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THE THEORY OF HEAT ENGINES

INCHLEY'S THEORY OF HEAT ENGINES

EDITED AND REVISED
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With Diagrams

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PREFACE TO THE FOURTH EDITION

IT has long been felt that a revised edition of William Inchley's "Theory of Heat Engines" was overdue, and the present book is an attempt to make good the defects which time has revealed in the original work.

Much new knowledge has been obtained since 1913, making considerable additions necessary, and to allow for these, without unduly increasing the size of the book, the chapters dealing with the kinematics and dynamics of engines, subjects already claiming whole books to themselves, have been omitted.

Addition alone, however, would not suffice. The emphasis on the chronological order in which developments have taken place, and the resulting sharp cleavages between the different types of engines, have tended to conceal the fundamental unity which underlies the whole subject, and the conceptions of thermodynamics have now been introduced in order to provide a logical basis upon which to build. No rigorous statement of thermodynamics has been attempted, but it is hoped that enough has been said to enable the student of engineering to visualise the mechanism of the processes which he seeks to control.

The classification of text-books as Elementary or Advanced has often given readers of the former an entirely false impression of security which has been rudely shattered when the real deviations from the simpler laws and theories have had to be considered. As such a classification is entirely arbitrary, any permanent separation of the subject on these lines has here been avoided; limits and deviations have been stated in their logical places, and the more difficult sections of the work follow those of a simpler nature. Naturally the student is advised to read the simpler sections first, and a selection of these has been marked with an asterisk for his guidance, but there, as in his wider reading, some discretion must be used. It is hoped that by the time the last paragraphs are read he will find but little which he will subsequently have to modify or unlearn, and that the book has achieved the purpose expressed by its first author of "giving in a complete and concise form the thermodynamical principles of the subject."

In the present edition the writer has brought together from many sources the figures, diagrams, and ideas which experience has shown to be specially helpful. The more important of these sources have been acknowledged in the text and it is hoped that the reader will turn to them

for further authoritative guidance, but many remain unrecorded. To the authors, named and unnamed, whose work has gone to the making of this book, and to the colleagues who have helped in many ways, the writer would express his gratitude and offer his sincere thanks.

H. WRIGHT BAKER.

ENGINEERING DEPARTMENT,
THE VICTORIA UNIVERSITY OF MANCHESTER,
September, 1938.

PREFACE TO NEW EDITION

THE Author, who held a Commission in the Reserve, was called up on the outbreak of war in August, 1914, and was killed in December, 1915. A new edition of his book is required, and as a tribute to the memory of a former colleague I have gone through his book and corrected a number of errors of various kinds which found their way into the first edition.

Thanks are given to Mr. Telford Petrie, to Mr. T. Bevan and others who have kindly taken the trouble to point out slips which they have come across in using the book.

A. MORLEY.

January, 1920.

Various amendments have been made, and an Appendix has been added.

A.M.

September, 1928.

CONTENTS

CHAP.		PAGE
I	THE LAWS OF STATE AND OF THERMODYNAMICS	
	General Statement	1
	The Three States of Matter	1
	The Laws of Thermodynamics, Gas Laws, etc.	3
	Molecular Theory	8
	Variations from the Simple Laws	11
II	LAWS OF EXPANSION AND COMPRESSION	
	Work done by a Gas during Expansion	14
	Relations between Pressure, Volume and Temperature during Expansion. Heat Reception or Rejection. Effect of n. Rate of Heat Reception	19
III	THEORETICAL HEAT ENGINE CYCLES	
	General Statement	29
	Graphical Representation of Work done during a " Closed Cycle " of Operations	30
	The Three Principal Cycles	31
	The Ratios of Compression and Expansion	38
	Comparison of Cycles	38
	Theoretical Importance of the " Constant Temperature " or " Carnot " Cycle	42
	Reversible Processes and Cycles	44
	Carnot's Principle	45
	Conditions for Maximum Thermal Efficiency	46
	Definition of Thermodynamic Temperature Scale	46
IV	FACTORS OF STATE. ENTROPY	
	General Statement	49
	Entropy of the Working Fluid, and Hot and Cold Bodies	50
	Entropy a Condition of State	50
	Other Definitions of Entropy. Section 1	53
	Other Definitions of Entropy. Section 2	54
	Representation of the Carnot Cycle	55

CHAP. PAGE

IV FACTORS OF STATE. ENTROPY—*continued*

General Equations for the Change of Entropy of a Perfect Gas when passing from One State to Another . . 56
Change of Entropy when the Operation takes place according to the General Law pv^n=Constant 59
Work done by an Expanding Gas, with reference to the Temperature Entropy Diagram 60
Clapeyron's Equation 61

V CURVES OF STATE

General Statement 64
The P.V. Diagram 64
The Tϕ or Temperature-Entropy Chart 67
Tϕ Curves for Various Fluids 69
Mollier, or Total Heat-Entropy Chart 69
Total Heat-Temperature and Pressure Diagrams . . . 73
Choice of Diagram 73

VI IRREVERSIBLE PROCESSES

Flow into Vacuum. 76
Throttling 76
Heat Transfer by Conduction 81
Friction 81
Fluid Friction 81

VII HOT AIR ENGINES

General Statement 85
Carnot or Constant Temperature Cycle 86
Stirling's Engine. Ericsson's Engine. Joule's Cycle . 86

VIII THEORY OF AIR COMPRESSORS AND MOTORS

General Statement 93
The Transmission of Power by Compressed Air . . 93
Methods of Reducing the Losses during Compression and Expansion 97
The Production of Compressed Air. General Statement . 97
The Single-Stage Compressor 97
Multi-Stage Compression. General Statement . . 101
Two-Stage Adiabatic Compressor 102
Three-Stage Adiabatic Compression 104
Air Motors 108
Two-Stage Adiabatic Expansion. Preheating . . . 109
Effect of Clearance 110
Turbo-Blowers and Compressors 112

CHAP. PAGE

IX THE PROPERTIES OF STEAM

Methods of Recording Pressures 118
Equations of State. Steam Tables 118
Generation of Steam at Constant Pressure . . . 119
Steam Tables and the Properties Enumerated therein . 121
Adiabatic Equations 125
Steam Diagrams 126
Some Mathematical and Graphical Calculations concerning Changes of State 127
Measurement of the Dryness of Steam 134
Mixtures of Air and Steam 139

XI STEAM ENGINE CYCLES

Cycle using Steam Non-Expansively 147
Carnot Cycle 149
Rankine Clausius Cycle. Efficiency in Terms of p and v . 150
The Rankine Cycle as a Reversible Cycle 155
Rankine Clausius Cycle. Efficiency in Terms of T and ϕ . 156
The Effect of using Superheated Steam 159
Efficiency of the Rankine Clausius Cycle using the Hϕ Diagram 160
Efficiency of the Rankine Clausius Cycle from Steam Tables 163
Engine in which the Steam is kept Dry and Saturated during Expansion 164
The Regenerative Steam Cycle 165
Note 166

XI THE RECIPROCATING STEAM ENGINE

General Statement 168
The Steam Engine Cycle 168
Comparison of Actual and Theoretical Indicator Diagrams . 169
Mean Effective Pressure. Diagram Factor 170
The Missing Quantity 173
Cyclic Condensation 173
Application of the Saturation Curve to an Actual Indicator Diagram 175
Application of Actual Indicator Diagram to Tϕ Chart . 179
Boulvin's Method of Drawing the Temperature-Entropy Diagram from the Indicator Diagram 180
The Heat Recovery Lines 182
Magnitude of Missing Quantity. Contributing Factors . 185
Methods of Increasing Engine Efficiency 186
The Uniflow Engine 186

CHAP. PAGE

XI THE RECIPROCATING STEAM ENGINE—*continued*

The Steam Jacket 187
The Use of Superheated Steam 189
Compound Expansion 191
Compound Engine without Intermediate Receiver . . 193
Compound Engine with Intermediate Receiver . . . 194
The Combination of Indicator Diagrams from a Compound Engine 196
Diagram Factor 199
Ratio of Cylinder Volumes for a Compound Engine . . 199
Governing of the Compound Engine 204
Effect of Throttling the Steam to the High-Pressure Cylinder on the Distribution of Work. Throttle Governing . 205
Initial Loads on the High-Pressure and Low-Pressure Pistons 207
The Regenerative Cycle 208

XII FLOW OF STEAM THROUGH ORIFICES AND NOZZLES

General Statement 211
The Velocity of Discharge 212
Weight of Steam Discharged. The Critical Pressure Ratio 214
Calculations of Throat and Exit Areas, assuming no Friction 217
Friction 220
Nozzle Shape and Size 223
Over and Under Expansion 224
Supersaturation 225
Convergent-Divergent Nozzle with Excessive Back-Pressure 226
Theory of the Injector 228

XIII THE STEAM TURBINE. DESIGN OF BLADING, ETC.

General Statement 235
Velocity Triangles for Single-Stage Turbines 236
Diagram for Velocity Compounded Impulse Stage assuming no Friction 243
Diagram for Velocity Compounded Impulse State, given the Value of K 244
Diagram for Velocity Compounded Impulse Stage, given the Blade Lengths 245
Velocity Diagram for Reaction Stage 248
Blade Speed and R.P.M. 249
Diagram Efficiency and Speed Ratio 249

CHAP. PAGE

XIII THE STEAM TURBINE. DESIGN OF BLADING, ETC.—*continued*

The Section of Turbine Blades 251
Pitch, Width, and Length of Blades 252
Blade Friction 253
Reheat Factor and Turbine Efficiencies 253
The Regenerative Principle 258
The Regenerative Principle applied to the Triple Expansion Engine or to Three Stages of Expansion in a Turbine . 261

XIV THE INTERNAL COMBUSTION ENGINE

General Statement 265
The Cycles Considered 266
The Ratio of Compression 273
The Influence of Maximum Permissible Pressure . . 275
Comparison of Actual and Theoretical Diagrams . . 277
Determination of the Value of n for the Compression and Expansion Curves 279
Crank Angle and Time-base Diagrams 282
The Tϕ Diagrams 283
Rate of Heat Reception or Rejection during Expansion and Compression 285
Actual and Theoretical Efficiencies. The Effect of Heat Losses 287
Effect of Jacket Water Temperature 290
The Calculation of Cycle Temperatures 291
Suction Temperature for a Gas Engine 291
The Suction Temperature for a Petrol Engine . . . 292
The Calculation of Compression Temperature . . . 294
Calculation of Maximum Temperature 295
Value of n for the Expansion Curve 295
Variations of Specific Heat 295
Effect of a Linear Variation of Specific Heat with Temperature on Formulæ for Heat Interchange and Adiabatic Expansion 297
Dissociation 299
The Process of Combustion in Air and Petrol Mixtures . 301
Turbulence 303
Effect of Residual Gases 304
Application of Knowledge of Flame Phenomena to Cylinder Head Design 304
Effects of Speed and Size 307
Combustion in the Compression Ignition Engine . . 307

CHAP. PAGE

XV MECHANICAL REFRIGERATORS AND REVERSED HEAT ENGINES

General Statement 312
Coefficient of Performance 312
The Reversed Heat Engine as Refrigerator. Cold Air Machines 313
Reversed Heat Engine as a Warming Machine . . . 317
The Use of the Reversed Heat Engine in Evaporation Problems 318
Refrigerators employing a Volatile Liquid . . . 319
Vapour Absorption Machine 319
Ideal Efficiency of Absorption Process 320
Intermittent Refrigeration 321
Continuous Action Absorption Machine 321
Vapour Compression Process 324
Irreversible Temperature Drop unavoidable in Practice . 330
Use of the Mollier Chart 333
To find the Dryness Fraction required before Compression to give Maximum Efficiency 335
Wet and Dry Compression 338
Two-Stage Throttling (Stokes' or Voorhees' Cycle) . . 339
Compound Compression Cycle (Linde, Lightfoot, or Wildhausen Cycle) 340
Choice of Refrigerants 340

XVI HEAT TRANSFER BY RADIATION, CONDUCTION AND CONVECTION

Heat Transmission by Radiation 343
Stefan's Law 344
Furnace Phenomena and " Black Body Condition " . . 345
Application of Radiation Theory 346
Radiation from Flames 346

Conduction of Heat through Solids

General Statement 347
Laws of Conduction 347
Flow of Heat through a Thin-Walled Tube . . . 348
Flow of Heat through a Thick-Walled Tube . . . 348
Equivalent Mean Radius of a Thick-Walled Pipe . . 348
Optimum Thickness of Pipe Lagging 349
General Equation for Conduction. Steady Conditions . 350
General Equation for a Periodic Fluctuation of Temperature 351
Dimensions of C and certain Related Values . . . 351
General Solution of Equation (6). Steady Temperature . 352

CHAP. PAGE

XVI HEAT TRANSFER BY RADIATION, CONDUCTION AND CONVECTION—*continued*

General Solution of Equation (6). Fluctuating Temperature 352
Interpretation of Formula 355
Heat Flow into Surface during Cyclic Fluctuation of Temperature 355
Graphical Estimation of Heat Loss 359
HEAT TRANSFER BY CONVECTION
Conduction of Heat through Pipes 363

XVII COMBUSTION

General Statement 370
Combustion Equations 371
The Calorific Value of Fuels 372
Estimation of Calorific Value from Chemical Analysis . 375
Calculation of Air required for Complete Combustion of Solid and Liquid Fuels 376
To calculate the Amount of Air required for the Combustion of a Gaseous Fuel 378
Flue and Exhaust Gas Analysis 379
The Examination of Exhaust and Flue Gas Analyses. Solid and Liquid Fuels 381
The Examination of Exhaust and Flue Gas Analyses. Gaseous Fuels 383
Approximate Method of finding Weight of Air Supplied per Pound of Coal 386
Producer Gas 386
Calculation of the Mean Specific Heat of the Flue Gases leaving a Boiler 393
Boiler Draught 394
Height of Chimney required to produce a Given Draught . 395

XVIII THE TESTING OF HEAT ENGINES AND PLANT

General Statement 399
Report on Tabulating the Results of Heat Engine Trials . 399
Steam Boiler Trial 401
Steam Engine Trial 409
Gas Engine Trial 413
Oil Engine Trial 420
Gas Producer Trial 423
Steam Turbine Trial 427

APPENDIX 433
ANSWERS TO EXAMPLES 437
INDEX 443

SYMBOLS, ETC.

The following are some of the symbols used :

A	The inverse of Joule's Equivalent $=\frac{1}{J}$.
c_v	Specific heat at constant volume, for air $=0{\cdot}1691$.
c_p	Specific heat at constant pressure, for air $=0{\cdot}2375$.
E	Internal energy. Alternative to I.
G	The universal gas constant $=\frac{R}{\text{Molecular weight}}=2779$ ft.-lb. per ° C. per mol. $=1543{\cdot}9$ ft.-lb. per ° F. per mol.
H	A quantity of heat, measured in B.Th.U. or C.H.U.
I	Internal energy. Alternative to E when applied to internal energy of evaporation.
J	Joule's Equivalent, or 778 ft.-lb. per B.Th.U. or 1400 ft.-lb. per C.H.U.
K	The bulk modulus of elasticity.
N	Modulus of rigidity.
N.T.P.	Normal temperature and pressure, i.e. 32° F. or 0° C. and 14·7 lb. per sq. in.
p	Pressure ; generally pounds per square foot.
q	Dryness fraction of wet vapour.
R	The constant in the gas equation $pv=RT$, for air 53·18 ft.-lb. per lb. per ° F. or 96·0 ft.-lb. per lb. per ° C.
s	Specific heat of solid or liquid.
S	Entropy. Alternative to ϕ.
S.C.F.	Standard cubic foot, used for the volumetric measurement of gases at N.T.P.
t and T	Temperature ° F. or ° C., and ° F. abs. or ° C. abs.
v	Volume, usually measured in cubic feet.
v_ω	Specific volume of water.
W	Work done, usually measured in foot-pounds.
α	Coefficient of volumetric thermal expansion, for a gas $=\frac{1}{273}$ per ° C., or $\frac{1}{492}$ per ° F.
γ	The ratio c_p/c_v. Value for air 1·401.
θ	See T.
ϕ	Entropy. Alternative to S.

The following values will be found of use:

1 Kg. per sq. cm.=14·233 lb. per sq. in.=1 continental atmosphere.
14·7 lb. per sq. in.=30 in. mercury=1 British atmosphere.
1 in. mercury=0·49 lb. per sq. in.=13·595 in. water.
Value of 1 lb. of air at N.T.P.=12·391 cu. ft.
42·42 B.Th.U./m.=1 H.P. 1 H.P.=0·746 kw.
55·9 B.Th.U./m.=1 kw.
Molecular weight of a gas in grammes occupies 22·41 litres, or in pounds 359 cu. ft. at N.T.P.

Chapter I

THE LAWS OF STATE AND OF THERMODYNAMICS

*GENERAL STATEMENT

In considering the production of mechanical energy by means of heat engines, the employment of mechanical energy or heat in refrigerators, and generally in the generation or control of heat, the engineer is guided by a number of experimental facts and "laws," some of which were stated in the early days of the physical sciences and are therefore subject to revision in the light of more exact researches. A number of these will now be considered.

*THE THREE STATES OF MATTER

Substances can exist in three states, as solids, liquids, or gases, provided that the conditions of pressure and temperature are suitable and that fundamental changes of the molecular structure do not occur. The phenomena commonly associated with the three states are as follows.

Solid State.—The substance offers resistance to changes of both size and shape, the resistance generally being of an elastic nature during the first stages of distortion. The resistance to change of size is then measured by the *bulk modulus of elasticity*, K, and the resistance to change of shape by the *modulus of rigidity*, N.

(For metals K lies between 9×10^6 and 31×10^6 lb. per sq. in.,
and N ,, $4{\cdot}5 \times 10^6$ and $13{\cdot}5 \times 10^6$,,)

The material expands slightly on heating, the *coefficient of expansion* depending on the material (for iron the coefficient of linear expansion is 0·0000108 per degree F. and the coefficient of volumetric expansion about three times this figure).

The absorption of heat by the material is accompanied by a rise of temperature, the *specific heat* being the ratio of the heat required to raise a given weight of the material through 1° compared with that required to raise a similar weight of water through the same temperature range, and is therefore independent of the temperature scale used. (The specific heat of iron is of the order of 0·11, and of ice 0·504.)

When the *melting-point* is reached the addition of heat results not in a rise of temperature but in the change of state from the solid to the

liquid. The heat required per pound of the substance to produce complete liquefaction is known as the *latent heat of fusion* (about 41 B.Th.U. for grey cast iron and 144 B.Th.U. for ice).

Liquid State.—The substance now offers resistance to change of size but not to gradual change of shape. The value of N is therefore zero. (The value of K for water is approximately 3×10^5 lb. per sq. in.)

The addition of heat produces a rise of temperature and a slight expansion, until a temperature called the *boiling-point* is reached, when a change to the vapour state occurs. This is accompanied by a large absorption of heat,[1] *the latent heat of evaporation*, and generally by a large increase of volume but not by a change of temperature. At the boiling-point liquid and vapour can exist together. The boiling-point increases with the pressure at which evaporation occurs.

Phenomena associated with the liquid state are the appearance of *surface tension* at the free surface of the liquid, and a tendency to *slow evaporation* at temperatures below the boiling-point.

State of Vapour and Gas.—A vapour will fill uniformly the vessel in which it is placed, irrespective of size, and will exercise a uniform pressure in all directions. No resistance is offered to gradual change of shape. The bulk modulus at constant temperature is dependent on, and equal to, the pressure p (measured in pounds per square foot).

[If the gas, originally at pressure p, is subjected to a pressure p_1, then by Boyle's law $p_1v_1=pv$. By definition K is the ratio of increase of stress to increase of strain, or

$$\mathrm{K}=\frac{p-p_1}{\frac{v_1-v}{v_1}}=\frac{p-p_1}{\frac{pv}{p_1}-v\Big/\frac{pv}{p_1}}=\frac{p-p_1}{\frac{v(p-p_1)}{p_1}\Big/\frac{pv}{p_1}}$$

$$=p.]$$

When the volume of the vapour is changed by compression, and work is done upon it, heat is produced, and the temperature tends to rise proportional to the work done. If the heat is removed at exactly the rate at which it is formed, *isothermal compression* occurs. If the heat is not removed the temperature will rise, and the work done by the external forces of compression will now not only compress the vapour, but also raise its internal energy. When no heat is removed or received, except as the result of work done, the process is said to be *adiabatic*.

Two values of the bulk modulus have now to be considered, the Bulk Moduli of Isothermal and Adiabatic compression.

The addition of heat is accompanied by a rise of temperature and either

[1] See critical temperature, p. 3.

(Note : Work is done when a force moves its point of application or when an expanding gas has its change of volume resisted. A force which does not move its point of application and a gas, whatever its pressure, which does not expand, can do no work.)

by an increase of pressure at constant volume (when no external work is done), or by an increase of volume at constant pressure (when the work done is the product of p and the change of volume).[1] In the first case the ratio $\frac{\text{heat received}}{\text{rise of temperature}}$ gives the *specific heat at constant volume*, in the second the *specific heat at constant pressure*; the difference between the two values is the equivalent of the external work done in the latter case.

The *coefficient of thermal expansion* is the same for all gases, and is 1/273 per degree C. or 1/492 per degree F.

(Strictly speaking, two Bulk Moduli and two specific heats should be considered for all three states, but the changes of volume corresponding to changes of pressure and temperature in the solid and liquid states are so small that for general purposes the values may be taken as coincident.)

As the pressure and temperature are increased at which evaporation occurs, the latent heat of evaporation decreases until the "*critical*" *temperature* for the substance is reached. At this temperature the latent heat is zero, the meniscus separating liquid and vapour vanishes, and the two states merge. Below this temperature the substance can be liquefied by the application of pressure alone, but above this liquefaction cannot be produced by pressure, however intense.

A vapour above its critical temperature is said to be a *gas*. The conditions for a *perfect gas* are approached more nearly as the temperature is raised above the critical value.

A "perfect gas" is one which satisfies exactly the laws of Boyle and Charles. In what follows gases will be assumed to be perfect unless a statement is made to the contrary.

*THE LAWS OF THERMODYNAMICS. GAS LAWS, ETC.

First Law of Thermodynamics.—This may be stated as follows: Heat and mechanical energy are mutually convertible and "Joule's Equivalent" is the rate of exchange.

If mechanical work is being converted into heat, this can be expressed in the form

$$W = JH$$

where W is the work done, measured in foot-pounds, H the quantity of heat evolved, measured in British thermal units, and J is Joule's Equivalent and equal to 778 ft.-lb. per B.Th.U. If H is measured in Centrigrade heat units the numerical value of J is 1400.

As will be seen later, the conversion of mechanical work into heat is a natural process which engineers have difficulty in restraining. The converse, the conversion of heat into work, is only effected with con-

[1] The case when neither p nor v remain constant is considered on p. 6.

siderable difficulty, and in practice much wastage of heat occurs in the process. The " efficiency " of conversion is often less than 20 per cent. and seldom more than about 45 per cent. (see Chapter III, *et seq.*).

Second Law of Thermodynamics.—The simplest statement of this law is : There is no process by which heat may be conveyed from one body to another at a higher temperature without the expenditure of energy supplied from an external source.

Boyle's Law.—If a given mass of gas is compressed at constant temperature the absolute pressure is inversely proportional to the volume. If p and v denote the pressure and volume respectively, then

$$p \propto 1/v \quad \text{or} \quad p \,.\, v = \text{constant}.$$

(" Absolute " pressures are recorded by such instruments as the mercury barometer, in which the moving element, in this case the mercury column, registers the difference in pressure between the pressure applied and a vacuum. Pressure gauges of the Bourdon type, and others which are subject to atmospheric pressure on one side of the element, record the difference of pressure between atmospheric and that which is applied. To obtain absolute pressures from such " gauge " pressures, a pressure corresponding to the height of the mercury barometer at the time must be added.)

Charles's Law, and the Absolute Zero of Temperature.—If the pressure of a given mass of gas is kept constant, equal increments of temperature will cause similar increments of volume, or $v_1 = v_0(1+\alpha t)$, where v_1 is the final volume, v_0 is the initial volume, usually taken at 0° C. or 32° F. for convenience, and t is the increment of temperature. " α " is called the " coefficient of (volumetric) thermal expansion," and, when referred to the volume at 0° C. or 32° F., has the values 1/273 and 1/492 respectively, and is constant for all perfect gases.

If now the temperature is kept constant at its new value and the volume restored by pressure to v_0, we have, from Boyle's law

$$\frac{v_1}{v_0} = \frac{p_1}{p_0} \quad \text{or} \quad \frac{v_0(1+\alpha t)}{v_0} = \frac{p_1}{p_0}$$

or $p_1 = p_0(1+\alpha t)$, α having the same value as before.

For example, at any given pressure 492 cu. ft. of gas measured at 32° F. will occupy 491 cu. ft. at 31° F., 460 cu. ft. at 0° F., and will approach zero volume when the temperature approximates to −460° F. Similarly, at any given volume the pressure of a gas approximates to zero at −460° F. The temperature −460° F. or −273° C., is called the *absolute zero of temperature.* (Though the above argument is based on the assumption that gases remain " perfect," whereas all actual gases change their physical

state at low temperatures, the conception of the absolute zero of temperature receives adequate support from other considerations.)

The temperature of a gas above absolute zero is spoken of as its absolute temperature. If t is a temperature on the Fahrenheit or Centigrade scale the corresponding temperature T on the absolute scale will be $t+460°$ F. abs. or $t+273°$ C. abs.

It follows that at constant pressure the volume of a gas is proportional to its absolute temperature, i.e. $v \alpha T$, and at constant volume the pressure is similarly proportioned, i.e. $p \alpha T$.

EXAMPLE.—The cylinder of an engine, when the piston is at top dead centre, contains gas at a pressure of 80 lb. per sq. in. gauge. Find the temperature to which it must be raised in order that the pressure may be increased to 800 lb. per sq. in. gauge. The initial temperature is 200° C.

This problem will first be worked out at length to illustrate the last statement.

From the equation $p_1=p_0(1+\alpha t)$ it will be seen that the pressure of the gas increases by an amount of $\frac{1}{273}$ of its pressure at 0° C. for every rise of 1° C.

The pressure at 0° C. in this case is given by

$$80+15=p_0\left(1+\frac{1}{273}\times 200\right)$$

or

$$p_0=\frac{95}{1+\frac{200}{273}} \text{ lb. per sq. in. abs.}$$

$$=95\times\frac{273}{473}=54{\cdot}8 \text{ lb. per sq. in. abs.} \quad . \quad (1)$$

The final rise in temperature t above 0° C. will be given by

$$800+15=54{\cdot}8\left(1+\frac{t}{273}\right)$$

or

$$\frac{815}{54{\cdot}8}=1+\frac{t}{273}$$

or

$$t=13{\cdot}87\times 273=3785° \text{ C.}$$

The final temperature is, then, 3785° C. or 4058° C. abs. From (1) it will be seen that, the volume being constant, the pressure is proportional to the absolute temperature, which at once suggests the much shorter solution :

As $p_2=815$ lb. per sq. in. absolute,

$p_1=95$ „ „ „

and $T_1 = 200 + 273 = 473°$ C. abs.

$$T_2 = T_1 \times \frac{p_2}{p_1} = 473 \times \frac{815}{95} = 4058° \text{ C. abs.}$$
$$= 3785° \text{ C.}$$

It will be noted that in the above working only ratios of pressures are involved, making it immaterial whether the unit is pounds per square inch or per square foot.

The constant-volume gas thermometer, which measures temperature in terms of the increase of pressure of certain gases when their volumes are kept constant, is used as the primary standard for defining temperatures. Nitrogen, hydrogen, and helium are generally used.

Combination of Laws of Boyle and Charles.—From Boyle's law $p \propto 1/v$ when T=constant, and from Charles's law $p \propto T$ when v=constant.

If a given mass of gas changes from the initial conditions of p_1, v_1, and T_1 to the final conditions of p_3, v_3, T_3, we can consider the change effected in two stages.

(1) The volume being maintained constant, the whole change of pressure is effected, after which T will have some intermediate value T_2, such that

$$p_1/p_3 = T_1/T_2$$

(2) The pressure now being constant the change of volume is made, when

$$v_1/v_3 = T_2/T_3$$

Multiplying these two equations $p_1v_1/p_3v_3 = T_1/T_3$

or
$$\frac{p_1v_1}{T_1} = \frac{p_3v_3}{T_3}$$

or $p \,.\, v = RT$, where R is a constant depending on whether the Fahrenheit or Centigrade scale is used.

For 1 lb. of dry air the numerical value of the constant R is 53·18 when T is measured on the Fahrenheit scale; it may be obtained as follows:

Consider 1 lb. of air at normal temperature and pressure (N.T.P.): under these conditions the weight of 1 cu. ft. of air is known to be 0·0807 lb., hence the volume of 1 lb. is 1/0·0807 or 12·391 cu. ft.

Standard atmospheric pressure is 14·7 × 144 or 2116 lb. per sq. ft., and the normal temperature is 32° F. or 492° F. abs., hence,

as
$$R = \frac{pv}{T}$$

$$R = \frac{2116 \times 12{\cdot}391}{492}$$
$$= 53{\cdot}18 \text{ ft.-lb. per lb. of air.}$$

Avogadro's Law.—Equal volumes of all gases, measured at the same temperature and pressure, contain the same number of molecules.

From this statement it follows that the volumes occupied by the molecular weights of all gases will be constant.

For the gramme molecular weight this volume is 22·41 litres, or for the pound molecular weight 359 cu. ft.

Dalton's Law.—The pressure exerted on the walls of the containing vessel by a mixture of gases which do not interact is equal to the sum of the pressures that would be exerted by the gases if they were present separately.

Joule's Law of Internal Energy, and the Specific Heat at Constant Volume.—If a given mass of gas is heated at constant volume, the energy supplied must be converted entirely into the form of internal energy, and give rise to an increase of temperature, the equation being $\delta H = C_v \delta T$, where C_v is the " specific heat at constant volume " and δT the increment of temperature.

Joule's Law of Internal Energy states that the internal energy of a given mass of gas is dependent upon its temperature only and is independent of the pressure or volume,

or $C_v \delta T$ is a function of T only, and C_v = constant.

The Specific Heat at Constant Pressure.—If a given quantity of gas is heated in such a way that its pressure remains constant, say in a cylinder closed by a piston the motion of which is resisted by a constant force, external work will be done by the expanding gas in moving the piston. The energy required to produce a rise of temperature from T_1 to T_2 will be greater than in the previous case by an amount equal to this work, and will be given by

$$\begin{aligned} C_p(T_2 - T_1) &= C_v(T_2 - T_1) + p(v_2 - v_1) \\ &= C_v(T_2 - T_1) + R(T_2 - T_1) \end{aligned}$$

Hence C_p, the " Specific Heat at Constant Pressure," is constant, both C_v and R being constant.[1] The ratio of the specific heats C_p/C_v is denoted by γ. It will be noted that a low value of γ indicates that the gas to which it applies has a relatively large capacity for storing internal energy.

The Meaning of R, Volumetric Heat.—It will be seen from the above equation that if $T_2 - T_1 = 1°$, then

$$C_p = C_v + R \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

[1] " In a gas the specific heat at constant pressure is independent of the pressure," is the " third gas law, due to Regnault, the laws of Boyle and Charles are the first and second.

and R is then the external work done in foot-pounds by 1 lb. of gas expanding at constant pressure while its temperature is raised 1°; i.e. if the pressure of 1 lb. of air is one atmosphere (or $14{\cdot}7 \times 144$ lb. per sq. ft.) and its volume at 32° F. is known to be 12·391 cu. ft., then, on heating through 1° F. the volume will be increased by one part in 492 (see p. 4), or by 0·02514 cu. ft., and the work done will be $0{\cdot}02514 \times 14{\cdot}7 \times 144$ ft.-lb. or 53·18 ft.-lb., as before. If the degree Centigrade is used, R is equal to 96 ft.-lb.

For hydrogen having a density of 0·00559 lb. per cu. ft. or a specific volume of 175·8 cu. ft., the work done per degree C., or $\bar{R}$, is

$$\frac{175{\cdot}8}{273} \times 14{\cdot}7 \times 144 = 1363 \text{ ft.-lb.}$$

Equation (1) may also be written

$$\frac{C_p}{C_v} - 1 = \frac{R}{C_v}$$

or

$$\gamma - 1 = \frac{R}{C_v} \quad \text{or} \quad C_v = \frac{R}{\gamma - 1}$$

For certain purposes it is convenient to consider the "volumetric heat," or the heat required to raise a stated volume of gas through one degree. The volume chosen may be either the "standard cubic foot" (i.e. a quantity occupying 1 cu. ft. at N.T.P.) or the volume occupied by the "pound molecule" or "mol." (i.e. 359 cu. ft.).

Now R is the work done when 1 lb. of gas has its temperature raised 1° at constant pressure. R × molecular weight will then be the corresponding work done by a quantity equal to the molecular weight in pounds, or, by Avogadro's law, by a quantity occupying 359 cu. ft. at N.T.P. As the coefficient of expansion with temperature of all perfect gases is constant, it follows that R × molecular weight is constant, or $R = G/m$, where "G" is the "Universal Gas Constant," and m is the pound molecular weight.

The numerical values of G are as follows:

2779 ft.-lb. or 1·985 C.H.U. per degree C 1543·9 ft.-lb. or 1·985 B.Th.U. per degree F	per pound molecule or 359 cu. ft. at N.T.P.
or 1·985 gramme calories per degree C.	per gramme molecule or 22·41 litres at N.T.P.

*MOLECULAR THEORY

Though the above-mentioned "laws" and phenomena have been studied separately over a long period of years, it is now known that they can be related by a single theory. Only a general sketch of the theory is given here, but this is of interest as offering a means of correlation, and

as providing an explanation of the deviations from the original conceptions which are now known to occur.

It is assumed that all substances are composed of small moving particles or molecules which, for the present purpose, can be considered as tiny elastic spheres possessing mass, and therefore exhibiting mutual attraction. It is also assumed that temperature is a manifestation of, and is proportional to, the mean molecular kinetic energy of translation. (A molecule may also possess kinetic energy of rotation, but this is not exhibited as temperature.)

In the solid state these molecules are drawn together by the forces of mutual attraction, but the gravitational bonds are strained by the jostling of the moving molecules against each other. The distances separating the molecules are insufficient to allow an interchange of positions, which are therefore permanently fixed and form a " space lattice."

As the temperature is raised the gravitational bonds will be further strained and the body as a whole will expand slightly. Conversely, a fall in temperature will be accompanied by contraction until finally, at the absolute zero of temperature, the molecules, having lost all their kinetic energy, will be drawn into a closely packed mass. External pressure, by restricting the range of movement of the molecules without reducing their velocities, and thereby increasing the number of molecular impacts of constant intensity, will produce elastic strain.

By supplying energy to the system when at a suitable temperature it is possible so to strain the gravitational bonds that the spaces separating the molecules increase and permit the passage of other molecules. The energy supplied will have been expended in the separation of the molecules and not in increasing their final translational energy or temperature, and will correspond to the " Latent Heat of Fusion."

The substance is now in the liquid state, the molecules being free to move among their fellows throughout the mass of the substance. They are still sufficiently close to each other to possess considerable gravitational adhesion, but the forces acting on any molecule will be in balance provided it is surrounded by other molecules. On reaching a boundary of the liquid the forces, now acting on one side of the molecule only, will tend to draw it again into the body of the liquid, giving the effect known as " surface tension."

Sufficient energy may be supplied to certain molecules to enable them to break the gravitational bonds entirely, as these decrease rapidly in strength with increasing distance and soon reach negligible proportions. These molecules will then escape from the surface and commence free flight through the space offered by the containing vessel. The breaking of the bonds will absorb energy without causing an increase in the final molecular velocity, giving the phenomena of the " latent heat of evaporation."

These " gaseous " molecules will continue in rectilinear flight until

their directions are changed by impact with the walls of the vessel or with other molecules. The sum of the impact forces exerted by all the molecules striking a given area of wall at any instant will give the "pressure" exerted by the gas, and, in view of the enormous number of molecules always involved,[1] and the frequency of the collisions,[2] the pressure will be exerted uniformly in all directions. A similar pressure (see also p. 12) is exerted throughout the gas by the forces of mutual impact between the particles.

The addition of heat to the gas results in increased molecular energy, part at least of which will be translational and will consequently be manifest as a rise of temperature.

First Law of Thermodynamics.—This law is now only the obvious statement that energy can be transferred from one body to another by elastic impact, the bodies concerned being, say, the moving molecules of a gas and the moving piston in the cylinder in which it is contained.

Second Law of Thermodynamics.—This law follows from the hypothesis that the kinetic energies of all molecules at the same temperature, whatever their mass, are constant. Were this not so then, if two gases having molecules of different masses were raised to the same temperature and then mixed without combination occurring, an interchange of energy would occur, resulting in a rise of temperature of one of the constituents and a fall of temperature of the other, a situation which, for a number of reasons, is impossible.

The second law is then equivalent to the statement that, upon elastic impact, the energy exchange is always from the body with the greater energy (or temperature) to the body with the less. Energy from an external source may be employed to reverse the positions of the giving and receiving bodies, but only by increasing the energy (and therefore the temperature) of the latter.

Boyle's Law.—If a cylinder fitted with a moving piston contains a constant number of molecules, the pressure upon the walls at any instant is due to bombardment by those molecules which have just reached them at that time. If the temperature, and therefore the velocity of bombardment, is constant, a decrease of volume will cause a proportionate increase in the molecular density, and therefore of the number of molecules approaching any given area of wall at any instant, and the pressure upon the walls will increase accordingly.

Further consideration is given to this law on p. 12.

Charles's Law.—If, with the above apparatus, the piston is stationary, then increasing the temperature will increase the energy of the individual

[1] Approximately $3{\cdot}03 \times 10^{23}$ in 1 litre of hydrogen at N.T.P.

[2] The velocity of travel is of the order of 1 mile per second.

molecules and produce increased pressure at each impact. The total pressure on a given area of surface can only be restored to its original value by a reduction of molecular density, that is by moving the piston outwards so that the volume occupied by the molecules is proportional to the temperature.

Avogadro's Law.—This follows from the facts that: (1) as the gases are at the same temperature, the molecular energies must be the same, or $\frac{1}{2}mv^2=\frac{1}{2}m'v'^2$, where m and m' are the masses of the molecules of the two gases, and v and v' their velocities; (2) as the pressures are equal the total forces of impact on unit area must be equal, or $mnv^2 = m'n'v'^2$, where n and n' are the numbers of molecules of each gas in unit volume.

(N.B.—The momentum of any molecule is the product mv, and the number of molecules reaching any portion of the wall in unit time will be proportional to the total number of molecules present and to their velocity. The pressure, which is proportional to the momentum reaching unit area of the wall in unit time, will then be given by mnv^2.)

Equations (1) and (2) can both be true only if $n=n'$.

Dalton's Law.—It has been assumed that in the gaseous state the molecules are so far separated that mutual attractions are negligible. Each molecule will exert pressure on the walls as though the other molecules did not exist, and the total pressures exerted by the molecules of each gas can therefore be considered separately.

Joule's Law of Internal Energy.—As the internal energy of a gas is entirely in the form of molecular kinetic energy, which can be varied only by changes of velocity, that is of temperature, Joule's law follows as a matter of course. (This statement is not vitiated by the fact that the molecules may have rotational as well as translational energy.)

VARIATIONS FROM THE SIMPLE LAWS

Evaporation at Temperatures below the Boiling-Point.—The tendency of a liquid to evaporate at temperatures below the boiling-point is due to the fact that, under the influence of impact the velocity of any individual molecule is constantly changing, though the mean behaviour of the innumerable molecules in the mass is constant. Individual molecules may therefore receive sufficient energy to liberate them from the surface of the liquid, but this can only be done at the expense of the energy of molecules remaining therein. The " latent heat of evaporation " for these molecules is acquired from the molecules still in the liquid state, and the liquid is thereby cooled.

As the velocity of any individual molecule is constantly being changed by impact, the behaviour of a substance as a whole must be related, not

to the square of the translational velocity of any particular molecule, but to the mean square of the velocities of all the molecules.

The Value of "γ."—As previously stated, the value of the specific heat at constant pressure exceeds that for the specific heat at constant volume by the amount of external work performed when 1 lb. of the gas is heated through 1° at constant pressure. The ratio of the specific heats is therefore the ratio

$$\frac{\text{energy absorbed internally}+\text{energy absorbed externally}}{\text{energy absorbed internally}},$$

and a low value of γ indicates that, of the total energy absorbed, a comparatively large proportion is absorbed internally.

On heating a gas with a complex molecular structure energy will be absorbed not only in increasing the translational velocity of the whole, which alone gives rise to the phenomena of temperature, but also in increasing the vibratory velocity of the particles making up the molecule. The total energy absorbed will be distributed between all the possible forms of motion, and the more complex the structure the greater will be the energy absorbed internally in increasing non-translational energy. The common gases can therefore be divided into groups according to their molecular structure as follows :

Monatomic gases, such as argon and helium, having a value of $\gamma = 1{\cdot}67$ approximately,

Diatomic gases, such as O_2, H_2, N_2, having a value of $\gamma = 1{\cdot}40$ approximately,

Triatomic gases, such as CO_2, H_2O, having a value of $\gamma = 1{\cdot}33$ approximately.

Variations from the Gas Laws.—The molecular theory provides explanations for the known variations from the simple gas laws, which hold, for most gases, over a limited range only.

Consider the equation $pv = RT$.

In deducing this equation it has been assumed that any gaseous molecule is free to travel throughout the space v, but this can only be so if the volume occupied by the molecules themselves is negligible. If "b" is the total volume of the molecules, the "free space" is not v but $(v-b)$. At atmospheric pressure the volume "b" for steam is approximately $1/1000v$, and may be taken, for present purposes, as equal to the volume of the water which could be obtained by condensation.

It has also been assumed that the pressure exerted on the walls of the vessel, as recorded by a "pressure gauge," is the same as that in the interior of the gas. This cannot be so as the molecules at the boundaries of the vessel are subject to molecular attraction on one side only, i.e. to a form of "surface tension" however small, (the attraction exerted by

the walls of the vessel can be shown not to affect the problem). The boundary pressure recorded is therefore less than the true internal pressure by an amount equal to this unbalanced molecular attraction. Now the mutual attraction between two bodies is proportional to the product of their masses, or, between two portions of a homogeneous fluid, to the square of the density, which, in turn, is inversely proportional to the square of the specific volume. The true pressure will therefore be equal to the recorded pressure plus a/v^2, where a is a constant.

$pv=\mathrm{RT}$ should therefore be modified to read $(p+a/v^2)(v-b)=\mathrm{RT}$, a form due to Van der Waals.

It has so far been assumed also that the form of the molecules remains unchanged, but this is known not to be always so. In the case of steam the normal triatomic molecules of superheated steam tend to group themselves together as the temperature approaches that of condensation. There will therefore be a shrinkage or "defect of volume," due to "coaggregation," which is dependent on temperature. Callendar gives the equation for steam, corresponding to the perfect gas equation $pv=\mathrm{RT}$, as $v-b=\mathrm{RT/P}-\mathrm{C}$, where C, the defect of volume, $=0{\cdot}4213\times\left(\frac{373{\cdot}1}{\mathrm{T}}\right)^{\frac{10}{3}}$ cu. ft. per lb., and "b" has the value ascribed to it earlier in this section. Still more accurately the equation is given by

$$v-b=\frac{\mathrm{RT}}{a\mathrm{P}}-\frac{\mathrm{C}}{1-\mathrm{Z}^2}$$

where Z is a function of P/T, as coaggregation will increase with P and be inversely proportional to T. This last equation covers with great accuracy a very wide range of conditions including the critical.

The Specific Heat of Water.—A somewhat similar allowance is found in the most recent equation for the total heat of water, for which the simple relation $h=\mathrm{S}_0(\mathrm{T}-32^\circ)$ is known not to have a high degree of accuracy. The molecules of water, though retaining the constant ratio of 2 : 1 between the numbers of atoms of hydrogen and oxygen, are known to form large and very openly spaced groups or compound molecules,[1] into the cavities of which the much smaller gaseous molecules can enter and remain "in solution." These latter will retain their latent heat, and the heat of the whole will be given by

$$h=\mathrm{S}_0(\mathrm{T}-32)+\frac{v\mathrm{L}}{\mathrm{V}-v}$$

the second term indicating that the water contains its own volume v of steam in solution, which now contributes the fraction $\frac{v}{\mathrm{V}-v}$ of latent heat, where V is the specific volume of steam at the temperature T.

[1] Callendar. Thomas Hawksley Lecture, *Proc.Inst.Mech.E.* Nov. 1929.

Chapter II

LAWS OF EXPANSION AND COMPRESSION

*WORK DONE BY A GAS DURING EXPANSION

Isothermal or Constant Temperature Expansion.—The law of the curve of isothermal expansion is derived from the formula $pv=RT$, or, in general, $pv=k$, where k is a constant depending on the temperature and the quantity of air considered (R refers to 1 lb. of air).

Suppose the expansion to take place from the initial state p_1, v_1, and T to the final state p_2, v_2, and T.

For a small change in volume δv during which the average pressure is p, the work done is given by

$$\delta W = p \times \delta v \text{ (see Fig. 1)}$$

Let W denote the work done during the expansion, then

$$W = \int_{v_1}^{v_2} p\,dv.$$

And since

$$pv = k = RT$$

$$p = k/v.$$

Therefore

$$W = \int_{v_1}^{v_2} \frac{k}{v} dv = k\left[\log_\epsilon v\right]_{v_1}^{v_2} = k \log_\epsilon \frac{v_2}{v_1}$$

or

$$W = p_1 v_1 \log_\epsilon \frac{v_2}{v_1} \quad \text{or} \quad RT \log_\epsilon \frac{v_2}{v_1}$$

which may be written,

$$W = p_1 v_1 \log_\epsilon r \quad \text{or} \quad RT \log_\epsilon r,$$

where r is the ratio of expansion, i.e. $\frac{\text{final volume}}{\text{initial volume}}$. The curve $pv=k$ is a rectangular hyperbola and symmetrical with regard to the axes of p and v.

The work done by the gas during expansion is equal to the work which would be done *on* the gas during compression, provided that the laws of expansion and compression were the same.

The General Case of Expansion.—In general, a gas will expand according to the law $pv^n=$constant, and a change of temperature will occur. (Isothermal expansion is a special case where $n=1$, and T=con-

stant, necessitating logarithmic integration.) Let the law of expansion be $pv^n=k$, the initial state p_1, v_1, and T_1, and the final state p_2, v_2, and T_2.

Then, as before, $W=\int_{v_1}^{v_2} p \cdot dv$

But $p=k/v^n$

$$\therefore W=\int_{v_1}^{v_2}\frac{k}{v^n}dv$$

$$=k\left[\frac{1}{1-n}\cdot v^{1-n}\right]_{v_1}^{v_2}$$

$$=\frac{k}{1-n}\{v_2^{1-n}-v_1^{1-n}\}$$

$$=\frac{p_1v_1^n}{1-n}\{v_2^{1-n}-v_1^{1-n}\} \quad . \quad . \quad . \quad . \quad (1)$$

and since $p_1v_1^n=p_2v_2^n$, equation (1) becomes

$$W=\frac{p_1v_1-p_2v_2}{n-1} \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

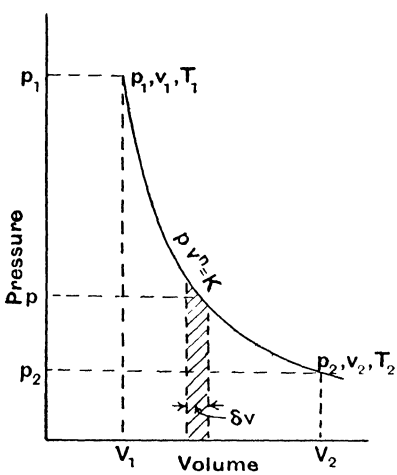


Fig. 1.

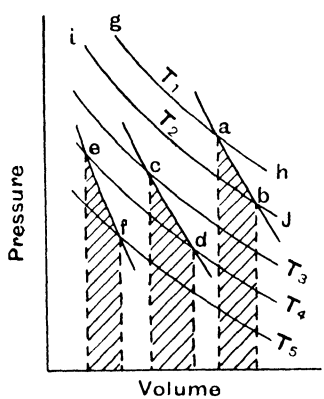


Fig. 2.

Also, for a perfect gas, $pv=RT$, and (2) may be written

$$W=\frac{RT_1-RT_2}{n-1} \quad \text{or} \quad \frac{R(T_1-T_2)}{n-1} \quad . \quad . \quad . \quad . \quad (3)$$

where T_1-T_2 is the change of temperature.

If, in Fig. 2, *ab*, *cd*, etc., are a series of expansion curves having the same value of "*n*," and *gh*, *ij*, etc., are a series of isothermals at temperatures T_1, T_2, T_3, etc., such that $T_1-T_2=T_2-T_3=T_3-T_4$, then the

areas shown shaded, representing the work done during expansions over equal temperature ranges, but from different initial temperatures, are all equal. The figure is not to scale.

The expression for work done may be put in several forms, as follows :

From equation (1)
$$W=\frac{p_1}{1-n}\left\{v_1^n\times v_2^{1-n}-v_1\right\}$$
$$=\frac{p_1v_1}{1-n}\left\{v_1^{n-1}\times v_2^{1-n}-1\right\}$$
$$=\frac{p_1v_1}{1-n}\left\{\left(\frac{v_1}{v_2}\right)^{n-1}-1\right\}$$
or
$$W=\frac{p_1v_1}{n-1}\left\{1-\left(\frac{v_1}{v_2}\right)^{n-1}\right\} \quad . \quad . \quad . \quad . \quad . \quad (4)$$

an expression in terms of the volume ratio.

Again,
$$p_1v_1^n=p_2v_2^n$$
or
$$\left(\frac{v_1}{v_2}\right)^n=\frac{p_2}{p_1} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

Substituting (5) in (4), we have—
$$W=\frac{p_1v_1}{n-1}\left\{1-\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}}\right\} \quad . \quad . \quad . \quad . \quad . \quad (6)$$

an expression in terms of the pressure ratio.

If all pressures are measured in pounds per square foot, and all volumes in cubic feet, the work done as calculated from either (2), (4), or (6) will be in foot-pounds.

The form given by equation (2) is that most generally used.

[1] EXAMPLE.—One cu. ft. of gas at a pressure of 300 lb. per sq. in. absolute expands to 60 lb. per sq. in. absolute, the law of expansion being $pv^{1\cdot2}=$constant. Find the volume at the end of the expansion, and the work done during expansion.

Now
$$p_1v_1^{1\cdot2}=p_2v_2^{1\cdot2}$$
$$v_2^{1\cdot2}=\frac{p_1}{p_2}v_1^{1\cdot2}=\frac{300}{60}\times1^{1\cdot2}=5$$
$$1\cdot2\log v_2=\log 5$$
from which
$$v_2=3\cdot823 \text{ cu. ft.}$$

[1] In solving numerical problems, the reader is warned against the habit of merely inserting numerical values in a symbolic formula which is either extracted complete from a text-book or quoted from memory. That way lies disaster. Wherever possible a start should be made from " first principles."

It must be assumed that the numerical examples worked in this book are to be taken in conjunction with the proofs of the equations given in the text, and with any statements of limitations which may apply.

Work done $$=\frac{p_1v_1-p_2v_2}{n-1}=\frac{144(300\times1-60\times3\cdot823)}{1\cdot2-1}$$

$$=50{,}820 \text{ ft.-lb.}$$

The work done might also be found without calculating v_2 by using equation (6), i.e.—

$$W=\frac{p_1v_1}{n-1}\left\{1-\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}}\right\}$$

$$=\frac{300\times144\times1}{0\cdot2}\left\{1-\left(\frac{1}{5}\right)^{\frac{0\cdot2}{1\cdot2}}\right\}$$

$$=216{,}000\left\{1-\left(\frac{1}{5}\right)^{\frac{1}{6}}\right\}$$

$$=50{,}820 \text{ ft.-lb. as before.}$$

Adiabatic Expansion.—When a gas expands without either receiving heat from or rejecting heat to any external body, as in a perfectly insulated cylinder, the work done must be performed at the expense of the internal energy of the gas, which in consequence becomes cooler.

If during the small expansion δv the temperature falls an amount δT,

$$p\delta v=-C_v\delta T$$

δT being essentially negative while δv is positive

or $$\frac{p}{C_v}=-\frac{\delta T}{\delta v}$$

or in the limit $$\frac{p}{C_v}=-\frac{dT}{dv} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Now for a perfect gas $pv=RT=(C_p-C_v)T$, and differentiating this with respect to v,

$$p+v\frac{dp}{dv}=\frac{dT}{dv}(C_p-C_v) \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Substituting the value of $\frac{dT}{dv}$ from (1) in (2) we have

$$p+v\frac{dp}{dv}=-\frac{p}{C_v}(C_p-C_v)$$

$$=-p\frac{C_p}{C_v}+p$$

or $$v\frac{dp}{dv}=-p\gamma$$

or $$\frac{dp}{p}=-\gamma\frac{dv}{v}$$

Integrating, we have

$$\log p = -\gamma \log v + \text{constant}$$

or $$\log p + \gamma \log v = \text{constant}$$

or $$pv^{\gamma} = \text{constant}.$$

This is a special case of the general form $pv^n = \text{constant}$ where $n = \gamma = C_p/C_v$.

Alternative Method.—This result may also be found as follows :—

In general $$W = \frac{R(T_1 - T_2)}{n-1}$$

Now if the gas changes in temperature from T_1 to T_2 its internal energy is diminished by the amount

$$C_v(T_1 - T_2) \text{ or, as } C_v = \frac{R}{\gamma - 1},$$

by the amount $$\frac{R}{\gamma - 1}(T_1 - T_2).$$

Hence equating the loss of internal energy to the work done, the condition for adiabatic expansion is secured when

$$\frac{R}{\gamma - 1}(T_1 - T_2) = \frac{R}{n-1}(T_1 - T_2)$$

that is when $n = \gamma$.

Hence the expansion or compression will be adiabatic when

$$pv^{\gamma} = \text{constant}.$$

EXAMPLE.—One lb. of dry air (volume 12·39 cu. ft.) at atmospheric pressure requires compressing to a pressure of 200 lb. per sq. in. abs. What will be the saving if the air is compressed isothermally instead of adiabatically ?

(1) *Isothermal compression.*—

$$p_1v_1 = p_2v_2 \qquad \therefore v_2 = \frac{p_1v_1}{p_2} = \frac{14{\cdot}7 \times 12{\cdot}39}{200} = 0{\cdot}91 \text{ cu. ft.}$$

$$\text{Work done on the gas} = p_1v_1 \log_{\epsilon}\left(\frac{v_1}{v_2}\right)$$

$$= 144 \times 14{\cdot}7 \times 12{\cdot}39 \log_{\epsilon} \frac{12{\cdot}39}{0{\cdot}91}$$

$$= 144 \times 14{\cdot}7 \times 12{\cdot}39 \log_{\epsilon} 13{\cdot}6$$

$$= 2116 \times 12{\cdot}39 \times 2{\cdot}607$$

$$= 68{,}320 \text{ ft.-lb.}$$

(2) *Adiabatic compression.*—Using equation (6)

$$W=\frac{2116\times 12{\cdot}39}{1{\cdot}4-1}\{1-13{\cdot}6^{\frac{0{\cdot}4}{1{\cdot}4}}\}=\frac{2116\times 12{\cdot}39}{0{\cdot}4}\{1-2{\cdot}108\}$$

$$=-72{,}630 \text{ ft.-lb.}$$

The negative sign simply means that work is done *on* the gas. Hence, by compressing isothermally we save 72,630−68,320=4400 ft.-lb.

EXAMPLE.—The temperature of 1 lb. of air is observed to fall from 600° F. to 300° F. while it expands adiabatically, doing 39,445 ft.-lb. of work. Find C_v and C_p.

Since the expansion is adiabatic, no heat is supplied to or taken away from the air during the process. Hence the work done will be equal to the loss of internal energy during the expansion, namely

$$C_v\times(\text{fall in temperature}).$$

Expressing both quantities in heat units we have

$$C_v(600-300)=\frac{39{,}445}{778}$$

$$C_v=\frac{39{,}445}{778\times 300}=0{\cdot}169 \text{ B.Th.U. per lb.}$$

Now $\dfrac{C_p}{C_v}=1{\cdot}4$ for air (p. 12)

hence $C_p=0{\cdot}169\times 1{\cdot}4=0{\cdot}237$ B.Th.U. per lb.

*RELATIONS BETWEEN PRESSURE, VOLUME, AND TEMPERATURE DURING THE EXPANSION OF A GAS ACCORDING TO THE LAW pv^n=CONSTANT

As $$p_1v_1^n=p_2v_2^n=\text{constant} \quad . \quad . \quad . \quad . \quad . \quad (1)$$

and since, for a perfect gas, $pv=RT$ or $\dfrac{pv}{T}=R$

$$\frac{p_1v_1}{T_1}=\frac{p_2v_2}{T_2}$$

or $$\frac{p_2v_2}{p_1v_1}=\frac{T_2}{T_1} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

From (1) $$\frac{p_2}{p_1}=\left(\frac{v_1}{v_2}\right)^n \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Substituting from (3) in (2) gives

$$\frac{T_2}{T_1}=\left(\frac{v_1}{v_2}\right)^n\times\frac{v_2}{v_1}=\left(\frac{v_1}{v_2}\right)^{n-1} \quad . \quad . \quad . \quad . \quad . \quad (4)$$

Again, from (1) $$\frac{v_2}{v_1}=\left(\frac{p_1}{p_2}\right)^{\frac{1}{n}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

Substituting from (5) in (2) gives

$$\frac{T_2}{T_1}=\frac{p_2}{p_1}\times\left(\frac{p_1}{p_2}\right)^{\frac{1}{n}}=\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} \quad . \quad . \quad . \quad . \quad (6)$$

Hence we may write the expression for work done, equation (6), p. 16,

$$W=\frac{p_1v_1}{n-1}\left\{1-\frac{T_2}{T_1}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

*GENERAL CASE OF EXPANSION OR COMPRESSION

Changes of pressure and volume, however produced, can be reproduced by the use of adiabatic and isothermal processes only; i.e. the expansion from *a* to *b* (see Fig. 3) can be effected in two stages: (1) by adiabatic

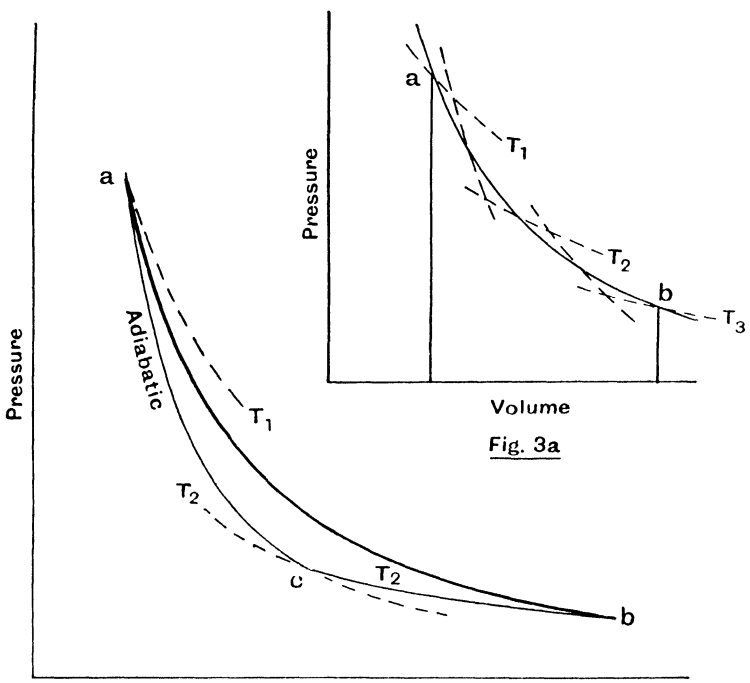


Fig. 3.

expansion from *a* to *c* over the full temperature range, and (2) from *c* to *b* by isothermal expansion.

By increasing the number of stages in the manner shown in Fig. 3*a*,

the area beneath the curve *ab* can be simulated to any required degree of accuracy.

This method of substitution is often of value when making calculations.

*FORMULÆ FOR COMPRESSION

The formulæ for compressions are identical with the corresponding formulæ for expansions. It should be noted, however, that p_1 refers to the higher pressure, i.e. the final pressure during compression and the initial pressure during expansion.

*HEAT RECEIVED OR REJECTED DURING AN EXPANSION OF THE FORM pv^n=CONSTANT

If a gas expands according to the law pv^n=constant, when n is not equal to γ, the heat received or rejected can be calculated as follows. Let p_1, v_1, and T_1 be the initial conditions, and p_2, v_2, and T_2 the final conditions.

Then if no heat were received or rejected as the gas expands from v_1 to v_2, we would have

$$\text{Work done} = \frac{1}{\gamma - 1}(p_1 v_1 - p_2 v_2) \quad . \quad . \quad . \quad . \quad (1)$$

If the expansion is according to the law pv^n=constant, the work done

$$= \frac{1}{n-1}(p_1 v_1 - p_2 v_2) \quad . \quad . \quad . \quad . \quad . \quad (2)$$

The difference between (1) and (2), by the law of conservation of energy, must be equivalent to the energy received or rejected during the expansion

or $$H = \frac{1}{n-1}(p_1 v_1 - p_2 v_2) - \frac{1}{\gamma - 1}(p_1 v_1 - p_2 v_2)$$

where H is in units of mechanical energy

$$= (p_1 v_1 - p_2 v_2)\left(\frac{1}{n-1} - \frac{1}{\gamma - 1}\right)$$

$$= \frac{1}{n-1}(p_1 v_1 - p_2 v_2)\left(\frac{\gamma - n}{\gamma - 1}\right)$$

$$= \text{work done} \times \frac{\gamma - n}{\gamma - 1} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

EXAMPLE.—Ten cu. ft. of air at 90 lb. per sq. in. abs. and at 65° F. are expanded to four times the original volume, the law of expansion

being $pv^{1\cdot25}$=constant. Given C_v=130·3 ft.-lb. per lb., and C_p=183·4 ft.-lb. per lb. : find—

(1) The temperature of air at the end of expansion.

(2) The work done in ft.-lb.

(3) The amount of heat which must have been given by, or been rejected to, an external source during the cycle.

(1) $$\frac{T_2}{T_1}=\left(\frac{v_1}{v_2}\right)^{1\cdot25-1}=\left(\frac{1}{4}\right)^{\frac{1}{4}}=\left(\frac{1}{2}\right)^{\frac{1}{2}}=\frac{1}{\sqrt{2}}=0\cdot707$$

$$\therefore T_2=0\cdot707T_1=0\cdot707(65+460)$$

$$=0\cdot707\times525=371\cdot2^\circ \text{ F. absolute}$$

$\therefore t_2^\circ$ F.=371·2−460=−88·8° F.=temperature of air at the end of expansion.

(2) Now $p_1v_1^{1\cdot25}=p_2v_2^{1\cdot25}$

$$\therefore p_2=p_1\times\left(\frac{v_1}{v_2}\right)^{1\cdot25}=90\times\left(\frac{1}{4}\right)^{\frac{5}{4}}$$

$$=90\times(0\cdot707)^5=15\cdot84 \text{ lb. per sq. in.}$$

$$\therefore \text{Work done } W=\frac{p_1v_1-p_2v_2}{n-1}$$

$$=144\cdot\frac{90\times10-15\cdot84\times40}{0\cdot25}$$

$$=576\times(900-633\cdot6)=576\times266\cdot4=153{,}446 \text{ ft.-lb.}$$

(3) From equation (3), p. 21, heat given to air=H=W $\cdot\frac{\gamma-n}{\gamma-1}$

$$=153{,}446\times\frac{1\cdot4-1\cdot25}{1\cdot4-1}$$

$$=153{,}446\times\frac{3}{8}=57{,}542 \text{ ft.-lb.}$$

$$=73\cdot9 \text{ B.Th.U.}$$

*EFFECT OF n ON THE SLOPE OF THE EXPANSION OR COMPRESSION CURVE

The slope of the curve is given by $\frac{dp}{dv}$, and since $pv^n=k$,

$$p=\frac{k}{v^n}$$

and

$$\frac{dp}{dv}=-\frac{kn}{v^{n+1}}=-\frac{np}{v}.$$

From this expression we see that as n increases, $\frac{dp}{dv}$ increases also, the effect on the slope of the curve being as shown in Fig. 4.

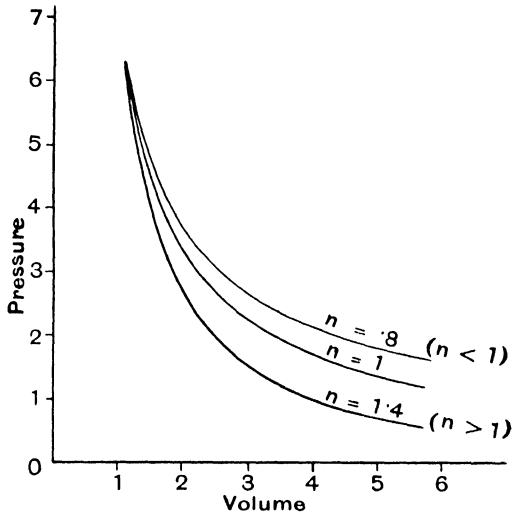


Fig. 4.

*RATE OF HEAT RECEPTION OR REJECTION ASSUMING CONSTANT SPECIFIC HEATS

Let dp be an indefinitely small change of pressure accompanying an indefinitely small change in volume dv. Then $\frac{dp}{dv}$ is the rate of change of pressure with respect to volume.

Also, if dH be the small quantity of heat given to the gas during the above small changes of pressure and volume, and dT the corresponding small change in temperature, then $\frac{d\mathrm{H}}{dv}$ will represent the rate at which the expanding gas receives heat per unit change in volume.

Now for a perfect gas we have, for any kind of expansion

$$pv=\mathrm{RT}$$

or

$$\mathrm{T}=\frac{pv}{\mathrm{R}}$$

Differentiating we get

$$\frac{d\mathrm{T}}{dv}=\frac{1}{\mathrm{R}}\left(p+v\frac{dp}{dv}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

But heat supplied = work done + increase in internal energy,

or $$dH = pdv + C_v dT$$

and $$\frac{dH}{dv} = p + C_v \frac{dT}{dv} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Substituting in (2) the value $C_v = \frac{R}{\gamma - 1}$, and the value of $\frac{dT}{dv}$ from (1), we have

$$\frac{dH}{dv} = p + \frac{R}{\gamma - 1} \cdot \frac{1}{R}\left(p + v\frac{dp}{dv}\right)$$

$$= p + \frac{1}{\gamma - 1}\left(p + v\frac{dp}{dv}\right)$$

$$= \frac{1}{\gamma - 1}\left(\gamma p + v\frac{dp}{dv}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

If now the expansion takes place according to the general law $pv^n = k$, we have

$$\frac{dp}{dv} = -\frac{kn}{v^{n+1}} \quad \text{or} \quad kv^{-n} \times -nv^{-1}$$

But $$kv^{-n} = p$$

Hence $$\frac{dp}{dv} = -\frac{np}{v}.$$

Substituting this value of $\frac{dp}{dv}$ in (3) we have

$$\frac{dH}{dv} = \frac{1}{\gamma - 1}(\gamma p - np)$$

or $$\frac{dH}{dv} = p \cdot \frac{\gamma - n}{\gamma - 1} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

This equation satisfies the condition that for adiabatic expansion or compression, during which $n = \gamma$, $\frac{dH}{dv} = 0$.

Alternative Proof.—The above result may be obtained directly from equation (3) of last section by differentiation, i.e. for a small change of volume dv the work done is $p \cdot \delta v$

or $$\delta H = \frac{\gamma - n}{\gamma - 1} p \delta v$$

Hence $$\frac{dH}{dv} = \frac{\gamma - n}{\gamma - 1} p.$$

Now $\frac{dH}{dv}$ represents the rate of heat reception per unit change in volume, hence the rate of heat reception per second will be given by

$$\frac{dH}{dv} \times \text{volume swept out by piston in cubic feet per second.}$$

If n is less than γ, the rate of heat reception $\frac{dH}{dv}$ will be positive; whereas if n is greater than γ, the rate of reception $\frac{dH}{dv}$ will be negative. Hence, if the expansion curve is less steep than the adiabatic, the gas is receiving heat during the expansion; but if the curve is steeper than the adiabatic, i.e. if n is greater than γ, the gas is losing heat. If the expansion is isothermal ($n=1$) the heat received is equal to the thermal equivalent of the work.

If the gas is compressed isothermally, an amount of heat equal to the work done on the gas must be rejected, and, if the gas is being compressed and n is less than γ, the rate of heat rejection will be positive, i.e. heat must be taken from the gas. If, however, n is greater than γ the rate of heat rejection will be negative, and the gas will be receiving heat during the compression.

Heat may be lost during expansion or compression by heat flow through the walls or ends of the vessel containing the gas, i.e. to the "cooling water" of a petrol engine. The reception of heat during a change of volume may be either through the cylinder walls, as in the case of a "steam-jacketed" cylinder of a steam engine, or by the generation of heat within the cylinder, as in the case of delayed combustion in an "internal combustion engine." An approximate analysis of the heat losses or gains during expansion or compression in an actual engine can readily be made from a study of the "indicator card" (see p. 175).

EXAMPLE.—If at a point of an expansion curve for air the pressure is 85 lb. per sq. in. (gauge) and the value of n is (*a*) 1·28, (*b*) 1·50, find the volumetric rate of heat reception or rejection.

Equation (4) gives $$\frac{dH}{dv} = p\frac{\gamma - n}{\gamma - 1}$$

$$= (85+15)\frac{1{\cdot}4-n}{0{\cdot}4}.$$

In case (*a*) $$\frac{dH}{dv} = 100 \times \frac{0{\cdot}12}{0{\cdot}4} = 30 \text{ B.Th.U. per cu. ft.}$$

In case (*b*) $$\frac{dH}{dv} = 100 \times -\frac{0{\cdot}1}{0{\cdot}4} = -25 \text{ B.Th.U. per cu. ft.}$$

In the former heat is received, in the latter it is rejected.

If the volume is changing at the rate of 10 cu. ft. per minute the heat interchanges will be at the rate of 300 and −250 B.Th.U. per minute.

*COLLECTION OF THE FORMULÆ PROVED ABOVE

$$C_v = C_p - R = \frac{R}{\gamma - 1} \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

For isothermal or hyperbolic expansion,

$$pv = \text{constant} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

and

$$W = p_1 v_1 \log_\epsilon r \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

For expansion other than isothermal

$$pv^n = \text{constant} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

$$\frac{p_2}{p_1} = \left(\frac{v_1}{v_2}\right)^n \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

$$\frac{v_2}{v_1} = \left(\frac{p_1}{p_2}\right)^{1/n} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

$$\frac{T_2}{T_1} = \left(\frac{v_1}{v_2}\right)^{n-1} \text{ or } \left(\frac{p_2}{p_1}\right)^{n-1/n} \quad . \quad . \quad . \quad (7)$$

$$W = \frac{p_1 v_1 - p_2 v_2}{n-1} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

$$= \frac{p_1 v_1}{n-1}\left\{1 - \left(\frac{v_1}{v_2}\right)^{n-1}\right\} \quad . \quad . \quad . \quad . \quad . \quad (9)$$

$$= \frac{p_1 v_1}{n-1}\left\{1 - \left(\frac{p_2}{p_1}\right)^{n-1/n}\right\} \quad . \quad . \quad . \quad . \quad (10)$$

$$= \frac{p_1 v_1}{n-1}\left\{1 - \frac{T_2}{T_1}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

$$H = \text{work done} \times \frac{\gamma - n}{\gamma - 1} \quad . \quad . \quad . \quad . \quad . \quad (12)$$

$$\frac{dH}{dv} = p \cdot \frac{\gamma - n}{\gamma - 1} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

For adiabatic expansion the value $n = \gamma$ is inserted in equations (4) to (11), and the values of H and $\frac{dH}{dv}$ are zero (equations (12), (13)).

EXAMPLE.—If 1 cu. ft. of air, at 100 lb. per sq. in. pressure (abs.), expands adiabatically, find the work done if the volume, pressure, and temperature ratios are successively equal to 2 : 1.

From equation (9) $$W=\frac{100\times144\times1}{1\cdot4-1}\left\{1-\left(\frac{1}{2}\right)^{0\cdot4}\right\}$$
$$=36{,}000\ (1-0\cdot7585)$$
$$=8780 \text{ ft.-lb.}$$

From equation (10) $$W=36{,}000\ (1-0\cdot5^{\frac{0\cdot4}{1\cdot4}})$$
$$=36{,}000\ (1-0\cdot767)$$
$$=8385 \text{ ft.-lb.}$$

From equation (11) $$W=36{,}000\ (1-\tfrac{1}{2})$$
$$=18{,}000 \text{ ft.-lb.}$$

EXAMPLES ON CHAPTER II

1. Find the volume of 3 lb. of air when at a pressure of 70 lb. per sq. in. abs. and at a temperature of 75° F. [Take $C_p=183\cdot4$ ft.-lb. and $C_v=130\cdot2$ft.-lb.]

2. If 1 lb. of air at 30° F. has its volume doubled at constant atmospheric pressure, what is its final temperature? How much external work is done during the expansion and how much heat must be supplied during the expansion? [$C_p=0\cdot2375$ B.Th.U.]

3. A cylinder contains 0·5 cu. ft. of gas at 15 lb. per sq. in. abs. Find the work expended in compressing it to a pressure of 90 lb. per sq. in. abs., the law of compression being $pv^{1\cdot35}=$constant.

4. The area of an engine piston is 100 sq. in. If the length of cylinder occupied by gas is 18 in. when the pressure is 120 lb. per sq. in. abs., find the work done by the gas in driving the piston through a distance of 2 ft. Take the law of expansion as $pv^{1\cdot5}=$constant.

5. Find the work done by the gas in Question 4, if the gas is kept at constant temperature during the expansion.

6. In a gas engine cylinder 5 cu. ft. of gas and air at 14·7 lb. per sq. in. abs. are compressed into a clearance space of 1 cu. ft. If the compression is adiabatic, (*a*) what is the pressure at the end of the compression stroke? and (*b*) how many foot-pounds of work must be expended in the compression of the charge? [$\gamma=1\cdot4$.]

7. If 0·1 lb. of gas occupying 0·5 cu. ft. is expanded in a cylinder at a constant pressure of 150 lb. per sq. in. abs. until its volume is 1 cu. ft., and is then expanded adiabatically to 5 cu. ft., find the temperature of the gas, (*a*) at the end of the constant pressure stage, (*b*) at the end of the adiabatic expansion, and calculate the heat expended and the work done during each portion of the process. [Take $C_p=198$ ft.-lb. and $C_v=144$ ft.-lb.]

8. The temperature of the mixture of gas and air in a gas engine at the end of the admission stroke is 90° F. and the pressure 15 lb. per sq. in. abs. The clearance volume is 4·6 cu. ft., and the total volume of clearance plus piston displacement is 12 cu. ft. Assuming adiabatic compression $pv^{1\cdot4}=$constant, determine the temperature at the end of the compression stroke.

If the pressure after ignition is 240 lb. per sq. in., find the temperature in the cylinder.

9. In a certain oil engine the piston displacement is 0·395 cu. ft., the volume of the clearance space 0·210 cu. ft., and the pressure of the charge at the instant compression begins is 13 lb. per sq. in. abs. Find the compression pressure and the temperature reached at the end of the compression stroke if the temperature of the charge at the beginning of compression is 260° F. Assume the law of compression to be $pv^{1\cdot39}=$constant.

10. If 20 cu. ft. of dry air are compressed adiabatically from 15 lb. per sq. in. bs. and 60° F. to 225 lb. per sq. in. abs., find the temperature after the compression and the work expended. [Take $\gamma=1{\cdot}4$.]

11. If 13 cu. ft. of air at 60° F. and 200 lb. per sq. in. abs. are expanded to a pressure of 25 lb. per sq. in. absolute, calculate the final volume and the work done during the expansion (*a*) if the expansion is isothermal, (*b*) if the expansion is adiabatic.

12. The law of the expansion curve of a gas engine indicator diagram is found to be $pv^{1{\cdot}57}=$constant. Assuming $\frac{C_p}{C_v}=1{\cdot}37$ find the rate of heat reception $\frac{dH}{dv}$. If the law of the compression curve is $pv^{1{\cdot}25}=$constant, what is the rate of heat reception during compression? If the piston sweeps at 600 cu. ft. per minute when the pressure on the expansion curve is 150 lb. per sq. in. abs., what is the rate of heat reception per second at this instant?

13. Air at 15 lb. per sq. in. abs. is drawn into a cylinder and compressed adiabatically to 135 lb. per sq. in. abs., and is then expelled at this pressure into a receiver; its original temperature was 50° F. In the receiver the compressed air cools to 50° F., and in order to maintain a uniform pressure in the receiver an equal weight of compressed air is drawn off constantly and expanded isothermally down to 15 lb. per sq. in. abs. Calculate (*a*) the work spent per cubic foot of air in the compressor; (*b*) the work done per cubic foot of air in expanding; (*c*) the temperature of the air as it enters the receiver.

14. If 1 lb. of air occupying 3 cu. ft. at 15,950 lb. per sq. ft., and absolute temperature 900° F., expands at constant temperature to a volume of 12 cu. ft., find its pressure after expansion, and the heat taken in.

15. If 42·46 cu. ft. of air at 676 lb. per sq. ft. pressure and absolute temperature 539° F., be compressed isothermally to 10·62 cu. ft., what is its pressure after compression, the work done on it, and the heat removed?

Chapter III

THEORETICAL HEAT ENGINE CYCLES

*GENERAL STATEMENT

The object of a heat engine is to convert heat energy into mechanical energy. For this purpose a " working fluid " at a high temperature and pressure is expanded, generally behind a moving piston in a cylinder, during which process work is done on the flywheel or other parts of the plant at the expense of the energy of the fluid.

As will be seen later, only a portion of the energy of the working fluid can be converted into mechanical energy, that which remains unconverted, a residue of the process, having to be discharged to some other body (normally to the air or to cooling or " condensing " water). During this process the working fluid either contracts in volume or suffers a fall of pressure.

During the final operations of the cycle the fluid is restored to its original conditions of pressure, volume, and temperature at the expense of the energy already stored in the flywheel, and by the addition of heat. The cycle is then said to be " closed."

In most practical engines the working fluid undergoes some of the necessary processes during a temporary withdrawal from the cylinder, as is the case with steam engines ; in others, i.e. internal combustion engines, it is more convenient to substitute a fresh supply of working fluid at some point in each cycle rather than re-condition the old. Such variations, however, are made for convenience and do not radically affect the main problem. In the steam turbine the energy liberated during expansion is expended in creating a high velocity of the steam itself, the kinetic energy of the steam being subsequently transferred to the turbine rotor and shaft. The replacement of the " moving piston " by " moving steam " causes no fundamental alteration to the problem, however.

Three organs are therefore essential : (1) a " hot body " or source of heat, (2) a " cold body " to which unconverted thermal energy can be discharged, and (3) a " working fluid " which returns at the end of each cycle to its original condition.

The principal processes of thermodynamic interest through which the working fluid can pass are as follows :

(1) adiabatic expansion and compression ;

(2) heat reception or rejection at constant temperature (isothermal expansion and compression) ;

(3) heat reception or rejection at constant volume ;
(4) heat reception or rejection at constant pressure.

In practice the processes are usually complex, but the actual conditions will be discussed separately in later chapters.

A complete cycle built up from such processes can be represented by a graph plotted with any two of the factors of state as ordinates. Pressure and volume are most commonly chosen, and a graph using these ordinates is known as the " Theoretical Indicator Card."

*GRAPHICAL REPRESENTATION OF WORK DONE DURING A " CLOSED CYCLE " OF OPERATIONS

Let Fig. 5 represent the pressure-volume diagram, having ordinates representing the pressures of the working fluid, and abscissæ the volumes.

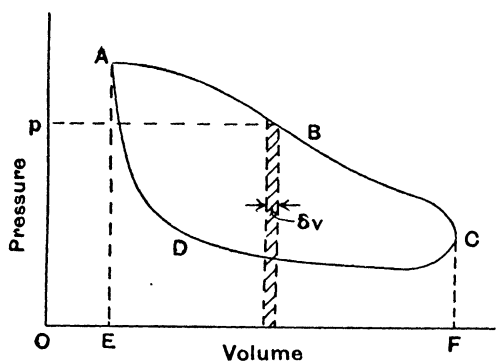


Fig. 5.

The distance OE represents the initial volume of the working fluid, and EA the initial pressure. When the fluid expands along any curve such as ABC to a volume OF and pressure FC, the area under this curve, EABCF, will represent the work done by the fluid during the expansion. If p is measured in pounds per square foot and v is in cubic feet, the work done will be measured in foot-pounds.

Let compression then take place along the curve CDA, till the working fluid is in its original condition of pressure, volume, and temperature. The work done *on* the fluid will now be represented by the area EADCF.

The net amount of work done *by* the fluid is therefore the area ABCD.

For any small change of volume during which the pressure may be considered constant, the work done is equal to $p\delta v$ or $p \times A \times \delta l$, where A is the area of the piston and δl the small distance travelled by it. It should be noted that if p is in pounds per square foot and A is in square feet, the total pressure on the piston is $p \times A$ lb., and is the same as if p had been in pounds per square inch and A in square inches. For the

work done to be measured in foot-pounds l must be in feet. The net work, represented by area ABCD, is imparted to the piston during the outward, or working, stroke only. The *mean effective pressure*, i.e. m.e.p., of the working fluid on the piston is then given by P, where P × area of piston × length of stroke = area ABCD

$$\text{or } P = \frac{\text{area of } p.v. \text{ diagram in foot-pounds}}{\text{area of piston in square feet} \times \text{length of stroke in feet}}$$

For practical purposes P is usually expressed in pounds per square inch.

EXAMPLE.—The area of the indicator diagram for a single acting engine is 2 sq. in. The scale of pressure is 100 lb. per sq. in. per in. and the length of the diagram is 2·5 in. The engine cylinder is 6 in. diameter and 12 in. stroke. Find the mean effective pressure and the indicated horsepower if the engine runs at 500 r.p.m.

The area of the piston is 0·1962 sq. ft. and the stroke is 1 ft., so that 2·5 in. on the diagram along the axis of V represents 0·1962 cu. ft., and as the vertical axis represents 100 × 144 lb. per sq. ft. per inch, the area given represents $2 \times 100 \times 144 \times \frac{0{\cdot}1962}{2{\cdot}5}$ ft.-lb. The mean effective pressure is then $\text{M.E.P.} = \frac{200 \times 144 \times \frac{0{\cdot}1962}{2{\cdot}5}}{0{\cdot}1962 \times 1}$ lb. per sq. ft.

It will be seen that the area appears both in the numerator and the denominator, leaving $\text{M.E.P.} = \frac{200 \times 144}{2{\cdot}5}$ lb. per sq. ft.

or
$$\frac{200}{2{\cdot}5} = 80 \text{ lb. per sq. in.} \quad . \quad . \quad . \quad . \quad . \quad (1)$$

$$\text{Work done per minute} = 80 \times 144 \times 0{\cdot}1962 \times 1 \times 500 \text{ ft.-lb.}$$

$$\text{and I.H.P.} = \frac{80 \times 144 \times 0{\cdot}1962 \times 500}{33{,}000} = 34{\cdot}2$$

It will be seen that the M.E.P. can be obtained in pounds per square inch as follows :

M.E.P. = area of diagram in square inches × (the pressure equivalent to 1 in. in height of the diagram in pounds per square inch) × (the piston travel in feet corresponding to 1 in. in length of the diagram).

*THE THREE PRINCIPAL CYCLES

The principal cycles to be considered differ in the conditions under which heat is received and rejected by the working fluid, and are named after these conditions, i.e.—

The constant temperature, or Carnot cycle, so called after Carnot, who conceived it in 1824.

The constant volume cycle.
The constant pressure cycle.

Other cycles are considered in Chapters VII and XIV.

The Carnot or Constant Temperature Cycle.—Fig. 6 shows the pressure-volume diagram of an engine working on this cycle. Starting at point *a*, the gas expands isothermally from volume v_a to volume v_b, while heat is supplied to keep the temperature constant at T_1. The supply of heat is then cut off, and the gas expands adiabatically to volume v_c, the temperature falling to T_2. On the return stroke of the piston the gas is compressed isothermally from volume v_c to volume v_d, heat being removed to keep the temperature constant at T_2. The cycle is completed by compressing adiabatically from volume v_d and temperature T_2 to

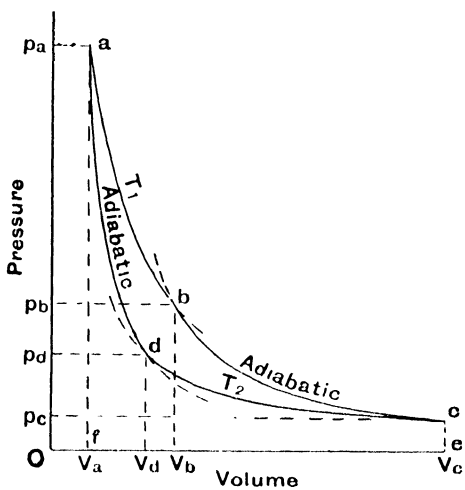


Fig. 6.

volume v_a and temperature T_1. A closed cycle is thus obtained, the pressure, volume, and temperature of the gas being the same at the end as at the beginning of the cycle.

To find the point at which Adiabatic Compression must be commenced.—The point *d* must be so chosen that an adiabatic drawn through it will pass through the starting-point *a*.

During the adiabatic expansion stage from *b* to *c*, we have

$$T_1/T_2=(v_c/v_b)^{\gamma-1}$$

Also during the adiabatic compression stage from *d* to *a*, we have

$$T_1/T_2=(v_d/v_a)^{\gamma-1}$$

Hence $$(v_c/v_b)^{\gamma-1}=(v_d/v_a)^{\gamma-1}$$

and therefore $$v_c/v_d=v_b/v_a$$

or the ratio of isothermal compression must be the same as the ratio of isothermal expansion.

Efficiency of the Cycle.—Let r denote the ratio of isothermal expansion and compression, then we have :

during the stage *ab* Heat taken in $= RT_1 \log_\epsilon r$ = work done *by* the gas
,, ,, *bc* No heat taken in or rejected
,, ,, *cd* Heat rejected $= RT_2 \log_\epsilon r$ = work done *on* the gas
,, ,, *da* No heat taken in or rejected

$$\text{Now efficiency} = \frac{\text{heat converted into mechanical work}}{\text{heat supplied}}$$

$$= \frac{RT_1 \log_\epsilon r - RT_2 \log_\epsilon r}{RT_1 \log_\epsilon r}$$

$$= \frac{T_1 - T_2}{T_1}, \text{ i.e. } \frac{\text{Range of Temperature}}{\text{Initial Temperature}}$$

EXAMPLE.—If a perfect air engine works on the Carnot cycle between temperature limits of 600° F. and 60° F., calculate its efficiency, and the ratio of the adiabatic expansion.

Here $T_1 = 600 + 460 = 1060°$ F. abs.

$T_2 = 60 + 460 = 520°$ F. abs.

$$\therefore \text{Efficiency} = \frac{540}{1060} = 0{\cdot}509$$

Now for adiabatic expansion

$$\frac{T_2}{T_1} = \left(\frac{v_1}{v_2}\right)^{\gamma - 1}$$

$$\therefore \frac{520}{1060} = \left(\frac{1}{r}\right)^{0{\cdot}4}$$

$$-0{\cdot}4 \log r = \log 520 - \log 1060$$

$$= 2{\cdot}7160 - 3{\cdot}0253$$

$$= -0{\cdot}3093$$

$$\therefore \log r = \frac{0{\cdot}3093}{0{\cdot}4} = 0{\cdot}7732 = \log 5{\cdot}93$$

therefore ratio of adiabatic expansion $= 5{\cdot}93$.

EXAMPLE.—A perfect engine working on the Carnot cycle is to develop a constant output, but the maximum temperature is to be varied. Show how the efficiency of the engine varies with the maximum temperature.

As the output of the engine is to remain constant, $T_1 - T_2$ must also

be constant. If $T_1 - T_2$ is, say, 500° C., then the efficiency will vary with the maximum temperature as follows :

$T_1 = 3000°$ C. $= 3273°$ C. abs. $\eta = 15{\cdot}3$ per cent.
$T_1 = 2000°$ C. $= 2273°$ C. abs. $\eta = 22{\cdot}1$ per cent.
$T_1 = 1000°$ C. $= 1273°$ C. abs. $\eta = 39{\cdot}3$ per cent.
$T_1 = 500°$ C. $= 773°$ C. abs. $\eta = 64{\cdot}7$ per cent.

It should be noted that in an actual engine the efficiency will tend to approach the above values more closely as the maximum temperature falls, owing to the corresponding reduction of the amount of heat lost by radiation and conduction. In the first case cited above heat will be discharged from the engine at the very high temperature of 2500° C., in the last the discharge will be at 0° C., or slightly below the normal minimum value (about 15° C.) for practical working.

EXAMPLE.—In the Carnot cycle shown in Fig. 10 the maximum cylinder volume is 25 cu. ft., the volume before adiabatic compression is 10 cu. ft., the lower temperature 17° C., and the maximum pressure 500 lb. per sq. in. abs. Check the maximum temperature, the mean effective pressure, and the efficiency.

The method of calculation is as follows. The pressure at d is given by

$$\frac{p_d}{p_c} = \frac{v_c}{v_d} = 2{\cdot}5.$$

The pressure at a determines the maximum temperature, i.e.—

$$\frac{T_a}{T_d} = \left(\frac{p_a}{p_d}\right)^{n-1/n} \quad \text{and} \quad T_a = 806° \text{ C. abs.}$$

The volume at a is given by

$$\frac{v_a}{v_d} = \left(\frac{p_d}{p_a}\right)^{1/n} = \text{ratio of adiabatic compression} = 1/r$$

$$\text{and the work done} = R' \log_\epsilon r(T_a - T_d)$$

The value of R' in heat units is given by $\frac{53{\cdot}18}{778} \times$ weight of air present, the latter term being obtainable from the fact that at 17° C. and 15 lb. per sq. in. pressure the volume of the air is 25 cu. ft., and that at N.T.P. 1 cu. ft. weighs 0·0807 lb. The mean effective pressure is then

$$P_m = \frac{\text{work done}}{25} \text{ lb. per sq. in.}$$

and the efficiency is given by the expression

$$\eta = \frac{T_1 - T_2}{T_1} = \frac{533 - 290}{533} = 64 \text{ per cent.}$$

The Constant Temperature Steam Cycle.—A cycle formed by two isothermals and two adiabatics can be applied also to the simple steam engine.

The operations are : (1) evaporation of water in the cylinder at constant temperature and hence at constant pressure ; (2) adiabatic expansion of

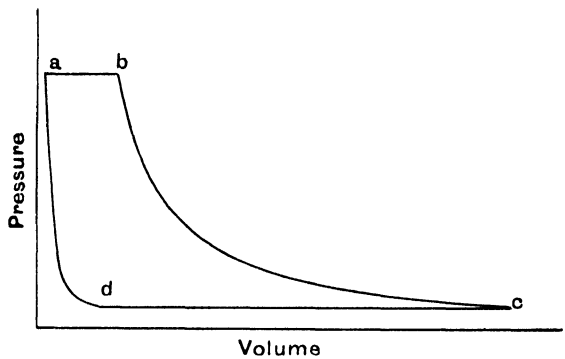


Fig. 7.

steam ; (3) partial condensation of steam in the cylinder at constant temperature and hence at constant pressure ; (4) adiabatic compression of the steam and water mixture, resulting in the completion of condensation.

The cycle is shown in Fig. 7.

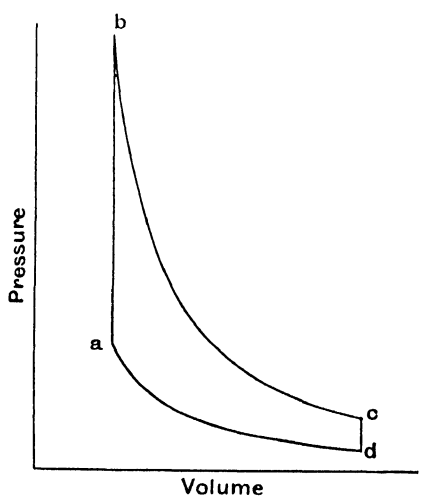


Fig. 8.

The Constant Volume Cycle.—Fig. 8 shows the pressure volume card for this cycle. Starting at point *a* the working fluid receives heat

and increases in temperature and pressure while the volume remains constant. At b the supply of heat is cut off and the fluid then expands adiabatically to c. From c to d heat is removed from the fluid, which falls in temperature and pressure while the volume is constant, and at d adiabatic compression is started which restores the fluid to its original condition at a.

If the temperatures at a, b, c, and d are T_a, T_b, T_c, and T_d,

Heat supplied at constant volume $= C_v(T_b - T_a)$

Heat rejected at constant volume $= C_v(T_c - T_d)$

Heat converted into work $= C_v(T_b - T_a) - C_v(T_c - T_d)$

$$\text{Efficiency} = \frac{C_v(T_b - T_a) - C_v(T_c - T_d)}{C_v(T_b - T_a)}$$

$$= 1 - \frac{T_c - T_d}{T_b - T_a}.$$

Now, since expansion and compression are adiabatic and over the same range of volume

$$\frac{T_c}{T_b} = \left(\frac{v_b}{v_c}\right)^{\gamma-1} = \frac{T_d}{T_a}$$

$$\frac{T_c}{T_b} = \frac{T_d}{T_a} \quad \text{or} \quad T_d = T_a{}^{T_c/T_b}$$

$$\text{and efficiency} = 1 - \frac{T_c - T_a{}^{T_c/T_b}}{T_b - T_a}$$

$$= 1 - \frac{T_c T_b - T_c T_a}{T_b(T_b - T_a)}$$

$$= 1 - \frac{T_c}{T_b} = 1 - \frac{T_d}{T_a} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

But

$$\frac{T_c}{T_b} = \frac{T_d}{T_a} = \left(\frac{v_a}{v_d}\right)^{\gamma-1} = \left(\frac{v_b}{v_c}\right)^{\gamma-1} = \left(\frac{1}{r}\right)^{\gamma-1} \quad . \quad . \quad . \quad (2)$$

and

$$\eta = 1 - \left(\frac{1}{r}\right)^{\gamma-1}$$

where r is the common value of the compression and expansion ratios.

From equation (1) $\eta = 1 - T_c/T_b$. It will be seen, however, that while T_b is the maximum temperature, T_c is inherently greater than the minimum temperature, which occurs at point d. The efficiency is therefore essentially less than that of the constant temperature cycle, for which the efficiency is given by

$$1 - \frac{\text{minimum temperature}}{\text{maximum temperature}}.$$

The effect of increasing the Compression Ratio is shown in Fig. 124, and is discussed in Chapter XIV.

The Constant Pressure Cycle.—Fig. 9 shows the pressure-volume card for this cycle. Starting at point *a* the working fluid expands, at the same time receiving heat to maintain the pressure constant. At *b* the supply of heat is cut off, and the fluid then expands adiabatically to

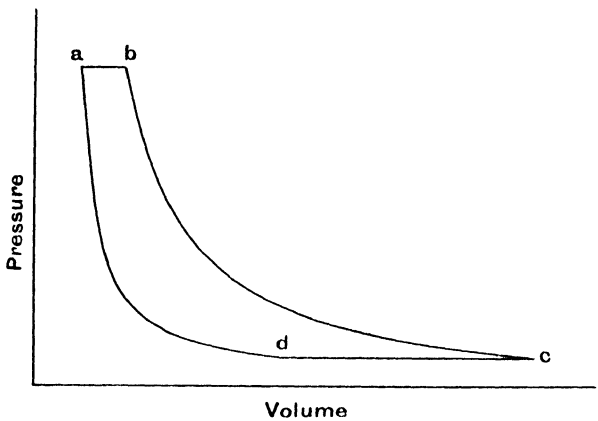


Fig. 9.

c. From *c* to *d* heat is removed from the fluid, which contracts at constant pressure. At *d* adiabatic compression commences and restores the fluid to its original condition.

If the temperatures at *a*, *b*, *c*, *d* are given by T_a, T_b, T_c, and T_d,

Heat supplied at constant pressure $= C_p(T_b - T_a)$

Heat rejected at constant pressure $= C_p(T_c - T_d)$

Heat converted into work $= C_p(T_b - T_a) - C_p(T_c - T_d)$

$$\text{and efficiency} = \frac{C_p(T_b - T_a) - C_p(T_c - T_d)}{C_p(T_b - T_a)}$$

$$= 1 - \frac{T_c - T_d}{T_b - T_a}.$$

The expansion and compression curves being adiabatic and over the same range of pressure,

$$\frac{T_c}{T_b} = \left(\frac{p}{p_b}\right)^{\gamma - 1/\gamma} = \frac{T_d}{T_a} = \left(\frac{v_b}{v_c}\right)^{\gamma - 1} = \left(\frac{1}{r}\right)^{\gamma - 1}$$

and the equation for efficiency reduces, as in the case of the constant volume cycle, to $1-(1/r)^{\gamma-1}$, where r is the common ratio of expansion and compression.

*THE RATIOS OF COMPRESSION AND EXPANSION

Though in many engines the ratios of compression and total expansion [1] are equal, as must be the case when using the normal arrangement of crank and connecting rod, this is not necessarily so, Atkinson's engine and Humphrey's pump being the best known exceptions.

In spite of the emphasis commonly laid on the value of the " Ratio of Compression," it must be noted that the efficiency of an engine is fundamentally dependent on the " ratio of expansion," i.e. on the change of temperature which can be brought about by adiabatic expansion. Where the process of compression is used in an engine it must be considered as akin to the use of a condenser with a steam engine, i.e. as a means of increasing the " ratio of expansion " and not as a device desirable in itself.

*COMPARISON OF CYCLES

In considering the practical values of cycles of operation, it is not sufficient to compare only the theoretical efficiencies. For an engine of given size and speed the output, neglecting friction, etc., will be proportional to the mean effective pressure, while the strength and weight of the parts will have to be proportional to the maximum cylinder pressure.

The following diagrams are of interest as providing a comparison of the above three cycles on a pressure basis. The maximum pressure in each case is 500 lb. per sq. in., and the compression ratios in the two latter are such as are used in practice.

It is at once obvious that the Constant Temperature cycle is impracticable, the ratio of maximum pressure above atmospheric to mean effective pressure being 81, necessitating an extremely large and heavy engine for a given output; while for the other cycles it is 4·6 and 8·9 respectively.

The constant pressure cycle chosen shows to advantage with regard to efficiency when compared with the constant-volume cycle on account of the higher compression ratio normally employed (see p. 269).

It will be noted that as the compression ratio of the constant-volume cycle is increased, the gain in pressure *ab* due to the reception of heat constantly decreases owing to the limitation imposed on the maximum pressure. As the compression ratio approaches 12·24 the efficiency will rise towards the figure of 64 per cent. shown by the constant pressure cycle, but the heat received, and consequently the output of the engine, will tend to zero.

As will be seen later (p. 268) the " toe " *c'cd* of the " constant pressure "

[1] Total expansion is here used to differentiate between the changes of volume due to the total outward movement of the piston and that portion occupied by adiabatic expansion, see p. 268.

diagram is omitted in practice, allowing a reduction of cylinder volume of nearly 60 per cent., and an increase of the mean effective pressure to 117 lb. per sq. in. The efficiency is decreased 8 per cent. thereby, as, though the " ratio of compression " remains constant, the " ratio of expansion " is reduced from 12·25 : 1 to 5 : 1 (see p. 268).

EXAMPLE.—An engine using pure dry air as a working fluid, operates on the Carnot cycle. The clearance volume is 1/13 of the swept volume, and when the piston is at the outer dead centre the pressure and temperature of the cylinder contents are 15 lb. per sq. in. and 150° F.

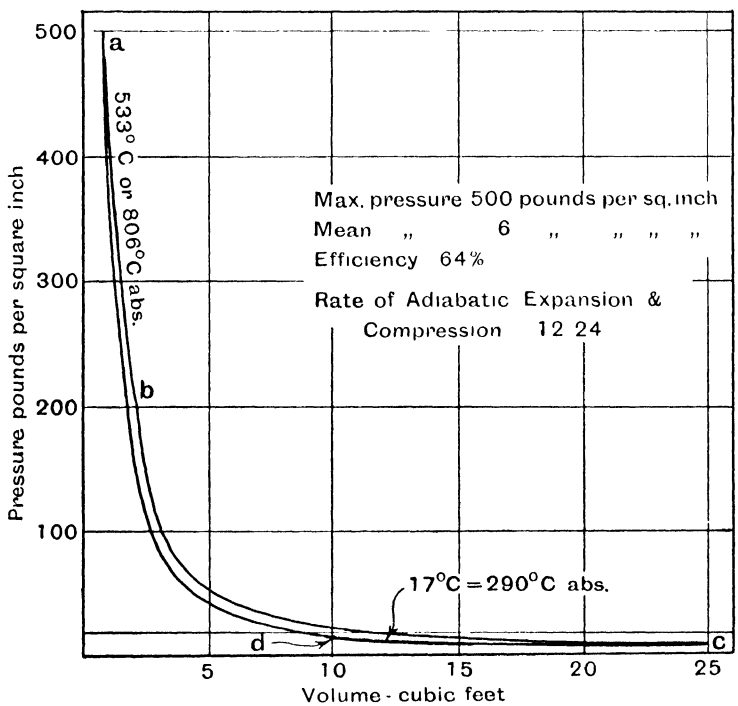


Fig. 10.—CONSTANT TEMPERATURE OR " CARNOT " CYCLE.

Determine the pressure, temperature, and percentage stroke at the principal points on the cycle in order that the highest possible mean effective pressure may be developed. What will then be the efficiency and the mean effective pressure ?

The problem amounts to dividing the stroke volume between the isothermal and adiabatic operations, to give the highest M.E.P., the limiting cases being (1) when the isothermal and (2) when the adiabatic operations occupy the whole stroke volume. In both cases, the M.E.P. will be zero.

In the second case, where the adiabatic through c passes also through a, we have $pv^{1\cdot4}=15\times(14)^{1\cdot4}\times144=600\times144$, taking the clearance volume as 1 cu. ft.

And at a where $v_a=1$, and $p_a=600\times144$ lb. per sq. ft.

$$T_a=T_c\frac{p_av_a}{p_cv_c}=\frac{600\times144\times1}{15\times144\times14}\times610=1743^\circ \text{ abs.}$$

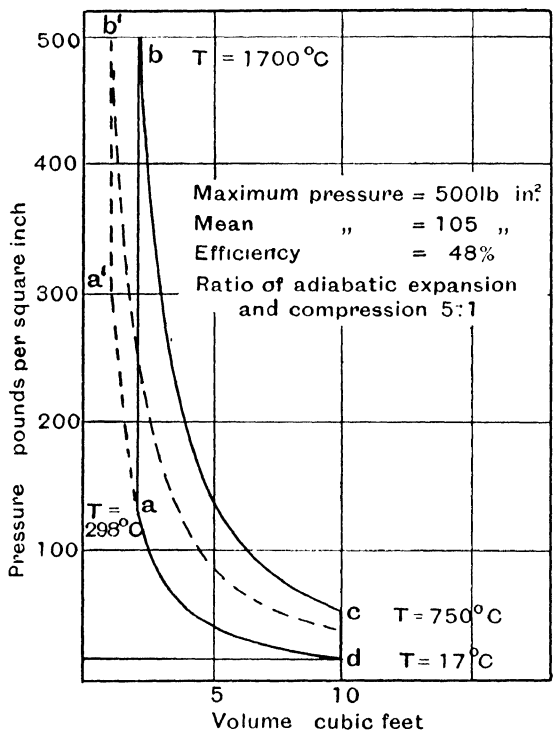


Fig. 11.—Constant Volume Cycle.

The upper limits of temperature and pressure are thus 1743° F. and 600 lb. per sq. in., and the efficiency will be $\frac{1743-(150+460)}{1743}=65$ per cent., though the output approaches zero.

In the first case, where the isothermal through c passes through a, as $p_c=15$ lb. per sq. in. and $v_c=14$ cu. ft., so that after isothermal compression to $v=1$,

$$p=210 \text{ lb. per sq. in.} \quad . \quad . \quad . \quad . \quad . \quad (1)$$

The temperatures of the cycle will be constant, giving an efficiency of zero and no output.

Between these limits there will be some position for a giving a maximum M.E.P.

The work done $= R(T_1 - T_2) \log_\epsilon \frac{v_c}{v_d}$

or $$W = RT_2\left\{\left(\frac{v_d}{v_a}\right)^{n-1} - 1\right\}\log_\epsilon \frac{v_c}{v_d}$$

$$= RT_2\{v_d^{n-1} - 1\} \log_\epsilon \frac{14}{v_d}, \quad \text{as } v_a = 1$$

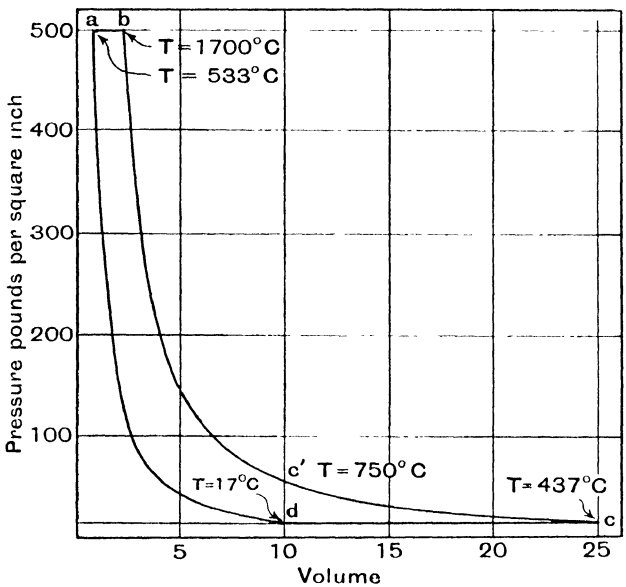


Fig. 12.—Constant Pressure Cycle.

Maximum pressure = 500 lb.
Mean ,, = 56 lb.
Efficiency = 64 per cent.

Ratio of adiabatic expansion and compression = 12 : 24.
(For cycle $abc'd$ mean pressure = 117 lb., efficiency, 56 per cent.)

and $$\frac{dW}{d(v_d)} = RT_2\{(n-1)v_d^{n-2}\} \log_\epsilon \frac{14}{v_d} + RT_2\{v_d^{n-1} - 1\} \times \frac{v_d}{14} \times \frac{14(-1)}{v_d^2}.$$

For W to be a maximum this must be equated to zero, or

$$0 = (n-1)v_d^{n-2} \log_\epsilon \frac{14}{v_d} - (v_d^{n-1} - 1)\frac{1}{v_d}$$

$$= 0{\cdot}4v_d^{-0{\cdot}6} \log_\epsilon \frac{14}{v_d} - (v_d^{0{\cdot}4} - 1)\frac{1}{v_d}.$$

Multiplying by v_d

$$0=0{\cdot}4v_d^{0{\cdot}4}\log_\epsilon\frac{14}{v_d}-v_d^{0{\cdot}4}+1.$$

By trial it will be found that the solution is given by $v_d=4{\cdot}5$.

Now d lies on an isothermal through C, for which $pv=210$ from (1), or

$$p_d=\frac{210}{4{\cdot}5}=46{\cdot}7 \text{ lb. per sq. in.}$$

The adiabatic through d to a will then give $pv^{1{\cdot}4}=46{\cdot}7\times(4{\cdot}5)^{1{\cdot}4}=382$, and at a, when $v_a=1$, $p_a=382$.

For the point b we have

$$\frac{v_b}{v_a}=\frac{v_c}{v_d} \quad \text{or} \quad v_b=\frac{1\times14}{4{\cdot}5}=3{\cdot}08$$

and

$$p_b=\frac{382}{3{\cdot}08}=124 \text{ lb. per sq. in.}$$

Also

$$\frac{T_1}{T_2}=\left(\frac{v_d}{v_a}\right)^{n-1} \quad \text{or} \quad \frac{T_1}{610}=(4{\cdot}5)^{0{\cdot}4}=1{\cdot}82$$

and

$$T_1=1110^\circ \text{ F. abs.}$$

$$\text{Efficiency}=\frac{1110-610}{1110}=45 \text{ per cent.}$$

$$\text{Work done per pound}=R(T_1-T_2)\log_\epsilon r$$

$$=53{\cdot}2(1110-610)\log_\epsilon\frac{14}{4{\cdot}5}$$

$$=30{,}150 \text{ ft.-lb.}$$

The volume per pound at 15 lb. per sq. in. and 150° F.

$$=\frac{53{\cdot}2\times610}{15\times144}=15{\cdot}01 \text{ cu. ft.}$$

$$\text{The M.E.P.} \quad =\frac{30{,}150}{\frac{13}{14}\times15{\cdot}01\times144}=\frac{30{,}150}{2008}=15 \text{ lb. per sq. in.}$$

giving a ratio of maximum to mean effective pressure of $\frac{382}{15}=25{\cdot}5:1$.

*THEORETICAL IMPORTANCE OF THE "CONSTANT TEMPERATURE" OR "CARNOT" CYCLE

Though this cycle shows an efficiency of only about 65 per cent., it will be shown in a later paragraph that this is the most efficient cycle which can be conceived.

The fact that of 100 heat units supplied only 65 are converted into mechanical energy does not contravene Joule's law. Every foot-pound of mechanical energy developed by the engine causes the complete disappearance of 1/778 B.Th.U. of heat energy. The engine is, however, inherently unable to utilise more than a certain proportion of the heat supplied to it, the remainder being neither converted nor destroyed, but discharged, still unchanged, as heat energy. An illustration, due to Reynolds, will be found of value.

A series of weights, W (Fig. 13), may be raised from ground level *a* to the level *b* of a shelf at height *h*, by either the direct or indirect application of suitable forces. In the first case a light chain C may be attached to

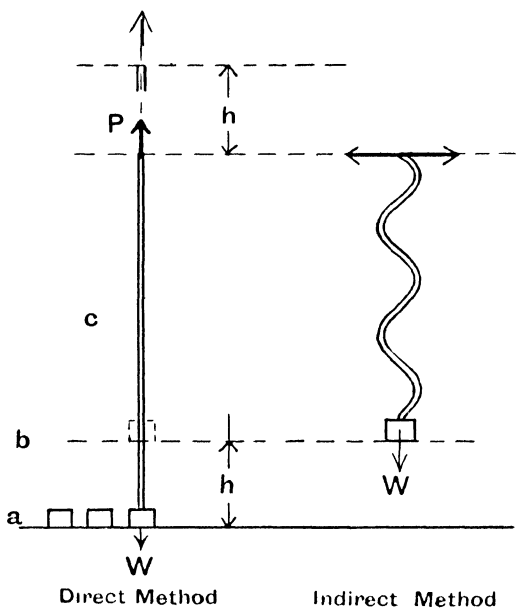


Fig. 13.

one of the weights and a force P, equal to W, applied as shown. By moving the upper end of the chain through a height equal to *h* the weight will be raised the requisite distance, and the energy supplied will be P . *h*, and is theoretically equal to the work done, i.e., W . *h*. The weight is therefore raised with an efficiency of 100 per cent., and the others may be subsequently so raised by the simple expedient of lowering the chain after each raised weight has been detached at the upper level. (If the chain is heavy its weight can be balanced by a simple counterpoise, and the theoretical efficiency will not be affected.)

In the indirect method, the upper end of a vertical heavy chain attached to the first weight can be moved rapidly backwards and forwards, or

with a circular motion, in a horizontal plane. A sinuous motion of the chain will be produced and the weight will be raised. The energy supplied, however, must be greater than $W \cdot h$, as kinetic energy has been imparted to the links of the chain. After the first weight has been raised and detached it will be necessary to disperse the kinetic energy remaining in the links, a process naturally effected by its conversion into frictional heat between the links, before the lower end can again be attached to a weight on the ground, thus completing a " cycle."

The analogy is clear when it is remembered that in the case of the heat engine energy is supplied indirectly to the piston through the medium of indiscriminately moving and vibrating molecules (the links of the chain). The heat supplied is transferred to the molecules as kinetic energy of translation, and these in turn part with some of their energy to the piston. In order to complete the cycle and restore the working fluid to its original condition, the unconverted energy must be removed from the system, by its discharge as heat to some suitable " cold body."

Only if the working fluid could be expanded adiabatically to the zero of absolute temperature could all the heat supplied be converted into work and the efficiency of the engine become 100 per cent. This being utterly impracticable,[1] heat, as such, must be discharged as a waste product at a temperature T_2 to some sink of heat. The amount of energy so rejected, i.e. the " incurred waste," will obviously be dependent on the value of T_2, the " minimum available temperature," and the importance of keeping T_2 at as low a value as possible, will be seen. This matter will be referred to on pp. 54 and 55.

*REVERSIBLE PROCESSES AND CYCLES

If two bodies in contact are originally at the same temperature, a small rise of temperature of one will cause a flow of heat to the other, while a small fall of temperature will cause flow in the reverse direction. If the necessary variation of temperature can be considered infinitely small the process is said to be reversible. Similarly, under ideal conditions, the operations by which heat and work are mutually convertible are reversible, even though it may not be possible to convert the whole of the heat supply into work. Processes involving friction, loss of heat by radiation, etc., are not reversible.

The cycles so far considered are built up of such reversible processes,

[1] A weight possesses an absolute potential energy due to its attraction towards the centre of the earth, but the only portion of this which can be utilised is that liberated while falling from a height on to the earth's surface. More could be liberated if a well were dug into which the weight could be allowed to fall, but the proposition is obviously unsound. It would be similarly possible to produce a sink of heat below T_2 by the employment of a refrigerator, but the latter would have to restore the waste heat to T_2 before it could be discharged, in the same way that soil from the well could only be discharged at ground-level.

and are therefore also reversible, i.e. the Carnot cycle " reversed " would take in heat at T_2, and, after absorbing mechanical energy, would deliver an increased quantity of heat to the hot body at T_1. Referring to Fig. 6, the gas would expand adiabatically from *a* to *d*, and isothermally from *d* to *c*, doing a total amount of work *adcef*. It would then be compressed from *c* to *b*, and from *b* to *a*, absorbing work *cbafe*. The net work absorbed by the reversed engine would be exactly equal to the net work produced if the engine were running " direct," and the heat interchanges would be exactly equal in magnitude, but opposite with regard to the direction of flow.

*CARNOT'S PRINCIPLE

The principle that no cycle can be more efficient than the Carnot can be shown as follows.

If two engines working on this cycle, and of equal size, are coupled together mechanically so that one tends to drive the other in a direction opposite to its normal direction of rotation, then it will be found that the power developed by the first is exactly the amount absorbed by the second.[1] The first will deliver, as waste, to the cold body, a quantity of heat exactly equal to that received by the second during isothermal expansion, and the heat absorbed from the hot body by the first will be exactly replaced by the second. Under ideal conditions the engines would continue to rotate, without producing any net effects.

If now the first engine could be made more efficient,[2] the two engines would continue to produce and absorb the same mechanical energy, and the heat interchanges with the cold body would still balance, but while

[1] The heat interchanges and work done will be as follows :

Direct-acting engine.			
Heat received from body at T_1 $=RT_1 \log_\epsilon r$ . .	(1)	Body at T_1 loses heat $RT_1 \log_\epsilon r$.	(1)
Heat rejected to body at T_2 $=RT_2 \log_\epsilon r$ . .	(2)	and also gains heat $RT_1 \log_\epsilon r$.	(5)
Net work done by engine $=R \log_\epsilon r(T_1-T_2)$.	(3)		
Reversed engine.			
Heat received from body at T_2 $=RT_2 \log_\epsilon r$ . .	(4)	Body at T_2 gains heat $RT_2 \log_\epsilon r$.	(2)
Heat rejected to body at T_1 $=RT_1 \log_\epsilon r$ . .	(5)	and also loses heat $RT_2 \log_\epsilon r$.	(4)
Net work done on engine $=R \log_\epsilon r(T_1-T_2)$	(6)	Net work done=zero	(3 and 6)

[2] With a direct acting engine of increased efficiency the work shown in item (3) could be obtained with a smaller consumption of heat than $RT_1 \log_\epsilon r$, so that item (1) would then read, say, 90 per cent. of $RT_1 \log_\epsilon r$.

The net work done would still be zero, and the heats received and rejected by the body at T_2 would be equal, but the body at T_1 would receive $RT_1 \log_\epsilon r$ heat units and part with only 90 per cent. of $RT_1 \log_\epsilon r$, or would have a net gain, per cycle, of 10 per cent. of $RT_1 \log_\epsilon r$.

the heat delivered to the hot body by the second would be $RT_1 \log_\epsilon r$, the energy absorbed from that body and used to drive the first would be less than this. The only net effect would be the continuous accumulation of heat in the hot body at temperature T_1 without the intervention of any external energy. As this is contrary to the Second Law of Thermodynamics, it must be assumed that no engine of this enhanced efficiency is possible. Similarly, any engine with a greater efficiency when reversed than the Carnot is equally impossible.

(Note : The Carnot cycle is not the most perfect cycle, as the Stirling and other regenerative cycles have the same efficiency, assuming perfect regeneration. No cycle has a higher efficiency, however.)

*CONDITIONS FOR MAXIMUM THERMAL EFFICIENCY

These may be summarised as follows :

(1) The engine must take in heat only at the highest possible temperature and reject the residue of heat only at the lowest.

(Although by using superheated steam in a steam engine the upper limit of temperature is raised, yet as the steam has taken in the greater part of its heat at the comparatively low temperature of evaporation, the thermodynamic efficiency is small, i.e. on expanding, the steam will rapidly lose the small portion of its energy represented by the superheat, and will fall in temperature correspondingly, the bulk of its energy being only available at the temperature of condensation.)

Though in practice some temperature difference must exist between the hot body and the working fluid, and between the working fluid and the cold body in order to cause heat transference, the differences must be kept as small as possible.

(2) All free or imperfectly resisted expansion, i.e. throttling, must be avoided.

(3) All loss of heat by radiation and conduction must be prevented.

*DEFINITION OF THERMODYNAMIC TEMPERATURE SCALE

Let $A_1B_1C_1D_1$, $A_2B_2C_2D_2$, etc. (Fig. 14), represent a series of isothermal curves T_1, T_2, T_3, etc., such that $T_1-T_2=T_2-T_3=T_3-T_4=\delta T$. Let these be cut by the adiabatic curves $A_1A_2A_3A_4$, $B_1B_2B_3B_4$, etc., which are chosen so that

$$\frac{v_{B_1}}{v_{A_1}}=\frac{v_{C_1}}{v_{B_1}}=\frac{v_{D_1}}{v_{C_1}}=r$$

Each figure such as $A_1B_1B_2A_2$, i.e. a, can be taken as the indicator diagram of an engine working on the Carnot cycle over a range of temperature δT.

For the Carnot diagrams *a*, *b*, and *c*, between the *same* adiabatics, the ratios of isothermal expansion and compression must be equal (see p. 19), i.e.

$$\frac{v_{B_1}}{v_{A_1}}=\frac{v_{B_2}}{v_{A_2}}=\frac{v_{B_3}}{v_{A_3}}, \text{ etc.}=r=\frac{v_{C_1}}{v_{B_1}}=\frac{v_{D_1}}{v_{C_1}}, \text{ etc.}$$

so that r is constant for all of the indicator diagrams shown in the figure.

The work done by each engine is then given by

$$\begin{aligned} W &= RT_1 \log_\epsilon r - RT_2 \log_\epsilon r \\ &= K(T_1 - T_2), \text{ where } K = R \log_\epsilon r \\ &= \text{constant.} \end{aligned}$$

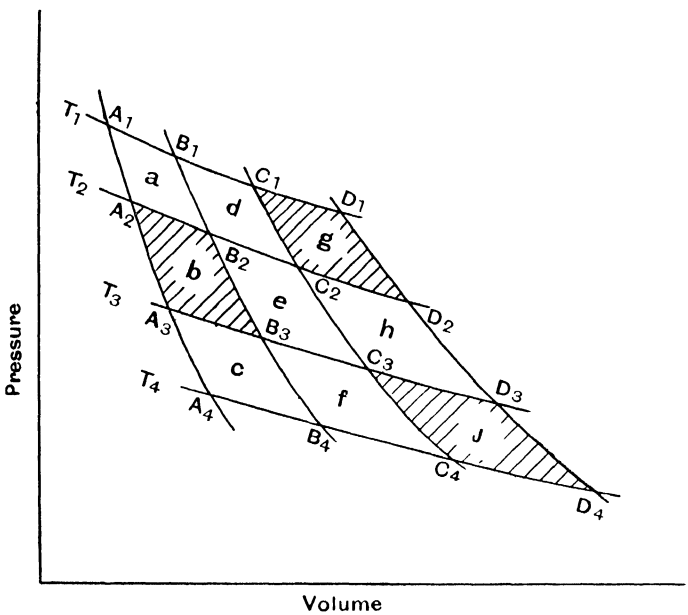


Fig. 14.

Each engine working over the range T_1 to T_2 will reject an amount of heat $RT_2 \log_\epsilon r$, which will be exactly equal in amount to the heat absorbed by one of the engines working over the range T_2 to T_3, and so on.

If, therefore, we conceive a series of perfect heat engines each passing on its waste heat to the next below it on the temperature scale, and all developing the same power, then the temperature ranges over which the engines work must be equal.

The divisions of the temperature scale can thus be defined dynamically, and will correspond with those for the gas thermometer using a perfect gas.

It should be noted that we are here defining equal *temperature differences.* The indicator diagrams *a*, *d*, or *g* will do equally well for the first engine of the series, and *b*, *e*, or *h* for the second engine. The series of engines may therefore be considered as using the indicator cards *g*, *b*, and *j* equally well as the cards *a*, *b*, and *c* or *a*, *e*, and *j*.

EXAMPLES ON CHAPTER III

1. The temperature limits for a Carnot cycle are 780° F. and 50° F. Estimate the efficiency and the ratio of adiabatic expansion.

2. Draw to scale a graph showing the variation of the efficiency of the constant-volume cycle with expansion ratio, and estimate the theoretical efficiencies for ratios of $3\frac{1}{2} : 1$, $4\frac{1}{2} : 1$, $7 : 1$, and $16 : 1$.

3. Plot a graph showing the variation of compression pressure and compression temperature for the constant volume cycle, and for values of the compression ratio between 2 : 1 and 20 : 1, assuming the minimum values of temperature and pressure during the cycle to be 212° F. and 14·7 lb. per sq. in. (abs.).

4. Check the values shown in Figs. 10, 11, and 12.

5. Plot a graph showing the variation of the efficiency of the constant pressure cycle with expansion ratio. Assume a constant compression ratio of 16 : 1, a constant maximum pressure of 500 lb. per sq. in., and minimum cycle temperatures and pressures of 212° F. and 14·7 lb. per sq. in. (abs.).

Chapter IV

FACTORS OF STATE. ENTROPY

*GENERAL STATEMENT

It has been seen in a previous section, dealing with the laws of Boyle and Charles, that p, v, and T, for a given quantity of fluid, are not independent variables, but that the state of the fluid can be completely specified in terms of any two of them. It is therefore possible to represent the state of the fluid upon a graph having two ordinates chosen, as may be most convenient for the problem in hand, from p, v, and T. Combinations of these quantities may also be used as ordinates, provided that only two factors are involved, i.e. the product $p \,.\, v$ plotted against p (the product $p \,.\, v$ plotted against T is equivalent to plotting T against itself). Other quantities dependent on p, v, and T, such as internal and total energy, can also be employed to advantage.

The graph will be equally determinate if the increments of these quantities relative to some arbitrary zero are plotted instead of the absolute values (i.e. temperatures in degrees C. instead of degrees C. absolute).

One of the most useful combinations is $\delta H/T$ plotted against T where δH is an increment of heat received and T is the absolute temperature of reception. (In practice the increment δH of heat content, and not the absolute value, is of interest.) The quantity $\delta H/T$ is expressed as $\delta\phi$, where $\delta\phi$ represents the increment of the quantity ϕ corresponding to the small heat reception δH, and is called the " change of entropy."

The interest of this quantity to engineers lies largely in the fact that during adiabatic operations $\delta H = 0$, so that $\phi = \text{constant}$. If ϕ is chosen as one of the co-ordinates of the state of a fluid, an adiabatic change will be represented by a line parallel to the axis of ϕ.

The following alternative symbols are commonly used :

$$S = \phi \quad Q = H \quad \text{and} \quad \theta = T$$

(It should be noted that though heat and mechanical energy are essentially of the same nature, i.e. heat is molecular kinetic energy, the reception of heat by a gas is not necessarily equivalent to the reception of the same amount of energy from a mechanical source. Though no *heat* is received or lost by the working fluid during an adiabatic operation, changes of p, v, and T all occur, work is done, and the process is reversible. An operation during which no *work* is received or lost by the fluid is essentially one carried out at constant volume, as when a fluid flows from a

container of fixed volume into an initially vacuous space likewise of constant volume, when, for a perfect gas, T=constant. In this case the process is utterly irreversible.)

The value of $\delta\phi$ is independent of the temperature scale used.

*ENTROPY OF THE WORKING FLUID AND HOT AND COLD BODIES

Though the entropy of a working fluid is as much a condition factor as p, v, or T, it is sometimes convenient to relate it to the hot and cold bodies also, i.e. if the working fluid of a perfect engine receives heat from the hot body the former suffers an increase of heat δH_1 and an increase of entropy $\delta H_1/T_1$, while the hot body loses heat δH_1 and also entropy $\delta H_1/T_1$. Similarly the working fluid will lose heat δH_2 to the cold body and the entropy will decrease by the amount $\delta H_2/T_2$. Both these quantities are gained by the cold body.

For the Carnot cycle, which is completely reversible, the heat received from the hot body is $RT_1 \log_\epsilon r$, and the interchange of entropy will be $RT_1 \log_\epsilon r/T_1 = R \log_\epsilon r$. The heat discharged to the cold body is $RT_2 \log_\epsilon r$ and the interchange of entropy will be $RT_2 \log_\epsilon r/T_2 = R \log_\epsilon r$.

In this case both the total energy and the entropy of the system remain constant.

For the same amount of work to be performed by a non-reversible cycle an additional quantity of heat H_3 must be received from the hot body and, by the law of the conservation of energy, an exactly similar amount of heat must be rejected to the cold body. The additional loss of entropy by the hot body will be H_3/T_1, and the corresponding gain by the cold body will be H_3/T_2.

As, however, T_1 is essentially greater than T_2, H_3/T_1 is, of necessity, less than H_3/T_2, so that though the total energy of the system remains constant there is an increase of entropy.

The increase of entropy of a system during any operation is then a measure of the wastage incurred, i.e. a measure of the inefficiency of the process compared with the corresponding ideal or reversible process.

For many purposes it is sufficient to consider only the changes of entropy of the working fluid during a cycle.

*ENTROPY A CONDITION OF STATE

It remains to be shown that the entropy of a body depends solely upon its condition, and is quite independent of the steps by which that condition has been reached.

Consider the working fluid in the Carnot cycle.

$$\text{Heat received} = \delta H_1 = RT_1 \log_\epsilon r$$
$$\text{Heat rejected} = \delta H_2 = RT_2 \log_\epsilon r$$

or $$\frac{\delta H_1}{\delta H_2}=\frac{T_1}{T_2} \quad \text{and} \quad \frac{\delta H_1}{T_1}=\frac{\delta H_2}{T_2}$$

or $$\frac{\delta H_1}{T_1}-\frac{\delta H_2}{T_2}=0$$

(All quantities of heat are essentially positive, the negative sign indicating the rejection as opposed to the positive sign showing the reception of heat.)

Any closed reversible cycle can be built up as a series of Carnot cycles as shown in Fig. 15.

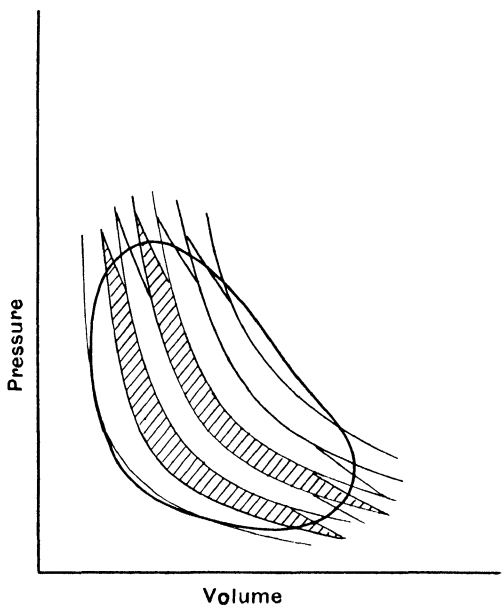


Fig. 15.

For the first of these

$$\delta H_1/T_1-\delta H_2/T_2=0$$

for the second $$\frac{\delta H_1'}{T_1'}-\frac{\delta H_2'}{T_2'}=0$$

or for the whole cycle $$\sum\frac{\delta H_1}{T_1}-\sum\frac{\delta H_2}{T_2}=0$$

or $$\delta H/T=0$$

This is the " Integral of Clausius."

Consider now the closed cycle ACBC′ (Fig. 16). Let the cycle consist of two parts, from A to B through C and the return through C′. Then the Integral of Clausius gives

$$\int_A^A \frac{dH}{T} = \int_A^B \frac{dH}{T} + \int_B^A \frac{dH}{T} = 0 \quad \text{. . . . .} \quad (1)$$

through C through C′

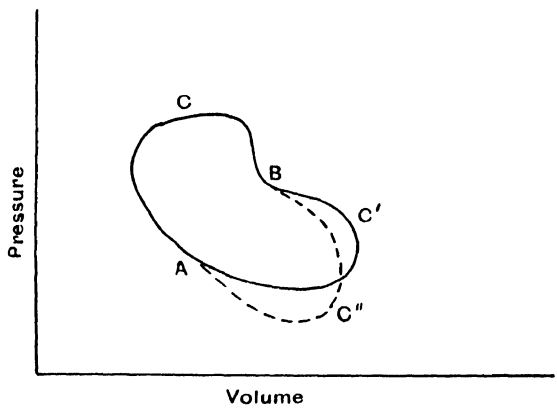


Fig. 16.

If the direction of one of these operations is reversed the sign will be reversed, i.e.

$$\int_A^B \frac{dH}{T} = -\int_B^A \frac{dH}{T}$$

through C through C′

giving, from (1)

$$\int_A^B \frac{dH}{T} - \int_A^B \frac{dH}{T} = 0$$

through C through C′

or

$$\int_A^B \frac{dH}{T} = \int_A^B \frac{dH}{T}$$

through C through C′

or the difference in the entropies at the states A and B is dependent only on the states, and is not in any way affected by the manner in which the change of state has been effected. In contrast to this the heat received during the change from A to B through C will differ from that during the change from A to B through C′ by the heat equivalent of the area ACBC′.

The absolute value of entropy can only be calculated for any state

when the properties of the fluid are determined down to the absolute zero of temperature. In engineering, however, only changes of entropy are required, and these are taken relative to 0° C. or 32° F.

*OTHER DEFINITIONS OF ENTROPY. SECTION 1

The definition derivable from the previous consideration, " Entropy is that quantity which remains constant during adiabatic operations," though accurate and convenient, is far too vague, and creates no mental picture as do the other conditions of state p, v, and T.

Other definitions have been attempted, but the student is warned that for the most part they apply only to certain ideal processes, and that outside a very limited range, usually unspecified, they are deplorably inaccurate.

One of the simplest partial definitions of entropy, " *Entropy is the co-ordinate with temperature of heat,*" is based upon a reasonable desire to obtain a graphical representation of heat. On the $p.v.$ diagram, or " work " chart, the change of the co-ordinate with P corresponding to the work area δW is obviously $\delta W/P=\delta V$.

Exactly corresponding to this chart is the Heat chart on which changes of heat are represented by areas and one ordinate is temperature. The other ordinate will obviously be $\delta H/T=\delta\phi$.

The closed cycle on the $p.v.$ diagram will have an exact counterpart on the $T\phi$ diagram (Fig. 17), and each point a or b on the one will have

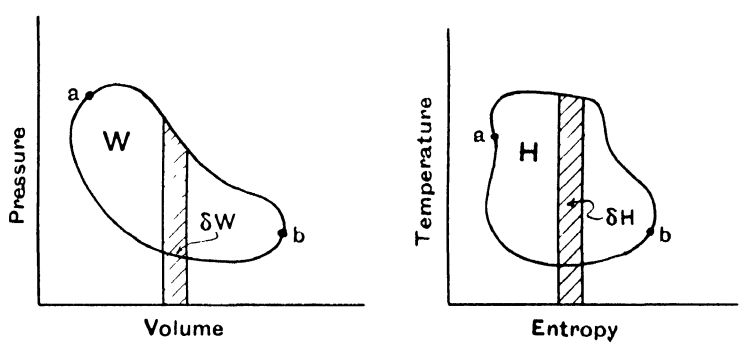


Fig. 17.

a corresponding point on the other. With the $T\phi$ chart in mind the engineer is able to think in terms of heat without the necessity of expressing it first in terms of work, nor are the two necessarily synonymous, especially when the processes considered are irreversible.

OTHER DEFINITIONS OF ENTROPY. SECTION 2

A serious defect of the above definition must be borne in mind. Let *abcd* (Fig. 18) represent the heat content of unit mass of a perfect gas at temperature T_1. If this gas is now compressed *adiabatically* till its temperature reaches $T_2=2T_1$ the heat content will become *efcd* which is twice *abcd*, and the entropy *ef*, i.e. the width of the diagram, will remain constant. If the gas is not perfect, but has a specific heat which increases with temperature, the heat content at T_2 will be greater than twice that at T_1. This can only be obtained by representing the new heat content by *eghd*, i.e. by increasing the entropy by the amount *ch*.

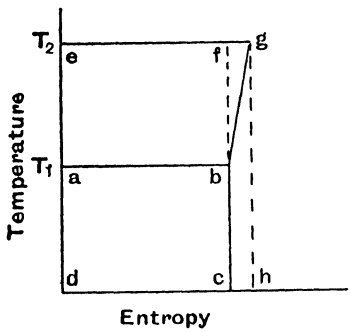


Fig. 18.

Obviously, if the entropy is constant during adiabatic operations the areas cannot represent heat unless the gas is " perfect," and in fact the idea must be discarded in such cases, i.e. when dealing with actual gases at high temperatures.

The conception that areas on the Tϕ chart represent heat arises from an unjustifiable extension of the original definition of entropy given by Clausius, which states that " Increase of entropy in any thermal operation is a measure of the energy degraded in the operation " (see Murray's " Dictionary "). Thermal energy is available for conversion into work when it exists in a body at a temperature T_1 which is higher than the minimum available temperature T_2, and is " degraded " or rendered unavailable for further use when transferred to a body at T_2. The total energy of a body is therefore a function of its temperature, which may have any value, whereas the unavailable energy is a function only of T_2, the " minimum available temperature," i.e. the total energy of a gas at T_1 is T_1 multiplied by the mean specific heat between 0° and T_1, which latter changes with changing values of T_1. On the other hand, the " degraded " energy is the product of the single temperature T_2, and the single value of the mean specific heat from 0° to T_2.

Applying this conception it will be seen that " degraded " energy (or energy not used with the maximum possible efficiency) can be represented by an area *abcd* (Fig. 19) of which T_2 is one ordinate and H/T_2 is the other, and that increases of " degraded " energy can similarly be represented by *befc* having the ordinates T_2 and H'/T_2.

Except for ideal operations it is *not* permissible to consider *da* is part of a *scale* of temperatures extending to T_1, and to assume that areas such as *aa'b'b* represent *available* energy.

The word " entropy " is derived from the Greek, and means " conversion," i.e. the change of " entropy " during any operation is a measure of the imperfection of the conversion of heat into work.

For further information the student is referred to " Steam Turbines," Stodola, Vol. 1, p. 33 *et seq.*, Vol. 2, p. 1319 *et seq.*; " Entropy and Probability," Barnes, *Engineering*, November 12, 1926; and " Anatomy of Science," Lewis.

In what follows it can be taken that, unless a statement is made to the contrary, the working fluid has a constant specific heat, and that all processes are reversible, in which case any of the above definitions will be satisfactory.

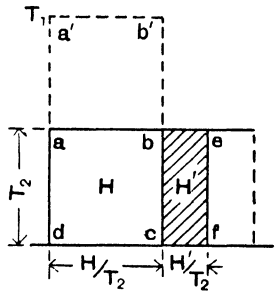


Fig. 19.

(The above conception demands that at zero temperature there should be no energy remaining in the body, and that its entropy should then be zero. This implies that ϕ, which is given by $C_p \log_\epsilon T$, and which approaches infinity if C_p is constant, must be modified by a corresponding approach to zero of the value of C_p. This reduction in the value of C_p is an established fact and leads to the Third Law of Thermodynamics (Nernst's)—" The entropy of every solid or liquid body has at absolute zero the value zero "—i.e. there is no " unknown constant " to be added in the integration $\phi = G\int_0^T \frac{C_p dT}{T}$.)

*REPRESENTATION OF THE CARNOT CYCLE

This cycle is represented by the diagrams of Fig. 20, the closed figures being bounded by two isothermals (T=constant), and two abiabatics (ϕ=constant). The points, A, B, C, D, on the *p.v.* diagram correspond exactly with the points *a*, *b*, *c*, *d* on the Tϕ diagram.

The efficiency of the cycle can be calculated at once from the Tϕ diagram :

Heat received=*aefd* Heat rejected=*befc*

Heat converted into work=*abcd*

$$\text{Efficiency} = \frac{abcd}{aefd} = \frac{ab}{ae} = \frac{T_1 - T_2}{T_1}.$$

If H_1 is the heat received from the hot body, and W=work done during the cycle, then

$$W = H_1 \cdot \frac{T_1 - T_2}{T_1} \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

and the heat discharged to the cold body is $H_2=H_1-W$

$$=H_1\left(\frac{T_1-T_1+T_2}{T_1}\right)$$

$$=H_1\frac{T_2}{T_1}$$

or
$$\frac{H_2}{T_2}=\frac{H_1}{T_1}$$

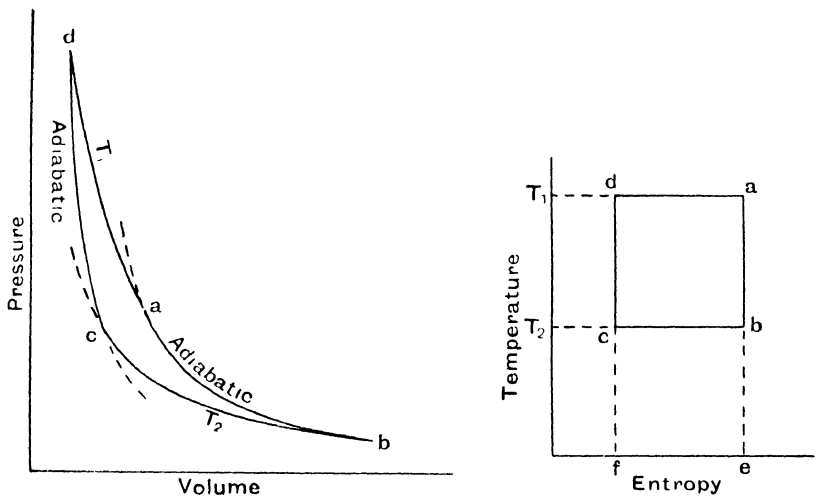


Fig. 20.

or the gain of entropy of the working fluid during heat reception in a perfect heat engine is equal to the loss of entropy during heat rejection (i.e. there is no degradation of energy when heat is converted into work with the utmost possible efficiency).

If a perfect heat engine works through a temperature range of 1°, then $W=H_1/T_1$ from (1) above, or the gain of entropy of the working fluid during heat reception is equal to the work done.

*GENERAL EQUATIONS FOR THE CHANGE OF ENTROPY OF A PERFECT GAS WHEN PASSING FROM ONE STATE TO ANOTHER

(1) If a pound of a substance is raised in temperature from T_1 to T_2, the gain of entropy will be $\Sigma\delta H/T=\Sigma C\delta T/T$ where C is the specific heat

or
$$\phi_2-\phi_1=\int_{T_1}^{T_2}\frac{C\,dT}{T}=C\log_\epsilon\frac{T_2}{T_1}.$$

If the substance is a gas this must be taken as $C_p \log_\epsilon T_2/T_1$ or $C_v \log_\epsilon T_2/T_1$, according to whether the gas is heated at constant pressure or at constant volume.

(2) To calculate the change of entropy with change of state from the condition p_1, v_1, T_1 to condition p_2, v_2, T_2.

Since the energy equation for a perfect gas is given by

$$H=C_v(T_1-T_2)+p(v_2-v_1)$$

we have, for a small change,

$$\delta H=C_v\delta T+p\delta v \quad . \quad . \quad . \quad . \quad . \quad . \quad (1$$

Dividing both sides of the equation by T we have

$$\frac{\delta H}{T}=C_v\frac{\delta T}{T}+\frac{p}{T}\delta v$$

or in the limit

$$\frac{dH}{T}=C_v\cdot\frac{dT}{T}+\frac{p}{T}dv \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Now $$pv=RT \quad \text{or} \quad \frac{p}{T}=\frac{R}{v} \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Substituting (3) in (2) we have

$$\frac{dH}{T}=C_v\frac{dT}{T}+R\frac{dv}{v}.$$

Integrating we have

$$\phi_2-\phi_1=\int_{T_1}^{T_2}\frac{dH}{T}=C_v\int_{T_1}^{T_2}\frac{dT}{T}+R\int_{v_1}^{v_2}\frac{dv}{v}$$

or gain of entropy $$\phi_2-\phi_1=C_v\log_\epsilon\frac{T_2}{T_1}+R\log_\epsilon\frac{v_2}{v_1} \quad . \quad . \quad . \quad . \quad (4)$$

Now $R=C_p-C_v$, hence

$$\phi_2-\phi_1=C_v\log_\epsilon\frac{T_2}{T_1}+(C_p-C_v)\log_\epsilon\frac{v_2}{v_1} \quad . \quad . \quad . \quad (5)$$

$$=C_v\left(\log\frac{T_2}{T_1}-\log_\epsilon\frac{v_2}{v_1}\right)+C_p\log_\epsilon\frac{v_2}{v_1}$$

$$=C_v\log_\epsilon\left(\frac{T_2}{T_1}\times\frac{v_1}{v_2}\right)+C_p\log_\epsilon\frac{v_2}{v_1} \quad . \quad . \quad . \quad (6)$$

Also, since

$$\frac{p_1v_1}{T_1}=\frac{p_2v_2}{T_2}$$

$$\frac{p_2}{p_1}=\frac{T_2}{T_1}\times\frac{v_1}{v_2} \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

Substituting (7) in (6) we have

$$\phi_2-\phi_1=C_v \log_\epsilon \frac{p_2}{p_1}+C_p \log_\epsilon \frac{v_2}{v_1} \quad . \quad . \quad . \quad . \quad (8)$$

Another expression can be found for the change of entropy as follows :

Substituting $C_v=C_p-R$ in (4) we have

$$\begin{aligned}\phi_2-\phi_1&=(C_p-R) \log_\epsilon \frac{T_2}{T_1}+R \log_\epsilon \frac{v_2}{v_1}\\ &=C_p \log_\epsilon \frac{T_2}{T_1}-R\left(\log_\epsilon \frac{T_2}{T_1}-\log_\epsilon \frac{v_2}{v_1}\right)\\ &=C_p \log_\epsilon \frac{T_2}{T_1}-R \log_\epsilon \left(\frac{T_2}{T_1}\times\frac{v_1}{v_2}\right)\\ &=C_p \log_\epsilon \frac{T_2}{T_1}-R \log_\epsilon \frac{p_2}{p_1} \quad \text{from (7)} \quad . \quad . \quad . \quad . \quad (9)\end{aligned}$$

Hence, in calculating the change of entropy of a perfect gas when passing from the state p_1, v_1, T_1, to the state p_2, v_2, T_2, equations (4), (8), or (9) may be used, i.e.

$$\phi_2-\phi_1=C_v \log_\epsilon \frac{T_2}{T_1}+R \log_\epsilon \frac{v_2}{v_1} \quad . \quad . \quad . \quad . \quad . \quad (4)$$

$$=C_v \log_\epsilon \frac{p_2}{p_1}+C_p \log_\epsilon \frac{v_2}{v_1} \quad . \quad . \quad . \quad . \quad . \quad (8)$$

$$=C_p \log_\epsilon \frac{T_2}{T_1}-R \log_\epsilon \frac{p_2}{p_1} \quad . \quad . \quad . \quad . \quad . \quad (9)$$

If British units are used, R, C_p, and C_v should all be expressed either in terms of B.Th.U. or in C.H.U., v_2 and v_1 in cubic feet, and p_2 and p_1 in pounds per square foot ; the change of entropy will then be measured in " units of entropy," and will be independent of the temperature scale used. No name has been officially adopted for the unit of entropy.

Special Cases.—In the case of an *isothermal* change of state from p_1 and v_1 to p_2 and v_2, $T_1=T_2$, and equation (4) becomes

$$\phi_2-\phi_1=R \log_\epsilon \frac{v_2}{v_1} \quad \text{or} \quad (C_p-C_v) \log_\epsilon \frac{v_2}{v_1} \quad . \quad . \quad (10)$$

If the change takes place at *constant volume*, $v_2=v_1$ and (4) becomes

$$\phi_2-\phi_1=C_v \log_\epsilon \frac{T_2}{T_1} \quad . \quad . \quad . \quad . \quad . \quad (11)$$

If the change takes place at *constant pressure*, $p_2=p_1$ and (8) becomes

$$\phi_2-\phi_1=C_p \log_\epsilon \frac{v_2}{v_1} \quad . \quad . \quad . \quad . \quad . \quad (12)$$

*CHANGE OF ENTROPY WHEN THE OPERATION TAKES PLACE ACCORDING TO THE GENERAL LAW pv^n =CONSTANT

It has been shown on p. 21 that the heat supplied to or taken from a gas is given by

$$\delta H=\frac{\gamma-n}{\gamma-1}\times \text{work done.}$$

Considering a small change in state we may write

$$\delta H=\frac{\gamma-n}{\gamma-1}\times p\delta v.$$

Dividing by T we have

$$\frac{\delta H}{T}=\frac{\gamma-n}{\gamma-1}\times\frac{p}{T}\delta v$$

or since $\frac{p}{T}=\frac{R}{v}$ from (3) we have in the limit

$$\frac{dH}{T}=\frac{\gamma-n}{\gamma-1}\times R\frac{dv}{v}.$$

Integrating we have

$$\int_{T_1}^{T_2}\frac{dH}{T}=R\cdot\frac{\gamma-n}{\gamma-1}\int_{v_1}^{v_2}\frac{dv}{v}$$

or

$$\phi_2-\phi_1=R\cdot\frac{\gamma-n}{\gamma-1}\cdot\log_\epsilon\frac{v_2}{v_1}.$$

But

$$\frac{v_2}{v_1}=\left(\frac{T_1}{T_2}\right)^{1/n-1} \text{ from p. 19}$$

and

$$R=C_v(\gamma-1) \quad \text{from p. 7}$$

hence

$$\phi_2-\phi_1=R\cdot\frac{\gamma-n}{\gamma-1}\cdot\log_\epsilon\left(\frac{T_1}{T_2}\right)^{1/n-1}$$

$$=C_v(\gamma-1)\cdot\frac{\gamma-n}{\gamma-1}\cdot\frac{1}{n-1}\log_\epsilon\frac{T_1}{T_2}$$

$$=C_v\cdot\frac{\gamma-n}{n-1}\log_\epsilon\frac{T_1}{T_2} \quad . \quad . \quad . \quad . \quad . \quad (13)$$

EXAMPLE.—Find the change in entropy when 1 lb. of air at 32° F. and atmospheric pressure changes in volume to 2 cu. ft. with a temperature of 540° F. given $C_p=0{\cdot}2375$ and $C_v=0{\cdot}1691$.

Here $p_1=14{\cdot}7\times144=2116$ lb. per sq. ft., and, for 1 lb. of air

$$R=C_p-C_v=0{\cdot}2375-0{\cdot}1691=0{\cdot}0684 \text{ B.Th.U. or } 53{\cdot}18 \text{ ft.-lb.}$$

Hence, as $p_1v_1=RT_1$, and $T_1=32+460=492°$ F. abs.

$$v_1=\frac{53{\cdot}18\times 492}{2116}=12{\cdot}39 \text{ cu. ft.}$$

Also it is known that $T_2=540+460=1000°$ F. abs. and $v_2=2$ cu. ft. Using equation (4)

$$\phi_2-\phi_1=C_v \log_\epsilon \frac{T_2}{T_1}+R \log_\epsilon \frac{v_2}{v_1}$$

$$=0{\cdot}1691 \log_\epsilon \frac{1000}{492}+0{\cdot}0684 \log_\epsilon \frac{2}{12{\cdot}39}$$

$$=0{\cdot}1196-0{\cdot}1247=-0{\cdot}0051 \text{ units of entropy.}$$

Alternatively, as $p_2v_2=RT_2$

$$p_2=\frac{RT_2}{v_2}=\frac{53{\cdot}18\times 1000}{2}=26{,}500 \text{ lb. per sq. ft.}$$

and $$\phi_2-\phi_1=C_v \log_\epsilon \frac{p_2}{p_1}+C_p \log_\epsilon \frac{v_2}{v_1} \quad \text{(equation 8)}$$

$$=0{\cdot}1691 \log_\epsilon \frac{26{,}590}{2116}+0{\cdot}2375 \log_\epsilon \frac{2}{12{\cdot}39}$$

$$=0{\cdot}4280-0{\cdot}4331=-0{\cdot}0051 \text{ units.}$$

Or as $$\phi_2-\phi_1=C_p \log_\epsilon \frac{T_2}{T_1}-R \log_\epsilon \frac{p_2}{p_1} \quad \text{(equation 9)}$$

$$=0{\cdot}2375 \log_\epsilon \frac{1000}{493}-0{\cdot}0684 \log_\epsilon \frac{26{,}590}{2116}$$

$$=0{\cdot}1680-0{\cdot}1731=-0{\cdot}0051 \text{ units.}$$

The gain of entropy $(\phi_2-\phi_1)$ being negative means that during the change of state the air *loses* entropy by the amount 0·0051 units.

*WORK DONE BY AN EXPANDING GAS, WITH REFERENCE TO THE TEMPERATURE ENTROPY DIAGRAM

Let T_1 and T_2 be the initial and final temperatures, and let AB (Fig. 21) represent the temperature-entropy curve for the expansion. Further, suppose δH is a small quantity of heat supplied at any temperature T, and giving rise to a small change of entropy $\delta\phi$.

Then $$\delta\phi=\frac{\delta H}{T}$$

$$T\delta\phi=\delta H \quad \text{or} \quad \delta(Td\phi)=\delta H.$$

The area of an elementary strip of the diagram of height T and width

$\delta\phi$ represents the small quantity δH of heat supplied, and the total area under the curve AB is

$$\int_{\phi_1}^{\phi_2} T d\phi = H$$

where H is the total heat supplied during the expansion.

Similarly if the gas is compressed from temperature T_2 to temperature T_1, it will reject an amount of heat equal to the above.

Suppose now the gas undergoes a complete cycle of changes so that

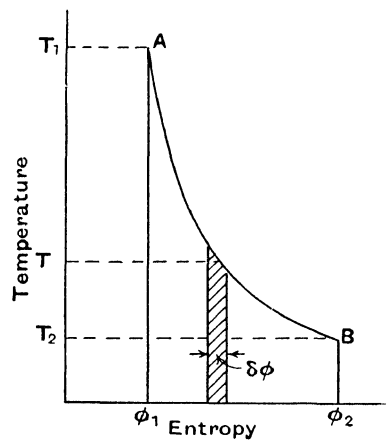


Fig. 21.

its temperature, pressure, and volume are the same at the end as at the beginning of the cycle. The temperature-entropy diagram will then form a closed figure as in the case given in Fig. 17.

Let H_1 be the amount of heat supplied to the gas, and H_2 the amount rejected by it during the cycle, then, in order to get the area of the closed figure we must integrate over the whole cycle, and

$$\int T d\phi = H_1 - H_2.$$

Now $H_1 - H_2$ is equal to the heat converted into work, hence we see that the area of the $T\phi$ diagram represents in B.Th.U. the work done in a closed cycle.

An extension of this relationship leads to Clapeyron's equation.

CLAPEYRON'S EQUATION

This equation connects the latent heat of steam, the increase in volume during evaporation, and the slope of the p . T curve.

If 1 lb. of water at pressure p_a is raised in temperature to the boiling-

point T_b and then evaporated, after which it is expanded in the dry state to its original temperature and finally condensed, the cycle of operation will be as shown on the PV and $T\phi$ diagrams in Fig. 22. If the varia-

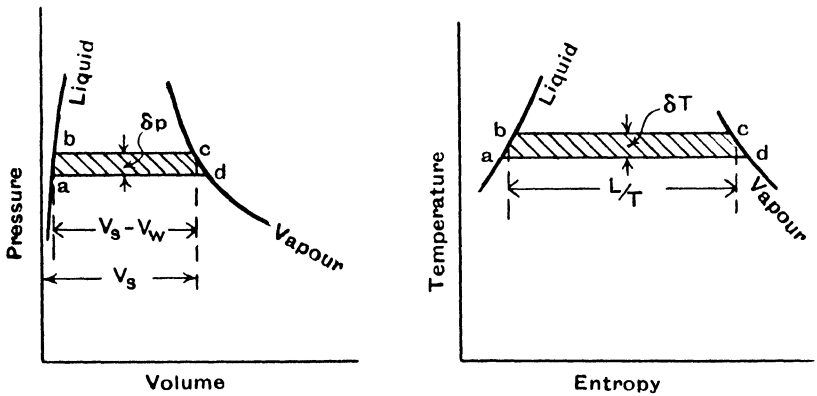


Fig. 22.

tions of pressure and temperature are small the work done will be given by $(v_s - v_w)\delta p$, where v_s=specific volume of steam and v_w=specific volume of water at pressure p_a. The corresponding heat interchange will be $\frac{L}{T}\delta T$. In the limit these two correspond exactly, or

$$(v_s - v_w)dp = \frac{JL\,dT}{T}$$

where J is Joule's Equivalent,

or
$$L/T = \frac{1}{J}(v_s - v_w)\frac{dp}{dT} = \phi_s - \phi_w \quad . \quad . \quad . \quad . \quad (1)$$

The term $\frac{dp}{dT}$ is the rate of increase of p with saturation temperature. Empirical values for v, L, etc., should be consistent with this equation, which can also be used for the determination of one of the quantities when the remainder are known.

The same reasoning is valid for any reversible change of phase which occurs isothermally at constant pressure; for instance, it may be used to find the change in the freezing-point of water due to change in pressure. The following example will illustrate this—

It is known that 1 lb. of water at 32° F. changes in volume from 0·016 cu. ft. to 0·0174 cu. ft. on solidifying, and gives out its latent heat, 144 B.Th.U.

From (1)
$$\frac{dT}{dp} = -(V - w)\frac{T}{JL}$$

where V=volume of 1 lb. of ice in cubic feet at 32° F., or 493° abs.

and w=volume of 1 lb. of water in cubic feet at 32° F., or 493° abs.

$$\therefore \frac{d\mathrm{T}}{dp} = -\frac{(0{\cdot}0174 - 0{\cdot}016)493}{778 \times 144} = -0{\cdot}0000065$$

Hence at atmospheric pressure when dp=2116 lb. per sq. ft.

$$d\mathrm{T} = -2116 \times 0{\cdot}0000065$$
$$= -0{\cdot}0135° \text{ F.}$$

i.e. the freezing-point of water changes 0·0135° F. for every atmosphere change in pressure, so that at a pressure of 11 atmospheres, assuming $\frac{d\mathrm{T}}{dp}$ to remain constant, the freezing-point would be 31·865° F.

EXAMPLES ON CHAPTER IV

1. If 1 lb. of gas occupying 3 cu. ft. at 110·7 lb. per sq. in. and 440° F. expands at constant temperature to a volume of 12 cu. ft., find its gain in entropy.

2. If 42·46 cu. ft. of gas at a pressure of 4·695 lb. per sq. in. and 80° F. be compressed isothermally to 10·62 cu. ft., what is the loss of entropy?

3. Ten cu. ft. of gas at 65° F. and 75 lb. per sq. in. gauge pressure are expanded to four times the original volume according to the law $pv^{1{\cdot}25}$=constant. If C_v=130·2 ft.-lb., and γ=1·4, find the change of entropy.

4. Calculate the gain of entropy when 1 lb. of water at 60° F. is converted into steam at 296° F. and then superheated to 500° F. (Assume C_p=0·5 and L=1114 −0·71° F.)

5. Given the following data, estimate the volume of 1 lb. of dry saturated steam at 100 lb. per sq. in. abs., using Clapeyron's equation, p. 62.

p lb. per sq. in. abs. .	90	95	103	110
Temperature, ° F. . .	320·3	324·1	330·0	334·8

Take the latent heat at 100 lb. per sq. in. as 888 B.Th.U.

Chapter V

CURVES OF STATE

*GENERAL STATEMENT

As indicated in Chapter IV, the states which can be assumed by a substance can be expressed in terms of any two independent variables, selected from a considerable number according to convenience.

The commonest combinations will now be considered, with special reference to steam.

*THE P.V. DIAGRAM

Imagine a cylinder, containing 1 lb. of water, to be fitted with a movable piston, and to stand upon a body of constant temperature so that the cylinder and its contents can be kept at that temperature while the capacity is slowly changed. Let the constant temperature be 100° C., and let the water be subject to a very high initial pressure, the initial condition point being given by A_1 (Fig. 23). If the piston is very slowly withdrawn the water will expand elastically while its pressure falls to 14·7 lb. per sq. in. when the state is shown by the point B_1. The curve A_1B_1 will be nearly, but not quite, vertical.

If, while the temperature is maintained at 100° C., the volume is allowed to increase, evaporation will gradually take place at constant pressure along B_1C_1 until completed at C_1. Any further increase of volume will be followed by a reduction of pressure along the curve C_1D_1, represented approximately by the equation pv^n=constant, on which all points to the right of C_1 represent conditions of superheated vapour.

Similar isothermal curves such as $A_2B_2C_2D_2$ can be drawn for other temperatures, but it will be noted that as the temperature is increased the points B_1B_2, etc., move slowly upwards and to the right, and the increase of volume during evaporation, B_1C_1, B_2C_2, etc., becomes markedly less.

At the critical temperature of 374° C. the length BC, corresponding to the process of evaporation, vanishes altogether and is replaced by a point of inflexion X in the isothermal curve, which is shown by E. At still higher temperatures the curves of state rapidly tend to become smooth, as the curve G, the fluid now being far above the critical temperature and in the state of a permanent gas.

The curves XB_2B_1 . . ., XC_2C_1 . . . are known as the liquid and

vapour boundaries. Between them exists a state of partial evaporation, both steam and water existing together. To the left of the curve $A_3XB_2B_1$ the liquid phase only can exist. In the area bounded by $D_3XC_2C_1$ the state is that of a superheated vapour, and above the curve A_3XD_3 the state is that of a " permanent " gas, more or less perfect. (A " permanent " gas is a gas above its critical temperature, so that it cannot be liquefied by pressure alone. A " perfect " gas is one which strictly obeys the laws of Boyle and Charles.)

At the critical point X the discontinuity between the liquid and vapour

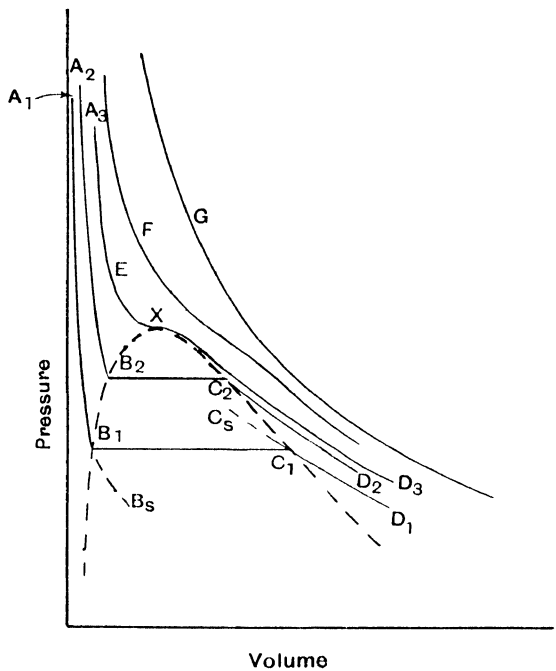


Fig. 23.

states finally disappears ; this is the point at which the three values for V in Van der Wall's equation, which is a cubic in V, should coincide.

The extensions of the curve A_1B_1 to B_s, and of D_1C_1 to C_s, will be referred to on p. 225.

It is sometimes convenient to trace other quality lines upon the P.V. diagram.

Constant Dryness Fraction Lines.—If, during evaporation at pressure P_a only one-half or one-quarter of the process had been completed, the " dryness fraction " would have been given by 0·5 or 0·25, and the change of volume would have been 0·5 or 0·25 of BC.

Any line of constant dryness fraction will therefore divide all such lines as BC, Fig. 24, in the same ratio and will be as shown by Xq_1, Xq_2, and Xq_3.

Lines of Constant Total Heat and of Entropy.—These can be obtained by direct plotting from the steam-tables (see p. 120), and take the course indicated by the light and chain dotted lines H_1, H_2 and ϕ_1, ϕ_2. In the gas area lines of constant temperature will also be lines of constant total heat.

Lines of constant superheat can also be drawn as *s*, *s* by adding constant increments of temperature to the right of the vapour boundary,

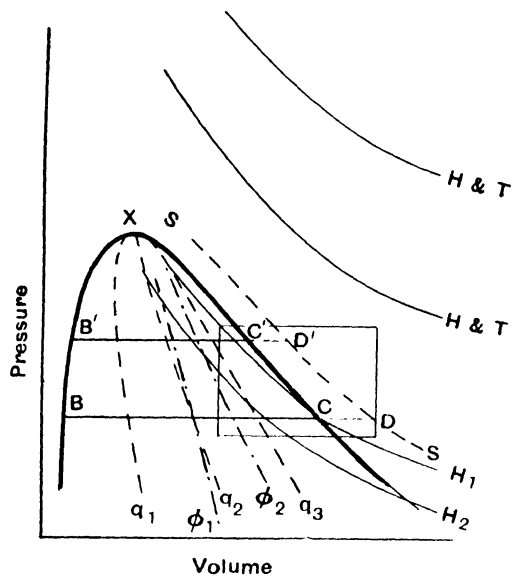


Fig. 24.

i.e. by making D, D′, etc., equal increments of temperature hotter than C, C′, etc.

If the condition points of steam performing a closed cycle are traced upon the chart, the area enclosed is the work in foot-pounds which would be performed by a perfect engine using that cycle.

Diagrams of the same nature can be obtained for other working fluids.

In practice, interest is concentrated on certain portions of the diagram only. For steam the area enclosed in the rectangular area marked on Fig. 24 is usually all that is required, while for air, for which the critical temperature and pressure are −146° C. and 45 atmospheres, or for gas engines the portion required lies entirely towards the top right-hand

corner, far from the vapour boundary and critical temperature. For carbon dioxide, which is used as a refrigerant and for which the critical temperature and pressure are 30·9° C. and 74 atmospheres, the area within and close to the liquid and vapour boundaries in the region of the point X is of interest.

*THE Tφ OR TEMPERATURE-ENTROPY CHART

The general nature of this chart can best be followed by considering 1 lb. of water which is heated at constant pressure.

Starting at a low temperature at A, Fig. 25, the increase of this factor

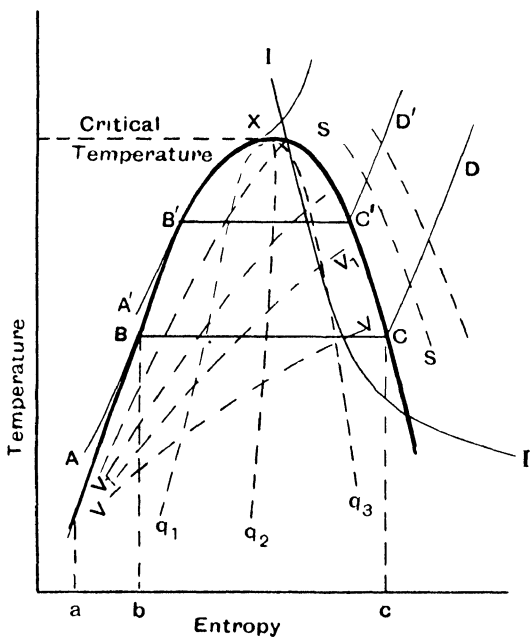


Fig. 25.

at constant pressure from T_1 to T_2 will be accompanied by an increase of entropy of an amount $C_p \log_\epsilon T_2/T_1$, and, provided the critical temperature is not approached, C_p for water may be taken as unity. At the temperature corresponding to the boiling-point for the given pressure, i.e. at the point B, evaporation will commence, and up to this stage the increased heat content of the water will be given by the area

$$aABb = C_p(T_B - T_A) = T_B - T_A.$$

Evaporation will then proceed at constant pressure and temperature, the state point moving meanwhile from B to C, the latter marking the

completion of the process. The gain in entropy will be L_B/T_B where L_B is the latent heat corresponding to the temperature T_B, and the area $bBCc$ will be L_B.

Further heating will cause a rise of temperature and an increase of entropy, following the curve CD, for which

$$\phi_D - \phi_C = C_p \log_\epsilon \frac{T_D}{T_C} = 0{\cdot}48 \log_\epsilon \frac{T_D}{T_C} \text{ approximately.}$$

This curve in the superheated region will therefore be roughly twice as steep as the curve AB.

Curves similar to ABCD can be obtained for a series of higher pressures, but it should be noted that as the critical temperature and pressure are reached the simple equations stated above have to be much modified to allow for the greatly increased compressibility of the water and reduced compressibility of the vapour.

The points BB′, etc., and CC′, etc., can now be joined to form the liquid and vapour boundaries, meeting at X, as before. The area below T_C and to the left of the liquid boundary represents the liquid phase, and below T_C and to the right of the vapour boundary will be found the vapour phase. All above T_C is the phase of "permanent" gas, and between the boundaries the liquid and vapour phases can co-exist. For water at normal pressures the curves AB virtually coincide with the liquid boundary.

Constant quality lines can be obtained, as before, by joining points representing equal fractional divisions of lines such as BC. They will be as shown by Xq_1, Xq_2, etc.

Constant Volume Lines.—If the volume of 1 lb. of dry steam at temperature T_B is, say, 10 cu. ft., then, if during evaporation only 0·1 lb. or 0·2 lb. be evaporated, the corresponding points reached on the diagram would be 0·1 or 0·2 of the distance from B to C (as 0·1 or 0·2 of the latent heat would have been absorbed and the dryness fraction would be given by 0·1, 0·2, etc.). Constant volume lines can therefore be obtained in the mixed phase area by dividing lines such as BC into equal parts to the number of the cubic feet occupied by 1 lb. of steam at the given pressure. Numbering the points of subdivision 0, 1, 2, 3, etc., starting from the ends B, B_1, etc., upon the liquid boundary, the points with the same numbers can then be joined, and will thus give constant volume lines such as vv, $v'v'$, etc. By reference to steam tables the lines can be continued into the vapour phase area.

Lines of constant heat, such as I, I, can be determined by calculating, from the figures given in the steam tables, the dryness fraction or the temperature of superheat of steam at different pressures but having some

specified total heat. The state points so obtained can then be plotted, and, when joined, will give one of the lines desired.

Lines of constant superheat can be obtained in a similar manner and follow the direction shown, *s*, *s*.

*T.S. CURVES FOR VARIOUS FLUIDS

Curves for air, carbon dioxide, and steam are shown plotted to scale in Figs. 26, 27, and 28. The two former are taken from " Technical

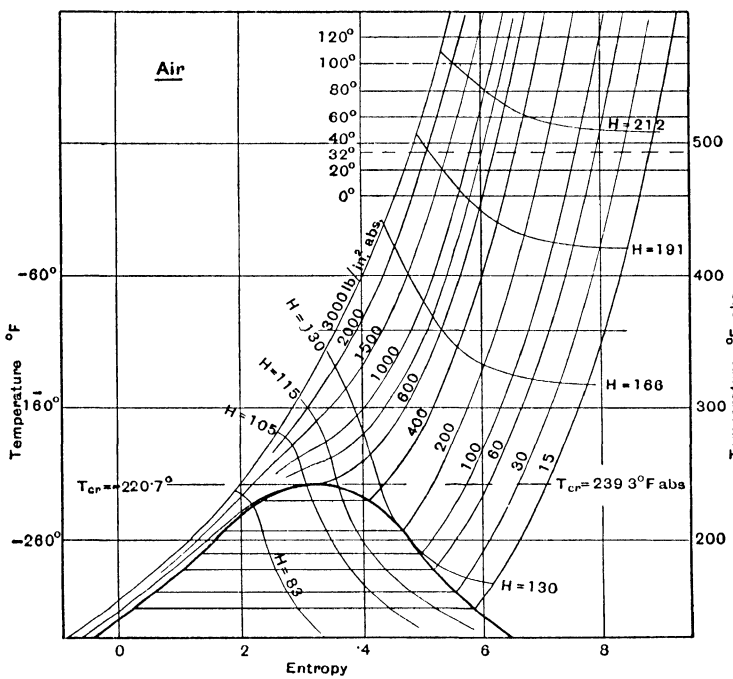


Fig. 26.

Thermodynamics " by Schüle, and the latter will be found accurately plotted to a large scale in " Steam Tables " by Keenan and Keyes.

*MOLLIER, OR TOTAL HEAT—ENTROPY CHART

This is probably the chart most commonly used (see Fig. 29). Tracing a line of constant pressure, starting from the point where the total heat is H_A, the total heat and entropy will both increase from A to B, at which point evaporation will commence. During evaporation both quantities

will increase proportionately, and BC will be a straight line (i.e. the increase of H is qL, and the increase of ϕ is qL/T, T being constant). Beyond C the vapour will become superheated, and the constant pressure curve will be as shown by CD.

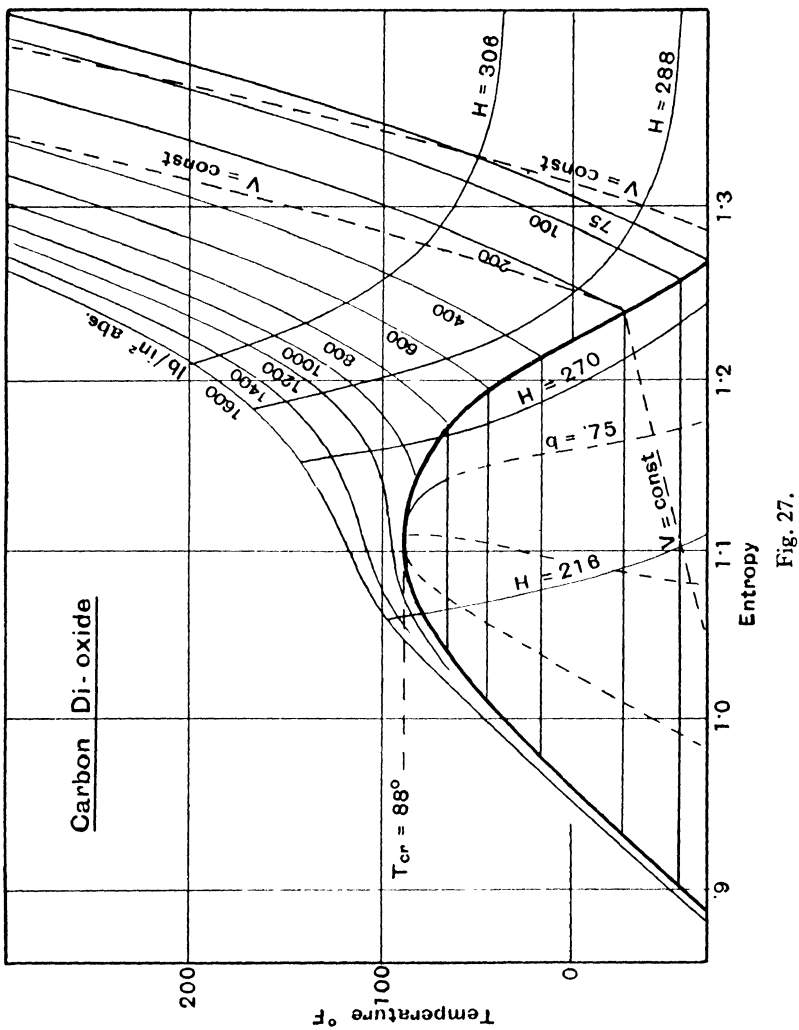


Fig. 27.

The slope at any point on a line of constant pressure will be given by $dH/d\phi$ which, by definition, is equal to T. If therefore two points, x and y, on a line of constant pressure, are taken close to but on opposite sides of the liquid or vapour boundaries, the temperatures at x and y

will be given by the slopes of the curve at these points. As x and y approach each other the temperatures at these points will tend to the same value, i.e. to T_B or T_C, so that there will be no sudden change in the slope of the curve ABCD at the points B and C.

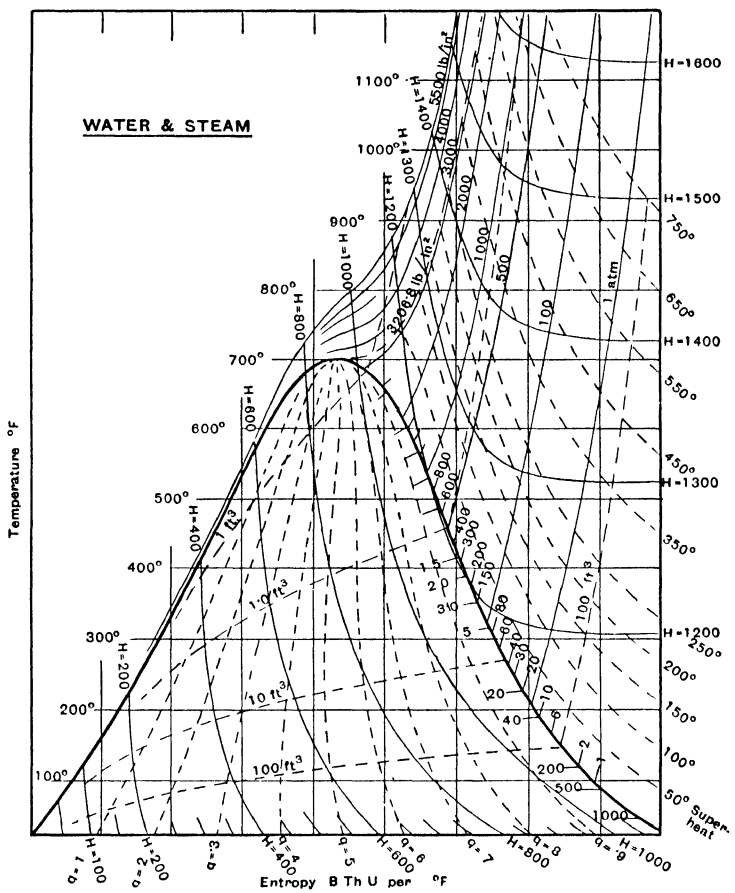


Fig. 28.

As before, liquid and vapour boundaries XB'B, XC'C can be traced, and lines of constant quality, volume, etc., may be drawn.

It is of interest to note that as the total heat increases but little with increasing pressure, the vapour boundary will only rise very slightly from right to left as the pressure increases.

The portion of the diagram normally used for steam calculations is shown by Fig. 55.

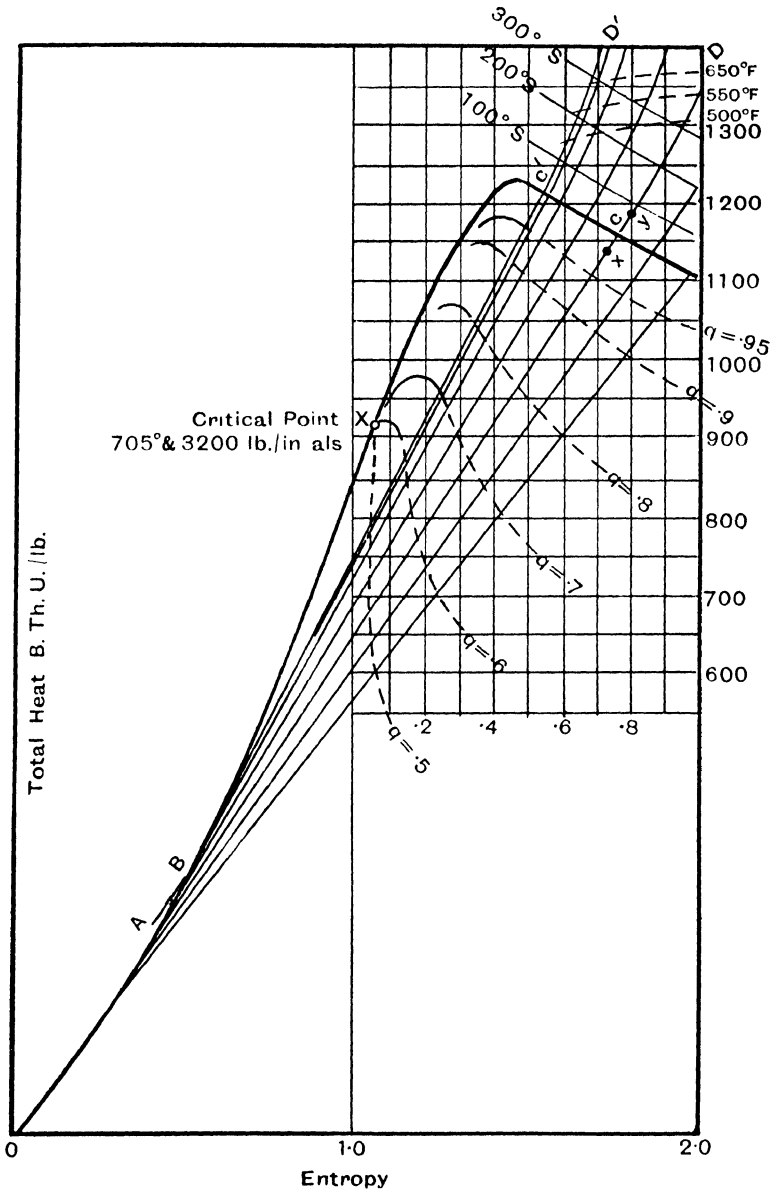

300°
200°
100°
D'
D
650°F
550°F
500°F
1300
1200
1100
1000
900
800
700
600
q = ·95
q = ·9
q = ·8
q = ·7
q = ·6
q = ·5
Critical Point
705° & 3200 lb./in als
X
A
B
·2
·4
·6
·8
Total Heat B. Th. U./lb.
0
1·0
2·0
Entropy

Fig. 29.

*TOTAL HEAT—TEMPERATURE AND PRESSURE DIAGRAMS

For certain purposes in steam engine calculations this diagram is of considerable value. The portion normally required is shown in Fig. 56.

*CHOICE OF DIAGRAM

As the diagrams all represent the same transformations of the given fluid the inherent " family resemblance " should be noted. A recollection of the general form of the curves will often be useful even though charts drawn to scale are not consulted.

Many problems, especially those connected with steam and refrigerants, can readily be solved by the use of diagrams of state, and these are available, plotted to an adequately large scale, in the printed form. The choice of the most convenient diagram is a matter of importance.

The P.V. diagram is used for the comparison of actual " indicator diagrams " for reciprocating engines with the corresponding theoretical diagrams. The area of the P.V. card is required for the determination of the indicated horse-power. The properties of the working fluid other than its pressure and volume may be read from the chart, but it will be found that adiabatic and isothermal curves are not easy to distinguish at high and low pressures, and intersect at inconveniently small angles, making accurate reading of the chart difficult. In order to measure work as an area on the diagram the scales of P and V must be uniform, and this results in much local crowding of the lines representing other quantities, and precludes the attainment of uniform proportionate accuracy. In the dry region the lines of constant total heat almost coincide with isothermals, the coincidence becoming more complete as the gas reaches the " perfect " state.

The $T\phi$ Diagram.—Like the P.V. diagram this is very instructive for theoretical purposes, and on the whole can be read with much greater convenience and accuracy, as isothermals and adiabatics intersect at right angles. Lines of p and v, however, intersect at sharp angles in the dry regions, making accurate reading difficult, and the scales of properties other than T and ϕ (the numerical value of the latter not often being required) are far from uniform.

$H\phi$ Diagram.—This is generally the most convenient of all the diagrams, and can be obtained ready printed on a large scale for steam and refrigerants. The heat drop during adiabatic operations can be read very simply on a uniform open scale. There is no sudden change of direction of the p lines on crossing the liquid or vapour boundaries, and the diagram is therefore comparatively simple to follow. Lines of constant volume are not easy to distinguish from lines of constant pressure, how-

ever, and in the revised diagram based on Callendar's figures the former are printed in red.

For refrigerants the expedient of rotating the axis of ϕ is frequently adopted, as measurements of areas on the diagram are not required, and the diagram is thereby opened out considerably in regions where accurate reading would otherwise be difficult. Such a diagram for CO_2 is shown on p. 334.

H log p Diagram.—The use of the value of log p in place of its simple numerical value has the advantage that equal *ratios* of expansion, which are of equal importance in practice, are represented by equal intervals. The lines of constant volume are readily shown on the diagram. Adiabatic lines, unlike those on the Mollier diagram, are not straight, but the curvature is slight, and there is no sudden change of direction on crossing the saturation line, as $d\mathrm{H}/d\mathrm{P}=a\mathrm{V}$ when ϕ is constant.

EXAMPLE.—The great ease with which many problems can be solved by means of the diagrams may be illustrated by a number of examples, using Fig. 28.

(1) Find the volume of 1 lb. of steam at 200° F. and dryness fraction 0·3. [*Ans.* 10 cu. ft.]

(2) Steam at 500° F. is expanded adiabatically to 200° F. If the initial dryness fraction is (*a*) 1, (*b*) 0·75, (*c*) 0·55, (*d*) 0·2, find the final dryness fraction. [*Ans.* (*a*) 0·77, (*b*) 0·64, (*c*) 0·55, (*d*) 0·37.]

Note that moderately dry steam tends to become wet, while very wet steam tends to become dry.

(3) Steam at 500° F. and dryness 0·72 is throttled (at constant total heat) to a temperature of 200° F. Find the final dryness fraction. [*Ans.* 0·85.]

(4) Steam at 2000 lb. per sq. in. and with 30° superheat is throttled. State the changes in its condition. [*Ans.* From 2000 lb. to 1000 lb. the superheat is reduced, the steam becoming dry at the latter pressure. At 400 lb. the steam has attained a dryness fraction of 0·98, but becomes dry again when 230 lb. is reached.

Below 230 lb. per sq. in. the steam rapidly superheats, attaining 50° superheat at 58 lb., 100° superheat at 15 lb., and 150° superheat at 4·5 lb.]

These and similar examples should be checked on large-scale Mollier and other diagrams, until the reader is fully conversant with the general arrangement of the curves.

Fig. 153 should also be replotted with the axis of entropy horizontal, instead of inclined at an angle of 72°, in order to note the effect of the obliquity upon the clarity of the diagram.

EXAMPLES ON CHAPTER V

By means of an entropy chart determine :

(*a*) The dryness fraction after expansion when dry saturated steam at 180 lb. per sq. in. abs. expands isentropically to 15 lb. per sq. in. abs.

(*b*) The dryness after expansion when steam at 280° F. and of dryness 0·8 expands isentropically to 110° F.

(*c*) Steam at 100 lb. per sq. in. abs. is superheated 200° F. and then expands isentropically down to 4 lb. per sq. in. abs. Find its dryness after expansion.

(*d*) What must be the final pressure in (*c*) in order that the steam may be just dry and saturated after expansion ?

(*e*) Determine the final quality of steam when 50 B.Th.U. are extracted at constant pressure, the initial condition being given by (1) $p=200$ lb. per sq. in. abs. and $q=1$; (2) $p=200$ lb. per sq. in. abs. and superheat is 50° F. ; (3) $p=200$ lb. per sq. in. and superheat is 100° F. Also determine the final quality in case (2) when the heat is extracted at constant volume. (Callendar's diagram.)

(*f*) Determine the final quality when steam at 16 lb. per sq. in. abs. and $q=0{\cdot}95$ is compressed adiabatically to (1) 40 lb. per sq. in. ; (2) 150 lb. per sq. in.

(*g*) Find from Callendar's Mollier chart the final condition and the change of total heat when steam at 100 lb. per sq. in. abs. and $q=0{\cdot}95$ is increased 10 per cent. in volume at constant pressure.

(*h*) Determine the final quality when steam at 200 lb. per sq. in. and $q=0{\cdot}98$ is throttled (at constant heat) to (1) 100 lb. per sq. in. ; (2) 50 lb. per sq. in.

Chapter VI

IRREVERSIBLE PROCESSES

Though irreversible processes are theoretically inefficient, their use is unavoidable in practice. A number of cases will now be considered.

FLOW INTO VACUUM

If a vessel containing a gas or vapour is put into communication with another which has been evacuated, the fluid will distribute itself between the two vessels. No external work whatever will be done, and the process will be one of *constant internal energy.*

If the fluid is a perfect gas this will be represented by a line of constant temperature, i.e. a standard isothermal on the *p.v.* chart, or a horizontal line of the Tϕ diagram.

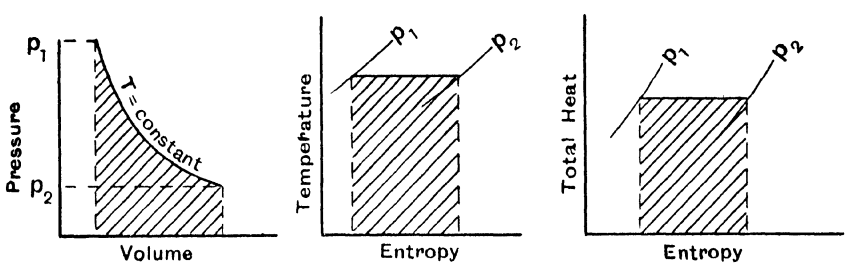


Fig. 30

N.B.—$E = C_v \times t$; $H = C_p \times t$.

For imperfect gases molecular changes, i.e. the reduction of compound molecules, etc., are involved, and a small drop in temperature—the Joule Thompson Cooling Effect—will occur.

In all cases the areas beneath the curves represent work wasted, not useful work done. The curves shown in Fig. 30 are for perfect gases.

THROTTLING

This process is involved in such cases as that of steam escaping through a small orifice into the atmosphere, when its passage is impeded only by the constant atmospheric pressure, or in the " throttling calorimeter," etc.

Let 1 lb. of gas or vapour in cylinder (1) be subject to a constant pressure p_1, and be forced through an orifice into cylinder (2) where its entry is resisted by the constant pressure p_2.

The total work done *on* the steam will be given by

$$p_1A_1l_1-p_2A_2l_2=p_1v_1-p_2v_2.$$

If during the process the internal energy has changed from E_1 to E_2

$$E_2=E_1+\frac{p_1v_1-p_2v_2}{J}$$

or

$$E_2+\frac{p_2v_2}{J}=E_1+\frac{p_1v_1}{J}$$

or the Total Energy—also referred to as the Total Heat or Enthalphy—is constant.

It should be noted that as throttling is a single process, not a cycle of

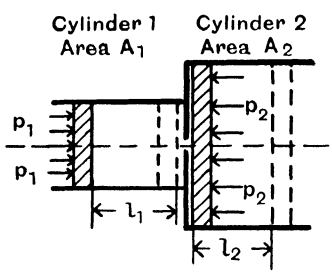


Fig. 31

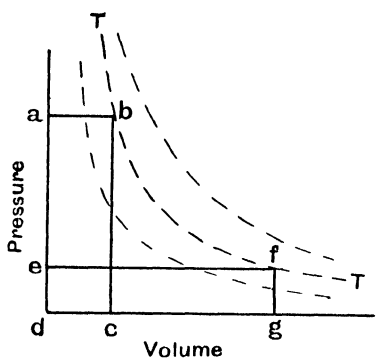


Fig. 32

operations, it will not be represented on the diagrams of state by a closed figure.

Perfect Gas.—The change can be shown for a perfect gas as follows: Fig. 32 represents the superimposed indicator diagrams for the two cylinders. The displacement of the gas from the first cylinder is shown by *ab*, and the work done on the gas by the area *abcd*. The pressure in the second cylinder being constant at p_2 the work done by the gas is given by area *efgd*.

If the two quantities of work are equal, *b* and *f* will lie on a rectangular hyperbola, which is also a curve of constant temperature T for a perfect gas. In this case the Total Energy or Total Heat is given by $C_p \times$ temperature (the internal energy is given by $C_v \times$ temperature), and as the Total Energy at *b* must be equal to that at *f*, it follows that the temperatures must also be equal, and the work done on a perfect gas must equal the work which is performed by it.

It should be noted that the area beneath the isothermal curve from *b* to *f* does not enter the discussion, and that there is a change of energy to the kinetic form and back again between these points. On the Tϕ chart, as total energy=$C_p \times T$, the initial and final states will be on a line of constant temperature *bf*, and similarly on the Hϕ chart (see Fig. 33).

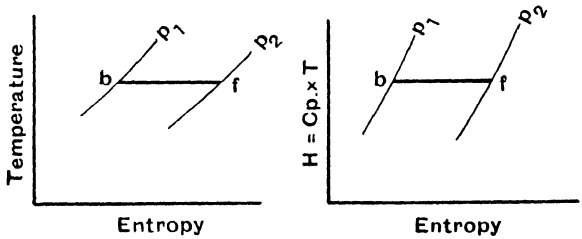


Fig. 33

It will be obvious that as there is *no* gain or loss of work as a whole the area under the curve *bf* does *not* represent useful work done, or the equivalent in heat units of work done.

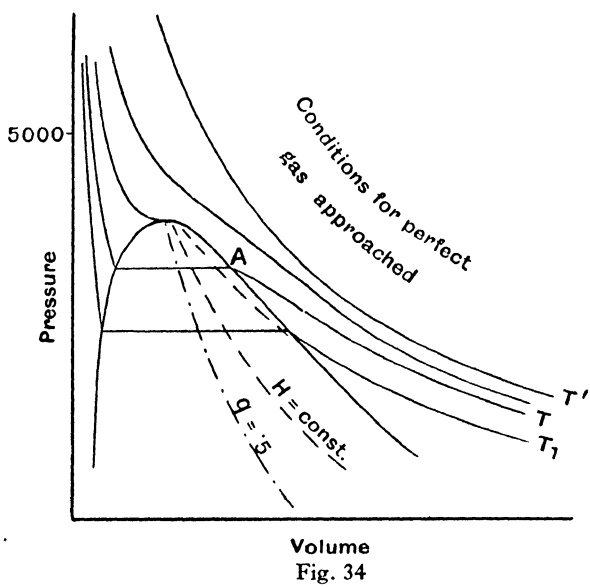


Fig. 34

Condensible Vapours and Imperfect Gases.—It should be remembered that the curves representing the changes of state of perfect gases are merely limiting cases of the general curves of state for fluids when the critical pressure and temperature have been greatly exceeded. For steam and other condensible vapours reference must be made to the complete curves of state, Figs. 26, 27, and 28, where the conditions

approximate to those of a perfect gas are indicated in the top right corners.

On the $p.v.$ diagram (Fig. 34) two lines of constant total heat are shown in the mixed phase area. These are generally slightly less steep than the curves of constant quality, showing that the steam, on throttling, tends to become drier, with the absorption of latent heat, except in the region above A where the reverse is the case. Upon reaching the vapour boundary these lines tend to flow into the curves of constant temperature, and distinction becomes more difficult as the extent of superheating increases. A much clearer impression is obtained from the Tϕ and Hϕ charts, Figs. 35 and 36.

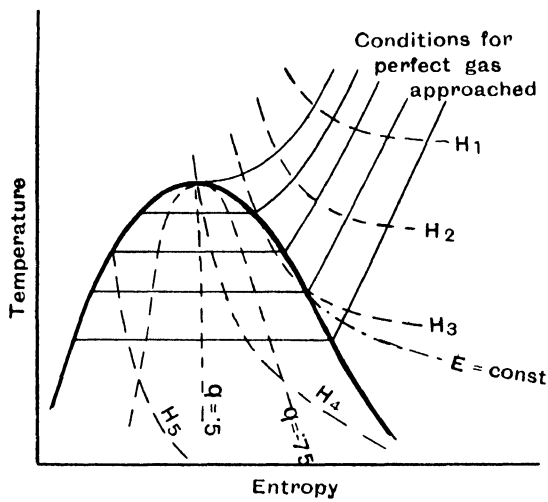


Fig. 35

It will be seen that except near the critical point steam tends to become drier or more superheated on throttling, and that in all cases there is a fall in temperature during the process. It will be noticed also how far the constant Total Heat lines deviate in general from being lines of constant temperature as required by perfect gases.

On calculating, from the steam tables, the volume occupied by the steam after throttling, it will be found that the product p_1v_1 is generally greater than p_2v_2, or the work done by the steam after passing the restriction is less than the work previously performed upon it, i.e.—

if p_1=200 lb. per sq. in., q=1, v_1=2·288, T_1=382° and p_1v_1= 457·6×144 ft.-lb.

p_2=10 lb. per sq. in., v_2=45·58, T_2=310° and p_2v_2=455·8×144 ft.-lb.

Exceptions will be found in the region of high pressure marked A, where throttling tends to produce condensation; i.e. throttling from 1000 lb.

to 450 lb. causes an increase of $p \cdot v$ of about 10 per cent., whereas throttling from 450 lb. to 10 lb. causes a fall of approximately the same amount.

To summarise this and the previous process it may be stated that :

The process of flow into a vacuum is one of constant internal energy, and no work is involved.

For a perfect gas the total energy (as $p_1v_1=p_2v_2$) and the temperature will be constant.

For an imperfect gas or condensible vapour the total energy generally falls and the temperature always falls.

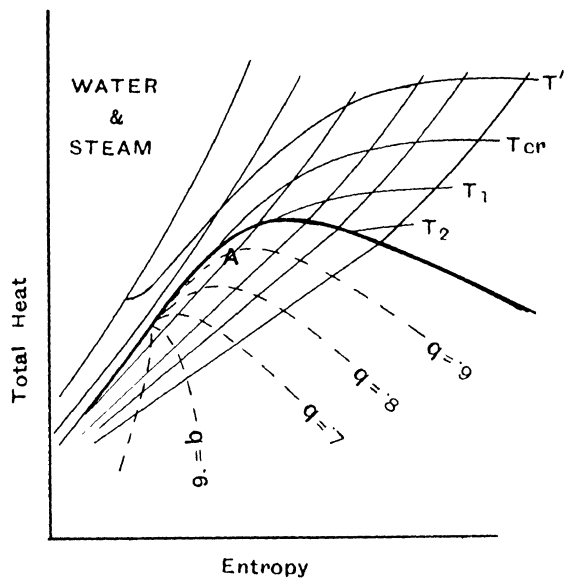


Fig. 36

(Curves for E=constant on the Tϕ chart, as indicated in Fig. 35, are of the same nature as those for H but more steep.)

The throttling process is one of constant Total Energy.

For a perfect gas the internal energy and temperature will also be constant and the net work done will be zero.

For an imperfect gas or condensible vapour the temperature always falls and the net work done, though generally negative, may be positive or zero.

(Note : The values for Total Heat given in the Steam Tables are for dry steam or for steam at a stated superheat, and corrections must be applied for the effect of wetness or for variations of superheat. Similar corrections must be made to the values of v.)

HEAT TRANSFER BY CONDUCTION

If a quantity of heat Q is conducted between two bodies at temperatures T_1 and T_2, then

$$\text{Loss of entropy of first body} = \frac{Q}{T_1}$$

$$\text{Gain of entropy of second body} = \frac{Q}{T_2}$$

As T_1 is essentially greater than T_2 there will be a gain of entropy on the whole

or
$$\phi_2 - \phi_1 = Q\left(\frac{1}{T_2} - \frac{1}{T_1}\right)$$

$$= Q\left(\frac{T_1 - T_2}{T_2 T_1}\right).$$

As the heat Q has been discharged at T_2 the net loss of energy is $(\phi_2 - \phi_1)T_2 = Q\frac{T_1 - T_2}{T_1}$, which is the whole of the work which could have been performed by an engine working on the Carnot cycle between temperatures T_1 and T_2; or the change of entropy multiplied by the minimum available temperature measures the energy degraded in the operation (see p. 54). Thermodynamically no useful work is performed. In practice, heat transfer by conduction plays a useful part in many operations, as in the transmission of heat through a boiler plate from the flue gases to the water, but $T_1 - T_2$ should be kept as small as possible consistent with the transfer of sufficient heat in a given time.

FRICTION

All work expended in friction will reappear as heat in the surrounding bodies.

If Q is the energy expended and T_2 is the room temperature, the net gain of entropy will be Q/T_2.

FLUID FRICTION

If steam expands in a suitably designed nozzle from p_1 to p_2, the loss of total heat per pound, $H_1 - H_2$, should result in an exactly equivalent gain in kinetic energy of the steam itself, and its velocity is calculable, i.e. $J(H_1 - H_2) = \frac{1}{2g}V^2$ where V is the velocity of the issuing jet of steam.

In all practical cases friction occurs between the steam and the nozzle,

and between the steam particles themselves. This will result in a reduction of the final velocity below that calculated, and a corresponding increase in the final heat of the steam above its theoretical value. The steam is said to be reheated, and the efficiency of the nozzle is given by

$$\frac{\text{Actual drop in Total Heat}}{\text{Theoretical drop in Total Heat}}$$

Consider the representation of this process on the *p.v.* diagram, Fig. 37. In both cases the steam will expand from pressure p_1 to pressure p_2 If *c* indicates the volume of 1 lb. of steam the theoretical expansion is along the adiabatic *cd*, and the useful work done will be the area *ecdf*.

With friction the final pressure will still be p_2, but the heat content—

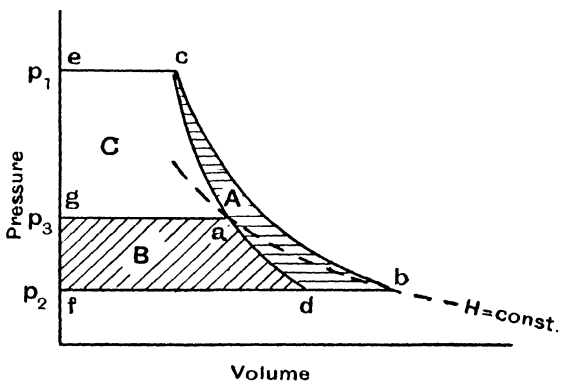


Fig. 37

and consequently the volume—of the steam will be increased beyond the values given by point *d*. If the final volume is given by *b*, expansion will have been along *cb*. The total area *ecbf* will be larger than *ecdf*, even though energy has been wasted in friction.

The useful work done in producing Kinetic Energy will be the equivalent of the actual Total Heat lost. If then through *b* we draw *ba*, a line of constant total heat, to cut the adiabatic *cd* in *a*, there would be the same total heat drop, and therefore the same useful work done, by expanding adiabatically from *c* to *a*, as by imperfect expansion from *c* to *b*. In both cases the useful work would be represented by the area C=*ecag*.

The lost portion of the useful work which could have been done by adiabatic expansion is then the area B. The total work expended in friction will be A+B, which is greater by the area A than the useful work lost.

The same argument can be followed on the Tϕ diagram (Fig. 38), drawn for superheated steam.

The ideal adiabatic expansion is now given by a line of constant

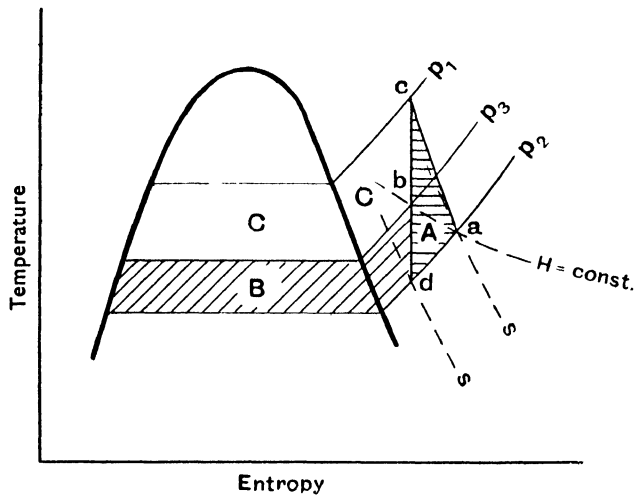


Fig. 38

entropy *cd*, whereas the actual expansion will be along some line *ca*, the final degree of superheat at *a* being greater than that at *d* by reason of the frictional reheating.

On the Mollier chart (Fig. 39) ideal expansion will again be from *c* to

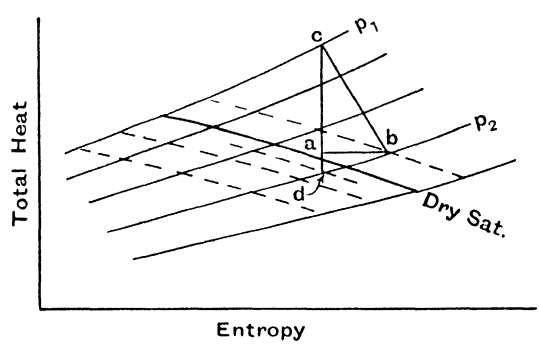


Fig. 39

d along a line of constant entropy from p_1 to p_2. Actual expansion will occur from *c* to *b*, where the degree of superheat is greater than at *d*. The line *ba*, of constant total heat, will now be horizontal.

The length *ca* is the heat usefully employed, *cd* the heat ideally avail-

able, *ad* is the loss of useful energy, and *ca*/*cd* is the efficiency of the nozzle.

The total friction work can only be found by calculating the " ineffective work " shown in previous diagrams by the area A:—

i.e. if the expansion is from 300 lb. and 300° superheat to 40 lb. with a nozzle efficiency of 75 per cent., then, from the diagram

cd=200 B.Th.U.

ca=75 per cent. of 200 B.Th.U.=150 B.Th.U.

Loss of KE=50 B.Th.U.

Also the volume at d with adiabatic expansion is 10·49 cu. ft.

and ,, ,, b ,, imperfect ,, 12·13 cu. ft.

The ineffective work is therefore

$$\frac{40\times144\times(12\cdot13-10\cdot49)}{778}\text{ B.Th.U.}=12\cdot1\text{ B.Th.U.}$$

and the total friction work=area A+B=50+12·1=62·1 B.Th.U.

It will be seen that, with the exception of the " total frictional work " which is seldom required, the Mollier diagram gives both the greatest accuracy and the least trouble when solving such problems.

It cannot be too clearly emphasised that, except for ideal operations, areas on the $p.v.$ and Tϕ diagrams do *not* entirely or necessarily represent useful work done.

Chapter VII

HOT AIR ENGINES

*GENERAL STATEMENT

The Hot Air Engine is a " Prime Mover " in that it derives its energy directly from the heat of combustion of a fuel, which may be solid, liquid, or gaseous. There are two classes : (1) the external combustion engine, in which the working fluid, air, is heated by the combustion of fuel outside the cylinder or vessel in which it is contained, the heat passing through the walls of the container, the method being the same as that by which water in a boiler is heated by an external furnace ; (2) the internal combustion engine, in which fuel is mixed with the air inside the cylinder and is burnt there, so that the supply of heat is not required to pass through the metal walls.

As was seen in the case of the Carnot cycle, the output of a heat engine increases as the temperature drop due to the performance of work is increased. In the external combustion engine the maximum temperature is restricted by the necessity of keeping the heating plates below the softening temperature, say 700° C., or a dull red heat, whereas in the internal combustion engine the walls may be maintained at a temperature of approximately 100° C. by water cooling while the temperature inside the cylinder may reach 2500°. In this case the working fluid at the end of expansion consists of the products of combustion, and must be replaced by fresh air and fuel after each cycle.

The name " Hot Air Engine " is normally applied to the external combustion group only, the second group being divided into gas, petrol, and oil engines, according to the nature of the fuel used. The first group only will be dealt with in this chapter.

Unlike the steam engine, a " Hot Air Engine " employs a working fluid which does not undergo a cyclic change from the liquid to the vapour state, so that pressure and temperature are independent variables, and high temperatures can be attained at convenient pressures instead of at the very high pressures which necessarily accompany high temperature steam. At the temperatures attained in practice only the simple gas laws have to be considered, and the engine is therefore free from the theoretical and practical complexities which are associated with steam engines, and from the complications met with in the case of the internal combustion engine for which the working fluid, by reason of the much greater

temperatures attained, and the nature of the products of combustion, behaves in a manner far removed from that of a " perfect " gas.

*CARNOT OR CONSTANT TEMPERATURE CYCLE

This cycle, described on p. 32, though of great theoretical interest, has never been employed in an engine owing chiefly to the very high ratio of maximum to mean effective pressure.

It should be noted that, from a theoretical point of view, the adiabatic operations serve the purpose of varying the temperature of the working fluid, between the upper and lower limits T_1 and T_2, without loss or gain of heat, so that condition (1), p. 46, is fulfilled.

*STIRLING ENGINE

In 1816 Dr. Stirling invented the " regenerator," which fulfilled the same purpose, of bridging reversibly between the isothermals, as the adiabatic operations of Carnot's cycle, while obviating the enormous piston displacements which the latter would have required.

The " regenerator " consists of a long vessel loosely packed with heat absorbing material, past which the working fluid can be made to flow, first in one direction and then the other. When in operation a uniform temperature gradient is set up in the absorbent material, from T_1 at one end of the vessel to T_2 at the other. On passing air at T_1 through the vessel from the hot to the cold end, heat is gradually lost by the air and temporarily stored in the packing, the final temperature of the air being approximately T_2. If the process is reversed the air will emerge at approximately T_1, having reclaimed from the packing the amount of heat previously stored, except for such a small amount of heat as, in practice, is lost from the surface of the vessel, and which must be replaced from the hot body. Were it possible to make the above approximate adjustments exact, then each step in the passage through the regenerator, and the passage as a whole, would become theoretically reversible.

The *p.v.* and $T\phi$ diagrams for Stirling's cycle are as shown in Fig. 40. The cycle is assumed to be performed by 1 lb. of air.

Stage AB.—Air at T_1 expands isothermally in the engine cylinder through a ratio r, taking in an amount of heat equivalent to the work done, or to $RT_1 \log_\epsilon r$. The energy stored up in the flywheel of the engine during this stroke carries the engine through the remainder of the cycle.

Stage BC.—The air passes at constant volume through the regenerator, its temperature falling to T_2 and the pressure undergoing a corresponding decrease. The amount of heat stored in the regenerator$=C_v(T_1-T_2)$.

Stage CD.—The air is now brought into contact with a cold body at temperature T_2, and is compressed isothermally by the engine piston to

its original volume. The amount of heat rejected by the air is equal to the work done or to $RT_2 \log_\epsilon r$.

Stage DA.—The air is now passed through the regenerator at constant volume in the reverse direction, its temperature rising from T_2 to T_1. The amount of heat taken in from the regenerator $= eC_v(T_1 - T_2)$, where e is the efficiency of the regenerator. The deficit of heat owing to imperfections of the regenerator is $C_v(T_1 - T_2)(1 - e)$.

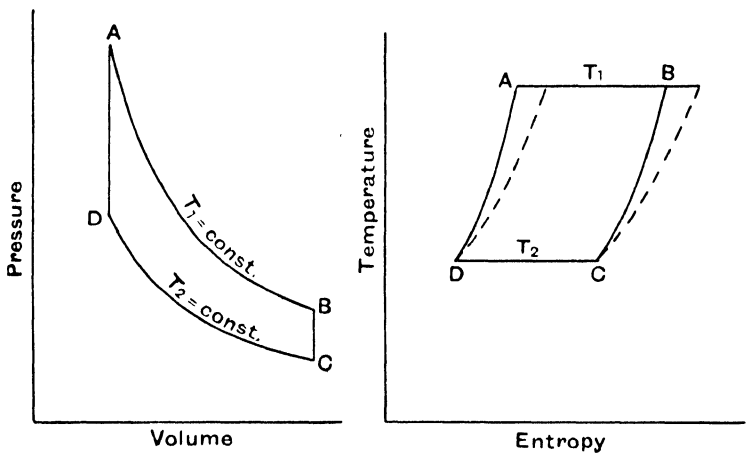


Fig. 40

$$\text{The efficiency of the cycle} = \frac{\text{work done}}{\text{heat supplied}}$$

Now the heat supplied $= RT_1 \log_\epsilon r + C_v(T_1 - T_2)(1 - e)$

and the heat rejected $= RT_2 \log_\epsilon r + C_v(T_1 - T_2)(1 - e)$

Hence the work done = heat supplied − heat rejected

$$= R(T_1 - T_2) \log_\epsilon r$$

and the efficiency

$$= \frac{R(T_1 - T_2) \log_\epsilon r}{RT_1 \log_\epsilon r + C_v(T_1 - T_2)(1 - e)} \quad . \quad (1)$$

If the regenerator could be made completely efficient, i.e. if $e = 1$,

$$\text{Efficiency of cycle} = \frac{R(T_1 - T_2) \log_\epsilon r}{RT_1 \log_\epsilon r} = \frac{T_1 - T_2}{T_1} \quad . \quad . \quad (2)$$

The actual value of e is between 90 and 95 per cent.

At a Dundee foundry in 1845 a double-acting Stirling engine, having a cylinder 16 in. in diameter and of 4 ft. stroke, and running at 28 r.p.m., gave about 50 I.H.P. with a consumption of 1·7 lb. of coal per I.H.P.

hour. The working temperatures were $t_1=650°$ F., and $t_2=150°$ F., and the ideal efficiency was therefore

$$\frac{650-150}{650+460}=0{\cdot}45 \text{ or } 45 \text{ per cent.}$$

This result was reduced to 0·3 by practical imperfections, and the coal consumption per B.H.P. per hour was about 2·7 lb. After three years' working the engine was abandoned owing to the burning out of the heating surface.

In order to obtain a rapid rate of heat transmission and a large output for a given size of cylinder, the air was initially compressed to a maximum pressure of 240 lb. per sq. in., the mean effective pressure then being 37·7 lb. per sq. in.

EXAMPLE.—A Stirling air engine works between temperatures of 800° F. and 90° F., the ratio of isothermal expansion being 2. Calculate the ideal efficiency when (*a*) the engine is fitted with a perfect regenerator; (*b*) when efficiency of regenerator is 0·9. Take $C_p=0{\cdot}2375$ and $C_v=0{\cdot}1691$.

Here $T_1=800+460=1260°$ F. abs.

$T_2=90+460=550°$ F. abs.

and $R=C_p-C_v=0{\cdot}2375-0{\cdot}1691=0{\cdot}0684$ B.Th.U. per lb.

(*a*) $$\text{Efficiency}=\frac{T_1-T_2}{T_1}=\frac{1260-550}{1260}=\frac{710}{1260}=0{\cdot}563$$

(*b*) Using equation

$$\text{Efficiency}=\frac{R(T_1-T_2)\log_\epsilon r}{RT_1\log_\epsilon r+(1-e)C_v(T_1-T_2)}$$

$$=\frac{0{\cdot}0684\times 710\times 0{\cdot}6931}{0{\cdot}0684\times 1260\times 0{\cdot}6931+0{\cdot}1\times 0{\cdot}1691\times 710}$$

$$=\frac{33{\cdot}67}{59{\cdot}74+12{\cdot}00}=\frac{33{\cdot}67}{71{\cdot}74}=0{\cdot}469.$$

*ERICSSON'S AIR ENGINE WITH REGENERATOR

This engine differed from Stirling's engine in that the passage of air through the regenerator was affected at constant pressure. Fig. 41 shows the *p.v.* diagram.

The Tϕ diagram is shown dotted in Fig. 40, C_p being substituted for C_v in the formula used when plotting the lines BC and DA.

The calculation of the theoretical efficiency is identical with that for Stirling's engine if C_p be substituted for C_v in the term involving the efficiency of the regenerator.

As fitted in the vessel "Ericsson" which plied on the Thames about

1852, the engine consisted of four working cylinders of 14-ft. bore and 6-ft. stroke, each with its own compressor cylinder. The engine speed was 9 r.p.m. and the mean effective pressure 2·12 lb. per sq. in. The maximum pressure was 22 lb. per sq. in.

As will be seen in Chapter XVI, the transmission of heat from furnace gases to air through a metal plate presents many difficulties, and though both the above engines gave reasonable efficiencies they were discarded owing to the burning of the heating surfaces.

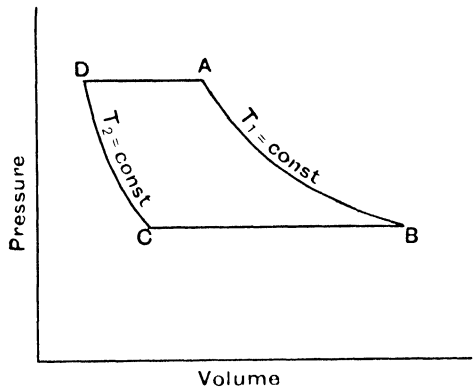


Fig. 41

EXAMPLE.—In the " Ericsson's " engine the temperature limits were 122° F. and 414° F., piston displacement per pound of air=22 cu. ft., ratio of expansion 1·5. Revs. per minute 9. Diameters of cylinders 14 ft., stroke 6 ft. Calculate: (1) Work done per pound of air per stroke; (2) thermal efficiency of engines (assuming efficiency of regenerator=0·9); (3) mean effective pressure; (4) indicated horse-power.

Here $T_1=414+460=874°$ F. abs.

(1) Work done per pound of air per stroke $=R \log_\epsilon r(T_1-T_2)$

and as $R=0{\cdot}0684$ B.Th.U. per lb. $=53{\cdot}2$ ft.-lb. per lb.

Work done $=53{\cdot}2\times0{\cdot}4055\times(414-122)$

$=6300$ ft.-lb.

(2) Heat supplied $=RT_1 \log_\epsilon r+(1-e)C_p(T_1-T_2)$

$=53{\cdot}2\times874\times0{\cdot}4055+0{\cdot}1\times184{\cdot}8\times292$

$=18{,}860+5396$

$=24{,}256$ ft.-lb.

Of the above quantity of heat, notice that the heat wasted owing to imperfection of the regenerator is 5396 ft.-lb., or the equivalent of 85 per cent. of the work done.

$$\text{Efficiency}=\frac{6300}{24{,}256}=0{\cdot}255.$$

Note.—If the efficiency of the regenerator is unity the engine efficiency is

$$\frac{6300}{18{,}860}=0{\cdot}33.$$

(3) *Mean effective pressure* $= \dfrac{\text{work done per pound per stroke}}{\text{volume swept through by piston}}$

$$= \frac{6300}{22}$$

$$= 286 \text{ lb. per sq. ft.}$$

$$= 2 \text{ lb. per sq. in. (approx.).}$$

(4) Area of each cylinder $= 0{\cdot}7854 \times 14^2 = 154$ sq. ft.

$\therefore$ Joint area of the four pistons $= 154 \times 4$

$\therefore$ Work done per minute $= 286 \times 154 \times 4 \times 6 \times 9$ ft.-lb.

$\therefore$ Indicated horse-power $= \dfrac{286 \times 154 \times 4 \times 6 \times 9}{33{,}000}$

$$= 288 \text{ I.H.P.}$$

*JOULE'S AIR ENGINE. THE CONSTANT PRESSURE CYCLE

In 1851 Dr. Joule proposed to use a cycle in which the heat was received and rejected at constant pressure. Though no engine was constructed to use this cycle, the reversed cycle was used extensively in

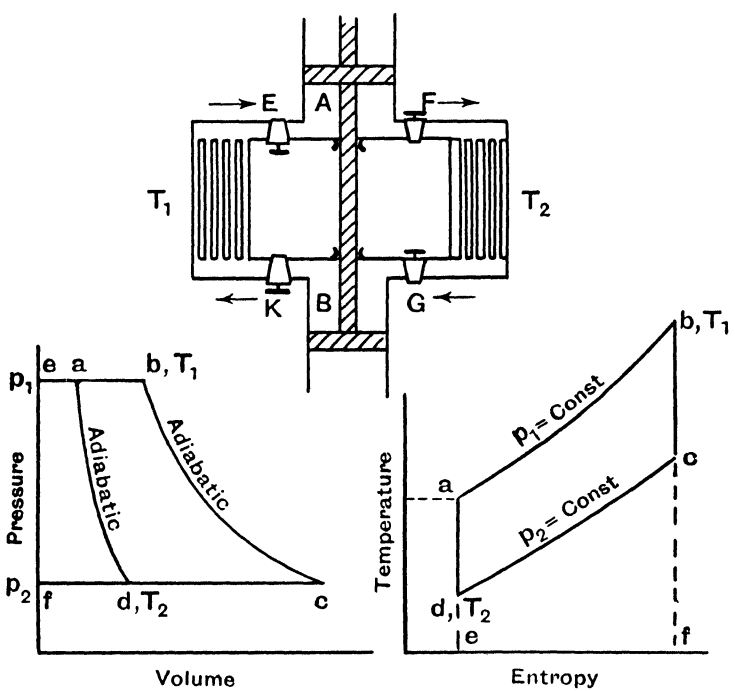


Fig. 42

refrigeration for a number of years (Chapter XV). Fig. 42 shows a diagrammatic sketch of the engine, and also the *p.v.* and Tϕ diagrams.

H is a chamber heated by a furnace to T_1, C is a similar chamber which is cooled externally, say by water, to T_2. A and B are a working or expansion cylinder and a pump cylinder respectively The chambers are large compared with the cylinders, so that the pressures in H and C are virtually unchanged by a single discharge of air into, or reception of air from, either A or B, or by the heating or cooling of a single charge.

The cycle of operations is as follows :

(1) Air having been heated in H to temperature T_1 and at pressure p_1 (point b) passes through valve E into A, where it is allowed to expand adiabatically (from *b* to *c*) until the pressure falls to p_2 and the temperature to T_c. The air is then discharged through F into C, where it is cooled at constant pressure to the lowest permissible temperature T_2. The *p.v.* diagram for cylinder A is *ebcf*.

(2) At the same time a similar weight of air is drawn from C, through G, at pressure p_2 and temperature T_2, into the compressor cylinder, where the pressure is raised to p_1 and the temperature to T_a by adiabatic compression. After compression it passes through K into H, where it is heated from T_a to T_1 at constant pressure. The *p.v.* diagram for cylinder B is *fdae*.

The net work done during the cycle is *ebcf*—*fdae*=*abcd*, measured in foot-pounds on the *p.v.* card, or *abcd* in heat units if the Tϕ diagram is used.

The efficiency can be calculated as follows :

$$\text{Heat taken in} = Q_H = C_p(T_b - T_a) \quad . \quad . \quad . \quad . \quad (1)$$

$$\text{Heat rejected} = Q_C = C_p(T_c - T_d) \quad . \quad . \quad . \quad . \quad (2)$$

and efficiency

$$= \frac{Q_H - Q_C}{Q_H}$$

$$= 1 - \frac{Q_C}{Q_H} = 1 - \frac{T_c - T_d}{T_b - T_a} \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Now, since the adiabatic expansion and compression both take place between the same terminal pressures, the ratios of expansion and compression are equal. Calling this common ratio *r* we have

$$\frac{T_a}{T_d} = \frac{T_b}{T_c} = r^{\gamma - 1} \text{ (see p. 19)}$$

Hence

$$\frac{T_b}{T_a} = \frac{T_c}{T_d}.$$

Subtracting 1 from each side of this equation

$$\frac{T_b - T_a}{T_a} = \frac{T_c - T_d}{T_d} \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

or

$$\frac{T_c - T_d}{T_b - T_a} = \frac{T_d}{T_a}$$

and the efficiency $$=1-\frac{T_d}{T_a}=1-\frac{T_c}{T_b}$$

or $$=\frac{T_a-T_d}{T_a}=\frac{T_b-T_c}{T_b}.$$

Now though T_b is the maximum temperature T_1, T_c is necessarily greater than the minimum temperature T_2, so that the efficiency is less than the value $\frac{T_1-T_2}{T_1}$ given by the Carnot cycle.

By using a single cylinder for both expansion and compression, and heating and cooling the air in the cylinder itself, instead of in H and C, the cycle closely resembles that used in certain classes of oil engines (see Chapter XIV).

EXAMPLES ON CHAPTER VII

1. In a Stirling air engine working between temperatures of 700° F. and 80° F., the ratio of isothermal expansion is 2. Calculate the ideal efficiency when (*a*) the engine is fitted with a perfect regenerator ; (*b*) when the efficiency of the regenerator is 0·9. Take $C_p=0{\cdot}2375$ and $C_v=0{\cdot}1691$.

2. Compare the efficiencies of : (*a*) A Stirling engine with perfect regenerator in which the maximum pressure is 140 lb. per sq. in. abs. and minimum pressure 15 lb. per sq. in. abs., and limits of temperature 750° F. and 70° F. ; and (*b*) a perfectly reversible steam engine working between the same limits of pressure.

3. If in a Joule air engine the ratio of expansion be 3, calculate the efficiency.

4. What will be the efficiency of the engine in question (1) if no regenerator is fitted ?

5. A Stirling engine, with perfect regenerator, works between pressures of 135 lb. per sq. in. abs., and 15 lb. per sq. in. abs., and temperatures 550° F. and 50° F. respectively. Calculate the mean effective pressure on the piston.

6. An air engine works on an ideal cycle in which heat is received at constant pressure, and rejected at constant volume. The pressure at the end of the suction stroke is 14 lb. per sq. in. abs., and temperature 40° C. The ratio of compression is 13 and the ratio of expansion 6·5. If the expansion and compression curves are adiabatic $pv^{1\cdot4}=$constant, find the mean pressure for the cycle and its efficiency.

7. An ideal air engine works on the following cycle : air is taken in at atmospheric pressure 14·7 lb. per sq. in. and at temperature 55° F., and is then compressed adiabatically so that at the end of the stroke the pressure is 550 lb. per sq. in. abs. Heat is then taken in at constant pressure and the air then expands adiabatically, the ratio of expansion being 5. The air is exhausted at the end of the expansion stroke, the heat being rejected at constant volume. Estimate the efficiency. (Take $C_p=0{\cdot}2375$ and $C_v=0{\cdot}1691$.)

8. An air engine works on an ideal cycle in which heat is received at constant pressure, and rejected at constant volume. The pressure before compression is 14 lb. per sq. in. abs., the ratio of compression 15·3 : 1, and the ratio of expansion 7·0 : 1. If the law of compression and expansion is $pv^{1\cdot4}=$constant, find the mean pressure of the cycle. If the temperature before compression is 60° F. estimate the efficiency, taking $C_p=0{\cdot}2375$ and $C_v=0{\cdot}1691$.

Chapter VIII

THEORY OF AIR COMPRESSORS AND MOTORS

*GENERAL STATEMENT

Though air compressors and motors are not strictly " heat engines " the theory underlying their action is closely akin to that of the air engine, and for this reason, and because of their great importance either for the transmission of power, or as auxiliaries used in its generation, they are included here.

*THE TRANSMISSION OF POWER BY COMPRESSED AIR

In general, this method of power transmission is employed because of its convenience rather than on account of its efficiency. Three units are involved :

(1) The compressor, by which the pressure of air is raised to, say, 100 lb. per sq. in.

(2) The receiver, or storage reservoir, and the transmission line which may be wholly or in part flexible.

(3) The air motor, which may resemble a small steam engine, using compressed air instead of water vapour under pressure, or may be of simpler form and incorporated in the body of a pneumatic riveter or other hand tool.

If it were possible to compress, transmit, and expand the air isothermally no heat losses would occur, the work done on the air during compression and by the air during expansion would be equal, and the efficiency of the system, neglecting friction losses, would be unity.

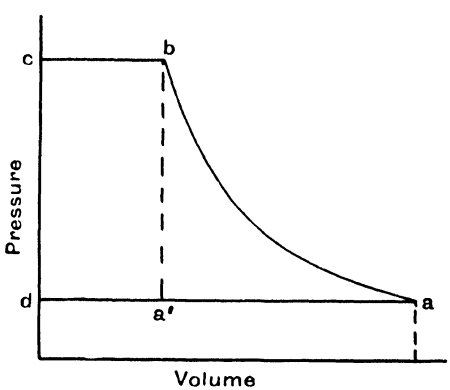


Fig. 43

Fig. 43 represents the theoretical *p.v.* diagrams involved. Air is drawn into the compressor from *d* to *a*, is compressed isothermally from *a* to *b*, and completely discharged into the receiver from *b* to *c*. The commencement, at

d, of the next outward stroke causes the inlet valve to open. The work done on the air per cycle will be given by the area *abcd*.

The same figure represents the action in the expansion cylinder, *cb* being the admission of air at constant pressure, *ba* its subsequent isothermal expansion, and *ad* its complete discharge into the atmosphere. The work done by the air per cycle will be the area *cbad*.

In practice isothermal compression and expansion are not realised, owing to the difficulties of extracting or adding heat at sufficiently great rates to maintain a constant temperature during compression and expansion, air having a very low thermal conductivity (see also film effect, p. 362). Though in some cases precautions are taken to make the operations as nearly isothermal as possible, as will be shown later, they normally approach fairly closely to the adiabatic form. In this case the air becomes

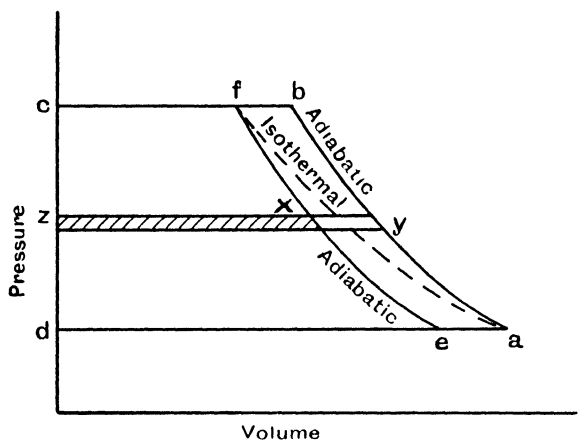


Fig. 44

heated on compression, but owing to the practical necessity of temporary storage or of transmission through considerable lengths of pipe, this heat is lost by radiation and conduction, and the air is at approximately atmospheric temperature once more before entering the motor. If, when utilising the high-pressure air, the motor works non-expansively (as in the case of most hand tools) there will be a sudden fall of temperature and pressure when the exhaust valve opens, and the diagram will be *cba'd*. If expansion is employed the temperature in the cylinder will gradually fall. In either case any moisture present will tend to be deposited, and may even freeze and block the passages or lock the valves of the machine. Certain precautions may have to be taken to prevent stoppages due to this cause.

Fig. 44 shows the combined *p.v.* diagrams assuming adiabatic compression and expansion. Figure *dabc* now represents the compressor

cycle, the work done on the air being given by the area of *dabc* instead of *dafc*, which represents the work which would have been done had the compression been isothermal.

Though the volume of air exhausted to the receiver is given by *bc*, this is reduced, by cooling, to *cf* by the time the motor cylinder is reached. Expansion will take place from *f* to *e*, and the work done will be given by *cfed*.

The overall efficiency will be given by $\frac{\text{area } cfed}{\text{area } dabc}$, and the energy lost through non-adiabatic working will be given by area *fbae*.

Now
$$\frac{T_b}{T_a}=\left(\frac{P_b}{P_a}\right)^{\gamma-1/\gamma}=\left(\frac{P_f}{P_e}\right)^{\gamma-1/\gamma}=\frac{T_f}{T_e} \quad . \quad . \quad . \quad (1)$$

and
$$\frac{V_f}{V_b}=\frac{T_f}{T_b}=\frac{T_a}{T_b}=\left(\frac{P_a}{P_b}\right)^{\gamma-1/\gamma}$$

But from p. 19, as *ab* and *ef* are adiabatics,

$$\frac{V_f}{V_b}=\frac{V_e}{V_a}=\frac{V_x}{V_y}=\text{constant},$$

where ZX, ZY are two elementary strips across the expansion and compression curves and at the same pressure.

Now the two work areas are the integrals of strips such as ZX and ZY, so that

$$\frac{cfed}{dabc}=\frac{V_x}{V_y}=\frac{V_f}{V_b}=\left(\frac{P_a}{P_b}\right)^{\gamma-1/\gamma}$$
$$=\left(\frac{1}{r}\right)^{\gamma-1/\gamma}$$

where r is the ratio of the maximum to the minimum pressure.

EXAMPLE.—The rapid decrease of efficiency $\left(\text{i.e. of } \frac{cfed}{dabc}\right)$ with increasing pressure ratios can be seen from the following figures :

r	$\frac{1}{r}$	$\left(\frac{1}{r}\right)^{\gamma-1/\gamma}=\left(\frac{1}{r}\right)^{0\cdot286}$
2	0·5	. . . 0·819
5	0·2	. . . 0·631
10	0·1	. . . 0·518
20	0·05	. . . 0·415
50	0·02	. . . 0·326
100	0·01	. . . 0·268
200	0·005	. . . 0·220

It will be seen that for the higher ratios some improved form of operation is almost a necessity.

EXAMPLE.—Air at 60° F. and 15 lb. per sq. in. pressure is drawn into a cylinder and compressed adiabatically to a pressure of 75 lb. above its original pressure, and is then expelled at this pressure into a receiver. In the receiver the compressed air cools down to its original temperature, and, in order to maintain a uniform pressure in the receiver, an equal weight of compressed air is drawn off and expanded isothermally in a working cylinder down to 15 lb. pressure. Calculate (*a*) the work spent per lb. of air in the compressor; (*b*) the work done per lb. of air in expanding; (*c*) the temperature of the air as it enters the receiver.

(*a*) Now, considering 1 cu. ft. of " free " air at 60° F.,

$$p_1v_1^{1\cdot4}=p_2v_2^{1\cdot4}$$

$$\therefore \left(\frac{v_2}{v_1}\right)^{1\cdot4}=\frac{p_1}{p_2}=\frac{15}{90}$$

$$\therefore 1\cdot4 \log v_2-1\cdot4 \log v_1 = \log 15-\log 90$$

$$\therefore 1\cdot4 \log v_2=1\cdot1761-1\cdot9542=-0\cdot7781$$

$$\therefore \log v_2=-\frac{0\cdot7781}{1\cdot4}=-0\cdot5559=\bar{1}\cdot4441$$

$$\therefore v_2=0\cdot2781 \text{ cu. ft.}$$

$$\textit{Work done per cycle} =p_2v_2-p_1v_1+\frac{p_2v_2-p_1v_1}{1\cdot4-1}=3\cdot5(p_2v_2-p_1v_1)$$

$$=3\cdot5\times144(90\times0\cdot2781-15)$$

$$=3\cdot5\times144(25\cdot029-15)$$

$$=3\cdot5\times144\times10\cdot029=5065 \text{ ft.-lb.}$$

Taking the volume of 1 lb. of air at 60° F.=13 cu. ft., i.e. $12\cdot39\times\frac{520}{492}$ cu. ft., we have—

Work done per pound of air=$5065\times13=65{,}845$ ft.-lb.

(*b*) Work done per cubic foot in expanding

$$=pv \log_\epsilon r=pv \log_\epsilon \frac{90}{15}$$

$$=15\times144\times2\cdot303\times0\cdot7782$$

$$=3870 \text{ ft.-lb.}$$

$\therefore$ Work done per pound=$3870\times13=50{,}310$ ft.-lb.

(*c*) $$\frac{T_2}{T_1}=\left(\frac{p^2}{p_1}\right)^{\frac{1\cdot4-1}{1\cdot4}}=\left(\frac{90}{15}\right)^{\frac{2}{7}}=6^{\frac{2}{7}}$$

And $T_1=60+460=520°$ abs.

$$\therefore T_2=520\times 6^{\frac{2}{7}}$$

$$\log T_2=\log 520+\tfrac{2}{7}\log 6=2{\cdot}7160+\tfrac{2}{7}\times 0{\cdot}7782$$

$$=2{\cdot}7160+0{\cdot}2223$$

$$\therefore \log T_2=2{\cdot}9383$$

$$\therefore T_2=867{\cdot}6° \text{ abs.}=867{\cdot}6-460=407{\cdot}6° \text{ F.}$$

*METHODS OF REDUCING THE LOSSES DURING COMPRESSION AND EXPANSION

It is obvious from Fig. 44 that these losses will be reduced by making the compression and expansions curves *ab*, *fe*, coincide as closely as possible with the isothermal *af*.

By fitting a water jacket to the cylinder a slight improvement can be effected, but by injecting a spray of water directly into the cylinder to absorb the heat generated during compression or to supply heat during expansion a marked advantage can be gained. For adiabatic compression the value of n, in the equation $pv^n=$constant, is 1·4, and for isothermal compression the value is 1·0. With spray injection an intermediate value of approximately 1·2 can be obtained, and the compression curve will lie between *ab* and *af*. For many purposes the presence of water mixed with the air is objectionable, however. Multi-stage operation will be considered in a separate paragraph.

*THE PRODUCTION OF COMPRESSED AIR. GENERAL STATEMENT

The operations involved in the production of compressed air will now be considered in more detail, and a brief note only will then be given concerning its utilisation, as the theory applying to the two cases is very similar.

It will be assumed, unless otherwise stated, that the actual compression curves are adiabatics, though under certain circumstances an improvement can be made by the use of spray injection. The equations will be affected by the nature of the compression curve only to the extent of the substitution of $n=1{\cdot}2$ or some other suitable value in place of $\gamma=1{\cdot}4$.

*THE SINGLE-STAGE COMPRESSOR

The *p.v.* diagram is shown in Fig. 45 for 1 lb. of air. Three processes are involved, as follows :

Suction from D to A, during which the work done by the air$=p_1v_1$.

Compression from A to B, when the work done on the air $=\frac{p_2v_2-p_1v_1}{\gamma-1}$

and the temperature rises to

$$T_2=T_1\cdot\left(\frac{P_2}{P_1}\right)^{\gamma-1/\gamma} \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Exhaust from B to C, while the work done on the air $=p_2v_2$.

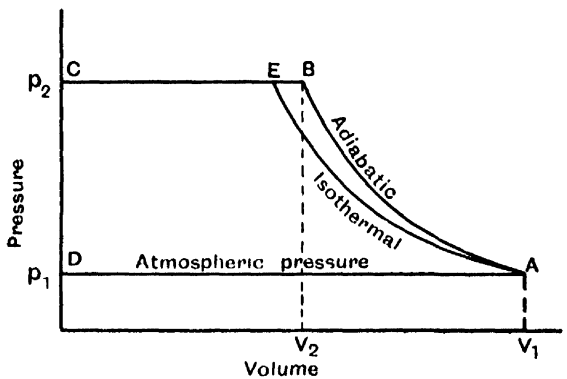


Fig. 45

Hence the total work done per pound of air per cycle during suction, adiabatic compression, and delivery, is given by the expression

$$W_{n=\gamma}=\frac{p_2v_2-p_1v_1}{\gamma-1}+p_2v_2-p_1v_1$$

$$=\frac{\gamma}{\gamma-1}(p_2v_2-p_1v_1) \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

or γ times the work of compression only

$$=R\cdot\frac{\gamma}{\gamma-1}(T_2-T_1) \text{ since } pv=RT \quad . \quad . \quad . \quad (3)$$

Substituting in (3) the value of T_2 from (1), we have

$$W_{n=\gamma}=R\cdot\frac{\gamma T_1}{\gamma-1}\left[\left(\frac{p_2}{p_1}\right)^{\gamma-1/\gamma}-1\right] \quad . \quad . \quad . \quad . \quad (4)$$

If, however, the compression is isothermal, the work done is

$$W_{n=1}=p_1v_1\log_\epsilon\frac{v_1}{v_2}+p_2v_2-p_1v_1=p_1v_1\log_\epsilon\frac{p_2}{p_1}$$

or

$$=RT_1\log_\epsilon\frac{p_2}{p_1} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

Treating the isothermal compressor as the ideal case, since it wastes no energy in uselessly heating the air, and calling its efficiency unity, the efficiency of the adiabatic compressor is

$$\text{Efficiency } \eta_{n=\gamma}=\frac{\mathrm{RT_1}\log_\epsilon \frac{p_2}{p_1}}{\mathrm{R}\cdot \mathrm{T_1}\cdot\frac{\gamma}{\gamma-1}\left[\left(\frac{p_2}{p_1}\right)^{\gamma-1/\gamma}-1\right]}$$

If r=ratio of compression pressures$=\frac{p_2}{p_1}$, we have

$$\eta_{n=\gamma}=\frac{\mathrm{RT_1}\log_\epsilon r}{\mathrm{RT_1}\frac{\gamma}{\gamma-1}[r^{\gamma-1/\gamma}-1]}=\frac{\log_\epsilon r}{\frac{\gamma}{\gamma-1}(r^{\gamma-1/\gamma}-1)} \quad . \quad . \quad (6)$$

It is convenient to express equation (3) in a simpler form, which can be done by substituting $\mathrm{C}_p/\mathrm{C}_v$ for γ and $\mathrm{C}_p-\mathrm{C}_v$ for R.

Hence
$$\mathrm{W}=(\mathrm{C}_p-\mathrm{C}_v)\frac{\mathrm{C}_p}{\mathrm{C}_v}\Big/\frac{\mathrm{C}_p-\mathrm{C}_v}{\mathrm{C}_v}\cdot(\mathrm{T_2}-\mathrm{T_1})$$
$$=\mathrm{C}_p(\mathrm{T_2}-\mathrm{T_1}).$$

EXAMPLE.[1]—12·39 cu. ft. of air at atmospheric pressure are compressed to a pressure of 200 lb. per sq. in. abs. Find the quantity of heat which must be added to or taken away from the air during the operation: (*a*) when the compression is isothermal; (*b*) when the compression is according to the law $pv^{1\cdot2}$=constant.

(*a*) The work done during isothermal compression is given by the equation

$$\mathrm{W}=p_1v_1\log_\epsilon r=p_2v_2\log_\epsilon r$$

Here $p_2=14\cdot7\times144$ lb. per sq. ft., and $v_2=12\cdot39$ cu. ft.

Also
$$\frac{v_2}{v_1}=r=\frac{p_1}{p_2}=\frac{200}{14\cdot7}=13\cdot6, \text{ and } \log_\epsilon r=2\cdot62.$$

Hence $\mathrm{W}=14\cdot7\times144\times12\cdot39\times2\cdot62$ ft.-lb.$=68{,}320$ ft.-lb.

The work done on the gas during compression is thus equal to 68,320

[1] In working examples on motors and compressors it is convenient to take p_1 as the higher pressure and p_2 as the lower, irrespective of which comes first. Thus $\frac{n}{n-1}(p_1v_1-p_2v_2)$ gives the work done *by* the gas during expansive working, and when applied to a compressor problem the value will be negative, or the work is done *on* the gas: i.e. the formula expresses the area of the *p.v.* diagram according to a certain convention, and the specific values applied determine whether the operation is one of expansion (positive value) or compression (negative value).

ft.-lb. Hence the heat taken away from the gas during compression is equal to 68,320 ft.-lb., or

$$\frac{68,320}{778}=87{\cdot}8 \text{ B.Th.U.} \quad \text{or} \quad \frac{68,320}{1400}=48{\cdot}4 \text{ C.H.U.}$$

(*b*) Work done $=\dfrac{p_1v_1}{n-1}\left\{1-\left(\dfrac{p_2}{p_1}\right)^{n-1/n}\right\}$

$$=\frac{14{\cdot}7\times 144\times 12{\cdot}39}{1{\cdot}2-1}\left\{1-\left(\frac{200}{14{\cdot}7}\right)^{1{\cdot}2-1/1{\cdot}2}\right\}$$

$$=\frac{2116\times 12{\cdot}39}{0{\cdot}2}\{1-(13{\cdot}6)^{1/6}\}$$

$$=131,086(1-1{\cdot}545)$$

$$=-71,440 \text{ ft.-lb.}$$

Hence the work done on the gas during the compression is 71,400 ft.-lb.

From equation (12), p. 26, the heat taken away from the gas during the compression is

$$\text{H}=\frac{\gamma-n}{\gamma-1}\times \text{work done}$$

$$=\frac{1{\cdot}4-1{\cdot}2}{1{\cdot}4-1}\times 71,440$$

$$=\frac{0{\cdot}2}{0{\cdot}4}\times 71,440$$

$$=35,720 \text{ ft.-lb. or } 45{\cdot}9 \text{ B.Th.U. or } 25{\cdot}5 \text{ C.H.U.}$$

EXAMPLE.—Calculate the size of the cylinder required for a double-acting air compressor of thirty-five indicated horse-power working as a simple compressor with spray injection, the law of compression being $pv^{1{\cdot}2}=$constant, the ratio of compression being from 15 lb. per sq. in. abs. to 120 lb. per sq. in. abs. Revolutions per minute=110, and average piston speed 560 ft. per minute. Neglect clearance.

If p_1 and p_2 are the higher and lower pressures respectively,

$$\text{Work done per pound of air}=p_1v_1-p_2v_2+\frac{p_1v_1-p_2v_2}{n-1}$$

$$\therefore \text{ Mean effective pressure}=p_1\frac{v_1}{v_2}-p_2+\frac{p_1\dfrac{v_1}{v_2}-p_2}{n-1} \quad . \quad (1)$$

Also $$p_1v_1^{\,n}=p_2v_2^{\,n}$$

$$\therefore \frac{v_2}{v_1}=\left(\frac{p_1}{p_2}\right)^{\frac{1}{n}}=\left(\frac{120}{15}\right)^{\frac{1}{1{\cdot}2}}=8^{\frac{5}{6}}$$

$$\therefore \log \frac{v_2}{v_1} = \tfrac{5}{6} \log 8 = \tfrac{5}{6} \times 0{\cdot}9031 = 0{\cdot}7526$$

$$\therefore \frac{v_2}{v_1} = 5{\cdot}657$$

Substituting this value of $\frac{v_2}{v_1}$ in equation (1), we have

$$\text{Mean effective pressure} = \frac{120}{5{\cdot}657} - 15 \times \frac{\frac{120}{5{\cdot}657} - 15}{0{\cdot}2}$$

$$= (21{\cdot}21 - 15) \times \frac{1{\cdot}2}{0{\cdot}2} = 37{\cdot}26 \text{ lb. per sq. in.}$$

Let A = area of cylinder in square inches

then $37{\cdot}26 \times 560 \times A = 35 \times 33{,}000$

$$\therefore A = \frac{35 \times 33{,}000}{37{\cdot}26 \times 560} = 55{\cdot}37 \text{ square inches.}$$

$$\therefore \text{Diameter} = \sqrt{55{\cdot}37} \div 0{\cdot}7854 = 8{\cdot}40 \text{ inches.}$$

Since revolutions per minute = 110, and there are 220 working strokes per minute (the compressor being double-acting), the stroke is

$$560 \div 220 = 2{\cdot}545 \text{ ft.} = 2 \text{ ft. } 6\tfrac{1}{2} \text{ in.}$$

*MULTI-STAGE COMPRESSION. GENERAL STATEMENT

This method of compression is very commonly employed, especially when the final pressure required is high. Not only is the work of compression much reduced, but there is an additional advantage which will be considered in the paragraph concerning clearance.

Fig. 46 represents the *p.v.* diagram for a two-stage compressor, and Fig. 47 that for a three-stage machine.

In the former case the machine is fitted with two cylinders, usually operated from the same crankshaft, and having volumes given by V_1 and V_2. In practice the strokes of the pistons are usually equal, but the areas are in the ratio of V_1/V_2.

The air is first compressed in the larger cylinder, for which the diagram is shown by FABG, and is then exhausted at pressure p_2 into an " intercooler " (intermediate receiver and cooler). Here the air is cooled to its original temperature T_A by means of cold water, and contracts from volume V_B to volume V_C. It is now drawn into the smaller cylinder and compressed to its final pressure p_3, after which it is discharged into the final receiver and ultimately cools to the original temperature T_A,

shrinking meanwhile to volume V_K, ACK being an isothermal line. The diagram for the second cylinder is GCDE.

In the three-stage compressor three cylinders and two inter-coolers are used.

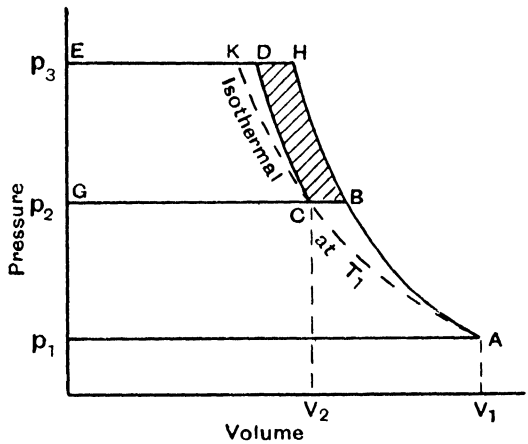


Fig. 46

The saving effected by these machines when compared with a single-stage compressor is shown shaded in Figs. 46 and 47.

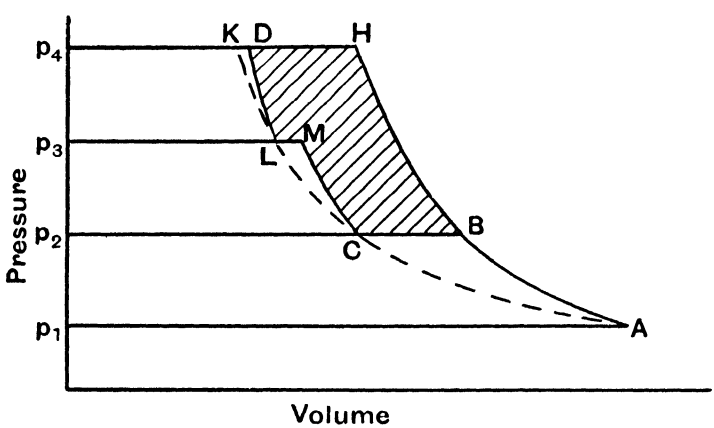


Fig. 47

*TWO-STAGE ADIABATIC COMPRESSOR

The *p.v.* diagram is shown in Fig. 46. The work done per pound of air per cycle is now

W=area ABGF+area CDEG

$$=RT_1\frac{\gamma}{\gamma-1}\left[\left(\frac{p_2}{p_1}\right)^{\gamma-1/\gamma}-1\right]+RT_1\frac{\gamma}{\gamma-1}\left[\left(\frac{p_3}{p_2}\right)^{\gamma-1/\gamma}-1\right] \quad . \quad (1a)$$

(Note that $T_A=T_C=T_K=T_1$)

$$=RT_1\frac{\gamma}{\gamma-1}\left[\left(\frac{p_2}{p_1}\right)^{\gamma-1/\gamma}+\left(\frac{p_3}{p_2}\right)^{\gamma-1/\gamma}-2\right] \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Differentiating (1) with respect to p_2, and equating to zero to find the minimum value, we have, since p_1 and p_3 are constants,

$$\frac{\gamma-1}{\gamma}\left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}-1}\times\frac{1}{p_1}+\frac{\gamma-1}{\gamma}\left(\frac{p_3}{p_2}\right)^{\frac{\gamma-1}{\gamma}-1}\times-\frac{p_3}{p_2^2}=0$$

or

$$\frac{\gamma-1}{\gamma}\cdot p_2{}^{-1/\gamma}\cdot\frac{p_1{}^{1/\gamma}}{p_1}+\frac{1-\gamma}{\gamma}\cdot p_3{}^{1-1/\gamma}\cdot p_2{}^{1/\gamma-2}=0$$

Dividing by $p_2{}^{-1/\gamma}$ we have

$$\frac{\gamma-1}{\gamma}p_1{}^{1-\gamma/\gamma}=\frac{\gamma-1}{\gamma}p_3{}^{\gamma-1/\gamma}\cdot p_2{}^{2(1-\gamma)/\gamma}$$

Therefore

$$p_2{}^{2(1-\gamma)/\gamma}=\frac{p_1{}^{1-\gamma/\gamma}}{p_3{}^{\gamma-1/\gamma}}=p_1{}^{1-\gamma/\gamma}p_3{}^{1-\gamma/\gamma}$$

or

$$p_2=\sqrt{p_1p_3}, \quad \text{or} \quad \frac{p_2}{p_1}=\frac{p_3}{p_2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

This is the condition for least work, and, when applied to equation (1a), shows that the same amount of work is done in both stages. When this condition is satisfied then

$$\frac{V_1}{V_2}=\frac{p_2}{p_1}=\sqrt{\frac{p_3}{p_1}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Also since

$$\frac{T_2}{T_1}=\left(\frac{p_2}{p_1}\right)^{\gamma-1/\gamma}$$

and

$$\frac{T_3}{T_1}=\left(\frac{p_3}{p_2}\right)^{\gamma-1\,\gamma}$$

(Note that $T_c=T_1$)

$$\frac{T_2}{T_1}=\frac{T_3}{T_1} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

Equation (4) shows that the ratio of the initial and final temperatures during compression is the same for each stage when the work of compression is a minimum.

Substituting from (2) in (1) we have the statement of the minimum work done

or

$$W=RT_1\frac{2\gamma}{\gamma-1}\left[\left(\frac{p_3}{p_1}\right)^{\gamma-1/2\gamma}-1\right] \quad . \quad . \quad . \quad . \quad (5)$$

In this case $r=\frac{p_3}{p_1}$, and the efficiency is given by

$$\eta=\frac{\log_\epsilon r}{\frac{2\gamma}{\gamma-1}(r^{\gamma-1/2\gamma}-1)} \quad . \quad . \quad . \quad . \quad . \quad (6)$$

(Note that for points on the isothermal curve the pressure ratio

$$\frac{p_3}{p_1}=\frac{v_1}{v_3}=r.)$$

*THREE-STAGE ADIABATIC COMPRESSION

The $p.v.$ diagram is shown in Fig. 48.

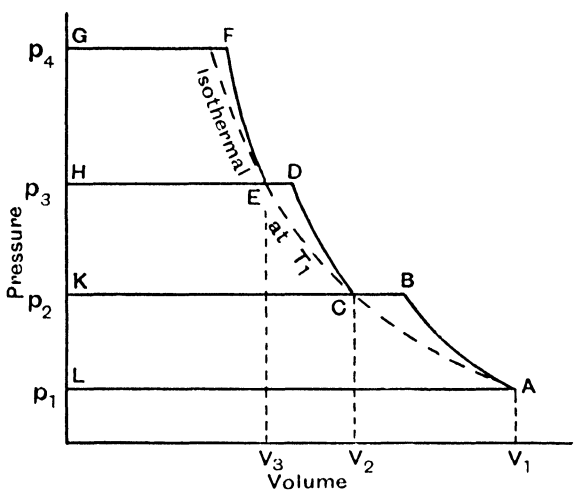


Fig. 48

The work done per pound of air per cycle is now

W=area ABKL+area CDHK+area EFGH

$$=RT_1\frac{\gamma}{\gamma-1}\left[\left(\frac{p_2}{p_1}\right)^{\gamma-1/\gamma}-1\right]+RT_1\frac{\gamma}{\gamma-1}\left[\left(\frac{p_3}{p_2}\right)^{\gamma-1/\gamma}-1\right]$$

$$+RT_1\frac{\gamma}{\gamma-1}\left[\left(\frac{p_4}{p_3}\right)^{\gamma-1/\gamma}-1\right] \quad . \quad . \quad . \quad . \quad (1)$$

or $$W=RT_1\cdot\frac{\gamma}{\gamma-1}\left[\left(\frac{p_2}{p_1}\right)^{\gamma-1/\gamma}+\left(\frac{p_3}{p_2}\right)^{\gamma-1/\gamma}+\left(\frac{p_4}{p_3}\right)^{\gamma-1/\gamma}-3\right] . \quad (2)$$

For the least work done the same conditions will hold as for the two-stage process, so that

$$\frac{p_2}{p_1}=\frac{p_3}{p_2}=\frac{p_4}{p_3}$$

or, generally, $=\frac{p_n}{p_{n-1}}$, where n=number of stages, and the ratio of the temperatures during each stage will be the same.

Hence $$\frac{p_2}{p_1}=\frac{p_4}{p_3}$$

$$p_2=\frac{p_1p_4}{p_3}=\frac{p_1p_4}{\sqrt{p_2p_4}}=\frac{p_1p_4^{\frac{1}{2}}}{p_2^{\frac{1}{2}}}$$

Therefore $$p_2^{3/2}=p_1\cdot p_4^{\frac{1}{2}}$$

or
$$\frac{p_2}{p_1}=\left(\frac{p_4}{p_1}\right)^{1/3}=\frac{p_3}{p_2}=\frac{p_4}{p_3} \quad . \quad . \quad . \quad . \quad (3)$$

Substituting from (3) in (2) we have

$$W=RT_1\cdot\frac{3\gamma}{\gamma-1}\left[\left(\frac{p_4}{p_1}\right)^{\gamma-1/3\gamma}-1\right] \quad . \quad . \quad . \quad (4)$$

and the efficiency is $$\eta=\frac{\log_\epsilon r}{\frac{3\gamma}{\gamma-1}[r^{\gamma-1/3\gamma}-1]} \text{ where } r=\frac{p_4}{p_1} \quad . \quad . \quad . \quad (5)$$

The ratio of the volumes of the cylinders will be

$$\frac{V_1}{V_2}=\frac{V_2}{V_3}=\left(\frac{p_4}{p_1}\right)^{1/3} \quad \text{or} \quad \sqrt[3]{\frac{p_4}{p_1}}.$$

EXAMPLE.—An air compressor is required to compress from one to ten atmospheres. Estimate the amount of work which must be expended for each pound of air compressed in each of the following cases. Calculate also the efficiency in each case, the initial temperature being 60° F.

Case I. Simple compressor with adiabatic compression.

Work done per pound of air $=\frac{n}{n-1}(p_1v_1-p_2v_2)$

$$=RT_2\frac{n}{n-1}\left\{\left(\frac{p_1}{p_2}\right)^{\frac{n-1}{n}}-1\right\}$$

$$=\frac{53{\cdot}18\times1{\cdot}4\times520}{1{\cdot}4-1}\left[\left(\frac{10}{1}\right)^{\frac{1{\cdot}4-1}{1{\cdot}4}}-1\right]$$

$$=\frac{53{\cdot}18\times1{\cdot}4\times520}{0{\cdot}4}[1{\cdot}931-1]=90{,}260 \text{ ft.-lb.}$$

The isothermal work would be $RT_2 \log_\epsilon r = RT_2 \log_\epsilon 10$.

and $$\eta = \frac{\log_\epsilon 10}{\frac{1{\cdot}4}{1{\cdot}4-1}\left(10^{\frac{1{\cdot}4-1}{1{\cdot}4}}-1\right)} = \frac{2{\cdot}303}{3{\cdot}5\times 0{\cdot}931} = 0{\cdot}706, \text{ or } 70{\cdot}6 \text{ per cent.}$$

Case II. Simple compressor with spray injection.

Here $n = 1{\cdot}2$.

$$\text{Work per pound} = \frac{53{\cdot}18\times 1{\cdot}2\times 520}{0{\cdot}2}\left[10^{\frac{0{\cdot}2}{1{\cdot}2}}-1\right]$$

$$= 53{\cdot}18\times 520\times 6\times 0{\cdot}466 = 77{,}110 \text{ ft.-lb.}$$

$$\eta = \frac{\log_\epsilon 10}{1{\cdot}2\times\left(10^{\frac{0{\cdot}2}{1{\cdot}2}}-1\right)} = \frac{2{\cdot}303}{6\times 0{\cdot}466}$$

$$= 0{\cdot}823 \text{ or } 82{\cdot}3 \text{ per cent.}$$

Case III. Two-stage adiabatic compression.

By equation (5), p. 103,

$$W = \frac{53{\cdot}18\times 520\times 2\times 1{\cdot}4}{1{\cdot}4-1}\left[10^{\frac{0{\cdot}4}{2{\cdot}8}}-1\right]$$

$$= \frac{53{\cdot}18\times 521\times 2{\cdot}8}{0{\cdot}4}\times 0{\cdot}39 = 75{,}630 \text{ ft.-lb.}$$

By equation (6), p. 104,

$$\eta = \frac{\log_\epsilon 10}{\frac{2{\cdot}8}{0{\cdot}4}\times 0{\cdot}39} = \frac{2{\cdot}303}{7\times 0{\cdot}39} = 0{\cdot}843, \text{ or } 84{\cdot}3 \text{ per cent.}$$

Case IV. Two-stage compression with spray injection.

Again $n = 1{\cdot}2$.

$$W = \frac{53{\cdot}18\times 520\times 2{\cdot}4}{0{\cdot}2}\left[10^{\frac{0{\cdot}2}{2{\cdot}4}}-1\right] = 53{\cdot}18\times 520\times 12\times 0{\cdot}212$$

$$= 70{,}470 \text{ ft.-lb.}$$

and $$\eta = \frac{2{\cdot}303}{12\times 0{\cdot}212} = 0{\cdot}915, \text{ or } 91{\cdot}5 \text{ per cent.}$$

Case V. Three-stage compressor with adiabatic compression.

By equation (4), p. 105,

$$W = \frac{53{\cdot}18\times 520\times 4{\cdot}2}{0{\cdot}4}\left[10^{\frac{0{\cdot}4}{4{\cdot}2}}-1\right] = 53{\cdot}18\times 520\times 10{\cdot}5\times 0{\cdot}246$$

$$= 71{,}550 \text{ ft.-lb.}$$

By equation (5), p. 105,

$$\eta=\frac{2\cdot303}{10\cdot5\times0\cdot246}=0\cdot891, \text{ or } 89\cdot1 \text{ per cent.}$$

Case VI. Three-stage compression with spray injection.

As $n=1\cdot2$.

$$W=\frac{53\cdot18\times520\times3\cdot6}{0\cdot2}\left[10^{\frac{0\cdot2}{3\cdot6}}-1\right]=53\cdot18\times520\times18\times0\cdot136$$

$$=67{,}810 \text{ ft.-lb.}$$

and $$\eta=\frac{2\cdot303}{18\times0\cdot136}=0\cdot940, \text{ or } 94\cdot0 \text{ per cent.}$$

EXAMPLE.—A single-acting tandem two-stage air compressor is to be designed to supply the blast air for a Diesel engine. The compressor is to run at 250 r.p.m., to have a stroke of 6 in., and to deliver $\frac{1}{2}$ lb. of air per minute at a pressure of 1000 lb. per sq. in. An intercooler is fitted between the stages, and the law of compression may be taken as $pv^{1\cdot3}=K$. Find the most suitable diameters for the low and high-pressure cylinders, and the horse-power absorbed by the compressor, neglecting friction and clearance effects. The suction temperature and pressure are to be 150° F. and 14 lb. per sq. in.

The intermediate pressure $p_m=\sqrt{p_1p_2}=\sqrt{1400}=118\cdot3$ lb. per sq. in. (note, p_1 is the high pressure and p_2 the suction pressure). The volume of the low-pressure cylinder per pound is

$$v_2=\frac{53\cdot18\times610}{14\times144}=16\cdot09 \text{ cu. ft.}$$

The volume to be delivered per stroke is

$$\frac{0\cdot5}{250}\times16\cdot09=0\cdot03217 \text{ cu. ft.}$$

As the stroke is 6 in. the volume swept per stroke

$$=\frac{\pi d^2}{4}\times6 \text{ cu. in.}$$

or $$\frac{\pi d^2}{4}\times6=0\cdot03217\times12\times144$$

Hence low-pressure diameter $$=\sqrt{\frac{0\cdot03217\times1728\times4}{6\pi}}=3\cdot435 \text{ in.}$$

For the diameter of the high-pressure cylinder, the air having been cooled to the original temperature,

$$p_m v_1 = p_2 v_2 \quad \text{or} \quad v_1 = \frac{6\pi d_1^2}{4} \text{ cu. in.}$$

therefore $$\frac{6\pi d_1^2}{4} = \frac{6\pi d_2^2}{4} \times \frac{14}{118{\cdot}3}$$

Hence diameter of high-pressure cylinder $= 3{\cdot}435\sqrt{\frac{14}{118{\cdot}3}} = 1{\cdot}182$ in.

If v_m is the volume of air delivered from the low-pressure cylinder, work done in low-pressure cycle

$$= \frac{n}{n-1}(p_m v_m - p_2 v_2)$$

$$= \frac{n}{n-1}(p_2 v_2)\left[\left(\frac{p_m}{p_2}\right)^{n-1/n} - 1\right] \text{ ft.-lb.}$$

The work done in the low-pressure cylinder per pound is

$$W = \frac{1{\cdot}3}{0{\cdot}3} \times 14 \times 144 \times 16{\cdot}09\left[\left(\frac{118{\cdot}3}{14}\right)^{0{\cdot}3/1{\cdot}3} - 1\right]$$

$$= 89{,}550 \text{ ft.-lb.}$$

Hence the work done in both cylinders per $\frac{1}{2}$ lb. of air

$$= 2 \times \tfrac{1}{2} \times 89{,}550 \text{ ft.-lb. per minute}$$

and $$\text{horse-power} = \frac{89{,}550}{33{,}000} = 2{\cdot}713.$$

*AIR MOTORS

For small hand tools the air from the high pressure reservoir is used non-expansively, a process which, though uneconomical, has the advantage of simplicity.

For larger machines the air can be expanded in a single stage, either adiabatically or with a coefficient $n = 1{\cdot}2$ approximately if a water spray is used, the purpose in this case being to warm the air during expansion. The diagram for single stage expansion is shown in Fig. 49. The loss of work owing to non-isothermal expansion is shown shaded.

Here the work done per pound $= W = \frac{\gamma}{\gamma - 1}(p_1 v_1 - p_2 v_2)$ . . . (1)

$$= \frac{\gamma}{\gamma - 1} \cdot R(T_1 - T_2) \quad . . \quad (2)$$

Now, $$T_2 = T_1\left(\frac{p_2}{p_1}\right)^{\gamma - 1/\gamma} \quad . . . . . \quad (3)$$

Hence, $$W = RT_1 \frac{\gamma}{\gamma - 1}\left[1 - \left(\frac{p_2}{p_1}\right)^{\gamma - 1/\gamma}\right] \quad . . . \quad (4)$$

The work done during isothermal expansion would be

$$W = RT_1 \log_\epsilon \frac{p_1}{p_2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

so that the efficiency of the simple adiabatic motor is

$$\eta = \frac{\frac{\gamma}{\gamma-1}\left[1-\left(\frac{p_2}{p_1}\right)^{\gamma-1/\gamma}\right]}{\log_\epsilon \frac{p_1}{p_2}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

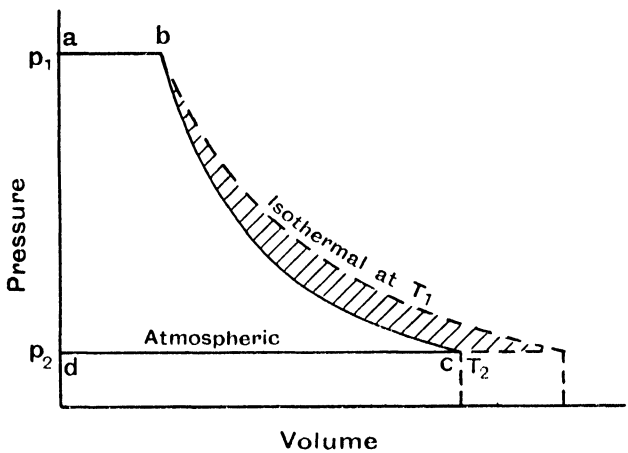


Fig. 49

(N.B. In the formula $W = \frac{\gamma}{\gamma-1}(p_1v_1 - p_2v_2)$ the first term is that which includes the higher pressure. This is the initial pressure for a motor, but the final pressure for a compressor.)

TWO-STAGE ADIABATIC EXPANSION. PREHEATING

The *p.v.* diagram is shown in Fig. 50. Multi-stage expansion may also be employed, the formulæ being deduced as for the compressor.

Both from a thermodynamic and a practical point of view, a better method than that considered above is to preheat the air to some temperature T_1 so that at the end of expansion the temperature has fallen approximately to atmospheric. Intermediate heating is then carried out to this initial temperature. The *p.v.* diagram is as shown in Fig. 50, but T_1 is now above atmospheric temperature, which is only attained at the end of each expansion, *i.e.* as the isothermal T_2.

The work done will be given by

$$W_1+W_2=RT_1\frac{2\gamma}{\gamma-1}\left[1-\left(\frac{p_3}{p_1}\right)^{\gamma-1/\gamma}\right]$$

and is therefore dependent on T_1, but the efficiency, as compared with a single-stage isothermal motor *working at the temperature of preheat* T_1, will be the same as in the previous case, and will be independent of this temperature.

The principle employed is very similar to that used in two-stage com-

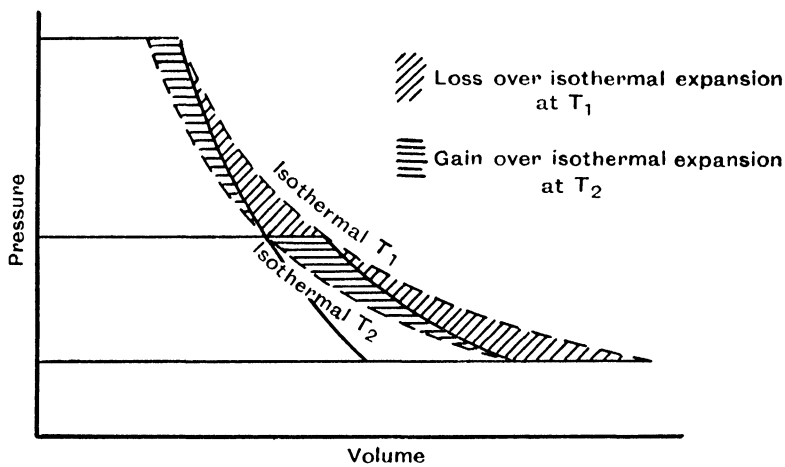


Fig. 50

pression, but in this case heat is supplied to the intermediate receiver to restore the temperature to T_1.

The condition for maximum output again occurs when $p_2=p_1p_3$, i.e. when the same work is done in both stages, and the efficiency is given by

$$\eta=\frac{\log_\epsilon r}{\frac{\gamma}{\gamma-1}\left[1-\left(\frac{1}{r}\right)^{\gamma-1/2\gamma}\right]} \text{ where } r=\frac{p_1}{p_3} \quad . \quad . \quad . \quad (1)$$

*EFFECT OF CLEARANCE

The clearance in the compressor cylinder should be as small as possible since its effect is to reduce the output. This is illustrated in Fig. 51.

The discharge of air from the compressor cylinder commences at *b*, but, instead of continuing to *k*, is completed only as far as the point *c*, the volume *ck* representing the clearance necessarily left between the piston and cylinder head to prevent mechanical contact with the head or valves. On the following outstroke of the piston the pressure will not at once fall

to atmospheric pressure, as the residual gas will expand, nearly adiabatically, along the curve *cd*, and the suction valve will not open to admit a fresh charge till the point *d* is reached. The effective portion of the volume swept by the piston will then be $(v_a - v_d)$ instead of $(v_a - v_k)$.

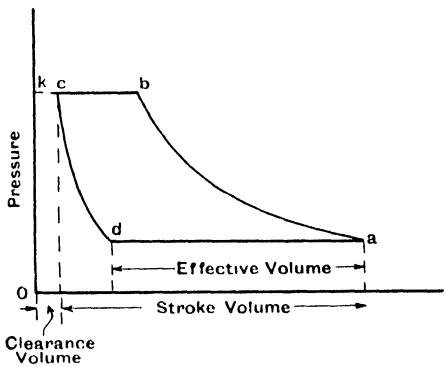


Fig. 51

The Volumetric Efficiency of an air compressor may be defined as the ratio of the volume of air at atmospheric temperature and pressure actually drawn in per stroke when compared with the volume swept by the piston. In the case contemplated in Fig. 51 this would be the ratio of $(v_a - v_d)$ to $(v_a - v_k)$.

As an extreme illustration, and assuming isothermal expansion for simplicity, if the clearance volume *ck* is $\frac{1}{10}$ of the swept volume and the maximum pressure is 10 atmospheres, *d* will coincide with *a*, and no charge will be drawn in.

The reduction of volumetric efficiency, whether caused by the clearance space or by restriction to flow provided by valves or passages, involves a corresponding increase in the number of strokes required for a given output, and hence of the energy wasted in mechanical friction and windage.

A great advantage of the multi-stage compressor using the same piston stroke for all cylinders is that the clearance volume of the high-pressure (or small volume) cylinders can be made extremely small.

EXAMPLE.—In a single-acting, single-stage compressor of 6-in. bore and 6-in. stroke, running at 100 r.p.m., the clearance volume is one-fiftieth of the swept volume. Find the weight of air delivered by the compressor per minute against a pressure of 20 atmospheres, and the horse-power required, neglecting mechanical losses. Assume the suction temperature and pressure to be 65° F. and 14·7 lb. per sq. in., and that compression and expansion take place according to the law $pv^{1\cdot2} = K$.

If the figures 4, 3, 2, and 1 refer to points on the indicator diagram corresponding to the beginning and end of expansion of the residual charge, and the beginning and end of compression, then

$$\text{Volume swept} = v_2 - v_4 = \frac{\pi \times (0\cdot5)^2}{4} \times 0\cdot5 = 0\cdot09818 \text{ cu. ft.}$$

$$\text{and the clearance volume} = v_4 = \frac{0\cdot09818}{50} = 0\cdot001963 \text{ cu. ft.}$$

Hence $v_2 = 0\cdot09818 + 0\cdot0019636 = 0\cdot10014$ cu. ft.

The residual charge will expand from v_4 to v_3, where

$$p_3 v_3^{1\cdot2} = p_4 v_4^{1\cdot2}$$

and $$v_3 = \left(\frac{p_4}{p_3}\right)^{1/1\cdot2} \times v_4 = 20^{1/1\cdot2} \times 0{\cdot}001963 = 0{\cdot}02383 \text{ cu. ft.}$$

The volume of air delivered per stroke at suction temperature and pressure is

$$0{\cdot}09818 - 0{\cdot}02383 = 0{\cdot}07435 \text{ cu. ft.}$$

At 14·7 lb. per sq. in. and 65° F., as $pv = RT$

$$v = \frac{53{\cdot}2 \times 525}{14{\cdot}7 \times 144} = 13{\cdot}195 \text{ cu. ft.}$$

and the weight of air per minute $= \dfrac{7{\cdot}435}{13{\cdot}195} = 0{\cdot}5661$ lb. per min.

The horse-power required is obtained as follows :

Work per stroke $= \dfrac{n}{n-1}\{(p_1v_1 - p_2v_2) - (p_4v_4 - p_3v_3)\}$ ft.-lb.

or $$W = \frac{1{\cdot}2}{0{\cdot}2} \times 144 \times 14{\cdot}7[(20v_1 - v_2) - (20v_4 - v_3)]$$

Now $$p_1 v_1^{1\cdot2} = p_2 v_2^{1\cdot2}$$

or $$\frac{v_2}{v_1} = \left(\frac{p_1}{p_2}\right)^{1/1\cdot2} = \left(\frac{p_4}{p_3}\right)^{1/1\cdot2} = \frac{v_3}{v_4}$$

or $$v_1 = v_2\left(\frac{v_4}{v_3}\right) = \frac{0{\cdot}001963}{0{\cdot}02383} \times 0{\cdot}10014 = 0{\cdot}00825 \text{ cu. ft.}$$

Hence work per stroke $= 6 \times 144 \times 14{\cdot}7[(20 \times 0{\cdot}00825 - 0{\cdot}10014) - (20 \times 0{\cdot}001963 - 0{\cdot}02383)]$

$= 628{\cdot}7$ ft.-lb.

The horse-power required, neglecting friction, is then

$$\frac{628{\cdot}7 \times 100}{33{,}000} = 1{\cdot}905$$

TURBO BLOWERS AND COMPRESSORS

Where large quantities of air are required at low pressures, as for forced-draught furnaces, or at pressures not exceeding about 120 lb. per sq. in., turbo blowers or centrifugal compressors can be used with advantage. Their design and use is dealt with in " Turbo Blowers and Compressors " by Kearton, and in numerous papers, but certain thermodynamic considerations will be given here.

Fig. 52 shows a section through a large compressor. Owing to the rapid rotation of the rotor A, the air, which enters the machine by an annular passage surrounding the shaft, will be thrown outwards between the rotor blades *a*, and will leave the periphery with increased pressure and at a high velocity. The air then passes into the outer portion of the casing or volute chamber B, where as much of the kinetic energy as possible is converted into the equivalent pressure energy. Guide vanes *b* are sometimes provided for the control of the direction of flow. Of the total pressure rise about 75 per cent. is attained in the rotor, the remainder representing the gain in the volute chamber. The air may be led through a number of compressor units or " stages," arranged in series, to obtain a cumulative pressure effect.

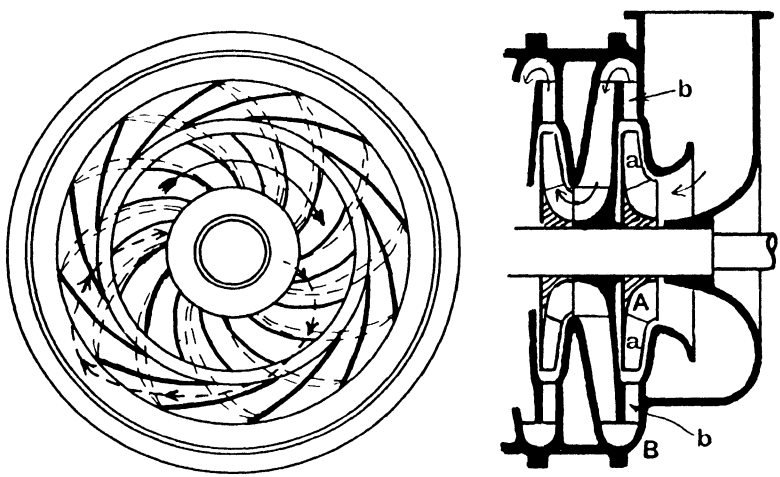


Fig. 52

Axial flow compressors with impellors of the " air-screw " type can also be employed for certain purposes.

Owing to the very high velocities attained by the air (peripheral speeds for rotors vary from 600 ft. per second to 1000 ft. per second in special cases) the rate of pressure rise is extremely rapid and there are heavy frictional losses due to surface effects and to eddy formation, both of which cause the generation of heat. The air will tend therefore to rise in temperature even more than would be the case were compression adiabatic, and the value of n may be between 1·57 and 1·69 for a single stage of compression. For a multi-stage machine with intercoolers the value for the whole machine may be about 1·15. Where high pressures are required ten or more stages, with intercoolers, may be provided.

In the case of the reciprocating compressor the theoretical $p.v.$ diagram is used merely to illustrate the problems which are then solved mathe-

matically, but with turbo compressors graphical methods of calculation may be convenient. For this purpose the *p.v.* diagram is not suitable owing to the very small angles at which the adiabatic and isothermal curves intersect, and the Tϕ diagram is preferable.

Tϕ Diagram for Air.—It will be seen by reference to Fig. 35 and the equations on p. 58, that the portion of the diagram now of interest consists of a series of similar constant pressure lines p_1, p_2, p_3, for which

$$\phi_b - \phi_a = C_p \log_\epsilon \frac{T_b}{T_a}$$

One such curve having been plotted, a template can be made and used for drawing the other curves by merely sliding it parallel to the axis of ϕ. The distance separating two pressure curves p_1 and p_2 at any given temperature is given by $ac = -\frac{R}{J} \log_\epsilon \frac{p_2}{p_1}$ (Fig. 53).

If 1 lb. of air is compressed adiabatically from pressure p_1 and absolute temperature T_1 to pressure p_2, the initial and final condition points will be given by *c* and *b* on a line of constant entropy. No heat is received from or rejected to any external body during compression, nor is heat generated by internal friction, and the final temperature will be T_2.

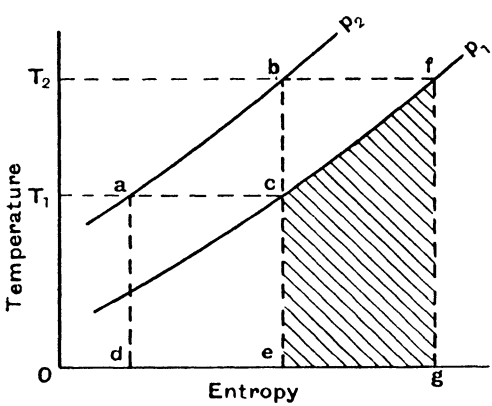


Fig. 53

By the equation on p. 99, the total work done on the air per pound during suction, compression, and delivery is given by $W = C_p(T_2 - T_1)$, i.e. the work required to raise the temperature of the air from T_1 to T_2 at constant pressure. This latter quantity is equal to the area *ecfg*, or, as the pressure curves are all similar, the work done in adiabatic compression from *c* to *b* is equal to the area *dabce*.

(It should be noted that for this portion of the diagram of state the total heat of the gas is given by C_p . T, so that by multiplying the vertical ordinate by C_p we obtain the Mollier diagram.)

Isothermal compression to pressure P_2 and T_1 changes the state point from *c* to *a*, and the work done is then the area *dace*, which also represents the heat removed during compression, so that the excess energy required for adiabatic compression is then the area *abc*.

Owing to the short time required for compression, the heat losses

during this process will be small or negligible, but surface friction, and shock and eddy losses due to imperfect guidance of air in the blades (see also steam turbine losses), will usually cause the air to leave the stage of

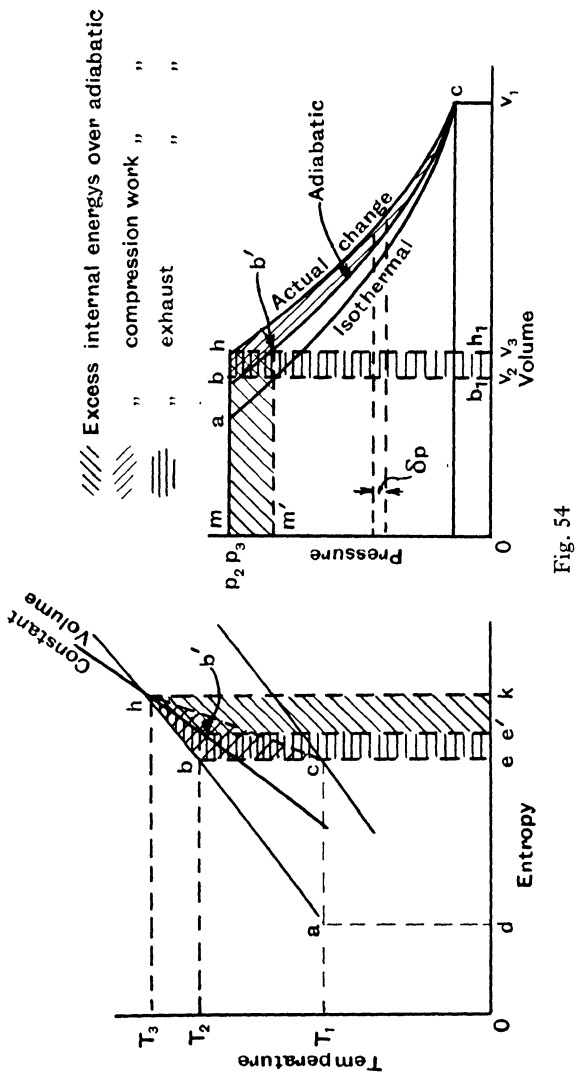


Fig. 54

the compressor at a higher temperature and consequently with a greater specific volume than had the compression been purely adiabatic. In this case the final condition point will be given by a point such as *h* (Fig. 54). The total work done in excess of that required for adiabatic

compression will now be the area under the curve *bh*, or *ebhk*, and the "adiabatic efficiency" will be $\frac{\text{area under } ab}{\text{area under } ah} = \frac{dabe}{dahk}$. This is usually of the order of 70 per cent. The total work done in excess of that required for adiabatic compression is *ebhk* on the Tϕ diagram, or $C_p(T_3-T_2)$. This is made up of two components, internal and external energy. To represent the former a line of constant volume *hb'* can be plotted through *h* according to the equation $\phi_h-\phi_{b'}=C_v \log_\epsilon T_3/T_2$, *b'* being on the line of constant temperature T_2 through C. The gain of internal energy in excess of that acquired during adiabatic compression will then be the area under *b'h*, or

$$c'b'hk=C_v(T_3-T_2) \quad . \quad . \quad . \quad . \quad . \quad (1)$$

The remaining area *ebhb'e'* must therefore represent the increase of external energy. Referring now to the *p.v.* diagram the excess work done during the discharge of the gas is

$$b_1bhh_1=p_2(v_3-v_2) \quad . \quad . \quad . \quad . \quad . \quad (2)$$

But $$v_3=v_2\frac{T_3}{T_2} \quad \text{or} \quad v_3-v_2=v_2\frac{(T_3-T_2)}{T_2}$$

or excess work done during discharge $=R(T_3-T_2)=b_1bhh_1$.

Somewhat similar cases are considered in the chapter on Irreversible Processes.

Where compression is effected in a number of stages each similar to the above, the air should be cooled at constant pressure after each stage along the curve *hb*, if possible as far as the initial temperature point *a*. If this is done the condition curve for the compressor as a whole will lie to the left of the adiabatic line *cb*.

For large compressors the pressure rise per stage is of the order of 6 lb. per sq. in., giving about sixteen stages for a rise of pressure of 100 lb. Capacities of 45,000 cu. ft. of free air per minute can be dealt with at this pressure in a single machine, while 85,000 cu. ft. per minute have been compressed up to 30 lb. per sq. in.

EXAMPLES ON CHAPTER VIII

1. In a simple compressed air installation the compressor draws in air from the atmosphere, and compresses it adiabatically to twelve atmospheres. The temperature of the atmosphere is 60° F. In the mains the air is cooled at constant pressure to its original temperature, and is then delivered to the motor, where, after cut-off, it expands adiabatically to atmospheric pressure. Estimate the efficiency.

2. Estimate the B.H.P. of the engine required to drive an air compressor that takes in 260 cu. ft. of air per minute at 60° F. and at atmospheric pressure, and compresses it adiabatically in one stage to ten atmospheres. Assume the mechanical efficiency of the compressor to be 86 per cent. and neglect all losses due to clearance, etc.

3. An engine is supplied with compressed air at 90 lb. per sq. in. abs. and at 65° F. The air is expanded, according to the law $pv^{1\cdot3}$=constant, down to 15 lb. abs., and then exhausted at that pressure. Determine the pounds of air that will be used per hour per I.H.P., and calculate the temperature of the air at the end of expansion. Neglect losses due to clearance, etc.

4. An ideal air compressor works between pressures of one and twelve atmospheres, with adiabatic compression, the initial temperature being 17° C. Estimate the amount of work which must be expended per pound of air compressed in the following cases :

(*a*) Single-stage compression.
(*b*) Two-stage compression.
(*c*) Three-stage compression.

In (*b*) and (*c*) how much heat must be extracted in the intercoolers per pound of air compressed ? (C_p=0·2375.)

5. Solve Problem 4, if by spray injection the law of the compression curves is $pv^{1\cdot2}$=constant.

6. Calculate the size of cylinder required for a double-acting air compressor of 50 I.H.P., working as a simple compressor, at 120 r.p.m., the law of compression being $pv^{1\cdot2}$=constant and the ratio of compression pressures being from 15 lb. per sq. in. abs. to 150 lb. abs. Take an average piston speed of 600 ft. per minute.

7. A single-acting, three-stage compressor is to be designed to deliver 1¼ lb. of air per minute at a pressure of 900 lb. per sq. in. abs. when running at 200 r.p.m. An efficient intercooler is fitted in which the air is cooled to suction temperature after compression in the low-pressure and intermediate stages. Assuming that the cylinder sizes are arranged to give maximum efficiency, the suction temperature and pressure being 100° F. and 14 lb. per sq. in. abs. respectively, and the mechanical efficiency is 85 per cent., calculate the horse-power required to drive the compressor, and the swept volume of the three cylinders. The law of compression may be taken as $PV^{1\cdot2}$=K, and clearance effects may be neglected.

8. A single-acting, single-stage compressor is required to deliver 0·5 lb. of air per minute at a pressure of 200 lb. per sq. in. abs. The bore and stroke of the compressor are 6 in. and 5 in. respectively, and the clearance volume is 3 per cent. of the swept volume. Assuming that the temperature and pressure at the end of the suction stroke are 85° F. and 14 lb. per sq. in. abs., calculate the speed at which the compressor must be run, and the horse-power required. The law of expansion and compression may be taken as $PV^{1\cdot2}$=K, and the mechanical efficiency as 90 per cent.

Chapter IX

THE PROPERTIES OF STEAM AND OF MIXTURES OF STEAM AND AIR

*METHOD OF RECORDING PRESSURES

Steam pressures may be recorded either as " absolute " pressures or as pressures in excess of or less than atmospheric pressure, the latter being designated " gauge " pressures and varying slightly with the barometric height.

Gauge pressures are recorded by all instruments in which the operating element is subject to steam pressure on one side and atmospheric pressure on the other, i.e. Bourdon type gauges, or mercury columns subject to atmospheric pressure at one end. Absolute pressures can only be recorded by instruments not subject to atmospheric pressure, i.e. a Bourdon pressure gauge if the case were evacuated, or by a mercury barometer subject at the lower end to the pressure to be measured. For the purposes of calculation absolute pressures are almost invariably required.

If the atmospheric pressure on a given day is 14·3 lb. per sq. in. abs. (14·70 lb. per sq. in.=30 in. of mercury), and the gauge pressure of steam is 100 lb. per sq. in., the absolute pressure is 114·3 lb. per sq. in.

If the " vacuum " in a condenser is 27 in. of mercury on a day when the barometer stands at 29 in., the absolute pressure is equivalent to 2 in. of mercury. The vacuum corrected to the " standard " atmospheric pressure of 30 in. of mercury is 30 in.−2 in.=28 in. of mercury.

For theoretical calculations involving symbols only pressures will usually be considered as expressed in pounds per square foot. For calculations involving actual figures, pounds per square inch absolute will be quoted unless otherwise stated.

*EQUATION OF STATE. STEAM TABLES

Many equations have been evolved to express the relationships between the various properties of steam, and have proved of immense value in the preparation of the Steam Tables in which these properties are tabulated, especially in interpolating between values empirically established. As will be seen from the nature of the curves of state already reproduced (Fig. 28, etc.) it is not to be expected that equations having a wide range

of application will be of a simple form, and care must be taken in their use not to exceed the range to which they adequately apply.

For most purposes direct reference will be made to the Steam Tables and diagrams [1] which are available in very convenient form. For details concerning the methods of preparation of the tables reference should be made to the original sources indicated in the references given below and to Professor Callendar's " Properties of Steam " (1920) and his papers before the Royal Society (1928) and the Institution of Mechanical Engineers (1929, etc.).

*GENERATION OF STEAM AT CONSTANT PRESSURE

This is the process employed in a boiler in normal operation, when the " feed water " is pumped into the boiler at the same (gravametric) rate at which the steam is generated and withdrawn. A number of processes are or may be involved :

(1) **Feeding the Boiler.**—This involves forcing the feed water into the boiler against the boiler pressure. If p_b and p_h are the pressures of boiler and hot well (or supply of feed water), and v_w the specific volume of the feed water, assumed constant, the work done will be

$$\frac{(p_b - p_h)v_w}{J} \text{ B.Th.U.}$$

In most cases this amounts only to a small fraction of 1 B.Th.U. per pound.

(2) **Raising the Temperature of the Water** from the temperature of the feed t_h° F. to the boiling-point, t° F.—The value of t will depend upon the pressure (which remains constant throughout) to which the water is subjected. If the specific heat of water be assumed constant and equal to unity the sensible head added will be

$$t - t_h \text{ B.Th.U. per pound.}$$

(The specific heat of water is not quite constant, being about 1·01 at 32° F., falling to 0·99 at about 100° F., and then gradually rising to 1·01 at 300° F. and 1·03 at 600° F. For approximate calculations the value of unity may be used, but for greater accuracy the values shown in the Steam Tables should be applied.)

(3) **Evaporating.**—If the application of heat be continued steam will be formed at constant temperature t° F. until the water has evaporated.

During the process of steam generation the steam and water are in

[1] " Thermodynamic Properties of Steam," Keenan and Keyes, pub. John Wiley, 1936, in °F. units. " Enlarged Steam Tables," Callendar, in °C. or °F. units. Other tables are also available.

thermal equilibrium, the steam is said to be "saturated," and t is the "temperature of saturation" corresponding to pressure p. If the steam contains water-particles in suspension, as is always the case in a simple boiler due to the bursting of the steam bubbles at the free surface of the water, it is said to be "wet," the "dryness fraction" being given by $\frac{\text{weight of actual steam}}{\text{total weight of steam and water}}$. This is denoted by q.

When the water has been completely evaporated q will be unity and the steam is said to be "dry saturated."

(4) **Superheating.**—If more heat be added to dry saturated steam after removing it from contact with water its temperature will rise to, say, t_1° F. The quantity of heat so added will be approximately equal to $C_p(t_1-t)$ B.Th.U., where C_p is the mean specific heat of the steam at constant pressure for the given range of temperature, and has the approximate value of 0·48, but this varies widely and is a function of both pressure and temperature. Such steam is said to be "superheated," and to have t_1-t degrees of superheat. Highly superheated steam approaches the condition of a perfect gas.

The heat added in stages (2) and (4) is said to be "sensible," in that its reception is marked by a rise of temperature which can be recognised (by the incautious) by the sense of touch. The heat added in stage (3) causes no rise of temperature, and, using the ancient phraseology, is said to be "latent."

*STEAM TABLES AND THE PROPERTIES ENUMERATED THEREIN

The numerical values of the properties of steam and water which are likely to be required will be found set out in convenient form in tables of the properties of steam. In these either the Fahrenheit or Centigrade degree and heat unit are used. The heat contents of water and steam are taken, for convenience, as the contents in excess of that of water at 32° F. or 0° C., the entropy being similarly calculated.

The following are quantities usually enumerated :

(1) Pressure p in pounds per square inch absolute or, for low pressures, in inches of mercury.

(2) Temperature t, ° F. or ° C.

(3) Specific volume v, in cubic feet per pound, for both liquid and vapour.

(4) Density or $1/v$ in pounds per cubic foot.

(5) Heat (or enthalpy) of the liquid, which is the heat required to raise 1 lb. of water from 32° F. (or 0° C.) and at room pressure to temperature t and pressure p. Allowances for variations of specific heat have been made in computing the tables. According to this definition the

heat of the liquid includes both the *heat* supplied to raise the temperature, and the *work* done in raising the pressure of the water. (Callendar, " Properties of Steam," pp. 24–30.)

(6) Latent heat (or enthalpy) of evaporation. This is the heat required to completely convert 1 lb. of water at $t°$ to dry saturated steam at the same temperature and at constant pressure. If q is the " dryness fraction " of the steam the latent heat content will be qL. An approximate value of L applicable over a limited range is given by $L=1437-0{\cdot}7t$ B.Th.U., from which it will be seen that L decreases as t increases.

(7) Total heat (or enthalpy) of the saturated vapour H. This is the heat required to convert 1 lb. of water at 32° F. (or 0° C.) and atmospheric pressure to dry saturated steam at temperature t and the corresponding pressure, or $H=h+L$. H is given approximately by $H=1082+0{\cdot}3T$ B.Th.U., or $1114-0{\cdot}7t$ °F., showing that H increases with T.

(8) Internal energy of evaporation I. During the evaporation of steam at constant pressure external work is done owing to the increase in volume from v_w, the volume of the water, to v, the volume of the steam or $W=A\,.\,144p(v-v_w)$.[1]

The increase of internal (or molecular) energy during evaporation will therefore be given by $I=L-144Ap(v-v_w)$.

(9) Internal energy of steam E. This is the increase of internal energy during the production of steam from water at 32° (or 0° C.) and atmospheric pressure, during which the external work $144Apv$ is performed, or $E=H-144Ap\,.\,v$. For steam of dryness fraction q,

$$E=h+qL-qAP(v-v_w).$$

(10) Entropy of water, ϕ (sometimes S, n, or θ). This has been obtained by an integration of the form

$$\phi=\int_{32°}^{t}\frac{c}{T}dt, \text{ variation of the specific heat } c \text{ being allowed.}$$

(11) Entropy of evaporation, given by L/T.

(12) Entropy of steam, given by the sum of the last two quantities.

(13) Entropy of superheated steam—also indicated by n—given by the entropy of saturated steam plus the increase of entropy due to superheating, allowing for the variation of the specific heat at constant pressure of superheated steam.

(The subscripts f, g, and fg, used by some authors, refer to conditions on the liquid boundary, the vapour boundary, and to the process of evaporation, i.e. h_f is the liquid heat, h_{fg} the latent heat, and h_g the total heat.)

EXAMPLE.—Calculate the total heat of evaporation, and the internal

[1] $A=\frac{1}{J}$.

energy of 1 lb. of saturated steam at a pressure of 100 lb. per sq. in. abs., (*a*) when the steam is dry, and (*b*) when the dryness fraction of the steam is 0·8. Given temperature of saturation 328° F., the specific volume of dry saturated steam at this pressure and temperature 4·229 cu. ft., and the value of the latent heat as 888·6 B.Th.U.[1]

(*a*) Total heat $H=h+L$

$=(328-32)+888{\cdot}6$

$=1184{\cdot}6$ B.Th.U. (from Tables, 1187·2)

External work done $E=\frac{P}{J}(v-v_w)$.

Here $P=144\times 100$ lb. per sq. ft., and the volume of 1 lb. of water (V_w) is 0·016 cu. ft., hence

$$E=\frac{144\times 100}{778}(4{\cdot}229-0{\cdot}016)$$

$$=\frac{144\times 100}{778}\times 4{\cdot}213$$

$=77{\cdot}9$ B.Th.U.

$\therefore$ Internal energy $I=H-E$

$=1184{\cdot}6-77{\cdot}9$

$=1106{\cdot}7$ B.Th.U. (from Tables, 1104·0 B.Th.U.)

(*b*) $H=h+qL$

$=(328-32)+0{\cdot}8\times 888{\cdot}6$

$=296+710{\cdot}8$

$=1006{\cdot}8$ B.Th.U.

External work done $E=\frac{Pq}{J}(v-v_w)$

$$=\frac{144\times 100\times 0{\cdot}8}{778}\times 4{\cdot}213$$

$=62{\cdot}3$ B.Th.U.

Internal energy $I=H-E$

$=1006{\cdot}8-62{\cdot}3$

$=944{\cdot}5$ B.Th.U.

EXAMPLE.—Calculate from first principles the entropy of 1 lb. of water

[1] If the actual value of the latent heat is not available, the approximate value may be obtained from the equation

Latent heat $=1114-0{\cdot}7\times$ temperature in ° F.

The student should note the errors involved by the use of this and other equations over a wide range of temperatures.

and 1 lb. of steam at the following temperatures: 110° F., 212° F., 280° F., 320° F., and 366° F., assuming L to be given by the equation 1114−0·7 T° F.

At 110° F., the absolute temperature is 110+461=571°.

$$\therefore \phi_w = \log_\epsilon \tfrac{571}{493} = 2{\cdot}3026 \log_{10} \tfrac{571}{493}$$
$$= 2{\cdot}3026 \times 0{\cdot}0638 = 0{\cdot}1469$$

Similarly we find

at 212° F. $\phi_w = 2{\cdot}3026 \log_{10} \tfrac{673}{493} = 2{\cdot}3026 \times 0{\cdot}1352 = 0{\cdot}3113$
,, 280° F. $\phi_w = 2{\cdot}3026 \log_{10} \tfrac{744}{493} = 2{\cdot}3026 \times 0{\cdot}1788 = 0{\cdot}4117$
,, 320° F. $\phi_w = 2{\cdot}3026 \log_{10} \tfrac{781}{493} = 2{\cdot}3026 \times 0{\cdot}1999 = 0{\cdot}4606$
,, 366° F. $\phi_w = 2{\cdot}3026 \log_{10} \tfrac{826}{493} = 2{\cdot}3026 \times 0{\cdot}2242 = 0{\cdot}5163$

Now, gain of entropy during evaporation $= \frac{L}{T}$, hence we have

$L_{110} = 1114 - 0{\cdot}7 \times 110 = 1114 - 77 = 1037 \quad \therefore \frac{L}{T} = \frac{1037}{571} = 1{\cdot}8161$

$L_{212} = 1114 - 0{\cdot}7 \times 212 = 1114 - 148 = 966 \quad \therefore \frac{L}{T} = \frac{966}{673} = 1{\cdot}4353$

$L_{280} = 1114 - 0{\cdot}7 \times 280 = 1114 - 196 = 918 \quad \therefore \frac{L}{T} = \frac{918}{744} = 1{\cdot}2338$

$L_{320} = 1114 - 0{\cdot}7 \times 320 = 1114 - 224 = 890 \quad \therefore \frac{L}{T} = \frac{890}{781} = 1{\cdot}1382$

$L_{366} = 1114 - 0{\cdot}7 \times 366 = 1114 - 256 = 858 \quad \therefore \frac{L}{T} = \frac{858}{826} = 1{\cdot}0384$

Hence we have

at 110° F. $\phi_s = 0{\cdot}1469 + 1{\cdot}8161 = 2{\cdot}0230$
,, 212° F. $\phi_s = 0{\cdot}3113 + 1{\cdot}4353 = 1{\cdot}7466$
,, 280° F. $\phi_s = 0{\cdot}4117 + 1{\cdot}2338 = 1{\cdot}6455$
,, 320° F. $\phi_s = 0{\cdot}4606 + 1{\cdot}1382 = 1{\cdot}5988$
,, 366° F. $\phi_s = 0{\cdot}5163 + 1{\cdot}0384 = 1{\cdot}5447$

The values of L, ϕ_w and ϕ_g given by Keenan and Keyes are as follows:

	L	$\phi_w = S_w$	$\phi_s = S_g$
T=110° F.	1031·7	0·1471	1·9577
T=212° F.	970·3	0·3120	1·7566
T=280° F.	924·7	0·4096	1·6597
T=320° F.	894·9	0·4637	1·6115
T=366° F.	857·1	0·5235	1·5165)

EXAMPLE.—In a test of a condensing plant the following data were obtained :

(*a*) Steam condensed per hour, 1552 lb.

(*b*) Temperature of exhaust vapour entering condenser, 105° F.

(*c*) Circulating water per minute, 476 lb.

(*d*) Temperature of circulating water as it enters the condenser, 60° F.

(*e*) Temperature of circulating water as it leaves the condenser, 90° F.

(*f*) Temperature of air pump discharge, 95° F.

Calculate the dryness fraction of the exhaust steam as it enters the condenser.

Latent heat at 105° F.=1034·3 B.Th.U.

Heat lost by the steam per minute

$$=\frac{1552}{60}\{(105-95)+q(1034{\cdot}3 \text{ B.Th.U.})\}$$

Heat gained by the circulating water per minute=476(90−60) B.Th.U.

Assuming no loss of heat, the heat lost by the steam will be equal to the heat gained by the water, hence

$$258{\cdot}6+q\,.\,26730=476(90-60)$$

and $$q=52{\cdot}4 \text{ per cent.}$$

EXAMPLE.—Steam at 110 lb. per sq. in. abs., and containing 12 per cent. moisture, is expanded to five times its initial volume. The pressure is then found to be 22·5 lb. per sq. in.

Find the final quality, final total heat per pound, and the final internal energy.

At 110 lb. per sq. in. the specific volume is 4·047 cu. ft.

The volume when $q=0{\cdot}88$ is therefore $0{\cdot}88\times 4{\cdot}047$,

and the final volume per pound $=5\times 0{\cdot}88\times 4{\cdot}047$ cu. ft.

$$=17{\cdot}81 \text{ cu. ft.}$$

At 22·5 lb. per sq. in. the specific volume is 17·995 cu. ft.

The final quality will therefore be $\frac{17{\cdot}81}{17{\cdot}995}=99$ per cent.

The final total heat $=q\mathrm{L}+h$

$$=0{\cdot}99(955{\cdot}9)+202{\cdot}5=1148{\cdot}8 \text{ B.Th.U. per lb.}$$

and the final internal energy

$$=q\mathrm{L}+h-\frac{\mathrm{P}(qv-v_w)}{\mathrm{J}}$$

$$=1148{\cdot}8-\frac{22{\cdot}5\times 144(0{\cdot}99\times 17{\cdot}99-0{\cdot}016)}{778}$$

$$=1074{\cdot}0 \text{ B.Th.U. per lb.}$$

(Note, the final internal energy is *not* $q\times$ the specific internal energy at 22·5 lb.)

*ADIABATIC EQUATIONS

The adiabatic equations for a gas, in terms of the ratio of the specific heats, no longer hold for a condensing vapour or one nearing that state. For steam the following equations may be used.

(1) $\frac{P}{T^{13/3}}$=constant, and (2) $P(v-b)^{1\cdot3}$=constant, where b is the volume of 1 lb. of water at the pressure and temperature of evaporation.

For many practical purposes the equation $PV^{1\cdot3}$=constant can be applied over a wide range of superheat with a high degree of accuracy (Callendar, *I.M.E.*, 1929; *Proc. Royal Soc.*, 1928). For steam which is initially "dry" the equation $PV^{1\cdot13}$=constant is usually used. (These values can easily be deduced by plotting the values of log p and log v from the Mollier chart when n will be the slope of the approximately straight line obtained.)

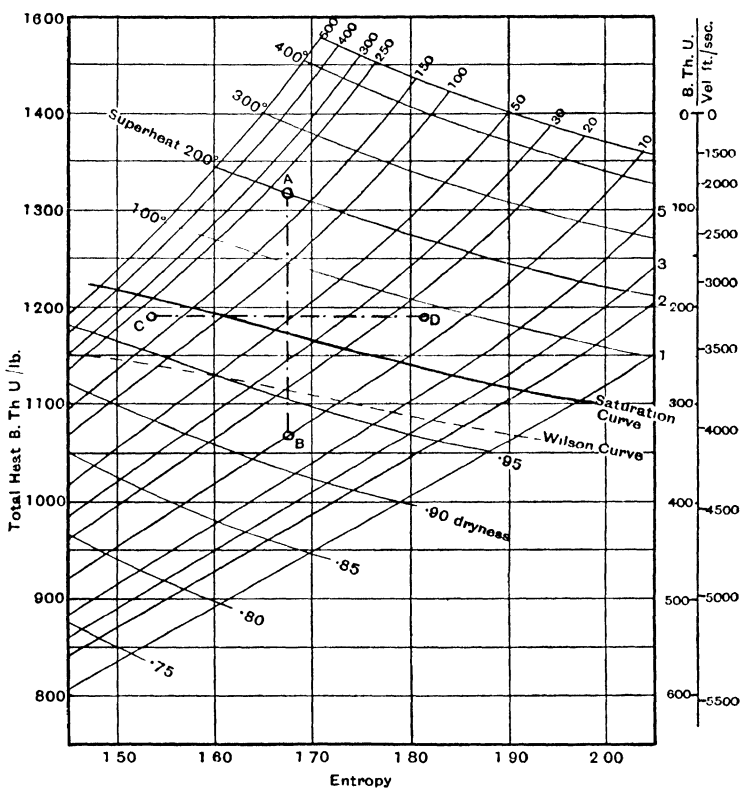


Fig. 55

*STEAM DIAGRAMS

The most useful charts for the solution of practical problems are the Total Heat-Entropy or Mollier diagram, and the Total Heat-Pressure and Temperature diagram. Large scale prints of both of these are avail-

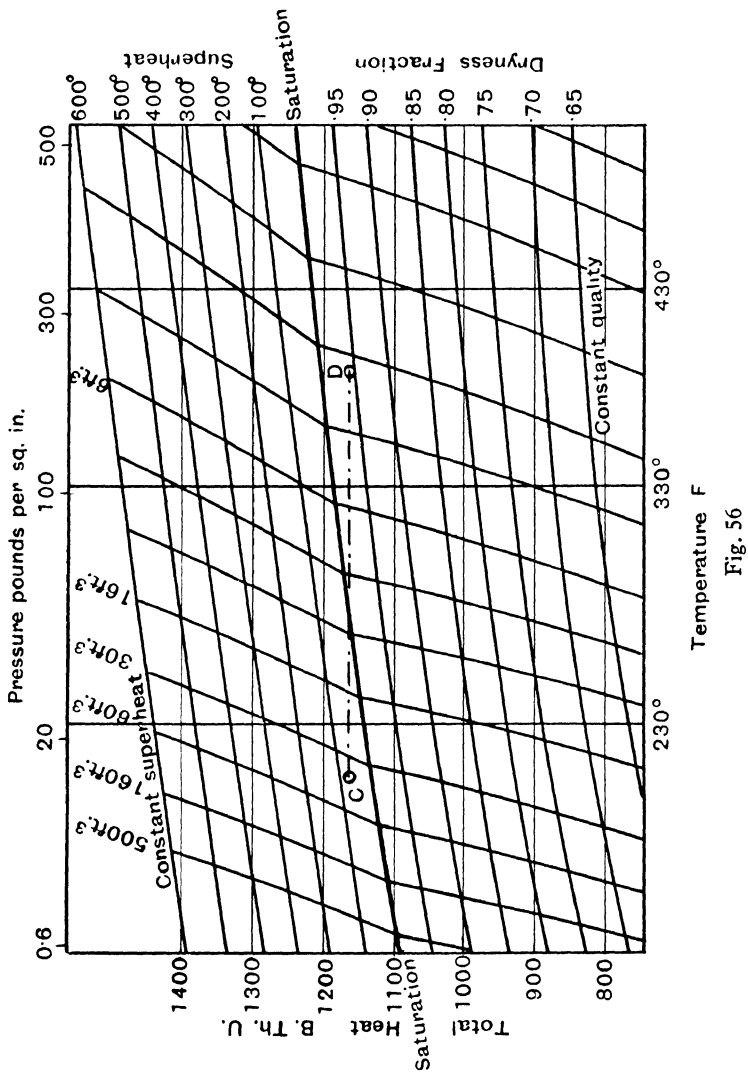


Fig. 56

able, and are outlined in Figs. 55 and 56. The $T\phi$ diagram is given in Fig. 28, and is supplied, on a large scale, with Keenan and Keyes's book. Only portions of the complete diagrams are given, as indicated on

p. 66, but these will usually be found to cover an adequate range of conditions.

A scale frequently included on the Hϕ diagram is of convenience in the design of steam nozzles. This shows the relation between any given number of B.Th.U. and the velocity which would be attained by 1 lb. of steam were this heat energy entirely converted into kinetic energy.

If V is the velocity attained then

$$x \text{ B.Th.U.} = x \,.\, 778 \text{ ft.-lb.} = \tfrac{1}{2} \text{ lb. mass } V^2$$

or

$$V = 2 \times 32{\cdot}2 \times 778x \text{ ft per second.}$$

The use of the diagrams is indicated in subsequent sections.

EXAMPLE.—Solve the following problems by use of the Tϕ and H.p. diagrams.

(*a*) A supply of steam is drawn through a reducing valve from a steam pipe conveying steam at 150 lb. per sq. in. abs. and of quality 0·95. At what pressure should the reducing valve be set to discharge dry saturated steam ? (15 lb. per sq. in. abs.)

(*b*) Steam at 260 lb. per sq. in. abs. and of 90° superheat, is expanded adiabatically to 11 lb. per sq. in. abs. What are the final quality and the ratio of expansion ? (q=0·87 ; r=14·65.)

*SOME MATHEMATICAL AND GRAPHICAL CALCULATIONS CONCERNING CHANGES OF STATE

(1) **Calculation of the Dryness of Steam after Isentropic or Adiabatic Expansion.**—*Suppose the steam to be initially dry and saturated* at temperature T, its condition being represented by the point J′ in the temperature-entropy diagram, Fig. 57. During expansion to T_2 there is no reception or rejection of heat, the difference between the initial and final internal energies being equal to the work done during the expansion. The expansion curve on the entropy diagram will be represented by the straight vertical line J′*f*, the point *f* representing the condition of the steam at the temperature T_2 after expansion.

The dryness fraction after expansion will therefore be

$$q_2 = \frac{af}{ae}$$

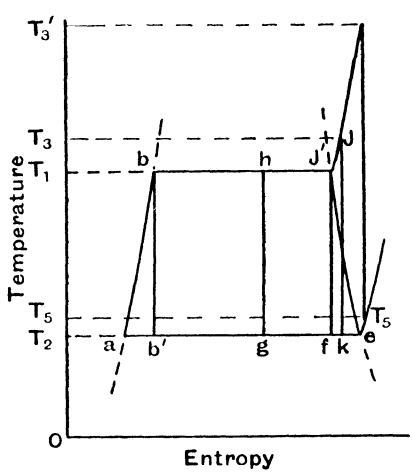


Fig. 57

Again, *if the steam was originally wet* its condition would be represented

by the point h, such that the initial dryness fraction $=\frac{bh}{bg'}=q_1$, and at the end of expansion, the dryness fraction would be

$$q_2=\frac{ag}{ae}$$

Then the entropy of the steam at $T_1=\log_\epsilon \frac{T_1}{492}+\frac{x_1L_1}{T_1}$ . . . . . (1)

and entropy of the steam at $T_2=\log_\epsilon \frac{T_2}{492}+\frac{x_2L_2}{T_2}$ . . . . . (2)

But the entropy remains constant throughout the expansion

$$\therefore \log_\epsilon \frac{T_1}{492}+\frac{x_1L_1}{T_1}=\log_\epsilon \frac{T_2}{492}+\frac{x_2L_2}{T_2}$$

$$\frac{x_2L_2}{T_2}=\log_\epsilon \frac{T_1}{T_2}+\frac{x_1L_1}{T_1}$$

or

$$x_2=\frac{T_2}{L_2}\left(\frac{x_1L_1}{T_1}+\log_\epsilon \frac{T_1}{T_2}\right) \quad . \quad . \quad . \quad . \quad (3)$$

For steam superheated to T_3, T_1 being the temperature of saturation, the initial entropy of steam at T_3

$$=\log_\epsilon \frac{T_1}{492}+\frac{L_1}{T_1}+C_p\left(\log_\epsilon \frac{T_3}{T_1}\right).$$

In general this will be equal to the final entropy of

$$\log_\epsilon \frac{T_2}{492}+\begin{cases}\frac{q_2L_2}{T_2} \text{ for steam finally wet, or}\\ \frac{L_2}{T_2}+C_p \log_\epsilon \frac{T_3}{T_2} \text{ for steam finally superheated.}\end{cases}$$

The second alternative is taken should the value of q_2 given by the equation be greater than unity.

The dryness fraction can be scaled at once from the Tϕ or Hϕ diagrams should these be available to a sufficiently large scale. The line AB in Figs. 55 and 56 shows the adiabatic expansion of steam from 200 lb. per sq. in. and 100° superheat to 10 lb. per sq. in. The final dryness fraction is 0·882.

EXAMPLE.—Determine—

(*a*) The dryness fraction at the end of expansion when 1 lb. of dry saturated steam at 366° F. expands adiabatically to 225° F.

(*b*) The dryness fraction at the end of expansion when 1 lb. of wet steam of dryness fraction 0·75 and temperature 380° F. expands adiabatically to 240° F. Assume L=1114+0·7T° F.

(*a*) $366° \text{ F.} = 366+460 = 826° \text{ abs.} = T_1$

$225° \text{ F.} = 225+460 = 685° \text{ abs.} = T_2$

$L_{366} = 1114 - 0{\cdot}7 \times 366 = 1114 - 256{\cdot}2 = 857{\cdot}8$

$L_{225} = 1114 - 0{\cdot}7 \times 225 = 1114 - 157{\cdot}5 = 956{\cdot}5$

Let q = dryness fraction after expansion $= \frac{af}{ae}$ (Fig. 57),

then
$$q = \frac{ab'+b'f}{ae} = \frac{\log_\epsilon \frac{826}{685} + \frac{857{\cdot}8}{827}}{\frac{956{\cdot}5}{685}} = \frac{0{\cdot}1870+1{\cdot}0372}{1{\cdot}3943}$$
$$= 0{\cdot}878$$

(*b*) $T_1 = 380+460 = 840°$ and $T_2 = 240+460 = 700°$ F. abs.

$L_{380} = 1114 - 0{\cdot}7 \times 380 = 1114 - 266 = 848$

$L_{240} = 1114 - 0{\cdot}7 \times 240 = 1114 - 168 = 946$

In Fig. 57 $\frac{bh}{bj'} = 0{\cdot}75$

and final dryness fraction
$$= \frac{ag}{ae} = \frac{ab'+b'g}{ae} = \frac{ab'+0{\cdot}75bj'}{ae} = \frac{\log_\epsilon \frac{841}{700} + 0{\cdot}75 \times \frac{848}{840}}{\frac{946}{701}}$$
$$= \frac{0{\cdot}1821+0{\cdot}7562}{1{\cdot}3495} = 0{\cdot}695.$$

The student should check the above results by means of the temperature-entropy and Mollier diagrams.

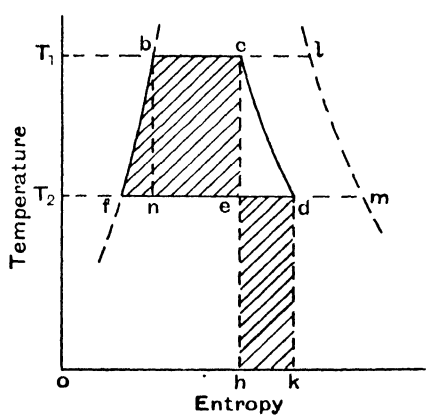


Fig. 58

(2) **Calculation of Dryness Fraction after Throttling or "Wire Drawing."** — As is shown on p. 77 this process is essentially one in which the total heat remains constant. There are two cases to be considered. In the first, the steam after throttling may be either wet or dry saturated ; in the second case the steam may be initially nearly dry or superheated—it will then be superheated after throttling.

First Case, in which the Steam is not Superheated after Throttling.—

Let the steam have an initial dryness fraction q_1, and let throttling take place between absolute temperatures T_1 and T_2. The temperature-entropy diagram is shown in Fig. 58. The condition of the steam before throttling is represented by the point c such that

$$q_1=\frac{bc}{bl}$$

The condition of the steam after throttling will be represented by the point d, such that

$$q_2=\frac{fd}{fm}$$

The length ed will represent the gain in entropy, which may be calculated as follows :

Entropy at T_1 reckoned from water at T_2

$$=fe=fn+bc$$

or

$$fe=\log_\epsilon \frac{T_1}{T_2}+\frac{q_1L_1}{T_1}$$

Entropy at T_2 reckoned from water at T_2

$$=fd=\frac{q_2L_2}{T_2}$$

$\therefore$ gain of entropy $=ed=fd-fe$

$$=\frac{q_2L_2}{T_2}-\frac{q_1L_1}{T_1}-\log_\epsilon \frac{T_1}{T_2} \quad . \quad . \quad . \quad . \quad (1)$$

But since the heat content remains the same, the total heat after throttling will be the same as the total heat before throttling, i.e.

$$T_2-492+q_2L_2=T_1-492+q_1L_1$$

$$\therefore\ q_2=\frac{T_1-T_2+q_1L_1}{L_2}$$

The gain of entropy may be found by substituting this value in equation (1).

$$ed=\frac{L_2}{T_2}\left(\frac{T_1-T_2+q_1L_1}{L_2}\right)-\frac{q_1L_1}{T_1}-\log_\epsilon \frac{T_1}{T_2}$$

$$=\frac{T_1-T_2+q_1L_1}{T_2}-\frac{q_1L_1}{T_1}-\log_\epsilon \frac{T_1}{T_2}$$

$$=\left(\frac{T_1-T_2}{T_2}\right)\left(1+\frac{q_1L_1}{T_1}\right)-\log_\epsilon \frac{T_1}{T_2} \quad . \quad . \quad . \quad . \quad (2)$$

Alternative Method of calculating Gain of Entropy.—The above result may be obtained by a more direct method as follows :

Reckoning all heat quantities from the absolute zero of temperature we have

Total heat per pound before throttling=area *ofbch*

Total heat per pound after throttling=area *ofdk*

$\therefore$ area *ofbch*=area *ofdk*

and since the area *ofeh* is common, it follows that

area *fbce*=area *edhk*

Now the area *fbce* represents the work done per pound of steam in the Rankine cycle (see p. 150), namely

$$(T_1-T_2)\left(1+\frac{q_1L_1}{T_1}\right)-T_2\log_\epsilon\frac{T_1}{T_2}\text{ B.Th.U.}$$

$$\therefore \text{ area } edkh=ed\times T_2=(T_1-T_2)\left(1+\frac{q_1L_1}{T_1}\right)-T_2\log_\epsilon\frac{T_1}{T_2}$$

or
$$ed=\frac{T_1-T_2}{T_2}\left(1+\frac{q_1L_1}{T_1}\right)-\log_\epsilon\frac{T_1}{T_2}\text{ as above.}$$

Second Case when the Steam is Dry Saturated before Throttling.—The condition of the steam at pressure p_1 and temperature T_1 before throttling is represented by the point *c* on the saturated steam line, Fig. 59. After throttling down to a pressure p_2 the condition of the steam is represented by the point *d* on this constant pressure line and in the superheated region of the temperature-entropy diagram.

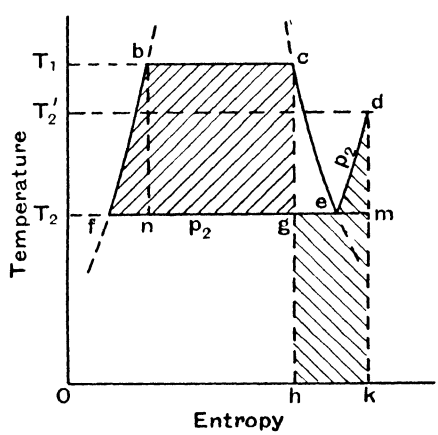


Fig. 59

The length *gm* represents the gain in entropy and may be found as follows :

Entropy at pressure p_1 reckoned from water at T_2

$$=fg=fn+bc$$

$$=\log_\epsilon\frac{T_1}{T_2}+\frac{L_1}{T_1}$$

Entropy at pressure p_2 reckoned from water at T_2

$$=fm=fe+em$$

$$=\frac{L_2}{T_2}+0{\cdot}48\log_\epsilon\frac{T_2'}{T_2}$$

where T_2' is the final temperature of the superheated steam after throttling.

$\therefore$ gain of entropy $= gm = fm - fg$

$$= \frac{L_2}{T_2} + 0{\cdot}48 \log_\epsilon \frac{T'_2}{T_2} - \log_\epsilon \frac{T_1}{T_2} - \frac{L_1}{T_1} \quad . \quad (2)$$

Now the total heat after throttling is equal to the total heat before throttling, i.e.

$$H_2 + 0{\cdot}48(T_2' - T_2) = T_1 - 492 + L_1 \quad . \quad . \quad . \quad . \quad (3)$$

where H_2 denotes the total heat per pound of dry saturated steam at pressure p_2, obtainable from steam tables, or in their absence from

$$H_2 = T_2 - 492 + L_2$$

From equation (3) the final temperature of the steam can be calculated and the substitution of its value in (2) gives the gain of entropy.

It should be noticed that reckoning the total heat from the absolute zero of temperature,

Total heat per pound before throttling = area *ofbch*

Total heat per pound after throttling = area *ofedk*

and since the area *ofgh* is common it follows that

area *fbcg* = area *gedkh*.

EXAMPLE.—Boiler steam of dryness 0·97 and at 340 lb. per sq. in. abs. (t=429° F.) is wire-drawn to 200 lb. per sq. in. abs. (t=382° F.). Find the final dryness fraction.

At 340 lb. h=406·66 and L=797·1, or total heat at dryness 0·97 is $406{\cdot}6 + 0{\cdot}97 \times 797{\cdot}1 = 1179{\cdot}7$ B.Th.U.

At 200 lb. h=355·36 and L=843·0.

As the total heat remains constant during throttling

$$1179{\cdot}7 = 355{\cdot}36 - q \,.\, 843{\cdot}0$$

or the dryness fraction after throttling is given by

$$q = \frac{824{\cdot}0}{843{\cdot}0} = 0{\cdot}978$$

It should be noted that at pressures as low as 300 lb. per sq. in. there are appreciable differences between the values obtained from the Keenan and Keyes, Marks and Davies, and Callendar Tables, and the reader is recommended to compare the saturation curves shown on the Mollier charts published with them. In the above question the divergence leads to a discrepancy of nearly 1 per cent. in the dryness fraction.

The question can be solved most rapidly by means of the Mollier chart.

EXAMPLE.—Find the gain of entropy when dry steam at a pressure of 210 lb. per sq. in. abs. is wire-drawn [1] to a pressure of 30 lb. per sq. in. abs.

The temperature after throttling is given by

$$H_2+0{\cdot}48(T_2'-T_2)=T_1-492+L_1$$

Substituting the values of H_2, T_2, T_1 and L_1 from steam tables

$$1163{\cdot}9+0{\cdot}48T_2'-0{\cdot}48\times 710{\cdot}3=846-492+839{\cdot}6$$

from which $$T_2'=770^\circ \text{ F. abs.}$$

The gain of entropy is then given by

$$\frac{L_2}{T_2}+0{\cdot}48\log_\epsilon\frac{T_2'}{T_2}-\log_\epsilon\frac{T_1}{T_2}-\frac{L_1}{T_1}$$

or

$$\frac{945{\cdot}1}{710{\cdot}3}+0{\cdot}48\log_\epsilon\frac{770}{710{\cdot}3}-\log_\epsilon\frac{846}{710{\cdot}3}-\frac{839{\cdot}6}{846}$$

$$=1{\cdot}3305+0{\cdot}0388-0{\cdot}1749-0{\cdot}9924$$

$$=0{\cdot}202$$

EXAMPLE.—Dry saturated steam is wire-drawn from a pressure of 140 lb. per sq. in. abs. down to atmospheric pressure (14·7 lb. per sq. in. abs.). The temperature is then found to be 304·8° F.

Find the mean specific heat of steam at atmospheric pressure between 212° F. and 304·8° F.

Total heat per pound of dry steam at 140 lb. per sq. in. = 1192·2 B.Th.U.

Total heat per pound of dry steam at 14·7 lb. per sq. in. = 1150·4 B.Th.U.

Hence, as during wire-drawing the total heat remains unchanged, there is available for superheating the atmospheric steam

$$1192{\cdot}2-1150{\cdot}4 \text{ B.Th.U.}=41{\cdot}8 \text{ B.Th.U.}$$

If s is the specific heat required, then

$$41{\cdot}8=s(304{\cdot}8-212)$$

or

$$s=\frac{41{\cdot}8}{92{\cdot}8}=0{\cdot}45 \text{ B.Th.U. per lb.}$$

The dryness or degree of superheat of steam after throttling can be read at once from the diagrams, especially from those having H as an ordinate. In Figs. 55 and 56 the line CD represents the throttling of steam of dryness fraction 0·98 at 200 lb. per sq. in. to a pressure of 15 lb. per sq. in. It will be seen that the final condition is 80° superheat. The fall of temperature, obtained from the steam tables or from Fig. 56, is from 381·9° F. to 273·2° F.

[1] I.e. "throttled."

*MEASUREMENT OF THE DRYNESS OF STEAM

There are several methods in use for determining the dryness fraction of steam, some of which are more accurate than others. The greatest difficulty experienced, in all cases, is that of obtaining a representative sample of the steam to be tested.

Method 1. Tank Calorimeter.—The simplest method is to blow a certain weight of steam into a known weight of water, and to measure the rise in temperature produced. Then by equating the heat lost by the steam to the heat gained by the water the dryness fraction can be calculated.

Let w=weight of steam condensed in pounds.
q=dryness fraction of the steam.
W=weight of water in pounds.
t_1=initial temperature of the water in ° F.
t_2=final temperature of the water in ° F.
W′=weight of vessel containing the water in pounds.
s=specific heat of the material of which the vessel is made.
Then heat lost by steam=$w(q\mathrm{L}+t-t_2)$ B.Th.U.
(where t° F. is the temperature of the steam under test)

and heat gained by vessel and water=$(\mathrm{W}+s\mathrm{W}')(t_2-t_1)$ B.Th.U.

Equating these two quantities we have

$$w(q\mathrm{L}+t-t_2)=(\mathrm{W}+s\mathrm{W}')(t_2-t_1) \quad . \quad . \quad . \quad . \quad (1)$$

an equation from which q may be calculated.

The above theory assumes that the specific heat of water is constant and equal to unity, which of course is not quite true. For all practical purposes, however, the error involved is negligible, particularly in view of the uncertainty with regard to the sample of steam taken. If instead of t, t_1, and t_2 in (1) we write h, h_1, and h_2, the sensible heats at these temperatures, we have

$$w(q\mathrm{L}+h-h_2)=\mathrm{W}(h_2-h_1)+s\mathrm{W}'(t_2-t_1) \quad . \quad . \quad . \quad (2)$$

an equation which is accurate.

EXAMPLE.—Steam at 85 lb. per sq. in. abs. is blown into a tank containing 200 lb. of water at a temperature of 60° F. The final increase of water in the tank amounts to 10 lb., and the final temperature is 110° F. What was the initial quality of the steam ?

For steam at 85 lb. per sq. in. L=897·1 and h=286·3
For water at 110° F. h=77·94
For water at 60° F. h=28·08

Hence $10(q\times 897{\cdot}1+286{\cdot}3-77{\cdot}94)=W(77{\cdot}94-28{\cdot}08)$

$$8971q+2083{\cdot}6=200\times 49{\cdot}86$$

and $$q=\frac{7898{\cdot}4}{8971}=88{\cdot}0 \text{ per cent.}$$

Had the tank weighed 50 lb. and been made of steel having a specific heat of 0·117, then

$$8971q+2083{\cdot}6=(200+5{\cdot}85)\times 49{\cdot}86$$

and $$q=91{\cdot}3 \text{ per cent.}$$

Method 2. Condensing Calorimeter.—This method is similar to the above, but the steam and water are not allowed to come into contact. The steam under test flows through a pipe (or series of tubes) around which water is circulated. The inlet and outlet temperatures of the water are measured, together with the weight of water flowing through the instrument in any convenient time, and the weight of steam condensed in the same time. By equating the heat lost by the steam and the heat gained by the water the dryness fraction of the steam may be calculated.

Let w=weight of steam condensed in pounds per minute.
t'=temperature of condensed steam in ° F.
W=weight of cooling water in pounds per minute.
t=temperature of the steam in ° F.
t_1=inlet temperature of cooling water in ° F.
t_2=outlet temperature of cooling water in ° F.

Then, the heat lost by the steam $=w(q\mathrm{L}+h-h')$ B.Th.U.

and heat gained by the water $=\mathrm{W}(h_2-h_1)$ B.Th.U.

$$\therefore\ w(q\mathrm{L}+h-h')=\mathrm{W}(h_2-h_1) \quad . \quad . \quad . \quad . \quad . \quad (1)$$

or, neglecting the variation in the specific heat of water,

$$w(q\mathrm{L}+t-t')=\mathrm{W}(t_2-t_1) \quad . \quad . \quad . \quad . \quad . \quad (2)$$

As the process is continuous no change occurs in the temperature of the containing vessels, so that no allowance for this has to be made.

The temperature of the outer vessel should be kept low to minimise radiation losses.

Method 3. Separating Calorimeter.—When steam is very wet, a rough estimate of the dryness fraction can be made by passing the steam through a vessel designed to produce a rapid rotary motion during which the heavy water particles are thrown out of suspension by centrifugal force, the water being collected and weighed. The steam is subsequently condensed and weighed.

If W_s=weight of steam after treatment

and W=weight of water thrown out of suspension

the initial weight of steam=W_s+W

$$\text{and dryness fraction}=\frac{W_s}{W_s+W}$$

The separation is frequently very imperfect, especially if the water particles are small. It may be a valuable adjunct to a throttling calorimeter, however.

Method 4. Throttling Calorimeter. On p. 66 it has been shown that except at very high pressures steam becomes drier after throttling, and, in certain cases, superheated. This principle is utilised in the throttling calorimeter, which enables the dryness of steam to be conveniently, accurately, and continuously measured.

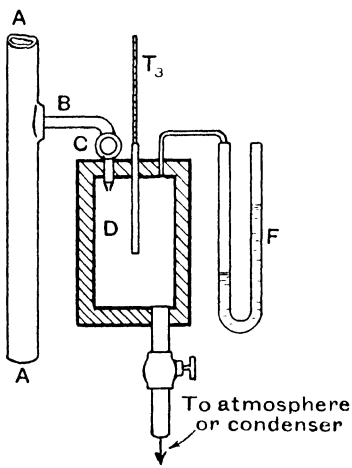


Fig. 60

Fig. 60 shows the arrangement. The pipe B is screwed into the main steam pipe A, and steam, admitted through the valve C, expands through a small hole into the calorimeter D. The thermometer E gives the temperature of the steam after the expansion, while the manometer F gives the pressure (above atmospheric). It will be seen that if the steam in the main pipe is nearly dry, it will become slightly superheated after expansion, the degree of the superheating being obtained by subtracting the temperature of saturation corresponding to pressure p_2 from that recorded by thermometer E.

Let t_3=reading of thermometer E.

t_2=temperature of saturation corresponding to pressure p_2 after expansion (obtained from steam tables).

h_1=sensible heat at pressure p_1 before expansion.

L_1=latent heat of steam at pressure p_1.

Then, if q=dryness fraction required,

$$h_1+qL_1=H_2+0{\cdot}48(t_3-t_2) \quad . \quad . \quad . \quad . \quad . \quad (1)$$

It will be found that steam with a dryness fraction less than about 0·95 cannot be superheated by throttling to atmospheric pressure, and unless this is achieved the instrument gives no indication as to quality.

EXAMPLES ON MIXTURES OF STEAM OR STEAM AND WATER, ETC.

EXAMPLE.—Ten lb. of steam at 100 lb. per sq. in. abs. pressure and 0·8 dry is mixed with 90 lb. of water at 40° F. Find the temperature of the resulting mixture. What would be the temperature had the steam been superheated 10° ?

If T° F. is the resultant temperature then

(*a*) Heat lost by steam

$=0{\cdot}8\times$ latent heat per pound $+10\times$ sensible heat per pound

$=0{\cdot}8\times 888+10[327{\cdot}8-T]=[10{,}382-10T]$ B.Th.U.

Heat taken up by water $=90(T-40)$ B.Th.U.

$=90T-3600.$

Equating these values $10{,}382-10T=90T-3600$

or $T=139{\cdot}8°$

(*b*) Heat lost by steam $=10[$Total heat above $32°-(T-32)]$ B.Th.U.

$=10[1192-(T-32)]$ B.Th.U.

or $90T-3600=11{,}920-10T$

or $T=158{\cdot}4°$ F.

EXAMPLE.—A vessel of 4 cu. ft. capacity is full of steam at 20 lb. per sq. in. pressure absolute and dryness fraction 0·8. It is coupled to a pipe delivering steam at 200 lb. per sq. in. and of dryness 0·9. Find the weight of steam admitted, and the final dryness fraction.

At 20 lb. per sq. in. we have

Latent heat	$=960$ B.Th.U. per lb.
Liquid heat	$=196{\cdot}1$ B.Th.U. per lb.
Specific volume	$=20{\cdot}08$ cu. ft.

The volume occupied by steam 0·8 dry is 16·06 cu. ft.

$$\text{and the weight of steam present} = \frac{4}{16{\cdot}06} = 0{\cdot}2490 \text{ lb.}$$

Let W = weight of steam admitted and q = final quality of the mixture. Then the total heat (from 32°) of the components before mixing are

$$0{\cdot}249[0{\cdot}8\times 960+196{\cdot}1]+W[843{\cdot}2\times 0{\cdot}9+354{\cdot}9] \quad . \quad . \quad (1)$$

as at 200 lb. per sq. in.

Latent heat	$=843{\cdot}2$ B.Th.U.
Liquid heat	$=354{\cdot}9$ B.Th.U.
Specific volume	$=2{\cdot}29$ cu. ft.

The total heat of the final mixture is

$$(W+0{\cdot}249)(q\times 843{\cdot}2+354{\cdot}9) \text{ B.Th.U.} \quad . \quad . \quad . \quad (2)$$

Equating (1) and (2),

$$240+1113{\cdot}78W=(W+0{\cdot}249)(843{\cdot}2q+354{\cdot}9)$$

Also

$$q=\frac{\text{Volume of steam present}}{\text{Volume present had it been dry saturated}}=\frac{4}{(W+0{\cdot}249)}\times 2{\cdot}29$$

or
$$240+1113{\cdot}78W=(W+0{\cdot}249)\left[\frac{843{\cdot}2\times 4}{(W+0{\cdot}249)2{\cdot}29}+354{\cdot}9\right]$$

or
$$W=1{\cdot}7415$$

and
$$q=\frac{4}{(W+0{\cdot}249)\times 2{\cdot}29}=0{\cdot}8775$$

EXAMPLE ON DALTON'S LAW.—In a condenser of 40 cu. ft. capacity the vacuum was 26·5 in. of mercury whilst the temperature was 112° F. If the barometric pressure was 29·7″ Hg, what was the weight of air present in the condenser ?

The actual pressure inside the condenser=29·7−26·5=3·2″ Hg. At 112° F. the partial pressure of saturated steam=2·74″ Hg and the partial pressure of air is therefore 0·46″ Hg.

(1 lb. per sq. in.=2·036″ Hg)

For air PV=RT, where R=53·2 ft.-lb. per lb.

Hence the specific volume under the conditions attained in the condenser is

$$\frac{53{\cdot}2(112+460)\times 2{\cdot}036}{0{\cdot}46\times 144}=935{\cdot}2 \text{ cu. ft.}$$

and the weight of air in the condenser is $\frac{40}{935}=0{\cdot}04278$ lb.

MIXTURES OF AIR AND STEAM

The knowledge of the greatest weight of water vapour which can exist as vapour with a given weight of dry air under given conditions of temperature and pressure is often required in boiler and internal combustion engine trials, ventilating problems, etc.

The calculations depend on two laws first discovered by Dalton, i.e.—

(1) The pressure exerted by, and the quantity of, the vapour required to saturate a given space (i.e. exists as saturated steam) at any given temperature, are the same whether the space is filled by a gas or is a vacuum.

(2) The pressure exerted by a mixture of a gas and a vapour, of two vapours, or of two gases, is the sum of the pressures which each would exert if it occupied the same space alone, assuming the constituents do not interact.

Suppose we have a volume of V cu. ft. of a mixture of air and water vapour, in which the quantity of vapour is the greatest that can exist, i.e. the air is saturated with vapour, or in other words the vapour exists as saturated steam. Let P be the absolute pressure in inches Hg, and T the temperature in ° F. From law (1) above, the pressure exerted by the vapour is the same as if no air were present. From steam tables we find that for temperature T the pressure of saturated steam is, say, p_s, and its density in pounds per cubic foot is, say, ρ_s. Then in the mixture we shall have V cu. ft. of steam at temperature T, pressure p_s, and density ρ_s. The total weight of this steam is evidently $V \times \rho_s$. Also from law (2), since the pressure of the vapour is p_s, the pressure of the air must be $P-p_s$. Its volume is V and temperature T.

Now weight of 1 cu. ft. air at N.T.P.$=0{\cdot}0807$ lb.

,, ,, V ,, ,, $=0{\cdot}0807V$ lb.

weight of V cu. ft. air at pressure $(P-p_s)$ and temperature T

$$=\frac{P-p_s}{29{\cdot}9}\times\frac{492}{460+T}\times 0{\cdot}0807V$$

and the weight of V cu. ft. of the vapour was seen to be $V\rho_s$ lb.

Therefore weight of vapour which can exist with 1 lb. dry air under the given conditions

$$=w=\frac{V\rho_s}{0{\cdot}0807\times\frac{492}{460+T}\times\frac{P-p_s}{29{\cdot}9}\times V}$$

$$w=0{\cdot}754\frac{\rho_s(460+T)}{P-p_s} \quad \left(\text{i.e. } \frac{29{\cdot}9}{492\times 0{\cdot}0807}=0{\cdot}754\right)$$

EXAMPLE.— $P=29{\cdot}9''$ Hg. $T=120°$ F.

Then $p_s=3{\cdot}438''$ Hg. $\rho_s=0{\cdot}004924$

$$w=\frac{0{\cdot}754\times 0{\cdot}004924\times 580}{26{\cdot}462}$$

$$=0{\cdot}0813 \text{ lb.}$$

A curve is plotted in Fig. 61 showing w for various temperatures and for $P=29{\cdot}9$ in. The value of w varies inversely as $P-p_s$.

If we wish to know the maximum weight of vapour which can exist with the volume of air which would occupy 1 cu. ft. *if dry* and at $29{\cdot}9''$ Hg, we proceed as follows :

We have seen that $V\rho_s$ lb. of vapour can exist with V cu. ft. of air at pressure $P-p_s$ and temperature T. The volume this air would occupy at $29{\cdot}9''$ Hg

$$=V\times\frac{P-p_s}{29{\cdot}9}$$

Therefore for a volume of air which would occupy 1 cu. ft. if dry, at 29·9 in. there corresponds a weight of vapour

$$=\frac{V\rho_s}{V\times\frac{P-p_s}{29\cdot9}}$$

Therefore $$w'=\frac{29\cdot9\rho_s}{P-p_s}$$

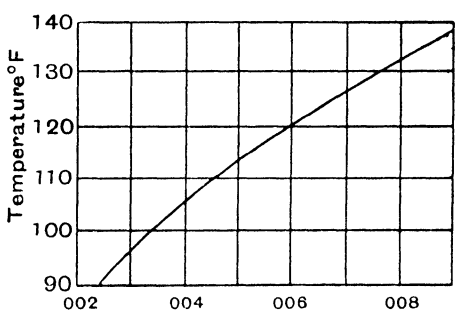
EXAMPLE.—P=29·9″ T=120°

Then $p_s=3\cdot438$ $\rho_s=0\cdot0049$

$$w'=\frac{29\cdot9\times0\cdot0049}{26\cdot462}=0\cdot0057$$

Dew Point.—Suppose that a weight w_2 of vapour is associated with 1 lb. of dry air, the mixture having a temperature T and pressure P.

If w_2 is less than w, i.e. $0\cdot754\frac{\rho_s(460+T)}{P-p_s}$, then the air is not saturated with vapour, and the vapour exists as superheated steam. If, however,



Weight of water (lb.) carried by the air which would occupy 1 cu. ft. if dry and at 29·9 in. mercury pressure.

Actual air pressure = barometric pressure − vapour pressure p_A at temperature T_A.

Actual weight carried per cubic foot = weight from curve $\times\frac{29.9}{p_A}$.

Fig. 61

the temperature falls, a point is reached at which $w_2=w$, and at this temperature w_2 is the maximum quantity of vapour which can exist as vapour with the air. Any further diminution of temperature (for the same pressure) will result in some of the vapour condensing. This temperature is called the *dew point*, and it is the temperature which satisfies the equation

$$w_2=0\cdot754\frac{\rho_s(460+T)}{P-p_s}$$

EXAMPLE.—In a boiler trial it is found that the weight of dry flue gas per minute=20 lb. and the weight of vapour formed by combustion per minute=1 lb. Pressure in flue=29·9 in.

Then $w_2=1/20=0{\cdot}05$ lb. per lb. dry gas.

It will be found from the steam tables that for T=105° F. the values of ρ_s and p_s satisfy the above equation; therefore 105° is the dew point, and if the gases are cooled below this temperature, vapour will condense. This may happen locally if the feed water in the economiser tubes is cold. (N.B. This temperature may be read off directly from the curves of Fig. 170.)

Humidity.—Water vapour is always present in the atmosphere in varying amounts. Usually the quantity of water present is less than that which would be required to saturate the air at the ordinary temperature, and a knowledge of the actual quantity of water present is often required, e.g. in producer trials. The ratio of the weight of vapour actually present in a given volume to the weight which would be present in the same volume if the air were saturated at the same temperature is known as the *humidity* or, more accurately, the *relative humidity* of the air, and is expressed as a percentage. When the humidity is less than 100 per cent. the vapour present is superheated, and will therefore obey Boyle's Law so long as the state of saturation is not passed.

If w=weight of vapour present in 1 cu. ft. of air, p=pressure of this vapour } at temperature t

and w_s=weight of vapour present in 1 cu. ft. of saturated air, p_s=pressure of this vapour } at same temperature t

Then
$$\frac{w}{w_s}=\frac{p}{p_s}$$

and we see that the relative humidity is the ratio of the actual vapour pressure to the saturation pressure for the same temperature.

Let T be the actual temperature of the air and vapour mixture. If this air were saturated by the vapour, the vapour pressure would be p_s, and the weight of 1 cu. ft. of the vapour (which is the same as the weight of vapour present in 1 cu. ft. of the saturated air and vapour mixture) would be w_s, which can be obtained from the steam tables.

Now suppose 1 cu. ft. of the unsaturated air to be cooled down to a temperature T_0 corresponding to its dew point, the pressure remaining the same. Its volume is reduced to $\frac{460+T_0}{460+T}$ cu. ft., and the air is now of course saturated. The weight of vapour present in 1 cu. ft. of this saturated air at T_0 is w_0 (from steam tables) and therefore the weight present in

$$\frac{460+T_0}{460+T} \text{ cu. ft.} = \frac{460+T_0}{460+T} \times w_0 \text{ lb.}$$

But this is the quantity w which was present in 1 cu. ft. of the unsaturated air, since no vapour can have been lost or gained.

Therefore relative humidity at

$$T = \frac{w}{w_s} = \frac{460+T_0}{460+T} \times \frac{w_0}{w_s}$$

To determine this we therefore require to know the dew point. This can be obtained by means of a hygrometer, the commonest form of which is the wet and dry bulb thermometer, and Glaisher's tables have been constructed giving the dew point and also the relative humidity for various readings of the wet and dry bulbs. (The dew-point temperatures have been determined experimentally by comparison with an absolute hygrometer such as Daniell's.)

EXAMPLE.—Dry bulb 60°. Wet bulb 55°.

Then $T_0 = 50{\cdot}6$ from Glaisher's tables

and $w_0 = 0{\cdot}000608$, $w_s = 0{\cdot}000828$ } from steam tables.

Relative humidity at

$$T = \frac{510{\cdot}6}{520} \times \frac{0{\cdot}00060}{0{\cdot}000828} = 71 \text{ per cent.}$$

which is the figure also given in Glaisher's tables. Therefore the actual weight of vapour present in 1 cu. ft. of the unsaturated air at 60° F.

$$= w_s \times \frac{71}{100} = 0{\cdot}000588 \text{ lb.}$$

The pressure exerted by this vapour will be

$$\frac{71}{100} \times p_s = \frac{71}{100} \times 0{\cdot}522'' \text{ Hg} = 0{\cdot}371'' \text{ Hg}$$

If the atmospheric pressure is, say, 29·5″ Hg, the pressure of the dry air will be $29{\cdot}5 - 0{\cdot}371 = 29{\cdot}129''$ Hg.

The weight of 1 cu. ft. of this dry air will be

$$\frac{29{\cdot}129}{29{\cdot}9} \times 0{\cdot}0807 = 0{\cdot}0785 \text{ lb.}$$

Therefore for each 1 lb. of dry air there will be

$$\frac{0{\cdot}000588}{0{\cdot}0785} \text{ lb. vapour} = 0{\cdot}0075 \text{ lb. vapour,}$$

and for each 1 lb. of air and vapour there will be

$$\frac{0{\cdot}0075}{1{\cdot}0075} \text{ lb. vapour.}$$

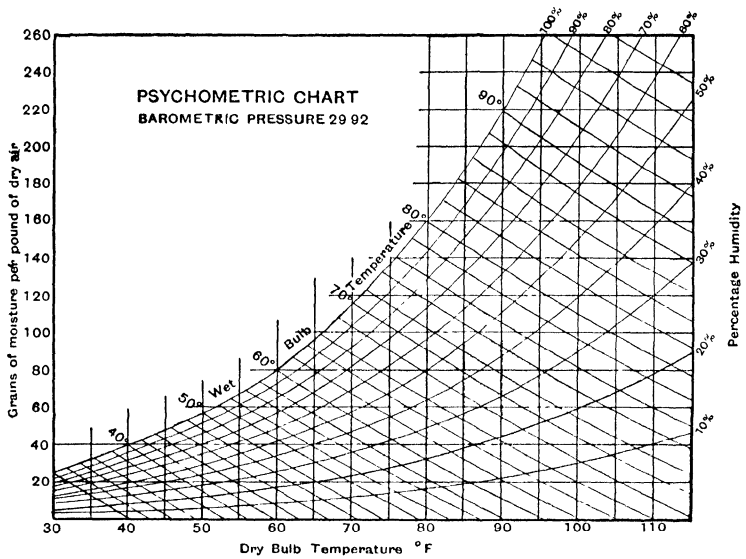


Fig. 62

Fig. 62 [1] shows the values obtained from Glaisher's tables in a convenient form. The ordinate chosen is the dry-bulb temperature, and the relative humidity and moisture content can be read at once.

EXAMPLES ON CHAPTER IX

1. Calculate the work done when 1 lb. of steam expands adiabatically from 80 lb. per sq. in. abs. to 15 lb. per sq. in. abs., (*a*) when the steam is originally dry saturated, and (*b*) when the dryness fraction is originally 0·9.

2. Steam 30 per cent. wet at 100 lb. per sq. in. abs. expands adiabatically to 20 lb. per sq. in. abs. Find its wetness after expansion. If the expansion can be represented by pv^n=constant, find n.

3. A steam engine works non-expansively, the initial pressure being 120 lb. per sq. in. abs., and the exhaust pressure 5 lb. per sq. in. abs. (t=162·3° F.). Estimate the work done per pound of steam and the efficiency of the engine by calculation, and from a temperature-entropy chart, (*a*) when the steam is initially dry saturated, and (*b*) when the initial dryness fraction is 0·8. [Given, volume of 1 lb. of dry saturated steam at 120 lb. abs.=3·726 cu. ft., and temperature=341° F.]

4. In an engine cylinder the clearance volume is 2 cu. ft. The boiler pressure is 100 lb. per sq. in. abs. and the pressure in the cylinder at the instant steam is

[1] " Heating and Air Cooling," Allen and Wilkes, and similar publications.

admitted is 15 lb. per sq. in. abs. The dryness fraction of the boiler steam is 0·9 and that of the steam shut in the clearance 0·95. Find the amount of steam admitted, and the dryness fraction, when the pressure in the clearance reaches 100 lb. abs. Neglect all losses and assume the cylinder to be a non-conductor of heat. Given—

Pressure	h	L	Specific Volume
100	298·3	888	4·429
15	181·0	969·7	26·27

5. Boiler steam of dryness fraction 0·97 and at a pressure of 340 lb. per sq. in. abs. (t=429° F.) is wire-drawn to 200 lb. per sq. in. (t=382° F.) abs. Calculate the dryness fraction on the engine side of the reducing valve.

6. Steam at a pressure of 315 lb. per sq. in. abs. and 0·95 dry is wire-drawn to 265 lb. per sq. in. abs. Find the final dryness fraction. If the steam is initially dry saturated, find the final condition.

7. In a test on a condensing plant the following results were obtained : (*a*) steam condensed per hour 2100 lb. ; (*b*) temperature of exhaust steam entering the condenser 126° F. ; (*c*) weight of circulating water used per minute 510 lb. ; (*d*) temperature of circulating water as it enters and leaves the condenser 55° F. and 90° F. respectively ; (*e*) temperature of the air pump discharge 95° F. Calculate the dryness fraction of the exhaust steam as it enters the condenser.

8. In a test with a throttling calorimeter the following data were obtained : Pressure in main steam pipe 153 lb. per sq. in. abs. (t=360 F. and L=862) ; temperature after wire-drawing 240° F. ; pressure after wire-drawing 17·19 lb. per sq. in. abs. (t=220° F., H=1153). Estimate the dryness of the steam in the main steam pipe. Assume the specific heat, C_p=0·5.

9. The following data were obtained in a test with a combined throttling and separating calorimeter : Water from separator 1·5 lb., steam condensed after wire-drawing 32 lb. Steam temperature before wire-drawing 340° F., after wire-drawing 225° F. ; pressure after wire-drawing 15 lb. per sq. in. abs. (t=213, H=1150·7). Estimate the dryness of the steam. Assume specific heat, C_p=0·5.

10. The pressure in a steam main is 103 lb. per sq. in. (gauge). A combined separating and throttling calorimeter is used to determine the dryness fraction. The observations recorded are : Water drawn from separator 1·5 lb. ; weight of steam condensed after wire-drawing 32 lb. ; temperature in the calorimeter after throttling 225° F. ; pressure in the calorimeter 15 lb. per sq. in. Find the quality of the steam in the main.

11. Using steam tables, calculate the dryness fraction of steam from the following observations taken from a throttling calorimeter : Pressure in main steam pipe 80·3 lb. per sq. in. gauge ; temperature of steam in calorimeter 260° F. ; manometer reading 3 in. of mercury ; barometer reading 29 in.

12. A cylindrical vessel of 8 cu. ft. capacity is connected to a steam pipe supplying 90 per cent. dry steam at 45 lb. per sq. in. abs. pressure. If the vessel initially contains steam at 15 lb. per sq. in. abs. and of quality 85 per cent. dry, calculate the weight of steam which will be admitted when the valve to the steam pipe is opened, and the final quality of steam in the vessel.

13. In a steam boiler 9·5 lb. of steam are generated per pound of coal burnt. The boiler pressure is 155·3 lb. per sq. in. gauge and the temperature of the feed water 90° F. If the dryness fraction of the steam is 0·98, and the calorific value of the coal is 14,500 B.Th.U. per lb., calculate the efficiency of the boiler.

14. Steam at a pressure of 150 lb. per sq. in. abs. is superheated 60° F. and then expands isentropically. At what pressure will the steam become dry and saturated ?

15. Steam at 200 lb. per sq. in. abs. (t=382° F.) is superheated 100° F. It is then passed through a reducing valve and has its pressure reduced to 15 lb. per sq. in. abs. (t=213° F.). Determine the temperature and condition of the steam after wire-drawing and calculate, without using steam tables, the change of entropy. Take the average specific heat as 0·5.

16. Steam passing to an engine has a pressure of 150 lb. per sq. in. abs. (t=358° F.) and a temperature of 600° F. when in the main steam pipe. Before entering the engine it is throttled down to 100 lb. per sq. in. abs. (t=328° F.), and is then expanded adiabatically down to 25 lb. per sq. in. abs. (t=240° F.). Determine, without using steam tables, the temperature and condition of the steam before and after expansion.

17. Steam at a temperature of 330° F. and of dryness 0·9 is wire-drawn to 190° F. Calculate the gain of entropy.

18. Steam at a pressure of 220 lb. per sq. in. abs. (t=390° F.) and of dryness 0·97 is wire-drawn to a pressure of 13 lb. per sq. in. abs. Determine the condition of the steam after throttling and the gain of entropy. Given

p	t° F.	L
220	390	836·2
13	205·9	974·2

19. Solve Problem 18 if the steam is initially dry and saturated at 220 lb. abs.

20. Solve Problems 14 to 19 inclusive by means of a total heat-entropy (Mollier) diagram.

Chapter X

STEAM ENGINE CYCLES

*CYCLE USING STEAM NON-EXPANSIVELY

This cycle is approximately that used in the Newcomen engine. The *p.v.* and Tϕ diagrams are shown in Fig. 63.

Assume 1 lb. of water at pressure p_2 and temperature T_2 to be pumped into a boiler where the pressure is maintained at the value p_1. This

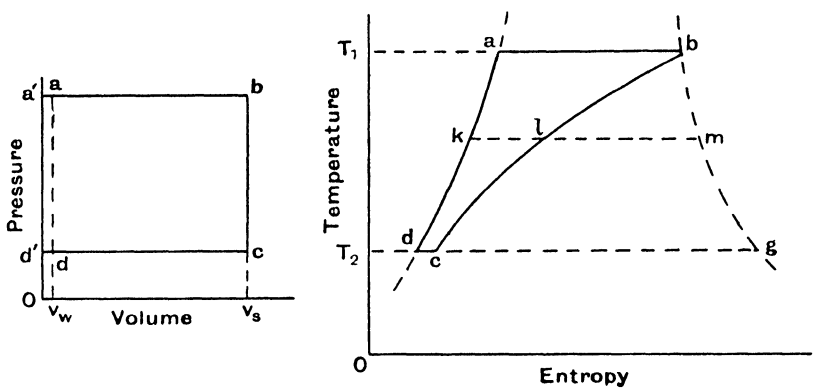


Fig. 63.

water is now heated to temperature T_1, the boiling temperature corresponding to p_1. The further addition of heat causes evaporation at constant pressure along the line *ab*, until dryness is reached at *b*. Heat is now withdrawn from the steam, which condenses at constant volume along the line *bc* until a dryness fraction q_c is attained at pressure p_2. Finally condensation is completed at constant pressure along the curve *cd*. Referring to the *p.v.* chart the work done by the steam in the cylinder is *a'bcd'*, and that done by the pump is *d'daa'*. If the very slight compressibility of water is neglected *da* becomes a line of constant volume v_w.

Assuming the specific heat of water is 1,

then the heat received by the water$=T_1-T_2$

and the work performed by the pump$=(p_1-p_2)v_w$.

The total energy supplied to the water

$$=(T_1-T_2)+\frac{(p_1-p_2)}{J}v_w$$
$$=h_1-h_2$$

Heat received during evaporation $=L_1$

The total work done $=a'bcd'-d'daa'$
$$=(v_s-v_w)(p_1-p_2) \text{ ft.-lb.}$$

and
$$\text{Efficiency}=\frac{(v_s-v_w)(p_1-p_2)}{J\{L_1+T_1-T_2+(p_1-p_2)v_w\}}$$

or, more accurately,
$$=\frac{(v_s-v_w)(p_1-p_2)}{J(H_1-h_2)},$$

as H and h take into account the variations of specific heat and volume of water.

On the Tϕ chart, except in the region of very high temperatures and pressures, the curve representing the heating of water from T_2 to T_1 at constant pressure may be taken as coincident with the liquid boundary (p. 68). (Contrast this case with that of heating liquid CO_2 at constant pressure, Fig. 153.) The curve bc is one of constant volume, the volume being that occupied by 1 lb. of dry steam at pressure p_1.

If v_2 is the specific volume of steam at p_2, then the dryness fraction at point c will be given by $cd/dg=v_1/v_2$.

The diagram for any particular case can be readily set off on Fig. 25, and the dryness fraction read.

The efficiency is of the order of 5 per cent., and the cycle is only used in engines of small power where considerations of simplicity may outweigh those of efficiency.

EXAMPLE.—A steam engine using saturated steam works non-expansively, the initial pressure being 100 lb. per sq. in. abs. ($t=327{\cdot}6°$ F.) and the final pressure 15 lb. per sq. in. abs. ($t=213°$ F.). Estimate its theoretical efficiency, given that the specific volume of steam at 100 lb. is 4·34 cu. ft., and the volume of 1 lb. of water is 0·016 cu. ft.

Now,
$$\text{Efficiency}=\frac{\text{work done}}{\text{heat energy absorbed}}$$
$$=\frac{(p_1-p_2)(v_s-v_w)}{J(H_1-h_2)}$$

From the tables $H_1=1186{\cdot}3$ B.Th.U.

and $h_2=181$ B.Th.U.

$$\therefore \text{efficiency}=\frac{144(100-15)(4{\cdot}34-0{\cdot}016)}{778\times(1186{\cdot}3-181)}$$
$$=\frac{144\times85\times4{\cdot}324}{778\times1005}=0{\cdot}068 \text{ or } 6{\cdot}8 \text{ per cent.}$$

N.B. Note that the Carnot efficiency working between the same limits of temperature would be—

$$\frac{327{\cdot}6-213}{327{\cdot}6+460}=\frac{114{\cdot}6}{787{\cdot}6}=0{\cdot}145 \text{ or } 14{\cdot}5 \text{ per cent.}$$

*CARNOT CYCLE

As shown on p. 32, the diagram for this "ideal" cycle is bounded by two isothermals and two adiabatics, the isothermals in the case of saturated steam being also lines of constant pressure.

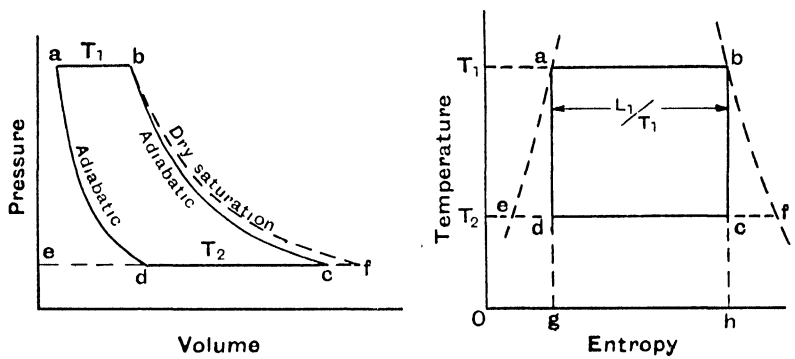


Fig. 64.

The *p.v.* and Tϕ diagrams are shown in Fig. 64. The operations are as follows :

(1) Water of specific volume v_w is evaporated to dryness along *ab*, when its volume is v_s.

(2) The steam is expanded adiabatically along *bc*, its dryness fraction at *c* being given by *ce/fe*.

(3) The steam is partially condensed at temperature T_2 along *cd*.

(4) Condensation is completed by adiabatic compression, the point *d* where compression commences being selected so that full condensation is attained at the point *a*.

N.B. Adiabatic compression of fairly dry steam increases the dryness, while similar compression of very wet steam tends to produce complete condensation.

The efficiency, as shown by the Tϕ diagram, will be given, as before, by

$$\frac{abcd}{abhg}=\frac{T_1-T_2}{T_1}$$

The work done during the cycle will be $L_1\left(\frac{T_1-T_2}{T_1}\right)$, L_1 being the only heat supplied during the cycle.

The efficiency of a perfect engine using this cycle, and with a lower temperature limit of 520° F. abs., or 60° F., is shown below for a number of initial pressures.

Absolute pressure pounds per square inch	Temperature T_1	Efficiency $\frac{T_1-T_2}{T_1}$
40	728·17	0·284
100	788·63	0·339
200	842·64	0·381
300	878·50	0·407

Pressures of 400 lb. per sq. in. are now common, and steam has been generated at much higher pressures (Munzinger, *I.Mech.E.*, 1936), but only under very exceptional circumstances can theoretical efficiencies be expected which are much in excess of those shown. It should be noted that the attainment of high initial temperatures by the use of superheated steam is not as advantageous as might be expected, as all the heat has not been taken in, nor is it all available, at the highest temperature of superheat.

The lower limit of T_2 is set by the temperature of the available condensing water and is about 60° F., the minimum value obviously being the freezing-point, 32° F.

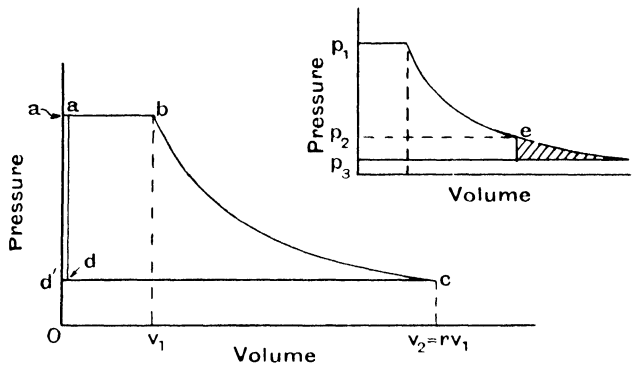
From a thermodynamic point of view much the most serious loss of efficiency is inherent in the boiler, where a very large irreversible drop of temperature occurs between the furnace gases and the steam generated. It is because of this loss that the internal combustion engine appears in such a favourable light.

*RANKINE CLAUSIUS CYCLE. EFFICIENCY IN TERMS OF p AND v

The disadvantage of the Carnot cycle is that condensation within the cylinder would be an extremely slow process, involving a large drop of temperature of the material of the cylinder itself, and a correspondingly great absorption of heat at the commencement of evaporation. To obviate these difficulties the steam is either exhausted at constant pressure into a condenser of large capacity where compression cannot be carried out, or discharged to the atmosphere in the case of non-condensing engines. Evaporation is always effected in a separate boiler, which is assumed to be fed from the condenser, i.e. the " hot well " discharge is returned to the boiler. In non-condensing engines heat quantities are calculated with reference to the temperature of the boiler feed-water.

The Rankine cycle, in which final compression is not used, is an attempt to attain a practical approximation to the Carnot.

The indicator diagram of an engine working on this cycle (assuming no clearance) is shown in Fig. 65. *ab* represents the admission of the high-



Figs. 65 and 65A

pressure boiler steam, *b* is the point of cut-off, *bc* is the expansion line (adiabatic), and *cd* the exhaust to the condenser.

Let p_1=initial pressure in pounds per square foot at *a*
p_2=final or exhaust pressure at *c* and *d*
v_1= initial volume (at *b*) before expansion
v_2=the volume after expansion

Then the work done per pound during admission

$$=p_1v_1 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

and the work done per pound during expansion

$$=\frac{p_1v_1-p_2v_2}{n-1} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Work done *on* the steam during exhaust

$$=p_2v_2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Work done by the feed pump

$$=(p_1-p_2)v_w \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

Heat supplied

$$=(H_1-h_2) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

Hence the net work done per pound of steam

$$=p_1v_1+\frac{p_1v_1-p_2v_2}{n-1}-p_2v_2-(p_1-p_2)v_w$$

$$=\frac{n}{n-1}(p_1v_1-p_2v_2)-(p_1-p_2)v_w \quad . \quad . \quad . \quad . \quad . \quad (6)$$

and the efficiency $=\dfrac{\text{net amount of work done}}{\text{heat supplied}}$

$$=\frac{\dfrac{n}{n-1}\left(\dfrac{p_1v_1-p_2v_2}{n-1}\right)-(p_1-v_2)v_w}{\mathrm{J}(\mathrm{H}_1-h_2)} \quad . \quad . \quad . \quad . \quad (7)$$

This is also considered by a different method on p. 156.

The work done in the cylinder $=p_1v_1+\dfrac{p_1v_1-p_2v_2}{n-1}-p_2v_2$

$$=\frac{n}{n-1}(p_1v_1-p_2v_2)$$

For many purposes the work done by the pump, $(p_1-p_2)v_w$, can be neglected, being of the order of 0·1 per cent. of the total energy supplied.

The above reasoning is for complete expansion, i.e. the steam expands until its pressure is the same as that of the condenser.

Incomplete expansion is also illustrated in Fig. 65a, the expansion being stopped at some point e. This results in a loss of work represented by the toe of the diagram shown shaded.

In this case, if p_2 is the pressure after expansion to e and p_3 is the pressure in the condenser, the efficiency becomes

$$=\frac{p_1v_1+\dfrac{p_1v_1-p_2v_2}{n-1}-p_3v_2-(p_1-p_3)v_w}{\mathrm{J}(\mathrm{H}_1-h_2)} \quad . \quad . \quad . \quad (8)$$

EXAMPLE.—If a steam engine works between 100 and 15 lb. per sq. in. abs., as in Example on p. 148, but with complete adiabatic expansion, i.e. on the Rankine cycle, estimate its efficiency, the clearance being neglected.

The law of expansion is $pv^{1\cdot135}=\text{constant}$.

In this case $\quad \text{Efficiency}=\dfrac{\dfrac{n}{n-1}(p_1v_1-p_2rv_1)-(p_1-p_2)v_w}{\mathrm{J}(\mathrm{H}_1-h_2)}$.

To find the ratio of expansion r

Since $\quad p_1{v_1}^{1\cdot135}=p_2{v_2}^{1\cdot135}$

$$\therefore \frac{p_1}{p_2}=\left(\frac{v_2}{v_1}\right)^{1\cdot135}=r^{1\cdot135}$$

$$\therefore 1\cdot135 \log r=\log 100-\log 15=2\cdot000-1\cdot1761=0\cdot8239$$

$$\therefore \log r=\frac{0\cdot8239}{1\cdot135}=0\cdot7259$$

$$\therefore r=5\cdot32$$

$\therefore$ efficiency

$$=\frac{\frac{1\cdot135}{0\cdot135}(100\times144\times4\cdot34-15\times144\times5\cdot32\times4\cdot34)-(100-15)144\times0\cdot016}{778(1186\cdot3-181)}$$

$$=\frac{104{,}690}{778\times1005}=0\cdot135 \text{ or } 13\cdot5 \text{ per cent.}$$

The above result may be checked by the expression for the Rankine efficiency in terms of heat units given on p. 157, i.e.—

$$\text{efficiency}=\frac{(T_1-T_2)\left(1+\frac{L_1}{T_1}\right)-T_2\log_\epsilon\frac{T_1}{T_2}}{L_1+T_1-T_2}$$

Here $L_1=888\cdot0$ B.Th.U.

$T_1=327\cdot8+460=787\cdot8°$ F. abs.

$T_2=213+460=673°$ F. abs.

$L_1+T_1-T_2=888+114\cdot8=1003$

Substituting these values

$$\text{efficiency}=\frac{114\cdot8\left(1+\frac{888}{788\cdot8}\right)-674\times2\cdot303\times\log_{10}\frac{787\cdot8}{673}}{1003}$$

$$=\frac{137\cdot8}{1003}=0\cdot137 \text{ or } 13\cdot7 \text{ per cent.}$$

The most rapid method of solution is by means of the Mollier diagram, as shown on p. 160.

EXAMPLE.—In a Stirling's engine, fitted with a perfect regenerator, the maximum pressure is 140 lb. per sq. in. abs. and the minimum 15 lb. per sq. in. abs., the upper and lower temperatures being 650° F. and 60° F. A perfectly reversible steam engine is supplied with dry saturated steam and works between the same limits of pressure. Compare their efficiencies, and if the piston speed and stroke be the same in each engine, compare the diameters of the cylinders for equal power.

Given temperature of saturation at 140 lb. abs.=352° F., and at 15 lb. abs.=213° F., and specific volume at 15 lb. abs.=25·85.

Stirling engine—

$$\text{Efficiency}=\frac{650-60}{650+460}=\frac{590}{1110}=0\cdot531 \quad \text{or} \quad 53\cdot1 \text{ per cent.}$$

Steam engine—As the engine is " perfectly reversible "

$$\text{Efficiency}=\frac{352\cdot8-213}{352\cdot8+460}=\frac{139\cdot8}{812\cdot8}=0\cdot171 \quad \text{or} \quad 17\cdot1 \text{ per cent.}$$

Since the piston speeds and strokes are the same, the ratio of the areas

of the cylinders will be inversely as the ratio of the mean effective pressures for equal power.

To find the mean effective pressures (P_m).

Stirling engine : $T_1=1110°$; $T_2=520°$ F. abs.

Let A, B, C, D indicate the points on the indicator diagram before and after isothermal expansion and before and after compression.

Then $p_b=p_c\times\frac{1110}{520}=15\times\frac{1110}{520}=31{\cdot}98$ (say 32 lb. per sq. in.)

Now $p_a v_a = p_b v_b$

$$\therefore \frac{v_b}{v_a}=r=\frac{p_a}{p_b}=\frac{140}{32}=4{\cdot}37$$

$$\therefore p_m=\frac{\text{area of diagram}}{\text{length of diagram}}=\frac{p_a v_a \log_\epsilon r - p_b v_b \log_\epsilon r}{v_b-v_a}$$

$$=\frac{140 v_a \log_\epsilon r - 15\times 4{\cdot}37 v_a \log_\epsilon r}{v_b-v_a}=\frac{v_a(140-15\times 4{\cdot}37)\log_\epsilon 4{\cdot}37}{4{\cdot}37 v_b - v_a}$$

$$\therefore p_m=\frac{140-65{\cdot}55}{3{\cdot}37}\times 2{\cdot}303\times 0{\cdot}6405$$

$$=\frac{74{\cdot}45\times 2{\cdot}303\times 0{\cdot}6405}{3{\cdot}37}=\frac{109{\cdot}818}{3{\cdot}37}=32{\cdot}58 \text{ lb. per sq. in.}$$

Steam engine : $T_1=813{\cdot}8°$; $T_2=674°$.

Let q_2=dryness fraction after expansion.

Then length of diagram for 1 lb. of steam$=25{\cdot}85\times q_2$.

To find q_2.—For adiabatic expansion we have the equation

$$q_2=\frac{T_2}{L_2}\left(\frac{L_1}{T_1}+\log_\epsilon \frac{T_1}{T_2}\right)$$

$L_1=1114-0{\cdot}7\times 352{\cdot}8=1114-246{\cdot}96=867{\cdot}04$ B.Th.U.

$L_2=1114-0{\cdot}7\times 213=1114-149{\cdot}1=964{\cdot}9$ B.Th.U.

$$\therefore q_2=\frac{674}{964{\cdot}9}\left(\frac{867{\cdot}04}{813{\cdot}8}+2{\cdot}303\times\log_{10}\frac{813{\cdot}8}{674}\right)$$

$$=\frac{674}{964{\cdot}9}\left\{\frac{867{\cdot}04}{813{\cdot}8}+2{\cdot}303(2{\cdot}9105-2{\cdot}8287)\right\}$$

$$=\frac{674}{964{\cdot}9}(1{\cdot}065+0{\cdot}188)=\frac{674\times 1{\cdot}253}{964{\cdot}9}=0{\cdot}866.$$

(q_2 may also be found directly from the Tϕ and Mollier diagrams.)

$\therefore$ neglecting the volume of the water, the volume after expansion is $25{\cdot}85\times 0{\cdot}866=22{\cdot}38$ cu. ft.

Work done=area of Tϕ diagram. This is a rectangle of height $(813{\cdot}8-674)°$

and $W\frac{L_1}{T_1}$ or $\frac{867{\cdot}04}{813{\cdot}8}$ units of entropy in width

or $=778(813{\cdot}8-674)\times\frac{867{\cdot}04}{813{\cdot}8}$ ft.-lb.

$$\therefore p_m=\frac{778\times139{\cdot}8\times\frac{867{\cdot}04}{813{\cdot}8}}{22{\cdot}38\times144}\text{ lb. per sq. in.}$$

$$=\frac{778\times139{\cdot}8\times1{\cdot}065}{22{\cdot}38\times144}=35{\cdot}96\text{ lb. per sq. in.}$$

Hence $$\frac{\text{area of Stirling}}{\text{area of steam}}=\frac{35{\cdot}96}{32{\cdot}58}=\frac{1{\cdot}104}{1}$$

$$\therefore \frac{\text{diameter of Stirling}}{\text{diameter of steam}}=\sqrt{1{\cdot}104}=\frac{1{\cdot}05}{1}$$

THE RANKINE CYCLE AS A REVERSIBLE CYCLE

The efficiency of the Rankine cycle will fall below that of the Carnot cycle solely because the reversible adiabatic compression of the latter is replaced by a process in which heat is taken in, to heat the water, over a range of temperature varying from T_2 to T_1, otherwise the cycles are identical. The Rankine cycle therefore represents the ideal performance of an engine using a separate condenser.

The work done during the cycle, and the efficiency of operation, can be calculated on this basis :

The engine takes in the greater part of its heat at the upper temperature limit T_1, but some is taken in between T_2 and T_1. The cycle may be made reversible if the feed water takes up its heat in an infinite number of instalments at temperatures ranging from T_2 to T_1 from a series of bodies having the same range of temperature. In each little instalment the expression

$$\frac{T-T_2}{T}$$

represents the efficiency of the possible transformation into work of the small quantity of heat, say δQ, which is taken in by the working substance at any particular temperature T. The amount of heat converted into work for each instalment is therefore

$$\frac{\delta Q(T-T_2)}{T}$$

and the total quantity of heat converted into work will be

$$\sum\frac{\delta Q(T-T_2)}{T} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

During this stage of the cycle the whole heat taken in is sensible from T_2 to T_1, and equals h_1-h_2. The total heat taken in per pound of working substance is the sensible heat plus the latent heat L_1 at temperature T_1.

Then the total work done per pound, expressed in heat units, is—

$$W=\int_{T_2}^{T_1} dQ\frac{(T-T_2)}{T}+L_1\frac{(T_1-T_2)}{T_1}$$

$$=\int_{T_2}^{T_1} dQ-T_2\int_{h_2}^{h_1}\frac{dQ}{T}+L_1\frac{(T_1-T_2)}{T_1} \quad . \quad . \quad . \quad . \quad (1)$$

Assuming the specific heat of water to be constant and equal to unity, in which case $dQ=dT$,

$$W=(T_1-T_2)-T_2\log_\epsilon\frac{T_1}{T_2}+L_1\frac{(T_1-T_2)}{T_1}$$

or $$W=(T_1-T_2)\left(1+\frac{L_1}{T_1}\right)-T_2\log_\epsilon\frac{T_1}{T_2}\text{ heat units per lb. } . \quad . \quad (2)$$

Now total heat supplied

$$=H_1-h_2=L_1+h_1-h_2=L_1+T_1-T_2 \text{ approximately}$$

$$\therefore \text{ efficiency}=\frac{(T_1-T_2)\left(1+\frac{L_1}{T_1}\right)-T_2\log_\epsilon\frac{T_1}{T_2}}{L_1+T_1-T_2} \quad . \quad . \quad . \quad (3)$$

If the steam be initially wet, and of dryness fraction q_1, the work done per pound becomes

$$W=T_1-T_2-T_2\log_\epsilon\frac{T_1}{T_2}+q_1L_1\frac{(T_1-T_2)}{T_1} \quad . \quad . \quad . \quad . \quad (4)$$

or $$W=(T_1-T_2)\left(1+\frac{q_1L_1}{T_1}\right)-T_2\log_\epsilon\frac{T_1}{T_2}\text{ heat units } . \quad . \quad (5)$$

*RANKINE CLAUSIUS CYCLE. EFFICIENCY IN TERMS OF T AND ϕ

Commencing with 1 lb. of water at temperature T_2, represented by the point *a* on the diagram (Fig. 66), the water is heated to T_1 along the line *ab*, the gain of entropy being

$$fg=C_p\log_\epsilon\frac{T_1}{T_2}=\log_\epsilon\frac{T_1}{T_2} \quad (C_p \text{ for water being assumed unity})$$

The water at temperature T_1 is now turned into steam at T_1 along the line *bc*, the gain of entropy *bc* or *gk* being $\frac{L_1}{T_1}$, where L_1 is the latent heat

at temperature T_1. The steam then expands adiabatically along *cd*, down to the original temperature T_2, when condensation follows along *da* to the original condition of water at temperature T_2. The work done

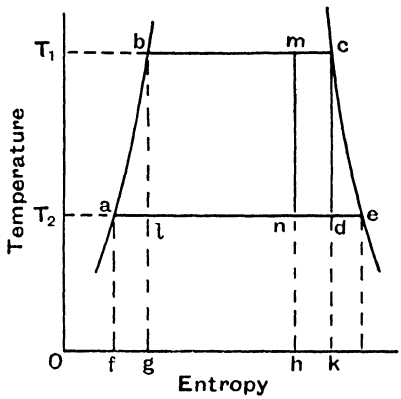


Fig. 66.

during the cycle is represented by the area *abcd* and the total amount of heat supplied is represented by the area *fabck*.

The efficiency of the cycle is therefore

$$\frac{\text{Heat converted into work}}{\text{Heat supplied}} = \frac{\text{area } abcd}{\text{area } fabck}$$

The area *abcd* = area *fabg* − area *falg* + area *lbcd*

$$= C_p(T_1-T_2) - T_2 \times \log_\epsilon \frac{T_1}{T_2} + \frac{L_1}{T_1} \times (T_1-T_2)$$

and taking $C_p = 1$

$$= (T_1-T_2)\left(1+\frac{L_1}{T_1}\right) - T_2 \log_\epsilon \frac{T_1}{T_2} \quad . \quad . \quad . \quad . \quad (1)$$

a result already obtained.

The area *fabck* = area *fabg* + area *gbck*

$$= T_1 - T_2 + T_1 \times \frac{L_1}{T_1}$$

$$= L_1 + T_1 - T_2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

The efficiency is therefore

$$\frac{(T_1-T_2)\left(1+\frac{L_1}{T_1}\right) - T_2 \log_\epsilon \frac{T_1}{T_2}}{L_1+T_1-T_2} \quad . \quad . \quad . \quad . \quad (3)$$

which may also be written

$$1-\frac{T_2\left(\frac{L_1}{T_1}+\log_\epsilon\frac{T_1}{T_2}\right)}{L_1+T_1-T_2} \quad . \quad . \quad . \quad . \quad . \quad (3A)$$

If the steam is initially wet, having dryness fraction $\frac{bm}{bc}$, the adiabatic expansion will take place along *mn*, and the work done per pound of steam will be represented by the area *abmn* and the total amount of heat supplied will be represented by the area *fabmh*.

The area *abmn* = area *fabg* − area *falg* + area *lbmn*

$$=(T_1-T_2)-T_2\times\log_\epsilon\frac{T_1}{T_2}+\frac{q_1L_1}{T_1}(T_1-T_2)$$

$$=(T_1-T_2)\left(1+\frac{q_1L_1}{T_1}\right)-T_2\log_\epsilon\frac{T_1}{T_2} \quad . \quad . \quad . \quad . \quad (4)$$

The area *fabmh* = area *fabg* + area *gbmh*

$$=(T_1-T_2)+T_1\times\frac{q_1L_1}{T_1}$$

$$=q_1L_1+T_1-T_2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

The efficiency is therefore

$$\frac{(T_1-T_2)\left(1+\frac{q_1L_1}{T_1}\right)-T_2\log_\epsilon\frac{T_1}{T_2}}{q_1L_1+T_1-T_2} \quad . \quad . \quad . \quad . \quad (6)$$

It will be seen from the diagram (Fig. 66) that the engine working on the Rankine-Clausius cycle does more work per pound of steam than the Carnot engine, the excess being represented by the area *abl*. To do this greater amount of work, however, the engine has to take in a proportionately larger quantity of heat represented by the area *falg*, and is therefore less efficient, which fact has already been shown on p. 155.

If the line for dry saturated steam *ce* be drawn, the dryness fraction of the steam at any point of the expansion can be found as on p. 68, the dryness fraction at the end of the expansion being $\frac{ad}{ae}$. With the engine working on Carnot's cycle the final stage of operations is adiabatic compression from T_2 to T_1. This is represented in Fig. 66 by the line *lb*, and it is obvious that the compression must be commenced when the steam has a dryness fraction $\frac{al}{ae}$.

*THE EFFECT OF USING SUPERHEATED STEAM

Let the steam be superheated to a temperature T_3. Fig. 67 shows the $T\phi$ diagram. The adiabatic expansion is now represented by the vertical line *de*, and if the ratio of expansion is large enough, or if the temperature of the superheat is not too high, the line *de* will cut the saturated steam line at some point *p*, at which point the steam will be just dry and saturated. Further expansion down to *e* results in wet steam of dryness *ae/af*. The total amount of work done per pound of superheated steam is represented by the area *abcde* and the increase of work done due to superheating is represented by the area *gcde*. The extra amount of heat which is supplied to obtain this increase of work done is represented by the area *mcdn*, and the efficiency of the conversion of this heat into work is given by

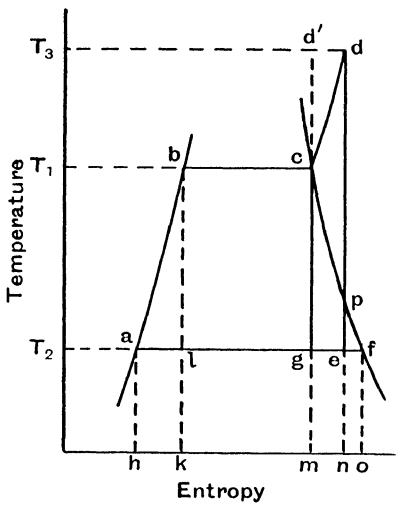


Fig. 67.

$$\frac{\text{area } gcde}{\text{area } mcdn}$$

which is greater than the efficiency when using saturated steam at T_1, namely,

$$\frac{\text{area } abcg}{\text{area } habcm}$$

because the additional heat *mcdn* is received at a much higher average temperature than that at which the other portion of the heat is received. It is, however, less than the efficiency would have been had all the heat been taken in a temperature T_3, i.e. than

$$\frac{gd'de}{md'dn}$$

The total work done per pound of superheated steam is represented by the area

$$abcde = \text{area } habk - \text{area } halk + \text{area } lbcg + \text{area } mcdn - \text{area } mgen$$

$$= (T_1 - T_2) - T_2 \log_\epsilon \frac{T_1}{T_2} + (T_1 - T_2) \times \frac{L_1}{T_1}$$

$$+ C_p(T_3 - T_1) - C_p T_2 \log_\epsilon \frac{T_3}{T_1}$$

$$=(T_1-T_2)\left(1+\frac{L_1}{T_1}\right)-T_2\log_\epsilon\frac{T_1}{T_2}+C_p(T_3-T_1)-C_pT_2\log_\epsilon\frac{T_3}{T_1}$$

$$=(T_1-T_2)\left(1+\frac{L_1}{T_1}\right)+C_p(T_3-T_1)+T_2\left(\log_\epsilon\frac{T_1}{T_2}+C_p\log_\epsilon\frac{T_3}{T_1}\right) \quad (1)$$

The total amount of heat supplied is represented by the area

$$habcdn=\text{area } habcm+\text{area } mcdn$$
$$=L_1+(T_1-T_2)+C_p(T_3-T_1) \quad . \quad . \quad . \quad . \quad (2)$$

The efficiency is therefore

$$\frac{(T_1-T_2)\left(1+\frac{L_1}{T_1}\right)+C_p(T_3-T_1)-T_2\left(\log_\epsilon\frac{T_1}{T_2}+C_p\log_\epsilon\frac{T_3}{T_1}\right)}{L_1+T_1-T_2+C_p(T_3-T_1)} \quad . \quad (3)$$

when C_p is the specific heat of steam at constant pressure.

As the value of C_p is not constant, a more accurate and much more rapid method of finding the work done and the efficiency is given in the next section.

The indirect effects of using superheated steam are considered on p. 189 *et seq.*

*EFFICIENCY OF THE RANKINE CLAUSIUS CYCLE USING THE Hϕ DIAGRAM

The efficiency of the cycle, whatever the initial quality of the steam, can be obtained at once from this diagram.

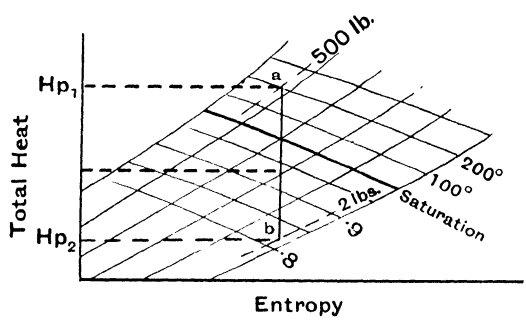


Fig. 68.

The initial condition point *a* is found by the intersection of the line of initial pressure (say 500 lb. per sq. in.) with the line of initial quality (say 200° superheat), and H_{p_1}, the initial total heat of the steam, is read on the scale of ordinates. From *a* a line of constant entropy is drawn

to intersect the line of final pressure (2 lb.) in b, and H_{p_2} is read as before.

Then the work done $= H_{p_1} - H_{p_2}$

and the total heat supplied above $T_2 = H_{p_1} - h_2$

The efficiency is given by $\dfrac{H_{p_1} - H_{p_2}}{H_{p_1} - h_2}$

The final dryness fraction will be seen to be 0·815.

EXAMPLE.—A steam engine using dry saturated steam at a pressure of 100 lb. per sq. in. abs. works under the following conditions: (1) with complete adiabatic expansion to a pressure of 5 lb. per sq. in., (2) with adiabatic expansion to 5 lb. per sq. in. and exhaust at 2 lb. per sq. in., (3) with complete adiabatic expansion to 2 lb. per sq. in., (4) with complete adiabatic expansion to atmospheric pressure.

Determine the work done and the efficiency in each case, using the Mollier chart, and compare the mean effective pressures.

The salient values are

$p_1 = 100$ $H_1 = 1191{\cdot}8$

$p_2 = 5$ $H_2 = 984{\cdot}0$ $h_2 = 130{\cdot}1$ $v_2 = 63$ cu. ft.

$p_3 = 2$ $H_3 = 933$ $h_3 = 94{\cdot}0$

(1) Work done $= 1191{\cdot}8 - 984{\cdot}0 = 207{\cdot}8$ B.Th.U.

$$\text{Efficiency} = \frac{207{\cdot}8}{1191{\cdot}8 - 130{\cdot}1} = 19{\cdot}58 \text{ per cent.}$$

$$\text{M.E.P.} = K \times \frac{\text{work}}{\text{volume}} = 3{\cdot}30K$$

(2) $$\text{Work done} = 207{\cdot}8 + \frac{(p_3 - p_2)63}{778} = 207{\cdot}8 + \frac{3 \times 63}{778}$$

$$= 242{\cdot}8 \text{ B.Th.U.}$$

Heat received $= 1191{\cdot}8 - 94{\cdot}0$ B.Th.U.

$$\text{Efficiency} = \frac{242{\cdot}8}{1097{\cdot}8} = 22{\cdot}12 \text{ per cent.}$$

$$\text{M.E.P.} = K \times 3{\cdot}85$$

(3) Work done $= 1191{\cdot}8 - 933 = 258{\cdot}8$ B.Th.U.

$$\text{Efficiency} = \frac{258{\cdot}8}{1097{\cdot}8} = 23{\cdot}58 \text{ per cent.}$$

$$\text{M.E.P.} = K \times 1{\cdot}725$$

In the latter case the final volume per pound is 150 cu. ft. as against 63 cu. ft. in the former.

(4) For adiabatic expansion to atmospheric pressure, followed by exhaust at that pressure, the work done, efficiency, and final volume are 139·8 B.Th.U., 13·98 per cent., and 26 cu. ft., and the M.E.P.$=K\times 5{\cdot}37$.

EXAMPLE.—The high-pressure cylinder of a steam engine receives dry steam at 200 lb. per sq. in. abs. and 80° F. superheat, admission occurring during 25 per cent. of the stroke. If the pressure at release is 36 lb. per sq. in. while the final back pressure is 34 lb. per sq. in., find the cylinder feed per I.H.P. per hour and the heat received or rejected during expansion per pound of cylinder feed.

At 200 lb. per sq. in. and 80° superheat, $v_1=2{\cdot}61$ cu. ft. and $H_1=1246{\cdot}5$ B.Th.U.

At 36 lb. per sq. in., $v_2=4\times 2{\cdot}61=10{\cdot}44$ cu. ft., while the specific volume is 11·58

or
$$q_2=\frac{10{\cdot}44}{11{\cdot}58}=90{\cdot}15 \text{ per cent.}$$

To find the value of n

$$\left(\frac{v_2}{v_1}\right)^n=\frac{p_1}{p_2}$$

or $$n(\log v_2-\log v_1)=\log p_1-\log p_2$$

or $$n(1{\cdot}0187-0{\cdot}4166)=2{\cdot}3010-1{\cdot}5563$$

or
$$n=\frac{0{\cdot}7447}{0{\cdot}6021}=1{\cdot}237$$

Work per pound of cylinder feed

$$=\frac{n}{n-1}(p_1v_1-p_2v_2)+(p_2-p_3)v_2$$

$$=\frac{1{\cdot}237}{0{\cdot}237}(200\times 2{\cdot}61-36\times 10{\cdot}44)\times 144+2\times 10{\cdot}44\times 144 \text{ ft. lb.}$$

$$=109{,}850+3007=112{,}860 \text{ ft.-lb.}$$

The cylinder feed per I.H.P. hour is $\dfrac{33{,}000\times 60}{112{,}860}=17{\cdot}55$ lb.,

and Heat to or from steam during expansion $=H_J$

where $$H_J=\frac{1}{n-1}(p_1v_1-p_2v_2)\frac{I}{J}-H_1+H_2$$

$$=\frac{109{,}850}{778}-1246{\cdot}5+0{\cdot}9015\times 937{\cdot}7+229{\cdot}6=-30{\cdot}7 \text{ B.Th.U.}$$

or the loss of heat$=30{\cdot}7$ B.Th.U. per pound of cylinder feed.

EXAMPLE.—Compare the efficiencies of Rankine cycles working with steam at 500° F. and a back pressure of 2 lb. per sq. in. when (1) the steam is superheated 120° and (2) when the steam is dry saturated.

In the first case the pressure is 200 lb. per sq. in., in the second it is 684 lb. per sq. in.

and the total heats of the supply steam are 1274 and 1227 B.Th.U.

" " after expansion are 945 " 844 "

The work done is 329 " 383 "

The efficiencies are

$$\frac{329}{1274-94} \text{ and } \frac{383}{1227-94} \quad \text{or} \quad 29 \text{ per cent. and } 33 \text{ per cent.}$$

showing that the cycle using superheated steam is less efficient than the cycle using saturated steam at the same temperature.

*EFFICIENCY OF THE RANKINE CLAUSIUS CYCLE FROM STEAM TABLES

When the Hϕ diagram is not available on a sufficiently large scale the calculation may readily be made from the tables.

If the steam is initially wet, the heat supplied

$$= h_1 + q_1 L_1 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

and heat after expansion $= h_2 + q_2 L_2$

$$= h_2 + \text{area } glnh + falg \text{ (see Fig. 66)}$$

$$= h_2 + T_2 \cdot q_1(bc) + falg$$

$$= h_2 + T_2 q_1(\phi_c - \phi_b) + (\phi_{w_1} - \phi_{w_2})T_2$$

$$= h_2 + T_2 q_1(\phi_{s_1} - \phi_{w_1}) + (\phi_{w_1} - \phi_{w_2})T_2 \quad . \quad . \quad . \quad (2)$$

where ϕ_{s_1} is the entropy of steam at T_1, ϕ_{w_1} that of water at T_1, and ϕ_{w_2} that of water at T_2.

This statement, though apparently complicated, is simple to evaluate in practice, as all the quantities required are given in the tables.

$$\text{Work done} = (1) - (2)$$

and

$$\text{efficiency} = \frac{(1)-(2)}{(1)-h_2}$$

If the steam is initially superheated and if tables giving the total heat and entropy for different values of superheat are available, the total heat in the supply steam n is obtained directly, and that of the steam after expansion can be found as soon as q_2 is known. The final dryness of the steam is given by

$$\frac{ae}{af} = \frac{ae}{\phi_{s_2} - \phi_{w_2}} \text{ (Fig. 67)}$$

But the entropy at e = entropy at d

or

$$q_2 = \frac{\phi_{s_3} - \phi_{w_2}}{\phi_{s_2} - \phi_{w_2}}$$

where w refers to water and s to steam, and the suffixes 1, 2, and 3 to the conditions at temperatures 1, 2, and 3.

All these quantities are obtainable directly or by interpolation from the tables.

Then as before

Work done = change of total heat

= total heat h_3 of superheated steam $-(h_2+q_2L_2)$

$$\text{and efficiency} = \frac{\text{work done}}{\text{total heat of superheated steam} - h_2}$$

*ENGINE IN WHICH THE STEAM IS KEPT DRY AND SATURATED DURING EXPANSION

This is partly secured in practice by applying a steam jacket to the engine cylinder. The expansion curve, now the saturation curve, follows the approximate law $pv^{\frac{16}{15}} = \text{constant}$. Fig. 69 shows the theoretical

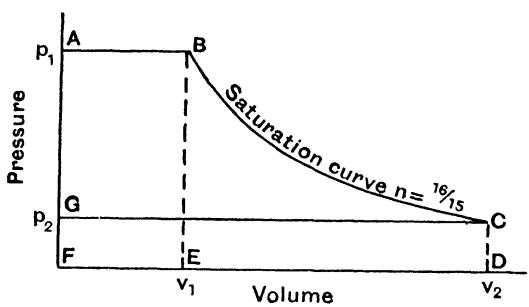


Fig. 69.

indicator diagram from which it will be seen that the work done during admission

$$= \text{area ABEF} = p_1v_1 \quad . \quad . \quad . \quad . \quad . \quad (1)$$

The work done during expansion from v_1 to v_2

$$= \text{area BCDE} = \frac{p_1v_1 - p_2v_2}{\frac{16}{15} - 1} = 15(p_1v_1 - p_2v_2) \quad . \quad . \quad . \quad (2)$$

Work expended during exhaust

$$= \text{area GCDF} = p_2v_2 \quad . \quad . \quad . \quad . \quad . \quad (3)$$

$\therefore$ net amount of work done by the steam

$$= p_1v_1 + 15(p_1v_1 - p_2v_2) - p_2v_2$$

$$= 15(p_1v_1 - p_2v_2) \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

Again—

$$\text{Heat received from the boiler} = H_1 - h_2 \quad . \quad . \quad . \quad (5)$$

$$\text{Heat rejected to the condenser} = H_2 - h_2 \quad . \quad . \quad . \quad (6)$$

Let H_j=heat supplied by the steam jacket.
Then heat received—heat rejected=work done, or

$$J(H_1 - H_2 + H_j) = 15(p_1v_1 - p_2v_2)$$

or
$$H_j = \frac{15(p_1v_1 - p_2v_2)}{J} - (H_1 - H_2) \quad . \quad . \quad . \quad . \quad (7)$$

and the efficiency is

$$\frac{15(p_1v_1 - p_2v_2)}{J(H_1 - h_2 + H_j)} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

THE REGENERATIVE STEAM CYCLE

It has been shown that the Rankine cycle has a lower efficiency than the Carnot, as the liquid heat between T_2 and T_1 is not taken in at the higher temperature. The liquid heat concerned is shown by area *eabf* in Fig. 70.

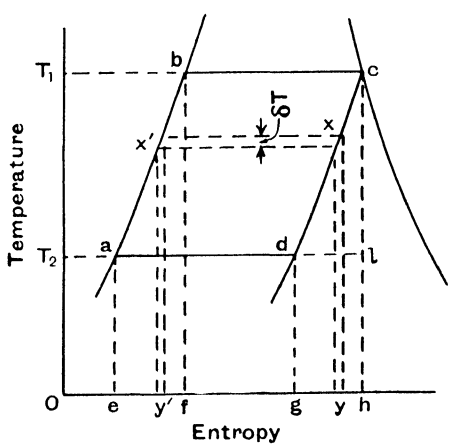


Fig. 70.

Let *cd* be drawn parallel to *ba* so that *gdch*=*eabf*. If, instead of expanding adiabatically along *cl*, the steam is constrained to expand along *cd* by the withdrawal of heat during expansion, then the heat so withdrawn (*gdch*) will be exactly that required to heat the water from T_2 to T_1. If heat, represented by the elementary strip *xy*, be removed while the steam falls over the element of temperature δT, it can be used to supply the quantity, represented by the elementary strip *x'y'*, which is exactly enough to heat the water over the range δT. Heat is therefore removed from the steam and imparted to the water *without* a fall in temperature, and we have a perfect regenerative process corresponding to that employed in the Stirling air engine. (Only note *ab* and *cd* are *not* lines of constant volume.)

The efficiency of the cycle will thus be that of a perfectly reversible cycle and will be given by

$$\frac{T_1 - T_2}{T_1}$$

That the efficiency will be improved can be seen from the fact that if the small amount of heat *xy* at T were retained in the cylinder during further expansion, only a small portion could be converted into mechanical work and the rest would be irretrievably rejected. In the regenerative process, however, the heat which normally would be supplied to heat the boiler feed is reduced to zero. The output of the engine is slightly reduced but the efficiency is increased.

The process of withdrawing heat is normally effected by drawing off or " bleeding " small quantities of steam to heat the feed-water heater.

This matter will be referred to on p. 258.

*NOTE

The cycles so far considered concern the conditions of the steam and not the mechanism by which it is caused to do work. They are therefore applicable alike to the Reciprocating Engine, in which the steam presses directly on a piston which drives the crank and flywheel, or to the Steam Turbine, in which the steam first converts its heat energy into kinetic energy of the steam itself, prior to conversion into mechanical work at the turbine shaft.

EXAMPLES ON CHAPTER X

1. Steam at 260 lb. per sq. in. abs. and of 90° superheat is expanded adiabatically to 11 lb. per sq. in. abs. during the performance of a Rankine cycle. What is the net work done per pound during the cycle, and what is the efficiency of the cycle reckoned from exhaust temperature ?

2. Calculate the work done when 1 lb. of steam expands adiabatically from 150 lb. per sq. in. abs. to 16 lb. per sq. in. abs., (*a*) when the steam is initially dry saturated, (*b*) when the initial dryness is 0·8.

3. Estimate the work done per cubic foot of steam in the following cases :

(*a*) When there is no clearance and no compression.

(*b*) When the clearance is 0·5 cu. ft. and with no compression.

(*c*) When the clearance is 0·5 cu. ft. and the compression pressure 50 lb. per sq. in. abs.

(*d*) When the clearance is 0·5 cu. ft. and the compression equal to the initial steam pressure.

In each case assume an initial steam pressure of 100 lb. per sq. in. abs., cut-off at $\frac{1}{4}$ stroke, hyperbolic expansion, and a back pressure of 25 lb. per sq. in.

4. An engine using dry saturated steam works on the Rankine cycle between temperature limits of 350° F. and 140° F. Estimate the work done per pound of steam and the efficiency of the cycle.

5. Solve Problem 4 if the initial dryness fraction of the steam is 0·85.

6. A steam engine requires 300 B.Th.U. per minute per horse-power when working between temperature limits of 390 °F. and 110° F. What is the ratio of its thermal efficiency to that of an ideal engine working between the same temperature limits (*a*) on the Rankine cycle, (*b*) on the Carnot cycle ?

7. Estimate the pounds of steam required per hour per horse-power by an engine working on the Rankine cycle between temperatures of 330° F. and 210° F.

8. Superheated steam at 180 lb. per sq. in. abs. (t=373° F.), and temperature 520° F., expands adiabatically down to a pressure of 6 lb. per sq. in. abs. (t=170° F.).

Assuming that the Rankine cycle is followed by this steam, determine the weight of steam required per hour per horse-power, and the dryness of the steam after expansion ($C_p=0{\cdot}5$).

9. An engine is supplied with superheated steam at 120 lb. per sq. in. abs. ($t=341°$ F.) and exhausts at 4 lb. abs. ($t=155°$ F.). Taking the mean specific heat of the steam as 0·5, find the superheat which must be given to the steam at the higher pressure in order that after adiabatic expansion to the lower pressure it may be just dry and saturated. An engine works on the Rankine cycle with that degree of superheat and between the above pressures. Find its thermal efficiency and the work done per pound of steam.

10. In a Stirling engine fitted with a perfect regenerator, the maximum pressure is 135 lb. per sq. in. abs. and the minimum 15 lb. per sq. in. abs., the upper and lower temperatures being 600° F. and 80° F. A perfectly reversible steam engine uses dry saturated steam between the same limits of pressure : compare their efficiencies. If the piston speed and stroke be the same in each engine, compare the diameters of the cylinders for equal power. [Given, temperature of steam at 135 lb. abs.=350° F., and at 15 lb. abs.=213° F., and specific volume at 15 lb. abs.=26·27 cu. ft.]

11. An engine is fitted with steam jackets so that the steam remains dry and saturated throughout the expansion. If the initial temperature is 400° F. and the final back-pressure temperature is 110° F., calculate the heat supplied by the jackets per pound of working steam and the efficiency of the engine.

12. Estimate the weight of steam required per hour per horse-power by an engine working between temperature limits of 200° C. and 60° C., (*a*) on the Rankine cycle, (*b*) when by the use of steam jackets the steam remains dry and saturated throughout the expansion.

13. In a compound steam engine the admission pressure to the high-pressure cylinder is 170 lb. per sq. in. abs. (L=855 B.Th.U. and $t=368{\cdot}5°$ F.), and the exhaust pressure in the low-pressure cylinder is 2 lb. abs. ($t=126°$ F.). The back pressure of the high-pressure cylinder and the admission pressure to the low-pressure cylinder is 50 lb. abs. (=281° F.). The feed water is heated in two stages : in the first stage, steam for feed heating is taken from the low-pressure steam chest, and in the second stage boiler steam is used. Assuming complete adiabatic expansion and no heat loss, estimate the efficiency of the engine, and the gain due to feed heating.

14. If in Problem 13 the temperature of the feed water leaving the feed heater was 200° F., feed heating being obtained by the first stage only, estimate the efficiency of the engine.

15. In a condensing engine the back pressure is 2·5 lb. per sq. in. abs., the release pressure 6 lb. per sq. in. abs., the boiler pressure 80 lb. per sq. in. abs., and the steam is 80 per cent. dry at release.

If the steam is expanded (1) according to the law $pv^{1{\cdot}13}=k$, (2) with constant dryness (3) hyperbolically, find the dryness at cut-off, the work done per pound, the heat rejected, the efficiency of the cycle, the heat received up to cut-off, and the heat received during expansion. Assume the condensate is fed back to the boiler.

Chapter XI

THE RECIPROCATING STEAM ENGINE

*GENERAL STATEMENT

Only those engines which use steam expansively will be considered in this chapter. Certain paragraphs will apply to non-expansion engines also, but such engines are employed only in very small sizes and under circumstances which make thermodynamic efficiency a matter of secondary importance.

As the thermal efficiency of an engine depends on the ratio of expansion employed, which in turn is dependent on the minimum pressure available, engines may be roughly classified under three headings : (1) Atmospheric engines which discharge directly into the air (the normal steam locomotive is typical). (2) Condensing engines in which a very low final pressure is obtained by condensing the steam with cold water. (3) Back-pressure engines which discharge their steam into plant requiring low-pressure steam, such as evaporating pans, etc., the dead steam from the engine becoming the live steam for some form of process work. In this chapter condensing engines will in general be considered.

*THE STEAM ENGINE CYCLE

Fig. 71 shows the theoretical $p.v.$ and $T\phi$ diagrams for this cycle. AB represents the admission of the steam, at full boiler pressure p_1, up

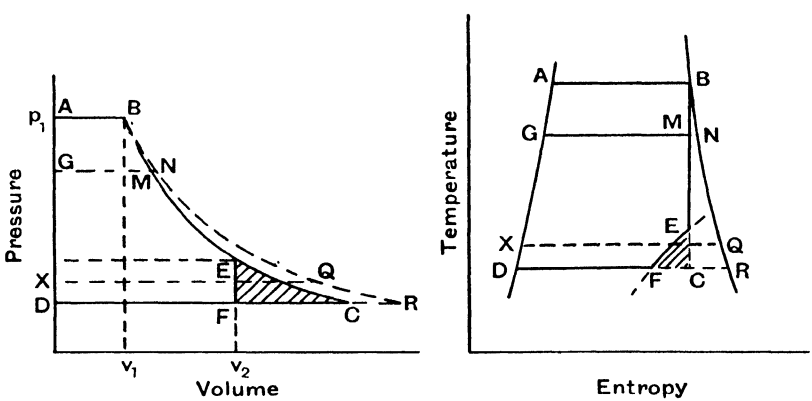


Fig. 71.

to the point of cut-off B. BC represents the curve of adiabatic expansion to the back pressure p_b, and CD the exhaust stroke at that pressure. BR shows the curve of expansion of dry steam, so that the dryness fraction of the steam at any pressure P_g is given by GM/GN.

The equation for BR on the *p.v.* diagram is given to a fair degree of accuracy by $pv^{16/15}$=constant.

Fig. 72 shows a typical diagram taken from an actual engine. It is necessary to see to what extent the diagrams are in agreement.

*COMPARISON OF ACTUAL AND THEORETICAL INDICATOR DIAGRAMS

A certain number of obvious differences will be noted.

(1) Though it is theoretically desirable to expand the steam in the cylinder to the lowest possible pressure, a limit is imposed in practice by the very large volume occupied by low-pressure steam. This would

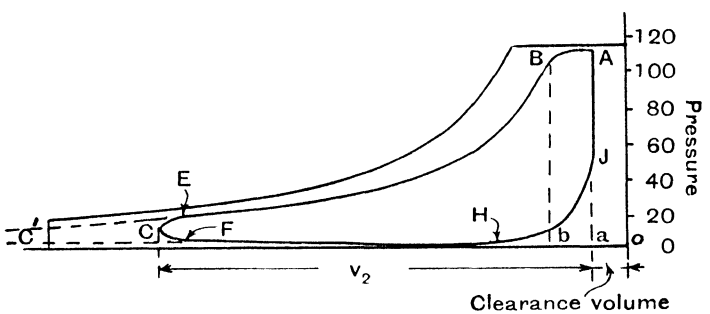


Fig. 72.

necessitate a cylinder so large that frictional losses would be excessive. By stopping expansion in the cylinder and opening the exhaust port at some point E, a rapid drop in pressure occurs, resulting in the loss of the " toe " ECF of the diagram, Fig. 71, but also reducing by a large amount the cylinder volume required.

If *r*, the cut-off ratio, is given by the ratio of AB/DF (i.e. the volume displaced by the piston during admission/the total volume displaced), the work done, *in the cylinder*, is

$$p_1v_1+\frac{p_1v_1-p_2rv_1}{n-1}-p_brv_1 \text{ (p. 152)}$$

or

$$p_1v_1+\frac{p_1v_1-p_2v_2}{n-1}-p_bv_2$$

(2) For mechanical reasons the cylinder volume can never be reduced to zero, a small " clearance volume " (*oa*, Fig. 72) having to be allowed both to prevent contact between the piston and cylinder head, and because

the valves cannot be built flush with the inner surface of the cylinder. The clearance volume will vary with the size of cylinder and the type of valves employed.

(3) The admission pressure must be lower than the boiler pressure in order to cause an adequate flow of steam towards the cylinder and through the inlet ports. The necessary allowance depends upon the length and size of pipe employed, the type of valve, and the speed of the engine. It should not in any case exceed 10 per cent. of the boiler pressure. Such throttling will tend to reheat the steam, and in testing the performance of an engine the quality of the steam should be measured as close to the steam-chest as possible. Owing to restrictions imposed by the port areas the admission line will always drop towards B as the piston gains speed from the top dead centre.

(4) Owing to restrictions at the exhaust ports, etc., the pressure during exhaust will be some 2 or 3 lb. per sq. in. higher than the pressure in the condenser (or the atmosphere if exhaust is into the air).

(5) As the inlet valve cannot close instantaneously, the " cut-off " at B will not be sharp, the rounding depending on the type of valve and valve gear employed.

(6) From E to F a considerable quantity of steam has to leave the cylinder, requiring an appreciable time. It is necessary therefore to open the exhaust valve before the end of the outward stroke, the fall in pressure being as shown by ECF (Fig. 72). Similarly, the admission valve is opened slightly before the end of the stroke.

(7) Owing to the necessity of providing " clearance space " at the end of the return stroke, the exhaust valve is closed at some point H. To fail to do so would result in having a pocket of cold low-pressure steam at the end of the stroke with which the hot boiler steam would mix, and while the pressure in the clearance space was rising to full admission pressure considerable throttling would occur with resulting waste of energy.

By closing the exhaust valve at H the steam remaining in the cylinder is compressed along the curve HJ. This reduces the throttling when the inlet valve opens, and also reduces initial condensation. Compression is in some cases effected up to full admission pressure. It also serves the purpose of " cushioning " or bringing the moving parts gradually to rest at the top dead centre, and so tending to prevent reversal of loading on the connecting rod bearings.

*MEAN EFFECTIVE PRESSURE. DIAGRAM FACTOR

For reasons stated on p. 174, the actual expansion curve for steam in a reciprocating engine approaches that given by the hyperbolic equation pv=constant. Under these circumstances, if $r=v_2/v_1$ and no cushioning is used, the work done per cycle is given by

$$W=144(p_1v_1+p_1v_1 \log_\epsilon r-p_bv_2) \text{ ft.-lb.},$$

and the mean effective pressure—or the mean pressure acting on the piston during the working stroke—by

$$p_m=\frac{p_1v_1+p_1v_1\log_\epsilon r-p_bv_2}{v_2}=\frac{p_1v_1}{v_2}(1+\log_\epsilon r)-p_b$$

$$=\frac{p_1}{r}(1+\log_\epsilon r)-p_b$$

an expression which is simply evaluated in any particular case.

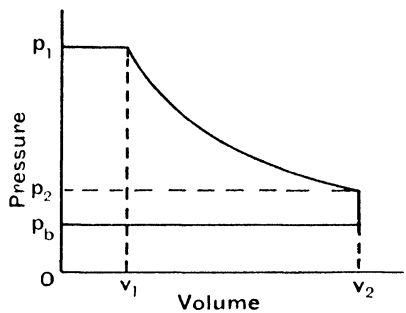


Fig. 73.

In practice the actual mean effective pressure is always less than this, and can be expressed by

$$p_m=e\left\{\frac{p_1}{r}(1+\log_\epsilon r)-p_b\right\}$$

where e is called the " diagram factor," and is less than 1.

If L=length of stroke in feet, A=piston area in square inches, and N=number of working strokes per minute, then the

$$\text{Indicated Horse-power}=\frac{p_m\,\text{LAN}}{33{,}000}$$

(Note : if n=number of revolutions per minute of the engine, then $N=n$ for a single-acting engine, and $N=2n$ for a double-acting engine.)

The " diagram factor " depends on the type of engine, the ratio of expansion, and speed, and whether jacketing or superheating are employed. The normal range is from about 0·63 to 0·86, being higher for the more perfect engines.[1]

EXAMPLE.—The diameter of a steam-engine cylinder is 6 in. and the stroke 12 in. If the initial pressure is 100 lb. per sq. in. abs. and cut-off is $\frac{1}{4}$ stroke, find the mean effective pressure, neglecting clearance, the back pressure being 3 lb. per sq. in. abs.

If the clearance is $\frac{1}{10}$ of the piston displacement, find the theoretical mean effective pressure and the I.H.P. if there are 400 working strokes

[1] For original discussion, see Mellanby, *Proc.I.Mech.E.*, 1905 ; and C. H. Innes, *Practical Engineer*, June 17, 1892.

per minute (i.e. the engine is double-acting and runs at 200 r.p.m.). Assume hyperbolic expansion.

If the initial and final pressures and volumes are given by p_1, v_1, p_2, v_2

$$\text{Work done} = p_1 v_1 \left(1 + \log_\epsilon \frac{v_2}{v_1}\right) - 3v_2$$

and
$$p_m = \frac{100}{4}(1 + \log_\epsilon 4) - 3$$
$$= 25(1 + 1{\cdot}3862) - 3$$
$$= 25 \times 2{\cdot}3862 - 3 \doteq 59{\cdot}65 - 3 = 56{\cdot}65 \text{ lb. per sq. in.}$$

The actual ratio $r = \dfrac{1+0{\cdot}1}{0{\cdot}25+0{\cdot}1} = \dfrac{1{\cdot}1}{0{\cdot}35} = 3{\cdot}14$

therefore
$$p_m = \frac{100}{4}\{1 + (1 + 0{\cdot}1 \times 4) \log_\epsilon 3{\cdot}14\} - 3$$
$$= \frac{100}{4}(1 + 1{\cdot}60) - 3 = \frac{100 \times 2{\cdot}6}{3{\cdot}14} - 3$$
$$= 65 - 3$$
$$= 62 \text{ lb. per sq. in.}$$

Therefore
$$\text{I.H.P.} = \frac{62 \times 1 \times (0{\cdot}7854 \times 36) \times 400}{33{,}000} = 21{\cdot}4$$

EXAMPLE.—Find the diameter of the steam-engine cylinder needed to develop 100 I.H.P. when the piston speed is 600 ft. per minute, initial steam pressure 150 lb. per sq. in. abs., back pressure 15 lb. per sq. in. abs., cut-off $\frac{1}{5}$ of stroke, clearance volume 8 per cent. of piston displacement, and the effect of early release, compression, etc., reduces the actual mean effective pressure to 90 per cent. of the theoretical.

Assuming hyperbolic expansion $W = p_1 v_1 + p_1 v_1 \log_\epsilon r - p_b v_2$

and
$$p_m = \frac{p_1}{r}\left\{1 + (1 + cr) \log_\epsilon \frac{1+c}{r+c}\right\} - p_b$$

where $r = 5$, $c = 0{\cdot}08$

$$\therefore\ p_m = \frac{150}{5}\left\{1 + (1 + 0{\cdot}08 \times 5) \log_\epsilon \frac{1{\cdot}08}{\frac{1}{5} + 0{\cdot}08}\right\} - 15$$
$$= 30 \times 2{\cdot}89 - 15 = 87 - 15 = 72 \text{ lb. per sq. in.}$$

Let A = area of cylinder required

then
$$600 \times 72 \times \tfrac{9}{10} \times \text{A} = 100 \times 33{,}000$$
$$\therefore\ \text{A} = \frac{100 \times 33{,}000}{54 \times 72 \times 10} = 84{\cdot}9 \text{ sq. in.}$$
$$\therefore\ \text{diameter} = \sqrt{\frac{84{\cdot}9}{0{\cdot}7854}} = 10{\cdot}4, \text{ say } 10\tfrac{1}{2} \text{ in.}$$

*THE MISSING QUANTITY

By far the most serious discrepancy between actual and theoretical indicator cards is due to the fact that there appears to be a marked difference between the weight of dry steam in the cylinder during expansion and the quantity theoretically present. This difference is called the " missing quantity," and is considered in the sections which follow.

*CYCLIC CONDENSATION

No actual cylinder walls are made of non-conducting materials. Even provided that no appreciable loss of heat occurs from the outside of the cylinder, which may be suitably lagged or steam-jacketed, and that it has reached a stable mean temperature, the material of the cylinder itself will undergo cyclic temperature changes. During the later stages of expansion and throughout the exhaust stroke the walls will be in contact with steam of low pressure and temperature, and will be cooled thereby. The incoming steam will, in part, condense on the chilled walls, which will begin to rise in temperature, and at some point during expansion the steam, now rapidly cooling, will fall to the temperature of the walls and will subsequently receive heat from them. The heat so received by the steam will be taken in at too late a position in the stroke to be of much value so far as its conversion into work is concerned. It is usually assumed that the steam still remaining in the cylinder when the exhaust valve closes is dry.

Actual measurements of the cyclic fluctuations of temperature of cylinder walls, the mathematical considerations of Chapter XVI, and the " blanketing " effect of stationary films stated on p. 362, show that the actual rise and fall of the temperature of the cylinder material, even at the working surface, is very much less than that of the working steam, and that such fluctuations of temperature are confined to the neighbourhood of the working surfaces only and do not penetrate effectively into the material. The amount of cyclic condensation and re-evaporation so caused must therefore be small.

EXAMPLE.—If an engine is supplied with 18 lb. of dry cylinder feed per minute at a pressure of 200 lb. per sq. in. abs., the back pressure is 15 lb. and the missing quantity is 30 per cent., find the weight of cast iron which, fluctuating in temperature over the full range of steam temperatures, would account for the condensation of the missing quantity.

At 200 lb. per sq. in. $H=1198{\cdot}1$ and $t=381{\cdot}9°$ F.

At 15 lb. per sq. in. $h=181{\cdot}0$ and $t_1=213°$ F.

Heat loss by steam during condensation $=1017{\cdot}1$ B.Th.U. per lb.

Taking the specific heat of cast iron as 0·130, then the weight of cast iron required would be given by

$$W\times 0{\cdot}130\times 168{\cdot}9=1017{\cdot}1\times 18\times\frac{30}{100}$$

or

$$W=250 \text{ lb.}$$

As the inner surface fluctuation of the cylinder walls is known to be of the order of only some 10° F., and the mean fluctuation of temperature of the cylinder block cannot exceed about 1° F., the actual weight of the block to account for the missing quantity by condensation would be at east 168·9 times the above or about 20 tons, a totally absurd quantity.

The effect of initial condensation during admission, and subsequent re-evaporation during expansion and exhaust, is to depress the initial stages of the expansion curve and raise the final pressures.

Assuming dry boiler steam the adiabatic curve is given approximately by $pv^{1·135}$=constant, the curve of dry saturation by $pv^{\frac{1·6}{1·5}}$=constant, while the actual curve for steam which is partially condensed during admission and becomes drier during expansion lies between this curve and the hyperbolic curve pv=constant.

EXAMPLE.—The high-pressure cylinder of a double-acting engine running at 100 r.p.m. has an effective area of 39 sq. in. and a stroke of 24 in., and the clearance volume is 7 per cent. of the stroke volume. During a trial the cylinder feed was 18 lb. per minute. The indicator diagram showed that cut-off and release occurred at 30 and 90 per cent. of the outstroke while compression commenced at 70 per cent. of the return stroke, the indicated pressures at cut-off, release, and the end of compression being 205, 78, and 44 lb. per sq. in. abs.

Find the quality of the steam at cut-off and release.

$$\text{The swept volume}=\frac{39}{144}\times 2=0·5416 \text{ cu. ft.}$$

and $$\text{the clearance volume}=\frac{7}{100}\times 0·5416=0·0379 \text{ cu. ft.}$$

$$\text{Total volume at cut-off}=0·1625+0·0379=0·2004 \text{ cu. ft.}$$

$$\text{Total volume at release}=0·4874+0·0379=0·5253 \text{ cu. ft.}$$

$$\text{Total volume after compression}=0·1625+0·0379=0·2004 \text{ cu. ft.}$$

$$\text{The cylinder feed per stroke}=\frac{18}{200}=0·090 \text{ lb.}$$

The specific volume at 44 lb. per sq. in.=9·59 cu. ft.

$$\text{Hence the weight of the cushion steam}=\frac{0·2004}{9·59}=0·0209 \text{ lb. per stroke}$$

At cut-off

$$\text{Weight of steam present}=0·090+0·0209=0·1109 \text{ lb.}$$

The specific volume at 205 lb. per sq. in.=2·24 cu. ft.

The actual volume at 205 lb. per sq. in.=0·2004 cu. ft.

and $$\text{quality at cut-off}=\frac{0·2004}{0·1109\times 2·24}=80·67 \text{ per cent.}$$

At release—

Weight of steam present = 0·1109 lb.

The specific volume at 78 lb. per sq. in. = 5·6 cu. ft.

The actual volume = 0·5253 cu. ft.

and the quality at release $= \frac{0{\cdot}5253}{0{\cdot}1109 \times 5{\cdot}6} = 84{\cdot}59$ per cent.

*APPLICATION OF THE SATURATION CURVE TO AN ACTUAL INDICATOR DIAGRAM

This curve represents the expansion curve which might be attained in a steam-engine cylinder if the whole contents of the cylinder were present throughout the stroke as dry saturated steam.

Fig. 74 shows an indicator card to which have been added the axes of zero pressure and volume. The former is drawn at a distance below the

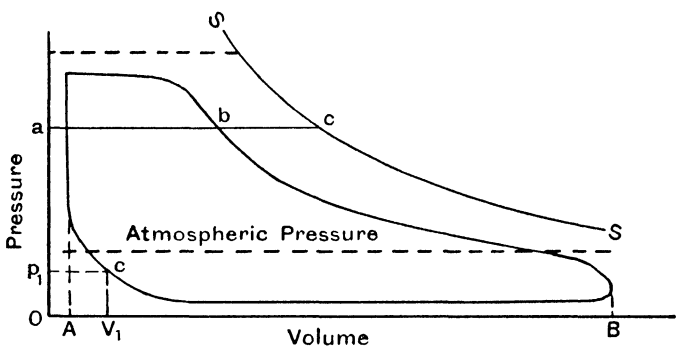


Fig. 74.

atmospheric line equal to the equivalent of the barometric height when the card was taken, and OA represents the clearance volume measured to the same scale on which AB represents the volume swept by the piston. The clearance volume can be obtained by direct measurement, or by estimation from the drawings of the cylinder, or an approximation can be obtained from the card itself (see p. 281).

The total steam present in the cylinder during expansion should be (1) the weight of steam passing through the engine per stroke, called the *cylinder feed*, and (2) the weight of steam contained in the clearance space before admission, called the *cushion steam*.

The cylinder feed is determined experimentally, say by running the engine for an hour, and condensing and weighing the exhaust steam; or by measuring the water supplied to the boiler and making allowance for any gain or loss of water in the boiler, as recorded by the water gauge, during the time of the trial. From the measured steam consumption

and the number of strokes made, the cylinder feed in pounds per stroke is easily calculated.

The weight of cushion steam is found from the indicator diagram as follows :

Any convenient point such as C, Fig. 74, is selected on the compression curve and the pressure p_1, in pounds per square inch absolute, and the indicated volume v_1 in cubic feet, are measured. If V_1 denotes the specific volume, in cubic feet, of 1 lb. of dry saturated steam at pressure p_1, then, assuming the cushion steam to be dry and saturated at point C,

$$\text{weight of cushion steam} = w = \frac{v_1}{V_1} \text{ lb.}$$

Let W = measured cylinder feed in pounds per stroke.

Then, assuming no leakage past the valves or piston, the total weight of steam present in the cylinder during expansion will be $w+W$ lb.

The saturation curve is obtained by plotting on the indicator diagram

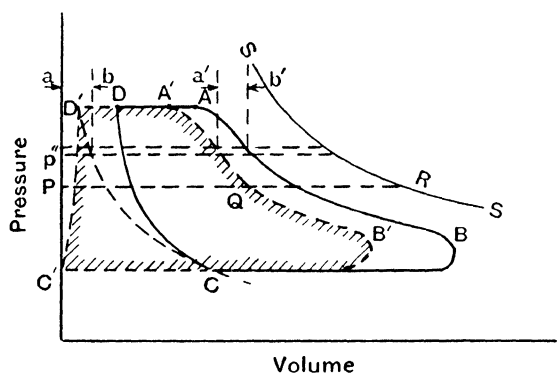


Fig. 75.

with the help of steam tables, a curve for $w+W$ lb. of steam as shown by SS.

Conversely the saturation curve can be plotted for 1 lb. of steam, and the indicator card redrawn with the volumes multiplied in the ratio of

$$\frac{1}{w+W}.$$

A further alternative, which is useful in cases where multiple expansion is used, and where the cushion steam per cylinder, and, possibly, the feed per cylinder are not the same, is to plot the curves for 1 lb. of cylinder feed. This involves modification of the diagram as follows. Let C, Fig. 75, represent the point of closing of the exhaust valve. Then, assuming the cushion steam to be dry saturated at this point, its weight w can be found, and the curve CD′ drawn to represent the compression

of w lb. of steam which remain dry and saturated (the assumption of dry saturation involves only slight errors). At any pressure p'' the volume occupied by the cushion steam will then be ab, and the volume occupied by $w+W$ lb. will be ab'. If $a'b'=ab$ is now subtracted as shown, the length aa' will be the volume occupied by W lb. of steam. The curve A'B' can be drawn by measurements made at a number of different pressures. As the whole of the work done in the cylinder is performed by the cylinder feed, it is conventional to add to the left side of the diagram amounts equal to those removed from the right. The diagram will then be as shown by D'A'B'C', and will have its original area. If no leakage of cushion steam or other irregularity occurs, and if compression is carried out to the full feed-pressure, the curve C'D' should lie along the pressure axis.

The curve D'A'B'C' can now be scaled to represent 1 lb. instead of W lb. of cylinder feed, and the saturation curve SS for 1 lb. of steam can be added to this scale.

Then at any pressure P during expansion the dryness fraction will be given by PQ/PR.

The dryness fraction may also be expressed as

$$\frac{\text{indicated weight}}{\text{total weight present in the cylinder}}$$

$$=\frac{\text{indicated volume at pressure } p \text{ (cu. ft.)} \times \text{density (in lb. per cu. ft.) at } p}{\text{weight of cushion steam } (w \text{ lb.}) + \text{weight of cylinder feed (W lb.)}}$$

It will be found in practice that the indicated weight of steam after cut-off is less than the measured steam consumption plus cushion steam; the difference is known as the missing quantity and is represented on the above diagram by the length QR. Owing to re-evaporation during expansion the missing quantity is almost always less at release than cut-off.

It must be noted that only during expansion, between points A and B, is the full weight of steam present in the cylinder, and it is only over this portion of the curve that the "dryness fraction" can be obtained.

EXAMPLE.—A calibrated indicator diagram is shown in Fig. 76. The measured steam consumption was 1344 lb. per hour at a speed of 200 r.p.m. The engine is double-acting and the card shown is an average diagram from both sides of the piston. Estimate the dryness fractions and missing quantities at cut-off and release, and deduce the interchange of heat per pound of steam between the steam and cylinder walls.

From the figure it is seen that at the point C on the compression curve when the pressure is 20 lb. per sq. in. abs., the indicated volume is 0·098 cu. ft. From steam tables the volume of 1 lb. of dry saturated steam at this pressure is 20 cu. ft.

Hence weight of cushion steam $=\frac{0{\cdot}098}{20}=0{\cdot}0049$ lb.

Cylinder feed per stroke $=\frac{1344}{200\times 2\times 60}=0{\cdot}056$ lb.

$\therefore$ Total weight of steam present during expansion $=0{\cdot}0049+0{\cdot}056$
$=0{\cdot}0609$ lb.

At cut-off (point A) the pressure is 76 lb. abs., the indicated volume is 0·166 cu. ft., and the specific volume (from steam tables) 5·74 cu. ft.

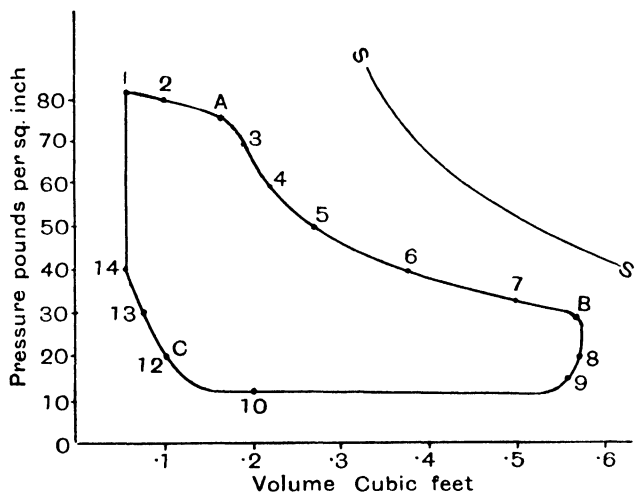


Fig. 76.

per lb. If, therefore, the steam were dry and saturated its volume at cut-off would be

$$5{\cdot}74\times 0{\cdot}0609=0{\cdot}349 \text{ cu. ft.}$$

Hence dryness fraction at cut-off $=\frac{0{\cdot}166}{0{\cdot}349}=0{\cdot}475$ or 47·5 per cent.

Indicated weight at cut-off $=\frac{0{\cdot}166}{5{\cdot}74}=0{\cdot}0289$ lb.

$\therefore$ missing quantity $=0{\cdot}0609-0{\cdot}0289$
$=0{\cdot}032$ lb. per stroke
$=0{\cdot}032\times 2\times 200\times 60$
$=768$ lb. per hour

At release (point B) the pressure is 30 lb. abs., the indicated volume

0·575 cu. ft., and the specific volume (from steam tables) 13·7 cu. ft. per lb. The indicated weight at release will therefore be

$$\frac{0{\cdot}575}{13{\cdot}7}=0{\cdot}042 \text{ lb.}$$

Hence dryness fraction at release $=\dfrac{0{\cdot}042}{0{\cdot}0609}=0{\cdot}690$ or 69 per cent.

Missing quantity at release $=0{\cdot}0609-0{\cdot}0420$

$=0{\cdot}0189$ lb. per stroke

$=0{\cdot}0189\times2\times200\times60$

$=453$ lb. per hour

From the diagram the mean pressure during expansion from A to B is 36 lb. per sq. in., hence the work done by 0·0609 lb. of steam is

Mean pressure (pounds per square foot) × change in volume (cubic feet).

$=36\times144(0{\cdot}575-0{\cdot}166)$

$=2128$ ft.-lb.

$\therefore$ work done per pound of steam $=\dfrac{2128}{0{\cdot}0609\times778}=44{\cdot}9$ B.Th.U.

At cut-off the pressure is 76 lb. abs. (L=903, $t=308°$ F.).

$\therefore$ heat per pound at cut-off $=308-32+0{\cdot}475\times903$

$=276+429$

$=705$ B.Th.U.

At release the pressure is 30 lb. abs. (L=945, $t=250°$ F.).

$\therefore$ heat per pound at release $=250-32+0{\cdot}690\times945$

$=218+652$

$=870$ B.Th.U.

Let H_j=heat received per pound from the cylinder walls between cut-off and release, then assuming no heat losses

$$705+H_j=870+44{\cdot}9=914{\cdot}9$$

$$\therefore H_j=914{\cdot}9-705$$

$=209{\cdot}9$ B.Th.U.

*APPLICATION OF ACTUAL INDICATOR DIAGRAM TO Tϕ CHART

On account of the greater relative obliquity of expansion curves on the Tϕ chart compared with those on the P.V. diagram, the former gives a much clearer impression of the actual occurrences in the cylinder.

Referring to the case considered in the last example, the pressures and

volumes occupied by the steam at a number of points round the cycle are measured. As the standard Tϕ diagram is drawn (Fig. 28) for 1 lb. of steam, the volumes taken from the indicator diagram are multiplied by $\frac{1}{w+W}$ to give the volume occupied were the cylinder contents during expansion equal to 1 lb. (Should the leakage be known, correction for this should be made. See *Proc.I.Mech.E.*, 1905, p. 239.)

The temperature-entropy diagram is most conveniently drawn by laying a piece of tracing paper over the Tϕ chart and plotting the points directly on it. Thus, the position on the chart corresponding to point 1 is the intersection of the constant-pressure line for 82 lb. per sq. in., with the constant volume line for 0·90 cu. ft., and similarly for all the other points. The complete temperature-entropy diagram, corresponding to the indicator diagram shown in Fig. 76, is thus drawn, and is as shown in Fig. 77.

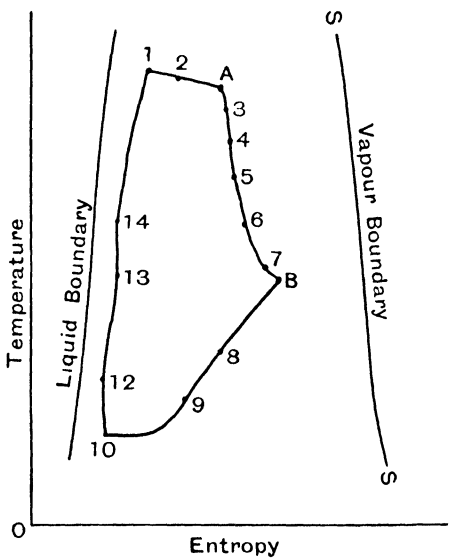


Fig. 77.

When the temperature-entropy diagram is drawn in this, or any other way, the expansion line clearly shows the nature of the interchange of heat between the steam and the cylinder walls. Referring to Fig. 77 it will be seen that throughout the expansion from A to B there is a gain of entropy, and therefore the cylinder walls are supplying heat to the steam, and as re-evaporation continues, the dryness fraction of the steam increases as shown.

*BOULVIN'S METHOD OF DRAWING THE TEMPERATURE-ENTROPY DIAGRAM FROM THE INDICATOR DIAGRAM

The axes of pressure temperature, volume and entropy are drawn as in Fig. 78, intersecting at O. Along OV a scale of volumes, in cubic feet for 1 lb. of steam, is set off, its length being made to represent at least the volume of 1 lb. of steam at the lowest pressure on the indicator diagram. On OT a scale of temperature and along OP a scale of pressure are set off; then by the aid of steam tables the temperature-pressure curve for saturated steam is plotted as shown in the quadrant POT.

In the quadrant POV the saturation curve for 1 lb. of steam is drawn, using steam tables. On the actual indicator diagram the saturation curve for the weight of steam present during expansion is drawn by the method already explained on p. 175. The actual indicator diagram is then transferred to the quadrant POV, so that its relationship to the saturation curve already drawn therein is the same as its relationship to its own

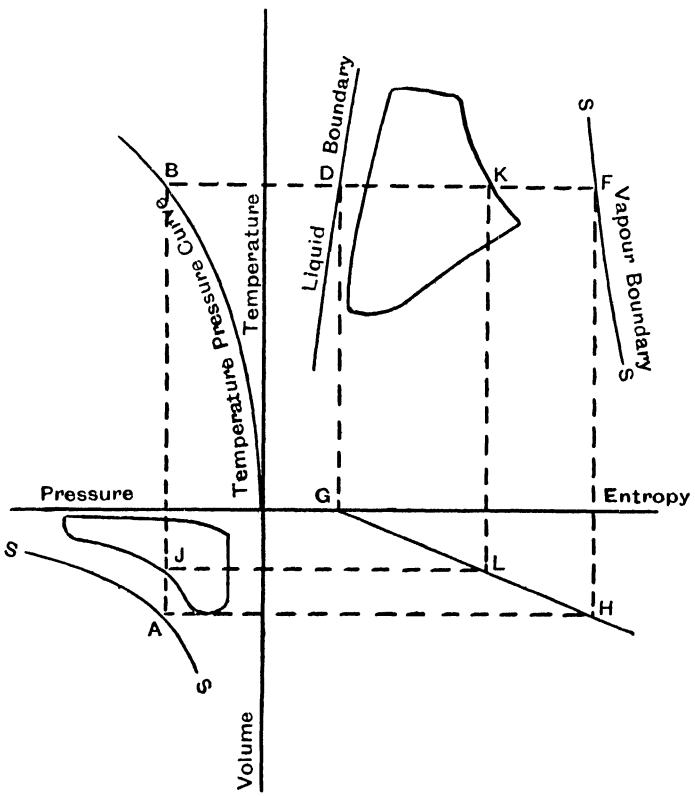


Fig. 78.

saturation curve, i.e. the volumes are adjusted to correspond to 1 lb. of steam.

In the quadrant TOE the steam and water lines of the temperature-entropy diagram are drawn for 1 lb. of steam by the method of p. 67, or these lines are transferred directly from a temperature-entropy chart.

Any convenient point A on the saturation curve is taken and the line AB perpendicular to the pressure axis is drawn. A line of constant temperature BF cuts the entropy lines in D and F. DG is projected vertically to cut the axis of entropy in G, and FH to cut the constant

volume line from A in H. HG is joined ; from J a constant volume line is drawn to cut HG in L, and from L a line of constant entropy is drawn to cut DF in K. Then K is a point on the Tϕ diagram required. This process is repeated for different points on the indicator diagram, and in this manner the indicator diagram is transferred to the temperature-entropy chart.

THE HEAT RECOVERY LINES

Fig. 79 shows a diagram obtained by Mr. Bryan Donkin and reproduced in " the Theta Phi Diagram " (Golding, Technical Pub. Co., Ltd.).

If adiabatic expansion had occurred from the point of cut-off B the diagram would have followed the line BF'. Two alternatives, allowing for condensation, are represented by the lines B*r* and BR.

(1) *The Priming Water Heat-recovery Line Br.*—This assumes that the condensation, as derived from the dryness fraction AB/AC, had occurred prior to the entry of the steam into the cylinder, and that the " missing quantity " had therefore entered the cylinder as hot water at temperature T_1. During expansion down to temperature T_2 an amount of heat, given by $(1-q)\phi(T_1-T_2)$, would be available for the re-evaporation of moisture in the cylinder. Now the heat so available in 1 lb. of water is given by the area ADE and the corresponding entropy by ED. If F then divides ED in the ratio in which B divides AC, the gain in entropy available from the priming water would be FD. If F'*r* is then made equal to FD, the expansion curve should lie along B*r*.

(2) *The Condensation Water Heat-recovery Line BR.*—This line assumes that the full quantity of dry steam has entered the cylinder and that the " missing quantity " has condensed there prior to cut-off at B, also that the heat is recovered from the walls, etc., later in the stroke. If the weight of water present at cut-off be multiplied by the latent heat of steam at T_1 and the product divided by the mean absolute temperature of the cycle, the result will be the amount of entropy which would be added to that possessed by the stream present at cut-off if all the heat of condensation were returned by the walls.

If from *r* is set off

$$rR=\frac{(1-q)L_1}{\frac{T_1+T_2}{2}}$$

and if B and R are joined, then the theoretical expansion will follow the line BR.

The relation of the actual expansion curve to the curves BF', B*r*, and BR is instructive. If no external heat is received from the jackets the expansion curve should lie between the B*r* and BR lines. Its actual

relation provides a useful clue to the processes occurring in the cylinder. The diagram shown is for a steam-jacketed cylinder.

EXAMPLE.—Dry steam is admitted to an engine cylinder at 84 lb. per sq. in. abs. (t=315° F., L=898 B.Th.U., specific volume=5·22 cu. ft.) and the condensation during admission is 25 per cent. of the whole steam

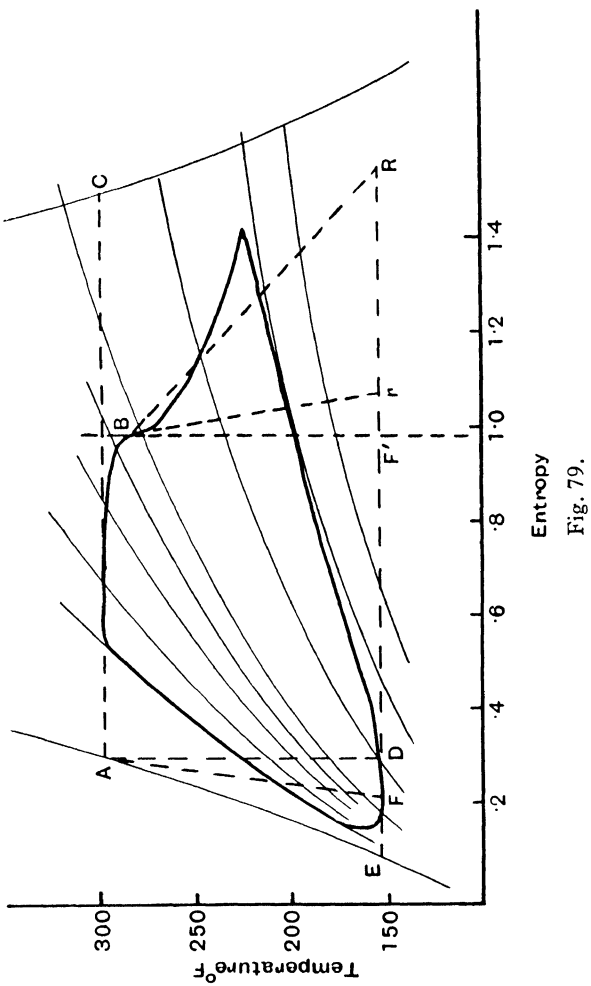


Fig. 79.

supply. During expansion one-half of the heat absorbed by the cylinder walls during admission is returned to the steam at a uniform rate as the temperature falls. If the expansion be complete and the back pressure be 8 lb. per sq. in. abs. (t=183° F., L=988 B.Th.U., v=47·3 cu. ft.), find the dryness fraction at the end of expansion. Also, assuming the

exhaust steam homogeneous in quality, find its dryness fraction. Neglect clearance, heat losses due to radiation and conduction, and assume the specific heat of water to be constant and equal to unity.

From steam tables we find the following :

Pressure	Temperature, ° F.	Latent heat	Specific volume	Entropy	
				Water	Evaporation
84	315	898	5·22	0·4579	1·1581
8	183	988	47·3	0·2673	1·5380

Since the condensation during admission is 25 per cent., it follows that the dryness fraction of the steam at cut-off is 0·75.

The heat absorbed per pound of steam by the cylinder walls during admission is

$$\frac{898}{4}=224{\cdot}5 \text{ B.Th.U.}$$

the heat returned during expansion while the temperature falls 132° F.

$$=\frac{224{\cdot}5}{2}=112{\cdot}25 \text{ B.Th.U.}$$

$$\therefore \text{ rate of heat return}=\frac{112{\cdot}25}{132}$$

$$=0{\cdot}85 \text{ B.Th.U. per } ^\circ \text{ F. fall in temperature}$$

$$\therefore \delta H=0{\cdot}85\delta T$$

or

$$\frac{\delta H}{T}=\delta\phi=0{\cdot}85\frac{\delta T}{T}$$

Hence total gain of entropy during expansion

$$=\int_{643}^{775}0{\cdot}85\frac{dT}{T}$$

$$=0{\cdot}85 \log_\epsilon \tfrac{775}{643}$$

$$=0{\cdot}85\times 0{\cdot}187=0{\cdot}1589 \text{ units of entropy}$$

Had expansion been adiabatic the initial and final entropies would have been equal. If A represents the condition point on the Tϕ diagram on the liquid boundary at 183° F., the initial entropy above that at A will be given by (0·4579−0·2673)+(0·75×1·581), and the final entropy, owing to the regain of heat, will be greater than this by the amount 0·1589 units,

or

$$0{\cdot}1906+0{\cdot}8596+0{\cdot}1589=1{\cdot}2091 \text{ units.}$$

Hence dryness after expansion $= \frac{1 \cdot 2091}{1 \cdot 538}$

$= 0 \cdot 786$ or 78·6 per cent.

During exhaust the cylinder walls must return the remainder of the absorbed heat to the steam, namely, 112·25 B.Th.U. Hence the further rise in entropy during exhaust will be

$$\frac{112 \cdot 25}{643} = 0 \cdot 1745 \text{ units}$$

and the dryness fraction at the end of exhaust will be

$$= \frac{1 \cdot 2091 + 0 \cdot 1745}{1 \cdot 538}$$

$= 0 \cdot 90$ or 90 per cent.

*MAGNITUDE OF MISSING QUANTITY. CONTRIBUTING FACTORS

The " missing quantity " is seldom less than 15 per cent. and may be very much greater under adverse conditions. Under these circumstances, and in view of the fact that the fluctuation of even the inner surface of the cylinder walls is only of the order of 10° F. (Callendar and Nicholson, *Proc.I.C.E.*, vol. cxxxi, 1898) the cyclic condensation theory is quite inadequate to provide a full explanation of the loss.

A possible source of loss is leakage past the piston and valves, which will cause the weight of fluid in the cylinder during the expansion to be constantly decreasing, so that the determination of dryness fraction by the method on p. 175 will be inaccurate.

With the older types of slide valve appreciable leakage may be brought about in the following way. By the movement of the valve fairly large areas of cool metal are continually being exposed to the live steam, which will partially condense upon them. On the return movement the valve will tend to ride over the film of moisture, which will pass, as a lubricant, beneath the valve face and at once enter the exhaust passages. Here, under the influence of the low pressure, the film will evaporate, and in doing so will cool the metal upon which it has been deposited. It will be seen then that the leakage thus accounted for is a leakage of water, and not of steam, the actual weight of water passing a given leakage path being twenty to fifty times greater than the corresponding weight of dry steam which could be passed.

The amount of valve leakage will obviously depend upon the type of valve employed, and separate inlet and exhaust valves would appear to have an advantage over the slide valve in this respect. In support of the above theory it appears that leakage is reduced by using superheated

steam or jacketing the cylinder, by the use of ample lubricating oil, and ample valve overlap, all of which tend to reduce condensation. Speed appears to have a similar effect, and would certainly reduce the cyclic condensation.

An adequate explanation for the missing quantity under modern conditions does not appear to be available.[1]

*METHODS OF INCREASING ENGINE EFFICIENCY

A number of methods of increasing the efficiency of actual engines are available, some of which have effects far greater than would at first appear to be probable. Most of these achieve their object largely by reducing cylinder condensation. They will be considered in subsequent paragraphs.

*THE UNIFLOW ENGINE

The employment of separate inlet and exhaust valves in place of the common slide valve will tend not only to prevent direct leakage from the steam chest to the exhaust, but will considerably reduce the cyclic fluctuations of valve-metal temperature and so decrease cyclic condensation. A further improvement is to place the valves at opposite ends of the piston stroke, as is done in the case of the Uniflow engine shown diagrammatically in Fig. 80. The cylinder is double-acting and has

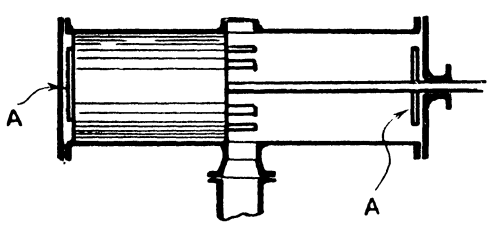


Fig. 80.

mechanically operated inlet valves A, A, at the extreme ends. The exhaust ports are placed centrally and are uncovered by the piston at the ends of its strokes. It will be seen that there is no scour of low-temperature steam against the hot ends of the cylinder at any point of the cycle, and that the metal near the exhaust ports is never subjected to a high temperature. An appreciable increase in economy is effected as a result of decreased condensation.

[1] See also First Report of Steam Engine Research Committee, *Proc.I.Mech.E.*, 1905; Lobley in *The Engineer*, Feb. 9, 1912; "Effect of Steam Jacketing of a Compound Engine," *Proc.I.Mech.E.*, 1905; Mellanby, "Surface Condensation in Steam Cylinders," Dec. 21, 1911.

If the engine is used to expand steam to the pressure of a condenser (say 1·5 lb. per sq. in. abs.) the clearance space A must be kept to $1\frac{1}{2}$ or 2 per cent. of the stroke volume. Should failure of the vacuum then occur very high pressures would result and special relief valves have to be provided to prevent damage under these circumstances.

The *p.v.* diagram for one end of the cylinder is shown in Fig. 81.[1]

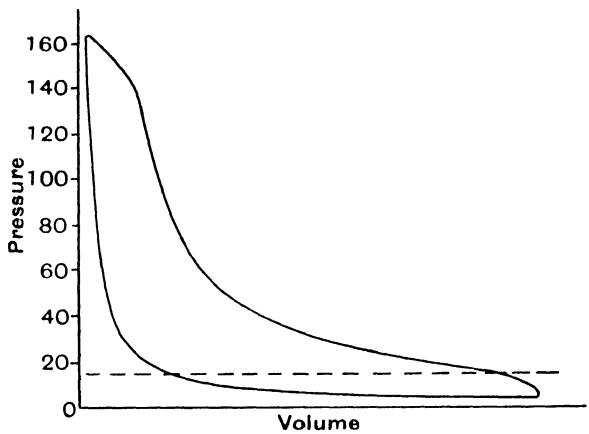


Fig. 81.

*THE STEAM JACKET

Initial condensation of the steam can be largely reduced by passing live steam through a jacket surrounding the cylinder, and as heat will flow into the cylinder during the expansion and exhaust strokes the dryness of the steam will be further increased during these stages.

The area of the *p.v.* diagram for 1 lb. of cylinder feed will be increased, but it must be remembered that the total steam consumption of the engine will now include the steam condensed in the jackets. In many cases the application of a steam-jacket materially improves the performance of an engine, and this is specially marked with engines of low efficiency. With high-efficiency and compound engines the effect of the jacket upon the efficiency is usually not marked.

The total heat supplied by the jackets can be deduced in practice by calculating the heat lost by an amount of live steam equal in weight to the water discharged from the jacket drain.

The heat supplied during expansion may be found approximately from the indicator diagram as follows (see Fig. 82).

[1] See also Papers by H. Pilling, Manchester Assn. of Engineers, 1920, and F. W. Perry, *Proc.I.Mech.E.*, July, 1920.

Work done during expansion $=\frac{1}{n-1}(p_1v_1-p_2v_2)\times\frac{1}{J}$ heat units.

Heat in steam at B $=h_1+q_1L_1\times$ total weight of steam present at B $=(h_1+q_1L_1)W$.

Heat in steam at C $=h_2+q_2L_2\times W$, assuming no leakage.

Then, if Q = heat from jacket during expansion

$$Q+W(h_1+q_1L_1)=\frac{1}{(n-1)J}(p_1v_1-p_2v_2)+W(h_2+q_2L_2)$$

or

$$Q=\frac{1}{(n-1)J}(p_1v_1-p_2v_2)+h_2+q_2L_2-h_1-q_1L_1$$

From the diagram the weight of cushion steam can be estimated, and this, added to the measured cylinder feed per stroke will give the total weight of cylinder contents during expansion, and the dryness fractions q_1 and q_2 can be obtained. The diagram also gives the pressures p_1 and p_2 and the volumes v_1 and v_2, and from these the value of the exponent n can be found. Q may now be calculated, using the above data.[1]

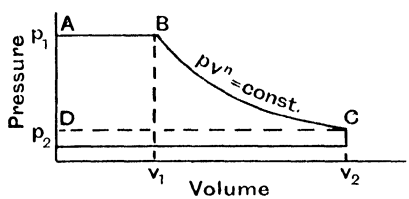


Fig. 82.

EXAMPLE.—The high-pressure cylinder of a jacketed steam engine is supplied with steam at 200 lb. per sq. in. abs. and 98 per cent. dry. Cut-off occurs at 25 per cent. of the stroke, the indicated pressure at release is 44 lb. per sq. in. abs., and the exhaust back pressure is 41 lb. per sq. in. abs. Find the net work done per pound of cylinder feed, the heat received from the jackets during expansion per pound of cylinder feed, and the efficiency reckoned from the exhaust temperature. Neglect clearance effects.

At 200 lb. per sq. in. specific volume = 2·29 cu. ft., and actual volume per pound = 0·98 × 2·29 = 2·44 cu. ft.

At release $v_2=4v_1=8\cdot977$ cu. ft., and as the specific volume is 9·59 cu. ft. the quality at release $=\frac{8\cdot97}{9\cdot59}=93\cdot61$ per cent.

To find the value of n during expansion

$$p_1v_1^n=p_2v_2^n \quad \text{or} \quad n=\frac{\log p_1/p_2}{\log v_2/v_1}=\frac{\log 200/44}{\log 4}=1\cdot092$$

[1] See also the following papers: Institution of Mechanical Engineers, *Proceedings*. Steam-Jacket Research Committee's Reports: First Report, 1889, p. 703; Second Report, 1892, p. 418; Third Report, 1894, p. 535. Steam-Engine Research Committee's First Report, 1905, p. 171. "Effects of Steam-Jacketing upon the Efficiency of a Horizontal Compound Steam Engine," by Professor A. L. Mellanby, 1905, p. 519. "The Triple Expansion Engine and Engine Trials at the Whitworth Engineering Laboratory, Owens College, Manchester," by Professor O. Reynolds. *Proc. Inst. C.E.*, vol. xcix, p. 152.

The net work done is $\frac{n}{n-1}(p_1v_1-p_2v_2)+(p_2-p_3)v_2$

$$=\frac{1\cdot092}{0\cdot092}(200\times1-44\times4)\times144\times2\cdot442+3\times144\times8\cdot977 \text{ ft.-lb.}$$

$=95{,}920$ ft.-lb. per lb.

The heat from the jackets during expansion :

$$H_j=\frac{1}{n-1}(p_1v_1-p_2v_2)+H_2-p_2v_2-H_1+p_1v_1$$

$$=\frac{1}{n-1}(p_1v_1-p_2v_2)-H_1+H_2 \text{ approximately}$$

$$=\frac{92{,}040}{778}-(0\cdot98\times843\cdot2+354\cdot9)+(0\cdot9361\times929\cdot2+240)$$

$$=118\cdot3-1181+870+242$$

$=49\cdot3$ B.Th.U. per lb. of cylinder feed.

The efficiency, calculated from the temperature of exhaust, when the liquid heat is 237·6 B.Th.U. per lb.

$$=\frac{95{,}920}{778\times(1181-238+49\cdot3)}=\frac{95{,}920}{778\times992\cdot3}=12\cdot43 \text{ per cent.}$$

Note that the heat supplied in the jackets must be debited to the engine. If the cylinder is jacketed with live steam the steam consumed in the jackets will be $\frac{49\cdot3}{943}=5\cdot2$ per cent. of that consumed in the cylinder or roughly 5 per cent. of the total consumption.

*THE USE OF SUPERHEATED STEAM

The use of superheated steam serves a double purpose. In the first case it enables steam to be used at a high temperature without the necessity of providing for the very high pressures which would be involved if saturated steam at that temperature were used. The thermodynamic efficiency can thus be improved while retaining moderate pressures, but the gain accounted for in this way does not exceed about 2 per cent. (from, say, 26 to 28 per cent.), the superheat being available only as the temperature falls below the maximum.

By far the most marked effect is produced by the reduction or elimination of condensation, with its accompanying leakage loss, as the whole of the superheat of the entering steam must be removed before condensation can commence.

The steam at release will also be much drier, and the greatly reduced amount of condensate on the walls, which will evaporate during the

exhaust stroke at the expense of the heat of the exposed surfaces, will result in considerably higher cylinder temperatures and, probably, in superheated cushion steam at the commencement of the cycle. It should be noted also that the heat transfer from steam to metal is much reduced if the surface is dry.

It is to be expected from the above remarks that when a sufficient

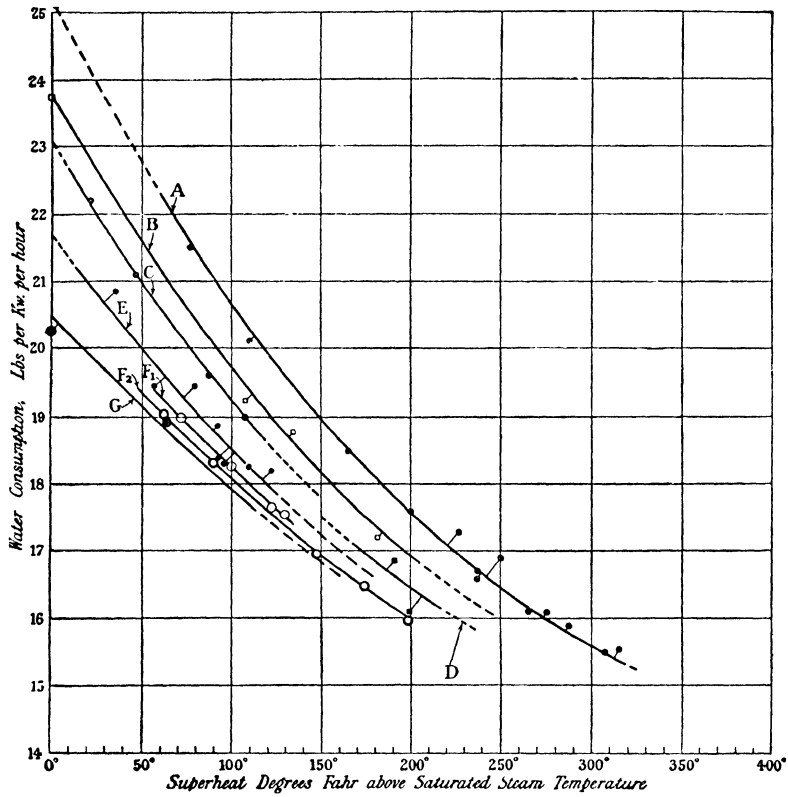


Fig. 83.—Non-jacketed Quick-revolution Triple-expansion Condensing Engines, using Superheated.

degree of superheat is employed to prevent cylinder condensation, the gain due to still further superheating will be small. That this is so seems to be borne out by tests reported by R. T. Smith in the First Report of the Steam Engine Research Committee,[1] from which the following particulars are taken.

Fig. 83 shows the effect of varying degrees of superheat on the steam consumption per kilowatt hour of seven engines, each coupled to a

[1] *Proc.I.Mech.E.*, 1905, p. 300.

dynamo, the output ranging from 220 to 1500 kw. They were all non-jacketed condensing engines, and were all tested at full load, one of them (F_2) being also tested at three-quarter load. The interesting result is, that if all the curves are produced sufficiently far, they will be found to meet very nearly in one point, namely 400° of superheat, showing that if one could only use enough superheat all engines of this type, of whatever size, would be about equally efficient. From a large number of experiments on the engine marked A in Fig. 83, a series of curves have been drawn as in Fig. 84, giving the pounds of steam per B.H.P. hour passing through the engine at all loads up to full load with saturated steam, and also for 50° to 350° F. superheat. These curves get flatter as the superheat increases, showing that, when sufficient superheat is used, an engine of this type tends to become equally efficient at all loads.

The following details relate to Fig. 83.

Set	Kw. output of generator coupled to engine	Load at test	Stop valve steam pressure. Pounds per square inch	Vacuum at engine. Inches of mercury
A	208	Full	155	26
B	220	,,	175	25
C	308	,,	190	25
D	362	,,	162	25·8
E	500	,,	150	26
F_1	700	,,	190	27
F_2	580	¾	192	27
G	1456	Full	183	26

In view of the fact that the effect of jackets and superheated steam is largely to prevent condensation, it follows that the full advantage of each expedient will not be realised when both are employed together.

For further particulars on the use of superheated steam the reader is referred to the following papers : Institution of Marine Engineers, " Marine Engines and Superheated Steam," by Mr. A. F. White, in the *Marine Engineer and Naval Architect*, Dec. 1909. Institution of Naval Architects, " Superheaters in Marine Boilers," by Mr. Harold E. Yarrow, read March 28, 1912, and reproduced in *Engineering* of April 5, 1912. *Proc.Inst.C.E.*, vol. cxxviii, " Superheated Steam Engine Trials," by Professor W. Ripper.

*COMPOUND EXPANSION

In order to take full advantage of the high boiler pressures and the high condenser vacua available in modern practice a large ratio of expansion must be used. If the expansion is carried out successively in two or more cylinders in series, each passing on its exhaust to the next, a number of advantages may be gained, as follows :

(1) The high-pressure steam is admitted to the high-pressure cylinder

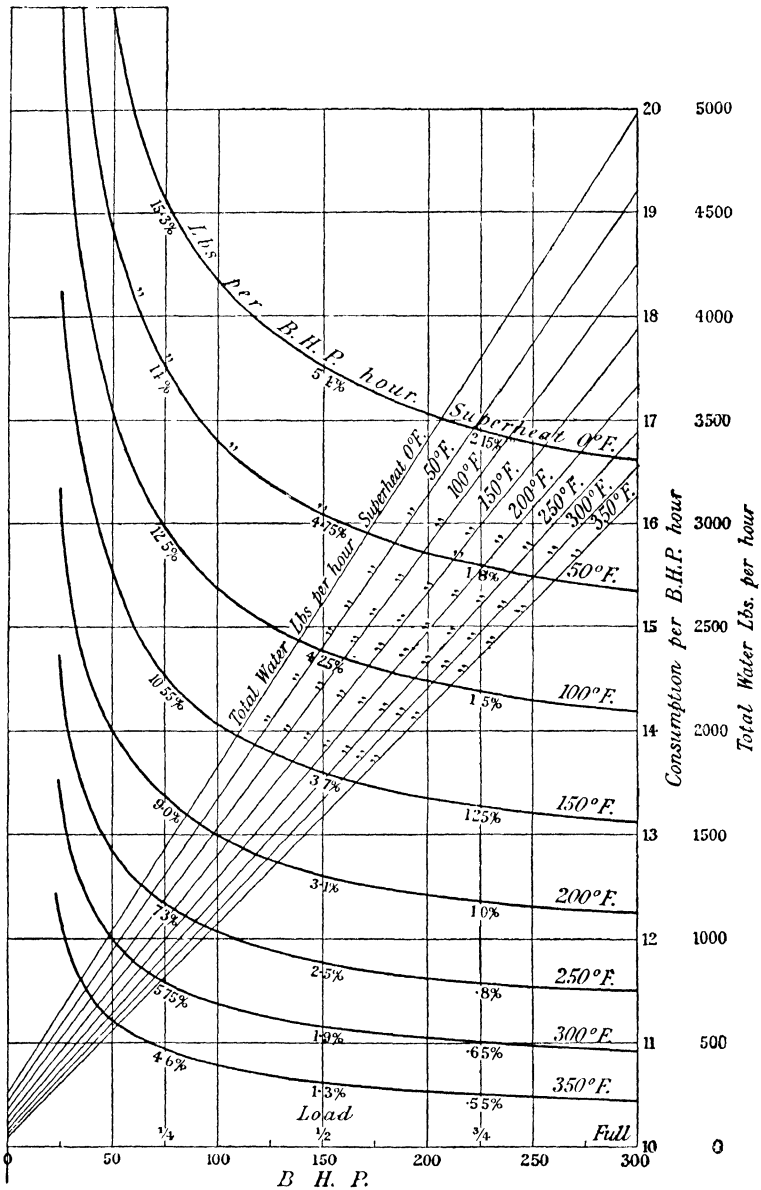


Fig. 84.—Non-jacketed Quick-revolution Triple-expansion Engine.

The percentage figures indicate the increase in pounds of water per B.H.P.-hour over full load consumption.

only, which, as it does not deal with the later stages of expansion, is of small size, and can be made of adequate strength without undue weight. The large low-pressure cylinder and piston (and also the intermediate-pressure cylinders and pistons if used), are never subjected to full boiler pressure, and can therefore be of much lighter construction than would be the case were the whole expansion carried out in one cylinder, as in the case of a simple engine.

(2) The action of the high-pressure steam on entering the cylinder is largely affected by the condition and quantity of the cushion steam in the clearance space between the piston and cylinder head. This action can be most readily controlled if the clearance space is small, and this in turn can be most easily arranged if the high-pressure cylinder is of small diameter.

(3) As the total pressure drop is divided between two or more cylinders the pressure causing leakage past pistons and valves is correspondingly reduced.

(4) As the total temperature range of the steam is distributed over two or more cylinders the cyclic fluctuations of metal temperature, and hence cyclic condensation, are reduced, thus carrying one stage further the advantageous action of the Uniflow engine.

(5) The steam condensed on the high-pressure cylinder walls is largely evaporated during the exhaust stroke, and augments the supply of " live " steam to subsequent cylinders, i.e. the steam expanding in the low-pressure cylinder is drier at a given pressure than would be the case if the expansion occurred in a single cylinder.

(6) Owing to the permissible lightening of the reciprocating parts, engine vibration and friction are reduced.

(7) The use of two, three, or four cylinders enables great improvements to be made with regard to engine balance and turning moment, and a heavy fly-wheel, which is always accompanied by large bearing losses, is not required.

(8) Feed-water heating by " bled " steam may be effected (see p. 165).

Compound expansion is normally carried out using two, three, or four cylinders in series, i.e. a high-pressure and a low-pressure cylinder, with one or two intermediate cylinders if desired. The terms " compound," " triple expansion," and " quadruple expansion " are applied. The cylinders may be arranged side by side or in tandem, though more than two cylinders in line is not usual.

*COMPOUND ENGINE WITHOUT INTERMEDIATE RECEIVER

In this type of engine the high-pressure cylinder exhausts directly into the low-pressure cylinder, which is always in direct communication with the exhaust port of the former and has no separate inlet valve and conse-

quently no " cut-off." For convenience a " tandem " arrangement is assumed.

The theoretical diagrams for the two cylinders, assuming no clearance, are as shown in Fig. 85. *ab* shows admission in the high-pressure cylinder, *b* being the point of cut-off, and *bc* shows the first stage of expansion. During the movement of both pistons to the right, the low-pressure cylinder is exhausting along *gh*. At *c* the high-pressure exhaust valve opens, and communication is at once established with the low-pressure cylinder. As the pistons now move towards the left the steam occupies both the reducing volume in the high-pressure cylinder and the expanding volume in the low-pressure cylinder, the exhaust and expansion curves being shown respectively by *cd* and *ef*. At any instant the total volume is given by $pq+rs$, and the combined diagram for the two cylinders will be as shown by *a'b'fgh*, which is exactly equivalent to the diagram

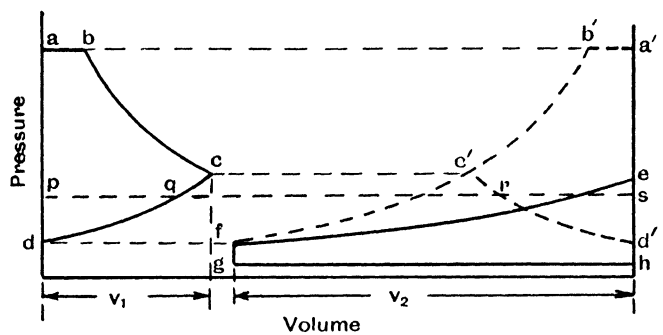


Fig. 85.

had the steam been admitted into and expanded in the low-pressure cylinder only.[1] If *x* is the point of intersection of *c'd'* and *ef*, the area *exd'* is duplicated on the superimposed diagram, so that the areas *exd'* and *fc'x* must be equal.

The disadvantage of this arrangement is that the high-pressure cylinder is subject to the full range of pressure and temperature down to the point of release in the low-pressure cylinder, and condensation in this cylinder may be expected to be high. The engine must also be arranged either with cranks in line (or one crank with cylinders in tandem), or at 180°, giving a very irregular turning moment.

*COMPOUND ENGINE WITH INTERMEDIATE RECEIVER

By providing a " receiver " or storage capacity between the cylinders it is possible to put the cranks at any desired angle, as the exhaust steam

[1] The " mean effective pressure referred to the low-pressure cylinder " is given by the area *ha'b'fg* divided by v_2.

from the high-pressure cylinder can, in whole or in part, be stored until it is required by the low-pressure cylinder. This is now provided with separate admission valves and has its own cut-off. The theoretical diagram, Fig. 86, is as shown on the assumption that the receiver is of very large capacity. *ab*, *bc*, and *cd* represent admission, expansion, and exhaust in the high-pressure cylinder, and during the latter the steam is discharged at constant pressure into the receiver. *ef*, *fg*, *ghj*, represent admission, expansion, and exhaust from the low-pressure cylinder, and, assuming that no pressure loss occurs owing to the intermediate storage process, the pressure along *ef* will equal the pressure along *cd*, and volume *ef* will equal *cd*. Owing to the storage of the steam the strokes *cd* and *ef* do not need to occur simultaneously, and the cranks may be placed at any angle desired.

The combined diagram is *a'b'fghj*, and is, as before, the same as if the whole expansion had occurred in the low-pressure cylinder.

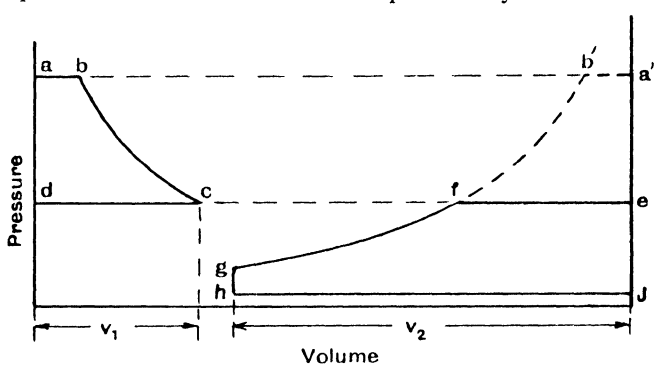


Fig. 86.

It will be noted that, in addition to better turning moment, etc., the initial condensation will be considerably reduced, compared with the previous arrangement, as the high-pressure cylinder is never subjected to low pressures and temperatures.

Fig. 87 shows the corresponding diagrams for an engine fitted with a small receiver only, a tandem arrangement of cylinders being assumed for convenience. Here the first stage *cc'* of the high-pressure exhaust will show a falling pressure as steam is removed from the receiver by the low-pressure cylinder. At *f* (corresponding to *c'*) cut-off occurs in the low-pressure cylinder, and the high-pressure exhaust pressure will rise, shown by *c'd*, as the total volume of receiver and high-pressure cylinder decreases.

It will be seen that the combined diagrams overlap in part but leave a corresponding area unaccounted for. The point of cut-off in the low-pressure cylinder must be chosen so that the initial and final pressures in the receiver—corresponding to pressures at *c* and *d*—will be equal.

For other crank or cylinder arrangements the diagram becomes more complicated.

In general the high-pressure exhaust port, low-pressure valve chest, and the intermediate piping, provide all the storage capacity required. Also, in addition to the usual alterations required to be made to the theoretical diagram owing to clearance spaces, non-instantaneous closing

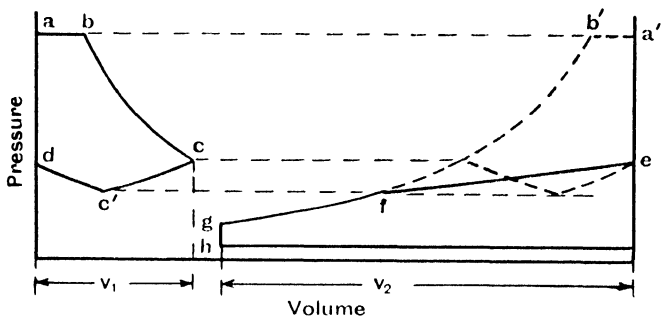


Fig. 87.

of valves, etc., it is necessary to allow a small pressure drop between the high-pressure and low-pressure cylinders in order to ensure sufficiently rapid movement of the steam. The actual high-pressure exhaust and low-pressure admission curves obtained with an indicator will be found to differ considerably from the theoretical curves *cc'd* and *ef*.

*THE COMBINATION OF INDICATOR DIAGRAMS FROM A COMPOUND ENGINE

The indicator diagrams for a large horizontal compound engine developing 1415 I.H.P. are shown in Fig. 88. In order to trace the action of the steam in the engine as a whole these diagrams must be combined.

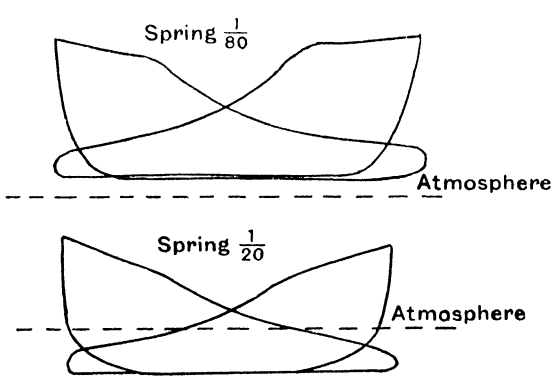


Fig. 88.

Average indicator diagrams must first be constructed representing the mean diagrams for both ends of the high-pressure and low-pressure cylinders, as shown by Fig. 89. The saturation curves S′S′ are then

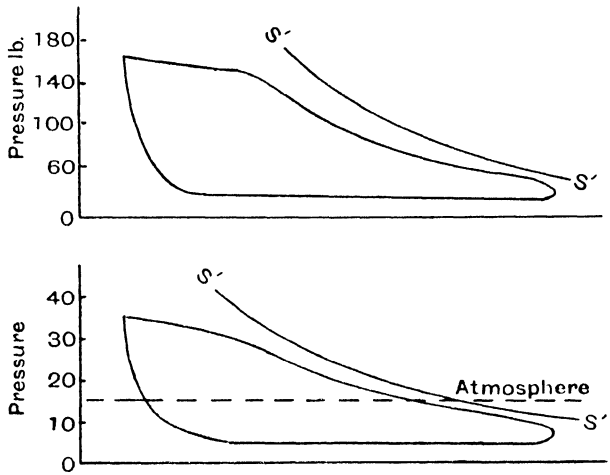


Fig. 89.

drawn one on each diagram by the method already explained on p. 175, and the clearance volume is also shown.

Next a convenient distance AB, Fig. 90, is set off to represent the

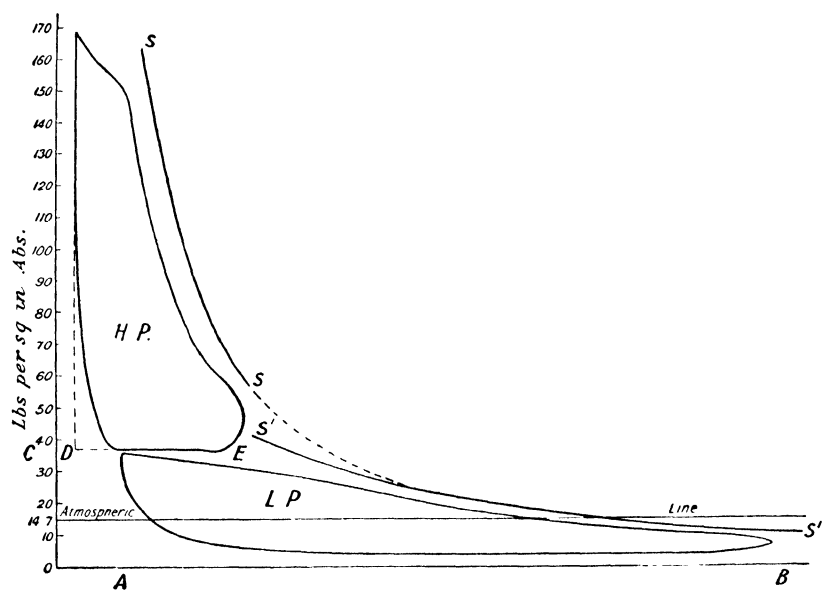


Fig. 90.

displacement of the low-pressure piston (i.e. area of cylinder × stroke), and OA to represent to the same scale the clearance volume; then choosing a convenient scale of pressures the mean diagram from the low-pressure cylinder is replotted together with its saturation curve S′S′. Next, to the same scale of volumes, CD is set off to represent the clearance volume of the high-pressure cylinder, and DE the stroke volume of that cylinder; and the mean diagram from the high-pressure cylinder is replotted together with its saturation curve SS.

In the combined diagram drawn in Fig. 90 the saturation curves SS and S′S′ do not form one continuous curve as the total weight of steam present in the high-pressure cylinder during expansion is not the same

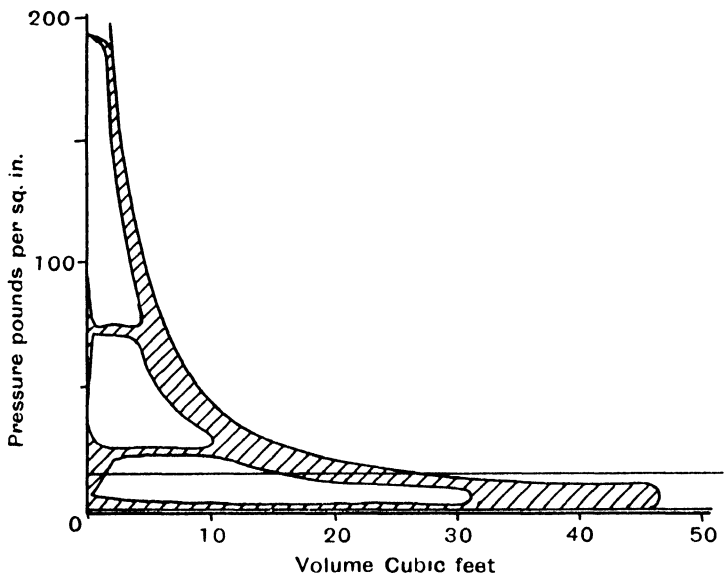


Fig. 91.

as that in the low-pressure cylinder, the difference being due to unequal weights of cushion steam in the two cylinders. As a rule the weight of cushion steam is less in the low-pressure than in the high-pressure cylinder, and this causes the saturation curve of the former to fall inside that for the latter.

The diagram is clarified considerably if the effect of cushion steam is eliminated from each part of the diagram by the method given on p. 176 when the diagrams are corrected to represent the expansion of W lb. (or 1 lb.) of cylinder feed. Fig. 91 shows such a combined diagram for a three-cylinder compound engine. For other such diagrams, see *Proc.I.Mech.E.*, 1927, vols. 1 and 2.

*DIAGRAM FACTOR

The following table [1] gives the diagram factor K for a number of different classes of engine for which the initial pressure (p), ratio of expansion (r), clearance volume expressed in terms of stroke volume (c), mean piston speed (s), and mechanical efficiency (η) are also shown. Both c and r are referred to the low-pressure cylinder volume in the case of compound engines.

Type of engine	p	r	c	s	η	K
A. Horizontal engines—						
(1) Simple (slide-valve)—						
Non-condensing . .	100	2	0·12	500	85	0·80
Condensing	80	4	0·10	500	85	0·75
(2) Compound condensing (Corliss or drop-valve) .	165	12	0·05	600	90	0·70
Ditto, with superheat .	165	12	0·05	600	90	0·65
(3) Uniflow condensing—						
(Drop-valve)	165	10–20	0·03	750	88	0·76
Ditto, with superheat .	165	10–20	0·03	750	88	0·71
B. Vertical engines—						
(4) Simple condensing .	100	3	—	600	88	0·76
(5) Compound condensing with superheat . . .	165	12	—	750	90	0·60
(6) Triple expansion condensing with superheat	220	16	—	750	92	0·55
(7) Marine condensing—						
Compound	—	8	—	—	—	0·70
Triple-expansion . .	—	12	—	—	—	0·625

*RATIO OF CYLINDER VOLUMES FOR A COMPOUND ENGINE

For many purposes it is desirable to distribute the total work equally between the two cylinders. In this case if the overall ratio of expansion is given, the requisite volume of the low-pressure cylinder can be calculated on the assumption that this cylinder alone is used. The ideal hypothetical diagram will then be as shown by *abef*, Fig. 92.

If *dc* represents the boundary between the high-pressure and low-pressure diagrams,

area *abcd*=*dcef*

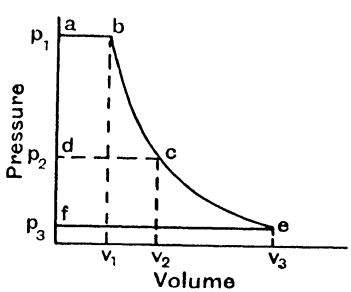


Fig. 92.

[1] Petrie, " Modern Practice in Heat Engines."

or $$\frac{n}{n-1}(p_1v_1-p_2v_2)=\frac{n}{n-1}(p_2v_2-p_3v_3)$$

or $$p_1v_1+p_3v_3=2p_2v_2 \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Now $v_3=v_1(p_1/p_3)^{1/n}$ and $v_2=v_1(p_1/p_2)^{1/n}$
and substituting in (1)

$$p_1v_1+p_3v_1(p_1/p_3)^{1/n}=2p_2v_1(p_1/p_2)^{1/n}$$

and $$p_1+p_3(p_1/p_3)^{1/n}=2p_2(p_1/p_2)^{1/n}$$

or $$p_1+p_1^{1/n}p_3^{1-1/n}=2p_1^{1/n}p_2^{1-1/n} \quad . \quad . \quad . \quad . \quad (2)$$

or $$p_1^{1-1/n}+p_3^{1-1/n}=2p_2^{1-1/n}$$

from which the value of p_2 can be calculated.

Alternatively p_2 and p_3 may be expressed in terms of p_1, in which case

$$p_1v_1+p_1(v_1/v_3)^nv_3=2p_1(v_1/v_2)^nv_2$$

and $$v_1+v_1^nv_3^{1-n}=2v_1^nv_2^{1-n}$$

or $$v_1^{1-n}+v_3^{1-n}=2v_2^{1-n}$$

or $$v_2=\frac{\sqrt[1-n]{v_1^{1-n}+v_3^{1-n}}}{2}$$

For hyperbolic expansion

$$p_1v_1 \log_\epsilon v_2/v_1=p_2v_2 \log_\epsilon v_3/v_2$$

or $$v_2/v_1=v_3/v_2$$

and $$v_2^2=v_1v_3 \quad \text{or} \quad p_2^2=p_1p_3$$

If $v_3/v_1=\mathrm{R}$, the total ratio of expansion, and $v_3/v_2=r$, the ratio of the two cylinder volumes, then the cut-off in the high-pressure cylinder must occur at the fraction of the stroke given by R/r.

The volume ratio calculated above may be varied to suit known conditions or when a closer approximation to the indicator cards is available.

Example.—In a two-cylinder compound engine, the admission pressure to the high-pressure cylinder is 105 lb. abs., cut-off 0·6 stroke. The release pressure in the low-pressure cylinder is 12 lb. abs. and the condenser pressure 3 lb. abs. If the initial loads on the pistons are equal and the curve of expansion is $pv^{1\cdot2}=$constant, estimate the cylinder volume ratio, the mean pressure in the receiver, the point of cut-off in the low-pressure cylinder, and the ratio of the work done in the two cylinders.

Let R=total ratio of expansion, then, assuming a continuous expansion curve and a volume at cut-off of 1 cu. ft.

$$105\times1=12\times\mathrm{R}^{1\cdot2}$$

from which $\mathrm{R}=6\cdot095$

$$\therefore \text{ cylinder ratio}=6\cdot095\times0\cdot6=3\cdot657$$

Let x=mean receiver pressure in pounds per square inch absolute.
For equal initial loads,

$$105-x=3\cdot657(x-3)$$
$$x=24\cdot9 \text{ lb. abs.}$$

Let r=ratio of expansion in low-pressure cylinder, then

$$24\cdot9\times1=12\times r^{1\cdot2}$$

from which $r=1\cdot838$

$$\therefore \text{ cut-off in low-pressure cylinder}=\frac{1}{1\cdot838}=0\cdot544 \text{ of the stroke.}$$

Let p_2=the absolute release pressure in high-pressure cylinder

$$105\times1=p_2\times\left(\frac{1}{0\cdot6}\right)^{1\cdot2}$$

from which $p_2=56\cdot87$ lb. per sq. in. abs.

It should be noticed that there is a drop in pressure between the high-pressure release and the mean receiver pressure of amount $56\cdot87-24\cdot9=31\cdot97$ lb. per sq. in.

To find the Distribution of Work between the Cylinders.—The mean effective pressure in the high-pressure cylinder is

$$\frac{105\times1+\dfrac{105\times1-56\cdot87\times\dfrac{1}{0\cdot6}}{1\cdot2-1}}{\dfrac{1}{0\cdot6}}-24\cdot9$$
$$=\frac{105+51}{1\cdot66}-24\cdot9$$
$$=68\cdot8 \text{ lb. per sq. in.}$$

The mean effective pressure in the low-pressure cylinder is

$$\frac{24\cdot9\times1+\dfrac{24\cdot9\times1-12\times1\cdot838}{1\cdot2-1}}{1\cdot838}-3$$
$$=\frac{24\cdot9+14\cdot2}{1\cdot838}-3$$
$$=18\cdot25 \text{ lb. per sq. in.}$$

Hence,

$$\frac{\text{work done in high-pressure cylinder}}{\text{work done in low-pressure cylinder}}=\frac{68\cdot8}{3\cdot657\times18\cdot25}=1\cdot03$$

In this example, the approximately equal distribution of work and the

equal initial loads are only obtained by the " drop " between the high-pressure release and the receiver.

EXAMPLE.—Determine the cylinder diameters of a horizontal compound steam engine to develop 600 I.H.P. under the following conditions : Pressure in steam chest 155 lb. per sq. in. abs., vacuum 26 in., number of expansions 12, diagram factor 0·82, piston speed 650 ft. per minute, cut-off in high-pressure cylinder $\frac{1}{3}$ stroke. Determine also the point of cut-off in the low-pressure cylinder, and compare the work done in the two cylinders when the initial loads are approximately equal.

The cylinder ratio will be $L=\frac{R}{r}=\frac{12}{3}=4$.

Hence if A_2 and A_1 denote the areas (in square inches) of the low-pressure and high-pressure cylinders respectively,

$$\frac{A_2}{A_1}=4, \text{ the strokes being made equal}$$

Referred to the low-pressure cylinder

$$\begin{aligned} p_m &= 0{\cdot}82\{\tfrac{15}{12}^{5}(1+\log_\epsilon 12)-2\} \\ &= 0{\cdot}82\{\tfrac{15}{12}^{5}\times 3{\cdot}48-2\} \\ &= 0{\cdot}82\times 43 = 35{\cdot}3 \text{ lb. per sq. in.} \end{aligned}$$

Hence

$$\frac{A_2\times 35{\cdot}3\times 650}{33{,}000}=600$$

$$A_2=\frac{600\times 33{,}000}{35{\cdot}3\times 650}$$

$$=861 \text{ sq. in.}$$

or

$$d_2=\sqrt{\frac{861}{0{\cdot}7854}}=33 \text{ in.}$$

Since $A_2=4A_1$, it follows that $d_2=2d_1$, hence the diameter of the high-pressure cylinder will be 16·5 in.

Let x=mean receiver pressure in pounds per square inch absolute.

Then $155-x=4(x-2)$

and $x=32{\cdot}6$ lb. per sq. in.

If r is the number of expansions in the low-pressure cylinder,

$$32{\cdot}6\times 1=\tfrac{15}{12}^{5}\times r$$

$$r=2{\cdot}5$$

$\therefore$ cut-off in low-pressure is $\frac{1}{2{\cdot}5}$ or 0·4 of the stroke.

The mean effective pressure in the high-pressure cylinder will be—

$$\begin{aligned}&0{\cdot}82\{\tfrac{155}{3}(1+\log_\epsilon 3)-32{\cdot}6\}\\&=0{\cdot}82\{\tfrac{155}{3}\times 2{\cdot}097-32{\cdot}6\}\\&=0{\cdot}82\times 75{\cdot}7\\&=62 \text{ lb. per sq. in.}\end{aligned}$$

The mean effective pressure in the low-pressure cylinder will be—

$$\begin{aligned}&0{\cdot}82\left\{\frac{32{\cdot}6}{2{\cdot}5}(1+\log_\epsilon 2{\cdot}5)-4\right\}\\&=0{\cdot}82\left\{\frac{32{\cdot}6}{2{\cdot}5}\times 1{\cdot}915\right\}\\&=0{\cdot}82\times 21\\&=17{\cdot}3 \text{ lb. per sq. in.}\end{aligned}$$

Hence,

$$\frac{\text{work done in high-pressure cylinder}}{\text{work done in low-pressure cylinder}}=\frac{62}{4\times 17{\cdot}3}=\frac{1}{1{\cdot}11}$$

EXAMPLE.—Estimate the diameters of the cylinders required for a quadruple expansion marine engine to develop 12,000 I.H.P. with a piston speed of 960 ft. per minute. Pressure in steam chest 210 lb. per sq. in. gauge, condenser pressure 26″ Hg, number of expansions 14. Assume a ratio of low-pressure to high-pressure cylinder volumes of 9, and a diagram factor 0·65. Find also the point of cut-off in the high-pressure cylinder.

The mean effective pressure referred to the low-pressure cylinder will be

$$\begin{aligned}p_m&=0{\cdot}65\{\tfrac{225}{14}(1+\log_\epsilon 14)-2\}\\&=0{\cdot}65\{\tfrac{225}{14}\times 3{\cdot}639-2\}\\&=0{\cdot}65\{58{\cdot}5-2\}\\&=0{\cdot}65\times 56{\cdot}5\\&=36{\cdot}7 \text{ lb. per sq. in.}\end{aligned}$$

Hence, if A be the area of the low-pressure cylinder,

$$\begin{aligned}A\times 36{\cdot}7\times 960&=12{,}000\times 33{,}000\\A&=\frac{12{,}000\times 33{,}000}{36{\cdot}7\times 960}\\&=11{,}240 \text{ sq. in.}\\\therefore\ d&=\sqrt{\frac{11{,}240}{0{\cdot}7854}}=119{\cdot}6 \text{ in.}\end{aligned}$$

and diameter of high-pressure cylinder

$$=\frac{119{\cdot}6}{3}=39{\cdot}5 \text{ in.}$$

Taking a ratio of cylinder volumes of

$$1 : 2{\cdot}1 : 4{\cdot}4 : 9$$

Diameter of first intermediate cylinder $= 39{\cdot}5 \times \sqrt{2{\cdot}1} = 58$ in.

,, second ,, ,, $= 39{\cdot}5 \times \sqrt{4{\cdot}4} = 83$ in.

The cut-off in the high-pressure cylinder will be—

$$\frac{\text{ratio of low-pressure to high-pressure volumes}}{\text{total number of expansions}} = \frac{9}{14} \text{ of the stroke}$$

*GOVERNING OF THE COMPOUND ENGINE

Two types of governing are available, and depend respectively for their actions on variations of the cut-off in the high-pressure cylinder, and on throttling. Their effects may be seen from the hypothetical combined diagrams shown in Figs. 93–95.

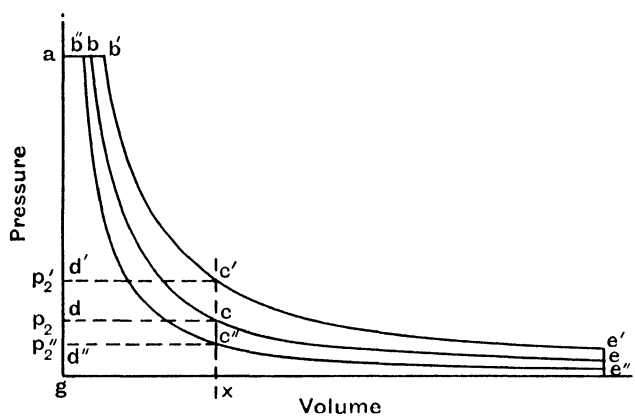


Fig. 93.

Cut-off Governing.—In Fig. 93 *abcd*, *dcefg* represent the high-pressure and low-pressure diagrams on the assumption of an equal distribution of work. The total work done will depend on the total weight of steam passing through the engine. By making the cut-off in the high-pressure cylinder later or earlier the expansion curve will be changed from *bce* to *b′c′e′* or *b″c″e″*, and the total work done will be given by the areas under these curves. The distribution of work between the cylinders will be altered, however. The cut-off ratio in the low-pressure cylinder remaining constant at *gx*/*gf*, the separation line *dc* between the two diagrams will change to *d′c′* or *d″c″*. A later cut-off in the high-pressure cylinder will raise the pressure in the receiver between the cylinders from p_2 to p_2', and will throw an increasing proportion of the total load (now augmented) on to the low-pressure

cylinder. An earlier cut-off will have the opposite effect, and when running light the work done in the low-pressure cylinder may become negligible.

Varying the cut-off in the low-pressure cylinder will not affect the total amount of steam passing through the engine, or the total work

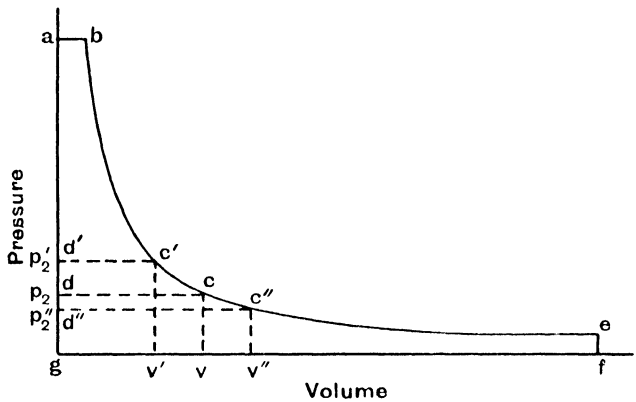


Fig. 94.

done. As shown in Fig. 94, if the low-pressure cut-off is made earlier, at c' instead of c, the intermediate pressure will be increased from p_2 to p_2', and an amount of work represented by $d'c'cd$ will be transferred from the high-pressure to the low-pressure cylinder. A later low-pressure cut-off will decrease the load on that cylinder.

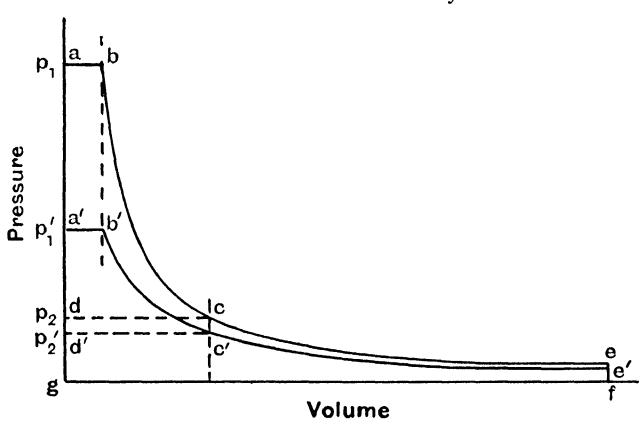


Fig. 95.

***Effect of Throttling the Steam to the High-Pressure Cylinder on the Distribution of Work. Throttle Governing.**—Consider the combined indicator diagrams shown in Fig. 95, in which *abcd* represents the work done in the high-pressure cylinder and *dcefg* the work done

in the low-pressure cylinder. Suppose that in order to meet a reduced load on the engine the steam is throttled down to the pressure p_1', the cut-off in each cylinder remaining unaltered. Then $a'b'c'd'$ will represent the high-pressure diagram and $d'c'e'fg$ the low-pressure diagram; the work done in the high-pressure cylinder will be much less than before, but the work done in the low-pressure cylinder will be little altered.

If, therefore, the engine is governed by throttling the power developed in the low-pressure cylinder will remain nearly constant, and when running on light load the power developed in the high-pressure cylinder may be very small. This is the converse to what happens with cut-off governing acting on the high-pressure cylinder.

That the steam consumption of an engine governed by throttling is almost a linear function of the Indicated Horse-Power can be seen at

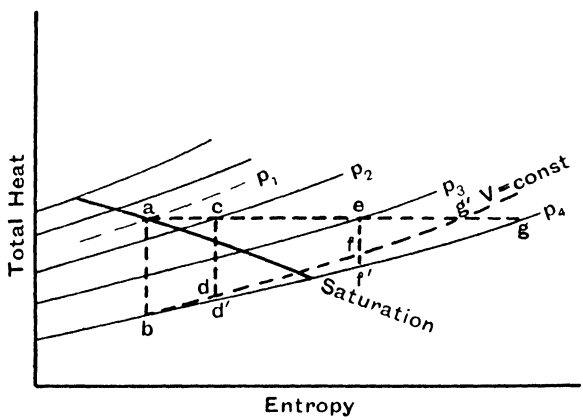


Fig. 96.

once from the Mollier chart, Fig. 96. Let the boiler pressure be given by p_1, and the full load initial condition point by a. If the expansion is adiabatic, the final condition will be given by point b. If the steam, in response to a decreased load, is now throttled to c the subsequent expansion will be to d on the constant volume line through b. Similarly throttling to e will be followed by expansion to f. The back pressure p_b, and stroke volume v_2 being constant, the term $p_b v_2$, representing the work done on the exhaust stroke, will be constant for all loads.

Now for all points in the mixed phase region, i.e. to the left of the saturation curve, lines of constant volume are straight, and except in very extreme cases the final condition points will lie in this area. It will be seen then that as throttling proceeds from a towards g' the work done per pound of steam, and consequently the steam consumption per I.H.P., will decrease in a linear manner, a fact which, demonstrated by Willans by other methods, is known as the Willans' Straight Line Law.

*INITIAL LOADS ON THE HIGH-PRESSURE AND LOW-PRESSURE PISTONS

The net load on a piston is the difference between the total forces exerted by the steam on its two sides. Thus on the high-pressure piston the initial load will be (neglecting the area of the piston rod),

Area of high-pressure piston × (initial steam pressure − receiver pressure)

and on the low-pressure piston

Area of low-pressure piston × (receiver pressure − back pressure).

It will frequently happen that if equal distribution of work is allowed between the cylinders, the initial loads will be unequal, and vice versa. The ratio of cylinder volumes should be so chosen that both the work done in the two cylinders and the initial loads on the pistons are approximately equal.

Suppose the initial steam pressure in the high-pressure cylinder is 100 lb. per sq. in. abs., the total number of expansions 10, the back pressure 4 lb. per sq. in. abs., and the ratio of cylinder volumes 3.

Let x be the receiver pressure in pounds per square inch absolute. Then, since the area of the low-pressure cylinder will be three times that of the high-pressure cylinder for equal strokes, for equal initial loads we have

$$100-p_r=3(p_r-4), \text{ where } p_r \text{ is the receiver pressure}$$

$$\text{therefore } p_r=28 \text{ lb. per sq. in. abs.}$$

Assuming complete hyperbolic expansion, the ratio of expansion in the high-pressure cylinder will be

$$\tfrac{100}{28}=3{\cdot}57$$

and in the low-pressure cylinder it will be 3, giving a pressure at exhaust opening of $28\div 3=9{\cdot}33$ lb. per sq. in. and a total ratio of expansion of $3{\cdot}57\times 3=10{\cdot}7$.

The mean effective pressure in the high-pressure cylinder will be

$$\frac{100}{3{\cdot}57}(1+\log_\epsilon 3{\cdot}57)-28$$

$$=28(1+1{\cdot}27)-28$$

$$=35{\cdot}36 \text{ lb. per sq. in.}$$

The mean effective pressure in the low-pressure cylinder will be

$$\tfrac{28}{3}(1+\log_\epsilon 3)-4$$

$$=9{\cdot}33(1+1{\cdot}097)-4$$

$$=15{\cdot}59 \text{ lb. per sq. in.}$$

Hence, the ratio

$$\frac{\text{work done in high-pressure cylinder}}{\text{work done in low-pressure cylinder}}=\frac{35{\cdot}36}{3\times 15{\cdot}59}=\frac{1}{1{\cdot}32}$$

THE REGENERATIVE CYCLE

The regenerative cycle, which can be applied to compound engines, is considered in detail on pp. 258, 259, 260.

An example will illustrate a simple case.

EXAMPLE.—In a compound steam engine, the admission pressure to the high-pressure cylinder is 150 lb. per sq. in. abs., and the exhaust pressure in the low-pressure cylinder is 4 lb. per sq. in. abs. The back pressure in the high-pressure cylinder and the admission pressure of the low-pressure cylinder are 45 lb. abs. Steam for feed heating is drawn from the low-pressure steam chest. Assuming complete adiabatic expansion and no heat losses, estimate the efficiency of the engine, and the gain due to feed heating.

From steam tables it is found that—

at 150 lb. abs. $L_1=863$ and $T_1=358+460=818°$ F. abs.
,, 45 ,, $T_3=274+460=734°$ F. abs.
,, 4 ,, $T_2=153+460=613°$ F. abs.

From (3), p. 259, $$w=\frac{1-\frac{613}{734}}{\log_\epsilon \frac{818}{734}+\frac{863}{818}}=\frac{1-0\cdot8351}{0\cdot1084+1\cdot0550}$$

$$w=\frac{0\cdot1649}{1\cdot1634}=0\cdot1417 \text{ lb.}$$

and from (5)

$$\eta=1-\frac{613\left(\log_\epsilon \frac{818}{613}+\frac{863}{818}\right)}{1\cdot1417(863+818-734)}$$

$$=1-\frac{613(0\cdot2885+1\cdot0550)}{1\cdot1417\times 947}$$

$$=1-0\cdot761$$

$$=0\cdot239 \text{ or } 23\cdot9 \text{ per cent.}$$

Without feed heating, and working on the Rankine cycle between temperature limits T_1 and T_2, the efficiency will be by (6),

$$1-\frac{613\left(\log_\epsilon \frac{618}{613}+\frac{863}{818}\right)}{863+818-613}$$

$$=1-0\cdot772$$

$$=0\cdot228 \text{ or } 22\cdot8 \text{ per cent.}$$

The increased efficiency due to feed heating is therefore $23\cdot9-22\cdot8$ $=1\cdot1$ per cent., or the percentage saving is $100\times\frac{1\cdot1}{22\cdot8}$

$$=4\cdot8 \text{ per cent.}$$

EXAMPLES ON CHAPTER XI

1. Measurements from an indicator diagram taken on a steam engine show that at a certain instant the volume of the steam is 1·95 cu. ft. and the pressure 70 lb. per sq. in. abs. If the actual weight of steam in the cylinder is 0·5 lb., estimate the dryness of the steam at that instant.

2. The diameter of a steam engine cylinder is 40 in., and of the piston rod 5 in., and the stroke is 5 ft. The mean effective pressure on the head end of the piston is 40 lb. per sq. in., and on the crank end 42 lb. per sq. in. If the speed of the engine is 120 r.p.m., what is the indicated horse-power ?

3. A locomotive has two double-acting cylinders, the admission pressure is 150 lb. per sq. in. abs., and the exhaust pressure 18 lb. per sq. in. abs. The cylinder diameters are 17 in., stroke 26 in., and the diameter of the driving wheels 6 ft. Find the tractive effort and the indicated horse-power when running at 40 miles an hour, the cut-off being 0·4. Allow a diagram factor of 0·9 and a mechanical efficiency of 80 per cent.

4. The following particulars are obtained from an indicator diagram taken from the high-pressure cylinder of a compound steam engine fitted with Corliss valves :

Cut-off $\frac{1}{4}$ stroke ; at a point on the compression curve the pressure was 59 lb. abs., and indicated volume 4 cu. ft.

Pressure at a point on expansion curve just after cut-off was 155 lb. abs., and indicated volume 7·2 cu. ft.

Pressure at $\frac{1}{2}$ stroke on expansion curve = 112 lb. abs.

Pressure at release = 62 lb. abs., and indicated volume = 17·5 cu. ft.

The diameter of the cylinder is 28 in. and stroke 4 ft., the clearance volume being 7 per cent. of the stroke volume and the cylinder feed 2·58 lb. per stroke with 150 working strokes per minute. Estimate the dryness fraction and missing quantity in pounds per hour (*a*) at cut-off, (*b*) at $\frac{1}{2}$ stroke, (*c*) at release.

5. Dry steam is admitted to an engine cylinder at 60 lb. per sq. in. abs., and the condensation during admission is 20 per cent. of the whole steam supply. During expansion three-quarters of the heat absorbed by the cylinder walls during admission is returned to the steam at a uniform rate as the temperature falls. If the expansion be complete and the back pressure be 4 lb. per sq. in. abs., find the dryness fraction at the end of expansion. Neglect clearance and all heat losses.

6. A steam engine cylinder is $33\frac{1}{8}$ in. diameter and the piston has a stroke of 3 ft. 3 in. The engine develops 600 I.H.P. at 100 r.p.m. Assuming a diagram factor of 0·82 what is the ratio of expansion if the initial steam pressure is 155 lb. per sq. in. abs. and the back pressure 2 lb. per sq. in. abs. ?

7. The main engines of a vessel are supplied with steam at 200 lb. abs. ($t = 382°$ F.) and use 17 lb. of steam per I.H.P. per hour, the temperature of the condenser water being 120° F. An auxiliary engine supplied with steam from the same boiler and exhausting into the atmosphere uses 25 lb. of steam per I.H.P. per hour. The main engines develop 6000 I.H.P. and the auxiliary engine 120 I.H.P. Find the actual efficiency (*a*) of the main engines alone, (*b*) of the whole plant when the auxiliary engine discharges into the hot well of the main engines.

8. The diameter of a steam-engine cylinder is 10 in. and the stroke 1 ft. If the initial pressure is 100 lb. per sq. in. abs. and cut-off is at $\frac{1}{4}$ stroke, find the theoretical mean effective pressure, neglecting clearance, the back pressure being 4 lb. abs.

If the clearance is 0·125 of the piston displacement find the probable I.H.P. if the engine runs at 250 r.p.m. [Assume a diagram factor of 0·85.]

9. Find the diameter of a steam-engine cylinder required to develop 80 I.H.P. with a piston speed of 650 ft. per minute. The initial steam pressure is 120 lb. per sq. in. abs., back pressure 2 lb. abs., cut-off $\frac{1}{3}$ stroke, clearance volume 7 per cent. of the piston displacement. Assuem a diagram factor of 0·9.

10. A double-acting single-cylinder engine of cylinder diameter 14 in. and stroke 24 in., runs at 120 r.p.m. and develops 90 I.H.P. The initial steam pressure is 90 lb. per sq. in. abs. and the back pressure 18 lb. abs. Taking a diagram factor of 0·8, find the ratio of expansion.

11. In a two-cylinder compound engine, the admission pressure to the high-pressure cylinder is 80 lb. per sq. in. abs., cut-off 0·5 stroke. The release pressure in the low-pressure cylinder is 8 lb. per sq. in. abs. and the condenser pressure 2 lb. abs. Assuming hyperbolic expansion and equal initial loads on the pistons, estimate the ratio of cylinder volumes, the mean pressure in the receiver, the point of cut-off in the low-pressure cylinder, and the ratio of the work done in the two cylinders.

12. Solve Question 11 if, instead of hyperbolic expansion, the law of expansion is $pv^{1\cdot15}$=constant.

13. Determine the cylinder diameters of a horizontal compound steam engine to develop 500 I.H.P. under the following conditions : Pressure in steam chest 140 lb. per sq. in. abs., vacuum 26 in., number of expansions 10, diagram factor 0·80, piston speed 600 ft. per minute, cut-off in high-pressure cylinder 0·35 stroke. Determine also the point of cut-off in the low-pressure cylinder and compare the work done in the two cylinders when the initial loads are equal.

14. In a two-cylinder compound engine the ratio of cylinder volumes is 5 and the total number of expansions is 10. The initial steam pressure is 100 lb. per sq. in. abs. and the back pressure 4 lb. per sq. in. abs. Assuming continuous hyperbolic expansion and equal distribution of work between the two cylinders, compare the initial loads on the pistons.

15. A triple expansion engine is required to develop 5000 I.H.P. at 90 r.p.m. under the following conditions : Pressure in high-pressure steam chest 200 lb. per sq. in. abs., cut-off in high-pressure cylinder 0·7 stroke, average piston speed 720 ft. per minute, vacuum 28 in. Using a cylinder ratio of 1 : 3 : 7·5 and a diagram factor 0·63, determine the dimensions of the cylinders.

16. If the initial loads on the pistons are equal estimate the mean receiver pressures for the engine in Question 15.

Chapter XII

FLOW OF STEAM THROUGH ORIFICES AND NOZZLES

*GENERAL STATEMENT

When steam flows through an orifice from a zone of high pressure to a zone of low pressure a portion of its heat energy is converted into the kinetic form in the issuing stream, i.e. the steam performs work upon itself instead of upon the piston of an engine. If friction is negligible three steps are inherent in the process.

(1) Admission of steam into the high-pressure zone. The work done on the steam is p_1v_1, and a similar volume of steam is forced through the orifice to make room for the fresh charge.

(2) Expansion of the steam in the immediate neighbourhood of the orifice while the pressure changes from p_1 to p_2, the work done being

$$\frac{1}{n-1}(p_1v_1-p_2v_2)$$

(3) Displacement of steam from the low-pressure zone by an equal volume discharged from the orifice, the latter expending work to the amount p_2v_2.

The net work done in creating kinetic energy of the steam is therefore

$$p_1v_1+\frac{1}{n-1}(p_1v_1-p_2v_2)-p_2v_2=\frac{n}{n-1}(p_1v_1-p_2v_2)$$

which is exactly equivalent to that done during the Rankine cycle.

In the design of orifices and nozzles the principal factors to be considered are :

(1) The actual velocity of discharge ;
(2) The minimum area required to pass a given amount of steam per second ;
(3) The area of exit, if differing from the above ;
(4) The general shape of the nozzle.

EXAMPLE.—Dry saturated steam expands from an initial pressure of 200 lb. per sq. in. abs. down to a condenser pressure of 1 lb. per sq. in. abs. Find the maximum amount of work possible per pound of steam. With the same initial pressure, what would be the pressure after complete

expansion in order that the work done may be half as much as the previous case ?

$$\text{Work done} = \frac{n}{n-1} \cdot p_1 v_1 \left\{ 1 - \left(\frac{p_2}{p_1} \right)^{\frac{n-1}{n}} \right\}$$

From steam tables we find $v_1 = 2{\cdot}29$ cu. ft., and taking $n = 1{\cdot}135$

$$\begin{aligned} \text{work done} &= \frac{1{\cdot}135}{0{\cdot}135} \times 200 \times 144 \times 2{\cdot}29 \left\{ 1 - \left(\frac{1}{200} \right)^{\frac{0{\cdot}135}{1{\cdot}135}} \right\} \\ &= 554{,}600\{1 - 0{\cdot}5326\} \\ &= 554{,}600 \times 0{\cdot}4674 \\ &= 259{,}200 \text{ ft.-lb.} \end{aligned}$$

Let p be the pressure after expansion in the second case, then

$$\frac{1{\cdot}135}{0{\cdot}135} \times 200 \times 144 \times 2{\cdot}29 \left\{ 1 - \left(\frac{p}{p_1} \right)^{\frac{0{\cdot}135}{1{\cdot}135}} \right\} = \frac{259{,}200}{2}$$

$$554{,}600 \left\{ 1 - \left(\frac{p}{p_1} \right)^{\frac{0{\cdot}135}{1{\cdot}135}} \right\} = 129{,}600$$

$$1 - \left(\frac{p}{p_1} \right)^{\frac{0{\cdot}135}{1{\cdot}135}} = \frac{129{,}600}{554{,}600} = 0{\cdot}2336$$

$$\therefore \frac{p}{p_1} = (0{\cdot}7664)^{\frac{1{\cdot}135}{0{\cdot}135}} = 0{\cdot}1067$$

$$\therefore p = 200 \times 0{\cdot}1067$$

$$= 21{\cdot}34 \text{ lb. per sq. in. abs.}$$

The latter case is approximately that of a non-condensing engine discharging to the atmosphere, the former that of a steam turbine working in conjunction with a good condenser. The enormous gain due to the use of the low-pressure energy of the steam will be noted.

A reciprocating engine working over the full pressure range would have to be heavily compounded in order to prevent an excessive loss due to cyclic condensation and leakage.

*THE VELOCITY OF DISCHARGE

Velocity in terms of Pressure.—The gain of kinetic energy is equal to the work done, or, assuming the velocity of approach to be negligible,

$$\frac{v^2}{2g} = \frac{n}{n-1}(p_1 v_1 - p_2 v_2) = \frac{n}{n-1} p_1 v_1 \left\{ 1 - \left(\frac{p_2}{p_1} \right)^{n-1/n} \right\}$$

or

$$V = \sqrt{2g \frac{n}{n-1} p_1 v_1 \left\{ 1 - \left(\frac{p_2}{p_1} \right)^{n-1/n} \right\}} \quad \ldots \ldots \quad (1)$$

Velocity in terms of Heat Content.—As the work done is equal to the change of heat content

$$\frac{V^2}{2gJ}=h_1+q_1L_1-(h_2-q_2L_2)$$

But as the expansion is adiabatic

$$q_2=\frac{T_2}{L_2}\left(\frac{q_1L_1}{T_1}+\log_\epsilon\frac{T_1}{T_2}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2a)$$

or

$$\frac{V^2}{2gJ}=h_1-h_2+q_1L_1-T_2\left(\frac{q_1L_1}{T_1}+\log_\epsilon\frac{T_1}{T_2}\right)$$

If h_1-h_2 can be taken as equal to (T_1-T_2)

$$\frac{V^2}{2gJ}=(T_1-T_2)\left(1+\frac{q_1L_1}{T_1}\right)-T_2\log_\epsilon\frac{T_1}{T_2}$$

or

$$V=\sqrt{2gJ\left\{(T_1-T_2)\left(1+\frac{q_1L_1}{T_1}\right)-T_2\log_\epsilon\frac{T_1}{T_2}\right\}} \quad . \quad (2)$$

EXAMPLE.—Dry steam expands through a nozzle from a pressure of 200 lb. down to 140 lb. per sq. in. abs.

Assuming the flow to be frictionless and adiabatic, estimate the velocity of the steam jet.

(1) Equating the kinetic energy to the change of total heat

Kinetic energy $\frac{V^2}{2gJ}=(h_1+L_1)-(h_2+q_2L_2)$ and q_2 will be found to be 0·974

$$=(1198\cdot1)-(324\cdot6+0\cdot974\times867\cdot6)$$
$$=1198\cdot1-1168\cdot6$$
$$=29\cdot5 \text{ B.Th.U.}$$

therefore $V=\sqrt{64\cdot4\times778\times29\cdot5}$

$$=1215 \text{ ft. per second.}$$

(2) Equating the kinetic energy to the area of the $T\phi$ diagram

$$\frac{V^2}{2gJ}=(T_1-T_2)\left(1+\frac{L_1}{T_1}\right)-T_2\log_\epsilon\frac{T_1}{T_2}$$

or

$$=(842-813)\left(1+\frac{843}{842}\right)-813\log_\epsilon\frac{842}{813}$$
$$=29\times2-28\cdot45$$
$$=29\cdot55 \text{ B.Th.U.}$$

which agrees very closely with the value obtained above.

(3) Equating the kinetic energy to the area of the $p.v.$ diagram for the Rankine cycle,

$$V=\sqrt{64{\cdot}4\times\frac{1{\cdot}135}{1{\cdot}135-1}\cdot 200\times 144\times 2{\cdot}29\left\{1-\left(\frac{140}{200}\right)^{0{\cdot}135/1{\cdot}135}\right\}}$$

$$=\sqrt{64{\cdot}4\times\frac{1{\cdot}135}{0{\cdot}135}\times 28{,}800\times 2{\cdot}29(1-0{\cdot}960)}$$

$$=1207 \text{ ft. per second.}$$

(4) By means of the Mollier diagram the velocity is read off directly and found to be 1200 ft. per second.

It will, of course, be realised that the four methods employed above are essentially the same, and differ only in the form in which the central facts are presented.

*WEIGHT OF STEAM DISCHARGED. THE CRITICAL PRESSURE RATIO

If A is the minimum area of the steam-jet, i.e. the area of the orifice if this is suitably bell-mouthed on the high-pressure side to maintain contact with the steam, and v_1 and v_2 are the volumes occupied by the steam at pressures p_1 and p_2, then

$$W=\frac{AV}{v_2}$$

But
$$v_2=v_1\left(\frac{p_1}{p_2}\right)^{1/n}$$

or
$$W=\frac{A}{v_1\left(\frac{p_1}{p_2}\right)^{1/n}}\sqrt{2g\frac{n}{n-1}p_1v_1\left\{1-\left(\frac{p_2}{p_1}\right)^{n-1/n}\right\}}$$

$$=A\sqrt{2g\frac{n}{n-1}\cdot\frac{p_1}{v_1}\left\{\frac{p_2}{p_1}^{2/n}-\left(\frac{p_2}{p_1}\right)^{1+1/n}\right\}} \quad . \quad . \quad . \quad (3)$$

This expression will have a maximum value when $\left(\frac{p_2}{p_1}\right)^{2/n}-\left(\frac{p_2}{p_1}\right)^{1+1/n}$ is a maximum

or when
$$\frac{2}{n}\left(\frac{p_2}{p_1}\right)^{(2-n)/n}=\left(1+\frac{1}{n}\right)\left(\frac{p_2}{p_1}\right)^{1/n} \text{ by differentiation}$$

or when
$$\frac{2}{n}\left(\frac{p_2}{p_1}\right)^{(1-n)/n}=1+1/n$$

or
$$\frac{p_2}{p_1}=\left\{\frac{n+1}{n}\times\frac{n}{2}\right\}^{n(1-n)}$$

$$=\left\{\frac{2}{n+1}\right\}^{n(n-1)} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

The value of $\frac{p_2}{p_1}$ from equation (4) varies with the coefficient n in the equation pv^n=constant and has the following approximate values :

For superheated steam (also for supersaturated steam)

$$n=1{\cdot}30 \quad \text{and} \quad p_2=0{\cdot}546p_1$$

For steam initially dry

$$n=1{\cdot}135 \quad \text{and} \quad p_2=0{\cdot}578p_1$$

For very wet steam

$$n=1{\cdot}113 \quad \text{and} \quad p_2=0{\cdot}582p_1$$

For approximate calculations the critical ratio can be taken as 0·58 for steam initially dry, in which case $V_{max.}=5{\cdot}85\sqrt{p_1v_1}$ or $70{\cdot}2\sqrt{p_1v_1}$ if p_1 is measured in pounds per square foot and pounds per square inch respectively.

For air $n=1{\cdot}40$ and $p_2=0{\cdot}527p_1$.

No such critical pressure ratio is found in practice in the case of liquids, but its existence for highly expansible media can be explained in the

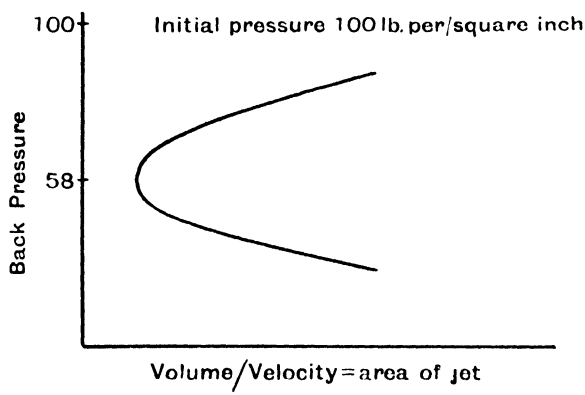


Fig. 97.

following manner. Assuming an initial pressure of, say, 100 lb. and various values of p_2, the values of the final volume occupied by the steam and also its velocity can be obtained. This is most easily done by the use of the Mollier chart, p. 125. The area of cross-section of a steam jet having these values of volume and velocity can be calculated by dividing the former by the latter. If values of p_2 and area are then plotted, as in Fig. 97, it will be seen that as p_2 decreases the velocity increases at first more rapidly than the volume, and the area of the jet will decrease. When p_2 is approximately 58 lb. a minimum area is required, but for lower values the volume will tend to increase more rapidly than the velocity and the steam will " choke " the orifice.

A more important explanation can be visualised as follows. If a sudden small impulse is given to the steam at the orifice a local pressure wave is initiated, which travels from the orifice with the velocity of sound in the low-pressure steam. No other speed of travel is possible for this wave. So long as p_2 is greater than $0{\cdot}58p_1$, the jet of steam follows the pressure wave at a lower velocity, and is therefore always discharging into a zone behind the wave where the pressure is p_2. At the critical pressure ratio the velocity of the steam reaches that of sound and the jet travels with the pressure wave; any attempt to increase the jet velocity results merely in increasing the intensity of the pressure wave, which now acts, as far as the steam jet is concerned, as a *relatively* immovable obstruction barring more rapid progress.

It may be easier to think of a wave of low pressure p_2 travelling towards the orifice along the jet which is moving away from it. When the jet moves away from the orifice with the velocity of sound the low-pressure wave becomes stationary in space and cannot proceed towards the orifice to relieve the local pressure there.

A zone of high pressure is therefore built up at the orifice after the acoustic velocity is reached, and the pressure drop across the orifice itself becomes $p_1-0{\cdot}58p_1$ approximately, whatever the value of p_2 may be. As the pressure wave travels away from the orifice in all directions the steam following in its wake will endeavour to do likewise, flow will become disorderly instead of having a single velocity component axial with the nozzle, and the energy will be dispersed.

If this dispersal is prevented, by confining the steam between walls which diverge only sufficiently to allow for the rapidly increasing volume, the column of steam so confined at any instant will be subjected to pressures of $0{\cdot}58p_1$ and p_2 at its two ends and will be driven forward as an expanding projectile at a speed dependent only on the values of p_1 and p_2. If the nozzle is suitably designed the jet will emerge at a pressure p_2 and with a velocity which, except for slight divergence, will be axial with the nozzle and will be greater than the velocity of sound.

Orifices or nozzles for vapours and gases can therefore be divided into two categories: (1) convergent, for use when $p_2 >$ about $0{\cdot}58p_1$; (2) convergent-divergent, for use when $p_2 <$ about $0{\cdot}58p_1$. In both cases the convergent portions are usually similar, the edge of the orifice being rounded so that the steam flows easily to the smallest section without sudden changes of direction or the formation of eddies. The length of the approach side of the orifice should be short, to reduce surface friction, and normally about $\frac{1}{8}$ in. will be found adequate. It should be remembered that convergence of the walls of a passage tends to stabilise flow so that changes of area may then be rapid, but that divergence has the opposite effect, necessitating gradual increases.

*CALCULATIONS OF THROAT AND EXIT AREAS, ASSUMING NO FRICTION

When the initial and final pressures and the initial quality of steam are known the nozzle areas required are readily calculated for the discharge of 1 lb., and may subsequently be modified for other quantities of steam.

(1) *When the Final Pressure=or>the Critical.*—From equation (2), p. 213, the velocity of discharge can be calculated, and the final volume

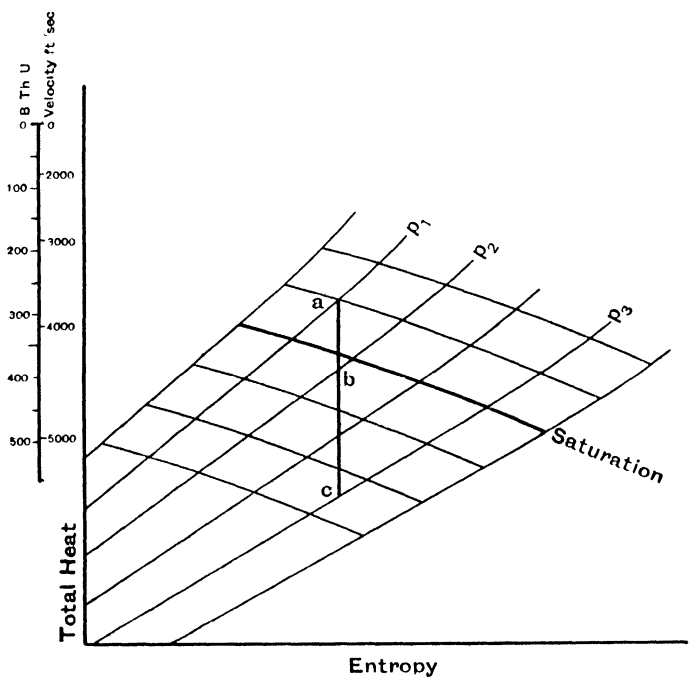


Fig. 98.

of the steam can be obtained from the steam tables and using equation (2a), or the equivalent equation for use with steam which remains superheated. The area of the throat for a discharge of 1 lb. of steam per second is then given by $\frac{\text{final volume}}{\text{final velocity}}$, the areas being in square feet.

The use of a large scale Mollier chart greatly facilitates the work as follows.

The initial condition point is found at *a*, and a line of constant entropy is drawn from *a* to cut the line of pressure p_2 in *b*.

$H_a - H_b$ now gives the heat which is converted into kinetic energy. If the Marks and Davis diagram is used the velocity equivalent to $H_1 - H_2$ can be read from the conversion scale which is attached, otherwise it may be obtained from the equation

$$\frac{V^2}{2g} = J(H_a - H_b)$$

or

$$V = \sqrt{2gJ(H_a - H_b)}$$

The position of b determines the final quality of the steam. The final volume may be obtained either from the steam tables, or read direct from the chart if constant-volume lines have been traced (see Callendar's $T\phi$ chart).

The volume divided by the velocity will now give the required area in square feet.

(2) *When the Final Pressure is less than the Critical.*—In this case two areas will be required, the first being the area at the throat where the pressure has the critical value, the second being the final exit area. For the first of these the point b is chosen so that p_2/p_1 has the critical value, and the adiabatic is then extended to the final pressure line at c. The final velocity is then derived from the heat drop $H_a - H_c$ and the final volume from the known quality at c. The exit area of the nozzle will be given by the quotient $\frac{\text{exit volume}}{\text{exit velocity}}$. The throat area is obtained, as before, from values at a and b.

The areas so calculated are for a discharge of 1 lb. of steam per second, and should be modified proportionately if a quantity other than this is to be employed.

If a simple nozzle is to be used it is customary to round the entrance edge leading to the throat for a length of about $\frac{1}{8}$ in., and to make the remainder of the nozzle of such a length that the included angle is about 12°. (See Fig. 99; see also p. 221).

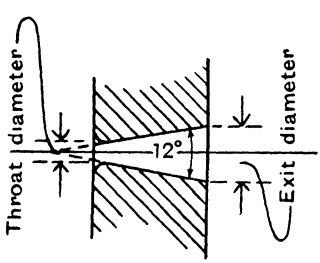


Fig. 99.

(In estimating the velocities of the jet by means of the conversion scale shown in Fig. 98, care should be taken to obtain the final velocity from $H_a - H_c$. If the final velocity is taken as the velocity equivalent to $H_a - H_b$ + the velocity equivalent of $H_b - H_c$, the contracted portion of the velocity scale will be used twice, and serious errors will result.)

EXAMPLE.—Dry steam at a pressure of 200 lb. per sq. in. is to expand to 5 lb. per sq. in. abs. Determine the principal dimensions of the nozzle if the discharge is to be 60 lb. per minute.

Here $n=1{\cdot}135$, and at the throat

$$\frac{p_2}{p_1}=\left\{\frac{2}{n+1}\right\}^{n/n-1}$$

At the Throat.—

From equation (3)

$$W=A_t\sqrt{2g\frac{n}{n-1}\frac{p_1}{v_1}\left\{0{\cdot}577^{\frac{2}{1{\cdot}135}}-0{\cdot}577^{\frac{2{\cdot}135}{1{\cdot}135}}\right\}}$$

$$=A_t\sqrt{2g\frac{n}{n-1}\frac{p_1}{v_1}\{0{\cdot}0235\}}$$

$$=A_t\sqrt{543\frac{p_1}{v_1}\{0{\cdot}0235\}}$$

$$=3{\cdot}60A_t\sqrt{\frac{p_1}{v_1}}, \text{ where } p_1 \text{ is in lb. per sq. ft.} \quad . \quad (1)$$

$$=43{\cdot}2A_t\sqrt{\frac{P_1}{v_1}}, \text{ where } P_1 \text{ is in lb. per sq. in.}$$

At Exit.—

$$W=A_e\sqrt{64{\cdot}4\times\frac{1{\cdot}135}{0{\cdot}135}\frac{p_1}{v_1}\left\{\left(\frac{p_3}{p_1}\right)^{2/n}-\left(\frac{p_3}{p_1}\right)^{1+1/n}\right\}}$$

or

$$\frac{A_e}{A_t}=\frac{3{\cdot}60}{\sqrt{64{\cdot}4\times\frac{1{\cdot}135}{0{\cdot}135}\times\left\{\left(\frac{p_3}{p_1}\right)^{2/n}-\left(\frac{p_3}{p_1}\right)^{1+1/n}\right\}}}$$

$$=\frac{0{\cdot}155}{\sqrt{\left(\frac{p_3}{p_1}\right)^{2/n}-\left(\frac{p_3}{p_1}\right)^{1+1/n}}}$$

$$\frac{A_e}{A_t}=\frac{0{\cdot}1155}{\sqrt{\left(\frac{5}{200}\right)^{2/1{\cdot}135}-\left(\frac{5}{200}\right)^{2{\cdot}135/1{\cdot}135}}}$$

$$=6{\cdot}81$$

or $A_t=2{\cdot}417$ sq. in.

Alternative Solution.—From the Mollier diagram the heat drop is found to be 253 B.Th.U., the velocity at discharge to be 3560 ft. per second, and the dryness fraction 0·815. From steam tables the volume of 1 lb. of dry steam at 5 lb. abs. is 73·33 cu. ft., hence the volume per pound of steam as discharged is (neglecting the volume of the water)—

$$0{\cdot}815\times 73{\cdot}33 \text{ cu. ft.}$$

hence $3560A_2 = 0{\cdot}815 \times 73{\cdot}33$

$$A_2 = \frac{0{\cdot}815 \times 73{\cdot}33}{3560}$$

$= 0{\cdot}01678$ sq. ft. or 2·416 sq. in.

FRICTION

The condition and area of the nozzle surface, the velocity and quality of the steam, etc., all contribute to a loss of kinetic energy which may be covered, for convenience, by the general term " frictional loss." For very short convergent nozzles the loss may in some cases be negligible, but for divergent nozzles of appreciable length the losses will vary from 4 to 15 per cent., i.e. the efficiency will be between 96 and 85 per cent. These losses of kinetic energy will cause the steam at exit to have a higher dryness fraction—or degree of superheat—than would otherwise be the case, the increased heat content being known as the " reheat."

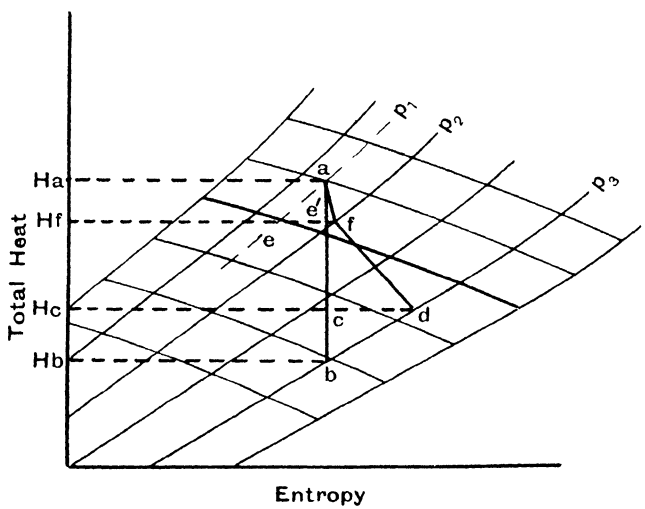


Fig. 100.

The design of a nozzle will obviously be affected by its probable efficiency, and the calculations outlined in the last section will require to be modified as follows :

In Fig. 100 *ab* represents the adiabatic expansion of steam from the condition point *a* on the pressure line p_1, to the pressure p_3. If *ac* is made such that *ac*/*ab*=the nozzle efficiency, then *cb* will represent the reheat, and $H_a - H_c$ the heat equivalent of the kinetic energy actually generated.

Now the final condition point must also lie on the line p_3, i.e. at *d*, the

point of intersection of the lines H_c and p_3, and the final area must be calculated from the volume corresponding to the condition at *d* and the velocity equivalent to $H_a - H_c$.

If $p_3 < 0{\cdot}58p_1$ the adiabatic throat drop would be given by *ae*, where *e* is on *ab* and also on the line $p_2 = 0{\cdot}58p_1$. If the efficiency η_t of the converging portion of the nozzle is known, then if $\frac{ae'}{ae} = \eta_t$, and the condition of the steam at the throat will be given by the point *f* on the pressure line p_2 and for which $H_f = H_e'$. The condition curve for the steam will then be *afd*.

The following approximate values of the friction loss may serve as a guide :

Loss to throat	2–3 per cent.
,, in divergent portion of a nozzle 2 in. long,	4–5 ,,
,, ,, ,, ,, 4 in. ,,	6–8 ,,
,, ,, ,, ,, 6 in. ,,	10–15 ,,

That the friction loss is largely a surface effect can be seen if dry saturated steam is expanded in a glass nozzle. The steam near the centre of the jet will become cloudy owing to the presence of tiny droplets formed as the steam expands, while the steam near the sides will remain clear, the reheat preventing condensation.

The length of the nozzle is reached as a compromise between a long nozzle with very slow divergence, giving an almost cylindrical jet at the expense of heavy frictional losses, and a short one with a rapid divergence and low friction loss but with a markedly divergent jet in which the outermost streams are not at a favourable angle for efficient utilisation in turbine blading. A very rapid divergence also tends to make the jet leave the sides of the nozzle and set up return-flow currents along the walls greatly increasing the total losses. Though inclusive angles of 20° are occasionally used, 8° to 12° are more normal.

EXAMPLE.—An impulse turbine of the de Laval type is to develop 250 H.P. with a probable consumption of 15·5 lb. of steam per H.P. hour, the initial pressure being 180 lb. and the exhaust 2 lb. per sq. in. abs. Taking the diameter at the throat of each nozzle as $\frac{1}{4}$ inch, find the number of nozzles required. Assuming that 12 per cent. of the heat drop is lost in the diverging part of the nozzle, find the diameter at the exit of the nozzle and the quality of the steam, which is to be fully expanded as it leaves the nozzle.

The weight discharged per second from each nozzle will be

$$W = 43{\cdot}2A\sqrt{\frac{P_1}{v_1}}$$

$$W = \frac{43{\cdot}2}{144} \times \frac{\pi}{4} \times \left(\frac{1}{4}\right)^2 \sqrt{\frac{180}{2{\cdot}53}} = 0{\cdot}1213 \text{ lb.}$$

$$\text{Total steam required per second} = \frac{250 \times 15{\cdot}5}{60 \times 60} = 1{\cdot}076 \text{ lb.}$$

$$\text{Number of nozzles} = \frac{1{\cdot}076}{0{\cdot}1213} = 8{\cdot}8, \text{ say 9 nozzles}$$

If the expansion were frictionless and adiabatic from 180 lb. to 2 lb. abs., the kinetic energy of the jet at exit from the nozzle would be

$$\frac{V^2}{2g} = \frac{n}{n-1} \cdot p_1 v_1 \left\{ 1 - \left(\frac{p_2}{p_1} \right)^{\frac{n-1}{n}} \right\}$$

Since 12 per cent. of this heat drop is unconverted, the kinetic energy will be 88 per cent. of the above, namely—

$$\frac{V_2}{2g} = 0{\cdot}88 \times \frac{1{\cdot}13}{0{\cdot}13} \times 144 \times 180 \times 2{\cdot}53 \left\{ 1 - \left(\frac{2}{180} \right)^{\frac{0{\cdot}13}{1{\cdot}13}} \right\}$$

$$= 202{,}700 \text{ ft.-lb., or } 260 \text{ B.Th.U.}$$

and the velocity

$$V = \sqrt{64{\cdot}4 \times 202{,}700} = 3610 \text{ ft. per second}$$

From steam tables we find the total heat of dry saturated steam at 180 lb. abs. is 1196·4 B.Th.U.

Hence the total heat per pound after expansion to 2 lb. abs. is

$$h_2 + q_2 L_2 = 1196{\cdot}4 - 260 = 936{\cdot}4$$

Substituting for h_2 and L_2 from steam tables

$$94 + q_2 \times 1021 = 936{\cdot}4$$

$$q_2 = 0{\cdot}825$$

$$\text{Volume per pound} = 0{\cdot}825 \times 173{\cdot}5 \text{ cu. ft.}$$

$$\therefore \text{ area of nozzle at exit} = \frac{0{\cdot}1213 \times 0{\cdot}825 \times 173{\cdot}5}{3610}$$

$$= 0{\cdot}00481 \text{ sq. ft.}$$

$$= 0{\cdot}693 \text{ sq. in.}$$

$$\therefore \text{ diameter of nozzle at exit} = \sqrt{\frac{0{\cdot}693}{0{\cdot}7854}} = 0{\cdot}939 \text{ in.}$$

Alternative Method.—By means of the Mollier chart the following figures may be obtained, referring to 1 lb. of steam (see also Fig. 100).

$H_a = 1203{\cdot}2$ B.Th.U.

$H_e = 1157{\cdot}0$ B.Th.U., where $p_e = 0{\cdot}58 p_1 = 104$ lb. per sq. in., and the corresponding velocity is 1500 ft. per sec.

$q_e = 96{\cdot}1$ per cent.

$v_e = 4{\cdot}20$ cu. ft.

$H_b = 906{\cdot}0$ B.Th.U.

H_c=941·6 B.Th.U., where $H_a-H_c=0{\cdot}88(H_a-H_b)$, and the corresponding velocity is 3612 ft. per sec.

q_d=83·1 per cent.

v_d=143 cu. ft.

The area per pound per second at throat $=\frac{4{\cdot}20}{1500}\times 144=0{\cdot}403$ sq. in.

" " " exit $=\frac{143}{3612}\times 144=5{\cdot}71$ sq. in.

As the area of one nozzle is 0·0491 sq. in. at the throat, the number of nozzles is

$$\frac{0{\cdot}403\times 1{\cdot}076}{0{\cdot}0491}=8{\cdot}8, \text{ say } 9.$$

The exit area per nozzle $=\frac{5{\cdot}71\times 1{\cdot}076}{9}=0{\cdot}884$ sq. in.

and exit diameter=0·942 in.

For other frictional coefficients or distributions the necessary modifications will be obvious.

NOZZLE SHAPE AND SIZE

A number of additional matters have to be taken into consideration in designing the nozzles for steam turbines. The straight conical nozzle has the advantage that it may be produced with a high degree of accuracy and finish by a simple rimering process. A jet of rectangular section would fill more satisfactorily the passages between the rotor blades, but accuracy and finish are not easy to attain and the surface area, which is largely responsible for friction, is large. With such nozzles it is customary to keep the radial width constant (see Fig. 101), the full divergence being allowed on the other transverse dimension. There is therefore no tendency of the issuing jet to diverge beyond the top and bottom of the rotor blades.

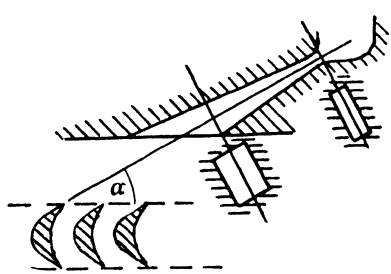


Fig. 101.

The maximum diameter or radial height of the nozzles is determined by the height of the blades. At the high-pressure end of the turbine this is of the order of 1·5 to 2 per cent. of the mean blade-ring diameter, but increases at the low-pressure end to 8 to 12 per cent., and occasionally up to 25 per cent. This factor determines whether the requisite nozzle area should be provided by a few nozzles of large diameter, or a large number with smaller dimensions.

The angle of inclination of the nozzle to the plane of rotation of the turbine blades should be kept as small as possible, but it will usually be found that an angle of less than 20° is not practicable, though occasionally values of about 12° are found at the high-pressure stage.

Owing to this obliquity, and the desirability of guiding the steam as far as possible towards the blades, the nozzle is not cut off at right angles to its axis, but is carried forward on the trailing edge. If this is done by extending the cone, Fig. 101, the steam will be over-expanded on that side. On the other hand, if a cylindrical extension is employed compression shock will occur at the point A, Fig. 102, where the outer stream-

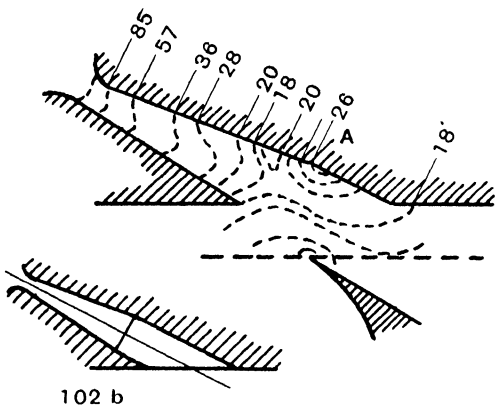


Fig. 102.

lines impinge on the extended wall. This figure shows the lines of constant pressure for the nozzle and the tip of one blade. It will be seen that the high pressure located at A will tend to cause deflection of the jet away from that point, and the angle made by the issuing jet with the plane of rotation of the blades will be greater than *a*, Fig. 101. Angles of deviation as high as 9° and 51° on the two sides of the jet have been measured by Stodola. A cylindrical extension, as shown by *b*, Fig. 102, is used in the de Laval turbine.

*OVER AND UNDER EXPANSION

It will be seen from preceding and following sections that many uncertainties exist as to correctness of the calculated areas. Both too large and too small an area at exit will cause a reduction of the final velocity, but it has been shown that the losses caused by under-expansion are markedly less than those for over-expansion, and, while the latter are entirely irrecoverable, the former may, at least in part, be made good in the blading. It is probable that a 10 per cent. under-expansion causes a loss of velocity of about 0·3 per cent. only, and this reduction of area is frequently allowed.

SUPERSATURATION

It would normally be expected that the actual discharge from a nozzle would be slightly less than that given by theory, but careful experiments show that the converse is often true, the actual discharge through a simple orifice of steam which is initially dry being from 2 to 5 per cent. in excess of the theoretical amount. An examination of the conditions affecting the quality of the steam is therefore desirable.

It has so far been assumed that equilibrium between the liquid and vapour phases of steam is established instantaneously when conditions given by the saturation curve are reached. At least three conditions are involved.

(1) Condensation is normally started round any tiny nuclei which may be present, usually tiny dust particles which are always present in commercial steam in sufficient quantities. In the absence of such particles condensation is considerably delayed, and the temperature of the steam continues to fall, the condition being then known as supersaturated. When a certain degree of supersaturation has been reached it appears that the presence of foreign particles is no longer necessary, and that equilibrium can then be attained completely and virtually instantaneously. As much as 70–90° F. supercooling may occur, though it is extremely doubtful whether such an intense degree of supercooling is ever attained in practice due to shortage of nuclei. The lowest probable limit of supersaturation is given by a line running nearly parallel to the saturation curve on the Hϕ chart (see Fig. 55) and known as the Wilson line.

(2) A certain time interval is necessary in which the molecules may collect to form droplets. The passage of steam through a short convergent nozzle is of the order of one ten-thousandth of a second, which may be quite inadequate.

(3) Though steam and a flat surface of water are in equilibrium at the same temperature this is not the case when the water forms a drop. If O, Fig. 103, is a molecule forming part of the water surface *aa*, it will be under the influence of a surface-tension effect due to the fact that all the molecular forces acting upon it do so from below. If *r* is the maximum radius of effective molecular attraction then all the molecules influencing O will lie within the hemisphere *bbb* drawn from O with *r* as radius. If O is part of a drop, however, the molecules must lie also within the surface *coc* of the drop, and the molecules acting upon O will be reduced in number as the space represented by the shaded area in Fig. 103 is no longer a portion of the liquid. This effect becomes more marked as the drops decrease in size, and is considerable at the start of condensation when the droplets are of the order of 1/10,000 inch in diameter.

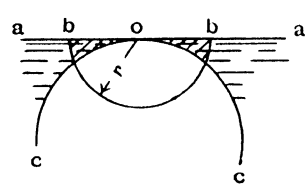


Fig. 103.

(As a drop is reduced in size by evaporation, and finally is completely evaporated, the potential energy represented by the surface tension must be dissipated, and will be distributed among the escaping steam molecules which will therefore have imparted to them a velocity greater than that represented by their initial temperature. The temperature of evaporation of a minute drop, and conversely its temperature of formation, will therefore be less than might at first be expected.)

The relative importance of the above three factors is in doubt, and also the amount of supersaturation which is likely to occur in any particular case. It is probable that the highly turbulent motion set up at the throat of a convergent-divergent nozzle greatly accelerates the attainment of equilibrium and that supersaturation is not likely to be found in the divergent portion. If, however, the condition point at the throat falls between the saturation curve and the Wilson line the steam should be treated as a supercooled vapour. Its general behaviour will be akin to that of superheated steam and its volume should be calculated from the equation $pv^{1\cdot3}=$constant. When necessary the total heat may be obtained from the equation

$$H_3=\frac{p(v-0\cdot0123)}{1\cdot2464}+835\cdot2 \quad \text{(Goudie, p. 227)}$$

The lines of constant pressure on the Mollier diagram can now be extended from the superheat region across the saturation line as far as the Wilson curve. Below the Wilson curve the diagram will remain unchanged and at the curve the lines of constant pressure will be broken. A diagram including lines of supersaturation is supplied with Goudie's " Steam Turbines."

Immediately after equilibrium has been reached the relation between pressure and volume will be given by $pv^{1\cdot13}=$constant.

CONVERGENT-DIVERGENT NOZZLE WITH EXCESSIVE BACK-PRESSURE

If a convergent-divergent nozzle is designed for a given back-pressure p_2, the pressure distribution along the nozzle will be as indicated by the smooth line in Fig. 104.

If a higher back-pressure is used the discharge from the nozzle will decrease, but will not be proportionate to the back-pressure. The action of the nozzle will now approach that of the Venturi meter, the pressure at the throat tending to be less than that at exit, and as the pressure drop between the nozzle entrance and the throat governs the rate of discharge this will remain larger than might be expected. Pressure curves for such conditions are as shown by the curves of Fig. 104, and will be distorted by the presence of stationary waves as soon as the acoustic velocity is exceeded.

Under such circumstances the jet tends to leave the walls after the throat, and eddies are set up which greatly increase the friction losses and cause considerable reheating of the steam.

In large steam accumulators such nozzles—or diffusers—may be

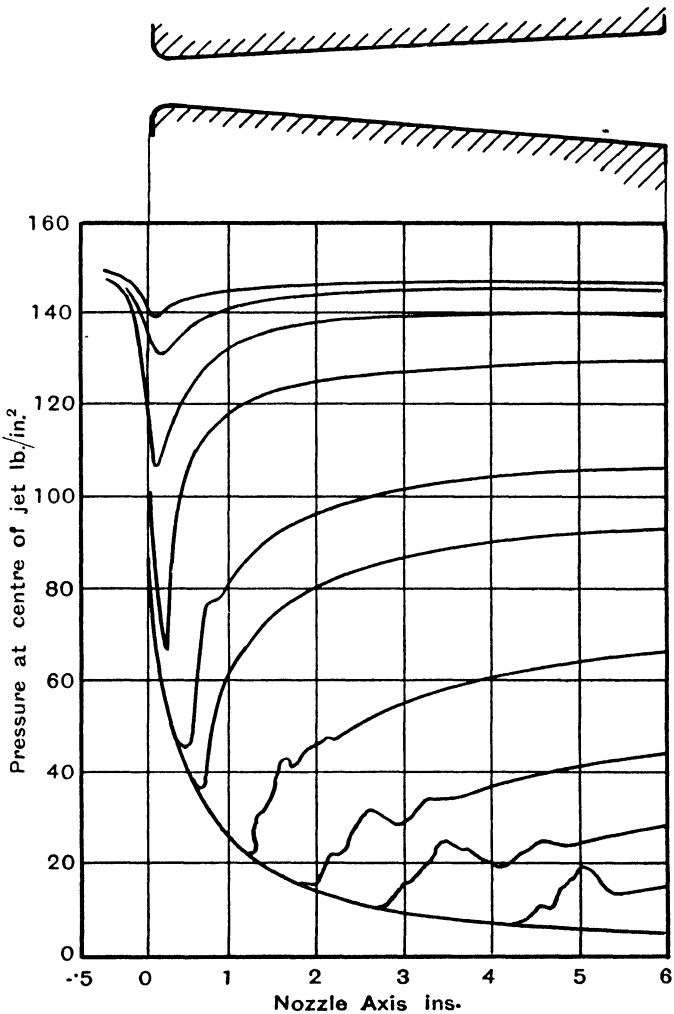


Fig. 104.

incorporated in the pipe-lines by which the steam is withdrawn. Normally the total pressure drop over the nozzle will be very small, as shown by the upper curve of Fig. 104, the divergence allowing for a considerable regain of pressure energy after the throat. Should the pipe-line on the

discharge side of the nozzle be broken the back-pressure will at once fall to atmospheric pressure—p_2—and the rate of discharge of the accumulator will be automatically limited to the maximum discharge at the throat. A simple orifice would restrict the maximum flow in a similar manner, but as there could be no regain of pressure energy the total drop of pressure during normal working would be excessive.

THEORY OF THE INJECTOR

The action of an injector will be discussed with reference to Fig. 105. The steam used for working the injector expands through a conical

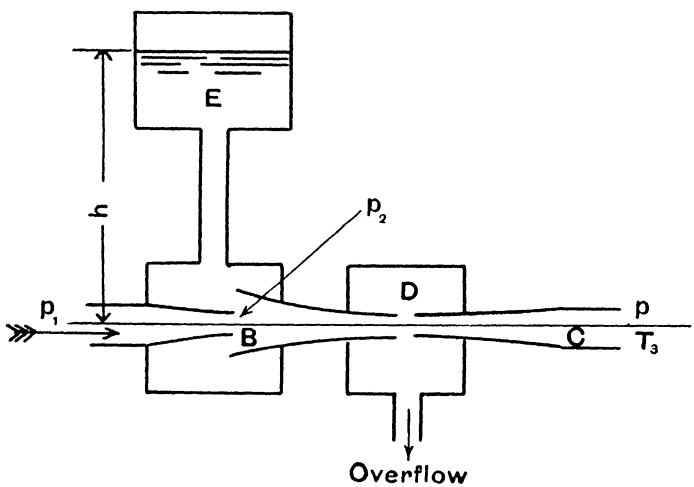


Fig. 105.

nozzle, issuing therefrom with a high velocity, and, coming into contact with cold water flowing in from the feed tank E, is condensed in the convergent combining tube or cone B. The resulting jet of water enters the divergent delivery tube or cone C, and at its smallest cross-section is moving with its maximum velocity. The kinetic energy of the jet of water is then converted into pressure energy in its passage along the delivery tube, its pressure increasing as its velocity decreases, until on leaving the tube the pressure is greater than the boiler pressure and the water enters the boiler. An outlet is provided at D through which any excess of water may overflow when starting.

Let V be the velocity of the steam jet in feet per second, p_1 the initial steam pressure, and p_2 the pressure in the jet just outside the steam nozzle where contact occurs between the steam and the entering water. Then

V may be calculated by any of the methods given above. If the condition for maximum discharge can be assumed then

$$V=70{\cdot}2\sqrt{P_1 v_1}$$

where P_1 is in pounds per square inch.

Weight of Water per Pound of Steam.—Let W be the number of pounds of water drawn from the feed tank per pound of steam, h the head of water in feet on the injector, p the boiler pressure in pounds per square foot, and H the height, in feet, of the boiler feed check valve above the delivery cone of the injector. Then neglecting losses the least velocity of the jet V_j entering the delivery cone will be given by

$$\frac{V_j^2}{2g}=\frac{p}{62{\cdot}4}+H \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

$\frac{p}{62{\cdot}4}$ being the height in feet of a water column equivalent to the boiler pressure (as 1 cu. ft. of water weighs 62·4 lb.).

In actual practice p in (1) should be taken about 20 per cent. greater than the absolute boiler pressure to ensure that the injector works properly.

The velocity with which the water will flow into the injector under the head of h feet will be

$$\sqrt{2gh}$$

the momentum of W lb. of this water will be

$$\frac{W}{g}\sqrt{2gh}$$

the momentum of 1 lb. of steam moving with velocity V will be

$$\frac{V}{g}$$

and the momentum of the resulting jet

$$\frac{(W+1)V_j}{g}$$

Hence, equating the momenta before and after combination we have

$$\frac{(W+1)V_j}{g}=\frac{V}{g}+\frac{W}{g}\sqrt{2gh}$$

or

$$V_j=\frac{V}{W+1}+\frac{W}{W+1}\cdot\sqrt{2gh} \quad . \quad . \quad . \quad (2)$$

If the water is not supplied under pressure to the injector but the feed

tank is h feet *below* the injector, as is the case of injectors of the lifting type, equation (2) becomes

$$V_j=\frac{V}{W+1}-\frac{W}{W+1}\sqrt{2gh} \quad . \quad . \quad . \quad . \quad (2A)$$

In most cases the term $\frac{W}{W+1}\sqrt{2gh}$ is so small that it may be neglected.

By substituting in (2) the value of V already found, the value of W may be estimated.

Estimation of the Feed Temperature.—Let t be the temperature of the water in the feed tank and t_3 the temperature of the delivery from the injector, i.e. the feed temperature to the boiler. Then per pound of steam used

$$\text{Kinetic energy of the jet}=\frac{(W+1)}{2g}\cdot\frac{V_j^2}{J} \text{ in heat units.}$$

The heat gained by W lb. of water $=W(t_3-t)$

and the heat lost by 1 lb. of steam $=q_1L_1+(t_1-t_3)$

equating the heat lost by the steam to the heat gained by the water, we get

$$q_1L_1+(t_1-t_3)=W(t_3-t)+\frac{(W+1)}{2g}\cdot\frac{V_j^2}{J} \quad . \quad . \quad . \quad (3)$$

from which t_3 may be estimated.

The kinetic energy of the jet is usually so small in comparison with the other items in (3) that for practical purposes it may be neglected.

Area of Steam Nozzle.—The dryness fraction, q_2, of the steam at pressure p_2 (or $0{\cdot}58p_1$) is found from a temperature-entropy diagram, or by calculation from

$$q_2=\frac{T_2}{L_2}\left(\frac{q_1L_1}{T_1}+\log_\epsilon\frac{T_1}{T_2}\right)$$

Let w lb. be the weight of steam used per second and v_2 the specific volume at pressure p_2, then, neglecting the volume of the water it contains, its volume will be

$$w\times q_2v_2$$

$$\text{and the area of the steam nozzle}=\frac{w\times q_2v_2}{V} \quad . \quad . \quad . \quad (4)$$

Area of Water-discharge Orifice.—The quantity of water drawn from the feed tank per second will be $w\times W$ lb., or

$$\frac{w\,.\,W}{62{\cdot}4} \text{ cu. ft.}$$

hence,

$$\text{area of the discharge end of the combining nozzle}=\frac{w\,.\,W+w}{62{\cdot}4V} \quad . \quad (5)$$

EXAMPLE.—Calculate the area of the orifices of a live steam injector to take 1000 gallons of water per hour from a feed tank to a boiler, the absolute steam pressure being 165 lb. per sq. in. The steam supplied to the injector may be assumed dry, the pressure at the throat of the steam orifice 0·6 of the boiler pressure, and the temperature of the water in the feed or suction tank 60° F.

The pressure in the steam jet $=0{\cdot}6\times165=99$ lb. abs.

From steam tables we find—

p	t	v	L	H	T
165	366	2·753	856·8	1195·0	826
99	327	4·47	886·6	1186·2	787

From

$$V=\sqrt{64{\cdot}4\times778\left\{(826-787)\left(1+\frac{856{\cdot}8}{826}\right)-787\log_\epsilon\frac{826}{787}\right\}}$$

$=1420$ ft. per second equation (2), p. 229

The dryness fraction $q_2=\dfrac{787}{886{\cdot}6}\left(\dfrac{856{\cdot}8}{826}+\log_\epsilon\dfrac{826}{787}\right)$

$=0{\cdot}963$ equation (2a), p. 230

(*Note :* Both V and q_2 could be obtained directly from a Mollier Diagram.)

From (1), p. 229, we have, neglecting H

$$\frac{V_j^2}{64{\cdot}4}=\frac{1{\cdot}2\times165\times144}{62{\cdot}4}$$

from which $V_j=171$ ft. per second

Neglecting the second term on the right-hand side of (2), i.e. if h is small

$$171=\frac{1420}{W+1}$$

$$W=\frac{1420-171}{171}=7{\cdot}30 \text{ lb.}$$

The feed water drawn from the feed tank per hour is 10,000 lb. $=\dfrac{10{,}000}{3600}$ lb. per second, hence the weight of steam used per second is

$$\frac{10{,}000}{3600\times7{\cdot}30} \text{ lb.}$$

and by (4), p. 230, the area of the steam nozzle will be

$$\frac{10{,}000}{3600\times 7\cdot 30}\times\frac{0\cdot 963\times 4\cdot 47}{1420}$$
$$=0\cdot 00115 \text{ sq. ft.}$$
$$=0\cdot 00115\times 144=0\cdot 166 \text{ sq. in.}$$

The discharge from the injector per hour will be

$$10{,}000+\frac{10{,}000}{7\cdot 3}$$
$$=11{,}700 \text{ lb.}$$
$$=\frac{11{,}700}{3600} \text{ lb. per second}$$

and by (5), p. 230, the area of the water discharge orifice will be

$$\frac{11{,}700}{3600\times 62\cdot 4\times 171}$$
$$=0\cdot 000304 \text{ sq. ft.}$$
$$=0\cdot 0438 \text{ sq. in.}$$

The feed temperature will be

$$856\cdot 8+(366-t_3)=7\cdot 30(t_3-60)+\frac{8\cdot 30}{64\cdot 4}\cdot\frac{171\times 171}{778}, \text{ or } t_3=199^\circ \text{ F.}$$

$$\frac{\text{Volume of steam removed from boiler}}{\text{Volume of water replaced}}=\frac{110}{1} \text{ approx.}$$

Example.—Calculate the diameter of the orifices for an injector to deliver 1200 gallons of water per hour into a boiler containing steam at 60 lb. per sq. in. abs. The steam supplied to the injector may be assumed dry, the pressure in the steam orifice 0·6 of the absolute boiler pressure, the temperature of water in the suction tank 100° F., and the temperature of the feed water 180° F.

Taking the necessary data from steam tables we have

$$V=\sqrt{64\cdot 4\times 778\left\{(753-721)\left(1+\frac{914\cdot 9}{753}\right)-721\log_\epsilon\frac{753}{721}\right\}}$$
$$=1400 \text{ ft. per second}$$

The dryness fraction $q_2=\frac{721}{937\cdot 7}\left(\frac{914\cdot 9}{753}+\log_\epsilon\frac{753}{721}\right)$
$$=0\cdot 970$$

The feed temperature is here given as 180° F., the weight of water per pound of steam may therefore be estimated as follows:—

From (3), and neglecting the kinetic energy of the issuing jet,

$$914\cdot 9+293-180=W(180-100)$$
$$W=12\cdot 84 \text{ lb.}$$

The velocity of the jet, neglecting the second term on the right-hand side of (2), is

$$V_j = \frac{1400}{13{\cdot}84} = 101 \text{ ft. per second}$$

If the weight of water drawn from the suction feed tank is assumed to be 12,000 lb. per hour, the weight of steam used per second will be

$$\frac{12{,}000}{12{\cdot}84 \times 3600} \text{ lb.}$$

and by (4) the area of the steam orifice will be

$$\frac{12{,}000}{12{\cdot}84 \times 3600} \times \frac{0{\cdot}970 \times 11{\cdot}58}{1400} \times 144$$

$$= 0{\cdot}295 \text{ sq. in.}$$

$$\text{hence the diameter} = \sqrt{\frac{0{\cdot}295}{0{\cdot}7854}} = 0{\cdot}613 \text{ in.}$$

The discharge from the injector per hour will be

$$12{,}000 + \frac{12{,}000}{12{\cdot}84}$$

$$= 12{,}934 \text{ lb.}$$

$$= \frac{12{,}934}{3600} \text{ lb. per second}$$

and by (5) the area of the water discharge orifice will be

$$\frac{12{,}934}{3600 \times 62{\cdot}4 \times 101} \times 144$$

$$= 0{\cdot}0821 \text{ sq. in.}$$

$$\text{hence the diameter} = \sqrt{\frac{0{\cdot}0821}{0{\cdot}7854}} = 0{\cdot}323 \text{ in.}$$

The above design is based upon the feed water temperature, and a check upon the pressure of the feed water is necessary. The pressure of the water at the feed check valve will be from (1),

$$\frac{101 \times 101}{64{\cdot}4} = \frac{p \times 144}{62{\cdot}4}$$

$$p = \frac{101 \times 101 \times 62{\cdot}4}{144 \times 64{\cdot}4} = 68{\cdot}6 \text{ lb. per sq. in. abs.}$$

Since the absolute boiler pressure is 60 lb. per sq. in. it is evident that the injector will work against this pressure.

EXAMPLES ON CHAPTER XII

1. Boiler steam of dryness fraction 0·98 and pressure 150 lb. per sq. in. abs. expands through a nozzle down to a pressure of 100 lb. abs. Assuming the flow to be frictionless and adiabatic, estimate the velocity and dryness fraction of the steam jet.

2. Dry steam at a pressure of 180 lb. per sq. in. abs. expands through a properly designed nozzle down to a pressure of 3 lb. abs. Determine the areas of the throat and discharge end of the nozzle to discharge 3000 lb. per hour, and state the dryness fraction at these places on the assumption that the flow is frictionless and adiabatic.

3. Superheated steam at a pressure of 200 lb. abs. and with 100° F. superheat (volume per pound = 2·68 cu. ft.) expands through a nozzle down to 15 lb. abs. Determine the principal dimensions of the nozzle to discharge 3600 lb. per hour, and state the condition of the steam in the throat and at the discharge end. Assume frictionless and adiabatic flow.

4. An exhaust steam injector is to be used for feeding a locomotive boiler in which the steam pressure is 200 lb. abs. If the pressure of the exhaust steam for working the injector is 17 lb. abs. and its dryness fraction is 0·85, estimate the weight of water that can be pumped per pound of steam, the area of the steam and water discharge orifices, and the feed temperature, if the weight of water taken from the feed tank is 10,000 lb. per hour at a temperature of 50° F.

5. A simple impulse turbine is fitted with convergent-divergent nozzles having throat diameters of $\frac{1}{4}$ in. Calculate the other dimensions if the initial pressure is 150 lb. per sq. in. abs., superheat 180° F., and the exhaust pressure 2 lb. per sq. in. abs.

If the machine is to develop 200 B.H.P. on an estimated consumption of 16 lb. per B.H.P. hour, find the number of nozzles required. The probable efficiency is 0·84.

6. At a certain stage of a pressure compounded impulse turbine dry steam at a pressure of 100 lb. per sq. in. abs. enters the nozzle with a velocity of 300 ft. per second.

The pressure in the wheel chamber is 68 lb. per sq. in. abs. Calculate the total exit area of the arc of nozzle required to pass 13·9 lb. per second, assuming a nozzle efficiency of 0·9.

Chapter XIII

THE STEAM TURBINE. DESIGN OF BLADING, ETC.

*GENERAL STATEMENT

In the reciprocating steam engine the transformation of energy is effected by the action of a static pressure upon the piston, the dynamic pressure being negligible. In the turbine a double transformation occurs, the heat energy being first converted into kinetic energy which is subsequently transformed into mechanical work on the rotor blades. The pressure upon the rotating parts is due almost solely to the change of momentum of the steam in its passage through the blade or nozzle channels. Except in the pure reaction turbine two elements are required, the nozzle, in which the first transformation takes place, and which may be either stationary or rotating, and the blading, in which the second conversion occurs. The functions of nozzle and blade may be shared.

Any section of a turbine in which the complete conversion takes place is known as a " stage."

Turbines may be classed as follows :

(1) Impulse, in which the nozzles are fixed, and which resemble the hydraulic Pelton wheel in general principles.

(2) Pure reaction, in which the nozzles rotate, and of which " Barker's mill " is the prototype. This type is not used in practice.

(3) Reaction, where the first phase of conversion occurs equally in both fixed and rotary nozzles, the blading sharing the nozzle function.

(4) Combination, where certain stages—always the high-pressure stages—are of the impulse type, and the remainder of the reaction.

In addition to the above general classifications, many sub-divisions are recognised, two of which are as follows :

(*a*) Pressure Compounded—in which the total pressure drop is distributed over a number of stages placed in series. In each stage the velocity of the jet produced is much less than would otherwise be the case, and the peripheral velocity of the wheel, which is a function of the velocity of the jets, is thus kept within convenient limits.

(*b*) Velocity Compounded—in which the full conversion of the kinetic energy of the jets into work done upon the rotor is made in several steps

so that the peripheral velocity of the moving blades is reduced. Fig. 106 shows a section of the nozzles and blading of a turbine having two pressure stages, each with three stages of velocity compounding. It will be seen that the velocity of the steam leaving the nozzles is gradually reduced as work is done upon the three sets of moving blades; fixed

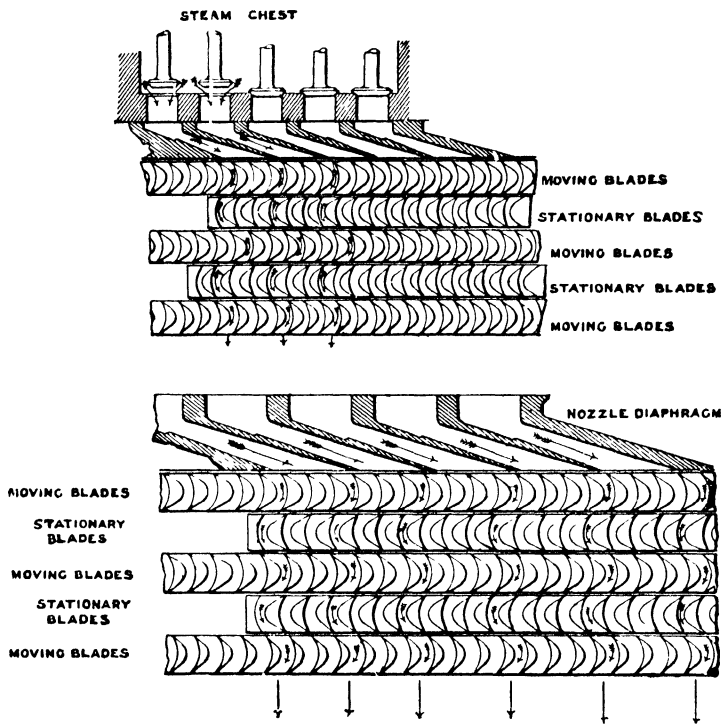


Fig. 106.

blades are used to redirect the steam into successive rows of blades upon the rotor.

In the Ljungström turbine the stationary blades are replaced by rings of moving blades, the two rotors moving in opposite directions at comparatively low speeds. The flow of the steam in this case is radial, the more usual axial direction not being practical for this arrangement where more than two rows of blades are concerned.

*VELOCITY TRIANGLES FOR SINGLE-STAGE TURBINES

The object of the turbine blading is to convert as much as possible of the kinetic energy of the steam into work upon the rotor. The final velocity of the steam should therefore be as low as possible, and parallel to the axis of the turbine (except, of course, for radial-flow machines).

The following notations will be used :

Subscripts a and r refer to " absolute " and " relative " respectively.

Subscripts i and o refer to inlet and outlet values.

Blade angles of moving blades at inlet and outlet are given by θ_i and θ_o degrees and of fixed blades by $0_i'$ and θ_o'.

Inclination of jet to plane of wheel is given by α_i and α_o degrees.

Absolute steam velocities at inlet and outlet are given by V_{a_i}, V_{a_o}.

Relative steam velocities at inlet and outlet are given by V_{r_i}, V_{r_o}.

Velocities of whirl of steam at inlet and outlet are given by V_{w_i}, V_{w_o}.

Angular velocity of blading ω radians per second.

Peripheral velocity of blading u ft. per second.

Mean radius of blading r ft.

Work done on blading E_b in ft.-lb.

$$\text{The Speed Ratio} = \frac{\text{blade velocity}}{\text{jet velocity}} = \rho.$$

The velocity ratio is given by K, where $V_o = KV_i$.

Blade length at inlet and outlet, l_i, l_o.

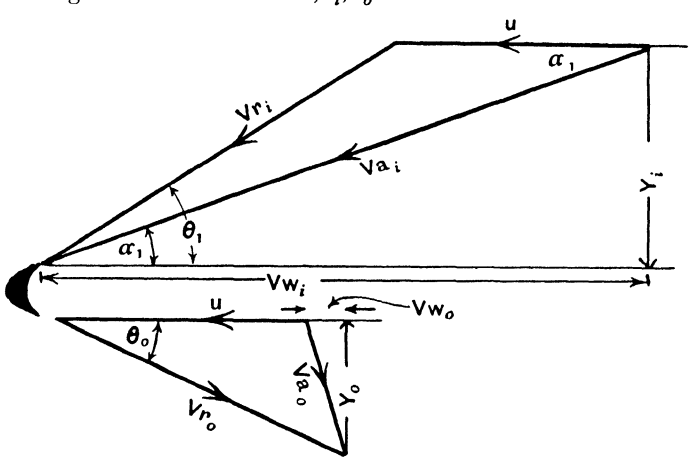


Fig. 107.

The pressure exerted by the steam upon the blading at any stage is the component of the change of momentum per second of the steam relative to the blading, measured in the direction of motion of the blades. The work done upon the blades is then the pressure upon the blades in the direction of their motion multiplied by the mean peripheral velocity of the blades, or the moment of the change of momentum per second multiplied by the angular velocity.

Fig. 107 shows the velocity triangle for a single stage axial flow impulse turbine.

V_{a_i} is the velocity of the steam leaving the nozzle, and has an inclination α_i to the plane of rotation. Compounding this with u, the velocity of the wheel, gives V_{r_i}, the velocity of the jet relative to the blade. The angle θ_i is then the necessary inlet angle of the blade. Similarly, if V_{r_o} is the relative velocity of the steam leaving the blade, then V_{a_o} will be the final absolute velocity, obtained by compounding V_{r_o} with u. The initial and final velocities of whirl will be as shown by V_{w_i} and V_{w_o}.

If there is no frictional or other loss in the blade channel $V_{r_i}=V_{r_o}$. In all practical cases there will be a loss of velocity owing to friction, and $V_{r_o}=KV_{r_i}$, where K is a velocity coefficient and is less than 1.

A more convenient arrangement of the velocity triangles is shown in Fig. 108.

It will be seen that the initial and final velocities of the steam are drawn from C, the initial and final jet angles lying to the left and right of that

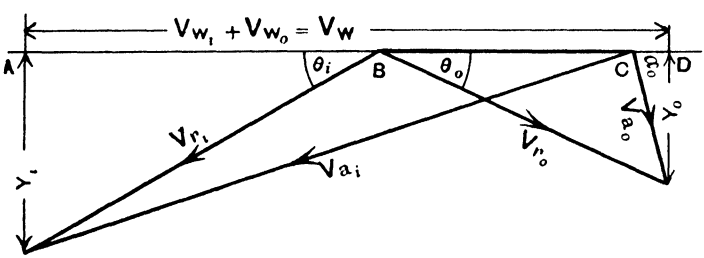


Fig. 108.

point, while the corresponding inlet and outlet velocities relative to the blades, and the inlet and outlet blade angles are determined by lines radiating from B. The total change in the velocity of whirl will be given by AD.

It will be seen that the pressure on the blading per pound of steam is given either by

$$1/g(V_{a_i}\cos\alpha_i+V_{a_o}\cos\alpha_o)=1/g\times AD$$

or by

$$1/g(V_{r_i}\cos\theta_i+V_{r_o}\cos\theta_o)=1/g\times AD$$

i.e. by either the change in absolute or the change in relative velocities. This must be so unless there are losses of energy from the steam between the exit from the nozzle and the entrance to the blading.

The torque on the rotor shaft per pound of steam will then be

$$T=\frac{r}{g}(V_{a_i}\cos\alpha_i+V_{a_o}\cos\alpha_o)$$

and the work done will be $E_b = T \times \omega = \frac{\omega r}{g}(V_{w_i} + V_{\omega_o})$

$$= \frac{u}{g}(V_{w_i} + V_{\omega_o})$$

$$= \frac{u}{g} V_w$$

where V_w is the change in velocity of whirl.

The axial thrust per pound will be given by $\frac{1}{g}(Y_i - Y_o)$.

In many cases the graphical method of calculation is employed, but where this is not desired the equations now to be deduced may be used. Reference should be made to Fig. 108. The quantities V_{a_i}, u, α_i and θ_i, K and ρ are normally fixed.

To calculate V_{r_i}

$$V_{a_i} \cos \alpha_i = V_{r_i} \cos \theta_i + u$$

or
$$V_{r_i} = \frac{V_{a_i} \cos \alpha_i - u}{\cos \theta_i} \quad . \quad . \quad . \quad . \quad . \quad (1)$$

To calculate V_w

$$V_{r_i} \cos \theta_i + V_{r_o} \cos \theta_o = V_w$$

or
$$V_{r_i}(\cos \theta_i + K \cos \theta_o) = V_w$$

and
$$V_w = (V_{a_i} \cos \alpha_i - u)\left(1 + \frac{K \cos \theta_o}{\cos \theta_i}\right) \quad . \quad . \quad . \quad (2)$$

To calculate E_b

$$E_b = \frac{u}{g} V_w = \frac{u}{g}(V_{a_i} \cos \alpha_i - u)\left(1 + \frac{K \cos \theta_o}{\cos \theta_i}\right)$$

foot-pounds per pound per second . . . . . (3)

If ρ is the ratio $\frac{u}{V_{a_i}}$

$$E_b = \frac{u^2}{g} C\left(\frac{\cos \alpha_i}{\rho} - 1\right) \quad . \quad . \quad . \quad . \quad . \quad (4)$$

where
$$C = \left(1 + \frac{K \cos \theta_o}{\cos \theta_i}\right) \quad . \quad . \quad . \quad . \quad . \quad (5)$$

The value of θ_o is usually fixed arbitrarily, and θ_i is determined by V_{a_i}, u, and α_i

or
$$\tan \theta_i = \frac{V_{a_i} \sin \alpha_i}{V_{a_i} \cos \alpha_i - u}$$

$$= \frac{\sin \alpha_i}{\cos \alpha_i - \rho}$$

and from these the value of C may be obtained.

To calculate the horse-power expended on the blades

$$\text{H.P.}=\frac{E_b}{550}=\frac{u^2C}{550g}\left(\frac{\cos\alpha_i}{\rho}-1\right) \quad . \quad . \quad . \quad . \quad . \quad (6)$$

the heat converted into work on the blades is

$$\left.\begin{aligned} h_b&=\frac{u^2C}{550Jg}\left(\frac{\cos\alpha_i}{\rho}-1\right)\\ &=\frac{E_b}{778}\text{ B.Th.U. per lb. per sec.}\end{aligned}\right\} \quad . \quad . \quad . \quad (7)$$

To calculate the axial components of the absolute velocities

$$\left.\begin{aligned} Y_i&=(V_{a_i}\cos\alpha_i-u)\tan\theta_i\\ &=u\left(\frac{\cos\alpha_i}{\rho}-1\right)\tan\theta_i\end{aligned}\right\} \quad . \quad . \quad . \quad . \quad (8)$$

and $$Y_o=(V_w-V_{a_i}\cos\alpha_i+u)\tan\theta_o$$

Substituting from (2)

$$\begin{aligned} Y_o&=\{(V_{a_i}\cos\alpha_i-u)C-(V_{a_i}\cos\alpha_i-u)\}\tan\theta_o\\ &=(V_{a_i}\cos\alpha_i-u)(C-1)\tan\theta_o \quad . \quad . \quad . \quad . \quad . \quad (9)\\ &=(C-1)u\left(\frac{\cos\alpha_i}{\rho}-1\right)\tan\theta_o\end{aligned}$$

and as $$C-1=\frac{K\cos\theta_o}{\cos\theta_i} \quad \text{from (5)}$$

$$Y_o=\frac{K\sin\theta_o}{\cos\theta_i}u\left(\frac{\cos\alpha_i}{\rho}-1\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

To calculate α_o

$$\begin{aligned} V_{w_o}=(V_w-V_{a_i}\cos\alpha_i)&=C(V_{a_i}\cos\alpha_i-u)-V_{a_i}\cos\alpha_i \quad \text{from (2)}\\ &=V_{a_i}\cos\alpha_i(C-1)-Cu \quad . \quad . \quad . \quad (11)\end{aligned}$$

But $$\tan\alpha_o=\frac{Y_o}{V_{w_o}}=\frac{(C-1)(V_{a_i}\cos\alpha_i-u)\tan\theta_o}{(C-1)V_{a_i}\cos\alpha_i-Cu} \quad \text{from (9) and (11)}$$

or $$\tan\alpha_o=\frac{(V_{a_i}\cos\alpha_i-u)\tan\theta_o}{V_{a_i}\cos\alpha_i-\dfrac{u}{1-1/C}}$$

$$=\frac{\left(\dfrac{\cos\alpha_i}{\rho}-1\right)\tan\theta_o}{\dfrac{\cos\alpha_i}{\rho}-\dfrac{1}{1-1/C}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

To calculate V_{a_o} and the residual energy

$$V_{a_o}=\frac{Y_o}{\sin \alpha_o}$$

and

$$V_{a_o}^2=V_{r_o}^2+u^2-2V_{r_o}u\cos\theta_o$$
$$=(KV_{r_i})^2+u^2-2uKV_{r_i}\cos\theta_o \quad . \quad . \quad (13)$$

from which the kinetic energy finally discharged can be calculated.

EXAMPLE.—The steam chest pressure in a De Laval turbine is 200 lb. per sq. in. abs., and the exhaust pressure 5 lb. per sq. in. abs., the steam being initially dry and saturated. The peripheral speed of the blades is 1200 ft. per second, and the nozzle is inclined 20° to the direction of motion of the blades. Estimate the angles of the blades, the work done on the blades per second per pound of steam, the absolute velocity of the steam at discharge from the blades, and the efficiency of the turbine. Neglect frictional resistances and assume adiabatic flow. If the steam consumption is 3600 lb. per hour, estimate the horse-power of the turbine.

From the Mollier diagram we find the heat drop in the nozzle to be 253 B.Th.U. and the velocity at entrance to the wheel

$$\sqrt{778\times 253\times 64\cdot4}=3560 \text{ ft. per second}$$

The velocity diagram is shown in Figs. 107 or 108. On setting this out to scale, θ_i is found to be 30°, and $V_{r_i}=2470$ ft. per second. The De Laval blades are made symmetrical with equal inlet and outlet angles, hence, making $\theta_i=\theta_o$ and $V_{r_i}=V_{r_o}$, the absolute velocity at discharge is found to be 1530 ft. per second.

The work done per second per pound of steam is

$$E_b=\frac{1200}{32\cdot2}(2470\cos 30°+2470\cos 30°)$$
$$=\frac{2400\times 2470\times 0\cdot866}{32\cdot2}$$
$$=159{,}400 \text{ ft.-lb.}$$

The efficiency will be

$$\frac{(3560)^2-(1530)^2}{(3560)^2}=0\cdot815$$

The value of the inlet and outlet angles and the absolute velocity at discharge may be obtained by calculation as follows :

$$V_{r_i}^2=(1200)^2+(3560)^2-2\times 1200\times 3560\cos 20°$$
$$=10^6(1\cdot44+12\cdot6736-8\cdot0288)$$
$$=10^6\times 6\cdot0848$$

$\therefore V_{r_i}=2466$ ft. per second, which agrees with the result found above.

Also $$\frac{\sin\theta}{3560}=\frac{\sin 20^\circ}{2466}$$

$$\sin\theta=\frac{0{\cdot}342\times 3560}{2466}=0{\cdot}4935$$

$$\therefore\ \theta=\sin^{-1}0{\cdot}4935=29{\cdot}6^\circ$$

and $$V_{a_o}{}^2=(1200)^2+(2466)^2-2\times 1200\times 2466\cos 29{\cdot}6^\circ$$

$$=10^6(1{\cdot}44+6{\cdot}0848-5{\cdot}1486)$$

$$=10^6\times 2{\cdot}3562$$

$$V_{a_o}=1535 \text{ ft. per second as above}$$

The work done per second is

$$\frac{3600}{60\times 60}\times 159{,}400 \text{ ft.-lb.}$$

and the horse-power $$=\frac{159{,}400}{550}=290 \text{ H.P.}$$

EXAMPLE.—In the turbine given in the last example, find the blade angles, work done per second per pound of steam, the speed of the wheel, and the efficiency, if the efficiency is to be the greatest possible.

Here $a=20^\circ$, $V_i=3560$ ft.

and $$u=\frac{3560}{2}\cos 20^\circ \text{ for maximum efficiency (p. 250)}$$

$$=1780\times 0{\cdot}9397$$

$$=1673 \text{ ft. per second}$$

and $$\tan\theta_i=2\tan 20^\circ$$

$$=2\times 0{\cdot}364$$

$$=0{\cdot}728$$

therefore $$\theta_i=\tan^{-1}0{\cdot}728=36^\circ \text{ nearly}$$

Also $$E_b=\tfrac{1}{2}\times\frac{(3560\times 0{\cdot}9397)^2}{32{\cdot}2}$$

$$=173{,}800 \text{ ft.-lb.}$$

and the efficiency $$=\frac{E_b}{\frac{1}{2\times 32{\cdot}2}(3560)}=(\cos 20^\circ)^2$$

$$=(0{\cdot}9397)^2$$

$$=0{\cdot}883$$

*DIAGRAM FOR VELOCITY COMPOUNDED IMPULSE STAGE ASSUMING NO FRICTION

The extended diagram is shown in Fig. 109. In order to distinguish the velocity stages the subscripts 1 and 2 are added to the symbols,

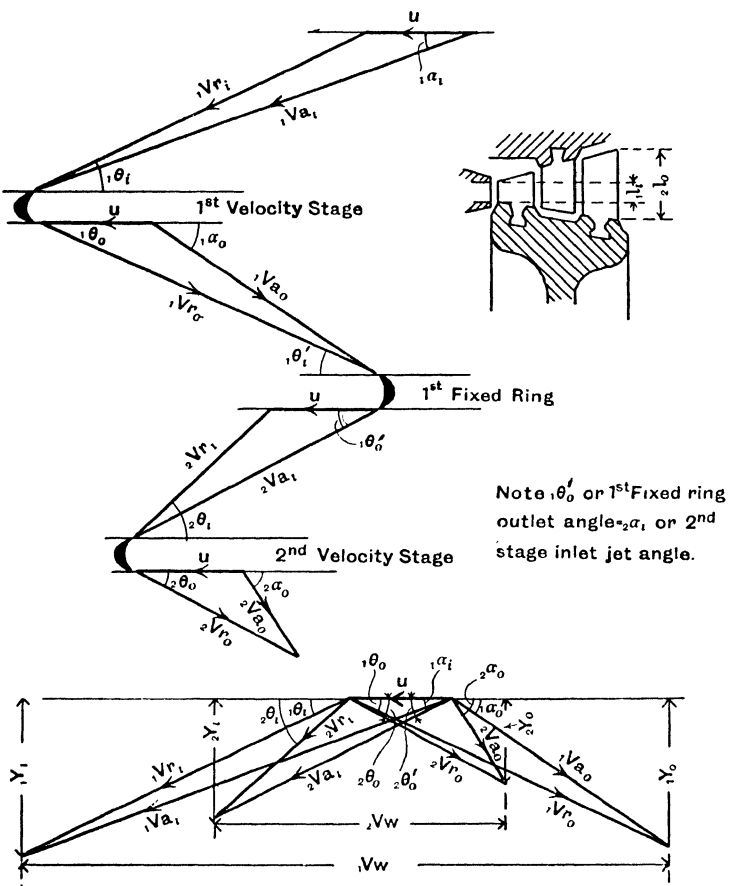


Fig. 109.

denoting the first and second moving rings and the first and second fixed rings. Guide blade angles are denoted by dashes. (It will greatly assist the reader if the symbols are read as follows : ${}_1V_{r_i}$ as first stage relative inlet velocity ; ${}_2V_{a_o}$ as second stage absolute outlet velocity.)

The mean blade speed will be the same for both stages. It will also be noted that the absolute velocity at entrance ${}_1V_{a_i}$ is the velocity of the steam leaving the nozzle, that the absolute velocity of exit from the first

moving ring ${}_1V_{a_o}$, is the entrance velocity to the fixed or guide ring, and that the outlet velocity from that ring is the absolute velocity ${}_2V_{a_i}$ at which the steam is discharged into the second moving ring.

The following quantities are normally given, the jet angle ${}_1\alpha_i$ and velocity ${}_1V_{a_i}$, and either the mean blade speed or the speed ratio ρ. The exit angles ${}_1\theta_o$, ${}_1\theta_o'$, and ${}_2\theta_o$ are usually arbitrarily chosen.

It will be seen that the velocity triangle at the entrance to the first set of moving blades is determined by two sides and the included angle, and that the closing side ${}_1V_{r_i}$ provides the second side ${}_1V_{r_o}$ of the outlet triangle, as

$${}_1V_{r_o}={}_1V_{r_i}$$

The contracted diagram is also shown. As before, it will be noted that all absolute velocity lines radiate from C, and all relative lines from B, and that inlet and outlet angles are measured to the left and right of these points respectively.

In this diagram the velocities of whirl of the two stages are given by ${}_1V_w$ and ${}_2V_w$, so that the work done upon the blades per pound is

$$E_b=u/g\times({}_1V_w+{}_2V_w).$$

The velocities of the steam in an axial direction are given by ${}_1Y_i$, ${}_1Y_o$, ${}_2Y_i$ and ${}_2Y_o$, and the axial thrusts on the two rings per pound of steam per second will be $1/g\times({}_1Y_i-{}_1Y_o)$ and $1/g\times({}_2Y_i-{}_2Y_o)$ respectively, or

$$\text{Total axial thrust}=1/g\times({}_1Y_i+{}_2Y_i-{}_1Y_o-{}_2Y_o)$$

If there is no friction the volume V_3 of the steam will remain constant, and the product of axial velocity$\times$(area of steam passages measured perpendicular to this direction) must remain constant. This necessitates a successive lengthening of the blades as shown in Fig. 107, so that

$${}_1l_i : {}_1l_o : {}_2l_i : {}_2l_o=\frac{1}{{}_1Y_i} : \frac{1}{{}_1Y_o} : \frac{1}{{}_2Y_i} : \frac{1}{{}_2Y_o},$$

the outlet length of the first stage and the inlet length of the second determining the inlet and outlet lengths of the fixed blade. It is customary to adjust the blade angles so that the increase of blade length is linear.

DIAGRAM FOR VELOCITY COMPOUNDED IMPULSE STAGE, GIVEN THE VALUE OF K

Fig. 110 shows the previous diagrams modified to allow for friction.

Here ${}_1V_{r_o}=K_1V_{r_i}$, ${}_2V_{a_i}=K_1V_{a_o}$, and ${}_2V_{r_o}=K_2V_{r_i}$

The value of K may be taken as constant, or may vary for each row of blades, the probable values in the second case being of the order of 0·8, 0·78, and 0·76. As all velocities will be decreased by friction the blade lengths will have to be adjusted to suit the new axial velocities.

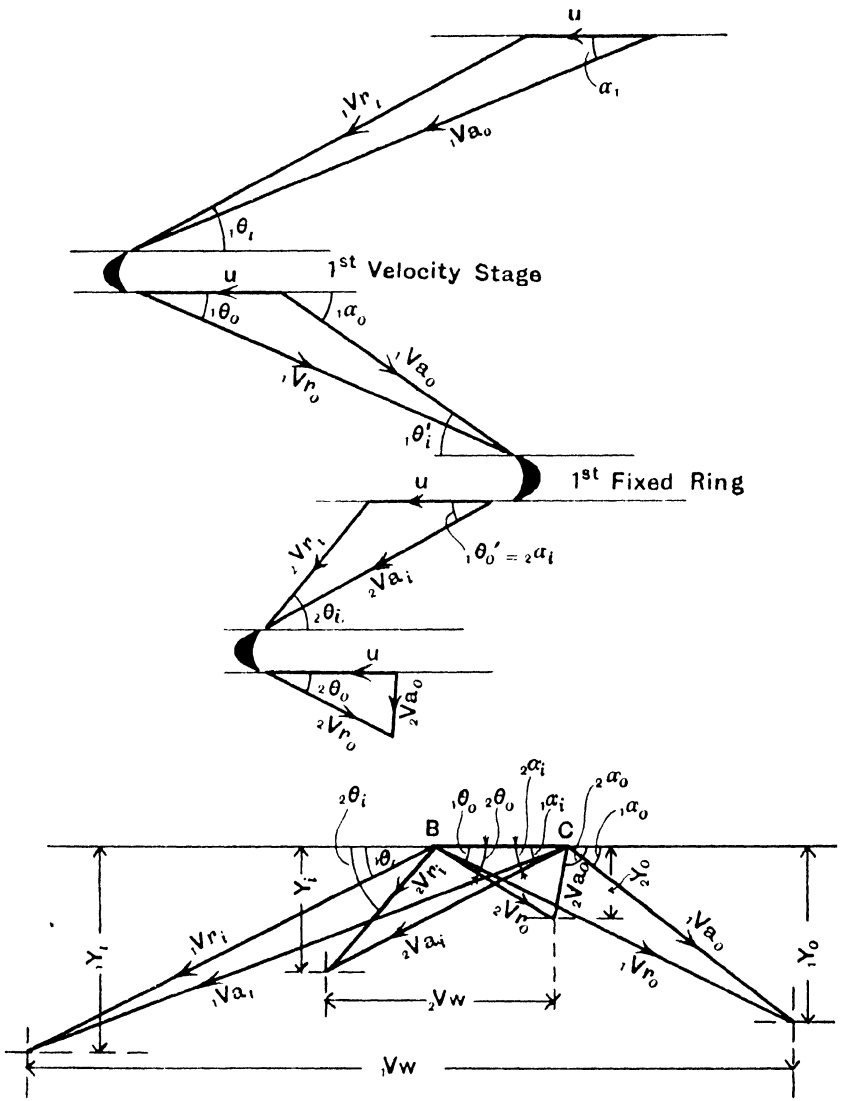


Fig. 110.

DIAGRAM FOR VELOCITY COMPOUNDED IMPULSE STAGE, GIVEN THE BLADE LENGTHS

Owing to the uncertainty existing concerning the values of K it may be equally satisfactory to assume the height ratio. For the two-velocity stage this varies between 2·5 and 3. Taking the former value, and refer-

ring to the inset of Fig. 109, the relative lengths of the blades will be as follows :

$$ {}_1l_i : {}_1l_o : {}_2l_i : {}_2l_o = 1 : 1{\cdot}5 : 2{\cdot}0 : 2{\cdot}5 = {}_2Y_o : {}_2Y_i : {}_1Y_o : {}_1Y_i $$

Referring to Fig. 111 the first velocity triangle BCE can now be drawn for the entrance to the first stage, and this will determine the value AE of the initial axial velocity ${}_1Y_i$. Taking AE as 2·5 units the dotted lines parallel to AD may now be drawn at distances from AD of 2·0, 1·5, and 1 unit. The apices E, F, G, H of the velocity triangles must now lie on these lines, i.e. the point F is fixed by the point of intersection with the line 2 units from AD of a line drawn through B at an angle of inclination of ${}_1\theta_o$ degrees, and similarly for the other points.

The values of the remaining velocities, and consequently of the velocity ratios, are therefore fixed if the height ratios and exit angles are known,

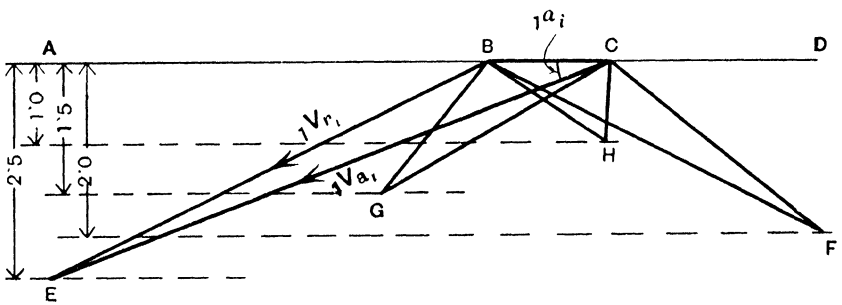


Fig. 111.

or, conversely, the exit angles could be determined given the velocities, or velocity ratios, and the height ratio.

EXAMPLE.—A stage of a Curtis turbine has a wheel with two moving rings. The jet angle is 20°, and the exit angles of the blades are, first moving ring 22°, fixed blades 24°, second moving ring 35°. The blade speed is 455 ft. per second, the height ratio 2·2, and the speed ratio 0·2.

Find the work done on the blading per pound of steam, and the H.P. expended. Find also the entrance angles of the blades, the velocity coefficients, and the total axial thrust.

The velocity diagram can be drawn as in Fig. 109 from the following data :

$$ \text{BC}=445, \text{ and } {}_1\alpha_i'=20^\circ, \text{ and } {}_1v_{a_i}=\frac{455}{0{\cdot}2}=2275 $$

These determine the first inlet triangle, from which ${}_1y_i=778$, ${}_1v_{r_i}=1860$, and ${}_1\theta_i=25^\circ$.

For the outlet triangle we have BC=445, ${}_1\theta_o=22^\circ$

and $$ {}_1y_o=778\times\frac{1}{\text{height ratio for blade}}. $$

The overall height ratio being 2·2 : 1, distributed over two moving and one fixed blades, the height ratios for the blades, allowing for a linear increase, are given by

$${}_1l_i : {}_1l_o,\ {}_1l_o : {}_2l_i,\ \text{and}\ {}_2l_i : {}_2l_o\ \text{(see Fig. 109)}$$

or $$1 : 1{\cdot}4,\ 1{\cdot}4 : 1{\cdot}8,\ \text{and}\ 1{\cdot}8 : 2{\cdot}2$$

Hence, $${}_1y_o = 778 \times \frac{1}{1{\cdot}4} = 555$$

The following values are obtained from this triangle :

$${}_1v_{r_o} = 1480,\ v_{a_o} = 1070,\ {}_1\alpha_o = 31°.$$

The height ratio of the fixed blades gives ${}_2y_i = 432$, and as ${}_2\theta_o = 24°$ and BC is known, the second inlet triangle, and subsequently the second outlet triangle, can be obtained giving the following values

$${}_2v_{a_i} = 1060,\ {}_2v_{r_i} = 672,\ {}_2\theta_i = 40°$$

and, as $${}_2y_o = 354,\ {}_2v_{r_o} = 615,\ {}_2v_{a_o} = 358,\ \text{and}\ {}_2\alpha_o = 83°,$$

${}_1v_w$ will be found to be 3058 ft. per second and ${}_2v_w = 1020$ ft. per second.

From these figures the required values are rapidly obtained, i.e.

$$E_b = \frac{u}{g}({}_1v_w + {}_2v_w)$$

$$= \frac{455 \times 4078}{32{\cdot}2} = 57{,}600 \text{ ft.-lb. per pound, or } 74{\cdot}1 \text{ B.Th.U. per lb.}$$

The horse-power is then $\frac{E_b}{550} = \frac{57{,}600}{550} = 105$ per pound of steam per second.

The thrust on each ring being $\frac{1}{g}(y_i - y_o)$

we have the total thrust $= \frac{1}{32{\cdot}2}\{(778-555)-(432-354)\}$

$= 9{\cdot}35$ lb. per pound of steam per sec.

The velocity coefficients for the first and second moving rings are

$$K_1 = \frac{{}_1v_{r_o}}{{}_1v_{r_i}} = \frac{1480}{1860} = 0{\cdot}796$$

$$K_2 = \frac{615}{672} = 0{\cdot}915$$

Considering the rings entirely separately the velocity coefficients should probably be of the order of 0·8 and 0·76 respectively, and it would appear that to obtain the abnormally high value of 0·915 the steam would have to be considerably restricted volumetrically in its passage through the

fixed and second row of blades, tending to give a reaction effect with the conversion of an unanticipated pressure drop across the blades into velocity energy.

*VELOCITY DIAGRAM FOR REACTION STAGE

In a reaction stage there is an increase of relative velocity between entrance and exit, due to the pressure drop in the blade channels. The fixed and moving rings have identical blades, and the heat drop is equally divided between the fixed and moving rings. It is conventional to make

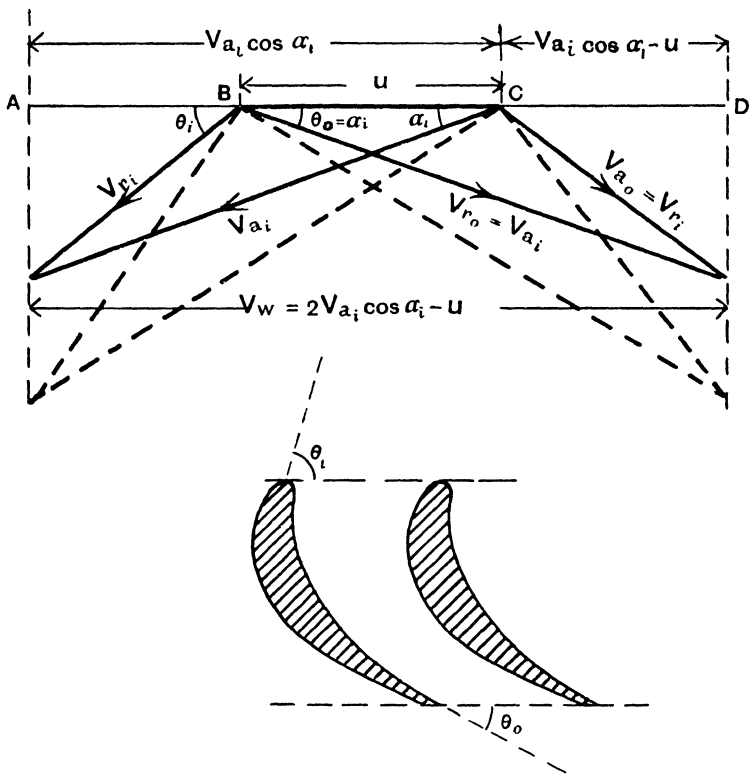


Fig. 112.

the blade length constant for each step of the rotor, in which case $Y_i = Y_o$, and as $\alpha_i = \theta_o$ the inlet and outlet triangles will be identical.

The velocity of whirl V_w will be given by $V_w = {}_2V_{a_i} \cos \theta - u$

and
$$E_b = \frac{u}{g}(2V_{a_i} \cos \theta - u) \quad \text{(see Fig. 112)}$$
$$= \frac{u^2}{g}\left(\frac{2 \cos \theta}{\rho} - 1\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

The constant entrance angle for each ring is given by

$$\tan \theta_i = \frac{V_{a_i} \sin \theta}{V_{a_i} \cos \theta - u} = \frac{\sin \theta}{\cos \theta - \rho} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

The simplicity of the equations for this type of blading enable the necessary particulars to be obtained more readily by direct than by graphical calculation.

The dynamic axial load upon the blading is theoretically zero, as $Y_i = Y_o$, and in practice is less than 2 per cent. of the total thrust. This latter is due almost entirely to the static difference of pressure at the two ends of the rotor. If A is the annular area between the rotor and drums, the thrust due to a difference of pressure of $p_i - p_o$ at the ends of one step of the rotor will be $(p_i - p_o)$A. As the pressure drop is divided approximately equally between the fixed and moving blades, the thrust upon the rotor will be $\frac{1}{2}(p_i - p_o)$A. This is usually balanced by steam pressure applied to a dummy piston secured to the rotor shaft, or by a thrust block.

In the reaction turbine the pressure drop is distributed over a large number of stages arranged along the " drum " and a corresponding gradual increase of specific volume of the stream occurs, necessitating an increase in the area of the blade channels measured perpendicular to the axis of the machine. This is normally obtained by increasing the lengths of successive groups of blades, and by " stepping up " the drum diameter towards the low-pressure end, as, by the latter, the necessary area can be provided without undue increase of blade length. The increased drum diameter also increases the torque due to the steam pressure. The total work done on the blading, and the total axial thrust, must be obtained by adding the effects obtained separately for each group of blades and for each " step " of the drum.

*BLADE SPEED AND R.P.M.

The maximum peripheral velocity of the rotor is determined by the strength of the disc or drum material. For mild steel the velocity is of the order of 380 ft. per second, and for nickel steel about 424 ft. per second. The speed of rotation varies with the size of the unit and the nature of the final drive. For electrical generation in large units and with direct drive 1500 r.p.m. is normal, with 3000 r.p.m. for plants of 12,000 kw. or under. For geared turbines speeds of 4000–8000 r.p.m. are employed.

DIAGRAM EFFICIENCY AND SPEED RATIO

The diagram efficiency is the ratio between the work theoretically done on a blade ring and the kinetic energy of the jet, and can be derived directly from the velocity diagram.

The diagram efficiency is then given by

$$\eta_d = \frac{E_b}{\frac{V_i^2}{2g}}$$

For the simple impulse stage $\eta_d = \frac{2uV_w}{V_{a_i}^2}$ equation (3), p. 239

For a velocity compounded impulse stage $\eta_d = \frac{2u}{V_{a_i}^2}\Sigma V_w$ p. 244

and for the reaction stage $\eta_d = \frac{2u^2}{V_{a_i}^2}\left(\frac{2\cos\theta}{\rho} - 1\right)$ equation (1), p. 248

For a constant steam velocity V_{a_i} and given blade angles, there is in each case a value of ρ which will give maximum values for both work done and diagram factor.

The work done in the impulse stage, by equation (4), p. 239,

$$E_b = \frac{u^2 C}{g}\left(\frac{\cos\alpha_i}{\rho} - 1\right)$$

and since $\frac{u}{V_{a_i}} = \rho$ $E_b = \frac{V_{a_i}^2 \rho^2 C}{g}\left(\frac{\cos\alpha_i}{\rho} - 1\right)$

$$= \frac{V_{a_i}^2 \rho \cos\alpha_i C}{g} - \frac{V_{a_i}^2 \rho^2 C}{g}$$

For E_b to have a maximum value $\frac{dE_b}{d\rho} = 0$

or $\cos\alpha_i - 2\rho = 0$

and $\rho = \frac{1}{2}\cos\alpha_i$

or $u = \frac{1}{2}V_{a_i}\cos\alpha_i$

The normal value of α_i is 20°, in which case

$$\cos\alpha_i = 0{\cdot}94 \text{ and } u = \frac{0{\cdot}94}{2}V_{a_i} = 0{\cdot}47V_{a_i}$$

In the case of the reaction stage

$$E_b = \frac{2V_{a_i}^2 \rho\cos\theta}{g} - \frac{V_{a_i}^2 \rho^2}{g} \text{ from equation (1), p. 248}$$

and $\frac{dE_b}{d\rho} = 0$ when $2\cos\theta - 2\rho = 0$

or $\rho = \cos\theta$

For $\alpha_i = 20°$ $u = V_{a_i}\cos\theta = 0{\cdot}94V\alpha_i$, and V_{a_o} will be perpendicular to u.

The following values may be taken in practice :

Simple Impulse, $\rho=0{\cdot}47$.

Velocity Compounded Impulse, two, three, and four moving rings, 0·235, 0·156, and 0·117.

Reaction 0·94.

It should be noted that if the above values are exceeded, that is, if the turbine is " overspeeded," the jet will exert a pressure on the back of the blades and tend thereby to reduce the torque. " Underspeeding " will be less detrimental, as though the speed of the wheel will be reduced, the pressure upon the blades will be increased.

THE SECTION OF TURBINE BLADES

Fig. 113, A, shows the normal section of blades for an impulse turbine, θ_i and θ_o being the inlet and outlet angles determined with reference to the velocity triangle. The inner face of the blade is cut to a radius r determined after the overall width w of the blade has been fixed, the same centre of curvature being used to describe the back of the adjacent blade. A channel of constant radial width is the result.

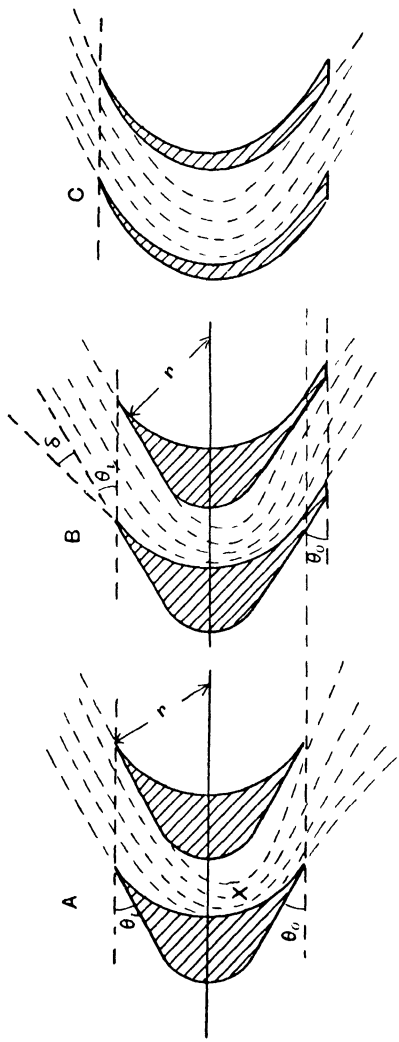


Fig. 113.

Two modifications are recommended to allow for practical difficulties : The entering edge of the bucket may be strengthened by increasing the angle θ_i of the inner curve by a small amount δ, Fig. 113, B, which may have the value of about 3°, this being found less detrimental than decreasing the exterior angle by the same amount (which would have a similar effect to overspeeding).

The outlet edge may be extended, as shown in B. This has the double effect of stiffening the blades and improving the highly important guidance of the steam leaving the blade. It will be seen from these figures that the steam does not fill the channels between the blades in a uniform manner, but is compressed by centrifugal and inertia forces towards a point X on the outside of the curve of the stream and towards the leaving edge. A zone of low pressure and high turbulence is thus left against the back of the blades, and the stream will expand as it moves from X towards the outlet of the blade, causing the final mean jet angle to be appreciably less than the designed value θ_o. The parallel extension allows to some extent for the guidance of the steam after re-expansion has taken place.

Curved sheet-metal buckets, as shown at C, are also extensively used. Though allowing considerable expansion of the steam passage along the blade centre-line, the performance, at least at low pressures, does not appear to be inferior to that of section A.

PITCH, WIDTH, AND LENGTH OF BLADES

The number of blades to be provided in any ring should be just sufficient to give adequate guidance to the steam. Blades in excess of this number merely increase the total friction owing to the increased surface provided. It would appear that the pitch should not be less than $\frac{3}{8}$ in. or more than 1 in., and that a value equal to the radius of the face of the blade is suitable in many cases.

The width of the blading is usually determined by convention, three factors having to be considered, i.e. the rigidity of the blade, and the provision of a suitable radius of curvature of the face with adequate but not excessive surface area. In the Parsons turbine, with a very large number of reaction stages, the axial width and the pitch vary from about 0·24 in. width and 0·165 in. pitch at the high pressure end, to 0·39 in. width and 0·24 in. pitch about the centre, and to 1 in. width and 0·55–0·63 in. pitch for the low-pressure stages. In the normal impulse wheel the width varies from a minimum of 0·6 in. to 1·0 in. for blades about 10 in. long, and to 1·2 in. or 1·4 in. for very long blades up to 24 in. in length. The ratio of length to axial width is usually about 15.

The length of the blade is determined by a number of factors. These include the ratio of blade length to mean blade ring diameter, as if this is excessive the pitch of the blades will vary excessively between the roots and tips, resulting in either too small a pitch with excessive friction at the roots or too wide a spacing with unsatisfactory guidance at the tips. The rigidity and strength of the blade to resist centrifugal and dynamic stresses and vibration are of primary importance.

The minimum length of blade for impulse turbines should not be less than $\frac{3}{8}$ in., and is usually from 1·5 to 2 per cent. of the mean ring diameter.

In the Parsons turbine this value should not be less than 3 per cent. in order that leakage over the tips of the blades may not be excessive.

In the Parsons turbine full peripheral admission is necessary from the first stage onward and should be attained in the impulse turbine at as early a stage as is practicable in order that the dynamic loading of the blades may be steady, impulsive loading tending to cause vibration.

For more complete discussions of blade dimensions, etc., the reader is referred to Goudie's " Steam Turbines," and Stodola's " Steam and Gas Turbines."

BLADE FRICTION

The term Friction as applied to blading is a general term covering a large number of factors, as is the case of nozzle friction. These include :

(1) Shock at entry owing to the necessary thickness of the blades.

(2) Shock of the entering steam upon dead steam carried round in the blade channels, especially if full peripheral admission is not employed.

(3) Surface friction proper.

(4) Centrifugal compression and subsequent re-expansion owing to the curvature of the blade faces.

(5) Centrifugal compression away from the axis of the machine and secondary circulation set up thereby in the blade channels.

(6) Inefficient guidance, attributable to several sources.

The overall coefficient is not known with accuracy, but for a normal impulse turbine the stage coefficient may vary from 38 per cent. at the high-pressure end of the machine to 30 per cent. towards the outlet.

REHEAT FACTOR AND TURBINE EFFICIENCIES

In the single-stage turbine the overall efficiency is not of a high order, chiefly because the friction in the blading reduces the efficiency of conversion of the kinetic energy to a figure of the order of 70 per cent. The heat generated by friction is discharged in the exhaust steam.

In the multi-stage turbine friction loss in the first stage represents a total loss to that stage only, as the friction heat is passed on in the live steam used in the second and subsequent stages, and is partly converted into work there. The multi-stage turbine is therefore one of the group of machines in which the overall efficiency may be greater than the efficiencies of the parts.

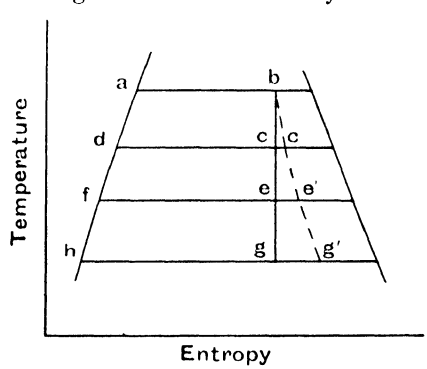


Fig. 114.

This effect is illustrated for a three-stage turbine in Fig. 114. Assuming

adiabatic frictionless expansion, the Tϕ diagrams for the three stages will be represented by *abcd*, *dcef*, and *fegh*. If friction is present, however, the steam will leave the stages with larger heat contents and higher dryness fractions than are indicated by points *c*, *e*, and *g*, the new condition points being *c'* and *e'* and *g'*, and the new condition curve *bc'e'g'*. It will, of course, be remembered that the increase in the areas of the two state diagrams does not correspond to an increase in effective work done; the reverse is in fact the case as friction renders the action irreversible.

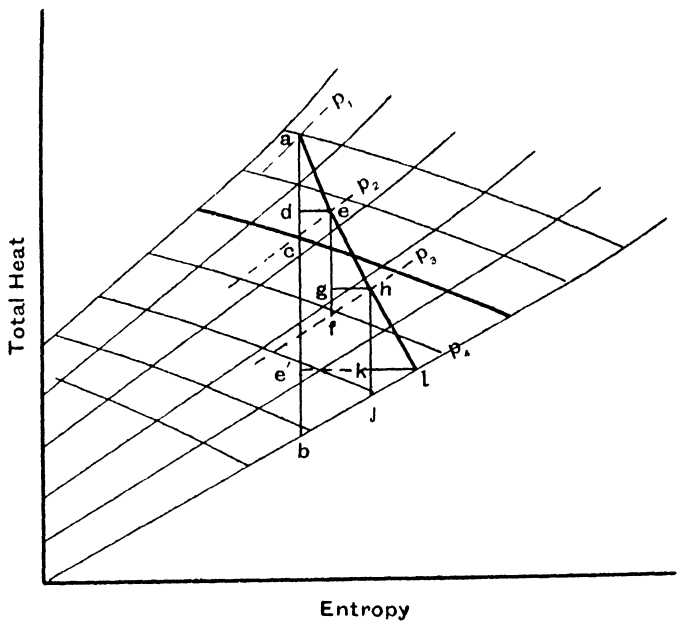


Fig. 115.

The total heat to be distributed between the stages is represented by *abg'h*, instead of *abgh*, the ratio between the two, $\frac{abg'h}{abgh}$, being known as the Reheat Factor, and *abg'h* as the "cumulative heat."

The effect of reheat is illustrated for this turbine on the Mollier diagram shown in Fig. 115.

Let η_s be the efficiency of each stage, or Stage Efficiency and R=the reheat factor for the turbine, the initial and final pressures being p_1 and p_4.

$H_a - H_b$ is the adiabatic heat drop, and the cumulative heat will be $R(H_a - H_b)$, where R has a value of the order of 1·04.

The heat drop per stage (assumed uniform) will be $\frac{R(H_a - H_b)}{3} = h_s$.

From the initial condition point a the distance $ac=h_s$ is measured, and $\frac{ad}{ac}$ is made equal to η_s. The final condition point for the first stage must then lie on the line of pressure through c and the line of constant heat through d, i.e. at e.

The initial condition point for the second stage is now e, from which $ef=h_s$ and $eg=ef \cdot \eta_s$ are measured, leading to the condition point h, which gives the properties of steam entering the third stage. Making $hj=h_s$ and $\frac{hk}{hj}=\eta_s$, the construction leads to the final condition point l of the steam leaving the turbine.

The reheat factor allows for the divergence of the constant pressure lines on the Hϕ chart, as if these were parallel the above construction could only be applied were R=1.

The internal efficiency η_i, i.e. the efficiency compared with an adiabatic operation, is then given by $\frac{ae'}{ab}$.

It will be seen that the success of the above construction depends on the proper selection of the values of R and η_s, which are not independent variables. The relation between the value of R and the efficiencies is obtained as follows :

$$\text{Internal efficiency } \eta_i = \frac{ae'}{ab}$$

$$= \frac{\text{actual heat drop}}{\text{adiabatic heat drop}}$$

$$= \frac{\Sigma h_s}{H_a}$$

where h_s and H_a are the actual heat drop per stage and the adiabatic heat drop

$$= \frac{\eta_s \Sigma h \text{ cumulative}}{H_a}$$

$$= \frac{\eta_s R \cdot H_a}{H_a}$$

or

$$\eta_i = R \cdot \eta_s$$

Experimental values of η_s are found to vary with the speed-ratio, and are normally shown plotted on a base of the theoretical value of the speed ratio ρ, or ρ_t, which is given by $\frac{u}{V_o'}$, where V_o' is the theoretical velocity at exit from the nozzle. A series of curves plotted in this manner are shown in Fig. 116, for three, two and one velocity stages, the latter being the right-hand curve.

That the efficiency of the turbine is dependent on the velocity ratio and not upon the individual values of the velocities can be shown as follows :

$$\eta_i = \frac{\text{work done}}{\text{energy available}}$$

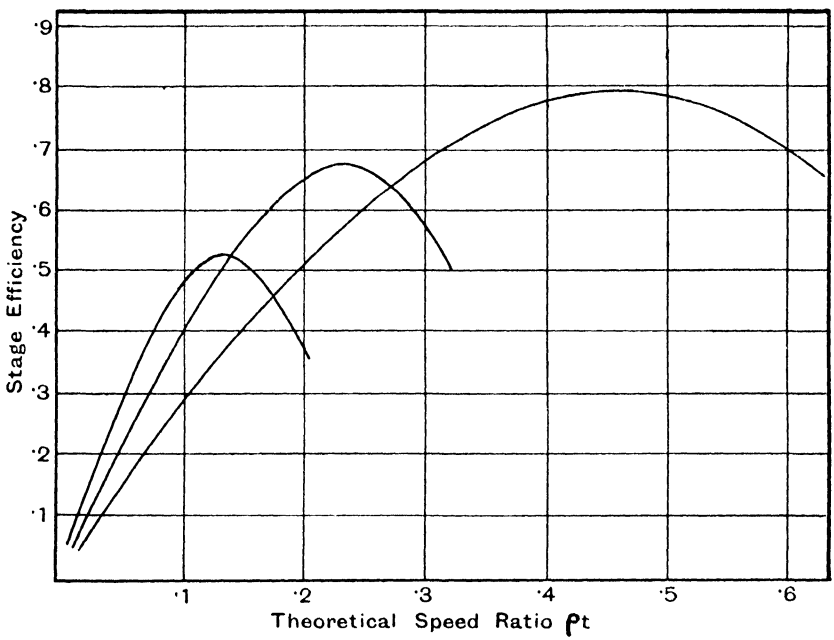


Fig. 116.

Taking the value of E_b from equation (3), p. 239, and writing energy available

$= V_{a_t}^2/2g$, where V_{a_t} is the theoretical velocity of discharge from the nozzle

$= \dfrac{V_a^2}{K^2 \cdot 2g}$, where V_a is the actual velocity of discharge.

$$\eta_i = \frac{\frac{u}{g}(V_{a_i}\cos\alpha_i - u)\left(1+\frac{K\cos\theta_o}{\cos\theta_i}\right)}{V_{a_i}^2/2gK^2}$$

$$= \frac{u}{V_a}\cdot 2K^2\left(\cos\alpha_i - \frac{u}{V_{a_i}}\right)\left(1+\frac{K\cos\theta_o}{\cos\theta_i}\right)$$

an expression in which the velocities appear only in the ratio $\dfrac{u}{V_a}$.

EXAMPLE.—A four-stage velocity compounded turbine is to work between the limits of 185 lb. per sq. in. abs. at 200° F. superheat, and 0·75 lb. per sq. in. abs. Find from the Mollier diagram the probable values of the initial stage pressures and the qualities of the steam, assuming that the average stage efficiency is 0·67.

In this case the stage efficiency and reheat factor are inter-related, but insufficient information is given to determine R. The method of procedure is therefore to assume a reasonable value of R, and proceed if necessary by a series of approximations. The probable value of R in this case will be about 1·04.

Referring to Fig. 115, the position of a may be marked on the Mollier chart (i.e. 185 lb. per sq. in. and 200° superheat), and also the point b, on an adiabatic through a, and on the line $p=0{\cdot}75$ lb. per sq. in.

Then H_a-H_b will be found to be 392 B.Th.U.

The cumulative heat will then be $H_c=1{\cdot}04\times 392=407{\cdot}68$ B.Th.U. per lb. and the average heat drop per stage is $\frac{407{\cdot}68}{4}=101{\cdot}92$ B.Th.U. per lb., say 102·0 B.Th.U. per lb.

The net heat drop per stage is then $h_s=0{\cdot}67\times 102=68{\cdot}34$. Referring again to Fig. 115, $ac=102$, giving the lower pressure of the first stage as 63 lb. per sq. in.

Also $ad=68{\cdot}34$, and e will be on the total heat line through d and the constant pressure line for 63 lb. per sq. in.

The quality at e will be found to be 123° F. superheat.

Continuing the process for the three remaining stages, the following values will be obtained :

Stage .	1	2	3	4	Exhaust
Pressure	185	63	18	4·2	0·75 lb. per sq. in. abs.
Quality .	200° F. s.	123° F. s.	50° F. s.	$q=0{\cdot}984$	0·947

It will be seen that as construction proceeds progressively down the diagram from a, an excessive stage heat drop, the effect of the selection of too high a value of R, will result in e being found on a pressure line below that specified as the exhaust pressure, and the construction will have to be recommenced with a lower value.

In the above case the total heats in the inlet and exhaust steam are 1315 and 1042 B.Th.U. per lb., and the internal efficiency will be

$$\frac{1315-1042}{392}=\frac{273}{392}=0{\cdot}696$$

THE REGENERATIVE PRINCIPLE

For practical reasons the perfect regenerative cycle discussed on p. 208 is never attained in practice, and a close approximation to it is only attempted in the steam turbine.

The modified cycle applied to a compound engine or two-stage turbine is shown in the Tϕ diagram, Fig. 117. *abcd* represents the diagram for the high-pressure cylinder, *cd* showing the exhaust into the intermediate receiver. Here a portion of steam represented by the change in volume from *c* to *e* is removed to heat the feed-water, the remainder going through the cycle *defg* in the low-pressure cylinder.

It must be remembered that this diagram is built upon the basis of 1 lb. of steam, and that the amount of steam passing through the two

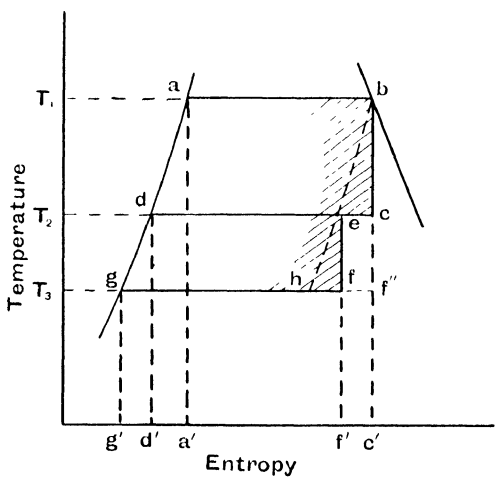


Fig. 117.

cylinders is different. The diagram thus modified, as by the change in volume *ce* which is effected by withdrawal of steam and not by condensation, must therefore be taken as representing heat quantities and *not* the actual conditions of the steam at the various points, i.e. if 1 lb. of steam passes through the high-pressure cylinder and *w* lb. of steam are extracted from the receiver, the heat lost by the receiver will be the area *cef'c'*, but the remaining steam will still have the quality given by the point *c*, *not* by the point *e*.

The calculations of work done and of efficiency are most conveniently performed if $1+w$ lb. are assumed to be supplied to the high-pressure

[1] " Stage " may here have a different meaning from that given on p. 235. The stages may consist of the high-pressure and low-pressure sections of a turbine.

cylinder, and w lb. is extracted at T_2, leaving 1 lb. to pass through the low-pressure cylinder. The quantities involved are readily found without reference to the diagram, Fig. 117, to avoid the complexities stated above. Let R be the heat removed from w lb. of extracted steam and passed to the 1 lb. of feed-water initially at T_3, which results from the final condensation process. To the feed thus raised to temperature T_2 is now added w lb. of water at T_2 condensed in the feed heater, leaving $1+w$ lb. to pass on to the boiler.

The heat received from outside the system is that required to change $1+w$ lb. of water at T_2 to steam at T_1, and the heat rejected from the system is the latent heat in 1 lb. of steam at temperature T_3 and of a dryness fraction obtained by adiabatic expansion from T_1.

Thus work done=heat supplied−heat rejected

and $$\eta=\frac{\text{work done}}{\text{heat supplied}}$$

$$\text{Now work done}=W=(1+w)(L_1+T_1-T_2)-q_3L_3 \quad . \quad . \quad (1)$$

the value of q_3 being given by

$$\frac{q_3L_3}{T_3}=\log_\epsilon\frac{T_1}{T_3}+\frac{L_1}{T_1} \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Also for w lb. performing a closed cycle between T_1 and T_2 and giving up the heat R during condensation at T_2,

$$w\log_\epsilon\frac{T_1}{T_2}+w\frac{L_1}{T_1}-\frac{R}{T_2}=0$$

i.e. the entropy of the closed cycle is constant,

or $$w=\frac{R/T_2}{\log_\epsilon T_1/T_2+L_1/T_1}$$

But $$R=T_2-T_3$$

or $$w=\frac{1-T_3/T_2}{\log_\epsilon T_1/T_2+L_1/T_1} \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Substituting the value of q_3 from (2) in the equation (1) and deducing the efficiency

$$\eta=\frac{(1+w)(L_1+T_1-T_2)-T_3(\log_\epsilon T_1/T_3+L_1/T_1)}{(1+w)(L_1+T_1-T_2)}$$

$$=1-\frac{1}{1+w}\cdot\frac{T_3(\log_\epsilon T_1/T_3+L_1/T_1)}{L_1+T_1-T_2} \quad . \quad . \quad . \quad . \quad . \quad (4)$$

where w is given by equation (3).

If steam tables are available equation (4) can be solved with greater ease and accuracy if written in the form

$$\eta = 1 - \frac{T_3(\phi_{s_1} - \phi_{w_3})}{(1+w)(H_1 - h_2)} \quad . \quad . \quad . \quad . \quad . \quad (5)$$

where $$w = \frac{\frac{h_2 - h_3}{T_2}}{\phi_{s_1} - \phi_{w_2}}$$

The corresponding formula for the Rankine cycle is

$$\eta = 1 - \frac{T_3(\phi_{s_1} - \phi_{w_3})}{H_1 - h_3}$$

The actual efficiency will be slightly less as the feed cannot be heated in practice quite to T_2.

EXAMPLE.—A steam turbine works between pressures of 200 lb. per sq. in. abs. and 2 lb. per sq. in. abs., and steam is bled off at 58 lb. per sq. in. abs. for feed heating. Calculate the efficiency of the cycle and compare it with that of the Rankine cycle.

Using the values obtained from the steam tables

$$\text{Work done} = \text{heat supplied} - \text{heat rejected}$$
$$= (1+w)(L_1 + h_1 - h_2) - q_3 L_3 \quad . \quad . \quad . \quad (1)$$

q_3 being the final quality after adiabatic expansion, and is found, from the Mollier diagram, to be 0·786

or $$W = (1+w)(843{\cdot}2 + 354{\cdot}9 - 259{\cdot}8) - 0{\cdot}786 \times 1021$$

or $$W = (1+w)(938{\cdot}3) - 803$$

Now $$R = h_2 - h_3 = 259{\cdot}8 - 94{\cdot}0.$$

The entropy gained by the feed, reckoned above T_2, is then

$$\frac{259{\cdot}8 - 94{\cdot}0}{290{\cdot}5 + 460} = 0{\cdot}221 \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

where $$T_2 = 290{\cdot}5 + 460° \text{ F. abs.}$$

The loss of entropy from w lb. of steam during the discharge of heat in the closed cycle from T_1 to T_2 is

$$w(\phi_{h_1} + \phi_{L_1} - \phi_{h_2})$$
$$w(0{\cdot}5437 + 1{\cdot}0019 - 0{\cdot}4242) = 0{\cdot}221 \text{ from (2)}$$

or $$w = 0{\cdot}197 \text{ lb.}$$

Substituting this value in (1) and also the values $L_1 = 843{\cdot}2$, $h_1 = 354{\cdot}9$, $h_2 = 259{\cdot}8$

$$\eta = \frac{\text{work done}}{\text{heat received}} = \frac{1{\cdot}197 \times 938{\cdot}3 - 803}{1{\cdot}197(843{\cdot}2 + 354{\cdot}9 - 259{\cdot}8)} = 34{\cdot}1 \text{ per cent.}$$

The corresponding Rankine $\eta = \dfrac{1198{\cdot}1 - 897}{1198{\cdot}1 - 94{\cdot}2}$

where $H_1 = 1198{\cdot}1$

$q_3 L_3 = 897$

$h_3 = 94{\cdot}2$

$= 27{\cdot}2$ per cent.

THE REGENERATIVE PRINCIPLE APPLIED TO THE TRIPLE EXPANSION ENGINE OR TO THREE STAGES OF EXPANSION IN A TURBINE

The ideal cycle for this principle is shown on the temperature-entropy diagram (Fig. 118). *bl* is the curve of perfect regeneration, *bc* expansion

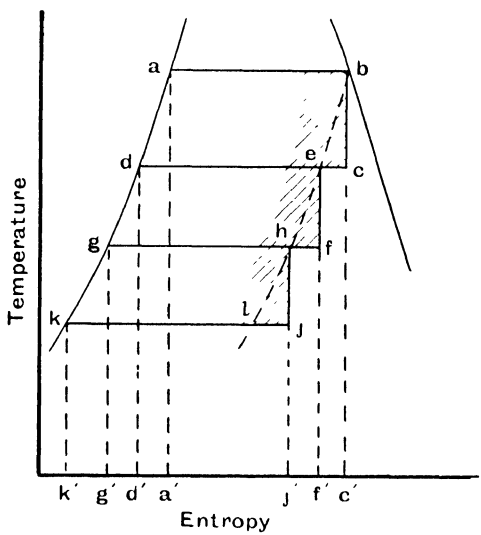


Fig. 118.

in the high-pressure cylinder, *ef* expansion in the intermediate cylinder, and *hj* the expansion in the low-pressure cylinder. The gross amount of heat supplied is represented by the area *k′kabc′*, and since an amount of heat represented by the area *j′hfecc′* is returned to the feed, the net amount of heat supplied is shown by the area *k′kabcefhj′*. The work done is represented by the shaded area, and the efficiency is

$$\eta = \frac{\text{area } kabcefhj}{\text{area } k'kabcefhj'}$$

The caution given in the last section regarding the use of the diagram is relevant in this case also.

Let w_1=weight of steam taken from the first receiver, between the high-pressure and intermediate cylinders, and T_2 its temperature.

R_1=heat supplied by w_1 to the feed.

w_2=weight of steam taken from the second receiver between the intermediate and low-pressure cylinders, and T_3 its temperature.

R_2=heat supplied by w_2 to the feed.

As before, work done=heat supplied−heat rejected

$$=(1+w_1+w_2)(L_1+T_1-T_2)-\text{heat discharged to condenser}$$

$$=(1+w_1+w_2)(L_1+T_1-T_2)-q_4L_4 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

and

$$\frac{q_4L_4}{T_4}=\log_\epsilon \frac{T_1}{T_4}+\frac{L_1}{T_1} \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

For w_1 lb. of steam working between T_1 and T_2 and giving up the heat R_1 during complete condensation at temperature T_2

$$w_1 \log_\epsilon \frac{T_1}{T_2}+w_1\frac{L_1}{T_1}-\frac{R_1}{T_2}=0 \quad . \quad . \quad . \quad . \quad (3)$$

and for w_2 lb. of steam working between T_1 and T_3 and giving up the heat R_2 during condensation at temperature T_3

$$w_2 \log_\epsilon \frac{T_1}{T_3}+w_2\frac{L_1}{T_1}-\frac{R_2}{T_3}=0 \quad . \quad . \quad . \quad . \quad . \quad (4)$$

But $R_2=T_3-T_4$

or

$$w_2=\frac{1-\frac{T_4}{T_3}}{\log_\epsilon \frac{T_1}{T_3}+\frac{L_1}{T_1}} \text{ from (4)} \quad . \quad . \quad . \quad . \quad . \quad (5)$$

and as

$$R_1=(1+w_2)(T_2-T_3)$$

$$w_1=\frac{(1+w_2)\left(1-\frac{T_3}{T_2}\right)}{\log_\epsilon \frac{T_1}{T_3}+\frac{L_1}{T_1}} \text{ from (3)} \quad . \quad . \quad . \quad (6)$$

Substituting the value of q_4 from (2) in the equation for efficiency

$$\eta=\frac{(1+w_1+w_2)(L_1+T_1-T_2)-q_4L_4}{(1+w_1+w_2)(L_1+T_1-T_2)}$$

becomes

$$\eta=1-\frac{T_4\left(\log_\epsilon \frac{T_1}{T_4}+\frac{L_1}{T_1}\right)}{(1+w_1+w_2)(L_1+T_1-T_2)} \quad . \quad . \quad . \quad . \quad (7)$$

where the values of w_1 and w_2 are given by equations (6) and (5).

If steam tables are available

$$w_2=\frac{h_3-h_4}{T_3(\phi_{s_1}-\phi_{w_3})} \text{ from (5)}$$

and

$$w_1=\frac{(1+w_2)(h_2-h_3)}{T_2(\phi_{s_1}-\phi_{w_3})} \text{ from (6)}$$

and the equation (7) for efficiency, becomes

$$\eta=1-\frac{T_4(\phi_{s_1}-\phi_{w_4})}{(1+w_1+w_2)(H_1-h_2)}$$

The modification required if the steam is not initially dry will be obvious.

Theoretically it would be of value to draw steam for feed heating from between every two stages of a multi-stage turbine, but in practice the extraction is made at not more than three to five points on account of the additional mechanical complications involved.

EXAMPLES ON CHAPTER XIII

1. The steam chest pressure in a De Laval turbine is 140 lb. per sq. in. abs., and the exhaust pressure 3 lb. per sq. in. abs., the steam being initially dry and saturated. The peripheral speed of the blades is 1200 ft. per second, and the nozzles are inclined 20° to the direction of motion of the blades. Estimate the angle of the blades, the work done on the blades per second per pound of steam, the absolute velocity of the steam at discharge from the blades and the efficiency of the blades. Neglect frictional losses and assume adiabatic flow.

2. In the turbine given in Example 1 find the blade angles, work done per pound of steam per second, the speed of the blades, and the efficiency if the efficiency is to be the greatest possible. Neglect all losses.

3. Steam of initial pressure 140 lb. per sq. in. abs. and with 160° F. of superheat is supplied to a De Laval turbine with exhaust pressure of 3 lb. per sq. in. abs.; the nozzles are inclined 20° to the direction of motion of the blades and the peripheral speed of the blades is 1200 ft. per second. Estimate the angle of the blades, the efficiency of the blades, and the horse-power developed if the steam consumption is 1800 lb. per hour.

4. Solve Example 1, if the effect of friction is such that the relative velocity of the steam at exit from the blades is 0·9 of the velocity at inlet (i.e. a velocity coefficient of 0·9).

5. One stage of an impulse turbine of the Curtis type has one set of nozzles and three rotating and two stationary sets of blades. Dry saturated steam is expanded in the first set of nozzles from a pressure of 200 lb. per sq. in. abs. to 40 lb. abs. The nozzles are inclined 20° to the direction of motion of the running blades, which have a speed of 500 ft. per second. Determine the angles of the three moving and two stationary sets of blades in this stage, and give the absolute velocity of the steam as it leaves the last set of running blades. Make the inlet and exit angles of all the moving blades the same in each ring.

6. In one stage of a Curtis turbine steam issues from the nozzles with a velocity of 2500 ft. per second. There are two moving rings and one fixed ring of blades in the stage, and the mean peripheral speed of the blades is 500 ft. per second. The exit angles are as follows: From nozzle 20°; from first moving ring 22°; from fixed ring 20°; from second moving ring 24°.

Find the horse-power in the stage per pound of steam per second. Assume a velocity coefficient of 0·8 for the fixed and moving rings.

7. The two nozzles of a single-stage impulse turbine are each 0·20 in. diameter at the throat, and are inclined at an angle of 20° to the path of the blades. The steam chest pressure is 35 lb. per sq. in. abs., the steam being 95 per cent. dry. If 15 per cent. of the total available heat drop is spent in friction, etc. in the diverging part of the nozzles, and the condenser pressure is 2·5 lb. per sq. in. abs., calculate the diameter of the nozzles at discharge, the angle of the blade tips (this angle to be the same at entry and exit), and the horse-power of the turbine. Mechanical losses may be neglected.

8. The blade ring of a stage of an impulse turbine has a mean diameter of 54 in. The outlet blade angle is 45° and the inlet jet angle 25°, the velocity of the jet being 1410 ft. per second. The rotational speed is 2400 r.p.m. and $K=0{\cdot}88$.

Calculate the inlet angle of the blades, and find the horse-power per pound, the axial thrust, and the heat value of the energy discharged.

9. A steam turbine works between pressures of 200 lb. per sq. in. abs. and 2 lb. per sq. in. abs., the steam being initially dry. By plotting values of efficiency from equation (5), p. 260, find the optimum pressure at which the turbine should be tapped in order to supply a single feed-water heater.

10. A triple expansion engine is supplied with steam at 225 lb. per sq. in. abs. and 554° F., the intermediate and condenser pressures being 75, 24, and 22 lb. per sq. in. abs. Find the thermal efficiency and the theoretical consumption (1) when operating on the ideal Rankine cycle, (2) when steam is bled from the intermediate receivers for feed heating, the heater drains being discharged to the hot well.

Chapter XIV

THE INTERNAL COMBUSTION ENGINE

*GENERAL STATEMENT

(For a complete discussion of many of the more involved matters treated in this chapter the reader is referred to " The Internal Combustion Engine," Pye, Oxford, and the Empire Motor Fuels Committee Report, *Proc.I.Aut.E.*, vol. xviii, pt. 1.)

In Chapter III are described a number of thermodynamic cycles for use with a perfect gas as working fluid, and in Chapter VI the employment of certain of these cycles in actual engines is considered on the assumption that the working fluid is air. Though the theoretical efficiency of these engines was good their failure in service could be attributed mainly to the following facts. The maximum temperature of the cycle was, of necessity, less than the safe working temperature of the heating surfaces, the specific heat and density of the working fluid were low, the rate of internal diffusion of heat through the gas was small, and the presence of two stationary gas films on the sides of the heating surfaces made the rate of transference of heat very slow. The engines were therefore of slow speed and great size considering their power, and were generally abandoned after the burning and collapse of the walls of their heating chambers.

By the intimate mixture of the requisite amount of fuel with the air, and its combustion within the cylinder, very high temperatures can be attained and the rate of heating is made independent of internal diffusion. The blanketing of the cylinder walls by the stationary gas films protects them from heat to such an extent that a simple system of air or water cooling keeps them adequately cool and prevents destruction of lubricating oil, while at the same time greatly reducing the heat losses from the working fluid. Many of the faults attendant upon external combustion thus prove the salvation of the internal combustion engine.

The combustion of the fuel within the cylinder naturally raises certain complications. The working fluid is now no longer air, which closely obeys the simple gas laws, but a mixture of gases resulting from combustion and having properties which are often far from " perfect." The final change of pressure or volume theoretically brought about by cooling the working fluid is achieved by exhausting the charge, more or less completely, to the atmosphere, and substituting in its place a fresh cool charge of air and fuel. This exhausting and recharging of the cylinder

following every expansion stroke necessitates a pumping action which can be performed either in the working cylinder, by the addition of two pumping strokes per cycle, making a complete cycle of four strokes, or in a separate cylinder or in the crank-case beneath the working piston, in which case only two strokes of the working piston are required to complete the cycle. These cycles are known as " four-stroke " and " two-stroke " cycles respectively.

A number of cycles have been employed at different times during the development of the internal combustion engine, and several have either been discarded or have had very limited use. Full descriptions of these early cycles are given in many works on I.C. engines, and to these the student is referred for details. Only the constant volume, constant pressure and compound (or " compression ignition "—abbreviated to C.I.) cycles will be considered here. Both historically and for practical purposes a further subdivision has been made into gas engines, oil engines, Diesel and compression ignition engines (sometimes called semi-Diesel), the last two invariably using oil fuel; but it should be noted that the distinctions are dependent on the nature of the fuel used and the method of its employment, and that the cycles themselves can and should be considered together. In the following sections the four-stroke cycle is considered unless the contrary is stated, the evacuation of the working cylinder and its recharge being performed by two additional strokes of the working piston itself.

*THE CYCLES CONSIDERED

The three cycles to be considered are as follows :

(1) **Constant Volume Cycle.**—This cycle, shown in Fig. 40, Chapter III, with the addition of the pumping strokes of exhaust *de* and suction *ed*, both at nearly atmospheric pressure, see Fig. 119, forms the " Otto " cycle upon which, until recently, the vast majority of engines have operated. The cycle is used exclusively for " gas " and " petrol " engines.

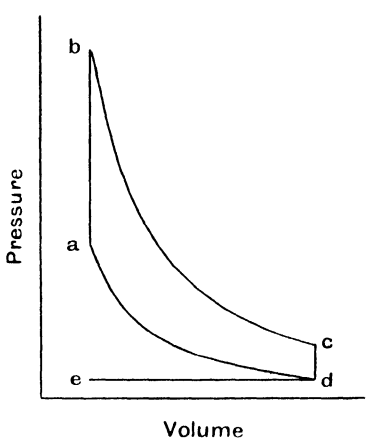


Fig. 119.

As the pumping strokes do not involve heat interchanges and are assumed to be at atmospheric pressure, the formula for the efficiency will correspond to that given in Chapter III, and is

$$\eta = 1-(1/r)^{\gamma-1}$$

EXAMPLE.—A four-stroke petrol engine has a compression ratio of 5 : 1,

the suction pressure and temperature are 14 lb. per sq. in. abs. and 250° F. respectively, and the maximum temperature is 4400° F. If the equations of the compression and expansion curves are $pv^{1\cdot35}=K$ and $pv^{1\cdot28}=K$, calculate the indicated M.E.P. What is the efficiency of a perfect engine working on the corresponding air cycle ?

Here $p_d=14$ lb. per sq. in. and $T_d=250+460=710°$ F.

For the compression stroke $p_a=p_d\left(\frac{v_d}{v_a}\right)^n=14\times(5)^{1\cdot35}$

$=14\times8\cdot78=123$ lb. per sq. in.

and $$T_a=T_d\frac{p_av_a}{p_dv_d}=710\times8\cdot78\times\frac{1}{5}=1247° \text{ F. abs.}$$

For the combustion process $T_b=4860°$ F. abs.

and $$\frac{p_b}{T_b}=\frac{p_a}{T_a} \text{ or } p_b=p_a\frac{T_b}{T_a}=123\times\frac{4860}{1247}=479 \text{ lb. per sq. in.}$$

For the expansion process

$$p_c=p_b\left(\frac{v_b}{v_c}\right)^{1\cdot28}=p_b\left(\frac{1}{5^{1\cdot28}}\right)=\frac{p_b}{7\cdot839}=61\cdot1 \text{ lb. per sq. in.}$$

$$\text{I.M.E.P.}=144\left\{\frac{p_bv_b-p_cv_c}{0\cdot28}-\frac{p_av_a-p_dv_d}{0\cdot35}\right\}\frac{1}{v_4-v_3} \text{ lb. per sq. ft.}$$

$$=144\left\{\frac{v_b}{0\cdot28}(479-5\times61\cdot1)-\frac{v_b}{0\cdot35}(123-14\times5)\right\}\frac{1}{4v_b} \text{ lb. per sq. ft.}$$

$=117$ lb. per sq. in.

Theoretical $\eta=1-\left(\frac{1}{r}\right)^{\gamma-1}$

$=1-\left(\frac{1}{5}\right)^{0\cdot408}$ i.e. $\gamma=1\cdot408$

$=48\cdot1$ per cent.

EXAMPLE.—A petrol engine uses 0·6 lb. of fuel per B.H.P. per hour, the calorific value of the fuel being 19,000 B.Th.U. per lb. The compression ratio is 5 : 1. Find the efficiency of the engine relative to the corresponding air cycle.

Heat equivalent of 0·6 lb. of fuel $=0\cdot6\times19{,}000$ B.Th.U.

and $$\text{heat supply per minute}=\frac{0\cdot6\times19{,}000}{60} \text{ B.Th.U.}$$

But 1 H.P. $=42\cdot42$ B.Th.U. per minute

and $$\text{efficiency of engine}=\frac{42\cdot42\times60}{0\cdot6\times19{,}000}=22\cdot33 \text{ per cent.}$$

$$\text{The air cycle efficiency} = 1 - \left(\frac{1}{5}\right)^{1\cdot4-1} = 1 - (0\cdot2)^{0\cdot4} = 1 - 0\cdot5252$$
$$= 47\cdot47$$

The relative efficiency is $\frac{22\cdot33}{47\cdot47}$ or 47·02 per cent.

(2) **The Constant Pressure Cycle.**—This cycle, when used in practice, is modified not only by the addition of pumping strokes, but by the omission of the " toe " of the diagram, thus making the heating-and-expansion stroke equal in length to the compression stroke.

That some loss of theoretical efficiency is incurred by this omission can be seen at once from Fig. 120 when it is remembered that the determining factor is the " ratio of expansion." If the diagram shown is divided up by a number of additional adiabatic lines, the small diagrams

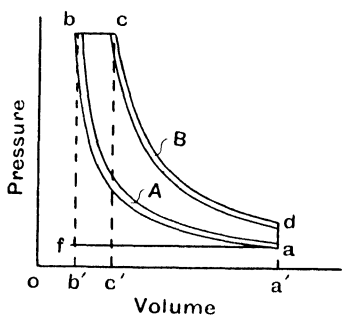


Fig. 120.

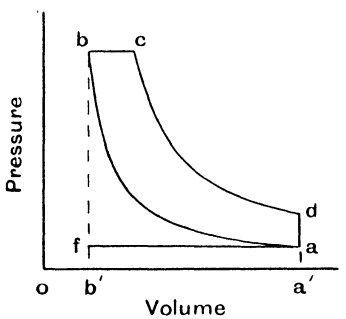


Fig. 121.

obtained, such as A and B, will all represent possible constant pressure cycles from which the " toes " have been removed. It will be seen that though in cycle A the expansion ratio is given by $\frac{oa'}{ob'}$, or roughly 10 : 1, the cycle B has an expansion ratio of only $\frac{oa'}{oc'}$, or roughly 3 : 1. The efficiency of the whole cycle will be a mean of the values for the separate elementary cycles of which it can be considered as composed, and will therefore be appreciably less than the value were the expansion ratio to remain $\frac{oa'}{ob'}$.

The cycle in this form, and when the fuel is sprayed into the cylinder by means of an air blast, is associated with the name of Diesel. In this case air only is drawn into the cylinder from *f* to *a* and compressed from *a* to *b*. The fuel, which always takes the form of a heavy oil, though powdered fuels have been suggested, is first admitted at the point *b* in

the form of a fine spray, the particles of which at once ignite on coming into contact with the air already greatly heated by compression. Injection of the fuel is continued for the portion of the stroke shown by *bc*, the point *c* being known as the "cut-off," after which more or less adiabatic expansion occurs. Injection and atomisation of the fuel is brought about by a blast of air at a pressure considerably in excess of that of compression.

Formula for the Efficiency of the Diesel Cycle.—Referring to Fig. 121

$$\text{Heat received} = C_p(T_c - T_b)$$

$$\text{heat rejected} = C_v(T_d - T_a)$$

and

$$\eta = 1 - \frac{C_v(T_d - T_a)}{C_p(T_c - T_b)} \quad . \quad . \quad . \quad . \quad (1)$$

Expressing the temperatures in terms of T_a

$T_b = T_a r^{\gamma-1}$, where r is the compression ratio $\frac{oa'}{ob'}$

$$T_c = T_b \times \frac{V_c}{V_b} = T_b \times \beta, \text{ where } \beta = \frac{V_c}{V_b}$$

$$= \beta T_a r^{\gamma-1}$$

$$T_d = T_c \left(\frac{V_c}{V_d}\right)^{\gamma-1} = \beta T_a r^{\gamma-1} \left(\frac{\beta}{r}\right)^{\gamma-1}$$

Substituting these values in (1)

$$\eta = 1 - \frac{C_v\left\{\beta r^{\gamma-1}\left(\frac{\beta}{r}\right)^{\gamma-1} - 1\right\}}{C_p(\beta r^{\gamma-1} - r^{\gamma-1})}$$

$$= 1 - \left(\frac{1}{r}\right)^{\gamma-1} \frac{\beta^{\gamma} - 1}{\gamma(\beta - 1)}$$

It will be seen that the efficiency is dependent both on the compression ratio and on the value of β. For a given compression ratio, the higher the temperature at C after combustion, the greater will be the value of β, and as this is of necessity greater than unity, the efficiency will decrease as the maximum temperature increases.

EXAMPLE.—In a Diesel engine the compression ratio is 14·1, and the equation of the expansion and compression curves may be taken as $pv^{1.30} = K$. Combustion occurs during 12 per cent. of the working stroke, and the suction temperature and pressure are 200° F. and 14 lb. per sq. in. respectively. Calculate the maximum temperature and pressure attained, the pressure at release, and the M.E.P.

Referring to Fig. 121

For the compression stroke $p_b v_b^n = p_a v_a^n$

or
$$p_b = p_a\left(\frac{v_a}{v_b}\right)^n$$
$$= 14 \times (14)^{1\cdot3} = 14 \times 30\cdot89$$
$$= 432 \text{ lb. per sq. in.}$$

and
$$T_b = \frac{p_b v_b}{p_a v_a} T_a = \frac{432\cdot5}{14} \times \frac{1}{14} \times 660$$
$$= 1456° \text{ F. abs.} = 996° \text{ F.}$$

(N.B. A solid object at a temperature of 1000° F. or 600° C. is just visibly " red hot " when seen in the dark.)

The maximum temperature $T_c = T_b \times \frac{v_c}{v_b} = 1456 \times 2\cdot56 = 3728°$ F. abs.
$$= 3268° \text{ F.}$$

The release pressure is given by $p_c v_c^n = p_d v_d^n$

or
$$p_c = p_d\left(\frac{v_d}{v_c}\right)^n$$
$$= p_d\left(\frac{14}{2\cdot56}\right)^{1\cdot3} = 9\cdot105 p_d$$

or
$$p_d = \frac{432\cdot5}{9\cdot105} = 47\cdot5$$

Then I.M.E.P. $= \dfrac{\frac{1}{n-1}\{(p_c v_c - p_d v_d) - (p_b v_b - p_a v_a)\} + p_b(v_c - v_b)}{v_a - v_b}$

$$= \frac{\frac{1}{0\cdot3}\{(432\cdot5 \times 2\cdot56 - 47\cdot5 \times 14) - (432\cdot5 \times 1 - 14 \times 14)\} + 432\cdot5 \times 1\cdot56}{13}$$

$= 104\cdot5$ lb per sq. in.

The η is given by $1 - \left(\frac{1}{r}\right)^{\gamma - n} \frac{\beta^\gamma - 1}{\gamma(\beta - 1)}$, where $\beta = \frac{v_c}{v_b}$

Here if $v_b = 1$, $v_c = 1 + \frac{12}{100} \times 14 = 1 + 1\cdot68 = 2\cdot68$

or
$$\beta = 2\cdot68$$

Hence
$$\eta = 1 - \left(\frac{1}{14}\right)^{0\cdot3} \times \left(\frac{2\cdot68^{1\cdot3} - 1}{1\cdot3 \times 1\cdot68}\right)$$
$$= 1 - 0\cdot454 \times \frac{2\cdot6}{2\cdot8} = 1 - 0\cdot454 \times 1\cdot19$$
$$= 1 - 0\cdot542$$
$$= 45\cdot8 \text{ per cent.}$$

The item 1·19 represents a " correction factor " applied to the efficiency of the ideal constant pressure cycle (i.e. with " toe ").

(3) **The Composite or " Compression Ignition " Cycle.**—This is a combination of the above two cycles. Air is drawn into the cylinder along the suction stroke *fa*, Fig. 122, and compressed from *a* to *b*, the compression ratio being of the order of 12 : 1, and the final temperature sufficient to cause ignition of the fuel oil. This is injected into the cylinder between the points *b* and *d* by some form of mechanical pump. The early stages of combustion occur at constant volume from *b* to *c*, but the later stages are prolonged until the piston has moved a short distance from top dead centre, giving a line of nearly constant pressure on the card up to the point *d*.

This cycle has the advantage of " auto ignition," as in the case of the Diesel, but without the complexities attendant on the use of compressed

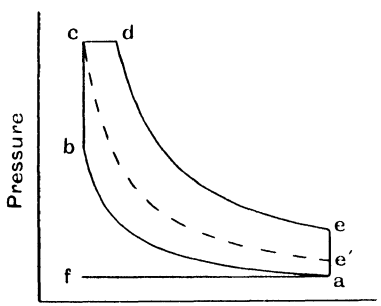


Fig. 122.

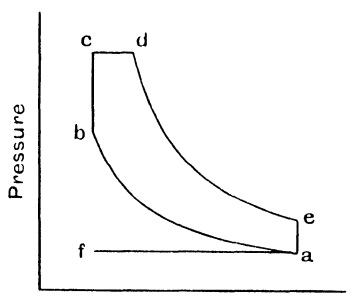


Fig. 123.

air, while, by the use of a lower compression ratio a constant volume effect *abce'*—with corresponding improvement of the efficiency—can be attained without excessively high maximum pressures. It has been found moreover that, at high speeds of revolution, the combustion of the fuel is such that, even in the constant pressure cycle, a considerable portion of the combustion (which is far from instantaneous) does in fact occur at almost constant volume, and the diagram exhibits a peak at dead centre with no well-marked constant pressure phase. These matters will be considered in more detail in later sections.

Formula for the Efficiency of the Composite Cycle.—In this cycle the heat is received in two stages, from *b* to *c* at constant volume, and from *c* to *d* at constant pressure (see Fig. 123). The rejection of heat is at constant volume.

Hence heat received $= C_v(T_c - T_b) + C_p(T_d - T_c)$

and heat rejected $= C_v(T_e - T_a)$

The exhaust and suction strokes are assumed to be at atmospheric pressure and do not affect the calculation.

Then
$$\eta=1-\frac{C_v(T_e-T_a)}{C_v(T_c-T_b)+C_p(T_d-T_c)} \quad . \quad . \quad . \quad . \quad (1)$$

Expressing all temperatures in terms of T_a

$$T_b=T_a r^{\gamma-1}, \text{ where } r=\frac{V_a}{V_b}$$

If
$$\frac{p_c}{p_b}=\alpha=\frac{T_c}{T_b}$$
$$T_c=T_b \cdot \alpha=\alpha T_a r^{\gamma-1}$$

Also
$$T_d=T_c\beta, \text{ where } \frac{V_d}{V_c}=\beta$$
$$=\alpha\beta T_a r^{\gamma-1}$$

and
$$T_e=T_d\left(\frac{V_d}{V_e}\right)^{\gamma-1}=T_d\left(\frac{\beta}{r}\right)^{\gamma-1}$$
$$=T_a\left(\frac{\beta}{r}\right)^{\gamma-1}\alpha\beta r^{\gamma-1}$$

Substituting these values in (1)

$$\eta=1-\frac{C_v\left\{\left(\frac{\beta}{r}\right)^{\gamma-1}\alpha\beta r^{\gamma-1}-1\right\}}{C_v(\alpha r^{\gamma-1}-r^{\gamma-1})+C_p(\alpha\beta r^{\gamma-1}-\alpha r^{\gamma-1})}$$
$$=1-\frac{C_v(\alpha\beta^{\gamma}-1)}{r^{\gamma-1}\{C_v(\alpha-1)+\alpha C_p(\beta-1)\}}$$
$$=1-\left(\frac{1}{r}\right)^{\gamma-1}\frac{\alpha\beta^{\gamma}-1}{(\alpha-1)+\alpha\gamma(\beta-1)}$$

If $\alpha=1$ this formula is identical with that for the Diesel cycle. The values of $\alpha=p_c/p_b$ and $\beta=V_d/V_c$ can be determined approximately in any given case from the indicator diagram if such is available. It should be remembered, however, that as combustion continues for some time after the closing of the fuel value, the point *d* cannot be predetermined accurately from the valve setting, and is not clearly defined in any case.

EXAMPLE.—A compression ignition engine with a compression ratio of 11 : 1 has suction pressures and temperatures of 14 lb. per sq. in. and 200° F.

At light load the diagram is virtually that of the constant volume cycle, and the maximum pressure is 800 lb. per sq. in.

If the fuel supply is increased by 100 per cent. and the injection is timed so that there is no increase of maximum pressure, calculate the

" correction factor " to be applied to the value of the efficiency of the corresponding constant volume cycle, and the theoretical efficiency. Take C_v as 0·1691 and C_p as 0·2375.

For the compression stroke $p_b v_b^n = p_a v_a^n$

or $$p_b = 14 \times \left(\frac{11}{1}\right)^{1\cdot4}$$

$$= 400 \text{ lb. per sq. in.}$$

Also $$\frac{p_b v_b}{T_b} = \frac{p_a v_a}{T_a} \text{ or } T_b = T_a \times \frac{400 \times 1}{14 \times 11} = 660 \times 2\cdot06$$

$$= 1713° \text{ F. abs.}$$

During combustion at constant volume $T_c = T_b \times \frac{800}{400}$

$$= 3426° \text{ F. abs.}$$

and the heat received $= C_v(T_c - T_b)$

$$= 0\cdot1691(3426 - 1360) = 350\cdot0 \text{ B.Th.U. per lb. of air.}$$

With increased fuel supply the heat to be added per pound of air.

$$= 350\cdot0 \text{ B.Th.U.}$$

or $$C_p(T_d - T_c) = 350$$

or $$0\cdot2375(T_d - 3440) = 350\cdot0$$

and $$T_d = 4913° \text{ F. abs.}$$

Also $$\frac{v_d}{v_c} = \frac{4913}{3440} = 1\cdot427 = \beta$$

and as $$\alpha = \frac{p_c}{p_b} = \frac{800}{318} = 2\cdot51$$

$$\eta = 1 - \left(\frac{1}{11}\right)^{0\cdot3} \times \frac{2\cdot51 \times (1\cdot427)^{1\cdot3} - 1}{1\cdot51 + 2\cdot51 \times 1\cdot3 \times 0\cdot427} = 1 - (0\cdot487) \times 1\cdot026$$

$$= 50 \text{ per cent.}$$

Air cycle efficiency = 51·3 per cent.

*THE RATIO OF COMPRESSION

It will be seen from Chapter III that the efficiency of the cycles under consideration is dependent upon the ratio of expansion (or of compression if the two are equal), and not upon the maximum temperature. The latter, however, is relevant as the output of a given engine is the product of the efficiency and the amount of heat received, the latter being a function of the maximum temperature. Though an engine of high compression ratio and low maximum temperature would have a high theoretical efficiency, the output would be small.

Fig. 124 shows the theoretical or " air cycle " efficiency plotted to a base of compression and expansion ratio. The very rapid rise of efficiency at the smaller ratios will be noted, and the much more gradual increase at the higher values.

In practice two factors limit the maximum permissible value of the compression ratio. In the first case the maximum pressure attained in the cylinder has to be considered in connection with the strength and weight of the engine parts, it being noted that at the higher compression ratios the maximum compression pressure increases at an extremely rapid rate, as shown in Fig. 125.

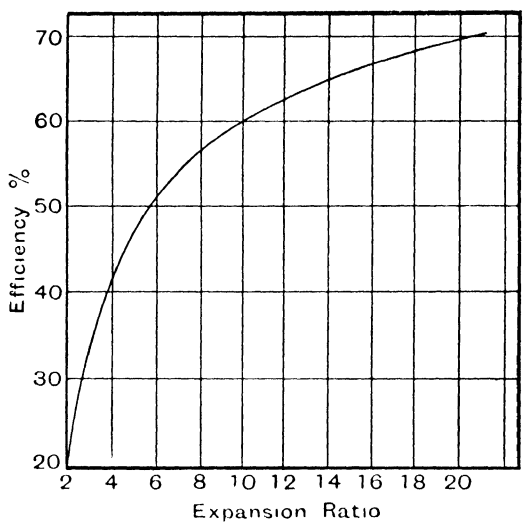


Fig. 124.

The second factor is provided by the nature of the fuel and the method of its use. If the mixing of fuel and air is effected prior to compression, it is essential to insure that spontaneous ignition does not occur during that process, even though the presence of hot exhaust valves, etc., are taken into account.[1] In the " gas " engine, where a mixture with air of producer or coal gas is employed, the compression ratio is limited to about 7 : 1, and in the " petrol " engine to about the same value, except in the case of engines running at extremely high speeds when the initial and final compression pressures and temperatures are below the normal owing to the poor " breathing capacity " at such speeds. The nature of the " gas " or " petrol " also effects the permissible ratio of compression, " doped " petrols permitting rather higher ratios. In the Diesel and compression ignition engines, in which air only is compressed, ratios of

[1] An exhaust valve often operates at a dull red heat.

12–16 : 1 and 10–12 : 1 are used respectively, but owing to the lower efficiency of constant pressure combustion the effective ratios of expansion are distinctly less. The Diesel cycle at normal full load and with a compression ratio of 16 : 1 has an efficiency approximately equal to that of a constant-volume cycle with a compression ratio of 9 : 1.

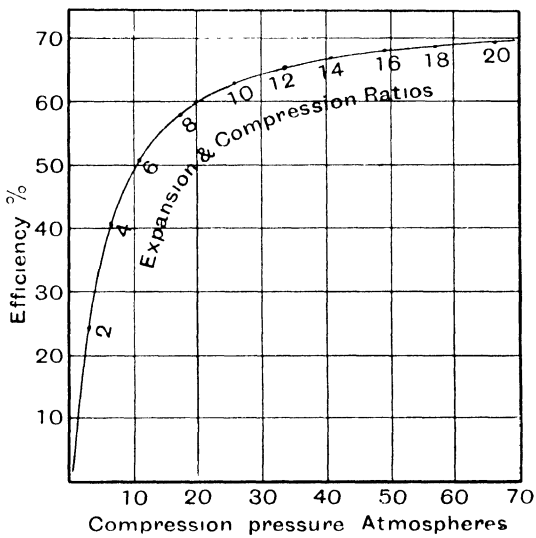


Fig. 125.

THE INFLUENCE OF MAXIMUM PERMISSIBLE PRESSURE

This factor is one of great importance when considering the nature of the cycle to be employed, and the efficiency which may be obtainable in practice. It will be obvious from Fig. 125 that at the higher ratios of compression a slight increase in efficiency can only be obtained at the cost of an altogether disproportionate increase in the maximum compression pressure, and a corresponding increase in the weight of many of the components of the engine and in the mechanical losses.

Fig. 126 shows two diagrams drawn on the assumption that the cylinder pressure must not exceed 1000 lb. per sq. in. $a'b'c'd$ is a constant volume cycle with a compression ratio of 10 : 1, the compression pressure at a' is 366 lb. per sq. in., and heat is added at constant volume from a' to b' until the maximum permissible pressure is reached at b'. $abcd$ is a cycle with a compression ratio of slightly over 20 : 1. As the compression pressure alone now reaches 1000 lb. it is obvious that heat can only be added at constant pressure, and the figure has been drawn on the assumption that the same amount of heat is added in both cycles. For inter-

mediate compression ratios, rising from 10 : 1 to 20 : 1, the amount of heat which could be added at constant volume would steadily diminish, the increasing remainder being added at constant pressure.

Were successively higher compression ratios than 10 : 1 to be used for the constant volume cycle the permissible heat reception would decrease rapidly, until at the ratio of 20 : 1 no heat whatever could be received.

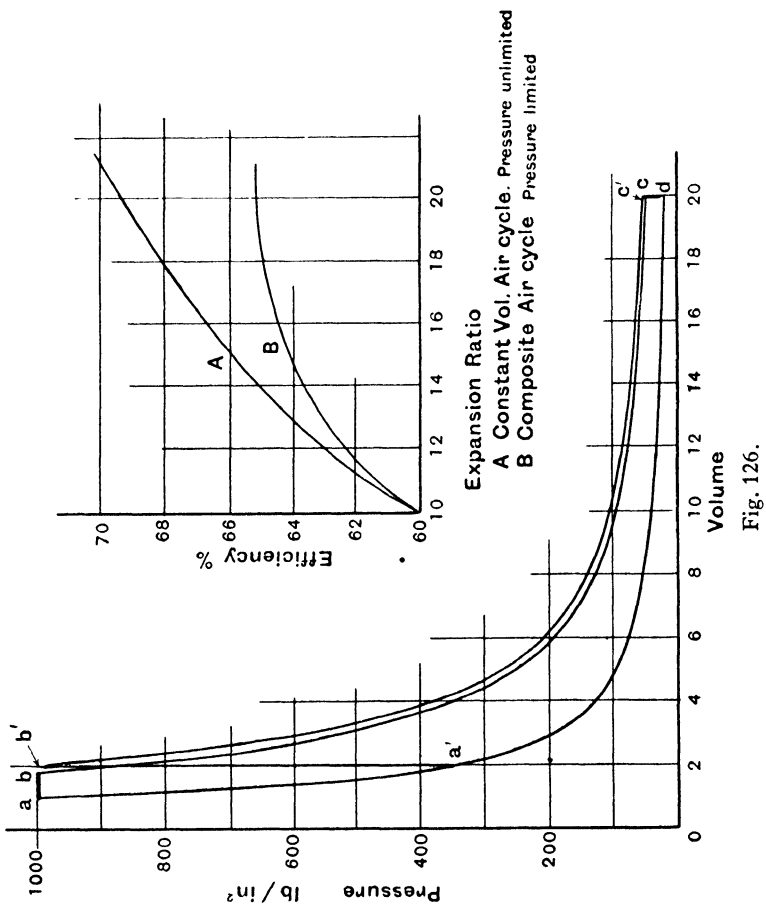


Fig. 126.

At lower ratios than 10 : 1 the heat reception could be augmented but the efficiency would decrease.

The following table, taken from " The Internal Combustion Engine," by Pye, gives the values calculated for a range of composite cycles on the suppositions stated above, i.e. a maximum pressure of 1000 lb. per sq. in., and a constant heat input.

Ratio of compression r	Compression		$\alpha = \frac{P_{max.}}{P_{compn.}}$	T_a	T_b	$\beta = \frac{T_b}{T_a}$	Efficiencies		r/β
	Temp.	Pressure					Composite	Const. vol.[1]	
10	933	366	2·735	2550	2550	1	0·60	0·60	10·00
12	1000	470	2·13	2130	2479	1·165	0·623	0·628	10·30
15	1093	645	1·55	1694	2422	1·434	0·640	0·658	10·45
18	1175	830	1·205	1415	2402	1·70	0·648	0·683	10·58
20	1223	962	1·04	1272	2396	1·88	0·651	0·695	10·63

[1] With unrestricted maximum pressures.

The efficiencies are plotted in Fig. 126. It will be seen that when the maximum pressure is limited the efficiency rises very little for compression ratios above about 15, as can be anticipated from the last column where the value of r/β indicates that the ratio of expansion is almost constant.

When the properties of the actual working fluid in an engine are taken into account, Pye has shown that with a ratio of 15 and a maximum pressure of 915 the theoretical efficiency is 0·508. With unlimited maximum pressure the corresponding maximum efficiency, assuming the constant volume cycle to be used, is 0·55. The increase of 4·2 per cent. in the efficiency is only obtained, however, by an increase of maximum pressure from 915 lb. to 1850 lb. per sq. in.

The effect of the dilution of the incoming charge by the burnt gases remaining in the compression space at the end of the exhaust stroke, which becomes increasingly serious at low compression ratios, is considered on pp. 291 and 304.

COMPARISON OF ACTUAL AND THEORETICAL DIAGRAMS

In general form the actual and theoretical indicator diagrams for the internal combustion cycles do not differ greatly for engines of slow speed. At high speeds a close comparison becomes almost impossible, first, because of the difficulty of obtaining accurate records of extremely rapid pressure changes with an instrument sufficiently robust to withstand accidental detonation pressures, and second, because of the difficulty of phasing the instrument with the engine. The pencil indicator becomes useless at speeds above a few hundred revolutions per minute; optical indicators, though capable of recording at higher speeds, are not free from inertia effects, and, unless built with the piston face or diaphragm flush with the cylinder wall, are subject to serious surges of pressure and other disturbances set up in the connecting passages and chambers; while the electrical indicator employing the cathode-ray tube is not fully developed.

Considerable variations are also found between the records of succeeding cycles,[1] probably due to slight changes in the metering of the fuel and to other more subtle causes. Indicators of the " Farnborough " type are available which give average pressures over a large number of cycles, but the details of performance are lost during the averaging process.

When allowances have been made for experimental inaccuracies, the following deviations from the theoretical pressure volume card are normally found.

(1) The pressure in the cylinder during the suction stroke is slightly less than atmospheric, the depression depending on the speed of the engine, the size and number of inlet valves, the valve timing, and the general layout of the induction system.

The temperature at the end of the suction stroke will, however, be considerably higher than that of the atmosphere, partly owing to the heat received from the hot valves and passages, but chiefly due to the admixture with the fresh charge of a considerable quantity of the products of the previous combustion which remain in the combustion space. For a petrol engine having a compression ratio of 7 : 1 the normal temperature before compression is of the order of 100° C. ; for a ratio of 5 : 1 the temperature will rise, owing to the larger quantity of residual gas, to about 120° C., and will be rather lower than 100° C. for values of compression ratio greater than 7 : 1. These figures allow for cooling by evaporation of the petrol before the closing of the inlet valve, amounting to some 21° C. For benzole the corresponding figure is about 26° C., and for alcohol about 86° C.

The weight of charge induced will thus be less than the theoretical amount, and its volume when reduced to atmospheric temperature and pressure and divided by the piston displacement, is known as the " volumetric efficiency," and is a measure of the " breathing capacity " of the engine. By taking full advantage of the inertia effects in the induction and exhaust pipes it is possible to increase this value appreciably. The volumetric efficiency is always obtained by direct measurement of the charge. (See also Pye, p. 117.)

(2) The compression curve is slightly below the adiabatic, owing to the loss of heat to the cylinder walls and piston, especially during the later stages of the stroke.

On the other hand, if, owing to the high speed of the engine, combustion has to be initiated appreciably before inner dead centre, a slight additional pressure rise may occur as that position is approached.

(3) The maximum temperatures (and in the case of the constant volume cycle the maximum pressures) attained will always fall considerably below the theoretical values owing, as will be shown later, to changes in the specific heat of the working fluid and to " dissociation."

[1] " Notes on the Dalby Watson form of Optical Indicator," Baker, *Proc.I.Mech.E.*, 1923.

A " rounding off " of the peak pressures is normally present, and the cut-off in the Diesel and compression ignition cycles is usually ill-defined.

(4) The expansion curve normally exhibits the strange anomaly of lying above the adiabatic, in spite of the heat losses which take place to the cylinder walls and to the piston. This has been attributable to " after burning " or delay in the completion of combustion, to the slow burning of weaker portions of the mixture or of gases in proximity to the cool walls, etc., but changes of specific heat and " reassociation " to be discussed later, p. 299, probably provide the explanation.

(5) The opening of the exhaust valve in advance of the bottom dead centre, in order to lower the cylinder pressure adequately before the exhaust stroke, results in a rounding of the toe of the diagram. The pressure during this stroke is on the whole somewhat above atmospheric pressure, though surges in the exhaust system may cause marked fluctuation of pressure both above and below atmospheric.

The " negative loop " formed by the suction and exhaust curves represents the work done in charging and discharging the cylinder. The area of this loop should be subtracted from the area of the remainder of the diagram when calculating the indicated horse-power.

DETERMINATION OF THE VALUE OF n FOR THE COMPRESSION AND EXPANSION CURVES

Two methods are available, for use when the clearance volume is and is not known.

(1) *When the clearance volume is known or can be obtained either by direct measurement, or from the drawings of the engine.*

As the expansion and compression curves follow laws of the nature pv^n=constant, $\log p + n \log v$=constant, which is the equation of a straight line. From the actual curves the values of p (absolute) and v (including clearance volume) are scaled off at a number of points, and the values of $\log p$ and $\log v$ are plotted on rectangular co-ordinates as shown in Fig. 127.

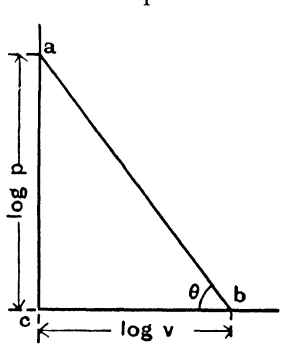


Fig. 127.

The slope of the line, given by $\tan \theta$ or ac/bc, will be the value of n.

It should be noted that the change of the values of p from pounds per square foot absolute to pounds per square inch absolute, or the substitution of lengths scaled from the $p.v.$ diagram in place of the equivalent pressures or volumes, will not affect the slope of the curve, and much work may be saved by the use of the simpler measurements.

Approximate method of finding the clearance volume and the index n from the indicator diagram.

Let Fig. 128 represent the expansion or compression curve. Take

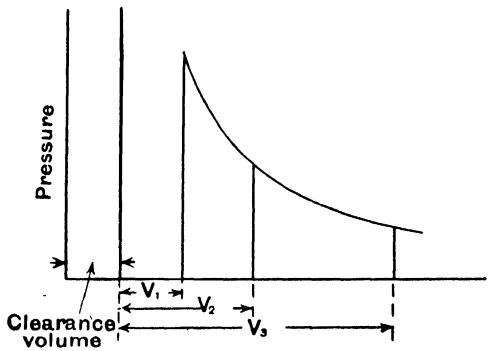


Fig. 128.

any two points 1 and 3, and choose an intermediate point 2 such that $p_2=\sqrt{p_1p_3}$, or the pressure at 2 is the geometric mean of the pressures at 1 and 3.

Let c=clearance (to be found)

p_1 and v_1 be the pressure and stroke volume at point 1

p_2 and v_2 ,, ,, ,, ,, 2

p_3 and v_3 ,, ,, ,, ,, 3

Then, since $p_2=\sqrt{p_1p_3}$

$$\therefore \frac{p_2}{p_1}=\frac{p_3}{p_2}=\text{constant}=k \text{ say} \quad . \quad . \quad . \quad . \quad . \quad (1)$$

If v is the indicated stroke volume, then

The law of curve is $p(v+c)^n=\text{constant}=b$ say

$$\therefore p^{\frac{1}{n}}(v+c)=b^{\frac{1}{n}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

or
$$v=-c+b^{\frac{1}{n}}p^{-\frac{1}{n}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Writing $m=\frac{1}{n}$ for convenience, the volumes at points 1, 2, and 3 are—

$$v_1=-c+b^m{p_1}^{-m} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

$$v_2=-c+b^m{p_2}^{-m} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

$$v_3=-c+b^m{p_3}^{-m} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

From (5) and (6) we have

$$v_3-v_2=b^m({p_3}^{-m}-{p_2}^{-m}) \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

From (4) and (5) we have

$$v_2-v_1=b^m(p_2^{-m}-p_1^{-m}) \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

Hence from (7) and (8)

$$\frac{v_3-v_2}{v_2-v_1}=\frac{p_3^{-m}-p_2^{-m}}{p_2^{-m}-p_1^{-m}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

But by (1) $p_3=kp_2$

and $p_2=kp_1$

$\therefore$ Substituting these values of p_3 and p_2 in (9) we have—

$$\frac{v_3-v_2}{v_2-v_1}=\frac{k^{-m}p_2^{-m}-p_2^{-m}}{k^{-m}p_1^{-m}-p_1^{-m}}$$

$$=\frac{p_2^{-m}}{p_1^{-m}}\left\{\frac{k^{-m}-1}{k^{-m}-1}\right\}$$

$$=\left(\frac{p_2}{p_1}\right)^{-m}=\left(\frac{p_1}{p_2}\right)^{m}=k^{\frac{1}{n}}.$$

$$\therefore \frac{v_3-v_2}{v_2-v_1}=k^{\frac{1}{n}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

Taking logs we have

$$\log\frac{v_3-v_2}{v_2-v_1}=\frac{1}{n}\log k$$

or

$$n=\frac{\log k}{\log\dfrac{v_3-v_2}{v_2-v_1}} \quad . \quad . \quad . \quad . \quad . \quad (11)$$

Equation (11) gives n in terms of $k=\dfrac{p_2}{p_1}=\dfrac{p_3}{p_2}$ and v_3, v_2, and v_1, all of which are known.

To find the Clearance.—Plotting v vertically and $p^{-\frac{1}{n}}$ horizontally, then by equation (3) the intercept on the axis of y will give the clearance c as shown in Fig. 129, equation (3) being the equation to a straight line. The method of determining the clearance volume by filling with a measured quantity of liquid is very greatly to be preferred as being far more accurate than the above method.

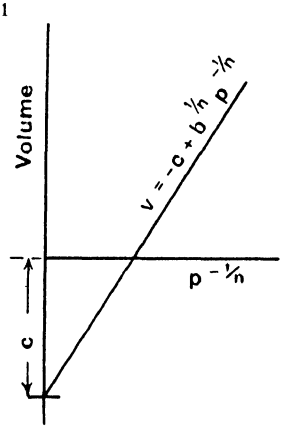


Fig. 129.

CRANK ANGLE AND TIME-BASE DIAGRAMS

Owing to the very great difficulties of obtaining, at all but slow rates of revolution, an accurate record of pressure phased exactly with an accurate reproduction of the piston displacement, little use is made of the indicator card merely to study the I.H.P., however useful that might be. It is unfortunate that, as the piston is virtually stationary during most of the process of combustion, the *p.v.* card gives little information relating to combustion phenomena. By setting the indicator driving-gear 90 angular degrees out of phase with the engine crank the dead centre of the engine is made to occur at the centre of the card, and a diagram of the form shown in Fig. 130 is obtained. Here ignition is

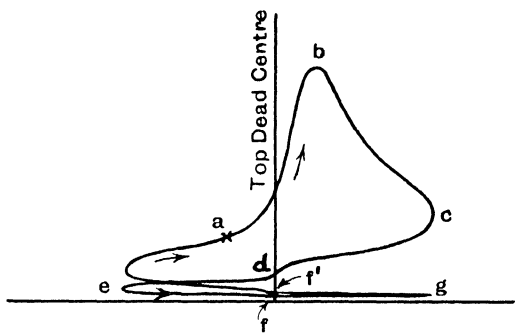


Fig. 130.

commenced at point *a*, well before T.D.C., in order that the peak pressure *b* shall occur early in the expansion stroke. This latter is represented by *bcd*, the exhaust stroke by *def*, suction by *fgf'*, and compression from *f'* to T.D.C.

The diagram is of considerable interest as it enables the process of combustion to be studied with considerable exactness, but the horizontal scale is variable and the difficulties of accurate phasing remain.

Diagrams both simpler to obtain, and of greater accuracy, can be produced either on a crank-angle base (see the Farnborough indicator) or on a time base, the latter being very simply applicable to indicators of the cathode-ray type.

Fig. 131 shows a diagram on a crank-angle base, the letters indicating

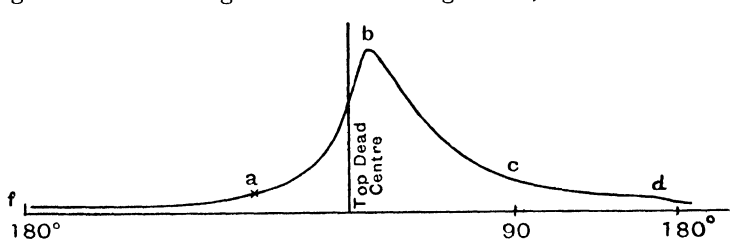


Fig. 131.

points corresponding to those on Fig. 130. This form of diagram enables the whole cycle to be studied with the full accuracy warranted by the pressure recording, and is of very great value. The time-base card is exactly similar to the above provided that the engine runs with no appreciable cyclic variation of speed.

THE Tϕ DIAGRAM

The theoretical Tϕ diagrams for the three cycles considered will be built from lines of constant volume, pressure, and entropy, and are shown diagrammatically in Fig. 132.

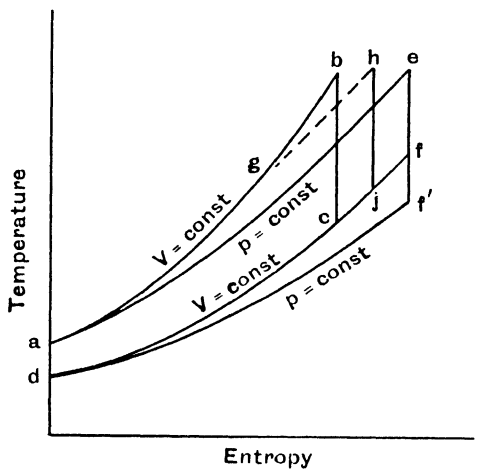


Fig. 132.

Here *da* represents the gain in temperature during adiabatic compression from the suction temperature at the point *d*.

From both *a* and *d* are drawn lines of constant volume *ab*, *df*, and of constant pressure *ae* and *df'*. The theoretical equations for these curves are

$$(1)\quad \phi_2-\phi_1=C_v \log_\epsilon \frac{T_1}{T_2}$$

where $\frac{T_1}{T_2}$ is the ratio of the absolute temperatures,

and

$$(2)\quad \phi_2-\phi_1=C_p \log_\epsilon \frac{T_1}{T_2}.$$

It will be remembered, from p. 114, that all lines of constant volume are similar except for a displacement parallel to the axis of entropy, the same applying to lines of constant pressure.

Adiabatic expansion will be represented by lines parallel to the axis of T.

The constant volume cycle will be given by *ab* (heat reception at constant volume), *bc* (adiabatic expansion), *cd* (exhaust or cooling at constant volume), and *da* (adiabatic compression).

The full constant pressure cycle with constant pressure exhaust will be given by the figure *aef'd*, and the Diesel cycle, with constant volume exhaust, by *aefd*. These two cycles have been drawn, for convenience, with the same maximum temperature.

The composite or C.I. diagram, similarly limited, will show heat

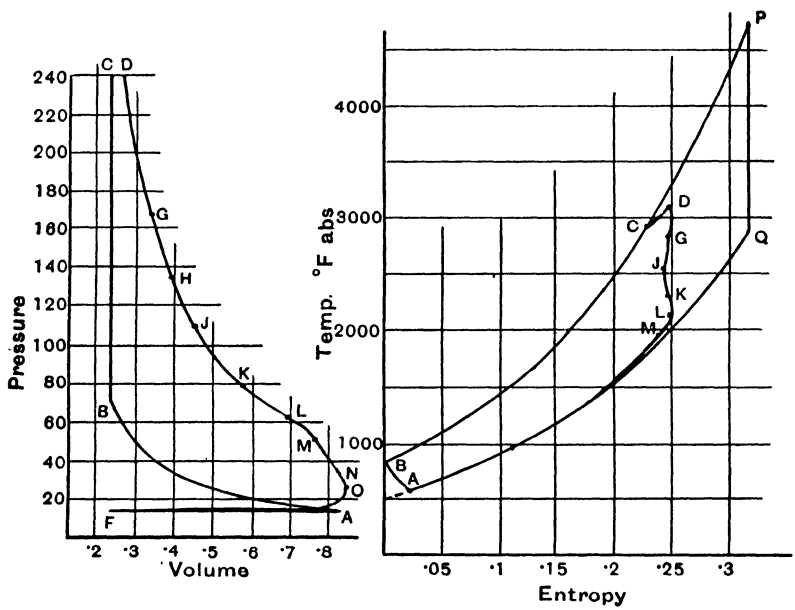


Fig. 133.

reception at constant volume from *a* to *g*, and at constant pressure from *g* to *h*, the whole cycle being shown by *aghjd*.

These diagrams have all been drawn for a common compression ratio, i.e. for a common rise of temperature *da* during compression. This condition is unlikely to be met in practice, as shown on p. 275, but the necessary adjustments, which would add considerably to the complexity of the figure, are not difficult to apply.

The discrepancies between the theoretical and actual *p.v.* diagrams will be reflected in the corresponding Tϕ diagrams, the most noticeable deviations being with regard to the maximum temperatures.

Fig. 133 from Golding's " Theta-Phi (i.e. Tϕ) Diagram " shows *p.v.* and Tϕ diagrams for a gas engine, the lettering on the two figures indicating

corresponding points. On the Tϕ diagram the heat loss during compression is given by the area under the curve AB. The curves of constant volume through B and A are extended to P and Q, where T_p is the final temperature after combustion, as calculated from the known calorific value and weights of fuel and air, and assuming the working fluid to be a perfect gas which, as shown later, is very far from being true.

The Tϕ diagram has little practical application for several reasons :—

(1) It implies a knowledge of the temperatures throughout the cycle, though these are not usually known or accurately calculable.

(2) The working fluid is not air, but either a mixture of a number of gases resulting from combustion or, during compression, of air or air mixed with various combustible gases or vapours. The actual mixtures used change not only with changes of fuel, but also with operating conditions.

(3) The mixture forming the working fluid is far from being a perfect gas, and normally has a specific heat which rises rapidly with temperature. The simple equations (1) and (2) on p. 283 do not then apply and the corresponding accurate expressions are cumbersome. Moreover, as the specific heat varies, the areas on the diagram are not numerically proportional to the corresponding temperature changes.

The comparatively extreme simplicity of the problems presented by the use of steam will be obvious.

*RATE OF HEAT RECEPTION OR REJECTION DURING EXPANSION AND COMPRESSION

If the laws of expansion and compression can be determined from an indicator card, as shown above, and if the specific heats of the working fluid are known, the volumetric rate of heat reception or rejection per cubic foot or per second can be estimated, as indicated on p. 24, from the equation $\frac{dH}{dv}=p\frac{\gamma-n}{\gamma-1}$.

Normally, $n<\gamma$ for compression, indicating heat loss, and $n>\gamma$ during expansion, indicating that though heat is being lost to the jackets it is being more than replaced owing to factors not yet discussed (see p. 299).

EXAMPLE.—The cylinder of a gas engine is 9½-in. bore, 19-in. stroke, and clearance volume 272 cu. in. The barometric pressure is 15 lb. per sq. in. The temperature at the beginning of compression is 180° F., $C_p=0{\cdot}24$, $C_v=0{\cdot}17$, and the law of expansion curve is $pv^{1{\cdot}292}=$constant.

The pressure at a point A early in the expansion stroke is 475 lb. gauge, the displacement volume being 0·023 cu. ft., and at the opening of the exhaust, B, the pressure is 50 lb. gauge and the displacement volume 0·719 cu. ft.

Calculate (*a*) the temperatures at the above points, (*b*) the heat given

to or taken from the gases between them, and (*c*) the rate of heat reception between them.

(*a*) The clearance volume=272 cu. in. or $\frac{272}{1728}$=0·157 cu. ft.

The piston displacement=$\frac{0{\cdot}7854\times(9{\cdot}5)^2}{144}\times\frac{19}{12}$=0·779 cu. ft.

and the volume at the beginning of compression

=0·157+0·779=0·936 cu. ft.

At A p_a=475+15=490 lb. per sq. in. abs.
and v_a=0·157+0·023=0·180 cu. ft.
At B p_b=50+15=65 lb. per sq. in. abs.
and v_b=0·157+0·719=0·876 cu. ft.

As $$\frac{pv}{T}=\frac{p_av_a}{T_a}$$

i.e. $$\frac{15\times144\times0{\cdot}936}{180+460}=\frac{490\times144\times0{\cdot}18}{T_a}$$

Therefore $$T_a=\frac{490\times0{\cdot}18\times640}{15\times0{\cdot}936}=4026°\text{ abs.}=3565°\text{ F.}$$

Also as $$\frac{15\times0{\cdot}936}{640}=\frac{65\times0{\cdot}876}{T_b}$$

Therefore $$T_b=\frac{640\times65\times0{\cdot}876}{15\times0{\cdot}936}=2599°\text{ abs.}=2138°\text{ F.}$$

The calculated temperature (maximum) at A=3565° F.
,, ,, ,, ,, B=2138° F.

(*b*) The heat added is H=work done$\times\frac{\gamma-n}{\gamma-1}$

In this example $$\gamma=\frac{C_p}{C_v}=\frac{0{\cdot}24}{0{\cdot}17}=1{\cdot}41$$

and work done from A to B=$\frac{p_av_a-p_bv_b}{n-1}$ ft.-lb.

$$=\frac{490\times0{\cdot}18-65\times0{\cdot}876}{1{\cdot}292-1}\times144$$

$$=\frac{88{\cdot}20-56{\cdot}94}{0{\cdot}292}\times144=\frac{31{\cdot}26\times144}{0{\cdot}292}$$

=15,430 ft.-lb.

$$=\frac{15{,}430}{778}=19{\cdot}85\text{ B.Th.U.}$$

Therefore heat added during expansion $=19{\cdot}85\times\frac{1{\cdot}41-1{\cdot}292}{0{\cdot}292}$

$$=+19{\cdot}85\times\frac{0{\cdot}118}{0{\cdot}292}=7{\cdot}75 \text{ B.Th.U.}$$

(*c*) Rate of heat reception $\frac{d\mathrm{H}}{dv}=p\times\frac{1{\cdot}41-1{\cdot}292}{0{\cdot}292}=0{\cdot}405p.$

ACTUAL AND THEORETICAL EFFICIENCIES. THE EFFECT OF HEAT LOSSES

If the performance of an actual engine in first-class order is determined for different compression ratios, it will be found that the efficiency calculated on the I.H.P. follows very closely the form of the curve given by $\eta=1-\left(\frac{1}{r}\right)^{\gamma-1}$, but that for all ratios the actual efficiency is approximately 70 per cent. of that for the theoretical " air cycle." Several major factors may influence the discrepancy, and the more obvious of these will now be considered.

In any actual engine a considerable loss of heat takes place from the working fluid by transference through the cylinder walls. This loss is incurred solely because it is impracticable to use walls of a non-conducting material, and, in theory at least, is preventable. Such preventable losses differ fundamentally from the exhaust loss which plays, at least in large measure, a fundamental part in any cycle working with a minimum temperature above the absolute zero or with a restricted cylinder capacity.

In low-compression engines, in which large areas of cylinder wall are exposed to the heated gases, the heat lost to the jacket water may considerably exceed in amount the heat equivalent of the B.H.P. In high efficiency engines with compact combustion chambers the jacket loss may vary from about 60 to 90 per cent. of the B.H.P. A close analysis of this loss is desirable to determine what increase in B.H.P. might result from the partial or total suppression of " jacket " losses. It should be remembered in this connection that the theoretical efficiency of an engine is dependent on the compression ratio and not upon the maximum temperature, so that by retaining a high compression ratio and successively lowering the temperature of combustion the theoretical efficiency could be maintained (though the output of the engine would fall), and the heat losses would simultaneously be reduced and might reach almost negligible proportions. Such an engine has in fact been produced,[1] though the difficulties of attaining low-temperature combustion are considerable.

[1] Ricardo, *Automobile Engineer*, Sept., 1922.

The heat-balance produced as the result of a bench test normally takes the form

Heat supplied = I.H.P. + jacket loss + exhaust and radiation (by difference), or

= B.H.P. + friction + jacket loss + exhaust and radiation (by difference) (i.e. "exhaust and radiation" are obtained by subtracting the I.H.P. and jacket loss from the heat supplied).

This analysis is in many ways very unsatisfactory, for, though the heat supplied (i.e. weight of fuel consumed × calorific value) can be obtained with a considerable degree of accuracy the other items are open to doubt for the following reasons.

I.H.P.—Even at slow speeds the indication of I.C. engines is far from satisfactory, the minimum error likely to be met being of the order of 5 per cent., and the maximum, at high speeds, very much greater.

B.H.P.—This can be obtained with considerable accuracy by using an electric or hydraulic dynamometer.

Friction.—This cannot be obtained satisfactorily as the difference of I.H.P. and B.H.P. owing to the probable errors in the I.H.P., a 5 per cent. error in the latter resulting in an error of about five times that amount in the friction, which is about one-fifth of the I.H.P.

It may be considered, with moderate accuracy, as the power required to motor the engine at full speed and when hot. It should be remembered, however, that the piston will then be lubricated with fresh oil which has not been subjected to the heat of combustion, and the friction will therefore be considerably less than normal. To offset this, however, the pumping losses will generally be excessive when motoring, owing to the absence of the scavenging effect automatically produced by the surge of high-pressure gases through the exhaust system.

An approximation to radiation and friction losses may also be obtained by subtracting the (measured) exhaust and jacket losses from the heat supplied in a zero load trial at full speed and jacket temperature.

Exhaust Loss.—An approximation to this heat loss can be made by estimating, from the indicator card, the temperature at the instant of the opening of the exhaust valve, and assuming the specific heat of the gases. The method is generally unsatisfactory, however, and an exhaust gas calorimeter should be used wherever possible. A thermometer placed in the exhaust pipe is almost valueless for this purpose owing to the irregularity of the flow of the gas, and probable radiation or conduction losses. The exhaust loss obtained "by difference" will include the calorific value of any fuel only partially burnt, a case very frequently met in petrol engine work (see p. 300).

Jacket Loss.—Though the heat rejected in the jacket water is easy to determine by direct measurement, the quantity so measured includes a

considerable portion of the heat generated by piston friction, which, as it has already figured under friction, is therefore counted twice. The jacket loss nearly always includes heat lost by the exhaust gases as they leave the cylinder, the exhaust bend being normally enclosed in the same jacket as the cylinder. A separate exhaust-bend jacket has been employed on occasion.[1]

The importance of the remaining loss depends upon the way in which the heat transfer has been distributed throughout the cycle.

Using the best methods available it is possible to draw up a fairly accurate heat balance in which all the items have been obtained by, or as the result of, direct measurement. Such a balance, for a petrol engine with 5 : 1 compression ratio, is as follows :

I.H.P., i.e. B.H.P.+friction, 32 per cent. Jacket loss, 28 per cent. Exhaust and radiation, 40 per cent.

The second item includes :

Heat lost by radiation, conduction, and convection during combustion (*a*),
,, during expansion (*b*),
,, after opening of the exhaust valve and during the exhaust stroke (*c*),
,, at exhaust bend (*d*),
,, due to engine friction (*e*).

Considering these items separately we have the following facts :

(*a*) The temperature will be high—say 2300° C. to 2500° C., and turbulence will be intense, giving large losses. Any heat saved before expansion commences could be converted into work with a theoretical efficiency given by $1-\left(\frac{1}{r}\right)^{\gamma-1}$, or 40 per cent. approximately.

(*b*) The importance of this loss is dependent on the manner of its distribution throughout the stroke. At first the temperatures will be high but the exposed wall areas will be small, and later on the position will be reversed, though the final temperature will be higher than might be anticipated, about 1650° C., owing to dissociation and later recombination of the gaseous mixture (see p. 299). The probable efficiency of conversion is not greater than 20 per cent.

(*c*) and (*d*) Temperatures will be low but exposed areas large. The loss at the exhaust bend, owing to the high rate of scour, may be as high as 50 per cent. of all jacket losses. The working stroke being over, none of the heat could be conserved and utilised.

(*e*) Friction losses are non-recoverable and probably form an appreciable portion of jacket losses.

[1] See Gibson, *Proceedings I.Mech.E.*, 1915.

Referring to the heat balance quoted above, we then have the following probable values :

	Probable proportion of heat supply, per cent.	Possible efficiency of conversion, per cent.	Possible conversion to I.H.P. Percentage of heat supply
Item (*a*)	6	40	2·4
Item (*b*)	7	20	1·4
Items (*c*) and (*d*) . .	15	none	none
	Total 28		Total 3·8

The maximum possible increase of I.H.P., were the "jacket" losses to be entirely suppressed, would then be from 32 to 35·8 per cent., or 12 per cent. This neglects the fact that the suppression of jacket losses would be accompanied by higher temperatures and higher "dissociation" losses, bringing the gain of I.H.P. to not more than 10 per cent.

EFFECT OF JACKET WATER TEMPERATURE

It would appear that jacket losses could be reduced by increasing the temperature of the jacket and so reducing radiation and conduction losses.

Taking 22° C. and 150° C. as extreme values for the temperature of the cooling medium, the maximum temperature difference between this and the heated gases would be of the order of 2000–22° C., and the minimum about 2000–150° C., a variation of about 4 per cent. Taking the gain of I.H.P. due to the total suppression of jacket loss as 10 per cent., then the extreme variation of temperature can only be expected to give a direct saving of the order of 4 per cent. of 10 per cent., or 0·4 per cent. of the I.H.P.

Three secondary effects will be produced, however.

(1) Effect on Volumetric Efficiency. The charge, on its entry into the cylinder, and before the inlet valve is closed, comes into contact with the heated walls and expands, thereby reducing the volumetric efficiency. Taking the rise of temperature of the charge due to this cause to be 30° C. with well-cooled walls, and 44° C. with hot walls, and assuming the mean temperature of the charge in the latter case to be 116° C. (389° C. abs.), the hotter walls will correspond with a change of volume of $\frac{389-375}{375} \times 100$ per cent., or 3·7 per cent., and the change in volumetric efficiency will be a decrease of this amount.

(2) An increase in temperature of the cylinder walls will produce a

corresponding decrease in the viscosity of the lubricating oil and an appreciable reduction of piston friction. The resulting increase of I.H.P. may be about 5 per cent., but is largely dependent on the type of piston.

(3) Dissociation will be slightly increased at higher jacket and cycle temperatures.

The net effect of raising the jacket temperature is largely determined by item No. 2, and normally shows a gain of I.H.P.

THE CALCULATION OF CYCLE TEMPERATURES

The calculation of the temperatures attained during the cycle is dependent primarily on the temperature of the charge before the commencement of compression. This in turn is determined by the temperature of the air, the temperature and relative quantity of the gaseous fuel, if used, the temperature and relative quantity of the residual gases remaining in the combustion chamber at the end of the exhaust stroke, and the possible cooling effect brought about by the evaporation of volatile fuel. The simplest case is that of the gas engine, as the third item does not then apply.

SUCTION TEMPERATURE FOR A GAS ENGINE

Let the suffixes a, g, and r refer to air, gas, and residuals, and let the symbols p, v, and t refer to the actual measurements of pressure, volume, and temperature (absolute). Let ρ indicate density (pounds for cubic foot) at n.t.p., and P_o, V_o, and T_o the pressure, volume, and temperature of the mixture when reduced to n.t.p., i.e. $P_o = 14{\cdot}7 \times 144$ lb. per cu. ft., and $T_o = 273°$ C. abs.

The volume of air measured at p_a and t_a is then v_a, and when reduced to n.t.p. $= v_a \frac{p_a}{t_a} \times \frac{T_o}{P_o}$. Similar values may be obtained for the other constituents of the mixture.

The total volume at n.t.p. is given by

$$V_o = \frac{T_o}{P_o}\left(\frac{p_a}{t_a}v_a + \frac{p_g}{t_g}v_g + \frac{p_r}{t_r}v_r\right)$$

where

$$\frac{T_o}{P_o} = 0{\cdot}1288$$

The mass of the cylinder contents will be

$$W_o = 0{\cdot}1288\left(\frac{p_a}{t_a}v_a\rho_a + \frac{p_g}{t_g}v_g\rho_g + \frac{p_r}{t_r}v_r\rho_r\right)$$

The specific volume of the charge will be

$$V_{sp.}=\frac{V_o}{W_o}$$

and the value of R for the mixture will be

$$R_m=\frac{P_o V_{sp.}}{T_o}=7{\cdot}77V_{sp.} \text{ ft.-lb. per lb. per degree}$$

The absolute suction temperature will then be given by

$$T_s=\frac{\text{pressure at end of suction stroke} \times \text{total cylinder volume per pound}}{R_m}$$

The suction pressure will be almost atmospheric unless the air supply has been throttled for governing purposes. The temperature T_r has been measured for a gas engine by Callendar, using a special thermometer arranged for periodic immersion. The temperature recorded by him for a slow-speed gas engine was 444° C. An approximate value can be obtained from the readings of an exhaust gas calorimeter.

When the suction temperature has been estimated the remaining cycle temperatures can be obtained approximately when the following facts are known :

(1) the value of n for the compression stroke, normally about 1·37 ;
(2) the pressures before and after combustion ;
(3) the value of n for the expansion stroke.

All these values may be obtained from the indicator card.

THE SUCTION TEMPERATURE FOR A PETROL ENGINE

The estimation of the suction temperature for a petrol engine is more complicated than that for a gas engine both on account of the nature of the fuel, and the very severe conditions of temperature and heat flow normally encountered.

The exhaust temperature with normal petrol mixtures will be about 850° C.

The fuel being a liquid which—with the exception of alcohol—is completely evaporated before the closing of the inlet valve, will absorb the necessary latent heat of evaporation either from the charge of air, or from the heated ports, valves, and walls of the cylinder. Assuming the former case the drop in temperature of the air can be determined readily from a knowledge of the mixture strength and the latent heat of evaporation of the fuel.

The mixture strength is usually determined experimentally as follows : The petrol, of known specific gravity, is measured volumetrically on its way to the engine, while the air at known temperature and pressure is

measured either with a gasometer, a meter, or more conveniently by an " air box " (the latter being a box of large capacity from which the air is drawn on its way to the engine, and which is fitted with a calibrated orifice for the admission of air, the pressure drop across the orifice determining the rate of flow). From these measurements the air-fuel ratio by weight may be determined.

The normal air-petrol mixture giving theoretically complete combustion according to the simple chemical formula is about 14·5 : 1 by weight, while maximum power demands a somewhat richer mixture of about 12 : 1. The latent heat of normal petrol is about 75 C.H.U. per lb., for benzene about 95 C.H.U. per lb., and for alcohol about 225 C.H.U. per lb.

The fall in temperature of the air due to evaporation will then be given by

$$\frac{\text{Latent heat of fuel per pound}}{\text{air : fuel ratio}} \times \frac{1}{0{\cdot}2413 \text{ C.H.U. per lb.}}$$

where 0·2413 C.H.U. per lb. is the specific heat of air.

The resulting temperature drop of the air charge will then be about 21° C. for normal petrol, 26° C. for benzol, and about 86° C. for alcohol, for which slightly stronger mixtures are required.

Unfortunately, the problem is complicated by an unmeasured but considerable flow of heat from the hot valves, etc., during aspiration, which brings the temperature of the charge to a considerably higher value than anticipated. An estimate of the temperature of the charge immediately before the closing of the inlet valve, and before admixture with the residuals (including the effect of fuel evaporation), can be made on the assumption that the specific heats of the charge and residuals are equal. Under these circumstances there will be no change of volume of the whole during mixing, and the charge and residuals can be considered as still occupying, after mixture, volumes equal to the stroke volume and clearance volume respectively.

If T_1 is the temperature of the air as measured, usually about 15° C. or 288° C. abs., then the temperature of the air after aspiration but before mixing will be given by T_2, where T_1/T_2=volumetric efficiency. The volumetric efficiency is normally between 82 and 92 per cent., giving a temperature before mixing of between 77° C. and 40° C. in the cases given. The value of 60° C. will be sufficiently close for general use. The supposition of the equality of specific heats is not correct, but it will be found that if an allowance is made for variations—the volumetric heat of the charge being about 20·5 ft.-lb. and for the residuals about 23·5 ft.-lb. per standard cubic foot—the estimated suction temperature before mixture will only be lowered by about 4° C. (N.B. For the purposes of calculation Dr. Pye compares the volume which would be occupied by the charge at n.t.p. with the stroke volume ; his values for volumetric

efficiency are therefore appreciably lower than those based on the actual air temperature.)

The final temperature attained after mixing, and at the end of the suction stroke, can then be obtained as follows :

Taking the previous values of C_v per standard cubic foot, i.e. the mean value for residuals up to 850° C. as about 23·5 ft.-lb., and the mean value for charge up to 100° C. as about 20·5 ft.-lb., and assuming the pressures to be atmospheric, we have :

$$\text{Volume of charge at n.t.p.} = v_c \times \frac{T_o}{t_c}$$

$$\text{Volume of residuals at n.t.p.} = v_r \times \frac{T_o}{t_r}$$

where v_c and t_c, v_r and t_r are the measured volumes and temperatures of the charge and residuals.

As the heat received by the charge=the heat lost by residuals, we have

$$v_c \frac{T_o}{t_c}(T_s - T_c)20{\cdot}5 = v_r \frac{T_o}{t_r}(t_r - T_s)23{\cdot}5$$

giving the temperature T_s of the mixture.

The value of T_s will obviously be affected by the ratio of v_c to v_r, i.e. by the compression ratio.

The following approximate values are representative :

Compression ratio 4 : 1 . . .	T_s=139° C.
5 : 1 . . .	=122° C.
6 : 1 . . .	=111° C.
7 : 1 . . .	=102° C.

These figures neglect the fact that the temperature of the residuals after expansion will be lower when the higher compression ratios are used. It should be noted also that in the case of alcohol, which has a very high latent heat, evaporation is not completed before the inlet valve closes and the cycle temperatures are all reduced as a consequence.

THE CALCULATION OF COMPRESSION TEMPERATURE

For the perfect air cycle the temperature after compression may be calculated from the suction temperature on the assumption that the law of compression is $pv^{1\cdot4}$=constant. In engines compressing a charge of air and gas the specific heats will not normally depart far from those for air alone, the gases concerned being largely diatomic. In the case of the petrol engine the vapour consists largely of heavy compound hydrocarbon molecules, and the resulting value for γ will be approximately 1·34.

This value will be affected by three variables : (1) the strength of the

mixture, and the consequent relative proportion of complex to diatomic molecules; (2) the speed of the engine, and consequently the time during which heat losses may occur; and (3) the degree of turbulence in the cylinder, which will affect the relative rate of movement of the gases over the exposed metal surfaces. Taking these factors into account a value of 1·33 will normally be found reliable. For gas engines the value of 1·37 may be taken, and for high-speed engines compressing air only the value will be about 1·33 for small cylinders and 1·35 for large.

CALCULATION OF MAXIMUM TEMPERATURE

A very rough approximation to the value of the maximum temperature can be found by equating the calorific value of the known weight of fuel to the product of the rise in temperature of the products of combustion, their weight, and their mean specific heat.

A number of additional factors should be noted.

(1) If the mixture is over-rich in fuel, as is usually the case in petrol engines, the whole of the fuel supply cannot be adequately burnt, leading to the formation of appreciable quantities of carbon monoxide, with a consequent loss of heat.

(2) The specific heat of the products of combustion increases rapidly with temperature so that a mean value must be taken.

(3) The effect of " dissociation," to be considered on p. 299, may be very marked, and may result in a lowering of the maximum temperature by about 300° C.

A closer approximation may be made by considering the rise of pressure due to combustion, if this value can be obtained from an indicator card.

VALUE OF n FOR THE EXPANSION CURVE

The process of expansion is, in practice, complicated by a number of factors, including heat loss—which involves questions of speed and turbulence—specific heat variations, and the effects of previous " dissociation " and of reassociation. For a petrol engine the value of $n=1{\cdot}258$ for " correct " mixtures and $n=1{\cdot}296$ for mixtures 20 per cent. " weak " may generally be used.

VARIATIONS OF SPECIFIC HEAT

It has been noted in previous articles that in the case of an actual engine the efficiency and the maximum temperatures attained fall appreciably below those anticipated from the application of the simple gas laws. One factor, which accounts in part for the above discrepancies, is the known variation with temperature of the specific heat of the gases forming the working fluid.

The nature of this variation is shown in the following table of volu-

metric heats, where the mean values over certain stated ranges of temperature are shown in foot-pounds per standard cubic foot. (To convert to C.H.U. per mol. divide by 3·90.)

100° up to	500°	1000°	1500°	2000°	2500°	3000°	Increase, per cent.
Diatomic gases .	20·2	20·6	21·5	22·4	23·4	24·6	22
Water vapour	24·4	27·1	29·8	32·8	37·9	43·6	38
CO_2 . .	32·2	37·25	39·25	40·9	42·4	42·7	33

The extreme variations shown are expressed in the final column as percentages of the first values, from which it will be seen that the variations are far from being negligible.

EXAMPLE.—Find the fractional change of efficiency of the constant volume cycle, assuming $\gamma=1{\cdot}40$ and $r=5{\cdot}96$, corresponding to a 1 per cent. increase in C_v.

$$\text{Efficiency}=E=1-\left(\frac{1}{r}\right)^{\gamma-1}, \text{ where } \gamma=\frac{C_p}{C_v}$$

Now since $$C_p-C_v=R$$

$$\gamma-1=\frac{R}{C_v}$$

$$\therefore E=1-\left(\frac{1}{r}\right)^{\frac{R}{C_v}}, \text{ or } 1-E=\left(\frac{1}{r}\right)^{\frac{R}{C_v}}, \text{ or } \frac{1}{1-E}=(r)^{\frac{R}{C_v}}$$

Differentiating with respect to C_v, we have

$$-\frac{1}{1-E}\cdot\frac{dE}{dC_v}=\frac{R}{C_v^2}\cdot\log_\epsilon r$$

$$\therefore \frac{dE}{dC_v}=-\frac{R(1-E)}{C_v^2}\cdot\log_\epsilon r \quad . \quad . \quad . \quad (1)$$

Or since $$1-E=\left(\frac{1}{r}\right)^{\frac{R}{C_v}}$$

$$\frac{dE}{dC_v}=-\frac{R}{C_v^2}\cdot\left(\frac{1}{r}\right)^{\frac{R}{C_v}}\cdot\log_\epsilon r \quad . \quad . \quad . \quad (2)$$

(1) may also be written

$$dE=-\frac{dC_v}{C_v^2}\{R(1-E)\cdot\log_\epsilon r\}$$

$$\text{i.e. } \frac{dE}{E}=-\frac{dC_v}{C_v}\left\{\frac{R}{C_v}\cdot\frac{1-E}{E}\log_\epsilon r\right\}=-\frac{dC_v}{C_v}\left\{(\gamma-1)\frac{1-E}{E}\log_\epsilon r\right\} \quad . \quad (3)$$

In this case efficiency $E=1-\left(\frac{1}{5\cdot96}\right)^{0\cdot4}=0\cdot499$

$$\therefore \frac{dE}{E}=-\frac{1}{100}\left\{0\cdot4\times\frac{1-0\cdot499}{0\cdot499}\times 2\cdot303\ \log_{10} 5\cdot96\right\}$$

$$=-\frac{1}{100}\left\{\frac{0\cdot4\times0\cdot501}{0\cdot499}\times2\cdot303\times0\cdot7752\right\}$$

$$=-\frac{1}{100}\{0\cdot717\}$$

$$=-0\cdot717 \text{ per cent.}$$

i.e. the efficiency would decrease 0·717 per cent. as the result of C_v increasing 1 per cent.

EFFECT OF A LINEAR VARIATION OF SPECIFIC HEAT WITH TEMPERATURE ON FORMULÆ FOR HEAT INTERCHANGE AND ADIABATIC EXPANSION

The formula $J\frac{dH}{dv}=\frac{\gamma-n}{\gamma-1}p$ has already been proved for constant specific heat.

Let $C_p=\alpha+ST.$

Then as $C_p-C_v=\text{constant}$

$C_v=\beta+ST$

and $R=J\,.\,(\alpha-\beta)$

Let $\alpha/\beta=c$

Then as $\delta H=C_v\delta T+\frac{p\delta v}{J}$

$$J\frac{\delta H}{\delta v}=JC_v\frac{\delta T}{\delta v}+p \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

But as $pv=RT$, or $T=1/R\times(pv)$, and $R=J(\alpha-\beta)$

$$\frac{\delta T}{\delta v}=\frac{1}{J(\alpha-\beta)}\left(p+v\frac{\delta p}{dv}\right) \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Substituting in (1)

$$J\frac{\delta H}{\delta v}=\frac{\beta+ST}{\alpha-\beta}\left(p+v\frac{\delta p}{\delta v}\right)+p$$

$$=p\left(1+\frac{\beta+ST}{\alpha-\beta}\right)+v\frac{\delta p}{\delta v}\cdot\frac{\beta+ST}{\alpha-\beta}$$

$$=\frac{1}{\alpha-\beta}\left\{\alpha p+STp+\beta v\frac{\delta p}{\delta v}+STv\frac{\delta p}{\delta v}\right\} \quad . \quad . \quad . \quad (3)$$

Multiplying outside the bracket by β and dividing inside by the same quantity,

$$J\frac{\delta H}{dv}=\frac{1}{c-1}\left\{cp+v\frac{\delta p}{\delta v}\right\}+\frac{ST}{\alpha-\beta}\left\{p+v\frac{\delta p}{\delta v}\right\} \quad . \quad . \quad . \quad (4)$$

But as $pv^n=K$

$$v\frac{\delta p}{\delta v}=-np$$

or $$J\frac{\delta H}{\delta v}=\frac{c-n}{c-1}p+\frac{ST}{\alpha-\beta}\{p-np\}=\left\{\frac{c-n}{c-1}-\frac{n-1}{\alpha-\beta}ST\right\}p$$

or $\frac{\delta H}{\delta v}\propto p$ when T=constant, and is a linear function of T when $p=$ constant.

The formula for adiabatic expansion is derived from equation (3)

or $$\alpha p+STp+\beta v\frac{\delta p}{\delta v}+STv\frac{\delta p}{\delta v}=0$$

or, collecting terms in $\frac{\delta p}{\delta v}$ and dividing by $(\beta+ST)$

$$v\frac{\delta p}{\delta v}+\frac{\alpha+ST}{\beta+ST}p=0$$

or $$\frac{\delta p}{p}+\frac{\alpha+ST}{\beta+ST}\frac{\delta v}{v}=0$$

or, dividing by $\beta+ST$,

$$\beta\frac{\delta p}{p}+\alpha\frac{\delta v}{v}+ST\left(\frac{\delta p}{p}+\frac{\delta v}{v}\right)=0 \quad . \quad . \quad . \quad . \quad (5)$$

But as $pv=RT$ we have, by differentiation with respect to T

$$\frac{p}{T}\frac{\delta v}{\delta T}+\frac{v(\delta p/\delta T\,.\,T-p)}{T^2}=0$$

or $$\frac{p}{T}\frac{\delta v}{\delta T}+\frac{v}{T}\frac{\delta p}{\delta T}-\frac{vp}{T^2}=0$$

or $$\frac{\delta v}{v}+\frac{\delta p}{p}=\frac{ST}{T}$$

Substituting in (5)

$$\beta\frac{\delta p}{p}+\alpha\frac{\delta v}{v}+ST\frac{\delta T}{T}=0$$

or $$\beta\log p+\alpha\log v+ST=\text{constant}$$

or $$p^\beta v^\alpha e^{ST}=\text{constant}.$$

It will be seen by reference to articles by S. Lees (*Engineering*, Jan., 1915) and W. J. Walker (*Phil. Mag.*, Sept., 1917) that the application of the

equation for even a simple linear variation of specific heat to equations for the various cycles becomes an extremely cumbersome process. More complex laws do, in fact, hold for the gases forming the working fluid, especially for the triatomic gases, and it becomes desirable to work from curves of internal energy and temperature and by indirect methods rather than by an extension of the methods already considered.

DISSOCIATION

The phenomena of "dissociation" is of primary importance in all problems concerning the temperatures and efficiencies of high duty internal combustion engines.

It appears that at high temperatures the bonds which bind simple molecules together to form complex molecules become strained to such an extent that a certain proportion of the molecules, depending on the temperature, actually disrupt, and molecules of the original form will then be found existing side by side with the products of decomposition of similar molecules. In other words, the molecules tend to "dissociate." The process is the reverse of combination by combustion, during which heat is liberated, and is accompanied by an absorption of heat proportional to the amount of dissociation effected. Heat is, of course, an extremely powerful agent for the bringing about of decomposition.

The molecules with which we are chiefly concerned are H_2O and CO_2. In these cases the equations for dissociation, i.e. $2H_2O \rightleftharpoons 2H_2+O_2$ and $2CO_2 \rightleftharpoons 2CO+O_2$, show a change in volume during the process, 2 volumes of gas occupying 3 volumes after decomposition. It will be obvious, therefore, that the effect of high pressure, which naturally opposes any increase in volume, will be to retard dissociation.

The following table shows the percentage of the original gas which dissociates at various temperatures and pressures :

CO_2	at 1500° C.	and 1 atm.	0·1	per cent.	at 1500° C.	and 100 atm.	0·01	per cent.
	,, 3000° C.	,,	54·8	,,	,, 3000° C.	,,	16·9	,,
H_2O	,, 1500° C.	,,	0·04	,,	,, 1500° C.	,,	0·004	,,
	,, 3000° C.	,,	28·4	,,	,, 3000° C.	,,	3·33	,,

The dissociated molecules will only remain in this condition so long as the temperature remains high, and as this falls, as during adiabatic expansion, they will again reunite to form molecules of the original type, with the corresponding liberation of heat. This completion of the combustion process, occurring in an actual engine, will only be achieved at a comparatively late stage of the expansion stroke, and too late for the heat to be utilised to full advantage. The whole process will explain in large measure the drop in maximum temperature (as much as 300° C. can be attributed to this cause) and the raising of the expansion curve above the adiabatic.

In the case of H_2O at any given temperature equilibrium is attained between the dissociated and parent molecules when the partial pressure of the oxygen in the H_2O molecules is balanced by the partial pressure of the oxygen in the atmosphere surrounding them. This latter partial pressure may be due either to the oxygen formed by dissociation of the H_2O, or to oxygen formed in any other way. It will be obvious therefore that dissociation can be very largely prevented by surrounding the H_2O molecules with an atmosphere already rich in oxygen. The same statement applies in the case of carbon dioxide.

An extremely interesting fact now emerges. As both H_2O and CO_2

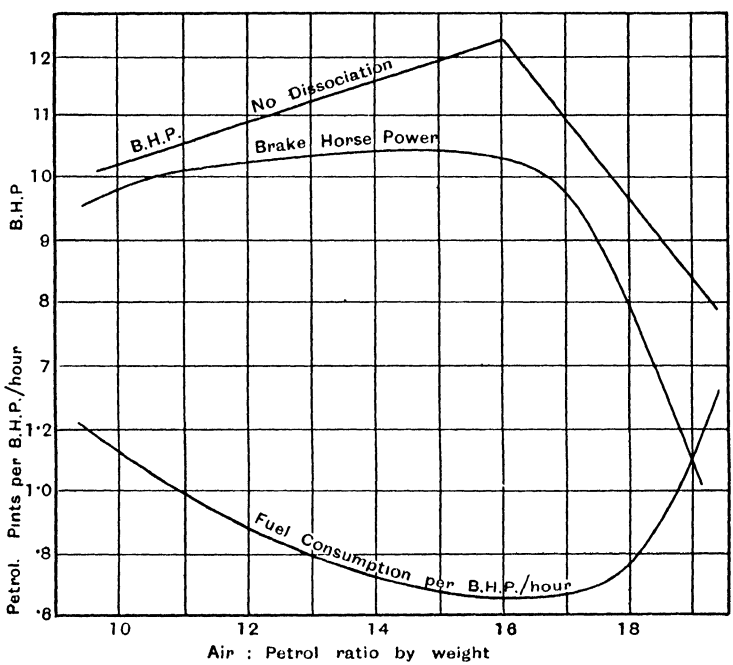


Fig. 134.

liberate O_2 upon dissociation, then, in a mixture of these gases the O_2 liberated by one will tend to prevent the decomposition of the other—i.e. the dissociation of such a mixture is different from the dissociation of the components considered separately. In the same way, as carbon monoxide may be formed in the cylinder of an engine either as the result of dissociation, or as the direct result of supplying an excess of fuel, the use of an over-rich mixture will tend to suppress dissociation and to cause the full calorific value to be obtained from such fuel as is burnt. The result is the well-known phenomena that a petrol engine is deficient in power at the " correct " mixture, when dissociation is unimpeded, but that the

power rises slightly as the mixture becomes richer in fuel and is maintained constant over a very wide range of over-rich mixtures.

It must not be supposed that the intentional production of CO to prevent its unintentional formation is an efficient proceeding, however useful it may be in maintaining maximum power over a wide range, generally an inefficiently controlled wide range, of mixture strengths. This is shown by the lower curve of Fig. 134 giving the petrol consumption in terms of pints per B.H.P. hour, the corresponding B.H.P. curve, and an indication of the form the latter curve would take were there no dissociation.

THE PROCESS OF COMBUSTION IN AIR AND PETROL MIXTURES

In order to understand some of the phenomena occurring in a petrol engine the complex process of combustion must be further understood. In what follows a stagnant charge will be assumed.

After the activation of a minute quantity of the combustible mixture in the immediate neighbourhood of the sparking-plug points, two well-marked phases of combustion can be traced.

(1) The tiny nucleus of flame endeavours to grow as an expanding ball of fire, gaining heat during the process from the combustion of the gases which previously were just outside it. At the same time heat is lost, largely by radiation, and the continued existence of the flame is dependent upon the balance of heat being in its favour. If the flame is not quenched at an early stage the increasing volume : surface ratio gives increasing stability.

(2) The " delay " period of approximate heat equilibrium having passed, the ball of flame proceeds to grow with increasing rapidity, retaining its form as far as may be permitted by the walls of the containing vessel.

At the flame front intense heat is generated, causing a local rise of pressure of considerable magnitude, and a wave of pressure is set up which travels away from its point of origin with the speed of sound, and, on reaching the walls of the containing vessel, is reflected and re-reflected until its energy is absorbed, this being accompanied by a general rise of pressure and temperature. The unburnt gases are thus compressed, nearly adiabatically, by the burning charge.

This compression of the unburnt charge has a secondary effect, as the gases beyond the flame front are brought nearer to their temperature of combustion, and the speed of travel of the flame is thus increased in a cumulative manner. The flame, now firmly established, proceeds to grow with great rapidity until, under circumstances favourable to the given mixture, the flame front is travelling with the velocity of sound. The " explosion " is then said to have become a " detonation," and as

the flame is travelling as fast as the pressure waves to which it gives rise, the local pressure has no means of release and therefore builds up to a very high value. Such a detonation wave, arriving at a boundary wall, produces a hammer blow, and is accompanied by the same sharp, metallic " pink " which would accompany a blow by a steel hammer. If the wall is of insufficient strength to withstand the blow it is usually shattered into many fragments. It must be remembered that these intense local pressures can be produced even in the case of " explosions " nominally at atmospheric pressure. The heat liberated and the general pressure produced in the containing vessel after a detonation are the same as if the combustion had been comparatively slow, but the local and transient effects may be entirely different. The usefulness of a fuel for employment in a high duty internal combustion engine is primarily determined by its tendency to detonate.

It appears that at low initial pressures the detonation wave is produced only after the flame has travelled a certain distance. In the case of methane and oxygen at atmospheric temperature and pressure this may be as small as $2\frac{1}{2}$ in., and for electrolytic gas 12 in. For the mixtures normally found in I.C. engines the distance is appreciably greater.

One factor of great importance in this connection is the presence in the cylinder head of local hot zones, produced by the presence of exhaust valve heads, overheated sparking plugs, etc. The effect of hot-spots may be such that the arrival of a pressure wave in advance of the flame front, as is the case with normal comparatively slow combustion, may bring the explosive mixture in such localities to a temperature above that required for ignition, and detonation may be initiated in the locality though the flame front is still some way off.

" Detonation " must not be confused with pre-ignition. Detonation follows ignition with regard to time. Pre-ignition is the ignition of a charge before the time arranged, and may be initiated during the compression stroke by the presence of an overheated sparking plug, or particle of glowing carbon. The flame spreads in the normal way, but the first, or delay, phase of ignition may have passed well before the end of the compression stroke. The compression of the now vigorously burning gases gives rise to very high temperatures and pressures which rapidly aggravate the overheating of the offending hot-spots, and tend to bring the engine quickly to a standstill. The abnormal pressures cause heavy strains on the engine parts and the violent treatment to which they are subjected gives rise to the well-known " knock," a sound very different from the metallic ring or " pink " caused by detonation.

Detonation has a tendency to remove carbonaceous deposits in the cylinder head, and in a mild form is not seriously detrimental, but if severe or long continued will cause overheating of plug-points, etc., and lead to pre-ignition.

TURBULENCE

Superimposed upon the effects described above are those due to turbulence, that is to the movements, either orderly or indiscriminate, produced in the gases either by the charging of the cylinder (induction turbulence) or by the motion of the piston (compression turbulence). They have the primary effect of breaking up the flame in its early stages and distributing the fragments through the combustible mixture, thus initiating the spread of flame from many centres. Were turbulence not present in the cylinder of an I.C. engine combustion would be so slow that none but the lowest rates of revolution would be possible, as is seen in Fig. 135, showing a normal explosion and one of an exactly similar mixture but with turbulence reduced to a minimum. As the effect of turbulence in a normal engine is adequate at low speeds and is nearly

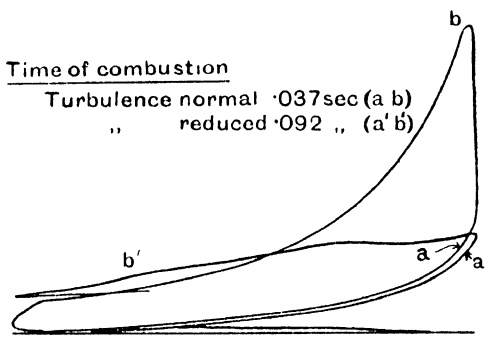


Fig. 135.

proportional to the rate of revolution, it follows that the speed of combustion will not prohibit the attainment of high engine speeds. By the suitable design of the induction port and cylinder head the turbulence may be increased to any extent required. It is generally desirable, in the case of a well-designed high-duty petrol engine, to provide such turbulence as will give a rate of pressure rise of 20–30 lb. per sq. in. per degree of crank angle, a sharper rise giving " rough " running owing to springing of the engine parts.

A high degree of turbulence has a number of secondary effects, the most important of which is the reduction of the tendency of a fuel to detonate, owing to the more adequate distribution throughout the whole body of the charge of the heat derived from heated valve heads, etc. Heat losses to the walls and piston head will be appreciably increased, but the effect of the former is to some extent counteracted by the relatively small wall area normally exposed in cylinders using a high degree of turbulence.

EFFECT OF RESIDUAL GASES

The possible effects of varying either the quality or the quantity of the residual gases remaining in the combustion space at the end of the compression stroke must be borne in mind when considering the effects which may be anticipated from scavenging, supercharging, etc., if applied to a high-duty petrol engine.

It has been shown on p. 292 that the temperature at the end of the suction stroke is affected by the admixture of hot residual gases with the fresh charge of air and fuel, and that the lower the compression ratio the larger this effect will be. A second effect, however, is that as the ratio of compression is decreased the specific heat of the mixture is increased owing to the increasing numbers of molecules of CO_2 and H_2O present, these having high specific heats. An increase of 1 per cent. by weight in the quantity of the residual exhaust gases may lower the final flame temperature owing to this cause by as much as 20°–25° C., or an amount equal to that produced by a variation in compression ratio from 5 : 1 to 4 : 1. A reduction of the amount of exhaust dilution, or the removal of residuals altogether, by scavenging, will, under given circumstances, greatly increase the tendency to detonation, and the addition of cooled exhaust gas through the carburettor has been used by Ricardo to suppress detonation.

These facts have a great importance only in engines for high duty, in which the fuel is always utilised as near to its detonation point as is practicable. They illustrate the importance of considering all the factors involved in the complex processes of combustion.

APPLICATION OF KNOWLEDGE OF FLAME PHENOMENA TO CYLINDER HEAD DESIGN

By the application of a knowledge of flame phenomena it has become possible to produce combustion of a type most fitted for a particular purpose, and considerable improvements in efficiency and very great increases of specific output have been obtained. The three primary conditions to be observed are :

(1) A minimum distance of flame travel ;
(2) A suitable degree of turbulence ;
(3) Absence of hot-spots, especially in pockets where the gas tends to stagnate.

Fig. 136 shows the old-fashioned T-head cylinder, which provides about the worst possible conditions, i.e. maximum distance of flame travel, low turbulence, and a hot exhaust valve head in a " pocket." Additional disadvantages are the bad flow of the gases past the valve heads, inadequate cooling of the valve seats on the cylinder side, and the very large surface : volume ratio, giving excessive heat losses.

The worst position for the sparking plug is above the inlet valve at A, as the compression of the unburnt charge near the exhaust valve by the burning of the charge near A is extremely likely to initiate detonation. By placing the plug at B a considerable improvement is effected as compression of the unburnt charge is then in the neighbourhood of the cool inlet valve. The presence of uncooled valve caps often makes matters worse.

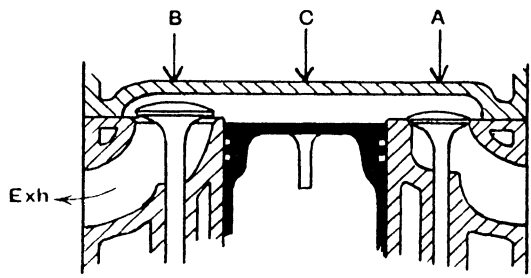


Fig. 136.

The highest useful compression ratio for normal petrol with this type of head is about 3·6 : 1.

By placing both valves on one side of the cylinder the L head is obtained, which is a considerable improvement on the former owing to the shortening of the maximum flame path. The maximum compression ratio is about 4·5.

In the hemispherical head, Fig. 137, a very compact arrangement is

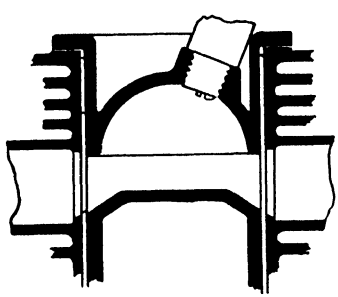


Fig. 137.

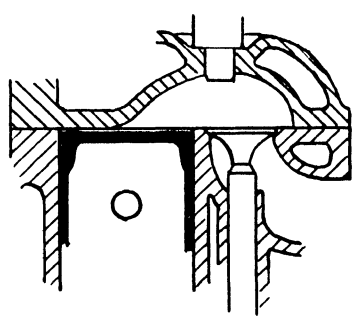


Fig. 138.

obtained, with a minimum distance for flame travel, an entire absence of pockets, and a form well arranged to maintain induction swirl as long as possible. The small surface : volume ratio will be noted. A sleeve valve engine is shown. A similar arrangement but with overhead valves is frequently used.

In the Ricardo head, Fig. 138, a very compact form of chamber is

obtained while retaining the side-valve arrangement. The very small clearance between the piston and a portion of the head is such that detonation is precluded in this region owing to the large cooling effect provided. In this design a high degree of turbulence is produced by the movement of the piston just before its highest position is reached.

It has been found that even with a robust engine having an exceptionally well supported crankshaft, etc., turbulence must not be increased beyond that which will give a rate of pressure rise of about 30 lb. per sq. in. per degree of crank angle. If the pressure is gradually applied at first, till all spring of the parts is taken up, a much greater rate of rise may be used

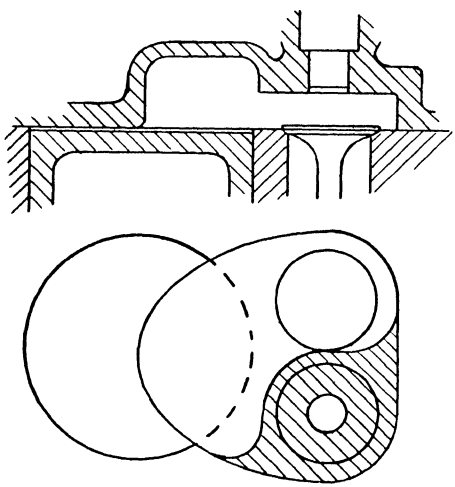


Fig. 139.

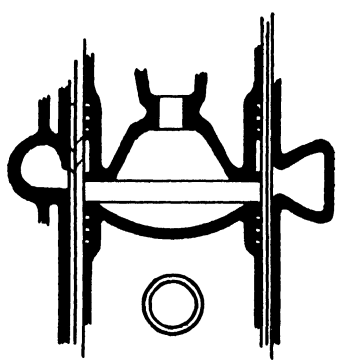


Fig. 140.

subsequently. The " shock-absorber " head shown in Fig. 139 has resulted.

About 15 per cent. of the total charge is compressed into a shallow pocket, in which turbulence rapidly dies down, and where ignition is initiated. The early stages of combustion are therefore gradual, the time required from the passing of the spark to the instant when the flame reaches the edge of the pocket being about 30 crank angle degrees. During the last stage of this slow combustion the flame front leaves the pocket and is ejected into the deep portion of the combustion head in which induction and compression turbulence have been well maintained. The combustion of the remaining 85 per cent. of the charge requires only about 8°–12° of crank angle, giving a final rate of pressure-rise of 45 lb. per sq. in. per degree without " roughness."

The sleeve-valve engine, Fig. 140, presents many attractive features. Among these are the compactness of the combustion chamber with a minimum flame-travel distance and an entire absence of hot-spots. The

degree of turbulence is controllable over a very wide range by means of vanes set in the induction passages. The compression ratio for normal speeds of running may be between 5·4 : 1 and 6 : 1.

EFFECTS OF SPEED AND SIZE

One effect of increasing engine speed is to lower the volumetric efficiency owing to the restriction offered by the valve passages, etc. The suction pressure, and consequently the cycle temperatures, will be reduced thereby, resulting in a decreased tendency to detonation, while the increased turbulence will have the same effect. An increase in speed from 1200 to 1600 r.p.m. may increase the highest useful compression ratio from, say, 4·5 : 1 to 5 : 1.

Exceptionally high speeds, such as are attained by racing machines, permit of much higher ratios.

The size of cylinder used has a direct effect on the permissible compression ratio owing to the variations produced in the maximum distance to be traversed by the flame. The following values indicate the compression ratios which may be used with similar sleeve-valve cylinders of different dimensions :

Bore 2¾ in.	7·9 : 1
3½ in.	7·3 : 1
5½ in.	6·2 : 1
8½ in.	5·4 : 1

COMBUSTION IN THE COMPRESSION IGNITION ENGINE

The process of combustion in these engines is more complex than in engines using a gaseous mixture. The main problem is that of bringing the oil fuel into intimate contact with the air required for its complete combustion, remembering that much of the combustion takes place from the surface of the fuel particles before their complete vaporisation, so that a high relative speed must be maintained in order to sweep away the products of combustion.

With air injection a very highly atomised spray may be attained, the injection air also considerably increasing the state of turbulence in the engine. For all except slow speeds, however, a mechanically operated injection pump is used.

The normal combustion process can be considered as consisting of three phases, as shown in Fig. 141.

(1) The " delay " period during which ignition is initiated by the combustion of the finest droplets in the first portion of fuel to enter the cylinder. Combustion spreads very slowly at first, though more rapidly than in the petrol engine owing to the higher compression temperatures employed.

(2) The general and largely uncontrolled spread of the flame, aided by turbulence.

These two stages are generally effected during the early stages of injection, and before the bulk of the fuel has entered the cylinder.

(3) The temperature throughout the combustion chamber having been greatly increased during stages (1) and (2), the remainder of the fuel burns as it passes the nozzle, the succeeding temperature and pressure rises being under direct mechanical control, and very largely determined by the relative velocity of the air and fuel particles.

As the duration of phase (1) is approximately constant with regard to time, and is therefore independent of engine speed, while the injection pump is directly driven by the engine and therefore gives a discharge

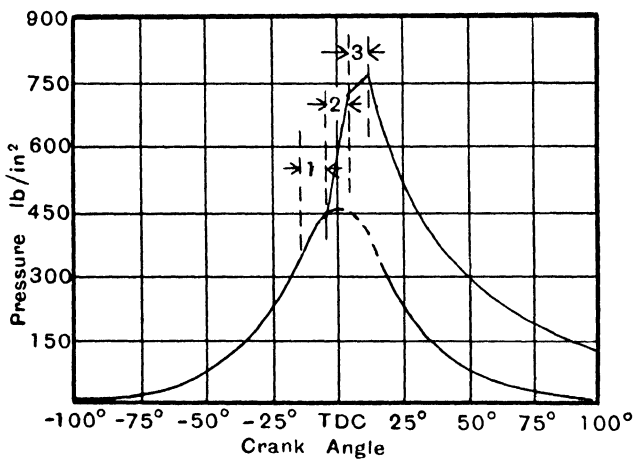


Fig. 141.

dependent on engine speed but not on time, it follows that as engine speeds increase the proportion of the total fuel entering the cylinder during the " delay " period increases, thus emphasising stage (2) and increasing the maximum pressure, the cycle tending to approach the " constant volume " cycle. It may be desirable to decrease the speed of injection at first to maintain adequate control of the maximum pressure.

A compression ratio of about 12 : 1 is the minimum which will give reliable auto-ignition under all circumstances.

It should be noted that in the petrol engine, where all the fuel is present throughout the process of combustion, it is desirable to use a fuel with a high auto-ignition temperature to reduce the tendency to pre-ignition and detonation. In the C.I. engine a low auto-ignition temperature is desirable in order to reduce the " delay " period, and give as full control as possible of the second phase of ignition. A petrol may therefore be

" doped " to decrease the tendency to detonate, but a fuel oil to produce exactly the opposite effect.

Fig. 142 shows some of the many types of combustion chamber employed.

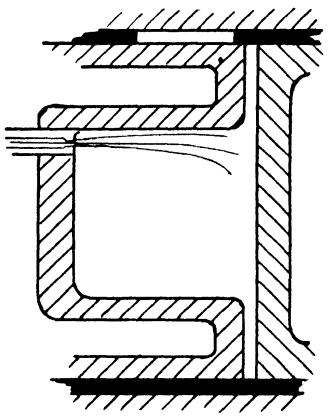


Fig. 142B.

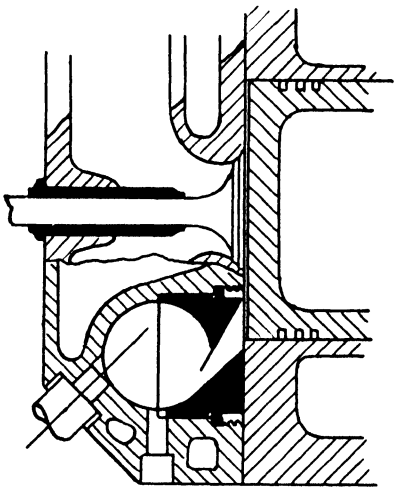


Fig. 142D

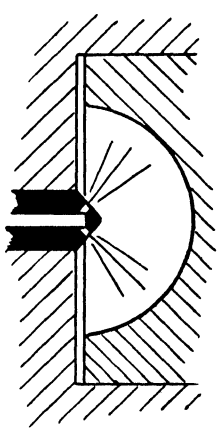


Fig. 142A.

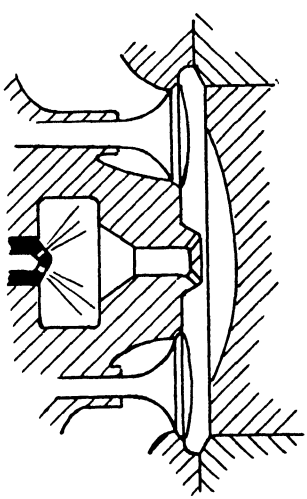


Fig. 142C.

A. In this type a comparatively low degree of turbulence is used, intimate mixture of the air and fuel being obtained by driving the fuel particles at high velocity into the charge of air.

B is a representative of the reverse process. Here a high degree of

induction swirl is produced, generally by means of a sleeve-valve with suitably inclined ports, and is greatly intensified by the compression of the whirling charge into the " pot " of smaller diameter than the cylinder. A second form of turbulence is finally superimposed upon the first as the piston reaches top dead centre and displaces the air from beneath the outer ring of the cylinder head. The fuel is injected as a single jet down one side of the combustion chamber, so that the air swirl brings the air to and removes the products of combustion from the fuel.

C represents the " pre-combustion chamber " design. Here a part of the air charge enters the small central pre-combustion chamber through a " pepper-pot " with a large number of small holes, and meets the atomised fuel. The first two phases of combustion develop in this chamber, but during the second the remaining fuel and highly-heated air are driven violently through the pepper-pot by the pressure set up by combustion and pass into the combustion chamber, to mix with the bulk of the air charge. The velocity of ejection is greatly in excess of any which can be produced by mechanical means.

The throttling of the gas flow by the pepper-pot is a serious disadvantage, and also the tendency of the fuel to " crack " and form carbonaceous deposits in the chamber.

D represents quite a different arrangement. Here practically all the air is compressed through a narrow neck into a small chamber which may be either in the cylinder head as shown, or in the piston crown. The injection of fuel into the chamber, where turbulence is very intense, is commenced rather slowly, so that phases (1) and (2) are well advanced before more than a small amount of fuel has been injected. The downward movement of the piston then causes an intense rush of air and burning fuel through the narrow neck resulting in very intimate mixing. The larger portion of the combustion, or phase (3), occurs outside the chamber, as is indicated by the fact that the latter does not overheat.

EXAMPLES ON CHAPTER XIV

1. A gas engine works on the ideal Otto cycle. The clearance volume is 272 cu. in., the cylinder is 9·5 in. diameter, and the stroke 19 in. At the end of the suction stroke the pressure is 13 lb. per sq. in. abs., and the temperature of the charge is 100° C. Estimate the ideal " air standard " efficiency, and the temperature and pressure at the end of the compression stroke ($\gamma=1{\cdot}4$).

2. Calculate (*a*) the ideal efficiency of a gas engine working on the Otto cycle when the compression pressure is 135 lb. per sq. in. above atmospheric, and (*b*) of an engine working on the same cycle with a compression pressure of 65 lb. per sq. in. above atmospheric. Assume the pressure during the suction stroke to be 14 lb. per sq. in. abs. in each case, and the expansion and compression to be adiabatic with $\gamma=1{\cdot}38$.

3. The engine in Question 1 receives 0·0836 cu. ft. of gas per suction stroke at 0° C., and 14·7 lb. per sq. in. abs., the density of the gas being 0·03 lb. per cu. ft., and its calorific value 550 B.Th.U. per cu. ft. The strength of the mixture is

1 of gas to 10 of air by volume, and its temperature at the end of the suction stroke is 100° C., the pressure being 14·7 lb. per sq. in. abs. Find

(*a*) Weight of the charge drawn in (take weight of 1 cu. ft. of air at N.T.P. as 0·0807 lb.).
(*b*) Pressure and temperature at the end of compression (take $\gamma = 1{\cdot}38$).
(*c*) Rise of temperature during explosion (neglect jacket loss and take $C_v = 0{\cdot}18$).
(*d*) Pressure at the end of the explosion.
(*e*) Temperature and pressure at the end of expansion.
(*f*) Efficiency of the cycle.

4. In an engine working on the Otto cycle the following temperatures were measured : Suction 210° F., at end of compression 590° F., maximum 2093° F., at end of expansion 1594° F. Estimate the compression ratio and the ideal efficiency. Assume the law of compression is $pv^{1{\cdot}445} = \text{constant}$.

5. The following figures are taken from the expansion curve of a 100 B.H.P. gas engine indicator diagram, the engine having a clearance volume of 0·6035 cu. ft., and piston displacement 3·1528 cu. ft., the volume v representing the total volume of the gases :

v (cu. ft.)	0·6035	0·7610	0·9188	1·234	1·549	1·865	2·180	2·495	2·810	3·126	3·441
p lb. per sq. in. abs.	400	279	229	155	121	92	72	60	50	42	35

Estimate the law of the expansion curve.

6. A petrol engine working at a compression ratio of 4·5 to 1 shows an indicated maximum pressure of 450 lb. per sq. in. abs. If the equations of the compression and expansion curves are $PV^{1{\cdot}35} = K$ and $PV^{1{\cdot}28} = K$ respectively, calculate the M.E.P. and the temperature at the end of the expansion stroke. Assume the suction temperature and pressure to be 200° F. and 14·5 lb. per sq. in. abs. What would be the efficiency of a perfect engine using the corresponding standard air-cycle at the same compression ratio.

7. A supercharged four-stroke single cylinder petrol engine has a compression ratio of 4·5 to 1. The temperature and pressure at the end of the suction stroke are 250° F. and 28 lb. per sq. in. abs. respectively. Assuming that the maximum temperature is 2200° C., that the law of expansion and compression is $PV^{1{\cdot}28} = K$, and that combustion occurs entirely at constant volume, calculate the indicated M.E.P., and the pressure at the end of the exhaust stroke.

If the engine uses 0·625 lb. of benzole (calorific value 18,600 B.Th.U. per lb.) per B.H.P. per hour, what is its thermal efficiency relative to that of a perfect air engine at the same compression ratio ?

8. A four-cylinder four-stroke petrol engine is to be designed to develop 55 I.H.P. at a speed of 3000 r.p.m. The bore and stroke are to be identical, the compression ratio is to be 5·5 to 1, and the law of compression and expansion may be taken as $PV^{1{\cdot}28} = K$. Assuming the suction temperature and pressure to be 100° C. and 14 lb. per sq. in. abs. respectively, and that on ignition the cylinder pressure rises instantaneously to 3·5 times the compression pressure, calculate the diameter of the cylinders.

What would be the efficiency of a perfect engine employing the corresponding ideal air-cycle ?

Chapter XV

MECHANICAL REFRIGERATORS AND REVERSED HEAT ENGINES

GENERAL STATEMENT

Refrigeration is the process by which heat is removed from a body in order to reduce its temperature below that of its surroundings. This removal cannot be carried out by the simple process of conduction as the heat concerned must be transferred from a low-temperature body to another at a higher temperature, a process which, according to the Second Law of Thermodynamics, can only be effected by the expenditure of energy from an external source.

The process involves four stages and the use of a " refrigerant " which may be either (1) a gas, or (2) a volatile liquid. The stages are as follows :

(1) The refrigerant is cooled to a temperature suitably below the temperature of its surroundings, either by causing it to expand and so convert some of its thermal energy into mechanical work (refrigerants of groups (1) and (2)), or by throttling (refrigerants of group (2) only).

(2) Heat is transferred by conduction from the body to be cooled to the refrigerant.

(3) The temperature of the refrigerant is raised by compression, external mechanical energy being absorbed.

(4) Heat is transferred from the refrigerant by conduction to a body at the upper temperature—normaally a supply of cold water.

The process is usually—but not always—continuous.

In the case of plants in which the chilling process has to be effective over a large area, it is often convenient to substitute two sub-stages for stage (2), the heat from the body to be cooled being first transferred to a brine solution (having a low freezing-point) which may then be pumped a considerable distance to a compact and centrally placed refrigerator unit, where the heat is transferred to the refrigerant and finally discharged.

COEFFICIENT OF PERFORMANCE

A refrigerator can be considered as a " heat pump " in which mechanical work is used to extract heat from a body having a low temperature.

The ratio $\frac{\text{heat extracted}}{\text{heat equivalent of work done}}$ is known as the " coefficient of performance," and corresponds to the " efficiency " of a heat engine, but, while the latter has a value essentially less than unity, the former, as will be shown, may exceed this value.

THE REVERSED HEAT ENGINE AS REFRIGERATOR. COLD AIR MACHINES

The " hot air " engines considered in Chapter VII can be employed as cold-air refrigerators if their cycles are reversed.

Reversed Carnot Cycle.—This being the simplest of the ideally perfect cycles is of considerable theoretical interest, though its employment in practice is precluded by exactly those factors which prevent its use in the non-reversed condition.

Fig. 143 shows the $p.v.$ and $T\phi$ diagrams of this cycle, T_1 being the

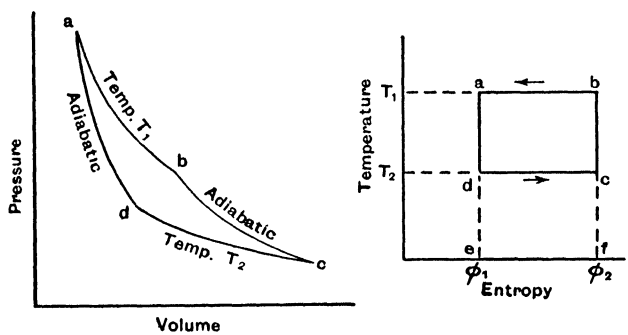


Fig. 143.

temperature of the sink of heat—or cooling water—and T_2 the temperature of the refrigerator cold chamber. Starting at point a the air is cooled by adiabatic expansion to T_2, which is reached at point d. The air is then expanded isothermally in contact with the body to be cooled, doing work and taking in an equivalent amount of heat at temperature T_2. From c to b the temperature of the air is raised by adiabatic compression to the temperature T_1 of the cooling water in contact with which it is then compressed isothermally, the mechanical energy absorbed in this latter process being discharged as heat to the water.

The coefficient of performance is best found from the $T\phi$ diagram. The heat received from the cold body is given by area $cdef$, and the work done by $adcb$.

The coefficient of performance is then

$$\frac{cdef}{adcb}=\frac{T_2 \cdot dc}{(T_1-T_2)dc}=\frac{T_2}{T_1-T_2}$$ (note that the temperatures are " absolute ").

This increases as the range of temperature T_1-T_2 decreases, and may exceed unity. It should be noted in this connection that the mechanical work done on the refrigerant is required merely to raise the temperature factor of the heat energy which already exists in any quantity and without value in the refrigerator cold chamber. In the theoretical case when $T_1-T_2=0$, or $T_1=T_2$, the coefficient of performance becomes infinite, which is merely the equivalent of the theoretical statement that heat can flow in any quantity from one body to another at the same temperature, without necessitating the use of external energy. The heat discharged to the hot body (i.e. the cooling or condensing water at the temperature of its supply) is available by means of, and not as the equivalent of, the mechanical work done.

EXAMPLE.—Find the least horse-power required by a perfect reversed heat engine that will make 900 lb. of ice per hour at 27° F. from water at 70° F. Take the latent heat of ice as 142 B.Th.U. per lb. and the specific heat as 0·5.

$$\text{Coefficient of performance}=\frac{T_2}{T_1-T_2}=\frac{460+27}{70-27}=\frac{487}{43}=11\cdot33.$$

Heat extracted from 1 lb. of water at 70° F. to produce 1 lb. of ice at 27° F. will be

$$(70-32)+142+0\cdot5(32-27)$$
$$=38+142+2\cdot5=182\cdot5 \text{ B.Th.U.}$$

Hence the least horse-power will be

$$\frac{182\cdot5\times900}{2545\times11\cdot33}=5\cdot7.$$

Reversed Stirling Cycle.—This cycle was employed in the Kirk refrigerator, and overcame the difficulties presented by the huge cylinder volumes necessitated by the Carnot cycle.

The efficiency, by reference to p. 87, will be found to be

$$\frac{RT_2\log_\epsilon r-(1-e)C_v(T_1-T_2)}{R(T_1-T_2)\log_\epsilon r}$$

Assuming a regenerator efficiency of unity this expression becomes

$$\frac{T_2}{T_1-T_2}$$

Reversed Joule Cycle.—Though theoretically less interesting than the reversed Carnot cycle, the reversed Joule cycle, used in the Bell-Coleman and Haslam and Hall refrigerators, was responsible for much of the pioneer work in the frozen-meat trade.

In this cycle the pressures in the cooler and cold chamber are constant. The cycle is as follows (see Fig. 144) :

(1) Air at the minimum temperature available in the cooler A is drawn into cylinder C and expanded to the pressure of the cold chamber R into which it is then exhausted. Upon entering the latter the temperature is considerably below the temperature in the chamber. The cycle in the expansion cylinder is shown on the *p.v.* diagram by *eadf*.

(2) The air receives heat at constant pressure from the cold chamber and rises in temperature to T_2, the process being shown by *dc*.

(3) The air is then removed by the compressor E and its pressure raised by adiabatic compression to the pressure of the cooler, the temperature, however, being then greater than the minimum temperature of the latter. Compression follows the curve *cb*.

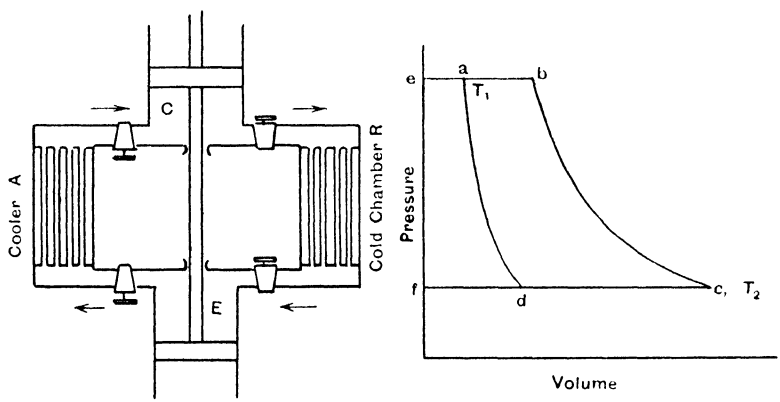


Fig. 144.

(4) The air is finally exhausted into the cooler and cooled at constant pressure to its original condition along curve *ba*.

By reference to the method of calculation used on p. 90 it will be found that the coefficient of performance is given by $\frac{T_d}{T_a - T_d}$, which is essentially lower than the value $\frac{T_2}{T_1 - T_2}$ for the Carnot cycle as T_d is essentially less than T_2.

Owing to the practical necessity of using a wide temperature range to secure reasonably rapid transference of heat the actual coefficient of performance was only of the order of $\frac{1}{2}$ to $\frac{3}{4}$.

In this machine considerable difficulties were experienced owing to the formation of snow which, produced by the freezing of moisture drawn in from the cold chamber, blocked the air ducts, etc. This was overcome to a large extent by expanding the air in two stages, the first reducing the

temperature to about 35° F., when nearly all the moisture condensed and could be drained away before the final expansion to temperatures below the freezing-point. A similar effect was also produced by employing a heat interchanger in which the compressed air from the cooler A parts with heat to the air coming from the cold chamber R. The air on its way to the expansion cylinder is thus cooled to a low temperature and deposits most of its moisture in the heat interchanger which can be drained.

EXAMPLE.—If the compression pressure of a reversed Joule heat engine be 45 lb. per sq. in. gauge and the suction pressure 15 lb. per sq. in. abs., find the lowest temperature produced in the engine, the air being cooled at the highest temperature by circulating water at the temperature of the atmosphere, which is 60° F.

The maximum temperature $T_a=460+60=520°$ F. abs., and T_d is the lowest temperature.

Now
$$\frac{T_a}{T_d}=\left(\frac{p_a}{p_d}\right)^{\frac{\gamma-1}{\gamma}}$$
$$=\left(\frac{45+15}{15}\right)^{\frac{1\cdot4-1}{1\cdot4}}$$
$$=4^{\frac{2}{7}}$$
$$\therefore T_d=\frac{520}{4^{\frac{2}{7}}}$$
$$T_d=350° \text{ abs. or } 350-460=-110° \text{ F.}$$

If the temperature of the refrigerating chamber is 0° F.=460° F. abs.

$$\frac{v_c}{v_d}=\frac{460}{350}=1\cdot312=\frac{v_b}{v_a}=\frac{T_b}{520}$$

or
$$T_b=683° \text{ F. abs. or } 223° \text{ F.}$$

The coefficient of performance is therefore

$$\frac{350°}{520-350}=\frac{350}{170}=2\cdot05$$

The corresponding Carnot coefficient would be

$$\frac{460}{520-460}=\frac{460}{60}=7\cdot67$$

the overall temperature range being much less than with the Joule machine, though the cylinder capacity would be much greater.

REVERSED HEAT ENGINE AS A WARMING MACHINE

A machine of the Bell-Coleman type may be used for this purpose, as was first pointed out by Lord Kelvin in 1852. The machine would take in air from the atmosphere, expand it down to a lower temperature and pressure and then allow its temperature to rise again by contact with the external air. After this the air would be compressed once more to atmospheric pressure, its temperature being thereby raised, preparatory to being delivered into the room to be warmed.

Let H_2=heat taken from the atmosphere at temperature T_2,
H_1=heat delivered to the room at temperature T_1,
W=work expended in heat units.

Then
$$\frac{H_1}{W}=\frac{H_1}{H_1-H_2}=\frac{T_1}{T_1-T_2} \quad . \quad . \quad . \quad . \quad . \quad (1)$$

When the range of temperature T_1-T_2 is small, H_1 may be many times greater than W, i.e. a large amount of heat may be raised through a small range of temperature with little expenditure of mechanical work.

In the case of one large auditorium refrigerating plant which has been installed for the purpose of cooling the air during the summer, is reversed during the winter and is used as a warming machine.

EXAMPLE.—An oil engine uses 0·5 lb. of oil per B.H.P. per hour, the calorific value of the oil being 20,000 B.Th.U. per lb. If it drives a reversed heat engine which takes in air at 40° F. and delivers it at 55° F., how much heat will be given to the air per B.H.P. hour if the reversed heat engine works at 80 per cent. of the ideal efficiency?

The ratio of heat to work is given by

$$H_1=W\cdot\frac{T_1}{T_1-T_2}\times\frac{80}{100}$$

$W=1$ B.H.P. hour$=2545$ B.Th.U.

Therefore
$$H_1=2545\times\frac{460+55}{55-40}\times 0{\cdot}8$$
$$=69{,}900 \text{ B.Th.U.}$$

The heat which would be available were the oil burnt in a lamp would be 10,000 B.Th.U., or about one-seventh of the above value. In addition, the heat discharged in the jacket water and in the exhaust of the engine could be used directly for heating purposes.

The heat supplied to the engine is $\frac{10{,}000}{60}$ B.Th.U. per minute per B.H.P.

giving an efficiency of $\frac{42{\cdot}42}{10{,}000}\times 60$ or 25·4 per cent.

The remaining heat, or 7460 B.Th.U. per hour per B.H.P., is available directly for heating purposes, bringing the total to 77,360 B.Th.U. per H.P. hour.

THE USE OF THE REVERSED HEAT ENGINE IN EVAPORATION PROBLEMS

An interesting example of the reversed heat engine is found in connection with certain evaporation problems. Diagram 1, Fig. 145, illustrates a simple evaporator heated by live steam. The solution to be concentrated enters at A, and the steam formed by evaporation leaves at B, approximately 1 lb. of steam being evaporated from the solution for every pound of live steam supplied.

The dead steam has nearly the same heat content per pound as the live steam, and differs from it in quality only to the extent that its temperature and pressure of condensation are slightly lower.

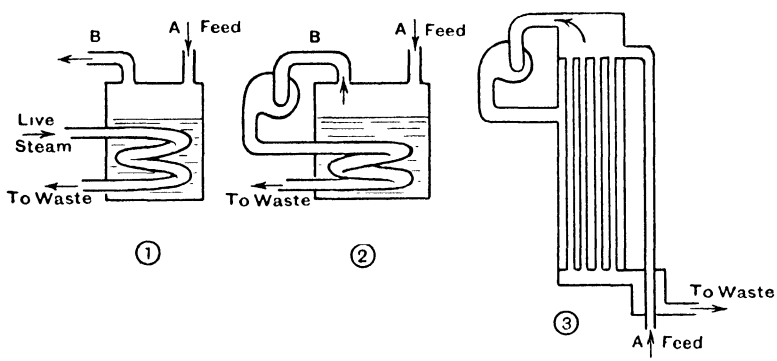


Fig. 145.

An alternative arrangement is shown in diagram 2 where the steam formed by evaporation is compressed, approximately adiabatically, till its temperature of condensation is sufficiently above the temperature of evaporation of the solution to allow a reasonable rate of heat transfer. A centrifugal compressor is used. The steam is then returned to the apparatus as live steam where it effects a further concentration of the solution.

It will be seen that the mechanical energy supplied to the compressor serves three purposes : (1) To make good the difference in heat content between the solution supplied at A and the condensate leaving the drain ; (2) to supply a suitable temperature difference between the " live " steam and the solution ; (3) to replace heat lost by radiation, etc.

By suitable design the temperature difference required for a reasonable rate of heat transfer may be made very small, and figures as low as 3° to 10° C. are quoted, with a temperature difference between A and the

drain of 5° C. These figures apply to the "falling film" type of evaporator fitted with a heat interchanger as shown in (3). The liquid to be evaporated is pumped to the top of the nest of tubes and traverses the heating surfaces in a downward direction as a thin clinging film.[1]

(An alternative is the "multiple effect" apparatus where two or more simple evaporators are arranged in series, passing on steam from one to the next, the temperature difference required being provided by a progressive lowering of the pressures in the stages.)

REFRIGERATORS EMPLOYING A VOLATILE LIQUID

General Statement.—For many years dentists and surgeons have used the method of "freezing" to produce local anæsthesia. A few drops of ether are placed on the part to be treated and its rapid evaporation, at the expense of the heat of that part, produces the numbing required. The same basic idea applies to vapour refrigerators employing a volatile liquid, but facilities have to be provided for the recovery of the refrigerant.

Two types of machine are distinguishable, and depend for their action on processes of absorption and compression respectively.

VAPOUR ABSORPTION MACHINES

In these machines the vapour produced by the evaporation of the refrigerant in the cold chamber passes over into a vessel containing some material in which it is absorbed, and which, by the removal of the vapour as it is formed, maintains a constant low pressure, facilitating further evaporation. The refrigerant is subsequently liberated in the vapour state by the direct application of heat, and at such a pressure that condensation can be effected at the temperature of the air or by cold water.

The coefficient of performance is given by

$$\frac{\text{heat absorbed during evaporation}}{\text{heat required to liberate refrigerant from absorbent}}$$

The quantity of heat stated in the denominator is dependent on the process employed.

If ammonia is used as the refrigerant its vapour may be dissolved in water, and this process can be considered as taking place in two stages :

(1) The liquefaction of the ammonia, accompanied by the liberation of its latent heat ;

(2) Mixing of the liquids, again accompanied by liberation of heat (as is the case when water and sulphuric acid are mixed).

The heat liberated during the first stage is approximately 5030 cals.

[1] For further details reference can be made to "Evaporating Apparatus," Hausbrand and Heastie, pub. Benn.

per gm.-molecule, and in the second stage 3400 cals. per gm.-molecule. During the subsequent recovery of ammonia vapour the processes will be reversed.

Assuming the latent heat of ammonia to remain constant over the range of temperature employed, the coefficient of performance will be approximately

$$\frac{5030}{5030+3400}=0{\cdot}6$$

If ammonia vapour be dissolved in a powdered absorbent which thereby becomes liquid (as is the case of anhydrous sulphocyanide of ammonium) the stages of absorption are:

(1) Liquefaction of the ammonia accompanied by the liberation of the latent heat of evaporation.

(2) Liquefaction of the absorbent accompanied by the absorption of its latent heat of fusion.

(3) Mixing of the liquids, generally with liberation of heat.

The heat involved in stage (3) is usually not great, and stages (1) and (2) are complementary.

The coefficient of performance is then

$$\frac{\text{latent heat of evaporation of ammonia}}{\text{latent heat of evaporation of ammonia}-\text{latent heat of fusion of absorbent}+\text{heat of mixing}}$$

This may be appreciably greater than unity.

IDEAL EFFICIENCY OF ABSORPTION PROCESS

These processes may be considered as employing heat (*sic*) from a high-temperature source to elevate heat from a low-temperature source, the total heat being discharged at an intermediate temperature. If the high, intermediate, and low temperatures are T_1, T_2, and T_3, then if Q_1 heat units are taken in at T_1, the work which could be done by the Carnot cycle over the range T_1 to T_2 will be

$$Q_1\frac{T_1-T_2}{T_1}$$

This work, applied to a reversed Carnot cycle working over the range T_2 to T_3, could extract Q_3 units of heat from the cold body such that

$$Q_3=\text{work done}\times\frac{T_3}{T_2-T_3}=Q_1\left(\frac{T_1-T_2}{T_1}\right)\times\left(\frac{T_3}{T_2-T_3}\right)$$

No process could give a higher ratio of Q_3 to Q_1 than

$$\frac{T_3(T_1-T_2)}{T_1(T_2-T_3)}$$

EXAMPLE.—Heat from a gas flame at a temperature of 1000° C. is used in a perfect absorption machine, the refrigerating chamber being at 0° C. and the temperature of heat discharge 20° C. Find the coefficient of performance.

In this case $T_1=1000+273=1273°$ C. abs.

$T_2=20+273=293°$ C. abs.

$T_3=0°$ C.$=273°$ C. abs.

The coefficient of performance will be given by

$$\frac{T_3(T_1-T_2)}{T_1(T_2-T_3)}=\frac{273\times 980}{1273\times 20}=8{\cdot}9$$

INTERMITTENT REFRIGERATION

The absorption machine, for very small capacities, may take the form of two small metal vessels connected by a short length of pipe, and containing liquid ammonia and a powdered absorbent respectively. The ammonia end is inserted into the cold chamber from which heat is gradually taken up, causing the liquid to evaporate. The ammonia vapour formed passes over to, and dissolves in, the absorbent, heat being liberated and dispersed to the air.

After evaporation is complete the apparatus is removed and the vessel containing the absorbent is heated. Ammonia vapour at a moderately high pressure is liberated and passes over into the vessel first occupied, where it condenses under the action of high pressure and room temperature. The cycle may now be repeated.

For small domestic refrigerators the element is generally removed, heated, and replaced once each day.

CONTINUOUS ACTION ABSORPTION MACHINE

The adoption of a continuous cycle of operations involves separating the plant into two sections, the "low side" in which evaporation is induced by a steadily maintained low pressure, and a "high side" in which reclamation of the refrigerant in a liquid state is attained under the influence of high pressure and heat.

The refrigerant most generally used is ammonia, with water as the solvent. Fig. 146 shows a general arrangement of the plant.

Low-pressure liquid ammonia [1] and a small quantity of ammonia vapour enter the refrigerating chamber B from A, and sufficient heat is

[1] The difference between liquid ammonia and "ammoniacal liquor" or solution of ammonia in water should be noted. The former is a colourless liquid exerting a vapour pressure of about 100 lb. per sq. in. at room temperature, and is highly dangerous unless skilfully handled; the latter in its commonest form is "household ammonia."

received to cause the complete evaporation of the liquid. The vapour passes into the chamber C where it comes into contact with cool water, in which it dissolves, with the liberation of heat. The concentrated solution is drawn from C by the pump D and discharged into the separator G. Here, by the addition of heat, ammonia vapour is liberated and passes to the condenser H, the high-pressure liquid ammonia collecting at J. Between J and A a throttle valve is situated and is regulated to maintain the necessary pressure difference between the " high " and " low " sides.

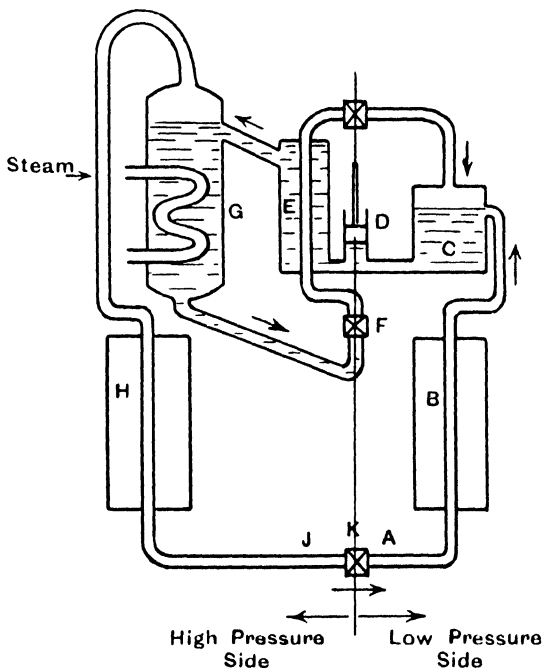


Fig. 146.

During the throttling process a certain amount of evaporation occurs (see p. 79), but this is unavoidable.

It remains only to remove from the bottom of the separator the hot water from which the ammonia has been liberated, reduce its pressure by means of the throttle valve F, and, after cooling it, return it to the absorber to receive a fresh charge of ammonia. The heat which has to be removed is transferred in the heat interchanger E to the strong cool solution which is on its way to the separator.

It should be noted that the high and low pressures refer essentially only to the pressures of the ammonia, and that the pressure exerted by the ammonia is dependent upon temperature alone and would be quite

unaffected by any admixture with inert gases (Dalton's Law of Partial Pressures). An ingenious application of this fact will be found in the " Electrolux " refrigerator shown in Fig. 147.

Here again strong solution of ammonia is heated in the separator G, from which ammonia vapour at the " high " pressure passes to condenser H. The liquid ammonia passes round the trap K and enters the refrigeration chamber B, where it enters an atmosphere of inert hydrogen under a pressure very little lower than that maintained on the high-pressure side, the partial pressure of the ammonia completing the equilibrium on the two sides of the trap K. The liquid ammonia, in effect subjected to

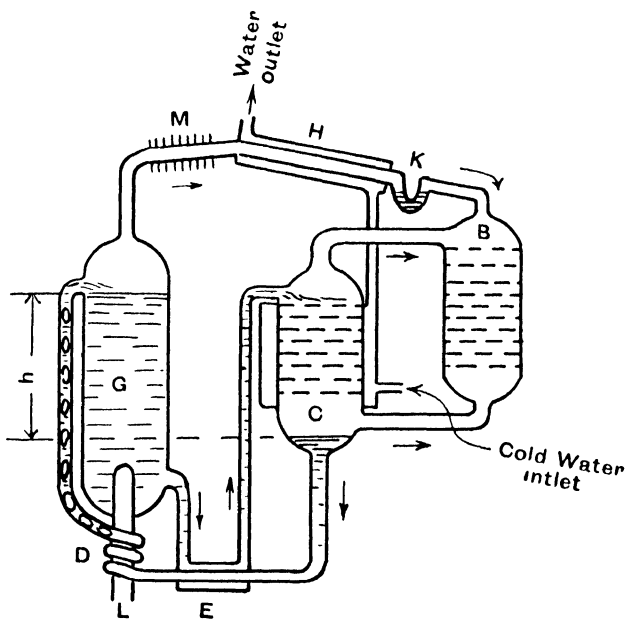


Fig. 147.

a low pressure equal to the " high " pressure minus the partial pressure of the hydrogen, now evaporates, and takes up its latent heat from the cold chamber.

The heavy mixture of ammonia vapour and hydrogen falls in the refrigerating vessel B and passes by means of the lower tube into the absorber C, where it meets water coming from G which has been cooled in the interchanger E. The ammonia is absorbed, the strong solution passing through E on its way to the separator G, while the hydrogen rises and passes back into B.

As the *total* pressures on the " high " and " low " pressure sides are equal, the pressure against which the liquid has to be forced in order that

it may enter G is only that due to the difference in level of the liquids in G and C. The requisite pumping action may be produced by applying a small amount of heat from the heating element L to the coil D containing the strong solution. Bubbles of vapour are liberated, reducing the total density of the rising column of liquid on the principle of the " air-lift " pump, and setting up a natural circulation in the system. A cooler M is provided prior to the condenser H, in which the small quantity of water vapour which is liberated along with the ammonia is condensed and returned to the separator.

It will be seen that, even after complete inversion of the whole apparatus, the hydrogen will be rapidly transferred to B and C when heat is applied at L. Apart from the pipes for the inlet and outlet of cooling water, the apparatus is hermetically sealed after being charged, and there are no moving parts whatever.

VAPOUR COMPRESSION PROCESS

In these machines the rise of temperature necessary to cause condensation of the evaporated refrigerant is supplied in the form of work during adiabatic compression.

Fig. 148 shows the general arrangement of the plant and also the

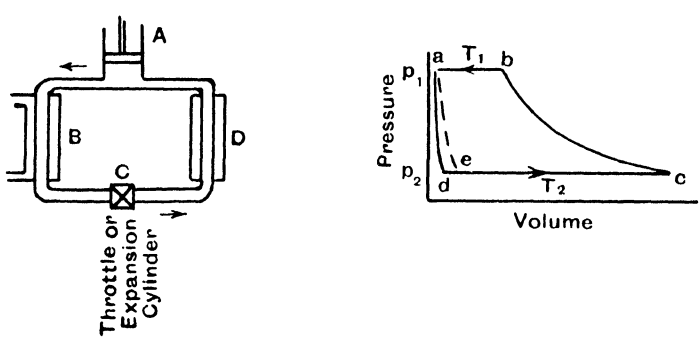


Fig. 148.

p.v. diagram for the cycle. Refrigerant, which enters the refrigerating chamber D from C as a liquid at low pressure, is evaporated along *ec*, and absorbs its latent heat of evaporation at temperature T_2. The vapour is then removed by the pump A, which raises the temperature and pressure by adiabatic compression (curve *cb*) and forces it into the condenser B, where the latent heat at T_1 is removed. The liquid gravitates to C, which consists of either a small expansion cylinder or (almost invariably in practice) a throttle valve, and here the pressure is reduced either by adiabatic expansion or by throttling to such a value that evaporation can subsequently occur at temperature T_2.

During expansion or throttling a small amount of evaporation will take place. In the former case the curve of adiabatic expansion *ad* will be followed, and work will be done and can be used to help in the driving of the compressor. In the latter case a curve *ae* of constant total heat will be followed, but the area *aed* does *not* represent useful work, as the process is completely irreversible.

If the refrigerant is evaporated to dryness at the point *c* then the proportion of the latent heat which is usefully collected from the refrigerating chamber will be $\frac{dc}{v_c}$ and $\frac{ec}{v_c}$ for the two processes. The vapour formed in expansion or throttling will pass through the refrigerating chamber as an inert substance. It is customary for evaporation to be incomplete at *c*, in which case, if q_c is the final dryness fraction, the loss of heat-absorbing capacity will be

$$\frac{v_d \text{ or } v_e}{v_c} \times q_c L_2$$

The Tϕ diagrams for the two processes are given in Fig. 149.

(1) *Compression to the dry saturation point, and pressure reduction by expansion cylinder.*—The figure will be bounded by two lines of constant temperature, T_1 during condensation and T_2 during evaporation, and by lines of adiabatic compression *cb* and expansion *ad*. It is assumed for the moment that the vapour is just sufficiently wet at *c* to be completely dried by compression.

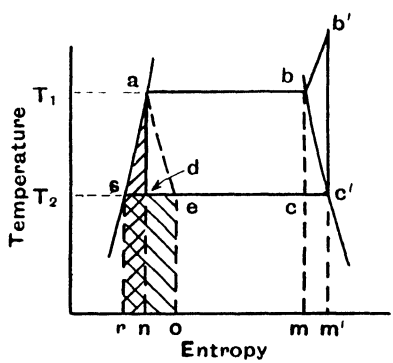


Fig. 149.

The heat received from the cold chamber is that required to evaporate 1 lb. of refrigerant in the condition given by point *d* to the condition given by *c*, and this is the area *dcmn*.

The heat rejected to the condenser is the latent heat $L_1 = abmn$;

The mechanical work done $= abmn - dcmn = abcd$; and

The coefficient of performance $= \frac{T_2}{T_1 - T_2}$.

This agrees with the coefficient for the reversed Carnot cycle, and has a constant value for all positions of *c* provided the vapour is not superheated after compression. In this case the work area will be *abb'c'd*, which shows a greater rate of increase than the area *dc'm'n* representing the heat received from the cold chamber.

EXAMPLE.—An ammonia compression refrigerating machine has to do an amount of refrigeration equal to the production of 25 tons of ice at

32° F. per 24 hours. If the temperature limits in the compressor are 75° F. and −5° F., calculate the horse-power of the compressor on the assumption that the cycle is a perfect one. The initial temperature of the water may be taken as 32° F.

The coefficient of performance of a perfect machine $=\frac{T_2}{T_1-T_2}$

$$=\frac{455}{80}=5{\cdot}687$$

Taking the latent heat of ice as 142 B.Th.U. per lb.,

the heat to be extracted per minute $=\frac{25\times 2240\times 142}{24\times 60}=5560$ B.Th.U.

Now $$\frac{H}{W}=5{\cdot}687$$

$\therefore$ work done per minute, $W=\frac{5560}{5{\cdot}687}\times 778$ ft.-lb.

and horse-power $=\frac{5560\times 778}{5{\cdot}687\times 33{,}000}=22{\cdot}4$ H.P.

Note.—An actual machine will have a coefficient of performance of from 60 to 70 per cent. of the ideal. Hence the B.H.P. of the engine driving the above compression would have to be about $\frac{22{\cdot}4}{0{\cdot}6}=37$ B.H.P.

(2) *Compression to the dry saturation point, and pressure reduction by throttling.*—The throttling process is represented by the line of constant total heat *ae*.

The total heat before throttling, reckoned from liquid at T_2, is represented by *rsan*, and after throttling by *rseo*.

The area *sdnr* is common to both these areas, so that

$$sad=ndeo$$

But if C_p=specific heat of liquid refrigerant, then $rsan=C_p(T_1-T_2)$

Hence $$C_p(T_1-T_2)=se\times T_2$$

or $$se=\frac{C_p(T_1-T_2)}{T_2}$$

The heat discharged to the condenser $=abmn$
and the heat received during evaporation $=ecmo$
Hence the work done $=abcd+ndeo$

$$=abcd+sad=sabc$$

The coefficient of performance $=\frac{ecmo}{sabc}$

But $ecmo = T_2 \times ec$

$$= T_2(sd + dc - se)$$

$$= T_2\left(C_p \log_\epsilon \frac{T_1}{T_2} + \frac{L_1}{T_1} - \frac{C_p(T_1 - T_2)}{T_2}\right) \quad \ldots\ldots (1)$$

and $sabc = rsabm - rscm$

$$= (rsan + nabm) - rscm$$

$$= C_p(T_1 - T_2) + \frac{L_1}{T_1} \times T_1 - T_2\left(C_p \log_\epsilon \frac{T_1}{T_2} + \frac{L_1}{T_1}\right)$$

$$= (T_1 - T_2)\left(C_p + \frac{L_1}{T_1}\right) - T_2 C_p \log_\epsilon \frac{T_1}{T_2} \quad \ldots\ldots (2)$$

$$=$$ the work done in the Rankine cycle ;

and coefficient of performance, from (1) and (2), is

$$\frac{C_p T_2 \log_\epsilon \frac{T_1}{T_2} + \frac{L_1}{T_1} T_2 - C_p(T_1 - T_2)}{(T_1 - T_2)\left(C_p + \frac{L_1}{T_1}\right) - T_2 C_p \log_\epsilon \frac{T_1}{T_2}}$$

The same line of argument can be followed if the vapour is superheated after compression.

EXAMPLE.—In an ammonia compression refrigerating machine the temperature in the refrigerator is 14° F., and after compression 86° F. In the cooler the vapour is condensed at 86° F. and then passes through an expansion valve into the refrigerator. Estimate the coefficient of performance when the vapour at the end of compression is, (*a*) just dry and saturated, (*b*) 90 per cent. dry. Take the latent heat of ammonia at 86° F. as 490·5 B.Th.U. per lb. and the specific heat of the liquid as 1·12.

(*a*) Heat extracted from refrigerating chamber per pound

$$= C_p T_2 \log_\epsilon \frac{T_1}{T_2} + \frac{L_1}{T_1} \cdot T_2 - C_p(T_1 - T_2)$$

here $T_1 = 86 + 460 = 546°$ abs.

$T_2 = 14 + 460 = 474°$ abs.

$L_1 = 490{\cdot}5$

$$\therefore \text{ heat extracted} = 1{\cdot}12 \times 474 \times \log_\epsilon \frac{546}{474} + \frac{490{\cdot}5 \times 474}{546} - 1{\cdot}12(546 - 474)$$

$$= 75{\cdot}08 + 425{\cdot}65 - 80{\cdot}64$$

$$= 420{\cdot}09 \text{ B.Th.U.}$$

$$\text{Work done} = (T_1 - T_2)\left(C_p + \frac{L_1}{T_1}\right) - T_2 \cdot C_p \log_\epsilon \frac{T_1}{T_2}$$

$$= (546 - 474)\left(1{\cdot}12 + \frac{490{\cdot}5}{546}\right) - 474 \times 1{\cdot}12 \log_\epsilon \frac{546}{474}$$

$$= 72 \times 2{\cdot}02 - 75{\cdot}08$$

$$= 70{\cdot}36 \text{ B.Th.U.}$$

$$\therefore \text{ coefficient of performance} = \frac{420{\cdot}09}{70{\cdot}36} = 5{\cdot}97$$

(*b*) $$\text{Heat extracted} = C_p T_2 \log_\epsilon \frac{T_1}{T_2} + \frac{q_1 L_1}{T_1} \cdot T_2 - C_p(T_1 - T_2)$$

$$= 75{\cdot}08 + 0{\cdot}9 \times 425{\cdot}65 - 80{\cdot}64$$

$$= 377{\cdot}52 \text{ B.Th.U.}$$

$$\text{Work done} = (T_1 - T_2)\left(C_p + \frac{q_1 L_1}{T_1}\right) - T_2 \cdot C_p \log_\epsilon \frac{T_1}{T_2}$$

$$= 72(1{\cdot}12 + 0{\cdot}81) - 75{\cdot}08$$

$$= 63{\cdot}88 \text{ B.Th.U.}$$

$$\therefore \text{ coefficient of performance} = \frac{377{\cdot}52}{63{\cdot}88} = 5{\cdot}90$$

EXAMPLE.—Solve the last problem when an expansion cylinder is used instead of an expansion valve.

(*a*) $$\text{Heat abstracted} = \frac{L_1 T_2}{T_1} = \frac{490{\cdot}5 \times 474}{546} = 425{\cdot}65 \text{ B.Th.U.}$$

$$\text{Work done} = \frac{L_1}{T_1}(T_1 - T_2) = \frac{490{\cdot}5 \times 72}{546} = 64{\cdot}69 \text{ B.Th.U.}$$

$$\text{Coefficient of performance} = \frac{425{\cdot}65}{64{\cdot}69} = 6{\cdot}58$$

(*b*) $$\text{Heat extracted} = 0{\cdot}9 \times 425{\cdot}65 = 383{\cdot}08 \text{ B.Th.U.}$$

$$\text{work done} = 0{\cdot}9 \times 64{\cdot}69 = 58{\cdot}22 \text{ B.Th.U.}$$

$$\text{coefficient of performance} = \frac{383{\cdot}08}{58{\cdot}22} = 6{\cdot}58.$$

Note.—The coefficient of performance in this case is simply $\frac{T_2}{T_1 - T_2}$ and there is no necessity to work out separately the heat extracted and the work done.

EXAMPLE.—Consider the machine taken in the last examples, but let the vapour be just dry and saturated when compression begins. Estimate the coefficient of performance. (Take $c_{p_v} = 0{\cdot}508$ and $L_2 = 577{\cdot}4$ B.Th.U. per lb.)

The temperature after compression (T_3) is given by

$$c_{p_v} \log_\epsilon \frac{T_3}{T_1} = \frac{L_2}{T_2} - c_p \log_\epsilon \frac{T_1}{T_2} - \frac{L_1}{T_1}$$

where T_2 is the temperature of evaporation and T_1 the temperature of condensation, and c_{p_v} is the specific heat of the vapour.

$$0{\cdot}508 \times \log_\epsilon \frac{T_3}{546} = \frac{577{\cdot}4}{474} - 1{\cdot}12 \times \log_\epsilon \frac{546}{474} - \frac{490{\cdot}5}{546}$$

$$0{\cdot}508 \times 2{\cdot}303 \log_{10} \frac{T_3}{546} = \frac{577{\cdot}4}{474} - 1{\cdot}12 \times 2{\cdot}303 \log_{10} \frac{546}{474} - \frac{490{\cdot}5}{546}$$

$$1{\cdot}17 \log_{10} \frac{T_3}{546} = 1{\cdot}218 - 0{\cdot}158 - 0{\cdot}898$$

$$= 0{\cdot}162$$

$$\therefore \log_{10} T_3 - \log_{10} 546 = \frac{0{\cdot}162}{1{\cdot}17} = 0{\cdot}1384$$

from which $T_3 = 750{\cdot}9°$ abs.

With expansion valve—

$$\text{heat extracted} = L_2 - c_p(T_1 - T_2)$$

$$= 577{\cdot}4 - 1{\cdot}12 \times 72$$

$$= 496{\cdot}76 \text{ B.Th.U.}$$

and work done

$$= (T_1 - T_2)\left(c_p + \frac{L_1}{T_1}\right) - T_2 \cdot c_p \log_\epsilon \frac{T_1}{T_2} + c_{p_v}\left(T_3 - T_1 - T_2 \log_\epsilon \frac{T_3}{T_1}\right)$$

$$= 70{\cdot}36 + 0{\cdot}508\left(750{\cdot}9 - 546 - 474 \log_\epsilon \frac{750{\cdot}9}{546}\right)$$

$$= 70{\cdot}36 + 27{\cdot}33$$

$$= 97{\cdot}69 \text{ B.Th.U.}$$

$$\therefore \text{coefficient of performance} = \frac{496{\cdot}76}{97{\cdot}69} = 5{\cdot}08$$

With expansion cylinder—

$$\text{heat extracted} = \frac{L_1 T_2}{T_1} + T_2 \cdot c_{p_v} \log_\epsilon \frac{T_3}{T_1}$$

$$= \frac{490{\cdot}5 \times 474}{546} + 474 \times 0{\cdot}508 \log_\epsilon \frac{750{\cdot}9}{546}$$

$$= 425{\cdot}65 + 76{\cdot}76$$

$$= 502{\cdot}41 \text{ B.Th.U.}$$

work done—

$$=\frac{L_1}{T_1}(T_1-T_2)+c_{p_v}(T_3-T_1)-T_2 \cdot c_{p_v} \log_\epsilon \frac{T_3}{T_1}$$

$$=\frac{490{\cdot}5\times 72}{546}+0{\cdot}508(780{\cdot}9-546)-474\times 0{\cdot}508 \log_\epsilon \frac{750{\cdot}9}{546}$$

$$=64{\cdot}69+27{\cdot}33$$

$$=92{\cdot}02 \text{ B.Th.U.}$$

$$\therefore \text{ coefficient of performance}=\frac{502{\cdot}41}{92{\cdot}02}=5{\cdot}45$$

For convenience in comparison the results obtained in the above examples are tabulated below.

	State at end of compression	Heat abstracted B.Th.U. per lb.	Work done B.Th.U. per lb.	Coefficient of performance
Expansion valve .	Dry and saturated	420·09	70·36	5·97
,, ,, .	90 per cent. dry	377·52	63·88	5·90
,, ,, .	Superheated to 750·9° abs.	496·76	97·69	5·08
Expansion cylinder	Dry and saturated	425·65	64·69	6·58
,, ,, .	90 per cent. dry	383·08	58·22	6·58
,, ,, .	Superheated to 750·9° abs.	502·41	92·02	5·45

It will be seen that the coefficient of performance is greatest in the first case when the refrigerating agent used is dry and saturated at the end of compression. The effect of allowing superheating to take place during compression increases the amount of refrigeration at the expense of a greatly increased amount of work done in driving the compressor, the result being a reduced coefficient of performance. The heat of compression also raises the temperature of the compressor walls, and on the entrance of the next charge of cold vapour, heat is absorbed by it and the vapour expands. The result is that a smaller charge is taken in and there is less refrigeration per cycle, although, as shown in the table, the refrigeration per pound is greater (see also p. 335).

IRREVERSIBLE TEMPERATURE DROP UNAVOIDABLE IN PRACTICE

In order that the requisite quantities of heat may be transferred to or from the refrigerant in a reasonable time it is necessary that appreciable temperature differences should exist between it and the receiver or source

of heat. Especially is this so for refrigerants other than air which have to be kept, often under pressure, in sealed vessels through the walls of which the heat must be transferred. If a chilled brine circuit is employed to transfer to the refrigerant heat collected over a large area, two allowances of temperature differences must be made, (1) between the cold chamber and the brine, (2) between the brine and the refrigerant.

It will be seen from the equation

$$\text{Coefficient of performance} = T_2(T_1 - T_2)$$

that the total temperature range should be kept as small as possible consistent with reasonable speed of operation. The temperature allowance for each heat transference is a matter for experience and compromise, but 4° C. to 6° C. are commonly allowed.

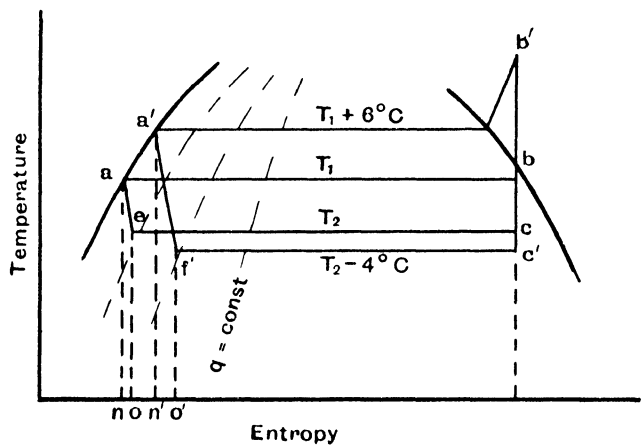


Fig. 150.

If it is required to produce ice at 0° C., the minimum temperature of the condensing water being 15° C. and brine circulation being used, the net range of temperature of the refrigerant is then not 15°, but of the order of $(15+6°)-(0-4°-4°)=29°$, an increase which appreciably affects the coefficient of performance.

The result of this increase of temperature range is indicated on the Mollier diagram, Fig. 150, where an allowance of 6° is made for the temperature drop between ammonia vapour and condensing water, and 4° between the cold chamber and evaporating ammonia. *aecb* shows the simple diagram without allowances, but when these are made the compression must be extended from *b* to *b′* in the superheat zone, and condensation is completed at *a′*. *a′f′* is the new curve of throttling, and it will be seen that the dryness fraction at *f′* is markedly greater than at *e*.

The heat received from the cold chamber per pound of refrigerant

circulated is now the area under $f'c'$ (instead of ec) and the work done is $a'f'c'b'+a'f'o'n'$ (instead of $aecb+aeon$). The loss of efficiency is obvious.

By the addition of a small cooling coil between the condenser and throttle valve, and served by fresh cooling water, it is possible to reduce the temperature of the liquid refrigerant almost to T_1, as the liquid heat

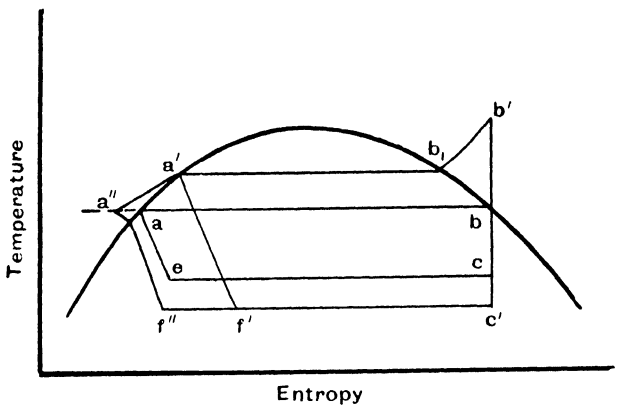


Fig. 151.

is only a small portion of the total heat, and the rate of heat transfer where two liquids are involved is greater than when one of the liquids is replaced by a condensing vapour.

The three cases are shown in Fig. 151 for a refrigerator using carbon

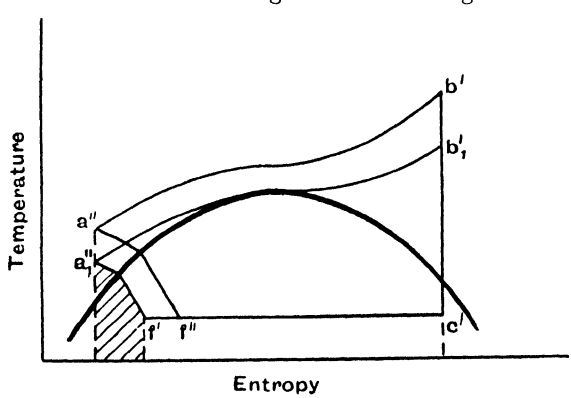


Fig. 152.

dioxide. For this refrigerant the critical temperature is only 31·3° C., and the deviation between the constant-pressure lines and the liquid boundary in this region is marked. Constant-pressure cooling and condensing will now be along the line $b'b_1a'a''$, not along $b'b_1a'a$.

(1) *aecba* is the ideal cycle ;
(2) $a'f'c'b'b_1a'$ is the cycle with temperature drop allowance ; and
(3) $a''f''c'b'b_1a'a''$ is the cycle with cooling of the liquid refrigerant to T_1. (Note the line of constant total heat is broken at the liquid boundary.)

Fig. 152 shows similar curves for CO_2 when the heat of the vapour is extracted at and above the critical temperature, the cycle in other respects being similar to (2) above.

The resulting decrease in refrigerating effect and great increase of power consumed will be obvious. (N.B. The work done is the area of the closed figure+the area under the throttling curve $a_1''f'$ or $a''f''$.) See Example, p. 327.

USE OF THE MOLLIER CHART

On account of the ease with which throttling (at constant total heat) and adiabatic compression can be represented on the Mollier chart, and the fact that quantities of heat can be scaled directly upon it, this chart is of great value. By the use of oblique co-ordinates the scales can be made conveniently open, and a high degree of accuracy can be obtained, while lines of constant pressure are continuous curves and so are easy to trace.

In Fig. 153 a diagram for carbon dioxide is shown upon this chart. It has been assumed that condensation will take place at 27° C. and evaporation at −8° C.

Tracing the curves for these temperatures from the right-hand top corner of the diagram to their point of contact with the vapour boundary, it will be found that they correspond to condensation and vaporisation pressures of 950 lb. and 400 lb. per sq. in. respectively. A piece of tracing paper laid over the diagram enables these pressure lines to be marked.

Assuming (see next section) that the vapour is drawn from the cold chamber when the dryness fraction is 0·85, the point c' can at once be found, and compression will be along a line $c'b'$ (oblique) of constant entropy which will cut the line of $p=950$ lb. at the point b', the vapour then having a temperature of 34° or 7° of superheat.

The removal of superheat, condensation, and cooling of the liquid to, say, 15° C. follow the line of constant pressure $p=950$, and are shown by $b'b_1$, b_1a', $a'a''$. Throttling is effected along the horizontal line $a''f''$.

The salient quantities involved in the cycle are then obtained as follows :

(1) Heat equivalent of work done per pound during compression
$=H_b'-H_c'$.

Superheat removed per pound $=H_b'-H_{b_1}$
Latent heat ,, ,, $=H_{b_1}-H_a'$
Liquid heat ,, ,, $=H_a'-H_a''$

(2) Total heat to cooling and condensing water$=H_b'-H_a''$.

(3) Heat removed from cold chamber per pound$=H_c'-H_f''$.

From (3) and a knowledge of the total heat to be removed per hour, the number of pounds of CO_2 to be circulated per minute can be obtained. Let this be n. From (1) and the value of n, the power required to drive the compressor can be found.

From n, the dryness fraction at c', and the specific volume at 400 lb. per sq. in. pressure (from tables), the volume swept by the compressor

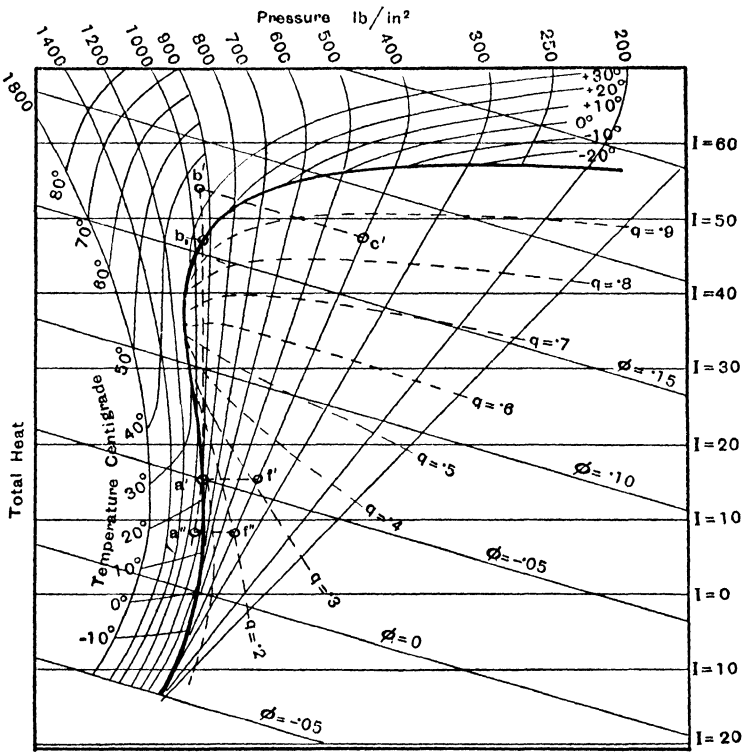


Fig. 153.

cylinder per hour can be calculated. If the revolutions per minute of the compressor are assumed the corresponding cubic capacity of the cylinder can be obtained.

From (2) and n, and the permissible rise in temperature of the "condensing" water, the quantity of water required per hour can be calculated. The direction of circulation of cooling water is important, the coldest water being required for cooling the liquid while an appreciably higher temperature will be effective in reducing the superheat. If there is no

liquid cooling the throttling action will be along $a'f'$ instead of $a''f''$; no alteration will be made to the amount of power consumed, but the effective work done will be reduced in the ratio of $f'c'/f''c'$.

TO FIND THE DRYNESS FRACTION REQUIRED BEFORE COMPRESSION TO GIVE MAXIMUM EFFICIENCY

The diagram can be set off on the Mollier chart as before, except that the position of the compression line cb is unknown.

Referring to Fig. 154, from f draw a tangent to the line of maximum

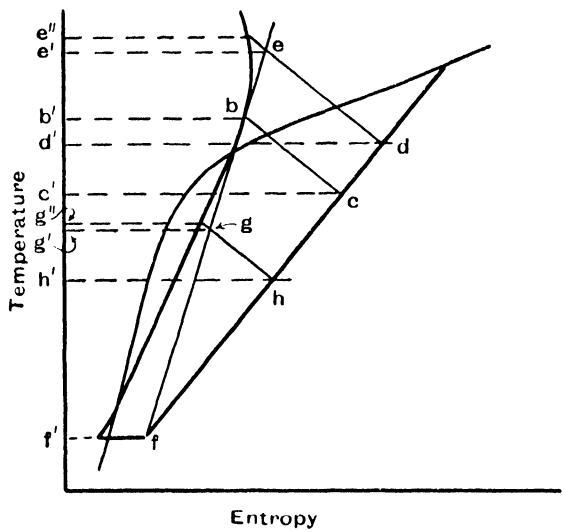


Fig. 154.

pressure, touching it at b. Draw bc, a line of constant entropy. Then the position of c gives the requisite dryness fraction.

The proof of this construction is as follows :

The heat to be added per pound during compression along cb is $c'b'$.

The heat extracted during evaporation along fc is $f'c'$.

The coefficient of performance is therefore $\frac{f'c'}{c'b'}$.

Take any points h and d on the evaporation line and on opposite sides of c, and through them draw lines of constant entropy to cut fb produced in g and e. Then as the triangles fhg, fcb, and fde are similar,

$$\frac{f'c'}{c'b'}=\frac{f'h'}{h'g'}=\frac{f'd'}{d'e'}$$

But *hg* and *de* must both be *extended* to meet the line of constant pressure condensation, or the coefficient of performance $\frac{f'c'}{c'b'} > \frac{f'h'}{h'g' \text{ extended}}$ and also $> \frac{f'd'}{d'e' \text{ extended}}$.

The tangent point *b* therefore determines the condition for maximum efficiency.

EXAMPLE.—In an ammonia refrigerator evaporation occurs at 14° F. and condensation at 86° F. Find the coefficient of performance if the vapour is (1) dry, (2) 90 per cent. dry before compression, (3) as (2), but using an expansion cylinder.

Case 1. $\begin{cases} T_1 = 86° \text{ F.} = 30{\cdot}5° \text{ C.} \\ T_2 = 14° \text{ F.} = -10° \text{ C.} \end{cases}$

Heat before evaporation	=34 C.H.U./lb.	$q=0{\cdot}15$
Heat after evaporation	=301 ,,	$q=1$
Heat after compression	=351 ,,	63·5° superheat (i.e. 94° C.)

Heat received =301–34 =267 C.H.U./lb.
Compression heat=351–301=50 ,,

$$\text{Coefficient of performance} = \frac{267}{50} = 5{\cdot}34$$

Case 2. Dryness before compression=0·9

Heat before evaporation	=34 C.H.U./lb.	$q=0{\cdot}15$
Heat after evaporation	=270 ,,	$q=0{\cdot}9$
Heat after compression	=310·5 ,,	$q=1$

$$\text{Coefficient of performance} = \frac{270-34}{310{\cdot}5-270} = \frac{236}{40{\cdot}5} = 5{\cdot}83$$

This coincides very closely with the maximum coefficient obtained by the tangent construction, see p. .

Case 3. $\begin{cases} \text{Dryness before compression} = 0{\cdot}9 \\ \text{Expansion cylinder used.} \end{cases}$

Heat before evaporation	=30·0
Heat after evaporation	=270
Heat after compression	=310·5

$$\text{Coefficient of performance} = \frac{270-30}{310{\cdot}5-270} = \frac{240}{40{\cdot}5} = 5{\cdot}93$$

EXAMPLE.—Solve the refrigeration problem stated above, using carbon

dioxide as the refrigerant. Find also the coefficient of performance if the liquid, after condensation, is cooled to 15° C.

As before $\begin{cases} T_1 = 30{\cdot}5^\circ \text{ C.} \\ T_2 = -10{\cdot}0^\circ \text{ C.} \end{cases}$

Case 1. Heat before evaporation = 27·4 C.H.U./lb. $q = 0{\cdot}53$
Heat after evaporation = 57·0 ,, $q = 1$
Heat after compression = 67·2 ,, Temp. 68° C.
Coefficient of performance $= \dfrac{57{\cdot}0 - 27{\cdot}4}{67{\cdot}2 - 57{\cdot}0} = \dfrac{29{\cdot}6}{10{\cdot}2} = 2{\cdot}90$

Case 2. Dryness before compression = 0·9
Heat before evaporation = 27·4 C.H.U./lb.
Heat after evaporation = 51·0 ,,
Heat after compression = 59·2 ,, Temp. 69° C.
Coefficient of performance $= \dfrac{51{\cdot}0 - 27{\cdot}4}{59{\cdot}2 - 51{\cdot}0} = \dfrac{23{\cdot}6}{8{\cdot}2} = 2{\cdot}88$

Note that the tangent construction shows a maximum efficiency when the maximum temperature is about 78°.

Case 3. As Case 2, with expansion cylinder.
Heat before evaporation = 23·6 C.H.U./lb.
Heat after evaporation = 51·0 ,,
Heat after compression = 59·2 ,,
Coefficient of performance $= \dfrac{51{\cdot}0 - 23{\cdot}6}{59{\cdot}2 - 51{\cdot}0} = \dfrac{27{\cdot}4}{8{\cdot}2} = 3{\cdot}34$

Case 4. As Case 2, but with liquid cooling to 15° C.
Heat before evaporation = 9·5 C.H.U./lb.
Heat after evaporation = 51·0 ,,
Heat after compression = 59·2 ,,
Coefficient of performance $= \dfrac{41{\cdot}5}{8{\cdot}2} = 5{\cdot}06$

EXAMPLE.—Ice at −10° C. is to be made from water at 15° C. A carbon dioxide refrigerator with brine circulation is to be used. The maximum temperature of the cooling water is 20° C.

Calculate the probable coefficient of performance, the compressor I.H.P., and the quantity of cooling water, if 10 tons of ice are required per 24 hours.

Heat to be extracted per minute

$$= \frac{10 \times 2240}{24 \times 60} \text{ (liquid heat + latent heat + heat from ice)}$$

$$= 15{\cdot}5(1 \times 15 + 80 + 0{\cdot}502 \times 10) \text{ C.H.U.}$$

(Specific heat of water=1. Specific heat of ice=0·502. Latent heat of ice=80·0 C.H.U. per lb.)

Heat to be extracted=15·5(100)=1550 C.H.U. per minute.

Allowing 4° C. heat drop across the brine piping walls, and 6° C. across the walls of the condenser, we have: minimum temperature of refrigerant=−10−4−4=−18° C.

Temperature of condensation=20+6=26° C.

The cycle is now set out on the chart, using the tangent method to find the condition for optimum performance.

Heat before evaporation	=13·3 C.H.U. per lb.	
Heat after evaporation	=50·2 ,,	
Heat after compression	=60·0 ,,	
Heat after removal of superheat	=48·0 C.H.U. per lb.	46·7
Heat after removal of latent heat	=21·9 ,,	
Heat after removal of liquid heat	=13·3 ,,	

Coefficient of performance $= \frac{50·2-13·3}{60-50·2} = \frac{36·9}{9·8} = 3·77$

Circulation of refrigerant required per minute $= \frac{1550}{36·9} = 42$ lb.

Compressor power required $= \frac{42 \times 9·8}{23·53} = 17·45$ H.P.

Cooling water required—allowing for a temperature rise of 5° C., i.e. from 15° to 20° C.—

$$= \frac{46·7}{5} \times 42 = 392 \text{ lb. per minute}$$

The actual compressor power will require to be increased by an amount depending on the efficiency of the compressor and its driving mechanism. No allowance has been made for heat leakage.

WET AND DRY COMPRESSION

If compression is completed before dryness is reached it is said to be "wet"; if dryness or superheating are attained the compression is "dry."

In the former case the refrigerant never reaches a temperature above that of condensation, the cylinder remains comparatively cool, and the volumetric efficiency should therefore be high. On the other hand any liquid remaining in the clearance space of the cylinder after compression will seriously reduce the volumetric efficiency by evaporating during the early portion of the suction stroke, in addition to which lubrication and corrosion troubles are likely to arise.

Excessive superheating, by increasing the temperature of the cylinder, will reduce the volumetric efficiency, and will also cause a reduction of performance by increasing the range of temperature over which the machine is expected to work.

TWO-STAGE THROTTLING (STOKES OR VOORHEES CYCLE)

During the throttling process a certain amount of evaporation occurs (see the line of constant dryness fraction through f', Fig. 153), the vapour so formed passing through the refrigerating chamber without taking up heat, occupying valuable space in the compressor, and requiring to be compressed over the whole pressure range. The difficulties presented are only serious in the case of carbon dioxide.

In the Stokes or Voorhees cycle, illustrated in Fig. 155, throttling is

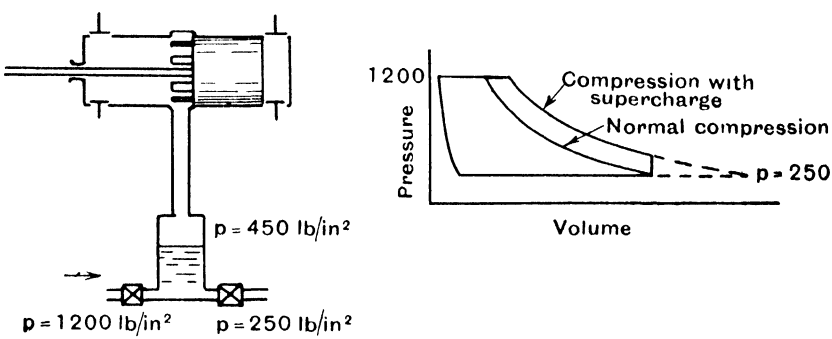


Fig. 155.

carried out in two stages, roughly from 1200 to 450 lb., and from 450 to 250 lb. per sq. in. Though the total amount of vapour formed during throttling is unchanged the major portion is liberated during the first and largest pressure drop, and is separated from the liquid in a vessel placed between the two throttle valves.

At the end of the suction stroke, when the cylinder contains very little more vapour than that actually formed during useful evaporation in the cold chamber, the piston uncovers a central ring of ports and allows the separated vapour to enter as a supercharge, the total amount of vapour being thus accommodated without further increase of the volumetric capacity of the cylinder.

As in this process the full cubic capacity of the compressor is occupied, at the low pressure, by vapour which has been formed almost entirely by useful evaporation in the cold chamber, the refrigerating effect for a given size of compressor is considerably higher than would otherwise be the case. The B.H.P. required is increased by an amount corre-

sponding to the area between the compression curves, but as the vapour formed during the first stage of throttling is compressed over part of the total range of pressure instead of the whole, the increase in horse-power is at a less rate than the increase in refrigerating capacity, and the coefficient of performance is improved.

COMPOUND COMPRESSION CYCLE. (LINDE, LIGHTFOOT, OR WILDHAUSEN CYCLE)

In the cycle last considered the irreversible process of throttling the vapour between the separator and cylinder still remains. By compressing the vapour from the refrigerator in two stages (i.e. 250 lb. to 450 lb., and 450 lb. to 1200 lb., to correspond with the case shown in Fig. 155) the vapour from the separator can be added to the vapour discharged at 450 lb. from the low-pressure cylinder, without appreciable pressure drop and prior to the final compression. The coefficient of performance will increase owing to the elimination of the irreversible process.

It should be noted that in both of the last-mentioned cycles the mass of refrigerant circulating through the various parts of the plant is not constant, as, though the condenser and first throttle deal with all, the second throttle and cold chamber only receive part of the whole. The cycles cannot therefore be represented simply on the $H\phi$ or other charts, which deal only with a constant quantity of 1 lb. The quality of the refrigerant at each stage can be found from the chart, but the quantities of heat involved can only be determined after the quantity of refrigerant at each stage has been determined. The complete calculation needs considerable care both with regard to quantities and to the effects obtained on mixing two quantities of refrigerant which may differ in pressure, temperature, and dryness.

CHOICE OF REFRIGERANTS

A large number of refrigerants are available for various uses. A choice has to be made after considering a number of factors, amongst the most obvious being the freezing-point, the liquid and latent heats over the working range, the maximum pressures likely to be attained, and the specific volume at low temperatures. Other factors are the action, if any, on metals, packing materials, etc., both separately and in combination with moisture, and the liability, in case of accident or leakage, to cause damage to surrounding apparatus or goods, to form an explosive mixture, or to give rise to risks of asphyxiation or other danger to attendants, etc. Much of the data concerning refrigerants is still incomplete.

The commonest refrigerants are as follows :

	Pressure lb. per sq. in.		Latent heat		Specific heat liquid	Specific volume at 0° C.	Volume cu. ft. per min. per ton, −15° to 30° C.	Coefficient of performance about −15° to 30° C.	Critical temperature ° C.	Efficiency relative to ideal cycle
	0° C.	20° C.	0° C.	20° C.						
Ammonia, NH_3 .	61·8	125	535	572	1·02	4·71	3·44	4·85	133	0·845
CO_2	525	864	102	62	0·98	0·160	0·98	2·56	31·8	0·447
SO_2	22·5	47·6	162	147·5	0·40	3·54	9·24	4·74	156·7	0·825
H_2O	0·09	0·53	1092	1067	1·0	3333	1972	4·10	375	0·715
CCl_2F_2 [1] . . .	45·06	82·3	37·0	34·5	0·212	0·908	5·83	4·72	101	0·823
Methyl chloride, CH_3Cl . .	36·7	71·0	96·9	91·7	0·375	2·69	6·09	4·67	257·6	0·817

[1] Dichlorodifluoromethane or " Freon " F12, a normally non-toxic refrigerent largely used for air-conditioning.

Ammonia has a powerful action on brass and copper, is slightly explosive, and extremely poisonous, but any leakage at once becomes obvious owing to the very pungent smell. The coefficient of performance is high, and the plant works at moderate pressure.

Carbon Dioxide is used owing to the absence of smell, deleterious action in general, and the risk of combustion, and the almost complete absence of toxic effects even in high concentrations. The plant is small but works at high pressures, and is used chiefly on board ships and in confined spaces. The critical temperature may be exceeded in the tropics.

Sulphur Dioxide.—Works at a low pressure, is poisonous, and has a deleterious action on certain metals and packings in the presence of moisture.

Water can only be used for chilling purposes above 0° C. Its enormous specific volume at low pressures makes it imperative to use a steam jet-pump or rotary compressor for compression purposes.

Details of these and other refrigerants may be found—among other places—in the Refrigerating Data Book, Am. Soc. of Refrigerating Engineers.

True-to-scale charts of NH_3, CO_2, SO_2, CCl_2F_2, and CH_3Cl will shortly be available from The Institution of Mechanical Engineers.

EXAMPLES ON CHAPTER XV

1. By means of a reversed perfect heat engine, ice at 32° F. is to be made from water at 67° F., the temperature of the brine or freezing mixture being 12° F. How many pounds of ice at 32° F. can be made per I.H.P. hour ? (Latent heat of ice 142 B.Th.U. per lb.)

2. If the compression pressure in a Bell-Coleman refrigerating machine is 60 lb. per sq. in. gauge and the suction pressure 15 lb. abs., find the lowest temperature produced in the machine if the air after compression is cooled to 60° F. What is the coefficient of performance, and how much ice can be made from and at 32° F. per I.H.P. hour ?

3. Find the least horse-power of a perfect reversed heat engine that will make 1200 lb. of ice per hour at 25° F. from water at 60° F. (Take specific heat of ice as 0·5 and latent heat 142 B.Th.U. per lb.)

4. In an ammonia refrigerating machine the temperature in the refrigerator is 15° F. and after compression 90° F. In the cooler the vapour is condensed at 90° F. and then passes through an expansion valve. Calculate the coefficient of performance when the vapour at the end of adiabatic compression is (*a*) just dry and saturated ; (*b*) 85 per cent. dry. Take the specific heat of liquid ammonia as 1·1, and the latent heat of vaporisation as $566-0{\cdot}8t°$ F.

5. Solve Problem 4 when an expansion cylinder is used instead of an expansion valve.

6. If in Problem 4 the ammonia is just dry and saturated at the beginning of compression, estimate the coefficient of performance (*a*) when an expansion valve is used, and (*b*) when an expansion cylinder is used. (Assume $C_p=0{\cdot}508$.)

7. A vapour compression machine has to produce 50 tons of ice at 28° F. per day of 24 hours from water at 50° F. If the temperature limits in the compressor are 80° F. and 10° F., calculate the horse-power of the compressor on the assumption that the cycle is a perfect one.

8. Work out the problem stated in the example on p. 337, if ammonia is the refrigerant.

Chapter XVI

HEAT TRANSFERENCE BY RADIATION, CONDUCTION, AND CONVECTION

*HEAT TRANSMISSION BY RADIATION

The transmission of energy by radiation between bodies at different temperatures plays a far greater part in many engineering processes than is usually supposed.

The radiation of light and heat from a body at a high temperature is a matter of common experience. If a suitable prism is placed in a beam of light coming from such a body refraction occurs, light of short wave-length being deflected through a greater angle than that of long wave-length.

If a Bolometer (an instrument for absorbing and measuring radiant energy) is traversed across the emergent ray, it will be possible to

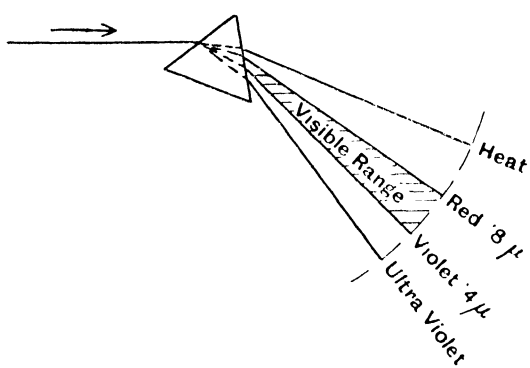


Fig. 156.

measure the radiant energy corresponding to the various wave-lengths. This will be found to extend continuously not only over the visible range, but over a very much wider range from the very short ultra-violet waves to the long-wave heat rays. It will be seen therefore that the heated body is radiating energy, only a small part of which gives the effect of light, though all of it partakes of the same nature. Radiant heat, like light, can be reflected, refracted, transmitted, absorbed, and polarised.

A *Black Body*, which is closely approximated to by a block of metal coated with "lamp-black," is one which absorbs all radiations falling upon it, and is entirely opaque and non-reflecting to all wave-lengths. The rise of temperature of such a body when exposed to energy radiation can be used, under suitable conditions, for measuring the intensity of the radiant energy.

STEFAN'S LAW

Fig. 157 shows the intensity of the radiations from such a body for a wide range of wave-lengths. The total radiant energy, shown by the areas under the curves, varies as the fourth power of the absolute temperature, i.e. the total energy emitted in unit time from unit area of a black body at absolute temperature T° F. is given by σT^4, where σ is Stefan's constant and has the value $17{\cdot}3 \times 10^{-10}$, i.e.

$$H = 17{\cdot}3 \times 10^{-10} T^4 \ \frac{\text{B.Th.U.}}{\text{sq. ft. per hour}}$$

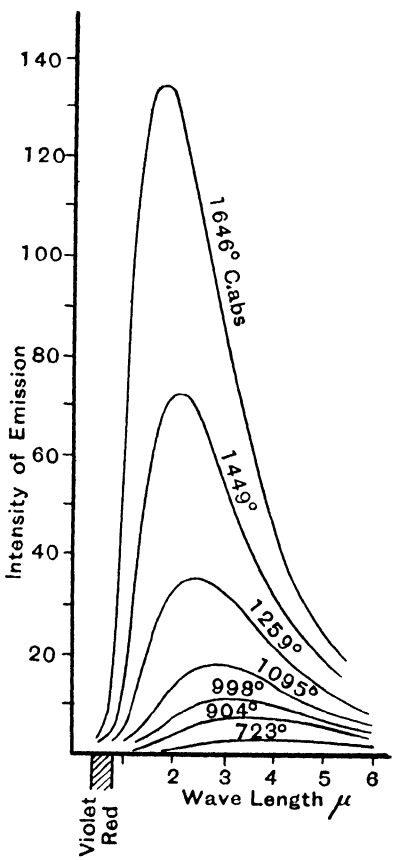


Fig. 157.

Where two black bodies at temperatures T_1 and T_2 radiate to each other, the energy radiated by the first will be proportional to T_1^4, and that by the second to T_2^4. The radiant energy passing from one to the other will therefore be proportional to $(T_1^4 - T_2^4)$. The second term is frequently negligible when compared with the first.

The temperature of a heated "black body" can be estimated with considerable accuracy by measuring either the total radiation, by means of a total-radiation pyrometer, or the intensity of radiation in the visible range, by means of a pyroscope. (The design and action of such instruments will be found in "Heat Transmission by Radiation, Convection and Conduction," Royds; "The Dictionary of Applied Physics," Vol. 1, or books on heat.)

For bodies other than perfectly "black" bodies the emissivity will

vary with the nature of the body, its temperature, and the portion of the wave-band in which measurements are made. A body which approaches but does not achieve " blackness " is said to be " grey."

Taking the emission of a black body as 1, the following are the approximate relative emissivities of representative substances :

	Heat waves	Light waves
Lamp black	0·94	0·98
Non-metallic paint, independent of colour	0·90–0·95	
Brick, various	0·45–0·60	0·72
Copper, oxidised black	0·78	
Aluminium paint	0·40–0·69	
Nickel polished	0·06	0·04
Aluminium polished	0·05	
Silver	0·01	0·11

For a vertical black surface in air with surroundings at 60° F., the radiation loss, as expressed as a percentage of the total loss by radiation, conduction, and convection, is approximately as follows :

At 100° F.	57	per cent.
500° F.	67	,,
1000° F.	82	,,
1500° F.	90	,,
2000° F.	94	,,

*FURNACE PHENOMENA AND " BLACK BODY CONDITION "

As a black body has no capacity to transmit or reflect radiant waves the radiations proceeding from it when heated must come from the body itself, whatever its surroundings may be.

If a number of different materials are put with a black body in a heated muffle furnace and allowed to attain the furnace temperature, it will be found, on inspection through a small aperture, that the radiations from all the materials are identical, both qualitatively and quantitatively, with those from the black body, and can therefore be used to determine the temperature. The objects are said to be " under black body conditions."

The rays proceeding from the black body can only come from that body itself, and in this case the furnace walls merely prevent cooling by radiation. Burnished silver will reflect a large percentage of the rays falling on it from the walls of the furnace ; its own power of emission must in consequence be very poor. Clear glass will transmit light waves from the back of the furnace, and in consequence will be found, on being

rapidly removed from the furnace and examined in the dark, to have only a very poor luminosity of its own. Green glass, which will absorb the red rays from the back of the furnace, will be found to have a bright red glow when seen in the dark, while, conversely, heated red glass will be found to shine green.

It follows therefore that for all wave-lengths of radiant energy, opaque bodies with high reflective powers have correspondingly low powers of emission, and are of considerable value when used to coat bodies from which heat losses have to be prevented.

*APPLICATION OF RADIATION THEORY

The black body conditions which are almost perfectly provided by tempering furnaces, etc., are of great value as providing radiations which can be used for measuring the temperatures required for heat-treatment. The back of the head of an exhaust valve in a high-speed petrol engine, when viewed through a small hole in the exhaust pipe close to the engine, is sufficiently " black " to enable a close estimate of its temperature to be made by means of a pyroscope while the engine is running.

In modern boiler and furnace design considerable use is made of " arches " or curved reflecting surfaces of special firebrick, to concentrate the heat radiations on to selected portions of the heating surface or on to the cooler portions of the grate.

In the case of a heated fluid transmitted through a pipe it will usually be found that the walls of the pipe are appreciably cooler than the fluid itself. A thermometer inserted into the pipe will then receive heat by transmission and conduction from the fluid, but will also lose considerable energy by radiation to the walls. It is therefore highly important, when accurate readings are required, to shield the thermometer bulb, and this may be done by placing a couple of small cylinders or a spiral roll of tin-foil to surround it without touching. Failure to do so may cause errors amounting in some cases to over 100° C.

RADIATION FROM FLAMES

As the gaseous molecules forming a flame are relatively widely spaced, few of the radiant beams emanating from any given molecule will be obstructed in their course by other molecules. Radiations will therefore proceed from the body of the flame, not from its exposed surfaces only as is the case of solids.

The radiation in any given direction will increase with increasing flame thickness measured in that direction until the radiations from the farther molecules are seriously obstructed by the increasing number of molecules in front of them. This stage is normally reached with a flame thickness of about 4 ft.

The radiation actually proceeds from the burnt gases, in general from H_2O and CO_2 molecules, and will be independent of the nature of the uncombined gases, except for the small effect of obstruction and absorption which varies for different gases.

A luminous flame, containing very small highly heated particles of solid carbon, may radiate more than twice as much energy as a similar flame in which these are not present.

CONDUCTION OF HEAT THROUGH SOLIDS

*GENERAL STATEMENT

The process by which heat is transferred through a solid body appears to depend upon the presence of " free " electrons, that is, of certain electrons which are associated with the molecules of the material as a whole but not with any particular molecule. These free electrons also provide the means whereby electricity is conducted. It follows that materials which are conductors of heat will also be conductors of electricity, and insulating bodies, which have no such electrons, will conduct neither.

*LAWS OF CONDUCTION

If two opposite faces of a rectangular block of uniform material are maintained at different temperatures heat will flow from the hotter to the colder side.

The heat transferred will be proportional to (1) the area A of the block measured transversely to the direction of flow, (2) $1/x$, where x is the distance between the isothermal faces considered, (3) the time t during which the flow occurs, (4) the temperature difference θ between the planes, and (5) the conductivity K of the material.

K is defined as the heat conducted through a cube of unit face in unit time when a temperature difference of 1° is maintained between opposite faces. Its value will be the same whether the heat and temperature are measured in C.H.U. and ° C. or B.Th.U. and ° F.

Hence
$$H=\frac{KA(\theta_1-\theta_2)t}{x} \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Or, for a small element of material

$$H=-K\frac{\delta\theta}{\delta x}\delta A\delta t \quad . \quad . \quad . \quad . \quad . \quad . \quad (1a)$$

If θ_1, the temperature of the hotter face, is maintained constant, the temperature θ_2 of the second face will decrease with increasing values of x, making $\frac{\delta\theta}{\delta x}$ essentially negative. The additional negative sign is then

employed so that heat quantities are positive when flowing in the direction giving increasing values of x.

*FLOW OF HEAT THROUGH A THIN-WALLED TUBE

This case will be illustrated by an example.

A boiler fire-tube 2 in. outside diameter and 0·1 in. thick transmits 6000 B.Th.U. per sq. ft. per hour. If K=0·01 B.Th.U. for a cube of 1 ft. side per ° F. per second, calculate the temperature difference between the inner and outer faces.

From equation (1) $H_1=6000=0{\cdot}01\times\dfrac{\theta_1-\theta_2}{0{\cdot}1/12}\times 1\times 3600$ in ° F.

or $$\theta_1-\theta_2=1{\cdot}4^\circ \text{ F.}$$

FLOW OF HEAT THROUGH A THICK-WALLED TUBE

For the conditions given in Example 1 find $\theta_1-\theta_2$ assuming the thickness is not small compared with the radius. The cross-section of metal through which the heat is transmitted increases as the heat flows outwards. Consider the tube made up of a number of elementary concentric cylinders of thickness δr and length l. The total heat flowing through all these cylinders is the same, and is given by

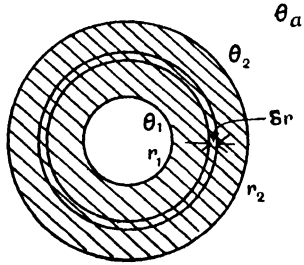


Fig. 158

$$H=K\,.\,2\pi rl\frac{\delta\theta}{\delta r}$$

But
$$\theta_2-\theta_1=\int_{r_1}^{r_2}\delta\theta=-\frac{H}{K\,.\,2\pi l}\int_{r_1}^{r_2}\frac{\delta r}{r}$$
$$=-\frac{H}{K\,.\,2\pi l}\log_\epsilon\frac{r_2}{r_1} \quad . \quad . \quad . \quad (1)$$

If the stated heat flow H_1 is measured in terms of the inner surface then:

$$H_1=\frac{H}{2\pi r_1 l} \quad \text{and} \quad \theta_1-\theta_2=\frac{H_1 r_1}{K}\log_\epsilon\frac{r_2}{r_1} \quad . \quad . \quad . \quad (2)$$
$$=1{\cdot}32^\circ \text{ F.}$$

*EQUIVALENT MEAN RADIUS OF A THICK-WALLED PIPE

When dealing with thick-walled pipes calculations may sometimes be simplified by finding the equivalent mean radius r_m of a corresponding thin-walled pipe.

For such a pipe $H=\dfrac{K \cdot 2\pi r_m(\theta_2-\theta_1)}{r_2-r_1}$ per unit length

whereas for the actual pipe (thick-walled) $H=\dfrac{2\pi K(\theta_2-\theta_1)}{\log_\epsilon r_2/r_1}$ from (1) above.

Equating these we have $r_m=\dfrac{r_2-r_1}{2{\cdot}303 \log_{10} r_2/r_1}$

r_m being the " logarithmic mean radius."

EXAMPLE.—Find the equivalent thickness of a layer of insulating material 2 in. thick surrounding a pipe 2 in. in diameter.

Here $r_2=3$ in., $r_1=1$ in.

and $$r_m=\frac{2}{2{\cdot}303 \log_{10} 3}=1{\cdot}84 \text{ in.}$$

OPTIMUM THICKNESS OF PIPE LAGGING

If Fig. 158 represents the lagging round a steam pipe of radius r_1, and θ_a is the temperature of the surrounding air, the heat passing through the lagging will be as given in equation (2), and this must equal the heat lost from the surface. If E is the emissivity (by radiation, convection, and conduction) of the surface per square foot per degree in unit time, then

$$H_1 2\pi r_1 l=E(\theta_2-\theta_a)2\pi r_2 l$$

or $$H_1=E(\theta_2-\theta_a)r_2/r_1$$

or $$\theta_2=\frac{H_1 r_1}{E r_2}+\theta_a \quad \ldots\ldots\ldots \quad (3)$$

Substituting this value in (2)

$$H_1=\frac{\theta_1-\theta_a}{r_1\left\{\dfrac{1}{Er_2}+\dfrac{1}{K}\log_\epsilon \dfrac{r_2}{r_1}\right\}} \quad \ldots\ldots\ldots \quad (4)$$

It will be seen from the figure that as r_2 increases the areas of successive layers of lagging increase, so that less resistance is offered by each succeeding layer of thickness δr. Moreover, the external area from which the heat is finally lost also increases.

The value of r_2/r_1 for the minimum value of H_1 can be found by differentiating equation (4) with respect to r_2 and equating to zero.

This will give

$$\frac{d}{dr_2}\left(\frac{1}{Er_2}+\frac{1}{K}\log_\epsilon \frac{r_2}{r_1}\right)=0$$

or a minimum when $Er_2=Kr_1$.

If E be taken as 0·35 $\frac{\text{B.Th.U.}}{\text{sq. ft. . hr.}}$ or 0·0000097 $\frac{\text{B.Th.U.}}{\text{sq. ft. . sec.}}$, and K for asbestos as 0·000032, then minimum loss will occur when $r_2=3{\cdot}3r_1$.

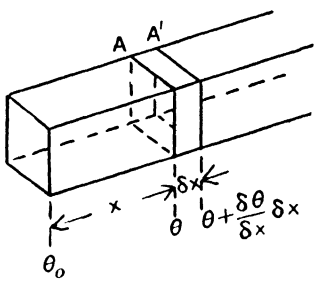


Fig. 159.

GENERAL EQUATION FOR CONDUCTION. STEADY CONDITIONS

Consider a uniform bar heated at the end $x=0$ and losing heat from the surface (see Fig. 159). Let θ be the difference in temperature between the bar, at section A, and its surroundings. The heat flow at A in unit time is

$$H_x=-KA\frac{\delta\theta}{\delta x} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

and at A′

$$H_{x+\delta x}=\frac{-KA\delta\left(\theta+\frac{\delta\theta}{\delta x}\delta x\right)}{dx}$$

$$=-KA\left(\frac{\delta\theta}{\delta x}+\frac{\delta^2\theta}{\delta x^2}\delta x\right) \quad . \quad . \quad . \quad . \quad . \quad (2)$$

The difference in heat flow at the two sections, in the limit when δx is small, must be equal the loss of heat from the surface, and will be

$$H_{x+\delta x}-H_x=-KA\frac{\delta^2\theta}{\delta x^2}dx=-Ep\theta dx$$

where E is the emissivity of the surface (i.e. E=heat loss per unit area in unit time, for 1° difference of temperature between surface and surroundings). $\frac{d\theta}{dx}$ is essentially negative.

Hence $$KA\frac{\delta^2\theta}{\delta x^2}=Ep\theta$$

or $$\frac{\delta^2\theta}{\delta x^2}=u^2\theta, \text{ where } u^2=\frac{Ep}{KA} \quad . \quad . \quad . \quad . \quad (3)$$

The solution to this, easily checked by reverse calculation, is of the nature $\theta=a\epsilon^{ux}+b\epsilon^{-ux}$, where a and b are constants . . . . (4)

Substituting the limiting values

$\theta=0$ when $x=\infty$ $\quad(0=a\,.\,\infty+0)$

$\theta=\theta_o$ when $x=0$ $\quad(b\epsilon^{u\cdot o}=\theta_o)$

then $a=0$ and $b=\theta_o$

or $\theta=\theta_o\epsilon^{-ux}$,

showing that the temperature along the bar decreases logarithmically.

GENERAL EQUATION FOR A PERIODIC FLUCTUATION OF TEMPERATURE

This equation is required for the solution of such problems as the heat flow through the walls of an engine cylinder which are subject to small cyclic variations of temperature corresponding with the fluctuations of the temperature of the working-fluid.

Two conditions should be noted, (1) as the cross-section of the " bar " increases the problem approaches that of heat flow through a sheet, and the " edge " area from which heat can be dissipated becomes relatively unimportant. When two opposite edges of the sheet are folded together, thus forming a cylinder, loss from the edges becomes zero and the fitting of covers effectively prevents loss from the ends of the " barrel." The problem is then equivalent to that applying either to a sheet of infinite area, or to a bar having no emissivity from the sides. (2) Heat flow will be alternatively towards and away from each section of the material, and heat flow past any section will produce a temporary storage of heat (with consequent rise of temperature) in the section beyond it.

If S=specific heat of bar, and ρ its density, also if t=time from some fixed moment, and H the heat flow in unit time, then

$$(H_x - H_{x+\delta x})\delta t = (E p \theta \delta x)\delta t + A\rho S \delta x \delta \theta \quad . \quad . \quad . \quad (5)$$

and if E=0 we have, as before,

$$KA\frac{d^2\theta}{dx^2} = A\rho S\frac{d\theta}{dt} \text{ or } K\frac{d^2\theta}{dx^2} = \rho S\frac{d\theta}{dt}$$

and

$$C\frac{d^2\theta}{dx^2} = \frac{d\theta}{dt} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

where $C = \frac{K}{\rho S}$ and is called the " diffusivity." C measures the ratio of the capacities of the material to transmit and to store heat.

DIMENSIONS OF C AND CERTAIN RELATED VALUES

If M, H, L, θ, and T represent units of mass, heat, length, temperature, and time, then K will have the dimensional form

$$\frac{HL}{L^2\theta T} = \frac{H}{L\theta T}$$

ρ will be given by $\frac{M}{L^3}$, and S by $\frac{H}{M\theta}$

or as $C = \frac{K}{\rho S}$ its dimensions will be

$$\frac{H}{L\theta T} \times \frac{L^3}{M} \times \frac{M\theta}{H} = \frac{L^2}{T}$$

or geometrically similar bodies of the same material will undergo the same temperature changes in times proportional to the square of their linear dimensions.

The approximate properties of certain common materials are given below :

	Conductivity B.Th.U. per sq. ft. per second per ° F. per ft. thickness	Specific heat	Density lb. per cu. ft.	C.
Aluminium	0·024	0·222	160	0·00075
Copper	0·048	0·093	557	0·000928
Bronze	0·0048	—	—	—
Cast iron	0·0077	0·12	440	0·000146
Tin	0·01	0·055	455	0·00040
Asbestos	0·000032	—	150	—
Powdered coal, felt, etc.	0·000007	—	—	—
Ebonite, Fats, oils, etc. (about)	0·000026	—	—	—

GENERAL SOLUTION OF EQUATION (6). STEADY TEMPERATURE

Here $\frac{d\theta}{d\mathrm{T}}=0$, and by double integration of (6) $\theta=\mathrm{A}+\mathrm{B}x$, A and B being constants.

Now when $x=0$, $\theta=\theta_o$, and when $x=l$, $\theta=\theta_l$.

Hence $\mathrm{A}=\theta_o$ and $\mathrm{B}=\frac{\theta_l-\theta_o}{l}$

The temperature at any point x along the bar (with no emissivity) is then given by

$$\theta_x=\theta_o+\frac{\theta_l-\theta_o}{l}x \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

GENERAL SOLUTION OF EQUATION (6). FLUCTUATING TEMPERATURE

Let the variation of temperature take a simple harmonic form, so that the temperature at the end $x=0$ is given by $(\phi+a \sin \omega\mathrm{T})$, ϕ being the mean temperature above the surroundings, and the second term showing the periodic fluctuation above and below the mean. The maximum range of temperature at the end $x=0$ will then be $2a$. If emissivity can be neglected the only term of interest is

$$\theta=a \sin \omega\mathrm{T} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

Two conditions now offer themselves for the solution of the general equation

$$C d^2\theta/dx^2 = d\theta/dT \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

(1) Should the rise and fall of temperature occur simultaneously at all parts of the bar, the temperature at any point of the bar would be a function of depth and time only, suggesting the solution $\theta = X \sin \omega T$, where X is a function of x only.

Substituting in (6) $\omega X \cos \omega T = C\frac{d^2X}{dx^2} \sin \omega T$

or
$$\frac{d^2X}{dx^2} = \frac{\omega}{C} X \cot \omega T$$

Hence X would be a function of x and T, which is inadmissible.

(2) It would appear that there may be a progressive time lag along the bar owing to the fact that each section will have to store up heat as its temperature rises preparatory to transmitting heat to the next section.

A possible modification will then be

$\theta = X \sin (\omega T + px)$, px representing a phase lag depending on x . (9)

Substituting in (6) we have

$$\omega X \cos (\omega T + px) = C\left[\frac{d^2X}{dx^2} \sin (\omega T + px) + 2p\frac{dX}{dx} \cos (\omega T + px) - p^2 X \sin (\omega T + px)\right]$$

or by rearrangement

$$C\left[\sin (\omega T + px)\left\{\frac{d^2X}{dx^2} - p^2X\right\}\right] + \cos (\omega T + px)\left\{2pC\frac{dX}{dx} - \omega X\right\} = 0$$

This will be true for all values of x and T if

$$\frac{d^2X}{dx^2} - p^2X = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

and
$$2pC\frac{dX}{dx} - \omega X = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

In both of these cases X is a function of x only.

The form of (11), when modified to $\frac{dX}{dx} = \frac{\omega}{2pC} X$, suggests a solution of the form $X = e^x$

$$\text{(in which case } \frac{dX}{dx} = e^x = X\text{)}$$

If
$$X = Ae^{\frac{\omega}{2pC}x} \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

we have $$\frac{dX}{dx}=A\frac{\omega}{2pC}e^{\frac{\omega}{2pC}x}$$

$$=\frac{\omega}{2pC}X, \text{ satisfying equation (11)}$$

From (12) when $x=0$, $X=A$

or from (9) $$\theta=A\sin(\omega T+px)$$

But the maximum temperature variation at the heated end of the bar is given by a, when $x=0$ and $\sin \omega T=1$

or $$X=ae^{\frac{\omega x}{2pC}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

Substituting this value in equation (10)

$$\frac{a\omega^2}{4p^2C^2}e^{\frac{\omega x}{2pC}}-p^2ae^{\frac{\omega x}{2pC}}=0$$

or $$p^2=\pm\frac{\omega}{2C}$$

For real values of p the negative sign is inadmissible and

$$p^2=\frac{\omega}{2C}$$

or $$p=\pm\sqrt{\frac{\omega}{2C}}$$

Substituting in (13) $X=ae^{\pm\sqrt{\frac{\omega}{2C}}x}$

But when x is large θ must obviously fall to zero, so that the $+$ sign is inadmissible.

Hence $$X=ae^{-\sqrt{\frac{\omega}{2C}}x}$$

and $$\theta=ae^{-\sqrt{\frac{\omega}{2C}}\cdot x}\sin\left(\omega T-\sqrt{\frac{\omega}{2C}}x\right)$$

If $$\alpha=\sqrt{\frac{\omega}{2C}}=\sqrt{\frac{\omega\rho S}{2K}}$$

$$\theta=ae^{-\alpha x}\sin(\omega T-\alpha x)$$

or $$\underset{(1)}{\theta}=\underset{(1)\ (2)}{ae^{-\alpha x}}\sin(\underset{(3)}{2\pi n}\underset{(3)}{T}-\underset{(4)}{\alpha x}) \quad . \quad . \quad . \quad . \quad (14)$$

where n is the number of cycles in unit time.

INTERPRETATION OF FORMULA

(1) The maximum temperature fluctuation at the face of the bar is $\pm a$ (factor 1, equation (14)).

(2) The maximum temperature fluctuation at any other point of the bar varies with the depth and is given by $\pm ae^{-\alpha x}$ (factors 1 and 2).

(3) For all values of x the period of a complete cycle of temperatures is constant and $=\frac{2\pi}{\omega}$ or $\frac{1}{n}$, where n is the number of cycles in unit time (factors 3 and 4).

(4) The phase lag varies with the depth, and $=\alpha x$, where $\alpha=\sqrt{\frac{\omega}{2C}}$.

(5) As $\sin \omega T=\sin(\omega T-2\pi)$, the temperature lags a complete cycle and is in phase with the surface temperature when $\alpha x=2\pi$ or $x=\frac{2\pi}{\alpha}$.

When $x=\pi/\alpha$ the two temperature variations are exactly out of phase.

(6) From (5) the length of the heat wave is $x=2\pi/\alpha$, and, substituting in (2) above, the fluctuation of temperature will be seen to decrease in the ratio of $1 : e^{-2\pi}=1/536$ for each wave-length of penetration. But as $\alpha \propto \sqrt{\omega}$ the wave-length is least for rapid fluctuations, so that the temperature variations at a given depth will decrease most rapidly with short period cycles.

(7) The velocity of propagation of the heat wave is $\frac{\omega}{2\pi} \times$ wave-length

$$=\frac{\omega}{2\pi}\times\frac{2\pi}{\alpha}=\sqrt{2C\omega}$$

The velocity of propagation will be a maximum for short period cycles, though the effective penetration will be least.

HEAT FLOW INTO SURFACE DURING CYCLIC FLUCTUATION OF TEMPERATURE

If H is the rate of heat flow at any instant per unit of surface area, measured in B.Th.U. per second, then $H=-K\frac{d\theta}{dx}$, where $\frac{d\theta}{dx}$ is the temperature gradient at the surface (equation (1*a*), p. 347).

But
$$d\theta/dx=\frac{d}{dx}[ae^{-\alpha x}\sin(\omega T-\alpha x)]$$
$$=-\alpha ae^{-\alpha x}\sin(\omega T-\alpha x)-\alpha ae^{-\alpha x}.\cos(\omega T-\alpha x)$$
$$=-\alpha ae^{-\alpha x}\{\sin(\omega T-\alpha x)+\cos(\omega T-\alpha x)\}$$

At the surface, when $x=0$

$$H=-K\frac{d\theta}{dx}=K\alpha a(\sin \omega T+\cos \omega T) \quad . \quad . \quad . \quad (15)$$

Now $H=0$ when $(\sin \omega T+\cos \omega T)=0$

or when $\omega T=-\pi/4$ and $3\pi/4$

Between these intervals the heat flow in time δT will be

$$-K\frac{d\theta}{dx}\delta T$$

or

$$-K\frac{d\theta}{dx}\frac{\delta(\omega T)}{\omega} \quad . \quad . \quad . \quad . \quad . \quad . \quad (16)$$

The heat flow per unit area per cycle, from (15) and (16), will be

$$H_C=\int_{-\pi/4}^{3\pi/4} K\alpha a(\sin \omega T+\cos \omega T)\frac{\delta\omega T}{\omega}$$

$$=\frac{K\alpha a}{\omega}\Big[-\cos \omega T+\sin \omega T\Big]_{-\pi/4}^{3\pi/4}$$

$$=\frac{K\alpha a}{\omega}\left[\left(\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}\right)-\left(-\frac{1}{\sqrt{2}}-\frac{1}{\sqrt{2}}\right)\right]$$

$$=2\sqrt{2}\frac{K\alpha a}{\omega}, \text{ where } \alpha=\sqrt{\frac{\omega\rho S}{2K}}$$

For cast iron $K=0{\cdot}0077$, $\rho=440$, and $S=0{\cdot}12$.

Hence $\alpha=58{\cdot}6\sqrt{\omega}$

and $$H=2\sqrt{2}\times 0{\cdot}0077\times 58{\cdot}6\frac{a}{\sqrt{\omega}}=\frac{1{\cdot}27a}{\sqrt{\omega}}$$

$$=\frac{0{\cdot}51}{\sqrt{n}} \text{ if } n \text{ is the number of cycles per second.}$$

EXAMPLE.—If the surface fluctuation of a cast-iron cylinder is $\pm 10°$ C. and the number of cycles per second is 1, find the fluctuation at a depth of $\frac{1}{2}$ in., the wave-length, and the heat flow into the surface.

Here $x=\frac{1}{2}$ in. $=0{\cdot}0417$ ft.

$$\alpha=\sqrt{\frac{\omega}{2C}}=58{\cdot}6\sqrt{\omega}=58{\cdot}6\sqrt{2\pi} \text{ radians per second}$$

Hence $$\theta=\pm 10e^{-58\,6\sqrt{2\pi}\times 0{\cdot}0417}$$

$$=\pm 10\frac{1}{\epsilon^{6{\cdot}84}}$$

$$=\pm 10\times\frac{1}{920}=0{\cdot}0186°$$

Wave-length $=\frac{2\pi}{\alpha}=\frac{2\pi}{146{\cdot}7}$ ft.$=\frac{2\pi\times 12}{146{\cdot}7}=0{\cdot}513$ in.

Heat flow per cycle$=\frac{0{\cdot}51}{\sqrt{n}}=0{\cdot}51$ B.Th.U.

Equation for Three-dimensional Heat Flow.—Consider a small rectangular block of material having sides of length δx, δy, and δz, and let θ be the temperature at its centre. The temperatures of the two faces *abcd*, *efgh* perpendicular to the axis of x will then be

$$\theta+\tfrac{1}{2}\frac{\delta\theta}{\delta x}\delta x \quad \text{and} \quad \theta-\tfrac{1}{2}\frac{\delta\theta}{\delta x}\delta x$$

where $\frac{\delta\theta}{\delta x}$ is the temperature gradient parallel to the axis of x.

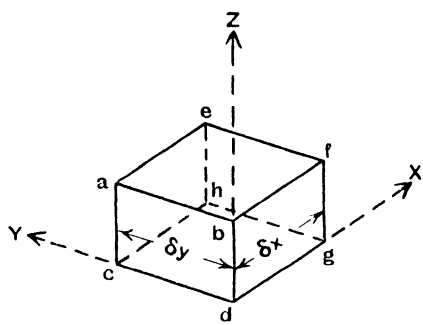


Fig. 160.

From equation (2), p. 350, the heat flow at the two faces will be

$$-\mathrm{K}\delta y\delta z\frac{\delta}{\delta x}\left(\theta+\tfrac{1}{2}\frac{\delta\theta}{\delta x}\delta x\right)=-\mathrm{K}\delta y\delta z\left(\frac{\delta\theta}{\delta x}+\frac{\delta^2\theta}{\delta x^2}\delta x\right)$$

and

$$-\mathrm{K}\delta y\delta z\frac{\delta}{\delta x}\left(\theta-\tfrac{1}{2}\frac{\delta\theta}{\delta x}\delta x\right)=-\mathrm{K}\delta y\delta z\left(\frac{\delta\theta}{\delta x}-\frac{\delta^2\theta}{\delta x^2}\delta x\right)$$

The difference between these two quantities will be the heat accumulated in the block in virtue of the temperature fluctuation in the direction X

and is

$$\mathrm{K}\frac{\delta^2\theta}{\delta x^2}\delta x\delta y\delta z$$

The total heat accumulated per second, in virtue of the temperature gradients in the direction of all three axes will be

$$\mathrm{K}\frac{\delta^2\theta}{\delta x^2}\delta x\delta y\delta z+\mathrm{K}\frac{\delta^2\theta}{\delta y^2}\delta x\delta y\delta z+\mathrm{K}\frac{\delta^2\theta}{\delta z^2}\delta x\delta y\delta z$$

$$=\text{rise of temp. per sec.}\times\text{mass}\times\text{sp. heat}=\frac{\delta\theta}{\delta\mathrm{T}}\rho s\delta x\delta y\delta z$$

or

$$\mathrm{K}\left(\frac{\delta^2\theta}{\delta x^2}+\frac{\delta^2\theta}{\delta y^2}+\frac{\delta^2\theta}{\delta z^2}\right)=\rho s\frac{\delta\theta}{\delta\mathrm{T}}$$

For steady flow in three dimensions $\frac{\delta\theta}{d\mathrm{T}}=0$

or

$$\frac{d^2\theta}{dx^2}+\frac{d^2\theta}{dy^2}+\frac{d^2\theta}{dz^2}=0$$

The general solution to the equation is of the form

$$\theta=\alpha-\beta(x^2+y^2-2z^2)$$

or
$$\theta=\alpha-\beta(r^2-2z^2) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

if the material takes the form of a cylinder of radius r, Fig. 161.

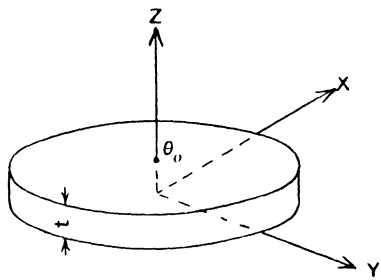


Fig. 161.

When Z=0 and $\frac{d\theta}{dZ}=0$, we have the condition for zero flow into the *lower* face.

If heat now flows into the upper face it will do so in the direction of Z decreasing, or $h=\mathrm{K}\frac{d\theta}{dZ}$ (note sign).

At the upper face $\frac{d\theta}{dZ}=4\beta t$ from (1), where Z=t, the thickness of the cylinder

or
$$h=\mathrm{K}\,.\,4\beta t \quad \text{and} \quad \beta=\frac{h}{4\mathrm{K}t}$$

Then from (1)
$$\theta=\alpha-\frac{h}{4\mathrm{K}t}(r^2-2t^2)$$

At the centre of the upper face $\theta_o=\alpha-\frac{h}{4\mathrm{K}t}\,.\,(-2t^2)$

$$=\alpha+\frac{ht}{2\mathrm{K}}$$

or
$$\alpha=\theta_o-\frac{ht}{2\mathrm{K}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

In general then
$$\theta=\theta_o-\frac{ht}{2\mathrm{K}}-\frac{h}{4\mathrm{K}t}(r^2-2z^2)$$

and
$$\theta_o-\theta=\frac{h}{2\mathrm{K}}\left[t+\frac{r^2-2z^2}{2t}\right]$$

$$=\frac{h}{4\mathrm{K}t}[2t^2+r^2-2z^2].$$

The following temperature gradients will then exist in the cylinder (see Fig. 162) :

$$T_A - T_B = \frac{ht}{2K}$$

$$T_A - T_C = \frac{hr^2}{4tK}$$

$$T_A - T_D = \frac{h}{4tK}(t^2 + 2r^2)$$

Also the surface temperature gradients, $\frac{d\theta}{dt}$, are zero for face DBD and $4t$=constant from (1) for the upper face, giving zero heat flow into the former and uniform heat flow into the latter.

These conditions hold closely for the cylindrical head of a piston, which receives heat over the working face and distributes it to the walls. Except for high speed I.C. engines which lose a considerable amount of heat from the lower face of the piston to the oil spray and air in the crank case, the heat loss on the lower face DBD is generally negligible.[1]

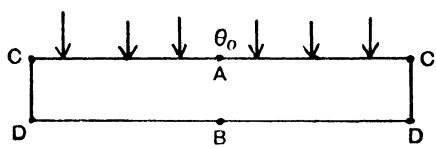


Fig. 162.

EXAMPLE.—If the temperatures at the centre and edge of a cast-iron piston crown having a diameter of 6 in. and a thickness of 1 in., are 230° and 550° C., find the rate of heat flow.

Here $T_{centre} - T_{edge} = 220°\text{ C.} = \frac{h \times 9}{144 \times 1 \times 0{\cdot}0077}$

or $$h = \frac{220 \times 12 \times 0{\cdot}0077}{9}$$

$$= 2{\cdot}26 \text{ C.H.U. per sq. ft.-sec.}$$

GRAPHICAL ESTIMATION OF HEAT LOSS

In the case of two-dimensional flow in a body of any shape with given boundary conditions a graphical method of procedure is often convenient.

This is based on two assumptions : (1) that heat flow is of necessity perpendicular to isothermal planes ; (2) that if lines of heat flow and isothermals are drawn for any case to such a scale that one of the figures formed by their intersection is a square, all the figures will approximate to squares, i.e. if ABCD is a block of uniform material through which heat is flowing from the uniformly heated face AB to a similarly heated

[1] Appendix to "Piston Temperature in a Sleeve Valve Oil Engine," *Inst.Mech.E.*, Sept. 1937.

face DC, the isothermals will be as shown by *aa*, *bb*, *cc*, Fig. 163, and the lines of heat flow by 11, 22, 33, etc. If the scales to which the figure is drawn are chosen so that a to $b=1$ to 2 then all the figures formed, in the limit when *ab* is very small, will be squares also.

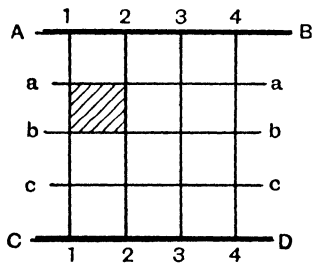


Fig. 163.

The following case is taken from the Proc. Fifth International Congress on Refrigeration, 1929.

To obtain an estimate of the increase in leakage of heat through a 12-in. thickness of insulation applied against the side of a ship due to a series of metal ribs projecting 8 in. into the insulation and spaced 27 in. apart.

In Fig. 164, AB represents the inner surface of the insulation extending half-way from one rib to the next. CD is the side of the ship and DE is the rib in contact with it.

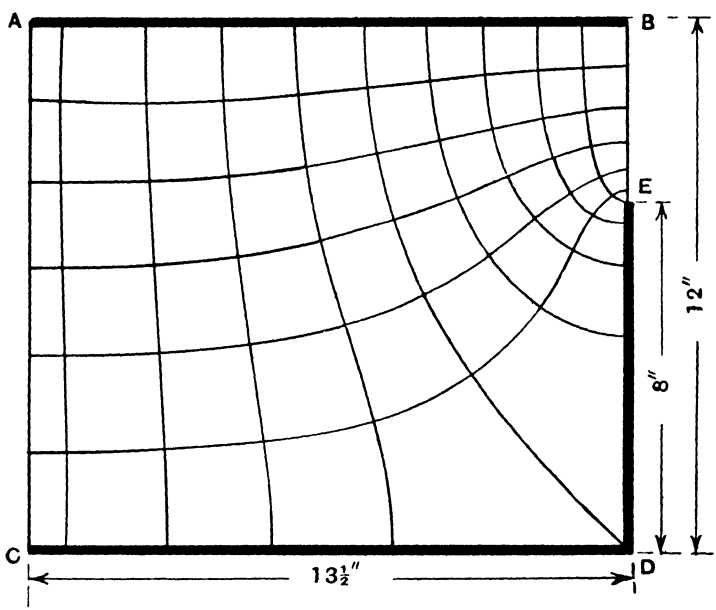


Fig. 164.

Here AB and CDE are the extreme isothermal surfaces, and five other isothermals have been inserted between them in a manner which seems suitable, remembering that as the insulation is continuous beyond the limits AC and BE the isothermals will be perpendicular to these lines. Lines of heat flow are drawn by eye to intersect these at right angles, and

to complete squares. (It is probable that the isothermals first drawn will be found to make the latter operation unsatisfactory, but a little experience will soon show how successive adjustments may be made to give a reliable solution.) It will be found that there are 9·3 heat flow bands between A and B.

Had there been no rib the isothermal and heat flow lines would all be straight, and the number of isothermal bands in AC being six, the number in AB would have been $6\times\frac{27/2}{12}=6{\cdot}7$. The presence of the rib will therefore have increased the heat flow from 6·7 to 9·3 or about 40 per cent.

The application of this method to a piston head of irregular shape will be found in *Proc.I.Mech.E.*, 1934–5, vol. 29, p. 312.

HEAT TRANSFER BY CONVECTION

Convection is the process by which heat is transmitted from a surface to a liquid or gas, radiation being neglected. It is therefore due to (1) the natural internal diffusion in the fluid, which is dependent on the temperature, and (2) the mass movement of the fluid, dependent on its velocity v and density ρ.

Convection is said to be " natural " when the movement is due to differences of density of the fluid, brought about by variations of temperature, and " forced " when the motion is due to outside forces compared with which the natural convection forces are negligible.

Reynolds expressed the heat transfer per unit area in unit time as

$$\mathrm{H}=\alpha t+\beta\rho v\,.\,t,$$

where α and β are constants. This can also be expressed as

$$\mathrm{H}=\left(\alpha+\beta\frac{m}{a}\right)t,$$

where m is the mass flowing in unit time, and a is the area of cross-section of the stream or $m=av\rho$.

It is obvious that two other factors must be considered : (1) the " hydraulic mean depth " of the fluid, or

$$\frac{\text{area of cross-section of stream}}{\text{length of contact surface perpendicular to flow}}$$

as this will determine the proportion of molecules in contact with the surface at any time, (2) the nature of flow, whether streamline or turbulent.

The experimental consideration of conductivity is extremely complicated. The fluid motion is often turbulent, or turbulent in part, and the heat transfer depends on many factors, such as the density, specific heat, conductivity and viscosity of the fluid, and the size, shape, and temperature of the surface. Experimental determination is further complicated as radiation cannot usually be eliminated.

In the actual transfer of heat from one fluid to another through a metal plate, the following factors must be considered :

(1) Beyond a short distance on either side of the plate the flow may be either streamline or turbulent, and the transfer will depend both on natural diffusion (or conduction when the fluid is considered stationary) and on the motion of the fluid itself.

(2) The fluid immediately on each side of the plate forms a stationary layer through which the heat is transmitted by diffusion alone. The thickness of the layer can be reduced by increasing the velocity of flow.

(3) The resistance of the plate.

(4) The receptivity (=1/emissivity) of the surface through which heat enters the plate, and the emissivity of that by which it leaves.

If heat passes from water to air through an iron plate, the films of water and air (factor 2) provide approximately 2 per cent. and 97 per cent. of the total resistance. When both faces are wet each film will contribute approximately 48·5 per cent.

Changes of emissivity of the surfaces may affect the total heat transmission to the extent of 75 per cent. and 16–30 per cent. for gases and liquids respectively.

The heat loss from a surface to a liquid will usually increase considerably when boiling commences owing to the increased stirring action set up by the bubbles.

Approximate values of total heat transfer expressed in $\frac{\text{B.Th.U.}}{\text{ft.}^2\text{hr.}^\circ\text{F.}}$ are as follows :

From a vertical plate to boiling water at 212° F. : stirred, 1400 ; not stirred, 450–1000.

From a condensing vapour through a pipe submerged in water, 40–45.

Through a double tube condenser, velocity of water flow 3 ft. per second, 100–120.

From a brine cooling pipe, 35–40.

From a brine pipe in air, 2–2·5.

Air cooler with fan, 6–7.

A full consideration of the problems will be found in " Calculation of Heat Transmission," H.M. Stationery Office. The following general deductions appear to be provisionally established :

Forced Convection.—(1) The heat loss from a hot surface is proportional to the temperature difference between the surface and the tranquil fluid.

(2) For streamline flow the loss is a function of velocity and linear dimensions.

(3) For turbulent flow in pipes the heat loss is proportional to the velocity.

Natural Convection.—(1) The heat loss from a hot surface is proportional to $T^{5/4}$.

(2) The heat loss is proportional to $p^{0.45}$, where p is the pressure of the gas.

(3) For bodies over 40 cm. in linear dimensions the scale effect is unimportant, and the heat loss per unit area is approximately

$$3{\cdot}33T^{5/4}\ \frac{\text{B.Th.U.}}{\text{ft.}^2\ ^\circ\text{F. hr.}}$$

CONDUCTION OF HEAT THROUGH PIPES

Heat interchangers can be classified as follows :

(1) Parallel flow, where the fluids flow in the same direction over the separating wall.

(2) Counter-flow, where the directions are opposite.

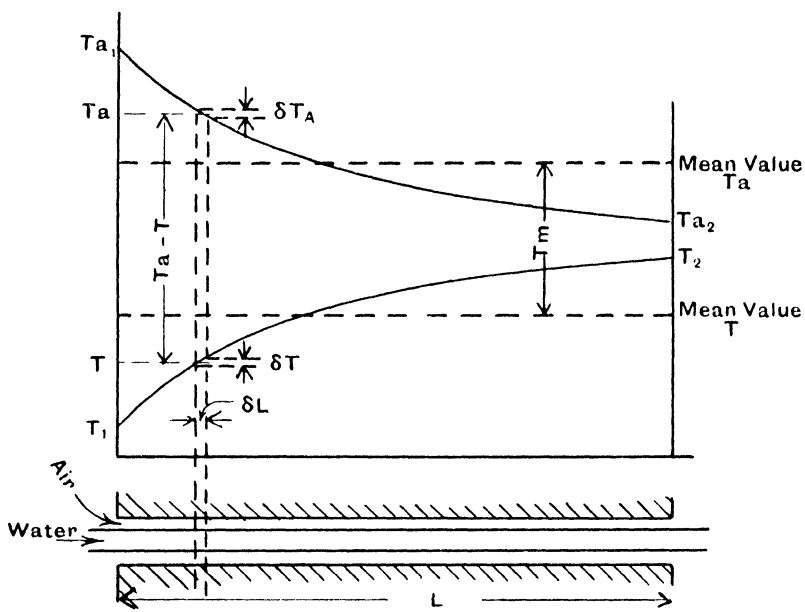


Fig. 165.

(3) Evaporators or condensers, where one fluid remains at a constant temperature while changing its state.

(4) Mixed flow, where one of the fluids takes an irregular direction with respect to the other.

In order to calculate the heat interchanged it is necessary to calculate the mean temperature difference between the fluids.

Parallel Flow Cooler.—Consider an interchanger of length L, Fig. 165, in which W_a lb. of air per hour are cooled by W lb. of water.

Let C_p and 1 be the specific heats of air and water, and K the coefficient of heat transmission in B.Th.U. per foot length of pipe per hour per ° F.

Then $$K(T_a-T)\delta L=W_aC_p\delta T_a=W\delta T$$

or $$L=\frac{W_aC_p}{K}\int_{T_{a_2}}^{T_{a_1}}\frac{dT_a}{T_a-T} \quad . \quad . \quad . \quad . \quad . \quad (1)$$

and $$L=\frac{W}{K}\int_{T_1}^{T_2}\frac{dT}{T_a-T} \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

and after any interval of time

$$W_aC_p(T_{a_1}-T_a)=W(T-T_1)$$

or $$T_a=T_{a_1}+\frac{T_1-T}{W_aC_p}W \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Substituting from (3) in (2)

$$L=\frac{W}{K}\int_{T_1}^{T_2}\frac{dt}{T_{a_1}+\frac{W}{W_aC_p}(T_1-T)-T}$$

$$=\frac{W}{K}\int_{T_1}^{T_2}\frac{dt}{\left(T_{a_1}+\frac{W}{W_aC_p}T_1\right)-T\left(1+\frac{W}{W_aC_p}\right)}$$

This is of the form

$$\frac{W}{K}\int_{T_1}^{T_2}\frac{dt}{a-bt}=-\frac{W}{Kb}\Big[\log_\epsilon(a-bt)\Big]_{T_1}^{T_2}$$

or $$L=\frac{W}{K}\left(\frac{1}{1+\frac{W}{W_aC_p}}\right)\log_\epsilon\left[T_{a_1}+\frac{WT_1}{W_aC_p}-\left(1+\frac{W}{W_aC_p}\right)T\right]_{T_2}^{T_1}$$

the limits being reversed to change the sign of the integral.

$$\therefore L=\frac{W}{K}\left(\frac{1}{1+\frac{W}{W_aC_p}}\right)\log_\epsilon\left[\frac{T_{a_1}+\frac{WT_1}{W_aC_p}-\left(1+\frac{W}{W_aC_p}\right)T_1}{T_{a_2}+\frac{WT_2}{W_aC_p}-\left(1+\frac{W}{W_aC_p}\right)T_2}\right]$$

$$=\frac{W}{K}\left(\frac{1}{1+\frac{W}{W_aC_p}}\right)\log_\epsilon\frac{T_{a_1}-T_1}{T_{a_2}-T_2} \quad . \quad . \quad (4)$$

Now for the whole transfer

$LKT_m=W(T_2-T_1)$, where T_m is the mean temperature difference (5)

Substituting the value of L from (5) in (4)

$$T_m=\frac{(T_2-T_1)(W+W_aC_p)}{W_aC_p \log_\epsilon \frac{T_{a_1}-T_1}{T_{a_2}-T_2}} \quad . \quad . \quad . \quad . \quad . \quad (6)$$

But, for the whole transfer also, $W(T_2-T_1)=W_aC_p(T_{a_1}-T_{a_2})$

or $$W_aC_p=W\frac{T_2-T_1}{T_{a_1}-T_{a_2}}$$

and substituting the value of W_aC_p in (6)

$$T_m=\frac{W(T_2-T_1)\left(1+\frac{T_2-T_1}{T_{a_1}-T_{a_2}}\right)}{\frac{W(T_2-T_1)}{(T_{a_1}-T_{a_2})} \log_\epsilon \frac{T_{a_1}-T_1}{T_{a_2}-T_2}}$$

$$=\frac{T_{a_1}-T_{a_2}+T_2-T_1}{\log_\epsilon \frac{T_{a_1}-T_1}{T_{a_2}-T_2}}$$

$$=\frac{\text{Initial temperature difference}-\text{final temperature difference}}{2{\cdot}30\times\log_{10}\left(\frac{\text{Initial temperature difference}}{\text{Final temperature difference}}\right)}$$

Counter Flow Cooler.—As before

$$K(T_a-T)\delta L=W_aC_p\delta T_a=W\delta T$$

or $$L=\frac{W_aC_p}{K}\int_{T_{a_1}}^{T_{a_2}}\frac{dT_a}{T_a-T} \quad . \quad . \quad . \quad . \quad . \quad (1)$$

and $$L=\frac{W}{K}\int_{T_2}^{T_1}\frac{dt}{T_a-T} \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

(Note change of limits as the direction of flow of water is opposite to that of L increasing.)

Also $W_aC_p(T_{a_1}-T_a)=W(T_2-T_1)$ or $T_a=T_{a_1}+\left(\frac{T_2-T}{W_aC_p}\right)W$. (3)

Substituting the value of T_a from (3) in (2)

$$L=\frac{W}{K}\int_{T_2}^{T_1}\frac{dt}{T_{a_1}+\left(\frac{T_2-T}{W_aC_p}\right)W-T}$$

or
$$L=\frac{W}{K}\int_{T_2}^{T_1}\frac{dt}{\frac{W_aC_pT_{a_1}-T_2W}{W_aC_p}-\left(1+\frac{W}{W_aC_p}\right)T}$$
$$=\frac{W}{K\left(1+\frac{W}{W_aC_p}\right)}\log_\epsilon\left[T_{a_1}+\frac{T_2W}{W_aC_p}-\left(1+\frac{W}{W_aC_p}\right)T\right]_{T_1}^{T_2}$$

the limits being reversed as δT is essentially negative.

Hence
$$L=\frac{W}{K}\left(1+\frac{W}{W_aC_p}\right)\log_\epsilon\frac{T_{a_1}-T_2}{T_{a_2}-T_1}$$

As $T_{a_1}-T_2$ and $T_{a_2}-T_1$ are the initial and final temperature differences, this equation is identical with equation (4) in the last case and the equation for T_m is exactly as before.

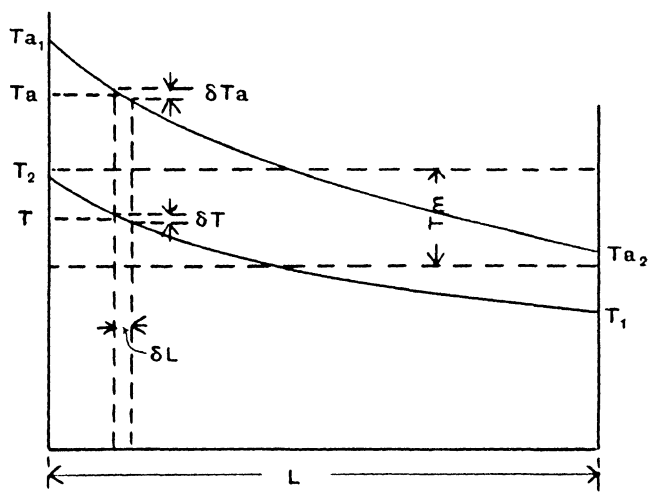


Fig. 166.

(N.B. The initial temperature difference is the temperature difference at the end of the cooler from which L is measured, i.e. the initial difference in space, not time.)

EXAMPLE.—If water ranging from 15° to 25° C. in temperature is to be used to cool a gas from 100° C. to 30° C., compare the lengths of similar parallel and counter-flow coolers which might be used.

For the parallel-flow cooler

$$T_m=\frac{(100-15)-(30-25)}{2{\cdot}30\log_{10}\frac{100-15}{30-25}}$$

$$=\frac{85-5}{2{\cdot}30\log_{10}\frac{85}{5}}=\frac{80}{2{\cdot}30\log_{10}17}=\frac{80}{2{\cdot}30\times1{\cdot}230}$$

$$=28{\cdot}3^\circ\text{ C.}$$

For the counter-flow cooler

$$T_m=\frac{(100-25)-(30-15)}{2{\cdot}30\log_{10}\frac{100-25}{30-15}}=\frac{75-15}{2{\cdot}30\log_{10}\frac{75}{15}}$$

$$=\frac{60}{2{\cdot}30\times0{\cdot}698}=37{\cdot}4^\circ\text{ C.}$$

The counter-flow cooler will therefore be $\frac{28{\cdot}3}{37{\cdot}4}$ or 76 per cent. of the length of the parallel-flow cooler.

The weight of water to be circulated will be given by

$$\text{weight of gas to be cooled}\times\frac{\text{sp. heat water}}{\text{sp. heat gas}}$$

If P is the perimeter of the cooling pipes, L the length, and K a suitable conductivity factor—say 2·5 heat units/ft.^{2} hr. per degree,

Then $P\times L\times K\times T_m=$weight of gas per hour$\times$specific heat of gas$\times$temperature range of gas

or $$L=\frac{\text{heat to be extracted per hour}}{P\,.\,K\,.\,T_m}\text{ feet.}$$

Condenser or Evaporator.—Here

$$W\delta T=K\delta L(T_s-T)$$

or $$\frac{\delta T}{T_s-T}=\frac{K\delta L}{W}$$

or $$L=\frac{W}{K}\log_\epsilon\frac{T_s-T_2}{T_s-T_1}$$

For the whole heat transfer $LKT_m=W(T_2-T_1)$

or $$T_m=\frac{W}{K}\frac{T_2-T_1}{\frac{W}{K}\log_\epsilon\frac{T_s-T_2}{T_s-T_1}}$$

Adding and subtracting T_s and changing signs throughout

$$T_m=\frac{-T_2+T_s+T_1-T_s}{-\log_\epsilon\frac{T_s-T_2}{T_s-T_1}}$$

$$\text{or} \quad T_m = \frac{\text{Initial temperature difference} - \text{final temperature difference}}{2{\cdot}30 \log_{10} \dfrac{\text{Initial temperature difference}}{\text{Final temperature difference}}}$$

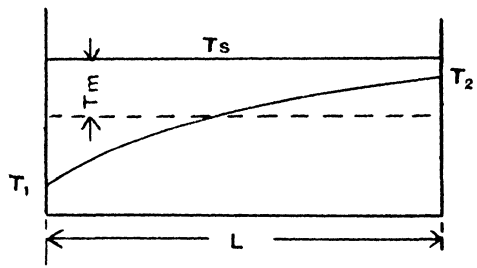


Fig. 167.

Mixed Flow Coolers.—Where the flow is quite indiscriminate the value of the mean temperature difference will be

$$T_m = \left(\frac{T_{a1} + T_{a2}}{2}\right) - \left(\frac{T_1 + T_2}{2}\right)$$

In cases where the flow is partly directional the method of estimating T_m will have to be determined by the circumstances.

EXAMPLES ON CHAPTER XVI

1. In order to determine the amount of heat lost by radiation from a metal surface, a cast-iron bar of square section 4 in. × 4 in. was heated at one end. When a steady condition was attained the temperatures were read from thermometers placed at different distances along the bar. Obtain a formula by means of which the amount of heat radiated per square foot per degree difference of temperature between the temperature of the bar and atmosphere can be calculated. Determine the actual amount of radiation from the figures given below :

Distance from end in inches	0	6	12	21	30	41
Temperature (° F.) . . .	235	171	131·5	97·9	80·5	69·2

Conductivity of cast iron, 5·4 B.Th.U. per sq. ft. per minute per ° F. per inch thick. Temperature of atmosphere, 59·5° F.

2. In a surface condenser with 6000 sq. ft. of cooling surface it was found that 58,000 lb. of steam were condensed per hour. The average temperature of the steam was 132° F., and of the circulating water 80° F. The tubes were of brass 0·05 in. thick and of such a conductivity that 25 B.Th.U. could pass per minute through a plate 1 sq. ft. in area 1 in. thick, per degree difference in temperature between the two surfaces. Calculate the temperatures of the metal on the steam and water sides of the tubes. Assume that when steam is in contact with a metal surface the rate of condensation is $0{\cdot}74(T - t)$ B.Th.U. per sq. ft. per second where T is the temperature of the steam and t the temperature of the metal, and latent heat at 132° F. = 1020 B.Th.U. per lb.

3. If the surface fluctuation of an aluminium plate is $\pm 10^\circ$ C. and the number of cycles per second is 60, find the length of the heat wave, and the heat flow per cycle into the surface.

4. An aluminium piston 6 in. in diameter and with a uniform crown thickness of $\frac{3}{4}$ in. receives heat at the rate of 2·5 C.H.U. per sq. ft. per second. If the temperature at the lower edge of the under surface is 220 °C., find the temperatures at the centres of the two faces.

5. Show that a 3-in. diameter steam pipe covered with lagging $1\frac{1}{2}$ in. thick will loose approximately $\frac{2}{5}$ of the heat lost by a similar pipe encased in lagging of square external section and measuring 6 in. along the sides.

Chapter XVII

COMBUSTION

*GENERAL STATEMENT

The heat which is the motive power of all heat engines is obtained, except in certain very exceptional cases, from the combustion of some form of fuel, which may be solid, liquid, or gaseous, and which, entering into chemical combination with the oxygen of the air, liberates heat during the process.

It should be noted that the oxygen has exactly the same importance as the fuel, and when either is exhausted the process of combustion stops. It has been seen in the case of the petrol engine that it is sometimes desirable to use an " over-rich " mixture, and that the heat produced reaches its limit when the oxygen supply is exhausted. It is as important to obtain an adequate supply of oxygen and to distribute it correctly as it is to obtain a suitable supply and distribution of fuel, even though the former can be obtained for nothing while the latter is costly.

Though a too lavish supply of fuel is expensive an excessive liberality with regard to air may be equally wasteful, as it must be remembered that the desirable element, oxygen, is accompanied by approximately three times its own weight of nitrogen. This nitrogen, which is almost completely inactive, is drawn into the combustion zone, where it occupies valuable room, is heating owing to the activity of the oxygen and fuel, and is finally discharged with its cargo of precious heat, having served no useful purpose except possibly that of preventing too great a rise of temperature during combustion.

*COMBUSTION EQUATIONS

The combustible elements in all normal fuels, whether solid, liquid, or gaseous, are carbon, hydrogen, sulphur, and compounds of carbon, hydrogen, and oxygen. As the fumes resulting from the combustion of sulphur are deleterious to metals, etc., especially in the presence of moisture, fuels are usually as free from this element as possible, and the traces present contribute very little to the total calorific value.

Table I shows the atomic and molecular weights of the commoner substances which have to be considered in connection with combustion problems. The reader is warned concerning the common, and often

convenient, practice of substituting the chemical symbol of a substance in place of the name, i.e. H refers to one atom of hydrogen, which can only exist in the combined state and the symbol therefore should not be applied to indicate hydrogen gas. H_2 refers to one molecule of free hydrogen or two atoms of hydrogen combined. Only when loosely used can H_2 indicate hydrogen in general.

TABLE I

	Atom		Molecule	
	Symbol	Atomic weight	Symbol	Molecular weight
Hydrogen	H	1	H_2	2
Oxygen	O	16	O_2	32
Nitrogen	N	14	N_2	28
Carbon	C	12	C	12
Sulphur	S	32	S	32
Water		—	H_2O	18
Carbon monoxide		—	CO	28
Carbon dioxide		—	CO_2	44
Sulphur dioxide		—	SO_2	64
Marsh gas (methane)		—	CH_4	16
Ethylene		—	C_2H_4	28

The complete combustion of carbon can be expressed by

$$C+O_2=CO_2$$

indicating that one molecule of carbon combines with one molecule of oxygen to give one molecule of carbon dioxide. This can be converted to show the relative weights of the reacting substances by multiplying by the molecular weights,

i.e. 12 lb. carbon+32 lb. oxygen=44 lb. carbon dioxide,

and as there is no change of mass the equation must be numerically consistent.

As the molecular weight of a gas in pounds occupies 359 cu. ft. at N.T.P. the equation may be written

12 lb. carbon+359 S.C.F. oxygen=359 S.C.F. carbon dioxide.

Neglecting the volume of the carbon there will be no shrinkage on combustion.

On the other hand the equation

$$2CO+O_2=2CO_2$$

shows that 2 volumes of carbon dioxide+1 volume oxygen become

2 volumes of CO_2, in which case the shrinkage is one-third of the total original volume or half the volume of carbon monoxide.

In the case of the combustion of hydrogen

2 volumes of hydrogen + 1 volume of oxygen = 2 volumes of steam,

showing a shrinkage of one-third, but if the steam condenses the volume occupied by the resulting water will be negligible and the shrinkage will be almost complete.

The following are the equations most frequently employed :

Carbon	$C+O_2=CO_2$
Carbon (incomplete combustion)	$2C+O_2=2CO$
Sulphur	$S+O_2=SO_2$
Hydrogen	$2H_2+O_2=2H_2O$
Carbon monoxide	$2CO+O_2=2CO_2$
Methane	$CH_4+2O_2=CO_2+2H_2O$
Ethylene	$C_2H_4+3O_2=2CO_2+2H_2O$

*THE CALORIFIC VALUE OF FUELS

Wherever possible the calorific value of a fuel, whether solid, liquid, or gaseous, should be obtained from the results of a direct test in a suitable " bomb " or gas calorimeter. In this case a sample of the fuel at room temperature is burnt with either air or oxygen, and the products of combustion are cooled to the original temperature. This ensures (1) that any moisture originally contained in the fuel will be condensed and will return to its original state, the heat required to evaporate it during the high temperature stages of combustion being restored to the calorimeter before the completion of the test ; and (2) that the latent heat of moisture formed by combustion will be deposited in the calorimeter. The test should therefore give an accurate measurement of the total heat liberated by combustion, and the value obtained is known as the *Higher Calorific Value.*

It is advisable in many cases to base calculations of performance or efficiency upon this higher calorific value. When used in determining the heat balance of an engine, for instance, the higher calorific value represents the total assets, and must be balanced against the power generated, the frictional losses, and the sum of all the heat losses in the exhaust, by radiation, etc., the final statement being complete.

While it is desirable to charge to the account of an engine or boiler all the heat which it should, in reason, be expected to utilise, it may not be fair to charge it with the losses which are necessarily incurred when an unsuitable fuel is used. For instance, if a fuel contains sulphur the fumes will be moderately inactive while in the gaseous form, but will produce an extremely corrosive acid when absorbed in moisture, and it is therefore necessary to discharge the products of combustion at a temperature

above the dew-point, thus causing the loss of the heat in the steam formed by combustion. A similar corrosive action is produced by nitrous fumes in an internal combustion engine, these being absorbed in the moisture deposited on the walls when starting from cold. Also, it may not be desirable to charge to the discredit of a boiler the latent heat, necessarily discharged to the atmosphere, which is formed by the combustion of wet fuel. The above are rather defects of the fuel than of the plant.

Under such circumstances the *Lower Calorific Value* may be used, this being the higher calorific value minus the heat content of the vapour discharged.

As the efficiency of a unit is higher when calculated on the lower calorific value than when the higher calorific value is used, great care must be taken to state clearly which quantity has been employed.

The desire to give full credit to the performance of some part of the plant must not be allowed to conceal the fact that all losses have to be paid for, and must therefore be charged to *some* account, and if it is arguable that a given fuel is not suitable for use in a given engine or boiler it may equally well be considered, for other purposes, that the engine or boiler are not suitable for the fuel. There are two distinct points of view, and care is required not to confuse them.

A somewhat similar difficulty arises in the case of volatile liquid fuels such as petrol. When tested in a calorimeter some of the heat of combustion is absorbed in the evaporation process, the total heat recorded being the heat of combustion minus the latent heat. When such a fuel (other than alcohol, see p. 292) is supplied to an engine, however, it is safe to assume that evaporation takes place partly in the carburettor and induction pipe, and partly at the cost of the otherwise wasted heat in the inlet port and valve, and is in any case complete before the closing of the latter. The latent heat is not, in fact, supplied directly at the expense of the heat of combustion, so that the heat available in the cylinder is in reality the calorific value (i.e. the heat of combustion minus the latent heat) plus the latent heat, or the total heat of combustion.

A difficulty in determining the lower calorific value is that of deciding the amount of heat to be deducted from the higher calorific value. Were the steam to be discharged at atmospheric pressure, the heat lost per pound of water vapour would be the latent heat, or 970 B.Th.U. per lb., and this value is used by some authorities. On the other hand, if the potential energy of the steam is taken into account, the Total Heat or 1122 B.Th.U. per lb. should be allowed, while Schüle uses a value of 1080 B.Th.U. per lb., or approximately the total heat of steam at room temperature, and the Heat Engine Trials Report [1] adopts the figure of 1055 B.Th.U. Actually the steam exerts only a partial pressure in the

[1] " Report on the Tabulation of Heat Engine Trials Reports." Pub. William Clowes, 1927.

exhaust, and is almost certainly present as a superheated vapour, the heat content of which will vary with each trial. In order to avoid additional measurements and elaborate calculations it is conventional to use one or other of the above figures in all cases—there appears to be no regularity with regard to the choice—but it will be found that the extreme error involved affects the ultimate result by an amount very seldom exceeding 1 per cent., which is not excessive when compared with other experimental errors.

In order to avoid confusion the " Heat Engine Trials Report " of the Institution of Civil Engineers recommends that the higher calorific value should always be used. Such a recommendation tends towards simplicity and clarity, and may be very desirable in general practice. The other view-point is, however, of interest.[1]

(For further details, see p. 399, also Pye, p. 53.)

When gaseous fuels are used it is convenient to express the calorific value in B.Th.U. per standard cubic foot of gas, or per standard cubic foot of the combustible mixture. Table II gives the approximate calorific values of a number of fuels or fuel constituents.

TABLE II

Fuel	Higher calorific value, B.Th.U.		Lower calorific value, B.Th.U.	
	per lb.	per standard cu. ft.	per lb.	per standard cu. ft.
Carbon, burnt to carbon dioxide	14,540		14,540[2]	
Carbon, burnt to carbon monoxide	4,372		4,372[2]	
Sulphur	4,000		4,000[2]	
Hydrogen	61,520	345	51,800	288
Carbon monoxide	4,391	342	4,391[2]	342[2]
Methane (CH_4)	23,830	1,062	21,430	954
Ethylene (C_2H_4)	21,420	1,674	19,900	1,561
Town gas		504		450
Blast furnace gas				85–70
Paraffin			18,890	
Petrol			18,920	
Alcohol			11,480	
Diesel oil	20,400		19,200	
Nixons Navigation coal	15,400		15,010	
Welsh steam coal	14,490			
Anthracite	15,200			
Coke	13,600			

[1] For descriptions of calorimeters, details of fuel analysis, etc., see " Fuel, Gaseous, Liquid, and Solid," Coste and Andrews. Pub. Griffin.

[2] Dry fuels not containing hydrogen have one calorific value only. All wet fuels or fuels containing hydrogen have two values on account of the final moisture content.

The very high calorific value of hydrogen on a weight basis should be noted, and also the small amount of heat liberated when carbon is burnt to carbon monoxide instead of carbon dioxide.

*ESTIMATION OF CALORIFIC VALUE FROM CHEMICAL ANALYSIS

The calorific value of a solid or liquid fuel may be obtained approximately from a chemical analysis of a dried sample. The analysis usually obtained gives the weights in 1 lb. of dry sample of carbon, hydrogen, sulphur, nitrogen, ash, and oxygen. Of these elements only the first three contribute to the calorific value, while the contribution of the third is usually negligible.

Now when 1 lb. of carbon is completely burnt to carbon dioxide 14,540 B.Th.U. are liberated, and when 1 lb. of hydrogen is completely burnt to water 61,520 B.Th.U. are liberated. In the latter case, if the 9 lb. of water vapour which are formed are not allowed to condense, the heat liberated will be 51,800 B.Th.U. approximately.

If, therefore, 1 lb. of fuel contains 0·86 lb. carbon and 0·04 lb. hydrogen, the higher calorific value will be $0{\cdot}86\times 14{,}540+0{\cdot}04\times 61{,}520=14{,}960$ B.Th.U., and the lower calorific value will be $0{\cdot}86\times 14{,}540+0{\cdot}04\times 51{,}800=14{,}570$ B.Th.U.

If the fuel contains oxygen it is assumed that this is already in combination with the hydrogen present, so that, as oxygen combines with one-eighth of its weight of hydrogen, the amount of free hydrogen will be

$$\text{Weight of hydrogen}-\frac{\text{weight of oxygen}}{8}$$

In the above case if 0·16 lb. of oxygen is present the higher calorific value is taken as $0{\cdot}86\times 14{,}540+\left(0{\cdot}04-\frac{0{\cdot}16}{8}\right)\times 61{,}520=13{,}730$ B.Th.U.

The process is not of great accuracy for two reasons: (1) It assumes that the carbon and hydrogen exist in the fuel in the free state whereas in reality they are chemically combined. The assumption is specially unsatisfactory in the case of oil fuels which contain highly complex compounds of these elements. (2) The oxygen is not obtained by direct measurement, but by difference, i.e. by subtracting the weights of carbon, hydrogen, sulphur, nitrogen, and ash from the original weight of the coal. The figure for oxygen therefore contains all the errors of the whole analysis. The calorific value obtained by the above method is usually too low when compared with the results of calorimetric tests.

The calorific value for a gaseous fuel may be obtained in a similar manner, but as the analysis is by volume the heat values of the constituents must be reckoned in B.Th.U. per cubic foot. In case the figures required are not available it is useful to remember that the pound

molecular weight of a gas occupies 359 standard cubic feet, so that 1 lb. occupies $\frac{359}{\text{molecular weight}}$ S.C.F. The analysis is both simpler and more satisfactory than that for a solid fuel.

*CALCULATION OF AIR REQUIRED FOR COMPLETE COMBUSTION OF SOLID AND LIQUID FUELS

For perfect combustion the amount of air which should theoretically be supplied is that quantity which contains just enough oxygen for the complete combustion of the constituents of the fuel, it being assumed that these exist independently, an assumption obviously not fully justified in the case of solid fuels as the carbon, hydrogen, oxygen, and sulphur are already combined to some extent.

The composition of air may be taken as follows :

By weight—oxygen, 23 per cent. ; nitrogen, 77 per cent.
By volume ,, 21 per cent. ; ,, 79 per cent.

Owing to the simplicity of the essential processes, and the ease with which formulæ may be misapplied, the calculations contained in this and succeeding paragraphs will be based on actual examples and not on symbolic values.

To calculate the Amount of Air required for the Complete Combustion of Welsh Steam Coal.

Let the analysis by weight be as follows :

Carbon, 83·8 per cent. ; hydrogen, 4·8 per cent. ; nitrogen, 1·0 per cent. ; sulphur, 1·4 per cent. ; ash, 4·9 per cent. ; oxygen (by difference), 4·1 per cent.

To burn the Carbon.

The formula is $C+O_2=CO_2$

or 12 lb. carbon+32 lb. oxygen=44 lb. carbon dioxide ;

or 0·838 lb. carbon will require $0{\cdot}838\times\frac{32}{12}$ or 2·23 lb. oxygen,

and will produce $0{\cdot}838\times\frac{44}{12}$ or 3·072 lb. carbon dioxide . . . (1)

To burn the Hydrogen.

The formula is $2H_2+O_2=2H_2O$

or 4 lb. hydrogen+32 lb. oxygen=36 lb. water.

It is assumed, however, that the oxygen is already combined with the hydrogen, so that the free hydrogen is $\left(0{\cdot}048-\frac{0{\cdot}041}{8}\right)=0{\cdot}043$ lb. (see

p. 375)

0·043 lb. free hydrogen will then require $0{\cdot}043 \times \frac{32}{4}$ or 0·344 lb. oxygen,

and will produce $0{\cdot}043 \times \frac{36}{4}$ or 0·387 lb. of steam or water . . (2)

To burn the Sulphur.

$S + O_2 = SO_2$

or 32 lb. sulphur + 32 lb. oxygen = 64 lb. sulphur dioxide,

or 0·014 lb. sulphur will require 0·014 lb. oxygen and will produce 0·028lb. sulphur dioxide . . (3)

The nitrogen and ash are incombustible.

The total oxygen required per pound of fuel is then

2·23 + 0·344 + 0·014 = 2·588 lb., which will be contained in

$2{\cdot}588 \times \frac{100}{23}$ or 10·23 lb. air.

The air supply per pound of fuel is therefore 10·23 lb.

Analysis of the Products of Combustion in the above Example.

The products of combustion will then consist of

3·072 lb. carbon dioxide, from (1)
0·387 lb. steam, from (2)
0·028 lb. sulphur dioxide, from (3)
0·010 lb. nitrogen, from the fuel
$\frac{77}{100} \times 10{\cdot}23$ or 7·878 lb. nitrogen, from the air
0·49 lb. ash, from the fuel.

At room temperature the steam will condense, and will absorb the sulphur dioxide so that the gaseous products will consist of

3·072 lb. carbon dioxide
7·888 lb. nitrogen
10·960 lb. total weight

and the analysis of the gaseous product by weight will be 28·0 per cent. carbon dioxide and 72·0 per cent. nitrogen.

The proportions by volume will be $\frac{28}{44} = 0{\cdot}636$ parts carbon dioxide

and $\frac{72}{28} = 2{\cdot}57$ parts nitrogen

Total . . 3·21 parts

and the analysis by volume will be 19·8 per cent. carbon dioxide
80·2 per cent. nitrogen

*TO CALCULATE THE AMOUNT OF AIR REQUIRED FOR THE COMBUSTION OF A GASEOUS FUEL

The analysis of a gaseous fuel is always stated in terms of volume. Let the analysis of the fuel be

Hydrogen	46 per cent.
Methane	37·5 ,,
Carbon monoxide	7·5 ,,
Nitrogen	4·5 ,,
Water vapour	2·0 ,,
Ethylene	2·5 ,,

Dealing with the constituents separately we have—

Hydrogen : $2H_2+O_2=2H_2O$

or 2 cu. ft. hydrogen+1 cu. ft. oxygen=2 cu. ft. water vapour,

and 0·46 cu. ft. hydrogen will require 0·23 cu. ft. oxygen to give 0·46 cu. ft. water vapour.

Methane : $CH_4+2O_2=CO_2+2H_2O$

or 1 cu. ft. methane+2 cu. ft. oxygen=1 cu. ft. carbon dioxide+2 cu. ft. water vapour,

or 0·375 cu. ft. methane+0·75 cu. ft. oxygen=0·375 cu. ft. carbon dioxide+0·75 cu. ft. water vapour.

Carbon Monoxide : $2CO+O_2=2CO_2$

or 2 cu. ft. carbon monoxide+1 cu. ft. oxygen=2 cu. ft. carbon dioxide,

or 0·075 cu. ft. carbon monoxide+0·0375 ft. oxygen=0·075 cu. ft. carbon dioxide.

Ethylene : $C_2H_4+3O_2=2CO_2+2H_2O$

or 1 cu. ft. ethylene+3 cu. ft. oxygen=2 cu. ft. carbon dioxide+2 cu. ft. water vapour,

or 0·025 cu. ft. ethylene+0·075 cu. ft. oxygen=0·05 cu. ft. carbon dioxide+0·05 cu. ft. water vapour,

or 1 cu. ft. of fuel gas will require
0·23+0·75+0·0375+0·075=1·0925 cu. ft. oxygen

$$\text{or } 1{\cdot}0925\times\frac{100}{21}=5{\cdot}202 \text{ cu. ft. air} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Note.—If there is any oxygen in the fuel gas this should be deducted from the total oxygen required. The oxygen in a gaseous fuel is obtained by direct analysis and not by difference.

The products of combustion will be—

Carbon dioxide $=0{\cdot}375+0{\cdot}075+0{\cdot}05 \quad =0{\cdot}500$ cu. ft.

Water vapour $=0{\cdot}46+0{\cdot}75+0{\cdot}05+0{\cdot}020=1{\cdot}28$ cu. ft.

Nitrogen $=0{\cdot}045+5{\cdot}202\times\frac{79}{100} \quad =4{\cdot}155$ cu. ft.

$=\overline{5{\cdot}935}$ cu. ft. total

The analysis will be carbon dioxide $\frac{0{\cdot}500}{5{\cdot}935}\times 100=$ 8·4 per cent.
Water vapour = 21·6 ,,
Nitrogen = 70·0 ,,

If the vapour is allowed to condense the total volume will be 4·655 cu. ft., and the analysis of the dry gas will be carbon dioxide 10·7 per cent., nitrogen 89·3 per cent.

The volume of the mixture of gas and air before combustion will be $5{\cdot}202+1=6{\cdot}202$ cu. ft., i.e. the sum of the volumes of air (from 1) and gas, and the contraction after combustion will be—

for the wet gas $6{\cdot}202-5{\cdot}935=0{\cdot}267$ cu. ft. or 4·5 per cent.
and for the dry gas $6{\cdot}202-4{\cdot}655=1{\cdot}547$ or 24·9 per cent.

Note.—The percentages of CO_2 shown in this and the previous analysis are maximum values, as any dilution of the products of combustion, either by the use of excess air or excess fuel, will decrease the values given.

*FLUE AND EXHAUST GAS ANALYSIS

In order to find whether the combustion of a fuel is satisfactory it is usual to analyse a sample of the products of combustion. This is performed at room temperature so that the water vapour has almost entirely condensed, and in doing so has absorbed the sulphurous fumes.

The analysis is made by volume, the following being the reagents normally used : [1]

(1) For carbon dioxide determination, caustic potash solution.
(2) For oxygen determination, either alkaline pyrogallol solution or yellow phosphorus.
(3) For carbon monoxide, ammoniacal cuprous chloride solution.

It is essential that the absorptions be carried out in the above order, as cuprous chloride reacts readily with both carbon monoxide and oxygen. Care must be taken to ensure that the sample analysed is representative, and is not contaminated, or diluted by the leakage of air, before the analysis is made.

[1] " Fuel, Gaseous, Liquid and Solid," Coste and Andrews.

The records of such an analysis are given below.

	Vol. c.c.	Difference	Per cent.
Original volume, measured at atmospheric pressure and temperature .	100	—	—
Volume after reaction with potash .	88	12	12 (CO_2) [1]
Volume after reaction with phosphorus or pyrogallol	80·5	7·5	7·5 (O_2)
Volume after reaction with cuprous chloride	80·2	0·3	0·3 (CO)
	Nitrogen by difference .		80·2 (N_2)

[1] For convenience the symbols are used qualitatively.

Typical analyses of flue gases obtained from the combustion of coal are as follows :

Gas	Per cent. by volume		
	(1)	(2)	(3)
CO_2	15·3	9·0	4·0
CO	0	10·3	5·0
O_2	4·1	0·6	11·7
N_2	80·6	80·1	79·3

Analysis (1) shows no carbon monoxide and a good percentage of CO_2. The combustion is good, with only a very moderate excess of air.

Analysis (2) shows a very high carbon monoxide value and very little excess oxygen. The air supply is obviously quite inadequate.

Analysis (3) shows a high carbon monoxide value coupled with much excess oxygen. The combustion zone appears to have been starved of air, but there would also seem to have been considerable air leakage, either through the fire bars, due to very bad stoking, or into the flues.

It must be remembered that the interpretation of an analysis of the products of combustion cannot be made without a knowledge of the analysis of the fuel, a good analysis for one type of fuel being a bad analysis for another. The physical state of the fuel must also be considered. A very intimate mixture with air is possible for a gaseous fuel, so that almost perfect combustion can be achieved. With coal burnt on an ordinary grate, however, the interstices between the lumps are of such a size that some air must pass without coming into close contact with the fuel, and an attempt to reduce the excess air below a certain value will

merely result in incomplete combustion. The firing of pulverised fuel or of a spray of heavy oil will also require excess air owing to the impossibility of a perfect distribution of the fuel throughout the air. On the other hand, a petrol engine will only develop full power when dissociation is suppressed by the direct formation of carbon monoxide.

*THE EXAMINATION OF EXHAUST AND FLUE GAS ANALYSES. SOLID AND LIQUID FUELS

To find the Air actually supplied per Pound of Coal.

This is calculated by comparing the carbon content of the coal with that of the flue gases.

Let the (dry) analyses be

Coal, 0·84 lb. carbon per cent.
0·056 lb. hydrogen per cent.
0·05 lb. ash per cent.
(by weight)

Exhaust gas, CO_2 10·5 per cent.
CO 0·5 per cent.
O_2 8·0 per cent.
N_2 81·0 per cent.
(by volume)

The weight of carbon per pound of dry flue gas can be calculated as follows : First, the flue gas analysis must be converted to an analysis by weight.

	Analysis by volume	Relative weight
	per cent.	
CO_2	10·5	$10·5 \times 44 = 462$
CO	0·5	$0·5 \times 28 = 14$
O_2	8·0	$8 \times 32 = 256$
N_2	81·0	$81 \times 28 = 2268$
Total .	100	3000

Hence in 3000 lb. of dry gas there are

$$\frac{12}{44} \times 462 + \frac{12}{28} \times 14 = 132 \text{ lb. of carbon} \quad . \quad . \quad . \quad (1)$$

or the weight of dry gas containing 0·84 lb. carbon (supplied by 1 lb. of coal)

$$= 3000 \times \frac{0·84}{132} = 19·09 \text{ lb.}$$

In addition to the dry gas the hydrogen in the coal will give

$$9 \times 0·056 = 0·504 \text{ lb. steam,}$$

and the total products of combustion per pound of coal will be

$$19{\cdot}09+0{\cdot}504 \text{ lb.}=19{\cdot}59 \text{ lb.}$$

These will come from (1) the air supply, (2) the coal, excluding the ash, or the air supply per pound of coal $=19{\cdot}59-(1-0{\cdot}05)=18{\cdot}64$ lb.

To find the air theoretically required.

The oxygen required to burn 0·056 lb. H_2 (from 1 lb. coal)

$$=0{\cdot}056\times 8=0{\cdot}448 \text{ lb.}$$

The oxygen required to burn 0·84 lb. $C=\frac{32}{12}\times 0{\cdot}84=2{\cdot}24$ lb.

Total 2·69 lb.

and weight of air required per pound of coal $=11{\cdot}69$ lb.

The excess air supplied per pound of coal is therefore

$$18{\cdot}64-11{\cdot}69=6{\cdot}95 \text{ lb., or 59 per cent.}$$

To find the heat loss through the formation of carbon monoxide.

In 3000 lb. of dry gas there are 14 lb. of carbon monoxide containing $\frac{12}{28}\times 14=6$ lb. carbon, the total carbon content being 132 lb. from (1).

Or, from 1 lb. of coal having a carbon content of 0·84 lb., $\frac{6}{132}\times 0{\cdot}84$ or 0·038 lb. of carbon are burnt to form CO.

The calorific value of carbon is as follows (see p. 374) :

When burnt to carbon dioxide 14,540 B.Th.U. per lb.
,, ,, monoxide 4,372 ,, ,,

or loss per pound on burning to carbon monoxide $=10{,}168$,
and for the combustion in question the loss will be

$$0{\cdot}038\times 10{,}168=386{\cdot}3 \text{ B.Th.U. per lb.}$$

If the calorific value of the coal is 15,450 B.Th.U. per lb., the loss amounts to 2·5 per cent.

The density of the dry exhaust gas can readily be found from the Table, p. 381.

If the gases shown in the second column had been entirely hydrogen the figure corresponding to the Relative Weights in column 3 would have been $100\times 2=200$.

The density of the gas relative to hydrogen is therefore $\frac{3000}{200}$, and as the density of hydrogen is 0·0056 lb. per cu. ft. at N.T.P. (i.e. the molecular weight in pounds occupies 359 cu. ft.), the density of the dry gas will be $\frac{3000}{200}\times 0{\cdot}0056=0{\cdot}0840$ lb. per cu. ft.

*THE EXAMINATION OF EXHAUST AND FLUE GAS ANALYSES. GASEOUS FUELS

The examination is most conveniently carried out on the lines indicated above, as the changes in volume due to combustion do not then enter the calculation.

Let the analysis of the exhaust gas be CO_2=6·77 per cent., O_2= 8·81 per cent., N_2=84·42 per cent., and the analysis of the fuel gas be as shown in column 2 of Table III, both analyses being by volume. First, it is necessary to find the weight of carbon per pound of gaseous fuel.

TABLE III

	Analysis by volume	Relative weights	Relative weight of carbon	Relative weight of moisture
(1) CH_4	19·2	16×19·2=307·2	12×19·2	4×19·2×9
(2) C_2H_4	2·2	28× 2·2= 61·6	24× 2·2	4× 2·2×9
(3) H_2	39·3	2×39·3= 78·6		78·6×9
(4) CO	17·0	28×17·0=476	12×17·0	
(5) CO_2	5·2	44× 5·2=228·8	12× 5·2	
(6) O_2	1·2	32× 1·2= 38·4		
(7) N_2	15·9	28×15·9=445·2		
		Total . 1635·8 lb.	549·6 lb.	1477·8 lb.

Column 3, showing the relative weights, is obtained by multiplying the relative volumes by the molecular weights, column 4 by multiplying the relative weights by the proportion of the weight of carbon to the total weight of the constituent (i.e. in the first line $\frac{12}{16}$ of the weight of CH_4 is carbon or 12×19·2 lb.). The fifth column is nine times the weight of the hydrogen (i.e. in the first line $\frac{4}{16}$ of 307·2×9).

From columns 3 and 4 the weight of carbon per pound of gas is

$$\frac{549{\cdot}6}{1635{\cdot}8}=0{\cdot}3358 \text{ lb.} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

and from columns 5 and 3 the weight of vapour per pound of gas is

$$\frac{1477{\cdot}8}{1635{\cdot}8}=0{\cdot}903 \text{ lb.} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

The density of the supply gas will be

$$\frac{0{\cdot}0056}{200}\times 1635{\cdot}8=0{\cdot}0458 \text{ lb. per cu. ft.} \quad . \quad . \quad (3)$$

Statement (1) above gives the same information as is given by the gravimetric analysis of a solid fuel, and the argument now follows the previous lines.

From the exhaust gas analysis :

	Volumetric analysis	Relative weights
CO_2	6·77	44 × 6·77 = 298
O_2	8·81	32 × 8·81 = 281·9
N_2	84·42	28 × 84·42 = 2361
		Total . . 2940·9

Hence in 2940·9 lb. of dry gas there are $\frac{12}{44} \times 298$ lb. carbon, or the weight of carbon per pound of dry gas

$$= \frac{\frac{12}{44} \times 289}{2940 \cdot 9} = 0 \cdot 02763 \text{ lb.} \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

and weight of dry exhaust gas per pound of supply gas

$$= \frac{0 \cdot 3358}{0 \cdot 02763} = 12 \cdot 15 \text{ lb. from (1) and (4)}$$

Hence the total products of combustion per pound of supply gas

$$= 12 \cdot 15 + 0 \cdot 903 = 13 \cdot 05 \text{ lb. from (1) and (2)}$$

Hence the air supplied per pound of gas

$$= 13 \cdot 05 - 1 = 12 \cdot 05 \text{ lb} \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

and the air supply per cubic foot of supply gas

$$= 12 \cdot 05 \times 0 \cdot 0458 = 0 \cdot 5520 \text{ lb.} \quad . \quad . \quad . \quad . \quad . \quad (6)$$

(from (5) and (3))

Or, as 1 lb. of air at N.T.P. occupies 12·36 cu. ft., the volume of air per cubic foot of gas

$$= 0 \cdot 5520 \times 12 \cdot 36 = 6 \cdot 821 \text{ cu. ft.} \quad . \quad . \quad . \quad . \quad . \quad (7)$$

The Theoretical Supply of Air.

This can be obtained, by weight, as follows :

The oxygen required for 1635·8 lb.

$$= \frac{64}{16} \times 307 \cdot 2 + \frac{96}{28} \times 61 \cdot 6 + 8\left\{78 \cdot 6 - \frac{38 \cdot 4^{1}}{8}\right\} + \frac{16}{28} \times 476 = 2302 \text{ lb.} \quad (8)$$

(1) Table III (2) (3) & (6) (4)

[1] See also p. 375.

(i.e. $CH_4+2O_2=2CO_2+2H_2O$, Rel. wts. 16 and 64 } or 64 lb. of oxygen are required to burn 16 lb. of CH_4, and so on.

1 lb. of supply gas therefore requires $\frac{2302}{1635{\cdot}8}=1{\cdot}408$ lb. oxygen from (8),

and the air supply per pound of gas $=\frac{100}{23}\times 1{\cdot}408=6{\cdot}11$ lb.,

or volume of air per cubic foot of gas

$$=\frac{12{\cdot}36\times 6{\cdot}11}{1/0{\cdot}0458}=3{\cdot}46 \text{ cu. ft., from (3)}$$

(the volume of 1 lb. air at N.T.P.=12·36 cu. ft.)

The excess air supplied is therefore $\frac{6{\cdot}821-3{\cdot}46}{3{\cdot}46}\times 100=97$ per cent.

(The above analyses actually apply to a gas engine running fairly light, with a " Hit and Miss " Governor.)

The Theoretical Exhaust Gas Analysis for Complete Combustion.

1635·8 lb. of gas will give

$$\frac{44}{16}\times 307{\cdot}2+\frac{88}{28}\times 61{\cdot}6+\frac{44}{28}\times 476+228{\cdot}8=2015{\cdot}8 \text{ lb. } CO_2$$

(1) Table III (2) (4) (5)

The nitrogen content will be that originally in the gas, i.e. 445·2 lb. (7) plus the nitrogen in the air supply or $\frac{77}{23}\times 2302=8152{\cdot}2$ lb.

(from equation (8))

The analysis will then be as follows :

	Analysis by weight.	Relative volume.	Per cent. volume.
CO_2	2015·8	$\frac{2015{\cdot}8}{44}=45{\cdot}8$	13·59
N_2	8152·2	$\frac{8152{\cdot}2}{28}=291{\cdot}2$	86·41
		Total 337·0	

These will be the maximum figures for nitrogen and carbon dioxide content in the exhaust gases obtained from the given fuel gas.

*APPROXIMATE METHOD OF FINDING WEIGHT OF AIR SUPPLIED PER POUND OF COAL

In many cases the gases leaving a boiler are used to heat the feed-water, in which case the heat lost by the gases will equal the heat gained by the water, if other losses can be neglected.

If W_F=weight of feed water passing per hour,

W_G=the unknown weight of the flue gas per hour,

T_{F_1} and T_{F_2}=the initial and final temperatures of the feed,

and T_{G_1} and T_{G_2}=the initial and final temperatures of the gas,

and S is the specific heat of the flue gas (about 0·237 B.Th.U. per lb.),

Then $W_F(T_{F_2}-T_{F_1})=W_G\times 0{\cdot}237(T_{G_1}-T_{G_2})$

or
$$W_G=\frac{W_F(T_{F_2}-T_{F_1})}{0{\cdot}237(T_{G_1}-T_{G_2})}$$

If the consumption of coal per hour is W_C

Then the weight of air per hour=weight of flue gas per hour−(W_C−ash)

or weight of air per pound of coal

$$=\left[\frac{\frac{W_F(T_{F_2}-T_{F_1})}{0{\cdot}237(T_{G_1}-T_{G_2})}-W_C+\text{ash}}{W_C}\right]\text{lb.}$$

PRODUCER GAS

For some purposes the direct combustion of a solid fuel is undesirable and the production from it of a combustible gas may be advisable, even if at the cost of a certain initial loss. Such a gas may be distributed conveniently from a central station for use in a number of places where its combustion is simply controlled, or it may be found that a greater overall efficiency is obtainable from an internal combustion engine and gas producer than from a coal-fired boiler and steam engine.

In its simplest form the process consists of the production of carbon monoxide from coal or other carbonatious fuel by combustion with a very limited air supply, according to the formula

$$2C+O_2=2CO \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

In this process 4372 B.Th.U. are liberated per pound of carbon whereas 14,540 B.Th.U. are liberated during complete combustion. The calorific value of the gas per pound of carbon will thus be 14,540−4372 or 10,168 B.Th.U., giving a possible efficiency of $\frac{10,168}{14,540}$, or 70 per cent., the remaining 30 per cent. being discharged as heat in the gas. As the resulting gas will be a mixture of carbon monoxide and the nitrogen of

the air used during the reaction, the calorific value will be low, and of the order of 110–125 B.Th.U. per standard cubic foot.

The temperature for the reaction is high—about 2500° F.—and is liable to give trouble owing to the formation of clinker. In order to reduce the tendency to clinker, to reduce the heat losses and to increase the efficiency by the utilisation of some of the otherwise waste heat, steam may be generated and injected along with the air. In this case two reactions are possible, as follows :

If the temperature exceeds about 1830° F.

$$H_2O+C=H_2+CO \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

and if the temperature is less than about 1100° F.

$$2H_2O+C=2H_2+CO_2 \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Both reactions absorb heat, thus lowering the temperature, and the gas is enriched both by the presence of hydrogen and the reduction in the amount of total nitrogen. The absorption of heat in the reactions amounts to 4300 and 2820 B.Th.U. per lb. of carbon respectively.

For a given weight of carbon the first of these reactions (i.e. equation (2)) shows the production of one volume of hydrogen and one volume of carbon monoxide having calorific values of 345 and 342 B.Th.U. per cu. ft., or a mean of 343·5 B.Th.U. per cu. ft. ; and for the same weight of carbon the second reaction (equation (3)) shows two volumes of hydrogen and one of carbon dioxide, the mixture having a calorific value of $\frac{2\times 345}{3}=229$ B.Th.U. per cu. ft. It is obvious therefore that the first reaction is preferable to the second, both on the grounds of the reduction of heat wastage and the quality of the gas produced.

Case I. Dry Air only admitted.—Two reactions are involved in this process, the formulæ being

$$C+O_2=CO_2$$

and

$$CO_2+C=2CO$$

The first reaction takes place at the grate-level of the producer, the resulting carbon monoxide being subsequently reduced according to the second equation in its passage through the fuel bed, provided the temperature is maintained above about 1560° F. Above 2200° F. the reduction is virtually complete, while below 900° F. the reaction is reversed.

The complete reaction is expressed by

$$2C+O_2=2CO \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

As already indicated, the calorific value of the gas amounts to 70 per cent. of the heat value of the carbon, the remaining 30 per cent. being lost by

radiation, conduction, and by heat in the gas, which is usually cooled before use.

The volume of gas produced per pound of carbon is obtained as follows :

From equation (4), 24 lb. carbon require 359 cu. ft. oxygen to form 2×359 cu. ft. CO.

The final gas mixture will therefore consist of $2\times359=$ 718 cu. ft. CO

and $\frac{79}{21}\times359=1350$ cu. ft. N_2

Total 2068 cu. ft.

or $\frac{2068}{24}=86{\cdot}1$ cu. ft. per lb. of carbon

The analysis will be CO, 34·7 per cent.

N_2, 65·3 ,,

and the calorific value per S.C.F. will be $\frac{34{\cdot}7}{100}\times342=118{\cdot}7$ B.Th.U.

Case II.—If the reaction is such that the reactions given by equations (1) and (3) occur together, i.e. at temperatures below about 1100° F.,

Then considering first the reaction

$$C+2H_2O=CO_2+2H_2$$

the heat liberated by the complete combustion of 12 lb. carbon is

$$12\times14{,}540=174{,}500 \text{ B.Th.U.}$$

and the heat required to evaporate 36 lb. of water

$$=36\times1120\,[1]=40{,}320 \text{ B.Th.U.}$$

So that the net heat liberated per 12 lb. of carbon consumed

$$=174{,}500-40{,}320=134{,}200 \text{ B.Th.U.}$$

Also the heat in 4 lb. hydrogen

$$=4\times61{,}520=246{,}000 \text{ B.Th.U. (higher calorific value)}$$
$$=4\times51{,}800=207{,}200 \text{ B.Th.U. (lower calorific value)}$$

It is therefore obvious that as the heating value of the gas is 207,200 B.Th.U. and the available heat is only 134,180 B.Th.U., a total of $207{,}200-134{,}200=73{,}000$ B.Th.U., or $\frac{73{,}000}{12}=6180$ B.Th.U. per lb. of carbon must be supplied in order to maintain the reaction.

[1] 1120 = total heat of steam at 14·7 lb., from 62° F.

Considering now the reaction

$$2C+O_2=2CO$$

1 lb. of carbon burning to carbon monoxide produces 4372 B.Th.U.

Thus each pound of carbon taking part in this reaction will allow $\frac{4372}{6180}=0{\cdot}706$ lb. carbon to take part in the former;

i.e. for each 2 lb. of carbon acting according to equation

$$2C+O_2=2CO$$

1·412 lb. of carbon can take part in equation

$$C+2H_2O=CO_2+2H_2$$

The relevant factors are set out in Table I on p. 390. It will be noted that in the last line and column the lower calorific value of hydrogen is taken. Were the higher value chosen it would indicate that the total heat produced by the final combustion of the gas could be retained and utilised in the producer itself, i.e. the steam from combustion could be utilised to replace the water shown in the second column. In practice this is never done. The discrepancy in the final heat values shown is due to uncertainty with regard to calorific values, but amounts in this case to only about $\frac{1}{10}$ per cent.

The *efficiency* of the process $=\frac{538{,}500}{594{,}600}=90{\cdot}5$ per cent.

The composition of the gas by volume is

$CO_2=$	506·8 cu. ft.	or 14·1	per cent.
$CO=$	718 ,,	20·0	,,
$H_2=$	1014 ,,	28·2	,,
$N_2=$	1348 ,,	37·6	,,
Total	3587		

The calorific value of the gas

$$=\frac{538{,}500}{2066+506{\cdot}8+1014}=\frac{538{,}500}{3580{\cdot}8}=150{\cdot}4 \text{ B.Th.U. per S.C.F.}$$

The weight of air required per pound of carbon $=\frac{139}{40{\cdot}94}=3{\cdot}395$ lb.

The weight of water per pound of carbon $=\frac{50{\cdot}83}{40{\cdot}94}=1{\cdot}242$ lb.

and *The volume of gas per pound of carbon* $=\frac{3580{\cdot}8}{40{\cdot}94}=87{\cdot}1$ S.C.F.

TABLE I

	Carbon	Water	Oxygen and Nitrogen	Carbon Monoxide and Nitrogen	Carbon Dioxide	Hydrogen
Equation 2 2 lb. C Equation 3 1·412 lb. C	2C 1·412 C	+2·824 H_2O	$+O_2$ = and Nitrogen =	2CO and Nitrogen	1·412 (CO_2)	+2·824 (H_2)
Weights in lb.	24+ 1·412×12= 16·94 or 40·94 (C)	− 2·824×18 = 50·83 (H_2O)	32 O_2+ $\frac{77}{33}\times 32$ or 107(N_2) = 139 (mixture)	56 (CO) +107(N_2) = 163 (mixture)	1·412×44 = 62·13 (CO_2)	2·824×2 = 5·65 (H_2)
Volumes in cub. ft.	—	—	359 (O_2)+ $\frac{79}{21}\times 359$ or 1348 (N_2) = 1709 (mixture)	2×359 (CO)+ 1348 (N_2) = 2066 (mixture)	1·412×359 = 506·8 (CO_2)	2×824×359 = 1014 (H_2)
Heat, B.Th.U.	40·94×14,540 = 594,600	−50·83×1120 = −56,920	—	56×4391 = 245,800	—	5·65×51,800 = 292,700
	537,700			538,500		

TABLE II

	Carbon	Water	Oxygen and Nitrogen	Carbon Monoxide and Nitrogen	Hydrogen
Equation 1 2 lb. C Equation 2 1·453 lb. C	2C 1·453 C	1·453 H_2O	O_2 (+Nitrogen)	=2CO (+Nitrogen) 1·453 CO	1·453 H_2
Weights, lb.	12×3·453 = 41·45 (C)	18×1·453 = 26·14 (H_2O)	32 (O_2)+ 107 (N_2)= 139 (mixture)	107 (N_2)+ 3·453×28=96·7 (CO) or 203·7 (mixture)	1·453×2 = 2·906 (H_2)
Volume, cub. ft.	—	—	359 (O_2)+ 1350 (N_2) 1709 (mixture)	3·453×359= 1239 (CO)+ 1350 (N_2) 2589 (mixture)	1·453×359 = 521·8 (H_2)
Heat, B.Th.U.	41·45×14,540 = 602,500	−26·14×1120 = −29,280		96·7×4391 = 424,700	2·906×51,800 = 150,600
	573,220			575,300	

Case III.—If the temperature is above about 1100° F. the reactions follow equations (1) and (2), i.e.

$$2C+O_2=2CO \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

and

$$C+H_2O=CO+H_2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Proceeding as before, from equation (2) we have

$$12 \text{ lb. } C+18 \text{ lb. } H_2O=28 \text{ lb. } CO+2 \text{ lb. } H_2$$
$$=359 \text{ cu. ft. } CO+359 \text{ cu. ft. } H_2$$

The heat liberated by the carbon is $12\times14{,}540=174{,}500$ B.Th.U.
and the heat absorbed by evaporation $=18\times1120=20{,}160$,,
or the total heat liberated is $=154{,}300$,,

Meanwhile the calorific value of the gas is

$$28\times4391+2\times51{,}800=226{,}500 \text{ B.Th.U.}$$

or the heat to be supplied to maintain the reaction

$$=\frac{226{,}500-154{,}300}{12}=6020 \text{ lb. per lb. carbon.}$$

But 1 lb. of carbon burning according to equation (1) produces 4372 B.Th.U. or for each 2 lb. of carbon burning in this way sufficient heat is produced to allow $\frac{2\times4372}{6020}=1{\cdot}453$ lb. to react according to equation (2).

The relative factors are set out in Table II, p. 391.

The efficiency$=\frac{575{,}300}{602{,}500}=95{\cdot}4$ per cent.

The composition of the gas is $CO=1239$ cu. ft. or 39·9 per cent. by volume
$H_2=521{\cdot}8$,, 16·8 ,, ,,
$N_2=1350$,, 43·4 ,, ,,
Total 3110 ,,

The calorific value$=\frac{575{,}300}{3110}=185{\cdot}0$ B.Th.U. per S.C.F.

The weight of air per pound of carbon $=\frac{139}{41{\cdot}45}=3{\cdot}35$ lb.

The weight of water per pound of carbon$=\frac{26{\cdot}14}{41{\cdot}45}=0{\cdot}631$ lb.

The volume of gas per pound of coal $=\frac{3110}{41{\cdot}45}=75{\cdot}0$ S.C.F.

This shows the highest efficiency of the three cases considered.

*CALCULATION OF THE MEAN SPECIFIC HEAT OF THE FLUE GASES LEAVING A BOILER

The calculation requires the analysis of the fuel as well as of the flue gases, as the latter is of the dry gases only. The method will be best illustrated by means of a numerical example. Taking the following analyses :

Analysis of Fuel by Weight

C	87·30 per cent.
H	0·78 ,,
Ash	8·27 ,,
Other matters	2·36 ,,

Analysis of Dry Flue Gases by Volume

CO_2	9·88 per cent.
CO	0·05 ,,
O_2	9·82 ,,
N_2	80·25 ,,

The first step in the calculation is to convert the analysis of the dry flue gases by volume (the gases are always analysed by volume) into the analysis by weight. By multiplying each of the volume proportions by the corresponding molecular weight, adding the products so obtained, and then dividing each separate product by the sum of all the products, the proportion by weight of each gas present may be obtained.

Constituent	Analysis by volume		Molecular weight				Analysis by weight
CO_2	0·0988	×	44	=	4·348	=	0·145
CO	0·0005	×	28	=	0·014	=	0·0005
O_2	0·0982	×	32	=	3·142	=	0·1055
N_2	0·8025	×	28	=	22·470	=	0·749
				Total	29·974		1·000

Therefore in 1 lb. of dry flue gases there are

$$0{\cdot}145\times\frac{12}{44}+0{\cdot}0005\times\frac{12}{28}\text{ lb. of carbon}$$

$$=0{\cdot}0395+0{\cdot}0002\text{ lb. of carbon}$$

$$=0{\cdot}0397\text{ lb. of carbon}$$

Hence the weight of dry flue gases per pound of dry fuel will be

$$\frac{0{\cdot}873}{0{\cdot}039}=22{\cdot}0\text{ lb.}$$

But the 0·0078 lb. of hydrogen in the fuel produces $0{\cdot}0078 \times 9 = 0{\cdot}070$ lb. of steam, hence the actual weight of each constituent in the flue gases from 1 lb. of fuel is as shown in column 3 below.

Constituent	Weight in pounds per pound of dry fuel burned	Weight in pounds of each constituent in 1 lb. of flue gases
CO_2	$22{\cdot}0 \times 0{\cdot}145 = 3{\cdot}190$	$\frac{3{\cdot}190}{22{\cdot}07} = 0{\cdot}1445$
CO	$22{\cdot}0 \times 0{\cdot}0005 = 0{\cdot}011$	$\frac{0{\cdot}011}{22{\cdot}07} = 0{\cdot}0005$
O_2	$22{\cdot}0 \times 0{\cdot}1055 = 2{\cdot}321$	$\frac{2\,321}{22{\cdot}07} = 0{\cdot}1052$
N_2	$22{\cdot}0 \times 0{\cdot}749 = 16{\cdot}478$	$\frac{16{\cdot}478}{22{\cdot}07} = 0{\cdot}7467$
H_2O	—— 0·070	$\frac{0{\cdot}07}{22{\cdot}07} = 0{\cdot}0031$
	22·070	1·0000

The specific heats at constant pressure of the above gases may be taken as :

$$CO_2 = 0{\cdot}216$$
$$CO = 0{\cdot}245$$
$$O_2 = 0{\cdot}218$$
$$N_2 = 0{\cdot}244$$
$$H_2O = 0{\cdot}480$$

Hence the mean specific heat of the flue gases is :

$$(0{\cdot}1445 \times 0{\cdot}216) + (0{\cdot}0005 \times 0{\cdot}245) + (0{\cdot}1052 \times 0{\cdot}218) + (0{\cdot}7467 \times 0{\cdot}244) + (0{\cdot}0031 \times 0{\cdot}48) = 0{\cdot}238$$

*BOILER DRAUGHT

It is necessary to maintain a difference of pressure above and below the firegrate in order to supply the quantity of air required for combustion. This difference of pressure is known as the draught, and may be produced either by means of

(*a*) A chimney (natural draught) ;

(*b*) Steam jets (induced or forced draught) ;

(*c*) Fans, which may either draw the gases from the flues (induced draught) or blow air under pressure into the ash-pit, or a closed boiler-room from which the air supply is drawn (forced draught).

The theoretical velocity of the gases produced by a draught or difference of pressure above and below the firegrate is given by the equation

$$v^2 = 2gl$$

where l is the height in feet of a column of air which could be supported by the given pressure difference.

Let h be the draught in inches of water.

Then, since a head of 1 in. of water is equivalent to a pressure of 5·198 lb. per sq. ft., and 1 cu. ft. of air at N.T.P. weighs 0·0807 lb.

$$l=\frac{5\cdot198h}{0\cdot0807}$$

$$\text{and } v^2=2\times32\cdot2\times\frac{5\cdot198h}{0\cdot0807}$$

$$v^2=4148h$$

$$v=\sqrt{4148h} \text{ ft. per second} \quad . \quad . \quad . \quad . \quad . \quad (1)$$

The actual velocity of the gases will be less than that given by (1) because of the frictional resistance offered to their passage along the flues.

*HEIGHT OF CHIMNEY REQUIRED TO PRODUCE A GIVEN DRAUGHT

Let h=required draught in inches of water,
H=height of chimney above the firegrate in feet,
T_1=absolute temperature inside the chimney (assumed constant),
T_2=absolute temperature outside the chimney (assumed constant),
n=number of pounds of air supplied per pound of fuel burned,
A=cross-sectional area of chimney in square feet,

then the difference between the weight of a column of external air equivalent in volume to the interior of the chimney, and the weight of the same volume of the gases which are in the chimney, is equal to the draught pressure, i.e.

Weight of A×H cu. ft. of external air—weight of A×H cu. ft. of gases in chimney=5·198h×A

$$\left(\frac{\text{A}\times\text{H}\times0\cdot0807\times492}{T_2}\right)-\left(\text{A}\times\text{H}\times\frac{n+1}{n}\times\frac{0\cdot0807\times492}{T_1}\right)=5\cdot198h\times\text{A}$$

$$\text{H}\times0\cdot0807\times492\left(\frac{1}{T_2}-\frac{n+1}{n}\cdot\frac{1}{T_1}\right)=5\cdot198h$$

$$\text{H}=\frac{5\cdot198h}{0\cdot0807\times492}\times\frac{nT_1T_2}{nT_1-(n+1)T_2}$$

$$\text{H}=0\cdot13h\times\frac{nT_1T_2}{nT_1-(n+1)T_2} \quad . \quad . \quad . \quad . \quad . \quad (2)$$

In the above theory the variation in the density due to the slightly reduced pressure inside the chimney is neglected. If in addition we

neglect the increased density due to the products of combustion, and assume the contents of the chimney to consist of air at atmospheric pressure

$$H = 0{\cdot}13h \times \frac{T_1 T_2}{T_1 - T_2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

and

$$h = \frac{H}{0{\cdot}13}\left(\frac{1}{T_2} - \frac{1}{T_1}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

In actual practice the effect of the frictional resistance offered to the passage of the air through the firebars, fire, flues, and chimney is to reduce the draught h below the value obtained from equation (4); also, the temperature of the gases inside the chimney is diminishing for every foot of its height. In other words, the height of chimney required to produce a given draught is greater than that given by equation (3).

EXAMPLES ON CHAPTER XVII

1. The analysis by weight of a certain coal is C 80 per cent., H_2 5 per cent., S 0·5 per cent.: estimate the theoretical quantity of air required for the complete combustion of 1 lb. of the coal. If 20 lb. of air are supplied per pound of coal and the combustion is complete, estimate the analysis of the flue gases by weight.

2. A producer gas has the following analysis by volume: H_2 18·73 per cent., CO 25·07 per cent., CO_2 5·2 per cent., N_2 51 per cent. Estimate the minimum quantity of air required for the complete combustion of 1 cu. ft. of the gas, the percentage contraction in volume after combustion, and the composition of the products of combustion.

3. The analysis (by weight) of the fuel used in a boiler trial was C 88 per cent., H_2 3·6 per cent., O_2 4·8 per cent., ash 3·6 per cent., and the volumetric analysis of the dry flue gases was CO_2 10·9 per cent., CO 1·0 per cent., O_2 7·1 per cent., N_2 81 per cent. Estimate the mean specific heat of the flue gases, and the quantity of heat carried away by the flue gases per pound of fuel burned if the temperature of the gases is 550° F., and of the air in boiler house 50° F.

4. The flue gas analysis by volume in a boiler trial was CO_2 10·5 per cent., CO 1 per cent., O_2 8 per cent., N_2 80·5 per cent., and the coal analysis as burned was C 82 per cent., H_2 4·2 per cent., O_2 4·8 per cent., other matters 9 per cent. Calculate the following items in the heat balance per pound of coal, the temperature of the flue gases being 600° F. and the temperature of the air supply 60° F.:

(*a*) Heat carried away by products of combustion, average specific heat 0·24.

(*b*) Heat carried away by excess air, average specific heat 0·238.

(*c*) Heat lost by incomplete combustion.

5. In a boiler trial the fuel analysis, dry coal as burned, was C 85 per cent., H_2 4 per cent., O_2 7 per cent., ash, etc., 4 per cent., and the flue gas analysis by weight was CO_2 11 per cent., CO 1·5 per cent., O_2 7·1 per cent., N_2 80·4 per cent. The temperature of the flue gases leaving the boiler was 600° F., and the boiler house temperature was 70° F. Estimate per pound of coal—

(*a*) The proportion of carbon burned to CO and the heat lost through this imperfect combustion, expressing the latter as a percentage of the available heat in the fuel.

(*b*) The heat carried away in the flue gases per pound of coal burned, the mean specific heat of the flue gases being taken as 0·24.

6. In a boiler trial the fuel analysis of the dry coal as burned was C=83 per cent., H_2=4 per cent., O_2=8 per cent., ash, etc.=5 per cent.; the volumetric analysis of the flue gases was CO_2=10 per cent., CO=1·7 per cent., O_2=8·1 per

cent., N_2=80·2 per cent. The temperature of the flue gases was 600° F. and of the boiler house 80° F. Find—

(1) The proportion of C burned to CO, and the heat lost through imperfect combustion, expressing the latter as a percentage of the heat in the fuel.

(2) The heat carried away in the flue gases per pound of fuel burned, the average specific heat being taken as 0·24.

7. In a trial of a boiler fitted with an economiser the following volumetric analyses of the gases entering and leaving the economiser were made :

	Leaving	Entering
CO_2	7·9 per cent.	8·3 per cent.
CO	nil ,,	nil ,,
O_2	11·5 ,,	11·4 ,,
N_2	80·6 ,,	80·3 ,,

The temperatures of the flue gases on entering and leaving the economiser were 350° C. and 180° C. respectively. Temperatures of water on entering and leaving economiser were 15° C. and 115° C. Weight of feed water per hour 10,000 lb., weight of coal used per hour 1060 lb. Carbon in 1 lb. of coal 0·8 lb. Assuming the average specific heat of the gases to be 0·25, estimate per pound of coal burned—

(*a*) The air leakage into the economiser.

(*b*) The heat lost by the gases in passing through the economiser.

(*c*) The heat gained by the feed water.

8. Estimate the minimum height of chimney required to produce a draft of $\frac{5}{8}$ in. of water if 24 lb. of air are supplied per pound of fuel burned, the mean temperature of the gases inside the chimney being 600° F. and the temperature of the external air 80° F.

9. A sample of coal gas has the following analysis by volume : H_2 46 per cent., marsh gas CH_4 39·5 per cent., olefiant gas C_2H_4 2·53 per cent., tetrylene C_4H_8 1·27 per cent., CO 7·5 per cent., N_2 0·5 per cent., water vapour H_2O 2 per cent. Calculate

(*a*) The volume of air required for the complete combustion of 1 cu. ft. of the gas.

(*b*) The higher calorific value in B.Th.U. per cubic foot.

(*c*) The lower calorific value in B.Th.U. per cubic foot.

Assume the calorific values of the above constituents the same as given in the table on p. 374. (C_4H_8 Higher Value=3060.)

10. The gas used in a gas engine trial was tested in a Junker calorimeter and the following results were obtained :

Gas burned in calorimeter	2·13 cu. ft.
Pressure of gas supplied	2·1 in. of water
Barometer	29·92 in. of mercury
Temperature of gas	53° F. (11·7 C.)
Weight of water heated by gas	50·3 lb.
Temperature of water at inlet	47·6° F. (8·7° C.)
Temperature of water at outlet	72·4° F. (22·4° C.)
Steam condensed during test	0·116 lb.

Determine the higher and lower calorific values per cubic foot at 60° F. and 30″ of mercury pressure (sp. gr. mercury 13·6).

11. A sample of oil contains 86 per cent. by weight of carbon and 14 per cent. of hydrogen. Estimate the minimum quantity of air required for the complete combustion of 1 lb.

12. The analysis of a sample of ———————— steam coal by weight is C 87·8 per cent., H_2 4·10 per cent., the remainder being ash. etc. If 18 lb. of air are supplied per pound of coal and the combustion is complete, estimate the composition of the products by weight.

13. The volumetric analysis of a sample of coal gas is as follows : CH_4=39·5 per cent., CO=7·5 per cent., N_2=5 per cent., water vapour 2 per cent. When gas of this quality was used in a gas engine the exhaust gas analysis gave 10 per cent. by volume of O_2. Estimate the quantity of air used per cubic foot of gas, and the contraction in volume after combustion.

14. The analysis of a certain oil fuel used in a boiler trial was C 86 per cent., H_2 14 per cent., and the volumetric analysis of the flue gases was CO_2=9·4 per cent., CO=1 per cent., O_2=10·1 per cent., N_2=79·5 per cent. Estimate, per pound of fuel burnt, the heat carried away by the products of combustion, and by the excess air, the temperature of the flue gases being 600° F., and of the air supply 60° F.

15. In a trial of a Lancashire boiler with economiser the following results were obtained :

Volumetric analyses of the flue gases on entering and leaving the economiser—

	Entering	Leaving
CO_2 . . .	8·3 per cent.	6·2 per cent.
CO . . .	0·4 ,,	0·3 ,,
O	11·2 ,,	13·7 ,,
N	80·1 ,,	79·8 ,,
Total . .	100·0	100·0

Temperatures of the flue gases on entering and leaving the economiser, 642° F. and 335° F.

Temperatures of feed water on entering and leaving the economiser, 134° F. and 234° F.

Weight of feed water per hour, 7370 lb.

Weight of coal stoked per hour, 1000 lb.

Per pound of dry fuel stoked the carbon burned was 0·735 lb., and the weight of the flue gases, including moisture, entering the economiser was found to be 21·5 lb.

The average specific heat of the gases may be taken as 0·25.

Calculate, per pound of fuel stoked—

(*a*) The air leakage into the economiser.

(*b*) The heat lost by the gases in passing through the economiser.

(*c*) The heat gained by the feed water.

16. The flue gases from a boiler pass around the tubes of an economiser. The temperature of the gases entering the economiser is 315° C., and on leaving it 149° C. The amount of feed water passing through the tubes is 90 lb. per minute ; the temperature of the water entering the economiser is 38° C., and on leaving 115° C.

If the boiler evaporates 10 lb. of water per pound of coal, find approximately the weight of air supplied per pound of coal burned. Assume that the specific heat of the gases is 0·25.

17. A sample of oil used in an oil engine trial was tested in a Mahler-Cook bomb calorimeter and the following results were obtained :

Weight of oil taken=1·090 grams.

Total weight of water, including water equivalent of calorimeter, 2800 grams.

Corrected rise of temperature of the water=4·26° C.

Determine the higher calorific value of the oil.

18. The volumetric analysis of a producer gas supplied to an engine is CO_2 7·66 per cent., CO 20·27 per cent., H_2 20·19 per cent., CH_4 2·778 per cent., N_2 47·1 per cent. The exhaust gases contained 10 per cent. of oxygen by volume. Estimate the quantity of air actually supplied per cubic foot of gas and the contraction in volume in the engine cylinder due to combustion.

Chapter XVIII

THE TESTING OF HEAT ENGINES AND PLANT

*GENERAL STATEMENT

The tests carried out on heat engines, etc., can be classified as either Industrial or Scientific.

Industrial Tests.—Under this heading come most of the tests carried out by manufacturers during the development of plant, the tests on the plant before delivery, and those carried out by the purchaser after its installation. In the latter cases it is usual to make only such measurements as are required for the determination of the efficiency, output, and performance, including such items as consumption of lubricating oil. The method of measurement should be as direct as possible, the apparatus simple and robust, and the whole test should be such that it can be carried out in a reasonable time by a competent engineer with a minimum of highly trained assistance.

Heat and other losses are usually only measured when these serve as a check on the principal items required.

Scientific Tests.—These tests may include the whole plant, or be confined to specific portions only, and are required for the elucidation of certain problems, the accumulation of information of value to designers and others, and for the checking of theories.

The information required is almost invariably of a much higher order of accuracy than that provided by the industrial test; time and cost are of secondary importance, and the apparatus required may become, of necessity, extremely elaborate, and may require great manipulative and technical skill on the part of the operator and his assistants. In such tests the measurement and analysis of losses may become matters of primary concern.

In all cases it is of the utmost importance that the engineer in charge should have a sound knowledge of the problems and difficulties to be faced, and of the qualities and accuracies of his apparatus, and the recalibration of all measuring instruments should receive special attention.

*REPORT ON TABULATING THE RESULTS OF HEAT ENGINE TRIALS

In 1927, a Committee of the Institution of Civil Engineers presented a Report (Wm. Clowes & Sons, 5*s.*) on the Tabulation of the Results

of Heat Engine Trials. In this Report detailed recommendations are made concerning the carrying out and recording of tests, the procedure suggested leading, in many cases, towards simplicity and clarity. In particular, it is recommended that the heat interchanges should be stated as input and output quantities instead of, as formerly, as the difference of two values stated with reference to a given datum, and that the higher calorific value alone should be used.

For scientific purposes a standard code is usually quite inadequate, and the special point of view of the investigator may make it undesirable to adopt the methods of calculation indicated above.

The reader is referred to the Report for details concerning industrial tests. In the following sections are given the results of tests on various types of plant, and notes on the method of calculation. The reader is advised to re-calculate one or more of these tests according to the recommendations in the above report, in order to contrast the methods used.

*STEAM PLANT TRIAL

I. BOILER

Date. . . . . . *Name* . .

OBSERVATIONS.

Description of Plant. .....Boiler, with economiser and feed heater.
Conditions of Trial.—Saturated steam. Induced draught.
Duration of Trial, 1¾ hours.[1] Barometer, 29·65 in. mercury=14·53 lb. per sq. in.
Grate area, 11 sq. ft. Heating surface : Boiler, 653 sq. ft.—superheater, nil ; Economiser, 320 sq. ft.

(1) **Fuel.** Description. .....Coal.

Analysis (by wt.) :—		Calorific Value :—	
Carbon . . .	0·86		
Hydrogen . .	0·04	Higher . .	15,400 B.Th.U.
Ash	0·07	Lower . .	15,010 B.Th.U.
Moisture . .	0·007	Carbon Value . . .	1·06
			1·03
Residue (by diff.)			
Total . .	1·00	Cost per ton . . .	36*s.* 6*d.*

Weight of Fuel supplied per min. . . 3·0 lb.
Weight of Ash collected per min. . . . — lb.

(2) **Flue Gas.**

Analysis (by vol.) :—		Temperature :—	
CO_2	11·7		
O	5·2	Behind Boiler .	T_8 437° F.
CO	0·2	Before Economiser	T_9 394° F.
N (by diff.) . .	82·9	After Economiser	T_{10} 219° F.
		Temperature of Air	T_A 64·7° F.
Total . .	100·0		

Draught Gauges :—(in. water).
Boiler Flue Fan Inlet . . Chimney Base .

[1] This trial was of very short duration. The Committee of the I.C.E. recommend that, after warming up, a boiler should be run under test conditions for 1 hour before the trial to insure stability, and that the duration of the trial should be such that the coal consumption per square foot of grate should be not less than 25 lb. per in. of thickness of the fire, with a minimum of 4 hours. The plant should then be run for one more hour under the same conditions to ensure that the final conditions were stable. For shorter trials the varying thickness of the fire may introduce serious errors owing to the difficulty of estimating the thickness of the coal bed.

(3) **Feed Water.**

Weight of Water pumped per min.	29·17 lb.
Gain of Water in Boiler per min.	+1·15 lb.
Leakage of Water per min.	nil lb.
Weight of Water Evaporated per min.	28 02 lb.
Temperature of Water entering Economiser T_5.	57·1° F.
,, ,, leaving ,, T_6.	141·1° F.

(4) **Steam.**

Pressure of Boiler Steam (gauge)	198 lb. sq. in.
,, ,, ,, (absolute) p_B	212·5 lb. sq. in.
Saturation Temp. corresponding to pressure, T_B	387° F.
Actual Temperature of Steam leaving Boiler, T'_B	—° F.
Degrees of Superheat	—° F.
Dryness Fraction of Steam	96%

(5) **Auxiliaries.** Fan current . . 5·38 amps. at 400 v. Feed Pump Current . . . . . 4·07 amps. at 200 v

(6) **DEDUCTIONS.**

Weight of Water pumped per lb. fuel	9·72 lb.
,, ,, evaporated ,,	9·34 lb.
Latent Heat of Steam at p_B (per lb. steam)	838·7 B.Th.U.
Heat in Superheat	— B.Th.U.
Liquid Heat T_6 to T_B ,, ,,	251·4 B.Th.U.
Total Heat T_6 to T_B ,, (q=0·96)	1056·9 B.Th.U.

Fan Power	2152 watts
Feed Pump Power	814 watts

(7) **Flue Gas.**

Weight of Dry Flue Gas per lb. Fuel W_G	18·11 lb.
Density 0·0842 lb. cu. ft. Sp. Ht.	0·235 B.Th.U. per lb.
Weight of Vapour in Gas per lb. fuel W_V	0·367 lb.
Total H't of Vapour (from T_A) per lb. Vpr. at T_8	1223 B.ThU.
,, ,, ,, ,, ,,	T_9 1203 B.Th.U.
,, ,, ,, ,, ,,	T_{10} 1121 B.Th.U.
Weight of Air supplied per lb. Fuel	17·55 lb.
,, ,, required for complete combustion	11·35 lb.
Excess Air supplied (% of Air required)	54·7%

(8) **Heat Balance, Boiler, and Superheater** (per lb. fuel.)—From Air Temperature·

Received :—	B.Th.U.	%
In Fuel (Higher Value)	15,400	100·0
Total	15,400	100·0

Discharged :—	B.Th.U.	%
Through Boiler Heating Surface	9,962	64·7
,, Superheater ,, ,,	—	—
Total	9,962	64·7
In Dry Flue Gas	1,592	10·3
In Vapour of Combustion, etc.	449	2·9
In Imperfect Combustion	147	1·0
In Radiation, etc.	3,250	21·1
Total	15,400	100·0

(9) **Heat Balance, Economiser** (per lb. fuel). From Air Temperature.

Received :—		B.Th.U.	%	Discharged :—	B.Th.U.	%
In Gas entering Economiser.	Dry gas	1401	76	Through Economiser Heating Surface	816	44·3
	Vapour	442	24	In Gas leaving Economiser. Dry gas	654	35·5
				Vapour	412	22·4
				In Radiation, etc.	−39	−2·1
Total . .		1843	100·0	Total . .	1843	100·0

(10) **Evaporation, etc.**	Gross	All. for Aux.
Corrected evaporation per lb. fuel . . lb.	9·43	8·37
Equivalent evaporation from & at 212° F. lb.	10·31	9·16
Eq. evap. from & at 212° per lb. Carbon Value . . . . . lb.	10·01	8·90

Water Evap. per sq. ft. Heating Surface per hr. .	2·60 lb.
Fuel Burnt per sq. ft. grate per hour	10·7 lb.
Heat Transmitted per sq. ft. Boiler Surface per min.	45·8 B.Th.U.
,, ,, ,, Economiser, do. .	7·65 B.Th.U.
Estimated steam to drive Auxiliaries	1·06 lb.

(11) **Efficiencies on Lower Calorific Value**	Gross	All. for Aux. per cent.
Thermal Efficiency, Boiler and Superheater	66·4	58·9
Economiser . . .	57·0	55·5
Combined Plant .	71·8	63·8

(12) **Efficiencies on Higher Calorific Value**	Gross	All. for Aux.
Thermal Efficiency, Boiler and Superheater . . .	64·7	63·0
Economiser . .	55·5	54·1
Combined Plant .	70·0	68·2

EXPLANATION OF ITEMS IN ABOVE REPORT

Table 1.

Higher calorific value, obtained by direct measurement in a bomb calorimeter (see p. 372).

Lower calorific value=higher calorific value−1055 (weight of moisture+9×weight of hydrogen) (see p. 375).

Carbon value=$\frac{\text{Calorific value of fuel}}{\text{Calorific value of carbon}}$. This can be taken as either the higher or lower calorific value. The calorific value of carbon is 14,540.

Table 3. *Gain of Water in Boiler per minute.*

This is estimated from the rise or fall of water in the gauge-glass during the trial, and the area of the drum at water level.

Item 4 is item 1−item 2.

Table 7.

Weight of dry flue gas per pound fuel (see p. 381).

Density of dry flue gas per pound fuel (see p. 382).

Specific heat of dry flue gas per pound fuel (see p. 393).

Where greater accuracy is required the variation of specific heat with temperature should be considered. This will be found to amount to an increase of about 4 per cent. of the value at normal temperature for each rise of 100° F.

Where many trials with one fuel are involved a series of curves as shown in Fig. 168 may be plotted, giving the variation of specific heat with temperature and carbon-dioxide content for a coal containing 4% H_2. The essential figures may be found in the Dictionary of Applied Physics, Dugald Clerk's "Gas Engines," etc. Such a table is suitable for one fuel only on account of the shrinkage produced by the combustion of the hydrogen.

Weight of vapour per pound of fuel=9×weight of hydrogen+moisture.

Weight of air supplied per pound of fuel=weight of dry gas per pound fuel+weight of vapour per pound fuel−(1−weight of ash per pound fuel) (see p. 381).

Weight of air required for complete combustion (see p. 382).

Table 8.

The temperature of the air is taken as base, and heat quantities are calculated in B.Th.U. per pound of fuel as fired, i.e. not dried.

Heat passing through boiler heating surface=weight of water pumped per pound of fuel×liquid heat from T_6 to T_B+weight of water evaporated per pound of fuel×latent heat at P_B.

Heat passing through superheater heating surface=weight of water evaporated per pound of fuel×heat per lb. of steam as superheat.

Heat in dry flue gas=weight of gas × ($T_8 - T_A$) × specific heat of gas.

Heat in vapour in flue gas=weight of vapour × total heat per pound at T_8.

Heat lost by imperfect combustion, i.e. by the formation of CO (see p. 382).

If an estimate can be made of the weight of unburnt carbon in the ash an additional loss is incurred equal to the estimated weight of carbon in the ash per pound of fuel × 14,540 B.Th.U.

Table 9. Economiser.

The specific heats of the gases for the particular ranges of temperature should be taken where accuracy is required.

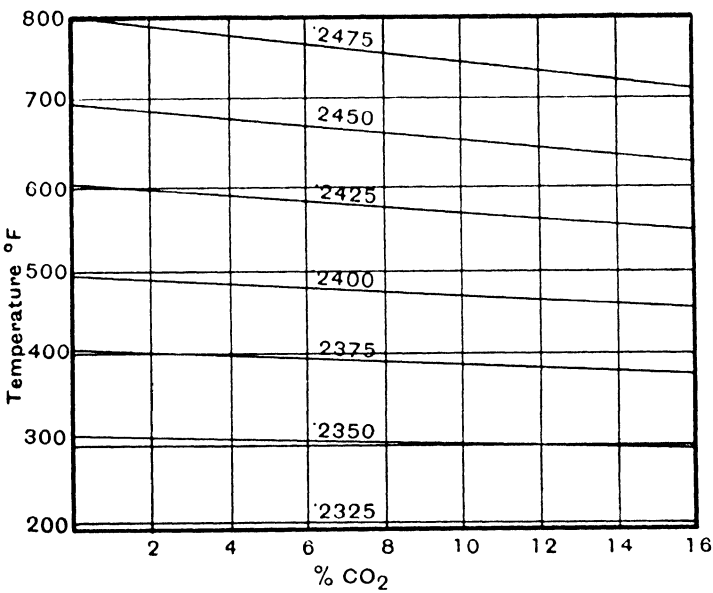


Fig. 168.

(1) Heat received in dry gas=weight of gas × mean specific heat from T_9 to T_A × ($T_9 - T_A$).

(2) Heat received in vapour=weight of vapour × total heat per pound at T_9.

(3) Heat passing through economiser heating surface=feed water pumped × ($T_6 - T_5$).

(4) Heat leaving economiser in dry gas=weight of gas × specific heat of gas × ($T_{10} - T_A$).

(5) Heat leaving economiser in vapour=weight of vapour × total heat at T_{10} from T_A.

Table 10. *Corrected Evaporation.*

Where the level of water in the boiler is different at the beginning and end of the trial, the heat passing through the heating surface does not represent the heat expended in evaporating the water. For instance, if there is a gain of water in the boiler part of the heat supplied has been utilised in heating up this excess from T_6 to T_5 but not evaporating it.

The corrected evaporation is the weight of steam which would have been produced by the same heat passing through the surfaces if the level of the water had remained constant, and

$$= \frac{\text{Total heat passing through heating surface of boiler per pound fuel}}{\text{Heat per pound steam from } T_6 \text{ to temperature of boiler}}$$

This value will be taken for the remainder of the evaporation calculations.

Equivalent Evaporation from and at 212°.—The purpose of this item is to provide a common basis for comparison of the performances of boilers working under different conditions. It is the evaporation which would be obtained if the feed water were supplied at 212° F. and converted into steam at 212° F., and

$$= \frac{\text{Corrected evaporation per lb. fuel} \times \text{heat per lb. steam from } T_6 \text{ to } T_B}{\text{Latent heat of steam at } 212° \ (=966)}$$

Equivalent evaporation per pound of carbon value is obtained by dividing the above by the carbon value per pound of fuel. This item provides a basis for comparison of boilers using different qualities of fuel.

Water evaporated per square foot of heating surface per hour

$$= \frac{\text{Corrected evaporation per pound of fuel} \times \text{fuel per hour}}{\text{Area of boiler heating surface}}$$

Allowance for Auxiliaries.—When the auxiliaries are supplied with power from external sources, an allowance must be made in the efficiency and evaporation figures. It is required to determine the evaporation available for power supply if the power for these auxiliaries were taken from the boiler itself.

(1) The most convenient method from the standpoint of a scientific test is to assume that if the auxiliaries were steam-driven, they would have a thermal efficiency of 5 per cent.,[1] so that the heat which would be supplied to them

$$= H_a = 20 \times \text{heat equivalent of power, in B.Th.U.}$$

and amount of steam used

$$= W_a = \frac{H_a}{\text{Heat given to each pound of steam in boiler}}$$

[1] The figure of 17 per cent. recommended by the Engine Trials Reports Committee is a compromise over an enormous range of possibilities, and its use is unjustified in this case.

In calculating the corrected evaporation per pound of fuel allowing for auxiliaries, W_a must be deducted from weight of water evaporated per pound of fuel in obtaining the heat passing through the boiler heating surface. This new value is then used in obtaining similar results for equivalent evaporation from and at 212° F.

(2) For a purely commercial test where the cost of running is the important factor, the cost of (say) auxiliary electrical power could be obtained, and also that of the steam evaporated, and the relative value calculated.

Thus, if 1 lb. of steam costs, say, 0·02*d.* and the cost per minute of electrical power is 0·05*d.*, then the equivalent quantity of steam used per minute in auxiliaries=0·05/0·02=2½ lb. This must be subtracted from evaporated weight to give net evaporation.

Thermal Efficiency.

Boiler and Superheater.—These cannot be separated when the superheater is not independently fired, as it is not known how much of the heat of the fuel goes through the boiler heating surface and how much goes to the superheater.

$$\text{Efficiency} = \frac{\text{Heat passing through boiler and superheater surface per pound of fuel}}{\text{Heat in 1 lb. of fuel (higher or lower calorific values)}}$$

Allowance for auxiliaries—deduct H_a from numerator.

Economiser efficiency=the ratio of the heat actually passing through the surface to the heat which could be obtained from the flue gas by cooling it down to the temperature of the air, but without condensing the vapour.

Heat passing through surface H_S=feed water pumped × (T_6-T_5).

Heat in gas H_G=weight of dry gas × specific heat × (T_G-T_A)+weight of vapour × (total heat at T_9 from T_A−total heat at T_{10} from T_A (at atmospheric pressure)).

$$\text{Then economiser efficiency} = \frac{H_S}{H_G}$$

Efficiency of Combined Plant.

$$\text{Efficiency} = \frac{\text{Heat passing through boiler, superheater and economiser surfaces}}{\text{Heat in 1 lb. of fuel (higher or lower calorific values)}}$$

Allowance for auxiliaries as in boiler and superheater.

Remarks.—In the particular trial for which the figures are given the durations of the trial and of the preparatory period were much too short to obtain fully satisfactory results.

*STEAM ENGINE TRIALS

In conjunction with the boiler trial just considered a trial was conducted on a Corliss Tandem Compound Engine fitted with a condenser for which the air pump was driven from the common piston rod. A prony brake was used to absorb the power.

The following are the relevant data :

	Dia., in.	Stroke, in.	Clearance, per cent.	Effective area, sq. in.	Wt. of reciprocating pts.	Tube surf. of condenser	Brake	
							Zero reading	Rad.
H.P. cylinder	$7\frac{1}{2}$	24	$6\frac{1}{2}$	39	lb.	sq. ft.	lb.	
H.P. piston rod	$2\frac{7}{8}$							
L.P. cylinder	$15\frac{1}{2}$	24	$3\frac{1}{4}$	174	966	220	152	792
L.P. piston rod	$2\frac{1}{4}$							
Air pump cylinder	$7\frac{1}{2}$	24		44·3				
Air pump rod	$1\frac{1}{2}$							

STEAM PLANT TRIAL

II. ENGINE

Date . . .

OBSERVATIONS.

Description of Plant.—Tandem Compound Engine with —— Boiler.
Conditions of Trial.—Saturated Steam. Steam in all Jackets. ¾ Full Load.
Duration of Trial, 1¾ hours. Barometer 29·65 in. mercury = 14·53 lb. per sq. in. Air Temp., 64·7° F.

	lb. per min.
Gain of Water in Boiler ±	+1·15
Steam taken from Boiler .	28·02
Hot Well discharge . .	25·88
Cylinder Feed (*a*) . . .	26·53
Jacket Steam (*b*) . . .	0·73
Engine leakage collected (*c*) } Other leakages (difference) }	1·3

Condensing Water.

Quantity per min.	1090 lb.
Initial Temp. .	66·6° F.
Final Temp. .	88·1° F.
Head	21·5 ft.

Hot Well.

Quantity per min.	25·88 lb.
Temperature .	126·7° F.
Air pump M.E.P.	1·7 lb. per sq. in.

Feed Water Heater.

Jacket Steam Condensed per min.	0·73 lb.
Temp. of ,, ,, ,,	75·7° F.
Feed water pumped per min.	29·17 lb.
Initial Temp. of Feed .	49·0° F.
Final ,, ,, .	55·7° F.

Steam Pressures.	Gauge	Abs.
At Stop Valve P_E . .	195	209·5
,, H.P. Steam Chest .	193·5	208
,, L.P. ,, ,, .	25·3	39·8
Condenser Vacuum	24·6 in.	
Do. Corr. to 30″ Bar	24·95 in.	
Abs. Press. in Condsr.	2·47 lb. sq. in.	

Steam Temperatures.

At Stop Valve . T'_E	385·8° F.
(Superheat at S.V.) . .	— ° F.
(Dryness fraction at S.V. .	96·0%)
Saturated Steam at p_E, T_E	385·8° F.
At H.P. Steam Chest .	385·2° F.
At L.P. ,, ,, .	267° F.
In Exhaust Pipe . . T_C	134·3° F.

Power.	H.P.	L.P.	
Cut off, Percentage of Stroke	23·4	33·3	
Max. Indicated Pressure lb. sq. in. abs.	189·8	30·0	gauge
Min. ,, ,, ,,	26·4	−11·0	gauge
M.E.P. Front . . lb. sq. in.	78·9	25·6	
,, Back . . . ,,	79·8	26·1	
M.E.P. Average . . ,,	79·3	25·8	

Revolutions per min.	99·4
Brake Load (net)	523·8 lb.

DEDUCTIONS.

Air Pump H.P.	0·45
Estimated Circulating Pump H.P.	1·77
Ratio of Condensing Water to Steam Condensed . . .	42·1

Total Heat per lb. Steam at T'_E from 32° F. . .	1165 B.Th.U.
,, ,, ,, ,, T_C . . .	1063 B.Th.U.
Steam Consumption per min. = (*a*) + (*b*) . . .	27·26 lb.
Heat in this Steam per min. (T_C as base) . . .	29,000 B.Th.U.
Equivalent Consumption per min. at 1100 B.Th.U./lb.	26·36 lb.

RESULTS.

Power.	I.H.P.	B.H.P.	Mech. Effy.
H.P. Cylinder	37·2		
L.P. ,,	54·0		
Total	91·2	78·4	86·4

Net Useful H.P. obtained . . . . 76·6

Steam Consumption.

	Per I.H.P.	Per net B.H.P.
Consumption per hour . . . . . lb.	17·9	21·4
Equivalent Consumption per hour at 1100 B.Th.U. per lb. ,,	17·4	20·7
Heat received per minute (T_C as base) . . . . B.Th.U.	318	378

$$\text{Mechanical Efficiency} = \left\{ \frac{\text{B.H.P.}}{\text{I.H.P.} - \text{Air pump H.P.}} \right\}$$

Heat Balance. (per min. 32° F. Base.)

Received :—	B.Th.U.	%	Discharged :—	B.Th.U.	%
			In measured I.H.P.	3,870	12·2
In Cylinder	30,920	97·3	In B.H.P.	3,327	10·5
In Jackets	850	2·7	,, Air Pump H.P.	19	0·1
			,, Friction (by difference)	524	1·6
			,, Condensing Water	23,430	73·8
			,, Heating Feed Water	195	0·6
			,, Condensed Jacket Steam	32	0·1
			,, Hot Well	2,450	7·7
			,, Radiation, etc.	1,793	5·6
Total	31,770	100·0	Total	31,770	100·0

Efficiencies. Thermal Efficiency on I.H.P. (T_C as base) . . . . . 13·3
,, ,, net B.H.P. ,, ... 11·2

	Carnot	Rankine	Dry Steam
Efficiency of Cycle T'_E to T_C	29·8	26·5	25·1
Relative efficiency of Engine (on net B.H.P.)	37·6	42·3	44·6

Remarks.

COMBINED PLANT.

		On net B.H.P.
Fuel Burnt per hour per I.H.P.	1·98 lb.	
,, ,, ,, net B.H.P.	2·35 lb.	
Cost per hour per net B.H.P.	0·46 pence	
Combined Thermal Efficiency, Higher Calorific Value		7·02%
,, ,, ,, Lower Calorific Value		7·2%

Condenser Vacuum.—In order to eliminate the effect of fluctuations in barometer it is usual to give the condenser vacuum corrected to a 30-in. barometer.

This is equal to 30 inches—(actual barometer reading—observed vacuum).

The condenser vacuum really means the difference of pressure between the interior of the condenser and the outside atmosphere expressed in inches of mercury. The absolute pressure in the condenser (inches of mercury)=actual barometer reading—observed condenser vacuum.

This can be converted to pounds per square inch by multiplying by 0·49. Care must be taken to distinguish between the above quantities.

Net Brake Load.—This is the actual load against which the engine is working. The zero load of the brake is the load when no weights are on the hanger.

Net Load=Actual Load+Zero Load—Spring Balance Reading.

Air Pump H.P.—This can only be obtained when separately driven. If the drive is electric this$=\frac{\text{volts} \times \text{amps.}}{746}$.

Circulating Pump H.P.—It is part of the duty of the engine to circulate its own condensing water. If this is done separately the power used must be determined, and debited, along with the power used in a separately driven air pump, against gross B.H.P., to obtain net B.H.P.

Where the flow is under gravity, equivalent power may be estimated as follows :

Work done per minute on water$=W_c \times$ head at condenser in feet. Assuming a pump efficiency of 0·4,

$$\text{H.P. supplied to pump } \frac{W_c \times \text{head in feet}}{0{\cdot}4 \times 33{,}000}.$$

Steam Consumption.—If the leakage is not collected, the steam consumption must be taken as the Hot Well discharge plus the Condensed Jacket Steam. Any leakage collected at the engine must be added to this.

The equivalent consumption at 1100 B.Th.U. per lb. is used for purposes of comparison of engines using steam at different temperatures and pressures, and

$$=\frac{\text{Heat in steam per minute } (T_c \text{ as base})}{1100}$$

The figure 1100 was suggested by the Institution of Civil Engineers because it is the amount of heat per pound of steam in the case of a condensing engine with an admission pressure of 160 lb. per sq. in., and a back pressure of 2 lb. per sq. in., both absolute, these being usual pressures for such condensing engines.

Power.—$\text{I.H.P.} = \dfrac{2\,\text{P.L.A.N.}}{33{,}000}$ where P = M.E.P. (mean)
L = stroke (feet)
A = mean area of piston (sq. in.)
N = R.P.M.

$$\text{B.H.P.} = \frac{\text{Moment applied} \times \text{angle turned through per minute}}{33{,}000}$$

$$= \frac{\text{Net brake load} \times \text{brake rad.} \times 2\pi n}{33{,}000}$$

Net useful B.H.P. = Gross B.H.P. − H.P. in air pump (if separate) − H.P. in circulating water (if separate).

HEAT BALANCE (32° F. as base). All quantities per minute

Heat Received.—In cylinder = hot well discharge × total heat per pound of steam at T_E.

In jackets = jacket steam condensed × total heat per pound of steam at T_E.

Heat Discharged.—In $\text{H.P.} = \text{H.P.} \times \dfrac{33{,}000}{778}$.

In condensing water = weight of condensing water per minute × rise in temperature.

In heating feed water = feed water pumped × rise in temperature = $F_1(T_4 - T_3)$.

In condensed jacket steam = weight of jacket steam condensed × $(T_J - 32°)$.

In hot well = hot well discharge per minute × $(T_H - 32°)$.

EFFICIENCIES (T_C as base)

$$\text{Thermal efficiency on} \begin{Bmatrix} \text{I.H.P.} \\ \text{B.H.P.} \end{Bmatrix} = \frac{\text{Heat equivalent of} \begin{Bmatrix} \text{I.H.P.} \\ \text{B.H.P.} \end{Bmatrix}}{\text{Total heat received per minute}}$$

Efficiency of Carnot Cycle $= \dfrac{T'_E - T_C}{T'_E}$, where temperatures are absolute.

Efficiency of Rankine Cycle, generally from the Mollier diagram (see p. 160).

Efficiency of perfectly jacketed engine (dry steam cycle) (see p. 164), and readily obtained from the Mollier diagram.

COMBINED PLANT

Net B.H.P. = Measured B.H.P. − (air pump H.P. + condenser H.P. + feed pump H.P. + fan H.P.).

Combined thermal efficiency on net B.H.P.

$$= \frac{\text{Heat equivalent of net B.H.P. per minute}}{\text{Heat supplied in fuel per minute}}$$

*GAS ENGINE TRIAL.

Date . . . . . .

OBSERVATIONS.

Description of Plant. —— Gas Engine.
Conditions of Trial.—¾ Full Load. Town Gas.
Duration of Trial, 70 min. Barometer, 29·77 in. Mercury = 14·58 lb./sq. in.

Fuel. Description.—Town Gas.

Analysis by volume :

CH_4	19·2
C_2H_4	2·2
H_2	39·3
CO	17·0
CO_2	5·2
O_2	1·2
N_2	15·9
	100

Calorific Value : Higher	464 B.Th.U./cu. ft.
Lower	413 ,, ,,
Weight of moisture condensed in calorimeter per cu. ft. gas	0·048 lb.
Cost per 1000 cu. ft.	3*s.* 6*d.*
Fuel supplied per min. (by meter)	10·6 cu. ft.
Pressure in supply pipe	0·147 in. mercury
Temperature in supply pipe	65·8° F.
Air supply (by measurement)	6·71 lb. per min.

Exhaust Gas.

Analysis by volume :

	All Strokes	Power Strokes only
CO_2	6·0	11·8
O_2	11·2	5·0
CO	0	0
N_2	82·8	83·2
	100·0	100·0

Temp. of gas leaving Calorimeter, T_E	112·2° F.
Temp. of Air	66·8° F.

Power.

Indicator Spring	250 lb. per in.
Compression Pressure	95 lb. per sq. in.
Max. Indicated Pressure	295 ,, ,,
M.E.P.	74·0 lb./sq. in.
Explosions per min.	84·4
Brake Ld. (net)	172·5 lb.
R.P.M.	206·1

Jacket Water.

Lb. per min.	29·5
Initial Temp.	64·9° F.
Final Temp.	113·2° F.

Exhaust Calorimeter.

Weight of water supplied per min.	24·4 lb.
Weight of water collected per min.	24·5 lb.
Initial Temperature	64·9° F.
Final Temperature	114·7° F.

DEDUCTIONS.

Power.

I.H.P.	28·5
B.H.P.	19·8
Mech. Efficiency	69·4%

Volume of gas supplied per min. at N.T.P.	9·90 cu. ft.
Density	0·0458 lb. per cu. ft.
Weight of gas supplied per min.	0·4535 lb.

Weight of Dry Exhaust Gas per min. (all strokes)	6·23 lb.
Specific Heat	0·01903 B.Th.U. per cu. ft.
,, ,,	0·231 B.Th.U. per lb.
Total Heat Steam at T_E	1074 B.Th.U./lb.
Weight of vapour required to saturate 1 lb. of dry air at T_E	0·065 lb.

Heat Balance. (B.Th.U. per min. T_A as base.)

	B.Th.U.	%				B.Th.U.	%
Received :—			Discharged :—				
In Supply Gas (Higher Value)	4630	100	In I.H.P.			1211	26·2
			,, Friction			371	8·0
			In B.H.P.			840	18·2
			,, Jacket Water			1425	30·6
			,, Exhaust :—				
			Cooling Water	1220			
			Dry Exhaust Gas	65			
			Water Vapour in Exhaust Gas	434			
			Water (liquid) carried over by exh. gas				
					1719	1719	37·2
			In Radiation, etc., and Error			275	6·0
Total	4630		Total			4630	100

Fuel Consumption.

	Per I.H.P.	Per B.H.P.
Volume of gas per hour	20·8	30
Cost per hour	0·87*d.*	1·26*d.*
B.Th.U./min. (lower cal. value)	143·3	206·3

Air Supply.

Volume of air supplied per cu. ft. gas in power stroke	3·99 cu. ft.
Volume of air theoretically required per cu. ft. gas	3·46 cu. ft.
Volume of excess air supplied per cu. ft. gas	0·53 cu. ft.
Excess air supplied (as % of air theoretically required)	15·3%

Efficiencies. Ratio of Compression. 4·93 to 1.

Theoretical Efficiency of Perfect Air Engine on Otto Cycle . . . . . 46·8%

	On Higher Value		On Lower Value	
	I.H.P.	B.H.P.	I.H.P.	B.H.P.
Thermal Efficiency	26·3	18·3	29·6	20·6
Relative Efficiency	56·2	39·1	63·3	44·0

With the engine under consideration two unusual testing appliances are in use : (1) a device for taking samples of the exhaust gases during power strokes only, and (2) a " wet " exhaust gas calorimeter. As already indicated (p. 288) the use of a thermometer in a pocket near the exhaust port is almost valueless except for crude comparative work on a particular engine. Where possible the use of a dry, rather than a wet, calorimeter is to be preferred as being both simpler and more accurate.

The details of the trial are considered in the following notes.

Exhaust Gas Analysis.—This gas engine is governed by the " hit and miss " method, so that instead of a power stroke occurring every two revolutions (as would be the case with governing by varying the quality or quantity of the mixture) the number of power strokes, or explosions, will be less than half the number of revolutions, depending on the load, and separate counters are fitted. From certain cycles therefore, in which no charge of gas has been admitted, the exhaust will consist of air and a trace of residual gases only. If a sample of the exhaust gas is drawn continuously from the exhaust pipe during a trial, it will not represent the true products of combustion, but will be a mixture of the latter with a certain quantity of air. It will not therefore be possible to calculate from the analysis of this gas the quantity of air actually supplied for the purpose of combustion. Hence a sample of the true products of combustion (i.e. eliminating the idle cycles) is required, and to enable this to be obtained a special valve is fitted in the exhaust pipe which is opened by a mechanism only after a power stroke has taken place.

Two samples of exhaust gas are thus collected : (*a*) an average sample of *all* the exhaust gas (i.e. including the idle strokes). The analysis of this sample will be required in calculations of the *heat discharged in the exhaust* ; (*b*) a sample of the true products of combustion, the analysis of which will be required in the calculation of the *air supplied for combustion.*

Exhaust Calorimeter.—The temperature of the gases leaving the engine cylinder is reduced by injecting water. The quantity of water supplied is measured by a venturi meter. The water comes into intimate contact with the hot gases, and a portion of the water is converted into vapour, saturating them and much reducing the temperature. From a knowledge of the final temperature of the exhaust gas, the weight of water carried over as *vapour* can be calculated. Owing to the rush of the exhaust gases through the calorimeter, a certain quantity of water is also probably carried away as *water* in the form of minute droplets. The remainder of the water collects in the bottom of the calorimeter and is syphoned over into a measuring tank.

Quantity of water supplied per minute (1)=Quantity collected per minute (2)+Quantity passing over as *water* per minute (3)+Quantity passing over as *vapour* per minute (4)—Water vapour produced by combustion per minute (5).

All these are known except (3) which can therefore be deduced.
These quantities will be considered under " Deductions."

The following are the relevant details :

Cylinder diameter, 11 in.
Stroke, 19·04 in.
Clearance volume, 460·4 cu. in.
Radius of brake arm, 2·92 ft.

Power.—The indicated horse-power, with this form of governing, must be calculated from the number of explosions per minute, not from half the number of revolutions.

In this case $$\text{I.H.P.} = \frac{\text{P} \cdot l \cdot \text{A} \cdot n}{33{,}000}$$

where P=M.E.P., l=stroke in feet, A=area of piston in square inches, and n is the number of explosions.

Volume of Supply Gas at N.T.P.—The calorific value of gas is given in B.Th.U. per cubic foot at N.T.P. (32° F., with the barometer at 30 in.

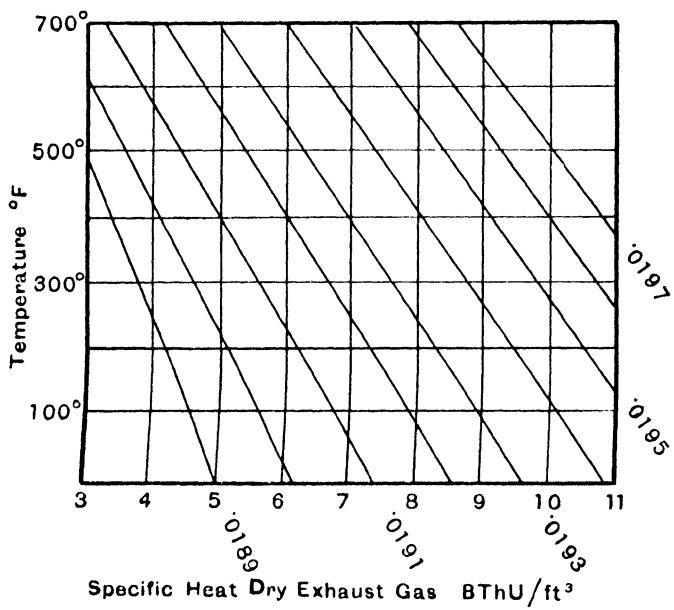


Fig. 169.

Hg). It is therefore necessary to calculate the volume of gas that would have been supplied to the engine under these conditions. The pressure of gas in the main is measured by a water gauge.

If p = actual pressure of gas (in. Hg) = atmospheric pressure (ins. Hg) + gauge pressure of gas (in. Hg).

t = temperature of gas ° F.

volume of gas at N.T.P. = volume of gas as shown by meter

$$\times \frac{p}{30} \times \frac{492}{460} \times t$$

Density of supply gas and of dry exhaust gas (see p. 382).

Weight of dry exhaust gas per minute (see p. 383).

Weight of vapour formed by combustion per minute (see p. 383).

Specific Heat.—This will vary with the temperature and the amount of CO_2 present, as shown on pp. 295, 393.

Except where unusually accurate values are required, the diagram, Fig. 169, may be used, the horizontal ordinate being % CO_2 and the oblique lines specific heat, or the specific heat at N.T.P. may be calculated, as shown on p. 393, and increased by 4 per cent. per 100° F. temperature rise.

Weight of water vapour to saturate 1 *lb. of dry air at exhaust temperature.*

This may be read direct from Fig. 170. See also p. 144.

Heat Balance.—In order to avoid the necessity of taking into account the sensible heat of supply gas, the heat balance is worked out to the temperature of the air as base.

Heat received in supply gas per minute = volume of supply gas at N.T.P. per minute × *higher* calorific value.

Heat discharged in I.H.P. and B.H.P. calculated as in Steam Engine Trial.

Heat discharged in friction is the difference between heat discharged in I.H.P. and B.H.P.

Heat discharged in jacket water = weight of jacket water per minute × rise in temperature.

Heat Discharged in Exhaust.—In cooling water = weight of cooling water collected × rise in temperature.

In dry exhaust gas = weight of dry exhaust gas per minute × specific heat × (temperature of exhaust − temperature of air).

In water vapour in exhaust gas = weight of vapour required to saturate 1 lb. at temperature of exhaust gas × weight of dry exhaust gas per minute × total heat of saturated steam at temperature of exhaust above temperature of air.

In water (liquid) carried over by exhaust gas = weight of water carried over per minute × (temperature of exhaust gas − temperature of air).

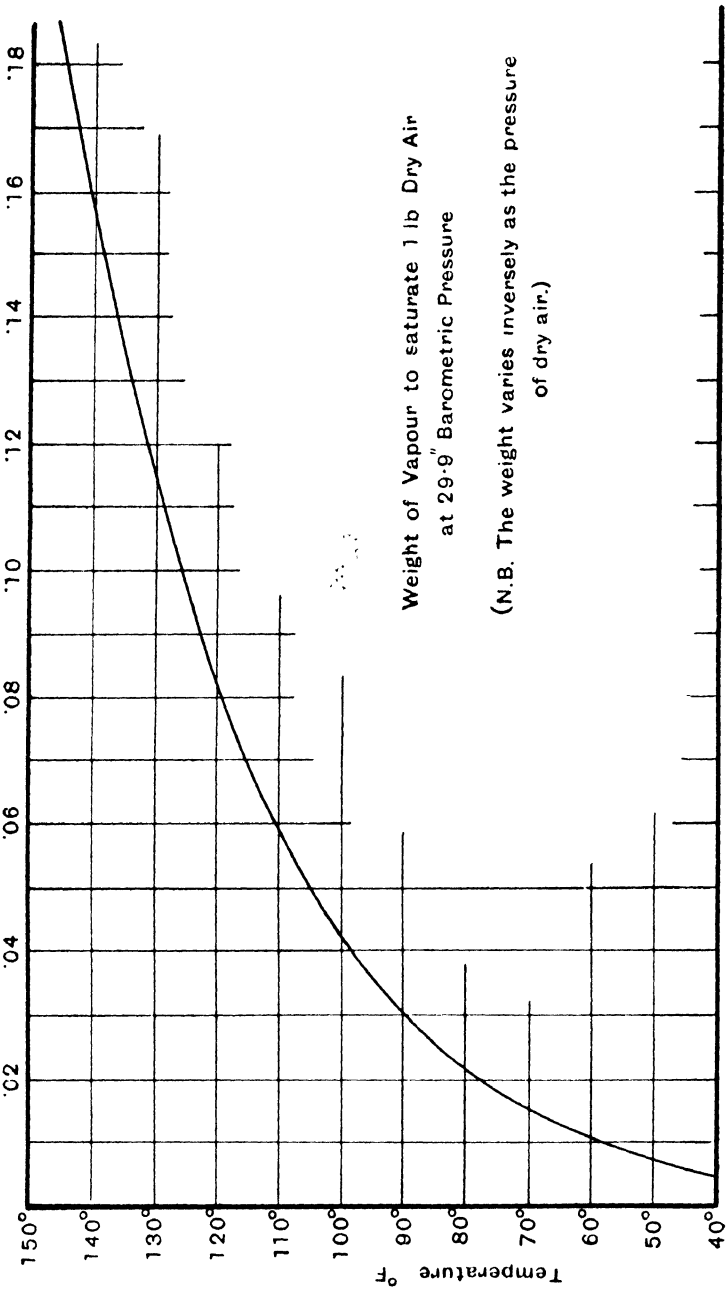

Weight of Vapour to saturate 1 lb Dry Air
at 29·9" Barometric Pressure
(N.B. The weight varies inversely as the pressure
of dry air.)
Temperature °F
150°
140°
130°
120°
110°
100°
90°
80°
70°
60°
50°
40°
·02
·04
·06
·08
·10
·12
·14
·16
·18
Weight of Vapour lbs.

Fig. 170

Air Supply.—For these calculations the analysis of the true products of combustion are required (see p. 415).

$$\text{Thermal efficiency} = \frac{\text{Heat discharged per minute in H.P.}}{\text{Heat supplied per minute in gas}}$$

Theoretical efficiency of perfect air engine on Otto cycle

$$= 1 - \left(\frac{1}{r}\right)^{\gamma - 1}$$

where r = compression ratio

$\gamma = 1{\cdot}40$

*OIL ENGINE TRIAL.

Date .. ..

OBSERVATIONS.

Description of Plant. —— Single Cylinder Vertical Diesel Engine.
Conditions of Trial.—¾ Full Load.
Duration of Trial, 70 min. Barometer, 29 59 in. Mercury=14·5 lb. per sq. in.

Fuel. Description.—

Analysis by weight :			
C	85·8	Calorific Value : Higher . .	20,440 B.Th.U. per lb.
H	12·9	Lower . .	19,215 ,,
Residue . .	—	Cost of fuel per ton	
		,, ,, 40 gallons	£1 4 6
	100·0	Weight of fuel supplied per min.	0·292 lb.
Sp. Gr. . .	0·844	Blast pressure (gauge)	823 lb. per sq. in.

Exhaust Gas.

Analysis by volume :

CO_2	O	CO	N	Total
5·8	12·5	—	81·7	100·0

Temperature of Gas leaving Calr. (T_F) . 687° F.
Temperature of Air (T_A) . . . . . 63° F.

Power.

Indicator Spring	360 lb. per in.		
Compression Pressure	516 lb./sq. in.		
Maximum Indicated Pressure	516 lb./sq. in.		
M.E.P. . . . 79·7 lb./sq. in.	Brake Ld. (net) . .	193·7 lb.	
Explosions per min. . . . —	R.P.M.	252	

Jacket Water.

Lb. per min. .	27·8
Initial Temp..	66° F.
Final Temp. .	119·1° F.

Exhaust Calorimeter.

(Not used)

Weight of water supplied per min.	— lb.
Initial Temperature .	— ° F.
Final Temperature . .	— ° F.

DEDUCTIONS.

Power I.H.P.	52·4
B.H.P.	35·1
Mechanical Efficiency . . .	67·1%

Weight of Dry Exhaust gas per lb. fuel	36·3 lb.
,, ,, ,, min.	10·6 lb.
Density ,, ,, lb. per cu. ft.	0·0824
Specific Heat of Dry Exhaust Gas at (T_F) .	0·01937 B.Th.U./cu. ft.=0·235 B.Th.U. per lb.
Total heat superheated vapour at (T_F) from (T_A)	1342 B.Th.U. per lb.
Weight of water vapour formed by combustion per lb. fuel	1·161 lb.
,, ,, ,, ,, ,, per min.	0·339 lb.

Heat Balance. (B.Th.U. per min. T_A as base.)

Received :	B.Th.U.	%	Discharged :		B.Th.U.	%
In Fuel	5970	100·0	In I.H.P.		2220	37·2
(Higher Cal. Value)	—	—	,, Mechanical losses		730	12·2
			In B.H.P.		1490	25·0
			,, Jacket Water		1476	24·7
			,, Exhaust :—			
			Cooling Water	—		
			Dry Exhaust Gas	1554		
			Water vapour in Exh. Gas	455		
				2009	2009	37·2
			In Radiation, etc.		265	4·4
Total	5970	100·0	Total		5970	100·0

Fuel Consumption.	Per I.H.P.	Per B.H.P.
Weight of fuel per hour	0·335	0·499
Cost per hour	0·292*d.*	0·435*d.*
B.Th.U. per min. (lower cal. val.)	107	160

Air Supply.	
Weight of air supplied per lb. fuel	36·5 lb.
,, ,, theoretically required per lb. fuel	14·4 lb.
Weight of excess air supplied per lb. fuel	22·1 lb.
Excess air supplied (as % of air theoretically required)	153%

Efficiencies. Ratio of Compression.—14 to 1.

Theoretical Efficiency of Perfect Air Engine . . . . . 65·9%

	On Higher Cal. Val.		On Lower Cal. Val.	
	I.H.P.	B.H.P.	I.H.P.	B.H.P.
Thermal Efficiency	37·3	24·9	39·7	26·5
Relative Efficiency	56·7	37·6	60·3	40·2

Oil Engine Trial.—The following are the relevant details of the single cylinder Diesel engine tested :

Cylinder diameter	12 in.
Stroke	18¼ in.
Clearance volume (per cent. cylinder volume)	6·55 per cent.
Radius of brake weight and spring balance .	3·78 ft.

Exhaust Gas Analysis.—As the action of the governor is to vary the quantity of fuel injected at the end of each compression stroke, an average sample drawn continuously from the exhaust pipe can be used to calculate both the heat discharged in the exhaust, and the air supplied for combustion. The procedure is therefore simpler than in the case of the gas engine. It should be noted that the Diesel or C.I. engine is capable of giving satisfactory combustion whatever the amount of excess air, as only the air in the immediate neighbourhood of the droplets of fuel is concerned. In the case of the gas engine, in which the whole of the air and fuel are intimately and uniformly mixed before combustion, a large excess of air reduces the speed of combustion and may even make ignition impossible. In engines which attain complete mixture before combustion, it is therefore necessary to govern either by reducing the supplies of both fuel and air, or by the introduction of idling strokes during which no fuel is admitted.

Heat Balance. Heat Discharged in Exhaust.—This engine normally employs a " dry " exhaust gas calorimeter, the gases and cooling water being kept separate, thus greatly simplifying the calculations, and increasing their accuracy.

The heat discharged in the cooling water refers to the weight of water supplied to this calorimeter × the rise of temperature of the water.

*GAS PRODUCER TRIAL.

Date . . . . . *Name* . . . . .

OBSERVATIONS.

Description of Plant. —— Suction Gas Producer.
Conditions of Trial.—With ..... Gas Engine.
Duration of Trial, 1 hour 26 mins. 35 secs. Barometer, 30·0 in. Mercury=14·7.

FUEL. Description.—Anthracite (cost 60*s.* per ton).

Analysis (by wt.) :—

C	0·83
H	0·03
Moisture	0·04
Ash	0·08
	—

Calorific Value :—
Higher . 14,800 B.Th.U./lb.
Lower . 14,470 B.Th.U./lb.
Wt. of carbon per lb. ash from Producer . — lb.

Wt. of fuel supplied per min. . . . . 0·429 lb.

VAPORISER.

Water supplied per min. . . . . 0·269 lb.
Temperature . . . . . 65° F.

SCRUBBER.

Water supplied per min. . . . . 21·4 lb.
Initial Temp. . . . . 52° F.
Final Temp. . . . . 79·7° F.

POWER GAS.

Analysis (by vol.) :—

CH_4	2·8
C_2H_4	—
H_2	13·25
CO	21·8
CO_2	4·55
O_2	0·21
N_2	57·39
	100

Calorific Value per cu. ft.
Higher . 125 B.Th.U.
Lower . 119 B.Th.U.

Water collected in Calorimeter per cu. ft. of gas. . 0·006 lb.

Final Temperature of gas . . . 61° F.

AIR SUPPLY.

Vol. of air per min. by meter . . . . . 20·5 cu. ft.
Temperature Dry bulb . . . . . 65·4° F.
Wet bulb . . . . . 57·3° F.
Relative humidity 59%

DEDUCTIONS.

Wt. of gas produced per lb. fuel	5·92 lb.
Density of gas	0·0699 lb./cu. ft.
Vol. of gas produced per lb. fuel at N.T.P.	84·7 cu. ft.
Vapour supplied in air per lb. fuel	0·0279 lb.
Vapour formed from constituents of 1 lb. fuel	0·31 lb.
Vapour from combustion of gas produced from 1 lb. fuel	0·805 lb.
Wt. of water decomposed in generator per lb. fuel	0·467 lb.
Wt. of water supplied to vaporiser per lb. fuel	0·627 lb.
Percentage of vaporiser water decomposed in generator	74·5%

HEAT BALANCE :—Per lb. fuel, from Air Temperature.

Received.	B.Th.U.	%
In Fuel (Higher Value)	14,800	99·8
In Vaporiser water	—	—
In Vapour in air	29	0·2
	14,829	100

Discharged.	B.Th.U.	%
In gas (Calorific Value (Higher))	10,580	71·3
In gas (Sensible heat)	−7	—
In Scrubber water	1,381	9·3
In radiation, etc.	2,875	19·4
	14,829	100

Thermal Efficiency of Producer (on Lower Calorific Value) . . . . . 69·7%

Deductions

Weight of gas produced per pound of fuel (see p. 382) . . . (1)

Density of gas (see p. 382) . . . . . (2)

Volume of gas produced per pound of fuel at N.T.P.=(1)÷(2) (3)

Weight of water vapour in air supply.—From the readings of the wet and dry bulb thermometers, and from Glaisher's Tables, which are given in a concise form in Fig. 144, the humidity of the air is found. Let this be h per cent. Then weight of vapour present in 1 cu. ft. of the air

$$=S\times\frac{h}{100}$$

where S=weight of 1 cu. ft. of saturated steam at temperature of air.

Weight of vapour in air per lb. fuel=weight of vapour in 1 cu. ft.× volume of air by meter per lb. fuel.

Weight of vapour formed from the constituents of the fuel per pound of fuel.

This is 9× weight of hydrogen+moisture=9×0·03+0·04=0·31 . (4)

Weight of vapour which would be produced on combustion of the gas per pound of fuel.—This is obtained from the analysis of the gas as follows :

	Per cent. by volume	Relative weight		Weight of water vapour
CH_4	2·8	2·8 ×16=	44·8	36/16 of 44·8=100·7
H_2	13·25	13·25× 2=	26·5	9×26·5=238·4
CO	21·8	21·8 ×28=	610·0	
CO_2	4·55	4·55×44=	200·0	Total . 339·1
O_2	0·21	0·21×32=	6·7	
N_2	57·39	57·39×28=	1607·0	
		Total . .	2495	

or weight of water vapour $=\frac{339\cdot1}{2495}$ lb. per lb. gas

or $\frac{339\cdot1}{2495}\times5\cdot92=0\cdot805$ lb. from (1) . . (5)

Weight of water decomposed in generator per pound of fuel=weight from gas−(weight from air+weight from fuel)=0·805−(0·0279+0·31)− =0·467.

Heat Balance.—This will be worked out per pound of fuel as supplied, taking the temperature of the air as base.

Heat supplied in fuel=higher calorific value.

Heat supplied in vapour in air=weight of vapour in air per pound of fuel× latent heat at temperature of air.

Heat supplied in water to generator=weight of water per pound of fuel × temperature of water − temperature of air.

Heat discharged in gas=volume of dry gas at N.T.P. per pound fuel × higher calorific value.

Heat discharged in scrubber water=weight of scrubber water per pound of fuel × (final temperature of water − initial temperature of water).

Thermal Efficiency of Producer.—This is the ratio of the potential heat produced in the gas to that supplied in the fuel. For this it is usual to take lower calorific values.

$$\text{Efficiency} = \frac{\text{Volume of gas per pound of fuel} \times \text{lower calorific value per cubic foot}}{\text{Lower calorific value of fuel per pound}}$$

*STEAM PLANT TRIAL.

TURBINE.

Date . . . . . .

Description of Plant. —— Seven-stage Impulse Turbine.

Conditions of Trial.—Superheated Steam. Two-thirds Full Load.

Duration, 70 mins. Barometer, 29·37 in. Hg = 14·39 lb./sq. in. Air Temperature, 63·1° F.

		Condenser.		Hot Well.		Oil Cooler.	
Water pumped/min.	26·41 lb.	Cooling Water/min.	905 lb.	Discharge/min.	25·1 lb.	Water/min.	7·2 lb.
Gain in boiler/min.	+0·88 lb.	Initial Temp.	58·6° F.	Temperature	95° F.	Initial Temp.	51·3° F.
Steam supplied/min.	25·53 lb.	Final Temp.	84·9° F.	Air pump Current Amps.	1·9	Final Temp.	80·1° F.
Hot well discharge	25·1 lb.	Head	11 ft.	Air pump Volts	400	Quantity of Oil in sump	200 lb.
Leakage Collected	Nil lb.	Mean Temp. in Condenser	108·6° F.			Oil temp. at start	88·6° F.
Other Leakage	0·43 lb.					Oil temp. at end	88·4° F.

Pressures.	Gauge	Abs.
At Stop Valve (P_E)	171·5	185·9
Exhaust Pipe	−13·2	1·2
Condenser Vacuum		26·9 in. Hg
,, ,, corrected to 30 in.		27·53 in. Hg

Temperatures—° F.	
Stop Valve (T_E')	461
Sat. steam at P_E	375·7
Degrees Superheat	85·3
In Exhaust Pipe (T_C)	108·6

Power.	
Speed	2983
Net Load	51·5 lb.
Brake arm	27 in.

DEDUCTIONS.	H.P.	K.W.
Air Pump power	1·02	0·76
Estimated Circulating Pump power	0·75	0·56
Ratio of Condensing Water to Steam condensed	36	

Total Heat per lb. of Steam at T_E' from 32°	1249
,, ,, ,, ,, from T_C	1172
Steam Consumption per minute	25·31
Heat in this Steam (T_C as base)	29,700
Equivalent Consumption at 1100 B.Th.U. per lb.	27·0

Power.

Measured B.H.P.	65·8
Net useful B.H.P.	64·0
H.P. given to Oil	4·9
Equivalent I.H.P.	70·7

Steam Consumption.

	Per net B.H.P.	Per net K.W.
Measured Consumption per hour	23·72	35·8
Equiv. Consumption at 1100 B.Th.U., per hour	25·3	38·2
Heat used per minute from T_C	464	700

Heat Balance (per min., from 32° F.)

	B.Th.U.	%
Received :—		
In Steam	31,600	—
Total	31,600	100

Discharged :—	B.Th.U.	%
In B.H.P.	2,792	8·8
In Oil	207	0·7
In Condensing Water	23,800	75·3
In Hot well	1,581	5·0
In Radiation, error, etc.	3,220	10·2
Total	31,600	100

Efficiencies (T_C as base).

Mechanical Efficiency	93·1
Turbine Efficiency	45
Overall Thermal Efficiency (on Net B.H.P.)	9·1
Efficiency of Rankine Cycle between T_E' and T_C	28·9
Efficiency relative to Rankine Cycle (on Net B.H.P.)	31·5%

Vacuum.

Pressure corresponding to Condenser Temperature	2·49 in. Hg
Pressure observed in Condenser 1·21 lb./sq. in. (abs.)	= 2·47 in. Hg
Pressure of Air in Condenser	—
% Vacuum on 30 in. Barometer	91·8

Turbine Trial.—This is the test of a small seven-stage impulse turbine driving a swinging-field electrical dynamometer, the B.H.P. being determined from the net brake load, the brake arm radius, and the revolutions per minute. The aim pump is separately driven.

Most of the heat calculations are the same as for the engine trial.

Power.—Net useful B.H.P.=measured B.H.P.—air pump H.P.—condenser H.P.

$$\text{H.P. given to oil} = \text{heat given to oil} \times \frac{778}{33{,}000} = W_0(T_4 - T_3) \times \frac{778}{33{,}000}$$

provided that temperature of oil itself is the same at beginning and end of trial. This should be secured by regulating the quantity of the cooling water. Otherwise, if there is a difference of temperature, the quantity of heat given to oil=weight of oil × specific heat × rise in temperature.

Equivalent I.H.P.=measured B.H.P.+H.P. given to oil.

Efficiencies. (T_C as base.)

$$\text{Mechanical efficiency} = \frac{\text{Measured B.H.P.}}{\text{Equivalent I.H.P.}}$$

Turbine Efficiency.—The heat actually used in turbine=H_6 when

H_6=heat received—heat discharged in condenser and hot well.

$$\text{Turbine efficiency} = \frac{\text{Measured B.H.P.}}{H_6}$$

$$\text{Overall thermal efficiency on B.H.P.} = \frac{\text{Heat in net B.H.P.}}{\text{Heat received (from } T_C)}$$

NOTE.—It is again emphasized that the figures given in the above tables are to illustrate the method of calculation only, the duration of the trials having been too short to permit a high degree of accuracy to be obtained in certain of the measurements.

EXAMPLES ON CHAPTER XVIII

1. In a boiler trial 3600 lb. of coal were consumed in 24 hours. The weight of water evaporated was 28,800 lb., mean steam pressure by gauge 95 lb. The coal contained 3 per cent. of moisture and 3·9 per cent. of ash by analysis. Determine the efficiency of the boiler and the equivalent evaporation from and at 212° F., (1) per pound of dry coal, (2) per pound of combustible. Feed temperature 95° F., total heat of 1 lb. of steam at 110 lb. per sq. in. abs.=1184 B.Th.U., calorific value of 1 lb. of the *dry* coal 13,000 B.Th.U.

2. The following data were obtained during a boiler trial :

Feed water per hour	10,115 lb.
Temperature of feed to boiler	174° F.
Steam pressure (pounds per square inch absolute)	170
Moisture in 1 lb. of steam	0·019 lb.
Coal fired per hour	1074 lb.
Dry coal per hour	1054 lb.
Calorific value of dried coal	14,000 B.Th.U. per pound

Analysis of dried coal	C 88%, H_2 3·6%, ash 3·6%, other matters 4·8%
Calorific value of ashes	900 B.Th.U. per pound
Weight of ashes per hour	38 lb.
Analysis of flue gases by volume	CO_2 10·9%, CO 1·0%, O_2 7·1%, N_2 81·0%
Temperature of flue gases leaving boiler	600° F.
Temperature of air in boiler house	60° F.

Draw up a heat account for this boiler.

3. In a trial of a jacketed engine the steam chest pressure was 145 lb. per sq. in. abs., the cylinder feed was 29 lb. per minute, and the jacket feed was 3·2 lb. per minute, the feed and jacket steam being 5 per cent. wet. The circulating water was 550 lb. per minute, inlet temperature 55° F., outlet temperature 104·3° F. The feed temperature was 125° F., and the I.H.P. 110. Draw up a heat balance and find also the thermal efficiency.

4. The following data were obtained from a trial on a steam engine :

Air pump discharge per hour	6417 lb.
Weight of steam used in jackets per hour	1079 lb.
Temperature of jacket drainage	352° F.
Pressure of steam at boiler side of stop valve (pounds per square inch absolute)	139·0
Moisture in steam at boiler side of stop valve (dry saturated)	nil
Temperature of exhaust steam	119° F.
Indicated horse-power	494·3
Circulating water per hour	87,300 lb.
Inlet temperature of circulating water	33·2° F.
Outlet ,, ,, ,,	91·6° F.

Draw up a heat account for this engine and in addition calculate—
(*a*) Steam consumption per I.H.P. hour.
(*b*) Thermal efficiency of the engine.
(*c*) Heat theoretically required per minute per I.H.P. by an engine working on the Rankine cycle between the above temperatures.
(*d*) Efficiency ratio or coefficient of performance.

5. The following data were obtained from trials run on three different boilers A, B, and C, the same coal of calorific value 14,000 B.Th.U. per pound being used in each trial. Calculate for each boiler, (*a*) the equivalent evaporation per pound of coal from and at 212° F., (*b*) the efficiency of the boiler.

	A	B	C
Steam pressure (pounds per square inch absolute)	140	180	160
Temperature of saturation, ° F.	353·1	373·1	363·6
Feed temperature, ° F.	50	65	100
Dryness of steam, per cent.	98	98·5	—
Temperature of superheated steam, ° F.	—	—	500
Water evaporated per pound of coal under working conditions	9·2	8·8	8·5

For boiler C take the specific heat of steam to be 0·5.

6. Draw up an approximate heat balance from the following particulars obtained from a trial on a Diesel oil engine :

Duration of trial	one hour
Average speed	157 r.p.m.
Indicated horse-power	126·6
Brake horse-power	87·2
Total oil consumed	40·58 lb.
Calorific value of oil	19,300 B.Th.U. per lb.

Analysis of oil	C=85% ; H=13·5% ; incombustible, 1·5%
Analysis of exhaust gases by volume	CO_2=4·3% ; CO=nil ; O_2=14·8% ; N_2=80·9%
Temperature of exhaust	492° F.
Temperature of engine room	60° F.
Cooling water per minute	43·0 lb.
Inlet temperature	49° F.
Outlet temperature	126·5° F.

7. The following particulars were obtained during a trial on a 25 B.H.P. Campbell gas engine :

Duration of trial, one hour ; total revolutions of engine=13,602 ; total number of explosions, 4620 ; net load on brake, 277 lb. ; mean effective pressure on piston, 106 lb. per sq. in. ; gas consumption as registered by meter, 455·5 cu. ft. ; lower calorific value=592 B.Th.U. per cu. ft. at N.T.P. ; pressure of gas, 771 mm. ; temperature of gas passing through meter, 15° C. ; diameter of cylinder, 9½ in. ; stroke, 19 in. ; effective circumference of brake, 12·8 ft. Work out (*a*) the indicated horse-power ; (*b*) brake horse-power ; (*c*) mechanical efficiency; (*d*) thermal efficiency ; (*e*) overall efficiency.

8. In a test with the above engine working on suction gas the following data were obtained :

Duration of trial, 6 hours ; average speed of engine, 224 r.p.m. ; average explosions per minute, 96·2 ; mean effective pressure, 75·4 lb. per sq. in. ; effective load on brake, 252 lb. ; coal consumption, 20·3 lb. per hour ; calorific value of coal (lower value), 15,020 B.Th.U. per lb. Work out (*a*) I.H.P. ; (*b*) mechanical efficiency ; (*c*) overall efficiency ; (*d*) thermal efficiency of producer.

9. The following particulars were obtained from a trial of a four-stroke cycle oil engine :

Duration of trial, 40 minutes ; oil used, 12·80 lb. ; total revolutions, 8142 ; jacket water, 738 lb. ; rise of temperature of jacket water, 74° F. ; mean effective pressure in cylinder, 96 lb. per sq. in. ; torque due to brake load, 786 lb.-ft. ; lower calorific value of oil, 17,000 B.Th.U. per lb. ; area of piston, 113 sq. in. ; stroke, 18¼ in.

Find (*a*) the indicated and brake horse-powers ; (*b*) the oil used per I.H.P. and per B.H.P. per hour ; (*c*) the heat converted into indicated work per minute ; (*d*) the heat rejected in the jacket water per minute ; (*e*) the heat lost in friction, exhaust gases, etc., per minute.

10. The following particulars were obtained from trials of a four-stroke cycle oil engine : Cylinder diameter, 12 in. ; stroke, 18¼ in. ; diameter of brake wheel, 9 ft. 0¾ in. Draw to a base of B.H.P., curves showing mechanical efficiency and oil used per B.H.P. hour.

Average revolutions per minute	199	202	203	204	204	205
Brake load (pounds)	225	195	156	113	60	0
Mean effective pressure (pounds per square inch)	107	97	87	74	60	42
Oil per hour	20·7	16·3	13·5	10·7	10·0	9·5

11. The following data were obtained during a trial of a gas engine : Duration of trial, one hour.

Total revolutions of engine	13,600
Total number of explosions	6,240
Net load on brake	350 lb.
Mean effective pressure on piston	110 lb. per sq. in.
Gas consumption as registered by meter	625 cu. ft.
Lower calorific value of gas at N.T.P.	550 B.Th.U. per cu. ft.
Pressure of gas passing meter	770 mm.
Temperature of gas passing meter	17° C.

Diameter of cylinder, 9½ in.; stroke, 19 in.; effective circumference of brake wheel, 12·8 ft.; clearance volume, 272 cu. in. Estimate: (*a*) indicated horse-power; (*b*) brake horse-power; (*c*) mechanical efficiency; (*d*) thermal efficiency (on I.H.P.); (*e*) overall efficiency; (*f*) efficiency ratio referred to the "Air Standard" cycle engine.

12. The following data were obtained from a gas engine trial: Brake horse-power, 20·9; gas consumption per hour, 331 cu. ft. at N.T.P.; lower calorific value of gas, 561 B.Th.U. per cu. ft. at N.T.P.; higher calorific value, 622 B.Th.U. per cu. ft. at N.T.P.; volumetric analysis of the gas, per cent., CH_4 33·73, C_2H_4 4·74, H_2 41·29, CO 7·13, N_2 10·22, CO_2 2·62, O_2 0·27. Temperature of air in engine-room, 48·4° F.; difference between wet and dry bulb thermometers, 3° F.; volume of air supplied per hour measured by anemometer, 3209 cu. ft.; atmospheric pressure, 14·58 lb. per sq. in.

Estimate:

(*a*) Overall efficiency of the engine.
(*b*) Heat carried into engine by the air supply per hour.
(*c*) Heat carried into engine by the gas supply per hour.

13. The analysis by weight of a sample of crude petroleum used in an oil engine gives C 85 per cent., H_2 13·5 per cent., incombustible matter 1·5 per cent. The volumetric analysis of the exhaust gases gives CO_2 7 per cent., O_2 11·3 per cent., and N_2 81·7 per cent. The engine uses 0·333 lb. of oil per I.H.P. hour, and 14·8 lb. of water per I.H.P. per hour pass through the jacket. The rise in temperature of the jacket water is 52° C. The temperature of the air at the end of suction is 30° C., and the temperature of the exhaust is 384° C. The lower calorific value of the oil is 10,720 C.H.U. per lb., and the specific heat of the exhaust gases 0·25.

From the above data draw up a heat balance for the engine.

APPENDIX

PROPERTIES OF SATURATED STEAM

Calculated from Professor H. L. Callendar's " Characteristic Equations," and compiled from the " Callendar Steam Tables," published by Mr. Edward Arnold, by whose permission they are reproduced.

Pressure lb./in.^{2} abs. p	Temperature ° F. t	Sp. vol. ft.^{3}/lb. v	Heat in B.Th.U.			Entropy.		
			Sensible h	Latent L	Total H	Water ϕ_w	Evaporation ϕ_e	Total ϕ
0·1	34·86	2940·0	2·86	1068·19	1071·05	0·0057	2·1605	2·1662
0·2	53·04	1524·0	21·01	1058·65	1079·66	0·0417	2·0651	2·1068
0·3	64·38	1038·0	32·30	1052·69	1084·99	0·0635	2·0092	2·0727
0·4	72·79	790·7	40·69	1048·26	1088·95	0·0794	1·9688	2·0482
0·5	79·54	640·5	47·42	1044·70	1092·12	0·0922	1·9377	2·0299
0·6	85·17	539·1	53·03	1041·71	1094·74	0·1025	1·9123	2·0148
0·7	90·05	466·2	57·89	1039·10	1096·99	0·1113	1·8905	2·0018
0·8	94·36	411·1	62·19	1036·80	1098·99	0·1191	1·8715	1·9906
0·9	98·30	367·9	66·09	1034·75	1100·84	0·1262	1·8548	1·9810
1·0	101·74	333·1	69·54	1032·89	1102·43	0·1323	1·8401	1·9724
1·2	107·91	280·6	75·71	1029·56	1105·27	0·1432	1·8143	1·9575
1·4	113·25	242·7	81·06	1026·68	1107·74	0·1526	1·7923	1·9449
1·6	117·98	214·0	85·77	1024·13	1109·90	0·1607	1·7732	1·9339
1·8	122·23	191·6	90·03	1021·82	1111·85	0·1683	1·7562	1·9245
2·0	126·08	173·5	93·89	1019·72	1113·61	0·1747	1·7412	1·9159
2·2	129·64	158·7	97·43	1017·77	1115·20	0·1807	1·7273	1·9080
2·4	132·92	146·4	100·68	1015·97	1116·65	0·1863	1·7148	1·9011
2·6	135·94	135·6	103·72	1014·33	1118·05	0·1914	1·7033	1·8947
2·8	138·82	126·5	106·58	1012·77	1119·35	0·1962	1·6926	1·8888
3·0	141·50	118·6	109·27	1011·29	1120·56	0·2007	1·6826	1·8833
3·2	144·04	111·6	111·80	1009·89	1121·69	0·2049	1·6732	1·8781
3·4	146·44	105·4	114·21	1008·56	1122·77	0·2089	1·6642	1·8731
3·6	148·73	99·93	116·50	1007·28	1123·78	0·2127	1·6558	1·8685
3·8	150·92	95·00	118·70	1006·07	1124·77	0·2162	1·6479	1·8641
4·0	153·01	90·54	120·79	1004·90	1125·69	0·2197	1·6043	1·8600
4·5	157·83	81·11	125·63	1002·18	1127·81	0·2275	1·6230	1·8505
5·0	162·28	73·44	130·07	999·68	1129·75	0·2346	1·6075	1·8421
5·5	166·32	67·19	134·13	997·38	1131·51	0·2411	1·5935	1·8346
6·0	170·09	61·91	137·90	995·25	1133·15	0·2472	1·5805	1·8277
6·5	173·60	57·44	141·41	993·26	1134·67	0·2528	1·5685	1·8213
7·0	176·88	53·59	144·70	991·37	1136·07	0·2579	1·5574	1·8153
7·5	179·98	50·24	147·80	989·58	1137·38	0·2628	1·5472	1·8100
8·0	182·91	47·30	150·76	987·87	1138·63	0·2673	1·5376	1·8049
8·5	185·68	44·69	153·55	986·27	1139·82	0·2716	1·5285	1·8001
9·0	188·31	42·36	156·20	984·74	1140·94	0·2757	1·5199	1·7956
9·5	190·84	40·27	158·71	983·28	1141·99	0·2797	1·5118	1·7915
10·0	193·25	38·39	161·12	981·90	1143·02	0·2833	1·5041	1·7874

PROPERTIES OF SATURATED STEAM—*continued.*

Pressure lb./in.² abs. p	Temperature ° F. t	Sp. vol. ft.³/lb. v	Heat in B.Th.U.			Entropy.		
			Sensible h	Latent L	Total H	Water ϕ_w	Evaporation ϕ_e	Total ϕ
11·0	197·78	35·11	165·69	979·20	1144·89	0·2903	1·4896	1·7799
12	201·99	32·37	169·94	976·70	1146·64	0·2967	1·4764	1·7731
13	205·92	30·03	173·88	974·36	1148·24	0·3027	1·4642	1·7669
14	209·59	28·02	177·59	972·21	1149·80	0·3081	1·4530	1·7611
15	213·05	26·27	181·05	970·11	1151·16	0·3134	1·4423	1·7557
16	216·34	24·73	184·40	968·08	1152·48	0·3184	1·4322	1·7506
17	219·46	23·37	187·55	966·17	1153·72	0·3230	1·4228	1·7458
18	222·42	22·16	190·53	964·35	1154·88	0·3274	1·4140	1·7414
19	225·24	21·06	193·38	962·62	1156·00	0·3316	1·4057	1·7373
20	227·97	20·08	196·11	960·97	1157·08	0·3356	1·3977	1·7333
22	233·08	18·37	201·30	957·76	1159·06	0·3430	1·3828	1·7258
24	237·81	16·93	206·11	954·77	1160·88	0·3499	1·3690	1·7189
26	242·24	15·71	210·56	951·97	1162·53	0·3563	1·3563	1·7126
28	246·40	14·66	214·78	949·36	1164·14	0·3622	1·3447	1·7069
30	250·30	13·74	218·76	946·82	1165·58	0·3679	1·3337	1·7016
32	254·03	12·94	222·53	944·41	1166·94	0·3732	1·3234	1·6966
34	257·56	12·22	226·13	942·11	1168·24	0·3782	1·3137	1·6919
36	260·91	11·59	229·53	939·91	1169·44	0·3830	1·3044	1·6874
38	264·13	11·02	232·81	937·80	1170·61	0·3874	1·2957	1·6831
40	267·21	10·50	235·94	935·77	1171·71	0·3918	1·2874	1·6792
42	270·16	10·03	238·99	933·77	1172·76	0·3959	1·2795	1·6754
44	273·00	9·603	241·90	931·84	1173·74	0·3999	1·2720	1·6719
46	275·74	9·212	244·70	930·00	1174·70	0·4037	1·2648	1·6685
48	278·38	8·853	247·41	928·21	1175·62	0·4073	1·2578	1·6651
50	280·94	8·520	250·00	926·48	1176·48	0·4109	1·2511	1·6620
52	283·41	8·213	252·55	924·80	1177·35	0·4142	1·2447	1·6589
54	285·82	7·928	255·00	923·15	1178·15	0·4176	1·2385	1·6561
56	288·16	7·663	257·40	921·51	1178·91	0·4208	1·2325	1·6533
58	290·42	7·415	259·74	919·93	1179·67	0·4239	1·2267	1·6506
60	292·61	7·184	262·00	918·39	1180·39	0·4269	1·2210	1·6479
62	294·76	6·966	264·20	916·88	1181·08	0·4298	1·2155	1·6453
64	296·85	6·761	266·36	915·43	1181·79	0·4327	1·2102	1·6429
66	298·88	6·571	278·42	914·02	1182·44	0·4354	1·2051	1·6405
68	300·88	6·388	270·47	912·60	1183·07	0·4381	1·2001	1·6382
70	302·83	6·218	272·49	911·21	1183·70	0·4407	1·1952	1·6359
72	304·72	6·056	274·43	909·86	1184·29	0·4433	1·1904	1·6337
74	306·60	5·902	276·37	908·53	1184·90	0·4458	1·1857	1·6315
76	308·40	5·757	278·24	907·22	1185·46	0·4482	1·1812	1·6294
78	310·19	5·618	280·11	905·92	1186·03	0·4506	1·1769	1·6275
80	311·93	5·487	281·90	904·66	1186·56	0·4530	1·1726	1·6256
82	313·65	5·362	283·66	903·42	1187·08	0·4553	1·1684	1·6237
84	315·32	5·241	285·40	902·19	1187·59	0·4575	1·1643	1·6218
86	316·98	5·127	287·12	900·99	1188·11	0·4597	1·1603	1·6200
88	318·60	5·018	288·80	899·80	1188·60	0·4619	1·1564	1·6183
90	320·16	4·913	290·43	898·63	1189·06	0·4640	1·1525	1·6165
92	321·73	4·813	292·05	897·48	1189·53	0·4661	1·1487	1·6148
94	323·28	4·717	293·67	896·33	1190·00	0·4681	1·1450	1·6131
96	324·79	4·624	295·24	895·19	1190·43	0·4701	1·1414	1·6115
98	326·26	4·535	296·77	894·09	1190·86	0·4720	1·1378	1·6098
100	327·70	4·451	298·27	893·01	1191·28	0·4739	1·1343	1·6082
105	331·25	4·251	301·98	890·31	1192·29	0·4786	1·1258	1·6044

PROPERTIES OF SATURATED STEAM—*continued.*

Pressure lb./in.² abs. p	Temperature ° F. t	Sp. vol. ft.³/lb. v	Heat in B.Th.U.			Entropy.		
			Sensible h	Latent L	Total H	Water ϕ_w	Evaporation ϕ_e	Total ϕ
110	334·67	4·070	305·55	887·72	1193·27	0·4831	1·1176	1·6007
115	337·96	3·903	309·00	885·19	1194·19	0·4874	1·1098	1·5972
120	341·15	3·751	312·34	882·72	1195·06	0·4915	1·1023	1·5938
125	344·25	3·609	315·60	880·31	1195·91	0·4955	1·0951	1·5906
130	347·23	3·479	318·72	877·97	1196·69	0·4994	1·0882	1·5876
135	350·13	3·358	321·79	875·70	1197·49	0·5032	1·0815	1·5847
140	352·96	3·245	324·76	873·48	1198·24	0·5068	1·0750	1·5818
145	355·69	3·140	327·66	871·32	1198·98	0·5103	1·0687	1·5790
150	358·36	3·041	330·46	869·22	1199·68	0·5138	1·0627	1·5765
155	360·95	2·949	333·22	867·13	1200·35	0·5172	1·0568	1·5740
160	363·48	2·862	335·89	865·10	1200·99	0·5204	1·0511	1·5715
165	365·97	2·781	338·51	863·10	1201·61	0·5236	1·0455	1·5691
170	368·39	2·703	341·10	861·12	1202·22	0·5266	1·0400	1·5666
175	370·74	2·631	343·61	859·18	1202·79	0·5296	1·0347	1·5643
180	373·06	2·562	346·08	857·27	1203·35	0·5326	1·0294	1·5620
185	375·33	2·496	348·51	855·38	1203·89	0·5355	1·0243	1·5598
190	377·55	2·435	350·90	853·54	1204·44	0·5383	1·0194	1·5577
195	379·72	2·376	353·22	851·72	1204·94	0·5411	1·0147	1·5558
200	381·83	2·320	355·48	849·96	1205·44	0·5437	1·0101	1·5538
210	385·98	2·216	359·87	846·47	1206·36	0·5490	1·0012	1·5502
220	389·97	2·120	364·18	843·08	1207·26	0·5540	0·9925	1·5465
230	393·84	2·034	368·35	839·79	1208·14	0·5588	0·9841	1·5429
240	397·56	1·954	372·38	836·57	1208·95	0·5635	0·9760	1·5395
250	401·18	1·880	376·31	833·42	1209·73	0·5680	0·9682	1·5362
260	404·67	1·811	380·11	830·36	1210·47	0·5724	0·9608	1·5332
270	408·08	1·748	383·81	827·37	1211·18	0·5767	0·9536	1·5303
280	411·39	1·689	387·42	824·43	1211·85	0·5808	0·9466	1·5274
290	414·63	1·634	390·95	821·55	1212·50	0·5848	0·9398	1·5246
300	417·78	1·583	394·40	818·73	1213·13	0·5887	0·9332	1·5219
310	420·84	1·534	397·76	815·96	1213·72	0·5924	0·9268	1·5192
320	423·82	1·489	401·08	813·24	1214·32	0·5961	0·9206	1·5167
330	426·74	1·446	404·31	810·57	1214·88	0·5997	0·9145	1·5142
340	429·60	1·406	407·44	807·97	1215·41	0·6023	0·9086	1·5109
350	432·41	1·368	410·55	805·39	1215·94	0·6067	0·9029	1·5096
400	445·55	1·206	425·19	793·13	1218·32	0·6228	0·8763	1·4991
450	457·55	1·079	438·67	781·68	1220·35	0·6374	0·8523	1·4897
500	468·63	0·977	451·19	770·96	1222·15	0·6508	0·8306	1·4814

ANSWERS

Chapter II

1. 8·47 cu. ft. **2.** 524° F.; 26,170 ft.-lb.; 116·86 B.Th.U.
3. 1824 ft.-lb. **4.** 12,440 ft.-lb. **5.** 15,255 ft.-lb.
6. (*a*) 139·9 lb. per sq. in.; (*b*) 23,900 ft.-lb.
7. (*a*) 3540° F.; heat expended = 39,600 ft.-lb.; work done = 10,800 ft.-lb.
(*b*) 1728° F.; work done = loss of internal energy = 26,093 ft.-lb.
8. 347° F.; 2913° F. **9.** 56·54 lb. per sq. in.; 634° F.
10. 667° F.; 126,170 ft.-lb.
11. (*a*) 104 cu. ft.; 778,500 ft.-lb.; (*b*) 57·4 cu. ft.; 26,093 ft.-lb.
12. $-0{\cdot}54p$; $+0{\cdot}32p$; −150 B.Th.U. per second.
13. (*a*) 6600 ft.-lb.; (*b*) 4750 ft.-lb.; (*c*) 495° F.
14. 27·7 lb. per sq. in.; 85·23 B.Th.U.
15. 2704 lb. per sq. ft.; 39,720 ft.-lb.; 51·05 B.Th.U.

Chapter III

1. 0·588; 9·2 : 1. **2.** See Fig. 122.

Chapter IV

1. 0 0947 units. **2.** 0 0947 units. **3.** 0·0348 units.
4. 1 693 units. **5.** 4·33 cu. ft.

Chapter V

(*a*) 0·860. (*b*) 0 688. (*c*) 0 912. (*d*) 23 lb. per sq. in. abs.
(*e*) $q = 0{\cdot}941$; $q = 0{\cdot}976$; 13° s; $q = 0{\cdot}979$. (*f*) $q = 1$; $s = 213°$ F.
(*g*) 25° s; 60 B.Th.U. (*h*) $q = 0{\cdot}997$; $s = 23°$ F.

Chapter VII

1. (*a*) 0·534; (*b*) 0 449. **2.** (*a*) 0·562; (*b*) 0·172. **3.** 0·355.
4. 0·184. **5.** 28·54 lb. per sq. in. **6.** 86·0 lb. per sq. in.; 0·581.
7. 0·553. **8.** 98·1 lb. per sq. in.; 60·6 per cent.

Chapter VIII

1. 49·2 per cent. **2.** 63·6. **3.** 48·3 lb.; −113° F.
4. (*a*) 100,655 ft.-lb.; (*b*) 83,410 ft.-lb.; (*c*) 78,050 ft.-lb., in first cooler 18·28 C.H.U., in second cooler 18·28 C.H.U.
5. (*a*) 85,690 ft.-lb.; (*b*) 76,840 ft.-lb.; (*c*) 74,166 ft.-lb., in first and second coolers, 10·2 C.H.U.
6. 9·12 in. diameter, 2 ft. 6 in. stroke.
7. 6·16 H.P.; 0·0923 cu. ft.; 0·0242 cu. ft.; 0·00636 cu. ft.
8. 116 r.p.m.; 1·634 H.P.

Chapter IX

1. (*a*) 108·5 B.Th.U.; (*b*) 98·3 B.Th.U.
2. Dryness fraction 0·666; $n = 1{\cdot}10$.
3. (*a*) 79·02 B.Th.U.; 7·5 per cent.; (*b*) 63·15 B.Th.U.; 7·1 per cent.

4. 0·420 lb.; 0·900. **5.** 0·977. **6.** (1) $q=0{\cdot}956$; (2) 2·98° superheat.
7. 0·467. **8.** 0·968. **9.** 0·920. **10.** 91·9 per cent.
11. 0·987. **12.** 0·62 lb.; 87 per cent. **13.** 74·05 per cent.
14. 90 lb. per sq. in. abs.
15. Temperature = 414° F., superheat 201° F.; gain of entropy 0·276 units.
16. Before expansion temperature = 588° F., superheat = 260° F. After expansion, temperature = 286° F., superheat = 46° F. **17.** 0·237 units.
18. Temperature = 247·9° F., superheat = 42° F.; 0·2952 units.
19. Temperature = 298·1° F., superheat = 92·2° F.; 0·30 units.

Chapter X

1. 242 B.Th.U.; $\eta = 22{\cdot}15$ per cent. **2.** 121·2 B.Th.U.
3. (*a*) 19,958 ft.-lb.; (*b*) 17,260 ft.-lb.; (*c*) 18,150 ft.-lb.; (*d*) 19,150 ft.-lb.
4. 255·3 B.Th.U.; 23·6 per cent. **5.** 221·5 B.Th.U.; 23·3 per cent.
6. (*a*) 0·483; (*b*) 0·428. **7.** 17·63 lb. **8.** 9·92 lb.; 0·874.
9. 596° F.; 26·2 per cent.; 357·48 B.Th.U.
10. Stirling, 49·0 per cent.; steam, 16·9 per cent.

$$\frac{\text{Diameter of Stirling}}{\text{Diameter of steam}} = \frac{1{\cdot}11}{1}$$

11. 300·9 B.Th.U.; 0·272.
12. (*a*) 8·82 lb.; (*b*) 9·6 lb. (including jacket steam).
13. 27·6 per cent.; without feed heating, 26·4 per cent. **14.** 26·8 per cent.
15.

	Dryness at cut-off per cent.	Work, B.Th.U./lb.	Heat rejected, B.Th.U./lb.	η per cent.	Heat received to cut-off B.Th.U./lb.	Heat received during expansion, B.Th.U./lb.
(1)	88·79	190·2	832·2	20·03	991·5	20·9
(2)	80	188·2	832·2	18·4	900·5	119·2
(3)	68	174·6	832·2	17·35	792·5	214·3

Chapter XI

1. 63 per cent. **2.** 1858·5. **3.** 7310 lb.; I.H.P. = 974.
4. (*a*) 0·807, 5310; (*b*) 0·804, 5400; (*c*) 0·826, 4770. **5.** 0·841. **6.** 12.
7. (*a*) 13·50 per cent.; (*b*) 13·74 per cent. **8.** 80·65 lb. per sq. in.; 85·1.
9. 8·12 in. **10.** 2·60. **11.** 5; 15 lb. per sq. in. abs.; $\frac{8}{15}$; $\frac{\text{H.P.}}{\text{L.P.}} = \frac{1}{1{\cdot}04}$.
12. 3·702; 18·58 lb. per sq. in. abs.; 0·480; $\frac{\text{H.P.}}{\text{L.P.}} = \frac{1}{1{\cdot}02}$.
13. L.P. = 31·42 in.; H.P. = 16·8 in.; 0·428 stroke; $\frac{\text{H.P.}}{\text{L.P.}} = \frac{1}{1{\cdot}229}$.
14. Equal. **15.** 31·6 in., 54·6 in., 86·5 in., stroke 4 ft.
16. 64·3 lb. per sq. in.; 19·09 lb. per sq. in.

Chapter XII

1. 1260 ft. per second; 0·951.
2. Area of throat 0·329 sq. in., $q = 0{\cdot}96$; area of discharge end 3·069 sq. in., $q = 0{\cdot}802$.
3. Area of throat 0·367 sq. in., of discharge end 1·066 sq. in. Condition of steam, in throat superheat = 24° F., at discharge end $q = 0{\cdot}90$.
4. 5·8 lb.; steam orifice 1·7 sq. in.; discharge orifice 0·0397 sq. in.; feed temperature 192° F.
5. Exit diameter 0·88 in., length, 3 in.; for a 12° cone angle, number of nozzles 9.
6. 10·03 sq. in.

Chapter XIII

1. 29·5° ; 162,660 ft.-lb. ; 1575 ft. per second ; 0·860.
2. 36° (nearly) ; 177,690 ft.-lb. ; 1690 ft. per second ; 0·883.
3. 28·8° ; 0·789 ; 159·3 H.P.
4. 29·5° ; 154,540 ft.-lb. ; 1345 ft. per second ; 0·860.
5. Moving blades, inlet angle=exit angle=24·5°. Stationary blades, first set, inlet angle=34°, exit angle=17·3° ; second set, inlet angle=43°, exit angle=7°.
6. 116·1 H.P. **7.** 0·375 in. diameter ; 36° 3′ ; 5·823 H.P.
8. θ_i=40°, 41·3 H.P., 0·615 lb., 8·52 B.Th.U. **9.** 45 lb. per sq. in. approx.
10. 28 per cent., 7·55 lb. per I.H.P. hour ; 30 per cent., 8·3 lb. per I.H.P. hour.

Chapter XIV

1. 51 per cent. ; 488° C. ; 157·9 lb. per sq. in. abs.
2. (*a*) 48 per cent. ; 38·1 per cent.
3. (*a*) 0·0393 lb. ; (*b*) 158 lb. per sq. in. abs. and 488° C. ; (*c*) 3616° ; (*d*) 907 lb. per sq. in. abs. ; (*e*) 1948° C. and 77·4 lb. per sq. in. abs. ; (*f*) 51 per cent.
4. 2·741 : 1 ; 36·2 per cent. **5.** $pv^{1\cdot323}=200\cdot2$.
6. 120·7 ; 2530° F. ; 45·83 per cent.
7. 210 lb. per sq. in. ; 115·2 lb. per sq. in. Relative efficiency, 46·8 per cent.
8. 3·66 ; 50·1 per cent.

Chapter XV

1. 123·3 lb. **2.** −131·7° F. ; 1·712 ; 30·6 lb. **3.** 5·90.
4. (*a*) 5·74 ; (*b*) 5·66. **5.** (*a*) 6·33 ; (*b*) 6·33.
6. (*a*) 5·38 ; (*b*) 5·9. **7.** 44·24.
8. Coefficient of performance, 5·46 ; refrigerant per min.=6·17 lb. ; compressor power, 12·1 H.P. ; cooling water, 366 lb. per min.

Chapter XVI

2. On steam side 128·3° F., on water side 127·98° F.
3. 0·151 in. **4.** Upper 273° C. ; lower 270° C.

Chapter XVII

1. 11·04 lb. ; CO_2 14·06, H_2O 2·15, N_2 73·87, O_2 9·92 per cent.
2. 1·0424 cu. ft. ; 10·6 per cent. ; CO_2 16·60, H_2O 10·21, N_2 73·19 per cent.
3. 0·2417 ; 2292 B.Th.U.
4. (*a*) 1535 B.Th.U. ; (*b*) 830 B.Th.U. ; (*c*) 723 B.Th.U.
5. (*a*) 17·5 per cent. ; 1502 B.Th.U.=10·77 per cent. ; (*b*) 3020 B.Th.U.
6. 8·9 per cent. ; 2271 B.Th.U.
7. (*a*) 1·28 lb. ; (*b*) 1039 C.H.U. ; (*c*) 943 C.H.U. **8.** 93·5 ft.
9. (*a*) 5·76 cu. ft. ; (*b*) 687·9 ; (*c*) 621·3.
10. 576·4 ; 524·7 B.Th.U. per cu. ft. **11.** 14·84 lb.
12. CO_2 17 per cent. ; steam 1·95 per cent. ; N_2 73·30 per cent. ; O_2 7·77 per cent.
13. 9·12 cu. ft. ; 10·9 per cent. **14.** 2202 B.Th.U., 651 B.Th.U.
15. 6·84 lb. ; 1650 B.Th.U. ; 737 B.Th.U. **16.** 17·5 lb.
17. 10,943 calories per gram, or 19,697 B.Th.U. per lb.
18. 2·915 cu. ft. ; 0·845 cu. ft.

Chapter XVIII

1. 71·0 per cent.; (1) 9·52 lb.; (2) 9·93.

2.

	B.Th.U.	Per cent.
Total heat value of 1 lb. of dried fuel . . .	14,000	100·00
Heat transferred to water (thermal efficiency) .	9,955	71·11
Heat carried away by products of combustion .	1,603	11·46
Heat carried away by excess air	860	6·24
Heat lost in evaporating and superheating moisture in the fuel	25	0·02
Heat lost by incomplete combustion . . .	749	5·45
Heat lost by unburnt carbon in ashes . . .	32	0·03
Unaccounted for	776	5·69
Total . . .	14,000	100·00

3.

	B.Th.U.	Per cent.
Gross heat supply entering engine per minute	37,480	100·00
Heat equivalent of I.H.P.	4,665	12·44
Heat leaving engine in jacket drain	1,037	2·76
Heat leaving engine in exhaust steam . . .	29,812	79·54
Unaccounted for	1,966	5·26
Total . . .	37,480	100·00

Thermal efficiency = 13·52 per cent.

4.

	B.Th.U.	Per cent.
Gross heat supply entering engine per minute	148,880	100·00
Heat equivalent of I.H.P.	20,962	14·07
Heat leaving engine in jacket drain	5,760	3·87
Heat leaving engine in exhaust steam . . .	93,838	63·03
Unaccounted for	28,320	19·03
Total . . .	148,880	100·00

(*a*) 15·16 lb.; (*b*) 15·19 per cent.; (*c*) 163 B.Th.U.; (*d*) 0·584.

5.

	A	B	C
(*a*)	10·98	10·45	10·48 lb.
(*b*)	75·81	72·17	72·32 per cent.

6.

	B.Th.U.	Per cent.
Heat in 1 lb. of oil	19,300	100·0
Heat equivalent of I.H.P.	7,938	41·12
Heat rejected to cooling water	4,929	25·53
Heat rejected in exhaust	5,314	27·53
Unaccounted for	1,119	5·82
Total . . .	19,300	100·00

7. (*a*) 28·2 ; (*b*) 24·37 ; (*c*) 86·4 per cent. ; (*d*) 27·68 per cent. ; (*e*) 23·92.
8. (*a*) 25·1 ; (*b*) 21·9 ; (*c*) 18·2 per cent. ; (*d*) 20·9 per cent.
9. (*a*) I.H.P.=50·9, B.H.P.=30·45 ; (*b*) 0·37 lb., 0·63 lb. ; (*c*) 2158 B.Th.U. ; (*d*) 1365 B.Th.U. ; (*e*) 1917 B.Th.U.
11. (*a*) 39·0 ; (*b*) 30·76 ; (*c*) 78·8 per cent. ; (*d*) 30·0 per cent. ; (*e*) 23·8 per cent. ; (*f*) 0·59.
12. (*a*) 28·6 per cent. ; (*b*) by steam 970 B.Th.U., by dry air 1501 B.Th.U. : total=2471 B.Th.U. ; (*c*) 99 B.Th.U.

13.

	C.H.U.	Per cent.
Heat in 1 lb. of oil	10,720	100·00
Heat converted into work (thermal efficiency) .	4,242	39·57
Heat rejected to cooling water	2,309	21·54
Heat rejected to exhaust	2,743	25·58
Unaccounted for	1,426	13·31
Total . . .	10,720	100·00

INDEX

Air, composition of, 376
—, curve of state for, 69
— engines, 85 *et seq.*
—, moisture to saturate. See Steam and air mixtures, also, 418
Ammonia, 321, 341
Analysis of flue and exhaust gas, 379 *et seq.*
Atomic weight, 371
Auxiliaries, allowance for, 406
Avogadro, law of, 7, 11

Bell-Coleman refrigerator, 314
Black body condition, 345
Blading, design of, 235 *et seq.*
Blanketing, 265, 362
Blower turbo, 112 *et seq.*
Body, black, 344
—, grey, 345
Boiler draught, 394
— test, 401
Boiling-point, 2
Boyle, law of, 4, 6, 10
Brine, circulation of, 312, 331

$C=\frac{K}{\rho S}$, 351
Callendar, 13, 74, 119, 433
Calorific value, 372 *et seq.*
Calorimeter, steam, 134 *et seq.*
—, exhaust gas, 288, 415, 422
Carbon dioxide, curve of state for, 70, 334
— —, properties of, 341 *et seq.*
Carnot, principle of, 45
—, cycle of. See Cycles.
Charles, law of, 4, 6, 10
Charts, of state, general, 64 *et seq.*
—, —, choice of, 73
— for CO_2, 334
— for steam, 71, 72, 125, 126
—, Glaisher's, 144
—, psychometric, 144
Chimney, height of, 395
Clapeyron, equation of, 61
Clausius, integral of, 51
Clearance, 110, 279
Coaggregation, 13
Coefficient of expansion, 1, 3
Combustion, 370 *et seq.*
—, process of, in I.C. engines, 301 *et seq.*
Compression, adiabatic, 2
—, formulæ for, 21
— ignition engines, 271, 307
—, isothermal, 2
— ratio, 38, 273, 305, 307, 308
—, wet and dry, 338
Compressors, air, 93 *et seq.*
Condensation, cyclic, 173 *et seq.*
Conduction, heat transfer by, 81, 347 *et seq.*
Conductivity, table of, 352
Convection, 361 *et seq.*
Coolers, 363 *et seq.*
Co-ordinates, oblique, 73, 334
Curves of state. See Charts.
Cushion, steam, 170, 175.
Cycles, Bell-Coleman, 314
—, Carnot, 31, 32, 35, 42, 55, 86, 149
—, closed, 29, 30
—, comparison of, 38, 276
—, composite, 271
—, compression-ignition, 271, 307
—, constant pressure, 37, 41, 268
—, — temperature. See Carnot cycle, also, 35
—, — volume, 35, 266
—, Diesel, 268
—, dry steam, 164
—, Ericsson, 88
—, Joule, 90, 314
—, Kirk, 314
—, Lightfoot, Linde, 340
—, non-reversible, 50
—, Otto, 266
—, Rankine-Clausius, 150, 156, 160
—, regenerative. See Stirling and Ericsson cycles, also, 165, 208, 261
—, reversed, 44, 312 *et seq.*
—, steam, 35, 147 *et seq.*, 164, 165, 261
—, Stirling, 46, 86, 314
—, Stokes, Vorhees, 339
—, Wildhausen, 340
Cylinder-head design, 305

Dalton, law of, 7, 11, also 139, 323
Density, table of, 352
Detonation, 301, 309

Dew-point, 141
Diagram efficiency, 249
— factor, 170, 199
Diagrams of state. See Charts.
— —, choice of, 73
Diesel cycle, 268
Diffuser, 227
Diffusivity, 351
Dissociation, 299
Draught, 394
Dryness fraction, 65, 129, 134 *et seq.*
— —, optimum, 335

Efficiency, condition for maximum, 46
—, internal, 255
—, stage, 254
Elasticity, 1 *et seq.*
Electrolux, 323
Emissivity, 345 *et seq.*
Engines, gas, I.C. oil and petrol, 85, and Chaps. XIV and XVIII
—, steam, Chaps. XI and XVIII
—, —, compound, 193 *et seq.*
—, —, uniflow, 186
—, —, warming, 317
Enthalphy, 77
Entropy, 49 *et seq.*
Ericsson engine, 88
Evaporation below boiling-point, 11
—, latent heat of, 2, 9 *et seq.*
— corrected, 406
— equivalent, 406
Evaporators, 318
Expansion, adiabatic, 2, 14 *et seq.*
—, coefficient of thermal, 1, 3, 4
—, compound, 109, 193
—, isothermal, 2, 14 *et seq.*
—, over- and under-, 224
—, ratio of, 38, 268

Flame, propagation of, 301
—, radiation from, 346
Freon, 341
Friction, 81, 115, 220, 227, 253, 288
Fusion, latent heat of, 2, 9

Gamma, values of, 12
Gas, constant, the universal, 8
— laws, 4, 12
—, perfect, 3, 65
—, permanent, 65
—, producer, 386, 423
Glaisher's tables, 143
Governing, steam engine, 204
—, I.C. engines, 422

Haslam and Hall refrigerator, 314
Heat, conduction of, 347 *et seq.*
Heat, cumulative, 254
— Engine Trials Report, 373, 374
— losses, 76 *et seq.*, 287, 359
— reception during expansion, 21, 23
— recovery lines, 182
—, radiation of, 343
—, specific, table of, 352
—, —, calculation of, 393, 405, 416
—, —, variation of, 54, 295
—, total, 77
—, transference of, 343 *et seq.*
—, volumetric, 7
Humidity, 142

Indicator diagram, 169, 196, 277, 282
Injector, 228
Irreversibility, 76, 116

Jacket, steam, 164, 187
—, water, 97, 287 *et seq.*
Joule, cycle of, 90, 314
—, equivalent of, 3
—, law of, 7, 11
Joule-Thompson effect, 76

K { Velocity ratio, 237
  { Coefficient of conductivity, 347
Kirk refrigerator, 314

Lagging, 349, 360
Latent heat of fusion, 2, 9
— — evaporation, 2, 9
Laws of thermodynamics, 10, 55. See also Avogadro, Boyle, Charles, Conduction, Dalton, Joule, Nernst, Reynault, Stefan, Willans.
Leakage of heat, 185. See also Heat losses
Lightfoot cycle, 340
Linde cycle, 340

Matter, states of, 1 *et seq.*
Melting-point, 1
Missing quantity, 177
Mol, 8
Molecular theory, 8
— weight, 371
Mollier chart, 69, 72, 125, 334
Motor, air, 93 *et seq.*

n, 22
—, value of, for air, 97
—, —, — steam, 125, 174, 215
—, —, — I.C. engines, 279, 295
Negative loop, 279
Nernst, law of, 55
Nozzle, steam, 211 *et seq.*

Orifice, flow through, 211 *et seq.*
Otto cycle, 266

Performance, coefficient of, 313
Pink, 302
Pound—molecule, 8
Preheating for air motor, 109
Preignition, 302
Pressure, absolute, 4
—, maximum permissible, 275
—, mean effective, 31, 170
— ratio, critical, 214
Priming, 182
Process, irreversible, 76 *et seq.*
—, reversible, 44

Quantity, missing, 173

R, 6, 7
Radiation, 343
Rankine-Clausius cycle, 150, 155, 160
Refrigerants, choice of, 341
Regenerative cycle, 208, 261
Regenerator, 86, 88, 165, 261
Reheat factor, 253, 291
Residual gases in I.C. engine, 304
Reynault, law of, 7
Reynolds, 43, 361
Ricardo, 287, 305
Roughness in I.C. engine, 306

Speed ratio, 237, 249
Specific heat, 1, 3, 7, 54, 295, 352, 393 *et seq.*
State, factors of, 49 *et seq.*
—, curves of, 64 *et seq.*
—, see also Curves.
Steam and air mixtures, 139, 418
— cushion, 170, 175
— cycles, 35, 147, 165, 261
Steam, mixtures of, with air, 118
—, properties of, 118 *et seq.*, 433
—, superheated, 46, 159, 189
— tables, 69, 120, 433
Stefan, law of, 344
Stirling cycle, 86
Stokes cycle, 339
Supersaturation, 65, 225

Temperature, critical, 3, 65, 332
—, cyclic fluctuation of, 173, 351
—, scale of, thermodynamic, 46
— suction, 278, 291
Tests of engines, etc., 398 *et seq.*
Thermodynamics, laws of, 4, 10, 55
Throttling, 76, 80, 129, 136, 325
Turbine, 166, 427, and chaps. xii and xiii
Turbo blower, 112, 318
Turbulence, 303

Uniflow engine, 186

Vacuum, flow into, 76, 80
Van der Waals, 13, 65
Vapour to saturate air, 418. See also Steam and air mixtures.
Variables, independent, of state, 49
Velocity, peripheral, 113
— ratio, 237
Volume, defect of, 13
— occupied by mol. wt. of gas, 7
Volumetric heat, 7
Vorhees cycle, 339

Water, specific heat of, 13, 119, 433
Wildhausen cycle, 340
Willans, law of, 206
Wilson line, 225
Wiredrawing. See Throttling.

THE END