

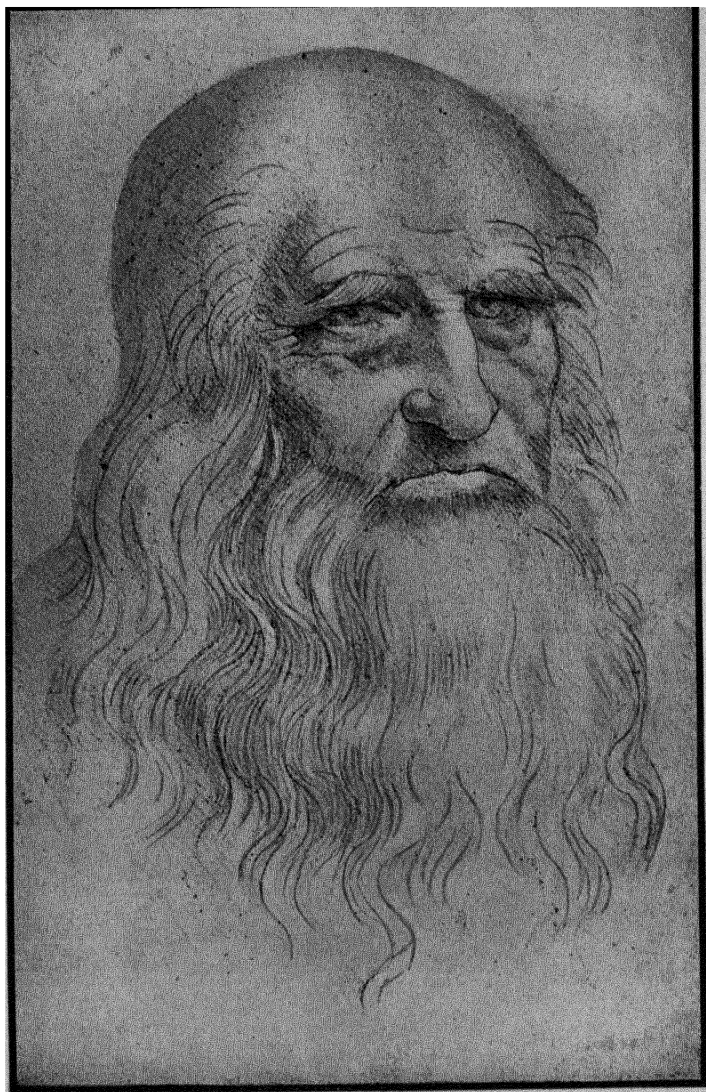
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THE MECHANICAL INVESTIGATIONS
OF LEONARDO DA VINCI



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LEONARDO DA VINCI (1452-1519). From a crayon portrait by himself (Royal Library, Turin).

[Frontispiece.]

THE MECHANICAL INVESTIGATIONS OF LEONARDO DA VINCI

By

IVOR B. HART

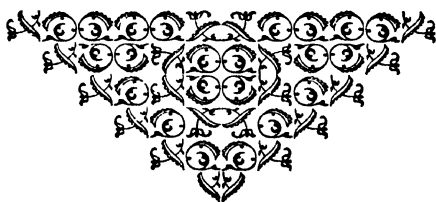
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AUTHOR'S PREFACE

IT has long been a matter for reproach that this country has shared but little in the researches which have increasingly been undertaken on the Continent into the amazing scientific activities of that great representative of the Italian Renaissance, Leonardo da Vinci. It is hoped that this book may help a little towards removing that reproach.

The author's primary object was to make a detailed study of the nature and the value of Leonardo's contribution to the study of aeronautics. The study of flight is, however, inevitably linked up with that of mechanics, and, as a consequence, the whole field of our philosopher's work in mechanics generally has been surveyed.

So far as the author is aware, this is the first work of its kind to appear in English. Much of the ground in mechanics has been covered before by Duhem and Seailles in France, and by Werner, Grothus, and Feldhaus in Germany, and to all of these writers the author is indebted. The section on Leonardo's aeronautical work is, the author believes, quite new, and is the most complete of its kind. Indeed, it is probably just to claim for the translation of Leonardo's manuscript *On the Flight of Birds* that it is the *only* translation of any complete manuscript by Leonardo in English. Apart from this, Paul Richter's famous collection of extracts from the note-books still remains the chief book of reference on da Vinci (as a scientist) in our own language.

As a preliminary to the study of Leonardo's mechanical

investigations, the author has included a detailed discussion of da Vinci's sources of the past, together with the nature of the contemporary scientific influences brought to bear upon him during his lifetime. There is also included a study of the general atmosphere of mechanical science in the fifteenth century.

Chapters VII. and VIII. originally appeared as papers in the *Journal of the Royal Aeronautical Society*. Chapter V. was read as a paper to the Newcomen Society for the Study of the History of Technology and Engineering, and an abbreviated version of Chapter VI. has appeared in the American monthly, *The Open Court*. To all of these the author is indebted for permission to republish in book form, whilst to the first his thanks are due for the loan of the illustrations of the chapters on aeronautics. He is also indebted to the Clarendon Press for the loan of some five plates and illustrations.

In submitting the English rendering of the manuscript, *On the Flight of Birds*, the author desires to express his sincere thanks to Mr. E. D. West, who ungrudgingly and freely helped with his knowledge and advice, and to W. Laidler for his assistance with the illustrations. It should be remarked, in connection with the illustrations, that as far as possible the atmosphere of the originals has been preserved. Finally, the author's gratitude is due to his friend and teacher, Dr. Charles Singer, but for whose continuous advice, interest, and encouragement this work could never have been accomplished.

IVOR B. HART.

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THE
MECHANICAL INVESTIGATIONS
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Chapter I

*General Introduction—The Characteristic of
Leonardo's Manuscripts*

LEONARDO DA VINCI was a great man. The same is true, but in a very different sense, of such men as Descartes and Newton. These latter have left behind them a mass of published writings which tell of brilliant discoveries and record research consistently conceived and carried out. They were philosophers and teachers first and last.

Da Vinci was neither teacher nor writer in this sense. He held no academic appointment, and he recorded no schemes of research. Living at a period when publication, though perhaps difficult, was nevertheless not impossible, he published nothing. Yet his notes teem with scientific discussion, and show at times an absorbing interest in the problems of science and philosophy covering an amazingly wide range—almost encyclopædic in extent. His note-books form, in fact, a vast scientific scrap-heap from which has been salved clear proofs of an ingenuity and a capacity for original thinking which alone would

suffice to place him in the forefront of the world's greatest thinkers.

His title to our careful consideration goes, however, much further than this. Study of his manuscripts affords evidence of Leonardo's familiarity with a number of facts, principles, and laws, the discoveries of which have been attributed to men who lived after him. This is an arresting fact of great significance, and forms the main justification for this present study, in so far as the subject of mechanics, and in particular of aeronautical science, is concerned. In the case of many of these laws of science, it is true that the vague terms used by Leonardo indicate a groping at the truth rather than a specific enunciation of fact. Nevertheless, the advance beyond the science of his times, which his writings portend, is well marked, and is clearly a contribution to the more purely modern period of scientific development. As a typical example of this vagueness of language, used in anticipation of a fundamental fact of modern mechanics, we may quote the principle of the parallelogram of forces. This is a subject which had been attacked by Aristotle in his *Questiones Mechanicæ*,¹ in the particular case of a body acted upon simultaneously by two forces at right angles. Aristotle's conclusion is sounder than the argument he employed in its establishment. He shows that if two forces, acting upon a body in directions at right-angles to each other, can be represented by two straight lines drawn in the direction of the two forces, and of lengths proportional to the magnitudes of those forces, then the diagonal of the completed rectangle will represent the direction and quantity of the resulting motion.

¹ Translation by E. G. Forster (Oxford, 1913), cap. i., 848b, line 15.

Leonardo's treatment was more general. He did not confine himself to the special case of the right-angle. In his manuscript on the *Flight of Birds* we meet the following passage :¹

The bird being in the act of falling in the direction of its open wing with a force of 4 (Fig. 1), and the wind which strikes it underneath with a force of 2, makes its course straight : we will say then that the descent of such a bird will be by the mean line, between the level of the course of the wind and the obliquity in which the bird was with the force 4.

As : let the obliquity of such a bird be the line $a d c$, and let the wind be $b a$: I say, if the bird $a d c$ had a force 4, and the wind $b a$ had a force of 2, that the bird would not go in the direction of the wind in f nor by its obliquity in g , but would fall by the mean line $d e$,

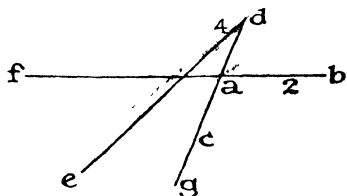


FIG. 1.

and one proves it thus :

There is here none of the precise enunciation of the Peripatetic. Yet as an insight into a truth of fundamental import in modern mechanics, based as it is upon careful observation, it must command both our respect and our attention.

What does the world owe to Leonardo da Vinci ? To what extent has he merited our gratitude ? However great may be the mentality of a man, however important he intrinsic value of his work and of his writings, unless his work and these writings have been offered to the world

¹ *Codice sul Volo degli Uccelli*, Leonardo da Vinci, 1505, fol. 6 (5) r.

for the edification of mankind, or at least until this man has taken the necessary steps to pass on to the civilisation about him something of his wisdom and his understanding, mankind owes him nothing. In the case of Leonardo da Vinci, there is before everything else his incomparable art. Inheriting as he did the glorious traditions of the Italian Renaissance, and living as he did in the full crest of that wonderful wave which had so effectively transformed the outlook of Western mankind, the whole objects, aims, hopes and achievements of the Renaissance seem to have been epitomised in his being. As a painter and a sculptor, he preached a mission of universality in art which brought into his range the whole of creation, and no doubt resulted initially in involving him in those scientific and philosophic studies a portion of which it is our purpose here to consider.

We are not, however, concerned with Leonardo the artist. Our interest is in Leonardo the scientist, and more particularly in Leonardo the student of mechanics. In this connection it is distinctly unfortunate that his voluminous manuscripts were so little known to the world of letters and of science for so long after his death. Da Vinci died in 1519, yet Venturi's first discussion of his manuscripts¹ was dated 1797—an interval of over 250 years—whilst the main flood of publication of da Vinci's writings and the serious study of their worth are matters of living memory, and are indeed by no means completed.

As to the intrinsic value of Leonardo's writings, the wonderful output by those who have studied him during the past fifty years is eloquent enough. As to the

¹ J. B. Venturi : *Essai sur les ouvrages physico-mathématique de Léonardo da Vinci, lu à la première classe de l'Institut National des Sciences et des Arts* (Paris, 1797).

peculiar force of circumstances which caused for centuries the loss of the manuscripts to the world, we shall have occasion to refer at length later.¹ What, however, we must ask ourselves now is, whether it was Leonardo's desire or intention to pass on to the world around him in his own time, and to posterity in the future, the fruits of his scientific labours.

One of the most obvious and, at the same time, the most striking of the characteristics of the manuscripts as a whole tends to answer this question in the negative. We refer to his invariable habit, from the age of twenty-one years onwards, of writing his notes from right to left, in the inverted "mirrored" style, which was for so long a trial and a puzzle to his commentators.

Vasari, the earliest biographer of Leonardo, tells us that ² "he wrote backwards, in rude characters, and with the left hand, so that any one who is not practised in reading them, cannot understand them," whilst on the question of the actual difficulties of the task of deciphering, we may note the following passage by Dr. Jean Paul Richter, who dedicated many valuable years to his noteworthy undertaking : ³

The handwriting is so peculiar that it requires considerable practice to read even a few detached phrases, much more to solve with any certainty the numerous difficulties of alternative readings, and to master the sense as a connected whole. . . . The aid of a mirror in reading reversed handwriting appears to me available for only a first experimental reading. Speaking from my own experience, the

¹ See Chap. VIII.

² *Lives of the Painters*, Giorgio Vasari, first edition (Florence, 1550).

³ *The Literary Works of Leonardo da Vinci*, J. Paul Richter (London, 1883).

persistent use of it is too fatiguing and inconvenient to be practically advisable, considering the enormous mass of manuscripts to be deciphered.

To all this we must add the further complication of the free use by Leonardo of a complicated host of abbreviations, and the omission of all punctuation. On the surface, it does indeed seem as though da Vinci desired no publicity of any kind for his labours.

Why should he have written thus? The reason must have been strong, since the practice of writing from right to left was of lifelong duration. It is held by some that da Vinci began this habit as a consequence of Eastern suggestion, and this is not impossible, at least as a factor in the situation. That da Vinci ever came under Eastern influences is, however, improbable. Controversy on this point has mainly centred round passages in which Leonardo describes himself as travelling (probably somewhere between 1482 and 1486) among the mountains of Armenia by order of the Sultan of Egypt, at a time when the Ottomans and the Egyptians were contending for the possession of those regions. Govi has argued¹ that the information embodying the passage was obtained from some other Italian who was the real traveller, and da Vinci's references to the East,² including his maps, lend countenance to this view. On the other hand, there exists no other positive evidence of an Eastern sojourn, and against it is the whole weight of common-sense inference from

¹ G. Govi *Tras. della Reale Accademia dei Lincei*, vol. v., p. 3.

² See *J. P. R.*, vol. 2, ch. 17, Nos. 1001-1112, pp. 223-270, including Nos. 1091-1092, The Red Sea; 1093-1098, on the Nile; 1099-1100, on the Customs of the Orientals; 1105-1106, on the Caspian Sea; 1108, on the Dardanelles; 1109, on Constantinople; 1110, on the Euphrates; and 1111, on Central Asia. See also p. 45, below.

the known biographical facts. Leonardo, however, was naturally a left-handed person. Such a person does in fact write mirror-wise quite naturally. The tendency to do so is instinctive. It should be noted that left-handedness is now definitely known to have a physiological basis, the explanation being that the normal structure of the brain, whereby the left lobe controls the right half of the human body, and the right lobe the left half, is reversed. The function of writing is normally controlled by the left lobe, and consequently in the case of people suffering such abnormality, this function is transferred to the right lobe, producing an instinct to "left-handedness." Why, then, if this was the case with Leonardo, did he write "normally" as a youth? The reason would be that his father would have behaved in the way usual even to-day towards left-handed children—he probably suppressed the natural tendency in his son, and caused him to cultivate writing with the right hand; and probably the latent artist in Leonardo enabled him to write freely with both hands.

It is interesting to note that we have further evidence on this point in the shade lines we sometimes meet amongst da Vinci's sketches. The person who is normally right-handed instinctively shades from right to left downwards, thus / / / /, whilst the normally left-handed person instinctively shades from left to right downwards, thus \ \ \ \. It is this latter type which is met with so frequently in da Vinci's sketches, and the inference is very clear.

We have now to consider the next point, namely, the reversal or "mirroring" of each letter in the manuscript. The practice undoubtedly savours of secrecy. Nevertheless, precautions of secrecy in the amassing of notes need

not imply absence of intention of future publicity. It does on the whole seem probable that da Vinci intended sooner or later to make an attempt to sort out his notes, and possibly to recast them in a form suitable for publication. The following oft-quoted passage is significant in several respects:

Begun at Florence at the house of Piero di Braccio Martello on the 22nd day of March, 1508 : this is a collection without order drawn from many papers that I have here copied, hoping later to put them in their right order, according to the subjects which they treat. I fear that I must repeat myself frequently ; do not blame me for this, reader, because the subjects are numerous and the memory is unable to have them all present, and say : " I will not write this because I wrote it before." And if I wished to avoid falling into this fault, it would be necessary in every case when I wanted to copy that, not to repeat myself, I should read over all that had gone before : and all the more since the intervals are long between one time of writing and the next. ¹

There is here implied not only the intention to collate his notes, but also that these notes are to be read by the general public. He addresses the reader : he takes his presence for granted. " Let no man who is not a mathematician read the elements of my work," wrote he ² on another occasion. The whole tone of his manuscripts, indeed, is that of the teacher addressing the taught; of the philosopher expounding his views to the reading public.

Having regard to this intention, it is not difficult to justify the deliberate difficulties with which Leonardo

¹ Br. M. I. a.

² W. An IV. 163b.

must have endowed the reading of his note-books. Until such time as publication under his own signature could give him the undisputed title to originality, there was the obvious need for protection against plagiarism. Until he could re-edit and collate his note-books into their final form suitable for the press, precautions of some kind were clearly indicated. Further, writing freely and honestly as he did whatever was actually in his mind, it was inevitable that passages would frequently occur, the publicity of which would inflict upon him serious trouble. Throughout all the political changes of his times, the authority of the Church was always sufficiently adequate to make dangerous the pronouncements of sentiments or dogmas contrary to its teachings. Consequently although da Vinci never closely associated himself with the Church, and indeed rather held himself aloof from it, he must have fully appreciated the fact that there was for him the alternative of either restraint in expression, or protection against the injudicious publicity of what he wrote ; and nothing could be further from da Vinci's nature than restraint. Spontaneity of expression was of the very essence of his being.

Fitting in with these considerations we have one other factor. There was in Leonardo a natural bent for the mystification of his fellows. He was almost a practical joker. "One will carry snow in summer, taken from the high crests of the mountains, and one will let it fall in places of festival in the summer time," wrote he on one occasion.¹

"He used often to cause the guts of a bullock to be carefully freed of its fat, and cleaned, and in this

¹ Leonardo da Vinci, *Codice sul Volo degli Uccelli*, fol. 14 (13) r.

way to become so fine, that it could be held in the palm of the hand : and having placed in another room a pair of blacksmith's bellows, to which he fixed one end of these guts, and blowing into them, filled the room, which was very large, whereby whoever was therein was constrained to retire into a corner.¹

He loved to consort with mystics, necromancers, and practisers of magic, and indeed was himself suspected, wrongly, of mystic practices. What more natural than that such a man, realising the advisability of some form of secrecy in his writings, should develop some such system of script as that which, in fact, he did adopt ? Finally, we may dismiss briefly the point with regard to the innumerable abbreviations with which his manuscripts teem. This is a practice common to all prolific note-takers, from the university undergraduate upwards, and is perfectly consistent with what one would expect in a man who wrote upwards of five thousand pages of notes. Nor was it a practice exclusive to Leonardo, as those who have to deal with manuscripts of the fifteenth and sixteenth centuries well know.

We may take it, then, that Leonardo da Vinci fully intended that his writings should ultimately see the light of day. Unfortunately, he never got past the stage of note-taking. The intention may have been frequently in his mind, yet he never translated it into action. Instead he wrote more notes and yet more notes. Critics have declared that he lacked the concentration of purpose in this, as in his incompleted works of art, which would have enabled him to narrow his energies to the detailed

¹ Vasari : *Lives of the Painters—Leonardo da Vinci* (Florence, 1550).

treatment of a specific subject. It would probably be more correct to say that, whatever may have been the extent of his power of concentration, the encyclopædic vigour of his mentality created for his pen a never-ending stream of comments and impressions which he was powerless to resist. The accusation of superficiality and lack of purpose, which it was customary to level against Leonardo, is nowadays happily coming to be regarded as premature. "We rightly value concentration. The map of human knowledge is now so elaborate that no man can hope to follow with profit more than one or two of its countless paths. This is the age of the specialist and the monograph. But in the fifteenth century the whole field of natural science was pure virgin soil, over which any pioneer was bound to range far and wide in making his surveys."¹ By their fruits shall ye know them. Leonardo da Vinci bequeathed to mankind thousands of pages of notes—many of them, alas, lost for ever—the labours of over thirty years. In these days of specialisation, the specialist in a host of important branches of learning has adequate material in these fifteenth and early sixteenth century writings to provide food for careful thought, and frequently, as in mechanics, to call for a serious review of historical values. What better tribute to this mediæval forerunner of modern science need we offer?

There is one more point to which it is but fair to allude by way of introduction to this work. We have established that although Leonardo never published a book, he obviously intended ultimately to reduce his notes to a form suitable for publication. This is a fact

¹ C. J. Holmes: Annual Master Mind Lecture, *Proceedings British Academy*, vol. ix.

which ought not to be ignored in setting ourselves the task of estimating the place and value of Leonardo da Vinci in the history of science.

It is the commonsense experience of every writer and thinker that the finished product of the published book is vastly superior to the collection of notes on which it is usually based.

This is a mere commonplace, which it is not necessary to elaborate further. It is, however, undeniable that students of Leonardo in the past have not allowed for this sufficiently. We have to go beyond the notes themselves, and we have to ask ourselves the question : " Suppose Leonardo had written a treatise on mechanics. Of what value, in the light of his notes on the subject, would such a treatise have been ? How far would it have been an original contribution to the knowledge of the day, and to the development of the future ? "

In this work it is proposed to confine attention to the mechanical investigations only of Leonardo da Vinci, including in particular his researches on flight. To the preliminary studies leading up to these we now proceed.

Chapter II

Mechanical Science in the Fifteenth Century

I

L EONARDO DA VINCI lived in the late fifteenth and early sixteenth centuries. We shall be in a better position to view his work in a true perspective if we consider the general conditions of mechanical science in his day.

The intellectual revival which succeeded the Dark Ages may be regarded as having dawned during the eleventh century. Gradually, the lost works of the ancients had been recovered, Latin translations of Aristotle had become available, and by the thirteenth century, what passed for Aristotelian philosophy had begun to be merged into Christian theology. The day of the Scholastics had opened. The Aristotelianism of the Scholastic Middle Ages was, however, a very different thing from that form of philosophy which we derive from modern critical editions of the works of Aristotle. Nowhere are the differences greater than in the treatment of the more strictly scientific works. For the Schoolmen, however, the mere pronouncements of the Peripatetic were what mattered. Initiative and independence were non-existent in the domain of physics. Observation was uncalled for, and the only people who in any way pretended to the practice of experiment were the alchemists, who, inspired by the example of the Arabian philosophers, vainly sought the *elixir vite* and the elusive secret of trans-

mutation, only to receive contempt or suspicion as magicians.

Then came the Renaissance proper, with its revival of the Greek language, the final passing of the Eastern Empire, the diminution of the power of the papacy, the invention of printing, and the historic voyages of Columbus. Beginning in Italy early in the fifteenth century, it swept over the whole of Europe, shaping, however, a different course in each country, and finding its final expression, so far as science is concerned, in the philosophy of the experimental method. The early phase, in which reliance upon the Schoolmen was substituted for the pronouncements of the ancients, in its turn gave way to the final overthrow of dogma and the direct appeal to experiment.

In Italy, the birthplace of the Renaissance, the new spirit of humanism, inaugurated by Petrarch (1304-1374), found its fuller development in the efforts of Marsilio Ficino (1433-1499).¹ Its chief mission was the reconciliation of knowledge with Christianity. Yet in all this science was but little touched.

It was at first the day of letters—of the *Divine Comedy* of Dante, and of the *Decameron* of Boccaccio in the fourteenth century, and of the fifteenth century imitators of the style of the ancients, such as Bruno's and Poggio's

¹ The impression has developed in recent years that the humanistic outlook became a very general feature of the early Renaissance in Italy. A word of caution is therefore necessary. Humanism as an everyday attitude of the general world of culture was a much later growth, even in Florence. The pioneers were there and the movement was given a healthy beginning, but there were probably not more than a few score of genuine humanist teachers in the whole country at any one time. Their teaching was, however, so vigorous and able that it has been easy for its early influence to have become exaggerated.

histories of Florence after the style of Livy, and the *Camaldunensis* of Landino, and the *De Voluptate* of Valla after the style of Cicero.

Luckily when art, following letters, came under the influence of the revival, there was little to draw upon. As a consequence there was a much more vigorous display of originality, producing the typical *cinquecento* humanism in painting of Raphael, da Vinci, Titian, and Corregio, and in sculpture of Donatello, Michelangelo, and Sansovino, and in architecture of Bramanti, Omodeo, and the Venetian Lombardi. So we come to science, which, touched last of all by the Renaissance, curiously enough received its first impetus in a warfare against the logicians and materialists through the medium of men of art—Alberti, da Vinci, Toscanelli, da Porta, and others. These addressed themselves in a small way to the design of scientific instruments, though in the case of da Vinci there was developed an attack on Nature the breadth of whose range is little short of astonishing. Yet during the first half of the Renaissance the awakening of science from its long slumber was only beginning. The true era of experimental and mechanical science was *after* the fifteenth century, and as is usual with the early histories of the various branches of the subject, the development was not initially a result of abstract inquiry, but was the outcome of the practical needs of the times. Consequently, fifteenth century mechanical science was an empirical study of the applications to the problems of navigation, military engineering, hydraulics, architecture and general technology.

So far as mechanics is concerned, the study of dynamics was practically unknown, whilst in statics very little advance had been made from the days of Aristotle and

Archimedes. The position of general technology in the fifteenth century does, however, call for further comment. Even the most cursory glance through the pages of da Vinci's note-books suffices to show an amazing range of technological devices and machines of all conceivable kinds. However much these may have been the product of this great man's brains, it is impossible to believe that there was not in existence a tradition of technology from which Leonardo was able to draw a certain amount of inspiration. Such indeed there was. Konrad Kyeser, Johannes Fontana, Jacopo Mariano, Francesco di Giorgio Martini, and Nicholas of Cusa¹ are only some, amongst many, technical writers of this period. Some at least of these had available for study the writings of such technological scientists of antiquity as Vitruvius, Heron, Ctesibius and Pappus.

Manuscripts of Vitruvius began to be available in print in the fifteenth century (his work had been lost sight of during the Middle Ages, and was rediscovered by Poggio at St. Gall in the fifteenth century). Leonardo frequently refers to this important writer, and his debt to Vitruvius was certainly very great. Of the fifteenth-century technologists, Konrad Kyeser of Eichstädt, in particular, left voluminous manuscripts early in the fifteenth century which were copied and recopied many times, and became

¹ Nicholas Cusanus (1401-1464), an important fifteenth-century philosopher, whose writings in mechanical science are mainly to be found in his *De Staticis Experimentis* and in his *Reparatis Calendarii* (1436). He was probably influenced by the newly-won manuscripts of Vitruvius, and in his turn influenced Regiomontanus (1436-1476). An English translation of *De Staticis Experimentis*, together with a summary of the life of Nicholas of Cusa by H. Viets, is to be found in *Annals of Medical History*, vol. iv., No. 2, June, 1922. A general account of him is given by E. Vansteenberghe, *Le Cardinal Nicolas de Cues*, Paris, 1920.

the standard work of his day on the subject. There is little doubt but that to some extent, then, Leonardo da Vinci must have been indebted to Kyser and his contemporaries for some, at any rate, of his technological inspiration.¹ The fact remains, nevertheless, that in the main, technology as da Vinci found it was largely an empirical study which had scarcely yet earned the name of an applied science. It would not, however, be right to assume from this that there was in the fifteenth century no such thing as mechanical science. To this aspect of the problem we now turn.

II

The study of the historical aspect of the mechanical sciences is impossible without due consideration as to the use of instruments of precision. In some respects, progress in the experimental sciences is concomitant with the development of accuracy in measurement. The two are necessarily closely allied, and have given rise to the term "exact sciences" which has grown into use in modern times. A brief consideration of the fifteenth-century position regarding the use of instruments of measurement is therefore called for, and it is proposed to consider these with reference to the allied group of subjects comprising astronomy, navigation, topography, and time.

As a preliminary consideration it should of course be pointed out that a minimum of accuracy in exact measurement will have been determined, at any given period of history, by the ordinary standards of length, weight and capacity in use at the time, and by the normal means supplied by the trading classes in employing these stan-

¹ See F. M. Feldhaus, *Leonardo da Vinci, der Techniker und Erfinder* (Jena, 1913), pp. 8 et seq.

dards. There were such standards in the fifteenth century in Italy as elsewhere, and there were the usual rules, measures, balances and weights for employing them. Balances, both of the equal-arm and of the steelyard type, date right back to early Roman times, but scientifically they were little used, except by the alchemists, and there was little about their use in our period calling for any special comment.

We now turn to the previously mentioned scientific activities more closely connected with exact measurement, beginning with Astronomy.

After the Dark Ages, observatories were set up by Arabian Caliphs at Damascus, at Baghdad (A.D. 829), on the heights of Mokattam near Cairo (about A.D. 1000), and by the Mongol Khans at Maragha, in N.W. Persia (c. 1240), and elsewhere. It was not, however, till the fifteenth century that observational astronomy became a progressive study. During this century Ulugh Beg founded an observatory of importance at Samarkand (c. 1420), whilst in Europe the first observatory was erected in 1472, at Nuremberg, by Bernard Walther (1430-1504), a German scientist of independent means. He collaborated with the famous Regiomontanus, and was the first to employ astronomical clocks driven by weights. Further developments, notably by Tycho Brahe, carry us beyond the limits of our period.

Allied, from the point of view of mechanical science, to the problems of astronomical measurement, we have the subject of practical navigation. Here, however, we come upon very barren soil, for navigation as a practical science was in a crude and primitive form in the fifteenth century. So far as the ancients were concerned, although they possibly realised that

the practical problems of fixing positions afloat were similar to those ashore, the probability is that the unsteadiness of their boats prevented them from developing this subject. Apart from the introduction of the mariner's compass during the fourteenth century (the magnet had, of course, been known long before this, though not used in ships), the true pioneers of scientific navigation were the Portuguese. In the fifteenth century Prince Henry of Portugal systematised the study of nautical science, and his successor, John II., followed this up in 1482 by setting up a committee on navigation. This resulted in the drawing up of solar declination tables, in the improvement of the instruments of the day, and in the organisation of proper instructional courses for pilots—courses which included a study of Sacrobosco's¹ recently printed *De Sphæra Mundi* and Ptolemy's *Almagest*.² The determination of longitude was both unknown and un hoped for, and the entire equipment of the fifteenth-century navigator comprised either a cross-staff or an astrolabe, a magnetic compass, tables of declination of the sun, and of pole-star altitude corrections, possibly a very crude chart, and what was known as a traverse board.

We may therefore summarise the instruments in use

¹ Sacrobosco (John Holywood, d. circa 1250). The *Tractatus de Sphæra* was first printed at Ferrara in 1472, and achieved a phenomenal circulation. It was the second astronomical work to be printed. By A.D. 1500 twenty editions had been printed and forty more had been issued by 1647, when the last edition appeared from Leyden.

² The translation into Latin of Ptolemy's *Almagest* from the original Greek was planned by George Purbach (1423–1461). His early death prompted his friend and pupil Johann Muller (Regiomontanus, 1436–1476) to continue this and other tasks (e.g., the reform of the Alphonsine tables), and this resulted in his *Epitome in Ptolemæi Almagestum*, completed in 1463, but first printed in Venice in 1496. An earlier but unreliable translation was by George of Trebizond. A mediæval translation existed, but had been lost.

during the fifteenth century which can in any way be regarded as being of interest to us in this work, although chiefly employed in astronomy, navigation, and surveying, as follows :

1. The astrolabe.
2. The armillary sphere.
3. The mural quadrant.
4. The cross-staff.
5. The clock.
6. The dioptra.

We propose offering a few brief remarks on each in turn.

1. The Astrolabe.¹—Literally meaning “to take a star,” this instrument was in fact used for measuring angular distances between any two objects. In principle, a graduated circular plate (made of copper or tin) has a datum line, AB , and a rotating index OC pivoted at O , and carrying two sight holes SS (Fig. 2). The instrument

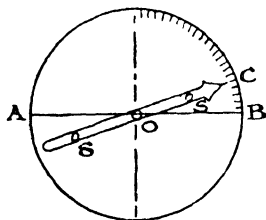


FIG. 2.

is held in the plane of the two objects (*e.g.*, stars), so that AB points towards one of them. OC is now rotated to point to the other object, and the angle is read on the instrument. This instrument was used in one form or another from the time of Eratosthenes onwards, and although originally intended for astronomical purposes, its use was gradually extended, until by the fifteenth century it was made in all sizes for a variety of purposes, including surveying and navigation and land travel. Christopher Columbus used a simple modification

¹ See J. Frank, *Zur Geschichte des Astrolabs* (Erlangen, 1920).

known as the *mariner's astrolabe*, the design of Martin Behaim (c. 1480). Three observers were necessary—one to hold the instrument, another to sight it, and a third to read the result; and what with the unsteadiness of the ship and the personal equation thrice involved, the results were not very reliable.

2. The Armillary Sphere.¹—This was of common origin with the astrolabe,² beginning with the armilla, a ring fixed in the plane of the equator, and developing by the addition of other rings into a sort of graduated model of the celestial sphere, one circle representing the horizon, another (graduated) the meridian, a third the equator, a fourth the ecliptic, and so on. They were of little use for quantitative work during the fifteenth century, little advance having been made in their design until Tycho Brahe modified them during the sixteenth century. Nicholas of Cusa was perhaps the most efficient fifteenth-century designer of such instruments.

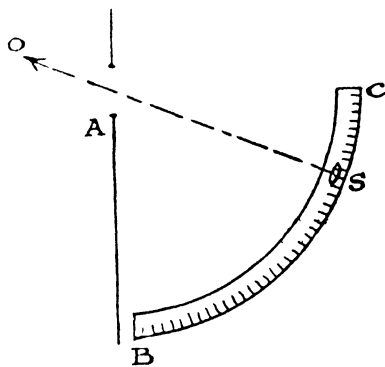


FIG. 3.

3. The Mural Quadrant.—This was an astronomical forerunner of the transit circle. Its principle is shown

¹ A very fine collection of these and other instruments of astronomical and general measurement of historical interest has recently been presented to the University of Oxford by Mr. Lewis Evans, F.S.A., and is at present housed in the Bodleian Library. There is also a valuable collection in the History of Science Museum at Magdeburg in Germany.

² See F. Nolte, "Die Armillarsphäre," in *Abhandlungen zur Geschichte der Naturwissenschaften und der Medizin*, Heft 11 (Erlangen, 1922).

diagrammatically in Fig. 3. The quadrant was mounted with reference to an aperture *A* in the observatory wall, *C* being on a level with *A*. A travelling sight *S* was swung round until *S* and *A* were in alignment with the object *O*, and the altitude was at once read off on the scale.

4. The Cross-staff.—This, again, was an instrument in general use in astronomy, navigation and topography,

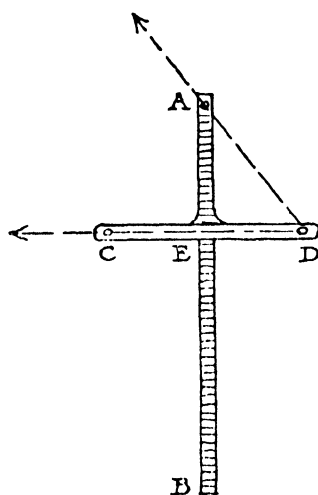


FIG. 4.

for the purpose of measuring angles subtended by objects. The theory is extremely simple. A graduated rod *AB* (Fig. 4) could slide into various positions at right angles to a shorter rod *CD*. There were sight holes at *A*, *C* and *D*. Sights *D* and *C* were first brought into alignment with one object, and kept there, and the position of *AB* was then adjusted until the sight holes *D* and *A* came into alignment with the other object. The reading of *AB* at *E* was then taken, and *DE* being of

constant length, a reference to a table of angles gave the corresponding angle.

5. The Measurement of Time and Velocity : The Clock.—Up to the time of the fifteenth century, little improvement had been effected in the measurement of time, the clepsydræ (water clocks) and sand-glasses remaining in general use throughout this period. Clocks based on mechanical power are recorded here and there as far back as the thirteenth and the fourteenth cen-

turies.¹ The earliest had no dials, and recorded hours by automatic figures striking bells. These were succeeded by dial clocks having an hour hand only. Nevertheless, the century of our study saw the inauguration of one innovation of importance in mechanical time measurement, for Bernhard Walther, to whom we have already referred as the pupil and collaborator of Regiomontanus, tells us that on the 16th of January, 1484, on the instant of the rising of the planet Mercury, he set in motion for the *purposes of scientific measurement* for the first time a clock driven by a hanging weight. The hour wheel was fitted with fifty-six teeth, each therefore representing a fraction more than a minute. At sunrise one hour and thirty-five teeth had passed, from which he concluded that Mercury rose one hour and thirty-seven minutes before sunrise. In spite of Walther's splendid lead, the measurement of time intervals for experimental purposes remained crude, and it is interesting to note as a typical example of the experimental conditions which obtained in the days of Leonardo da Vinci what was the method used for ascertaining the speed of a ship. A piece of wood was flung into the sea from the bows of the ship. Two observers stood on deck at a known distance apart, and the time taken by the wood in moving through this distance was estimated. Or, again, a certain "chanty" was begun at a given instant, and the distance travelled by the piece of wood was noted at the completion of the tune. The approximate time required for the singing of the chanty being known, the necessary speed was at once found.

One can readily understand that such crude methods

¹ *Old Clocks and Watches and their Makers*, F. J. Britten (London, 1911). See also *Time Measurement*, L. Bolton (London, 1924).

would scarcely find favour with Leonardo da Vinci. He certainly gave the matter of clockwork some attention, for we find in one of his note-books¹ a record of mechanism of clock springs which is not without interest. He shows a sketch (see Fig. 5) to which he attaches the accompanying note :

Four springs for a clock in such a way that when one has completed its run, the other begins. During the revolution of the first the second remains fixed. The first fastens itself upon the second by means of a screw, and when it has screwed itself quite firmly, the second spring takes up the same movement, and so on with the remainder.

The sketch is so crude and the description so brief that it is difficult to follow the writer's intentions. His

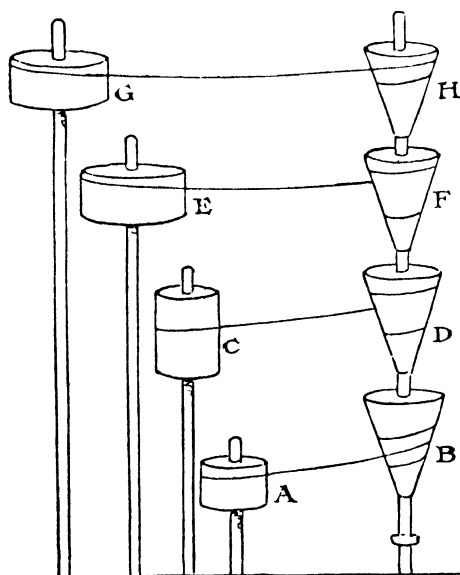


FIG. 5.

object is clearly to design a scheme which will operate for as long a time as possible without re-winding. His system consists of a series of drums, one obliquely above the other, each connected by means of gut with a conical "worm" working on a "worm shaft." Possibly beginning

¹ MS. B., fol. 50 v.

from the bottom, the pull of the gut from the drum *A* gradually rotates the cone *B* on the right, and in so doing screws the cone *D* into *B*. This screwing becomes completed at the moment when the drum *A* is completely unwound, and the pull of the gut *C D* now brings the drum *C* into play, rotating *D* and screwing it into *F*, and so on.

We do not know whether this design was actually put to practical use. Leonardo da Vinci was, however, himself directly interested in the subject of the estimation of the speed of a ship, and in one of his notebooks¹ we find the following paragraph :

On Movements. To know how much a Ship advances in an Hour.—The ancients used various devices to ascertain the distance gone by a ship each hour, among which Vitruvius gives one in his work on Architecture² which is just as fallacious as all the others ; and this is a mill-wheel which touches the waves of the sea at one end and in each complete revolution describes a straight line which represents the circumference of the wheel extended to straightness. But this invention is of no worth excepting on the smooth and motionless surface of lakes. But if the water moves together with the ship at an equal rate, then the wheel remains motionless ; and if the motion of the water is more or less rapid than that of the ship, then neither has the wheel the same motion as that of the ship, so that this invention is of but little use.³

There is another method tried by experiment with a known distance between one island and another ; and this is done by a board or under the pressure of wind which strikes on it with more or less swift-

¹ MS. G., fol. 54 r.

² Vitruvius : *De Architectura*, lib. x., c. 14.

³ This paragraph is of special interest as an example of the facts of "relative motion," all too rare at this time.

ness. This is in Battista Alberti. Battista Alberti's method is made by experiment on a known distance between one island and another. But such an invention does not succeed excepting on a ship like the one on which the experiment was made . . . but my method will serve for any ship, whether with oars or sails ; and whether small or large, or broad or long, or high or low, it always serves.

It is regrettable that there is no trace of what this method actually was. The above paragraph seems to suggest, however, that it was probably based on the principle of the anemometer. This view is strengthened by the fact that Leonardo was the inventor of such a wind-measuring in-

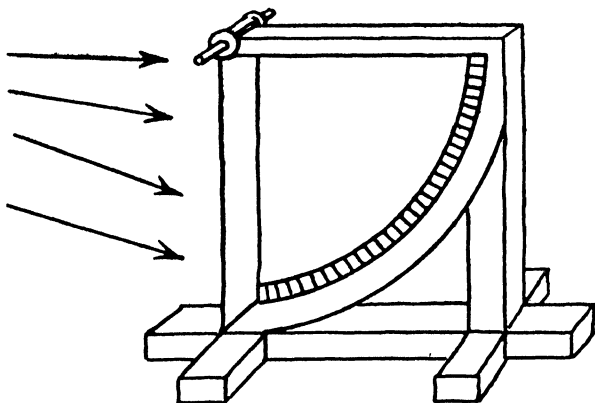
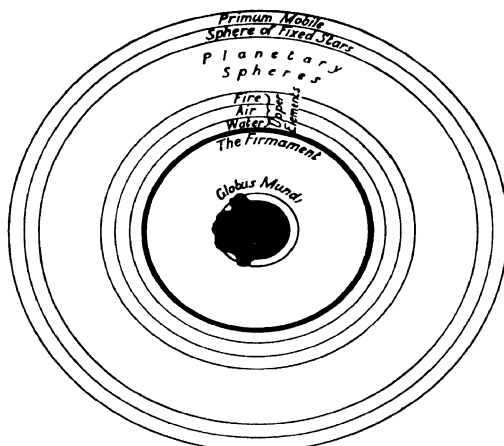


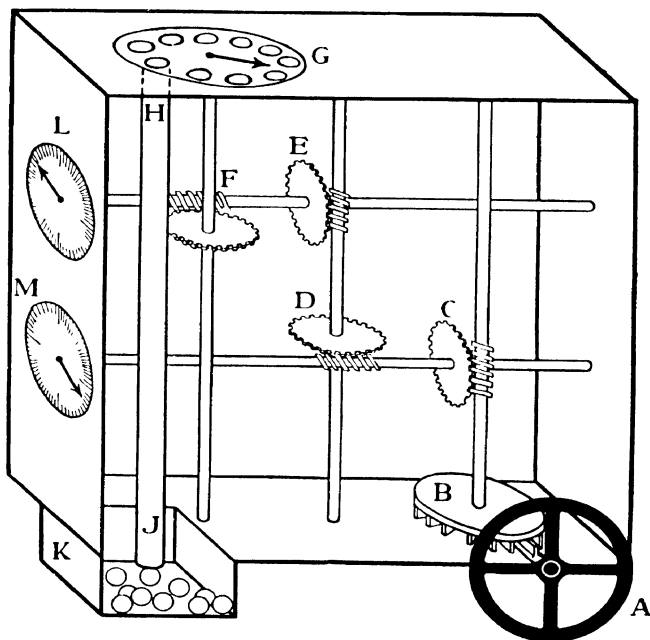
FIG. 6.

strument. In the *Codex Atlanticus* we find a sketch of a framework carrying a thin rectangular tin plate capable of swinging about a horizontal pin. The lower end moves over a graduated quadrant, and the swing will naturally be a function of the wind-force (shown in the sketch (Fig. 6) by four lines) acting upon it.¹ Accompanying the sketch

¹ The earliest recorded instance of the invention of the anemometer is to be found in the *Philosophical Transactions of the Royal Society*, vol. xi., p. 444 (1667). The inventor's name is not given.



Scheme of the Universe, as developed by the early mediæval followers of Aristotle. (See footnote, p 86)



"Taxicab," or odometer, as described by Vitruvius.—From C. Singer,
"Science," in Bailey's *Legacy of Rome* (Oxford, 1923)

[To face page 27.]

is the following note: "Here an index is necessary for showing the hours, points, and minutes. It is to measure how great a distance one covers with the current of the wind."¹ Clearly, however, to meet Leonardo's claims, such an instrument needs standardising, and then must be used on a ship in the absence of wind, or alternatively, first when the ship is at rest, and later when the ship is in motion.

There is, however, an instrument mentioned by da Vinci for measuring distances — the *hodometer* (in effect a revolution counter), which, while of general applicability to land measurement, he may yet have had in mind for use at sea. In any case the description of the instrument is properly in its place in this

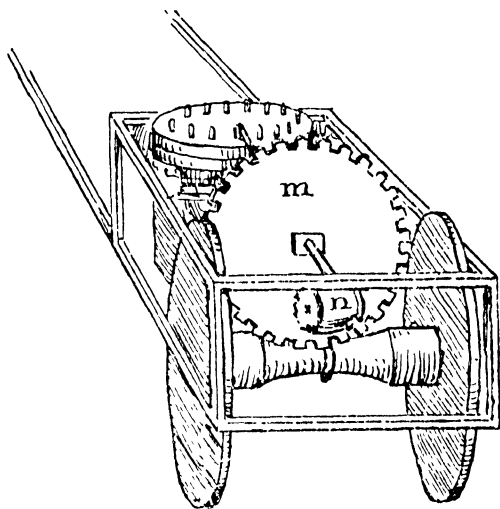


FIG. 7.

chapter as an example of attempts at exact measurement in the fifteenth century. Leonardo derives his inspiration presumably from Vitruvius.² The sketch (Fig. 7) and accompanying note again occur in the *Codex Atlanticus*.³ The note begins as follows :

¹ *Codex Atlan.*, fol. 249 v.a.

² Vitruvius: *De Architectura*, lib. x., c. 9, in which a single-wheeled *hodometer* is described.

³ *Codex Atlan.*, fol. 1 r.

If the wheel of a cart is to have a circumference of 10 yards, then the diameter will be $3\frac{1}{2}$ yards.¹ . . . If the wheel has made a complete revolution, it will have measured 10 yards of the way, which is the three-hundredth part of a mile (miglia), which is 3,000 yards long, and the wheel *M* will have only moved one tooth further. This has 300 teeth, and the consequence is that the cart will have just covered a mile when the wheel has made a complete revolution ; but then the wheel *F* will have only moved one tooth, and the wheel *N* has done the same. This announces, by means of an indicator, each mile covered, instead of the hour as in the case of the watch. On the other hand, the wheel *F*, instead of reading by means of a pointer, can cause the ear to hear a sound by means of a small stone which is allowed to fall into a vessel suitable for emitting a note when struck.

Presumably the wheel of the vehicle is geared to the central toothed wheel in such a way that one revolution of the former produces a displacement of one tooth in the latter, and this in its turn is similarly geared to the wheels *F* and *N*. The sketch from this point of view is not very clear. With regard to the applicability of this scheme to the movement of a ship, we know that Leonardo paid considerable attention to the problem of paddle attachments to boats,² and we may at least say with safety that it was quite within the bounds of possibility for him to have intended some such hodometer attachment to these boats. Nevertheless it is, of course, more probable that

¶¹ This is incorrect, of course. Note a familiar practice, Leonardo's, in not reducing fractions to their lowest terms.

¶² *Codex Atl.* fol. 319 v.a.; fol. 333 v.; fol. 344 b.; fol. 374 v.b.; fol. 384 r.b.; MS. B., fol. 83 r., etc.

he had in view the anemometer method for the estimation of the speed of ships.

6. **The Dioptra.**—So far as general surveying is concerned, it must be confessed that the position in the fifteenth century was little further advanced in the matter of instruments beyond that which obtained in the days of Hero of Alexandria. That is to say, the dioptra, which was the staple instrument of the ancient engineers and surveyors, probably persisted throughout the Middle Ages with very little change.¹ Indeed it may almost be claimed that in principle it has persisted even to modern times, for after all, apart from the addition of telescopic vision and refinements of adjustment and workmanship and graduation, it is not far wrong to say that the dioptra of the Greeks, with its levelling-staff accessories, has become the theodolite and the levelling instrument of to-day.

It was certainly the theodolite of the sixteenth century, and a brief reference to the instrument will therefore not be out of place. The figure (Fig. 8) (purely diagrammatic) indicates the salient features. The plate *P* is graduated in degrees round the rim, and carries the sights *S S* with the pointer *P P* at right angles to them. Elevation control² is obtained from the toothed wheel *A*

¹ Hero's dioptra was perhaps known to the Middle Ages through the agency of translations from the Arabic.

² It should be remarked that it is by no means clear from the obscure records which exist whether Hero of Alexandria, and the engineers and scientists of his days, did in fact utilise the toothed wheel *A* as a means of obtaining elevation. It is certain that they used this wheel for adjustments to obtain a true horizontal, and it is probable that the fifteenth-century engineers used it for the measurement of angles of elevation. The true inventor of the theodolite (he was the first to use the term "theodolitus") was Leonard Digges, in 1571, the essentially new features in his instrument being: (*a*) the attachment of a magnetic compass for

actuated by the drum and handle *B*, the rotation of each tooth being equivalent to a definitely known angle of elevation, whilst control of the horizontal rotation of the upper part of the apparatus into any desired direction is derived from the worm and worm wheel *C*. The levelling staff used in connection with this instrument calls for

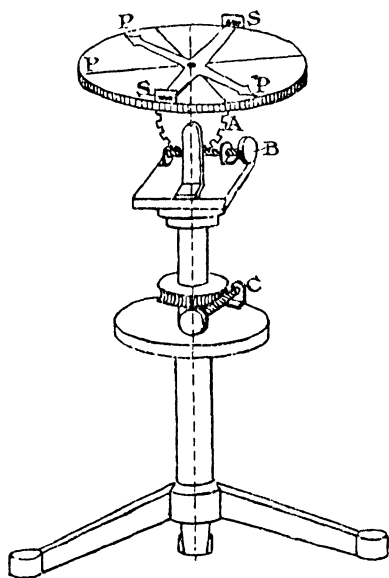


FIG. 8.

little comment. It was practically a long graduated rod, and carried a plumb line to indicate when it was in a vertical position. A pulley-wheel was mounted on the top, and from it there could be raised or lowered a disc marked with a horizontal index. This was moved up or down until the man operating at the sights signalled that he had obtained alignment with his cross-wires, and the readings and bearings were then taken. For the purposes of water levelling it should perhaps be pointed out

that the top plate *P* was removed, and a suitable modification (on the principle of the water-level) was substituted.

Some very interesting notes by Leonardo da Vinci on

control in orientation; and (*b*) a definite mechanism for elevation control and measurement. See L. Digges, *Pantometria* (London, 1571); also R. C. Skyring-Walters, *Transactions of the Newcomen Society*, vol. ii. (1921-22), p. 45; also E. A. Reeves, *Maps and Map Making*, pp. 9, 30.

altimetry and levelling are to be found in the *Codex Atlanticus*.¹ For example, in order to find the height of a mountain, Leonardo devised an instrument, a sketch (Fig. 9) of which is here shown. Supported on a vertical rod FC (there are prongs at D to enable the instrument to be stuck into the ground) at a height of about two yards is a horizontal graduated measure AB , about three yards long. This is hinged at A to a similar rule AE , whose angle EAB can be varied. Behind AE and also resting on FD is an upright rule CF , notched at the top. AE can be swung into any desired position by means of the waxed thread GCH . Leonardo illustrates the method of applying this instrument to

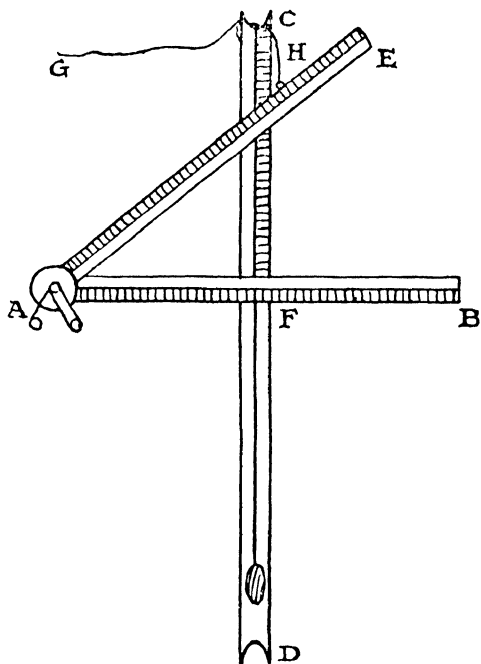


FIG. 9.

the determination of the height of a mountain by means of the sketch (Fig. 10) here shown.

Angles of elevation are taken from two positions at a known distance apart, and the required result is calculated by trigonometry.

¹ *Codex Atlan.*, fol. 131 r.a.

Leonardo follows this up in the same note-book¹ with a note on levelling. "If you wish to level a plain of many miles accurately, then set about it in the way here indicated," he writes. His sketch (Fig. 11) shows a series of

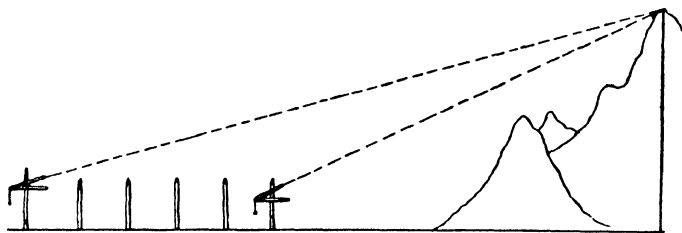


FIG. 10.

alignment poles at 400 braccia intervals, the first of them being placed at a distance of 200 braccia from the levelling instrument (obviously a sort of dioptra). An auxiliary diagram shows by its details of the poles that the observations are to be taken by night. Mounted on the alignment poles are oil lamps, and Leonardo ensures

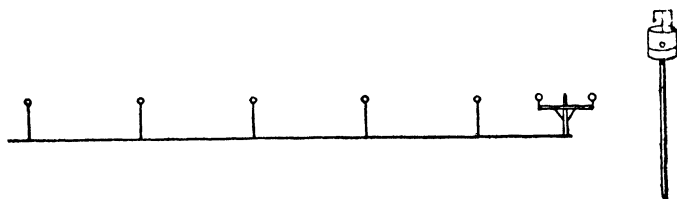


FIG. 11.

that the lights from all these lamps are maintained at the same height by the simple device of an oil feed. This consists of an oil vessel above each lamp, from which oil flows, drop by drop, into the lamp at a rate which exactly compensates for the consumption in the lamp. In

¹ *Codex Atlan.*, fol. 131 r.a.

another note-book ¹ Leonardo deals with the problem of finding how much higher is the elevation of one plane above that of another adjacent to it. The sketch (Fig. 12) and the accompanying note are as follows :

If you wish to know how much one plane is higher than another, do thus : Place your level between the two planes of which you wish to know the depression of the one below the other ; place on the lower

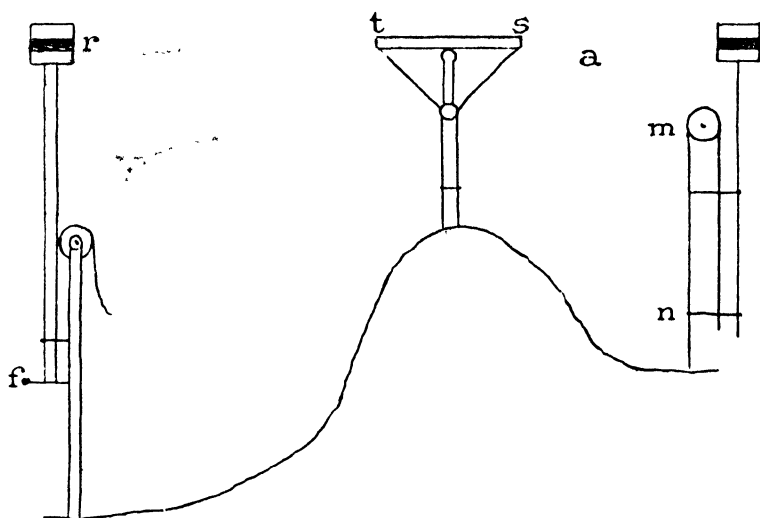


FIG. 12.

position the graduated rod, and raise or lower it until your eye *A* and the two uprights *S* and *T* are in the same horizontal as the upper black bands made on the drum *R*. Mark the rod below, at the point *F*, with a nail. Then carry the rod to the other side, and do the same as at first. Look through the instrument the other way *T S*, and put

¹ MS. B., fol. 65 v.

in another nail. If the first nail is N and the second is M , then the difference is the difference in levels.

III

This, then, was the general atmosphere of instrumental science in da Vinci's day. It was scarcely calculated to encourage the would-be experimentalist. We have of necessity confined our remarks to the instruments, such as they were, in a very narrow field of subjects. There is, alas, no option, for it does no injustice to the records of those times to say that outside these subjects there were practically no scientific instruments whatever. We have pointed out that so far as mechanics is concerned dynamics was a stillborn science, and that the subject of statics had not advanced since the days of Archimedes. In all those activities which are the inevitable accompaniments of organised communities—in architecture, in engineering, and in fortification and siege craft—the fifteenth century did indeed witness much progress, but that it was scientific progress is saying too much. Rather was it an empirical response to the needs of the period. It was a product of the arts and crafts rather than of the sciences. Accordingly, it is of little assistance to us to inquire, for example, whether the introduction of locks and lock gates may be attributed to the fourteenth century Dutch engineers or to the fifteenth-century Italians ; or what were the practical and political considerations which governed the fifteenth-century innovations in fortification and siege craft ; or what were the artistic and other factors which produced the typical Renaissance architecture of fifteenth-century Italy through the genius of such men as Brunelleschi, Alberti, Bramante, and San Gallo. Such problems do

not come within our scope. They do not help us to appreciate the conditions under which the mechanical investigations of Leonardo da Vinci were carried out, even though he himself was interested almost as no other was in all these works. Obviously, the true call to experimental science was yet to come, and we shall see later that come it did, and from no other in this century than he whose mechanical researches it is our business to consider.

Chapter III

Contemporary Scientific Influences

I.—YOUTH—FLORENCE

IT is our purpose in the following two chapters to consider, as far as possible, the sources whence Leonardo derived his scientific inspiration and his materials. Two considerations are here involved. We have first the personal influences and scientific associations with which Leonardo was surrounded, and secondly, the written records of the philosophers of the past to which he had access. The latter of these considerations was naturally dependent upon the former. The extent to which Leonardo would endeavour to delve into the works of the past would largely be determined by the extent to which a spirit of scientific inquiry was fostered in him by those who were in a position to influence his mind and his development, and by the opportunities that were available for him to pursue that inquiry.

In the previous chapter we discussed the general nature of the scientific atmosphere of the fifteenth century. Let us now consider more closely some of the personalities connected with this atmosphere in Italy, confining ourselves only to those with whom Leonardo was concerned, both in his upbringing and in the years of his maturity.

Leonardo, born in 1452, was brought up in his father's household, and in consequence of his early evidence of a

taste for drawing, was apprenticed at the age of fourteen to the artist Andrea Verrocchio (1435-1488).

Here we are confronted with an important factor in the determination of da Vinci's future career, for we must consider what was the general trend of the Florentine culture amidst which he spent his early youth. At the height of its renaissance in art and literature, dominated by the Platonic teachings, which were so emphatic a characteristic of the Italian revival of learning, there was a complacent self-sufficiency in the attitude to science which did not augur well for future progress. The doctrine of macrocosm and microcosm yet went unchallenged. The Ptolemaic system of the universe, with its toll of epicycles, or the alternative scheme of concentric spheres, were all-satisfying. They supplied a cosmic scheme which appeared complete. There appeared to be nothing left to whet the appetite of the curious. For the people of Florence the philosophical spirit had to take a different turn. Again, the Greek influence of the Platonic school manifested itself. In the place of that natural science which was lacking, we find developed a taste for the abstract inquiry of the mathematician. The studies and the faculty for criticism which were characteristic of the Greek peoples came to the fore, except that perhaps to geometry there was added a taste for arithmetic which was foreign to the Greek era after the days of the Pythagoreans. So we find that the great interests of the *cinquecento* philosophers lay in the direction of arithmetic, geometry, cosmography and perspective.

It has been regarded as a matter for remark, that in fact some of the chief exponents of these subjects in the Florentine school of this period were the great artists—

men like Piero della Francesca¹ (c. 1416–1492), Filippo Brunellesco² (1379–1446), and Leon Battista Alberti³ (1404–1472). This, however, was perfectly natural. For the ideal practice of art, there is something mathematical in the development of the necessary sense of proportion ; there is a clear necessity for a knowledge of that branch of optics known at that time as perspective ; and above all, the artist could achieve success only in proportion to his appreciation of a knowledge of anatomy. Indeed, so far as anatomy is concerned, it may be argued that the history of its development runs side by side with the history of the artistic treatment of the living model ; each helped the other. Finally—and none showed this perhaps more than da Vinci himself—both art and anatomy were advanced by a proper understanding of the laws of movement and of balance. Hence the spur on the part of the artist for a study of—or, at least, an interest in—the science of mechanics. It was in such an atmosphere as this that Leonardo received his early training. Indeed, he was steeped in it from the outset. It is probable that as a lad, before the days of his apprenticeship to Verrocchio, he went to the Florentine *Scuola d'Abbaco*, where such mathematics as was available was taught.⁴ Here he came under the care of a mathematician, Benedetto del'Abbaco (b. 1432), sometimes

¹ Francesca was a painter of the Umbrian school. He received a scientific education. A manuscript on perspective in the Ambrosian Library of Milan has been proved to have been written by him. He is important to us, also, from the fact that two of his disciples, Fra Luca Pacioli and Bartolomeo della Gatta, were later friends and contemporaries of Leonardo da Vinci (see p. 47).

² Or Brunelleschi. He was a famous student of Roman sculpture, and was one of the first to apply the scientific laws of perspective to his art.

³ See pp. 40, 41.

⁴ O. Werner : *Zur Physik Leonardo da Vinci*, p. 5 (Erlangen, 1910).

referred to as Benedetto Aritmetico.¹ Vasari records that Leonardo was too clever for his teachers, but whether this be so or no, it is clear that the first germs of his scientific future were absorbed from them. A brief reference to Benedetto has been found amongst da Vinci's note-books.² From this school there emanated a series of papers known as the *Trattato d'Abbaco*, which are preserved in the Italian libraries. Here, too, Leonardo must have come in contact with one of the best-known mathematical text-books of those times, the *Liber Abaci* of Leonardo of Pisa, the famous thirteenth-century arithmetician, as well as with the works of Euclid.

We have already mentioned that at the age of fourteen da Vinci was apprenticed to the goldsmith and artist Verrocchio. Verrocchio, without being in any sense great, except perhaps as a sculptor, was at least famous, and was certainly, from the point of view we have just developed, truly representative of the Florentine culture of his days. Sculptor, painter, musician and goldsmith, he was at the same time something of an engineer, and was distinctly well known as a mathematician and student of perspective. To this it is only necessary to add that Leonardo not only remained apprenticed to Verrocchio during the impressionable years of adolescence, from fourteen to twenty, but that he continued to live with his master for some years beyond this period until 1476, when domestic developments in his father's household rendered it advisable for da Vinci to depend in future upon his own resources. Everything, therefore, in the circumstances of Leonardo's upbringing made for the

¹ See also E. Solmi : *Leonardo da Vinci* (Berlin, 1908), p. 15.

² *Codex Atlan.*, fol. 12 v.a.

fostering of the wide interests he found in his master. Verrocchio's tastes and abilities naturally brought him into contact with all his contemporaries of a like calibre. From all these da Vinci must have learnt. Consequently, when he left his master to carve out a career of independence for himself, it was certainly with the seeds of scientific curiosity truly well sown.

Let us consider briefly the claim of those personalities with whom Leonardo must at this period have been acquainted. One of the earliest of these—since he left Florence when the subject of our study was but twenty years of age—was Giovanni Johannes Agiropulo (1416–1486). He is important as having probably been the first seriously to interest Leonardo in the writings of Aristotle. A brief mention of him is to be found in the *Codex Atlanticus*.¹ Agiropulo was a Greek exile of distinguished attainments. He taught Greek in Florence from 1456 to 1471, a period of fifteen years. His lectures were very popular and were attended by all the intellectuals of the city, and amongst these no doubt was included da Vinci. Agiropulo was the translator into Italian of a number of Aristotle's works, among them being the *Physica* and *De Cælo*.²

The chief remaining personal influences during Leonardo's youth were Leon Battista Alberti (1404–1472) and Paolo dal' Pozzo Toscanelli (1397–1482). The former of these, Alberti, was born in Venice, but became a canon in the Metropolitan Church of Florence. His duties were light and left him much time in which to pursue his remarkably varied interests. He was a painter, poet, musician, architect, and philosopher. His scientific interests led him to the improvement of the hitherto

¹ *Codex Atlan.*, fol. 12 v.a.

² Solmi, pp. 14, 16.

crudely conceived camera obscura, to the formulation of a means of measuring the depths of the sea beds, to the invention of a form of hygrometer, and to the devising of a mechanism for raising sunken wreckage. His scientific writings included *Trattato della Pittura*, *Opere Volgari*, *Ars Ædificatoria*, *Ludi matematici*, and *Liber navis*. While it is problematical as to whether Leonardo actually knew Alberti, he certainly knew of his writings and read them. References to them have been found in the Leicester Manuscript¹ and in Manuscripts F. and G. of the collection at the Institute of France.²

Finally, we come to Paolo Toscanelli, the great Florentine physicist of the fifteenth century. He was a student of medicine, mathematics, astronomy, and geography. As a geographer he has been credited with having, by correspondence in 1474, inspired Columbus to his great Western project, though this has recently been questioned by critics. There is no doubt that Leonardo knew him. The reference in the *Codex Atlanticus*,³ amongst other names already mentioned, to "Maestro pagholo medici" (Master Paolo the Doctor) can be to none other, and da Vinci, if his cosmographical views were not influenced by Toscanelli (and Seailles has thrown doubt on this),⁴ must at least have been inspired by him, as by the others we have mentioned, to a love of the study of Nature and of natural phenomena such as must

¹ MS. Leic., fol. 13 a. See also J. Paul Richter, vol. ii., para. 1472.

² MS. F., fol. 8 r.; MS. G., fol. 54 r. A reference in MS. F. (cover) which has been regarded by Muntz, C. Ravaisson-Mollien, Werner, and others as an allusion to L. B. Alberti, has been shown by Duhem (*Études sur Leonardo da Vinci*, vol. i., pp. 10, 11) to refer to Albert of Saxony.

³ *Codex Atlan.*, fol. 12 v.a.

⁴ Seailles: *Léonard de Vince, l'Artiste et le Savant* (Paris, 1906), p. 251.

have been a spur to his vivid intellect and imagination in later manhood. Leonardo may be said to have established a position for himself by the year 1472, when at the age of twenty he was enrolled as a member of the painters' guild of St. Luke. Although he remained with Verrocchio some years longer, by 1482 he was working independently under the patronage of Lorenzo the Magnificent. It was at about this period that Leonardo probably threw himself energetically into his first serious scientific studies. Probably a study of architecture on the one hand and of civil and military engineering on the other would have led him insensibly and instinctively into various problems in mechanics and hydraulics generally, and his painting would of course have induced serious attention to anatomy and perspective. With his energy, youth, imagination and originality, he must have made a prominent free-lance student. His essentially modern outlook had already begun to develop. Agiropulo had, as we have seen, brought him into contact with the ancients. But this philosopher had left Florence by 1472, and by now da Vinci was able to bring to bear upon the memory of Agiropulo's influence a critical judgment which, side by side with his bent for observation and experiment, led him to develop that spirit of independence towards, and disagreement with the blind dependence upon ancient authority which was characteristic of his contemporaries. "Any one who in discussion relies upon authority uses, not his understanding, but his memory,"¹ says he in the *Codex Atlanticus*; and again:

Many will think that they may reasonably blame me by alleging that my proofs are opposed to the

¹ *Codex Atlan.*, fol. 75 r.a.

authority of certain men held in the highest reverence by their inexperienced judgments, not considering that my works are the issue of pure and simple experience, which is the one true mistress.¹

Incidentally, it is generally recognised that portions, at least, of the *Codex Atlanticus* were written from 1483 onwards, though probably not until Leonardo had left Florence.

With all this scientific interest diverting him from the more purely money-making side of his art, it may readily be understood that Leonardo was none too well off. When, therefore, Ludovico Sforza (il Moro) invited him to the Court of Milan in or about 1483, there must clearly have been no hesitation on the part of Leonardo in accepting the offer. With his departure from Florence the first chapter in da Vinci's life closed.

II.—MILAN

The letter written by da Vinci to Ludovico Sforza commending himself to the Court of Milan is well known, but is worth quoting, since it gives us a clear indication of the scope of Leonardo's activities and interests whilst at Florence. The letter reads thus : ²

Having, most illustrious Sir, seen and considered the experiments of all those who profess to be masters in the art of invention of the apparatus of war, and having found that their instruments do not differ materially from those in general use, I venture, without wishing injury to any one, to make known

¹ *Codex Atlan.*, fol. 117 b.

² *Codex Atlan.*, fol. 391 r.

to your Excellency certain secrets of my own, briefly enumerated as follows :

1. I have a process for the construction of very light bridges, capable of easy transport, by means of which the enemy may be pursued and put to flight ; and of others more solid, which will resist both fire and sword, and which are easily lowered or raised. I know also of a means to burn and destroy hostile bridges.

2. In case of the investment of a place, I know how to drain moats and construct scaling ladders and other such apparatus.

3. Item ; if, by reason of its elevation or strength, it is not possible to bombard a hostile position, I have a means of destruction by mining provided the foundations of the fortress are not of rock.

4. I know also how to make light cannon easy of transport capable of ejecting inflammable matter, the smoke from which would cause terror, destruction, and confusion among the enemy. [Note here a fifteenth-century forecast of smoke-screens and poison gas.]

5. Item ; by means of narrow and tortuous subterranean tunnels, dug without noise, I am able to create a passage to inaccessible places, even under rivers.

6. Item ; I know how to construct secure and covered wagons for the transport of guns into the enemy's lines, and not to be impeded by ever so dense a mass, and behind which the infantry can follow without danger. [A possible vision of the modern tank ?]

7. I can make cannon, mortars, and engines of fire, etc., of form both useful and beautiful, and different from those at present in use.

8. Or if the use of cannon happens to be impracticable, I can replace them by catapults and other

admirable projecting weapons at present unknown ; in short, where such is the case, I am able to devise endless means of attack.

9. And, if the combat should be at sea, I have numerous most powerful engines both for attack and defence ; and ships which are both gun-proof and fire-proof ; and also powders and inflammables.

10. In times of peace, I believe that I can compete with anyone in architecture, and in the construction of both public and private monuments, and in the building of canals. I am able to execute statues in marble, bronze, and clay ; in painting I can do as well as anyone else. In particular, I will undertake to execute the bronze horse in the eternal memory of your father and of the very illustrious house of Sforza, and if any of the above-mentioned things appear to you impossible or impracticable, I will offer to make an attempt at it in your park or in any other place which Your Excellency may please to choose, to which I commend myself in all humility.

It may be argued that the letter lacks modesty. A sufficient reply to this is that we have here an artist who puts his art last on his list of qualifications. Da Vinci was in effect seeking employment, and such a suppliant naturally stresses those qualities which he considers most needed. These were troublous times for the Court of Milan, and the impending differences between it and Venice were a sufficient index to Leonardo as to the wisdom of stress upon his qualifications as a military engineer.

Nevertheless the first years following his departure from Florence were shrouded in uncertainty as to his movements, and it is to this period that those who believe that da Vinci travelled to the East assign these

wanderings.¹ It is a theory which may be dismissed however. Although there is definite evidence of his presence at Milan in 1487,² it is fairly certain that da Vinci must have been there some considerable time before this.

The year 1490 is important for our purpose. In that year Leonardo was called as consultant regarding certain architectural problems connected with the Cathedral at Pavia, some sixteen miles from Milan. Pavia was an old seat of learning in whose university Ludovico Sforza had taken a personal interest, and here Leonardo made the most of his time, browsing in the libraries, meeting his learned contemporaries, and generally proceeding steadily and deeply with the many problems of scientific research in which he had become so interested. That he was already at work on his note-books is clear from the reference in Manuscript C.—a treatise chiefly on light and shade, which reads, “on the 23rd day of April, 1490, I commenced this book and recommenced the horse.”³

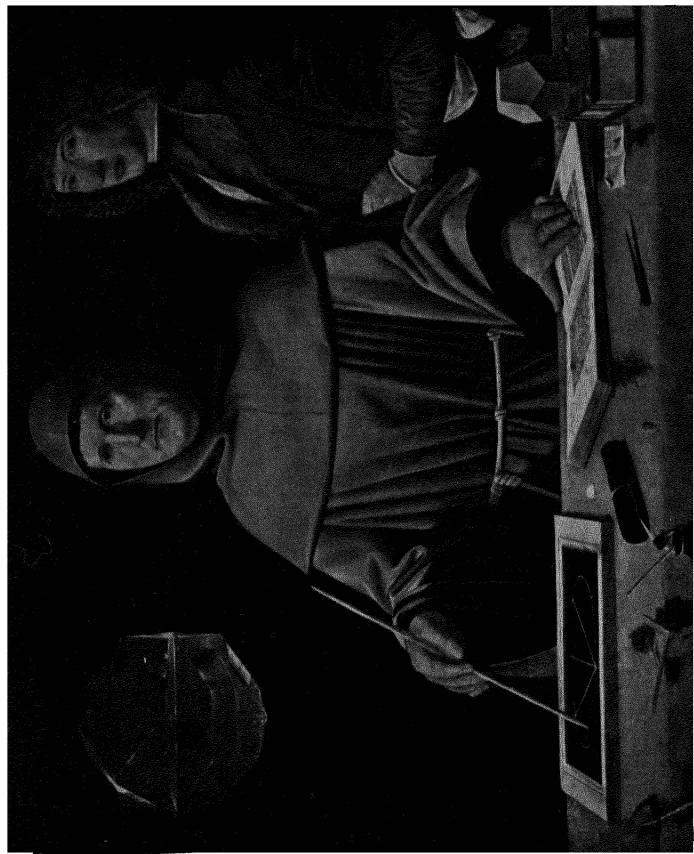
The scientific atmosphere of Milan was very different from that which had prevailed in Florence.

“North Italy played the part of leaven in the case of Leonardo ; it was contact with the Milanese that drew out his scientific powers. His new fellow-citizens had neither the lofty culture of the Florentines nor their spiritual aspirations. They adhered, in fact, to doctrines diametrically opposed to those of the Tuscans. To the altar of Plato they opposed that of Aristotle, and of Avicenna and Averroes, his Arab

¹ See Chap. I., p. 6.

² It is very significant that in the very valuable *Documenti e Memorie riguardante la Vita e le Opere di Leonardo da Vinci*, arranged chronologically by Luca Beltrami (Milan, 1919), there is a distinct gap in the records between 1483 and 1487.

³ MS. C., fol. 1.



Anderson Photo.

FRA LUCA PACIOLI, friend and contemporary of Leonardo.

From a painting in the *Museo Nazionale* at Naples ascribed to Jacopo de' Barbari.

[To face page 46.]

interpreters. These were the true initiators of Leonardo. They taught him to observe as well as to reflect, and to alternate experiment with abstract thinking.”¹

Later, in 1494, we find da Vinci once more at Pavia, renewing his associations of 1490, and, as we know, making a particular study of the Latin and vernacular grammars, the better to delve into the writings of those ancients to whom he frequently refers in his manuscripts. The advent of printing was beginning to have its influence on the world of learning, and of this da Vinci took full advantage.² Leonardo remained at Milan altogether for a period of sixteen years, that is to say, until 1499. The chief scientific characters with whom he appears to have associated during this time were Fra Luca Pacioli, the mathematician, Bartolomeo della Gatta, his friend, Fazio Cardano, the father of the famous Jerome Cardan, Peruzzi, an astrologer and mathematician, Jacopo Andrea, Bramante, Giacomo Andrea di Ferrari, Pietro Monti Alberghetti, and Martelli.

Of these Fra Luca Pacioli was easily the most famous and the most important. Born at Borgo San Sepolcro in Tuscany about the middle of the fifteenth century (he is sometimes known as Lucas di Borgo), he was a minorite friar, and lectured on mathematics at Rome, Pisa, and Venice, afterwards becoming the first occupant of a chair in mathematics at Milan founded by Sforza. He died at

¹ E. Muntz : *Leonardo da Vinci* (London, 1893), vol. ii., p. 51.

² In 1476 a Greek Grammar by C. Lascaris had appeared, *Celsus* was printed in 1478, Craston's (Johannes Placentini) Latin-Greek Lexicon became available in 1480, *Serenus Samonicus* and *Apuleius* by 1484, whilst in 1490 there appeared a Latin translation of Galen. Other volumes of importance soon followed.

Florence about 1510. His chief work, printed at Venice in 1494, was called *Summa de arithmetico, geometria, proportioni e proporzionalita*. It was in fact the earliest printed text-book on arithmetic and algebra, and was founded on the writings of the thirteenth-century mathematician Leonardo of Pisa. Da Vinci had bought a copy of this book at Pavia on its appearance, and when shortly after Pacioli came to Milan, the two were naturally at once drawn to each other. They became close friends, and Leonardo was able to help Pacioli very materially in the preparation of his next work, *De divina proportione*. When Leonardo left Milan to return to his native Florence, it was in the company of his friend.

Fazio Cardano (1445-1524) was a learned jurist of Milan who was also well known as a student of mathematics. He was born in 1445,¹ and had graduated both in medicine and in law. Mathematics was his hobby, and this took practical shape of great importance in 1480, when he edited for the first time the hitherto unprinted manuscript, *Perspectiva Communis*, of John Peckham, Archbishop of Canterbury. The *Perspectiva* was a treatise on optics, and as will be seen in the next chapter, was studied by Leonardo da Vinci.

Fazio was intimate with both Leonardo and the friend of da Vinci's old age, Melzi, and in view of the fact that his natural son, Jerome Cardan, the famous mathematician, was thereby enabled to become acquainted with the general nature of da Vinci's manuscripts, this association is of some importance.

Bramante (c. 1444-1514), whose real name was Donato

¹ His horoscope was published in *Libelli V. De supplemento Almanach, etc.* (ed. Norimberg, 1547), p. 121. See also H. Morley, *The Life of Girolamo Cardan of Milan* (London, 1854), pp. 1-5.

d'Augnolo, was a famous painter and architect. He came to Milan in 1476, and remained there till 1499, when the trend of political events drove him, as it did so many others, to Rome. He was naturally interested in mechanics and science generally, and Leonardo must have met him frequently.

References and notes in da Vinci's manuscripts offer continual indications of his desire to acquire knowledge and information from all sources—a visit to a library here, the loan of a book there, or a talk with an interested contemporary on some point of mutual interest. One trend in his mental development is clearly brought out by a reference in the Windsor manuscript (mainly about mathematical and anatomical topics). Brought up, as he must have been, in the days of the Ptolemaic doctrine of a geocentric universe, we find in this MS. a clear indication that at Milan he had come to the conclusion that Ptolemy was wrong. The note reads *il sole no si muovo* ("the sun does not move").¹ An index as to the date of this pronouncement is afforded by the fact that in the same note-book we read a note, dated April, 1489, in which he speaks of writing "about the human figure."²

It has been thought by some writers that Leonardo was at this time at the head of something in the nature of a school or academy. This view is untenable. There is nothing to substantiate it apart from the words *Achademia Leonardi Vinci* on various whimsical engravings. The whole trend of Leonardo's personality was that of a "free-lance" student. We know, too, that in fact he lived very quietly and in a very unassuming

¹ MS. Windsor, fol. 132 a. See also J. P. Richter, vol. ii., p. 152.

² MS. Windsor I., fol. 1 a.; MS. B., fol. 42 r.

way at Milan. He shared a house with his pupils and apprentices, amongst whom were Antonio Boltraffio, Ambrogio Preda, Marco d'Oggionno, and his personal attendant and friend, Andrea Salai. Constant references in the note-books show the extremely modest nature of his personal expenses. If such an academy, as has been hinted at, did in fact exist, there would surely have been some direct reference to it amongst the hosts of purely personal references with which the note-books abound. But none such are to be found. Further, it is pertinent to point out that Leonardo was so frequently absent from Milan on engineering, irrigation, or architectural missions, that it would surely have been extremely difficult for him efficiently to carry on such an academy. We are bound, therefore, to dismiss the suggestion as extremely improbable.

III.—LATER MOVEMENTS

Towards the year 1499, political events at Milan began to move rapidly. The enmity of France, in league with Venice, was constituting a serious menace to Ludovico, who left for Innsbruck in September, 1499, with a view to enlisting the sympathies of the Swiss and the Germans. In his absence Louis XII. of France invaded and captured Milan. For Leonardo the future seemed full of uncertainties. He had but recently returned from a tour of duty in the inspection and maintenance of the canal and waterway systems of the duchy. There was a possibility, but it was merely a possibility, that Sforza, who was preparing a counterstroke, might drive out the invading army. Leonardo decided to await developments on "neutral" territory, and so, accompanied by his friend Pacioli, he left Milan in December,

1499, after a stay of sixteen years, and set out for Florence. As we know, Ludovico Sforza's attack failed, he himself being taken prisoner, and the city was sacked of its great treasures of Art, Leonardo's famous equestrian model being destroyed.

Travelling to Venice *via* Mantua, Leonardo and Pacioli spent their time chiefly in the study of mathematics and cosmography, until the news of the final overthrow of Ludovico drove them further afield, and they moved on to Florence.

Here, as ever, Leonardo da Vinci is the man of science. Problems of Nature engross him even to the neglect of his art. He undertakes a commission to paint an altar piece for the Church of the Annunziata, but is too much absorbed in his investigations to go on with it. Thus we find him interested in the tidal problems of the Caspian Sea, in the canalisation of the waters of the Arno, and in various other engineering schemes. It was at this period that he came into contact with Andrea Sansovino, a brother artist of great repute, whose chief scientific interest was in the direction of cosmography.

In the spring of 1502 we find Leonardo taking service under Cesare Borgia, Duke of Valentinois, as chief engineer, and travelling, in the course of his duties, over various parts of Central Italy. On his return to Florence in March, 1503, the famous battle piece of the Anghieri began to occupy his attention in rivalry with Michelangelo, as a result of which a new impulse of enthusiasm in art began to spread over Florence. Nevertheless, as had happened so often before, the fresco was never completed, and very few fragments indeed of the original cartoon are extant to-day.

This failure probably had much to do with his accept-

ance, in 1506, of an invitation extended to him on behalf of Louis XII. of France to return to Milan, and in 1507 he was formally appointed painter and engineer-in-ordinary to the French King. Here he stayed for seven years, litigation in connection with his father's death in 1504 recalling him periodically to Florence. He was again able to renew old scientific associations and to make fresh ones. In particular, mention should be made of the gifted young anatomist, Marc Antonio della Torre. Very little is known of della Torre, but his relations with Leonardo have been investigated.¹ Descended from a noble family, he was born at Verona about the year 1483. His father, Jerome della Torre, had himself taught medicine at Padua, and here in due course the son followed. Later, he moved from Padua to Venice, and finally from Venice to Pavia. His chief interest was in anatomy. A close student of Galen, he had little patience for the later teaching of the Arabians. Unlike many of his contemporaries, he did not limit himself to mere book study. Whenever opportunity offered, he practised dissection, and so studied his subject first hand. A student of brilliant promise, his life was cut short in his thirtieth year from a fever he contracted in 1512. His death was an undoubted loss to the science of anatomy.

The available details of della Torre's life are very scanty, but it is clear that at Pavia a close friendship developed between him and Leonardo da Vinci. The latter's deep interest in anatomy, together with his natural ability in the pictorial representation of anatomical parts, must have appealed strongly to the young professor, and the two

¹ Karl F. H. Marx: *Über Marc Antonio della Torre und Leonardo da Vinci, die Begründer der bildlichen Anatomie* (Göttingen, 1849).

must have helped and influenced each other very considerably.

Leonardo was now approaching his sixtieth year, and although full of vigour, was beginning to feel the need for peace and freedom from external strife. Political changes at Milan in 1512 did not lend themselves to such tranquillity, and a year later our philosopher left the city once more for Rome, and later, in the summer of 1515, he accepted an invitation from Francis I. of France, Louis XII.'s successor, to take up his residence in the castle of St. Cloux, near Amboise. Here he spent the remainder of his days. He died on May 2nd, 1519.

Chapter IV

Leonardo's Sources

I.—INTRODUCTION

WE are now in a position to understand the facilities which existed in Leonardo's days for a study of the works of the philosophers who lived before him. We have already pointed out the essentially modernist trend of da Vinci's method. He was no scholastic. Removed from their crudities of language, his manuscripts indicate clearly his interpretation of scientific method. Seailles¹ has summed this up as follows : (1) to observe a phenomenon ; (2) to reproduce it artificially if possible ; (3) to discover some relationship between its various factors or features ; (4) to apply measurement to these relationships ; and (5) to deduce a law. Such a sequence will surely stand the rigid test of scientific practice to-day.

But let us consider the materials upon which Leonardo worked. The difficulties with which we are faced at the outset are due to the fact that he lived at a time when it was not easy to differentiate between books and manuscripts which were original from those which were more or less inefficient translations.² Yet in a certain sense this does not matter greatly. Leonardo's was an intellect so far superior to those of his days that although he was

¹ G. Seailles : *Léonard de Vince, l'artiste et le savant* (Paris, 1905), p. 219.

² D'Adda : *Leonardo da Vinci e la sua libreria* (Milan, 1873), discusses this problem at some length.

admittedly and inevitably influenced by such great philosophers of the past as Aristotle, Galen, Archimedes, and Jordanus Nemorarius, his fearless individuality and original genius were such as to make for an independent outlook of the most fruitful kind. The tragedy of it is that the circumstances which resulted in the scattering and loss of his manuscripts should have prevented him in his turn from exerting more influence on the course of scientific history after him.

Leonardo's reading must have been wide and generous in the extreme. Werner has set forth in alphabetical order a list of no fewer than seventy-two names of the various writers of the Classical and Middle Ages with whose works Leonardo must have been acquainted.¹ As a typical example we may quote the following from a page in one of da Vinci's note-books² :

Horace has written : of the speed of the heavens.	Concave mirrors. Books of Venice. Vernacular and Latin vocabulary. Bohemian knives. Vitruvius. Meteorologica. Archimedes : de centro gravitatis. Allesandro Benedetto's "Anatomy." The "Dante" of Niccolo della Croce. Albertuccio and Marliano De calculatione. Albert : De coelo et mundo —by Fra Bernardino.	Aristotle's "philosophy." Messer Octavius Pallavicino for his Vitruvius.
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Clearly Leonardo had cast his net far and wide, and a systematic study of his sources is probably best achieved

¹ O. Werner : *Zur Physik Leonardo da Vinci* (Erlangen, 1910), pp. 22, 23.

² MS. F. (inside cover).

from a chronological standpoint. Obviously, also, of the seventy-two names of authors traced in his writings, the majority call for no special attention. They merely indicate the range of da Vinci's reading. We are concerned rather with those who influenced him in the sense of modifying what might otherwise have been his views.

II.—ARISTOTLE AND ARCHIMEDES

Beginning then with the classical era of philosophy, we may say that da Vinci's sources were Aristotle, Euclid and Archimedes in mechanics and mathematics, Galen in physiology, Pliny in natural history, and Ptolemy in cosmography. These were more or less fundamental. Other teachers must also have influenced him. Thus Anaxagoras is specifically mentioned by Leonardo in the *Codex Atlanticus*.¹ The reference was perhaps obtained from Lucretius.

Aristotle, as might be expected, plays a great part in the building up of da Vinci's outlook. Direct references abound in his manuscripts.² From the point of view of mechanics, we may regard Aristotle's work as the starting point of a chain of thought which played an important part in the evolution of the subject up to the days of Leonardo da Vinci. Aristotle made no distinction between a theory of equilibrium and a theory of movement. That was a development which came after him. His standpoint was that of treating generally of mechanisms from the point of view of the movements which are produced in

¹ *Codex Atlan.*, fol. 385 v.c.

² E.g., *Codex Atlan.*, fol. 97 v.a.; fol. 123 v.a.; fol. 89 v.a.; MS. D., fol. 10 v.; MS. M., fol. 62 r.; MS. J., fol. 130 v.; MS. F., inside cover; fol. 26 r.; MS. Brit. Mus., fol. 191 r.

them. When in fact there are no such movements, he regarded the mechanism as being in equilibrium. What Aristotle called the "motive power" which moves a body he measured by the product of the weight moved and its velocity. Aristotle used the term "weight" very generally where we use the term "mass." There was no distinction between the two.

It followed from this conception of motive power that the same power would move a heavy body slowly, and a light body quickly, the velocities produced being, for the same power, inversely proportional to the weights. This principle, generally applicable to all mechanisms, he applied to the lever

(Fig. 13) by showing that whilst the large mass M moved a distance x , m moved through X . Aristotle deduces that "the

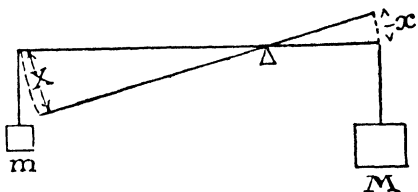


FIG. 13.

weight which is moved is to the weight which moves in the inverse ratio of the lengths of the arms of the lever ; always, in fact, a weight will move as much more easily as it is further from the fulcrum." ¹ This sums up Aristotle's most important contribution to the history of mechanics. Duhem regards it, indeed, as "la graine d'où sortiront par un developpement vingt fois seculaire les puissantes ramifications du Principe des vitesses virtuelles." ² Aristotle's influence upon Leonardo is shown clearly in the following pronouncement : ³

¹ *Mechanica Problematica*, Δ.

² P. Duhem : *Origine de la Statique* (Paris, 1905), vol. i., p. 8.

³ MS. F., fol. 126 r. See also MS. F., fol. 51 v., where much of the above is repeated.

Firstly : if a force moves a body for a given time over a given distance, the same force will move half the mass in the same time through twice the distance.

Secondly : or again the same force will move half this mass through the same distance in half this time.

Thirdly : and half this force will move half the body through the same distance in the same time.

Fourthly : and this force will move twice the mass through the same distance in twice the time ; and one thousand times the mass in one thousand times the time, through the same distance.

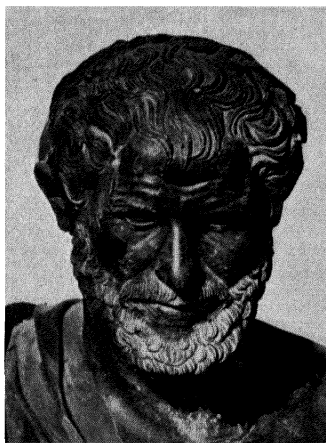
Fifthly : and the half of this force will move the whole of this mass through half the distance in the same time, and one hundred times the mass through one hundredth of the distance in the same time.

Sixthly : and if two separate forces move two separate bodies in so much time through so much distance, the same forces together would move the same bodies together all this distance in the same time, because the original proportions always remain the same.

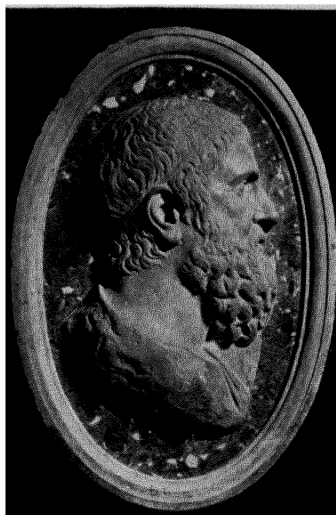
For the next step in the logical development of Leonardo's outlook in mechanics we turn to Archimedes (c. 287–212 B.C.), the Syracusan mathematician and philosopher. Archimedes' chief contributions to mechanics were : (1) *On the Equilibrium of Planes and the Centres of Gravity of Planes*—a work which, being of much greater scientific merit than that of Aristotle on the same subject, may be considered to mark the true beginnings of the science of mechanics ; and (2) *On the Floating Bodies*—a sound exposition of the fundamental principles of hydrostatics.

We have already seen that the former of these must have been studied by Leonardo da Vinci, though it had not yet been printed.¹ It consists of two books, the first

¹ MS. F., inside cover.



ARISTOTLE. From Herculaneum; probably copied from work of the fourth century B.C.



Anderson Photo.

ARCHIMEDES. From a cameo of the first century A.D. in the Capitoline Museum, Rome.



ROGER BACON, with a pupil, taking observations from a window. From a fifteenth-century MS. of his *De Retardatione Senectutis*, Bodleian Library. (See p. 70.)
[To face page 58.]

containing seven postulates and fifteen propositions, together with demonstrations of the type employed to-day for finding the centres of gravity of systems of two weights, and of parallelograms, triangles, and trapezia. The second book builds up in ten propositions a study of the centres of gravity of parabolic segments and other more complicated areas.

The general structure of Archimedes' writings is Euclidean in its characteristics. Thus he begins by postulating certain self-evident requirements, such as :

1. Equal masses suspended at equal distances are in equilibrium.

2. Equal masses suspended at unequal distances are not in equilibrium, that which is suspended at the greater distance being carried lower.

3. If masses suspended at certain distances are in equilibrium, and a weight is added to one of them, then equilibrium is destroyed, the side to which the weight has been added being carried lower.

4. If masses suspended at certain distances are in equilibrium, and a weight is subtracted from one of them, then equilibrium is destroyed, the side from which the weight is not subtracted being carried lower.

Following on seven of these we have the fifteen propositions deduced logically from them. Of these, two deal with the equilibrium of the straight lever, and read as follows :

Proposition 6.—Two commensurable masses are in equilibrium when they are reciprocally proportional to the distances at which they are suspended.

Proposition 7.—Two incommensurable masses are in equilibrium when they are reciprocally proportional to the distances at which they are suspended.

These are the essential contributions of Archimedes to our subject.

III.—EUCLID

We come next to the great geometer, Euclid, who was probably a contemporary of Archimedes, having been born somewhere about the year 300 B.C. Euclid's influence upon Leonardo and his predecessors of the Middle Ages has been the subject of considerable controversy. The basis of the discussion lies in the fact, that whilst most Greek writers make no reference to Euclid on the subject of mechanics, the inference being that there was none to make, we find numerous such references amongst the Arabic writers on statics. In the year 1851, Dr. F. Wöpkcke drew attention to *The Book of Euclid on the Balance*.¹ This was Manuscript No. 9522 in the Arabian Section of the Bibliothèque Nationale, Paris, the copy in question having been written at Chîrâz in the year A.D. 970. This work had been attributed to Beni Moûchâ by some, but Wöpkcke advanced the view that the author was Euclid himself. The work contains four propositions which are developed in characteristic Euclidean fashion, due allowance being made for the inevitable maltreatment due to the additions and emendations of writers and translators, other than the original. F. Hultsch² has, however, adduced a further argument in favour of the Greek origin of the manuscript, which is of interest. He remarks upon the fact that in the Arabic version, the letters used to indicate the points in the figures illustrating the text are

¹ F. Wöpkcke: *Notice sur des Traductions Arabes de deux ouvrages perdus d'Euclide* (*Journal Asiatique*, fourth series, vol. xviii., p. 217), 1851.

² F. Hultsch: *Pappi Alexandrini collectiones quæ supersunt* (Berlin Weidmann, 1878).

used in the order a, b, g, d, e, z, h and t. This he compares with the first letters of the Greek alphabet, α , β , γ , δ , ϵ , ζ , η , and θ . The inference is clear, and has received the support of Duhem.¹ Heiberg, on the other hand, opposes the Euclidean view.² For us, however, one of the chief points of interest in connection with this manuscript is the fact that the axioms with which the work opens are to be found in Leonardo's note-books. These axioms, four in number, lay down various conditions under which a lever or "balance" will remain horizontal. For example, the first axiom states that if two equal weights are placed at the ends of a uniform balance with the fulcrum at the mid-point, the balance will remain horizontal. In succeeding axioms the conditions are varied, until in axiom 4 we read that if a number of weights are suspended on both sides of the balance so as to keep the arms horizontal (*i.e.*, so as to be in equilibrium), then if one of the weights on one side of the fulcrum be moved, say, to the right, it is possible to maintain the arms in a horizontal position by suitably moving another weight on the same side of the fulcrum to the left. That Leonardo must have read and have been influenced by this work is clear from the fact that we meet with the second of these axioms in the *Codex Atlanticus*,³ whilst the fourth axiom is met with in two different note-books.⁴ Interestingly enough, the first axiom is also to be found in the writings of Jordanus Nemorarius,⁵ a famous mathematician of the Middle

¹ P. Duhem : *Origine de la Statique* (Paris, 1905), vol. i., pp. 62-67.

² Heiberg : *Literargeschichtliche Studien uber Euklid* (Leipzig, 1882).

³ *Codex Atlan.*, fol. 145 v.a.

⁴ MS. A., fol. 5 r.; MS. E., fol. 33 r.

⁵ *Liber Jordani Nemorarii de ponderibus propositiones XIII.*, prop. iii.

Ages, to whom we shall shortly refer as one of Leonardo's chief sources.

There are two other works, or rather fragments of works, which have been attributed to Euclid. One of these, the *Liber Charastonis*, we will leave for consideration under the heading of Leonardo's Arabic sources.¹ The other is a fragment called *De canonis*, of which two copies, MS. 8680 A and MS. 7378 A, exist in the Bibliothèque Nationale of France. The name of the translator is unknown, but Duhem² regards the work as a direct translation from a Greek text, and as a supplement to the *Liber Charastonis*. The subject of the manuscript is a discussion of the problem, "What weight must be suspended at the extremity of the shorter arm of an 'unequal' balance to counteract the effect of the excess of weight of the longer arm?" In other words, "How can the balance be treated so as to regard the arms as weightless?"

We must not forget, however, that Leonardo da Vinci was not alone concerned with Euclid's writings, assuming they existed, on the subject of mechanics. References to Euclid's *Elements* are to be found in Manuscript E.,³ but of equal importance from our standpoint is the influence exerted on Leonardo by Euclid's *Optics*.⁴ Here we find the teaching that "the visual rays proceed in straight lines, leaving a certain space between them. The figure

¹ See p. 67.

² P. Duhem: *Origine de la Statique* (Paris, 1905), vol. i., p. 17.

³ E.g., MS. E., fol. 10 r.; fol. 29 v.; fol. 30 r., etc.

⁴ The authenticity of this work is discussed in J. L. Heiberg and H. Menge, *Euclidis Opera Omnia* (Leipzig, 1885-95), vol. vii., 1895: "Euclidis optica opticorum recens Theonis, Catoptrica cum Scholiis antiquis." See also Heiberg's *Literargeschichtliche Studien über Euklid* (Leipzig, 1882).

formed by the visual rays is a cone having its apex at the eye and its base at the object seen. Only those things are not seen on which they do not impinge.”¹ Two other works, *Catoptrica* and *De speculis*, which were attributed to Euclid and which found an extensive circulation in the Middle Ages, have been proved by Heiberg² and Vogl³ to be spurious. Most of the references made by Leonardo da Vinci to Euclid's optical writings are to be found in the *Codex Atlanticus*.⁴

IV.—FURTHER GREEK INFLUENCES

Several other Greek philosophers call for consideration in connection with Leonardo da Vinci. Claudius Ptolemaeus (*circa* A.D. second century) is a notable example. He is specifically referred to in the Ashburnham Manuscript.⁵ His famous work, *Great System*, translated into Arabic as the *Almagest*, and thence into Latin, was regarded as the astronomical system prior to Copernicus. In addition he was the great geographer of antiquity, but more especially his work in optics, *Opticorum sermones quinque*, exerted a profound influence in the days that followed him. His studies on reflection and refraction followed naturally on that of Euclid and of a later thinker, Cleomedes, and in their turn formed the basis

¹ A recent translation is by G. Ovio, *L'Ottica di Euclide* (Milan, 1918).

² Heiberg: *Literargeschichtliche Studien, etc.*, p. 148.

³ S. Vogl: *Über die (Pseudo) Euclidische Schrift de Speculis* in the *Archiv für Geschichte der Naturwissenschaften und der Technik*, vol. i., p. 419 (Leipzig, 1909).

⁴ *Codex Atlan.*, fol. 90 v.a.; fol. 148 v.b.; fol. 174 r.a.; fol. 176 r.d.; fol. 183 v.a.; fol. 250 r.a.

⁵ MS. Ash., fol. 10 r.

of further development by the famous Alhazen in the eleventh century.

Hero of Alexandria is also of importance. His work was largely a series of applications of mechanical principles already put forward by Archimedes to such devices as the balance, toothed wheels, pulley blocks, wedges and screws. His influence upon Leonardo has been worked out by W. Schmidt,¹ and Leonardo da Vinci often quotes him. It is clear that he knew of the *Pneumatica*. Nevertheless Duhem² has questioned the possibility of a very serious influence, on the ground that it was improbable that a sufficiently full manuscript of Hero's work was available to Leonardo da Vinci at that time. It seems clear, however, that to one whose note-books teem with sketches and notes on machines and mechanical devices of all kinds, as did Leonardo's, the sort of work which interested Hero must have struck an extremely sympathetic note. There is some probability, too, that Leonardo knew of the writings of Pappus, the great mathematician of the third century A.D., of Alexandria. Pappus makes frequent references to Hero, and thus we have here a further source of indirect influence by Hero on Leonardo da Vinci. Hero was the first specifically to declare that the angle of incidence of light is equal to the angle of refraction.

Galen, an Asiatic Greek of the second century, and a contemporary of Ptolemy, is another figure of importance in connection with Leonardo. He studied medicine at Alexandria, but later practised at Rome, and acted as physician to the Emperor Marcus Aurelius. He

¹ W. Schmidt : *Leonardo da Vinci und Hero von Alexandria, Bibliotheca Mathematica*, third series, vol. iii., No. 180, pp. 180-187 (Leipzig, 1902).

² P. Duhem : *Origine de la Statique*, p. 86.

taught a physiological system of the human body which received general acceptance for centuries afterwards, and in particular his views as to the nature and action of the heart, the blood vessels and the pulse, remained practically unchallenged until Leonardo da Vinci, to whom Galen's works were familiar, took it upon himself to criticise them. He established two important points of difference. Galen taught that what he called the venal artery (our pulmonary vein) conveys air direct to the heart. Leonardo showed this to be incorrect. Secondly, Galen considered the heart to possess two cavities only—the ventricles. Leonardo proved it to possess four, the ventricles and the auricles, and he was able to show that the valves allow the blood to pass in one direction only.

V.—ROMAN INFLUENCES

As is well known, science amongst the Romans was very scanty and ineffectual as compared with that of their Greek predecessors.¹ Nevertheless, although the Romans lacked originality as abstract thinkers, the few writers they produced in the field of science exerted a very real influence upon the days that followed. Of these we may quote Marcus Terentius Varro (127–116 B.C.) with his *Rerum rusticarum libri III.*, Plinius Secundus (A.D. 3–79) with his very copious *Naturalis historia*, and Lucius Annæus Seneca (3 B.C.—A.D. 65), with his *Questiones naturales*. Vitruvius (Marcus Vitruvius Pollio), the famous architect and engineer of the Augustan era, only differs from the others in that his monumental treatise, *De architectura libri decem*, dedicated to the Emperor

¹ For a discussion of this, see C. Singer on *Roman Science* in Bailey's *Legacy of Rome* (Oxford, 1923).

Augustus, was lost for a long time, and was rediscovered in the fifteenth century. With the possible exception of Seneca, however, all were known to Leonardo, whose references are unmistakable, through the medium of editions in the Italian vernacular of his day.

Of Varro's *Rerum rusticarum* little need here be said. It deals with agriculture, cattle, and other farm animals, bees, fishes, and certain wild animals. Pliny's *Natural History*, a work in thirty-seven books, though much more extensive, is on no higher plane. Intellectual observer and exhaustive reader rather than critical scientist, Pliny roamed far and wide. Cosmology, geography, anthropology, zoology, botany, medicine, mineralogy, and art all come within his ken, and throughout his main theme is that Nature is intended to serve man. The work is almost entirely a copy, so far as material is concerned, from almost a multitude of Greek and Roman writers who preceded him.¹ Similar in this respect of borrowed material is Seneca's *Questiones naturales*, a work dealing chiefly with astronomy, meteorology, and the subject of earthquakes and allied phenomena.

The work of Vitruvius is very different. Dealing, as does the *Architecture*, with a definitely applied science, the evidence of the originality of a capable and ingenious architect and engineer is very real. Yet even here the historical and theoretical portions are largely copied from earlier Greek authors, whom, however, he clearly indicates as his sources. Italy in the fifteenth century was, as we have seen in the previous chapter, the centre of a

¹ M. Girolamo Calvi, in his *L'Archivio storico lombardo* (1898), argues that Leonardo da Vinci's notes on animals were derived from (1) Pliny's *Natural History*; (2) an anonymous work, *Le Fiore di Virtu*, written prior to 1323; and (3) book iii. of *L'Acerba*, by Cecco d'Ascoli (d. 1327).

very active band of brilliant architects and engineers, which included Bramante, Michelangelo, Palladio, and Leonardo da Vinci himself. All of these were careful students of Vitruvius, whose work is, therefore, from the point of view of this chapter, of the utmost importance.

VI.—ARABIC INFLUENCES

Turning to Arabic influences, we have first to consider a work, the *Liber Charastonis*, to which reference has already been made in connection with Euclid.¹ This work, a number of copies of which are extant, is associated with the name of Thâbit ibn Kurrah, a well-known Arabic geometer and commentator.² He was born in A.D. 836 in Mesopotamia, and taught mathematics at Baghdad. He made a diligent study of the Greek language, and translated and commented upon such writers as Euclid, Archimedes, Appolonius of Perga, and Ptolemy. He also wrote originally upon mathematical, astronomical, and astrological topics. He died in Baghdad in the year A.D. 901.

The author of the *Liber Charastonis*, supposed by some to be Euclid, is not known, and a discussion has developed³ as to whether "Charaston" was in fact the name of the author, or was derived from the Arabic rendering of the Greek term for "balance" (χαριστήριον). The latter theory has received the support of both Heiberg and Curtze.

¹ See p. 62.

² Details of his career are to be found in Wustenfelf, *Geschichte der Arabischen Ärzte und Naturforscher* (Göttingen, 1840).

³ M. Curtze: *Das Abgebliche Werk des Euklides über die Wage* (1874). See also Heiberg, *Literargeschichtliche Studien über Euklid* (Leipzig, 1883), p. 11.

Quite a number of Arabic philosophers served to influence Leonardo through their writings. Of these Jacob al Kindi, who wrote and translated extensively from the Greek on mathematics, meteorology, astronomy, and optics during the latter half of the eighth and the first half of the ninth centuries, may be quoted as an example. He exerted a definite influence, not only on da Vinci, but also, through his *De aspectibus*, upon many of the scholastics. It is recorded in the *Codex Atlanticus*¹ that Fazio Cardano possessed a copy of *Le proportioni d'Alchindi*, and that Leonardo requested the loan of it.

Another example is the well-known writer on anatomy and medicine, Avicenna (Ibn Sina A.D. 980—1037), who comes in for frequent mention in the note-books of da Vinci.² The references seem to indicate that Leonardo used a Latin translation of Avicenna's *De anima*, but he was also, through the medium of Mondino da Luzzi,³ familiar with the well-known fourteenth-century studies in anatomy based on the *Canon* of Avicenna. It was mentioned in the previous chapter⁴ how da Vinci, in collaboration with the ill-fated but brilliant young Marc Antonio della Torre, attacked the anatomical outlook of Arabian philosophy, with its lack of dissection, and in this respect Avicenna came in for his share of criticism.

Avicenna was also an important worker in the field of optics. He was a contemporary of Alhazen, and his

¹ *Codex Atlan.*, fol. 225 r.b.

² MS. F., cover; MS. Windsor An. IV., fol. 151 b. and fol. 167. See also J. P. Richter, *The Literary Works of Leonardo da Vinci* (London, 1883), paras. 1434, 1482.

³ G. Seailles: *Leonardo da Vinci, l'Artiste et le Savant* (Paris, 1892), p. 278.

⁴ See p. 52.

optical writings, apart from that contained in a small work *On Physics*, are scattered amongst many volumes on diverse subjects, and include a description of the anatomy of the eye.

Alhazen (Ibn al Haitham A.D. 965—1038), although nowhere mentioned by da Vinci, calls also for attention in these pages, since even if Leonardo had never seen his works, such writers as Avicenna and Witelo, who did influence our philosopher, were themselves intimately dependent upon Alhazen for their own sources. Nevertheless, Werner¹ has advanced a theory, resulting from a reference to a phrase, "the seventh of the fourth," used by Leonardo in connection with a problem on light rays, that da Vinci must in fact have handled either a Latin or an Italian translation of Alhazen. Werner based this view on the fact that in Risner's² edition of Alhazen, the same problem is dealt with under the reference, "The fourth of the seventh." Alhazen was a Mesopotamian, born at Basra. He considerably improved upon Ptolemy's studies on refraction, and did excellent quantitative work. He also studied the optical effects of curved mirrors, and advanced a much improved theory of the structure of the eye. His work has been ably discussed by Wiedemann.³

Finally, we have to mention Witelo, a Polish writer of the first half of the thirteenth century, to whom Leonardo refers in his note-books.⁴ He is mentioned here amongst the Arabic writers on account of the fact that although he

¹ O. Werner: *Zur Physik Leonardo da Vinci* (Erlangen, 1910), p. 29.

² Risner: *Opticæ thesauriis Alhazeni* (Basle, 1572), p. 235.

³ E. Wiedemann: *Zu Ibn al Haitam's Optik*, in the *Archiv für die Geschichte der Naturwissenschaften und der Technik*, iii., p. 1 (Leipzig, 1910).

⁴ MS. S. K. Mus., fol. 55 r.

knew no Arabic himself, he was a close student of both Alhazen and Avicenna through Latin translations.

VII.—LATER SOURCES

We come finally to the series of later writers in Latin of the thirteenth and fourteenth centuries, with whose works Leonardo was familiar, and by whom he must have been influenced. These include the philosophers of the Franciscan school, Roger Bacon and John of Peckham ; the thirteenth-century mathematician, Jordanus Nemorarius, and his anonymous successor and disciple, the "forerunner of Leonardo" ; and in the fourteenth century, Biagio Pelacani (Blaise of Parma), and Albert of Saxony.

Roger Bacon's influence upon Leonardo is clear and unmistakable, and it is fitting that this should be so, in view of the fact that the former stood very much in the same relationship to those about him in the thirteenth century in England as did the latter to those about him in the fifteenth and early sixteenth centuries in Italy. Both were incomparably ahead of their times. Both were isolated disciples of the doctrine of experimental inquiry.

We find Bacon specifically mentioned in the manuscript in the possession of the British Museum.¹ But quite apart from this reference, a distinct thread of similarity runs through the treatment of optical problems in the two cases. Roger Bacon was born in 1214 and died in 1294, and it is interesting to note, in view of what has already been said, that his mathematical ideas were based on Latin translations from Arabian works, particularly through the

¹ MS. Brit. Mus., fol. 71 b.

medium of Witelo. Hence we may trace through him a further influence upon Leonardo of both Avicenna and Alhazen. Of Roger Bacon it may be said that he was the first in the West at least to endeavour specifically to apply the laws of reflection and refraction to the artificial improvement of the power of vision. He had sufficient knowledge of the properties of lenses to enable him to foretell the telescope and compound microscope, and it is probable that he actually made a simple microscope.

John Peckham, Archbishop of Canterbury (1240–1292), was a contemporary of Roger Bacon, but his work, partly derived no doubt from Bacon himself, but mainly perhaps from Alhazen, was of a much poorer quality. In spite of this, his *Perspectiva communis* achieved a wide publicity, and was in general use as a text-book, and was often printed in the fifteenth and sixteenth centuries. We find no direct reference to Peckham in da Vinci's manuscripts, but we have already drawn attention to the friendship which existed between Leonardo and Fazio Cardano, the first editor of and commentator upon Peckham's *Perspectiva communis*.¹ Cardano's book was published in Milan in 1482, and that it must have been read by Leonardo follows definitely from the fact that the *Codex Atlanticus* contains an almost literal copy of the introduction to Cardano's book.²

We come next to another thirteenth-century philosopher, Jordanus Nemorarius. In him we have one of the most important of Leonardo's sources in mechanics. Of his works the *De ponderibus* is quoted directly in one of the

¹ *Perspectiva Communis d. Johannes Archiepiscopi Cantauriensi . . . ad unguem castigata per . . . Facium Cardanum* (Milan, 1482).

² *Codex Atlan.*, fol. 203 a.

Windsor manuscripts,¹ but, quite apart from such direct references,² evidences of similarity between Jordanus and da Vinci occur constantly.³ A point of importance in this connection is the fact that we see here a further evidence of the passing on to the Italian Renaissance of the Greek tradition in science, since Jordanus was essentially a follower of the Greek school in general, and of Aristotle, Archimedes, and Euclid in particular. Jordanus, although not so great intrinsically as his Italian contemporary, Leonardo of Pisa, nevertheless had a great following, and Fra Luca Pacioli, Leonardo's friend and colleague at Milan, mentions him frequently in his writings. Details of his life are very scanty. Jordanus was born⁴ at Borgentreich, near Warburg, and entered the Dominican Order in Paris in 1220. He was a prolific writer, chiefly in mathematics and mechanics. His works included *De numeris datis* (quoted by Regiomontanus), the *Liber Jordani de Astrolabis*, *De triangulis*, the *Tractatus de sphaera*, the *Arithmetica decem libris demonstrata*, *De speculis*, and *De ponderibus*.

Following on this philosopher was an anonymous writer who was evidently a close student of the writings of Jordanus, since his work, called by Duhem a *Commentaire Peripaticien*, was modelled after him. The fourth book of this work deals with dynamics, and there are evidences that Leonardo was acquainted with it.

¹ J. P. Richter: *The Literary Works of Leonardo da Vinci*, vol. ii., para. 1436.

² See also *Codex Atlan.*, fol. 1 v.a.; fol. 37 v.a.

³ This has been very fully dealt with by P. Duhem in his *Origine de la Statique* (Paris, 1905).

⁴ M. Cantor: *Vorlesungen uber Geschichte der Mathematik* (Leipzig, 1894).

Duhem, indeed, refers to this writer as the "forerunner of Leonardo da Vinci."

Evidence of this anonymous source is supplied in the Ashburnham manuscript,¹ which contains a discussion of a theorem by a fourteenth-century writer, Blaise of Parma (Biagio Pelacani). This theorem occurs in Pelacani's *Tractatus de ponderibus secundum Magistrum Blasium de Parma*,² and was frankly derived from the school of Jordanus above referred to. Blaise of Parma was a medical graduate at the University of Pavia in 1374. He taught astrology at Bologna from 1378-1384, moved on to Padua till 1388, and then returned to Bologna and Pavia again till 1407. He then migrated to Paris, where he taught till 1411. He died at Padua in the year 1416. His work was well known to Leonardo and his contemporaries. Pacioli and Cardano both

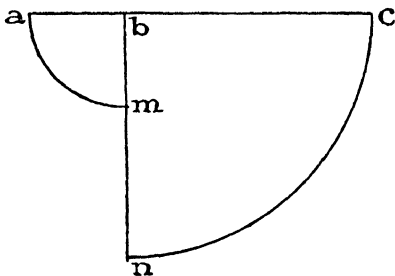


FIG. 14.

refer to him. A direct reference by Leonardo is to be found in a fragment of two pages preserved in the Bibliothèque Nationale.³ Leonardo reproduces a sketch (Fig. 14) from Blaise's book, and adds the following comment: "Pelacani says that the longer arm of this balance will fall quicker than the shorter, because its descent describes a quarter-circle," etc. Leonardo then draws another figure, and proceeds to correct Parma's false deduction.

¹ MSS. Ash. I., fol. 2 v.

² Bibliothèque Nationale, MS. 10,252.

³ Bibliothèque Nationale, MS. 2,038 (Italian), fol. 2 v.

Finally, we have to refer to another writer of the fourteenth century, Albert of Saxony, a scholastic whose works, there are sound reasons for believing, Leonardo studied. If the reader will refer back to the quotation given at the beginning of this chapter of the list of borrowings contained in the inside cover of Manuscript F., he will see the name "Albertuccio." It has been suggested that this refers to Leon Battista Alberti, who died in 1472, and whom Leonardo possibly knew.¹ Duhem has, however, shown that this view is incorrect, and that the reference is most probably to Albert of Saxony.² He points out that in the sixteenth century certainly Albert of Saxony was commonly referred to as Albertuccio. The succeeding reference on the same manuscript cover to "*Albert, De cælo et mundo*," strengthens this view, since Albert of Saxony wrote a work called *Questiones in libros de cælo et mundo*. Further, he also wrote a *Tractatus proportionum*, the second book of which is the *Tractatus de proportionem velocitatum in motu*, and this book received specific mention by Leonardo da Vinci in Manuscript I.³ in the following terms: "Albert of Saxony says in his *De proportionem* that if," etc. Duhem is clearly justified, therefore, in his conclusion that Leonardo must have studied at least two of the works of Albert of Saxony.

Details of Albert's career are somewhat scanty. He appears to have studied in Paris, and to have taught there at the Sorbonne from 1350-1361, after which he retired to the curacies of the parishes of St. Côme and St. Damien. One copy of his *De Cælo et Mundo* is dated 1378.

¹ See Chap. III., p. 40.

² P. Duhem: *Les Origines de la Statique*, vol. i., pp. 10 et seq.

³ MS. I., fol. 120 r.

Chapter V

Leonardo da Vinci's Mechanics—Dynamics

I.—INTRODUCTION

WE now proceed to consider the various contributions by Leonardo da Vinci in the field of mechanics—a study to which he referred as the “paradise of the mathematical sciences.”¹ His notes on this subject are found scattered throughout his writing, with a frequency and a persistence which show clearly the importance which he attached to it. They have at various times been considered by a number of students, notably by Grothus² and Schuster³ (and more superficially by Seailles⁴) for theoretical mechanics, and by Feldhaus⁵ in the study of Leonardo's applications of mechanical principles to mechanisms. By virtue of the nature of the notebooks, and of the circumstances under which they were compiled, there is always, of course, the danger of reading into da Vinci's notes and sketches an ordered and a systematic scheme of mechanics which did not in fact exist in his mind. Nevertheless, the constant repetitions in

¹ MS. E., fol. 8 v.

² H. Grothus: *Leonardo da Vinci als Ingenieur und Philosoph*. (Berlin, 1874).

³ F. Schuster: *Zur Mechanik Leonardo da Vincis* (Erlangen, 1910).

⁴ G. Seailles: *Leonardo da Vinci, l'Artiste et le Savant* (Paris, 1906).

⁵ F. M. Feldhaus: *Leonardo da Vinci als Techniker und Erfinder* (Jena, 1913).

various forms of important statements and experiments do speak for themselves with a definiteness which must not be denied, and to this we must repeat the plea stated earlier that the final judgment of a philosopher must depend, not upon his casual notes, jotted down at various odd moments, but upon the connected writings derived from these notes. We have already pointed out that this was definitely Leonardo's intention, but that he never in fact reached this stage. As a consequence we are entitled to collate his materials, and to set them forth in such a manner as would enable us to estimate their intrinsic value, both in the light of the contemporary knowledge of his own days and of those which followed in the normal course of the historical development of mechanics.

At the outset it may be stated frankly that Leonardo suffered badly from the want of a precise and an accurate scientific vocabulary. Indeed, to the student of modern mechanics, accustomed to attach specific meanings to specific terms, there must be an instinctive tendency to condemn outright much of da Vinci's scientific writing under the general charges of "vagueness" and "looseness of thought and expression." All the modern ideas of force, motion, mass, inertia, work, moment, etc., are constantly to be found among the note-books of our philosopher, but they are clouded by a phraseology which is rarely precise, which is frequently puzzling, and which is seldom rigid. Yet such a condemnation as this would surely be most unjust. The notion of rigidity in scientific thought had no place in the fifteenth century, nor, indeed, in any preceding period. We except, of course, the science of pure mathematics, which from its very nature demanded rigidity of treatment. It may not, perhaps,

be considered a diversion in this connection (since in da Vinci's days the conditions were not altogether dissimilar) to draw attention to the Greek era in which a beautiful logic and rigidity of geometrical science developed side by side with an extravagant range of loose speculation and vagueness of nomenclature in those realms of general philosophy which became the forerunner of our modern science.

Looseness of expression is frequently the result of ignorance, and it is not an unknown device as a cloak of assumed wisdom to conceal ignorance. This was a device typical of the encyclopædists of the Dark Ages. No one can accuse Leonardo da Vinci of belonging to this class of writer. There is, however, the other side of the picture—the looseness of expression due to a sheer inadequacy of words to convey new ideas, such as may occur to a writer who lives ahead of his times. It is to this side of the picture that Leonardo belongs. Many of his ideas, lost with the dispersal of his manuscripts, had to be re-discovered by others long after him. That these later philosophers had the benefit of a later and more complete terminology with which to clothe their discoveries is a fact that need neither detract from the credit due to Leonardo nor blind us to the difficult path of inadequate language through which he had to grope his way.

Yet one other factor requires mention in this connection. Clarity of thought in the exact sciences could not properly come into its own until the test of experiment had been recognised and practised. It is a commonplace in the history of science to say that little real headway was made until the advent of such pioneers of experimental science as Galileo in Italy and Gilbert in England.

These preachers, by inaugurating a new era in scientific research, heralded the downfall of the dogmatists, and paved the way for such an advance in accurate scientific knowledge as to render absolutely imperative an expansion of technical vocabulary. The rigidity of thought and of proof which was so striking a feature of Greek geometry became extended for all time to other sciences, thus establishing for mathematics that rôle of handmaiden to the sciences which Leonardo da Vinci clearly saw for it.¹

Now Leonardo, more than one hundred years before Galileo, was most definitely an experimentalist. If the great Tuscan philosopher was indeed the "Father of Experimental Philosophy," then da Vinci was its grandfather. "Experience" (*i.e.*, experiment), says he, "never deceives ; it is only our judgment which deceives us, promising from it the things which are not in its power."² The appeal to experiment pervades all his writings on scientific topics. What could be more specific as an enunciation of the experimental method, for example, than the following ? "Before making this case a general rule, test it by experiment two or three times, and see if the experience produces the same effect."³ Small wonder,

¹ MS. G., fol. 95 b. : "There is no certainty in sciences to which one of the mathematical sciences cannot be applied, or which are not in relationship with these mathematics." Again, in MS. W., An. III., fol. 241 a., we read : "The man who blames the supreme certainty of mathematics feeds on confusion, and can never silence the contradiction of sophistical sciences which lead to an eternal quackery." More particularly, referring to the subject under discussion in this present chapter, we find in MS. E., fol. 8 v., the following reference : "Mechanics is the paradise of the mathematical sciences, because here we come to the fruits of mathematics."

² *Codex Atlan.*, fol. 154 r. See also fols. 23 r., 119 r.

³ MS. A., fol. 47 r.

therefore, that his ideas and his discoveries outstripped his language. Living and thinking in terms of the late sixteenth and seventeenth centuries, he was yet compelled to express himself in the restricted language of the fifteenth. Vagueness of expression in the circumstances was inevitable.

For these reasons, therefore, it behoves us to approach the notes on mechanics in no hypercritical spirit, but to make due allowances by assuming for ourselves rather the *rôle* of the "man in the street," to whom ideas of scientific rigidity are foreign and are replaced by what he would call the "common sense of it."

II.—FORCE, ENERGY AND MOTION

Force and the Principle of Inertia.—Immediate examples of the vagueness of language, but not necessarily of thought, occur in the various references contained in Leonardo's manuscript to the term "force." Mechanics being in effect the study of rest and motion, da Vinci was quite clear as to the fundamental importance of this concept of force.

"Nothing whatever can be moved by itself," he tells us,¹ "but its motion is effected through another. This other is the force." We have here at one and the same time his conception of the term "force" and what we speak of to-day as the Principle of Inertia. It is by no means the only example. Thus, in his endeavour to bring out the non-material nature of force, he writes ² :

Force I define as a spiritual power, incorporeal and invisible, which with brief life is produced in those

¹ MS. A., fol. 21 v.

² MS. B., fol. 63 r.

bodies which, as the result of accidental violence, are brought out of their natural state and condition. I have said spiritual because in this force there is an active, incorporeal life ; and I call it invisible because the body in which it is created does not increase in weight or size, and of brief duration because it desires perpetually to subdue its cause, and when it is subdued it kills itself.

This passage contains, of course, much that sounds like nonsense if read in the language of modern science. But the scientific spirit is there. His analysis of what he regarded as the three " properties " of force—spirituality, invisibility, and brevity of duration—and his common-sense deduction as to the invariability of the " weight " (*i.e.*, of the mass) of a body before, during, and after the action of the force, are of a standard far beyond his days.

We have already mentioned the implication of the Principle of Inertia in Leonardo's understanding of the term " force." He says that a body which is at rest cannot move of itself. It is important to note that he extended this to the case of the inertia of a moving body. Thus in his manuscript *On the Flight of Birds*, in the course of a discussion on the sudden turning of a bird on to its side, he says, " And because all movement tends to maintenance, or rather all moved bodies continue to move as long as the impression of the force of their motors remains in them." ¹ Here, surely, we have a real flash of the scientific genius which forestalled Galileo (to whom the credit of the first enunciation of the Principle of Inertia is generally ascribed) by more than a hundred years. One can almost read in the above phraseology the classic

¹ *Sul Volo degli Uccelli*, fol. 13 (12) r.

First Law of Motion of Newton as it is taught to-day in schools.

The Law of Reactions.—The first was not the only one of the famous “ Three Laws ” which da Vinci either tacitly assumed or actually enunciated, although not, of course, in the precise or definite manner of his successors. It may reasonably be claimed that in its crudest sense—namely, that the effect of a force is proportional to the magnitude of the force acting—the Second Law of Motion is more or less implied in da Vinci's notes.

The Law of Reactions, however, certainly does find a definite place in the manuscripts. Thus in the *Codex Atlanticus* in the course of a note on the parachute we read : “ An object offers as much resistance to the air as the air does to the object.”¹ Although the statement here takes the form of a particular application, it is clearly of a character which leaves little doubt as to the writer's definite appreciation of a general law of reactions. A similar reference is to be found later in the same manuscript.²

Force and Motion.—Turning to the relationship between force and motion, it has already been pointed out in the previous chapter³ that da Vinci, following the teaching of Aristotle, laid down a series of enunciations in this connection.⁴ A reference to these will show that in effect he regards the intensity of a mechanical action (*i.e.*, of a force) to be proportional to the velocity which it communicates to a mobile body. He had not reached the stage of realising that it was the *acceleration*

¹ *Codex Atlan.*, fol. 372 b.

² *Ibid.*, fol. 1158 b.

³ Chap. IV., p. 57.

⁴ MS. F., fol. 26 r.; fol. 51 v.

which was in question, and not the velocity. As an example of this, let us test in the light of modern elementary dynamics the second of Leonardo's enunciations, namely, that "if a force moves a body in a given time over a given distance, the same force will move half this mass through the same distance in half the time."

Using the formula $F = Ma$, and denoting by a_1 the acceleration produced when the mass reduces to $\frac{M}{2}$, we have—

$$(1) F = Ma, \text{ giving } a = \frac{F}{M},$$

$$\text{and } (2) F = \frac{Ma_1}{2}, \text{ giving } a_1 = \frac{2F}{M},$$

whence $a_1 = 2a$.

Next let s be the distance through which the mass moves, mass M in time t and mass $\frac{M}{2} = M_1$ in time t_1 .

$$\text{Now } s = \frac{1}{2} a t^2, \text{ giving } t^2 = \frac{2s}{a}.$$

$$\text{Also } s = \frac{1}{2} a_1 t_1^2$$

$$= \frac{1}{2} \cdot 2 a t_1^2, \text{ giving } t_1^2 = \frac{s}{a}.$$

$$\text{Hence } \frac{t^2}{t_1^2} = \frac{2}{1}, \text{ giving } \frac{t}{t_1} = \frac{\sqrt{2}}{1},$$

and not $\frac{2}{1}$, as stated by Leonardo.

Falling Bodies.—We turn next to da Vinci's treatment of the subject of falling bodies. We shall see that here again our philosopher just fell short of the realisation of an important truth which might have carried him very

far. However, throughout all his scattered notes on this subject Leonardo the experimentalist stands out very clear.

If a weight of a hundred pounds falls ten times from the height of ten fathoms to one spot, and sinks in one fathom, how much would it sink in falling from the height of one hundred fathoms ? ¹

is a typical query. In another manuscript ² we read as follows :

Why does not the weight *O* remain in its place ? It does not remain because it has no resistance. Where will it move to ? It will move towards the centre (of the earth). And why by no other line ? Because a weight which has no support falls by the shortest route to the lowest point, which is the centre of the world. And why does the weight know how to find it by so short a line ? Because it is not independent and does not move about in various directions.

Here, then, we get a concise statement of Leonardo's philosophy of the falling body. It was his study experimentally of bodies falling freely under the action of their own weight which led him to the idea of a uniformly accelerated motion. A note in Manuscript M. is very significant in this connection :

In the air of uniform density, the heavy body which falls at each stage of time acquires a degree of movement more than the degree (of movement) of the preceding time.³

¹ MS. A., fol. 3 r.

² MS. Brit. Mus., fol. 175 a.

³ MS. M., fol. 44 v.

Here the idea of an acceleration is very clearly brought out. Later in the same manuscript we read :

If after the descent of a fathom by one ball you let fall another, you will find that at each stage of the fall the proportion of their velocities and powers will change.¹

Interestingly enough, in the same manuscript another passage² makes it clear that da Vinci was aware of the influence of the air. He discusses the fall of two equal weights, the one vertically above the other, and notes that the distance between them steadily diminishes as they fall until the bodies touch, his explanation being that, owing to the screening effect of the lower body upon the upper one, the resistance of the air retards the motion of the former, but not of the latter.

Leonardo da Vinci definitely sought to discover the true law of accelerations for falling bodies. In one note, for example, he enunciates his problem thus :

If a weight falls a distance of two hundred fathoms, by how much would it fall quicker in the second hundred fathoms than in the first ?³

Unfortunately he failed to find a correct answer to his question. The correct law for falling bodies is, of course, that the fall is proportional to the square of the time, but Leonardo concluded that the function was linear. In Manuscript M. we find the following passage, a portion of which has already been quoted above :

In the air of uniform density, the heavy body which falls at each stage of time acquires a degree of movement more than the degree (of movement) of the

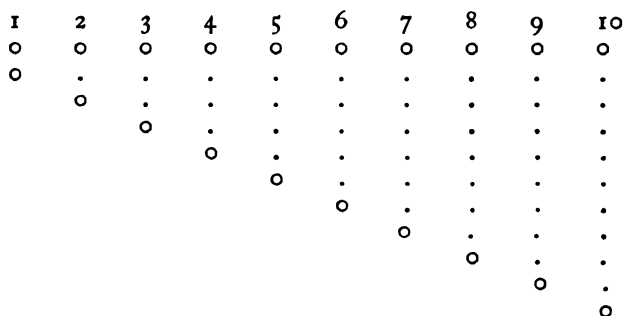
¹ MS. M., fol. 61 r.

² MS. A., fol. 43 v.

³ *Ibid.*, fol. 3 v.

preceding time, and likewise a degree of velocity more than the degree (of velocity) of the preceding time. *Then to each quantity doubled in time the length of the descent is doubled, likewise the velocity of the movement.*¹

We see from the sentence in italics that da Vinci regards (1) the velocity as proportional to the time (which is correct ; cf. the formula in kinematics, $v = gt$), and (2) the height of fall as also proportional to the time (which is incorrect). Additional interest is lent to the above passage by a sort of graphical representation of a velocity-time scheme adjoining it, with a time scale shown horizontally, and a velocity scale vertically, somewhat as follows :



The note against this scheme reads thus :

Here is shown how such a proportion as one quantity of time with another will have a quantity of movement with the other, and a quantity of speed with the other.

From some points of view it may fairly be claimed that this was the earliest example of the graphic method in the history of the mathematical sciences, and as such it calls for our respectful admiration.

Influence of the Earth's Rotation on Falling Bodies.—In the course of his investigations on the falling of heavy

¹ MS. M., fol. 44 v.

weights Leonardo shows us a very brilliant example of sound scientific deduction. He dropped a number of heavy weights from the top of a tower, and noticed that they did not meet the ground absolutely vertically beneath the starting point. There was, he thought, a small easterly deviation, and this he proceeded to explain by its true cause, namely, the rotation of the earth on its axis, or rather, in the language of the time, the movement of the surrounding spheres around the earth.

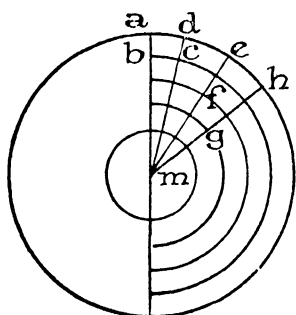


FIG. 15.

Actually, it seems impossible that in the relatively small fall of a body from the top of a tower such a deviation is capable of observation. The probability is that Leonardo *thought* he saw it, and was thus started on a train of thought which led him to believe that there *must* be such a deviation, so that it would never afterwards occur to him to

doubt the accuracy of his observations. This, after all is not an isolated example of the part played by the imagination in scientific discovery. Dalton offers another instance in the train of ideas and impressions that led up to his enunciation of the atomic theory. In Leonardo's case the passage in question occurs in Manuscript G. of the collection at the Institute of France, and is illustrated by a diagram similar to Fig. 15.

Let *A* be the body [he says ¹] which falls from the highest point of the sphere of fire ² in order to come

¹ MS. G., fols. 54 v.; 55 r.

² It should be remembered that the mediæval conception of the universe was that of the firmament around the *globus mundi* being surrounded by

to the mid-point of the earth m . I say that this weight, descending in a spiral, will not leave the straight line which follows its way towards the mid-point of the earth, because if the body goes from a in order to reach b , so will it go towards b and come to the position c . The point a in turning reaches d . If we now consider the position of the body, we see that it is always found in the straight line which, originally at a and now at d , joins it to the mid-point of the earth at m . If the body goes further towards f , then the point d will at the same time go to e . During the descent from f towards g the rotation of e is brought into the position of h . Thus the body descends to the earth, always out of the starting point. This is an accumulative movement. It is a combination of a straight and a curved line. It is a straight line because the body is always found on the shortest line which joins it from a to the centre of the earth. It is a curved line in itself and in every point of the path. Therefore a stone thrown from the height of a tower does not strike the wall of the tower until it reaches the earth.

The Compounding of Forces and Velocities.—A point of immediate importance in connection with the preceding note is that we are here afforded a definite example in Leonardo's manuscripts of the virtual compounding of two velocities or motions into a single resultant motion different from either of the constituent ones. Seailles¹ even goes so far as to regard it as an appreciation by Leonardo of the principle of the physical independence

"elemental" spheres (moving successively outwards) of water, air, and fire, and beyond these the planetary spheres, then the sphere of the fixed stars, and finally the *primum mobile*. (See Fig. on p. 53.)

¹ G. Seailles: *Leonardo da Vinci, l'Artiste et le Savant* (Paris, 1905), p. 229.

of forces and velocities, but this is perhaps straining the point beyond the stage of justification. The question, however, of whether or no we may fairly say of da Vinci

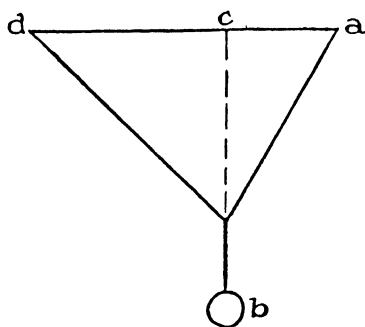


FIG. 16.

that he knew that of which to-day we speak as the principle of the parallelogram of forces and of velocities, is full of both interest and importance. Probably it is most true to say that he knew *something* of this principle ; that he was, in fact, on the fringe of its discovery. Duhem definitely takes the

opposite view,¹ and in support of this he quotes a passage which is accompanied in the manuscript by the diagram shown in Fig. 16, and which reads as follows :

The suspended weight at the angle will divide its weight between the two sides of the angle in the same proportion as that of the obliquity of their sides. Or such a weight will distribute itself between its supports in the same proportion as that of the two angles formed of the division of the angle where the weight is suspended. . . . Thus, the angle $a b d$ being cut by the line $e b$ and the angle $e b d$ being 9-11 of the angle $a b c$, the angle $a b e$ is 2-11 ; $a b$ is 9-11 of the weight, and $d b$ 2-11. ²

This, of course, is a faulty conclusion, but it does not follow that it is not due to faulty mathematics rather than to faulty mechanics. After all, judging by their absence from his note-books, Leonardo was probably none too

¹ P. Duhem : *Les Origines de la Statique* (Paris, 1903).

² MS. E., fol. 6 r.

happy with his trigonometrical ratios. But quite apart from any such considerations, we can hardly expect such a problem as the above, essentially one on the tensions produced in the two strings by the hanging weight, to be regarded by anybody in the fifteenth century, however brilliant, as an example of the combining of two forces into a single equivalent resultant. It is fundamentally either a deduction from the *converse* of the principle of the parallelogram of forces (to which we will refer in the next paragraph), or it may be looked upon as a specific case of the principle of the triangle of forces. We are bound to say, therefore, that quite apart from whether Duhem was right or wrong in his conclusion, the example he quotes in support of his contention is scarcely admissible.

It is in fact difficult to find a definite example in which da Vinci is led to inquire into the effect of two forces acting simultaneously upon a body. In Chapter I., however, we had occasion to mention one such note (see Fig. 1), the two forces in question being the weight of the bird and the force of the wind. But although Leonardo neither constructs nor mentions a parallelogram, he at least definitely speaks of a resultant effect—he mixes his forces and his velocities quite freely—intermediate between the two.

Again, the example quoted in the previous paragraph of the deviation from the vertical of a freely falling body due to rotation of the earth on its axis is also definite enough as proof of da Vinci's appreciation of the fact that a body acted upon by two different motions simultaneously takes up a resultant path differing from either. But this, probably, is as far as we may dare go. To say less would be unjust—to say more would be to state that which is not properly substantiated by evidence.

The Resolution of Forces.—But what of the converse problem—that of the resolution of a single force or velocity into two components? References in Manuscript G. are most illuminating in this connection. In the course of a discussion on the motion of a weight sliding down an inclined plane, we read :

The weight which descends obliquely divides its weight in two different aspects. One proves it thus : Let ab (Fig. 17) move according to the obliquity abc . I say that the weight of the body ab divides its gravity in two aspects, that is, according to the line bc and according to the line mn . Why and how much the weight is greater for one than for the other aspect, and what obliquity is that which divides the two weights in equal parts, would be in the weight of the body.¹

It is difficult to deny for Leonardo, in the face of this note,

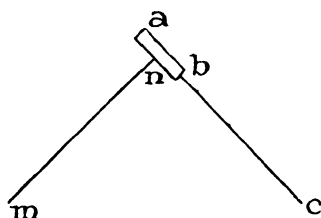


FIG. 17.

an understanding of the resolving of a force into two components in different directions ; but again we are unable to say that he knew in the complete form of to-day the "principle of the resolution of forces," either as such, or as a converse of the "principle of the parallelogram of forces."

Motion down an Inclined Plane.—It will be convenient to consider at this stage more generally the way in which Leonardo da Vinci dealt with the problem of the motion of a body down an inclined plane, a subject not

¹ MS. G., fols. 75 r., 76 r.

successfully attempted by any of his predecessors. Notes on this problem are to be found in the *Codex Atlanticus* and in Manuscript A. He first considers a sphere on a horizontal plane, and points out that the perpendicular through the centre of gravity passes through the point of contact, there being no resultant motion. Thus we read :

The perfect sphere placed on a perfect plane, will have no movement whatever, if you do not give it any. And the reason of it is that all its parts are equally distant from the centre ; consequently it always remains in equilibrium, and the balance of its equal arms of weight and length remain without movement.¹

He next passes on to a sphere on an inclined plane, and shows in the accompanying figure that the vertical line through the centre of gravity lies on one side of the point of contact, and the body is driven down the plane by the action of force. Leonardo realises that as the angle of

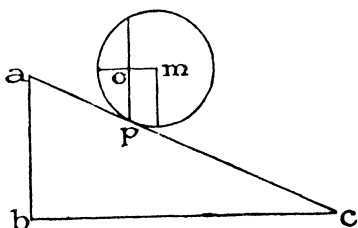


FIG. 18.

inclination of the plane increases, so do both : (1) the distance between the point of contact and the vertical line through the centre of gravity (*i.e.*, the distance *om* in Fig. 18), and (2) the speed of movement of the sphere down the plane. He assumes (presumably from general observation), that this speed of movement is proportional to *om*. "The greater is the distance between *m* and *p*," he writes, "the greater will be its speed."² It would not,

¹ MS. A., fol. 21 v.

² *Ibid.*, fol. 52 r.

of course, be correct to speak of this motion down the plane as a "velocity," since in fact the motion is accelerated. So keen an observer as Leonardo will naturally have realised this fully, and as is seen from the passage quoted below, he uses the term "rapidity of movement." Leonardo's conclusion may be given in his own words :

The spherical body (Fig. 18) will take up a movement which is more rapid as its contact with the point where it runs will be more distant from the perpendicular from the central line. As much as $a b$ is less than $a c$, so much will the spherical body fall more slowly down the line $a c$.¹

It is improbable that da Vinci, in the phrase "as much as $a b$ is less than $a c$," had in his mind the idea of a difference. It is more likely he was thinking of the *ratio* of $a b$ to $a c$. Accordingly, his conclusion really amounts to the statement that

$$\frac{\text{rapidity of movement of sphere down inclined plane}}{\text{rapidity of movement of the sphere freely falling}}$$

$$= \frac{\text{the height of the fall } a b}{\text{the length of the plane } a c}.$$

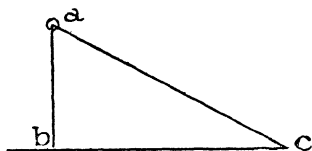


FIG. 19.

This is strikingly borne out by another passage in the *Codex Atlanticus*, which reads : "The fall of the body a on the line

$a c$ requires a longer time in proportion to the fall $a b$ as $a c$ is to $a b$ " (Fig. 19). A comparison of the facts of the motion of a sphere down a smooth inclined plane, as we know them to-day, clearly reflects, therefore, the greatest credit upon our fifteenth-century philosopher.

¹ MS. A., fol. 52^r.

The Principle of Work.—We next turn to a consideration of Leonardo's writings regarding that of which we speak nowadays as the "principle of virtual velocities." Inasmuch as this is generally considered in text-books from the standpoint of forces in equilibrium, it may perhaps more properly be regarded as belonging to the realms of statics ; but our philosopher himself makes no hard and fast distinctions in his notes as between statics and dynamics, and it will on the whole be more convenient for us to discuss the matter at this stage, especially as some aspects of it undoubtedly come within the sphere of the subject of this chapter.

It need scarcely be said at the outset that da Vinci never made use of the term "work" in the modern technical sense. Nor did he have any clear ideas as to the importance of the product of force and distance. Nevertheless, he did appreciate the fact of a value in, and a measure of, what we might speak of as the *achievement* of a force. We have already alluded to the series of enunciations regarding forces which occurs in Manuscript F.¹ The last of these reads as follows :

If a force carries a weight in a certain time through a definite distance, the same force will carry half the body in the same time through double the path.²

There are other pronouncements in the same strain. They contain the germs—and, indeed, the live and very healthy germs—of the present-day principle of work, or the principle of virtual velocities. There is, in effect, a

¹ See Chap. IV., p. 57.

² MS. F., fol. 51 v.

definite limit to the results of a given effort, and this effort is not alone a question of the magnitude of the force, but also of the distance, in any given time, through which it acts. If the one be increased, it can only be at the expense of the other.

Again, in the *Codex Atlanticus*, in the course of a note on the use of a machine for the movement of heavy bodies, Leonardo writes :

When the moving part exerts more movement in the same time than the moved body, then it (*i.e.*, the power) has more force than the moving body, and it will move more quickly than the body itself. When the part which is the moving part has less speed than the moved part, then it will have the less force than the moved body.¹

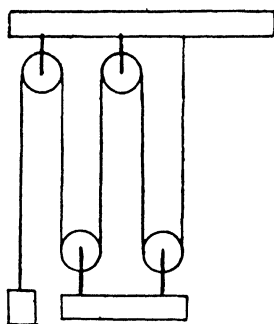


FIG. 20.

Again we see the contention, in effect, that the forces which are in equilibrium in a machine are in inverse proportion to their virtual velocities.

There is to be found in Manuscript E. a beautifully clear application of this, albeit typical of many others, in what is really a discussion on the mechanical advantage of a pulley system, illustrated in the note by a sketch (Fig. 20).

The powers [writes Leonardo] that the cords interposed between the pulleys receive from their movement are between them in the same proportion

¹ *Codex Atlan.*, fol. 185r.

as are between the speeds of their movements. The movements made by the cords on their pulleys, the movement of the last cord is in the same proportion with the first as that of the number of the cords ; that is to say, that if there are five, the first cord moving one yard, the last will move a fifth of a yard ; and if there are six, this last cord would have the movement of one-sixth of a yard, and so on to infinity. The proportion which the movement of the force of the pulleys will have with the movement of the weights raised by the pulleys will be such as that of the weights raised by these pulleys with the weight of the force.¹

Perpetual Motion.—Intimately linked up with this principle of work is the age-old myth of perpetual motion. If the principle of work be true, then the achievement of perpetual motion is impossible. On this matter, Leonardo had no illusions whatever. There were, however, many contemporary with him, vain ancestors of our present-day would-be inventors of the perpetual motion machine, who thought otherwise, and with these da Vinci had no patience.

Oh speculators on perpetual motion [he writes] how many vain projects of the like character you have created ! Go and be the companions of the searchers after gold ! ”²

Leonardo understood clearly the limitations implied by the principle of work.

No inanimate object will move of its own accord ; consequently when in motion it will be moved by unequal power, unequal that is in time and velocity,

¹ MS. E., fol. 20 r.

² MS. S.K., fols. 122, 67 a.

or unequal in weight ; and when the impulse of the first motive power ceases, the second will cease abruptly.¹

In other words, a given expenditure of power produces only a finite result, and if further results are required, then a further expenditure of power is necessary. Later in the same manuscript we find a practical illustration of this, in a note on the water-wheel.

If a wheel is moved for a moment by a quantity of water, and if this water is not added to either by flowing or by quantity, or by a greater fall, the function of this water is finished.²

With such lucid conceptions, we need scarcely be surprised, therefore, that Leonardo da Vinci writes "*Satanasso*" against comments on "perpetual motion" in his notes, and that he consigns the "perpetual motion" dreamers with a gesture to their true place among the ranks of the large army of necromancers, searchers after the philosopher's stone, and the other quacks of those times.

Impulsive Forces.—We conclude our survey of the dynamics of Leonardo da Vinci with a brief comment upon his notes on the subject of impulsive forces. That he realised that these forces of percussion, as he called them, were at all different from forces in general is a fact which in itself calls for our appreciation ; but there was certainly something more in his understanding of this class of forces than the bald fact of their existence. It is clear from his references in the *Codex Atlanticus*³ and else-

¹ MS. A., fol. 2 v.

² *Ibid.*, fol. 30 r.

³ *Codex Atlan.*, fols. 28, 47, 64, etc.

where that he carried out a number of experiments both on the collision of bodies, and upon the recoil of spherical bodies from plane surfaces. His two chief conclusions are on the subject of the glancing impact of smooth spherical bodies on smooth planes. The first is that a blow applied normally to a surface has a greater effect on the plane than one applied at an angle, and that of two such inclined blows, the one whose angle of incidence is the smaller produces the greater effect.

The blow [he writes] will be less powerful than its impulse according as the angle of its percussion is nearer the right angle.¹

The second conclusion is that for a rebounding body, the angle of incidence is equal to the angle of rebound. This is embodied in a note headed "Percussion," which reads as follows :

The line of percussion (incidence) and that of rebound (reflection) are placed at the middle of equal angles. All blows struck on an object rebound by an angle equal to that of the percussion.

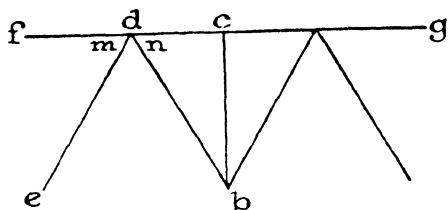


FIG. 21.

This proposition appears clearly ; in effect, if you strike a wall with a sphere, it will rebound behind by an angle equal to that of the percussion. Thus if the ball *b* (Fig. 21) is thrown on *c* it will return by the line *c b*, because it is constrained to fall on the wall *f g* at equal angles (*i.e.*, normally) ; and if you throw it by the line *b d*, it will return back

¹ MS. A., fol. 21 v.

by the line de , and thus the line of percussion (incidence) and the line of rebound make an angle on the wall fg situated at the middle of two equal angles, as one sees the angle d between the angles m and n .¹

Conclusion.—We have now passed in review the chief contributions of Leonardo da Vinci to one of the two main branches of the science of mechanics. It will be seen in the succeeding chapter that the field of statics claimed even a larger share of his attention. Nevertheless the reader is reminded that whereas up to the time of Leonardo the history of mechanics was virtually a history of statics, the study of bodies in motion was almost entirely a new one. In this respect, as in so many others, our philosopher was a true pioneer. Dynamics had made such very little headway since the Greek days of speculation on theories of gravity chiefly because the influence of the Peripatetics was persistently against experiment—a situation with a distinct element of irony in view of the fact that their greatest master, Aristotle, was himself so keen an observer of nature. From this attitude, as we have seen, Leonardo broke completely away. Paying due respect to the wisdom of the ancients, who indeed influenced his views very profoundly, as was shown in Chapter IV., he yet maintained the claims of the test of experiment. Naturally a genius, professionally an artist and an engineer, he was bound to break new ground. To those who, surveying the subject matter of the previous pages of this chapter, are tempted to regard the dynamics of Leonardo as superficial and bare, we would answer that the wonder is not that he did so little, but that he accomplished so much. For of the whole of it, it may justly be said that it constituted pure research in a virgin field.

¹ MS. A., fol. 19 r.

Chapter VI

Leonardo da Vinci's Mechanics—Statics

INTRODUCTION.—Da Vinci's *statics* covers a very wide field. In a sense this is not surprising. Unlike dynamics, the way had been pointed by his predecessors. He had at hand, as was shown in Chapter IV., the fruits of the labours of Aristotle, of Archimedes, of Jordanus Nemorarius and his anonymous successor, and of others ; and he applied these materials in his own way with his usual vigour and independence. If the range of da Vinci's statics was wide, it was also, within the possible limits of his days, thorough. Nevertheless, in submitting this survey of Leonardo's notes on this second of the main branches of mechanics to the reader, it is necessary to point out that the same prefatory caution as was offered regarding Leonardo's dynamics, as to the attitude of criticism to be adopted towards the general faults of language and nomenclature, necessarily applies with equal force to his statics.

Centres of Gravity.—Referring back to the summary in Chapter IV.¹ of the fundamental contributions to mechanics by Aristotle and Archimedes (the latter of whom, by virtue of the scientific superiority of his works, is the more important of the two), it will be obvious that these centred chiefly round the lever laws, or the laws of the balance, and the conception of centres of gravity. Naturally, therefore, the traditional approach to the

¹ Chapter IV., pp. 56 *et seq.*

whole subject by all later writers was through these fundamentals, and in this respect Leonardo was no exception.

The law of the balance, imperfectly presented by Aristotle, received its real scientific blessing at the hands of the great Syracusan philosopher. Its demonstration was based on certain definitions and axioms,¹ amongst which is the statement that in every body there is a definite point called *a centre of gravity*, at which we may suppose the weight of the body to be collected.

A consideration, therefore, of Leonardo's pronouncements upon the subject of centres of gravity constitutes a suitable starting point for our study. This was a subject of peculiar importance and interest to da Vinci. It touched upon matters vital to his professional career. As an engineer he realised its importance in connection with the stability of structures and of the machines he was called upon to devise ; as an artist he was constantly concerned with the balance of the human frame. References to the centre of gravity are therefore plentiful.² Yet it is difficult to find any actual attempt at definition. This, however, need not be surprising. The conception of the centre of gravity is one which has grown up literally through the ages. It had penetrated all existing writings on mechanics. It was, so to speak, a commonplace of ancient and mediæval science. Let us remember, further, that da Vinci left us not a complete treatise, but a compendium of notes. Where he might have attempted a

¹ These have been ably presented by J. M. Child in a recent paper, "Archimedes' Principles of the Balance, and Some Criticisms upon It," in C. Singer's *Studies in the History and Method of Science*, vol. ii., p. 490 (Oxford, 1921).

² *Codex Atlan.*, fol. 86 r.a. ; MS. A., fol. 5 r. ; MS. G., fol. 78 v. ; MS. H., fol. 105 r. ; MS. M., fol. 37 r.

general definition in a text-book, one can understand its omission from a collection of notes. Nevertheless, that Leonardo regarded the conception of centre of gravity as fundamental to mechanics is clear enough from his remark that

Mechanical science is very noble and useful beyond all others, for by its means all animated bodies which have movement perform their operations ; which movement proceeds from their centre of gravity. This is situated at the centre, except with unequal (distribution of) weight.¹

It is clear, too, that he studied the subject experimentally. In Manuscript B. is shown a sketch of a suspended weight (Fig. 22) with the note, "The centre of all suspended weights is established under its support,"² and in a similar sketch of a suspended artificial bird, we read "This is done to find the centre of gravity of a bird."³ A number of da Vinci's notes on this subject occur as incidental to his studies on the poise of the human figure. They bring out his appreciation of the need for a due distribution of weight about the "axis" (*i.e.*, the vertical line through the centre of gravity) under such varying circumstances as standing, sitting, kneeling, walking up and down hill, mounting stairs and ladders, and so on. The following is quoted as a typical example :

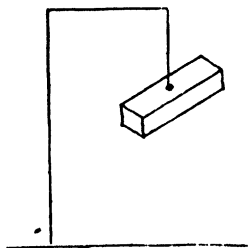


FIG. 22.

¹ *Sul Volo degli Uccelli*, fol. 3 r.

² MS. B., fol. 18 v.

³ *Sul Volo degli Uccelli*, fol. 16 (15) v.

A sitting man [we read] cannot raise himself if that part of his body which is in front of his axis does not weigh more than that which is behind that axis without using his arms. A man who is mounting any slope finds that he must involuntarily throw more weight forward—that is, in front of the axis and not behind it. Hence a man will always involuntarily throw the greater weight towards any point whither he desires to move than in any other direction. A man who runs down hill throws the axis on to his heels, and then on the points of his feet.¹

It is worthy of note that Leonardo was aware of the possibility of the centre of gravity of a body being actually outside itself. Thus, in the course of a long note in the *Codex Atlanticus* attached to a sketch (Fig. 23), we read :

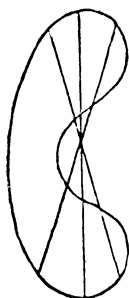


FIG. 23.

Occasionally the centre of gravity is to be found outside of the body, that is to say, not within the weight of the matter, that is to say, in the air.²

So far we may say that, apart from the experimental aspect, we see here little that is really an advance on what had been done before.

There is one aspect of the whole question of centres of gravity, however, for which claims of pioneer work may justly be made on behalf of Leonardo, namely, in the finding of the centres of gravity of solid figures. It has already been stated³ that Archimedes made a thorough study of the centres of gravity of *plane* surfaces in his *Treatise on the Equilibrium of Planes and of their Centres of Gravity*,

¹ MS. A., fol. 28 v.

² *Codex Atlan.*, fol. 153 v.b.

³ Chapter IV., p. 58.

deducing his results on Euclidian lines. Up to the time of da Vinci, however, no one appears to have considered mathematically the problem of the centre of gravity of the solids. It is of peculiar interest, therefore, to find in Manuscript F. the following note accompanied by a sketch of two

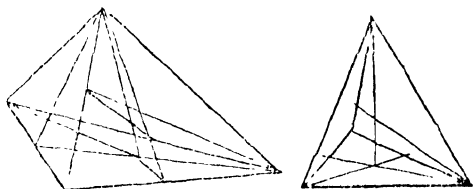


FIG. 24.

figures (Fig. 24) from which it is clear that Leonardo certainly considered the case of the tetrahedron :

The centre of gravity of a pyramid is in the fourth of its axis, towards the base, and if you divide the axis in four equal parts, and as you cut between two of the axes of this pyramid, one such intersection comes out at the above-mentioned quarter.¹

Apart from this note on the two figures we have no further guide as to how da Vinci arrived at his result. The treatment appears to be modern enough, and it embodies a result which formal history has hitherto attributed to Commandin and Maurolycus in the middle of the sixteenth century.²

Leonardo's work on this subject was not undertaken in vain. Duhem³ has shown in a discussion on da Vinci's influence upon his successors that the sixteenth century Spaniard, Juan Bautista Villalpando (1552-1608) took up the study of centres of gravity, and clearly borrowed freely, though without acknowledgement, from Leo-

¹ MS. F., fol. 51 r.

² See *Libri-Histoire des Sciences Mathématiques en Italie*, p. 40 (Paris, 1841). See also Duhem, *Etudes sur Leonardo da Vinci*, vol. ii., pp. 35-36.

³ Duhem, *ibid.*, p. 80.

nardo's writings. The importance of this lies in the fact that Villalpando was freely quoted in this, as in other subjects, by the well-known sixteenth century commentator in mathematics, Father Mersenne, in his widely-read *Mechanicorum Libri*. Similar remarks apply equally to Bernardino Baldi.¹

The Principle of the Lever.—We turn next to the

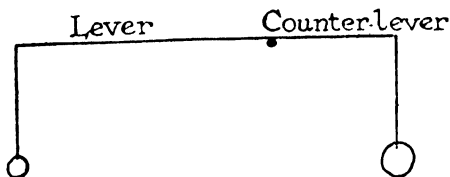


FIG. 25.

second of the two chief legacies of antiquity to the statics of the Middle Ages, namely, the principle of the lever. We find this stated by Leonardo in its simplest form in Manuscript A. He speaks of the long arm as the lever, and the short arm as the counter-lever (Fig. 25), and his note reads :

The weight attached to the extremes of the lever made of any material whatever will lift up at the extremity of a counter-lever a weight superior to itself by the same proportion as is the counter-lever to the lever.²

Nothing could be simpler than this. The fact that the principle is more clumsily expressed, as for example, in the *Codex Atlanticus*,³ need not seriously concern us. Schuster stresses the fact that in this collection Leonardo expresses the relationship mathematically in an unnecessarily complicated form,⁴ but inasmuch as he also ex-

¹ Duhem, *Etudes sur Leonardo da Vinci*, vol. ii., p. 101.

² MS. A., fol. 47 r.

³ *Codex Atlan.*, fol. 369 v.b.

⁴ Schuster : *Zur Mechanik Leonardo da Vinci* (Erlangen, 1915), p. 34.

pressed it as above in its simplest form, all cause for doubt as to the clear-mindedness of our philosopher is removed. He realises completely the direct consequences of the principle. In Manuscript A., to which we have already alluded, we find a sketch (strongly reminiscent of the modern elementary text-book in mechanics) illustrating the practical utility of the lever (Fig. 26) with the remark, "10 lbs. at the end of a lever will do the same as 20 lbs. at the mid point, and as 40 lbs. at the fourth part." Leonardo was not alone influenced by Archimedes in his treatment of the balance. We have already referred to the other influences.¹ Wöpcke quotes

from an Arabic work of Greek origin² a series of four axioms, embodying various conditions under which a variously

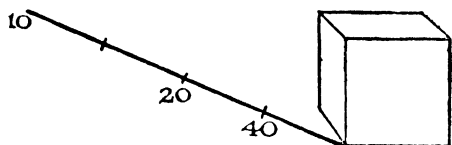


FIG. 26.

loaded lever remains horizontal. Of these, the second (which gives in effect the lever principle in its simplest form) is to be found in the *Codex Atlanticus*,³ and the fourth (which states that in a beam loaded with a number of weights to produce equilibrium, if one weight on one side of the fulcrum is moved inwards, and another on the same side is moved outwards by a suitable amount, the beam remains horizontal) occur in both Manuscripts A. and E.⁴ It is interesting to note that with all Leon-

¹ Chapter IV., p. .

² Wöpcke: *Das Buch des Euklid über die Wage* (Berlin, 1851). See also Duhem's *Etudes*, vol. i., p. 65.

³ *Codex Atlan.*, fol. 154 v.a. It also occurs in the *De ponderibus* of Jordanus Nemorarius.

⁴ MS. A., fol. 5 r.; MS. E., fol. 33 r.

ardo's clarity of thought on this subject, he occasionally blundered over simple, but not unimportant points. In a sketch ¹ somewhat suggestive of the steelyard ² (Fig. 27), we find a short arm, at the extremity of which is a suspended weight, and on the other side of the fulcrum an arm eight times as long, with divisions marked as shown. The lever is assumed to be heavy, and the problem Leonardo sets himself is to ascertain what weight he must suspend at the extremity of the short arm in order to counteract the effect of the heavier arm, given that each section of the balance weighs 1 lb. A simple calculation

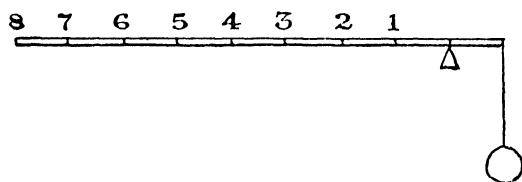


FIG. 27.

argument lies in his unfortunate view that the weight of each section acts at the *outer end* and not at the mid

point. On this basis, he says, in effect, that the weight of the short arm is cancelled out by section 1 of the long arm, so that the weight must equal the sum of 2 lbs. (to balance the 1 lb. of section 2 at twice the distance), and 3 lbs. (to balance the 1 lb. of section 3 at three times the distance), and so on to the 8th section at seven times the distance, *i.e.*, $2 + 3 + 4 + 5 + 6 + 7 + 8 = 35$ lbs. Happily, this clearly did not satisfy the philosopher, since a later sketch is accompanied by the correcting remark that the weight must act in the middle of each portion.

¹ MS. A., fol. 57 v.

² It is worthy of note that the actual discussion of the Roman steelyard as such is absent from Leonardo's manuscripts.

In the light of this self-correction we must not condemn Leonardo. The inference is rather to remember that the materials left to us are notes and memoranda *only*, jotted down as they occurred to the mind of the writer, so that faults were often cancelled out as occasion arose in later notes.

The Bent Arm Lever: Conception of Potential Arm.—Leonardo's next step, in what we may regard as his logical scheme, is of the greatest importance, since upon it hinges a number of applications which undoubtedly carried our philosopher very far. This concerns what, in effect, is the modern bent arm lever. So far the lever or balance has been straight and the weights perpendicular to it. What if one arm is now bent relative to the other, so that the corresponding weight is inclined to it at some angle? The fundamental experiment upon which Leonardo bases this problem has often been quoted, and is illustrated in Fig. 28.¹

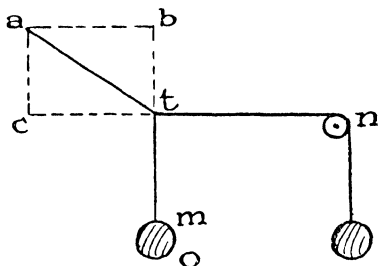


FIG. 28.

A bar at is pivoted at a , and has a weight o suspended from t at m . A second weight is also attached to t by means of a horizontal cord tn passing over a pulley n . The problem is to find the ratio of the weights depending from t and n for the equilibrium of the rod at .

Leonardo regards this, in effect, as a lever problem in which the lever arm for the weight o is not at , but what he calls the *potential arm or potential lever* ab , and for the

¹ MS. E., fol. 65 r.

weight at n the lever arm is the potential arm ac . He also speaks of these potential arms as *real* (i.e., in the sense of effective arms), and the lines ctn and btm (i.e., the real lines with the cord extensions) as *semi-real*. Alternative terms used by Leonardo are "spiritual lines" and "corporeal lines." Leonardo's conclusion, therefore, becomes that, as with the simple balance, the ratio of the two weights will be inversely as the ratio of their potential arms.

How did da Vinci arrive at this conception? The

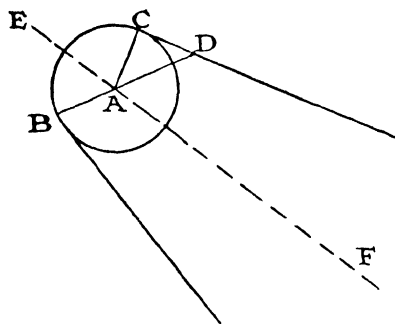


FIG. 29.

point, in view of the many problems upon which it is based, is one of importance. Mach¹ has suggested the following as Leonardo's train of thought. Imagine a string laid round a pulley A (Fig. 29), and subject to equal tension on both sides. The portion in con-

tact being BC , EF will be a line of symmetry, and the system will be in equilibrium. But it is to be noted that the only essential parts of the pulley are limited by the two rigid radii, AB and AC . These suffice to determine the form of the motion of the points of application of the two strings, and the rest can be cut away without disturbing equilibrium. Hence, although the radius BA produced will cut the string at D , the lever arm for the right-hand force is not AD , but the "potential lever"

¹ E. Mach: *The Science of Mechanics* (English edition), p. 21 (London, 1911).

AC. The view offered in this reasoning is naturally not impossible, but it seems improbable as a natural line of approach. It belongs rather to the sequence of thoughts which might be developed *after* the conception of the "potential arm" than to those which preceded it. A more plausible origin of the conception is, however, suggested by a note which appears in Manuscript E., accompanied by a sketch (Fig. 30), which reads: "The junction of the appendices of the balances with the arms of these balances is always a potential rectangle, and is not able to be real if these arms are oblique,"¹ and again, "The real arms of the balance are longer than the potential arms, and as much more as they are nearer the centre of the world"² (*i.e.*, nearer the vertical through the fulcrum). Finally, on the next page we read: "And always the real arms will not have in themselves the potential arms if they are not in the position of equality."³ Here, surely, we have a more natural approach to Leonardo's important conception of the potential arms. The swinging of the arm of the balance has at all times been a familiar sight, and with da Vinci's powerful imagination, it is pertinent to believe in his quick ability to seize upon the significance of the diminishing perpendicular distance between the two suspended weights.

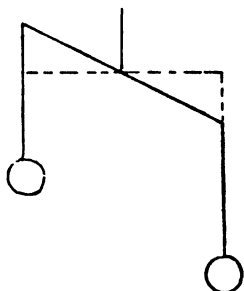


FIG. 30.

The Conception of Moments.—An interesting and important problem next arises. In view of the fact that

¹ MS. E., fol. 64 r.

² *Ibid.*, fol. 65 v.

³ *Ibid.*, fol. 65 v.

the conception of the potential arm involves both the weight or force factor, and the perpendicular distance on to its line of action from the point of suspension or

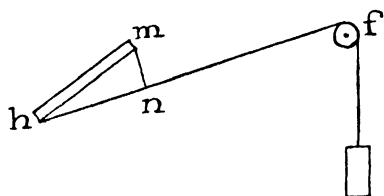


FIG. 31.

fulcrum, did Leonardo knowingly have in his mind the idea of moments as we understand it to-day? Some controversy has not unnaturally developed regarding this point.

Duhem,¹ and with him Mach,² favours the view that Leonardo understood and employed the idea of moments; Schuster³ takes the opposite view. We are bound to express the view here that the balance of argument is distinctly against

the statement that Leonardo knew of the idea of the moment of a force about a point. Duhem's case is an ingenious one. He cites first a passage from Manuscript I., which, referring to a sketch (Fig. 31)

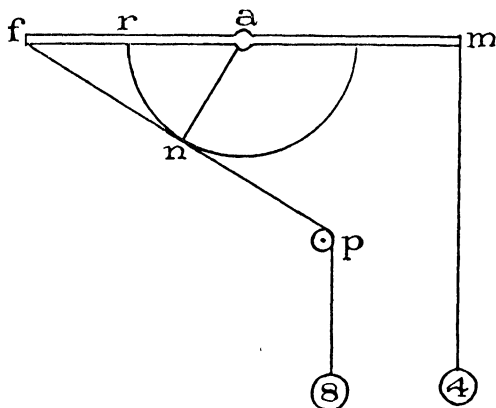


FIG. 32.

of a force applied by cord to take a mass out of the vertical, read as follows :

¹ Duhem : *Etudes*, vol. i., p. 143.

² E. Mach : *The Science of Mechanics*, p. 20; supp. vol., p. 7.

³ Schuster : *Zur Mechanik, etc.*, p. 43.

To know to each degree of movement the extent of the force of the power which moves and of the cause of the thing moved. Make as you see in $m n$ (i.e., drop a perpendicular on to the line of the force which moves) with $f h$.¹

The point Duhem makes here is the dropping of the perpendicular. He now quotes Leonardo's use of a tangential circle, of which he speaks as the "circumvoluble." Thus, in Manuscript M.² we find a diagram (Fig. 32) of a lever $f m$ having a weight of 4 lbs. suspended vertically from m , and one of 8 lbs. at an inclined direction $f p$ through the use of the pulley p . Leonardo clearly indicates his use of the perpendicular to the line of action of the force by his employment of the "circumvoluble."

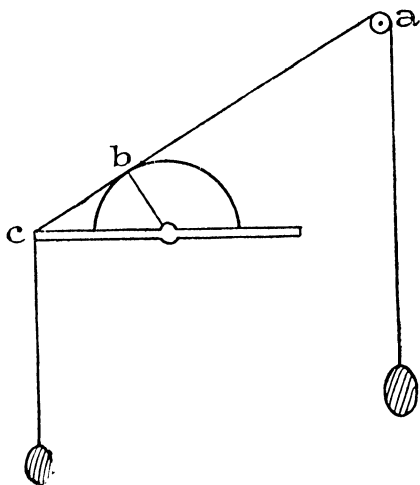


FIG. 33.

This Leonardo stresses by a variety of conditions, as illustrated by Fig. 33³ and Fig. 34, against the latter of which we read :

In some part, as would be, for example, the cord $n c$ it makes no difference, because one always employs a line which falls perpendicularly from

¹ MS. I., fol. 30 r.

² MS. M., fol. 40 r.

³ *Ibid.*, fol. 50 r.

the centre of the balance and the line of the cord, *i.e.*, the line mf .¹

It emerges clearly from all the above that Leonardo da Vinci appreciated the significance of the perpendicular on to the line of action of the force. Where we cannot agree with Duhem, however, is in the contention that it follows from this that Leonardo both used the product of force and distance, and attached to this product the significance of

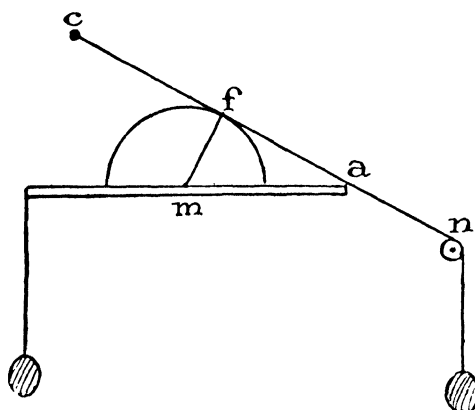


FIG. 34.

the measure of the turning power of the force about the fulcrum. The essential alternatives are really as between the general use by Leonardo of *either* the factor of proportion *or* of the product of force and distance. The principle of the balance was to the effect that for equilibrium the *ratio* of the weights was inversely as the lengths of the arms (or of the potential arms in the case of inclined forces). The principle of moments applied to the balance would express the fact that the *product* of the weight into the length of the arm was the same on both sides. These are the two alternatives, and the whole case against Leonardo's conception of the idea of moments lies in the fact that, wherever possible, Leonardo employs the ratio factor and not the product factor. In the light of this undoubted

¹ MS. M., fol. 50 v.

fact, it is therefore difficult to believe that he was the originator of the conception of moments.

The Function of the Pulley and the Length of the String.—In connection with the general theme of the potential balance arm, we find interspersed through the Manuscripts ¹ a number of variations of Figs. 33 and 34. Common to all these figures we have a graduated balance, with the arms horizontal and with the fulcrum at the midpoint. Also in every case we find at one extremity a

weight hanging directly downwards, and at the centre a "circumvoluble" circle. The variations occur at the other extremity. The cord comes directly from it at some angle, tangentially to a "circum-

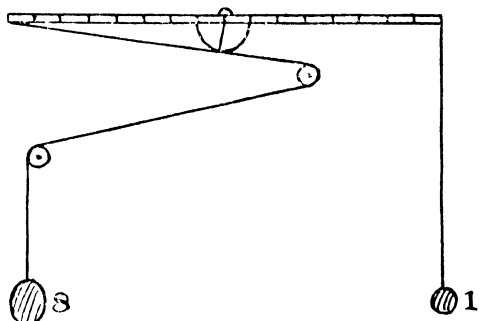


FIG. 35.

voluble," and thence by varying pulley connections to the second hanging weight. Two extreme cases are illustrated in Figs. 35 and 36. In Fig. 35 the radius of the circumvoluble and, therefore, the potential arm, is one-eighth of the real arm, and hence, *in spite of the interposition of two pulleys*, the weights are as 8 to 1 for equilibrium.

In Fig. 36 there are six pulleys, with the potential arm one quarter of the real arm, and the weights are as 8 to 2. As Leonardo puts it, "In the same relationship in which m n stands to m s stand inversely their powers." ²

¹ E.g., *Codex Atlan.*, fol. 100 r.b.

² MS. M., fol. 38 r.

Clearly Leonardo understood (1) that neither the length of the cord nor the number of pulleys had the slightest influence on the relationship between the powers, and (2) the arrangement of pulleys was purely a matter of convenience. We shall shortly have occasion to refer again to the function of the pulley.

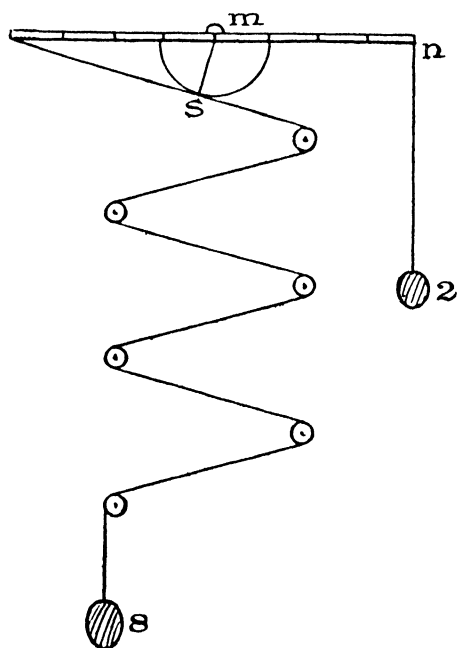


FIG. 36.

Leonardo's Applications of the Lever Principle. — We are now in a position to consider the various applications by Leonardo of the principle of the lever to mechanisms in general. As a preliminary we may consider the class of problem in which a suspended weight is deflected out of the vertical by the action of a disturbing force inclined to it at an angle. In connection with this it is interest-

ing to observe in passing that da Vinci extended to angles the same nomenclature of "real" and "potential" that he employed in the case of lines. Thus in the *Codex Atlanticus*¹ occurs a figure of a weight *e* (Fig. 37) suspended from two strings *c b* and *b a*. The angle *a b c* is the *real* angle

¹ *Codex Atlan.*, fol. 271 v.

while the angle $a d b$ is the potential angle, and according as the real angle is obtuse (e.g., Fig. 37) or acute, the potential angle is outside or inside the real angle. Turn-

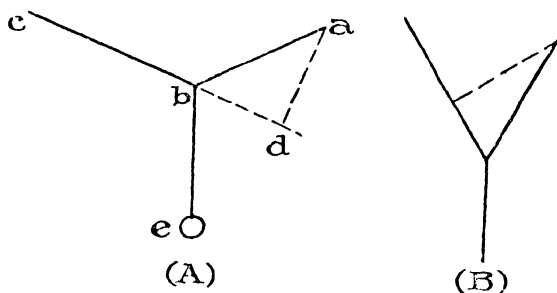


FIG. 37.

ing then to Fig. 38¹ we have a weight of 4 lbs. at the end of a cord deflected from the vertical $a b$ into the position $a n$ by a tangential force $n f$ applied by the weight of 1 lb. passing over the pulley f .

The angle $a n f$ is therefore a right angle. Leonardo shows the position of equilibrium as such that each of the four divisions $b d$ of $a b$ is equal to $a c$. This is in direct accordance with the lever laws, since the potential arm for the 1-lb. weight is $a n$, and that for the 4-lb. weight is

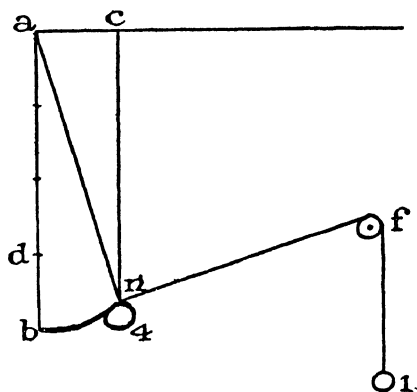


FIG. 38.

$a c$, so that $4 : 1 = a n (= a b) : a c$. Indeed, in a further sketch Leonardo traces the increase in the necessary deflecting load along $n f$ from zero in the

¹ *Codex Atl.*, fol. 268 v.b. See also fol. 365 v.a.

philosopher dealt with the dynamical aspects of this problem. Stevinus¹ has received the tribute of orthodox scientific history as having introduced the first complete treatment of the subject.² Indeed, it is but just to state that inasmuch as he was ignorant of the writings of Leonardo da Vinci he deserves full credit for his work, especially having regard to the completeness and the logic of his treatment. Nevertheless, Leonardo, some seventy or eighty years before him, had also tackled the problem. He, however, shows little more than a few detached notes and sketches and his treatment is incomplete and undoubtedly lacks

the clearness of his Dutch successor. Interspersed throughout the various note-books we find sketches—with and without accompanying notes

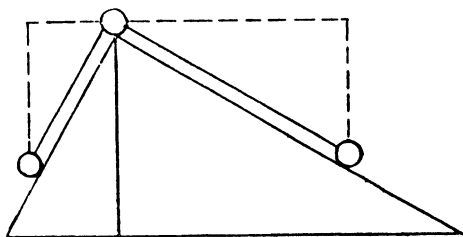


FIG. 40.

—showing a double inclined plane with two weights connected by a pulley at the top. His linking up of this problem with that of the lever is clearly shown in a sketch in Manuscript G.³ (Fig. 40) in which we see added to the usual diagram of the double inclined plane, what we might call an “equivalent simple lever,” the two weights being common to the two systems. Clearly he regards the ratio of the weights for equilibrium as equal to the

¹ Simon Stevinus of Bruges (1548–1620).

² Thus in Cox's *Mechanics* (Cambridge, 1904), p. 41, we read, “His discovery constitutes the second important step in the historical development of mechanics.”

³ MS. G., fol. 49 r. See also MS. E., fol. 1 v.

ratio of the bases. Again, in the manuscript *On the Flight of Birds*¹ we find an elaborate diagram (Fig. 41) in which the values are so proportioned that the perpen-

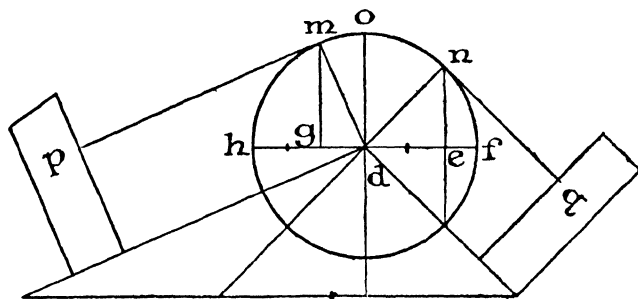


FIG. 41.

dicular $m g$ on $h d$ makes $g d$ equal to one-third of $h d$, and $d e$ two-thirds of $d f$. We read against this sketch as follows: "The weight q , because of the right angle n

above $d f$ at the point e , weighs two-thirds (*i.e.*, referring to the pull on the rope $n q$) of its natural weight, which was 3 lbs., and is a force of 2 lbs." The application of the principle of the lever is clear. With d as the fulcrum, the potential arm for the pull in the cord $n q$ is $n d$ (equal to $d f$) and that for the weight q (considered at n) is $d e$. Hence the pull is to the weight as 2 is to 3; and similarly for the other side of the figure.

Miscellaneous Practical Applications.—Certain simple

¹ See Chap. VIII.

practical applications of the lever principle may next fittingly claim our attention. Thus we find references to the Chinese windlass in the *Codex Atlanticus*. Against one sketch (Fig.

42) he writes :

"If one pulls at the arm of the windlass ac with the cord bc , it is just the same as turning a balance ab ." ¹ On the

same page we find

a similar mechanism for winding heavy loads (Fig. 43), represented as a lever with unequal arms. The lever

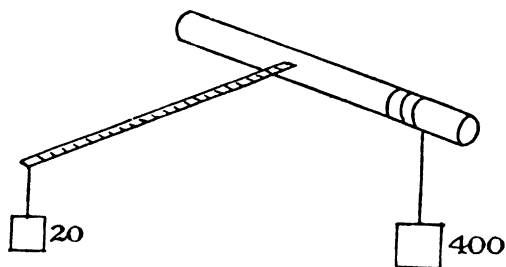


FIG. 43.

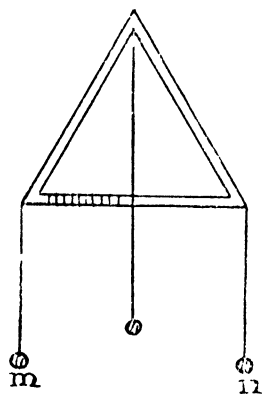


FIG. 44.

arm on which the force acts has a length equal to nineteen times the radius of a winding cylinder, thus providing a leverage of one in twenty. In Manuscript A.² is to be found an ingenious device for weighing bodies, in which the principle of the lever is directly applied. It consists of an equilateral triangular frame (Fig. 44) of wooden mouldings suspended from one corner. The base is graduated in such a way as to indicate automatically the weight of an object m suspended at one extremity, in terms of weight n at the other extremity. A variation

¹ *Codex Atlan.*, fol. 25 r.b.

² MS. A., fol. 52 r

of this is also to be found in a sketch ¹ (Fig. 45) unaccompanied by any note, in which the triangle is replaced by a semi-circle, with the right half presumably weighted to counteract the effect of a scale-pan depending from the left extremity of the diameter.

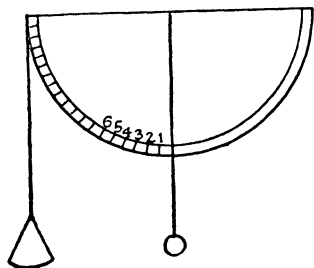


FIG. 45.

The left quadrant is again graduated to read the load in the scale-pan directly. Leonardo also applied the principle of the balance to the design of hygrometers. Hygrometry was a subject which attracted some attention in the fifteenth century,

and both Nicholas of Cusa in Germany and Leon Battista Alberti in Italy (the latter of whom, as we have seen, was known to Leonardo) had devised various forms of hygrometers. Leonardo was in this respect little more than a follower, but his designs were both ingenious and interesting. In one form ² (Fig. 46) we see a balance whose arm is triangular, in the left scale-pan of which is a bullet of cotton, and in the right a bullet of wax, arranged to balance in dry weather. The absorption of moisture in humid atmospheres by the cotton bullet was then clearly indicated by the depression of the left scale-pan. A second form of hygrometer is found later in the same manuscript (Fig. 47).³ A

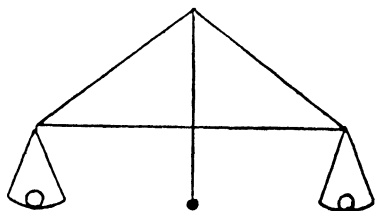


FIG. 46.

¹ MS. 2038, Bibliothèque Nationale, Paris.

² *Codex Atl.*, fol. 18 v.b.

³ *Ibid.*, fol. 249 v.a.

balanced rod carries the absorbent bullet on the left extremity and the dry bullet on the right, and the absorption of moisture now draws the left arm down over a graduated quadrant of a circle. The graduations indicate in effect the relative humidity of the atmosphere.

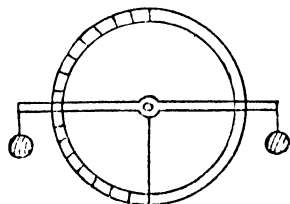
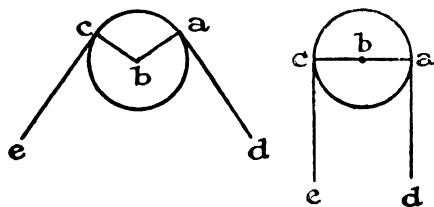


FIG. 47.

The Simple Pulley.—In view of Leonardo's activities in the rôle of engineer, it is not surprising that his theoretical investigations in those branches of mechanics which are capable of easy practical application were peculiarly successful. This especially applies to his studies on pulley systems. Here, however, we must make due allowance for the fact that he had at hand a rich store of achievement handed down from the mechanics



(B)

(A)

FIG. 48.

and engineering of antiquity. Aristotle, Archimedes, Vitruvius, Heron, Ctesibius, Pappus and others had freely contributed to the subject, and Leonardo was acquainted with

their work. Accordingly we find in the summation of his notes a very thorough discussion of the whole subject, and a clear grasp of the practical possibilities which arise from it. The theoretical side of Leonardo's work on pulley systems has been fully summarised by Schuster.¹ Throughout it is evident that the treatment is based on

¹ Schuster: *Zur Mechanik, etc.*, pp. 114-132.

the important conceptions of (1) the lever laws and (2) the principle of virtual velocities. The initial position is simply stated by da Vinci in the *Codex Atlanticus* in a note (accompanied by two sketches (Fig. 48)) which reads :

The line of movement is ab ; the line of the force is da . The line of the movement in the balance is the distance of the middle points of the pulleys from their circumferences, especially from the direction of the force which acts on the wheels as a tangent, *i.e.*, ab . At the point of contact of the rope from which the weights act with the wheel-circle right angles continually arise between this rope and the radius of contact.¹

The simple pulley is therefore in effect a simple balance, as in *A*, or a virtual balance with potential arms, as in *B*. The interposition of a fixed pulley as producing no change in the mechanical advantage (a term, of course, not employed by Leonardo) is next indicated by Fig. 49,²

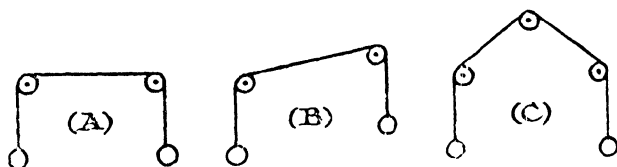


FIG. 49.

which illustrates the type of sketch frequently seen in the note-books unaccompanied by text.

We have next to ask ourselves whether Leonardo appreciated the simple function of a pulley as a means of changing the direction of a force. Here, unfortunately, we find him distinctly at fault. Referring to Fig. 50,

¹ *Codex Atlan.*, fol. 149 r.a.

² MS. L., fol. 16 r.

he asks, "Which of the ropes of, on and om have more stretch, and further, how much, and why?"¹ Leonardo must clearly have been confusing this with the problem of the inclined plane; because, he must have argued, with the changing inclination of the plane, the pull on the rope supporting the body on the plane steadily changes, so will the pull on the rope passing over a single pulley change with its angle of inclination to the vertical.

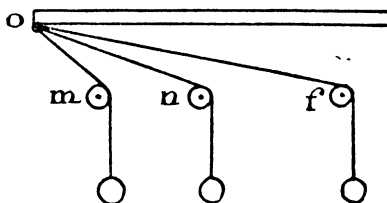


FIG. 50.

Fig. 51² shows the error in reasoning still more strikingly, the tension in the horizontal portion of the rope being given as 8 lbs. with a 4-lb. load on each pulley. We are, therefore, unable to say that da Vinci properly understood the simple

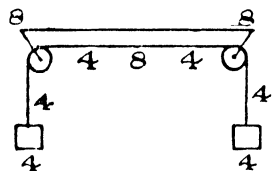


FIG. 51.

function of change of direction of a force, which all simple pulleys possess. Allied to this error on the part of Leonardo, we find a simple misconception regarding the pull exerted by a pulley on its supports. It will be noticed in Fig. 51 that these fastenings are inclined at an angle of 45 degrees, and therefore receive a pull of $4\sqrt{2}$ lbs. Leonardo gives it as 8 lbs.

Let us next consider Leonardo's interpretation of the purposes of the *movable* pulley. Here, happily, we are on more fortunate ground. Attached to a simple sketch in

¹ *Codex Atlan.*, fol. 346 v.d.

² *Ibid.*, fol. 321 r.a.

the *Codex Atlanticus* (Fig. 52) is the note "Pay heed that g is the half of the weight h , and that the path of g is twice as great as that of h ." Nothing could be more explicit, both from the point of view of mechanical advantage and of velocity ratio.

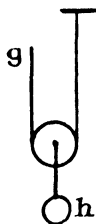


FIG. 52.

As a direct application of this property of the movable pulley, we may quote again from the *Codex Atlanticus*, from which Fig. 53 is taken. Here we have a system of two fixed and two movable pulleys. Leonardo speaks of the end of the rope carrying the power as the "arganica," and the end of the rope carrying the cord as the "retinente." Referring to the sketch, he writes :

If one divides the burden lifted through the pulley tackle by the number of pulleys, one obtains a weight that, fastened at the "arganica," makes equilibrium.¹

Here, then, is the statement regarding the mechanical advantage obtained by this system. Continuing, however, he says,

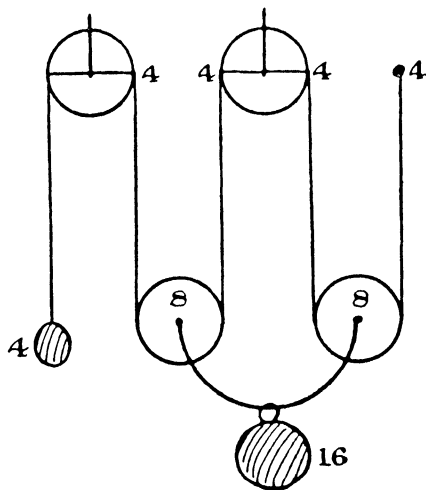


FIG. 53.

The path of the arganica which lifts the burden is so many times as long as the path of the burden

¹ *Codex Atlan.*, fol. 321 v.a.

which becomes lifted through the pulley tackle as the number of pulleys,

which completes the statement regarding the velocity ratio. Fig. 54 illustrates an extreme application of the property of the movable pulley in a pretty scheme of suspension from a framework of two rings of pulleys, the upper set fixed and the lower movable, producing a very large mechanical advantage.

Pulley Systems.—It is interesting to note, in turning to the various pulley systems with which Leonardo was familiar, that practically all forms of pulley combinations such as we meet in the modern text-book are to be found in one place or together among the various manuscripts. Fig. 55¹ shows a typical example, with four fixed and four movable pulleys, and Leonardo reduces the discussion to the study of the tension in the rope: "The

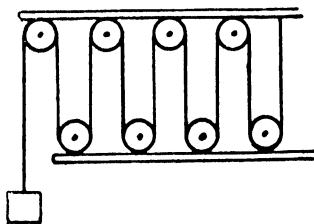


FIG. 55.

weight divides itself into eight pieces of rope. The ninth opposed to this simply holds the equipoise of the eight." A curious case is presented by a note in Manuscript A. accompanying a sketch of a system of two fixed and two movable

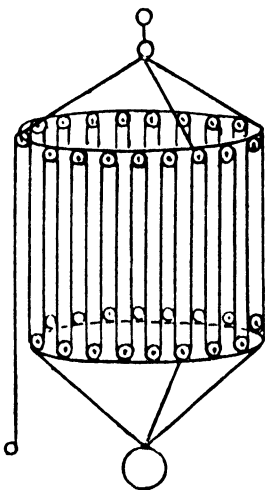


FIG. 54.

two fixed and two movable pulleys (Fig. 56). The burden is 20 lbs., and the power

¹ *Codex Atlan.*, fol. 104 v.b.

should clearly be 5 lbs. Yet Leonardo makes it 6 lbs. His note reads :

If the burden borne be 20 lbs., then I say let 10 lbs. act on the pulley *l* and 10 lbs. on the pulley *k*, these being the points of suspension of the 20 lbs. load. That is to say that *o* takes 5 lbs. off from *l*, and *p* also takes 5 lbs. from *l*, and 5 lbs. from *k*. Finally, *k* transmits 5 lbs. to *q*. If one wishes to overcome the 5 lbs., one must apply at *x* an opposing weight of 6 lbs. So long as one applies 6 lbs. at the

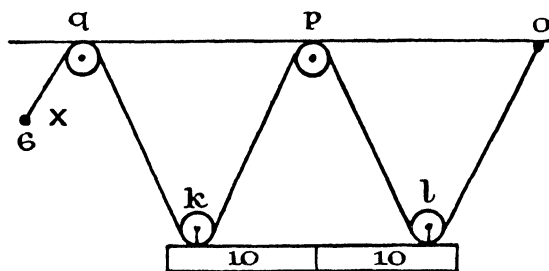


FIG. 56.

extreme point against the 5 lbs. at *x*, and so long as each of the four pieces of string which bear the 20 lbs. themselves only experiences the 5 lbs. pull, then, because the active extra weight on the rope *q x* finds nothing to equal it in the opposing effective pieces of rope, the tension will be overcome, and movement will result.¹

Clearly the point here is that whilst the theoretical value for the power is recognised by Leonardo as 5 lbs., nevertheless frictional and other resistances require an addition to this value for motion to ensue. At the same time it is just a little difficult to appreciate exactly what was in his

¹ MS. A., fol. 62 r.

mind regarding this extra pound pull. On the previous page in the same note-book,¹ for example, Leonardo deals with the same system of pulleys, except that he now has three movable pulleys (Fig. 57). The discussion proceeds as follows :

On movement and force.—Among one and the same causes of moved bodies which receive greater velocity, more loss is caused to the moving body. For example, in the case of the blocks *a g* and *b h* let the lower one be raised to the line *m n*. I maintain that necessarily in this case the piece of rope *g h* so falls that the point *g* comes into the position of *h*.

He then goes on to point out that for the balk to move to *m n* the pieces of rope *m b o*, *p d q*, etc., must all pass

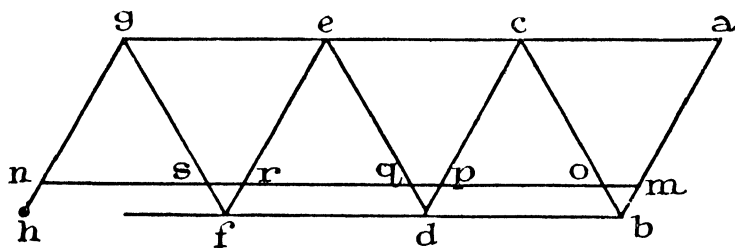


FIG. 57.

round and below *g*, so that the power applied at *h* moves through a big distance. He refers to the movement involved by any point as the "fatigue," and he points out that the fatigue at *h* is greater than that at *f*, that at *f* greater than that at *d*, and so on. We are unable to say that the term "fatigue" was employed as a technical term to express linear displacement, and it is to be feared that here Leonardo was introducing physiological con-

¹ MS. A., fol. 61 v.

siderations which ought to have been foreign to the abstract theory of the pulley system under consideration.

The use of sheaves of pulleys is frequently met with in da Vinci's manuscripts. Fig. 58 shows one such example.¹ Other pulley systems are illustrated freely in the *Codex Atlanticus*. One such (Fig. 59) is of especial interest on account of the clear enunciation of the relationship between mechanical advantage and velocity ratio given in the text beside the drawing. It will be noticed that the figure, in so far as the scheme of tensions in the ropes and the relationship between power and load are concerned, is accurate and complete. The note

FIG. 58.

reads :

Just as one can here find a rule of diminishing force for the mover,

so one can also lay down a rule for the increase in the velocity of the movement. The path of m stands in proportion to that of n as the weight n is to the weight of m .²

For further illustrations of the extent and variety of

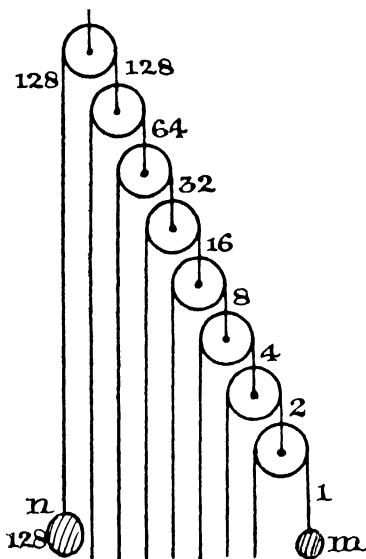


FIG. 59.

¹ *Codex Atlan.*, fol. 141 v.a.

² *Ibid.*, fol. 120 v.c.

pulley systems dealt with by Leonardo, we may refer to yet another page in the *Codex Atlanticus*,¹ in which occurs a number of diagrams (Fig. 60, A, B and C). These are self explanatory, which may possibly account for the fact that they are unaccompanied by any notes in the text.

The Motion of Systems of Connected Weights.—One of the most interesting aspects of Leonardo's work on pulleys is to be found in his discussion of the motion which

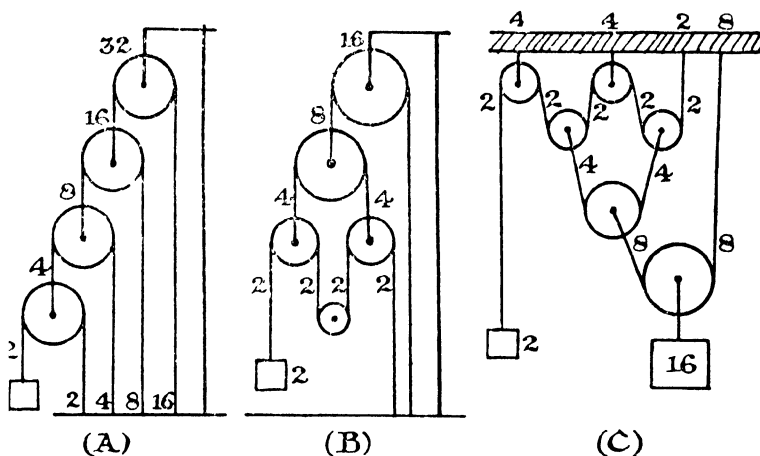


FIG. 60.

must ensue when two different weights are connected by a string passing over a pulley. A number of sketches show that this class of problem interested our philosopher. Naturally, a correct solution, being dependent upon a knowledge of the value of the acceleration due to gravity, such as was unknown in his days, was beyond him. Nevertheless that he even recognised the existence of the problem and attempted a solution is all to his credit. Certain broad and admittedly ill-defined facts regarding

¹ *Codex Atlan.*, fol. 153 r.a.

such systems did emerge from his experiments. Thus he realised that whilst the pull on the pulley was equal to the sum of the weights in the case when no motion ensues, in the case of unequal weights setting up a resulting motion the pull on the pulley was *less* than the sum of the two weights.¹ Fig. 61 illustrates a very interesting example of this type of problem.² On the left of a beam is suspended a weight of 2 lbs., and the other extremity *m* carries a pulley over which passes a string carrying weights of 2 lbs. and 1 lb. respectively. Leonardo's contention is that with such a system the beam remains horizontal, *i.e.*,

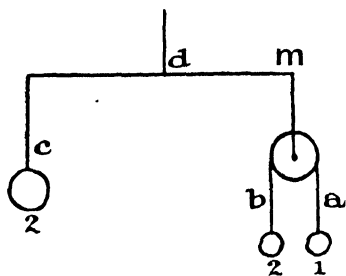


FIG. 61.

although the 2 lbs. weight at *b* falls and the 1 lb. weight at *a* rises, the resulting pull on the point *m* is still 2 lbs., and the beam is in equilibrium.

Of course he is wrong, since the tension along

$$a\ b \text{ is } \frac{2\ w_1 w_2}{w_1 + w_2} = \frac{2 \times 1 \times 2}{3} = 1\frac{2}{3} \text{ lb.,}$$

if the weight of the pulley be neglected, thus giving a pull of $1\frac{2}{3}$ lbs. on *m*, so that *m* is depressed and *c* rises. Leonardo specifically points out, however, that the weight *c* must in all cases be less than the sum of the weights suspended from the pulley, and in this respect he is right.

The Transmission of Power.—Turning next to the general subject of the transmission of power, we come again to a field which, by the fifteenth century, was

¹ *Codex Atlan.*, fol. 249 r.; fol. 323 r.a.; MS. G., fol. 95 v., etc.

² MS. No. 2057, Bibliothèque Nationale, fol. 13 r.

already full of historical associations.¹ Indeed, from the time when Aristotle discussed the gearing of three wheels in rough contact onwards,² the theory of power transmission, mainly through toothed gearing, progressed rapidly, and to the accompaniment of an equally rapid development in practice. Hence not only do we find innumerable examples of machines, structures, cranes, and all kinds of practical mechanism freely drawn and described throughout Leonardo's note-books, some of them indeed extremely elaborate and complicated in construction, but we also meet with a number of theoretical discussions. The ordinary type of toothed wheels in gear with each other is naturally very frequent, but in addition Leonardo had a partiality³ for the method of transmission illustrated in Fig. 62, in which a wheel *A* is made to rotate by a number of pieces projecting from it at equal intervals being made to engage in the spaces between the cylindrical uprights of a rotating spindle *B*. By

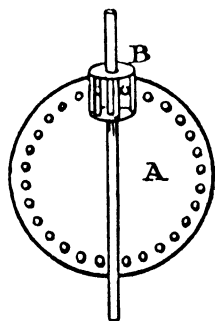


FIG. 62.

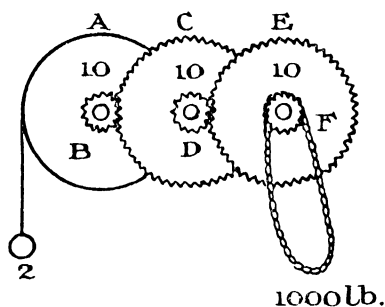


FIG. 63.

this means power is transmitted through a right angle.

¹ A very comprehensive summary of this history is to be seen in Feldhaus' *Die Geschichtliche Entwicklung des Zahnrades in Theorie und Praxis* (Berlin, 1911).

² Aristotle: *Mechanica*, chap. i.

³ MS. H., fol. 86 v.; MS. G., fol. 26 v.; MS. G., fol. 27 r., etc.

The general scheme of Leonardo's treatment can perhaps be illustrated by a typical example (Fig. 63) taken from the *Codex Atlanticus*.¹ The scheme is as follows : There are three large wheels, *A*, *C* and *E*, of which *C* and

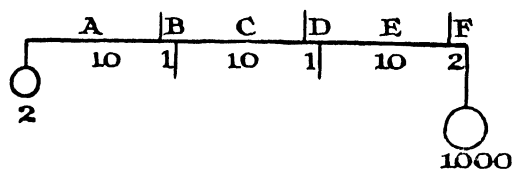


FIG. 64.

E are toothed. *A* has no teeth, but carries the power-load at the extremity of a cord. The

axles *B*, *D* and *F* of each of *A*, *C* and *E* are also toothed, and the radii of the axle wheels *B* and *D* are one-tenth of those of the larger wheels *A* and *C*, whilst that of *F* is *one-fifth* of *E*. *F* carries an endless chain, along which the lifted load is distributed. Leonardo's treatment is very simple, and is based entirely on the principle of the lever. He reduces the system to what he calls an "interrupted balance," which is

in effect a sequence of equivalent simple levers working from left to right, as shown in Fig. 64. In this we see that with a

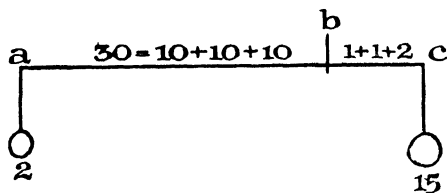


FIG. 65.

power of 2 lbs. the system *AB* gives an equivalent load of 20 lbs., the system *CD* an equivalent load of 200 lbs., and *EF* a final load of 1,000 lbs. So far the reasoning is perfect. Where, however, Leonardo now goes astray is in introducing the further (and incorrect) notion of an equivalent *uninterrupted* balance (Fig. 65), the left arm of which is equal to the sum of the "interrupted" left

¹ *Codex Atlan.*, fol. 153 v.d.

arms of Fig. 64 (*i.e.*, $10 + 10 + 10 = 30$), and the right arm of which is equal to the sum of the right arms of the "interrupted" scheme (*i.e.*, $1 + 1 + 2 = 4$). This, for a power of 2 lbs. as before, gives a load $\frac{2 \times 30}{4} = 15$ lbs.

Reverting to the original value of 1,000 lbs. Leonardo now points out that 15 lbs. is contained in the 1,000 lbs. $66\frac{2}{3}$ times, and thus concludes that the movement of the heavy weight is $66\frac{2}{3}$ times as slow as that of the power. In view of the fact that Leonardo had shown himself to be quite clear as to the relationship between velocity ratio and mechanical advantage, it is difficult to understand why he should have brought in this incorrect and unnecessary notion of the "uninterrupted balance."

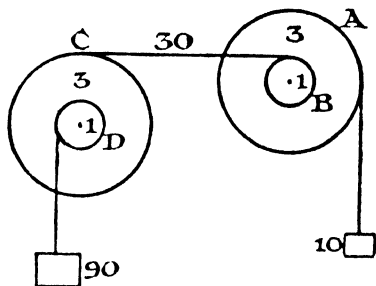


FIG. 66.

We offer one more illustration of Leonardo's applications of the principle of the lever to the transmission of power in Fig. 66.¹ Here we have a combination of two wheels and axles, each pair of which has a radii ratio of 3 to 1. The power being 10 lbs., Leonardo readily deduces the load to be 90 lbs.

Reactions on Supports of Loaded Bodies.—We turn next to another branch of inquiry in statics—that which belongs to-day to the strength of structures. Leonardo was again naturally interested as an engineer and architect, but it was not alone in such rôles as these that he approached the subject. His study of bird-flight also

¹ MS. J., fol. 132 r.

brought him into contact with this class of problem, and in his manuscript *On the Flight of Birds* he asks :

In which part of the under surface of the width of the bird does the wing press the air more than in any other part of the length of the wings ? ¹

So he begins with the consideration of a rigid structure supported at equal intervals.

All bodies which do not bend [he says] will exert equal pressures on all of the supports that are equally distant from the centre of gravity, the centre being the middle of the substance of such a body. One proves how the above-mentioned weights exert equal

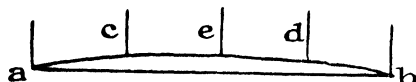


FIG. 67.

pressure on all its supports ; let us assume that it is 4 lbs., and that it is sustained by the supports *a b* (Fig. 67). I say that the body being unhindered in its fall except by the two supports *a b*, that these supports will sustain equal parts of this weight, that is to say, 2 and 2, and the same thing would apply to the two supports *c d*. If the three other supports were not there, and if the middle one at *e* only remained, it would support the whole of the weight. ²

He then goes on to point out that this would not apply to the structure if it were not uniform. A similar example is to be seen in the *Codex Atlanticus*.³ A sketch (Fig. 68) shows a uniform beam of 6 lbs. weight, and the reactions at the ends are given as 3 lbs. each. Adjoining this,⁴ we see how he next considers what happens when one support is

¹ *Sul Volo degli Uccelli*, fol. 4 v.

² *Sul Volo degli Uccelli*, fol. 4 v.

³ *Ibid.*, fol. 101 r.a.

⁴ See also *Codex Atlan.*, fol. 185 r.a.

moved inward two divisions (Fig. 69), and he correctly shows the reactions to be $4\frac{1}{2}$ lbs. and $1\frac{1}{2}$ lbs. respectively ; and the problem is later expressed in more general terms ¹ by the assertion that if one support is kept unaltered at one end, and the other moved towards it, the pull on the moved support steadily increases, whilst that in the fixed one equally diminishes. He gives a number

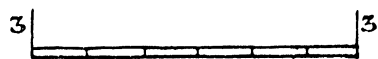


FIG. 68.

of calculations in illustration of this, many of which are incorrect. Some, however, are quite correct,² so that the errors are not due to faulty principles, but, as happens so frequently with da Vinci, to faulty mathematics.

Strength of Loaded Struts.—Standing in a class by themselves are Leonardo's studies of the loads which vertical struts, pillars, etc., are capable of sustaining.

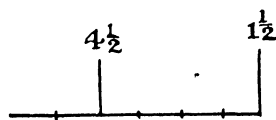


FIG. 69.

Most of his notes on this subject are to be found in Manuscript A. of the collection at the Institute of France, though others also occur in the *Codex Atlanticus*, and they constitute the first scientific attempt of their kind. All previous efforts were frankly empirical. Of da Vinci's, we can at least say that it attempted scientifically to combine theory with practice. He begins early in Manuscript A. by pointing out that a number of pillars or supports held together are stronger collectively than a single "equivalent" pillar.

Many little supports held together [*i.e.*, in a bundle, he writes] are capable of bearing a greater

¹ *Codex Atlan.*, fol. 141 v.a.

² *E.g.*, *Ibid.*, fol. 132 v.b.

load than if they are separated from each other. Of 1,000 such rushes of the same thickness and length which are separated from one another, each one will bend if you stick it upright and load it with a common weight. And if you bind them together with cords so that they touch each other, they will be able to carry a weight such that each single rush is in the position of supporting twelve times more weight than formerly.¹

Later, in the same manuscript,² he insists that this increase in the carrying capacity is entirely dependent upon the firmness with which the bundle is bound together, so that if the connection is loose, the total load possible becomes merely equal to the sum of the loads of which each rush is separately capable, instead of a multiple of that sum.

It is interesting to note the nature of the experiments carried out by Leonardo in verification of the above. He applies a vertical load on a tightened iron wire strip, and increases the load until a plumb line shows the wire to begin to bend. He then repeats the experiment with two wires bound together, and then with three, and so on. We quote in his own words :

Make the following experiment. Take two pieces of iron wire which have been stretched in a four-cornered wire drawer, and fasten one of them below with two supports, and load it above with a given weight. Notice exactly when it begins to incline, and further investigate with a thread which carries a plumb at which weight this incline occurs. Next double the iron wire, bind the two with a fine

¹ MS. A., fol. 3 v.

² *Ibid.*, fol. 46 v.

silk cord, and see whether this investigation agrees with what I have maintained. And similarly repeat the experiment with a fourfold increase, and so often as desired, continuing to bind the new ones with silk.¹

Turning next to a comparison of the loading capabilities of two struts of equal height, but of different cross section (Fig. 70), Leonardo writes : " A support

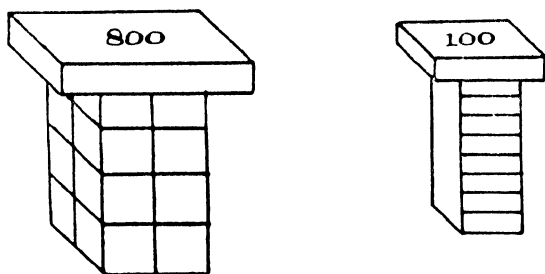


FIG. 70.

with twice the diameter (width and depth ?) will carry eight times as much weight as another, both having the same height."² His proof of this is not very convincing, but he concludes in effect, that the ability to carry a load is directly proportional to the area of the carrying surface, and inversely proportional to the relation of the height to the diameter. Schuster³ expresses Leonardo's results generally by the formula

$$\text{Possible load} = \frac{\text{carrying surface}}{\text{height}} \times \text{constant.}$$

Applying this to Fig. 70, we see that there is a factor

¹ MS. A., fol. 47 r.

² *Ibid.*, fol. 47 r.

³ Schuster : *Zur Mechanik, etc.*, p. 136.

of 4 in respect of surface areas, and an inverse factor of $\frac{1}{2}$ in respect of the denominator, giving one load 4 times the other. The figure gives 8 as the final factor. Possibly Leonardo regards his constant as 2. More probably his application of this rule is again frequently faulty in arithmetic. Thus he quotes one example, in which the comparison of loads shows according to the rule $320 : 90$, whereas he makes it $320 : 60$, a note in the text giving the product of 15 and 6 as 60 !

Leonardo now passes on to a comparison of the carrying capacities of struts of equal cross-section, but of different

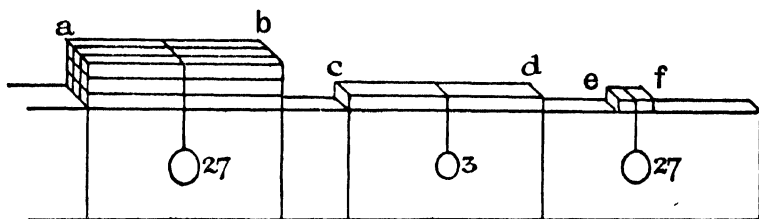


FIG. 71.

heights, and concludes that these are inversely as their heights. "So often as the short staff is contained in the longer one," he writes, "so much greater is the load which it can carry than the longer one."¹

Loaded Beams.—Finally, we may refer briefly to Leonardo's work on loaded beams, the chief notes on which are to be found in the *Codex Atlanticus*. On one page of this we find a sketch (Fig. 71) with the following note :

You will find the same power of support by binding together 9 balks as by the ninth part of one of these balks *a b*, which consists of 9 balks, carries

¹ MS. A., fol. 49 r. See also fol. 48 v.

27 units of weight. Further, cd which is the ninth part of the same in cross-section, carries 3; thereby can ef , which is the ninth part of the length of cd , carry 27 units of weight, since it is 9 times as short as the former.¹

Whatever else we may say of this result, it does embody certain generally correct features regarding the influence of length, breadth and depth. Leonardo, however, proceeds later to an actual discussion of the amount of deflection produced in the beams, which is also interesting (Fig. 72). He compares the deflection of

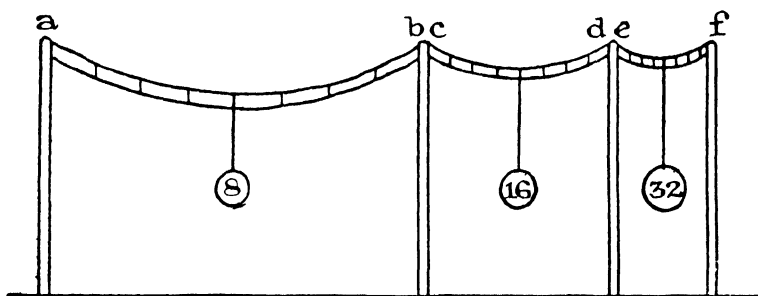


FIG. 72.

three beams, of which cd is twice as long as ef , and ab twice as long as cd . His note proceeds as follows :

If ab bends through an eighth of its length when loaded with 8 units, then cd , if it is, as I believe twice as strong as ab , will not bend itself through an eighth of its length with a smaller weight than 16 units; since it is half as long as ab . Similarly will ef , since it is half as long as cd , be twice as strong and will bend one-eighth of its length with a load of 32 units. One must also note that the

¹ *Codex Atlan.*, fol. 152 r.b.

beam cd which is twice as strong as ab on a load double that of ab will not bend $\frac{1}{8}$ of its length but, speaking much more exactly, will bend $\frac{1}{16}$ of its length.

The formula for the deflection x of a beam loaded in the middle and supported at the ends is given by $x = \frac{W l^3}{4 E b d^3}$, where W is the load, l the length, b the breadth, and d the depth of the beam. An application of this to the case cited by Leonardo in Fig. 72 will show that the correction he offers in the last paragraph of the note is right, the former result being wrong. Again, however, we must stress the point that although he accomplished little in the way of accurate results, he was the first man to attempt the problem scientifically, and to that extent, at least, he deserved considerable credit.

Friction.—We conclude our study of Leonardo's researches in mechanics with a note on his experiments on friction. Here, undoubtedly, the experimental scientist in da Vinci showed itself with distinction. It is regrettable that his work was lost to posterity. It was incomplete, and in some respects incorrect. But Amontons in 1699, and Coulomb after him, had to cover much the same ground through ignorance of his work, where otherwise they might have gone much further than they did. Leonardo's writings on friction are almost exclusively to be found in the *Codex Atlanticus*.¹ His experiments were similar in scope to those which obtain to-day, namely, on the sliding of a given surface over both horizontal and inclined planes by means of a weight passing over a pulley, and he carefully distinguished between sliding

¹ *Codex Atlan.*, fols. 16, 47, 64, etc.

friction and rolling friction ("which, moving forward by infinitely small steps, touches rather than rubs").¹

Leonardo's experiments led him to conclude that the amount of friction was independent of the areas of the surfaces in contact; that artificial smoothing or lubricating reduces friction; that for bodies polished or smoothed to an equal degree, the friction is proportional to the pressure between them, and that on a horizontal polished surface, the resistance to friction by all bodies is with a force equal to one-quarter of the weight. This last result is most interesting. It recognises a definite co-efficient of friction, but with a value common to all bodies and equal to .25. Nevertheless, in the general importance of the conclusions as a whole—since they afford us the first presentation in scientific history of any laws of friction whatever—we may well excuse Leonardo this incorrect result.

Conclusion.—In the foregoing two chapters we have endeavoured to summarise as completely as possible the main achievements of Leonardo da Vinci in mechanical science. In so doing we have endeavoured to confine ourselves to a survey of broad scientific principles rather than to a detailed and exhaustive scheduling of notes and experiments. Such notes and experiments as we have quoted have been introduced rather for the purposes of illustrating the particular principle or theme under discussion. Indeed, so prolific was our philosopher in the writing of his notes, that to attempt anything more than this would be an almost overwhelming task. At the same time it must be pointed out that we have purposely omitted a treatment of Leonardo's work in hydrostatics

¹ *Codex Atlan.*, fol. 195 r.

and hydraulics on the one hand, and in mechanical technology on the other. Each of these offers a very wide and very important field in itself, and it is hoped to reserve their survey for a special and later study elsewhere. We have seen how, of the two branches of dynamics and statics into which we have divided Leonardo's work in mechanics, the former, although perhaps smaller in extent than the statics, offered more that was purely new and original. In statics Leonardo's rôle was rather of "carrying on." Nevertheless, we may regard him as having been conspicuously successful in both rôles. He "carried on" where work had already been done, he created where there had been nothing upon which to build. In so doing we may claim for him that he was a genius, and inasmuch as his work breathed the whole spirit and atmosphere of true scientific method, he was a scientific genius. Nor was his work entirely lost to posterity. The devoted labours, spread over many years, of Pierre Duhem ¹ in the works of which we have already so frequently quoted, have shown only too conclusively that Leonardo's influence in the world of mechanical science was, indirectly perhaps, but decisively and effectively nevertheless, handed down through Jerome Cardan to Bernardino Baldi, thence to Roberval (and through him to Descartes) and to Galileo.

¹ P. Duhem : *Des Etudes de Leonardo da Vinci*, vol. i., pp. 12 et seq.

Chapter VII

Leonardo da Vinci as a Pioneer of Aviation

FLIGHT BEFORE LEONARDO.—Aeronautics is a very young science. From the point of view of the achievement of "heavier-than-air" flight, it is but little more than a quarter of a century since the late Professor Langley, secretary to the Smithsonian Institution at Washington, successfully flew his *model* aeroplane for half-mile distances over the River Potomac. He called it an "aerodrome," and it measured 12 feet from tip to tip, weighed 30 lbs., and carried a steam engine and boiler weighing 7 lbs. The brothers Wright achieved the first motor-driven, *man-carrying* flight in an aeroplane for a period lasting but a portion of a minute as recently as December, 1903. Yet young as is aeronautics as a science, as an aspiration and a philosopher's dream we may claim for it the age of many centuries. Aeronautics has, in fact, a history, and although it is inevitable that we must accord to this history a much longer period of legend than obtains with most of the sciences, it is very real nevertheless.

Among those who have contributed effectively to the sum of ideas concerning flight, no figure is more striking than Leonardo da Vinci, who has this much at least in common with the science of aeronautics—that he has only recently come into his own.

Dreamers of air conquest pass before us throughout the panorama of the early history of civilisation. We see

them in the winged statuary of the Egyptians and we read of them in stories of ancient Greek mythology. Ever since man has learnt to think, the challenge of the air has tantalised him. He has ever regarded as an anomaly and an incongruity that the ability to fly should be granted to birds and yet denied to him. And so we read of such legendary figures as Icarus and Perseus and Hermes, and of such entertaining but improbable stories as of Simon Magus, who, with the aid of some demon colleagues, essayed a short-lived flight and finished with a broken neck ; of Abaris (as related by Diodorus of Sicily) and his flight round the world on a golden arrow (somewhat reminiscent of old Mother Shipton and of the nursery pictures of the cow jumping over the moon) ; and of the story of Aulus Gallius in his *Attic Nights* of Archytas and his mechanical pigeon of wood: "To wit, it was thus suspended by balancing and was animated by an occult and enclosed aura of spirit." ¹

Later we have the more circumstantial story of the Saracen of Constantinople who, in the presence of both the Emperor Comnenus and the Sultan of the Turks and a vast concourse of people, attempted to fly round the hippodrome at Constantinople. Wearing a long white robe braced with rods with which to catch the breeze, he took his station at the top of a tower and leaned into the wind. But "the weight of his body having more power to drag him down than the artificial wings had to sustain him, he broke his bones, and his evil plight was such that he did not long survive." ²

A like fate befel a similar attempt in the year 1065 by

¹ Aulus Gallius : *Noctes Atticæ*, lib. x., cap. xii.

² *Histoire de Constantinople* (Cousin, Paris).

Oliver Malmesbury.¹ Then there is Roger Bacon's classical thirteenth-century prophecy that—

an instrument may be made to fly withal if one sit in the midst of the instrument, and do turn an engine, by which the wings, being artificially composed, may beat the air after the manner of a flying bird. . . .

In the fifteenth century we come to records concerning the famous astronomer Regiomontanus. We are told of him that in his workshop at Nuremburg was an automaton in "perpetual" motion, and that he made an artificial "fly" which, "taking its flight from his hand, would fly round the room," and at last, as if weary, would return to his master's hand, and that he fabricated an eagle which, on the Emperor's approach to the city, he sent out, high in the air, a great way to meet him, and that it kept him company to the gates of the city. Shorn of all the inevitable additions of credulous narrators the probability is that Regiomontanus, who was of a mechanical turn of mind, fashioned a clockwork contrivance which, more by luck possibly than by design, acted as a glider when released. Finally, there are records of experiments by an obscure contemporary of Leonardo, one Giovanni Baptista Danti, who essayed a flight at Perugia towards the close of the fifteenth century by means of a contrivance of wings which worked "with a horrible hissing sound." It was perhaps an imitation of one of the machines which Leonardo himself designed.

Da Vinci and His Problem of Flight.—Leonardo da Vinci was born in 1452 and died in 1519. It is therefore

¹ *Historiæ Anglicanæ Scriptores*, x., Twysden.

obvious from what we have said that in attempting to tackle seriously the problem of flight he owed next to nothing either to his contemporaries or to those who lived before his day. Careful study of his manuscripts make it clear to all fair-minded students that, in the truest sense of the term, da Vinci was not only a real pioneer of the science of flight, but was also the first pioneer. With all his many interests and activities, he yet gave, at intervals during thirty years of his life, a close consideration to the problem. It fascinated and held him. His ambition appears to have been to achieve a flight from the summit of Monte Ceceri (the name of a bird), situated a little to the north and east of Fiesole. No doubt the name appealed to the artist in him as being peculiarly appropriate.

From the mountain which bears the name of the great bird, the famous bird will take its flight, and will fill the world with its great fame,¹

he writes ; and again,

The great bird [he frequently refers to his conception of a flying machine as a bird] will take its first flight on the back of the great bird, and filling the world with stupor and all writings with renown and bringing glory to the next where it was born.²

These are extravagant words which carry their own significance as to the frame of mind of him who penned them, but, coming as they do from one whom we know to have been imbued with the scientific spirit, they have an added importance. It is not surprising, therefore,

¹ *Sul Volo degli Uccelli*, fol. 18 v.

² *Ibid.*, inside back cover.

that we meet with his discussions on flight both as early as the year 1490, during his residence at Milan,¹ and as late as 1514, whilst at Rome.² His main contribution to the subject, the note-book *Sul Volo degli Uccelli* (*On the Flight of Birds*), was written at Florence in 1505.

There were certain fundamental ideas which clearly guided da Vinci through the whole course of his investigations. Of these the most important was the time-old view that the imitation of the bird was the right line to adopt, and that its study would reveal the true secrets of flight.

A bird [writes he] is an instrument working according to mathematical law, an instrument which it is within the capacity of man to reproduce with all its movements, though not with a corresponding degree of strength, for it is deficient in the power of maintaining equilibrium. We may therefore say that such an instrument constructed by man is lacking in nothing except the life of the bird, and this life must needs be supplied from that of man.³

He had no illusions as to the comparison between a bird using its own living members and accessories and a man using wings and accessories which have no life in themselves :

The life which resides in the birds' members will, without doubt, better conform to their needs than will that of man which is separated from them, and especially in the almost imperceptible movements which preserve equilibrium. But since we see that the bird is equipped for many obvious varieties of

¹ MS. B. of the Collection at the Institute of France.

² MS. E. of the Collection at the Institute of France.

³ *Codex Atlan.*, 161 r.a.

movements, we are able from this experience to deduce that the most rudimentary of understanding and that he will to a great extent be able to provide against the destruction of that instrument of which he has himself become the living principle and the propeller.¹

Another fundamentally sound principle laid down by Leonardo should also be noted, namely, the need for study of the medium in which flight is carried out.

To attain to the true science of the movement of birds in the air it is necessary to give first the science of the winds, which we will establish by means of the movements of water. This science will be a degree [means] of arriving at the knowledge of the winged creatures in the air and in the wind.²

Was da Vinci attempting the impossible? He lived centuries before the days of mechanical power. The problem before him was, therefore, the attachment to the human frame of contrivances to be worked by muscular energy only, in conjunction with such natural assistance as could be derived with understanding from air currents. The history of flight before the days of steam, oil and gas engines includes many similar attempts after Leonardo's times, all unsuccessful, and even up to very recent years we should perhaps have dismissed the idea as incapable of achievement. The time has now come to abandon this attitude. That which Leonardo sought to achieve over 400 years ago, and for which many an intrepid adventurer has since laid down his life, has to some extent been accomplished.

¹ *Codex Atlan.*, fol. 161 r.a.

² MS. E., fol. 54 r.

To-day aeroplane flights in machines without mechanical power, on the glider principle, have been successfully carried out. One of the consequences of the Treaty of Versailles was that Germany was forbidden to build aeroplanes fitted with engines. German inventors were thus driven to the old problem of pre-engine days, and with that superior knowledge of the principles of flight which was of necessity denied to Leonardo da Vinci, they have been the pioneers in the finding of a solution. This solution is not of the type for which da Vinci worked. There is no attachment to the human frame of a scheme of mechanical wings to be operated by muscular energy. It has been rather a case of a modern type of aeroplane without an engine, deriving its power from a stiffening breeze, and operated by a man, and capable of both manœuvre and control. To this extent the problem that Leonardo set himself has actually been solved.

It is important to notice that throughout Leonardo faces his problem in a true scientific spirit. He looks to those beings who do fly, he studies them carefully, their physiology, their anatomy, the medium in which they exist ; he studies their movements through a vast variety of conditions. To every effect which he observes he looks for a cause, and to that cause he applies modifying conditions. He seeks and makes deductions, always with a view to obtaining control.

There are two very interesting contributions to the general subject of air conquest for which we are indebted to da Vinci. They are not only important in themselves, but serve also to show another aspect of the real man of science in the object of our study. The first of these is Leonardo's invention of the parachute. In the *Codex Atlanticus* we read :

An object offers as much resistance to the air as the air does to the object. [Note here a fifteenth century statement of the law of reactions.] You may see that the beating of its wings against the air supports a heavy eagle in the highest and rarest atmosphere, close to the sphere of elemental fire.¹ Again you may see the air in motion over the sea fill the swelling sails and drive heavily-laden ships. From these instances and the reasons given, a man with wings large enough and duly connected might learn to overcome the resistance of the air, and by conquering it, succeed in subjugating it and rising above it.²

Here, clearly, we have the principle of the parachute, and Leonardo includes in his manuscript the figure here shown (Fig. 73) together with the accompanying explanation :

If a man have a tent roof of calked linen 12 braccia broad [roughly a bracci equals a yard] and twelve braccia high, he will be able to let himself fall from any great height without danger to himself.

There is evidence that nearly a century (c. 1595) after da Vinci's invention, and probably as a direct result of it, Veranzio, a Venetian, modified the original design, using a sort of square sail extended by four rods of equal size and having four cords attached at the corners.³ Nevertheless, the first real descent in a parachute was not made till 1783, when Lenormand carried out a successful experiment from an observatory at Montpellier.

¹ Referring, of course, to the fifteenth century conception of the outer universe.

² *Codex Atlan.*, fol. 372.

³ *A History of Aeronautics* (Vivian, London, 1921).

The other contribution is da Vinci's virtual discovery of the "lighter-than-air" principle. Leonardo was aware of the now familiar principle of the decreasing density of the atmosphere with altitude. "The air," he remarks, "has greater density when nearer to water and greater rarity when nearer to the cold region, and mid-way between these it is purer."¹

Further, he knew of the principle of the fire balloon. Vasari tells us² that he made figures of thin wax of strange shapes and filled them with warm air, causing them to "fly" through the air to the great surprise of on-lookers. Why then was Leonardo not

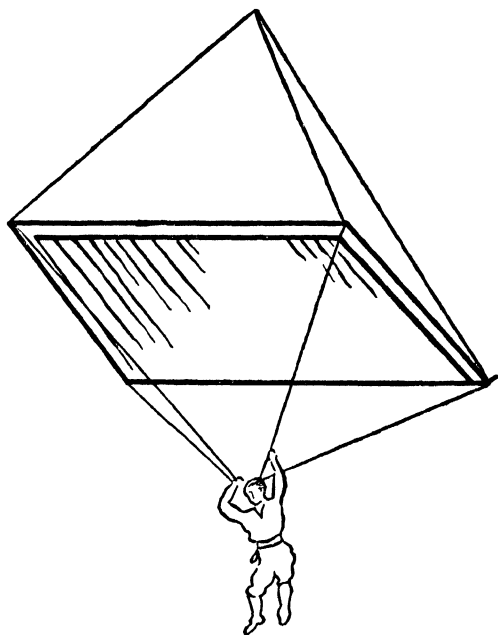


FIG. 73.

diverted to the study of balloons and aerostatics? So much simpler in basic theory is this than the sister study of aerodynamics that as soon as hydrogen was discovered the science of ballooning monopolised the whole attention of students of aeronautics, to the detri-

¹ *Codex Atl.*, fol. 161 r.a.

² *Lives of the Painters* (Vasari, Venice, 1550).

ment of heavier-than-air design. Having regard to da Vinci's intellect, his practical bent and his opportunities, he could undoubtedly have made the fire balloon and the parachute the starting point of a very successful venture in the field of aerostatics. But this had no appeal for him. In a balloon there is neither life nor control. In the "bird" of da Vinci's imagination there was both, and with this goal clear in his mind there was no room for diversion. We have here a clear instance of the influence of scientific purpose. One less imbued with the true scientific spirit would have lost sight of the original problem in the joy of a new discovery off the beaten track. That this was not so with Leonardo is a fact which must affect our estimate of his scientific worth.

Da Vinci and the Flight of Birds.—Let us now turn to some extracts from da Vinci's note-books dealing with the flight of birds. Leonardo was very clear as to the need for a proper scheme of approach to his problem. In another manuscript we read ¹ :

I have divided the treatise on birds into four books ; the first treats of their flight by beating their wings ; the second treats of flight without beating their wings and with the help of the wind ; the third treats of flight in general, such as that of birds, bats, fishes, animals and insects ; the last of the mechanism of this movement.

Again, there occurs the following passage :

Whether birds when continually descending without beating their wings will proceed a greater distance in one sustained curve. . . . Whether when they wish to pass in flight from one spot to

¹ MS. K., fol. 3 r.

another they will go more quickly by making impetuous, headlong movements, then rising up with reactive movement and again making a fresh descent, and so continuing. To speak of this subject you must explain in the first book the nature of the resistance of the air, in the second the anatomy of the bird and of its wings, in the third the method of working of the wings in their various movements, in the fourth the power of the wings and of the tail at such time as the wings are not being moved and the wind is favourable, to serve as a guide in different movements.¹

Leonardo fully recognised the rôle of centres of gravity as being fundamental in aeronautics, as in technics generally. At the outset we read in his *Flight of Birds* that

mechanical science is very noble and useful beyond all others, for by its means all animated bodies which have movement perform their operations ; which movement proceeds from their centre of gravity. This is situated at the centre, except with unequal [distribution of] weight.²

Later in the same note-book³ we meet with a most interesting little sketch of a "bird" suspended from a bracket by means of pulleys (Fig. 74), together with the note : " This is done to find the centre of gravity of the bird without which this instrument would have but little value."

Da Vinci next interests himself in the problems of wind pressure. For a bird to be sustained in the air during horizontal flight there must be an upward pressure

¹ MS. F., fol. 41 v.

² *Sul Volo degli Uccelli*, fol. 3 r.

³ *Ibid.*, fol. 16 (15) v.

over a moving wing surface. Accordingly he asks, "In which part of the under-surface of the width of the bird does the wing press the air more than in any other part

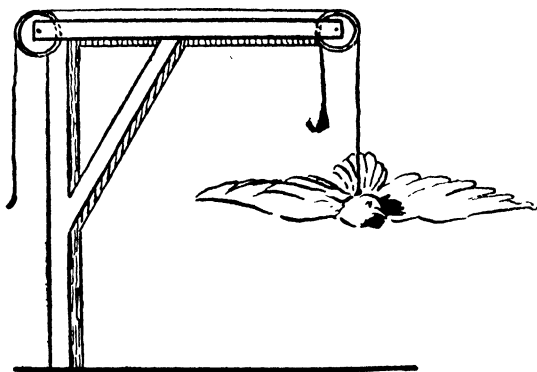


FIG. 74.

of the length of the wings?"¹ (Fig. 75). Developing his theme further, he continues :

All bodies which do not bend, whatever their size or weight, exert equal pressures on all the supports that are equi-distant from the centre of gravity, this centre being at the middle of the substance of such a body.

He then considers the case of a stiff surface loaded at



FIG. 75.

equal intervals with equal weights, and next the case of a flexible body or surface loaded in various ways, always

¹ *Sul Volo degli Uccelli*, fol. 4 v.

bearing in mind the point that "the heavier portion always guides the movement."

How is the necessary "wind pressure" required to support the bird's weight during flight to be obtained? Da Vinci finds his answer in the beating of the wings, and he studies with care the local variations in pressure in the neighbourhood of the wing surface caused by the "beating" movements. "The properties of the air are such that it may become condensed or rarefied," says he. He realises clearly that support must come from air condensation below the wing. "Unless the movement of



FIG. 76.

the wing which presses the air is swifter than the movement of the air so pressed, the air will not become condensed beneath the wing, and in consequence the bird will not support itself above the air. . . .

"That part of the air which is nearest to the wing which presses on it will have the greatest density."¹ He makes a similar point elsewhere in his notes.

When the bird shall be in the position *a n c* (Fig. 76) and shall wish to rise, it will elevate the shoulders *m* and *o*, and will thus be in the position *b m n o d*, and the air will be pressed between the sides and the point of the wings so that it will be

¹ *Codex Atlan.*, fol. 161 r.a.

condensed and cause an upward movement and give rise to an impetus in the air, which impetus of the air will push the bird upwards by its condensation.¹

He further develops the function of the beating of wings in this connection as follows :

The simple movement possessed by the wings of birds is easier, as they are raised and lowered. This ease of movement is born of two causes, of which the first is that the lowering weight raises up the wings a little by themselves ; the second is that the wings being convex above and concave below, the air escapes more easily the percussion of the wing with elevation than with lowering, since the air in the concavity of the wing engenders more easily its condensation than its escape.²

In this connection it is almost startling to notice that even the essentially modern subject of "streamlines" attracted Leonardo's penetrating attention.

That part of the air nearest the wing [he writes] will most resemble in its movement the movement of the wing which presses on it ; and that part will be more stable which is further from the said wing. That part of the air which is the nearest to the wing which presses on it will have the greatest density.³

One of the most important aspects of the "heavier-than-air" theory is that concerned with the relationship between centre of gravity and centre of pressure. In any

¹ *Sul Volo degli Uccelli*, fol. 13 (12) r.

² MS. E., fol. 39 r.

³ *Codex Atlan.*, fol. 161 r.a.

given machine the centre of gravity is a fixed point, but the centre of pressure being dependent upon a number of varying factors, changes with the conditions of the flight. It shifts with the varying angles formed by the wing surfaces to the direction of the air opposing it, and its position can be calculated for any given position and for any given amount of wing surface from the ratio of what are termed the lift coefficient and drag coefficient respectively (in effect two components of the resultant air pressure upon the given surface) and the turning moment about some selected point. In the modern machine, longitudinal stability is assured by having the centre of gravity suitably placed and coincident with the centre of pressure and the thrust of the propeller. Obviously, however, a shift in the centre of pressure from this point will produce a "turning moment" about the new position of the point, and will disturb the equilibrium of the machine. Such changes constantly occur during flight and are necessary to effective manoeuvre, but so long as they are *controlled* changes, they are all to the good.

This important factor of the centre of pressure, or centre of resistance as we may alternatively call it, was known to da Vinci, though naturally in the absence of exact mathematical knowledge he could deal with it only in a very general way. Nevertheless, his notes indicate remarkable accuracy of observation combined with scientific intuition.

When a bird which is in equilibrium throws the centre of resistance of the wings behind the centre of gravity, then it will descend with its head downwards. A bird which finds itself in equilibrium will have the centre of resistance of the wings more

forward than the bird's centre of gravity ; and such a bird will fall with its tail turned towards the earth.¹

Looking at the matter from the point of view of manœuvre, we have the following passages :

When the bird sinks, then the centre of gravity is outside the centre of its resistance ; as if the centre of gravity were on the line *ab* (Fig. 77) and the centre of resistance on the line *cd*. And if the bird wishes to rise, then the centre of its gravity remains

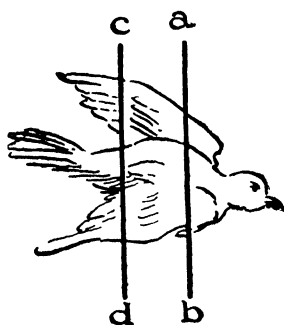


FIG. 77.

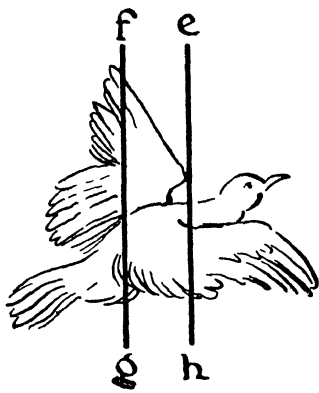


FIG. 78.

behind the centre of its resistance. As if the centre of gravity mentioned might be in *fg*, the centre of resistance would be *eh*² (Fig. 78).

We quote yet once more from the same note-book :

The descent of a bird will be always by that end nearer to its centre of gravity. The heavier part of a bird descending will always be in front of its centre of pressure. When, without the assist-

¹ *Sul Volo degli Uccelli*, fol. 8 (7) v.

² *Ibid.*, fol. 16 (15) v.

ance of the wind and without beating its wings, the bird remains in the air in the position of equilibrium, this shows that the centre of gravity is coincident with the centre of pressure.¹

The Function of the Tail.—The function of the tail comes in for Leonardo's consideration and gives rise to very important conclusions. The subject affords particular interest from the point of view of scientific method. Leonardo first carries out close observations on the various uses to which a bird appears to put its tail. He makes deductions as to the functions of the tail, and then proceeds to design models in illustration of these functions. Nothing could be more sound or more essentially modern than the scientific outlook here involved, quite apart from the truth or otherwise of the conclusions arrived at. It is unfortunate that comparison with the corresponding functions in the modern aeroplane is difficult, but it must be remembered that whilst in the case of the bird there is merely a tail, in the case of the modern aeroplane we have tailplane, rudder and fin, each with their separate functions. On the other hand, we must also bear in mind the interesting physiological fact that the bird, by virtue of its internal structure, can so control its air sacs as to be able at will to shift its centre of gravity within certain limits. This undoubtedly provides the bird with a mechanism for ensuring stability and control that is denied to the artificial machine.

We will first quote passages illustrative of da Vinci's general observations on the use of the tail in birds, *e.g.* :

The opening and lowering of the tail and the simultaneous spreading out of the wings to their full

¹ *Sul Volo degli Uccelli*, fol. 8 (7) v.

extent arrests the swift movement of birds. When birds, in descending, approach the ground and the head is below the tail, they lower the tail, spread it wide open, and take short strokes with the wings ; as a consequence the head becomes higher than the tail and the speed is checked so that the bird alights on the ground without any shock. [Note here, in passing, the excellent treatment of the subject of landing, which finds so delightful a counterpart in the corresponding manœuvre in modern aviation practice.] In all changes which birds make in their lines of movement, they spread out their tails.¹

On the next page of the same note-book we read : “ The speed of birds is checked by the opening and spreading out of the tail.² ” Further observations are recorded in the *Codex Atlanticus* :

Beneath the wings the down and feathers are plentiful, and at the ends of the wings and tail the tips of the feathers are flexible or capable of being bent, whilst those on the front of the wings, where it strikes the air, are firm.³

And again,

Beginnings of things are oft the cause of great results. Thus we may see a small almost imperceptible movement of the rudder turn a ship of marvellous size and loaded with a very heavy cargo, and that, too, amid such a weight of water as presses on its every beam, and in the teeth of the impetuous winds enveloping its mighty sails. So in those birds which can support themselves above the course of

¹ MS. L., fol. 58 v.

² *Ibid.*, fol. 59 v.

³ *Codex Atlan.*, fol. 308 v.b.

the winds without beating their wings, a slight movement of wing or tail, serving them to enter either below or above the wind, suffices to prevent their fall.¹

We turn next to examples illustrative of da Vinci's combination of observation with deduction. In the notebook *Sul volo degli Uccelli* we read :

The second shaft in the opposite portion, beyond the centre of gravity of the bird, and that is the tail, which, if it is struck by the wind underneath, since it is beyond the said centre it will cause the forward portion of the bird to lower. And if this tail be struck on top the forward portion of the bird will rise. And if this tail twist a little and turn its under-surface obliquely to the right wing the anterior portion of the bird is turned to the right side. And if it turn this side obliquely of the undersurface of the tail to the left wing, it (the bird) will be turned with its forward portion toward the left side, and in each of the two manners the bird will sink.

But if the tail, placed obliquely, be struck by the wind on its upper surface, the bird will be turned, in turning it (the tail) slowly from this side where the superior surface of the tail shows its obliquity.²

Again, in Manuscript K. occurs the following passage :

When the bird rises up by assistance of the wind without beating its wings, it spreads out and raises its wings so that they form an arch with concave side towards the sky and it receives the wind under its wings continuously, in its movements to and fro, and this would cause it to turn right over if it were

¹ *Codex Atlan.*, fol. 308 v.b.

² *Sul Volo degli Uccelli*, fol. 14 (13) v.

not that the point of its tail is turned to the wind as it enters beneath the wind ; and this afterwards by its power of resistance acts to prevent the said movement of turning over, because the wings are restrained by the tail in such a way that their various parts are of equal power, and so the tail becomes partly lowered and the bird is raised forward slightly.¹

Perhaps one of the most interesting passages of this type is the following, which shows clearly da Vinci's interpretation of the functions of the tail with reference to that consideration of the shifting of the centre of pressure to which we have already drawn attention :

If the bird fall tail first, by throwing the tail back it will restore itself to the place of equilibrium, and if it throws it forward it will overturn itself.²

These, then, are da Vinci's observations and deductions. What did he do in the way of experiment to support them ? In one manuscript³ occurs the following highly interesting passage, affording another of those references to Leonardo's personal experiments :

Suppose that here then is a body suspended, which resembles that of a bird, and that its tail is twisted to an angle of various different degrees ; you will be able by means of this to deduce a general rule as to the various twists and turns in the movements of the birds occasioned by the bending of their tails.

The Movement of the Wings.—It will be convenient

¹ MS. K., fol. 10 v.

² *Sul Volo degli Uccelli*, fol. 8 (7) v.

³ MS. L., fol. 61 v.

at this stage to consider Leonardo da Vinci's observations on the movement of the wings during "beating" flight (as distinct from "soaring" flight) and the criticisms to which these observations have been subjected. It will be helpful if we point out at once that the current theory as to wing movements has emerged as a consequence of the careful researches of the late J. B. Pettigrew. The results of these researches were presented in a series of memoirs¹ in the year 1867. We may summarise his results in his own words as follows :

That quadrupeds walk, and fishes swim, and insects, bats and birds fly by figure-of-eight movements.

That the flipper of the sea bear, the swimming wing of the penguin and the wing of the insect, bat and bird, are screws *structurally* and resemble the blade of an ordinary screw propeller.

That these organs are screws *functionally*, from their twisting and untwisting, and from their rotating in the direction of their length when they are made to oscillate.

That they have a reciprocating action, and reverse their planes more or less completely at every stroke.

That the wing describes a *figure-of-eight* track in space when the flying animal is artificially fixed.

That the wing, when the flying animal is progressing at a high speed in a horizontal direction, describes a *looped* and then a *waved track*, from the

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- ¹ 1. "On the Various Modes of Flight in Relation to Aeronautics," J. Bell Pettigrew (*Proc. Royal Inst.*, March, 1867).
2. "On the Mechanical Appliances by which Flight is attained in the Animal Kingdom," J. Bell Pettigrew (*Trans. Linn. Soc.*, June, 1867).
3. "On the Psychology of Wings" (*Trans. Roy. Soc., Edin.*, August, 1867).

fact that the figure-of-eight is gradually opened out or unravelled as the animal advances.

That the wing acts after the manner of a kite, both during the down and up strokes.¹

These conclusions apply equally to both insect and bird, but naturally the loops of the figure-of-eight will be differently proportioned in the different species. Again, during progressive flight the loops made by the wing of the insect, owing to the more oblique stroke, are more horizontal than those made by the wing of the bat and bird.

The principle is, however, in both cases the same, the loops ultimately terminating in a waved track. The impulse is communicated to the insect wing at the heavy parts of the loops *a b c d e f g h i j k l m n o* of Fig. 79 (*a*); the waved tracks being indicated at *p q r s t* of the same figure. The recoil obtained from the air is represented at corresponding letters of Fig. 79 (*v*), the body of the insect being carried along the curve indicated by the dotted line. The impulse is communicated to the wing of the bat and the bird at the heavy part of the loops *a b c d e f g h i j k l m n o* of Fig. 79 (*c*), the waved track being indicated at *p s t u v w* of this figure. When the horizontal speed attained is high, the wing is successively and rapidly brought into contact with innumerable columns of undisturbed air. It consequently is a matter of indifference whether the wing is carried at high speed against undisturbed air, or whether it operates upon air travelling at high speed (as, *e.g.*, the artificial currents produced by the rapidly reciprocating action of the wing); the result is the same in both cases, inasmuch as a certain

¹ *Animal Locomotion*, p. 16, J. B. Pettigrew (London, 1873).

quantity of air is worked up under the wing and the necessary degree of support and progression extracted from it. It is, therefore, that as the horizontal speed of

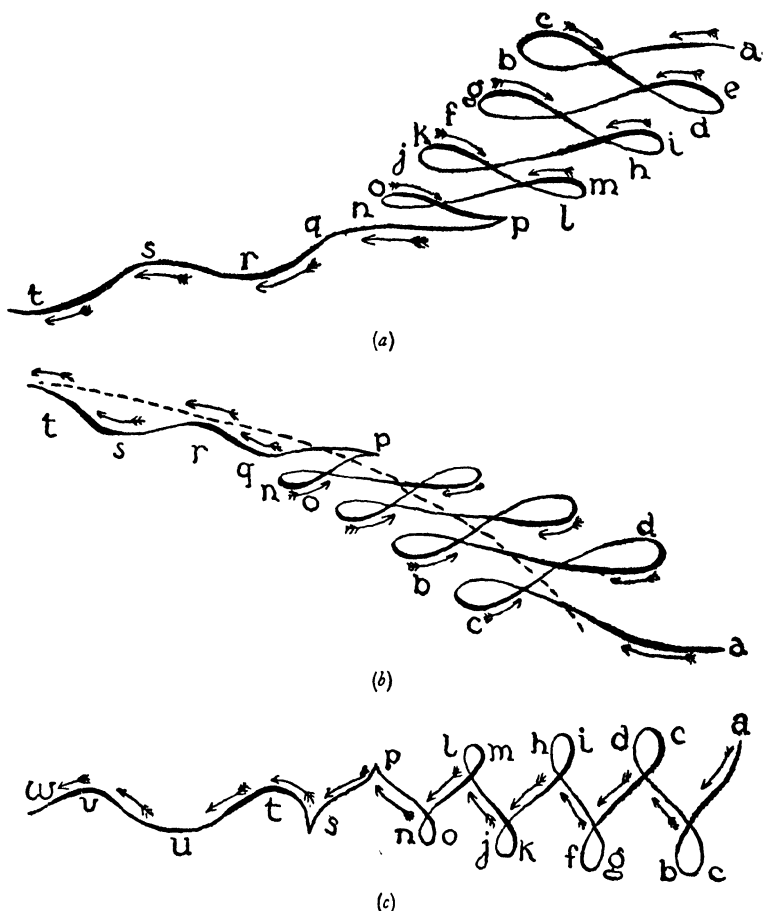


FIG. 79.

the body increases the reciprocating action of the wing decreases, and *vice versâ*.¹

¹ *Animal Locomotion*, p. 44, J. B. Pettigrew (London, 1873).

What then is the actual nature of the stroke of a wing during the progressive movement of a bird in horizontal flight? One further quotation from Pettigrew will serve to make this clear.

It is a condition of natural wings, and of artificial wings constructed on the principle of living wings, that when forcibly elevated or depressed, even in a strictly vertical direction, they inevitably dart forward. If, for example, the wing is suddenly depressed in a *vertical direction*, as represented at *ab* (Fig. 80), it at once darts downwards and forwards in a curve to *c*, thus converting the down stroke into a *down oblique forward stroke*. If, again,

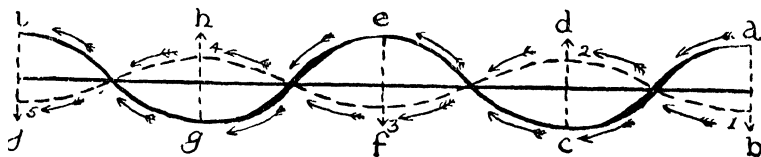


FIG. 80.

the wing be suddenly elevated in a strictly vertical direction, as at *cd*, the wing as certainly darts upwards and forwards in a curve to *e*, thus converting the vertical up stroke into an *upward oblique forward stroke*.¹

Clearly, then, *the wing strikes downwards and forwards during the down stroke, and upwards and forwards during the up stroke*, and this was strikingly confirmed two years after Pettigrew's statement of the theory by Marey,² who devised an instrument by means of which he was able to obtain an automatic self-registration of the wing movements for both stationary and progressive flight.

¹ *Animal Locomotion*, p. 157, J. B. Pettigrew (London, 1873).

² *Revue des cours scientifiques de la France et de l'Etranger*, 1869.

The importance of a true understanding of these movements to the student of both natural and artificial flight is beyond question. This must have been evident to Leonardo da Vinci, and references in his note-books clearly show this. It must be confessed, however, that the observations show our pioneer to be at fault. Three instances occur, one in Manuscript L. and two in the note-book *Sul volo degli Uccelli*. In the former we read as follows :

The birds which fly swiftly, keeping at the same distance above the ground, are in the habit of beating their wings downwards and behind them, downwards to the extent necessary to prevent the bird from descending and behind when they wish to advance with greater speed.¹

In the latter we have the following passage :

If, in the descent of the bird, it rows backwards with its wings, the bird will move rapidly, and this happens because the wings strike in the air which successively flows behind the bird, to fill the void that it has left.²

These two passages clearly indicate the inaccuracy of da Vinci in the light of the facts as they have been presented by Pettigrew. The third passage shows, however, that he was not wholly wrong and that for at least a part of the movement his observations were sound.

But when the wing recovers a new force with its return *upwards and forwards*, then the big finger of the wing puts itself in a straight line with the other fingers, and thus with its cutting extremity

¹ MS. L., fol. 59 v.

² *Sul Volo degli Uccelli*, fol. 12 (11) r.

divides the air, by some movement, high or low that the bird would rise.¹

On the whole, the balance of argument undoubtedly shows that on this subject Leonardo was at fault.

Soaring Flight.—We turn now to the subject of soaring flight—that is to say, flight without the beating of wings—in so far as it was developed by Leonardo da Vinci. In this, more than in any other branch of the study of natural flight, Leonardo excelled in the care and accuracy of his observations.

There are still problems of interest connected with the soaring flight of birds awaiting solution. It was known to Leonardo that the power of birds in keeping poised in the air for long periods with apparently very little effort is aided to some extent by the existence of varying air currents at different levels. He also knew that some of the features of the flight and buoyancy of birds are due to their instinctive knowledge and utilisation of air movements which normally appear almost too fine for detection. It is in the attempt at such detection that experiments upon the subject of soaring flight are being carried out to-day, and it is interesting to note that the possibilities have emerged largely as a result of the recent successful experiments on man-propelled machines. The spaces of the sky, in which no apparent motion is visible to the human eye, are full of a movement that the instinct of a bird can apprehend and utilise. It is known that air currents are influenced by ground obstacles, such as hills and woods, while the very colouring of the countryside, which absorbs the solar heat unevenly, gives rise to wind deflections. That

¹ *Sul Volo degli Uccelli*, fol. 14 (13) v.

da Vinci was to some extent aware of the influence of ground contour on air currents is evident from the following passage :

Nature has so provided that all the large birds can stay at so great an elevation that the wind which increases their flight may be of straight course and powerful. If their flight were low, among mountains where the wind goes wandering and is perpetually full of eddies and whirlwinds they would be unable to find any spot of shelter by reason of the fury of the icy blasts among the narrow defiles of the mountains, nor would they be able to so guide themselves with their great winds as to avoid being dashed upon the cliffs and high rocks and trees, and this would sometimes prove to be the cause of their destruction.¹

In 1908 the late Sir Hiram Maxim published his well-known book on *Artificial and Natural Flight*, the second chapter of which was devoted to Air Currents and the Flight of Birds. It contains no reference to Leonardo's note-book, *Sul Volo degli Uccelli*. A comparison of the two books, the one written almost exactly four hundred years after the other, shows in many respects a very remarkable similarity of statements and a coincidence of views. For example, we read in Maxim :

I have often observed the flight of hawks and eagles. They seem to glide through the air with hardly any movement of their wings. Sometimes, however, they stop and hold themselves in a stationary position directly over a certain spot, carefully watching something on the earth immediately below. In such cases they often work their wings with great

¹ *Codex Atlan.*, fol. 308 v.b.

rapidity, evidently expending an enormous amount of energy. When, however, they cease to hover and commence to move again through the air, they appear to keep themselves at the same height with an almost imperceptible expenditure of power.¹

And again, later, we meet with the following passage :

I have often noticed that gulls are able to follow a ship without any apparent exertion ; they simply balance themselves on an ascending column of air, where they seem to be quite as much at ease as they would have been roosting on a solid support. If, however, they are driven out of their position, they generally commence at once to work their passage. If anything is thrown overboard which is too heavy for them to lift, the ship soon leaves them behind, and in order to catch up with it again they move their wings very much as other birds do ; but when once established in the ascending column of air, they manage to keep up with the ship by doing little or no work. In a calm or head wind we find them directly aft of the ship ; if the wind is from the port side, they may always be found on the starboard quarter, and *vice versa*.²

Again, later in the chapter we read of

Soaring birds which practically live upon the wing and, by some very delicate sense of touch, are able to feel the exact condition of the air . . . and I have no doubt that the air cells, which are known to be very numerous and to abound throughout the bodies of birds, are so sensitive as to enable soaring birds to know at once whether they are in an ascending or a descending column of air.³

¹ Maxim, *Artificial and Natural Flight*, p. 13 (London, 1908).

² *Ibid.*, p. 20.

³ *Ibid.*, p. 23.

It is obvious here that the contention is that soaring birds are capable of periods of "rest" by supporting themselves on those columns of ascending air which are so plentifully produced in nature by the ordinary processes of convection. One further quotation will serve to complete the summary of the position and to enable us to make comparisons with the corresponding entries in Leonardo da Vinci's note-books.

Suppose [says Maxim] that the local influence which causes the up and down motion of the air should be sufficiently great to cause the air to rise at the rate of two miles an hour, and that the wind at the same time should be blowing at the rate of ten miles an hour, the motion of the air would then be the resultant of these two velocities. In other words, it would be blowing up an incline of 1 in 5. Suppose now that a bird should be able to so adjust its wings that it advanced five miles in falling one mile through a perfectly calm atmosphere, it would then be able to sustain itself in an inclined wind, such as I have described, without any movement at all of its wings. If it were possible to adjust its wings in such a manner that it could advance six miles by falling through one mile of air, it would then be able to sustain itself in an inclined wind, such as I have described, without any movement at all of its wings. If it were possible to adjust its wings in such a manner that it could advance six miles by falling through one mile of air, it would then be able to rise as relates to the earth, while in reality falling as relates to the surrounding air.¹

We turn now to compare these statements with the following passages from da Vinci's note-books :

¹ Maxim, *Artificial and Natural Flight*, p. 19.

The imperceptible flutterings of the wings without any actual stroke keeps the bird poised and motionless amid the moving air.¹ The kite and other birds that move their wings very little seek the air currents and when the wind is blowing high up then they will be seen at a great height and if the wind blows low down they remain low down. When there is little wind the kite beats several times with its wings during flight so that it may rise and obtain impetus, with which impetus descending a little, it goes a long distance without beating its wings ; and when it has dropped somewhat it repeats the movements and so successively ; and this drop without beating the wings serves as a means of repose in the air after the fatigue of the said beating of the wings.²

And again,

The helms which are on the shoulders of the wings are necessary when the bird in its flight without beating its wings wishes to maintain itself in part of a tract of air, upon which it is either slipping down or rising, and when it wishes to bend either upwards or downwards or to right or left.³

The similarity of treatment with Maxim is strikingly indicated, too, by the following further passages from the manuscript *On the Flight of Birds* :

Here the big fingers of the wings are those which hold the bird still in the air against the movement of the wind ; that is to say, the wind moves on which it supports itself without beating its wings and the bird does not change its place. The reason

¹ *Codex Atlan.*, fol. 308 v.b.

² *Sul Volo degli Uccelli*, fol. 6 (5) v.

³ MS. K., fol. 7 v.

of this is that the bird arranges its wings on such an obliquity that the wind, which strikes it underneath, does not cause a wedge of such a nature as would lift it up, but lifts it up however just as much as its weight would press it down ; that is to say, if the bird fall with a force of two the wind would lift it up with another force of two and thus . . . the bird remains in its place without rising or falling.¹

Later in the same note-book we meet with the following passage:

Let us say, that the impetus be as 6, and that the bird weigh 6, and that in the middle of the movement the impetus become 3, and that the weight still remains 6 ; here the bird would sink by half-movement, that is to say, by the diameter of the square and the wing oblique with the contrary aspect, also by the diameter of this square does not allow such a weight to sink, neither does the weight permit the bird to rise ; consequently it moves in a straight line. That is to say, the descent of the bird during the said half-movement would be by the line ab ² (Fig. 81).

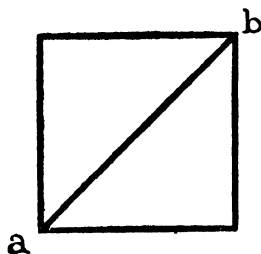


FIG. 81.

No study of the soaring of birds can be considered in any sense complete without due treatment of the large circular sweeping movements which form so familiar a spectacle of the skies, and these happily come in for a large share of attention in Leonardo's note-books. Thus, in the *Codex Atlanticus* we read :

¹ *Sul Volo degli Uccelli*, fol. 14 (13) r.

² *Ibid.*, fol. 15 (14) v.

The ways in which birds rise, without beating their wings but by circles, with the help of the wind, are of two kinds—simple and complex. The simple comprise those in which in their advancing movement they travel above the wind, receiving its buffeting from beneath, and so finish the reverse movement against the wind. The complex movement by which birds rise is also circular, and consists of an advancing and reverse movement against the direction of the wind in a course which takes the form of a half circle, and of an advancing and reverse movement which follows the course of the wind.

The simple circular movement of rising without beating the wings will always occur when there is great agitation of the winds, and this being the case, it follows that the bird in so rising is also carried a considerable distance by the force of the wind. And the complex movement will be found to occur when there are light winds, for experience shows that in these complex movements the bird rises through the air without being carried too far by the wind in the direction in which it is travelling.¹

In the manuscript *Sul Volo degli Uccelli* this theme is further developed with minute completeness,² in the course of which we meet with the following summary :

I conclude that the mounting of the bird without the beating of wings is caused by nothing other than its circular movement which, when it starts from the arrival of the wind, sinks until it reaches the place where the reflex movement begins, after which and so circulating, it has described a semi-circle and its face turned to the wind, and follows the reflex

¹ *Codex Atlan.*, fol. 308 r.b.

² *Sul Volo degli Uccelli*, fol. 7 (6) r.; fol. 13 (12) r. and v.; fol. 15 (14) v.

movement on the wind still circulating until, with the help of the wind, it makes its greatest height between its lowest and the arrival of the wind and is left with the left wing to the wind ; and from this greatest height again circulating, it descends to the last incident movement, being left with the right wing to the wind. As if to say, the wind goes from *a* to *c* (Fig. 82) and the bird moves from and sinks from *a b c* and in *c* it makes the reflex movement as in *c d a* and by the favour of the wind it is much higher at the end of the reflex movement, which end of the reflex movement is started perpendicularly over the said commencement of the incident movement.¹

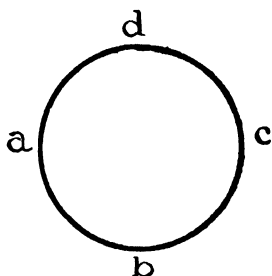


FIG. 82.

Manœuvre.—We may now turn to the question of manœuvre and control, a subject to which da Vinci gave the closest attention. He went far beyond the stage of mere observation. As befits a man with an essentially scientific outlook, he considered practically every type of manœuvre for which one would look in an artificial machine ; he closely observed the corresponding procedure in the case of birds, and then brought scientific principles to bear upon their physical explanation. In connection with the main topics so far considered in this chapter, as, for example, in the section on the functions of the tail, we have of necessity already met with a number of references of this kind. The examples are neverthe-

¹ *Sul Volo degli Uccelli*, fol. 15 (14) r.

less by no means exhausted, and we now proceed to indicate some others. As an instance, let us consider the manœuvre of turning. The necessity for "banking" was well known to da Vinci, who tells us that "when the

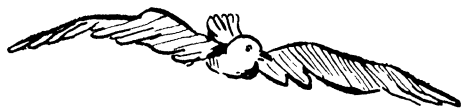


FIG. 83.

bird wishes to turn to the right or the left by beating its wings then it will beat lower with the wing on the side to which it wishes to turn (Fig. 83), and thus the bird will turn the movement behind the impetus of the wing which moves most."¹

Other observations on this manœuvre in birds are as follows :

The bird when it wishes to turn does not beat its wings with equal movement, but moves the one which makes the convex of the circle it describes more than that which makes the concave of the circle.²

And further on in the same manuscript :

The bird beats its wings repeatedly on one side only when it wishes to turn round while one wing is held stationary ; and this it does by taking a stroke with the wing in the direction of the tail, like a man rowing in a boat with two oars, who takes many strokes on that side from which he wishes to escape, and keeps the other fixed,³

Whilst the case of rapid turning in a high wind is dealt with as follows :

¹ *Sul Volo degli Uccelli*, fol. 6 (5) r.

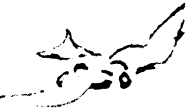
² MS. K., fol. 4 v.

³ *Ibid.*, fol. 7 r.

וְהָיָה כִּי יִשְׁמַע ה' אֶת הַקּוֹל
וְהָיָה כִּי יִשְׁמַע ה' אֶת הַקּוֹל
וְהָיָה כִּי יִשְׁמַע ה' אֶת הַקּוֹל



וְהָיָה כִּי יִשְׁמַע ה' אֶת הַקּוֹל
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וְהָיָה כִּי יִשְׁמַע ה' אֶת הַקּוֹל
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A facsimile page from Leonardo's *Manuscript on the Flight of Birds*.

[To face page 176.]

When the bird is carried along by the wind and wishes to turn quickly towards it, it will then enter beneath the wind with the wing turned towards it ; and then with the feathers of the tail turned towards the wind, it will enter upon it, and so by the help of the wind striking upon its tail it turns much more rapidly.¹

There is yet one further reference to the manœuvre of sudden turning to be found in the note-book *On the Flight of Birds*. This calls for special attention, on account of the clear indication it contains as to Leonardo's acquaintanceship with the principle of inertia. Two facts in this connection should be carefully borne in mind. Firstly, that in the fifteenth century the science of dynamics had hardly been born ; and, secondly, that according to accepted histories of mechanics,² the so-called law of inertia was discovered by Galileo, and was presented by him in his *Discorsi e dimostrazioni matematiche* in 1638. In the light of these facts, the following entry in Leonardo da Vinci's note-book is most remarkable.

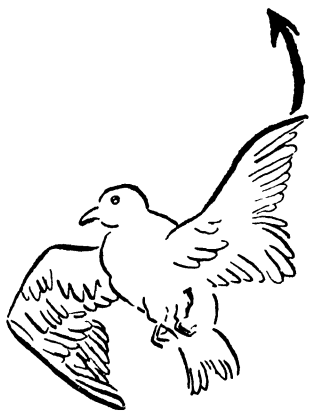


FIG. 84.

When a bird wishes to turn itself suddenly on one of its sides, then it quickly pushes the point of the wing on this side towards its tail (Fig. 84), and

¹ MS. K., fol. 9 v.

² E.g., *Science of Mechanics*, E. Mach. (English edition), p. 141. See also Chap. V.

because all movement tends to maintenance, or rather, all moved bodies continue to move as long as the impression of the force of their motors remain in them, the movement then of such a wing thrust with violence towards the tail reserving still at the end a part of the said impression, not being able by itself to follow the movement commenced at first, will move all the bird with itself, until the impetus of the moved air may be consumed. The tail pushes with its face, and the wind struck by it, makes the bird move suddenly in the contrary direction.¹

Turning to the manœuvre of rising, we have the following interesting paragraph :

When the bird by the beating of its wing wishes to rise, it raises the shoulders and it beats the points of the wings towards itself, and so condenses the air which lies between the points of the wings and the breasts. The tension (of the condensed air) lifts up the bird.²

This is elsewhere developed further from the point of view of stability.

When the bird rises up by assistance of the wind without beating its wings, it spreads out and raises its wings to form an arch with the concave side towards the sky, and it receives the wind under its wings continually in its movements to and fro. This would cause it to turn right over if it were not that the point of its tail is turned to the wind as it enters beneath the wing, and this afterwards by the power of resistance acts to prevent the said movement of turning over, because the wings are restrained

¹ *Sul Volo degli Uccelli*, fol. 13 (12) r.

² *Ibid.*, fol. 6 (5) v.

by the tail in such a way that their various parts are of equal power, and so the tail becomes partly lowered and the bird is raised forward slightly.¹

One of the most interesting features of the note-book *On the Flight of Birds* is the series of passages which deal



FIG. 85.

with problems of stability and control under a variety of conditions. Leonardo discusses, for example, the behaviour of birds on meeting with a gust of wind.

If the point of the wing be struck by the wind, and this wind enter under such a point (Fig. 85) then the bird would be liable to be upset unless it uses one of two remedies, that is to say, either that it force suddenly such a point under the wind or that it lower the opposite wing, from the middle on.²

This latter alternative, in effect, is designed to create a restoring moment. Leonardo frequently makes use of



FIG. 86.

the varying extent of wing surface presented to a wind as a method of maintaining control and equilibrium. The following passage is a striking illustration of this :

When the wind strikes the bird under its course from its centre of gravity towards this wing (Fig. 86)

¹ MS. K., fol. 10 v.

² *Sul Volo degli Uccelli*, fol. 7 (6) v.

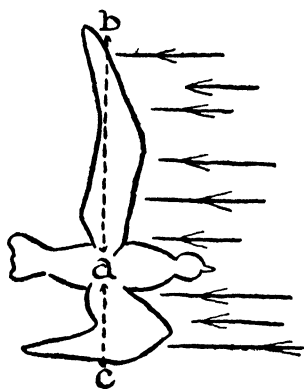


FIG. 87.

bird that it offers little resistance in comparison with b .¹

In the same note-book there is to be found an illuminating little passage worthy of notice in this connection, since it shows how sincerely scientific was Leonardo's treatment of problems of wing span in relation to wind pressure.

That the wing does not utilise all the pressure of the air [says he], and that this is true, note that the interstices of the primary feather are spaces much wider than the width of the feathers themselves ; therefore you who study flying should not calculate on the entire surface of the wing, but note the different varieties of wings for all flying creatures.²

¹ *Sul Volo degli Uccelli*, fol. 10 (9) r.

² *Ibid.*, fol. 9 (8) v.

In thus drawing a clear distinction between *apparent* wing surface and *effective* wing surface, he gives us clear evidence of a true appreciation of values. He was able, in fact, to probe beneath the surface of his problem.

In Manuscript L. there occurs an interesting example dealing with "head gusts."

When the wind is about to throw the bird backwards then the bird draws together the shoulders of its wings, so that its weight is massed more to the front than it was at first, and consequently the part that is heaviest is first in its descent while in addition the tail is spread out and bent down.¹

The following passage is also interesting as being typical of da Vinci's general treatment of problems of equilibrium during flight :

If the wing and the tail are too much on the wind, lower half the opposite wing, and therewith receive the force of the wind and equilibrium will be restored.

And if the wing and the tail were under the wind raise the opposite wing and it will be corrected to your desire, provided that such wing as is raised be less oblique than that which is opposite it.

And if the wing and the breast are on the wind, one must depress half the opposite wing, which will be struck by the wind and forced upwards, which will right itself.

And if the bird is so that its hind quarters are on the wind then the tail ought to be forced under the wind, and thus one will be able to equalise the powers.

¹ MS. L., fol. 62 r.

But if the bird has its hind quarters under the wind let it enter with the tail on the wind, and it will right itself.¹

Safety Precautions in Natural and Artificial Flight.—We next turn to examples of Leonardo da Vinci's comments on the subject of safety precautions in flight. He had no illusions as to the dangers attendant upon attempts at artificial flight. Consequently it is no matter for surprise that references to safety devices, both in the design of apparatus and in the governing principles of control, are to be found fairly plentifully in his note-books. In particular he clearly realised that which modern aviators universally regard as almost a first principle in safe flying, namely, maintaining as high an altitude as possible. There are very few accidents to apparatus, or troubles due to sudden gusts, resulting in the temporary upsetting of the equilibrium of a machine, which cannot be corrected, given sufficient time and manœuvring space. The restoration of control, in the machine of to-day, is largely a matter of skilful pilotage. But this restoration does take time, and if during this period the machine is falling then disaster must ensue unless a high altitude has at first been attained. "High flying is safe flying" is the accepted principle to-day, and it was clearly so accepted by Leonardo da Vinci four hundred years ago. In his *Sul Volo degli Uccelli* we read :

The movement of the bird ought always to be above the clouds lest the wing should be moistened to disclose a broader view and to avoid the peril of wind revolutions among mountain gorges, which are always full of whirlings and turnings of the

¹ *Sul Volo degli Uccelli*, fol. 9 (8) r.

winds. Beyond that, if the bird should turn somersault, you would have more time for righting it, with the directions already given, before it reaches the ground.¹

In the next page of the same note-book Leonardo brings this point out again, more specifically with respect to artificial flight, and he deduces the best materials advisable for the construction of the artificial limbs. He condemns the use of iron fittings and argues in favour of joints of strong tanned leather with "nerves" or "sinews" of raw silk cord. The note is illustrated by a "limb" or wing, and reads as follows :

The said bird ought, with the assistance of the wind, to rise to a great height, and that will be its safety ; even should it experience all the above-mentioned revolutions, it has time enough to recover its balance provided that its limbs be very strong so that they may safely resist the fury and vigour of the descent with the above-mentioned defences, its joints of strong tanned leather and its nerves of raw silk cord of great strength ; and let not anyone hamper himself with fittings of iron, because they burst easily in twisting, or waste away, for which reason they are not to be used.²

On the assumption—and with Leonardo this amounted almost to an article of faith—that man is capable of flight, he wisely advises as much freedom from encumbrances as possible.

A man with wings should [he writes] be free from the waist upwards in order to balance himself as he

¹ *Sul Volo degli Uccelli*, fol. 7 (6) v.

² *Ibid.*, fol. 8 (7) r.

does in a boat in order that his centre of gravity and that of the instrument might be able to balance and change when necessity required it, according to the change in the centre of its resistance.¹

Fig. 88 shows in a modified form his diagram in illustration of this point.

Another interesting passage, later on in the same notebook, is headed, "To escape the Peril of Ruin," and reads as follows :

The ruin of such instruments may happen in two

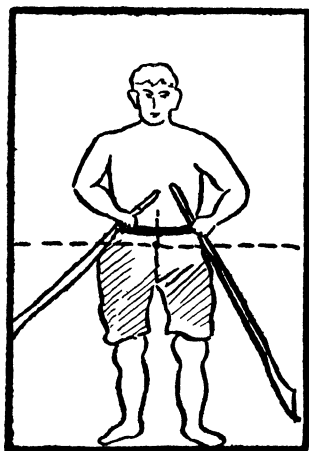


FIG. 88.

ways, of which the first is that the instrument break ; the second is that the instrument turn on its edge or near its edge, because it ought always to descend by a great obliquity or almost by the line of equality. . . . As to the protection of the instrument from rupture, one avoids it by making it of the greatest strength by no matter what line it may be able to turn itself, that is to say, either by the edge falling either with the head or the tail first, or even with the point of the right or left wing, or by the

halves or quarters of the said lines as the design shows (referring to a figure accompanying the note). As to turning by any side whatever of the edge, one ought to prevent it even in the beginning by making the instrument in such a fashion that in descending under whatever aspect

¹ *Sul Volo degli Uccelli*, fol. 6 (5) r.

that may come about the prevention should be ready ; and this will be done in giving its centre of gravity on the centre of the weight carried by it, always in a straight line, and one of its centres very distant from the other ; that is to say, that in an instrument 30 braccia wide these centres may be 4 braccia apart and that one, as mentioned, be situated under the other and the heavier underneath, so that in the descent the heavier portion might always be the guiding portion of the movement.¹

Finally, there is one further note hidden away in the corner of a page in which da Vinci, in anticipation of a fall by the fifteenth-century airman, provides the means for absorbing the shock of collision with the ground. Referring to a sketch of some wine skins strung together, he writes :

Bags, where a man, falling from a height of 6 braccia would not injure himself, falling either in water or on land ; and that these bags, fastened together in the fashion of beads, are surrounded by others.²

On the reverse side of this same folio is the comment :

If you fall, see that you strike the ground with the double wine skin that you hold under you.³

Leonardo da Vinci's Flying Machines.—We come finally to da Vinci's notes on flying machines. We have seen the general nature of our philosopher's observations and experiments and of the arguments and principles he brought to bear upon them. How did he apply these to

¹ *Sul Volo degli Uccelli*, fol. 13 (12) v.

² *Ibid.*, fol. 17 (16) r.

³ *Ibid.*, fol. 17 (16) v.

the design of suitable mechanism wherewith to achieve for mankind his ambition of mastery over the air? He lay down very specifically a principle for general guidance in the design of wings.

You are to remember [wrote he] that your bird [*i.e.*, flying machine] ought not to imitate anything but the bat (Fig. 89), because the membranes form an armour or liaison to the armour, that is to say, strength to the wings. And if you imitate the wings of the feathered birds, the wings are more powerful in bone and nerve, through being pervious; that



FIG. 89.

is to say, the feathers are disunited and permeable to the air. But the bat is aided by the membrane which binds the whole and is not pervious.¹

This was sound advice and finds its final expression in modern aviation design. It is but a small step from the impervious membrane of a bat's wing to the doped fabric of an aeroplane wing. Yet how far did Leonardo follow this advice himself? Here was a line of attack on his practical problem which, in the light of modern developments, would appear to have contained the germs of some possible success. Yet in fact he appears to have neglected

¹ *Sul Volo degli Uccelli*, fol. 16 (15) r.

it, and it is disappointing that the sketches and notes in his manuscripts show Leonardo to be occupied chiefly with the idea of the substitution of "jointed" oars for wings. An almost isolated exception occurs in one note-book in which we meet with a sketch of an artificial wing (Fig. 90), obviously modelled after the wing of a bat.

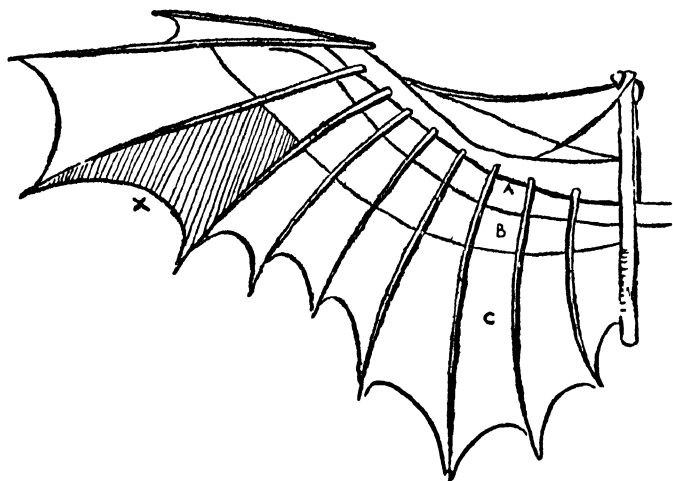


FIG. 90.

Against the shaded portion *X* he writes : " Make the meshes of this fibre of $\frac{1}{8}$ in. width." Further details of construction are given with reference to the letters *A*, *B* and *C* in the diagram as follows :

A will be of blades of deal, which has threads (?) and is light. *B* will be of fustian, on which will be glued the feathers, so as not to be easily pervious to the air. *C* should be of starched light silk, and in order to test it, you may make it of thin pasteboard.¹

¹ MS. B., fol. 74 r.

Presumably Leonardo abandoned this type of wing as impracticable. Possibly he may have concluded that the wind resistance created by such an expanse of wing surface was far too much for the muscular energy of the human frame to cope with, but if this had been the case, one would have supposed the scientific alternative would have been to have reduced the area of surface accordingly. It seems probable that Leonardo attempted some experiments with a mechanical wing, and if so it is difficult to believe that the relationship between wing surface and

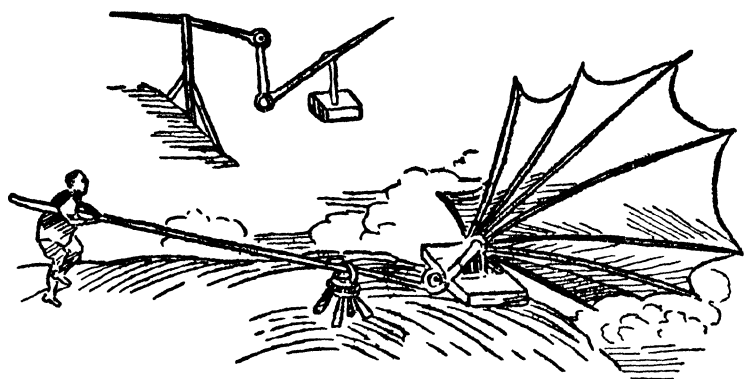


FIG. 91.

wind resistance did not claim his attention. Thus we meet with a sketch of a man operating a mechanical wing (Fig. 91) accompanied by the following explanatory note :

If you wish to see the true test of the wings, take some pasteboard strengthened with fibre, and fitted with cane ribs, a wing of the width and length of 20 braccia at least, and fix it on a board (sheet-pile) of 200 pounds weight ; ¹ it will produce, as shown in

¹ Sketches explanatory of these parts are shown in MS. B., fol. 77 v.

the figure, an effective force. And if the board of 200 lbs. is lifted before the wing falls, the test is good, but see to it also that the force be prompt (to act) and if the above-mentioned effect is not obtained, lose no more time on it.¹

We conclude from Leonardo's note-books—and these are almost our sole materials for judgment—not only that he either abandoned or neglected the idea of a “bat's wing” surface, but also that he came to no settled conclusions as to the use of a “jointed oar.” Regarding it from the modern point of view, it is obvious that the air displacement created by the movement of a form of “oar” through the air must of necessity be utterly inadequate for the generation of a sufficient force to support a human being plus mechanical attachments. Consequently one need not be surprised at the variety, the vagueness, and the incompleteness of design displayed in many sketches and notes in Manuscript B. and in the *Sul Volo degli Uccelli*. They reflect the artist and the visionary vainly striving to link up with the engineer and the scientist. Da Vinci's scientific intuition was wonderful. His “sensing” of laws of nature was one of the most remarkable traits in his personality, and it carried him far—certainly far beyond his contemporaries. But there are serious limitations to this type of personality. Between the sensing of a law and an exact knowledge of its comprehension and its consequences lies a wide gulf of mathematical expression and equipment, and in this gulf it was inevitable that Leonardo would grope and flounder with little chance of success.

As a consequence the flying machines of Leonardo da

¹ MS. B., fol. 88 v.

Vinci are mainly of artistic and historic interest. Fig. 92 shows a typical example of a da Vinci wing—the left half only of the “machine.” The following note accompanies the sketch :

a b c arranges that in rising the part *m n* is promptly raised ; *d e f* arranges that in descending *m n* is promptly lowered, and the wing (thus) fulfils its purpose ; *r t* lowers the wing with the foot, that

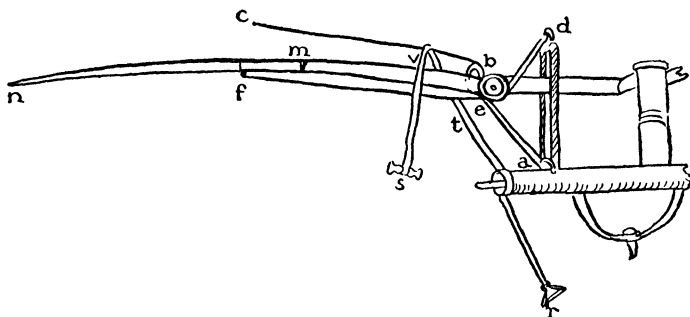


FIG. 92.

is to say, by extending the legs ; *v s* raises the wing by the hand and the turn.¹

Fig. 93 shows one of da Vinci's flying machines. The aviator is lying on a plank in front of which is a sort of pole. Fixed to this pole we see a rounded iron shank, and the jointed “oars” or wings are attached to this shank. The wings are operated by the feet by means of stirrups *c* and *d*, the right foot lowering the wing and the left raising it. The design is difficult to follow, but that Leonardo intended it seriously is evidenced by the following interesting paragraph : “ You will experiment with this instrument on a lake, and you will

¹ MS. B., fol. 73 v.

carry engirdled a long wine skin, so that in falling you will come to no harm.¹

Other designs² show double systems of oars, and he frequently favours the use of two ladders at the base of the machine to represent the feet of the "bird" and to serve the purpose of climbing into his instrument.

There is, however, one further design which calls for

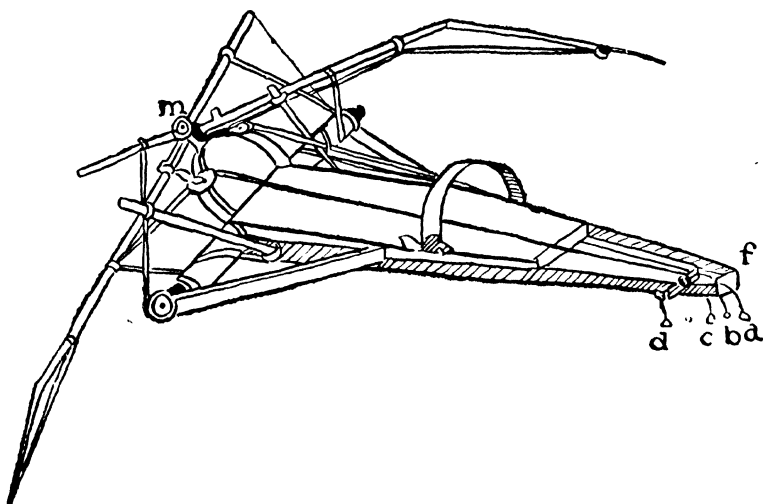


FIG. 93.

comment, since it constitutes a very real contribution to the science of aviation. It shows, in fact, that da Vinci may fairly lay claim to the invention of the helicopter ; that is to say, to a type of machine capable of vertical movement upwards and downwards, and of hovering in any given position for any required time. Leonardo's sketch is outlined in Fig. 94. It consists of a mechanism

¹ MS. B., fol. 74 v.

² E.g., MS. B., fol. 80 r.

furnished with a helical wheel. This wheel is actuated by a twisted spring which, when released so as to unwind itself, sets the helical wheel in rapid rotation. Referring to this instrument, Leonardo comments,

I say that if this instrument made with a helix is well made, that is to say, of flaxen linen, of which

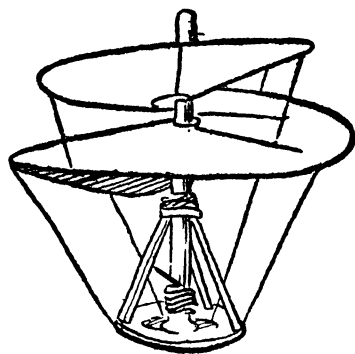


FIG. 94.

one has closed the pores with starch, and is turned with great speed, the said helix is able to make a screw in the air, and to climb high. Take the example of a wide and thin ruler and directed violently into the air ; you will see that your arm will be guided by the line of the edge of the said board. . . . One is able to make a little model of this of cardboard,

whose axis should be of thin sheet iron, twisted with force ; on freeing this, it causes the helix to turn.¹

Here, then, beyond doubt, and for the first time in history, we have the principle of the helicopter.

What are we to conclude from all these activities ? We have reviewed at some length the full scope of Leonardo da Vinci's researches and investigations in the field of aviation, and it is impossible to withhold our admiration for their breadth and their thoroughness. That he failed to achieve flight in no wise detracts from the value of his work. It is doubtful indeed if he ever even made the attempt himself. Jerome Cardan, the mathematician, whose father was a contemporary of

¹ MS. B., fol. 83 v.

Leonardo, who knew him and his work, tells us in his *de Subtilitate* that "Leonardo da Vinci also attempted to fly, but misfortune befel him from it. He was a great painter." This appears to be the only reference to an actual attempt. Nevertheless, Leonardo's work in aviation was real enough, and having regard to the limitations imposed upon him by the knowledge of his days, it was *scientific*.

Chapter VIII

Leonardo da Vinci's Manuscript on the Flight of Birds

INTRODUCTION.—The manuscript by Leonardo da Vinci, of which a complete English translation is here offered, is known as the *Codice sul Volo degli Uccelli e Varie Altre Materie* (*Codex on the Flight of Birds and other Matters*). It is a small note-book of some thirty pages, and measures about 8·4 inches by 6 inches. It was written at Florence in the year 1505, between March 14th¹ and April 15th.² A close study of this manuscript has been made by G. Piumati³ in a very fine edition containing an excellent facsimile of the original note-book, together with (*a*) a printed copy folio by folio of the old Italian script in which it was written, (*b*) a rendering into modern Italian, and (*c*) a French translation by C. Ravaisson-Mollien.

When da Vinci died at Amboise in the south of France, in 1519, he left a will bequeathing his manuscripts, instruments, paintings and sketches to the friend of his old age, Francesco Melzi, who was with him at his death. Melzi returned to his home in Milan, carrying this bequest with him. Here, in his villa at Vaprio, the manuscripts were guarded with tender and jealous care

¹ Amoretti : *Memorie storiche su la vita, gli studi, e le opere di Leonardo da Vinci* (Milan, 1840), p. 99.

² See MS., fol. 18 v., below.

³ *Codice sul Volo degli Uccelli e Varie Altre Materie*, Pubblicato da Teodoro Sabachnikoff. Trascrizioni e note di Giovanni Piumati. (Paris, 1893.)

for the next fifty years. Melzi, however, died in 1570, and it is from this time onwards that the tragic dispersal of the manuscripts may be said to have taken place. Attempts which, during Melzi's lifetime, had been made to obtain access to the note-books (notably by Alphonso I. of Ferrara and by Vasari, Leonardo's earliest biographer), and which had failed, were now renewed, this time with success.

Francesco's heirs were not interested in science, art, or letters, and when the family tutor, Lelio Gavardi di Asola, conceived the idea of disposing of some thirteen volumes of the manuscripts, he met with little difficulty. However, the Grand Duke Francesco of Florence, to whom it was intended to sell these volumes, died at this juncture, and Lelio Gavardi took them instead to Pisa. Fortunately, the man whom he approached, Ambrosio Mazzenta,¹ dissuaded Gavardi from pursuing his dishonest intentions, and the latter left the volumes with Mazzenta to be restored to the Melzi family. Orazio Melzi, the rightful owner, in his lack of interest, however, not only permitted Mazzenta to keep the volumes as a reward for his trouble, but told him further that he could help himself to any of the remaining bundles of matter lying about the attics at Valprio.

Others, too, had heard of the spoils to be gathered in from these attics, and they readily availed themselves of Orazio's free invitations of "help yourself." Amongst those seeking to acquire Leonardo's manuscripts was Pompeo Leoni, a sculptor and friend of Philip II. of Spain.

¹ The chief source of our knowledge regarding the history of these manuscripts is derived from *Alcune memorie di Giovanni Ambrogio Mazzenta intorno a Leonardo da Vinci e a suoi manoscritti, con illustrazioni del Prof. Gilberto Govi* (Il Buonarroti).

It was his plan to seek favour from his royal master in exchange for these manuscripts, and accordingly he approached Orazio Melzi with promises of reward from King Philip if he would secure the return of the thirteen volumes given to Mazzenta. As it happened, Ambrosio Mazzenta had by now (1590) taken vows with the Barnabite Order, and had passed on his thirteen volumes to his brother Guido. It was Guido, therefore, who was approached by Orazio Melzi, and as a result seven of the volumes (one of which contained the small note-book on the flight of birds) were returned and handed on to Leoni. Of the remaining six, three were subsequently acquired by Leoni on the death of one of the Mazzentas, and another volume was given in 1603 to Cardinal Federico Borromeo, the founder of the famous Ambrosian Library at Milan (1609). The remaining two have long been completely lost sight of, although we know that one of them was given to Ambrogio Figino, a painter, and has been traced through various owners up to 1759 ; and the other was given to Duke Charles Emmanuel of Savoy, and was probably destroyed in an outbreak of fire at the Royal Library at Turin about the year 1667.

Actually Pompeo Leoni did not give his manuscripts to King Philip II. of Spain. Not only did he keep them himself until his death, but he went so far as to cut out portions of them to form, together with a number of loose drawings, a single large volume which, on account of its size (it contains 402 sheets and more than 1,700 drawings and sketches), he called the *Codice Atlantico* (*Codex Atlanticus*). Leoni died in 1610 and the manuscripts passed into the possession of his heir, Cleodoro Calchi, who in his turn sold some of them, including the *Codex Atlanticus*, to S. Galeazzo Arconati in 1625. Arconati kept these

until 1636, in which year he presented to the Ambrosian Library at Milan the *Codex Atlanticus* together with eleven other volumes of da Vinci's manuscripts, thus making, together with the original volume presented by the founder of the library in 1603, a collection of thirteen volumes. To these a further one was added in 1647 by Count Orazio Archinto. One was probably subsequently stolen, since its description coincides with that of a volume bought in 1750 by Carlo Trivulzio. The period of restoration may now be said to have begun.

In Arconati's deed of gift, making over these volumes to the library, we find the first authentic record of the existence of the manuscript on the flight of birds. Inside the cover of the third volume of the gift we read: "At the end of this book is another little volume of various mathematical figures and of birds, of eighteen pages sewn in the same page in parchment."¹

For the next chapter in the story of the wanderings of these manuscripts we come to the year 1796, when Napoleon Buonaparte, during his invasion of Italy, commandeered various works of art, including the thirteen volumes of Leonardo's manuscripts, and these were transported to the library of the Institute of France in Paris, where according to the official records they arrived on the 25th of November. Here the volumes were carefully examined by Venturi and to him the world is indebted for the first real description of the manuscripts.² For purposes of reference, Venturi assigned to each manu-

¹ I Manoscritti di Leonardo da Vinci: *Codice sul Volo degli Uccelli*, etc., Pubblicato da Teodoro Sabachnikoff. Trascrizioni e note di Giovanni Piumati, pp. 28 and 29 (Paris, 1893).

² J. B. Venturi: *Essai sur les ouvrages physico-mathématiques de Leonard de Vinci* (Paris, 1796).

script a capital letter beginning with A, and this nomenclature has since received general acceptance.

As a result of political changes, in the year 1815 representations were made to France on behalf of Lombardy for the return of da Vinci's manuscripts to Milan. For some reason or other, although France consented to this restoration, only the *Codex Atlanticus* (which alone of all these volumes was housed in the Bibliothèque Nationale instead of in the library of the Institute of France) was returned to the Ambrosian Library, and the remainder have been kept in Paris ever since.

The particular volume to which we have already referred as containing an appendix of the note-book *On the Flight of Birds* was lettered "B" in Venturi's series. Venturi, however, subsequently considered the note-book of sufficient importance to be regarded as a fourteenth volume in itself and he lettered it "N" accordingly.

The later history of our small note-book is interesting in the extreme. In the year 1848 it was discovered that certain manuscripts, and amongst them the appendix to Manuscript B., were missing. That they were stolen is fairly certain, and that the culprit was Professor Giacomo Libri is indicated first by the fact that prior to 1848 he had had frequent access to these manuscripts and, secondly, that he was in undoubted possession of them in 1867. In December of that year Count Giacomo Manzoni of Lugo, whilst on a visit to Florence, was shown the manuscript amongst others "acquired" by Libri, and bought it from him a year later. Libri also disposed of some of the manuscript to Lord Ashburnham in England, but these latter were returned to France in 1888.

On the death of Count Manzoni, in 1889, the note-book *On the Flight of Birds* passed to his heirs, and from them it was acquired in 1892 by M. Theodor Sabachnikoff, and the volume associated with his name and that of Piumati is the honourable result.

The note-book has since passed into the possession of the Library of Turin.

The genuineness of the manuscript on the flight of birds is beyond all dispute. Not only does it bear all the unmistakable characteristics of Leonardo da Vinci's general work, script, mode of presentation and style, but the note-book is autographed with the writer's name on the cover. Such doubts as have been thrown by M. Ludwig,¹ not so much upon the authorship as upon the originality of the paging as it at present exists, have been effectively disposed of by Piumati.² Although a second set of page numbers has been written in by some one subsequent to the original writer, Leonardo's numbering remains intact and unmistakable.

The reason for this duplication of numbering is to be found in the fact that the manuscript on the flight of birds is not complete. Comparison with Venturi³ and Amoretti,⁴ who were the first translators of some of the passages from Leonardo's works, show clearly that pages which were in the note-book originally are not there now. The pages in question are folios 1, 2, 10 and 17. Page 18 was also originally missing, but happily was rediscovered

¹ M. Ludwig: *Leonardo da Vinci, Das Buch von der Malerei*, ii., Band, p. 5.

² Sabachnikoff: *Sul Volo degli Uccelli*, etc., p. 35.

³ J. B. Venturi: *Essai sur les ouvrages*, etc., p. 18.

⁴ Amoretti: *Memorie storiche su la vita, gli studi, e le opere di Leonardo da Vinci* (Milan, 1804), p. 99.

in England and secured by M. Th. Sabachnikoff in time to be included as an appendix to his book.¹ Its identity was assured from its dimensions and general agreement in every respect with the remainder of the note-book. The number of the page, absent through mutilation, has been identified from references to Venturi and Amoretti.

It should be remarked that although the primary purpose of the translation is to further the study of Leonardo da Vinci's contributions to aeronautical science, every note in the manuscript has been included, even though not relevant to this primary purpose. Not only is this done for the sake of completeness, but it is also hoped that the result may be of interest to students of scientific history in general, and to students of Leonardo da Vinci in particular. It is a matter for regret that it has not been found possible to include all the diagrams and sketches in the original manuscript. Nevertheless, all those which seriously bear upon the text are given.

“ON THE FLIGHT OF BIRDS.”

TRANSLATION OF A MS. BY LEONARDO DA VINCI.

NOTE.—*In the translation, text enclosed in brackets thus [] has been interpolated by the author ; text enclosed in brackets thus () denotes matter written in by Leonardo and then erased.*

[Cover—Interior [1].]

To stamp medals.—Boil emery mixed with brandy, or iron-scales with vinegar or ashes of walnut leaves, or ashes of straw finely triturated.

The diamond is crushed enveloped (within) in lead,

¹ Sabachnikoff, p. 147.

and beaten with a hammer, and the lead spread out several times, and refolded (and *r*) and kept wrapped in paper so that the powder be not spilled and then melt the lead and the powder will rise to the surface of the molten lead, which may be then rubbed between two steel plates, so that it powder well, and then wash it with nitric

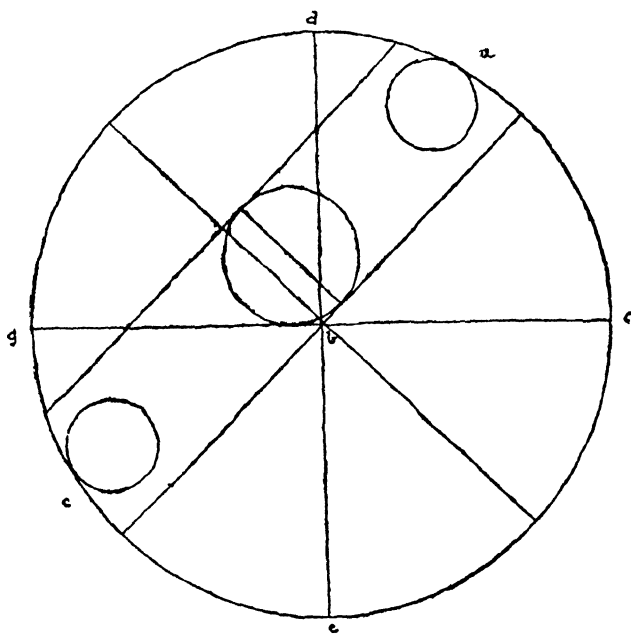


FIG. 95.

acid, and the blackness of the iron will dissolve and leave the powder clean.

Emery in large pieces is broken by putting it on a cloth folded (in) many times and then striking it sideways with the hammer ; thus it will break into flakes, little by little and then is easily crushed ; if you put it on an anvil, you will never break it, being so large.

Who powders enamel, should do so on plates of tempered steel, with a steel pestle, and then put it in nitric acid, which dissolves all the steel which is used up and mixed with this enamel making it black, so that it remains purified and clean ; if you powder it on porphyry, this porphyry wastes, is mixed with the enamel and spoils it,

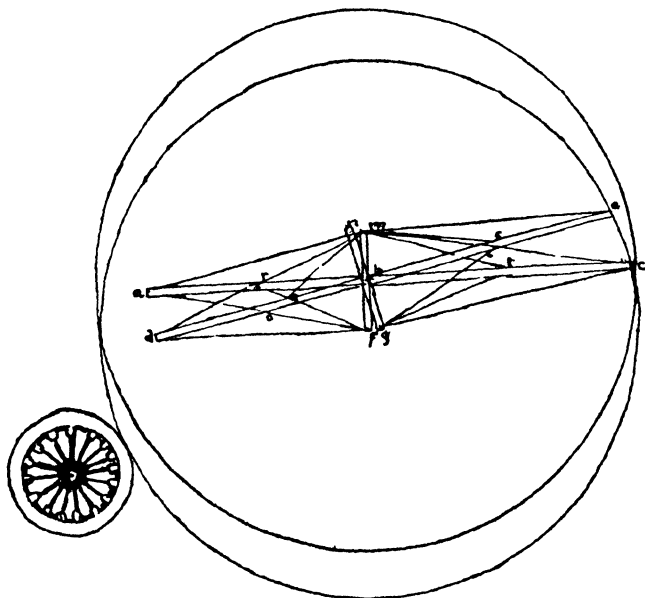


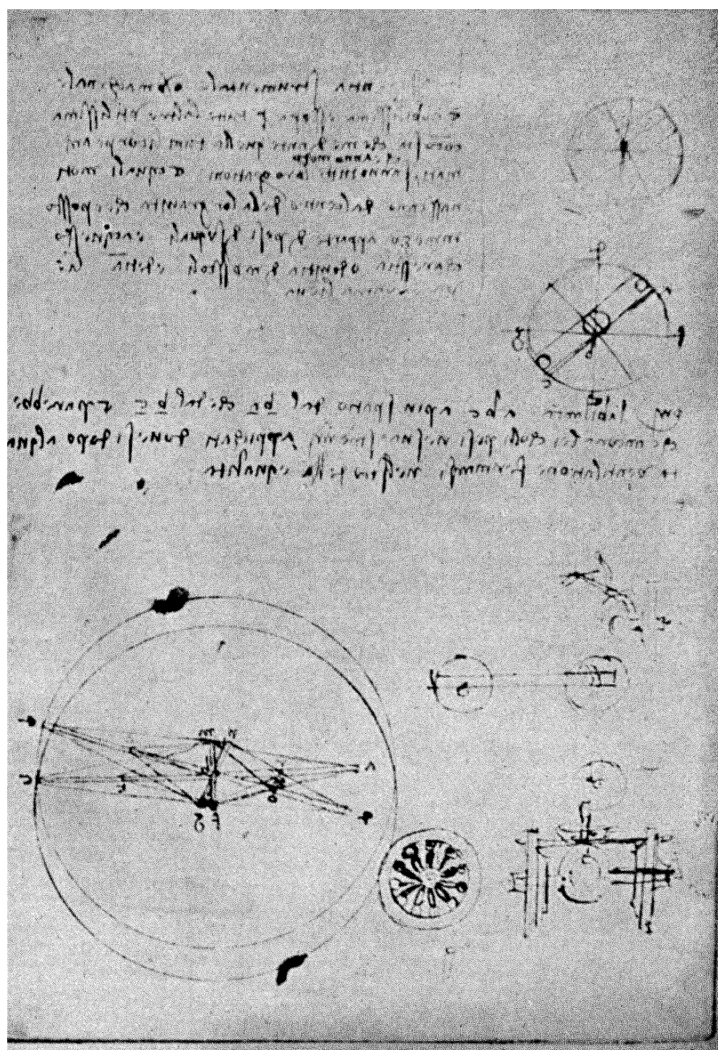
FIG. 96.

and nitric acid will never remove it (*s*), because it is not able to dissolve such porphyry.

If you wish to make a beautiful blue colour, resolve the enamel made with tartar and then remove the salt. Brass vitrified makes a beautiful red.

[Fol. —3r.]

Instrumental or rather mechanical science is very noble, and useful beyond (*p*) all others, see that, by its



A facsimile page from Leonardo's *Manuscript on the Flight of Birds*.

[To face page 202.]

means, all animated bodies which have movement, perform their operations ; which movements proceed from their centre of gravity, which is situated at the centre, except with unequal weights, and the latter has paucity or abundance of muscles, and also, lever and counter-lever.

Here the balance abc [Figs. 95 and 96] has more space in ba than bc , and it would appear that it also, with weights attached to its extremities would, after several oscillations, stop at the place of equality.

[Fol. 3—v.]

Different figures give in their obliquity different weights.

(If the rod (equipoised) is (no fixed) able to revolve with one of its extremities round its other extremity. . . . The end of the rod will be prevented from revolv(ing) round its other extremity, to which will be fixed a straight cord, which will be attached to the opposite end under the centre of the above mentioned revolution.) At the end of this rod circular movement will be prevented round its other extremity by this cord, which in a straight line is fixed under the centre of the said circle and is attached to the extremity of the rod. As if the rod were the line $p q$ [Figs. 97 and 98] and the end which is prevented from moving in a circle $q m$ be the end q , and the straight cord attached under the

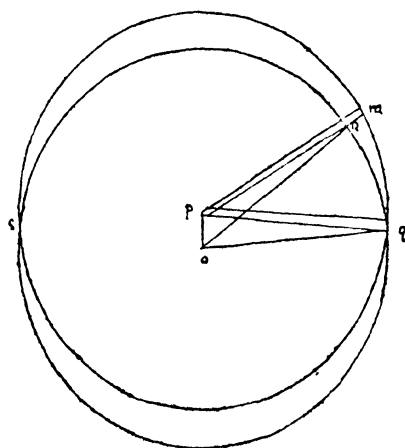


FIG. 97.

The objector says here, that the rod $p m$ will bend until it makes with its extremities the length of the cord on [which it] will en(t)er.

[Fol. 4 verso.]



FIG. 101.

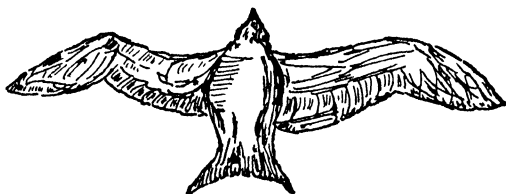


FIG. 102.

I ask in which part of the under-surface of the width of the bird, the wing presses the air more than in any part of the length of the wings. [Fig. 101.]

All bodies which do not bend, though they may be each in itself, of different sizes and weights, they will exert equal pressures on all of the supports that are equally distant from the centre of gravity, this centre being at the middle of the substance of such a body. [Fig. 103.]

Those feathers that are farthest from their points of attachment will be the most flexible. Therefore, the tips of the feathers of the wings will always be higher than their roots, so that we may reasonably say that the bones of the wing will always be lower in the depression of the wings than any part of the wing: and in the elevation these wing bones will be higher than any part of the wing. Because the heavier portion always guides the movement. [Fig. 102.]

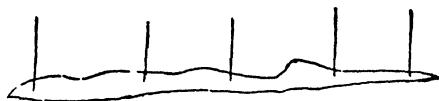


FIG. 103.

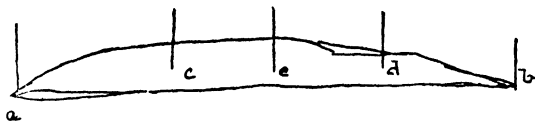


FIG. 104.

One proves how the above-mentioned weight exerts equal pressure on its supports : let us assume that it is 4 lbs. (I say) and that it is sustained by the support *a b*. I say that the body being unhindered in its fall except by the two supports *a b*, that these supports will sustain equal parts of this weight that is to say 2 and 2 and the same thing would apply to the 2 supports *c d*, if the 3 other supports were not there : and if the middle one at *e* only remained it would support the whole of the weight. [Fig. 104.]

But if the said body be flexible with different sizes (volumes) and weights still let the centre of gravity be at the centre of the substance it does not follow that the support which is nearest to the centre of gravity or to another inequality of gravity may not be more charged with weight than that which is over some lighter portions.

[Fol. 6 [5] r.]

A man with wings should be free from the waist upwards in order to balance himself as he does in a boat in order that his centre of gravity and that of the instrument might be able to balance and change when necessity required it according to the change in the centre of its resistance.

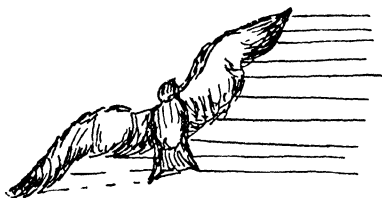


FIG. 105.

(Falling.) The bird being in the act of falling in the direction of its open wings [Fig. 105] with a force of 4 and the wind which strikes it underneath with a force of 2 makes its course straight : we will say then that the descent of such a bird will be by the mean line between the level of the course of the wind and the obliquity in which the bird was with the force 4. As : let the obliquity of such a bird be the line *a d c*, and let the wind be

b a ; I say, if the bird *a d c* had a force 4 and the wind *b a* had a force 2 that the bird would not go in the direction of the wind (either) in *f* nor by its obliquity in *g* but would fall by the mean line *a e* [Fig. 106] ; and one proves it thus.

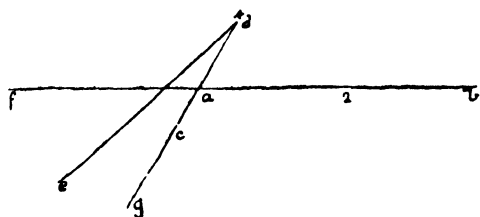


FIG. 106.

And if a bird make such an oblique descent [Fig. 107] with a force of 4 and if the wind that follows it have a force of 8. . . .

When the bird wishes to turn to the right or the left

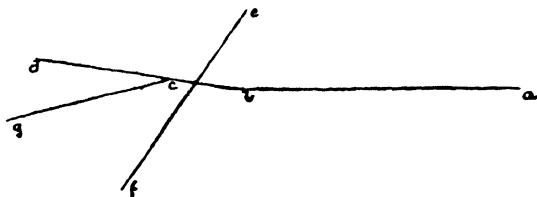


FIG. 107.

by beating its wings then it will beat lower with the wing on the side to which it wishes to turn and thus *the bird will turn the movement behind the impetus of the wing which moves most.* [Fig. 108.]

[Continued in Fol. 6 [5] recto.]

[Fol. 6 [5] v.]

and makes the reflex movement under the wind on the opposite side.



FIG. 108.

When the bird by the beating of its wings wishes to

rise, it raises the shoulders and it beats the points of the wings toward itself, and so condenses the air which lies between the points of the wings and the breast of the bird, the tension of which lifts up the bird.

The kite and other birds that move their wings very little seek the air currents, and when the wind is blowing high up then they will be seen at a great height and if the wind blows low down they remain low.

When there is little wind the kite beats several times with its wings during flight so that it may rise and obtain impetus, with which impetus inclining a little it goes a long distance without beating its wings ; and when it has dropped [vol-planed] somewhat it repeats the movements and so successively : and this drop without beating the wings serves as a means of repose in the air after fatigue of the said beating of the wings. All birds that fly by spasms raise themselves by the beating of the wings and when they come down [vol-plane] they rest because they do not beat with their wings in descending.

[Fol. 7 [6] r.]

Concerning the 4 movements reflex and incident with varying aspects of the wind made by birds.

The oblique descent of birds, being made against the wind will be made under the wind and their reflex movement will be made on the wind.

But if such an incident [downward] movement is made to the east, the wind blowing from the north, then the north wing [upward] will remain under the wind, and in the reflex movement will do the same, so that at the end of this reflection the bird will find itself with its front to the north.

And if the wind descend to the south the north wind prevailing, it will make its descent on the wind, and its reflex motion will be under the wind : but this gives rise to a long discussion, which will be given in its due place because here it would appear that the bird would not be able to make a reflex movement.

When the bird makes its reflex movement against and on the wind then it will rise much more than it should from its natural impetus, seeing that it has the assistance of the wind which entering under it, serves as a wedge. But when it has completed its rise it will have used up its impetus and there will remain only the assistance of the wind which would turn it upside down because it strikes it on the breast and if it did not depress the right or left wing which will cause it to turn to the right or left, descending in a semi-circle.

[Fol. 7 [6] v.]

The movement of the bird ought always to be above the clouds lest the wing should be moistened to disclose

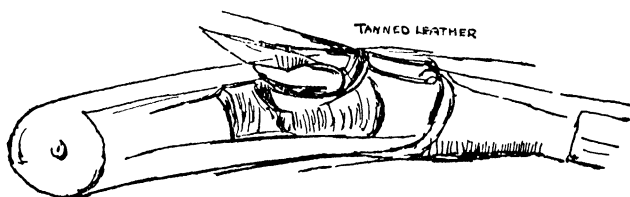


FIG. 109.

a broader view and to avoid the peril of wind revolutions among mountain gorges, which are always full of whirlings and turnings of the winds. Beyond that, if the bird should turn somersault, you would have more time for righting it, with the directions already given before it reaches the ground.

If the point of the wing be struck by the wind and this wind enter under such a point, then the bird would be liable to be upset unless it use one of two remedies, that is to say : either that it force suddenly such a point under the wind or that it lower the opposite wing, from the middle on.

a b c d [Fig. 110] are 4 upper nerves to raise the wing (*b*) and act as strongly as the lower nerves *e f g h* on account of the overturning of the bird so that they might resist above as below although a single one of tanned leather thick and wide [Fig. 109] might perchance suffice ; however, in the end, we must leave that to experience.

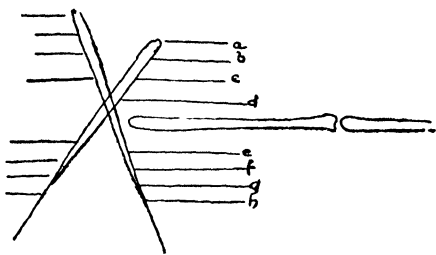


FIG. 110.

[Fol. 8 [7] r.]

The said bird ought, with the assistance of the wind to rise to a great height, and that will be its safety, even

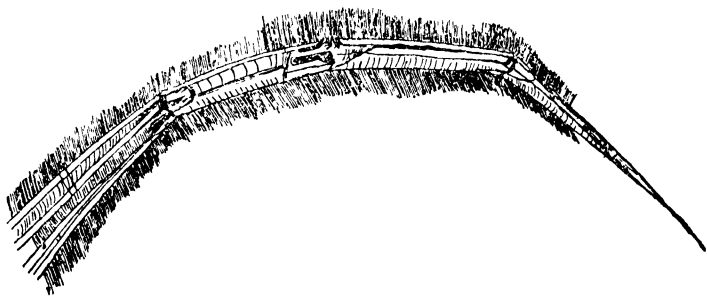


FIG. 111.

should it experience all the above mentioned revolutions, it has time enough to recover its balance provided that

its limbs be very strong so that they may safely resist the fury and vigour of the descent with the above-mentioned defences, its joints of strong tanned leather and its nerves [sinews] of raw silk cord of great strength; and let not anyone hamper himself with fittings of iron, because they burst easily in twisting, or waste away, for which reason they are not to be used.

The nerve [sinew] *a* [Fig. 111] serving to stretch out the wing, should be of thick tanned leather, so that if the bird should be turned upside down, it would be able to

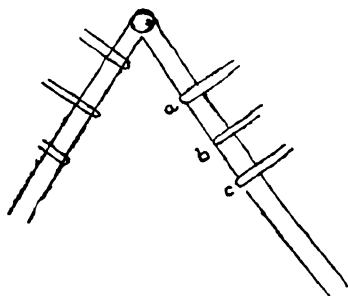


FIG. 112.

overcome the fury of the wind which would strike in the wing and tend to close it, because that would cause the ruin of such a bird; but for greater security you will make precisely the same nerve outside as inside, and you will be beyond all suspicion and peril.

a b c [Fig. 112] are the attachments of the nerves of the 3 junctions of the fingers of the wings; *d* is the position of the motor of the lever *a d* which moves the wing.

[Fol. 8 [7] v.]

When the edge of the point of the wing is opposed to the edge of the wind for an instant, I put this wing under or on this edge of the wind and the same thing happens to the point or sides of the tail, and similarly to the shaft of the shoulders of the wing.

The descent of a bird will be always by that (part *w*) extremity which will be nearest to its centre of gravity.

The heaviest part of a bird descending will always be in front of its centre of resistance.

3rd. When without the assistance of the wind the bird remains in the air without beating its wings, in the position of equilibrium this shows that the centre of gravity is coincident with the centre of resistance.

4th. The heaviest part of the bird which descends with the head underneath, will never be above or level with the height of the lightest part.

If the bird fall tail first, by throwing the tail back it will restore itself to the place of equilibrium, and if it throws it forward it will overturn itself.

1st. When the bird which is in the position of equilibrium shall act so that the centre of resistance of the wings be behind the centre of gravity, then such a bird will descend head first.

2nd. And that bird which being in a position of equilibrium shall have the centre of resistance of the wings more forward than the centre of gravity of the bird then such a bird will fall with the tail turned towards the ground.

[Fol. 9 [8] r.]

If the wing and the tail are too much on the wind, lower half the opposite wing, and therewith receive the force of the wind and equilibrium will be restored.

And if the wing and the tail were under the wind raise the opposite wing and it will be corrected to your desire, provided that such a wing as is raised be less oblique than (the) that which is opposite to it.

And if the wing and the breast are on the wind, one must depress half the opposite wing, which will be struck by the wind and forced upwards which will right itself.

And if the bird is so that its hind quarters are on the wind, then the tail ought to be forced under the wind, and thus one will be able to equalise the powers.

But if the bird has its hind quarters under the wing (raising the tail) let it enter with the tail on the wind, and it will right itself.

[Fol. 9 [8] v.]

When the bird is on the wind turning the beak with the bust (s) to the wind then the bird would be turned over

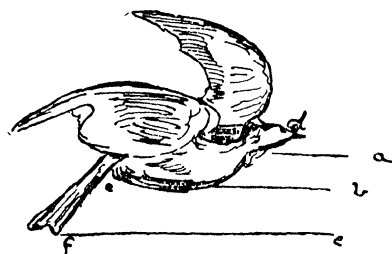


FIG. 113.

by the wind, if it had not depressed the tail, and received in the tail a large portion of the wind : thus doing it cannot be turned over. One proves it by the first of the mechanical elements which shows that things placed in

balance which are struck beyond their centre of gravity, force down the opposite parts placed on this side of the centre before-mentioned. For example, let the quantity [surface] of the bird be def [Fig. 113] and its centre of revolution be e and the wind which strikes it be $abde$ and $bcef$: I say that a greater volume of wind strikes in ef the tail of the bird beyond the centre of revolution, than in de on this side of the said centre : and for this cause the said bird cannot be upset above all, while holding its wings to the wind by the edge.

And if the bird is so that its length is under the wind it is liable to be turned upside down by the wind if it does not immediately raise its tail up. One proves it thus :— Let the length of the bird be dnf [Fig. 114], n is the

centre of its revolution : I say that dn is struck by a greater amount of wind than nf ; and for this reason dn will respond to the course of the wind, giving place to it, and moving downwards, thus raising the bird to the position of equality.

That the wing does not utilise all the pressure of the air and that this is true, note that the interstices of the primary feathers are spaces much wider than the width of the feathers themselves : therefore you who study flying should not calculate on the entire surface of the wing, and note the different varieties of wings for all flying creatures.

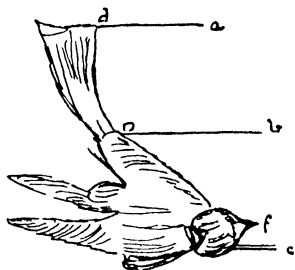


FIG. 114.

[Fol. 10 [9] r.]

When the wind strikes the bird under its course from its centre of gravity towards this wind, then such a bird will turn itself with its spine to the wind, and if the wind were more powerful below than above then the bird would turn upside down, *if it did not immediately take care to draw in the under wing and stretch out the over wing ; in this manner it rights itself and returns to the position of equilibrium.*

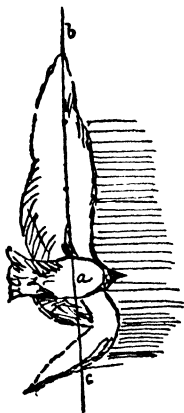


FIG. 115.

One proves it thus :—Let ac [Fig. 115] be the wing withdrawn under the bird, and ab be the extended wing. I say that the forces of the wind that strike the 2 wings will have the same proportion (ratio) as that of their extensions, that is to say ab to ac . It is

true that c is wider than b ; but it is so near the centre of gravity of the bird that it offers little resistance in comparison with b .

But when the bird is struck under the wind beneath

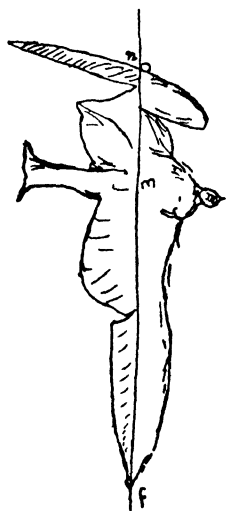


FIG. 116.

one of its wings, then it would be possible for the wind to upset it, unless, immediately after it had turned with its breast to the wind, it stretched the opposite wing towards the earth, and shortened the wing which had been first struck by the wind, which wing remains superior and thus it will restore itself to equilibrium. One

proves it by the 4th of the 3rd that is to say, *that object is the more affected which is attacked by the greater force* ; again by the 5th of the 3rd that is to say *this support resists less and is situated farther from its fixed point* ; again by the 4th of the 3rd *between winds of equal force [velocity] that will have the greater force [power] which has the greater volume* and that one will strike with a greater volume which finds a greater object ; so that mf being longer than mn , mf will obey the wind. [Fig. 116.]

[Fol. 10 [9] v.]

Struck from above, (up) the force of the wind which strikes it from above, has not full play, seeing that the wedge of wind, which is separated from the middle of the shoulder downwards [beneath], lifts the wing up with almost as much force as that exerted by the upper wind to depress the wing. One proves it : let fbd (and baf)

[illegible][illegible][illegible]

[Fig. 117] be the shoulder of the wing : and $efcd$ (and) the sum total of the wind which strikes this shoulder of the wing, of which wind half is $abcd$ which strikes from the summit of the shoulder b down to d : and because the line bd of this shoulder is oblique, the wind $abcd$ forms a wedge at the contact bd and raises the shoulder : and the upper wind $abef$ which strikes the obliquity bf makes a wedge there and pushes

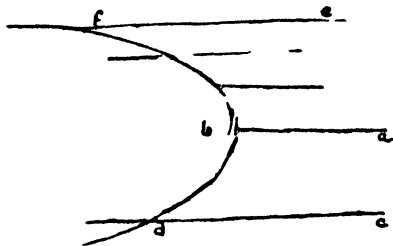


FIG. 117.

the wing down, with the result that these 2 said contraries prevent the shoulder from immediately entering beneath or above the arrival [course] of the bird, according to whichever its necessity requires : whence this necessity is met by putting a shaft on the round shoulder which acts as a buckler and immediately cuts the wind in

such manner as the necessity of the bird demands as one shows in mn [Fig. 118].

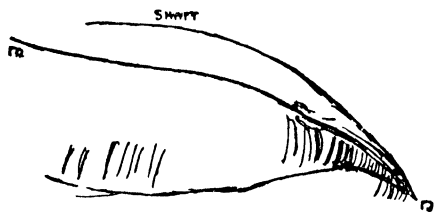


FIG. 118.

But if the wind strike the bird on the right or left wing, then it must enter below or

above such a wind with the point of the wing that is struck by the wind [Fig. 119], which change occupies as much space as is taken up by the points of the wings : which change being under the wind, the bird turns with its beak to the wind and if it is on the wind, the bird will turn with the tail at will : and here arises the peril that

the bird will turn upside down, if nature had not provided for it by placing the weight of the body of the bird

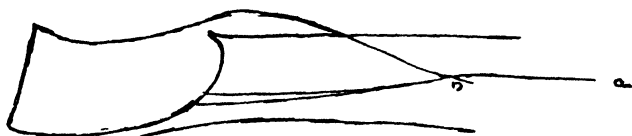


FIG. 119.

lower than the place of extension of the wings as one will show here.

[Fol. 12 [11] r.]

And lying is in such ill repute, that if it is said very great things concerning God it would (detract from the grace of the) Deity ; and truth is of such excellence that if it praised trifles it would make them noble.

Without doubt, there is a similar proportion of truth to untruth as there is of light to darkness and this truth is in itself of such excellence that even if it be spread over humble and base matters, it exceeds beyond comparison

the incertitudes and lies spread over great and very high discourses ; because that still though our spirit may have lying for its fifth element, nevertheless, the truth of things remains as the supreme nutriment for fine intelligences ; but not for wandering humours.

The incident movements with their reflex movements are of 4 kinds, of which one, incident and reflex, is rectilinear, having lines of equal obliquity ; the other is again

But you who live by visions the sophistries and rogueries of boasters (please you more) in great and uncertain things please you more than those certain, natural and not of such great height.

rectilinear but the obliquities are different ; the 3rd has the incident movement rectilinear and the reflex curvilinear ; the 4th has the incident movement curvilinear and the reflex a straight line. Of these straights and curvilinears each of them is divided into 2 parts, because the first may have its incident movement rectilinear directly opposite to the chord of the arc made by the curvilinear reflex movement, and again this reflex may bend to the right or to the left of the incident rectilinear movement.

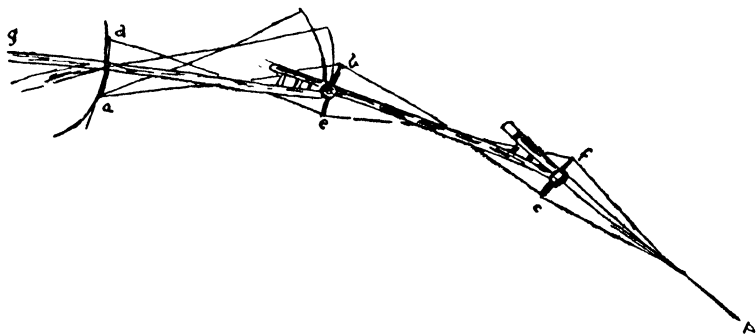


FIG. 120.

When the bird flies by beating its wings it does not fully extend its wings, because the points of the wings would be too distant from the lever and the nerves [sinews] which move it.

If in (lowering) the descent of the bird it rows backward with its wings, the bird will move rapidly ; and this happens because the wings strike in the air which successively flows behind the bird, to fill the void that it has left.

[Fol. 12 [11] v.]

When *g* (*i*) descends *p* rises. [Fig. 120.]

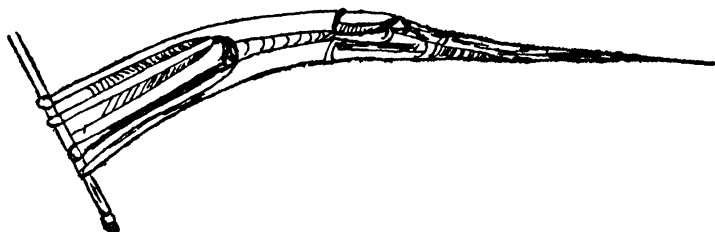


FIG. 121.

[Outline of a leaf here.]

[Fol. 13 [12] r.]

[Continuation of Fol. 9 v.] such a movement will bend and make it a semi-circle ; then such a bird will be at the end of this movement with its beak turned towards whence such a reflexion began : which, if it be made against the arrival of the wind the end of the reflex movement would be made much higher than was the commencement of the incident movement, and this is the manner in which the bird rises, without beating its wings and in circulating ; and the remainder of the said circumference would finish (with) by the sense of the wind, by incident movement always with one of the wings low, and similarly one side of the tail ; and it then makes a reflex movement towards the flight of the wind and remains at the end with the beak turned to the flight of this wind and then makes incident and reflex over again, against the wind always circulating.

When the bird wishes to turn itself suddenly on one of its sides, then it quickly pushes the point of the wing on this side towards its tail, and because *all movement tends to its maintenance* or rather, *all moved bodies continue to move as long as the impression of the force of their motors remains in them*, the movement then of such a wing thrust

with violence towards the tail reserving still at the end a part of the said impression, not being able by itself to follow the movement commenced at first, will move all the bird with itself, until the impetus of the moved air may be consumed.

The tail pushes with its face, and the wind struck by it, makes the bird move suddenly in the contrary direction.

When the bird shall be in the position *a n c* [Fig. 122]



FIG. 122.

and shall wish to rise, it will elevate the shoulders *m o* and will thus be in the position *b m n o d* and the air will be pressed between the sides and the point of the wings, so that it will be condensed and cause an upward movement, and give rise to an impetus in the air, which impetus of the air will push the bird upwards by its condensation.

[Fol. 13 [12] v.]

To escape the peril of ruin.

The ruin of such instruments [machines] may happen in two ways, of which the first is that the instrument break; the second is that the instrument turn on its edge, or near its edge because it ought always to descend by a great obliquity or almost by the line of equality. . . . As to the protection of the instrument from rupture, one avoids it by making it of the greatest strength by no matter what line it may be able to turn itself, that is to say : either by the edge (or sense about the) falling

either with the head or the tail first, or even with the point of the right or left wing, or by the halves or quarters of the said lines as the design shows. As to turning by any side whatever of the edge, one ought to prevent it even in the beginning by making the instrument in such a fashion that in descending, under whatever aspect that may come about the prevention should be ready ; and this will be done in giving its centre of gravity on the centre of the weight carried by it, always in a straight line, and one of the centres very distant from the other ; that is to say, that in an instrument 30 fathoms wide these centres may be 4 fathoms apart, and that one, as mentioned, be situated under the other and the heavier underneath, so that in the descent, the heavier portion might always be the guiding portion of the movement. Further than that, if the bird should fall head first (1) with such obliquity as would turn it upside down, the latter would not be able to happen, because the lighter portion would be under the heavier, a thing which is impossible in a long descent as one proved in the 4th of the mechanical elements.

And if the bird fall with the head underneath (with the body) with part obliquity of the body turned to the earth, then the sides of the wings underneath ought to turn flat against the earth and the tail be raised towards the loins (back), and that the head and the underneath of the jaws be turned also towards the earth, whence its reflex movement will commence immediately in such a bird, which will cast it back towards the sky ; on this account such a bird will, at the end of this reflexion begin to fall backwards, unless in its rise, it lowers one of its wings a little, which

[Continued on Fol. 12 recto.]

[Fol. 14 [13] r.]

Here, the big fingers of the wings are those which hold the bird still in the air against the movement of the wind : that is to say, the wind moves on which it supports itself without beating its wings and the bird does not change its place. The reason of this is that the bird arranges its wings on such an obliquity that the wind, which strikes it underneath does not cause a wedge of such a nature as would lift it up, but lifts it up however just as much as its weight would press it down, that is to say : if the bird fall with a force of 2 the wind would lift it up with another force of 2 and because equal things do not surpass one another, this bird remains in its place without rising or falling. It remains for us to speak of movement which drives it neither forward nor backward : and this happens if the wind accompanies it or rather pushes it out of its place with a force of 4, and the bird with the same force inclining with the said obliquity against the wind : here also the powers being equal such a bird will not move forward nor be driven back, the wind being equal. But because the movement and powers of the winds are variable, and the obliquities of the wings ought not to be changed, for if the wind should increase, and it overcame the obliquity, in order not to be pushed upwards by this wind. . . .

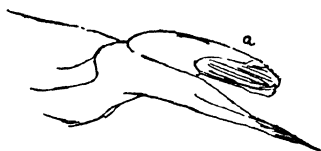


FIG. 123.

The wind does not enter, in the above-mentioned cases, as a wedge under the oblique wings, but reaches the wing only at the edge which tries to descend against the wind, whence it strikes the edge of the shoulder, which

shoulder acts as a buckler for the rest of the wing : and here the descent of the wings would not have any defence if the great finger *a* [Fig. 123] were not there which then faces front, and itself receives all the force of the wind directly on it or less than directly, according to the more or less great force of the wind.

[In Margin.]

One will carry snow in summer, taken from the high crests of the mountains and one will let it fall in places of festival in the summer time.

[Fol. 14 [13] v.]

The big (thumb) finger *n* [Fig. 124] of the hand *m n* is that which, when the hand is lowered, is lowered more

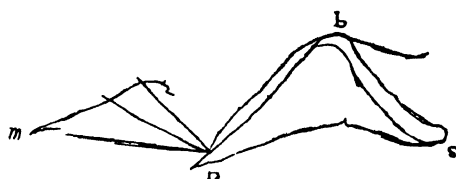


FIG. 124.

than the hand, so that it closes and prevents the escape of the air pressed by the descent of the hand, so that in this place the air is condensed

and resists the oars of the wings and therefore nature has provided such a finger with a bone of great strength, to which strong nerves [sinews] and short feathers are joined. The short feathers have greater strength than the other feathers of the wings of birds will have, because that by them the bird supports itself on the air already condensed with all the force of the wing and of its strength, because it is that by which the bird is moved

forward : and this finger does the same work for the wings that the claws do for the cat when it climbs on the trees.

But when the wing recovers a new force with its return upwards and forwards, then the big finger of the wing puts itself in a straight line with the other fingers, and thus with its cutting extremity divides the air and does the work of a shaft which always cuts the air by some movement, high or low, that the bird would rise.

The second shaft in the opposite portion, beyond the centre of gravity of the bird, and that is the tail which, if it is struck by the wind underneath, since it is beyond the said centre, it will cause the forward portion of the bird to lower. And if this tail be struck on top the forward portion of the bird will rise. And if this tail twist a little, and turn its under surface obliquely to the right wing, the anterior portion of the bird is turned to the right side. And if it turn this side obliquely of the under surface of the tail to the left wing, it (the bird) will be turned with its forward portion toward the left side : and in each of the two manners the bird will sink. . . . 4

-vity

[In Margin.]

4. But if the tail, placed obliquely, be struck by the wind on its upper surface, the bird (it) will be turned, in turning the tail slowly from this side where the superior surface of the tail shows its obliquity.

[Fol. 15 [14] r.]

The pivot of the shoulders of birds is that which is turned by the muscles of the chest and of the backbone ; and it is there that the discretion arises of lowering or raising the elbow according to the will or necessity of the animal which is moving.

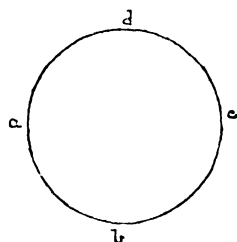


FIG. 125.

I conclude that the mounting of the bird without the beating of wings, is caused by nothing other than its circular movement which, when it starts from the arrival of the wind sinks until it reaches the place where

the reflex movement begins after which and so circulating, it has described a semi-circle and its face turned to the wind, and follows the reflex movement, on the wind still circulating until, with the help of the wind it makes its greatest height between its lowest and the arrival of the wind and is left with the left wing to the wind ; and from this greatest height again circulating, it descends to the last incident movement being left with the right wing to the wind. As if to say :—

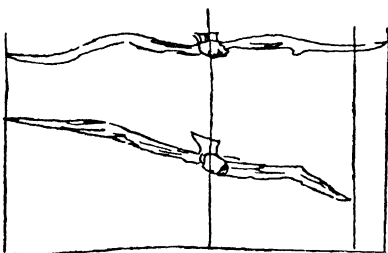


FIG. 126.

The wind goes from *a* to *c*

(Fig. 125] and the bird moves from *a* and sinks from *a b c* and in *c* it makes the reflex movement as in *c d a*, and by the favour [help] of the wind, it is much higher at the end of the reflex movement, which end of the reflex movement is started perpendicularly over the said commencement of the incident movement.

The equal resistance of the bird's wings always springs from the fact that they [the wings] are equally distant, at their extremities, from the centre of gravity of any bird. [Fig. 126.]

But when one of the extremities of the wings is nearer the centre of gravity than the other extremity, then the bird will descend from that side where the extremity of the wing is nearer the centre of gravity.
[The last syllable *-vity* is found on Fol. 14 [13] v.]

[Fol. 15 [14] v.]

The hand of the wing is that which gives the impetus ; and then the elbow is turned with the edge forward so as not to hinder the movement which created the impetus ; and when this impetus is afterwards created, the elbow is lowered and made oblique and being oblique it makes the air on which it lies as in the form of a wedge on which the wing raises itself, and if the movement of the bird were not so performed, during the time that the wing moves forward the bird would sink towards the end of the impetus ; but it is not able to sink because as much as the impetus lacks so much the percussion of the elbow resists this descent and lifts up the bird.

Let us say, that the impetus be as 6 and that the bird weigh 6 and that in the middle of the movement the impetus become 3 and that the weight still remain 6 ; here the bird would sink by half-movement, that is to say, by the diameter of the square, and the wing oblique with the contrary aspect, also by the diameter of this square does not allow such a weight to sink, neither does the weight permit the bird to rise ; consequently it moves in a straight line. That is to say : the descent of the bird during the said half-movement would be by the line *a b*

[Fig. 127] and, because of the obliquity of the wings with contrary aspect would have to rise by the line $d c$ [Fig. 128]; whence, from the above-mentioned causes, it moves on the place of equality $e f$. [Fig. 129.]

The elbows of the animal are not lowered just at the commencement because in the principal flight of the impetus the bird would jump upwards, but they are

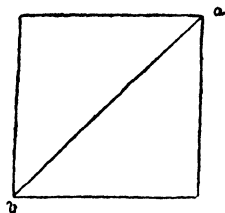


FIG. 127.

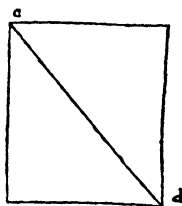


FIG. 128.

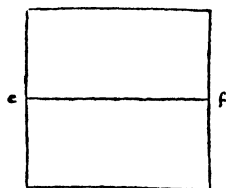


FIG. 129.

lowered just so much as is necessary to prevent the descent according to the will and discretion of the bird.

When the bird wishes to glide suddenly upwards it lowers the elbows immediately after it has created the impetus.

But if it wishes to descend, it holds the elbows up firmly after the creation of the impetus.

[Fol. 16 [15] r.]

You are to remember that your bird ought not to imitate anything but the bat, because, the membranes [web] form an armour or liaison to the armour, that is to say, mistress of the wings.

And if you imitate the wings of the feathered birds, the wings are more powerful in bone and nerve, through being pervious : that is to say, the feathers are disunited permeable to the air. But the bat is aided by the membrane which binds the whole and is not pervious.

Of the method of balancing oneself.

It is always the heaviest portion of the body which guides its movement.

Then the bird finding itself in the position *a b* [Fig. 130], *a* being lighter than *b*, where the motive power is disposed it will always keep above *b*, so that it will never happen that *a* will precede *b* except by accident, which will not last long.

The bird which has to rise without beating its wings, places itself obliquely against the wind, presenting the wings to the latter with its elbows in front, with the centre of its gravity more towards the wind than the centre of the wings, whence it happens that if the obliquity of the bird would sink with a force of 2 and that the wind strikes it with a force of 3, this movement obeys the 3 and not the 2.



FIG. 130.

[Fol. 16 [15] v.]

This [Fig. 131] is done to find the centre of gravity

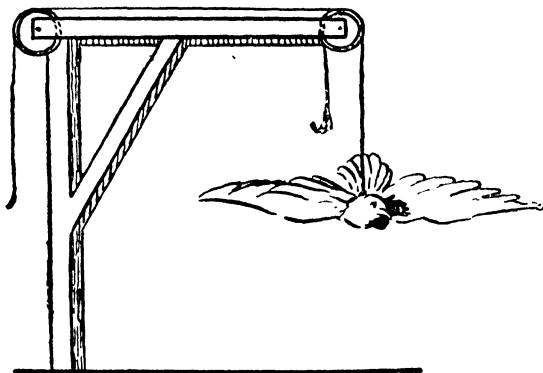


FIG. 131.

of the bird, without which instrument, this instrument would have little value.

When the bird sinks, then the centre of gravity of the bird is outside the centre of its resistance ; as if the centre of gravity were on the line $a b$ and the centre of resistance on the line $c d$. [Fig. 132.]

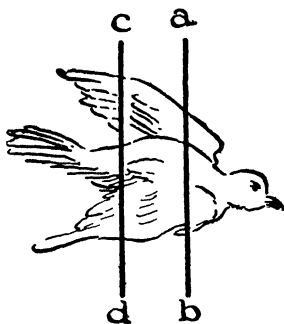


FIG. 132.

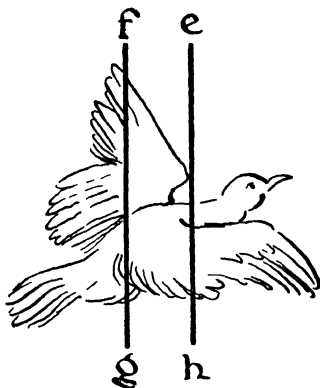


FIG. 133.

And if the bird wishes to rise, then the centre of its gravity remains behind the centre of its resistance.

As if the centre of gravity mentioned might be in $f g$, the centre of resistance would be in $e h$. [Fig. 133.]

The bird is able to rest in the air without holding its wings in the place of equality because not having its centre of gravity at the middle of the pivot, as balances have, it is not constrained by necessity to hold its wings at an equal height, as in the said balances. But if the wings are outside this place of equality then the bird will descend by the line of obliquity of these wings ; and if the obliquity is compound, that is to say, double, as for example, that the wings decline to the south, and that the obliquity of the head and tail declines to the east, then the bird will descend with the obliquity to the south-east. And if the obliquity of the bird be double that of the wings

then the bird will descend in the middle between the south-east and the east, and the obliquity of its movement will be between the two said obliquities.

[Fol. 17 [16] r.]

Persuasion in haste (or persuasion of the enterprise) *which removes the objections.*

If you say that the nerves and muscles of the bird are, beyond comparison, of greater power than those of man seeing that all the flesh of so many muscles and pulps of the breast is made for the benefit and augmentation of the movement of the wings, with this bone of the breast all in one piece which provides the bird with great power, with the wings a tissue of great nerves and other very strong ligaments of cartilage and a skin made very strong with diverse muscles; here one replies that so great a force is provided for power because that beyond just sustaining itself by its wings it must at will double and triple the movement in order to escape pursuit or to follow its prey, whence, in such an effect it is necessary for it to double or triple its force, and beyond that, carry as much weight in its claws, in the air, as equals its own weight: as one sees the falcon carry the duck, and the eagle the hare, by which thing it is very well demonstrated where such super-abundant force is distributed: but it

Wine-skins, where a man, falling from a height of 6 fathoms, would not injure himself, falling either in water or on land: and that these wine-skins, fastened together in the fashion of beads, are surrounded by others.

Man, also, has a greater amount of strength in his legs, than is necessary for his weight: and that this is true, place a man upright on some mud and note how deeply his foot sinks. Then place another man on his back and you will see how much deeper he sinks. Then, having removed the man from his back, make him leap straight up in the air, as far as he can, and you will find the imprint of the foot deeper from the leap, than with the man on his back: therefore it is here proved in two manners, that a man has more than double the force that is requisite to sustain himself

needs little force to sustain itself and to balance them on the currents of wind and to direct the shaft in its paths : a little movement of the wings is sufficient and as much more slow movement as the bird is larger.

[Fol. 17 [16] v.]

If you fall, see that you strike the ground with the double wine-skin that you hold under you.

Because the wings have to row below and behind in

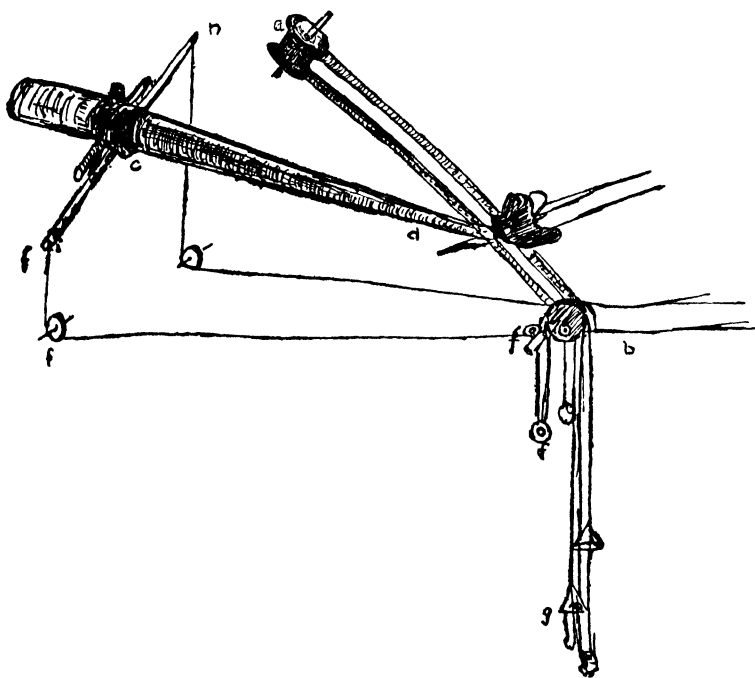


FIG. 134.

order to keep the instrument up, and that it may move forward, the movement of the lever *c d* [Fig. 134] is made by an oblique way, guided by the strap *a b*.

I could make it so that the foot which presses the stirrup *g* might be that which beyond its ordinary function would draw down the lever *p*. But this would not be according to our design because we require that the lever *f* may rise or sink before the stirrup *g* move from its place, in order that the wing in throwing itself forward, or lifting itself up (during the time in which the impetus, already acquired is moving the bird forward, without the beating of the wings) may be able to put the wings in the air by the edge, because if that were not done, the face of the wings would strike the air, hinder the movement, and prevent the impetus from carrying the bird forward.

[Fol. 18 (recto).]

Always in the elevation of the hand, the elbow is lowered and presses the air, and in the lowering of this hand the elbow is raised and remains by the edge lest it hinder the movement by means of the air that strikes therein.

The lowering of the elbow at the time when the bird moves the wings forward by the edge a little on the wind guided by the impetus already acquired causes the wind to strike under this elbow and form a wedge on which the bird with the said impetus, without beating its wings, mounts up and if the bird is 3 lbs. and the breast 1-3 the width of the wings, the wing will feel but 2-3 of the weight of such a bird.

The hand feels a great fatigue towards the big finger, or rather

The palm of the hand goes from *a* to *b* always between angles almost equal declining and pressing the air, and at *b* turns immediately by the edge and goes behind rising by the line *c d*, arrived at *d* it is turned suddenly in front and goes sinking by the line *a b* and in turning does so always around the centre of its width. [Fig. 135.]

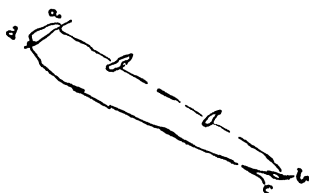


FIG. 135.

The recoil of the hand, by the edge will be made with great speed and the pressure behind,

shaft of the wing, because it is this part which strikes the air.

in front, will be made with that speed which the last power of the motor requires.

The course of the point of the fingers is not the same going as returning but the return is through a higher line: and under this is the figure made by the superior and inferior line, and oval with a long and narrow curve.

[Fol. 18 (verso).]

1505 Tuesday evening on the 14th of April Laurent came to live with me, he says he is 17 years old. And on the 15th of the said April I had 15 florins of gold from the chamberlain of San Maria Nova.

From the mountain, which bears the name of the great bird, the famous bird will take its flight, and will fill the world with its great fame.

To raise a tree by *p* and *r s* sustain. [Fig. 136.]

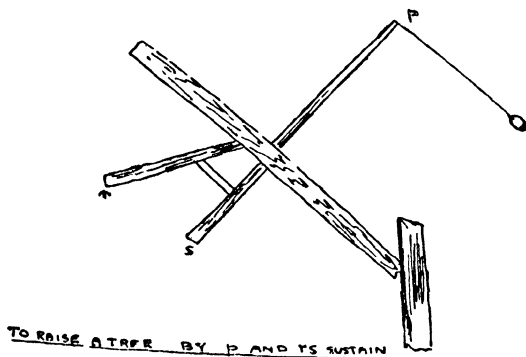


FIG. 136.

[Back Cover—Inside.]

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husks	44	
in straw	23	
key	·6	148
myself	28	111
chicken	·2	—

037

111 (8)

(11)

·28

83

The great bird will take its first flight, on the back of its great swan, and filling the universe with stupor, filling all writings with its renown, and glory eternal to the nest where it was born.

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