C_p, C_v and R in order to estimate the value of the mechanical equivalent of heat. He assumed, however, that the internal energy of a gas does not change during free or unresisted expansion, an assumption which was not valid until it had been confirmed by Joule's experiment (p. 422).

EXERCISES ON CHAPTER XXXII.

1. Give a brief explanation of the reasons for stating that the absolute temperature and the mean of the squares of the velocities of the molecules in a gas are proportional.

2. The total kinetic energy of the molecules of a gas and the absolute temperature are related. State the relation and explain it briefly.

3. Make use of the kinetic theory of gases to explain what is meant by absolute zero of temperature.

4. State Avogadro's law. What deduction can be made regarding the relation of the mean of the squares of the speeds of the molecules of different gases under like conditions of pressure and temperature.

5. What is meant by the "internal energy" of a gas, and how is it measured in practice?

6. Describe Joule's experiment on the free expansion of a gas. State the result and the inference which may be made.

7. Work is done on a piston by air at a constant absolute pressure of 14·7 lb. per sq. inch. How much work is done (*a*) per cubic foot of air, (*b*) per pound of air admitted to the cylinder? Take one pound of air to occupy a volume of 12·5 cubic feet.

8. Draw a diagram of work done during the expansion of 500 c.c. of air at an absolute pressure of 6 kilograms wt. per square centimetre. The final volume is 3000 c.c., and Boyle's law is followed. Scale of pressure,

1 cm. height to 1 kilogram wt. per square cm.; scale of volume, 1 cm. to 500 c.c. Find the area of the diagram, and hence calculate the work done.

9. In heating a building, 300,000 cubic feet of air at the temperature of 10° C. and 1 atmosphere pressure enter the heating appliance per hour; the temperature is raised to 16° C., and the pressure remains constant. Calculate the quantity of heat required per hour, given that the specific heat of air at constant pressure is 0·237 and that one cubic foot of air at 0° C. and 1 atmosphere pressure has a mass of 0·0807 pound.

10. In Question 9, calculate the quantity of heat which is used in doing external work while the air is being heated.

11. The specific heat of air at constant pressure is 0·237 and is 1·4 times that at constant volume. Find the heat required to raise the temperature of 100 kilograms of air from 0° to 100° C. at constant volume.

12. Explain why the specific heat of a gas at constant pressure is greater than its specific heat at constant volume. Hence show that $C_p - C_v = p/dJT$, assuming that no internal work is done when a gas expands. (d = the density of the gas at pressure p and absolute temperature T.) Calculate the value of J, taking $C_p = 0{\cdot}238$, $C_p/C_v = 1{\cdot}41$, and the density of air at normal temperature and pressure = 0·001293 gram per c.c. Bombay Univ.

13. The specific heat of hydrogen at constant pressure is 3·402 calories per gram, and the density at 0° C. and 76 cm. of mercury is 0·08987 gram per 1000 c.c. Take J = 42,000,000 ergs, and calculate the specific heat at constant volume. Find also the ratio of the specific heats.

14. Explain the meaning of the term "mechanical equivalent of heat." Which is the greater, the specific heat of a gas at constant pressure, or the specific heat at constant volume; and why? L.U.

15. Explain briefly the relation which is supposed to exist, according to the kinetic theory of matter, between the velocity of the molecules of a gas, its temperature, and the pressure it exerts against the walls of a containing vessel. How is it deduced from this theory that two different gases contain the same number of molecules per unit volume at the same temperature and pressure? Madras Univ.

CHAPTER XXXIII

THE EXPANSION AND COMPRESSION OF GASES IN PRACTICE

Isothermal and adiabatic expansion.—In **isothermal expansion** and **compression** of a gas, the operation takes place without change in the temperature. In **adiabatic expansion** and **compression**, no heat is allowed to enter or leave the gas during the operation. Strictly speaking, both methods of expansion and compression must be **reversible**, *e.g.* if expansion is going on, there may be certain alterations in the conditions of pressure, volume, temperature and internal energy; these conditions must be capable of exact reproduction in the reverse order if expansion be stopped and compression substituted.

Neither isothermal nor adiabatic operations in a gas can be realised perfectly in practice on account of reasons explained below, but they serve as useful standards for comparison with practical cases

Heat must be supplied during isothermal expansion.—It has been seen that no appreciable change in temperature occurs when a gas is allowed to expand freely (p. 422). In this method of expansion no external work is done, and the internal energy of the gas remains unaltered. External work is done, however, if a gas expands in a cylinder driving a piston which offers resistance. This external work may be done at the expense of some of the internal heat energy of the gas, in which case a fall in the temperature must occur; hence such an operation cannot be isothermal. If isothermal operations are to be secured, there must be no change in the temperature, and therefore no change in the internal energy. Hence, as none of the internal stock of energy is available for doing the external work, it follows that heat must be supplied continuously to the gas in quantity sufficient to perform the external work. The total heat thus supplied during the expansion must be equivalent to the total external work done.

Let W = the total external work done.

Then $\frac{W}{J}$ = the heat which must be supplied.

Practical difficulties in isothermal operations.—The supplying of the requisite quantity of heat during isothermal expansion constitutes a practical difficulty which cannot be overcome perfectly. There is no material known of which the cylinder might be constructed that will permit heat to pass rapidly enough into the gas as expansion proceeds. Further, heat will not flow from the walls of the cylinder into the gas unless the walls are at a temperature higher than that of the gas, and this heat, supplied at the boundary, has to be distributed through the entire volume of gas. A fairly close approximation to isothermal expansion may be obtained by causing the piston to move at a very slow rate, thus giving ample time for heat to enter through the cylinder walls and to distribute itself throughout the gas.

In isothermal compression external work is done upon the gas, and the internal energy will be increased unless heat is abstracted from the gas. Anyone who has used an ordinary tyre inflator has noticed the rise in temperature of the discharge end of the inflator. Isothermal compression is the converse of isothermal expansion, and heat must be removed from the gas in quantity equivalent to the external work done on the gas during compression, *i.e.* to W/J.

Practical difficulties in adiabatic operations.—The principal difficulty which prevents the realisation of adiabatic operations are due to the non-existence of any material which will absolutely prevent any leakage of heat, either inwards or outwards. The cylinder would require to be made of a material having perfect non-conducting qualities, and also having no capacity for heat.

From what has been said regarding isothermal operations, it will be understood that the store of internal energy will diminish during adiabatic expansion, which will therefore be accompanied by a fall in temperature. The converse takes place during adiabatic compression. Internal heat energy to an amount equal to the external work done by the gas disappears from the gas during adiabatic expansion; during adiabatic compression the internal energy is increased by an amount equal to the external work done on the gas.

In a cylinder made of any ordinary metal, the quantity of heat which can flow into, or through the walls, depends upon the time allowed for the flow to take place. An approximation to adiabatic expansion or compression can be obtained by conducting the operation very quickly; but little heat will then enter or escape. The compressions and expansions which take place in sound waves occur so rapidly that the changes are adiabatic.

Laws of expansion.—In isothermal operations performed on a perfect gas, Boyle's law is followed, viz.

$$pv = \text{a constant}$$

In adiabatic operations it may be shown that the law followed is

$$pv^{\gamma} = \text{a constant},$$

where γ is the ratio of the two specific heats of the gas, viz. $C_p \div C_v$.

In practice the law followed is of the form

$$pv^{n} = \text{a constant},$$

where the index n generally falls between the values 1 (the index for isothermal operations) and γ. The value of γ varies from 1·67 for the monatomic gases such as argon, mercury vapour, etc., to nearly 1 for gases having highly complex molecules.

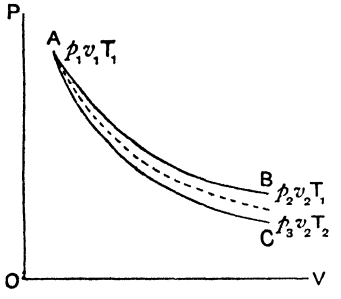


FIG. 415.—Expansion curves for a gas

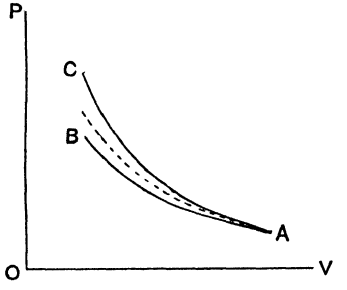


FIG 416.—Compression curves for a gas

In the pressure-volume diagram shown in Fig. 415 a given mass of gas has been taken under initial conditions p_1, v_1, T_1, and the point A has been plotted to represent these conditions. The curve AB represents isothermal expansion at constant temperature T_1, ending at B where the conditions are p_2, v_2 and T_1. AC represents adiabatic expansion; this curve falls below AB, since the temperature is falling continuously, and therefore the pressure will be lower at any volume corresponding to a selected point on AB. The terminal conditions at C are p_3, v_2 and T_2. The actual expansion curve obtained in any practical cylinder would generally fall between AB and AC, as is shown dotted in Fig. 415.

In compression operations (Fig. 416) and starting at A with given conditions of pressure, volume and temperature, isothermal compression is represented by AB and adiabatic compression by AC.

Since the temperature is rising along AC, it follows that the pressure at corresponding volumes will be higher than in isothermal compression; hence AC lies above AB. The practical compression curve would generally fall between AB and AC, as is shown by the dotted line in Fig. 416.

Air-exhausting pumps.—Pumps of this type are used for withdrawing air or other gases from closed vessels. A laboratory pump is shown in Fig. 417. The cylinder A is fitted with a piston B, which is rendered tight against air leakage by means of a leather ring. The piston has a valve which opens upwards, permitting the gas to pass from the lower to the upper side of the piston, but not *vice versa*. A similar valve is fitted at C. The piston rod D is worked by a lever (not shown) attached to the top end. The vessel to be exhausted is connected by rubber tubing to E. The gases enter at E, pass through F, and may enter the cylinder through a main passage G or through a by-pass H. The exit orifice to the atmosphere is at K.

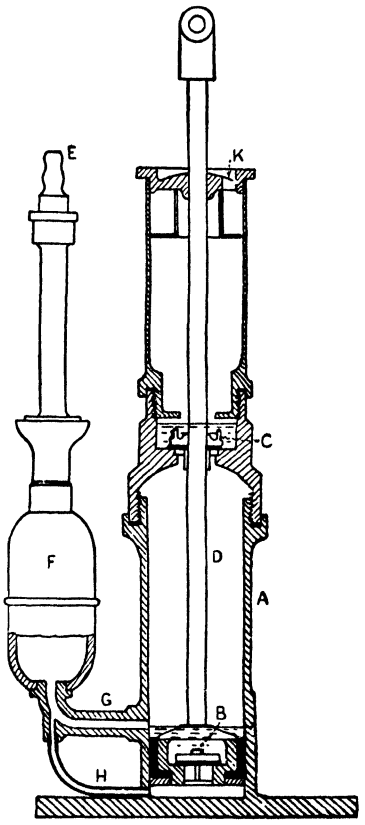


FIG. 417.—Air-exhausting pump.

Starting with the piston at the bottom of the stroke, the vessel to be exhausted is in communication with both sides of the piston through G and H; the pressures on the top and bottom of the piston are therefore equal, and the piston may be moved easily. Immediately the piston passes the opening of G, it begins to compress the gas in the space between the valve C and the piston. Compression goes on until the pressure of the enclosed gas is equal to that of the atmosphere (neglecting the weight of C), when the valve C lifts, and the gas is discharged through C and K during the remainder of the stroke.

The piston cannot quite arrive at the top of the cylinder, and hence all the gas will not be discharged Complete discharge can be obtained by introducing some oil, which lies on the top of the piston. As the piston approaches the top of the stroke, the oil fills ultimately the whole of the upper part of the cylinder, thus driving the entire gaseous contents through C Some oil will find its way through C, and will remain on the top of this valve during the next downward stroke.

During the upward stroke just described, more gas has been flowing into the lower part of A through G and H. The piston now descends, C closes and the space above B becomes more rarefied than that below; hence the valve in the piston opens, and the descent is completed with equal pressures on both top and bottom On the piston passing G, the function of the by-pass H is to permit the escape of any gas or liquid which may be trapped between the piston and the bottom of the cylinder; the piston may thus be pushed to the cylinder bottom, and is then ready to repeat the action.

During each upward stroke the volume of air removed is equal to that of the portion of the cylinder lying above G, this volume is at the pressure existing in the vessel under exhaustion at the beginning of the stroke considered.

Let V = the volume of the vessel up to the opening of G into the cylinder (Fig. 417).

v = the volume of the cylinder between B and C.

p_1 = the initial pressure in the vessel, taken as equal to that of the atmosphere.

Assuming that the temperature remains constant, we may find the pressure after any number of strokes by applying Boyle's law. Thus, the piston being at the bottom, the total volume is $(V+v)$; during the first upward stroke, the volume removed is v at pressure p_1. While this stroke is proceeding, the air in the receiver expands from V at p_1 to $(V+v)$ at p_2.

$$\therefore\ p_1V = p_2(V+v),$$

$$p_2 = \left(\frac{V}{V+v}\right)p_1. \qquad \text{.....(1)}$$

This gives the pressure at the end of the first upward stroke. There is no change in pressure during the next downward stroke; hence we have the piston again at the bottom, and the volume is $(V+v)$ at p_2. During the second upward stroke a volume v is removed

at pressure p_2, and the air in the vessel expands from V at p_2 to $(V+v)$ at p_3; $\therefore p_2V = p_3(V+v)$,

$$p_3 = \left(\frac{V}{V+v}\right) p_2 = \left(\frac{V}{V+v}\right)^2 p_1. \quad \text{.....(2)}$$

Similarly, at the end of the nth upward stroke the pressure is given by

$$p = \left(\frac{V}{V+v}\right)^n p_1. \quad \text{.....(3)}$$

Mercurial air pump.—The pump shown in Fig. 418 is used for exhausting the bulb A. A is connected at B to an inverted U tube CBD, having a bore of about 1 mm. Another tube EF has a funnel at its top end, and is connected by rubber tube FC to the U tube. Mercury is poured into the funnel, and the flow through the tubes is regulated by means of a pinch-cock G. The limb BD should be about one metre in length. In action the mercury passing the branch at B separates into drops, and after descending BD is discharged into a beaker. Air from the bulb A fills the spaces between the drops, and is swept down the tube by the descending mercury. As exhaustion proceeds, the air in A becomes rarefied, the spaces between the drops become smaller, and finally the column of mercury in BD is equal to the height of the barometer. The conditions in A are then similar to those in a Torricellian vacuum.

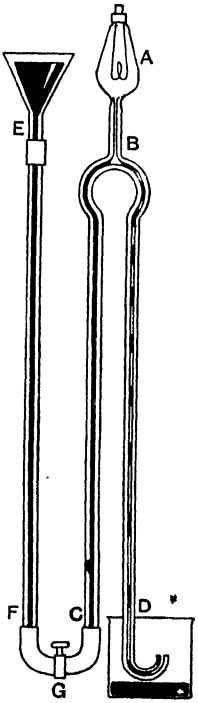


FIG. 418.—Mercurial air pump.

Gaede's molecular air pump.* — However smooth the surface of any solid may appear to be, there are two kinds of irregularities, viz., mechanical and molecular. The mechanical irregularities may be reduced by skilful workmanship, but the molecular irregularities cannot be so reduced. The formation of a film of gas, adhering to the surface of a solid in contact with the gas, is probably due to these molecular irregularities. If the surface be in motion, the adhering film moves with it, and in turn drags the adjacent layers of gas.

* See *Nature*, Vol. 90, January 23, 1913, p. 574.

The principle of the **Gaede molecular air pump** may be understood by reference to Fig. 419. A cylinder A rotates clockwise in a case B, and B has two openings n and m connected by a slot. The gas is dragged by the cylinder from n to m, and consequently a difference of pressure is established between n and m. This pressure difference is proportional to the speed of rotation and also to the internal friction of the gas. The latter is independent of the pressure, and hence the difference in pressure produced should also be independent of the pressure. This is true for relatively high pressures, but if it continued to be true down to the lowest pressures, we should be able to create an absolute vacuum by exhausting initially with another pump at n (Fig. 419) to a pressure lower than the constant difference of pressure between n and m. This is no longer the case for pressures lower than 0·001 mm. of mercury. If the surface of the cylinder A had a velocity greater than the molecular velocity, we would obtain an absolute vacuum, but such speeds are impossible in practice. However, at these low pressures the ratio of the pressures at m and n remains constant independently of the pressure, and it has been found that

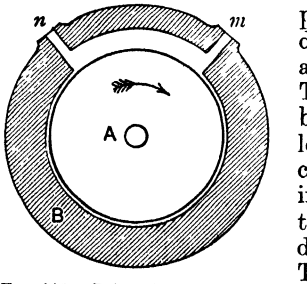


FIG. 419.—Principle of the Gaede molecular air pump.

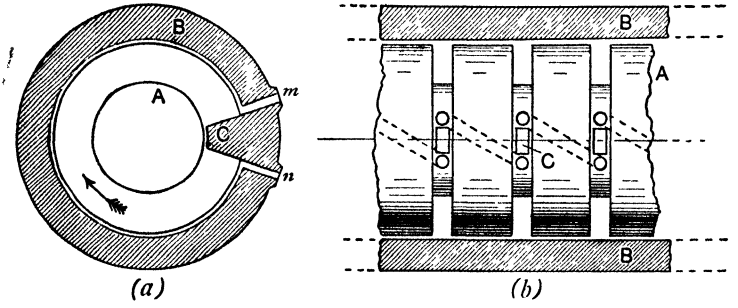


FIG 420.—Construction of the Gaede molecular air pump

speeds of 8000 to 12,000 revolutions per minute are sufficient to give a vacuum better than that produced by any other type of pump.

In practice the pump is constructed as shown in Fig. 420. The cylinder A is grooved, and a tongue C projects from the case into the groove; this is equivalent to a very long slot in the case. In

order to increase the efficiency, several grooves are cut in A, and are connected with one another so that the low-pressure side of one is the high-pressure side of the next (Fig. 420 *b*); there is thus a number of pumps in series. A preliminary pump is used in order to reduce the initial pressure to a few millimetres of mercury. The Gaede pump deals effectively with vapours as well as gases, since there is no compression during the removal and therefore no condensation of the vapours. An external view of the pump is given in Fig. 421.

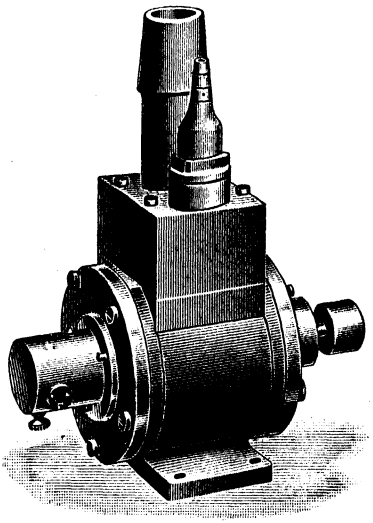


FIG. 421.—Gaede molecular air pump.

Very low pressures can be obtained by first exhausting the vessel as thoroughly as possible by means of a pump. A tube containing coconut charcoal, and in communication with the vessel, is immersed in a bath of liquid air. The remainder of the gas in the apparatus condenses in, and is absorbed by, the charcoal, thus producing a very low pressure.

M'Leod's pressure gauge.—The pressure gauge illustrated in Fig. 422 is suitable for measuring low pressures. A vertical tube ABCD is sealed at its top end; the portion AB is of small bore and there is a large bulb between B and C. The lower end is connected by flexible tubing to a mercury cistern E furnished with a tap. A branch at C leads to another tube FG, of the same bore as AB in order to avoid capillary effects, and thence to the vessel in which the pressure is to be measured.

With the mercury level adjusted so that the branch at C is just closed, the volume V contained between A and C is known from a previous calibration, as are also the volumes between A and various levels of the mercury in AB. A scale of volumes is attached to AB, zero of the scale being at A.

In using the gauge, the level of the mercury is first adjusted so as to be lower than C; the whole of the space above the mercury surface is filled with gas at the same pressure as that in the vessel. The cistern E is then raised slowly, and the mercury level in CD rises. On reaching C, the mercury seals the gas in ABC. Further

raising of the cistern ultimately produces the result shown in Fig. 422, in which the mercury surface is at K, and the entrapped gas has been compressed to a volume V′ as shown by the scale of volumes. The mercury level in FG is now at L, and is still subjected to the pressure which it is intended to measure; let this pressure be p mm. of mercury, and let h mm. be the difference in levels at K and L. Then the pressure of the volume V′ of gas in AK is $(p+h)$ mm. of mercury.

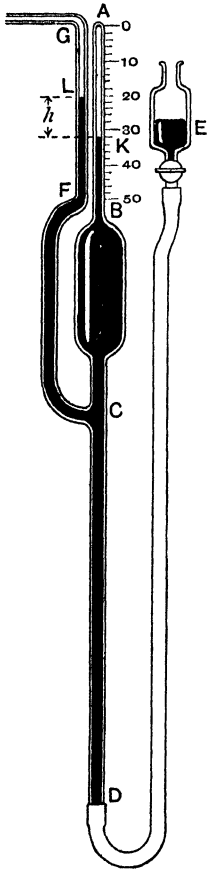


FIG. 422.—M'Leod's pressure gauge.

Assuming that there has been no change in the temperature of the room while the observations have been made, and that sufficient time has been given to permit the compressed gas in AK to return to its original temperature, we may apply Boyle's law. Thus:

$$pV=(p+h)V'=pV'+hV',$$

$$\therefore\ p(V-V')=hV',$$

or,
$$p=\frac{V'h}{V-V'}.$$

EXAMPLE.—In a M'Leod pressure gauge, V=50 c.c.; in measuring the pressure in an exhausted vessel, h was 8 mm. and V′ was 0·2 c.c. Find the pressure.

$$p=\frac{0\cdot2\times8}{50-0\cdot2}=\frac{1\cdot6}{49\cdot8}$$

$$=0\cdot0321 \text{ mm. of mercury.}$$

Action in an air compressor.—The use of compressed air for operating certain machines has become of great importance. The principal parts of an air compressor may be understood by reference to Fig. 423. A cylinder A is fitted with a piston B, which is driven up and down in the cylinder by means of a connecting rod CD, which is connected to the piston by a pin at C, and to a revolving crank ED. The crank is fixed to a shaft E, which is driven by some outside source of power, such as a steam engine or an electro motor.

At the top of the cylinder is a suction valve F, which opens when the piston is moving downwards, and this permits air from the atmosphere to flow into the cylinder. During the upward move-

ment of the piston the suction valve F is closed, and the air in the cylinder is compressed and delivered through a discharge valve G into a receiver not shown in Fig. 423. The discharge valve opens at the instant when the rising pressure in the cylinder has reached the pressure in the receiver, or a pressure slightly higher. Pipes connect the receiver to the machines to be driven.

The compression is kept as nearly isothermal as possible by means of water circulating in a water jacket H, which surrounds the cylinder. Cold water enters the jacket at K and is discharged at L. The practical objects of the cold-water jacket are twofold: (*a*) The working parts of the cylinder would otherwise become excessively hot, and damage would probably result. (*b*) Air discharged hot into the receiver will cool there by conduction of heat through the receiver walls into the atmosphere; this heat is wasted, and represents mechanical work done by the source of power on the piston of the compressor. Less power will be required to drive the machine if the temperature in the cylinder is prevented from rising considerably during compression.

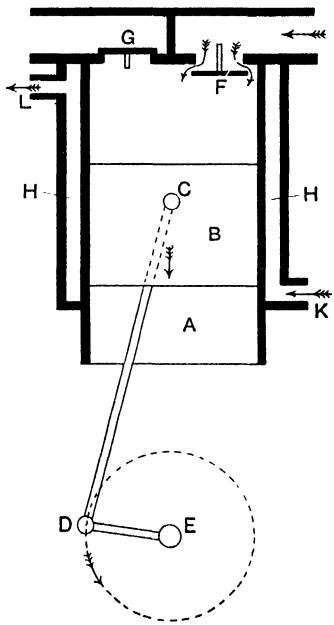


FIG. 423.—Section of an air compressor.

Of course heat is carried away by the water circulating in the jacket and is thus wasted, but it is more economical of power to abstract this heat from the air while still in the cylinder rather than from the air after it has passed into the receiver.

Diagram of work done in an air compressor.—A pressure-volume diagram for the air compressor is shown in Fig. 424. Starting with the piston at the bottom of the stroke, the cylinder is full of air at atmospheric pressure p_1, the volume being v_1; these conditions are plotted at A. As the piston moves upwards, compression (approximately isothermal) of the air takes place along the curve AB. Both

valves F and G (Fig. 423) are closed during the compression. When the pressure reaches the receiver pressure, p_2, the discharge valve G opens, and delivery into the receiver occurs at constant pressure p_2; this is shown in Fig. 424 by the horizontal line BC.

Delivery of air stops when the piston reaches the top end of the stroke. Had the whole of the contents of the cylinder been expelled into the receiver, the pressure in the cylinder would drop instantly to that of the atmosphere when the piston starts to move downwards. Total expulsion is not possible, since practical conditions require that there should be a small clearance volume, v_c, between the piston when at the top of the stroke, and the cover. Hence, at the beginning of the downward stroke there is a volume of air v_c at a pressure p_2. Expansion of this air takes place along the curve CD, and at D the pressure has fallen to p_1. The suction valve now opens, permitting a fresh supply of air to enter the cylinder, as shown by the horizontal line DA.

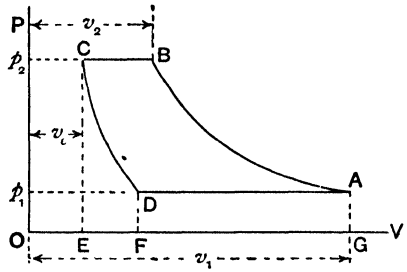


FIG. 424.—Diagram of work done in compressing air.

Assuming isothermal compression, and applying Boyle's law, we have

$$p_1v_1 = p_2v_2; \quad \therefore \ v_2 = \frac{p_1}{p_2}v_1.$$

Also, Air delivered into the receiver $= (v_2 - v_c)$, at pressure p_2.

Let v be the volume of this air when reduced to atmospheric pressure p_1, then

$$p_1v = p_2(v_2 - v_c),$$

$$v = \frac{p_2}{p_1}(v_2 - v_c) = \frac{p_2}{p_1}\left(\frac{p_1}{p_2}v_1 - v_c\right)$$

$$= v_1 - \frac{p_2}{p_1}v_c.$$

Thus a portion only of the total volume v_1 reaches the receiver.

The total work done on the air during the compression and delivery is represented by the area ABCEGA (Fig. 424). During the downward stroke of the piston the air in the cylinder does work on the piston to an amount represented by the area CDAGEC. Therefore the net work which must be supplied to the compressor during the two strokes of the piston (one upwards and one downwards) is represented by the difference in these areas, viz. ABCDA.

Action in charging an air receiver.—Reference is made to Fig. 425, in which A is an air compressor in which the piston travels

between B and G. Air enters the cylinder from the atmosphere through a valve C, and is discharged through a valve D and pipe DE into the receiver F, which initially contains air at atmospheric pressure p_1. At the beginning of the first compression stroke, the piston is at B and the cylinder is full of air at atmospheric pressure. In the pressure-volume diagram, *al* is a horizontal line representing pressures equal to p_1, and *b* is the point corresponding to the initial conditions in the cylinder. On pushing the piston inwards, the discharge valve D opens practically at once, since the pressures are equal on both sides of it, and air will be delivered into the receiver throughout the whole stroke; this first compression stroke is represented by the curve *bc*, and the piston comes to rest at G. In this stroke the initial volume v_1 is the total volume of air in the receiver, pipe and cylinder up to the piston at B. The final volume v_2 is the total volume in the receiver, pipe and cylinder up to the piston at G. Assuming isothermal compression, and applying Boyle's law, we have

$$p_1v_1=p_2v_2; \quad \therefore\ p_2=\frac{p_1v_1}{v_2}.$$

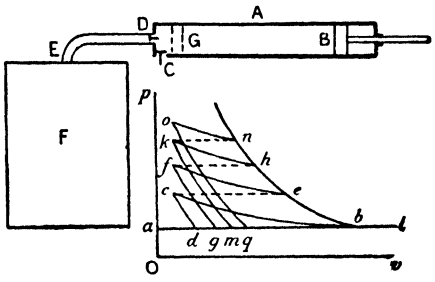


FIG. 425.—Action in charging an air receiver.

During the early part of the first return stroke, the air in the clearance volume of the cylinder, under initial conditions represented by the point *c*, expands down to atmospheric pressure along the curve *cd*. The remainder of the return stroke then takes place along the line *db*.

During the early part of the second compression stroke of the piston, both valves C and D are closed, since the pressure is rising, but is not yet equal to the pressure p_2 in the receiver. Hence the initial volume being dealt with is the volume in the compressor cylinder only, and the pressure therefore rises at a more rapid rate, as is shown by the line *be* (Fig. 425). At *e*, which is at the same height (or pressure) as *c*, the discharge valve D opens, and the remainder of the stroke is completed by compressing the total volume of air in the receiver, pipe and cylinder. This part of the stroke is shown by the curve *ef*; the rate of rise of pressure is lower in this stage than in the earlier part of the stroke, owing to the larger volume of air being compressed. Expansion from *f* back to atmospheric pressure takes place along the curve *fg*, and intake of fresh air along *gb* follows as before. Other two successive strokes are shown in Fig. 425 by the curves *bhkmb* and *bnoqb*. The

operations are repeated until the desired pressure is reached in the receiver.

The action of pumping up a bicycle tyre is similar to that described above, with a slight difference owing to the volume of the receiver (the rubber tyre) not being constant, but increasing to some extent as the charging process goes on. The pressure-volume diagram will be altered in this respect only—the curves *bc*, *ef*, *hk*, etc., will not rise so steeply; the other portions of the diagram in Fig. 425 will be unaltered.

Bell-Coleman refrigerating machine.—The earliest commercially successful refrigerating machines operated by taking advantage of the heating and cooling that occurs when a gas is compressed and expanded adiabatically, or approximately so. The action in the Bell-Coleman refrigerating machine may be understood by reference to Fig. 426. A is the chamber which is to be kept at a low temperature. A pump B draws air from this chamber and compresses it; during this operation the temperature of the air rises. The pump delivers the hot air at a pressure of 3·5 to 4 atmospheres into a pipe coil C, which is kept cool by means of cold water circulating round it. The air, after cooling in C, is fed into a motor cylinder D, where it is allowed to expand, doing work in driving a piston, and thus assisting the pump piston in B. During the expansion the air falls in temperature, and at the end of expansion the air is delivered at low temperature into the refrigerator chamber A, where it again takes in some heat from the walls of the chamber, and from the mutton and other substances being chilled. A steam engine, or some other source of power, is used to drive the pump.

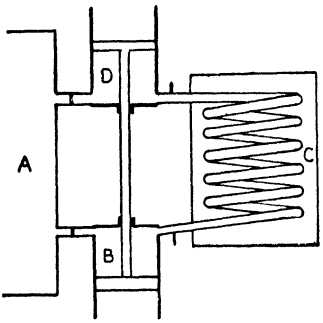


FIG. 426.—Arrangement of the Bell-Coleman refrigerating machine

EXERCISES ON CHAPTER XXXIII.

1. Define isothermal and adiabatic expansion of a gas. Explain how these operations may be realised approximately.

2. Some gas is contained in a cylinder fitted with a piston, and does work on the piston whilst expanding. If the expansion is isothermal, heat

must be supplied. Explain this, and state what quantity of heat is required in order to maintain constant the temperature.

3. A metal syringe has a nozzle adapted to receive a small piece of tinder. If the piston of the syringe be pushed in very rapidly, the tinder may be ignited. Give a full explanation of this.

4. 880 cubic inches of air at a pressure of 90 lb. wt. per sq. inch absolute are expanded to a volume of 3520 cubic inches. Calculate the final pressure (*a*) if the expansion is isothermal, (*b*) if the expansion follows the law $pv^{1\cdot4}$ = a constant. (*c*) if the law is $pv^{1\cdot2}$ = a constant.

5. In Question 4 (*b*), find the final temperature of the air if the initial temperature is 40° C.

6. Make an outline sketch of the principal parts of a machine for compressing air and describe the action.

7. A bicycle tyre has a capacity of 200 cubic inches when fully inflated, the pressure being then 2·5 atmospheres (absolute). If the tyre initially is quite flat, calculate the volume of air at atmospheric pressure required to inflate it.

8. An air receiver has a capacity of 20 cubic feet, and contains air at one atmosphere absolute pressure. How many cubic feet of air at atmospheric pressure must be pumped into the receiver in order to attain a pressure of 6 atmospheres (absolute) Assume that the temperature does not alter. Sketch approximate pressure-volume diagrams for the first three strokes of the pump.

9. A vessel to be exhausted of air contains 2400 c.c. at a pressure of 76 cm. of mercury absolute. The air-pump is similar to that shown in Fig. 417 and removes 160 c.c. of air during the first upward stroke. What will be the pressure in the vessel at the end of the fifth upward stroke ?

10. The cylinder of an air compressor is 7 inches in diameter, and the piston has a stroke of 10 inches. When the piston is at the end of the stroke, the clearance volume is 5 cubic inches. Air is taken in at a pressure of one atmosphere and is compressed isothermally to 6 atmospheres (both absolute pressures) before being discharged into a receiver. What volume of air, measured at atmospheric pressure, is discharged each stroke ? Sketch a diagram showing the action.

11. Give an outline sketch and explain the action of a refrigerating machine using air.

12. What are adiabatic and isothermal changes ? Explain why the barrel of an ordinary bicycle pump becomes heated when air is pumped into a tyre. Allahabad Univ.

13. Give an account, with a sketch, of some form of gas-pump suitable for the attainment of very low pressures ; and of a gauge which will measure such low pressures. Madras Univ.

14. Explain the action of a mercury air-pump. Show how to find the pressure of the air in the receiver of a simple air-pump at the end of ten strokes of the piston, the volume of the barrel and the receiver being 50 c.c. and 200 c.c. respectively. If the valve at the bottom of the barrel is a metal disc of area 10·6561 sq. inch, and its weight is 1·16 oz., find after how many strokes maximum exhaustion will be reached, assuming the atmospheric pressure to be equal to 14·4 lb. per sq. inch. Presidency College.

15. In a M'Leod pressure gauge (Fig. 422, p. 436), the tube AB has a bore of 1 mm. and is graduated in mm. The total volume between A and C is 100 c.c. In measuring the pressure in a vessel to which this gauge is connected, the following readings were taken: Level of the mercury at K, 56·3 mm.; difference in levels at K and L, 4·6 mm. Find the pressure in the vessel.

16. In Question 10, assume that the compressed air in the clearance space expands in accordance with Boyle's law, and calculate the distance travelled by the piston during the suction stroke before the suction valve opens.

CHAPTER XXXIV

CHANGE OF STATE

Change of state from solid to liquid.—A solid may be conceived as a collection of molecules which preserve their relative mean positions in ordinary circumstances. Each molecule may vibrate in a comparatively small space, but does not leave that space. If heat be imparted continuously to a solid, a temperature is reached at which the molecular motions have increased to such an extent that cohesion is no longer possible. At this temperature a change of state from solid to liquid occurs. That cohesion has broken down is shown by the ease with which the blade of a knife may be passed through water compared with the difficulty experienced in cutting a block of ice. The molecules make extended excursions in a liquid, and currents of molecules are set up easily.

Melting point.—The **melting point** of a substance is the temperature at which change of state from solid to liquid occurs. This temperature is generally the same as that at which solidification of the same substance takes place—a temperature which is called the **freezing** point of the substance. Different substances have different melting points; thus, ice melts at a temperature much lower than paraffin wax.

The melting points of some substances are well defined, and are thus determined easily. Others, such as glass and wrought iron, have an intermediate plastic stage in which the material can be worked into different shapes. Some substances expand when freezing, others contract. Thus the volume of a mass of ice is greater than that of the water from which it was formed (p. 340). Cast iron expands on solidification, a fact which enables sharp, accurate castings to be made; the molten metal is poured into a mould and fills it completely during solidification. Paraffin wax is an example of a substance which contracts while solidifying.

Influence of pressure upon the melting point of a substance.—Water freezes at 0° C. when the pressure is one atmosphere. If the pressure is increased, the expansion which must take place during solidification is prevented partially and the freezing point is lowered; the substance thus remains liquid while the temperature is lowered below 0° C. In general, substances which expand in solidifying have their freezing points lowered by increase in pressure; others which show contraction in freezing have the freezing points raised by increase of pressure. Thus the melting point of ice is lowered by about 0·0072° C. for each increase in pressure of one atmosphere. Paraffin wax melts at 46·3° C. at a pressure of one atmosphere, and at 49·9° C. at 100 atmospheres.

That the freezing point of water is lowered by increase of pressure was proved experimentally by Lord Kelvin, who applied pressure to ice contained in a closed glass vessel by means of a screw tapped into a hole in the cover of the vessel. The vessel contained a thermometer in a case so as to protect it from the effects of the high pressure. The following simple experiment illustrates the same fact.

EXPT. 93.—**The freezing point of water is lowered by pressure.** Let a block of ice rest on two supports; attach a heavy weight to a loop of copper or iron wire and pass the loop round the ice. The pressure of the wire on the ice causes the freezing point to be lowered, and the ice melts under the wire. Ultimately the wire passes completely through the block. During the passage of the wire, the water formed on the lower side of the wire passes round the wire to the upper side, and being relieved of pressure, freezes again. Thus the block of ice at the end of the experiment is still one solid body.

In skating on ice, if the skates are sharp and in good order, the pressure on the sharp edge is sufficient to cause momentary melting of the ice under the edge. Thus a person may be said to be skating on water.

EXPT. 94.—**Determination of the melting point of a substance.** The following method may be used for substances having comparatively low melting points, such as paraffin wax, sulphur, etc. A small quantity of the substance is enclosed in a short piece of thin glass tube of small bore; the tube is then fastened to the stem of a thermometer, near the bulb (Fig. 427). Both are then placed in a test tube fitted with a cork to keep the thermometer steady. The test tube contains some liquid which may be warmed, and has a wire stirrer fitted. The liquid chosen should have a boiling point higher than the melting point of the substance under examination; water may be used for finding the melting point of wax, and oil, or sulphuric acid, in determining the melting point of sulphur.

Heat the test tube gently and stir constantly until the substance is observed to melt. Note the temperature and allow the tube to cool. Note the temperature at which solidification is observed to occur. Repeat the

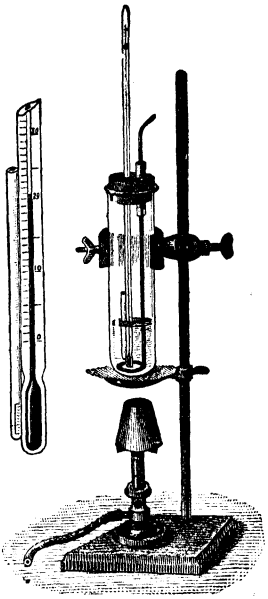


FIG. 427.—Apparatus for determining melting points.

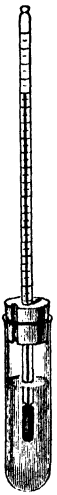


FIG. 428.—Melting point by cooling.

operations several times and take the mean temperature as the melting point of the substance.

EXPT. 95.—**Melting points by cooling experiments.** Referring to Fig. 428, a test tube contains some paraffin wax or naphthalene, and is fitted with a cork and thermometer. The cork has a groove cut up one side, so that it does not fit air-tight. Heat the tube until the substance is melted, and raise the temperature about 10° C. higher than the melting point. Clamp the test tube at some height above the table, and observe the temperature every half-minute during cooling. Continue until the substance has solidified and cooled considerably below the freezing point.

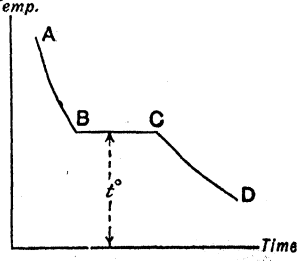


FIG. 429.—Cooling curve, showing the temperature of solidification.

Plot temperature and time, giving a graph resembling Fig. 429. AB shows the fall in

temperature of the liquid; BC indicates steady temperature conditions while solidification is taking place, and CD shows the further fall in temperature of the solid substance. The melting point is t, and is shown by the height of the horizontal line BC.

This method of cooling is much employed in the investigation of metallic alloys, and gives valuable information regarding the temperatures at which various constituents of the alloy change into the solid state.

Latent heat of fusion.—In testing the freezing point of a thermometer (p. 316), it has been observed that the temperature remained steady whilst the ice was melting. Expt. 95 illustrates the same fact with other substances. Since melting and freezing take some time to complete, it is evident that heat is entering the substance during liquefaction and is leaving it during solidification. The **latent heat of fusion** of a substance is the quantity of heat which must be imparted at constant temperature to unit mass of the substance in the solid state in order to effect the change of state from solid to liquid.

EXPT. 96.—**Latent heat of fusion of ice.** Weigh a copper calorimeter; pour in about 300 c.c. of water; weigh again and thus ascertain the mass of the water. Take a piece of ice weighing about 50 grams; wrap it in blotting paper in order to remove moisture. Take the temperature of the water in the calorimeter; and drop the dry ice into the water. Stir gently until the ice has disappeared and note the temperature at this instant. Weigh again in order to find the mass of ice used.

Let m = the mass of the calorimeter, in grams.
s = the specific heat of its material.
m_1 = the mass of water used, in grams.
m_2 = the mass of ice used, in grams.
t_1 = the initial temperature of the water, deg. Cent.
t_2 = the final temperature, deg. Cent.
L = the latent heat of fusion of ice, in calories.

The ice has taken in latent heat while melting, and the resulting water has been raised in temperature from 0° to t_2° C. Assuming that the heat taken up by the ice and resulting water is equal to that given up by the water originally in the calorimeter and by the material of the calorimeter, we have

$$m_2(\mathrm{L} + t_2) = (m_1 + ms)(t_1 - t_2);$$

$$\mathrm{L} = \left(\frac{m_1 + ms}{m_2}\right)(t_1 - t_2) - t_2.$$

The latent heat of fusion of ice is about 80 calories per gram. Compare this value with the result of the experiment.

EXPT. 97.—**Latent heat of fusion of paraffin wax.** This experiment is carried out in the same manner as Expt. 96, excepting that melted paraffin wax is substituted for the ice. The calculation differs somewhat, as account must be taken of the heat given up by the liquid wax while cooling to its freezing point. The latent heat given up while solidification is taking place, and the heat given up by the solid wax while cooling to the final temperature of the mixture.

Let m_1 = the mass of water in the calorimeter, in grams.
m_2 = the water equivalent of the calorimeter, in grams.
m_3 = the mass of wax used, in grams.
t_1 = the initial temperature of the liquid wax, deg. Cent.
t_2 = the freezing point of the wax, deg. Cent.
t_3 = the initial temperature of the water, deg. Cent.
t_4 = the final temperature of the mixture, deg. Cent.
s_1 = the specific heat of the wax in the liquid state.
s_2 = the specific heat of the wax in the solid state.
L = the latent heat of the wax, in calories.

The specific heats s_1 and s_2 may be obtained by application of the methods explained in Chapter XXVI. Assuming that the heat given up by the wax is equal to the heat taken up by the water and by the material of the calorimeter, we have:

$$m_3 s_1(t_1 - t_2) + m_3 \mathrm{L} + m_3 s_2(t_2 - t_4) = (m_1 + m_2)(t_4 - t_3)$$

or,
$$\mathrm{L} = \frac{(m_1 + m_2)(t_4 - t_3) - m_3\{s_1(t_1 - t_2) + s_2(t_2 - t_4)\}}{m_3}.$$

Freezing points of solutions.—When a solid is dissolving in a liquid, *e.g.* common salt in water, the solid takes in latent heat of fusion; this heat is derived from the store of heat in the liquid, and hence a cooling effect is produced. If the action is not merely the dissolving of the solid in the liquid, but includes chemical combination of the two substances, heat may be generated by the chemical process, and there may be a rise in temperature. An example of this occurs when a stick of caustic potash is dropped into water, considerable rise in temperature taking place.

The freezing point of a solution is always lower than that of the solvent. Thus, if equal weights of ammonium nitrate and water, both at 0° C., be brought together, the temperature of the resulting liquid will be found to be about −15° C., a temperature which renders the mixture useful in cooling operations. Another useful freezing mixture is produced by mixing equal weights of common salt and snow or pounded ice.

Determination of specific heats by the Bunsen ice calorimeter.—This method is useful when a small quantity of the substance only is available. The apparatus employed (Fig. 430) consists of a test tube A fused into a bulb B. A tube C leads from the bottom of B and has an iron coupling D, which serves as a connection for another fine bore tube S. S has a millimetre scale attached to it. The part of B surrounding A contains pure water which has been boiled, and the lower part of the bulb B and the whole of the tube CD and part of S contain pure mercury. The position of the end of the mercury in S may be adjusted by pushing the tube to a greater or less extent into the collar D. Some of the water in B is frozen by a process of circulating chilled alcohol into and out of the test tube A, or by evaporating ether in the test tube. The whole apparatus is then immersed in fresh pure snow. The shell of ice round A may be from 6 to 10 mm. thick.

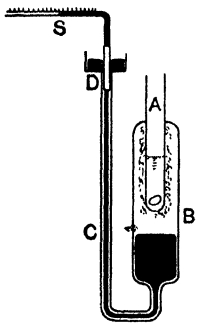


FIG. 430.—Diagram of Bunsen's ice calorimeter.

The apparatus is calibrated by introducing some pure water into A. If the mass of this water is m and its initial temperature t, then it gives out mt units of heat in falling to 0° C. In consequence of this, some of the surrounding ice melts and shrinks in volume, and the mercury in S recedes. Let the recession amount to n scale divisions, then one scale division of movement corresponds to mt/n units of heat. It will be noted that the principle relied upon for the measurement is that of the latent heat of ice, combined with the change of volume which takes place when ice melts.

A fragment of the substance to be tested is now heated in a heater (p. 347) and dropped into some water already in A. A plug of cotton-wool at the bottom of A prevents fracture of the tube. Let m_1 and t_1 be the mass and initial temperature of the substance. Suppose the mercury recedes u_1 scale divisions while the substance is cooling to 0°, and let q be the quantity of heat corresponding to 1 scale division. Then, if s is the specific heat of the substance,

$$m_1 s t_1 = u_1 q,$$

$$s = \frac{u_1 q}{m_1 t_1}.$$

The position of the mercury in S does not remain constant when the instrument is not in use; hence a correction has to be applied. Observe the movement of the mercury during half an hour before the experiment and again for half an hour after finishing the experiment. Let the movements be respectively u_a scale divisions in t_a

minutes and u_3 scale divisions in t_3 minutes. Then the average movement is given by

$$\text{Average rate of variation} = \tfrac{1}{2}\left(\frac{u_2}{t_2} + \frac{u_3}{t_3}\right).$$

The correction to be applied in the experiment is this quantity multiplied by the duration of the experiment in minutes.

Change of state from liquid to vapour.—In liquids, collisions between the molecules will be frequent, as there is but little space in which a molecule can move. Heat imparted to the liquid increases the speed of the molecules, thus enabling the liquid to store the additional energy. The surface film (p. 298) prevents the escape of most of the molecules, but those molecules which happen to have a speed considerably higher than the mean speed when in the neighbourhood of the surface may break through the surface film and escape. Molecules which have escaped in this manner mingle with the gas over the liquid and behave in the same way as gas molecules. The action is called **evaporation**, and the accumulated escaped molecules are called a **vapour**. If the liquid is contained in an open vessel, evaporation would lead in time to the total disappearance of the liquid.

The process of evaporation may be hastened by raising the temperature of the liquid, thus increasing the mean speed of the molecules and so enabling them to break more easily through the surface film.

Evaporation in a closed vessel.—Suppose there to be some liquid in a closed vessel from which air and all gases, other than vapour formed from the liquid, have been extracted. Molecules will be escaping continually from the liquid; other molecules in the vapour will occasionally strike the surface of the liquid, and will penetrate the surface film, thus rejoining the liquid; the latter operation is called **condensation**. Conditions will be attained ultimately in which the number of molecules escaping from the liquid per second is balanced exactly by the number of molecules returning per second. There is then a definite number of molecules per c.c. in the vapour space of the vessel, and the vapour is said to be **saturated**. A **saturated space** is one in which no greater number of molecules can be maintained under the existing conditions.

Saturated conditions are attained very quickly in a closed vessel such as has been assumed above. In fact, the space available for vapour is always saturated. Both liquid and vapour will be at the

same temperature. Raising the temperature of the vessel and its contents causes an increase in the mean speed of both liquid and vapour molecules. The effect of the former is to render easier the escape of more molecules, and of the latter, to increase the pressure on the walls of the vessel due to the bombardment of vapour molecules. At any stated temperature, the saturated vapour space contains a definite number of molecules per unit volume, having a definite mean speed, and hence producing a definite pressure on the walls of the vessel. Hence the pressure of a given saturated vapour at a given temperature is constant.

Saturated vapour.—Reduction of the capacity of the vapour space available in a closed vessel will not lead to any increase in the pressure of the vapour, provided the temperature of the vessel and its contents be maintained constant. Saturated vapour at a given temperature can have one pressure only, and reduction of the capacity of the vapour space will therefore produce condensation of some of the vapour. If such reduction be continued, the whole of the vapour condenses ultimately, the effect being completed when the total capacity of the vessel is sufficient to accommodate only the given quantity of liquid at the given temperature.

Saturation conditions in any space, of whatever capacity, are independent of the volume, but there must be sufficient liquid available to form enough vapour to saturate the space. If the quantity of liquid be insufficient, the whole of it will be evaporated before saturation of the space occurs; the vapour is then said to be **unsaturated.**

Superheated vapour.—If saturated vapour be conducted away from a closed vessel by means of a pipe, and if heat be then imparted to it without the pressure being allowed to rise, the temperature will increase, accompanied by an increase in the mean speed of the molecules, and the vapour is found to behave more like a perfect gas, *i.e.* it obeys the laws of Boyle and Charles. Such vapour is no longer in a condition in which it may be very easily converted into the liquid state, and is said to be **superheated vapour.** The term is synonymous with the term unsaturated vapour.

EXPT. 98.—**Maximum vapour pressure at the temperature of the room.** Arrange two barometer tubes A and B (Fig. 431) as previously di ected (p. 259). Bend the point of a small pipette C, and charge it with the liquid whose vapour pressure is to be examined. Put the point of the pipette

under the open mouth of B, and by blowing introduce a very small quantity of the liquid; this will rise to the surface of the mercury at D, and if the quantity of liquid is small enough, the whole of it will be evaporated. The space above the mercury in B thus becomes filled with unsaturated vapour, and the level of the mercury at D falls a little owing to the pressure exerted by this vapour. Introduce another small quantity of the liquid, when again the whole of it may be evaporated, and the level of the mercury falls further, indicating an increase in the pressure in B. Continue the process until sufficient liquid is introduced to produce a very thin layer of liquid lying on the mercury surface at D. The space in B is then full of saturated vapour at the temperature of the room, which should be noted. The pressure exerted by the saturated vapour is obtained by measuring the difference in levels of the surfaces of the mercury in A and B.

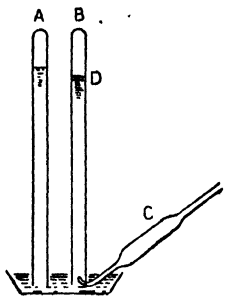


FIG. 431.—Vapour pressure at the temperature of the room.

Verify the fact that no increase in pressure occurs when another small quantity of the liquid is introduced into B. The absence of further fall of the mercury level at D shows that the pressure exerted by the vapour when saturated at the temperature of the room is constant.

The pressure exerted by a vapour when saturated at a given temperature is the maximum pressure which the vapour can exert when at this temperature, and is called the **maximum vapour pressure.**

EXPT. 99.—**The maximum vapour pressure is independent of the volume or the space occupied.** In the apparatus shown in Fig. 431, use a mercury bath deep enough to permit the tube B to be raised, or lowered a few centimetres. Having charged B with sufficient liquid to obtain saturated conditions and measured the difference in mercury levels, lower B in the bath. It will be found that the effect is to diminish the volume of the vapour space; some of the vapour condenses into the liquid state, and on measuring the difference in levels of the mercury in A and B, this will be found to be the same as at first. Now elevate B, thus increasing the vapour space; this should not be carried to excess, or the whole of the liquid will be evaporated and the vapour will be unsaturated. Again measure the difference in mercury level and compare with the initial reading. The result shows that, so long as the vapour is saturated, the pressure at the constant temperature of the room remains unaltered.

EXPT. 100.—**Maximum pressure of aqueous vapour at lower temperatures.** In Fig. 432, ABC is a closed bent tube having a bulb at C. The portion BD and part of the bulb contains mercury; the portion AB is a Torricellian

vacuum (p. 259). The bulb contains some water lying on the surface of the mercury and the remainder of the bulb contains vapour of water. A scale is attached to the straight part of the tube, and the bulb and bent part of the tube are immersed in a beaker of water. The water in the beaker may be heated by means of a bunsen flame, and its temperature is measured by a thermometer E, placed near the bulb. The straight part of the tube and the scale are shielded from the flame and from the beaker.

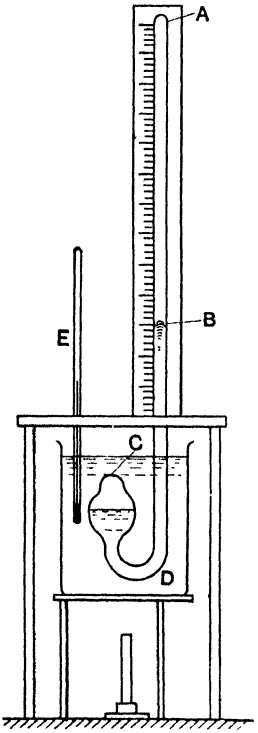


FIG. 432.—Maximum vapour pressure at lower temperatures

The aqueous vapour in the bulb is saturated so long as there is any water in the bulb, and its pressure is equivalent to the mercury head between the surfaces in B and C. The level in B is read directly on the scale; that in the bulb is taken by means of a metal rod 20 cm. long, arranged to slide up and down along the scale. The lower end of the rod is adjusted to the mercury level in C, and the top end of the rod is read on the scale. The head of mercury is then equal to the scale reading at B minus scale reading at the top end of the rod plus 20 cm.

Maintain steady the temperature of the water in the beaker for a few minutes so as to ensure that the bulb and its contents are at this temperature. Read the temperature and the mercury head as directed above. The result gives approximately the maximum aqueous vapour pressure at the measured temperature.

Since the whole of the mercury column is not at the same temperature, a correction should be applied. Measure the height of the mercury column from B down to the surface of the water in the beaker (the metal rod will assist in taking this measurement); let this be h_a cm. Measure also the difference in levels of the water surface in the beaker and the mercury surface in the bulb; let this be h_w cm. Let the observed temperature of the air in the room be t_a deg. Cent., and that of the water in the beaker t_w. Let β be the coefficient of expansion of mercury (0·000181). Then the corrected head h_0 is given by:

$$h_0 = h_a + h_w - \beta \left\{ h_w t_w + h_a \left(\frac{t_w + t_a}{2} \right) \right\}.$$

Repeat the experiment at intervals of about 10° C. up to the range

of the instrument. Plot a graph showing the relation of the maximum vapour pressure and temperature of aqueous vapour.

EXPT. 101.—**Maximum pressure of aqueous vapour at higher temperatures.** The apparatus employed is shown in Fig. 433. The bent tube at A contains mercury standing in both limbs, and the space in the closed shorter limb contains some water and water vapour only. The tube is immersed in a beaker B containing glycerine, the temperature of which can be raised, with care, considerably above 100° C. The tube A is connected to a reservoir C containing air, and the reservoir is connected to a U gauge D containing mercury, and also to an air pump (not shown in Fig. 433) by means of which air may be forced into the apparatus.

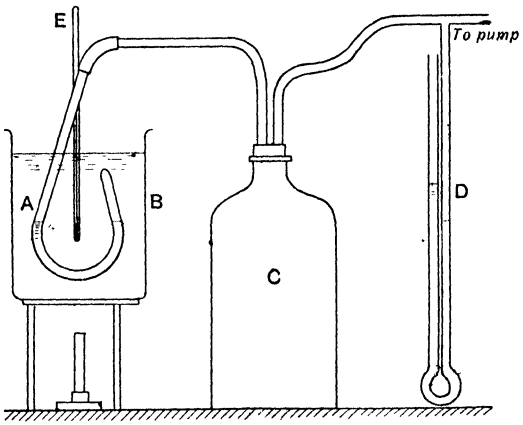


FIG. 433.—Maximum vapour pressure at higher temperatures.

The glycerine bath is brought to the temperature of 100° C., as shown by the thermometer E, and the mercury surfaces in the limbs of A are brought to the same level. During this operation the pump is disconnected and the connection is left open to the atmosphere; hence the pressure on both surfaces in A will be that due to the atmosphere, and will be found by reading the barometer. Connect the pump again and raise the temperature of the bath to say 105° C.; pump in a sufficient quantity of air to restore the mercury to the same levels in A; this level had been disturbed by the increasing aqueous vapour pressure in the shorter limb. When the conditions are steady, read the difference in levels in the gauge D, and obtain the maximum vapour pressure corresponding to the temperature in the bath by adding this difference to the barometer reading.

Repeat the experiment at several different temperatures, and plot a graph showing the relation of maximum vapour pressure and temperature.

Boiling point of a liquid.—Let an open vessel contain some water, and let the pressure of the atmosphere be 76 cm. of mercury. Let heat be imparted continuously to the water, when vapour will be given off constantly. Saturated aqueous vapour under a pressure of 76 cm. of mercury has a temperature of 100° C. Hence, when the water reaches this temperature, it becomes possible for bubbles of saturated water vapour to form just underneath the surface of the water. The water near the bottom of the vessel is at a pressure greater than that of the atmosphere by an amount equal to the head of water in the vessel. Therefore, if the temperature of the water be raised slightly above 100° C., in fact to the saturation temperature corresponding to the pressure near the bottom of the vessel, bubbles of vapour can form at the bottom. These ascend to the surface, enlarging as they travel owing to the diminishing pressure, and disengage themselves from the liquid on reaching the surface. The water is then said to **boil**, and the formation of bubbles in the water is called **ebullition**. The temperature at which a liquid boils under standard atmospheric pressure of 76 cm. of mercury is called the **boiling point** of the liquid, and is **the temperature at which the maximum vapour pressure of the substance is equal to standard atmospheric pressure.**

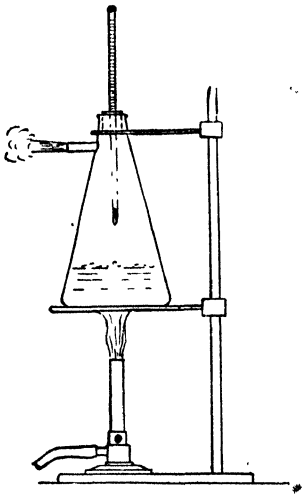


FIG. 434.—Boiling point of a liquid.

EXPT. 102.—**Boiling points of solutions.** Arrange apparatus as shown in Fig. 434. See that the flask is perfectly clean, and introduce some tap water. Heat the water; note the ascending bubbles of air which are liberated when the water becomes hot. When the boiling point is nearly reached, some bubbles of vapour may form near the bottom and collapse before reaching the surface owing to the colder water in the neighbourhood of the surface. When ebullition is taking place freely, note the temperature of the vapour. Push the thermometer down into the water, and verify that the temperature of the boiling water is very nearly equal to that of the vapour.

Boiling with bumping sometimes occurs; a short quiescent period is followed by the formation of a very large bubble which raises the mass

of water; the water falls a[illegible]. If this occurs, the thermometer will probably [illegible] of the water has risen a little. Some pieces of b[illegible]ped into the vessel will remedy bumping by promoting the [illegible]small bubbles at many parts of the liquid.

Since the bubbles of vapour are under saturated [illegible]ns, it follows that the temperature must remain steady throughout the whole period of boiling.

Prepare a number of solutions of common salt in water, say 2, 4, 6, etc. per cent. of salt. Introduce one of these into the vessel (Fig. 434) and have the thermometer in the vapour space. Bring the solution to boiling and note that the thermometer reads the same as if pure water had been in the vessel. Now lower the thermometer into the liquid, when it will be found that the temperature of the liquid is higher than the boiling point of pure water. It follows that the boiling point of a solution should be determined with the bulb of the thermometer immersed in the liquid. Find the boiling points of the remaining solutions. The temperatures

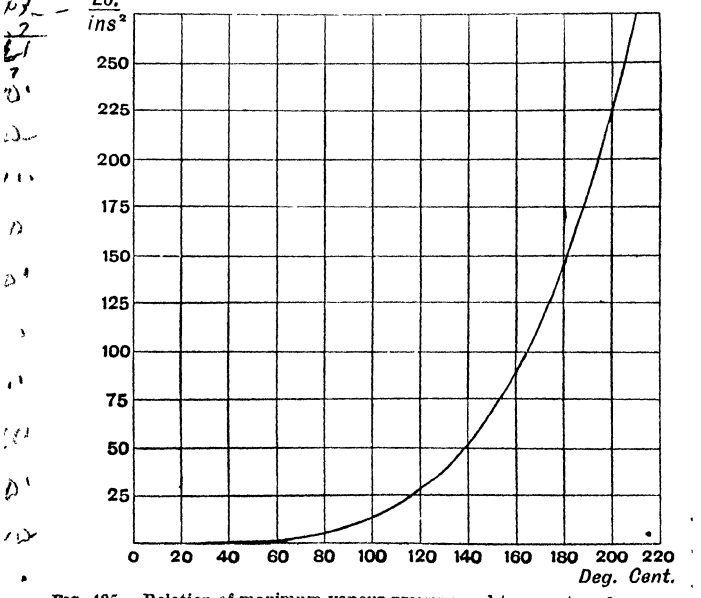


FIG. 435.—Relation of maximum vapour pressure and temperature for aqueous vapour.

should be noted immediately after the solution starts to boil, as the process of evaporation obviously concentrates the solution. Plot a graph showing the relation of boiling points and percentage of salt present.

Influence of pressure on the temperature at which boiling occurs.—Since a liquid does not boil until the temperature reaches the saturation temperature corresponding to the pressure to which the liquid is subjected, it follows that increase in pressure will raise the temperature at which boiling occurs. Thus water under a pressure of one atmosphere boils at 100° C ; if the pressure be raised to 6 atmospheres, the boiling point is raised to about 160° C. ; at a pressure of 0·1 atmosphere water boils at a temperature of about 47° C. In Fig. 435 is shown a graph indicating the boiling points of water at various pressures.

Exercises on Chapter XXXIV.

1. What is meant by the melting point of a substance ? Give some instances of substances which exhibit different phenomena during the process of melting, or freezing.

2. What are the effects of increased pressure on the melting point of a substance ? Give instances.

3. Describe experiments by means of which the melting point of a given substance may be determined.

4. Define the term " latent heat of fusion " of a substance.

5. How would you determine by experiment the latent heat of ice ?

6. A piece of ice at 0° C. weighing 83 2 grams is mixed with 364 c.c. of water at 20° C. in a calorimeter having a water equivalent of 42 grams. Calculate the final temperature. (The latent heat of ice is 80 calories per gram.)

7. How much heat must be abstracted from a ton of water at 15° C. in order to convert it into ice at −6° C. ? The specific heat of ice is 0·502.

8. Describe how you would carry out an experiment to determine the latent heat of fusion of paraffin wax. Explain how to reduce the results.

9. Certain solids when mixed with water produce a rise in temperature, others a fall. Explain this, and give examples. Give instances of freezing mixtures.

10. Taking the data of Question 7, find the work equivalent to the heat abstracted. If one ton of ice is made per hour, what horse-power is required theoretically ?

11. Describe some method for measuring the latent heat of ice, pointing out the precautions to be taken to obtain an accurate result.

Into a copper calorimeter weighing 100 grams and containing 100 grams of water at 16° C.. there are placed 20 grams of ice at −10° C. Will the ice all melt, and, if so, what will be the final temperature of the mixture ? (Specific heat of ice = 0·5 ; specific heat of copper = 0·094.) Sen. Cam. Loc.

12. The melting point of tin is 232° C., its latent heat of fusion is 14 calories, and its specific heat is ·055. How many units of heat are required to melt 100 grams of tin originally at 20° C. ?

13. How would you obtain the melting point of a substance from an inspection of its cooling curve ? Hard paraffin wax melts at about 54° C.; draw a rough cooling curve for a quantity of this wax cooling from 80° C. to 30° C.

14. A mixture of crushed ice and water is poured into a vessel containing a thermometer. What will be the effect on the reading of the thermometer :

(*a*) when more water is poured in ;
(*b*) when more ice is put in ;
(*c*) when a handful of salt is stirred in ?

Give reasons in each case for your answer. Adelaide University.

15. Define the terms water equivalent, latent heat of fusion. Explain what becomes of the heat absorbed by a body in the process of melting.

In a copper calorimeter weighing 50 grams there are 200 grams of water at a temperature of 20° C. 20 grams of dry ice are added, and stirred well ; the final temperature is 11° C. Find the latent heat of fusion of ice. The specific heat of copper is 0·095. Tasmania Univ.

16. Give a brief description of the process of evaporation (*a*) in an open vessel, (*b*) in a closed vessel.

17. Distinguish between a saturated and unsaturated vapour. A closed vessel containing initially no substance in the gaseous state is maintained at constant temperature, and a small quantity of liquid at the same temperature is introduced into the vessel. Will the whole of the liquid be evaporated ? Give reasons for your answer.

18. Calculate the quantity of heat required to superheat one pound of aqueous vapour from 100° to 150° C. at the constant absolute pressure of one atmosphere. The specific heat is 0·502. If the initial volume is 26·75 cubic feet, find the final volume, assuming that the laws of perfect gases are followed.

19. Describe how you would find the maximum vapour pressure of alcohol at the temperature of the room. Does the volume of the vapour space provided in the apparatus make any difference in the result ?

20. In an experiment on the maximum pressure of aqueous vapour, using the apparatus shown in Fig. 432 (p. 452), the following readings were obtained : Temperature of the water in the beaker, 80° C. ; difference in levels of the mercury in the bulb and the water in the beaker, 5·2 cms. ; difference in levels of the mercury in the tube and the water in the beaker, 30·6 cms. ; temperature of the surrounding atmosphere, 20° C. Find the vapour pressure. The coefficient of expansion of mercury is 0 00018.

21. Describe an experiment for determining the maximum vapour pressure of aqueous vapour at absolute pressures from about 1 to 2 atmospheres. State the condition which must be satisfied if a liquid is to remain unchanged in presence of its vapour.

22. Define the " boiling point " of a liquid. Some tap water is contained in an open glass vessel ; the initial temperature is about 15° C. and the temperature is gradually raised to the highest possible degree. Describe clearly what may be observed in the vessel. Supposing the water had contained some salt in solution, would this have had any effect on the final temperature ? Explain the phenomena of boiling with bumping.

23. Distinguish carefully between saturated and unsaturated vapours. Into a cylinder exhausted of air and provided with a piston there is introduced just enough water to saturate the space at 20° C. Describe what happens under the following conditions :

(*a*) The volume of the space is increased by pulling out the piston.
(*b*) The volume is diminished by pushing the piston down.
(*c*) The volume remaining as at first, the temperature is increased to 30° C.
(*d*) The temperature falls to 10° C. Calcutta Univ.

24. Taking the density of ice at 0° C. to be 0·917, find what fraction of a block of ice will be immersed when floating in (*a*) fresh water (density = 1), (*b*) sea water (density = 1 03).

25. If the end of the mercury column in a Bunsen ice calorimeter recedes through a volume of 0 2 c.c. when a mass of 3 5 grams of a substance at 98° C. is placed in the test tube, find the specific heat of the substance. Take the density of ice at 0° C. to be 0·917, and the latent heat of water to be 80 calories per gram.

26. Describe Bunsen's ice calorimeter and explain how you would use the instrument to determine the specific heat of a substance. A mass of 0·96 gram of a substance was heated to 100° C. and dropped into the calorimeter. The thread of mercury retreated through a distance of 8·3 mm. in the capillary tube of 1 sq. mm. section. If 1 gram of water expands by 0·0918 c.c. on freezing and evolves 80 calories, calculate the specific heat of the substance. Bombay Univ.

CHAPTER XXXV

PROPERTIES OF VAPOURS—*Continued*

Pressure of a mixture of a vapour and perfect gas.—Dalton first stated the law, and Regnault proved experimentally, that **the pressure which a vapour exerts is very nearly independent of the presence of any other gas or vapour present,** provided there is no chemical action. In a space occupied by a mixture of a vapour and perfect gas which do not react chemically on each other, each exerts the pressure which it would produce if it alone occupied the space, and the total pressure is equal to the sum of these pressures. The law followed is thus the same as for a mixture of two perfect gases (p. 410).

If a closed vessel contains a perfect gas and also some liquid, the whole being at constant temperature, steady conditions will be attained in the space above the liquid when the space is saturated with vapour evaporated from the liquid. The pressure of the vapour is then that corresponding to the given steady temperature, and the total pressure is the sum of this maximum vapour pressure and the pressure exerted by the perfect gas. Increase in the temperature will raise the total pressure, owing partly to the increase in the pressure exerted by the perfect gas, and partly to the increase in the saturation pressure of the vapour.

Experiments on the pressure exerted by a given vapour in a closed vessel must always be arranged in such a way as to exclude from the space any air, or other gas or vapour, since it is now evident that the presence of such would lead to false readings of vapour pressure.

Isothermal line for a mixture of a saturated vapour and perfect gas.—In Fig. 436, AB is the isothermal line in a pressure-volume diagram for a given quantity of a perfect gas at a given temperature, and may be drawn by applying Boyle's law (p. 398). The isothermal line for the same gas when mixed with a saturated vapour

at the same constant temperature may be deduced by adding vertically the constant pressure of the vapour. Thus AC = BD = EF = the constant vapour pressure, and the isothermal line CD is drawn through the points C, F and D. It will be noted that, for points corresponding to equal pressures, BA rises more steeply than DC. If a perfect gas be compressed in a cylinder fitted with a piston, no condensation occurs, and the pressure increases at fairly rapid rates. If a mixture of a perfect gas and a saturated vapour be compressed in the same cylinder, some of the vapour condenses as compression proceeds; the volume of the resulting liquid is negligible, and the pressure increases at less rapid rates than would be the case if the vapour were absent.

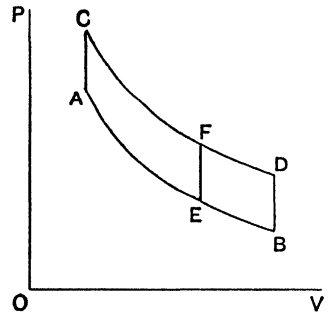
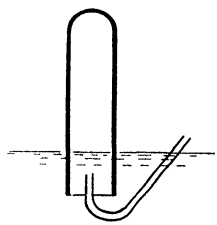


FIG. 436.—Isothermal line for a mixture of saturated vapour and gas

Collection of gases over water.—When gases are collected over water (Fig. 437), the contents of the graduated collecting vessel consist of a mixture of the gas collected and aqueous vapour saturated at the temperature of collection. The pressure of the gas collected will be equal to that of the atmosphere, as shown by the barometer, if the collecting vessel be adjusted vertically before reading the volume, so that the water surfaces inside and outside the vessel are at the same level.


FIG 437.—Collection of a gas over water

Let v_1 = the measured volume of the mixture, in c.c.

v_2 = the volume of dry gas collected, at standard temperature and pressure, in c.c.

h_a = the barometer reading, in cm. of mercury.

$t°$ C. = the temperature of the water.

h_v = the pressure of saturated aqueous vapour at temperature $t°$ (from the Table, p. 533), in cm. of mercury.

The sum of the pressures of the gas and the aqueous vapour is equal to h_a; hence

Pressure of the measured volume of dry gas at $t° = h_a - h_v$.

Applying the law $p_1v_1/T_1 = p_2v_2/T_2$, we have

$$\frac{(h_a - h_v)v_1}{t+273} = \frac{76v_2}{273},$$

or

$$v_2 = \frac{273(h_a - h_v)v_1}{76(t+273)}.$$

Vapour density.—Strictly speaking, the density of a substance is the mass of unit volume (p. 4). In dealing with substances in the gaseous state, the density may be measured as the mass in grams per litre (1000 c.c.). **Vapour density** is generally understood to mean the ratio of the mass of a given volume of the substance in the gaseous state, under stated conditions of pressure and temperature, to the mass of an equal volume of dry air at the same pressure and temperature. Sometimes dry hydrogen, or oxygen, is taken instead of air in making the comparison. The densities of these standard gases at 0° C. and 76 cm. of mercury are as follows:

Dry air,	0·001293 gram per c.c.
Dry hydrogen,	0·00008987 gram per c.c.
Dry oxygen,	0·001429 gram per c.c.

EXPT. 103.—**Vapour density of an unsaturated vapour by the method of Dumas.** A glass bulb having a finely drawn neck is employed (Fig. 438). At first the bulb contains air only at the temperature and pressure of the room, and is weighed. This weight represents the weight of the material of the bulb (supposed to be obtained by weighing *in vacuo*) diminished by the buoyancy of the air displaced by the material of the flask.

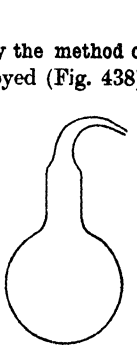


FIG. 438.—Dumas's bulb.

Let w_1 = the observed weight in air, in grams.

w_v = the weight of the material of the bulb which would be obtained *in vacuo*, in grams.

w_a = the buoyancy, in grams, of the air displaced by the material of the bulb.

t° C. = the temperature of the room.

h_a = the barometer reading, cms. of mercury.

Then, $w_1 + w_a = w_v$,

or $w_1 = w_v - w_a$ .....(1)

About 2 c.c. of the substance to be examined (alcohol, say) is now introduced into the bulb, which is then placed in a bath of water and held down by means of a metal frame. The water is boiled until the liquid in the bulb has disappeared entirely. The bulb is now full of alcohol vapour only, at the temperature of 100° C. and under a pressure h_a cm. of

mercury. This temperature is considerably above the saturation temperature of alcohol vapour at ordinary atmospheric pressure, and the vapour is therefore unsaturated. The neck of the bulb is now sealed hermetically, and the bulb is allowed to cool to the temperature of the room. It is then weighed again; the result represents the weight of the material of the bulb *in vacuo*, together with the weight of the vapour which filled the bulb at 100° C., diminished by the buoyancy of the air displaced by the outside volume of the whole bulb.

Let w_2 = the observed weight of bulb and vapour, in grams.

w_A = the buoyancy in grams of the air displaced by the closed bulb.

w_v = the weight in grams of the contained vapour, or the liquid condensed from the vapour.

Then, $$w_2 + w_A = w_b + w_v,$$

or $$w_2 = w_b + w_v - w_A. \quad \text{.....(2)}$$

From (1) and (2), $$w_2 - w_1 = w_v - (w_A - w_a). \quad \text{.....(3)}$$

Now $(w_A - w_a)$ is the weight of air at t° C. and h_a cm. of mercury which would occupy the interior volume of the bulb. The interior volume of the bulb is now determined by immersing the closed flask in water and breaking off the neck. When full of water, the bulb is weighed again (the piece of glass broken off must be included).

Let w_3 = the observed weight, in grams.

Then, Weight of contained water $= (w_3 - w_1)$ grams.

$\therefore$ Volume of interior of bulb $= (w_3 - w_1)$ c.c. .....(4)

To obtain the weight of the air contained by the bulb at temperature t and pressure h_a, let d' be the density of air under these conditions, and d_0' its density at normal temperature and pressure (p. 461). Then, from equation (5) (p. 412),

$$d' = \frac{273 h_a d_0'}{76(t+273)};$$

$\therefore$ Weight of the contained air $= (w_3 - w_1)\dfrac{273 h_a d_0'}{76(t+273)}$. .....(5)

Substituting this for $(w_A - w_a)$ in (3), we have

$$w_2 - w_1 = w_v - \frac{273 h_a d_0'(w_3 - w_1)}{76(t+273)},$$

or $$w_v = w_2 - w_1 + \frac{273 h_a d_0'(w_3 - w_1)}{76(t+273)} \text{ grams.} \quad \text{.....(6)}$$

The volume occupied by this mass of vapour being $(w_3 - w_1)$ c.c. at 100° C. and h_a cm. of mercury, we have for the vapour density of alcohol under these conditions

$$d_{100} = \frac{w_2 - w_1}{w_3 - w_1} + \frac{273 h_a d_0'}{76(t+273)} \text{ grams per c.c.} \quad \text{.....(7)}$$

The density of hydrogen at normal temperature and pressure is 0·00008987 gram per c.c., and at 100° C. and h_a cm. of mercury is

$$d_H = \frac{273h_a}{76(100+273)} \times 0{\cdot}00008987$$

$$= 0{\cdot}0000008654\ h_a \text{ grams per c.c.} \quad \ldots\ldots(8)$$

Hence the vapour density of alcohol at 100° C. and h_a cm. of mercury, with hydrogen as standard, is

$$D_{100} = \frac{d_{100}}{0{\cdot}0000008654h_a}. \quad \ldots\ldots(9)$$

Assuming that the alcohol vapour follows the laws of a perfect gas, the density at normal temperature and pressure is given by

$$d_0 = \frac{373 \times 76}{273h_a} \cdot d_{100}, \text{ (p. 412)} \quad \ldots\ldots(10)$$

and the vapour density under these conditions with hydrogen as standard is

$$D_0 = \frac{d_0}{0{\cdot}00008987}. \quad \ldots\ldots(11)$$

Expt. 104.—**Vapour density of an unsaturated vapour by Victor Meyer's method.** The apparatus employed is shown in Fig. 439. A is a glass tube having a bulb at its lower end. The upper end is connected by a rubber tube having a clip B to a short piece of glass tube which is open to the atmosphere. A branch tube C, having an enlargement, connects A to a graduated gas jar D, inverted over a trough of water. A is placed inside another glass or copper vessel E, which contains some water; the water is boiled and the steam escapes through F. The bulb of A can thus be maintained at 100° C.

The liquid to be tested (say alcohol) is contained in a small phial shown enlarged at G; the phial has a ground glass stopper which can be forced out easily when the pressure in the interior of the phial exceeds that of the atmosphere by a small amount. The phial is inserted at the top end of A after the water in E has been boiling quietly for a few minutes, and the clip is instantly closed. The phial drops down A, and its fall is arrested by a cushion of asbestos fibre in the bulb, which prevents fracture. The stopper of the phial is blown out almost immediately by the pressure of the vapour, and the alcohol evaporates rapidly. The vapour formed drives air out of A through the branch C into the jar D; the volume of air thus measured gives, after corrections have been applied, the volume of the vapour.

Let w_1 = the weight of the phial and stopper when full of air, in grams.
w_2 = the same when containing alcohol, in grams.
V = the volume of air collected in c.c., at h_a and t.
h_a = the barometer reading, in cm. of mercury.
t° C. = the temperature of the room.

The weight of the vapour formed is equal to the weight of alcohol in the phial, hence

$$\text{Mass of the vapour} = w_2 - w_1 \text{ grams.} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

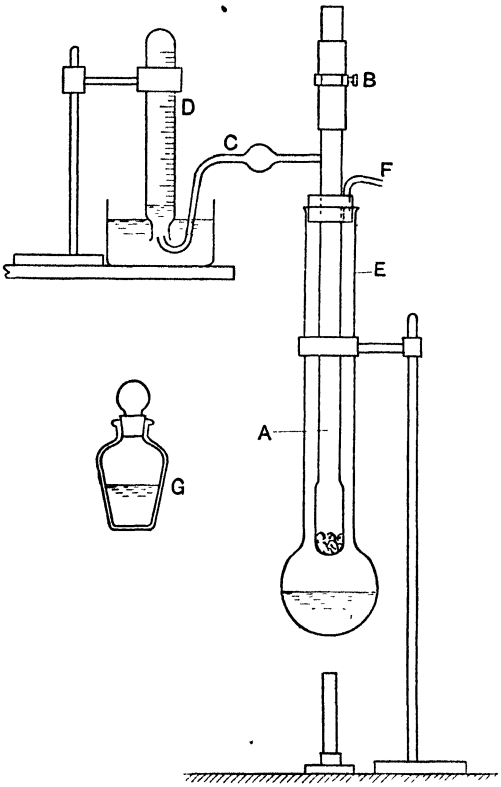


FIG. 439.—Victor Meyer's apparatus for vapour density determinations.

The contents of the graduated jar consist of a mixture of air and saturated water vapour (p. 460). Obtain from the Table (p. 533) the pressure h_v of saturated water vapour at temperature t. The pressure of dry air at temperature t which would occupy the volume V if the aqueous vapour were absent is $(h_a - h_v)$ cm. of mercury. Applying Boyle's law to obtain the volume V′ of this air at pressure h_a and temperature t, we have

$$(h_a - h_v)\mathrm{V} = h_a \mathrm{V}';$$

$$\therefore \ \mathrm{V}' = \left(\frac{h_a - h_v}{h_a}\right)\mathrm{V} \text{ c.c.} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

From (1) and (2), the density of the alcohol vapour at 100° C. and h_a cm. of mercury is

$$d_{100} = \frac{(w_2 - w_1)h_a}{(h_a - h_v)V} \text{ grams per c.c. } \ldots\ldots\ldots\ldots\ldots\ldots (3)$$

The vapour formed in the tube A is at a temperature considerably above the saturation temperature, and is therefore unsaturated vapour. Assuming that it follows the laws of perfect gases, the density at 0° C. and 76 cm. of mercury may be obtained from equation (5), (p. 412),

$$d_0 = \frac{76(t+273)}{273h_a} d_{100} \text{ grams per c.c. } \ldots\ldots\ldots\ldots\ldots\ldots (4)$$

And the density relative to hydrogen, both being at normal temperature and pressure, is

$$D_0 = \frac{d_0}{0{\cdot}00008987}. \ldots\ldots\ldots\ldots\ldots\ldots (5)$$

Specific volume of saturated vapours.—The density of saturated vapours cannot be determined easily by experiment; generally

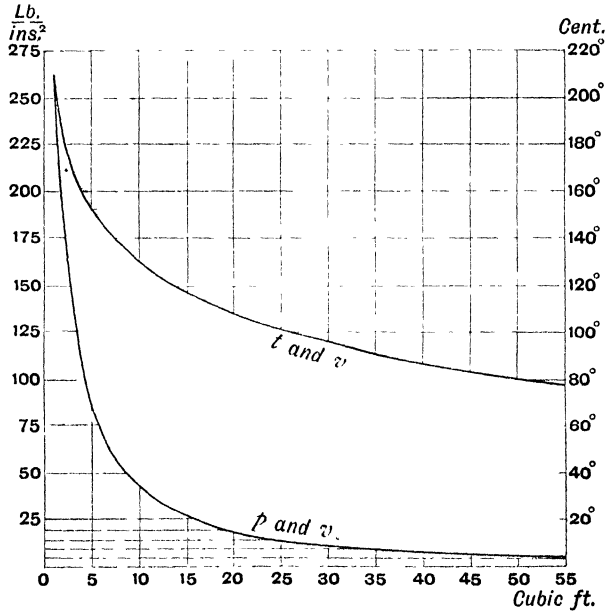


FIG. 440.—Relation of specific volume, pressure and temperature of saturated aqueous vapour.

indirect methods are employed, by means of which the density may be calculated from other experimental data.

The volume occupied by unit mass of a substance in the gaseous state, under given conditions of pressure and temperature, is called the **specific volume.** The specific volume of a saturated vapour is defined when the temperature has been stated, since the saturated vapour of a given substance at a stated temperature can exist at one pressure only. If d is the density of a saturated vapour in grams per cubic centimetre, the specific volume is $1/d$ cubic centimetres per gram.

The graph given in Fig. 410 shows the specific volume of saturated aqueous vapour at various pressures and temperatures.

Latent heat of vaporisation.—If a mass of liquid changes state from liquid to gaseous at constant pressure, there is no change in

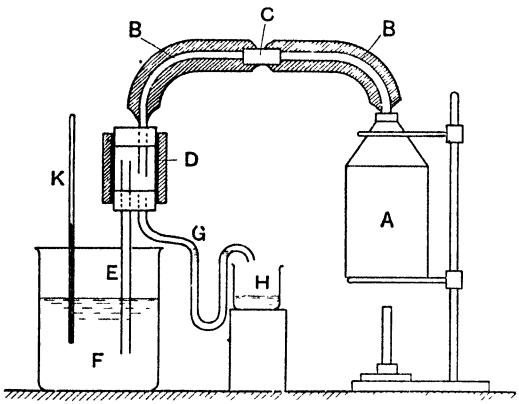


FIG. 441.—Apparatus for determining the latent heat of vaporisation of water

temperature. The **latent heat of vaporisation** of a substance is the heat which must be imparted at constant temperature to unit mass of the substance in the liquid state in order to effect the change of state from liquid to gaseous.

EXPT. 105.—**Latent heat of vaporisation of water, boiling under a pressure of one atmosphere.** In Fig. 441, a small copper boiler A contains some water which can be heated by means of a bunsen flame. The resulting steam flows out of the boiler through a tube B, consisting of two parts connected at C by a short piece of rubber tube. B is "lagged," *i.e.* heat insulated, by winding strips of flannel round the tube; this helps to prevent condensation of the steam flowing through the tube. The tube B leads into a separator D, consisting of a short piece of wide bore glass tube having

rubber stoppers at both top and bottom. The tube B reaches nearly to the bottom stopper. Another tube E leads from D into a measured quantity of water contained in a calorimeter F. A tube G, bent as shown, is pushed partly into the lower stopper of D, and drains any water from D into a beaker H. There will always be some water in G which will serve to seal the apparatus and will prevent steam being blown through G. The arrangement helps to prevent water from entering the calorimeter, and renders the entering steam as dry as possible. The temperature of the water in the calorimeter is observed by a thermometer K.

Weigh the empty calorimeter; pour in some water and weigh again; by differences find the mass of the water. While this is being done, the water in the boiler is being heated. Let steam discharge freely from the mouth of the tube E into the atmosphere for three or four minutes. Take the temperature of the water in the calorimeter; put the tube E into the calorimeter (the rubber joint at C assists this operation) so that its mouth is below the surface of the water. A crackling noise will be heard, caused by the collapse of steam bubbles undergoing rapid condensation. Continue until the water in the calorimeter has risen in temperature about 20 Centigrade degrees. Remove the tube E; read the temperature of the water; weigh the calorimeter and its contents, and by differences find the mass of the steam which has been condensed.

Let m = the mass of the empty calorimeter, in grams.

s = the specific heat of its material.

m_1 = the mass of the water used, in grams.

m_2 = the mass of the steam condensed, in grams.

t_1° C. = the initial temperature of the water.

t_2° C. = the final temperature.

L = the latent heat of vaporisation of saturated aqueous vapour at 100° C.

The steam has given up latent heat in condensing into water at 100° C., and the resulting water has given up additional heat in cooling from 100° C. to t_2. Assuming that the total heat thus given up is equal to the heat taken up by the mass m_1 of water and the material of the calorimeter, we have

$$m_2\{L + (100 - t_2)\} = (m_1 + ms)(t_2 - t_1).$$

$$\therefore L = \left(\frac{m_1 + ms}{m_2}\right)(t_2 - t_1) - (100 - t_2).$$

This result is in calories per gram mass of steam.

Careful experiments show that one gram of steam at a pressure of one atmosphere and at 100° C. has a latent heat of 539 calories. Compare the result of your experiment with this number.

The graph in Fig. 442 shows the latent heat of saturated aqueous vapour at other temperatures, ranging from 0° C. to 210° C. It will be noted that the latent heat decreases as the temperature is increased.

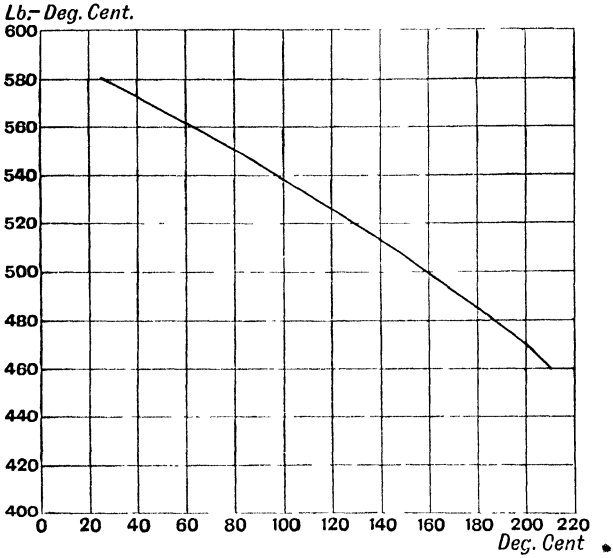


FIG. 442.— Relation of latent heat and temperature of saturated aqueous vapour.

EXPT. 106.—**Freezing water by evaporation of ether.** Stand a beaker containing some ether in a small pool of water spilled on the bench. Use a glass tube and foot-bellows to blow air through the ether. Rapid evaporation of the ether will take place and the heat will become latent so quickly that the water will freeze.

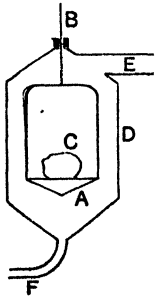


FIG. 443.—Diagram of Joly's steam calorimeter.

Joly's steam calorimeter.—In this apparatus (Fig. 443) a pan A is suspended by means of a fine wire B from one arm of a delicate balance. C is a substance whose specific heat is to be determined. The arrangement is enclosed in a chamber D, which can be supplied with steam from a boiler through a pipe E; the pipe F drains off the water of condensation. A plug of plaster of Paris is fitted at G in order to prevent water of condensation from interfering with the free vertical movements of the suspending

wire B; a small coil of platinum wire heated electrically prevents condensation of steam on the wire beyond the plug.

EXPT. 107.—**Specific heat of a substance by Joly's method.** With air only in the chamber, place the substance in the pan, and determine its mass by weighing. Read the temperature as shown by a thermometer in the chamber. Admit steam suddenly so as to fill the chamber quickly with saturated steam, then reduce the steam supply so as to avoid steam currents which would interfere with the following weighing. In a few minutes, the condensation which is taking place inside the chamber ceases, and the substance arrives at the temperature of the steam. Weigh again, thus determining the mass of steam which has been condensed on the substance and pan.

Let m_1 = the mass of the substance, in grams.

m_2 = the mass of the steam condensed, in grams.

t_1° C. = the initial temperature of the chamber.

t_2° C. = the temperature of the steam, obtained by reading the thermometer in the chamber, or by consulting the Table (p. 533).

L = the latent heat of the steam, in calories.

s = the specific heat of the substance.

c = the capacity for heat of the pan (p. 345).

Then,
$$m_1 s(t_2 - t_1) + c(t_2 - t_1) = m_2 \mathrm{L}.$$
$$m_1 s(t_2 - t_1) = m_2 \mathrm{L} - c(t_2 - t_1).$$
$$s = \frac{m_2 \mathrm{L}}{m_1(t_2 - t_1)} - \frac{c}{m_1}.$$

Make another experiment with the pan empty; a similar calculation will then give the value of the capacity for heat of the pan. Insert this value in the above equation and calculate the value of s. In accurate work, corrections are applied for the buoyancies of the air and steam in the weighing operations.

The **differential Joly calorimeter** is used for determining the specific heat of gases at constant volume. The apparatus (Fig 444) is arranged so as to eliminate corrections as far as possible. Two equal copper globes are suspended from the opposite arms of the balance, and hang in the steam chamber. One globe is filled with the gas under test and the other is exhausted. Small pans are fitted to the globes in order to catch the water condensed on their surfaces. Greater condensation takes place on the globe containing gas, and the excess gives the data required in calculating the quantity of heat necessary in order to raise the temperature of the enclosed gas.

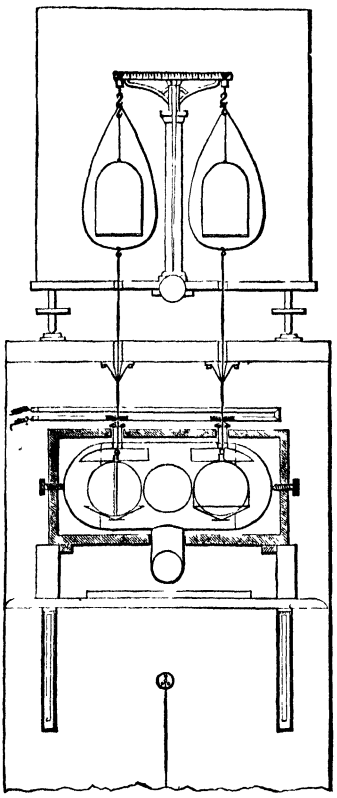


FIG 444.—Differential Joly calorimeter.

Let m_1 = the mass of the gas filling the sphere.

m_2 = the difference between the masses of the steam condensed on the globes.

t_1 = the initial temperature.

t_2 = the temperature of the steam.

p_1 = the initial pressure of the gas.

p_2 = the final pressure of the gas.

L = the latent heat of the steam.

s = the specific heat of the gas at constant volume for the temperature range t_1 to t_2, and the pressure range p_1 to p_2

Then $m_1 s(t_2 - t_1) = m_2 \mathrm{L}$,

$$s = \frac{m_2 \mathrm{L}}{m_1(t_2 - t_1)}.$$

EXERCISES ON CHAPTER XXXV.

1. State Dalton's law for the pressure of a mixed gas and vapour. A closed vessel contains air saturated with aqueous vapour; the temperature is 70° C. and the absolute pressure inside the vessel is 78·2 cm. of mercury. Find the pressure exerted by the air in the vessel.

2. Draw the isothermal line for 4 cubic feet of dry air at 15 lb. wt. per square inch absolute pressure and 94° C. when compressed to a volume of one cubic foot. If the air is saturated with aqueous vapour, draw the isothermal line for the mixture.

3. Some hydrogen is collected over water; the measured volume is 245 c.c.; the temperature of the water is 16° C.; the barometer reads 75·43 cm. of mercury. Find the volume of dry hydrogen at 0° C. and 76 cm. of mercury.

4. In a test on the vapour density of alcohol, using the method of Dumas (p. 461), the following observations were made: Weight of bulb, 9·77 grams; weight of the bulb and vapour filling it at 100° C., 9·889 grams; weight of the bulb when full of water, 141·65 grams; temperature of the room, 15° C.; barometric height, 75·1 cm. Find the vapour density of alcohol at 100° C. and 75·1 cm. pressure, using hydrogen as the standard.

5. Describe an experiment for determining the vapour density of a given substance.

6. Using Victor Meyer's method (p. 463), the following readings were taken in an experiment on the vapour density of alcohol: Weight of the empty phial, 1·415 grams; weight of the phial containing alcohol, 1·738 grams; volume of air collected, 171·2 c.c.; barometric height, 76·29 cm.; temperature of air, 15·2° C. Find the vapour density of alcohol at 0° C. and 76 cm. Use hydrogen as the standard.

7. Take the required data from the Table (p. 534), and plot graphs showing the relation of specific volume and (*a*) pressure, (*b*) temperature for saturated aqueous vapour.

8. Define latent heat of vaporisation. Describe an experiment for determining the latent heat of vaporisation of water at ordinary boiling point.

9. Find the quantity of heat which must be supplied to one pound of water at 20° C. in order to convert it into dry saturated steam at a pressure of 146 lb. wt. per square inch absolute. Take the quantities required from the Table on p. 534.

10. A tank contains 20 gallons of water at 30° C. which is heated by discharging dry saturated steam at atmospheric pressure into it. If the final temperature is 80° C., find the weight of the steam used. Consult the Table on p. 534 for any quantities required.

11. What is meant by the statement, "The maximum pressure of aqueous vapour at 15° C. is 13 mm."?

Some nitrogen is collected in a tube over water at 15° C. and is found to occupy 50 c.c., the gaseous pressure in the tube being 743 mm. Calculate the volume which the dry nitrogen would have at 0° C. and 760 mm. pressure. Sen. Cam. Loc.

12. Explain the meaning of latent heat. How would you determine the latent heat of fusion of ice? Explain the cooling effect of a fan.
Calcutta Univ.

13. Explain how you would determine the latent heat of steam. Would you expect it to depend in any way on the temperature at which water is boiled? 7 grams of ice float in water in a calorimeter of thermal capacity 5 calories. If, when 4·5 grams of steam (at 100° C.) are passed into the calorimeter, the final temperature becomes 50° C., how much water was there in the calorimeter? (Latent heat of steam at 100° C. = 540.)
Calcutta Univ.

14. Define "vapour density." Describe how to find the vapour density of a liquid, using the method of Dumas. Prove the formula.

15. Describe Victor Meyer's method of finding the vapour density of a liquid. Prove the formula. Bombay Univ.

16. Draw a curve indicating generally the change in maximum vapour pressure of water between 0° and 100° C.

Given that, at any temperature, the maximum vapour pressure is slightly diminished upon dissolving a salt in water, show that the boiling point of the solution at any pressure is higher than that of pure water. L.U.

17. Describe the properties of saturated and non-saturated vapours.

A barometer tube dipping into a mercury reservoir contains a mixture of air and saturated vapour above a column of mercury which is 70 centimetres above the level in the reservoir, the atmospheric pressure being 76 centimetres. What is the height of the mercury column when the tube is depressed so as to reduce the volume occupied by the air to one half its original value, the pressure of the saturated vapour being 1·5 centimetres? L.U.

18. Give the definitions of the expressions, specific heat, latent heat of fusion, latent heat of vaporisation, thermal conductivity.

Fifty grams of steam at 100° C. are passed into a mixture of 100 grams of ice and 200 grams of water at 0° C. Find the rise of temperature, the latent heat of vaporisation of water at 100° C. being 537 and the latent heat of fusion of ice 80. Sydney Univ.

19. Describe how the specific heat of a body may be found by the Joly steam calorimeter. Given the following observations, find the specific heat of the sample of calcite: Preliminary experiment; mass of the dry pan in the steam calorimeter, 28·395 grams; mass of pan and condensed steam, 28·565 grams. Calcite experiment; mass of the dry pan and dry calcite, 41·46 grams; mass of pan, calcite and condensed steam, 42·016 grams. Initial temperatures in both experiments, 24° C.

20. Describe an experiment for determining the specific heat of a gas at constant volume.

CHAPTER XXXVI

ATMOSPHERIC CONDITIONS. HYGROMETRY

Evaporation from free surfaces of water.—Evaporation takes place constantly at all temperatures from open surfaces of water. Molecules escape from the surface of the liquid and may be carried away by currents of air, so that there is no chance of return to the liquid. The continual loss of liquid from open surfaces of water is compensated by the return of rain water contained in rivers and streams. Increase in temperature of the water will increase the quantity evaporated in a given time. Increase in the speed of the wind also causes increased rates of evaporation by further diminishing the opportunities of return to the liquid. A familiar illustration of this fact occurs in the drying of the wet interior of a tube by blowing air into it by means of bellows.

The general result of evaporation from open sheets of water is that the atmosphere always contains aqueous vapour.

Different liquids contained in open vessels under like conditions possess different rates of evaporation. Liquids which evaporate more easily are said to be **volatile**. Ether, alcohol and petrol are examples of volatile liquids.

Mist, cloud, dew.—Our perceptions regarding the dryness of the atmosphere depend on its **hygrometric state**, a quantity which may be expressed as the ratio of the mass of water vapour present per cubic centimetre of air to the mass of water vapour which would be required to saturate one cubic centimetre of air if the temperature remained unaltered. If the air be not saturated with water vapour at a given temperature, it may become so if the temperature falls; less vapour is required to produce saturation at low temperatures. Hence the presence of a cold body in an atmosphere which has not quite reached saturation point may cool the air in its immediate vicinity sufficiently to produce saturation or over-saturation. Some

of the vapour will then condense on the surface of the cold body, producing the phenomenon of **dew**.

If a large body of the atmosphere be cooled slowly, saturation point will be reached throughout the mass simultaneously. The atmosphere is charged more or less with particles of dust and condensation takes place on each dust particle, giving rise to a **mist**. The fogs of large towns are caused by condensation of this kind on soot particles floating in the atmosphere.

Clouds are caused by similar condensation of water vapour in the upper strata of the atmosphere; heated air containing water vapour, but above the saturation point, becomes colder as it ascends and expands, and the water vapour condenses to a mist or cloud on the saturation point being reached.

Evaporation of snow and ice.—Supposing that a solid exposed to the atmosphere is undergoing the process of melting, and that the maximum vapour pressure of the substance at the temperature of the atmosphere is equal to, or greater than, the pressure of the atmosphere, then it is impossible that the substance can remain in the liquid state, and the phenomenon is presented of the substance changing direct from the solid to the gaseous state. The process is called **sublimation**, iodine among other substances possesses this property. Snow and ice evaporate slowly; in Arctic regions this is the only method of evaporation possible. There is an appreciable vapour pressure of water substance below 0° C, and consequently ice can pass directly from the state of solid to that of vapour without any intermediate liquid state.

Hoar-frost is deposited instead of dew if the temperature of the slowly-cooling atmosphere reaches the freezing point of water before saturation occurs. The process of formation of hoar-frost and of snow does not consist in the freezing of dew, but the direct change of state from aqueous vapour into the solid form.

Ventilation.—The ordinary atmosphere contains carbonic acid gas to the amount of 3 or 4 parts in 10,000 parts of air. Indoors the proportion is usually higher, especially if many persons be present in a room. An adult person at rest produces about 0·6 cubic feet of carbonic acid gas per hour, and the conditions in a room become objectionable unless these exhalations are diluted largely by a plentiful supply of air admitted to the room. Should the proportions exceed 10 parts of carbonic acid gas in 10,000 of air, there will be discomfort. In ventilation problems a limit of 6 to 7 parts of

carbonic acid gas in 10,000 of air is aimed at. To dilute sufficiently the gas expelled by an adult requires from 1800 to 3600 cubic feet of air per hour, depending upon the particular conditions existing in the building.

In ordinary houses, open doors, windows, and the chimney of open fireplaces generally suffice. None of these are very effective; and it should be noted that fireplaces produce the peculiar condition that a greater number of persons in the room will require more energetic ventilation, produced by urging the fire, despite the fact that the people by their presence are assisting to raise the temperature of the room.

In mechanical methods of ventilation, a fan is employed to propel air into the room, or to withdraw air from it. Fresh air is led into the room by means of properly arranged ducts. The incoming air may be warmed by passing it over pipes through which steam or hot water circulates, and may be brought to a proper hygrometric state by passing it over water, or by spraying water into it.

Dew point.—The temperature at which dew begins to form when the atmosphere undergoes cooling is called the **dew point.** Experiments on the determination of the dew point are based generally on the principle of cooling artificially a portion of the atmosphere until it is observed that dew is commencing to form. The cooling is carried out at constant pressure—that shown by the barometer—and Charles's law may be assumed to be followed by the vapour molecules until the temperature of saturation is reached.

The perception of dampness in the atmosphere is a consequence of a near approach to saturation conditions, under which evaporation goes on very slowly. The actual quantity of aqueous vapour present is only one of the factors to consider; warm air may contain much more aqueous vapour than colder air, and yet may convey the impression of dryness because the atmosphere is not nearly saturated.

Relative humidity may be defined as the ratio of the pressure of the aqueous vapour actually present at the existing temperature to the pressure of the vapour which would be present if the vapour were saturated at the same temperature.

Let t_1° C. = the observed temperature of the atmosphere.

t_2° C. = the dew point.

p_1 = the pressure of aqueous vapour when saturated at t_1 (see Table, p. 533).

p_2 = the pressure of aqueous vapour when saturated at t_2 (see Table).

Assuming that the barometric pressure has remained constant in the process of determining the dew point by cooling a portion of the atmosphere, p_2 is the pressure of the aqueous vapour actually present in the atmosphere. Hence

$$\text{Relative humidity} = \frac{p_2}{p_1}.$$

Inspection of the following short Table shows that the masses per cubic metre at two given temperatures have nearly the same ratio as the pressures corresponding to the same temperatures. Hence it is often sufficiently accurate to take

$$\text{Relative humidity} = \frac{\text{mass of water vapour per unit volume of air at observed temp.}}{\text{mass of water vapour per unit volume of saturated air at same temp.}}.$$

PROPERTIES OF SATURATED AQUEOUS VAPOUR.

Temp. ; deg. Cent. - -	0	5	10	15	20
Pressure of sat. vapour, mm. of mercury -	4·58	6·54	9·20	12·78	17·51
Mass of sat. vapour ; grams per cub. metre -	4·84	6·76	9·33	12·71	17·12
Temp. ; deg. Cent. - -	25	30	35	40	
Pressure of sat. vapour ; mm. of mercury -	23·69	31·71	42·02	55·13	
Mass of sat. vapour ; grams per cub. metre -	22·80	30·04	39·18	50·7	

Hygrometry is the determination of the state of the atmosphere as regards the hygrometric state, dew point, etc.

EXPT. 108.—**Determination of the dew point by Regnault's hygrometer.** A form of Regnault's apparatus is illustrated in Fig. 445. A brightly polished silver vessel A contains some ether B and has a tube C dipping into the ether. Air may be pumped into the ether through C by means of a perfume spray bulb D ; this air accelerates evaporation from the ether and the resultant mixture of air and vapour escapes through a tube E. The evaporation of the ether cools the vessel and also the atmosphere in

the immediate neighbourhood, and will ultimately cause saturated conditions to be attained by the air near the vessel. A film of dew will then be deposited on the bright surface of the vessel. The temperature at which dew appears may be read by means of the thermometer F. Another similar but empty silver vessel G is mounted on the same stand and aids, by comparison of the surfaces, the detection of the first appearance of dew.

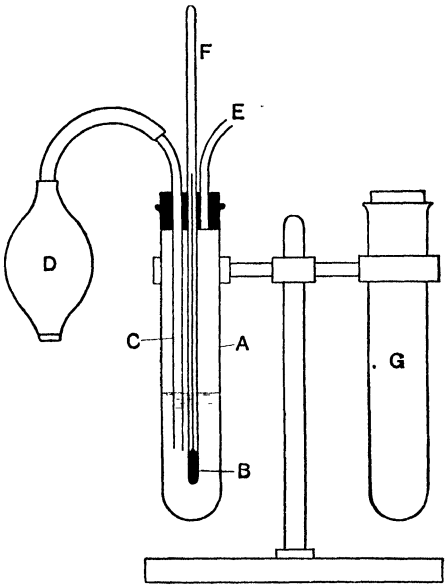


FIG. 445.—Regnault's hygrometer.

The temperature of the atmosphere in the room is read by another thermometer. A large sheet of glass should be interposed between the observer and the instrument in order to prevent his breath temporarily increasing the amount of moisture present.

Lower the temperature slowly and observe the reading of the thermometer F when dew appears. Allow the temperature to rise and note the reading of F when the dew disappears. Repeat several times and take the mean temperature as the dew point. Note the temperature of the atmosphere in the room in the neighbourhood of the apparatus.

Calculate the relative humidity of the atmosphere at the time of the experiment (p. 475).

EXPT. 109.—**Determination of the dew point by Daniell's hygrometer.** This instrument (Fig. 446) consists of two glass bulbs A and D connected by

a tube C, from which all air has been exhausted. A contains some ether, the remainder of the interior contains ether vapour only. A thermometer B is situated inside A, its bulb dipping into the ether. The internal surface of A is blackened, or gilt, in order to enable the formation of dew on the external surface to be observed easily. The thermometer E enables the temperature of the atmosphere to be observed. The bulb D is covered with muslin, and is saturated with some ether. Rapid evaporation of this ether takes place, and consequently the bulb D is cooled. The ether vapour inside D is thus cooled and condensed, and more vapour is evaporated from the ether in A to take its place. Thus the bulb A is cooled slowly, and presently dew appears on the surface. Take similar readings to those observed with the Regnault hygrometer and estimate the dew point and the relative humidity. The sheet of glass should be employed as before between the observer and the instrument.

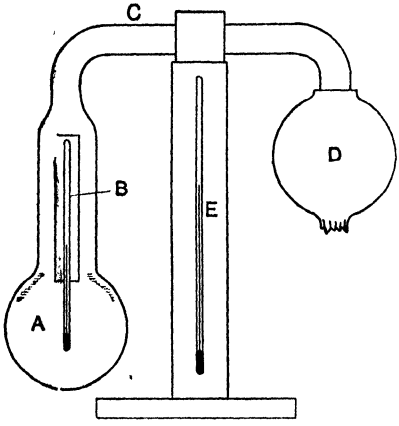


FIG 446.—Daniell's hygrometer.

Wet and dry bulb method. —The pressure of the aqueous vapour present in the atmosphere, and hence the dew point, may be found by the **wet and dry bulb thermometer method.** Two thermometers are arranged on a stand ; one is an ordinary thermometer and shows the temperature of the atmosphere. The bulb of the other thermometer has a piece of clean lamp wick wrapped round it, which is led to a small basin of water ; this keeps the bulb moistened. If there be much aqueous vapour present in the atmosphere, there will be but little evaporation from the moisture near the bulb of the thermometer, and therefore only a small cooling effect. The difference in reading of the thermometers will thus depend on the hygroscopic state of the atmosphere. Both thermometers are read when steady conditions have been attained, and the aqueous vapour pressure is found by use of specially constructed tables. The method does not give very accurate results.

EXPT. 110.—Chemical hygrometer. The apparatus required is shown in Fig. 447. A and B are drying tubes charged with phosphorus pentoxide ; C is also charged with the same substance and serves to seal the drying tubes from the vessel D. D is a large stone jar furnished with a tap at the

bottom. By filling D with water and then making the connections as shown, a volume of air equal to the capacity of D will be drawn through the apparatus, entering at E. D is called an aspirator. Practically all moisture in the entering air is absorbed in A; any escaping A will be absorbed in B. C absorbs any moisture which might be travelling backwards from the vessel D when the flow of water from the tap is stopped. The temperature of the air entering the apparatus is observed by the thermometer G.

Pour water into D until it is quite full. Disconnect A and B at H and weigh both together in order to determine the mass m_1 grams. Connect up again and run the water entirely out of D through the tap; read the

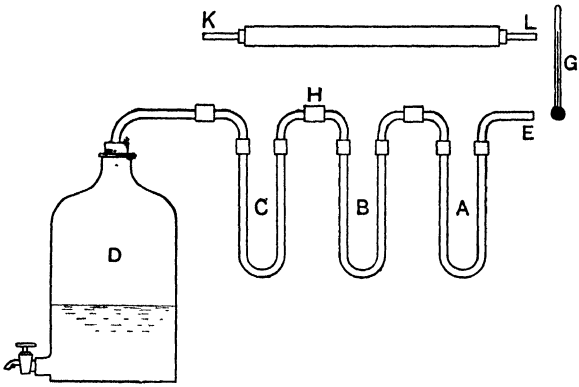


FIG 447.—Chemical hygrometer.

thermometer G while the water is escaping. Disconnect at H and again weigh A and B together; let the mass be m_2 grams. Then the mass of moisture absorbed is $(m_2 - m_1)$ grams.

Fill D again and make the connections. KL is a wide bore glass tube (Fig. 446) having a rubber stopper at each end fitted with short pieces of tube of smaller bore. The tube is charged with broken pumice and water; any air passed through this tube will become saturated with water vapour. Connect the tube K to the apparatus at E and repeat the operations described above. Let m_3 be the final mass of A and B together, then $(m_3 - m_2)$ will be the weight of saturated aqueous vapour which can be held by air at the existing temperature.

Since the volume of air passing through the apparatus is practically the same in both experiments, the relative humidity of the atmosphere γ (p. 476), obtained by dividing the actual mass of vapour per cubic centimetre of air by the mass of vapour required to saturate one cubic centimetre of air, is given by

$$\gamma = \frac{m_2 - m_1}{m_3 - m_2}.$$

Exercises on Chapter XXXVI.

1. State the cause of the presence of aqueous vapour in the atmosphere. Describe the formation of dew, mist, fog and clouds.

2. There are six adults in a room measuring 16 feet by 12 feet by 10 feet, and each requires 2500 cubic feet of air per hour. Find the number of times per hour that the air in the room should be changed. Supposing each person exhales 0·6 cubic foot of carbon dioxide gas per hour and that the air in the room contains at first 4 parts of this gas per 10,000, in what time will the proportion reach 10 parts per 10,000 if ventilation be absent entirely?

3. Define the dew point. State the conditions upon which it depends. What is meant by "relative humidity"?

4. Describe Regnault's hygrometer. In an experiment with this instrument, the mean of three tests gave the temperature of dew formation 9·6° C., and of the disappearance of dew 10·5° C. The temperature of the atmosphere was 18° C. What is the dew point? Find also the relative humidity.

5. Describe the Daniell hygrometer. In an experiment with this instrument, the dew point was found to be 11·13° C. and the temperature of the atmosphere was 19° C. Find the relative humidity.

6. Describe the wet and dry bulb method of determining the hygrometric state of the atmosphere.

7. Describe the method of carrying out an experiment with a chemical hygrometer. The following readings were obtained with an instrument of this type: Weight of U tubes at first, 85·48 grams; weight of U tubes together with water vapour from ordinary air, 85·61 grams; weight of U tubes together with water vapour from saturated air, 85·88 grams; temperature of air, 18° C. Find the hygrometric state and the dew point.

8. Describe an experiment to determine the dew point. What do you understand by a table of saturation pressures of aqueous vapour? Explain how relative humidity of the air can be determined by the dew point and such a table. Panjab Univ.

9. Calculate the dew point when the air is ⅔ saturated with water vapour, the temperature being 15° C., given that for pressures of 7, 9, 11, 13 mm. of mercury, the corresponding boiling-points of water are 6°, 10°, 13°, and 15° C. respectively. Sen. Cam. Loc.

10. What is meant by "sublimation"? Explain how hoar-frost is formed.

11. Calculate the mass of 7·6 litres of moist air at 27° C. given that the dew point is 15° C., and the barometric height 762·75 mm. Calculate also the humidity of the air. Vapour pressure of water at 27° C. and 15° C. =25·5 mm. and 12·75 mm. respectively. Bombay Univ.

CHAPTER LXI

MAGNETISATION

Lodestone.—The peculiar properties of a mineral called **lodestone**, found in the neighbourhood of Magnesia in Asia Minor, were known in very early times, and consist in its power of attracting fragments of the same material, and of setting in one particular direction when suspended. This mineral is now known as **magnetite**, and is an oxide of iron. If a piece of magnetite be dipped into iron filings it will be found that they adhere to it, more particularly at certain places. There are in general two such places on any piece of magnetite where the filings adhere in greatest quantity. On suspending a piece of magnetite in a stirrup by means of a piece of silk, it will be found that it will not rest in just any position, but only in such a position that the line joining the two places where the filings adhere in greatest quantity, points north and south.

Magnets.—One of the most important properties of these natural magnets is that they can communicate their properties to pieces of steel. Thus, if a steel knitting needle be stroked from one end to the other with a piece of magnetite, using for the place of contact that at which the filings adhere most freely, the needle will now possess the property of attracting iron filings, and of setting north and south when suspended. The effect may be increased by stroking the needle several times, but always with the same part of the magnetite and in the same direction.

Such a needle is said to be a **magnet.** Magnets are now made of bars of steel, which are many times more powerful than the needle described, and their method of manufacture will be considered later. Their essential properties, however, differ in no way from the magnetised needle; they are simply more powerful, and for this reason are generally used in experimental work.

EXPT. 161.—**Magnetisation of needle.** Dip a new knitting needle into iron filings and notice that they do not adhere to it. Place it in a thin wire stirrup S (Fig. 708) supported by a single silk fibre. Note that the needle will remain in any direction in which it is placed. Now remove the needle and stroke it from one end to the other with the end of a bar magnet. Dip the needle into iron filings; it will be found that the filings adhere at the ends, but not to the middle portion. Replace it in the stirrup, and notice that it will no longer remain in just any direction, but always sets north and south

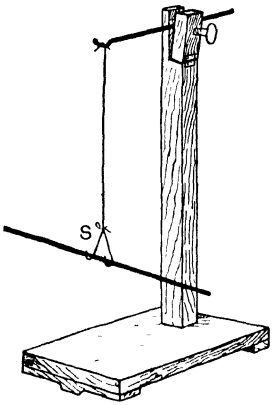


FIG 708.—Suspended magnetised needle

Magnetic poles.—On dipping a bar magnet into iron filings it is found that they adhere most freely at and near the ends These places are called **poles.** It was seen in Expt. 161 that the magnetised needle has poles, one at each end If the end which points to the north be now marked by placing a piece of paper on it, it will be observed that this end always points north, the other end cannot be made to rest pointing north. Thus, one pole always points north, and is called the **north-seeking** or **N** pole The other is the **south-seeking** or **S** pole

Force between poles.—Magnetic poles always exert forces upon each other. The force between any pair of poles depends upon their distance apart, becoming greater as they approach each other, but it is universally true that **N poles repel each other and S poles repel each other, while a N and a S pole attract each other. Thus there is a repulsion between poles of a like kind and an attraction between unlike poles.**

EXPT. 162.—**Forces between poles.** Magnetise two knitting needles and suspend them in turn in the stirrup as in Fig. 708; in each case mark the N pole. Now, with one of them suspended, bring a pole of the other near a pole of the suspended needle, trying all the four pairs of poles in the same way in turn. It will be found that like poles repel each other while unlike poles attract each other.

Molecular theory of magnetisation.—In early times many theories were suggested to account for the facts described, but one theory was suggested which has been modified and improved until at the

present time it is held universally. This is, that every magnetic substance, or substance capable of being rendered a magnet, consists of smaller parts which are themselves magnets. These minute magnets, which are sometimes called *molecular magnets*, are not arranged in any one particular direction when the substance is unmagnetised, but are orientated indiscriminately in all directions. The act of magnetisation consists in arranging them in one direction. Fig. 709 (*a*) represents diagrammatically the arrangement in an unmagnetised bar of iron, the strokes representing diagrammatically the molecular magnets, the arrow-heads being the N poles; (*b*) represents the same bar when magnetised It will be noticed that in (*b*) the little *n* and *s* poles of the molecular magnets are close to each other within the material, but at the N end are the free *n* poles without any corresponding *s* poles, and at the S end we have the free *s* poles This explains very well the reason for the poles of a bar magnet being situated at the ends and not in the middle of the bar.

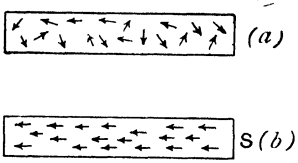


FIG 709 —Magnetisation

The term *magnetic molecule* requires some consideration. We do not mean the chemical molecule or necessarily the atom; in fact, at this stage we must not endeavour to state precisely their nature, but merely to consider that they are extremely small bodies, each having a N and S pole, and free to turn in any direction into which external magnets tend to set them.

EXPT. 163.—**Poles produced on magnetising a needle.** Mark one end of a knitting needle and stroke it several times from the unmarked to the marked end, with the N pole of a bar magnet Then suspend it and note that the unmarked end points north. Repeat with the S pole of the bar magnet, and note that the marked end now points north. Repeat, stroking from the marked to the unmarked end. From these results it will be seen that the end at which the applied pole of the bar magnet leaves the needle has always the opposite polarity to that by which it is stroked.

Evidence for molecular theory.—The last experiment is easily explained on the molecular theory of magnetisation. For on reference to Fig. 709 it will be seen that on stroking the bar from, say, left to right with the N pole of a bar magnet, the S poles of the molecular magnets will turn towards the N pole of the bar magnet, and, since the latter leaves at the right-hand end, this end will be a S pole.

Again, **if a magnet be broken in the middle**, new poles will appear at the freshly produced ends, as in Fig. 710 (*b*). This is easily

explained, for the act of breaking leaves a set of molecular S poles to the left of the gap, and of N poles at the right. A further breaking will produce additional poles, as in Fig. 710 (*c*)

The production of a magnet may be imitated by placing a ***test-tube of steel filings*** with one end near the pole of a strong bar magnet. On removing the tube **without shaking**, and testing it by hanging it near a *suspended magnet* (Fig 708), it will be found that the ends of the tube of filings are poles, that which was placed near the pole of the bar magnet being opposite in kind to the pole of the bar magnet The fragments of steel become permanent magnets and act like the molecular magnets in the bar. On shaking the tube all traces of poles at the ends disappear.

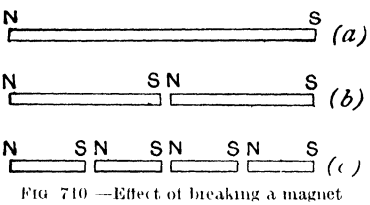


Fig 710 —Effect of breaking a magnet

If an unmagnetised bar of steel be placed with one end on the pole of a bar magnet and **tapped with a hammer**, it will be found to become a fairly strong permanent magnet If the bar be afterwards tapped, its magnetism is gradually lost The tapping assists the arrangement of the molecular magnets in the first part of the experiment when the bar magnet is near, and destroys their arrangement in the second part when the bar magnet is absent

Magnetisation disappears at high temperatures. This fact may be shown by heating a magnetised knitting needle to red heat by a long burner which will heat the whole of it at the same time, and then allowing it to cool in an east and west direction (the reason for this will be seen later) On testing the needle with iron filings, and by bringing it near a suspended needle, it will be found to be unmagnetised.

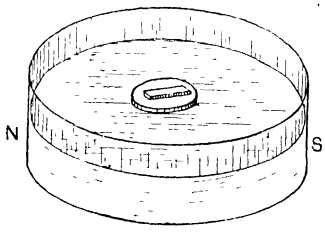


Fig 711 —Magnet floated on water

The amounts of N and of S pole on a magnet are equal. —Perhaps the strongest evidence in favour of the molecular theory of magnetisation lies in the fact that on any piece of iron or steel **the total amount of N pole is always equal to the total amount of S pole**, because the act of magnetisation does not involve the putting of pole on the bar or needle, but merely the arrangement of the molecular magnets as in Fig. 709 This may be proved by floating a bar magnet upon a piece of wood on water. The magnet is then free to move upon the surface in any direction. It will be found that the magnet rotates until it points north and south, but it does

not move bodily either north or south. Thus, the forces on the N and S poles are equal and opposite, giving rise to a couple but not to a resultant force of translation. This shows that the total amounts of N and of S pole are equal.

The **phenomenon of saturation** also points to the truth of the molecular theory. It will be seen later that a piece of iron or steel cannot be magnetised to more than a certain amount, when all the molecular magnets have been turned into the same direction it is obviously impossible to increase the magnetisation any further.

Soft iron and steel.—The chief magnetic difference between iron and steel is that iron is easily magnetised and readily demagnetised, while steel is not so easily magnetised, but retains its magnetisation much more than does iron.

Thus, if the pole of a strong bar magnet be applied to one end of a bar of soft iron, the bar of iron itself becomes a strong magnet

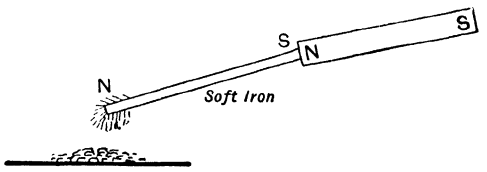


FIG 712 Magnetisation of soft iron

and will pick up a considerable amount of iron filings. On removing the bar magnet the filings immediately drop off the iron rod. Also if its polarity be tested by bringing its poles near those of a suspended needle, they will be found to be arranged as in Fig. 712.

On repeating this experiment with a steel rod it will be found that it is not nearly so strongly magnetised by contact with the magnet as was the iron rod, but on removing the magnet the steel rod is found to retain its magnetisation.

It is thus seen that the presence of a magnet always magnetises a piece of soft iron, and may magnetise a piece of steel if the previous magnetisation of the steel is sufficiently feeble. Hence, if a piece of soft iron be brought near to the N pole of a suspended magnet, the nearer parts of the iron become a S pole, as would be expected from the molecular theory, and there will now be an attraction between the iron and the N pole of the needle. This is characteristic of soft iron. The presence of a magnet produces such poles in the iron that **there is always an attraction between the iron and the magnet**. In the case of steel which is already magnetised, and has therefore so-called **permanent magnetisation**, there may be an attraction between it and a magnet, or there may be repulsion, as was seen on p. 770

Inverse-square law. In all cases where an effect is radially and uniformly distributed with respect to a point, the effect falls off inversely as the square of the distance from the point. We have already seen (p. 544) such a distribution in the case of light radiated from a point. The effect on a point magnetic pole due to another follows the same law. It is usual to attempt to prove this inverse-square law in the case of each phenomenon to which it applies, and on p. 786 we shall do so for the law of force between magnetic poles. For the present, however, it will be taken for granted. Thus, **the force between two point poles varies inversely as their distance apart**, or, expressed mathematically,

$$F \propto \frac{m_1 m_2}{d^2}$$

That the force between the poles varies directly as the product of their strengths m_1 and m_2, follows from the fact that it is independent of the presence of other poles. Thus, there is a certain force between two given poles A and B, if a third pole, C, be added to A, we have a force between (A + C) and B which is equal to the sum of the separate forces between A and B and between C and B. If the process be continued, A, B, C, etc., being all unit poles, we see that the force between any two compound poles is proportional to the product of the number of units of pole in each.

Unit pole. In the relation $F \propto m_1 m_2/d^2$, where F is the force in dynes between the two poles m_1 and m_2 situated d centimetres apart, let the poles be chosen of such a strength that when they are equal, and d is one centimetre, F is one dyne. Each of these is then taken as the **unit of magnetic pole**, and, measuring m_1 and m_2 in terms of these units, we may then write our equation,

$$\text{Force} = \frac{m_1 \times m_2}{d^2} \text{ dynes.}$$

Thus, **the unit magnetic pole may be defined as such a one, that when placed one centimetre from an equal pole, the force between them is one dyne, the poles being situated in air.**

EXAMPLE.—Magnetic N poles of strength 50 and 90 units are placed at the corners B and C of an equilateral triangle ABC of side 10 cm. If a S pole of strength 80 be placed at A, find the resultant force on A.

$$F = \frac{m_1 m_2}{d^2}.$$

For poles A and B, $F = \frac{50 \times 80}{10^2} = 40$ dynes.

" " A and C. $F = \frac{80 \times 90}{10^2} = 72$ dynes.

Since both these forces are attractions, draw AD to scale equal to 40 and AE equal to 72. Complete the parallelogram ADFE, and the diagonal AF then represents the resultant force on A. Its magnitude may be calculated from the relation on p. 82.

Thus,

$$AF = \sqrt{(72)^2 + (40)^2 + (2 \times 72 \times 40 \times \cos 60^\circ)}$$
$$= \sqrt{5184 + 1600 + 2880}$$
$$= \sqrt{9664} = 98 \text{ dynes.}$$

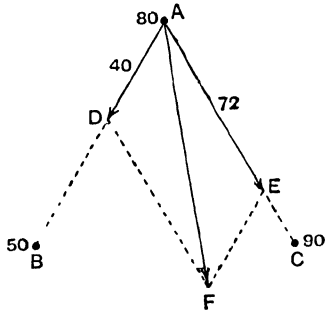


FIG 713.—Forces between magnetic poles.

EXERCISES ON CHAPTER LXI.

1. Describe how you would magnetise a knitting needle so that one particular end should be a N pole. How would you test this when made?

2. How would you magnetise a needle so that it should have a N pole at each end, and a S pole in the middle?

3. What kinds of forces are exerted between magnetic poles? Describe how you would demonstrate the proof of your statements.

4. A soft iron rod is held with an end near the south pole of a bar magnet. Give a diagram illustrating the magnetisation that the bar will acquire, and explain why it should become magnetised in this way.

5. A magnetic pole of strength 180 units is situated in line with a magnetised knitting needle, and at a distance of 30 centimetres from its middle point. If the length of the needle is 20 cm. and its strength of pole 40 units, find the force on the given pole.

6. Find the force on a magnetic pole of strength 40 units, situated at a distance of 30 cm. from each end of a magnetised needle whose length is 20 cm. and strength of pole 30 units.

7. Find the force between two magnets placed in the same straight line with a distance of 18 cm. between their middle points, if one has length 12 cm. and pole strength 60, and the other, length 6 cm. and pole strength 45 units.

8. Two magnetised knitting needles, each of mass 4 grams, are suspended with their two S poles together and the N pole ends hanging downwards. The magnets are free to turn about the S poles, and the N poles move apart until the distance between them is 4 cm. Find the pole strength of each needle, assuming that the N poles are situated 20 cm. below the S poles, and the centres of gravity are 10 cm. from the S poles.

9. A knitting needle is magnetised and then broken into four pieces equal in length. What will be the magnetic condition of the different pieces? What view of the nature of a magnet does such an experiment suggest?

10. State the law of action between magnet poles.

Two north poles repel one another with a force of 2·4 dynes when their distance apart is 2 cm. What will be their distance apart when the force is 3·6 dynes? Find also their repulsive force when their distance apart is 3 cm.

ter LXI. certain effects
. Thus, there is a region
... its influence may be detected.
... case of powerful magnets, and restricted
... The term mag-
... sometimes applied in a
...eral manner to this region, but
the term has further a definite and
more restricted meaning.

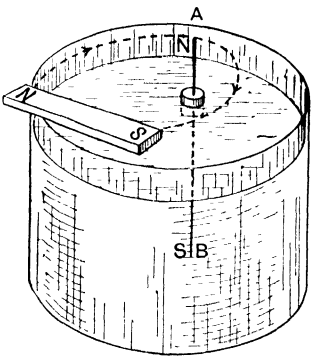


FIG. 714.—Experiment to illustrate magnetic field.

At every point near a magnet, or system of magnets, a magnetic pole would experience a force in some definite direction, and if the pole is free to move it would travel in this direction. This direction is called the **direction of the magnetic field** at that point. Although free poles cannot be obtained, their effect may be illustrated roughly by passing a long magnetised needle through a cork and floating it on water (Fig. 714). On bringing a bar magnet NS near, and on a level with the upper pole A, the force on this will be much greater than that acting upon the lower pole B. If A be a N pole, it will, on starting near N of the bar magnet, be driven along a curved path, and eventually reach S. Thus, at each point of its path it experiences a force driving it in the direction of the magnetic field at that point.

Again, a suspended needle, or compass needle, experiences two forces, one on its N pole in the direction of the magnetic field, and

the other on its S pole in the opposite direction These in general form a couple (Fig. 723) tending to twist it into the direction of the magnetic field, but when it attains this direction, the couple becomes *zero and it is then in equilibrium.* Thus, a suspended needle or compass needle, if small enough, comes to rest in the direction of the magnetic field Of course, if the needle is large, its two poles may be in places where the direction of the field is not the same, and the finding of its position of equilibrium is not then so easy.

Lines of force. A line whose direction at each point is that of the magnetic field at that point is called a **magnetic line of force** Thus, the magnetic pole A (Fig 714) traces in its motion, a line of force. A line of force may also be defined as the path along which a N pole, perfectly free to move, would travel As a perfectly free N pole cannot be obtained in practice, magnetic lines of force are usually obtained by means of a small compass needle, which, as we have seen, always comes to rest in the direction of the magnetic field.

EXPT. 164 —**Lines of force of a bar magnet.** Place a bar magnet on a sheet of drawing paper, and draw in its outline. Divide up this outline

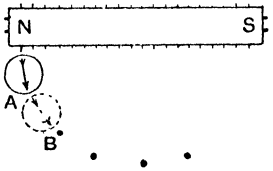


FIG. 715 —Plotting lines of force

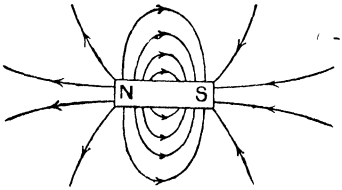


FIG 716 —Lines of force of a bar magnet

by a number of approximately equidistant marks Place a small compass needle with one pole as nearly as possible upon one of these marks, and make a mark A (Fig. 715) on the paper at the other pole of the compass. Now place the first end of the compass on A and make another mark B, and so on, until the line traced out reaches the edge of the paper or returns to the magnet. Draw an even curve through the points obtained, and mark it with an arrow to show the direction in which the compass pointed. Repeat the process until every mark upon the outline of the magnet is either the beginning or end of a line of force. The lines of force in Fig. 716 were obtained in this way.

Note.—Lines of force arise on a N pole and end on a S pole ; also no two lines can meet or cross each other, for if they did it would mean that the compass would have two directions at the same time, where the lines cross, which is absurd

EXPT. 165.—**Lines of force of two bar magnets, unlike poles together** ace two bar magnets on a sheet of drawing paper as in Fig. 717, anc ap out the lines of force as in Experiment 164.

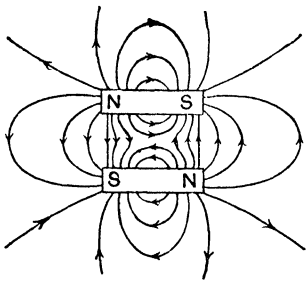


FIG. 717.—Lines of force of two bar magnets, unlike poles together.

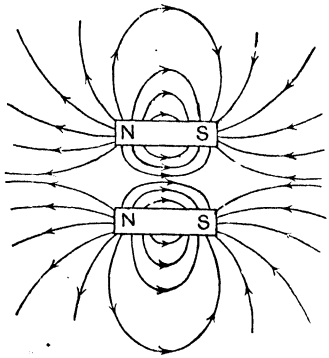


FIG. 718. Lines of force of two bar magnets, like poles together.

EXPT. 166.—**Lines of force of two bar magnets, like poles together.** Repeat Experiment 165, but with the N poles together as in Fig. 718.

EXPT. 167.—**Lines of force by means of iron filings.** Place a piece of rawing paper over the magnets in Expts. 164, 165 and 166, and sprinkle

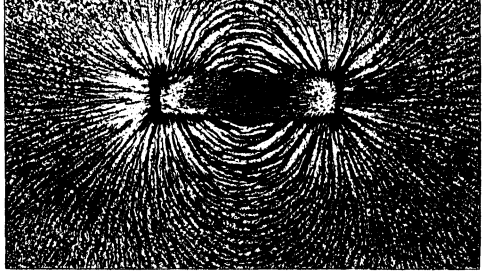


FIG. 719.—Lines of force by means of iron filings.

ron filings on the paper. Tap the paper gently until the filings arrange hemselves in lines, as in Fig. 719. Each filing becomes a magnet in the ield of the bar magnet, and the filings then hang together in chains. By oating the paper with molten paraffin wax before the experiment and llowing it to cool, a smooth surface is presented to the filings. After hey have taken their proper arrangement the lines can be rendered

permanent by passing a bunsen flame over the sheet, which melts the wax and imbeds the filings. Figs 719, 720 and 721 were obtained in this way.

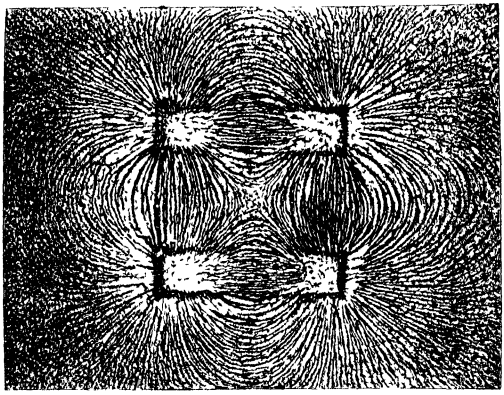


FIG 720 – Lines of force by means of iron filings

Strength of magnetic field, or magnetic intensity.—Having seen that the magnetic field has a definite direction at every point, its

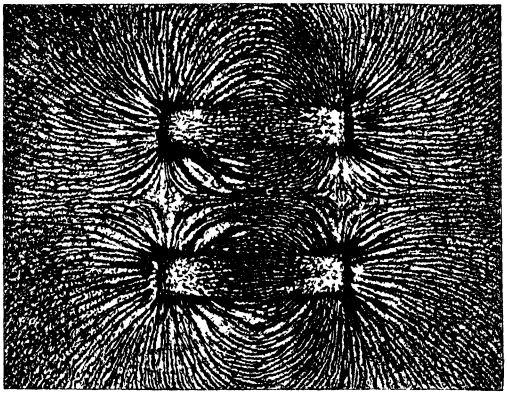


FIG 721 – Lines of force by means of iron filings

strength must now be considered It is fairly obvious from Figs. 716-721 that the field is stronger where the lines of force are packed closely together than where they are far apart. In order, however, to define exactly the strength of field at any point, a unit pole must be imagined to be placed at the point This pole experiences a

force, which force is taken as the measure of the strength of field. Thus, the **strength of field, or magnetic intensity at any point is the force that would be exerted on a unit N pole situated at this point** This quantity is usually denoted by the letter H Thus, the force on a pole m, situated in a magnetic field of intensity H, is mH dynes.

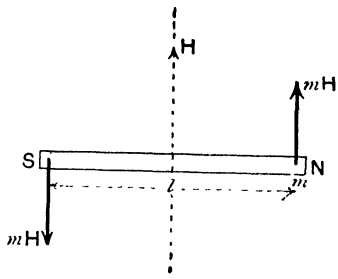


FIG 722.—Couple acting on a magnet

Magnetic moment of a magnet. - Consider a magnet NS (Fig 722) having a pole of strength m at either end, to be situated at right angles to a magnetic field of intensity H Then each pole experiences a force mH dynes, these forces act in opposite directions, and therefore constitute a couple (p. 125) whose turning moment is Hml, where l is the length of the magnet or distance between the poles This couple consists of two parts, the field H and the part ml belonging to the magnet The quantity ml is called the **magnetic moment** M of the magnet.

Hence, Couple = HM

As a rule magnets do not have the two poles situated exactly at the ends, and hence both m and l are indefinite quantities. Still, the magnetic moment M is not indefinite, for it may be measured by mechanical means, by determining the couple necessary to hold the magnet at right angles to a given field. Thus, the magnetic moment may also be defined as **the couple required to hold a magnet at right angles to a field of unit intensity.**

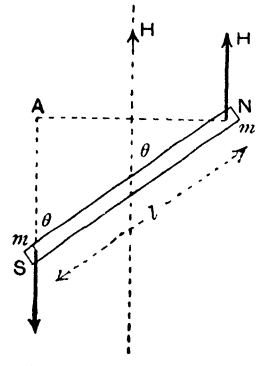


FIG 723.—Couple acting on a magnet inclined to a magnetic field

Couple acting on a magnet in any position. - If the magnet be inclined at an angle θ to the field H (Fig. 723) the force on each pole is still Hm, but the perpendicular distance between the forces is

AN = $l \sin \theta$:

$\therefore$ couple = H$ml \sin \theta$

= HM $\sin \theta$.

From this it is seen that when $\theta = 90°$, the couple is HM, as found above, and when $\theta = 0°$ the couple is zero, since $\sin 0° = 0$. Hence,

the only position in which a freely suspended magnet is in equilibrium in a given field is when its direction is coincident with that of the field

Field due to a bar magnet.—The general form of the field due to a bar magnet is shown in Fig 716 The strength of field at certain points may be calculated without difficulty Thus, take a point P (Fig 724) on the line passing through the poles of the magnet of pole strength m. Let l be half the length of the magnet and d the distance of P from its middle point. Then the distance of P from N is $(d-l)$, and from S the distance is $(d+l)$.

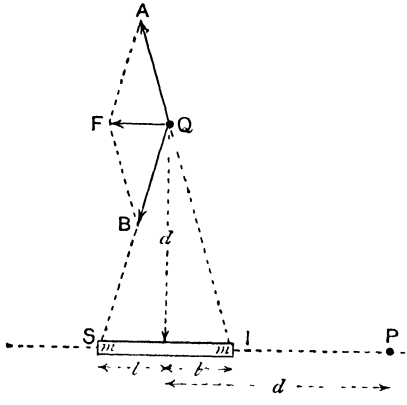


FIG 724.—Strength of field due to a bar magnet by calculation

Imagine a unit N pole placed at P

Force on unit pole due to N $= \dfrac{m}{(d-l)^2}$

Force on unit pole due to S $= \dfrac{m}{(d+l)^2}$.

Since these two forces are in the same line but have opposite senses, we have

$$\text{Resultant force} = \frac{m}{(d-l)^2} - \frac{m}{(d+l)^2}$$

$$= m\,\frac{(d+l)^2-(d-l)^2}{(d^2-l^2)^2}$$

$$= \frac{4ml\ d}{(d^2-l^2)^2}.$$

This is the force on unit pole, and is therefore the strength of the field at P Also, since $2ml$ is the magnetic moment M of the magnet,

$$\text{Strength of field at P} = \frac{2\mathrm{M}d}{(d^2-l^2)^2}.$$

If P is at a considerable distance from the magnet, l^2 may be neglected in comparison with d^2, and the strength of field is then $2\mathrm{M}/d^3$.

At a point Q on the line bisecting the magnet at right angles (Fig. 724), the strength of field may be determined in a somewhat

similar manner, by placing a unit N pole at Q and finding the force acting on it. Thus,

$$\text{Force due to N} = \frac{m}{QN^2},$$

$$\text{,, ,, S} = \frac{m}{QS^2}.$$

Representing these forces by QA and QB, the resultant force will be represented by QF

From the similarity of the triangles FBQ and NQS,

$$\frac{FQ}{BQ} = \frac{NS}{QS}.$$

Now, $BQ = \frac{m}{QS^2}$, and $NS = 2l$;

$$\therefore FQ = \frac{2ml}{QS^3}.$$

Also, since $QS^2 = d^2 + l^2$, $QS^3 = (d^2 + l^2)^{\frac{3}{2}}$;

$$\therefore FQ = \frac{2ml}{(d^2 + l^2)^{\frac{3}{2}}} = \frac{M}{(d^2 + l^2)^{\frac{3}{2}}}$$

which is therefore the strength of field at Q due to the magnet.

If d is so great compared with l, that l^2 may be neglected in comparison with d^2, the strength of field is M/d^3.

Neutral point. In plotting the lines of a force of a magnet (Fig 716), it is the resultant of the fields due to the magnet and the earth that has been found Near the magnet, the earth's field has little effect, as it is feeble in comparison with that of the magnet At greater distances, however, the earth's field is more important, and at very great distances from the magnet the earth's field only is of consequence It will therefore happen that, at some points, the fields of the earth and the magnet are equal, and if they are also opposite in direction, the resultant is zero. The compass needle would set in any direction at such points. They are called **neutral points.**

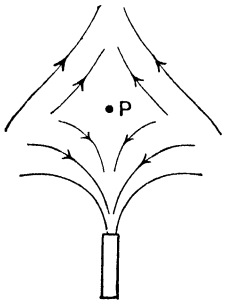


FIG. 725.—Neutral point.

EXPT. 168.—**Neutral point.** Place the bar magnet on a sheet of drawing paper with its S pole pointing north. Plot the lines of force diverging from the S pole. It will be found that they are of the form shown in Fig. 725 in the neighbourhood of some such point as P. Draw the lines nearer and nearer to P until its position is determined. Measure the distance d from P to the

middle of the magnet, and, taking the earth's field strength as 0·18, calculate the magnetic moment of the magnet from the relation $2M/d^3 = 0{\cdot}18$; or use the more exact relation

$$\frac{2Md}{(d^2 - l^2)^2} = 0{\cdot}18.$$

The magnetometer.—It has already been seen that a suspended needle points north and south, that is, it sets in the direction of the earth's magnetic field If, however, a magnet be brought near the needle, it may be deflected from the true north and south position, it will set along the resultant magnetic field due to the earth and the magnet

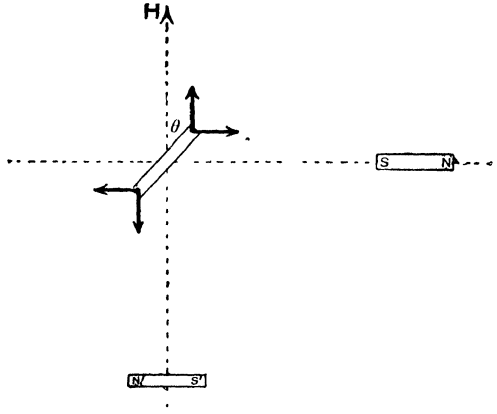


FIG 726 —Diagram of the magnetometer

Let the field due to the magnet NS be at right angles to that due to the earth, as in Fig. 726. The suspended magnet will take up some such position as shown, making an angle θ with the earth's field

Hence, the couple due to the earth's field is $Hm \sin\theta$ (where m is the magnetic moment of the suspended magnet), and tends to rotate it into the direction of the earth's field (p 781) In a similar manner the deflecting couple due to the magnet's field F, is $Fm \cos\theta$, and the suspended magnet is in equilibrium when these two couples, which tend to rotate it in opposite directions, are equal,

$$\therefore Fm \cos\theta = Hm \sin\theta ;$$

$$\therefore \frac{F}{H} = \tan\theta.$$

For the magnet NS in the position shown, which is known as the "**end on**" position, $F = 2M/d^3$ (p. 782), so that

$$\frac{2M}{d^3} = H \tan \theta, \quad \text{or} \quad \frac{M}{H} = \frac{d^3}{2} \tan \theta.$$

If the magnet is placed at N′S′, then $F = M/d^3$, and

$$\frac{M}{d^3} = H \tan \theta, \quad \text{or} \quad \frac{M}{H} = d^3 \tan \theta.$$

This is known as the "**broadside**" position.

The **magnetometer** has a variety of forms, a common one being shown in Fig. 727. The magnetic needle may be supported on a

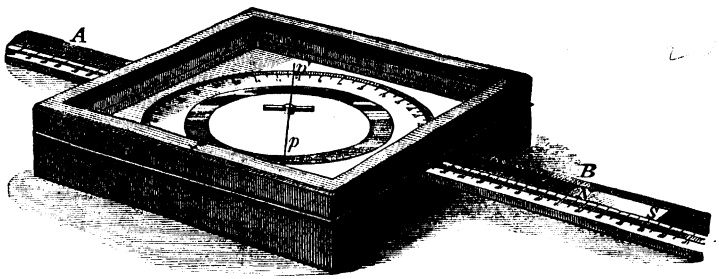


FIG. 727.—The magnetometer.

needle point, or suspended by a fibre. It is provided with a long pointer *pp′*, so that its position with respect to a horizontal circular scale may be read to half a degree or less.

In Fig. 727 the magnet producing the deflection is shown in the **end on** position at B, and is either east or west of the suspended needle. For the **broadside** position the magnetometer must be turned round so that the magnet is either north or south of the suspended needle. Its distance may be measured upon the linear scale.

Use of the magnetometer.—The magnetometer may be used for comparing magnetic fields, magnetic moments, or finding the ratio of the values of a magnetic moment and a field. In all cases the procedure is as follows:

(i) The instrument is turned round, and levelled if necessary, until both ends of the pointer are at 0° on the scale when there is no deflecting magnet near.

(ii) The magnet B is placed with its middle point at a certain distance *d* from the needle, and the deflection indicated by **both**

ends of the pointer observed This is to correct for want of symmetry of the scale and pointer

(iii) If the magnet was east of the needle in the first position, it is now removed to an equal distance west of it, or *vice versa*, and the deflection again observed

This is to correct for the fact that the zero of the linear scale may not be exactly at the middle of the suspended needle

(iv) The magnet is now turned over so that the end which pointed east now points west, and all the observations repeated The magnet may not be symmetrical in its polarity, but this is corrected for by this reversal of its direction

There are now eight readings of the deflection, and the mean of the eight is the deflection, free from the above errors. The readings may with advantage be recorded as follows

	Deflection	
Magnet E of needle, N pole pointing E - -		
,, ,, ,, ,, W - -		
Magnet W of needle, N pole pointing W - -		
,, ,, ,, ,, E - -		
Distance d = cm	Mean deflection θ = .	

Expt. 169.—To prove the relation $\frac{M}{H} = \frac{(d^2 - l^2)^2}{2d} \tan\theta$ for the end on position

Follow the above instructions, with the magnet at 50 cm. from the needle in the end on position, and observe the mean deflection. Repeat at distances of 45 cm., 40 cm., and 35 cm. In each case calculate the value of $(d^2 - l^2) \tan\theta / 2d$, and show that this is approximately constant; for each distance calculate the percentage error introduced in using $(d^3 \tan\theta)/2$ instead of the more exact relation.

Expt. 170.—To prove the relation $\frac{M}{H} = (d^2 + l^2)^{\frac{3}{2}} \tan\theta$ for the broadside position.

Repeat the last experiment, using the magnet in the broadside instead of the end on position. Calculate $(d^2 + l^2)^{\frac{3}{2}} \tan\theta$ for each distance, and find the percentage error at each distance if $d^3 \tan\theta$ were used.

Proof of inverse square law.—It will have been noticed by the student that the formula for the strength of field due to a magnet was calculated on p 782, by assuming the inverse square law to be true: hence, the validity of the results depends upon the truth of

the law, and the constancy of the values for M/H in the magnetometer experiments is a good indication of the truth of the law.

Comparison of magnetic moments. By employing different magnets in the magnetometer experiment, the ratio of their magnetic moments may be found. Using the first magnet of magnetic moment M_1 and obtaining the corresponding mean deflection θ_1, as on p 786, for distance d_1.

$$\frac{M_1}{H} = \frac{d_1^3}{2} \tan \theta_1,$$

and for the second magnet,

$$\frac{M_2}{H} = \frac{d_2^3}{2} \tan \theta_2;$$

$$\therefore \frac{M_1}{M_2} = \frac{d_1^3 \tan \theta_1}{d_2^3 \tan \theta_2}$$

If more exact results are required, the longer expressions for M/H must be used.

EXPT. 171.—**To compare magnetic moments.**—Find $d^3 \tan \theta$ for each of two magnets in turn, using three distances d, in each case, and obtain the ratio M_1/M_2 as above.

Comparison of magnetic fields. The strengths of magnetic field at two places may be compared by performing the magnetometer experiment with the same magnet at both places. Thus, at the first place let H_1 be the strength of field,

then,
$$\frac{M}{H_1} = \frac{d_1^3}{2} \tan \theta_1.$$

At the second place, where H_2 is the magnetic field,

$$\frac{M}{H_2} = \frac{d_2^3}{2} \tan \theta_2;$$

$$\therefore \frac{H_1}{H_2} = \frac{d_2^3 \tan \theta_2}{d_1^3 \tan \theta_1}.$$

Equivalent length of magnet. Since the poles of a magnet are not merely at its ends, but are spread over an appreciable part of the sides, it is not strictly correct to take l as half the actual length of the magnet in the above cases. For every magnet, however, there must be some equivalent length l, such that $2ml$ is the moment of the magnet, and it is this value of l that should be used in the above experiments.

This equivalent length l may be found for any magnet by making

observations and finding $(d^2 - l^2)^2 (\tan\theta)/2d$ for two distances d_1 and d_2. These results should be equal, hence

$$\frac{(d_1^2 - l^2)^2}{2d_1} \tan\theta_1 = \frac{(d_2^2 - l^2)^2}{2d_2} \tan\theta_2,$$

and, since d_1, d_2, $\tan\theta_1$ and $\tan\theta_2$ are now known, l may be calculated.

The following instructive method, due to Mr E. Edser, may be employed. Using the broadside position, we have,

$$\frac{M}{H} = (d^2 + l^2)^2 \tan\theta,$$

or

$$d^2 + l^2 = \left(\frac{M}{H}\right)^{\frac{2}{3}} \frac{1}{\tan^{\frac{2}{3}}\theta} = \left(\frac{M}{H}\right)^{\frac{2}{3}} \cot^{\frac{2}{3}}\theta$$

If a series of values of d and θ be observed, d^2 and $\cot^{\frac{2}{3}}\theta$ may be calculated, and if plotted in the form of a graph, will give a straight line, such as AB (Fig. 728). This line may be produced backwards until it cuts the d^2 axis in the point C, for which point $\cot^{\frac{2}{3}}\theta = 0$, and therefore $d^2 + l^2 = 0$, so that OC is numerically equal to l^2. By taking the square root l is found.

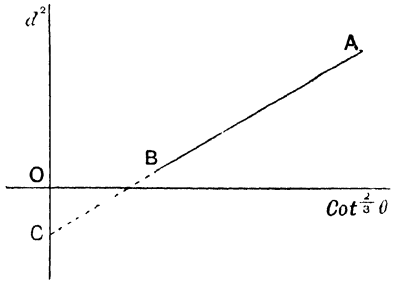


FIG. 728.—Graph for finding the equivalent length of a magnet.

EXPT. 172.—**To find the equivalent length of a magnet.** Perform a set of readings as in Expt. 170, but plot d^2 and $\cot^{\frac{2}{3}}\theta$ as in Fig. 728. Produce the line AB to cut the axis in C. Note the length of OC, find its square root l, and double it to obtain the equivalent length of the magnet. Measure its actual length and evaluate the ratio

$$\frac{\text{equivalent length of magnet}}{\text{real length of magnet}}$$

Vibration of a suspended magnet. – It will have been noticed that comparisons of magnetic moments and strengths of field may be made by means of the magnetometer, but that neither can be determined in absolute measure. To do this, some other relation between these quantities must be found. This relation is given by the equation for the time of oscillation of a suspended magnet, vibrating with small amplitude in a magnetic field. This relation is

$$T = 2\pi \sqrt{\frac{I}{MH}},$$

where T is the time of one complete oscillation and I is the **moment of inertia** (p 200) of the magnet, M and H have the same meanings as before. The moment of inertia is related to rotational motion, exactly as **mass** is to linear motion, and its value for a rectangular bar magnet about an axis passing through the centre of mass is found to be Mass $\times (l^2+b^2)/12$, where l is the length and b the breadth of the bar. For a circular bar magnet $\mathrm{I} = \mathrm{Mass} \times \left(\frac{l^2}{12} + \frac{r^2}{4}\right)$, where r is the radius of the circle.

Comparison of magnetic fields.—Since the moment of inertia of any body with respect to a given axis is constant, a knowledge of the time of vibration of a magnet affords a good method for the comparison of field strengths. For if T_1 is the time of vibration at a place where the field strength is H_1, and T_2 is the time where the field strength is H_2,

$$\mathsf{T}_1 = 2\pi\sqrt{\frac{\mathrm{I}}{\mathsf{MH}_1}}, \quad \text{and} \quad \mathsf{T}_2 = 2\pi\sqrt{\frac{\mathrm{I}}{\mathsf{MH}_2}},$$

$$\therefore \frac{\mathsf{T}_1}{\mathsf{T}_2} = \sqrt{\frac{\mathsf{H}_2}{\mathsf{H}_1}}, \quad \text{or} \quad \frac{\mathsf{H}_1}{\mathsf{H}_2} = \frac{\mathsf{T}_2^2}{\mathsf{T}_1^2}.$$

As a rule, the numbers of vibrations n_1 and n_2 in a given time t are observed, then $t = n_1\mathsf{T}_1 = n_2\mathsf{T}_2$:

$$\therefore \frac{\mathsf{H}_1}{\mathsf{H}_2} = \frac{n_1^2}{n_2^2},$$

where n_1 and n_2 are the numbers of vibrations made in the same time at two places where the field strengths are H_1 and H_2 respectively.

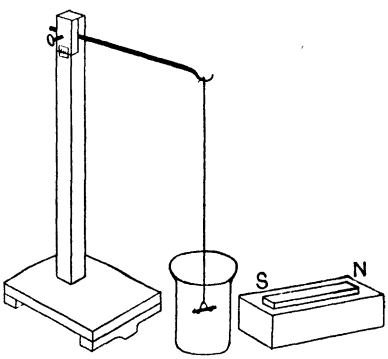


FIG. 729.

EXPT. 173.—**Plotting fields by means of vibrations.** Suspend a piece of magnetised knitting needle or clock-spring about 2 cm. long, by means of a silk fibre, protecting it from draughts by means of a beaker, as in Fig. 729. Either north or south of the needle place the bar magnet NS, pointing north and south, with its nearest pole 10 cm. from the needle. Count the number of vibrations n made by the needle in one minute. Repeat at distances of

15, 20, 25, and 30 cm., and again when the magnet is taken away, so that the needle vibrates in the earth's field alone. Tabulate the result as follows:

Distance	n	n'	$(F+H)\propto n^2$	F
15				
20				
∞			0·18	0

In the fourth column is the resultant field due to magnet and earth $(F+H)$; the values for this field are found from the fact that n^2 is proportional to the resultant field, and with the magnet absent (or at infinity) the earth's field alone is assumed to be 0·18. The last column, F, is the field due to the magnet alone, and is obtained by taking 0·18 from each of the values of $(F+H)$. Plot a curve with distances from the magnet as abscissae and F as ordinates.

Repeat the experiment with the magnet east or west of the needle and still pointing north and south, measuring the distance from the middle of the magnet.

Determination of earth's magnetic field.—From the magnetometer experiment (Expt. 169) we obtain the relation $\frac{M}{H} = \frac{d^3}{2}\tan\theta$, and from the vibration experiment (p. 789) $T = 2\pi\sqrt{\frac{I}{MH}}$, or $MH = \frac{4\pi^2 I}{T^2}$. Combining these equations we obtain $MH \times \frac{M}{H} = M^2$, or $MH \div \frac{M}{H} = H^2$, and thus both M and H may be found in absolute measure.

EXPT. 174.—**Determination of H.** By means of the magnetometer find the value of $\frac{1}{2}d^3\tan\theta\,(=M/H)$, as in Expt. 169. Now suspend the bar magnet at the place previously occupied by the magnetometer needle, employing a paper stirrup hung by a silk thread, or the finest cotton that will carry the weight of the magnet. Take the time for fifty oscillations of the magnet by means of a stop-watch, and so obtain T, the time of one oscillation. Care should be taken that the magnet oscillates only a few degrees on either side of its equilibrium position, and **complete** oscillations must be taken, that is, the time from passing any given position to next passing the **same position in the same direction.** Now weigh the magnet, measure its length and breadth, and calculate I, the moment of inertia (p. 789). $4\pi^2 I/T^2\,(=MH)$ should now be found, and combining this with the result of the magnetometer experiment, M and H should be calculated.

EXERCISES ON CHAPTER LXII.

1. Describe how the strengths of two magnetic fields may be compared by means of a vibrating magnet.

2. Define the magnetic moment of a magnet. Describe how the magnetic moments of two magnets may be compared by a deflection method.

3. Show that the magnetic intensity due to a short bar magnet of moment M at a distance d from the mid-point along a line at right angles to the magnetic axis is approximately M/d^3.

4. A small magnet oscillating in the earth's magnetic field makes 20 vibrations in 150 seconds. It is then placed due north of a bar magnet, which is lying in the magnetic meridian with its north pole pointing north, and is then found to make 20 vibrations in 80 seconds. Find the strength of magnetic field, at the position of the oscillating magnet, due to the bar magnet. (H = 0·18.)

5. A small suspended magnet makes 10 oscillations in 35 seconds in the earth's field alone. On bringing a bar magnet with its S. end pointing north to a point due east of the suspended needle, the needle makes 20 oscillations in 55 seconds. Find the time of oscillation of the suspended needle if the bar magnet is now reversed pole for pole.

6. Define the magnetic moment and the magnetic axis of a magnet, and describe how you would determine one of them experimentally.

7. Define the terms *unit magnetic pole* and *strength of a magnetic field at a given point.*

Calculate the field due to a bar magnet, 10 cm. long, and having a pole strength of 100 units, at a point 20 cm. from each pole. L.U.

8. What factors determine the time of oscillation of a magnet when free to swing in a horizontal plane?

Two bar magnets are bound together side by side and suspended so as to oscillate in a horizontal plane. The time of swing is 12 seconds when like poles are together and 16 seconds when the direction of one magnet is reversed. Compare the moments of the magnets. L.U.

9. How would you define (*a*) unit magnetic pole, and (*b*) the moment of a magnet?

A thin magnet 20 cm. long, having its north-seeking end pointing south, just balances the earth's field (H = 0·2 C.G.S.) at distances of 10 cm. from its poles. Find its magnetic moment. L.U.

10. Explain the method of comparing the intensities of magnetic fields by observations of the times of oscillation of a magnetic needle.

A small magnet vibrating horizontally in the earth's field has a period of 4 seconds. When another magnet is brought near to it, 50 swings take 160 seconds. Compare the strength of the field due to the magnet with the earth's horizontal field, assuming that the two fields are in the same direction or in opposite directions. L.U.

11. What is the law of force between two magnetic poles? The moment of a magnet is 200 units, and the poles are 10 cm. apart. Find the force

which this magnet exerts upon a pole of strength 10 units placed upon its axis at a distance of 25 cm. from the mid point of the magnet
L.U.

12 Describe some method of measuring the horizontal component of the earth's magnetic field.

13. A bar magnet is 8 cm. long, and has its magnetic poles concentrated at its ends. Determine graphically the direction of the magnetic field at points distant from the poles as follows. (*a*) 4 cm. from the N. pole and 9 cm. from the S. pole; (*b*) 6 cm. from the N pole and 8 cm. from the S. pole, (*c*) 7 cm from the N. pole and 5 cm from the S pole.
Adelaide University.

14. What is the *magnetic moment* of a magnet?
A short magnet is placed in a horizontal plane with its axis parallel to the meridian and its N-seeking pole pointing to the south It is found that at a point on the axis of the magnet 50 cm to the south of its mid-point the resultant horizontal field is zero. If the intensity of the earth's horizontal field is 0 20 dyne per unit pole, calculate the magnetic moment of the magnet. L.U.

15. Define Magnetic Field, Strength of Field, Lines of Force, Magnetic Moment, Unit Pole.
What is the couple acting on a magnet 12 cm long, of pole strength 7 units, placed at an angle of 60° with the direction of a field of strength 0 017 unit? Punjab University.

CHAPTER LXIII

TERRESTRIAL MAGNETISM

Earth's resultant magnetic field.—In Chapter LXII the magnetic field has been treated as though it were horizontal, because it is only the horizontal component of the field that is active in making a compass needle point north and south. That the needle itself remains horizontal is not surprising, for it is always adjusted in its stirrup, or on its needle-point, until it assumes a horizontal position.

In order to determine whether the earth's magnetic field is really horizontal, it is necessary to balance the needle carefully **before it is magnetised.** It is then magnetised and mounted so that it is free to rotate in a vertical plane (p 797) It will then be found that it does not set horizontally; in the northern hemisphere the N end of the needle dips downwards, and in the southern hemisphere the S end dips downwards. The resultant magnetic field is therefore inclined to the horizontal.

Again, if a soft iron rod be placed north and south, it will be magnetised by the earth's field, the end pointing north becoming a N pole Also, if it be placed vertically, in the northern hemisphere, the lower end will become a N pole; further, if the rod be placed parallel to the direction in which the dip needle sets, it will be still more strongly magnetised. A slight tapping assists these effects. If an unmagnetised bar of steel be used, a vigorous tapping with a hammer will be necessary for it to become magnetised.

EXPT. 175.—**Magnetisation of a bar in the earth's field.** Place a soft iron bar, about 18 in. long and $\frac{1}{4}$ in. diameter, horizontally north and south and tap it gently. Bring a compass needle near each end in turn, and test its polarity. Repeat with the bar arranged vertically, and test the polarity.

Place the bar in a vertical plane situated N and S, the length of the bar being inclined at about 60° to the horizontal Tap it gently and test the polarity of its ends with a compass needle.

Magnetic declination and dip. It is well known that a compass needle does not point true geographical north and south, thus the direction of the earth's magnetic field is not in general horizontal, neither is it in the geographical meridian If the vertical plane AB (Fig 730) be taken to represent the geographical meridian, or plane passing through the point of observation and containing the axis of rotation of the earth, the **magnetic meridian,** or vertical plane containing the axis of a freely suspended magnet, is inclined to AB, and may be represented by the plane CDEF. The angle GCD between the geographical and magnetic meridians is called the **magnetic declination,** or in nautical language, the **magnetic variation,** or the **variation of the compass**

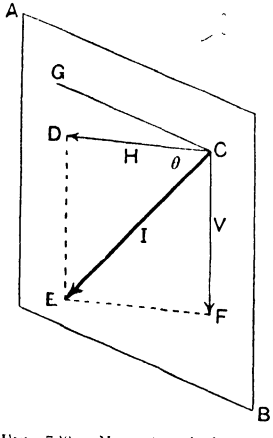


FIG 730 —Magnetic declination and dip

The angle DCE between the resultant direction of the earth's magnetic field and the horizontal is called the **magnetic dip**, it is the angle of dip of a magnetised needle which is free to rotate in the plane of the magnetic meridian

The resultant magnetic intensity CE, represented by I, may be resolved into two components, H and V, one of which is horizontal and the other vertical Since the triangles CDE and EFC have right angles at D and F, we have

$$\tan\theta = \frac{V}{H}, \quad \text{and} \quad I^2 = H^2 + V^2$$

These quantities, the declination, dip, H, V and I are known as the **magnetic elements** at any point on the earth's surface, and if the declination and any two of the other elements are known, the whole of them may be calculated

The three elements usually measured at any place are the **declination, dip,** and the **horizontal component of the earth's field, H.** The measurement of H has already been described.

Measurement of declination. To determine the declination at any place, the position of the geographical meridian must be found, and also that of the magnetic meridian. The geographical meridian is found by astronomical means, by observing the direction of the sun at a known time, its direction at this particular time being known in terms of the longitude of the place, from astronomical tables. To determine the direction of the magnetic meridian, a suspended needle or compass is employed, but it must be so arranged that when the direction of its axis has been observed, it may be turned over and suspended from the other side and a new observation made. The true direction of the magnetic meridian is obtained by bisecting the angle between the two observed positions of the geometrical axis of the magnet.

The reason for this may best be understood by considering what we mean by the **magnetic axis of a magnet**. In the case of a thin magnetised needle there is little doubt as to the meaning of the term, it is a line joining the poles. But magnets are not as a rule



FIG 731. Magnetic axis of a magnet

of this simple form. A common type is shown in Fig. 731. The poles are spread over the ends, and if the position of the resultant point poles be defined in a manner somewhat similar to that employed in finding the centre of gravity (p. 107), the line joining these resultant poles is the magnetic axis. The magnet, when suspended, will then set with the magnetic axis in the direction of the magnetic field. The positions of these resultant point poles cannot conveniently be found, but it may be noticed that a suspended magnet always sets with its magnetic axis in the direction of the magnetic field. Hence, we may define the **magnetic axis of a magnet as that line in it which always takes the direction of the magnetic field when the magnet is freely suspended in the field and is allowed to come to rest.**

Thus, if CD is the magnetic axis of the magnet AB in Fig. 731, when the magnet is suspended and comes to rest, CD has the direction of EF, the magnetic meridian. On turning the magnet over and suspending it from the other side, the magnetic axis CD will still be in the meridian, but A′B′ is the position of the magnet. The magnetic meridian EF then bisects the angle between AB and A′B′.

EXPT. 176.—**Determination of the magnetic meridian and axis of a magnet.** Take two equal discs of cardboard and paste them together with several parallel magnetic needles between them. Select two points on the

circumference diametrically opposite each other and join them by a thick ruled diameter on each side of the disc. Suspend the disc above, but near to, a sheet of drawing paper; it comes to rest with the ruled diameter in some such position as AB (Fig. 732). Mark the position of AB on the paper. Now suspend the disc from the other side. The ruled diameter comes to rest in the position A′B′. Bisect the angle between AB and A′B′ by the line EF, which is then the direction of the magnetic meridian, and may be checked by suspending a magnetised knitting needle in place of the disc. The position of the magnetic axis CD of the disc coincides with EF.

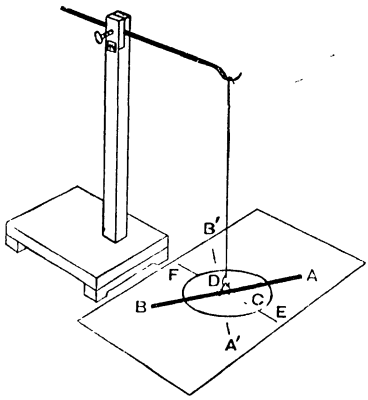


FIG. 732.—Determination of the magnetic meridian

It should be noticed that, even in the extreme case of a disc whose direction of magnetisation is unsuspected, the magnetic meridian can be found by taking the two positions of rest of some arbitrary fixed mark on the disc, the disc being suspended from its two sides in turn.

Measurement of magnetic dip. The dip is usually measured by an instrument known as the **dip circle**. This consists of a vertical circle (Fig. 733) at the centre of which the dip needle is suspended by a fine steel axle resting upon two agate knife-edges. The circle and supports carrying the knife-edges can rotate about a vertical axis, the azimuth (*i.e.* the angle between the plane of the circle and some fixed plane of reference) being observed by the horizontal circular scale and verniers. The magnet can be raised from the knife-edges, or lowered on to them, by means of two V supports, not shown in Fig. 733, the ends of the magnet's axle resting in the V's. In this way the axis of the magnet may always be brought back to the centre of the circle, and any sticking of the axle on the knife-edges may be prevented.

To use the dip circle, it is first levelled by means of the spirit-level and the levelling screws. It is then turned about its vertical axis until the magnet sets vertically, both ends pointing to 90° upon the scale. The plane of the circle is then at right angles to the magnetic meridian, and by rotating the instrument through 90° by

the horizontal scale, the plane of the vertical circle and of rotation

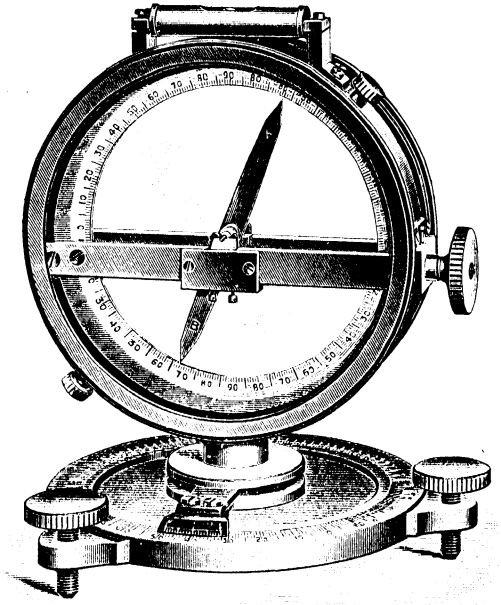


FIG. 733. The dip circle.

of the needle is brought into the magnetic meridian. The reason for this adjustment is, that when the plane of rotation of the magnet is at right angles to the magnetic meridian, the horizontal component of the earth's field H, being parallel to the axis of rotation, does not produce any couple tending to rotate the magnet about this axis (Fig. 734). The vertical component V then sets the magnet vertically.

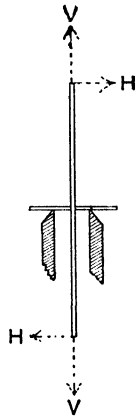


FIG. 734.—Supports of the dipping needle.

In the instrument shown in Fig. 733, the points of the needle are very close to the divisions of the vertical circular scale, and very small error is made in reading their position. By estimation, the readings can be made to about one-tenth of a degree. If greater accuracy is required, the points of the needle are observed by means of low-power microscopes, carried on a framework

which rotates about an axis having the same position in space as the dipping needle The microscopes are provided with cross-wires, and their positions are observed by means of vernicrs moving over the vertical circular scale

Having placed the dip circle with its plane in the magnetic meridian, there are four sources of error to be corrected for, involving the making of sixteen observations

(i) **The axis of rotation of the magnet may not be at the centre of the scale,** as shown to an exaggerated extent in Fig 735. Both ends of the pointer are therefore observed, and the mean value of the dip is then free from this error

(ii) **The zero line of the circle may not be truly horizontal** This would make the dip appear to be too great or too small, as is seen

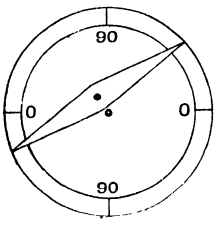


FIG 735.—Eccentric error

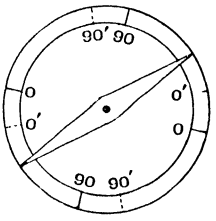


FIG 736 Levelling error.

for the positions O O and O′—O′ respectively in Fig. 736. On rotating the dip circle through 180° about its vertical axis, as measured by the horizontal scale (Fig 733), this error is reversed and the readings of (i) are repeated

(iii) **The magnetic axis of the magnet may not coincide with its geometric axis** This source of error has been discussed on p 795 The magnet is reversed on its bearings, and the previous readings (i) and (ii) repeated

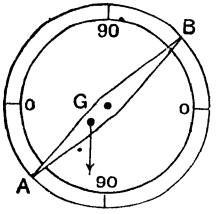


FIG 737.—Gravitational error

(iv) **The centre of gravity of the needle may not lie in the axis of rotation of the magnet** In this case there would be a couple due to gravity, which would either increase or decrease the dip In Fig 737 the end A of the magnet is the heavier, the centre of gravity being at G The magnet must now be remagnetised, so that the end B dips downwards, and as the previous readings all indicate too great a value for the dip, on now repeating them they will indicate too small a value. Thus, in all, 16 readings are taken, and

may be tabulated as follows, the mean being taken as the true dip :

		Reading of Upper end	Reading of Lower end
End A dipping	Circle facing E - - - -		
	,, ,, W - - -		
	Magnet reversed on bearings		
	Circle facing W - -		
	,, ,, E - - - -		
End B dipping	Circle facing E - - - -		
	,, ,, W - -		
	Magnet reversed on bearings		
	Circle facing W - - -		
	,, ,, E - - - -		
	Total - -		
Mean value of dip =			

EXPT. 177.—**To measure the magnetic dip.** By means of the dip circle, follow the instructions given above, and determine the angle of dip.

Magnetic maps.—Observations of the magnetic declination, dip, and horizontal intensity have been made at a great number of places on the earth's surface, and the results represented for convenience upon maps This may be done in a variety of ways, but the commonest is to draw lines through points upon the map for which the value of any particular element is the same.

Isogonals.—**Isogonals** are lines passing through points for which the magnetic declination has the same value In Fig 738 the isogonals (shown in full and heavy dotted lines) all pass through the geographical north and south poles There are also two other points through which they pass. One is situated in latitude about 73° 31′ N., longitude 96° 43′ W., and is called the **magnetic north pole,** and the other in latitude about 72° 21′ S., longitude 155° 16′ E., and is called the **magnetic south pole.**

One **agonic line,** or line at every point of which the declination is zero, or the compass points due north, runs through the American Continent, and the other through Europe, Arabia, the Indian Ocean and Australia. At all points in the "Atlantic" space between these two agonic lines the declination is westerly (full line), that is, the compass points west of geographical north. In the "Pacific" space the declination is easterly (heavy dotted line), except within

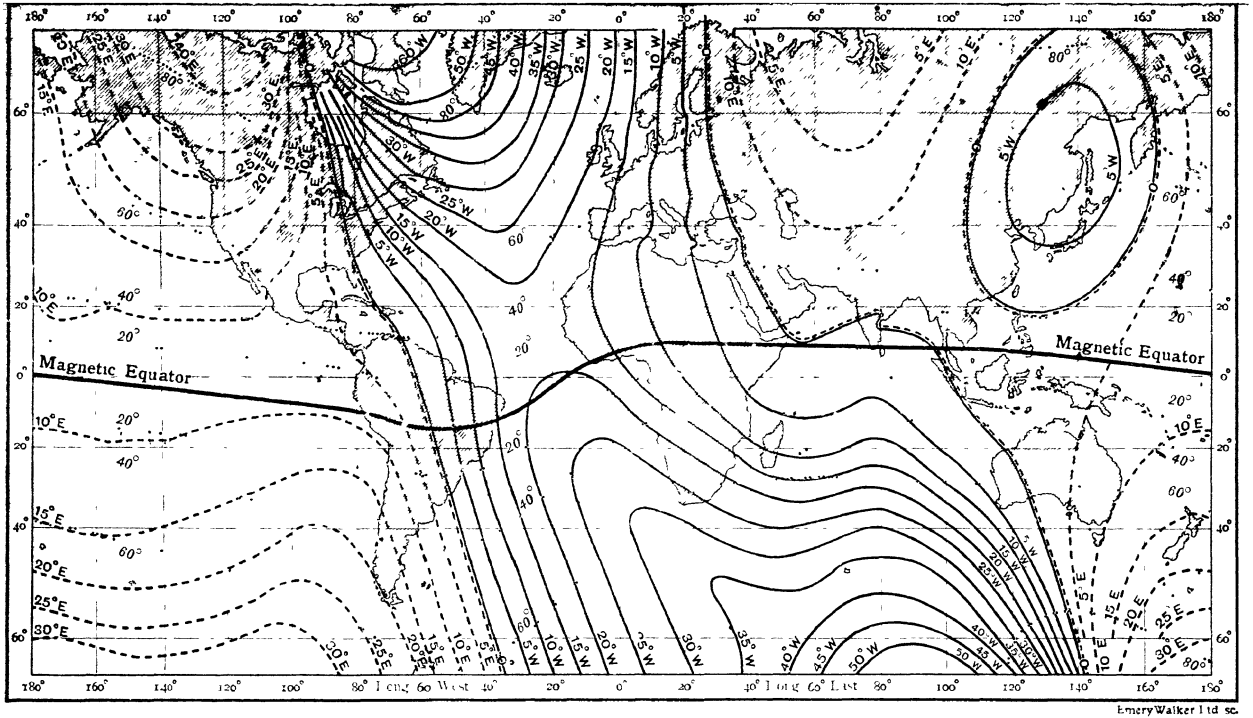


FIG 738.—Map of the world showing the isogonals and isoclinals.

an oval agonic line passing through China and Siberia, called the **Siberian Oval**, within which the declination is westerly.

It will thus be seen that the isogonal lines have somewhat the character of lines of longitude, in that their general tendency is to run from north to south. The irregularities, however, are very great.

Isoclinals. -Lines for which the dip has the same value at every point are called **isoclinals.** These are shown in light dotted lines in Fig 738. The line of no dip, or the **magnetic equator,** follows the general course of the geographical equator. It is, however, south of the geographical equator in America, and north in Africa At all points upon this line, the dip-needle remains horizontal The isoclinals resemble the parallels of latitude, and form closed curves about the magnetic poles At the magnetic poles the dip-needle sets vertically, with the N pole downwards at the north pole and the S pole downwards at the south pole.

Isodynamic lines.—Lines passing through points for which the horizontal intensity of the earth's magnetic field is the same are called **isodynamic lines.** The horizontal intensity is zero at the magnetic poles, and increases to a maximum at the magnetic equator Thus the isodynamic lines have a similar shape to the isoclinals, but must not be confused with them.

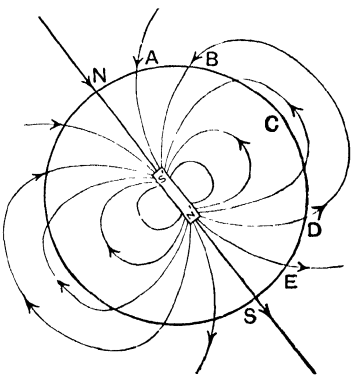


FIG. 739.—Magnetic condition of the earth

The earth as a magnet.—The cause of the earth's magnetisation has been the subject of a great deal of speculation. We shall here content ourselves with saying that the magnetisation is not entirely due either to internal, or to external causes, but to both. However, the general character of the earth's magnetic field may be represented roughly by imagining a large internal magnet. In order to represent the earth with correct magnetic polarity, the S pole S′ of this internal magnet N′S′ (Fig 739) must be situated under the magnetic N pole of the earth. The lines of force due to such an internal magnet are shown in the figure. This state of magnetisation may be imitated by taking a disc of cardboard to represent the earth and laying upon it a bar magnet in the position N′S′. A compass needle carried

round the disc will set perpendicularly to the circle on reaching N. At A the dip will be less, and will be still smaller at B, until at C which represents the magnetic equator, the dip will be zero Similarly, at D and E the S pole of the needle will dip and at S the dip is 90°.

Such a magnetic distribution is far from being a true one, but it gives a rough idea of the actual state of affairs Attempts have been made, by adding a smaller auxiliary magnet, to imitate more closely the earth's magnetic condition, but no representation by a small number of magnets can produce a field presenting all the irregularities of the earth's field

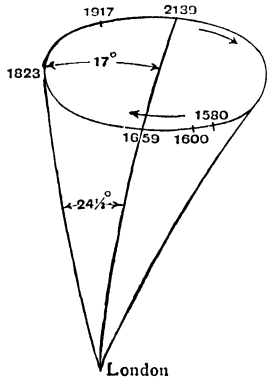


FIG. 740 -Secular variation in the earth's magnetic field

Variations in the earth's magnetic field. - The form of the earth's magnetic field is never twice the same, it is continually varying Not only is it undergoing minute irregular variations at every place, but there is a continual periodic change which repeats itself daily, known as the **daily variation** There are also annual changes, and upon all these there is imposed a **secular variation**, which is large in amount and goes through a cycle of change in about a thousand years

Secular variation. - The earliest record of the magnetic declination is that of the year 1580, when its value was 11° 15′ E. at London Successive changes are recorded in later years, the easterly declination diminishing and becoming zero in 1659. At this time the compass needle pointed true north at London The declination then became westerly, increasing to 24½° W in 1823, since which time it has been decreasing This change can be very well represented by considering the magnetic poles to rotate round the geographical poles, the magnetic north pole describing a circle of 17° radius about the geographical north pole, the direction of rotation being clockwise, as shown in Fig 740. A computation from the changes already recorded shows that the declination may be expected again to be zero at London in the year 2139, which is 480 years from the last occasion. The magnetic north pole will then have described half its circular path, so that the time required for its complete circle is 960 years. The magnetic conditions all over the world go through an approximate cycle in this time

Annual variation.—The declination also goes through a very

small cycle once a year, and this annual cycle is in opposite directions in the northern and southern hemispheres. The declination is about $2\frac{1}{4}'$ east of its mean position in August at London, and the same amount west of its mean position in February.

Daily variation.—All the magnetic elements undergo fairly regular daily changes, but as these are small, special instruments are necessary to record them. They can be recorded only at fixed observatories, the portable magnetometers generally used in magnetic surveys not being sufficiently sensitive for their observation. A mirror is attached to the magnetic needle, and a beam of light reflected from it is brought to a focus upon a sheet of photographically sensitive paper wrapped round a drum. This drum is rotated by clockwork at uniform speed, so that the motion of the paper is at right angles to the variation to be recorded. Thus, for recording variations in declination, the axis of the drum must be horizontal, so that the motion of the sensitive paper takes place vertically, the movement of the spot of light corresponding to changes in declination taking place horizontally.

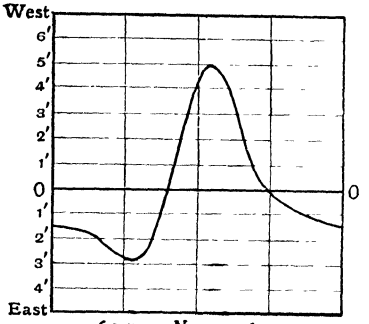


FIG. 741.—Daily variation in declination

The curve in Fig. 741 is of the type usually obtained for the daily variation in declination. The line O—O indicates the mean position of the spot of light. It will be seen that there is a maximum variation of 3′ E. from the mean position just before 8.0 a.m., and 5′ W. at about 1.0 p.m. These variations, however, are not the same from day to day; on specially 'quiet' days they are less than the above, and on days of considerable magnetic disturbance they are greater.

It has been shown that the daily variations are due probably to electric currents in the upper regions of the atmosphere, but their explanation is by no means complete.

Eleven-year period.—Apart from the irregularities which occur, the daily variation undergoes a cyclic variation in magnitude, closely allied to the frequency of occurrence of sun-spots. Thus at a sun-spot maximum the daily variation is greatest, and *vice versa*. Sun-spots go through a periodic change in occurrence once in approximately eleven years. The daily variation in the magnetic elements have been compared with the intensity of sun-spot occurrence since 1855, and the coincidence in the variation of one with the other is remarkable.

Magnetic storms.—It often happens that the suspended needles at the magnetic observatories over considerable parts of the earth are suddenly and violently affected at the same time. These sudden disturbances, called **magnetic storms,** cannot be foretold; they apparently occur quite irregularly. It may, however, be noticed that the sudden appearance of a large sun-spot is usually accompanied by a magnetic storm. Also a prominent display of the Aurora Borealis generally occurs with magnetic storms, although there may be magnetic storms when there is no aurora.

The connection between magnetic storms and the aurora and sun-spots makes it appear likely that the sun emits rays similar to the kathode rays of the vacuum tube (Chap. LXXX.), which, on reaching the earth's atmosphere render it a conductor of electricity, the electric currents then produced being accompanied by magnetic fields (Chap. LXV.). The form of the aurora is just what would be expected from analogy with the vacuum tube, when these kathode rays enter the earth's magnetic field.

The magnetic compass. Probably the magnetic compass finds its most important application in connection with the navigation of ships. The true geographical bearing, as well as the latitude and longitude, may be determined by astronomical observations, but these are tedious and are generally performed only once a day. The actual steering of the ship is generally performed according to the compass, or magnetic, bearing. The Kelvin compass card is the one most commonly used. It consists of a light disc of aluminium or mica on which the points of the compass are drawn, the magnetic part being composed of several light steel magnets. The whole is supported upon an agate cup resting on a needle point, and in the best compasses is immersed in methylated spirit, to buoy up the card and so take part of its weight off the needle point. The liquid also serves to reduce any oscillations of the compass that would otherwise be troublesome. Passing through the axis of support is the mid line of the ship, indicated by two lines or marks, one fore and the other aft of the card, thus enabling the bearing of the ship according to the compass to be observed directly.

It follows, of course, that in order to know the geographical bearing of the ship, the declination (or magnetic variation) must be added to, or subtracted from, its magnetic bearing. The value of this at all parts of the world is given on charts supplied by the Admiralty.

The magnetic compass has also great importance in the steering

of air-craft. An aeroplane compass of the Creagh-Osborne type is shown in Fig. 742. The bowl is of spherical form and is mounted in such a manner as to minimise vibration. The compass card has several steel magnets, and is provided with a vertical mica ring, on which the points of the compass are marked in luminous radium paint. A window at the back of the bowl enables the pilot to read

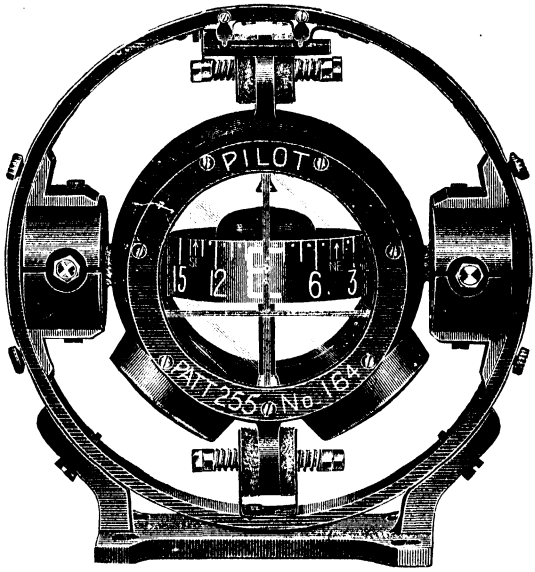


FIG. 742.—The Creagh-Osborne aeroplane compass.

the course by means of the scale on the compass card. The bowl contains liquid to buoy up the card, part of which consists of a hollow float. The liquid thus takes part of the weight off the agate point suspension, and also helps to damp the vibrations of the card.

Ship's magnetisation.—Masses of iron or steel situated near the compass will, of course, affect its reading; and since modern ships are built almost entirely of these materials, the errors introduced and the corrections necessitated are considerable.

These errors are of many kinds and cannot all be considered here in detail, but the three principal errors will now be dealt with.

Semicircular deviation.—Most iron ships are permanently magnetised, thus the ship itself is a large magnet, and as it takes different

courses it will have different effects upon its compass. This permanent magnetisation is acquired at the time of building the ship, the magnetic axis being in the magnetic meridian, that part of the ship which lies northwards being a N pole (p 793) Consider the line NS to be the magnetic axis of the ship (Fig. 743). In the position shown, that is, with the N pole west of the magnetic meridian AB, the compass ns is evidently deflected so that n is east of the meridian This applies for any position of the ship when N is west of the meridian When N is east of the magnetic meridian the deviation of the compass is westerly Hence, as the ship rotates through a complete circle, the deviation due to the permanent magnetisation is east for half the circle and west for the other half. For this reason this is called **semicircular deviation**

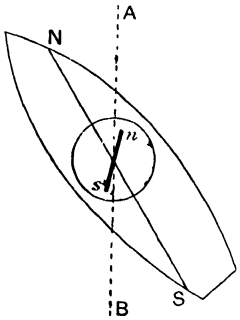


FIG 743 —Permanent magnetisation of a ship

There is another cause of semicircular deviation, namely, masses of soft iron situated with their greatest length vertically. A vertical soft iron column will be magnetised by the vertical component of the earth's field (p 793) and in the northern hemisphere, the N pole will be at the bottom and the S pole at the top If situated as in

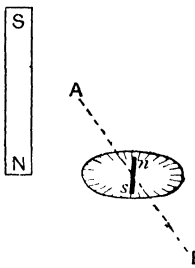


FIG. 744 —Vertically disposed soft iron.

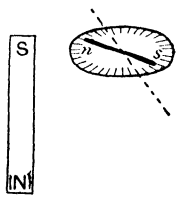


FIG. 745.—Vertically disposed soft iron.

Fig. 744 with the N pole near the compass, the deviation is east when the magnet is west of the compass and west when the magnet is east of the compass But as the ship rotates, the deviation is east for half the rotation and west for the other half. The direction of the deviation is reversed if the upper end of the bar is level with the compass, as in Fig 745, and, of course, the directions are all the opposite to those in the earth's southern hemisphere, where the N pole of the iron bar would be at the top.

Although there may be many such masses of iron in a ship, the resultant effect will always give rise to a semicircular deviation.

Quadrantal deviation.—When the soft iron is disposed horizontally its effect is more complicated, as its direction of magnetisation changes as the ship rotates Thus, suppose the bars to be disposed as in Fig. 746 (*a*). The position of the N and S poles are as shown, and the deviation produced is west. On rotation of the ship till the bars are in position (*b*) the deviation will be east. A further rotation through 90° will reproduce case (*a*), for it must be remembered that we are dealing with soft iron, and its magnetisation depends on its position in the earth's magnetic field. The bars have changed places with respect to the compass, but their polarity has changed too, so that the deviation is again west. A further rotation through 90° produces again case (*b*). Thus the deviation changes in direction four times during one rotation of the ship, and is therefore only constant in sign for a rotation of 90°. It is therefore called **quadrantal deviation.**

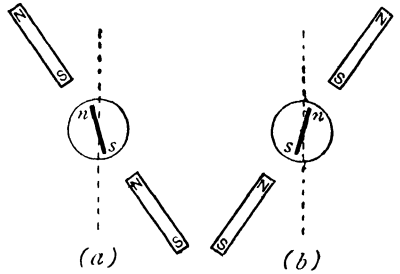


FIG. 746.—Horizontally disposed soft iron.

Swinging the ship.—Owing to the complexity of the errors due to the ship's magnetisation they cannot be foretold, and their resultant must be found by direct observation. This is called **swinging the ship.** The ship is swung round into a number of positions, and in each position the true magnetic bearing and the observed compass bearing are observed. The difference is the error of the compass, due to the ship's magnetisation A table or diagram is then constructed for future use, so that for any observed compass bearing the correction to be applied in order to obtain the real magnetic bearing is known To this must then be applied the declination, in order to obtain the geographical bearing.

Methods of correcting for ship's magnetisation.—Although there is no method of compensating completely the disturbances of the compass due to the ship's magnetisation, yet they may be partially compensated in several ways. To compensate for the **quadrantal deviation** two hollow soft iron spheres are placed one on each side of the compass, and on a level with it. If FF (Fig. 747) be the fore and aft line of the ship, and AB the direction of the magnetic meridian, the spheres NS, NS are placed as shown. Since they become magnetised as indicated, they will, by themselves, produce a deviation in an easterly direction. But reference to Fig. 746 shows that

the quadrantal deviation is usually westerly for this position of the ship. Hence the spheres placed at a suitable distance will compensate for the quadrantal deviation From p. 807 it will be seen that the deviation produced by the spheres alone is quadrantal. With spheres 5 inches in diameter a quadrantal deviation of about 2° is corrected when their centres are 9 inches from the compass.

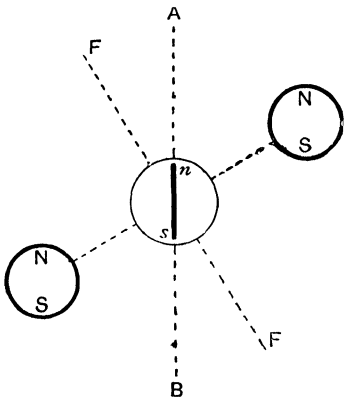


FIG 747 — Soft iron compensators

The **semicircular deviation** due to permanent magnetisation is corrected by fixing a number of small permanent magnets near the compass, their correct number and position being found by trial The part of the semicircular deviation due to vertical soft iron is got rid of by fixing a vertical soft iron bar in front of, or behind, the compass This is called a **Flinder's bar**

MEAN VALUES OF THE MAGNETIC ELEMENTS AT GREENWICH.

Year	Declination W.		H.	Dip		
	°	′		°	′	″
1914	15	6·3	0·18518	66	51	13
1915	14	56·5	0·18508	66	51	50
1916	14	16·9	0·18191	66	52	45

EXERCISES ON CHAPTER LXIII

1. What are the magnetic elements which are usually measured in order to find the state of the earth's field at any point ? How are they related to each other ?

2. Describe the precautions necessary in finding the direction of the magnetic meridian

3. A horizontal bar magnet is brought near a dip-needle in the northern hemisphere, the bar being in the plane of rotation of the needle, and the N pole pointing south. Describe the effect on the observed dip (*a*) when the magnet is immediately north of the needle, and (*b*) when it is vertically over the needle.

4. Describe a dip circle and explain its action.
A dip circle is slowly rotated about a vertical axis. Describe and explain the behaviour of the needle during one complete turn.
Sen. Camb. Loc.

5 Define the angle of dip and explain how it may be measured. Describe roughly how the angle of dip varies from place to place on the earth's surface. L.U.

6. Explain how to determine (*a*) the vertical component of the earth's magnetic force, (*b*) small changes in the magnitude of the horizontal component. L.U.

7. Assuming that the magnetism of the earth is due to a small magnet placed at its centre, find expressions for the horizontal component and the dip of the field at any point of the earth's surface in terms of the magnetic latitude. L.U.

8. A dip-needle placed so that it can oscillate in the meridian is found to make 35 oscillations per minute in a locality where the dip is 60°. At another locality, where the dip is 45°, it is found that the needle makes 40 oscillations per minute. Assuming that the needle is constant in magnetic condition, find (*a*) the ratio of the earth's total intensities, and (*b*) the ratio of the horizontal components of the earth's magnetic field at the two places. L.U.

9. Define declination and dip (in reference to the earth's magnetism). How are they determined ?
Find the resultant force of the earth's magnetism at a place at which the dip is 30° and the horizontal force is 0·18. Calcutta University.

10. Define the angle of magnetic dip and describe a method of measuring it.
A dip circle is placed so that the needle sets vertical. The circle is then rotated through an angle A about a vertical axis and the dip measured in this position. Show how the dip as observed is related to the true dip and to the angle A. L.U.

11. How is the horizontal component of the earth's magnetic force determined in absolute measure ?
At a place A the total magnetic intensity is 0 5 and the dip is 64° ; at a place B the total intensity is 0·6 and the dip is 72°. If a magnet vibrating horizontally at A makes 20 oscillations in a minute how many oscillations would it make in the same time at B ? University of Bombay.

CHAPTER LXIV

MAGNETIC PROPERTIES OF MATERIALS

Intensity of magnetisation.—In order to study the magnetic properties of a given material, something more must be known than the magnetic moment of a bar of the material, for this evidently depends, amongst other things, upon the size of the bar. Dividing the magnetic moment by the volume, a quantity is obtained which gives the average intensity of magnetisation of the material If the body be uniformly magnetised, the magnetic moment per cubic centimetre is the same whatever part of the body be chosen It is called the **intensity of magnetisation** of the body. Thus, for a uniformly magnetised body,

$$\text{Intensity of magnetisation} = \frac{\text{magnetic moment}}{\text{volume}}$$

or,

$$I = \frac{M}{V}.$$

There is another way of representing this quantity: imagine a uniformly magnetised bar (Fig. 748), let l be the length, and a the area of each end Let σ be the amount of pole per unit area of each end. Then the amount of pole on each end is $a\sigma$, and the magnetic moment is $la\sigma$ (p. 781). Now the volume is la;

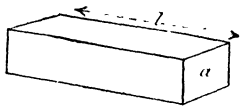


FIG. 748.—Uniformly magnetised bar

$$\therefore \text{ Intensity of magnetisation} = \frac{la\sigma}{la} = \sigma.$$

Thus, the intensity of magnetisation may also be defined as **the amount of pole per unit area of face, the face being taken at right angles to the direction of magnetisation**; or,

$$I = \sigma.$$

Although bodies are not, as a rule, magnetised uniformly, the above definitions of intensity of magnetisation hold for any volume

sufficiently small for the magnetisation throughout it to be considered uniform.

Magnetic susceptibility.—Magnetisation is always acquired on account of the magnetic body being placed in a magnetic field, and the intensity of magnetisation acquired depends upon the strength of the field, and the nature of the body. The ratio of the intensity of magnetisation (I) produced, to the strength of the field (H) producing it, is called the **magnetic susceptibility** (k) of the material, thus,

$$\frac{\mathrm{I}}{\mathrm{H}} = k, \quad \text{or} \quad \mathrm{I} = k\mathrm{H}$$

The magnetic susceptibility of most magnetic materials varies with the magnetising field in a complex manner. Its study will be left until later (p. 821).

Magnetic permeability. We must now reconsider the equation given on p. 774 for the force between magnetic poles, namely,

$$\text{Force} = \frac{m_1 m_2}{d^2} \text{ dynes.}$$

This equation is strictly true only when the poles are situated in empty space, and very nearly true when they are situated in air, or any other non-magnetic material. If the poles are situated within a magnetic material, the force is quite different. It is, however, still proportional to both pole strengths and varies inversely as the square of their distance apart A quantity (μ) is now introduced into the equation to render it correct.

Thus, the equation

$$\text{Force} = \frac{m_1 m_2}{\mu d^2} \text{ dynes}$$

applies to the force when the poles are situated in any material, where μ is called the **magnetic permeability** of the material. This quantity, like the susceptibility, is not constant for any material; its variations are studied on p. 821.

Field strength represented by lines of force.—On referring to the diagrams of lines of force (pp 779, 780) it becomes obvious that the strength of field is greater where the lines are packed closely together, and weaker where they are spread apart. This suggests the possibility of representing the magnetic field quantitatively, as well as merely in direction, by means of these lines. It can be shown by reasoning, beyond the scope of the present book, that if a surface be taken round a magnet, and through every unit of area of this surface a

number of lines of force be drawn, numerically equal to the strength of field at that unit of area, these lines when continued will, by their number per unit area, represent everywhere the strength of the field.

This quantitative representation of a magnetic field by means of lines of force is very useful, as it helps us to visualise the field itself. Thus, where there is one line of force per square centimetre the strength of field is unity, and where the strength of field is H, there are H lines of force per square centimetre, the area always being taken at right angles to the direction of the field at the place considered.

Hence the total number of lines of force crossing an area of A sq cm is AH, where H is the strength of field, provided that this is uniform. The total number of lines crossing any area is sometimes called a **magnetic flux.**

Magnetic induction.—We must now introduce a new term, **magnetic induction,** into our magnetic considerations, a term which has often been loosely and improperly employed.

Consider two magnetic poles, m_1 and m_2, d centimetres apart. The force between them, when in air or in a vacuum, is $m_1 m_2 / d^2$ dynes, and if m_2 is a unit pole, the force m_1/d^2 upon it is called the strength of field (H) due to the pole m_1 (p. 781). If, however, the medium in which the poles are situated has permeability μ the force between the poles is $m_1 m_2/\mu d^2$ dynes, and the strength of field (H) due to m_1 is $m_1/\mu d^2$. Thus the strength of field due to a pole depends upon the medium in which it is situated. It is most important, as we shall see later, to have some quantity which is fixed for a given pole and distance, independently of the medium. This quantity is called the **magnetic induction** (B), and we see from the above that it must be μ times the strength of field, or $\mathsf{B} = \mu \mathsf{H}$.

Now, for a distance d from a given pole,

$$\mathsf{H} = \frac{m}{\mu d^2}; \quad \therefore \ \mathsf{B} = \mu \cdot \frac{m}{\mu d^2} = \frac{m}{d^2}.$$

B is therefore independent of the medium, thus fulfilling the condition required. Thus, **the definition of magnetic induction is that it is μ times the strength of magnetic field at any point** where H is defined as on p. 781 and μ as defined on p. 811.

Magnetic lines of induction.—In plotting lines of force (p. 778) the process always begins at the surface of the magnet, in fact the lines may be said to arise upon N poles and end on S poles. These

lines, however, are continuous with others within the magnet, so that each line in its entire path forms a closed circuit (Fig. 749); but within the magnet they are not lines of force, that is to say, they do not by their number per unit area, represent the strength of field, or force that would be exerted on a unit pole. These are **magnetic lines of induction**, and may, or may not, lie wholly or partly within a magnetic material When in air or other non-magnetic material they are also lines of force; that is, they represent the strength of field by their number per unit area, but do not do so within a magnetic material The number of these lines per square centimetre, however, represents the magnetic induction at the place considered, whatever the nature of the medium may be. The relation of their number per unit area to the strength of field within a magnetic material will be found on p 816.

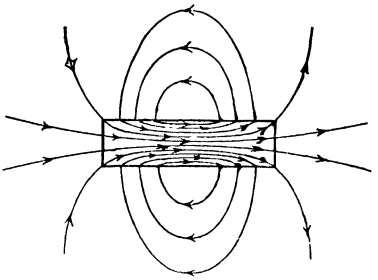


FIG. 749.—Lines of induction

Gauss's law. There is a convenient theorem due to Gauss which is derived from the inverse square law, and is of great service in solving magnetic problems. Its proof is beyond the scope of this work, but it may be stated in the following simplified manner. **The number of lines of magnetic induction arising upon any N pole or ending on a S pole is equal to 4π times the strength of the pole.** This applies to any pole, whatever may be the medium in which it is situated, but when it is situated in air, the lines of induction are also lines of force Hence for a magnetic N pole of strength m, situated in air, the number of lines of force arising upon it is $4\pi m$.

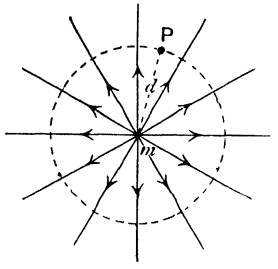


FIG. 750.—Magnetic field due to a single pole

Application of Gauss's law to a single pole.—Consider a N pole of strength m, and let us find the strength of field at a distance d from it. Let P be a point at distance d cm. from m (Fig. 750). Through P describe a sphere with centre m. The magnetic field around a single pole is symmetrically distributed, that is, the same number of lines of force will pass through each square centimetre of the sphere. Now, from Gauss's law, the total number of lines is $4\pi m$, and since the area of the sphere is $4\pi d^2$ sq. cm., the number of lines per sq. cm.

is $4\pi m/4\pi d^2 = m/d^2$, and this we already know to be the strength of field in air at a distance d (p. 812).

Field due to a plane sheet of magnetic pole. Whenever the distribution of magnetic field is symmetrical, Gauss's law can be applied to find the strength of magnetic field. Imagine a plane sheet of N pole, of infinite extent, the amount of pole per unit area being σ. The lines of force pass out uniformly in both directions from such a sheet.

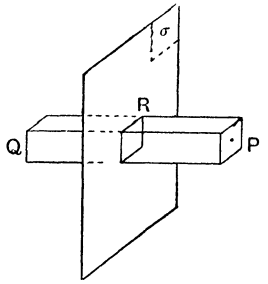


FIG 751.—Magnetic field due to a plane polar sheet

To find the strength of field at any point P (Fig. 751) draw a unit of area through P parallel to the plane polar sheet. Through the boundary of this unit area draw lines perpendicular to the sheet, so as to form a prism which cuts the sheet in unit area at R, and close the prism by another plane at Q. The amount of pole which lies within the prism is that upon the unit area at R, which is σ. Hence, by Gauss's law, $4\pi\sigma$ is the number of lines passing outwards from R. The field being everywhere perpendicular to the sheet, half the lines pass through the unit area at P, and the other half through Q. Hence the number of lines per square centimetre, or the strength of field at P, is $2\pi\sigma$.

It will be seen that the strength of field is independent of the distance of P from the sheet, provided that the sheet is so large that the lines pass out from it uniformly.

Field near the end of a bar magnet.—Consider the space between the poles of two bar magnets placed close together as in Fig. 752. If the faces are sufficiently close together, the magnetic field at a point P between them is uniform, and each polar face may, for all practical purposes, be considered to be infinite. Let I be the intensity of magnetisation of each magnet. The amount of pole per square centimetre of each face is then I (p. 810), and the strength of field at P due to the polar sheet N is 2πI (from above), and that due to S is also 2πI. These, however, are both directed from N to S, so that the resultant strength of field at P is 4πI, and this is the force on a unit pole when situated at P.

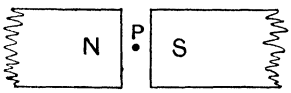


FIG 752.—Magnetic field between two plane poles.

It will be noticed that the strength of magnetic field at P is independent of the distance apart of the polar faces N and S, provided that these faces are of such an extent that the field in the space between them is uniform.

Force between plane poles in contact. In Fig. 752 the magnetic field due to either polar face is $2\pi I$, and when the poles are very close together, each is situated in the field due to the other. Thus, the field due to N is $2\pi I$, and each square centimetre of S has an amount of pole I upon it, and therefore experiences a force

$$2\pi I \times I = 2\pi I^2.$$

Hence if the two poles are in contact there is a force $2\pi I^2$ for each square centimetre of area in contact, causing them to adhere together.

It is not necessary that the two magnets in contact should be permanent magnets, one may be a permanent magnet and the other a piece of soft iron. In this case the iron becomes magnetised in the field due to the magnet, the force between the two polar faces being the same as before, provided that the two intensities of magnetisation are the same. If they are not, the force per square centimetre will, of course, be $2\pi I_1 I_2$, where I_1 and I_2 are the respective intensities of magnetisation on the two sides of the plane of contact.

Since lines of induction are continuous, and for two plane parallel faces they are at right angles to the faces, it follows that the value of B must be the same on both sides of the gap. It will be seen on p. 816 that $B = 4\pi I$, so that the force between the faces may be represented by $B^2/8\pi$ instead of $2\pi I^2$. This result is of value in finding the lifting power of electromagnets upon masses of iron, and in the case of the telephone (Chap. LXXVIII.), where the force between a small electromagnet and an iron disc is used in the transmission of sounds.

Magnetic induction in iron.—We must now find the value of the induction (B) in the interior of a mass of iron. The strength of field there (H) is the force upon unit pole situated within the iron. This is due to external causes; the magnetisation of the iron itself does not affect it, except in so far as the free poles at the ends of the magnet may modify it, because the molecular magnets constituting the iron consist of poles situated so close together that at any appreciable distance from them, their effect is zero. The effect of this external field (H) has been seen (p. 771) to cause an alignment of the molecular magnets, and consequently there will be lines of induction running the length of the magnet, due to these molecular magnets. These lines will, of course, leave at the N end of the magnet, since there they pass from the free N ends of the molecular magnets and will contribute to the external magnetic field. In the interior of the iron they pass immediately from the N ends of one molecular magnet to the S end of the next. This

state of affairs is represented diagrammatically in Fig 753, where the original field H is represented in dotted lines, and the induction due to the magnetisation of the iron by continuous lines

In order to find the total value of the induction due to the magnetisation of the iron, consider a section of it in the inter-molecular spaces at P. If I be the intensity of magnetisation of the iron, then on each side of the section there is an amount of pole I units per square centimetre, N on one side and S on the other, and the number of lines crossing this square centimetre is $4\pi I$ (p 814). Hence, the total number of lines of induction per square centimetre (B) is made up of H due to the original field, and $4\pi I$ due to the magnetisation of the iron, or $B = H + 4\pi I$

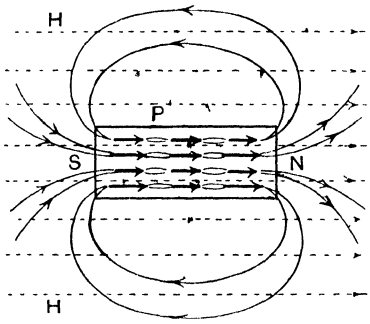


FIG 753.—Lines of induction due to a magnetic material

In the case of a permanent magnet there may not be any magnetising field, in which case $B = 4\pi I$

The above expression may be written differently, for, on dividing through by H, we get

$$\frac{B}{H} = 1 + 4\pi \frac{I}{H},$$

or

$$\mu = 1 + 4\pi k,$$

μ being the magnetic permeability of the material, and k its magnetic susceptibility, as defined on p. 811

The right-hand side of the equation $B = H + 4\pi I$ is represented by the lines drawn in Fig 753, the two sets of lines being drawn in the same diagram. Within the iron we see that their resultant, or sum represents B. At external points the two sets are not in the same direction at every point, and in order to obtain their resultant, the two fields must be compounded by the ordinary law for the addition of vector quantities. This is not always easy, particularly for a rectangular bar, but the general arrangement is exhibited in Fig. 754. It will be noticed on examining this diagram of the resultant field, that the effect of placing the iron bar in a uniform field is to cause the lines of induction to converge upon it, producing

a concentration of the lines at points such as A and B, and a separation at C and D Thus, at A and B the field is strengthened by the presence of the specimen, while at C and D it is weakened Within the iron the induction is greatest, but the strength of field is not increased Within a medium of permeability μ, $H = B/\mu$, so that although B is great, μ is also great, in fact B depends upon μ, so that H has its original value, apart from any disturbance due to the poles at the ends of the specimen

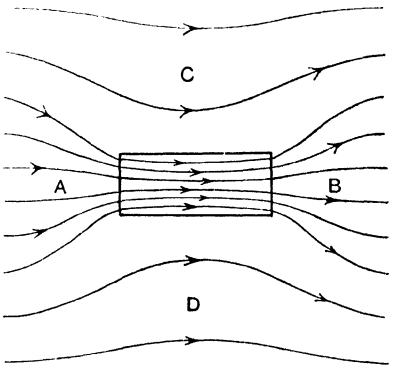


FIG 754.—Resultant lines of induction for a magnetic body in a magnetic field

Keepers.—The above discussion is only an approximation to the truth, for it will be seen that the poles at the ends of the specimen produce a field within it, of opposite direction to the original magnetising field H. This field **tends to demagnetise the bar,** with an effect which is greater, the shorter the bar. With long thin bars this demagnetising effect is small, but with short bars it becomes important. It is for this reason that permanent magnets are provided with soft iron **keepers** when not in use Thus two bar magnets NS (Fig. 755) when not in use are packed in a box with the poles of opposite kind adjacent to each other. A soft iron keeper is placed at each end, and the poles N′S′ produced in these keepers will produce opposite fields to those of the bar magnets. Hence the **demagnetising** effect of the poles on the bars is removed by the **magnetising** field of the poles on the keepers.

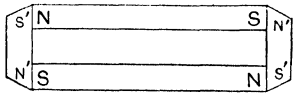


FIG 755.—Keepers for a pair of bar magnets

Measurement of magnetic susceptibility and permeability.—In order to measure the magnetic susceptibility and permeability of any material, it must be placed in a magnetic field of known strength, and must be of such a form that no poles are produced, or else that the poles, if produced, shall have as little demagnetising effect as possible. This is usually effected by using the material in the form of a long thin wire. The magnetising field is nearly always produced by means of a coil of wire through which an electric current is flowing. A straight coil of this kind (Fig. 756) produces a uniform magnetic

field of strength $H = 0{\cdot}4\pi n \times$ (current) in its interior (Chap LXXV). Hence the magnetising field (H) is known, and it only remains to find the magnetic moment of the specimen. This may be carried out by means of the magnetometer. AB and CD (Fig 757) are two coils of length about 12 cm , each consisting of about 400 turns of No 22 copper wire They are placed one on either side of the magnetometer needle, and are so connected together that when an electric current flows through them, they produce equal and opposite effects upon the needle. This may be attained by moving one of them nearer to, or further from the needle, until there is no movement of the needle, whatever the current in the coils may be

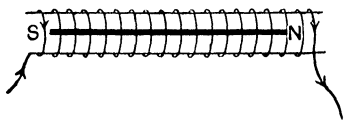


FIG 756 —Magnetising coil

The specimen to be experimented upon may be a piece of knitting needle about 8 cm long and diameter 2·5 mm When placed in one of the coils, it becomes a magnet (Fig 756), and it is our object to determine its magnetic moment M, for when this is known we can calculate from it the intensity of magnetisation. It is first necessary to find the effective length of the magnet, l, and the strength of the earth's field H_1. This is effected by placing the specimen in the coil and magnetising it with the strongest current available, and then stopping the current, when the specimen will remain magnetised Take the specimen out of the coil, and with its middle point at a distance d_1 cm from the needle, the deflection θ_1 is obtained, and from the relation on p 786 we have

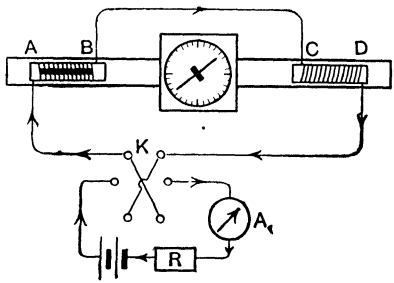


FIG 757 —Magnetometer arranged to measure intensity of magnetisation

$$\frac{M}{H_1} = \frac{(d_1^2 - l^2)^2}{2d_1} \tan\theta_1$$

The specimen is now brought nearer to the needle, to a distance d_2 cm., and the new deflection θ_2 observed , when

$$\frac{M}{H_1} = \frac{(d_2^2 - l^2)^2}{2d_2} \tan\theta_2.$$

Hence,
$$\frac{(d_1^2 - l^2)^2}{2d_1} \tan\theta_1 = \frac{(d_2^2 - l^2)^2}{2d_2} \tan\theta_2,$$

or,
$$\frac{d_1^2 - l^2}{d_2^2 - l^2} = \sqrt{\frac{d_1 \tan\theta_2}{d_2 \tan\theta_1}},$$

and since d_1, d_2, θ_1 and θ_2 are all known, l may be found.

Next, the magnet is suspended in a vibration box or beaker, and its time of oscillation T found (p. 788).

Then,
$$T = 2\pi\sqrt{\frac{I_1}{MH_1}}, \quad \text{or} \quad MH_1 = \frac{4\pi^2 I_1^2}{T^2},$$

where I_1 is the moment of inertia of the magnet (p. 789).

Combining this with the relation
$$\frac{M}{H_1} = \frac{(d_2^2 - l^2)^2}{2d_2} \tan\theta_2,$$

we have
$$H_1^2 = \frac{4\pi^2 I_1^2}{T^2} \cdot \frac{2d_2}{(d_2^2 - l^2)^2} \frac{1}{\tan\theta_2},$$

or
$$H_1 = \frac{2\pi I_1}{T(d_2^2 - l^2)} \sqrt{\frac{2d_2}{\tan\theta_2}},$$

whence H_1 is known.

Then in any further experiment on the deflection, at distance d,
$$M = \frac{(d^2 - l^2)^2}{2d} \cdot H_1 \tan\theta.$$

and, since the intensity of magnetisation I is M/la, where a is the area of cross section of the specimen,
$$I = \frac{(d^2 - l^2)^2}{2d \,.\, l \,.\, a} H_1 \tan\theta,$$

or,
$$I = K_1 \tan\theta,$$

where
$$K_1 = \frac{(d^2 - l^2)^2 H_1}{2dla}.$$

Every quantity in this last expression is known, so that the constant K_1 is found, and the deflection θ at once gives us the intensity of magnetisation of the specimen. With a specimen of the dimensions given, the demagnetising effect of its own poles is great (p. 817) and involves a large correction. If greater accuracy is required a long thin wire must be used and the reflecting magnetometer employed.

EXPT. 178.—**Determination of intensity of magnetisation.** Using a piece of steel knitting needle, find the deflections θ_1 and θ_2 at distances d_1 and d_2 and so obtain l (p. 818). Then find the time of vibration T and obtain H_1. Hence obtain the constant $K_1 = \frac{(d^2 - l^2)^2}{2dla} H_1$.

Replace the specimen in the coil AB (Fig. 757) and adjust the rheostat R until the largest current available is indicated by the ammeter A_1. Read the deflection θ. Then diminish the current and again read θ. Repeat this, taking a number of readings down to zero current. Reverse by means of the key K, and increase the current up to its greatest negative value; then reduce it again to zero by steps, taking observations of θ on the way. Then reverse again and proceed to the greatest positive current, noting the values of θ as before. Make a table of the observations as *below. The first two columns give the values of the current and* θ. Then fill in the third column with the values of $\tan\theta$, and the fourth with the values of I, that is, $K_1 \tan\theta$.

Current	θ	$\tan\theta$	I	H

The values of H, the magnetising field, are put in the last column, being obtained from the relation $H = 0{\cdot}4\pi n \times$ (current) (p. 818).

Plot, on squared paper, the values of H and I. This will give a curve of the form *abcdefa* (Fig. 760) and shows a cycle of magnetisation

If an unmagnetised piece of knitting needle exactly like the specimen be used, the observations may be started at zero instead of maximum current, so that the part O*a* of the curve is obtained; or, if preferred, the cycle of readings may be taken before the standardising observations are made, in which case only one specimen is required.

A further experiment may be made upon a piece of soft iron, using the same value of K_1 as before, provided that the specimen has the same dimensions as the steel specimen used in making the standardising measurements.

It should be noted that the demagnetising effect of the poles upon the specimen (p. 817) has been neglected. With the dimensions of specimen given (p. 818) this introduces an error of about 10 per cent. in the value of H for the greatest readings, and as a first approximation is neglected. The more refined methods in which this correction is eliminated are beyond the scope of this book.

B-H curves.—The curve in Fig. 760 gives the relation between I and H. If that for B and H is required the values of B must be calculated from those of I, remembering that $B = H + 4\pi I$.

For many purposes the cycle of magnetisation (Fig. 760) is not required, but only the values of the induction with continually increasing magnetising field. Such a curve is given in Fig. 758 for soft iron. It will be noticed that for small values of H, B increases slowly,

as from O to *a*. From *a* to *b* the curve is very steep, the value of B rising rapidly as H increases. After this the increase in B is slow, and eventually the part *bc* becomes very nearly a straight line, inclined at a small angle to the axis OH. The corresponding values of the permeability μ, obtained from the equation $B = \mu H$, are plotted on the same diagram. It will be seen that μ starts with a constant small value, rises to a maximum at *d*, and then falls gradually to a small value, constantly approaching the limiting value $\mu = 1$ for very high magnetising fields.

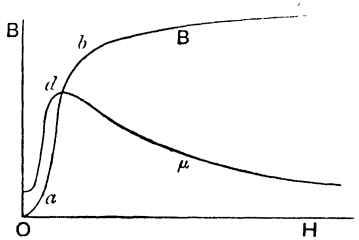


FIG 758. B-H and μ-H graphs.

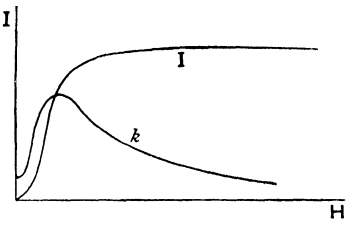


FIG 759. I-H and *k*-H graphs.

I-H curves: saturation.—On reducing the curve of B (Fig. 758) to that of I by the relation $B = H + 4\pi I$, it will be seen that there is a general similarity between the two, but in this case the curve of I (Fig. 759) gradually becomes horizontal. It would only become strictly horizontal for infinite values of H. For the highest fields at which measurements have been made, I = 1610 when H = 15530, and the curve is very nearly horizontal. In this case the value of *k* has become very nearly zero. Since the curve of I eventually becomes horizontal for very high fields, the iron is said to have reached **saturation.** This is to be expected, from the molecular theory of magnetisation (p. 771), for if all the molecular magnets have been turned into the direction of the magnetising field, there is no possibility of producing further magnetisation. The relative magnetic properties of iron, steel, nickel, and cobalt are given in Fig. 763.

Hysteresis.—On examining a typical curve representing a cycle of magnetisation, several things may be noticed. As the magnetising field H increases, the intensity of magnetisation I increases, as shown from O to *a* (Fig. 760). On decreasing H, the value of I remains greater than when H was rising, and the path *ab* is followed. The value *Ob* of the intensity of magnetisation when the magnetising field has dropped to zero is called the **residual magnetism.** This must not be confused with what is usually called permanent magnetism, which is the magnetisation persisting in a piece of steel although it may be subjected to a variety of treatment.

On reversing H and increasing its value to Oc, the value of I is brought to zero. This reverse field Oc required to reduce I to zero is called the **coercive force.** On further increasing the negative value of H, the path *cd* is followed On diminishing and reversing, the completion *defa* of the curve is obtained It will be seen that the descending branch of the curve always lies above the ascending branch Further, the zero value of H is reached at an earlier part of the cycle than the zero value of I. Owing to this lagging of I behind H in the cycle the phenomenon has been called **hysteresis,** from the Greek word ὑστερέω "to lag behind"

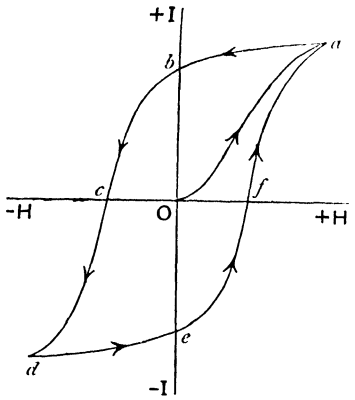


FIG 760 —Cycle of magnetisation

Iron, steel, nickel, cobalt.—The relative magnetic properties of iron and steel may be studied by reference to Fig 761 Iron has a greater residual magnetism than steel, but a smaller coercive force. The curve representing the cycle in the case of iron is more upright

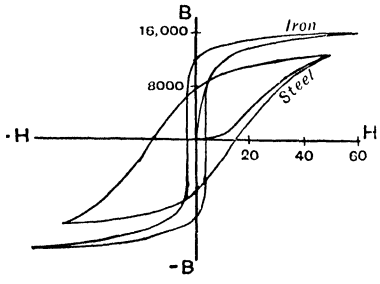


FIG 761 —Cycles of magnetisation for iron and steel

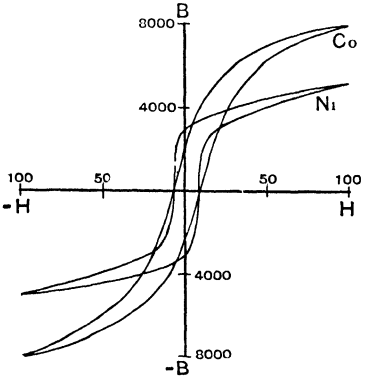


FIG 762.—Cycles of magnetisation for nickel and cobalt

and has a smaller area than that for steel. In Fig 762 are cycles for nickel and cobalt

In order to take a specimen through a cycle of magnetisation work must be done: for every cubic centimetre of the material,

the work for a complete cycle is equal to the area, to scale, enclosed by the I -H curve. Thus, it will be seen that more work is required to take a specimen of steel through a magnetic cycle than in the case of a specimen having the same size and consisting of iron. As this work is converted into heat in the specimen, steel will become heated to a greater extent than iron when subjected to reversals of magnetisation. This fact is important in the design of electrical machinery, as this energy is wasted; consequently the parts subjected to rapid reversals of magnetisation are usually made of soft iron.

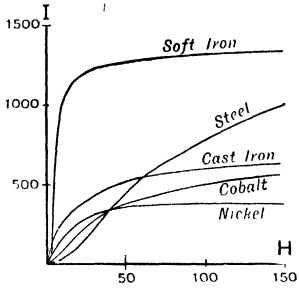


FIG. 763.—Magnetic properties.

Demagnetisation of steel.—It is frequently necessary to demagnetise a piece of steel. This can, of course, be performed by heating it up to a red heat and allowing it to cool, but a more satisfactory method is desirable. Heating the steel changes its character, and in the case of a delicate body like the hair-spring of a watch, the elastic properties of the body would be destroyed.

An examination of Fig. 760 would, at first sight, lead us to suppose that by stopping at the point *c* in the cycle, the body would then be demagnetised But this is not the case, as there is still a magnetising field Oc. and on removing this field, the body will be found to be magnetised. The only satisfactory method is to take the body through a succession of magnetic cycles, continually decreasing in magnitude, until the cycles become so small that the magnetic field is practically zero. Thus a watch whose hair-spring has been magnetised accidentally, may be very much improved, if not cured, by placing it inside a coil in which an alternating current is flowing. The current should be great at first, and then be gradually diminished to zero.

Ewing's molecular theory of magnetisation.—The **molecular theory of magnetisation** has been accepted for a long time, but in its simple form certain difficulties arise. For example, how is it that a magnetic field, however weak, does not set all the molecular magnets in its own direction, and so produce saturation? To get over this difficulty it must be supposed that the molecular magnets are not perfectly free to turn, and former investigators in the subject assumed that there is some friction opposing the turning of the molecular magnets. Sir James Ewing showed that the observed magnetic effects could all be explained, without introducing the idea

of friction, by means of the magnetic effects of the neighbouring molecular magnets upon each other He imitated a piece of magnetic material by means of a group of compass needles. The explanation may be simplified by considering a group of four such magnets, but it must be remembered that actually there must be an infinite number arranged in all possible ways

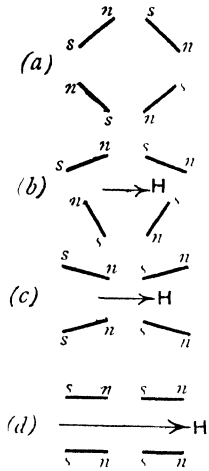


FIG 764.—Magnetisation of a group of four magnets

A group of four magnets arranged as in Fig 764 (*a*) would set as shown when there is no magnetising field The N and S poles of consecutive magnets would be so close together that there would be no external magnetic effect produced. This corresponds to the unmagnetised condition A weak magnetising field H would rotate the magnets slightly into its own direction as in (*b*), but would not break up the group. On increasing H, a point will be reached at which the lower two magnets will swing round to the position shown in (*c*) Thus, a small increase in H will correspond to a large increase in the intensity of magnetisation, being the change indicated

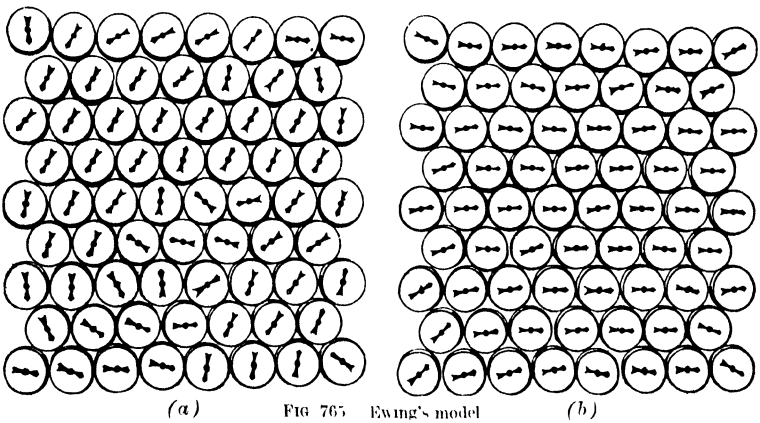


FIG 765 Ewing's model

by the part *ab* of the curve in Fig 758. Any further increase in H can only set the magnets slightly more in line, as in Fig. 764 (*d*), the part *bc* of the curve in Fig. 758 being realised

Remembering that an actual piece of iron consists of a vast number of molecular magnets which arrange themselves in groups and lines of all degrees of stability, it will be seen that these groups will not all break up at the same time, and the flowing curve of Fig. 758 will be the result. Fig. 765 (*a*) is from a photograph of a group of compasses which are not subject to a magnetising field, and (*b*) the same group when subjected to a moderately strong magnetising field.

Paramagnetic and diamagnetic substances.—The three substances —iron, nickel, and cobalt —are so vastly more magnetic than any other substances that they are placed in a group by themselves; they are said to be **ferromagnetic.** The permeability of iron may be as great as 2000, that of nickel 300, and of cobalt 250. No other substance has permeability approaching these, in fact its

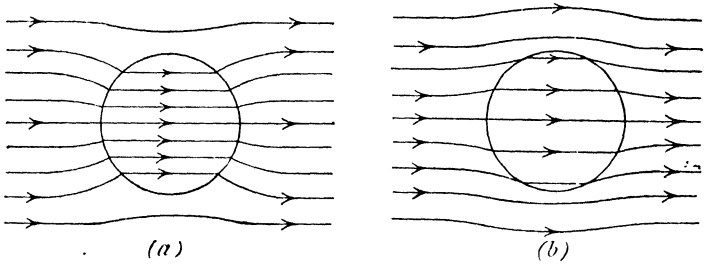


FIG. 766.—Paramagnetic and diamagnetic substances

value in all other cases is very nearly unity. Nevertheless, nearly all substances have feeble magnetic properties. The magnetic property of these substances is better expressed in terms of the susceptibility than permeability. Thus the magnetic susceptibility of platinum is $+1{\cdot}32 \times 10^{-6}$, of aluminium $+0{\cdot}65 \times 10^{-6}$, water $-0{\cdot}8 \times 10^{-6}$, copper $-0{\cdot}087 \times 10^{-6}$, and bismuth $-1{\cdot}4 \times 10^{-6}$.

For some substances the magnetic susceptibility is positive and for others negative. When it is positive the substance is said to be **paramagnetic,** and when negative the substance is **diamagnetic.** By employing the relation $\mu = 1 + 4\pi k$, we see that the permeability of platinum is 1·000017 and of bismuth 0·99996. Thus **paramagnetic substances have a permeability greater than unity and diamagnetic substances a permeability less than unity.**

Fig. 766 (*a*) represents the distribution of the lines of induction for a sphere of a paramagnetic substance placed in a uniform magnetic field, and (*b*) represents the case of a diamagnetic substance.

Whether a substance is paramagnetic or diamagnetic may be determined by its behaviour in a strong magnetic field. Thus, a

rod of the substance, suspended in a strong magnetic field, will set with its length in the direction of the field, if the substance is ferromagnetic or paramagnetic, but if the substance is diamagnetic the rod will set with its length at right angles to the field Thus a piece of iron or platinum suspended in a magnetic field will set as shown in Fig 767 (*a*), but a piece of bismuth as in (*b*)



FIG 767 —Para- and diamagnetism

Further, a piece of a ferromagnetic or paramagnetic substance tends to move from the weaker to the stronger parts of a magnetic field, as has been seen by the manner in which iron filings cling to a magnet On the contrary, a piece of diamagnetic substance tends to move from the stronger to the weaker parts of the field, but the effect is so small even in the case of the most diamagnetic substances that special arrangements are necessary in order to observe it.

* **The magnetic circuit.** –A magnetic field may be represented completely by means of lines of induction (p. 816) This gives rise to a means of calculating the magnetic field and induction in several important practical cases. Through the boundary of any area S_1 situated at *a* (Fig 768) and at right angles to the direction of the induction, draw lines of induction Since lines of induction are closed curves, they will enclose a tube *abcd* when produced in both directions The lines crossing the area S_1 at *a* will also cross S_2 at *b*, S_3 at *c*, and so on, for no lines of induction pass into or out of the tube Now the number of lines per unit area at *a* is B_1 (the value of the induction there), so that the whole number crossing S_1 is B_1S_1. Also the number crossing S_2 is B_2S_2, and so on But as these are all the same,

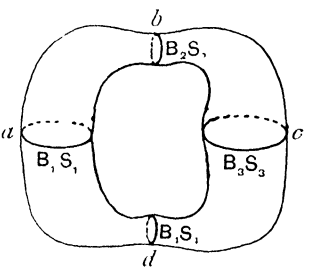


FIG. 768 —Magnetic circuit

$$B_1S_1 = B_2S_2 = B_3S_3 = B_4S_4$$

In other words, the quantity BS is the same for every section of the tube Such a closed tube of induction is called a **magnetic circuit** It is the property of a magnetic circuit that the quantity BS, sometimes called the **magnetic flux**, or the total number of lines of induction, is the same for every section.

* To be omitted until after reading Chapter LXXV.

The magnetic circuit may be made up of a number of parts, having different cross-sections and different permeabilities. Thus, consider the core of an electro-magnet (Fig. 769). The magnetic circuit will be very nearly that indicated by the dotted lines. Let the yoke have an effective length l_1 and cross section S_1, and be constructed of iron of permeability μ_1. The quantity $l_1/\mu_1 S_1$ is called the **magnetic resistance** of this part of the circuit. Similarly $l_2/\mu_2 S_2$ is the magnetic resistance of each limb, and $l_3/\mu_3 S_3$ that of each pole piece. The magnetic resistance of the air gap between the pole pieces is l_4/S_4, since the permeability of air is unity. Now **the magnetic resistance of the whole circuit is the sum of the magnetic resistances of its separate parts.** Thus, for the whole circuit

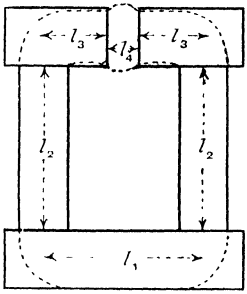


FIG. 769 — Core of an electro-magnet.

$$\text{Magnetic resistance} = \frac{l_1}{\mu_1 S_1} + \frac{2l_2}{\mu_2 S_2} + \frac{2l_3}{\mu_3 S_3} + \frac{l_4}{S_4}.$$

The **product of the total induction or magnetic flux and the magnetic resistance is called the magneto-motive force in the circuit,** usually written M.M.F.

Thus, $$(\text{Magnetic flux}) \times (\text{magnetic resistance}) = \text{M.M.F.},$$

or, $$\text{Magnetic flux} = \frac{\text{M M F.}}{\text{magnetic resistance}}.$$

In the case of an electro-magnet the M.M.F. is due to an electric current flowing in coils surrounding the limbs (Fig. 933). **The product of the current in amperes** (p. 813) **and the total number of turns is called the number of ampere-turns**; further,

$$\text{M.M.F.} = \frac{4\pi \times (\text{ampere-turns})}{10}.$$

Thus, for a given electro-magnet, if the number of ampere-turns be known, the M.M.F. may be found. Then, knowing the magnetic resistance, the magnetic flux can be calculated. Dividing this by the area of the air gap, the value of B, or the strength of magnetic field in the gap can be found.

EXAMPLE.—An iron ring of circumference 50 cm. and cross-section 0·5 sq. cm. is wound with 400 turns of wire in which a current of 1·5 amperes flows. At one part of the ring is an air gap 2 mm. wide. Find the strength of magnetic field in this gap, if the permeability of the iron is 500.

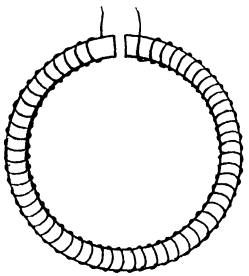


FIG. 770.—Circular magnetic circuit.

Number of ampere turns $= 400 \times 1{\cdot}5$

$= 600$;

$$\text{M M F} = \frac{4\pi \times 600}{10} = 240\pi$$

Again, Magnetic resistance of the iron ring

$$= \frac{50}{500 \times 0{\cdot}5} = 0{\cdot}2,$$

Magnetic resistance of the air gap $= \frac{0{\cdot}2}{0{\cdot}5} = 0{\cdot}4$;

Total magnetic resistance $= 0{\cdot}2 + 0{\cdot}4 = 0{\cdot}6$.

Now magnetic flux $= BS = \frac{\text{M M F}}{\text{magnetic resistance}} = \frac{240\pi}{0{\cdot}6} = 1257.$

This is also the value of BS for the air gap;

$$B = \frac{1257}{0{\cdot}5} = 2514 \text{ c g s units.}$$

But the permeability of the air is unity, so that the strength of field in the air gap is also 2514 c g s units.

EXERCISES ON CHAPTER LXIV.

1. Define intensity of magnetisation. How is it measured, and how does it vary in iron with the magnetising force? L.U.

2. Define "intensity of magnetisation" and "magnetic susceptibility." A hollow iron mast 12 metres high, having external diameter 30 cm and internal diameter 20 cm, is magnetised by the vertical component of the earth's magnetic field. Taking the intensity of this to be 0·40 unit, and the susceptibility of the iron to be 8·0, calculate the magnetic moment of the mast, and its effect upon the time of vibration of a compass needle placed 4 metres north of the foot of the mast, neglecting the effect of the pole at the top, and taking $H = 0{\cdot}2$. L.U.

3. Give two separate definitions of intensity of magnetisation. Find an expression for the force between the poles of two bar magnets placed face to face and in contact.

4. Calculate the strength of field near a plane sheet of magnetic pole of strength 8·5 per sq. cm.

A gap is cut in a ring magnet, the intensity of magnetisation of which is 80. Find the strength of magnetic field in the gap.

5. Give a brief account of the molecular theory of magnetism.

6. What is a magnetic circuit?

An electro-magnet of ring form has a cross-section of 10 sq. cm., a length of 70 cm. measured along the circumference, and is excited by a current of 5 amperes flowing in 600 turns of wire. The length of the air gap being 1 cm., find the strength of magnetic field in this gap, if the permeability of the iron is 500.

7. Define intensity of magnetisation.

The maximum intensity of permanent magnetisation in a steel bar 10 cm. long by 1 cm. square has been found to be 225 c.g.s. units.

Find the tangent of the greatest deflexion of a magnetometer which such a magnet could cause if the centre of the needle were 30 cm. east of the centre of the magnet. ($H = 0.18$ c.g.s.) L.U.

8. Define "magnetic moment" and "intensity of magnetisation."

Find the strength of magnetic field at a point 50 cm. from the middle of a piece of magnetised steel wire 5 cm. long and 2 mm. in diameter, if the intensity of magnetisation of the steel is 200, the point being situated on the axis of the magnet. L.U.

9. A cylindrical magnet of 1 sq. cm. section and with poles 20 cm. apart, suspended by a vertical thread, makes 20 complete vibrations in 88 secs. at a place where the earth's magnetic horizontal intensity is 0·25. If the moment of inertia of the magnet is 245, find the magnetic moment, pole strength and intensity of magnetisation. Univ. of Sydney.

10. A piece of iron wire 36 cm. long and 2 mm. in diameter is magnetised in the direction of its length by a field of strength 25 c.g.s. units. If the magnetic susceptibility of the material is 52, calculate the magnetic moment and the strength of field due to the wire, at a point 80 cm. from it and situated on the line bisecting it at right angles.

11. A bar of steel of length 23 cm., breadth 1·2 cm. and thickness 0·5 cm. is placed in a magnetic field of strength 7·5 units and parallel to its length. Find the magnetic moment of the bar if its permeability is 640.

12. Find the magnetic moment and strength of pole of a bar of iron of length 10 cm. and cross-section 0·5 sq. cm., if it is uniformly magnetised in the direction of its length to an intensity of 500.

13. Two long soft iron rods of area of cross-section 2·5 sq. cm. are placed end to end and in contact. They are situated in a long solenoid having 15 turns per cm. in which a current of 1·5 ampere is flowing. If the permeability of the iron is 150, what is the force required to separate the rods?

CHAPTER LXV

THE ELECTRIC CURRENT

Effects of current.—In certain circumstances a wire may exhibit characteristic phenomena, thus, there may be a magnetic field associated with it, although the material of the wire is not itself a magnetic substance. Also the wire may become hot, without any apparent application of heat to it, and, further, if the wire be cut and the two ends dipped into a solution of sulphuric acid in water, bubbles of gas, oxygen and hydrogen, will appear upon the ends immersed. Without, for the moment, considering how the wire may be brought into this condition, we shall study these effects. When they are observed we say that **an electric current is flowing in the wire**.

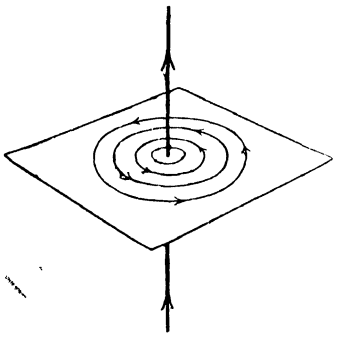


FIG. 771.—Magnetic lines of force due to a straight current.

Magnetic field due to an electric current.—In the case of a long straight wire carrying an electric current, the magnetic field in its neighbourhood is in the form of circles having their centres upon the wire and their planes perpendicular to it. The magnetic lines of force may be traced as on p. 778 by passing a straight wire through a piece of cardboard, and using a compass needle to find the direction of the magnetic field at various points. Or iron filings may be sprinkled upon the card; on tapping the card the filings will set themselves in approximate circles (Fig. 771).

If the wire be in the form of a circle, the form of the field in a plane at right angles to the circle is as shown in Fig. 772.

EXPT. 179.—**Magnetic field due to a circular current** Pass a current round the circular coil (Fig. 772), which passes through a horizontal board. Cut holes in a piece of drawing paper to fit the coil and place it upon the board. Trace the magnetic lines of force by means of a compass needle as on p. 778. Note that the lines close to the limbs of the coil approximate to circles.

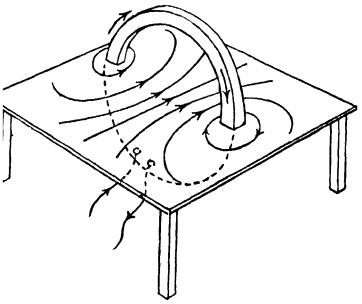


FIG 772.—Magnetic field due to a circular current.

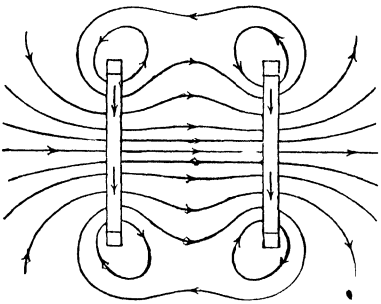


FIG. 773.—Magnetic field due to a double coil

EXPT. 180.—**Magnetic field due to a double coil.** Pass a current in the same direction round two parallel circular coils (Fig. 773). Fit a piece of drawing paper to the coils, and plot the magnetic lines of force as in the last experiment. Note that for a considerable space in the middle of the field the lines are nearly parallel; that is, the magnetic field is very nearly uniform.

Direction of current.—In the preceding experiments there is nothing to indicate the direction in which the electric current is flowing in the wire or coil. In fact, the direction of flow is defined conventionally in the following manner. Let the wire be taken in the **right** hand with the fingers pointing in the direction of the magnetic lines of force; then the **thumb** points in the direction of the current (Fig. 774). This will be found to be in accordance with the arrows in Figs. 771 and 772.

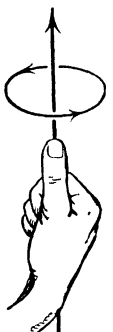


FIG 774.—Direction of current

EXPT. 181.—**Direction of magnetic field due to a current.** Join one end of a piece of copper wire to the zinc pole of a Daniell's cell, and the other end to the copper pole. The current then flows in the wire from the copper to the zinc. Place part of the wire horizontally with the current flowing from north to south and situated immediately over a compass needle. Note the direction of deflection of the N pole

of the compass. Place the wire under the compass and again note the direction of deflection Continue according to the following table, and enter the results as shown :

Wire	Current flowing,	Position of wire	Deflection of N pole of compass
Horizontal	north to south	above compass	E
,,	south to north	,, ,,	
,,	,, ,,	below compass	
,,	north to south	,, ,,	
Vertical	downwards	N of compass	
,,	,,	S ,,	
,,	upwards	,, ,,	
,,	,,	N ,,	

Verify the fact that these results are in accordance with the rule for the direction of current given on p. 831.

EXPT. 182.—**Magnetisation of iron by an electric current** Take a cylindrical bar of soft iron, about 1 cm diameter, and wrap round it about 20 or 30 turns of insulated copper wire Pass a current through the wire It will be found that the rod becomes a strong magnet while the current is flowing. Test with iron filings, and find the polarity of each end by means of a compass needle. Reverse the direction of the current and again find the polarity of the ends. Show that the polarity is in accordance with Fig. 773.

EXPT. 183—**Heating effect of a current** Connect the terminals of a storage cell by means of a piece of bare platinoid wire, No. 22, about a metre long, and note that the wire becomes warm. If the wire be shortened it becomes hotter. Replace the platinoid wire by a piece of iron wire, size No. 30. This wire also becomes very hot, and if shortened to a few centimetres becomes red hot and eventually fuses.

Sharpen two carbon rods to points and bind a fine piece of bare copper wire round each rod. Join each copper wire to one terminal of a battery of secondary cells. On touching the two carbon points together they will become red hot or even white hot at the tips.

EXPT. 184.—**Chemical effect of a current** Dip the ends of the two carbon rods of the last experiment into a vessel of water containing a few drops of sulphuric acid. Bubbles will form on each of the rods and will rise up and burst at the surface of the liquid. Note that at the carbon rod connected to the **black pole** of the battery the bubbles are about twice as copious as those on the rod connected to the **red pole** If the bubbles be collected in inverted test tubes, it will be found that the gas liberated at the carbon rod connected to the black pole is hydrogen, while that from the rod connected to the red pole is oxygen.

Dip the rods into a solution of copper sulphate. Copper is deposited on the carbon attached to the black pole of the battery, while bubbles appear upon that attached to the red pole.

Measurement of electric current.—Like all other physical quantities an electric current may be measured by the effect it produces. Thus for its measurement, the magnetic effect, the heating effect, and the electrolytic or chemical effect are all available, and any one of them might be chosen to define the magnitude of the current. If the current be taken as proportional to the heat produced in a given time, very grave difficulties would be encountered in its future study. This is consequently abandoned. Use of the chemical effect involves the measurement of time, as the effect goes on continually, being greater for a large than for a small time. On the other hand, for a given current the magnetic field is constant, being independent of time. It is therefore chosen as the most satisfactory property of the current for the purpose of measurement. We will therefore consider that **an electric current is proportional to the strength of the magnetic field which accompanies it.**

Unit of current.—In order to define a unit of current in terms of a strength of magnetic field, it is desirable to take such a shape for the conductor carrying the current that the strength of field is proportional to the length of the conductor as well as to the current. This can only be the case when the current is in the form of a circle, and the point at which the magnetic field is considered is the centre of the circle.

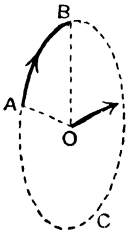


FIG. 775.—Magnetic field due to unit current.

Unit current is such that when flowing in an arc of a circle of unit radius, the arc having unit length, the magnetic field at the centre has unit strength.

Thus, if ABC (Fig. 775) be a circle of 1 cm. radius, and AB be an arc of this circle having length 1 cm., the magnetic field due to the current in this arc AB has unit strength at the centre O.

Of course, it is not usual to attempt to produce a current flowing in a portion of a circle such as AB; the circuit consists in practice of a complete circle, the length of which is 2π cm. when the radius is 1 cm. The strength of field due to the complete circle is therefore 2π units, in fact, we may define a unit current as a current of strength such that when flowing in a complete circle

of 1 cm. radius, it produces a magnetic field of strength 2π at the centre.

Strength of magnetic field at the centre of any circular current.—It has now been seen that the strength of magnetic field is proportional to the current (i) flowing in a circle, and to the length of the arc (l), and it remains to be seen how it depends upon the radius of the circle. This will not be proved experimentally here, but it has been conclusively shown that, other things being equal, the strength of field varies inversely as the square of the radius Thus, the complete expression is,

$$\text{Strength of magnetic field, } \mathsf{H} \propto \frac{li}{r^2}$$

Since, however, $\mathsf{H}=1$, when $i=1$, $l=1$, and $r=1$, from the definition of unit current (p. 833), we may now write $\mathsf{H}=li/r^2$

Again, for one complete circle, $l=2\pi r$, or for a circle consisting of n complete turns of wire, $l=2\pi nr$, hence,

$$\mathsf{H}=\frac{2\pi nri}{r^2}=\frac{2\pi ni}{r}$$

where r is in centimetres and i in units of current

The tangent galvanometer. From the preceding discussion we see that the measurement of electric current depends upon the measurement of magnetic field. For the measurement of magnetic field the magnetometer was devised (p. 784) and this instrument may now be adapted for the purpose of measuring current. It is then called the **tangent galvanometer**, owing to the tangent law which we saw to hold in the case of the magnetometer.

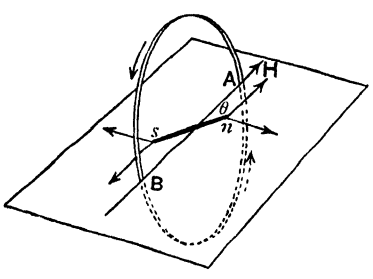


FIG. 776.—The tangent galvanometer

A vertical circular coil AB (Fig. 776) carries the current, and this produces a magnetic field of strength $2\pi ni/r$, at the centre, the direction of field being at right angles to the plane of the circle. The plane of the circle is therefore placed in the magnetic meridian, so that a suspended magnet ns experiences a couple twisting it out of the meridian, while the earth's horizontal component of magnetic field H tends to retain it in the meridian. This is exactly the condition that held in the case of the magnetometer (p. 784), and the

suspended magnet will come to rest at some angle θ to the meridian (Fig. 777) when the restoring and deflecting couples are equal.

The restoring couple is $\mathsf{H}m \sin\theta$, where m is the magnetic moment of the suspended magnet; also,

$$\text{Deflecting couple} = \frac{2\pi ni}{r} m \cos\theta;$$

$$\therefore \frac{2\pi ni}{r} m \cos\theta = \mathsf{H}m \sin\theta,$$

or,
$$i = \frac{\mathsf{H}r}{2\pi n} \tan\theta.$$

Thus, if H, r, and n be known and θ be observed, the strength of current, i, can be calculated.

Even when H is not known the tangent galvanometer may be usefully employed for comparing currents, in which case the equation is usually written, $i = k \tan\theta$.

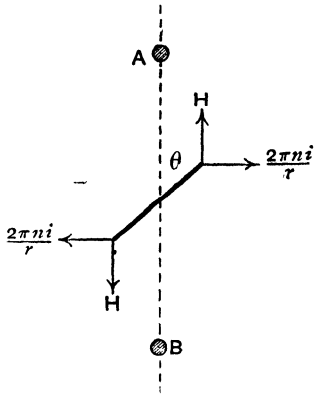


FIG. 777.—Diagram for the tangent galvanometer

The quantity k is called the **reduction factor of the galvanometer.**

When two currents i_1 and i_2 are to be compared, the deflections θ_1 and θ_2 which they produce are observed; then

$$i_1 = k \tan\theta_1,$$
$$i_2 = k \tan\theta_2;$$
$$\therefore \frac{i_1}{i_2} = \frac{\tan\theta_1}{\tan\theta_2}.$$

Use of the tangent galvanometer.—A common form of the galvanometer has a coil consisting of a number of turns wound upon a circular frame, the ends of the coils being attached to the terminals upon the base of the instrument. Three levelling screws are provided and enable the instrument to be adjusted so that the suspended needle may be at the centre of the coil. The deflections are observed by means of a long light pointer, attached to the magnet, the end of the pointer moving over a large horizontal circular scale as in the magnetometer (Fig. 727). To avoid parallax in making the readings, the scale is pasted upon a piece of plate-glass mirror, in order that the eye may be placed so that the pointer and its image in the mirror coincide. The eye is then normally over the scale and the reading is free from error due to parallax.

It should be noted that the suspended magnet must be small, so that the magnetic field due to the coil may be considered to be uniform over the whole of the magnet, and its value, therefore,

equal to that at the centre of the coil. An inspection of Fig. 772 will show that the magnetic field is sensibly uniform over a small space only at the centre of the coil, so that, should the magnet be too large, it is not in a uniform field and the poles of the magnet are no longer situated in a magnetic field whose value is $2\pi m/r$.

Adjustments of the tangent galvanometer.– (i) The fibre supporting the needle must be free from torsion, otherwise there is a disturbing couple due to this cause. If the magnet is fairly strongly magnetised and the fibre sufficiently thin, this disturbing effect may be eliminated by removing the magnet and replacing it by a small piece of wire of equal weight. When this comes to rest, replace the magnet.

(ii) The needle must be at the centre of the scale. This adjustment is effected by means of levelling screws, and is sufficiently accurate when the needle is in the centre of the hole in the middle of the plate glass, and the pointer is symmetrically situated with respect to the scale.

(iii) The plane of the coil must be in the meridian. The whole instrument is rotated until the ends of the pointer are at the zero marks on the scale.

(iv) The pointer may not be at right angles to the magnetic axis of the magnet, in which case the plane of the coil is not in the magnetic meridian, and the tangent law does not hold good. This error may be eliminated by suspending the magnet and pointer the other way up, and seeing whether the pointer still comes to rest in the same position. When this is the case the pointer and needles are at right angles (p. 795), but if it comes to rest in a new position, one must be twisted with respect to the other, and another trial must be made. Fig. 778 shows how this may be done. The system is suspended by a loop A from a hook attached to the fibre. When the system is reversed it is suspended from the loop B. The framework, made of copper wire, is sufficiently flexible for the twisting of the magnet with respect to the pointer to be performed.

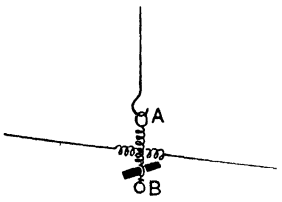


FIG. 778.—Suspension of the needle of a tangent galvanometer

When this adjustment is completed, (iii) must be performed again.

Method of reading.—(i) Having made the above adjustments, pass the current through the coil; the deflection should not be less than 10° or greater than 60°. Both ends of the pointer must be observed, in order to correct for any want of centring of the needle with respect to the scale.

(ii) The current is now reversed and the deflections on the other

side of the zero observed. This eliminates any small want of symmetry of the needle with respect to the coil.

These measurements should be repeated with each of the currents to be measured or compared, the results being recorded as under :

	Deflections					
	E end of pointer.	W end of pointer.	Current reversed E	Current reversed W.	Mean θ.	tan θ.
1st current -						
2nd current -						

Expt. 185.—**Measurement of current by the tangent galvanometer.** Make the adjustments described on p. 836. Pass the current from a Daniell's cell through the tangent galvanometer, making the readings of deflection and recording them as above. Count the number of turns in the coils and measure their mean diameter. Calculate the current from the equation $i = Hr \tan \theta / 2\pi n$, using a previously determined value of H, the earth's field.

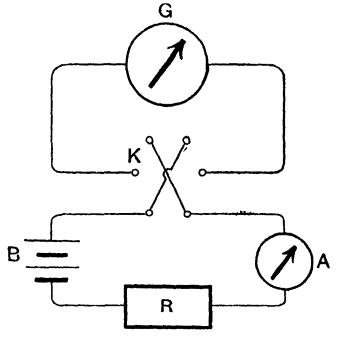


Fig. 779.—Use of the tangent galvanometer.

Expt. 186.—**Measurement of H by means of the tangent galvanometer.** For this purpose some other method of measuring the current is necessary. An ammeter of some form to be described later (p. 870) may be used. Connect up as in Fig. 779, G being the tangent galvanometer, A the ammeter, R an adjustable resistance, B a battery and K a key which reverses the current in the tangent galvanometer without reversing it in the rest of the circuit. Starting with a small current, observe θ, and record as above, writing down the value of the current as observed by the ammeter in the current column. Then repeat with a slightly larger current, proceeding until ten values of the current have been employed. Plot on a curve the values of i and tan θ, when it will be found that the points lie very nearly on a straight line. Draw a straight line with a ruler to lie evenly amongst the points. The equation $i = Hr \tan \theta / 2\pi n$ may then be written, $H = 2\pi ni / r \tan \theta$, and the values of n and r measured as before.

The value of $i/\tan\theta$ may be taken from the graph, but it must be noticed that the current as recorded by the ammeter must be divided by 10, for a reason given later (p. 843). In this way the value of H is obtained.

EXERCISES ON CHAPTER LXV.

1. Describe how you would show that an electric current is flowing in a given wire, giving a method for finding its direction.

2. Explain why it is necessary that the suspended magnet in a tangent galvanometer should be of small dimensions.

3. Calculate the strength of field at the centre of a circular coil of 30 turns of diameter 18 cm. due to a current of 5 C.G.S. units flowing in it.

4. A circular coil of 10 turns and radius 8 cm. is placed with its plane at right angles to the magnetic meridian. If a suspended magnet at its centre makes 18 vibrations per minute with the current in one direction and 30 vibrations per minute when the direction of the current is reversed, what is the strength of the current, given that the field due to the coil is greater than H? (Take H = 0·18.)

5. Describe the principle of the tangent galvanometer. Why is the deflection not proportional to the current?

6. Explain why the plane of the coil of the tangent galvanometer must be in the magnetic meridian.
What is the relation between the strength of the electric current and that of the magnetic force due to it at the centre of the coil?

7. Two tangent galvanometers, A and B, are identical in construction, except for the number of turns in the coil. They are connected in series and a current is sent through them. The deflection in A is 45°, and that in B is 31°. Calculate the ratio of the numbers of turns in the two instruments. (Tan 31° = 0·60.) L.U.

8. Describe with the aid of diagrams the construction of a tangent galvanometer, give the theory of the instrument, and show that it may be used to determine the strength of a current in absolute measure.
The coil of a tangent galvanometer has a radius of 15 cm. and contains 50 turns. Assuming that it is only used to measure currents which give a deflection less than 60° and greater than 1°, determine the range of current in amperes for which it is available. L.U.

9. A current flowing through a tangent galvanometer consisting of 10 turns of wire of radius 8 cm. produces a deflection of 45° when the instrument is in a position where H = 0·18 dyne per unit pole. What alterations would you make in the instrument so that it would give this same deflection for a current of one thousandth of an ampere? L.U.

10. A current of 3 amperes is flowing in a coil consisting of 5 turns of wire each of 10 cm. radius.
Calculate the magnetic field at the centre of the coil, stating the units in which it is expressed. If a magnetic pole of strength m is placed at the centre of the coil, what force acts on the coil when the current is flowing? L.U.

11. Define the c.g.s. unit of current.

A circular coil of wire of 30 turns and radius 20 cm. is placed with its plane vertical and at right angles to the magnetic meridian. When no current flows in the coil, a magnetic needle at its centre makes 15 vibrations per minute; but when a current flows, the needle is reversed in direction and makes 50 vibrations per minute. Taking the horizontal component of the earth's magnetic field as 0·2, calculate the current.

12. Describe, and explain the use of, the tangent galvanometer.

If in a given instrument a current of 5 ampere causes a deflection of 45°, what currents would be indicated by deflections of 30° and 60° respectively?

CHAPTER LXVI

POTENTIAL DIFFERENCE, RESISTANCE, AND WORK

Analogy between an electric current and the flow of liquid in a tube. There are certain important points of resemblance between the electric current and the flow of water in a tube. These arise from the fact that in both cases some motive power is necessary to maintain the flow, and the energy of the supply eventually becomes heat in the circuit. The student is particularly warned against pushing the analogy too far. For example, in the case of an electric current there is no fluid passing along the wire, and, on the other hand, the flow of water in a tube which is not horizontal is partly due to the weight of the water. Also, when the water is flowing it possesses kinetic energy derived from the source supplying the energy to maintain the current. If, however, we confine our attention to narrow horizontal tubes the analogy serves a useful purpose.

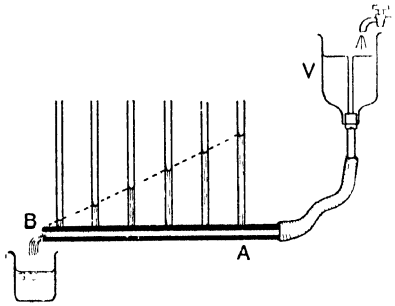


FIG. 780.—Flow of water in a horizontal narrow tube

When a steady flow of water is maintained in a narrow horizontal tube into which vertical tubes are fixed to act as pressure gauges (Fig. 780), it will be noticed that the pressure falls uniformly along the tube. It is a maximum at the end A, and falls uniformly to the pressure of the atmosphere at the open end B. This fall of pressure is due to frictional resistance. By raising or lowering the vessel V, the flow of water may be altered, but the current of water, as measured by the amount passing through the tube in unit time, is

proportional to the difference of pressure between the ends of the horizontal tube.

In the case of the electric current, the quantity analogous to difference of pressure is called **potential difference**, or p.d. Further, **the potential difference is proportional to the current**, and for a uniform conductor, the potential difference over equal lengths of conductor is the same.

There is a further resemblance between the two cases, for the work done in maintaining both currents is converted into heat, the **quantity of heat produced in unit time being in both cases proportional to the flow and to the difference of pressure, or potential difference.** The measurement of potential difference is deferred until later.

For convenience, the analogous quantities in the two cases are collected into the following table:

Flow of Liquid	Electric Current
(i) Flow, or volume of liquid crossing any section of pipe per second.	Electric current.
(ii) Flow $\propto$ difference of pressure between ends	Current $\propto$ potential difference between ends of conductor.
(iii) Uniform fall of pressure along pipe	Uniform fall of potential along conductor.
(iv) Quantity of liquid = flow $\times$ time.	Quantity of electricity = current $\times$ time.
(v) Amount of work converted into heat per second $\propto$ flow $\times$ difference of pressure.	Amount of work converted into heat per second $\propto$ current $\times$ p.d.

Unit of potential difference.—Analogy (v) in the above table serves to define the unit of potential difference. The relation of p.d. to current depends, of course, upon the nature of the conductor. For a given conductor, any current may flow, depending upon the p.d. between its ends. Thus, for a long and thin conductor, the p.d. corresponding to a given current will be greater than for a thicker conductor of the same material, and the rate of working to maintain this current is greater in the former case. We will

define our unit of potential difference, as **that potential difference which, when maintaining unit current, performs one unit of work per second.** Measuring all our potential differences in terms of this unit, the analogy (v) may now be expressed as follows :

Rate of working in conductor = current × p d , ergs per second
$= \iota \times e$, ergs per second,

or, total work performed in time t seconds
= current × p d × t
$= \iota \times e \times t$ ergs.

Ohm's law. The relation between the current in a conductor and the potential difference between its ends is indicated in analogy (ii). It was first given by G S Ohm, and is known as **Ohm's law.** It may be stated as follows :

For a given conductor, the ratio of the potential difference between its ends to the current flowing in it is constant; or, p d current = constant.

For this law to hold, the conductor must remain under constant conditions ; for example, its temperature must not vary. A satisfactory proof of Ohm's law is beyond the scope of this work.

Resistance.—The name **resistance** is given to the ratio of p d to current for any conductor ; thus,

$$\frac{\text{p d}}{\text{current}} = \text{resistance.}$$

or,
$$\frac{e}{\iota} = r, \quad e = \iota r.$$

A conductor therefore has unit resistance if the p d. between its ends is unity when unit current flows in it.

Since the rate of working in any circuit is known from the above equation, in terms of the p.d and current, it may now be expressed in terms of any two of the three quantities - p d., current, and resistance.

Thus, Rate of working $= e\iota = \iota^2 r = \frac{e^2}{r}$ ergs per second

This work performed in maintaining the current appears as heat in the conductor. With a small current, the amount of heat produced per second may be so small that it leaks away by conduction, etc., so rapidly that the temperature of the wire is not appreciably raised. It may happen, however, that the rate of production of heat is so great that the conductor is considerably raised in temperature, as

in the case of the electric incandescent lamp. In fact, pieces of fine wire, called **fuses**, are generally placed in lighting and power circuits, which, for the limiting safe current, are raised to such a temperature that they melt, and so break the circuit. Such fuses are generally pieces of tin wire, since tin melts at a fairly low temperature, but they are sometimes made of copper.

The amount of heat produced in a circuit in a given time may be found from a knowledge of the fact that $4{\cdot}2 \times 10^7$ ergs (approximately) when converted into heat become 1 calorie (p. 355).

Thus, Heat produced in circuit

$$= \frac{ei}{4{\cdot}2 \times 10^7} = \frac{i^2 r}{4{\cdot}2 \times 10^7} \text{ calories per second.}$$

It will be noticed that for a circuit of given resistance, the heat produced per second is proportional to the square of the current and not simply to the current. This is the reason why the heating effect was not chosen to define the unit of current (p. 833).

Practical units.—The units defined above have been chosen on account of their simple relation to the mechanical units of force and work. For practical purposes these units are of inconvenient size, and others are therefore selected which can be obtained easily from the absolute units, and represent, by convenient numbers, the ordinary quantities to be measured.

The ampere.—One tenth of the absolute unit of current is taken as the practical unit and is called the **ampere**, after Ampère, who did a great amount of work in the study of electric and magnetic phenomena. We shall in future use I to denote a current when measured in amperes, and i when measured in absolute units. Thus, for any given current $\mathrm{I} = 10i$, since the number of amperes is ten times the number of absolute units in its measurement.

Hence the magnetic field (H) at the centre of a circular coil (p. 834) is given by

$$\mathrm{H} = \frac{2\pi n i}{r} = \frac{2\pi n \mathrm{I}}{10r}.$$

The volt.—The absolute unit of potential difference is an extremely small quantity; hence the practical unit is taken to be 100000000, or 10^8 times as great. It is called the **volt**, after Volta, the discoverer of the 'voltaic' pile.

Thus, $e = 10^8\mathrm{E}$, where E is a given p.d. measured in volts, and e the same measured in absolute units.

The Ohm.—Having chosen two of our new units of convenient size, we are no longer at liberty to choose the others, but must derive them from those already fixed. Thus, for a conductor to have one

practical unit of resistance, there must be a p d of 1 volt between its ends when the current in it is 1 ampere This unit of resistance is called the **ohm.** Thus if R is the resistance of a conductor in ohms,

$$\frac{E}{I} = R. \quad \text{or} \quad E = IR.$$

But, $E = e \times 10^{-8}$ and $I = 10\iota$;

$$\therefore R = \frac{E}{I} = \frac{e \times 10^{-8}}{10\iota} = \frac{e}{\iota} \times 10^{-9} = r \times 10^{-9}.$$

Hence, $r = 10^9 R$, and a given resistance has 10^9 as many absolute units in it as it has ohms Thus the ohm is equal to 10^9 absolute units of resistance

Hence there are two equations which may be used in the calculation, of current, potential difference, and resistance, namely, $E/I = R$, when the ampere, volt, and ohm are the units employed, and $e/\iota = r$ when the absolute units are used The former is by far the more common method.

The watt.—On p 842 it was seen that the unit of p.d. is so chosen that the product of p d and current is the rate of working in a circuit, expressed in ergs per second It is desirable to give a name to the rate of working in a circuit when the current is 1 ampere and the p.d. 1 volt. This is called the **watt.**

Thus, Rate of working $= E \times I$ watts

$$= I^2 \times R \text{ watts.}$$

Now for given current and p d , $I = 10\iota$ and $E = e \times 10^{-8}$;

$$\therefore \text{ rate of working} = e \times 10^{-8} \times 10\iota \text{ watts}$$

$$= e\iota \times 10^{-7} \text{ watts}$$

$$= e\iota \text{ ergs per second.}$$

Hence it follows that the given rate of working, namely $e\iota$ ergs per second, is only $e\iota \times 10^{-7}$ watts, so that 1 watt $= 10^7$ ergs per second.

It likewise follows that,

$$\text{Rate of working} = E \times I \times 10^7 \text{ ergs per second.}$$

Again, since $4{\cdot}2 \times 10^7$ ergs, when converted into heat, give rise to 1 calorie,

Rate of production of heat in conductor

$$= \frac{EI \times 10^7}{4{\cdot}2 \times 10^7}$$

$$= EI \times 0{\cdot}24 \text{ calories per second}$$

$$= I^2R \times 0{\cdot}24 \text{ calories per second.}$$

It is useful to note (p. 172) that since 1 horse-power is 33,000 foot-pounds per minute, by converting foot-pounds to ergs, we find that

$$1 \text{ horse-power} = 746 \text{ watts.}$$

Hence to find the horse-power expended on any circuit we have

$$\text{Rate of working} = EI \text{ watts}$$
$$= \frac{EI}{746} \text{ horse-power.}$$

EXAMPLE 1.—Find the heat produced in a conductor in five minutes, when a current of 3·5 amperes is flowing, the p.d. between the ends of the conductor being 20 volts.

$$\text{Rate of working} = 3{\cdot}5 \times 20 \text{ watts}$$
$$= 3{\cdot}5 \times 20 \times 0{\cdot}24 \text{ calories per second.}$$
$$\therefore \text{ Total heat produced} = 3{\cdot}5 \times 20 \times 0{\cdot}24 \times 300 \text{ calories}$$
$$= 5040 \text{ calories.}$$

EXAMPLE 2.—Find the horse-power required to maintain a current of 3 amperes in a 220 volt incandescent lamp.

$$\text{Rate of working} = 3 \times 220 \text{ watts}$$
$$= \frac{3 \times 220}{746}$$
$$= 0{\cdot}885 \text{ horse-power.}$$

Units of work and energy.—The absolute unit of work being the erg, while the practical unit of rate of working is the watt, or 10^7 ergs per second, it is useful to employ a practical unit of work corresponding to the watt. This is the work done when a rate of working of 1 watt is maintained for 1 second, and is called the **joule**. Thus the watt is equivalent to 1 joule per second, while the joule itself is equal to 10^7 ergs. Hence for any conductor carrying current,

$$\text{Work done} = E \times I \times t \text{ joules,}$$

where t is the time for which the current flows, measured in seconds.

For the commercial supply of electrical energy, the joule is too small for convenience, and another derived unit is employed. This is the **kilowatt-hour**, and is the work done when a rate of working of 1 kilowatt, or 1000 watts, is maintained for 1 hour. The kilowatt-hour is the legal unit for the supply of electrical energy and is called the **Board of Trade unit.**

$$1 \text{ kilowatt-hour} = 1000 \text{ watts for 1 hour}$$
$$= 1000 \times 3600 \text{ joules}$$
$$= 3{\cdot}6 \times 10^6 \text{ joules.}$$

EXERCISES ON CHAPTER LXVI.

1. Define the units of potential difference and resistance.

2. State Ohm's law and give an account of what you mean by the 'resistance' of a conductor

3. Find the potential difference required to maintain a current of 2·5 amperes in a conductor of 23 ohms resistance, and the rate of working in the conductor.

4. State the relation of the ampere and the volt to the absolute units of current and p.d., and find the number of ergs performed in a circuit in 10 minutes when a current of 0·3 ampere flows in a conductor when the p.d. between its ends is 40 volts.

5. Find the horse-power required to maintain a 30 candle-power incandescent lamp of 1·5 watts per candle-power, and the current in the lamp when on a 200 volt circuit.

6. A wire conveying an electric current of 0·5 ampere has a resistance of 1·6 ohm per metre. If 75 cm. of this wire be placed in 650 grams of water, find the rise of temperature in 15 minutes

(1 joule = 0·24 calorie)

7. Find an expression for the heat developed by an electrical current in a wire.

The current through a resistance coil which is connected to the terminals of a 100 volt circuit, raises the temperature of 1 litre of water 10 degrees in one minute. What is the resistance of the coil? L.U.

8. How would you concentrate the production of heat in one part of an electric circuit carrying an electric current? Illustrate your answer by reference to an electric lamp and its leads. State definitely the laws which relate to the production of heat in the circuit. L.U.

9. How is the heat produced in a conductor related to the potential difference between the ends of the conductor and the current flowing in it?

If the heating effect in a certain resistance box endangers the constancy of the coils when the energy used in them exceeds 0·0001 watt per ohm, find the limiting safe voltage applied to the box when the resistance 1·5 ohm is being used, and also when 2500 ohms resistance is being used. L.U.

10. What is the law of heat developed by an electric current?

A glow lamp is immersed in a litre of water, and when the current is switched on the temperature of the water rises at the rate of one degree in 5 minutes. If the current through the lamp is $\frac{1}{2}$ ampere, what is the voltage of the lamp, and what is its resistance?

(42 million ergs., or 4·2 joules = 1 calorie.) L.U.

11. What are the laws relating to the development of heat in a conductor during the passage of an electric current through it?

A current of electricity of 2 amperes is passed for 1 hour through an electric lamp the resistance of which is 52 ohms. Calculate the amount of energy dissipated as heat, and the difference of potential between the terminals of the lamp. L.U.

12. The lighting of a room requires 300 candle-power and the lamps supplied have an efficiency of 1·5 watts per c.p. What is the cost of lighting the room for 24 hours if the cost of supply is 3d. per Board of Trade Unit?

13. The lighting installation of a building consists of 120 incandescent lamps each having a resistance of 65 ohms and requiring a p.d. of 100 volts. Find the rate of working in kilowatts and in horse-power required to maintain them incandescent.

14. If a piece of wire carrying a current is immersed in 600 grams of water and the rise in temperature is 5° C. per minute, calculate the current, if the resistance of the wire is 2·5 ohms.

15. Twenty metal filament lamps, each of 32 candle power and taking 1·4 watts per candle, are installed in a house and supplied at 250 volts. What is the total current taken when the lamps are all lighted, and the cost per hour if electricity is supplied at fourpence per Board of Trade unit? C.G.

CHAPTER LXVII

ELECTRICAL CIRCUITS · ELECTROMOTIVE FORCE

Conductors in series.—Conductors are said to be arranged **in series** when one current flows through all of them, as in Fig 781. The current enters at A and leaves at D, and is the same in all the conductors Calling this current I amperes, the potential difference between A and B is IR_1 volts Similarly, the p d between B and C is IR_2 volts, and between C and D is IR_3 volts

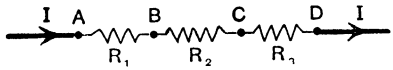


FIG 781 —Conductors in series

If R be the total or effective resistance between A and D, the p.d. between A and D is IR.

Hence, $$IR = IR_1 + IR_2 + IR_3,$$

or $$R = R_1 + R_2 + R_3.$$

Thus, **for conductors in series, the effective resistance is the sum of the separate resistances,** and for any number of conductors,

$$R = R_1 + R_2 + R_3 + R_4 + \text{. ....} \quad (1)$$

Conductors in parallel. -When several conductors are joined between two points so that the current divides between them, they are said to be **in parallel,** as in Fig. 782. The current I, entering at A, divides into three parts, I_1, I_2 and I_3 which unite again at B.

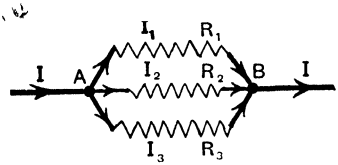


FIG 782 —Conductors in parallel

$$I = I_1 + I_2 + I_3.$$

Now taking the conductors separately we have,

p.d. between A and B $= I_1R_1 = I_2R_2 = I_3R_3 =$ say e ;

$$\therefore\ I_1 = \frac{e}{R_1},\ I_2 = \frac{e}{R_2} \quad \text{and} \quad I_3 = \frac{e}{R_3}.$$

Again, if R be the equivalent resistance between A and B,

p.d. between A and B = IR = e;

$$\therefore \quad I = \frac{e}{R},$$

where I is the total current. Since this is equal to the sum of the three currents I_1, I_2 and I_3,

$$\frac{e}{R} = \frac{e}{R_1} + \frac{e}{R_2} + \frac{e}{R_3},$$

or,

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3},$$

Thus, **when conductors are in parallel, the reciprocal of the effective resistance is the sum of the reciprocals of the separate resistances**, and for any number of conductors,

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4} + \ldots \ldots \quad (2)$$

To find the **current in each branch**, notice that $I_1 = \frac{e}{R_1}$, and that $I = \frac{e}{R}$.

Thus,

$$I_1 = I \cdot \frac{R}{R_1}, \quad \ldots \ldots \quad (3)$$

or, **the current in any branch is equal to the main current multiplied by the combined resistance and divided by the resistance in that branch.**

When there are only two conductors in parallel, the current divides into two parts, inversely as the resistances of the two branches. Thus:

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{R_1 + R_2}{R_1 R_2};$$

$$\therefore \quad R = \frac{R_1 R_2}{R_1 + R_2}.$$

Now,

$$I_1 = I \frac{R}{R_1}, \quad \text{and} \quad I_2 = I \frac{R}{R_2};$$

$$\therefore \quad I_1 = I \cdot \frac{R_2}{R_1 + R_2}, \quad \text{and} \quad I_2 = I \cdot \frac{R_1}{R_1 + R_2};$$

or

$$\frac{I_1}{I_2} = \frac{R_2}{R_1}. \quad \ldots \ldots \quad (4)$$

Conductance.—The reciprocal of the resistance is sometimes called the **conductance.**

Thus,
$$\text{Resistance} = \frac{\text{p d.}}{\text{current}},$$
$$\text{Conductance} = \frac{\text{current}}{\text{p.d.}},$$
$$\therefore \text{ Resistance} = \frac{1}{\text{conductance}},$$

or
$$R = \frac{1}{K}, \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5)$$

where K is taken to be the conductance of the conductor. This enables us to write the relation for conductors in parallel in a simpler manner.

For
$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots$$

But
$$\frac{1}{R} = K, \quad \frac{1}{R_1} = K_1, \quad \frac{1}{R_2} = K_2, \text{ etc. ;}$$
$$\therefore \quad K = K_1 + K_2 + K_3 + \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (6)$$

Or, **the effective conductance of a number of conductors in parallel is equal to the sum of the separate conductances**

Specific resistance, or resistivity. In dealing with conductors of different materials, it is desirable to define in some way the specific properties of each material Two conductors of exactly the same dimensions will not have the same resistance if they are made of different materials.

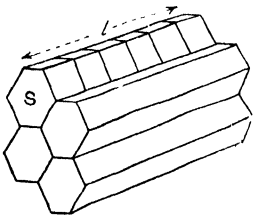


FIG 783.—Conductor of uniform cross-section

The specific resistance of any material is the resistance of a conductor of this material, having unit length and unit area of cross-section. The shape of the cross-section is immaterial, provided that it is uniform throughout the length of the conductor.

Let the rod S in Fig. 783 be supposed to have a length of 1 centimetre and an area of cross-section of 1 square centimetre If its resistance be S this is the specific resistance, or **resistivity**, of the material of the rod. The section has been drawn hexagonal to emphasise the fact that it need not necessarily be square. If l of these unit blocks be placed in series the resistance is lS (p. 848). Thus a conductor of

length l and unit cross-section has resistance $l\mathsf{S}$, where S is the resistivity of the material.

To find the effect of varying the cross-section, imagine several similar rods of unit cross-section placed in parallel. If there are 4 such rods, as in Fig. 783,

$$\frac{1}{\mathsf{R}} = \frac{1}{l\mathsf{S}} + \frac{1}{l\mathsf{S}} + \frac{1}{l\mathsf{S}} + \frac{1}{l\mathsf{S}} = \frac{4}{l\mathsf{S}},$$

or, if there are a rods, a being then the area of cross-section of the whole conductor,

$$\frac{1}{\mathsf{R}} = \frac{a}{l\mathsf{S}},$$

or

$$\mathsf{R} = \frac{\mathsf{S}l}{a}. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (7)$$

Thus, **the resistance of a uniform conductor is its resistivity multiplied by the length of the conductor and divided by the area of cross-section.** The method of measuring resistivity is given on p. 889.

TABLE OF RESISTIVITIES.

	Resistivity at 0° C.	Coefficient of increase of resistivity
Aluminium	$3{\cdot}00 \times 10^{-6}$	0·00423
Copper	$1{\cdot}59 \times 10^{-6}$	0·00428
Gold	$2{\cdot}17 \times 10^{-6}$	0·00377
Iron	$8{\cdot}85 \times 10^{-6}$	0·00625
Mercury	$9{\cdot}407 \times 10^{-5}$	0·000879
Platinum	$1{\cdot}17 \times 10^{-5}$	0·003669
Silver	$1{\cdot}54 \times 10^{-6}$	0·00400
Manganin	$4{\cdot}76 \times 10^{-5}$	0·000018
Platinoid	$4{\cdot}24 \times 10^{-5}$	0·00025
Platinum silver	$2{\cdot}26 \times 10^{-5}$	0·000344

EXAMPLE 1.—Find the resistance of a copper wire of length 180 metres and diameter 0·5 mm., given that the resistivity of copper is 0·0000016.

Area of cross-section $= \pi \times 0{\cdot}025^2$ sq. cm.;

length $= 18000$ cm.

$$\therefore \text{resistance} = \frac{\mathsf{S}l}{a}$$

$$= \frac{0{\cdot}0000016 \times 18000}{\pi \times 0{\cdot}025^2}$$

$$= \underline{14{\cdot}7} \text{ ohm.}$$

EXAMPLE 2.—On maintaining a p d. of 100 volts between the two faces of a sheet of badly conducting material it is found that a current of 0 25 ampere flows through it. Find the resistivity of the material if the area of the sheet is 120 sq. cm. and its thickness 2 mm

$$\text{Resistance of sheet} = \frac{100}{0\ 25} \text{ohms.}$$

Now, $$R = \frac{Sl}{a}, \quad \text{or} \quad S = \frac{Ra}{l},$$

$a = 120$ sq. cm. and $l = 0\ 2$ cm.

$$S = \frac{100 \times 120}{0\ 25 \times 0\ 2}$$

$$= 240000 \text{ ohms per unit conductor.}$$

Conductivity.—The reciprocal of the resistivity is called the **conductivity** of any material It is thus the conductance of a conductor, made of the material, whose length is 1 cm and area of cross-section 1 sq. cm. The conductivity of various materials may thus be calculated from their resistivities given on p 851

Electromotive force.—Up to the present, only currents in inactive conductors have been considered By an inactive conductor is meant one which carries the current but does not in any way contribute to the energy required to maintain the current. In fact energy is always dissipated in such a conductor, at a rate measured by the product of the current and the p d between the ends of the conductor. In a circuit made up entirely of such inactive conductors no current would flow To produce a current some source of electrical energy in the circuit is required, in other words, an **electromotive force** is necessary.

Since a current only flows in complete circuits, and there is a drop of potential along the conductors in which there is a current, it follows that there must be a step up in the potential somewhere, for the potential cannot drop all along a complete circuit. This step up in the potential occurs at some place, or places, where energy of some other form is converted into electrical energy. It may be due to a voltaic cell or battery, to a dynamo, or to a variation of temperature at different parts of the circuit, or to certain other causes.

In all cases, however, the total rate of performing work in the conductors comprising the circuit must be equal to the rate at which energy is given to the circuit from the source of electrical energy.

When unit current is flowing in a circuit, the rate at which electrical energy is drawn from the source and dissipated in the circuit is called the Electromotive Force (e.m.f.) in the circuit. Hence, for a circuit in which the electromotive force is e and the current i,

$$\text{Rate of working} = ei \text{ ergs per second.}$$

Or, in practical units,

$$\text{Rate of working} = \mathsf{EI} \text{ watts.} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (8)$$

Thus, electromotive force is measured in the same units as potential difference, that is, the practical unit is the volt. The distinction between the two quantities may be made clearer by considering the hydraulic analogy (p. 841) to apply to a complete circuit. Let the horizontal tube (Fig. 780) be bent round and joined upon itself so that the water in it forms a complete circuit. There will then be no flow, as there is no source of energy in the circuit. Now imagine a pump, centrifugal or otherwise, introduced at some point of the circuit. The rate of supply of energy by the pump to the water in the tube in maintaining a circulation corresponding to unit current, is analogous to the electromotive force in the electrical circuit. If any two points in the circuit be considered, there is a difference of pressure between them, and this difference of pressure represents, for unit flow, the rate of dissipation of energy between these two points. This is analogous to the potential difference between two points in a conductor in the case of an electric current.

Ohm's law applied to a whole circuit.—Consider a circuit consisting of a number of conductors in series, say R_1, R_2 and R_3. Let I be the current in the circuit, and E the electromotive force at some place in the circuit, maintaining the current. Then, for the whole circuit, the rate of working is EI watts. Again, for the separate conductors, the rates of dissipation of energy are $\mathsf{I}^2\mathsf{R}_1$, $\mathsf{I}^2\mathsf{R}_2$ and $\mathsf{I}^2\mathsf{R}_3$ watts (p. 841).

Hence, $$\mathsf{EI} = \mathsf{I}^2\mathsf{R}_1 + \mathsf{I}^2\mathsf{R}_2 + \mathsf{I}^2\mathsf{R}_3,$$

or, $$\frac{\mathsf{E}}{\mathsf{I}} = \mathsf{R}_1 + \mathsf{R}_2 + \mathsf{R}_3. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (9)$$

Thus, for the whole circuit we might say that the ratio of electromotive force to current is constant, and this constant is identified as the total resistance of the circuit.

Or, for practical purposes, we may use the equation

$$\frac{E}{R_1 + R_2 + R_3} = I, \quad \ldots \ldots \ldots \ldots \ldots \ldots \quad (10)$$

to calculate the current in any circuit when we know the electromotive force in it and the resistances of its several parts.

Diagram of circuit.— The various quantities applying to an electrical circuit may be represented conveniently upon a diagram of potential differences and resistances. Thus, let the circuit (Fig. 784 (*a*)) consist

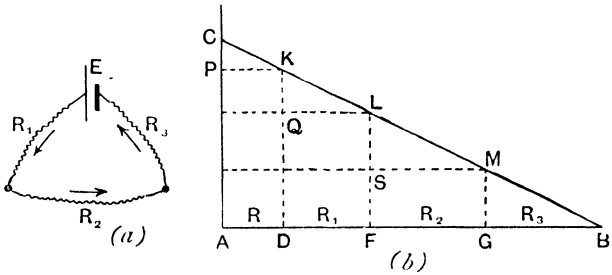


FIG 784.—Diagram of an electrical circuit.

of a cell of electromotive force E and internal resistance R, in series with resistances R_1, R_2 and R_3. Set off to scale along AB (Fig. 784 (*b*)) the resistances in order, and the e.m.f. E along AC. Then the current in the circuit is $\frac{E}{R + R_1 + R_2 + R_3}$. Join the points C and B, then the current I in the circuit is represented by CA/AB.

Note also that MG represents the p.d. between the ends of the resistance R_3, for

$$MG = R_3(CA/AB)$$
$$= R_3 I.$$

Similarly, LS and KQ represent to scale the p.d.'s between the ends of the resistances R_2 and R_1 respectively.

Note that CP does not represent the p.d. between the terminals of the cell. It is the p.d. corresponding to the resistance of the cell and the current; but, between the terminals, the source of electromotive force exists. The p.d. between the cell terminals is represented on the diagram by AP, or DK, because this is the p.d. for the external resistance $R_1 + R_2 + R_3$ and this is not complicated by the existence of any source of electromotive force in these conductors.

Potential difference between the terminals of a cell.—It was seen on p. 854 that the electromotive force of a cell is not in general the same thing as the potential difference between its terminals. Thus, in Fig. 784 (*b*), the e.m.f. is represented by AC, and the p.d. between the terminals by DK. Consider, however, the case of a cell placed in series with a very high resistance. The p.d. between the terminals of the cell being KD, this is less than the e.m.f. of the cell by the small amount CP. On very greatly increasing the external resistance, the current becomes less, but the p.d. between the terminals of the cell becomes DK′ (Fig. 785). As the external resistance approaches infinity, the line CB′ becomes horizontal, taking the position CB″. In this limiting condition the p.d. between the terminals is DK″ and is equal to AC the e.m.f. of the cell. When the external resistance is infinite, the current is zero, and the cell is said to be **on open circuit.** Thus **when the cell is on open circuit the p.d. between its terminals is equal to the e.m.f. of the cell.**

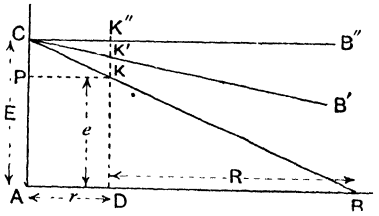


FIG. 785.—Potential difference between the terminals of a cell

Thus, if E is the electromotive force of a cell or dynamo, and r its internal resistance, R the external resistance of the circuit, and e the potential difference between the terminals of the cell or dynamo,

$$\text{Current in circuit} = \frac{E}{r+R};$$

$$\therefore \text{ p.d. between terminals} = \frac{E}{r+R}\,R = e;$$

$$\therefore ER = er + eR,$$

$$\frac{E-e}{e} = \frac{r}{R}. \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (11)$$

The voltage E of the cell may therefore be considered to consist of two parts,—e, that required to maintain the current in the external resistance R, and $E-e$, that required to maintain the current in the cell or dynamo itself. Further, these voltages are proportional to the resistances of the corresponding parts of the circuit.

Maximum current obtainable from a cell or dynamo.—Obviously the greatest current is obtainable when the resistance external to the source of electromotive force is zero. Then $I = E/r$, where r is the internal resistance.

A Daniell's cell (p. 911) has an e.m.f. of about 1·1 volts and its internal resistance is usually of the order of half an ohm. Hence

the greatest current that such a cell can produce is $1{\cdot}1/0{\cdot}5 = 2{\cdot}2$ amperes. When great currents are required, a number of these cells may be arranged in parallel. If, say, four cells be joined in parallel (Fig. 786), the combined internal resistance is only one-quarter of that of one cell, say 0·125 ohm. Hence the greatest current such a set of cells can produce is $1{\cdot}1/0{\cdot}125 = 8{\cdot}8$ amperes.

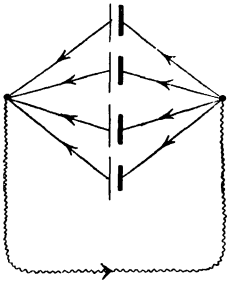


FIG. 786. Cells in parallel.

For the production of large currents by means of cells, it is usual to employ secondary cells. These have an e.m.f. of about 2·1 volts, and a very small internal resistance. For an internal resistance of 0·01 ohm, the maximum current would be 210 amperes. Such a large current is rarely necessary, and is probably not allowable, since the cell would in all likelihood be injured by it.

In the case of dynamos, the internal resistance is usually very small, so that large currents may be obtained, and the heating in the dynamo produced by this current is small.

EXAMPLE.—The terminals of a cell of e.m.f. 1·5 volts and internal resistance 3 ohms are joined by two conductors in parallel having resistances 10 and 15 ohms respectively. Find the p.d. between the terminals, and the current in each conductor.

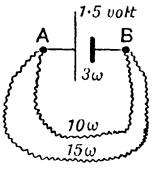


FIG. 787.

Let A and B (Fig. 787) be the terminals.

Combined external resistance, R, is given by

$$\frac{1}{R} = \frac{1}{10} + \frac{1}{15};$$

$$R = 6 \text{ ohms};$$

$$\text{Current} = \frac{1{\cdot}5}{3+6} = \frac{1{\cdot}5}{9} \text{ ampere};$$

$$\text{p.d. between terminals} = \frac{1{\cdot}5}{9} \times 6 = \underline{1} \text{ volt.}$$

$$\text{Current in 10 } \omega \text{ conductor} = \frac{1{\cdot}5}{9} \times \frac{6}{10} \quad \text{(p. 849)}$$

$$= \underline{0{\cdot}1} \text{ ampere.}$$

$$\text{Current in 15 } \omega \text{ conductor} = \frac{1{\cdot}5}{9} \times \frac{6}{15} = \underline{0{\cdot}0667} \text{ ampere.}$$

Insulation Resistance. The materials used for protecting wires, cables, and parts of electrical machinery from leakage of current from them, must have very high resistivity, and are called **insulators.**

Among these may be noticed guttapercha ($S = 2 \times 10^9$), mica ($S = 9 \times 10^{15}$), ebonite ($S = 2 \times 10^{15}$), paraffin wax ($S = 3 \times 10^{18}$), and quartz ($S = 1\ 2 \times 10^{14}$). The values of the resistivity (S) given are only approximate.

In dealing with cables it should be noticed that any leakage of current takes place through the insulation from the conductor inside to the conducting sheath outside, so that the total or effective resistance decreases as the length of the cable increases. Thus the effective insulation resistance varies inversely as the length of the cable the sections being in parallel.

Temperature Coefficient of Resistance.—It may be taken as a general rule that metallic conductors increase in resistivity with rise in temperature, while for electrolytes (Chap. LXX.), carbon, and most insulators, the resistivity falls with rise of temperature. For the metals the equation

$$R_t = R_0(1 + \alpha t + \beta t^2)$$

gives the resistance R_t at $t°$ C., in terms of the resistance R_0 at 0° C. for a very wide range of temperature, where α and β are constants for each metal. The constant β is so small that for moderate ranges of temperature, say from 0° to 100° C., it may be neglected, and the equation becomes

$$R_t = R_0(1 + \alpha t).$$

The quantity α is called the **coefficient of increase of resistivity**, and its value is given for several metals in the table on p. 851.

It will be seen later (Chap. LXXVIII.), that the resistance of a carbon filament lamp is about half of the value for the lamp when cold. The resistivity of guttapercha falls very rapidly with rise of temperature, falling about 50 per cent. in value for every rise of 5° C. at ordinary temperatures. The insulation resistance of mica changes very little with temperature.

Exercises on Chapter LXVII.

1. Explain the terms 'series' and 'parallel' in connection with electric circuits.

Show that if n similar conductors are arranged in series the combined resistance is n^2 times as great as when they are arranged in parallel.

2. Conductors of 5, 10, and 15 ohms resistance respectively are arranged in parallel. Find their effective resistance.

3. A current of 3 amperes flows through three conductors in parallel whose resistances are 1, 2, and 3 ohms respectively. Find the current in each conductor.

4. Find the resistance of the wire that must be joined in parallel with a wire of 10·5 ohms, in order that the combined resistance shall be 10 ohms.

5. *Calculate the specific resistance* of the material of a wire of diameter 0 5 mm. if a length of 6 metres of it has a resistance of 1 5 ohm.

6. Find the resistance of 20 metres of copper wire of diameter 0 1 mm.

7. Find the number of calories produced in 5 minutes in a copper wire of diameter 0·5 mm. and length 150 cm. in which a current of 4 amperes is flowing.

8. Explain what is meant by the internal resistance of a battery.
One battery A has an e.m.f. of 2 volts and an internal resistance of 1 ohm. Another battery B has an e.m.f. of $2\frac{1}{2}$ volts and internal resistance 2 ohms. A wire is found of such resistance that the current passing is the same whether it connects the poles of one battery or the other Find the current and the resistance. Sen. Camb. Loc.

9. How does the difference of potential between the poles of a battery change when the poles are connected by a wire ?
A battery whose resistance is 5 ohms and voltage 8 is put in series with a resistance of 6 ohms. Calculate the current strength and the potential difference at the poles of the battery.

10. State the law of production of heat in the circuit of a battery.
A battery has an internal resistance of 4 ohms and its poles are connected by two wires in parallel, of resistances 3 and 5 ohms respectively ; compare the quantities of heat generated in a given time in the battery and in the pair of wires

11. State Ohm's law, defining the meaning of all the electrical quantities concerned.
A battery of cells with an electromotive force of 6 volts is connected in series with a coil of wire of resistance 12 ohms. An electrostatic voltmeter connected to the battery terminals indicates only 4 volts. Explain this result, and state what information can be deduced from it. L.U.

12. Four cells each of 1 5 volts e.m.f. and 2 ohms internal resistance, are used to send a current through a single wire of 2 ohms resistance. The cells are arranged (*a*) all in series, (*b*) in two parallel groups of two in series, and (*c*) all in parallel. Calculate the current in the wire in each case. L.U.

13. A battery of 20 volts e.m.f. and 4 ohms resistance is joined in parallel with another of 20 volts and 3 ohms to send a current through an external resistance of 10 ohms.
Calculate the current through each battery. L.U.

14. Two cells of e.m.f. 1·1 volts and 1 3 volts, and internal resistances 0·4 ohm and 0 6 ohm respectively, are connected in series with a galvanometer whose resistance is 4 ohms. Calculate the current with the cells (*a*) helping, and (*b*) opposing each other.

15. Two cells, each of e.m.f. 1 5 volts and resistance 5 ohms, are joined in series with a resistance box and a resistance coil of 10 ohms. What resistance must be unplugged in the box in order that the difference of potential between the ends of the 10 ohm coil may be 0 1 volt ?
Madras University.

16. A battery of e.m.f. 3 volts and internal resistance 5 ohms drives a current through an external resistance of 30 ohms, causing a certain heating effect in the external resistance.

What alteration must we make in the external resistance in order that the heating effect in it may be double of that in the original case ?

Madras University.

17. The poles of a given constant battery are joined by a wire of 1 ohm resistance, and the potential difference between them is then 1 volt. A second wire of 3 ohms resistance is now joined in parallel with the first, and the potential difference between the poles of the battery falls to 0·9 volt.

Find the electromotive force and the internal resistance of the battery.

L.U.

18. Define the ampere, the volt, and the watt.

A battery supplies current to 250 incandescent lamps in parallel, the resistance of each lamp being 300 ohms. If the potential difference between the lamp terminals is 120 volts, but rises to 122 volts when 100 lamps are switched off, calculate the internal resistance of the battery, assuming that of the leads to be negligible. Also find the watts absorbed by each lamp in the first case, assuming that the resistance of each lamp remains constant. L.U.

19. The terminals of a battery of 3 cells connected in series, each of electromotive force 1·5 volts and internal resistance 2 ohms, are connected by two conductors in parallel whose resistances are 2 and 3 ohms respectively. Find the potential difference between the terminals of the battery, and the current in each of the parallel conductors. L.U.

20. The current for 150 incandescent lamps, each having 120 ohms resistance when running in parallel on a p.d. of 100 volts is maintained by a dynamo. If 10 per cent. of the power produced by the dynamo is wasted in the leads, what must be the output of the machine in horse-power ?

21. Give the meaning of the term ' temperature coefficient of resistance.' What is the effect of rise of temperature upon the resistivity of (*a*) copper, (*b*) manganin, (*c*) carbon, (*d*) mica, and (*e*) guttapercha ?

22. The insulation resistance of a cable between two stations A and C is 15000 ohms. If the insulation resistance between A and an intermediate station B is 20000 ohms, find the insulation resistance of the section between B and C.

CHAPTER LXVIII

GALVANOMETERS, AMMETERS, AND VOLTMETERS

Sensitiveness of galvanometers. The only galvanometer considered up to the present has been the tangent galvanometer. This is used for the measurement of current, or for the comparison of currents. Modifications, however, are necessary in order to meet the variety of purposes to which current measurers are put, and these modifications are made along two distinct lines. For some purposes it is necessary to make the instrument much more sensitive than the tangent galvanometer, as when extremely small currents are to be detected or measured. The name **galvanometer** is generally confined to this type of instrument, and the deflection does not as a rule indicate any fixed scale of current; the scale of the instrument has to be **calibrated** when necessary.

When comparatively large currents are to be measured, and rapidity of reading is desirable, instruments indicating the current directly in amperes upon a fixed scale are employed. These are called **ammeters.**

Referring to the equation for the tangent galvanometer (p. 835) we saw that

$$i = \frac{\mathsf{H}r}{2\pi n} \tan\theta, \text{ in absolute units,}$$

or,

$$\mathrm{I} = \frac{10\mathsf{H}r}{2\pi n} \tan\theta, \text{ in amperes.}$$

Thus, in order to make the instrument more sensitive, that is, to give readable deflections for smaller currents, the quantity $\mathsf{H}r/n$ must be made as small as possible. This may be done by increasing n, the number of turns of wire in the coil, by diminishing r, the radius of the turns, and by diminishing H, the controlling magnetic field. Also, improved methods of measuring θ, the deflection, are employed, so that much smaller values can be measured accurately than is possible with the pointer and scale of Fig. 791.

The term **sensitiveness**, or **sensibility**, of a galvanometer is the deflection produced by 1 micro-ampere, that is, 10^{-6} ampere. This point will be dealt with later (p. 865). The current required to produce a deflection of one scale division is called the **figure of merit** of the galvanometer

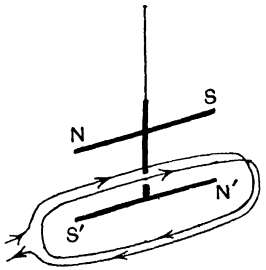


FIG. 788.—Astatic magnetic system.

Astatic couple.—In all sensitive galvanometers, the number of turns of wire in the coil is much greater than in the tangent galvanometer, and these turns are of smaller radius. There is a method of reducing the effect of the controlling magnetic field that further increases the sensitiveness. Two magnets NS and N′S′ (Fig. 788) are attached to the same vertical rigid stem, and thus form a couple which, when suspended, experiences a very small directive influence by the earth's magnetic field. If the magnets are vertically over each other and have exactly equal magnetic moments, the couples acting on them due to the earth's field are always equal and opposite, so that there is no resultant couple tending to make them turn towards the meridian. This perfect equality cannot be obtained, and, in fact, is not desirable. But when the two magnets are nearly alike, the directive effect of the earth's magnetic field is much less than when one magnet alone is used. A given current then produces greater deflection and the sensitiveness is increased. The coil may be wound so that it surrounds one of the magnets (Fig. 788), or two coils wound in opposite directions (Fig. 789) may be used. It will be seen that both coils produce deflection in the same direction, so that the sensitiveness is further increased.

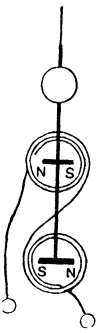


FIG. 789.—Galvanometer with astatic magnets

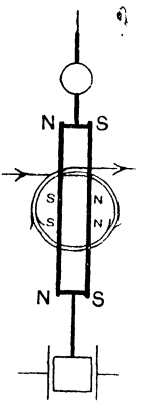


FIG. 790.—Magnetic system of the Broca galvanometer

In some modern forms of galvanometers, as in the case of the "Broca" type, an astatic arrangement of magnets is employed, but with the magnets vertical (Fig. 790). The vertical magnets

NssN and SnnS have poles ss and nn at their middle parts. The coil is arranged with these middle poles at its centre.

In addition to rendering the galvanometer more sensitive, the astatic arrangement of magnets makes it less liable to disturbances by varying external magnetic fields. This is important, as in towns, the proximity of electric trams and railways causes considerable disturbances in the magnetic field.

Controlling magnets.—The controlling magnetic field can be varied by means of a **controlling magnet** which may be attached to the instrument as in Fig 795, or in the case of a simple galvanometer (Fig. 791) may be placed on the bench near it.

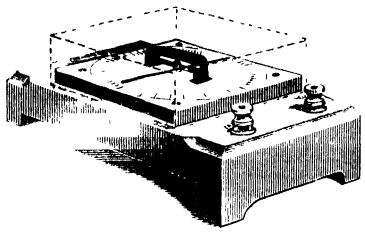


FIG 791.—Simple galvanometer

The controlling magnet has two uses. By means of it the controlling field may be strengthened or weakened. The former makes the galvanometer less sensitive and the latter makes it more sensitive. To strengthen the controlling field, the magnet is so placed that its field is in the same direction as the earth's field ; to weaken it, the magnet must be reversed pole for pole. The second use of the controlling magnet is to bring the suspended magnet into its zero position, so that the deflection is 0° when there is no current. This is effected by a slight rotation of the controlling magnet.

Calibration of galvanometer scale. It must be noted that in designing a galvanometer for increased sensitiveness the tangent law has been abandoned. The current is no longer proportional to the tangent of the deflection (compare p 835), since the coil is no longer large in comparison with the magnet When the deflection is small, say less than 5°, the current is very nearly proportional to the deflection. If, however, larger deflections are used, the scale of the galvanometer must be specially calibrated. This is done by passing a series of known currents through the galvanometer, and noting the deflection in each case The results are then plotted in the form of a graph, which is preserved for future use, and enables the current for any observed deflection to be found.

EXPT. 187.—**Calibration of a simple galvanometer** Connect a simple galvanometer G, a secondary cell E, an adjustable resistance box R (p. 880), and a reversing key ABCD, in series, with a resistance of 2100 ohms in the box (Fig. 792).

Neglecting the resistances of the cell and galvanometer, we have

$$\text{Current} = \frac{2 \cdot 1}{2100}$$
$$= 0 \cdot 001 \text{ ampere}$$
$$= 1 \text{ milliampere.}$$

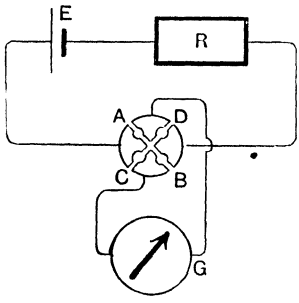


FIG 792 —Calibration of a galvanometer

Read both ends of the pointer of the galvanometer, when the plugs in the key are in A and B. Then remove the plugs to C and D; this reverses the current in the galvanometer. Read both ends of the pointer again. The mean of the four readings is the value of the deflection corresponding to a current of 1 milliampere.

Repeat with a resistance of 1050 ohms. The current is then 2 milliamperes. Continue, with the resistances given in the table, recording the readings in the form indicated below.

Resistance	Current	Readings of ends of pointer				Mean deflection
2100 ohms	1 milliampere					
1050 ,,	2 milliamperes					
700 ,,	3 ,,					
525 ,,	4 ,,					
420 ,,	5 ,,					
350 ,,	6 ,,					
300 ,,	7 ,,					
262 ,,	8 ,,					
233 ,,	9 ,,					
210 ,,	10 ,,					

The readings may be extended to values above 10 milliamperes, or below 1 milliampere, according to the sensitiveness of the galvanometer.

Plot the current and mean deflection on squared paper in the form of a graph.

EXPT. 188.—**Use of a controlling magnet.** Connect up a circuit as in Expt. 187, and adjust the resistance until a deflection of 40° is obtained. This resistance must not then be altered. Stop the current by removing one of the plugs from the key. Place a bar magnet upon the bench at a distance of 30 cm. from the needle of the galvanometer (Fig. 793) with its N pole pointing north.

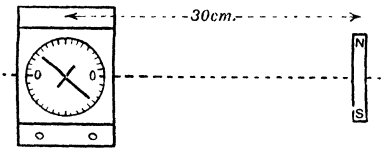


FIG. 793.—Experiment with controlling magnet.

Rotate the magnet slightly until the pointer is again at zero. Put in the plug-key and observe the deflection, taking the four readings, as in the last experiment. Then stop the current and reverse the magnet so that its S pole points north. Start the current and again observe the deflection. Repeat, with the magnet at 25 cm., then at 20 cm., and so on, continuing until, with the N pole pointing north position, the galvanometer needle swings round. For the N pole pointing south position, the readings may be continued until the magnet lies underneath the galvanometer. Readings should be recorded as in the last experiment.

Draw two graphs connecting deflection and distance of magnet. Note that the current has been the same throughout, so that for one direction of the magnet, the galvanometer becomes more sensitive as the magnet approaches; for the other direction it gets less sensitive.

Mirror or reflecting galvanometer.—For measuring deflection, the pointer (Fig. 791) is very clumsy, the error of observation being of the order of half a degree. This error is at least 1 per cent, and may be a higher percentage of the deflection. It may be greatly reduced by the following modification A small mirror M, usually concave, of 1 metre radius of curvature, is attached to the suspended system of magnets (Fig. 789). A beam of light from the lamp L falls upon M and is reflected to the scale S. Figure 794 is partly diagrammatic, the supports of the lamp and scale not being shown These vary a great deal in design, but the essentials are nearly constant in type. On the front lens L is a vertical scratch, and this being at a distance of 1 metre from the mirror M an image of the lens and scratch is produced upon the semi-transparent scale S, which is also at a distance of a metre from M. The lens L merely serves as a condenser for the light from the incandescent electric lamp situated within the holder.

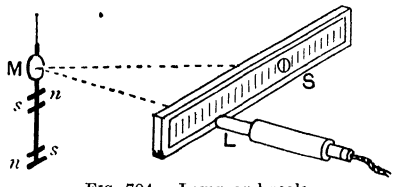


FIG 794.—Lamp and scale

The advantages of this optical method of measuring the deflection are fourfold: (i) the pointer, being a beam of light, is weightless, and therefore does not add to the inertia of the suspended system, or to the strength necessary in the suspending fibre; (ii) the angular deflection of the beam of light is twice that of the mirror (p. 557), thus doubling the observed deflection; (iii) the beam of light is

much longer than is allowable for a mechanical pointer; and (iv) there is no parallax to avoid, the image falling upon the scale itself.

A good type of **reflecting galvanometer** is shown in Fig. 795.

In adjusting the lamp and scale for use, it is first necessary to direct the beam of light upon the mirror; then to withdraw, or push in, the lens in its sliding tube, until the spot is sharply focussed upon the scale; and lastly by means of the controlling magnets, or otherwise, to bring the spot of light to the middle of the scale. The scale itself should be symmetrically situated with respect to the galvanometer and beam of light. If it is not situated with its zero on the normal to the face of the galvanometer, or is not itself in a plane at right angles to the normal, the deflections on opposite sides of zero, for the same current will be unequal. Hence, it is necessary to pass a small current through the galvanometer and to observe the deflection; then to reverse the current and observe the deflection on the other side of zero. If these are not equal, adjust the scale until, on making the two observations, the deflections are equal.

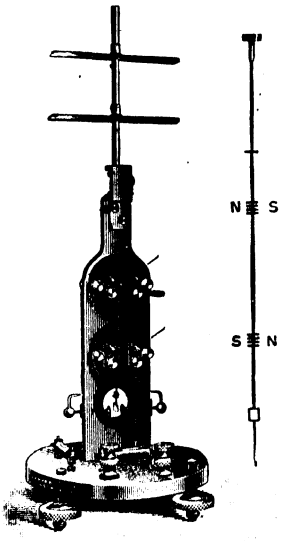


Fig. 795. Reflecting galvanometer.

The scale is generally divided in millimetres, the convention being to express the sensitiveness of a galvanometer in terms of millimetres deflection with the scale at a distance of 1 metre from the mirror, for a current of 1 micro-ampere. Or, the **figure of merit** is the current in micro-amperes that will produce a deflection of 1 millimetre with the scale at a distance of 1 metre. It does not follow that the galvanometer with the highest sensitiveness, expressed in this way, is always the best. The time of swing of the suspended system and the resistance of the galvanometer have also to be considered; but these considerations are too complicated for a full discussion here.

A millimetre deflection, with the scale at 1 metre, corresponds to an angular deflection of 0·001 radian, or 0·0573 degree of the beam of light, or half this, that is 0·0286°, for the suspended system. Readings with a pointer cannot be determined accurately to less than 1°, and thus the accuracy of observation has been increased 1/0·0286, or roughly 30 times, by substituting the beam of light

for the pointer. This is quite distinct from any accuracy gained in the design of the electrical parts of the galvanometer.

EXPT. 189.—**Sensitiveness of a reflecting galvanometer** After adjusting the lamp and scale, as described on p 865, connect the reflecting galvanometer G in series with a Daniell's cell E, and a high resistance R (Fig 796), which should include coils of at least a million ohms. With plugs in A and B of the reversing key, adjust the resistance until about 10 mm. deflection is produced Reverse the current by placing the plugs in C and D, and observe the reverse deflection. Repeat with other resistances and record in a table, for each case, the resistance, the current in micro-amperes calculated from the relation I 1000000E R, taking the e.m.f. of the Daniell's cell as 1 1 volts, also the deflections and mean deflection. Plot in the form of a graph the current and mean deflection, and from the graph obtain the figure of merit and the sensitiveness of the galvanometer. If the resistance of the galvanometer be known, the p.d in microvolts between its terminals corresponding to deflection of one division may also be found.

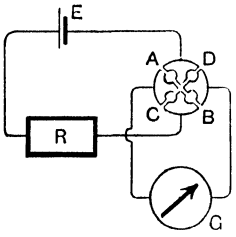


FIG 796 —Determination of the sensitiveness of a galvanometer

Suspended coil galvanometer. - In recent years suspended magnet galvanometers have been largely replaced by those of the suspended coil type. In Chap LXXV it will be shown that when a coil carrying a current is suspended in a magnetic field, it experiences a couple whose magnitude is proportional to the current. The coil CC (Fig. 797) consists of a number of turns of fine wire and is usually rectangular, but sometimes circular, in shape. It is suspended between two massive soft iron pole pieces N and S fixed to a permanent magnet which is commonly of the horse-shoe type. The coil is suspended by a fine phosphor-bronze strip F, which also serves to bring in the current, which leaves by the loosely coiled phosphor-bronze strip G When the current passes, the couple rotates the coil until the opposite couple due to the twist in the strip

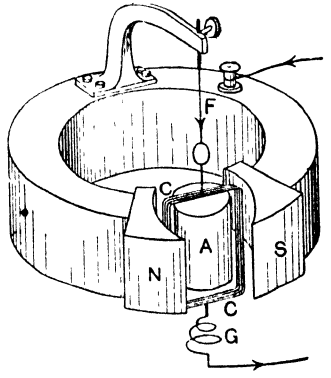


FIG 797 —Suspended coil galvanometer.

F brings it to rest. The control in this case is therefore mechanical, and not magnetic, as in the suspended magnet galvanometer.

The ends of the soft iron pole pieces are hollowed out and between them is situated a soft iron cylinder A (Fig. 798). Thus the sides of the suspended coil move in the cylindrical space between the cylinder and the pole pieces. In this space the magnetic field is radial, and as the coil rotates, the sides of the coil remain situated in a magnetic field of constant strength. With this arrangement the couple, and therefore the current, is proportional to the deflection. Hence such galvanometers, if properly designed, have a **linear scale**, that is, the deflection is proportional

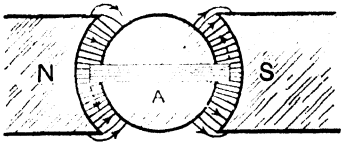


FIG. 798.—Magnetic field of the suspended coil galvanometer.

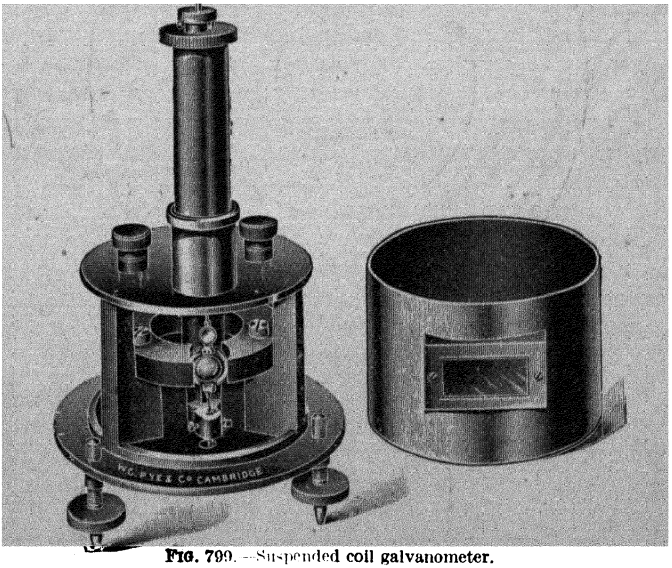


FIG. 799.—Suspended coil galvanometer.

to the current over quite a large range. Without the cylinder A the field would no longer be radial, so that when the coil rotates it would be differently situated with respect to the magnetic field. The law connecting deflection and current would then be more complex.

In Fig. 799 a suspended coil galvanometer is illustrated whose

form is similar to that of Fig. 797, the chief difference being that one coil is circular while the other is rectangular. Another type

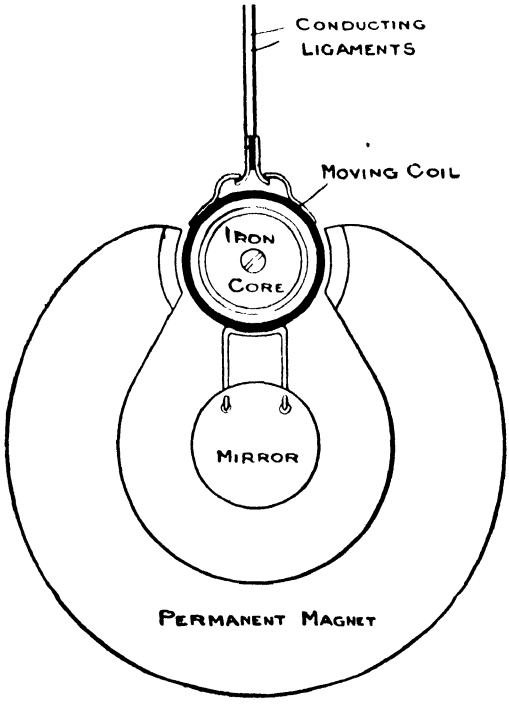


FIG. 800.—Suspended coil galvanometer with bifilar suspension

of suspension is shown in Fig. 800. The magnet is vertical and the coil is circular. The most important difference between this and the previous type is in the suspension, which is bifilar. The current comes down one phosphor-bronze strip, passes round the coil, and goes up the other strip.

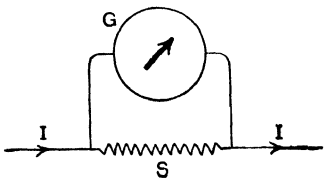


FIG. 801.—Galvanometer shunt.

Shunts.—Another method of varying the sensitiveness of a galvanometer, according to the use to which it is to be put, consists in placing a conductor in parallel with it to carry part of the current. Such a conductor is called a **shunt**. If S

be the resistance of the shunt (Fig. 801) and G that of the galvanometer, the current I in the main circuit divides into two. If the current in the galvanometer is I_G, and that in the shunt I_S, then,

$$I_G = I\frac{S}{G+S}, \quad \text{and} \quad I_S = I\frac{G}{G+S}$$

With the older types of galvanometer, shunts were supplied by the makers, frequently in boxes of three, and having resistances $\frac{1}{9}$, $\frac{1}{99}$, and $\frac{1}{999}$ of that of the galvanometer. By placing the plug in the suitable place the corresponding shunt is used.

Thus with the resistance $S = G/99$; the current in the galvanometer is given by

$$I_G = I\frac{\frac{G}{99}}{G+\frac{G}{99}} = \frac{I}{100}.$$

One-hundredth of the main current now flows in the galvanometer and the sensitiveness is thus reduced to one-hundredth of its value when unshunted. Similarly, with the resistance G/9 its sensitiveness is one-tenth and with G/999, one-thousandth.

Universal shunt.—Owing to the inconvenience of requiring a box of shunts for each galvanometer, the **universal shunt** is now generally employed Its advantage lies in the fact that it can be attached to any galvanometer. This instrument consists of a very high resistance, AB (Fig. 802), which is connected to the terminals of the galvanometer. AB should have at least 100 times the resistance of the galvanometer so that it does not appreciably reduce its sensitiveness. The main current enters at A, and if it leaves at B, the current in the galvanometer is IR/(G + R). If now the point at which the current leaves be transferred from B to C, the resistance of AC being R/n, the two circuits in parallel between A and C have resistances of R/n and (G + R − R/n) respectively. Hence the current in the galvanometer is given by

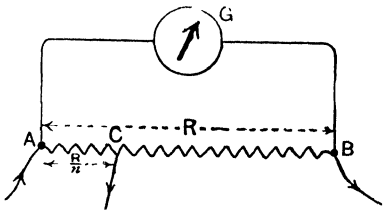


FIG. 802.—Universal shunt.

$$I_G = \frac{\frac{R}{n}}{\left(G+R-\frac{R}{n}\right)+\frac{R}{n}} = I\frac{R}{n(G+R)}.$$

That is, by transferring the external connection from B to C the current in the galvanometer is reduced to one-nth of its original value

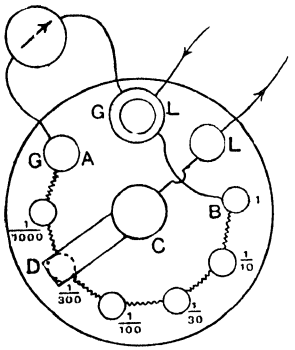


FIG. 803.—Universal shunt box.

By fixing a number of points on AB corresponding to C, having different values for n, the shunt may be used to give corresponding degrees of sensitiveness to the galvanometer. In Fig 803 the connections of a common form of universal shunt are given, where the current may be $\frac{I}{1000}, \frac{I}{300}, \frac{I}{100}, \frac{I}{30}, \frac{I}{10}$ or I, according to which of the terminal blocks the rotating arm CD is brought into contact with

It must be remembered that if the shunt be changed during an experiment, the effective resistance of the circuit is changed. This, however, is not usually of great importance, and may be allowed for, or compensated by extra resistances in series, if necessary.

Ammeters.—The distinction between a galvanometer and an **ammeter**, or amperemeter, is that the latter is provided with a fixed scale calibrated to read current directly in amperes, or some simple fractions of an ampere. Most **milliammeters** are moving coil galvanometers with a pointer moving over a scale calibrated to read thousandths of an ampere. A typical arrangement is illustrated in Fig 804. The permanent magnet, with its pole pieces and the soft iron cylinder, is arranged exactly as described on p. 866 for the galvanometer. The coil, however, is pivoted between needle points, and the control is exerted by the spiral spring S, which in some instruments also serves as one of the leads for bringing the current to the moving coil. The long, light,

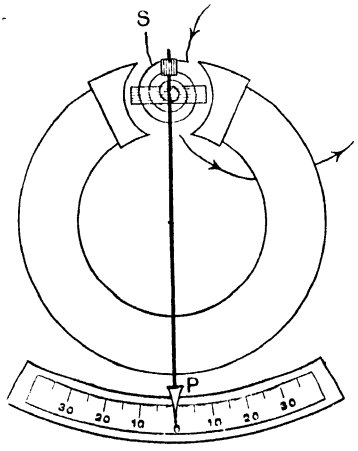


FIG 804.—Milliammeter

balanced pointer P is carried by the moving coil, and serves to indicate the deflection upon the scale. The scale is so graduated

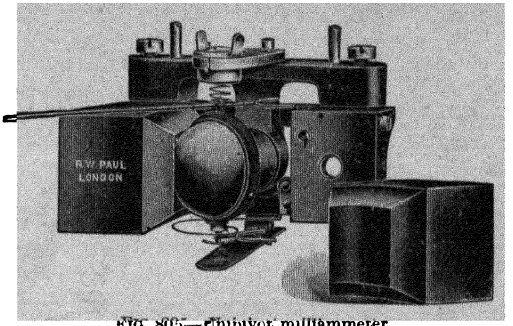


FIG. 805.—Unipivot milliammeter.

that the readings indicate milliamperes. In the form shown, the zero is at the middle of the scale, so that currents, in either direction, up to 35 milliamperes can be measured. An excellent form of

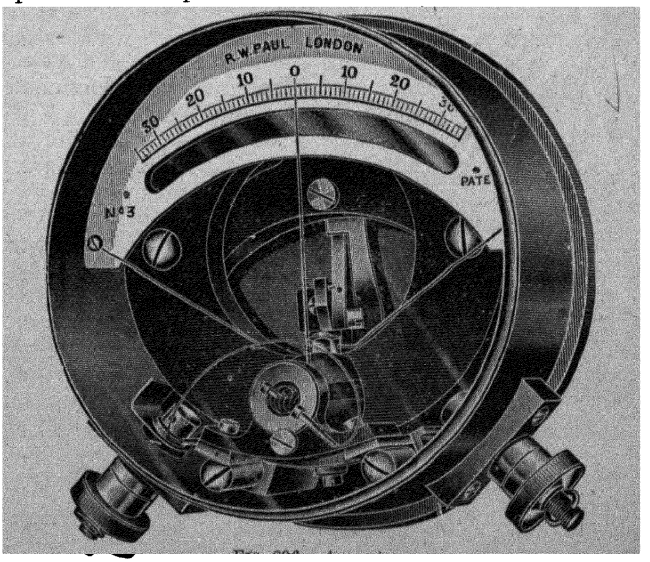


FIG. 806.—Ammeter.

suspension is given in Fig. 805, where the actual suspension is made by a needle point resting upon an agate cup situated at the centre of a soft iron sphere. The coil in this case is circular instead of

rectangular. As there is only one point of suspension this is called the **unipivot** form

The same form of instrument is employed for measuring greater currents, but the main current is carried by a shunt, the resistance of the shunt being so adjusted that the scale reads directly in amperes.

If the resistance of the milliammeter be, say, 10 ohms, and it is required that the given scale shall give amperes instead of milliamperes, the shunt must carry $\frac{999}{1000}$ of the main current while the coil carries $\frac{1}{1000}$. Hence the shunt must have $\frac{1}{999}$ of the resistance of the coil which in this case will therefore be $\frac{10}{999}$ ohm. This method of determining the range of the instrument by means of a shunt is convenient, as it enables the galvanometer part to be constructed of some standard form, the only variation in pattern being that of the shunt, which is usually placed inside the case of the instrument. An ammeter of this type, designed to read up to 35 amperes, is shown in Fig. 806.

Hot-wire ammeter. The galvanometer type of ammeter described above depends upon the mutual action of an electric current and a magnetic field. There are also **hot-wire ammeters** which are designed to make use of the heating effect of a current, and these may be calibrated so that the numbers on the scale give amperes direct. The main current I passes through a shunt S (Fig. 807) placed in parallel with a fine platinum-iridium wire W, through which a small fraction of the current passes. A fibre is attached at A, and after taking a turn round the axle B of the pointer, is attached to a stretched spring C. This keeps all the wires taut. On the current passing, W is heated and therefore expands. Owing to its sagging, the spring C is able to pull the fibre CBA forward and in so doing rotates the axle B, thus moving the pointer over the scale. The value of the scale deflections must first be fixed by comparison with some standard ammeter, so that they will afterwards indicate amperes. When small currents are to be measured, that is, currents below 0·3 ampere, the shunt may be dispensed with, but for higher reading instruments the shunt is necessary.

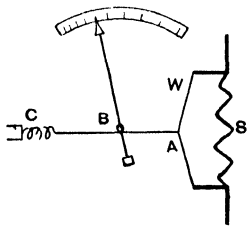


FIG. 807.—Hot-wire ammeter.

Hot-wire instruments are very liable to change of zero due to slipping of the axle and fibre, and also to change of temperature of the frame. The latter is usually compensated as well as possible by constructing the framework which supports the wire W of such material, or materials, that its coefficient of expansion shall be the same as that of the wire. Change of temperature of the entire

instrument does not then tend to tighten or slacken the wire. For some purposes the hot-wire ammeter has this advantage, that owing to the heating in the wire being independent of the direction of the current, the instrument will give a reading when the current is alternating. Owing to the rapid reversals in direction, an alternating current would not produce any deflection upon the galvanometer type of ammeter Its value, as indicated by the hot-wire ammeter, is called the **virtual current.**

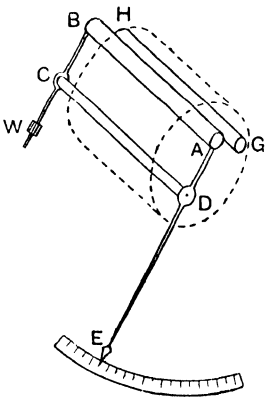


FIG 808 —Soft iron ammeter of the repulsion type

Soft iron ammeters. Another type of ammeter frequently used is that in which soft iron is magnetised by the current passing in a coil or solenoid. There are two forms of **soft iron ammeters.** In one of them (Fig 808) two parallel soft iron rods AB and GH are magnetised by the current flowing in the solenoid, to the axis of which they are parallel. To prevent confusion, this solenoid is only represented by a dotted outline in the diagram Whatever may be the direction of the current in the solenoid, the poles at A and G are of the same kind, and there is a repulsion between them. Similarly, there is repulsion between B and H. These repulsions rotate the framework ABCD which is pivoted on jewelled points at C and D. The pointer DE then indicates the current upon the scale E, whose divisions are fixed by comparison with some standard ammeter. The control in this case is due to gravity, the small weight W being adjusted in position until the pointer is on the zero of the scale when there is no current.

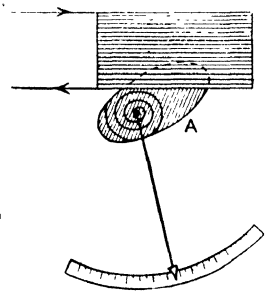


FIG. 809 —Soft iron ammeter

Another form of the soft iron instrument is shown in Fig. 809. An oval-shaped piece of soft iron sheet A is pivoted at a point to the left of the axis of the solenoid carrying the current. When the current flows, the soft iron is magnetised in such a manner that the force between it and the coil tends to draw the iron into the coil. As A is mounted eccentrically, this force causes it to rotate. It is shown with a spring control in Fig. 809, but some forms of the instrument have a gravity control, as in Fig. 808. The coil

of potential of 120 millivolts between the terminals gives a deflection from one end of the scale to the other. What resistances must be provided as shunts in order that the instrument may register currents (*a*) from 0 to 5 amperes, (*b*) from 0 to 100 amperes? L.U.

4. You are provided with a sensitive galvanometer of 600 ohms resistance, and are asked to measure a current, the strength of which is known to be not more than three amperes. If the largest current that can be indicated by the unshunted galvanometer is 1/1000 ampere, what resistance would you apply as a shunt? If the shunted galvanometer then indicates a current of 0·0009 ampere, what is the actual current in the main circuit? L.U.

5. A galvanometer whose resistance is 300 ohms is so shunted that only 0·01 of the total current flows in it. What is the resistance of (*a*) the shunt, and (*b*) of the galvanometer and shunt combined? Allahabad University.

6. Describe a method of measuring the resistance of a battery.
When two batteries, A and B, are joined in turn to a galvanometer it is found that A gives the greater current, but when another galvanometer is employed B gives the greater current. Explain how this may occur. L.U.

7. Describe any form of galvanometer suitable for detecting very small electric currents, explaining particularly the way in which the sensitiveness is secured. L.U.

8. A galvanometer having a resistance of 40 ohms gives a deflection of one scale division for a current of 1/1001 ampere. Find the magnitude of the resistance required, and show how it must be connected, to change the galvanometer into: (*a*) an ammeter reading 1 ampere per scale division, (*b*) a voltmeter reading 1 volt per scale division. Bombay University.

9. A current circuit consists of a cell of e.m.f. 1·5 volts and internal resistance 1 ohm, a coil of resistance 10 ohms, and a galvanometer of resistance 50 ohms in series. What is the current through the galvanometer? What is the current in the galvanometer when a shunt of 5 ohms is connected across its terminals? L.U.

10. An instrument of resistance 4·5 ohms reads milliamperes. Find what resistance must be placed (*a*) in series with it in order to convert it into an instrument reading volts, and (*b*) the resistance in parallel with it in order that it shall read amperes.

11. An instrument of resistance 15 ohms, gives a reading of 25 when a current of 32 milliamperes flows in it. Find the resistance that must be placed in series with it in order to convert it into a voltmeter.

12. Describe the construction of a moving coil galvanometer and explain its action.
How can it be adapted—
(i) for use as a voltmeter?
(ii) for measuring currents of widely varying magnitude?

13. Describe the hot wire ammeter, and also some method for adapting it for use as a voltmeter.

14. Mention the principles on which the action of different types of voltmeters is based, and state the conditions for which each type is most suitable. C.G.

15. Describe, with sketches, some form of voltmeter suitable for a 220-volt continuous current circuit. Show how this voltmeter, and also an ammeter, would be connected to this circuit. C.G.

CHAPTER LXIX

MEASUREMENT OF ELECTROMOTIVE FORCE AND RESISTANCE

Resistance by ammeter and voltmeter.—The most direct method of measuring the resistance of a conductor is to determine the potential difference between its ends by means of a voltmeter, while the current in it is observed by means of an ammeter This method, although not capable of great accuracy, is rapid and convenient The only difficulty likely to arise is due to the fact that instruments of suitable range may not be available For a conductor such as an incandescent lamp, on a 100 volt supply, the voltmeter must, of course, have a range of 100 volts and the ammeter should read up to 2 or 3 amperes. For conductors through which heavy currents must not be passed, milliammeters and millivoltmeters will be required

EXPT. 190.—**Resistance of an incandescent lamp.** Connect the lamp in series with an ammeter A (Fig. 813), including a rheostat or adjustable resistance R in the circuit. Connect a voltmeter V across the lamp terminals. First with the resistance of R zero, read the current and p.d. Then increase R, and again read the current and p.d. Continue the process until the lamp no longer emits light. Tabulate the results, making four columns, one each for the current, the p d., the power in watts and the resistance in ohms, remembering that watts = p.d. × current, and ohms = p.d. current. Plot a graph showing the relation between watts and ohms.

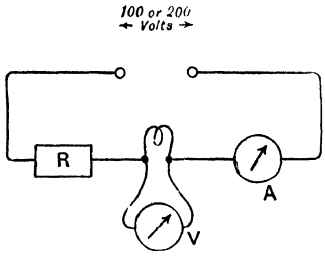


FIG 813.—Measurement of resistance

Repeat the experiment with another lamp. If the first one was a carbon filament lamp, now use a metal filament lamp, and *vice versa*.

EXPT. 191.—**Resistance of a coil.** Join the coil in series with a Daniell's cell and a milliammeter. Connect a millivoltmeter to the ends of the coil as in Fig. 813. Using the readings of the instruments, calculate the resistance of the coil from the relation $R = E/I$. Note that if the ranges of the instruments are unsuitable, an extra resistance may have to be included in series with the coil.

Standard resistances.—For the purposes of electrical measurement it is necessary that the instruments used should all be compared with some fixed standards. The determination of these fixed standards in accordance with the definitions in Chapter LXVI is not an easy matter, and its description is beyond the scope of this book. From experiments it has been found that the ohm is the resistance of a column of mercury of length 106·300 cm., of uniform cross-section and having a mass of 14·4521 grams, when its temperature is 0° C. This corresponds to a cross-section of 1 square millimetre; but since the mass of the mercury is much more easily measured with accuracy than the bore of the tube in which it is contained, the mass rather than the cross-section is defined. This definition of the ohm has been rendered legal in this country.

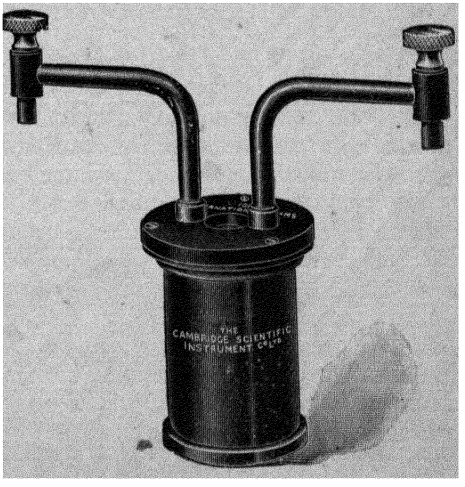


FIG. 814.—Standard ohm coil.

For practical use, several makers supply 1 ohm standards which have been standardised at the National Physical Laboratory. One form of standard ohm is shown in Fig. 814. The wire comprising the conductor is of platinum-silver and is inclosed in a brass case. The ends of the wire are soldered to stout brass terminals which are well amalgamated at the ends; when in use they rest in mercury cups. This makes certain that the resistance of the contacts is very small. A hole running down the axis is provided, for the insertion of a thermometer. The whole is then inserted in an oil bath to ensure constancy of temperature. The certificate issued with the coil states the temperature at which it is correct.

Resistance boxes. -It would be very inconvenient in experimental work if the resistances employed had to be made up of coils of the form shown in Fig 814. They are therefore joined together in boxes. Each coil is wound upon a bobbin B (Fig 815) attached to an ebonite sheet. The wire is first doubled and then wound upon the bobbin, the free ends being soldered, one to each of the brass blocks Between each pair of brass blocks a hole, slightly conical, is bored, into which fits a brass plug attached to an ebonite handle D. When all the plugs are out, the coils are all in series and the total resistance in the box is the sum of the separate resistances of the coils. By inserting any plug, the corresponding coil is short circuited, so that when all the plugs are in, the total resistance is practically zero By making the coils to have respectively 1, 2, 3, 4, 10, 20, 30, 40 100, 200, 300, 400, 1000 2000, 3000 and 4000 ohms, any resistance from 0 to 11110 may be used. In some cases fractions of an ohm are also supplied In Fig 816 is shown a view of such a resistance box. There are many forms of resistance box

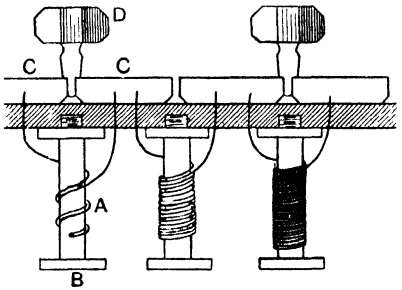


FIG. 815.—Arrangement of the coils in a resistance box

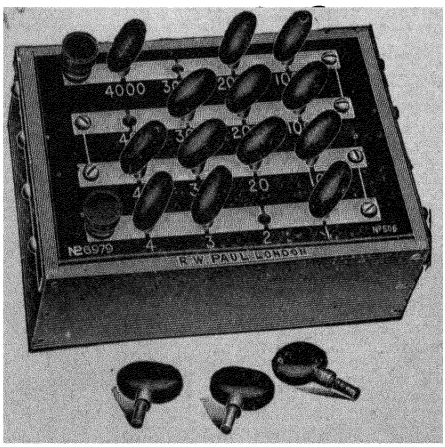


FIG. 816 —Resistance box.

Comparison of electromotive forces.- The simplest method of comparing the e.m f.'s of two cells is to find the current which each would produce in a given circuit On placing one of the cells of e m f E_1, in series with a resistance box R and a galvanometer G (Fig 817) the current is $E_1/(G+R)$, and the deflection observed may be written θ_1 On replacing the cell E_1 by another cell whose e m f. is E_2, the current is $E_2/(G+R)$ and the deflection is θ_2. If the deflections are proportional to the

currents, $E_1/E_2 = \theta_1/\theta_2$, or if a tangent galvanometer be employed, $E_1/E_2 = \tan\theta_1/\tan\theta_2$. Or, again, if a simple galvanometer, which has previously been calibrated, is used, the currents may be taken from the calibration curve (p. 863) and then $E_1/E_2 = I_1/I_2$.

Another method is to obtain the same deflection with each cell in turn, by varying the resistance R. Then, since the current in each case is the same, $E_1/(G + R_1) = E_2/(G + R_2)$. When R_1 and R_2 are great in comparison with the resistance G of the galvanometer,

$$\frac{E_1}{E_2} = \frac{R_1}{R_2}.$$

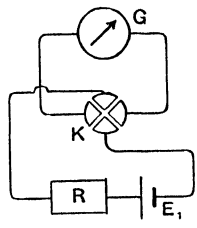


FIG. 817.—Comparison of e.m.f.'s

EXPT. 192.—**Comparison of e.m.f.'s (constant resistance).** Connect the two cells to be compared, in turn, in series with a galvanometer and resistance box, using a reversing key K (Fig. 817), so that the current in the galvanometer may be reversed. Adjust R, when the cell is a Daniell's cell, until the deflection is a reasonable amount, and read the galvanometer deflection as in Expt. 187. Replace the Daniell's cell by another cell, say a Leclanché cell, and again read the deflection. If the galvanometer is of the tangent form, calculate the ratio of the e.m.f.'s from the relation $E_1/E_2 = \tan\theta_1/\tan\theta_2$. If it is a calibrated simple galvanometer, find the currents I_1 and I_2 corresponding to the deflections θ_1 and θ_2 (p. 863) and use the relation $E_1/E_2 = I_1/I_2$. Taking the e.m.f. of the Daniell's cell as 1·1 volt, find the e.m.f. of the other cell.

EXPT. 193.—**Comparison of e.m.f.'s (constant deflection)** Connect up the circuit as in Expt. 192, noting the deflection and recording the resistance R_1. Replace the cell by the other with which it is to be compared, and adjust the resistance until the same deflection as before is obtained. Calling this resistance R_2, we have

$$\frac{E_1}{E_2} = \frac{R_1}{R_2}.$$

Sum and difference method.—In both the above methods, the resistances of the cells and of the galvanometer have been assumed to be negligible. A method which is independent of these resistances consists in joining the two cells in series with the galvanometer and resistance box, (i) both acting in the same direction, and (ii) acting in opposition. The resistance in the circuit is then the same in both cases, so that

$$\frac{E_1 + E_2}{E_1 - E_2} = \frac{I_1}{I_2},$$

or

$$\frac{E_1}{E_2} = \frac{I_1 + I_2}{I_1 - I_2}.$$

With the tangent galvanometer this equation becomes

$$\frac{E_1}{E_2} = \frac{\tan\theta_1 + \tan\theta_2}{\tan\theta_1 - \tan\theta_2}$$

EXPT. 194.—**Comparison of e m f's (sum and difference)** Connect two cells, say a Daniell's and a dry cell, in series with a tangent galvanometer and a resistance box, employing a reversing key as in Fig 817. Find the mean deflection, reading both ends of the pointer with the current first in one direction and then in the other. Let the mean deflection be θ_1. Reverse the Daniell's cell, and repeat the experiment, obtaining the deflection θ_2. Then,

$$\frac{E_1}{E_2} \quad \tan\theta_1 + \tan\theta_2$$
$$\tan\theta_1 - \tan\theta_2$$

Taking the e.m.f of the Daniell's cell as 1 1 volt, find that of the dry cell.

Comparison of resistances. – A method closely resembling that of Expt. 192 may be used for comparing resistances, provided that these are not small. The circuit is made up as in Fig 817 but with the unknown resistance R_1 in series, R being at first zero The deflection is noted, and the unknown resistance is removed The coils in the box R are then unplugged until the deflection, and therefore the current, is the same as before The e m f being constant, the total resistance is therefore the same as before; hence the unknown resistance removed from the circuit must be equal to the resistance due to the standard box

For the measurement of very high resistances, of the order of several million ohms, another substitution method is frequently employed The resistance is placed in series with a sensitive reflecting galvanometer and a cell, and the deflection observed The unknown resistance R_1 is then replaced by a standard resistance of 100000, or 1000000 ohms, R_2, and the deflection again observed Since the deflections are proportional to the current, and the current is inversely proportional to the resistance,

$$\frac{R_1}{R_2} = \frac{\theta_2}{\theta_1}.$$

With these high resistances, it is clear that the resistances of the cell and galvanometer may certainly be ignored.

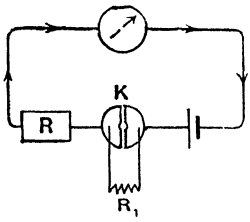


FIG. 818.—Measurement of resistance

EXPT. 195.—**Measurement of resistance (simple substitution)** Connect the resistance box R, the galvanometer and cell in series (Fig. 818). Join the unknown resistance R_1, which may be an incandescent lamp (cold), or a resistance coil, to the terminals of the plug key K Observe the deflection when all the plugs are

in the resistance box R so that its resistance is zero. If the deflection is too great, it may be reduced by placing a piece of platinoid wire across the galvanometer terminals as shunt (p. 868) or by placing a bar magnet as a controlling magnet near the galvanometer (Expt. 188, p. 863). Now place the plug in the key K, to short circuit the resistance R_1. Remove plugs from the box R until the original deflection is restored. The resistance of the box is now equal to the resistance of R_1.

EXPT. 196.—**To make a high resistance.** Obtain a piece of ground glass about 2 cm. × 10 cm. Make a thick black coating of graphite over a considerable area at each end by rubbing with a lead pencil, and then draw a firm line AB (Fig. 819) with the pencil, to connect the blackened areas. Upon each blackened area lay a piece of tinfoil, and upon this a piece of fine copper wire C bent into a flat spiral, to serve as a conductor to bring the current to the conducting line AB. Place a piece of plain glass upon the ground glass and clamp the two together. After testing to see whether the carbon line has a suitable resistance (Expt. 197) the edges may be cemented with sealing wax to make the arrangement permanent.

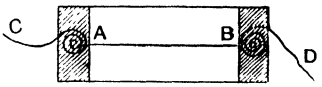


FIG. 819.—High resistance

EXPT. 197.—**High resistance (simple substitution).** Make a circuit of the high resistance R made in Expt. 196, a cell E_1, and a reflecting galvanometer G, employing the reversing key K (Fig. 817). Observe the deflection, reverse the current, and again observe the deflection; take θ_1 to be the mean of the two. Replace R by a standard megohm (10^6 ohms) and repeat, taking the mean deflection to be θ_2.

Then,
$$\frac{R}{10^6} = \frac{\theta_2}{\theta_1}$$

or
$$R = \frac{\theta_2}{\theta_1} \times 10^6 \text{ ohms.}$$

Resistance of galvanometer. The most satisfactory method of measuring the resistance of a galvanometer is to clamp the moving part, and treat the instrument as an ordinary conductor, measuring its resistance by the method on p. 888 or that on p. 889. There are, however, several methods of finding the resistance, in which no other galvanometer is used.

With an instrument of resistance 5 to 20 ohms, a simple circuit is made of the galvanometer, a resistance box and a Daniell's cell (Fig. 817). The deflection θ_1 being observed, with 10 ohms resistance in the box, this is now changed to 20 ohms and θ_2 observed. Then, since the e.m.f. in the circuit is the same in both cases,

$$\frac{G+10}{G+20} = \frac{\theta_2}{\theta_1},$$

from which G can be found. The resistance of the cell has been neglected; but in the case of a Daniell's cell, if this is not known accurately, it may be taken as about 1 ohm. The value of G obtained will, of course, include the battery resistance. It will be seen that this method is not of great precision.

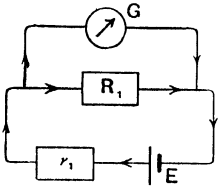


FIG. 820.—Measurement of galvanometer resistance.

When the resistance of the galvanometer is over 20 ohms the following method may be used. A high resistance r_1 (Fig. 820) is placed in series with the cell and a moderate resistance R_1 in parallel with the galvanometer. The current in the battery circuit is then

$$I_B = \frac{E}{r_1 + \frac{R_1 G}{R_1 + G}},$$

and that in the galvanometer is obtained by multiplying this by $\frac{R_1}{R_1 + G}$, it is therefore

$$I_G = \frac{E}{r_1 + \frac{R_1 G}{R_1 + G}} \cdot \frac{R_1}{R_1 + G}$$

The two resistances are then changed to r_2 and R_2, which give the same current in the galvanometer and hence the same deflection.

Then,
$$I_G = \frac{E}{r_1 + \frac{R_1 G}{R_1 + G}} \cdot \frac{R_1}{R_1 + G} = \frac{E}{r_2 + \frac{R_2 G}{R_2 + G}} \cdot \frac{R_2}{R_2 + G},$$

$$\therefore \frac{R_1}{(R_1 + G) r_1 + R_1 G} = \frac{R_2}{(R_2 + G) r_2 + R_2 G},$$

from which
$$G = \frac{R_1 R_2 (r_1 - r_2)}{R_1 r_2 - R_2 r_1}.$$

Since the resistance of the cell is always small in comparison with r_1 and r_2 it may be neglected without introducing any sensible error.

EXPT. 198.—**Resistance of a galvanometer (simple deflection).** Connect the galvanometer, cell and resistance box as in Fig. 817. Find the mean deflection with resistance 10 ohms in the box, and again with 20 ohms. Then, as on p. 883, $(G + 10)/(G + 20) = \theta_2/\theta_1$ if the galvanometer is direct reading and $(G + 10)/(G + 20) = \tan \theta_2 / \tan \theta_1$ if it is a tangent galvanometer, otherwise the values of I_1 and I_2 must be obtained from a calibration curve (p. 863).

EXPT. 199.—**Resistance of galvanometer (shunt method).** Connect two resistance boxes, a cell and a galvanometer, as in Fig. 820. Adjust the

resistances to give a reasonable deflection. Change them to two other values which give the same deflection, and calculate the galvanometer resistance G, from the equation,

$$G = \frac{R_1 R_2 (r_1 - r_2)}{R_1 r_2 - R_2 r_1}.$$

Repeat with another value of the deflection, and take the mean value of G.

Wheatstone's bridge.—By far the most accurate and convenient method of comparing resistances is that due to Wheatstone. This method makes use of the principle of the divided circuit, which in this case is known as **Wheatstone's bridge.** Consider the two circuits in parallel ACB and ADB (Fig. 821). The current I entering at A divides into two parts, whose values are inversely as the resistances of the two branches. Hence the current in ACB is

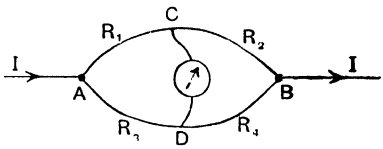


FIG. 821.—Wheatstone's bridge.

$$I_C = I \frac{R_3 + R_4}{R_1 + R_2 + R_3 + R_4},$$

and that in ADB is $I_D = I \dfrac{R_1 + R_2}{R_1 + R_2 + R_3 + R_4}$.

Thus the p.d. between A and C is

$$E_{AC} = R_1 I \frac{R_3 + R_4}{R_1 + R_2 + R_3 + R_4},$$

and that between A and D is

$$E_{AD} = R_3 I \frac{R_1 + R_2}{R_1 + R_2 + R_3 + R_4}.$$

If E_{AC} and E_{AD} are equal, there is the same p.d. between A and C as between A and D, and hence there is no p.d. between C and D. If C and D are then connected by a conductor there will be no current in this conductor. The condition for this is, that

$$R_1 I \frac{R_3 + R_4}{R_1 + R_2 + R_3 + R_4} = R_3 I \frac{R_1 + R_2}{R_1 + R_2 + R_3 + R_4},$$

or $$R_1(R_3 + R_4) = R_3(R_1 + R_2),$$

$$R_1 R_3 + R_1 R_4 = R_1 R_3 + R_2 R_3;$$

$$\therefore R_1 R_4 = R_2 R_3,$$

or $$\frac{R_1}{R_2} = \frac{R_3}{R_4}.$$

Thus the four resistances form a proportion when there is no current between C and D, although a current is flowing through the system from A to B. This condition is found experimentally by connecting a galvanometer between C and D. With a sensitive galvanometer, very small currents, and therefore minute potential differences, between C and D may be detected. For this reason very accurate balances between the resistances may be obtained. Since the deflection is zero for a perfect balance, this is called a **zero or null method.**

It should be noticed that if the current enters at C and leaves at D, the galvanometer being connected to A and B, the same condition will hold for zero deflection, that is,

$$\frac{R_1}{R_2} = \frac{R_3}{R_4}$$

EXPT. 200.—**Wheatstone's bridge.** Screw four double terminals A, B, C and D into a board (Fig. 822). Connect C and D to a simple galvanometer, and A and B to a cell.

(1) Connect the ends of a piece of platinoid wire 1 metre long to A and B and clamp its middle point in the terminal C. Connect a similar piece of platinoid wire to A and B, and find by trial the point of it which, when touched to the terminal D, produces no deflection upon the galvanometer. Measure the lengths of the parts AD and DB. Show that the ratio of lengths AC CB is equal to the ratio AD DB.

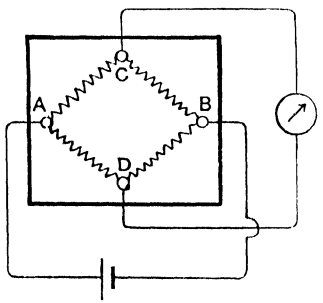


FIG. 822.—Experiment for demonstrating the Wheatstone's bridge relation.

(2) Make AC = 20 cm. and CB = 80 cm. and repeat. Show that the ratios are again equal.

(3) Make AC = 30 cm. and CB = 70 cm. and repeat.

(4) Replace ADB by a piece of fine copper wire and repeat, showing that the ratios must be equal whatever the actual resistances may be.

(5) Make AC = 60 cm. and CB = 30 cm. of platinoid wire. In AD place an unknown resistance coil of 2 or 3 ohms. In BD place a piece of platinoid wire and adjust its length by trial until the galvanometer deflection is zero, so that a balance is obtained. Knowing the ratio of CB to AC, and therefore the ratio of the resistances DB and AD, find the length of platinoid wire which has the same resistance as the coil in AD.

The metre bridge.—For the purposes of practical measurements of resistance, the Wheatstone's bridge is constructed in several convenient forms. One of the commonest forms is the **metre bridge**, so called because a wire AB one metre long (Fig. 823) is stretched between two fixed copper strips, the wire comprising two arms of the Wheatstone's bridge. The parts AC, DE and FB are stout copper strips, which have negligible resistance. If two conductors are placed in the gaps at D and E, a cell connected to C and F, and a galvanometer to L and to a movable point M on the wire, it will be seen that the four resistances R_1, R_2, R_3, and R_4 comprise a Wheatstone's bridge. The point of contact M on the wire is found, which gives zero galvanometer deflection. Then, $R_1/R_2 = R_3/R_4$. But if the wire is uniform, $R_3/R_4 = L_1/L_2$, where L_1 and L_2 are the corresponding distances of M from A and B. Hence for a balance, $R_1/R_2 = L_1/L_2$. L_1 and L_2 are measured by means of a scale fixed alongside of the wire, and hence the ratio of R_1 to R_2 becomes known. If R_1 is a standard coil, the resistance of R_2 in ohms can be calculated.

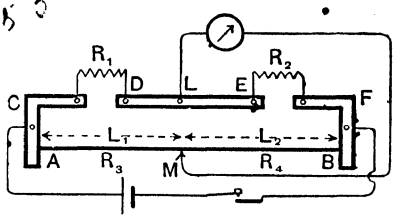


FIG. 823.—Diagram of the metre bridge.

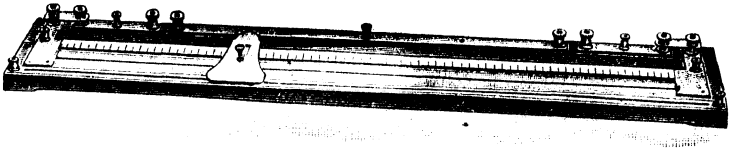


FIG. 824.—The metre bridge.

It is usual to employ a tapping key in the cell circuit so that the current only flows while the test is being made. The contact at M is usually a tapping contact, and **the practice should always be followed of making the battery circuit before the galvanometer circuit.** The reason for this will be understood later (Chap. LXXVI.). Also the cell and galvanometer may be interchanged (p. 886). For some reasons it is desirable to connect the cell to the wire contact M, and the galvanometer to the fixed points C and F, as will be explained in Chap. LXXIX.

The actual form the metre bridge takes in practice is shown in Fig. 824. The sliding contact and heavy terminals should be noted.

EXPT. 201.—**Resistance by metre bridge.** Place a resistance box in the gap D of the metre bridge (Fig. 823), and in E put a coil of unknown resistance. Having connected up the cell and galvanometer as shown, unplug the 10 ohm coil of the box and move the slider M until, on making contact, there is no galvanometer deflection. Observe the lengths L_1 and L_2, and calculate the resistance in E from the relation, Resistance $= 10\ L_2/L_1$. If this resistance is less than 10 ohms, repeat the experiment, using respectively 5 ohms, 2 ohms and 1 ohm in the box at D. If the resistance is more than 10 ohms, repeat, using 20, 50 and 100 ohms at D. In each case calculate the resistance at E and notice that the most accurate result is obtained when the balance is such that the point M is near the middle of the bridge wire.

The post-office box.—Any three resistance boxes may be used as the three arms, R_1, R_2 and R_3 of the Wheatstone's net (Fig. 825 (*a*))

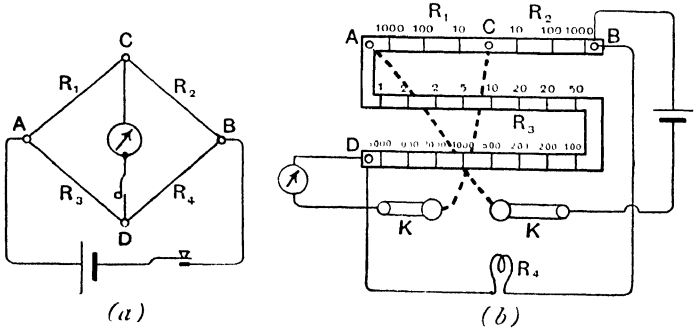


FIG 825.—The post-office box

where R_4 is the unknown resistance to be determined. Such sets of resistances are frequently made all in one box, one such arrangement is shown diagrammatically in Fig. 825 (*b*). By comparison of (*a*) and (*b*) it will be seen that R_1 or AC consists of three coils of respectively 10, 100 and 1000 ohms resistance Similarly, CB consists of three such coils These are known as the **ratio arms**, and it will be noticed that ratios from $\frac{1000}{10}$ to $\frac{10}{1000}$ can be obtained The arm AD is an ordinary set of resistances which, by unplugging, can be adjusted to have any resistance from 1 ohm to 11110 ohms. The fourth arm DB is the unknown resistance, which is therefore connected to D and B The two tapping keys K are also included in the box, one being in the cell circuit and the other in that of the galvanometer. The connections of these keys vary with different

patterns, and must be made out by the student for each box used. The whole set is known as the **post-office box.** It will be seen that for any given ratio of R_1 to R_2, the resistance R_3 may be varied until a balance is obtained, when the unknown resistance R_4 can be calculated. When R_4 is small it is advisable to make

$$R_1/R_2 = 1000/10 = 100/1,$$

and when R_4 is large to make

$$R_1/R_2 = 10/1000 = 1/100,$$

so that full use of the adjustable arm R_3 may always be made. In this way resistances ranging from 0·1 ohm to 1000000 ohms may be measured to within an error of 1 per cent., and intermediate resistances to 0·1 per cent.

EXPT. 202.—**Resistance by post-office box.** Connect up the post-office box, as in Fig. 825, using an unknown resistance coil as R_4. Beginning with 100 ohms in each ratio arm, find the resistance R_3 which produces a balance, and calculate R_4. If this is very small, or very large, alter the ratio arms suitably and redetermine R_4. In all cases the bridge is most sensitive when all four arms are of the same order of resistance. Measure the resistances of several coils and of an incandescent lamp.

EXPT. 203.—**Specific resistance** Set up the post-office box, as in Fig. 825, using a piece of platinoid wire, at least a metre long, as resistance R_4. Measure its resistance as in Expt. 202. With a metre scale find the length of the wire between the terminals, and with a micrometer gauge determine its diameter in several places and take the mean. Calculate the area of cross-section a of the wire, and, calling its length l, and resistance R, find the specific resistance of the platinoid from the relation

$$S = \frac{Ra}{l} \quad \text{(p. 851)}.$$

Repeat the experiment, using a piece of fine copper wire, and find the specific resistance of the copper.

EXPT. 204.—**Temperature coefficient of resistance.** Take leads P and Q (Fig. 826) from a coil of fine copper wire to the terminals B and D of the post-office box (Fig. 825). The fine copper wire is immersed in paraffin oil contained in a test-tube. The test-tube is surrounded by water contained in a beaker, so that its temperature can be raised gradually. A thermometer, passing through the stopper, enables the temperature of the oil, and therefore of the copper wire, to be observed. Starting at the temperature of the room, measure the resistance of the coil. Then raise the temperature about 5° C., and after making sure that this has become

steady, measure the resistance again. Repeat this at intervals of about 5°, up to 100° C. Record the results in the form of a table, and draw a graph connecting temperature and resistance. The points should be very nearly on a straight line. Draw a straight line with a ruler to lie evenly amongst them, and produce it backwards so that the resistance of the wire at 0° C. can be found. Calling this R_0, and R_{100} the resistance at 100° C., calculate the coefficient of increase of resistance α from the relation $R_{100} = R_0 (1 + \alpha t)$. It is from methods such as this that the temperature coefficients of the metals given on p. 851 have been found.

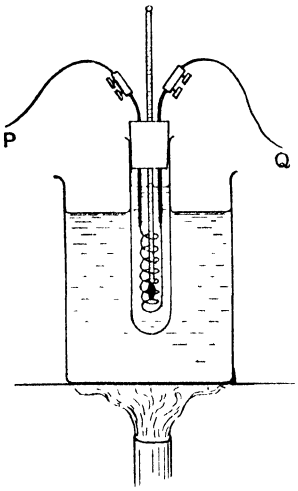


FIG. 826.—Measurement of temperature coefficient of resistance

The potentiometer.—Provided that a wire is uniform and that a steady current flows in it, there is a uniform fall of potential along it, that is, the potential difference between any two points on it is proportional to the length of wire between those two points. This is made use of in the **potentiometer** for the comparison of electromotive forces and potential differences. A current produced by a cell C, preferably a secondary cell (p. 916) flows in a uniform wire AB (Fig. 827). A cell E has one terminal connected to A, and the other through a galvanometer to a movable contact D. When the cell is on open circuit, the p.d. between its terminals is equal to its e.m.f. (p. 855). For some point D upon the wire, the p.d. between A and D is equal to that between the cell terminals on open circuit. If the terminal be then connected to the wire at D, nothing will happen in the cell circuit, as the p.d. between A and D due to the current in AB is exactly the same as that due to the cell E. No change is then made by making contact at D, and the galvanometer will not be affected. If the correct point

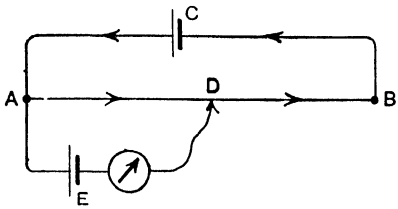


FIG. 827.—Diagram of the potentiometer

be not found, a current will flow in one direction or the other through E and the galvanometer on making contact. Hence for a correct balance, the e.m f. of E is equal to the p d. between A and D and this is proportional to the length of wire AD, which equals, say, l_1. Let E_1 be the e m f of E Replace this cell by another of e.m f. E_2, and find the length of wire l_2 for a balance.

Then, $$\frac{E_1}{E_2} = \frac{l_1}{l_2}.$$

Thus the two e.m f 's are compared

This method is excellent for several reasons. It is a null method (p. 886), and therefore does not depend upon measuring a galvanometer deflection The length of the wire AB may be several metres, and since the adjustment is easily made to within 1 millimetre, considerable accuracy is obtainable; also the cell E is not producing current when the adjustment is correct, and its internal resistance is thus immaterial

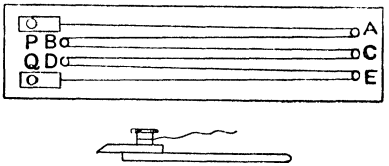


FIG 828 The potentiometer.

A convenient and simple form of potentiometer may be constructed by pasting a piece of paper square-ruled in centimetres and millimetres upon a board After placing nails or screws at the points ABCD and E (Fig. 828), and terminals soldered to copper strips at P and Q, a piece of platinoid wire is passed round the screws and soldered to the copper strips at P and Q. The millimetre squares serve to measure lengths of wire from P or Q. A convenient form of contact maker is also shown. It is a wooden handle having a brass strip, filed to a sharp edge, screwed to it, a terminal being previously attached.

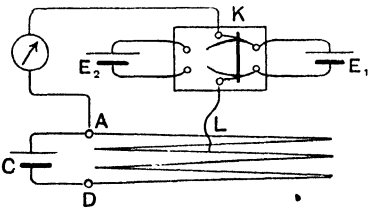


FIG 829.—Comparison of e m.f.'s by potentiometer

EXPT. 205.—**Comparison of e.m.f.'s by potentiometer.** Connect a secondary cell C (Fig. 829) in series with the potentiometer wire ALD. If E_1 and E_2 are the cells whose e.m.f.'s are to be compared, join one to each side of the rocking key K, and join the galvanometer and terminal A of the potentiometer to one of the

middle terminals of K, the movable contact wire L being joined to the other. With the rocker of K on one side, find the length l_1 of potentiometer wire, as measured from A which gives a balance. Rock the key to the other side and find the length l_2 for the other cell. Then $E_1 E_2 = l_1 l_2$. If E_2 be a Daniell's cell, then $E_1 = 1 \cdot 1 \, l_1/l_2$ volts. Repeat, placing another cell in the position E_1 and find its e.m.f. in the same way.

Range of usefulness of the potentiometer.—Another great advantage of the potentiometer consists in the very great range of electromotive forces that it may be used to compare. It may be made so sensitive that e.m.f.'s of the order of millivolts or less may be compared by means of it. Also, by using auxiliary resistances with it, e.m.f.'s up to thousands of volts may be measured.

In order to make the potentiometer more sensitive, all that is necessary is to reduce the current in it, so that the p.d. over the whole wire, instead of being about 2 volts, is only a small fraction of a volt. This may be done by placing a resistance in series with it, in the manner employed in measuring thermal electromotive forces (Chap. LXXIX.).

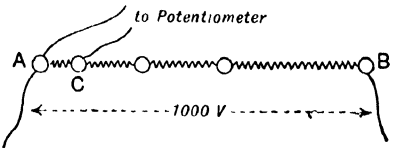


FIG. 830.—Subdivided resistance.

For the measurement of high electromotive forces, a **subdivided resistance** must be used. Thus, the resistance AB (Fig. 830) may be 10,000 ohms, and if the e.m.f. to be measured is of the order of 1000 volts, it may be applied to the points A and B. The p.d. between two points A and C, such that the resistance of AC is one-thousandth of AB is then of the order of 1 volt and may be measured by the potentiometer. The e.m.f. applied to AB may then be obtained by multiplying by 1000.

Zero error of the potentiometer.—It may happen that the resistance of the joint and terminal at P (Fig. 828) is not negligible, and it will then follow that lengths of wire measured by the scale are not strictly proportional to the resistances between the terminal and the movable point of contact. The proper correction to be applied to the scale readings may easily be found, and is called the **zero error** of the instrument. Thus, if AB and BC (Fig. 831) be two resistances in series in which a steady current is flowing, the p.d. between A and C is the sum of the p.d.'s between AB and BC.

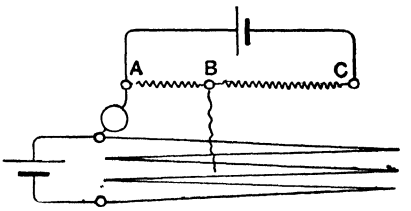


FIG. 831.—Measurement of the zero error of potentiometer.

On finding lengths of potentiometer wire l_1, l_2 and l_3 corresponding to these three p.d.'s, $l_3 = l_1 + l_2$ if there is no zero error. If, however, there is a zero error α, each length must have α added to it in order to make it proportional to the p d. being measured. The relation between the three is then

$$l_3 + \alpha = (l_1 + \alpha) + (l_2 + \alpha),$$

or,

$$\alpha = l_3 - l_1 - l_2.$$

α is the correction that must be added to the potentiometer readings. It may be either positive or negative.

EXPT. 206.—**Zero error of potentiometer** Place two resistance boxes AB and BC (Fig. 831) in series with a Daniell's cell; with 50 ohms in AB and 100 ohms in BC, measure the length of wire l_1 for the p.d. between A and B, l_2 for that between B and C, and l_3 for that between A and C. Then if α be the zero error of the potentiometer, $\alpha = l_3 - l_1 - l_2$.

EXPT. 207.—**Comparison of resistances by potentiometer** For the comparison of low resistances the potentiometer is convenient. Join the two low resistances AB and CD in series, and pass a current through them (Fig. 832). If the resistances are 0·1 ohm or less, at least 1 ampere should be employed. Find the length of potentiometer wire l_1 corresponding to the p.d. between A and B. Then remove the connections from A and B to C and D and again find the corresponding length of potentiometer wire.

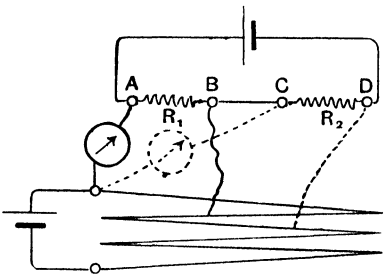


FIG 832.—Comparison of resistances.

Then,

$$\frac{R_1}{P_2} = \frac{l_1}{l_2}.$$

If the resistance of either AB or CD in ohms be known, that of the other can be found.

Current measurement by potentiometer.—Currents of any magnitude can be measured by means of the potentiometer, both accurately and conveniently. Hence this method is frequently employed for the calibrating of ammeters. The current I to be measured is caused to flow through a standard low resistance R. This should have a resistance of 0·1 ohm for a current of 10 amperes, 0·01 ohm for a current of 100 amperes, and so on, so that the p.d. between its terminals is of the order of 1 volt. The arrangement is shown in

Fig. 833 With the key K on one side, the length of potentiometer wire l_1 for the p.d over R is found. Then the rocker of the key is placed over to the other side and the length of wire l_2 for the standard cell E is found. Since the p.d. between the terminals of R is IR, we have

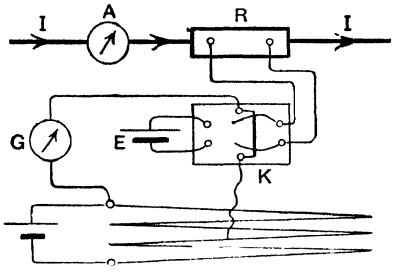


FIG 833.—Current by potentiometer.

$$\frac{IR}{E} = \frac{l_1}{l_2},$$

or,
$$I = \frac{E}{R} \cdot \frac{l_1}{l_2}.$$

In this way the current I is found in terms of the two standards E and R. For rough measurements E may be a Daniell's cell whose e.m.f. is taken as 1·1 volt, but for more exact measurements the cadmium cell (p. 914) or the Latimer-Clark cell (p. 913) should be used.

EXPT. 208.—**Calibration of an ammeter by means of a potentiometer.** Place the ammeter A in series with the standard resistance R. A rheostat or an incandescent lamp should be placed in the current circuit, the lamp being used when 100 volt supply is employed. Measure the length l_1 of wire corresponding to the p.d. between the terminals of R, and the length l_2 for the cell E, as above. Increase the current step by step, each time making the above two observations and reading the ammeter. Record the results in a table, making a fourth column for the current calculated from the relation $I = E/l_2 R/l_1$. After reaching the end of the ammeter scale, plot the current and ammeter readings in the form of a graph. If the ammeter is graduated to read amperes, the error for each observation should be found and plotted against the ammeter readings, thus giving a curve for future use with this ammeter.

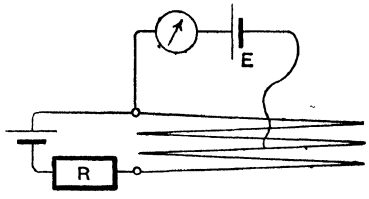


FIG 834.—Calibration of a potentiometer.

EXPT. 209.—**Calibration of a potentiometer to read directly in volts.** Connect up the potentiometer and standard cell E as in Fig. 834, including an adjustable rheostat R in the potentiometer circuit. If the standard cell is a Daniell, place the movable contact at the point 110 cm. upon the wire. Adjust the rheostat R until a balance is obtained. Then 110 cm. of wire corresponds to 1·1 volt, so that 1 cm. corresponds to 0·01 volt. This calibration is convenient

when a number of voltages are to be measured, as the instrument is now direct reading. When great accuracy is required a better form of standard cell than the Daniell must be used. For the cadmium cell the length of wire should be 101·8 cm., and for the Latimer-Clark cell 145·3 when 100 cm. of wire is to correspond to 1 volt.

Resistance box potentiometer.—A very convenient form of potentiometer may be constructed of two similar resistance boxes The two boxes AB and CD are joined in series with a secondary cell E (Fig. 835) One of the cells whose e.m.f. is required is placed at E_1 in series with the galvanometer G. At first all the plugs are taken out of the box CD while all the plugs are in the box AB. The adjustment is made by removing plugs from AB and placing them in the corresponding positions in CD until the galvanometer deflection is zero.

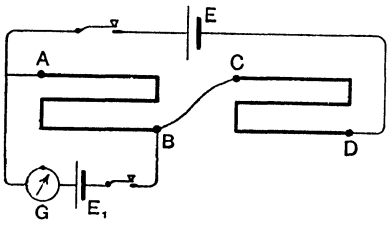


FIG 835.—Resistance box potentiometer.

Thus the resistance in the circuit ABCD remains constant, and the current in it, therefore, remains constant also. The e.m.f. E_1 is then proportional to the resistance R_1 in AB, when the adjustment is correct. E_1 is now replaced by E_2 and the corresponding resistance R_2 in AB, for a balance is found Then

$$\frac{E_1}{E_2} = \frac{R_1}{R_2}$$

Measurement of the internal resistance of a cell.—The potentiometer affords a means of measuring the internal resistance of a cell. On measuring the p.d. between the terminals of the cell, first on open circuit, and then when it is producing current in an external resistance of known value, the internal resistance of the cell can be calculated. These two quantities are represented by E and e in equation (11) (p. 855), where r is the internal resistance of the cell and R the external resistance joined between its terminals. The absolute values of E and e are not required, the lengths of potentiometer wire employed when the balance is obtained in the two cases serving instead. Calling these lengths L and l, equation

$$\frac{E-e}{e} = \frac{r}{R}, \quad \text{becomes} \quad \frac{L-l}{l} = \frac{r}{R}.$$

Since all the quantities except r are known, r can now be calculated.

EXPT. 210.—**Internal resistance of a cell.** Connect up the potentiometer and galvanometer as in Fig. 827, using a secondary cell for C, and

the cell (a Daniell's cell) whose internal resistance is required for E. Obtain the length AD = L when the cell E is on open circuit Now join a resistance box across the terminals of E, using 20 ohms as the resistance in the box. Measure the length of wire l which now gives a balance.

Calculate the internal resistance r of the cell from the equation

$$\frac{L-l}{l} = \frac{r}{20}$$

Repeat the experiment using values of R equal to 15, 10, 5, 2 and 1 ohms. and in each case calculate the value of r.

Repeat the experiment with another form of cell, such as a dry cell, or a Leclanché.

EXERCISES ON CHAPTER LXIX

1. Enunciate Ohm's Law and explain its meaning.

A battery is connected to a tangent galvanometer of resistance 9 ohms and produces a deflection of 60°. An extra resistance of 7 ohms is then placed in the circuit and the deflection falls to 45° Calculate the resistance of the batte y. Sen. Camb. Loc

2. Two voltaic cells A and B are connected in series, and form a simple circuit with a galvanometer. The current indicated is 2 4 amperes. The cell A is then reversed so as to oppose B, and the current observed is 0 6 ampere in the same direction as before. Calculate the ratio of the electromotive forces of A and B. L U.

3. Explain the use of a potentiometer in measuring (*a*) potential difference, (*b*) current. What apparatus would you want in each case ? L U

4. Explain how the e.m.f.'s of a number of cells can be compared by the aid of a potentiometer.

The terminals of a cell with an e.m.f. of two volts and with a negligible internal resistance, are joined by two coils in series. One coil has a resistance of 1 ohm, while the other has a variable resistance which may be denoted by R. What must be the value of R in order that a Leclanché with an e.m.f. of 1 58 volts can be connected in parallel with the 1 ohm coil, in such a way that no current flows through the Leclanché ? Give a diagram of the arrangement. L.U.

5. Explain the Wheatstone's bridge method of comparing resistances.

If the resistance of a wire of length 120 cm. and diameter 0 4 mm. is found to be 2 5 ohms, what is the specific resistance of the material ? L.U.

6. Two cells, when connected in series, give rise to a deflection of 45° on being joined to the terminals of a tangent galvanometer, and to a deflection of 30° when connected so as to oppose each other. Compare the electromotive forces of the cells. L.U.

7. What do the terms " electromotive force " and " internal resistance ' mean as applied to a voltaic cell ?

If the difference of potential between the poles of such a cell when no current is flowing is 1·4 volts, and this becomes reduced to 1 1 volts when

the poles are joined by a wire of 5 ohms resistance, find the internal resistance of the cell. L.U.

8. Describe some method of measuring the resistance of a galvanometer. A galvanometer has a shunt of resistance 30 ohms, and the rest of the circuit comprises a cell and a resistance of 450 ohms. If the shunt is changed to 20 ohms and the external resistance decreased from 450 ohms to 350 ohms, the galvanometer indicates the same current as before. Find the resistance of the galvanometer.

9. A cell of e.m.f., 1·5 volts and resistance 2 ohms, maintains a current in an external resistance of 6·5 ohms., find the p.d. between the terminals of the cell.

10. A circuit consists of a cell of e.m.f. 1 72 volts and resistance 1 5 ohms, together with an external resistance of 4·2 ohms. Calculate the amount of heat developed in the external resistance and in the cell in one minute.

11. A cell having an e.m.f. of 1 6 volts and internal resistance 0·5 ohm is placed in series with a tangent galvanometer whose coil consists of 15 turns of radius 18 cm., and the deflection is observed to be 12°. Calculate the resistance of the galvanometer, taking $H = 0\ 18$.

12. A piece of wire, 165 cm. long and 0·82 mm. in diameter, is used as an unknown resistance with the post office box. If the ratio arms have resistances 1000 : 10 and the third (Fig. 825) is 85 ohms when a balance is obtained, find the specific resistance of the wire.

13. Explain how the internal resistance of a cell may be determined. A length of potentiometer wire of 155 cm. balances the e.m.f. of the cell on open circuit, and a length of 135 cm. when the cell has a conductor of resistance 8 ohms connected between its terminals. Calculate the internal resistance of the cell.

14. Describe how the errors in the scale of an ammeter may be determined by the use of a potentiometer. What would be a suitable value of the standard resistance employed in this standardising experiment if the ammeter reads up to 50 amperes.

15. Describe how the zero error of a potentiometer may be measured. If in Fig. 831 the length of wire to balance the p.d. over AB is 36 5 cm., over BC 29 4 cm., and over AC 66 3 cm., what is the zero error ? What fault in construction in the instrument could such a zero error be due to ?

CHAPTER LXX

ELECTROLYSIS: CELLS AND BATTERIES

Introduction to electrolysis.—Reference to the fact that the passage of an electric current through certain liquids, mostly solutions, is accompanied by chemical effects has already been made (p 830) A more complete account of this phenomenon must now be given. It has been known for a very long time that some liquids are capable of carrying a current while others are not. Faraday, who first systematically studied this branch of electrical phenomena, called those liquids which would carry a current **electrolytes.** Water to which a small amount of an acid, or salt, has been added is a moderately good conductor, and is hence an electrolyte. On the other hand, paraffin oil is a good insulator, and is therefore not an electrolyte. Pure water is not an electrolyte.

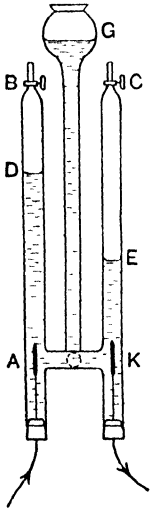


FIG 836.—Electrolysis of water

The conductors which bring the current into and lead it from the electrolyte are called **electrodes.** Thus, if two electrodes consisting of pieces of platinum foil be dipped into an electrolyte consisting of water to which a few drops of sulphuric acid have been added, and the electrodes connected respectively to the poles of a battery of a few cells in series, a current will flow through the water. While the current is flowing it may be noticed that bubbles form on the electrodes. If the experiment be performed with the apparatus shown in Fig. 836, consisting of two vertical graduated tubes AB and KC, united at their bases by the short piece AK, the gases formed on the electrodes may be caught in the tubes and measured.

The electrode at which the current enters the electrolyte is called the **anode**, and that by which it leaves is the **kathode**. It will be found that the gas evolved at the kathode K has twice the volume of that evolved at the anode in the same time. On opening the tap C, the gas in CE escapes, and if a light be applied, the escaping gas burns It is hydrogen. The gas in BD as it escapes will ignite a glowing taper or splinter of wood, showing that it is oxygen. Thus hydrogen is evolved at the kathode, oxygen at the anode, and the two are in the proportion in which they combine to form water. The significance of this fact will be seen shortly.

In the event of the electrolyte being a solution of a salt, the metal is liberated at the kathode and the acid radicle or element at the anode. These substances are deposited upon the electrode if they can exist in a stable form in water, but in many cases there is some reaction with the water. When the current passes between platinum plates immersed in a solution of copper sulphate, metallic copper is deposited upon the kathode, since there is no reaction between the copper and the water. At the anode, the acid radicle SO_4 is liberated. This cannot exist alone, so with the water forms sulphuric acid, oxygen being liberated in the gaseous form. This reaction follows the equation :

$$2SO_4 + 2H_2O = 2H_2SO_4 + O_2.$$

A similar reaction occurs when the current passes through acidulated water The hydrogen of the sulphuric acid is liberated at the kathode and SO_4 at the anode. This, with the water, forms sulphuric acid and oxygen as above. Faraday called the materials liberated in electrolysis **ions.** This name is still sometimes used, but is also employed in a more particular sense (p 1034).

EXPT. 211.—**Examples in electrolysis** (1) Dip two carbon rods (arc-lamp carbons) into water slightly acidulated with sulphuric acid. Bind a piece of bare copper wire round each of the rods at the part not immersed, and join one wire to each pole of a battery of a few secondary cells. Test the direction of the current by means of a compass needle and so determine which carbon rod is the anode and which the kathode.

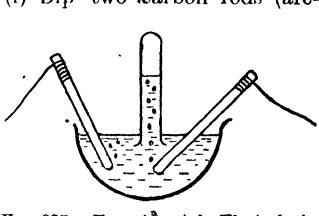


FIG. 837.—Experiment in Electrolysis.

Note that bubbles are formed on both rods, but more copiously at the kathode. Collect some of the gas from each electrode by means of

inverted test-tubes (Fig. 837) and test the gas, showing that one is hydrogen and the other oxygen.

(ii) Replace the dilute acid by a solution of copper sulphate and observe that when the current flows, a red deposit of copper is formed on the carbon rod which is acting as kathode and bubbles of gas form on the anode.

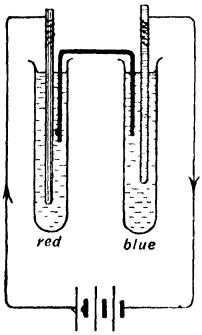


FIG 838 —Electrolysis of sodium chloride

(iii) Using two lead plates as electrodes, pass a current through a solution of lead acetate. A beautiful fern-like deposit of metallic lead forms upon the kathode

(iv) Place each carbon rod in a test-tube containing a solution of sodium chloride to which a few drops of litmus have been added, and unite the tubes by means of a strip of blotting paper. Observe that the litmus in the anode tube (Fig. 838) turns red. This is due to the formation of hydrochloric acid. Chlorine is liberated, which, with the water, forms hydrochloric acid, which turns the litmus red.

$$4Cl + 2H_2O = 4HCl + O_2.$$

At the kathode, sodium is liberated, which dissolves in the water, forming sodium hydroxide, which keeps the litmus blue.

$$2Na + 2H_2O = 2NaOH + H_2.$$

Hydrogen escapes in bubbles at the kathode and oxygen at the anode.

Laws of electrolysis.—The quantitative laws of electrolysis were first given by Faraday and are known by his name. In order to

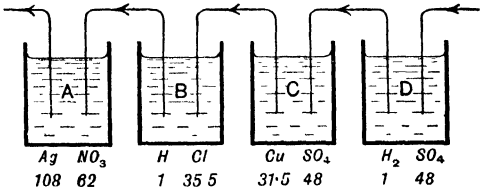


FIG 839 —Laws of electrolysis.

understand them let us consider the case of a current flowing through a number of electrolytic cells A, B, C and D in series (Fig. 839). Let all the electrodes be platinum or carbon, so that there are no secondary actions of the ions with the electrodes to consider. For a moment we will disregard all secondary reactions and concentrate our attention upon the materials liberated by the process of electrolysis, leaving the consideration of their ultimate condition for a time.

Suppose that A contains a solution of silver nitrate, B one of hydrochloric acid, C of copper sulphate and D of sulphuric acid. From what we have already seen, there will be liberated at the respective kathodes, silver, hydrogen, copper and hydrogen; and at the anodes NO_3, Cl, SO_4 and SO_4. It is a matter of experiment to determine how much of each substance is liberated by any current in a given time. The **first law of electrolysis states that the mass of any substance liberated is proportional to the current flowing and to the time for which it flows; that is, it is proportional to the product of current and time**, or,

$$\text{Mass liberated} \propto \text{current} \times \text{time}.$$

It will be shown later (p. 902) that the product of current and time represents a quantity of electricity, in fact, the first law of electrolysis is usually stated, that **the mass of any substance liberated is proportional to the quantity of electricity passing through the electrolyte.** Thus, when a current of 10 amperes flows for 30 seconds, the same quantity of electricity passes as when a current of 5 amperes flows for 60 seconds, and in each case the amount of electrolysis occurring is the same.

The relation of the mass of substance liberated to the chemical nature of the substance must now be considered. Suppose that the current flows until 1 gram of hydrogen has been liberated in B (Fig. 839), then, since the current and time of flow are the same for all the cells, 1 gram of hydrogen also is liberated in D. Now, in B, 1 gr. of hydrogen is combined with 35·5 gr. of chlorine to form hydrochloric acid. Hence, when 1 gr. of hydrogen is liberated at the kathode, 35·5 gr. of chlorine will be liberated at the anode. Similarly, in D, 48 gr. of SO_4 will be liberated in the same way. Again, 48 gr. of SO_4 is liberated in C, and hence the equivalent amount of copper, that is, 31·5 gr., is liberated. In A there will be 108 gr. of silver and 62 gr. of NO_3 liberated.

For any other current, or time of flow, the whole of these quantities will vary in the same ratio, so that we may state that **the masses of various substances liberated are proportional to their chemical equivalents.** In the case of an acid radicle the chemical equivalent is the mass of the substance that would combine with 1 gr. of hydrogen, and in the case of the base it is the mass that would replace 1 gr. of hydrogen in an acid to form the salt.

Thus, with the monovalent substances the chemical equivalent is the atomic weight; with divalent substances, half the atomic weight, and so on. The two laws of electrolysis may thus be stated formally as follows:

Faraday's laws of electrolysis:

(i) **The mass of any substance liberated in electrolysis is proportional to the quantity of electricity passing through the electrolyte (or to the current and the time for which the current flows)**

(ii) **The masses of different substances liberated by a given current in a given time are proportional to the chemical equivalents of the substances**

The coulomb. -When a current of one absolute unit flows for one second, one absolute unit of quantity of electricity passes along the conductor Since, however, the current is usually measured in amperes, it is convenient to give a name to the quantity of electricity which passes along the conductor when 1 ampere flows for 1 second. It is called one **coulomb**, and is the practical unit of quantity of electricity, upon the same system as that upon which the ampere, volt and ohm were devised.

The ampere being one-tenth of the absolute unit of current we see that the coulomb is one-tenth of the absolute unit of quantity of electricity.

Electro-chemical equivalent.—From Faraday's laws it is evident that if the mass of some one substance liberated by a given current in a known time can be determined experimentally, then the corresponding mass of any other substance for any current and time may be calculated It does not follow that every substance when liberated can be collected and weighed with accuracy Hence, for the purpose of exact experimental work it is necessary to choose the substance with care The choice falls upon silver, as this has a very high chemical equivalent, and, when deposited from silver nitrate solution by means of a moderate current, forms a hard metallic deposit of great purity.

The mass of any substance liberated by unit current in unit time is called its electro-chemical equivalent. The electro-chemical equivalent of silver has been found to be 0·0011183 That is, a current of 1 ampere flowing for 1 second, or the passage of 1 coulomb, liberates 0·0011183 gram of silver. From this, the electro-chemical equivalents of the other substances may be calculated, using the knowledge of their chemical equivalents. Thus, the atomic weight of silver being 107·88, it follows that the electro-chemical equivalent of **hydrogen is**

$$z_H = \frac{0{\cdot}0011183 \times 1{\cdot}008}{107{\cdot}88} = 0{\cdot}00001045;$$

also the electro-chemical equivalent of copper is

$$z_{Cu} = 0{\cdot}0011183 \times \frac{63{\cdot}57}{2 \times 107{\cdot}88} = 0{\cdot}0003295.$$

In this case the atomic weight is 63·57, but since copper is usually divalent, its chemical equivalent is $\frac{1}{2}$ (63·57).

TABLE OF SOME ATOMIC WEIGHTS AND ELECTRO-CHEMICAL EQUIVALENTS.

	Atomic weight O = 16	Valency.	Electro-chemical equivalent
Aluminium - - (Al)	27·1	3	0·0000936
Bromine - - (Br)	79·92	1	0·0008285
Chlorine - - (Cl)	35·46	1	0·0003676
Copper - - - (Cu)	63·57	1 or 2	0·0003295
Gold - - - (Au)	197·2	3	0·0006814
Hydrogen - - (H)	1·008	1	0·00001045
Iron - - - (Fe)	55·81	2 or 3	— —
Lead - - - (Pb)	207·10	2	0·001073
Oxygen - - - (O)	16·0	2	0·00008293
Platinum - - (Pt)	195·2	4	0·0005059
Potassium - - (K)	39·10	1	0·0004053
Silver - - - (Ag)	107·88	1	0·0011183
Sodium - - - (Na)	23·00	1	0·0002381
Zinc - - - (Zn)	65·37	2	0·0003388

Voltameters. Owing to the exactness of Faraday's laws of electrolysis and hence of our knowledge of certain electro-chemical equivalents, it follows that electrolytic methods may be employed for the accurate measurement of current. Knowing the electro-chemical equivalent of any substance, a suitable solution of it is used as an electrolyte, and the current to be measured is passed through the electrolyte for a known time. Since the amount of deposit is proportional to the time for which the current passes, a small current may be employed, the time being made large enough to produce a fair amount of deposit. In fact, large currents must not be employed, since with large currents, in many cases, the deposit, if metallic, is friable and easily detached from the electrode.

If z be the electro-chemical equivalent of the material employed, this is the mass liberated by 1 ampere in 1 second, and it follows from Faraday's laws (p. 902) that the total mass of the deposit is given by the equation, $W = Izt$,

where I is the current in amperes and t the time in seconds for which the current flows.

An instrument designed to make use of this principle for the measurement of currents is known as a **voltameter.** There are many forms of voltameter, but the variation in type is only such as is necessary for the accurate determination of the mass of the ion liberated.

Copper voltameter.—In the case of the **copper voltameter,** a strong solution of copper sulphate, with a few drops of sulphuric acid added, is used as the electrolyte, the electrodes being copper plates This gives a very cheap and useful form of the instrument, by means of which currents of the order of one or two amperes may be measured with a fair degree of accuracy. From the simplicity of its form it is readily made in any laboratory The measurement of any given current requires considerable time, and this voltameter is therefore chiefly used in the standardisation and calibration of ammeters and tangent galvanometers

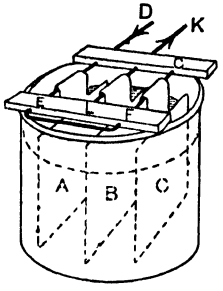


FIG. 840.—Copper voltameter.

The anode is in the form of two thin copper sheets A and C (Fig. 840) hanging one on either side of a similar copper sheet B which serves as kathode. With this arrangement the deposit occurs on both sides of the kathode and thus the maximum area is utilised. A bent brass or copper rod DEFG serves to support the plates A and C, and a similar rod KL supports the kathode B.

In order to measure the current, the circuit is arranged as in Fig. 841. The voltameter V is joined in series with the ammeter A, the battery B, a rheostat R and a key K, the kathode having previously been well cleaned with emery paper. On closing the key, the rheostat must be adjusted until the current has a suitable value, say, 1 ampere. No more than $\frac{1}{50}$ ampere per available square centimetre of kathode surface should be used, or the deposited copper will not be hard and compact, and is liable to be washed off the plate.

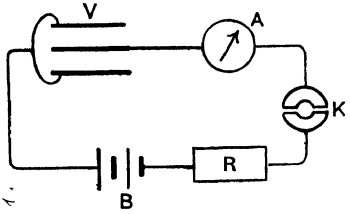


FIG. 841.—Calibration of an ammeter by electrolysis

The circuit is then broken at K and the kathode removed, washed in clean water, dried and carefully weighed. The drying may be performed by waving the plate about **some distance above** the flame of a spirit lamp. The plate must not be heated too much, or it is liable

to oxidise. The kathode is then replaced and the key closed, the instant of performing this being carefully noted on a watch or clock and recorded. All the time that the current is flowing it should be kept constant by means of the rheostat. If considerable variations occur, the observations so far are wasted and must be repeated. After an interval of about half an hour, or longer for smaller currents, the circuit is broken at a carefully noted time, and the kathode again removed, washed, dried and weighed. The observations may be recorded as follows :

Reading of ammeter - - =		amperes.	
Time of starting current - - =	h.	m.	s.
,, stopping ,, - - =	h.	m.	s.
Interval for which current flowed =		m.	s.
$\therefore t =$		seconds	
Weight of kathode at start - =		grams.	
,, ,, stop - =		grs.	
Gain in weight of kathode - =		grs. = W.	

Since $W = Izt$,

$$\text{Current} = \frac{W}{0{\cdot}0003295 \times t}$$

Error of ammeter = .

EXPT. 212.—**Calibration of an ammeter by a copper voltameter.** Arrange a circuit with ammeter and voltmeter included as in Fig. 841. Find the error of the instrument for one of the low readings as directed above. Repeat for larger currents, the last determination of the error being made with the largest ammeter scale reading. Plot a graph having ammeter readings as abscissae and errors as ordinates.

Silver voltameter.—For ordinary purposes the copper voltameter is sufficiently accurate, but for the highest order of accuracy the **silver voltameter** must be employed.

This usually takes the form of a platinum basin (Fig. 842) which has been previously cleaned with nitric acid, dried and weighed. It contains the electrolyte consisting of a solution of silver nitrate (15 gr. of $AgNO_3$ to 100 c.c. of water). Suspended in the solution, by means of platinum wire, is a plate of silver which constitutes the anode. From the reactions described on p. 901 it will be seen that silver is deposited upon the platinum

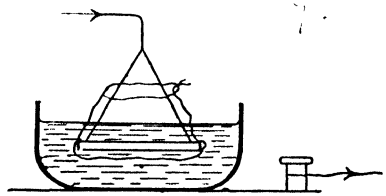


FIG. 842 —Silver voltameter.

basin, and also silver goes into solution at the anode, so that the strength of the solution remains constant. The silver anode is wrapped in a piece of blotting paper, which serves to catch any impurities liberated as the silver dissolves away, and prevents them falling on the platinum basin The principle of the use of the silver voltameter is identical with that for the copper voltameter, the mass of silver deposited in a known time being found and the current calculated from the relation,

$$\text{Current} = \frac{W}{0{\cdot}0011183 \times t}.$$

Water voltameter.—It is possible to use the graduated tubes in Fig. 836 as a voltameter, by collecting the hydrogen liberated by the current in a given time. The gas collected is not at the atmospheric pressure, since the level of the water at G differs from that at E. On correcting for this, the volume must be reduced to that for standard pressure and temperature, taking into account the saturated aqueous vapour present (p. 460). Then on multiplying by the density of hydrogen (0·0000899 gr per c c) the mass of hydrogen is known and the equation

$$\text{Current} = \frac{W}{0{\cdot}00001045 \times t},$$

may be used to find the current.

Owing to the difficulty of determining the volume of the gas with accuracy, and the number of corrections to be applied, the method is inferior to those in which the amount of ion liberated is found by direct weighing.

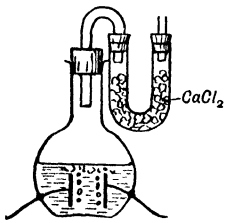


FIG 843.—Water voltameter.

The method may be modified, in order to make it depend upon direct weighings, by electrolysing the dilute acid solution in a flask provided with sealed-in platinum terminals (Fig. 843). The oxygen and hydrogen escape together, and since they are insoluble they pass away into the air. On weighing the flask before and after passing the current for a known time, the weight of water decomposed is known. Any moisture carried off by the escaping gases is caught by a drying tube through which they pass before leaving, which tube must, of course, be weighed with the flask. Since the oxygen and hydrogen escaping are both weighed, the electro-chemical equivalent used in calculating the current is the sum of those of hydrogen and oxygen, *i.e.*

$$Z_{H_2O} = 0{\cdot}00001045 + 0{\cdot}00008293 = 0{\cdot}00009338.$$

Thus,

$$\text{Current} = \frac{W}{0{\cdot}00009338 \times t}.$$

EXPT. 213.—**Water voltameter.** Perform the calibration of an ammeter as in Expt. 212, using the water voltameter (Fig. 843) instead of the copper voltameter.

Theory of electrolysis.—It would be out of place to attempt to give in this volume a comprehensive account of the modern theory of electrolysis, but certain important facts may be considered. The laws of electrolysis certainly suggest that the electricity is carried through the solution by the atoms in it, all monovalent atoms carrying the same amount of electricity, divalent atoms twice that amount, and so on. Faraday considered the primary decomposition occurring, when the current passed, to be that of the water which is acting as solvent, the deposition of the dissolved substances being due to the secondary actions of the oxygen and hydrogen at the electrodes. It is now considered that the primary action is the decomposition of the dissolved substance, and that in cases where the hydrogen and oxygen are liberated, as when a solution of sulphuric acid is electrolysed (p 898) it is the secondary action of the SO_4 and the water that causes the liberation of the oxygen.

$$2H_2O + 2SO_4 = 2H_2SO_4 + O_2.$$

It is a significant fact that pure water is a non-conductor.

The question then arises as to whether, in the case of a simple substance such as potassium chloride, the passage of the current causes a splitting up of the molecules, or whether these are already dissociated into potassium and chlorine in the solution and are merely directed towards the kathode and anode when the current flows.

The first consideration would lead us to think that Ohm's law would not be true for electrolytes, but the second is consistent with the truth of Ohm's law. Since Ohm's law is true for electrolytes this strengthens our belief in the dissociation theory.

If the first alternative were true, then no current would flow until the electromotive force had a sufficient value to disrupt the molecules, and any excess of electromotive force would freely cause current. Since, however, the current is proportional to the p.d. in the electrolyte, probably the e.m.f. does not cause the splitting up of the molecules, but merely drives the constituents of the already dissociated molecules in one direction or the other.

Evidence from widely differing sources leads to the belief that in any electrolyte, some of the molecules of the dissolved substance are

dissociated, the atoms then having positive or negative electric charges, according to their nature. Such atoms, possessing electric charges, are called **ions** and have very different properties to the neutral atoms of the same substance ordinarily met with. Thus, in a given solution of potassium chloride a certain percentage of the molecules will be dissociated into ions. These may be designated by $\overset{+}{K}$ and $\overset{-}{Cl}$ When the external source of electromotive force, such as the battery, is connected to the electrodes, one is raised to a higher potential than the other. There is thus an electric field between them, and we shall see on p 925 that in such an electric field positive charges are urged in one direction and negative charges in the other It will thus be seen from Fig 844 that the $\overset{+}{K}$ ions having positive charges are driven towards the kathode On reaching it they give up their positive charges to the kathode and resume their ordinary qualities. The potassium would then dissolve in the water in the usual way. Similarly the $\overset{-}{Cl}$ ions on arriving at the anode give up their negative charges and resume their ordinary form. These streams of ions in the electrolyte constitute the current When there is no dissociation the liquid is non-conducting Metallic ions such as potassium, silver, copper, hydrogen, etc., have positive charges, and hence are driven towards the kathode, while, on the other hand, the acid radicles have negative charges, and are driven towards the anode.

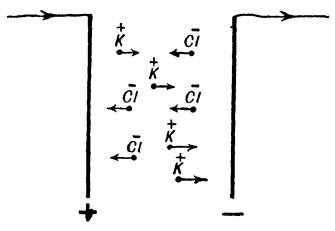


FIG 844.—Electrolytic conduction

The distinction between the chemical dissociation that occurs at high temperature and the electrolytic dissociation that occurs in solutions may well be illustrated by the case of ammonium chloride. At high temperature this substance dissociates according to the equation

$$NH_4Cl = NH_3 + HCl.$$

In solution the dissociation is represented by the equation

$$NH_4Cl = \overset{+}{N}H_4 + \overset{-}{Cl}.$$

Electroplating.—Electrolytic processes are used for many industrial purposes. The deposition of silver or gold upon the baser and more corrodible metals is well known. For these purposes solutions which give a hard metallic deposit are essential. In the case of silver plating, a solution of the double cyanide of silver and potassium is used as electrolyte. On adding potassium cyanide to a solution of silver nitrate, silver cyanide is precipitated, which however

dissolves when excess of potassium cyanide is added. The articles to be silver plated must first be thoroughly cleaned with wet sand or emery, afterwards being dipped in strong caustic soda to remove all traces of grease, and then washed before being placed in the electrolytic bath. The articles are then suspended by conducting holders and the current passed in such a direction that the article is kathode. The current must not be excessive, or the deposit will not be hard and adherent. After sufficient deposition has taken place, the article is washed and finally burnished to give it the familiar bright metallic finish

For gold plating the process is similar to the above, but the solution is obtained by dissolving gold fulminate, or sometimes gold chloride, in potassium cyanide.

Electrotyping.—In some printing processes a copper reproduction of each page of type is made. A coating of wax is placed upon the set-up type and forced upon it by hydraulic pressure, the type having been previously sprinkled over with powdered graphite to prevent the wax adhering. After removal from the type, the wax mould is coated with powdered graphite to render its surface conducting. To increase this conductivity the surface is washed and a solution of copper sulphate poured upon it. Iron filings are then sprinkled upon it, which displace copper from the solution, so that in a few minutes a very thin layer of copper is deposited upon the surface. The mould is then placed in an electrolytic bath of copper sulphate solution, the electrical contact being so made that the mould is kathode. In the course of an hour or so a layer of copper is formed which is a good copy of the original type. On removing the wax and pouring molten type metal into the copper shell to stiffen it, the block used for the actual printing is obtained.

Simple voltaic cell.—It is well known that a piece of zinc immersed in a dilute solution of sulphuric acid is dissolved with the formation of zinc sulphate and the liberation of hydrogen.

$$H_2SO_4 + Zn = ZnSO_4 + H_2.$$

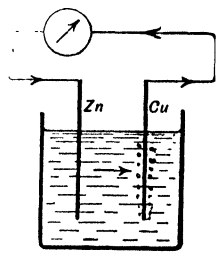


FIG. 845.—Simple cell.

The hydrogen liberated forms bubbles on the surface of the zinc. If a rod of copper be now immersed by the side of the zinc and the two connected externally through a galvanometer, as in Fig. 845, it will be observed that a current will flow in the external circuit from the copper to the zinc, and at the same time the bubbles of hydrogen will appear upon the copper instead of the zinc. The zinc still dissolves, with formation of zinc sulphate, and it is the

energy liberated from this reaction which is available for the maintenance of the current.

Remembering that hydrogen appears at the kathode and SO_4 at the anode, we see at once that the current leaves the cell at the copper electrode The copper is called the **positive pole** of the cell and the zinc the **negative pole.** The copper may be replaced by carbon or platinum with similar results. The original voltaic cell due to Volta was of this type, the arrangement alone being different. Alternate discs of zinc and copper are laid upon each other in a column with pieces of cloth moistened with dilute acid in the following order Beginning at the bottom (Fig 846) is a copper disc, then cloth, then zinc Again comes copper, cloth, zinc and so on. This forms a pile consisting of simple cells of copper-acid-zinc in series. It was by means of such a pile that Volta first demonstrated many of the effects of an electric current.

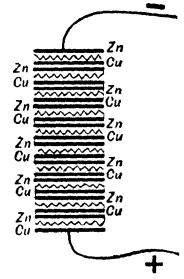


FIG 846 —Voltaic pile

Polarisation. —A simple cell of the above type (Fig. 845) is of little practical utility since in use the current drops rapidly. The cause of this is the layer of hydrogen bubbles which collects upon the copper. There are two deleterious effects of this layer of bubbles. In the first place, the effective area of the copper in contact with the solution is diminished, thereby increasing the resistance of the cell In the second place, the presence of the hydrogen produces an electromotive force which tends to send a current through the cell in the opposite direction to the current actually passing. Hence the effective electromotive force of the cell is diminished by the amount of this e m f which is called a **polarisation e.m.f.**

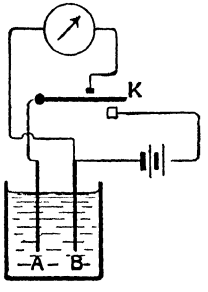


FIG 847 — Polarisation e m f

The presence of this **back e.m.f.** may easily be demonstrated in the following way Two platinum sheets A and B are immersed in dilute sulphuric acid (Fig 847) When the key K is depressed a current passes from A to B. A then becomes coated with bubbles of oxygen and B with hydrogen bubbles. On raising the key, the battery becomes disconnected and the galvanometer

connected to the plates A and B. It will then be noticed that a current flows for a short time, in fact as long as the bubbles remain. The electromotive force is due to the presence of these gases; they play a similar part in forming a cell, to the copper and zinc in the simple cell. The plate on which the oxygen is deposited is the positive pole of the cell and that on which the hydrogen is deposited is the negative pole.

Daniell's cell.—Although cells are never used now for the production of large currents, they are frequently used when small currents are required. Zinc is almost universally employed for the negative pole. Pure zinc does not dissolve when immersed in dilute sulphuric acid unless the battery circuit is complete and a current flows Commercial zinc, however, does dissolve and would consequently waste away when the cell is not in use. The reason for this is that pieces of impurity in the zinc, chiefly iron, form local cells with the bulk of the zinc, which therefore dissolves away. To get over this difficulty the zincs are always rubbed over with mercury, forming an amalgam which presents a surface of uniform composition to the solution and thus prevents local action.

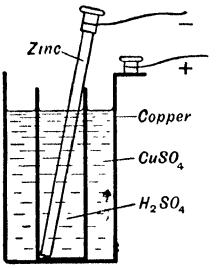


FIG. 848.—Daniell's cell.

To overcome the difficulty of polarisation many devices are used. In the **Daniell's cell** the positive pole is copper and the electrolyte in contact with it is a strong solution of copper sulphate. Sometimes the outer pot is of copper and itself forms the electrode, as in Fig. 848. Sometimes an earthenware jar is used, and a copper plate immersed in the copper sulphate solution. Inside this is a porous unglazed earthenware pot which contains a dilute solution of sulphuric acid, or sometimes zinc sulphate, and in this is immersed the amalgamated zinc rod. The object of the porous pot is to keep the copper sulphate solution and the dilute acid from mixing while allowing contact between them within the pores of the earthenware.

On completing the external circuit, the current flows from zinc to copper within the cell. Thus, at the anode zinc is dissolved,

$$Zn + SO_4 = ZnSO_4.$$

At the same time copper is deposited on the kathode,

$$CuSO_4 = Cu + SO_4.$$

At the junction of the two solutions hydrogen ions from the zinc side and SO_4 ions from the copper side form sulphuric acid:

$$\overset{+}{H_2} + \overset{-}{SO_4} = H_2SO_4.$$

There is consequently no hydrogen liberated upon the copper,

and hence no polarisation. The e m f. of the Daniell's cell is fairly constant and is about 1·1 volt.

Leclanché cell.—In the case of the Daniell's cell there is no polarisation, because copper from the solution is deposited upon the copper positive plate. Other cells are rendered non-polarising by the addition of some oxidising reagent to remove the hydrogen as it is liberated. Thus, in the bichromate cell, potassium bichromate is added to the sulphuric acid solution and carbon is used as the positive plate. In the Bunsen's and Grove's cells strong nitric acid is the oxidising reagent, and surrounds the positive plate, which is of carbon or platinum But the only cell of this type used to any extent at the present time is the **Leclanché cell**, because there are no objectionable fumes from it and it gives a fair current for a short time, and recovers its original e m f after a period of rest.

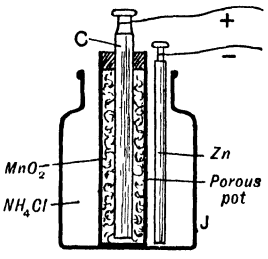


FIG 849.—Leclanché cell

The liquid of the Leclanché cell is a saturated solution of ammonium chloride (sal ammoniac) contained in a glass jar J (Fig 849). In this is immersed the negative pole, consisting of an amalgamated zinc rod. The positive pole is a carbon rod, around which is closely packed a mixture of manganese dioxide and crushed gas carbon, contained in a porous pot which is cemented with pitch at the top The sal ammoniac solution diffuses through the porous pot and the manganese dioxide mixture and reaches the carbon rod. When the current passes, zinc is dissolved at the negative pole according to the equation

$$2Cl + Zn = ZnCl_2$$

The ions of NH_4 are liberated at the positive pole and there form ammonia (NH_3) and hydrogen,

$$2(NH_4) = 2NH_3 + H_2.$$

The ammonia escapes and the hydrogen is gradually oxidised by the manganese dioxide, $2MnO_2 + H_2 = Mn_2O_3 + H_2O$

On the disappearance of the hydrogen the cell regains its original e m f. of about 1·5 volt.

On account of its power of recovery after use, the Leclanché cell is largely used for telephones and electric bells.

Dry cells.—For the sake of portability many forms of Leclanché cell have been constructed in which there is no free liquid present. In most of these there is a paste containing manganese dioxide surrounding a carbon rod. This is in contact with a layer of sawdust, or in some cases plaster of paris, saturated with sal-ammoniac. The

whole is contained in a zinc case which forms the negative electrode. Sometimes the outer case is of millboard, a zinc rod serving as electrode.

Standard cells.—Cells in which polarisation occurs are useless as standards of electromotive force, since they polarise, and therefore vary in electromotive force, directly a current flows. On referring again to the Daniell's cell (p 911) it is seen that there is no polarisation, and the electrodes do not change in character when the current flows. The reason for this is that the solution in contact with each electrode contains as active ion the same metal as the electrode itself. Also, if the cell runs for some time, zinc is dissolved, forming zinc sulphate, and copper sulphate is decomposed with deposition of copper. If a current be now sent through the cell in the reverse direction by any external means, copper is redissolved and zinc is deposited. Thus a reverse current can bring the cell back to its original condition. Such a cell is said to be **reversible.** Reversible cells have fairly constant electromotive force. The Daniell's cell may, for rough purposes, be used as a standard of approximately 1·1 volt. More reliable standards must, however, be used for exact measurements. Such reliable standards are the **Latimer Clark cell** and the **Cadmium or Weston cell.**

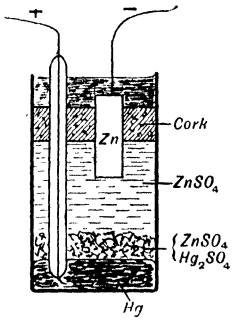


FIG 850.—Latimer Clark cell.

Latimer Clark cell.—In this cell a pool of mercury forms the positive and a pure zinc rod the negative electrode. Upon the mercury (Fig 850) rests a layer of paste made of mercurous sulphate and a saturated solution of zinc sulphate. Upon this rests a saturated solution of zinc sulphate, into which dips the zinc electrode. In order to make sure that the solution is saturated, some crystals of zinc sulphate are placed upon the top of the paste. Contact is made with the mercury by means of a platinum wire sealed into a glass tube, so that the tip of the wire projects into the mercury. The glass tube and the zinc electrode are kept in place by a cork, and the vessel is sealed up by a layer of marine glue.

Since the negative electrode, zinc, is in a solution of zinc sulphate, and the positive electrode, mercury, is in contact with mercurous sulphate, it will be seen that the cell is of the reversible type. If made carefully with pure chemicals, its electromotive force at any given temperature is very constant. It is given by the equation

$$\text{e.m.f.} = 1{\cdot}433 - 0{\cdot}0012\,(t - 15) \text{ volt},$$

where t is the centigrade temperature Thus, at 15° C its electromotive force is 1·433 volt, and that at any other temperature may be calculated

Cadmium or Weston cell.—This is a modification of the Clark cell and is constructed by replacing the zinc by cadmium amalgam and using cadmium sulphate instead of zinc sulphate. The form of the cell adopted by the National Physical Laboratory is given in Fig 851 The electromotive force of this cell is given by

$$\text{e m f.} = 1{\cdot}0183 - 0{\cdot}0000406\,(t - 20) \text{ volt}$$

at t° C. Thus, at 20° C the electromotive force is 1·0183 volt. Owing to its very small variation of voltage with temperature this cell has been adopted as the international standard of electromotive force

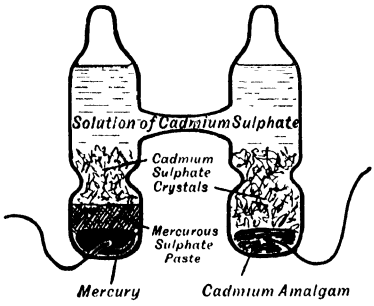


FIG 851.—Standard cadmium cell

In using the Latimer Clark or the cadmium cell great care must be taken not to allow more than a very minute current to pass through it For this reason it is desirable, when using the cell, to keep a resistance of several thousands of ohms permanently in series with it.

Source of energy and e.m.f. of cell.—It has already been indicated (p. 910) that the source of energy of the current maintained by a cell is the chemical action occurring in it Let us consider the case of the Daniell's cell When 1 ampere flows for 1 second through the cell, 0·00003388 gr. of zinc is dissolved and 0·0003295 gr. of copper deposited. Now when 1 gr. of zinc is dissolved in sulphuric acid in a calorimeter, energy to the extent of about 1630 calories is liberated. This energy for 1 amp flowing for 1 sec is $1630 \times 0{\cdot}0003388 = 0{\cdot}553$ calories $= 0{\cdot}553 \times 4{\cdot}2 \times 10^7$ ergs $= 2{\cdot}32 \times 10^7$ ergs, and this is the energy liberated On the other hand, 1 gr of copper dissolved in sulphuric acid liberates 881 calories, so that 1 amp. in 1 sec. requires an amount of energy of

$$881 \times 0{\cdot}0003295 \times 4{\cdot}2 \times 10^7 = 1{\cdot}22 \times 10^7 \text{ ergs}$$

for the liberation of the copper. The balance of energy

$$(2{\cdot}32 \times 10^7) - (1{\cdot}22 \times 10^7) = 1{\cdot}1 \times 10^7 \text{ ergs}$$

is available for driving the current.

Now, the work done in maintaining the current is $E \times I \times 10^7$ ergs per second, which becomes E ergs per second when $I = 1$ amp.

$$\therefore E \times 10^7 = 1{\cdot}1 \times 10^7,$$

$$E = 1{\cdot}1 \text{ volt.}$$

This reasoning rests entirely upon the assumption that **the energy liberated from the chemical changes is entirely converted into energy of electrical current.** The closeness of the result to the known value of the electromotive force shows that in the case of the Daniell's cell this assumption is very nearly justified. But this is not necessarily always the case. In fact, in some cells some of the energy is liberated directly in the form of heat, and only the remainder is available for maintaining the current. Such cells become hotter when running, and decrease in electromotive force as the temperature rises, as in the case of the Latimer Clark cell (p. 913), and the Cadmium or Weston cell (p. 914). On the other hand, in some cells heat energy is drawn directly from the cell and goes to increase the energy available for maintaining the current. The cell then becomes colder when running, and its electromotive force increases with rise in temperature.

Minimum e.m.f. in electrolysis.—When polarisation occurs, an opposing electromotive force is brought into existence (p. 910). In the case of the electrolysis of water this is of importance, as it follows that any cell whose electromotive force is less than this is unable to maintain the electrolysing current. In the case of water the polarisation e.m.f. may be calculated. When 1 gram of water is formed by the combustion of hydrogen in oxygen there is a liberation of heat to the extent of about 3800 calories (p. 361). We may take this as the amount of energy required to separate the hydrogen from the oxygen for one gram of water. If 1 ampere flows through the electrolytic cell for 1 second 0·00009338 gr. of water is decomposed (p. 906). Hence the energy required for this decomposition is

$$3800 \times 0{\cdot}00009338 \times 4{\cdot}2 \times 10^7 \text{ ergs} = 1{\cdot}49 \times 10^7 \text{ ergs.}$$

Again, the work done in opposition to the polarisation e.m.f. (E) is $E \times I \times 10^7$ ergs per second for I amperes, or $E \times 10^7$ ergs per second for 1 ampere;

$$\therefore E = 1{\cdot}49 \text{ volt.}$$

This is the minimum electromotive force necessary to maintain the current, and it is thus seen why a single Daniell's cell is not sufficient for the electrolysis of water. It would, of course, start a current, but when polarisation begins, the back e.m.f. due to it

rises until it equals the e m f of the Daniell, when, of course, the current will cease

Secondary or storage cells.—In the case of the cells already described, some substance is used up chemically in the production of the current This is expensive, and moreover these cells cannot be used for the production of large currents In 1859 Planté succeeded in making a cell in which energy put into the cell as electric current can be stored up and subsequently drawn again from the cell in the form of current. Such cells are called **secondary** or **storage** cells, or sometimes **accumulators.**

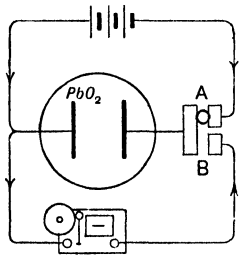


FIG 852.—Principle of the secondary cell

Two lead plates are used as electrodes in a solution of sulphuric acid On the first passage of current through the cell, oxygen is liberated at the anode, which oxidises the surface of the lead plate, forming PbO_2 The hydrogen bubbles away at the kathode

On stopping the current and joining the lead plates by a conductor, a current will pass for a time from the oxidised plate (+) to the unoxidised plate (−) through the external circuit If the experiment be performed as shown in Fig. 852 the key must be placed in A for the charging current to flow, and in B for the discharge. which may be used to ring an electric bell

When the discharge takes place, the oxidised plate is reduced by the hydrogen liberated and forms PbO which, with the sulphuric acid, forms $PbSO_4$ The current passes until the negative plate is oxidised and forms $PbSO_4$, and ceases when both plates have reached the same condition.

If the charging and discharging be repeated for a number of times it will be found that the storing capacity of the cell becomes greater. This is generally carried out not merely by letting the cell discharge, but by reversing the current in it until the kathode is reduced to lead. This reduced lead forms a soft layer upon the plate, known as **spongy lead**, and with repeated reversals of current this spongy lead forms a deeper and deeper layer, and the storage capacity of the cell increases. This process is called '**forming**' the plate.

When the plates have been properly 'formed' the processes of charging and discharging may be represented approximately by the following equations :

Charging.

Positive Plate.

$$PbSO_4 + SO_4 + 2H_2O = PbO_2 + 2H_2SO_4.$$

Negative Plate.

$$PbSO_4 + H_2 = Pb + H_2SO_4.$$

Discharging.

Positive Plate.

$$PbO_2 + H_2SO_4 + H_2 = PbSO_4 + 2H_2O.$$

Negative Plate.

$$Pb + H_2SO_4 + O = PbSO_4 + H_2O.$$

It will be seen that during the process of charging, sulphuric acid is formed, and consequently the density of the electrolyte rises.

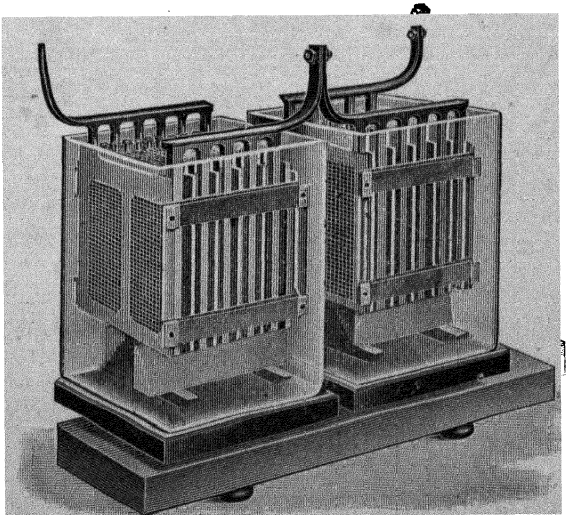


FIG. 852. Secondary cells.

During discharge, sulphuric acid disappears and the density of the electrolyte falls. Observation of the density of the 'acid' gives the best indication of the condition of the cell. It should not exceed 1·21 when the cell is fully charged, nor fall below 1·15 when it is discharged.

The secondary cell of this type has a remarkably constant electromotive force of about 2·1 volts throughout the greater period of its discharge It is usually constructed of a series of parallel lead plates, alternate plates being connected to one of two lead strips at the top which form the leads for the current (Fig. 853). Owing to the high conductivity of the electrolyte and the large area of the plates and their closeness together, the internal resistance is very small, and hence considerable currents can be obtained from them For a 100 volt supply circuit a battery of 50 to 55 of such cells would be used in series

Faure, or paste cell. The secondary cell of the Planté type is costly to produce on account of the long 'forming' required. In order to cheapen the production, Faure constructed the plates in the form of a network or grid, into the interspaces of which a paste consisting of red lead (Pb_3O_4) and sulphuric acid is pressed This forms lead oxide and lead sulphate on both plates,

$$Pb_3O_4 + 2H_2SO_4 = PbO_2 + 2PbSO_4 + 2H_2O$$

On passing the charging current, the $PbSO_4$ on the positive plate is oxidised to PbO_2,

$$PbSO_4 + O + H_2O = PbO_2 + H_2SO_4,$$

while at the negative plate the oxide and sulphate are reduced to spongy lead,

$$PbSO_4 + H_2 = Pb + H_2SO_4,$$

$$PbO_2 + 2H_2 = Pb + 2H_2O$$

Thus the first charging produces the requisite layer of spongy lead, in this way obviating the lengthy and costly process of 'forming'

Paste plates, however, are not so durable as 'formed' plates Some makers use 'formed' plates for the positive and paste plates for the negative Many varieties of both kinds are on the market.

EXERCISES ON CHAPTER LXX.

1. State the laws of electrolysis, and explain what is meant by the electro-chemical equivalent of an element.

Describe how you would determine the electro-chemical equivalent of copper. Sen. Camb Loc.

2. Give an account of the laws of electrolysis; and explain what you mean by electropositive and electronegative elements.

It is found that in 1 minute 40 seconds a certain current deposits 0·112 gm. of silver; and in twice the time 0·081 gm. of potassium. Given that the chemical equivalent of silver is 108, find that of potassium. L.U.

3. What is meant by the *polarisation* of a voltaic cell, and how may this effect be exhibited ?

What, also, is *local action* in such a cell, and how may it be diminished ?

L.U.

4. What is *electrolysis* ? Describe an experiment in which occurs what is commonly called the 'electrolysis of water.' Give your view of the appropriateness or otherwise of this way of describing the experiment and its results. L.U.

5. Explain how the electromotive force of a Daniell cell can be calculated from the chemical changes that take place therein when a current flows. How can the electromotive force of two cells be compared by experiment ?

L.U.

6. A water voltameter, a conductor in a calorimeter, and a tangent galvanometer are connected in series. A current causes an evolution of 10 cubic centimetres of hydrogen, a rise of 4° C. in the same time, and a deflection of 10° in the galvanometer. The current is then doubled. Describe the effect in each part of the circuit.

7. State the laws of electrolysis.

A circuit includes a silver voltameter and a tangent galvanometer of 20 turns of 16 cm. diameter. If the galvanometer shows a steady deflection of 45° and if ·115 gm. of silver is deposited in 15 minutes, find the strength of the earth's horizontal magnetic field.

(Electro-chemical equivalent of silver = 001118 gram per coulomb.)

L.U.

8. State Faraday's laws of electrolysis.

Point out the most important differences between electrical conduction in metals and in solutions. L.U.

9. State and explain Faraday's laws of electrolysis. A tangent galvanometer was joined in series with a battery and a silver voltameter. The deflection of the needle was 45°, and in the course of an hour the mass of silver deposited was 0·1052 gr. Given that the electro-chemical equivalent of silver is 0·001118, calculate the constant of the galvanometer.

L.U.

10. Describe the parts of a storage cell or accumulator, and state the changes that occur in them during the process of charging and discharging. Why is it important that the voltage of the cell should not be allowed to fall below 1·9 ? L.U.

11. A copper voltameter and a wire of resistance 2 8 ohms immersed in 350 grams of water are in series and a current passes through them. If 0 86 gr. of copper is deposited in 18 minutes, find the rise in temperature of the water in half-an-hour.

12. The coil of a tangent galvanometer, having 12 turns and radius 15 cm., is placed in series with a copper voltameter. If the deflection is 60° and H = 0 18, calculate the amount of copper deposited in 15 minutes.

13. Describe the Leclanché cell and give an account of the chemical reactions occurring in it. For what purposes is it chiefly used and why ?

14. Give a description with sketch of either the cadmium cell or the Latimer Clark cell. For what purpose is such a cell used, and what precautions must be taken in using it ?

15. A current is passed for 45 minutes through acidulated water and the hydrogen liberated is dried and collected over mercury. If the volume of hydrogen at 68 cm. pressure and 15° C. is 430 cc. and an ammeter through which the current also passes reads 1 2 amp., what is the error of the ammeter ?

CHAPTER LXXI

STATIC ELECTRICITY: ELECTRIC CHARGES

Charge of electricity.— In the previous chapters electric currents have been treated, the name itself implying something flowing along the conductor, to which the name electricity or electric charge is given. Its properties at rest are quite different from those exhibited when it is flowing in the form of a current. Thus, there is no magnetic field due to it, no heat produced, and no electrolysis unless it is moving.

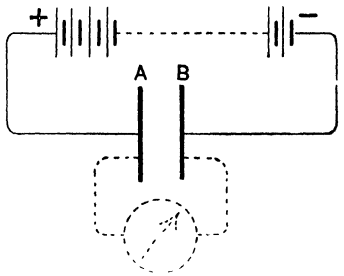


FIG. 854.- Existence of electric charge.

If a great many cells be joined together in series, and the positive pole connected to a conductor A (Fig. 851) and the negative to B, no current flows until A and B are connected together by a conductor. If, however, A and B are joined to the poles of the battery, and the battery connections **broken** and the two **afterwards** connected by a wire, a current will flow through the wire **for a short time**, and may be detected if a delicate galvanometer be in series. For the success of the experiment 100 or more cells are necessary in the battery, and the conductors A and B should be large plates, close together but not touching. Otherwise the current will be too small to be detected.

The explanation is that the battery tries to produce a current, that is, to cause electricity to circulate in the circuit. Since there is no complete circuit, electricity, or electric charge accumulates upon A and B. That on A is said to be positive electricity (+) and that on B negative electricity (−). On breaking the battery connections these charges remain, and on connecting A and B the charges flow through the connecting conductor until they are used up, their flow constituting the current.

Forces acting on charges.—In the above case, the current flows from A to B This means that the positive charge on A flows through the conductor to B, or the negative charge flows from B to A, or both processes go on. On connecting A and B to the battery, the electromotive force of the battery causes a difference of potential (p 852) between A and B. Also, we know that the positive charge upon A is acted on by a force driving it towards B, and will travel if a conducting path is provided for it Similarly, the negative charge on B is urged towards A, and the question arises, can these forces be discovered without joining the two by a conductor ?

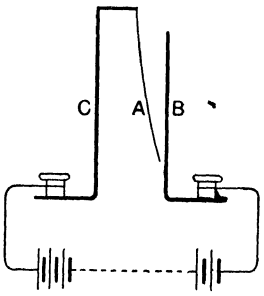


FIG 855.—Attraction between opposite charges

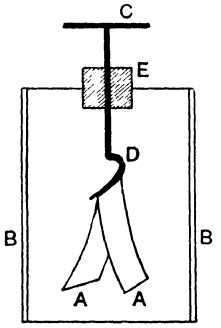


FIG 856.—Gold-leaf electroscope

Let B be a fixed piece of metal (Fig 855) and A a strip of gold leaf suspended from a metal carrier C Then, on connecting C and B to the battery, it will be noticed that the gold leaf A is displaced towards the fixed conductor. The same thing happens if the poles of the battery are interchanged. Thus A experiences a force urging it towards B, and B a force urging it towards A.

We may thus say that **positive and negative charges attract each other, or that a positive charge experiences a force driving it from points of higher to points of lower potential, and a negative charge experiences a force driving it from points of lower to points of higher potential.** Both statements amount to the same thing, as we shall see later. It is advisable to place a piece of paper or a piece of mica between A and B, so that if they happen to touch no harm will be done.

Electroscope.—There is another convenient form of apparatus for exhibiting the forces on charges, which is commonly used for their detection. Two strips of gold leaf AA (Fig. 856) hang from

a wire support CD, which passes through a stopper E in a box. E is made of sulphur or paraffin wax, which is non-conducting. BB are two strips of tinfoil pasted on to the sides of the box, which should have glass windows at the back and front to allow observation of the gold leaves. On connecting C to one pole of the battery and B to the other, a slight divergence of the leaves may be detected, since each of the leaves A is urged towards the nearest strip B, as explained on p. 922. The leaves are driven apart, and we may say if we choose, that the like charges on the leaves repel each other. This apparatus is called a **gold-leaf electroscope**, although the leaves may be made of any thin metallic foil.

Production of electric charges by friction.—Electric charges are nearly always produced when two substances are rubbed together, but if the substances are conducting, the charges escape, and therefore cannot be detected. If, however, a rod of ebonite be rubbed with a piece of fur, both substances being non-conductors, the ebonite will be found to have a negative charge and the fur a positive charge. Both the fur and the ebonite must be thoroughly dry; if they are damp the charges escape. The charges may be detected by bringing the bodies in turn into contact with the cap of an electroscope, when they impart part of their charge to the electroscope and the leaves diverge. Glass rubbed with silk exhibits a similar phenomenon, the glass acquiring a positive and the silk a negative charge. The method of deciding the sign of the charge is given later (p. 934).

Conductors and insulators.—It was seen in Chap. LXVII that substances differ very much in electrical conductivity. Thus the metals are excellent conductors, while wood, cotton, etc., are such poor conductors that it might almost be said that they will not carry a current of electricity. This is not strictly true, but the current they will carry is so minute that it can hardly be detected by any of the means used for detecting a current. Nevertheless, if an electroscope be charged (p. 934) and the cap touched with one end of a wooden rod held in the hand it will be found that the leaves soon collapse, showing that the rod has conducted the charge away. The same result follows if a cotton thread be used instead of the wooden rod.

If, however, a stick of sulphur or paraffin wax be used, the charge will not sensibly leak away through it. Substances which will not allow a charge to pass through them at all are called **insulators**. Thus wood, cotton, the human body, etc., are very feeble conductors, but sulphur, paraffin wax, sealing wax, silk, lead-glass. fused quartz, amber, etc., are insulators, provided that their surfaces are kept clean and dry.

The electricity circulating in a conductor, and constituting the electric current produced by a battery, is identical with that collected on the surface of a body by friction, but whereas the battery can produce larger quantities of charge at low potential, only very small quantities can be produced by friction, but these are at relatively enormous potentials Evidence of this fact may be found by bringing the hand near the surface of an ebonite rod which has been rubbed with fur A spark will take place between the hand and the ebonite. This means that the difference of potential between the two is sufficient to cause a current to flow, for an instant, through the air between them, and from our experience of batteries we know that the potential difference required to make electric charge jump an air gap is very great.

Law of force between charges. We saw on p 922 that charges of opposite kinds attract each other Now, in considering the electroscope (p 922) it was observed that the gold leaves move apart when they have charges of the same sign, and hence we may consider that they repel each other. Thus we may describe the behaviour of charges to each other by saying that **charges of the same kind repel each other, and charges of opposite kinds attract each other.** This is closely analogous to the behaviour of magnetic poles (p 770) and may be exhibited in a similar manner

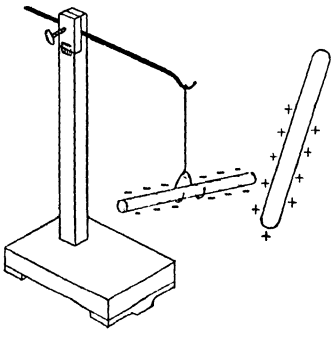


FIG 857.—Force between charges

Rub a piece of ebonite rod with fur and suspend it in a stirrup by means of a **silk** fibre (Fig. 857) Bring near it a glass rod rubbed with silk. It will be observed that the glass attracts the ebonite. If the silk be thoroughly dry, it will, after rubbing the glass, be found to repel the ebonite Thus the charges of like sign repel each other and unlike charges attract each other

The law of force between charges is also similar to that between magnetic poles. That is, **for two small charges, the force between them varies inversely as the square of their distance apart**

There is no means of proving the law of force directly by a single experiment. Coulomb, using a torsion balance, proved it very roughly, but this proof is far from satisfactory. The best evidence

of the truth of the law is that innumerable calculations based upon it are found to be in accordance with experimentally ascertained facts.

The force between two charges also depends upon their magnitudes, and the treatment of the force between charges may begin with the relation

$$\text{Force} \propto \frac{q_1 \times q_2}{d^2},$$

where q_1 and q_2 are the magnitudes of the charges, and d is their distance apart

Unit of electric charge.—The above law holds whatever the unit in which the charges are measured, but in order to obtain the force in absolute measure, the charges q_1 and q_2 must be given in terms of some definite unit. It is usual to choose the unit of charge so that when the distance between unit charges is 1 centimetre, the force between them is 1 dyne. Hence **the unit charge is such that when placed one centimetre from an equal charge the force between them is 1 dyne.** Thus the force in dynes between any two charges q_1 and q_2 is now given by the relation.

$$F = \frac{q_1 q_2}{d^2} \text{ dynes.}$$

EXAMPLE.—Find the force upon a positive charge of 16 units when placed midway between a positive charge of 10 units and a negative charge of 20 units situated 8 cm. apart.

Force on $+16$ due to $+10 = \frac{16 \times 10}{4^2} = 10$ dynes.

Force on $+16$ due to $-20 = \frac{16 \times 20}{4^2} = 20$ dynes;

$\therefore$ **Total force on charge $+16 = 10 + 20 = 30$ dynes.**

The above definition of unit charge assumes that the charges are situated in a vacuum, and is sufficiently accurate for all ordinary purposes if the changes are situated in air. When situated in other media the force will not be the same (p. 951).

Electric field.—As in the case of magnetism, it is of the greatest convenience to be able to map out the **electric field** at any point (p. 778). The direction of the field at any point is, of course, the direction in which a positive charge situated at that point would be urged. In fact, **the field at any point may be completely defined in magnitude and direction by the force upon unit positive charge situated at that point.** Hence, in order to calculate the strength of field at any

point, due to a number of charges, we must imagine a unit positive charge placed at that point, and find the resultant of the forces on it due to the separate charges The term **electric intensity** is also used to denote this quantity, namely, the force on unit positive charge.

EXAMPLE.—Calculate the strength of field, or electric intensity, at a point situated at equal distances of 5 cm. from two equal charges of 50 units, one of which is positive and the other negative, their distance apart being 8 cm.

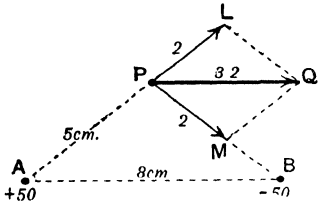


FIG 858.—Problem on electric fields

Let P (Fig. 858) be the point at which the electric intensity is required. Imagine a unit positive charge to be situated at P. Then the electric intensity at P, due to $+50$ units $= 50/5^2 = 2$, and may be represented by PL. Also, that due to -50 is also 2, and may be represented by PM. The resultant PQ is then the required electrical intensity and is obviously parallel to AB From the similar triangles PLQ and APB,

$$\frac{PQ}{PL} = \frac{AB}{AP'}$$

or,

$$\frac{PQ}{2} = \frac{8}{5};$$

$$PQ = \frac{2 \times 8}{5} = \underline{3\,2} \text{ dynes.}$$

Lines of force.—Since a positive charge situated in an electric field is subject to a force in the direction of the field, it will, if free to move, travel along some path in the field. The **line along which a free positive charge would move is called an electric line of force.** The whole electric field in the neighbourhood of electric charges may be considered to be mapped out by lines of force, exactly as in the magnetic case (p. 779). For two small equally and oppositely charged spheres the lines of force are as shown in Fig. 859. All the lines of force arise upon the positive charge and end on the

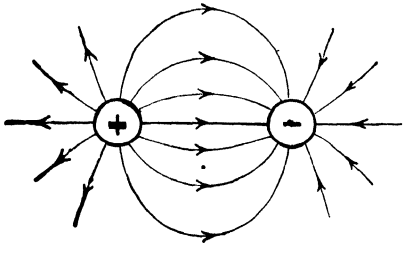


FIG 859.—Lines of force, two unlike charges.

negative charge, for a free positive charge would be repelled by the positively charged sphere and attracted by the other. When the spheres are both charged positively the lines of force are as shown in Fig. 860. The lines then all arise upon the spheres. They must, however, end somewhere, and in this case will end upon the walls of the room. The case of a single charged conducting

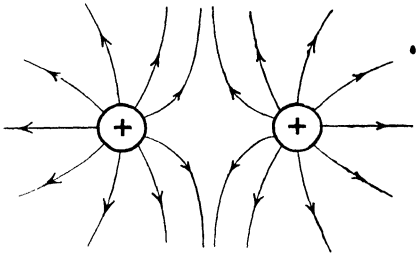


FIG. 860.—Lines of force; two positive charges.

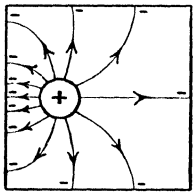


FIG. 861.—Lines of force, positive charge in a conducting enclosure

sphere situated in a cubical space is indicated in Fig. 861. It should be noticed that the lines of force always meet the conducting surfaces upon which the charges are situated so that the angles made are right angles.

It will also be seen that where the electric intensity is greatest, that is, near the charges, the lines of force are more densely packed. In fact, the convention of representing the electric intensity by the number of lines of force per square centimetre may be adopted here, as in the case of the magnetic field (p. 811).

EXERCISES ON CHAPTER LXXI.

1. Describe the gold-leaf electroscope, and the method of using it to detect, and to ascertain the nature of, an electric charge.

2. State and explain the law of electric forces between two point charges. Calculate the position of the point in the neighbourhood of two point charges of +50 and −18 units situated 40 cm. apart, where a third charge would experience no resultant force. L.U.

3. Two small spheres 30 cm. apart have charges $5q$ and $-3q$. Show the distribution of lines of force, before and after the spheres are connected by a wire; and calculate the force exerted between them in each case. L.U.

4. Define unit quantity of electricity.

Small spheres carrying charges +5, +10, +5, and −5 units are placed in order at the angular points of a square ABCD of 10 cm. side. Calculate the force on a charge of +2 units placed at the intersection of the diagonals of the square. L.U.

5. Equal and opposite charges of electricity are placed at two points l cm. apart. Compare the electric intensity midway between the points with that at a distance r cm. from the centre, (a) on the production of the line joining the points, (b) on a line through the centre at right angles to that joining the points. L.U.

6. Describe any two experiments to illustrate the essential identity of frictional and current electricity. What are the differences in the two under ordinary conditions ? L.U.

7. Define unit quantity of electricity.
A small conductor carries a charge of 10 units. Show, by means of a graph, the variation of the force which it exerts on a unit charge, as the latter is moved from 1 cm. distance to 10 cm distance. L.U.

8. Define unit charge of electricity. Two small charged and insulated conducting spheres repel each other with a force of F dynes. Find the repulsion if the charge on one sphere is trebled and the distance between them is also trebled. Allahabad University.

CHAPTER LXXII

POTENTIAL: CAPACITY

Work done in moving a charge.—Whenever a charge of electricity is situated in an electric field it experiences a force, just as a piece of matter in a gravitational field experiences a force. In order to move the charge in opposition to this force, work must be expended upon it, exactly as to lift a body against gravitational force, work is necessary. On the other hand, when the charge is allowed to move in the direction of the force, work is done upon it; similarly, when gravity causes a body to move from higher to lower level, it does work upon it. The analogy between the two cases, gravitational and electrical, is mathematically very close. The law of inverse squares holds in both cases.

Potential.—In the gravitational case, a body when free to move will always move from higher to lower level; in the same way, a positive charge of electricity will, if free to move, travel from a point of higher to a point of lower potential. Potential, then, is analogous to level, and determines the direction in which a charge will travel when free to move. It thus determines the direction of the force upon the charge. Hence, **a positive charge experiences a force tending to drive it from points of higher to points of lower potential, that is, down the gradient of potential.** On the other hand, a negative charge is urged up the gradient of potential, that is, from points of lower to points of higher potential. There is nothing in the gravitational case corresponding to negative electricity.

We define the difference of potential between two points as **the work done in moving a unit positive charge from one point to the other.** Thus, difference of potential is measured in ergs per unit charge. Suppose that a positive charge of 10 units is moved from one place to another where the potential is 20 units higher, work

to the amount of $10 \times 20 = 200$ ergs must be done upon it. If the field moves the charge in the reverse direction it will do work to the extent of 200 ergs upon the charge.

Potential due to electric charge.—In most cases it is very difficult to calculate the electrical potential due to a system of charges, but in some cases it is easy. When there is only one charge to consider, we can imagine a unit positive charge carried from one point to another in its neighbourhood, and since we know the force on it at each point, we can find the work done, and therefore the difference of potential between the two points

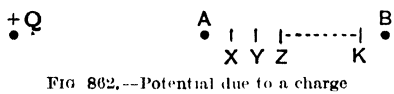


FIG 862.—Potential due to a charge

We will find the difference of potential between A and B (Fig 862) due to the positive charge Q. Call the distance of A from Q, a, from B to Q, b, and so on; then,

$$\text{Force on unit positive charge at A} = \frac{Q}{a^2}.$$

$$\text{,, ,, ,, ,, B} = \frac{Q}{b^2}.$$

The path from A to B must now be imagined to consist of a number of very short steps, AX, XY, YZ, . . , and KB.

Now for the step AX the force on unit charge at A is Q/a^2 and that at X is Q/x^2. Since these forces are very nearly equal, the step being small, we may, without sensible error, take the average force over the path AX to be Q/ax.

∴ Work done in moving unit charge from A to X

$$= \text{force} \times \text{distance}$$

$$= \frac{Q}{ax}(x - a) = \frac{Q}{a} - \frac{Q}{x}.$$

Similarly, for path XY, work done $= \frac{Q}{x} - \frac{Q}{y}$.

,, YZ ,, $= \frac{Q}{y} - \frac{Q}{z}$

etc., etc.,

,, KB ,, $= \frac{Q}{k} - \frac{Q}{b}$.

Then, in going over the whole path AB,

$$\text{Work done} = \frac{Q}{a} - \frac{Q}{x} + \frac{Q}{x} - \frac{Q}{y} + \frac{Q}{y} - \frac{Q}{z} + \frac{Q}{z} - \ldots - \frac{Q}{k} + \frac{Q}{k} - \frac{Q}{b}$$

$$= \frac{Q}{a} - \frac{Q}{b}.$$

However many steps there are in the path from A to B, the only terms remaining in the expression for the total work done in going from A to B are Q/a and Q/b. Hence, if we make the steps infinitely great in number, each step being then infinitesimal, we are justified in writing Q/ax for the average between Q/a^2 and Q/x^2.

Hence **the difference of potential between A and B due to the charge Q is** $\frac{Q}{a} - \frac{Q}{b}$.

There is no means of determining absolute potential, any more than there is of fixing absolute level. For convenience levels are reckoned from mean sea-level. In a similar way we may choose any convenient place to measure potential from. At an infinite distance from the charge q, the force on unit charge due to it is zero, and the charge could be moved about at an infinite distance without doing any work upon it; hence all points at infinity are at the same potential. It is therefore convenient to choose the zero of potential to be that at infinity. The difference of potential between the point A and one at infinity is therefore obtained by putting $b = \infty$, so that

$$\frac{Q}{a} - \frac{Q}{b} = \frac{Q}{a} - \frac{Q}{\infty} = \frac{Q}{a} - 0 = \frac{Q}{a}.$$

Thus, reckoning from the potential at infinity as zero, we should say that the absolute potential at A due to the charge $+Q$ is $+Q/a$.

For some purposes it is convenient to consider the potential of the earth to be zero.

EXAMPLE.—Find the potential at a point 5 cm. distant from each of two charges of $+50$ and -50 units respectively. Also find the potential at a point 10 cm. from the charge $+50$ and 20 cm. from the charge -50.

Referring to Fig. 858.

Potential at P due to $+50 = +\frac{50}{5} = +10$.

„ P $-50 = -\frac{50}{5} = -10$;

∴ potential at P $= 50 - 50 = \underline{0}$.

Again, potential at point 10 cm. from charge $+50 = +\frac{50}{10} = +5$.

„ „ „ 20 „ „ $-50 = -\frac{50}{20} = -2{\cdot}5$;

∴ resultant potential $= +5 - 2{\cdot}5$

$= \underline{2{\cdot}5}$ units.

Equipotential surfaces.—For any given distribution of charge there will be a number of points at which the potential is the same. **A surface drawn through points having the same potential is called an**

equipotential surface. The meaning of the term indicates that all points of an equipotential surface are at the same potential.

It follows from the nature of such a surface that a charge may be moved about upon it without the performance of work, since there is no difference of potential between points on the surface. Further, as regards static electricity, **the surface of a conductor must be an equipotential surface.** For if this were not so, there would be a difference of potential between certain points on the surface, and then a current would flow from the point of higher to that of lower potential, and the conditions would thus not be statical.

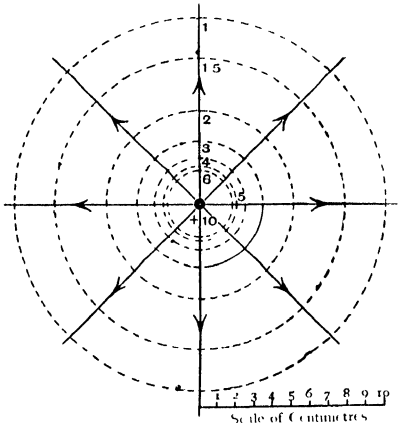


FIG. 863.—Equipotential surfaces and lines of force

It is easy to find the equipotential surfaces near a single charge. For all points at the same distance from it are at the same potential ($v = Q/r$), and thus the equipotential surfaces are spheres, having the charge as centre. The equipotential surfaces and lines of force for a charge of +10 units are shown in Fig 863

Since no work is done in moving a charge along an equipotential surface, it follows that there is no force on the charge in the direction of the surface Therefore, **the direction of the electrical field must always be at right angles to every equipotential surface.** If this were not the case there would be a component of the field parallel to the surface It follows that lines of force always meet conducting surfaces at right angles, as may be seen on examining Fig 863.

Charging a conductor by influence.—In the earlier experiments with the electroscope (p 923) this was charged by touching it with the rubbed ebonite rod. There is a better way than this. Let a positively charged body Q (Fig. 864) be brought near to a conductor AB supported upon a glass rod in order to insulate it. The presence of Q raises the potential everywhere in its neighbourhood ; hence if AB were originally at the potential of earth it will now have a positive potential. But the nearer parts A will be at a higher potential than the more distant parts B($V = Q/r$). Hence a current will flow from A to B, which means that the end B will acquire an excess of positive charge and A of negative charge. This flow will go on until all parts

of AB are brought to the same potential, since when the charges are at rest, the surface of a conductor must be an equipotential surface. Let the conductor be now connected to earth. A current will flow to earth, since the conductor is at a higher potential than earth, owing to the presence of Q. The flow will continue until AB is reduced to the potential of earth (zero) and AB will then retain just sufficient negative charge everywhere to produce a negative potential equal to the positive potential Q, so that the actual potential, being the sum of these, is zero. The distribution of charge will be somewhat as indicated in Fig. 865 (*a*).

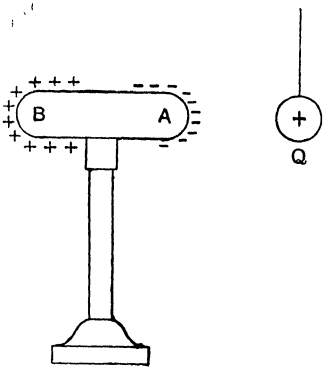


FIG. 864.—Effect of a charged body near a conductor.

Now let the earth connection be broken, and then let Q be removed. The negative charge on AB cannot get away, since AB is now insulated. The charge will distribute itself over the body AB (Fig. 865 (*b*)), which will now have a negative potential due to this negative charge.

Attraction of light bodies.—It is now easy to explain the earliest known electrical phenomenon, namely that a piece of amber rubbed with cloth will attract light bodies, such as small pieces of paper or pith. The amber represents the charged body Q in Fig. 864 and the conductor AB the light body. The presence of the charged body causes a redistribution of charge upon the light body, the charge nearer to Q being opposite in kind to that of Q. The attraction between the unlike charges is greater than the repulsion between the like charges, since these are at greater distance from each other. If the body is then light enough, and free to move, it will, as a whole, be attracted, and move towards Q. The experiment may easily be performed with a rod of ebonite or sealing-wax. The Greek name for amber is **elektron**, and it is from this that the name 'electricity' arises.

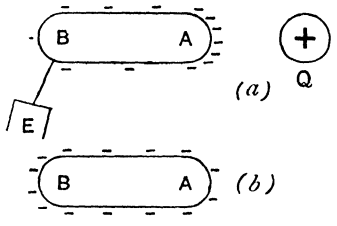


FIG. 865.—Charging by influence

If Q be a charged conductor, the light bodies after striking it will be repelled from it, since they will then be charged by conduction from Q, and having then the same kind of charge, repulsion results.

EXPT. 214.—**The electroscope.** See that the electroscope (p. 922) is perfectly dry. Rub an ebonite rod with fur and bring it near the electroscope. Observe that the leaves diverge. While the rod is still near, momentarily touch the cap of the electroscope with the finger. The leaves collapse because the potential is reduced to zero. Now take away the rod. The leaves diverge because the potential due to the charge upon them is unbalanced by that due to the charge upon the rod. Since the rubbed ebonite has a negative charge, that upon the electroscope is positive.

Thoroughly dry the pieces of fur, silk, and cloth. Also rods of ebonite, sealing wax, glass, shellac, and a brass tube having a glass handle. Take each rod in turn and rub it with each rubber, and in each case test the charge upon the rod and the rubber by bringing each in turn near the electroscope. Note that since the electroscope has a positive charge, its potential is positive, and on bringing a positive charge near it, its potential is raised still further and the leaves diverge more. Bringing a negative charge near it lowers its potential and the leaves collapse.

Tabulate the results as follows:

Rod	Sign of charge	Rubbed with	Sign of charge
Ebonite	−	Fur	+
Glass		Silk	
Brass		Fur	
.....	.....	.....	.....
.....	.....	.....	.....
Etc.	Etc.	Etc.	Etc.

Faraday's ice-pail experiments.—Several very important facts concerning electrified conductors were established by Faraday, hollow conducting cans (such as ice-pails) being used

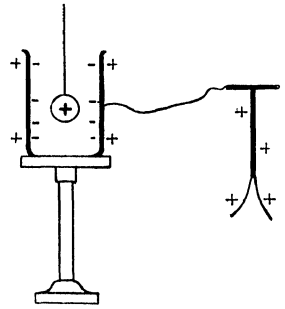


FIG 866.—Faraday's ice-pail experiment Distribution of charge

I. The can, or ice-pail, is placed upon an insulating stand and connected to an electroscope. A charged conductor suspended by a silk thread is then lowered into the can. Assuming that the charge is positive, the can and electroscope will now have a positive potential, and the leaves diverge. The distribution of charge is indicated by signs, and follows that indicated on p. 933. When the body is well inside the can, it may be

moved about without altering the divergence of the leaves of the electroscope. On removing the body the leaves collapse, as the potential then returns to zero.

If, however, while inside, the body be allowed to touch the inside of the can, no alteration of the leaves is observed ; and on removing the body there is still no alteration, the leaves remain diverged and the body, if tested, will be found to have entirely lost its charge. Thus the charge on the body has gone entirely to the can, its own positive charge and the negative charge on the inside of the can having neutralised each other, being, therefore, equal in amount.

Also, since the body, on being removed, after touching the inside of the can, is uncharged, it shows that **no charge resides upon the inside of a hollow conductor.**

II. Let the electroscope be disconnected from the can and positively charged, and let a positive charge be also given to the can, as in the last experiment.

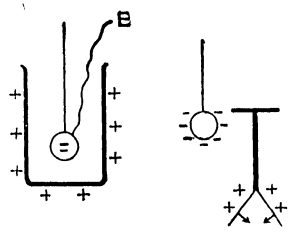


FIG 867 Faraday's ice-pail experiment Distribution of potential

On lowering the suspended uncharged body into the can and then connecting it to earth for a moment by touching it by a wire held in the hand, its potential is lowered, from that of the space inside the can, to zero, a current flowing to earth, thus leaving the body negatively charged. This negative charge may be demonstrated by bringing the body near the positively charged electroscope, when the leaves will wholly or partially collapse (Fig. 867).

It can be shown that the charge produced by earthing the body inside the can is always the same, wherever the body may be situated within the can, provided that it is not near the opening. This shows that the potential is the same throughout the whole of the space in the interior of the can, except near the opening. In fact, **the whole of the space within a closed conductor is at the same potential, and this is the same as the potential of the conductor.**

Proof plane.—It has already been seen that when a conductor is hollow, the charge resides entirely upon the outside. This fact may be demonstrated much more satisfactorily by means of **a proof plane.** This is a small metal plate carried upon an insulating handle. It may be made to coincide approximately with the

surface, the charge upon which it is required to examine Two forms of proof plane A and B are shown in Fig. 868. On placing the proof plane upon the surface, it becomes part of the surface and the

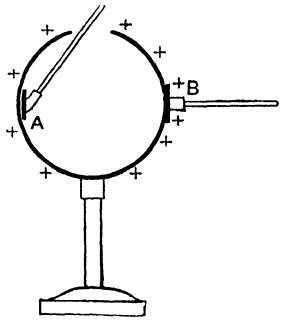


FIG 868.—Proof plane.

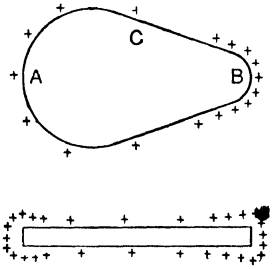


FIG 869 — Distribution of charge on a conductor

charge is shared with it. On taking the proof plane away it carries the charge upon it, and on bringing it near a charged electroscope the charge carried may be examined. When using the hollow conductor in Fig. 868 no charge can be taken from A, but charge can be taken from B; thus showing that, except in the neighbourhood of the opening, no charge resides upon the inside of a charged conductor.

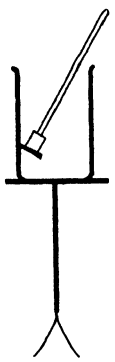


FIG 870—Faraday cylinder.

Distribution of charge on a conductor.—It is only upon a spherical conductor within a concentric conducting space that charge is ever distributed uniformly In the case of a conductor such as AB (Fig 869) the distribution of charge is somewhat as indicated. Where the surface has different curvatures at different parts, the accumulation of charge is greater where the curvature of the surface is greater This may be demonstrated by means of the proof plane, by touching various parts of the surface with it, and examining the charge by means of the electroscope

To obtain some idea of the relative magnitudes of the charges from different parts of the surface, it does not suffice merely to bring the charged proof plane near the electroscope. *Its charge must be given up* completely to the electroscope and the divergence of the leaves noted. For this purpose a can, or hollow vessel, is placed upon the cap of the electroscope

(Fig. 870) and the proof plane with its charge put inside the can, contact with the can being made. It is then certain that the whole charge is given up to the can and electroscope (p. 935). This vessel is sometimes known as the **Faraday cylinder.** If the experiment be performed after touching the conductor (Fig. 869), first at A, then at B, and then at C, it will be found that the surface density of the charge is greatest at B and least at C.

Discharge from points.—Since the density of the charge on a conductor is greater the greater the curvature, it will be seen that at a point the density should become infinite. An absolute point cannot, of course, be obtained, but at a fairly sharp point the density of the charge does become very great. In the neighbourhood of a great density of charge the electric field becomes very great (p. 955). Now air is, as a rule, non-conducting, but for a very strong

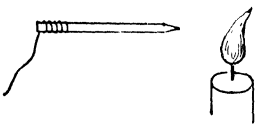


FIG. 871.—Discharge from a point

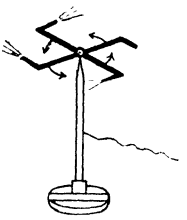


FIG. 872.—Demonstration of electric wind.

electric field this ceases to be the case, and the air becomes a conductor (Chap. LXXX). Hence the charge in this strong field passes away from the point, the carrier of the charge being the air itself, which streams away from the point. This stream is sometimes called an '**electric wind.**' It may be exhibited by connecting a pointed wire to the conductor of an electrical machine (p. 960), which produces a quantity of charge rapidly. The air now streams from the point and may be detected by a candle flame near the point (Fig. 871). A further illustration of this may be given by making four pointed wires into a little wheel (Fig. 872) with the points directed all the same way. On balancing the wheel on a needle point and charging it strongly by an electrical machine, the air streams from the points. The momentum given to the air has its counterpart in momentum given to the wheel, which thus rotates as shown.

Capacity.—Since the presence of an electric charge changes the potential at all points in its neighbourhood, it is obvious that, when a positive charge is placed upon a conductor, its potential is raised. Similarly, a negative charge placed upon a conductor lowers its

potential. Unless otherwise stated, it will be assumed that the charge placed upon a conductor is positive.

In certain cases it is possible to calculate the change of potential of a conductor produced by placing a given charge upon it. But when the conductor has not a simple geometrical form the potential due to a given charge can only be determined by experiment. However, in all cases there is a definite relation between the charge and the potential produced by that charge, and we shall call the ratio of the charge placed upon a conductor to the change in the potential produced by that charge, the **capacity** of the conductor.

Thus, if C is the capacity, Q the charge, and V the potential of the conductor due to that charge, then

$$C = \frac{Q}{V} \quad \ldots\ldots\ldots\

Concentric spheres.— It was seen on p. 935 that when a charged conductor is situated within a hollow conductor, an equal charge of opposite kind is situated upon the inner surface of the hollow conductor. Thus, if a charge $+Q$ be situated upon the sphere A (Fig. 874) of radius a cm. there will be produced a charge $-Q$ upon the inner surface of the concentric sphere B of radius b. This sphere being earthed, its potential is, of course, zero. In fact, the charge upon B must be such that the potential is zero. Now the potential of B due to the charge $+Q$ upon A is $+Q/b$, and the potential due to its own charge is $-Q/b$, the resultant potential of B being zero.

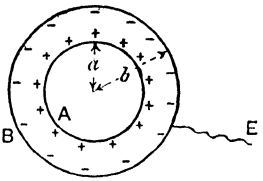


FIG. 874.—Concentric spheres.

Again, the potential of A due to the charge $+Q$ upon it is $+Q/a$. Remembering that the potential within a closed conductor due to the charge upon it is uniform and equal to that of the conductor (p. 935) we see that the potential inside B due to the charge $-Q$ upon it is $-Q/b$.

Hence, resultant potential of $A = \frac{Q}{a} - \frac{Q}{b}$

$$= Q\left(\frac{b-a}{ab}\right).$$

Now charge upon A is Q; hence, by definition

$$\text{Capacity of A} = \frac{Q}{Q\left(\frac{b-a}{ab}\right)} = \frac{ab}{b-a}. \quad \ldots\ldots\ldots\ldots\ldots\ldots (3)$$

Thus the effect of surrounding a sphere by an earthed concentric sphere is to increase its capacity from a to $\frac{ab}{b-a}$. This may be written $\frac{a}{1-a/b}$, which is very nearly equal to a if b is very great; but as b becomes smaller the capacity increases until, when b is very nearly equal to a, the capacity becomes very great.

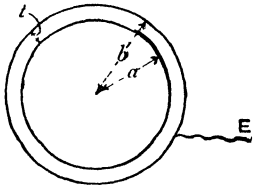


FIG. 875.—Nearly equal spheres

Another way of expressing the capacity when the spheres are of nearly equal radius is of importance. The thickness of the air space is $b - a = t$ (Fig. 875).

Then, $$\text{Capacity of sphere} = \frac{ab}{b-a} = \frac{ab}{t}.$$

When b is very nearly equal to a, a^2 may be written for ab.

$$\text{Capacity} = \frac{a^2}{t}.$$

But the whole area of the sphere is $4\pi a^2$;

$$\therefore \text{ Capacity per square centimetre} = \frac{a^2}{t} \cdot \frac{1}{4\pi a^2}$$

$$= \frac{1}{4\pi t} \quad \ldots\ldots\ldots\ldots\ldots\ldots (4)$$

Parallel plates. Equation (4) only becomes strictly valid when the radius of the sphere is infinite. But in this case the surface is plane. Hence the capacity per square centimetre of a plate A, at a distance t from an earthed parallel plate B, is $1/4\pi t$, and for an area of A square centimetres,

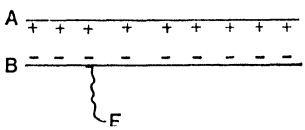


FIG 876 -- Parallel plates

$$\text{Capacity} = \frac{A}{4\pi t} \quad \ldots\ldots\ldots\ldots\ldots\ldots (5)$$

Of course the plates must be of such a great extent that the area considered is not near the edges. Near the edge of any actual pair of parallel plates the charge is not distributed uniformly and the capacity is no longer given by the above simple expression.

It will be noticed that the capacity of the insulated plate increases, the nearer the earthed plate is brought to it. This may be demonstrated by connecting the insulated plate A (Fig. 877) to the electroscope and charging it. The leaves diverge to an extent corresponding to the potential of A. On bringing the earthed plate B nearer to A the divergence of the leaves will decrease, showing that the charge upon A no longer raises it to such a high potential as before. Thus the capacity of A must have increased. On removing B to its original distance the leaves diverge again to their first position, showing that the potential has again risen and the capacity therefore decreased. The arrangement is called a **condenser.**

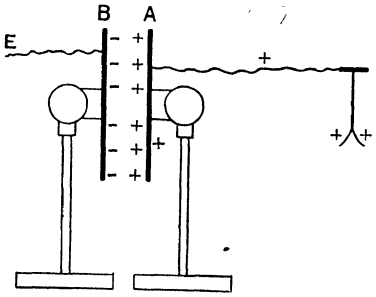


FIG 877 —Plate condenser

Leyden jar. —A convenient form of conductor having large capacity may be made by coating a glass jar A (Fig. 878) with a layer of tinfoil C inside, and another layer B outside. These layers form

approximately parallel sheets, of which C is insulated and B is earthed by standing on the table or being held in the hand. D is a wire conductor which makes contact with C, and is provided with a knob for making contact with external bodies. The capacity of C is much greater than if there were no earthed parallel conductor B. On touching D to the conductor of an electrical machine (p. 961) C will acquire a considerable charge, and on short circuiting D and B by a pair of discharging tongs held by a glass handle, a considerable spark will occur when the air gap becomes small enough. Both the glass jar A and the glass handle of the discharging tongs should be varnished with shellac varnish in order to render them good insulators. The apparatus is called a **Leyden jar.**

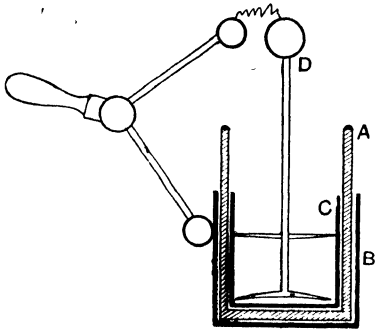


FIG. 878.—Leyden jar.

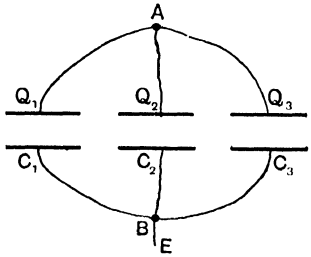


FIG. 879.—Condensers in parallel.

Condensers in parallel.—Let a number of condensers, C_1, C_2, C_3, etc., be joined in parallel between two points A and B (Fig. 879). To find the resulting capacity, imagine a charge Q_1 situated upon the insulated plate of C_1, Q_2 upon that of C_2, etc. Then the potential of A is $Q_1/C_1 = Q_2/C_2 = Q_3/C_3 = V$, these three potentials being the same, since the insulated conductors are joined together;

$$\therefore\ Q_1 = C_1V,\quad Q_2 = C_2V,\quad \text{and}\quad Q_3 = C_3V.$$

Now the total charge Q is

$$Q_1 + Q_2 + Q_3 = C_1V + C_2V + C_3V = Q,$$

$$\therefore\ \frac{Q}{V} = C_1 + C_2 + C_3.$$

But $\frac{Q}{V}$ is the resultant capacity C;

$$\therefore\ C = C_1 + C_2 + C_3. \qquad \text{.....(1)}$$

Thus, **when condensers are joined in parallel the resulting capacity is the sum of the separate capacities.**

Condensers in series.—Let the condensers be connected in series, as in Fig. 880, the last plate of the last condenser being earthed.

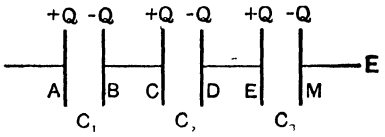


FIG. 880.—Condensers in series

Since the opposite plates of any condenser have equal and opposite charges, when charge $+Q$ is given to A, B will have charge $-Q$. But B and C together are insulated from other conductors, and their total amount of charge, if zero at first, must remain zero. Therefore, charge on $C = +Q$. Similarly, that on D or M is $-Q$, and that on E, $+Q$. Now,

$$\text{Difference of potential between A and B} = \frac{Q}{C_1}.$$

$$\text{,, ,, C and D} = \frac{Q}{C_2}$$

$$\text{,, ,, E and M} = \frac{Q}{C_3},$$

$\therefore$ Total difference of potential between A and M

$$= \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3} = V$$

But if C is the resultant capacity between A and M,

$$V = \frac{Q}{C};$$

$$\therefore \frac{Q}{C} = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3},$$

and

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

Therefore, **for condensers in series (sometimes said to be in cascade) the reciprocal of the resultant capacity is the sum of the reciprocals of the separate capacities.**

Energy of charge.—From the occurrence of a spark when the Leyden jar is discharged, it may be inferred that the jar has energy stored in it when charged. In fact, whenever a conductor is discharged, work is being done, a strict measure of which is the energy dissipated in the form of heat, or otherwise, when the current flows. It is possible to calculate the energy associated with any charge when we know the potential at the place occupied by the charge.

Let us imagine that a conductor is at first at zero potential and without charge. From the definition of potential (p. 929) it will then be seen that to bring up a very small charge from infinity and put it on the conductor requires an amount of work depending upon the potential of the conductor. It is zero if the potential is at first zero. But after placing the charge upon the conductor its potential will no longer be zero. If the charge upon the body and the potential be plotted in the form of a graph (Fig 881), then to bring an additional small charge q, when the potential is v, the work is $q \times v$, and is represented by the rectangular strip of height v and width q. Continuing this process for the whole increase of charge from zero up to Q, the work done is represented by the sum of all such strips as qv, and is the area OAB when the strips are made sufficiently narrow. Since the potential is proportional to the charge, OB is a straight line, and therefore the total work done is $\frac{1}{2}$OA × AB But if OA is the final charge Q, and AB the final potential V,

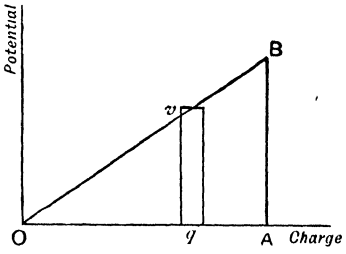


FIG. 881.—Energy diagram.

$$\text{Work done, or energy of charge} = \tfrac{1}{2}QV \text{ ergs.}$$

Remembering that $Q = CV$, we have

$$\text{Energy of charge} = \tfrac{1}{2}QV = \tfrac{1}{2}CV^2 = \tfrac{1}{2}\frac{Q^2}{C} \text{ ergs.}$$

This energy is available to produce the current when the conductor is discharged. In most cases it appears entirely in the form of heat in the wire carrying the current, or in the case of a spark discharge, owing to the high temperature, light and also sound may be produced.

Loss of energy on sharing charge between two conductors.—Let the charge Q be situated upon a conductor A of capacity C_1. Then on connecting A to a second conductor B of capacity C_2, the charge becomes shared between A and B by some of it flowing from A to B. At first the potential of A was Q/C_1, but when connected to B the potential is $Q/(C_1+C_2)$, which is less than Q/C_1. Hence the same charge Q is situated upon a conductor of less potential than previously, and the energy must therefore be less. The energy has been used up in driving the charge through the conductor connecting A and B, which is, of course, heated in the process.

Practical unit of capacity (the farad).—In practice it is useful to have a unit of capacity founded upon the volt and the coulomb.

Thus, a capacity such that a charge of 1 coulomb raises the potential to 1 volt is said to be **1 farad.** The farad is rather large for ordinary purposes, so that the standards usually constructed for laboratory purposes are one-millionth of a farad This is called a micro-farad ;

$$\therefore \ 1 \text{ farad} = 10^6 \text{ micro-farads.}$$

The farad is founded upon the electro-magnetic system of units, being derived from the volt and ampere There is, however, a relation between this unit and the electrostatic unit of capacity, which cannot be discussed here. Suffice it to say that

$$1 \text{ farad} = 9 \times 10^{11} \text{ electrostatic units of capacity.}$$

EXERCISES ON CHAPTER LXXII

1. Describe a Leyden jar and explain its action.

If you were given two Leyden jars, a means of charging them at a constant potential, and a gold-leaf electroscope, how would you determine which jar has the greater capacity ? Sen. Camb. Loc.

2. An insulated sphere of radius 26 cm is surrounded by a concentric earthed sphere of radius 30 cm Find the charge required to raise the potential of the insulated sphere to 30 units, and also the energy of the charge.

3. How would you show that there is no electric charge on the inner surface of a charged insulated hollow conductor ?

A conductor A has a capacity of 10 and a potential of 50 ; another conductor B is of capacity 6 and potential 65. Calculate the charges on A and B after they have been connected by a long thin wire.

4. Describe how it may be shown experimentally that there is no free charge on the inner surface of a hollow charged conductor.

Two equal spheres of water, having equal and similar charges, coalesce to form a larger sphere. If no charge is lost, how will the surface density of electrification change ? L.U.

5. What is meant by the electrical capacity of a condenser, and on what does it depend.

Two Leyden jars of capacity C_1 and C_2 are respectively connected (*a*) in parallel, (*b*) in series ; find the resultant capacity in each case. L.U.

6. Two concentric metal spheres are insulated from earth and from one another and a charge of $+e$ is given to the inner sphere. What will be the electrical condition of the outer sphere ? How will it be changed (*a*) by connecting the outer sphere to earth momentarily, and (*b*) by afterwards connecting the inner sphere to earth ? L.U.

7. Explain how the distribution of a charge of electricity over the surface of a conductor depends on its shape, and describe how you would verify your statements.

Explain the action of the pointed rods used as collectors on electrical influence machines. L.U.

8. Show how the energy of a charged condenser depends on its charge and the potential difference between the plates.

What is the energy of a sphere of radius r when charged with Q units of electricity? L.U.

9. Define electrostatic potential.

Obtain an expression for the potential at a point due to a spherical conductor charged with q units of electricity.

Two small conductors 30 cm. apart have positive charges of 10 and 20 units respectively. Calculate (a) the force, (b) the potential, at a point midway between them. L.U.

10. Define the term "capacity of a conductor."

Calculate the capacity of an insulated conducting sphere 24 cm. in diameter, surrounded by a concentric earthed sphere 26 cm. in diameter.

What is the energy of a charge of 5 electrostatic units communicated to the insulated sphere? L.U.

11. Define the electrostatic and the electromagnetic units of capacity.

Three equal condensers joined in parallel and connected to a cell of e.m.f. 2 volts produce 1·8 microcoulombs on discharge through a galvanometer. What would they produce if joined in series or cascade? What is the capacity of each condenser? L.U.

12. A condenser whose capacity is 3020 units has a potential of 35 units. Calculate the potential when this condenser is connected to another of capacity 4530 units.

13. The insulated conductor of a spherical condenser A has a charge of +40 units and that of a similar condenser B has a charge of +25 units. The spheres of A have radii 18 cm. and 20 cm. and those of B 45 cm. and 50 cm. Find the direction of flow of the current on connecting the insulated sphere of A to that of B.

14. In Question 13, calculate the total energies of the charges before and after connecting the insulated spheres together.

15. A parallel plate condenser has area 35 sq. cm., a charge of 50 units, and the distance between its plates is 3 mm. Calculate the energy of the charge.

CHAPTER LXXIII

THE ELECTROMETER · DIELECTRICS

Gold-leaf electroscope used as an electrometer.— An **electrometer,** as its name implies, is an instrument for measuring electrical potential, whereas the ordinary gold-leaf electroscope as usually employed will only indicate a rise or a fall of potential, without measuring it The electroscope, however, has been made into an instrument capable of fair accuracy for measuring potentials by Mr C T. R. Wilson

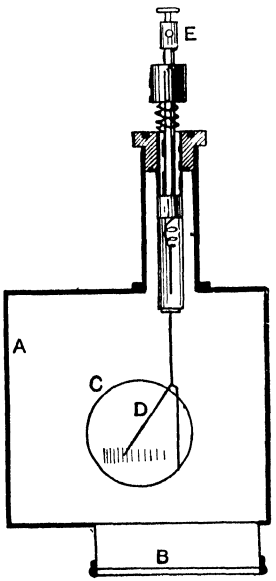


FIG 882 —The Wilson electroscope

The leaf of gold, or aluminium D, is supported by a wire (Fig 882) and connection can be made between it and the conductor E by means of a piece of wire carried by a spring. On depressing E, the wire comes into contact with the wire supporting the leaf, making metallic connection. The tube carrying the conductors is insulated by an ebonite plug. The leaves are surrounded by the brass box A, having a circular window through which they are observed by a short-focus telescope having a finely divided scale in the eyepiece. The position of the image of the leaf on the eyepiece scale indicates the potential of the leaf. It is necessary, however, to calibrate the scale by applying known potentials and noting the positions of the leaf, before any readings of potential can be made by the instrument.

Quadrant electrometer.—For accurate measurement of small potentials the **quadrant electrometer** is much superior to the gold-leaf electroscope. This consists of four hollow quadrants AABB (Fig. 883) with a paddle-shaped conductor C (sometimes called the 'needle') hanging within them. This paddle is carried by a wire support W to which a small concave mirror is attached, the whole being suspended by a quartz fibre. This arrangement enables the deflection of the paddle to be observed, exactly as in the case of the reflecting galvanometer (p. 864). In Fig. 884 the arrangement of a quadrant electrometer is shown. The quadrants Q, are supported upon amber blocks A, and two of the quadrants are drawn back so that the paddle inside them can be seen. F is the quartz fibre and M the mirror. K is a conductor through which the paddle can be charged before any measurements are made.

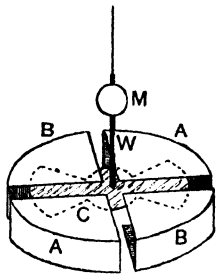


FIG. 883.—The quadrant of the electrometer.

When in use the paddle hangs symmetrically between the quadrants when A, B and C are all at zero potential (Fig. 883). The quadrants AA are connected together by a wire so that they must always have the same potential, which we shall call V_A; similarly, BB are connected together, and their potential will be called

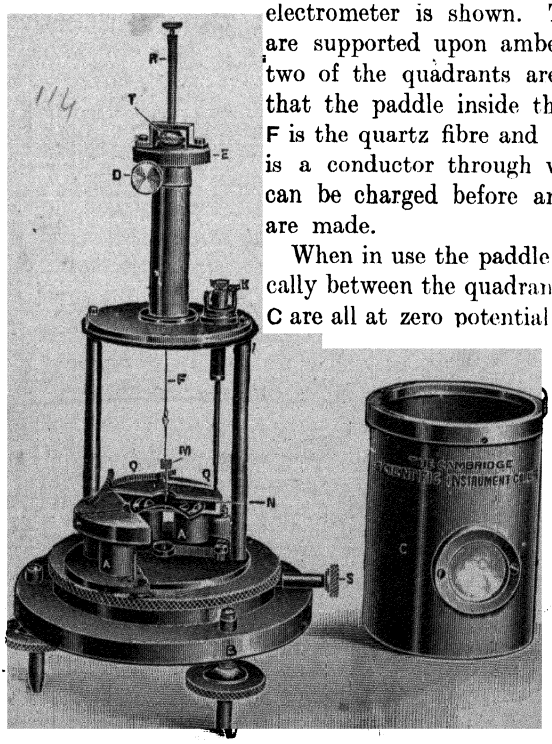


FIG. 884.—The quadrant electrometer.

V_B. It can then be shown that with these potentials the needle will be deflected from its zero position by an amount θ, where

$$\theta \propto (V_A - V_B)\left(V_C - \frac{V_A + V_B}{2}\right),$$

V_C being the potential of the paddle In practice V_C is always a much higher potential than V_A or V_B, so that when these are small the term $\left(V_C - \frac{V_A + V_B}{2}\right)$ is practically constant, and we may write

$$\theta = K (V_A - V_B),$$

where K is now a constant, to be determined by experiment It is thus seen that **the deflection is proportional to the difference of potential between the pairs of quadrants.**

That the needle will undergo a deflection may be seen from Fig. 885. Suppose the paddle to be positively charged and the pair

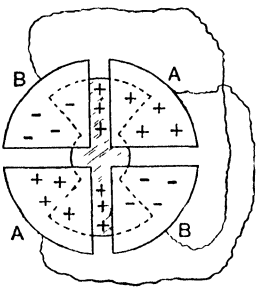


FIG. 885.—Charges on the quadrants

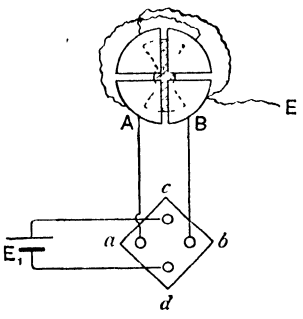


FIG 886.—Use of the electrometer.

of quadrants AA positively and BB negatively charged. It will be noticed that the upper half of the paddle in the figure will experience a force driving it towards the left, and the lower half a force driving it towards the right. This is equivalent to a couple, and the paddle will rotate until the opposite couple due to the twist in the suspension fibre brings it to rest.

Note that if the quadrants B are connected to earth they may be taken to be at zero potential, and then the deflection is proportional to the potential of A, thus $\theta = KV_A$.

EXPT. 215.—**Comparison of e.m.f.'s of cells.** Connect the case of the electrometer to earth by means of a piece of copper wire bound round one of the supply water-pipes in the laboratory or outside. Connect

the A quadrant to (a) one of four holes (a, b, c and d) (Fig. 886) drilled in a piece of paraffin wax and filled with mercury. Connect B to b, taking care that in each case the connecting wires are well insulated. Connect the paddle of the electrometer for a moment to one terminal of a 100 volt supply, the other terminal being earthed. These connections should only be made by the student under supervision. Now let B be also connected to earth. To adjust the instrument, join a and b by a short wire connection. All the quadrants are now earthed. The instrument should now be levelled and the spot of light adjusted to zero by turning the suspension head. When all is ready, join one of the cells, E_1, whose e.m.f.'s are to be compared, to c and d. Place connectors to join a and c and also b and d, and observe the deflection. Now remove the connectors to join a and d, b and c. The deflection is reversed and should be again noted.

Replace the cell E_1 by another E_2 and repeat the observations. Tabulate the results as follows:

Name of cell	Deflection to right	Deflection to left.	Mean deflection (θ)	E.m.f. of cell

Then, $E_1/E_2 = \theta_1/\theta_2$. If one of the cells be a standard, the e.m.f. of the other can be calculated.

Electrostatic voltmeter.—For certain purposes the needle of the electrometer is connected to one pair of quadrants, and has then the potential of that pair. Let it be connected to the A quadrants. Then $V_C = V_A$, and the expression for the deflection,

$$\theta \propto (V_A - V_B)\left(V_C - \frac{V_A + V_B}{2}\right)$$

becomes

$$\theta \propto (V_A - V_B)^2.$$

The points between which it is required to find the difference of potential are then connected to A and B and the deflection is proportional to the square of the difference of potential. Moreover, since the deflection is proportional to the square, it is in the same direction whether $V_A - V_B$ is positive or negative. Hence, since the deflection is always in one direction, the instrument may be used on alternating-current circuits. If the scale be calibrated to read volts, then on an alternating-current circuit it reads **virtual volts.**

A quadrant electrometer is not mechanically strong enough to be used as a voltmeter. It is therefore generally made with a stout

suspension, and to obtain sufficient sensitiveness, the axle (Fig. 887) has a number of paddles attached, which hang in the spaces between a number of quadrants. A pointer is attached, which moves over a scale calibrated in volts. Such an instrument is usually called a **multicellular electrostatic voltmeter.**

One great advantage of the electrostatic voltmeter lies in the fact that on a continuous-current circuit no current flows in the instrument. In this respect it is an ideal voltmeter (p. 875).

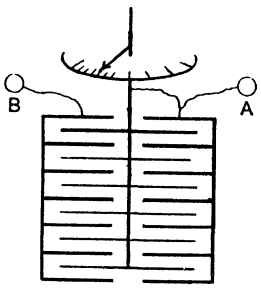


FIG. 887.—Electrostatic voltmeter.

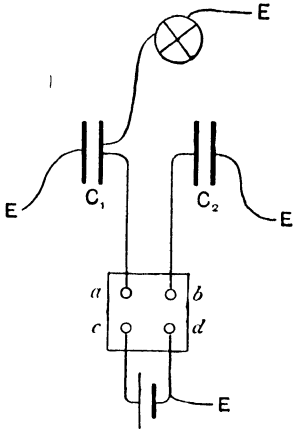


FIG. 888.—Comparison of capacities.

Comparison of capacities.—There are many ways of comparing capacities, but the following is one of the simplest. On giving a charge $+Q$ to the condenser of capacity C_1 (Fig. 888) its potential V_1 is Q/C_1. Now let the second condenser of capacity C_2 be connected to the first, so that the charge is shared between them. Their potentials must now become the same, a charge $+ q$ passing from the first to the second condenser. This lowers the potential of the first from V_1 to V_2, the final common potential;

$$\therefore V_1 - V_2 = \frac{q}{C_1}.$$

At the same time the potential of the second condenser is raised from zero to V_2;

$$\therefore V_2 = \frac{q}{C_2};$$

$$\therefore C_1(V_1 - V_2) = C_2 V_2,$$

or,

$$\frac{C_1}{C_2} = \frac{V_2}{V_1 - V_2}.$$

If the potentials V_1 and V_2 are measured by means of the quadrant electrometer, the ratio of the capacities can thus be found.

EXPT. 216.—**Comparison of capacities.** Set up the electrometer as in Expt. 215. Connect the insulated plate of each condenser to one terminal *a*, *b* of the key, the other two terminals, *c* and *d*, being joined to a cell. The earth connections are shown in Fig. 888. Connect *a* and *c*, thus charging the condenser C_1 to potential V_1, and observe the electrometer deflection θ_1, while terminal *b* is earthed. Insulate *b* and remove the connector from *c* to *b*, so that the charge in C_1 is shared with C_2, and again note the deflection θ_2. Then, since the deflections are proportional to the potentials,

$$\frac{C_1}{C_2} = \frac{V_2}{V_1 - V_2} = \frac{\theta_2}{\theta_1 - \theta_2}.$$

Dielectrics.—In all the discussions so far, it has been considered that the charged conductors have been surrounded by air. The air, however, might be replaced by any non-conducting medium without altering the distribution of the charges. If the medium between the charges were conducting, the charges would at once move in the direction of the electrical field at each point, and would continue to move until the potential throughout the whole of the medium became uniform. Thus there would no longer be any electrical field in the medium. Hence there cannot be any statical condition involving an electrical field in a conductor. But when the charges are separated by a non-conductor the charges cannot move and the electrostatic field may therefore persist permanently in the medium. For this reason non-conductors are frequently called **dielectrics**, or media in which there can be an electric field, without the charges being caused to move.

Dielectric constant.—The definition of unit charge and the law of force between charges (p. 925), are, strictly speaking, only valid when the charges are situated in a vacuum. But for most practical purposes, the replacing of vacuum by air would not produce an appreciable difference. While true for most gases besides air, this is certainly not true for liquid or solid dielectrics.

If two charges, q_1 and q_2, be immersed in a dielectric, such as, say, paraffin oil, the force between them will be less than in air, and must be represented by the modified equation :

$$\text{Force} = \frac{q_1 q_2}{k d^2} \text{ dynes},$$

where k is some number depending upon the nature of the medium.

This quantity (k) is called the **dielectric constant**, or **specific inductive capacity** of the medium. For solid paraffin the dielectric constant is about 2·3, and therefore, for charges immersed in it, the force between them is $q_1q_2/2{\cdot}3d^2$ dynes. The following is a table of the dielectric constants of several important substances:

Substance	Dielectric constant (k)
Ordinary glass	8·15
Plate glass	4·67
Ebonite	3·15
Sulphur	3·84
Mica	6·61
Paraffin (solid)	2·3
Petroleum	2·0
Water	80
Air (at 76 cm. pressure)	1·0006
Hydrogen (76 cm. press.)	1·0003

Effect of dielectric upon potential. On referring to p. 930 it will be seen that the potential at any point due to a charge + Q is obtained by finding the work done in transferring a unit charge from infinity to the point. In order to find the work done in any small step of this transference, the average force is multiplied by the distance. When the dielectric is other than air or vacuum, the distances are, of course, all unchanged but every force is diminished in the ratio $1/k$; that is, it becomes $1/k$ of what it was for air. Hence the average force for the step Ax (Fig. 862) becomes Q/kax instead of Q/ax, and the work becomes

$$\frac{Q}{kax}(x-a)=\frac{Q}{ka}-\frac{Q}{kx}.$$

This factor $1/k$ enters into every term in the series, the sum of which gives the work done. Therefore

$$\text{Work done}=\frac{Q}{ka}-\frac{Q}{kb}.$$

Considering B to be at infinity, the potential at A is Q/ka. Hence **the effect of changing air for a dielectric of constant k, is to reduce the potential to 1/k of its previous value.**

Effect of dielectric upon capacity.—It is now easy to see that the dielectric surrounding a conductor affects its capacity. For on putting a charge + Q upon the conductor, the potential at any point

with air as dielectric would be, say, V, but with the dielectric of constant k instead of air the potential is V/k. Thus the capacity will, by definition (p. 938), be $Q \div V/k = kQ/V$. That is, **it is k times the capacity with air as dielectric.** It follows that

$$\text{Capacity of sphere} = kr;$$

$$\text{Capacity of concentric spheres} = \frac{kab}{b-a};$$

$$\text{Capacity of parallel plates} = \frac{k\mathsf{A}}{4\pi t}.$$

In the case of the sphere, the dielectric must extend uniformly to a great distance, practically infinity, all round the sphere. On the other hand, for the concentric spheres and the parallel plates, the dielectric need only occupy the space between the earthed plate and the insulated plate; because this is the only region occupied by the electric field under consideration. It is only the region occupied by the electric field that need be filled with the dielectric.

EXPT. 217.—**Effect of the dielectric upon capacity.** Connect one of two insulated parallel plates A (Fig. 889) to the gold-leaf electroscope and charge it. Earth the other plate B. The divergence of the leaves now corresponds to the potential due to the charge upon A. Now introduce a slab of glass or ebonite between the plates. Note that the leaves partially collapse. This shows that the potential of A has fallen, the charge upon it does not now raise it to the original potential, so that its capacity must have increased. Remove the slab and note that the leaves return to their original divergence. The effect of introducing the slab of dielectric is therefore to increase the capacity of the condenser.

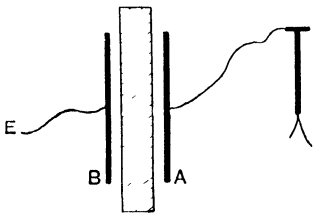


FIG. 889.—Condenser with dielectric slab.

Capacity of Leyden jar. If the glass of the Leyden jar (Fig. 878) be not very thick, the inner and outer coatings may be considered to be parallel plates. Let the total area of the inner tinfoil be A sq. cm. and the thickness of the glass be t cm., then the capacity of the jar is $k\mathsf{A}/4\pi t$. Taking the dielectric constant of the glass as 8, the capacity will be $8\mathsf{A}/4\pi t$. Thus with an area of inner tinfoil of 750 sq. cm. and a thickness of glass 2 mm., the capacity would be $\frac{8 \times 750}{4\pi \times 0{\cdot}2} = 2380$ units approximately. Thus the capacity would be equal to that of a sphere of radius 2380 cm., or 47·6 metres diameter.

EXAMPLE.—Find the capacity of a condenser consisting of two rectangular sheets of tinfoil 30 cm. × 20 cm. separated by a sheet of mica 0·2 mm. thick, taking the dielectric constant of mica as 6·6.

$$\text{Capacity} = \frac{kA}{4\pi t} = \frac{6{\cdot}6 \times 30 \times 20}{4\pi \times 0{\cdot}02} = \underline{15750} \text{ units.}$$

Measurement of dielectric constant.—From the effect of the dielectric upon the capacity of a condenser (p. 953) it is possible to derive a method for measuring dielectric constants. If the capacity C of a condenser, having air as dielectric, can be measured, and then the capacity C_1 with some other medium as dielectric, we have

$$C_1 = kC, \quad \text{or} \quad k = \frac{C_1}{C},$$

where k is the dielectric constant of the medium. It is not necessary to measure the capacities in absolute measure; the **ratio** of the capacities with the medium and with air as dielectric only need be found. Faraday was the first to employ this method, and by means of it he investigated the properties of several insulators. Two spherical condensers, AB, A′B′ (Fig. 890), as nearly as possible alike, are employed. The two insulated conductors A and A′ are first joined together and charged. Since they must now be at the same potential, they will have equal charges if the capacities are the same. On insulating B and B′ and connecting A′ to B and A to B′ the respective positive and negative charges neutralise each other if they are equal, so that no charge remains. If the remaining charges are zero then the two capacities must be equal. The space inside A′B′ is now filled with the dielectric shellac, or sulphur, etc., which is poured into the space in a molten condition. The ratio of the capacities C of AB and C′ of A′B′ is now measured as on p. 950. The value C_1/C is the dielectric constant k.

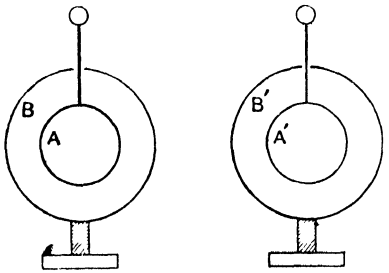


FIG. 890.—Faraday's method of measuring k.

Faraday only half filled A′B′ with the dielectric, in which case

$$C' = \frac{C}{2} + \frac{kC}{2} = \frac{C}{2}(1+k), \qquad \therefore \; k = 2\frac{C'}{C} - 1.$$

The quadrant electrometer was not invented until after Faraday's time. He employed a calibrated electroscope similar to the gold-leaf electroscope, but having two light suspended pith-balls instead of the gold leaves. In this way he found, for shellac, $C_1 = 1{\cdot}5C$;

$$\therefore\ k = (2 \times 1{\cdot}5) - 1 = 2.$$

Parallel-plate method for measuring k.—For solid substances Faraday's method is not applicable unless the substance has a low melting point. For glass, ebonite, etc., the parallel-plate method is necessary. The principle of Expt. 217 is employed. Before this can be followed, it is necessary to find the capacity of a parallel-plate condenser partially filled with dielectric. Let A be the insulated and B the earthed plate (Fig. 891). Let t be the distance apart of the plates, and h the thickness of the slab of dielectric introduced between them. On p. 940 it was seen that the capacity of the condenser without the slab is $A/4\pi t$, and if $+\sigma$ and $-\sigma$ are the amounts of charge per square centimetre of the plates, $A\sigma$ is the charge upon A ;

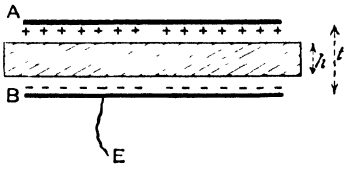


FIG. 891.—Measurement of k

$$\therefore \text{ Difference of potential between A and B} = \frac{Q}{C} = \frac{A\sigma}{\frac{A}{4\pi t}}$$

$$= 4\pi\sigma t.$$

But the difference of potential between the plates is equal to the force on unit charge × distance ;

$$\therefore \text{ Force on unit charge} \times t = 4\pi\sigma t,$$

or, $$\text{Force on unit charge} = 4\pi\sigma. \quad \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

This is the force on unit charge in air or vacuum, due to the charges $+\sigma$ and $-\sigma$.

Inside the slab, the force on unit charge will be $4\pi\sigma/k$, outside it it is still $4\pi\sigma$;

$\therefore$ Work done in carrying unit charge through the slab

$$= \frac{4\pi\sigma}{k} \cdot h,$$

and, Work done in carrying unit charge through the air space $(t - h)$

$$= 4\pi\sigma\,(t - h)\,;$$

$\therefore$ Work done in carrying unit charge from B to A

$$-\frac{4\pi\sigma}{k}h+4\pi\sigma(t-h)$$

$$=4\pi\sigma\left\{t-h+\frac{h}{k}\right\}$$

But this is also the difference of potential between the plates due to the charge $A\sigma$ upon A ;

$$\therefore \text{Capacity}=\frac{A\sigma}{4\pi\sigma\left\{t-h+\frac{h}{k}\right\}}=\frac{A}{1\pi\left\{t-\left(h-\frac{h}{k}\right)\right\}} \quad \ldots\ldots\ldots (2)$$

Thus the capacity has changed from $A/1\pi t$ to the value given in (2) by introduction of the slab. The same change in capacity might have been produced by bringing B nearer to A by the amount $(h-h/k)$ instead of introducing the slab

This change in value of t gives us a method of measuring k Let A be connected to the electrometer or electroscope with the slab in, and the deflection or divergence noted If the slab be then withdrawn the deflection will increase. Move B nearer until the original deflection is restored, so that the capacity regains its original value. This travel of B must be measured , let it be l.

Then $$l=h-\frac{h}{k},$$

or, $$\frac{l}{h}=1-\frac{1}{k},$$

and $$k=\frac{h}{h-l} \quad \ldots\ldots\ldots\ldots (3)$$

h and l being known, k may be found.

Exercises on Chapter LXXIII.

1. What is meant by the electric capacity of an insulated conductor ? Describe an experiment you would make to show that the capacity of an insulated conductor is increased when a second conductor connected to earth is brought near it. Sen. Camb. Loc.

2. Distinguish between the electric potential and the energy of a charged conductor. An isolated sphere of radius 8 cm. receives a charge of 720 E.S. units. Show how to calculate the potential and the energy when the dielectric is air, and also when the medium has a dielectric coefficient 2·5. L.U.

3. Define dielectric constant and describe how it may be measured in the case of a liquid such as paraffin oil.

4. Describe one form of electrometer and describe its action. L.U.

5. How does the energy of a charged condenser depend on the substance between the plates ?
What conclusions do you draw regarding the source of the energy of electrical charges ? L.U.

6. Calculate the capacity of a spherical condenser of which the radii of the spheres are 15 6 and 16 8 cm. respectively. What is the energy of the charge if the insulated sphere be raised to a potential of 80 units ?

7. A Leyden jar has a diameter of 15 cm., a depth of tinfoil of 18 cm. and thickness of glass 2 5 mm. If the value of k for the glass is 6 4, find the capacity of the jar.

8. Explain why the capacity of a condenser is changed when the dielectric is changed from air to some other substance.

9. A condenser consists of eleven rectangular pieces of tinfoil each measuring 15 cm. × 20 cm. all joined together, with ten similar pieces of tinfoil joined together and alternating with the first set. If the tinfoils are separated by sheets of mica of thickness 0 2 mm. whose specific inductive capacity is 6·28, what is the capacity of the condenser ? Also find the amount of work necessary to put a charge of 100 electrostatic units upon it. L.U.

10. An air condenser with plates 10 centimetres square and half a centimetre apart is charged with 100 electrostatic units of electricity. Find the loss of electric energy when it is plunged under oil of specific inductive capacity 2. L.U.

11. Two parallel plate condensers, A and B, have the following dimensions : (A) area of plate 35 sq. cm., thickness of dielectric 3 mm. ; (B) area 75 sq. cm., thickness of dielectric 5 mm. The dielectric of A has a constant 6 2 and that of B 7 5. If A receives a charge of +80 units and B a charge of +70 units, find the direction of flow of the current on connecting A to B.

CHAPTER LXXIV

ELECTRICAL MACHINES

The electrophorus.—Any apparatus for the production of an unlimited supply of electrical charge may be called an electrical machine. The **electrophorus** can hardly be given the name of 'machine,' but it represents a type from which more efficient machines have been developed. It consists of a sheet of ebonite B (Fig. 892) which can be rubbed with a piece of fur, giving it a negative charge. Upon this, a brass plate A, carried by an insulating handle, can be placed On earthing A by touching it with the finger or a wire, it becomes positively charged, the negative charge upon it escaping to earth (p. 933) On lifting A by its insulating handle, the positive charge is carried with it and is available for use.

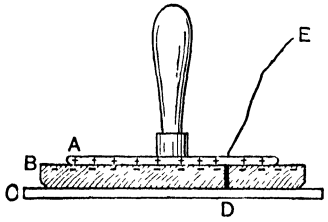


FIG 892.—The electrophorus

The ebonite sheet B may be mounted upon a metal sole-plate C, through which passes a metal pin D that makes contact with A automatically when this is placed upon the ebonite and so earths it. This, however, is not essential; the earthing may be done by hand.

The plates A and B are only in actual contact at a few points, so that the charge upon B diminishes with extreme slowness Thus the amount of positive charge produced by the electrophorus is almost unlimited Since this charge has considerable energy it is of interest to note the source of this energy After earthing the plate A it is, of course, at zero potential, but has a positive charge. The positive potential due to this charge is, of course, equal to the negative potential due to the negative charge upon B. When A is lifted off B work must be done in opposition to the force between

the opposite charges. This work is the source of the energy of the positive charge, for as A is removed from the neighbourhood of the negative charge its resultant potential will rise, and eventually, when A is far from B, become that due to its own positive charge alone.

The water dropper.—Two tin cans, A and B (Fig. 893), with the bottoms removed, are placed vertically above two others, C and D, in which funnels or pieces of gauze are fixed. A water supply ends in two jets, E and F, one situated within each of the cans, A and B. The jets must be so regulated that they break up into drops while still situated within the cans A and B. These drops in falling must make contact with the interiors of the cans C and D. A and C are connected together, as are also B and D.

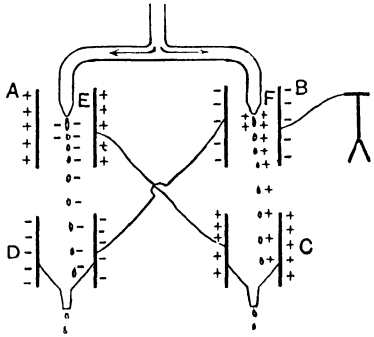


FIG. 893.—The water dropper

To begin with, a small charge is given to one pair of cans; let a positive charge be given to A. This makes the potential of A positive, and since the interior of a hollow conductor acquires the potential of the conductor (p. 935) the jet is in a region of positive potential. But since the jet is earthed, through the water pipes, a current flows to earth, leaving the jet negatively charged. As it breaks up into drops, the negative charge is imprisoned upon the drops and is carried to D, where, on the drop making contact with the interior, the charge passes to the outside, and is, of course, shared with B. Upon the other side, B being now negatively charged, a similar process goes on, positive charge being carried to C and A. The higher the charges the more rapid the process, so that once started, the accumulation of charge goes on more and more rapidly.

The source of energy of these accumulated charges is the work done by gravity in pulling the negatively charged drops at E out of the neighbourhood of the positive charge upon A, and the positively charged drops at F out of the neighbourhood of B's negative charge.

In setting up the apparatus, the cans may be fixed to a wooden framework by means of sealing wax, and the length of the jet must be adjusted until the accumulation of charges begins. There is generally enough charge upon the apparatus to start the process without the necessity for putting any upon the cans.

Kelvin replenisher. —This type of influence machine is almost the same, electrically, as the water dropper, but it is mechanically constructed so that the carriers, which are the counterpart of the drops, are carried round an axis continually. The conductors A and B (Fig. 894) act both as inductors, in this respect resembling A and B in the water dropper, and also as collectors like the cans C and D The carriers E and F are fixed upon an insulating arm which is so mounted that it can be made to rotate freely about the axis O.

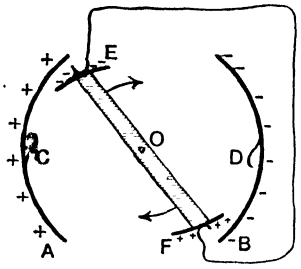


FIG 894.—Kelvin replenisher

Imagine A to be given a small positive charge to begin with. As the carriers rotate in the direction of the arrows they come simultaneously into contact with two light strips of metal connected by a wire, at the position shown in the diagram This connects E and F electrically But E being nearer than F to the positive charge upon A, is at a higher potential than F, so that a current flows from E to F, leaving E negatively charged and F positively. As the motion continues, E and F leave the contact pieces and carry away the charges Soon E arrives in contact with the metal tongue D and gives up its negative charge to the conductor B. At the same time F comes in contact with C and gives up its positive charge to A. The process is then repeated during the next half turn. Owing to the collection of negative charge upon B and positive charge upon A the charges produced upon E and F become greater and greater as time goes on, and the rate of accumulation of charge becomes more rapid.

Wimshurst influence machine. If we could imagine the conductors A and B of the Kelvin replenisher to rotate in the opposite direction to the carriers E and F, we should get twice the number of operations in a given time This alteration is accomplished in the Wimshurst machine, together with a further multiplication produced by increasing the number of carriers and inductors. These parts, in this machine, are exactly like each other in form They consist of metal sectors, fixed to glass or ebonite discs which rotate in opposite directions As the electrical arrangements cannot very well be shown in a picture of these discs, they are represented by two concentric circles in Fig 895 The plates, with their sectors, are supposed to rotate in the directions shown by the arrows.

As in the previous two cases, some charge must be given to one set of sectors in order to start the action, but we will describe the state of affairs after the discs have made a few turns. At A and B are two wire brushes, connected by a conducting rod, which touch

simultaneously two diametrically opposite sectors upon one disc. At this moment charges of opposite sign upon the sectors on the other

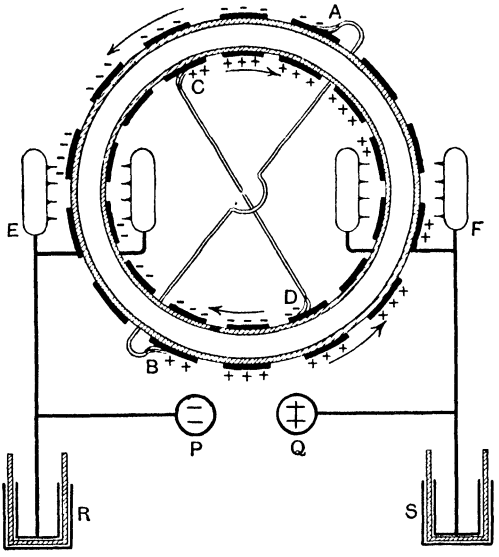


FIG 895.—Diagram of the Wimshurst machine

disc are passing, so that the sector at A will receive a negative charge and that at B a positive charge. These charges are eventually given up, the negative to the collector, at E and the positive to the collector F. Before arriving at the collectors they play a similar part to the sectors passing the brushes C and D, giving those in contact with C a positive charge and those at D a negative charge. It will thus be seen whence the charges passing A and B were derived.

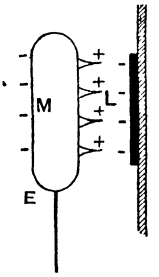


FIG 896.—Collection of the charges.

The collection of the charges at E and F requires notice. Consider Fig. 896. Several points fixed upon the collector are directed towards the sector, which is supposed to have a negative charge. This causes the points L to be at lower potential than the further parts M of the conductor, the consequence being that positive charge flows to the points and negative charge to M. The effect of the points is to cause this positive charge to flow from them, as described on p. 937, and the stream of positive charge falling upon the sector

neutralises its negative charge. Hence the sector passes away uncharged, while a corresponding negative charge remains upon the collector.

Two discharge knobs, P and Q (Fig. 895), are connected to the collectors, and on the difference of potential between them rising to a sufficient amount, owing to the accumulation of the opposite charges, a spark discharge will take place Most Wimshurst machines are provided with Leyden jars, R and S, one connected to each

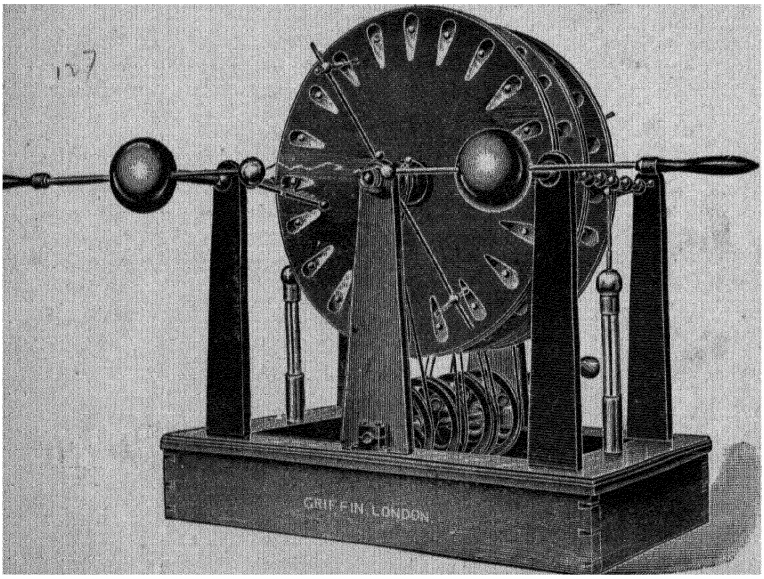


FIG 897.—The Wimshurst influence machine

collector. Their function is to increase the capacity so that a greater accumulation of charge takes place before the difference of potential between P and Q is sufficient to cause a discharge. The discharge will then be much more violent than when only a small amount of charge has collected.

In Fig 897 the general appearance of the Wimshurst machine is shown It has six plates, alternate plates being connected together so that adjacent plates rotate in opposite directions.

Some makers enclose the machine in an outer case which is filled with coal gas as it is thought that the machine then works more efficiently than in air

Influence machines are sometimes used for working X-ray tubes,

but they are not so convenient or reliable as the induction coil. There are several other forms of influence machine, but those described above will serve as types

EXERCISES ON CHAPTER LXXIV.

1. A charged ebonite rod is brought near to a pin, which is fixed to the knob of an electroscope, and the rod is then removed. State and explain what may be observed.

Explain what practical use is made of the effect observed.

Sen. Camb. Loc.

2. Explain the action of some machine for producing electrostatic charge, indicating clearly the source of the energy of the charge obtained.

L.U.

3. Explain the action of a Wimshurst machine.

Show how the potential and charge of a sector vary during a revolution.

4. Describe the action of the electrophorus, and explain why the amount of charge that can be given to any insulated conductor by means of it is limited.

5. Describe the action of (*a*) the water dropper, or (*b*) the Kelvin replenisher.

CHAPTER LXXV

ELECTROMAGNETICS

Work done in carrying a magnet pole round an electric current.—It was seen in Chapter LXV that the system of units developed in connection with electric currents is founded upon the relation between the current and its magnetic field. The field in a certain case (p. 833) is taken as a measure of the current, and we may thus say that the strength of magnetic field at any point is proportional to the current to which it is due. This leads, in a manner which cannot here be stated, to the fact that the work done in carrying a magnetic pole once round a conductor in which a current is flowing is proportional to the current. Further, if the current is measured in absolute units (p. 833) **the work done in carrying a unit magnetic pole once round the current is 4π times the strength of the current.**

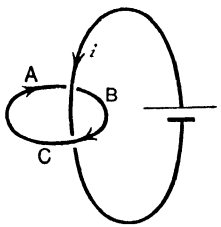


FIG. 898.—Path linked with current.

In taking the magnetic pole round the current in this way, its path must be linked with the current circuit. In Fig. 898 if ABC be the path of the magnetic pole, we know from p. 831 that the magnetic field due to the current is in such a direction that the pole, if a N pole, would be urged along the path, and work is thus done upon it. If it be carried round in the direction CBA, the current's magnetic field opposes the motion, and energy must be expended by some outside agency to effect this motion. In either case, however,

$$\text{Work done} = 4\pi i \text{ ergs.}$$

Magnetic field due to a long straight current.—We will now apply this relation to calculate the magnetic field due to a current in several important cases. The first case is that of a current flowing in a straight wire, of sufficient length to be considered infinitely long. The magnetic lines of force due to such a current are circles

whose centres lie upon the wire. Hence, from symmetry, it is obvious that the strength of magnetic field is the same all round any given circle having its centre upon, and its plane perpendicular to, the wire.

To find the strength of magnetic field H at a point P (Fig. 899) at distance r from the wire, remember that the force on a unit magnetic pole at P is H dynes, and is tangential to the circle (p. 830). As the pole is carried round this circle,

$$\text{Work done} = \text{force} \times \text{distance}$$
$$= \mathsf{H} \times 2\pi r \text{ ergs.}$$

But from the law given above,

$$\text{Work done} = 4\pi i \text{ ergs ;}$$
$$\therefore\ 2\pi \mathsf{H} r = 4\pi i ;$$
$$\therefore\ \mathsf{H} = \frac{2i}{r} \text{ gauss.} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

It is thus seen that the strength of magnetic field due to a long straight current is inversely proportional to the distance from the current. The unit of magnetic field is defined on p. 781 as the force on unit pole, and is sometimes called the **gauss.**

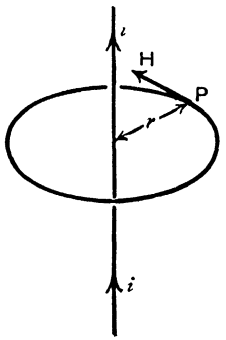


FIG 899.—Magnetic field due to a straight current

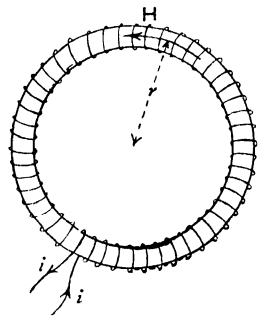


FIG 900.—Endless solenoid

Magnetic field inside an endless solenoid. A circular ring upon which a wire is uniformly wound, so that the magnetic field is everywhere in the direction of the circumference of the ring, is called an **endless solenoid.** If there are n turns per centimetre length of the solenoid, measured circumferentially, the total number of turns is $2\pi rn$, where r is the radius (Fig. 900). Thus, for a current i in the wire, a unit magnetic pole carried round the curved axis of the solenoid passes $2\pi rn$ times round the current, and the work done is therefore $4\pi(2\pi rni)$ ergs. But if H is the strength of magnetic field,

this is the force on the unit pole, and the work done upon the pole in making a complete circuit along this path is $2\pi r\mathsf{H}$;

$$\therefore\ 2\pi r\mathsf{H} = 4\pi(2\pi r n i),$$

or,
$$\mathsf{H} = 4\pi n i \text{ gauss} \quad . \quad . \quad . \quad ..(2)$$

If the current be given in amperes, the strength of field is then

$$\mathsf{H} = \frac{4\pi n \mathrm{I}}{10} \text{ gauss} \quad . \quad (3)$$

Unless the circular path of the magnetic pole has the mean radius of the solenoid, the expression for the field is not so simple as the above, for the field is not quite uniform over the cross-section of the coil. If, however, the section of the solenoid is small compared with the radius r, this want of uniformity is negligible, and when r is very great, it may be entirely ignored

Magnetic field inside a long straight solenoid. It will be noticed that the radius r does not enter into the expression $4\pi n i$ for the magnetic field inside the endless solenoid If then r be made very great without altering n the number of turns per centimetre, the magnetic field remains the same On continuing to increase the radius, the solenoid eventually becomes straight. Hence the strength of magnetic field inside an infinitely long straight solenoid is $4\pi n i$.

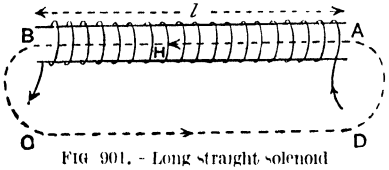


FIG 901. - Long straight solenoid

This may be proved independently by considering a long straight solenoid of length l, having n turns per centimetre For a path ABCD (Fig 901), the work done in carrying a unit pole from A to B is $\mathsf{H}l$ ergs. For the rest of the closed path BCDA, the field is nearly zero if the solenoid is very long, so that the work for this part of the path is zero ,

$$\therefore \text{ Work done for the closed path} = \mathsf{H}l = 4\pi n l i,$$

or,
$$\mathsf{H} = 4\pi n i \text{ gauss.} \quad \ldots\ldots\ldots\ldots (4)$$

The only difficulty in the above reasoning arises from the indefiniteness near the ends, but it must be remembered that, strictly speaking, the field is only $4\pi n i$ for a very long solenoid, in which case the path near the ends is an insignificant part of the whole path.

Magnetic field due to a short solenoid.—The above law cannot be applied to the case of a short solenoid, as it is impossible to find a complete path, enclosing the current, along which the field has constant strength. Fig 902 illustrates the form of the magnetic field for a short solenoid. The lines of force spread out from the

ends, and have even begun to spread out within the coil On comparison with the lines of force due to a bar magnet (p 778) it will be seen that the external magnetic field due to the solenoid is very similar to that of the magnet. There is the equivalent of a N pole at one end and a S pole at the other.

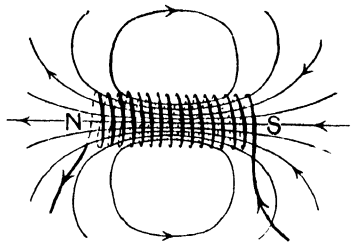


FIG 902 —Magnetic field of a short solenoid

EXPT. 218.—**Magnetic effect of a short solenoid.** Wind some cotton-covered copper wire upon a piece of brass tube, 10 cm. long and about 1 cm. diameter. Pass a current of 1 or 2 amperes through the coil. By means of a suspended magnet, as in Expt. 162, find the N and S poles of the solenoid, and show that this agrees with the law for relation of direction of current to that of magnetic field given on p. 831.

EXPT. 219.—**Force between solenoids.** Wind another solenoid as in the last experiment, but bring the free ends of wire to the middle of the solenoid and tie them firmly with cotton, taking care to keep them insulated from each other. Attach to each free end a piece of copper wire A and B, and suspend the coil as in Fig. 903, so that the current may enter at A and leave by B. Bring the pole of a bar magnet near each end of the solenoid in turn, and show by the attractions and repulsions that each end of the solenoid is a magnetic pole, one end N and the other S. Verify the statement that **on looking at the end of the coil at which the current travels in an anti-clockwise direction that end is a N pole.** The end at which the current flows clockwise is a S pole (Fig. 902).

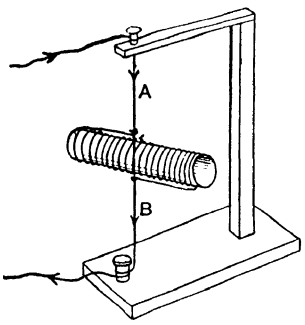


FIG 903 —Magnetic effect of a solenoid

Repeat the experiment, using the coil of Expt. 218 in place of the bar magnet.

Place a piece of iron rod in each coil and show that the poles are now very much stronger, but of the same kind as before.

Force on a straight current in a magnetic field.—The results of Expts. 218 and 219 show that electric-current circuits behave as magnets. It is therefore evident that a conductor carrying a current

will experience a force when situated in a magnetic field. The laws governing this force are as follows :

(i) The force on the conductor is at right angles to the plane containing the direction of the current and that of the magnetic field. It is in the direction shown in Fig. 904, which may be remembered by the following **Left-Hand Rule.** Place the thumb, fore-finger and middle finger of the **left hand** mutually at right angles to each

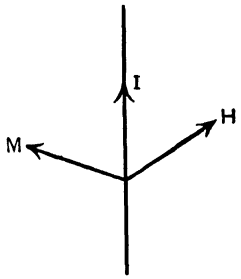


FIG. 904.—Force on electric current in a magnetic field

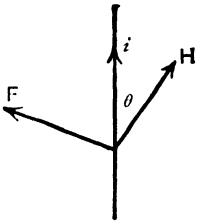


FIG. 905.—Force on current inclined to magnetic field

other. Let the middle finger point in the direction of the current I and the Fore-finger in the direction of the magnetic Field, then the thuMb points in the direction of Motion in which the circuit is urged.

(ii) When the magnetic field and the current are at right angles to each other, the force acting on every centimetre of the conductor is Hi dynes. Or, if the current is measured in amperes, the force per centimetre of conductor is $HI/10$ dynes.

If the current and magnetic field are inclined to each other at an angle θ, the force per centimetre of the conductor is then $Hi \sin \theta$ dynes. When $\theta = 0$, that is, when the current and field have the same direction, there is no force on the conductor since $\sin \theta = 0$. When, however, $\theta = 90°$, $\sin \theta = 1$, and the force is Hi dynes, as given above.

Couple acting on a rectangular coil in a magnetic field.—Let ABCD (Fig. 906 (*a*)) be a rectangular coil, situated in a magnetic field. The forces on the sides AB and CD are equal, opposite, and in the same straight line, and therefore cancel each other out. The force on the side AD, however, is $F = liH$ dynes, where l is the length of AD, i the current and H the strength of magnetic field. There is an equal and opposite force on BC, and the two give rise to a couple, whose

moment $= lb\mathsf{H} \times \mathsf{EB}$, where EB is the perpendicular distance between the forces. Fig. 906 (*b*) is a plan of the coil, in which it will be seen that if the normal to the coil makes an angle θ' with the direction of the magnetic field, then the angle $\mathsf{EAB} = \theta'$ and

$$\mathsf{EB} = \mathsf{AB} \sin \theta'.$$

Calling AB, the breadth of the coil, b, we have

$$\text{Couple} = \mathsf{H}.lb \sin \theta'$$

But $$l \times b = \text{area of coil} = \mathsf{A},$$

Then $$\text{Couple} = \mathsf{H}\ \mathsf{A}i \sin \theta'.$$

Comparing this with the expression for the couple on a magnet situated in a magnetic field (p. 781), that is, $\mathsf{HM} \sin \theta'$, we see that as regards the magnetic field, the coil may be considered to have a magnetic moment equal to $\mathsf{A}i$. This result is quite general for plane circuits, whatever the shape of the coil. Thus, for a circular coil of radius r cm., in which a current i flows, the magnetic moment is $\pi r^2 i$.

It will be seen that magnetic lines of force due to the coil (Fig. 906) emerge at the front face and enter at the back. Thus, the front face is a N polar face, which will correspond to the direction of rotation shown.

The following is a convenient rule for finding the direction of rotation of a current circuit situated in a magnetic field: **the forces acting in the circuit are in such a direction that the circuit moves to embrace the greatest number of magnetic lines of force.** This rule is quite general and applies to flexible as well as to rigid current circuits.

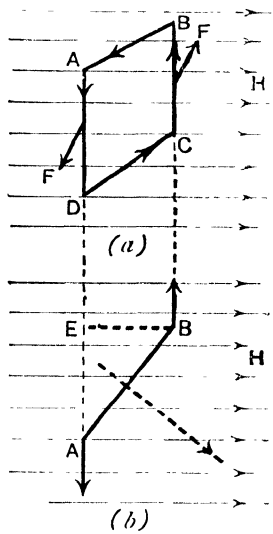


FIG. 906.—Coil in a magnetic field.

Suspended coil galvanometer. One of the most important uses to which the above fact is put is in the construction of the modern type of galvanometer (p. 866).

If the magnetic field between the poles (Fig. 797) be uniform, and of strength H, the couple acting on the coil is $n\mathsf{A}i.\mathsf{H}$, where n is the number of turns in it, provided that the plane of the coil is parallel to the field. In other positions the couple is $n\mathsf{A}i\mathsf{H}\cos\theta$, where θ is the angle between the magnetic field and the plane of the coil. The coil then rotates until the twist in the suspension

produces an equal and opposite couple Since the latter couple is proportional to the angle of twist, it may be written $c\theta$, where c is a constant depending on the length, thickness and material of the strip. Hence, for equilibrium,

$$nAiH\cos\theta = c\theta ;$$

$$\therefore\ i = \frac{c}{nAiH}\cdot\frac{\theta}{\cos\theta} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ (1)$$

For very small deflections $\cos\theta = 1$, and the current is proportional to the deflection, but for large deflections the current is proportional to $\theta/\cos\theta$, which is so complicated an expression that it is not of much use.

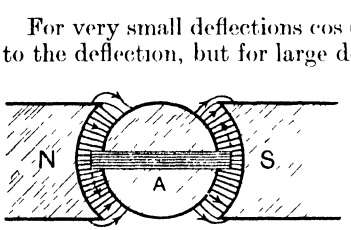


FIG 907 —Magnetic field of the suspended coil galvanometer

To get over this difficulty the soft iron cylinder A (Fig 907) is placed between the pole pieces, so that the sides of the coil move in the cylindrical space between the pole pieces and the soft iron cylinder This ensures that the permanent magnetic field shall be radial, and therefore the sides of the coil, even when deflected, are still in a field of the same strength Also, the force on the sides is still at right angles to the plane of the coil Hence the couple is always $nAiH$, unless, of course, the deflection be so great that the sides of the coil come near the edges of the pole pieces

Thus, with the soft iron cylinder present,

$$nAiH = c\theta ;$$

$$\therefore\ i = \frac{c}{nAH}\theta. \qquad \ldots\ldots\ldots\ldots\ (2)$$

The current is therefore proportional to the deflection. This is of great convenience in practice, and renders this arrangement particularly serviceable for the construction of ammeters (p 870)

Force between currents.—Since a current has a magnetic field in its neighbourhood, and a current in a magnetic field experiences a force, it follows that two current circuits exert forces upon each other. Indeed, we have seen in Expt. 219, p. 967, that this is the case

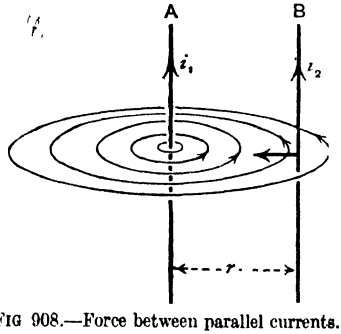


FIG 908.—Force between parallel currents.

Let A and B (Fig. 908) be two

long straight parallel wires carrying currents i_1 and i_2 in the same direction. If the distance between them be r centimetres, the strength of magnetic field at B due to A is $2i_1/r$ (p. 965). B being situated in this magnetic field experiences a force

$$F = \frac{2i_1}{r} \times i_2 = \frac{2i_1 i_2}{r},$$

per centimetre of its length. By applying the left-hand rule (p. 968) it will be seen that the direction of this force is such that B is urged towards A.

By a similar reasoning it may be shown that A is urged towards B with a force $2i_2 i_1/r$ per centimetre of its length.

Further, we see that **currents in the same direction attract each other, and currents in opposite directions repel each other.**

When the circuits are not of such a simple form, it may not be easy to calculate the force between them, but **for any given position the force is always proportional to the product of the current strengths** Many current measuring instruments are based upon this principle.

Kelvin current balance. The current to be measured passes through six coils CLMABD in series (Fig. 909). A, B, C and D are fixed, while L and M are attached to an arm which can rotate about an axis GH. A scale is also attached to the arm and carries a weight W which can be caused to slide along this scale. The arm and scale must be so balanced that when the weight W is on the zero mark the beam is horizontal, as indicated by a pointer, when no current is flowing. The coils are so connected that when the current flows, the forces between A and L, C and L are in such a direction that L is pushed downwards. Similarly M is pushed upwards. The arm thus rotates. It is brought back to its equilibrium position by sliding the weight W to the right, which introduces a couple equal and opposite to that due to the current. The scale is so graduated that the movement of the weight upon it gives a direct reading of the current. This scale, however, is not equally spaced, for the couple is proportional to the product of the current strengths in adjacent coils. Since, however, the current in these is the same, the couple is proportional

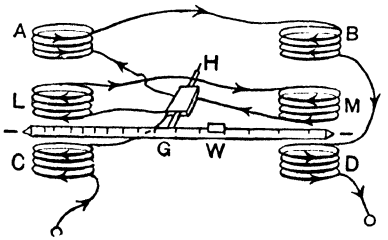


FIG 909.—Diagram of the Kelvin current balance

to i^2. But the couple is also proportional to the linear displacement of the weight; $\therefore$ Displacement $\propto i^2$.

Before being sent out by the instrument maker, the marks upon the scale are so placed that the scale reads directly in current.

By changing the weight, the range of current to be measured may be varied.

It should be noted that if the direction of the current be reversed, **it is reversed in all the coils,** so that the direction of every force is the same as before. Hence the direction of the current is immaterial, which is essential if the instrument is intended to measure alternating currents.

EXPT. 220.—**Calibration of an ammeter by the Kelvin current balance.** Place the ammeter to be calibrated in series with the current balance and an adjustable rheostat. Set the movable weight W upon one of the fixed marks on the scale of the balance, and adjust the rheostat until the current is such that the movable arm returns to its zero position. Observe the readings of both ammeter and current balance. Repeat for other currents and record the results in a table as below:

Ammeter reading	Position of weight on movable arm	Current

The current for each position of the movable weight is given by the instrument maker for each weight, in a table supplied.

Kelvin watt balance. A similarly designed instrument has been made for the direct measurement of the power absorbed in any circuit. In this case the coils are differently connected, but the method of measuring the couple by displacing the sliding weight is the same as before.

Suppose that it is desired to measure the power in watts absorbed in the lamp PQ (Fig. 910). A, B, C and D, the fixed coils, are then placed in series with PQ, so that the current I in the lamp also flows through these coils. In this case the movable coils LM consist of a great many turns of fine wire, so that they have considerable resistance. They are joined in parallel with the lamp, so that the current in them is proportional to the p.d. E between the lamp terminals. Hence, the force between A and L is proportional to $E \times I$, that is, it is proportional to the product of the strengths of the currents in A and L. Similarly, all the other forces between the coils are proportional to

EI, therefore the couple acting on the movable arm is also proportional to EI. Thus, the displacement of the sliding weight necessary to restore equilibrium is proportional to EI. But EI is the power in watts used in driving the current through the lamp. Therefore, the arm on which the movable weight slides may be calibrated so that the instrument reads directly in watts.

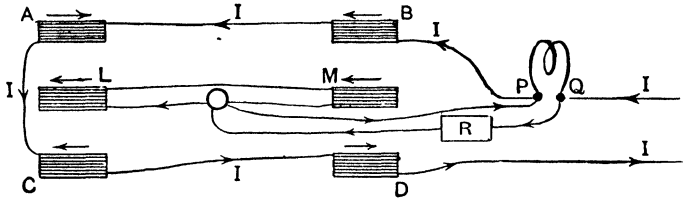


FIG. 910.—Diagram of the Kelvin watt balance.

Owing to the fact that the movable coils LM are placed in parallel with the lamp, their resistance must be high, for the same reason that the resistance of a voltmeter must be high (p. 875). For this reason it is customary to introduce an extra fixed resistance R (Fig. 910) into the movable coil circuit, and the wattmeter is usually calibrated with this high resistance included in the shunt circuit.

EXPT. 221.—**Measurement of watts absorbed by an electric lamp.** Set up the wattmeter and connect it to the lamp as shown in Fig. 910. Move the sliding weight until equilibrium is attained and record the power in watts. If possible, measure the candle-power of the lamp at the same time (p. 547) and obtain the watts per candle-power of the lamp. Repeat for several other lamps.

Siemens' electrodynamometer. — The principle of the **electrodynamometer** is the same as that of the Kelvin current balance, but the design of the apparatus is different. The couple is here measured by the twist in a spring instead of by means of a movable weight. Two coils ABCD and PQRS (Fig. 911), approximately rectangular in form, are connected in series. PQRS is fixed, but ABCD is suspended so that it can rotate about a vertical axis. In rotating it twists the spring whose upper end is attached to a pointer at the torsion head T. When the current flows, the forces

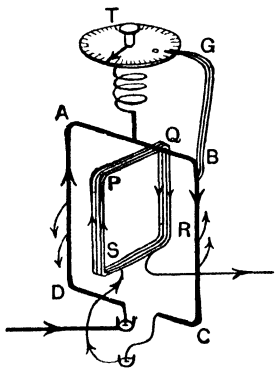


FIG. 911.—Diagram of Siemens' electrodynamometer

between the coils cause rotation of the suspended coil in the direction of the arrows It is then brought back to its original position by rotating the pointer to which the upper end of the spring is attached. The number of degrees twist thus given to the spring is a measure of the couple required to maintain the suspended coil in its original position This position is indicated by the pointer G, attached to the movable coil.

Each force acting between the adjacent sides of the two coils is proportional to the product of the currents in them But since these currents are both the same, namely, the current I to be measured, each force is proportional to I^2, and the couple is proportional to I^2 If the twist in the spring, as measured by the torsion head, is θ, the couple is also proportional to this

$$\therefore \theta = kI^2,$$

or,

$$I = K\sqrt{\theta},$$

where k and K are constants whose values must be found if the current is to be measured in amperes If the direction of the current is reversed, that in both coils is reversed, so that the forces are in the same direction as before Hence the reading does not depend upon the direction of the current, and the instrument is applicable for use with alternating currents

Some types of this instrument are designed for use as wattmeters, when one coil is placed in series and the other in parallel with the apparatus in which the power is to be measured The arrangement is similar to that of Fig 910, but it should be remembered that the coil used as shunt must have a very high resistance

EXPT. 222 —**To calibrate the Siemens' electrodynamometer** Connect the instrument in series with a standard ammeter and an adjustable rheostat. Turn the pointer T (Fig. 911) until the indicator G is at the zero of the scale when no current is flowing. If T is not at the zero of the scale, its position must be recorded, and this zero error applied to all readings taken with the instrument. Pass the current. Then G moves from its zero position, and must be brought back to it by rotating T. Note the new position of T and also the ammeter reading for the current. Repeat for other currents, until the whole scale has been traversed. Record the results in a table.

Current.	θ, uncorrected for zero error	θ, corrected for zero	$\sqrt{\theta}$

Plot a graph connecting current and θ. This will be a parabola. Plot

another graph connecting current and $\sqrt{\theta}$. This should be a straight line. From the latter graph find the constant K of the instrument from the relation

$$I = K\sqrt{\theta}.$$

Exercises on Chapter LXXV.

1. State the law for the work done when a magnetic pole is carried round a closed path linked once with a current circuit, and apply the law to calculate the strength of magnetic field at distance r from a long straight wire carrying a current.

2. Calculate the strength of magnetic field inside a long straight solenoid, and state why the result is inapplicable to the case of a short solenoid.

What is the strength of magnetic field inside a solenoid of length 60 cm. having 300 turns in which a current of 1·2 amperes is flowing?

3. A straight solenoid of length 75 cm. consists of 600 turns in which a current of 0·2 ampere flows. What is the magnetic flux through the solenoid, if its radius is 1·8 cm.?

4. An iron ring forms a closed magnetic circuit, having a mean length of 30 centimetres, and a section of 1·5 sq. cm. On the ring are wound 100 turns of insulated wire, and 1 ampere in the wire gives a total flux of 12,000 lines in the ring. Find the permeability of the iron. C.G.

5. Describe, giving sketches, two forms of current-measuring instruments, one depending on the interaction of currents and magnets, the other of current and currents. Explain the action of each instrument. L.U.

6. A coil of a single turn of wire in the form of a rectangle of height 15 cm. and width 8 cm. is suspended in a horizontal magnetic field of strength 2·5. Draw a diagram indicating the forces acting on each side of the rectangle, and calculate the couple acting upon it when its plane makes an angle of 45° with the magnetic field and a current of 20 c.g.s. units flow in the wire. L.U.

7. Find an expression for the current flowing in a suspended coil galvanometer (*a*) when no soft iron cylinder is used, (*b*) when a soft iron cylinder is situated between the pole pieces.

8. Describe some form of wattmeter and the method of using it to measure the watts per candle power for an incandescent lamp.

9. State in as general a form as possible the quantitative laws relating to the magnetic forces due to electric currents.

A current of one ampere flows round a wire bent into a circle of 20 cm. in diameter. An equal current flows round a circle of 2 cm. in diameter suspended at the centre of the larger coil. What couple is required to hold the small circuit with its plane at right angles to that of the large one? L.U.

10. Two long straight parallel wires carry currents of 10 and 15 amperes respectively, and are situated 12 cm. apart. Find the force on 5 cm. length of each wire.

11. Describe some form of suspended coil galvanometer and obtain an expression for the deflection in terms of the current and the constants of the instrument. Why is a soft iron cylinder usually placed between the pole pieces?

12. A circular coil of 18 turns of radius 12 cm carries a current of 3·5 amperes. Calculate the value of the couple required to maintain it with its plane parallel to a magnetic field of strength 25 gauss.

Plot a gi

CHAPTER LXXVI

ELECTROMAGNETICS (*CONTINUED*): MUTUAL AND SELF-INDUCTION

Electromotive force due to cutting across a magnetic field.—Whenever there is relative motion between a conductor and a magnetic field, there is an electromotive force in the conductor. This electromotive force is of very great importance, its existence being the foundation for an enormous number of applications of electricity, the electric dynamo, for example, depending upon it. A few simple experiments demonstrate at once the presence of this electromotive force. If a straight conductor (Fig. 912) be moved through the field of an electromagnet in a direction indicated by the arrow M, and a galvanometer be placed in series with the conductor, the galvanometer will indicate a current flowing, **so long as the conductor is moving across the magnetic field.** If the direction of motion of the conductor be reversed, the direction of the current is also reversed. When the conductor is part of a closed circuit, a current will flow, but the primary effect of the motion across the magnetic field is to produce an electromotive force. The resulting current depends, of course, upon the resistance of the circuit.

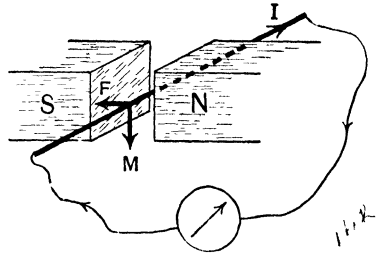


FIG. 912.—Conductor cutting across a magnetic field.

The direction of the electromotive force and current is given by the **Right-Hand Rule,** similar in form to the left-hand rule given on p. 968. Place the thumb, fore-finger and middle finger of the **right hand** mutually at right angles to each other. Let the Fore-finger point

in the direction of the magnetic Field, and the thuMb in the direction of Motion; then the mIddle finger will point in the direction of the current I, or the electromotive force.

A further experiment illustrates this production of electromotive force. Let a magnet NS (Fig. 913) be brought up to a coil of wire connected in series with a galvanometer. While the magnet approaches the coil there will be a deflection of the galvanometer, which ceases directly the magnet comes to rest. A reverse current is produced on withdrawing the magnet, or on using the S pole of the magnet instead of the N pole

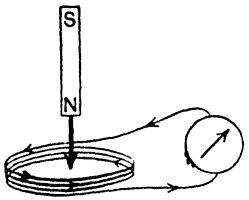


FIG. 913.—Induced e m f

If, instead of moving the magnet, the coil be advanced towards it, the effects are exactly the same as though the coil were at rest and the magnet moved. Thus the electromotive force depends only upon the **relative motion** of the magnet and the coil

EXPT. 223.—**Induced electromotive forces.** Connect a coil or solenoid of about 100 turns of wire in series with a galvanometer. Push a magnet into the coil, N pole first, and observe the direction of the kick of the galvanometer. After letting the magnet remain in the coil until the galvanometer needle comes to rest, withdraw the magnet quickly from the coil and again note the direction of the kick Repeat, using the S pole of the magnet. Again repeat, keeping the magnet at rest and moving the coil on to the magnet. Tabulate the results as follows:

Magnet advancing to coil	Direction of kick	Magnet withdrawn from coil	Direction of kick
N pole		N pole	
S ,,		S ,,	
Coil advancing to magnet			
N pole		N pole	
S ,,		S ,,	

Value of e.m.f. due to cutting magnetic flux.—In any case in which magnetic flux is cutting across a conductor, **the value of the electromotive force is the rate of cutting of magnetic flux, or the amount of**

magnetic flux cut per second. Magnetic flux being usually measured in terms of magnetic lines of induction, we may say that **the electromotive force in a conductor is equal to the number of magnetic lines of induction cut per second.**

Thus, if a conductor cuts a number of magnetic lines of induction in t seconds,

$$\text{e.m.f.} = \frac{\text{number of lines of induction cut}}{t} \text{ c.g.s. units.}$$

$$= \frac{\text{number of lines of induction cut}}{10^8 \times t} \text{ volts.}$$

Since in air the magnetic induction is identical with the magnetic field, lines of induction are lines of force, and hence,

$$\text{e.m.f.} = \frac{\text{number of lines of force cut}}{10^8 \times t} \text{ volts.}$$

In the cases with which we shall deal, in which the conductor is situated in air, or at any rate in a medium of unit permeability, we shall always refer to lines of force rather than lines of induction, but if the permeability should differ from unity the above distinction must be observed.

Application to a closed circuit.—On applying the above rule to every part of a closed circuit, a simple rule for the whole circuit may be found. **The electromotive force in a closed circuit is equal to the rate of change of the magnetic flux (number of lines of magnetic induction) threaded through the circuit.** To find the direction of the electromotive force, look along the lines of induction at the circuit (Fig. 914), then **if the number of lines threaded through the circuit is increasing the direction of the current is anticlockwise, if the number is decreasing the current is clockwise.**

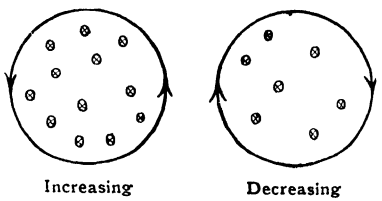


FIG. 914.—Rule for direction of e m f.

Again, if the circuit consist of a number of turns, the total electromotive force is that given by the above relation multiplied by the number of turns, since the above relation applies to each turn.

EXAMPLE.—If the number of lines of induction threaded with a coil of 1500 turns is 10,000 and if this induction is removed at a constant rate

in one-tenth of a second; find the momentary electromotive force in the coil.

$$\text{e.m.f. for each turn} = \frac{10000}{10^8 \times 0{\cdot}1} \text{ volts};$$

$$\therefore \text{ e.m.f. for 1500 turns} = \frac{1500 \times 10000}{10^8 \times 0\,1} \text{ volts}$$

$$= 1\,5 \text{ volt.}$$

Mutual induction.—It has now been seen that the electromotive force in a closed circuit depends upon the fact that there is a change in the amount of magnetic flux passing through it The electromotive force depends only upon this flux and not at all upon the origin of the flux. Thus it may be due to a magnet, or it may, on the other hand, be due to a current in its neighbourhood If the current in a coil A (Fig 915) be started, magnetic lines of force due to it become established, and in doing so, some of them cut into the coil B. With the direction of current and field as shown, the momentary current in B, as seen from A, will be anticlockwise (p 979). The current in B only lasts while the current in A is growing. Directly the current in A ceases to grow, the current in B stops. On stopping the current in A, the magnetic lines due to it disappear, and there is a momentary current in B in a clockwise direction. It is usual in an arrangement like this to call A the **primary circuit** and B the **secondary circuit** It will then be seen that for two neighbouring circuits, while the current in the primary circuit is growing there is an induced current and e.m f. in the secondary circuit, **opposite in direction to that in the primary.** While the current in the primary is **diminishing** the induced current and e.m.f. are in **the same direction** as the current in the primary.

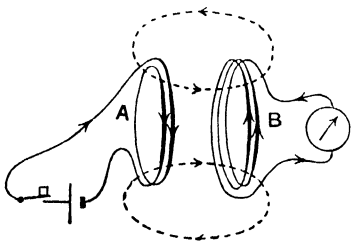


FIG 915.—Mutual induction

This effect of one circuit upon another is called **mutual induction. The coefficient of mutual induction, or the mutual inductance of two circuits, is the magnetic flux linked with the secondary, when unit current flows in the primary.**

If this magnetic flux be removed by the current in the primary ceasing to flow, and if this change takes place uniformly in one second,

the electromotive force in the secondary is numerically equal to the amount of flux which has disappeared, that is, **the mutual inductance of two coils is the e.m.f. in the secondary, when the current in the primary changes at the rate of one unit per second.** It does not matter which of the two coils is primary and which secondary, the mutual inductance is the same in either case.

If the e.m.f. in the secondary is 1 volt when the current changes at the rate of 1 ampere per second the mutual inductance is 1 henry.

Since the volt is 10^8 absolute c.g.s. units and the ampere is 10^{-1} absolute c.g.s. unit, it follows that

$$1 \text{ henry} = \frac{10^8}{10^{-1}} = 10^9 \text{ absolute c.g.s. units of inductance.}$$

Foucault or eddy currents.—When a mass of metal is situated in the neighbourhood of a changing current, it acts as a secondary circuit, electromotive forces and currents being developed in it. The electromotive force is great when the rate of change of magnetic flux is considerable, and the current further depends upon the conductivity of the material. As a good example, take the iron core of an electromagnet, represented diagrammatically in Fig. 916. A few of the magnetic lines of induction are shown. These, in becoming established, cut the iron core and produce considerable momentary currents which circulate as shown by the arrow. These currents are considerable, for the conductivity of the mass of iron is great. Such currents are called **eddy currents**, or **Foucault currents.**

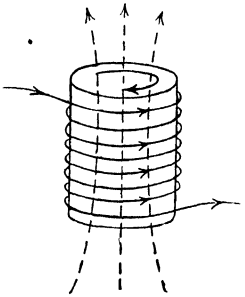


FIG 916.—Eddy currents

Part of the energy of these currents becomes dissipated into heat in the iron; for when the primary current ceases to grow, these eddy currents stop. In fact, the iron core acts as a very low resistance secondary circuit. This heat means so much energy wasted. It is not of great consequence in starting or stopping the current once only, but in some cases, such as the core of an armature of a dynamo (p. 992) or of a transformer (p. 984), the magnetic flux is reversed many times per second, and the waste of energy and heating of the iron are highly objectionable.

To diminish the amount of eddy current in such cases the iron core is built up of laminae, or of wires, each lamina or wire having a thin coat of iron oxide upon it which insulates it from its neighbours.

The direction of lamination is so chosen that a maximum of resistance is opposed to the eddy currents The core in such a case as Fig. 916 would therefore be built up of sheets (or wires) parallel to the axis of the magnetising coil. Such lamination enormously reduces the eddy currents and the consequent energy losses and heating.

Lenz's law.—Since a current in a magnetic field experiences a force, it follows that whenever induced currents arise, forces come into play between these and the field which causes them which may be a moving magnet or a changing current By applying the laws already given, the direction of these forces can be found, but there is a simple general expression of them known as **Lenz's law** which enables us to give the direction of such forces at once.

When a circuit and a magnetic field move with respect to each other, the induced currents bring about forces which always tend to oppose the motion.

Let us apply this law to the case of a magnet NS (Fig. 917) approaching a solenoid AB The law on p 979 tells us that the induced current, as seen from the magnet, is anticlockwise. Thus there is a N pole of the solenoid at A (p 967). This repels the N pole of the magnet, and thus **opposes the motion** of the magnet.

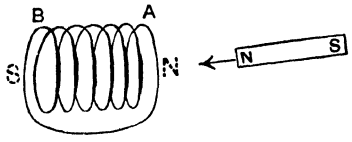


FIG. 917.—Application of Lenz's law

Whatever may be the nature of the motion of the magnet or coil, the same opposition to the motion will be brought about.

This effect has many important applications. For example, if it is desired to render the motion of the suspended coil of a galvanometer dead-beat, the coil is wound upon a copper frame. If the coil oscillates, this copper frame cuts the magnetic field of the permanent magnets, and according to Lenz's law, forces are brought into play which quickly bring the coil to rest. A useful method of bringing the coil of a galvanometer to rest when it is not provided with a copper frame, is to short circuit the terminals of the galvanometer. As the coil oscillates, induced currents then flow in it and it is rapidly brought to rest. In fact, the coil itself acts as the copper frame just described.

Many modern forms of electro-motor also depend upon this phenomenon. A mass of metal mounted upon an axle is subjected

to a rotating magnetic field By Lenz's law the currents induced in the metal, by their magnetic action, tend to oppose the relative motion of the conductor and the rotating magnetic field. The conductor is thus dragged round after the field.

Induction coil.—It is clear from the definition of mutual inductance (p. 981) that for a given rate of change of current in the primary circuit, the greater the mutual inductance of the two coils, the greater will be the electromotive force produced in the secondary. In the case of the **induction coil**, the mutual inductance is made so great that the electromotive force in the secondary amounts to hundreds of thousands of volts. The mutual inductance is made large by providing an iron core for the coils, and by making the number of turns in the secondary coil very great.

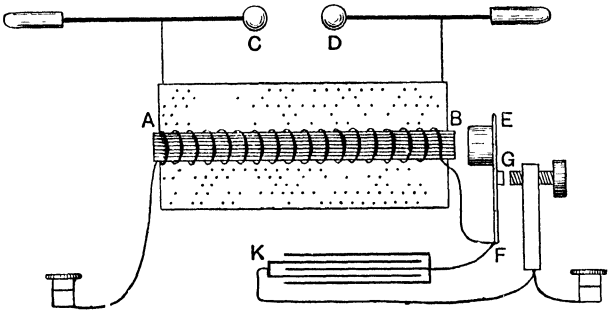


FIG 918.—Induction coil

The arrangement of the circuits in the induction coil is shown diagrammatically in Fig. 918. Upon the soft iron core AB, consisting of a bundle of soft iron wires, the primary coil, consisting of a few layers of thick wire, is wound. The secondary coil, consisting of many thousands of turns of fine wire, is shown in section, the ends being connected to the sparking terminals CD.

In a circuit such as the primary, the current does not become established or die away instantaneously. It dies away, however, more quickly than it is established, and consequently the rate of change of magnetic flux through the secondary is greater at the 'break' of the primary circuit than at the 'make.' A higher electromotive force is therefore produced at the break of the primary than at the make. In practice the electromotive force at 'break' is the only one high enough to cause the secondary current to jump the air gap at CD.

In order to render the making and breaking automatic, a spring EF is usually provided. The soft iron head E is pulled towards the

iron core AB when this is magnetised by the primary current. This breaks the contact at G, where both contact surfaces are faced with platinum, to diminish the sparking that occurs whenever the circuit is broken. When the break occurs, the iron core becomes demagnetised and the pull on E ceases. The spring then flies back, making contact again at G, and so the process is repeated. The number of makes and breaks occurring per second depends upon the stiffness of the spring and the mass of the iron head E.

Another form of interrupter sometimes employed consists of a rapidly rotating toothed wheel, against which a mercury jet is directed. The primary current is broken every time a tooth leaves the jet

A commutator for reversing the direction of the primary current is usually provided, but this is not shown in Fig 918

For the purpose of increasing the efficiency of the induction coil, a tinfoil condenser is usually connected across the spark gap. The effect of this is to increase considerably the length of spark obtainable from the coil. The function of this condenser is complicated, but its effect is best understood by considering that on the break occurring at G, the current does not merely fall to zero, but proceeds to flow in the opposite direction for a time, the current flowing into the condenser. Thus, instead of a mere stoppage of the current at the break there is an actual reversal of it Hence the electromotive force in the secondary lasts for a longer time. The drop of current also occurs more quickly, and both effects increase the efficiency of the coil

The transformer.—It will have been noticed in the case of the induction coil, that the primary current is supplied at low voltage from probably a few secondary cells, while the current in the secondary circuit is at very high voltage. Of course the secondary current is correspondingly smaller than the primary current. In the supply of alternating currents it is often desirable to be able to convert from one voltage to another. This is done by means of the **transformer** A 'step-up' transformer is one in which the voltage is changed from low to high. In a 'step-down' transformer the reverse is the case. Let the alternating current enter at the terminals AB (Fig. 919) of the coil wound upon a laminated soft iron core, which in this case has the form of a ring. Considering this as the primary, the secondary consists of a great number of turns of fine wire having terminals CD.

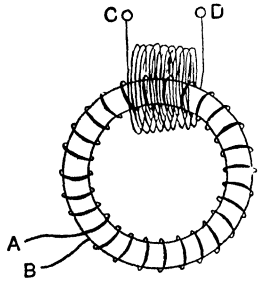


FIG 919.—Circuits of the transformer

An alternating current may be represented by a curve, such as

the thick line curve in Fig. 920. At points such as A, C and E the current is changing most rapidly, and the magnetic flux through the secondary is also changing rapidly. Hence the secondary e.m.f. is great. It is positive at A and E and negative at C. At B, D and F

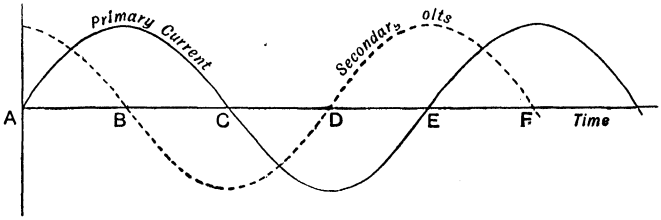


FIG. 920.—Current and e.m.f. curves for the transformer.

the primary current is for a moment constant and the secondary e.m.f. is zero. The dotted curve represents the secondary e.m.f. It should be noticed that the curves are not drawn to scale.

For a step-down transformer CD (Fig. 919) would be the primary and AB the secondary. In this case small current at high voltage would be supplied and large current at small voltage obtained from the transformer.

The ratio of the primary and secondary voltages is approximately the same as that of the number of turns in the primary and secondary coils.

Thus,

$$\frac{\text{primary voltage}}{\text{secondary voltage}} = \frac{\text{number of primary turns}}{\text{number of secondary turns}}.$$

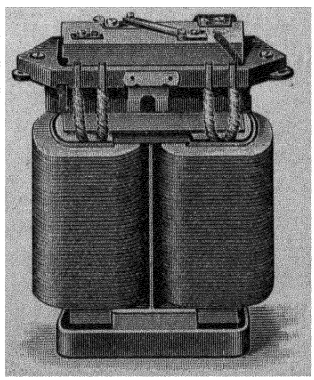


FIG. 921. The transformer.

The use of the transformer is to cheapen the transmission of electric current over long distance; the small current at high voltage only requires thin copper wires to transmit it, while for the same rate of transmission of energy at low voltage, the current must be great and stout copper leads are then necessary. Hence it is cheaper to transmit at high voltage and transform down to low voltage at the place where the current is being used. An illustration of a transformer is given in Fig. 921.

Self-induction.—The law that the changing magnetic flux threaded through a circuit causes an electromotive force, applies to the

circuit in which the current is changing as well as to neighbouring circuits. Consider the circuit ABC (Fig. 922) in which the current is supposed to be growing. The magnetic flux through the circuit is increasing, and by applying the rule on p. 979 we see that the e.m.f due to this acts in the direction CBA That is, it is in the opposite direction to the current. The current, therefore, is less at any given instant than it would be if this induced e.m.f were absent.

On the other hand, if the current is decreasing, then the magnetic flux is also decreasing, and the induced e.m.f. acts in the same direction as the current, and therefore tends to prevent it from dying away. The current, therefore, decays more slowly than would be the case if no such e.m.f. were present.

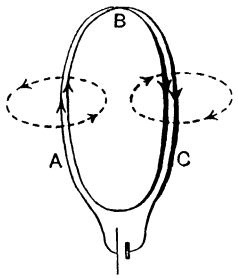


FIG 922.—Self-induction

This effect is called **self-induction**, and its general effect is to oppose any change in the current, and hence to make the current change more slowly than it would if there were no self-induction

The electromotive force, being equal to the rate of change of magnetic flux, is greater, the greater the rate at which the current dies away. Hence on breaking a circuit the rate of change is very great, and is generally sufficient to produce an e.m.f. that will cause the current to jump the gap for an instant. This is the cause of the spark that occurs when a current circuit is broken. With a large magnetic flux, as in the case of an electromagnet, the e.m.f. due to self-induction may be so great that the spark is very violent.

The measurement of self-induction is similar to that of mutual induction (p. 981), being the e.m.f. in the circuit due to unit rate of change of current. It is therefore measured in henrys. The value is called the **coefficient of self-induction**, or the **self-inductance.**

The effect of self-induction on Wheatstone's bridge measurements may be important if precautions are not taken. Thus, on closing the battery circuit the currents in the branches may not grow at the same rate. There will then be a momentary kick of the galvanometer, although the balance may be perfect for steady currents. It is for this reason that the battery key is always closed before the galvanometer key (p. 887). This ensures that the currents will become steady before the galvanometer is joined to the circuit.

In order to render he disturbing effect as small as possible, resistance coils are usually wound **non-inductively.** That is, the wire is first doubled and then wound upon the bobbin (p. 880). In this way the current going and returning is so nearly in the same position that the magnetic field due to it is extremely small, and hence the self-inductance is small.

Exercises on Chapter LXXVI.

1. Write an account of the theory and construction of a moving coil galvanometer. Why is the coil sometimes enclosed in a thin silver tube? In what circumstances would the presence of the silver tube be disadvantageous? L.U.

2. Describe two forms of interrupter that may be used with an induction coil.

Discuss the action that takes place in the induction coil, and the characteristics of the currents in the two coils.

What is the function of the condenser sometimes employed? L.U.

3. Explain the terms mutual inductance and self-inductance. How may a resistance coil be wound so that it shall have extremely small self-inductance?

4. Explain the construction and action of a transformer L.U.

5. Give the laws of production of an electromotive force in a circuit when this is cutting across a magnetic field.

A closed coil of wire rotates slowly about a vertical axis, and a magnetic needle is suspended at its centre. As the coil rotates, it cuts the earth's magnetic field. Represent by a curve, or other diagram, the deflecting couple acting on the needle during one rotation of the coil. L.U.

6. A copper disc of radius 18 cm. rotates with its axle parallel to a magnetic field of strength 20 c.g.s. units. If the disc makes 250 revolutions per minute, find what e.m.f. acting along a radius will be developed.

7. A solenoid of length 50 cm. consisting of 1000 turns of radius 3 cm. carries a current of 0·6 amp. A secondary coil of 500 turns is wound upon the middle part of the solenoid. Calculate the average e.m.f. in the secondary coil if the primary current falls to zero in 0·001 sec.

8. Explain why the resistance coils used in connection with a Wheatstone's bridge should have very small self-inductance. If the coils are known to have considerable self-inductance, what precaution must be taken in making the test.

CHAPTER LXXVII

THE DYNAMO AND MOTOR

Coil rotating in magnetic field.—Consider a plane coil of wire mounted so that it can rotate about an axle O (Fig. 923) at right angles to a uniform magnetic field of strength H. As the coil rotates it cuts across the field and an electromotive force is produced in it. When the plane of the coil is at right angles to the field as at CD, the e.m.f. is zero, since the edges of the coil are not in this position cutting across the field. At EF, where the plane of the coil is parallel to the field, the e.m.f. is a maximum, for the edges of the coil are now cutting perpendicularly across the lines of force.

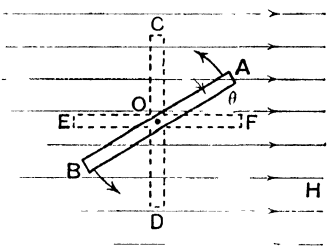


FIG. 923.—Rotating coil

For a position AB, where the coil makes an angle θ with the magnetic field H, only the component of the field H sin θ at right angles to the

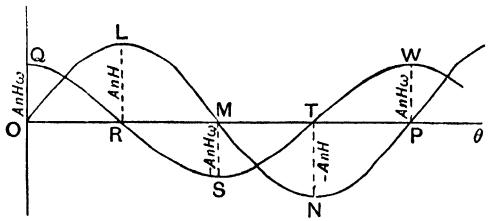


FIG. 924.—Curves of magnetic flux and e.m.f. for a rotating coil

coil passes through it. Hence the number of lines passing through the coil, if it consists of n turns of area A sq. cm. each, is $AnH \sin \theta$. If this be plotted in the form of a graph for different values of θ as the coil rotates, we get the curve OLMNP (Fig. 924). O corresponds

to the position EF (Fig. 923) and L to position CD As the coil rotates with uniform angular velocity, the e.m.f. is proportional to the rate of change of the flux, which is given by the slope of the curve OLMNP. This is greatest at O, M and P, and zero at L and N. It is well known that the rate of change of a sine curve OLMNP is a cosine curve QRSTW. Hence the curve QRSTW represents the e.m.f. in the coil rotating with uniform angular velocity

In the neighbourhood of O, M and P, θ is very small, and we may write θ for $\sin\theta$. If the time in which θ is described is t, then the rate of change of $AnH\theta$ is $AnH\theta/t = AnH\omega$, where $\omega = \theta/t$, the angular velocity of rotation of the coil. Hence $AnH\omega$ is the maximum e.m.f. in the coil. The equation for the e.m.f is therefore

$$\text{e.m f.} = AnH\omega \cos\theta.$$

It will be seen that the direction of the e.m.f in the coil is reversed twice in every revolution. If the ends of the coil are connected to metallic rings A and B (Fig. 925), with metal brushes C and D touching them, there will be an alternating e.m.f. acting in the circuit joined to C and D. This arrangement of slip-rings with brushes touching them permits continuous rotation of the coil. The alternating-current (A.C.) dynamo is built upon this principle.

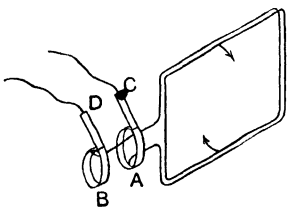


FIG. 925.—Coil producing an alternating e m f.

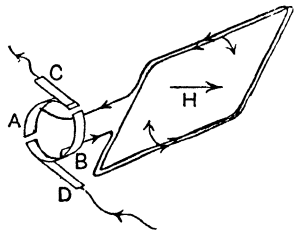


FIG. 926.—Rotating coil with commutator

Principle of the direct-current dynamo.—When the current in the external circuit is required to be always in the same direction, that is, to be **direct current** (D.C.), the slip-rings (Fig. 925) must be modified in such a way that the connection is reversed every half revolution. This reversal is obtained by using one ring, split into two parts diametrically, the two halves, A and B (Fig. 926), being mounted on a non-conducting axle. One end of the coil is connected to A and the other to B. The brushes C and D, connected to the external circuit, bear against the split ring. This arrangement is called a **commutator.** The current is now always in the same direction in the

external circuit. An inspection of Fig. 926 shows that the reversal of the direction of the e.m.f. in the coil takes place when the coil is vertical. It will be seen that the descending side of the coil is always in connection with the brush C and the ascending branch with the brush D. Hence the e.m.f. is always directed the same way round the external circuit.

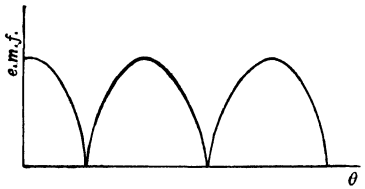


FIG. 927.—e.m.f. curve when a commutator is used

The electromotive force produced by a single coil of this type is always in the same direction. It is not, however, constant in value. The commutator has merely changed the direction of the reverse halves of the curve of e.m.f. so that for the external circuit the e.m.f. curve will be of the form shown in Fig. 927.

Gramme ring armature.—In a dynamo for actual use, the e.m.f. must be as nearly steady as possible. Hence the unevennesses of

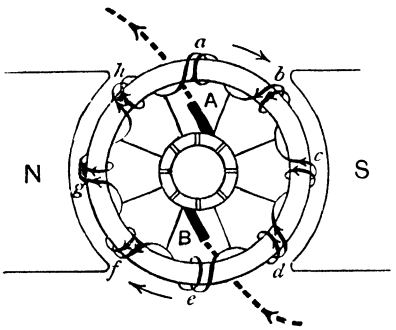


FIG. 928.—Gramme ring armature

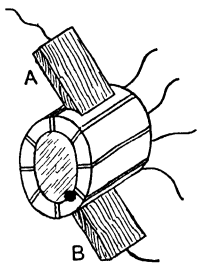


FIG. 929.—Commutator and brushes

the e.m.f. curve for a single coil must be smoothed out in some way. The rotating coil or coils of the dynamo is called the **armature**, and the **Gramme ring** form of armature produces a much more nearly constant e.m.f. than is possible with a single coil. A number of coils are wound continuously upon an iron ring. In Fig. 928 there are eight such coils, *a*, *b*, *c*, *d*, *e*, *f*, *g* and *h*. The commutator is also in eight sections, and each bar of the commutator is connected to the junction of a pair of the coils in the ring. The brushes A and B bear upon the commutator as shown. A better view of the commutator and brushes is given in Fig. 929. The commutator segments are made of hard drawn copper, and are insulated from each other by means of sheet mica, which wears down, by the rubbing of the brushes, at the same

rate as the copper. The segments are also insulated from the shaft by means of mica, and are held in position by two nuts (not shown), from which also they are insulated. The brushes are generally made of copper or brass gauze compressed into a block; but they are sometimes of hard carbon.

N and S are the poles of the field magnet (Fig. 928).

When the armature rotates in the direction shown (Fig. 928) it can be seen, by applying the rule on p. 979, that the e.m.f. in every coil is as indicated by the arrows. The coils *c* and *g*, at the instant shown, have maximum e.m.f., while at this same instant there is no e.m.f. in *a* and *b*. It should be noticed that the currents in *b*, *c* and *d* are all flowing towards the commutator bar in contact with the brush A, and that those in *f*, *g* and *h* are also flowing towards A, and the current in the external circuit will therefore flow from A to B.

At the instant given, the coils *a* and *b* are on the point of being short circuited; but since there is no e.m.f. in them this does no

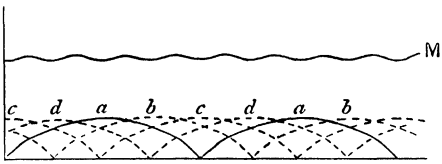


FIG. 930.—Resultant e.m.f. in the gramme ring

harm. Further, since *b*, *c* and *d* are in series, the resulting e.m.f. will be the sum of the separate e.m.f.'s, and the same applies to *f*, *g* and *h*. As the armature rotates, the e.m.f. in each coil will vary as shown in Fig. 927. Thus the e.m f. in coil *a* will correspond to the thick line in Fig. 930. The e.m.f.'s for *b*, *c* and *d* are drawn dotted, to avoid confusion. At any instant the resultant e.m.f. between A and B is the sum of the e.m.f.'s for the four coils in one side of the armature. These e.m.f.'s are added up in Fig. 930 and the resultant curve LM is obtained. It is then seen that with eight coils in the armature the e.m.f. is very nearly steady. With a greater number of coils the e.m.f. would be still more nearly constant.

Lead of the brushes.—It will be remembered that each coil in the armature is short circuited twice in a revolution, as the two commutator segments to which its ends are attached come under a brush. Hence it is necessary that this should take place when there is no e.m.f. in the coil, or else the short circuiting will mean a big current in it, with consequent heating of the coil and sparking when the circuit is broken an instant later. If the magnetic field in which

the coil rotates were symmetrical about a vertical plane, as it is when the armature is at rest (Fig. 931) the brushes would have to be placed at the ends of a vertical diameter. When the current flows

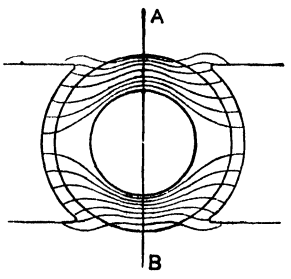


FIG 931.—Magnetic field with armature at rest

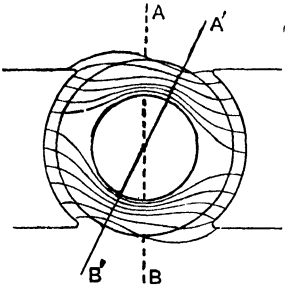


FIG 932.—Magnetic field when current flows in armature

in the armature, however, it produces a component of the magnetic field perpendicular to that of the field magnets, and the resultant field is as shown in Fig. 932.

The brushes must now be placed at the ends of the diameter A'B'. The coil short circuited by the brush will then be moving parallel to the magnetic field, and the e.m.f. in it is zero, and excessive sparking will be avoided. The angle by which the line joining the brushes must be advanced from AB to A'B' to stop sparking is called the **lead of the brushes.**

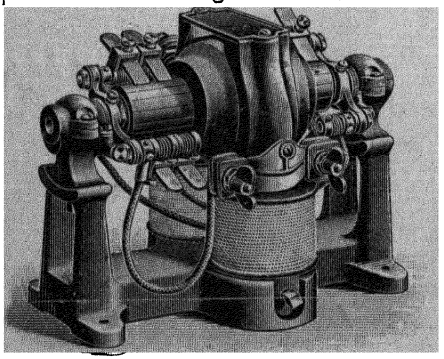


FIG. 933.—Direct current dynamo

To facilitate this adjustment the brushes are carried in brush holders (Fig. 933) attached to a framework which can be rotated through any desired angle, the position for sparkless running being found by trial.

Drum armature.—In the ring armature the conductors lying outside the iron core cut the magnetic field as the armature rotates, but those lying on the inside of the ring do not cut any appreciable magnetic field, and therefore they do not give rise to any e.m.f. They are merely idle conductors, and for this reason, the ring form of armature is only used in certain small machines. The simple

rotating coil (Fig. 926) does not have any idle conductors except at the ends, but the electromotive force in it is not sufficiently steady for industrial purposes. If, however, a number of such coils in series arranged at various angles to each other can be employed, their resultant e.m.f. will be much more nearly constant than when one is used alone. The connecting up of such coils presents difficulties, but the arrangement is practically realised in the **drum armature** (Fig. 934).

A number of straight copper conductors, properly insulated, lie in slots cut in the cylindrical surface of the armature, and are properly insulated from it. The simple case of eight such conductors will now be considered, although of course the number would be much greater in practice. There is also great variety in the methods of connecting the conductors to each other and to the commutator bars. When there are eight conductors *a*, *b*, *c*, *d*, *e*, *f*, *g* and *h* (Fig. 934) there will be four commutator bars A, B, C and D. With the direction of rotation shown, the e.m.f. in *b*, *c* and *d* is from back to front of the armature, while in *f*, *g* and *h* it is from front to back. The conductors are joined to the commutator bars as indicated by the thick lines, while the dotted lines show how the conductors are joined together at the back of the armature.

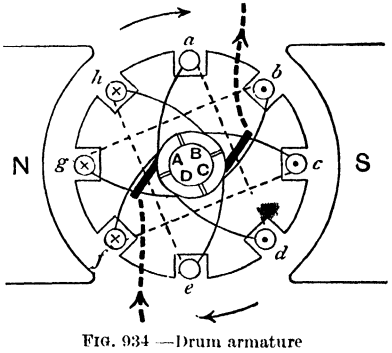


FIG. 934.—Drum armature

Starting with the bar A, the current enters the armature from the external circuit and divides into two circuits in parallel. Thus one part passes to *a*, and flowing down this comes at the back of the armature to *d*, thence it passes across the front, down *g* and across the back to *b*, and then to the brush at C. It then flows out to the external circuit. The other part of the current flows from A to *f*, thence by way of *c*, *h* and *e* to the brush at C.

We see, then, that there are two circuits in parallel. Also, as the brushes pass from one commutator bar to the next, one coil is short circuited and the brushes must have such an angle of lead that the e.m.f. in this coil is zero at this particular moment (p. 992).

Electromotive force in armature.—Since there are two sets of conductors in parallel, the e.m.f. of the dynamo is equal to the e.m.f. in either one of these sets. In the simple case of eight conductors, there will be four in each set, but in practice the number will, of course, be very much greater.

Let F be the total magnetic flux, or number of lines of force that each conductor cuts across in passing from top to bottom of the field. Again, let Z be the total number of conductors round the armature Then $\frac{1}{2}Z$ is the number of conductors in series. Each conductor cuts F lines in a half turn, so that if the armature makes N revolutions per second, it makes 2N half revolutions per second, and 2NF is the number of lines cut per second by each conductor It is therefore the average e.m.f in each conductor. As the number of conductors in series is $\frac{1}{2}Z$ $2NF \times \frac{1}{2}Z = FNZ$ is the e m f. between brush and brush. This is, of course, in absolute c.g.s. units of e.m.f.

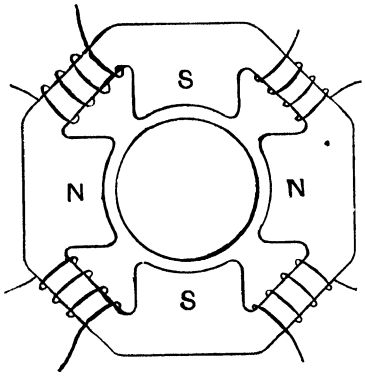


FIG 935 —Four-pole field magnet.

$\therefore$ e.m f. of machine
$= FNZ \times 10^{-8}$ volts

In some machines a number of pairs of poles are arranged round the circumference of the armature, and the above result gives the e m f for **each pair of poles.** For the resultant e.m.f. we must multiply by the number of pairs of poles.

Field magnets.—For the production of the magnetic field of the dynamo, powerful field magnets are necessary These have various

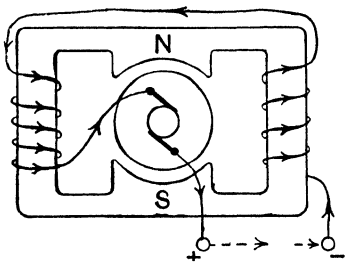


FIG. 936 —Series wound field magnet

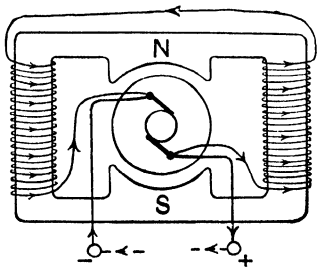


FIG 937 —Shunt wound field magnet.

forms, one form being that given in Fig. 936. Here the current flowing in the field coils, which causes the magnetisation, flows also in the external circuit. The field coils are thus in series with the armature and the external circuit. When this arrangement is adopted the dynamo is said to be '**series wound**' Series winding has this advantage, that as the current in the external circuit increases,

the magnetisation of the field magnets is thereby increased. There is usually enough residual magnetisation in the iron cores of the field magnets to produce a feeble current when the machine starts, provided that the external circuit is closed. The voltage and current then mount up rapidly as the magnetisation increases. The series winding consists of few turns of low resistance, since the resistance of the coils is included in the internal resistance of the machine.

In Fig. 937 the field coils consist of a great many turns of fine wire and are placed in parallel with the armature and external circuit. For this reason this is called a '**shunt wound machine.**'

If the resistance of the external circuit decreases, so that the external circuit takes more current, the current in the field coils drops, and hence the e.m f. of the machine falls. The shunt wound machine is, however, useful for charging accumulators; since on connecting the machine to the accumulators, current flows in the field coils and produces excitation of the field magnet in the proper direction. Also, as the cells become more fully charged, their e.m.f. rises, and hence more current flows in the shunt coils, thereby increasing the magnetisation and the e.m.f. of the dynamo. Series winding must not be exclusively employed in a machine for cell charging, as on connecting the machine to the cells, they cause a current which produces the magnetisation in the wrong direction in the field magnets, and the e.m.f. of the dynamo becomes reversed, with disaster to both dynamo and battery.

Characteristic curve of a dynamo.—Considerable information can be deduced from a curve of current and voltage for a dynamo running at constant speed. Such a curve is called a **characteristic curve** for the machine. On running the machine at constant speed with an ammeter in series and a voltmeter across the machine terminals, current and voltage can be observed. On varying the external load the current can be changed. OAB (Fig. 938) is the type of curve obtained for a series wound machine. The curve rises at first owing to the rapid increase in magnetisation of the field coils with increasing current. At high currents it drops slightly, as the external resistance becomes less in proportion to the internal resistance of the machine.

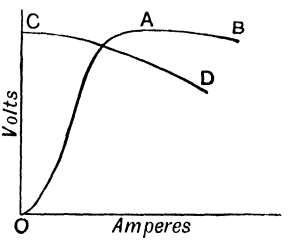


FIG. 938.—Characteristic curve of a dynamo

For a shunt wound machine the characteristic is of the form CD (Fig. 938). The e.m.f. is greatest when there is no external current, the whole current then going through the field coils. As the external

current becomes greater, that flowing in the shunt coils gets less, and the e.m.f. therefore drops.

By suitably combining these types of winding, a machine having a nearly flat characteristic may be produced, which therefore has nearly constant voltage at all loads. The machine has an ordinary shunt winding consisting of many turns of fine wire, but upon the outside of this a few series turns are wound. Such an arrangement is called **compound winding.**

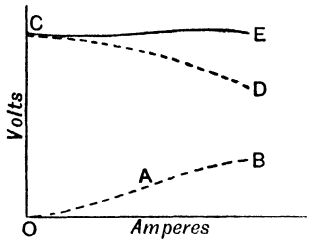


FIG. 939.—Characteristic curve of a compound wound dynamo

If the curve OAB (Fig. 939) gives the e.m f. due to the series turns, and CD that due to the shunt turns, the curve CE, which is the sum of the other two, will be the characteristic when both windings are employed.

Such compound wound dynamos are largely used for purposes of public supply, and in all cases where it is desired to maintain automatically a constant voltage of supply at all loads.

Efficiency of dynamos.—There are several points of view from which the efficiency of a dynamo may be regarded The fraction of the electric power generated in the machine which is available in the external circuit is called the **electrical efficiency.** Thus,

$$\text{Electrical efficiency} = \frac{\text{watts in external circuit}}{\text{total watts generated}} \quad \ldots\ldots(1)$$

This quantity will, of course, vary with the current produced, and will be determined by the p d between terminals E, the current I, and the resistances of the armature R_a, and of the field coils, R_m for a series machine, or R_s for a shunt machine.

Thus, for a series machine,

$$\text{Electrical efficiency} = \frac{EI}{EI + I^2(R_a + R_m)} \quad \ldots\ldots(2)$$

and for a shunt machine,

$$\text{Electrical efficiency} = \frac{EI}{EI + \frac{E^2}{R_s} + \left(I + \frac{E}{R_s}\right)^2 R_a} \quad \ldots\ldots(3)$$

The user of the dynamo is usually chiefly concerned with its **commercial efficiency**, which may be expressed as follows :

$$\text{Commercial efficiency} = \frac{\text{watts in external circuit}}{\text{mechanical power supplied to dynamo}} \quad \ldots(4)$$

The mechanical power supplied to the dynamo is used in supplying the energy in the form of current, and also in providing for certain

unavoidable losses, such as friction at the bearings, magnetic hysteresis losses in the armature core (p. 823), etc. If these losses constitute a total of W watts, the equation for a series machine becomes,

$$\text{Commercial efficiency} = \frac{EI}{EI + I^2(R_a + R_m) + W}, \quad \ldots\ldots\ldots\ldots(5)$$

with a corresponding equation for the shunt machine.

To the mechanical engineer the chief question of interest in the dynamo is the ratio of the total watts generated to the mechanical power supplied, which is called the **mechanical efficiency.** Thus,

$$\text{Mechanical efficiency} = \frac{\text{total watts generated}}{\text{mechanical power supplied}}. \quad \ldots\ldots\ldots\ldots(6)$$

It will further be seen that

$$\text{Mechanical efficiency} = \frac{\text{commercial efficiency}}{\text{electrical efficiency}}. \quad \ldots\ldots\ldots\ldots(7)$$

Electromotors.—Although it is a general rule that any dynamo can be run as a motor, still the design of the motor is usually different from that of the dynamo, to suit the purposes to which the motor is to be put. That a dynamo will run as a motor if supplied with current may be seen from Fig. 940, which is similar to Fig. 934. In this case, however, current produced by some external source enters at the brush A and leaves at C. Hence the current flows in the conductors *a*, *b*, *c*, etc., exactly as in the previous case. That is, in *b*, *c* and *d* the current flows from back to front and in *f*, *g* and *h* from front to back. Now, taking into account the direction of the magnetic field and applying the **left-hand** rule (p. 968), it will be seen that the forces upon the armature conductors are in such a direction that the armature is urged to rotate in an anticlockwise direction. Hence the brushes must be set for this direction of rotation.

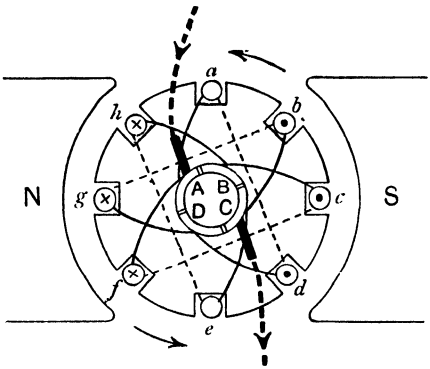


FIG. 940.—Drum wound motor armature.

It should further be noted, that if the directions of the current in the armature and in the field magnets be both reversed, the

direction of rotation will remain the same. In order to reverse the direction of rotation of the motor **either** the direction of the field, **or** that of the current in the armature must be reversed, **but not both.**

For the direction of rotation of the motor to be the same as that for the dynamo, the field having the same direction, the current in the armature is in the **opposite** direction in the motor to that in the dynamo, and the brushes will require to have a **lag**, if sparkless running is to be attained, instead of the **lead** required in the case of the dynamo (p. 992).

Back e.m.f. in motor.—In the conductors of the armature of a motor, an e m f. is produced, because these conductors cut across the magnetic field Its direction is such that it opposes the current driving the motor It therefore tends to reduce the current in the armature and is known as a **back e.m.f.** Hence the current in the armature may be calculated from the expression $(E - E')/R$, where E is the e.m.f. applied to the armature terminals, E' the back e.m.f in the armature, and R its resistance. The power IE watts is thus applied to the armature, and of this $I(E - E') = I^2R$ is wasted as heat in the armature, while IE' is the power in watts converted into work in turning the armature

It is clear that E' depends upon the speed of rotation of the motor. When the motor is at rest $E' = 0$, and if the e m f E were then applied to it, a very great current would flow, sufficiently great to injure the insulation of the conductors and perhaps melt the solder used in making the joints. For this reason it is necessary to employ a **starting resistance** in series with the motor. By means of a switch, the coils of the starting resistance can be cut out one at a time, so that the current never rises to an excessive amount. As the speed rises, the back e.m.f. increases, and at full running speed the starting resistance is entirely cut out.

Efficiency of motors.—Just as in the case of the dynamo (p. 996) the efficiency of a motor may be considered from several points of view. Three of the most important are as follows :

Electrical efficiency

$$= \frac{\text{Electrical power spent in producing rotation}}{\text{Total electrical power supplied}}. \quad \ldots\ldots(1)$$

Commercial efficiency

$$= \frac{\text{Brake output}}{\text{Total electrical power supplied}}. \quad (2)$$

Mechanical efficiency

$$= \frac{\text{Brake output}}{\text{Electrical power spent in producing rotation}} \quad \ldots\ldots(3)$$

$$= \frac{\text{commercial efficiency}}{\text{electrical efficiency}} \quad \ldots\ldots(4)$$

The brake output is the power which the motor is capable of giving out for driving purposes, and may be measured by means of a brake (p. 513).

If the back e.m.f. E' be known (p. 998), these efficiencies may be simply expressed; for $E'I$ is the rate at which electrical power is converted into mechanical power, while EI is the total electrical power supplied to the motor, E being the p.d. between terminals;

$$\text{Electrical efficiency} = \frac{E'I}{EI} = \frac{E'}{E} \text{ (for a series motor)} \quad \ldots\ldots(5)$$

$$= \frac{I_a E'}{(I_s + I_a)E} \text{ (for a shunt motor)}, \quad \ldots\ldots(6)$$

where I_a and I_s are the currents in the armature and the shunt coils respectively.

Again, if W is the waste of power in friction, hysteresis loss, etc.,

$$\text{Commercial efficiency} = \frac{I_a E' - W}{I_a E} \text{ (for a series motor)} \quad \ldots\ldots(7)$$

$$= \frac{I_a E' - W}{(I_s + I_a)E} \text{ (for a shunt motor).} \quad \ldots\ldots(8)$$

The most profitable speed at which to run a motor is not necessarily the speed at which its efficiency is greatest. For, take a series motor in which the back e.m.f. is E' at any given speed. This, of course, depends upon the speed, increasing with it, and R is the resistance of the machine.

Now, electrical efficiency $= E'/E$, and the greatest possible value of this is unity, when, of course, $E' = E$, and since $I = (E - E')/R$ (p. 998), $I = 0$. This means that the efficiency increases as the speed increases; but as the speed for which $E' = E$ is approached, the current becomes so small that the output of the motor is negligible.

To find the speed for greatest output, note that,

$$\text{Electrical power spent in producing rotation} = E'I$$
$$= I(E - RI)$$
$$= IE - I^2R.$$

Draw a line OB (Fig. 941) such that $CB/OC = R$, then the greatest possible current that the motor will take is $E/R = OC$, if $CB = E$.

Now complete the rectangle OABC For any other current OH, the electrical power spent in producing rotation is represented by the area of the rectangle AF, for

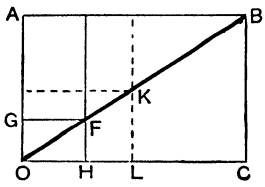


FIG. 941 —Diagram for motor efficiency

$$IE - I^2R = (OH \times OA) - \left(OH^2 \times \frac{FH}{OH}\right)$$
$$= (OH \times OA) - (OH \times FH)$$
$$= \text{area of rectangle AF.}$$

This area is very small when the current I represented by OH is small, and vanishes when $I = 0$. It vanishes again when $I = OC$, since in this case the motor is at rest, and it has its maximum value equal to the area AK when the current has the value OL, which is half its maximum value. Also, the speed for greatest output is half the speed for which the back e.m.f. would equal the applied e.m.f. if the field be constant, for

$$\frac{E - E'}{R} = \frac{1}{2}\frac{E}{R}.$$

or,
$$E' = \tfrac{1}{2}E.$$

Alternators.—It was seen (p. 988) that a single coil rotating in a magnetic field gives rise to an alternating electromotive force. To produce a sufficiently high e.m.f. for any useful purpose, the coil would require to have an immense number of turns, or else its rate of rotation would have to be very high. Also, in using a single turn, a large amount of space would be wasted In order to get over these difficulties, many coils are arranged in series round a cylindrical core, and pole pieces lie outside the coils. The **alternator** shown in Fig. 942 has four poles and four rings. The field magnets in the case of an alternator must be separately excited by means of a continuous current. In small machines this current is supplied by a battery, but in large machines a separate direct-

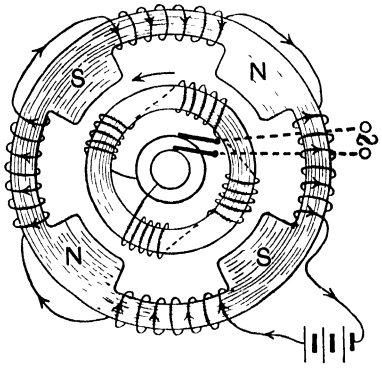


FIG 942 —Four-pole alternator.

current dynamo is used for producing the field current. The regulation of the alternating voltage to secure constancy is performed by altering the resistance in the field circuit by means of a rheostat.

The e.m.f. in the coils depends only upon the relative motion of the coils and the field magnet. In the case given (Fig. 942) the field magnets are fixed, and the coils rotate in the direction of the arrow. Some machines, however, are designed so that the coils are fixed and the field magnets rotate. For the e.m.f. in the above case to remain unchanged, the magnets would have to rotate in a direction opposite to that indicated by the arrow. Whichever part is fixed is called the **stator.** The rotating part is called the **rotor.**

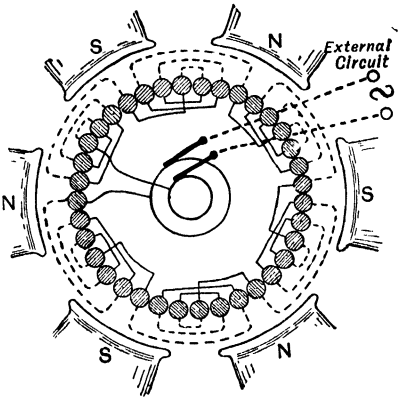


FIG 943.—Rotor connections for a six-pole alternator.

As was seen on p. 992, the ring form of armature is wasteful in copper, hence it is desirable to arrange the conductors upon the outside only of the iron core. In Fig. 943 an arrangement of thirty-six conductors round the surface of the rotor is shown. These are supposed to be connected at the front as shown by the thick lines and at the back by the dotted lines. There are three pairs of poles in the stator. It will be seen that all the conductors of the rotor are in series and that they are grouped into sets in such a way that the e.m.f. in the sets at any one instant is the same, but in passing from one

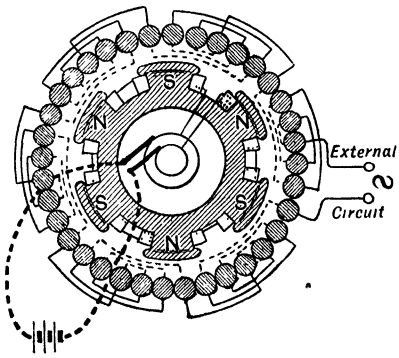


FIG. 944.—Stator connections for a six-pole alternator.

pole to the next of the same kind a complete alternation in e.m.f. is made.

In Fig. 944 is shown a similar arrangement, but with the field magnets as rotor, and the conductors, in which the alternating e.m.f is being produced, as stator. The field is shown as though it were excited by a battery, but a separate direct-current dynamo is usually employed for this purpose.

EXERCISES ON CHAPTER LXXVII

1. Explain the principle of the dynamo, and give sketches of (*a*) a series-wound, (*b*) a shunt-wound dynamo.

2. Give the laws of production of an electromotive force in a circuit when this is cutting across a magnetic field.

A closed coil of wire rotates slowly about a vertical axis, and a magnetic needle is suspended at its centre. As the coil rotates it cuts the earth's magnetic field. Represent by a curve, or other diagram, the deflecting couple acting on the needle during one revolution of the coil. L.U.

3. Describe some form of commutator by which the alternating e.m.f. in a coil rotating in a magnetic field may give rise to a current always in the same direction in an external circuit.

4. Describe the Gramme ring armature, and account for the fact that the brushes must be given a 'lead,' in order to obtain sparkless running

5. Give a sketch of the connections of a drum armature, and trace the current through *one* set of conductors from one brush to the other.

6. What is 'compound winding'? Account for the advantages obtained by means of it.

7. Describe some form of alternator, giving the connections of the conductors of the armature.

8. An electromotor is supplied with current from the mains at a fixed difference of potential. When heavily loaded and running slowly the current passing is about 5 amperes. As the load is progressively diminished and the speed increased the current diminishes, attaining its minimum value when the speed of the motor is greatest. Explain how this occurs.
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9. A continuous current shunt motor on being installed is found to rotate in the wrong direction. Describe what you would do in order to reverse the direction of rotation, and give reasons. C.G.

10. Describe some form of armature for a continuous current dynamo, stating the material of which each part is made and the reason why that material is used. Why is the armature core made up of a number of thin plates instead of a solid cylinder? C.G.

11. What is the "counter" or "back" electromotive force of a motor and on what does it depend? What change takes place in the speed of a series motor if the load is reduced? C.G.

12. In what forms may the efficiency of an electro-motor be expressed? How will the efficiency depend upon the speed in the case of a series motor?

13. Describe some form of electro-motor, and state how you would calculate the horse-power, neglecting friction and other losses, given that there are n straight conductors in the armature, each of which cuts across a magnetic field consisting altogether of 20,000 lines, N times per second, if 1 ampere flows in each conductor. L.U.

14. Find the electrical and commercial efficiencies of a series dynamo, if p.d. at terminals is 125 volts, current = 100 amperes, resistance of dynamo is 0·04 ohms and power supplied is 15·5 kilowatts.

15. Explain the terms electrical, commercial, and mechanical efficiencies of a dynamo.

Calculate the electrical and mechanical efficiency of a shunt dynamo when the p.d. between terminals is 150 volts, the external current 70 amperes, if the resistance of the armature is 0·06 and of the shunt coils 30 ohms, and the power wasted in friction etc. is 500 watts.

CHAPTER LXXVIII

THE TELEGRAPH, TELEPHONE, AND ELECTRIC LAMPS

The electric telegraph.—One of the most important applications of the electric current is its use for conveying messages. When the message is conveyed by means of a number of intermittent currents passing along a wire or cable from the transmitting to the receiving station, the arrangement is generally called a **telegraph.** A simple form of telegraph is shown in Fig. 945. A magnetic needle NS

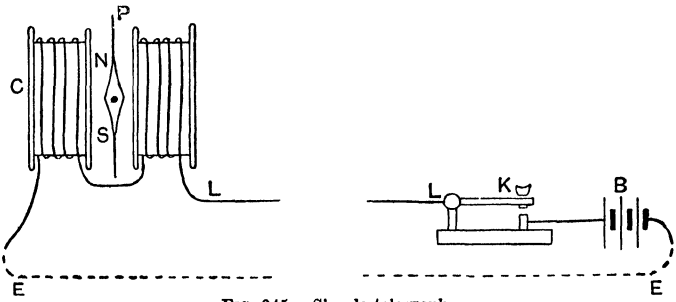


FIG. 945.—Simple telegraph.

is pivoted on the axis of a coil C, the arrangement being a simple form of galvanometer Attached to the axis of rotation of the needle is a pointer P, so that the movement of the needle may be seen from outside the coil. This simple galvanometer, or indicator, is situated at the receiving station. At the transmitting station is a battery in circuit with a key K. The circuit between the two stations is completed by the line or cable LL, the return part of the circuit being either another line EE, or in most cases the earth itself. For this purpose a plate is buried in the earth at each station and the end of the circuit E connected to the plate.

It will thus be seen that on closing the key K, the needle and pointer are deflected, and by a code of signals any desired message

may be transmitted. The code most frequently used is the **Morse code.** A long depression of the key corresponds to a dash and a short one to a dot. The Morse code is as follows ·

E	.	T	—				
I	..	M	— —				
S	...	O	— — —				
H	....						
A	.—	N	—.				
U	..—	D	—..				
V	...—	B	—...				
W	.— —	G	— —.				
C	—.—.						
R	.—.	K	—.—				
L	.—..	Y	—.— —				
F	..—.	Q	— —.—				
P	.— —.	X	—..—				
J	. — — —						
Z	— —..						

Short numerals preceded by FI.		Long numerals	
1	.—	1	.— — — —
2	..—	2	..— — —
3	...—	3	...— —
4	....—	4	....—
5	.	5	.....
6	—....	6	—....
7	— —...	7	— —...
8	— ..	8	— — —..
9	— .	9	— — — —.
0	—	0	— — — — —

Followed by FF

With the arrangement shown in Fig. 945 it is only possible to send messages in one direction. It must therefore be understood that the apparatus is duplicated, so that at each end there is a transmitting battery and key, and also a receiving indicator. The line can only be used for sending messages in one direction at a time.

Sounders and ink-writers.—It is generally preferable in receiving messages to take them by ear rather than by sight. Thus the pointer P (Fig. 945) strikes a bell when the needle is deflected. Also, it is usually arranged that the current in the line can be sent in one direction or the reverse so that the deflections are to right or left, right corresponding to a dash and left to a dot in the code. If there are two bells of different tones, one struck by P when the deflection is to the right and the other when it is to the left, this does away with the necessity of watching the instrument. The observer recognises the dots and dashes by the tones of the two bells.

There are also special **sounders** that can be used in conjunction with the galvanometer at the receiving station. The Morse sounder is used in this way. A two-pole electromagnet A, only one pole of which can be seen in the diagram (Fig. 946), attracts a soft iron armature D when the current from the line flows in its coils. This pulls down the arm to which D is attached, the sound being produced by the screw C coming into contact with a stop. The screw B

enables the distance of the soft iron D above the pole face to be adjusted to the proper amount. When the current ceases the arm is pulled up again by a spring which is not shown in the diagram

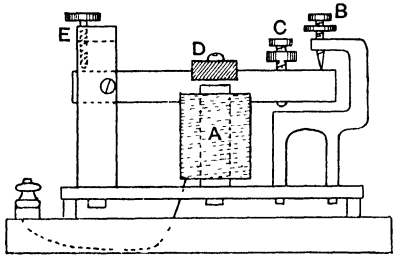


FIG 946.—Morse sounder

In the Morse ink-writer a wheel which is kept inked is carried by the pivoted arm. When the current from the line flows through the coils of the electromagnet the arm is pulled down and the inky wheel is brought into contact with a strip of paper which is driven past it by clockwork. A dash then corresponds to a line inked by the wheel upon this paper, a dot being a very short line.

For the purpose of signalling, a small **buzzer** is often employed. This is a small electro-magnet with a soft iron armature arranged as a make and break in the same manner as the interrupter of the induction coil (Fig. 918) A long buzz is used for a dash and a short one for a dot

Duplex telegraphy.—In the simple system described above, the line can only be used for transmission of messages in one direction at a time. This means very low efficiency for the line, so that it is desirable to increase the efficiency by arranging that messages can be sent in both directions at the same time. This is called a **duplex** arrangement. There are also **multiplex** arrangements, but only the duplex telegraph will be described here. There are two widely used duplex systems, namely, the **differential** system and the **bridge** system

The **differential** method is illustrated in Fig. 947. G_1 and G_2 are the galvanometers at the two stations. These galvanometers are wound in two similar sections so that the current from the battery B_1 will flow through these two sections in opposite directions when the key K_1 is depressed. If these two currents are equal, then the effect upon the galvanometer G_1 is zero. In order to make the currents equal, a resistance R_1 is placed in series with one of the coils, and the line in series with the other, the resistance R_1 being so adjusted that it is equal to the joint resistance of the line and the instruments at the far end. It follows that, on depressing K_1, current goes to the line, but the instrument G_1 is not affected. However, the instrument G_2 at the other end of the line is affected by this current, since it flows through the first coil and the key K_2 to earth, and also partly through the other coil and the resistance R_2. But in

this case it flows through the two coils in series, so that the effect is the sum of the two, and the instrument G_2 therefore indicates the signals from the first station.

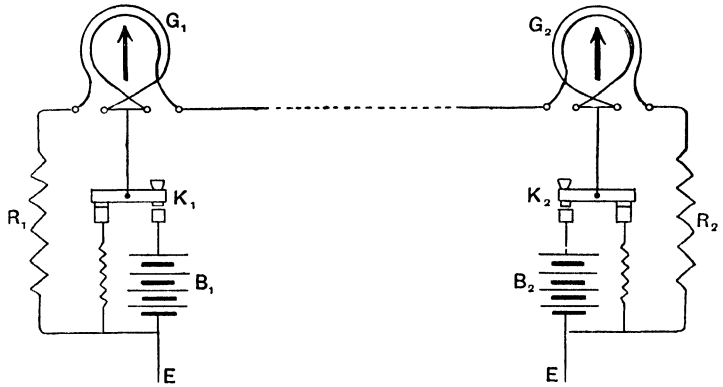


FIG 947.—Duplex telegraph (differential).

With this arrangement the line is available for sending messages in both directions simultaneously, since at either station a depression of the key does not affect the galvanometer at that station, but does affect the galvanometer at the other end of the line. This differential system is chiefly used for land telegraphs.

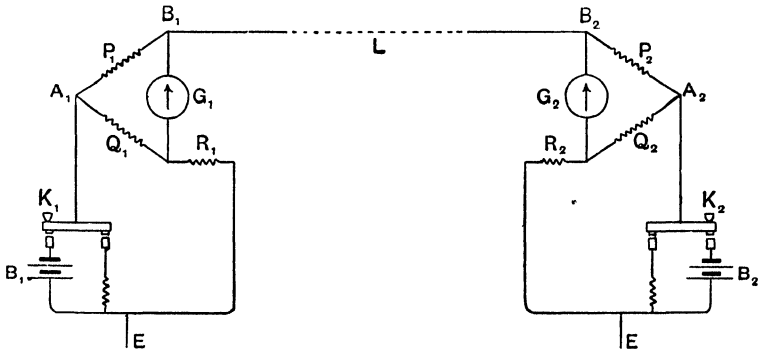


FIG 948.—Duplex telegraph (Wheatstone's bridge)

The **bridge method** is employed for submarine cables. At one station the battery B_1 (Fig. 948) is connected by means of the key to one corner, A_1, of a Wheatstone's bridge. The four arms of the bridge are P_1, Q_1, the line L, and the resistance R_1.

Then, if R_1 is so adjusted that

$$\frac{P_1}{Q_1} = \frac{L}{R_1},$$

no current flows through the galvanometer G_1 on depressing K_1. The current from the line on reaching B_2 will pass to earth, partly through P_2 and partly through G_2, so that the signal is recorded at this end. A similar bridge arrangement is at each end of the line, so that messages can be sent simultaneously in both directions

The galvanometer must be of a sensitive type, either a reflecting galvanometer, or the siphon recorder described below being used.

Relays.—For working an ink-writing machine, or a sounder, the currents from the line are, in many cases, too feeble. A relay is then employed. This consists of an electromagnet whose coils are in the circuit of the current at the receiver These coils consist of a great many turns of wire so that a feeble current is able to magnetise the core The arrangement of the Post Office standard relay is shown in Fig 949.

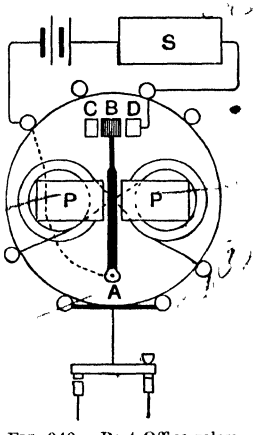


FIG. 949.—Post Office relay

PP are the soft iron pole pieces of the electromagnet, the coils of which are wound differentially and connected up in the same way as the galvanometer coils in Fig 947. At A is pivoted a soft iron arm which lies between P and P. This soft iron arm is maintained permanently magnetised by a steel permanent magnet, so that it is deflected to left or right according to the direction of the current from the line. Mounted upon the same axle as the soft iron arm is a light lever ending in the contact piece B, which makes contact with C or D. This closes the local circuit, consisting of a battery and the sounder or ink-writer S. Thus the current in the line, which is not itself strong enough to work the sounder, can close the local circuit by means of the much more sensitive relay, and thus enable the strong local current to work the sounder.

Siphon recorder.—In long-distance submarine telegraphy the currents are so feeble that a very sensitive recorder is necessary. The **siphon recorder** due to Lord Kelvin is commonly employed. It is really a suspended coil galvanometer. The coil A (Fig. 950) is suspended between the poles of a strong permanent magnet. *ab* and *cd* are two fibres connecting the suspended coil to a small block which can turn about the wire *ef* as axis. *s* is a fine tube fixed to the block *bd*, having one end dipping into an ink-pot and the other

resting upon the paper at p, which is caused by rollers to travel under it. It will then be seen that a current in A will cause the point p to travel laterally upon the paper and so record the signal. g is a fibre attached to s at one end and to a vibrator at the other. This causes the point p to be continually lifted off and allowed to touch the paper, producing a fine dotted line as the paper travels forward. This reduces to a great extent the friction between p and the paper.

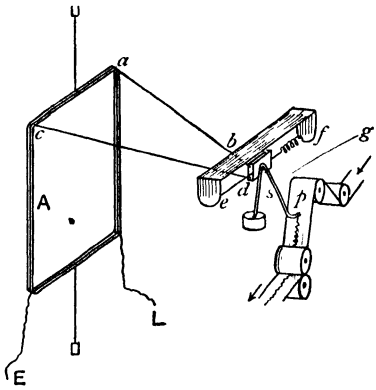


FIG 950.—Siphon recorder

The telephone.—Sounds arising at one place may be reproduced at another place by the arrangement due to A. Graham Bell known as the **telephone.** The sound waves arriving at the thin sheet-iron diaphragm A (Fig. 951) causing it to vibrate, a compression of the air (p. 682) driving it inwards and a rarefaction causing it to bulge outwards. Behind the diaphragm is a permanent horseshoe magnet NS, provided with a pair of soft iron pole pieces which nearly but do not quite touch the diaphragm. Coils having a great number of turns are situated upon these pole pieces, and are in series with the line LL, and a similar instrument acting as receiver at the other end. Magnetic flux, therefore, passes from the N pole of the permanent magnet through one pole piece, and by way of the soft iron diaphragm to the other pole piece, and the S pole of the magnet. The less the air space between the pole pieces and the diaphragm the greater will be this magnetic flux, and since this magnetic flux is threaded through the coils upon the pole pieces, it follows that any variation in position of the diaphragm causes an alteration in the magnetic flux, and hence produces an electromotive force in the coils. As the diaphragm follows the compressions and rarefactions of the air, a corresponding variation of the e.m.f. and therefore of current in the coils and in the line will take place.

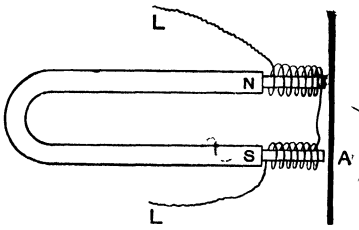


FIG 951.—The Bell telephone.

On arrival at the receiving end, the current passes through a similar

pair of coils upon the pole pieces, and the variation in magnetic flux causes a varying attraction between the pole pieces and the soft iron diaphragm. Hence the diaphragm of the receiver copies the motion of the transmitter and in doing so sets the air in motion, and reproduces the sounds which caused the motion of the diaphragm of the transmitter.

It is clear that the permanent magnet of the transmitter is necessary, otherwise there would be no magnetic flux at all, and consequently no current produced by the movement of the diaphragm. That the permanent magnet of the receiver is necessary is not at first sight obvious, since the varying current would produce a varying magnetic flux even without the presence of the permanent magnet. But it must be remembered that the movement of the diaphragm of the receiver depends upon the **variation** in the pull of the magnets upon it. Now, the pull per square centimetre is $B^2/8\pi$, where B is the magnetic induction (p. 815). If, now, B changes by the small amount b, it becomes $B+b$, and the pull is $(B+b)^2/8\pi$, or $(B^2+2Bb+b^2)/8\pi$. The increase of the pull is therefore $(2Bb+b^2)/8\pi$, or $2Bb/8\pi$ approx. Thus it is proportional to the increase in induction b, **and also to the value of the induction** B. Hence the permanent magnet maintains a fairly high value of B. This must not, however, be too high, for if the iron approaches saturation, the current will produce very little alteration in B, as will be seen from the curve, Fig. 763, p. 823.

The carbon microphone.—The Bell receiver can be used as a transmitter, as described above. But, owing to the feebleness of the currents produced, the intensity of the sound produced by the receiver is small. For this reason the **microphone** is nearly always used for the transmitter. This employs the fact that the resistance of a carbon contact is considerable, but is highly sensitive to mechanical disturbance. Thus, if a pointed carbon rod C (Fig. 952) is supported by two carbon blocks A and B, and the circuit is completed through a few cells and a Bell receiver, any disturbance of the carbon rod causes a loud noise to be heard in the receiver The carbon contacts at A and B vary so much in resistance with any slight mechanical disturbance, that the current in the circuit varies sufficiently to produce a loud sound in the receiver.

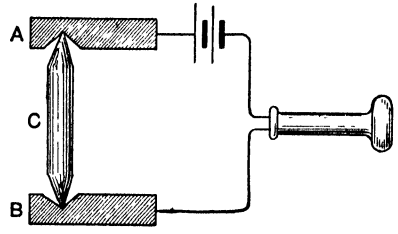


FIG 952.—Carbon microphone

In telephoning, single carbon contacts are not used, but granulated carbon is placed between a carbon diaphragm and a carbon

plate. The arrangement of the transmitter is given in Fig. 953. D is a thin carbon diaphragm which vibrates in accordance with the sound waves arriving by the mouthpiece M. A is a carbon block having corrugations or protuberances. Between D and A granules of carbon, G, are situated. The movement of the diaphragm thus causes corresponding changes in the resistance of the carbon contacts between D and A, and therefore corresponding variations of the current going to the receiver by way of the line L.

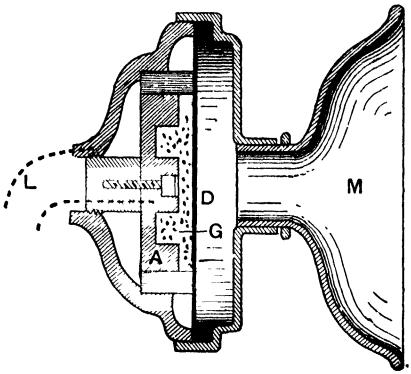


FIG. 953.—Carbon microphone transmitter.

Use of a transformer, or induction coil.—In order to keep the resistance of the microphone circuit low, so that any variation in resistance of the carbon contacts shall be as large a proportion of the total resistance as possible, and so produce large variations in current, it is usual to use a transformer or induction coil, as shown in Fig. 954. C is the induction coil, the low resistance primary of which is in series with the microphone M and a battery. The secondary, consisting of many turns, is in series with the line and the receiver at the other end of the line. In the position shown, the receiver R is supposed to be in use. When not in use it is hung up on the key K, which then throws the induction coil out of circuit and joins the line to the key K_2, and the distant station can then call up, since the line is in series with the electric bell B. To call up the distant station, the button of the key K_2 must be pressed. This puts the battery in series with the line and the electric bell at the distant station.

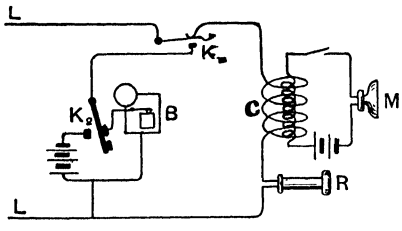


FIG. 954.—Transformer in telephone circuit

Arc lamps.—Whenever an electric circuit is suddenly broken a spark occurs, owing to the inductance in the circuit (p. 986). As a rule this spark is very soon quenched; but if the electromotive force in the circuit is sufficiently high, and the materials where the

break occurs are suitable, the current persists, the spark becoming transformed into an arc. The most suitable material for forming the arc is carbon, since this does not melt, even at the very high temperature of the arc.

The arc between carbon rods cannot exist permanently unless the e.m.f. in the circuit exceeds a certain value, about 40 volts.

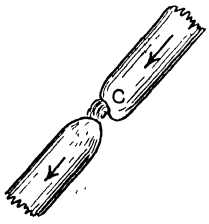


FIG 955.—Electric arc

When the carbons are allowed to touch, and are then drawn apart, the positive carbon soon burns out into the form of a crater C (Fig 955). Both positive and negative carbons burn away, but the positive more rapidly than the negative. There is a small area within the crater, at which the temperature is the highest, and reaches 3500° C. to 4000° C. It is from this point that most of the light of the arc is emitted The crater forms most readily if the positive carbon has a soft core, and for this reason the positive carbons are usually 'cored,' the negative carbons being solid.

It is usual to run arc lamps with a resistance in series. Thus, to run a 10 ampere arc lamp on a 100 volt circuit, a resistance must be placed in series with it. To calculate the resistance required, remember that 40 volts is required to maintain the arc, apart from its resistance. Then, $(100-40)/10=6$ ohms resistance must be placed in series.

Owing to the very high temperature of the arc, it has now many industrial applications. Silica may be fused in it; welding of iron and steel is performed by it; and the calcium in lime can be caused to unite with carbon forming the calcium carbide used for the production of acetylene gas.

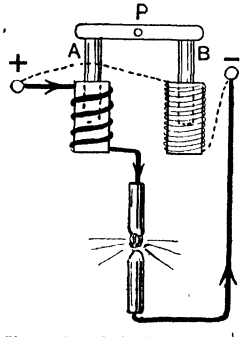


FIG. 956.—Principle of the automatic air lamp

Automatic arc lamp.—There are many arrangements for the maintenance of the arc, but that illustrated in Fig. 956 is a good example. Two soft iron plungers, A and B, are attached to a rocking arm pivoted at P. A lies partly within a solenoid through which the main current in the arc passes, while B has a high resistance and is in parallel with the arc. Before switching on the current, the carbons are in contact, but on completing the circuit the current flows in the series coil through the arc. This pulls down the plunger

A, which by a suitable mechanism draws the carbons apart, and the arc is 'struck.' As the carbons burn away, the resistance of the arc increases so that more current will flow in the shunt coil and the plunger B is pulled down. This releases a brake and allows the mechanism to bring the arcs closer together. If the mechanism, which is not shown in the diagram, be properly adjusted, the differential effect of these two processes keeps the carbon the proper distance apart.

Incandescent lamps.—For the purpose of ordinary lighting, the **incandescent or glow lamp** is the most convenient. It consists of a filament of fairly high resistance which is rendered incandescent by the heat produced by the current in it. The filament is attached at the ends to pieces of platinum wire sealed into the walls of a glass bulb. The air is exhausted from the bulb containing the filament, for two reasons; the air would cause oxidisation of the filament when hot, and would also remove heat so rapidly by conduction and convection that the lamp would have very low efficiency.

The older type of incandescent lamp has a **carbon filament.** The filament is constructed by dissolving cotton wool in zinc chloride solution, to form a paste, which is squirted through small holes into alcohol. On remaining in the alcohol for several hours the filaments harden. They are then removed, cleaned, and bent into the shape required for the lamp. This is done upon a carbon "former," and they are then packed round with carbon powder and heated to a temperature of about 550° C. in a carbon crucible, which converts them into the well known hard carbon filaments. The final treatment is the **flashing,** which consists in heating in benzine vapour by means of a current. Where the filament is thin, the temperature rises most and the benzine vapour is decomposed, and carbon is deposited upon the filament, thus rendering the thickness uniform. The filament is then fixed in its bulb which is exhausted and sealed off.

Carbon filament lamps have now been largely superseded by **metallic filament lamps,** the chief advantage of which is their high efficiency. The efficiency of the carbon lamp varies from about 4 watts per candle power for small lamps to 2·5 watts per candle power for large lamps, while the metallic filament lamp can be run at a higher temperature, and has an efficiency of about 1·0 to 1·5 watts per candle power. Recently, lamps taking only 0·5 watt per candle power have been produced, but these are only made in candle powers of 300 and upwards.

The metallic filaments have been constructed of various metals. Platinum is unsuitable as its temperature when running is not much below the melting point. Filaments constructed of the metal tantalum have proved of considerable efficiency. The later lamps,

of which the Osram lamp is a type, have filaments of tungsten, annealed and drawn into wire.

Resistance and efficiency of incandescent lamps.—From Expt. 190, p. 878, it will be seen that the resistance of a carbon filament lamp falls as the temperature rises. Carbon is, in this respect, unlike the metallic conductors, whose resistance increases with rise in temperature. The resistance of carbon when white hot is about half that when cold. Thus a carbon filament lamp whose resistance when incandescent is 150 ohms, will have a resistance of about 300 ohms when cold. It follows that the luminosity of carbon filament lamps varies considerably for small fluctuations in the voltage of the supply. A slight drop in voltage reduces the current in the lamp and the consequent cooling causes an increase in resistance with further drop in current. The reverse effect is observed with a slight rise in voltage. On the other hand metallic filaments are much more steady in running, as a slight increase in voltage produces a rise in temperature and therefore an increase in resistance in the filament. This prevents any considerable rise in the current.

The term **efficiency of a lamp**, in its common use, is not logically applied. It is usually given in terms of **watts per candle power.** Thus the higher the number of watts per candle power, the less will be the real efficiency of the lamp. The measurement of candle power has already been described (p 547), and to determine the efficiency, the watts must be measured (p. 878 or 973) at the same time.

The **life of the lamp** is also of great importance; it should be at least 1000 hours for an efficient lamp. The watts per candle power in the case of a carbon filament lamp rises throughout the life of the lamp, owing to deposition of carbon upon the glass bulb with consequent absorption of light and lowering of the candle power. In the metallic filament lamp, there is a slight drop in watts in the first 100 hours with gradual increase afterwards, but the change is not nearly so great as in the carbon lamp.

EXERCISES ON CHAPTER LXXVIII.

1. Describe some simple form of telegraph for the transmission of messages electrically.

2. Explain clearly the use of 'Relays' in the Morse telegraph, giving a diagram showing the connections. Allahabad University.

3. In what ways does the presence of a permanent magnet in a telephone receiver enhance its working efficiency and how is the result brought about? C.G.

4. Give an account of some system in which messages may be sent in both directions simultaneously through a telegraphic cable.

5. Give a skeleton diagram and explain the working of

(*a*) A single current differential duplex circuit;
(*b*) A single current bridge duplex circuit. C.G.

6. Describe some arrangement for the automatic feeding of an arc lamp.

7. Sketch and describe the construction of any type of galvanometer used on a telegraph circuit, and state its simple function. C.G.

8. Describe briefly the process of making the carbon filament for an incandescent lamp. What is the reason for the process of "flashing"?

9. Describe some form of incandescent electric lamp, giving figures for volts, amperes, and candle power, and state how the resistance varies with change of temperature. C.G.

10. Why are metal filament lamps more efficient than carbon filament lamps? A metal filament lamp costing 2s. 3d. gives 25 c.p. with 30 watts, and is discarded after 500 hours; a carbon filament lamp costing 8d. gives 25 c.p. with 100 watts and is discarded after 800 hours. Find the cost per 1,000 candle hours, inclusive of lamp renewals, in each case, the price of current being $1\frac{1}{2}$d. per B.O.T. unit. C.G.

CHAPTER LXXIX

THERMO-ELECTRICITY

Thermo-electric couples.—When all parts of a circuit, composed entirely of metals, are at the same temperature, there is no resultant electromotive force in the circuit and therefore no current. If, however, any point at which two different metals are in contact be at a different temperature to the rest of the circuit, there will be an electromotive force, and if the circuit be complete, a current will flow in it.

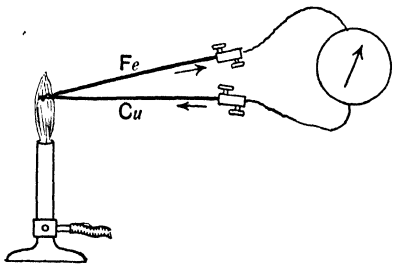


FIG. 957.—Copper-iron thermal couple.

This may easily be shown by joining a piece of copper and a piece of iron wire (Fig. 957), either by twisting them together, or, better still, by soldering them, the free ends being connected to a sensitive galvanometer. Such a pair of metals is called a **thermal couple**. On heating the copper-iron junction a deflection will be observed, showing that a current is flowing, and therefore that an electromotive force exists in the circuit. This **thermo-electric effect** was discovered by Seebeck in 1826, and bears his name. He arranged a number of metals in a list so that the current flows across the hot junction from the earlier to the later metal in the list. Amongst these metals were:

> Bismuth (Bi), Platinum (Pt), Copper (Cu), Lead (Pb), Tin (Sn), Silver (Ag), Zinc (Zn), Iron (Fe), Antimony (Sb), Tellurium (Te).

That the electromotive force, for a copper-iron couple, is not proportional to the excess of temperature of the hot junction over the

rest of the circuit may easily be seen. For, as the temperature rises the current gets greater for a time and then ceases to grow. After this it decreases to zero and eventually becomes reversed, after which it gets greater and greater in this reverse direction.

Measurement of thermo-electric e.m.f.—If the deflection produced by a known e.m.f. in the circuit of Fig. 957 be determined, the galvanometer deflections can then be converted into volts or micro-volts. A more satisfactory way of determining the relation between e.m.f. and temperature consists in employing a potentiometer method. The two similar resistance boxes A and B are placed in series with a cell C (Fig. 958) of known e.m.f. A Daniell's cell will do, if great accuracy is not required, the e.m.f. being taken as 1·1 volt. In making the measurements, the circuit ABC must have constant resistance, so that the current may remain constant. This is attained by starting with all the plugs out of B and only transferring plugs from A to their similar positions in B for producing a balance.

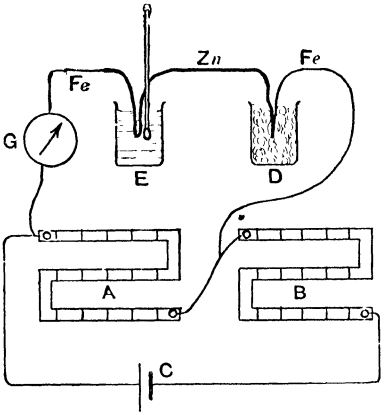


FIG 958.—Potentiometer method of measuring thermo-electric e.m.f.

The thermo-electric couple is connected across the box A, the galvanometer G being in its circuit. One junction of the couple is situated in ice at D to keep its temperature constant, the other junction being raised to various temperatures in E. This is the potentiometer arrangement (p. 895) in which the pair of boxes A and B replace the stretched wire of Fig. 827. The e.m.f. in the thermo-electric couple is proportional to the resistance in A when the galvanometer deflection is zero, or

$$\frac{\text{e.m.f. of couple}}{\text{e.m.f. of cell C}} = \frac{\text{resistance in A}}{\text{total resistance in A and B}}.$$

The resistance of the cell may be neglected in comparison with that in A and B, which is usually of the order of 10,000 ohms. It is desirable, if possible, to attain a fairly high temperature for the hot junction. For the zinc-iron couple, the vessel E may be a crucible containing solder with the couple in an oil capsule. The e.m.f. is usually small, and is therefore conveniently measured in micro-volts.

The curve in Fig. 959 is that for a zinc-iron couple, one junction being at 0° C.

Thermo-electric diagram.—It will be seen that to include the e.m.f.'s for all possible pairs of metals upon one diagram, such as

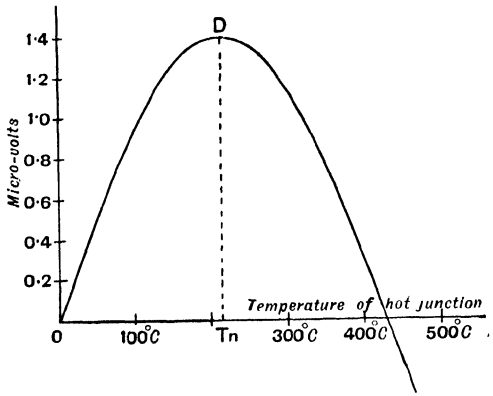


FIG 959.—e m f -temperature curve for zinc-iron couple.

Fig 959, would lead to great complexity. Another method of representation due to Prof. Tait is therefore usually adopted. Instead of plotting e.m.f. against temperature, **the e m.f. with unit difference of temperature between the junctions** is plotted. Thus, in Fig. 960, the ordinate tC represents the e m f. in the circuit of a couple when one junction is half a degree below $t°$ and the other junction is half a degree above $t°$, so that the average temperature is $t°$. For nearly all metals in pairs, such a curve for any pair is a straight line AB.

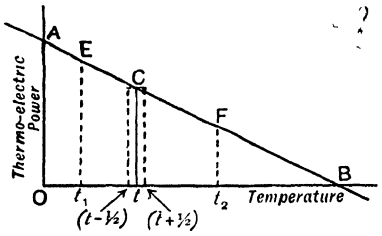


FIG. 960.—Principle of the thermo-electric diagram

On such a diagram, the e.m f. with one junction at $t_1°$, and the other $t_2°$, the area Et_1t_2F represents the corresponding e.m.f. in the circuit. For, the area of a strip such as tC is equal to the e.m.f., since the width of the strip is unity. In adding up such strips between t_1E and t_2F, the whole area is the sum of the areas of the corresponding strips, and thus represents the total e.m.f.

For a reason to be given later, curves, such as AB, are drawn for all the metals, using lead as one element of the couple. When this is done, a diagram such as Fig. 961 is obtained. This is known as the **thermo-electric diagram**, and the ordinate of each point on a line

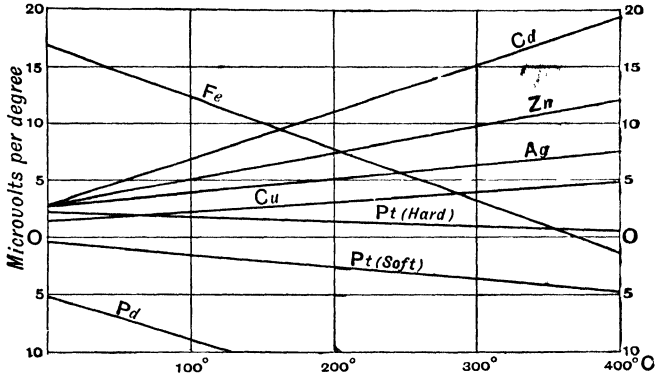


FIG. 961.—Thermo-electric diagram.

is known as the **thermo-electric power** of the metal at each temperature. The advantage of such a diagram as this, lies in the fact that the e.m.f. for any pair of metals for any difference of temperature between the junctions can be found.

Let us suppose that $A_1B_1C_1DE_1$ (Fig. 962) is the thermo-electric line for one metal and $A_2B_2C_2DE_2$ that for another metal, plotted with respect to lead. Then, for temperatures represented by B and C, the e.m.f. for a couple consisting of the first metal and lead is represented by the area BB_1C_1C. Again, the e.m.f. for the second metal and lead is represented by the area BB_2C_2C. Hence the e.m.f. for a couple consisting of the first metal and the second is represented by the difference of these two areas, that is, by area $B_1B_2C_2C_1$.

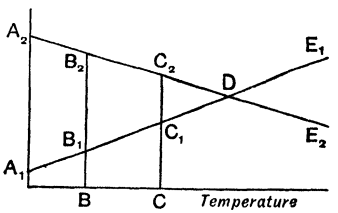


FIG. 962.—Explanation of the thermo-electric diagram

Neutral point.—We can see from the thermo-electric diagram why there should be a diminution, followed by a reversal, of the e.m.f. in the case of a thermal couple (p. 1017). The thermo-electric diagram is so drawn that for any pair of thermo-electric lines such as $A_1B_1C_1$ and $A_2B_2C_2$ (Fig. 963) the e.m.f. acts one particular way round the circuit. This direction is anticlockwise. Thus for the two given

metals, if the junctions are at temperatures A and B the e.m.f. is represented by the area $A_1B_1B_2A_2$ and acts across the hot junction from the metal 1 to the metal 2.

Since the thermo-electric lines are in general inclined to each other, they intersect at some point such as D. Hence, if the temperature of the hot junction be gradually raised, the lower junction being kept at fixed temperature, the area $A_1B_1B_2A_2$ gradually increases, which means that the e.m.f. in the couple increases. But the rate of increase gets less as the point D is approached. When the hot junction is at a temperature above that represented by the point D, as at C, the area DC_2C_1 must be deducted from A_1DA_2 in order to obtain the e.m.f. in the circuit. It will be seen from the arrows that the e.m.f.

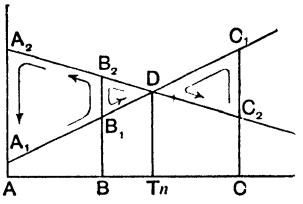


FIG. 963.—Neutral point

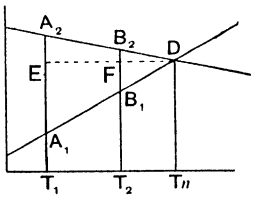


FIG. 964.—Calculation of e.m.f. from the thermo-electric diagram

corresponding to the part DC_2C_1 must be deducted from A_1DA_2 in order to obtain the e.m.f. in the circuit. The arrows show that the e.m.f. corresponding to the part DC_2C_1 is acting the opposite way round the couple to that corresponding to A_1DA_2. D therefore corresponds to the turning point D of the e.m.f. curve in Fig. 959. The temperature corresponding to the point D is called the **neutral temperature**, T_n, for the two given metals.

Thus, **when the temperature of one junction passes the neutral temperature, the e.m.f. begins to decrease.** Also, if one junction is as much above the neutral temperature as the other is below it, the e.m.f. in the circuit is zero. Again, when the difference of temperature between T_n and C (Fig. 963) becomes greater than the difference between A and T_n, the e.m.f. becomes reversed.

It is possible to find from the thermo-electric diagram, an equation for the e.m.f. in any couple, in terms of the neutral temperature and one other constant, depending upon the metals forming the couple. All that is necessary is to find the area of $A_1B_1B_2A_2$ (Fig. 964). Thus,

$$\text{Area of triangle } A_1A_2D = \tfrac{1}{2}A_1A_2 \times ED.$$

$$\text{Area of } B_1B_2D = \tfrac{1}{2}B_1B_2 \times FD;$$

$$\therefore \text{ e.m.f.} = \tfrac{1}{2}(A_1A_2 \times ED) - \tfrac{1}{2}(B_1B_2 \times FD).$$

Again, from similar triangles, the sides A_1A_2 and B_1B_2 are proportional to ED and FD respectively.

$$\therefore \text{ e.m.f.} \propto (ED^2 - FD^2).$$

But $$ED = T_n - T_1 \quad \text{and} \quad FD = T_n - T_2;$$

$$\therefore \text{ e.m.f.} \propto (T_n - T_1)^2 - (T_n - T_2)^2$$
$$\propto T_1^2 - 2T_1T_n - T_2^2 + 2T_2T_n$$
$$\propto (T_1^2 - T_2^2) - 2T_n(T_1 - T_2)$$
$$\propto (T_1 - T_2)(T_1 + T_2 - 2T_n),$$

or $$\text{e.m.f.} = K(T_1 - T_2)\left(\frac{T_1 + T_2}{2} - T_n\right),$$

where K is some constant, which together with T_n must be found by experiment for each pair of metals.

The above equation shows that **the e.m.f. in any couple is proportional to the difference of temperature of the junctions and also to the difference between the neutral temperature and the average temperature of the junctions.** It thus appears that **the e.m.f. vanishes either when the junctions are at the same temperature,** or when **their average is the neutral temperature.**

Peltier effect.—When we seek for the origin of the energy required to maintain the current in a thermo-electric couple, we see that since the metals are unchanged in any way, the heat supplied by external agencies, such as the bunsen burner (p. 1016), is the only source of energy available. It is natural then to turn to the junction of the two metals as the place where heat energy is converted into current energy. This implies that there is an electromotive force acting across the junction from one metal to the other. It is clear that if both junctions are at the same temperature the e.m.f.'s at the two junctions are directed in opposite ways round the circuit and the resultant e.m.f. is zero. But if the e.m.f. at the junction changes with temperature, it follows that when there is a difference of temperature between the junctions, these e.m.f.'s do not balance, and there will be some resultant e.m.f. available for driving a current.

This e.m.f. at the junction of two metals was discovered by Peltier and is known as the **Peltier coefficient.** Like all other electromotive forces (p. 853), it implies a reversible condition. If the current flows in the direction in which the e.m.f. tends to drive it, heat energy is converted into electrical energy, but if it flows in the

reverse direction, electrical energy is converted into heat. Thus, at a copper-iron junction the e.m f causes a current to flow from copper to iron at the hot junction, that is, where heat is being absorbed. This is the state of affairs in the upper diagram (Fig. 965). Indeed, if the current be caused to flow by applying some external source of e.m.f., such as a cell, heat will still be absorbed at this junction, and if this heat is not supplied by some agent, such as a burner, the heat is absorbed from the metal itself, which is thereby cooled. In the lower diagram the current is reversed, and in this case the junction becomes warmed.

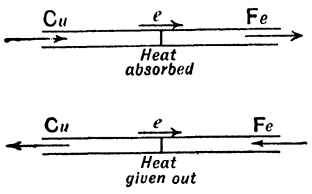


FIG 965.—Peltier effect

If the copper be replaced by bismuth and the iron by antimony, the same effect will follow, but it is much greater in this case, since a bismuth-antimony junction has a greater Peltier coefficient than one of any other pair of the common metals.

Even in the case of bismuth and antimony, the Peltier coefficient is only of the order of 0·03 volt, so that for a current of 1 ampere the work done at the junction per sec. is only $0{\cdot}03 \times 1$ joule or $0{\cdot}03 \times 0{\cdot}24 = 0{\cdot}007$ calorie. The amount of warming or cooling is therefore not easy to observe. It may, however, be detected by soldering a bismuth bar between two antimony bars, as in Fig. 966, and passing a few amperes through. Then one junction is cooled and the other warmed, as may be shown by means of two similar pieces of fine platinum wire, A and B, wrapped round near the junctions. The difference in temperature produced by the Peltier effect causes one platinum wire to have a different resistance from the other (p. 851). Hence, if the two resistances are balanced upon a metre bridge before the main current flows, the heating and cooling will disturb the bridge balance, and we can see which junction is warmed and which cooled. With the current as shown in Fig. 966, A is warmed and B cooled.

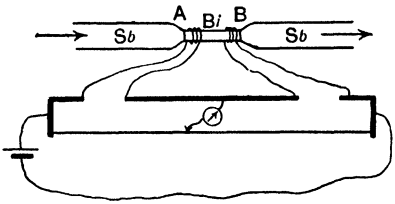


FIG 966.—Demonstration of the Peltier effec

This heating and cooling, known as the **Peltier effect**, must not be confused with the heating produced by all currents due to the resistance of the conductor. The latter is proportional to the square of the current, and is not reversible. If r be the resistance of the junction, i^2r is the energy in ergs converted into heat per second. Then, if π be the Peltier coefficient, πi is the energy in ergs converted from electrical energy to heat or *vice versa* in one second. Thus the total heating of any junction per sec. is $i^2r \pm \pi i$ ergs. The positive or negative sign must be taken, according to the direction of the current. In observing the Peltier effect, i^2r is made small by making r as small as possible, although in the above experiment i^2r will be approximately the same for both junctions, so that the difference of temperature is due to the Peltier effect only.

Thomson effect.—It was pointed out by Lord Kelvin (then Sir Wm. Thomson), on theoretical grounds, that there must be thermo-electric effects in a couple other than those occurring at the junctions. He found that **there is an electromotive force between different parts of the same metal when at different temperatures.** Thus, in copper there is an e.m.f. acting from the parts of lower to those of higher temperature, and in iron from parts of higher to those of lower temperature. Hence, if a copper bar be heated in the middle and a current passed through it, heat is absorbed as the current flows from colder to hotter parts, and given out as the current flows from hotter to colder parts. In fact, the current behaves exactly as a stream of liquid would do on account of its specific heat. On the other hand, in the case of iron the current gives out heat in passing from colder to hotter parts and absorbs heat in passing from hotter to colder parts. It is sometimes said that in iron the current behaves like a liquid of negative specific heat. The metals, cadmium, zinc, and silver behave like copper, while palladium behaves like iron. In fact, the direction of the slope of the thermo-electric line (Fig. 961) gives the sign of the Thomson effect.

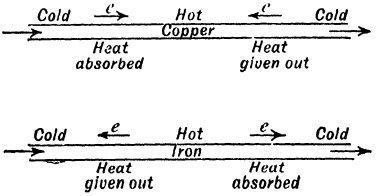


FIG. 967.—Thomson effect.

In lead the Thomson effect is zero, and it is for this reason that thermo-electric powers are always plotted with respect to lead (p. 1019).

Both the Peltier effects and the Thomson effects may be identified upon the thermo-electric diagram. For, if the temperature be

reckoned from the absolute zero (p. 402) and A_1B_1, A_2B_2 (Fig. 968) are the thermo-electric lines for two metals, we have already seen that the e.m.f. of the couple is represented by the area $A_1B_1B_2A_2$. Further, it may be shown from thermo dynamic reasoning that the Peltier coefficients at the two junctions are represented by the areas HA_1A_2E and GB_1B_2F respectively. Also the e.m.f. due to the Thomson effect in the first metal is represented by HA_1B_1G and in the second by FB_2A_2E.

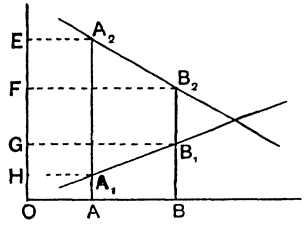


FIG 968.—Peltier and Thomson effects upon the thermo-electric diagram

Also,

$$FB_2A_2E + GB_1B_2F + HA_1B_1G - HA_1A_2E = A_1B_1B_2A_2.$$

That is, **the resultant e.m.f. in the couple is the algebraic sum of all the Peltier and Thomson e.m.f.'s.**

The thermopile.—These electromotive forces due to thermal effects are used for the measurement of radiant heat. In the **thermopile** a number of rods of antimony and bismuth are connected in series, the two metals alternating (Fig. 969). One set of junctions, B_1, is

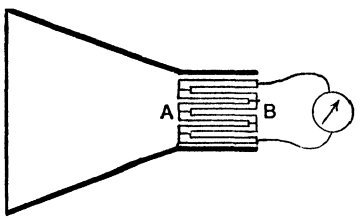


FIG 969.—The thermopile

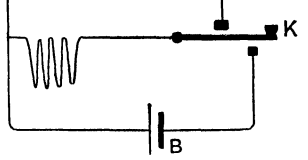


FIG. 970.—Demonstration of thermal e m f 's by the thermopile

protected from external variations in temperature by means of a brass cap, while the radiant energy to be measured is allowed to fall upon the other set of junctions A. The junctions A, therefore, rise slightly in temperature, and the resulting electromotive force is equal to that for one bismuth-antimony couple multiplied by the number of couples. The resistance of the thermopile being small, it is desirable to use a low-resistance galvanometer to obtain as large a current as possible. The galvanometer in series with the thermopile then gives a deflection proportional to the difference of temperature of the junctions.

By means of the thermopile, the existence of the thermal e.m.f.'s may be shown indirectly. Connecting it up to a cell B, galvanometer G, and a key K, as shown in Fig. 970, then on depressing the key a current flows through the thermopile. Owing to the Peltier effect, one set of junctions becomes warmed and the other set cooled. On raising the key the battery is cut out, and the galvanometer placed in series with the thermopile. The difference of temperature between the junctions produced by the current in the first part of the experiment now causes the thermo-electromotive force, and a current will flow in the galvanometer until the junctions are all brought again to uniform temperature.

Radio-micrometer.—In the radio-micrometer, due to Prof. C. V Boys, the thermo-electric couple and the galvanometer are combined in one instrument. Two small bars, one of antimony and the other of bismuth, touch at their lower ends, the upper being connected

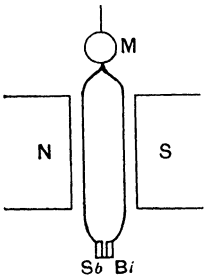


FIG. 971.—Boys's radio-micrometer

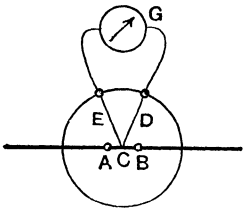


FIG. 972.—Thermo-milliammeter.

through a loop of wire which hangs in the field of a powerful magnet NS (Fig. 971). The radiation to be detected falls upon the bismuth-antimony junction, and the resulting current in the coil, which is suspended by a quartz fibre and is provided with a mirror M, produces a large deflection for a very small amount of radiant energy.

Thermo-milliammeter.—Several forms of **thermal galvanometer, or thermo-milliammeter,** have come into use. The current to be measured passes through a fixed wire of constantan AB (Fig. 972), which therefore becomes heated. A couple consisting of bismuth and tellurium has one junction soldered to the constantan wire at C, the other ends being connected to a galvanometer G by means of the leads E and D. The galvanometer scale can be calibrated by sending known currents through AB, and observing the deflection. The instrument is very sensitive and can be used equally well for

direct or alternating currents The sensitiveness is increased by enclosing the thermo-electric part of the apparatus in an exhausted glass bulb

Pyrometers.—It will be seen, on referring to Fig. 959, that on attempting to use the thermal electromotive force for the measurement of temperature, the result is ambiguous, for each e.m.f. corresponds to two different temperatures. If, however, we could find two metals for which the thermo-electric lines are parallel, or very nearly parallel, it would follow from the reasoning on p. 1020 that the e.m.f.-temperature curve would be a straight line, and each e m f would correspond to one particular temperature. The couple can then be placed in series with a galvanometer or milliammeter, and if one junction be kept at constant temperature the scale of the galvanometer can be so calibrated that it indicates the temperature of the other junction. Such an arrangement, when used for measuring high temperatures, such as furnace temperatures, is called a **thermo-electric pyrometer** The couple for this purpose usually consists of platinum and rhodium, or of platinum and an alloy of platinum and rhodium. In Fig 973 such a pyrometer is shown. The couple is placed in an iron or porcelain tube, and the scale of the reading instrument is graduated in degrees, up to 1000° or 2000° C.

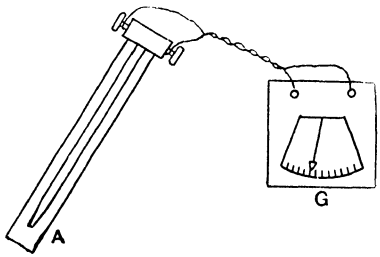


FIG 973.—Thermo-electric pyrometer

EXERCISES ON CHAPTER LXXIX

1. What is meant by a thermo-electric couple? Describe the construction of a thermopile. State whether you would select a galvanometer of high resistance or one of low resistance for use with the thermopile, giving reasons for your answer. L.U.

2. Describe some method of measuring the e.m.f. of a given thermo-electric couple, and its variation with temperature.

3. How does the thermo-electric electromotive force for a given pair of metals vary as the temperature of one junction is raised while the other is kept at constant temperature?

4. Describe the Seebeck, the Peltier and the Thomson effects.

5. Show how thermo-electric powers may be represented on a diagram, and describe how the various thermal e.m.f.'s in a circuit may be represented by areas upon this diagram.

6. Give an account of thermo-electric inversion and neutral temperature.

7. Describe some form of thermo-electric pyrometer for measuring high temperatures.

8. Describe the Peltier effect, and give an account of some method by which its existence and sign may be determined.

9. What is the meaning of the term thermo-electric power? In what units is it measured?

CHAPTER LXXX

CURRENT IN GASES X-RAYS: RADIOACTIVITY

Electric spark.—The fact that the air space between two conductors may become conducting has been mentioned several times With air at atmospheric pressure, the difference of potential that must exist between the conductors before any current passes is considerable. It depends, moreover, upon the distance apart of the conductors and also upon their shape. It was seen on p. 937 that the discharge takes place most readily from points. Since the shape of the conductor is of such importance in determining whether the discharge will take place, it is usual in making measurements upon the sparking potential to employ spheres as terminals.

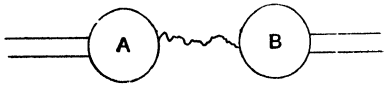


FIG. 974.—Electric sparks

As the difference of potential between two knobs A and B (Fig. 974) is caused to rise by connecting them to an electrical machine or induction coil, the first form of the discharge to be observed is nearly silent A faint hissing noise is heard, and in the dark a faint violet coloured glow can be seen. This is called the **brush discharge.** On further raising the potential, a point is eventually reached at which a sudden crackling discharge takes place. This consists of a succession of discharges, each one being accompanied by a luminous streak of light between the knobs, of a form almost exactly like a flash of lightning. This is called the **spark discharge.**

The presence of points or roughness upon the conductors facilitates the brush discharge. Hence the use of **lightning conductors** to protect buildings. For, the earth being at a considerably different potential from the thunder cloud above, the lightning conductor, which consists of a strip of copper, earthed at its lower extremity and ending in a number of sharp points above, facilitates a quiet or brush

discharge between earth and the cloud. This reduces the difference of potential and lessens the probability of a disruptive discharge taking place.

Current in gas at low pressure.—If the pressure of the air is continually reduced, the character of the discharge undergoes a number of changes. At about one-half to one-third of the atmospheric pressure the crackling nature of the discharge ceases, and it assumes a silent streamer-like form, and becomes coloured. The colour depends upon the nature of the gas in the tube, being a pinkish colour in the case of air. With further reduction in pressure, the discharge spreads out until it fills the tube, and at this stage it is highly luminous. This is the form known as the Geisler tube, and the difference of potential between the ends of the tube is very much less than that for the discharge at atmospheric pressure.

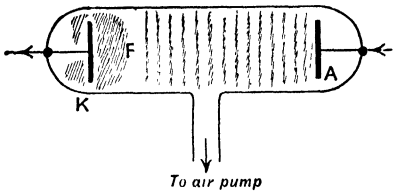


FIG. 975.—Discharge at moderately low pressure.

At this stage, a want of symmetry in the discharge becomes apparent. Near the electrode at which the current leaves, that is, at the kathode K (Fig. 975), a dark space appears and this becomes more pronounced as the pressure is reduced. It is called the **Faraday dark space**, F, and is separated from the kathode by a bluish glow. From the Faraday dark space to the anode A and extending quite up to it is the **positive column.**

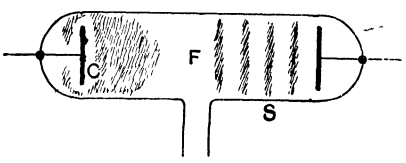


FIG 976.—Discharge at low pressure

When the pressure falls to about 0·1 mm. of mercury the appearance is somewhat as shown in Fig. 976. The kathode glow has increased in size and is seen to be separated from the kathode by a dark space C of constant thickness. This is called the **Crookes dark space.** The positive column is seen to consist of disc-shaped **striations** S. On reduction of pressure, the scale of the whole phenomenon grows, the place from which it grows being the kathode. Hence the Crookes dark space, the kathode glow, and the Faraday dark space all increase in size. The striations also become larger and more distinct, but, of course, fewer in number, as they always extend as far as the anode however near or distant that may be.

It will easily be seen that on further reduction of pressure, say to 0·01 mm., there will not be room in an ordinary vacuum tube

for any positive column at all The Crookes dark space will then occupy the greater part of the tube and only part of the kathode glow will be present.

Kathode rays.—In the Crookes dark space several phenomena may be observed The boundary of this dark space is always luminous. When the boundary is the gas in the tube, the luminescent gas constitutes the kathode glow, but when the dark space extends to the walls of the tube, a bright fluorescence is produced. This is usually coloured, the colour depending upon the nature of the material. With soda glass the wall of the tube fluoresces a bright green, as may be seen in the case of the X-ray tube (p. 1033). With lead glass the fluorescense is blue. Many minerals give characteristic colours, corundum is a bright crimson and zinc sulphide a bright blue

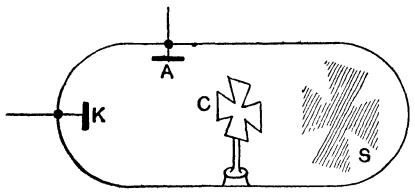


FIG 977.—Kathode rays stopped by metallic screen

Whatever it is that produces this effect is travelling in straight lines from the kathode and can be stopped by dense bodies; for a metallic screen C (Fig 977) placed in front of the kathode casts a shadow upon the walls of the tube The name **kathode rays** is given to the emission from the kathode which produces these effects.

Mechanical effect of kathode rays. If the kathode rays fall upon a little mill wheel W having light mica vanes, the wheel is caused to rotate when one half of it is shielded from the rays. Thus in Fig. 978 if A is kathode the rotation is as shown, but if B is kathode the rotation is in the opposite direction The explanation of this is not quite so simple as might at first appear, but it is undoubtedly due to the rays from the kathode falling upon the vanes.

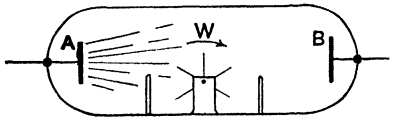


FIG 978.—Mechanical effect of kathode rays.

Heating effect of kathode rays.—When falling upon a surface, the kathode rays always heat the material upon which they fall. If the rays, which are emitted normally from the kathode, are

concentrated upon a small area of a platinum surface (Fig. 982), the platinum may be raised to red heat.

Electrical charge carried by kathode rays.—On catching the kathode rays in a hollow metallic vessel, placed within the vacuum tube, the vessel acquires a negative charge. Thus, if the vessel V (Fig. 979) be connected to an electrometer or electroscope, it will be found to acquire a negative charge when the kathode rays enter it. Thus, whatever their nature may be, the kathode rays are certainly accompanied by a stream of negative electricity.

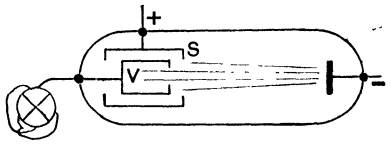


FIG. 979.—Electric charge carried by kathode rays

Effect of magnetic field upon kathode rays.—Kathode rays are easily deflected by a magnetic field. They are driven in a direction at right angles to both their own path and the magnetic field. This may easily be shown by allowing the beam of kathode rays passing through a narrow slot in the screen S (Fig. 980) to pass along a screen AB covered with a layer of zinc sulphide. The zinc sulphide fluoresces a bright blue in the kathode rays. Their path is therefore marked by a straight streak upon the screen. On advancing the N pole of a magnet towards the vacuum tube the beam becomes curved downwards. Considering the kathode rays to be a stream of charged particles emitted by the kathode, the direction of their deflection, as indicated by the **left-hand rule** given on p. 968, shows that the charges must be **negative** electricity. This confirms the conclusion from the last experiment (Fig. 979).

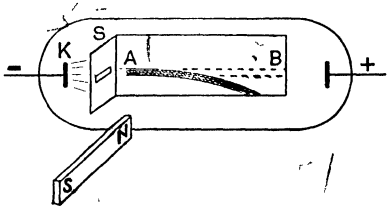


FIG 980.—Deflection of kathode rays in a magnetic field.

Nature of the kathode rays,—electrons.—It is beyond the scope of this work to consider the methods by which the nature of the kathode rays was established. It is, however, of interest to note that they are now considered to consist of particles whose mass is $\frac{1}{1860}$ of that of an atom of hydrogen (*i.e.* $8{\cdot}8 \times 10^{-28}$ gr.), and with each particle is associated a negative charge of $1{\cdot}57 \times 10^{-20}$ electromagnetic units. They are now universally called **electrons.**

Owing to the negative charge of the electron it has an acceleration in the electric field applied to the tube, which acceleration is very great owing to the small mass of the electron. Since their discovery

it has been found that they are fundamental constituents of all atoms and play a part in phenomena as widely separated as the conduction of electricity and the emission of light.

The mass and charge of the electron are constant, whatever may be the material of the electrode or gas in the tube, in fact, the charge of an electron seems to be the ultimate and indivisible unit of electricity.

Canal rays —Positive particles.—Electrons, owing to their negative charges, experience forces driving them away from the kathode. There are, however, **positively charged particles which travel towards the kathode.** Since these positive particles travel towards the kathode and eventually strike it, they were not discovered until a perforated kathode was used. This is a kathode having a number of holes or canals bored through it (Fig. 981). Many of the positive particles, travelling towards the kathode pass through these canals, and form faintly luminous streamers called **positive rays** or **canal rays** upon the side remote from the anode.

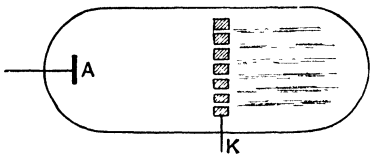


FIG 981.—Canal rays

These positive rays can be deflected only by very powerful magnetic fields. The direction of deflection is then found to be **opposite to that of the kathode rays**, which indicates that their charges are positive. They are much more complex than electrons, and have masses depending upon the nature of the gas in the tube. The smallest mass of any particle is equal to the mass of an atom of hydrogen, and their masses are proportional to the atomic weights of the various elements. Their charges are always multiples of that of the electron, the lowest being equivalent to an electronic charge, but, of course, of opposite sign. It seems probable that the positive rays consist of ordinary atoms which have lost one or more electrons.

Röntgen rays, or X-rays.—One of the most important properties of the kathode rays is that whenever they strike any material substance a new form of radiation is emitted. This radiation has great penetrating power and produces photographic and electrical effects It was discovered by Röntgen and was called by him **x rays.** A modern form of X-ray tube is shown in Fig. 982. It is exhausted until the Crookes dark space is larger than the tube itself, so that there is no kathode glow present

The kathode K is of aluminium and is concave, so that the kathode

rays come to a focus at a small spot upon a sheet of platinum B placed at an angle of 45° to the axis of the kathode. B is sometimes called the antikathode and is connected to an aluminium anode A. The presence of A is not essential.

The X-rays arise from the small spot upon B, upon which the kathode rays impinge. They have so great a penetrative power that they pass through the walls of the tube. Their presence may be detected by the fluorescence they produce in certain substances. Barium platino-cyanide is one of the best substances for this purpose,

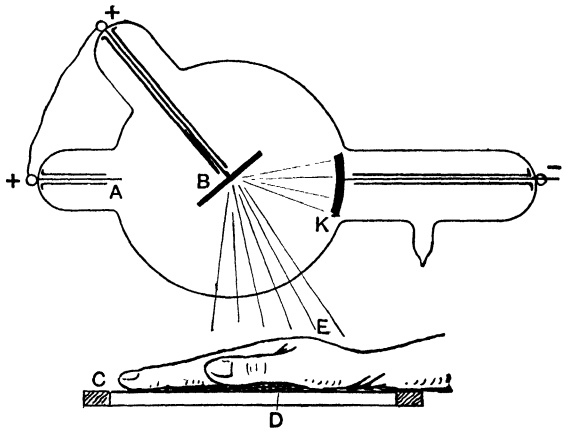


FIG 982 —Production of X-rays

and it is usually spread upon a cardboard screen C. On looking at the side D upon which the fluorescent material is spread, a pale blue fluorescence due to the X-rays may be seen.

The penetrative power of the X-rays depends upon the vacuum in the tube and also upon the density of the material through which they are passing. Thus the higher the vacuum, the greater the penetrative power of the X-rays. With very high vacuum and great penetrative power of the X-rays, these are said to be **hard** X-rays. The reverse are said to be **soft** X-rays. Since the penetrative power depends upon the density of the material, a body such as E (Fig. 982), placed in the path of the X-rays will cast a shadow upon the screen, indicating the variation in density of structure of the body. If the body E be the human hand, the bones being dense, obstruct the rays and cast a dense shadow. The fleshy parts are

not so dense, and the parts of the screen illuminated by the rays passing through these parts are brighter.

If the screen C be replaced by a photographic plate, the plate is acted upon by the rays to an extent depending upon the intensity of the rays. On developing the plate a negative of the object is

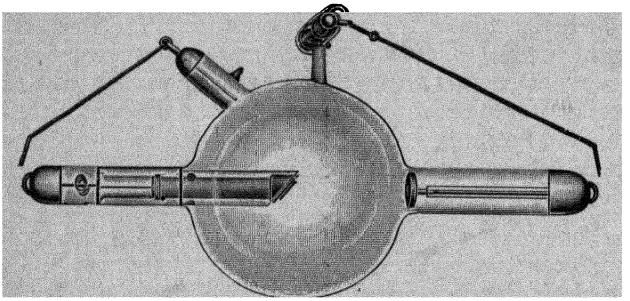


FIG 983.—X-ray tube

obtained and a positive may be printed from it in the ordinary way. Such a photograph is called a **radiograph.** A modern form of X-ray tube is shown in Fig. 983.

Ionisation produced by X-rays.—Even more important than their fluorescent and photographic effects is the power which X-rays possess of rendering the gas through which they pass a conductor of electricity. If a battery consisting of a number of cells be joined to two conductors between which is situated a gas, an appreciable current will flow through the gas when X-rays are passing through it The reason is that the X-rays have the power of liberating electrons from the atoms of the gas. These electrons, having negative charges, the remaining parts of the atoms are therefore positively charged, and these **ions,** as they are called, experience forces in the electric field, causing the positive ions to move towards the kathode and the negative ions towards the anode. These drifts of charges constitute an electric current. The process is called **ionisation** and the gas is said to be **ionised.**

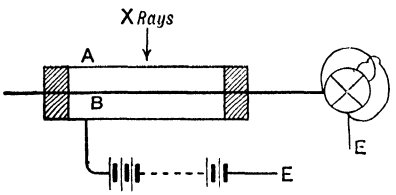


FIG 984.—Ionisation current

An ionisation current may be exhibited by insulating a wire B (Fig. 984) which passes axially down a thin aluminium tube A, also

insulated. B is connected to one pair of quadrants of an electrometer, the other pair being earthed. A is connected to one terminal of a battery of cells of which the other terminal is earthed. Starting with B at zero potential, by momentary earthing, it will be seen that on passing a beam of X-rays through the tube A, the needle of the electrometer will have a continually increasing deflection. This shows that a current is passing through the gas and is charging the wire B and the quadrants of the electrometer.

Nature of X-rays.—It is now known that X-rays are identical in character with light, that is, they consist of a wave motion. But whereas the shortest wave-length of light waves is about $0{\cdot}4 \times 10^{-4}$ cm, that of X-rays is of the order of 10^{-8} cm. For this reason their penetrative power is much greater than that of light waves. The wave-length of X-rays has been determined by observing their mode of reflection from certain crystals, and has given rise to a more intimate knowledge of the nature of crystalline structure than had hitherto been attainable.

Secondary X-rays.—When X-rays fall upon any material, other X-rays are emitted. These are of various types. Some are merely the incident X-rays scattered by the material, but others are of an extremely homogeneous form, and have a wave-length characteristic of the material upon which the primary X-rays fall. There are two types of these homogeneous secondary X-rays, one the "**Series K**" being very '**hard**' or penetrating, and the other "**Series L**" being '**soft**' or less penetrating. The quality of secondary X-rays emitted depends only upon the material emitting them and not upon the quality of the primary X-rays which cause their emission, but the primary X-rays must always be 'harder' than the secondary rays emitted. The higher the atomic weight of the element constituting the material, the harder will be the secondary X-rays emitted.

Radioactivity. There are a few rare substances which emit rays which are in some respect similar to X-rays. That is, they can penetrate ordinary materials and can produce fluorescence, photographic effects and ionisation. Such substances are said to be **radioactive**, and are uranium, radium, thorium and actinium. Of these substances radium is by far the most active, but the property of radioactivity was discovered by observations on uranium.

Of the various effects produced by the rays emitted by radioactive substances, that of ionisation lends itself best to a study of the properties of different substances. To measure the ionisation, the gold-leaf electroscope is very convenient, and several forms of this instrument have been devised for the purpose. That described

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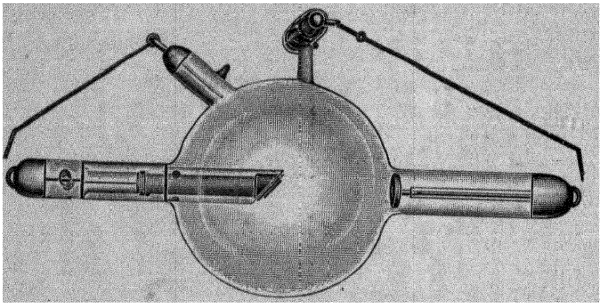


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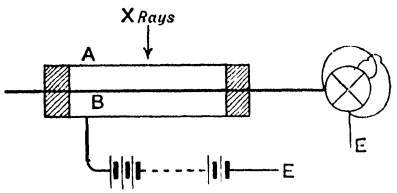


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on p 946 (Fig 882) was devised by C T. R. Wilson. The motion of the leaf D is observed by means of a short focus telescope with a scale in the eyepiece The rate at which the image of the leaf passes the divisions of the scale is then a measure of the rate of leak to earth of the charge upon the leaf, and this in turn is a measure of the conductivity of the air, or the ionisation due to the rays entering the chamber.

If a minute amount of radium be situated below B, the leaf will collapse rapidly, but if some salt of uranium or thorium be spread upon a piece of paper and placed below B the leaves will collapse comparatively slowly.



FIG 985.—Effect produced by thorium upon a photographic plate.

The photographic effect of the rays from thorium may be shown in an interesting manner. If an ordinary incandescent gas mantle be placed in contact with a photographic plate and left for about a week, then on developing the plate the pattern of the mantle will be seen The mantle is impregnated with thorium and the radiation from the thorium affects the plate. Fig. 985 has been obtained in this way, the letters being produced by placing a piece of tinfoil, from which the letters have been cut, between the mantle and the plate This tinfoil shields the plate from the rays

α β and γ rays. These rays emitted by a radioactive substance, are complex The y may be distinguished one from the other, by measuring by means of an electroscope their penetrating power for various layers of material Some thorium oxide may be spread upon a sheet of glass and placed immediately under the aperture B of the electroscope (Fig 882) the cover being removed The time

required for the leaf to pass from some particular scale division to another is then noted. A thin sheet of paper is then placed upon the thorium oxide and the observation repeated. Other layers of paper are then added, and it will be found that the ionisation is cut down to a small fraction by the first sheet, say to $\frac{1}{4}$, but the second sheet will not produce much further reduction. The reason for this is that some of the rays have very little penetrating power and are nearly all absorbed by the first layer of paper. These are called the **α rays.**

If now a sheet of paper be placed upon the thorium oxide, to cut off the α rays, and a similar experiment be performed by adding layers of thin aluminium foil, it will be found that the first layer produces a disproportionately large decrease in the ionisation. This shows as before that the remaining rays, when the α rays have been removed are still complex. The most easily absorbable of these remaining rays are called **β rays** and the most penetrating of all are called the **γ rays.**

The following table illustrates the relative penetrating powers of the α, β and γ rays

Rays	Thickness of aluminium required to reduce the intensity to one-half	Relative penetrating power
α	0·0005 cm.	1
β	0·05 cm.	100
γ	8 cm.	10000

The methods by which the properties of these rays have been studied cannot here be entered into, but it should be noted that the α rays are of the same nature as the positive rays in the discharge tube, the β rays are like the kathode rays and the γ rays are identical in kind with the X-rays. Thus the α rays consist of positively charged particles, emitted with a velocity which sometimes reaches $2{\cdot}5 \times 10^9$ cm. per sec. The β rays consist of electrons with velocity, in some cases, as high as $2{\cdot}85 \times 10^{10}$ cm. per sec, which is very near the velocity of light.

The spinthariscope.—Many minerals fluoresce when α rays fall upon them. Thus, the diamond exhibits a blue fluorescence which enables a true stone to be distinguished from a false one. Zinc sulphide fluoresces brightly in α rays. This fact has been made use of in constructing the **spinthariscope**, which is a thin layer of zinc sulphide behind which a speck of radium bromide or some other material which emits α rays is placed. The zinc sulphide is examined by a short focus lens, and a bright and continuous

shower of sparks is seen when the observation is made in the dark. Each flash is due to the rupture of a minute crystal of zinc sulphide when struck by an α ray particle.

Emission of heat by radium.—It is a well-established fact that radioactive substances emit heat. In the case of radium bromide, the temperature is always about 2° C. above that of its surroundings. The rate of emission of heat has been measured by means of the Bunsen's ice calorimeter (p 118), and it has been found that 1 gram of radium emits 100 calories per hour, and that the rate of emission is constant whatever the temperature of the radium That there is an immense store of energy in radioactive materials is evident from the fact that during its whole life 1 gram of radium emits about 10^{10} calories. This store of energy accounts to some extent for the energy continually emitted from the sun in the form of heat

Radioactive changes. -An examination of the changes occurring in a radioactive material has shown that the process of radioactivity accompanies a change in nature of the substance Thus the α ray particle has an atomic weight of about 4 and is probably a helium atom, for it has been hown that radium is continually producing the element helium.

As a type of the change that goes on in a radioactive substance, let us take the case of radium. A new substance can be separated from radium on heating it or dissolving it in water. This substance is a gas at ordinary temperatures, but can be condensed at the temperature of liquid air It is called **radium emanation.** The emanation decays to half its quantity in 3·85 days, and in doing so changes in turn to radium A, radium B, Ra C, Ra D, Ra E and Ra F. The change from radium to the emanation is accompanied by the emission of α and β rays That from the radium emanation to Ra A by the emission of α rays, from Ra A to Ra B by α rays, Ra B to Ra C by β rays, Ra C to Ra D by α, β and γ rays The change from Ra D to Ra E occurs without the emission of rays Ra E to Ra F by β rays, and Ra F in changing to some unknown product by emission of α rays The decay to half its quantity for the successive substances, radium, etc, to Ra F occur respectively in about 2000 years, 3·85 days, 3·0 minutes, 26·7 minutes, 19·5 minutes, 15 years, 4·8 days and 140 days.

The other radioactive materials undergo similar changes, some being more and others less complicated than those of radium. It is interesting to note that in the course of its changes, a uranium atom emits three α ray particles, and since each has an atomic weight 4, this would reduce the atomic weight of uranium from 238·5 to 226·5, and the atomic weight of radium is 225. Further, a radium atom during its changes emits five α ray particles, which would bring its atomic weight to 206·5, and that of lead is 206·9. It is a significant

fact that minerals containing uranium and radium always contain a large proportion of lead.

Whether it will be found that all elements are radioactive and are undergoing changes similar to those of the radioactive materials, but vastly slower, only the future can decide.

EXERCISES ON CHAPTER LXXX.

1. Describe the changes which occur in an electric spark in air as the pressure of the air is gradually reduced to a high vacuum.

2. Give an account of kathode rays and the effects they produce. What are canal rays ?

3. Describe the effect of a magnetic field upon kathode rays, and deduce some property of these rays.

4. Describe the production of X-rays and state what you know regarding their nature.

5. Give some account of radioactivity, and describe how its intensity may be measured.

6. Give a short account of the radioactive changes which occur in the case of some one substance.

7. Describe the apparatus you would use for obtaining Rontgen rays, and show how you would arrange the apparatus to obtain a radiograph of the hand. L.U.

CHAPTER LXXXI

WIRELESS TELEGRAPHY

Radio-telegraphy.— There are many methods of signalling in which a code of dots and dashes, or short and long sounds, such as the Morse Code (p 1005) is employed. In ordinary telegraphy a buzzer, sounder, or galvanometer at the receiving station is connected with a battery and key at the sending station, by means of a wire, or pair of wires, so that the circuit may be completed by the key at the sending station. But it is essential to this method that the stations should be connected by the wire or cable, which in many cases has a length of several thousand miles. In **radio-telegraphy**—or, as it is generally called, **wireless telegraphy**—this wire or cable is dispensed with, a series of electro-magnetic waves being given out at the sending station, which on arriving at the receiving station actuate a telephone, producing long or short sounds corresponding to dashes and dots, according to the length of each series of waves.

In the middle of the last century Lord Kelvin showed by calculation, that if a charged condenser (p. 940) be discharged by connecting the plates by a conductor of sufficiently small resistance, the charge does not merely disappear It surges backwards and forwards between the plates, just as the water in a **U** tube does when disturbed, the discharge being then said to be oscillatory. It was shown afterwards by experiment that this is the case

The next step is the Electro-magnetic theory of James Clerke Maxwell, according to which any sudden alteration in the electrical field anywhere causes a disturbance to travel outwards through space with the velocity of light (3×10^{10} cm. per sec.) It follows that the oscillatory field due to the discharge of a condenser causes waves to travel outwards The presence of these waves was at a later date demonstrated by Hertz and also by Lodge. It only remained to find a sensitive detector of these waves, and this was discovered by Branley, and took the form of a tube of loosely

packed metal filings, whose conductivity was enormously increased when the electro-magnetic waves fell on them. The process of signalling here briefly sketched was adapted to practical telegraphy by Marconi and others.

Oscillatory discharge.—On p. 926 the electric lines of force are given for the case in which two spheres have opposite electric charges. Suppose that these spheres are now connected by a wire AB (Fig. 986 (*a*)). A being at a higher potential than B, a current immediately begins to flow from A to B, that is, the positive charges or positive ends of the electric lines of force travel from A to B and

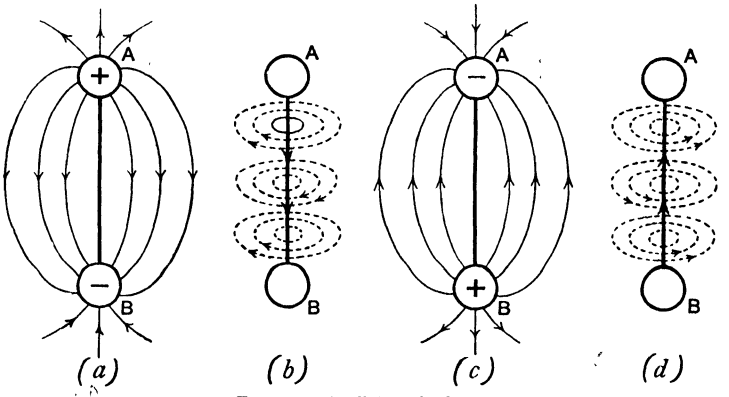


FIG. 986.—Oscillatory discharge.

the negative ends from B to A This is of course accompanied by the production of magnetic lines of force (p. 830), which are circles surrounding the wire (Fig. 986 (*b*)). These grow until the current has reached its greatest value, at which instant the electric charges and electric lines of force have just disappeared. The magnetic lines of force now begin to collapse upon the wire, and in so doing produce an e.m.f. which causes the current to continue flowing from A to B until the magnetic lines of force have all collapsed and disappeared. This continued current means that B is acquiring a positive charge and A a negative charge (Fig. 986 (*c*)), and at the end of this stage the charges and electric field are exactly the reverse of those in Fig. 986 (*a*), except that there may be a diminution of charge due to energy being dissipated in the heat produced by the current in the conductor, and also by any radiation to be described below.

The return current then immediately begins, and is shown at its greatest value in Fig 986 (*d*).

The discharge in this case is said to be **oscillatory**, and it is easily seen that the energy exists alternately in the form of electric field and magnetic field. If there is no dissipation of energy, or if energy is supplied continuously by some source, the curve representing the amount of electric charge at succeeding intervals of time is a sine curve, shown dotted in Fig. 987, and the oscillation is said to be **undamped**. If, on the other hand, the energy dies away, the oscillation is said to be **damped**, and is shown by the continuous line curve in Fig. 987 In the case of wireless telegraphy the frequency of oscillation may vary from 2×10^4 to 3×10^6 complete oscillations per second

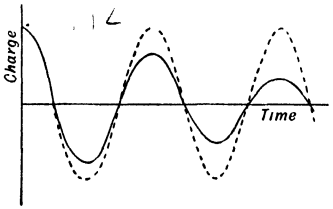


FIG. 987 – Damped and undamped oscillations

Radiation – It must be understood that the process of radiation can only be described with completeness in terms of mathematics of a higher order than is required for the rest of this book, but the following explanation may help the student to some understanding of

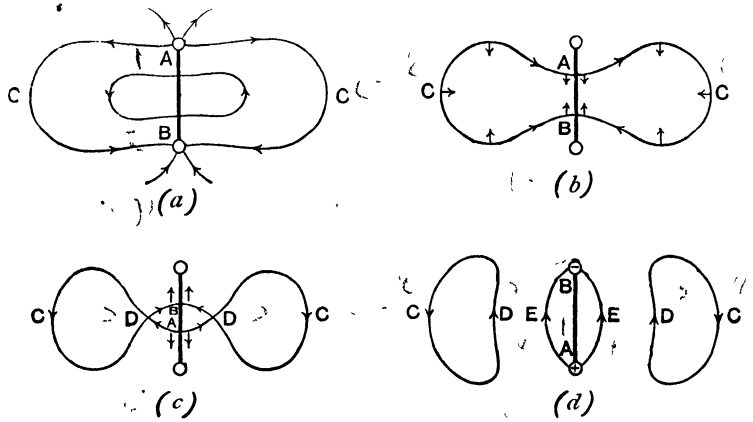


FIG. 988.—Production of electric waves.

the manner in which electric waves arise. When the difference of potential between the ends of the conductor is very great, the electric lines of force may be similar to ACB (Fig. 988 (*a*)), and if the con-

ductivity of the conducting bridge AB, which is usually a spark (p. 1028), is great, the ends of the lines of force travel as in Fig. 988 (*b*) and may reach each other during the discharge, and even cross, before the part C reaches the conductor, thus forming a loop DC (Fig. 988 (*c*)). The intersection D is a point of instability and the lines immediately break, forming a closed loop DC and a shortened line AEB (Fig. 988 (*d*)). The loop DC travels outwards, and the part AEB continues the oscillation previously described.

In wireless telegraphy the conductor is of considerable length, in order to produce waves of sufficient size, and is then called an **aerial** or **antenna**. Aerials are of many forms, but their function is the same, that is, to increase the size of wave produced. It should be remembered that only half of each complete loop is produced by the

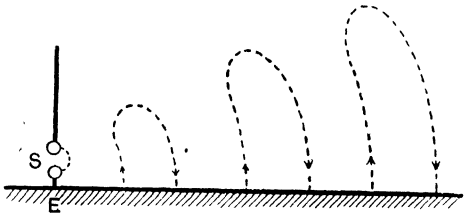


FIG. 989.—Waves from an aerial.

aerial, the other part, which would be produced if the aerial were symmetrical about the spark gap S (Fig. 989), being suppressed by the earth. A few waves emitted by an aerial are illustrated in Fig 989.

The waves are transmitted with the velocity of light, that is 3×10^{10} cm. per sec. as has been noted above . in fact, they are waves of the same character as light waves, but are of vastly greater length. The average wave-length of light is about 6×10^{-5} cm., while those used in wireless telegraphy have a wave-length varying from 100 metres to 15,000 metres.

The usual relation between frequency (n), wave-length (λ), and velocity (v) holds here as in the case of all other waves (p. 680), that is,

$$\text{velocity} = \text{wave-length} \times \text{frequency},$$

or

$$v = \lambda n.$$

As an example, consider a case in which the frequency of oscillation is 120,000. Then, since the velocity is 3×10^{10} cm. per sec.

$$3 \times 10^{10} = \lambda \times 120{,}000,$$

$$\lambda = 2{\cdot}5 \times 10^{5} \text{ cm.}$$

That is, the wave-length is 2500 metres.

Production of oscillations. There are several methods of producing electric oscillations in the aerial, but that most commonly used consists in raising the p d. between two conductors to such a high value, by means of an induction coil (p 983), that a spark occurs. The spark gap may be in the aerial itself (Fig. 990), which shows an **open oscillatory circuit,** or the secondary coil of a transformer may be in the aerial (Fig 991), which is an example of a **closed**

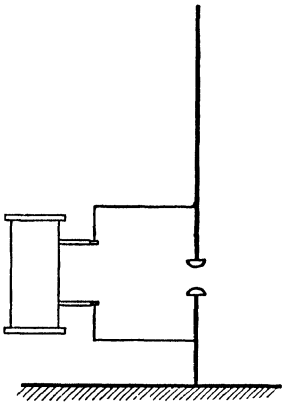


FIG. 990.—Open oscillatory circuit.

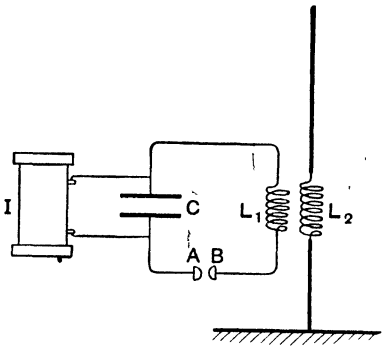


FIG. 991.—Closed oscillatory circuit.

oscillatory circuit. In this case the induction coil **I** raises the p.d. between A and B to such a magnitude that a spark occurs between A and B. This is accompanied by oscillations in the circuit consisting of the spark gap AB, the condenser C and the primary coil L_1 of the transformer L_1L_2, sometimes called a **jiggar.** This oscillation is really an alternating current, so that an alternating e m f is produced (p. 985) in the secondary coil L_2. Thus for every spark at AB a train of waves will be given out into space by the aerial. Such a train of waves is of the damped type (Fig 987), because the energy corresponding to each discharge of the condenser C is rapidly radiated outwards from the aerial in the form of waves.

The frequency of oscillation in any circuit in which the resistance is small is given by the equation

$$n = \frac{1}{2\pi\sqrt{LC}},$$

where L is the self-inductance (p. 986) and C the capacity (p. 943)

in the circuit. As an example, let the capacity be 6×10^{-9} farads or 0·006 micro-farads, and the self-inductance 80 micro-henrys or 8×10^{-5} henrys.

Then
$$n = \frac{1}{2\pi\sqrt{6 \times 10^{-9} \times 8 \times 10^{-5}}}$$
$$= \frac{1}{2\pi\sqrt{48 \times 10^{-14}}} = 2{\cdot}3 \times 10^{5}$$
$$\therefore \text{ wave-length} = \frac{3 \times 10^{10}}{2{\cdot}3 \times 10^{5}} \text{ cm.}$$
$$= 1300 \text{ metres.}$$

Receiving.— It should be remembered that an electric line of force is really the representation of an electric field (p. 926) and that a difference of potential exists between different points on a line of force. Hence the arrival of an electric wave at a conductor causes differences of potential between the parts of a conductor and gives rise to an electric current. Thus, imagine waves travelling from left to right to arrive at the conductor AB (Fig. 992).

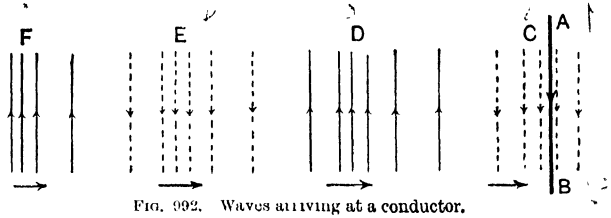


FIG. 992. Waves arriving at a conductor.

On the arrival of the part C or E of the wave, there will be a current in the conductor from A to B while the parts D and F will produce current from B to A. Alternating current will therefore flow in AB during the arrival of electric waves. It might perhaps be thought that by including a telephone receiver in AB a sound would be heard ; but a moment's consideration will show that this is not the case. The frequency of the waves employed in wireless telegraphy is such that any note produced would be of too high a pitch to be audible (p. 672). Moreover, the oscillations are too rapid for the diaphragm of the telephone to follow to any appreciable extent. Some other means, such as the **coherer** or the **rectifier**, must be employed for their detection.

The Coherer – The **coherer** is the earliest device used in wireless telegraphy, and consists of a tube AB (Fig. 993) into which the conductors C and D enter, the gap between which is completed by a small quantity of metal filings E, usually silver. When the feeble current produced in the aerial F by the waves received, passes through the coherer, its resistance is reduced to such an extent that the current in the local circuit due to the cell H becomes large enough to actuate the galvanometer, or sounder (p. 1006), or a relay G (p. 1008). Thus long or short trains of waves give rise to dashes or dots according to the Morse system (p. 1005).

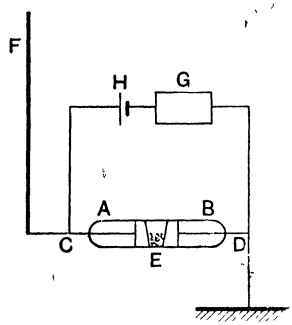


FIG. 993.—Coherer

One great drawback to the coherer is that it does not automatically "decohere"; that is, the conductivity remains comparatively high after the train of waves has ceased. It recovers, however, its normal high resistance when subjected to mechanical disturbance, such as tapping, and this necessitates the employment of some device for tapping it to cause it to decohere after each train of waves.

Crystal detector It will be remembered that the waves, and therefore the oscillatory currents, in the receiving aerial are too rapid to produce any effect in the telephone. If, however, the negative halves of the waves, B, D and F (Fig. 994) could be suppressed, the positive halves A, C and E, which follow each other very rapidly, would combine to produce a pull on the diaphragm of the telephone. This can be effected by the **crystal detector**.

A crystal of suitable substance, carborundum, zincite, or galena is placed with one of its sharp angles in contact with a metal plate.

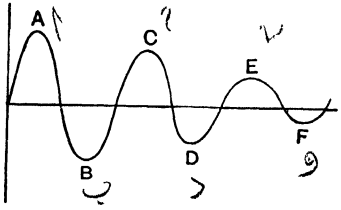


FIG. 994.—Damped wave.

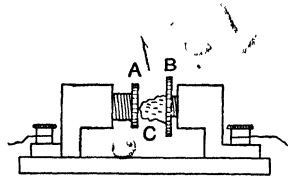


FIG. 995.—Crystal detector.

This conducts much more freely for a current in one direction than for a current in the reverse direction. The crystal C (Fig. 995)

is embedded in solder in the brass holder B, and can be screwed forward in order to make contact with the metallic plate A. It is impossible to say from inspection whether any particular crystal will be a good rectifier. Numbers of crystals are tried and the bad ones rejected. In Fig 996 the crystal detector C is shown in series with the telephone T, and this circuit is placed in parallel with the inductance I which is in series with the aerial A. Sometimes it is advantageous to place the detector circuit in parallel with the capacity C_1 instead of with the inductance Whenever a train of waves arrives at the receiving station, a rapidly intermittent current will pass in the telephone and a short sound will be heard, one corresponding to each spark at the sending station. This will build up into a buzz or hum whose duration is equal to that for which the sending key is pressed and the sparks produced.

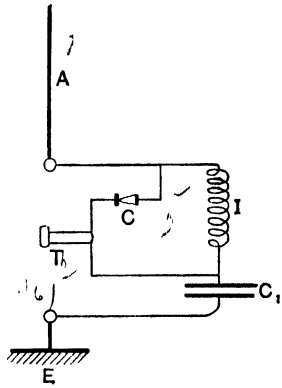


FIG. 996.—Crystal detector circuit.

It frequently happens that the rectifying power of the crystal is greatest when there is a constant e.m.f. acting across the contact. The reason for this may be seen by examining the curve connecting current across the contact and the p.d. (Fig. 997). This curve is usually concave upwards as shown. If then the constant potential difference O*a* be applied by means of a battery and *ab* and *ac* be the positive and negative values of the p.d. due to the waves received, *ab* causes an increase *ef* in the current through the crystal and *ac* causes a decrease *gh*. As the former is greater than the latter the resultant is an excess of current in one direction, which is the condition for a sound to be heard in the telephone.

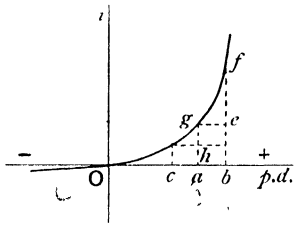


FIG. 997.—Rectification.

In order to apply this steady p.d., an auxiliary battery with a potentiometer arrangement is employed. The battery B and resistance AC (Fig. 998) form a closed circuit so that there is a drop of potential from A to C. One of the telephone leads makes contact at a point of AC which can be varied, so that a greater or less fraction

of the whole p d between A and C can be included in the crystal circuit. The point of maximum distinctness of the signals is found by trial. This diagram also shows how the variations in p.d. in the

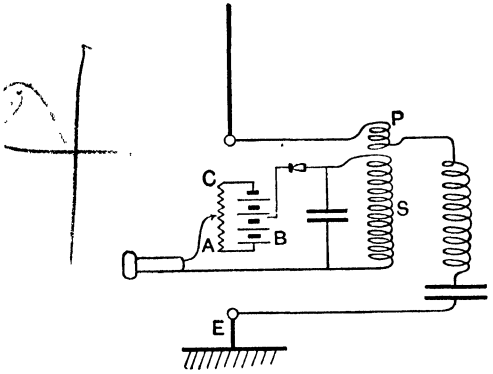


FIG 998.—Potentiometer arrangement for crystal.

receiver circuit may be increased from those in the aerial circuit by means of the transformer, whose primary, P, is in the aerial circuit and secondary, S, in the detector circuit.

Rectifying valve.—It is well known that hot bodies emit electrons (p. 1031), which are small charges of negative electricity. If the hot body is at a higher potential than surrounding bodies, these electrons

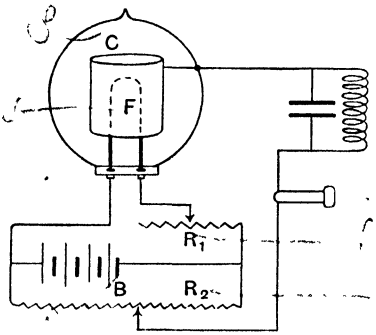


FIG 999.—Rectifying valve.

will not pass away from it but will tend to remain upon it. On the other hand, if it is at a lower potential than surrounding bodies, the electrons, being negative charges, will be driven away from it. This stream of **negative** charges **from** it constitutes a **positive** electric

current **towards** it. Hence the current passes through the surrounding space more readily in one direction than in the other, and the arrangement therefore forms a **rectifying valve** whose function is similar to that of the crystal detector. There are many forms of the apparatus, one being shown in Fig. 999. The hollow cylindrical conductor C surrounds the carbon filament F of an incandescent lamp. The battery B serves to maintain the filament in a state of incandescence, and may also be used in conjunction with the resistance R_2 to maintain the necessary p.d. between F and C to obtain the best rectification. In fact, the valve plays a part similar to the crystal contact in the case of the crystal detector.

Amplifying valve or Audion.- In an incandescent lamp employed as a valve, the gas is almost entirely exhausted from it, and any current which passes on to the filament does so on account of the electrons emitted from it. If then a wire gauze or grid G (Fig. 1000)

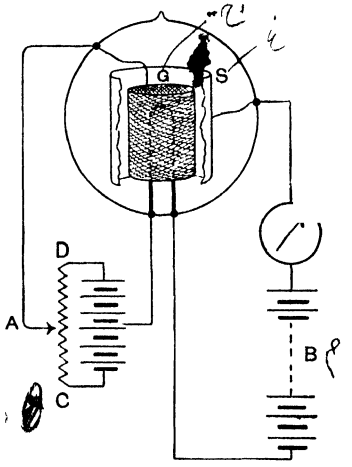


FIG. 1000.

surrounds the filament, the electrons emitted will charge the gauze negatively and the passage of them will nearly cease; the space round the filament is then practically a non-conductor for positive current towards the filament. This effect is increased if the grid be made at a negative potential with respect to the filament by bringing the movable contact A to the negative C. The battery B is then unable to send current from the sheath S to the filament.

If, however, A be raised until the grid is at a positive potential with respect to the filament, electrons will now pass freely towards the grid and reach the sheath, which means that the battery B can now produce considerable current. A small negative potential of grid over filament therefore nearly stops the current from B and a small positive potential allows a large current. This amplification is just of the kind necessary to enable feeble oscillations in the aerial to produce a large effect in the telephone. The arrangement is shown in Fig. 1001 E and F are the condenser terminals and the

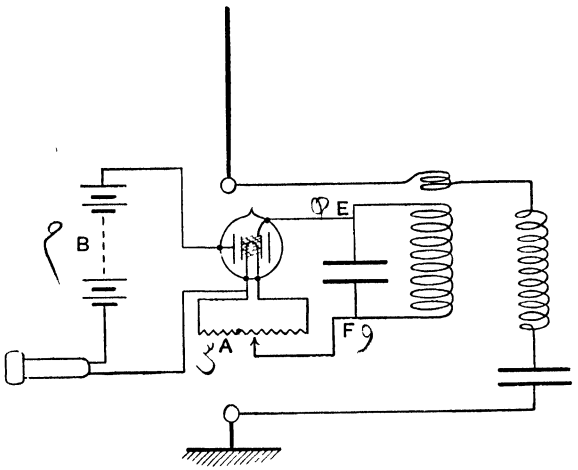


FIG. 1001.—Receiving circuit with amplifying valve.

small oscillatory current produces alternations in p.d. between the grid and the filament, which in turn allow large unidirectional currents in the telephone. In Figs. 1000 and 1001 the battery for maintaining the incandescence of the filament is not shown. The battery B in the telephone circuit should have an e.m.f. of about 100 volts.

Tuning and resonance.—It was seen on p. 1044 that every electrical circuit has a natural frequency of oscillation of current in it. If an oscillating e.m.f. acts upon a circuit, the oscillating current produced will be greatest when the frequency of the applied e.m.f. is the same as the natural frequency of oscillation for the circuit. In other words, the circuit **resounds** to the applied e.m.f. just as a vibrating body resounds to an impulse of suitable frequency (p. 713). This

resonance plays a most important part in wireless telegraphy. The oscillatory circuit of the aerial must be tuned to the frequency of the received waves, and the secondary circuit (Fig. 998) must be tuned to the same frequency. This applies to all the oscillatory circuits used. The method of tuning is to vary the inductance or capacity or both, in each circuit, but the means of doing this and the results obtained are beyond the scope of this work. It may, however, be noted that the possibility of picking out the signals from one station and ignoring those from others rests upon this principle of tuning and resonance.

EXERCISES ON CHAPTER LXXXI.

1. Give a short account of the development of wireless telegraphy.

2. Describe the processes occurring during the oscillatory discharge of a condenser.

3. Give the difference between an open oscillatory circuit and a closed oscillatory circuit, for the production of electro-magnetic waves.

4. Describe some form of receiver for the detection of minute oscillatory currents.

	0	1	2	3	4	5	6	7	8	9	1 2 3	4 5 6	7 8 9
10	0000	0043	0086	0128	0170						4 9 13	17 21 26	30 34 38
						0212	0253	0294	0334	0374	4 8 12	16 20 24	28 32 36
11	0414	0453	0492	0531	0569						4 8 12	15 19 23	27 31 35
						0607	0645	0682	0719	0755	4 7 11	15 19 22	26 30 33
12	0792	0828	0864	0899	0934						3 7 11	14 18 21	25 28 32
						0969	1004	1038	1072	1106	3 7 10	14 17 20	24 27 31
13	1139	1173	1206	1239	1271						3 7 10	13 16 20	23 26 30
						1303	1335	1367	1399	1430	3 7 10	13 16 19	22 25 29
14	1461	1492	1523	1553	1584						3 6 9	12 15 19	22 25 28
						1614	1644	1673	1703	1732	3 6 9	12 15 17	20 23 26
15	1761	1790	1818	1847	1875						3 6 9	11 14 17	20 23 26
						1903	1931	1959	1987	2014	3 6 8	11 14 17	19 22 25
16	2041	2068	2095	2122	2148						3 5 8	11 14 16	19 22 24
						2175	2201	2227	2253	2279	3 5 8	10 13 16	18 21 23
17	2304	2330	2355	2380	2405						3 5 8	10 13 15	18 20 23
						2430	2455	2480	2504	2529	2 5 7	10 12 15	17 20 22
18	2553	2577	2601	2625	2648						2 5 7	9 12 14	16 19 21
						2672	2695	2718	2742	2765	2 5 7	9 11 14	16 18 21
19	2788	2810	2833	2856	2878						2 4 7	9 11 13	16 18 20
						2900	2923	2945	2967	2989	2 4 6	8 11 13	15 17 19
20	3010	3032	3054	3075	3096	3118	3139	3160	3181	3201	2 4 6	8 11 13	15 17 19
21	3222	3243	3263	3284	3304	3324	3345	3365	3385	3404	2 4 6	8 10 12	14 16 18
22	3424	3444	3464	3483	3502	3522	3541	3560	3579	3598	2 4 6	8 10 12	14 15 17
23	3617	3636	3655	3674	3692	3711	3729	3747	3766	3784	2 4 6	7 9 11	13 15 17
24	3802	3820	3838	3856	3874	3892	3909	3927	3945	3962	2 4 5	7 9 11	12 14 16
25	3979	3997	4014	4031	4048	4065	4082	4099	4116	4133	2 3 5	7 9 10	12 14 15
26	4150	4166	4183	4200	4216	4232	4249	4265	4281	4298	2 3 5	7 8 10	11 13 15
27	4314	4330	4346	4362	4378	4393	4409	4425	4440	4456	2 3 5	6 8 9	11 13 14
28	4472	4487	4502	4518	4533	4548	4564	4579	4594	4609	2 3 5	6 8 9	11 12 14
29	4624	4639	4654	4669	4683	4698	4713	4728	4742	4757	1 3 4	6 7 9	10 12 13
30	4771	4786	4800	4814	4829	4843	4857	4871	4886	4900	1 3 4	6 7 9	10 11 13
31	4914	4928	4942	4955	4969	4983	4997	5011	5024	5038	1 3 4	6 7 8	10 11 12
32	5051	5065	5079	5092	5105	5119	5132	5145	5159	5172	1 3 4	5 7 8	9 11 12
33	5185	5198	5211	5224	5237	5250	5263	5276	5289	5302	1 3 4	5 6 8	9 10 12
34	5315	5328	5340	5353	5366	5378	5391	5403	5416	5428	1 3 4	5 6 8	9 10 11
35	5441	5453	5465	5478	5490	5502	5514	5527	5539	5551	1 2 4	5 6 7	9 10 11
36	5563	5575	5587	5599	5611	5623	5635	5647	5658	5670	1 2 4	5 6 7	8 10 11
37	5682	5694	5705	5717	5729	5740	5752	5763	5775	5786	1 2 3	5 6 7	8 9 10
38	5798	5809	5821	5832	5843	5855	5866	5877	5888	5899	1 2 3	5 6 7	8 9 10
39	5911	5922	5933	5944	5955	5966	5977	5988	5999	6010	1 2 3	4 5 7	8 9 10
40	6021	6031	6042	6053	6064	6075	6085	6096	6107	6117	1 2 3	4 5 6	8 9 10
41	6128	6138	6149	6160	6170	6180	6191	6201	6212	6222	1 2 3	4 5 6	7 8 9
42	6232	6243	6253	6263	6274	6284	6294	6304	6314	6325	1 2 3	4 5 6	7 8 9
43	6335	6345	6355	6365	6375	6385	6395	6405	6415	6425	1 2 3	4 5 6	7 8 9
44	6435	6444	6454	6464	6474	6484	6493	6503	6513	6522	1 2 3	4 5 6	7 8 9
45	6532	6542	6551	6561	6571	6580	6590	6599	6609	6618	1 2 3	4 5 6	7 8 9
46	6628	6637	6646	6656	6665	6675	6684	6693	6702	6712	1 2 3	4 5 6	7 7 8
47	6721	6730	6739	6749	6758	6767	6776	6785	6794	6803	1 2 3	4 5 5	6 7 8
48	6812	6821	6830	6839	6848	6857	6866	6875	6884	6893	1 2 3	4 4 5	6 7 8
49	6902	6911	6920	6928	6937	6946	6955	6964	6972	6981	1 2 3	4 4 5	6 7 8

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
50	6990	6998	7007	7016	7024	7033	7042	7050	7059	7067	1	2	3	3	4	5	6	7	8
51	7076	7084	7093	7101	7110	7118	7126	7135	7143	7152	1	2	3	3	4	5	6	7	8
52	7160	7168	7177	7185	7193	7202	7210	7218	7226	7235	1	2	2	3	4	5	6	7	7
53	7243	7251	7259	7267	7275	7284	7292	7300	7308	7316	1	2	2	3	4	5	6	6	7
54	7324	7332	7340	7348	7356	7364	7372	7380	7388	7396	1	2	2	3	4	5	6	6	7
55	7404	7412	7419	7427	7435	7443	7451	7459	7466	7474	1	2	2	3	4	5	5	6	7
56	7482	7490	7497	7505	7513	7520	7528	7536	7543	7551	1	2	2	3	4	5	5	6	7
57	7559	7566	7574	7582	7589	7597	7604	7612	7619	7627	1	2	2	3	4	5	5	6	7
58	7634	7642	7649	7657	7664	7672	7679	7686	7694	7701	1	1	2	3	4	4	5	6	7
59	7709	7716	7723	7731	7738	7745	7752	7760	7767	7774	1	1	2	3	4	4	5	6	7
60	7782	7789	7796	7803	7810	7818	7825	7832	7839	7846	1	1	2	3	4	4	5	6	6
61	7853	7860	7868	7875	7882	7889	7896	7903	7910	7917	1	1	2	3	4	4	5	6	6
62	7924	7931	7938	7945	7952	7959	7966	7973	7980	7987	1	1	2	3	3	4	5	6	6
63	7993	8000	8007	8014	8021	8028	8035	8041	8048	8055	1	1	2	3	3	4	5	5	6
64	8062	8069	8075	8082	8089	8096	8102	8109	8116	8122	1	1	2	3	3	4	5	5	6
65	8129	8136	8142	8149	8156	8162	8169	8176	8182	8189	1	1	2	3	3	4	5	5	6
66	8195	8202	8209	8215	8222	8228	8235	8241	8248	8254	1	1	2	3	3	4	5	5	6
67	8261	8267	8274	8280	8287	8293	8299	8306	8312	8319	1	1	2	3	3	4	5	5	6
68	8325	8331	8338	8344	8351	8357	8363	8370	8376	8382	1	1	2	3	3	4	4	5	6
69	8388	8395	8401	8407	8414	8420	8426	8432	8439	8445	1	1	2	2	3	4	4	5	6
70	8451	8457	8463	8470	8476	8482	8488	8494	8500	8506	1	1	2	2	3	4	4	5	6
71	8513	8519	8525	8531	8537	8543	8549	8555	8561	8567	1	1	2	2	3	4	4	5	5
72	8573	8579	8585	8591	8597	8603	8609	8615	8621	8627	1	1	2	2	3	4	4	5	5
73	8633	8639	8645	8651	8657	8663	8669	8675	8681	8686	1	1	2	2	3	4	4	5	5
74	8692	8698	8704	8710	8716	8722	8727	8733	8739	8745	1	1	2	2	3	4	4	5	5
75	8751	8756	8762	8768	8774	8779	8785	8791	8797	8802	1	1	2	2	3	3	4	5	5
76	8808	8814	8820	8825	8831	8837	8842	8848	8854	8859	1	1	2	2	3	3	4	5	5
77	8865	8871	8876	8882	8887	8893	8899	8904	8910	8915	1	1	2	2	3	3	4	4	5
78	8921	8927	8932	8938	8943	8949	8954	8960	8965	8971	1	1	2	2	3	3	4	4	5
79	8976	8982	8987	8993	8998	9004	9009	9015	9020	9025	1	1	2	2	3	3	4	4	5
80	9031	9036	9042	9047	9053	9058	9063	9069	9074	9079	1	1	2	2	3	3	4	4	5
81	9085	9090	9096	9101	9106	9112	9117	9122	9128	9133	1	1	2	2	3	3	4	4	5
82	9138	9143	9149	9154	9159	9165	9170	9175	9180	9186	1	1	2	2	3	3	4	4	5
83	9191	9196	9201	9206	9212	9217	9222	9227	9232	9238	1	1	2	2	3	3	4	4	5
84	9243	9248	9253	9258	9263	9269	9274	9279	9284	9289	1	1	2	2	3	3	4	4	5
85	9294	9299	9304	9309	9315	9320	9325	9330	9335	9340	1	1	2	2	3	3	4	4	5
86	9345	9350	9355	9360	9365	9370	9375	9380	9385	9390	1	1	2	2	3	3	4	4	5
87	9395	9400	9405	9410	9415	9420	9425	9430	9435	9440	0	1	1	2	2	3	3	4	4
88	9445	9450	9455	9460	9465	9469	9474	9479	9484	9489	0	1	1	2	2	3	3	4	4
89	9494	9499	9504	9509	9513	9518	9523	9528	9533	9538	0	1	1	2	2	3	3	4	4
90	9542	9547	9552	9557	9562	9566	9571	9576	9581	9586	0	1	1	2	2	3	3	4	4
91	9590	9595	9600	9605	9609	9614	9619	9624	9628	9633	0	1	1	2	2	3	3	4	4
92	9638	9643	9647	9652	9657	9661	9666	9671	9675	9680	0	1	1	2	2	3	3	4	4
93	9685	9689	9694	9699	9703	9708	9713	9717	9722	9727	0	1	1	2	2	3	3	4	4
94	9731	9736	9741	9745	9750	9754	9759	9763	9768	9773	0	1	1	2	2	3	3	4	4
95	9777	9782	9786	9791	9795	9800	9805	9809	9814	9818	0	1	1	2	2	3	3	4	4
96	9823	9827	9832	9836	9841	9845	9850	9854	9859	9863	0	1	1	2	2	3	3	4	4
97	9868	9872	9877	9881	9886	9890	9894	9899	9903	9908	0	1	1	2	2	3	3	4	4
98	9912	9917	9921	9926	9930	9934	9939	9943	9948	9952	0	1	1	2	2	3	3	4	4
99	9956	9961	9965	9969	9974	9978	9983	9987	9991	9996	0	1	1	2	2	3	3	3	4

	0	1	2	3	4	5	6	7	8	9	1 2 3	4 5 6	7 8 9
·00	1000	1002	1005	1007	1009	1012	1014	1016	1019	1021	0 0 1	1 1 1	2 2 2
01	1023	1026	1028	1030	1033	1035	1038	1040	1042	1045	0 0 1	1 1 1	2 2 2
02	1047	1050	1052	1054	1057	1059	1062	1064	1067	1069	0 0 1	1 1 1	2 2 2
03	1072	1074	1076	1079	1081	1084	1086	1089	1091	1094	0 0 1	1 1 1	2 2 2
04	1096	1099	1102	1104	1107	1109	1112	1114	1117	1119	0 1 1	1 1 2	2 2 2
05	1122	1125	1127	1130	1132	1135	1138	1140	1143	1146	0 1 1	1 1 2	2 2 2
·06	1148	1151	1153	1156	1159	1161	1164	1167	1169	1172	0 1 1	1 1 2	2 2 2
07	1175	1178	1180	1183	1186	1189	1191	1194	1197	1199	0 1 1	1 1 2	2 2 2
08	1202	1205	1208	1211	1213	1216	1219	1222	1225	1227	0 1 1	1 1 2	2 2 3
·09	1230	1233	1236	1239	1242	1245	1247	1250	1253	1256	0 1 1	1 1 2	2 2 3
10	1259	1262	1265	1268	1271	1274	1276	1279	1282	1285	0 1 1	1 1 2	2 2 3
11	1288	1291	1294	1297	1300	1303	1306	1309	1312	1315	0 1 1	1 2 2	2 2 3
12	1318	1321	1324	1327	1330	1334	1337	1340	1343	1346	0 1 1	1 2 2	2 2 3
·13	1349	1352	1355	1358	1361	1365	1368	1371	1374	1377	0 1 1	1 2 2	2 3 3
·14	1380	1384	1387	1390	1393	1396	1400	1403	1406	1409	0 1 1	1 2 2	2 3 3
·15	1413	1416	1419	1422	1426	1429	1432	1435	1439	1442	0 1 1	1 2 2	2 3 3
·16	1445	1449	1452	1455	1459	1462	1466	1469	1472	1476	0 1 1	1 2 2	2 3 3
·17	1479	1483	1486	1489	1493	1496	1500	1503	1507	1510	0 1 1	1 2 2	2 3 3
·18	1514	1517	1521	1524	1528	1531	1535	1538	1542	1545	0 1 1	1 2 2	2 3 3
·19	1549	1552	1556	1560	1563	1567	1570	1574	1578	1581	0 1 1	1 2 2	3 3 3
·20	1585	1589	1592	1596	1600	1603	1607	1611	1614	1618	0 1 1	1 2 2	3 3 3
·21	1622	1626	1629	1633	1637	1641	1644	1648	1652	1656	0 1 1	2 2 2	3 3 3
·22	1660	1663	1667	1671	1675	1679	1683	1687	1690	1694	0 1 1	2 2 2	3 3 3
·23	1698	1702	1706	1710	1714	1718	1722	1726	1730	1734	0 1 1	2 2 2	3 3 4
·24	1738	1742	1746	1750	1754	1758	1762	1766	1770	1774	0 1 1	2 2 2	3 3 4
·25	1778	1782	1786	1791	1795	1799	1803	1807	1811	1816	0 1 1	2 2 2	3 3 4
·26	1820	1824	1828	1832	1837	1841	1845	1849	1854	1858	0 1 1	2 2 3	3 3 4
·27	1862	1866	1871	1875	1879	1884	1888	1892	1897	1901	0 1 1	2 2 3	3 3 4
·28	1905	1910	1914	1919	1923	1928	1932	1936	1941	1945	0 1 1	2 2 3	3 4 4
·29	1950	1954	1959	1963	1968	1972	1977	1982	1986	1991	0 1 1	2 2 3	3 4 4
·30	1995	2000	2004	2009	2014	2018	2023	2028	2032	2037	0 1 1	2 2 3	3 4 4
·31	2042	2046	2051	2056	2061	2065	2070	2075	2080	2084	0 1 1	2 2 3	3 4 4
·32	2089	2094	2099	2104	2109	2113	2118	2123	2128	2133	0 1 1	2 2 3	3 4 4
·33	2138	2143	2148	2153	2158	2163	2168	2173	2178	2183	0 1 1	2 2 3	3 4 4
·34	2188	2193	2198	2203	2208	2213	2218	2223	2228	2234	1 1 2	2 3 3	4 4 5
·35	2239	2244	2249	2254	2259	2265	2270	2275	2280	2286	1 1 2	2 3 3	4 4 5
·36	2291	2296	2301	2307	2312	2317	2323	2328	2333	2339	1 1 2	2 3 3	4 4 5
·37	2344	2350	2355	2360	2366	2371	2377	2382	2388	2393	1 1 2	2 3 3	4 4 5
·38	2399	2404	2410	2415	2421	2427	2432	2438	2443	2449	1 1 2	2 3 3	4 4 5
·39	2455	2460	2466	2472	2477	2483	2489	2495	2500	2506	1 1 2	2 3 3	4 5 5
·40	2512	2518	2523	2529	2535	2541	2547	2553	2559	2564	1 1 2	2 3 4	4 5 5
·41	2570	2576	2582	2588	2594	2600	2606	2612	2618	2624	1 1 2	2 3 4	4 5 5
·42	2630	2636	2642	2649	2655	2661	2667	2673	2679	2685	1 1 2	2 3 4	4 5 6
·43	2692	2698	2704	2710	2716	2723	2729	2735	2742	2748	1 1 2	3 3 4	4 5 6
·44	2754	2761	2767	2773	2780	2786	2793	2799	2805	2812	1 1 2	3 3 4	4 5 6
45	2818	2825	2831	2838	2844	2851	2858	2864	2871	2877	1 1 2	3 3 4	5 5 6
·46	2884	2891	2897	2904	2911	2917	2924	2931	2938	2944	1 1 2	3 3 4	5 5 6
·47	2951	2958	2965	2972	2979	2985	2992	2999	3006	3013	1 1 2	3 3 4	5 5 6
·48	3020	3027	3034	3041	3048	3055	3062	3069	3076	3083	1 1 2	3 4 4	5 6 6
·49	3090	3097	3105	3112	3119	3126	3133	3141	3148	3155	1 1 2	3 4 4	5 6 6

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
·50	3162	3170	3177	3184	3192	3199	3206	3214	3221	3228	1	1	2	3	4	4	5	6	7
·51	3236	3243	3251	3258	3266	3273	3281	3289	3296	3304	1	2	2	3	4	5	5	6	7
·52	3311	3319	3327	3334	3342	3350	3357	3365	3373	3381	1	2	2	3	4	5	5	6	7
·53	3388	3396	3404	3412	3420	3428	3436	3443	3451	3459	1	2	2	3	4	5	6	6	7
·54	3467	3475	3483	3491	3499	3508	3516	3524	3532	3540	1	2	2	3	4	5	6	6	7
·55	3548	3556	3565	3573	3581	3589	3597	3606	3614	3622	1	2	2	3	4	5	6	7	7
·56	3631	3639	3648	3656	3664	3673	3681	3690	3698	3707	1	2	3	3	4	5	6	7	8
·57	3715	3724	3733	3741	3750	3758	3767	3776	3784	3793	1	2	3	3	4	5	6	7	8
·58	3802	3811	3819	3828	3837	3846	3855	3864	3873	3882	1	2	3	4	4	5	6	7	8
·59	3890	3899	3908	3917	3926	3936	3945	3954	3963	3972	1	2	3	4	5	5	6	7	8
·60	3981	3990	3999	4009	4018	4027	4036	4046	4055	4064	1	2	3	4	5	6	6	7	8
·61	4074	4083	4093	4102	4111	4121	4130	4140	4150	4159	1	2	3	4	5	6	7	8	9
·62	4169	4178	4188	4198	4207	4217	4227	4236	4246	4256	1	2	3	4	5	6	7	8	9
·63	4266	4276	4285	4295	4305	4315	4325	4335	4345	4355	1	2	3	4	5	6	7	8	9
·64	4365	4375	4385	4395	4406	4416	4426	4436	4446	4457	1	2	3	4	5	6	7	8	9
·65	4467	4477	4487	4498	4508	4519	4529	4539	4550	4560	1	2	3	4	5	6	7	8	9
·66	4571	4581	4592	4603	4613	4624	4634	4645	4656	4667	1	2	3	4	5	6	7	9	10
·67	4677	4688	4699	4710	4721	4732	4742	4753	4764	4775	1	2	3	4	5	7	8	9	10
·68	4786	4797	4808	4819	4831	4842	4853	4864	4875	4887	1	2	3	4	6	7	8	9	10
·69	4898	4909	4920	4932	4943	4955	4966	4977	4989	5000	1	2	3	5	6	7	8	9	10
·70	5012	5023	5035	5047	5058	5070	5082	5093	5105	5117	1	2	4	5	6	7	8	9	11
·71	5129	5140	5152	5164	5176	5188	5200	5212	5224	5236	1	2	4	5	6	7	8	10	11
·72	5248	5260	5272	5284	5297	5309	5321	5333	5346	5358	1	2	4	5	6	7	9	10	11
·73	5370	5383	5395	5408	5420	5433	5445	5458	5470	5483	1	3	4	5	6	8	9	10	11
·74	5495	5508	5521	5534	5546	5559	5572	5585	5598	5610	1	3	4	5	6	8	9	10	12
·75	5623	5636	5649	5662	5675	5689	5702	5715	5728	5741	1	3	4	5	7	8	9	10	12
·76	5754	5768	5781	5794	5808	5821	5834	5848	5861	5875	1	3	4	5	7	8	9	11	12
·77	5888	5902	5916	5929	5943	5957	5970	5984	5998	6012	1	3	4	5	7	8	10	11	12
·78	6026	6039	6053	6067	6081	6095	6109	6124	6138	6152	1	3	4	6	7	8	10	11	13
·79	6166	6180	6194	6209	6223	6237	6252	6266	6281	6295	1	3	4	6	7	9	10	11	13
·80	6310	6324	6339	6353	6368	6383	6397	6412	6427	6442	1	3	4	6	7	9	10	12	13
·81	6457	6471	6486	6501	6516	6531	6546	6561	6577	6592	2	3	5	6	8	9	11	12	14
·82	6607	6622	6637	6653	6668	6683	6699	6714	6730	6745	2	3	5	6	8	9	11	12	14
·83	6761	6776	6792	6808	6823	6839	6855	6871	6887	6902	2	3	5	6	8	9	11	13	14
·84	6918	6934	6950	6966	6982	6998	7015	7031	7047	7063	2	3	5	6	8	10	11	13	15
·85	7079	7096	7112	7129	7145	7161	7178	7194	7211	7228	2	3	5	7	8	10	12	13	15
·86	7244	7261	7278	7295	7311	7328	7345	7362	7379	7396	2	3	5	7	8	10	12	13	15
·87	7413	7430	7447	7464	7482	7499	7516	7534	7551	7568	2	3	5	7	9	10	12	14	16
·88	7586	7603	7621	7638	7656	7674	7691	7709	7727	7745	2	4	5	7	9	11	12	14	16
·89	7762	7780	7798	7816	7834	7852	7870	7889	7907	7925	2	4	5	7	9	11	13	14	16
·90	7943	7962	7980	7998	8017	8035	8054	8072	8091	8110	2	4	6	7	9	11	13	15	17
·91	8128	8147	8166	8185	8204	8222	8241	8260	8279	8299	2	4	6	8	9	11	13	15	17
·92	8318	8337	8356	8375	8395	8414	8433	8453	8472	8492	2	4	6	8	10	12	14	15	17
·93	8511	8531	8551	8570	8590	8610	8630	8650	8670	8690	2	4	6	8	10	12	14	16	18
·94	8710	8730	8750	8770	8790	8810	8831	8851	8872	8892	2	4	6	8	10	12	14	16	18
·95	8913	8933	8954	8974	8995	9016	9036	9057	9078	9099	2	4	6	8	10	12	15	17	19
·96	9120	9141	9162	9183	9204	9226	9247	9268	9290	9311	2	4	6	8	11	13	15	17	19
·97	9333	9354	9376	9397	9419	9441	9462	9484	9506	9528	2	4	7	9	11	13	15	17	20
·98	9550	9572	9594	9616	9638	9661	9683	9705	9727	9750	2	4	7	9	11	13	16	18	20
·99	9772	9795	9817	9840	9863	9886	9908	9931	9954	9977	2	5	7	9	11	14	16	18	20

TRIGONOMETRICAL TABLE.

Angle.	Radians	Sine.	Tangent.	Cotangent.	Cosine		
0°	0	0	0	∞	1	1·5708	90°
1	·0175	·0175	·0175	57 2900	·9998	1 5533	89
2	0349	·0349	·0349	28·6363	·9994	1 5359	88
3	·0524	·0523	·0524	19·0811	·9986	1·5184	87
4	0698	·0698	·0699	14 3006	·9976	1 5010	86
5	0873	·0872	·0875	11 4301	·9962	1 4835	85
6	1047	·1045	·1051	9 5144	·9945	1 4661	84
7	1222	·1219	·1228	8 1443	·9925	1 4486	83
8	·1396	·1392	·1405	7 1154	·9903	1 4312	82
9	·1571	·1564	·1584	6 3138	·9877	1·4137	81
10	·1745	·1736	·1763	5 6713	·9848	1 3963	80
11	·1920	·1908	·1944	5 1446	·9816	1 3788	79
12	·2094	·2079	·2126	4 7046	·9781	1 3614	78
13	·2269	·2250	·2309	4 3315	·9744	1 3439	77
14	·2443	·2419	·2493	4 0108	·9703	1 3265	76
15	·2618	·2588	·2679	3·7321	·9659	1 3090	75
16	·2793	·2756	·2867	3 4874	·9613	1 2915	74
17	·2967	·2924	·3057	3 2709	·9563	1·2741	73
18	·3142	·3090	·3249	3 0777	·9511	1 2566	72
19	·3316	·3256	·3443	2 9042	·9455	1 2392	71
20	·3491	·3420	·3640	2 7475	·9397	1·2217	70
21	·3665	·3584	·3839	2 6051	·9336	1 2043	69
22	·3840	·3746	·4040	2 4751	·9272	1 1868	68
23	·4014	·3907	·4245	2 3559	·9205	1·1694	67
24	·4189	·4067	·4452	2 2460	·9135	1 1519	66
25	·4363	·4226	·4663	2 1445	·9063	1 1345	65
26	·4538	·4384	·4877	2 0503	·8988	1 1170	64
27	·4712	·4540	·5095	1 9626	·8910	1 0996	63
28	·4887	·4695	·5317	1 8807	·8830	1 0821	62
29	·5061	·4848	·5543	1·8040	·8746	1 0647	61
30	·5236	·5000	·5774	1·7321	·8660	1 0472	60
31	·5411	·5150	·6009	1·6643	·8572	1 0297	59
32	·5585	·5299	·6249	1 6003	·8480	1 0123	58
33	·5760	·5446	·6494	1 5399	·8387	9948	57
34	·5934	·5592	·6745	1·4826	·8290	·9774	56
35	·6109	5736	·7002	1·4281	·8192	·0599	55
36	·6283	·5878	·7265	1·3764	·8090	9425	54
37	·6458	·6018	·7536	1·3270	·7986	9250	53
38	·6632	·6157	·7813	1·2799	·7880	·9076	52
39	·6807	·6293	·8098	1 2349	·7771	8901	51
40	·6981	·6428	·8391	1·1918	·7660	·8727	50
41	·7156	·6561	8693	1·1504	·7547	·8552	49
42	·7330	·6691	·9004	1·1106	·7431	8378	48
43	·7505	·6820	·9325	1 0724	·7314	·8203	47
44	·7679	·6947	·9657	1 0355	·7193	·8029	46
45	·7854	·7071	1 0000	1·0000	·7071	·7854	45
		Cosine.	Cotangent.	Tangent.	Sine	Radians.	Angle.

ANSWERS

PART I. DYNAMICS

CHAPTER I. p. 11.

1. Miles × 1·609 = kilometres ; 5·129 kilometres.
2. 9 ft. 7·75 in. **3.** 7·298 sq. in.
4. 154 sq. cm. ; 1·54 grams wt. **5.** 381·9 cub. in. ; 99·45 pounds.
6. 19,500 lb. wt. **7.** 4,400 sq. cm. **8.** 1,200 sq. ft.
9. 17·49 lb. wt. **10.** 3·055 lb. wt. **11.** 8·710 inches.
12. 1·125 : 0·96 : 1. **13.** 2·347.

CHAPTER II. p. 24.

1. Length of forward reading vernier 1·2 inches ; vernier has 25 divisions.
Length of backward reading vernier 1·3 inches ; vernier has 25 divisions
2. Length of vernier, 59 circle divisions ; vernier has 30 divisions.
3. 20 divisions on thimble scale. **4.** 250 divisions.
5. 5·013 cm. **6.** 81·05 cm. **7.** 0·0660 mm.
8. 0·288 lb. wt. per cub. inch. **11.** 2·6356 mm.
12. 24·467 mm. ; 7,668 cub. mm.

CHAPTER III. p. 37.

3. 49·94 cm. at 19° 53′ east of north.
4. 3·101 inches at 64° 8′ to OX. **5.** 12·91 ft./sec.
6. 0·729 mile. **7.** $\frac{1}{3}$ mile. **8.** 26·67 miles/hour.
9. 32·73 miles/hour. **10.** 0·9778 feet/sec.^{2}
11. −0·02778 metre/sec.^{2} **12.** −106·7 feet/sec.^{2}
13. 0·447 metres/sec.^{2} **14.** 18·75 miles/hour ; 0·0651 mile.
15. 0·2291 mile. **16.** 367·1 seconds ; 415·1 seconds.
17. −1·613 feet/sec.^{2} **18.** 31·32 metres/sec. ; 3·19 seconds.
19. 98·28 feet/sec. ; 6·104 seconds. **20.** 100·6 feet.
21. 101·2 feet. **22.** 116·6 feet/sec. ; 204·9 feet.

23. 400 feet ; 181·3 feet/sec. (taking $g = 32$ feet/sec ^{2}).

24. 1 second ; 48 feet above the ground.

25.

No of body - - -	1	2	3	4	5	6	7	8
Distance from top, in feet -	196	144	100	64	36	16	4	0

Relative velocity = 16 feet/sec., downwards.

26. Acceleration is not uniform.

27. 139 feet ; 94 32 feet/sec. ; 4 947 seconds **29** $\frac{1}{3}$ mile

30.

Accel., ft /sec ^{2} -	80	70	60	35	0	−10	−35	−60	−70	−70
Displacement, ft	1 6	4 6	7 2	9 1	9 8	9 6	8 7	6 8	4 2	1 4

Total displacement = 63 feet

CHAPTER IV. p. 50.

1. 17·32 cm /sec at 30° ; 10 cm /sec at 60° ; zero ; 14·14 cm /sec at 315°.

2. 5 563 feet/sec ; 2 248 feet/sec ; 53 93 seconds.

3. 1,612 feet/sec at 7° 8′ to the horizontal.

4. 18·05 feet/sec. at 57° 48′ east of north.

5. 24 18 feet/sec. at 6° 58′ to the vertical

6. (*a*) 65° 33′ to the vertical ; (*b*) 62° 51′ and (*c*) 67° 48′ to the vertical.

7. 4·106 miles/hour at 76° 56′ with the direction of the rails.

8. 36° 52′ with the edge of the platform ; 5·999 feet/sec.

9. 22 36 feet/sec from a point 10° 18′ east of south

10. 25 miles/hour at 36° 52′ east of north.

11. 8·544 miles/hour at 20° 33′ east of north ; 20° 33′ west of south.

12. 77 16 miles/hour ; 55° 46′ west of north.

13. 60 miles/hour at 120° to Ox ; 1·732 miles.

14. 40° 33′ east of north ; 35° 38′ east of south.

16. (*a*) Relative velocity, 16 22 knots ; direction of motion 17° 6′ east of north ; (*b*) Relative velocity, 4·988 knots ; direction of motion 72° 54′ east of north

17. 10·62 metres/sec at 86° 58′ west of north ; 0 531 metres/sec ^{2}

18. 7·727 feet/sec. in a direction bisecting the angle included between the straight portions of the pipe.

19. 4·77 feet/sec at 26° 59′ to the original direction of motion.

20. 720 cm./sec.^{2} **21.** 6·453 feet/sec.^{2}

22. 64·49 feet/sec. at 82° 53′ to the horizontal.

23.

Time, sec. -	0	0·1	0·2	0·3	0·4	0·5
x feet - -	0	0·8	1·6	2·4	3·2	4·0
y feet - -	0	0·16	0·64	1·44	2·56	4·0

24. 1,500 feet; 1° 54′.

25.

Angle.	30°	40°	45°	50°	60°
Horiz range, ft - -	130,982	148,951	151,250	148,951	130,982
Time, seconds - -	68·75	88 38	97 23	105·3	119·1
Greatest height, ft.- -	18,906	31,248	37,812	44,269	56,711

26. 26° 48′. **27.** Angle of elevation, 4° 22′, or 27° 10′.

CHAPTER V. p. 64.

1. 9·425 radians/sec. **2.** 0·1047 radian/sec.
3. 286 4 revs./min. **4.** 9·546 revs./min.
5. 12 radians/sec. **6.** 0·873 radians/sec.^{2}
7. -4 radians/sec.^{2} **8.** -1 radian/sec.^{2}
9. 78·57 seconds; 98·21 revs. **10.** 750 radians.
11. 0·0633 radian/sec.^{2} **12.** 15·09 radians/sec.; 720 revs.
13. 32 inches. **14.** 60 revs./min. **15.** 71·55; 281·7.
16. 40·74 revs./min. **17.** 1 : 8. **18.** 24 teeth. **19.** $\frac{v \times ON}{OP^2}$.

CHAPTER VI. p. 73.

1. 675 poundals. **2.** 3·816 cm./sec.^{2}
3. Dynes $\times$ 0·000072328 = poundals; poundals $\times$ 13,826 = dynes.
4. 34 06 poundals. **5.** 4·141 tons wt.
6. (*a*) 1000 lb. wt.; (*b*) 937·9 lb. wt.; (*c*) 1062·1 lb. wt.
7. 2·236 tons wt.
8. 160 lb. wt.; 80 lb. wt.; he would have a downward acceleration of 0·5 g.
9. 2·927 feet/sec.; 29 27 poundals; that the table offers a resistance to sliding equal to 1 lb. wt.; 1 lb. wt.
10. 280·3 cm./sec.^{2}; 630,600 dynes.
11. Close agreement; actual a : theoretical $a = 2{\cdot}785 : 2{\cdot}8$.
12. 4·5 miles; 28·7 lb. wt. **13.** 264 ton-feet/sec.; 0·4099 ton wt.
14. 1,800,000 pound-feet/sec.; 5,590,000 lb. wt.
16. 0·976 feet/sec.; arithmetical sum of momenta = 2·013 pound-feet/sec.
17. 23·06 pound-feet/sec.; at 116° 34′ to initial direction of motion.
18. 30 ounce-feet/sec.; 5 feet/sec.; 1·035 feet.

19. 40° 54′; 33·58 pound-feet/sec

20. (*a*) 2 236 seconds; (*b*) 3·354 seconds (taking $g=32$ feet/sec.^{2}).

22. 40 feet/sec.; 30°

CHAPTER VII. p. 90.

1. 10·58 lb. wt at 19° 8′ to the 8 lb force.

2. 6 928 lb wt at 30° to the 8 lb. force.

3. 1·951 lb. wt at 41° 13′ to the resultant.

4. 27° 40′ between 7 lb. and 10 lb.; 40° 32′ between 5 lb. and 10 lb.

5. 5·292 lb. wt.; 48° 36′. **6.** 3·085 lb. wt; 40° 30′.

7.

Angle, degrees - -	165	170	174	178	180
Equilibrant, lb. wt -	2 610	1·744	1·046	0 3500	0

8. (*a*) P = 1·2856 lb wt; R = 1 532 lb wt; (*b*) P = 1 6782 lb wt; R = 2·6108 lb wt.; (*c*) P = 1·368 lb. wt.; R = 1·064 lb wt.; (*d*) P = 1·485 lb. wt; R = 2·274 lb. wt.

9. 13·61 *m* poundals; 13·61 feet/sec.^{2}; 1·084 seconds.

10. 5·571 lb. wt; 3·571 lb wt

11. 2·996 lb. wt at 26° 27′ to the vertical. **12.** 102° 38′; 120°.

13. Coordinates of the weight are 3·2 and −2·4 feet; 41·67 lb. wt.

14. 89·94 lb. wt.; 113·7 lb. wt at 34° 36′ to AC.

15. 0·16 ton wt. in AB; 0 89 ton wt. in AC.

16. 5·28 tons wt.; 2·36 tons wt. **17.** 7 78 tons wt; 4·86 tons wt.

18. 19 24 tons wt.; 7·76 tons wt. **19.** 1 lb. wt. in direction from O to D.

20. P = 28·28, S = 45 95, V = 17 67, all in tons wt.

21. 1·732 tons wt.; 11 928 tons wt

22. $P = W \sin\theta$; $P\cos\alpha + Q\cos\beta + R\cos\gamma = W\sin\theta$;
$P\sin\alpha + Q\sin\beta + R\sin\gamma = 0$.

24.

Distance of knot from A, feet	1	2	3	4	5	6	7	8	9	10	11
Tension, lb. wt. -	4	4	3·85	3·65	3 35	3 05	2·5	1·85	0·67	0	0

25. 16,000 lb. wt; 8000 lb. wt.

CHAPTER VIII. p. 103.

1.

Angle, degrees - - -	0	30	60	90	120	150	180
Turning moment, lb.-inches	0	70	121·2	140	121·2	70	0

2. Two positions differing by 180°; OA makes 26° 34′ with the vertical through O.

3. 14 lb. wt., falling between the given forces at 5·143 inches from the 8 lb. wt.
4. 2 lb. wt., falling outside the given forces; of same sense as, and distant 36 inches from the 8 lb. wt.
5. At 0·667 foot from the pivot, on the side opposite to the 12 lb. wt.
6. 10·95 inches from A.
7. 1·667 tons wt.; 0·833 ton wt.
8. 1,425 lb. wt.; 3,150 lb. wt.
9. 20 kilograms at 59·5 cm. from A.
10. 2·309 lb. wt. at 30° to the vertical; 1·527 lb. wt. at 49° 6′ to the vertical.
11. 2·506 lb wt., vertical; 1·856 lb. wt. at 47° 29′ to the vertical.
12. 23·53 lb. wt.; 423·53 lb. wt.
13. Reaction at A = 220·6 lb. wt. at 24° 56′ to the vertical; reaction at B = 93 lb. wt., horizontal.
14. 96·22 lb. wt.; 178·2 lb. wt. at 32° 41′ to the vertical.
15. Reaction at A = 2·267 tons wt.; reaction at B = 25·73 tons wt.
16.

Distance from left-hand support, feet	2	4	6	8	10
Reaction of left-hand support, lb. wt.	125	100	75	50	25
Reaction of right-hand support, lb. wt.	25	50	75	100	125

17.

Distance of A, feet -	12	10	8	6	4	2	0
Reaction, lb. wt. -	0	25	50	100	150	200	250

18. 742 lb. wt., at 5·622 feet from bow.
19. Both spring balances are attached at points on the rod between the loads; one is at 3 inches from the 3 lb. wt., and the other is at 7 inches from the 2 lb. wt.

CHAPTER IX. p. 121.

1. 4750 lb. wt.; 1750 lb. wt.
2. 400 lb. wt.
3. 19·2 feet from A.
4. 16·67 lb. wt.
5. $\bar{x} = 3·083$ inches; $\bar{y} = 4·583$ inches.
6. G is 2·37 inches from A and 2·65 inches from B; 0·81 lb. wt.
7. In the median, 4·33 inches from the 18 inches side.
8. Coordinates of G from centre of plate, 0·72 and 0·37 inches.
9. $\bar{x} = 3·32$ inches; $\bar{y} = 4·51$ inches; $\bar{z} = 6·49$ inches.
10. 133·3 and 266·7 lb. wt.; 400 lb. wt.; 0 and 800 lb. wt.
11. 600 lb. wt.
12. 26° 34′
13. Taking the origin at D, $\bar{x} = 5·59$ feet; $\bar{y} = 6·54$ feet; P = 654,000 lb. wt.
15. 5·825 inches.
16. AB makes 65° with the vertical.
17. 1·72 inches from AB.
18. 0·55949 lb. wt.; 0·00001 lb. wt. too much.
19. 237·1 grams wt.
21. 51 degrees.
22. In the radius bisecting the quadrant, at $\frac{4\sqrt{2}}{3} \cdot \frac{r}{\pi}$ from the centre of the circle.

23. The zero mark on FDE is $\left(\frac{W_0 c - W_1 d}{w}\right)$ from F; the length of graduation corresponding to unit load in the scale pan is c/w; $c = 0{\cdot}25$ inch; $W_1 = 0{\cdot}25(W_0 - 1)$.

24. $\frac{3}{8}s$ and $\frac{7}{16}s$, where s = the side of the square.

25. 22° 37′. **26.** 83·6 inches.

CHAPTER X. p. 136.

1. Two opposing couples; a force of 400 lb. wt along each long edge; a force of 133·3 lb wt. along each short edge.

2. Top hinge, upward pull of 75 lb wt. away from the door at 36° 52′ to the vertical; bottom hinge, upward push of 75 lb. wt. towards the door at 36° 52′ to the vertical

3. A vertical force of 5 tons wt. in the axis, and a couple of 40 ton inches

4. 112,000 lb wt acting vertically at the centre of the base, and a couple of 186,700 lb -feet

5. 20 lb. wt. at B, at 30° to AB produced.

6. The system reduces to a couple, having a moment represented by $2\triangle ABC$.

7. R = 2·828 lb wt., at 45° to the sides of the square, and acting at a point 2 feet from CD produced and 3 feet from AD produced.

8. 36 96 lb. wt. at A, at 23° 6′ to the vertical; 14 5 lb wt at B, horizontal.

9. 825·3 lb. wt. at B; 950 2 lb. wt. at A, at 62° 10′ to the horizontal.

10.

θ, degrees	45	30	15	5
P, lb. wt.	12·5	21·65	46 65	142·9
S, lb wt	2·5	2 5	2 5	2·5
Q, lb. wt.	17·68	25·00	48·30	143·4

11.

θ, degrees	(a)			(b)		
	P, lb. wt	S, lb wt	Q, lb. wt.	P, lb. wt.	S, lb.wt.	Q, lb wt.
45	10·00	0	14·14	8·75	−1·25	12·37
30	17 32	0	20 00	15·16	−1·25	17 50
15	37 32	0	38·64	32·66	−1·25	33·81
5	114 3	0	114·7	100·0	−1·25	100·4

12. 184·8 lb. wt. at 39° 40′ to the horizontal.

13.

x, feet	4	8	12	16	19
P, lb. wt.	35·80	53·12	70·44	87·76	100·75

The graph is a straight line.

14. 33·33 lb. wt.

15. 2·828 lb. wt., parallel to CA and passing through a point on CD produced at twice the side of the square from D.

16. 3749 feet from the end having the rope inclined at 60°.

17. Reaction $= \frac{1}{2}W$, horizontal. **19.** 22·66 lb. wt.

20. $lW/2a$; $W\sqrt{4a^2 - l^2}/2a$.

21. 5 lb. wt.; 5·176 lb. wt. compression; 4·226 lb. wt. tension; 2·887 lb. wt. tension.

22. $T = 14{\cdot}43$ lb. wt.; $R_x = 14{\cdot}43$ lb. wt.; $R_y = 7{\cdot}5$ lb. wt.

25.

Bar	AB	BC	CD	DE	EF	FA	CF
Force, lb. wt.	11·54	23·09	23·09	11·54	23·09	23·09	23·09
Nature of force	Pull	Pull	Pull	Pull	Pull	Pull	Push

CHAPTER XI. p. 150.

1. R acts downwards towards the right, at 40° to AD, and passes through a point 5·33 feet from A on AD produced; R = 8·65 lb. wt.

2. 1,500 lb. wt. at 4·47 feet from P.

3. $R_A = 5{\cdot}44$ tons wt; $R_B = 5{\cdot}56$ tons wt.

4.

Bar	AB	AC	BC	BE	CD	CE	DE
Force, lb wt.	460	460	0	680	310	540	78
Nature of force	Push	Push	—	Push	Push	Push	Pull

Reactions: $R_D = 300$ lb. wt. vertical; $R_E = 1{,}040$ lb. wt. at 35° to vertical.

5. 4·08 lb. wt. horizontal; 4·5 lb. wt., 5·68 lb. wt., 7·2 lb. wt., at angles to the horizontal of 26·5, 45, 56 degrees respectively.

6.

Bar	1, 2	2, 3	4, 5	1, 4	2, 4	2, 5	3, 5
Force, lb. wt.	500	700	600	707	141	141	990
Nature of force	Pull	Pull	Push	Push	Pull	Push	Push

Reactions: At 1, 500 lb. wt., at 3, 700 lb. wt.

7. 2·72 lb. wt. passing through a point whose coordinates with respect to A are (−1·45, 4·5) inches, and making 54° 58′ with AB.

8. AB, 10 cwt.; AD, 17·3 cwt.; both pulls. AC, 11·6 cwt.; BC, 23·2 cwt.; CD, 20 cwt.; all pushes. Reaction at A, 11·5 cwt.; reaction at B, 15 3 cwt.

9. Outside D, at 66° to horizontal; between A and B, at 51° to horizontal; outside A, at 63° to horizontal.

10.

Bar	BF	FA	AE	EC	BD	DC	FD	AD	DE
Force, tons wt.	2 38	1 7	1·7	3·05	1 6	2·05	0 67	1·52	1·35
Nature of force	Push	Push	Push	Push	Pull	Pull	Push	Pull	Push

Reactions : $R_B = 2\ 75$ tons wt ; $R_C = 2\ 25$ tons wt.

11.

Bar	DA	AF	CB	BE	CD	AB	EF	BD	AE
Force, in terms of W = 1	0 45	0 9	0 9	0 45	0·45	0·45	0 45	0 62	0 62
Nature of force	Pull	Pull	Pull	Pull	Pull	Pull	Pull	Push	Push

12. $\frac{1}{4}(W + W')$ cot ABC.

13.

Bar	1, 2	2, 4	4, 5	5, 7	2, 3	5, 6
Force, lb wt.	4610	4250	4700	5240	710	1080
Nature of force	Push	Push	Push	Push	Push	Push

Bar	1, 3	3, 6	6, 7	3, 4	4, 6
Force, lb. wt.	4180	2430	4730	1900	2450
Nature of force	Pull	Pull	Pull	Pull	Pull

Reactions : At 1, 1800 lb. wt. ; at 7, 2,600 lb. wt.

CHAPTER XII. p. 164.

1. 2·8 tons wt. per sq. inch. **2.** 12·5 tons wt.

3. 1·784 inches. **4.** 86·62 tons wt. **5.** 1·697 tons wt. per sq. inch.

6. 0·000806. **7.** 0·000417. **8.** 0·0556 inch.

9. 3,600 lb. wt. per sq. inch ; 0·000125 ; 28,800,000 lb. wt. per sq. inch.

10. 6·11 tons wt per sq. inch ; 0 000456 ; 0 046 inch.

11. 13,440,000 lb. wt. per sq. inch. **12.** 0·0485 inch.

13. 0·0794 cubic inch. **14.** 12,520,000 lb. wt. per sq. inch.

15. Bending moments : at middle, 15 ton-feet ; at each 1 ton load, 10 ton-feet. Shearing force 1 ton wt.

16.

Distance of section from wall, ft.	0	2	4	6	8
Bending moment, lb.-ft.	1,600	900	400	100	0
Shearing force, lb. wt.	400	300	200	100	0

17. 30,190,000 lb. wt. per sq. inch. **18.** 15,000 lb. ft. ; 18,750 lb. ft.

19. $R_A = 50$ lb. wt. ; $R_B = 40$ lb. wt.

Distance from A, ft. - -	1	3	5	7
Bending moment, lb.-ft. -	50	110	80	0
Shearing force, lb. wt. -	50	10	−40	−40

CHAPTER XIII. p. 180.

1. 4,032,000 foot-lb. **2.** 4,752,000 foot-lb. **3.** 697,000 foot-lb.

4. 24,550,000 foot-lb. **5.** 18,000,000 foot-lb. **6.** 49 7 foot-lb.

7. 265,800 foot-tons ; 100·7 tons wt. **8.** 1·559 tons wt.

9. 298·1 lb. wt. **10.** 0·64 H.P. **11.** 60·61 H.P ; 101 H.P.

12. 10·96 H.P. ; 48·1 amperes. **13.** $\mu = 0{\cdot}267$; $\phi = 14° 57'$. **14.** 3·28 feet.

15.

θ degrees -	0	15	30	45	60	75
P, lb. wt. -	0·25W	0·242W	0·253W	0 284W	0·350W	0·500W
Work, foot-lb.	0 25W	0·234W	0·219W	0·201W	0·175W	0·129W

16.

θ degrees -	0	15	30	45	60	75	90
P, lb wt. -	0·250W	0·500W	0·716W	0 884W	0·991W	1·031W	W
Work, ft -lb.	∞	1·932W	1·432W	1·250W	1·145W	1·067W	W

17.

θ degrees -	0	15	30	45	60	75	90
P, lb. wt. -	0·250W	0·559W	0·966W	1·667W	3·460W	59·4W	∞
Work, ft.-lb.	∞	2·086W	1·673W	1·667W	1 996W	15·92W	—

$P = \infty$ when $\theta = 76°$ nearly.

18. 18·21 feet/sec.^{2} **19.** 643 lb. wt. **20.** 14·5 H.P. **21.** 1,796 lb. wt.

22. (Energy wasted in impact = 50 ft.-cwt., see p. 235) ; pile is driven 10 inches.

24. 3,366 lb wt ; 62·4 miles/hour.

25. (*a*) Times are equal ; (*b*) $t_A/t_B = 0{\cdot}5$.

26. 1 in 55·8. **27.** ml^2/t^2 ; ml^2/t^3 ; 8·96.

28. (*a*) 17·2 feet/sec. ; (*b*) 12·15 feet/sec. ; 46·6 lb. wt. **29.** 36 inch-lb.

30. 5·367 feet/sec.^{2} ; 268·3 poundals. **32.** $w(\frac{1}{2}\tan\theta - \mu)$.

CHAPTER XIV. p. 195.

1. Velocity ratio = 3 ; mechanical advantage = 2·143 ; effect of friction = 60 lb. wt. ; efficiency = 71·43 per cent.

2. 5·625 lb. wt. ; 16 ; 20·83 lb. wt.

3. 1·067 lb. wt. ; 5·714 lb. wt. **4.** 20 ; 420 lb. wt. ; 14.

5. 32 ; 27·5 ; 90 lb. wt. ; 85·9 per cent.

6. 48 ; 936 lb wt ; 31 2 ; 504 lb wt.

7. 3,140 degrees ; 187,900 inch-lb. **8.** 18·75 lb. wt. ; 100 per cent.

9. Neglecting friction, 40 lb. wt ; taking account of friction, 74·3 lb. wt.

10. $P = \frac{1}{15}W + 7\frac{2}{3}$; $18\frac{2}{3}$ lb. wt. ; 29·5 per cent.

12. $P_n = \frac{W}{2^n} + \left(\frac{2^n - 1}{2^n}\right)w$, where w = the weight of each pulley, n = the number of pulleys, and friction is neglected ; 161 lb. wt. ; 160 lb. wt., assuming that there is no fixed pulley attached to the beam.

13. 377·1 ; 93·33 ; 24·75 per cent **14.** 12·37 per cent.

15. Work done = $W(H + \mu B)$; mechanical advantage = $L/2H$.

16. Mechanical advantage, neglecting friction = number of ropes passing from the upper to the lower block ; H.P. = 11·07.

CHAPTER XV. p. 213.

1. 260 3 lb.-feet. **2.** 25,143,000 dyne-cm.

3. 45·96 pound and foot units.

4. (*a*) 0·1778 ; (*b*) 4·8 ; (*c*) 1·2 ; all in pound and foot units.

5. (*a*) 0·364 ; (*b*) 0·182 ; (*c*) 0·546 ; all in pound and foot units.

6. (*a*) 0 5625 ; (*b*) 0·2812 ; (*c*) 1·406 ; (*d*) 1·687 ; (*e*) 0·8437 ; all in pound and foot units

7. (*a*) 2 ; (*b*) 4 5 ; (*c*) 0·5 ; (*d*) 1·125 ; (*e*) 1·625 ; (*f*) 6 5 ; all in pound and foot units.

8. 859·2 pound and foot units. **9.** 21,270 pound and foot units.

10. 23·57 ; 82·49 ; both in pound and foot units.

11. 6·78 ton-feet. **12.** 12 49 pound and foot units.

13. 7,071 pound, foot and sec. units ; 0·366 lb.-feet.

14. 2·514 pound, foot and sec. units ; 1042 revs./min.

15. 166,000 foot-lb. ; 1,660 foot-lb.

16. (*a*) 8·31 foot-lb. ; (*b*) 9·62 foot-lb ; (*c*) 17·93 foot-lb.

17. Kinetic energy of translation = 2·325 foot-lb. ; kinetic energy of rotation = 1·163 foot-lb.

18. 1·872 feet/sec.^{2} ; 7·488 radians/sec.^{2}

19. A reaches the bottom first. **20.** $\bar{x} = 6{\cdot}03$ inches ; $\bar{y} = 6{\cdot}05$ inches.

21. $2\pi^2 n^2 I$ absolute units ; 16·3 pound and foot units.

22. $0{\cdot}69\sqrt{g}$ radians/sec. **23.** 18·33 feet.

26. 122,500 pound and foot units ; 4,900 pounds.

CHAPTER XVI. p. 230.

1. 1,325 lb. wt. **2.** 3,270 lb. wt. **3.** 16·7 poundals.

4. 230 lb.-feet. **5.** 1·691 tons wt. ; 7·971 tons wt. ; 12·029 tons wt.

6. 49° 33′. **7.** 19° 39′ ; 64·2 lb. wt. ; 0·357.

8. 20·95 feet/sec. ; 438·8 feet/sec.^{2}

9. 0·2319 second ; 18·45 cm ; 13,550 cm /sec.^{2}

10. 0·2038 foot ; 8·64 seconds gain per day.

11. 7·211 inches ; 0·5882 radian.

12.

Revs./min. -	20	40	60	80	100	120
H feet - -	7·33	1·83	0·814	0·457	0·293	0·203

13. 0·904 second ; 96·6 poundals ; 64·4 revs./min.

14. 0·0513 foot. **15.** 7·196 pounds. **16.** 171 and 189 revs./min.

17. 5 37 inches **18.** 27·3 miles/hour. **19.** 98·29 feet/sec.

20. $v = \sqrt{ga}\tan\alpha$.

21. Velocity $= p\mathrm{A}\sin(pt - a)$; 1·95 feet/sec. ; 0·07 second nearly.

23. 0·328 second. **24.** 0 452 second. **25.** 2 019 revs./sec.

27. 0·15 foot/sec. ; 0·268 foot/sec.^{2} ; 3 503 seconds.

28. Pull in upper cord, 5·817 lb. wt. ; in lower cord, 4·404 lb. wt.

CHAPTER XVII. p. 241.

1. 7·143 feet/sec ; 22·86 foot-poundals.

2. − 1·429 feet/sec (same sense as B) ; 365·7 foot-poundals.

3. $v_A = 9{\cdot}086$ metres/sec. ; $v_B = 11{\cdot}89$ metres/sec. ; $69{\cdot}3 \times 10^6$ ergs.

4. $v_A = -2{\cdot}571$ metres/sec. ; $v_B = 11{\cdot}43$ metres/sec. ; $1{,}747 \times 10^6$ ergs.

5. (*a*) $v_A = 8{\cdot}571$ metres/sec. ; $v_B = 12{\cdot}57$ metres/sec. ;
(*b*) $v_A = -5{\cdot}143$ metres/sec. ; $v_B = 14{\cdot}86$ metres/sec.

6. − 144 feet/sec. ; − 78 feet/sec.

7. Heights in feet : 5·76, 3·69, 2·36, 1·50 ; energy wasted, 0·664 foot-lb.

8. 20·58 feet/sec., at 35° 47′ to the normal to the plane.

9.

Angle, degrees - -	0	30	45	60	90
Forces, lb. wt. - -	0	5·39	7·62	9·34	10·78

10. m/M ; 69,320 lb. wt. **13.** 1,155 feet/sec. **14.** 12 lb. wt. ; 20 foot-lb.

CHAPTER XVIII. p. 255.

3. 333·2 grams wt./sq. cm. **4.** 4,693 lb. wt./ sq. inch.

5. 156 and 260 lb wt./sq. foot

7. 2 544 feet ; 15 01 lb. wt /sq. inch. **8.** 34 feet.

10. 1,000 lb. wt. ; 500 lb. wt. ; 250 lb. wt.

11. 4,978 lb. wt. **12.** 4,563 lb. wt.

13. 15,000 lb -wt. ; 15,910 lb wt ; 9,000 lb. wt.

14. 1,000 lb wt. ; 37,500 lb. wt.

15. 6,154 grams wt. **16.** 1·113 lb wt. ; 1·855 lb. wt.

17. AB, 1,125 lb wt. ; BC, 1,299 lb. wt. ; end, 162·4 lb. wt. ; depth, 1·5 feet

18. 16,500 lb. wt. ; 2 06 feet below the top of the door.

19. 1,963 lb. wt. at a depth of 10 02 feet.

20. 450 lb wt. ; 495 lb wt. ; 45 lb. wt. at a depth of 2 feet.

21. 22,500 lb. wt ; 3 27 feet.

22. 52,500 lb. wt. ; 22,780 lb. wt. ; 57,220 lb. wt. at 23° 27′ to the vertical ; 12·23 feet from B.

23. 16,875 lb. wt. at 3·8 feet from the bottom.

24. ml/t^2 ; m/lt^2 ; ml^2/t^2 ; 14 51 lb wt./sq inch.

25. 20·83 lb. wt. **26.** 21,600 lb. wt. ; 33,250 lb. wt. **27.** 3,633 lb. wt.

CHAPTER XIX. p. 271.

1. 356 lb. wt. **2.** 56,340 lb. wt. ; 40,570 lb. wt. ; 27,045 lb. wt.

3. 56·25 ; 3,142 lb. wt. ; 176,700 lb. wt.

4. 2,765 foot-lb. **5.** 71·6 gallons per hour.

6. (*a*) 100,800 foot-lb. ; (*b*) 1,613 foot-lb. ; 806,500 foot-lb.

7. 294,300 ergs **8.** 926 lb. wt /sq. inch.

9. 26,950 lb. wt. ; 5,544 cubic inches ; 323,400 foot-lb.

10. 4,950 lb. wt. ; 59,400 foot-lb.

11. 4·725 horse-power ; 3·071 horse-power.

12. 164·6 lb. wt./sq. inch. **13.** 1,636 c.c. **14.** 9·67 inches.

15. 2,567 cubic feet. **16.** 1,153 lb. wt. **17.** 42·5 feet.

18. 4·036 inches ; 2,045 lb. wt.

CHAPTER XX. p. 283.

1. 11,160 tons wt. ; 390,600 cubic feet. **2.** 286·7 sq. feet.

3. 8,270 lb. wt. **4.** 7·454 lb. wt. **5.** 3·555 lb. wt.

7. 8·23. **8.** 0·0289 lb. wt. **9.** 7·68.

10. 1,166 cubic inches ; 28·12 lb. wt. **11.** 7·00.

12. 1·807 lb. wt. **13.** 173·7 tons wt. **14.** 8·59 ; 0·2167.

15. 8·76 ; 135·9 cm. **16.** 8·69. **17.** 1·2 cm.

18. 23·53 cm. **19.** 0·864 ; 0·8698. **22.** 27·2 cubic inches

CHAPTER XXI. p. 295.

4. 85·37 foot-lb.

6. (*a*) 40 ; 33·9 ; 0·559 ; (*b*) 6 ; 67·9 ; 0·559 ; all in foot-lb.

7. 9 33 lb. wt /sq. inch. **9.** 12·5 lb. wt./sq. inch.

11. 24 07 feet/sec. ; 23·3 feet/sec.

12. 0·0262 cubic feet/sec.

13. 15·57 feet/sec. ; 0·8 inch ; 0·0543 cubic feet/sec.

14. 45,000 lb. wt. ; 17·72 horse-power.

16. 146,400 foot-lb. ; 213 horse-power.

CHAPTER XXII. p. 308.

2. 73·7 dynes/cm. **3.** 5·958 cm. **5.** 2·35 mm.

6. 2·10 mm. **7.** 74·3 dynes/cm. **9.** 3·2 minutes.

PART II. HEAT

CHAPTER XXIII. p. 322.

4. (*a*) 284° F. ; (*b*) 21° F. ; (*c*) −459·4° F.

5. (*a*) 37·78° C. ; (*b*) −12·22° C. ; (*c*) −51·11° C.

6. −40° C. = −40° F. **9.** −0·52° F. **12.** −1·6° F.

17. Either the Centigrade thermometer should read 43·33°, or the Fahrenheit thermometer should read 113°.

CHAPTER XXIV. p. 331.

2. 1·963 inches. **3.** $10·2 \times 10^{-6}$. **4.** 0·01397 inch.

5. 188·9 cm. **6.** 12·0099 sq. feet.

8. True length, 2,000 feet ; recorded length, 2000·119 feet ; error, +0·119 foot.

9. 40·087 cm.

11. 42,057 lb. wt. (this is probably in excess of the elastic limit, see p. 155).

12. 41,980 lb. wt. (see note to Answer 11). **13.** 0·3305 cubic inch.

CHAPTER XXV. p. 341.

1. 8·403 grams per c.c. **2.** 0·00000936. **3.** 7·78 grams per c.c
4. 0·02118 c.c. **5.** 0 00000667. **7.** 76·062 cm
8. 2 624 c.c. **9.** 0 000006. **10.** 0·0000489
13. 389,600 cub. feet ; 39,600 cub. feet. **15.** 18·02 cm.

CHAPTER XXVI. p. 350.

1. 2,545 lb.-deg.-Fah. units ; 641,390 calories.
2. 432·2 lb -deg -Cent. units ; 196,060 calories.
3. 1·504 lb -deg -Cent units ; 0 0376 pound.
4. 1,752,000 lb.-deg -Cent. units
5. 41 4° C **6.** 0 0991. **7.** 390 7° C.
8. 0·4933. **9.** 0·6 nearly. **12.** 30·55° C
13. 936 : 837. **14.** 0 601.

CHAPTER XXVII. p 364.

1. 1,980,000 foot-lb. ; 1,414 lb.-deg.-Cent. ; 2,545 lb.-deg.-Fah. ; 641,400 calories.
3. 60 6 minutes. **4.** 748 4 lb -deg.-Cent **8.** 4·46 per cent
11. Heat per penny, in lb.-deg -Cent : Coal, 67,880 ; petrol, 3,285 ; lighting gas, 8,333.
12. 19,320 lb.-deg -Cent. **16.** 41 5×10^6 ergs
17. 6,728 lb.-deg.-Cent. per pound. **18.** 10,920 lb.-deg.-Cent. per pound.

CHAPTER XXVIII. p. 378.

7. $40{\cdot}32 \times 10^6$ calories per hour. **8.** 262·1 calories per hour.
9. 104·92° C. **13.** 411,500 calories per hour.
16. Temperature of faces in contact, 72° C. ; 6·667 ; 15·24.

CHAPTER XXIX. p. 388.

15. 0·7264.

CHAPTER XXX. p. 399.

2. 1,011 grams wt./sq. cm. **3.** 1,764 lb. wt./sq. inch.
6. 76·025 cm. ; 1·00035. **7.** 29·39 inches of mercury.
10. 75, 50, 37·5, 30, 25, all in lb wt./sq. inch.
13. 24·87 metres. **15.** 23·4 c.c. **16.** 58·2 cm.

CHAPTER XXXI. p. 416.

1. 266·3° absolute (Cent.). **2.** 441·8 cubic feet. **3.** 530·6° C.
6. 96·08 ; 1,387. **7.** $2{\cdot}87 \times 10^6$. **8.** 947·8° C.
9. 47·12 litres **11.** 122·3° C. **12.** $41\ 45 \times 10^6$.
13. 3·512 grams **14.** 578·4 lb. wt. **16.** 0·00128 grams/c.c.
17. 6·93 tons wt. **18.** 308·6 cm. of mercury. **19.** 28·1° C.

CHAPTER XXXII. p. 426.

7. (*a*) 2,117 foot-lb ; (*b*) 26,460 foot-lb.
8. 10·75 sq cm. ; 5,376 cm.-kilograms.
9. 33,210 lb.-deg.-Cent **10.** 9,617 lb.-deg.-Cent.
11. $1{\cdot}69 \times 10^6$ calories **12.** $41\ 6 \times 10^6$ ergs.
13. 2·418 ; 1 407.

CHAPTER XXXIII. p. 440.

4. (*a*) 22 5 ; (*b*) 12·91 ; (*c*) 17·05 ; all in lb. wt./sq. in.
5. −93·5° C. **7.** 500 cubic inches. **8.** 100 cubic feet.
9. 55 cm of mercury **10.** 360 cubic inches.
14. 1·547 lb. wt./sq. inch ; 8. **15.** 0·00203 mm. **16.** 0·65 inch.

CHAPTER XXXIV. p. 456.

6. 3° C. nearly. **7.** 219,500 lb.-deg.-Cent.
10. 307×10^6 foot-lb. ; 155 H.P. **11.** Yes ; 0·39° C.
12. 2,566 calories. **15.** 81·13 calories.
18. 25·1 lb.-deg.-Cent ; 30·3 cubic feet.
20. 35·45 cm. of mercury. **24.** (*a*) 0 083 ; (*b*) 0·1098.
25. 0·515. **26.** 0 0753.

CHAPTER XXXV. p. 470.

1. 54 85 cm. of mercury.
2. Pressure of the aqueous vapour is 11 8 lb. wt./sq. inch.
3. 226 c.c. **4.** 32·5. **6.** 22·5
9. 646 3 lb.-deg.-Cent. **10.** 17·9 lb. wt.
11. 45·5 c.c. **13.** 29·9 grams. **17.** 65·5 cm.
18. 68·1° C. **19.** 0·209.

CHAPTER XXXVI. p. 480.

2. 7·81 ; 19·2 minutes. **4.** 10·05° C. ; 0·597.
5. 0·603 **7.** 48 2 per cent. ; 6·8° C.
9. 9·4° C. **11.** 8·923 grams ; 0·5.

CHAPTER XXXVII. p. 492.

1. 45·15 calories; 453·8 calories. **14.** 8·93 lb.-deg.-Cent.
15. 0 00776 grams/c.c.; 7·76/872.

CHAPTER XXXVIII. p. 500.

1. 20·28 per cent. **6.** 22·07 per cent.; 15,450,000 foot-lb.

CHAPTER XXXIX. p. 518.

3. 73·3 per cent. **4.** 10·23 per cent.
7. 15·9 per cent; 21·7 per cent **12.** 473 H.P.
13. 4 8 B H.P.; 5·71 I.H P; 0 91 H.P.
16. 2·78° C. per cm.; 0·268.

CHAPTER XL. p. 531.

3. 6·64. **4.** 5 39. **5.** 21 3 per cent.
11. 14·82 per cent. **12.** 169·5. **13.** 133 tons.

PART III. LIGHT

CHAPTER XLI. p. 542.

4. Annular. **5.** 2·6 inches. **6.** 3·33 cm.; 10·67 cm. **8.** 5·6 cm.

CHAPTER XLII. p. 550.

4. Electricity costs twice as much as gas. **5.** 1920 candle-power.
6. 58 6 cm from the 32 c.p. lamp.
7. At 1·414 feet on other side of screen.
8. 81 candle-power. **9.** 2 feet. **10.** 11·42 per cent.
11. 164 candle-power.

CHAPTER XLIII. p. 560.

4. 2 ft. 10 in. **6.** 60°. **8.** 55°.
12. $\Delta = n\pi - 2(i_1 + i_2 + i_3 + \text{etc.})$.
13. Two mirrors including an angle of 70°. **14.** 270°.

CHAPTER XLIV. p. 571.

2. 79·4 cm. **5.** +51·4 cm.; 4·28 cm., inverted.
7. (*a*) +100 cm.; 10 cm. diam., inverted; (*b*) −60 cm.; 10 cm. diam., upright.
8. 2·5 cm, inverted; +22·5 cm.

9. Distance of rod, +45 cm.; distance of image, +22·5 cm., inverted.

10. (*a*) +27 cm.; (*b*) +13·5 cm.

11. +5·33 inches; 0·667 inch. **13.** +11·11 cm.

14. Final image 6 cm. behind convex mirror, 6 cm. long, and inverted.

15. Image formed by one reflection 14 cm., and by two reflections 38 cm behind plane mirror: $r = 70$ cm.

16. 515·7 cm. **17.** −46·67 cm.

CHAPTER XLV. p. 585.

3. $r = 28° 8'$. **4.** 4·5 feet nearly. **8.** 1·467.

9. $r_1 = 13° 11'$; $r_2 = 36° 49'$; $i_2 = 64°$. **11.** 38° 41'. **13.** 1·667 cm.

CHAPTER XLVI. p. 600.

1. 2·8 inches, inverted, at −11·9 inches from lens.

2. +24 cm. from lens. **4.** 23·7 cm., or 126·2 cm. from source.

5. On axis, on side of lens opposite to object, pointing away from lens; 3·33 cm. long; head −20 cm. from lens

6. Distance of object from nearer lens must be greater than $2f$.

7. 4 cm.; 4 cm., inverted.

8. $r = -6·6$ inches; −3 or −1·5 inches. **9.** −20 cm.

0. $v = -28$ cm. beyond second lens; image real, inverted, and 3·2 cm. high.

1. −60 cm. **12.** Infinity; infinity.

3. Image inverted and virtual, twice size of object; +6 inches from the 6 inch lens.

4. At 5·68 cm. from surface of sphere, on radius through object.

5. Image virtual; object +5 cm. from lens.

6. Virtual image, erect; object at +2·667 inches from lens; real image, inverted; object at +5·33 inches from lens.

7. −10 cm.; −6·667 cm.

CHAPTER XLVII. p. 620.

2. +8·18 inches focal length. **5.** 2.

6. −22·15 inches focal length. **8.** +100 cm. focal length; 8·7 cm.

9. +1·067 cm; 89. **10.** −9·6 inches.

1. +3·75 cm.; 4. **13.** 6. **14.** Lenses 20 cm. apart; 4.

5. (i) 15; 32 inches; (ii) 17·5; 31·71 inches.

6. −11·6 inches focal length. **17.** −33·33 cm. focal length.

CHAPTER XLVIII. p. 634.

7. 2·1°. **8.** $D = (\mu - 1)\alpha$. **13.** +160·7 cm.; −140·5 cm.

1. ∞; 111·4 cm.; 111·4 cm.; 64·37 cm.

CHAPTER L. p. 654.

6. 65·1°.

CHAPTER LI. p. 660.

2. 201600 miles/sec **6.** 8 333 revs /sec **7.** 9 26 revs /sec.

PART IV. SOUND

CHAPTER LII. p. 668

4. 365·3

CHAPTER LIII. p. 677.

6. 880 **8.** $599\frac{1}{15}$ before loading $599\frac{9}{10}$ after loading.

9. $2{\cdot}75 \times 10^5$ ergs

CHAPTER LIV. p 687.

2. 440. **4.** 238 1; 142 8 cm **7.** 18330; 36·7; 8 97.

CHAPTER LV. p. 704.

2. 4495 ft and 3 75 feet per sec. **4.** (*a*) No; (*b*) yes.

5. 2240 ft **6.** $n_1 : n_2 = (V+v) : (V-v)$. **8.** 4 98.

9. 517 ft. **10.** 50 miles per hr

CHAPTER LVI. p. 717.

7. (1) 2; (2) 4 **10.** 1037

11. 256, 1st diff.; 1280 1st sum; 1536 and 1024 self-comb.

13. 260. **14.** 517.

CHAPTER LVIII. p. 741.

4. 23·26. **5.** 2 : 3. **6.** $1 \cdot 2\sqrt{3}$.

7. Tension, 25 : 4; length, 2 : 5. **8.** 3 52 per sec **9.** 1 308 gram.

10. 42 2. **13.** 1 : 9. **14.** 5 kilos. wt **16.** 49·5

CHAPTER LIX. p. 756

1. 64·4 cm. and 329 8 metres per sec.

2. 2 : 1. **3.** 95·3° C. **5.** 19·4° C. open pipe.

6. 186·7, 373·3, 560, 746·7 open pipe; 93·3, 280, 466·7, 653·3 closed pipe.

7. 1·009. **10.** Open. **11.** 1105·5 ft. per sec.

13. 110·5. **16.** Wide pipe to be shortened by 6 cm. **18.** 5 : 8.

PART V. MAGNETISM AND ELECTRICITY

CHAPTER LXI. p. 775.

5. 13·5 dynes. **6.** 8/9 dyne, parallel to needle. **7.** ±18·91 dynes.
8. 56·4 units. **10.** 1·633 cm. ; 1·067 dynes.

CHAPTER LXII. p. 791.

4. 0·453 C.G.S. units. **5.** 5·68 sec. **7.** 1/8 C.G.S. unit.
8. 25 : 7. **9.** 450 C.G.S. units.
10. 9 : 16 (same direction) ; 41 : 16 (opposite direction). **11.** 0·278 dyne.
13. (*a*) $80\frac{1}{2}°$; (*b*) 36° ; (*c*) −32° with axis of magnet.
14. 12500 C.G.S. units. **15.** 1·236 C.G.S. unit.

CHAPTER LXIII. p. 808.

7. tan (dip) = 2 cot (magnetic latitude).
8. (*a*) 0·766 ; (*b*) 0 541. **9.** 0·208 C.G.S. unit.
10. tan (true dip) = sin A tan (observed dip). **11.** 18·4.

CHAPTER LXIV. p. 828.

2. Magnetic moment = $1·508 \times 10^6$ C.G.S. units ; time changes in ratio 1 : 0·98.
4. 53·39 C.G.S. units ; 1005 4 C.G.S. units. **6.** 3307 C.G.S. units.
7. 0·926. **8.** $5·03 \times 10^{-4}$ C.G.S. unit.
9. M = 2000 ; $m = 100$; I = 100 C.G.S. units.
10. M = 1470 C.G.S. units ; 0·00267 C.G.S. unit.
11. 5260 C.G.S. units. **12.** M = 2500 C.G.S. units ; $m = 250$ C.G.S. units.
13. $1·766 \times 10^6$ dynes.

CHAPTER LXV. p. 838.

3. 104·7 C.G.S. units. **4.** 0·0487 absolute C.G.S. unit.
7. $n_A : n_B = 10 : 6$. **8.** 0·01504 to 0·1489 amperes.
9. Increase number of turns to 2292. **10.** 0·943 C.G.S. unit ; $0·943m$ dynes.
11. 0·0231 absolute C.G.S. unit. **12.** 0·288 amp. ; 0·866 amp.

CHAPTER LXVI. p. 846.

3. 57·5 volts ; 143·7 watts. **4.** $7·2 \times 10^{10}$ ergs.
5. 0·0603 H.P. ; 0·225 amp. **6.** 0·997° C. **7.** 14·4 ohms.
9. 0·015 volt ; 25 volts. **10.** 27·8 volts ; 55·6 ohms.
11. $7·488 \times 10^5$ joules ; 104 volts. **12.** 2*s.* 8·4*d.*
13. 18·4 kilowatts ; 2·47 H.P. **14.** 9·13 amp.
15. 3·58 amp. ; 3·58*d.*

CHAPTER LXVII. p. 857.

2. 2·73 ohms. **3.** 1 6364, 0 8182, 0·5454 amp.
4. 210 ohms. **5.** $4{\cdot}91 \times 10^{-6}$. **6.** 40·5 ohms.
7. 140 calories. **8.** 3 ohms ; 0·5 amp.
9. 0·727 amp ; 4·36 volts. **10.** 32 : 15.
11. $I = \frac{1}{3}$ amp ; resistance of battery = 6 ohms.
12. (*a*) 0 6 amp ; (*b*) 0 75 amp ; (*c*) 0 6 amp.
13. 0 976, 0·732 amp **14.** (*a*) 0·48 amp ; (*b*) 0 04 amp **15.** 280 ohms
16. Reduce external resistance to $6\frac{2}{3}$ or $3\frac{1}{4}$ ohms. **17.** 0 5 ohm ; 1·5 volt.
18. 0·0513 ohm ; 48 watts. **19.** 0 75 volt, 0·375, 0·25 amp.
20. 18 6 H.P. **22.** 60000 ohms.

CHAPTER LXVIII. p. 875.

1. 3 37 cm. **2.** 343·3 ohms. **3.** 0 0240, 0·00120 ohm.
4. 0·2001 ohm ; 2·7 amp. **5.** 3 03 ohms ; 3 ohms.
8. (*a*) 0 04 ohm in parallel with G ; (*b*) 961 ohms in series with G.
9. 24 6, 8·77 milliamp **10.** (*a*) 995 5 ohms ; (*b*) 0 004504 ohm.
11. 766 2 ohms.

CHAPTER LXIX. p. 896.

1. 0 5628 ohm. **2.** $E_A : E_B = 3 : 5$. **4.** 0·266 ohm.
5. 0·0000262. **6.** 3·733 : 1. **7.** 1 36 ohm.
8. G = 40 ohms. **9.** 1·15 volt.
10. 5 505 calories ; 1·966 calorie. **11.** 21·38 ohms.
12. $2{\cdot}72 \times 10^{-5}$. **13.** 1·185 ohm. **14.** 0·02 ohm. **15.** +0·4 cm.

CHAPTER LXX. p. 918.

2. 39·05. **6.** 19° 25′. **7.** 0·1795 C.G.S. unit.
9. 0·0261. **11.** 20·2° C. **12.** 0·1838 gram.
15. −0·04.

CHAPTER LXXI. p. 927.

2. 60 cm. from −18 and on side opposite to +50.
3. $q^2/60$ dynes, attraction ; $q^2/900$ dynes, repulsion.
4. 0·6 dyne, from B towards D.
5. (*a*) $4(r^2 - \frac{1}{4}l^2)^2/rl^3$; (*b*) $8(r^2 + \frac{1}{4}l^2)^{\frac{3}{2}}/l^3$. **8.** F/3 dynes, repulsion.

CHAPTER LXXII. p. 944.

2. Q = 5850 C.G.S. units ; 87750 ergs.
3. $Q_A = 556{\cdot}25$; $Q_B = 333{\cdot}75$. **4.** $\sigma_1 : \sigma_2 = 1 : \sqrt{2}$.
5. (*a*) $C_1 + C_2$; (*b*) $C_1C_2/(C_1 + C_2)$.

6. At first, charges $+e$ outside, $-e$ inside, potential $= +e/r_2$; (*a*) charges 0 outside, $-e$ inside, potential $=0$; (*b*) charges $-(r_2-r_1)e/r_2$ outside, $-r_1e/r_2$ inside, potential $=(r_1-r_2)e/r_2^2$, where r_1 and r_2 are the radii of the inner and outer spheres respectively.

8. $Q^2/2r$ units. **9.** (*a*) 0·0444 dyne; (*b*) +2 c.g.s. units.

10. 312 c.g.s. units; 0·040 erg.

11. 0·2 microcoulomb; 0·3 microfarad.

12. 14 units **13.** From A to B,

14. 5·139 ergs; 3·354 ergs. **15.** 134·4 ergs.

Chapter LXXIII. p. 956.

2. 90 c.g.s. units, 32400 ergs in air; 36 c.g.s. units, 12960 ergs in the other medium.

6. 218·4 c.g.s. units; 698880 ergs. **7.** 2088 c.g.s. units.

9. 150000 c.g.s. units; 0·0333 erg. **10.** 157·1 ergs.

11. From A to B.

Chapter LXXV. p. 975.

2. 7·55 gauss. **3.** 20·5 c.g.s. units. **4.** 1910.

6. 4240 c.g.s. units. **9.** 0·0197 c.g.s. unit.

10. 1·25 dyne. **12.** $7{\cdot}12 \times 10^4$ c.g.s. units.

Chapter LXXVI. p. 987.

6. $8{\cdot}48 \times 10^{-4}$ volt. **7.** 2·13 volts.

Chapter LXXVII. p. 1002.

13. H.P. $=0{\cdot}0268\ n$N. **14.** 0·969; 0·806. **15.** 0·906; 0·958.

Chapter LXXVIII. p. 1014.

10. 3·96 pence; 6·4 pence.

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