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INTERMEDIATE PHYSICS

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PREFACE TO THE THIRD EDITION

THE demand for a third edition has given the author an opportunity to add articles on Lactometer, Bicycle pump, Cinema pictures, Dynamos, Motors, X-rays, etc. The account of these important applications is necessarily brief and inadequate. But the fact that a college student studying Physics should have some acquaintance with such important topics of scientific and even popular interest is enough to justify the additions however inadequate they may be.

While bringing out this new edition, the author has tried to cover as wide a field as possible, but, in spite of his best attempts, the book cannot be called "complete". And it is doubtful whether "completeness" is desirable in any book, for it is admitted on all hands that every student should have as exhaustive a reading as his time permits. The author, therefore, hopes that his friends will excuse him, if they do not find all that they would like to see in this book.

The author expresses his thanks to his several friends for their valuable criticisms.

Any suggestions for its improvement or for its better adaptation to the requirements of (Intermediate) students will be gratefully received.

G. L. DATTA.

D.A.-V. COLLEGE, LAHORE,
May 1928.

PREFACE TO THE FIRST EDITION

In these days of numerous text-books the author who dares still further to increase the number owes to the public an explanation in which he should set forth his purpose. The following few lines are written accordingly :—

The author felt while lecturing to the Intermediate students that there was no single book on the market which could meet the requirements of good as well as weak students. At this stage the students cannot read a good many books themselves to form a clear notion of the subject and much less to gain general information.

The aim of the author has been to prepare a text-book adapted to the needs of the Intermediate students of the Indian Universities. He has not merely, keeping the treatment elementary, included all the fundamental principles of Physics, but has also tried to put in as much general information as possible.

An attempt has been made throughout the book to mention mostly those experiments which a student would perform himself in the Laboratory, and hence hints about practical work have been given here and there.

The author makes no claim for originality in this book. He has contributed nothing of his own save the labour of a careful selection and condensation.

Due care has been taken not to overload the book with a mass of examples. At first solved examples have been given and then only as many other examples follow as should be worked out by every student.

A student who wants to pass the examination only may safely omit all sections printed in small type.

It is hoped that this book will be found of value as a class book for the Intermediate students.

The author is especially grateful to L. Brij Lal Pury, B.A., LL.B., for his kindness in reading and correcting the proof-sheets and for seeing the book through the press.

G. L. DATTA.

LAHORE,
February 1924.

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INTRODUCTION

1. Study of Nature.—Hold a piece of stone under the surface of water. When let free, it sinks to the bottom. Next try a piece of cork; instead of sinking down, it rises to the surface. This holds good not only in the case of the pieces of stone and cork which have been experimentally seen to sink or float, but of all pieces of stones and corks, great or small. Given a piece of cork never tried before, we feel certain that it will rise to the surface when placed in water. What makes us so sure of the fact? We know that millions of pieces of cork have at different times been tried and not one of them has ever been known to behave otherwise. There is no reason why a new and untried piece may behave differently; in other words, we *infer* that Nature treats alike all pieces of cork when placed in water.

Let us pass on to another still more familiar example. The sun in the evening appears to sink below the horizon and so to vanish from our sight; but we are always certain that he will reappear the next morning.

These two illustrations are simply particular cases of the general principle that *what the forces of Nature have been found to do once, they will, under the same circumstances, do again.* This principle is known as the **Uniformity of Nature.**

The ancients at a very early age acquired from what they observed a certain amount of knowledge of the laws which regulated the phenomena around them; this knowledge was not only superficial and imperfect, but was desultory and unsystematised as well. Those people who by careful observation of and by close reflection on the causes of natural events classified the phenomena in a systematic manner were called Philosophers (*i.e.* lovers of wisdom). The mass of knowledge gathered by them was called *Natural Philosophy.*

They included in the Natural Philosophy *Astronomy, Botany, Zoology, Chemistry, Medicine, Mechanics,* and even *Astrology.* Nowadays the term is very much restricted in its

scope. It stands only for that branch of science which deals with the general relations and properties of the three fundamental quantities—*Space*, *Time*, and *Matter*. This branch of science is more usually called now by the name of **Physics.** It includes *Mechanics*, *Heat*, *Light*, *Sound*, *Frictional Electricity*, *Magnetism*, and *Current Electricity*.

We shall consider each of these branches in the course of the following pages.

Before we begin doing so, let us consider briefly what matter is.

2. Matter.—Whatever can affect one or more of our senses, —that is to say, the existence of whatsoever can be recognised by sight, touch, taste, smell, or hearing—we call by the name of matter. It is, however, only a working definition. When we come to analyse the question carefully, we arrive at the conclusion that, although we know many of the properties of matter, our knowledge of the thing itself is very limited. We know indeed very little of its real nature.

Matter is generally found to exist in three states—*solid*, *liquid*, and *gaseous*. Some of the properties common to matter in all its states are given below.

(1) **Matter is indestructible and uncreatable.**—Very careful experiments have shown that under no circumstances whatsoever does the quantity of matter undergo a change; it does not increase, neither does it diminish, though it might assume a different shape. This experimental result, which forms the basis of the science of chemistry, is known as **Conservation of Matter.**

(2) **Extension.**—This is another fundamental property of matter. It simply means that every piece of matter must occupy some space which cannot be occupied by any other piece of matter at the same time. There are a good many apparent exceptions to this statement. For instance, a lump of sugar takes up water into itself without any appreciable increase in volume. But the riddle is solved if we remember that the structure of matter in general (sugar in this case) is not continuous. It is supposed to be made up of minute particles called molecules, separated by inter-spaces. The real volume or the space occupied by the matter is much less than its *apparent volume*. It is into these inter-spaces that water penetrates.

(3) **Porosity.**—The fact that matter is porous can be easily proved experimentally. On mixing 50 c.c. of alcohol with 50 c.c. of water, the volume is found to be 97 c.c. instead of 100 c.c.,

which could not be the case if there were no pores in the liquids. In 1640, Bacon, on compressing a hollow sphere of lead filled with water, found that water oozed out and appeared on the outer surface, showing thereby that lead is porous. Florentine at a later date proved in a similar manner that silver is porous.

(4) **Divisibility.**—All portions of matter with which we are acquainted can be split up into yet smaller portions. Let us suppose we are given a piece of table salt. We can divide it into smaller and smaller pieces. The smallest visible particle which can be obtained by mechanical grinding is capable of still further sub-division. A question naturally arises: supposing we had suitable means at our disposal to sub-divide a body, will there be no limit to this divisibility? Investigations have led us to this result (remember it is only an inference) that there is a *limit* to this sub-division, and that further sub-division is secured at the cost of changing the nature of the substance. *This smallest portion of the body is called the* **molecule.** If we still continue the sub-division of our salt we split the molecule into its constituents, sodium and chlorine forming the elementary atoms. Hence we suppose that for any particular kind of matter there is a limit to which it can be sub-divided, the limit being reached when we arrive at molecules. It may be of interest to note that the diameter of a molecule is roughly equal to $\frac{1}{2500000}$ millimetre; it is about 600 times smaller than the diameter of the smallest visible particle.

(5) **Mutual attraction.**—All particles of matter attract one another. When the particles are at a distance from each other, the force of attraction is said to be *gravitation*. If the particles be the molecules of one and the same substance, the force of attraction is said to be *Cohesion*; whereas if the molecules be of different substances, such as glass and water, the force of attraction is called *Adhesion*.

(6) **Elasticity.**—When a body is acted upon by a number of external forces which change or tend to change its volume, forces are called into play which resist the change. As soon as the external forces are withdrawn, the body tries to regain its original volume. This *power of recovery is called elasticity.* Solids resist change of shape in addition to change of volume. Liquids and gases resist change of volume only, and hence they are said to possess volume elasticity.*

(7) **Matter is the vehicle of energy.**—Energy, be it kinetic or potential, is always found associated with matter. As a

* It is treated more fully in Chapter IX.

matter of fact it is this daily experience of ours which has led us to suppose that the space beyond our atmosphere is filled with *Ether* (a hypothetical substance possessing elasticity, etc.), because otherwise we cannot explain transference of light or heat energy from the celestial bodies to our planet.

(8) **Matter possesses inertia.**—Matter is found to be incapable of changing its state of rest or motion unless some external force acts upon it. This inertness or inactivity is called *inertia*. We shall deal with it in detail later on.

In addition to these general properties there are others which are quite different for different kinds of matter or for the same kind of matter in different states, as for instance *tenacity*, *ductility*, *malleability*, etc., for solids; *capillarity*, *viscosity*, etc., for liquids; *diffusion*, *expansion with decrease of pressure*, etc., for gases.

Different bodies contain different quantities of matter in them. This is expressed by saying that their masses are different. The mass of a body means *the quantity of matter contained in it*.

3. Structure of matter.—We have already remarked above that matter is made up of discrete particles called molecules, separated by inter-spaces. This hypothesis not only enables us to explain the apparently exceptional behaviour of sugar taking up water into itself, but also to explain such phenomena as compression, diffusion, solution, and change of volume with pressure, etc. Without this hypothesis all these phenomena are quite unintelligible. We shall briefly consider the molecular structure of matter in different states, taking it for granted that molecules are hard, elastic spheres.

In gases the average separation of molecules is very large compared with the dimensions of a molecule, which results in almost complete absence of cohesion. Consequently a molecule is quite free in its motion. It is supposed to be flying about in all directions with a great velocity. During its motion it collides with other molecules. Each molecule during the period between two consecutive collisions is free from the influence of the surrounding molecules. It is on account of this free motion of the molecules that a gas always endeavours to expand. When the molecules strike against the walls of the containing vessel, they exert force (*i.e.* pressure) on the walls.

In liquids molecules are supposed to be so close together that although a single molecule moves about, yet collisions

are so frequent that it is never free; *i.e.* away from the influence of the other molecules.

Perhaps it will be interesting to know the numerical data concerning the molecules. The number of molecules in one c.c. of hydrogen gas at atmospheric pressure and at 0° C. is 27×10^{18}. A molecule of H_2 gas flies about with a velocity of 1840 metres per second (or 3600 miles per hour). It suffers about 10,000,000,000 (*i.e.* 10^{10}) collisions per second.

Knowing that a cubic centimetre of water when changed into the gaseous state occupies 1600 c.c., it is quite evident that a molecule in the gaseous state is $(1600)^{1/3}$, *i.e.* twelve times as far apart as in the liquid state.

In the case of solids the motion is still more restricted. A molecule is supposed to oscillate to and fro round its mean position, which it never leaves. This is confirmed by the fact that carefully preserved jewels engraved by the ancients still preserve their original sharpness of outline.

4. Measurement of fundamental quantities.—Physics is an exact science, and it deals with the measurement of different quantities. How much the measurement of different quantities means to Science will be clear from a remark made by Maxwell:

"The most important step in the progress of every science is the measurement of quantities. Those whose curiosity is satisfied with observing what happens have occasionally done service by directing the attention of others to the phenomena they have seen; but it is to those who endeavour to find out *how much* there is of anything that we owe all the great advances in our knowledge."

In order to measure any quantity we must fix upon a certain amount of the same physical quantity as our unit. The given quantity is then measured in terms of this unit, and to convey a complete notion about it to another all that we have to say is that it is so many units. We see thus that the complete statement of the result of measurement of a physical quantity consists of two parts: first, the pure number, which states the number of times the unit is contained in the given quantity; and second, the name of the unit which has been employed. It should be clearly grasped that the unit fixed upon is simply conventional; there is no particular reason why we should not coin our own unit. So far as we ourselves are concerned, we are perfectly justified in doing so, but the difficulty arises when we want to express ourselves to others. In order that they may completely understand us, we would

have to explain to them our unit. To remove this difficulty, certain international units have been fixed upon which are used by all the civilised nations.

Let us take an example. Suppose I want to tell a man at Calcutta how much money I can spare for him. To do so I must first choose the unit in which I want to express myself and then the number of units which my amount contains Let the amount be Rs. 100. When I write this to him, he at once understands how much I can spare for him, for he understands what a rupee is. Otherwise I would have to explain to him the connection between the rupee and the coin with which he is familiar. This shows that by sticking to the international standards we cut short the explanations. Before we take up these standards it will be better to consider the relation between the number and the size of the unit in terms of which the quantity is measured. Let us go back once more to our 100 rupees. We could express the same amount as 1600 annas or 6400 pice. This shows clearly that as the size of the unit becomes smaller the number of times the unit is contained in the given quantity becomes greater, and *vice versa*. We can express the same thing as follows :

The numerical value (i.e. *measure of the quantity*) *is inversely proportional to the unit which is employed to measure the quantity.*

Certain relations exist between different kinds of physical quantities, and it is found that by selecting the units in a certain number of cases it is possible by making use of these relations to fix the magnitude of the units for the rest of the physical quantities. The units that are thus chosen as the basis for the whole of the system of units are called ***fundamental units***, and the other units that depend upon these fundamental units as their basis are called ***derived units***. Fundamental units commonly employed are three in number: those of *length*, *mass*, and *time*. Other units can be derived from these three ; as, for instance, the unit area is defined as the area of the square whose side is of unit length.

Choice of fundamental units

5. Unit of Length.—In selecting the unit it should be borne in mind that it must be unalterable and that other standards which are supposed to be the copy of it may readily be compared to it. Bearing this in mind it will be seen that it becomes necessary that the standard unit must be kept in custody somewhere at a very safe place.

In the British Empire the unit of length is the Imperial yard, which is defined by the Act of Parliament as follows :

"The straight line or distance between the centres of the transverse lines on the two gold plugs in the bar deposited in the office of the Exchequer shall be the genuine standard yard at 62° Fahrenheit, and if lost, it shall be replaced by its copies."

The copies above referred to are those preserved at the Royal Mint, the Royal Society, the Royal Observatory, and the House of Parliament.

The yard is subdivided into three equal parts, each of which is called a foot (ft.). The foot is further subdivided into 12 parts called inches.

For measuring large distances multiple of the unit yard like a mile or a furlong is used.

The yard is quite an arbitrary unit, and is used only in the British Empire, and that too by the engineers and the commercial people. In scientific work the unit employed is the **metre.** This is the international unit.

It was defined by a law of the French Republic in 1795 as the distance between the centres of two lines engraved upon the polished surface of platinum-iridium bar at the temperature of melting ice. The bar is deposited in the national archives at Sèvres, near Paris.

It was originally intended to be one ten-millionth part of the distance from the North Pole to the Equator measured along the meridian passing through Paris and hence to be equal to an object existing in Nature. But as more accurate determinations of the quadrant of the earth's meridian were made, it was found that the metre is not exactly the ten-millionth part of the quadrant; it is too short by about ·2 mm. according to the latest measurement. Moreover, the length of the quadrant is changing, hence the standard is not the length of a natural object but the platinum-iridium bar, which is as arbitrary as the British yard.

The metre has its sub-multiples and multiples as follows :

1 decimetre equal to $\frac{1}{10}$th of a metre.
1 centimetre ,, $\frac{1}{100}$th ,,
1 millimetre ,, $\frac{1}{1000}$th ,,
1 kilometre ,, **1000** metres.

A metre is equal to 39 37 inches, or 1 inch is equal to 2 54 cm. approximately.

6. Unit of Mass.—There are two units of mass corresponding to the two units of length: one that is commonly used in the British Empire, and the other the international unit

that is used in scientific work. The first unit is called the Pound. It is the mass of a lump of platinum marked, "P.S. 1844 1 lb.", kept at the Standards Office at Westminster. It is sub-divided into 16 parts called ounces (oz.); for heavy masses, multiples of this unit like tons are used. One ton is equal to 2240 pounds.

The second unit is called the gramme, more often written as **gram.**

It is one-thousandth part of the mass of the piece of platinum-iridium called the "Kilogramme-International", which is kept at Sèvres, near Paris.

Originally it was intended that the kilogramme-international should be equal to the mass of 1000 c.c. of distilled water at 4° C., which is now known to be only approximately true. The variation, however, is so slight that in all calculations we neglect it unless very great accuracy is required.

The gram has its sub-multiples and multiples as follows:

1 decigram equal to $\frac{1}{10}$th of a gram.
1 centigram ,, $\frac{1}{100}$th ,,
1 milligram ,, $\frac{1}{1000}$th ,,
1 kilogram ,, **1000** grams.

A kilogram is equal to 2 2 pounds.

A cursory glance at the units mentioned will show that the British units are not well adapted for scientific work. The International system renders the calculations extremely simple, because to convert or reduce a unit to its multiples or sub-multiples, we have only to divide or multiply by some power of 10, and this can be done at once by moving the decimal point or adding to or taking away the cyphers. Moreover, on account of the simple relation which exists between the mass and volume in this system, we can easily calculate the volume of the body in cubic centimetres if we know its mass in grams.

7. Unit of Time.—The idea of time in its simplest form is the recognition of the succession of events. The setting or rising of the sun supplies an excellent example of the succession of events, even to a cursory observer. Hence no wonder that this event has been made use of in measuring time. From noon to noon is generally taken as the sequence of events. Owing to the motion of the earth round the sun the interval between the times when the sun is highest in the heavens on two successive days is not constant throughout the year, but differs slightly from day to day. The average value of this interval is called the Solar Day. The $\frac{1}{86400}$th part of the

solar day is called the mean solar second, or for shortness **second.** This mean solar second is the fundamental unit of time. Note the unit of time in both the systems is the same.

To sum up, we have two systems of units; in the first the units are, foot, pound, and second. This system is called briefly the F.P.S. system. In the other the units are centimetre, gram, and second. This second system is called the C.G.S. system.

It will be useful to remember that the quantities to be measured in this world can be divided into two classes: (i) those which have magnitude only, as for instance 50 rupees; and (ii) those which have direction in addition to the magnitude, as for instance force, velocity etc. The former quantities are called **Scalars**; the latter **Vectors.**

INTERMEDIATE PHYSICS

PART I

MECHANICS

CHAPTER I

8. Formerly Mechanics, which literally means "contrivances", dealt with the devices which were found helpful in lifting heavy bodies and moving them to any desired distance. In other words, it was the science of Machines. Nowadays this branch of science is called applied mechanics, and by **Mechanics** we understand that branch of science which deals with the state of rest or motion of the bodies. We can also define it as *that branch of science which deals with the effects of force or forces on matter.* Before going any further, let us first understand what is meant by the word Force.

9. Force.—When we make a muscular exertion to overcome some resistance we become conscious of the effort we are making, and we say we are exerting force. For instance, suppose we want to lift a weight of 20 seers from the ground. The earth attracts the weight downwards, but we want to raise it upwards, hence we have to overcome the downward pull of the earth, and in so doing we feel a certain sensation. It is this sensation which gives the idea that we are exerting force. In this example the effect of muscular exertion is to produce motion in a body. Now let an inanimate agent which cannot exert muscular effort also produce motion in the given body. Since the effect produced in both the cases is the same, therefore we infer that the cause producing this effect must also be similar. Hence the inanimate object is said to exert force. For example, the pull of the earth brings the weight of 20 seers down when it is released, *i.e.* it produces motion in the body just as we did when we exerted force; hence the earth is also said to exert force on bodies. Take next the case of an archer who bends his bow and shoots an arrow. Here motion is produced in the arrow by the force

of elasticity in the bent bow. In this case the bow produces motion in the arrow—an effect which our muscular force would produce. Hence the bow is said to exert force on the arrow. Note, the force may not necessarily produce motion; for instance, in the case of the arrow, so long as it is held by the hand it does not fly off although the bow is exerting force all the time. Thus force may be defined as *that which produces* or *tends to produce* motion in a body.

But the idea is not complete, because force instead of producing motion may destroy it. For instance, suppose a ball is moving; to stop it we have to exert muscular effort. In this case the force destroys motion. Similarly when we hold a weight in our hand the earth tends to produce motion by pulling it down, but the force exerted by our hand destroys this influence. If a weight heavier than the one we can support be placed in our hand, our effort will tend to sustain it, but it will be unsuccessful. Even in this case our hand is exerting force. These examples clearly show that we can recognise force not only when it produces or tends to produce motion, but also when it destroys or tends to destroy motion. Thus a more correct definition would be the following:

Force is that which produces or tends to produce, destroys or tends to destroy motion.

Let us go back to the primitive notion of force, that of making muscular exertion to overcome a certain resistance. It is clear from the very statement that if there were no resistance there would be no muscular exertion. This shows that in order that an agent may exert force it must meet with some opposition or resistance (opposite force). As a matter of fact *the force exerted is precisely equal to the amount of resistance.* This leads us to the important result that *forces always exist in pairs.*

Let us consider the case of a bullet fired from a gun. The bullet while passing through the air does not exert force to any appreciable extent. When it strikes the target, however, the force it exerts may be so great as to pierce it through. Here the bullet wants to pierce the target and the latter wants to stop it. The force exerted by the bullet is opposite in direction to that exerted by the target, the two constituting the pair. Such a pair of forces is called a **stress.**

10. Motion of a body.—It will be easier first to consider motion in abstract, that is, without any reference to the forces producing it. Before discussing motion of a body we must understand what is meant by saying that a body is at rest. A motor car is said to be at rest if it remains stationary in the

garage day after day. The man sitting in the moving car is said to be at rest with respect to the car if he does not move in it. The man actually, however, is not at rest, for he is sharing the motion of the car with respect to the earth.

Similarly by saying that the car is at rest in the garage, we mean that it does not move with respect to the house in which it is kept. But the house itself is sharing the motion of the earth, which is moving round the sun at the rate of 18 miles per second, hence the car also is moving at this rapid rate round the sun.

Thus all that we mean when we say a body is at rest is that it is not changing its position with respect to its surrounding objects. Hence *rest* is relative only. But if a body occupies one position at one instant and another position at the next moment with respect to its surrounding objects, it is said to be in motion. *Motion thus means simply change of position with respect to the surrounding objects.* It should be noted that motion also is relative; for instance, in the case of the moving car its motion with respect to the earth is known; but since the earth itself is moving with respect to the sun, the resultant motion of the car is not the one that we observe, but is compounded of two motions, one of the car and the second that of the earth itself. This does not end the matter, for we know that the sun along with the other planets, including the earth, is moving in space at the rate of 400,000 miles per day. If this motion be also taken into account the motion of the car becomes very complex. The heavens may also be moving with some speed for what we know. Thus we see that we do not know the *absolute* motion of a body; all that we can say is that it is moving at such-and-such speed with respect to certain bodies regarded as fixed. Hence

All motion and all rest is relative.

11. Velocity.—To describe fully the motion of a body we must know two things—Speed and Direction. *The speed of a body is its rate of change of position.* If it moves slowly its speed is small, but if it moves quickly its speed is great. To specify speed completely, we need state its magnitude only; we are not at all concerned with the direction in which the body moves. Suppose a man is walking on foot on the road running between Lahore and Amritsar. To know his speed all that we need know is how much distance he travels during a particular time. If he travels 4 miles in one hour, we say his speed is 4 miles an hour. But this statement does not completely decide his motion; for instance, we do not learn whether he is going from

Lahore to Amritsar or from Amritsar to Lahore. To give a complete idea we must mention which way he is going; that is to say, we must mention the direction as well. Both these ideas are included in the word ***Velocity, which means the rate at which a body moves along a particular line.*** In the above example, if we fix upon one direction, say Lahore to Amritsar, as positive, then when the man moves from Lahore to Amritsar, we say his velocity is plus 4 miles an hour ; but if, on the other hand, he moves from Amritsar to Lahore, we say his velocity is minus 4 miles an hour. Note that his speed is the same but velocity is opposite. It should be borne in mind that it is unimportant which direction is taken as positive ; but when once some particular direction is fixed upon as positive, all along we must stick to this convention.

12. Uniform and Variable Velocities.— *If a body moves over equal distances in equal intervals of time, however small, it is said to move with uniform speed,* and if in addition to the speed the direction also remains the same, it is said to move with *uniform velocity.* But if, on the other hand, sometimes it moves fast and sometimes slow, the speed, and therefore the velocity, is *variable.* If speed remains the same, but the direction changes, even then the velocity is variable. For instance, a railway engine moving in a circle might move with the same speed, but its velocity is undergoing a change of direction every moment, and hence it is variable. It is important to note that the *velocity is variable whether the rate of motion changes or direction changes.* In order to understand completely what we mean by uniform and variable velocities, let us consider some concrete cases.

1. Suppose a train leaves Peshawar for Calcutta and covers this distance in 50 hours, stopping at important stations, and suppose further that during every 12 hours it passes over 350 miles. Now, if we fix upon 12 hours as our interval of time, we shall have to say that its speed is uniform, for it travels equal distances during these intervals. But actually it is not the case, for it has been stopping at the important stations, and, moreover, over the bridges and within yards of the stations its speed has not been the same as at other places. Fixing one hour as our unit, therefore, we come to the conclusion that the train is not moving with a uniform velocity. Let a passenger further note the time the train takes in going from one mile-stone to the next for a distance of about 5 miles somewhere between two stations. If he finds that the time taken by the train is the same for covering each mile, then the train travels with a uniform speed over that distance. A surer test

than this even would be to note the time the train takes in going from one telegraph post to the next. If the time is the same the speed is certainly uniform during the period over which observations extend. If, in addition to it, the direction also does not change, the velocity is uniform.

2. Let us next look at the hands of a clock. We find that the minute hand completes one revolution during every hour. So that if we fix upon one hour as the unit, the speed is uniform, but actually it is not so. For if we watch closely the motion of the hand we shall find that it moves by jerks. When a minute is over it moves suddenly to the next mark. Thus fixing upon seconds as our unit, we find that speed is not uniform. Of course in this case even if the speed has been uniform velocity could not be so, because of the fact that the direction is changing constantly.

3. Let us pass on to another example. The earth rotates on its axis in a period of always the same duration, which is equal to the mean solar day. A point lying on it moves through equal distances in equal intervals, however small. In one minute a point on the equator moves through 17·3 miles. This is an example of a *most uniformly moving* body.

Another example of uniform motion would be the motion of sound in the atmosphere when there is no wind blowing.

We have learnt what we mean by uniform speed and uniform velocity; let us now consider the relation between the distance passed over, the velocity, and the time.

13. Relation between distance passed over, velocity, and time.—If a body is moving with a uniform velocity of u ft. per second it will cover u ft. in the first second; in the second second also it will pass over the same distance; hence in two seconds it will pass over $2u$ ft., in 3 seconds $3u$ ft., and so on. In general, if it moves for t seconds, it will pass over ut ft. It can be expressed mathematically as

$$S = ut, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

or

$$u = S/t,$$

where S is the distance passed over from the starting-point, u the velocity, and t the time for which the body has moved. The distance passed over will be given in ft. if u is measured in ft.; and in centimetres if the velocity is measured in centimetres. It is immaterial which units are used for distance or time, but when once they have been fixed upon, we must stick to them throughout a problem. Note, ***velocity is length per unit time, and hence it is not correct to speak of***

it only as so many ft. We must express it as so many ft. per second, or whatever unit is used.

14. Geometrical representation of velocity, and distance passed over.—Since a velocity is completely specified, if we mention both its speed and direction, it is obvious that it can be represented by a straight line, for the length of the straight line can represent the speed, and the direction of the line the direction of the velocity.

Fig. 1

For instance, when we say that the velocity is represented by the line AB (Fig. 1), we mean simply that the magnitude of the speed is given by the length of the line AB.* The order of the letters shows the direction of the motion; for instance, in this case it means that the body is moving from A to B. If we were to represent the velocity by BA, that would mean that the speed is the same but the direction is opposite. What has been said above about velocity is true in general of all the vector quantities. All of them can be completely represented by a straight line.

The space (S) passed over has been shown to be equal to ut, viz. equal to the product of two quantities u and t. In the language of geometry it means that we can represent space passed over by area † of a rectangle whose adjacent sides are u and t.

Bearing this in mind, let us represent along the axis of X, time, and along the axis of Y, velocity. Suppose further that OA (Fig. 2) represents 1 second and OG the velocity of 1 ft. per second. The space passed over in one second will be represented by the area of the figure $OAFG$, which is equal to 1 unit, for $OG = 1$ unit of velocity, and $OA = 1$ unit of time. The distance passed over in 3 seconds will be given by the area of the figure $OCDG$, which is equal to

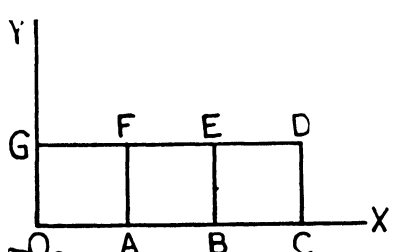


Fig. 2

* Fixing upon any convenient length to represent the unit of velocity, we can represent any speed we like by simply adjusting the length of the line AB.

† Note, it is because area also is the product of two quantities.

3 units. More generally, if the body moves for t seconds and the velocity is u ft. per second, the space passed over by the body would be given by the area (ut) of the rectangle whose one side is t units and the other side u units.

15. Variable Velocity.—When a body traverses unequal distances in equal intervals of time, it is said to move with a variable velocity. To measure the variable velocity at any point we determine the space passed over, including the point, in an infinitely small time. Suppose, for instance, we find that ds (an indefinitely small amount of space), including the point, is traversed in an infinitely small time dt, then the ratio $\frac{ds}{dt}$ gives the velocity at that particular point.

If a point moves with variable velocity and describes S ft. in time t seconds, we speak of $\frac{S}{t}$ as the *average velocity* of the body during the time t seconds. It is such that a body possessing it and moving uniformly would cover the same distance in t seconds as is covered by the body with the actual velocity.

16. Acceleration.—When the velocity of a body is variable, it is said to have acceleration. Just consider a mail train starting from a railway station. It begins to move at first slowly, and as it proceeds on and on, its speed goes on increasing till it attains its full speed. Consider next a cyclist starting for a ride who goes on increasing his speed until he reaches the fastest speed at which he can go. In both these examples the velocity has increased. ***The rate of change of velocity is called the acceleration.*** If velocity had decreased instead of being increased, even then the velocity would have had an acceleration, but only of a negative kind. Sometimes this negative acceleration is called *Retardation*. In the above example the magnitude of the velocity undergoes a change, but this may remain constant and only the direction may change; even then the body is said to have an acceleration. Note, *the rate of either change is called acceleration.*

17. Uniform acceleration.—The acceleration may be uniform or variable. It is said to be *uniform when the velocity increases by equal amounts in equal intervals of time, howsoever small.* But on the other hand, if the increase in velocity is not regular, the acceleration is said to be variable.

Remember the words *howsoever small.* Their importance will be made clear by the following examples:

(1) Let us suppose we are sitting in a 4-oared boat rowing 15 strokes to the minute. Suppose in 3 strokes it attains a velocity of 6 miles an hour. During each of the strokes the velocity increases by 2 miles an hour. One stroke takes 4 seconds, hence the velocity of the boat increases in each interval of 4 seconds by 2 miles an hour, and if this be our unit of equal interval, the acceleration is uniform; but as a matter of fact this is not the case, for the increase in velocity is brought about mostly when the oar is in water. Hence we see that the acceleration is not uniform in this case.

(2) Let us take next an example of a falling stone. Its velocity increases regularly however small the interval may be. This supplies us with an example of uniform acceleration.

Mathematically we can express uniform acceleration as

$$a = \frac{v - u}{t},$$

where v is the final velocity; u, the initial velocity; t, the time during which the change takes place; and a, the acceleration. This can be written as

$$v = u + at \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

We could write it down directly also. For the body starts with a velocity of u ft. per second and after one second its velocity becomes $u + a$; in the next second the velocity again increases by a ft. per second, hence after 2 seconds the final velocity is $u + 2a$, and so on. More generally after t seconds the velocity v will be $u + at$.

Note carefully that *acceleration is the increase in velocity introduced in a unit time.* Suppose, for instance, we are dealing with a railway train moving at the rate of 30 miles an hour, whose velocity becomes 45 miles an hour after one minute. Let us find out the acceleration.

In one minute the velocity is increased by 15 miles an hour;
or in one second ,, ,, ,, ,, ,, $15/60$ miles an hour;
,, ,, ,, ,, ,, ,, ,, ,, $\frac{1}{4} \times 1760$ yd. an hour;
,, ,, ,, ,, ,, ,, ,, ,, $\frac{1}{4} \times 1760 \times 3$ ft. an hour;
,, ,, ,, ,, ,, ,, ,, ,, $\frac{440 \times 3}{60}$ ft. a minute;
,, ,, ,, ,, ,, ,, ,, ,, $\frac{440 \times 3}{60 \times 60}$ ft. a second.

This is equal to an acceleration of 11/30 ft. per second per second. This example makes it clear that ***it is absurd to talk of an acceleration as so many feet per second,*** for

it would be then velocity only. Let us take one more example to clear this point up.

Example.—A train starts from rest and after one minute has a velocity of 30 miles an hour. Find the acceleration.

A velocity of 30 miles an hour is equal to 44 ft. per second. The change in velocity by 44 ft. per second has been brought about in 1 minute.

Hence in one second the change in velocity brought about is 11/15 ft. per second. We can say the same thing as that the velocity has increased by 11/15 ft. per second in one second, or that the acceleration is 11/15 ft. per second per second.

Before proceeding further the student should thoroughly grasp the meaning of the words ***per second per second.***

18. The Kinematic Formulae. — We have already seen that when a body moves with variable velocity the average velocity is S/t, where S is the space passed over during t seconds.

Let us see what this average velocity is equal to when the body moves with uniform acceleration, bearing in mind the explanation of average velocity given in § 15.

Let us represent time along the axis of X and velocity along the axis of Y, as before.

Suppose the body starts from rest, viz. suppose the velocity at the beginning is zero. At the end of one second (represented by OA) the velocity will be a feet per second, where a ft. is the acceleration per second per second. The space passed over will be represented numerically by the area OAH, which is equal to $\frac{1}{2}a \times 1$ or $\frac{1}{2}a$. Hence the space passed over is equal to $\frac{1}{2}a$ ft.

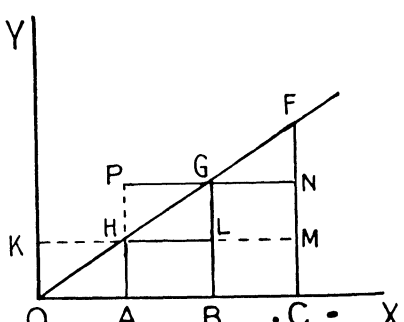


Fig. 3

After two seconds the velocity will be $2a$ ft. per second, and the space passed over during both the seconds, the first as well as the second, will be given by the area of the figure OBG, which is equal to $\frac{1}{2} \times 2a \times 2$, *i.e.* $2a$. Let us draw a line KHL parallel to the axis of X through H. The area of the rectangle $OKLB$ is also equal to $2a$, and therefore it also represents the space passed over in 2 seconds. In this

case it is supposed that the velocity instead of undergoing an increase remains the same throughout, the value being that which is at the middle of the time, or more generally half the sum of the initial and the final velocities.

We shall study now the distance passed over during the 2nd and 3rd second together. The velocity at the beginning of the 2nd second is a ft./sec. and at the end of 3rd second is $3a$ ft./sec. The space passed over is represented by the area of the figure $HACF$, which can be considered to be made up of two parts, a rectangle HC and a triangle HMF; the total area being $2a + \frac{1}{2} \times 2 \times 2a$ or $4a$. This is equal to the area of the rectangle $ACNP$, in which it is supposed as before that the velocity instead of undergoing a regular increase remains the same, the value being that which is at the middle of the time or half the sum of the initial and final velocities.

This shows that the *average velocity of a body moving with uniform acceleration during a given interval is equal to half the sum of the initial and final velocities.** It shows, moreover, that the space passed over by a body moving with uniform acceleration is given by the relation

$$S = \frac{u+v}{2} t \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

where u is the initial, v the final velocity, and t the time for which the body moves.

But from formula (2) we know that $v = u + at$; substituting this value of v in equation (3) we get

$$S = \frac{u + (u + at)}{2} t$$

$$= ut + \tfrac{1}{2}at^2 \quad . \quad . \quad . \quad (4)$$

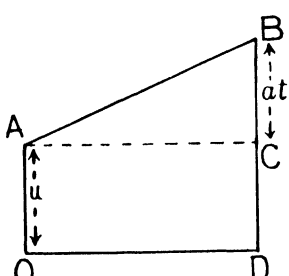


Fig. 4

We can get the same result directly from Fig. 4.

Draw at O, OA representing the velocity u, and let OD represent the time t. At D draw DB, representing $u + at$ or the final velocity. Join AB. The area of the figure $OABD$ (compare it to the figure $HACF$) will give the space passed over by the body in t sec.

The area of the figure is equal to the sum of the area of

* It is also equal to the velocity of the body at the middle of the interval.

the rectangle $OACD$ (ut) and of the ΔABC ($\frac{1}{2}at \times t = \frac{1}{2}at^2$) or $ut + \frac{1}{2}at^2$.

Note, when a = zero, the equation is reduced to $S = ut$, equation (1). Let us modify slightly equation (2); instead of writing

$$v = u + at, \text{ let us write it as}$$
$$v - u = at, \text{ and the equation (3) as}$$
$$v + u = \frac{2S}{t}.$$

Multiplying the corresponding sides of the equations we get

$$v^2 - u^2 = 2aS \quad . \quad . \quad . \quad . \quad . \quad (5)$$

a very useful relation for solving numerical problems.

19. To find the space passed over in the *n*th second.— Sometimes it becomes necessary for a man to know the space passed over by a body in any particular second, say for instance nth. One method is to find out the space passed over in n seconds and also in $n-1$ seconds; subtracting the latter from the former we get the result. Another method would be to determine the average velocity during that second and to get from it the space passed over. As a matter of fact the average velocity itself would be equal to the distance passed over, for the time is 1 second.

The velocity at the end of $(n-1)$th second is $u + (n-1)a$ ft. per sec.

The velocity at the end of n second is $u + na$ ft. per sec.

Average velocity is $u + (2n-1)\frac{a}{2}$ ft. per second.

The space passed over during the nth second is

$$\therefore \quad u + (2n-1)\frac{a}{2} \times 1, \text{ or } u + (2n-1)\frac{a}{2} \quad . \quad . \quad (6)$$

20. To sum up, we have arrived at the following relations:

$S = ut$, when the body is moving with uniform velocity of u ft. per second.

$v = u + at$, when the body is moving with uniform acceleration a ft. per second per second.

$S = \frac{u+v}{2}t$, when $\frac{u+v}{2}$ is the average velocity.

$S = ut + \frac{1}{2}at^2$, when a is the uniform acceleration and t the time for which the body moves.

$v^2 - u^2 = 2aS$, when v is the final velocity, u the initial velocity, a the acceleration, and S the space passed over.

$S = u + (2n - 1)\frac{a}{2}$ gives the space passed over in the nth second.

When initial velocity is zero, we get the following relations :

$$v = at.$$
$$S = \tfrac{1}{2}vt.$$
$$S = \tfrac{1}{2}at^2.$$
$$v^2 = 2as.$$
$$S = (2n - 1)\frac{a}{2}.$$

EXERCISES

When solving numerical examples it is useful to remember that a velocity of 60 miles per hour is equal to 88 ft./sec.

1. Find the speed of a point per minute at the equator due to the rotation of the earth on its axis if the earth's radius is 3963 miles.

The circumference at the equator $= 2\pi r$ or $2 \times 3{\cdot}1416 \times 3963 = 24{,}900$ miles.

This is the distance travelled by the point in one mean solar day, or in 1440 minutes.

$$\therefore \qquad v = \frac{24900}{1440} = 17\ 3 \text{ miles per minute.}$$

2. If a particle were moving at any instant at the rate of $7\frac{1}{2}$ ft. per second, after what time would its velocity be quadrupled if the acceleration be 10 ft. per second per second, and what distance would it describe ?

$u = 7\frac{1}{2}$ ft. per second.
$v = 30$ ft. per second.
$a = 10$ ft. per second per second.
$t = ?$
$S = ?$

$v^2 - u^2 = 2aS$ would give us at once S, for all other quantities are known.

$$900 - \frac{225}{4} = 2 \times 10S.$$

Therefore $$S = \frac{3600 - 225}{4 \times 20} = \frac{675}{16} = 42\frac{3}{16} \text{ ft.}$$

To know time we shall use the formula

$$v = u + at.$$

or $$30 = \frac{15}{2} + 10 \times t.$$

Therefore $$t = \frac{45}{2 \times 10} = \frac{9}{4} = 2\frac{1}{4} \text{ seconds.}$$

3. How long must a body travel with the acceleration of 32 ft. per second per second before it acquires the velocity of light? How far would it move in that time?

Velocity of light $= 186000 \times 1760 \times 3$ ft. per second.

$a = 32$ ft. per second per second; $t = ?$ and $u = 0$.

$v = u + at = at$ for $u = 0$.

$$\therefore \quad t = \frac{186000 \times 1760 \times 3}{32} \text{ seconds},$$

$$= \frac{186000 \times 1760 \times 3}{32 \times 60 \times 24 \times 60} \text{ days},$$

$$\text{or} \quad = \frac{465 \times 55}{72} \text{ days},$$

$$= 355 \text{ days } 5 \text{ hours}.$$

Space passed over or $S = \frac{1}{2}vt$, where v is the final velocity.

$$S = \frac{1}{2} \frac{186000 \times 465 \times 55 \times 60 \times 60 \times 24}{72} \text{ miles}$$

$$= 186 \times 465 \times 55 \times 6 \times 10^5 \text{ miles}$$

$$= 186 \times 465 \times 33 \times 10^6 \text{ miles}$$

$$= 2854170 \text{ million miles}.$$

4. A train going 64 ft. per second is brought up in 64 ft. to avoid a collision. What is the acceleration in yards per second per second; in yards per hour per hour, and in ft. per second per second?

$u = 64$ ft. per second.

$v = 0$.

$a = ?$

$S = 64$ ft.

Use $v^2 - u^2 = 2aS$. Since $v = 0$, hence it is reduced to $-u^2 = 2aS$, or

$$a = -\frac{u^2}{2S}$$

$$= -\frac{64 \times 64}{2 \times 64} = -32 \text{ ft. per second per second}$$

$$= -\frac{32}{3} \text{ yards per second per second}.$$

An acceleration of -32 ft. per second per second means that

In one second velocity of -32 ft. per second is added;
or in one hour the velocity of $-32 \times 60 \times 60$ ft. per second is added;

or in one hour the velocity of $-\frac{32\times60\times60}{3}$ yards per second is added;

" " " $-\frac{32\times60\times60\times60}{3}$ yards per minute is added;

" " " $-\frac{32\times60\times60\times60\times60}{3}$ yards per hour is added.

So we see that the acceleration of -32 ft. per second per second is equal to the acceleration of $-\frac{32\times60^4}{3}$ yards per hour per hour.

5. A body starts with a velocity of 5 ft. per second and has an acceleration of 2·5 ft. per second per second. Find its velocity after 3 seconds as well as the space passed over by it in that time. *Ans.* 12·5 ft. per second, 26·25 ft.

6. A particle is moving under uniform acceleration with a velocity of 100 ft. per second; at the end of 1 minute its velocity is 220 ft. per second. How far will it move in ten minutes, and what will then be its velocity?

Ans. 420000 ft. and 1300 ft. per second.

7. A body has an initial velocity of 25 ft. per second and an acceleration of 1 ft. per second per second, opposite to the direction of motion. At what time will it have moved over half the distance it moves through before coming to rest, and what will be its velocity then?

Ans. $25\left(1-\frac{1}{\sqrt{2}}\right)=7{\cdot}3$ sec.

$\frac{25}{\sqrt{2}}=17{\cdot}7$ ft./sec.

8. If the acceleration with which a body moves is 96 ft. per minute per minute, what is the acceleration in terms of centimetres per second per second?

Ans. ·813 cm./sec.^{2}.

9. A train going 40 miles an hour is brought up in 200 yards by the brakes to avoid a collision. What is the acceleration in miles per second per second, in miles per hour per second, and in ft. per second per second?

Ans. (1) $\frac{11}{20250}$ miles per second per second.

(2) $\frac{88}{45}$ miles per hour per second.

(3) $\frac{1936}{675}$ ft. per second per second.

10. A train moving 40 miles an hour has its speed reduced to 25 miles per hour in one minute 30 seconds. How far would it move in the meantime? *Ans.* $\frac{13}{16}$ miles.

11. A body describes 12 ft. in the 2nd second and 50 ft. in the 4th second of its motion. If the motion be uniformly accelerated, how far will it go in the next 3 seconds?

Ans. 264 ft.

CHAPTER II

BODIES FALLING VERTICALLY UNDER GRAVITY

21. It is a matter of common observation that, as a rule, bodies when left free in air fall to the earth. There are some exceptions to this observation; for instance, when a balloon filled with coal gas is released from the hand, instead of coming down, it goes up. These facts were known to the ancients as well. Before Galileo (1564-1643) the fact that as a rule bodies fall down to the earth when left free and that some of them (that is, light-ones), rise up, was explained on the hypothesis put forward by Aristotle that every body sought its "natural place"—an hypothesis which held the field for two thousand years. This natural place for the heavy bodies was below and for the light bodies above. To distinguish the motion of heavy bodies in the upward direction (as, for instance, when an iron ball is thrown upward) from their downward or "natural" motion, the former was called "violent". Another fact also, that some bodies fell more quickly than others, did not escape the notice of the ancients. The difference in the times which various bodies took in falling from the same height to the ground was supposed to be due to the difference in their nature. How vague seem these notions to us to-day! But all the same, they did hold the field so long. It required a genius like that of Galileo to shatter the absurdity of these notions. He showed clearly that there was no other natural place for bodies except the one on the ground, *i.e.* every body would fall to the ground except when there were other disturbing causes; as, for instance, resistance of air. The rising up of bodies was explained as due to the upward thrust of the air, which is equal to the weight of the air displaced by the balloon. (For a further explanation see § 118 Principle of Archimedes.) He further showed that the time taken by a body to fall from a particular height to the ground was independent of its nature and mass, quite the

contrary of Aristotle's view, who supposed that the time depended upon the nature of the body and was inversely proportional to its mass. For instance, he thought that a chhatack would take 160 times as long to reach the ground as a 10 seer weight would from the same height. Fancy, before Galileo, none thought of challenging this statement and

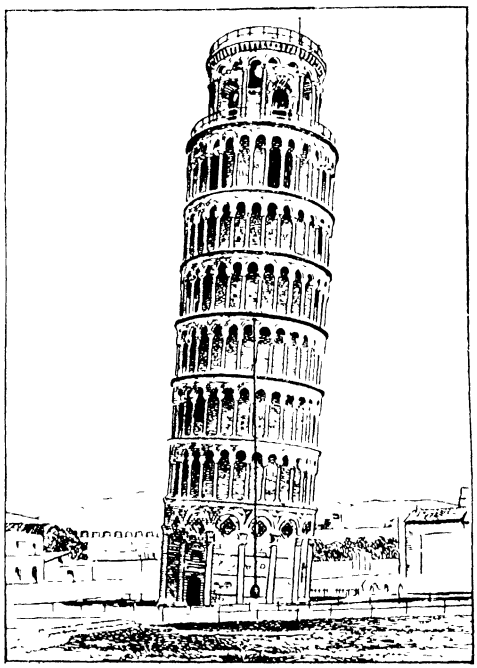


Fig. 5.—Leaning Tower of Pisa

proving it to be fallacious! It was Galileo who in his famous experiment of the leaning tower of Pisa (Fig. 5) shattered Aristotle's views once for all. He released spheres of lead small and great from the tower at the same time, and found that they fell on the ground simultaneously. Next he tried spheres of different materials; even then the time was approximately the same; whatever slight difference was observed was explained to be due to the resistance of air. He next tried bodies of different shapes, spheres, and discs, made up of the same material

and having the same weight. He found that times of fall were widely different. Now, the weights of the different bodies were the same and so was their nature, but still the times of fall were different. On Aristotle's views we could never explain this observation, but Galileo at once explained this as due to unequal resistance offered by air to the bodies; the disc having greater surface experienced greater resistance and therefore took a longer time to reach the ground. It was more conclusively proved a few years later by Newton, "the greatest man in the roll-call of Science, whose genius has impressed contemporaries and posterity alike as almost superhuman," by his famous *feather and guinea experiment.* He took a glass tube about 6 ft. long and 3 inches in diameter closed at one end. A guinea and a feather were then inserted, and the other end closed with an air-tight cap and a stop-cock. The tube was attached to an air-pump and the air was exhausted. The tube was detached and was inverted. The feather and guinea fell side by side, down the tube. This shows that all bodies (in vacuum) move towards the earth with velocities which are equal at the end of equal intervals of time. Since the bodies start from rest and possess equal velocities after equal intervals of time, this shows that *the acceleration is the same for all bodies in vacuum at the same place.* The acceleration is due to the action of gravity. Thus we come to the conclusion that the *pull of gravity has nothing to do with the nature of the material of the body.*

22. Value of the acceleration due to gravity.—We have said above that acceleration is the same for all bodies in vacuum at the same place. Mark the words *at the same place.* It shows that acceleration may be different at different places. As a matter of fact it is so. At the poles it is greatest and at the equator smallest. At the depth of an ocean it is greater than at the top of a mountain.

At the equator it is 32·04 ft./sec.^{2} or 978·1 cm./sec.^{2}, and at the poles it is 32·26 ft./sec.^{2} or 983·2 cm./sec.^{2}. In calculations it is usual to take the value of acceleration (generally called g) as 32 ft. per second per second, or 981 cm. per second per second.

As remarked above, the value of g is different at different places. The difference may arise from the difference in distance of the body from the centre of the earth, or it may arise from the various mechanisms used. We shall consider these later on.

Since the acceleration due to gravity is uniform, all the formulae that we have proved in the last chapter will hold

good in the case of bodies falling or rising vertically; we shall have to substitute g in place of a. The formulae will become as below:

$$v = u + gt,$$

$$S = \frac{u+v}{2}t,$$

$$= ut + \tfrac{1}{2}gt^2,$$

$$v^2 - u^2 = 2gS,$$

and the space passed over in nth second $= u + (2n - 1)\frac{g}{2}$.

One point should be noted here, and that is the sign of g. For instance, if the initial velocity of a body which is thrown upwards is taken as positive, g must be taken as negative. But if there is no initial velocity, direction of g might be considered as positive. *It is immaterial whether "g" is taken as positive or negative, but whatever sign is given to it, the same convention must be followed rigidly throughout one and the same problem.* We shall consider a few special cases of importance.

(1) *A body is projected upwards with a velocity u ft. per sec., to find the greatest height to which it will rise.*

When a body is thrown upwards, its velocity goes on constantly decreasing, due to the downward pull of the earth. At the point of greatest height its upward velocity must be zero.* Let us use the equation

$$v^2 - u^2 = 2gh,$$

bearing it in mind that u is upward and positive and g is downward and therefore negative; substitute zero for the final velocity v in the above equation. It reduces to

$$-u^2 = -2gh.$$

$$\therefore \quad h = \frac{u^2}{2g} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (i)$$

$$= \frac{u^2}{64},$$

where it is supposed that g is 32 ft. per second per second.

(2) *To find the time to reach the highest point.*

To find the time we shall make use of the same fact, that velocity is zero at the highest point. Let us use equation

$$v = u + gt.$$

* For after that the velocity changes its direction, and the body begins to move downwards instead of upwards.

Since u is positive, hence g must be negative. Bearing this in mind, we get

$$0 = u - gt.$$

or
$$t = \frac{u}{g} \quad . \quad . \quad .

(4) *To find the velocity of a body with which it falls on the ground when projected upwards with a velocity u ft. per second.*

Total time of journey $= 2u/g$. Making use of this fact, let us see what would be the velocity with which it falls on the ground. We know that

$$v = u + gt,$$

where u is positive and g is negative.

Substituting the value $\frac{2u}{g}$ for t we get

$$v = u - g\frac{2u}{g}$$
$$= -u.$$

This shows that the velocity with which a body falls on the ground is numerically equal to the velocity with which it is projected upwards.

EXERCISES

1. A bullet is fired vertically with a velocity of 1600 ft. per second. After how many seconds will it return to the earth ? What is the greatest height reached ?

$u = +1600$ ft. per second,
$g = -32$ ft. per second per second,
$t = ?$
$H = ?$
$S = 0.$

(1) Using equation $S = ut + \frac{1}{2} gt^2$ we get

$$0 = 1600 \times t - 16\ t^2.$$

Therefore $t = 100$ seconds.

(2) At the greatest height $v = 0$.

Therefore $v^2 - u^2 = 2gS$ reduces to $-u^2 = 2gS$,

or $$-1600 \times 1600 = -2 \times 32 \times H,$$

or $$H = \frac{1600 \times 1600}{2 \times 32} = 40{,}000 \text{ ft.}$$

2. A stone falls over a cliff and travels 80 ft. in the last second before it strikes the ground at the foot of the cliff. How high is the cliff ?

Here the space passed over in the last (call it nth) second is given. Using $S = (2n-1)\frac{g}{2}$ we can find out the value of n,

$$80 = (2n-1)\ 16.$$

Therefore $2n = 6,$

or $n = 3.$

Thus we see that the last second was the 3rd one ; *i.e.* the stone travels for 3 seconds in all. To find the height of the cliff we shall use equation $S = ut + \frac{1}{2} gt^2$. In this case the stone starts from rest, hence $u = 0$.

Therefore
$$S = \tfrac{1}{2} gt^2$$
$$= 16 \times 9$$
$$= 144 \text{ ft.}$$

3. A stone is dropped from a balloon moving vertically upwards with a uniform velocity of 48 ft. per second, and is found to reach the ground in 4 seconds. Find the height of the balloon (i) when the stone was dropped, (ii) when the stone reached the ground.

The stone was moving upwards along with the balloon with a velocity of 48 ft. per second, hence when it was dropped it had an upward velocity of 48 ft. per second.

$S = ?$
$t = 4$ sec.
$u = 48$ ft. per sec.
$g = -32$ ft./sec.^{2}

use $S = ut + \tfrac{1}{2} gt^2$,
or $S = 48t - 16t^2$
$= 48 \times 4 - 16 \times 16$
$= 192 - 256$
$= -64$ ft.

Just note that 64 ft. is the space passed over from the starting point in the downward direction which has been taken negative in this question. The actual distance passed over will be 36 ft. above the point where it was dropped, 36 ft. back from the highest point to the starting point, and 64 ft. downwards.

(ii) To find out the height of the balloon at the time when the stone reached the ground we have simply to make use of the relation,
$$S = ut$$
$$= 48 \times 4$$
$$= 192.$$

Since the balloon was already 64 ft. high at the time when the stone was dropped, and during the period of fall it has risen through 192 ft., therefore the total height from the ground $= 192 + 64 = 256$ ft.

4. A stone is dropped from a height of 288 ft., and another is at the same instant projected vertically up to meet it. They pass at half the height. How high will the second stone rise, and with what velocity does it start?

Let the stones meet after t seconds, and let the velocity with which the second stone is projected upwards be u ft. per second.

The first will come down during t seconds through
$$\tfrac{1}{2} gt^2 \text{ ft. or } 16t^2 \text{ ft.}$$
Therefore
$$16\, t^2 = 144,$$
or
$$t = 3 \text{ seconds.}$$

Let us first find the velocity with which the second stone is projected upwards. We are given the following data:

$S = 144$ ft.
$t = 3$ seconds
$g = -32$ ft./sec.^{2}
$u = ?$

use $S = ut + \frac{1}{2}gt^2$,
or $144 = u \times 3 - 16 \times 9$,
or $288 = u \times 3$;
$\therefore \quad u = \frac{288}{3}$ ft./sec., or 96 ft. per sec.

To find how high the second stone will rise we shall make use of the fact that at the highest point the velocity is zero. Using $v^2 - u^2 = 2gS$ we get

$$0 - 96 \times 96 = -2 \times 32 \times S;$$

therefore

$$S = \frac{96 \times 96}{2 \times 32}$$
$$= 144.$$

Thus we see that the second stone will rise through 144 ft.

5. If a body fell at a certain place 50 ft. in the 3rd second of its fall from rest, what will be the value of acceleration there? *Ans.* 20 ft. per second per second.

6. A body is projected upwards with a velocity of 120 ft. per second; what is the greatest height to which it will rise, and when will it be moving with a velocity of 40 ft. per second? *Ans.* 225 ft.; $2\frac{1}{2}$ or 5 seconds.

7. From a balloon which is ascending with a velocity of 80 ft. a second a stone is dropped, and is seen to strike the ground in 7 seconds. Find the height of the balloon at the moment of dropping the stone. *Ans.* 224 ft.

8. The intensity of gravity on Jupiter is 2·6 times as much as on the earth. How long will a body take to fall on Jupiter from a height of 167 ft.? *Ans.* 2 seconds.

9. A balloon is going up at the rate of 80 ft. per second, and when at the height of 800 ft. a stone is dropped. When, and with what velocity, does it hit the ground?

Ans. in 10 seconds; 240 ft. per second.

10. A stone is dropped from the top of a cliff 160 ft. high, at the same instant another is thrown upwards with a velocity of 80 ft./sec. from the bottom of the cliff. When and where will the two stones meet?

Ans. 2 seconds; 64 ft. from the top of the cliff.

CHAPTER III

COMPOSITION OF MOTIONS

23. As has been said already, all quantities can be conveniently divided into two groups. (1) Scalar quantities which possess magnitude only, as, for instance, 2 seers, 3 rupees, and so on, and (2) vector quantities which possess in addition to the magnitude, direction, as velocity or acceleration. To add the scalar quantities we need use arithmetical laws only; for instance, if we have two sums of money with us, 3 rupees and 5 rupees, we add them as $(5+3)=8$ Rs. But when we have to deal with two vector quantities, this sort of addition does not serve the purpose, except in the special case when the direction is the same.

In this chapter we shall deal with the addition of vector quantities.

24. Let us consider the motion of the point O. Suppose it moves through a distance OA, and when it reaches A it takes the direction AB till it reaches B. Its distance from O is not $OA+AB$, but OB. It would reach the same point B if it would first go along OC (equal to AB) and then along $CB(=OA)$. Thus we see that *the order in which the displacements are given to the point O is immaterial.* The point O would reach the point B if a single displacement OB is given to it. The single displacement which produces the same effect as the two separate displacements is called the resultant displacement. It is represented, as is seen from the above figure, by the diagonal

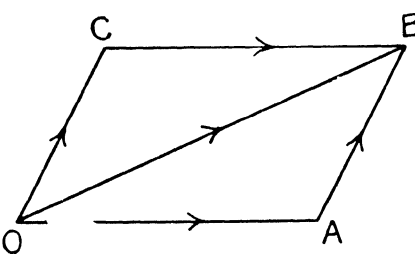


Fig. 5A

of the parallelogram constructed on the two lines representing the displacements as the adjacent sides.

25. When these two displacements take place simultaneously in the course of 1 second they represent velocities, and OB represents the resultant velocity, which is equivalent to them both. It is not very difficult to imagine a body possessing two velocities simultaneously. Suppose, for instance, a ball is thrown with a certain velocity in a horizontal direction by a passenger inside a railway carriage against the opposite side of the carriage when the train is at rest. Next, the same ball is thrown in the same direction with the same force when the train is in motion. It strikes the opposite side at the same point with the same velocity. This means that the motion of the ball is not in the least affected by the motion of the carriage, so far as its motion relative to the carriage is concerned. But the carriage in the meantime has moved forward, and in that onward movement the ball also has taken part. Its actual motion then, as regards the earth, has not been at right angles to the rails, but in a slanting direction. For instance, suppose OA represents u, the velocity of the train, and OC or AB represents v, the velocity with which the ball is thrown horizontally across the carriage. When the train is at rest the ball strikes the point C of the opposite side of the carriage, but when it is in motion the point C, where the ball strikes, is itself carried to B, hence we see that the ball with respect to the rails has travelled along OB. To be sure that the ball has actually moved along OB we shall consider where it lies after half a second. During this interval the train has gone over $\frac{1}{2}\ u$ ft., and the ball $\frac{1}{2}\ v$ ft., since both the velocities are uniform.* So we see it lies at a point $\frac{1}{2}\ u$ ft. along OA and $\frac{1}{2}\ v$ ft. along OC or AB. This point is evidently D, and it lies on OB. Similarly we can show

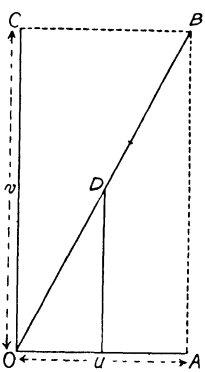


Fig. 6

* The acceleration due to gravity is neglected, for it will produce very little effect in very short periods with which we are dealing at this time. Suppose, for instance, OC is 8 ft. and that the velocity v of the ball is 120 ft. per sec.; it will take 1/15th of a second to go from O to C. During this time it will pass downwards over $\frac{1}{2}gt^2$ or $16 \times \frac{1}{225}$ inch or 0·8 of an inch, which is negligible in comparison to OA. Hence we suppose that the body moves horizontally with uniform velocity across the carriage.

that after $\frac{1}{4}$ or $\frac{1}{8}$ of a second the ball would be on the line OB. Generalising, we come to the conclusion that the ball actually travels along the diagonal OB of the parallelogram $OACB$.

This law of addition is called the **parallelogram law.** It may be stated thus :

If a particle possesses two uniform velocities simultaneously, on representing them in magnitude and direction by two lines drawn from a point, the resultant velocity is completely represented by the diagonal passing through the same point of the parallelogram constructed with the two above-said lines as adjacent sides.

It should be borne in mind that *both the velocities must be either towards or away from the point.*

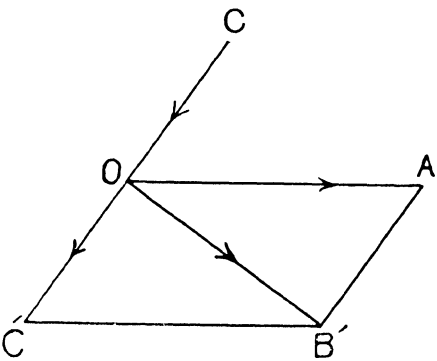


Fig. 7

Example.—Let us suppose that OA and OC are the two velocities, both away from the point (Fig. 5A) ; OB gives the resultant. If, on the other hand, one of the velocities was CO instead of OC, then in order to find out the resultant we must produce CO to C' where $OC' = CO$. Now the two adjacent sides of the parallelogram are OA and OC'. The resultant is given by OB'.

26. Graphical Method of determining the Resultant of Two Velocities.—Let us suppose we have two velocities, u and v, at an angle Θ. We want to find out their resultant. Draw OA to represent the velocity of u ft. per second. On the same scale draw OB, making an angle Θ with OA to represent the velocity of v ft. per second. Complete the parallelogram and measure the diagonal passing through O. It represents the resultant in

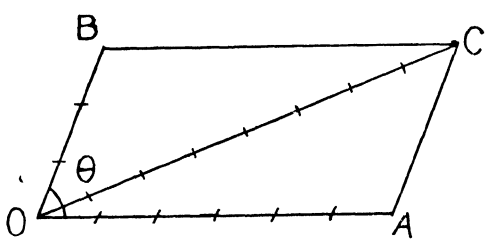


Fig. 8

magnitude (on the same scale) as well as direction. To get accurate results as large a scale as the paper permits should be used, because the larger the scale the greater the accuracy.

This is only an approximate method. The imperfections are due to (i) error in measuring angles and (ii) error in measuring distances. This method is useful in very complicated cases where the calculations are too tedious.

27. To find the Resultant by Calculation.—Let u and v be the two velocities as before, and Θ the angle between them. Complete the parallelogram $OABC$. OB evidently represents the resultant R. Drop BD perpendicular to OA produced from B. The angle $BAD = \angle COA = \Theta$.

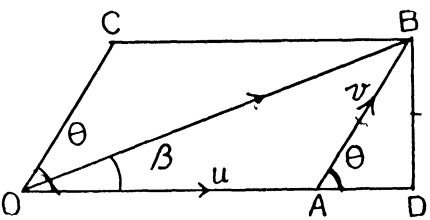


Fig. 9

Now $\frac{BD}{AB} = \sin \Theta$.

Therefore $BD = AB \sin \Theta$, or $v \sin \Theta$.

Similarly $AD = v \cos \Theta$.

Consider the triangle OBD. Since it is a right-angled Δ,

$\therefore$ $$OB^2 = OD^2 + BD^2,$$

or $$R^2 = (u + v \cos \Theta)^2 + (v \sin \Theta)^2$$
$$= u^2 + v^2 \cos^2 \Theta + 2uv \cos \Theta + v^2 \sin^2 \Theta$$
$$= u^2 + v^2 (\cos^2 \Theta + \sin^2 \Theta) + 2uv \cos \Theta$$
$$= u^2 + v^2 + 2uv \cos \Theta,$$

or $$\boldsymbol{R = \sqrt{u^2 + v^2 + 2uv \cos \theta}}.$$

This is a very useful relation; it at once enables us to find out the resultant of any two component velocities, u and v, acting at an angle Θ.

To know the direction of the resultant the following relation is useful. Let the resultant make an angle β with OA:

$$\tan \beta = \frac{BD}{OD} = \frac{v \sin \theta}{u + v \cos \theta}.$$

Let us now consider some special cases:

(1) When θ is zero, *i.e.* direction is the same,

$$R^2 = u^2 + v^2 + 2uv,$$

or $$R = u + v.$$

It is just what is expected, because when direction is the same the component velocities would simply be added up.

Therefore resultant has the same direction as the two velocities.

(2) When $\theta = 90°$, *i.e.* the components are rectangular,

$$R^2 = u^2 + v^2 + 2uv \times 0,$$

or
$$R^2 = u^2 + v^2,$$

as expected, for in a rectangle the diagonal will be given by this result. Moreover, when $u = v$, $R^2 = 2u^2$, and the direction is given by

$$\tan \beta = \frac{v}{u} = 1, \quad \text{or } \beta = 45°.$$

(3) When $\theta = 180°$, *i.e.* the components are in the opposite directions,

$$R^2 = u^2 + v^2 + 2uv \cos 180°$$
$$= u^2 + v^2 - 2uv,$$

or
$$R = u - v.$$

Thus we see that this relation at once enables us to find out the magnitude and the direction of the resultant.

When we have to deal with more than two velocities we proceed as follows. We find the resultant of any two velocities, and then the resultant of this (resultant) and the next velocity, and so on. The last resultant so found would be the resultant of all the velocities.

28. Triangle of Velocities.—Let us go back to Fig. 5A. We know that two velocities OA and OC are equivalent to a single velocity OB. Now $OABC$ is a parallelogram, hence $AB = OC$, and therefore AB represents the same velocity as OC. In the triangle OAB, the side OA represents one velocity, AB represents the second velocity, and OB (the third side) represents the resultant. Thus we see that if the two sides of a triangle be taken in order to represent the two velocities, the third side in the opposite order represents the resultant. This method of composition of velocities is called the **Triangle of Velocities.** It is defined as follows :

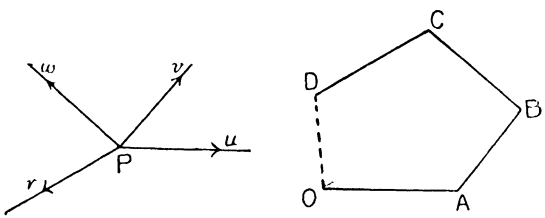
If a particle possesses two uniform velocities simultaneously, represented by the two sides OA, AB, of a triangle taken in order, they are equivalent to a velocity represented by the third side in the opposite order.

A very important corollary which follows directly from the triangle of velocities is that, if there be impressed on the particle three velocities simultaneously which can be represented by the three sides of a triangle taken in order, the particle will be at rest, for the third side is opposite in direction to the resultant of the other two.

29. Polygon of Velocities.—Let us try to find next the resultant of a number of velocities simultaneously impressed

upon a particle. The magnitude and direction of each velocity is known; let them be represented in magnitude and direction by u, v, w, and r, as shown in the Fig. 10 (a).

In order to find out the resultant let us draw OA parallel and equal in length to u. This will represent the velocity u. Let, further, AB be drawn parallel and equal to v. OB would represent the resultant of these two. Draw BC parallel to w and equal to it in length. OC would represent the resultant of OB and BC. Draw next CD parallel to r and equal in length to it. OD would represent the resultant of OC and CD. Since OC is equivalent to BC and OB or BC, OA and AB,



(a) Fig. 10 (b)

we see that OD is equivalent to all the four velocities impressed on the particle.

This method of compounding velocities is called the **Polygon of Velocities.** It can be stated as follows:

If a particle possesses velocities represented by the sides OA, AB, BC, CD, . . . KL of a polygon, the resultant velocity is represented by OL, the closing side of the polygon.

If L coincides with O, the polygon becomes a closed one, the resultant velocity vanishes, and the particle would be at rest.

In order to find the magnitude of the resultant we need represent a certain velocity by a definite length, then proceeding as explained above we find out the length of the resultant, and therefrom we find out its magnitude.

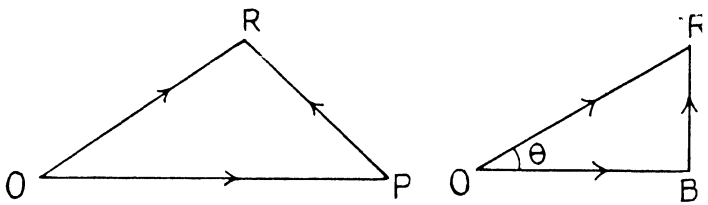
30. Resolution of Velocities.—So far we have been compounding two or more velocities into one; now let us see how to resolve a velocity into two components along definite directions. *Finding out the components of a velocity is called the* ***Resolution.*** Suppose OR is the velocity, and we want to resolve it into two components along OP and PR [Fig. 11 (a)]. Draw through O a line parallel to OP, and through R parallel

to RP, meeting each other at P. Now in the triangle OPR we see that OP and PR are equivalent to OR. Generally we require only rectangular components. Suppose OR is the velocity (equal to u ft. per second), and we want to resolve it into two components at right angles to one another, one of them along the line OB making an angle Θ with OR [Fig. 11 (b)]. Draw RB perpendicular to OB. Then the velocity OR is equivalent to OB and BR (from triangle law).

Also $$OB = OR \cos \Theta = u \cos \Theta,$$

and $$BR = OR \sin \Theta = u \sin \Theta.$$

The parallelogram, the triangle, and the polygon laws of velocities will also hold good in the case of uniform accelerations. Even the method of resolution explained above will hold good. Hence we shall not repeat them. But we shall



(a) Fig. 11 (b)

pass on to the composition of a uniform motion with an accelerated one.

31. To compound a Uniform Velocity with a Uniformly Accelerated Velocity.—Let a body be thrown with horizontal velocity of u ft./sec.; on account of gravity it will have acceleration downwards. Due to its horizontal velocity, it will pass over u ft. in the horizontal direction in one second, and due to gravity it will fall vertically through $\frac{1}{2}g$ or 16 ft. in one second. Notice that we are supposing that the body starts to fall down with zero velocity, for it has no velocity in the vertical direction to start with. After the *2nd* second the body is 2 u ft. from the starting point in the horizontal direction and 2 g or 64 ft. below the horizontal line, and hence it is at B (Fig. 12); after the 3rd second it is at 3 u ft. from the starting point and 4 5 g or 144 ft. below the horizontal line, and hence is at C, and so on. Plotting these points, we find that the curve joining them all is a parabola.

Just note that the horizontal velocity does not interfere with

the vertical velocity generated by the gravity. The two are quite independent of each other.

Let us suppose that a cannon ball is fired from a cannon with a velocity of 40 ft. per second, the cannon standing 16 ft. above the level of the ground. The ball would take one second to fall down, and in the meantime it would travel 40 ft. in the horizontal direction, hence it will fall on the earth at a distance of 40 ft. from the cannon. The path described by it would be a parabola.

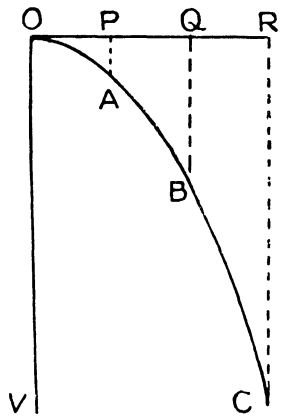


Fig. 12

32. Relative Velocity. — It has already been remarked that motion is relative. So far we have been considering motion of bodies with respect to their surroundings which we regarded as fixed, but which were not actually so. For instance, in the preceding article we have supposed that the path of the cannon-ball fired from a cannon with a horizontal velocity of 40 ft. per second is a parabola. It is so with respect to the earth. But what it is with respect to space is a difficult matter to tell. However, we are not the worse for it, for we are not required to know the path with respect to space. All that we want to know is the velocity with respect to the earth, considering it to be at rest.

As a matter of fact here we have been making use of a general principle unconsciously, that to find out the relative motion of one body with respect to another, be this second body at rest or in motion, it is convenient to regard it as fixed. We shall take some examples to clear this point up.

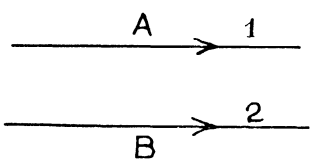


Fig. 13

(1) Consider the case of two trains moving on parallel rails in the same direction with equal velocities, say u ft. per second, and let two passengers, A and B, look at each other. A, if he be unconscious of his own motion, will regard B to be at rest. In other words, the velocity of B with respect to A is zero.

(2) Let the train No. 2 be moving faster than the train

No. 1 (suppose the train No. 2 is going at the rate of 30 miles an hour, and the train No. 1 at the rate of 25 miles per hour). The passenger B will seem to the passenger A to be moving at the rate of 5 miles an hour.

(3) Let the train No. 2 move in the opposite direction to train No. 1. The passenger B will seem to the passenger A to be moving at the rate of 55 miles per hour.

Before proceeding further let us analyse our conclusions from these examples. It would be seen that all along we have supposed A to be at rest, and then we have determined the velocity of B with respect to it. For instance, in the first example if we impress velocity of $-u$ ft. per sec. upon both the trains, train No. 1, and hence passenger A, is brought to rest, and so is the case with B. Therefore we say that B is at rest with respect to A. In the second example we impress upon train No. 1, and consequently upon passenger A, a velocity of -25 miles an hour, and similarly upon B, whose resultant velocity becomes 5 miles an hour. In the third example the resultant velocity of B becomes $-(30+25)$, or -55 miles per hour. From this we arrive at the following conclusion :

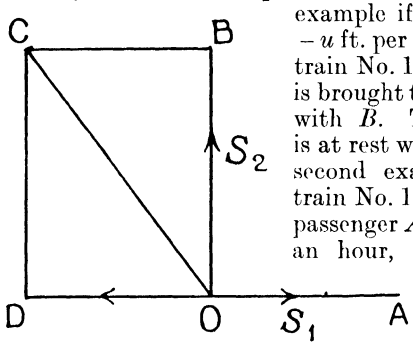


Fig. 14

That the relative velocity of a body B with respect to a body A, when both of them are in motion, is obtained by compounding with the velocity of B a velocity which is equal and opposite to that of A.

Let us take one more example. Suppose one ship is sailing due east at the rate of 12 miles per hour, and a second ship is sailing due north at the rate of 16 miles per hour, and we want to determine the relative velocity of the second ship with respect to the first.

Impress upon S_2 and S_1 both a velocity of -12 miles per hour in the direction of due west, *i.e.* AO. The ship S_1 would be brought to rest, and S_2 will appear to move along OC, about 37° west of north, with a velocity of 20 miles per hour (Fig. 14).

EXERCISES

1. A boat is rowed across a river at the rate of 3 miles per hour, the river is flowing at the rate of 4 miles per hour. Find the velocity of the boat.

Here on the boat two velocities are impressed, one 3 miles per hour and the other 4 miles per hour perpendicular to the first one. The resultant velocity is

$R = \sqrt{3^2 + 4^2} = 5$ miles per hour.

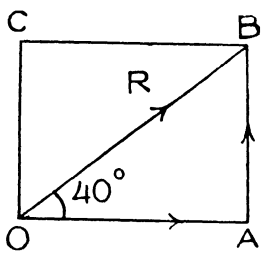


Fig. 15

2. Find the rectangular components of a velocity of 60 ft. per second, one of them should make an angle of 40° with the original velocity.

The component along OA is $R \cos 40°$, and along AB or OC is $R \sin 40°$.

Therefore the components are 60 cos 40°, or $60 \times 0\,766$—*i.e.* 45 960 ft. per second and 60 sin 40°, or $60 \times 0\,6428$—*i.e.* 38 5680 ft. per second.

3. A man is walking towards the east at the rate of 4 miles an hour in a shower of rainfall. The rain comes at an angle of 45° west of north with a velocity of $2\sqrt{2}$ miles per hour. At what angle does the rain strike the man ?

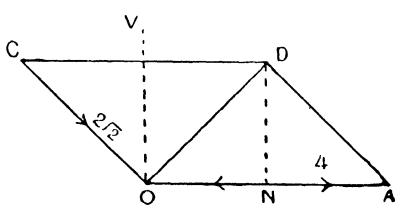


Fig. 16

Let the man go along OA and let the shower fall along CO. We want to determine the angle at which the shower will strike the man. Impress upon the man as well as the shower a velocity of 4 miles per hour in the direction AO. The man will be brought to rest, and the resultant of CO and AO will be DO. Hence the rain will appear to fall down along DO.

The angle $COA = 135°$,
The angle $DOA = \beta$,

$$\tan\beta = \frac{2\sqrt{2}\sin 135°}{4 + v\cos 135°}$$

$$= \frac{2\sqrt{2} \times \frac{1}{\sqrt{2}}}{4 - 2\sqrt{2}\,\frac{1}{\sqrt{2}}} = 1$$

$$\left\{\begin{array}{l}\cos 135° = \cos(90+45) \\ \qquad\quad = -\sin 45°, \\ \sin 135° = (90+45) \\ \qquad\quad = \sin 45°.\end{array}\right.$$

$$\therefore \qquad \beta = 45°.$$

We can prove it directly also. Drop perpendicular DN from D on OA. $\angle DAN = \angle ADN = 45°$, $\therefore DN = NA = 2$.

Since $OA = 4$, and $NA = 2$,

Hence $ON = 2$, therefore $\angle DON = 45°$.

4. A boat is rowed at right angles to the bank of a straight river at a pace half as fast again as the stream flows; it reaches the opposite bank 2 miles below the starting point. Find the breadth of the river and the distance rowed.

Ans. 3 miles, $\sqrt{13}$ miles.

5. A river 100 yards broad is running downwards at the rate of $\frac{1}{2}$ mile per hour, and a swimmer who can swim at the rate of 1 mile per hour wishes to reach a point just opposite. Along what line must he strike out, and how long will he take in crossing? *Ans.* 60° with the bank, 3·941 min.

6. Two equal velocities have a resultant equal to either. At what angle are they inclined to each other? *Ans.* 120°.

7. A velocity of 500 ft. per second is resolved into two components at right angles. One of these is 250 ft. per second; find the other. *Ans.* $250\sqrt{3}$ per second.

8. One of the rectangular components of a velocity of 60 miles per hour is a velocity of 30 miles per hour; find the other component. *Ans.* $30\sqrt{3}$ miles per hour.

9. Two trains start from a station with velocities of 25 and 40 miles per hour, along two lines inclined at an angle of 60°. Find their relative velocity and the distance they will be apart in 10 minutes. *Ans.* 35 miles, $5\frac{5}{6}$ miles.

10. A man is walking to the east at the rate of 2 miles per hour in a shower of rainfall. The rain appears to him to come down vertically at the rate of 2 miles an hour. Find the actual velocity and the direction of the rainfall.

Ans. $2\sqrt{2}$ miles per hour, 45° west of north.

CHAPTER IV

LAWS OF MOTION

33. So far we have been considering motion in the abstract without any reference to the matter possessing it or to the forces producing it. Now let us pass on to the motion of real bodies under the action of forces. The laws of the motion of bodies were first enunciated clearly by Newton. They run as follows :

First Law.—*Every body continues in its state of rest or of uniform motion in a straight line except in so far as it is compelled by impressed forces to change that state.*

Second Law.—*The rate of change of momentum is proportional to the impressed force, and takes place in the direction of the impressed force.*

Third Law.—*To every action there is an equal and opposite reaction.*

We shall consider them one by one.

34. First Law of Motion.—The first law contains two statements. The first of these, that a body continues in its state of rest except in so far as it is compelled by the impressed forces to change that state, seems to be self-evident. A book kept in an almirah in a library continues to be at the same place except when it is moved from there. The second part, that it will continue to move uniformly in the same straight line unless some other force acts upon it is rather difficult to understand in the beginning, because our everyday experience, that all moving bodies on the earth's surface show a tendency to stop, *seems* to contradict it. A little reflection will convince us, however, that the law is true enough, but that since all bodies in motion on the surface of the earth are in reality subject to frictional forces as well as resistance offered by the air, hence it is impossible to prove it experimentally. The more we eliminate the external forces the longer the

motion continues. For instance, when a ball is set rolling on earth it stops after traversing a short distance; if the same ball, on the other hand, be allowed to roll on a smooth surface it rolls over a longer distance (on a smooth sheet of ice the ball rolls over a much longer distance than over the earth's surface). A top rotates in air hardly for about 20 minutes, but in exhausted air it continues to rotate in the "sleeping" condition for more than 2 hours. These experiments lead us to the generalisation that if *all* the external forces were eliminated the body would continue in its state of motion *for ever*.

Our everyday experience also serves to deepen our faith in the truth of the law if we only carefully reflect upon what we see. For instance, a man who steps out of a rapidly moving train falls to the ground if he does not run forward, on account of the fact that he was participating in the motion of the train, and when he jumps out of it, his feet being arrested by the ground are brought to rest, but the upper part continues to move forward, and that is why he falls down.

The strongest proof, however, of the first law of motion as well as of the other two laws is indirect. On the basis of these laws the astronomical observations are predicted four years beforehand in the *Nautical Almanac*. The places of the sun, moon, and other satellites, the duration and the commencement of the eclipses, are all foretold to a wonderful degree of accuracy. Not even in a single case our confidence in the truth of these laws has ever been proved wrong, which shows that the laws underlying our calculations are without a flaw.

The property enunciated by the first law is called **Inertia.** It stands for the inability of a material body to change by itself its condition of rest or of uniform motion in a straight line. It is on account of this property that matter requires force to change its state. Sometimes the first law of motion is referred to as the *Law of Inertia*. It will not be out of place to mention that this was considered to be the sole test of matter by Newton. Inertia is a general property of matter, and it is made use of to measure the quantity of matter or mass in a given body.

The first law states that unless some force from without acts upon a body its motion continues unchanged.

The second law tells us how the motion will be changed when a force acts upon it.

35. The Second Law of Motion.—Before we deal with this law of motion, which says that the rate of change of momentum

is proportional to the impressed force and takes place in the direction of the force, it will be useful to understand what we mean by the term **Momentum.** Let us support a cricket ball on our hand. We are hardly conscious of the effort that we have to make to support it. Let us throw it up slowly and try to stop it when it comes back into our hand. This time the force required to stop it is greater. Throw it up with a great force and try to stop it when it comes down. Our hand will have to exert much greater force now to stop it. A bullet requires little force to support it, but when it is fired from a gun and strikes against the palm of our hand, it passes through, piercing it, which means that the hand cannot exert enough force to stop it. This leads us to the conclusion that the force required to stop a body depends upon its velocity.

Let us suppose next that a billiard ball and a cricket ball are moving with the same velocity. In order to stop the latter we require a much greater force than we do to stop the billiard ball. Combining both these aspects we come to the conclusion that the force exerted by a body depends upon two things—(*a*) its mass and (*b*) its velocity. Both these factors are included in the term momentum, which is defined as follows :

The momentum of a body is the quantity of motion it possesses, and is equal to the product of its mass and velocity.

The second law can be expressed mathematically as

$$F \propto \frac{mv - mu}{t},$$

or

$$\propto m\frac{(v-u)}{t}$$

where F is the force, m the mass of the body, and v and u denote the velocity at the end and beginning, and t the time during which the velocity changes.

Since $\frac{v-u}{t} = a$, the acceleration,

hence $$F \propto ma,$$

or $$F = k \times ma$$

where k is some constant.

In order to simplify matters, the unit of force adopted is such that it produces unit acceleration in unit mass. Under these circumstances we have

$$1 = k \times 1 \times 1$$

or $$k = 1.$$

Therefore the equation $F = kma$ is reduced to

$$\mathbf{F = ma,}$$

or, in other words, the force is equal to the product of the mass of a body and its acceleration.

Since mass and acceleration are measured differently in the F.P.S. and C.G.S. systems, there is a different unit of force in each system.

When unit of mass taken is 1 pound, and unit of acceleration 1 ft./sec.^{2}, the unit of force is called a **Poundal.** *It is that much force which produces in a mass of one pound acceleration of* 1 *ft. per second per second.*

When unit of mass taken is 1 gram, and unit of acceleration 1 cm./sec.^{2}, the unit of force is called a **Dyne.** *It is that much force which produces in a mass of one gram an acceleration of* 1 *cm. per second per second.*

The poundal and the dyne are termed the absolute units of force.

36. Weight.—We have already learnt that every body is pulled downwards by the earth. Due to this pull every body falls or tends to fall down on the earth, in other words, every body exerts a downward force which is called its **Weight.** This downward force or weight W is equal to the product of the mass of the body and the acceleration produced in it by the force of gravity;

i.e. $$W = mg$$

where m is the mass, and g the acceleration due to gravity. The weight W' of another body having m' as the mass would be

$$W' = m'g,$$

since the acceleration produced in a body is independent of its mass (see § 21).

Divide one by the other.

$$\frac{W}{W'} = \frac{mg}{m'g} = \frac{m}{m'}.$$

Thus we see that the weights of two bodies at the same place are in proportion to their masses. Of course the weight of a body is not a constant quantity, since g is different at different places. The weight of a body would be different at the poles from its weight at the equator.

37. Unfortunately, too frequently hopeless confusion is made by a beginner between these two terms *mass* and *weight.* The confusion arises on account of the fact that we compare

masses of bodies by *weighing*. For instance, a man goes to the bazaar to buy 10 lb. of sugar. The shopkeeper compares not the mass but the weight of the sugar with the weight of his standards. But, on the other hand, the buyer is thinking of the mass in his mind. He would be glad to have one handful more ; he does not bother himself about the force with which the earth pulls it down. He is thinking of the quantity all along and would wish very much to increase it. But when he carries it home, the inconvenience which he feels is due to the weight, and if it could be diminished without decreasing the mass the buyer would hail the proposal. Thus in this example in the first part we are concerned with the mass and in the second part with weight. This quick succession of the ideas of mass and weight, coupled with the shopkeeper's weighing, leads to confusion between the two notions. Another cause of confusion is the carelessness in the ordinary language, to use words like one pound both for mass as well as weight. We shall always make a distinction between these two notions, whenever there is any chance of confusion, by calling the weight as 1 lb. wt. and the mass as one pound.

To bring home to the student the difference between mass and weight we shall take one more example. Suppose we weigh out 201 pounds of gold at one of the poles with a spring balance and give it to a gentleman to hand it over to some banker in Colombo. The same piece of gold when weighed there with the same spring balance will be found to weigh about 200 pounds only. During this transaction the quantity (mass) of gold has remained the same, whereas the force with which it tends to come down has decreased. Similarly, if it were taken to a place 4000 miles above the surface of the earth, its weight would become 50 lb. In scientific language we say that this piece of gold tends to come down there with a force of 50 lb. wt. The mass or the quantity of gold does not change in the least. Let us next take it to the centre of the earth ; there it has no weight at all, for it is not being attracted in any direction now, but all the same its mass has not changed. (In this experiment we must use a spring balance.)

This example helps us to think of the mass as something different from the weight.

38. It should be noted that an ordinary pair of scales helps us to compare masses and not weights, for both the substances in the pans are equally affected by gravity. We make use of the relation $\frac{W}{W'} = \frac{m}{m'}$. But in the case of a spring balance it

is the weight that we measure, for the increase in the length of the spring depends upon the force with which it is pulled downwards. Hence with the help of this we compare weights and not masses.

In the above-said example of gold, if we had sent an ordinary pair of scales along with the weights from the pole to Colombo, no change in the weight would have been detected.

39. Gravitational Units of Force.—The absolute units of force, the poundal and the dyne, are too small to measure the actual forces which we come across in the world. Hence the engineers use the weights of one pound and of one gram as their units, these are called the **Gravitational Units.** Since the value of g is different at different places, the gravitational units are not constant. But the actual variation in the value of g is so small that for purposes of measuring we can ignore it. So the units adopted are:

1 *lb. wt.* $= g$ poundals $= 32$ poundals (approximately),
1 *gm. wt.* $= g$ dynes $= 981$ dynes (approximately).

We have already learnt that the velocity of a ball falling down constantly increases. Let us see why it is so. Sometimes it is supposed by the student that the distance of the body goes on decreasing from the centre of the earth, hence the force of gravity on it goes on increasing, and that it is due to this increase of force that the velocity changes. This idea is erroneous, for we have already seen that over the distances with which we are concerned the force, and hence the acceleration g, does not change appreciably. The force of gravitation is inversely proportional to the square of the distance between the two bodies, and in the presence of 4000 miles a distance of a few feet is negligible, hence the square of the distance remains practically constant for our purposes. The real cause is the constant generation of acceleration which is due to the constant application of the pull of the earth. Newton's second law says that acceleration produced is proportional to the force, hence, if force acts constantly, the acceleration would go on being generated.

40. Comparison of Masses.—One method of comparing masses has already been explained. It is based upon the fact that the weights are proportional to the masses. This is the method adopted by a shopkeeper, who balances the weights of the bodies with the weights of his standards and thereby compares the masses. A second method is the direct applica-

tion of the second law. Equal forces are impressed upon the two bodies whose masses we want to compare. The acceleration produced in each would be inversely proportional to its mass for

$$F = ma,$$

and if the mass be in one case m', and in the other m'', the acceleration, a' and a'', we know that

$$m' a' = m'' a''.$$

Therefore

$$\frac{m'}{m''} = \frac{a''}{a'}.$$

Thus we see that the masses are inversely proportional to the accelerations that are produced in them.

If

$$a' = a'',$$
$$m' = m''.$$

Thus to test the equality of two masses we have simply to impress upon them equal forces and to measure the acceleration produced in each; if it be the same the masses are equal.

41. Composition of Forces.—When forces more than one are acting upon a particle simultaneously, each one of them will produce the same effect which it would do if others were not present. We have already come across one example in which this point was very clear. That example was of a body projected horizontally with a certain velocity. The horizontal force produced its own effect, whereas the downward pull of the earth produced downward acceleration irrespective of the horizontal force. This is called the **Physical Independence of Forces.**

When two or more forces are acting upon a body, in general, it will move under their influence if they do not neutralise each other completely. The same motion may be produced by another single force which will be equivalent to them all. This last force is called the **Resultant** of these two or more forces. Since force has direction in addition to magnitude, it is a *vector* quantity. Therefore to find out the resultant of two or more forces we cannot add them up arithmetically. The method of compounding velocities will be applicable here. When we have two forces only, we might compound them either by parallelogram law or by triangle law, which are in this case called **Parallelogram Law of Forces** or **Triangle Law of Forces.** They run as follows:

44. Motion in a Circle.—Consider the motion of a particle which moves in a circle of radius r, with a uniform velocity v. Suppose it starts its motion from P (Fig. 19), and moves from P to Q in t sec. Evidently arc $PQ = vt$. If θ be the circular measure of the angle PMQ, obviously

$$r\theta = \text{arc } PQ = vt,$$

or
$$v = r \times \frac{\theta}{t}.$$

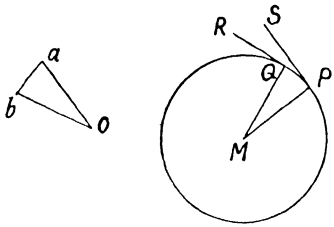


Fig. 19

Now $\frac{\theta}{t}$ is the angle described by the particle in one second, or, in other words, it is the **Angular Velocity** (generally written as ω). Substituting ω for $\frac{\theta}{t}$ we get

$$v = r\omega.$$

Question arises, What makes this particle move in a circle? For if there be no other force acting upon it, by Newton's first law of motion, it will move along PS. But since it moves in a circle, there must be some force acting upon it. Moreover, its motion is uniform, hence the force must be acting always perpendicularly to its path in order to have no component in the direction of motion.

Thus we see that *when a particle moves in a circle with a uniform velocity, it is subject to a force directed towards the centre.* This force is called the **Centripetal Force.**

Let us see what its magnitude is. From Newton's second law we know that $F = ma$, and if we know the acceleration and the mass of the particle we can determine the centripetal force. Note that the acceleration is producing the change in direction only.

Suppose the particle is at P to start with, and after an infinitely small interval of time, t, it is at Q. Evidently $PQ = vt$. At P the velocity is along PS, and at Q it is along QR. Draw Oa parallel to PS to represent the velocity v at the point P. Next draw Ob parallel to QR to represent the velocity v at Q. Evidently $Oa = Ob$.

By triangle of velocities Ob, the final velocity, is equal to the resultant of initial velocity Oa, and *the change in velocity* ab during the time t. Therefore acceleration is $\frac{ab}{t}$.

Since time t is very small, we can consider PQ as almost a straight line. Keeping this in view we can regard the sector MPQ as a Δ.

The two Δs, MPQ and Oab, are similar.

$$\therefore \quad \frac{PQ}{MP} = \frac{ab}{Oa},$$

or
$$\frac{vt}{r} = \frac{ab}{v},$$

or
$$\frac{ab}{t} = \frac{v^2}{r}.$$

But $\frac{ab}{t}$ is the acceleration.

$\therefore$ the acceleration $= \frac{v^2}{r} = r\omega^2$.

If the mass of the particle be m

$$\textbf{the centripetal force} = \frac{mv^2}{r} = mr\omega^2.$$

To see experimentally that without centripetal force a body cannot move in a circle, we have simply to whirl a small piece of stone tied to a string. As soon as the force of the hand stops, by breaking of the string or otherwise, the stone flies off tangentially.

45. Third Law of Motion. To every action there is an equal and opposite reaction. We have already seen in § 9 that forces always exist in pairs. For instance, a book that is lying on a table is pressing the table downwards, and the table is pressing it with an equal and opposite force upwards. The book cannot exert the force unless the table or some other thing presses it upwards. A bullet fired from a gun cannot exert force unless it meets with some resistance, and then even the force that it exerts is equal to the amount of the resistance that it meets. Air offers it resistance, it exerts equal and opposite force on it. The target offers it much greater resistance and the force exerted on it by the bullet also is much greater.

Unless there is this opposite force or reaction, we cannot have action. It is for this reason alone that we cannot cut a piece of paper with one blade of a pair of scissors; that we cannot drive a nail into a wooden block unless it is supported against something. In all these examples it is clear that forces must exist in pairs; *i.e.* action and reaction must exist side by side.

Before we take other examples let us understand it carefully that each constituent of this pair of forces is acting upon different bodies. For instance, the book is pressing the table downwards; this force is acting on the table, the reaction of the table acts upon the book. Whenever we want to discuss Newton's third law we should be very careful about this fact, *that to consider the motion of a particular body we must confine ourselves to the forces acting upon that very body alone.*

Now let us see why the book does not move. There are two forces acting upon it—one is the gravity, *i.e.* its weight; the second is the reaction of the table equal to the weight in the upward direction. These two opposite forces neutralise each other, and hence the book is at rest.

Let us take next the well-known example which has been discussed by Newton himself, that when the horse pulls the cart the cart pulls the horse backwards with an equal force. Beginners invariably ask the question, If it be so, why should they start at all? In order to understand this we must be clear as to what the objection is: whether we want to know why the cart moves, or why the horse moves forward instead of backward in which direction it is pulled by the cart, or why they both move.

So far as the cart goes, the problem is quite simple. There is the weight of the cart acting downwards, there is the reaction of the earth upon the cart in the upward direction; these forces are equal and opposite to each other, and hence they produce no effect, for if any one of them were greater than the other the cart would move in that direction.

The only other important force on the cart is the forward pull of the horse transmitted along the traces. On account of it, in accordance with Newton's second law of motion, the cart moves forward.

Let us next examine the motion of the horse. The horse is being pulled downward by the earth; and also there is upon him the reaction of the earth. Both these forces are equal and opposite, and hence they are cancelled away.

But let us see what happens when he pulls the cart forward by trying to 'go 'onward. He exerts force upon the earth tending to push it backward, the earth pushes him forward. There is also the backward pull of the cart. Thus there are two forces on the horse, one due to the cart in the backward direction and the other due to the earth in the forward direction, this latter is greater, and hence the horse moves forward.

Let us finally consider the motion of the cart and the

horse both together. The forces acting are the weight of the cart and the horse in the downward direction, and the upward reaction of the earth; these forces are equal and opposite, and hence they do not produce any effect. The forward pull on the cart and the backward pull on the horse cancel each other, so far as the system consisting of both of them is concerned. The only outstanding force on the *system* is the forward thrust of the earth when the horse tries to move forward. It is on account of this that the system moves forward.

It will be clear, from what has been said above, why it is that a man cannot raise himself by pulling at his own boot-straps or when seated in a chair by lifting its arms. It is because there is no external force acting on the system. All these examples show clearly that action and reaction are equal and opposite.

46. We have said that unless there is an external force acting on the system it will not move, but there seems to be an apparent exception in the case of a gun which is fired. At first it appears that without any external force the bullet is driven forward violently, so that there is no reaction in this case. Careful experiments tell us that this case is no exception to the third law. When the bullet goes forward the gun goes backward, the forces on the two are equal and opposite, but on account of the difference in the masses the motion produced is widely different. The momentum generated in the bullet is equal and opposite to the momentum generated in the gun. This principle is called the **Conservation of Momentum.** It may be stated as follows:

The algebraical sum of the momenta of a system of bodies along any straight line is not altered on account of their mutual actions upon each other.

This explains why a piece of stone when left free in air moves downward if the action and reaction are equal. No doubt when the earth pulls it downward the stone pulls the earth upward with such a force that the momentum of the earth is equal to the momentum of the stone. But since the mass of the earth is infinitely greater than that of the stone the velocity generated in the earth is very much smaller than the one generated in the stone.

To experimentally verify this principle we shall consider the case of a magnet floating horizontally in water. If a piece of soft iron smaller in size be floated also, it will be observed that the soft iron moves towards the magnet and the magnet

moves towards the soft iron piece, only the velocity generated in the magnet is smaller on account of its greater mass.

Thus, in short, we find that the constituents of a system might move, but that **the system as a whole** cannot change its position, or, in other words, C.G. of the system remains at the same place, unless some external force be acting upon it on account of the coexistence of action and (equal and opposite) reaction. The last sentence is very important, for it connects up the third law with the first law of motion. If action and reaction were not equal, the body would move *as a whole* (*i.e.* the C.G. of the system would change its position without the application of an external force), which the first law of motion denies. In other words, the first law of motion says clearly that internal forces cannot move the body as a whole; *i.e.* action and reaction are equal and opposite. Thus we see that the third law of motion is deducible from the first law of motion. Now let us see what is the relation between the first law and the second law.

The second law says that the rate of change of momentum is proportional to the impressed force and takes place in the direction of the force. In other words, it means that when there is no force there is no change of momentum; *i.e.* a body will not change its state either of rest or of uniform motion unless some force acts upon it—a fact which is called the first law of motion. Thus we see that the first law is contained in the second law.

We have already seen that the 3rd law is contained in the 1st law, and now we see that the 1st law is contained in the 2nd law; combining both these statements, we come to the conclusion that the laws 3rd and 1st are both contained in the 2nd law.

Hence *the 2nd law is* ***the*** *law of motion.*

EXERCISES

1. A train of 120 tons mass running 45 miles an hour is pulled up by the brakes in half a mile. What is the retarding force of the brakes in poundals and also in tons weight ?

Here $u = 45$ miles per hour $= 66$ ft. per second,
$S = \frac{1}{2}$ mile $= 2640$ ft ,
$v = 0$,
$a = ?$

Using $v^2 - u^2 = 2aS$

we get $$a = -\frac{u^2}{2S}$$

$$= -\frac{66 \times 66}{2 \times 2640} \text{ ft. per second per second.}$$

But $F = ma$ poundals.

$$\therefore \quad F = -\frac{120 \times 2240 \times 66 \times 66}{2 \times 2640}$$

$$= -99 \times 2240 \text{ poundals,}$$

or $$= -\frac{99 \times 2240}{32 \times 2240} \text{ tons weight}$$

$$= -\frac{99}{32} = -3{\cdot}09 \text{ tons weight.}$$

The force is thus $-221{,}760$ poundals or $-3{\cdot}09$ tons weight. The minus sign shows that the force is acting in the opposite direction to that of the positive direction of velocity; in other words, it produces retardation.

2. A 600-lb. cannon ball is fired from a gun weighing 12 tons with a velocity of 2000 ft. per second. If the gun is free to move, with what velocity will it move backwards ?

By the third law the momentum generated in the gun will be equal and opposite to that of the ball. Hence the velocity with which the gun will move will be given by the following equation :

$$v = -\frac{600}{12} \times \frac{2000}{2240} = -\frac{625}{14} = -44{\cdot}64 \text{ ft. per second,}$$

the negative sign shows that the gun will go in the opposite direction to that in which the ball moves.

3. A balloon is rising upwards with an acceleration of 4 ft. per second per second. Find the weight of a man as recorded by a weighing machine (or spring balance), who weighs 12 stones on earth.

Since there is no relative motion between the man and the balloon, the reaction of the balloon must be equal to the force with which the man presses the balloon downwards. Hence the weight of the man equals the reaction of the balloon, say R.

Now this reaction of the balloon in addition to overcoming the pull of gravity has to produce in man the acceleration of 4 ft. per second per second, or, in other words, the difference between the reaction and the pull of gravity must be equal to ma where m is the mass of the man and a the acceleration produced in him.

Therefore $R - mg = ma$,

or $R = m(g + a)$

$= m(32 + 4)$

$= 12 \times 14 \times 36$ poundals,

or $= \dfrac{12 \times 14 \times 36}{32 \times 14} = 13\frac{1}{2}$ stones weight.

If the balloon were to descend with this acceleration the reaction of the balloon would be smaller than the pull of gravity for the acceleration is downwards. In this case we shall have

$mg - R = ma$,

or $R = 12 \times 14 \times 28$ poundals,

or $= \dfrac{12 \times 14 \times 18}{32 \times 14} = \dfrac{21}{2} = 10\frac{1}{2}$ stones weight.

Note that if the balloon were moving either up or down with uniform velocity there would be no change in the weight of the man as recorded by the spring balance.

4. What force is required to stop a train of 100 tons going 30 miles an hour—(1) in half a minute, (2) in half a mile?

Ans. $4\frac{7}{12}$ tons, $1\frac{7}{48}$ tons.

5. A force of 8 lb. wt. acting on a certain mass for 3 seconds gives it a velocity of 6 ft. per second. Find the mass in lb.

Ans. 128 lb.

6. A mass of 10 lb. moving with a velocity of 25 ft. per

second is stopped by a uniform force in a distance of 50 ft. Find the force. *Ans.* $62\frac{1}{2}$ poundals.

7. The mass of a gun together with its carriage is 7 tons. It fires 100 lb. shot with a velocity of 1000 ft. per second. What is the velocity of the recoil of the gun ?

Ans. 6·38 ft. per second.

8. What do you understand by the term reaction ? A lift is rising with an acceleration of 8 ft. per second per second. What pressure would a man weighing 16 stones exert on the floor of the lift ? *Ans.* 20 stones wt.

9. A body whose mass is 12 lb. is moving with a velocity of 100 ft. per second. If a constant resistance equal to a weight of 15 lb. is applied to stop it, find how far will it travel before it comes to rest ? *Ans.* 125 feet.

CHAPTER V

THE INTENSITY OF GRAVITY

47. Modes of diluting it.—All bodies falling down freely are subject to an acceleration of 32 ft. per second per second approximately on the surface of the earth. Sometimes it becomes necessary to reduce the value of g. The methods employed can be divided into two classes: (1) by moving the body along an inclined plane, and (2) by making it move, in addition to itself, another body. We shall consider these methods one by one.

48. 1. Method.—Motion of a body along an inclined plane. An inclined plane is a plane surface inclined at an angle to the horizontal. We can represent it diagrammatically by a right-angled triangle whose each line stands for a surface. Let AC be the horizontal surface, and AB the plane surface inclined at an angle BAC or Θ to AC. BC is the height of the plane, which rises 1 in n; *i.e.* the height BC through which one moves is 1 ft. (yard or inch), in walking along AB n ft. (yards or inches).

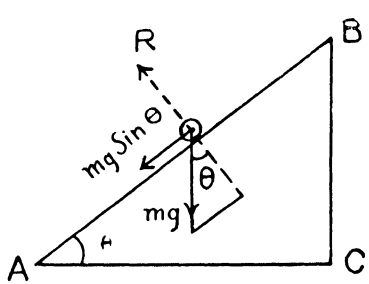


Fig. 20

Let us consider motion along such a plane. We shall suppose that the plane is a smooth one, which means simply that there is no resistance offered by the inclined plane parallel to its surface to a body moving along it, or, in other words, the reaction offered by the plane is perpendicular to its surface.

Suppose m is the mass moving down the inclined plane, and R the reaction of the inclined plane on the mass. The weight of the mass is mg (dynes, or poundals), acting vertically

downwards. Resolve it into two components, one along the plane and the second perpendicular to it. The component along the plane is $mg \sin \Theta$, and perpendicular to it is $mg \cos \Theta$. The forces perpendicular to the plane are R and $mg \cos \Theta$. Since there is no motion perpendicular to the plane, these two forces must be equal and opposite to each other, *i.e.*

$$R = mg \cos \Theta.$$

The only downward force is $mg \sin \Theta$; let us suppose it produces an acceleration a in the body. By second law of motion

$$mg \sin \Theta = ma,$$

or

$$a = g \sin \Theta. \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

The max. value of sin Θ is 1, and that will be the case when Θ is 90°, viz. when the body falls vertically; for all other angles sin Θ is less than 1, hence a is less than g. Thus we see that when a body moves down an inclined plane the acceleration is reduced; the amount to which it is reduced depends upon the inclination of the plane. Let us see next what will be the velocity of the body when it reaches the bottom of the plane l ft. long, starting from rest at the top.

$$v^2 = 2g \sin \Theta \times l = 2g \times l \sin \Theta = 2gh$$

where $l \sin \Theta = h$, the height of the plane.

But if the body were to fall down freely in the vertical direction, even then the velocity acquired would have been $2gh$. Hence we see that

The velocity acquired by a body in falling down along an inclined plane is equal to that which it would acquire if it were to fall down vertically through the same height as the height of the plane.

49. II. Method.—In this case we make the body move another body in addition to itself. We shall consider two such methods only.

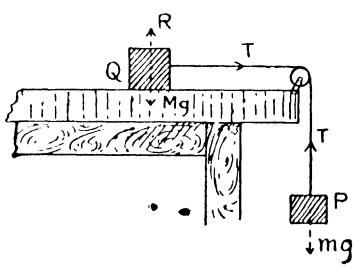


Fig. 21

Let us suppose that the body P in falling down is made to move a second body Q along a smooth horizontal table. The two are connected by an inextensible string passing over a pulley. The weight mg of the body P is acting downwards. The weight Mg of the body Q is also acting downwards, but the reaction of the table is acting upwards, and as there is no

motion of Q in the vertical direction, hence the two are equal. Now here we have a system consisting of two bodies P and Q which is being moved by the force mg units (dynes or poundals). The total mass moved is $m+M$.

Using the second law of motion, we get

$$(m+M)a = mg,$$

where a is the acceleration in the bodies P and Q,

therefore
$$a = \frac{m}{m+M}g \quad . \qquad (2)$$

The ratio $\frac{m}{M+m}$ can be made as small as we like, hence a, the acceleration, may be made as small as desired. For instance, suppose we want to make a only 4 ft./sec.^{2}, all what we have to do is to make $\frac{m}{M+m} = \frac{1}{8}$, *i.e.* to make $M = 7m$.

We have considered the two bodies to form a system, and therefrom we have calculated the acceleration. We could have done it also by considering the forces on each body separately. Let us consider first the body P. There are two forces acting on it: (1) mg downwards, (2) T upwards. It moves with an acceleration, say a ft/sec.^{2} downward ; writing the equation of motion we get

$$mg - T = ma \quad . \qquad . \quad . \quad . \quad . \quad . \quad (i)$$

The tension all over the string will be the same, if it is inextensible and the pulley is frictionless. On the body Q the only effective force is the horizontal force due to the tension of the string. Hence the body Q will move in the horizontal direction according to the relation

$$T = Ma. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (ii)$$

The acceleration would be the same because the two bodies are connected together with an inextensible string, one goes as much down as the other moves along the table in the same time. Substituting this value of T in the above equation, we get

$$mg - Ma = ma,$$

or
$$mg = (M+m)a,$$

or
$$a = \frac{m}{M+m}g \quad . \quad . \quad . \quad . \quad . \quad . \quad (iii)$$

This method enables us to find tension in the string if we know a, m and M. For (i) can be written as

$$T + ma = mg,$$

but $a = \frac{T}{M}$ from (ii), hence substituting it

we get $$T + \frac{m \times T}{M} = mg,$$

or $$T\frac{(m+M)}{M} = mg,$$

or $$T = \frac{Mmg}{M+m} \quad . \quad . \quad . \quad . \quad (3)$$

50. Let us consider the second method when the string connecting two bodies passes over a frictionless pulley. Let us suppose the two bodies are as before P and Q. Consider the motion of each separately. On the body Q there are two forces, Mg acting downwards, and T, the tension in the string, acting upwards. The body moves downwards with an acceleration a ft./sec.^{2}, which is given by the following equation :

$$Mg - T = Ma \quad . \quad . \quad . \quad (iv)$$

On the body P the two forces are mg downwards and T upwards. Under the action of these two forces the body P moves upwards with an acceleration a ft./sec.^{2}.

Hence $$T - mg = ma \quad . \quad . \quad . \quad (v)$$

Add the two above equations together.

$$Mg - mg = (M+m)\,a,$$

or $$a = \frac{M-m}{M+m}\,g \quad . \quad . \quad . \quad (4)$$

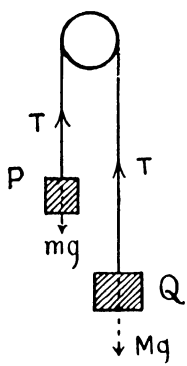


Fig. 21A

Now $\frac{M-m}{M+m}$ is necessarily smaller than 1, hence we see that a is smaller than g. For instance, if M be 17 lb. and m 15 lb. $a = \frac{2}{32} \times 32 = 2$ ft. per second per second. Thus we can make acceleration as small as we like.

Let us see what is the amount of tension in the string. Instead of adding equations (iv) and (v) let us divide one by the other,

$$\frac{Mg - T}{T - mg} = \frac{M}{m},$$

or $$mMg - mT = MT - Mmg,$$

or $$(M+m)T = 2\,Mmg.$$

Therefore $$T = \frac{2Mm}{M+m}g \quad . \quad . \quad . \quad (5)$$

Note the *pressure on the axle of the pulley is twice the tension* and not the sum of the weights of bodies, except when the masses are equal.

51. In the beginning of the chapter we have seen that when a body comes down a plane, the component of the downward force (weight) that is effective is $mg \sin \Theta$.

Now let us see what force is required to drag it up along the plane. Since the downward force along the plane is $mg \sin \Theta$, naturally any force greater than this would drag the body up; whereas if we want to raise it up vertically we would have to exert a force greater than mg. Thus we see that by exerting a smaller force we are enabled to raise a body up through a given height with the help of an inclined plane.

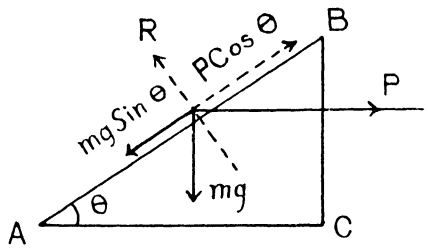


Fig. 22

But note *what we gain in force we lose in distance*, for now we have to travel along the length of the plane, which is greater than the height.

Let us apply the force horizontally instead of applying it along the plane. There are three forces acting on the body, the weight vertically downwards, the reaction perpendicular to the plane; and P in the horizontal direction. Since the body can move only along the plane, if the components of the force along the plane are equal and opposite, the body would be in equilibrium. R being at right angles to the plane will have no component along it. The component of weight along the plane is $mg \sin \Theta$ in the downward direction, and the component of power used along the plane in the upward direction is $P \cos \Theta$. So that the body may be in equilibrium, these two must be equal.

Therefore $$mg \sin \Theta = P \cos \Theta,$$

or $$P = mg \tan \Theta.$$

Thus we see that when we want to move a body up an inclined plane by applying a force horizontally, we must exert a force greater than $mg \tan \Theta$.

The reaction of the plane when the force is applied parallel to the plane is $mg \cos \Theta$, since there is no motion of the body at right angles to the plane. Now let us see what is the reaction of the plane on the body when the force is applied horizontally. There are three forces acting on the body. One is the reaction, the second is the component of the force P perpendicular to the plane, and the third is the component of the weight perpendicular to the plane, $mg \cos \Theta$. Both these latter forces are in the downward direction, hence

$$R = mg \cos \Theta + P \sin \Theta.$$

But $P = mg \tan \Theta$, substituting this value,

we get
$$R = mg \cos \Theta + mg \frac{\sin \Theta}{\cos \Theta} \times \sin \Theta$$
$$= \frac{mg}{\cos \Theta} (\cos^2 \Theta + \sin^2 \Theta)$$
$$= \frac{mg}{\cos \Theta}$$

since $\cos^2 \Theta + \sin^2 \Theta = 1$.

Before dealing with the determination of g let us understand what we mean by *Work* and *Energy*.

52. Work.—Ordinarily speaking any exertion is work; for instance, trying to lift a heavy mass from the floor, or holding a weight in our hand, are ordinary examples of work. But in science the word "work" stands for accomplishment, and not for the effort or fatigue. Let us consider a particular example. Suppose a building is under construction and some labourers are carrying bricks to the masons working on the roof of the first storey. The owner of the house naturally wants to get maximum work out of the labourers. He always praises that labourer who carries the largest number of bricks, because he is doing greater work than others. Suppose next that the different storeys are in the course of construction simultaneously. The man who goes to the highest storey with say 15 bricks is doing greater work each time than another who is carrying 15 bricks to the first storey. This example enables us to understand what we mean by work in the scientific sense of the term. It shows us that by work we mean simply causing of motion in any body by the application of force (in this case in bricks). The amount of work done depends upon two factors (i) the mass lifted up against a force (in this case gravity), and (ii) the distance through which the body is raised against the force. It should be noted that it is immaterial by which

way the labourer goes up, *i.e.* whether his ladder is slanting or vertical, the work done is the same. And secondly, the amount of work done does not in any way depend upon the time taken to do it. For instance, in the above example, if the owner of the house wants to make payment to the labourers according to the amount of work they do, he will not care to know the time that each labourer takes in taking the bricks from the ground-floor to the masons. All that he would note is how many bricks *in all* during the working hours the labourer has carried and through what height. One labourer may work slowly and steadily, and another may work energetically for some time and then may take rest. More generally, whenever a body is moved in opposition to certain forces acting on it, work is said to be done. To define work we say that *work is done by a force when its point of application moves.* As said before, the amount is proportional to the force as well as the distance through which the body moves. Or mathematically,

$$\textbf{\textit{Work} = \textit{Force} \times \textit{Distance}.}$$

The unit of work is different in the two systems; in *F.P.S.* or British system, it is **foot-poundal,** *i.e. unit of work is that much work which is done by a unit force* (1 *poundal*) *in moving the body through* 1 *ft. in the direction of the force.* In the *C.G.S.* system the unit of work is called the **erg,** *it is the amount of work done when a force of* 1 *dyne moves the body through* 1 *cm. in the direction of the force.* These two units are the absolute units of work. In practical life units adopted are different. In the *F.P.S.* system the practical unit is called the **Foot-pound,** which is the amount of work done by a force of 1 lb. wt., acting through a distance of 1 ft. In the *C.G.S.* system the practical unit is called the **Kilogramme-metre,** which is equal to the amount of work done when a kilogramme moves through a metre against the force of gravity.

53. Let us suppose now that the force makes an angle Θ with the direction of motion of the body. The effective component which causes motion is as seen from the figure $F \cos \Theta$, the other component $F \sin \Theta$ is at right angles to the direction of motion, hence it does not produce any effect. Therefore the work done is equal to $SF \cos \Theta$ where S is the distance through which the body moves. As an example of this kind of work, let us con-

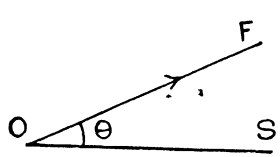


Fig. 23

sider the motion of a body up an inclined plane when the force is acting horizontally. The component of the force (P) acting along the plane is $P \cos \Theta$ (see § 51). The work done in moving the body from the bottom to the top of the plane is

$$lP \cos \Theta.$$

When Θ is zero, *i.e.* the direction of force is the same as the direction of displacement, the work done is $F \times S$, as said before. When Θ is 90°, the work done is zero, *i.e.* the force does not produce any effect.

54. Rate of Performing Work.—So far we have considered the amount of work done by an agent, not caring to know in how much time it is done. But most frequently a given amount of work is required to be done within a given time, and in order to do that we must employ an agent who can finish it within that time. This necessity introduces us to a second idea about work, *i.e. the rate of doing work* or **Power,** as it is generally called. It is measured by work done in a second.

The practical unit adopted by the British engineers is the one called the **Horse Power.** It was introduced by James Watt, who supposed that a horse could raise 33000 pounds one foot high in one minute of time. It is now known that very few horses are capable of working continuously at this rate for any length of time. However, this unit, *i.e.* the horse power or 33000 foot-pounds a minute or 550 foot-pounds in a second, is the one that is generally used. Another unit of power is also used. It is called a **Watt.** It is the power of doing 10^7 ergs of work in one second. It is useful to remember the connection between the two units.

$$\text{One } H.P. = 746 \text{ } watts.$$

55. Energy.—When work is done upon a body, it is found to have an increased power of doing work itself. It can produce physical changes in other bodies. It is therefore said to possess more energy than before. **Energy** usually is defined as *the power or capacity to do work.* It may take any one of the two forms: energy due to the motion of the body, or energy due to its position. The first kind is called the **Kinetic Energy,** and the second kind the **Potential Energy.**

56. Kinetic Energy.—As an example of kinetic energy, let us consider a stream of water. The water is made to flow down the slope by gravity. Now, by virtue of its motion, it is capable of doing more work than an equal mass of water at rest. For instance, it will set the water-mill in motion, if

made to fall upon the vanes of a turbine. Similarly a bullet shot from a gun is capable of piercing through obstacles.

57. How to Measure it.—When once a body is set in motion, it continues to move uniformly in a straight line until a force acts upon it. Suppose it is moving with a velocity of v ft. per second. Let it be brought to rest by a constant force F in a distance S. Before it stops it does FS units of work. Now, let us see how it is stopped. The force produces in it a retardation according to the formula

$$a = \frac{F}{m}.$$

Its initial velocity is v, and its final velocity is zero, the distance through which it moves before coming to rest is S, using the equation

$$v^2 = 2aS, \text{ where } a = \frac{F}{m}$$

we get

$$v^2 = 2\frac{F}{m}S,$$

or

$$\tfrac{1}{2}mv^2 = FS.$$

But FS is equal to the work done before the body stops, hence it is the measure of the energy possessed by it due to its motion. Thus we see that kinetic energy of a body is equal to

$$\tfrac{1}{2}mv^2.$$

58. Potential Energy.—As has been said above, the potential energy of a body is due to its position. Now either a body may change its position as a whole with respect to its surroundings, as, for instance, when a weight is moved from the ground to the roof of a house, or one part of the body may move with respect to the second part, as in the case of compression. In both cases it is found that the body possesses greater energy than what it did before the change of position was brought about. As an example of bodies possessing potential energy, let us consider a ball lying on the roof of a house. It is capable of doing greater amount of work than another ball lying on the ground. If when released from the roof it is allowed to fall on a pile on the ground, the pile will be driven into the ground. Similarly a watchspring when wound up possesses energy enough to set the watch going for a long period.

59. How to Measure it.—No formula can be given to express the potential energy of a body, so universally applicable as $\frac{1}{2}mv^2$ in the case of kinetic energy. But in certain cases the formula is very simple. For instance, in the case of the ball

lying on the roof, the potential energy is mgh, where mg is the weight and h the height of the roof above the ground. It should be noted that when the ball is lying on the ground, we suppose it possesses no energy, but it is fallacious, for it does possess potential energy—equal to the amount of work done in raising it from the centre of the earth to the surface, but this energy is of no use to us. Hence when we say that the body possesses no energy, we mean to say in fact that it possesses no *available* energy. Since we have to deal with the bodies raised to certain heights very frequently, we shall consider this case thoroughly. When the ball is on the roof of the house, the energy that it possesses is only potential, and is equal to mgh. When it is half-way down, half of the energy is kinetic and half potential. Let us see that it is so. So far as the potential energy is concerned, it is quite clear that it is equal to $\frac{1}{2}mgh$, because the height now has become from h, $\frac{h}{2}$. Let us see about the second part. The velocity at half the height will be

$$v^2 = 2gS = 2 \times \tfrac{1}{2} \times hg = gh.$$

The kinetic energy of the ball will be $\frac{1}{2}mv^2$ or $\frac{1}{2}mgh$, which is half the initial potential energy. The total energy is $\frac{1}{2}mgh + \frac{1}{2}mgh = mgh$, the same which the ball possessed when on the roof. When it is at $\frac{1}{4}$th the height, the potential energy is only $\frac{1}{4}mgh$, and the kinetic energy would be $\frac{1}{2}mv'^2$, where

$$v'^2 = 2g \times \tfrac{3}{4}h = \tfrac{3}{2}gh,$$

or in other words it is $\frac{1}{2}m \times \frac{3}{2}gh = \frac{3}{4}mgh$. The total energy again is the same as at the top. When the ball reaches the ground, although the (available) potential energy is zero, yet the kinetic energy even now is equal to the initial potential energy. The velocity with which the ball falls on the ground is given by

$$v^2 = 2gS = 2gh,$$

because $S = h$, the whole of the height. The $K.E. = \frac{1}{2}mv^2 = \frac{1}{2}m \,.\, 2gh = mgh$. This example shows that the total energy remains the same. This fact is very important, and it forms the basis of one of the most important laws—the law of **conservation of energy,** *which states that the sum-total of energy in this world is unalterable.* A word must be said about that case in which energy seems to be lost. For instance, when we haul a body up along a rough inclined plane the potential energy that it possesses on the top is not equal to the work

done upon it. It is on account of the fact that the energy spent in overcoming the friction is irrecoverable. It is changed into heat, which is not available to us. Almost in every operation part of energy is lost to us in the form of heat. In the example of the ball, the energy spent in overcoming the friction of air is lost. This loss of energy which cannot be made available is said to be **Dissipation of Energy.** As a consequence of this dissipation the temperature of the universe is slowly rising.

60. Measurement of g.—Any of the relations proved already in the articles on the dilution of the intensity of gravity might be used to find out g. For instance, when the body moves down an inclined plane, the acceleration becomes $g \sin \Theta$. Now let us note the time that the body takes in going down the plane from the top to the bottom; since it starts with zero velocity,

$$l = \tfrac{1}{2}at^2 = \tfrac{1}{2}g \sin \Theta \times t^2$$

we know l, we know t, we know Θ, hence we can find out g.

We can hang two masses m and M connected by a string over a pulley, and after noting the acceleration in them, find out what the g would be,

for $$a = \frac{M-m}{M+m}g.$$

The method consists in noting time which one of the masses takes in travelling a definite distance, and therefrom calculating a ($l = \frac{1}{2}at^2$) substituting this result in the above equation we get g. This is the principle of **Atwood's Machine,** which consists of a frictionless pulley, over which a string carrying two equal masses passes. A small slotted weight is slipped on to one of them; due to unequal weights on the two sides, the heavier body begins to move down and the other moves up. Time which one of the bodies takes to cover a definite distance is noted, and therefrom a is calculated. When a is known, g can be determined. But the simplest method of finding out g is by the method of *Pendulum.*

61. Pendulum.—*A simple pendulum consists of a heavy particle suspended by a weightless inextensible and flexible string.* In practice it is difficult to realise all these conditions, *i.e.* to have a perfectly weightless, inextensible and flexible string, and also to have a heavy particle, *i.e.* a bob whose mass is concentrated at the point of *C.G.* But this description is realised with sufficient accuracy in the case of a small lead sphere suspended by a fine, strong cotton thread. The distance

between the point of support and the centre of lead sphere is called the *length of the pendulum*. If such a pendulum be drawn aside and then let go, it begins to oscillate to and fro in a vertical plane, the bob describing an arc of a circle.

Suppose the bob is left free at B, Fig 24, it swings nearly to an equal distance on the other side of A (the position in which the pendulum was at rest). It goes on oscillating between these two extreme positions (B and C). It starts from B, goes to C and comes back to B. *This to and fro motion of the body from one extreme position to the opposite and back again is called* ***Vibration.*** Some people define a vibration to be the motion of a body from one extreme position to the other; this would correspond to our oscillation.

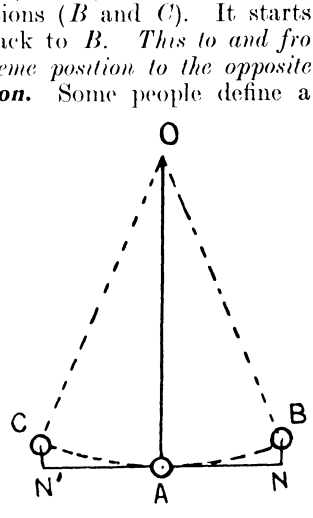


Fig. 24

The greatest distance travelled by the bob from its mean position or position of equilibrium is called the ***amplitude*** *of the vibration.*

The motion of such a pendulum is periodic, and the rate of vibration simply depends upon the length of the pendulum and the intensity of gravity. The time taken to complete one vibration is the same whatever may be the amplitude of the vibration.* The amplitude goes on decreasing on account of friction of air as time passes, but the time of one complete vibration remains the same. This fact is often expressed by saying that the movements of a pendulum are **isochronous.** *The time of one complete vibration,* i.e. *between two passages through the same point in the same direction is called the* ***Time-period*** *or the* ***Period.***

The time-period is independent of the nature of the material constituting the bob. The time of one complete vibration can be proved theoretically to be equal to

$$2\pi\sqrt{\frac{l}{g}},$$

where l is the length of the pendulum, and g the intensity of gravity. We know l, and we can determine the time-period; from these two we can find out g. To find out the time-

* Provided the angle AOB does not exceed 10°.

period it is advisable to note with a stop watch the time taken by 40 or 50 vibrations, and therefrom to find out time for one vibration. Let it be equal to t seconds. Measure the length from the point of suspension to the centre of gravity of the bob. Making use of the relation

$$t = 2\pi\sqrt{\frac{l}{g}},$$

or

$$g = \frac{4\pi^2 l}{t^2},$$

we can find out the value of g very easily. One point should be noted in this connection that, when we say a seconds-pendulum, we mean the one with such a length that the time for $\frac{1}{2}$ vibration is one second.

62. An interesting example of mutual conversion of potential energy into kinetic energy and kinetic energy into potential energy is afforded by the pendulum. It also explains beautifully what we mean by the dissipation of energy. The bob of the pendulum is at rest, when it is at the lowest position A, Fig. 24, the string is vertical in this condition. When the bob is displaced to B, it not only moves to the right but rises upward, for it moves in an arc of a circle with OA as radius and O as centre. In order to raise it through the height NB work must be done. Thus we see that when the bob is at B, work is done upon it, which is stored in it as potential energy. The bob will be able to do an equal amount of work in coming back to its original position. As soon as the bob is released at B, it begins to move with gradually increasing velocity, which becomes the greatest when the bob is at the lowest position. As a matter of fact, at this place the whole of the potential energy is changed into kinetic energy. It is this kinetic energy which does not let the bob come to rest at this lowest position, for a moving body will not come to rest unless some external force brings it to rest. It consequently moves over to the opposite side to an almost equidistant point C, where the whole of the kinetic energy is once more converted into potential energy. On account of the resistance of the air the available energy decreases in each swing, and hence the amplitude goes on decreasing. Thus we see that a part of the energy in each swing is converted into heat. This conversion into heat or degradation of energy goes on till the whole of the available energy which was originally stored in the pendulum is converted into heat.

EXERCISES

1. A mass M is drawn up a smooth inclined plane rising 2 in 3 by a string passing over a pulley at its top and attached to a mass m hanging vertically. If the acceleration is $10\frac{2}{3}$ ft./sec.^{2}, find the ratio of M to m.

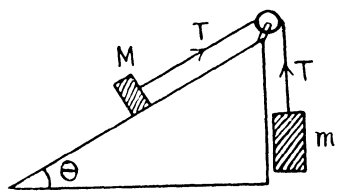


Fig. 25

Let the masses move with an acceleration a ft./sec.^{2}. Considering the motion of m we get

$$mg - T = ma,$$

or $$m(g-a) = T. \quad . \quad \text{(i)}$$

Taking the forces on M into consideration, we get

$$T - Mg \sin \Theta = Ma,$$

or $$T - Mg\tfrac{2}{3} = Ma, \text{ since } \sin \Theta = \tfrac{2}{3},$$

or $$3T - 2Mg = 3Ma,$$

or $$M(3a + 2g) = 3T \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{(ii)}$$

Dividing (ii) by (i) we get

$$\frac{M(3a+2g)}{m(g-a)} = 3.$$

Here $a = 10\frac{2}{3}$ ft./sec.^{2} and $g = 32$ ft./sec.^{2}.

Therefore $$\frac{M}{m} = \tfrac{2}{3},$$

or $$M : m :: 2 : 3.$$

2. One of the masses in an Atwood machine is 13 lb., and ascends from rest through $19\frac{1}{5}$ ft. in 3 seconds. What is the other mass ?

First let us find out what is the acceleration with which the masses move.

$$\tfrac{1}{2}at^2 = S,$$

or $$a = \tfrac{96}{5} \times \tfrac{2}{9} = \tfrac{64}{15} \text{ ft./sec.}^2,$$

but $$a = \frac{M-m}{M+m}g;$$

here $g = 32$ ft./sec^2, $a = \frac{64}{15}$ ft./sec^2, $m = 13$, M we want to know. Substituting these values we get

$$\frac{64}{15} = \frac{M-13}{M+13} \times 32,$$

or $$\frac{2}{15} = \frac{M-13}{M+13},$$

or $$2M + 26 = 15M - 13 \times 15,$$

or $$13M = 13 \times 15 + 26$$

$$= 13 \times 17.$$

Therefore $$M = 17 \text{ lb.}$$

3. A man cycles up a hill, whose slope is 1 in 20, at the rate of 4 miles an hour. The weight of the man and the machine is 187½ lb. What work per minute is he doing?

In one minute the man covers $\frac{1}{15}$ of a mile or 352 ft. The work that he does is FS where $S = 352$ ft. and $F = mg \sin \Theta$,*

or $$\frac{375}{2} \times \frac{1}{20} = \frac{75}{8} \text{ lb. wt.}$$

Therefore work done per minute $= \frac{75}{8} \times 352$ ft. pounds,

or $$\frac{75}{8} \times 352 \times \frac{1}{33000} = \frac{1}{10} \ H.P.$$

4. The mass of a train is 250 tons, and the resistance to its motion on a level line (friction) amounts to 15 lb. wt. per ton. Find the horse power of the engine which can maintain a speed of 40 miles per hour on the level.

Since the train is running on the level, hence the engine has to do no work against gravity, only it has to work against the friction in order to maintain the speed. The frictional forces are

$$15 \times 250 \text{ lb. wt.}$$

In one minute the engine goes over $\frac{40 \times 1760 \times 3}{60}$ or 3520 ft.

Hence the work done in foot lb. $= 15 \times 250 \times 3520$.

Therefore $H.P.$ required $= \frac{15 \times 250 \times 3520}{33000} = 400.$

* Note that the man is moving with no acceleration, hence $F = mg \sin \theta$.

5. Two bodies of 17 and 15 lb. respectively are connected by a string which passes over a smooth pulley. Find the acceleration with which they will move, and the tension of the string. *Ans.* 2 ft./sec^2, 510 poundals.

6. Two weights each of 5 lb. are tied to the two ends of a long cord which hangs over a pulley in a vertical plane. An additional weight of 2 lb. is suddenly placed on one of them. Find the acceleration of the weights and also the velocity and displacement after 3 seconds.

Ans. $5\frac{1}{3}$ ft./sec^2, 16 ft. per second ; 24 ft.

7. A body of 8 lb. mass is placed on a smooth plane inclining at 30° to the horizon, and is connected by a string passing over a smooth pulley at the top of the plane with a mass of 4 lb. Find the acceleration. *Ans.* 0.

8. Find the length of a seconds-pendulum at a place where $g = 981$ cm. per second per second.

(A seconds-pendulum will have 2 seconds as the time-period.) *Ans.* 99·36 cm.

9. How long will a man whose weight is 11 stones take in getting from the ground to the top of a steeple 400 ft. high by means of a ladder if he works at the rate of $\frac{4}{15}$ *H.P.* ?

Ans. 7 minutes.

10. A train of 10 tons moves up a rough incline of 1 in 100 at the rate of 25 miles per hour, the resistance offered by friction being 10 lb. wt. per ton. Find the *H.P.* of the engine.

Ans. 21·6 *H.P.*

CHAPTER VI

FORCES ON RIGID BODIES

63. So far we have been dealing with forces acting on particles or bodies so small that they could be treated as such. We shall now consider the effect of forces on bodies whose size is not very small. We shall suppose that they are rigid, viz. they do not undergo any change in shape under the action of forces. A rigid body is defined as follows :

A perfectly ***rigid*** *body is one the distance between any pair of particles of which remains unaltered, no matter what forces act on it.*

It is important to remember in this connection that in this world there is no body which is perfectly rigid. All bodies yield more or less to the action of forces, but the yielding is so slight that in practice almost all solids can be regarded as rigid.

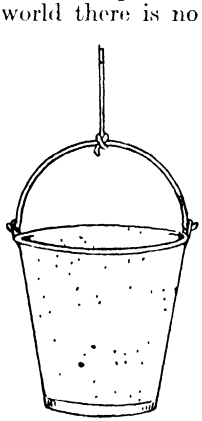


Fig. 26

A rigid body may have two kinds of motion, it may move bodily from one place to another, which motion is said to be **Translatory,** or it may rotate about a fixed line, which motion is called **Rotatory.** It may possess both of them simultaneously. In order to know what sort of motion is produced by a force in the rigid body, we must know the point of the application of the force. We did not care for this point so far because there could not be any rotation in a particle, but in dealing with forces on rigid bodies we must know the point or points of application of the force or forces in order to know the full effect.

Let us take an example of the motion of a bucket full of water. If the rope is tied just in the middle of the curved rod at the top, the bucket rises upwards without upsetting the

contents, but if, on the other hand, the rope is tied towards one end, the bucket rises, no doubt, but at the same time it goes round, upsetting the contents. Thus we see that the type of the motion produced in a body depends upon the point of application of the force.

64. Transmissibility of force.—Let the body be acted on at A by a force P in the direction AN. Consider any other point B lying on the line of action AN (produced backwards). Impress upon the body at B two opposite forces, Q and R, each equal to P; since they are equal and opposite, they do not produce any effect. Thus we see that the effect of the three forces P, Q and R will be the same as the effect of the single force P. As the forces P and Q are opposite, they may be considered to cancel each other, and the effect produced may be considered to be due to R alone. In other words, we have replaced the force P acting at A by an equal force R acting at another point in the same line of action. This is a general principle, that *the effect of a force upon a rigid body depends upon its magnitude and on the line along which it acts, but the effect is unaltered by changing its point of application to any other point on its line of action.* This is called the ***principle of transmissibility of forces.***

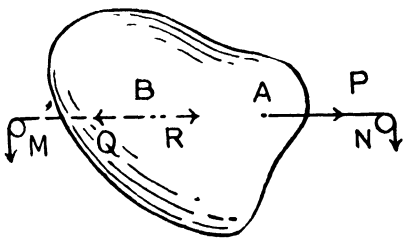


Fig. 27

This tells us that we need not know the actual point of application of the force, if we know its line of action.

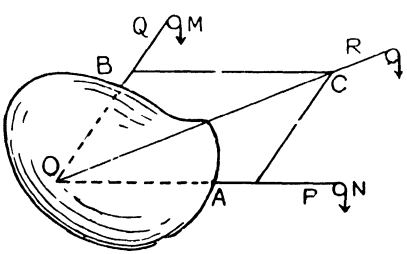


Fig. 28

65. Composition of forces in the same plane.—If the forces are in the same plane, we can use our parallelogram, triangle or polygon law of forces.

For suppose A and B are two points on which the forces P and Q act in the directions AN and BM. Produce these two lines to meet at O. Therefore P can be supposed by

the principle of transmissibility of forces to act at O. Similarly the force Q can be supposed to act at O. In other words, we have here two forces P and Q acting on the same particle O. We can compound them by the methods already explained. Represent the two forces by the sides of a parallelogram and complete it. The resultant must pass through this point O, and will be represented in magnitude as well as direction by the diagonal OC. If the number of forces be greater than two, we can use the polygon law of forces.

But let us see what will be the case when the two forces do not meet, in other words when they are parallel to each other. The above-mentioned construction will fail there. When all parallel forces are in the same direction they are said to be **like parallel forces,** but, on the other hand, when some of the forces are in one direction while others are in the other direction, the forces are said to be **unlike parallel forces.**

66. Composition of Parallel forces.—Suspend a metre rod from two spring balances in such a manner that it is horizontal, the balances being vertical; it will be seen that if the spring balances are equally distant from the middle point at which the weight (w) of the rod is acting, they will have equal readings, and each reading will give half the weight of the rod. The sum of the readings on both will be equal to w.

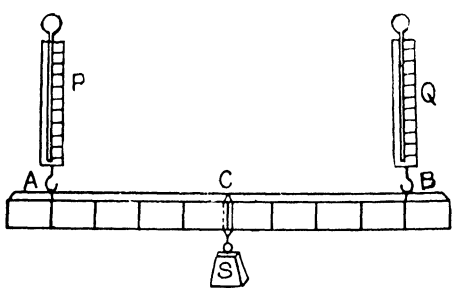


Fig. 29

Now suspend a weight S at the middle point C, the reading of each balance will increase by $\frac{S}{2}$. The sum of the readings will be $S+w$. If the spring balances are at different distances from the middle point, the nearer balance will read more and the other will read less, but the sum will be the same, *i.e.* $S+w$. Let us see what our experiment tells us. We have seen that in each case the upward forces are equal to the downward forces. For instance, suppose P and Q are the readings at the balances, and $S+w$ is the weight acting at the middle of the rod. Since these forces are in equilibrium, the resultant

R of P and Q must be equal to $S+w$ and act upwards. Since $S+w$ is vertical hence R must be vertical also, which means that it will be parallel to P and Q, for they are both vertical. Next let us see what is the relation between the reading of a spring balance and its distance from the resultant.

It will be seen that readings are such that

$$P \,.\, AC = Q \,.\, BC,$$

where P is the reading on one balance, AC is the distance of this upward force from the resultant, Q is the reading on the second balance and BC, its distance from the resultant, C being the point where the resultant acts. The above result can be expressed also as

$$\frac{P}{Q} = \frac{BC}{AC} \qquad \ldots\ldots \quad (a)$$

This enables us to find out the position of C, if we know P, Q and AB. In the language of Geometry, we can say that the resultant acts at a point which divides the distance between the points of application of the forces internally in the inverse ratio of the forces. To sum up, we have come to the following conclusions :

(1) *The resultant is parallel to each of the forces, and is equal to the sum of the two.*

(2) *The point at which the resultant acts divides internally the distance between the two forces in their inverse ratio.*

Since the three forces P, Q and $S+w$ are in equilibrium, any one of them can be considered as equal and opposite to the resultant of the other two. Suppose, for instance, that we want to compound, Q and $S+w$ acting at B and C. The resultant must be equal and opposite to P, in order that the rod may be in equilibrium. But the readings are such that $P+Q=S+w$, therefore $P=S+w-Q$. We can express it by saying that when the forces are unlike, the resultant is equal to their difference, and is parallel to them in direction.

It will be seen that

$$Q \,.\, AB = (S+w) \,.\, CA,$$

or

$$\frac{Q}{S+w} = \frac{CA}{AB} \qquad \ldots\ldots \quad (b)$$

This relation enables us to find out the position of A. In the language of Geometry, we can express it as that the resultant must divide externally the distance between the two forces in

their inverse ratio. Thus for unlike forces we come to the following conclusions :

(1) *The resultant is parallel to each of the two forces and is in the direction of the greater force, and is equal to the difference of the two forces.*

(2) *It divides externally the distance between the two forces in their inverse ratio.*

If there be more than two parallel forces, we should find out first the resultant of any two of them, and then compound this resultant with the third force, and so on. By repeating this process, we can find out the resultant of any number of parallel forces acting on a rigid body.

We can arrive at the same results theoretically as well.

67. Let P and Q be the two like forces to be compounded acting on the rod GH at points A and B. Let us impress at these points

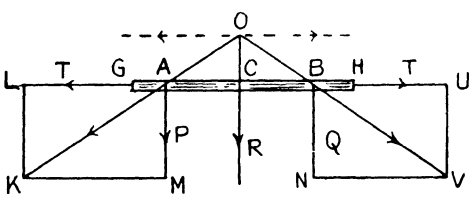


Fig. 30

two forces AL and BU equal to T in the opposite directions, the line of action being the same. So far as the rod is concerned, they will not produce any effect. Compound the forces P and T by the parallelogram law and let the resultant be AK, and similarly compound Q and T and let the resultant be BV. Produce both of them to meet at O. The resultant of these two resultants will give the resultant of the parallel forces. The simplest method of finding out the final resultant is to resolve each of the forces AK and BV into components along AL and AM in one case, and along BU and BN in the other case. The components would be P and T, and T and Q respectively. T, T will cancel away, whereas P and Q will be added, being in the same direction.

Thus we see that the resultant of two like parallel forces is given by the sum and is parallel to the forces. Transfer the point of application of R from O to C. Then the resultant of P and Q is $P+Q$ acting at C, which proves the law No. (1) given on page 83. To find out the position of C, consider first the triangles LAK and OAC ; they are similar, hence

$$\frac{AL}{LK} = \frac{AC}{OC} = \frac{T}{P},$$

therefore

$$P \,.\, AC = T \,.\, OC.$$

Next consider the triangles UBV and OBC. They are similar, hence

$$\frac{BU}{UV} = \frac{BC}{OC} = \frac{T}{Q}.$$

Therefore $$T \,.\, OC = Q \,.\, BC.$$

Since $P \,.\, AC = T \,.\, OC$, and $Q \,.\, BC$ is also equal to $T \,.\, OC$, hence

$$P \,.\, AC = Q \,.\, BC,$$

which proves that the point C at which the resultant acts divides internally the distance between the two forces in their inverse ratio.

When the forces are unlike, by similar construction it can be proved that the resultant is parallel to each of the forces and is equal to the difference of the two, acting in the direction of the greater force, and that its point of application divides externally the distance between the two forces in their inverse ratio.

Example.—If there be two forces, 5 lb. and 3 lb., acting at the end of a rod AB 4 ft. long in opposite directions, find the magnitude and position of the resultant.

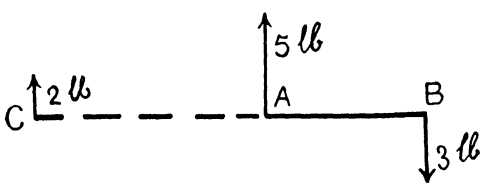


Fig. 31

Obviously the resultant R will be equal to $5 - 3 = 2$ lb. It will act in the direction of the greater force, viz. 5 lb., and will divide externally the distance between the forces in their inverse ratio.

Hence $$P \,.\, AC = Q \,.\, BC,$$

or $$5 \,.\, AC = 3 \,BC, \text{ where } AB = 4 \text{ ft.},$$

or $$AC = \tfrac{3}{5} \,BC.$$

To know the position of the point C, we have to remember simply that

$$BC = AC + AB$$
$$= \tfrac{3}{5} BC + 4;$$

therefore $$\tfrac{2}{5} BC = 4,$$

or $$BC = 10 \text{ feet},$$

and $$AC = 6 \text{ feet}.$$

68. Centre of parallel forces.—Let us suppose we have a metre rod on which two vertically downward forces are acting, one of them 3 lb. and the other 5 lb. (its weight is neglected). The resultant will be 8 lb. and will act in the downward direction at the point C, which will divide the distance AB internally in the ratio of the forces P and Q, *i.e.* $P \cdot AC = Q \cdot BC$. Now suppose P and Q, instead of acting vertically downward, act at an angle as shown in the figure. Even now the resultant will

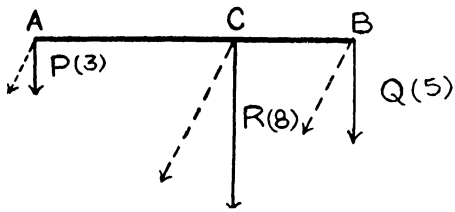


Fig. 32

be 8 lb., and the point at which it will act will divide the line AB internally in the inverse ratio of the forces, *i.e.* $P \cdot AC = Q \cdot BC$. In other words, the point where the resultant acts is independent of the inclination of the parallel forces P and Q. This point, which is fixed for a system of parallel forces, is called the *centre of parallel forces.*

Let us consider a body under the action of gravity. Each particle of it is being attracted by the earth, and consequently *there is a very large number of parallel forces* acting upon it.*

The centre of all these parallel forces which is a fixed point in the body is called the Centre of Gravity. This is defined as follows: *The resultant of all the parallel forces acting on the various particles of a body passes through a fixed point, whatever be the position of the body. This point is called the* ***Centre of Gravity*** *of the body.* In simpler words we can say that the centre of gravity of a body is a point at which the weight may be supposed to act. It should be noted that since there can be only one centre of parallel forces, hence there can be only one centre of gravity.

When a body is suspended freely, in a position of rest the centre of gravity and the point of suspension will be in the same

* We consider the forces to be parallel, for the lines joining the various particles of the body to the centre of the earth are parallel for our purposes, as 4000 miles compared to a few inches are infinite.

vertical line, for when the body is at rest, it must be under the action of two equal and opposite forces having the same line of action. One of these forces is the weight, which acts vertically downwards, and the second is the force due to suspension (be it the reaction at the point of suspension or the tension of the string), which must not only act vertically upwards, but must also pass through the fixed point (*C.G.*) so that the line of action may be the same.

69. To determine the C.G. of a body practically.—To find out the position of the *C.G.* of a body we shall make use of what has been said in the last article, that *when a body is suspended freely, its C.G. lies vertically below the point of suspension.* Let us take a triangular piece of cardboard and suspend it freely from one corner. The centre of gravity lies vertically below the point of suspension. Draw with the help of a plumb-line a vertical line on the cardboard piece. Then suspend it from another corner. The centre of gravity must lie below this point also. Draw a vertical line as before. Suspend it next from a third corner, and repeat the same process. The point where all the three lines meet is the centre of gravity. In the same way the *C.G.* of any other body may be determined.

70. To determine the C.G. of a body theoretically.—We shall take only two simple cases in order to illustrate the method. More than this is beyond the scope of this book.

(1) **The centre of gravity of a uniform rod.**—Let us suppose *AB* is the rod, and *G* its middle point. Suppose further that the rod is divided up into a large number of small parts equal in weight. Consider any two parts having the same weight and the same distance from *G*. Evidently their resultant will pass through *G*. Consider any other pair of parts equidistant from *G*, the resultant in their case also will pass through *G*. In this manner we can prove that the resultant of each pair will pass through *G*. Hence *G* is the centre of gravity of the rod. Thus we see that in the case of a uniform rod the centre of gravity is at the middle point of the rod.

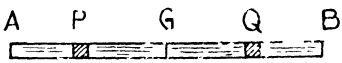


Fig. 33

(2) **The C.G. of a triangle.**—Let us join *AD*, where *D* is the middle point of *BC*. Draw lines *EF*, *HK*, etc., parallel to *BC*. We see that the triangle is divided up into thin strips. Consider a strip *EFHK*. It may be supposed to be a uniform rod. The line *AD* bisects the lines *EF*, *HK*, etc. Therefore the *C.G.* of the strip lies on the line *AD*. Similar is the case with other strips.

This means that the *C.G.* of the whole triangle is in the line *AD*. Remember in this case that *AD* is not uniformly loaded, the strips near the base possess greater weight, and hence the *C.G.* is not at the middle point. In order to find the position of the *C.G.*, now divide the triangle into strips parallel to *AB*. The *C.G.* lies on this median also. Evidently the point where the two medians meet corresponds to the *C.G.* of the triangle. We know from Geometry that the point where the medians meet is $\frac{1}{3}$ of the median, hence $DG = \frac{1}{3}AD$. Thus the point *G* is known.

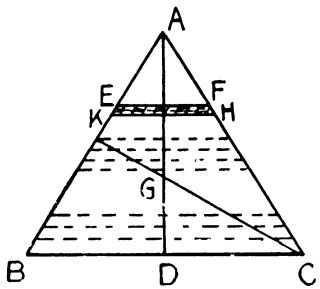


Fig. 34

By applying similar reasoning, *C.G.* of any regular body may be calculated. Some of the results are given below:

The *C.G.* of a parallelogram			is at	the point where the diagonals intersect.
,,	,,	cylinder	,, ,,	middle point of its axis.
,,	,,	pyramid	,, ,,	point which is $\frac{1}{4}$ of the axis from the base.
,,	,,	cube	,, ,,	point where the diagonals intersect.
,,	,,	disc	,, ,,	centre of the disc.

71. Moment.—When a rigid body is acted upon by forces, in general, translation and rotation both are produced. If the force is so applied that it passes through the *C.G.* of the body where the whole weight can be supposed to be acting, then the motion of the body is the same as of an ordinary particle, *i.e.* if there is no other opposing force, it moves bodily. Whereas if the force does not pass through *C.G.*, the motion of translation, as well as of rotation, is produced. If, further, a line in the body is fixed, the only motion possible is of rotation; the fixed line is called the axis of rotation. We have been considering so far the translatory motion; let us now consider the rotatory motion.

It is an everyday experience that to shut a door rotating on its hinges we require a much smaller force when it is applied at right angles to the door near the handle than when applied near the hinges. This shows that what a small force at a distance from the axis of rotation can do, a great force near the axis of rotation cannot achieve. Hence we see that in a rotatory motion we must take into consideration two factors, one the magnitude of the force,

and secondly, the distance between the line of action of the force and the axis of rotation.

The tendency of a force to produce rotation about a line (or the axis of rotation) is called its **moment,** *and it is equal to the product of the force and the shortest distance between the line of action of the force and the axis of rotation.* It should be understood clearly that in order to know the moment of a force we must take the perpendicular distance between the line of rotation, or more simply the fixed point and the line of action of the force, and not the ordinary distance.

For instance, suppose CD is a rigid body fixed at the point O, the vertical line passing through O being the axis of rotation. A force F_1, acting at D, which passes through O produces no rotation, on account of the fact that the perpendicular distance between the line of action and the fixed point is zero. Note the distance between the fixed point and the point of application is not zero, but is OD. Hence remember in dealing with moments take always the *perpendicular distance between the line of action of the force and the axis of rotation.* From the figure it is clear that F_2 and F_3 being equal, the former has a much greater tendency to turn the body round than F_3, an account of the fact that the perpendicular distance OG is much greater than OH.

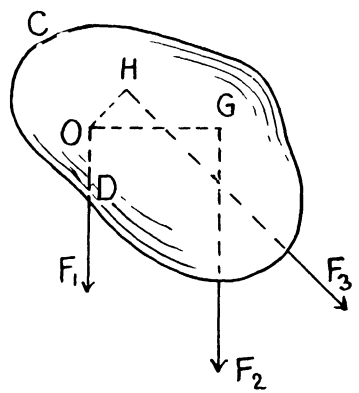


Fig. 35

We cannot express a moment completely unless we take into consideration the direction in which the force tends to produce rotation. Either the rotation may be clockwise or anti-clockwise. In order to understand fully the effect of the force we must know in what direction the force tends to turn the body. It is usual to call the anti-clockwise direction as positive and the clockwise direction as negative.

We can apply the principle of moments to determine the point of application of the resultant of parallel forces. Let us try to find out the resultant of a large number of parallel forces acting on a rigid body.

Suppose we want to find the resultant of a number of forces P, Q, R, etc., acting on the rod AB. In this example AB is at right angles to the forces, if it be not the case actually, instead of AB take into consideration a line at right angles to all the forces. The magnitude of the resultant is clearly

$$S = P + Q - R.$$

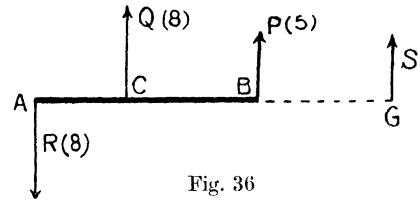


Fig. 36

To find the line of action of the resultant we shall make use of the principle of moments, *i.e.* the moment of the resultant about a point must be the same as the sum of the moments of the component forces about the same point. Suppose the resultant S passes through G. The moment of the resultant about the point A will be $= S \,.\, AG$. Hence the sum of the moments of the components also must be $= S \,.\, AG$.

Therefore
$$P \,.\, AB + Q \,.\, AC = S \,.\, AG$$
$$= (P + Q - R) \,.\, AG,$$

or
$$AG = \frac{P \,.\, AB + Q \,.\, AC}{P + Q - R}.$$

This gives us the distance of the line of action from the end A. Suppose $P = 5$ lb., $Q = 8$ lb., $R = 8$ lb., and $AC = 2$ ft. and $AB = 5$ ft.,

then
$$AG = \frac{25 + 16}{5} = \frac{41}{5} = 8\tfrac{1}{5} \text{ ft.}$$

72. Couples.—Let us take that case into consideration, when a pair of equal unlike parallel forces act on a rigid body at different points. Their resultant is evidently zero. Suppose, in the example given above, there is no force like P, the only forces are Q and R, therefore $S =$ zero. The distance AG, where the line of action of the resultant meets the line AC is infinity. Hence we see that although the resultant vanishes, its moment does not vanish, being equal to $0 \times \infty$. *Two parallel, unlike, and equal forces having different lines of action constitute a* ***couple.***

The perpendicular distance between the two forces is called the ***arm of the couple.***

Now let us try to find the moment of a couple. Suppose P and Q to be the forces and AB to be the arm. The moment

about the point A is $P \,.\, AB$, Q produces no effect; similarly about B is $Q \,.\, AB$ which is equal to $P \,.\, AB$, for $P = Q$. The moment will be the same about any other point also. Let us consider the moment about the point O. The moment of Q about O is $Q \,.\, AO$. The moment of P about O is $P \,.\, OB$, and is in the same direction as $Q \,.\, AO$. Hence the total moment is $P \,.\, OB + Q \,.\, AO$, *i.e.* $P \,.\, OB + P \,.\, AO$ or $P(OB + OA) = P \,.\, AB$. Thus we see that *the moment of a couple is equal to either force multiplied by the arm*

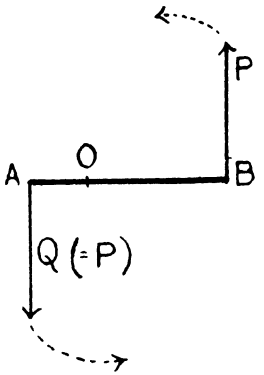


Fig. 37

Remember the effect of a couple acting on a body is to twist it round without moving it bodily as a whole.

73. Equilibrium.—When all the forces applied to a body produce no change in its state, they are said to be in equilibrium. It is important to note at the very outset that, in order that a set of forces may be in equilibrium, only no change of state or no acceleration is to be produced by the forces. It does not mean that the body is to be at rest necessarily. It only means, that if it is originally at rest it will continue to be so under the action of the forces, otherwise it will continue to move with uniform velocity. Thus we see that **equilibrium is zero acceleration.**

Now let us consider the **conditions of equilibrium** of a body when it is acted upon by a large number of forces *all acting in one plane*. Now either all the forces may be parallel, or they may meet in a point, or some of them may be parallel and others may meet in a point; and so that a body may be at rest under their influence the conditions will be different in each case. Let us take these cases one by one.

74. When all the forces are parallel.—If they are to be in equilibrium, naturally their resultant must be zero, *i.e.* $P + Q + R$, etc., with due consideration paid to the signs of the forces, must be zero. When it is so, the body cannot move bodily, in other words it can have no translatory motion, and further, it *may* have no rotatory motion. Mark the word may; it is not essential that, when the resultant is zero, the body should not rotate, for the parallel forces may be reduced

to a couple, which will have no resultant, but will have a moment all the same. Hence the general condition is *that the resultant should be zero, and in addition to it the moment about any point should be also zero.* If these two conditions are fulfilled, the body will be in equilibrium.

75. When all the forces meet in a point.—So that the forces meeting in a point may be in equilibrium, *they must be capable of being represented by the sides of a closed polygon* (triangle, if there are only three forces). When this is so, they have a zero resultant, hence the body has neither any tendency to move bodily nor to rotate about any fixed point.

76. When some forces are parallel, and others meet in a point.—The forces which meet in a point can be reduced to a resultant, and similarly the parallel forces; if the combined resultant of both is zero, the body will be in equilibrium. But it may so happen that all the forces may be reduced to a couple; even in that case the resultant is zero. So the conditions of equilibrium in this case also are *that the resultant should be zero, and in addition to it the moment of the forces about any point should be zero.*

The last two conditions are quite general. If these are satisfied, the body must be in equilibrium. Sometimes we are required to consider the conditions of equilibrium of a body in contact with other bodies. In this case the reaction also comes in. It is useful to remember in this connection that, when a body is smooth, the direction of reaction is perpendicular to its surface (see § 48).

77. Stable, Unstable and Neutral Equilibrium.—Let us consider three bodies lying on a smooth table: one a cube of wood; the second, an ellipsoid; and the third, a sphere. When the cube is raised from one side and is let go, it returns to its former state of rest. Such an equilibrium is called **Stable.** But when the ellipsoid standing upon its longer axis is slightly displaced, it moves further off from its original position; such an equilibrium is said to be **Unstable.** When, however, the sphere is slightly displaced, it neither comes back nor topples over, but simply rolls. Such an equilibrium is said to be **Neutral.** Why these three bodies behave differently will be clear from the following. When the cube is tilted up, the centre of gravity is raised, and the weight tends to bring it back to the original position, on account of the tendency of the gravity to bring the *C.G.* of the body to the lowest position. When, on the other hand, the ellipsoid is displaced, the moment of the weight here also tends to bring the *C.G.*

lower down, and the result is that the ellipsoid does not come back to its original position, for its *C.G.* to start with was the highest. In the case of a sphere since the *C.G.* of the body remains at the same height, it neither tends to come back to the original position nor to topple over.

This tells us that *so that a body may be in stable equilibrium, its C.G. must be as low as possible.* A boat, if the men are standing in it, is liable to be turned over more easily, than if all of them are sitting.

To know whether a body is in stable, unstable, or neutral equilibrium, we have simply to note whether the displacement

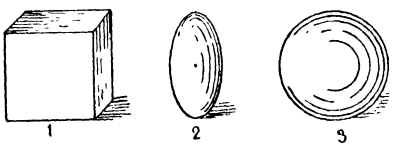


Fig. 38

tends to raise its *C.G.*, to lower it or to leave it unaffected. There is, however, another important point to remember, that a body may be in stable equilibrium for slight displacements, and in unstable equilibrium for violent displacements. For instance, suppose that we slightly displace a wooden cylinder standing on the end, it oscillates a little, and then comes to rest. If, however, it is subjected to a violent push, it falls on its side instead of regaining its first position. The reason is that on being slightly displaced, the *C.G.* still falls within the contour of the base, and so long as it is so, the body is in stable equilibrium. But as soon as the *C.G.* falls out of the base, the body is in unstable equilibrium.

EXERCISES

1. A ladder 10 ft. long, whose centre of gravity is 4 ft. from one end, is carried horizontally by two men, each carrying the same weight. One of them is at the heavier end. At what distance must the other man be from the other end ?

Let us suppose AB represents the ladder whose $C.G.$ is at G (4 ft. from B). Suppose w is the weight of the ladder, our object is to divide the load equally between the two men. In

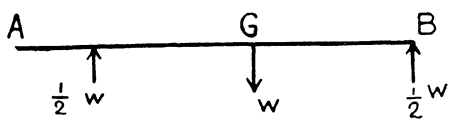


Fig. 39

order that it may be so the sum of moments of the forces applied by them about G must be zero. Hence

$$\tfrac{1}{2}w \times 4 = \tfrac{1}{2}wx.$$

Therefore $x = 4$ ft. from G.

Hence the distance of the man from A is 2 ft.

2. One end of a uniform ladder weighing 60 lb. rests against a smooth vertical wall at a height of 15 ft. above the ground, the foot of the ladder being 12 ft. from the wall. Find the pressure on the ground.

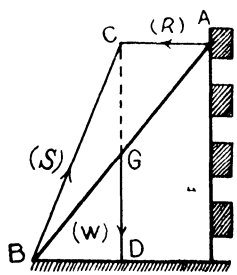


Fig. 40

The ladder is in equilibrium under the action of three forces, wt. acting at middle of the ladder, R the reaction of the wall perpendicular to it, and S the reaction of the ground. The weight (w) and reaction R, both of them, meet at C, hence the third force, *i.e.* the reaction of the earth, must also pass through C. The forces are proportional to the sides of the Δ BCD.

Since $CD = 15$ ft., $BD = 6$ ft.

Therefore $BC = \sqrt{225 + 36} = \sqrt{261}$,

$$S = \frac{60}{15} \times \sqrt{261} = 4\sqrt{261}$$

$$= 64{\cdot}62 \text{ lb.}$$

3. Find the $C.G.$ of a solid formed by two spheres touching each other, their radii being 6 and 9 cm., and their weights being 3 and 2 grams.

The weights may be supposed to be concentrated at their centres, hence the question is reduced to this that there are two weights 2 and 3 grams at A and B distant 15 cm. from each other, and we want to find out the centre of gravity. Let this point be G. The resultant (weight) will pass through this point. The moments of the weights about this point must be equal and opposite.

Therefore $2 \,.\, AG = 3 \,.\, BG$,

or $\frac{2}{3} = \frac{BG}{AG}$,

since $AG + BG = 15$, hence

$AG = 15 - BG$.

Therefore $\frac{2}{3} = \frac{BG}{15 - BG}$,

or $BG = 6$,

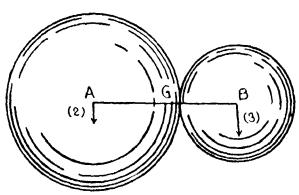


Fig. 41

i.e. the $C.G.$ passes through the point of contact.

4. A rod 10 ft. long, and weighing 100 lb., is supported at its two ends. A weight of 150 lb. is placed 3 ft. from one end. Find the pressure on the supports. *Ans.* 155 lb. and 95 lb.

5. Two men A and B carry a weight slung from a pole resting on their shoulders. If the maximum weight A can carry is 120 lb. and the maximum weight B can carry is 90 lb., what is the maximum load they can carry between them, and where must it be slung on the pole? The weight of the pole is to be neglected. *Ans.* 210 lb., $\frac{4}{7}$ of AB from B.

6. A uniform bar of length $3\frac{1}{2}$ ft. and weighing 4 lb. is supported on a smooth peg at one end and by a vertical string distant 6 inches from the other end. Find the tension of the string. *Ans.* 2·33 lb. wt.

7. A bar 8 ft. long, weighing 10 lb., inclined at an angle of 30° with one end against a wall (rough) rests across a smooth rail at a point 2 ft. away from that end. Find out

graphically the direction and the amount of the reaction of the wall.

Let AB be the rod, AG the wall, and AH the horizon. Let C denote the point where the bar rests against the rail. The bar is being acted upon by three forces, *weight* 10 lb. acting at the middle point E, the *reaction* R of the rail at C in the direction CD, and the *reaction* of the wall.

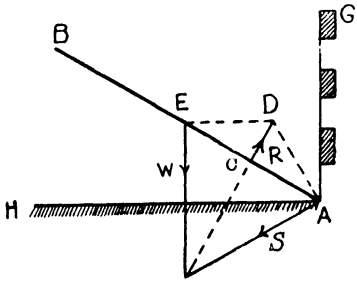


Fig. 42

Since there are only three forces acting upon the rod and it is in equilibrium, they all must pass through the same point, *i.e.* F where R and W meet. Join AF, this gives the direction of the reaction of the wall; it is inclined 30° to the horizon on the other side of the rail (can be measured graphically). It is clear even from the figure, for in triangle AFE, $AC = CE$, AC being 2 ft. and $AE = 4$ ft. The angle at C is a right angle, hence the two triangles ACF and CEF are congruent, and therefore angle $FAC = \angle FEC = 60°$. But $\angle HAE = 30°$, hence angle $HAF = 30°$. Let us find out the magnitude.

Let AE represent R. It is drawn perpendicular to FD. Erect one perpendicular to EF and another at A perpendicular to AF (representing S). These two meet at D. Triangle DEA represents the forces acting upon the rod. It is evident that $AD = DE = 10$ lb.

8. A ladder weighing 56 lb. and 30 ft. long rests against a smooth wall with its foot 15 feet from the bottom of the wall. Find the pressure on the ground and the wall if the $C.G.$ is $\frac{1}{3}$ of its length up from the bottom.

Ans. Pressure on the wall horizontal and 10·77 lb.
On the ground about 57 lb.

9. A square of side $2a$ is divided into 4 squares of side a by straight lines through its centre parallel to its sides. One of these squares is removed. Find the centre of gravity of the remainder.

Ans. $\frac{a}{6}\sqrt{2}$ from the centre along the diagonal.

10. The mass of the moon is $\frac{1}{39}$ of the mass of the earth; if the distance between the two be 240,000 miles, where would the $C.G.$ of the system be?

Ans. 6000 miles from the centre of the earth.

CHAPTER VII

MACHINES

78. *Any contrivance or appliance by means of which a force acting at a given point and in a given direction is made available at some other point, and in some other more convenient direction, is called a* ***Machine.*** It is, so to speak, a mediary by which energy is transmitted to some other point to perform useful work there. The term includes a numberless variety—from the knife which is used to mend a pencil or a pair of scissors which is used to cut a piece of cloth, to the mighty engine which propels a ship, weighing thousands of tons, through the pathless sea. It will be useful to point out in the very beginning that machines enable us only to *transmit* energy and not to increase it. The sum total of energy remains constant provided there is no dissipation on account of friction, etc. In other words, in a perfect machine the work done *on* it is equal to the work done *by* it. To put it in commercial language,

Input = Output.

This statement is generally called the **principle of work.**

If P denotes the power, a the distance through which the power acts, W the resistance, and b the distance through which the resistance acts, the principle of work can be written as

$$Pa = Wb.$$

It shows clearly that if, with a small force, we overcome a great resistance we exert the force through a greater distance and the resistance rises through a smaller distance. This point should never be lost sight of. This principle is often stated as *whatever is gained in power is lost in speed or distance.* But in spite of this fact machines are very useful, for they enable us either to overcome resistances which we could never do otherwise; as for instance, raising of a locomotive by a man with the help of a crane, or to convert a slow speed into a rapid one, as, for example, with the help of a bicycle. *The ratio of the resist-*

ance overcome to the effort applied is called the ***mechanical advantage*** $\left(=\frac{W}{P}\right)$, and *the ratio of the distance moved through by the power to that moved through by the resistance is termed the* ***velocity ratio*** $\left(=\frac{a}{b}\right)$. The **efficiency** (written as η) *of a machine is defined as the* $\frac{\text{useful work done}}{\text{whole work done}} = \frac{Wb}{Pa} = \eta$,

or
$$\frac{W}{P} = \eta \cdot \frac{a}{b},$$

viz. Mechanical advantage = efficiency × velocity ratio.

For an ideal machine efficiency = 1 ; hence in this case

Mechanical advantage = velocity ratio itself.

But for all actual machines efficiency is less than 1 (see § 102), hence velocity ratio is always greater than 1 for mechanical advantage equal to or greater than 1.

Machines are divided into two types, simple and compound. We shall briefly deal with simple machines only, because compound machines, however complex they may appear, when analysed, are found to be made up of the simple ones, and if the student understands the principles of the simple machines, he would have no difficulty in understanding the principle of any compound machine. The simple machines are six in number, (1) the **lever,** (2) the **wheel and axle,** (3) the **pulley,** (4) the **inclined plane,** (5) the **wedge,** and (6) the **screw.**

1. The Lever

79. A lever is a rigid bar, either straight or curved, which is capable of motion of rotation about a fixed point called the **fulcrum.** The levers are used to overcome resistances, hence it is important to remember that soft, flexible substances should not be used, for otherwise part of power would be lost in bending them. While dealing with levers we have to consider three things, the **power** applied, the **weight** overcome, and the **reaction** at the fulcrum. Since the lever is in equilibrium under the action of these three forces, they either must all meet in a point or be parallel to each other. The easiest way of getting the relation between the power and the weight is to find out the moment of these two about the fulcrum, and since the lever is in equilibrium, these moments must be equal and opposite.* Let a and b denote the perpendicular dis-

* The reaction at the fulcrum is not considered, for its moment about the fulcrum is zero.

tances of the direction of P (power), and W (weight), from the fulcrum, then since the moments about the fulcrum are equal and opposite,

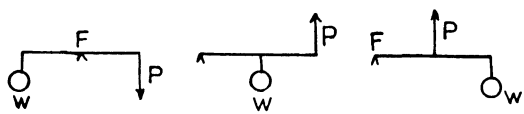
$$Pa = Wb,$$

or

$$\frac{W}{P} = \frac{a}{b}.$$

The perpendicular distances a and b are called the **arms** of the lever. The relation obtained above shows that if the arms are equal, $P = W$; *i.e. the power is equal to the weight*, but if the two arms are unequal, *the ratio of weight to power is inversely as the ratio of the arms of the lever*. It means, in other words, that if the arm a is 5 times the arm b, $W = 5P$, and so on, which shows that the resistance overcome may be made as great as we like by simply increasing the power-arm of the lever in comparison to the weight-arm. It was this fact which led Archimedes to remark "give me a lever long enough, and I shall move the world". The ratio $\frac{W}{P} = \frac{a}{b}$ is called the **mechanical advantage** of the lever.

The position of the fulcrum may be either, between the power and the weight, when the lever is said to be of the *first*



(1st order) (2nd order) (3rd order)

Fig. 43

order, or the power and the weight both may be on the same side of the fulcrum; if W (the weight) is nearer F (the fulcrum) than P (the power), the lever is of the *second order*; if, on the other hand, P is nearer F than W, the lever is of the *third order*.

The examples of the first order of levers are afforded by a balance, a pair of scissors (only in this case the two levers are combined together), a pair of pliers, etc. The examples of the second order of levers are met with in an oar,* nutcracker, a wheel-barrow, etc. The examples of the third order of levers are forearm,† used in lifting weights, the fire-tongs, knife, and fork, etc.

* The fulcrum being at the end of the blade which is kept at rest by the pressure of the water, the weight at the row-lock, and the power at the end of the oar.

† The weight being on the palm, the power at the middle of the arm, and the fulcrum at the elbow.

80. Balance.—Balance, which is simply a lever of the first order, is used very often in our experiments, hence it will be advisable to consider it in a somewhat greater detail than we have done other levers. Note it is used not as a machine but simply to compare masses. It consists of a metal beam AB (Fig. 44) turning about the point C. At the two ends A and B, pans are suspended with the help of knife-edges, so that they can swing freely round these points. When a mass is placed in a pan, it takes up such a position that the combined $C.G.$ of the mass and the pan is just below the point of suspension. Hence *it is immaterial where in the scale-pan the mass is placed.*

The requisites of a good balance are :

(1) **Truth,** (2) **Sensitiveness,** (3) **Stability.**

81. Truth.— *A balance is said to be true if the beam becomes horizontal when equal masses are placed in the scale-pans.* If both the scale-pans are removed, even then the beam must be horizontal. In order that this may be so the $C.G.$ of the beam must lie vertically below C, for then there is no moment of the

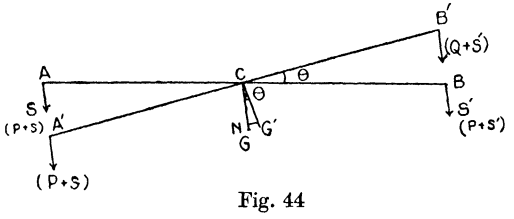


Fig. 44

weight about the point of suspension C, as it passes through that point. The conditions that a balance must fulfil in order to be true are that

(i) *The arms must be of equal length,* and

(ii) *The pans must have equal weights.*

82. We can derive these conditions easily from the definition of a true balance. Call the arm AC, a ; the arm CB, b ; the distance CG, h ; and let the pans weigh S' and S respectively, and let further two weights, P and Q, be placed in the pans (Fig. 44).

So that a balance may be true the beam must be horizontal when $P=Q$. Taking moments about the point C we get

$$(P+S)\,a=(P+S')\,b.$$

In a true balance the beam will be horizontal also when the scale-pans are empty. When this is so the following relation must hold good :

$$Sa=S'b.$$

Subtracting this from the former equation we get

$$Pa = Pb,$$

or

$$a = b,$$

which is our first condition; and since $Sa = S'b$, hence we must have also

$$S = S' ;$$

i.e. the pans must have equal weights, which is our second condition.

83. Sensitiveness.— *A balance is said to be sensitive if a very small difference in weights causes a great deflection.*

To study the sensitiveness of a balance we shall suppose it to be true; *i.e.* we shall suppose $S = S'$ and $b = a$. Place two weights P and Q in the pans, and let the beam be deflected through an angle Θ, viz., let $A'B'$ be the position in which it comes to rest under the action of forces. The centre of gravity (G) of the beam moves to G', hence the weight will also affect. Since the beam is in equilibrium, the moments about C must be equal and opposite

i.e. $$(P + S)a \cos \Theta = (Q + S)a \cos \Theta + Wh \sin \Theta,$$

or $$a(P - Q) \cos \Theta = Wh \sin \Theta,$$

or $$\tan \Theta = \frac{a(P - Q)}{W \times h}.$$

We want that for a small difference between the weights the deflection should be appreciable; in other words, that tan Θ should be great for a small difference $(P - Q)$ in the weights. In order that it may be so, a should be large, or W (*i.e.* the wt. of the beam) should be small, or h, *i.e.* the distance between the $C.G.$ of the beam and the point of suspension, should be small. In practice a cannot be made very large, for the balance otherwise becomes unmanageable, nor can the beam be made light, for it would be bent by heavy weights. So the only method to increase the sensitiveness is to make h, *i.e.* the distance between C and G, small.

84. Stability.— *A balance is said to be stable if it returns quickly to its position of equilibrium after being displaced.*

That it may be so, the moment tending to bring it back to the position of equilibrium should be as great as possible. When the pans are equally loaded, the only quantity tending to bring the beam back is $Wh \sin \Theta$. So that this moment may be as great as possible, for a fixed value of Θ, h should be made large. Note that by increasing W no advantage is

secured, for although the moment tending to bring back the beam to horizontal position becomes greater, the mass to be moved also becomes greater. Hence the only method to make the balance stable is to increase h, which reduces the sensitiveness. Thus we see that **stability is opposed to sensitiveness.**

85. To determine the Real Weight of a Body with a False Balance.—(1). The simplest method is one which is called the method of double-weighing. The body to be weighed is balanced by a counterpoise, say lead-shot or sand. Then the body is replaced by the standard weights till the beam is horizontal again. Since the standard weights are balancing the counterpoise of the given body, their mass is the same as that of the body.

(2). (*a*) Let the arms of the balance be unequal, say they are a and b, and let further the real weight of the body be w, which we want to determine. Place it in two pans turn by turn, and let the weights which balance it under the two conditions be x and y. Suppose when x balances w (the beam being horizontal) the distance of w from the fulcrum C is a and of x is b. Taking moments,* we get

$$wa = xb\ ;$$

and when y balances w, the perpendicular distance of w is b and of y is a,

therefore $$wb = ya.$$

Multiplying the corresponding sides of these two equations we get

$$w^2ab = xy\,.\,ab,$$

or $$w = \sqrt{xy}.$$

(*b*) Let the pans be unequal in weight, and say the weights are S and S', the arms being equal in length. Let, further, the real weight be w. Place it first in one pan, say

---

* It is supposed that the beam is horizontal when the pans are empty. Mathematically speaking, it means that

$$Sa = S'b$$

where S and S' are the weights of the corresponding pans. When the body is placed in one pan and is balanced, taking moments we get

$$(w+S)a = (x+S')b,$$

or subtracting the upper equation, we get

$$wa = xb.$$

Similarly $$wb = ya,$$

weighing S, and balance it with x. The moments about the point C will be equal and opposite,

$$(w + S)a = (x + S')a$$

or

$$w + S = x + S'$$

Now place it in the other pan and balance it with y. Taking the moments as before,

$$(w + S')a = (y + S)a,$$

therefore

$$w + S' = y + S$$

Adding the corresponding sides,

$$2w = x + y,$$

or

$$w = \tfrac{1}{2}(x + y)$$

86. Steelyard. - It is a special form of balance very useful for rough but rapid weighing. It consists of a beam AB

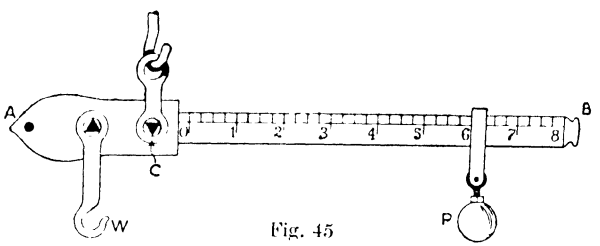


Fig. 45

with fulcrum (at C) very near one of the ends. The arm BC has a number of divisions, and over it moves a weight P. By simply changing the position of P the weight W can be balanced. The divisions along BC are marked first by suspending at the hook a known weight, and marking the point (with that weight) on the arm CB where P balances the given weight. By using different weights the arm CB is graduated. It should be clearly grasped that the graduations will be correct if P is constant. When it is changed the graduations must be changed correspondingly.

2. Wheel and Axle

87. We can raise a heavy body with the help of the lever, but the height to which it can be raised is limited, for at the most it can be equal to the weight-arm (which is shorter than the other arm in order to secure mechanical advantage greater than one). If the body is to be raised to very great heights, the fulcrum on which the lever rests must be raised con-

tinuously. In order to do this conveniently the lever is modified to what is called *wheel and axle.*

It consists of two cylinders of different diameters turning about a common fixed axis. The cylinder of bigger diameter is called the **wheel,** and the other one of smaller diameter is called the **axle.** The weight W is hung from the rope coiled round the axle. The power P is applied to the end of the second rope coiled round the wheel in the opposite direction, so that when the rope 2 is uncoiled the rope 1 is coiled round, raising thereby the weight. Frequently, instead of rope 2,

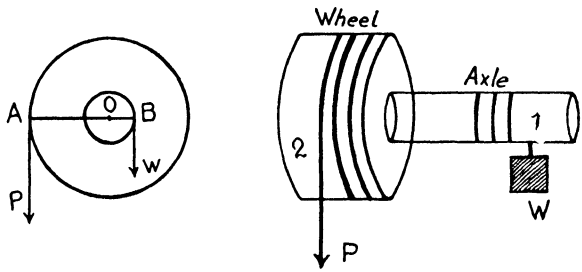


Fig. 46

handles projecting from the rim are used, as, for instance, in the case of the familiar device for raising water from the wells.

Let us now see what is the mechanical advantage in this case. The moment of the force P about the point O is $P \,.\, AO$ or Pa, and of W is $W \,.\, OB$ or Wb. So that there may be equilibrium Pa must be equal to Wb, *i.e.*

$$Pa = Wb;$$

or

$$\frac{W}{P} = \frac{a}{b} = \frac{\textit{radius of the wheel}}{\textit{radius of the axle}}.$$

First equation shows that the wheel and axle has the same principle as a straight lever. As a matter of fact, at any given instant AOB forms a straight lever ; but at the next moment this radius AO moves off and is replaced by another radius, and so on. This shows *that the wheel and axle is simply a continuous or perpetual lever.*

3. The Pulley

88. The pulley consists of two parts, sheave and block. The **sheave** is a circular disc or wheel with a groove cut in its circumference round which a string can pass. It is mounted on an axle which is fixed to a framework called the **block.** The

sheave rotates freely in the block. The block may be fixed or movable. Let us first consider the case when the block is fixed; the pulley is said to be a fixed pulley in this case.

(1) **The Single Fixed Pulley.**—When the pulley is used as fixed, no mechanical advantage is obtained, for it is simply working like a lever with equal arms. So that there may be equilibrium the moments of P and W about O must be equal and opposite, or mathematically

$$P \times OA = W \times OB.$$

But $OA = OB$ [being the radii],

hence $P = W$.

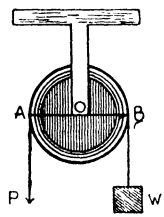


Fig. 47

Such a pulley enables us simply *to change the direction of the force*. To raise the weight W *up* we are pulling the rope *down*. Now let us pass on to the second case.

(2) **The Movable Pulley.**—In this case the weight is suspended from the pulley-block. One end of the string that passes round the pulley is fixed to a fixed support as at C, and at the other end the power is applied. We shall suppose throughout that there is no friction, and that therefore the tension is the same at all parts of the string. It may be remarked, however, in passing that the greater the size of the pulley the less the force required to overcome friction, so that while dealing with big pulleys the friction may be neglected. When the strings are vertical it is clear that the total upward force is $2P$, and if the weight of the pulley is to be neglected, for equilibrium

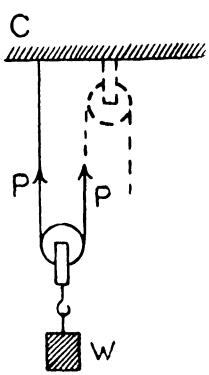


Fig. 48

$$2P = W,$$

or

$$\frac{W}{P} = 2.$$

Thus *the mechanical advantage is* 2.

If the weight of the pulley cannot be neglected, and is equal to w, the condition of equilibrium will be

$$2P = W + w.$$

$$\text{Mechanical advantage} = \frac{W}{P} = 2 - \frac{w}{P}.$$

This shows that the mechanical advantage is slightly less than 2.

If the direction of the force is to be changed, another fixed pulley in the dotted position should be used, which will not affect in the least the mechanical advantage.

In order to secure greater mechanical advantage than two, a combination of pulleys is used. There are three types of the combination which are in general use. They are called the first system, the second system, and the third system. We shall consider them one by one.

89. The First System of Pulleys. In this system each pulley is suspended by a separate string, one end of which is fixed to the beam and the other end to the block of the next pulley. The weight is suspended from the lowermost pulley. Note that *all pulleys are movable*. Let us establish the relation between P and W. We shall suppose the pulleys to be weightless. Start with the highest pulley, A_3 in this case. The total upward force is $2P$ and the downward force is the tension of the string going round A_2. Since the pulley A_3 is in equilibrium under the action of these forces, the tension in the lower string going round A_2 must be $2P$. Consider now the equilibrium of the pulley A_2. The total upward force is $2P+2P=4P$, and the downward force must be also $4P$, which gives us the tension in the string going round A_1. Similarly considering the equilibrium of A_1 we see that the total upward force is $4P+4P$, and the downward force is W. Because they are in equilibrium, therefore $W=8P=2^3P$. The mechanical advantage $\frac{W}{P}=2^3$ or 8.

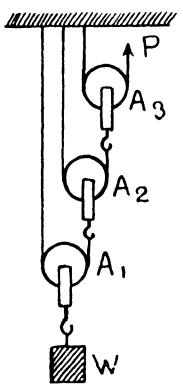


Fig. 49

If there were 4 movable pulleys the mechanical advantage would be 2^4 or 16. More generally, if there be n movable pulleys, the mechanical advantage would be given by

$$\frac{W}{P}=2^n.$$

90. If the weights of the pulleys cannot be neglected, then with each movable pulley the mechanical advantage will be less than two, hence the total mechanical advantage will be much less than 2^n. Let us consider it in detail. Consider the pulley A_3:

$$2P=T_2+w_3, \quad \text{or } T_2=2P-w_3.$$

Take the next pulley A_2:

$$2T_2 = T_1 + w_2.$$

Therefore
$$T_1 = 2(2P - w_3) - w_2$$
$$= 4P - 2w_3 - w_2.$$

Take now the pulley A_1:

$$2T_1 = w_1 + W.$$

Therefore
$$W = 2(4P - 2w_3 - w_2) - w_1,$$

or
$$W = 8P - 4w_3 - 2w_2 - w_1,$$

or
$$8P = W + w_1 + 2w_2 + 4w_3.$$

So that we see that the condition for equilibrium for the third pulley in terms of P and W is

$$2^3P = W + w_1 + 2w_2 + 2^2w_3.$$

Generalising for n pulleys, the condition becomes

$$2^nP = W + w_1 + 2w_2 + 2^2w_3 + \ldots + 2^{n-1}w_n$$

where w_1 is the weight of the lowest pulley, and w_n the weight of the uppermost pulley. In numerical questions it will be useful to remember that the highest pulley should be the lightest unless otherwise stated.

Mechanical advantage $= \dfrac{W}{P} = 2^n - \dfrac{w_1}{P} - \dfrac{2w_2}{P} - \ldots - \dfrac{2^{n-1}w_n}{P}$.

91. The Second System of Pulleys. In this system there are two blocks of pulleys, each block having a set of sheaves.

The string is fastened either to the upper block or to the lower block and is then passed round a movable and a fixed pulley in turn. In Fig. 50 both the types are shown. It should be noted that when the string is fastened to the upper block the number of sheaves must be the same in each block, but when it is fastened to the lower block the number of sheaves in the lower block will be one less than that in the upper block.

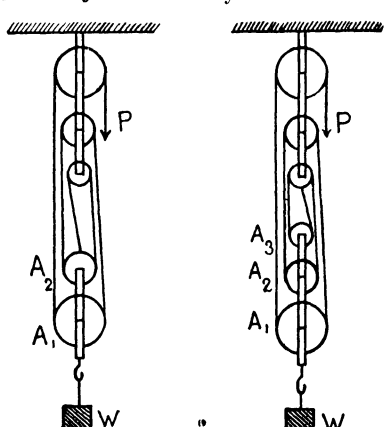


Fig. 50

The string finally must pass over a fixed pulley and be pulled downwards.

Let us consider the condition of equilibrium. The tension

round the string is P everywhere, and if there are n segments of the string which support the lower block, the upward force on it is nP; this must be equal to the weight supported, neglecting the weight of the pulleys themselves. Thus we see that in this case the mechanical advantage $\frac{W}{P}=n$.

It is important to remember that *n means the number of segments of the string which support the lower block.*

If the weight of the lower block cannot be neglected, let it be w. For equilibrium the total upward force must be equal to the downward force

$$nP = W + w,$$

or

$$\frac{W}{P} = n - \frac{w}{P},$$

or the mechanical advantage is less than it will be otherwise.

92. The Third System of Pulleys.—In this system the uppermost pulley is fixed, the one end of the string passing round it is fastened to a bar which supports the weight, and the other end supports the next pulley. One end of the string passing round this second pulley also goes to the bar supporting the weight, and so on. It is simply the 1st system of pulleys in the inverted position. The bar H will become the fixed beam in the 1st system and the weight will be supported by G.

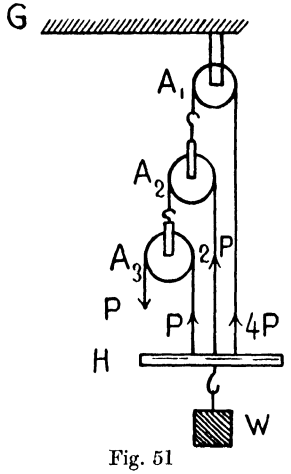


Fig. 51

To find out the relation between P and W let us begin at the lowest pulley, supposing that each pulley is weightless. The pull P produces the same pull on the other side of the string which is communicated to the bar H supporting weight. The tension in the rope round the second pulley will evidently be $2P$, and this is also communicated to the bar supporting the weight. The tension in the rope round the 3rd pulley is obviously $4P$. Thus the total upward force at the bar is $4P+2P+P=7P=(2^3-1)P$. This must be equal to W the weight (neglecting the weight of the pulleys), or

$$(2^3-1)P = W.$$

The mechanical advantage therefore $= \frac{W}{P} = 2^3 - 1$, or more generally when there are n pulleys in all, *the mechanical advantage will be* $2^n - 1$.

93. If the weight of each pulley is appreciable, let us see what effect it will produce. The tension in the second string will be $2P + w_3$, and the upward pull on the bar due to this string will be $2P + w_3$.

The tension in the third string will be $4P + 2w_3 + w_2$.

Thus the total tension will be $7P + 3w_3 + w_2$.

This must be equal to W, the weight supported.

The mechanical advantage $\frac{W}{P} = 7 + \frac{3w_3}{P} + \frac{w_2}{P}$.

Thus we see that the mechanical advantage is greater in this case when the pulleys have appreciable weight. Note that the weight of the fixed pulley does not produce any effect.

93(a). Application of the Principle of Work. — We have not cared to prove in any of the above cases the fact that what is gained in power is lost in distance. Let us see it in the third system of pulleys. Suppose weight rises through 1 ft., the string over A_1 will become slackened and will consequently lower the pulley A_2 by one foot making the string passing over it slack by 2 ft. (one foot on one side and one foot on the other side). But the weight also, by rising through one foot, made this string slack by one foot, hence the total slackening is 3 feet; in other words, the third pulley A_3 is lowered by $(2^2 - 1)$ or 3 feet. Let us see what effect it produces. The string passing over the third pulley is made slack by 6 feet, due to the lowering of the pulley A_3; and one foot is added to it by the rise of the weight, hence the total slackening is 7 ft. Thus we see that the point of application moves through 7 ft. or $(2^3 - 1)$ when weight moves through 1 ft., which shows clearly that what we gain in power we lose in distance. Similarly applying this principle to other systems we can see that the principle of work holds good throughout.

4. The Inclined Plane

94. We have already seen that it is possible, by a small force, to raise a heavy weight up an inclined plane. We found that a force $W \sin \Theta$ or $mg \sin \Theta$ keeps the body in equilibrium, when the force is applied along the plane, Θ being the angle of inclination. Mathematically we expressed it as $P = W \sin \Theta$.

Thus we see that the mechanical advantage is

$$\frac{W}{P} = \frac{1}{\sin \Theta}.$$

Since $\sin \Theta$ is always less than one (unless Θ is 90°), we see that the mechanical advantage is greater than one. But

note, when the force works along the whole length of the plane the weight rises through height only, which shows that what we gain in power we lose in distance.

When the force is applied horizontally we know that

$$P = W \tan \Theta,$$

therefore

$$\frac{W}{P} = \tan \Theta.$$

5. The Wedge

95. It is simply a double inclined plane made up of steel or some other hard material. It is generally used either to split wood, or for other similar purposes. Knife and chisel are most common examples of the wedge.

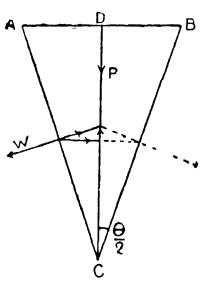


Fig. 52

In order to find the mechanical advantage we shall suppose that the angle of the wedge is Θ, and that it is driven into the wood with a force P, and further that it exerts on the wood forces W and W perpendicular to its own faces. (The wedge is symmetrical about the vertical line DC.) The wood naturally will exert an equal and opposite pressure W on the wedge. The vertical components of W and W will balance P, *i.e.*

$$2W \sin \frac{\Theta}{2} = P,$$

$$\therefore \quad \text{Mechanical advantage} = \frac{W}{P} = \frac{1}{2 \sin \frac{\Theta}{2}}.$$

It is supposed in the above calculations that there is no friction, but as a matter of fact without friction the wedge will not work; it will recoil or spring back as soon as it is driven inwards. The actual theory of the working of the wedge is very complicated, for we require to know not only the force of friction to be overcome, but also we have got to remember that the wedge is moved not by pressure or continuous action but by impact or percussion, and it is almost impossible to state accurately the proportion between the force of the blow and the resistance to be overcome.

6. The Screw

96. It is another form of inclined plane, the only difference being that it is *a movable inclined plane*, and, instead of going downwards directly, its motion is circular.

How the screw is formed from an inclined plane will easily be understood from Fig. 53.

Let $EFGB$ be a cylinder of diameter D. Cut a rt. angled triangle of paper with base equal to πD in length and height $= BC$. Wrap it round the cylinder, keeping the base AB at rt. angles to the axis; since AB is equal to the circumference, the points B and A will coincide. The point C will be directly above the points A and B. The hypotenuse will mark out on the cylinder a path of the form of a spiral. This path,

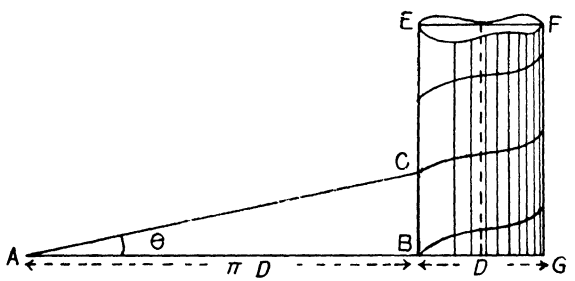


Fig. 53

which gives the form of the screw, is called the **thread** of the screw. The distance parallel to the axis of the cylinder between any two contiguous threads is evidently the height of the triangle, and is called the **pitch.** The angle BAC is called the angle of the screw. It is clear from the figure that the tangent of the angle of the screw *i.e.*

$$\tan \theta = \frac{\text{the pitch}}{\text{the circumference of the cylinder}}.$$

The thread is not a line in actual practice, as is supposed above, but has a definite thickness, on account of the fact that the screw is generally used to transmit a very great force.

If the thread is projected over the surface of the cylinder, the cylinder is called a *screw* or *a bolt*; but if the thread is made to run like a groove on the inner surface of a hollow cylinder, it is called a *nut*. Let us determine the mechanical advantage. When the nut is fixed and the bolt is moved forward, after one complete revolution, the screw or bolt,

is advanced by a distance equal to the pitch of the screw, let us call it h. To simplify matters, let us confine ourselves to a concrete case, say a **screw press.** Let the power P be exerted at one end and W the force exerted by the screw at the other end. Let, further, the lever which is fixed at the end of the screw where power is applied possess an arm l. In order to drive the screw downwards we have simply to rotate the lever.

In one complete revolution the force acts through a distance $2\pi l$; and therefore the work done is $P \times 2\pi l$. But in one complete revolution the screw goes downwards through h, the pitch of the screw; if at this end the screw exerts a force W, the work done is $W \times h$. From the principle of work we have

$$\therefore \quad P \times 2\pi l = Wh,$$

or

$$\frac{W}{P} = \frac{2\pi l}{h}.$$

How much mechanical advantage it means cannot be grasped from the formula written above. We shall consider one example.

Example. —A youth wants to exert a pressure of 22 tons with the help of a screw of $\frac{1}{16}$ inch pitch. The arm of the lever at the end of the screw is 14 inches. How much power should he apply?

Using the relation established above we get

$$\frac{W}{P} = \frac{2\pi l}{h}$$

$$\frac{22 \times 20 \times 112}{P} = \frac{2 \times \frac{22}{7} \times 14}{\frac{1}{16} \times \frac{1}{12}}$$

$$= 2 \times \frac{22}{7} \times \frac{14}{12} \times 16 \times 12$$

$$= 2 \times 22 \times 2 \times 16.$$

$$P = \frac{22 \times 20 \times 112}{2 \times 22 \times 2 \times 16} = 35 \text{ lb. wt.}$$

Thus we see that *by exerting a force of* 35 *lb. only he can make the effective pressure to be* 22 *tons—a pressure which a hundred people together will not be able to exert without the help of a machine.*

The mechanical advantage in a real screw is not so enormous, because friction consumes in general more than two-thirds of the work done on the screw. It should be remembered that this loss due to friction is not wholly a disadvantage, for it is friction which keeps the screw from turning backwards of itself.

EXERCISES

1. The arms of a false balance are in the ratio of 20 to 21. What will be the loss to a tradesman who places articles to be weighed at the end of the shorter arm if he is asked for 4 lb. of goods priced at 3s. per lb. ?

Let the weight of the article be W lb. ; taking the moments about O we get

$$W \times 20 = 4 \times 21,$$

therefore

$$W = \frac{4 \times 21}{20} = \frac{21}{5} \text{ lb.}$$

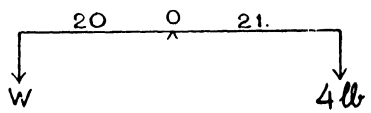


Fig. 54

Now since he is charging for 4 lb. only but he is giving away $4\frac{1}{5}$ lb., he loses the price for $\frac{1}{5}$ lb. or $7\frac{1}{2}d$.

2. There are three pulleys in a combination of the third system of pulleys ; neglecting the weights of the pulleys, find what pull must be exerted on the free end to raise a load of 1 cwt.

The ratio of W to P in this case is equal to

$$\frac{W}{P} = 2^n - 1$$

where $n = 3$, and $W = 1$ cwt. or 112 lb. ;

therefore $$\frac{112}{P} = 2^3 - 1 = 7,$$

therefore $$P = 16 \text{ lb.}$$

3. In the first system of pulleys with four equal movable pulleys a force equal to the weight of 20 lb. suffices to support 245 lb. *Calculate the weight of each pulley.*

Let the tension in second, third, and fourth string be called T_2, T_3, and T_4; let, further, the weights of the pulleys be called w_1. w_2, w_3, w_4.

Since pulley (4) is in equilibrium,

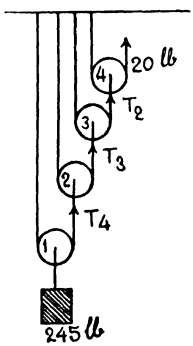


Fig. 55

therefore $T_2 = 40 - w_4$.

Similarly $T_3 = 2T_2 - w_3 = 80 - 2w_4 - w_3$.

and $T_1 = 2T_3 - w_2 = 160 - 4w_4 - 2w_3 - w_2$,

and lastly $2T_4 = w_1 + 245$;

therefore $2T_4 - w_1 = 245$,

or $320 - 8w_4 - 4w_3 - 2w_2 - w_1 = 245$.

Since $w_4 = w_3 = w_2 = w_1 = w$,

therefore $320 - 15w = 245$ lb.,

or $15w = 75$ lb.

or $w = 5$ lb.

4. An object is placed in one scale-pan of an ordinary balance and it is balanced by 20 lb. The same object is then put in the other scale-pan and now it takes 21 lb. to balance it. When both scale-pans are empty the scale-pans balance. What is the matter with the balance, and what is the true weight? *Ans.* 20·493 lb.

5. A man weighing 10 stones is supported in a well by means of a windlass, the arm and axle of which are 30 inches and 9 inches in radius respectively. (1) What force must be applied to support him? (2) to let him down with uniform velocity? *Ans.* 42 lb. in each case.

6. In the first system of pulleys what weight can an effort of 3 lb. support by means of three movable pulleys, (i) if the pulleys are weightless, (ii) if they weigh $\frac{1}{4}$, $\frac{1}{2}$, and 1 lb. respectively? *Ans.* 24 lb. and 21 lb.

7. In the second system of pulleys, if a force of 6 lb. just supports a weight of 28 lb. and a force of 8 lb. just supports a weight of 42 lb., find the number of pulleys and the weight of the lower block. *Ans.* 7 and 14 lb.

8. In a certain system of pulleys it is found that power descends 1 ft. while the weight rises 1 inch. What power will be required to raise a weight of 1 cwt.? *Ans.* $9\frac{1}{3}$ lb.

CHAPTER VIII

FRICTION

97. In the preceding chapters we have already used the term " friction " here and there without explaining it. Let us now see what we mean by this term. When a solid body moves over the surface of another solid body, as, for instance, a ball rolling on the ground, or when a body moves through a fluid, as a boat moving through water, its motion is *always* opposed by a force oppositely directed to the motion. The amount of this resisting force of course depends upon the nature of the pair of bodies concerned; in some cases it is small, in other cases great, but never zero. This opposing force is called the **force of friction** or, more simply, the **friction.**

When a body lying on the surface of another body is subjected to a force and no motion results, it is the force of friction which prevents the motion. *Friction is a self-adjusting force*; that is to say, only as much of it is called into play as is just sufficient to prevent motion. Let, for instance, a wooden trolley containing weights be pulled gently with the help of a string fixed to its side over the surface of a table. A resistance will be felt which will prevent the motion of the trolley. If the pull is stopped the force of friction also stops.

The amount of the force of friction which can be called into play is not, however, unlimited. If we gradually increase the pull on the trolley we find that, after a certain stage, the force of friction can no more balance the pull, with the result that the trolley begins to move. *This maximum force of resistance which can be called into play is called the* **limiting friction.**

Coulomb was the first physicist to investigate the motion of a body on the surface of another body. He deduced the following laws for the *limiting friction* from his experiments:

(1) *The magnitude of the limiting friction is directly proportional to the normal reaction between the two surfaces.*

(2) *The magnitude of the limiting friction between two bodies is independent of the area and the shape of the surfaces in contact, so long as the normal reaction remains the same.*

These laws are experimental and are therefore only approximately true. They cannot be accepted as rigorously accurate.

They, however, represent with a fair degree of accuracy the actual conditions of mechanical problems.

98. We can verify these laws in the laboratory easily. Let us suppose we have a well-planed wooden board B clamped on to the table. The board has a ball-bearing pulley D at one end. A small wooden tray A whose lower surface is well planed has a hook fixed in its side. It is placed on the board. A thread is fastened to the hook on one side and to a scale-pan on the other side. It passes over the pulley. The weights of the scale-pan and of the tray are known. Weights are added to the pan till the tray just begins to slide along the board on gently tapping.

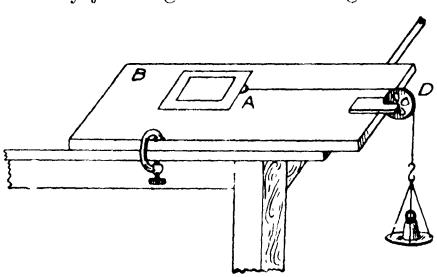


Fig. 56

Some weights are placed in the tray and again the experiment is tried. This process is repeated with three or four different weights. The normal reaction here is equal to the weight of the tray + additional weights, if any. The friction is equal to the weight of the scale-pan and the weights added to produce sliding. It will be seen from the results of the experiment that *limiting friction is always proportional to normal reaction*, viz. the ratio of the limiting friction to the normal reaction is always constant. In order to make it clear we shall take the data of an actual experiment.

EXPERIMENT : *Limiting Friction between Wood and Wood.*

The weight of the empty tray = 450 gm.
The weight of the scale-pan = 104 gm.

Additional Weights in Tray.	Weights added to the Pan just to produce Sliding.	Normal Reaction R.	Limiting Friction F.	Ratio $\frac{F}{R}$
0 gm.	46 gm.	450 gm.	150 gm.	0·33
400 ,,	168 ,,	850 ,,	272 ,,	0·32
800 ,,	313 ,,	1250 ,,	417 ,,	0·33

Mean ratio = 0·33 (approx.).

The experiment given above shows that the ratio remains practically the same whatever the normal reaction may be.

By taking trays of different sizes and shapes, but of the same material, and repeating the experiment it can be seen that the ratio $\frac{F}{R}$ is just the same.

This shows that the limiting friction depends upon the nature of the materials and not upon their size or shape.

If in place of a wooden board we clamp a sheet of glass we can find out the ratio $\frac{F}{R}$ for wood and glass. Similarly any other pair of surfaces can be tried.

99. This law, when expressed in mathematical language, can be written as

$$F \propto R$$

where F is the limiting friction and R the normal reaction.

Or
$$\frac{F}{R} = \text{a constant}$$
$$= \mu \text{ (say)},$$
or
$$F = \mu R.$$

This constant is called the **coefficient of friction.** It must be clearly understood that *this equation holds good only when sliding is just to take place.*

The constant μ is different for different pairs of surfaces. It depends upon the state of polish of the surfaces. It may be made very small by making the surfaces very smooth and by polishing them. But it should be noted that *we cannot get a pair of perfectly smooth bodies*—bodies for which μ is zero.

The following table gives the approximate values for the coefficient of friction for some important pairs :

Wood upon wood dry	0·25 to 0·5
,, ,, ,, soaped	0·04 to 0·2
,, ,, polished metal	0·5 to 0·6
Metals upon metals dry	0·15 to 0·20
,, ,, ,, wet	0·3

100. We can study the laws in another way also. Let us suppose that the board instead of being clamped on to the table is hinged so that the angle between the board and the table can be gradually increased. Imagine the wooden tray

to be standing on such a board which makes an angle of 0° with the table to start with. Raise the board gradually, gently tapping it from time to time, till the tray begins to slide down. Note the angle at which the sliding *first* occurs. It will be found that for any one pair of substances the angle at which the sliding just begins to take place is always the same. The weights in the tray may be altered, but the angle will be found to be always the same. Moreover, it will be found to be independent of the area in contact. The slipping always begins to take place just at the same angle whether the tray is small in size or big. This shows *that the angle at which slipping first takes place depends upon the nature of the two substances in contact.* This angle is called the **angle of friction.**

The angle of friction is different for different pairs of substances. For wood it varies from 10° to 25°; for iron

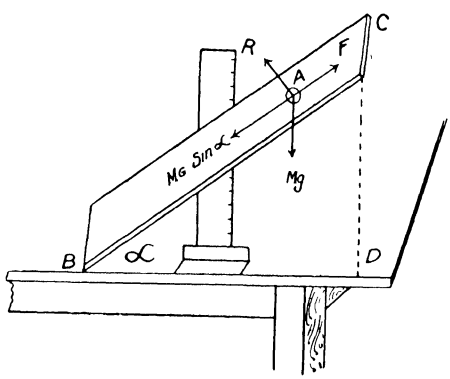


Fig. 57

on wood it varies from 10° to 30°, while for iron sliding on iron it varies from 9° to 12°.

Let us now study the relation which exists between the coefficient of friction and the angle of friction. Suppose Mg is the weight of the body A placed on the inclined plane, and α is the angle which the plane makes with the horizontal when the slipping first occurs.

The force of friction *opposes the slipping down the plane.* When the body just begins to slide down, at that time the component due to weight ($Mg \sin \alpha$) just overcomes the frictional force. In other words, *just* before the slipping takes place, the forces acting upon the body are in equilibrium.

Let in the position of equilibrium R be the normal reaction; F, the limiting friction acting along the plane; and Mg the weight.

Resolving along the plane we have

$$F = Mg \sin \alpha,$$

and resolving perpendicular to the plane we get

$$R = Mg \cos \alpha,$$

where α is the angle of friction.

By dividing the corresponding sides we get

$$\frac{F}{R} = \tan \alpha.$$

But $$\frac{F}{R} = \mu.$$

$\therefore$ $$\mu = \tan \alpha.$$

Thus we see that *the coefficient of friction is equal to the tangent of the angle of friction.*

101. Static and Dynamic Friction.—The force required to start a body to slide over the surface of another body is distinctly greater than the force necessary to maintain it sliding with a constant velocity. In other words, **starting friction,** which is very often called **static friction,** is greater than the **sliding friction,** which is generally called **dynamic friction.** What we have studied so far is static friction. We can define ***dynamic friction*** *as the force necessary to maintain a body in steady motion over the surface of another body when once the motion has been started.*

It is found experimentally that the (dynamic) frictional force is independent of the velocity provided the latter is not too great. It is clear that we shall have a second coefficient of friction corresponding to this sliding or dynamic friction. It can be expressed mathematically as

$$\mu_d = \frac{F_d}{R}$$

where μ_d = coefficient of dynamic friction, F_d is the frictional force when steady sliding has been attained, and R is the normal reaction between the two surfaces in contact.

Remember *coefficient of static friction is* ***invariably greater*** *than the coefficient of dynamic friction.*

Coefficient of Friction

Substances.	Static Coefficient.	Dynamic Coefficient.
Oak on oak	0·62	0·48
Wrought iron on cast iron	0·19	0·18
Cast iron on cast iron. .	0·16	0·15

101a. Fluid Friction.—So far we have been discussing solid friction. A discussion at length of fluid friction is beyond the scope of this book. It is interesting, however, to note that whereas solid friction for moderate speeds is independent of velocity, the resistance offered by a fluid to the motion of a solid through it increases with its velocity. The force of friction (*i.e.* resistance) for very slow speeds increases nearly as the velocity. For more rapidly moving bodies it increases nearly as the square of the velocity. For very high speeds it increases at a considerably greater rate. It is on account of this reason that the trains are not run very fast, because resistance of air, which is very small in comparison to the solid friction for small speeds, becomes many times more important than the solid friction. The overcoming of this extra resistance due to air adds considerably to the cost of coal required to run the engine.

102. Work done against Friction.—Whenever a body is moved over the surface of another body, part of the work done is spent up in overcoming friction. This part of the work done is always irrecoverable; that is to say, it is always lost to us for mechanical purposes. Either the surface over which a body moves may be horizontal or it may be inclined. We shall consider the work done against friction in both these cases.

(a) To find the work done in dragging a body along a rough horizontal plane.—Let us suppose μ is the coefficient of (dynamic) friction of the pair of bodies, and Mg is the weight of the body which is moved. The normal reaction will evidently be equal to Mg. The force of friction called into play will be equal to μMg. We know that

$$Work = force \times distance.$$

Substituting for force μMg we get

$$W = \mu Mg \times S$$

where W stands for the work done against friction and S stands for the distance through which the body is moved.

(b) To find the work done in dragging a body up a rough inclined plane.—Let us suppose we drag the body A up the inclined plane BC, the coefficient of (dynamic) friction being μ. Since friction is an opposing force, therefore when A is dragged up, the force of friction will act in the downward direction, and it will be equal to μR or $\mu Mg \cos \alpha$. Moreover, the weight Mg has also a component $Mg \sin \alpha$ acting in the downward direction along the plane. Therefore the total force acting along the plane in the downward direction is equal to $Mg \sin \alpha + \mu Mg \cos \alpha$, or $Mg(\sin \alpha + \mu \cos \alpha)$. The work done in dragging up the body from B to C,

$$= Mg(\sin \alpha + \mu \cos \alpha) BC *$$
$$= MgCD + \mu MgBD.$$

But $MgCD$ is equal to the work done in dragging the body up the smooth inclined plane, and $\mu MgBD$ is equal to the work done in moving the body against friction.

Thus we learn that some work must be done to overcome friction whenever a body is moved over another body. *This part of the work is lost to us for mechanical purposes.*

This principle holds good in the case of machines as well. We can never make a machine which has no frictional forces between its parts, nor can we make it weightless, hence due to these two causes some work is always lost to us. In other words, we always have to do more work than we can get back from the machine, viz. *the efficiency of a machine must be always less than unity* (see § 78).

* If a body be dragged down an inclined plane which is too rough for the body to slide down by itself, the work done is

$$Mg(\mu \cos \alpha - \sin \alpha)BC.$$

EXERCISES

1. Find the H.P. of an engine which is to move at the rate of 30 miles an hour, the weight of the engine and the load being 50 tons, and the resistance from friction 16 lb. wt. per ton.

We know that

$$W = \mu Mg \,.\, S$$

where μMg is the force of friction and S is the distance passed over.

Let us find out the work done in one minute.

The friction is equal to 16×50 lb. wt.

The distance passed over in one minute $= 44 \times 60$ ft.

$\therefore$ Work done $= 16 \times 50 \times 44 \times 60$ foot pounds per min.,

or $$= \frac{16 \times 50 \times 44 \times 60}{33{,}000} = 64 \text{ H.P.}$$

2. Find the H.P. of an engine which is to move at the rate of 20 miles per hour up an incline which rises 1 foot in 100, the weight of the engine and the load being 60 tons and the resistance from friction 12 lb. wt. per ton.

The force exerted by the engine should overcome resistance as well as drag the body up the plane.

The total force downward $= Mg \sin \alpha +$ Frictional force,

or in lb.-wt. $= \left\{ \frac{(60 \times 2240 \times 1)}{100} + (12 \times 60) \right\}$.

The engine moves through $= \frac{88}{3} \times 60$ ft. per minute.

The work done $= 60 \left\{ \frac{224 + 120}{10} \right\} \times \frac{88}{3} \times 60$ per minute

$$= 60 \times \frac{344}{10} \times \frac{88}{3} \times 60 \text{ per minute}$$

$$= 60 \times 344 \times 88 \times 2 \text{ per minute.}$$

The H.P. required $= \frac{60 \times 344 \times 88 \times 2}{33{,}000} = 110{\cdot}08.$

3. Find the work expended in moving a weight of 1 cwt. along a plane 200 yards long, the coefficient of friction between the weight and the plane being 0 43.

$$W = \mu Mg \,.\, S. \text{ foot-poundals}$$
$$= \mu \,.\, M \,.\, S. \text{ foot-lb.}$$
$$= 0\ 43 \times 112 \times 200 \times 3 \text{ ft. lb.}$$
$$= 28{,}896 \text{ ft. lb.}$$

4. Find the H.P. of an engine which is able in 4 minutes to generate in a train, of mass 100 tons, a velocity of 30 miles per hour on a level line, the resistance due to friction being equal to 8 lb. wt. per ton. *Ans.* $166\frac{2}{3}$ H.P.

5. A train, of mass 50 tons, is kept moving at the rate of 30 miles an hour on the level by an engine whose H.P. is 160. Find the resistance due to friction per ton.

Ans. 40 lb. wt. per ton.

6. A train of mass 100 tons, including the engine, is drawn up an incline of 3 in 500 at the rate of 40 miles per hour by an engine of 300 H.P. Find the resistance per ton due to friction.

Ans. 14 69 lb. wt. per ton.

7. A train of mass 50 tons is moving up an incline of 1 in 280 with a uniform velocity; the resistance due to friction is 16 lb. wt. per ton; if the H.P. of the engine be 100, find the velocity of the train. *Ans.* $31\frac{1}{4}$ miles/hour.

8. A horse drawing a load along a level road at a speed of 3 miles per hour does 4500 ft.-lb. of work in 3 minutes; what pull does the horse exert continuously in drawing the load? *Ans.* 5 7 lb. wt.

9. A mass of 80 lb. rests on a plane ($\mu = 0\ 15$) inclined at an angle of 30° to the horizon. Compare the forces required to draw it up the plane by a rope parallel to the plane and by a horizontal pull. *Ans.* 791 to 1000.

10. A mass of 30 lb. is resting on a rough horizontal plane and can be just moved by a force of 10 lb. wt. acting horizontally. Find the coefficient of friction. *Ans.* $\mu = 0\ 33$.

CHAPTER IX

FLUIDS

103. All natural bodies that we see or feel around us can be divided into three classes : *Solids, Liquids* and *Gases.*

From our everyday experience we get a fairly good idea of the general difference between them. We know, for instance, that a solid does not change its shape or size ; a sphere of steel will have the same shape, whether it is put into a bottle or is placed in a cup. Moreover, we cannot force our finger into it, nor can we stir it up. A liquid, like water, has no shape of its own. It at once takes up the shape of the vessel containing it. But it has, however, a volume of its own, which cannot be changed unless it is subjected to a considerable force. Besides, it can be easily stirred. A gas has neither shape nor volume of its own. It not only takes up the shape of the vessel containing it, but completely fills it up too. The hand can be moved more freely in it than in a liquid. In other words, the particles can move among themselves more easily in a gas than in a liquid.

But on closer examination we find that there are some bodies which are intermediate between these different states. For instance, cobbler's wax can be cast in the form of a bell which will ring, but if given sufficient time, say years, it will flow and take up the shape of the vessel containing it. Naturally a question arises, How are we to say in such a case whether the body is a solid or a liquid ? The answer which Physics gives is ; " Subject the body to the action of a deforming force, and if it has even the slightest tendency to return to its original shape when the force is removed, in other words, if it has the *Elasticity of form* (what is called *Rigidity*), it is solid, otherwise liquid or more generally fluid ". In simpler words, this means that *a fluid yields to any force, howsoever small, tending to change its shape or to produce a movement of*

its particles. This is sometimes expressed by saying that a fluid offers no resistance to a shearing stress.

Before we proceed further, let us first understand something about elasticity, on which our conception of the distinction between the solids and fluids is based.

104. Elasticity.—When a body is subjected to external forces, in general, it changes either its form or volume, and as soon as these forces are removed it recovers more or less its original condition. Some bodies recover completely, while others have a permanent change left in them. We express it by saying that the bodies which recover more completely are more elastic. In short, *the power of recovery is called* ***elasticity***.

Let us see how this recovery is brought about. As soon as the external forces begin to act upon a body without causing it to move, a strain is set up in it, viz. either a change of volume or a change of shape is produced, which means that there is a relative motion of the various particles of the body with respect to one another. By the relative displacement of the particles forces are called into play in the body which tend to bring it back to the original condition. *This restoring force called into action owing to the strain is called* ***stress***. By ***strain*** *is meant the change in volume or shape which a body undergoes when it is subjected to the external forces.* When the stress called into play is very great, the body is said to be very elastic; for instance, when an attempt is made to change the shape or volume of an iron sphere, very great forces are called into play which resist the change. Iron consequently is very elastic. In general the stress called into play is equal and opposite to the forces deforming the body, hence it will simplify matters if for stress we consider the deforming forces.

The elasticity of a body is measured by the relation,

$$\textbf{\textit{Elasticity}} = \frac{\textbf{\textit{stress}}}{\textbf{\textit{strain}}}.$$

It should be noted that the solids resist change of volume as well as change of shape, liquids resist change of volume to a smaller extent than the solids, but they offer absolutely no resistance to change of shape, whereas gases resist even the change of volume to a very much smaller extent than the liquids. The resistance to the change of volume is called *volume elasticity.* From what has been said above it follows that liquids and gases (fluids) possess volume elasticity only.

105. Let us consider the ratio $\frac{stress}{strain}$, for solids, liquids,

and gases. Consider a wire L cm. long of cross section a sq. cm. fastened at the upper end, carrying a pan at the lower end. Let a force of m grams wt., or F dynes be applied to the lower end, and let the change in length produced be l cm.

The strain $= \frac{\text{change in length}}{\text{original length}} = \frac{l}{L}$, and the stress $= \frac{F}{a}$.

Therefore elasticity (E) which is equal to

$$\frac{Stress}{Strain} = \frac{F}{a} \times \frac{L}{l} = \frac{FL}{al}.$$

It is sometimes called *Young's Modulus.*

Consider next a definite volume of a fluid, whether liquid or gaseous. What will be said in the following few lines will be applicable to both these states of matter. Suppose the volume of the fluid is V and the pressure is P dynes. On increasing the pressure to $P + dp$ where dp represents the increase in pressure, let the volume be reduced to $V - dv$ where dv denotes the change in volume.

Since strain $= \frac{\text{change in volume}}{\text{original volume}}$, hence it is equal to $\frac{dv}{V}$

The stress which is producing this change in volume is the increment of the pressure, *i.e.* dp.

Therefore $E = \frac{stress}{strain} = \frac{dp}{dv} \times V = V\frac{dp}{dv}$.

Example.—The elasticity of sea-water is 20,000 atmospheres; find the compression produced at a depth of 150×34 ft.

$E = \frac{stress}{strain} = V\frac{dp}{dv} = 20{,}000$ atmospheres. But $\frac{dv}{V}$ is the compression produced per c.c. of water.

Hence compression $= \frac{dv}{V} = \frac{dp}{20{,}000} = \frac{150 \times 34}{34 \times 20{,}000} = \frac{3}{400}$ 0·0075, for $dp = 150$ atmospheres, as a column of water 34 ft. deep produces the same pressure as one atmosphere. Thus we see from this example that the compression produced by a pressure of 150 atmospheres is 0·0075, or by one atmosphere $= \frac{0{\cdot}0075}{150} = 0{\cdot}00005$. This is called the *Coefficient of compressibility.* For gases, when the pressure is made equal to two atmospheres, the volume becomes half or, in other words, by an increase in pressure of one atmosphere, the volume is halved (at a pressure of one atmosphere). Thus the coefficient of compression is 0·5. The two figures 0·00005 and 0·5 enable us at once to compare the compressibility of liquids and gases. It is clear that in comparison the liquids are almost incompressible.

We have already studied the effect of forces on solids considering them as rigid bodies, now let us study the effect of forces on liquids and gases.

106. Viscosity. - We have said above that a fluid offers no resistance to a shearing stress, which, in other words, means that a fluid yields to a shearing stress, howsoever small the stress may be. But in how long a time the effect becomes perceptible depends upon the nature of the fluid. For instance, water and alcohol yield at once to the deforming forces, whereas honey and glycerine take comparatively a much longer time. Pitch offers a great resistance to a deforming force, so much so that, if the force is not applied for a long time, no effect is produced ; for example, if a piece of pitch be placed in a funnel it does not flow through it appreciably in some days even ; but if it be left there for some months, the pitch slowly makes its way down the funnel under the action of gravity. This is expressed by saying that the fluids are viscous to different extents. This property, which is due to the friction between the molecules of a fluid or, in other words, due to "internal friction", is called **viscosity**. We define it *as the temporary resistance offered by fluids to the deforming forces or the shearing stress.*

A perfect fluid is perfectly mobile ; it is not viscous. It flows quite rapidly even through a very narrow orifice, whereas a viscous fluid such as honey offers a good deal of resistance to the shearing stress, and hence flows slowly through the same narrow orifice. It is found that the velocity with which a fluid escapes by a long and narrow tube depends upon

(i) the difference of pressure on the two ends of the tube ;

(ii) the radius of the tube ;

(iii) the length of the tube ; and

(iv) on the nature of the fluid, or what is called the coefficient of viscosity.

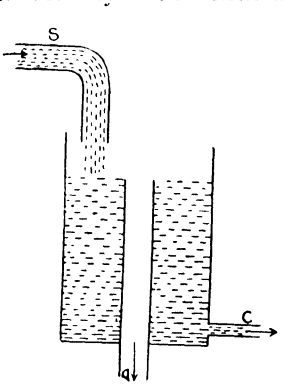


Fig. 58

In order to compare the coefficients of viscosity of different fluids the apparatus shown in Fig. 58 is used. It is a cylinder having a glass tube q passing through the middle of it to serve as an overflow. It has a capillary tubing C inserted into its side near the bottom. Through the tube S the fluid falls into the cylinder. This arrangement enables us to maintain a

constant difference of pressure between the two ends of the tube C. Note how much volume flows through the tube C in t seconds. Let it be V c.c.

Next, take the second fluid. Calculate first what should be the height of the column in order to get the same difference of pressure as before. Remember it will depend upon the density of the fluid: the greater the density the less the height required for the same pressure, and the smaller the density, the greater the height. Note the volume that flows through the tube in the same time, *i.e.* t seconds. Let it be V' c.c. The ratio of the coefficients of viscosity would be in the inverse ratio of these volumes. For instance, if the coefficient of viscosity in the first case be η, and in the second case η'; the ratio would be

$$\frac{\eta}{\eta'} = \frac{V'}{V}.$$

Although the gases seem to be perfectly mobile, yet it should be noted that they even do possess viscosity. It is on account of the viscosity that the wind can raise waves over the surface of water. The feather and guinea experiment also is based upon the fact that air is viscous. It is found that the coefficient of viscosity of a substance varies with the temperature. In the case of a liquid it diminishes with the rise of temperature, whereas in the case of a gas it increases. The following table gives the coefficients of viscosity at two temperatures:

Coefficients of Viscosity

C.G.S. Units

	0° C.	15° C.
Pitch	51×10^{10}	13×10^{10}
Water	0·0179	0·0114
Air	173×10^{-6}	181×10^{-6}
Hydrogen . . .	86×10^{-6}	89×10^{-6}

107. Surface Tension.—We know that the molecules of a substance possess cohesion, *i.e.* attract each other. If this force were destroyed, blocks of iron, rocks, houses, and in fact all solids would crumble to powder by their own weight.

To form an idea how strong this molecular attraction or cohesion is, it is enough to point out that the tensile strength of mild steel is as much as 30 tons per square inch.

In liquids the magnitude of this force is comparatively much less, but by no means negligible. It is on account of the cohesion only, that the liquids possess a definite free

surface, when placed in a vessel. Moreover, it is found that *the free surface behaves as if it were covered with an elastic skin or membrane, i.e.* it can support weight, etc. For instance, if a needle be greased and gently placed on the surface of water in a beaker, it keeps lying on the surface. On looking through the side just below the surface of water, it will be seen that the needle floats in a depression on the surface. The needle apparently floats upon an elastic membrane stretched over the surface of water.

Insects can often move on the surface of water without sinking. A depression is produced around their feet, and the elastic skin of the surface keeps them up in spite of their weight.

This force in the surface, which acts perpendicularly to the bounding surface, is called the ***surface tension.***

We can also define it as the force exerted across unit length of any line imagined in the surface of the liquid.

This force depends upon the nature of the liquid; for example, in case of mercury it is very great, and in case of alcohol very small. If a drop of alcohol be dropped over the surface of water near the floating needle in the above example, it will be seen that the needle at once moves off to one side. The drop of alcohol, where it strikes water, weakens the film, which by elasticity pulls itself away from that point, bringing along with it the needle.

The following table gives the surface tension of various liquids in contact with air :

Surface Tension

Alcohol,	26 dynes/cm.	Olive oil,	38 dynes/cm.
Benzol,	29 ,,	Petroleum,	32 ,,
Mercury,	450 ,,	Water,	74 ,,

The effects of this force are generally studied with the help of soap films. A metal ring is dipped in a soap solution. On removing, it carries a thin soap film with it. A small cotton loop, which has been previously moistened with the soap solution, is placed carefully on the film. This loop can be made to have any form, as shown at *B* (Fig. 59).

If, however, the film within *B* is pricked, it disappears from there, and the loop is pulled out into the form of a circle, as shown at *A*. If it is now deformed in any way, on being released it immediately springs back to the circular form. The reason why the film behaves in this way is that in the first case the tension acts equally on both the sides of the loop

(inside and outside); but when the film inside is broken, the force is acting only on one side and at right angles to the thread, hence the loop is pulled out into a circle.

The effect of the surface tension is to make the area of the free surface as small as possible. It is on account of this reason that the loop is pulled into the circular form. On account of the same reason the raindrops are spherical, the sphere being the body which has the least surface area for a given volume.

The reason why the free surface of a liquid is not so commonly observed to be spherical as surface tension leads us to expect

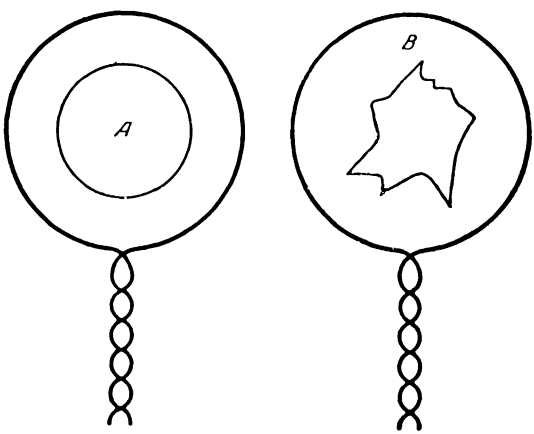


Fig. 59

is that ordinarily the force of gravity and other external forces are more influential in determining the shape of the free liquid than the molecular forces.

That it is actually so can be verified by eliminating the disturbing forces. If by adding alcohol to water a solution be made of the same density as common lubricating oil, it is found that when a large drop of oil is left free with the help of a pipette beneath the surface of solution, the oil floats there as a perfect sphere.

To explain how the surface tension is brought about, let us consider two molecules, P and Q; one of them, P, is well below the surface, and the other, Q, is just near the surface. As is clear from the figure, the particle P is pulled on all sides equally by the neighbouring molecules, whereas the particle

Q is mainly pulled downwards and laterally, for there are very few particles above it to pull it upwards. Similar is the case with all other particles which are near the surface. The result is that the molecules on the surface are bound together to form something like a stretched membrane over the surface of the liquid.

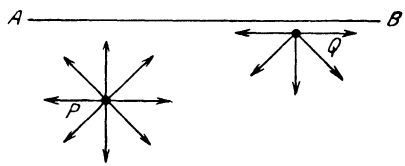


Fig. 60

108. An important effect of surface tension is to raise the liquids* in capillary tubes. The height to which a liquid rises in a tube depends upon the internal diameter of the tube ; the smaller the diameter, the greater the height to which the liquid rises. Here the weight of the column of the liquid above the level of the rest of the liquid is supported by the surface tension of the liquid round the edge of the meniscus.

Bearing this in mind, it is very easy to calculate the value of the surface tension, and also to get at the relation between the height to which a liquid rises above its level and the diameter of the capillary tube.

Let the inner diameter of the capillary tube be $2r$, the density of the liquid d, the height to which it rises h, and the value of surface tension per unit length T.

(i) The total force due to surface tension is $2\pi rT$.

The weight of the column of the liquid is πr^2hdg dynes. From what has been said above, it is clear that these two must be equal. Hence

$$2\pi rT = \pi r^2hdg,$$

or

$$T = \frac{rhdg}{2}.$$

(ii) The height h to which a liquid rises in a tube is given by

$$h = \frac{2T}{rdg}.$$

It is clear from the above relation that the height depends upon the inner diameter of the bore. For instance, when r becomes $\frac{1}{2}$, evidentally h becomes 2 times ; in general, the smaller the bore, the greater the height to which the liquid rises.

* If the magnitude of cohesive force between the molecules is less than that of the adhesive force, the liquid wets the walls of the vessel, and rises in the capillary tube, as, for instance, is the case with water in a glass vessel. On the other hand, if the cohesive force be greater than the adhesive force, the liquid does not wet the sides of the vessel, and is depressed below the general level of the liquid outside, as, for instance, is the case with mercury in a glass vessel.

109. The free surface of a liquid at rest is horizontal.—For if the surface were not perfectly horizontal, some parts will be higher than others. Let us suppose the free surface is represented by *ABCDEF* where the part *BCD* is higher than the part *DEF*.

Draw an inclined plane through *BD* as shown in the figure. The weight (*mg*) of the liquid lying above the plane *BD* will tend to make it slide down, and since liquids yield to a shearing stress, howsoever small, the liquid lying above

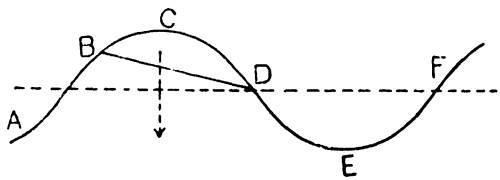


Fig. 61

BD will slide down to places which are lower in level. This means that the liquid is not at rest. This sliding down will take place whenever there is a difference in level, hence, so that a liquid may be at rest, its surface must be horizontal.

It should be noted that the surface will be plane, when we deal with comparatively small surfaces. A large expanse of water, for instance the surface of sea, possesses a curved surface—part of the spherical surface of the earth. A small surface, however, seems to be plane and is perpendicular to the radius of the earth, which line gives us the vertical direction at the place. Such a surface, as a matter of fact, is called horizontal. When we are dealing with a vast expanse, by saying that the surface is level and horizontal, all that we mean is that different parts of the surface are practically equidistant from the centre of the earth. Hence by saying that a free surface is horizontal, we mean that the surface is level in the sense explained above.

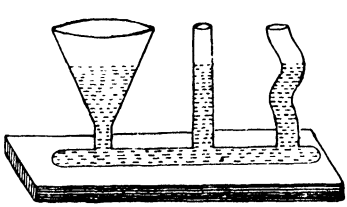


Fig. 62

The law that the free surface of a liquid at rest is horizontal is expressed by saying that *a liquid finds its own level*. It can be seen experimentally by taking tubes of various shapes connected together by a horizontal tube. When water is poured into any one tube it stands at the same level in all the

tubes. This is what is just expected, for a liquid at rest will have a horizontal or a level surface, and therefore the surface of water in the various tubes will form a part of the same level surface.

110. Fluid Pressure.—We have already seen that two solid bodies in contact press against each other, *i.e.* exert action and reaction. This pair of forces is called **stress,** and each constituent force is called a **thrust.** The thrust is exerted by a body in contact with another body whatever be its state, whether solid, liquid or gaseous. We have dealt with the thrust between two solid bodies (action and reaction) already, now let us study the thrust when one of them is a fluid. That fluids exert a thrust is familiar to everybody. We know, for instance, that if a man wants to dive into water, he cannot succeed by falling flat on it, for the surface of water offers resistance (upward thrust). In order to dive easily, the hands should clear the water and the body should follow.

But there is a difference between the thrust exerted by a solid body on another solid body and the thrust exerted by a fluid on a solid body. In the case of solid bodies, the direction of reaction depends upon the fact whether the surface is smooth or rough, while in the case of a fluid the thrust is always normal to the surface in contact with it. This is one of the fundamental properties of a fluid.

111. To prove that the thrust of a fluid at rest is always perpendicular to every surface in contact with it. — Let us suppose ABC is the surface in contact with the surface of the fluid (say water). Consider the stress at the point B. The thrust of the liquid will be equal, and opposite to the thrust of the surface at the point B. If the thrust of the liquid is not normal let it be along MB, consequently the thrust of the surface will be along BM. This force can be resolved into two components, one BN perpendicular to the surface and the other BL parallel to it. Since liquids cannot withstand even the slightest tangential force (*i.e.* shearing stress) the liquid will try to move under the influence of the component along BL, and therefore will be no longer at rest.

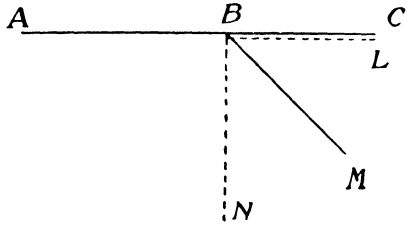


Fig. 63

Thus we see that, so that a fluid may be at rest, the thrust must be normal to the surface in contact with it.

112. Pressure.—Let us consider the surface in contact with the liquid at rest. We have seen that it will experience normal thrust. *The thrust that it experiences per unit area is called the* ***pressure.*** It will be uniform if the thrust on any two equal areas, howsoever small, be equal. When it is so, the resultant thrust on a surface will be equal evidently to *pressure* × *area*. Mathematically we can express the same thing as

$$P = \frac{F}{A}, \text{ or } F = PA,$$

where P is the pressure, F the resultant thrust, and A the area of the surface.

For quantitative measurement of pressure it is necessary to fix upon a unit of pressure.

Since pressure is a force per unit area, evidently the numerical measure of pressure will depend upon the unit of force and the unit of area used. Corresponding to the two systems of the units *C.G.S.* and *F.P.S.* we have two units. Either it may be expressed as dynes or grams weight per square centimetre or as poundals per square ft., or pounds-weight per square inch. Engineers use the last unit, *i.e.* pounds-weight per square inch. The atmosphere exerts a pressure of 14·7 pounds-weight per square inch.

113. Transmissibility of Fluid Pressure.—We have already studied one fundamental property of the fluids, namely, the pressure of a fluid at rest is perpendicular to the surface in contact with it; now we shall study the second property, which may be expressed thus :

The fluids transmit pressure equally in all directions.

This law can be verified very simply by taking a closed vessel (Fig. 64) fitted with four water-tight pistons, which, for simplicity, will be assumed to possess equal cross sections. Let us suppose they are at rest to start with. Push A inwards with a force of 1 lb. wt. It will be seen that in order to keep the other pistons at rest the same force must be applied to each one of them, which shows that increase of pressure applied at one point is transmitted equally to all other points. It should be carefully noted that *it is the* ***pressure*** *which is equally transmitted and not the thrust.* To be clear, let us suppose that the cross section of the piston B is two times that of A. On pushing the latter inwards with a force of 1 lb. wt. the piston B must be pushed with a force of two pounds. In

practice this principle is made use of in the **Bramah Press,** which consists essentially of two cylinders of different sizes fitted with water-tight pistons, and connected together with a pipe.

A general sketch of the machine is shown in Fig. 65. P is called the pump-plunger ; Q the press-plunger ; T is the pipe connecting the two cylinders ; S is the reservoir of water.

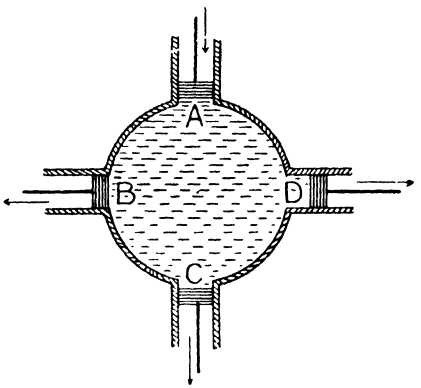


Fig. 64

Let us suppose the cross sections of the plungers P and Q to be a and b sq. inches respectively, and further, suppose P to be pressed down with a force R. The pressure on this side is $\frac{R}{a}$ lb. This pressure will be transmitted to the bigger cylinder. The thrust on the piston Q will be $\frac{R}{a} \times b$. Let it be W. Now it is clear that $R \times \frac{b}{a}$ is greater than R for $b > a$.

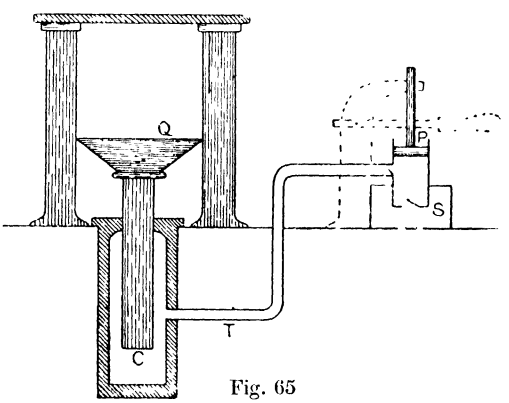


Fig. 65

Hence we see that a small downward force on P is transmitted as a great upward force at Q. By placing goods between Q and

the top of the frame work, they can be pressed, and hence the name *Bramah Press.* It should be remembered that whatever is gained in force is lost in distance, for P must work through a greater distance than Q in order that work done on the two sides may be equal.

$$\text{Mechanical advantage} = \frac{W}{R} = \frac{b}{a} \times R \times \frac{1}{R} = \frac{b}{a}.$$

If r' and r be the radii of the two pistons, then

$$\frac{b}{a} = \frac{\pi r'^2}{\pi r^2} = \frac{r'^2}{r^2}.$$

Further, if lever arrangement be used to press down the pump-plunger, the mechanical advantage will become still greater. Suppose the power-arm is L, and the resistance arm (*i.e.* distance of the axle of the piston from the fulcrum) is l, then the mechanical advantage of the lever is $\frac{L}{l}$.

If we exert a force F units on the lever, the downward force on the piston P is $\frac{FL}{l}$, the force on Q will be $\frac{b}{a} \times \frac{FL}{l}$, therefore the mechanical advantage will be $\frac{b}{a} \times \frac{L}{l}$ and not $\frac{b}{a}$ as before.

114. Pressure of a liquid varies directly with the depth.—Let AB be the free surface of the liquid in a vessel; we want to show that pressure at a point within the liquid depends upon its depth; for the sake of simplicity we shall neglect the atmospheric pressure on the surface, for it is common in all cases.

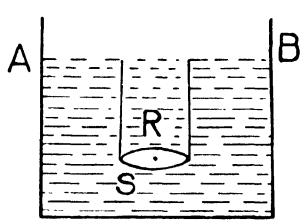


Fig. 66

Suppose we want to find out the pressure at the point R, h cm. below the free surface. Take a unit horizontal circular surface S round R and consider a vertical column of the liquid h cm. long and 1 sq. cm. cross section. The vertically downward pressure on S is due to the weight of this column of the liquid. Its weight evidently is proportional to the volume ($= h \times 1$), or in other words, it is numerically equal to $h \times d$ grams per sq. cm. where d is the weight of one cubic centimetre of the liquid. This shows that the pressure at a point is proportional to the depth.

A very important result follows directly from this. Since the free surface of a liquid is always horizontal, all points at an equal distance below this surface will have the same pressure; in other words, the pressure in a horizontal plane is same everywhere in the case of a liquid at rest.

115. Pascal verified the truth of this statement experimentally. His experiment can be performed by taking vases with equal bases but of different shapes, *a*, *b* and *c*. Their heights are equal. The vases are open on both the sides.

Hold the vase (*a*) with a clamp as shown in Fig. 67. *D* is a movable disc held against the lower edge of the vase by the string, whose other end is fastened to the hook of the pan of a balance. Put weights in the other pan, so that the disc is held against the lower edge. Go on pouring water into the vase till the disc *just* gets detached. This can be seen by the flow of water. Note the height and the weights. Try other vases, keeping the weights in the other pan constant; it will be found that the disc gets detached as soon as the liquid reaches the same height. This shows that the thrust on the base exerted by a liquid depends only upon the area of the base, the depth and the density of the liquid, and *not at all upon the volume of the liquid.*

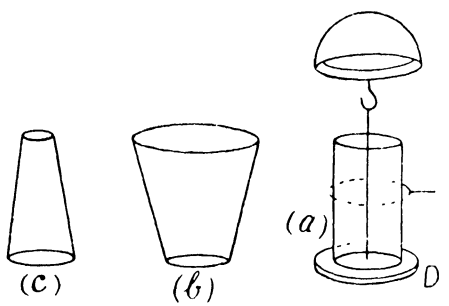


Fig. 67

116. Pascal's experiment looks at first sight puzzling to some extent, for the vases contain different amounts of the liquid, and one would suppose that in *b* the pressure would be greater, and in *c* smaller than in *a*.

Let us see theoretically how this can be explained. First consider the vase *a*, filled with water. The sides do not support any weight at all, for they, being vertical, will be pressed normally, *i.e.* horizontally, and will in turn press the liquid horizontally. In other words, the liquid experiences no vertical thrust on account of the reaction of the walls, which are, as a matter of fact, helpful in maintaining the shape of the water against the gravity which tends to make it flow. Thus the total thrust is due to the weight of the vertical

column of liquid, which is numerically equal to $h \times d \times s$, where h is the height, s is the cross section, and d is the weight of 1 c.c. of the liquid. The pressure is simply $h \times d$ grams per sq. cm.

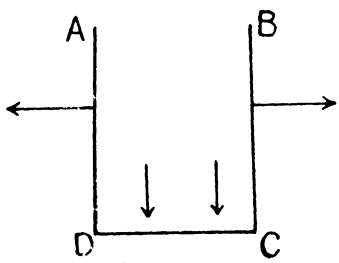


Fig. 68 (*a*)

Consider next the vase *b*. The sides will be pressed normally, and will exert pressure on the liquid in the opposite direction. The last can be resolved into two components, vertical and horizontal. Horizontal component does not produce any effect at all in the pressure at the base, but the vertical component does support some of the weight. In order to know how much weight is supported by the sides, proceed as follows.

Imagine that vase $ABCD$ is made up of two parts, cylinder $CDEF$ and ring-shaped vessel $ADCB$ excluding $CDEF$. The inner cylinder evidently is wholly supported by the base, and, as a matter of fact, it is in every way equal to the vase *a*. Therefore the thrust on the base is $h \times d \times s$. The ring-shaped mass is not at all supported by the base, as is evident from the figure ; it is supported by the sides only. Thus we see that in this case also the pressure is the same as before, *i.e.* $(h \times d)$.

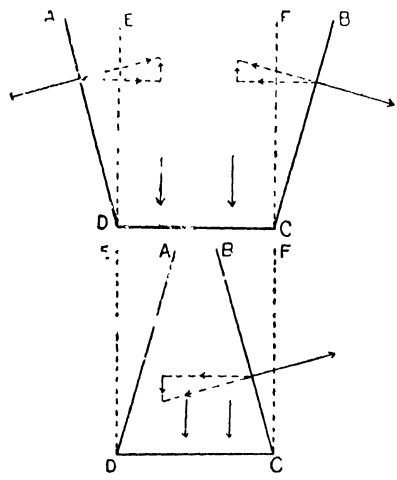


Fig. 68 (*b*, *c*)

Let us now consider the vase *c*. The sides will be pressed normally, and in turn will press the liquid in the opposite direction. The pressure of the sides on the liquid can be resolved as before into two components, vertical and horizontal. The latter need not be considered, as it does not produce any effect in the weight. The vertical component, instead of supporting any weight, presses downwards, which pressure is

communicated to the base by the principle of *transmissibility of fluid pressure*. The total thrust will evidently be greater than the actual weight of water. By following a similar reasoning which we followed in the second case, it can be proved that the thrust numerically is exactly the same as in the other cases.

117. Paradox.—The principle of Pascal's vases, viz. the pressure depends on the depth and not on the actual amount of water, is made use of in what is called Hydrostatic Bellows. It consists of two wooden discs connected together by a collapsible drum of leather or water-proof canvas to hold water. A tube opens into the cavity and rises on the other side to a height of 4·5 ft. or thereabouts. It ends in a funnel. A man may raise his own weight by pouring water down the tube slowly. It seems as if *a mere thread of water is supporting the weight of a man*. This is called **Hydrostatic Paradox.**

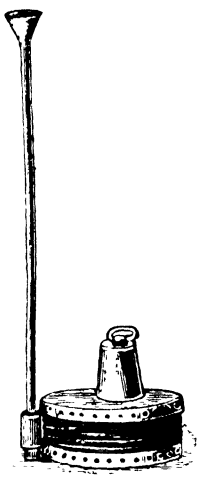


Fig. 68 (*d*)

The explanation, however, is very simple. The thrust on the top of the bellows due to water is $h \times d \times s$, where h is the height of water in the tube above the level of water in the bellows, d is the weight of 1 c.c. of water or 1 c. ft. of water, and s is the area of the top of the bellows in sq. cm. or in sq. ft. It is seen from the formula that all that we are concerned with *is the height of the column of water and not its cross section*. A numerical example will make it clear. Let us suppose the area of the top of the bellows is ·75 sq. ft. (it will be approximately so when the diameter is a foot), and the man who is to be supported has a weight of 150 lb. Now, so that the man may be supported, the thrust of water on the top of the bellows should be equal to 150 lb. Hence

$$h \times d \times s = 150,$$

or

$$h \times 62{\cdot}5 \times {\cdot}75 = 150,$$

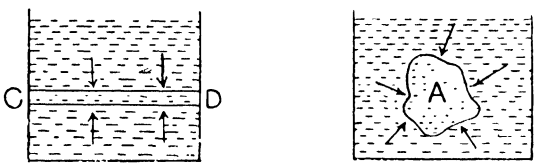
for s = ·75 sq. ft. and d = weight of 1 cubic foot of water = 62·5 lb.

$$\therefore h = \frac{150}{{\cdot}75 \times 62{\cdot}5} = 3{\cdot}2 \text{ feet.}$$

Thus we see that a small column of water, 3·2 ft. in height, will balance the man.

118. So far we have been dealing with the downward thrust or pressure exerted by the liquid. But there exists an upward pressure also in the fluid which is just equal and opposite to the downward pressure. Let *CD* be a thin layer of liquid at rest. The liquid lying above this layer will press it downwards, and if this were the only force, the layer would move downwards. But since it is at rest, an equal and opposite force must be acting upon it.

Instead of considering a plane like *CD*, let us consider an irregularly shaped mass *A*, in a liquid at rest. It will possess a certain weight, which we can suppose to act at its centre of gravity. Besides this force it is experiencing a



(i) Fig. 69 (ii)

normal thrust at every point. Since it is in equilibrium, the resultant of all the thrusts must be equal and opposite to its weight. Thus we come to the conclusion, that the resultant thrust of a liquid on any portion such as *A* is equal to the weight of the liquid filling the space *A*.

Now let us suppose that *A* is removed without disturbing the rest of the liquid, and in its place any other body having exactly the same volume and form is introduced. Since the rest of the liquid is the same, this new body will also experience the same upward pressure as *A* did. In other words, the upward thrust on a body is equal to the weight of the liquid displaced. This result is perfectly general, and is known as the **Principle of Archimedes.** It is stated as follows :

When a solid is immersed in a liquid, it experiences an upward thrust equal to the weight of the liquid displaced by it. Note the effect of this upward thrust is that the apparent weight of the body becomes less, the loss being equal to the weight of an equal volume of the liquid.

This upward thrust acts vertically. Moreover, it acts through the centre of gravity of the fluid displaced, usually called the *Centre of Buoyancy.*

119. Floating bodies.—The weight of the liquid displaced may be greater than, equal to, or less than the weight of the

solid body immersed in the liquid. If the weight of the solid body be w_s and that of equal volume of water be w, then evidently, by the Principle of Archimedes, the solid body loses w grams of weight, in other words, *its weight in water will be* $w_s - w$. The value of this expression will be negative, zero, or positive, just as w_s is smaller than, is equal to, or is greater than w.

(i) Let us first consider the case of *zero weight.* Obviously in this case $w_s = w$, *i.e.* the weight of the substance = weight of the liquid displaced. When such a substance is immersed in a liquid, its weight is just balanced by the upward thrust of the liquid and hence it just floats.

(ii) In the second case, the weight of the substance, *i.e.* $w_s - w$, *has a negative value*; it means that the body when immersed will experience a resultant upward thrust and will therefore rise up to the surface, and will begin to just float when the weight of the liquid displaced is equal to the weight of the body.

Suppose a piece of cork of any shape has a volume of 1 c.c. Since the weight of 1 c.c. of water is 1 gram and of 1 c.c. of cork is ·24 gram, therefore when this piece is immersed in water, it will have a weight equal to $-0{\cdot}76$ gram, which means simply that the upward thrust exceeds the downward pull by 0·76 gram. Under the influence of this resultant upward force, the cork will move upward and will float when the part immersed displaces 0·24 c.c. of water.

(iii) In the third case the value of the expression $w_s - w$ is positive, which means simply that when the body is immersed in the liquid, the downward pull on account of gravity is greater than the upward thrust. Under the influence of this resultant downward force, the body will move down. *Note the apparent weight of the body in the liquid will be* $w_s - w$.

If we consider a piece of iron which has a volume of 1 cubic centimetre, it will be pulled down with a force of 7·8 grams, whereas the upward thrust on it will be 1 gram. Evidently the downward pull is in excess by 6·8 gram, which is, so to speak, the weight of the iron piece in water. Under the influence of this apparent weight it will sink down.

From what has been said above, it is clear that so that a body might float, the necessary condition is that $w_s - w$ should be equal to zero. When expressed in other words, the condition becomes *that a floating body sinks in a liquid until it displaces a weight of the liquid equal to its own weight.*

The above condition that $w_s - w = 0$, or that the resultant

force acting on the floating body should be zero, is a condition for zero motion of translation. It is possible that the two forces, the upward thrust and the downward pull of gravity, may be equal and opposite but may act along different lines, and this may give rise to a couple which will produce rotation. For equilibrium, therefore, it is not enough that resultant should be zero, but also that the forces shall act along the same straight line so as to have no moment.

To sum up, we arrive at the following conditions for complete equilibrium of a floating body :

(1) The weight of the body should be equal to the weight of the liquid displaced.

(2) The centre of gravity of the body and that of the fluid displaced or centre of buoyancy must be in the same vertical line.

These conditions of equilibrium hold good when the body is at rest. If it returns to its original position after being slightly

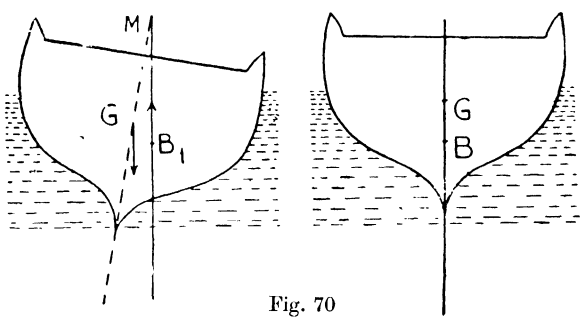


Fig. 70

displaced, the equilibrium is said to be **stable.** On the other hand, if it moves away from its original position, the equilibrium is said to be **unstable.**

Since the stability of equilibrium of floating bodies is very important on account of its application to ships, the following remarks perhaps will not be out of place. Let us consider a ship floating in water. Fig. 70 represents the section passing through G. the $C.G$. The centre of buoyancy is at B. As long as the ship floats upright the points B and G are in the same vertical line. Let us see what will be the tendency of the ship when it is inclined. The centre of buoyancy will shift to some other point, say B_1. The weight acts at G. Thus we get a couple acting on the ship tending to bring it back. Hence the equilibrium is *stable.*

It should be noted that M, the point of intersection of a vertical line through B_1, and the line passing through G and the original position of buoyancy is called **meta-centre.** It is a very important point. In this case it lies above G. If it were to lie below G, the couple would tend to move the body further away from the position of equilibrium, and hence the body would be in unstable equilibrium. In short, the condition of stable equilibrium is *that the meta-centre should fall above the C. G.*

CHAPTER X

DENSITY

120. If we take spheres of the same material but of different sizes, and weigh them, we shall find that the smallest sphere possesses the least weight. It is what is just expected. The bigger sphere will naturally possess greater weight, since it can be considered to be made up of a large number of smaller spheres, and its weight will evidently be the sum of the weights of the smaller spheres. This is sometimes expressed by saying that *the mass of a substance depends upon the volume.*

If we next take spheres of the same size, but of different materials, and weigh them, it will be seen that they possess different weights. Suppose, for instance, the spheres are made up of sandal-wood, marble, iron, brass, lead and gold. The spheres will be found to have weights in the order in which they are written; the gold sphere will be the heaviest, and the wooden sphere the lightest. This shows that the weight or mass of a body depends not only upon its volume, but also upon the nature of the substance.

Both these conditions can be included in the mathematical relation,

$$M = D \times V,$$

where M stands for the mass of the body, V for the volume, and D is a constant depending upon the nature of the material. It is called *Absolute Density*, or *density* of the substance. It is defined as the ratio of mass to volume, or, in simple words, it is the *mass of unit volume of the substance.* Mathematically we can say $D = \frac{M}{V}$. Evidently the numerical measure will depend upon the units of the mass and volume. The most commonly used units are those of the C.G.S. system. On this system, the density is equal to the mass of one c.c. of the substance.

The mass of 1 c.c. of water is one gram,* hence the density of water is 1 gram per c.c. In the F.P.S. system the density of water is 62·5 (or, more accurately, 62·43) lb. per c. ft.

While mentioning the density of a substance it is essential to mention the units, for the numerical measure depends upon the system used, as is clear from the density of water given above. Note that the density of one and the same substance can have many numerical values. To remember them all would lead to confusion. To overcome this difficulty the idea of relative density is introduced. Before taking it up we shall consider one example to clear up the underlying basis of this idea. Let us suppose we have a litre of water, its weight will be 1000 grams. The weight of a gold cube with sides 10 cm. will be 19,320 grams. The ratio of the weights or masses of equal volumes of gold and water is 19·32. Let us express the volume in cubic inches and weight or mass in lb. The volume of both water and gold will be 61·02 cubic inches. The weight of water will be 2·2 lb. and of gold 42·6 lb. The ratio of the weights or masses is, as before, 19·32. Thus we see that the ratio of the masses of equal volumes of different substances is the same whatever units are used. This simplifies the whole affair, for all that we are to do is to take a standard substance and to determine the ratio. Knowing this and the weight of the unit volume of the standard substance, we can find out the density of the substance in any units. For instance, suppose we want to find the density of iron in F.P.S. units. Since the ratio of weights of equal volumes of iron and water is 7·4,

$$\begin{aligned}\text{hence density of iron in F.P.S. units} &= 7{\cdot}4 \times \text{density of water in F.P.S. units}\\ &= 7{\cdot}4 \times 62{\cdot}43\\ &= 643{\cdot}6 \text{ pounds per cubic foot.}\end{aligned}$$

The standard substance that is universally adopted for solids and liquids is water at 4° C., because water is easily obtained in a pure condition everywhere. The ratio of mass of a certain volume of a substance to the mass of an equal volume of water is called the **relative density** or the **specific gravity.** It is defined as follows:

The specific gravity of a substance is the ratio of the mass of the body to the mass of an equal volume of water.

Since weight is proportional to the mass, we can define it as the ratio of the weight of the body to the weight of

* To be more accurate, it is equal to the mass of 1 c.c. of distilled water at 4° C.

an equal volume of water. Mathematically we can write it as

$$\text{Specific gravity} = \frac{\text{weight of the substance}}{\text{weight of an equal volume of water}} \quad \ldots \text{(i)}$$

$$= \frac{\text{mass of the substance}}{\text{mass of an equal volume of water}}$$

$$= \frac{\text{mass of unit volume of the substance}}{\text{mass of unit volume of water}}$$

$$= \frac{\text{density of the substance}}{\text{density of water}} \quad \ldots \ldots \text{(ii)}$$

Density of water is 1 in the C.G.S. system, hence in this system the specific gravity is numerically equal to the density of the substance. But when other units are used, the value of specific gravity will be quite different from density.

If we know the specific gravity and the volume of the substance we can at once find out the weight of the substance. For

$$\text{weight of the substance} = \text{specific gravity} \times \text{weight of an equal volume of water}$$
$$= \text{specific gravity} \times V \times \text{weight of unit volume of water.}$$

In the C.G.S. system the weight will be equal to specific gravity $\times$ V only. But if any other units are used we must multiply also by the weight of unit volume of water. For instance, if the unit of volume is a cubic foot we must multiply by 62 5.

For gases the standard substance adopted is the hydrogen gas. Whatever has been said about solids and liquids is applicable to gases as well. Only for water hydrogen is to be substituted.

121. Determination of Density.—We shall here briefly discuss the principles underlying the various methods without explaining the practical details, for which the student must refer to books on Practical Physics. In order to know the density of a substance we must find out its weight and its volume; the ratio of the two will give us the density. So far as the weight is concerned there is no difficulty. It can be determined with the help of a balance. But for determining the volume we shall have to use different methods depending upon the shape of the body, whether it is regular or irregular and also whether it is solid or liquid. Let us first consider solid bodies.

122. Density of a Solid Body.—Weigh it in air, and then afterwards find out its volume by any of the following methods:

(1) If it is a regular body, its volume can be calculated by making use of the formulæ of mensuration; for instance, if it is a sphere its volume will be $\frac{4}{3}\pi r^3$, or if it is a cylinder its volume will be $\pi r^2 \times h$, and so on.

(2) If it is irregular, its volume can be determined by putting it in a graduated cylinder which contains a liquid up to a certain mark. As soon as the body goes into the liquid it displaces the latter, and makes it rise in the cylinder. Noting the difference between the two graduations, we get the volume.

(3) The volume can be determined with the help of a burette and an ordinary cylinder. Paste a gummed paper at a particular height from the bottom of the cylinder. Run down liquid from the burette into the cylinder till it comes just up to the mark. Note the volume of liquid run down. Throw off this liquid and put in the irregular body, and again run down the liquid till it reaches the same mark. Note again the volume of the liquid run down. The difference between the two readings gives the volume.

(4) If the irregular solid be lighter than the liquid used, it is necessary to use a heavier body, whose volume is known, along with the given solid in order to make it sink. The combined volume can be determined by any of the two methods given above, and subtracting from this the known volume of the heavy body we get the volume of the irregular body. Knowing the weight and the volume we can find out the density at once,

for $$D = \frac{M}{V}.$$

123. Density of a Liquid.—The easiest method is to take a vessel whose capacity is known and weigh it. Fill it with the liquid and weigh it again. The difference gives the weight of the liquid. Dividing the weight by the volume we can find out the density at once. We shall now pass on to the various methods of finding out the specific gravity of various substances.

124. Determination of Specific Gravity.—In order to find out the specific gravity of a body we need only to find out its weight and the weight of equal volume of water; the ratio of the two weights gives us the specific gravity.

125. The Specific Gravity of a Solid Heavier than Water.— The simplest method to find out the specific gravity of a solid is to weigh it in air and then to weigh it in water; the loss in weight will, by the Principle of Archimedes, be equal to the weight of an equal volume of water. Dividing the weight in air by the loss in weight in water we get the specific gravity of the solid. To do this experimentally we generally use a **hydrostatic balance** (Fig. 71). As is evident from the figure, one of the pans is shorter than the other and is provided with a hook. The solid is suspended with a thread from the hook and is first weighed in air. Afterwards it is dipped in a beaker of water, and the loss in weight is noted. Let us suppose that its weight in air is a, and its weight in water is w, the loss in weight being $a - w$.

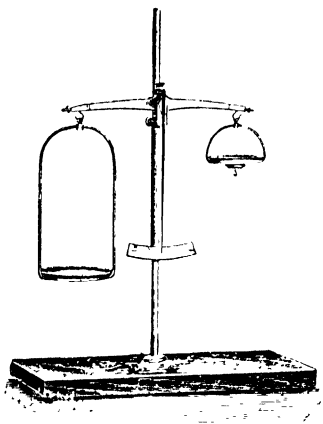


Fig. 71

$$\text{The specific gravity} = \frac{a}{a-w}.$$

126. The Specific Gravity of a Solid Lighter than Water.— A hint has already been given for such a solid while dealing with the determination of densities. A metal piece heavy enough to sink the solid is used along with the latter. This metal piece is called the ***sinker.*** It should be noted that a solid lighter than water will ordinarily float on the surface, and hence cannot be weighed inside water. Let

the weight of solid in air be $= a$,
the weight of sinker + solid in water be $= w$,
weight of the sinker in water be $= s$.
$\therefore$ the weight of the solid in water $= w - s$.
The loss in weight evidently is $= a - (w - s)$.

$$\text{Therefore the sp. gravity} = \frac{a}{a-w+s}.$$

In both the above-mentioned methods we have supposed

that the solid is insoluble in water. If it be soluble, we shall have to weigh it in some other liquid in which it is insoluble. The loss in weight will be equal to the weight of an equal volume of the liquid. If we know the density or sp. gravity of the liquid we can find out the volume at once. For instance,

Let the weight of the solid in air be $= a$,
the weight of the solid in liquid be $= l$,
and the density of the liquid be $= d$.

The loss in weight in the liquid is $a - l$, and therefore this is the weight of the equal volume of the liquid. Since its density is d, therefore the volume will be equal to $\frac{a-l}{d}$.

Hence the sp. gravity $= \frac{a \times d}{a-l} = \frac{a}{a-l} \times d$, viz. specific gravity with respect to the liquid $\times$ density of the liquid.

127. The Specific Gravity of a Liquid.—(i) We can use the hydrostatic balance for determining the sp. gravity of a liquid. Weigh a given piece of solid (say a brass cylinder) in air, and let the weight be a. Weigh it in water, and let the weight be w. Finally weigh it in the liquid, and let the weight be l. The loss in weight in water is evidently $a - w$, therefore this is the weight of the equal volume of water. Similarly the weight of the same volume of the liquid is $a - l$, hence the

$$\text{Sp. gravity} = \frac{a-l}{a-w}.$$

Fig. 72

For determining the sp. gravity of liquids and powdered solids there is another method which is not only more accurate, but also more convenient. In this method we use what is called a *specific gravity bottle.* It consists of a small bottle with a tightly fitting stopper having a fine hole passing through it.

(ii) Weigh the sp. gravity bottle alone. Fill it up completely with the liquid and push in the stopper. Wipe off the superfluous liquid that overflows through the hole. Weigh it again; the difference between the two weights gives the weight of the liquid filling the bottle; let it be l.

Pour off the liquid and dry the bottle with hot air, and fill

it next with water in the same manner as in the case of the liquid. Let the weight of water filling the bottle be w.

$$\text{The sp. gravity} = \frac{l}{w}.$$

It should be noted that in this case we have taken equal volumes of the two liquids.

128. The Specific Gravity of a Powder.—Proceed as before: first weigh the empty bottle, then fill it with water and weigh it. The difference in the weight gives the weight of water. Let it be w. The bottle is next emptied and is dried. Put in the solid and weigh. Subtract the weight of the bottle to get the weight of the solid; let it be a. Fill it up with water now, the solid being also in the bottle, and weigh it; the difference in this weight and the last weight (of solid along with the bottle) gives the weight of water filling the space *unoccupied* by the solid. Let us call it e. It is evident that the volume occupied by the solid is $w - e$.

Hence $$\text{Sp. gravity} = \frac{a}{w - e}.$$

If the solid be soluble in water—as, for instance, powdered common-salt—the method will have to be slightly modified. In place of water, use some other liquid like kerosene oil or turpentine in which the solid does not dissolve. The procedure will be as follows:

First of all, find out the relative density of the solid with respect to the liquid in the manner given below. Multiply this with the sp. gravity of the liquid, if it be known. Otherwise fill the sp. gravity bottle with water, and weigh. We already know the weight of the liquid filling the bottle. From these two observations it is quite easy to find out the sp. gravity of the liquid.

Let the weight of the solid in air be $= a$,
the weight of the liquid filling the bottle be $= l$,
the weight of the liquid filling the space unoccupied by the solid be $= e$,
and finally the weight of the water filling the bottle be $= w$.

The sp. gravity of the solid with respect to the liquid is

$$\frac{a}{l - e}.$$

and the sp. gravity of the solid with respect to water will be

$$\frac{a}{l-e} \times \frac{l}{w}.*$$

The sp. gravity of solid with respect to water = sp. gravity of solid with respect to liquid × sp. gravity of liquid with respect to water.

129. Hydrometers.—The method of determining the sp. gravity of a liquid with the help of a sp. gravity bottle is a very good process. For commercial purposes we do not require a very accurate method; but, on the other hand, we require a simple method which should take very little time. For this purpose we use generally a special type of instruments called **hydrometers.** They enable us to find the sp. gravity readily, although the results are not very accurate. They all depend upon the *principle of flotation*; *i.e.* when a body floats in a liquid its weight is equal to the weight of the liquid displaced. Evidently either the hydrometer may have a constant weight, in which case it will sink to different depths in different liquids, or its weight may be changed in order to sink it to the same level. Corresponding to these two alternatives there are two types of hydrometers.

1*st class*: Hydrometers of constant weight but of variable immersion; and

2*nd class*: Hydrometers of variable weight but of constant immersion.

We shall consider both these types one by one.

130. Hydrometers of Variable Immersion but of Constant Weight.—In order to understand the principle of this type of hydrometers let us consider a tube closed and loaded at the lower end so that it floats vertically in a liquid. Further, suppose there is a millimetre scale placed in it in such a manner that the zero of the scale begins at the bottom. Float it in different liquids; as the weight of the hydrometer remains unchanged,

* For sp. gravity of the solid with respect to water $= \frac{\text{weight of 1 c.c. of solid}}{\text{weight of 1 c.c. of water}}$

$= \frac{\text{wt. of 1 c.c. of solid}}{\text{wt. of 1 c.c. of liquid}} \times \frac{\text{wt. of 1 c.c. of liquid}}{\text{wt. of 1 c.c. of water}}$

= sp. gravity of solid with respect to liquid × sp. gravity of the liquid.

hence *in each case the weight of the fluid displaced will be the same,* but since the densities are different the volumes will be different. Let us suppose in one case the hydrometer sinks to a depth l, and in the second case to l'. If a be the cross section of the tube the volumes will be equal to al and al' respectively. Let the densities of the liquids be d and d'; evidently the mass of the liquids will be equal to

$$ald, \text{ and } al'd'.$$

But this must be equal, as explained above;

hence $$ald = al'd'$$

$$\therefore \quad \frac{d'}{d} = \frac{l}{l'}$$

If the first fluid be water, evidently d will be equal to one, and d' will give us the sp. gravity of the liquid. Let us take a numerical example to make it still clearer. Suppose the tube sinks to the graduation 100 in water, and to graduation 114 in turpentine. The first thing that strikes one is that turpentine must be lighter, since greater volume gives the same weight. The sp. gravity will be equal to $\frac{100}{114} = 0\,877$. Let us sink it next in a solution of $CuSo_4$ in water, and suppose it sinks to the graduation 85. The specific gravity will be $\frac{100}{85} = 1{\cdot}17$.

Such a hydrometer will not be very sensitive, for a change of 1 mm. in the position of flotation means a change of about 1 per cent. in the specific gravity; for instance, in the above example, if the tube sinks to 115 the specific gravity will be 0·869; this means that we cannot measure the sp. gravity very accurately with this tube. If, however, the tube were to sink to the division 1000 in water, and to 1145 in turpentine, the sp. gravity $= \frac{1000}{1145} = 0{\cdot}873$; suppose next it sinks to 1146, the specific gravity will be $\frac{1000}{1146} = 0{\cdot}872$. In other words, a change of 1 mm. will now mean a change of 1 per thousand in specific gravity. This is ten times more sensitive than the former hydrometer; but to have a long hydrometer is impossible. Hence it is usual to have only the stem straight, the rest of the hydrometer is made into the form of a long cylindrical bulb, ending in a spherical bulb.

The stem is not graduated in as simple a manner as explained above. The method of graduation is different according to different hydrometers. For instance, in the common hydrometer or Universal Hydrometer, the stem is graduated

in such a manner that the reading directly gives the sp. gravity. In Beaume's Hydrometer the scale is divided in such a manner that for liquids lighter than water the density is given by the formula

$$D = \frac{146{\cdot}3}{136{\cdot}3 + r}$$

where r is the reading on the scale.

For heavier liquids the density is given by the relation

$$D = \frac{146{\cdot}3}{146{\cdot}3 - r}.$$

Twaddel's hydrometers are also used in various laboratories. They are meant for fluids heavier than water. They are divided into a set of six. The stem is graduated in such a manner that the reading, divided by 200 and added to 1, gives the specific gravity of the liquid, *i.e.*

$$D = 1 + \frac{r}{200}.$$

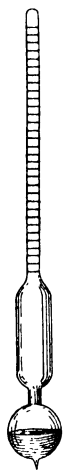


Fig. 73

131. Lactometer.—Perhaps the best known constant-weight hydrometer is the one used in testing milk. It is called a lactometer. The specific gravity of cow's milk is approximately 1 030.* The scale of sensitive lactometers is divided into 20 parts. By floating the lactometer in milk to be tested one can find out whether it is pure or not; for the addition of water makes it lighter, and lowers its density. In cheaper types of lactometers the scale is marked W, 1, 2, 3, and M (*i.e.* milk).

A lactometer does not give a conclusive test as to the quality of the milk, for skimmed milk is heavier than unskimmed milk on account of the removal of fat constituents, which are light; and by adding water to the skimmed milk we can make its density normal. Evidently the lactometer is unable to detect such a defect. It is only by determining both the amount of fat and specific gravity that one can determine the real quality of milk.

132. Hydrometers of Constant Immersion but of Variable Weight.—As the heading shows, in this case our hydrometer is made to sink just to the same level in different liquids by varying the weight. Let us take the most familiar hydrometer. It is made up of a hollow metallic cylinder, from the top of which projects a stem made up of a steel rod b, ending in a

* It varies from 1·027 to 1·035.

disc A. To the lower end of the cylinder is fixed a cone C loaded with lead in order to float the hydrometer vertically. A mark is made on the stem of the hydrometer, and it is always immersed to that level.

Let the weight of the hydrometer be H. Lower it into a cylinder full of water, and place weights in the disc or pan A so that it just sinks to the fixed mark on the stem. Let the extra weight be w. The weight of water displaced is $w + H$. Lower it next into the cylinder full of the liquid whose sp. gravity is to be determined. Add extra weights so that it sinks to the same mark in this liquid also. Let the extra weights be l. Evidently the weight of the liquid displaced is $H + l$.

Fig. 74

Since the volume is the same in both the cases, we get the following relation:

$$d \times V = H + w,$$

and

$$d' \times V = H + l$$

where d' stands for the density of the liquid, and d for that of water.

Therefore

$$\frac{d'}{d} = \frac{H + l}{H + w}.$$

Since $d = 1$,

therefore

$$d' = \frac{H + l}{H + w}.$$

This hydrometer can be used to determine the density of solids as well. Suppose we are given a piece of stone. To determine its density, proceed as follows. Make the hydrometer sink in water up to the mark with the help of extra weights; suppose we require w grams. Remove these weights from the disc, and put the solid on it and add extra weights to sink the hydrometer to the same mark. The total weight on the disc must be w. Subtracting the weights added in the second case from w we get the weight of the solid in air; let it be a. Next, place this solid in the lower pan; the weight will be less now since it is immersed in water. Add more weights in the upper pan so that it sinks again to the same level. Let the weight of the solid in water be m. The loss in weight is $a - m$, hence sp. gravity $= \frac{a}{a - m}$.

133. The U-Tube Method.—This method of comparing the densities of the liquids is very simple and at the same time very instructive. If one of the liquids be water, the ratio of densities will give us the sp. gravity of the liquid. Let us consider at first the liquids which do not mix. The U-tube

shown in Fig. 75 contains the liquids which do not mix. The surface of separation of the two liquids being at C: B and C are at the same level. Since the liquids are at rest, the pressure at the points B and C, which are in the same horizontal line, must be equal. Let the height above the separation surface at C be h_1 and above B be h_2, and let, moreover, the densities be d_1 and d_2. The pressure at the point C, due to the column of liquid lying above it, will be $h_1 \times d_1$. Similarly the pressure at B will be $h_2 \times d_2$. But, as has been said above, these must be equal.

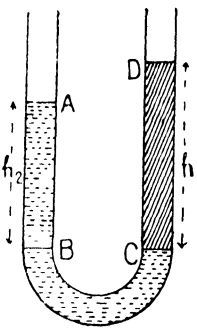


Fig. 75

Hence $h_1 d_1 = h_2 d_2$,*

or $$\frac{d_1}{d_2} = \frac{h_2}{h_1},$$

or *the densities are proportional inversely to the heights.*

If d_2 be the density of water, the ratio gives us the sp. gravity of the other liquid.

134. The Sp. Gravity of the Liquids which Mix.—1. A simple method will be to introduce some mercury first in the bend. Pour one of the liquids in one arm. Mercury will be pushed over the other side. Now pour the second liquid in the second arm till the mercury on two sides is at the same level. Measure heights above the surface of mercury on the two sides. Applying the same reasoning as we did in the preceding article, we get $\frac{d_1}{d_2} = \frac{h_2}{h_1}$.

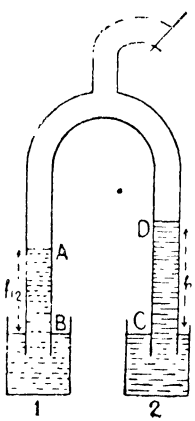


Fig. 76

If d_2 stands for the density of water the ratio $\frac{h_2}{h_1}$ gives us the specific gravity of the liquid.

2. We can also determine the sp. gravity of a liquid with the help of an inverted U-tube, having at the top a small tube with a rubber tubing which can be closed with a pinch-cock. The two ends dip into

* We have not taken into consideration the pressure due to the atmosphere. Since it is common on both sides, it does not make any difference.

beakers, one containing water and the other the liquid whose sp. gravity is to be determined. Suck out air and close the pinch-cock. Since the pressure in the tube decreases, the atmospheric pressure pushes up the liquids in the arms, and the less dense is pushed up to a greater height. The pressure of the gas at A and D is the same, and also the pressure at B and C is the same, being the atmospheric pressure. Hence the pressure due to column AB must be equal to the pressure due to the column CD. If d_1 and d_2 be the densities of the liquids in the arms CD and AB respectively, we get

$$h_1 \times d_1 = h_2 \times d_2,$$

or

$$\frac{d_1}{d_2} = \frac{h_2}{h_1}.$$

Suppose beaker 1 contains water, hence $d_2 = 1$, the density of water; therefore the ratio $\frac{h_2}{h_1}$ gives us the sp. gravity of the liquid.

Table of Densities

(in grams per c.c.)

Alcohol	0·79	Gold	19 3
Aluminium	2·58	Iron	7 4
Copper	8·9	Lead	11·3
Cork	0 24	Mercury	13 6
Glass	2 6	Oak wood	0 8
Glycerine	1·26	Silver	10 5

EXERCISES

1. Compare the density of water at the surface and at the bottom of a lake 100 metres deep, given that the volume elasticity is 22,000 atmospheres.

Let us find out the compression produced in one c.c. of water at the bottom; from that we will calculate the volume of 1 gram of water at the bottom. Since the density is inversely proportional to the volume, we shall be able to get the ratio of the densities, knowing the ratio of volumes.

$$E = \frac{dp}{dv/v} = \frac{100 \times 100}{76 \times 13{\cdot}6} \times \frac{1}{dv/v} = 22{,}000,$$

$$\therefore \quad \frac{dv}{v} = \frac{100 \times 100}{76 \times 13{\cdot}6} \times \frac{1}{22{,}000} = 0{\cdot}00044.$$

This is the compression produced; hence the volume of 1 gram of water at the bottom is 0·99956 and at the surface is 1 c.c. Therefore the ratio of densities is 0·99956.

2. The diameter of the press-plunger is 20 inches, and it is required to lift 100 tons. What should be the size of the pump-plunger if the mechanical advantage of the handle is 10 and the force that the man uses is 10 lb. wt. ?

It is evident that the force on the pump-plunger is 100 lb. wt. on account of the mechanical advantage of the lever. Let the radius be d,

$$\frac{100 \times 2240}{100} = \frac{10 \times 10}{d^2},$$

$$d^2 = \frac{10}{224},$$

or $$d = 0{\cdot}213,$$

or $$\text{diameter} = 0{\cdot}426 \text{ inch.}$$

3. A piece of wax weighs 18·03 grams in air. A piece of metal is found to weigh 17 03 grams in water. They both together weigh 15·03 grams in water. What is the sp. gravity of the wax ?

$$\text{Sp. gr.} = \frac{a}{a+s-w} = \frac{18{\cdot}03}{18{\cdot}03+17{\cdot}03-15{\cdot}23}$$
$$= \frac{18{\cdot}03}{19{\cdot}83}$$
$$= 0{\cdot}909.$$

4. Compare the density of water at the surface and at the bottom of an ocean 1 mile deep, given that volume elasticity is 20,000 atmospheres. *Ans.* 0 9925.

5. What is the total pressure on the bottom of a tank filled with water 4 ft. long, 2 ft. broad, and 2 ft. deep. Density of water may be taken as 62 5 pounds per cubic ft.
Ans. 1000 lb. wt.

6. If a resistance of one ton is overcome by a force of 5 lb. wt. applied to a Bramah press, and the diameters are in the ratio of 8 to 1, find the ratio of the arms of the lever employed to work the piston. *Ans.* 7 to 1.

7. Find the ratio of the areas of the pistons, if a force of 12 lb. wt. produces a thrust of 3 tons, the mechanical advantage of the lever being 7. *Ans.* 80 to 1.

8. A closed cylindrical vessel 6 ft. diameter, 15 ft. long, weighing 5000 lb., is floating in fresh water with the axis of the cylinder in the plane of the water surface. What weight is it carrying ? *Ans.* 8270 lb.

9. Find the length and sp. gravity of a cylinder which floats in water with 2 inches of its vertical axis out of the water, and with 6 inches out of the liquid whose specific gravity is 1 5. *Ans.* 14 inches and 0 857.

10. The height of a mercury barometer is 760 mm., while glycerine barometer stands at 813 cm. If the density of mercury is 13 6, find the density of glycerine. *Ans.* 1 27.

11. Water is supplied to a house from a reservoir 810 ft. above sea-level. Find in pounds per square inch the pressure in a pipe at the top of a house 60 ft. high above sea-level.
Ans. 325 52 lb. wt.

12. The weight of a submarine boat, which lies damaged and full of water at the bottom of the sea, is 200 tons. If the specific gravity of the material is 7 8, find the total pull which must be exerted by the lifting-chains in order to raise the boat from the bottom. (The sp. gravity of sea-water may be taken as 1·025.) *Ans.* 173·7 tons wt.

CHAPTER XI

THE PRESSURE OF THE ATMOSPHERE

135. Whatever has been said already about liquids in the two preceding chapters is applicable equally well to gases, with the exception of the free surface of the liquids, for a free surface is precisely what a gas can never have, on account of its being infinitely expansible. In addition to the properties which the gases possess in common with the liquids there are some properties which are peculiar to them alone.

In this chapter we propose to study the properties of gases, paying special attention to the pressure exerted by the atmospheric air.

That air possesses weight was first satisfactorily proved by Galileo in 1632. It is found experimentally that 1000 c.c. of dry air at a pressure of 760 mm. of mercury at 0° C., and at sea-level in latitude 45°, weigh 1·293 grams. If 1 c.c. of water be supposed to weigh one gram, as we have been doing all along, the sp. gravity of air with respect to water will be 0·001293.

Since air possesses weight, it must exert pressure on all surfaces with which it comes in contact, as we find the case to be with liquids. That air does exert pressure can be shown with the help of the following simple experiments.

(1) Place a closed bladder full of air under the receiver of an air-pump and exhaust air. It swells, on account of the pressure inside, till it bursts.

(2) **Torricelli's Experiment.**—In 1643 Torricelli, a pupil of Galileo, took a glass tube closed at one end, about a metre long and 1 cm. in diameter, and completely filled it with mercury. He closed the open end with the thumb, and inverted the tube into a cup of mercury, removing the thumb when the open end was below the surface of mercury in the cup. He found that mercury no longer filled the tube completely, but descended, leaving a clear space at the top. After a few oscillations it remained stationary, with its surface at a height of about

760 mm. above the surface of mercury in the cup. Since the pressure over a horizontal surface of liquid at rest is the same, the pressure at *A* (inside the tube) must be the same as outside the tube at the free surface of mercury. Moreover, pressure at *A* is due to the column of mercury *BA*, therefore this much downward pressure is being exerted by the atmosphere also.

Pascal, taking Torricelli's experiment as his basis, argued that since mercury is 13·6 times as heavy as water, therefore the atmosphere should support 34 ft. of water. He repeated Torricelli's experiment with a 46 ft. long tube, and actually obtained a 34 ft. high column of water.

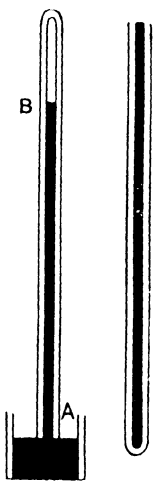


Fig. 77

To completely establish the fact that it was atmospheric pressure which was supporting the column of the liquid, Pascal said that the pressure should be less at the top of mountains, and therefore there the height of the column inside the tube also should be less; consequently he asked his brother-in-law, who lived at the top of a mountain nearly 1000 metres high in southern France, to repeat the experiment there. The brother-in-law found that the column of mercury supported was about 8 cm. less in height. Hence it was inferred that

The height of the column of mercury is a measure of the pressure of the atmosphere.

If the tube be furnished with a scale to read off the height of the column of mercury, the apparatus constitutes a **mercurial barometer.**

The space above *B* is almost a perfect vacuum. It is called *Torricellian Vacuum.* Strictly speaking, it contains a very minute quantity of mercury vapour, but it is so small that it may be conveniently neglected without introducing an appreciable error.* If the tube be lowered or inclined so that the top of the mercury column strikes the tube, a characteristic click is heard, showing thereby that there is no air.

The barometric height depends upon the climatic conditions of the place and its altitude. But, generally, by the term pressure of one atmosphere we understand the pressure equal to the weight of a column of mercury of unit area and 760 mm. high. This pressure is called the *standard or the normal*

* At ordinary temperature it is about 0·0005 mm.

pressure of one atmosphere. This corresponds to a pressure of 14·7 pounds per sq. inch, or 1·033 kilogrammes per sq. cm.

136. Probable height of the atmosphere.—If the density were not to decrease with the height, the weight of the air column considered above would correspond to a height of five miles. But we know that as we go higher up density decreases. Newton estimated air to be four times rarer at an elevation of $7\frac{1}{2}$ miles than at sea level, 10^6 times rarer at a height of 76 miles, and so on. It is impossible to lay down a definite limit beyond which the atmosphere does not extend. Even at a height of 125 miles the density of air is quite enough to offer so much resistance to the passage of meteors as suffices to raise their temperature to incandescence. It is quite safe to say that at least the atmosphere extends up to a height of more than 200 miles. The supposed limit is 500 miles. We shall now consider the two barometers that are generally used to read the pressure of the atmosphere.

137. Fortin's Barometer.—In the simple barometer like the one shown in Fig. 77, whenever the pressure increases, mercury is forced into the tube, with the result that the level of mercury in the cup below changes. The reverse happens whenever the pressure decreases. Although by making the cross-section of the tube small in comparison with the cross-section of the cup the rise or fall in the latter can be made very small, yet for accurate work this must be taken into consideration. In Fortin's Barometer the difficulty is removed by making the cup of leather, so that its bottom can be screwed up or down with the help of a screw. The zero point coincides with the tip of an ivory point *P*, and each time a reading is taken the surface of mercury is first made to touch this ivory point. Near the top in the copper tube enclosing the glass tube there is a slot in which slides up and down a vernier. At the back also there is a slot in which a brass plate connected with the vernier slides. The lower edge of the back-plate is just at the same level as the zero of the vernier. To

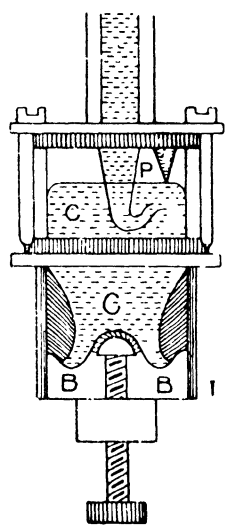


Fig. 78

take a reading the vernier is moved down till the lower edge of the vernier, the lower edge of the back-plate, and the top of the surface of mercury, and the eye are all at the same level. It gives readings accurate to $\frac{1}{20}$th of a mm., or $\frac{1}{500}$ of an inch.

138. Aneroid Barometer. — The trouble with Fortin's Barometer is that it is not portable, it cannot be carried from place to place. If, for instance, we want to measure the height of a place with the help of a barometer, Fortin's Barometer is practically useless. For such a purpose an Aneroid Barometer is used, which is called so because no liquid is used in its construction.

Fig. 79

It consists of a hollow cylindrical metal box, the lid of which is made up of thin corrugated metal plate. The inside of this box is exhausted by means of an air-pump, leaving more or less a perfect vacuum.

The lid is very elastic, and it readily yields to alterations in the atmospheric pressure. When the pressure increases, the lid is pressed inwards. *The amount of flexure is proportional to the change of pressure.*

When the pressure decreases, the lid moves out. This motion of the lid is magnified by means of a system of levers, and is then transmitted to a pointer which moves over a scale, which is graduated by comparison with the readings of a mercury barometer. The aneroid barometers have been made as small as watches. They can be made so delicate as to be able to detect a difference of a few feet, as, for instance, the height of the table from the ground.

To determine the heights accurately with the help of a barometer a very complex formula must be used, but for rough purposes it will suffice to know that the column falls down by about an inch for every rise of 900 ft. approximately.

139. The barometer is used by the meteorologists in forecasting changes in weather.

The principle underlying the science of Meteorology is that damp air being lighter exerts less pressure than dry air, the temperature, etc., being the same. This being so, it is clear that the more water vapour there is in the air, the lower will the barometer stand.

Now evidently when there is much water vapour in the air, there is more chance of rain. Hence the falling of the barometer, which indicates that the air contains much water vapour, is an index for the coming rain.

On the other hand, when the barometer stands high, this indicates that there is very little water vapour in the air, and hence very little, or practically no chance for the rain. In other words, the high barometer indicates fair weather.

To sum up we find that

(i) A fall of barometer indicates the approach of rain or storm.

(ii) A rising barometer indicates the approach of fair weather.

(iii) An unchanging high barometer indicates settled fair weather.

140. Boyle's Law.—All bodies should undergo a change in volume when subjected to a change of pressure, though it is only in the case of gases that the change is considerable ; in the case of solids and liquids the change is almost negligible. It was Boyle who, in 1662, first discovered the law connecting the pressure and volume of a gas. He found *that the volume of a gas varies inversely as the pressure, the temperature remaining the same.*

Mathematically it can be written as

$$P \propto \frac{1}{V}, \text{ or } PV = \text{a constant.}$$

The law can be proved experimentally with the help of the apparatus shown in Fig. 80. It consists of a glass tube B, about 20 cm. long and 1 cm. wide, closed at one end by a ground glass stop-cock, with the other end drawn out a little so that a rubber tube can be slipped on to it. A second piece of glass tubing A is attached to the other end of the rubber tube. The tubes B and A can be clamped in any position.

Open the stop-cock S, and pour mercury down the tube A. It will stand at the same height in both the tubes A and B. Close the stop-cock, enclosing thereby a quantity of gas at atmospheric pressure in tube B. Note the length of the enclosed column of air and read the barometer. Now raise the tube A ;

mercury will be seen to rise in both the tubes, but more rapidly in the tube A than in the tube B. Note the difference in level between mercury surfaces in the two tubes; let it be h cm. The pressure on the enclosed gas is atmospheric pressure (in cm.) $+ h$ cm. Measure the corresponding length of the enclosed column of air. Take several readings. If the tube be uniform in bore, the length of the air column will be proportional to the volume of the enclosed air. Hence for volume we can consider the length only. It will be found from the observations that the product of pressure and volume (or in this case length) has the same value in each case.

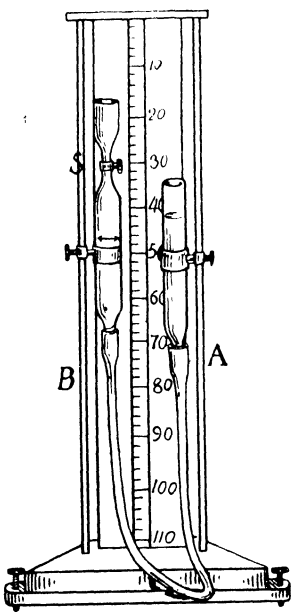


Fig. 80

Boyle verified it for a small range of pressures. The investigation was extended further by Amagat, who used as great a pressure as 3000 atmospheres. He found that the gases do not obey this law very rigorously over a wide range of pressures, but that for ordinary purposes Boyle's Law is quite accurate.

Since in the last experiment the quantity of the gas remains the same, it is obvious that with the decrease in volume, the density must increase. Let P be the initial pressure, v the initial volume, D the initial density, P' the final pressure, v' the final volume, D' the final density, and m the quantity of gas; we have

$$D = \frac{m}{v}, \text{ and } D' = \frac{m}{v'}.$$

But by Boyle's law $Pv = P'v'$,

or
$$\frac{P}{P'} = \frac{v'}{v} = \frac{m}{D'} \times \frac{D}{m} = \frac{D}{D'}.$$

Therefore
$$\frac{P}{P'} = \frac{D}{D'}, \text{ or } \frac{P}{D} = \frac{P'}{D'} = \text{a constant.}$$

If the pressure be kept constant and the temperature be

changed, it is found that the volume of a gas increases considerably. The quantitative relation between the increase in volume and the rise in temperature was first discovered by Charles. He found that *the volume of a given quantity of any gas increases by* $\frac{1}{273}$ *of its volume at* 0° C. *when it is heated through* 1° C. This is known as **Charles's Law.** We shall consider this law in detail in Heat.

Now we shall pass on to the pneumatic appliances: air-pumps, water-pumps, airships and aeroplanes.

141. Air-Pump.—An air-pump is used for exhausting air. Formerly it was found only in laboratories, but its use in creating partial vacuum in appliances like vacuum cleaners, incandescent lamps, etc., has made it of great commercial importance.

To understand the principle of an air-pump, we shall take a simple type whose sketch is shown in Fig. 81.

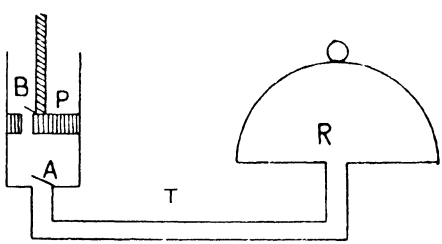


Fig. 81

It consists of a cylinder in which an air-tight piston moves up and down. It is connected to a metal plate over which the vessel to be exhausted is placed in such a manner that the contact between the edge of the vessel and the plate is air-tight. There are two valves *A* and *B*, as shown in the figure. Normally both of them remain closed on account of their own weight, but they open only upwards when pressed from below, and close all the more tightly if pressed from above.

To understand the action of the pump let us start with the piston at the bottom at *A*. When it is raised up, a partial vacuum is created in the cylinder. The valve *B* is pressed down by the atmosphere, whereas valve *A* is forced open by the air in the vessel *R* and the tube *T*. As a result of it, when the piston is at the top of the stroke, the air which at first filled the vessel *R* and the tube *T* now fills the cylinder as well.

During the down-stroke, the air between the piston and the

valve becomes compressed, and on account of increased pressure (due to compression) the valve A closes tightly, whereas B opens upwards and the air is forced out into the atmosphere. The next stroke of the piston causes B to close and A to open as in the first stroke, with the result that some air is sucked again from the vessel. During the downward stroke the air in the cylinder is thrown out, and so on. Every double-stroke, in short, removes some air, until finally the air left in the vessel is too weak to raise the valve A. It is easy to show theoretically that it is impossible to attain perfect exhaustion with any finite number of strokes, or what is the same thing, that perfect exhaustion would require infinite time.

Suppose the volume of the vessel R and the tube T connecting it to the cylinder up to A to be V, and the volume of the cylinder swept through by the piston in one upward stroke to be v. Let the initial pressure be P.

Let us start with the piston at the bottom, the volume of the gas is V. At the end of first up-stroke the volume becomes $V+v$; during the down-stroke the volume v is expelled. Evidently the fraction $\frac{V}{V+v}$ of the original quantity is left behind. Call it m_1. At the end of the next up-stroke the volume of m_1 quantity becomes $V+v$, and during the down-stroke the volume v is expelled, leaving behind $\frac{V}{V+v}$ of the quantity at the beginning of the second stroke or $\left(\frac{V}{V+v}\right)^2$ of the original quantity. After n strokes the quantity left behind would be $\left(\frac{V}{V+v}\right)^n$ of the original quantity. We know from Boyle's Law that the pressure is proportional to the density, which again is proportional to the mass of the gas, hence we see that the pressure of the gas is proportional to the mass.

To start with, the pressure was P, and after n strokes the pressure becomes $\left(\frac{V}{V+v}\right)^n \times P$. Since fraction $\left(\frac{V}{V+v}\right)^n$ can never become zero, the pressure inside therefore never becomes zero.

142. Compression Pump.—The object of a compression pump, or sometimes called compressor, is to pump air into a given vessel, so as to increase pressure within it. The mechanism of such a pump is practically the same as that of an air-

pump, *except that the valves open in the reverse direction*, *i.e.* downwards in place of upwards. The use of compressed air in industries makes it of very great importance.

To understand its principle we shall consider a bicycle pump, which is perhaps the most familiar compression pump. It consists of a barrel in which a piston moves up and down. A cup-shaped washer, made up of a leather piece which is slightly larger than the barrel of the pump, is fixed round the disc of the piston with the rim of the cup turned towards the bottom of the barrel. This washer acts as the valve corresponding to the valve *B* in the piston of the air-pump. When the piston is raised up, air rushes down the space between the rim of the washer and the walls of the barrel into the space below, as shown by the arrow heads in Fig. 82. During the down-stroke the increased pressure of the air below the piston presses the rim of the washer against the walls so tightly that no air can pass up. As the piston is pushed further down the pressure of the enclosed air opens the valve of the valve-tube, through which the compressed air makes its way into the bicycle tyre.

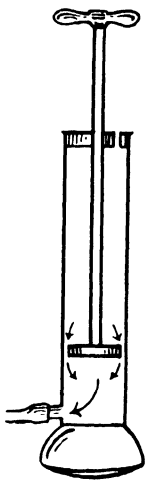


Fig. 82

Note the second valve is in the valve tube, which, too, opens downwards.

143. Water-Pump. —It is a machine used for raising water. It consists of a long vertical pipe of iron which dips in the well *W* at the lower end. At its upper end there is a valve *V* which opens upwards into a barrel or cylinder. There is a second valve in the piston, which also opens upwards. *S* is the outlet by which water passes out (Fig. 83).

Action of a water-pump in the first few strokes is to simply exhaust air from the pipe below the valve *V*. For instance, when the piston is raised up, a partial vacuum is created above *V*; the pressure on the lower surface of *V* being greater than on the upper surface, it opens upwards, allowing the air in the pipe to enter the cylinder. Due to this expansion the air in the pipe is at a reduced pressure. The atmospheric pressure on the water in the well consequently forces some water up the pipe. During the down-stroke the air in the cylinder below the piston is compressed, and on account of this increased pressure *V* closes and *P* opens upwards,

allowing the air in the cylinder to escape into the atmosphere. During the next up-stroke some more water is forced into the pipe, and so on, till all the air in the pipe is removed and water is pushed up through the valve V into the cylinder.

Since the water is raised in the pipe by the atmospheric pressure alone, it is clear that if the pump is to work, the pipe must not be longer than 34 ft. (the barometric height), otherwise water will not reach the valve V. Owing to

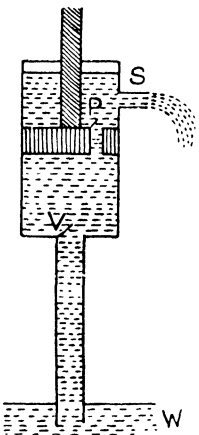


Fig. 83

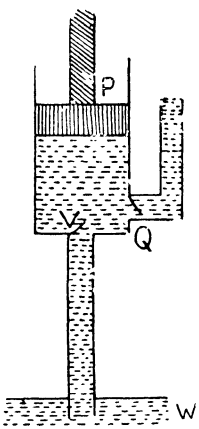


Fig. 84

mechanical imperfections it is not possible to raise water to a height more than 29 or 30 ft.

The **Force-Pump** is slightly different in construction (see Fig. 84). A side tube having a valve Q opening outwards is fitted to the bottom of the cylinder. Note there is no valve in the piston. The action up till V is just the same as in the water-pump, and hence the force-pump also is subject to the same limitations as the water-pump. When the water enters the cylinder it is forced up by the force of the piston during the downward stroke, the valve V being closed and Q open.

With the help of this pump, water may be forced to any height above the piston, but, as said above, the valve V at the bottom of the cylinder must be within 30 ft.

The most usual example of this type is the **Fire-Engine.**

144. Flying Machines.—The possibility of aerial flight was foreseen by the writers of the fourth century, but this problem

remained practically unsolved till very recent times. The airships and aeroplanes are no sudden advances in man's struggle with Nature, but rather the final yielding of defences which withstood his attacks for hundreds of years.

The first instrument which enabled man to rise into the air like a bird was the balloon. Nobody knows when the first balloon was sent up. But it is reported that at Pekin in the year 1306 a balloon was sent up at the coronation of the then Emperor of China. The first *authentic* account, however, dates back to the 5th of June 1783. On that day the two brothers, Joseph and Stephen Montgolfier, sons of a French paper manufacturer, succeeded in causing a large paper balloon filled with heated air to ascend to a height of 300 yards. On the 19th of September 1783, a large Montgolfier balloon ascended from Versailles, carrying with it for the first time living creatures, a sheep, a duck and a hen, and after a flight of ten minutes sank gently to the earth with its passengers uninjured.

Professor Charles of Paris substituted hydrogen for hot air. This not only removed the danger of serious accidents to which the hot air balloons were liable, but also increased their weight-lifting capacity. To make it clear let us suppose that the balloon has the capacity of an ordinary room. The weight of the air having the same volume will be about 112 lb. The hot air which is required to fill up the balloon so that the pressure inside is the same as outside will have a weight of 45 lb.; so that the balloon can lift a weight of 60 lb. or thereabouts in all (including its own weight). But now suppose that the balloon is filled up with hydrogen. The amount of H_2 gas which is required to fill up the balloon will be 8 lb., which means that the balloon can lift a weight of about 100 lb. in all, including its own weight.

One of the most remarkable ascents was made by two Englishmen named Glaisher and Coxwell in 1862. They attained a height of about seven miles—the greatest height ever attained with the help of a balloon. At this height the air was found to be very rare, so much so that Glaisher fainted. The temperature was lower than $-16°$ C. The barometric pressure was about 7 inches only.

We shall explain very briefly how the balloons were managed, so that the student might see the advantage of the modern flying machines over the older ones. A balloon is usually filled with the gas in place of hydrogen, because the former is cheaper and is more

easily obtained than the latter. The balloon is not filled with coal gas quite full, for the atmospheric pressure diminishes as it rises, and the gas inside by expanding may burst it. It is sufficient for the ascent if the weight of the air displaced exceeds that of the balloon along with the contents by 8 or 9 lb. This upward force remains nearly constant so long as the balloon is not quite fully distended by expansion of the gas inside. If, for example, the atmospheric pressure becomes $\frac{3}{4}$, the volume of the gas inside the balloon by Boyle's Law becomes $\frac{4}{3}$. The air displaced also becomes $\frac{4}{3}$; but since its density has become $\frac{3}{4}$, the weight, and consequently the upward force, remains the same. When the balloon becomes fully distended there can be no more increase in the volume, and hence the balloon rises for some distance only, because soon a stage comes when the weight of the balloon is equal to that of the displaced air. At this level the balloon begins to float and move in the horizontal direction in obedience to the currents of air prevailing at that level. From the barometer inside the balloon, it can be seen whether the balloon is ascending or not, and what height has been reached.

If the aeronaut wishes to descend, he has only to open a valve at the top of the balloon. The gas inside escapes and the balloon sinks. If he wants to make the speed slower or wants to rise up again, he does so by emptying out the bags of sand, which are placed in large numbers in the balloon.

A free balloon unprovided with any means of propulsion is at the mercy of the wind, and completely beyond the control of the aeronaut so far as direction is concerned. He may, by rising or falling to another level, find a favourable wind which will carry him in the desired direction, but under ordinary circumstances the probability of reaching a predetermined destination is quite remote. Steering rudders cannot be of any help here, *for there is no relative motion between the balloon and the air.* So it was recognised quite at an early date that if a sufficiently light and powerful engine capable of driving the balloon through the air in which it floated could be obtained, the problem of control would be solved. All the remarkable advances that have been made during the recent years in aerial navigation have been mostly due to the development of such an engine—the internal combustion engine of the motor car.

To sum up, there are two primary questions involved in the problem of aerial navigation :

1. *The upward movement of the machine against the action of gravity.*
2. *Its propulsion through air.*

In addition to these two primary questions there are a good many secondary questions, such as stability, control,

etc.; but we shall confine ourselves to the two primary questions only.

The flying machines of to-day are of two kinds:

(*a*) *Lighter-than-air*, or *air-ships*; and

(*b*) *Heavier-than-air*, or *aeroplanes*.

We shall very briefly refer to airships, and then pass on to the principle underlying aeroplanes.

145. Airships.—They are nothing else but balloons provided with a light engine of high power. Count Zeppelin's airships are most famous. His ships are cigar-shaped (Fig. 85). The framework is made up of aluminium or one of its alloys. Externally the framework is covered with a waterproof fabric. It is divided into a large number of compartments. Each compartment contains a gas-bag filled with hydrogen which gives the necessary buoyancy. The engines drive the propellers.

The description of two of Zeppelin's airships will be given here, just to enable the student to form some idea about the machines used.

(*a*) It was as large as an Atlantic liner—being 525 feet long and 54 ft. in diameter; he used eight engines, giving altogether 820 horse power, to propel the rudders. Its speed was 75 miles an hour. It could remain up in the air for four days and nights.

(*b*) The Zeppelin, which completed its first voyage in 1924 from Berlin to New York—a distance of 8153 kilometres—in 81 hours 17 min., is 200 metres long. The gas-bags have the total capacity of 70,000 cubic metres. The tonnage is 44,000 kilogrammes.

146. Aeroplanes. -As has been said above, the aeroplanes are heavier-than-air machines. To understand the principle on which they are based, it is necessary to understand how a kite flies.

If a kite is balanced in a wind it speedily attains a certain height, at which it remains so long as the wind does not drop. When at rest, let us see what forces are acting upon it. They are shown diagrammatically in Fig. 86.

(i) The weight of the kite.

(ii) The pressure of the wind on the lower surface of the kite.

(iii) The pull of the string.

Forces No. (i) and No. (iii) pull the kite downwards, whereas force No. (ii) tends to lift it. When the kite is at rest, it is in

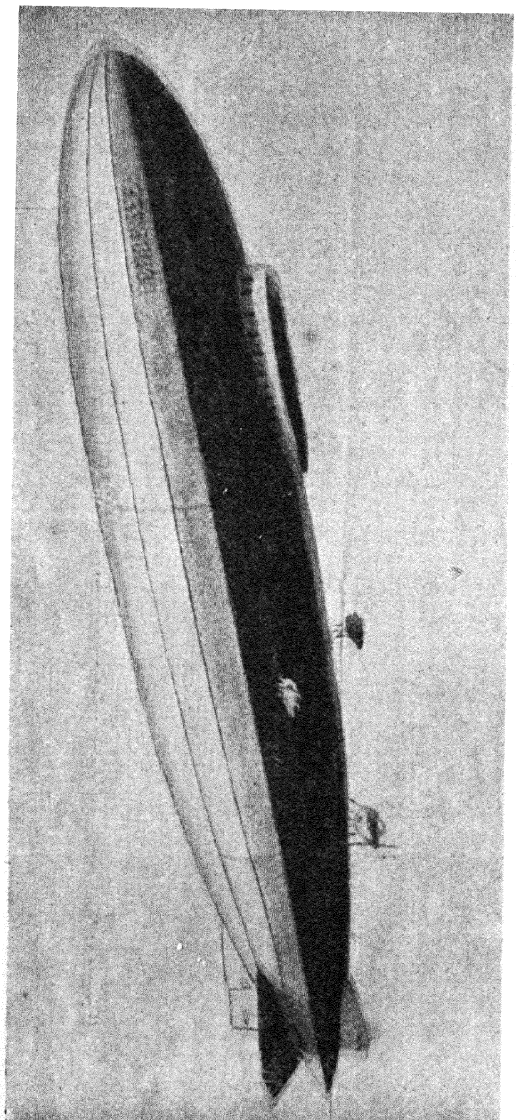


Fig. 85

(From Dürr's work, *Fünfundzwanzig Jahre Zeppelin-Luftschiffbau.* V.D.I.-Verlag G. m. b. H., Berlin SW 19 Germany.)

equilibrium under the action of these three forces. But if the pressure of the wind is very intense, it lifts the kite against the action of gravity and the pull of the string. The pressure of the wind is really a combination of two forces—(i) the ***drift***, tending to move the kite in the direction of the wind, and (ii) the ***lift***, tending to raise the kite vertically up. These two effects vary with the angle of the kite and the speed of the wind. When the effective speed is increased by the boy who flies the kite, by running against the wind (see Fig. 86), the kite goes higher. If the boy makes the effective speed less by running with the wind, the kite sinks lower. As the inclination of the kite is diminished, the vertical component, *i.e.* lift, increases, and the horizontal component, *i.e.* drift, decreases.

Now let us see how this principle helps us in making an aeroplane. Suppose a plane is held inclined to the horizontal against a wind. The latter would exert a pressure on the plane which, as in the case of the kite, can be resolved into two components, one tending to move the plane in the direction of the wind, and the other, tending to lift the plane vertically up. The first corresponds to what we have called the *drift*, and the second to the *lift*. The shape * of the plane should be such that the lift is as high as possible.

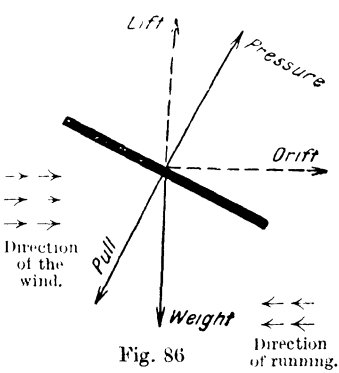


Fig. 86

In order to increase the lift, the effective speed of the wind should be increased, and the greater the speed, the greater the lift. It should be noted that the aeroplane rises only when the engine works. As soon as it stops, the aeroplane begins to glide down. It is because the lift depends upon the forward motion of the plane, *i.e.* upon the effective speed of the wind. This is the reason why the aeroplane is made to run along the ground before flying, because it will rise only when a certain speed is reached. Of course the speed depends upon the weight to be lifted.

* When a body forces its passage through the air, it produces whirls and eddies, to avoid which the shape of the surface of the body should coincide with the path of the particles displaced.

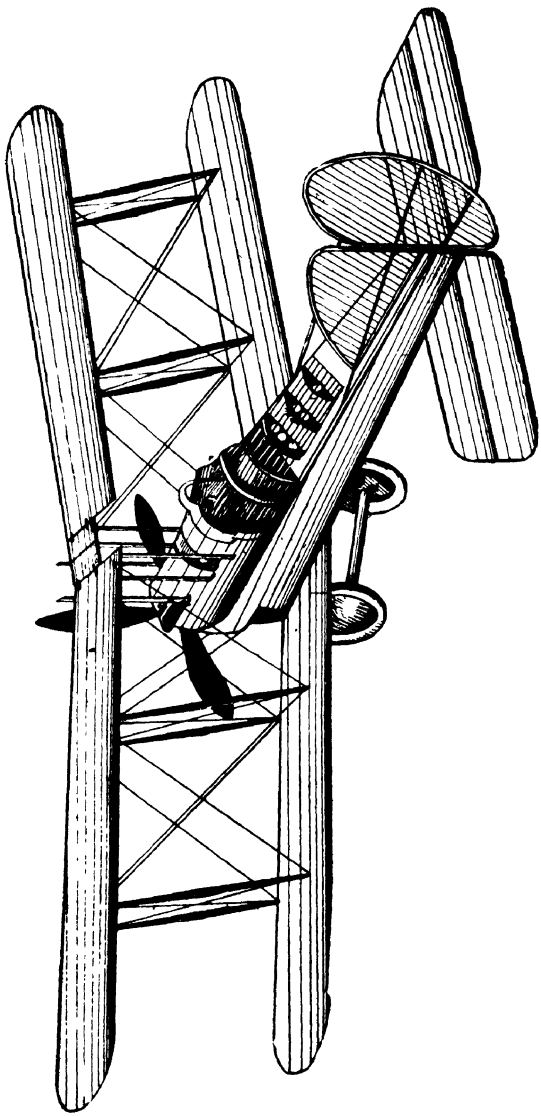


Fig. 87

The aeroplanes which are generally used have two planes, and are called biplane machines. The planes are curved downwards. They are kept apart by wooden struts which are about a foot wide; the struts have the maximum thickness (about 3 inches) along the middle line in the vertical plane, and taper towards the edges in order to reduce the resistance of the air to the motion of the aeroplane. Each plane is made up of two parts, hinged at the middle line, so that the angle of the plane might be varied by the pilot.

The propeller blades are generally made up of wood and are two in number. The diameter varies generally from 5 to 10 ft. The width is about a foot. They are generally placed at the front. The blades are curved in such a manner that when they revolve, the air pressure pushes them from behind. The horse power of the engine which revolves the propellers varies from 80 to 250.

There is a tail at the other end of the aeroplane. It is made up of three parts, one vertical plane in the middle, and two horizontal wings just at right angles to the central vertical plane (Fig. 87). Each of the horizontal wings is made up of two parts; one remains fixed, whereas the angle of inclination of the other with the horizontal can be varied. At first this second part is inclined a little below the horizontal. For rising up this part is tilted up, and for going down it is tilted down. For going to the right the central vertical plane is tilted to the right, and for turning to the left the vertical plane is tilted to the left.

To start the machine, two men first rotate the propeller blades, just to start the engine. When the blades begin to move very rapidly, they get aside and remove the obstacle placed before the wheels. The aeroplane then runs forward, and since the planes are curved downwards, the air as it rushes past creates a vacuum above the plane and pushes it from underneath. In the case of an ordinary military aeroplane, when the speed reaches about 40 miles an hour the lift on the plane is sufficiently intense to raise the machine high up in the air against the pull of gravity.

PART II

HEAT

CHAPTER I

147. Let us take a kettle half full of cold water and place it over fire. If we dip our hand in it from time to time, we find that it first becomes lukewarm, then warm, hot, very hot, and so on till at last it begins to boil. We can express the same fact by saying that water goes on getting hotter and hotter till at last it begins to change its state. To account for the change that takes place in the condition of water (from cold to hot), we say that something has passed from the fire to the water. This something which is responsible for this sensation of warmth is called **Heat.** In other words, *Heat is the agent which produces in us the sensation of warmth.*

We know, no doubt, by the sense of touch that the water in the kettle is getting hotter and hotter, but *how much* hot it is at a particular moment we cannot say. In other words, our sense of touch does not enable us to form a quantitative estimate of hotness, *i.e.* we cannot form an idea of the *degree of hotness.* This *degree of hotness* is called **Temperature** in scientific language. A hot body is said to possess a high temperature, and a cold body a low temperature. It is very important to understand the difference between the temperature and the heat. To make this difference clear we shall consider the following example. Take two exactly alike Bunsen burners and connect them to gas taps, their flames will be of the same height and character. Keep over one the kettle containing water, and over the other an iron ball. It will be noticed that the ball becomes red-hot very soon, whereas the water in the meantime is hardly affected. Since flames are similar, they have given out the same amount of heat, but the degree of hotness or temperature of the two substances is widely different; it is very much higher in the case of the iron ball. The iron ball when introduced into

the water would part with its heat, becoming colder thereby and raising the temperature of the water.

Thus we see that whereas heat is the agent which makes a body hot or cold, **the temperature is the *state* or *condition* of that body.** Moreover, we have seen that a substance higher in temperature parts with its heat to a body lower in temperature. Combining these two notions, we arrive at the following definition of temperature :

Temperature is that condition of a body on which its power to communicate heat to, or to receive heat from, another body (placed in contact with the first one) depends.

To make this distinction still more clear, consider the following hydrostatic analogy. Suppose we have two vessels, one small and the other big. Fill them both with water. The quantity of water in the bigger vessel will evidently be greater than in the smaller vessel. Place them at different levels and connect them up together. Water will begin to flow from the vessel at higher level to the other, irrespective of the fact whether the quantity in the higher vessel is smaller or greater. As a matter of fact, the *quantity* has nothing to do with the flow of water, else rivers would never flow down the mountains to the oceans, for the springs contain much less quantity of water than the oceans. So remember it is the ***level*** which determines the flow, and not the quantity. The rôle which is played by level in the case of the flow of liquids is played by temperature in the case of the flow of heat. Remember the level corresponds to the temperature, and the quantity of water to the quantity of heat.

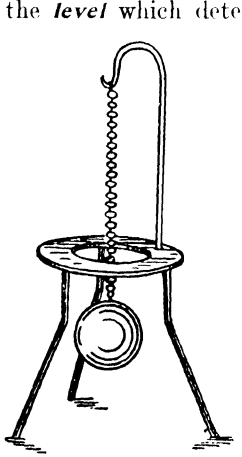


Fig. 1

148. Effects of Heat. (1) We have already learnt that the temperature of a body becomes higher when it is heated up. So obviously one effect is the **rise in temperature.**

(2) The second effect of heat is the **increase in size.** All bodies, with a few exceptions, expand when heated, though to very different extents. The liquids are more expansible than solids, whereas gases are more expansible than even liquids.

Experiments. (*a*) *Expansion of Solids.*—Consider *Gravesand's Ring and Ball* experiment. At the room temperature

the ball passes through the ring quite easily, but when it is heated it refuses to go through the ring, showing thereby that it has expanded. If the ball is allowed to remain there, it contracts as it cools, and the ring on the other hand expands a little, due to the contact of the hot ball, with the result that after some time the ball drops through.

(*b*) *Expansion of Liquids.*—Take a flask, fit it up with a cork through which a glass tube is passing (Fig. 2). Fill the flask with coloured water and force the tube down so that the water stands in the tube above the level of the cork. On heating this flask over a Bunsen burner, it will be seen that the column of coloured water begins to ascend, showing thereby that liquids expand when heated.

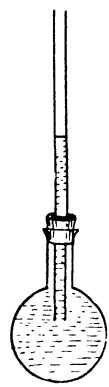


Fig. 2

(*c*) *Expansion of Gases.*—Take the above-mentioned flask fitted with a cork and a glass tube once more. Instead of filling it up with water, keep it empty. Hold it upside down this time, so that the open end of the tube is downwards. Immerse the tube in coloured water and mark the level to which it rises. Place your hand on the flask, the column of coloured water will be seen to fall in the tube, showing thereby that gas inside the flask has expanded.

(3) The third effect of heat is the **change of state.** It is a matter of common observation that ice when heated turns into water; in other words, from a solid state it is changed into a liquid state. This is true not only in the case of ice, but in the case of all solid substances, only we require different amounts of heat to bring about this change of state. The converse also is true, that is to say, by cooling a liquid can be changed into a solid.

The liquid state can be changed into gaseous by further application of heat. For instance, if water that we get from ice is heated further, it begins to boil and changes into steam, which is simply the gaseous form of water. The converse also is true, viz., by cooling a gas changes into a liquid.

(4) The fourth effect is that the **addition of heat accelerates the chemical action.** It is a well-known fact that substances combine with oxygen very easily at high temperatures; for instance, when steam is passed over heated iron filings, hydrogen is given out, and iron oxide is formed. Substances like magnesium, sulphur, wood, coal, etc., combine vigorously with oxygen when heated. They produce a large amount of heat during this oxidation, or combustion as it is more often

called. Sometimes instead of oxidation we get dissociation at high temperatures, as, for example, the splitting up of ammonium chloride into ammonia and hydrochloric acid.

(5) The fifth effect is that **heat modifies the physical properties of substances.** Hardness, rigidity, elasticity, etc., change with the rise of temperature. For instance, zinc which is brittle and hard when cold becomes soft and flexible when heated. Many substances on being heated suffer a change in colour, as, for example, mercuric iodide, which at ordinary temperatures is red but changes to yellow when heated. The conductivity of metals for heat and electricity is also affected by the increase in temperature. Under proper conditions heat may produce electricity even.

149. Thermometry.— We have already said in the beginning of the chapter that by the sense of touch we can compare two bodies and say which of them is relatively hotter than the other. But it must be noted, at the very outset, that it is true only in a general way, for the sensation is not always a reliable guide in determining the hotness or coldness of a body. For instance, let us suppose we have three beakers, one containing cold water, the second tepid water, and the third hot water. If one hand is put in cold water, and the other in hot water, and after some time both of them are put in tepid water, to one hand it will seem cold and to the other hot. Thus we see that we get cold as well as hot sensation from one and the same body. This shows that the sensation that we get depends upon the previous state of the hand.

Next, let us suppose that on a hot day while cycling on the road we touch the handle of the bicycle instead of the grips, we shall find that the former is too hot for the touch, whereas the grips can be held quite conveniently. At first sight it will seem that the handle is at a higher temperature than the grips, but as a matter of fact it is not so. What happens, however, is that the handle is a good conductor of heat, and when we touch any part of it, heat flows to the hand, not only from that part, but also from the neighbouring parts. Whereas in the case of a bad conductor the heat flows to the hand from that part only which is touched. Thus we see that the handle seems hotter, not because it is at a higher temperature, but because it is a good conductor of heat, and therefore the flow of heat to the hand is greater and more rapid than in the case of the grips.

Both these examples show that *the sensation cannot be relied upon as a safe guide for determining the hotness of a body.* Moreover, although we can say, by the sense of touch, if the condition of the hand is similar in both the cases, which of the

two bodies is hotter, we cannot estimate by *how much* one body is hotter than the other. In other words, we cannot make any quantitative comparison of temperatures.

150. For this purpose the effects produced by heat are more useful than the sensation of hotness. The effect that is generally made use of in measuring temperature is the *expansion*. We know that the length of a solid or the volume of a liquid or a gas depends upon its temperature; as it increases, the length or volume also increases. Solids expand by very small amounts, therefore they can be used to measure large intervals of temperature only. Gases, on the other hand, expand considerably with a slight increase of temperature, and hence can be used to measure very small intervals of temperature. But the fact that the volume of a gas depends, in addition to the temperature, upon the pressure to which it is subjected, brings in complications, and makes the use of gases in actual practice rather difficult. Mercury is the best substance for daily use, for it is not only very convenient, but is also tolerably accurate, as we shall see later on. *An instrument that is used for measuring temperatures is called a* ***Thermometer,*** and if mercury be used as the thermometric substance the instrument is called a mercury thermometer. Now let us see how to use a thermometer for measuring temperatures. To do it, we require first of all a standard temperature and then a scale, by means of which we might say that the temperature of the given body is so many degrees higher than the temperature of our standard substance.

To get at a standard temperature use is made of the fact that the physical state of a body depends upon temperature, and that, as long as the state does not completely change, the temperature does not rise. Ice is a very common substance, hence the temperature at which it melts is taken as the standard temperature. Next we require a scale. For this purpose we require another standard temperature or fixed point on the thermometer. The second fixed point corresponds to the temperature at which water boils under normal conditions (760 mm. pressure, sea level, and 45° latitude). These two fixed points are very convenient standards of reference, for ice and water can be obtained in a pure state very easily. The interval between these two fixed points is divided into a certain number of parts; for instance, if the lower fixed point is called 0°, the upper fixed point 100°, and the length between the two fixed points is divided into 100 equal parts, we get what is called the **Centigrade Scale.** If the upper point is called 80

instead of 100 and the interval is divided into 80 parts, we get what is called a **Reaumur Scale.** If, on the other hand, the lower temperature is called 32 and the upper 212, and the interval is divided into 180 parts, we get a **Fahrenheit Scale.** To convert the reading of one into another, remember that 0° Centigrade corresponds to 32° Fahrenheit and 0° Reaumur. If we call a particular reading *C.*, *F.*, and *R.* on the respective thermometers corresponding to the *same* temperature we have the following relation :

$$\frac{C}{100} = \frac{F-32}{180} = \frac{R}{80}.$$

This relation enables us to convert a reading on one scale into the other scales.

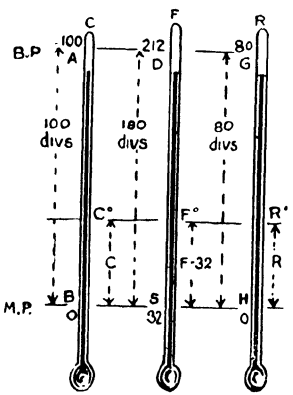


Fig. 3

151. Choice of a Thermometric substance. We have already remarked that solids have a very small expansibility, and hence they are used for measuring large differences of temperature ; gases are very expansible, and therefore they are used for measuring small differences of temperature. For ordinary purposes the solids and gases cannot be used conveniently for measuring temperature. We generally use liquids as a thermometric substance, although the best thermometric substance is air, as we shall see later on, but since an air-thermometer is not very convenient to work with, it is used only to standardise other thermometers. The requisites of a good thermometric substance are :

(1) It should be easily obtainable in pure form.

(2) It should be easily visible in capillary tubes.

(3) It should continue to remain liquid within a very wide range of temperatures ; in other words, it should possess a low freezing point and a high boiling point.

(4) It should be a good conductor of heat, so that when one part is heated, the other parts may be heated also to the same extent very soon.

(5) It should not wet the walls.

(6) The expansion should be uniform with increase of temperature.

(7) The substance should take a very small amount of heat from the body whose temperature is to be measured.

All these qualities are found *almost* satisfactorily in mercury, hence it is chosen as a thermometric substance.

152. Construction of a Mercury Thermometer.—A mercury thermometer generally consists of a bulb and a long capillary stem of uniform bore. On account of the contained air and the fineness of the bore the bulb cannot be filled directly by pouring down mercury. For filling, generally the capillary tube is made to end in a funnel. Mercury, pure and dry, is placed in it. The bulb is then heated, the air expands and passes out. As the bulb cools the air contracts, thus causing a vacuum. Mercury is forced inwards to fill up the vacuum. Repeating this process of alternate heating and cooling, we can fill up the bulb. The mercury in the bulb is next boiled to drive off the contained air and moisture. While the mercury is still at a very high temperature—higher than the highest temperature which it is to read—the tube is sealed off. A thermometer sealed off in this way is said to be **hermetically sealed.** The thermometer is allowed to cool very slowly. It is generally put aside for about two weeks so that it might recover its original volume. After this period its fixed points are determined. The stem between the two fixed points is coated with paraffin wax, and the divisions and numbers are marked with a needle-point so as to lay bare the glass. The stem is then exposed to the action of hydrofluoric acid, with the result that the exposed part becomes etched.

153. Determination of the fixed points.—(1) **Lower fixed point.**—The bulb and the part of the stem containing mercury is surrounded by ice, which has been washed off by distilled water and has been broken into small pieces. The position at which the column remains stationary for about ten minutes is marked. This gives the lower fixed point (or 0° on the Centigrade scale). Sometimes it is recommended to use a funnel so that melted water might be drained away, but there seems to be no advantage gained, for so long as there is plenty of ice present round the thermometer, the temperature will not change.

It is of extremely great importance to note, however, that the ice must be pure, and that water used to wash it must be distilled.

(2) **Upper fixed point.**—To determine this point the thermometer is placed in the apparatus shown in Fig. 4. It is called a **Hypsometer.** The thermometer is lowered until the whole of the stem is being played upon by steam, which rises up the inner cylinder round the stem of the thermometer, down the space between the inner and outer cylinder, and then escapes by the outlet. It should be carefully noted that the bulb must not dip in the boiling water, for the boiling point of water

is considerably altered if it contains impurities. The level of the column of mercury when it remains stationary for about ten minutes is marked.* At the same time the barometer is read, for the boiling point of pure water or the temperature of the steam depends upon pressure, as the following table shows:

Pressure	Temperature.	Pressure.	Temperature
700 mm.	97·71	750 mm.	99·63
720 mm.	98·49	755 mm.	99 82
735 mm.	99 07	760 mm.	100·00
740 mm.	99·25	765 mm.	100·18
745 mm.	99·44	770 mm.	100·36

Knowing the pressure we know from the table what the boiling point is. Thus we know both the fixed points.

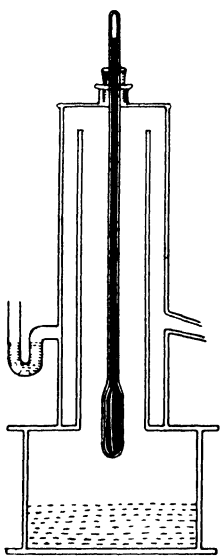


Fig. 4

Repeating the same process with an old thermometer we can verify the fixed points and determine the error, if there be any.

In order that a thermometer may be sensitive (*i.e.* able to detect small differences of temperature), it is necessary that the thread of mercury should move through a considerable length for 1° rise of temperature. In order that it may be so the bore should be very fine; the finer the bore of the tube with a given size of the bulb, the more sensitive a thermometer. The increase of the size of the bulb also makes the sensitiveness greater, for the amount of mercury contained in the bulb becomes greater, and consequently the *increase* in volume becomes greater. But the weight of the mercury filling the bulb may become too great for the latter to sustain, hence the size of the bulb cannot be increased very much.

* The water must not be made to boil too violently, otherwise the pressure of the steam in the hypsometer will exceed the atmospheric pressure. In order to know that the pressure inside is the same as outside, a glass U-tube which contains coloured water is connected to the hypsometer.

Note, in order that the readings should be reliable, the bore must be of a uniform diameter. To test whether the bore is uniform or not, a small thread of mercury is detached in the stem, and at various positions its length is read with the help of a vernier microscope. If the length is the same, the bore is uniform, otherwise not.

154. Errors of a Mercurial Thermometer.—(1) **Change of Zero.**—We have already remarked that a thermometer after being hermetically sealed is allowed to regain its original size, and that for this purpose it is kept aside for about two weeks. If the stem be graduated before the bulb contracts to its original size, due to contraction the volume of the bulb decreases, and consequently the column of mercury stands higher up than at 0° C. when immersed in ice. Suppose it stands at 0·8° C. Evidently to get the true temperature we should subtract ·8 from all readings. This error can be easily determined by placing the thermometer in pure ice washed by distilled water.

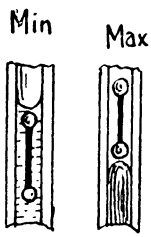
(2) **Recent Heating.**—In this case the bulb increases in size, and consequently the column of mercury stands lower down than at 0° C. when surrounded by ice. Suppose it stands at −0·8, to get the true temperature we must add ·8 to all readings. This error will occur when the thermometer has been subjected to a high temperature. It is temporary only.*

(3) **Exposure of the stem.**—While determining the high temperatures the whole of the stem cannot be immersed in the substance whose temperature is to be noted, say oil which is being heated over a flame. Part of the stem is exposed to the atmosphere, and hence its temperature is too low, with the result that reading on the whole is also low. In order to do away with this error, the whole of the stem should be immersed.

(4) **Inequality of the bore.**—As explained already, we graduate the stem by dividing the distance between the two fixed points into a definite number of parts equal in length. This method is accurate only if the bore is exactly uniform, otherwise we should detach a thread of mercury and, by noting down the lengths at different places, determine the error, which should be taken into account while taking the temperature.

* It is for this reason that the freezing point is marked before the boiling point while constructing a new thermometer. If the boiling point be determined first and then the freezing point soon after, the zero point marked would be too low.

(5) **Position and Pressure.**— The thermometer should be held in the position in which it was graduated, for due to the hydrostatic pressure there will otherwise be some difference. For instance, a thermometer which is graduated in the horizontal position will give too low readings when it is held vertically.



155. Maximum and Minimum Thermometers.—When we require to ascertain the highest temperature reached during a given time, we use what is called a **Maximum Thermometer.** It is an ordinary mercury thermometer, containing a small, light, steel index. With the help of a magnet it is moved down so that it just touches the surface of the mercury. As the mercury column moves up, it pushes the index before it. When the temperature decreases, the mercury column retreats, leaving behind the index, which thus *registers the highest temperature reached.*

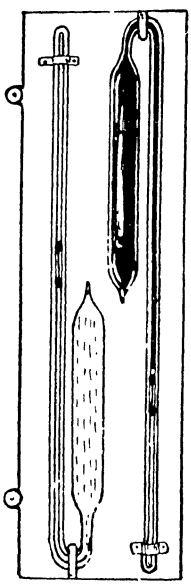


Fig. 5

When, on the other hand, we require to ascertain the lowest temperature reached, we use a **Minimum Thermometer** (Fig. 5). It is just like a Maximum Thermometer, with the difference that in this *alcohol* is used in place of mercury. As is seen from the figure, the concave surface of alcohol, due to surface tension, drags the index back as it falls, but leaves it unaffected while expanding. Thus *it registers the lowest temperature reached.*

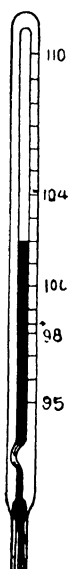


Fig. 6

156. The Clinical Thermometer.—It is a kind of maximum thermometer, in which in place of a steel index we have a constriction, which enables us to achieve the same object. The scale is Fahrenheit (Fig. 6).

To use the thermometer it is shaken at first, so that the mercury column stands low. It is placed in the armpit

or below the tongue by the patient; the mercury goes on passing through the constriction till the temperature of the patient is reached. When the thermometer is removed the column breaks at the constriction, leaving the upper portion intact in the stem.

CHAPTER II

EXPANSION

157. It was mentioned in the first chapter that all substances increase in size when heated. A few experiments were also given there to illustrate this effect of heat. There we merely dealt with the subject qualitatively, now we propose to deal with it quantitatively. We shall take up first the expansion of solids, then of liquids, and finally of gases.

158. Expansion of Solids.—That all solids, excepting a few alloys, expand when heated and contract when cooled is a well-known fact. We see numerous practical examples around us daily. For instance, in making railways a small space is left between the successive rails, so that on account of the expansion due to rise in temperature in summer they may not bend.

The iron tyre of the wheel which is always made a bit smaller in diameter, is heated and is slipped on to the wooden wheel, and is then submerged in water. On cooling the iron tyre contracts and holds the various parts firmly together.

The best way of loosening a glass-stopper when it has stuck fast in the neck of a glass bottle is to heat slightly the neck of the bottle turning it round in the flame. Due to the expansion of the neck, the stopper can easily be pulled out.

Riveting is another familiar example. Two plates which are to be fastened together are held fast whilst a hole is drilled through them both. A rivet is heated and passed through the hole. It is hammered until both ends have heads closely gripping the plates. The contraction of the rivet as it cools binds the plates together with a great force.

Solids, when heated, generally expand uniformly in all directions ; for example, if a metal cube be heated, its each side becomes longer, with the result that on the whole it is a bigger cube. Increase in length which each side undergoes is spoken of as *Linear Expansion*. Since each side becomes

longer, naturally the area of each face becomes greater, which means we have increase of surface, which is called *Surface* or *Superficial Expansion*.

As the cube becomes bigger on the whole, it means that there has been an increase in the volume, which is called the *Cubical Expansion*. We shall take these expansions one by one.

159. Linear Expansion.—While discussing linear expansion we are concerned with increase in length only, hence to simplify our treatment we shall confine ourselves to a thin rod. Experiment shows that when such a rod is heated, the expansion depends upon :

(1) *the increase in temperature, i.e.* the larger the increase in temperature the greater the increase in length ; for instance, if the increase be $\frac{1}{10}$th of a mm. for 1° rise of temperature, it will be $\frac{1}{2}$th of a mm. for 5° rise of temperature ;

(2) *the original length, i.e.* the longer the original length, the greater the increase in length ;

(3) *the nature of the material* ; some substances expand more than others.

To express it mathematically, suppose the length of a rod at 0° C. is L_0, and at t° C. it is L_t. Let α stand for the constant, which depends upon the nature of the material. The increase in length $L_t - L_0$, is given by the following relation :

$$L_t - L_0 = \alpha L_0 t, \quad . \quad . \quad . \quad . \quad (1)$$

or

$$\alpha = \frac{L_t - L_0}{L_0 t} \quad . \quad . \quad . \quad . \quad . \quad (2)$$

This constant α is called the **coefficient of linear expansion.** Let us see the significance of the formula (1). It means in words that the constant α or the *coefficient of linear expansion is the increase in length which a bar of unit length undergoes when its temperature is raised through* 1° C. We may define it also as follows :

It is the ratio of the increase in length produced by a rise of 1° C. *to the original length.*

The formula (1) can be written as

$$L_t = L_0 + \alpha L_0 t,$$

or

$$L_t = L_0(1 + \alpha t). \quad . \quad . \quad . \quad . \quad . \quad (3)$$

This formula enables us to find out the length at any temperature provided we know the original length, the rise in temperature, and the coefficient of linear expansion.

It should be noted that the coefficient of linear expansion

of different substances is different. To form an idea of the magnitude of the quantity with which we are dealing, a table is given below.

Coefficients of Linear Expansion

Glass . . .	·0000087	Brass . . .	·000019
Platinum . .	·0000089	Silver . . .	·000019
Iron . . .	·000012	Zinc . . .	·000026
Copper . .	·000017	Lead . . .	·000028

160. Determination of the Coefficient of Linear Expansion.

(1) To measure the linear expansion experimentally, a rod which has two vertical needle-points fixed to it near its ends is allowed to rest on rollers in a trough. The distance between the needle-points is measured very accurately. At first the rod is surrounded by ice, and two vernier microscopes are focussed upon these needles. The trough is heated so that the water is at a given temperature. With the help of the microscopes the displacement of each needle-point is measured. Adding them up we get the total increase in length of the rod. The object of using the rollers is to let the rod remain free to expand. Knowing the increase in length, the original length, and the rise in temperature with the help of formula (2), we can determine the coefficient of linear expansion.

(2) The laboratory method which is in common use may be understood by reference to Fig. 7.

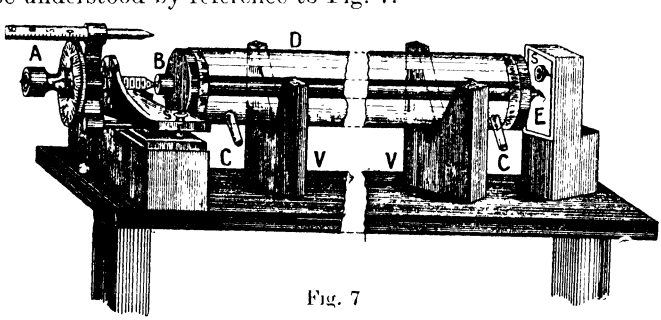


Fig. 7

A metal rod *B* about 50 cm. long and 0·5 cm. in diameter is enclosed in a tube *D* through which steam can be passed. The ends of the tube are closed by corks through which the ends of the rod *B* project. The temperature of the rod is measured with a thermometer. The tube *D* rests in two V-shaped supports *v*, *v*; one end of the rod is made pointed and the other flat. The pointed end rests against a *fixed* wooden

support, the face E of which is covered by a thin metal plate. To the flat end is applied the central foot of the spherometer A in such a way that the axis of the spherometer is in a straight line with the bar. By connecting one pole of a leclanche cell to the spherometer, and the other to the metal plate at E through a galvanometer, it is very easy to find when the central foot just touches the rod.

Measure the length of the rod at the room temperature and set it up as explained above; read the spherometer when the galvanometer needle *just* gives a throw. Turn *back* the head of the spherometer and pass steam till the rod acquires a steady temperature. The rod is thus heated to about 100° C. Turn the head of the spherometer till it makes contact with the rod. The difference between this and the first reading gives the *increase* in length of the bar. With this apparatus the change in length can be measured to about 0·005 mm. Now we know the original length, the rise in temperature, and the increase in length, hence we can find very easily the coefficient of linear expansion.

161. Superficial Expansion.—When a body is heated, it expands in length as well as in breadth or thickness. Knowing the increase in length and breadth or thickness, we can find out the increase in area of the face concerned. This increase in area is called the superficial expansion. Just as in the case of linear expansion, *the superficial expansion also is proportional to the increase in temperature, the original area, and the nature of the material.* Let us try to express it mathematically. Suppose S_0 is the surface at 0° C; heat it to t° C., and let the surface be S_t. Further let β be the constant depending upon the nature of the material. We can express the above-said relation mathematically as

$$S_t - S_0 = \beta S_0 t, \quad . \quad . \quad . \quad . \quad . \quad (1)$$

or

$$\beta = \frac{S_t - S_0}{S_0 t} \quad . \quad . \quad . \quad . \quad . \quad (5)$$

The constant β is called the **coefficient of superficial expansion.** *It is equal to the increase in unit area produced by* 1° C. *rise of temperature.*

Relation (4) can be written as

$$S_t = S_0(1 + \beta t) \quad . \quad . \quad . \quad . \quad . \quad (6)$$

This relation is very useful in solving numerical examples.

There exists a very simple relation between the coefficient of superficial expansion and the coefficient of linear expansion.

For the sake of simplicity consider a square plate $ABCD$, of side AB ($= L$). When heated through 1° C., suppose the plate becomes $AB'C'D'$. Let the coefficient of linear expansion be α, and that of superficial expansion be β. Each side will become $L(1+\alpha)$ (formula No. 3), and area will increase from L^2 to $L^2(1+\alpha)^2$.

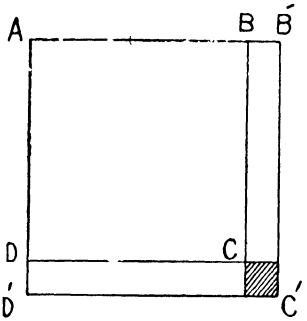


Fig. 8

The increase in area is naturally $L^2(1+\alpha)^2 - L^2$.

Using formula (5) we get the coefficient of superficial expansion, or

$$\beta = \frac{L^2(1+\alpha)^2 - L^2}{L^2 \times 1},$$

since t is 1.

Hence $\beta = (1+\alpha)^2 - 1 = 2\alpha + \alpha^2 = 2\alpha$,

neglecting α^2, which is too small for our purpose.*

Thus we see that for a homogeneous solid,

The coefficient of superficial expansion is twice the coefficient of linear expansion.

162. Cubical Expansion. When a homogeneous body is heated, it increases *uniformly* in all directions, and hence naturally its volume becomes greater. The increase in volume is called cubical expansion. Like the linear and superficial expansions this also depends upon three factors (i) *the original volume*; (ii) *the rise in temperature*; and (iii) *the constant depending on the nature of the material.* Let us suppose that the volume of a body at 0° C. is V_0, and that on heating it to t° C. it becomes V_t. Let further the constant depending upon the nature of the material be called γ. We can express the above-said relation mathematically as

$$V_t - V_0 = \gamma V_0 t. \quad . \quad . \quad . \quad . \quad . \quad (6)$$

or

$$\gamma = \frac{V_t - V_0}{V_0 t}. \quad . \quad . \quad . \quad . \quad (7)$$

This constant γ is called the **coefficient of cubical expansion.** It can be defined as follows:

The coefficient of cubical expansion is the increase in unit volume produced by 1° C. *rise of temperature.*

* Since α is a very small quantity, hence α^2 will be still smaller. Let us suppose α is ·00002, α^2 would be ·0000000004, which we can easily neglect in comparison with ·00002.

To study the relation which exists between the coefficient of cubical expansion and the coefficient of linear expansion, let us consider a cube.

Suppose one of its faces is represented by the square $ABCD$ at 0° C., the length of the side being L. Heat it through one degree and let the face be represented by $AB'C'D'$, each side being now equal to $(1+\alpha)L$.

The original volume was L^3, and the volume after heating $= L^3(1+\alpha)^3$.

Using formula (7) we get the coefficient of cubical expansion

or
$$\gamma = \frac{L^3(1+\alpha)^3 - L^3}{L^3 \cdot 1},$$

since $t = 1°$.

Hence
$$\begin{aligned}\gamma &= (1+\alpha)^3 - 1 \\ &= 3\alpha + 3\alpha^2 + \alpha^3 \\ &= 3\alpha,\end{aligned}$$

neglecting α^2 and α^3, which will be extremely small when compared with α. Thus we see that for a homogeneous body

The coefficient of cubical expansion is three times the coefficient of linear expansion.

163. Change of density with rise of temperature.—We have seen in Mechanics that density, $\rho = \frac{M}{V}$. This shows that if the mass of a body remains constant, the increase in volume will decrease the density of the substance; *i.e.*

$$V_0\rho_0 = V_t\rho_t = M,$$

where V_0 is the volume at 0° C., ρ_0 the density at 0° C., V_t the volume at $t°$ C., and ρ_t the density at $t°$ C.;

or
$$\frac{V_t}{V_0} = \frac{\rho_0}{\rho_t}.$$

But $V_t = V_0(1+\gamma t)$,

$\therefore$
$$\frac{V_0(1+\gamma t)}{V_0} = \frac{\rho_0}{\rho_t},$$

or
$$\frac{\rho_0}{\rho_t} = 1 + \gamma t \quad . \quad . \quad . \quad . \quad . \quad (8)$$

This relation is true for all kinds of matter, be they solids, liquids, or gases. But since for solids and liquids γ is small, in their case the formula

$$\frac{\rho_t}{\rho_0} = \frac{1}{1+\gamma t} \text{ is further reduced to } \frac{\rho_t}{\rho_0} = 1 - \gamma t.$$

164. Compensated Pendulum.—We have learnt in Mechanics that t, the time period of a pendulum, is given by

$$t = 2\pi\sqrt{\frac{l}{g}},$$

where l = the length of the pendulum,

and g = the acceleration due to gravity.

This shows that if l becomes larger, t becomes greater. If a pendulum of a clock is made of some metal, since the length will increase in summer, naturally t will become greater; in other words, the clock will run slower. In winter the opposite will happen: the pendulum will become smaller in size, t will become smaller, and therefore the clock will run faster. In order to prevent this, the pendulums are made in such a way that the distance between the point of suspension and the centre of gravity * of the pendulum remains the same.

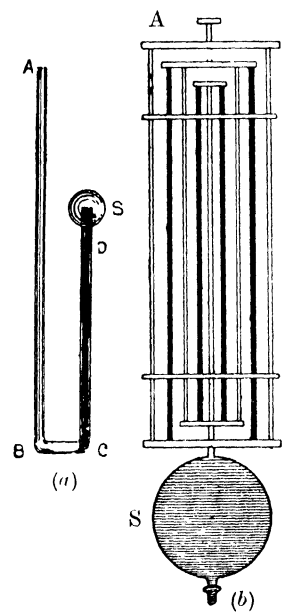


Fig. 8A

Let us see how this is achieved. It is clear from the table of the coefficients of linear expansion that the coefficient of linear expansion of brass is $\frac{1}{1}\frac{9}{2} = \frac{3}{5}$ (approx.) of that of iron. Let us suppose two rods, one of iron (AB) and the other of brass (CD), are of such lengths that their expansion is equal. The ratio of the lengths $\left(i.e.\ \frac{AB}{CD}\right)$ should be equal to approximately $\frac{3}{5}$. If AB be fixed at A, it will move downwards when heated, whereas CD will move upwards through the same distance, with the result that S will remain at the same place. If C.G. of the whole system were at the centre of S, the distance between A and C.G. of S would remain the same. In other words, if AS were the length of the pendulum it would not alter with change in weather. Since it is inconvenient to

* More accurately the centre of oscillation, which coincides very nearly with the C.G. of the pendulum.

place the bob at S, the form of an actual compensated pendulum is modified. The pendulum shown in Fig. 8A (*b*) is called *Harrison's Gridiron Pendulum.*

The brass rods are shown black, and the iron rods are represented by double lines. The iron rods expand downwards, and the brass rods expand upwards. It should be noted that the effect of five iron rods is the same as of three, and of four brass rods the same as of two. The lengths are arranged in such a manner that the distance AS remains unaltered. The two cross bars in the middle serve simply as guides. They are perforated with holes through which the various rods pass.

Clock pendulums are now generally constructed of invar—an alloy of nickel and steel (36 per cent nickel), which has an extremely small coefficient of expansion ($= \cdot 0000009$). The change in length of a pendulum constructed of this material when subjected to the ordinary fluctuations of temperature is negligible.

EXERCISES

1. When would the two thermometers Fahrenheit and Centigrade read the same temperature ?

Let that reading be x.

$$\therefore \quad \frac{x}{100} = \frac{x-32}{180},$$

or $$18x = 10x - 320,$$

or $$x = -40°.$$

2. Rails laid on a railway are 60 ft. in length ; what space must be left between each rail for alteration in length, assuming that they are laid down at 0° C., and that the highest summer temperature is 45° C., the coefficient of linear expansion for steel being ·000012 ?

Each rail would expand by $L_t - L_0$ or $L_0 \alpha t$,

or $$60 \times \cdot 000012 \times 45,$$

or $$\cdot 0324 \text{ ft.},$$

or $$\cdot 3888 \text{ inch.}$$

Since every rail would expand in both directions, hence one rail would go towards the next by ·1944 of an inch, the next would also come towards the former by this amount, hence the total distance to be left between two rails is ·4 inch approx.

3. A certain piece of sulphur is found to displace 48 c.c. of water at 0° C. What volume of water would it displace at 35° C. if the coefficient of linear expansion of sulphur is ·000077 ?

The volume evidently at 0° C. is 48 c.c.

Volume at 35° C. would be given by the formula

$$V_t = V_0(1 + \gamma t), \text{ where } V_0 = 48 \text{ c.c.}, \ t = 35°,$$

and $$\gamma = 3\alpha = 3 \times \cdot 000077$$

$$= \cdot 000231 = \cdot 00023 \text{ (app.)}.$$

Substituting the values we get

$$V_{35} = 48(1 + \cdot00023 \times 35)$$
$$= 48(1\cdot00805)$$
$$= 48\cdot3864 \text{ c.c.}$$

Since the volume of the piece of sulphur has become 48·3864, hence evidently it will displace 48·3864 c.c. of water.

4. At what temperature has the Fahrenheit thermometer the same reading as the Reaumur thermometer ?

Ans. $-25\cdot6$.

5. Prove the following formula for the conversion of Fahrenheit temperatures into Centigrade temperatures:

$$C + 40 = \tfrac{5}{9}(F + 40).$$

6. A brass rod is measured with a wooden scale at 25° C. and is found to be 255 cm. long. Find the length of the rod at 0° C. if the coefficient of linear expansion of brass is 0·000019.

Ans. 254·885 cm.

7. What space must be left between two rails 30 ft. long so as to allow for expansion between 0° C. and 40° C., assuming that the rails are laid down at 12° C., and further, that coefficient of linear expansion of steel is ·000012 ?

Ans. ·121 inch.

8. A clock which keeps correct time at 25° C. has a pendulum made of brass, whose coefficient of linear expansion is ·000019. How many seconds a day will it gain if the temperature falls to 0° C. ?

$\left(\text{Hint.} \quad \text{The time-period } t \text{ is given by } t = 2\pi\sqrt{\frac{l}{g}}\right)$.

Ans. 20·52 sec.

9. The density of silver at 0° C. is 10·31, and its coefficient of linear expansion is ·000019. Find its density at 150° C.

Ans. 10·22.

10. The volumes of two pieces of metal, iron, and brass are 702 and 700 c.c. respectively at 0° C. Find the temperature at which they will have the same volume, the coefficient of linear expansion being ·000012 and ·000018 respectively.

Ans. 159·7° C.

CHAPTER III

EXPANSION OF LIQUIDS

165. That liquids expand when heated was shown in the first chapter while discussing the effects of heat. If the experiment that was explained there be repeated, using different liquids, it will be seen that each liquid expands by a different amount. Since liquids do not possess any shape of their own, but always take up the shape of the vessel containing them, it is absurd to speak of their linear or superficial expansion. We get only the increase in volume, or what is called the cubical expansion. Owing to the fact that liquids must be kept in some vessel, another complication comes in. Before we can heat the liquid, we must heat the vessel as well, with the result that the capacity of the latter increases, and the liquid has to fill up this increase in volume before its expansion can be perceived. Consequently the *expansion that we observe* is the difference between the real expansion and the expansion of the vessel. Thus we see that in the case of liquids we have to deal with two kinds of expansion, the **real** and the **apparent.**

166. Relation of the coefficient of real and apparent expansions.—Let us consider the flask A filled with liquid up to mark P. Let the volume of the vessel be V up to mark P at 0° C. For the sake of simplicity suppose that heating takes place separately, *i.e.* at first only the vessel gets heated and then the liquid. Let the flask be heated to t° C.; its volume up to P will no longer be V, but will increase. If the coefficient of cubical expansion of glass be g, the volume of the flask up to P will be $V(1+gt)$, which means, in other words, that the volume increases by $V \times gt$. If the liquid does not expand, it must fall down, for its volume is V yet. Suppose it falls to the level Q. Evidently the volume of the portion of the neck between P and Q must be Vgt. Suppose now that the liquid is also heated to t° C., and that it comes

up to the mark R. If γ_r be the real cubical expansion of the liquid, the volume V of the liquid at 0° C. will become $V(1+\gamma_r t)$ at $t°$ C., and will reach up to R. Evidently the volume of the portion of the neck between Q and R is $V\gamma_r t$. But if we were not to take into consideration the expansion of the vessel, we would have taken PR, *i.e.* $(QR - PQ)$ or $V(\gamma_r - gt)$ as the expansion. In other words, the apparent expansion is $Vt(\gamma_r - g)$. The coefficient of apparent expansion or

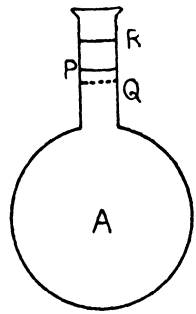


Fig. 9

$$\gamma_a = \frac{Vt(\gamma_r - g)}{Vt}$$

$$= \gamma_r - g,$$

or $$\gamma_r = \gamma_a + g.$$

Hence *the coefficient of real expansion of a liquid is equal to the sum of the coefficients of its apparent expansion and the cubical expansion of the material of the vessel.**

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* It should be noted that the proof given above is not rigorous. As a matter of fact, the coefficient of real expansion of a liquid is equal ***approximately*** to the sum of the coefficients of its apparent expansion and the cubical expansion of the vessel. Let us see how it is so. As has been said above, the graduations are true only at 0° C. Let V_0 denote the volume of the liquid and the vessel up to P at 0° C. Let, further, V_r be the real volume of the liquid and V_a its apparent volume as indicated by the flask.

$$V_r = V_0(1 + \gamma_r t),$$

and $$V_a = V_0(1 + \gamma_a t).$$

But V_a is the apparent volume of the vessel (*i.e.* the volume that is written on the vessel) up to the mark R at the temperature $t°$ C. Since each c.c. of the vessel has expanded to $1 + gt$, hence V_a c.c. have expanded to $V_a(1+gt)$, and this expression must evidently be equal to the real volume, *i.e.* V_r (the same as of the liquid, because it is filling it up to that mark).

$$V_r = V_a(1 + gt)$$

$$= V_0(1 + \gamma_a t)(1 + gt)$$

$$= V_0(1 + \gamma_r t).$$

Hence $$1 + \gamma_r t = (1 + \gamma_a t)(1 + gt)$$

$$= 1 + \gamma_a t + gt + \gamma_a g t^2,$$

or $$\gamma_r = \gamma_a + g + \gamma_a g t$$

$$= \gamma_a + g,$$

neglecting $\gamma_a gt$, which will be very *small* on account of being the product of two small quantities.

167. Determination of the coefficient of apparent expansion of liquids by the weight thermometer method.—If we neglect the increase in size of the vessel, the increase that we get in the volume of a liquid is the apparent expansion. Dividing this by the original volume and the rise of temperature we get the coefficient of apparent expansion. But in actual practice the weight thermometer method is used. A weight thermometer is usually a glass tube of the form shown in Fig. 10. It is weighed empty. Let the weight be w. Place below its capillary end a small dish containing mercury or some other liquid whose apparent expansion is to be determined. By alternately heating and cooling the thermometer, nearly fill it up with the liquid. Boil the liquid to remove the air and the moisture, keeping all along the capillary end below the surface of the liquid in the dish. Surround the rest of the tube with ice so that its temperature is 0° C. The thermometer is thus filled up with the liquid at 0° C.

Fig. 10

Place a fresh dish, say of weight a, below the capillary end, and heat the thermometer to t° C. Some of the liquid will overflow; let the weight of the liquid which has overflown be m. Weigh the thermometer with the remaining liquid, and subtract the weight w of the thermometer. Suppose the weight of the liquid remaining in the thermometer at t° C. is M. Let γ_a stand for the coefficient of the apparent expansion of the liquid.

Note, M grams of the liquid completely fill the thermometer at t° C., whereas $M+m$ grams completely fill it at 0° C. We see from what has been said above, that M grams when heated from 0° C. to t° C. expand so much that they occupy the volume occupied by $M+m$ grams at 0° C.; *i.e.* the increase in volume is equal to the volume occupied by m grams at 0° C., neglecting of course the expansion of the vessel. Since the volumes occupied by them at 0° C. are proportional to the weight, for volumes we can substitute the weights. Thus the apparent increase in volume is proportional to m; the original volume which increases when heated is proportional to M; the temperature range is t°. Hence the coefficient of apparent expansion or

$$\gamma_a = \frac{m}{Mt},$$

or

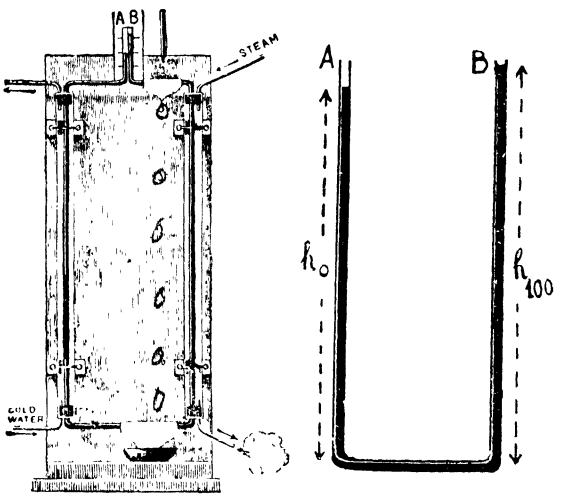
$$\frac{\text{Mass of liquid expelled}}{\text{Mass of liquid left behind at higher temperature} \times \text{rise of temperature}}$$

This formula is true enough, even if the lower temperature is not 0° C.

If it be desired to determine the real expansion of the liquid with the help of the weight thermometer, add to γ_a the coefficient of cubical expansion of the material of the vessel, for

$$\gamma_r = \gamma_a + g.$$

This is one method of determining the coefficient of real expansion of liquids. There is another direct method of



(*a*) Fig. 11 (*b*)

determining the coefficient of the real expansion, which we shall now consider.

168. Determination of the coefficient of real expansion of liquids by the method of Dulong and Petit.—In its essentials the apparatus used by Dulong and Petit is a U-tube. The method is based on the principle that if two liquids produce equal pressures, *i.e.* are in equilibrium, their heights are inversely as their densities. (See § 133.) Fig. 11 (*a*) shows the actual apparatus, and Fig. 11 (*b*) shows the rough sketch of the essentials.

A and B are the two arms of the U-tube. Each arm is surrounded by a bigger glass tube; the one surrounding A is filled with ice-cold water, and through the other steam is freely passed.

Both the arms contain the same liquid, but owing to difference of temperature the density is different on two sides. Let h_0, ρ_0 be the height and density of the liquid in the arm A, and h_{100}, ρ_{100} the height and density of the liquid in the arm B. Since the liquids are in equilibrium,

$$h_0 \cdot \rho_0 = h_{100} \cdot \rho_{100},$$

or

$$\frac{\rho_0}{\rho_{100}} = \frac{h_{100}}{h_0}.$$

But $\frac{\rho_0}{\rho_{100}} = 1 + \gamma 100$ (see § 163, formula 8), where γ is the coefficient of cubical (real) expansion of the liquid.

Hence

$$\frac{h_{100}}{h_0} = 1 + \gamma 100,$$

or

$$\gamma = \frac{h_{100} - h_0}{h_0 \cdot 100}.$$

As is clear from the formula, *the result is independent of the expansion of the tubes.* It gives directly the coefficient of absolute or real expansion of a liquid.

Regnault made a series of experiments and found that the coefficient of real expansion of mercury is not the same at all temperatures. His results are as follows:

Coefficient of real expansion between 0° and 100° C. = ·0001815
" " " 0° " 200° C. = ·0001841
" " " 0° " 300° C. = ·0001866

From this we see that the expansion of mercury is not very uniform with the rise of temperature.

169. Expansion of Water.—We have considered in general the expansion of liquids, but the expansion of water deserves special notice at our hands, since its expansion is quite peculiar. In the case of liquids in general, they go on increasing in volume, *i.e.* decreasing in density as the temperature is raised; but in the case of water it is found that if water be heated from 0° C. upwards, the volume goes on decreasing or density goes on increasing up till 4° C., after which water begins to expand or the density begins to decrease. *The density of water is maximum at* 4° C. To show it experimentally take a flask of 200

to 300 c.c. capacity fitted with a cork having two holes. Pass through one a glass tube of narrow bore and through the other a thermometer. Fix behind the glass tube a paper scale. Fill one-seventh of the flask with mercury and the rest with coloured water. Place the flask in a freezing mixture of ice and salt, and observe carefully the level of the coloured water in the tube; it will be noticed that at first the level falls rapidly, then slowly, and finally becomes steady. The thermometer will be found to register at this stage 4° C. As the cooling continues below 4° C. the level rises again, at first slowly, but more rapidly afterwards. It is clear from this experiment that the volume of the coloured water is minimum, and hence **the density maximum at 4° C.** This method of taking a flask and filling it up one-seventh with mercury enables us to study the real expansion of all liquids directly, for the coefficient of expansion of mercury is seven times that of glass, and hence the mercury expands as much as the glass flask, with the result that *the space in the flask unoccupied by mercury remains the same at all temperatures.*

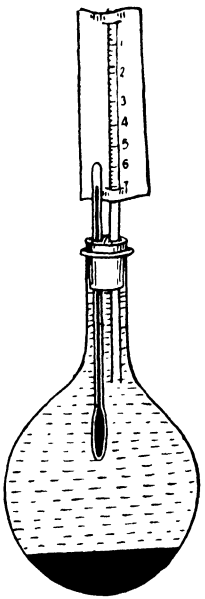


Fig. 12

The following table shows the volume of water at various temperatures between 0° to 100° C., the volume at 4° C. being taken as unity:

0° C. . . .	1·00013	50° C. . . .	1 01205
3° ,, . . .	1 00003	60° ,, . . .	1·01698
4° ,, . . .	1·00000	80° ,, . . .	1 02885
10° ,, . . .	1 00027	90° ,, . . .	1·03566
30° ,, . . .	1·00433	100° ,, . . .	1 04315

This shows that the mean coefficient of cubical expansion of water between 4° C. and 100° C. is ·000432.

The fact that water possesses maximum density at 4° C. plays a very important part in the economy of Nature. The cooling of water in winter goes on till the ponds and tanks are all at 4° C. Further cooling results in the water at the surface becoming lighter and therefore remaining at the top, leaving

the water at the bottom at 4° C. and hence unaffected. The cooling may be so much that the upper surface may entirely freeze. Since ice is a bad conductor of heat, it does not allow the lower layers to be cooled by the cold outside. This enables the aquatic animals to continue to live even in the most severe winter.

EXERCISES

1. A weight thermometer weighs 40 gm. when empty, and 490 gm. when filled with mercury at 0° C.; on heating it to 100° C., 6·85 gm. of mercury escape. Calculate the coefficient of expansion of glass, given that the coefficient of real expansion of mercury is ·000182.

$$\gamma_a = \frac{m}{Mt}$$
$$= \frac{6{\cdot}85}{443{\cdot}15 \times 100}$$
$$= 0{\cdot}0001546.$$

But $\gamma_r = \gamma_a + g,$

$$\therefore \quad g = \gamma_r - \gamma_a = {\cdot}000182 - {\cdot}0001546 = {\cdot}0000274.$$

2. A weight thermometer which contains 50 grams of mercury at 15° C. is heated to a certain temperature, and the mercury expelled is found to weigh ·954 gm. Find the temperature to which the thermometer was heated, the coefficient of apparent expansion of mercury being ·000155.

$${\cdot}000155 = \frac{{\cdot}945}{49{\cdot}055 \times (t - 15)},$$

$$\therefore \quad t - 15 = \frac{{\cdot}945}{49{\cdot}055 \times {\cdot}000155}$$
$$= 114{\cdot}3,$$

$$\therefore \quad t = 139{\cdot}3° \text{ C.}$$

3. Find what length of a glass tube 750 mm. long must be filled with mercury in order that the length of the tube unoccupied by mercury may be the same at all temperatures. The coefficient of expansion of mercury is ·000182, and of glass ·000026.

Note that the coefficient of expansion of mercury is seven times that of glass. Hence if the mercury be 1/7th of the volume of the

glass tube, it will expand as much as the glass tube would do and the expansion would not be visible. Since the tube has a uniform bore, its volume will be proportional to length. Hence the mercury must fill $\frac{1}{7}\frac{5}{0}$ mm. of the tube, or 107·14 mm.

4. How much mercury will overflow when a weight thermometer which contains 400 gm. of mercury at 0° C. is heated to 100° C., given that the apparent expansion of mercury in glass is $\frac{1}{6\,5\,0\,0}$? *Ans.* 6·0667 gm.

5. A glass weight thermometer contains 147·4 gm. of mercury at 10° C. At a temperature of 95° C., 2·02 gm. of mercury are expelled. Find the coefficient of expansion of mercury in glass. *Ans.* ·000163.

6. A glass weight thermometer contains 67 gm. of a liquid at 10° C. When the temperature is raised to 100° C. 1 gm. of liquid overflows. What is the coefficient of absolute expansion, assuming the coefficient of linear expansion of the glass to be ·000009? *Ans.* ·000179.

7. A weight thermometer which contains 1000 gm. of mercury at 0° C. is placed in an oil-bath and the mercury expelled is found to weigh 20 gm. Find the temperature of the bath. *Ans.* 131·7°.

8. In an experiment with Dulong and Petit's apparatus to find the expansion of a liquid, the column at 32° F. was 62·3 cm. high and at 212° F. it was 5·67 cm. higher. Find the coefficient of real expansion of the liquid for 1° C. *Ans.* ·00091.

9. The coefficient of real expansion of mercury is ·00018 and the coefficient of linear expansion of glass is ·000009. Find what length of a tube of glass 300 mm. long must be filled with mercury in order that the volume unoccupied by mercury may remain the same at all temperatures. *Ans.* 45 mm.

10. A long capillary tube of glass contains a thread of mercury at 10° C., the length of which according to a brass scale attached to the tube is found to be 150·56 cm. What will be the length recorded by the scale at 55° C.?

Coefficient of linear expansion of glass ·000009, of brass ·000019, and of absolute (cubical) expansion of mercury ·00018. *Ans.* 151·54 cm.

CHAPTER IV

EXPANSION OF GASES

170. In the case of solids and liquids we did not take into consideration the pressure to which they were subjected, for the simple reason that change of pressure does not produce any appreciable change in their volume. But the case of gases is quite different. The change of pressure must be taken into consideration, for we have already learnt in § 140 that the volume of a gas is inversely proportional to the pressure, temperature remaining the same. This shows that in the case of a gas when we want to study its expansion with rise of temperature we must take into consideration the pressure to which it is subjected. Moreover, on account of the fact that the gases are roughly twenty times as expansible as mercury and nine times as expansible as water, we cannot apply approximate relations as we did in the case of solids and liquids. We shall come to this point later on, where we shall consider it in detail.

Gases, as has been said, might change in volume, pressure, or temperature. In mathematical language we say that gases have three *variables*, p, v, and t. Now either all these quantities may be independent, or some relation may exist between them, which means simply that if we know any two of these quantities, we may calculate the third one. It is found that as a matter of fact such a relation does exist (see § 177). But before we consider this relation we shall discuss some experiments in which we shall keep one quantity constant, vary the second, and study the change produced in the third. We can do so in three ways :—

(i) We can vary p, keeping t constant, and study the change produced in v; or

(ii) We can vary t, keeping p constant, and study the change produced in v; or

(iii) We can vary t, keeping v constant, and study the change produced in p.

We shall take up these cases one by one.

171. The relation between pressure and volume of a gas at constant temperature.—We shall refer to it very briefly here, for we have already studied it in § 140. It was explained there that *the volume of a gas varies inversely as the pressure, provided the temperature remains constant, i.e.* the product of pressure and volume is constant, the temperature remaining the same. Mathematically we say that

$$pv = K.$$

This law is called **Boyle's law.**

It may be remarked here that the law says that the product of p and v remains constant, whatever the value of p may be. A gas which obeys this law perfectly is called a *perfect gas.* But experiments tell us that at higher pressure the product $p \times v$ no longer remains the same as at lower pressures. It decreases slightly in value. At an ordinary pressure the gases obey Boyle's law quite satisfactorily. The dry air, hydrogen, nitrogen, and oxygen are all very nearly perfect gases.

172. The relation between temperature and volume of a gas at constant pressure.—This relation corresponds to the relation that we have studied already in the case of solids and liquids, *i.e.* the cubical expansion with rise of temperature. It is found that not only the gases are the most expansible of all bodies, but that the expansion is most regular. To study experimentally the expansion of gases with rise of temperature, Regnault's apparatus sketched in Fig. 13 is used. The bulb A of known volume is connected by a thin glass tube to a U-tube $BCDE$ with unequal arms, the arm DE being longer than the arm CB. There is a tap at D which enables the experimenter to run down the liquid which is in the arm DE. There is a small capillary tubing t connected to the thin glass tube. To perform the experiment the interior of the bulb A is dried first by filling it with warm gas, then exhausting it,* filling it again and exhausting it once more, with the help of an air pump through the tube t. The vessel A is surrounded by another vessel M, which is filled with ice or ice-cold water; mercury is poured into the U-tube with the tap closed so that it comes to the level f in both the arms. The tube t is then sealed off. The

* The actual details as to how it can be done are too complicated to be discussed in this book. The procedure given seems to show the principle underlying the experiment rather than the actual details.

vessel M next is emptied and is filled with steam. The gas in A gets heated and consequently expands, pressing downwards the column of mercury in the arm BC. The level of mercury column in the other tube rises. So that the pressure may be atmospheric once more in the vessel A, the tap is opened and mercury allowed to run down, so that once more it is at the same level in both the arms as shown at f' and f''. The arm BC is graduated, hence we know the increase in volume, at the same time we know the original volume, and also we know the rise of temperature. From this we can find out the **coefficient of expansion of the gas** which is defined as *the increase in volume produced in unit volume at* 0° C. *when it is heated through* 1°, *the pressure remaining the same.* As a result of the experiment, it is found that the coefficient of the expansion of a gas is $\frac{1}{273}$, or ·00366 of its volume at 0° C. This result is true almost for all gases. It should be noted that the expansion that we get in this experiment is apparent expansion, but since the increase in the volume of the vessel is negligible in comparison with the increase in volume of the gas, we may suppose, with a fair degree of accuracy, that this expansion is equal to the real expansion.

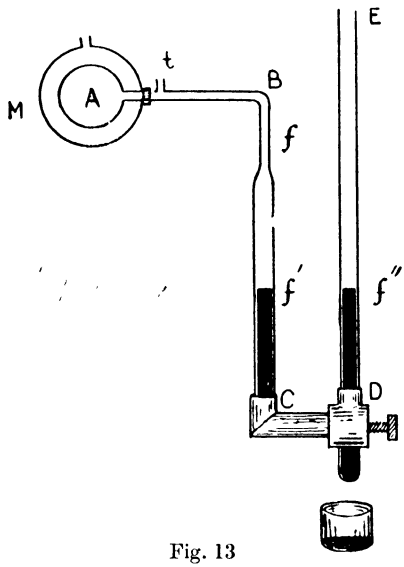


Fig. 13

To sum up, we have learnt that *if a given volume of gas be heated through* 1°, *it expands by* $\frac{1}{273}$ *of its volume at* 0° C., *provided the pressure is kept constant.* This law is known as **Charles's Law** or **Gay Lussac's Law.** As a matter of fact Charles discovered it in 1786, but he did not publish his results. Gay Lussac, however, discovered it independently in 1802.

173. The student should note the words **volume at 0° C.** in the above definition of Charles's law, for they play a very important part in the expansion of gases. In the formulæ

relating to the expansion of liquids and solids we did not specify any lower temperature. It was so because the expansion in their case was very small, so small that α^2, etc., could be neglected; if that were not the case, there also we would have used the lower temperature as 0° C.

Let us see why it is important in the case of gases and not in the case of solids or liquids. Suppose we have,

at 0° C. 273 c.c. of a gas,
at 1° C. it becomes 274 c.c. or $273\left(1+\frac{1}{273}\right)$,
at 2° C. ,, ,, 275 c.c. or $273\left(1+\frac{2}{273}\right)$,
at 100° C. ,, ,, 373 c.c. or $273\left(1+\frac{100}{273}\right)$,
at 101° C. ,, ,, 374 c.c. or $273\left(1+\frac{101}{273}\right)$,

or in general at t° C., the volume will be

$$V_t = V_0\left(1+\frac{t}{273}\right).$$

We see from what has been said above that the increase in volume is always $\frac{1}{273}$ of its volume at 0° C. for 1 degree rise of temperature, in the above example by 1 c.c., whether the gas is heated from 100° to 101° or from 0° to 1°. If we do not mind the words $\frac{1}{273}$ of its volume at 0° C., we get wrong results. For instance, volume of 373 c.c. at 100° C. would become at 101° C. 374·367, in place of 374 as shown below:

$$\begin{aligned} V_{101} &= 373\left(1+\tfrac{1}{273}\right) \\ &= 373+\tfrac{373}{273} \\ &= 373+1{\cdot}367 \\ &= 374{\cdot}367. \end{aligned}$$

If we were to calculate the coefficient of expansion from the increase in volume when the gas is heated from 100° to 101°, we would get the coefficient to be $\frac{1}{373}$ and not $\frac{1}{273}$ of the volume at 100°. This shows why we should remember the words *of its volume at* 0° C.

In the case of solids this point does not materially affect the result. Suppose we have a metre rod of copper which is 100 cm. at 0° C.,

at 1° it will become 100 (1 + ·000017) or 100·0017 cm.
at 100° ,, ,, 100 (1 + ·0017) or 100·1700 cm.
at 101° ,, ,, 100 (1 + ·001717) or 100·1717 cm.

Calculating directly $L_{101} = L_{100}(1+L)$
$= 100{\cdot}17\ (1+{\cdot}000017) = 100{\cdot}171703.$

The difference in the two calculations comes out to be ·000003 of a cm., which is too small for our purposes.

Next let us suppose we want to find the volume at temperature t_2 of a gas when the volume at temperature t_1 is given, the pressure remaining the same. From what has been said above, it is clear that we must know the volume at 0° C. before we can find out the volume at t_2° C. This is not always essential, however; we can proceed in a slightly different manner.

$$V_{t_1} = V_0\left(1 + \frac{t_1}{273}\right)$$

$$V_{t_2} = V_0\left(1 + \frac{t_2}{273}\right).$$

Dividing one by the other we get

$$\frac{V_{t_2}}{V_{t_1}} = \frac{1 + \frac{t_2}{273}}{1 + \frac{t_1}{273}} = \frac{273 + t_2}{273 + t_1},$$

or

$$V_{t_2} = V_{t_1} \times \frac{273 + t_2}{273 + t_1}.$$

174. The relation between pressure and temperature of a gas at constant volume.—When a gas is heated, keeping the volume same, it is found that the pressure increases. *The ratio of the increase of pressure to the pressure at* 0° C. *when the gas is heated from* 0° *to* 1°, *the volume remaining constant, is called the* **coefficient of increase of pressure.** To study the relation between pressure and temperature at constant volume the apparatus shown in Fig. 13 is used. The procedure is exactly the same which has been explained in § 172, with this difference only that instead of opening the tap to make the level of mercury same in both the arms, we pour in more mercury at E, so that the level of mercury in the tube BC may come once more to f. When it is so, the difference in the height of mercury columns is noted, and from that the total pressure to which the gas in the vessel is subjected is calculated. It is found, as a result of the experiment, that the coefficient of increase of pressure of a gas is $\frac{1}{273}$ of its pressure at 0° C. We find thus that the coefficient of expansion is equal to the coefficient of the increase in pressure.

To sum up, the relation between pressure and temperature at constant volume is as follows:

If a given volume of a gas be heated through 1° C. *its pressure increases by* $\frac{1}{273}$ *of its pressure at* 0° C., *provided the volume is kept constant.*

In mathematical language the relation would be expressed as

$$P_t = P_0(1 + \tfrac{1}{273}t).$$

Here also the student is warned to note carefully the words pressure at 0° C. or P_0. If pressure is to be calculated at a temperature t_2 when it is given at temperature t_1, then either we must first find out the pressure at 0° C., or we should use the following relation :

$$P_{t_2} = P_{t_1} \times \frac{273 + t_2}{273 + t_1}.*$$

We have seen above that the two coefficients — the coefficient of expansion and that of increase of pressure—are the same. We can prove the same thing theoretically.

174 (a). Let us start with a volume V_0 of a gas at a pressure P_0 and temperature 0° C. Heat it to t° C., keeping pressure constant, the new volume V_t will be given by

$$V_t = V_0(1 + ct)$$

where c is the coefficient of expansion of the gas. Now let us change the pressure so much (keeping the temperature constant) that the volume V_t may be reduced to V_0 once more. It is nothing else but a case of Boyle's law.

Hence $$P_0 V_t = P' V_0,$$

$$\therefore \quad P' = P_0 \frac{V_t}{V_0},$$

but $\dfrac{V_t}{V_0} = 1 + ct$, substituting this we get

$$P' = P_0(1 + c).$$

Thus we see that in the case of a gas which obeys Boyle's law the coefficient will be the same.

175. Air Thermometer.—This is the standard thermometer with which all other kinds of thermometers are compared. The reason why this form of thermometer is selected as a standard one is that the air (or as a matter of fact any gas) is a best thermometric substance, requisites of which we have already studied. Since a gas is 20 times more expansible than

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* This can be proved exactly in the same way in which

$$V_{t_2} = V_{t_1}\frac{273 + t_2}{273 + t_1}$$ was proved.

For $$P_{t_2} = P_0\left(1 + \frac{t_2}{273}\right) \text{ and } P_{t_1} = P_0\left(1 + \frac{t_1}{273}\right),$$

$$\therefore \quad \frac{P_{t_2}}{P_{t_1}} = \frac{1 + t_2/273}{1 + t_1/273} = \frac{273 + t_2}{273 + t_1}.$$

mercury, evidently an air thermometer is far more sensitive than a mercury thermometer; moreover air expands much more uniformly with the rise of temperature than mercury. The most important reason, however, of taking air thermometer as the standard thermometer is that its temperature scale agrees very closely with the scale derived from theoretical principles into which we cannot enter, because they are too advanced for this book.

The air thermometer may be either constant pressure air thermometer, or constant volume air thermometer, according as the pressure or volume is kept constant while measuring temperatures. In the first case the temperature is given by

$$t = \frac{V_t - V_0}{V_0} \times 273.*$$

In the second case we get,

$$t = \frac{P_t - P_0}{P_0} \times 273.$$

In practice the second type is used, for it is more convenient to work with than the first type. In the first type, if a volume of gas were to be heated through 100° the volume would increase by $\frac{100}{273}$ of the volume at 0° C. This increase in volume becomes extremely difficult to manage. There are, however, special types of air thermometers which can be used at constant pressure, but we shall not enter into a discussion of them.

176. Absolute Temperature.—Consider a constant pressure air thermometer. Suppose in place of the bulb A (Fig. 13) it consists of a long tube. Suppose further to start with we have 273 c.c. of air at 0° C. in this long tube at atmospheric pressure; mark the end where the mercury stands (f in Fig. 13). On heating to 100° C. the volume will become 373 c.c., and note the point where mercury stands; divide the tube between these two points into 100 equal parts, writing against the point f 0° C. Continue the graduations on the other side of the initial point f. The reading at the end of the tube will evidently be −273° C., which means that at this temperature the gas will have zero volume. The result is, of course, physically impossible, for a few degrees before this

* For $V_t = V_0\left(1 + \frac{t}{273}\right)$, $\therefore V_t - V_0 = V_0 \times \frac{t}{273}$,

or $$t = \frac{V_t - V_0}{V_0} \times 273.$$

temperature is reached the gas is liquefied, and hence it no longer obeys Charles's law.

If we regard this $-273°$ C. as zero and count the temperature from this upwards, naturally 0° C. will correspond to 273, and 100° C. to 373°, and in general $t°$ to $t+273°$.

This $-273°$ C.* is called the **Absolute zero,** and the scale of temperature mentioned above, *i.e.* $t+273$, is called the **Absolute scale** of temperatures. In solving questions relating to gases given at the end of this chapter the use of the absolute scale will be found to be very convenient.

We shall sum up once again the relations which we have proved already.

$$\frac{V_{t_2}}{V_{t_1}} = \frac{273+t_2}{273+t_1} = \frac{T_2}{T_1},$$

where T_2 and T_1 are absolute temperatures. This relation can be expressed more simply by $\frac{Vt_2}{T_2} = \frac{Vt_1}{T_1} = \frac{V}{T}$ = a constant. Thus we see that Charles's law leads us to the conclusion that

The volume of a given mass of a gas at constant pressure is proportional to its absolute temperature.

Similarly the law of pressure would be expressed as

$$\frac{P_{t_2}}{P_{t_1}} = \frac{273+t_2}{273+t_1} = \frac{T_2}{T_1}, \text{ or } \frac{P_{t_2}}{T_2} = \frac{P_{t_1}}{T_1},$$

or more simply $\frac{P}{T}$ = a constant, *i.e. the pressure of a given mass of any gas at constant volume is proportional to its absolute temperature.*

177. General relation between pressure volume and temperature of a gas.—Let V denote the volume of a given mass of gas, P its pressure and T its absolute temperature. We know from Charles's law that

$$V \propto T.$$

Further, we know from Boyle's law that

$$V \propto \frac{1}{P}.$$

Combining these two together we get

$$V \propto \frac{T}{P}, \text{ or } V = \frac{T}{P} \times \text{a constant},$$

* On Fahrenheit scale it is 459°·47.

or $$PV = RT,$$

or $$\frac{PV}{T} = \frac{P'V'}{T'} = R.$$

R is called the gas constant.

This equation is generally called the *gas equation*. It makes the calculations extremely simple.

EXERCISES

1. Compare the volumes of equal masses of air at 33° C. and −30° C., the pressures being the same.

This question means in simpler words that the temperature of a definite quantity of gas is changed from 33° C. to −30° C. at the same pressure; we have to compare the volumes,

$$\frac{V_{33}}{V_{-30}} = \frac{273+33}{273-30} = \frac{306}{243} = \frac{34}{27} = 1{\cdot}259.$$

2. A litre of dry air weighs 1·293 gm. at N.T.P. At what temperature will a litre of air weigh 1 gm., the pressure being 72 cm.?

At N.T.P. 1 gm. of air would have $\frac{1}{1{\cdot}293} \times 1000$ or 773·4 c.c. as the volume. The problem is reduced to this; what would be the temperature when 773·4 c.c. at N.T.P. become 1000 c.c. at 72 cm. pressure?

Applying
$$\frac{PV}{T} = \frac{P'V'}{T'},$$

we get
$$\frac{76 \times 773{\cdot}4}{273} = \frac{72 \times 1000}{T'},$$

or
$$T' = \frac{72 \times 1000 \times 273}{76 \times 773{\cdot}4},$$

or the temperature −61·4° C.

3. A steel cylinder placed in melting ice is filled with compressed oxygen at a pressure of 42 atmospheres; if the cylinder is now taken out of the freezing mixture and is allowed to stand in a room at 26° C., what will the pressure become?

$$P_t = P_0\left(1 + \frac{t}{273}\right) = 42\left(1 + \frac{26}{273}\right)$$

$$= 42 \times \frac{299}{273} = 46 \text{ atmospheres.}$$

4. Gas is contained in a vessel of 2 litres capacity at 0° C. and under a pressure of 74 cm. What does the pressure become when the temperature rises to 24° C. ?

Ans. 80·5 cm. of mercury.

5. Find the temperature at which the column of a given mass of air will be half as much again as it is at 15° C., the pressure being the same. *Ans.* 159° C.

6. The volume of a certain mass of gas is 145 c.c. at 17° C., and under a pressure of 72·5 cm. of mercury. What will be the volume if the temperature falls to 7° C. ? *Ans.* 140 c.c.

7. On heating a certain substance it is found that 380 c.c. of oxygen are given off, the temperature being 23° C. and the pressure 74 cm. What would be the volume of the oxygen measured at the normal pressure and temperature ?

Ans. 341·25 c.c.

8. 54·02 c.c. of a gas at a temperature of 22° C. and a pressure 74 cm. on cooling down to 0° C. became 49·3 c.c. the pressure being 75 cm. ; calculate the coefficient of the expansion of the gas. *Ans.* ·003685.

9. If 3000 c.c. of air at 0° C. expand by 11 c.c. for each degree rise of temperature, find the volume at 100° of a quantity of air which at 50° C. measures 100 c.c., the pressure remaining constant. *Ans.* 115·5 c.c.

10. A litre of hydrogen at N.T.P. weighs 0·9 gm. What is the weight of a litre of this gas at 27° C. and 740 mm. pressure ? *Ans.* 0·797 gm.

11. Find out the absolute zero and the coefficient of increase of volume of a gas on the Fahrenheit scale.

Ans. $-459°$ F. and $\frac{1}{491}$.

CHAPTER V

CALORIMETRY

178. After explaining the difference between heat and temperature in the first chapter, we studied the methods of measuring temperature and the changes in volume produced by heat without caring to know how much heat was utilised in the processes. Now we shall consider the subject of measurement *of the quantities of heat.* This subject is called **calorimetry.** Before we begin the subject proper, let us understand thoroughly what we mean by the words—the quantity of heat. Heat a kettle full of water to a high temperature. Take out of it a small quantity in a cup. The temperature of water in both the vessels is the same, but the quantity of heat is quite different in the two. It is obvious that the water in the kettle will give out much more heat as it cools to the atmospheric temperature than the water in the cup, for the water in the kettle can be regarded as so many distinct "cups," and each "cup" will give out as much heat as our first cup does. Of course the heat given out by water in the act of cooling is the same which it takes when it is heated through the same range of temperature. This illustration tells us clearly the significance of the words, the quantity of heat. We shall take one more example to make the meaning of the term still clearer. Let us suppose we have a block of ice in which a hollow is scooped out. Lower into it a ball of iron heated to the temperature of boiling water (taking care that it is not dipped directly into water). It will melt a certain amount of ice. Take next a ball of double the size and heat it similarly to the same temperature. When this is lowered into the hollow in the block, it melts double the amount of ice. It is obvious that the heat which ice requires to be melted comes from the hot balls, and further that the heat given out by the second ball is twice the heat given out by the first ball. We express the same thing in the language of Physics by saying that the quantity of heat given out by the second ball is

double of the quantity of heat given out by the first ball. This shows that the quantity of heat given out by a body in falling through a certain range of temperature depends upon its mass, *the greater the mass the greater the quantity of heat given out.*

Lower next into the hollow in ice one and the same ball twice, once after heating it to 100° C. and then to 200° C. It will be noticed that the ice melted in the second case is double in quantity of the ice melted in the first case. This shows that the amount of heat required by a body to be heated to a particular temperature depends upon the rise of temperature, *the greater the rise of temperature the greater the quantity of heat.*

Lower now into the hollow in ice balls of different materials, but of the same mass heated to the same temperature. It will be found that the amount of heat given out by each ball is different, showing thereby that heat given out by a body in falling through a certain range of temperature depends upon its *nature.*

We can express all the above-mentioned three conditions mathematically by the relation,

$$Q = SmΘ \quad . \quad . \quad . \quad . \quad . \quad (i)$$

where Q is the quantity of heat given out by a body, S is the constant depending upon its nature, m is the mass, and $Θ$ the fall in temperature.

Note this formula also represents the heat absorbed by the above body to get itself heated through $Θ$° C.

It is clear from what has been said above that heat is a measurable quantity. This being so, whatever has been said in mechanics about other measurable quantities like length, mass, etc., must hold good in this case as well. To measure length we had to fix upon a certain unit (we preferred international unit) in terms of which we could express the length; similarly about the mass we fixed upon a convenient unit as our standard. To measure the quantities of heat, too, we shall have to select a certain quantity as our unit. The unit generally adopted is what is called a **Calorie.**

A calorie is the amount of heat required to heat one gram of water through 1° C.* Of course the heat given out by one gram of water in cooling through 1° C. is also the same. Substituting

* To be exact we must fix upon the position of the degree also, for the amount of heat required to heat one gram from 0° to 1° is not the same as from 99° to 100°. The range which is generally fixed is from 17°·5 to 18°·5, but the variation is so slight that the position need not be remembered.

in formula (i) 1 gram for m and 1° for Θ, and 1 calorie for Q, we get $S = 1$. In other words, the constant is equal to unity in case of water.

179. Specific Heat.—We have seen already that equal masses of different substances in falling through the same range of temperature give out widely different amounts of heat. We expressed it by saying that the amount of heat given out by a body when falling through a certain range of temperature is proportional to a constant (S) depending upon the nature of the material. Let us study this constant in greater detail.

Suppose we take m grams of water and m grams of copper, and heat or cool them through Θ° C. The heat absorbed or given out by water will be given by $Q = m\Theta$ since $S = 1$. For copper it will be $Q' = m\Theta S$; dividing one by the other, we find that $S = \frac{Q'}{Q}$.

This constant S is called the **specific heat.** We define the term thus:

The specific heat of a substance is the ratio of the amount of heat required to raise the temperature through a certain range of a given mass of a substance to the amount of heat required to heat the same mass of water through the same range of temperature.

Numerically, in the terms of our unit, the specific heat is equal to the units of heat required to heat 1 gram of a substance through 1° C.

It is clear from formula (i) that the amount of heat required by a body to be heated through 1° is equal to mS. This quantity is called the **thermal capacity** of the body. It is defined as the amount of heat required to heat the given body through 1° C.

180. Before we begin the calorimetry proper, let us understand once for all the basis underlying all such experiments. Suppose 100 grams of water at 100° C. are mixed with 100 grams of water at 0° C.; it is found that the temperature of the mixture is very nearly 50° C. It is not exactly 50°, because some heat is absorbed by the vessel containing cold water. If it had not absorbed any heat, the temperature would have been exactly 50°. Now let us see what it means. The temperature of the hot water has fallen through 50°, therefore it has lost $100 \times 50 = 5000$ calories. The cold water has been heated through 50°, therefore it has gained $50 \times 100 = 5000$ calories. Thus we see that the heat which has been lost

by one body has been gained by the other. This is the underlying basis of all experiments dealing with calorimetry. If the student remembers this point all questions dealing with calorimetry will become extremely simple. Hence remember that **heat gained = heat lost.** In heat problems the student should note that this principle would be true only when there is no chemical action involved. For instance, if hot sulphuric acid be mixed with cold water the resultant temperature will be considerably modified by the heat given out during the chemical action, and the calculation cannot be made in a simple manner. Hence he is to avoid all such possibilities.

181. Water Equivalent of a Calorimeter. — The apparatus used in measuring the quantities of heat is called a *calorimeter*. Generally it consists of a copper vessel fitted with a stirrer and a thermometer. To protect it from the effects of the atmosphere it is placed inside a second copper vessel. Either it is suspended within the outer vessel with the help of hooks, or, as is more generally done, a felt pad is placed at the bottom of the outer vessel, and on the pad is placed the calorimeter. The rest of the space between the two vessels is packed with dry cotton. In order to minimise the effects due to radiation, the outer surface of C and the inner surface of F are polished. We shall treat this radiation of heat later on.

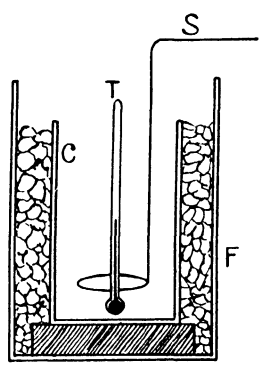

Fig. 14

Since liquids cannot be kept without some vessel, so whenever we mix a solid with a liquid or any two liquids, some heat necessarily is emitted or taken by the vessel containing the mixture, viz. the calorimeter, which may be regarded as an extra quantity of water. The amount of water to which a calorimeter is equivalent, or in other words *the amount of water which absorbs or emits the same amount of heat as is done by the calorimeter, is called the* ***water equivalent of the calorimeter.*** This may be found out either by experiment or by calculation.

To see experimentally how much heat is absorbed or emitted by the calorimeter proceed as follows: Take the empty calorimeter (C) along with stirrer, and let its water equivalent be w; weigh it, and let the weight be m. Fill it

about one-third with cold water. Reweigh it, subtracting the first weight get the weight of the cold water. Let it be m_1. Find out its temperature, let it be $t°$ C. Add hot water at temperature T, enough to fill the calorimeter about two-thirds. Keep on stirring all the while. Note the final temperature Θ. Weigh the calorimeter again, and find from it the weight of hot water added. Let the weight of hot water be m_2 grams.

Now hot water has lost heat, whereas cold water and the calorimeter both have gained heat.

The heat lost by hot water $= m_2(T - \Theta)$.

The heat gained by the cold water $= m_1(\Theta - t)$.

The heat gained by the water equivalent of the calorimeter $= w(\Theta - t)$.

Since *heat gained = heat lost*,

hence $$m_2(T - \Theta) = m_1(\Theta - t) + w(\Theta - t),$$

or $$w = \frac{m_2(T - \Theta) - m_1(\Theta - t)}{\Theta - t}$$

$$= \frac{m_2(T - \Theta)}{(\Theta - t)} - m_1.$$

If we know the specific heat of the material of which the calorimeter is made, evidently the heat gained by the calorimeter would be $mS(\Theta - t)$; in the above calculations we have taken $w(\Theta - t)$ as the heat absorbed by the calorimeter. Comparing the two we find $w = mS$ (*i.e.* = the thermal capacity of the calorimeter).

182. Determination of the Specific Heat.—There are three methods of finding out the specific heat. (1) One is by mixing the given substance at a high temperature with a second substance (whose specific heat is known) at low temperature, and calculating therefrom the specific heat of the first. This method is called the **Method of Mixtures.**

(2) The second method is applicable only to liquids. The given liquid is heated to a certain temperature and is allowed to cool. The rate at which it cools is observed, and is compared with the rate at which water cools under similar circumstances. From the rate of cooling the specific heat of the liquid is determined. This method is called the **Method of Cooling.** The details will be given later on.

(3) The given substance is heated to a certain temperature and is made to melt ice. Determining the amount of ice melted, the amount of heat given out is calculated, from which

the specific heat of the substance is determined. This method is called the **Method of Fusion of Ice.**

We shall consider at this stage the first two methods only, leaving the third method for a later stage.

183. (*a*) **Method of Mixtures.**—This method can be used for determining the specific heat of solids as well as liquids. At first we shall consider the method with reference to its application to solids. Suppose we want to find out the specific heat of brass nails. The first step is to heat the nails to a known temperature. The apparatus used for heating is called a heater (Fig. 15). Its mouth is fitted with a cork through which an iron tube passes. The nails are introduced into the tube in such a way that they surround the bulb and part of the stem of the thermometer, which is placed in the middle of the tube. Heat the water; when it begins to boil, steam is formed which plays upon the outer surface of the tube. The temperature of the substance inside the tube begins to rise. After some time the temperature becomes constant; this is so when the temperature of the nails is the same as that of the steam. When the temperature has been constant, say at $T°$ for about ten minutes, the nails are ready to be transferred. In the meanwhile a calorimeter, whose water equivalent is w, is weighed and is filled two-thirds with cold water. Suppose the weight of cold water is m and the temperature is $t°$ C. The brass nails are quickly transferred from the tube to the cold water and the water is stirred; the final temperature is noted carefully. Suppose it is $\Theta°$ C. The calorimeter is weighed once again, to find out the weight of the nails which have been dropped into the calorimeter. Let the weight be M.

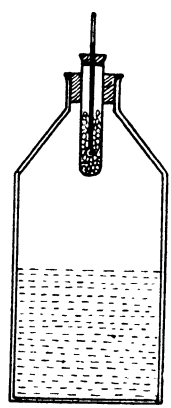

Fig. 15

Let us suppose that the specific heat of nails which we want to determine is S.

The heat lost by nails is $MS(T-\Theta)$. The heat absorbed by water and calorimeter is $(m+w)(\Theta-t)$.

$$\text{Heat lost} = \text{Heat gained.}$$

Hence $$MS(T-\Theta) = (m+w)(\Theta-t),$$

or $$S = \frac{(m+w)(\Theta-t)}{M(T-\Theta)}.$$

Precautions.—Detailed precautions will be found in some book on Practical Physics. Here we shall simply give very briefly the chief points to which attention should be paid while determining experimentally the specific heat.

(1) The substance should be always in small pieces.

(2) The temperature should become constant before the substance is transferred.

(3) The temperatures should be read accurately.

(4) The substance should be *rapidly* transferred to the calorimeter.

(5) The mixture should be thoroughly stirred.

(6) The *final temperature* of the mixture should be read carefully with a *half-degree* thermometer.

(7) The radiation should be avoided.

(8) The calorimeter should be screened from the burner which is used to heat the water.

To remove the error due to radiation, as was pointed out, the outer surface of the calorimeter and the inner surface of the outer vessel should be polished. Moreover, a hot substance always loses heat, *i.e.* radiates heat, and a cold substance always gains heat from its surroundings. If in our experiment we can so arrange that *during the first part of the experiment the temperature is as much lower than the temperature of the atmosphere as the final temperature is higher during the second part,* then the heat that it radiates in the second part would be compensated by the heat that it gains in the first part. The method consists in first taking the water at a lower temperature than the atmosphere and ending with a higher temperature. In order to form an idea as to the quantity of water and solid to be taken, a preliminary experiment should be performed.

This method can be used to determine the specific heat of liquids as well. The procedure is exactly the same as in the case of solids. The calculations are also made in exactly the same manner. The only difference is that in this case we know the specific heat of the solid in place of the liquid. While selecting a solid, it should be remembered that it should have no chemical action with the liquid.

184. (*b*) **Method of Cooling.**—If in a calorimeter a hot liquid, say water, is poured and temperature is recorded after every 30 seconds, and a curve is drawn showing the relation between the temperature and the time, a curve of the form shown in Fig. 16 is obtained.

Note the first point that this curve shows is that the greater the difference between the temperature of the liquid and that of the atmosphere, the greater the amount of heat lost during one second. This fact was first expressed in the form of a law

by Newton, and consequently it is called **Newton's law of cooling.*** It may be stated as :

The rate at which a given vessel loses heat is proportional to the difference in temperature between the vessel and its surroundings.

If hot water be allowed to cool in two or three calorimeters and cooling curves be drawn it will be seen that *the amount of*

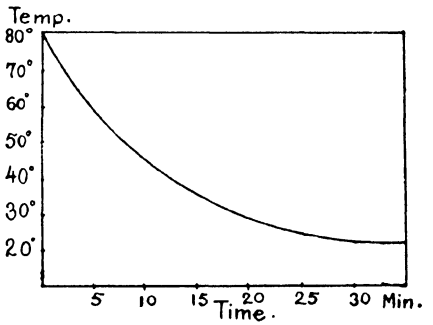

Fig. 16

heat lost per second depends upon the extent and nature of the radiating surface. In other words it means that the greater the surface the greater the amount of heat lost per second ; the more polished the surface the less the amount of heat lost per second.

Note, if the temperature difference and the extent and nature of the radiating surface be the same in any two cases, the rate of loss of heat will be the same; *i.e.* the rate of cooling is independent altogether of the nature of the liquid. This fact is made use of in finding out the specific heat of liquids. The given liquid and water both are heated approximately to the same temperature. Equal volumes of them are taken and placed in exactly similar vessels. They are allowed to cool under similar circumstances, and time is noted which each takes for falling through the same range of temperature, as for instance, from 65° to 55° C. Let us suppose the mass of the liquid is M, and of the same volume of water is m; and further, that the time taken by the liquid is T and by the

* Newton's law of cooling holds good only for small differences of temperature (like 50°, 60°, etc.). For larger differences the rate at which a body radiates out heat is proportional to the fourth power of the absolute temperature.

water is t, the water equivalent of the calorimeter being w. Let the specific heat of the liquid be S.

The heat lost in T seconds by the liquid is $M \times S \times 10$.

The heat lost by the calorimeter is $w \times 10$.

The rate of cooling is $\frac{M \times S \times 10 + w \times 10}{T}$ or $\frac{(MS + w)10}{T}$.

Similarly the rate of cooling of water is $\frac{(m + w)10}{t}$.

The rates must be equal,

$$\therefore \quad \frac{(MS + w)10}{T} = \frac{(m + w)10}{t},$$

$$\text{or} \quad (MS + w) = (m + w)\frac{T}{t},$$

$$\text{or} \quad S = \frac{(m + w)\frac{T}{t} - w}{M}.$$

Caution.— *It is the rate of cooling which is equal and not the rate of fall of temperature.*

185. Specific Heat of Gases.—The actual experimental details are too complicated to be mentioned here; but the principle, however, is just the same as mentioned above in the method of mixtures. The gases are heated in a bath kept at constant temperature, and they are passed through a spiral immersed in a certain mass of a liquid whose initial temperature is first noted. After a certain time the rise in temperature is noted. The mass of the gas that has been passed through the spiral is known from the change of pressure in the reservoir. We know the mass of the gas, we know the fall of temperature as well as the weight of the liquid, its specific heat and the rise in temperature. The only unknown quantity is the specific heat of the gas. Equating the heat lost to the heat gained we can determine the specific heat of the gas.

It should be noted, however, that gases may be heated either at constant pressure or at constant volume, and corresponding to these two methods of heating there are two specific heats. One is called the *specific heat at constant pressure*, and the other the *specific heat at constant volume.*

The specific heat at constant pressure is greater than the

specific heat at constant volume. For in the case of constant pressure the gas expands, and the heat that is supplied to the gas is utilised in heating the gas as well as in doing work in expansion against the external pressure.

Table of Specific Heats

Alcohol	0·61	Lead	0·031
Aluminium	0·202	Mercury (liquid)	0·033
Copper	0·095	Silver	0·056
Iron	0·109	Water	1·000
Ice	0·502	Zinc	0·0935
Kerosene oil	0·51	Turpentine	0·428

186. The advantages of high specific heat of water.—The specific heat of water is the greatest, *i.e.* 1 ; all other substances possess specific heat less than one. On account of this fact water is used for heating the rooms, etc. In Nature this fact plays a very important part. The oceans do not get heated so soon as the earth in summer, and hence they send a cool breeze towards the land, and in winter they do not get cooled so soon as the earth, and hence send a warm breeze towards the land. This is why the extremes of climate are absent in an island.

EXERCISES

1. Twenty grams of common salt at 91° C. are immersed in 250 gm. of turpentine oil at 13° C. whose specific heat is ·428. The final temperature was found to be 16° C. Find the specific heat of common salt.

Let the specific heat of common salt be S.

Heat given out by the common salt $= 20 \times S \times 75$.

Heat gained by turpentine $= 250 \times \cdot 428 \times 3$.

$$\therefore \quad 20 \times S \times 75 = 250 \times \cdot 428 \times 3,$$

$$\text{or} \quad S = \frac{250 \times \cdot 428 \times 3}{20 \times 75} = \cdot 214.$$

2. A ball of lead whose specific heat is ·031 and mass is 100 gm. is removed from a furnace and is immersed in 75 gm. of water at 0° C. The temperature of the mixture is 30° C. Find the temperature of the furnace. The heat absorbed by calorimeter is to be neglected.

Let the temperature be T° C.

Heat given out by lead $= 100(T - 30) \times \cdot 031$.

Heat taken by water $= 75(30 - 0) = 75 \times 30$.

Heat gained = heat lost.

$$\therefore \quad 75 \times 30 = 100(T - 30) \times \cdot 031,$$

$$\text{or} \quad T - 30 = \frac{75 \times 30}{3 \cdot 1},$$

$$\text{or} \quad T = 756° \text{ C. (approx.)}.$$

3. A piece of silver weighing 25·2 gm. is heated to 101·4° C. and dropped into 136·4 gm. of turpentine at 11·2° C. The temperature of mixture is 13° C. The water equivalent of the calorimeter is 6·2 gm. The specific heat of silver is ·056. Find the specific heat of turpentine.

Let the sp. heat of turpentine be S.

Heat given out by silver $= 25 \cdot 2 \times \cdot 056 \times 88 \cdot 4$.

Heat taken by turpentine $= 136{\cdot}4 \times S \times 1{\cdot}8 + 6{\cdot}2 \times 1{\cdot}8$.

Heat gained = heat lost.

$$\therefore \quad 136{\cdot}4 \times S \times 1{\cdot}8 + 6{\cdot}2 \times 1{\cdot}8 = 25{\cdot}2 \times {\cdot}056 \times 88{\cdot}4,$$

or
$$S = \frac{25{\cdot}2 \times {\cdot}056 \times 88{\cdot}4 - 6{\cdot}2 \times 1{\cdot}8}{136{\cdot}4 \times 1{\cdot}8}$$

$$= {\cdot}46.$$

4. 154 grams of a certain substance at 212° F. are dropped in a vessel containing 182 gm. of water at 15° C. The final temperature is 24° C. Find out the specific heat of the body. *Ans.* ·14.

5. A steam boiler is made of steel and weighs 10 tons. The specific heat of the material is ·12. The boiler contains 8 tons of water. Find the quantity of heat required to raise the temperature of the whole from 15° to 100° C., assuming no waste. *Ans.* 1,751,680 lb. deg.

6. A calorimeter whose water equivalent is 10 gm. is filled with 50 gm. of water at 80° C. and the time taken for the temperature to fall to 75° C. is 4 min. When filled with another liquid, the weight being 40 gm., the time taken for the same fall of temperature is 130 seconds. Find the specific heat of the liquid. *Ans.* ·56.

7. A piece of metal weighing 200 gm. is immersed in 120 gm. of water at 15° C. and the temperature rises to 37° C. If the specific heat of the metal is ·12, find its original temperature. *Ans.* 147° C.

8. 25 grams of turpentine are contained in a calorimeter of water equivalent 4·5 gm. at a temperature of 13° C. 30 gm of water at 80° C. are mixed with the turpentine, and the fin temperature is 57° C. Find the specific heat of turpentine. *Ans.* ·

9. The specific heat of ether and alcohol are respecti ly ·53 and ·61. If 48 grams of ether at 10° C. are mixed with 60 gm. of alcohol at 38° C., find the resultant temperature. *Ans.* 26·5°

10. A glass vessel of weight 6 oz. contains 2 lb. of wat glass and water being at 20°; 8 lb. of mercury at 45° added. What is the final temperature? The specifi of mercury and glass are given to be ·033 and ·2 resp y.

CHAPTER VI

CHANGE OF STATE

SOLID TO LIQUID

187. While explaining the general effects of heat in § 148 it was shown that the application of heat causes a rise in temperature and an increase in size, and if the heating is continued long enough it changes even the state of a body, *i.e.* a solid is turned into a liquid and a liquid into a gas. The changes in the reverse direction take place if instead of adding we subtract heat from the bodies. We shall treat these changes of state under the name of *fusion* and *solidification, evaporation* and *condensation*.

188. Fusion.—Fusion or melting is the passage of a solid body to the liquid state. This phenomenon is produced when the force of cohesion between the molecules which binds them close together becomes weak on account of great increase in the molecular motion. Since the force of cohesion between the molecules is different in different substances, evidently for every substance there is a different temperature at which its cohesive force becomes weak, and hence a different melting point. Some substances melt easily; others require very high temperatures. There are some which are incapable of fusion on account of want of sufficiently high temperature, such as carbon, for instance. Most of the substances, especially crystalline, possess well-defined melting points, and as soon as that temperature is reached the fusion begins to take place abruptly; while there are others, such as glass and wrought iron, which pass through a plastic stage; *i.e.* first become soft, and then they grow softer and softer as they are heated till they change into the liquid condition. This sort of fusion is said to be ***vitreous fusion***. Almost all amorphous bodies pass more or less through a plastic stage. They cannot be said to possess a sharp melting point. But in general the change of state is sufficiently sudden for yielding well-defined melting points.

We shall confine ourselves to the case of those substances which possess well-defined melting points, leaving out of consideration substances like glass, etc.

189. Before we take up the determination of the melting point, let us see what are the laws of fusion. The best way to study them is to take up a concrete case.

Suppose we take ice shavings in a beaker, at a very low temperature—say $-10°$ C.*—and heat them. At first the temperature is seen to rise with the application of heat. As soon as the temperature reaches 0° C. it becomes constant; any further supply of heat does not raise the temperature, but is utilised to change the state. The temperature remains constant so long as the whole of the ice is not changed into water. If the burner be removed from below and the liquid be stirred, it solidifies at the same temperature at which it melted, showing thereby that solidification is just the reverse process of melting.

If heating is continued even after melting, it is seen that the water begins to boil and thus begins to be turned into the gaseous state at about 100° C. The temperature once more becomes constant and the heat supplied to the liquid is spent up in changing its state.

The heat that is used up in changing the temperature of a body is called sensible heat. That heat always is not sensible and is used otherwise also is quite clear from what has been said above. We have seen that whenever a change of state is taking place, the temperature does not rise in spite of the fact that the substance is being heated just as before. *The heat which is used up in changing the state of a body without increasing its temperature is called* ***latent heat.*** The word latent comes from a Latin word meaning "lying hidden". It is called so because its addition cannot be detected with the help of a thermometer.

If the above experiment is performed carefully, it is seen that the volume changes during the change of state. In the case of ice a decrease in volume takes place. It can be easily seen if the experiment is modified. Take a flask (Fig. 17), put in it a few pieces of ice, and fill up the rest of the flask with tap water. Insert the cork, through which is passing a narrow glass tube having a graduated scale behind it, until the surface of water is high up the tube.

* In India we are accustomed to look upon ice as a substance at 0° C., the freezing point of water and melting point of ice. But such is far from being the case in Arctic regions or even in Siberia, where the winter is very severe and ice is many degrees below zero.

Heat the flask from below, and notice how the surface of the water falls as the ice melts. That it should be so is quite clear from the fact that ice is lighter than water,* and hence as it changes into water it must decrease in volume.

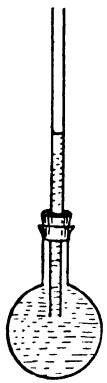

Fig. 17

We can see it in another way also. Take a small cast-iron bottle, fitted at the mouth with a screw to close it, with sides a quarter of an inch or a bit more in thickness. Fill it completely with water and close it tightly with the screw. Place it in a mixture of ice and common salt. After some time the bottle is seen to burst. This happens when the water inside is changed into ice. Since the volume of ice is greater than that of the same amount of water, it exerts so great a force on the walls of the bottle that they yield to the pressure.

Cast iron, bismuth, and antimony also expand on solidifying, and hence very sharp castings can be made of these metals. But gold, silver, copper, etc., contract on solidifying, and hence we cannot make any castings of these metals. They must be stamped with dies.

It is well to note here that as a rule substances contract on solidifying. Expansion on solidification is met with only in the case of a few substances.

190. Regelation.—That the melting point is affected by the change of pressure was shown experimentally by Lord Kelvin. He showed that as the pressure upon ice is increased, its melting point is lowered. He found that the freezing point is lowered by ·0072° C. for each increase of one atmosphere in pressure. It means, in other words, that under a pressure of 1000 atmospheres water will not freeze above the temperature of −7·2° C. (approximately). That melting point of ice is lowered by increase of pressure can be shown easily by a simple experiment.

Take a slab of ice and let it rest on two supports. Let a copper wire carrying two equal weights, say 14 lb. each, at its two ends be laid on the slab. The wire slowly cuts its way into the slab and after some time passes completely through it without leaving any permanent mark behind, except that some air bubbles can be seen. The slab is a

* Since it floats on the surface of water.

single piece as before, although the wire has passed through it. This phenomenon is called *Regelation* (literally re-freezing). Let us see what happens.

Under the wire the pressure increases on the ice; it lowers the melting point so that it can no longer remain as ice at 0° C. But before the ice melts it requires some heat. This comes from the immediate neighbouring part as well as from the copper wire, with the result that their temperature falls below zero. Now we have water below the wire. The wire cuts through the water, which goes above it where it freezes again, because there is no more increased pressure upon it and its temperature is below zero, being in touch with the wire and the neighbouring parts. It gives out heat which it absorbed while melting. This heat goes down to melt the ice immediately below the wire, and so on. This process continues till the wire passes completely through the slab.

The melting point is not always lowered under increased pressure. It is found that there are two classes of substances, one consisting of those whose melting point is lowered by increase of pressure; the second, of those whose melting point is raised by increase of pressure, as, for instance, wax. Bunsen found that a specimen of paraffin wax which melted at 46·3° C. under a pressure of one atmosphere melted under a pressure of 100 atmospheres at 49·9° C. To know the effect of increase of pressure on the melting point it is useful to remember as a general rule *that those substances which* ***expand*** *on freezing have their melting points* ***lowered*** *by increase of pressure, whereas those which* ***contract*** *on freezing or solidifying have their melting points* ***raised.***

191. Laws of Fusion.—We have studied fusion already; now let us see what are the laws of fusion. The experiments given above will help us in understanding the laws which govern the change from the solid to the liquid state.

I. *Every substance begins to fuse or melt at a certain fixed temperature, depending upon the pressure.*

II. *From the moment the fusion commences, the temperature remains constant until the fusion is complete.*

III. *Each substance undergoes a change of volume with the change of state.*

IV. *The substances which increase in volume on freezing have their melting points lowered by increase of pressure. Others which decrease in volume on solidification have their melting points raised by increase of pressure.*

192. Determination of the Melting Point.—Let us suppose we want to determine the melting point of paraffin wax. Melt it in a dish. Take a fine capillary tubing. Dip its one end in the melted wax and suck up. Seal the lower end. Attach this tube to a thermometer with rubber bands. Fix the thermometer in such a position with the help of a stand that the bulb along with the lower part of the stem and the capillary tube dips in a beaker of water which is heated from below. The water is stirred so that the temperature may remain uniform. At a certain stage the wax will melt and the opacity will disappear. Note this temperature roughly. Remove the burner from below, and let the water cool till the wax solidifies again. Begin to heat the water once more, and when that temperature (rough melting point) is near, turn the flame of the burner low, so that the temperature of the water rises very gradually. Keep on stirring the water; note the temperature at which the wax melts. Remove the burner and let the water cool, stirring it meanwhile. Note the temperature when it solidifies. Repeat this once again. Take the mean of four readings as the melting point.

There is a second method also of determining the melting point. It is called the **cooling curve method.** The substance is heated in a beaker to a temperature much above its melting

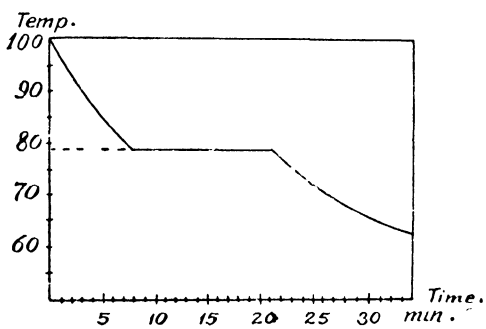

Fig. 18

point. The liquid is then poured into a calorimeter, where it is allowed to cool. Continue recording the temperature after every half a minute till the substance is not only solidified, but is also much below the freezing point.

Plot a curve showing the relation between the temperature and the time. The flat portion gives the melting point or the

freezing point, for that part corresponds to a constant temperature at the point when the change of state takes place. Those substances which pass more or less through a plastic stage do not give well-defined flat parts.

Table of Melting Points

Iridium	2230° C.	Naphthalene	80° C.
Platinum	1755° ,,	White wax	68° ,,
Silver	951° ,,	Paraffin wax	52° ,,
Tin	232° ,,	Butter	33° ,,
Cane sugar	160° ,,	Ice	0° ,,

193. Latent Heat.—We have said already that whenever change of state takes place, the temperature does not rise in spite of the fact that the substance is receiving heat. This heat, which cannot be detected by a thermometer, is called latent heat. Note, as a matter of fact latent heat is no heat at all, for when heat disappears, it ceases to exist as heat energy, having been transformed into molecular potential energy. This will be clear if we remember that the molecules of a liquid are farther apart than the molecules of a solid. Hence when we change a solid into a liquid, we force apart the molecules against their mutual attraction (*i.e.* cohesion). In doing so we do some work, the equivalent energy of which comes from the heat energy, which is said to become latent. Hence remember ***latent heat is no heat. It is work.*** Since there are two changes of state which a substance can undergo, evidently every substance will have two latent heats, one the latent heat of fusion, and the other the latent heat of vaporisation.

It is found that a given substance absorbs or gives out per unit mass a definite amount of heat while undergoing the change of state. The amount is different for each substance and for each change of state.

The **latent heat of fusion** *of a substance is defined as the amount of heat required to melt one gram of the substance without changing its temperature.* If L stands for the latent heat of fusion of a substance, then L = the number of units of heat required to melt one gram of the substance without change of temperature. If M grams of the substance be just melted without change of temperature the amount of heat used up will be $M \times L$ units. The **latent heat of vaporisation** of a substance is defined as *the amount of heat used up to convert one gram of the substance from liquid to gaseous state without changing the temperature.*

194. To find the latent heat of fusion of ice.—A calorimeter containing warm water is taken; the weight of the calorimeter (including stirrer) as well as warm water being known. Ice bits which have been washed with distilled water and dried afterwards with filter paper are added one by one, so that the final temperature of the water is as much below the atmospheric temperature as the temperature of warm water was above. The calorimeter is weighed again. Subtracting the weight of warm water and calorimeter we get the weight of ice added.

Let m be the weight of calorimeter and stirrer.
m_1 ,, weight of warm water.
m_2 ,, weight of ice added.
T ,, temperature of warm water.
Θ ,, the final temperature of water
S ,, sp. heat of material of the calorimeter.
L ,, the latent heat of ice.

Heat has been given out by warm water and calorimeter, and it has been taken up by ice, partly for melting and partly for getting the temperature of ice-cold water raised from 0° to Θ°.

Heat given out $= m_1(T-\Theta) + mS(T-\Theta)$
$= (m_1 + mS)(T-\Theta)$.
Heat taken up $= m_2L + m_2\Theta$.
But *heat gained* = *heat lost*.

$$\therefore \quad (m_1 + mS)(T-\Theta) = m_2L + m_2\Theta.$$

$$\therefore \quad L = \frac{(m_1 + mS)(T-\Theta) - m_2\Theta}{m_2}$$

$$= \frac{(m_1 + mS)(T-\Theta)}{m_2} - \Theta.$$

The latent heat of ice is found experimentally to be 80 units.

Latent Heats of Fusion

Ice	80 units.	Tin	14 units.
Paraffin wax	35 ,,	Sulphur	9·4 ,,
Zinc	28 ,,	Lead	5·9 ,,
Silver	21 ,,	Mercury	2·8 ,,

It is seen from the above table that the latent heat of fusion of ice, which is also called the latent heat of water, is very high, and it is well that it is so. If the latent heat of water were low, ice would melt very soon and disastrous

floods would result. Moreover, our ponds and lakes would freeze very much sooner than they do at present, destroying thereby the lives of aquatic animals.

195. Freezing Mixtures.—When a substance is dissolved in a liquid ordinarily only a physical change takes place, as, for instance, the solution of sodium sulphate in water. Since sodium sulphate is a solid, to change its state we must supply it with its latent heat. This heat if it does not come from outside is taken from the materials themselves, which are thereby cooled.* Such a mixture is called a **freezing mixture.** The usual freezing mixture which is used in the laboratories is salt and ice. If we take them in proper proportions (*i.e.* one part of salt and three parts of ice), as low a temperature as $-22°$ C.† may be produced. If three parts (by weight) of crystalline $CaCl_2$ are mixed with two parts by weight of ice, as low a temperature as $-55°$ C. may be obtained.

196. Ice Calorimeters.—The fact that one gram of ice at 0° C. requires 80 calories of heat to change into water at 0° C. is made use of in finding out the specific heat. There are a good many forms of ice calorimeters with the help of which we can determine the specific heat, but the principle in every case is the following: The body is heated to a high temperature, say $T°$ C., its mass being M. It is dropped into the ice calorimeter, where it falls to the temperature of ice. The heat given out by it is utilised in melting ice. The water which is obtained on melting is collected carefully; its weight is determined. Suppose the weight is m gm. m gm. of ice which have been melted have taken $80 \times m$ units of heat. They must have been given out by the hot body. But the heat given out by the body is MST.

$$\therefore \quad MST = 80\ m.$$

$$\therefore \quad S = \frac{80\ m}{TM}.$$

There are two types of calorimeters which are used for this purpose. One is the Lavoisier and Laplace's ice calorimeter, and the other is the Bunsen's ice calorimeter. We shall consider them individually.

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* It should be noted that the cooling is temporary only. It cannot last long.

† If an excess of salt is added to the ice, the low temperature is produced more quickly, but it does not affect the temperature of the freezing mixture. It is for this reason that we add an excess of common salt when we wish to freeze ice-cream quickly.

197. Lavoisier and Laplace's Ice Calorimeter.—The apparatus is shown in Fig. 19. It consists of an inner metallic vessel which is entirely surrounded with a chamber packed up with broken ice. In order that the ice in this chamber may not be melted by the external heat, another chamber which is also packed up with ice is provided. There are two taps through which water can be drained off. One of them is connected with the inner chamber and the other with the outer, as shown in the figure.

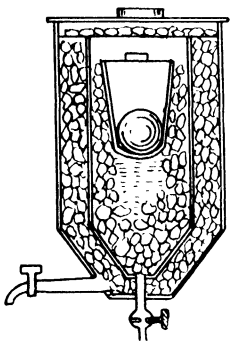


Fig. 19

The hot body is dropped into the inner vessel, and the weight of the water that is obtained from the inner chamber gives the weight of the ice melted; knowing the weight of the body which is dropped and its temperature, we can find out the specific heat.

This form of the ice calorimeter is, however, interesting from historical point of view only, for it does not yield accurate results.

198. Bunsen's Ice Calorimeter.—Bunsen's ice calorimeter is a very useful apparatus. It yields very accurate results and, moreover, can be used even when only a small quantity of the substance is available.

It consists of a thin glass test-tube *A* fused into a bigger glass tube *B*. At its lower end the tube *B* is connected to a U-tube. The U-tube is connected to a graduated capillary tubing *TT*.

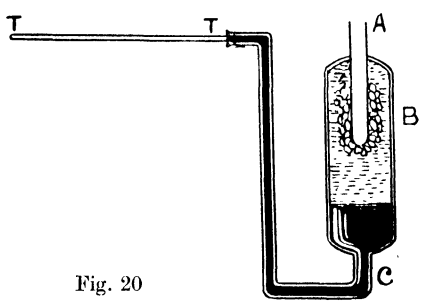


Fig. 20

To use the apparatus the tube *B* is filled partly with pure distilled water. The lower part of the U-tube is filled with mercury. The tube *TT* is filled with a coloured liquid. The whole apparatus is placed in a vessel containing pure ice so that its temperature may remain 0° C. throughout. Some of the water in the tube *B* is made to freeze by evaporating ether in the test-tube *A*. The column of the liquid in the

capillary tube is noted. The hot body whose specific heat is to be determined is dropped into *A*. It melts some of the ice in *B*. Since ice on melting decreases in volume, evidently the column of coloured liquid in the capillary tubing moves inwards. As the tube is graduated, we can find out how much change in volume is brought about. Knowing that 1 gram of ice when melted causes a decrease in volume by ·09 * c.c., we can at once determine how much ice has been melted. Knowing this, we calculate the amount of heat given out by the body. Applying the relation $S = \frac{80 \cdot m}{MT}$, we can find out the specific heat.

* We know that 1090 c.c. of ice at 0° C. yield 1000 c.c. of water at 0° C. The weight approximately will be 1000 grams. In other words, 1000 grams of ice on melting decrease by 90 c.c., or 1 gram of ice on melting suffers a decrease of ·09 c.c.

EXERCISES

1. Twenty grams of ice at $-10°$ C. are mixed with 240 grams of water at 100° C. Find the final temperature of the mixture. Specific heat of ice may be taken as ·5 and the latent heat of water as 80.

Let the final temperature be Θ.

Heat given out by the hot water $= 240 \times (100 - \Theta)$.

Heat taken up by ice is utilised, first in heating ice from $-10°$ C. to 0° C., next in melting it to water at 0° C., and finally to heat this ice-cold water to the temperature Θ.

Therefore heat taken up by ice

$$= 20 \times 10 \times \cdot 5 + 20L + 20 \times \Theta.$$

But heat gained = heat lost.

Hence $20 \times 10 \times \cdot 5 + 20 \times 80 + 20 \times \Theta = 240(100 - \Theta)$,

or $$5 + 80 + \Theta = 12(100 - \Theta),$$

or $$13\,\Theta = 1200 - 85 = 1115,$$

or $$\Theta = \frac{1115}{13} = 85{\cdot}8° \text{ C. (approximately)}.$$

2. Nine grams of ice at 0° C. are added to 121 grams of water contained in a brass calorimeter (specific heat ·09) weighing 35 grams. The initial temperature of water was 24° C. and the final temperature 17° C. Find out the latent heat.

Heat given out by water and calorimeter

$$= 121 \times 7 + \cdot 09 \times 35 \times 7 = 847 + 22{\cdot}05.$$

Heat taken up by ice $= 9 \times L + 9 \times 17 = 9L + 153$.

Therefore $$9L + 153 = 847 + 22{\cdot}05.$$

Therefore $$L = \frac{716{\cdot}05}{9} = 79{\cdot}56.$$

3. Ten grams of mercury at 100° C. are introduced into the inner tube of a Bunsen's ice calorimeter. The column of coloured liquid moves through 37 mm. in a tube 1 sq. mm. in

section. If 1000 c.c. of water become 1090 c.c. of ice, find the specific heat of mercury.

First of all let us see how much decrease in volume takes place on the melting of ice. It is obviously equal to $3{\cdot}7 \times \frac{1}{100}$ c.c. or ·037 c.c.

Now, since decrease of 90 c.c. would occur in 1000 gm. a decrease of ·037 c.c. corresponds to the melting of

$$\frac{1000}{90} \times {\cdot}037 = {\cdot}411 \text{ gm.}$$

The heat taken up by ice to melt $= {\cdot}411 \times 80 = 32{\cdot}88$ calories.

Heat given out by mercury $= 10 \times 100 \times S$.

Therefore $10 \times 100 \times S = 32{\cdot}88$,

or $S = {\cdot}03288$.

4. The specific heat of ice is ·5. How much heat will be required to change 20·4 gm. of ice at $-6{\cdot}7°$ C. to water at 14° C. ?

Ans. 1985·94 calories.

5. How many ounces of ice would melt if 3 lb. of water at 25° C. are mixed with 1·5 lb. of ice at $-10°$ C. ?

Ans. 13·5 oz.

6. Would a change in the thermometric scale affect the numerical value of the latent heat of a substance ? The specific heat of ice is ·5 and its latent heat of fusion is 80 units when the Centigrade scale is used. What are the corresponding numbers on the Fahrenheit scale ?

Ans. Yes ; sp. heat same ; latent heat 144 units.

7. 143 grams of water were obtained on introducing a kilogram of iron (specific heat ·114) at 100° into an ice calorimeter. Find out the latent heat of ice.

Ans. 79·7 units.

8. The density of ice is ·93 gm. per c.c. at 0° C., the latent heat being 80. A piece of metal weighing 100 gm. is heated to 75° and is placed in a Bunsen's calorimeter. The volume decreases by ·938 c.c. Find out the specific heat of the metal.

Ans. ·133.

9. What change of volume would be produced in an ice calorimeter by placing in it 2·5 gm. of a substance of sp. heat ·076 at a temperature of 100° C. ? *Ans.* ·0214 c.c.

CHAPTER VII

CHANGE OF STATE (*continued*)

LIQUID TO VAPOUR

199. Evaporation.—When a glass tumbler full of ice-cold water is brought into a hot crowded room and is placed on the table, its outside very soon becomes covered with drops of water. As the water in the tumbler becomes warmer, the "dew" gradually evaporates, and after some time, though the tumbler may still be much colder than the room, its surface becomes quite free of moisture. Evaporation is evidently taking place here at a temperature lower than that of the atmosphere.

Moreover, who has not observed that a wet cloth on being exposed to the air becomes dry in a short time? Such like observations have been made by every one of us. They all lead to the conclusion that *evaporation takes place at all temperatures.*

It is best to dispel in the very beginning the idea that boiling is the only change from the liquid to the gaseous state, and that this change takes place only at the boiling point. Remember the change from liquid to the gaseous state takes place at all temperatures. The only effect of temperature is to alter the rate at which evaporation takes place. The higher the temperature of the liquid the more rapid the evaporation.

200. Vapour.—When a liquid is turned into the gaseous state we say that the liquid has been changed into vapour. Before we proceed any further it will be useful to understand the distinction between a gas and a vapour. Strictly speaking, *a substance in the gaseous condition which can be liquefied by pressure alone is called a* ***vapour.*** In other words, a gas below its critical * temperature is a vapour, and a vapour above the

* Critical temperature is that temperature above which a gas cannot be liquefied to however great a pressure it may be subjected.

critical temperature is a gas. But according to this definition CO_2 at ordinary atmospheric temperature should be called a vapour instead of a gas, for its critical temperature is 31° C. Similarly SO_2 should be called a vapour, for its critical temperature is 157° C. Thus we see that in practice the word gas is being used for vapour. *Generally by vapour we mean the gaseous condition of those substances which are liquid at ordinary temperatures, such as alcohol, ether, water, etc.* It should be remembered, however, that the laws of gases are obeyed by vapours very imperfectly, when they are close to the temperature of the boiling point of the liquid. At temperatures far removed from the boiling point a vapour behaves just like a gas.

201. Vapour Pressure.—Since vapour is gaseous in character, evidently it should exert pressure in the same way as a gas does. Let us see experimentally if it is really so.

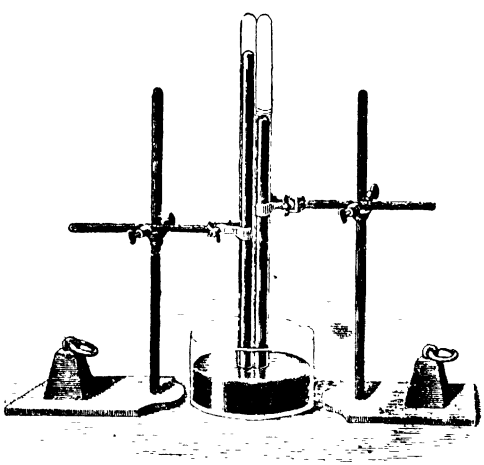


Fig. 21

Experiment.—Take two clean and dry barometer tubes. Fill them up as ordinary barometers with clean dry mercury and invert them in a dish containing mercury. Support both the tubes side by side. Measure the heights of the mercury column. With the help of a bent pipette introduce into one of the tubes two or three drops of water. The drops at once pass up the mercury column into the Torricellian vacuum and vaporise there. The column of mercury at once falls down.

The fall of the level shows that the vapour is exerting pressure. As more water is introduced drop by drop, it also quickly changes into vapour. The column of mercury falls down gradually until a limit is reached, when no further change of level takes place. If any drop is passed up at this stage, it lies at the top merely as liquid. The vapour at this stage is said to be **saturated,** and the pressure which it exerts is said to be **maximum vapour pressure,** or more simply **vapour pressure.**

Now introduce into the second tube ether drop by drop, till the space above the mercury column becomes saturated with the ether vapour. It will be found that the vapour pressure is widely different in the two cases. Suppose, for instance, that the temperature of the room is 20° C., the maximum vapour pressure of water will be 17 mm. and of ether 400 mm. This shows that *the vapour pressure depends upon the nature of the substance.*

Let us see now if the volume of vacuum and the temperature of the vapour produce any effect in the pressure. If barometer tubes of different lengths are taken, and are filled up and inverted in a dish containing mercury, it will be seen that in each case the height of mercury column is the same. Introduce into each tube ether, so that the space above the mercury becomes saturated, and measure the height of the mercury column. It will be seen that the height of the mercury column is the same in each case, the amount of liquid required being of course different in each case. Thus we see that *the vapour pressure is independent of the volume of the vacuum.*

Now pass a Bunsen flame up and down a tube containing saturated vapour in the Torricellian vacuum. It will be noticed that the pressure increases. If enough of the liquid be lying at the top of the column of mercury, then even on heating, the vapours will remain saturated, otherwise they will become unsaturated. As the tube is allowed to cool, the level of mercury column is seen to rise up gradually, till it comes to the original level once again. This shows that *the vapour pressure depends upon the temperature: the greater the temperature the greater the pressure.*

To sum up, we find—

1. That a liquid is converted into vapour when introduced into the vacuum; the amount required to saturate the space above mercury depends upon the volume and the temperature.

2. That the maximum vapour pressure depends upon the nature of the liquid.

3. That the pressure of the vapour of a given liquid is independent of the volume of the Torricellian vacuum, and depends upon the temperature only.

By the maximum vapour pressure or vapour pressure simply we mean the pressure exerted in a closed space by the vapour in contact with its own liquid.

202. We have been introducing so far the liquid into the vacuum, let us now study the case when the liquid is introduced into the space which already contains a vapour or a gas. The apparatus used is just similar to the one used before. In the vacuum either air or some other vapour is introduced first. The height of mercury column is noted. Next the drops of the given liquid are passed up to the top. The liquid vaporises, of course more slowly than it does in the vacuum. When the space can no longer hold any more liquid in the form of vapours, it is found that the pressure exerted by the vapour is the same as before; that is to say, the pressure of a vapour is independent of the presence of any other vapour in the same space, provided there is no chemical action between the two. Moreover, it is found that the total pressure exerted by the mixture is the sum of the pressures which the two constituent vapours would exert if they were enclosed in the same space at the same temperature. Dalton was the first to study these questions, and consequently the results explained are called **Dalton's laws.** They are as follows:

(1) *The maximum pressure exerted by a particular vapour in a closed space at a given temperature is independent of the presence of other vapours or gases having no chemical affinity for it.*

(2) *The total pressure exerted in a given space by a mixture of gases and vapours having no chemical affinity for each other is the sum of the pressures which the constituents would exert when enclosed in the same space at the same temperature.*

The first law is applicable to saturated vapours, whereas the second is applicable to both, saturated as well as unsaturated vapours.

203. Saturated and Unsaturated Vapours.—When a space can hold no more vapours the introduction of any more liquid in that space does not cause any appreciable fall of the mercury column, which means that the vapour is exerting maximum pressure. Evidently the density of the vapour is maximum. Vapour in this condition is called **saturated vapour.** A vapour in contact with its own liquid must always be saturated.

When a vapour exerts a pressure less than maximum pressure corresponding to that temperature, it is called **unsaturated vapour.**

Let us see how to distinguish the one from the other. The easiest way would be to see whether the vapour is present in contact with its own liquid or not. If the liquid is present there, the vapours are saturated, for the presence of the liquid is a sure proof that no more liquid can be vaporised. If it is not present, the vapour may be saturated or unsaturated. To decide that, proceed as follows :

Lower the tube containing the vapour, thereby decreasing the space above the mercury column. We have tried to compress the vapour or, in other words, to increase its density. Now, since the saturated vapour is already exerting maximum pressure and possesses maximum density, any attempt to increase the pressure or density must fail. So that if the vapour is saturated the decrease of volume should cause condensation. If, on the other hand, the vapour is unsaturated, the decrease in volume would cause increase in density or increase of pressure corresponding to Boyle's law. This will go on till the unsaturated vapour becomes saturated, when any further attempt to increase the pressure will fail, but will simply result in condensation of the vapour.

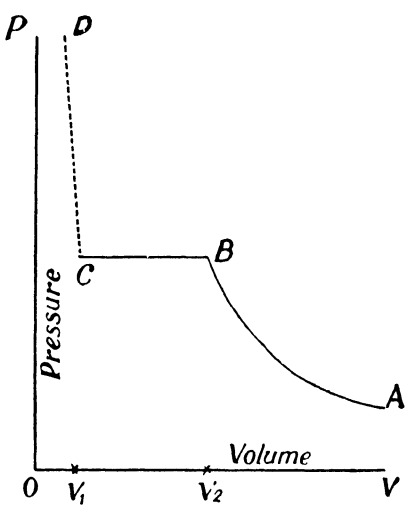


Fig. 22

Note, the increase in volume does not help us in making any decision, for if the volume is increased, the saturated vapour would also become unsaturated and would obey Boyle's law. Of course it is taken for granted that there is no liquid present at the top of the mercury column.

If a curve be plotted showing the relation between the pressure and volume of a vapour at a constant temperature, it will be of the form *ABCD* (Fig. 22). This curve represents in a very striking manner the behaviour of a vapour. For instance,

start with the unsaturated vapour having a pressure and a volume corresponding to the point *A*. On increasing the pressure the volume decreases in accordance with Boyle's law. The part *AB* represents the behaviour of unsaturated vapour.

At *BC* condensation begins, and without any appreciable change in pressure the change in volume is considerable. The part *BC* represents the behaviour of vapour in contact with the liquid (*i.e.* of saturated vapour).

The part *CD* represents the behaviour of the substance in the liquid state. Note that the curve is almost parallel to the pressure axis, showing thereby that liquids change very little in volume with pressure, *i.e.* their compressibility is very small.

Next, let us lower the temperature of the vapour by enclosing the tube in another tube of bigger bore and filling the space between the two with cold water. If a decrease in temperature causes condensation, the vapour will be saturated, for at a lower temperature less vapour is required to saturate the space than the amount actually present, and hence the surplus amount must condense.

If, on the other hand, it is unsaturated vapour it would contract in accordance with Charles's law and thereby cause a rise in the mercury column till the vapour becomes saturated, when any more cooling would cause condensation. Note, the increase in temperature does not help us in coming to any decision, for the rise in temperature would make the saturated vapour also unsaturated, in which case it would obey Charles's law. As before, it is taken for granted that there is no liquid present, for otherwise there is no necessity of all this experimentation.

Since we are concerned very much with pressure exerted by water vapour at different temperatures, a table is given below showing the maximum pressure at various temperatures.

Pressure of Water Vapour

Temperature.	Pressure.	Temperature.	Pressure.
0° C. . .	4·60 mm.	95° C. . .	633·7 mm.
10° ,, . .	9·16 ,,	99° ,, . .	733·2 ,,
20° ,, . .	17·36 ,,	100° ,, . .	760·0 mm. = 1 atmosphere.
30° ,, . .	31·51 ,,		
40° ,, . .	54·87 ,,	101° ,, . .	787·7 mm.
50° ,, . .	91·98 ,,	102° ,, . .	816·0 ,,
60° ,, . .	148·9 ,,	120·6° C. .	2 atmospheres.
70° ,, . .	233·3 ,,	134° C. . .	3 ,,
80° ,, . .	354·9 ,,	144° ,, . .	4 ,,
90° ,, . .	525·5 ,,		

204. Ebullition.—Take a flask containing water and heat it from below. Evaporation takes place at the free surface. As the temperature is raised, the evaporation becomes greater and greater. The following changes will be noted as water is heated from ordinary room temperature to the boiling point:

(1) Air which is dissolved in water forms bubbles on the inside of the walls of the flask; they will be seen to grow larger by degrees. Very minute bubbles will be shot off to the surface from the bigger bubbles.

(2) Next, small bubbles of steam are formed somewhere at the bottom and they begin to rise to the surface. Since the upper layers are colder, these bubbles of steam are condensed during their upward course. They condense with sharp clicks producing the sound which is called the "singing" of the kettle.

(3) After some time the bubbles cease to condense; on the other hand, they burst on the surface. This will happen when the temperature of water becomes uniform. At this stage the singing of the kettle ceases. The water now is boiling, or is in a state of ebullition.

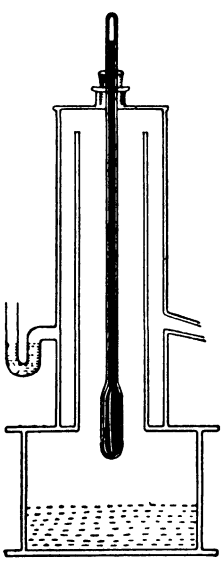


Fig. 23

(4) Howsoever long the boiling may be continued, the temperature does not rise. This constant temperature at which a liquid begins to boil is called the boiling point.

205. Determination of the Boiling Point.—To determine experimentally the boiling point of a liquid, take a hypsometer and introduce the liquid into it. Place the thermometer in such a manner that its bulb and stem are both played upon by the vapour given off by the liquid. *It is of extremely great importance to note that the thermometer is not to be dipped in the liquid*, for the temperature of the liquid will depend upon the state of its purity. At the same time read the barometer, for the boiling point depends upon the pressure, as will be seen from the experiment given below. When the reading of the thermometer remains constant for about ten minutes, the temperature

is noted. This is the boiling point of the liquid at the pressure noted. Let us now see the effect of change of pressure on the boiling point.

206. Influence of Pressure on the Boiling Point.—Take a round-bottomed flask of about 300 c.c. capacity. Fill it half with water. Heat it over a Bunsen flame to boil water, so as to drive out the air and fill the upper part with steam. Remove the burner and close the mouth with a cork just when the ebullition is about to cease. Turn the flask upside down and cool the bottom by pouring water over it. The water inside begins to boil, and the more you cool it the longer the boiling continues. Let us see what the significance of this result is. At the instant of closing the mouth of the flask, the pressure inside is equal to the atmospheric pressure. When water is poured over the bottom, some of the steam condenses and thereby the pressure inside decreases. Since water begins to boil again, this shows that the boiling point is lowered by the decrease of pressure. The more it is cooled, the more the pressure decreases and the lower the boiling point.

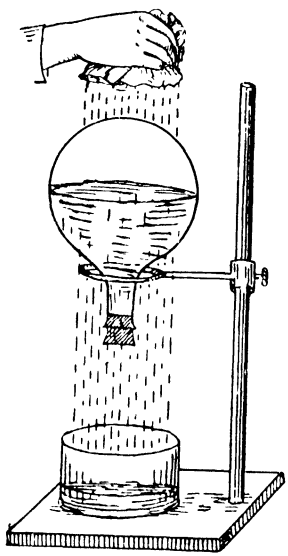


Fig. 24

On the other hand, the effect of increasing the pressure is to raise the boiling point. On the mountains water boils much below 100° C., with the result that cooking cannot be done properly. People consequently are compelled to cook their articles in a Papin's digester. The top of the digester is closed with a lid which is held in position by a screw. A valve in the lid which is closed by means of a lever carrying weight can be made to open at different pressures by simply changing the position of the weight along the lever. With this apparatus the boiling point may be raised to as high a temperature as 200° C.

207. Now let us see whether there is any relation between the external pressure and the maximum vapour pressure at the

boiling point. Take a J-tube, whose short limb is closed. Fill the tube with mercury to about 1 cm. from the open end; fill the remaining portion with water. The short tube as well as the bigger one is filled up now and there is no air left in the tube. Close the open end with the thumb and invert the tube. Water will pass into the shorter limb. Now take mercury out of the lower limb so that it stands just above the hand as shown in Fig. 25. Introduce this tube into the flask in which water is boiling.

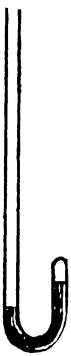


Fig. 25

As much water in the shorter limb is converted into steam as suffices to saturate the space with vapour, and since the temperature is that of the boiling point, the pressure inside is the maximum pressure at the boiling point.

It will be seen that *mercury stands in both the limbs at the same level.*

Thus we see that at the boiling point the external pressure is equal to the maximum pressure. In short, we may define the ***boiling point*** *as the temperature at which the maximum vapour pressure of the liquid is equal to the atmospheric pressure.*

Table of Boiling Points

Hydrogen . .	−238° C.	Oil of turpentine .	159° C.
Sulphur dioxide .	−10° ,,	Naphthalene . .	220° ,,
Ether	35° ,,	Sulphuric acid .	318° ,,
Alcohol . . .	78° ,,	Mercury . . .	357° ,,
Water	100° ,,	Sulphur . . .	440° ,,

208. Latent Heat of Steam.—In the preceding chapter (see §§ 189 and 193) we defined the latent heat of vaporisation as the amount of heat required to convert 1 gram of the liquid into the gaseous condition without rise of temperature. Since we are very much concerned with the boiling of water, let us see what is the latent heat of steam and how it is determined.

A heater is fitted with a sound cork through which a delivery tube bent upwards is passing. Cotton wool is wrapped round the tube so that it might not be cooled by the atmosphere. The longer limb of the tube passes into a steam trap. A second tube goes into the calorimeter fitted with a stirrer and a cover. That part of the steam which on its way to the calorimeter gets condensed must not be allowed to pass into the calorimeter,

for it is merely water at the boiling point. In order to get rid of this condensed part of steam, at first the tube is made to slope upwards so that the condensed water may run back to the heater, and as a further precaution a trap is used to catch the condensed water. A screen is placed just to shield the calorimeter from the direct heat of the flame.

The heater is half filled with water and is placed over a Bunsen burner to boil water. The calorimeter is weighed along with the stirrer; let its weight be m, the specific heat of its material being s. Fill the calorimeter half with water, and lower its temperature a little by adding ice. Let the lowered temperature be $t°$ C. and the weight of water be m_1 gm. Pass so much steam into it as may raise its temperature by about 20°. Stir the water and note the temperature; let it be $\Theta°$ C. Read the barometer and determine therefrom the temperature of the steam; let it be $T°$ C.; it will be very nearly equal to 100° C., hence we shall suppose that $T° = 100°$. Weighing the calorimeter now we can find out how much steam has been condensed. Let the weight be m_2 grams.

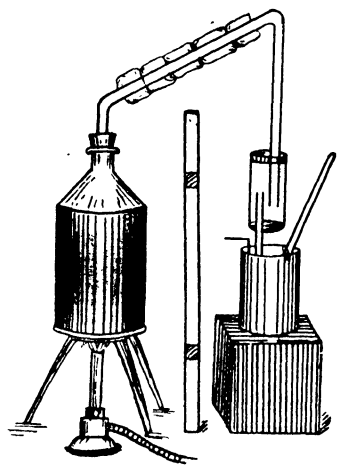


Fig. 26

We have to find out the latent heat of steam from the above data. Let it be L.

Heat given out by steam in changing its state to water at the boiling point is m_2L, and in cooling to the final temperature Θ, $m_2(100-\Theta)$ calories.

This has been used to raise the temperature of water and calorimeter. The heat taken up by them is

$$m_1(\Theta - t) + m \times s \times (\Theta - t),$$

$$\therefore \quad m_2L + m_2(100-\Theta) = (m_1 + ms)(\Theta - t),$$

or

$$L = \frac{(m_1 + ms)(\Theta - t)}{m_2} - (100 - \Theta).$$

The latent heat of steam at 100° C. is found to be 537 * calories.

Table of Latent Heats

Water	537	Carbon disulphide	87
Alcohol	202	Oil of turpentine	74
Ether	90	Iodine	24

209. Laws of Ebullition.—I. *Every liquid begins to boil at a certain definite temperature different for each substance depending upon the pressure.*

II. *From the moment the boiling commences, the temperature remains constant until the whole of the liquid is boiled off.*

III. *Unit mass of each substance requires a definite amount of heat to change its state from liquid to gaseous without change of temperature.*

IV. *The boiling point of a liquid is raised by increase of pressure and lowered by decrease of pressure.*

V. *There is an enormous increase in the volume attendant upon the change of state.*

210. Ebullition and Evaporation.—We have studied both ebullition (or boiling) as well as evaporation; let us now compare the two together. The ebullition takes place only at fixed temperature, but the evaporation takes place at all temperatures. The ebullition takes place throughout the mass of the liquid, whereas the evaporation takes place only at the free exposed surface. Whether a vessel is shallow or deep, the ebullition takes place with equal rapidity, but the evaporation is considerably affected by it. The shallower the vessel the greater the evaporation.

211. Conditions favourable for Evaporation.—If the liquid is enclosed in a space, the evaporation goes on till the space above the liquid becomes saturated with vapour, after which stage no more evaporation takes place. But if, on the other hand, the liquid is exposed to open air, the atmosphere cannot be saturated, hence evaporation goes on constantly. Let

* It should be noted that the latent heat of steam is different at different temperatures. Roughly it may be said that the lower the temperature the greater the latent heat. For instance, at 0° C. the latent heat is 606 calories. Regnault gives an empirical formula for the latent heat at different temperatures. He says that the total heat required to convert 1 gram of water at 0° C. to steam at T° C. is given by the relation

$$Q = 606{\cdot}5 + {\cdot}305\ T.$$

For instance, at 100° C., $Q = 606{\cdot}5 + 30{\cdot}5 = 637$; but 100 units are required to heat water from 0° to 100° C., hence $L = 537$. At 0° C. $Q = 606{\cdot}5$, since there is no rise of temperature. Hence $L = 606{\cdot}5$.

us see what are the conditions which make the evaporation rapid.

1. The first condition that makes the evaporation very rapid is *the large extent of the free surface of the liquid.* If we take equal amounts of water in a dish and in a cup and expose them both to the air, we find that the water in the dish disappears very much sooner than the water in the cup. This shows that the greater the free surface of the liquid the more rapid the evaporation.

2. The second condition which increases the rate of evaporation is *the renewal of air in contact with the liquid.* The layers lying above the liquid become almost saturated after some time, and therefore absorb the vapour very slowly, which means that evaporation is very slow. If we change rapidly the layer of air in contact with the liquid, the evaporation becomes very rapid ; for as soon as the layer becomes almost saturated, it is blown off and a fresh dry layer is brought in its place, which again takes vapour and goes away.

3. The higher the temperature of the air the greater the evaporation, for the greater the amount of vapour that it requires for saturation. Moreover, the higher the temperature of the liquid itself, the greater the rate of evaporation. Thus the third condition is *the high temperature of the liquid and of the air.*

4. Evaporation also depends upon the dryness of the air. The drier the air the more vapour it will require for saturation. Hence *the drier the air the more the evaporation.*

5. *The lower the pressure of the atmosphere the more rapid the evaporation*, for at a lower pressure the boiling point is very low, and hence the passing of the liquid into the gaseous state is also very rapid.

6. *The lower the boiling point of the liquid the greater the evaporation.* It is found that the more volatile liquids like ether, alcohol, etc., all possess low boiling points, and hence evaporate very quickly.

212. Cooling caused by Evaporation.—We have seen that whenever a change of state takes place heat is always absorbed. If heat is not supplied from external sources, it must be taken from the liquid itself. We found the same thing in the case of the freezing mixtures. Just corresponding to that we have here cooling caused by evaporation. Whenever evaporation takes place, heat required for the change of state is always taken from the liquid or from the surrounding objects if not

supplied from outside. For instance, when ether is poured over the hand, after a very short time it disappears, leaving the hand very cold. The cold produced may be made so great as to freeze the liquid itself by making the evaporation very rapid. We can prove it experimentally in a good many ways. We shall, however, consider only two experiments.

Experiments.—1. Take a shallow beaker of thin glass. Place it over a block of wood with a layer of water between the two. Pour ether into it, and blow into it vigorously. The ether evaporates rapidly, and by the time ether disappears the water is converted into ice. Here heat is taken continually from the water, with the result that it freezes.

2. Take a **cryophorus,** which consists of a glass tube having a bulb on each end. One of these bulbs is filled with a volatile liquid (as, for instance, water or ether) and the rest of the space contains nothing else except the vapour. The whole of the liquid is transferred to one bulb (here *B*). The other bulb is surrounded by the freezing mixture; the vapour in it condenses, thereby decreasing the pressure. Very rapid evaporation takes place in *B*, so as to send fresh vapours from *B* to *A*. For production of vapour heat is taken from the remaining liquid in *B*, with the result that it is cooled. The fresh vapour produced also is condensed, which causes more evaporation in *B*. This goes on till the liquid in *B* freezes.

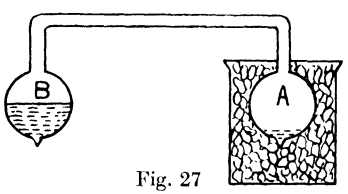


Fig. 27

Every day in our practical lives we come across the applications of this principle. The refreshing effect produced on a hot day by watering a road is due to this cooling produced by evaporation. Water evaporates off, leaving behind the road very cool. Why water remains cooler in the earthen pots is also due to this very reason. The earthen pots being porous, allow the water to come out of the pores where evaporation takes place, with the result that cold is produced.

EXERCISES

1. An aluminium calorimeter (sp. heat ·2) of mass 13·1 gm. contains 175 gm. of water at 8° C. If 5 gm. of steam at 100° C. are passed into the water, what will be the final temperature ? The latent heat of steam may be supposed to be 536.

Let the final temperature be Θ.

Heat given out by the steam $= 5(L + 100 - \Theta)$
$= 5(636 - \Theta)$.

Heat taken up by water and calorimeter
$= 175(\Theta - 8) + 13{\cdot}1 \times {\cdot}2 \times (\Theta - 8)$
$= (\Theta - 8)(175 + 2{\cdot}62)$
$= (\Theta - 8)(177{\cdot}62)$.

Therefore $3180 - 5\,\Theta = 177{\cdot}62 \times \Theta - 1420{\cdot}96$,

or $182{\cdot}62\,\Theta = 4600{\cdot}96$.

Therefore $\Theta = \dfrac{4600{\cdot}96}{182{\cdot}62} = 25{\cdot}2°$ C.

2. Seven grams of ice float in water in a calorimeter of thermal capacity 5 calories. If when steam 4·5 gm. in weight (at 100° C.) is passed into the calorimeter, the final temperature becomes 50° C., how much water was there in the calorimeter ? (Latent heat of steam = 540.)

Let the mass be M gm.

The heat given out by steam $= 4{\cdot}5 \times 540 + 4{\cdot}5 \times 50$
$= 2655$.

The heat taken up by water and calorimeter
$= (M + 5)50 + 7 \times 80 + 7 \times 50$
$= 50M + 1160$.

Heat gained = heat lost.

Therefore $2655 = 50M + 1160$.

Therefore $M = \dfrac{1495}{50}$
$= 29{\cdot}9$ gm.

3. The combustion of 1 lb. of coal generates sufficient heat to raise 8000 lb. of water through 1° C. How much coal will be required to convert 56 lb. of ice at 0° C. into steam at 100° C. ?

If in place of using grams we use pounds throughout, this will make no difference in the numerical results. Conversion is simply tedious and fruitless.

Heat consumed by 56 lb. of ice in passing into steam would be $56 \times 80 + 56 \times 100 + 56 \times 536 = 56 \times 716$.

Combustion of 1 lb. gives out so much heat as heats 8000 lb. through 1°; let us see how many pounds of coal we require for our purpose.

$$\frac{56 \times 716}{8000} = \frac{5012}{1000} = 5{\cdot}012 \text{ lb.}$$

4. 50 gm. of steam at 100° C. are passed into a mixture of 100 gm. of ice and 200 gm. of water. Find the rise of temperature produced. The water equivalent of the calorimeter = 15 gm. *Ans.* 65·3° C.

5. A calorimeter of iron weighing 149·41 gm. contains 251·64 gm. of water at 9° C. 25·83 gm. of steam are passed into the water; the final temperature rises to 64° C. Calculate the latent heat of steam.

(Take the specific heat of iron as ·113.) *Ans.* 536.

6. At a certain place water boils at 60° C. It was found there that 10 gm. of steam (at 60° C.) when passed into 600 gm. of water at 4° C. raise the temperature of water to 14·18° C. Find the latent heat of steam at 60° C. *Ans.* 565·4.

7. One lb. of coal gives out enough heat to evaporate 15 lb. of water at 100° C. How much heat does it give out in burning ? *Ans.* 8040 pound-degrees of heat.

8. One lb. of coal gives out enough heat to raise the temperature of 8240 lb. of water through 1° C. How many pounds of coal are required for converting 15·4 lb. of water at 100° to steam at 100° C. ? *Ans.* 1 lb.

9. An iron calorimeter contains 220·56 gm. of water at 13° C. When 23 gm. of steam are passed into the calorimeter, final temperature becomes 69° C. Find the weight of the calorimeter. The latent heat of steam is 539·4, and specific heat of iron is ·114. *Ans.* 120·27 gm.

10. 20 grams of steam at 100° C. are condensed in a calorimeter containing 200 gm. of water at 10° C. The temperature is raised to 64·43° C. The latent heat of steam is 536. Find the water equivalent of the calorimeter. *Ans.* 10·02 gm.

CHAPTER VIII

HYGROMETRY

213. Moisture in the Air. Evaporation is constantly going on at all temperatures from the surface of oceans, rivers, lakes, and all other wet bodies exposed to the air. The records of observation made during the past several years show that the evaporation from a free water surface amounts to as much as 88 inches in depth annually at the equator. Even in London, where the climate is so cold, the evaporation from a free water surface amounts to about 20 inches. This will just enable the student to form an idea about the amount of evaporation that is going on always at all places.

That moisture is always present in the atmosphere is easily shown by taking ice-cold water in a glass tumbler, whose outside becomes covered at once with drops of water, which shows that water vapour is always present in the atmosphere. The same thing is shown by placing outside exposed to the air any hygroscopic substance, say, for instance, calcium chloride. It becomes damp in a very short time owing to the absorption of moisture. *The science which deals with the dampness of the air is called* ***hygrometry.***

214. Relative Humidity.—We generally remark that the "air is very dry" when water evaporates freely, and in consequence a wet cloth dries rapidly, or we say that the "air is very moist" when wet things dry slowly. A layman would jump to the conclusion that when the air is dry it contains a small amount of moisture, and that when it is damp it contains a lot of moisture. But to a student of science the same remarks convey quite a different notion. He understands by these remarks that in the first case the air is far from saturation, and in the second case the air is almost saturated. But he cannot say how much water vapour is actually present in the atmosphere. As a matter of fact it is just possible that the actual amount of water vapour present in the atmosphere

in the " dry " condition may be much greater than the amount present in the " damp " condition. This tells us that the state of dryness or wetness of the air depends upon the fact how far removed is the air from saturation. In other words, we must consider two things when we talk about the weather (the state of atmosphere). The first is the actual amount of water vapour present in the air, and the second is the amount of water vapour required to saturate the air at the same temperature. It is *on the ratio of these two* that our idea of dryness or dampness chiefly depends and not upon the first alone.

The ratio is called the **relative humidity,** or more simply the **humidity,** of the air. It is defined as follows :

The relative humidity of the air at any time is the ratio of the mass of water vapour actually present in a given volume of air to the mass of water vapour required to saturate the same volume at the same temperature.

Mathematically we can express it as

$$h = \frac{m}{M},$$

where h = the relative humidity,

m = the mass of water vapour actually present in a given volume of the air,

and M = the mass of water vapour required to saturate the same volume with water vapour at the same temperature.

The mass of water vapour actually present in a volume is proportional to the density of the water vapour. Now, since the water vapour obeys Boyle's law (even up to saturation) the density is proportional to the pressure. Therefore humidity can be defined also as the ratio of the actual vapour pressure to the saturation pressure at that temperature. Mathematically we can express it as $h = \frac{p}{P}$, where p = the actual pressure and P = the saturation pressure at the same temperature.

215. The amount of water vapour actually present in the air can be determined by a **chemical hygrometer.**

It consists of an aspirator (Fig. 28) connected to a bottle containing pumice stones soaked in sulphuric acid. This is connected with the help of a glass tube to two U-tubes containing calcium chloride. The open end of the tube is stopped with a plug of wool. The tubes are weighed and their weight is noted. The purpose of the trap is to prevent any moisture

going from the aspirator to the U-tubes. Open the stop-cock of the aspirator, and at the same time remove the wool plug. The water will begin to flow out and the air will be sucked in. When four or five litres of air have passed, stop the cock. Disconnect the U-tubes and weigh them. The increase in weight gives the amount of water vapour present in the air. The

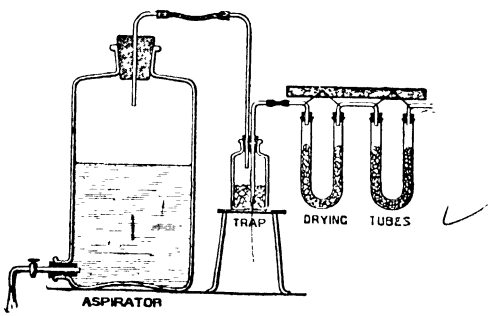


Fig. 28

volume can be determined from the weight of water that has escaped from the aspirator. Knowing the volume and the weight of the moisture present, we can determine the mass of water vapour present in a cubic metre of air. This is called the **absolute humidity** of the air.

We can find from the tables which are available the amount of water vapour required to saturate a cubic metre of air at the temperature of the experiment. *The ratio of the two gives the* ***relative humidity.*** Relative humidity is expressed sometimes as a fraction and sometimes as a percentage. For instance, let us suppose that a cubic metre of air contains 15 gm. of water vapour and the mass required to saturate a cubic metre at the same temperature is 25 gm., then the relative humidity may be expressed as 0·6 or 60 per cent.

216. Dew-point Hygrometer.—The above method gives the relative humidity accurately, but the method itself is rather tedious. There are other methods which enable us to determine the relative humidity in a very much simpler manner; we shall now consider them. Before we pass on to the consideration of the actual methods of determining relative humidity, let us understand first the principle underlying these hygrometers.

No doubt the moisture is always present in the air, but as

a general rule it is far from being enough to saturate the air. Let us cool a body gradually; at a certain temperature the amount of water vapour present would suffice to saturate the air. As soon as this temperature is reached, condensation begins and we say that dew is formed. *The temperature at which vapour actually present in the air is sufficient to saturate it is called the* ***dew-point.***

Now determine the dew-point of the air on a certain day. Consult the tables and find out the maximum vapour pressure at the dew-point. This pressure is equal to the actual pressure of the vapour. Find out from the tables the maximum pressure at the temperature of the air. The ratio of these two gives the relative humidity. So the method is to find out the dew-point. Knowing this we can find the maximum pressure at the dew-point and the maximum pressure at the atmospheric temperature from the tables and take the ratio.

These hygrometers which enable us to determine the dew-point are called **dew-point hygrometers.** We shall consider only two such hygrometers.

217. Daniell's Hygrometer. —It is essentially a cryophorus having ether and ether vapour inside it, instead of water and water vapour. The bulb *A* has inside it a thermometer dipping in ether. It has, moreover, a metallic strip going round it. The bulb *B* is covered with muslin and ether is poured over it. The ether evaporates very soon, producing cold which condenses the vapour in *B*. This causes more vapour to leave the bulb *A* to be condensed in turn. The constant evaporation in *A* results in the gradual fall of the temperature of the ether. As soon as dew is formed on the metallic strip the temperature is noted. The cooling is discontinued and the temperature at which dew disappears is noted. The mean of the two gives the dew-point. This hygrometer is not very satisfactory to work with on account of the following disadvantages:

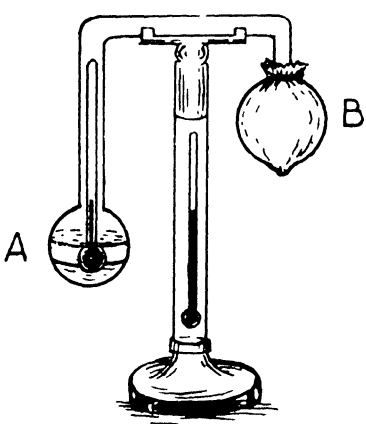


Fig. 29

(*a*) The rate of cooling cannot be regulated.

(*b*) The temperature of the metallic strip is not the same as of the liquid inside the bulb, owing to the glass being a bad conductor of heat.

(*c*) The temperature at which dew appears and disappears cannot be noted easily.

(*d*) The proximity of the observer disturbs the hygrometric state of the air near the apparatus.

These disadvantages are removed in Regnault's hygrometer, which we shall now consider.

218. Regnault's Hygrometer.—It consists of a glass test-tube (*A*) having the lower portion made up of silver (*B*). The

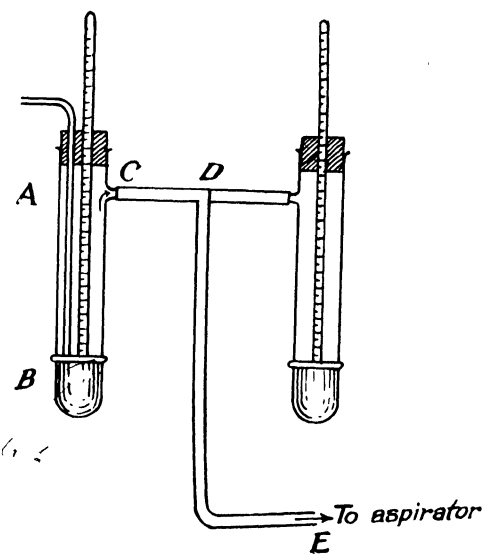


Fig. 30

outer surface of the silver part is polished. The mouth is fitted with a cork having two holes. Through one passes a thermometer and through the other a bent tube. A side tube *CDE* leaves the test-tube near the top. It is connected to an aspirator. There is a similar test-tube supported in the same stand. To work the hygrometer, ether is poured into the test-tube *AB*. A current of air is regulated with the help of the stop-cock of the aspirator. The air bubbles through

the ether and causes evaporation which results in cooling. The observations are made with the help of a telescope from a distance so that the breath of the observer may not disturb the hygrometric state. Moreover, the fixing up of the temperature at which dew appears is facilitated by the presence of the second tube, for we can contrast the two metallic strips, and as soon as we find B dim we note the temperature. Then the cock of the aspirator is stopped and the temperature at which dew disappears is noted. The mean of these two gives the dew-point from which the relative humidity is determined. This hygrometer is very elaborate, consequently it takes rather a long time to be set up, and, moreover, requires a good deal of attention. Hence it is not used by the meteorologists. They use the wet and the dry bulb hygrometer, which is extremely simple.

219. The Wet and Dry Bulb Hygrometer.—It consists of two thermometers mounted together. The bulb of one of the thermometers is wrapped with a muslin piece the lower end of which dips in a small beaker containing water. The temperature of the wet bulb thermometer falls, owing to the cold caused by evaporation. The difference between the readings of the thermometers evidently will depend upon the state of the air. If it be dry, the evaporation will be brisk and the difference will be great. On a damp day the difference will be comparatively small. In a general sort of way we can say that the greater the difference between the readings of the two thermometers the lower will be the dew-point.

If the atmosphere is saturated with vapour there will be no evaporation, and hence the readings on the two thermometers will be identical. In this case this very temperature would be the dew-point.

Tables have been prepared by means of which the relative humidity can be determined from the readings of dry bulb thermometer and the difference between the dry bulb and the wet bulb thermometers.

For detailed information about the tables the student should consult some book on Practical Physics.

EXERCISES

1. The dew-point on a certain day was found to be 12° C., the temperature of the air being 16°·5 C. Find its humidity.

The maximum vapour pressure at 12° C. = 10·46 mm. of (mercury)
" " " 16° C. = 13·64 mm. "
" " " 17° C. = 14·42 mm. "

The actual vapour pressure at 16·5° is equivalent to 10·46 mm. (of mercury).

The maximum pressure at 16·5° would be 14·03 mm. (of mercury).

$$\therefore \quad h = \frac{10{\cdot}46}{14{\cdot}03}$$

$$= 74{\cdot}5 \text{ per cent.}$$

2. On a certain day when the barometer was 760 mm. high, the temperature of the air was 20° C. and the relative humidity was 50 per cent. What fraction of the whole pressure of the air was due to the water vapour? The saturation pressure at 20° C. is 18 mm. (of mercury).

Since the relative humidity is 50 per cent, the actual vapour pressure is 9 mm. (of mercury). The whole pressure is 760 mm. Therefore the pressure of the water vapour is $\frac{9}{760}$ of the whole air pressure; that due to air alone is 751 mm.

3. Calculate the weight of 10 litres of air saturated with water vapour at 18° C. and 750 mm. pressure. The maximum vapour pressure at 18° C. is 15·46 mm. of mercury.

The pressure of air alone is $750 - 15{\cdot}46 = 734{\cdot}54$ mm. We *know that* 1 *litre of dry air at N.T.P. weighs* 1·293 *gm. and that mass of* 1 *litre of water vapour under the same conditions is* $\frac{5}{8}$ *of the mass of* 1 *litre of dry air.*

Moreover, we know that $\frac{PV}{T} = \frac{P'V'}{T'}$.

Substituting the values we get

$$\frac{10 \times 734{\cdot}54}{291} = \frac{760 \times V'}{273}.$$

$\therefore$ V' or the volume of 10 litres originally at 734·54 and 18° C. would become at N.T.P. $= \dfrac{10 \times 734{\cdot}54}{291} \times \dfrac{273}{760}$ litres.

The weight of the air alone $= \dfrac{10 \times 734{\cdot}54}{291} \times \dfrac{273 \times 1{\cdot}293}{760}$ gm.

$= 11{\cdot}73$ gm.

The weight of the water vapour $= \dfrac{10 \times 15{\cdot}46}{291} \times \dfrac{273}{760} \times \dfrac{5}{8} \times 1{\cdot}293$

$= {\cdot}153$ gm.

$\therefore$ Total weight $= 11{\cdot}73 + {\cdot}153 = 11{\cdot}883$ gm.

4. What is the relative humidity on a day when the temperature of the air is 14° C. and the dew-point is 5° C.? What is the actual vapour pressure on that day?

Given that the max. vapour pressure at 14° C. = 11·9 mm.
" " " " 6° C. = 7·0 mm.
" " " " 4° C. 6·1 mm.

Ans. 50 per cent; 6·55 mm.

5. Find out the relative humidity if 20 litres of air at 18° C. when passed through the drying tubes increase their weight by ·198 gm., given that 15·2 gm. of water vapour is required to saturate a cubic metre of air at 18° C.?

Ans. 65 per cent.

6. The amount of water given out by the lungs and skin may be taken as 426·2 gm. in 12 hours. How many cubic metres of half saturated air at 15° C. and 78 cm. will this quantity of water saturate? *Ans.* 64·9 cubic metres.

7. A quantity of hydrogen collected over water occupies 350 c.c., the temperature being 15° C. and the pressure 747·7 mm. of mercury. Calculate the volume of dry hydrogen at N.T.P.

(The max. pressure of water vapour at 15° C. = 12·7 mm.)

Ans. 320·7 c.c.

8. Find the mass of 5 litres of moist air at 18° C., the dew-point being 5° C. and the barometric height 759·5 mm.

The maximum vapour pressure at 18° C. = 15·4 mm.
" " " 5° C. = 6·5 mm.

Ans. 6·025 gm.

CHAPTER IX

TRANSMISSION OF HEAT

220. Suppose a heap of apples is to be transferred from one end to the other end of a room in which a row of men are standing. There are evidently three methods of doing so. The first method is that each man may come to the heap and take an apple from one end to the other. In this case the carrier himself moves along with the apples. The second method is that the man nearest to the heap may take apples one by one from there, and may hand over to the man next to him, who in turn will do the same thing, and so on. In this way also the apples would reach the other end. The third method is that the apples may be thrown directly from this end to the other end without affecting the men standing there.

Corresponding to each of these three methods there is a method of transmission of heat from one place to another. The heated particle may move itself and go over to the second place, or the heat may be handed over from particle to particle, or the heat may travel itself through the medium without affecting it. These methods are called respectively *Convection*, *Conduction*, and *Radiation*.

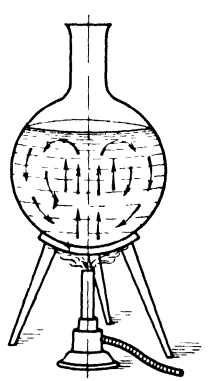


Fig. 31

221. Convection. — Take a round-bottomed flask; fill it partly with water. Throw some pieces of solid colouring matter, say crystals of potassium permanganate, in it. Heat it over a small flame. The water at the bottom, which becomes coloured on account of contact with the colouring matter, gets heated, and hence becomes lighter. It therefore rises upwards through the middle of the flask. At top it gets cooled by coming in contact with cold

water. The cold water being heavier, comes downwards from the top along the sides of the flask to take the place of the hot water, which in turn gets heated and rises upwards. Upward currents of heated water and downward currents of cold water are thus set up. These currents are called *convection currents*. They convey the heat from the hotter to the colder parts of the liquid by transferring bodily the heated particles.

It is evident from the above experiment that this mode of transmission can occur only in liquids and gases, for convection currents cannot be set up in solids. As a matter of fact this is the only important mode of heating fluids. Remember, ***Convection*** *is the transmission of heat from one part of the body to another by the actual motion of the heated particles.*

What has been said above about water is equally true of gases. They are also heated by convection currents. Ventilation is merely an application of the convection currents in gases. The air in the room becomes warmer on account of the fact that the dwelling rooms are always at a higher temperature than the free air outside. This warm air rises upwards and passes out through the ventilators. The cold fresh air comes into the room to take the place of the warm air through the doors and windows. Trade winds, land and sea breezes, are all convection currents in gases.

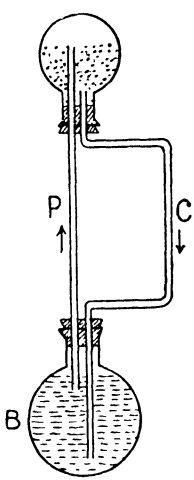


Fig. 32

How the convection currents in liquids are used practically for heating rooms can be illustrated experimentally in the laboratory by the apparatus shown in Fig. 32. When the flask *B* is heated from below, the water begins to circulate in the direction of the arrows. To make the movement visible, colour the water in the upper flask. The flask *B* corresponds in the actual arrangement to the boiler in the basement, and the pipe *C* to the pipe of the radiators in the rooms. The actual arrangement is complicated by the safety valves and overflow pipes, etc.

222. Conduction.—Hold one end of a rod in a Bunsen flame. After a short time the other end becomes too hot to be held. If we touch the rod at different parts we find that the nearer the part is to the end in the flame, the hotter it is. In this case the heated particles have not moved bodily from the

hot end to the cold end, but simply the heat has moved. Such a method of transmission of heat is called conduction.

It is defined as—

***Conduction** is the mode of transmission of heat from particle to particle in the direction of decrease of temperature, the particles not changing their positions.*

It should be noted that the intermediate parts become heated to the intermediate temperatures. This method takes place mostly in metals, as will be shown later on.

Different substances differ enormously in their power to allow heat to pass through them. Roughly the facility with which heat can flow along a body is said to be its conductivity. For instance, an iron rod allows heat to flow along it much more easily than a glass rod, which means that iron is a much better conductor than glass. In order to give a complete and exact definition we must take into consideration other factors as well. Let us consider a cube of a certain substance (Fig. 33). If we heat one of its faces, due to conduction, the opposite face will also become heated. The amount of heat that comes over to this face evidently depends upon the conductivity of the substance. The better the conducting power the more the heat that will pass over to the opposite face. But at the same time we must not lose sight of the fact that the amount of heat crossing over will depend upon the area of the face: the greater the area the greater the amount of heat passing through the cube. It depends also upon the difference of temperature between the two faces: the greater the difference the greater the flow of heat.

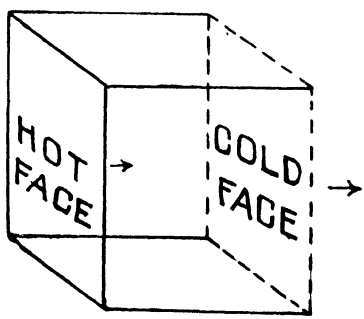


Fig. 33

The thickness also produces effect on the flow of heat: the greater the thickness the less the flow. Moreover, the quantity of heat passing through the cube is proportional to the time; for instance, in two minutes twice the quantity of heat flows as in one minute. Mathematically we can express all the above-said conditions as

$$Q = \frac{KA(\Theta_2 - \Theta_1)t}{r}, \quad . \quad . \quad . \quad . \quad . \quad (a),$$

where Q = the amount of heat that flows over to the opposite face.

A = Area of the face,

Θ_2 = Temperature of the hot face,

Θ_1 = Temperature of the cold face,

t = Time for which heat flows,

r = thickness,

and K = the constant depending upon the nature of the substance.

It is called the coefficient of thermal conductivity, or, more simply, the conductivity.

Formula (*a*) can be written as

$$K = \frac{Qr}{A(\Theta_2 - \Theta_1)t} \quad . \quad . \quad . \quad . \quad . \quad (b).$$

This formula helps us to find out the conductivity of a material. If r be 1 cm., A be 1 sq. cm., $\Theta_2 - \Theta_1 = 1°$ C., and $t = 1$ second, then $K = Q$. This shows that—

***Conductivity** of a substance is equal to the amount of heat that passes in one second through a centimetre cube whose faces are maintained at a difference of temperature of* 1° C.

To find out the value of the constant K or the conductivity of a material we take a thin parallelopiped of the material and let steam play upon one of its faces, the opposite face is kept in touch with ice. The heat that is transmitted is used up in melting the ice. From the weight of the water obtained we can calculate the amount of heat that has passed through the parallelopiped. Since we know $\Theta_2 - \Theta_1$, A, t, r, and Q, we can find out K. It should be noted in this case that we have supposed that the parallelopiped is not losing heat on other faces, that is to say, there is no lateral loss of heat. But in actual cases it always does take place. The method sketched above helps us to understand what we mean by conductivity and how to determine it roughly.

223. Flow of Heat along a Bar. —Take a long bar of iron. Let holes be drilled in it at regular intervals. Fill them up with mercury. Heat its one end to a constant temperature, say by keeping it immersed in a bath of molten lead. Place thermometers in the holes; the thermometer nearest the heated end begins to rise and is followed in succession by others. If heating is continued for a sufficiently long time,* all thermometers cease to rise; of course the temperature recorded by the nearest thermometer is the greatest. If the bar be long enough the thermometer at the other end will hardly be affected.

* It may be some hours.

The readings of the various thermometers will correspond to the curve drawn in Fig. 34. In the steady state, although the thermometers do not rise, it must not be supposed that no heat is being conducted from the hot end. As a matter of fact the bar as a whole is above the temperature of the atmosphere and is therefore losing heat by radiation as well as by

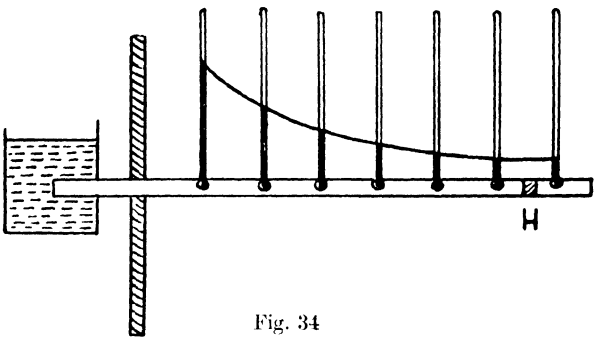


Fig. 34

convection. All that we mean by the steady state is that a part like *H* loses as much heat on account of radiation and convection at the lateral surface and conduction to the colder part as the amount of heat that it receives at the hotter face.

224. Let us analyse a little more thoroughly this flow of heat along the bar. We shall suppose for the sake of simplicity that the bar is divided into equal parts like *H*. The end in the molten lead rapidly attains the temperature of the source. The next part to this end receives heat by conduction, absorbs part of this heat to increase its own temperature, loses a small portion on its free exposed surface due to radiation and convection, and passes on the rest to the adjacent part. The same thing happens with this part, and so on. When this state of affairs has gone on for a long time, ultimately a stage is reached when the part ceases to absorb heat for raising its own temperature, but loses the whole of the heat that it has received either by radiation, etc., or by giving over to the adjacent part. *This state when there is no more absorption is called the* **stationary state.**

The stage previous to this steady or stationary state is called the **variable state.**

Let us now see what is the effect of conductivity on the

flow of heat along the bar. If the material of the bar is a good conductor it will allow heat to flow along it easily, so that the temperature of a part like *H* will be higher than what it will be when the material is a bad conductor, for in the latter case much of the heat will be lost before it reaches this part. But it should be noted that the rise of temperature of the part *H* depends not only upon the conductivity, but also upon the specific heat. For let us see what will be the case when the specific heat is different, other things being equal. Evidently the rise of temperature in that rod whose specific heat is low will be higher for the absorption of the same quantity of heat than in the other case. But when the steady state has been reached, *since there is no rise of temperature, specific heat does not produce any effect.* So that we see that the remark that we made above, that the temperature of the part *H* will be higher in the case of a good conductor than in the case of a bad conductor, is meant to be applied in the steady state. It is found that the other conditions being the same, the length which we have to travel along a bar to attain a certain fall of temperature is greater in the case of a good conductor than in the case of a bad conductor.

It is found theoretically that *the distance which we have to travel along different rods to attain the same fall of temperature is proportional to the square root of the conductivity of the substance.* Mathematically we can express it as

$$L \propto \sqrt{K},$$

or

$$L^2 = bK,$$

where b is a constant depending upon the conditions of the experiment.

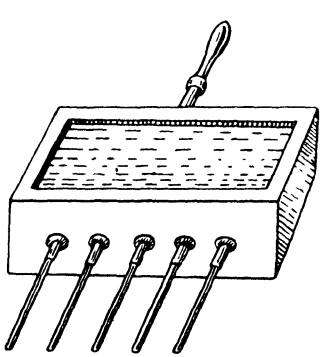


Fig. 35

This fact forms the basis of an experiment for comparing the conductivities of different substances.

225. Ingenhousz's Experiment.—Several rods of exactly the same dimensions but of different materials are fixed into the side of a rectangular metallic trough (Fig. 35). The trough is filled with water. The rods are coated with wax. The water is made to boil. The wax begins to melt on each rod. When no more wax melts—that is to say, when the steady state is

reached—the length of the melted portion is measured off in each case. The conductivities are proportional to the squares of these lengths.

Note, the quickness with which the wax melts is no sure guide to conductivity, for, as has been said already, the quickness depends also upon the specific heat. Thus we see that we must make observations only when the steady state has been reached.

As a result of experimental determinations it is found that each substance possesses different conductivity. That some are good and some are bad conductors is known to everybody. For instance, we know from our everyday experience that wood is a bad conductor of heat. If we put a silver spoon in a hot cup of tea, we find it becomes much hotter than the china cup itself.

226. The Davy Safety Lamp.—A mixture of air and an inflammable gas will not burn unless it is heated to a certain temperature known as *ignition temperature*. If somehow or other temperature of the mixture be kept below this ignition temperature, it will not take fire. Davy made use of the high conductivity of copper in the construction of his safety lamp. The principle underlying the construction of this lamp will be clear from the following: To see it experimentally, place a piece of wire-gauze a little above the tube of a Bunsen burner. Turn the gas on and set fire to the gas above the gauze. It will be noticed that the flame does not pass downwards, because the temperature of the mixture of the gas and air below the gauze does not reach the ignition point. The heat of the burning gas is conducted away by the gauze. *This property of wire-gauze of preventing the flame passing through it to the other side is the basis of Davy's safety lamp.* The flame is surrounded completely by wire-gauze of copper; this enables the lamp to burn quietly, even when the atmosphere contains an explosive gas. For the latter penetrates the gauze and burns inside with a blue flame, which is an indication of danger to the miner. But as the flame cannot pass through the gauze, the explosion which would otherwise have occurred is averted.

227. Conduction in Liquids.—Liquids, with the exception of mercury, are bad conductors of heat, and in order to study their conductivity experimentally we must avoid the complications arising due to (*a*) the convection currents and (*b*) the disturbing effects of the walls of the vessel in which they are enclosed. In order to get over the first difficulty, liquids are

heated from above, and to remove the second difficulty a vessel made up of a bad conducting material such as glass or wood is used.

228. Despretz's Method.—In order to compare the conductivities of liquids the same principle which is the basis of the Ingenhousz's experiment is made use of. Here we take a cylindrical wooden vessel B (Fig. 36), furnished on one side with a row of holes through which thermometers pass. At the top a metallic vessel A is placed. The wooden vessel is filled with the liquid whose conductivity we want to examine. Hot water at 100° C. is poured over into the copper vessel A. It is renewed after every five minutes. It is found experimentally that heat travels slowly downwards. When the steady state is reached, *i.e.* when the temperatures of the thermometers become constant (which might take as long a time as 40 hours) the temperatures indicated by various thermometers are noted.

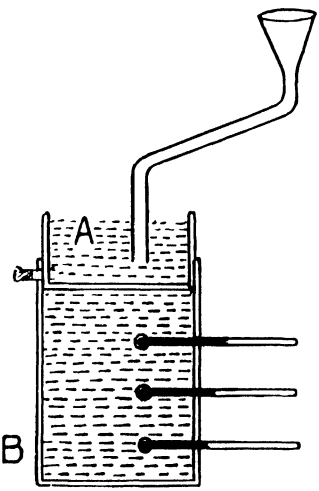


Fig. 36

Fill the wooden vessel with various liquids turn by turn, the hot water in A being in each case at the same temperature. The respective conductivities are proportional to the squares of the distances (downwards from the bottom of the vessel A) which give the same fall of temperature.

229. Conduction in Gases.—The experimental determination of the conductivity of gases is far more difficult than that of the liquids. Here we have not only to avoid the disturbance due to the convection currents and the effect of the vessel, but also the effect of radiation. Consequently the experiment must be performed with great care. We shall here explain only the outlines of the experiment which is performed to determine the conductivity of gases.

The method is to find out the rate of cooling ot a thermometer bulb first in a glass vessel exhausted of air and then in the same vessel when it is filled with gas. The first observation

tells us the error due to radiation. In order to remove the errors due to convection the gas is used at a very low pressure.

As a result of experiments it is found that the gases are very poor conductors of heat. To give an idea of their conductivity it may be mentioned that the conductivity of air is one-thousandth of copper.

That air is a very bad conductor is known almost to every one of us. The warmth of woollen clothing is mostly due to the presence of the air in the minute spaces in the cloth. People in Arctic regions make houses of ice to live in. In order to save themselves from the cold outside they make the houses double-walled, enclosing air between the walls. The air being a bad conductor, does not allow the cold from outside to go in.

Table of Conductivities

Substance.	K.	Substance.	K.
Silver	1·006	Platinum	0·166
Copper	0·916	Water	0·0014
Zinc	0·258	Wood	0·0006
Iron	0·158	Air	·000057

230. Radiation.—It is a matter of daily experience that we feel a sensation of warmth when we expose ourselves to the sun's rays. That the sensation is not due to the temperature of the air is clear from the fact that as soon as we place a screen between the sun and our body we do not feel the warmth. Hence evidently this sensation of warmth is due to the heat energy that comes from the sun to our planet, which is at a distance of 90,000,000 miles. Earth's atmosphere becomes thinner and thinner as we ascend upwards from its surface, ceasing altogether at a height of 200 to 500 miles. Above this there is merely empty space. Evidently we cannot have convection and conduction there, for there is no material medium. So we see that there is a third method of transmission of heat which requires no material medium for its propagation. This point was more definitely proved by Davy, who enclosed a thermometer, whose bulb was painted with lampblack, in a vessel at a distance from a platinum wire which could be electrically heated. The vessel was exhausted of air, and the wire was heated. The thermometer indicated an appreciable rise of temperature, showing thereby that the existence of material medium is not necessary for the transmission of heat.

Moreover, it is seen that the medium is not affected by this

mode of transmission of heat, for on screening ourselves we do not feel any warmth, which shows that the air around us is not sensibly affected.

The heat radiated travels in straight lines. This is shown by the simple fact that when we want to screen ourselves from the heat we place a small obstacle between the source and our body. If heat were not to travel in straight lines but in curves the heat rays would bend round the obstacle and fall upon our body.

Thus the third mode of propagation of heat, which is called radiation, may be defined as follows :

***Radiation** is the process by which heat is transmitted from one point to another in straight lines without heating the medium.*

It should be noted that all substances do not allow the heat radiation to pass through them with equal facility. Some allow the radiation to pass through them more readily than others.

Substances which allow heat radiation to pass through them without themselves being heated are called **diathermanous;** and those which absorb the radiations are called **athermanous.** No material substance, however, is *perfectly* diathermanous. Air is only approximately so.

Liquids and solids are mostly athermanous. It is not necessary that a body which does not allow light to pass through it must cut off heat rays as well. The solution of iodine in CS_2 is perfectly opaque to light rays, but is diathermanous to heat rays. Before we consider the properties of heat radiation we shall explain the method of detecting it.

If sunlight or the radiation from a piece of heated metal be allowed to fall on the bulb of an ordinary mercurial thermometer, it indicates a rise of temperature. If the bulb is coated with black paint* the rise of temperature is much more marked. Hence such a thermometer can be used to detect the heat radiation; but in actual practice we use the apparatus explained below.

231. Differential Air Thermoscope.— It consists of a glass tube bent twice at right angles terminating in equal bulbs containing air (Fig. 37). The bend of the tube contains a coloured liquid (generally coloured sulphuric acid is used, for it is neither volatile nor very dense and viscous). The quantity of the air in the bulbs is so arranged that when the bulbs are at the same temperature the liquid in the two arms stands at the same level.

* The black paint is made by mixing lampblack with alcohol in which a small amount of shellac has been dissolved.

Such an apparatus is very sensitive to the difference of temperature. If there be even a very slight difference it will be indicated by the difference in the level of the liquid in the arms. For the air in the bulb on which heat radiation is incident expands and causes the liquid to go down and move over to the other arm.

If one bulb be coated with lampblack the apparatus becomes still more sensitive.

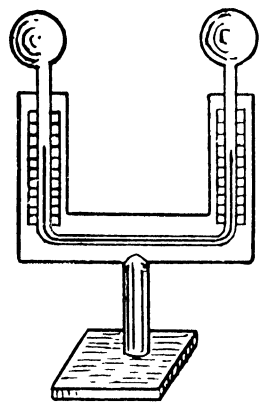


Fig. 37

232. Properties of Heat Radiation.

They should be studied after reading Light.

(1) *Thermal or heat radiation is transmitted through vacuum.*

We have already seen in § 230, how Davy proved it experimentally.

(2) *Thermal radiation travels in straight lines.*

To prove it experimentally, take two wooden screens, each one of them having a small hole. Arrange them one behind the other in such a way that the holes are at the same level.

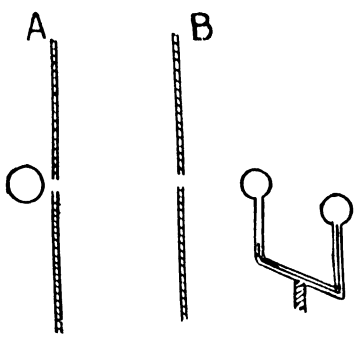


Fig. 38

Place in front of one (say *A*) a hot iron ball and behind the other (*B*) the black bulb of the differential thermoscope. The liquid moves at once. Now disturb the arrangement so that the holes are not at the same level, and note that the liquid column remains undisturbed.

(3) *Thermal radiation obeys inverse square law.*

It should be noted that heat is radiated in all directions and in straight lines. Keeping this in view, it is very easy to show that thermal radiation obeys the inverse square law. Suppose *A* is a hot ball of iron. It is emitting heat in all directions.

The radiation which falls on spherical surface *B* would fall after

some time on the surface C, whose radius is double and surface four times that of B. Thus we see that the amount of heat radiation falling on 1 sq. cm. becomes $\frac{1}{4}$th at double the distance. Similarly we can show that it will become $\frac{1}{9}$ at three times the distance. Thus we see that the heat radiation obeys the inverse square law.

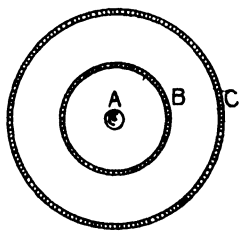


Fig. 39

(4) *Thermal radiation gets reflected from polished metallic surfaces obeying the laws of reflection.*

Two tubes hinged together are placed, as shown in Fig. 40, on a polished metallic plate M. At A a hot ball is placed, and at B the black bulb of the differential air thermoscope. It is found that the column of liquid remains unaffected so long as the tubes are not equally inclined. But as soon as the angles are equal the liquid moves down, showing

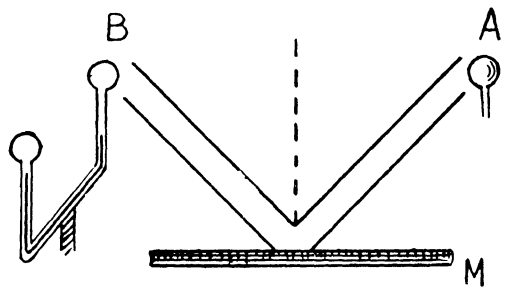


Fig. 40

thereby that the heat rays are reflected in such a manner that the angle of incidence is equal to the angle of reflection.

To show in a more striking manner that heat rays suffer reflection at metallic surfaces, take two concave mirrors, M_1 and M_2, mounted on stands. Arrange them at a distance of about five yards from each other. Place an iron ball A heated to redness at the principal focus of M_1 (Fig. 41). Place a piece of phosphorus at the adjustable stand, and adjust the height of the stand so that the phosphorus is a little below the principal focus of the other mirror M_2. The phosphorus is not affected. Raise the stand, and it will be seen that as soon as the phosphorus is at the principal focus of the mirror M_2, it at once takes fire. Let us see what happens. The heat radiations fall on M_1, get

reflected as a parallel beam, fall on M_2, and come to focus at the principal focus.

Very powerful effects may be produced by concentrating the sun's rays at a point with the help of a concave mirror ; with a mirror of 6 ft. aperture copper and silver can be melted.

(5) *Thermal radiation can be refracted.*

The easiest method of proving it is to hold a convex lens in the sun and to focus the sun's rays on a piece of black cloth. After a short time the cloth will catch fire, showing thereby that the thermal radiations can be refracted.

(6) *Thermal radiation does not affect the medium through which it passes.*

In the preceding experiment we have seen that at the principal focus of a lens a piece of cloth can be burnt, but if the

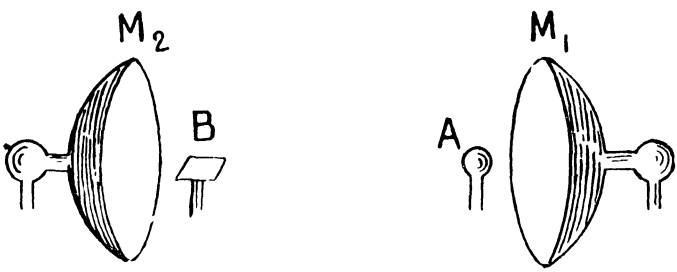


Fig. 41

lens itself be touched, it will be found to be practically unaffected. The slight rise in temperature is due to the absorption of the heat by glass, as it is not a perfect diathermanous.

(7) *Thermal radiation travels with the speed of light.*

It has been proved experimentally that so long as the total eclipse lasts no heat radiation reaches the earth, and that as soon as light comes to the earth the heat radiation comes along with the light, showing thereby that heat radiation travels with the same speed as light.

233. Reflecting and Absorbing Power.—When heat falls upon a body a portion of it is reflected ; the rest is used up in raising the temperature of the body. The proportion between the quantities reflected and absorbed is different with different substances. There are some which reflect more than they absorb ; they are said to be *good reflectors* (and *bad absorbers*),

while there are others which absorb more than they reflect; they are said to be *good absorbers* (and *bad reflectors*). To study the reflecting power experimentally let the rays emitted from a red-hot iron ball fall upon the mirror M. They come to focus at P. Just in front of P, towards M, interpose a plate of the substance whose reflecting or absorbing power is to be determined. The rays after reflection will come to focus at Q. Place there the black bulb of the thermoscope and note the difference produced in the level of the liquids. Keeping every other thing the same, use plates of different materials. The

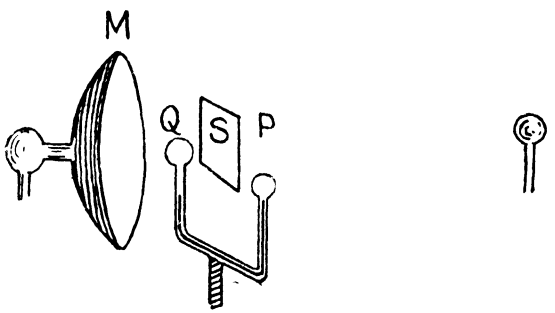


Fig. 42

substance which causes the greatest difference in level is the best reflector, and therefore the worst absorber.

It is found as a result of experiment that polished brass plate produces the greatest difference, whereas plates coated with lampblack and moisture do not practically affect the level, showing thereby that polished brass is the best reflector and therefore the worst absorber, and lampblack and water the worst reflectors and hence the best absorbers.

234. The Emissive or Radiating Power.—It is found that all bodies emit heat more or less which they contain. The quantity radiated per second by a particular body depends upon the nature and the temperature in addition to the area of the surface and the temperature of the atmosphere, etc. In order to compare the radiating powers of various substances we generally take unit area and difference of temperature between the body and its surroundings as 1° C. *The quantity of heat which a unit surface of a body emits in one second when the difference in temperature between the body and the surroundings is* 1° *C. is called the* ***radiating power*** *of the substance.*

To compare the radiating power of various bodies Leslie proceeded as follows:

He took a cube whose one side was covered with polished tin, the second with a white paper, the third with a glass plate, and the fourth with lampblack. It was placed at a distance from a concave mirror M (Fig. 43), at whose focus was placed the black bulb of the thermoscope. Boiling water was poured into the cube. The black side was first turned towards the reflector and the column of liquid at once went down considerably, indicating a large amount of increase of temperature. Next the

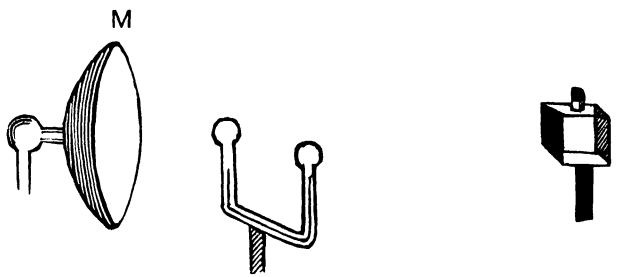


Fig. 43

paper side was turned; the increase of temperature was found to be less than in the first case. The glass side came next to the paper side in producing the rise of temperature. The polished tin surface produced the least rise of temperature.

Leslie concluded from such like experiments that lampblack is the best radiator, and polished metallic surfaces the worst radiators.

But we have already seen that lampblack is the best absorber, and the polished metallic surfaces are worst absorbers. Combining these two results we come to the conclusion that **good absorbers are good radiators.**

A very beautiful experiment can be arranged to show the relation between radiating and absorbing power.

In a differential thermoscope the glass bulbs are replaced by cylindrical metallic vessels having flat sides. Between them is placed a cylindrical metal canister which can be filled with hot water. The flat sides of this vessel are exactly equal to the flat sides of B and C. The dark faces are coated with lampblack, and the other faces are polished. We see from Fig. 44 that the black surface is opposite to the polished surface, and the polished surface is opposite to the black surface.

When A is filled with hot water, its polished face radiates towards the black face and its black face radiates towards the polished surface of the thermoscope.

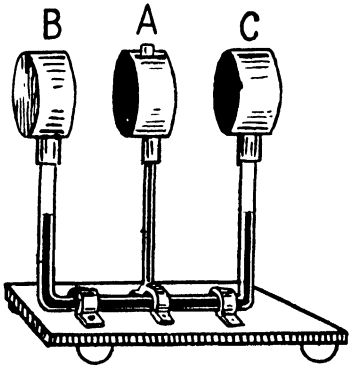


Fig. 44

With this arrangement it is found that the column of the liquid does not move, showing thereby that the great radiating power of the black face is balanced by the weak absorbing power of the polished face, and so on. If, however, the canister be turned round, the liquid moves at once, for now good absorber is opposite to good radiator and bad absorber opposite to bad radiator. This experiment shows clearly that good radiators are good absorbers.

235. Dewar Flask.—The fact that the radiation from a surface depends upon its nature, being greatest from the black surface and least from the polished surface, is made use of in the construction of a Dewar flask, in which a liquid can be kept either hot or cold without any fall or rise of temperature for a considerable period. The Dewar flask is ordinarily called the **thermos-flask.**

It is essentially a glass vessel with double walls. The space between the two walls is a vacuum, and the inner surface of the outer wall and the outer surface of the inner wall are silvered. The vacuum does not allow the conduction to take place, and the polished surfaces minimise the effects due to radiation. To make the effect of radiation still less, the glass vessel is further surrounded by a polished cover.

236. Prevost's Theory of Exchanges.—Consider a ball at a high temperature placed in an enclosure. The ball constantly loses heat on account of radiation and the enclosure gains heat, till both are at the same temperature. Now, when the temperature remains constant, we think that the ball is not radiating any energy. But surround this enclosure by another and fill the space between the two by ice, the ball will again begin to radiate heat energy, and its temperature will fall. To explain how the presence of a cold body which has no action directly on the ball makes it emit radiation,

and further, how the hot enclosure stops the body from losing heat, which it begins to lose again as soon as the temperature of the enclosure falls, *we assume that a body radiates heat energy at all times whatever be its temperature.* A cold body of course radiates less heat than a hot body.

On this theory it is very easy to explain the heat radiation from a body. It is supposed that a hot body falls in temperature because it gives out more energy than it receives from the cold bodies around it. When two bodies have the same temperature, radiation emitted by one is equal in amount to the radiation that it receives from the other. A cold body, on the other hand, is receiving more energy than it is giving out to others, and hence its temperature rises.

This theory of exchanges of heat radiation is known as **Prevost's theory of exchanges.**

EXERCISES

1. The thermal conductivity of felt is represented by ·000087. Calculate the heat lost in one hour through felt 1 cm. thick and 100 sq. cm. in area when its opposite faces are kept at a difference of 30° C.

We know that $K = \dfrac{Q \cdot r}{A(\Theta_2 - \Theta_1)t}$.

Substituting the various values we get

$$\cdot 000087 = \frac{Q \cdot 1}{100 \times 30 \times 60 \times 60}.$$

Therefore $$Q = \cdot 000087 \times 3600 \times 3000$$
$$= 8{\cdot}7 \times 108 = 993{\cdot}6 \text{ calories}$$

2. In an experiment it was found that the underground temperature in a sandstone district increased 1° C. for 27 metres descent. The amount of heat lost per hour per square kilometre of the earth's surface was $3{\cdot}6 \times 10^7$ units. What is the conductivity of the sandstone ?

$$K = \frac{Q \cdot r}{A\Theta(_2 - \Theta_1)t}$$
$$= \frac{3{\cdot}6 \times 10^7 \times 27 \times 100}{100000 \times 100000 \times 1 \times 60 \times 60}$$
$$= \frac{3{\cdot}6 \times 27}{36000} = \cdot 0027 \text{ C.G.S. units.}$$

3. A waterfall is 50 metres high. The water falls below on a perfectly non-conducting surface. How much warmer will it be after the fall ?

Let us see how much work is done by gravity on one gm. of water. The potential energy at the top will be *mgh*, or $1 \times 981 \times 50 \times 100$ ergs.

But we know that $4{\cdot}2 \times 10^7$ ergs of work must be expended in producing 1 calorie of heat, hence

$981 \times 5 \times 10^3$ will produce $\dfrac{981 \times 5 \times 10^3}{4{\cdot}2 \times 10^7}$,

or $\dfrac{981 \times 5}{4{\cdot}2 \times 10^4} = \cdot 117$ calorie (approximately).

Since the ground underneath is perfectly non-conducting, it does not take any heat from the water. Therefore every gram rises in temperature through ·117 C., the sp. heat being one.

4. The opposite faces of a metal plate 20 cm. thick are at a difference of temperature of 100° C., the area of the plate being 2 square metres. If the conductivity be ·2, find out how many units of heat will flow across the plate per second.

Ans. 20,000 units.

5. Calculate the quantity of heat which flows per hour through a window pane of glass 7 mm. thick, 1 sq. metre in area, the temperature difference between the two sides of the glass pane being 32° C. Take the thermal conductivity of glass to be ·0005. *Ans.* 822,857 calories.

6. Find out the conductivity of an iron plate 8 cm. thick and 1 sq. metre in area, the opposite faces of which are kept at a difference of temperature of 80° C., when 960,000 units of heat flow across the plate per minute. *Ans.* ·16.

7. The two faces of a metal plate 100 sq. cm. in area are kept at a temperature difference of 100° C. It is found that 18,000 units of heat pass across the plate in one hour. If the conductivity be ·02, what will be the thickness of the plate ?

Ans. 40 cm.

8. Find out the thermal conductivity of an iron plate 30 cm. long, 40 cm. broad, and 20 cm. thick, which is exposed on one side to steam at 100° C. and whose opposite side is in contact with ice, if 810 gm. of ice are melted in one minute.

Ans. ·18.

9. A raindrop falls on the ground from a height of 210 metres. What will be the increase in the temperature during fall ? Suppose that $4{\cdot}2 \times 10^7$ ergs produce 1 calorie of heat.

Ans. ·491° C. (approx.).

10. A block of ice at 0° C. is dropped into a well of water (the water also being at 0° C.) from a height. Find out what the height should be, so that $\frac{1}{100}$ of it may be melted. Suppose that $4{\cdot}2 \times 10^7$ ergs produce 1 calorie of heat.

Ans. 341·6 metres.

CHAPTER X

MECHANICAL EQUIVALENT OF HEAT AND STEAM ENGINE

237. In what we have studied up to this time we have ignored the question of the nature of heat. In describing the phenomena and in stating the laws governing them it has not been necessary for us to know anything more about heat than that it is something which is associated with the sensation of warmth (*i.e.* temperature), and which can be measured by the calorimeter. Now we shall pass on to the conception of the nature of heat. We shall consider two theories attempting to explain the nature of heat. One of them is the caloric theory, which is as old as the ancient Greeks, and the other is the modern one. It will not be possible to go into details as regards these theories; we shall have to content ourselves with their outlines merely.

238. Caloric Theory.—Heat, according to this theory, is considered to be an elastic, imponderable fluid called caloric. The particles of this fluid are supposed to repel each other and attract the particles of ordinary matter. Moreover, it is supposed to pervade all bodies. Further, it is considered to be indestructible and uncreatable. With these assumptions we can explain quite satisfactorily the ordinary phenomena of heat, as, for instance, the flow of heat from a body at a higher temperature to another at a lower temperature, or radiation of heat from the flames, or the expansion of bodies when heated. If we suppose further that the attraction of caloric for different kinds of matter is different, we can explain easily the fact that equal amounts of different substances require different amounts of heat to be heated through the same range of temperature. With other modifications, the phenomena of latent heat, the expansion or contraction on solidification, can be explained.

But the production of heat by friction cannot be explained

on this hypothesis. It was Lord Bacon who first called attention to this fact and made it one of the strongest points against this hypothesis.

In 1798 Count Rumford, while engaged in boring cannon at Munich, was struck by the considerable rise of temperature of the cannon, especially of the chips cut out by the boring tool. By immersing the cannon and the blunt boring tool under water he actually boiled water.

The upholders of the caloric theory explained this rise of temperature by supposing that the thermal capacity of a body is less in the form of a powder than in the form of a solid block. Rumford modified his experiment in such a way that the amount of metal abraded was very little, and even then he found a very considerable increase in the temperature of the block of metal. With Rumford the idea was that anything which could be produced without limit could not be material in nature. The fatal blow to this theory, however, was served by Davy, who showed that two pieces of ice can be melted by rubbing together. Now surely, since water contains a far greater amount of heat than ice, liquefaction cannot take place unless there is *generation of new heat*. According to caloric theory, the generation of new heat was impossible, and hence, Davy put forward the hypothesis that it was the work done against friction which was converted into heat.

Before we go further we shall study first the connection between the amount of work done and the heat produced.

239. Heat and Mechanical Work.— Dr. Joule of Manchester between 1843 and 1848 carried on a classic series of experiments and showed that a definite quantity of heat was always produced by a definite amount of mechanical work. It was he who first measured the amount of work which must be done to produce a given quantity of heat. His apparatus is shown diagrammatically in Fig. 45. It consists essentially of a copper calorimeter C to the sides of which the vanes V, V. . . . are soldered, and a set of paddles P, P . . . . carried by a spindle which is free to revolve about the axis ED.

The calorimeter contains a known mass of water, whose temperature is read with a very sensitive thermometer. When the weight T attached to the end of the cord wound round the drum B is allowed to fall, the spindle turns and the paddles move the water. The motion is stopped at once by the vanes, and is converted into heat. When the weight reaches the floor the drum B is disconnected from the spindle by unscrewing the pin S, and the cord is again wound round

it. The drum is fixed again upon the spindle without moving it, and the experiment is repeated again and again, till the rise of temperature is appreciable.

Without going into the corrections and precautions we shall explain here simply the method by which the calculation is made. Suppose the weight T lb. falls each time through a

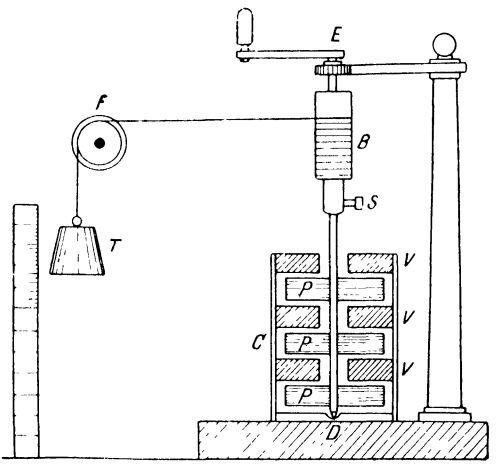


Fig. 45

vertical height h ft. and that the experiment is repeated n times. Evidently the total work done is nTh foot-pounds, call it W.

If the thermal capacity of the calorimeter, vanes, water, spindle and the paddles be M and the rise in temperature be $\Theta°$ C., the heat produced $= M\Theta$: call it H.

It is found that the ratio $\frac{W}{H}$ is always the same, *i.e.* $\frac{W}{H} =$ a constant.

Joule repeated the experiment a large number of times, sometimes using water in the calorimeter, sometimes mercury, and so on, but he found that the ratio in every case was the same. This means, in other words, that to produce one unit of heat the same amount of mechanical work must always be done. This amount of work is equal to the value of the constant. Joule found from his experiments that 772 foot-pounds of work must be done to produce heat enough to raise the temperature of 1 lb. of water through 1° Fahrenheit. More

recent determinations show that 778 foot-pounds of work must be done. In the C.G.S. units it is found that $4{\cdot}2 \times 10^7$ ergs of work must be done to produce 1 calorie of heat. This constant is called the **Joule's mechanical equivalent of heat.** It is generally denoted by J.

So we see that the constant $= J$. Hence we can write the above result as

$$W = JH.$$

The law that we have arrived at, that **the work done is proportional to the amount of heat produced,** is quite a general one; it is immaterial whether the work is done one way or the other.

On caloric theory conversion of heat into mechanical work is impossible, hence the idea that heat is a fluid must be given up once for all.

240. Modern Theory.—According to this theory the molecules of all bodies are in a state of continued agitation. In the case of solids they are supposed to be vibrating to and fro about their mean positions. The velocity with which a molecule vibrates depends upon its position in the body, for a molecule which is near the centre of a body comes into collision with neighbouring molecules more often than a molecule which is near the surface. But we can suppose that there is an average velocity for the molecules of a body at a particular temperature. Since each molecule is vibrating to and fro, it must have its own kinetic energy. The average velocity of a molecule of a body, and hence its kinetic energy, depends upon the temperature. With rise of temperature the velocity as well as the amplitude becomes greater. As kinetic energy depends upon the velocity, hence the kinetic energy of a molecule becomes greater. This shows that the molecules of a hotter body possess more energy than the molecules of a colder body. This theory, then, leads us to the conclusion that **heat is a form of energy.** On this theory there is no difficulty in explaining the generation of heat when two bodies are rubbed together. The work that is done against friction is transformed into molecular energy.

It will be well to point out here that the motion of the molecules is within so small a range that even a microscope of the highest power is unable to detect it.

Naturally a question will arise in the mind of the student: if heat is simply a form of energy due to the molecular motion, how is it propagated from a hot body to a cold body at a

distance? Before we answer this question, let us consider first a parallel case. When we move our hand up and down in water, waves are set up in it, and they begin to travel out from the place of disturbance. A straw lying at a distance from our hand begins to move up and down as soon as the waves reach there. In other words, we have transferred some energy from the hand to the straw at a distance. Just similar is the case with the propagation of heat. The molecules when they vibrate, set up the waves which are propagated from the hot body to the cold body. As soon as the waves fall upon the cold body, they are absorbed there, increasing its molecular energy and therefore raising its temperature. Just as we require water to convey the waves from our hand to the straw, similarly in the case of heat we require some medium to convey the wave disturbance from the hot body to the cold body. The scientists suppose that this whole universe is permeated with a hypothetical substance called **ether,** and it is this medium which allows the propagation of heat from one place to another.

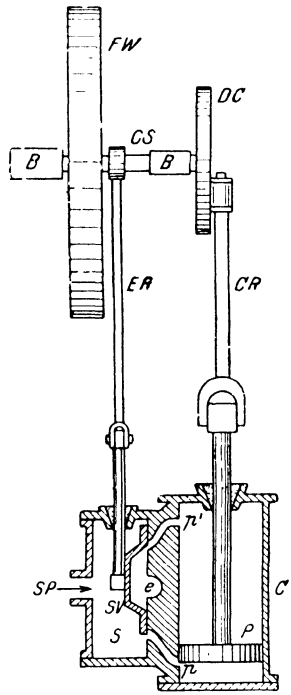
241. Principle of a Steam Engine.—A steam engine is a machine for converting heat energy obtained by the combustion of some fuel into mechanical work, steam being used for effecting the transformation. Though steam engines appear to have many complicated parts, the principle constituting the engine proper is very simple. Section of a simple steam engine is given in Fig. 46.

It is merely a cylinder C traversed by a steam-tight piston P, which divides it into two parts, upper and lower chamber. SP is a pipe which communicates with the boiler. Through this pipe the live steam comes from the boiler at a very high pressure (it may be as much as 10 atmospheres) into the steam chest S. From the steam chest the steam passes into the cylinder through the steam port p or p', whichever of them is open. Let us suppose that the piston, as shown in the figure, is just beginning the upward stroke, and the sliding valve has just begun to open the lower steam port p admitting steam into the cylinder. As the piston P moves up, the valve opens the port to its full extent, and the steam present above the piston is forced out through p' into the exhaust pipe e, which is called exhaust port. Moreover, the upward movement of the piston causes the flywheel FW to rotate. As the wheel rotates, it actuates by means of suitable connecting rods the sliding valve SV to move down and close the port p at an early stage of the upward stroke of the piston (it may be as much as $\frac{1}{4}$ or $\frac{1}{3}$),

and thereby cut off the supply of the steam to the cylinder. But since the steam has enormous elastic force, it *expands* and becomes cooler thereby. In expanding it pushes the piston through the rest of the stroke. When the piston is at the top the upper port p' is opened by the sliding valve, admitting the steam above the piston which pushes the piston down. When the piston begins to move down the dead steam from below is driven out through e. Thus we see that the steam acts on the two sides of the movable piston, alternately working it upwards and downwards in a cylinder.

Twice in each revolution the top of the connecting rod (CR) is at the same level as the crank-shaft (CS). At these positions, which are called the "Dead-centres," the piston exerts no turning effect on the shaft. In order to enable the crank-shaft to rotate smoothly without a jerking motion past these points, the **fly-wheel** is fixed to the shaft, the large inertia of which makes the motion quite regular.

It may be of interest to note that the steam engines are of two kinds, condensing and non-condensing. A condensing engine is one that exhausts its dead steam into a condenser for the purpose of being returned as hot water to the boiler to be again converted into steam. A non-condensing engine is one that allows the steam to escape altogether after it has passed once through the cylinder.



C = Cylinder
P = Steam-tight piston
S = Steam chest
SP = Steam pipe
SV = Slide valve
CR = Connecting rod
DC = Disc crank
B = Bearing
CS = Crank shaft
FW = Fly wheel
p and p' = Steam ports
e = Exhaust port
ER = Eccentric rod

Fig. 46

242. Work done by the Steam in an Engine.—Consider the motion of the piston of an engine. Evidently when it goes

up and down some work is done against the pressure of the steam. To calculate this work let us suppose the piston just begins its upward stroke. The pressure which the steam exerts on the lower side of the piston instead of remaining constant falls off continuously * as the piston moves up. Further, there is some pressure on the upper side of the piston as well, due to the pressure of the dead steam. These facts do not allow us to proceed in a simple manner to find out the work done by the steam. To simplify the calculations it is usual to suppose that there exists an average effective pressure on the piston acting in the upward direction during the upstroke, and in the downward direction during the downstroke. With this simplifying assumption, work which is done by the steam in an engine, or in other words, the horse power of an engine, is represented by the formula :

$$\frac{PAln}{33{,}000},$$

where P = the average effective pressure of steam *in lbs. per square inch*,
A = the area of the piston in *sq. inches*,
l = the length of the stroke measured one way *in feet*;
n = the number of *single* strokes per minute.

Example.—A steam engine has a mean effective pressure of 50 lbs. per sq. inch, the length of the stroke is six inches, the diameter of the piston is 5 inches, and the engine makes 500 revolutions (or 1000 single strokes) per minute. What is the H.P. of the engine ?

Using the relation given above, we get

$$50 \times \frac{3\,142 \times 25}{4} \times \tfrac{1}{2} \times 1000 \Big/ 33{,}000 = 14\,87.$$

* For as said above the steam is cut off at an early stage of the stroke, and as the steam expands the pressure of gas goes on falling.

PART III

SOUND

CHAPTER I

WAVE-MOTION

243. When a piece of stone is dropped into a pond of water ripples travel outwards from the point where it strikes the surface. To a cursory observer it seems that water is travelling out from that point, but actually it is not the case. If a piece of cork be thrown at the surface of the water it is found that when the waves reach it, it simply moves *up and down*, and this movement ceases when the waves have passed away. The position of the cork with respect to the centre of the disturbed area remains the same. This shows that it is not the water but the *disturbance* which travels outwards. This is an example of what is called **wave-motion.** It can be defined as *a form of disturbance which travels through a medium and is due to the repeated periodic motion of the parts of the medium about their mean positions, the motion being handed on from one part to the next.*

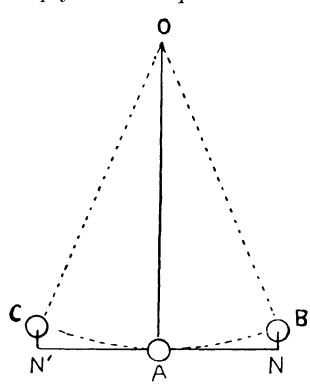


Fig. 1

Before dealing with wave-motion let us first study briefly the vibratory motion or the repeated periodic motion of a single particle. It is best done with the help of a simple pendulum (Fig. 1). A represents the bob, and OA the string. Pull the bob to one side, to B say, and let it go; it will swing

to nearly an equal distance on the other side, and will begin to oscillate between these two extreme positions (B and C).

The number of vibrations completed by the bob in one second is called the *Frequency.*

The fraction of the whole period which has elapsed since the body last passed through the mean position in the positive direction is called the ***Phase***. Suppose, for instance, we fix upon AB as the positive direction. The vibration will be completed when the bob starts from A, goes to B, comes back to A, goes on the other side to C, and once again comes back to A. The phase of the body when it is at B is $\frac{1}{4}$, or when it is at C is $\frac{3}{4}$. The phase may be expressed as a fraction of the time-period or as an angle which varies directly as the time. In simple cases with which we shall deal, it will be convenient to remember the phase as the fraction of the time-period.

Note the velocity of the bob at the extreme position (*i.e.* at B or C) is zero, and at the mean position (*i.e.* at A) it is maximum (see § 61). What holds good in connection with the motion of the pendulum also holds good in the case of the repeated or periodic motion of a single particle.

Before proceeding any further the student would do well to read carefully the motion of a pendulum explained in §§ 61 and 62.

244. Keeping these preliminary remarks about the periodic motion of a particle in view, let us study the ripples which are formed in water. As soon as the stone strikes the surface of water it causes a depression. The water tries to recover its original level (due to elasticity), and consequently flows into the depression. But owing to the kinetic energy of its particles it overshoots the mark, with the result that the depression is followed by an elevation which is again followed by a depression, and so on. In other words, as a result of this, water-particles begin to move up and down. Due to cohesion the neighbouring particles of water are also affected. The result is that they in turn also reproduce exactly the same motions. The disturbance is thus handed on to the neighbouring particles. This explains the wave-motion set up in the pond. From what has been said it is clear that *for the production of wave-motion we require a periodic motion of the particles.*

The very idea of the propagation of the disturbance involves the notion that the transmission requires time. For instance, when an elastic body suffers disturbance at one part, the whole of it does not become immediately affected, but on the other hand the disturbance is handed on from one layer of particles to

the next one. Suppose we give a blow with a hammer to one end of a wooden rod held horizontally. The blow causes compression at the place where it strikes, while the remaining part is in its normal condition. But the compression, however, quickly moves onwards. So soon as the force which produces the original disturbance is removed, the particles first affected try to resume their normal state, but in doing so, owing to the inertia, overshoot the mark and go to the other extreme, with the result that a wave of compression is followed by a wave of rarefaction, or, depression in case of water is followed by elevation. These opposite waves succeed each other rapidly; the amplitude, however, of each successive one being smaller than that of its predecessor. This goes on till the whole of the body resumes its normal state.

We have seen that the particles move about their mean positions while the disturbance moves forward. Now either the particles may move up and down at right angles to the direction in which the wave travels as in the case of water, or they may move to and fro about their mean position in the same direction in which the wave moves as in the case of the rod. In the first case the wave-motion is due to transverse vibrations, and the disturbance is said to be **transverse wave-motion;** and in the second case the wave-motion is due to longitudinal vibrations, and the disturbance is said to be **longitudinal wave-motion.** We have given one example of each kind; let the student think out for himself other examples of transverse and longitudinal wave-motions.

Analysis of Wave-motion

245. Transverse Waves.—Let us consider in a somewhat greater detail the resultant motion when a large number of particles of a medium are executing periodic motions, the phases of the neighbouring particles being related to each other by a definite relation which, in simpler language, means that the disturbance reaches the particles turn by turn and that the time taken by the disturbance in going from one particle to the next is the same for any two neighbouring particles.

Suppose we have a number of particles (say 9) arranged originally at equal intervals along the line AB, Fig. 2. Imagine that the transverse vibratory motion begins at A. This motion, on account of the cohesion between the particles, will be handed on from one particle to the other with a certain phase retardation, say $\frac{1}{8}$th of the time-period, *i.e.* the disturbance will

reach the next particle after $\frac{T}{8}$ sec. This phase-retardation will be the same for any pair of neighbouring particles.

Initially the particles are all on the same line AB as shown in Fig. 2 (i). After $\frac{T}{8}$ sec., the particle 1 is displaced upwards, whereas No. 2 is just affected (see Fig. 2 (ii)), for the disturbance

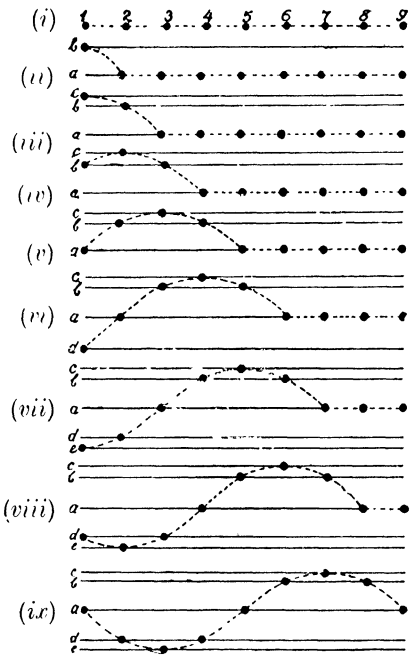


Fig. 2

reaches it after $\frac{T}{8}$ sec., as said above. It will be useful to point out here that the velocity of the particle is not uniform but on the other hand it is maximum at the mean position and zero at the point of maximum displacement. Due to this change in velocity the distance passed over by the particle during each fraction of $\frac{T}{8}$ sec. will not be the same. It will be greater when the particle is passing through the mean position

than when it is near the point of maximum displacement. Suppose for the sake of simplicity that during $\frac{T}{8}$ sec., when the particle is near its mean position, it travels three times the distance which it travels near the farthest point. Graphically, we can represent by drawing two lines *b* and *c* at such distances from the line *a* that the distance between *b* and *a* is three units and that between *c* and *a* is 4 units. Similar is the case on the lower side.

After a period $\frac{2T}{8}$ $\left(i.e.\ \frac{T}{4}\right)$ sec., Fig. 2 (iii), the first particle will go over to the maximum distance from the mean position, *i.e.* to the line *c*,* the second particle will just reach the line *b*, whereas the third one will be just affected. After $\frac{3T}{8}$ sec. the first particle will be on its return journey towards the mean position, and will be back on the line *b*, particle No. 2 will be at the maximum distance from the mean position, *i.e.* will be at the line *c*, and the particle No. 3 will be on the line *b* on its outward journey, whereas particle No. 4 will be just affected. This is shown in Fig. 2 (iv). When half the period is over, the particle No. 1 will be at its mean position but with a tendency to go downwards; the particle No. 2 will be on line *b* on its backward journey, the particle No. 3 will be at the maximum distance (line *c*), the particle No. 4 on the line *b*, but on the outward journey, whereas particle No. 5 will be just affected. Similarly, the changes in the position of the other particles can be traced very easily from the figures given. It will be seen that after a complete time-period the particle No. 1 is just back in its mean position with a tendency to move upwards, in which direction it started at first. Particle No. 9 is also just about to move upwards, these two particles are said to be in the *same phase*; and particles No. 1 and 5 are said to be in *opposite phases*, for although they have numerically the same speed, the direction of motion is opposite. The point at the maximum distance above the line of mean position is called a **crest**; and that at the maximum distance below the line of mean position is called **a trough.** The distance *between the two nearest particles in the same phase* (*as particles* 1 *and* 9) *is said to be the* ***wave-length.*** It is also equal to the distance between two nearest

* After $\frac{T}{4}$ second the particle must reach its extreme position, corresponding to *B* or *C* of the pendulum bob (Fig. 1).

crests or troughs. It is evident from what has been said above that the wave-length is the distance through which the disturbance travels during the time a particle makes one complete vibration. It is another way of defining it. Let us call the wave-length λ. We know that $\frac{1}{T} = n$, where n is the frequency and T is the time-period. We can very easily arrive at a relation between the velocity with which a wave travels and the wave-length and time-period or frequency of the body giving rise to wave motion,

for $$\text{velocity} = \frac{\text{distance travelled}}{\text{time taken}}$$

$$= \frac{\lambda}{T}$$

$$= n\lambda.$$

This is a very important relation for the wave-motion in general.

We have assumed that there are nine particles in the row, but actually there is an infinite number of particles; the increase in number of particles, however, affects in no way our reasoning; only it should be remembered that the pulse arrives at each point *a little later* than at the preceding one.*

246. Longitudinal Wave-motion.—It has already been said that when the particles vibrate to and fro about their mean position in the same direction in which the wave travels, the wave-motion is said to be longitudinal. It can be very nicely demonstrated by giving a slight tap to a long helix at one end, a wave of condensation will be seen to run along the length of the helix.

In gases, since there is no cohesion, the transverse waves are not formed at all. It is only the longitudinal type of wave-motion which the gases can transmit. To understand this kind of wave-motion, let us suppose that a number of particles, say 11, are arranged at equal distances along a line, the particles being capable of a to-and-fro motion along the line itself about their mean positions. Let us suppose, further, that each particle will begin its movement *a little later* than its

* A careful glance at the curves (Fig. 2) reveals the fact that they are not smooth, though actually the curves in the case of wave motion are very smooth. It is due to the fact that we have considered the average velocity during $\frac{T}{8}$ sec., whereas actually the velocity is different at each instant and goes on increasing or decreasing quite uniformly.

preceding one, in other words, that there is a definite phase relation between two neighbouring particles. Suppose this phase retardation is $\frac{1}{8}$th of the time-period as in the case of the transverse motion. This means simply that the next particle will begin its movement $\frac{T}{8}$ seconds later than its predecessor.

Before commencing the actual analysis it is important to remember what we pointed out in the preceding example, that a particle does not move with a uniform velocity; it passes over greater distances when passing through the mean position than when it is near the extreme position. For the sake of simplicity let us suppose as before, that during $\frac{T}{8}$ sec.

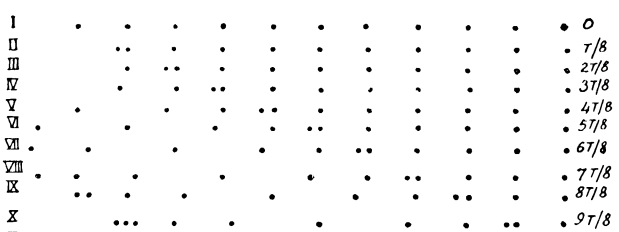


Fig. 3

when the particle is near its mean position, it travels over three times the distance which it travels when it is near the extreme position. Bearing this in mind, suppose particle No. 1 to be first displaced towards the right. At the end of $\frac{1}{8}$th of the time-period it will be at $\frac{3}{4}$ of the amplitude towards the right. Just at this time the second particle will be affected, see Fig. 3 (ii). After $\frac{2}{8}$th of the time-period is over, the particle No. 1 will be at the maximum distance on the right, whereas No. 2 will be at $\frac{3}{4}$th of the amplitude towards the right, and No. 3 will be just affected. It is shown in Fig. 3 (ii). After $2\,T/8$ has elapsed, the particle No. 1 will be on its return journey and will be at $\frac{3}{4}$th of the amplitude on the right from the mean position, particle No. 2 will be at the maximum distance, particle No. 3 at $\frac{3}{4}$th of the amplitude but on outward journey, whereas particle No. 4 will be just affected. In this way we can go on tracing the transmission of disturbance. A glance at Fig. 3 shows clearly that after a complete time-period, the disturbance has reached the particles 7, 8, and 9; 1 and 9

being in the same phase. It should be noted that in this case, instead of getting crests and troughs, we get crowding or **condensation** and **rarefaction** of the particles. As before, *wavelength is the distance between any two nearest particles in the same phase.* It is also equal to the distance between the centres of any two nearest condensations.

Note that the overcrowding is maximum near the centre of condensation. And in the case of rarefaction, the distance is maximum between two neighbouring particles in the centre of rarefaction.

During the time the particle No. 1 completes one vibration, the disturbance just reaches the particle No. 9, hence the distance between particles 1 and 9 gives one wavelength. Velocity of wave-motion $= \frac{\lambda}{T} = n\lambda$ as in the case of transverse wave-motion.

247. Since this is the type of wave-motion with which we shall be chiefly concerned in sound, it is advisable to lay a

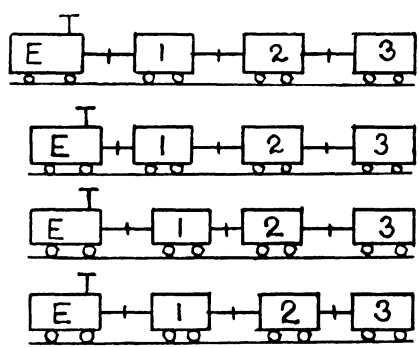


Fig. 4

greater emphasis on this mode of wave-motion. We shall take up one more instance to clear how a wave of compression or condensation travels forward. Suppose three carriages and one engine are coupled together, and that, to begin with, their buffers are in contact with each other. Since the buffers contain springs, when they are compressed by a push, they shorten up. The length of a carriage we shall take from buffer to buffer. If the engine moves forward through, say, half a metre, the buffer between it and the carriage No. 1 will be shortened.

Now as the spring shortens up it exerts a greater force on the carriage No. 1, tending to move it forward. When the carriage No. 1 moves forward, the spring on the side of the engine is relaxed to some extent, whereas the spring on the other side, *i.e.* towards the carriage No. 2, is compressed up. The carriage No. 1 will not stop here when the pressure on the two sides is equal, but will continue to move forward on account of the kinetic energy that it possesses, till the spring between (1) and (2) becomes compressed and the pressure of the spring brings it to rest. The carriage No. 2 will become compressed or shortened up and will repeat the same movement. Thus we see that the compression travels forward from one carriage to the next.

248. Let us carefully note the characteristics of this wave-motion. The first point to note is that it is the disturbance that travels forward and not any material particles or bodies. The second point is that the movement of a particle (or a body) begins just a little later than that of its predecessor. In other words, there is a definite phase-relation between two neighbouring particles (or bodies). The third point is that the velocity of the wave-motion is entirely distinct from the velocity of bodies transmitting it. For instance, in our example of carriage, the carriage moved through 50 cm., whereas the disturbance or wave during that very time moved through the complete length of the carriage itself.

CHAPTER II

PROPAGATION OF SOUND

249. Every body which is producing sound can be shown to be in a state of vibration, if suitable apparatus is used. As the vibration dies down, *i.e.* as the amplitude of vibration becomes smaller and smaller, the sound diminishes in loudness. This can be verified by a good many experiments. Ring a bicycle bell. Touch it with the tip of a finger. It will feel distinctly the vibratory movement of the bell. At the same time, since the finger stops the vibrations gradually, the sound goes on getting less and less loud. The vibratory motion of a sounding body is much too rapid to be seen directly by the eye, but it can be inferred in some cases from the blurred and indistinct appearance which the body presents to the eye, as for instance in the case of a stretched string when it vibrates. As the amplitude decreases, the intensity of sound also decreases. Strike a tuning-fork against any hard object, and let it touch your teeth, you will be able to perceive distinct strokes succeeding each other very rapidly. A very sensitive instrument for detecting the vibrations of a sounding body is what is called a pith-ball pendulum, which consists of a small pith-ball suspended by a thread (see Fig. 1, Electrostatics). Bring it near a sounding body; it will be repeatedly driven away as if it were receiving blows at regular intervals.

250. The vibrations of the body cannot be the immediate cause of the sensation of sound, but on the other hand it must be something in contact with the organ of hearing, the ear. With the help of a stretched membrane it is easy to show that it is set into vibrations where even a faint sound is audible. But how are those vibrations transmitted from the vibrating body to the membrane? Evidently the vibrating body sets the particles of the air around it into motion, which transmit their motion to the neighbouring set, and so on, till the disturbance reaches the ear and affects the

tympanic membrane; it is this physical action of the membrane which is the cause of the mental sensation called *sound.*

It is important to understand at the very outset that it is the disturbance which travels up to the ear, and nothing material. It is easy to infer from what has been said above, that for the propagation of sound an elastic material medium is essential. That sound can travel through solids, liquids, and gases we know from our daily experience. But in order to be sure that sound is propagated as a disturbance through *the medium* surrounding the sounding body, enclose the latter in a place which is devoid of matter. Take an electric bell and suspend it with the help of copper wires inside a bell jar mounted over the platform of an air-pump. Pass the wires through an india-rubber cork fitting into the mouth of the bell jar. Make the whole arrangement air-tight. Join the ends of the wires to the battery circuit containing a key. Press the key, the bell rings. Exhaust the air to some extent, and again complete the circuit, the sound of ringing will be heard even now, but it will be fainter than before. After a particular stage of exhaustion the sound will not be heard at all. This experiment proves conclusively that **material medium is required for transmission of sound.** It also shows us that the intensity of the sound heard depends upon the density of the air surrounding the bell. As the exhaustion proceeds the sound heard outside the bell-jar becomes fainter and fainter. By slightly modifying the experiment we can fill the jar by various gases, and try the effect produced upon the intensity of the sound heard. It is found as the result of experiments that when hydrogen and coal gas are surrounding the bell, the sound heard outside is very feeble; but when, on the other hand, the bell-jar is filled with CO_2 the sound heard outside on ringing the bell is very much louder. This result can be expressed by saying that the *intensity of sound depends upon the density of the medium in which it is originated.*

251. Sound transmitted by Liquids and Solids.—Take the resonance-box of a tuning-fork, at the top of which (the box) there is a hollow cylindrical projection into which can be fitted a metallic rod or a glass tube closed at one end. Fit up first the glass tube and fill it with water. Take the tuning-fork and fix with the help of wax a conical piece of cork to one of its prongs. Strike the tuning-fork and let the cork dip into the liquid. The sound will be immediately increased, showing thereby that the vibrations are transmitted by the liquid to the box, which intensifies the sound.

Fit up now the metallic rod, and after striking the fork against something hard, place it at the top of the rod; the sound will be increased immensely, as a matter of fact it will be louder than what we get in the case of a glass tube full of water. This experiment establishes that sound can be transmitted through liquids and solids.

It should be noted that sound does not pass from one medium to another of a very different density. It will be clear, if we remember that we can hear distinctly the sound made by the wheels of a train if we apply our ear to the rails when the train is at a distance of two or three miles, though we do not hear any sound in the air. It shows that sound passes from rails more readily to the bones than to the air.

252. Nature of Sound.—It has already been remarked that sound is produced by a body in motion, that it requires a definite medium to travel through, and finally, that whenever sound is audible, a tightly stretched membrane is set into vibrations. All these things show clearly that sound travels as a wave-motion through air, and since air cannot transmit transverse wave-motion owing to lack of cohesion, sound must travel as a longitudinal wave-motion. Let us see how these waves originate. Take a tuning fork and strike it with a wooden hammer to produce sound. The prongs will begin to vibrate to and fro.

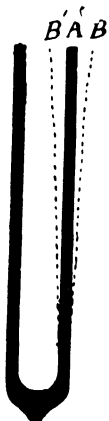


Fig. 5

We shall confine our attention only to the prong *A* for the sake of simplicity. When the prong *A* moves from *A* to the position *B*, it compresses the layer of air in front of it. Air being elastic tries to resist the compression and expands in the forward direction, and in so doing compresses the next layer and so on. In this way a wave of compression travels forward through the air. Again, when the prong returns from the position *B* and goes to B′, it leaves room behind it into which air can expand, so to speak, the pressure on the layer of the air in immediate contact with the prong is reduced, and hence it expands. This tendency for expansion is also transmitted forward through the air as a wave of rarefaction. Since the prong goes on vibrating, waves of compression and rarefaction are sent out in regular succession.

253. Velocity of Sound.—It was Newton who first showed

theoretically that the velocity of longitudinal waves, *i.e.* of pulses of compression and rarefaction in a homogeneous medium, is equal to the square-root of the elasticity of the medium divided by the square root of its density. Mathematically it can be expressed as

$$u = \sqrt{\frac{E}{\rho}},$$

where u = velocity of sound, E is the elasticity of the medium, and ρ is its density. Since sound travels as longitudinal waves, therefore with the help of this formula the velocity of sound in any substance can be determined. We have already seen in § 105 that E, *i.e.* $\frac{stress}{strain}$, is reduced to $v\frac{dp}{dv}$ in the case of *fluids*, where dp is the change in pressure and dv is the change in volume v.

In the case of a gas which obeys Boyle's law, this relation further reduces to $E = v\frac{dp}{dv} = P$ where P stands for the pressure. For by Boyle's law

$$\begin{aligned} Pv &= (P + dp)(v - dv) \\ &= Pv + v \times dp - P \times dv - dp \times dv. \end{aligned}$$

Now when the sound waves are passing through the atmosphere, the change in pressure and the change in volume are both extremely small, hence the product $dp \times dv$ is negligible for our purposes. Under these circumstances we get

$$Pv = Pv + v \times dp - P \times dv,$$

or

$$v \times dp = P \times dv,$$

or

$$\begin{aligned} P &= v\frac{dp}{dv} \\ &= E. \end{aligned}$$

Thus we see that in the case of a gas when the change in pressure and volume is very small, and when the temperature is constant (for otherwise Boyle's law is not applicable),

$$\boldsymbol{P = E}.$$

But when the temperature changes during the change of volume, elasticity is no longer equal to P. This will be clear if we consider a cylindrical vessel having unit cross-section and fitted with a piston. Suppose the volume of the gas enclosed between the bottom of the vessel and the piston gh is v c.c., when the force on the piston is P dynes per sq. cm. Decrease the pressure by dp (dynes) and let the piston rise to

$g'h'$, the increase in volume being equal to dv. Since the gas expands suddenly the temperature is cooled. As it takes up the atmospheric temperature, the volume increases in accordance with Charles's law. Hence the total change in volume is not dv, but something greater than this, say dv'. The ordinary (adiabatic*) elasticity will be $v\frac{dp}{dv}$, but the isothermal elasticity will be equal to $v\frac{dp}{dv'}$. Now since $dv' > dv$, isothermal elasticity is smaller than adiabatic. It is found that there is a constant relation between the two, *i.e.* $\frac{\text{adiabatic elasticity}}{\text{isothermal elasticity}}$ = a constant = γ (say).

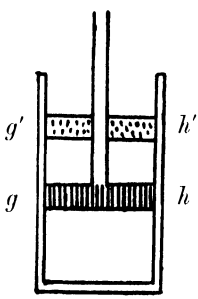


Fig. 6

Hence adiabatic elasticity $= \gamma \times$ isothermal elasticity $= \gamma \times P$,

for we have seen above that isothermal velocity is $= P$.

This constant is different for different gases; for air it is 1·42; for O_2, N_2, and H_2, etc., it is 1·4; but for argon it is 1·66; for Co_2 it is 1·30.

254. Velocity of Sound in Solids.—Applying Newton's formula $\left(i.e.\ u = \sqrt{\frac{E}{\rho}}\right)$ we can determine the velocity of sound in any substance. Since the elasticity of solids is very great it is clear that the velocity also in them must be very great when compared with the velocity in other substances. For instance, the velocity of sound in a steel wire whose elasticity is $2{\cdot}1 \times 10^{12}$ dynes per sq. cm., and whose density is 7·8, will be equal to

$$u = \sqrt{\frac{2{\cdot}1 \times 10^{12}}{7{\cdot}8}}$$

$$= \sqrt{26{\cdot}92 \times 10^{10}}$$

$$= 5189 \text{ metres/sec.},$$

or $5{\cdot}189 \times 10^5$ cm. per sec.

As we shall see later on *this is about* 4 *times the velocity of sound in water and about* 16 *times the velocity of sound in gases.*

* Adiabatic elasticity is that elasticity in which temperature changes are not taken into account, the initial and final temperatures are different.

Other indirect methods of determining the velocity of sound in solids will be taken up later on. It will suffice to say at this stage that velocity of sound as indirectly determined agrees with the theoretical calculation explained above. That solids transmit sound better than air can be very well realised from Airy's remark: "There are instances of people totally deaf to sounds produced in the air but who can hear the sound of a watch when held by the teeth; the sound being conveyed by the bony and the other portions of the head to the auditory nerves." Beethoven, who was one of the best musicians of Europe, was deaf, and used to hear some of his own compositions by means of a stick, one end of which rested against the sounding board of the piano, and the other against his teeth. The velocity of sound is different in different directions in the case of solids, which do not possess the same properties in all directions. For instance, in the case of wood the velocity is greatest along the fibres and least across them, owing to the fact that elasticity is greatest in the first direction.

255. Velocity of Sound in Liquids.—Theoretically we can determine the velocity of sound in liquids in the same way as we have done in the case of solids. Only in this case by elasticity we must understand volume elasticity. Let us calculate the velocity of sound in sea-water whose volume elasticity is $2{\cdot}23 \times 10^{10}$ dynes per sq. cm. and density is one approximately.

$$u = \sqrt{2{\cdot}23 \times 10^{10}}$$

$$= 1493 \text{ metres per second,}$$

or $$1{\cdot}493 \times 10^5 \text{ cm./sec.}$$

The experimental determination of the velocity of sound in water leads us to similar results. In the experiment given below it was found that velocity of sound in water is 1435 metres/second.

256. Experimental Determination of Velocity of Sound in Water.—It was first in 1826 that velocity of sound in water was measured by Colladon and Sturm, who moored two boats in the lake of Geneva, at a distance of 13,487 metres (more than 8 miles) apart. From one boat R a huge bell B was suspended in the water. It was struck by a hammer H, and the arrangement of striking was such that as soon as the stroke was made, a heap of powder was fired by a lighted fuse M. It was brought about by the lever A. As soon as it was lowered down the hammer struck against the bell, whereas M came down in contact with P, the powder.

The man at the receiving station Q noted the time of

observing the flash, and applied to his ear the trumpet E, whose mouth was covered by a membrane T to take up the vibrations. This trumpet was suspended in water from the

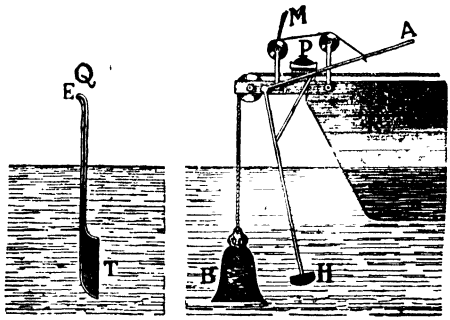


Fig. 7

second boat at Q, with its mouth towards the bell at R. It was found that the interval between seeing the flash and hearing the sound was 9·4 seconds.

$$\therefore \text{ velocity of sound} = \frac{13487}{9{\cdot}4}$$

$$= 1435 \text{ metres/second.}$$

257. Velocity of Sound in Air.—We have shown in § 253, that for gases which obey Boyle's law, isothermal elasticity is equal to pressure (measured in absolute units). Hence substituting P for E, Newton's formula for velocity of sound in air reduces to

$$u = \sqrt{\frac{P}{\rho}}.$$

Newton, when he used this relation, found that the results that he got were very low as compared with the results obtained experimentally. He obtained the value for velocity at 0° C., 280 metres/sec., as shown below :

$$u = \sqrt{\frac{76 \times 13{\cdot}6 \times 981}{{\cdot}001293}}$$

$$= 280 \text{ metres/sec.,}$$

or $$916 \text{ ft./sec. at } 0° \text{ C.}$$

The result is about 16 per cent less than that actually observed (323 metres per second or 1090 ft./second). The difference between the observed and calculated values is much

greater than any possible error of experiment, hence the conclusion that some assumption made in the theoretical formula $u=\sqrt{\frac{P}{\rho}}$ is erroneous.

It was the French mathematician Laplace who first pointed out the true cause of the discrepancy between the two results. He said that the compressions and rarefactions attending the passage of a sound wave were so sudden and of so short a duration, that in a medium of low thermal conductivity like air, they took place practically adiabatically.* In other words, air does not get time to radiate out heat which is produced due to compression and gain heat at the time of rarefaction which produces cooling. Hence we cannot use the value of isothermal elasticity ($E=P$) in our equation. But on the other hand, the value of adiabatic elasticity should be used. We have already seen in § 253 that the adiabatic elasticity = a constant × isothermal elasticity.

$$\begin{aligned}&=\gamma\times P,\\&=1\cdot42\times P \text{ for air.}\end{aligned}$$

Thus the velocity in air $=\sqrt{\frac{1\cdot42\times P}{\rho}}$

$$\begin{aligned}&=\sqrt{\frac{1\cdot42\times76\times13\cdot6\times981}{\cdot001293}}\\&=333\cdot7 \text{ metres/sec. at } 0^\circ \text{ C.},\end{aligned}$$

a result which agrees so well with the experimental values.

258. Discussion of the Formula.—The formula $u=\sqrt{\frac{\gamma\times P}{\rho}}$ leads us to some very simple, but important results.

(1) **Influence of Density.**—The formula tells us that *the velocity is inversely proportional to the square-root of the density*, and hence the velocity must be different in different gases. As a matter of fact, in hydrogen velocity should be four times the velocity in oxygen, which is found actually to be the case. In carbonic acid gas the velocity comes out to be

$$\begin{aligned}&=\sqrt{\frac{1\cdot3\times76\times13\cdot6\times981}{\cdot00198}}\\&=258 \text{ metres per second,}\end{aligned}$$

for $\gamma=1\cdot3$ and $\rho=\cdot00198$.

* The condensed part is at a higher temperature and the rarefied part is at a lower temperature, but note that the *average* temperature of the air, in spite of this local heating and cooling, remains the same.

Experimentally the velocity is found to be 259 metres per second.

Thus we see that this formula enables us to determine the velocity of sound in any gas very accurately.

(2) **Influence of Moisture.**—Since the density of water vapour is less than that of the air, the presence of moisture tends to decrease the density of the air on the whole, which results in the increase of the velocity. This shows that *the velocity of sound in air in damp weather must be greater than in dry weather.*

(3) **Influence of Pressure.**—The change in pressure does not produce any change in the velocity, for as soon as the pressure is increased, the density of the gas increases.

For Pv is constant by Boyle's law.

But $$v = \frac{m}{\rho},$$

where m is the mass and ρ is the density of the gas. Hence for v we can substitute m/ρ; doing so we get

$$P \times \frac{m}{\rho} = \text{a constant} = k,$$

or $$\frac{P}{\rho} = \frac{k}{m} = k' = (\text{a new constant}).$$

Thus we see that since $\frac{P}{\rho}$ is constant, any change in the pressure changes the density by the same ratio. Therefore pressure does not produce any change in the velocity of sound. Note that *no effect is produced when both the pressure and the density are changed in the same ratio, i.e.*, change of pressure unaccompanied by changes of temperature (as for instance a change in barometric pressure) will not affect the velocity of sound in a gas. But if only one of them were to be changed, the velocity of sound will be affected. For instance, let us suppose we heat the air inside a room after stopping all the outlets for communication with the external air. When we do so we do not change the volume of the gas, nor the mass, hence the density remains the same, but the pressure is being changed on account of heating, hence the velocity will be increased.

(4) **Influence of Temperature.**—When a gas is heated, pressure remaining the same its volume increases, *i.e.* density decreases. The decrease of density produces an increase in the velocity of sound; on the other hand, cooling the gas leads to a

decrease in the velocity of sound in the gas. Let us suppose that u_0 is the velocity at 0° C., u_t is the velocity of sound at t° C. and $u_{t'}$ at t'° C., ρ_0 is the density at 0° C., ρ_t density at t° C., and $\rho_{t'}$ at t'° C. P is the pressure, which we shall consider to remain constant. Let further v_0, v_t and $v_{t'}$ be the volumes of the gas we are dealing with at 0° C., t° C., and t'° C.

We know that

$$m = v_0\rho_0 = v_t\rho_t.$$

$$\therefore \quad \frac{\rho_0}{\rho_t} = \frac{v_t}{v_0} = \frac{v_0(1 + \frac{1}{273}t)}{v_0} = (1 + \tfrac{1}{273}t) = \frac{273 + t}{273} = \frac{T}{T_0},$$

where T and T_0 are the absolute temperatures corresponding to t° C. and 0° C. respectively,

$$\text{velocity at } 0° \text{ C.} = u_0 = \sqrt{\gamma \frac{P}{\rho_0}},$$

and ,, ,, t° C. $= u_t = \sqrt{\gamma \frac{P}{\rho_t}} = \sqrt{\gamma \frac{P}{\rho_0}\frac{T}{T_0}} = u_0\sqrt{\frac{T}{T_0}}.$

$$\therefore \quad \frac{u_t}{u_0} = \sqrt{\frac{T}{T_0}}.$$

Thus we see that the velocities at two temperatures are proportional to the square-root of the ratio of the absolute temperatures.

Now let us see what is approximately the increase in velocity for 1° C. rise of temperature.

$$u_t = u_0\sqrt{\frac{T}{T_0}} = u_0\sqrt{\frac{273 + t}{273}} = u_0\left(1 + \frac{t}{273}\right)^{\frac{1}{2}}$$

$$= u_0\left(1 + \tfrac{1}{2} \times \frac{t}{273}\right)* \text{ approx.} \qquad = u_0\left(1 + \frac{t}{546}\right) \text{ approx.}$$

But $u_0 = 33200$ cm. per second,

hence $u_t = 33200 + 61t$ approx.

This means that for one degree rise of temperature the increase in velocity is 61 cm. per second. If we were to express these results in ft./second, we would get the value of this increase as 2 ft./second.

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* Note, it is true only so long as t is small in comparison with 273, for otherwise we cannot apply the Binomial Theorem.

259. Experimental Determination of Velocity of Sound in Air. —That sound takes time to travel from one place to another is a matter of common experience. The flash of lightning is always *seen* before the thunder is *heard.* Similarly the flash of a cannon is always seen before the sound is heard. It is this fact which is made use of in determining the velocity of sound. At two stations near Paris, which were separated by 18,613 metres, guns were fired simultaneously; the time which elapsed between the perception of the flash and the hearing of the sound was noted at both the stations, in order to remove any error arising from the direction of the wind. As the mean of the observations it was found that the report took 54 6 seconds on the average to travel from one station to the other. Dividing the distance 18,613 by the time 54 6, the velocity of sound in air was determined. The temperature of the air was 16° C.

Hence
$$u = \frac{18613}{54\ 6}$$
$$= 341 \text{ metres/second.}$$

This is the velocity at 16° C.; allowing for the effect of temperature the velocity at 0° C. comes out to be 331 metres approximately, which agrees quite closely with our calculated value.

While performing this experiment the effect of the wind and the moisture must be eliminated. Above all the error due to the personal equation, viz. the error arising out of the fact that different individuals require different intervals, in seeing the flash and noting the time, and similarly hearing the report and noting the time must be allowed for. If the time taken were the same on both the occasions, *i.e.* between seeing the flash and noting the time and hearing the report and noting the time, no error will be introduced. But unfortunately these intervals are different, and hence some error always creeps in. In modern investigations observers are dispensed with altogether. The firing of the gun is recorded electrically at the sending station, and the time of hearing the sound is recorded also electrically at the receiving station. This gives us very accurately the time taken by the sound to travel from one station to the other.

The result of all these experimental determinations is that the velocity of sound in air at 0° C. is 332 metres per second or 1090 ft./sec.

EXERCISES

1. A bell is struck near the bottom of the sea, and 1·5 sec. later its sound reflected from the bottom is heard. If the depth be 2151 metres, find out the elasticity of the sea water.

$$\text{Velocity of sound} = \frac{215100 \times 2}{3}$$

$$= 143400 \text{ cm./sec.}$$

Now $u = \sqrt{\frac{E}{\rho}} = \sqrt{E}$; ρ being equal to 1 approximately.

$\therefore \quad E = u^2$

$= 2{\cdot}056 \times 10^{10}$ dynes per sq./cm.

2. Find at what temperature the velocity of sound in air is double the velocity of sound at 0° C.

$$\frac{u_t}{u_0} = \sqrt{\frac{273+t}{273}} = 2 = \sqrt{\frac{T}{273}}.$$

$\therefore \quad \frac{T}{273} = 4.$

$\therefore \quad T = 1092$ absolute or 819° C.

Caution.—Do not make use of the fact that the velocity increases by 2 ft./sec. for 1° C. rise in temperature, for this is true only when the rise in temperature is small in comparison with 273. See footnote on page 309.

3. Elasticity or Young's modulus of steel $= 2 \times 10^{12}$ dynes per sq. cm. The density of steel is 7·7 gm. per c.c., find the velocity of sound in it. *Ans.* 5095 metres/sec.

4. Find the temperature at which sound travels in hydrogen with the same velocity as in oxygen at 1000° C.

Ans. −193° C.

CHAPTER III

MUSICAL SOUNDS

260. Sounds which affect our ear are divided into two classes, noises and musical sounds. Before taking up the difference between the two, let us first consider a few sounds. Blow into an organ pipe, pluck a string of monochord, or play upon the flute. Next clap two flat blocks of wood together, listen to the ticks of a watch, or play at the same time entirely different tunes upon three or four musical instruments. It will be noticed that whereas the effect of the first set of sounds is pleasant on the ear, the effect of the second set is anything but pleasant. The sounds of the first set which produce a pleasing sensation are called **musical sounds,** and those of the second set are termed **noises.**

The physical difference between musical sounds and noises can be beautifully expressed graphically. In Fig. 8 curve (i) represents noises, whereas curve (ii) represents musical sounds. It is clear from the curve (i) that there is no regularity about

Fig. 8

it; in other words, the noise to which it corresponds is an irregular sound. The curve (ii), on the other hand, is uniform and regular, which shows that the sound which is represented by this curve is periodic; in other words, the body which produces it vibrates with perfect regularity. In addition to this periodicity, which is possible even in some cases of noises, as,

for instance, ticks of a watch, the sounds succeed each other rapidly. If the frequency is less than 16 per second, the sounds are perceived as separate. They do not produce a pleasing effect on the ear. It is important to note that *the only condition necessary for the production of musical sounds is that the sounds should succeed each other at regular intervals with sufficient rapidity.* No matter what the origin of the sound is; for instance, if the ticks of a watch could be made to succeed each other 30 or 40 times a second, the note would become musical, although very low. The humming of the bee or other insects seems musical on account of the vibrations being too rapid for the ear to perceive them as separate. One point more about these musical sounds. They do not change their character, *i.e.* the sound remains continuous with the same pitch and without any sudden change of amplitude for an appreciable interval of time [see curve (ii)]. To sum up, a ***musical sound*** *may be defined as one which is produced by a series of similar impulses following each other regularly at equal intervals and in quick succession with no sudden change in loudness.*

But noises, on the other hand, are either sounds of very short duration without any periodicity, or if they last at all for some appreciable time, their character is changing, as in curve (i). We shall deal with the musical sounds only in the following few pages.

Musical sounds may differ from each other in three respects: (i) in intensity or loudness, (ii) in pitch, and (iii) in quality. We shall consider each one of them separately.

261. Intensity or Loudness.—It is characteristic of all sounds, whether musical or noises. It is very easy for the ear to distinguish between loud and feeble sounds. The sensation of loudness of a sound is proportional to the amount of energy associated per unit area with the sound wave. To understand this point, consider the following experiment. Pluck the string of a monochord, and note its amplitude as well as the loudness of the sound it produces. Pluck it again, but with greater force. It will be noticed that it oscillates through a larger range, and, at the same time, produces a louder sound (though the frequency of the sound remains the same). As the amplitude of the string decreases, the loudness becomes less and less. It is just what is expected, for the greater the amplitude of the vibrations of the source of sound, the greater the amplitude of the air particles, and hence the greater the amount of the energy associated per unit area with the sound wave. We come to the same conclusion when we

consider the fact that the sound heard is feebler the farther is the hearer from the source. To understand the last point, consider a bell producing sound represented by A in Fig. 9. Energy is being transmitted from it to the surroundings. The disturbance is propagated equally on all sides, and hence it moves out as a spherical shell. There being no accumulation of energy, as much passes out of a shell like B or C as enters it. Consider two shells of the same thickness, one at a distance of say one metre and the second at a distance of two metres from the bell, the centre of the disturbance. The second shell has four times the surface as the first shell (the surface being equal to $4\pi r^2$, since r_2 becomes double of r_1, hence $r_2^2 = 4r_1^2$). If we take a third shell at a distance of three metres from the centre, it would have nine times the surface, and so on. The energy passing through the third shell is the same as associated with the second shell only some time before, as passed through the first shell. Since the surface is four times in the second shell and nine times in the third shell, we see that the energy per unit surface becomes $\frac{1}{4}$ at double the distance, and $\frac{1}{9}$ at three times the distance. But the energy associated with the unit surface would determine the loudness, hence we come to the conclusion that at double the distance the loudness becomes $\frac{1}{4}$, or more generally that *the intensity varies inversely as the square of the distance from the source.* The law of inverse squares holds good if the sound waves spread out uniformly in all directions. *Speaking tubes* and *megaphones* do not allow this spreading out of the waves, and hence the sound is heard even at a distance with almost the original intensity.

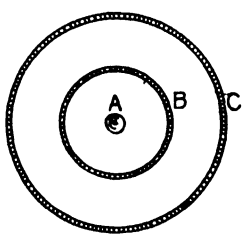


Fig. 9

Remember the loudness depends also on the *density of the medium* in which the sound is originated. We have already dealt with it in § 250, therefore we shall not consider it here at any length. We shall content ourselves by recapitulating that the denser the medium in which the sound is produced, the louder it is.

Note the energy associated with a sound wave is proportional to the square of the amplitude, as well as the square of the frequency of the note.

262. It will be clear if we take into consideration the energy of a vibrating body. For the sake of simplicity we shall go back to our

simple pendulum. We know that to move it from the position of rest to one side work is done upon it, storing thereby potential energy in it. When it is let go the potential energy is gradually converted into kinetic energy, at the lowest position the whole of the potential energy is changed into the kinetic energy, and due to this it cannot stop there, but moves to the other side, stops at the extreme position, where once again the kinetic energy is converted into potential energy. This mutual conversion goes on as the bob vibrates. Let us see how this energy is related to the amplitude. Supposing u is the maximum velocity, *i.e.* velocity which the bob possesses when it is at the lowest position, the whole of the energy being kinetic is equal to $\frac{1}{2}mu^2$, where m denotes the mass of the bob.

Now, the motion of a pendulum is an example of what is called a simple harmonic motion. Mathematically a simple harmonic motion can be considered as the projection of a uniform circular motion on a straight line, the amplitude being equal to the radius of the circle. Bearing this in mind, we can write down $u = a\omega$ where ω is the angular velocity and a is the radius of the circle. But $\omega = 2\pi/T$. $\therefore u = a \times 2\pi/T$, where T is the time-period, *i.e.* the period in which one complete vibration is finished.

$$\therefore \quad \text{Energy} = \tfrac{1}{2}mu^2 = \tfrac{1}{2}m \times \left(a \times \frac{2\pi}{T}\right)^2$$

$$= \tfrac{1}{2}m \,.\, a^2 \,.\, 4\pi^2 \,.\, \frac{1}{T^2}$$

$$= 2m \,.\, a^2 \,.\, \pi^2 \,.\, n^2,$$

where $\frac{1}{T} = n$, and a is the amplitude.

263. Pitch.—It depends on frequency; the greater the frequency the higher the pitch, and the lower the frequency the lower the pitch. In ordinary language a sound of a higher pitch is called shrill. It should be clearly understood that pitch is not the same thing as frequency, but that *it is a sensation depending upon frequency.* Since it is proportional to frequency and, in the case of noise, there being no periodicity, the term pitch is not applicable to it. It is only applicable to musical notes.

To make clear what is meant by pitch, it will be helpful to point out here that the voice of ladies is always of a higher pitch than that of men. The voice of children also has a very high pitch, though with the increase of age the pitch of their voice becomes less and less.

To show that the pitch depends upon the frequency, take the **Savart's wheel** and rotate it first slowly and then go on increasing its speed. Hold a piece of tin against the teeth of

the wheel. At first when the speed is low the taps can be heard as separate. As the speed increases the sound is blended together, and a note of low pitch is produced. With a still greater speed the pitch rises high. The effect produced upon the ear is not very pleasant, the sound produced being rather harsh.

That the pitch rises with the frequency can also be proved with the help of a **siren.** It consists of a circular disc *C* mounted on a vertical spindle (Fig. 10). The disc can rotate freely. It is perforated with a ring of holes at equal distances. They are bored obliquely. Just below the disc is the cap *B* of the wind box *E*. The cap is also perforated with the same number of holes as the disc, but these holes are inclined in the opposite direction. At the upper end of the spindle there is a worm (screw) which engages a pair of toothed wheels which help in counting the number of revolutions of the disc.

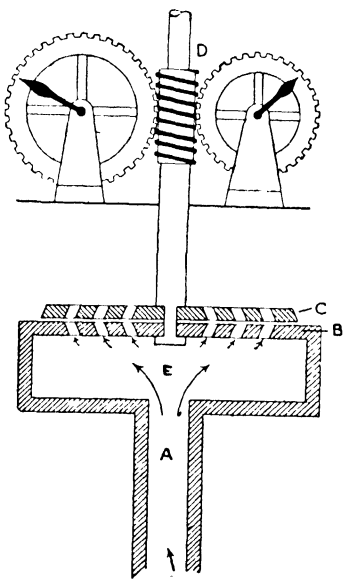


Fig. 10

The action of the siren is as follows. Air is forced into the wind box through *A* by means of a foot-bellows. Entering the box under pressure it makes its escape through the slanting holes of the cap where it has to pass in the opposite direction through the holes in the disc. Due to this inclination of the holes in the opposite direction the air strikes against the side of the holes in the upper plate and makes the upper plate rotate. Each time the upper ring of holes comes over the holes in lower plate a puff of air escapes and an impulse is given to the upper plate. If the disc is rotating with sufficient rapidity, these puffs will produce a musical note. By increasing the speed of rotation the frequency of the puffs occurring in a second can be increased, and it will be seen that as the frequency increases the pitch becomes high.

An everyday experience illustrates beautifully the change of pitch with frequency. When a railway engine is approaching the station its whistle becomes shriller and shriller, but when it is going out of the yard the whistle becomes lower and lower in pitch. It is due to the fact that when the engine is entering the yard the number of the waves falling upon the ear per second increases, and hence the frequency increases, while on the other hand, when it is going out the number decreases, and hence the pitch becomes lower. This principle is called **Doppler's Principle.** It may be stated as follows :

When a sonorous body is approaching the observer, its pitch becomes higher and higher, but when it is going away the pitch becomes lower and lower.

264. Measurement of the pitch.—The frequency of a given note can be determined by comparing it with either a note produced by Savart's wheel or by siren, or by comparing it with standard sources of known frequency.

1. Set the wheel in rotation and by changing the speed see when the two notes, one that of the given body and the other due to the wheel, are in unison, viz. are of the same pitch ; calculating the number of revolutions that the wheel completes in one second and multiplying by the number of teeth on the wheel, we get the frequency. But the instrument requires to be made extremely elaborate before it can yield an accurate determination of the frequency. Siren is easier, and is more suitable for such determinations. Let us suppose each plate of the siren (the disc as well as the cap) is pierced with h number of holes. During one complete revolution of the upper plate *any one* particular hole in it will pass over all the holes in the lower plate in turn, and therefore the coincidences will be h in number. At each coincidence a puff of wind escapes, and hence during one complete revolution h puffs are produced. If, moreover, the plate completes n revolutions in one second, which will be registered by the wheels in connection with the worm, the number of puffs produced per second is nh. To determine the frequency of the sound with a siren, disconnect the wheels and increase the speed of rotation of the disc till the sound produced is of the same pitch. Connect now the wheels and note the number of revolutions made per second ; on multiplying it by the number of holes, we get the frequency. The speed of rotation can be increased or decreased by changing the pressure on the bellows. The pressure is kept constant when the desired frequency is

reached. It is very difficult in practice to keep pressure in the bellows constant, hence actually the siren is driven by a small electric motor, whose speed can be regulated by a regulator.

2. Comparison with a standard source. The standard sources most generally used are the **tuning forks.** The note emitted by a fork is almost pure, and of a definite pitch, which is different for different forks. By comparing the given note with the note produced by the different forks we can fix upon the frequency, because the frequency of the fork that will produce the sound of the same pitch is known. If the note is not in unison with the note of a fork, but the frequency is somewhere near the frequency of the fork, beats will be produced. To understand the formation of beats let us consider the state of the medium which conveys the sound to our ears. It is set up into condensations and rarefactions. Due to the simultaneous production of disturbances by the two sources, the state of medium is determined by the resultant of the two disturbances. At some points condensation due to one may coincide with rarefaction due to the other. At such points the effect of one is neutralised by the other, and sound produced becomes faint. Where two condensations coincide the sound becomes louder. *These alternations of loudness and faintness are called* ***beats.*** The number of beats is always equal to the difference between the frequencies of the two notes. It is important to remember that the beats can be heard only when the frequencies are nearly equal to each other; if the difference is greater than 16, the beats cannot be heard as separate.

Tuning fork
Fig. 11

Knowing the frequency of the fork used, it is easy to know the frequency of the note. Let n be the number of beats. The frequency will be $N+n$, if the note is higher in pitch, otherwise $N-n$, where N is the frequency of the fork.

Another method of measuring the pitch we shall give later on.

265. The Musical Scale.—*When two or more notes are sounded together, their combination is called a* ***chord.*** *If the combination produces pleasing effect upon the ear, it is called* ***concord,*** *but if it produces disagreeable and jarring effect, it is called* ***discord.*** When two notes which produce concord are sounded one after the other, the pleasing effect is called **melody,** while if they are

sounded simultaneously the pleasing effect is called **harmony.** It is found that whether we deal with melody or harmony the combination of two sounds produces pleasing effect on the ear only if the ratio of the frequencies of the two sounds is simple, *i.e.* if the ratio is expressible as a fraction of two integers, neither of them being large. It should be noted that *the pleasing effect is not related to the difference between the frequencies but to the ratio. This ratio between the frequencies of two notes is called* ***interval.*** The simplest interval is 1 ; when it is so, the frequencies of the two notes are the same, and the notes are said to be in *unison.* The next simple interval is 2. It is called **octave.** When a note and its octave are sounded, either one after the other or simultaneously, a pleasing effect is produced. The ear recognises between a note and its octave six intermediate notes whose frequencies are well defined. These eight notes in all form what is called a **musical scale** or **gamut.** It is also known as the **major** or **diatonic scale.** The lowest note in the scale is called the **tonic.** The eight notes are generally represented by

C	D	E	F	G	A	B	C
Doh	ray	mi	fah	soh	lah	te	(Doh)
Sa	Re	Ga	Ma	pa	da	ni	(Sa)
	$\frac{9}{8}$	$\frac{10}{9}$	$\frac{16}{15}$	$\frac{9}{8}$	$\frac{10}{9}$	$\frac{9}{8}$	$\frac{16}{15}$

The intervals between consecutive notes are given above. It is clear that there are three intervals (fractions, ratios), $\frac{9}{8}$, $\frac{10}{9}$, and $\frac{16}{15}$. Each of them has received a different name.

They are called **major tone, minor tone,** and **semi tone** respectively.

To remember the intervals between different notes it will be useful to remember that the relative frequencies of the notes of the scale can be represented by

Sa	re	ga	ma	pa	da	ni	sa
24	27	30	32	36	40	45	48
	$\frac{9}{8}$	$\frac{10}{9}$	$\frac{16}{15}$	$\frac{9}{8}$	$\frac{10}{9}$	$\frac{9}{8}$	$\frac{16}{15}$

We can start with a note of any frequency as tonic and construct the scale, keeping the intervals the same.

Let us further suppose that we are given the interval between the notes *Sa, re,* and *re, ga,* and we want to determine the interval between *Sa* and *ga.* The interval between *sa* and *re* is $\frac{9}{8}$, and that between *re* and *ga* is $\frac{10}{9}$. From the relative

frequencies given above it is seen that the interval between *sa* and *ga* is $\frac{30}{24}$ or $\frac{5}{4}$. This is equal to

$$\frac{\cancel{9}}{\cancel{8}_{4}} \times \frac{\overset{5}{\cancel{10}}}{\cancel{9}} = \frac{5}{4}.$$

Thus we see that the interval between first and third is obtained *by multiplying* the intervals between first and second and second and third, and not *by adding*. Let us see what is the interval between first and eighth (the octave).

$$\frac{\cancel{9}}{\cancel{8}} \times \frac{\cancel{2}\,\cancel{10}}{\cancel{9}} \times \frac{\cancel{2}\,\cancel{16}}{\cancel{15}_{\cancel{3}}} \times \frac{\cancel{3}\,\cancel{9}}{\cancel{8}} \times \frac{\cancel{2}\,\cancel{10}}{\cancel{9}} \times \frac{\cancel{9}}{\cancel{8}} \times \frac{2\,\cancel{16}}{\cancel{15}_{5}} = 2.$$

Hence we see that *the sum of intervals is obtained by* ***multiplying them*** *and* ***not by adding*** *them arithmetically.*

266. Overtones.—It is very difficult indeed to get a pure note, *i.e.* corresponding to one particular frequency, from a musical instrument. Tuning fork can do so approximately. In general other notes are emitted along with the note sounded. Suppose, for instance, we produce a note having frequency n on an instrument. It is found that in addition to it other notes having frequencies $2n$, $3n$, $4n$, etc., also are produced. These are called **harmonics** or **overtones.** The lowest note is called the *fundamental.*

Each musical instrument produces different overtones, and it is the presence of these higher tones in addition to the fundamental which is the characteristic of each instrument. It is on account of these different overtones that we can distinguish the sound of one instrument from that of another, although we sound the same fundamental note. This peculiar property of a note is called the **quality** or **timbre.** Stroke called it **clang-tint**, for the latter name expresses far more beautifully the idea underlying this peculiar property than the word quality. We know that the colour depends upon the frequency* (the frequency for

* We know that $u = n\lambda$, where n is the frequency and λ is the wave-length, and u the velocity of light which is constant for all wave-lengths. This shows that $n = \frac{u}{\lambda}$. But we know further that the colour depends upon the wave-lengths, for blue light the wave-length is smaller and hence the frequency greater, and for red the wave-length is greater, and therefore the frequency is smaller.

blue light being about 8 billions per second and for red light about 4 billions). Similarly the high note corresponds to greater frequency and a low note to smaller frequency. Just as a pure colour has but one frequency, similarly a pure note (which is called a tone) has only but one rate of vibration. A compound colour, on the other hand, is produced by a mixture of one or more simple ones; similarly the sound produced by instruments is a mixture of three or four tones. Just as colour tells us whether light consists of one frequency or not, similarly the quality tells us whether the note produced is pure or not, hence the quality should be called clang-tint or colour of sound. We shall come back to the subject of overtones later on.

CHAPTER IV

TRANSVERSE VIBRATIONS OF STRINGS

267. By a *string* in sound we mean such a wire, or a cord of any material, which is perfectly flexible and uniform in cross-section. Moreover, it is assumed that while vibrating the string undergoes no change in length. All these conditions are quite satisfactorily fulfilled by thin metal wires or catgut. Let us study the vibrations of such a string when it is stretched between two fixed points.

When such a stretched string is plucked to one side it tends to return to its original position of rest. When let go it flies

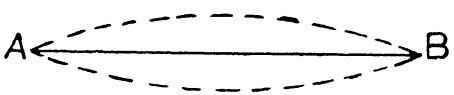


Fig. 12

back, but like a pendulum, owing to the inertia that it possesses, it overshoots the mark and goes over to the other side and goes on swinging between these two extreme positions. But in each vibration the amplitude goes on decreasing, hence after some time it stops. The vibrations thus produced are called **transverse vibrations.** In the above mentioned case the string vibrates as a whole. The fixed points where the amplitude is zero are called **nodes.** Note that the strain at these points is not zero ; on the other hand, it is maximum, for the string tries to swing and the reaction at the fixed points does not allow it to do so. The vibrating portion of the string between these two nodes is called **ventral segment.** The middle point where the amplitude is the greatest is called **antinode.** Note that the strain at the middle point is zero, for there is no reaction here to oppose the tendency of the string to vibrate. Nodes and antinodes can be defined as follows :

Node is a point in a vibrating body where the amplitude is zero and the strain is maximum; and

Antinode is a point where the amplitude is greatest and the strain is zero.

The note emitted by a string when vibrating in the above-said way is called its **fundamental,** which is defined as follows:

The fundamental note of a string is the note emitted by it when it vibrates as a whole in one segment.

This is not the only mode of vibration, it may vibrate in a good many other ways which we shall consider in § 269. For the present we shall confine ourselves to the laws of the vibrating strings, when they are emitting their fundamental note.

268. To study experimentally the laws, the instrument called the **sonometer**, Fig. 13 is used. It consists of a hollow

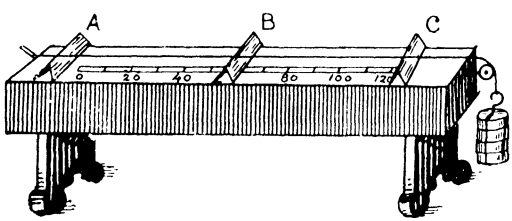


Fig. 13

sounding-box, provided with two fixed bridges and one movable bridge. One end of each string is secured to a fixed point at one end of the sounding-box, the other end passes over a pulley and carries a hook on which weights can be slipped on. At the top of the box a scale is fixed to measure the length of the string taken for vibration. Three or four strings—two of the same material but of different diameters, and the others of different materials—are generally fixed upon the board.

1. Pluck first one wire, keeping the distance, say 60 cm., between A and B, and the weight used suppose being 10 lb. Mark the note produced. Make the distance between A and B 30 cm., keeping the weight the same. Pluck it once more and mark the note produced. It will be found to be the octave of the first one. In other words, if their frequencies are determined it will be found that the frequency of the second note is double the frequency of the first note. It should be noted that every other thing remains the same. Thus we see, by halving the length, the frequency has been doubled or, more

generally, *the frequency is inversely proportional to the length of the string.*

2. Make the length between A and B 60 cm., and let the weight used be 10 lb. as before. Pluck the string and mark the note. Increase the weight now to 40 pounds and pluck the string again. The note produced this time will be octave of the first note, which means that the frequency has become double now. This shows that, by making the weight 4 times, the frequency is doubled or, more generally, *the frequency is directly proportional to the square root of the stretching force.*

3. Consider next two wires of different diameters: suppose they are 0·122 and 0·061 cm. thick. Take equal lengths, using the same stretching force. Pluck them both and mark the notes produced. The note produced by string which is 0·061 cm. thick will be octave of the note produced by the other. Thus we see that *the frequency is proportional inversely to the diameter of the string.*

4. Try now two strings of different materials but of the same diameter, same length, the same stretching force being used in the two cases. Pluck them and compare the frequencies of the notes produced. It will be seen that *the frequency is proportional inversely to the square root of the density of the string.*

All the abovesaid laws are included in the following relation:

$$n = \frac{1}{lD}\sqrt{\frac{F}{\pi\rho}}$$

where n is the frequency, l the length, D the diameter, F the stretching force, and ρ the density of the material.

The relation can be simplified by the following modification. The mass of a length of wire evidently = volume × density. Here volume $= l \times \pi \times \frac{D^2}{4}$; and density $= \rho$. Therefore mass of a unit length of wire $= \rho \times \pi \times \frac{D^2}{4}$; call it m. Substitute m in the abovesaid formula. It becomes

$$n = \frac{1}{2l}\sqrt{\frac{F}{m}}$$

where m is the mass of the unit length of the string. To sum up, we have arrived at the following laws:

1. **The frequency is inversely proportional to the length of the string.**

2. **It is directly proportional to the square root of the stretching force.**

3. **It is inversely proportional to the square root of the mass of a unit length of the string.**

This supplies us with another method of determining the frequency of a sonorous body. Only we have to tune in unison a particular string of the sonometer with the sound produced by the body, and substituting the value of l, F, and m in the formula $n = \frac{1}{2l}\sqrt{\frac{F}{m}}$ we can at once determine the frequency of the string, and thereby the frequency of the sounding body, since the tunes are in unison.

269. Overtones of a Stretched String.—When the two ends of the string are fixed they must be nodes, for there the

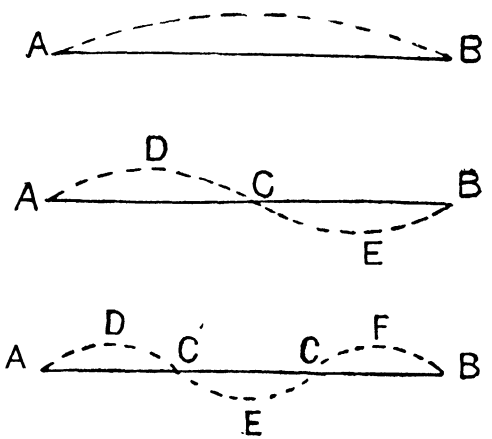


Fig. 14

amplitude must be zero. A string will have to vibrate always in such a manner that the two ends are the nodes. One way of vibrating (*i.e.* in one segment) we have already studied. But this is not the only way. Touch the string at the middle (C) and pluck it at D. The string will vibrate in two segments. Touch it at C', and again pluck at D. It will vibrate in three segments. Let us see what is the frequency of the notes produced when the string vibrates in this way. When it vibrates as a whole the note given out is fundamental. In the next case the string is divided up into two segments, and

we may consider now that only $\frac{1}{2}$ length of the string AB is producing note. By our laws of vibrating strings the frequency will be double. When it vibrates in three segments, in the same way we can show that the frequency will be three times, and so on.

So far we have been considering each mode separately, but in actual practice a string vibrates in all these modes simultaneously. In simple words, a string does not produce a pure note. When it is sounded for its fundamental, other overtones whose frequencies are in the ratio of 1, 2, 3, 4, etc., are present. Of course, the fundamental is very much louder than the other tones. It is on these overtones, as has been said in § 266, that the quality or timbre of a note depends. Different instruments, although sounding the same fundamental note, have different overtones, and it is on account of these overtones that their sounds can be perceived as distinct.

270. Melde's Experiment.—The laws of the vibrations of strings can in a very easy and striking manner be verified by means of the following experiment which is due to Melde.

A string about 1 metre long is fixed at one end to the prong of a tuning fork, while the other end passes over a pulley and is attached to a scale-pan. Suppose that the string is so fixed that the prong vibrates to and fro in the direction of the string. Let us start with the prong at its maximum displacement to the left. The string will occupy the lower dotted position. When the prong moves back to its mean position the string tightens. It becomes horizontal when the prong reaches its maximum distance towards the right. This shows that the string completes $\frac{1}{4}$ vibration while the prong completes $\frac{1}{2}$.

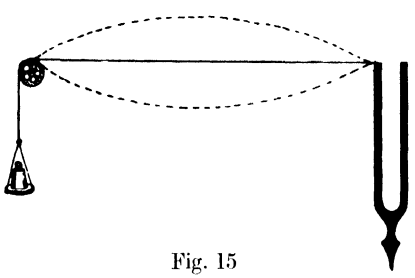


Fig. 15

Hence the frequency of the fork is double that of the string. We can find out the values for l, m, and f directly, and thereby calculate n, the frequency of the string, and compare it with the frequency of the fork.

CHAPTER V

REFLECTION OF SOUND WAVES RESONANCE AND LONGITUDINAL VIBRATIONS OF AIR COLUMNS AND RODS

271. Reflection of Sound Waves.—Since sound travels as longitudinal wave motion we need consider only the reflection of the longitudinal waves. We shall go back to our example of the trucks and the engine. Consider an engine coupled with three loaded trucks, the last truck standing against the siding

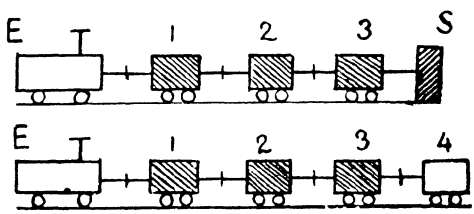


Fig. 16

S. Let the engine move forward. The buffers between *E* and truck No. 1 will be compressed up, with the result that there will be a force exerted on truck 1, which will consequently move forward, compressing the springs between 1 and 2. This wave of compression will pass on forward till at last the spring between the siding and truck No. 3 becomes shortened up. The shortening will exert a force on truck No. 3 in the backward direction; under the action of this force truck 3 moves backwards, shortening up the springs between 3 and 2, which causes the truck No. 2 to move backwards, and so on. This shows that the wave of compression is reflected back from the siding as a wave of compression. The same thing will happen if we were to suppose that in place of a siding

some other very heavy object is placed. That also will not move forward (appreciably) by the force of the spring.

This shows that *the longitudinal waves are reflected without change of type when reflection takes place at a heavy obstacle (or a denser medium)*. Sometimes this kind of reflection is said to be reflection without change of type, but with change of sign, for the direction of displacement of the truck is opposite to what it was before.

Next, consider that in place of a siding there is an empty truck. As soon as the springs between 3 and 4 become shortened up, the force due to shortening produces motion in truck No. 4. But since it is lighter, it moves through a greater distance than that through which the trucks 1, 2, and 3 moved. The result of this greater displacement is that it drags along with it the truck No. 3. Owing to the forward movement of 3 the springs between 2 and 3 become elongated. This causes the truck 2 to move forward, and so on, which, in other words, means that the wave of compression is reflected as a wave of rarefaction. Here the empty truck where reflection takes place is lighter than the loaded trucks; the same thing will happen if reflection takes place at a rarer medium.

This shows that *the longitudinal waves are reflected with change of type (but without change of sign), when reflection takes place at a rarer medium.*

The reflection of the longitudinal waves that has been discussed above applies to the reflection of sound waves. That sound waves suffer reflection is well-known to everybody. Echo, for instance, is nothing except reflection of sound waves from an obstacle.

272. Echoes.—Whenever sound waves are reflected from big obstacles, as, for example, a hill-side or a huge building, echoes are produced. If we stand at a fairly good distance (say 50 yards) from such an obstacle and fire a pistol, we hear the echo. If we come nearer than 56 ft. we can hear the echo no more. It is on account of the fact that the sensation of sound lasts for $\frac{1}{10}$th of a second after the original sound; and therefore, in order that the echo may be distinguishable from the original sound, it must reach the ear *later* by $\frac{1}{10}$th of a second after the sound has ceased. At ordinary temperatures the velocity of sound is 1120 ft. per second, therefore the total distance that the sound must travel before it falls on the ear should be 112 ft., or 56 ft. for going and 56 ft. for coming; in other words, the obstacle should be at a distance of 56 ft. at least.

This holds good only when the sound is of a very short duration. If we want to hear articulate sounds, it is found that the distance of the obstacle should be at least 112 ft. instead of 56 ft. It is so because a man cannot pronounce more than 5 syllables in one second distinctly. Moreover, the ear also cannot make out what is said if more than 5 syllables are uttered; hence on the ear more than 5 syllables should not fall in one second. In other words, so that we may hear the echo of a syllable distinctly, the sound should reach us $\frac{1}{5}$th of a second later than the original sound, *i.e.* it should in all cover 224 ft. before it falls on the ear. If at this distance, *i.e.* 112 ft., 2 syllables are spoken, only the last syllable will be heard distinctly. At a distance of 224 ft. from the obstacle 2 syllables will be distinguishable. Echo which enables us to hear only one syllable distinctly is called *monosyllabic*. That which enables us to hear two or more syllables is called *polysyllabic*.

273. Forced and Free Vibrations.—Consider a pendulum. At a given place it always has its own free time-period, depending upon the length. When disturbed, it begins to vibrate with its free period. But if, on the other hand, it be held in the hand and moved, it may be made to complete one vibration in any period we like. The vibrations of the first type are called **free** vibrations, but of the second type are called **forced** vibrations. In order to excite free vibrations we do not require any strong impulse. We can do so even by very slight impulses, provided they are repeated at the proper intervals (equal to the time-period of the body). A pendulum, for instance, may be set up into vibrations by blowing gently at the bob after regular intervals (equal to the time-period).

But we cannot make a body vibrate with any other period (than its natural period) with slight impulses. Let us consider a tuning fork. Strike it against something hard. Its sound becomes audible only when held near the ear. Bring it near the table and keep it there an inch or two above it. The sound is not increased. Put it over the table, the sound becomes intense. The reason of it is that when it is at a distance of 1 inch it cannot force the table to vibrate, but as soon as it is placed above it the table is *forced* to vibrate with the period of the fork. Since now much more powerful effect is being communicated to the air from the comparatively much larger surface of the table than the tuning fork alone could do, sound becomes louder. It should be noted, however, that, the amount of energy remaining the same, the sound

increases in loudness but decreases in duration; *i.e.* it lasts only for a short period.

All the stringed instruments owe the loud sound to the forced vibrations of the sounding-box.

Take two tuning forks mounted upon sounding-boxes adjusted to exactly the same pitch. Keep the open ends of the boxes towards each other. Sound one tuning fork, and after some time stop it. The other tuning fork will be heard to be producing sound. This effect is called *resonance*.

The term **resonance** *is applied to the phenomenon of vibrating motion produced in one body by the influence of another when the time periods of both are exactly the same.*

274. Resonance of Air Columns.—A column of air may be thrown into vibrations by a tuning fork, provided the time-period of the air column is the same as that of the fork. A tube of glass is taken and is immersed into a glass cylinder half-filled with water. A tuning fork after being struck against something hard is placed near the mouth. The tube is moved up and down till a reinforcement of the sound is heard. When it is so, the period of the air column is just the same as that of the fork.

Let us see how this resonance is brought about. Consider the lower prong of the fork (Fig. 17). Suppose it vibrates between limits d' and d''. Start with the prong at d'', about to move downwards. When it moves downwards a wave of compression is sent downwards. When this wave reaches B it is reflected from there (the denser medium) as a wave of compression. Let us suppose at the instant when the wave reaches the mouth of the tube the prong is just about to move from d' to d''. At the open end the reflection of the wave of compression again takes place. Since near the mouth of the cylinder the air is in the state of compression and outside the air is normal, the outside air is comparatively rarer. Therefore the reflection takes place at the rarer medium, consequently a wave of rare-

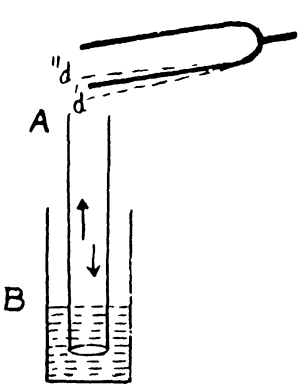


Fig. 17

faction goes down. But since the prong is also going from d' to d'', hence due to prong also a wave of rarefaction is being sent downwards. *These two waves reinforce each other.* The reinforced waves suffer reflection at B and come up to A just when the prong has reached d''. This shows that the time-period of air column is just the same as of the fork.

Moreover, it is seen that during the time the prong makes half a vibration the pulse or wave travels twice the length of the air column (once downwards and then upwards). Therefore the wave travels four times the length of the air column during the time the prong makes one complete vibration; this evidently is equal to the wave length of the note emitted. Thus we see that *the wave length of the note is equal to four times the length of the air column.*

This result enables us to determine the velocity of sound in air, for $u = n\lambda = 4nl$.

We know λ from the resonance experiment and n from the frequency of the tuning fork, hence we can find out u.

Lord Rayleigh came to the conclusion, on theoretical grounds, that the formula $u = 4nl$ is not accurate, for the reflection does not take place at the open end of the cylinder but slightly above it. He found, as a result of his investigation, that the length should be increased by $0{\cdot}3D$ where D is the diameter of the tube. The correct formula is

$$u = 4(l + 0{\cdot}3D)n.$$

This correction of $0{\cdot}3D$ is called **end-correction.**

275. Longitudinal Vibrations of Air Columns.—The method generally used to set the air columns in vibration is by the help of wind instruments—the simplest type of the wind instruments is called the **organ pipe,** as shown in Fig. 18. The mouthpiece is very much like a whistle. When we blow into such an organ pipe the air, after passing through the short tube, is forced against a bevelled surface and is set into vibrations. The pipe, acting as a resonator, selects out of these vibrations the note of its own frequency, and reinforces that.

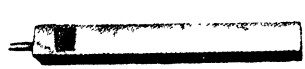


Fig. 18

The upper end of the pipe may be either open, in which case it is called an open pipe, or it may be closed, in which case it is called a closed pipe. Let us see the overtones produced in each case. It is well to remember that at the closed end there can be no motion of the air particles, hence *the closed end must always*

be a node; and that at the open end, where the air column communicates with the atmosphere, changes of density will be practically zero; *i.e.* strain will be practically zero, hence *open end must always be an antinode.*

276. Overtones of Pipes—

(*a*) **Open Pipe.**—As explained above, the two open ends must be antinodes.

1. The simplest method of vibrating is that the two ends should be antinodes, at the middle there being a node, for we

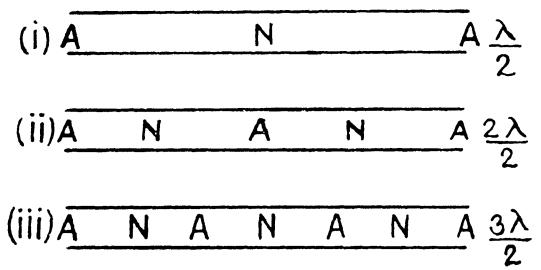


Fig. 19

cannot have two antinodes without a node intervening (Fig. 19 (i)). This is the fundamental note. Let its frequency be equal to n.

2. The next note, or the first overtone, is produced when the air column vibrates as shown in Fig. 19 (ii). Evidently the

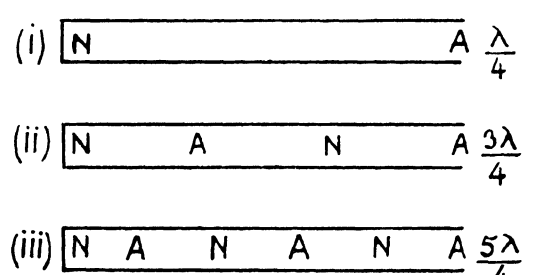


Fig. 20

frequency is double that of the fundamental—viz. the frequency of the first overtone is $2n$.

3. The next overtone is produced when the air column

vibrates as shown in Fig. 19 (iii). The air column is divided into 3 segments. The frequency will be $3n$, and so on. Thus the frequencies of possible overtones, together with the fundamental, are represented by the *natural numbers* 1, 2, 3, 4, etc.

(*b*) **Closed Pipes.**—As said above, the closed end must always be a node and the open end an antinode.

1. The simplest method of vibrating is that there should be a node at the closed end and an antinode at the open end. This is the fundamental note. Let the frequency be n.

2. The first overtone is produced when the air column vibrates as shown in Fig. 20 (ii). Evidently the frequency is $3n$.

3. The second overtone is produced when the air column vibrates as shown in Fig. 20 (iii). The frequency is $5n$, and so on.

Thus the frequencies of possible overtones, together with the fundamental, are represented by 1, 3, 5, etc., or *odd numbers*.

277. Longitudinal Vibrations of Rods.—A rod vibrates longitudinally in the same manner in which an air column does. If it is clamped in the middle and is made to vibrate longitudinally by passing a wet or resined cloth along it, the fundamental note produced corresponds to the vibration of the air column in an open pipe; *i.e.* with a node at the middle point and antinodes at the free ends. The overtones also are exactly similar. To find out the velocity of sound in a glass rod, proceed as follows:

Clamp the glass rod in the middle and pass a wet cloth along it to set it into vibration. The rod will vibrate like the air column in an open pipe; *i.e.* the fundamental note will be produced when the rod vibrates in such a manner; that its length corresponds to $\frac{\lambda}{2}$. Find out the frequency of the note emitted. Thus we know n and λ.

But $u = n\lambda$.

Hence the velocity is also known.

The rods may be made to vibrate longitudinally by clamping at one end. The vibrations then would correspond to closed pipes. In this case the fundamental note will be produced when the rod vibrates in such a manner that its length is equal to $\lambda/4$.

EXERCISES

1. Two strings, each 60 cm. long, are stretched, one by a weight of 4 lb. and the other by a weight of 9 lb. What is the interval between the notes which they produce ?

Since we are concerned with the interval it is immaterial whether we consider the force in pounds or grams, for we do not want the frequencies.

$$n = \frac{1}{2l}\sqrt{\frac{4}{m}},$$

$$n' = \frac{1}{2l}\sqrt{\frac{9}{m}},$$

$$\therefore \quad \frac{n}{n'} = \sqrt{\frac{4}{9}} = \frac{2}{3}.$$

2. A glass rod 35 cm. long is clamped at the middle and is rubbed with wet flannel. The frequency of the note produced is 6000. Find the velocity of sound in glass.

The rod will vibrate in one segment, and

therefore $l = \lambda/2,$

$\therefore$ $\lambda = 70$ cm.,

$$u = n\lambda$$
$$= 6000 \times 70 \text{ cm.}$$
$$= 420000 = 4{\cdot}2 \times 10^5 \text{ cm.}$$

3. A string is 75 cm. long and weighs 15 gm. With what force must it be stretched so that it may vibrate 256 times a second ?

$$n = \frac{1}{2l}\sqrt{\frac{F}{m}},$$

or $$n^2 = \frac{1}{4}\frac{F}{l^2 m},$$

or $$(256)^2 = \frac{1}{4 \times 75 \times 75} \times \frac{F}{1/5},$$

$$\therefore \quad 5F = \frac{256 \times 256 \times 4 \times 75 \times 75}{981 \times 1000} \text{ kilograms wt.}$$

or $F = 300{\cdot}6$ kilograms wt.

4. What will be the frequency of the note emitted by a string 100 cm. long when stretched by a weight of 25 kilograms, if 1 metre of the string is found to weigh 2·395 gm. ?

Ans. 160.

5. A string when stretched by a weight of 20 kilograms gives the note whose frequency is 256. Find the weight required to produce the note whose frequency is 384.

Ans. 45 kgm.

6. What will be the length of a closed pipe if the lowest note has a frequency of 256 at 15° C. ? *Ans.* 33·2 cm.

7. The frequency of the note given by an organ pipe is 312 at 15° C. At what temperature will the frequency be 320, supposing the pipe to remain unchanged in length ?

Ans. 30° C.

8. A glass rod 2·5 ft. long is clamped at the middle. It is set into longitudinal vibrations. What is the velocity of sound in glass if the frequency is 2590 ? *Ans.* 12,950 ft.

PART IV

LIGHT

CHAPTER I

278. On looking round the room you see a good many objects which on covering your eyes with the hands you see no longer. Also on shutting all the doors, stopping the chinks, etc., of the room, and painting the door panes black, you do not see them. This shows that there is something in the absence of which you cannot see things, you cannot become aware of your surroundings and environments. This something is called **Light.**

Note, light is the external physical cause of the sensation called *sight.* It in itself is not visible but it just makes the other things visible to us; that is to say, we do not directly observe the existence of light, but *infer* it in order to *explain* the formation of visual impressions.

279. From the earliest times need has been felt to understand satisfactorily the process of seeing things, to explain how we see objects that surround us, there being no direct connecting link between the object seen and the eye seeing it. Although nowadays little difficulty is experienced in explaining this fact, in ancient times a good many controversial opinions were held by the then eminent scientists. Nowadays we say that we require a source emitting light and the eye to see things; that is to say, given the eye, we require a luminous source to enable us to see things and know our surroundings. But in ancient times much doubt prevailed as to whether objects became visible by means of something emitted by them or by means of something issuing from the eye of the spectators. According to the opinion of Pythagoras, who lived about 500 B.C., and his followers, vision was caused by particles continually projecting from the surface of objects into the pupil of the eye; while Empedocles (444 B.C.) maintained

that vision was effected by means of something emitted from the eye itself which after meeting something else emanating from the object excited the sense of sight. Plato considered three elements to be necessary for vision. According to him, a visual stream of light or divine fire emitted by the eye itself entered into union with the light of the sun and the two together, meeting a third emanation from the object seen, completed the act of vision.

Later on it was realised that heat and light both are manifestations of energy, and therefore the problem of transmission of light from one object to another was simply reduced to the transmission of energy from the first to the second. The simplest method is to suppose that energy is carried from one object to the other by particles, just as bullets carry the energy from the gun to the target across the intervening space. This is the basic principle of **Newton's Corpuscular theory** of light. He assumed that a luminous body continually emits small imponderable particles of extreme minuteness in all directions, which travel through the space with the velocity of light. This theory at once accounts for such general phenomena as the rectilinear propagation and reflection of light, but some of its consequences are quite inconsistent with the observed facts; as, for instance, it leads us to the conclusion that light should travel faster in denser media like water and glass, whereas actually we find that it moves slower. This theory held the field for a very long time because Newton's name was linked with it. The other theory, *i.e.* wave theory, could not come into prominence as the name of the discoverer, Huyghens, was not so great as that of Newton.

280. Modern ideas. — But nowadays light is generally believed to have its origin in the vibrations of the particles of a luminous body. You have read already that heat is due to the vibrations of the molecules; so is the case with light too. Consider, for instance, an iron ball; its every molecule, as explained in § 240, is moving to and fro with a certain average velocity about a particular position called the mean position, which is different for different particles. As the temperature is raised this average velocity increases.

When it is very great, light is given out in addition to heat. Roughly to give an idea of the frequency of vibration of these molecules when they are emitting light, it will suffice to say that the frequency varies from 400 to 800 billions per second. This is just in accordance with other observations made. Consider the iron ball once more. At ordinary temperatures

you do not find anything peculiar about it, but when you heat it to, say, 200° C. and then touch it, you will find that it is not quite pleasant to try that; if you heat it to 500° C. it will begin to glow slightly and will appear dull red; at 800° C. it will appear cherry red; and at 1500° C. or above, dazzling white. From this it is evident that as the temperature rises, light emitted becomes more and more intense. In fact, this enables us to conclude that heat and light are similar phenomena, and that at higher temperatures they go hand in hand.

281. What is Light?—We have learnt how light originates; now we shall see what it is. When a molecule vibrates it sends out waves similar to the waves or ripples that you find in a pool when disturbed by the fall of a piece of stone. These waves are propagated through the space which is supposed to be filled up with **Ether** (see § 240) and reach other objects. Considering that light is nothing but very short waves, you at once see for yourself that it is impossible to see it; hence the statement that *light is invisible.*

282. Some bodies in this universe send out light of their own, such as the sun, or a candle-flame or a red-hot coal. They are called **Luminous**. But by far the larger number of bodies is non-luminous, because they do not send out any light of their own. They become visible in the presence of a luminous body. What happens is that light from the luminous body falls on the non-luminous one and is scattered by it; a portion of the incident light reaches our eyes and makes the body visible to us.

283. Terms used in connection with light.—*A substance through which light can pass is often spoken of as an* ***optical medium*** *or more simply a* ***medium,*** as for instance air, glass, water.

When a substance has, at all points and in all directions, the same properties, chemical composition, density, etc., it is said to be ***homogeneous***, as for instance good glass or water.

A substance which has different properties at different points is said to be ***heterogeneous.***

A body which allows light to pass through it easily and through which objects can be seen distinctly is said to be ***transparent***, *as* glass or water; *that which allows light to pass through it imperfectly and through which bodies cannot be seen distinctly is said to be* ***translucent***, as for instance oiled paper, ground-glass, etc. *A substance which does not allow light to pass through it at all is said to be* ***opaque***, as for instance stone, gold, etc.

It should be noted very carefully that all these terms are

relative; no substance is absolutely transparent; while, on the other hand, none is absolutely opaque. Take the case of water: it is transparent if you are concerned with moderately thick layers, but when you deal with very thick layers you will find that part of the light passing through (especially

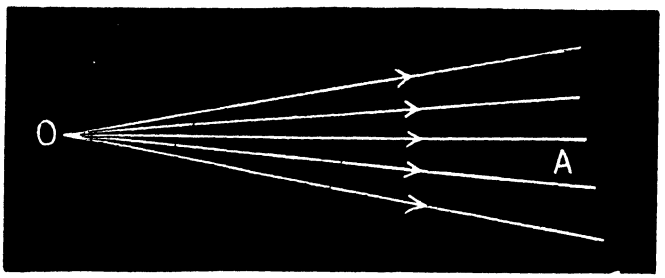


Fig. 1

the red-end of the spectrum) is absorbed. Even gold can be made transparent when you take a very thin leaf; and so on.

Experiments show that in a homogeneous medium light travels in straight lines—we shall consider in detail this question later on. *The path of light in a homogeneous medium*

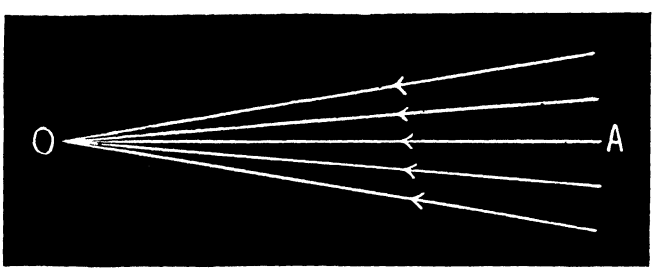


Fig. 2

is called a ***Ray.*** *A collection of rays issuing from one point is called a* ***Pencil of Rays.***

If light is travelling away from the point it is said to be a **divergent pencil** (Fig. 1).

But if, on the other hand, light is travelling towards the point, the pencil is said to be a **convergent** one (Fig. 2).

When the point O is situated at an infinite distance the pencil becomes parallel. A **beam** of light is a collection of adjacent rays. When the individual rays in a beam are all parallel, we call the beam a parallel one.

Generally no distinction is made between a pencil of rays and a beam of light. But as a matter of fact a beam of light is an assemblage of rays proceeding from various points of a luminous object.

284. Rectilinear Propagation of Light.—That light travels in straight lines in a homogeneous medium is taken for granted by every one of us, for we always assume that the body exists in the direction of the rays of light which enter our eyes from it. As a matter of fact, if we do not want to look at a particular object we interpose a small obstacle between the object and the eye. The rays of the sun entering a darkened room through a chink in a shutter are always seen to be straight. A most convincing proof of this fact, however, is the study of formation of shadows.

When light falls upon any opaque body it cannot penetrate into the space immediately behind the body, and *this space where light is either wholly or in part cut off by the opaque body is called the* ***shadow.***

In determining the extent and the shape of the shadow of a body two cases are to be distinguished: first, when the source of light is a single point; and secondly, when the body possesses an appreciable size.

285. Let us first consider the case of **a point source of light.**

Let S be the point source, MM' a solid sphere, and AQ a screen. The cone of rays SMM' is intercepted by the solid

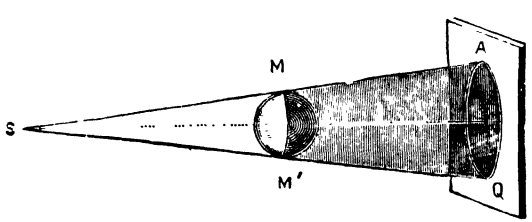


Fig. 3

sphere and hence no rays of light enter the region AQ on the screen. The section of the cone will be a circle if the screen is held at right angles to the axis of the cone. Looking at Fig. 3 it is at once clear that if light travels in straight lines

the shadow would be a circle, which is actually the case. Take another case, in which the obstacle is a square piece of cardboard. It will be noticed that the shadow of it also is square, and that further at a distance double that of the obstacle from the point, the area is four times that of the obstacle itself (Fig. 4), which at once suggests that light travels in

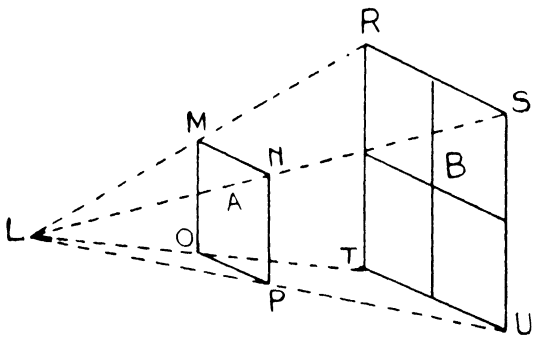


Fig. 4

straight lines. Notice that in this case the shadow is uniformly dark, and that the boundary line of the shadow is quite sharp.

286. Now take the second case, when the source has an appreciable size.

This can be further subdivided into two cases: first when the source is smaller than the opaque object, and second when the source is bigger than the obstacle.

I. **When the source is smaller than the obstacle.**—It is clear from Fig. 5 (*a*) that the shadow should not be quite uniform all over if light travels in straight lines, and it is actually found to be so. In fact, the central portion *GH* receives absolutely no light from any part of the source, and hence it is perfectly dark; but as we go out, we find that this portion is being partially illuminated. Consider, for instance, the point *L*; it receives light from the portion *A* to *N*, but none from *N* to *B*. Any point nearer *G* than *L* receives light from a portion smaller than *AN*, and so on; thus we find that as we move out from *G* to *L*, and then to *D*, more and more light falls upon the screen, *i.e.* the shadow becomes less and less dark. The appearance of the shadow on the screen is shown in Fig. 5 (*b*).

The perfectly black portion of the shadow is called **Umbra**, and the lighter part of the shadow is called **Penumbra**. The difference between the two portions should be clearly marked. The umbra is uniformly dark all over, but in the penumbra the intensity decreases from the centre towards the edges.

Another point that requires attention is the fact that as the distance of the screen increases, the umbra and the penumbra both increase in area, the distance of the obstacle remaining constant from the source.

II. **Source bigger in size than the obstacle.**—In this case also we get umbra and penumbra; but the only difference

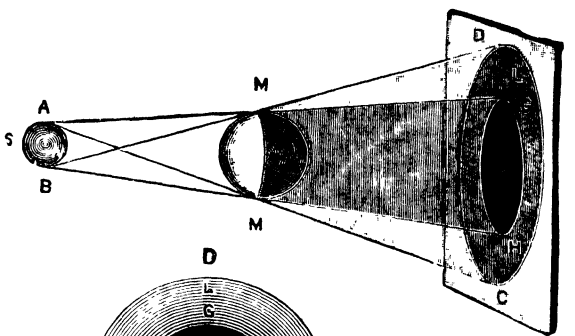


Fig. 5 (*a*)

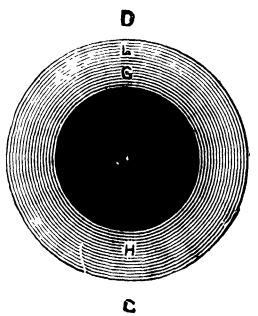


Fig. 5 (*b*)

is that umbra is convergent, and beyond a certain point there is no umbra to be seen at all. That is the reason why we do not see the shadow on the ground of a kite flying in the air. Note it is because the distance of the kite from the ground is very great and the shadow (penumbra) becomes too faint to be detected, umbra being altogether absent.

287. Eclipse.—The eclipses of the sun and the moon are simple illustrations of the formation of shadows. A *solar eclipse* is caused when the moon passes between the sun and

the earth. As the sun is much bigger than the moon, therefore the shadow will be of the type shown in Fig. 6.

The moon will cut off all or some of the sun's rays from certain parts of the earth's surface. In such parts the earth will be darkened and the sun will appear wholly (in umbral

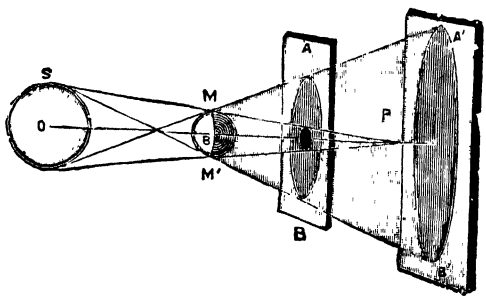


Fig. 6

region) or partially hidden (in penumbral region). This phenomenon is solar eclipse, which may be either total or partial.

Owing to the variations of the distances of the moon and the sun from the earth it is not always possible for the moon's umbra to touch the earth, as for instance at $A'B'$. At all points in the produced umbral cone, *i.e.* at a greater distance than P, the eclipse would appear to be annular,* *i.e.* only the rim of the sun's disc would be visible, the central part would be hidden.

Lunar Eclipse.—Similarly when the earth passes between the sun and the moon, all the three being in the same straight line, the earth acts like an obstacle and cuts off the rays of the sun before they reach the moon, and the latter may be either partially or wholly eclipsed.

288. Dimensions of the Shadow-cones.—The diameter or the length of a shadow-cone can be determined easily either

* To understand the difference between the total and annular eclipse, let us suppose that the observer is situated on the line joining the centres of the sun and the moon. Then if the angle subtended by the diameter of the moon at the earth be greater than the angle subtended by sun's diameter, the whole of the sun will appear to be hidden by the moon, and the eclipse will be total. If, on the other hand, the angle subtended by the sun's diameter be greater, the eclipse will appear to be *annular*.

The angle subtended by the diameter of the moon at earth varies from 28′ 48″ to 33′ 22″, and that subtended by the sun varies from 31′ 32″ to 32′ 36″. Hence both total and annular eclipses are possible.

by calculation or by graphical method. For instance, let us suppose we want to find out the distance of the moon from the earth when the umbra just touches it; if the distance of the sun from the earth is 93,000,000 miles, the diameter of the sun 864,000 miles, and the diameter of the moon 2160 miles.

Suppose S (Fig. 6) represents the sun, M the moon and $A'B'$ the earth. We want to find out BP. From similar triangles it is clear that

$$\frac{\text{Moon's diameter}}{\text{Sun's diameter}} = \frac{BP}{OP}, \text{ or } \frac{2,160}{864,000} = \frac{BP}{93,000,000},$$

or

$$\frac{1}{400} = \frac{BP}{93,000,000},$$

or

$$BP = \frac{93,000,000}{400} = 232,500 \text{ miles.}*$$

289. Pinhole Camera.—It consists of a box with a small hole on one side and a screen on the other side. This affords a very interesting proof of the rectilinear propagation of light. Let AB represent a luminous object placed in front of the hole of a pinhole camera. Each point of this luminous object may be thought of as a distinct and separate source of light. Consider the point A of this object. The light rays from this point diverge in straight lines in all directions. Some of them pass through the hole C and fall upon the screen at a point like A', which forms the image of the point A. Similarly B produces an image of itself at B'. The intermediate points between A and B have their images between A' and B'. Thus for every point in AB there is a corresponding point in the image $A'B'$. The assemblage of all such points constitutes the image $B'A'$ of the object AB.

From similarity of triangles it is clear that

$$\frac{B'A'}{BA} = \frac{v}{u},$$

which shows that the size of the image of an object ($B'A' = BA \times \frac{v}{u}$) depends upon the distances v and u only. Following are the peculiarities of the pinhole camera.

1. The image can be made as large as we please, by increasing v or decreasing u.

2. As u is decreased, the pencil of rays starting from a point on the object becomes more divergent, and therefore

* At distances greater than this the eclipse will appear to be annular.

a larger area on the screen is illuminated by the light from a particular point of the object. This results in overlapping of the images of contiguous points and hence there ensues a blurring of the image; consequently we cannot decrease u beyond a certain limit which must be determined experimentally.

Increasing v does not increase the blurring of the image, for with the increase of v the size of the image also increases and hence overlapping is not increased. But beyond a particular limit the image becomes too faint to be seen clearly.

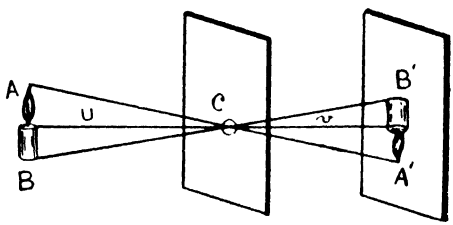


Fig. 7

3. By diminishing the size of the hole the definition can be improved up to a certain point, but if it be diminished beyond a certain value depending upon the values of u and v, the image is no longer reproduced faithfully, but on the other hand coloured rings make appearance.

If instead of one small aperture there be two near each other, two images will be seen, and if the apertures be sufficiently close to each other, the images will partly overlap and thus become blurred. This explains why we do not get distinct images with a large hole, for it can be considered to be made up of a large number of small holes, each one of which gives rise to an image, and all these on overlapping produce blurring.

4. The form of the image obtained is independent of the shape of the aperture. The latter merely affects the shape of the individual spots, but so far as the complete image of the object is concerned it does not produce any effect.

5. It requires no focussing, but gives images at all distances of the screen.

6. The pinhole camera gives a faithful and accurate picture of the buildings, whereas a lens, unless it is especially designed for taking photographs of buildings, will not give pictures free from defects (curvature, distortion, etc.)

CHAPTER II

PHOTOMETRY

290. It is a matter of common observation that the amount of light emitted by different sources is different; and before proceeding to describe the methods employed for comparing the amounts of light emitted by various sources, we must first consider how to define the amount of light emitted by a source.

291. Illuminating Power.—Let us suppose we have a point source O; it will evidently emit light uniformly in all directions. If a sphere of unit radius be described with O as centre, naturally all the light emitted by the source falls on it. Obviously the quantity of light which falls on a unit area of the sphere will be proportional to the quantity of light emitted by the source. If we keep the radius of the sphere the same, and use different sources, the quantities of light which fall on a unit area in a second will be proportional to the quantities of light emitted by the sources. This gives an idea how to compare the illuminating power of two sources. The illuminating power thus can be defined as:

The quantity of light which falls per second on a surface one sq. cm. in area at a distance of one centimetre from a source of light placed so as to be perpendicular to the rays, is called the illuminating power of the source.

292. Intensity of Illumination.—Consider now the surface which is being illuminated. Let L denote the quantity of light energy incident per second on a small element of the surface S. The amount of light L which is incident on the surface depends upon three factors—(i) the illuminating power of the source, (ii) the area of the surface, and (iii) on the position of the surface with regard to the source. For instance, the stronger the illuminating power of the source, the more the light falling upon the surface. Similarly the greater the surface the greater the amount of light incident on it. Again, the amount of light received by the surface

will depend on the direction in which light falls upon it as well as its distance from the source. If the surface be placed so that the rays strike it at right angles it will receive more light than if it were inclined to the direction of the rays.

Keeping this in view it is clear that the maximum brightness of a surface at a particular distance from the source will be obtained when the surface is placed normally to the rays of light. Note the brightness will be proportional to the amount of light falling upon per unit surface. The quantity of light which falls upon a unit surface in one second gives what is called the intensity of illumination at a point.

Definition.—*The intensity of illumination at a point is measured by the amount of light which falls per second on a surface one sq. cm. in area containing the point.*

Note that the intensity of illumination of a surface placed normally to the rays at a unit distance from the source gives the illuminating power.

293. The amount of light falling upon a surface, as has been remarked above, depends upon the angle of the inclination. To understand this more fully, let us consider the surface AB placed obliquely with reference to a cone of light OPQ (Fig. 8). The

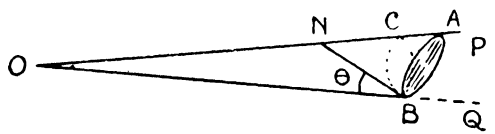


Fig. 8

amount of light that falls on BC will fall in its absence on AB. Evidently AB is greater than BC; as a matter of fact $BC = AB \cos \theta$, the angle ACB being a right angle; hence we see that the same amount of light which falls on BC now falls on a larger area; and if other conditions, the illuminating power of the source, the distance of the surface from the source, are constant, then the amount of light per unit area when the surface is oblique is less than when it is normal. Mathematically, suppose L is the amount of light falling upon BC, the amount of light per unit surface will be given by $\frac{L}{BC}$ and the amount of light falling upon a unit area of AB is $\frac{L}{AB}$, but $BC = AB \cos \theta$;

hence $$\frac{L}{AB} = \frac{L}{CB} \cos \theta.$$

Thus we see that the cosine law holds good, *i.e.* the amount of light falling upon a unit surface is proportional to the cosine of the

angle which the rays make with the normal to the surface. If it is 0° the amount of light is maximum, whereas if it is 90° the amount is zero.

294. Law of Inverse Square.—The intensity of light diminishes as we recede from the source of light and increases as we come nearer. Let us see how and according to what law the intensity of illumination varies with the distance from the source.

Consider a point-source uniformly emitting light in all directions, and imagine a sphere of unit radius to be drawn round that point as centre. The amount of light falling upon this sphere will after some time fall upon a second sphere, say at a distance of 2 cm. from the centre, there being no accumulation of light in the intervening space. But the surface of a sphere is equal to $4\pi r^2$, hence the surface of the second sphere is four times the surface of the first sphere, as r has become double. This shows that the amount of light falling on a unit surface of the second sphere is only $\frac{1}{4}$th ($=\frac{1}{2^2}$) of the amount falling on a unit surface of the first sphere. Similarly if we take a sphere at a distance $r=3$ cm., the light falling on a unit surface will be $\frac{1}{9}$th ($=\frac{1}{3^2}$) of the amount falling on the first.

Thus we see that the law is of inverse squares, i.e. *the intensity is inversely proportional to the square of the distance, provided the medium is perfectly transparent.* The same thing can be proved experimentally with the help of a wire pyramid.

Let us consider the cone of light *LMNOP* (Fig. 4) which just falls on a square piece of cardboard, *A*; the same cone of light will illuminate at double the distance the cardboard piece *B*. Geometry of the figure tells us clearly that the piece *B* is four times bigger in area than the piece *A*. This shows that at double the distance a given amount of light falls upon four times the surface, and therefore the intensity of illumination, *i.e.* the amount of light incident per second upon a unit surface, is only $\frac{1}{4}$th.

Mathematically it can be represented as

$$I = \frac{L}{r^2},$$

where I = intensity of illumination, L is the illuminating power of the source, and r is the distance.

295. Principle underlying Photometry.—The various methods of comparing the illuminating powers of the sources of light are based on the law of the inverse square.

By means of some suitable apparatus the two sources are

made to illuminate two neighbouring surfaces. The distance of one or other of the sources is adjusted until the two portions appear of equal brightness. When this is the case the intensity of illumination due to the two sources is the same. Now if L_1 and L_2 are the illuminating powers of the two sources r_1 and r_2, the distance of each from the surface they illuminate, the intensity of illumination due to each, is respectively

$$\frac{L_1}{r_1^2} \text{ and } \frac{L_2}{r_2^2}.$$

Thus when these two are equal, we have

$$\frac{L_1}{r_1^2} = \frac{L_2}{r_2^2},$$

or

$$\frac{L_1}{L_2} = \frac{r_1^2}{r_2^2}.$$

Hence the illuminating powers of two sources are directly proportional to the square of the distances at which they respectively produce equal intensity of illumination.

296. Unit of Intensity of Illumination.—Before proceeding further let us fix upon a convenient unit in terms of which the illuminating powers of various sources can be expressed.

The unit generally adopted (in England) is the standard candle, although it is most unsatisfactory and a variable one. It is a sperm (a sort of fat obtained from a species of whale) candle weighing six to the pound, burning at the rate of 120 grains (7·776 grams) of wax per hour.

The candle-power of any source is, then, the number of standard candles which would have the same illuminating power as the source.

Let us consider the various photometers used in practice for comparing the illuminating powers of various sources.

297. Rumford's Photometer.*—This consists of a rod of the shape of a lead pencil, which is mounted vertically before a screen. Fig. 9 (*b*) gives the plan of the arrangement: R is the rod, AC the screen, L_1 and L_2 the two sources.

BC is the shadow formed by L_1, and AB is formed by L_2. The distances from the screen are adjusted in such a way that the two shadows are just side by side and are equally dark. It should be noted that portion BC is being illuminated by L_2 and AB by L_1; and hence what we mean by saying

* This photometer possesses an advantage that it can be used in a room which is not quite dark. The extra light produces no effect.

that the two shadows are equally dark is that the two portions are equally illuminated by the two sources. Now measuring

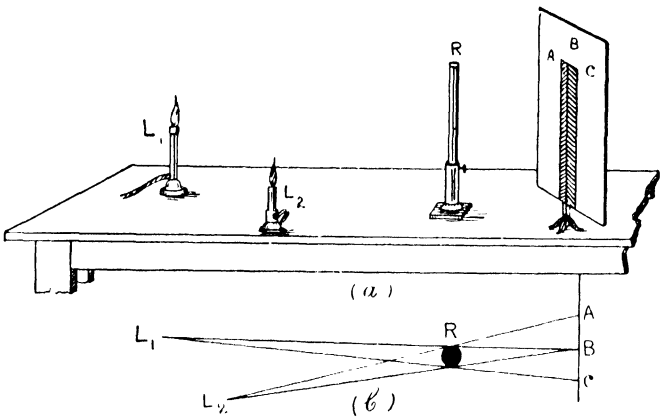


Fig. 9

the distances with a metre rod we can compare the illuminating powers of the sources.

For let r_1 and r_2 be the distances from the screen of L_1 and L_2, we have as before,

$$\frac{L_1}{r_1^2} = \frac{L_2}{r_2^2} \text{ or } L_2 = L_1 \frac{r_2^2}{r_1^2}.$$

Note, the screen should possess an unglazed surface so that nothing like a regular reflection might take place.

298. Bunsen Photometer.—It is made up of a circular piece of unglazed paper, in the middle of which a small portion has been rendered translucent by a drop of grease.* It is placed between two sources in such a way that the centre of the spot and the two sources are in the same straight line. It possesses the advantage of simplicity, but it is difficult to make a grease spot with a sharp boundary, and of course it is impossible to detect slight inequalities in the illumination when the boundary is not sharp.

The grease-spot is translucent and the surrounding paper is comparatively opaque. If the distance of the photometer is adjusted so that the grease spot appears of the same bright-

* The grease spot should have sharp edges, and it is better, on account of this reason, to make the spot star-shaped.

ness as the surrounding paper, the illuminating powers of the two sources are as the square of their distances from the photometer.

Let us take up the theory of this photometer. Look at a candle through the paper; the spot appears brighter than the rest of the paper, for it is translucent and allows more light to come through. Hold the paper against a dark background; the spot looks darker, for it allows more of the light which falls upon it to pass through than the rest of the paper, which scatters the incident light considerably and hence sends a large part of it to the eyes.

Now if the amount of light lost through the spot by transmission could be exactly made up by the light coming through

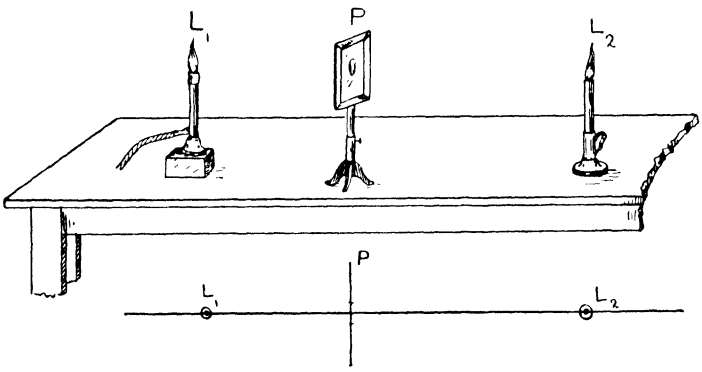


Fig. 10

it, the spot will become indistinguishable. This is done by placing two sources of light (say L_1 and L_2) on the opposite sides of the grease-spot, and so adjusting their distances from the photometer that the spot appears equally bright with the rest of the paper. Suppose these distances are d_1 and d_2 cm. Let the intensities of illumination on the two sides be I_1 and I_2. When we look at the photometer from the left side (*i.e.* the side of L_1) our eye receives mainly diffused light from the opaque portion and transmitted light from the spot. To be exact, let us suppose g and s are the fractions of the incident light which are scattered by the grease-spot and the opaque portion respectively. Evidently the fractions $(1-g)$ and $(1-s)$ are transmitted.

The light reaching our eye from the opaque portion is $I_1 s + I_2(1-s)$. The first part is due to the diffused light due

to L_1, and the second part is the transmitted light due to L_2.

The light reaching our eye from the grease-spot is $I_1 g + I_2 (1-g)$.

When the spot is equally bright with the rest of the paper,

$$I_1 s + I_2(1-s) = I_1 g + I_2(1-g),$$

or $$I_1(s-g) = I_2(s-g),$$

or $$I_1 = I_2,$$

or $$\frac{L_1}{r_1^2} = \frac{L_2}{r_2^2},$$

or $$L_2 = L_1 \frac{r_2^2}{r_1^2}.$$

Bunsen photometer is in practice very difficult to work with, for it is found that if the spot is made equally bright with the rest of the paper when viewed from one side, it is not at all so when viewed from the other side. So in actual working, the adjustment should be made, not for disappearance of the spot, but for the same contrast effect on the two sides; *i.e.* the spot should appear equally darker as compared to the rest of the paper on both the sides.

Frequently two mirrors, one on each side of the photometer, are fixed at an angle of 45°, to enable the observer to see both the sides simultaneously.

299. These historical photometers are not very accurate, and they are practically useless in comparing the illuminating powers of sources which emit lights of different colours. A more accurate apparatus than these two is the **Flicker photometer.**

A circular disc of stout white cardboard is divided into 8 equal sectors, and the alternate sectors are removed as shown at W (Fig. 11). The disc is then mounted on an axle through its centre and can be rotated in a vertical plane by a suitable arrangement. One of the sources of light, L_2, is placed so as to illuminate the disc in such a way that the light from the source falls on the sectors of the disc at an angle of 45°. A screen made up of the same stuff is placed just behind the disc at right angles to it and is illuminated by the second source, L_1, also at an angle of 45°. The disc is set in rotation and is viewed through a tube T having a small pinhole, placed in such a position that light enters it after reflection from the sectors as well as the screen. The eye looking through T first sees one of the sectors of W illuminated by L_2, and then as the disc moves on, the gap takes the place of the sector and the screen illuminated

by L_1 comes into view. The two screens are thus presented to the eye in rapid succession, and if the illumination is different, a very unpleasant flicker is produced. The distances of the two

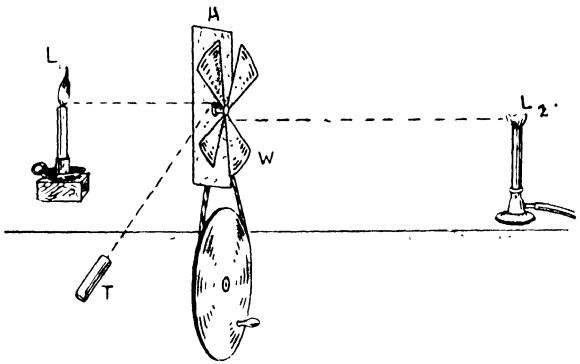


Fig. 11

sources from their respective screens are adjusted until the flicker disappears. The calculations are made as in the other case. As the sensation of flicker is independent of the colour of light, hence the photometer can be used when the sources emit lights of different colours.

CHAPTER III

REFLECTION OF LIGHT

300. When a ray of light travelling in one homogeneous medium meets the boundary of another homogeneous medium, in addition to the portion which is absorbed, it is split up into three parts :

(*a*) A part is reflected or sent back to the first medium in a certain direction according to certain laws.

(*b*) A part is scattered or diffused by the surface. It goes into no particular direction. It is by this light that a non-luminous surface becomes visible in the presence of a luminous body.

(*c*) The rest is transmitted into the second medium in a direction determined by definite laws. This part is said to suffer refraction.

When light falls upon an opaque body, a portion of it is always absorbed. (Generally no regular refraction takes place.) The rest of the incident light is sent back to the first medium both as scattered and reflected light; the ratio of these two quantities depends upon the nature of the surface and upon the angle of incidence. A polished surface may reflect as much as 98 per cent of the incident light.* But if the body is a rough one, mostly the light is scattered by it; very little is regularly reflected. A piece of paper, for instance, scatters 70 per cent of the incident light. It is not only the nature of the surface which governs the proportion between the regularly reflected and the scattered light, but also obliquity. At very oblique incidences even bad reflectors become good reflectors; for example, still water which at normal incidence reflects only 2 per cent of the incident light, reflects as much as 33 per cent at 80° (angle of incidence) and 72 per cent at 89° 30′ (angle of incidence).

* Such a surface will be almost invisible if it is not sending directly the reflected beam to the observer, owing to very little diffusion.

301. Definitions.—Any smooth surface from which light is reflected is called a **mirror**; but in practice, however, the term is restricted to polished surfaces of a definite geometrical form, as a plane, a part of a sphere, or a paraboloid, etc.

The **normal** to a reflecting surface at any point is a line drawn at right angles to the tangent plane to the surface at that point. If the surface is plane, the normal at any point is at right angles to the surface, and if spherical it is coincident with the radius drawn through that point.

The **angle of incidence** is the angle between the incident ray and the normal to the reflecting surface at that point of incidence.

The **angle of reflection** is the angle between the reflected ray and the normal to the surface at the point of incidence (the same point being the point of reflection).

The **plane of incidence** is the plane containing the normal and the incident ray. Similarly the **plane of reflection** is the plane containing the normal and the reflected ray.

When a beam of light is allowed to strike the surface of a piece of highly polished metal or glass, the light leaves the surface in accordance with definite laws known as the **laws of reflection**. They are two in number:

1. *The incident ray, the normal to the surface at the point of incidence, and the reflected ray lie in one plane.*

2. *The angle of reflection is equal to the angle of incidence.*

302. Experimental Proof.—Take a plate bent into a semicircle and divided into 180° with a slit at the middle. At the centre of the semicircle fix a rod in the vertical plane. Mount on it a small plane mirror, to which is attached a pointer which moves over the graduations. Hold a candle near the slit, and tilt the mirror so that the pointer makes an angle with the slit; it will be found that the reflected ray makes an equal angle with the pointer (which coincides with the normal to the mirror) on the other side. By tilting the mirror through different amounts it will be seen that the angle of incidence is always equal to the angle of reflection. If the incident beam be allowed to graze the bottom of the apparatus it will be seen that the reflected ray also grazes along the surface, thus proving the first law.

The laws can be proved also in another simple way. Fix a piece of white paper on a drawing board and mark a line AB on it. Support vertically a piece of polished plane mirror in the groove made in a side of a block of wood (Fig. 12).

Stand the mirror with its silvered surface on the line. Fix

a pin Q near the front surface of the mirror, and another, P, at a great distance from it, say 10 to 12 cm. in an oblique direction. Look into the mirror and view the images of these two pins; move the eye until the image of Q covers that of P. Keep the eye in this position, and fix two pins R and S at a distance of about 8 or 10 cm. from each other so that these two pins, together with the images of P and Q, appear in the same straight line. Remove the mirror and draw the lines PQ and SR, and produce them to meet at O. Draw ON as

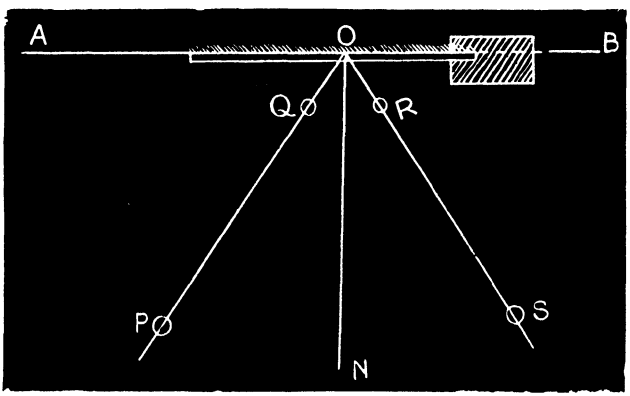


Fig. 12

the normal to the mirror at O. Measure the angle of incidence PON and of reflection SON; they will be found to be equal.

Repeat this three or four times, taking a different angle of incidence in each case; it will be found that in every case the angle of incidence is equal to the angle of reflection.

303. Image.— *If a pencil of rays diverging from a point is caused by reflection (or refraction) to converge to or to appear to diverge from some other point, this second point is called the image of the first point.* If the rays of light actually pass through the second point, the image is said to be **real**, but if the rays simply appear to do so, the image is called **virtual**.

An ***optical image*** *is, therefore, a point where rays of light either intersect or appear to do so.* Hence we need trace the path of any two rays by which it is formed in order to fix its position, because *where any two rays intersect*, the others also would meet there.

The image of an object is simply an assemblage of the

image-points corresponding to the various points which form the object; thus an assemblage of image-points will make up a more or less correct reproduction of the object.

It should be noted here that the picture formed by the pinhole camera is not an optical image at all, as the course of the rays of light is not changed in any way in passing through the pinhole and the rays do not intersect upon the screen. Moreover, in this case a picture will be formed on the screen whatever its distance from the pinhole, and not at some particular point only, as in the case of an optical image.

304. Image in a Plane Mirror.—When an illuminated object is placed before a plane mirror, the rays of light

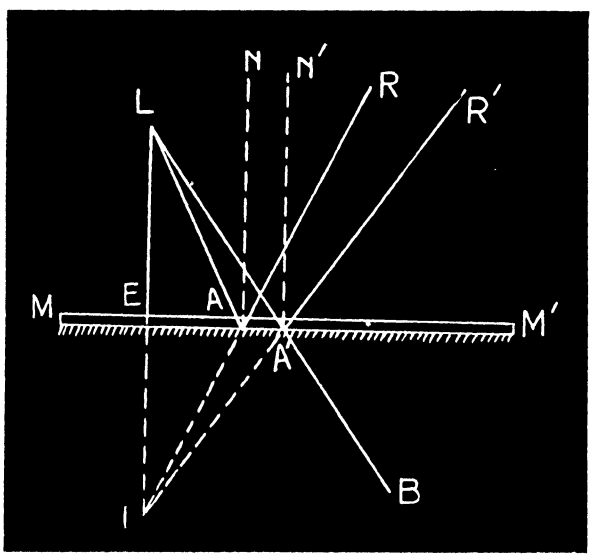


Fig. 13

emanate from it and after falling on the mirror are reflected to the eye. The image of the object is seen reflected in the mirror *in the direction of the rays which enter the eye.*

Let L denote the position of the luminous point and MM' that of the mirror. Consider the reflection of any ray LA (Fig. 13). Draw the normal AN at A, then according to the law of reflection the reflected ray AR will be on the other

side of AN, making the angle of reflection NAR = the angle of incidence LAN.

Draw LE perpendicular to MM' and produce RA to meet it in I, then LI and AN both being at right angles to MM' are parallel.

$$\angle RAN = \angle RIL, \text{ being corresponding angles,}$$

but $$\angle RAN = \angle NAL = \angle ALE,$$

$\therefore$ $$\angle RIL = \angle ALE.$$

But $\angle RIL$ is the same as $\angle AIE$; hence $\angle AIE = \angle ALE$. In Δs ALE and AEI,

$$AE \text{ is common,}$$

$$\angle ALE = \angle AIE,$$

and $$\angle AEL = \angle AEI, \text{ being rt. angles,}$$

hence the triangles are congruent.

$\therefore$ $$LE = EI,$$

i.e. the reflected ray if produced backwards intersects the normal LE at I, a point as far behind the mirror as the object is in front of it. It will be seen that the result is independent of the position of the point of incidence A. Hence all the rays appear to proceed from I after reflection.

Thus the image formed in a plane mirror is as far behind the mirror as the object is in front of it.

It should be noted, however, that as the rays do not actually pass through I, but only appear to do so, the image is a virtual one.

Taking any ray into consideration, it is clear that on account of reflection it is turned away from its original path; for instance, the ray $LA'B$ after reflection goes into the position $A'R'$, *i.e.* turns through an angle $180° - 2\ \angle LAN'$. This change in angle is called **deviation.**

Experimentally also we can establish this relation between the distance of the object and of the image from the mirror.

Take a plane mirror supported in a groove made in a side of a block of wood and stand it as before with its silvered side on the line AB. Fix vertically in front of it a pin, O. View the image I of the pin O along PQ, and fix vertically two pins P and Q, so that both of them are in the same straight line with I. Similarly view I along SR, and fix two pins R and S so that they are also in the same straight line with I. Remove the mirror and join PQ and SR. Since the image lies along PQ as well as along SR, its true position is the

point of intersection, I, of these two lines. It will be found that $OE = EI$.

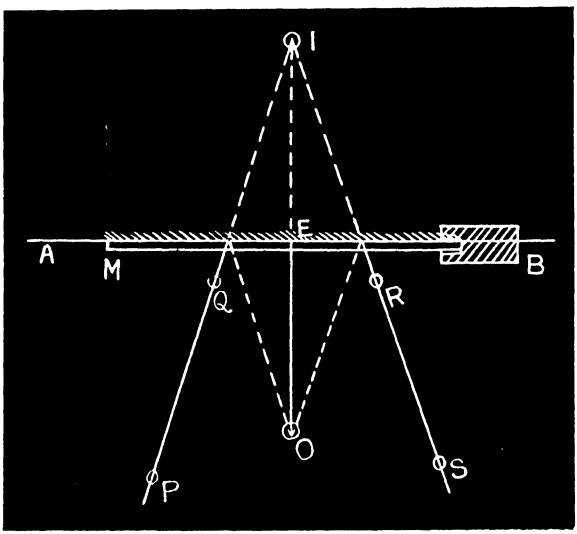


Fig. 14

305. There is a second method of locating the position of the image. It is based upon what is called **parallax**. Look at two objects at different distances from the eye in such a way that the nearer object hides the distant one from view (finger and a post or edge of the door will do). Move the eye sideways; the two objects will appear to shift relatively to each other. They will no more cover each other. *This apparent shift or separation of the two objects situated at different distances from the eye relatively to each other when the eye is moved sideways is known as Parallax.*

Once more place your eye in such a position that the objects are one behind the other. Now move your eye to the left and notice that the distant object moves in the same direction in which the eye moves, whereas the nearer object moves in the opposite direction. This is a very important result, for it enables one to know which of the two objects is distant. Remember that **the distant object appears to move in the same direction as the observer's eye.**

If the two objects be at the same distance from the eye

they will not separate out in whichever direction the eye may be moved.

Experiment.—Take a strip of plane mirror the central portion of which is unsilvered. Stand it on the line AB on the paper fixed on the drawing board as before (see § 304). Fix in front of it a pin at O. Through the lower portion of the mirror its image can be seen, whereas through the unsilvered portion the second pin which is to be fixed behind the mirror can be seen directly. Fix the second pin I in such a way that the image of O as seen through the lower portion of the mirror is just below the upper portion of the pin I seen through the unsilvered portion. Move the eye to one side, say to the right, and see which of the two moves to the right, and hence is more distant, whether the pin I or the image. Adjust the position of I till there is no parallax. Remove the mirror and join OI, cutting AB in E. Measure OE and EI. They will be found to be equal.

306. Reflection of a convergent pencil.—In the preceding article we have dealt with the reflection of a divergent pencil and have shown that after reflection it *appears to diverge* from its image. Now let us see how will a convergent pencil be reflected.

Let a converging pencil OL' be incident on the mirror MM' (Fig. 15). In the absence of the mirror the rays would converge to a point L; but owing to the mirror the rays are

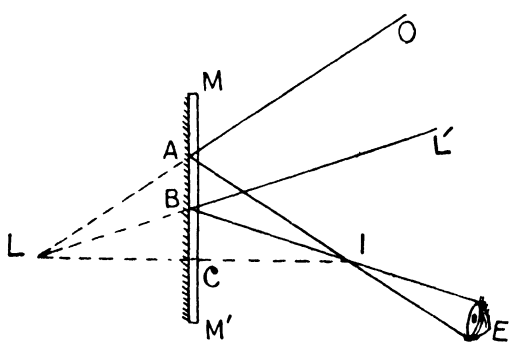


Fig. 15

reflected and converge to the point I, where $CI = CL$, which can be proved easily and in the same way as was done in the case of a diverging pencil.

In this case the eye placed at E sees the real image, because the rays actually pass through that point.

This case should be carefully distinguished, as *this is the only case in which the plane mirror gives a real image, for otherwise the plane mirror always gives a virtual image.*

307. Image of an object in front of a mirror.—Let MM' represent the trace of the mirror and OO' the object placed in front of it (Fig. 16). From O and O' drop ON, and $O'N'$ perpendiculars on MM' and produce them so that $ON = NI$ and $O'N' = N'I'$. Then I and I' are the images of O and O'. Proceeding in the same way, the image of all the points lying between O and O' can be determined, and thereby the complete image of the object can be constructed. It will be noticed that the image is upright, but is laterally inverted, viz. the right-hand side of the object is the left-hand side of the image.

308. To trace the course of rays by which image is seen.—Suppose OO' is the object and we want to trace the rays by

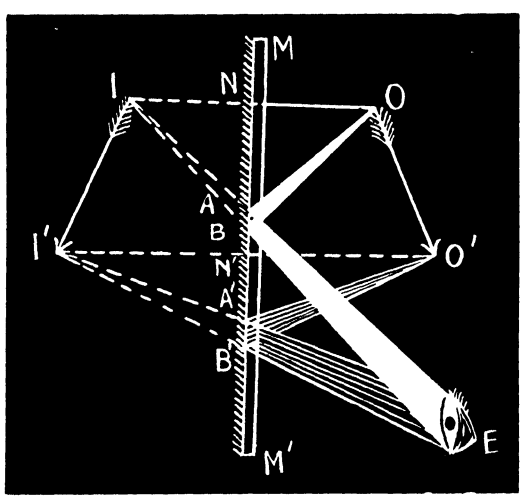


Fig. 16

which we see the point O'. Join I', the image of O', to the extremities of the eye. One ray cuts the mirror at A' and the other at B'; join O' to A' and B'. It is evident from Fig. 16 that the point O' is seen by the cone of rays comprised between $O'A'E$ and $O'B'E$.

Note, the principle of graphically tracing the path of rays which are employed in seeing any point of the object is to *join the image of the point to the extremities of the eye and then join the points of intersection of the mirror to the object point.*

It is clear from Fig. 16 that to see the image of OO' only the small portion AB' of the mirror is used. The rest of the mirror is useless. How much and which part of the mirror is used in seeing the image of an object depends upon the position of the object and the eye. It is well to remember that if the object is lying anywhere in front of the plane of the mirror it will give rise to an image though it may not be visible; for to see it we must place the eye in a suitable position. We shall make use of this fact later on.

309. Deviation produced by the rotation of the plane mirror.—If a ray of light falls upon a plane mirror and the

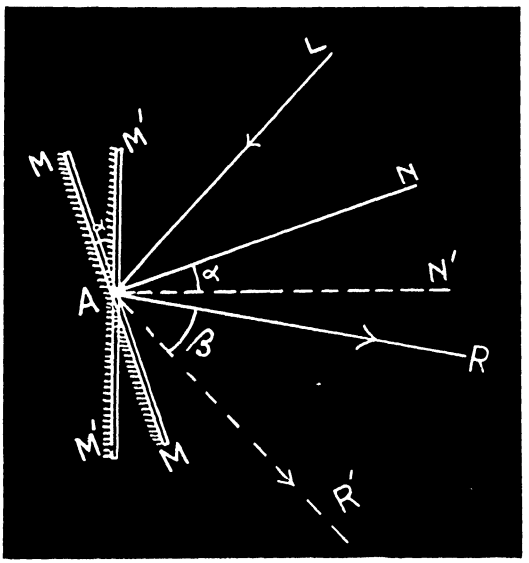


Fig. 17

mirror be turned through an angle, the reflected ray turns through double that angle.

Let LA (Fig. 17) be the incident ray falling upon the mirror MM. It will be reflected along AR, where

$$\angle LAN = \angle NAR \quad . \quad . \quad . \quad . \quad (i)$$

Let the mirror be now turned through an angle α and the reflected ray through β; the normal AN also will be turned through α. New angle of incidence LAN' = new angle of reflection $R'AN'$.

But $$\angle R'AN' = \angle R'AR + \angle RAN'$$
$$= \beta + \angle N'AR = \beta + (\angle NAR - \alpha).$$

Hence $$\beta + \angle NAR - \alpha = \angle LAN'$$
$$= \angle LAN + \alpha.$$

But $$\angle NAR = \angle LAN \quad . \quad . \quad . \quad .$$ by (i.)

Therefore we get
$$\alpha = \beta - \alpha,$$
or
$$\beta = 2\alpha.$$

Thus we see that *when the mirror moves through a certain angle, the reflected ray moves through double that angle.* This fact is very useful, and is employed in the measurement of small angles.

310. Successive reflections from two mirrors.—A pencil of light may undergo reflection at two or more plane mirrors

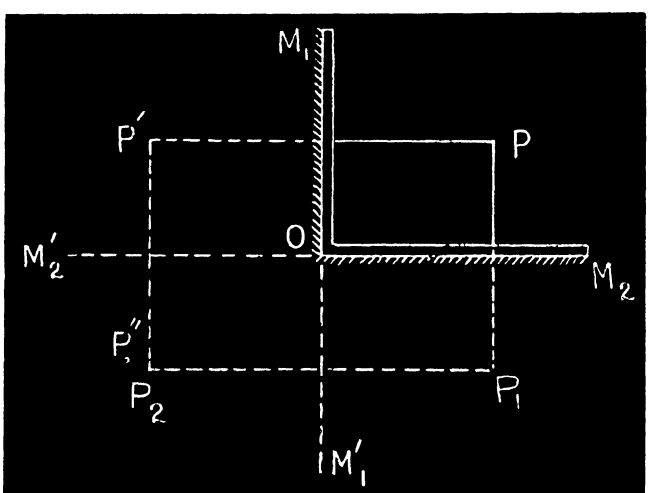


Fig. 18

before it reaches the eye. Let us consider the case when it is successively reflected from two mirrors. Imagine the object

P placed between two mirrors M_1 and M_2 (Fig. 18) inclined to each other at an angle. An image of P will be formed by each mirror, and if the image formed by M_1 lies in front of M_2, *i.e.* if it is anywhere in front of the plane of this mirror, an image of this image will be formed by M_2. Similarly if the image formed by M_2 lies in front of M_1, an image of this image is formed by M_1. These are said to be images of the *second order*. In precisely the same manner we can get images of the 3rd, 4th order, etc. This multiplication of images stops when a pair is formed in the space behind both the mirrors.

311. Mirrors inclined at right angles.—Let OM_1 and OM_2 (Fig. 18) represent two mirrors at right angles to each other. By direct reflection image P' is formed in OM_1, and image P_1 in OM_2. But P' lies in front of OM_2 and therefore an image P'' is formed by that mirror. Similarly P_1 lies in front of OM_1, and therefore gives rise to image P_2. It is easy to show from the geometry of the figure that these two images are coincident. Since both of them are at the back of the mirrors, further reflection is impossible. Thus in this case we get three images in all.

312. Mirrors inclined at any angle.—Let OM_1 and OM_2 as before represent the mirrors meeting at O. Let P be the

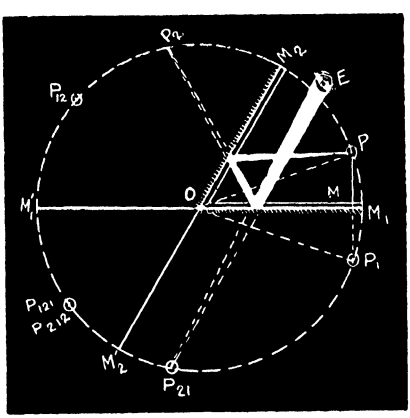


Fig. 19

luminous point lying anywhere between them. Draw $PM \perp$ to OM_1, and produce it to P_1, so that $PM = P_1M$. Then P_1 is the image of P. Now in triangles OPM and OP_1M, OM is common, $PM = P_1M$, and angles PMO and P_1MO are right angles, therefore $OP = OP_1$. Describe a circle with centre O

and radius OP cutting the mirrors in M_1 and M_2. P and P_1 both lie on this circle. We can show in the same way that P_2 lies on the circle. As a matter of fact, by repeating the same steps it can be proved that all the other images also lie on this circle. No further images will be formed when the image falls on the arc $M'_1M'_2$.

By drawing an accurate diagram it will be found that when the angle between the mirrors is 60° the number of images is five, and seven if it is 45°. The number of images continues to increase as the angle diminishes, and when it is zero, *i.e.* when the mirrors are parallel, the number of images theoretically should be infinite. In general, if two mirrors are inclined to each other at an angle θ (θ being an exact sub-multiple of π), they produce twice as many images, counting the object as one, as the angle between the mirrors is contained in 180°, *i.e.* the number of images n can be expressed mathematically by $\frac{2\pi}{\theta}$, including the object also. If the object be not included the number of images will be $\frac{2\pi}{\theta} - 1$.*

313. Parallel Mirrors.—Since the mirrors are parallel, the angle between them is zero, and therefore theoretically the number of images will be infinite, for $\theta = 0$ in the expression $\frac{2\pi}{\theta}$. The physical explanation of this fact is that owing to the mirrors being parallel, no image can be formed behind both; it is in front of one of the mirrors at least, and hence every image gives rise to another image and thus an infinite number of images is formed. But as reflection is never complete, hence intensity of light becomes less and less at each successive reflection, with the result that the images formed by a great number of reflections are too faint to be seen. Let AM_1 and BM_2 (Fig. 20) represent the trace of two mirrors M_1 and M_2 and O an object placed between them. Draw BOA normal to both the mirrors and produce it on both the sides. All the images will lie on this line, and their positions will depend upon the position of the object O between the mirrors and the distance between them.

* To be more precise, let us suppose $\frac{2\pi}{\theta} = n$; then when n is odd, the number of *images* is also n, except when the object is in the middle; in that case there are only $n - 1$ *images*.

When n is even, the number of images is $n - 1$ only, whether the object is in the middle or not.

As is evident from the figure, there will be two series of images, one beginning with reflection in M_1 and the other in M_2.

Place a candle between two parallel mirrors and observe the infinite train of images. Notice that the images become fainter and fainter as the number of reflections by which they are produced increases. At the same time they look smaller and smaller in size, whereas actually they are all of equal size. It is owing to the reason that the distance of these

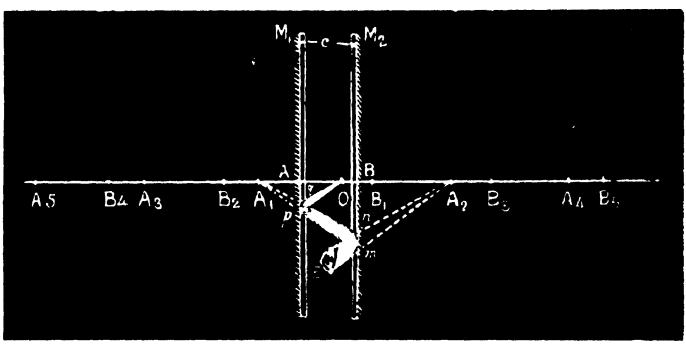


Fig. 20

images increases from the eye, and it is a well-known fact that if articles equal in size be placed at different distances from the eye the distant ones look smaller.

Let the distance between the mirrors be denoted by c and that between O and M_1 by a, and between O and M_2 by b, so that $a + b = c$.

Let us consider first the series beginning with the reflection in M_1.

The object will give rise to A_1 as the first image, where $AA_1 = AO = a$.

If its distance be measured from the object we get $OA_1 = 2a$.

A_1 is in front of M_2, hence it will give rise to image A_2, where as before—

$$A_1B = BA_2 = 2a + b \text{ (for } A_1B = A_1O + OB).$$

But $$OA_2 = OB + BA_2 = 2a + 2b = 2c.$$

Now A_2 will give rise to the image A_3,

where $AA_3 = AA_2 = 2c + a$ (for $AA_2 = AO + OA_2$).

$\therefore$ $$OA_3 = OA + AA_3 = 2c + 2a.$$

A_3 will give rise to the image A_4,
where $A_3B = BA_1 = 2c + 2a + b$.
$\therefore \quad OA_4 = OB + BA_3 = 2c + 2a + 2b = 4c$.
Similarly $OA_5 = 4c + 2a$
$(AA_5 = AA_4 = 4c + a\,;\ \therefore OA_5 = 4c + 2a)$.

Thus we see that the series can be written as

$OA_1 =$ zero $c + 2a$
$OA_2 = 2c$
$OA_3 = 2c + 2a$
$OA_4 = 4c$
$OA_5 = 4c + 2a$
$OA_6 = 6c$
$OA_7 = 6c + 2a$, and so on.

Similarly consider the second series arising out of the first reflection in M_2; we shall get

$OB_1 =$ zero $c + 2b$
$OB_2 = 2c$
$OB_3 = 2c + 2b$
$OB_4 = 4c$, and so on.

The series enables us at once to find out the distance of an image of any order from the object.

314. To trace the course of rays by which an image is seen.—Suppose we desire to trace the course of rays which enable us to see the image A_2. First trace this image back to O: A_2 is the image of A_1, A_1 is the image of O. Join the extremities of the aperture of the eye to A_2 by lines cutting M_2 at m, n; now join m, n to A_1 and join the points of intersection of these rays with M_1 (p, q) to O; it is with the help of the cone of rays comprised within $OqnE$ and $OpmE$ that we see the image A_2.

315. Kaleidoscope.—If the angle between the plane mirrors be 60°, five images will be seen, and these, with the object, will be arranged symmetrically with respect to the mirrors. This fact is made use of in the construction of the toy called kaleidoscope, which in its simplest form consists of two and sometimes three narrow strips of mirrors enclosed in a tube and inclined to each other at 60°. One end of the tube is formed by a piece of metal or cardboard with a hole at its centre; the other end of the tube is closed with a piece of ground-glass. A number of pieces of coloured glass, which rest on the ground-glass and can be shifted about by rotating the tube, are kept loosely in their places by a disc of plane glass. An observer looking through the hole along the axis of the tube sees a symmetrical pattern formed by those pieces and their images in the mirror. As the tube is rotated the pattern seen changes its arrangement.

316. Multiple images formed by a thick mirror.—Place a candle close in front of a thick glass mirror and look somewhat obliquely at the mirror. A number of reflections will be seen; of these, the image nearest the candle is fainter than the next one, which is the brightest of the series, while others gradually decrease in brightness. The first image is formed by the light which is reflected from the front surface of the mirror. Most of the incident light, however, penetrates the glass, being refracted at the first surface. On reaching the silver at the back it is all reflected to the front surface, where again most of it is refracted out, appearing to come from the second (*i.e.* the brightest) image. A part of it, however, is reflected back from the front surface of the glass and emerges after two or more reflections (at the back), giving rise to the other images; but the brightness decreases as the number of reflections increases.

317. Reflection at Spherical Mirrors.—The laws of reflection are true, whatever be the form of a reflecting surface. If the surface is not plane, the direction of its normal is different at each point, and hence the determination of the position of the image formed by reflection is more complicated than in the case of a plane surface. The mirrors which are generally used in scientific work, in addition to the plane ones, are spherical, *i.e.* they form a part of a spherical surface. They are either concave or convex according as the reflection takes place from the internal (towards the centre of the sphere) or the external (away from the centre of the sphere) surface.

The **centre of curvature** of a mirror is the centre of the sphere of which the mirror forms a part.

The radius of the sphere, or in other words the distance of the centre from any point on the mirror, is called the **radius of curvature.**

The middle point of the mirror is termed the **pole** of the mirror.

The diameter of the boundary, which is generally circular, is termed the **aperture** of the mirror.

The straight line joining the centre of curvature to the pole is called the **principal axis** of the mirror. Any other line passing through the centre of curvature is called the **secondary axis.**

A section of a mirror by a plane passing through the centre of curvature and the pole is termed the **principal section.**

318. Suppose that a luminous point P (Figs. 21 (*a*) and (*b*)) is placed on the axis of the mirror; the ray of light from P incident along the axis will be reflected back, for the line

PCO or *POC* is the normal to the mirror at *O*. If we join the point of incidence *M* where another ray *PM* strikes the mirror to the centre of curvature *C*, the line *CM* or *CM* produced will be the normal to the mirror at *M*; and hence if we make the angle *RMC* equal to the angle *PMC*, or in the case of a convex mirror the angle *RMN* equal to the angle *PMN*, the line *MR* will be the path of the reflected ray. Let *MR*, or in the case of a convex mirror *MR* produced backwards, cut the axis at *P'*, then *P'* will be the point of

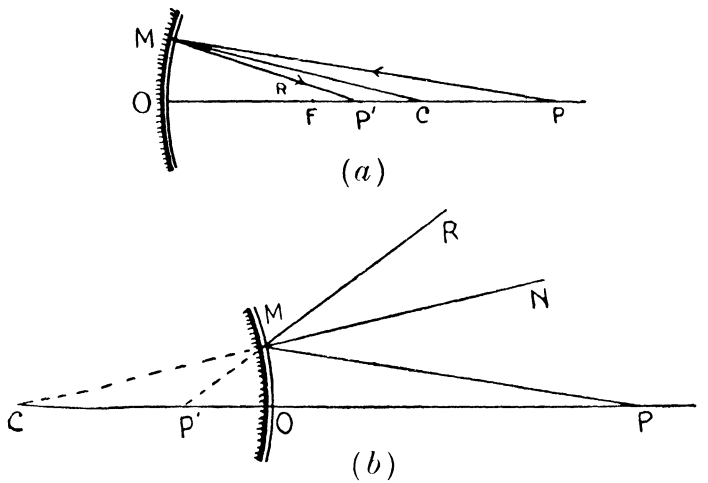


Fig. 21

intersection of the two rays incident at *O* and *M* after reflection. Thus *P'* will be the image of *P*, which may be real or virtual.

But if a parallel beam of light falls on a spherical mirror in a direction parallel to the principal axis (Fig. 22), the reflected beam converges to a point *F* on the principal axis. This point is called the **principal focus** of the mirror, and the distance *OF* is termed the **focal length** of the mirror.

In the case of spherical mirrors it is not strictly true to say that the reflected pencil meets exactly at a point; if the pencil is small, this is approximately the case, but with large pencils the outer rays are reflected to points nearer the mirror than the central ones. This failure of rays to converge accurately to focus at a point after reflection from a mirror is called **spherical aberration.**

319. The Position of Principal Focus.—Let PM be a ray incident on the mirror parallel to the principal axis (Fig. 22). It will be reflected along MF, where $\angle PMC = \angle CMF$. Now we want to find out the position of F.

$\angle PMC = \angle MCF$ . . . being alternate angles,

and $\angle PMC = \angle CMF$ . . . being angles of incidence and reflection;

$\therefore \quad \angle MCF = \angle CMF,$

hence $\quad CF = FM.$

Now if OM (the aperture) be small, FM will be approximately equal to FO,

$\therefore \quad FO = FC = \frac{r}{2},$

where r = radius of curvature.

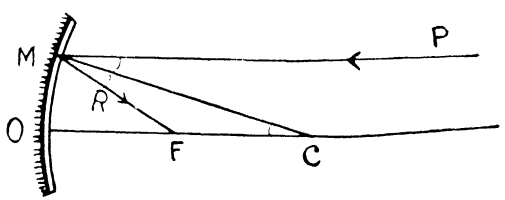


Fig. 22

*Thus the focal length of a spherical mirror is equal to half the radius of curvature of that mirror, provided the aperture of the mirror is small.**

Before dealing with images formed by the concave and convex mirrors it will be advisable to note the assumptions and conventions that are employed in these cases. The first assumption is that the aperture of the mirror is small; to put it trigonometrically, we assume that the angular aperture is so small that the angle itself is equal to its tangent. The second assumption is that light is not incident very obliquely, but on the other hand the axis of the incident pencil of light is nearly parallel to the principal axis of the mirror and the pencil is not very divergent, but is a narrow beam in order to avoid spherical aberration.

The conventions are that—

(*a*) the distances are measured along the axis from the pole of the mirror;

* The student should try to prove in a similar manner the corresponding proposition for a convex mirror.

(*b*) when the direction of measurement is opposite to that in which the incident light travels the distance is positive, otherwise negative.

Consequently the radius of curvature of a concave mirror is positive, whilst that of a convex mirror is negative.

These conventions and assumptions are very important, and they should be thoroughly grasped before proceeding further.

320. The Mirror Formula. — Let AOB represent the principal section of a concave mirror of which O is the pole and C is the centre of curvature. Let OC, the radius of curvature, be equal to r, a positive quantity according to our conventions. If P be the luminous point on the axis of the mirror, then according to the construction explained already in § 318, AI will be the reflected ray.

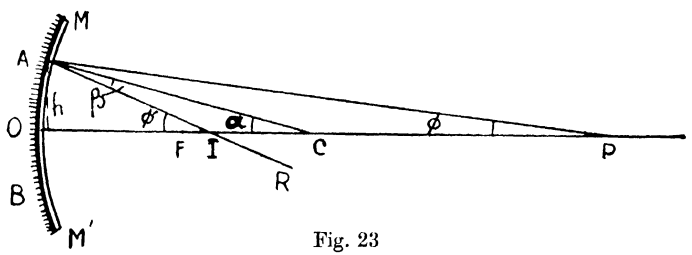


Fig. 23

To determine the position of the point I where the reflected ray cuts the axis, or, in other words, to determine the position of the image I, corresponding to the position P of the luminous point, proceed as under :

Let the distance of P from O (OP) be denoted by u,

" " I " " (OI) " " v,

and " " C " " (OC) " " r.

As has been said above, AI is the reflected ray,

$\therefore \quad \angle IAC = \angle CAP$ (AC being the normal).

Now in the ΔIAP, AC being the internal bisector of the angle IAP, will divide the base internally in the ratio of the sides :

$$\therefore \quad \frac{CP}{CI} = \frac{PA}{IA};$$

but as we have assumed the aperture to be very small,

$$PA = PO, \text{ and } IA = IO;$$

therefore under these circumstances

$$\frac{CP}{CI} = \frac{PO}{IO},$$

or

$$\frac{u-r}{r-v} = \frac{u}{v}.$$

Cross-multiplying, we get

$$vu - vr = ur - uv,$$

or $2\,vu = ur + rv$; divide by uvr, and we get

$$\frac{2}{r} = \frac{1}{v} + \frac{1}{u} = \frac{1}{f}\left(\text{as } f = \frac{r}{2}\right).$$

We can prove the same relation very simply making use of Trigonometry.

Let the angle APO (Fig. 23) be denoted by Φ, $\angle ACO$ by α,

$\angle PAC$ or $\angle CAI$ by β, and $\angle AIO$ by Φ'.

$\Phi' = \alpha + \beta$, but $\beta = \alpha - \Phi$ as $\alpha = \Phi + \beta$.

$\therefore$ $\Phi' = \alpha + \alpha - \Phi = 2\alpha - \Phi$.

Angles being small can be measured by tangents (vide our assumption).

$\therefore$ $$\frac{h}{v} + \frac{2h}{r} = \frac{h}{u} \text{ or } \frac{1}{v} + \frac{1}{u} = \frac{2}{r} = \frac{1}{f}.$$

Now let us consider reflection from a convex mirror (Fig. 24).

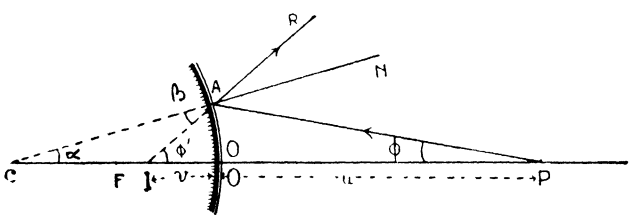


Fig. 24

Here AN is the external bisector of the angle PAR (the external angle of ΔPAI).

$\therefore$ $$\frac{CP}{IC} = \frac{PA}{IA} = \frac{PO}{IO},$$

$\therefore$ $$\frac{u-r}{-(r-v)} = \frac{u}{-v} \text{ or } \frac{u-r}{r-v} = \frac{u}{v},$$

as in the case of concave mirror. Simplifying, we get the same result in this case as well.

The relation $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$ holds good in the case of concave as well as convex mirrors. It is called the **mirror formula.**

The points P and I connected by this relation are said to be **conjugate foci,** because of the fact that either of them can be considered as the image of the other; that is to say, their positions are interchangeable.

Notice that in the case of the concave mirror the rays actually pass through I (hence the image is *real*), and that in the case of the convex mirror the rays simply appear to do so (hence the image is *virtual*).

321. Image of a small object placed on the axis of a spherical mirror.—So far we have been dealing with the image of a single luminous point, but now we shall proceed to find out the image of a small object placed on the axis of a mirror.

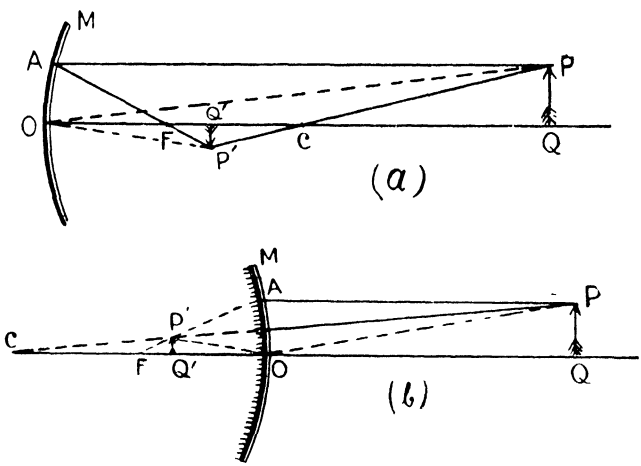


Fig. 25

The construction becomes very simple if we bear in mind the fact that the image of an object point will lie at the point where *any two* rays starting from the source intersect after reflection. The two rays generally taken are, one going parallel to the axis, which after reflection passes through the focus, and the other passing through the centre of curvature which, falling normally upon the mirror, retraces its path in the opposite direction after reflection. The intersection of these two rays gives the image corresponding to the object point.

Now let PQ be a small object. Each point of it can be considered as an object and a corresponding image can be determined. The collection of such images will be the image of the object. Consider first the point P; take the two rays as explained above, one going parallel to the axis which will pass after reflection at the mirror through F, and the second passing through C, which will return along the same path. Their intersection gives the image P'; similarly the image of any other point can be determined. It is seen at once from Fig. 25 that the image $P'Q'$ will be small in this case; following the same construction an image can be formed of any object lying anywhere on the axis.

322. Magnification.—The ratio of the size of the image to that of the object is said to be the **linear magnification.** To determine the magnification consider the two right-angled triangles POQ and $P'OQ'$.

$\angle POQ = \angle P'OQ'$, hence they are similar;

$$\therefore \quad \frac{P'Q'}{PQ} = \frac{OQ'}{OQ} = \frac{v}{u}.$$

Hence the ratio of the size of the image to that of the object is as the ratio of their distances from the mirror.

If we consider two similar Δ's OAF and $FP'Q'$ we get

$$\frac{I}{Q} = \frac{P'Q'}{PQ} = \frac{FQ'}{FO} = \frac{v-f}{f}.$$

The magnification can also be proved equal to $\frac{f}{u-f}$.

To prove this relation we should take a ray from P passing through F, which after reflection at the mirror would go parallel to the parallel axis.

Note, these relations are not independent of each other; on the other hand, they are all equal to each other. For instance, since

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}, \quad \therefore \quad \frac{u}{v} + \frac{u}{u} = \frac{u}{f},$$

or

$$1 + \frac{u}{v} = \frac{u}{f} \quad \text{or} \quad \frac{u}{v} = \frac{u}{f} - 1 = \frac{u-f}{f},$$

or

$$\frac{v}{u} = \frac{f}{u-f}.$$

Similarly by multiplying the mirror formula by v we can show that

$$\frac{v}{u} = \frac{v-f}{f}.$$

323. It is sometimes very convenient to measure the distance of the object and of the image from the principal focus instead of from the mirror. Let the distance between the object and the principal focus be called x, and that between the image and the principal focus x', these quantities being taken as positive if they are measured from F towards the object; we have for both types of mirrors, remembering that f is + for concave mirrors and negative for convex mirrors,

$$u = x + f, \text{ and } v = x' + f.$$

Substituting these values in the mirror formula we get

$$\frac{1}{f} = \frac{1}{(x+f)} + \frac{1}{(x'+f)},$$

or $$(x+f)(x'+f) = f(x+f) + f(x'+f),$$

or $$xx' = f^2.$$

This equation shows that the product of the distances, of the object from the focus and of the image from the focus is equal to the square of the focal length. It shows further that since f^2 is always positive, the product xx' must be positive; so that it may be so, each constituent must be of the same sign, i.e. *the image and the object must be on the same side of the principal focus.*

In order to find out the distance of the image it gives a very useful as well as a simple relation:

$$x' = \frac{f^2}{x}.$$

(I) Let us suppose first that the object is at infinity, $\therefore x = \infty$; hence

$$x' = \frac{f^2}{\infty} = 0,$$

i.e. *when the object is at infinity, the image is at the principal focus.*

(II) Next suppose the object is at the centre of curvature.

Here $$x = f,$$

hence $$x' = f,$$

i.e. *the image and the object are at the same distance* (viz. at the *centre of curvature*), *and are equal in size.*

(III) It is clear from this that if x be $> f$, x' must be smaller than f, i.e. *if the object lies between* ∞ *and* C, *the image will lie* (*real, and small in size*) *between the centre of curvature and the focus.* The reverse also will be true, viz. if the object lies

between the centre of curvature and the focus, the image will lie beyond the centre of curvature, and will be magnified.

(IV) Suppose further that the object is at the mirror, viz.

$$x = -f,$$

naturally

$$x' = -f,$$

meaning thereby that *both the object and the image are at the mirror, and are equal in size.* The image will in this case be virtual.

Thus we see that at a glance we can find out the relative position of the image corresponding to the position of an object. This formula is very useful in making numerical calculations.

Next consider for a moment the case of a convex mirror. In this case it should be remembered that although f is negative, yet f^2 is positive.

(i) When the object is at ∞ the image will be at F, erect, and virtual.

(ii) When the object lies between ∞ and the mirror, then x is less than infinity but greater than $+f$, therefore x' must be less than $+f$ (*i.e.* the image will be between F and O, and will be small, virtual, and erect).

(iii) Let the object be at O, the pole of the mirror, the image will be at O; it will be virtual and of the same size as the object.

Thus we see that *the image in the case of a convex mirror is between the pole and the focus and is virtual for all positions of the object in front of the mirror.*

Our remarks for the convex mirror hold good in case the object is a real one; but if, on the other hand, the object be virtual it is possible to get a real image, into the discussion of which we shall not enter. (Compare with the real image given by a plane mirror.)

324. It is very simple to trace the course of rays by which an image is seen. Let MOM' (Fig. 26) represent the principal section of a concave mirror, OQ the principal axis, and PQ the object lying on the axis. The image $P'Q'$ will lie also on the axis, and its position can be determined by the method already given. In order to trace the course of rays by which the image is seen we shall take up the rays from only one point of the image; the same construction can be applied to all other points. Let us suppose we want to trace the course of rays by which we see the point Q. Join Q' (the image of Q) to the extreme limits of the eye and produce these rays backwards to meet the mirror at A and A'. Join these

points to Q. The cone of light QAA' contains the rays which after reflection pass through Q' and then enter to the eye.

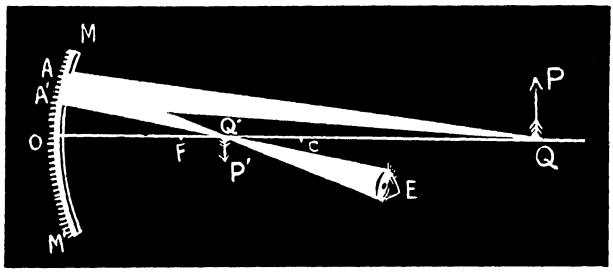


Fig. 26

Similarly we can trace the course of rays by which an image formed by a convex mirror is seen.

325. Experimental determination of the focal length of a concave mirror.—There are three methods which are commonly used for determining the focal length of a concave mirror.

1. The first is an approximate method, and consists in forming on a screen the image of a distant object, say the sun or a distant tree. The distance of the mirror from the image will give roughly the focal length.

2. Fix the mirror on an upright which is mounted on an optical bench. Place an upright carrying wire-gauze illuminated by a candle in front of the mirror. Place a third upright carrying a cardboard screen also before the mirror. The relative positions of the screen and the wire-gauze will depend upon the fact whether we require a smaller or a magnified image. In case of a smaller image the cardboard screen will be placed between the wire-gauze and the mirror. By moving about the cardboard screen a small well-defined image can be obtained upon it. Careful adjustment of heights of the wire-gauze and the screen should be made, for if the whole of light is cut off by the screen, no light is allowed to fall upon the mirror, and consequently no image will be obtained.

It is always desirable to take the illuminated wire-gauze as the object, as this enables us to say easily when the image is distinct and well-defined. In the case of the image of a bright object like candle it is very difficult to judge when it is most distinct. Measuring the distance of the object and of the image from the mirror, and applying the formula $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$, we

can determine the focal length. It must be noted, however, that the distance between the object and the mirror or the image and the mirror is usually measured by the distance between two fixed marks, one on each upright. Now, this distance may not necessarily be equal to the actual distance between the mirror and the object or the mirror and the screen. In order to know the difference, if there be any, the two uprights, say, carrying the mirror and the wire-gauze, are moved towards each other till the distance between them is equal to the length of an object of known length, say a knitting needle, whose length can be measured with sufficient accuracy with a scale. The distance between the fixed marks is read and the difference between these two distances, *i.e.* the length of the knitting needle and the distance between the fixed marks, gives what is called the **index error**. This must be taken into account when making measurements, and it should be added or subtracted as the actual length is greater or smaller than the length between the two fixed points.

Similarly the index error between the mirror and the screen should be determined and taken into consideration.

3. **Parallax Method.**—In this method two points should be borne in mind: the first is that if two objects at different distances from the eye be looked at so that they appear in the same straight line (one behind the other), on moving the eye sideways it will be found that they shift relatively to each other—*the distant object moving in the same direction in which the eye moves.* Secondly, *the image must be inverted.* Bearing these two facts in mind, we can proceed now to determine the focal length of a concave mirror.

Place a mounted needle (in an upright) in front of a concave mirror in such a position that the image seen in the mirror is inverted and is just above the needle. Move the eye and see whether the image and the needle seem to shift or not. If they do, move the position of the needle till a position is reached when the two (the image and the needle) do not seem to separate out with the moving of the eye. When this position is reached the distance of the needle and of the image is the same from the eye. This will be the case when the needle is at the centre of curvature, because then the image will also be there. Measure the distance between the needle and the mirror and take into consideration the index error. Half the corrected distance gives the focal length. This is, though simple, yet quite an accurate method.

EXERCISES

1. What will be the minimum size of the mirror which will enable a man 6 ft. high to see his full-length image when the top of the mirror is 6 ft. from the ground ?

Let AB be the man, EF the mirror, and CD his full-length image. So that he may see the image of part B, the minimum length of the mirror required is EF. But from the geometry of the figure it is clear that EF is $\frac{1}{2}$ AB, for ACD is a Δ, E is the middle point, and EF is parallel to CD, hence EF is $\frac{1}{2}$ CD or AB. Thus we learn that *to see a full-length image we require a mirror of half the size at least.*

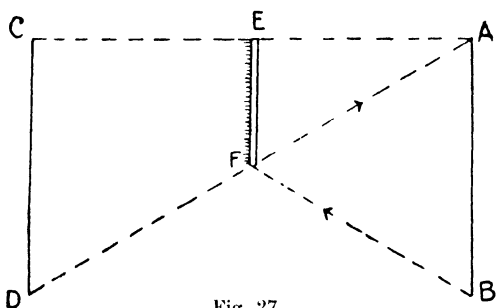


Fig. 27

2. What will be the angle between the mirrors when a ray incident on one parallel to the other after reflection at the second goes parallel to the first ?

Let OM_1 and OM_2 be the two mirrors, and let the angle between them be Θ. Let further the ray LA be incident on OM_1, parallel to OM_2; after reflection it falls on OM_2 and is reflected parallel to OM_1.

The angle of incidence NAL is $90-\Theta=NAB$, the angle of reflection. The angle $LAB=2(90-\Theta)$.

$$\angle M_2BC=\angle M_2OM_1=\Theta.$$

The angle of reflection $NBC = 90 - \Theta$

$= \angle ABN$, the angle of incidence,

$\therefore \quad \angle ABC = 2(90 - \Theta)$.

The angle $BCA = \angle CAM_1 = \angle M_2OM_1 = \Theta$.

The three angles LAB, ABC, and BCA of $\Delta\ ABC$ must be equal to 180°;

$\therefore \quad 4(90 - \Theta) + \Theta = 180°,$

$\therefore \quad 3\Theta = 180°,$

$\therefore \quad \Theta = 60°.$

Thus the angle between the mirrors must be 60°.

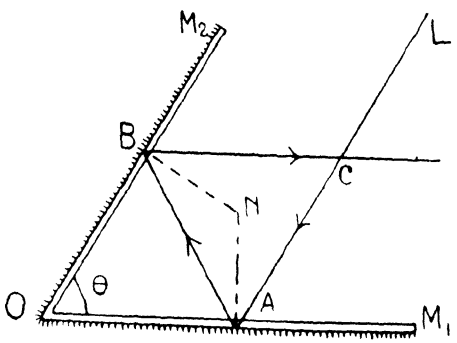


Fig. 28

3. In front of a concave mirror at a distance of 3 cm. is placed an object. The radius of curvature being 10 cm., find out the position of the image. What will be its nature?

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f},$$

$$\therefore \quad \frac{1}{v} + \frac{1}{3} = \frac{1}{5},$$

$$\therefore \quad v = -7{\cdot}5.$$

The image will be virtual, and erect. We could get the same result by applying the formula $xx' = f^2$; here $x = -2$ and $f = 5$; $\therefore x' = -12{\cdot}5$ or from the mirror $-7{\cdot}5$.

4. A convex spherical mirror has a radius of curvature of 10 cm. Find the position and size of the image when the

object is 1 cm. high and distant (i) 10 cm., (ii) 4 cm. from the surface of the mirror.

3·33 cm. behind.	2·22 cm. behind.
·33 cm. high.	·55 cm. high.
Virtual.	Virtual.
Erect.	Erect.

5. An image produced by a concave mirror is n times the size of the object. What is the distance of the latter from the mirror in terms of the focal length ?

$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f}, \text{ but } v=nu,$$

$$\therefore \quad \frac{1}{nu}+\frac{1}{u}=\frac{1}{f}$$

$$=\frac{1}{u}\left(1+\frac{1}{n}\right),$$

$$\therefore \quad u=f\left(\frac{n+1}{n}\right).$$

6. An object is placed 16 inches from the centre of curvature and 12 inches from the focus of a convex mirror. Find the nature and position of the image.

Ans. $v=-2\frac{1}{2}$ inches (virtual, erect, and diminished).

7. How far from a lamp must a concave spherical mirror of focal length 3 ft. be placed in order to throw an image on a screen 8 ft. from the lamp ? *Ans.* 4 ft.

8. The image formed by a convex spherical mirror of focal length 3 inches is $\frac{1}{4}$ in size as the object. Find the distance of the object from the mirror. *Ans.* 9 inches.

9. An eye placed 7 cm. in front of a mirror sees an image of itself 3 cm. behind the mirror. Find the nature of the mirror and its focal length. *Ans.* Convex, 5·25 cm.

10. An object is placed 2·5 inches in front of a concave spherical mirror of radius 3·75 inches. Find the position of the image and draw rays showing how it is formed.

Ans. 7·5 inches in front of mirror.

11. The image of an object placed at a point P in front of a concave spherical mirror is twice its size; when placed at Q the image is magnified three times. If PQ is 15 cm., find the radius of the mirror. *Ans.* 180 cm.

12. Prove that there are two positions for an object placed

in front of a concave spherical mirror so that the image may be of a given size. If the radius of a mirror is 25 cm. and the required magnification is 3, find the distance between the two positions of the object. *Ans.* $8\frac{1}{3}$ cm.

Hint. One position will be for virtual and magnified and the other for real and magnified image.

13. What is the difference between the image formed by a pinhole camera and a plane mirror ?

14. Prove that a ray incident at a point distant from the axis of a rotating mirror moves through double the angle through which the mirror moves.

CHAPTER IV

REFRACTION OF LIGHT

326. It has already been remarked that when a ray of light is incident obliquely on a surface of separation between two media, part of it goes into the second or new medium along a different path from the direction of the incident beam. This change in direction is called *refraction*, from a Latin word which means broken. It can be illustrated very simply by immersing a piece of stick in water, which seems broken or bent at the surface of separation between air and water. This is due to the refraction of the rays coming from the points of the stick below the surface of water.

As before, we shall call the point where the incident ray meets the surface of separation between the two media the point of incidence; also the angle of incidence, the plane of incidence and the normal, etc., will have the same significance. *The angle between the normal in the second medium and the refracted ray will be called the angle of refraction.*

327. Laws of Refraction.—When a ray of light is refracted in passing from one medium to another, the following laws hold good:

I. *The incident, the normal, and the refracted rays all lie in the same plane, that plane being perpendicular to the common surface separating the two media.*

II. *The ratio of the sine of the angle of incidence to the sine of the angle of refraction is constant for the same two media, for all angles of incidence.*

We can write it mathematically as

$$\frac{\sin i}{\sin r} = \text{a constant} = \mu \text{ (say)}.$$

This relation is called **Snell's law,** after the name of its discoverer, Willebrord Snellius. The constant is termed the

refractive index of the second medium with respect to the first; its value depending on the nature of the two media.

It should be borne in mind that in the case of refraction the conditions are much more complicated than in that of reflection, for, while in the case of the latter the direction of the reflected ray was independent of the nature of the reflecting surface, or of the medium in which light was travelling, or of the nature of light (colour), in the case of refraction the direction of the refracted ray depends on all these conditions.

The second law of refraction says that when μ is greater than unity, sin i must be greater than sin r, from which it follows that r must be smaller than i; *i.e.* the refracted ray must make an angle with the normal, which is smaller than the angle of incidence; in other words, the refracted ray must bend towards the normal. This is the case, for instance, when light passes from air into water or glass. But if, on the other hand, μ is less than one the converse will be true; that is to say, i will be smaller, and therefore the refracted ray will make with the normal an angle which is greater than the angle of incidence, *i.e.* the refracted ray will be bent away from the normal. In the first case, when r is smaller than i, the second medium is said to be *optically denser*; but in the second case, when the refracted ray is bent away from the normal, the medium is said to be *optically rarer* or less dense than the first.

Remember the general rule, that *when a beam of light enters obliquely a* ***denser*** *medium* it bends ***towards*** the normal.

It should be noted carefully that an optically denser medium may not necessarily possess greater density; for instance, turpentine is optically denser than water, whereas water possesses greater density than turpentine.

328. To verify the laws of refraction in a simple manner, take a glass slab and place it on a piece of paper fixed upon a wooden board. Mark with a pencil the boundary of the glass slab ($ABCD$). Fix on one side of the slab any two pins, P_1 and P_2, at least 10 cm. apart from each other. Looking through the other side, fix a pin, say P_3, in such a manner that it appears to be in the same line with the other two as seen through the slab. Fix one more pin P_4 in the same manner, so that this new pin also is in the same straight line with all the three pins P_1, P_2, and P_3. Fix the pins in such a manner that their heads seem to be at the same level. Remove the slab. Join P_1P_2 and P_3P_4, and let them when produced meet AB and CD at O and M respectively. Join OM. Then the path of the refracted ray inside

the slab is given by OM. Let NON' be the normal at O. Measure the angles NOF and MON', and find the ratio of their sines,

$$\frac{\sin \angle NOF}{\sin \angle MON'} = {}^{a}\mu_{b},$$

where a is the first medium and b the second medium.

Repeat the experiment, taking different angles of incidence; μ will always be found to be equal to about 1·5.

Produce FO to J. The original path of the ray is FOJ, but inside the glass slab it is OM. This shows that the path of

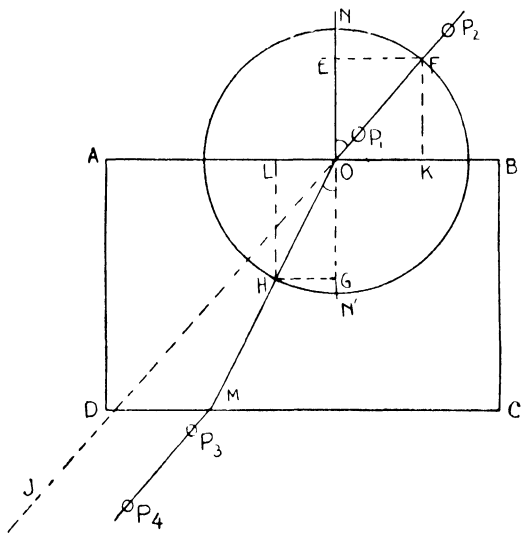


Fig. 29

the ray is deviated inside through an angle JOM, which is equal to the difference of the angle of incidence and the angle of refraction. Outside the slab the path of the emergent ray, *i.e.* MP_3P_4, is evidently parallel to the original path P_2P_1O.

If the heights be measured it will be found that all the pins have their heads at the same height above the board, proving thereby that the incident ray, normal, and the refracted ray, are all in the same plane.

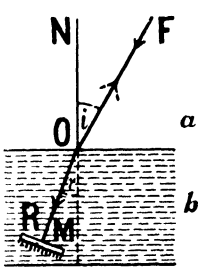
329. Graphical construction for the refracted ray.—Let P_2P_1, Fig. 29, be the direction of the incident ray meeting the refracting surface AB at O, and let NON' be the normal at O. Let

μ be the refractive index. With O as centre and any radius, say OF, describe a circle. Let the circle cut the incident ray at F. Drop FK perpendicular to the surface of separation AB.

Express μ, the refractive index, in the form of a fraction, say $\frac{a}{b}$. Divide OK into a parts and mark along OA, OL equal to b parts. From L draw a perpendicular to the surface AB, meeting the ⊙ at H. Join OH. Then OH gives the direction of the refracted ray,

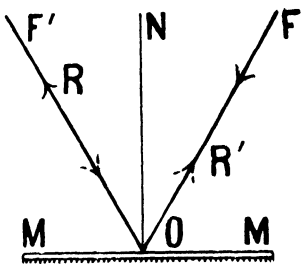
for
$$\frac{\sin i}{\sin r} = \frac{\frac{OK}{OF}}{\frac{OL}{OH}} = \frac{\frac{OK}{OF}}{\frac{OL}{OF}} = \frac{OK}{OL} = \frac{a}{b} = \mu.$$

Let us consider a concrete case. Suppose the refracting medium is glass of refractive index 1·5, express it as $\frac{3}{2}$. Divide



(i)

Fig. 30



(ii)

OK into three parts and mark OL equal to two parts, draw $LH \perp$ to AB. OH will represent the refracted ray (Fig. 29).

330. Reversibility of the path of Light.—Let FO be the incident ray meeting the interface between the two media at O, and OR be the refracted ray in the second medium (Fig. 30 (i)). If we reverse the ray RO by reflecting it normally with a mirror, the experiment shows that the reversed ray RO gives rise to a refracted ray travelling along OF. In the case of reflection too this principle holds good ; as a matter of fact it is much simpler then, because any one of the two rays (incident and reflected)

can be considered as the incident ray on account of their being symmetrical about the normal (Fig. 30 (ii)). Both these examples are simple illustrations of a general law :

If light, after suffering any number of reflections and refractions, has its final path reversed, it will travel back along the same path in the opposite direction.

331. Refractive Index.—If ${}^a\mu_b$ represents the refractive index of the second medium with respect to the first, then evidently

$${}^a\mu_b = \frac{\sin i}{\sin r},$$

where i is the angle of incidence in the first medium, and r the angle of refraction in the second medium (Fig. 30 (i)).

On reversing the ray OR, our angle of incidence now at O is r and angle of refraction is i, hence

$${}^b\mu_a = \frac{\sin r}{\sin i},$$

where ${}^b\mu_a$ is the refractive index of the first medium with respect to the second.

Multiplying together the corresponding sides, we get

$${}^a\mu_b \times {}^b\mu_a = \frac{\sin i}{\sin r} \times \frac{\sin r}{\sin i} = 1 ;$$

therefore

$${}^a\mu_b = \frac{1}{{}^b\mu_a}.$$

Thus *the index of refraction of medium* (a) *with respect to medium* (b) *is equal to the reciprocal of the index of refraction of* (b) *with respect to* (a).

332. Law of refraction for small angles of incidence.—When light is incident normally on a refracting surface, $i = 0$, and therefore $\sin i = 0$, but $\sin i = \mu \sin r$,

$$\therefore \quad \mu \sin r = 0, \text{ and hence } r = 0 ;$$

in other words, the light is transmitted normally without any refraction. When the angle is small, we may without any sensible error substitute the angle itself (measured in circular measure or radians) in place of the sine of the angle. In this case

$$i = \mu r.$$

333. Image of a point formed by refraction at a single plane surface. —Let AB represent the surface separating the two media, the refractive index of b with respect to a being μ (say).

Let P be the luminous point in the plane of the paper at a perpendicular distance OP from the surface. The ray PO in the plane of the paper incident normally will pass along ON without any refraction, but any other ray like PO_1 (also in the plane of the paper) incident at O_1 at a small angle i will be refracted. Let us consider this ray PO.

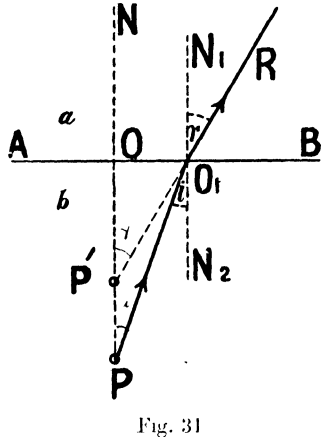


Fig. 31

Since NOP is $\parallel$ to $N_1O_1N_2$, hence $\angle OPO_1$ is equal to $\angle PO_1N_2 = i$. The refracted ray O_1R corresponding to the incident ray PO_1 must lie in the plane of the paper. Produce RO_1 backwards to meet OP at P'; the angle $OP'R$ will be equal to r, the angle of refraction.

Call OP, u and OP', v.

The refractive index of b with respect to a is μ, hence of a with respect to b will be

$$\frac{1}{\mu}\left(=\frac{\sin i}{\sin r}\right).$$

When the angle is small, $\sin\Theta = \Theta = \tan\Theta$; therefore, for the sine of the angle, tangent of the angle can be written.

Doing so, the relation, $\sin i = \frac{1}{\mu}\sin r$ becomes equal to

$$\frac{OO_1}{OP} = \frac{1}{\mu} \times \frac{OO_1}{OP'},$$

or
$$OP = \mu \times OP',$$

or
$$OP' = \frac{OP}{\mu},$$

or
$$v = \frac{u}{\mu},$$

which gives the distance of the image from the surface of separation between the two media.

If the second medium (b) be denser than the medium (a), as we have supposed above, μ, the refractive index of b with

respect to a, is greater than 1, *i.e.* $\frac{1}{\mu}$ is smaller than 1, and hence v is smaller than u.

This explains at once why a pond looks shallower than what it really is.

Our construction is true for the rays which are incident almost normally, for otherwise we cannot assume $\sin\Theta = \Theta = \tan\Theta$. For angles for which this limitation imposed by trigonometry is not true, the position of the image-point is complicated, and students are referred to somewhat more advanced treatises on Physics, as, for instance, text-book of Physics by Watson.

334. Image formed by refraction at a parallel plate.—Look at an object through a glass plate with parallel faces, placing the plate perpendicular to the line of vision. The object due to refraction will seem to be at a distance of $v = \mu u$,* from the surface of the plate towards the object (AB in Fig. 32), μ being the refractive index of the plate with respect to air, and u the distance of the object, say luminous point P, from AB along the normal.

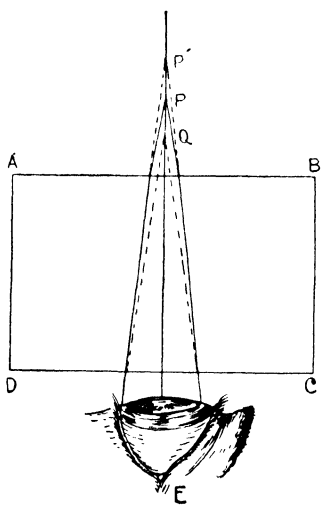


Fig. 32

The image P' will serve as an object as far as refraction at the second surface towards the observer is concerned. If the thickness of the glass plate be t, the distance of this virtual object P' will be $\mu u + t$ from the face CD. At CD the second refraction will take place, and the distance of the final image Q from CD therefore will be

$$v' = \frac{1}{\mu}(\mu u + t)\dagger = u + \frac{t}{\mu}.$$

The distance of P from CD is $u + t$ and the distance of the final image is $u + \frac{t}{\mu}$, hence it will be nearer than the object. The distance diminishes by

$$t - \frac{t}{\mu} \text{ or } \frac{t}{\mu}(\mu - 1).$$

* In this case the object is in the rarer medium, and hence $v = \mu u$. In the preceding article the object was in the denser medium.

† Refraction is from denser to rarer medium, and hence $\frac{1}{\mu}$ is used.

Thus the object seen through a plate is apparently nearer by an amount depending on the thickness and the refractive index of the plate.

335. Refraction through a plate with parallel sides.—Let AB and CD be the two parallel sides of a glass plate (say $ABCD$) of refractive index μ. The surfaces which are represented by AB and CD are perpendicular to the plane of paper (Fig. 33). A ray FO incident at O at an angle i will give rise to the refracted ray OR. If the angle of refraction RON' be denoted by r, it is given by the relation

$$\frac{\sin i}{\sin r} = \mu.$$

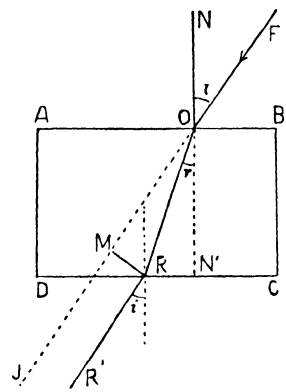


Fig. 33

The ray OR will meet the surface CD at an angle r, and as the path of light is reversible, hence corresponding to an angle of incidence r in the denser medium, the angle of refraction in the rarer medium will be i. This shows that RR', the emergent ray, will make an angle i with the normal to CD.

Thus we find that the emergent ray RR' will be parallel to the incident ray FO.

The ray RR', however, will be displaced laterally with respect to the incident ray FO, whose direction would have been FOJ if there had been no glass plate. From R draw a perpendicular RM to OJ, then RM gives the lateral displacement.

Suppose $ON' = t =$ thickness of the plate,

then $OR/ON' = \sec r$,

$\therefore$ $OR = ON' \times \sec r = t \sec r$.

But $RM/OR = \sin(i - r)$

$\therefore$ $RM = t \,.\, \sec r \,.\, \sin(i - r)$.

Thus it is seen that the lateral displacement is proportional to t; therefore by making t sufficiently small the lateral displacement can be made as small as we like. Also we see that the displacement increases as the angle of incidence increases, because r increases with the increase of i and sec r increases with the increase of r.

336. Refraction through a Compound Plate.—Next let us consider refraction through a compound plate made up of two plates of different substances having different refractive indices with a common interface CD (Fig. 34). Let a ray of light be incident along FO at O; it will enter into the first plate at an angle r_1, given by the relation

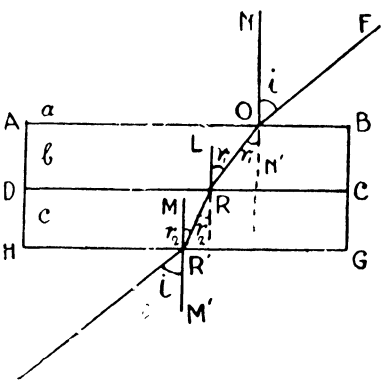


Fig. 34

$$\frac{\sin i}{\sin r_1} = {}^a\mu_b,$$

where ${}^a\mu_b$ is the refractive index of b (*i.e.* the material of the first plate) with respect to a.

It will meet the second plate at an angle r_1, and suppose r_2 is the angle of refraction here, it will be given by the relation

$$\frac{\sin r_1}{\sin r_2} = {}^b\mu_c,$$

where ${}^b\mu_c$ is the refractive index of the material of the second plate with respect to the material of the first.

Evidently it will be incident at an angle r_2 on the face GH at point R'; it will come out at an angle x, in accordance with the following relation:

$$\frac{\sin r_2}{\sin x} = {}^c\mu_a.$$

But we know *from experiment* that the ray comes out parallel through a compound plate with parallel sides provided the first and final media are the same; hence $x = i$, and therefore

$$\frac{\sin r_2}{\sin x} = \frac{\sin r_2}{\sin i} = {}^c\mu_a.$$

By multiplying the respective sides of the three relations mentioned above, we get

$$\frac{\sin i}{\sin r_1} \times \frac{\sin r_1}{\sin r_2} \times \frac{\sin r_2}{\sin i} = 1 = {}^a\mu_b \times {}^b\mu_c \times {}^c\mu_a;$$

$$\therefore \quad {}^a\mu_b \times {}^b\mu_c = \frac{1}{{}^c\mu_a},$$

or

$${}^a\mu_c = {}^a\mu_b \times {}^b\mu_c.$$

This is an important relation, and enables us to determine at once the relative index of refraction from a to c, given the indices of refraction from a to b and b to c. Suppose, for example, we want to determine the relative refractive index of water to turpentine.

The refractive index of water $= 1{\cdot}33 = {}^{a}\mu_{w}$.

The refractive index of turpentine $= 1{\cdot}46 = {}^{a}\mu_{t}$.

Refractive index of turpentine with respect to water $=$

$$
\begin{aligned}
{}^{w}\mu_{t} &= {}^{w}\mu_{a} \times {}^{a}\mu_{t} \\
&= \frac{1}{1{\cdot}33} \times 1{\cdot}46 \\
&= 1{\cdot}09.
\end{aligned}
$$

337. Total Internal Reflection. — Let us now consider refraction, keeping this in view that light is travelling from denser to a rarer medium, say from water to air. In this case it is refracted away from the normal, the angle of refraction (r) being greater than the angle of incidence (i). Note, in this case $\frac{\sin i}{\sin r}$ will be equal to $\frac{1}{\mu}$ and not μ, because μ is the refractive index of the denser medium with respect to the rarer one (water with respect to air). Consider that angle of incidence for which the angle of refraction is 90°; obviously in this case $\sin r = 1$;

hence
$$\sin i = \frac{1}{\mu},$$

Thus we come to the conclusion that for a particular value of the angle i, r becomes equal to 90°, *i.e.* the refracted ray just grazes along the surface. The value of this angle is given by the formula

$$i = \sin^{-1}\left(\frac{1}{\mu}\right),$$

where μ is the refractive index of the denser medium with respect to the rarer one.

This angle is called the **critical angle.**

When refraction takes place from denser into a rarer medium the critical angle is that angle of incidence for which the corresponding angle of refraction is 90°.

As soon as the angle of incidence becomes greater than the critical angle the ray is no longer refracted out, but is completely reflected back as *PMC* (Fig. 35). The ray is said to suffer total reflection.

Let us take the case of water, $\mu = 1{\cdot}33$, hence the critical angle

$$i = \sin^{-1}\left(\frac{1}{1{\cdot}33}\right) = 48^\circ\ 27'.$$

The critical angle for crown glass is 41° 10′.

The total internal reflection can be beautifully demonstrated by an experiment called " Fountain of fire ".

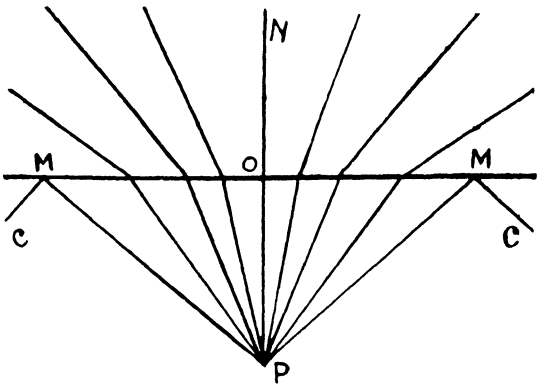


Fig. 35

A metallic rectangular vessel is taken, in one side (AD) of which the lens * L is fixed, opposite to which in the other side is a hole O. A beam of light is focussed by L upon the hole. The vessel is filled with water, and a jet begins to stream out of the hole in the form of a parabola. The angle of incidence on the interior surface of the jet being greater than the critical angle, the beam is reflected from side to side along the stream, with the result that the stream becomes brilliant with light; if a red glass be placed before the lens L the stream becomes red, and it seems as if fire is flowing out of the vessel; hence the name " Fountain of fire ".

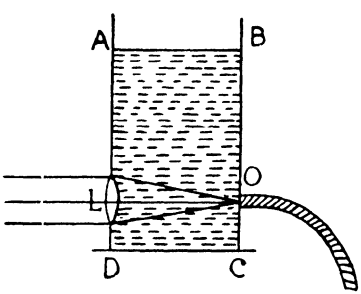


Fig. 36

The phenomenon of total reflection can be illustrated in

* The focal length of the lens should be equal to LO.

a very simple manner by holding a test-tube half filled with water above the level of the eyes and looking at the surface of separation of water and air from below. The surface appears bright, just like a polished sheet of silver.

The same phenomenon happens in the case of a prism whose angles are 45°, 45°, and 90°, when light is incident on one of the faces normally. For instance, if a ray be incident along LM, on the surface AB of the prism, it suffers no refraction, as it is incident normally on that face. The angle of incidence at the face represented by BC is 45°, hence there

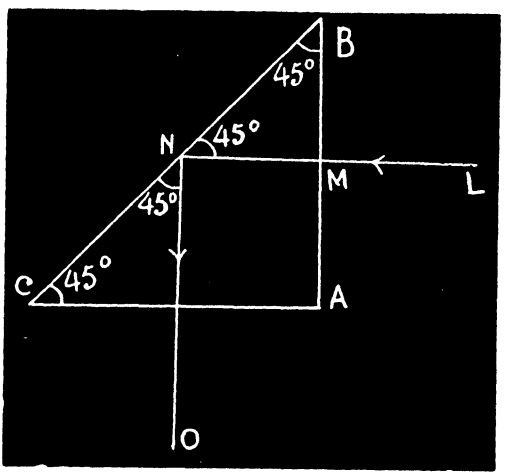


Fig. 37

will be no refraction (for critical angle of glass is about 41°). On the other hand the ray will be totally reflected and will meet normally the face AC, and hence will pass out without suffering any further refraction. Thus we see that the path is deviated through 90°, without any loss of light due to reflection. This prism is used for simply changing the direction of light through a right angle.

The brilliancy of the precious stones like the diamond and the ruby is due to their large refractive indices and consequently small critical angles. For instance, the critical angle for diamond is

$$i = \sin^{-1}\left(\frac{1}{2{\cdot}4}\right)$$
$$= 23° 41'.$$

When light enters a piece of diamond at any cut face, it cannot get out at most of the other faces on account of the critical angle being low, and therefore suffers total reflection again and again, making the faces look very brilliant.

338. Some interesting effects produced by refraction.—We shall consider a few of them only, as for instance the mirage, the effect on the altitude of the stars, and the twinkling or scintillation.

339. Mirage.—The mirage is an optical illusion; images of the distant objects and of the sky are seen reflected as if from a lake. This phenomenon is of most frequent occurrence in hot climates, and more especially on the sandy deserts. By contact with the hot sand the lower layers of air become so much heated that up to the height of a few feet the density increases upwards. The least dense layers are then the lowest. Rays of light from a distant object traverse layers which are gradually less refracting, because the refractive index of air becomes smaller and smaller as the density decreases.

The rays continue their path, being, however, more and more bent as they pass from one layer to the other, and at last fall on a stratum at an angle greater than the critical angle (the angle of incidence continually increases as the rarer layers are below). Here total reflection takes place, and the rays are bent upwards. They become bent more and more in traversing the denser layers above, and at last reach the observer's eye as if they came from a point as far below the reflecting layer as the object is above it. Thus he sees an inverted image of the object just as if it had been reflected from the surface of a calm lake.

340. The effect on altitude of Stars.—Owing to the presence of the atmosphere the rays of light proceeding from a celestial body suffer a change of path. Let us consider the atmosphere to be divided up into layers parallel to the surface of the globe. On account of the pressure exerted by upper layers on the lower ones, the latter are denser and therefore possess greater refractive index. Thus suppose a ray *Sa* (Fig. 38) meets the layer *AA'*; it will be bent towards the normal on refraction, and the same will happen at all subsequent refractions, with the result that the star will be seen in the direction of *Od* or *OS'*, whereas the actual direction is *OS*.

Hence the apparent altitude of a star is increased by refraction.

Note, the sun rises somewhat earlier, and sets somewhat later

on account of this effect. This makes the day seven or eight minutes longer.

341. The Twinkling or Scintillation.—The air near the ground is more or less disturbed by convection currents, and thus the refractive index is different at different points even on the same level. The rays of light from a star are bent, due to regular refraction in the different layers of atmosphere, as explained in § 340. Besides this change of direction the rays suffer deviation occasionally from their path in the atmosphere on account of passing through disturbed areas, and so the light

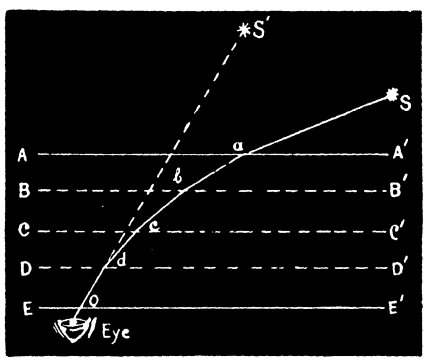


Fig. 38

from a star at a point sometimes appears concentrated and sometimes decreased in intensity. This explains why to an observer standing at a point the star appears to twinkle.

Prisms

342. A **prism** is a wedge-shaped body made up of a transparent medium bounded on two sides by plane surfaces meeting at an angle.

The two plane surfaces are called the **faces** of the prism; the angle between the two faces is called the refracting angle or simply the **angle of the prism,** and the line in which the two faces meet is called the **edge of the prism.**

A plane perpendicular to the two faces, and hence also to the edge, is called a **principal plane** of the prism.

The prisms generally used are triangular, *i.e.* their principal section is triangular in form.

343. Refraction through a Prism.—In dealing with refraction through a prism we shall confine ourselves to the case when

the rays of light pass through the prism in a principal plane; because this plane being perpendicular to both the faces, if incident ray lies in it, it continues in this plane both during its passage through the prism and after leaving the prism. In other cases, to trace the path we require knowledge of higher mathematics.

Let ABC (Fig. 39) represent the principal section of a prism the two faces of which are represented by AB and AC. BAC is the angle of the prism, and a straight line through A perpendicular to the plane of the paper is the edge.

Let GH be a ray incident on AB in the principal plane (here in this case the plane of the paper) at an angle i_1. If μ is

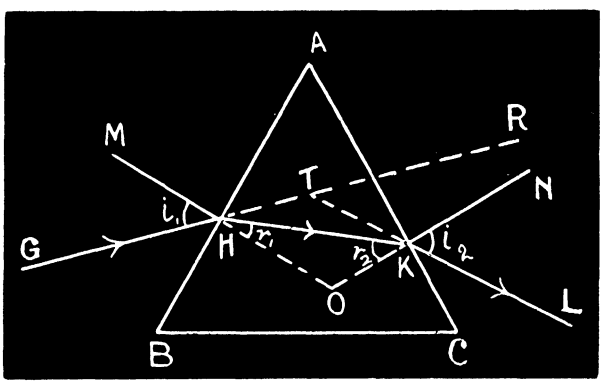


Fig. 39

the index of refraction of the material of the prism, the refracted ray HK will be inclined to the normal at an angle r_1 given by the relation

$$\frac{\sin i_1}{\sin r_1} = \mu.$$

If μ be greater than one, as is generally the case, r_1 will be smaller than i_1, *i.e.* the ray on refraction will be bent towards the thick end of the prism. Now let it be incident on the second face AC at an angle r_2, the ray KL refracted into the air will lie in the plane of the paper and will make an angle i_2 with the normal given by the relation

$$\frac{\sin i_2}{\sin r_2} = \mu.$$

The angle i_2 will be greater than r_2. It is clear from the figure that the emergent ray is no more parallel to the incident ray GH.

Produce LK, the emergent ray, backwards to meet the produced incident ray GHR at T. The angle RTL measures the deviation which has been produced in the path of the incident ray GH. This angle is called the **angle of deviation**; let it be noted by D.

It is easily seen from the figure that $\angle THK = i_1 - r_1$, and $\angle TKH = i_2 - r_2$. Then since $\angle RTK$ is the external angle of the ΔTHK, hence

$$\angle RTK = \angle THK + \angle TKH,$$

$$\therefore \quad D = i_1 - r_1 + i_2 - r_2.$$

But $r_1 + r_2 + O = 2$ rt. angles, being the three angles of ΔHKO and $O + A = 2$ rt. angles as in quadrilateral $OHAK$, OHA and OKA are rt. angles.

$$\therefore \quad r_1 + r_2 = A.$$

Hence
$$D = i_1 + i_2 - A,$$

or
$$D + A = i_1 + i_2,$$

a very important relation, that *the angle of incidence + the angle of emergence = the angle of the prism + the angle of deviation.*

344. Angle of minimum deviation.—The deviation produced in the path of the incident ray depends not only upon the material and the angle of the prism but also upon the angle of

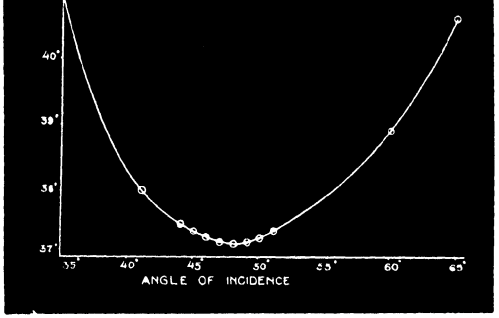


Fig. 40

incidence. It is found experimentally that with a given prism the angle of deviation alters with the angle of incidence, and that the *deviation is minimum for one particular value of the angle*

of incidence. This particular angle of incidence is different for different prisms (be they different on account of material, or angle). If we draw a curve showing the relation between the angle of incidence and the angle of deviation for a prism, it will be of the form shown in Fig. 40, which shows at a glance that the deviation is minimum for a particular angle of incidence, and that as the latter increases or decreases, the deviation increases only. The curve shows the relation between the deviation and the angle of incidence for a prism whose angle is 60° and the refractive index is 1·5.

345. In this particular position when the deviation is minimum the incident ray and the emergent ray make equal

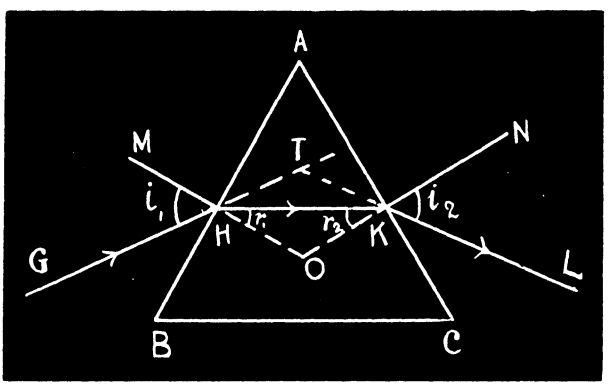


Fig. 41

angles with the normals at their respective faces, and the ray HK within the prism is equally inclined to the two faces; in other words, the ray passes symmetrically through the prism. As has been said, the experiment shows that *there is only one position of the prism with respect to the incident ray for which the deviation is minimum.* Now assuming this result as true, it is easy to show theoretically that the ray will pass symmetrically through the prism in this position.

For let i_1 and i_2 (Fig. 41) be unequal when the ray GH suffers minimum deviation. Reverse the emergent ray KL by placing a mirror at L normally, then the ray LK incident at an angle i_2 gives rise to the ray HG (by the principle of reversibility) emerging at an angle i_1; the deviation being the same as before, must have therefore a minimum value even now. Thus the ray

GH, incident at an angle i_1, suffers minimum deviation, and so does the ray LK, incident at an angle i_2. There are consequently two angles of incidence which give rise to minimum deviation, which is against the experience and our hypothesis with which we started that there is only one position for which a ray suffers minimum deviation. Hence i_1 and i_2, and consequently r_1 and r_2, must be equal.

Let $i_1 = i_2 = i$ and $r_1 = r_2 = r$ (say).

Obviously $\angle AKH = \angle AHK$, since each of them is $90° - r$.

$\therefore$ $AH = AK$.

Thus the ray passes symmetrically through the prism.

346. Refractive Index of a Prism.—When a ray suffers minimum deviation, it has been shown above that in that position $i_1 = i_2 = i$ and $r_1 = r_2 = r$.

$$A = r_1 + r_2 = 2r.$$

$\therefore$ $$r = \frac{A}{2}.$$

$$D = 2(i - r) = 2i - A.$$

$\therefore$ $$i = \frac{A + D}{2}.$$

Hence the relation $\mu = \frac{\sin i}{\sin r}$ can be written as—

$$\mu = \frac{\sin \frac{A + D}{2}}{\sin \frac{A}{2}},$$

where A is the angle of the prism and D the angle of minimum deviation.

Thus if we know the minimum deviation and the angle of the prism, the refractive index of the material composing the prism can at once be calculated.

347. Deviation produced by a Prism of small Angle.—If a ray of light falls at a small angle on the prism whose refracting angle is very small, then the angle of refraction inside also is small. We know that

$$D = i_1 + i_2 - A,$$

and $$\frac{\sin i_1}{\sin r_1} = \mu,$$

or $$i_1 = \mu r_1,$$

when angles are so small that the angles can be substituted for their sines.

Similarly $$i_2 = \mu r_2.$$

$$\therefore \quad D = \mu r_1 + \mu r_2 - A$$
$$= \mu A - A$$
$$= A(\mu - 1).$$

Thus we see that the deviation produced by a prism of a small angle is independent of the angle of incidence provided it is small.

348. Image formed by Refraction through a Prism.— When a ray passes through a prism symmetrically, the

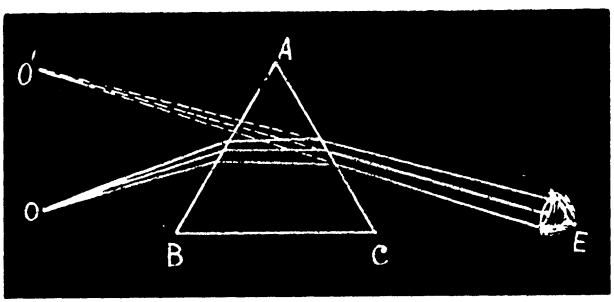


Fig. 42

deviation is minimum. Near this position a small change in the angle of incidence produces but little change in the magnitude of deviation. Hence the deviation which the rays undergo in passing through a prism, by paths near to that of minimum deviation, is practically the same, and very nearly equal to the minimum value.

Consequently if a small pencil of rays coming from a point O (Fig. 42) be passing through the prism in such a manner that the axis of the pencil suffers minimum deviation, then all the rays will suffer almost the same magnitude of deviation and on emergence will be inclined to one another at nearly the same angle as before incidence, and thus will appear to diverge from a single point O', which will be the virtual image of the object O.

Lenses

349. A **lens** is a portion of a refracting medium bounded by either two curved surfaces or by one curved and the other

plane surface. The straight line joining the centres of curvature of the surfaces is called the **axis of the lens**; or if one of the surfaces be plane, the axis is the straight line normal to it passing through the centre of curvature of the other. When the boundary of the lens is circular the diameter of the boundary is known as the **aperture** of the lens. A section of the lens through the axis is called the **principal section.**

Lenses are generally divided into two classes, convex and concave. Convex lenses are thicker in the middle, and thin towards the edges. Such lenses (provided their material has a higher refractive index than that of the medium outside) deviate the rays passing through them towards the axis, and

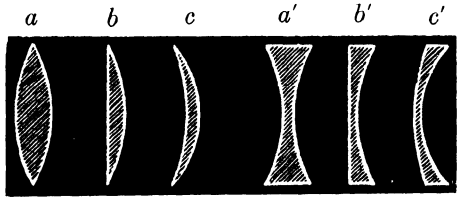


Fig. 43

are known as **converging** lenses. Of these there are three types:

1. Double convex (Fig. 43, *a*).
2. Plano-convex or convexo-plane (Fig. 43, *b*) according as light falls first on plane surface or on convex surface.
3. Concavo-convex or convexo-concave. It is known also as converging meniscus. (Fig. 43, *c*.)

Concave lenses are thin in the middle and thick towards the edges. Such lenses in a medium rarer than themselves deviate a ray away from the axis and hence are known as **diverging** lenses. But in a medium denser than themselves they behave as converging lenses. Corresponding to the three forms of the convex lenses we have in this case as well—

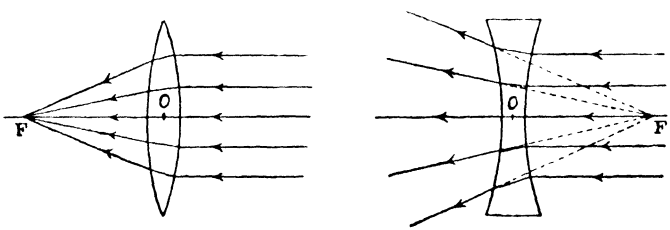
1. Double concave (Fig. 43, *a′*).
2. Plano-concave or concavo-plane (Fig. 43, *b′*), just as light falls first on plane or concave surface.
3. Convexo-concave or concavo-convex (Fig. 43, *c′*), known also as diverging meniscus.

If a pencil of parallel rays (Fig. 44, *a* and *b*) falls on a lens

in a direction parallel to its axis, the rays after passing through the lens either converge to or diverge from a point on the axis. This point is called the **principal focus** of the lens. The distance of the principal focus from the centre of the lens O is called the **focal length** of the lens.

Note, in the case of concave lens the principal focus lies on the same side as the incident beam, and hence the focal length is positive, and in the case of convex lens, the principal focus lies on the opposite side and hence the focal length is negative.

350. Optical Centre.—It is clear from Fig. 44 (a) and (b) that a ray travelling along the principal axis (since it falls normally on the surfaces of the lens) does not suffer any change in direction. But it is not the only direction in which



(a) (b)

Fig. 44

a ray of light can pass through a lens without suffering any change in its path. There is a remarkable point in a lens, called the **optical centre,** *which has this peculiar property, that all rays passing through it suffer no change in direction.*

Let C and C' be a double convex lens $FEGH$ (Fig. 45). Draw any radius CA of the spherical surface FEG; draw parallel to CA, $C'B$ a radius of the other spherical surface FHG. CA and $C'B$ represent the normals at A and B to the two surfaces of the lens. Join AB, cutting the principal axis at O. Let BA represent a ray passing through the lens. This ray makes equal angles with the normals at A and B since the latter are parallel by construction, hence the incident and emergent rays KA and BL also make equal angles with the normals, CA and $C'B$, and are therefore parallel.

Thus we see that a ray $LBAK$ suffers no change in direction while passing through a lens. But it does suffer a lateral displacement, the magnitude of which depends upon

the thickness of the lens. By taking a thin lens the lateral displacement can be made very small, and hence for such a lens we can say that a ray passing through the point O suffers no deviation but continues its course in the same straight line. *This point O, which possesses this peculiar property that a ray passing through it is parallel after emergence to the incident direction, is called the* ***optical centre.*** Its position can be found in the following way :

The two Δs OAC and OBC' are similar,

$$\therefore \quad \frac{OC}{OC'} = \frac{CA}{C'B} = \frac{R_1}{R_2},$$

which shows that the point O divides the line joining the

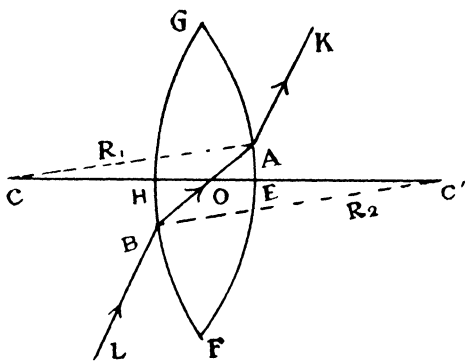


Fig. 45

centres of curvature of the surfaces in a definite ratio depending upon the radii, *hence it is a fixed point for any one lens.* The optical centre coincides with the geometrical centre in the case of a double convex or concave lens having surfaces of equal curvature. In the case of a meniscus it lies altogether outside the lens.

A line other than the principal axis passing through the optical centre is called a *secondary axis.*

351. Graphical Construction of Image formed by a Lens.— Let PQ represent the object placed on the axis of the lens. In order to get the position of the image of PQ, we shall regard PQ to be made up of a number of points, and explain how the image corresponding to a point can be obtained graphically. The same construction can be repeated for each point, and the

collection of the image points thus obtained will make the image of the object.

Consider the point P. Its image will be at the intersection of any two rays starting from it. Bearing in mind the pro-

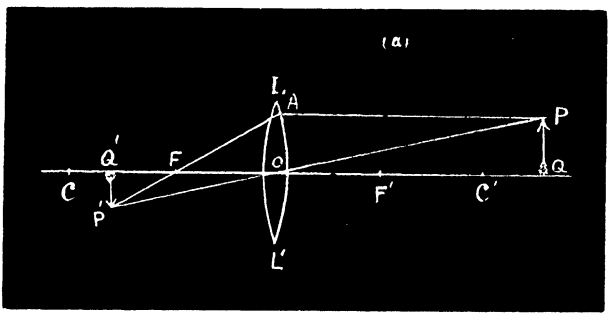


Fig. 46 (*a*)

perties of the optical centre and the principal focus, the construction becomes very easy. Take a ray parallel to the axis of the lens P; it will pass through F (the principal focus) after passing through the lens. Take next the ray passing through

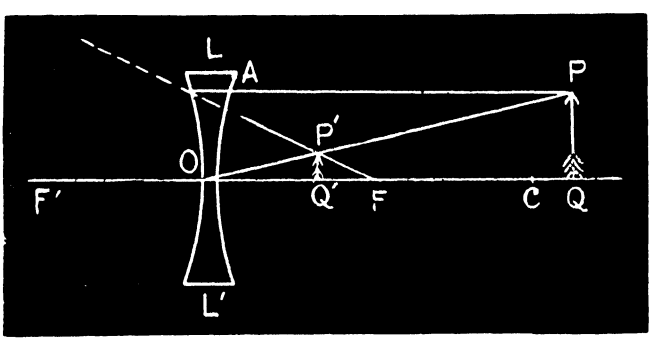


Fig. 46 (*b*)

O. It will pass on without any deviation; the intersection of these two rays at P' gives the image of P. Similarly the image of every other point can be found, and the collection of these image points will correspond to the image of PQ.

It is clear from Figs. 46 (*a*) and (*b*) that in the case of the convex lens the image is inverted, real, and on the other side of the lens, but in the case of the concave lens it is virtual, erect, and on the same side as the object.

352. Lens Formula.—Let LL' represent the lens Fig. 46 (*a*), QOF the principal axis, and PQ an object small in size lying on the axis of the lens. Suppose $P'Q'$ is the image of the object formed by the lens.

Δs POQ and $P'OQ'$ are similar,

$$\therefore \quad \frac{PQ}{P'Q'} = \frac{OQ}{OQ'}\text{; but } PQ = AO,$$

$$\therefore \quad \frac{PQ}{P'Q'} = \frac{AO}{P'Q'} = \frac{OQ}{OQ'}.$$

Now consider the Δs $P'Q'F$ and AOF. They are similar,

$$\text{hence} \quad \frac{AO}{P'Q'} = \frac{OF}{FQ'}.$$

$$\therefore \quad \frac{OQ}{OQ'} = \frac{OF}{FQ'}.$$

Call OQ, u; OQ', v; OF, f; we have (bearing in mind the convention of sign) for the convex lens

$$\frac{u}{-v} = \frac{-f}{-(v-f)} = \frac{f}{(v-f)};$$

$$\therefore \quad vu - fu = -fv,$$

$$\text{or} \quad vu = fu - fv.$$

$$\text{Divide by } uvf, \quad \frac{1}{f} = \frac{1}{v} - \frac{1}{u}.$$

Similarly for the concave lens we can write down

$$\frac{OQ}{OQ'} = \frac{OF}{FQ'},$$

$$\frac{u}{v} = \frac{f}{f-v},$$

$$\text{or} \quad fu - vu = fv,$$

$$\text{or} \quad vu = fu - fv.$$

$$\text{Divide by } uvf, \quad \frac{1}{f} = \frac{1}{v} - \frac{1}{u}.$$

So we get at the same formula.

Thus the relation $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$ (lens formula) is applicable to both the classes of the lens, be they convex or concave, provided due care is taken of the sign of the distance.

Any two points such as Q and Q' which are connected with this relation are said to be **conjugate foci.**

Let us consider this relation $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$.

When the object is at infinity $\frac{1}{u}=0$, and therefore $\frac{1}{v}=\frac{1}{f}$, *i.e.* the image is at the principal focus, exactly what was stated before, that the principal focus is a point where the rays parallel to the principal axis converge to or diverge from. While using the formula $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$, remember that the *focal length of a convex lens is negative, and of a concave lens is positive.*

353. A lens will have two principal foci, because the light may be incident on it from either direction; these foci are generally designated by F and F'. The formula

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$$

can be written as $uv-uf-vf,$

or as $f^2=uf-vf-uv+f^2$

or $=f(f+u)-v(f+u)$

$=(f+u)(f-v).$

Now let x and x' denote the distances of the object PQ from F' and of the image $P'Q'$ from F (Fig. 46). Therefore

$$x=QF'=f+u.$$

It is clear for the concave lens, but the student might have some difficulty in understanding it for the convex one, but he will understand it if he remembers that f is negative in this case and hence negative value is to be substituted. Similarly

$$x'=Q'F=(f-v).$$

Substituting these values we see that $xx'=(f+u)(f-v)=f^2$, a relation similar to what we got in the case of mirrors.

The relation between the distances of the object and the image is just similar to the relation in the case of mirrors. But it should be borne in mind that the *convex lens corresponds to the concave mirror* and the concave lens to the convex mirror.

We shall briefly state the position of the image for the various positions of the object.

354. Convex Lens.—1. When the object is situated at infinity the image is situated at the principal focus. It is a real image.

2. When the object is situated between $2f$ and infinity the image is situated between $2f$ and f. It is real, inverted, and small in size.

3. When the object is situated at $2f$ the image is also situated at $2f$. It is real, inverted, and equal in size.

4. When the object is situated between $2f$ and f the image is situated between $2f$ and infinity. It is inverted, real, and magnified in size.

5. When the object is situated between lens and f its image is magnified, virtual, and erect.

It should be carefully noted that in the case of lenses **we use $2f$ instead of centre of curvature,** because $2f$ is not generally equal to the radius of curvature. Focal length depends upon the curvature of the surfaces as well as the refractive index of the medium.

In the case of a concave lens the image of a *real* object is always virtual, erect, and diminished in size. It moves from the focus to the lens as the object moves from infinity to the lens. The image is always on the same side as the object.

355. Magnification.—By linear magnification as explained in § 322 we mean the ratio of the size of the image to that of the object.

It is clear from Fig. 46 that the

$\triangle$s PQO and $P'Q'O$ are similar,

$$\therefore \quad \frac{P'Q'}{PQ} = \frac{OQ'}{OQ} = \frac{v}{u}.*$$

Thus we see that the linear magnification is equal to the ratio of the distance of the image to that of the object from the lens, just as in the case of a mirror.

356. Combination of two or more thin Lenses.—Let us consider the case in which two thin lenses of focal lengths f_1 and f_2 are placed in contact with each other. Suppose an object is situated at the axis of the lenses at a distance of u cm. from the first lens, it will give rise to an image at a distance v_1 from the same lens, given by the formula

$$\frac{1}{v_1} - \frac{1}{u} = \frac{1}{f_1},$$

or

$$\frac{1}{v_1} = \frac{1}{u} + \frac{1}{f_1}.$$

* The linear magnification can also be proved equal to $\frac{f-v}{f}$ or $\frac{f}{u+f}$.

This image will serve as an object for the second lens. The distance of this new object will be v_1 from the second lens, if we neglect the thickness of the first lens. This will produce the final image say at a distance of v cm. given by the formula

$$\frac{1}{v} - \frac{1}{v_1} = \frac{1}{f_2},$$

or

$$\frac{1}{v} = \frac{1}{v_1} + \frac{1}{f_2} = \frac{1}{u} + \frac{1}{f_1} + \frac{1}{f_2}.$$

We can write it as

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{F}.$$

Thus we see that the combination of two thin lenses behaves just in the same way as a new lens of focal length F, where $\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2}$; that is to say, *the reciprocal of the focal length of an equivalent of a combination of two thin lenses is equal to the sum of the reciprocals of the focal lengths of the constituent lenses.* The result is true even if the number be larger than two.

357. Power of a Lens.—The equation given in the last article shows that for a number of lenses in contact with one another the algebraical sum of the reciprocals of their respective focal lengths is equal to the reciprocal of the focal length of the equivalent lens. This result suggests that it is more convenient to deal with the reciprocals of the focal lengths rather than with the focal lengths themselves, whenever we have to consider an equivalent of a coaxial system of lenses. *The reciprocal of the focal length of a lens is termed the converging power, or simply the* ***power*** *of the lens.* It is sometimes referred to as the dioptric strength of the lens, from the fact that the unit chosen by opticians is the power of a lens of 1 metre focal length which they call ***dioptre.*** Thus in order to get the power in conventional units we are simply to take the reciprocal of the focal length expressed in metres. The opticians regard the power of a convex lens as plus and that of a concave lens as minus. For instance, a lens of power +4 means a convex lens of focal length 25 cm. Compare carefully the sign of power with that of the focal length; for instance, the convex lens has a plus power but negative focal length.

358. Determination of the Focal Length of a Convex Lens.—(1) An approximate and rough method is to take a distant object as the source of light, and to get its distinct image on the screen. For this purpose it is advisable to take such

an object whose detailed structure can be seen, say a tree; in this case that position will be the focus when the leaves are seen most distinctly. Mount the lens on an optical bench and place behind that a screen mounted on an upright. By moving the screen to and fro get a distinct image and measure the actual distance between the lens and the screen; this will be the focal length approximately.

(2) Mount the wire-gauze, the lens, and the screen on an optical bench in such a way that the axis of the lens is parallel and the plane of the uprights is perpendicular to the bed of the bench. Illuminate the wire-gauze by a candle-flame, and adjust the relative positions of the uprights till a distinct image of the gauze is formed on the screen. Measure the distance of the object and the image from the lens, taking into consideration the index error as in the case of a concave mirror. Substitute the true distances with proper signs in the formula

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f},$$

and get f.

In this method a good many positions can be taken, for corresponding to any one position of the object there is a conjugate focus where the image will be situated. Hence three or four distances should be tried and the mean result taken. It is better to start with the object at one end of the optical bench and the screen at the other. Then move either of them and get the position of a distinct image.

(3) There are two methods to determine the focal length by parallax. The first is that in which we have only *one needle*. Suppose this needle is placed at the principal focus of the lens; the rays after refraction through the lens will come out as a parallel beam. Now, if they be allowed to fall perpendicularly on a plane mirror, they will be reflected back along the same path, and after passing through the lens an image will be formed at the principal focus, *i.e.* just above the needle. The image will be inverted. Adjusting the position of the needle and the lens, get the inverted image just vertically above the needle itself. Move the eye sideways and find out whether there is any parallax or not. If there be any, move the needle forwards or backwards just as it moves in the opposite direction or in the same direction in which the eye moves, till the parallax is removed. The distance of the needle from the lens gives the focal length.

In the second method we have *two needles*. The image of

one needle is made to fall just above the second needle, and the position of the latter adjusted till there is no parallax between it and the image of the other needle. This gives the conjugate focus corresponding to the position of the first needle. Measuring the true distances (after taking into account the index error) of the two needles from the lens and applying the formula

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f},$$

can be determined.

(4) **Displacement Method.**—Let A and B be two conjugate foci of a lens. If the object, say a wire-gauze, brightly illuminated, is placed at B and the lens at L_1, a distinct,

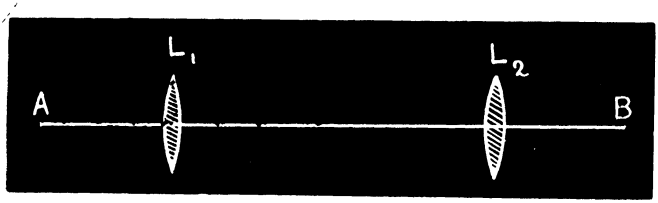


Fig. 47

diminished image falls upon the screen placed at A. A magnified image of the object will be obtained at the screen at A, if we move the lens to a position L_2 nearer B than A, where $BL_2 = AL_1$, and $AL_2 = BL_1$.*

Now let us denote the distance AB by l, L_1L_2 by d, AL_1 or BL_2 by m, and BL_1 or AL_2 by n.

As $$AB = AL_1 + BL_1,$$

$$\therefore \quad l = m + n, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (a)$$

$$L_1L_2 = d = AL_2 - AL_1 = n - m. \quad . \quad . \quad . \quad (b)$$

Adding up (a and b) we get

$$l + d = 2n,$$

$$\therefore \quad n = \frac{l+d}{2};$$

and subtracting (b from a) we get

$$m = \frac{l-d}{2}.$$

* This is practically the same thing as placing the object at A and the screen at B with lens at L_1.

But we know

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f},$$

and $$-AL_1 = -m = v, \quad BL_1 = n = u.$$

$$\therefore \quad -\frac{2}{l-d} - \frac{2}{l+d} = \frac{1}{f} = -\frac{4l}{l^2 - d^2}.$$

Thus we see that $$f = -\frac{l^2 - d^2}{4l}.$$

To get the focal length we have only to measure l, the distance between the object and the screen, and d, the distance through which the lens is moved.

359. Determination of the Focal Length of a Concave Lens. —In this case the image is always virtual and therefore cannot be received upon a screen, hence the experimental determination is slightly more difficult than in the first case.

But if we remember the formula

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2},$$

this case also becomes very easy. We need couple the concave lens with a convex lens of such power that the combination is equivalent to a single convex lens. $\frac{1}{F}$ can be found out by any of the methods explained already, and also the focal length of the convex lens alone. Substituting these values in the formula, the focal length of the concave lens can be determined.

EXERCISES

1. A mark is made on a piece of paper and a vernier microscope is focussed upon it. On placing a slab of glass 6 cm. thick on the mark, the microscope has to be raised by 2 cm. to bring the mark in focus again. Find out the refractive index of the glass.

We have already seen that as a result of vision through a plate of refractive index μ and thickness t the object seems to be nearer by $\frac{t}{\mu}(\mu - 1)$. Substituting the values we get

$$\frac{6}{\mu}(\mu - 1) = 2,$$

or $$6\mu - 6 = 2\mu,$$

$\therefore$ $$\mu = \frac{6}{4} = 1{\cdot}5.$$

2. How will you find the refractive index of a liquid by taking a *thin* layer of it in a concave mirror ?

Place the concave mirror horizontally on a table and adjust a needle at its centre of curvature by parallax method (see Art. 325). The distance of the needle from the mirror will be equal to the radius of curvature of the mirror. Let it be r cm. ($=AC$, Fig. 47, *a*).

Now put a thin layer of the liquid in the mirror and again adjust the position of the needle for no parallax. It will be found that this time the position of the needle is nearer the mirror. Let the distance be r' cm. ($=AC'$).

The refractive index of the liquid $= \frac{r}{r'} = \frac{AC}{AC'}$.

To prove it, consider the ray $C'B$. It suffers refraction at B, goes into the liquid along BD, suffers reflection at the mirror at D, and so that the image may be at C' (it must retrace its path), *i.e.* the ray BD in the liquid must be falling

normally on the mirror. In other words, BD is a normal to the mirror and hence it must pass through the centre of curvature C of the mirror. The angle of incidence evidently is

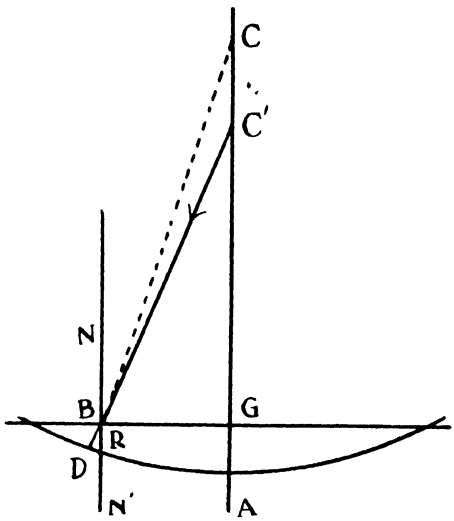


Fig. 47 (*a*)

$\angle C'BN$ and the angle of refraction $\angle DBN' = \angle CBN$,

$$\therefore \quad \mu = \frac{\sin C'BN}{\sin CBN} = \frac{BG/C'B}{BG/CB}$$

$$= \frac{CB}{C'B}.$$

As the depth of water is small and D is very near A, therefore $CB = CA$ and $C'B = C'A$;

hence $$\mu = \frac{CA}{C'A} = \frac{r}{r'}.$$

Note, the layer of the liquid must be *thin*, otherwise the approximations are not valid.

3. A lamp is placed at a distance of 8 ft. from the wall of a room. How far from the lamp must a convex lens of focal length 18 inches be placed in order to throw a distinct image of the lamp on the wall ?

Let the distance of the lens from the lamp be u ft. and the

distance of the wall from the lens be v ft.; it will be in this case equal to $(8-u)$ ft.

Now $\quad \frac{1}{v}-\frac{1}{u}=\frac{1}{f}$, (where) $v=8-u$,

$\therefore \quad -\frac{1}{8-u}-\frac{1}{u}=\frac{1}{f}=-\frac{2}{3}$,

or $\quad \frac{1}{8-u}+\frac{1}{u}=\frac{2}{3}$,

or $\quad \frac{u+8-u}{(8-u)u}=\frac{2}{3}$,

or $\quad 24=16u-2u^2$,

or $\quad u^2-8u+12=0$,

or $\quad (u-6)(u-2)=0$,

$\therefore \quad u=6$ ft. or 2 ft.

This shows that there are two positions of the lens; in one case the image will be magnified and in the other case small.

4. Find out the focal length of a thin lens which forms an image at a distance of 30·3 cm. behind the lens when the object is placed 91·1 cm. in front. *Ans.* $f=-22{\cdot}7$ cm.

5. A concave lens whose focal length is 12 inches is placed on the axis of a concave mirror of 12 inches radius at a distance of 6 inches from the mirror. An object is so placed that light from it passes through the lens, is reflected from the mirror, again passes through the lens, and forms an inverted image coincident with the object itself. Where must the object be placed? *Ans.* $u=12''$.

6. Show that when a ray of light is refracted through a prism in the minimum deviation position, its course in the prism is $\perp$ to the line bisecting the angle of the prism.

7. At what distance must an object be placed from a convex lens of 20 cm. focal length to obtain a real image four times the size of the object? *Ans.* 25 cm.

8. A convex lens of focal length 20 cm. is combined with a concave lens of focal length 40 cm. What is the nature and the focal length of the combination? *Ans.* 40 cm., convex.

9. A candle stands at a distance of 6 ft. from a wall. Where must a convex lens of 18 inches focal length be placed along

the perpendicular from the candle to the wall so as to produce upon the wall a distinct image of the candle ?

Ans. Half-way between the candle and the wall.

10. A concave lens has a focal length of 14 cm. Find by graphical method the position, size, and nature of the image formed of an object 1 cm. high when the latter is situated (i) 50 cm., (ii) 20 cm., from the lens.

Ans. 10·9 cm. on same side ·22 cm. high, virtual, erect; and 8·2 cm. on same side, ·41 cm. high, virtual, erect.

11. An object is gradually brought up to a convex lens from a great distance ; trace the changes in the position of the image. If the focal length of the lens is 6 inches what is the least distance between the object and its image ?

Ans. 2 ft.

12. What would have been the effect on the length of the day-time if there had been no atmosphere ?

CHAPTER V

OPTICAL INSTRUMENTS

360. The term optical instrument generally stands for any combination of lenses or of lenses, prisms, and mirrors. We shall consider only three types: (i) microscopes, which are used to obtain a magnified image of an object whose real dimensions are too small to make it distinctly visible to the naked eye; (ii) telescopes, by which very distant objects, whether celestial or terrestrial, are observed somewhat magnified; and (iii) projection instruments, in which the image, whether magnified or diminished, of any object is projected on a screen, either to be rendered visible to a crowd of spectators, or to get a permanent impression, such as the optical lantern or the camera. We shall consider first the last class, *Projection apparatus.*

361. The Photographic Camera.— The optical part of a camera consists of a combination of lenses called the objective, which forms a real, inverted image of an object situated in front of it. The objective is usually mounted in front of a box whose interior is darkened. At the back of this box there is a ground-glass screen which is replaced by a sensitive plate when taking a photograph. The sides of this box are usually made of folded leather to enable the objective to be moved forwards or backwards (and thereby to increase or decrease the distance between the objective and the screen) to accurately focus the image on the ground-glass screen. When this is done, a sensitive plate, *i.e.* one sensitive to the action of light, is placed in place of the screen. The objective is uncovered for a short time and then again is covered. The plate after being developed shows the inverted image of the object, with this difference, that whatever is bright in the object seems here dark, and whatever is dark in the object seems here transparent. This is called *the negative.* From this the photographs are printed on a sensitive paper, which are called the *prints* or the *positive.* In the print what is dark in the negative becomes bright.

361a. Cinema Pictures.—It is found that the visual impression on the retina lasts for about $\frac{1}{10}$th of a second after the stimulus which produced it has passed away. Now, if a series of photographs be taken upon a film of a moving object at a rate of more than 10 per second, and if these photographs be projected upon a screen at the same rate at which they were taken, the discontinuous pictures, due to the persistence of vision, seem to blend smoothly into each other and thereby give the effect of *moving pictures*.

Note, *what we see are not moving pictures in reality, but stationary pictures rapidly succeeding each other.*

To take the photographs for the moving pictures a special camera is used. Its shutter opens and closes automatically a number of times during a second and the film is moved past the lens with a jerky motion, so that it remains at rest during the time of exposure, but moves forward with a jerk while the shutter is closed. The pictures are generally taken at the rate of 16 per second, their size being 1 inch by $\frac{3}{4}$ inch. There being 16 pictures in a foot of the film, a reel of film, which is usually 1000 ft. long, contains 16,000 pictures.

362. Optical Lantern.—It consists of a series of lenses arranged with a view to throw on a screen a magnified image

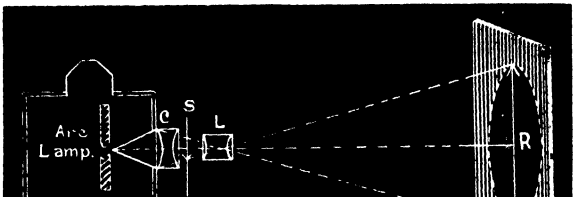


Fig. 48

of a photograph. The series of lenses can be divided into two systems: one system condenses the light upon the object and is called the *condenser*, and the other forms a magnified image on the screen and is called the *objective*. The source of light is usually the arc-light, because it is advisable to use a small source of light; as a matter of fact, the smaller the better. The source is placed almost just outside the focus of the condenser *C*, which consists of a pair of plano-convex lenses of short

focal length with convex-surfaces turned towards each other. The Condenser helps in bringing the more divergent rays from the source, after passing through the peripheral parts of the slide, to fall on the focussing lens. The cone of light emerging from the condenser falls upon the object, a transparent photographic slide, and then passes through the objective or the projecting lens L. The slide is placed as near the condenser as possible, and the tube carrying L is moved forwards or backwards to get a clearly defined image upon the screen. The object is placed just outside the focus of the objective, hence the image obtained is real, inverted, and magnified. Since the intensity of illumination of the image varies inversely as the square of the distance (LR), the best magnification that can be obtained depends ultimately on the illuminating power of the source.

363. Eye.—From an optical point of view the eye is practically a photographic camera with a lens possessing an automatic adjustment for focus in front and a sensitive film consisting of a network of nerve-fibres at the back. The eye is spherical in form and is surrounded, except in front, by a horny opaque coating called the **sclerotic.** It is tough and strong, and holds the ball of the eye in shape and forms the white of the eye. The front portion of this opaque layer is transparent and is slightly more convex than the rest of the eyeball. This portion is called the **cornea.**

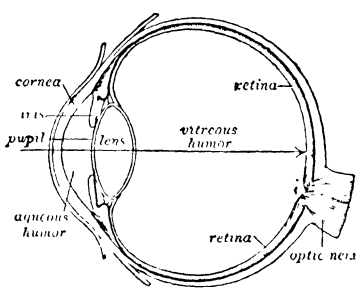


Fig. 49

The second coat inside the sclerotic is the **choroid** coat, a black, opaque membrane which serves to darken the interior of the eye. This has in front a contractile circular aperture called the **pupil.** This aperture is regulated by an opaque, coloured diaphragm called the **iris.** In strong light the diaphragm makes the aperture very small, while in feeble light it makes it very big.

Immediately behind the iris is the double-convex **lens** having surfaces of unequal curvatures. It is built up of concentric layers. The optical density of these layers increases as we go towards the centre. The lens is held in position by the **ciliary processes** which are attached round its circumference. The

ciliary muscles (which connect it with the sclerotic) help in changing the curvature of the surfaces of the lens, thus making it more or less convex at will. Investigations have shown that these muscles by contracting thicken the lens and thereby render more accurate the focus for objects near at hand. This power of adaptation, or of **accommodation** as it is called, is most marked in young children and gradually diminishes with age. The eye is divided by the lens and the ciliary muscles into two chambers: the **anterior chamber** between the cornea and the lens, which is filled with the **aqueous humor,** a watery liquid containing a little common salt in solution, and the **posterior chamber,** which is filled with a viscous mass of the highest transparency, called the **vitreous humor.**

The third coat is the retina—a delicate membrane profusely supplied with nerve-fibres and blood-vessels. This serves as the sensitive screen for the reception of the images formed by the optical system of the eye. The retina possesses two points of peculiar interest, the yellow spot and the blind spot.

(i) The **yellow spot** is situated at the centre of the retina. It is a slightly raised spot, having a minute depression called the **fovea centralis** at its summit. This fovea centralis is the most sensitive spot on the retina. It is upon this spot that the image of anything is brought that we wish to examine closely.

(ii) The second point *i* also is an eminence; it is situated where the optic nerve enters the eye, being a little nearer the nose than the optic axis of the eye. It is left uncovered by both the choroid and the retina. It is insensitive to light, and hence is called the **blind spot.**

This is, in general, the structure of the eye. Experience shows that an eye can see distinct images of objects at very different distances. For instance, we can see a distant tree through a window and also a scratch on the glass-pane, though not both distinctly at the same moment, because when the eye is adjusted to see the one clearly, the image of the other does not fall accurately upon the retina. An eye completely at rest seems adapted for seeing distant objects. The point for which the eye is focussed when at rest is termed the **far point** or **punctum remotum.** Evidently the far point is at an infinite distance from a normal eye. But with a slight effort, so slight indeed that we do not become even aware of it, we can see objects lying over a large range of distance. There is, however, a limit to this range of vision, as, for instance, the objects which are very near the eye cannot be seen distinctly. The nearest point to the eye at which a small object

can be clearly seen (say the head of a pin) is called the **near point** or **punctum proximum** of the eye. In the case of a normal eye it is at a distance of about 6 inches.

Within this wide range of accommodation, however, there is one point at which the vision is most distinct. This will be clear from the fact that when an object is long way off we cannot make out its small details, and neither can we do so when it is very near; the distance at which we can see an object most distinctly is called the distance of **distinct vision.** For a normal eye it is between 25 and 30 cm. or 10 and 12 inches.

363a. Vision.—When rays of light enter an eye from an object, first refraction occurs at the surface of cornea and crystalline lens. Since the refractive index of both the cornea and the aqueous humor is the same (1·337), we can regard the aqueous humor as extending up to the outermost surface of the cornea.

Second refraction occurs at the lens, the refractive index of which is 1·437.

Third refraction occurs when the rays of light emerge from the lens and enter the vitreous humor, whose refractive index is equal to that of the aqueous humor. After all these refractions an inverted * image is formed upon the retina. Now an object is seen distinctly when its image is formed upon the retina, hence it is clear from what is given above that the eye should see an object clearly at only one distance. It will be true if the curvature of the lens could not be changed. Owing to the power of accommodation, however, the curvature of the lens alters just as the object is near or distant from the eye.

If from any cause the eye cannot accommodate itself to various distances, the vision will be defective. The defects are generally three :

1. Short sight, or myopia ;
2. Long sight, or hypermetropia ; and
3. Astigmatism.

364. Myopia.—An eye which is myopic (*i.e.* short-sighted) cannot see distinctly distant objects. The eyeball is too long for the lens ; this may be due to either the oval shape of the eyeball itself or due to the lens becoming too convergent. Due to this the parallel rays are focussed in front of the retina, and the object is either not seen at all or seen

* But in spite of this inverted image, we see the objects erect. It is due to the mental interpretation of the retinal images.

very indistinctly. If in front of the eye we place a concave lens, this will to some extent counteract the convexity of the lens of the eye, and by taking concave lens of the proper power, the image will be formed *just* upon the retina.

Let us suppose that a short-sighted eye can see an object clearly lying up to N, and beyond that point the vision becomes indistinct; in other words, let his far-point lie at N.

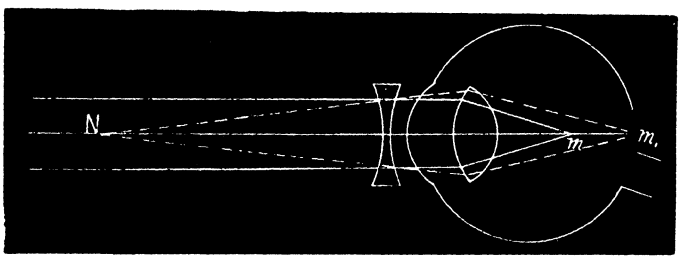


Fig. 50

Now, if we place a concave lens of such a focal length that the $\parallel$ rays are brought to focus at N, then it is evident that the distant object will have its image at N, and thus the eye will be able to see that distinctly. Suppose a short-sighted man's far-point is 20 cm. To correct his eye we should use a concave lens whose focal length is 20 cm. or whose power is 5,

for $$\frac{1}{v}-\frac{1}{u}=\frac{1}{f};\ \text{here } u=\infty,\ \therefore \frac{1}{v}=\frac{1}{f},$$

but $$v=20 \text{ cm.},\ \therefore f=20 \text{ cm.}$$

This shows that *the focal length of a lens required to correct a short-sighted eye must be equal to the distance of the far-point from the eye.*

Note, all other points lying between ∞ and N will be visible distinctly, for their image will be formed between the lens and N (see § 354).

365. Hypermetropia, or Long-sightedness.—In this case the near objects cannot be seen distinctly, due to the fact that even when the lens is curved to its utmost limit the image is formed behind the retina, which means that the eyeball has become too small for the lens or that the lens has become less convex. In this case, even the parallel rays are focussed behind the retina, so that to see the distant objects the eye has to be accommodated. This defect can be remedied by

placing a convex lens in front of the eye, which increases the total convexity. By selecting the suitable power of the lens the image of the near objects can be made to fall upon retina.

Let us suppose that a long-sighted eye cannot see objects clearly lying nearer than N', when N is the point of distinct vision for the normal eye.

The focal length of the spectacle lens must be such that

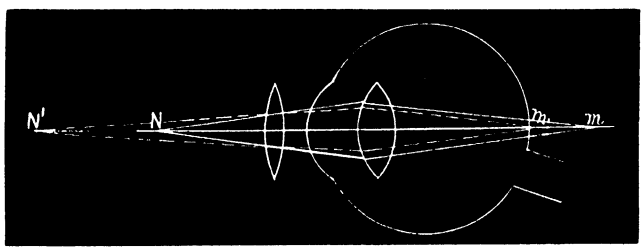


Fig. 51

an object lying at N should have its image at N' so that the object may be seen distinctly.

Let us suppose that the least distance of distinct vision for the unaided long-sighted eye is at a distance of 40 cm. from it, and we want to make objects clearly visible up to 25 cm. Then an object at a distance of 25 cm. must form an image in the convex lens at a distance of at least 40 cm. The focal-length of the lens which will form the image at a distance of 40 cm. will be given by $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$, where $v=40$ cm. and $u=25$ cm.

$$\therefore \quad \frac{1}{40}-\frac{1}{25}=\frac{1}{f}, \text{ or } f=-\frac{200}{3} \text{ cm. or } -\frac{2}{3} \text{ metres.}$$

The negative sign shows that we require a convex lens to remedy the defect. The power will be $+1{\cdot}5$ dioptres.

366. Astigmatism.—An astigmatic eye has different refractive powers in different planes. It is generally due to irregularity in the curvature of cornea; for instance, the cornea might have stronger curvature in the vertical than in the horizontal plane, with the result that the horizontal and vertical lines will not be in focus at the same time, and hence such an eye may be able to see clearly, for instance, the horizontal

wires of a network, and the vertical wires may be indistinct or curved. To remove this defect cylindrical lenses are used.

367. Binocular Vision.—In spite of the fact that we have two eyes and that when we fix them on an object each will form an image on its own retina, we see only one object, just as with two ears we hear only one sound. The simultaneous vision with two eyes enables us to see bodies with greater lustre. It will be clear from an experiment of threading a needle with one eye closed. It will be found more difficult to do so than when both eyes are open. It also enables us to judge the distances of the objects more accurately. This can be seen experimentally if we try to place a pin upon a small object in front of us, say a dot, keeping one eye closed.

Further, the images formed on the retinas of the two eyes are not exactly similar, due to the difference in position of the two eyes with respect to the object; the right eye sees more of the right-hand side and so does the left about the left-hand side. These two aspects combined give the idea of solidity. This is, as a matter of fact, the principle of the stereoscope.

368. The Stereoscope.—It is an ingenious although simple instrument which was invented by Sir C. Wheatstone and modified to its present form by Sir D. Brewster. It consists in its simplest form of two halves of a convex lens mounted in a framework in front of which two photographs are placed. These photographs are not exactly similar, but are obtained by the use of two cameras in two slightly different positions, the positions in which a person's two eyes would be if he were actually observing the object from about the same distance and position in which the camera is placed. Such two photographs will give the idea of solidity; that is to say, they will not appear flat, but will look like the actual solid things they represent. To understand the effect of the stereoscope it should be remembered, as said above, that the two images formed on the retina are not quite identical, for each corresponds to a different point of view. But when the two are looked at together, one as would be seen by the left eye and the other as seen by the right eye, the picture is seen in natural relief.

369. Telescopes. — They are used for seeing distinctly distant objects. A real image of the object is formed by a convex lens or a concave mirror, and is examined by a magnifying lens. There are several forms of telescopes, the details of construction in each case being adapted to the purpose for which that particular form is intended.

370. Astronomical Telescope.— In its simplest form it consists of a big convex lens O of long focal length (Fig. 52) mounted at one end of a brass tube, and a small convex lens E of short focal length mounted at the other end. The lens is fitted in a tube which slides inside the bigger brass tube. The lens O, which first receives the rays from a distant or a celestial body, is called the **object glass.** It may be a simple double convex lens, made up of two lenses, a convex and a concave. A real and inverted image of the object is formed at the focus F of the object glass O. The image will be real and inverted. The small lens E, which is simply a magnifying lens, may be either a single convex lens of a very short

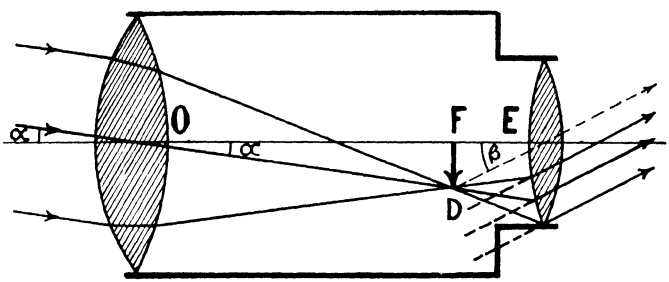


Fig. 52

focal length or a combination of two convex lenses. It is called an **eye-piece.** This is moved forwards or backwards in order to get the (inverted) image at the principal focus of the eye-piece so that an image might be seen at ∞. It should be remembered that a normal eye is focussed for parallel rays. When it is so done, the telescope is said to be in normal adjustment. But if, on the other hand, the image lies between the lens E and its focus, the eye will see through the eye-piece a virtual magnified image of the first inverted image. By moving the eye-piece backwards or forwards the final image can be seen at a distance of distinct vision. Note the final image will be inverted. But this does not make any difference so far as the celestial bodies are concerned.

Generally, on an astronomical telescope is fixed a small telescope, which is called the **finder.** It is used in fixing the direction of the telescope towards any particular star. As the finder has a larger field of view, it is more convenient to find the direction with it than with the bigger telescope.

When an image is highly magnified, it is proportionately diminished in brightness. And as the celestial objects which are to be observed through the telescope are generally faint, unless an arrangement is made to catch as much light from them as possible, they will not be observable and thus the number of observable stars will be very much limited. To do this the diameter of the object glass should be made very large. With such a telescope a number of stars which are invisible to the naked eye are made visible. There is another advantage of an object glass of a bigger diameter—it can show apart the stars which are very near to each other and which will be seen as one not only with the naked eye but even with a small telescope. The larger the object glass, the greater this power of showing apart two near objects. This power is called *resolving power*. One of the largest telescopes in the world is at the Michigan University. It has an object glass of 40 inches diameter of focal length 62 ft. In Paris there is one having an object glass of 50 inches diameter.

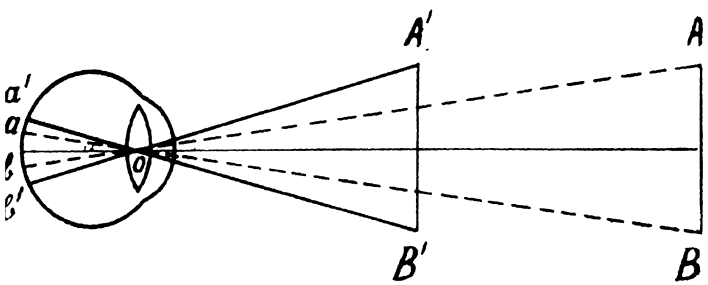


Fig. 53

371. Magnification.—Apparent size of an object depends upon the size of the image formed on the retina, and consequently on the angle *AOB* which the object subtends at the eye. From Fig. 53 it is evident that the angle *AOB* increases as the object is brought nearer to the eye. For instance when we look at a building from a distance of one mile, the angle under which we see it is very small, consequently the image is very small and we see nothing but its general form. At half a furlong it subtends a much larger angle; its image may occupy the whole of the retina, and we are able to make out the various details as doors, windows, and other smaller features. At a few yards only a part of the building occupies the whole of the retina, and that part subtends much greater angle than before; this enables us to make out still greater details, even to know what is placed inside the room, if the door, etc., be open. This example illustrates

fully what is meant by the angle subtended at the eye and the relation of the vision to the angle. From it we come to the conclusion that **the greater the angle under which an object is observed, the more magnified it appears;** consequently the *magnifying power of a telescope is the ratio of the angle which the distant object subtends at the eye to the angle subtended by the image as seen through the telescope.*

Let *DOB* (Fig. 52) represent the axis of the telescope. The image of the object will be formed at *F* by the lens *O*. It is clear from Fig. 52 that the parallel rays coming from the object make an angle α with the axis of the telescope; this, in other words, means that the object subtends an angle α at the object glass or since the length of the telescope is small in comparison with the distance of the object from the object glass, the angle subtended at the eye is also the same, *i.e.* α. The angle under which the object is observed is β.

Therefore magnification

$$= \frac{\angle \beta}{\angle \alpha}.$$

But when the angle is small, it is equal to its tangent.

$$\therefore \quad \text{Magnification} = \frac{\angle \beta}{\angle \alpha} = \frac{FD}{EF} \times \frac{FO}{FD} = \frac{FO}{EF}.$$

We know that *FO* is the focal length of the object glass and *EF* is less than the focal length of the eye-piece.* Thus we see that **the magnifying power of a telescope is as the ratio of the focal length of the object glass to that of the eye-piece.**

This leads us to the conclusion that the magnifying power of a telescope is greater the greater the focal length of the object glass, and the smaller that of the eye-piece.

Consequently **a telescope must have a long focus object glass and a short focus eye-piece.**

372. Reflecting Telescope.—It is only in recent times that good convex and concave lenses of large diameters have been successfully made; before this for astronomical observations a reflecting telescope was generally used. Reflecting telescopes have a concave mirror instead of an object glass; this produces a real image at its focus, wherefrom it is observed through a magnifying eye-piece. There are two types of such telescopes, one in which the axis of the mirror is coincident with the axis of the tube, and the other in which the axis of the mirror is slightly inclined to the axis of the tube.

Let us first consider the first type. In this type the axis of

* Generally the eye-piece is so adjusted that it gives the virtual image at the distance of distinct vision from the eye, *i.e.* at a distance of about 30 cm.

the tube, as has been said, coincides with the axis of the mirror. The image of the distant object is formed at the focus F. If the head be placed to view the image directly most of the light will be cut off. In order to avoid it a plane mirror N is placed in such a manner that the rays are reflected to one side, where they fall upon the eye-piece.

In the second type of the reflecting telescopes the axis of the mirror makes a small angle with the axis of the tube, hence the

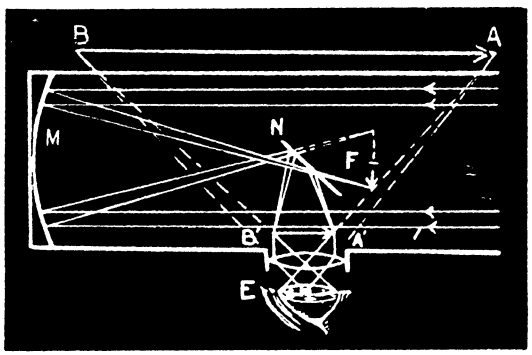


Fig. 54

image is thrown to one side by the mirror itself, and can be looked at through the eye-piece as before. There is no fundamental difference between the two types of the reflecting telescopes: the difference lies only in the method of observing the image.

The same remarks apply to the reflecting telescopes as to the refraction ones. Here also the magnifying power is equal to the ratio of the focal lengths. The diameter of the mirror must be large in order to gather as much light as possible. The diameter of the mirror of one of the biggest telescopes is 7 ft., its focal length is 54 ft., and its weight 9000 pounds.

373. Terrestrial Telescope.—In the astronomical telescope the image produced is inverted, and since in observing stars it does not matter whether the image is inverted or erect, no device is used for getting an erect image. But in observing terrestrial objects, it is very inconvenient if their images are inverted, hence a device for getting an erect image is employed. This can be done in two ways. In the first case the inverted image is reinverted, and this is done by interposing a convex lens

between the object glass and the eye-piece in such a way that the image I_1 formed by the object glass is at a distance of $2f$ from this auxiliary convex lens O' (see Fig. 55). The image I_2 formed by it will be inverted and equal in size to I_1. This is looked at through the eye-piece, which gives a virtual, magnified image of

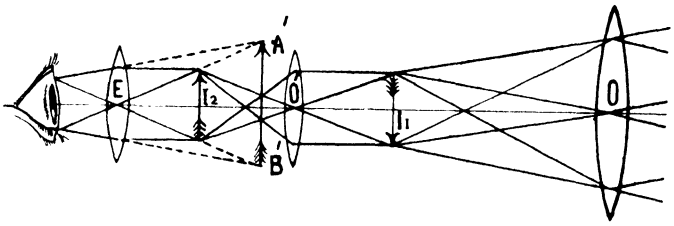


Fig. 55

it. The eye-piece produces a virtual, erect magnified image of the object, provided the image formed by the second convex lens O' lies between the focus of the eye-piece E and E itself.

In the second method a concave lens is used as an eye-piece ; such a telescope is called **Galileo's telescope.** Suppose O is the object glass and it produces an image I_1 at its own focus. Now

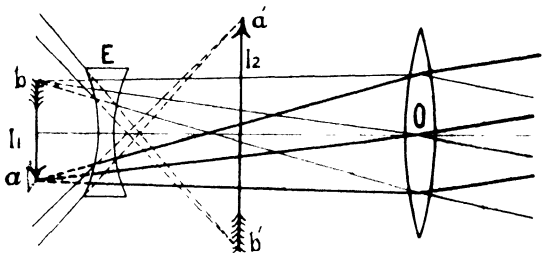


Fig. 55 (*a*)

interpose a concave lens between the object glass and I_1 in such a manner that I_1 is just outside the focus of the concave lens ; the rays after passing through the lens will become more divergent, and hence will appear to come from I_2 ($a'b'$). The image produced is inverted of the inverted image, *i.e.* the final image as seen through the concave lens is erect.

The magnifying power is the same as in the case of an astronomical telescope: the ratio of the focal length of the object glass to that of the eye-piece. Usually the distance between E, the concave lens, and I_1 is greater than the focal

length, and when it is so it gives rise to a virtual, magnified image towards O. By adjusting the position of the eye-piece the image I_2 can be obtained at the distance of distinct vision from the eye or at infinity. Evidently the total length of such a telescope is smaller than that of an ordinary terrestrial telescope.

374. The Simple Microscope or **Magnifying Glass.**—It has been explained already in § 371 that the apparent size of the object depends on the angle which it subtends at the eye. If the angle can be made larger, the body will also appear larger. As was pointed out, one method to increase the visual angle is

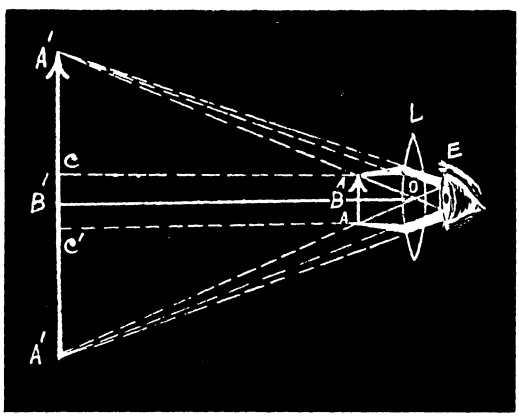


Fig. 56

to bring the object nearer the eye. But we know that we cannot decrease the distance below a certain limit (*i.e.* the limit of distinct vision), because after that the object is seen very indistinctly. Now, if a convex lens be interposed between the eye and the object in such a manner that the object lies between the lens and its focus, a virtual magnified image will be produced at a greater distance from the eye. Fig. 56 will help us to understand the effect produced. E is the eye, L is the lens, AB is half the object at a distance, say, of 5 cm. It is subtending a much greater angle at the eye than it will do when it is placed at a distance of 25 to 30 cm., the distance of distinct vision. With the help of the convex lens it appears at $A'B'$, and the apparent distance of the object as seen through the convex lens is increased. Now suppose $A'B'$ is at the distance of distinct vision, the object will appear very much

greater as it is subtending a greater angle at the eye, the angle being the same which the image is subtending. By a slight adjustment of the lens, *i.e.* by varying its distance from the object, the magnified image can be obtained at the distance of distinct vision.

375. Magnifying Power of a Convex Lens.—In calculating the magnifying power it is assumed that the image formed by the lens is at the distance of direct vision. Let us suppose it is d. As is seen from the figure, when the object is placed at the distance of distinct vision, the angle subtended by half of it is COB' at the centre of the lens, and if the eye is very close to the lens the angle subtended at the lens is the same as the angle subtended at the eye. So the magnification is

$$\frac{\angle A'OB'}{\angle COB'} \quad \text{or} \quad \frac{\angle AOB}{\angle COB}.$$

When angles are small, they are equal to their tangents. Of course it is supposed that object AB' is very small, so that our approximation is applicable. We get

$$\text{Magnification} = \frac{AB}{OB} \times \frac{B'O}{CB'}; \text{ but } CB' = AB,$$

$$\therefore \quad \text{,,} \quad = \frac{OB'}{OB} = \frac{d}{u},$$

where $OB' = d$, and $OB = u$, the distance of the object from the lens. Let, further, the focal length be f; then with the help of the formula $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$ we can determine the value of u in terms of f and d, only bearing in mind that v is equal to d here.

Thus
$$\frac{1}{u} = \frac{1}{d} - \frac{1}{f} = \frac{f-d}{fd},$$

$$\therefore \quad u = \frac{fd}{f-d}.$$

$$\text{Magnification} = \frac{d}{u} = \frac{(f-d)}{f} = 1 - \frac{d}{f},$$

where d is + and f is negative.

It is clear from the above expression that magnification increases as f decreases, hence *to obtain a great mangifying power a lens of a very short focal length must be taken.*

1. Calculate the magnification when a lens of focal length 4 cm. is used as a magnifier, the distance of distinct vision being 28 cm.

$$\text{Magnification} = 1 - \frac{d}{f}.$$

Here $d = +28$ cm. and $f = -4$ cm.

$$\therefore \quad \text{Magnification} = 1 - \frac{28}{-4} = 1 + 7 = 8.$$

2. Calculate the magnification produced by a convex lens of focal length 3 cm. when it is placed at a distance of 1 cm. from the eye, the distance of distinct vision being 28 cm.

The image will be produced at a distance of distinct vision and hence $OB' + OE = d$ (Fig. 56). The angle subtended by the image $A'B'$ at the eye is $\frac{A'B'}{OB' + OE}$ or $\frac{A'B'}{d}$, and the angle subtended by the object AB when at the distance of the distinct vision will be $\frac{CB'}{d}$ or $\frac{AB}{d}$, where d is the distance of distinct vision from the eye.

$$\therefore \text{Magnification} = \frac{A'B'}{d} \times \frac{d}{AB} = \frac{A'B'}{AB}$$

$$= \frac{OB'}{OB}$$

$$= \frac{v}{u}$$

$$= \frac{f - v}{f}$$

For $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$,

$$\therefore \frac{1}{u} = \frac{1}{v} - \frac{1}{f} = \frac{f - v}{fv},$$

$$\therefore \frac{v}{u} = \frac{f - v}{f}.$$

Now we know $f = -3$ cm. $v = 27$ cm., as the image is at the distance of distinct vision, and $d = 28$ cm.

Remembering that *f is negative*, we get in this case magnification $= 1 - \frac{27}{-3} = 1 + 9 = 10$.

376. The Compound Microscope.—With a simple microscope very high magnification cannot be produced. When a very high magnification is required, lenses are combined to form an instrument called Compound Microscope. In principle it is exactly similar to the astronomical telescope,* but the difference in the two instruments results from the adaptation of the object glass to view the nearer objects. Here we do not require the object glass to be a lens of a long focal length, but on the other hand a lens of a short focal length is needed.

The object (say AB) to be magnified is placed just outside the focus of the lens O. A real, inverted, and magnified image of the object is formed at $A'B'$. The second lens, or E the eye-piece, is placed at such a distance from the object glass, which is called **objective** in microscopes, that the image $A'B'$ falls between the principal focus and the lens E itself. This second lens forms a virtual magnified image $A''B''$ of $A'B'$.

But as the normal eye is adjusted for parallel rays, the

* Because the front lens forms a real image of the object in both cases.

position of E is adjusted in such a manner that $A'B'$ is at the principal focus of E. The theory given above explains the principle rather than the actual discussion of the path of the rays. The modern microscopes are very complicated. The objective consists of a number of lenses instead of only one; there may be as many as 6. The image produced by such a combination is seen through the eye-piece, which is made up generally of two lenses. This arrangement can give a magnification of 300 to 1000 diameters. In addition to this

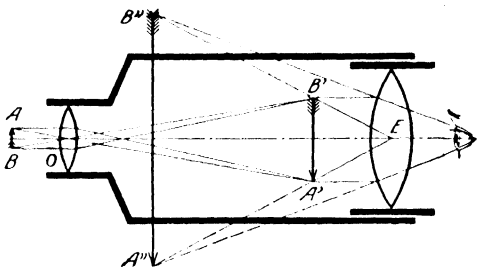


Fig. 57

there is a stage upon which the object which is to be viewed through the microscope is placed. Below the stage there is a mirror, which can be tilted to any angle to throw a strong beam of light upon the object. The necessity of strong illumination will be realised if we consider that the modern microscope gives us a linear magnification of 500 to 1000 times, which means a superficial magnification of 250,000 to 1,000,000 times. Linear magnification has been carried even further, but such a high magnification is seldom used, for it is secured at the cost of definiteness.

In modern microscopes the distance between the objective and the eye-piece is fixed, and a distinct image is obtained by adjusting the distance of the object from the objective.

CHAPTER VI

DISPERSION

377. Suppose a narrow slit is brightly illuminated by arc light or sun light, and a short image of it is thrown on a screen with the help of a convex lens. On interposing a prism between the slit and the lens with the refracting edge of the prism parallel to the slit, the image will no longer be formed at the former position. It will be deviated to one side, and in addition to that it will be noticed that the image is drawn out into a continuous broad band of colours. On

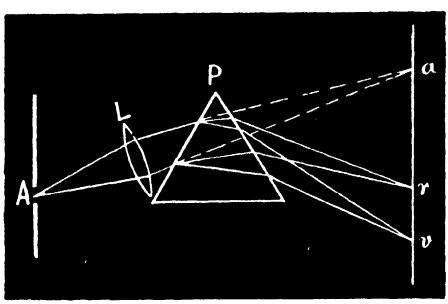


Fig. 58

one end the image is red, and as we pass from this to the opposite end, the colour changes successively through orange, yellow, green, blue, indigo, and violet (Fig. 58). The violet colour is at the end of the band farthest from the first position, whereas red is at the nearest end. This is sometimes expressed by saying that the red rays are least deviated and the violet rays are most deviated; the intermediate colours are arranged in the order given above. *Such a band of colours is called a* ***spectrum.*** If sun light be used as the source of light, this

band of colours is called the solar spectrum. *The splitting up of white light into its component colours is called* ***dispersion.*** Each component colour suffers deviation to a different extent; this is expressed by saying that each constituent colour has a different **refrangibility**; the most refrangible rays (violet) suffer the greatest deviation, while the least refrangible suffer the least deviation, as for instance red rays.

378. The explanation of this splitting up of white light into different colours was first given by Newton. He said that white light is not homogeneous, but consists of different constituents which are deviated by different amounts when passed through a prism. He pointed out clearly that the prism did not produce colours, but simply separated them. To substantiate his statement, he took two similar prisms of the same material, and placed them with their edges in the opposite directions, and their faces parallel. The light on the screen was no longer coloured, it was white, and the spectrum was gone. The second prism produced dispersion in the opposite direction to that produced by the first. The image of the slit fell near the original position; as a matter of fact, the two prisms together behaved like a plate (with parallel sides) and produced a lateral displacement only. This clearly proved that white light is made up of all these constituents.

He found that, on the other hand, when the prisms are placed with their edges parallel and in the same direction, the deviation is doubled.

If a red plate be placed before the slit, a red image of the slit is seen, the blue and green parts being absent. But if, on the other hand, a glass plate coloured deep blue is interposed before the slit, only a blue image is seen. The position, however, of these images (red and blue) is not the same. This can be very easily verified by covering the upper half of the slit with a deep red plate and the lower half with a deep blue. These results are very simply accounted for by Newton's explanation. When a red plate is held before the slit, all the constituents except those which form red end are absorbed, and thus we get red colour, and similarly, when a blue plate of glass is held before the slit we get the blue colour. And why they are not at the same place is explained simply by the fact that different colours possess different refrangibility.

379. Pure Spectrum.—The spectrum formed on the screen with the help of a prism alone is not pure, for any particular point in the spectrum is not being illuminated by a single colour.

It is clear from Fig. 59 that each ray produces a spectrum

at the screen; and consequently in the central region there is overlapping of the various colours. If the slit be wide and the light used be white, the central portion is white and the edges are coloured.

If the axis of the pencil of light falling upon the prism undergoes minimum deviation, then each colour seems to proceed from a virtual image like *V* and *R*. Thus a virtual spectrum is seen at *VR*. It is *almost pure*, as overlapping does not appreciably occur.

So even when on the screen the central portion is white, the spectrum as seen through the prism is pure. But to get

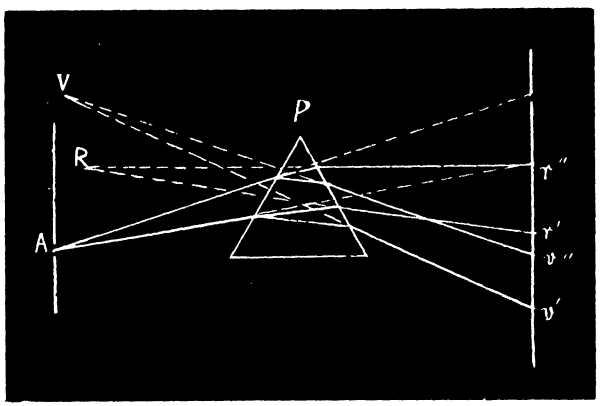


Fig. 59

a pure spectrum on the screen we must interpose a convex lens which should give us a real image on the screen of the virtual spectrum *VR*. Now when a lens *L* is interposed (Fig. 58), the rays of each particular constituent colour are focussed at different points on the screen, and hence the resulting spectrum is pure. *L* gives, when *P* is removed, a distinct image of the slit at *a*, and when the prism is interposed in the position of minimum deviation, the pencil of rays converging to *a* will be dispersed after passing through the prism. Real images of the slits are thus formed at these points by the light of each colour, and as each image is narrow and distinct there is no overlapping, and the spectrum obtained is pure. The lens may be interposed either between the prism and the screen or between the prism and the source.

A still better arrangement is to make the rays fall upon a prism as a parallel beam, so that when one ray passes through the prism in the minimum deviation position, all rays will pass suffering minimum deviation, and hence the spectrum will be still purer.

Thus we see that if we use two lenses and one prism, with proper adjustments we can get a very pure spectrum. This proper adjustment, as has been said, consists in making the incident-beam of light parallel, adjusting the prism in the minimum deviation position and using a second lens to focus the different colours on the screen. All these adjustments are easily made with the help of an instrument called **spectrometer.** It consists of three parts: (i) the *collimator*, which consists of a metal tube with a slit at one end and a lens at the other end, the length of the tube being equal to the focal length of the lens, so that the beam coming out is parallel; (ii) *turntable* for carrying the prism, which can be rotated about a vertical axis; (iii) a *telescope*, to view the spectrum distinctly. The place of the screen is taken up by the eye-piece.

380. Invisible Spectrum.—The spectrum we have been discussing so far is called visible on account of the fact that it can be seen with the eyes. But it is found that it is continuous on both sides; beyond the red end side it continues as long heat waves, which, though invisible, yet produce a strong heating effect. This part is called **infra-red.** It is constituted by longer wave-lengths than the red. The portion of the spectrum which extends beyond the violet end is called **ultra-violet.** It consists of extremely short wave-lengths. They are particularly active in producing decomposition in the salts of silver in certain substances. In order to study these invisible portions of the spectrum, we have to resort to special devices. To study the infra-red part a prism of such a material should be used as does not absorb that part; the one that is generally used is made up of rock-salt, because rock-salt does not absorb the heat waves, whereas a glass prism absorbs them considerably. It is found that the infra-red rays are less refrangible than the red rays. To study ultra-violet part quartz prism is used. It is found that this part consists of smaller wave-lengths, which are more refrangible than the violet rays.

381. Dispersion in a Lens.—A lens, whether convex or concave, can be considered to be made up of a number of prisms placed one above the other in the order of their refracting angles. In the case of a convex lens, for instance, the angles

are directed away from the axis: prism farthest away from the axis possesses the largest angle; but, on the other hand, in the case of concave lenses the angles are directed towards the axis. Now, if what has been said is true, the beam of light, on account of the action of the prisms in passing through a lens, should suffer dispersion. And as a matter of fact, it is found actually to be the case. When white light falls on a lens the violet colour comes to focus at *V* and red at *R*, as shown in Fig. 60. The image formed anywhere near *V* or *R* will be coloured at its edges. If the screen be placed anywhere

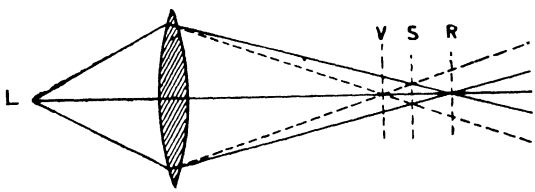


Fig. 60

near *V*, the outer edge will be coloured red, but near *R* it will be coloured violet. The best position, however, is somewhere midway between the two, as *S* in the figure. This defect is called **chromatic aberration.** To remedy this defect generally a combination of lenses, one convex and the other concave, is used; the chromatic aberration being in opposite directions for the two lenses, is neutralised.

382. Spectrum Analysis.—When we heat a piece of metal, we find that as the temperature rises, it passes in succession through different colours, beginning with the dull end.* As the temperature rises it emits waves of smaller and smaller lengths. If we allow the light emitted by the metal piece to pass through a prism in the minimum deviation position, and afterwards look at it through a telescope, we find that the different colours are added on, so to speak, to the spectrum as the temperature rises. When it is white-hot the spectrum is continuous, and consists of all the usual colours. But on the other hand, when the substance is in a gaseous condition, the spectrum emitted by it is quite different. It is no longer continuous. It consists of only a few bright lines here and there; of course the position of lines emitted by a particular substance is fixed. The line spectrum may be

* At about 600° C. it is red, at about 1200° C. it is somewhat white, but at 1500° C. it is dazzling white.

simple, as is the case of sodium vapour (of 2 lines), or complex, as in the case of iron which has more than 2000 lines. Since as small a quantity as one-millionth of a milligram of a substance will produce the characteristic spectrum of the substance, we have, therefore, in spectrum analysis a very delicate method of analysing substances. To do it experimentally, a salt of a substance is introduced into a flame, and then the spectrum is analysed with the help of a spectrometer. By noting the positions of the lines emitted by the substance, its identification can be established.

383. Absorption Spectra.—If a dark red plate of glass be placed before the slit, only a red image is seen on the screen; similarly if any other coloured plate be placed, it lets through only the rays of its own colour and absorbs the rest. This shows that substances absorb certain portions of the spectrum, producing absorption bands.

Kirchhoff found the relation between the emission and the absorption spectra. He found *that a substance which emits certain wave-lengths when heated, absorbs the same when cool.* This is a very important law. The experimental verification of this law is quite simple. If in a bunsen flame sodium salt be introduced, it emits two bright lines very close together called sodium *D* lines. If an arc lamp be placed behind it, so that the bunsen flame is between the arc light and the slit of the spectrometer, dark lines corresponding to the bright lines are seen, establishing thereby the truth of Kirchhoff's law.

384. Fraunhofer Lines.—When a solar spectrum is analysed with a spectrometer, it is found to be crossed by a large number of black lines. They were first discovered by Wollaston in 1802, but were rediscovered and carefully studied by Fraunhofer in 1814. Their explanation was given first by Kirchhoff. He first noticed that the sun is made up of a central sphere at a very high temperature (about 6000° C.), and is surrounded by a cooler atmosphere (at about 2000° C.) in which substances in gaseous condition are present. These vapours being at a lower temperature than the central portion, absorb the lines which they themselves would emit, and hence the solar spectrum is crossed by dark lines. As a matter of fact the composition of the sun has been found out by locating the position of these lines in the spectrum and then comparing them with the lines given out by the terrestrial elements. It is worth while to note that these dark lines are not absolutely so, they are simply dark in comparison. The background is the

spectrum due to a very bright source, and the dark lines are less bright, being emitted by a source at a lower temperature.*

385. Colours of Bodies.—A piece of cloth which appears blue when white light falls upon it also looks blue when it is held in the blue end of the spectrum, but in the red part it appears dark. The reason of it is simply this, that a blue body reflects only blue colour and absorbs the rest. Thus when it is held in the red part of the spectrum it appears dark, because it cannot reflect to the eye the red rays. Similarly a body looks green, because it absorbs all other colours except the green.

In the case of the transparent bodies the principle is just the same, only the body, instead of reflecting a particular colour, transmits that. Thus a red plate of glass appears red because it lets through the red rays and absorbs the rest, when white light falls upon it.

It should be noted further that a white body appears red in red light and blue in blue light, because it reflects every constituent equally well. As a matter of fact it is white because it reflects all the constituents.

In some cases the bodies possess *surface colour*; it is on account of the fact that they reflect certain colours more strongly than others; hence they appear to possess one colour when viewed by reflected light, and another when viewed by transmitted light.

386. Complementary Colours.—It is found that by suitably mixing two certain colours white colour can be produced. Such two colours are called *complementary colours*. For instance yellow and blue are complementary. If we whirl on a whirling table two discs, blue and yellow, we find the resultant colour to be somewhat like white. By adjusting the proper areas of the discs white colour can be imitated. There are three colours which cannot be imitated; they were shown to be red, green, and violet by Maxwell. They are called, therefore, *primary colours*.

Distinction must be made, however, between the *pigment* and the *spectrum colours*. It has been said that the green colour of the spectrum is primary, and therefore is inimitable

* The central portion of the sun, which is at 6000° C., emits continuous spectrum; suppose in the cooler atmosphere sodium vapour is present, obviously it will absorb the lines which it will emit itself, but all the same it will itself emit those very lines, and they being due to a body at a temperature of 2000° C., will be less bright than the rest of the spectrum, and hence in comparison they will appear dark.

to the eye. But in practice when we paint a paper blue and yellow, green colour is produced, and not white, as will be expected if they were the spectral colours. It is due to the fact that paints are not pure in colour, and therefore they allow other colours also to be reflected. For instance, the blue paint not only allows blue to be reflected but also a little of the green, on account of the fact that there is some impurity of green in its composition; and similarly the yellow paint allows, in addition to the yellow colour, the green to be reflected. Pure blue and yellow combine to form white, but the green is left outstanding. So we see that, if we take pure blue and yellow, the result will be not the production of the primary colour green, but of white colour.

CHAPTER VII

VELOCITY OF LIGHT

387. Light travels with such an inconceivable velocity that for long it was considered it does not take any time to go from one place to another. The first man to give a definite proof of the velocity of light was Römer—a Danish astronomer. He in 1676, in the observatory of Paris, deduced it from his study of the eclipses of an inner satellite of Jupiter. This satellite

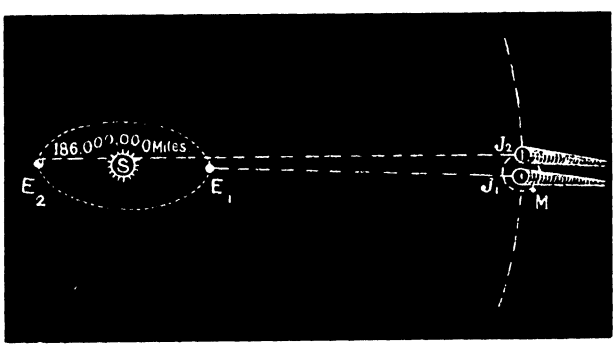


Fig. 61

appears to us to be eclipsed once during each revolution when it passes into the shadow of Jupiter. It disappears quite suddenly, and the interval between two successive eclipses can be determined accurately to half a minute. While the earth moves in that part of the orbit nearest Jupiter, its distance from that planet does not alter materially from day to day, and the interval between two consecutive eclipses is found to be constant.* Römer timed the eclipse when the earth was at E_1 (Fig. 61), *i.e.* nearest Jupiter, and from this time

* This occurs at an interval of 42 hours, 28 minutes, 36 seconds.

calculated the times of the successive eclipses on the assumption that the interval between two consecutive eclipses is constant. During the succeeding months he began to compare his observations with the calculations; he found that the observed times were *later* than the calculated times, and that the difference between these two times varied with the relative positions of the earth and Jupiter. He found, moreover, that the difference increased as the earth moved away from Jupiter, and reached a maximum when the distance between the earth and Jupiter was greatest (*i.e.* when the earth was at E_2). This was after a period of about ·545 years. The eclipse at E_2 was found to be 16 minutes and 36 seconds *later* than the time calculated from observations at E_1. Römer explained the fact by saying that since the distance of the earth was increasing from Jupiter, therefore light took longer time to reach the earth, and as no provision was made in the calculations for the increase of distance, hence the discrepancy between the two times. The diameter of earth's orbit is 185,600,000 miles, hence

$$\text{Velocity of light} = \frac{185{,}600{,}000}{996} \text{ miles per second}$$
$$= 186{,}000 \text{ miles per second or}$$
$$2{\cdot}996 \times 10^{10} \text{ cm. per sec.}$$

388. Fizeau's Method.—The first man who succeeded in measuring the velocity of light over terrestrial distance was Fizeau. His experiment depends upon the following principle.

If a toothed wheel be made to rotate very rapidly about its axis, the time taken by a tooth or a space to pass a particular point is extremely small. To form an idea it may be mentioned that the time taken by the tooth to pass a point may be made as small as $\frac{1}{20{,}000}$ of a second. During this time light will travel only 9·3 miles.

Let us consider the simple arrangement of a toothed wheel placed at a distance from a plane mirror. Suppose a source of light sends parallel rays through a space of the wheel, which fall on the mirror normally and are reflected back. An eye placed behind the wheel will see evidently the image of the source of light in the mirror, and if light travels with finite speed it will take *some* time to go from the wheel to the mirror and to come back to the wheel after reflection. If it does, it is possible, by rotating the wheel and so adjusting its speed that when the beam of light after passing through the space

comes back to the wheel, it finds there a tooth in place of space, and thus gets stopped; again the light will pass through the next space, but will be stopped by the next tooth, and so on. The result of this will be that the image of the source will no more be seen in the mirror. By doubling the speed, the light will pass through one space, and will fall on the next space after reflection and the image will be visible. Making the speed three times the light will again be cut off. This leads us to the conclusion that, *if light travels with a finite velocity, the speed of rotation should affect the image seen.* But if, on the other hand, light travels instantaneously, then, however great the speed of rotation of the wheel may be, we should always find the image.

Fizeau found in 1849 that the speed of the rotation of the wheel does influence the visibility of the image. His appar-

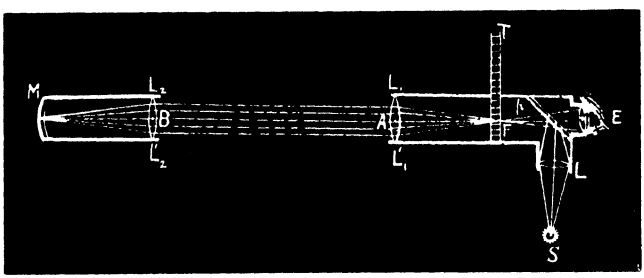


Fig. 62

atus is represented diagrammatically in Fig. 62. A pencil of light from the source S passes through a convex lens L, and, after reflection at a plate of glass at an angle of 45°, comes to focus in the space of the toothed wheel T at F, which is also the principal focus of the lens L_1L_1' (FA being its focal length). The light consequently emerges from L_1L_1' as a parallel beam. After traversing a distance of 4 or 5 miles it falls on a second lens L_2L_2', at whose principal focus a concave mirror M, having the radius of curvature equal to MB, is placed. The beam of light is brought to focus at M; it is reflected back * to fall upon L_2L_2', and to emerge from there as a parallel beam (BM being equal to the focal length of the

* It should be clearly understood that the central ray BM falls normally on the mirror and hence it will be reflected back, but the ray L_2M is reflected along $L_2'M$, and the ray $L_2'M$ is reflected along L_2M.

lens). After refraction through L_1L_1' the beam will come to focus at F, therefrom it will fall upon the plate of glass p; part of it being reflected, and the rest being transmitted to the eye at E.

When the wheel is rotated slowly, a flickering sensation is produced, due to the fact that light passes after reflection through the spaces and is cut off by the teeth. Now if the speed be increased so that in one second more than 10 images are seen, then owing to persistence of vision the flickering ceases, and in place of it a continuous image is seen; of course the brightness of the image is less than what it will be in the absence of the wheel. On increasing the speed to such an extent that the time taken by the light to go from F to M and back to F is just equal to the time taken by the next tooth to come into the position of the space, light is cut off, and the field of view becomes dark. What is actually seen in the experiment is that at first a bright star-like body makes its appearance, which gradually diminishes in brightness as the speed of rotation increases till at a *certain* speed it is entirely extinguished. If the speed of rotation be increased still further, the star makes its reappearance, attains its maximum brightness; this happens when the speed is double that of the first (*certain*) speed. As the speed becomes 3 times, the star again disappears, and so on.

Let the distance FM between the two stations be d, and the wheel make n revolutions per second when the first eclipse occurs, there being m teeth and m spaces on the wheel, the teeth and the spaces being of equal width. Further, if V stands for the velocity of light, the time taken by the light to go from F to M and to come back to F will obviously be

$$\frac{2d}{V}.$$

This will be equal to the time taken by the tooth to come into the place of space. Now as the wheel makes n revolutions per second, it will make one revolution in $\frac{1}{n}$ of a second. Further, the time taken by the tooth to come into the place of the space will be $\frac{1}{2m}$ of the time taken for one revolution or $\frac{1}{2mn}$ of a second. Thus

$$\frac{1}{2mn} = \frac{2d}{V}$$

or

$$V = 4dmn.$$

In Fizeau's actual experiment, d was equal to 8663 metres, the wheel had 720 teeth, and the number of revolutions made by the wheel per second when the star just disappeared for the first time was 12 6. Hence

$$V = 4 \times 8663 \times 720 \times 12{\cdot}6$$

$$= 314{,}400{,}000 \text{ metres per second.}$$

Cornu in 1874 improved upon this apparatus, and obtained the result as 300,330,000 metres per second, or 3×10^{10} cm. approximately.

PART V

FRICTIONAL ELECTRICITY

CHAPTER I

389. When amber is rubbed with silk it is found to possess the property of attracting small pieces of paper pithballs, feathers, and other light objects. It was first discovered by Thales in the year 600 B.C. that amber when rubbed becomes physically different. As in Greek it is called elektron, it was after this name that the agency which endowed amber, when rubbed, with the property of attracting the above-mentioned light bodies, was called **electricity** by Dr. Gilbert, physician to Queen Elizabeth; and the body was said to be **electrified**. Towards the end of the sixteenth century he showed that this property was shared by many other substances, such as glass, sealing wax, ebonite, sulphur, etc. But now it is known that **all** substances when suitably rubbed attract light bodies provided proper precautions are taken (see § 393).

When a substance is rubbed with a suitable material and is then found to possess the property of attracting light objects, it is said to be electrified. The unknown mysterious agent which is the cause of this state is called **electricity**. Bodies which are in their normal condition and do not exhibit this property are said to be **neutral**.

We shall not enter into the question of the real nature of this agent, electricity, but shall content ourselves at this stage with the remark that it is not like heat, light, and sound, a form of energy. We shall come to this point later on. Let us at first study briefly the properties of an electrified body.

390. If an ebonite rod is rubbed with flannel and is brought near some pieces of cork, it will be noticed that the pieces jump up to the rod, remain in contact with it for a moment, fall back to the table, jump up to the rod again, and so on. All light bodies behave in a similar manner. If the rod is held near a

pithball which is suspended by a *dry silk* thread like the bob of a pendulum (Fig. 1), the pithball is attracted towards the ebonite rod, touches it, and is then repelled by it. Now if a glass rod rubbed with silk be held near the same pithball, instead of being repelled it will be attracted; of course after coming in contact it will be repelled even now.

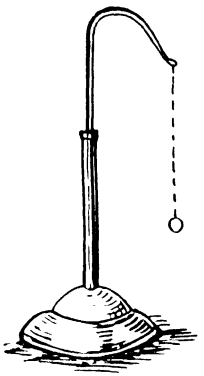


Fig. 1

Suspend an electrified ebonite rod in a wire stirrup by dry silk thread (Fig. 2), and bring near it a glass rod rubbed with silk. Notice the attraction. If, on the other hand, another ebonite rod rubbed with flannel is brought near, repulsion will be noticed.

If a charged glass rod be substituted for ebonite rod, it will be seen that another charged glass rod repels it.

If a wooden or metal bar is brought near a suspended charged rod it will be seen that the electrified rod rotates so as to approach the bar. If two rods oppositely charged be suspended in stirrups and are brought near each other, both will swing in a manner indicating mutual attraction. Similarly it can be shown that repulsion is mutual. The actual motion produced in the two, however, may not be equal, since one of them may possess a greater mass than the other.

From what has been said above we learn that—

(i) there are two kinds of electricity, one that is developed on glass when rubbed with silk, and the second that is developed on ebonite when rubbed with flannel;

(ii) similarly charged bodies repel each other, while dissimilarly charged bodies attract each other;

(iii) the attraction and repulsion are mutual.

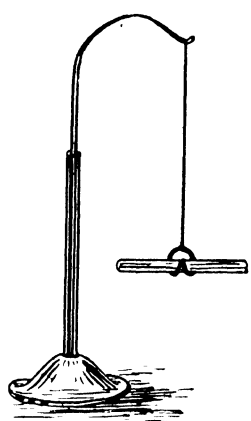


Fig. 2

391. It was Dufay who first observed these results. The kind of electricity excited on a glass rod when rubbed with silk

was called by him **vitreous** electricity, for in Latin glass is called *vitreum*, while that excited on ebonite by flannel was termed **resinous**. But these terms were discarded soon afterwards, because observers found that a body, say a glass rod, was not always charged with the same kind of electricity. Under different circumstances it became charged with different kinds; for instance, an ordinary glass rod rubbed with silk becomes charged with what is called vitreous electricity, but if the same rod is rubbed with fur it becomes charged with the second kind of electricity, *i.e.* resinous. Similar is the case with other bodies. Thus it is found that the kind of electricity produced depends upon two things: (i) the nature of the body which is rubbed, and (ii) the nature of the second body used for rubbing the former. For this reason the terms vitreous and resinous which were introduced had be to given up, as they had connection only with the nature of the body rubbed, and in their place the terms positive and negative, which have no connection absolutely with the nature of the body rubbed, were introduced by Benjamin Franklin. Nowadays we say that—

(*a*) Glass rod rubbed with silk is positively charged.

(*b*) Ebonite rod rubbed with flannel is negatively charged.

It should be noted that these terms are arbitrary. There is no reason, except custom, why we should call the glass rod rubbed with silk positively charged and why not negatively charged. As a matter of fact, if we were to call it negatively charged the same objection could be raised again. Hence there is no reason for calling one kind of electricity positive except that we have got to give it some name.

A list can be drawn up of various bodies in which out of any two on being rubbed together, the one standing higher up becomes positively charged and the other negatively.

It must be remembered, however, that the order will depend upon the nature of the specimen used, hence this is approximate only.

Fur	Silk	Sulphur
Flannel	Hand	Ebonite
Ivory	Metals	Leather coated with amalgam
Glass	Amber	

392. Simultaneous and equal development of the two kinds.—It is found that these opposite kinds of electricity are

always developed together; that when the glass rod is charged with positive electricity, the silk used for rubbing carries off the negative electricity, and that when the ebonite rod is charged negatively, the flannel carries off the positive electricity. This can be proved experimentally in the following way: Take an ebonite rod fitted with a fur cap to which a silk thread is attached. Charge the rod with electricity by turning it round for some time inside the cap. Present them together to the pithball pendulum and observe that there is no attraction.

Fig. 3

After removing the cap by the silk thread bring it near a pithball charged with a glass rod rubbed with silk, and notice the violent repulsion. The flannel, therefore, is charged positively. On presenting the rod to a neutral pithball attraction will be noticed, and on presenting it to a third pithball charged negatively, violent repulsion will be seen. So we see that not only the body rubbed becomes charged with electricity, but also the body with which it is rubbed becomes charged with equal and opposite kind.

This experiment proves conclusively that both kinds of charges are produced simultaneously and in equal quantities, one on the rubber and the other on the body rubbed.

393. Conductors and Insulators.—If a bar of metal be held in the hand and rubbed with fur or flannel, it does not acquire the properties of an electrified body, however long the rubbing may be continued. For a long time it was supposed that only a certain number of bodies were capable of being electrified. But later on it was found to be erroneous. *All* bodies can be electrified only if proper precautions are taken. In the case mentioned above, if the bar be fitted on to a glass rod or be provided with an ebonite handle, and while held by the handle be rubbed with flannel or cat skin, it is found to repel a charged ebonite rod, showing thereby that it is negatively charged.

Touch the brass rod while so electrified with the hand, the charge at once disappears; touch it next with ebonite,

glass, sulphur, fur, or flannel, the rod remains charged. In other words, electricity flows through the hand but not through ebonite, etc.

Let, further, a charged rod be supported in a stirrup suspended by a dry silk thread, and let it be tested from time to time. It will be found to gradually lose its electrification. The rate of loss will be found to depend on external circumstances; for example, if the air is moist, the rate of loss is very rapid; but if dry, the charge may remain on the rod for hours together. The rate of loss will be much more rapid than before if the rod is exposed to steam or is immersed in water. These facts show that dry air does not allow the electricity or the charge to be lost, whereas moisture tends to accelerate the loss.

Substances which allow electricity to flow along them, i.e. *offer little resistance to the flow of electricity, are called* ***conductors;*** *those which do not allow electricity to flow along them,* i.e. *offer a great resistance to the passage of electricity, are called* ***insulators.***

Let us once again electrify an ebonite and a metal rod, and touch each of them with a finger; the metal rod will be discharged, whereas the ebonite rod will hardly be affected. The reason is not far to seek. In the case of metals the charge distributes itself over the whole of the surface, because they offer practically no resistance to the flow of electricity which on touching passes through the hand of the operator to the earth. But in the case of insulators, *i.e.* ebonite, sulphur, etc., the charge is confined to the places rubbed and does not distribute itself over the whole of the surface; and on touching, only the portion touched loses its charge, while on the neighbouring portion the charge remains as before.

It should be noted very carefully that the terms conductors and insulators are comparative only, there being no perfectly conducting or insulating substance. An insulator may be considered as a very bad conductor, whereas a good conductor may be considered as a very bad insulator. There are many substances, as might be expected, such as wood, etc., which are neither good conductors nor good insulators. In short, different bodies have very different powers of conducting electricity, all being intermediate between a perfect conductor and a perfect insulator. In the following list the substances are arranged in order of their conducting powers.

Conductors or bad insulators	Partial conductors, partial insulators	Insulators or bad conductors
Silver		Oils
Copper	Cotton	Silk
Metals	Wood	Resin
Carbon	Marble	Sulphur
Acids	Paper	Ebonite
Water	Ivory	Amber
Vegetable		Glass
Body		Dry air

Absolutely pure water is a non-conductor, but it usually contains small quantities of dissolved matter which make it a tolerably good conductor. It is on account of the conductivity of water that if the silk supporting the pithball be made damp, the pithball does not retain its charge at all. Indeed, it is impossible to do experiments in electrostatics in a damp atmosphere.

394. To insulate Bodies.—The bodies occurring last in the list are worst conductors and hence good for insulating purposes. A body is said to be insulated when it is supported on or suspended from a non-conductor. Usually it is insulated by being placed on a support with glass feet, or on a cake of resin or ebonite, or by being suspended by a dry silk thread. No body, however, can insulate perfectly. All electrified bodies lose their electricity more or less rapidly through the supports on which they rest. Glass in itself is almost a perfect insulator, but it is somewhat hygroscopic, and the water vapour which condenses on its surface affords a passage to electricity. Sulphur is also one of the best insulators known, but it possesses a serious disadvantage in being brittle. Amber is, however, the best insulator for all practical purposes. Ebonite, if its surface is clean and dry, is a very efficient and convenient insulator.

A body which is connected to the earth by a conductor is said to be earth-connected. The earth, the body, and the connecting conductor form one big conductor, and the electricity present on the body spreads over the entire surface of the big conductor, including earth, with the result that the fraction of the charge remaining on the body is infinitesimal; in other words, the body becomes discharged. This point will again be taken up in the next chapter.

395. Theories of Electrification.—Several theories from time to time have been advanced to explain the phenomenon of electrification. We shall refer to them here very briefly.

1. **Two-fluid Theory.**—It was advanced by Symmer in the year 1759. He assumed three things: matter and two electric fluids, called positive and negative, corresponding to the two states of electrification. The degree of electrification was supposed to be the measure of the excess of positive electricity over the negative, or of negative over positive according to the sign of the electrification. The two similar fluids were supposed to repel and dissimilar ones to attract each other, and in this way the observed attraction and repulsion of electrified bodies was explained. In a normal body these fluids were supposed to be present in equal quantities. Further, the fluids were supposed to be inexhaustible and imponderable.

Thus on this theory in a neutral body the two fluids are present in equal quantities, thereby neutralising each other. But by friction or some other cause the positive fluid goes over to one body and the negative to the other, causing them to be charged with opposite kinds of electricity. Obviously, according to this theory, only one state of electrification can never be excited without a simultaneous and equal excitation of the other. This theory is successful in explaining ordinary phenomena, but it is inadequate in explaining modern facts.

2. **One-fluid Theory.**—Benjamin Franklin in the year 1749 propounded the one-fluid theory, according to which there is only one electric fluid, which he called positive. This fluid, according to him, could be associated with matter in different degrees. Corresponding to the normal state of matter, in which no electrical properties are exhibited, there is a definite amount of the electric fluid. When the ordinary normal distribution is distributed, the two states of electrification result, corresponding to the excess or deficit of this normal amount. A positively charged body, therefore, according to Franklin, has more and negatively charged body less than the normal amount. The particles of this fluid repel one another, while they attract the particles of matter.

The production of equal amounts of positive and negative electricity follow at once from it; for in rubbing two bodies together the total quantity of fluid remains the same; simply we transfer it from one body to the other; in other words, only a redistribution of the fluid takes place, one gains a certain amount and the other loses the same amount.

It should be noted that this theory points out clearly that in reality there are not two kinds of electricity, but simply two different *states* of electrification. This theory, although it is the oldest, did not become very popular, because

the two-fluid theory could explain the phenomena of induction, etc., more clearly.

There is much in Franklin's theory which approaches our modern theory only if the words positive and negative are interchanged. It is supposed that it is the negative fluid, so to speak, which goes over from one body to another.

3. **Modern Theory.**—The two theories which have just been mentioned rested on no sure experimental evidence except the ordinary experiments which they had to explain. The modern view of electricity, on the other hand, is based on an enormous mass of experimental evidence, to which contributions are made, not only by the phenomena of electrostatics, but also by the phenomena of almost every branch of Physics and Chemistry. The modern explanation of electricity resembles very closely Franklin's one-fluid theory. Simply we need suppose the electric fluid of Franklin's theory to be replaced by a crowd of small particles called **electrons**—all exactly similar and each having exactly the same charge permanently attached to it. The electrons are unthinkably small; the mass of each is about 8×10^{-28} gram, the size being of the order of 3×10^{-13} cm., *i.e.* $\frac{1}{2500}$ of that of H_2 atom. To grasp this relative proportion, remember that the size of an electron bears to the size of an atom the same ratio as a fly does to a big college hall.

When a body has an excess of these particles it is negatively charged, but when it has a deficit it is positively charged. When we electrify a body by rubbing it, say, with fur, all we do is to transfer some of the electrons from fur to the body, with the result that fur has a deficit and the body an excess of them. Since the mass of an electron is so small in comparison with the charge it carries, the change in mass of a charged body, which we would be led theoretically to expect, is much too small to be perceptible by any methods of measurement which are at our disposal.

Before closing this chapter it may be mentioned that modern work has also established the existence of positive electricity, but the mass of a unit of positive electricity, unlike electrons, is of atomic dimensions.

CHAPTER II

ELECTRICAL INDUCTION

396. If we bring a charged body in the neighbourhood of an insulated neutral conductor, say a copper cylinder, we find that electricity acting across the medium (by which the charged body is surrounded) attracts the opposite kind of electricity to the end near the charged body and repels that of the same kind to the distant end. Such electrical action is called *induction*, the charged body producing this action is called the *inducing* body, and the electricity produced is called the *induced* electricity.

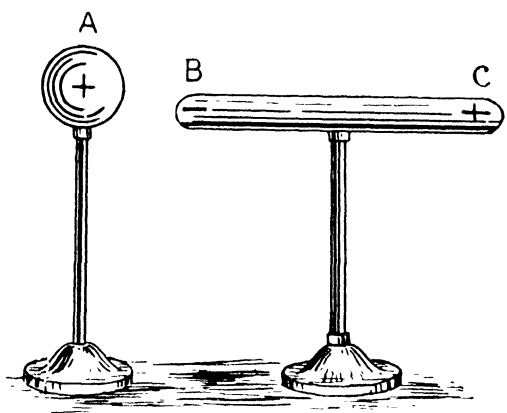


Fig. 4

Thus if A be an electrified body, say positively charged, and BC an insulated unelectrified body in its neighbourhood, the end B, *i.e.* the nearest end, is found to be negatively charged, and C, the distant end, positively. That the same kind of electricity goes over to the distant end can be shown

in a very simple way. Connect end C with a copper wire to a pithball; now since the pithball forms a part of the conductor, and that being the distant part, the positive charge goes over to it. Remove the copper wire with a glass rod. On bringing a positively charged glass rod near the pithball violent repulsion will be seen. This experiment can be modified slightly by substituting two insulated hollow brass spheres in contact with each other in place of a single conductor BC. As before, on B will be induced negative electricity and on C positive. Separate B and C and then remove A; on testing, B will be found charged negatively and C positively.

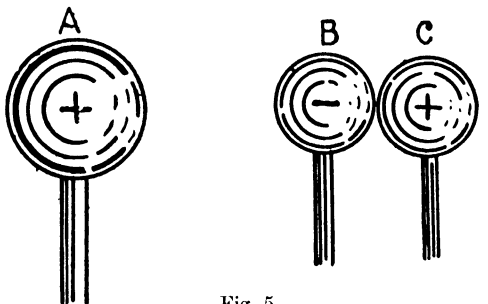


Fig. 5

From experiments similar to these we infer that a charged body induces electricity on all the conductors placed in its neighbourhood—*attracting the opposite kind of electricity nearest to it, and repelling the similar kind to the remote parts.*

The opposite kind of electricity which is induced on the near end on account of attraction between dissimilar charges is held captive, and hence is called **bound electricity**. The like or similar kind on account of repulsion is, however, sent to the distant part of the conductor. If that part be touched by the hand or be put in communication with the earth in some other way, then the earth forms the distant part of the conductor, and hence the similar kind flows to it. Since this similar kind of electricity is free to move to the distant part, it is called **free electricity**. Thus we learn that if, while under the influence of an electrified body an insulated conductor be touched or otherwise connected with the earth, the charge of the same kind as that of the inducing body escapes to the earth. Note, the conductor retains the charge of the opposite kind and it does not get discharged. As long as the inducing charge is near, this induced charge remains fixed or bound;

it does not even spread itself over the whole of the surface of the conductor. When, however, the inducing charge is removed to a distance, the induced charge becomes free, and it distributes itself over the whole of the conductor.

397. Electroscopes.—The instruments which are used to detect and determine the kind of electricity present on the charged bodies are called *Electroscopes.* A simple form of electroscope consists of a pithball suspended by a dry silk thread from a glass support, but this is a very crude form of the instrument. A more reliable form of the electroscope, and the one that is generally used in the experimental work, is the ***gold-leaf electroscope.*** It is a practical application of the principle that two similarly charged bodies repel one another. It consists of a brass rod at the top of which a metal disc with well-rounded edges is fixed. Its other end is flattened to fix two strips of gold leaves which, when the instrument is unelectrified, hang down parallel to each other. The brass rod is enclosed in a glass tube, the space between the walls of the glass tube and the brass rod being filled up with sulphur. The glass tube passes through a cork which fits into the mouth of a glass bell jar. Usually two strips of tinfoil run from the base of the jar up to the level of the gold leaves. The base, which is generally covered with tinfoil, is of wood, in which a circular groove is cut for the walls of the bell jar. In addition to all this, a cup containing sulphuric acid or $CaCl_2$ is placed inside the jar to keep the air inside dry. (It is not shown in the figure.)

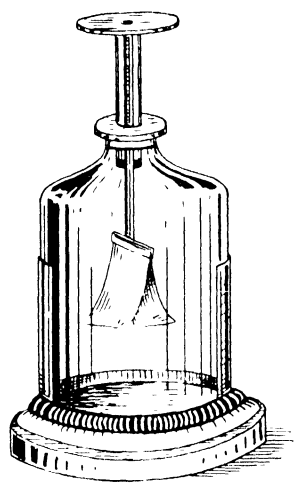


Fig. 6

When the disc is touched with a body charged with any kind of electricity, the leaves diverge, because the same kind of charge is communicated through the disc and the rod to the gold leaves, which repel each other on being charged with the same kind of electricity. On the other hand, if a charged rod be brought near the disc, the leaves diverge, because by induction they are charged with the same kind of electricity,

whereas the disc is charged with the opposite kind of electricity. In both the cases the leaves diverge; in the first case, however, they remain diverged, so long as they are not discharged, but in the second case they collapse so soon as the inducing charge is removed. In order to detect the presence of a charge all that we have to do is to bring the body which is to be tested near the disc of the electroscope; if the leaves diverge, it is charged, otherwise not. But in order to know the sign of the charge we must charge the leaves with this given charge and then later on test its sign. The usual method of charging the electroscope is not by touching it with a charged body but by induction, because of the fact that usually the charges are present on insulators, and hence by contact cannot be transferred to the disc to any appreciable extent. Suppose we are dealing with a charged glass rod; now by contact electricity will only flow from those parts which are actually touching the disc, but in the case of induction the whole of the charge present on the rod is effective in producing divergence. Hence *it is usual to charge the electroscope* ***by induction.***

398. Inductive process of charging an Electroscope.—From what has been said already in § 396 it is easy to infer that an

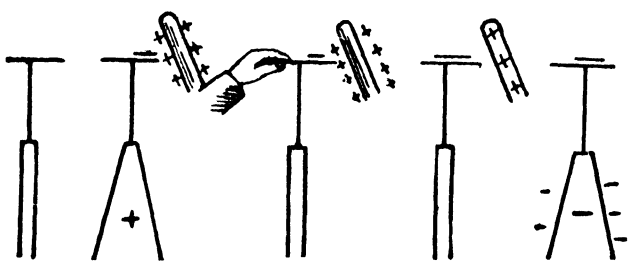


Fig. 7

electroscope is charged with the opposite kind of electricity to that of the inducing charge. Let us see in detail the method of charging. Suppose Fig. 7 (*a*) represents an electroscope when it is not affected by the inducing charge. Bring near the disc a glass rod charged with positive electricity; opposite kind of electricity, *i.e.* negative, will be induced on the disc, and the same kind, viz. positive, will go over to the leaves, which will diverge on account of repulsion between similarly charged bodies. This is shown in (*b*).

Touch the disc with the hand ; it is put into communication with the earth, which forms then the distant part of the conductor, and hence positive electricity will pass away to it. This is shown in (*c*). When the hand is removed as shown in (*d*) the negative electricity still remains on the disc, as it is bound by the positive electricity of the rod. The rod is then removed as shown in (*e*) and the leaves once more diverge, but this time with negative electricity and not with positive electricity as they did at first. Our original rod was charged with positive electricity, but the leaves are charged at the end of the process with negative electricity. Note, *by induction we charge the electroscope with the opposite kind of electricity.*

It should be remembered in this process of charging that the disc is to be touched while the rod is still there near the disc ; *the hand should be removed first and afterwards the rod.* It should in no case be touched after the rod is removed. This is very important to remember.

399. To determine the kind of electricity with which a body is charged all that we are to do is to bring it near a gold leaf electroscope charged with a known kind of electricity. If the leaves diverge more than before, *i.e.* if the divergence increases on the approach of the given body, then this body and the electroscope are both charged with the same kind of electricity. But if, on the other hand, we find decrease in the divergence, there are two conclusions to be drawn from it ; either the body is neutral, or it is oppositely electrified. In order to be sure, charge the electroscope with the opposite kind of electricity to that with which it was charged hitherto-fore and again bring the body near it. If the divergence now increases the body is charged with this kind of electricity, but if it decreases once again, then the body is neutral. Note, the decrease in divergence is not a sure test for determining the kind of charge, because it takes place in both the cases ; in one case, on account of the attraction between the charged electroscope and the neutral body, and in the second case on account of attraction between the opposite charges. Thus we come to the conclusion that increase in divergence, *i.e.* **repulsion, is the only sure test of electrification.** It tells us, further, the kind of electricity with which the body is charged.

The electroscope can be used to determine roughly the amount of electricity present on a body, for the larger the amount of electricity the greater the divergence it will produce in the leaves. If a semicircular scale be fixed below the leaves as

shown in Fig. 8, the position of the leaves can be noted. By comparing the divergence produced by different bodies we can compare the amounts of charges present on them.

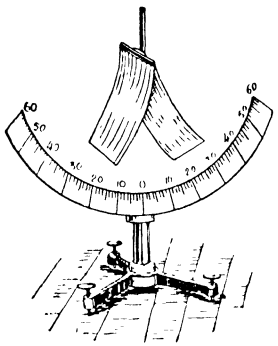


Fig. 8

400. If the charge be too great, the leaves might diverge too much, stick to the walls of the vessel, and thus might get ruptured. To avoid this possibility the two strips of tinfoil connected with the base and thus to the earth shown in Fig. 6 are used. As soon as the leaves touch them, they get discharged.

Sometimes the bodies are too strongly charged with electricity and therefore they cannot be brought near the electroscope because their approach might damage the leaves, or the bodies may be too big to be handled conveniently; in such cases a small instrument called **proof-plane** is used. It consists of a conducting metal disc mounted on an insulating handle (Fig. 9). Generally the disc is made up of copper and the handle of ebonite; but a cardboard piece covered with tinfoil with a handle of shellac or glass answers the purpose equally well.

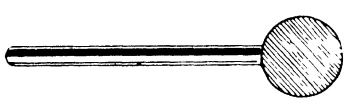


Fig. 9

When the disc is placed on the surface of a conductor whose electrification is to be tested, both form, electrically speaking, one body. On removing the proof-plane it carries off a small quantity of electricity along with it, It is then brought near the electroscope, and the sign of the charge is tested in the usual manner.

401. Induction precedes Attraction.—Let us once more go back to the phenomenon of a charged body attracting a neutral one. We know that a charged body induces the opposite kind of electricity on the near end and the same kind of electricity on the distant end of a neighbouring conductor. Let us suppose

we are dealing with an ebonite rod *a* and a pithball *cd* suspended by a dry silk thread. The negative charge on "*a*" induces positive charge on the side "*c*" of the pithball and repels negative to the side "*d*." The negative charge of "*a*" attracts the positive charge on "*c*" and repels the negative charge on "*d*"; but since the distance of the positive charge is smaller than the distance of the negative charge from the rod, hence on the whole the positive charge is more strongly attracted than the other is repelled; in other words, the attraction prevails over repulsion.

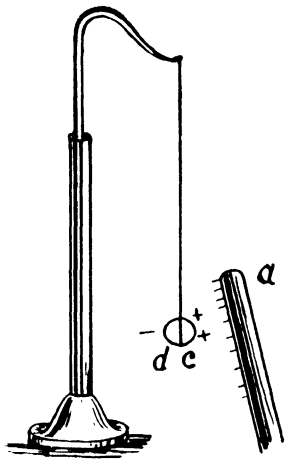


Fig. 10

402. To prove that total induced charge is equal to the inducing charge. — It was Faraday who first experimentally established the equality of the induced charge to the inducing charge. He used an ice-pail in his experiments, and hence they are generally called *Faraday's ice-pail* experiments.

Take a metal pail *A* (Fig. 11), closed at the bottom and open at the top, and support it on an insulating block, say of ebonite; connect its outside with an electroscope *B* at a distance. Charge a brass ball *C* suspended by a dry silk thread with positive electricity, and lower it into the pail. The leaves will be found to diverge. The divergence increases as the ball is lowered more and more into the pail; but after the ball has been lowered nearly, say, a third of the depth, no further divergence is perceptible on further lowering. Note, a limit to the divergence is reached when the ball is well within the pail; now

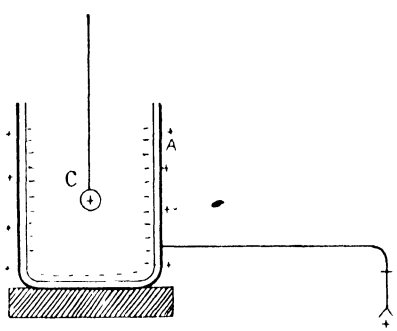


Fig. 11

take out the ball, the leaves will be seen to collapse. Let us try to understand what this experiment teaches. The ball is charged with positive electricity; when it is lowered into *A*, it induces negative charge on the inner surface of it, while a corresponding positive charge appears on the outer surface of the pail as well as on *B*. As a result of this action the gold leaves of the electroscope begin to diverge. The divergence increases as the ball *C* is lowered into the pail, for its inductive action on it goes on increasing and on the external objects goes on decreasing. When the ball is well within the pail, practically the whole of the inductive action is concentrated on the pail itself, and consequently further lowering does not alter the divergence. When *C* is taken out, the two kinds of charges neutralise each other, and the leaves collapse. This shows that *the induced positive and negative electricity are equal in amount.*

Next lower the ball once more into the pail and let it touch the bottom. If the leaves of the electroscope be carefully watched it will be noticed that they are not affected in the least by the contact. If the ball be removed after this, it will be found to be completely discharged. Now let us see what is the significance of this experimental result. When *C* touches the inner surface of the pail, the positive charge on the ball neutralises the negative charge on the inner surface; and since *C* is completely discharged and the charge on the external surface of the pail and the leaves remains quite undisturbed, hence the two opposite charges have just neutralised each other. Thus we see that the **total** induced charge is exactly equal to the inducing charge in magnitude. We have learnt two things: one that the induced charges, positive and negative, are equal in amount—this statement is true in all cases of induction—and the second that the total induced charge is equal to the inducing charge. The significance of the word *total* must be noted very carefully. Ordinarily the induced charge on a body is always less than the inducing charge, because the charged body acts inductively on all the bodies around it; those which are near it are affected strongly, while others which are farther off are affected slightly, but induction takes place on all of them. The induced charge on all of these bodies taken together is equal to the inducing charge. But in a special case, as in the experiment explained above, the effect on the surrounding objects can be almost completely done away with by surrounding the inducing charge by the body on which the charge is induced.

403. Faraday repeated the experiment with a series of ice-pails, placed one inside the other, but separated by insulating pads, as

shown in Fig. 12. When he lowered the ball *A* charged with electricity into the innermost pail *B*, the same effects as before were observed in *E*, the outermost pail. This illustrates another effect which is called **successive induction.** The charge on *A* is positive; it acts inductively on the pail *B*, with the result that the inner surface of this pail becomes negatively and the outer surface positively charged; this last positive charge acts inductively on the next pail *C*, and so on, and by the successive induction induced charges are developed on each pail.

It will be noticed that the inductive action on any one conductor is due to the *algebraic sum* of the charges in its interior; for example,

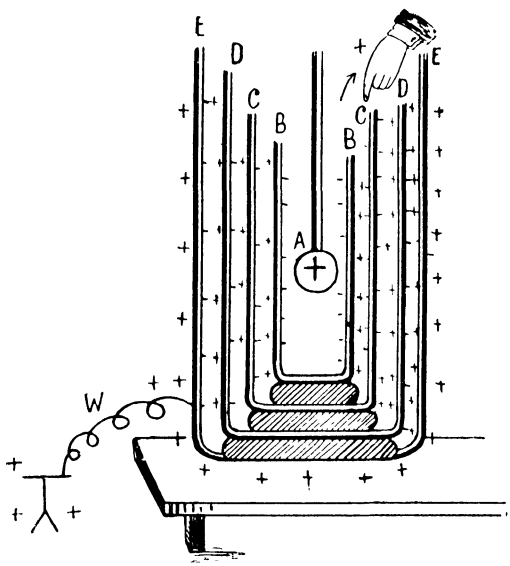


Fig. 12

in the case of pail *D* there are five equal charges in its interior, and the effective charge, *i.e.* the algebraic sum of the five, may be represented by that on *A* or by that on the outer surface of the pail *C*.

If the electroscope be connected to one of the pails and another, interior to that one, be earthed, the leaves will not diverge when *A* is lowered into the innermost pail. Suppose, for instance, we touch *C* with the hand, the positive electricity will flow to the earth, but the negative will remain on the inside, as it is bound by the positive of *B*. As the algebraic sum of charges inside *D* and *E* is equal to zero, there will be no inductive action on them. It is clear that the positive charge of *A*, negative and positive of *B*, and

negative of C, cannot produce any inductive action. The same thing can be expressed in other words by saying that *it is only* ***free*** *electricity that can act inductively.* This shows that the earth-connected conductor limits the field of induction, *viz.*, so to speak, acts as a screen.

404. To charge a vessel with any number of times the charge of a given body.—Let us suppose we have a ball C charged with positive electricity, and we want to charge the pail B with three times the charge of C.

Place a smaller vessel A within the given vessel B so as to be insulated from it. Place the electrified body within the inner vessel, taking care that the two do not touch each other. The charges on the outer side of A and B will be equal to that of C and of the same sign, but on their inner sides the charges will be of opposite signs. Now join A and B together. The positive of A will neutralise the negative of B, leaving the positive charge on B unaffected. Disconnect them.

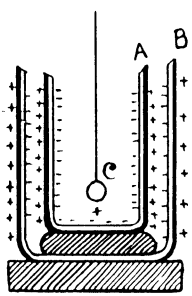


Fig. 13

Remove the electrified body; take out the inner vessel and discharge it, then put it back; place the electrified ball within it. Inductive action will take place and A will have positive electricity induced on the outer surface as before. On the inner side of B a negative charge and on its outer side an equal new positive charge will be induced. Connect once again the vessels. The outer vessel has now received a double charge. Repeat this process once more, and the outer vessel will be charged with three times the electricity. By repeating this process again and again, any number of charges, each equal to that of the electrified body, may be communicated to the outer vessel.

To charge the outer vessel with an opposite kind of electricity (*i.e.* negative), place the charged body within the smaller vessel, and put this vessel for a moment in connection with the earth; the charge residing on the outer surface will pass off to the earth. Remove the charged ball and let the inner vessel touch the outer one; the negative charge which was on A will go over to the outer vessel. Insulate the smaller vessel over again and repeat this process. By doing so a number of times the vessel can be charged with the desired amount of negative electricity.

405. Electricity resides on the Surface.—Electricity resides on the external surface of a conductor. A direct demonstration is afforded by the following experiment of Biot:

Take an insulated sphere of metal, charge it with electricity, and cover it with two hemispheres which fit the sphere exactly and are furnished with insulating handles (Fig. 14).

So long as the contact does not take place between the sphere and the enclosing shell formed by the hemispheres the charge remains on the sphere, but immediately the contact is made, the charge passes to the outer surface. On removing the hemispheres and testing them they will be found to be charged, while the sphere will be seen to be completely discharged. It must, however, be remarked that in order to perform this experiment successfully great care must be taken

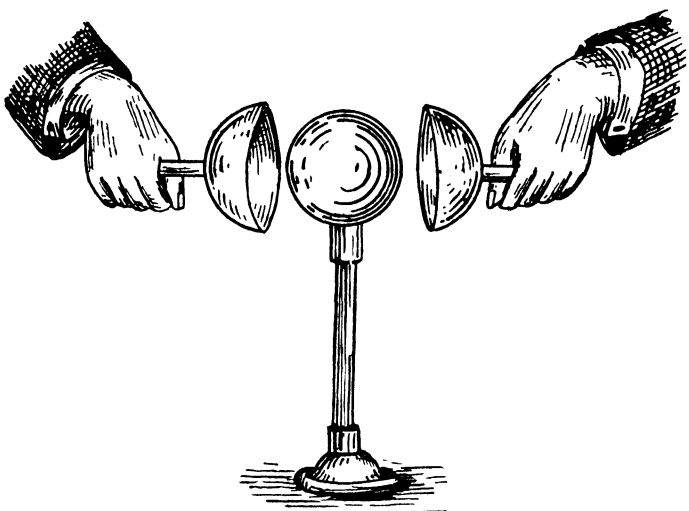


Fig. 14

in removing the hemispheres, for if their edges touch the sphere after the first separation, the sphere will become charged again.

Many other experiments have been devised in support of the fact that no charge resides on the inner surface. The following is a much simpler form of the experiment; a hollow insulated sphere (Fig. 15) with an orifice at the top is charged with electricity.

Apply a proof-plane to the interior surface of the sphere, and test by a gold leaf electroscope whether it is charged or not. It will be seen to show no trace of electricity. But if, on the contrary, the proof-plane be applied to the external surface of the sphere, it is found to be electrified.

This proves conclusively that the charge resides on the outer surface.

The same fact can be proved with Faraday's butterfly net. A metal ring is fixed upon an insulating stand. To this ring is attached a cone-shaped bag of fine linen, which is a conductor of electricity. A silk thread, attached to the apex of the cone, and extending both ways, enables the operator to turn the bag inside out as often as required without discharging it. When the bag is electrified, the application of the proof-plane shows that electricity resides on the outer but not on the inner surface. If the bag is turned inside out, the electricity passes from one to the other surface.

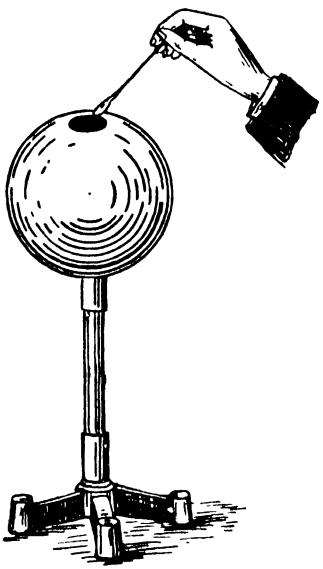


Fig. 15

406. Limitations of the Rule.—There are two exceptions to the rule that charge resides on the external surface of a conductor.

1. It does not hold good for electric currents. We shall see hereafter in connection with current electricity that the resistance which a wire of given length offers depends not upon its circumference, but upon its sectional area. A hollow wire does not conduct electricity so well as a solid wire of the same external diameter.

2. Electricity may be induced on the inner surface of a hollow conductor by the presence of an electrified body insulated from the conductor itself. Or suppose that a conducting body connected with the earth is introduced into the interior of a hollow charged conductor, so as to be partially surrounded by it, the body thus introduced will acquire an opposite charge by induction, and by the reciprocal action of this charge electricity will be induced on the inner surface of the hollow conductor.

407. Distribution of the charge on the surface of a conductor.—Charge strongly an insulated metal sphere. Test the distribution by applying a proof-plane at different points and then comparing the divergence produced in the leaves of a sensitive gold-leaf electroscope. If this experiment be

repeated several times for different points, the electroscope will show every time sensibly the same amount of divergence. Tyndall once in his lecture said: "Even when the greatest exactness of the most practised experimenter is brought into play, the spherical conductor is found to be equally charged at all points of its surface".

Take now a pear-shaped body (Fig. 16) and charge it. Test the state of electrification with a proof-plane as before. It will be found that the proof-plane collects a greater charge from the point than from the broad end, showing thereby that the amount of electricity present per unit surface at the point is greater than that present at the other end, for the electricity carried off by the proof-plane is simply that which resided on the part of the surface covered by it, as the proof-plane during the time of contact formed the part of the surface of the conductor. We must therefore conclude that equal areas on the different parts of the surface of a conductor do not have equal amounts of electricity upon them. But *the amount of charge on a unit area of surface is known as the* ***surface density*** of electrification, hence the same thing will be expressed by saying that the surface density is greater at the sharp end than at the broad end. It is further found that if the charge of the conductor be varied, the electricity residing upon different parts of the surface is changed in the same ratio. The ratio of the quantity of electricity on any two specified parts of the surface is in fact independent of the charge and depends only on the form of the conductor.

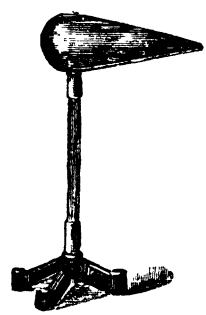


Fig. 16

This is expressed by saying that distribution is independent of the charge, but depends only on the form of the conductor.

Suppose a charge of Q units is uniformly distributed over a surface of A sq. cm., the charge per unit area obviously is $\frac{Q}{A}$ units. This is the mathematical expression for the *electric density*; it is usually written as

$$\sigma = \frac{Q}{A},$$

where σ = the surface density,
Q = the quantity of charge, and
A = the area of the surface.

We have presupposed that the distribution, and therefore density, is uniform, which is not, however, usually the case. It would be therefore more correct to say that the *surface density at a point on a conductor is the ratio of the quantity of electricity on a small area containing the point to that area, when the area is so small that the distribution over it may be looked upon as uniform.*

408. Electrical Density on differently-shaped Conductors.—It has already been remarked that electricity tends to accumulate on points. The distribution in general is not uniform; it depends upon—(1) the shape of the conductor; if it has points and corners, electricity accumulates there; (2) on its position relative to other conductors or charged bodies.

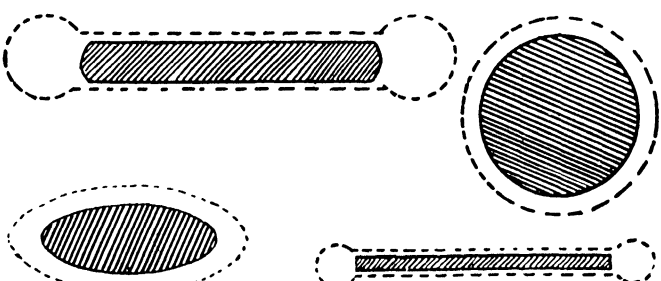


Fig. 17

To simplify the discussion it is usual to suppose that the body is at a distance from all other conductors; so to speak, it is isolated unless it is otherwise stated; in all such cases the distribution depends upon the shape only. As a general rule it should be remembered that the surface density becomes very great at places where the curvature of the surface is large. The density can be represented graphically by drawing round the boundary of the body dotted lines, the distance of the latter from the former representing the electric density at the point. Some of Coulomb's results are shown in Fig. 17.

409. Points.—Theoretically at a perfectly sharp edge, such, for example, as is formed by two planes meeting at an angle but not rounded off, the density must be infinite, and it

must also be infinite at a perfectly sharp point; for example, at the apex of a cone, however obtused, if not rounded off. But practically the points and edges of bodies are always rounded off; the microscope shows them merely as places of very sharp convexity (*i.e.* of very small radius of curvature), and hence the electric density at those places is really finite,

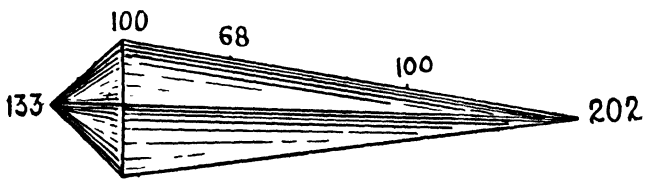


Fig. 18

but it is very great in comparison with the density at other points, and this is specially true of very acute points, such as the points of a fine needle. The following example, taken from *Frictional Electricity*, by Professor Riess of Berlin, will make very clear the effects of points on the distribution and consequent density of electricity. Two cones are placed together base to base; calling the density of the charge along the circular edge where the two bases join each other 100, the density at the apex of the blunt cone is 133, and that at the apex of the sharper cone 202. The other numbers give the density of the charges taken from the points at which they are written.

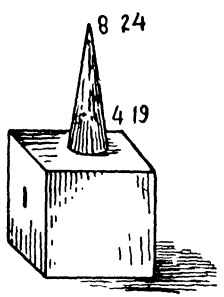


Fig. 19

Fig. 19 represents a cube with a cone placed upon it.

The density of charge on the face being 1, the density at the corners of the cube is 4 19, and at the apex of the cone 8 24; these numbers are far in excess of the density at the flat surface.

Riess found that he could deduce with great accuracy the sharpness of a point from the charge which it imparted to the proof-plane. He compared in this way the sharpness of various thorns with that of a fine sewing needle. The following is his result: Euphorbia thorn was sharper than the needle, gooseberry thorn of the same sharpness as the needle, while cactus, blackthorn, and rose fell

more and more behind the needle in sharpness. Calling, for example, the charge obtained from euphorbia 90, that obtained from the needle was 80 and from the rose only 53.

410. Action of Points.—Remembering that electricity is self-repulsive and that it heaps itself upon a point in the manner here shown, there will be little difficulty in understanding that when the charge of a conductor having a point is sufficiently strong, it will diffuse itself by streaming from the point. As a matter of fact, what happens is that the surface density at the point becomes very high, and the air particles round the point become charged and are repelled by the charge remaining on the conductor, while their places are taken up by fresh unelectrified particles. A kind of *electric wind*, so to speak, is set up; since each particle which is thus repelled carries with it a small charge, there is therefore a considerable loss or leakage of charge from the charged point. This principle is made use of in transferring an electric charge from one part of an apparatus to another, the charge being carried across the gap by the electric wind. We shall come back to it while dealing with electric machines. It should be noted here that conductors intended to retain a charge of electricity must have no points or edges and must be very smooth. If of considerable length in proportion to their breadth, they are usually made to terminate in large knobs.

CHAPTER III

THE FIELD OF ELECTRIC FORCE

411. Consider a charged body surrounded by air. If a point charge (*i.e.* a small body charged with electricity) be brought in its neighbourhood, it experiences a force either of repulsion or of attraction. *The space surrounding a charged body within which the influence of the charge extends is called its* ***electric field.*** To simplify matters a very small sphere carrying a unit positive charge is used to explore the field. At every point of the electric field there is a certain definite force with which the unit charge is repelled or attracted; this force is called the **electric force** at the point, and the direction in which the unit positive charge is urged is called the direction of the force. In any field of force if a free, small positive charge be placed, it will be urged in a definite direction at each point by the electric force, under the influence of which it will trace a continouus and smooth curve in the field. This curve is called a **line of force.** Geometrically, a line of force may be defined as *a line such that the tangent to it at any point gives the direction of the electric force at that point.* The positive direction of a line of force is the direction in which the positive charge will move. In the field due to a single charged sphere the lines of force are radial and straight, emanating from the centre of the sphere (Fig. 20). If, however, the field be due to two or more charged bodies, the lines of force will not be straight lines.

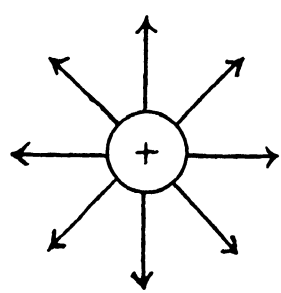


Fig. 20

Fig. 20 gives the lines of force in the case of a positively charged sphere placed in the centre of the room at a distance

from all other conductors. If it were negatively charged the lines of force will be exactly similar, only their direction will be opposite.

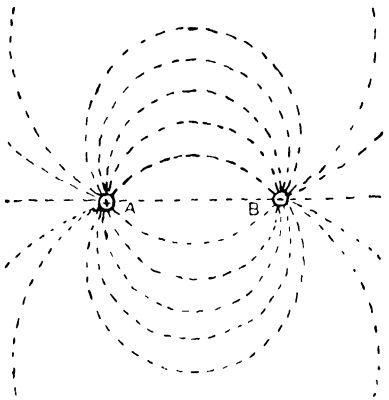


Fig. 21

Fig. 21 represents the case of two equal and opposite charges, whereas Fig. 22 represents the field for two equal and similar charges.

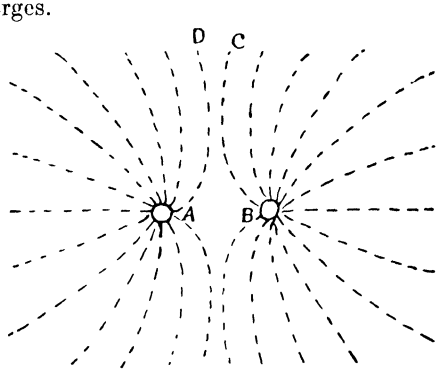


Fig. 22

The figures given represent the lines of force in one plane. But actually they are going in all sides.

Now we shall study some of the properties of these lines of force. Since the positive charge is repelled by a similar positive charge and is attracted by a negative charge,

evidently a line of force will start from a positive charge and end on a negative charge somewhere. So remember before you read further that *lines of force start from a positive charge and end on a negative charge somewhere.*

412. To explain attraction successfully Faraday made the supposition that the lines of force tend to *contract longitudinally.* Let us suppose that A and B are two oppositely charged bodies (Fig. 21). In accordance with what we have said in § 411, lines of force start from A and end on B; Faraday says they tend to contract, which means that they have a tendency to bring A and B closer together. In order to explain repulsion these lines of force are endowed with the property of *lateral repulsion.* Consider the two similarly charged bodies A and B (Fig. 22); AD and BC are two lines of force exerting lateral repulsion on each other, with the result that they tend to move A and B further apart.

If we take an area on the surface of a positively charged body, such that this surface contains a unit charge of electricity, and, starting from all points on the curve bounding this area, draw the lines of force, they will enclose a tube-shaped space which is called a unit *tube of force*; note, it begins from a unit charge. It will not be endless, but, on the other hand, will end on a negatively charged body and at this end also it will have a unit charge, but of a negative kind. Evidently the number of such tubes of force issuing from a body will be numerically equal to the charge upon it. In general, for tubes of force, lines of force passing along the axis are drawn. In these circumstances the number of these lines of force which leave the body represent the charge on it. If a body is charged with a unit of positive electricity, then only one line of force will leave its surface, while on the other hand, if it is charged with a unit of negative electricity one line of force only will end on it.

From what has been said of the lines of force it will be evident that no two lines of force can intersect, for intersection implies that at the point of intersection there are two directions for the resultant electric force, and this is impossible. We shall once more come back to this point, and see that the lines of force cannot intersect.

This conception of the lines and tubes of force due to Faraday is very useful in the study of electrostatics; it enables a man to picture to himself the physical processes which bring about the electric phenomena. For instance, it at once tells a man how the attractions and repulsions are brought into play, or why in induction an opposite and

equal charge must be present on some conductor or conductors separated from the parent conductor (on which the original charge is present), and so on.

But in spite of all this it should never be forgotten even for a moment that *these lines and tubes of force are imaginary; they have no actual existence.*

413. Force between Electrical Charges.—Let us suppose we have two small positively charged spheres separated by some definite distance. They will repel each other with a certain force. On doubling the charge on one of them the force of repulsion also is found to be doubled. If the charge on the second body is also doubled, then the magnitude of the repelling force is again doubled, *i.e.* by doubling the charge on each one of them the force is made four times. Similarly if one charge be made four times and the other three times, the force between them becomes twelve times, or in general *the force between two charged bodies varies as the product of their charges.*

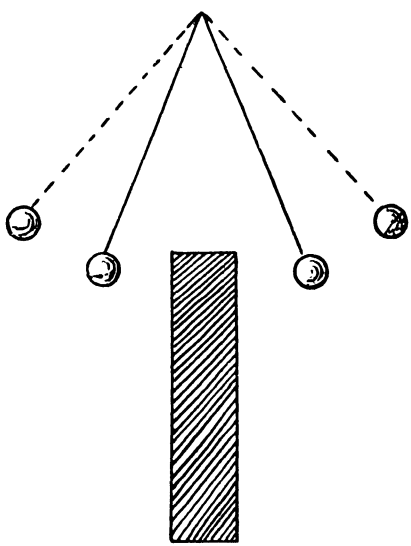


Fig. 23

Let us now consider the second condition. It is found that as the distance becomes smaller the force becomes greater. Coulomb established experimentally that by halving the distance the force becomes four times, and by reducing the distance to $\frac{1}{3}$, the force becomes nine times. In other words, we can say that *the force between two charged bodies varies inversely as the square of the distance between them.* This law is rigorously true only if the charges are supposed to be point ones, and to get actually point charges in the world is physically impossible. The law is, however, sufficiently correct if the distance between the charged bodies is very great when compared with their dimensions.

The nature of the medium also plays some part in the actual magnitude of the force between two charged particles separated by a definite distance. Let us suppose we are dealing with two pithball-pendulums suspended from the same point and charged with the same kind of electricity. They will repel each other with a certain force. The medium in this case is air. On interposing a glass or ebonite plate, it will be noticed that the divergence decreases very much; on withdrawing the plate, the balls fly apart to their original position (Fig. 23). Thus we see that while the charges remain the same, the force is appreciably altered when glass or ebonite is substituted for air as the medium between the charges. *This shows that the force depends upon the nature of the medium.*

414. It was Coulomb who first established all the above-mentioned relations. As a result of a series of experiments he enunciated the following law, which embodies the three relations explained above :—

The force between two charged bodies is proportional to the product of their charges, and is inversely proportional to the square of their distance apart, the force being one of repulsion or attraction according as the two charges are of the same or of opposite kinds.

In mathematical language we express the same thing as

$$F \propto \frac{qq'}{r^2},$$

where F is the force between the charged bodies, q and q' their charges, r their distance apart.

The same thing can be written as

$$F = \frac{1}{K}\frac{qq'}{r^2},$$

where K is a constant depending upon the nature of the medium.

Faraday, who was the first to point out the importance of the insulating medium (which he called dielectric) between the charges in electrical phenomena, called this constant K, depending upon the nature of the medium, the **specific inductive capacity** or **dielectric constant.** The greater the value of the specific inductive capacity the smaller the value of the force between the charges. Supposing this constant to be 1 for air, it is 6 for glass, 3 for ebonite, and 80 for water. In other words, if a glass plate is substituted, force becomes $\frac{1}{6}$, and when water is substituted, the force becomes $\frac{1}{80}$ of the force in air.

But since we shall deal with the charges placed in air, for our purposes the formula reduces to

$$F = \frac{qq'}{r^2} \text{ (dynes)}.$$

415. Coulomb's Torsion Balance.—Let us see now how these relations were experimentally verified by Coulomb. The torsion balance (Fig. 24) consists of a glass needle or a light rod of shellac (p) carrying at one end a gilt pithball (F) and at the other a counterpoise (not shown in the figure). It is suspended by a fine silver wire (w) from brass-head B. This lever or rod moves in a horizontal plane. The brass-head B fits into a glass tube, in which it can be easily rotated, and carries a pointer which moves over a brass disc D on which is cut a circular scale divided into degrees. This arrangement for suspension and control of wire is called **torsion head.** At the level of the lever round the circumference of the glass case is etched a scale divided into degrees. In the upper part of the case there is a hole through which an insulated gilt pithball (E) of the same size as F is introduced.

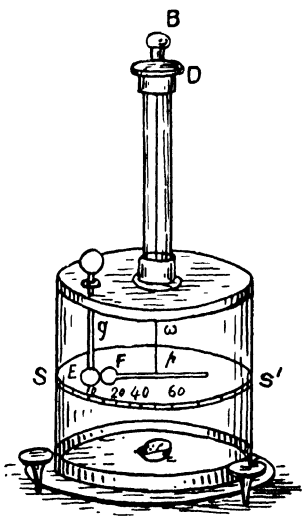


Fig. 24

Method of using the Instrument.—The torsion head is adjusted in such a position that the gilt pithballs F and E just touch each other. The glass rod carrying the pithball E is removed, and the pithball is charged. The rod is inserted into the instrument, and the pithball E is brought into contact with the pithball F. The charge is shared and mutual repulsion takes place. This causes the wire to become twisted. The force of repulsion becomes smaller as the distance between the pithballs increases, but the force of torsion becomes greater and it tends to bring back the ball; hence at a certain distance they balance each other. But we know from the laws of elasticity that the force is proportional to the angle of torsion. The angular distance is halved and the total torsion read. By comparing the torsion we compare the forces of repulsion. In order to understand clearly how this instrument is worked, consider the data of the following experiment. The original deviation of the ball was 36°. At this stage the force of repulsion was balanced by the torsion. The angular distance was halved by turning the torsion head. In order to make the angular distance between the

balls 18° the torsion head had to be turned through 126°, and for a further reduction to 9° it had to be turned through 567°. Now let us see what it means: when the angular distance is halved, the total torsion on the wire is $126° + 18° - 144°$, and when it is made $\frac{1}{4}$ of the original distance the total torsion is $567° + 9° - 576°$, so we find that when the distance is halved the torsion which will be equal and opposite to the force of repulsion becomes $\frac{144}{36}$, *i.e.* 4 times, and when the distance becomes $\frac{1}{4}$ the total torsion becomes $\frac{576}{36}$, *i.e.* 16 times. This shows clearly that when the distance becomes $\frac{1}{2}$ the force becomes 4 times, and when the distance becomes $\frac{1}{4}$ the force becomes 16 times. Thus we find that *force of repulsion varies inversely as the square of the distance.* Coulomb also proved by means of this instrument that the distance remaining constant, by doubling one of the charges the force becomes doubled, and in general that *the force of attraction or repulsion between two small electrified bodies is proportional to the product of their charges.* The experimental results obtained with the torsion balance are not so good as supposed above. On the other hand, they are very rough indeed, and in hands less experienced than those of Coulomb the torsion balance would never give satisfactory results. Its chief sources of error are:

(i) That the size of the gilt pithballs is not infinitely small, as is presupposed by the theory. This would not matter if the charges were uniformly distributed over the spheres, which is not the case. Owing to the repulsive action between them, the charges are concentrated on the farthest parts of the spheres.

(ii) There is a leakage of charge during the course of the experiment.

(iii) Charges are induced upon the case of the instrument.

(iv) The amounts of torsion and the distance apart of the balls cannot be measured very accurately.

It does not mean that if the torsion balance is not a perfect instrument the law of inverse square is untrue. As a matter of fact the best proof that can be given for any hypothesis is to calculate on its assumption certain results and then to see whether they are true or not, and the inverse square law is established very firmly indeed by this indirect method.

416. Quantity of Electricity.—Let us once more go back to the formula $F = \pm \frac{qq'}{r^2}$, where the medium is air and + sign means repulsion and − sign attraction.

If $q = q'$ we get $F = \frac{q^2}{r^2}$; and further, if F and r are both $= 1$, then $q = 1$ also. This gives us a method of measuring electricity. **Unit quantity of electricity is that quantity which, when placed in air at a unit distance from another equal similar quantity,**

repels it with a unit force. In the C.G.S. system of units this unit quantity of electricity is evidently that quantity which, when placed in air at a distance of one centimetre from a similar and equal quantity, repels it with a force of one dyne. Some idea of the magnitude of this unit quantity may be obtained by noting that if two pithballs, each weighing one gram and hung by a silk fibre one metre long without appreciable weight, are equally electrified so as to make them separate from each other by 10 cm., the charge on each ball must be about 70 units of electricity.

This unit is too small to be used in practice, and hence instead of this unit its multiple 3×10^9 times as large as this is used as the unit. This latter unit is called the **Coulomb.**

Electric force has already been defined as the force which a unit positive charge experiences at that point. In our formula for $F = \frac{qq'}{r^2}$ we have to suppose that $q' = 1$, and this will give us the electric force or the electric intensity at the point. It will be mathematically equal to

$$F = \frac{q}{r^2}.$$

More accurately *the* ***electric force*** *at a point is defined as the force which a unit charge would experience at the point, the charge on the particle being supposed to be so small that the distribution of electricity on the conductors in the field is not affected by its presence.*

It should be noted that electric force is a *vector* quantity, and hence in addition to the magnitude possesses a definite direction. If there be two or three electric forces acting on a charged body they cannot be added up arithmetically unless their directions are the same or parallel. In general their resultant can be determined by the parallelogram law of forces.

EXERCISES

✓1. Two equally charged spheres repel each other when their centres are half a metre apart with a force equal to the weight of 6 milligrams; what is the charge on each in electrostatic units?

$$\frac{qq'}{r^2} = F.$$

If the charge q is to be taken in *E.S.* units, F must be expressed in dynes and r in cm.

6 milligram wt. will be equal to $\frac{6}{1000} \times 981 = 5\,886$ dynes.

$\therefore$ $\frac{qq'}{r^2} = 5{\cdot}886$ or $\frac{q^2}{r^2} = 5{\cdot}886$, since $q = q'$,

or $\frac{q^2}{(50)^2} = 5{\cdot}886$, $\therefore q = \{(50)^2 \times 5\,886\}^{\frac{1}{2}}$

$= 121{\cdot}25$ *E.S.* units.

✗2. *ABCD* is a square of 2 cm. side, charges of + 4, − 4,

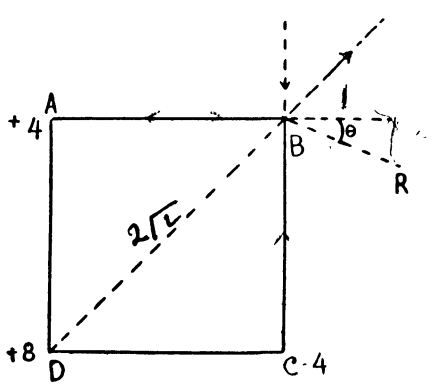


Fig. 25

and + 8 electrostatic units are placed at the corners *A*, *C*, and *D*. The medium is air, find the intensity of the field at *B*.

(i) Force at B due to charge at $A = \frac{4}{4} = +1$ along AB.

(ii) Force at B due to charge at $C = -1$ along BC.

(iii) Force at B due to charge at $D = \frac{8}{8} = 1$ along DB.

Now the combined force due to these three charges will be the resultant of the above said forces. The (iii) force can be resolved into two components along AB and BC at right angles to each other. The component along AB will be $\frac{1}{\sqrt{2}}$, and along BC will be $-\frac{1}{\sqrt{2}}$.

Therefore the two components of the resultant force are :

$$1+\frac{1}{\sqrt{2}} \text{ and } 1-\frac{1}{\sqrt{2}}.$$

$$R^2 = u^2+v^2 = 1+\tfrac{1}{2}+\frac{2}{\sqrt{2}}+1+\tfrac{1}{2}-\frac{2}{\sqrt{2}} = 3.$$

$$\therefore \quad R = \sqrt{3} \text{ dynes.}$$

The resultant will make an angle Θ with AB produced given by the relation

$$\tan\Theta = \frac{1-\frac{1}{\sqrt{2}}}{1+\frac{1}{\sqrt{2}}}.$$

3. Two small spheres, each of mass one decigram, are suspended from a point by threads, each 50 cm. long. They are equally charged and repel each other to a distance of 20 cm. If $g = 980$, what is the charge on each ?

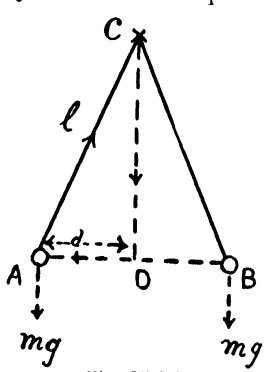


Fig. 25 (*a*)

Let us consider the sphere A charged with q units. There are three forces acting upon it, in the direction of the arrows. Their resultant is zero, as the sphere is at rest under the influence of these forces. A glance at the figure shows that the forces are parallel to the sides of the triangle ACD, hence the forces are proportional to the sides. The forces on A are :

(i) The force $\frac{1}{10}q$ dynes downwards.

(ii) The repulsion $\frac{q^2}{400}$ dynes.

(iii) The tension of the thread.

Since the forces are proportional to the sides, hence we have

$$\frac{q^2}{400} \Big/ \cdot 1 \times 980 = \frac{AD}{CD} = \frac{1}{\sqrt{24}}, \quad \therefore q^2 = \frac{400 \times 98}{\sqrt{24}}$$

$$= 89\,4 \text{ } E.S. \text{ units.}$$

4. The force of attraction between two small balls was 8 dynes when they were placed 6 cm. apart. What is the charge on each if the + charge was twice the negative ?

Ans. + 24, − 12 units.

5. Two insulated metal spheres charged respectively with + 5 units and − 5 units are placed one metre apart. What is the direction of the resultant electric force exerted on a unit + charge at a point one metre distant from the centres of each of the spheres ?

(Parallel to the line joining the centres of spheres.)

6. Three charges of + 10, + 20, − 10 units are placed at three corners of a square of 1 metre side. Find the intensity of the field at the fourth corner due to these three charges.

Ans. 0 00173 dyne.

7. Two equal positive charges of 10 units each are placed at the extremities of the diameter of a circle of 5 cm. radius. Find the intensity of the field at the middle point of the semi-circular arc joining the charges. *Ans.* 0·283 dyne.

8. Two small spheres, each of mass m grams, are suspended from a point by threads each l cm. long. They are equally charged and repel each other to a distance of $2d$ cm. What is the charge on each ?

Ans. $q = 2d\frac{\sqrt{l^2 - d^2}}{\sqrt{mg}}$.

9. Two small spherical pith-balls, each 1 decigram in weight, are suspended from the same point by threads 60 cm. long, and are equally charged so as to repel each other to a distance of 20 cm. Find the charge on each in *E.S.* units.

Ans. 8,1'4 units.

10. Eight equal metal cubes are placed at a distance from each other and given equal charges. Without being discharged

they are placed on an insulator and built up so as to form a single cube; compare the original density at a given point on one of the small cubes with the final density at the corresponding point on the large cube. What would the result have been if one of the small cubes had been accidentally discharged ?

Ans. 1 : 2 ; 4 : 7.

11. Two insulated spheres charged respectively with +1 and −1 units are placed at the two corners of an equilateral triangle with a side equal to 20 cm. What is the direction of the resultant force at the third corner ?

Ans. (parallel to the base).

12. Define unit quantity of electricity, and find the force exerted between two equal small spheres with centres 8 cm. apart, one charged with + 16 units, and the other with −20 units. *Ans.* 5 dynes.

CHAPTER VI

ELECTRICAL MACHINES

443. Any mechanical device designed to rapidly produce electrical charges on a large scale is termed an electrical machine. A peculiar feature of all machines which are mentioned below is their power of developing large potential differences, which may be as high as 30 to 50 thousand units (volts).

The electrical machines may be divided into two classes; one class depends for the production of electricity upon friction, and the other on induction. The first type is almost obsolete now; it has entirely been replaced by the second class. We shall consider both the types briefly.

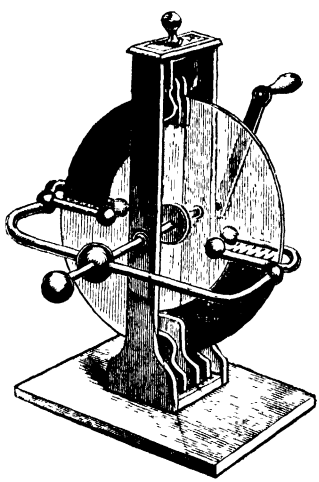


Fig. 43

444. The Plate-Machine.— It consists of two parts: (i) a plate of an insulator, either glass or ebonite, which is electrified by friction with a rubber; (ii) the prime conductor, which collects electricity. The plate revolves between two pairs of rubbers, placed at the upper and lower extremities of the vertical diameter of the plate. These rubbers are simply pads stuffed with horsehair, the leather being covered with an amalgam of one part of tin, two parts of zinc, and eight parts of mercury and some grease. To each of these rubbers a silk flap is attached. The plate revolves between the two

pads and has consequently both the sides excited. The prime conductor consists of a curved rod, to the front of which a knob is attached. The ends terminate in horseshoe-shaped rods furnished with rows of spikes between which the plate revolves. The prime conductor is attached to the framework of the machine, and is insulated from it by a glass rod which should be perfectly clean and dry.

The Action of the Machine.—Let us begin at the lower pads. When the plate is turned, the friction of the pads generates positive electricity on the plate and negative on the rubbers. The positive charge is carried round until it comes opposite the comb, where, on account of induction, negative electricity is induced on the points and the positive on the distant part of the prime conductor. The points set up a storm of negatively charged dust particles, which strike against the surface of the plate and neutralise it. Thus rendered neutral it comes under the influence of the pads at the top, where again the same steps are repeated; as a result of this the prime conductor becomes positively charged. The rubbers should be earth-connected, for otherwise the negative charge will accumulate on them, and finally will become sufficiently strong to prevent by attraction the positive electricity produced on the plate from reaching the combs and thereby stopping the work of the machine. So far, we have not explained the use of the silk flaps. When the plate is rotated the positive charge which is generated over the plate is separated from the negative one which is produced on the rubbers. Now, owing to attraction between the opposite kinds of charges, the positive charge tends to slip back towards the rubbers. The silk flap, however, acts somewhat as a rubber and is, moreover, somewhat negatively charged, hence the tendency of the positive charge to slip back is minimised. This form of the machine works satisfactorily only when the plate is thoroughly dry and the atmosphere as well is quite dry.

445. The second type is more convenient to work with. As has been said, this type depends upon induction for the supply of electric charges. The simplest machine of this type is an **electrophorus.** In this case, in the beginning a small charge is first produced by friction, and then by induction a large number of charges can be obtained. It consists essentially of a disc of ebonite (Fig. 44) and a brass plate to which is attached an insulating handle of glass. To use the electrophorus, place the ebonite disc on the table and charge it negatively by rubbing it with fur. The disc becomes

charged negatively. Place now the brass plate on the ebonite cake. Since the two opposite surfaces, the upper one of the ebonite cake and the lower one of the brass plate, are not quite plane, they touch each other at some points; at rest of the points an air-gap separates the two surfaces. Induction takes place and the lower side of the brass plate becomes positively charged and the upper side negatively. Remember that the points of the brass plate, which are just touching the ebonite cake, are neutralised, but on the whole the brass plate has a positive charge on the lower side and a negative charge on the upper one. Just touch the upper side of the brass plate for a moment, the negative electricity will pass over to the earth. Remove the brass plate from the disc with the help of the insulating handle; it will be found to be charged with positive electricity. Discharge it, and put it back again on the ebonite disc; touch it, and remove it as before; it will be found to be charged with positive electricity as before. This can be repeated a good many times. Thus charging the ebonite cake once only we can obtain a large number of charges from it. Since every charge has potential energy equal to $\frac{1}{2}QV$, a question naturally arises wherefrom this energy comes: we do not perform any extra work, but all the same, from one charge we are able to get a large number of charges. This surprise is removed if we remember that in separating the brass plate from the ebonite disc we do some work against the attraction of the two opposite charges and ***this work is converted into electrical energy.***

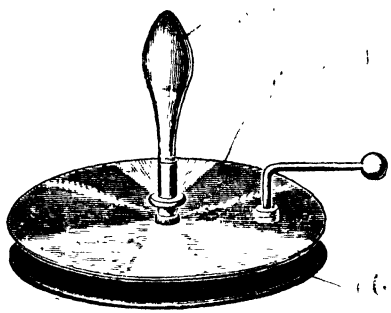


Fig. 44

Generally the ebonite disc is fitted into a metal sole on the lower side. To get over the necessity of touching the brass plate every time it is charged, a brass pin is placed in the middle of the disc, one end of which is in contact with the sole, and the other end is just above the surface of the disc, so that when the brass plate is placed upon the disc, it makes contact with the pin and therefore with the sole which is resting on the table.

446. Wimshurst Machine.—It is the most recent form of influence or induction machine. It generally consists of two ebonite or shellac varnished glass plates capable of rotation *in opposite directions* about a horizontal axis (Fig. 45). The sides of the plates remote from each other carry a series of strips of metal fixed in a circle round each plate. Lying across the front plate is a metal rod terminating in wire

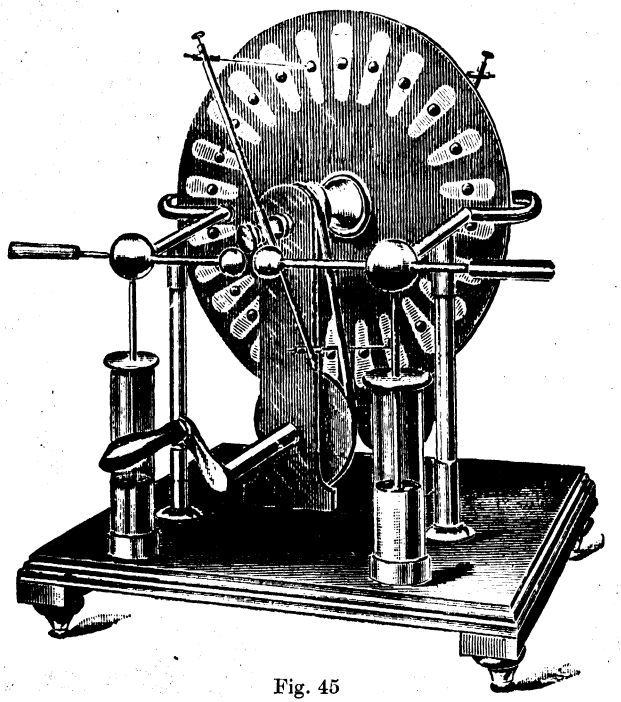


Fig. 45

brushes, making an angle of 45° with the combs. Across the back plate roughly at right angles to this rod is lying another rod which also terminates in wire brushes. These brushes come in contact with the strips in succession as the plates are revolved. The discharging part of the machine is in connection with two insulated horizontal combs. The distance between the knobs can be adjusted with the help of ebonite handles. Two Leyden jars are often supported on the base-board with their knobs touching the combs.

To understand the action of the machine we shall make reference to Fig. 45 (*a*).

If one of the sectors possesses a slightly different potential than the others, which is usually the case, it is sufficient to start the work of the machine. Let us imagine that a strip on the back plate, say *P*, possesses a slight + charge. It will induce negative charge on the strip *Q* opposite to it; the corresponding positive charge will go over to the strip *R*, which

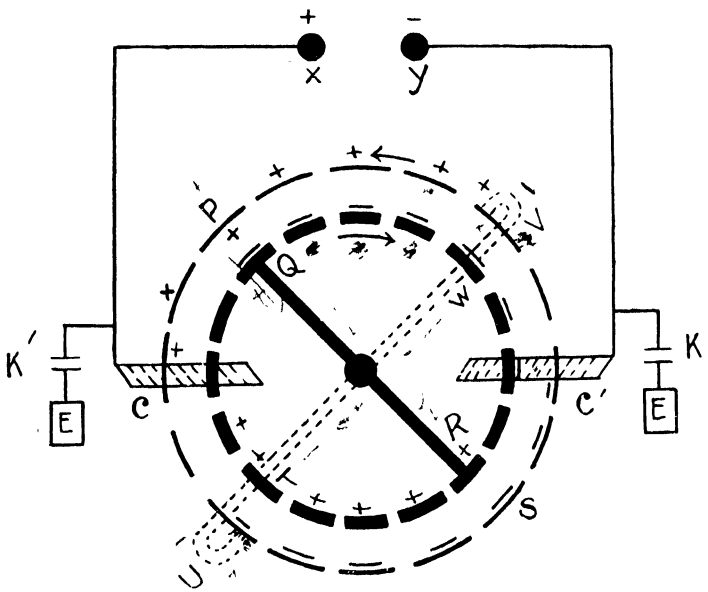


Fig. 45 (*a*)

is connected with the strip *Q* by the metal rod with brushes at the two ends. Let us confine our attention first to the front plate only; *Q* moves with its negative charge to the right, and on reaching *W*, acts inductively on *V*. The positive charge will remain on *V*, while the negative charge will go over to the distant part of the conductor, strip *U*, through the brush, rod, and the second brush. Passing *W* the strip moves into the comb, where again it acts inductively, causing the points to be positively and the discharger *Y* negatively charged. The dust particles near the positively charged points will become charged and an electric wind will be set up. These particles will neu-

tralise the charge on the sector, with the result that it will leave the comb uncharged. When this sector reaches R it will acquire a positive charge. Further, on coming at T it will induce a negative charge in the sector U; the corresponding positive charge will go over to the sector V. Passing T this strip enters into the comb, causing the points to be negatively and the discharger X positively charged. The sector will come out of the comb uncharged. At Q the sector again will become negatively charged. Now let us for a short time turn to the back plate. The sector P, which is positively charged, will move into the comb on the left, causing the points to be negatively and the discharger X positively charged as in the case of the front plate. On coming out of the comb it will be neutral. At U it will become charged negatively by induction, and on reaching S it will act inductively on R, causing it to be positively and Q negatively charged. Passing this point the sector moves into the comb on the right, making the points positive and the discharger Y negative, as in the case of the front plate. Thus we see that due to both the plates the discharger Y becomes negatively charged and X positively. If in the beginning there is no potential difference between the sectors, a sector like V can be charged by induction. Keep near it a charged ebonite rod; the sector V will be positively charged and U negatively. Rotate the plates and the action will start as explained above. Compound machines consisting of four, six, eight, and more plates are also constructed. The potential difference generated is extremely high, of the order of 50,000 volts. The quantity of electricity generated in a certain time depends upon the speed, but the potential difference is independent of it.

447. Experiments with Machines.—Bring the two knobs X and Y of the Wimshurst machine near each other, and work

Fig. 46

the machine. Owing to the opposite charges on the two knobs the medium (air) between them is strained. When the strain becomes sufficiently great, the insulation of the medium breaks down and the two kinds of electricities combine with a crack and a spark. The crack is due probably to the sudden expansion and compression of the air which takes place on account of the violent disturbance in it and the consequent heating of

the molecules in the path of the spark. The spark is due to the heat being so intense that the particles of dust, etc., in the path of the discharge are rendered incandescent. The shape of the spark varies with different conditions of the experiment. When the quantity of electricity is large and the distance between the knobs small, the spark is short, straight, and brilliant. As the distance increases, the spark becomes irregular and takes a branching form.

If a needle with one end pointed is fixed to the positive pole and the machine worked, the other discharger being at a distance, the discharge takes the appearance of a brush-like glow.

448. Multiplication of the Electric Spark.—The old electricians contrived several pieces of apparatus for multiplying the electric spark. The principle underlying the construction of all these contrivances is the same. We shall study only one such apparatus. It is called a luminous or spangled tube (Fig. 47). Small squares of tinfoil are arranged spirally along

Fig. 47

a cylindrical glass tube having a brass cap at each end. The first square is connected with one cap, and the last of the series with the other cap. There is a small distance between the corners of the squares. One end of the tube is put in communication with a pole of the machine and the other end is earthed. On working the machine a discharge passes along the tube, and sparks are seen simultaneously at all gaps between the successive squares.

449. Physiological Effect of the Spark.—When a strong spark is drawn by presenting the knuckle of a finger to the prime conductor of a powerful machine, a peculiar sensation is experienced. The same effect can be obtained from a Leyden jar when it is strongly charged up. The sensation is difficult to describe. It is a kind of painful shock, felt especially in the arm, and causing an involuntary bending of the elbow. If the shock is severe, a man may fall down senseless. Tyndall described a shock which he received accidentally in the following words :

" Some time ago I stood in this room with a charged battery

of 15 Leyden jars beside me. Through some awkwardness on my part I touched the wire leading from the battery and the discharge went through me. For a sensible interval life was absolutely blotted out, but there was no trace of pain. After a little time consciousness returned ; I saw confusedly both the audience and the apparatus, and concluded from this and from my own condition that I had received the shock. . . . This may be regarded as an experimental proof that people killed by lightning suffer no pain."

Electricity has frequently been resorted to for medical purposes. At first electrical machines were used and afterwards the Leyden jars, but nowadays both have been replaced by magneto-electric machines and induction coils.

450. Mechanical Properties of the Spark.—That the electric spark produces a violent disturbance in the medium is easily shown by means of Kinnersley's thermometer (Fig. 48), which consists of two glass tubes of unequal bore, the smaller being open at the top, while the larger one is completely closed with the exception of the side passage, by which it communicates with the smaller. The caps which close the ends of the large tube are traversed by rods terminating in knobs. The upper rod can be raised or lowered to vary the distance between the knobs. Both tubes are filled, to a height little below the lower knob, with a very mobile liquid such as coloured water. When the spark passes between the knobs, the liquid is projected with great violence and may rise to a height of several yards if the spark is very strong.

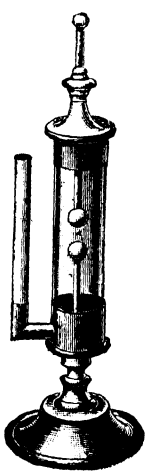


Fig. 48

451. Perforation of Glass.—In order to puncture glass a thin plate of it is placed on a glass cylinder. Along the axis of the cylinder passes up a metallic rod terminating in a point which just touches the plate. Another pointed rod exactly over this and insulated from it is lowered until it just touches the upper surface of the plate. A powerful spark from a Leyden jar or battery is passed between the two points, and if the experiment succeeds, a hole is produced by pulverisation of the glass. Sometimes the discharge takes place round the edges of the

glass instead of through its substance. To stop this a drop of oil is placed on the upper face of the plate at the point where the hole is to be made.

452. Chemical Effects of the Discharge.—That the discharge can produce chemical changes is readily shown by the following experiment. Soak a piece of blotting-paper in an emulsion of starch and potassium iodide, and place it on a plate of glass. Hold one corner in the hand and let the other corner be connected to one end of the discharger. On turning the machine iodine is liberated, producing a dark blue coloration on the piece of paper near the discharge. If *KI* solution alone is used the coloration produced is brown.

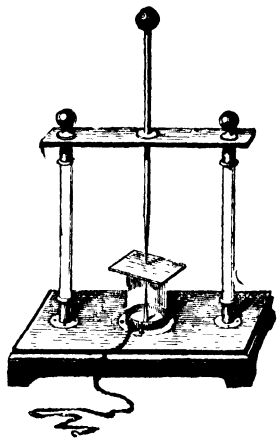


Fig. 49

453. Magnetising Effects.—Place a steel knitting needle across a strip of tinfoil which is lying over a plate of glass. Connect one end of the foil to the positive pole and the other to the negative pole. Work the machine for a short time; the needle will be found to be magnetised.

CHAPTER VII

ATMOSPHERIC ELECTRICITY

454. The early observers perceived that thunder and lightning were similar in nature to cracking and glow of the electric spark. One of them expressed himself thus: "If any one should take upon him to prove from a well-connected comparison of phenomena, that thunder is in the hands of nature what electricity is in ours, and that the wonders which we exhibit at our pleasure are but the little imitations of these great effects which frighten us, I avow that this idea, if it were well supported, would give me a great deal of pleasure." But it was reserved for Benjamin Franklin to establish the substantial identity of the lightning flash and the electric spark. He made an exhaustive comparison between the effects produced by both. He found that the lightning flash was of the same shape as the elongated electric spark, that it followed the path of least resistance and hence its appearance was zigzag; that it burnt, rent bodies asunder, and struck men blind. Franklin, the great physical philosopher, imitated all these effects, striking a pigeon blind, and killing a hen by the electric discharge. But the final proof was given by him when he collected electricity from the clouds. He flew a kite provided with a metal point in a field near Philadelphia. The kite was flown with ordinary hempen string at the end of which was attached a key, the key being separated from the observer by the silk thread. He then fixed the silk cord to a tree, and presented his knuckle to the key to see if he could get sparks. But he did not succeed, and was almost despairing of success, when rain poured down and wetted the thread, thus making it a good conductor. He got sparks from the key, and his emotion at the success of his experiment was so great that he could not refrain from shedding tears.

Lightning is an electric discharge taking place either between two clouds charged with opposite kinds of electricity or between

a charged cloud and the earth. We can regard a charged cloud and the earth's surface below it as the two coatings of a condenser and the intervening air as the dielectric. When the strain becomes too great for the air, a discharge passes. Generally the discharge assumes either of the two forms (i) *Forked lightning*, due to probably less resistance offered by some parts than by others, which gives rise to a zigzag flash—that being the path of least resistance, or (ii) *Sheet lightning*, which is probably the reflection from the surface of clouds of a distant discharge. The direct lightning in this case is hidden by an intervening cloud or horizon. It will be interesting to note that the duration of the flash is certainly less than ten-thousandth of a second. The brightness produced by the lightning is comparable to that produced by the sun, although actually it seems much less. The reason of it is that the full effect of any light on the eye is produced only when its duration is at least $\frac{1}{10}$ of a second, and if the duration is less than this, the impression lasts for the rest of the period, with the result that a man sees the average effect during $\frac{1}{10}$ of a second, which is the period of the persistence of vision. In the case of lightning the duration of the flash is less than ten-thousandth of a second, and for the rest of the time there is no flash, but the eye sees the average effect. From results like this it is concluded that a landscape continuously illuminated by a lightning would appear 10,000 times as bright as it actually appears to us during the flash.

It is easy to estimate the distance of the discharge. Since light travels at the rate of 186,000 miles per sec., therefore the flash is seen by our eye almost instantaneously. But the velocity of sound is 1120 ft. per sec. at ordinary temperatures; thus, noting the interval of time in seconds which elapses between the flash and the thunder and multiplying it by the velocity of sound, we can get the distance of the discharge.

455. Lightning Conductors.—To protect the buildings from the destructive effect of the lightning, lightning conductors were first suggested by Franklin. They consist of three parts: (1) a rod, usually of copper or of galvanised iron, with a pointed end which is projected above the highest point of the building; (2) an iron or copper rod or a flat strip connecting the first conductor with the ground; and (3) the earth connection, which must be a very good one. Generally the lower end of the conductor No. (2) terminates in several branches, each one of them passing either into a well or into moist earth. All

metal pipes outside the building must also make a good earth connection.

The action of the lightning conductor is two-fold—(i) it becomes charged with an opposite kind of electricity by induction, and sends forth a stream of oppositely charged dust and air particles to neutralise the cloud; and (ii) if a discharge does take place to afford it a path of least resistance, *i.e.* to allow it to pass to the earth without damaging the building.

Cases arise which we shall not consider where this type of conductor is useless. It is enough for us to know that there and everywhere, as a matter of fact, a network of conductors should surround the building, all parts of the network should be in good connection with moist earth. It should be remembered that all the year round the earth connection is to be between the moist earth and the conductors, and hence the plates should be buried to such a level that in summer even they are surrounded by damp soil.

Let us consider a charged cloud many miles long to be hanging over the earth. By induction the earth and other bodies below get charged with the opposite kind of electricity. When the strain becomes too great the insulation breaks down and the discharge takes place. The electrification of the earth is neutralised not merely at the point of the discharge, but even at a distance. A man under inductive influence at a distance from the point of discharge receives therefore a shock at the moment of abrupt neutralisation of the induced charge in his body. *This sudden neutralisation is called* ***return shock.***

456. Origin of the Electrification of the Clouds.—It will be best to say at the outset that the origin of the electrification of the clouds is not very clearly understood in these days. The explanations given are more or less of a speculative character. For some time past it has generally been supposed that electrification is derived from evaporation. This theory supposes that the water vapour rising from the surface of the water carries positive charge with it, leaving on the liquid behind negative charge. Experiments show that this is insufficient to explain satisfactorily all the observed phenomena.

Splashing of liquid also gives rise to electrification, the air around becoming negatively charged and the spray positively charged; salt water gives opposite results.

Other theories are also put forward to explain the cause of the atmospheric electricity, but none of them is satisfactory. Whatever may be the source of electricity, this much is certain *that each minute particle has a small charge.* Lord Rayleigh

has shown that water particles have a tendency to coalesce when slightly charged up. Let us consider what will be the effect of coalescence of a number of these water-globules.

Suppose 27 charged globules each of radius r unite to form one bigger globule of radius r'

$\frac{4}{3}\pi r^3 =$ the volume of a sphere.

Evidently the volume of this bigger globule (*i.e.* $\frac{4}{3}\pi r'^3$) will be 27 times the volume of one small globule (*i.e.* $\frac{4}{3}\pi r^3$),

or $$\frac{4}{3}\pi r'^3 = 27 \times \frac{4}{3}\pi r^3,$$

or $$r'^3 = 27r^3, \text{ or } r' = 3r.$$

Thus we see that the radius of the new globule is three times the radius of the smaller ones. Since they are all charged up with the same quantity of electricity, the total charge on the bigger globule is evidently 27 times. As the potential $= \frac{Q}{C}$, where $C =$ radius, hence we see that the potential is increased 9 times—an enormous increase. Thus we come to the conclusion that the coalescence leads to an increase in the potential. It is well to remark here that the actual increase in the potential of the cloud will not be so simple to calculate as this, for it depends upon the neighbouring charges also. But this gives us an idea as to how the potential increases with coalescence. This increase in the potential tends to increase the strain in the insulating medium (air) between the cloud and the earth, and at last it may become so great that a discharge may pass between the two.

PART VI

MAGNETISM

CHAPTER I

457. Magnetism can be defined as the property of attracting small pieces of iron; and the bodies which are endowed with this property are called **magnets.** This word magnetism is derived from Magnesia—a province in Asia Minor, where an iron ore was found in ancient times which had the peculiar property of attracting iron filings or iron nails. This ore is found very widely distributed over the surface of the earth—as in Sweden, Spain, North America, etc. It is called magnetite, and its composition is represented by the chemical formula Fe_3O_4. If a piece of magnetite be dipped in iron filings they cling to it, and more especially at corners, leaving the other places clear. There are in general two regions on any one piece of magnetite where the filings cling in greater quantity. These two regions are called the **poles.** On suspending suitably a piece of magnetite in a stirrup, it is seen that it does not rest just in any position, but, on the other hand, rests in a particular direction—the direction being such that the line joining the two poles points north and south. When displaced, it comes back to the same position. It was on account of this property that it was called a "*Lodestone*" or leading stone, as this property of constancy of direction of a freely suspended magnet could be used to steer a ship.

Let a long steel needle be rubbed with a pole of the lodestone several times, taking care that every time the same pole is used, and that it is moved in the same direction. The needle will be found to possess the above-mentioned two properties of attracting iron filings and of setting itself, when suspended, along north and south. We thus see that these properties of magnetite can be imparted to the steel needle. The process by which these properties are imparted is called

magnetisation, and the steel needle is called an **artificial magnet** in comparison to the **natural magnet,** the magnetite. Various forms of artificial magnets are used—they may be rectangular or cylindrical bars, or else they may be bent into a horseshoe form.

Hold a bar magnet in your hand and bring near its lower surface soft iron nails. Notice that near the middle the iron nails are not attracted, but as you move out the attraction gets greater and greater and correspondingly a larger number of nails can be supported; in other words, the attraction reaches its maximum value near the end, while it is zero at the middle, a result similar to the one we have already referred to in connection with the attraction of iron filings. *The end regions where the force of attraction is maximum are called poles,* as already explained, but strictly speaking the poles are only points, one near each end of the magnet. *The line joining these two poles is called a* ***magnetic axis.*** *A line perpendicular to the magnetic axis and passing through the middle of the magnet is called a* ***neutral line.*** *The vertical plane passing through the magnetic axis of a freely suspended magnet is called the* ***magnetic meridian*** *at the place.*

We have seen already that a magnet points north and south. This property enables us to distinguish the poles from each other. The pole which points towards north is called the **north-seeking pole** or **north pole,** while the other one is called the **south-seeking pole** or **south pole.** They are sometimes also called **positive** and **negative** poles. The north pole is generally marked, while the south pole is left unmarked.

458. Force between Poles.—Suspend a magnet freely in a stirrup, and mark with a piece of chalk its end which points towards north. Bring the north pole of another magnet near the marked end of the suspended magnet. Notice that repulsion takes place. Present the north pole towards the unmarked pole of the suspended magnet, and notice the attraction. Now repeat the same observations, but this time presenting the south pole towards the marked or north pole, and observe the attraction; if the south pole is presented to the south pole, repulsion is noticed between the two. Hence we learn that whereas both the poles have the power of attracting soft iron, they repel a similar pole and attract a dissimilar one. This is the *first law of magnetism.* It is stated as **like poles repel, unlike poles attract.**

459. Magnetic Induction.—If we take a bar of soft iron and present it to the pole of a sufficiently strong magnet, the

soft iron will be picked up and supported by it. If while one end adheres to the pole of the magnet the other be dipped into iron filings, they will be found to cling to it; on removing the magnet, the filings will drop off almost instantaneously. This magnetism, which exists in soft iron when in contact with a magnet, is called **induced magnetism.** It is not essential that the iron piece should be in contact with the magnet; on the other hand, even when it is at a distance, the magnetic induction takes place all right. Let us now study the law of induced polarity. Consider a magnet AB, in whose neigh-

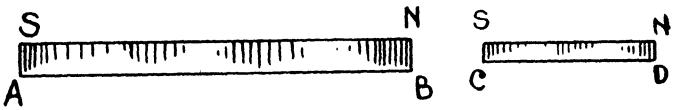


Fig. 1

bourhood a soft iron piece CD is placed. Bring near D a magnetic needle mounted on a pivot, and notice that its north pole is repelled whereas the south pole of the needle is attracted, showing thereby that:

If a bar be magnetised by magnetic induction, the nearest end to the inducing pole acquires the opposite polarity to that of the inducing pole, whereas the distant end acquires the same polarity. The induced magnetism on CD can also further induce magnetism on a piece of iron in its neighbourhood.

In the case of the soft iron, as soon as the magnet is removed the soft iron piece becomes demagnetised. But if instead of soft iron steel be used, it is found that although the strength of the induced magnetism is very much weaker, still it does not become demagnetised on the removal of the inducing magnet from its neighbourhood. In other words, steel retains the magnetism induced in it even when the inducing magnet is removed, whereas soft iron becomes altogether demagnetised. *This power of retaining magnetisation when the inducing magnet is removed is called* ***retentivity*** in scientific language. The aforesaid results can be expressed by saying that steel has a greater retentivity than soft iron.

460. Magnetic Substances.—We have already seen that soft iron and steel are acted upon by magnets, that is, they are attracted by them. Practically these are the only substances on which magnetic attraction is at all appreciable. Nickel and cobalt are but feebly attracted. *The substances which are*

susceptible of magnetic influence are called magnetic substances. It was shown by Faraday, with the help of very powerful magnets, that all substances are influenced by magnets, though the influence is very feeble in the case of other than the four substances mentioned above. It should be noted, however, that all the substances are not attracted by magnets. There is one distinct class of substances called **paramagnets** which are attracted by magnets such as soft iron, steel, etc.; whereas the second class of substances, such as bismuth and copper, etc., called **diamagnets,** are repelled by magnets.

Take a freely suspended magnet, and bring near its one pole an iron bar, attraction will be noticed; invert the iron bar, *i.e.* bring the other end near the same pole, again attraction will be noticed. Take it to the other pole, even now attraction will be observed. If we bring an opposite pole of a magnet near the pole of the suspended magnet, even then attraction will be observed. Now in both the cases, when the substance is a magnet or an unmagnetised piece of iron, attraction takes place. The question arises how to distinguish between them. Just as was pointed out in the case of frictional electricity that repulsion is the only sure test, the same is the case here. In the case of an unmagnetised body you always get attraction; but in the case of a magnet, in one case, when the poles are unlike, you get attraction, but in the other case, when the poles are similar, there is repulsion.

Thus the distinction between a magnet and an unmagnetised body is that the latter is attracted by both the poles of the magnet, whereas the former will be attracted by one pole and repelled by the other. *Thus the only sure proof of magnetism is* ***repulsion.***

Surgeons make use of this property of the attraction of iron pieces by a magnet in removing the fragments of iron from the eye where operation otherwise is difficult. Of course a special type of a magnet (*i.e.* an electro-magnet with a tapered cone) is used for this purpose.

461. Induction precedes Attraction.—We have noticed that an iron piece is attracted by a magnet, but we know also that opposite polarity to that of the inducing pole is induced on the nearer end of the iron piece when it comes in the neighbourhood of a magnet, the distant end acquiring the same polarity. In other words, it no longer remains unmagnetised. As opposite poles attract each other and similar poles repel, therefore in Fig. 2, where AB is a magnet and CD an iron piece, we find attraction between B and C and repulsion between B and D.

As the distance BC is smaller than BD, attraction is much greater than repulsion. But so far we have not taken into account the other end. Considering the magnet to be very long, A can be supposed to be at a great distance, and in comparison with AC, CD can be neglected; this being so, the effect of A can be looked upon as equal and opposite on C and D, and hence it may not be considered. Thus we find on the whole that attraction which an unmagnetised piece experiences is a consequence of induction. Instead of supposing that CD is unmagnetised let us suppose that it is *weakly* magnetised. It will be observed that even in this case the pole B will attract both ends, C as well as D. The explanation is simple. In the case when the polarity on the nearer end is negative there

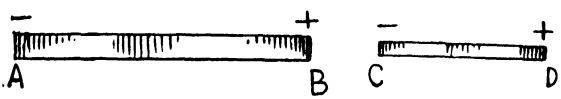


Fig. 2

should be attraction as a rule, but let us see what happens when the positive pole is placed near B. The strong pole B will act by induction and produce opposite polarity on the nearer end, which overpowers the permanent magnetism and this converts repulsion into attraction. But note the first movement is that of repulsion. It is in this respect that a weak magnet differs from an unmagnetised piece of iron; the latter is attracted from the very beginning, whereas the former first experiences repulsion and then attraction. So remember, in experiments with weak magnets it is the *first* movement which is to be observed.

462. Magnetic Poles cannot be separated.—It is very convenient to speak of a single pole when we are dealing with induction or magnetic force, but we should remember that we can never isolate a pole of one kind. In frictional electricity we have seen that we can separate the positive electricity from the negative electricity, but that is not the case in magnetism. If we take a magnetised knitting needle and break it into two pieces, we find that each of the pieces is a complete magnet; that is, each piece possesses a north pole and a south pole. Again, break one of the pieces into two—these both will be found as before complete magnets. This process may be repeated any number of times, but still the magnets obtained will be always complete. It is impossible to obtain a magnet with a single pole only. We infer from this, consequently, that

even the smallest molecule into which a magnet can be divided is a complete magnet in itself, and that a bar magnet is an assemblage of such molecules, the poles being arranged in such a manner that all the north-seeking poles are turned in one direction and the south-seeking poles in the other. If, therefore, the magnet be broken asunder at any point, one face of it will be the north-seeking pole and the other the south-seeking pole.

463. Theories of Magnetism.—If we have two equal magnets and place their opposite poles in contact, we get a magnet of double the length of either magnet, but the pole-strength remains the same. The two opposite poles in contact produce no effect upon external points. Instead of placing them with opposite poles in contact, if we place them side by side, or one above the other, so that the two north poles touch each other, and so do the south poles, we get a magnet of the same length but of double the power. Let us now consider a large number of molecular magnets placed with their opposite poles in contact

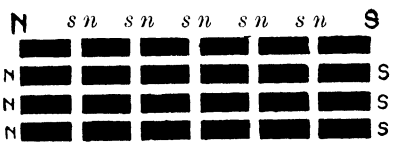


Fig. 3

as well as side by side (Fig. 3). Consider only one horizontal row. At all the points except the ends opposite poles are touching each other, hence no effect is produced on external points due to them. But the magnetism at the ends is free, hence it produces magnetic effect at all external points. Similar is the case with all the other rows. Their combined effect, so to speak, is to concentrate north polarity at one end and south polarity at the other end. This theory, which supposes the magnetism to be a molecular property, viz. that the molecules themselves are complete magnets, is called the **molecular theory of magnetism.** We have seen that in a magnet the molecules are arranged in a particular manner such that at one end north polarity is free and at the other the south; but what about an unmagnetised body? On this theory it is supposed that the molecules are arranged in all sorts of ways, there is absolutely no regularity, with the result that the effect of one molecule at an external point is neutralised by a neighbouring molecule. The process of magnetisation consists in swinging round more or

less the molecules, so that a large number of them point nearly in the same direction, with their north poles pointing one way and the south poles the other way. Evidently if this theory is true a limit must come beyond which a body cannot be more strongly magnetised, and that limit will be reached when all the north poles point in one direction and all the south poles in the opposite direction—a fact borne out by experiment. Even the regular arrangement can be roughly shown by an experiment.

Take a test tube which is partially filled up with steel filings. Place the tube horizontally and pass over it a magnet: observe that the filings set themselves end to end, placing their lengths along the direction in which the magnet is moved. Without disturbing the tube bring near it a magnetic needle, and see that the north pole of the needle is attracted by one end and repelled by the other. Now give a jerk to the tube, and again place it back in its old position and test it once more. No repulsion will be observed, showing thereby that the filings have lost their apparent magnetism. Another strong evidence in favour of the above-sketched molecular theory is the fact that on a piece of iron or steel the total amount of north pole is always equal to the total amount of south pole. This is just what our theory leads us to expect, because it has been already explained that when two equal magnets are placed with their opposite poles in contact, we get a magnet of the same strength but with double the length. The free magnetism at both ends is equal. Instead of considering two magnets consider a large number of molecular magnets having the same strength. When they are placed with opposite poles in contact with each other, the strength of the north pole at one end must be equal to that of the south pole at the other end. And this is exactly what we find practically. This can be proved by placing a magnet on a piece of wood which is floating in water. The magnet, though it is free to move in any direction, will simply rotate until it points north and south, and will not move bodily at all in any direction. This shows that the forces acting upon its two poles are equal and opposite in direction producing a couple, for otherwise there must be some tendency for bodily movement of translation. Since the force is proportional to the strength of a pole, the pole strength must be equal at the two ends. But the question arises how these molecules of a magnetic substance become magnets. Ampère supposed that each molecule had a current of electricity flowing round it. Later on we shall see that when the direction of the current

in a face is counter-clockwise, the face is a north pole, otherwise a south pole. So on this theory the molecules and therefore the currents are arranged irregularly before magnetisation, and the process of magnetisation consists in simply making the currents parallel to each other in such a way that when we look at one end they all seem to be circulating in one direction, and on the other end they appear to be circulating in the opposite direction.

464. Demagnetisation of a Magnet due to its own Poles.—Let us confine ourselves to a small element AB of a permanent magnet NS (Fig. 4). Due to magnetisation the north pole

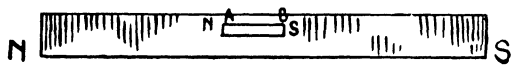


Fig. 4

will be arranged towards the left at A and the south pole of the element towards the right at B. But due to the inductive influence of free magnetism at the poles, at A south polarity will be induced and at B north, which is opposite to the polarity already present on the element AB. Thus we see that the free magnetism at the poles tends to demagnetise the magnet itself. This demagnetising effect can be removed if we arrange the bar magnets in pairs, and keep armatures or soft iron pieces across their ends. These iron pieces are called

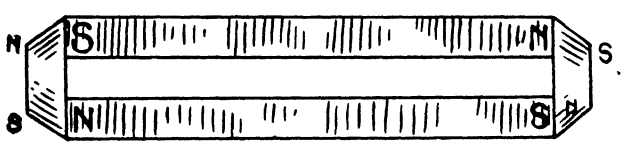


Fig. 5

keepers. The free pole of the magnet induces opposite polarity in the keeper, and thus at any point in the magnet the effect due to the free magnetism at the end of the magnet is neutralised by the induced polarity. By looking at Fig. 5 it will be seen that there is no free magnetism left anywhere. Moreover, these opposite poles attract each other, and so tend to preserve the arrangement of the molecules which was brought about during magnetisation. Thus we see that the *keepers not only reduce the tendency of the free magnetism at the poles to*

demagnetise the magnet, but also tend to prevent their magnetism from dissipating on account of accidents (jerks or the induction of a neighbouring magnet).

465. The Earth—a Huge Magnet.—The fact that a freely suspended magnet always points in one particular direction shows that there is a force acting upon it, which always brings it to rest in that position. Knowing that a magnet is influenced by another magnet, we *infer* that there must be some magnetic force acting upon the magnet which sets it in the direction north and south. This force is supposed to arise from the earth's magnetism: it is supposed that the earth is a huge magnet with its poles near the geographical north and south. Remembering that like poles repel each other and unlike poles attract, we infer that the polarity of the free magnetism near the north pole of the earth is opposite to that of the north-seeking pole, or, in other words, it is similar in nature to that of the south-seeking pole of a magnet. Similarly, the polarity of the south pole of the earth is similar in nature to that of the north-seeking pole of a magnet. The cause of the earth's magnetism is not known with any certainty, but the supposition of a large magnet with its poles near the geographical poles helps us to understand easily the various phenomena.

466. Methods of Magnetisation.—A bar of steel can be magnetised in any one of the following three ways:

(i) By inductive action of permanent magnets.
(ii) By inductive action of the earth.
(iii) By passing electric current round it.

Let us take the first method first.

467. Inductive Action of Permanent Magnets.—There are two methods in common use:

(*a*) Method of single touch; and
(*b*) Method of divided touch.

Method of single touch.—It consists merely in rubbing the bar to be magnetised several times lengthwise, and always in the same direction, with the pole of a strong magnet. Suppose *AB* is the bar to be magnetised. Place it flat on the table, hold a permanent magnet *NS* in the position indicated in Fig. 6, and rub it uniformly along the bar to the other end. When it reaches *B*, lift it off and carry it back at some distance above the rod into its original position. Repeat this process

several times, taking care that always the same pole is used and the magnet is drawn in the same direction along the bar. The end *B* will be a south pole and *A* a north pole. It should be

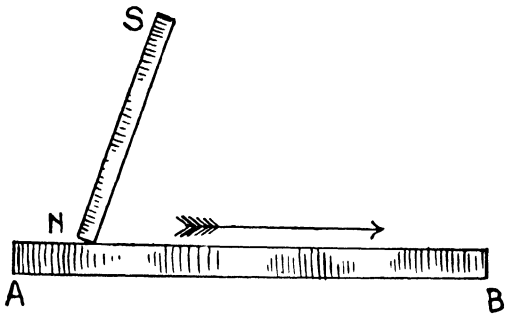


Fig. 6

remembered that *the end of the bar where the rubbing magnet leaves it is of opposite polarity to the rubbing pole.*

Divided touch.—It consists in fixing the bar to be magnetised over the poles of two magnets which are placed on a table with their unlike poles facing each other. Two permanent magnets of equal strength are taken and are placed at the centre of the bar, with their opposite poles in contact. They are then drawn slowly apart along the surface of the bar to the opposite ends. When the ends are reached they are lifted up and are placed

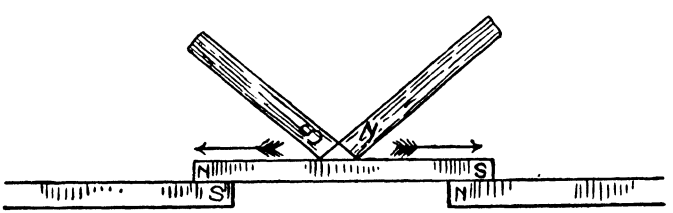


Fig. 7

back in their original position. Repeating this process several times the bar is magnetised. The end where the north pole leaves the bar is found to be the south pole, and the end where the south pole leaves it is found to be the north pole. The method gives a strong and even magnetism to moderately thin and long bars.

If the rubbing is performed carefully, we get only two poles,

one at each end, but when the rod is magnetised irregularly it exhibits more than two poles. They can be demonstrated by sprinkling iron filings over the magnet. The filings will be found to be clustering round several points along the length of the bar. These poles, which are not at the ends of the bar, are called **consequent poles.**

468. Magnetisation by Earth's Induction.—Since the earth behaves as a magnet (see § 465), it will therefore produce inductive effect on a soft iron piece. Place a soft iron knitting needle in the magnetic meridian in the vertical position at a place in the northern hemisphere. Strike it with a hammer several times. On bringing it near a magnetic needle it will be seen that the lower end repels the north pole of a magnetic needle and attracts the south pole, showing thereby that the lower end of the knitting needle has become a north-seeking pole. From what has been said of the earth it will occur to the student that if the same experiment were tried at a place in the southern hemisphere, polarity produced at the lower end will not be the same. In the northern hemisphere the lower end of the needle is pointing towards the north pole of the earth's magnet, whose polarity is similar to the south-seeking pole of an ordinary magnet. Hence by induction the lower end becomes the north-seeking pole. In the southern hemisphere the case will be opposite, and therefore the lower end there will have the south-seeking polarity induced upon it. To produce the strongest effect on the iron piece it is best to place it in such a manner that the lower end points towards the magnetic pole of the earth, instead of keeping it vertical.

469. Magnetisation by Electric Current.—The above-mentioned methods are seldom used in practice for magnetisation. Occasionally in the laboratory they might be used, especially the single touch method for magnetising a knitting needle, but generally the bodies are magnetised by an electric current.

Let us wind a coil of insulated copper wire round a glass tube, and join the ends to a battery. Place inside the glass tube a knitting needle, and pass a strong current through the coil for some time. The needle will be found to be magnetised. If it be of steel it will become a permanent magnet, but if it be a soft iron piece it will remain highly magnetised so long as the current flows round it, and will lose most of its magnetisation as soon as the current ceases. Such an arrangement, in which a soft iron core is taken and a current is passed round it, is called an **electro-magnet.** The core becomes a powerful magnet when the current is passed round it. The magnet obtained in this

way with a soft iron core is much more powerful than an ordinary steel permanent magnet. The general form of an electromagnet is that of a horseshoe. When magnets are made by an electric current, which end will be the north pole and which the

Fig. 8

south? The following rule will help in knowing the polarity: *Look at the end of the iron bar having copper wire coiled around along its length. If the direction of the current in the coil is counter-clockwise the end will be a north pole; if, on the other hand, the direction is clockwise, the end will be a south pole.*

470. Destruction of Magnetism.—So far we have been considering how to magnetise an iron piece, now let us see how to demagnetise it. The simplest method is to use it roughly, *i.e.* to strike it against something. This produces disturbance in the *molecular arrangement*. Another method is to heat the magnet to red-heat and then allow it to cool, placing it east and west, that is, perpendicular to the magnetic meridian, and therefore free from the earth's inductive influence. It is necessary to heat it strongly, for if it be only slightly heated it will regain its original magnetism. This method is satisfactory only when we are dealing with thin needles. The general method to demagnetise is to magnetise the piece in opposite direction and thereby to destroy its former magnetisation.

A magnet can be demagnetised also by placing it near a similar magnet in such a manner that like poles are adjacent. The tendency of each pole is to induce opposite polarity in the other, which naturally tends to weaken the strength of the magnets.

CHAPTER II

FIELD OF MAGNETIC FORCE

471. The pole of a magnetic needle is either repelled or attracted by the pole of a suspended magnet when the former is brought near the latter; similarly an iron nail is attracted by a magnet even when the nail is at some distance from it. This shows that there is a region or space round a magnet in which its influence can be felt, viz. in which it exerts magnetic force; this region is called its **magnetic field** or **field of magnetic force.** In order to know fully the magnetic field at any point we must know the magnitude as well as the direction of the force at that point. As we fixed upon certain conventions in the case of electrostatics to measure the magnitude and the direction of the electric force, we must do the same thing here as well, for the magnitude and direction will depend upon the strength and the kind of pole brought near the magnet. It is customary to suppose that a north pole of unit strength is placed at the point where we want to know all about the magnetic force. The force which it will experience gives us a measure of the magnitude. The direction in which it will move gives us the direction of the force.* In short, we can define *the strength of a magnetic field at any point as the force experienced by a unit north pole placed at that point, and the direction in which it will be urged as the direction of the force there.* This direction will be different at different points. Let us consider a unit north pole placed in the neighbourhood of a magnet. It will experience magnetic forces due to both the poles; one will attract it and the other will repel it, but it will move in the direction of the resultant of these two forces. As soon as the pole moves to the next point the direction of the resultant

* A unit south pole will move along the same path, but in the opposite direction. The direction in which a south pole will move is called *negative*, whereas the direction in which a unit north pole will move is called *positive*.

force changes, hence the pole will begin to move in the direction of the new resultant force; the same argument will hold good for all other points. Thus we see that the direction changes continuously; in other words, the pole will be urged along a curved line starting from the north pole and terminating at the south pole. *The path along which a north pole would move, if free to do so in a magnetic field, is called* ***a line of force.*** A line of force can be defined as *a curve, tangent to which at any point gives the direction of the force at that point.*

472. Mapping a Field of Force.—If an isolated pole could be placed in a field, the paths traced by it would give us the lines of force directly, but in practice it is impossible to isolate a pole, hence other methods have to be followed. The lines of force are supposed to run out from the north pole in all directions, but if we confine ourselves to one plane we can exhibit them experimentally. Lay a sheet of thin glass over a magnet. Cover it up with a piece of paper. Sprinkle iron filings over it with a sieve. On gently tapping, the filings arrange themselves in beautiful curves. Each separate filing under the inductive influence of the magnet becomes magnetised, and on tapping it settles down along a line of force, the north pole being away from the north pole of the inducing magnet. In this way the field can be mapped out roughly. The field is stronger where the filings are crowded together in the largest quantity. Permanent pictures can be made if the paper be coated with wax beforehand and, after the filings arrange themselves, by heating it with a flame. The wax melts and the filings get fixed up.

473. To exhibit the lines of force in a magnetic field by means of a small compass needle.—We have already defined a line of force as the path traced out by a north pole if free to travel. The path travelled by a south pole also will be the same, only the direction will be exactly opposite to that in which a north pole will move. Let us consider a line of force ABC more closely, starting from the north pole, and going out to the south (Fig. 9).

When we keep a small compass needle near the magnet, say at P (Fig. 9), the south pole will be urged towards BPA on account of attraction, and the north pole towards APB on account of repulsion; as a result of it the needle will arrange itself as shown in the figure. If the needle be *small*, the force of attraction will be equal to the force of repulsion, and hence

the needle will arrange itself along the direction of the field.* In other words, *a small compass needle arranges itself along a line of force.* Making use of this fact we can trace out the

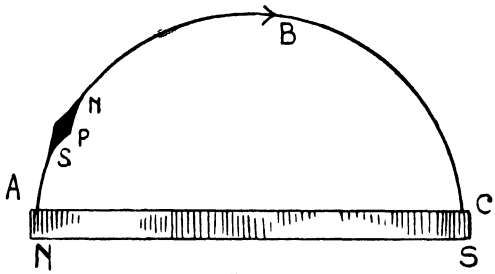


Fig. 9

lines of force more accurately than we can do with iron filings. The general method is to place the needle *NS* near a pole of the magnet, say the north pole, and mark the two ends of the needle. The compass is moved away in such a manner that *S* comes over *N*, and the new position for *N* is marked. This

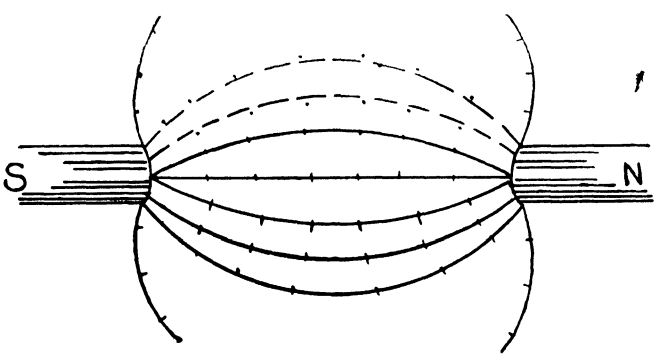


Fig. 10

is repeated, till we come to the other pole. A curve is drawn through the various points thus obtained. It represents a line of force. By starting from different positions we can map out the whole field of force (of course in one plane only) (see Fig. 10).

* The student will learn later on that when a needle is placed in a uniform field it will experience a couple tending to make it parallel to the direction of the field.

If we experimentally map out a field of force we find that we do not get, in all cases, the field of the magnet like the one sketched above, for the plain reason that the above field is due to two opposite magnetic poles only, but in practice we cannot get rid of the earth's field. Note, the field which we map out experimentally is the resultant field of the magnet and the earth. On account of this reason the form of the sketch that we get depends upon the position of the magnet with respect to the magnetic meridian. We shall consider

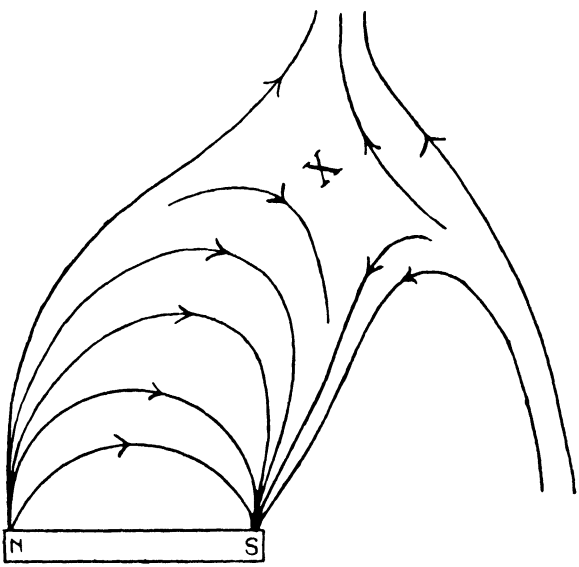


Fig. 11

the following three cases : (*a*) when the magnet is placed east and west, at right angles to the magnetic meridian ; (*b*) when the magnet is placed in the magnetic meridian with the south pole northwards and the north pole southwards ; and (*c*) when the magnet is placed in the magnetic meridian but with the north pole northwards and the south pole southwards.

(*a*) *Magnet is placed east and west.*

Fig. 11 shows the result when the south pole is towards the east. Near the magnet the lines of force are mainly due to the magnet itself, but as we move farther off the force due to the

magnet becomes weaker, whereas the force due to the earth's magnetic field remains almost the same. At greater distance, therefore, the field of the magnet will become almost negligible, and hence the lines of force will be parallel, as the field due to the earth can be supposed to be uniform * over the limited regions with which we are concerned. So we see that *near the magnet its own field is important and at a distance the earth's field.* We are naturally led to infer that at a particular point the two fields must be opposite to each other where they will neutralise. At this point there will be no magnetic force. Such a point is called a **neutral point.** Here the pole will have no tendency to move, and the compass needle will set in any direction. There will be two such points in the case above mentioned. One of them is shown at *X*. The positive direction of the lines of force due to the earth will be upwards, as in the geographical south pole we have north-seeking pole. The direction of the lines of force due to the magnet at the point *X* will be downwards. If the magnitude of the magnet's field is equal and opposite to the earth's field the point will be neutral.

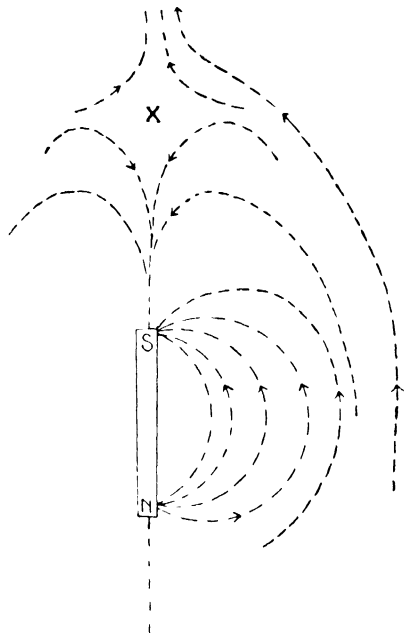


Fig. 12

* If the number of lines of force passing through an area is large the field is strong, and if the number is small the field is weak, as in the case of electrostatics. A change in the number of the lines of force will mean a change in the strength of the field. Since at a distance the lines of force are almost equidistant, therefore when we are considering a small region at a long distance from the magnet, the field can be supposed to be fairly uniform.

(*b*) *Magnet placed in magnetic meridian with the south pole northwards and the north pole southwards.*

The same remarks will apply to this case as well. Near the magnet the field of the magnet predominates and at distant points the earth's field. At a point like X (Fig. 12) the two fields are in opposite directions as shown by the arrows, hence X is a neutral point. There will be a second neutral point on the other side.

(*c*) *The magnet is placed in the magnetic meridian, but with the north pole towards the north and the south pole towards the south.*

In this case it will be seen that the force due to the earth's field is in the same direction as the force of the magnet but near

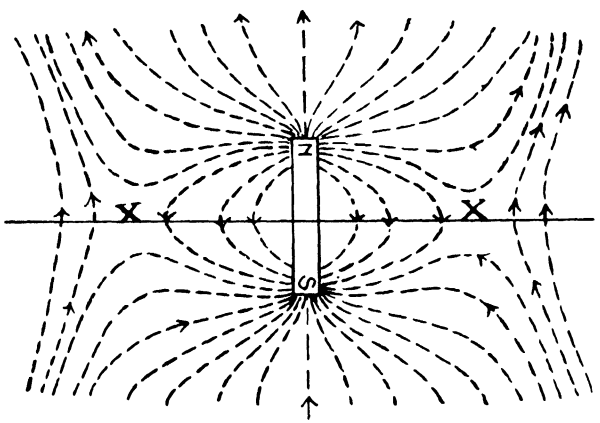


Fig. 13

the middle it is in the opposite direction ; hence the neutral points will be on the line at right angles to the length of the magnet (Fig. 13).

474. Position of the Poles.—It was taken as a supposition in a preceding article that the poles are situated at the ends, but this is not in reality the case. To find out the position of the poles, place the magnet AB on a cardboard. Place near A the small compass needle. Mark off the two ends, place it on the other side of the magnet, and mark similarly two points. Produce these two lines to meet at a point inside the magnet. This will give the position of the pole ; similarly the position of the other pole can be located. In the above-mentioned experiment we have neglected the effect of the earth's mag-

netism; in order to be more accurate we must arrange the position of the magnet in such a manner that the needle points north and south. This can be done easily by rotating the cardboard. In this position the earth's field does not produce any effect. At the same time the field of the magnet has also the same direction, for otherwise the needle would be deflected out of the meridian. Therefore in this case the direction given by the needle points towards the pole of the magnet. Proceed

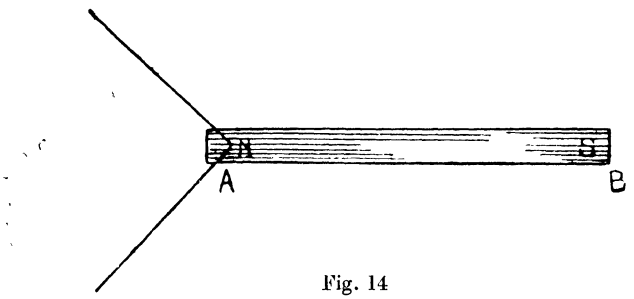


Fig. 14

on the other side in the same manner. Produce the lines and their point of intersection will give the true position of the pole. We have neglected the influence of the other pole of the magnet, which we are justified to do only if the magnet be a long one.

The actual length of a magnet is slightly greater than the length between the poles. *The product of the pole strength and the true length between the poles is called the* **moment of the magnet.** It is a very useful quantity, as will appear later on.

475. Laws of Magnetic Force.—The law that like poles repel and unlike poles attract each other is a qualitative one only. Coulomb was the first to determine the quantitative expression

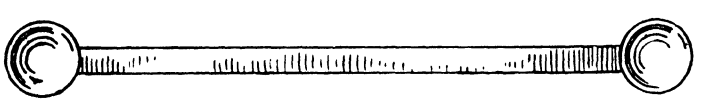


Fig. 15

for the magnetic force between the two poles. He performed experiments with his torsion balance, the theory of which has already been explained in § 415. In place of charged spheres he used long and thin ball-ended magnets (Fig. 15). In this case a correction had to be applied for the earth's magnetic

field which was not necessary in electrostatic experiment. He established the following laws :

1. *The force between the magnetic poles is directly proportional to the product of the pole strengths.*

2. *The force with which two magnetic poles attract or repel each other is inversely proportional to the square of the distance between them.*

Mathematically we can express these two relations as

$$F \propto \frac{m_1 m_2}{r^2}, \text{ or } F = \frac{1}{\mu} \frac{m_1 m_2}{r^2}.$$

where μ is the constant (corresponding to K) depending upon the medium. If the medium is air, the constant depending upon it is equal to unity. Hence for air the formula reduces to

$$F = \frac{m_1 m_2}{r^2}.$$

476. Unit Magnetic Pole.—Let us suppose that we have two exactly equal poles, placed at a unit distance apart; if the force between them be unity, *i.e.* one dyne, the above formula gives us

$$1 = m^2,$$

$$\therefore \quad m = \pm 1.$$

Thus we see that m is 1 when F is 1 dyne and $r = 1$ cm.

Definition.— *A magnetic pole has unit strength if it experiences a force of* 1 *dyne when placed at unit distance in air from an equal and similar pole.*

477. Intensity of the Field.—The force which a pole experiences at any point in a field depends upon its own strength. In order to compare the two fields we must take poles of the same strength in both the cases. So that different people may be able to compare their results together, it is necessary that the pole-strength adopted by them all be the same. In other words, for mutual comparison it is necessary to fix upon a definite unit which should be used by all. The usual convention is that the pole should be of unit strength. Under this condition *the magnetic force at any point in a field is equal to the force which a pole of unit strength experiences at that point.* This force is called the **magnetic intensity** at the point. Supposing that the field is due to a single pole of strength m, the magnetic intensity at a point distant r cm. from it will be equal to $\frac{m}{r^2}$, because $m_1 = 1$.

478. Magnetic Potential.—We have already explained the term potential in electrostatics. Its conception here is also similar. It can be defined as *equivalent to the work done in bringing a unit magnetic pole against the magnetic force from an infinite distance, up to the point at which the potential is to be determined.*

The potential due to a pole of strength m can be proved equal to $\frac{m}{r}$, where r is the distance of the point from the pole. The magnetic potential is connected with the magnetic intensity in the same way as the electrical potential is connected with the electric intensity. Mathematically $F = \frac{V_1 - V_2}{S}$, where V_1 and V_2 represent the potential at the two points and S the distance between them. Note, the two points should be so close to each other that the force may be considered uniform between them, otherwise the relation does not hold good.

It should be remembered that the potential is a *scalar* quantity, whereas the magnetic intensity is a *vector* quantity. Therefore the potential at a point due to the magnetic poles is to be added up simply, whereas in the case of the magnetic intensity the resultant must be determined with the help of parallelogram law of forces. It will be useful to compare all these formulae and definitions with those that are given in Electrostatics.

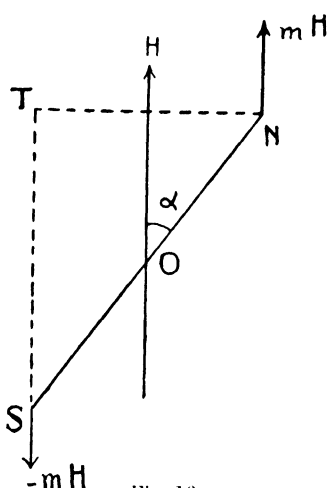


Fig. 16

479. Magnet in a Uniform Field.—As has been said in § 473 footnote, the field is uniform when the lines of force are parallel and the number of them per unit area is the same. We can express the same thing by saying that the field is uniform if the magnitude and the direction of the force is the same at every point of the field. The earth's magnetic field can be considered fairly uniform in the whole of a laboratory. In other words, a magnet suspended in a laboratory at a distance from other magnets is acted upon by a uniform field of force. Let us suspend a magnet NS making an angle α with the magnetic

meridian. Suppose that the magnetic intensity of the earth's field is H, then the force upon the north pole of the magnet NS of pole strength m will be mH, on the south pole the force will be $-mH$, *i.e.* it will be in the opposite direction. Thus we see that the forces at the two ends are equal and opposite, which means, in other words, that a couple is acting upon the magnet. This at once explains why the effect of the earth's magnetic field is simply *directive*. The moment of the couple $= mH \times NT$.

Denoting the length of the magnet by $2d$, the moment of the couple $= mH \times 2d \sin \alpha = MH \sin \alpha$, where α is the angle which the magnet makes with the magnetic meridian and M is the magnetic moment. Thus we see that the *couple acting on a magnet placed in a uniform field of strength* $H = MH \sin \alpha$, where α is the angle between the magnetic axis and the magnetic meridian.

The tendency of this couple is to rotate the magnet until it is parallel to the direction of the field in which position the couple vanishes, for then $\alpha = 0$ and therefore $\sin \alpha = 0$. The moment of the couple is maximum when $\alpha = 90$, for then $\sin \alpha = 1$.

480. Field due to a Bar Magnet.—We have already learnt that the space round the magnet is called the field of force and that throughout this region the magnetic force is perceptible. If we know the pole strength of the magnet we can calculate the magnetic force at any point in the field. Two cases require special consideration, hence we shall take them up; the method of calculation for any other point, however, will also be similar. One of the two cases is the force at any point on the axial line of a bar magnet, and the second is the force at any point lying on the magnetic equator.

481. Magnetic force at any point on the axial line of a Bar Magnet.—The axial line is simply the prolongation of the

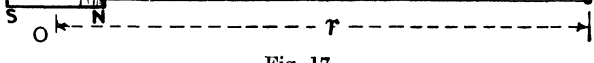


Fig. 17

magnetic axis. When the axial line passes through the point at which we wish to determine the field, the magnet is said to be in the **end-on** position with respect to that point. Let us calculate the magnetic force at the point P at a distance

of r cm. from the middle point O of a magnet NS (Fig. 17), where the latter is placed in the end-on position with respect to that point. If the length of the magnet be denoted by $2d$, it is evident that the distance $NP = r - d$ and $SP = r + d$.

Force on unit pole at P due to $N = \frac{m}{(r-d)^2}$ in direction NP.

Force on unit pole at P due to $S = -\frac{m}{(r+d)^2}$ along PN.

It is written – (ive) because it is in the opposite direction. These two forces are opposite although in the same straight line. The resultant will simply be equal to their difference.

Hence the resultant force on a unit pole placed at P

$$= \frac{m}{(r-d)^2} - \frac{m}{(r+d)^2}$$

$$= m\left\{\frac{1}{(r-d)^2} - \frac{1}{(r+d)^2}\right\} = m\left\{\frac{(r+d)^2 - (r-d)^2}{(r^2-d^2)^2}\right\}$$

$$= \frac{m \times 4rd}{(r^2-d^2)^2}$$

$$= \frac{m \times 2d \times 2r}{(r^2-d^2)^2}$$

$$= \frac{M \times 2r}{(r^2-d^2)^2}$$

$$= \frac{2Mr}{(r^2-d^2)^2}.$$

If P is at a very great distance from the magnet, d^2 may be neglected in comparison with r^2, and the strength of the field is reduced to $\frac{2M}{r^3}$.

It should be clearly understood that this approximation is true only if the distance of the point is very large as compared with half the length of the magnet itself.

482. Magnetic force at any point on the perpendicular to the axis through the middle point of the magnet.—The perpendicular drawn to the axis through the middle point of the magnet is called the **magnetic equator.** When a point at which we wish to calculate the force lies on the magnetic equator, the magnet is said to be in the **broadside-on** position with respect to that point.

Let NS be the magnet having poles of strength $\pm m$ units and Q the point at which we wish to calculate the intensity

lying on the magnetic equator at a distance r cm. from O. The length of the magnet will be denoted as before by $2d$.

$$NQ^2 = r^2 + d^2.$$

Magnetic force due to N at $Q = \frac{m}{r^2 + d^2}$ along NR.

Magnetic force due to S at $Q = \frac{m}{r^2 + d^2}$ along TS.

Let us resolve both these forces into rectangular components along the axis of the magnet and perpendicular to it. Draw

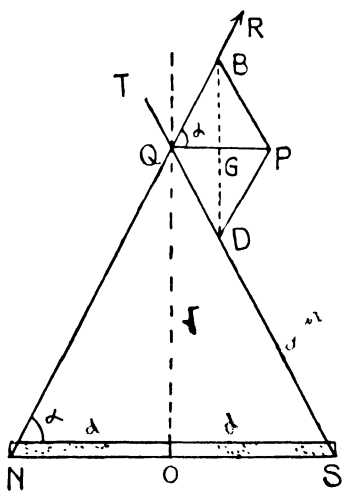


Fig. 18

QP parallel to the length of the magnet and at any point G draw a line BGD parallel to the magnetic equator. Let the angle QNO be denoted by α.

The components of force $\frac{m}{r^2 + d^2}$ due to N will be

$$\frac{m}{(r^2 + d^2)} \sin \alpha \text{ along } GB,$$

and

$$\frac{m}{(r^2 + d^2)} \cos \alpha \text{ along } QG.$$

Similarly the components of force $\frac{m}{r^2 + d^2}$ due to S will be

$$\frac{m}{(r^2 + d^2)} \sin \alpha \text{ along } GD,$$

and $\dfrac{m}{(r^2+d^2)} \cos \alpha$ along QG.

Adding up the components which are in the same direction we get the components of the resultant along these two rectangular axes.

The component is zero along DGB, and

$$\frac{2m}{(r^2+d^2)} \cos \alpha \text{ along } QG.$$

Now $\cos \alpha = \dfrac{NO}{NQ} = \dfrac{d}{(r^2+d^2)^{\frac{1}{2}}}$,

$$\therefore \quad \text{Force} = \frac{2m}{(r^2+d^2)} \times \frac{d}{(r^2+d^2)^{\frac{1}{2}}} = \frac{M}{(r^2+d^2)^{\frac{3}{2}}} \text{ along } QP.$$

If we suppose as before r to be very large as compared with d, d^2 will be negligible as compared with r^2, and the expression for the force will be simplified to

$$\frac{M}{r^3} \textit{ parallel to the axis.}$$

The same result can be arrived at by geometry.

The force at Q due to N is $\dfrac{m}{r^2+d^2}$ along QR.

The force at Q due to S is $\dfrac{m}{r^2+d^2}$ along TS.

Since these two forces are equal, their resultant will be equally inclined to them; in other words, it will bisect the angle between QR and QS, which means that it will be parallel to NS. The resultant can be easily determined.

Mark off QB equal to $\dfrac{m}{r^2+d^2}$ units $= QD$.

Complete the parallelogram; the resultant will be represented by QP.

Consider the two Δs QPD and NQS. They are similar, since their sides are parallel to each other and hence their angles are also equal.

$$\therefore \quad \frac{QP}{NS} = \frac{QD}{QS}.$$

or
$$QP = \frac{QD}{QS} NS$$
$$= \frac{m \times 2d}{(r^2+d^2)^{\frac{3}{2}}}$$
$$= \frac{M}{(r^2+d^2)^{\frac{1}{2}}}.$$

$\left\{ QD = \dfrac{m}{r^2+d^2},\ NS = 2d \right.$
$\left. \text{and } QS = (r^2+d^2)^{\frac{1}{2}}. \right.$

It will be simplified to $\frac{M}{r^3}$ parallel to the axis when d^2 is negligible in comparison with r^2.

483. Two magnetic fields at right angles to each other.—That a small magnetic needle when placed in a magnetic field sets itself along the direction of the field has already been explained in § 473. Let us see now what happens when the needle is subjected to two magnetic fields at right angles to each other in place of one. Let us suppose the magnetic needle NS with poles of strength $\pm m$ units, on being subjected to two fields H and F acting at right angles to each other comes to rest in a position making an angle α with the direction of H. The couple acting on the magnet due to H is given by the formula

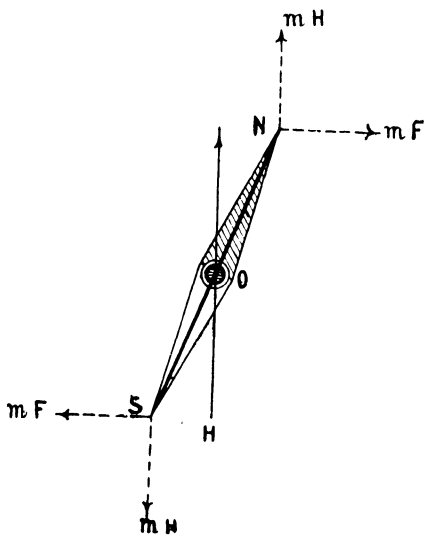


Fig. 19

$$MH \sin \alpha,$$

where M is the magnetic moment of the needle and α the angle between the needle and the direction of H.

The couple acting on the needle due to F is $MF \cos \alpha$. In the position in which the needle is at rest these two couples must be equal and opposite. Equating the two couples we get

$$MH \sin \alpha = MF \cos \alpha,$$

or

$$F = H \tan \alpha.$$

Now if we know H, and measure the angle through which the needle is deflected by the introduction of the field F, we can measure F. When F is not acting, the needle sets itself along H. Therefore in order to compare the two fields we need see the deflection of the needle only, and get the value of F from the formula $F = H \tan \alpha$.

Remember, *the relation* $\frac{F}{H} = \tan \alpha$ *is applicable only when the two fields are uniform and are perpendicular to each other.*

484. Magnetometer.—It is an instrument by means of which magnetic moments or magnetic fields can be compared. It consists of a short magnetic needle pivoted at the centre, which carries generally at right angle to it a non-magnetic aluminium pointer. It moves over a circular scale divided into degrees. Two long arms bearing millimetre scales pass out at right angles to the two sides of the wooden box. The zeroes of the scales are at the centre of the needle. To eliminate error due to parallax in reading the position of the needle, the scale is fixed on plane-glass mirror. The eye is placed in such a position that the pointer and its image are in the same straight line, and the graduation which is given by the line joining these two is taken as the correct reading.

To use the instrument, place it with its arms east and west, and the needle north and south; in other words, with the arms

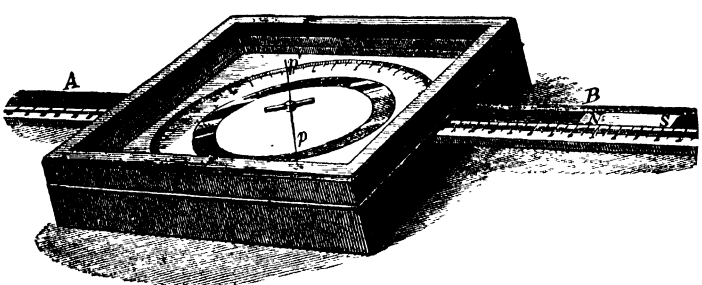


Fig. 20

at right angles to the needle and parallel to the pointer. Now let us suppose that we want to find out the field due to a magnet at a distance r cm. from it. Place it on the scale with its N pole at B (Fig. 20) in such a way that the centre of the magnet is at a distance of r cm. from the centre of the needle. Notice that we have two magnetic fields here, one due to the magnet and the other due to the earth; the two are, moreover, at right angles to each other. Hence using the formula $F = H \tan \alpha$, where H is the field due to the earth and α the angle which the needle makes with the earth's field or which the pointer makes with the 0° and 0° of the scale, we can calculate F. Four positions of the magnet must be tried for α. At first, with the

north pole towards the centre of the needle at a distance $r-d$ cm., then with the south pole towards the needle at $r-d$ cm. from the centre of the needle; and two similar observations on the other side of the needle. The mean of all these should be taken as the correct reading.

Since $F=H\tan\alpha=\frac{2M}{r^3}$, if we know H we can determine M, the magnetic moment. The magnet in this case is in the "end-on" position. For the "broadside-on" position the two wooden scales must be pointing also north and south, *i.e.* they must be parallel to the needle and at right angles to the pointer. The magnet is to be placed perpendicular to the axis of the scales. It will be noticed that if the same magnet be used and at the same distance, $F=H\tan\alpha'$ will be half of the former result, for in this case

$$F=H\tan\alpha'=\frac{M}{r^3}.$$

485. Gauss's method of proving Inverse Square Law.—The two expressions (i) $F=H\tan\alpha=\frac{2M}{r^3}$ for "end-on" position, and (ii) $F=H\tan\alpha'=\frac{M}{r^3}$ for "broadside-on" position, are obtained on the assumption that the inverse square law holds good for the magnetic force. These relations show that if a magnet is used in the two positions at the same distance, the ratio of the tangents of the deflection should be equal to 2, for

$$\frac{2M}{H}=r^3\tan\alpha \quad \ldots \text{ for "end-on" position,}$$

and $$\frac{M}{H}=r^3\tan\alpha' \quad \ldots \text{ for "broadside-on" position.}$$

Dividing one by the other, we get

$$\frac{\tan\alpha}{\tan\alpha'}=2.$$

Gauss showed experimentally that the ratio does come out to be equal to 2, proving thereby that the law on which the derivation of the relation was based must be true.

486. Comparison of Magnetic Moments.—Let us suppose we have two magnets A and B, whose magnetic moments we want to compare. Place the first magnet A in the end-on position at a distance r cm. from the centre of the needle, *the distance being measured from the centre of the needle to the*

centre of the magnet. Read the deflection on both the ends of the pointer. Turn the magnet round so that the north and south poles change their places with each other, and again read the pointer at both the ends. Move the magnet over to the other side of the needle, and place it at the same distance from the centre of the needle. Note the deflection at both the ends of the pointer. Interchange the positions of the north and south poles and again take the readings of the deflection. The mean of these eight readings gives the deflection a_1. Repeat with the second magnet B the same procedure, placing it at the same distance, and get the mean of eight readings; call it a_2. Calling the moments of A and B, M_1 and M_2, we can write

$$\frac{2M_1}{r^3} = H \tan a_1,$$

and

$$\frac{2M_2}{r^3} = H \tan a_2;$$

$$\therefore \quad \frac{M_1}{M_2} = \frac{\tan a_1}{\tan a_2}.$$

There is another method also of comparing the moments. Instead of keeping the distance constant we could alter the distance and keep the deflection the same. In this case

$$\frac{2M_1}{r_1^3} = H \tan a = \frac{2M_2}{r_2^3}.$$

$$\therefore \quad \frac{M_1}{M_2} = \frac{r_1^3}{r_2^3}.$$

It should be noted that the principle underlying our experiments with magnetometer is *that we must have two fields at right angles to each other and that when this is so, the tangent law* ($F = H$ *tan* a) *holds good.*

487. Oscillation Method.—There is a third method also of comparing the magnetic moments of two magnets. (It is found that if a freely suspended magnet be displaced slightly in a magnetic field it begins to vibrate. Although the amplitude of the vibrations goes on decreasing yet the time taken by each vibration is the same.) It can be seen experimentally with a stop-watch. The time taken in completing one vibration is given by the formula

$$t = 2\pi\sqrt{\frac{K}{MH}},$$

where t is the time period, K* a constant depending upon the mass and the shape of the magnet, M is the magnetic moment of the magnet, H the field in which it vibrates. Let us suppose that we have two similar magnets of the same shape and weight, whose magnetic moments we want to compare. Let us call their moments M_1 and M_2. Suspend them one after the other, and let them vibrate. Note the time for 50 vibrations in each case; dividing by 50, find time for one vibration. Knowing the time periods in the two cases we can compare the magnetic moments.

$$t_1 = 2\pi\sqrt{\frac{K}{M_1 H}}, \text{ or } t_1^2 = 4\pi^2 \frac{K}{M_1 H},$$

$$t_2 = 2\pi\sqrt{\frac{K}{M_2 H}}, \text{ or } t_2^2 = 4\pi^2 \frac{K}{M_2 H}.$$

Since the magnets are alike, K is the same in both the cases.

$$\therefore \quad \frac{M_1}{M_2} = \frac{t_2^2}{t_1^2}.$$

Thus we find that we can compare the magnetic moments by this method.

Fig. 21 shows the apparatus generally used for this experiment.

This enables us to compare the two magnetic fields also. Suppose we want to compare the intensity of two magnetic fields. Let a needle vibrate in both the fields; note the time it takes to complete 50 vibrations in each field, and let the time for one vibration in the two fields be t_1 and t_2 seconds respectively;

$$t_1^2 = 4\pi^2 \frac{K}{MH_1},$$

and

$$t_2^2 = 4\pi^2 \frac{K}{MH_2}.$$

$$\therefore \quad \frac{H_1}{H_2} = \frac{t_2^2}{t_1^2} = \frac{n_1^2}{n_2^2}.$$

* The constant K is called the **moment of inertia.** For a cylindrical bar magnet oscillating about an axis perpendicular to its length and passing through its centre of gravity $K = m\left(\frac{l^2}{12} + \frac{r^2}{4}\right)$, where m represents the mass, l the length, and r the radius. For a rectangular bar magnet $K = m\left(\frac{a^2 + b^2}{12}\right)$, where a represents the length, and b the breadth.

where n_1 and n_2 are the number of vibrations made in one second, for $n_1 = \frac{1}{t_1}$ and $n_2 = \frac{1}{t_2}$.

In order to find out the field due to a magnet, place it in such a manner that its field is parallel to that of the earth's field. Let a needle vibrate first in the earth's field ($= H$)

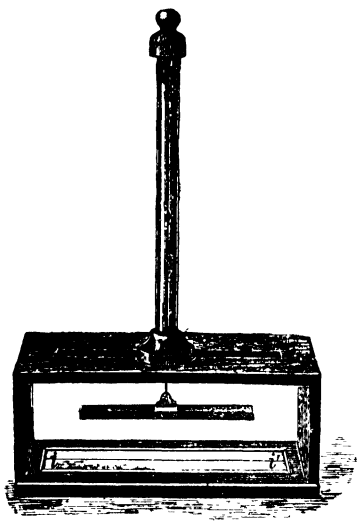


Fig. 21

alone and then in the combined field ($= H + F$). Substituting $H + F$ for H_2,

$$\frac{H + F}{H} = \frac{n_2^2}{n_1^2},$$

where n_1 is the number of vibrations made in one second in the earth's field alone, and n_2 is the number of vibrations in the combined field of the earth and the magnet.

Subtract 1 from both the sides, and we get

$$\frac{F}{H} = \frac{n_2^2 - n_1^2}{n_1^2},$$

or

$$F = H\frac{n_2^2 - n_1^2}{n_1^2}.$$

Thus F is found if H is known.

488. Absolute Value of H.—We have already seen that

$$\frac{2M}{r^3} = H \tan \alpha,$$

where α is the deflection produced by a magnet whose moment is M, when its centre is at a distance of r cm. from the centre of the needle of the magnetometer, the magnet being placed in the end-on position. Let the magnet now oscillate in the earth's field. We know that the time of one complete vibration is given by

$$t = 2\pi\sqrt{\frac{K}{MH}},$$

or

$$MH = 4\pi^2\frac{K}{t^2} = 4\pi^2 K n^2.$$

Now we have two equations,

$$\frac{M}{H} = \frac{r^3}{2} \tan \alpha, \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{(i)}$$

and

$$MH = 4\pi^2 K n^2. \quad . \quad . \quad . \quad . \quad . \quad \text{(ii)}$$

Divide one by the other,

$$H^2 = \frac{4\pi^2 K n^2 \times 2}{r^3 \tan \alpha} = \frac{8\pi^2 K n^2}{r^3 \tan \alpha}.$$

On the right-hand side everything is known: K can be calculated, n is observed, r is known also, and so is α.

Hence H can be determined.

EXERCISES

1. A magnetic pole of strength 180 units is situated in a line with a magnetised knitting needle, and at a distance of 30 cm. from its middle point. If the length of the needle is 20 cm. and its strength of pole 40 units, find the force on the given pole.

The force on unit pole on the axial line of a magnet at a distance r cm. is

$$\frac{2Mr}{(r^2-d^2)^2}.$$

Substitute the value for various quantities,

$$F=\frac{2\times 800\times 30}{(30^2-10^2)^2}=\frac{2\times 800\times 30}{800\times 800}=\frac{2\times 30}{800}=\frac{3}{40}.$$

This is the force on a unit north pole, but we have placed there a pole whose strength is 180 units.

$$\therefore \qquad \text{total force}=\frac{3}{40}\times 180=\frac{27}{2}=13\,5 \text{ dynes.}$$

2. Find the force on a magnetic pole of strength 40 units, situated at a distance of 30 cm. from each end of a magnetised needle whose length is 20 cm. and strength of pole 30 units.

The force on unit pole placed on the equatorial line at a distance r cm. from the middle point

$$=\frac{M}{(r^2+d^2)^{\frac{3}{2}}}$$

$$=\frac{600}{(30^2+10^2)^{\frac{3}{2}}}$$

$$=\frac{6}{100\sqrt{10}}.$$

$$\text{Force on 40 units}=\frac{6\times 40}{100\sqrt{10}}=\frac{12}{5\sqrt{10}} \text{ dyne.}$$

Using approximate formula,

$$\text{force on 40 units} = \frac{600+40}{27000} = \frac{8}{9} \text{ dyne.}$$

3. A small magnet vibrating horizontally in the earth's field has a period of 4 seconds; when another magnet is brought near to it, 50 vibrations take 160 seconds. Compare the strength of the field due to the magnet with the earth's horizontal field, assuming that the two fields are in the same direction or in opposite directions.

$$t_1 = 4 \text{ sec.}, \quad \therefore \frac{1}{t_1} = n_1 = \frac{1}{4}.$$

$$t_2 = \frac{160}{50} \text{ sec.}, \quad \therefore \frac{1}{t_2} = n_2 = \frac{5}{16}.$$

$$\frac{H_2}{H_1} = \frac{{n_2}^2}{{n_1}^2} = \frac{25 \times 16}{16 \times 16} = \frac{25}{16}.$$

When the two fields are in opposite directions, one of them will be $H_2 - H_1$, and the other H_1

$$\frac{H_2 - H_1}{H_1} = \frac{25}{16} - 1 = \frac{9}{16}.$$

In the other case, when they are in same direction, the ratio becomes

$$\frac{H_2 + H_1}{H_1} = \frac{25}{16} + 1 = \frac{41}{16}.$$

4. ABC is a right-angled triangle, and $AB = BC = 20$ cm.; poles of strengths 36 and 60 units respectively are placed at A and C. What is the intensity at B? *Ans.* 175 dynes.

5. A suspended magnet makes 10 oscillations in 117·6 seconds at a place where H, the horizontal intensity, is 0·189 dyne. How many complete oscillations will it make per minute at another place where $H = 0{\cdot}214$ dyne. *Ans.* 5·4.

6. The length of a magnet is 16 cm. and the strength of its pole is 15 units. Find the intensity at a point on the axis of the magnet at a distance of 16 cm. from its centre.

Ans. 0·208 dyne.

7. A magnet of moment 100 is suspended in the magnetic meridian. Find the couple acting on it when it is turned through an angle of 30° ($H = {\cdot}2$ dyne). *Ans.* 10.

CHAPTER III

TERRESTRIAL MAGNETISM

489. It has already been pointed out in a preceding article that the pointing of the magnetic needle in a definite direction —that of approximately north and south—shows that the earth is a huge magnet. Before Gilbert the setting of a freely suspended magnet in the north and south direction was attributed to the special attraction which the pole-star was supposed to possess for iron. It was he who first showed that the setting of the needle in this direction showed simply that there was a magnetic field existing over the surface of the earth, and that the needle simply sets itself in the direction of the magnetic force. This magnetic field is supposed to be due to the earth's magnetism. But what is this magnetism due to is still an unsolved problem. To simplify matters the earth's magnetism is supposed to be internal, viz. it is supposed that there is a long bar magnet placed inside the earth with its poles near the geographical north and south poles. In the preceding articles we have been taking it for granted that the lines of force due to the earth are horizontal because we find the magnetic needle to be resting horizontally; but this is not so as a matter of fact. Since the needle is pivoted at the centre, therefore it cannot move in a vertical plane. If the needle be suspended from its centre of gravity and be free to move in a vertical plane, it sets itself in a definite position in the northern hemisphere, the north pole pointing downwards (but not vertically downwards except at the poles), showing thereby that the earth's field is not horizontal but makes an angle with the horizon. In order to know completely the earth's field anywhere we must know three things—(1) the angle between the magnetic axis of a freely suspended needle, and the true geographical meridian, in other words, the angle between the magnetic and the geographical meridians; it is called **declination**; (2) the angle which a magnet suspended

freely from its centre of gravity makes with the horizon—this angle is called the **dip**; and (3) the magnitude of the **horizontal component** of the earth's field. The determination of (1) fixes the vertical plane in which the magnetic force acts; the determination of (2) fixes the direction of the force in that plane, and from (2) and (3) we can determine the total intensity of the field. Thus the three above-mentioned quantities enable us to know completely the direction as well as the magnitude of the earth's force at any point. *These three quantities are called the* ***magnetic elements*** *of a place.* Fig. 22 gives a very clear idea of these elements; the plane *POS* represents the geographical meridian at any place, the plane *QONT* represents the magnetic meridian, and the $\angle POQ$ gives the declination at the same place.

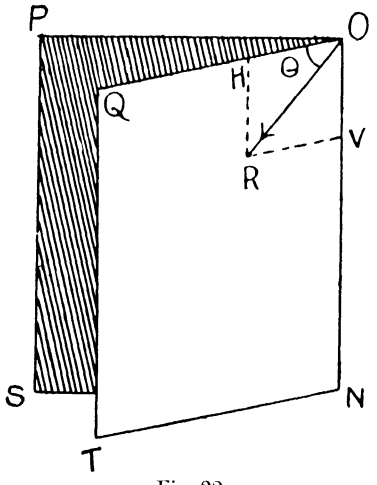


Fig. 22

OPQ represents a horizontal plane, and the angle between it and *OR*, the direction of the resultant field along which a magnetic needle suspended from its centre of gravity comes to rest, gives the dip. Supposing *OR* to be the total intensity, on resolving it into rectangular components we find that *OH* is the horizontal component and *OV* the vertical component. Now in order to know the total intensity we need know the horizontal component and the dip only. Calling the total intensity *R*, we find that

$$\frac{H}{R} = \cos \Theta,$$

$$\therefore \quad R = H/\cos \Theta.$$

Thus if we know *H* and Θ, we know *R*.

Let us consider the elements one by one.

490. Declination.—Declination has already been defined *as the angle between the magnetic and the geographical meridians*

at a place. This angle is different at different places. In Kew observatory the angle recorded in 1912 was 15° – 46·5′ west, in Calcutta in the same year it was 0° – 44·0′ east, whereas in Bombay it was 0° – 51·2′ east. The words west and east mean simply that the north pole of the needle is towards west or east of the geographical meridian. Suppose that *NS* (Geographical) gives the direction of the geographical north and south and *NS* (Magnetic) the direction of the needle in which it comes to rest. In this case the declination is towards the west.

To actually determine the declination at a place we must determine in the first place the geographical north and south, and in the second place the magnetic meridian. The determination of the geographical meridian belongs properly to astronomy, therefore here only a rough method will be described. Erect a thin vertical rod having a pointed top in an open space so that the sun can strike upon it. Watch the shadow of the top-point of the rod and note where it touches a circle before the noon and also after the noon. Connect these points on the circumference to the centre. Bisect the angle. This bisector is the true north and south line. Or note when the shadow is the shortest, the line of the shadow at that time will give the direction of the true north and the south line. To determine the magnetic meridian, take a magnetic needle suspended from its centre; the direction in which it points will give the direction of the magnetic north and south. Of course it is supposed that the geometrical axis coincides with the magnetic axis. If this be not the case, the needle will have to be turned upside down, *i.e.* turned through 180° and again the position noted, the mean of these two positions will give the actual direction of the magnetic axis. A vertical plane passing through this axis is called the **magnetic meridian.**

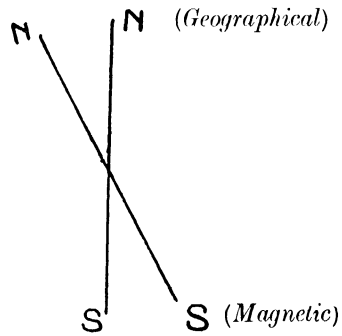


Fig. 23

ABCD (Fig. 24) is a magnetic needle, whose magnetic axis *ns* does not coincide with the geometrical axis *AC*. But on freely suspending it we mark *AC* as the axis and not *ns*. Turn it upside down, *ns* will point in the same direction, but *AC* will

be pointing in the direction of $A'C$. It is clear from Fig. 24 that by bisecting the angle between the axes AC and $A'C'$ we get the actual direction of ns.

Thus we know the magnetic meridian. Finding the angle between this meridian and the geographical meridian we determine the declination. We have already seen that declination is different at different places. A line joining those places which have the same declination is called an **isogonic line** or **isogonal.** A line joining those places where there is no declination is called an **agonic line.**

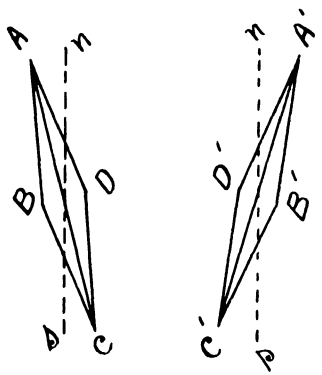


Fig. 24

Declination not only varies from place to place but varies at the same place with time as well. In 1580 the declination in London was 11° 15′ east, in 1657 it was 0°, in 1802 it was 24° 6′ west, in 1909 it was 16° 10 8′ west. Thus we see that from east it became zero, and then became westerly, increased to a maximum and again began to decrease. This slow change is often called the **secular change.**

Declination also varies with different seasons; in a particular season it is maximum, while in another it is minimum. This change is called **annual change.**

Daily Changes.—With sensitive instruments it has been observed that the declination has a daily variation also. During the whole day it does not remain constant; for instance in England the north pole of the needle moves to the west from 10 A.M. to 1 P.M. by about 5′ and then begins to move to the east, this goes on till morning, the maximum decrease being 4′.

In addition to the above-mentioned three changes there are sudden changes which are called **magnetic storms.** These storms accompany the appearance of unusually large sunspots or other natural phenomena, as earthquakes, etc.

491. Dip.—It has already been defined as *the angle between the direction of the resultant intensity of the earth's field and the horizon.* To get at the direction of the resultant intensity of the earth's field, we are to suspend freely a magnetic needle from its centre of gravity and to measure the angle between the

magnetic axis of the needle and the horizon. To measure the dip at a particular place an instrument called a **dip-circle** is used. Its essentials are : (i) a mounted magnetic needle free to rotate in a vertical plane ; (ii) a circular scale in front of which the needle moves ; and (iii) a circular scale near the base by which the rotation of the frame carrying the needle can be read. The needle turns freely about a horizontal axle of hard polished steel which passes through its centre of gravity. The

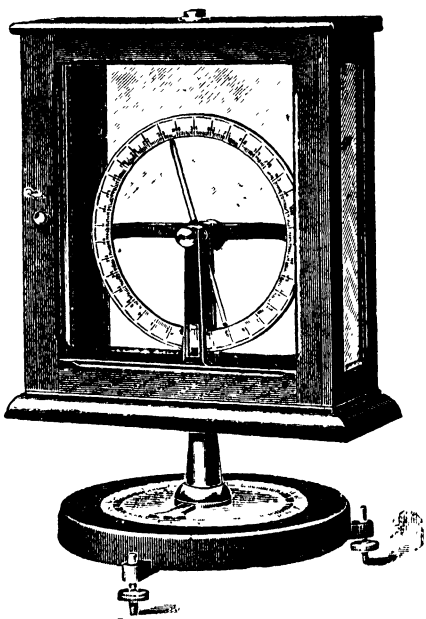


Fig. 25

axle rests on two polished agate knife-edges, and the angle which the needle makes with the horizon is read on the vertical circle. The instrument is provided with three levelling screws at the base.

492. To use the Dip Circle.—The first thing that we have to do is to set the magnetic needle in the magnetic meridian, *i.e.* the plane of the needle to be made coincident with the magnetic meridian at the place. At first the instrument is carefully levelled in order to make the needle turn quite freely about the axle. The needle and the vertical circle are turned about till

the needle stands quite vertical. When this is so the plane of the needle is perpendicular to the magnetic meridian. In this position the horizontal component of the earth does not produce any effect on the needle itself, but, on the other hand, produces pressure upon the bearings only. Since only the vertical component is producing an effect upon the needle, it sets itself exactly vertical. Turn the needle through 90° and it will be in the magnetic meridian. Read the angle which the needle makes with the horizontal line marked 0°, 0° on the scale. This gives the dip. But we have supposed that there is nothing wrong with the instrument. As a matter of fact this is rarely the case. The defects that may be present in the instrument are :

(i) *The error due to eccentricity*, viz. if the axle does not pass through the centre of the vertical scale. The actual centre of the circle is at *O*, but if the axle of our needle is at *O'*, it is clear that at the upper end our reading is too low, whereas at

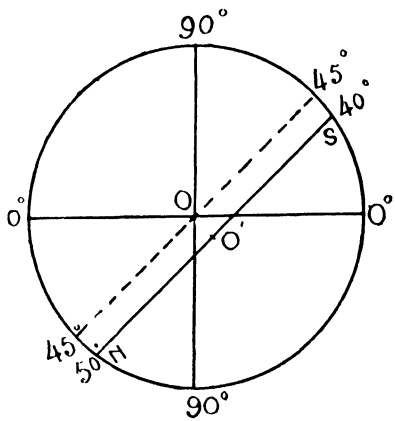


Fig. 26

the lower end it is too high (see Fig. 26). In order to remove this source of error, the needle must be read at both the ends.

(ii) *The error due to magnetic axis not being coincident with the geometrical axis of the needle.*—This error is the same with which we have dealt already in our discussion on declination. We are simply to reverse the needle so that the face which was to the east is now to the west, and take a fresh set of readings.

(iii) *The error arising from non-coincidence of the axle with the centre of gravity.*—To remove this error the needle is simply re-magnetised so that the south pole becomes the north pole and the north pole the south pole. After doing it, again a fresh set of readings is taken.

The mean of all these readings gives the dip. We have supposed in this that the 0° line of the vertical scale is truly horizontal. If this be not the case, the two ends of the needle are read first, then the instrument is turned through 180°, and again the two ends are read. In very careful measurements the mean of 16 readings is taken; these 16 readings are, 2 for (i), and 2 in the second position after turning through 180°. These 4 are to be repeated for No. (ii) which means in all 8, and these 8 observations are repeated after remagnetising.

All this has been written to inform the student about the elaborateness of this instrument. For complete discussion of these various sources of error the student is referred to some advanced treatise on the subject.

The value of the dip, like that of the declination, is different at different places. At the magnetic poles the needle stands vertical, *i.e.* the dip is 90°. At the magnetic equator the needle stands horizontal, *i.e.* the dip is zero. In the northern hemisphere the north pole dips downwards, but in the southern hemisphere it is the south pole which dips downwards. The value of the dip in Kew in 1912 was 66° 56 5′; in Bombay in the same year it was 23° 56·1′ (north pole downwards); in Melbourne in the year 1898 it was 67° 22 4′, south pole downwards.

A line joining the places having the same dip angle is called an **isoclinic line.** A line joining the places having no dip is called an **aclinic line.**

Dip is also subject to **secular changes,** its value alters after a very long time. For instance in London in 1576 it was 71° 51′, in 1676 it was 73° 30′, in 1723 it was 74° 42′, in 1802 it was 70° 36′, in 1909 it was 66° 59·7′. In addition to the secular changes, there are annual, daily, and irregular changes in the dip.

493. Horizontal Component.—Let us pass on now to the third magnetic element at a place, the value of horizontal component of the earth's field. A method of finding the absolute value of the horizontal component of the earth's field has already been given. A magnet is freely suspended at the place where we are to determine the horizontal magnetic

force. When it is displaced it begins to vibrate, and the time for one vibration is determined. The strength of the horizontal component of the field which alone is responsible for the vibrations can be calculated in the following manner:

The period of one vibration

$$t = 2\pi\sqrt{\frac{K}{MH}},$$

or

$$t^2 = 4\pi^2 \frac{K}{MH},$$

and further we know from the magnetometer, that

$$\frac{2M}{r^3} = H \tan \alpha,$$

when the magnet is in the end on position. Eliminating M we get the value of H.

$$H^2 = \frac{4\pi^2 \times Kn^2 \times 2}{r^3 \tan \alpha} = \frac{8\pi^2 k^2 n^2}{r^3 \tan \alpha}.$$

See § 488.

Knowing H, α, and Θ the angle of dip, we can find out the total intensity R at any place, for

$$R = \frac{H}{\cos \Theta}.$$

If we want to compare the total intensity at the places where the horizontal components are H and H', we get

$$R \cos \Theta = H,$$

and

$$R' \cos \Theta' = H',$$

where Θ and Θ' are angles of dip at the two places.

We know further that

$$\frac{H}{H'} = \frac{n^2}{n'^2} = \frac{R \cos \Theta}{R' \cos \Theta'},$$

or

$$\frac{R}{R'} = \frac{n^2 \cos \Theta'}{n'^2 \cos \Theta}.$$

494. Mariner's Compass.—The most important application of terrestrial magnetism is to the art of navigation. The only element with which the sailor is directly concerned in steering his vessel is the declination. The true geographical latitude and longitude may be determined by astronomical observations, but they are too tedious, hence the steering of the ship is directed by the compass alone. It consists of a cylindrical

case, which, to keep the compass in a horizontal position, is supported on gimbals -two concentric rings supported one within the other upon axes fixed at right angles, but in the same plane. The compass usually employed consists of a flat circular disc of aluminium or mica on which a scale is pasted, or drawn ; on its under surface are secured four to eight light magnetic needles, securing thereby a system of high magnetic moment, but of very small weight. In many of the modern compasses the card is floated on methylated spirit, which takes most of the weight off the needle point. The

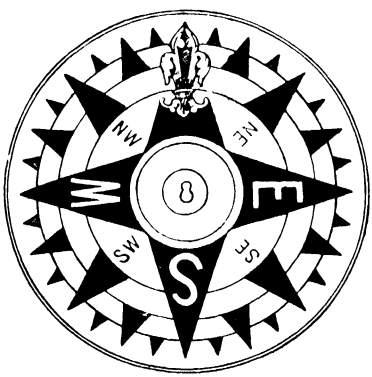


Fig. 27

liquid is useful in reducing the oscillations of the compass which would otherwise be troublesome. The card is divided into 32 points, Fig. 27. At first it is divided into 4 parts, N., E., S., and W. Each of these parts is again divided into two, giving N.E., S.E., S.W., and N.W. Again these eight parts are subdivided into two each, giving 16 parts. Every one of these sixteen parts is again divided into two, giving in all 32 parts. Let us now see how this compass is used in steering. The officer in charge of the ship is provided with a map on which the declination at all the important places is given. Suppose the declination at a place is 10° E, and the ship is to sail at an angle of 60°, west of north. So that it might go in the right direction the course of the ship should be 70° west of north pointed by the compass. On the other hand, suppose that the ship is to sail in the direction 60° east of north. In this case the needle is already pointing 10° east of

north, hence the direction of the ship should make an angle of 50° eastward, with the north pointed out by the needle. There are some disturbances due to the iron in the ship, which are very complicated in nature, hence we shall not go into them.

495. Astatic Pair of Needles.—We have seen already that a magnetic needle always sets itself in a particular direction, approximately that of N. and S. But in some cases we want the magnetic needle to stand in any position, in other words,

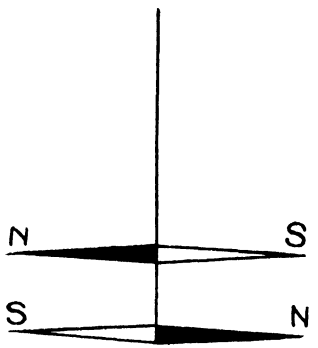


Fig. 28

we want that the earth should have no directive influence upon it. The usual method is to take a combination of two needles exactly similar in shape and size, of the same pole strength, fixed one above the other, in such a way that the north pole of the one is just above the south pole of the other, and similarly the south pole is above the north pole. If the pole strengths be equal, as has been supposed, the effect of the earth on the north pole of the one is counter-balanced by the effect of the earth on the south pole of the other. Such a pair of needles will stand in any desired position. It is difficult to make an exactly astatic pair, for the two needles cannot be easily magnetised to the same strength, and secondly, cannot be fixed parallel to each other with a very high degree of accuracy.

PART VII

CURRENT ELECTRICITY

CHAPTER I

496. It has already been explained that whenever two conductors at different potentials are connected together, a current of electricity flows from one to the other and during this act energy is always consumed, because there is always a loss of energy attendant upon such an exchange. It was also explained there that the loss of electricity appears as heat, etc. This current is a momentary one, because as soon as the potential is equalised, the current stops; but, if somehow or other this potential difference could be maintained, a continuous current would be obtained. It was Volta first who, towards the end of the eighteenth century, discovered a method of setting up and maintaining an electrical potential difference called after him a Voltaic cell. Till then the only source of electricity was a frictional machine, which was not only capricious in its working, but even allowed the charge to leak away. The Voltaic cell, on the other hand, furnished a steady source of electricity constantly available in all weathers, and requiring no special precautions to prevent its escape. Moreover, the electricity thus developed exhibited an entirely new set of phenomena, and opened up the way to such various and important applications that frictional electricity at once fell into the background.

497. Voltaic Cell.—Volta found that if two strips, one of copper (C) and the other of zinc (Z), be placed in a vessel containing dilute sulphuric acid and be connected by a wire (M), a current of electricity flowed continuously through the wire (Fig. 1); the direction of the current through this wire being from copper to zinc, *i.e.* from the higher to a lower potential. It is important to note that this is simply a convention—an accepted mode of speech, for we might with equal cogency

regard the current as a flow from a lower to a higher potential. Modern investigations have, as a matter of fact, established beyond doubt that an electric current consists of a transfer of electrons—negatively charged particles—from a lower potential to a higher potential. It is nevertheless convenient to stick to the generally recognised nomenclature, and speak of the current as flowing from a higher to a lower potential. Such a simple arrangement, as sketched above—a copper plate and a zinc plate placed in dilute sulphuric acid joined together by means of a wire, constitutes what is called a **voltaic cell.** There are three principal things in a cell as noted above—(1) a liquid called electrolyte; (2) a metal plate (like Zn) which is acted upon chemically by the electrolyte when the current passes through the cell; and (3) a metal plate which is not acted upon by the electrolyte. The last condition that one of the metals should be unaffected by the solution is not an essential condition, although it is best if it be so; still all that is necessary is, as we shall see later on, that the chemical action upon one be greater than upon the other. The plates are called **electrodes.** Before considering the explanation of such a cell, it is advisable to know that dilute sulphuric acid affects neither pure zinc nor copper. So long as the two plates are not connected with each other no action is observed. But as soon as the two plates are connected by a wire, bubbles are seen rising at the surface of the copper plate. If this gas be collected, it is found to be *Hydrogen.* Although no action is apparently observed at the surface of zinc, all the same it constantly dissolves with formation of zinc sulphate; and it is the energy liberated from this chemical reaction which maintains the current. Note, *it is this chemical action which is the source of electricity in this case.*

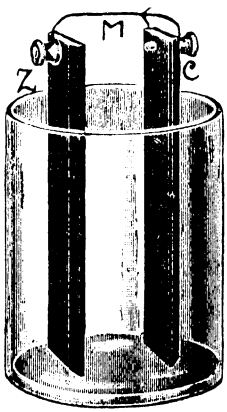


Fig. 1

498. Action of the Voltaic Cell.—Now let us try to understand how we get a current from the voltaic cell. When the plates of zinc and copper are immersed in dilute sulphuric acid, a difference of potential is set up between the two metals, and when they are connected together a current begins to flow.

That there is a difference of potential between the ends can be shown experimentally. The difference of potential is too small when a single cell is used to be demonstrated with an ordinary electroscope, but can be shown easily with a special type of electroscope called **condensing electroscope,** which is an ordinary electroscope, with the difference that the disc is rather large and is covered with varnish. An exactly similar plate, to which is fixed an insulating handle (like that of an electrophorus), is laid upon the disc of the electroscope and is earthed. Now the two plates together form a condenser with the insulating layer of varnish between them as a dielectric, and hence the capacity is increased, so that for a given difference of potential the electroscope takes a larger quantity of electricity. When the upper plate is removed the capacity decreases, and the same charge produces greater potential, and consequently a greater divergence.

So far we have been quite silent as to the question how this potential difference is brought about. There are two theories about it. We shall not enter into details; it will suffice for our purposes to say that out of the two theories—chemical and contact—the former has been credited with greater success. The contact theory—that mere contact of the metals brings about potential difference—forwarded by Volta was thrown into the background by Clausius, Ostwald, etc., who were staunch advocates of the chemical theory. We shall confine ourselves to the chemical theory. The briefest outline merely will be given here.

Nowadays it is supposed that the dilute solutions are dissociated, viz. some of the molecules are broken up into atoms or groups of atoms charged with opposite kinds of electricity. These charged atoms or groups of atoms are called **ions.** The number of molecules dissociated depends upon the strength of the solution: the weaker the solution the larger the percentage of the dissociated molecules. So we can look upon our acidulated water as split up to some extent into H_2 and O_2 atoms charged with opposite kinds of electricity. The H_2 atoms are supposed to be charged positively, and O_2 atoms negatively. Now, when zinc is dipped in water, the chemical affinity of the zinc for the oxygen causes the O_2 and Zn atoms to unite, the oxygen delivering up its negative charge to the zinc. Consequently the zinc plate becomes negatively charged, whereas the liquid becomes positively charged (due to the loss of the negative charge). As this action proceeds the surface of zinc becomes so much charged with negative electricity that the repulsion of O_2 atoms which are negatively charged becomes equal to the attraction due to

chemical affinity. When this is the case, the potential of the acidulated water with respect to the zinc is about +1·8 volts, or in other words, the potential of the Zn with respect to the electrolyte is −1·8 volts. If now a copper plate be placed in the same vessel of water, a similar action takes place at its surface; but since copper is chemically less active than zinc, the copper attains a potential of about −0·8 volt with respect to water. Since the potential of zinc with respect to water is −1·8 and that of copper is −·8, therefore *the resultant potential of copper is* +1·0 volt *higher than that of zinc*, and this is the potential difference between the poles of the cell.

Now suppose that the two plates are connected together. A current of electricity will evidently flow from the copper plate to the zinc plate through the wire; the copper plate will lose positive electricity and the zinc plate will gain it, which means, in other words, that the zinc plate will become less strongly charged with negative electricity, and therefore the equilibrium will be disturbed. It will consequently attract more oxygen in order to attain the equilibrium. Thus we see that in order to keep the Pot. difference the same more chemical action takes place, with the result that more zinc is consumed. Similar is the case at the copper plate. The hydrogen molecule which is set free when oxygen combines with zinc passes on from molecule to molecule of the electrolyte and at last appears at the copper plate.

499. Nomenclature.—The general nomenclature of the parts of the cell is rather confusing, and it is hopeless for any one to attempt to simplify it, because positive and negative are applied in diametrically opposite sense to the poles and plates (cathode, anode, etc.).

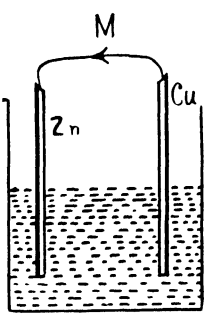


Fig. 2

It has already been said that the current flows through the wire from Cu to Zn. *The part of the copper plate outside the electrolyte is called the* ***positive pole*** of the cell, and *the part of the zinc plate outside the solution is called* the ***negative pole.*** The plate, or electrode as it is also sometimes called, on the whole is called ***positive*** *if it is acted on actively* (corresponding to zinc in the simple voltaic cell); it is called ***negative,*** *if it is unacted on or acted on less actively* (it corresponds to copper in our above-mentioned cell). The direction of the current in the

outer circuit is from the negative plate to the positive plate or from the positive pole to the negative pole (from Cu to Zn), whereas the direction in the inner circuit is from the positive plate to the negative, or from zinc to copper. Here we have used zinc and copper, but any two substances which are unequally acted upon by the electrolyte may be used as the electrodes. Thus we could use copper and platinum; in this case copper plate will behave as the positive and platinum plate as the negative. Similarly we can use zinc and platinum, zinc and carbon, and so on. The following list approximately indicates which of the two substances will become a negative plate and which a positive plate when the two are used in a cell. It should be remembered that the *one standing higher upon the list will become positive with respect to the one standing below*:

1. Zinc.	4. Copper.	7. Gold.
2. Lead.	5. Mercury.	8. Platinum.
3. Tin.	6. Silver.	9. Carbon.

The greater the distance between the two substances on the list the more efficient the cell formed; for instance, zinc and platinum is far more efficient than zinc and copper; but zinc and carbon is more efficient than zinc and platinum even. If we use gold and platinum we get almost no current.

The negative plate or where the current leaves is called the ***cathode,*** *and the positive plate, or where the current enters the electrolyte, is called the* ***anode.***

500. Local Action.—The consumption of zinc produces an electric current. If zinc is consumed without producing this useful current, it is wasted. When pure zinc is placed in dil. H_2SO_4, no action is noticed, but as soon as it is connected with another electrode, say of copper, action begins and H_2 gas is evolved. But, on the other hand, as soon as commercial zinc is dipped in it, bubbles are seen rising from its surface, which means that it is being consumed without producing any current. Now why should this commercial zinc give off H_2 even when it is not connected to another electrode? The reason is plain. It contains impurities, usually carbon and iron. And as soon as we dip the zinc plate in the acid, small cells are formed so to speak, with zinc and carbon or zinc and iron as electrodes, with the result that local currents are produced. These are of no use absolutely; they waste the zinc. This effect with the commercial zinc is termed *Local action.*

This can be avoided by amalgamating the zinc. In order

to do it, zinc is dipped in dil. H_2SO_4 to remove oxide, and then mercury is rubbed over the surface with a rag soaked in the dilute acid. The mercury rapidly spreads and combines with the zinc, forming a bright amalgam. Mercury is incapable of dissolving the impurities so that they are merely covered up. As the zinc is consumed the impurities fall down. Amalgamated zinc presents a surface of uniform composition to the electrolyte, and is not attacked so long as the circuit is not closed, *i.e.* so long as there is no current.

501. Polarisation.—So long as the chemical action goes on, the voltaic cell theoretically should continue to produce a current. But in practice if the strength of the current be observed, it is noticed that as time passes the strength of the current diminishes, and after some time it stops. The most important cause of the enfeebling of the current is due to the fact that the copper plate, or more generally the negative plate, becomes covered with a layer of hydrogen. This layer weakens the action of the cell in two ways—(1) the deposition of hydrogen * decreases the effective area of the plate and thereby increases the internal resistance of the cell; and (2) hydrogen, like zinc, is a readily oxidisable element and behaves in a cell in a manner similar to zinc, thus setting up a current in the opposite direction (H_2 to zinc inside the cell) to that sent by the cell (from zinc to Cu inside the cell). Owing to this, the resultant current produced by the cell is appreciably lessened. It also produces many other minor effects, as, for instance, of decomposing the zinc-sulphate in the liquid and causing a deposit of zinc on the copper plate. There are three methods of getting rid of this hydrogen; one is mechanical, the second is of oxidising it, and the third is of replacing it. We shall consider these methods one by one. There are three distinct types of cells in which these various methods are used.

502. I. Mechanical method of removing H_2.—The simplest method of removing the layer of hydrogen is to do it mechanically. Suppose we brush the copper plate every now and then, then there will be no layer of H_2 permanently deposited upon the plate, and therefore the cell will not become polarised. But to brush the plate repeatedly is a tedious task, therefore some device becomes necessary. Smee was the first to suggest that if the surface be made rough by depositing finely divided

* Note, the hydrogen being a non-conductor wherever the bubbles accumulate, that portion of the plate no more acts like a conductor.

platinum, these points will not allow hydrogen to accumulate upon them, but on the other hand will begin to discharge the gas quite freely. He makes use of this principle in the cell called after him.

Smee's Cell.—It consists of two amalgamated zinc plates on opposite sides of a silver plate covered with finely divided platinum. The silver plate is generally very thin, and is therefore supported by a cross-piece *E*. The two zinc plates (*A* and *B*) are connected together and form one pole, whereas the silver plate forms the second pole (Fig. 3). The liquid used is H_2SO_4 diluted with water. The action is similar to that of the simple cell. Smee's cell is not very satisfactory, for the current obtained from it is not constant.

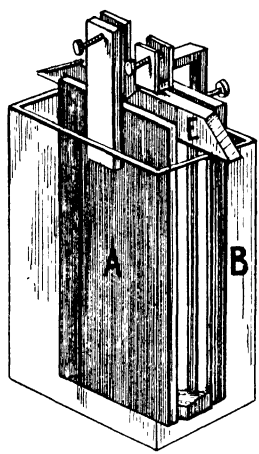


Fig. 3

503. II. Oxidising the Hydrogen.—The mechanical method of removing H_2 is not very satisfactory, but the oxidising method is quite good. The common cells which are based upon this principle are the following: Leclanche, Bichromate, Grove, and Bunsen.

(*a*) **Leclanche Cell.**—This cell consists of a glass vessel in which is placed a porous pot containing a *gas carbon* plate tightly packed round with fragments of the same gas carbon and *manganese dioxide*; the latter acts as a depolariser. The carbon plate projects outside the porous pot and has a lead socket attached to it, to which a binding screw is fixed. In the outer glass vessel there is a *zinc rod* immersed in a solution of *ammonium chloride*. When the cell is working, ammonium chloride attacks the zinc, forming $ZnCl_2$, and liberating hydrogen, which is slowly oxidised by MnO_2. The chemical reactions may be expressed thus:

$$2NH_4Cl + Zn = ZnCl_2 + 2NH_3 + 2H.$$

Ammonia escapes or dissolves in the solution, and hydrogen is oxidised by MnO_2 forming the lower oxide of manganese Mn_2O_3.

$$2MnO_2 + 2H = Mn_2O_3 + H_2O.$$

As is seen from these equations, the action of the cell depends upon the presence of MnO_2 which is being reduced to Mn_2O_3, so naturally the cell works so long as the whole of MnO_2 is not reduced, after which it ceases to work. Moreover, hydrogen is liberated more quickly than it is oxidised, so polarisation sets in and the current falls off. But when the cell is left to itself, the manganese dioxide becomes depolarised, and thus the cell again recovers its original strength. Hence

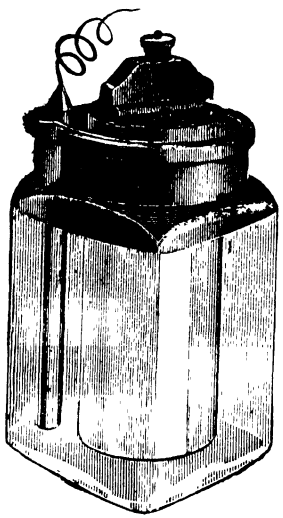


Fig. 4

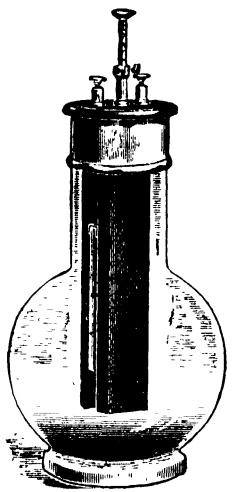


Fig. 5

it is very useful for intermittent currents such as are required for electric bells, telephones, etc.*

The electro-motive force of such a cell is about 1·45 volts. This cell possesses the following advantages: (1) no chemical action takes place so long as there is no current passing through it; (2) as there is only one kind of electrolyte, no diffusion takes place; (3) the recurring expenses are very low, for only the solution of ammonium chloride is to be changed occasionally.

(*b*) **Bichromate Cell.**—The glass vessel is usually of the form shown in Fig. 5. The positive pole is made up of two plates of hard carbon connected together at the top. The

* Commercial ammonium chloride should not be used, as the impurities it contains tend to increase the internal resistance of the cell. Generally the resistance of the commercial cell is from 4 to 5 ohms.

carbon used is a good conductor of electricity, and besides this, on account of its hardness, cannot be attacked by acids. The other plate is of zinc, and the electrolyte is a solution of potassium bichromate acidulated with sulphuric acid. The exact proportion of these substances is 100 grams $K_2Cr_2O_7$; 200 grams H_2SO_4, and 1200 grams water. The zinc plate is attached to a sliding rod by means of which it can be lowered into the solution when the cell is to be used. This is important and essential, for the zinc is attacked by the solution even when no current is passing through the cell, hence the desirability of withdrawing it from the liquid when not in use.

The chemical action in the cell is somewhat complicated. We can represent it approximately by the following equations :

The effect of adding H_2SO_4 is to form chromic acid as :

$$7H_2SO_4 + K_2Cr_2O_7 + H_2O = 2H_2CrO_4 + K_2SO_4 + 6H_2SO_4. \quad \text{(i)}$$

The action of H_2SO_4 on Zn is to form zinc sulphate and liberate H_2,

$$3H_2SO_4 + 3Zn = 3ZnSO_4 + 6H \quad . \quad . \quad . \quad \text{(ii)}$$

The chromic acid oxidises the hydrogen and is converted into chromic oxide and water

$$2H_2CrO_4 + 6H = Cr_2O_3 + 5H_2O. \quad . \quad . \quad . \quad \text{(iii)}$$

The chromic oxide so formed is acted upon by the excess of H_2SO_4 which is in the glass vessel according to

$$Cr_2O_3 + 3H_2SO_4 = Cr_2(SO_4)_3 + 3H_2O. \quad . \quad . \quad . \quad \text{(iv)}$$

In addition to the above-mentioned reactions there takes place a secondary reaction between the chromic sulphate of equation (iv), and potassium sulphate of (i)

$$K_2SO_4 + Cr_2(SO_4)_3 = K_2Cr_2(SO_4)_4.$$

This product of the secondary reaction is called chrome-alum. It is altogether a waste-product and it crystallises on the plates and weakens the cell. Hence in the modern form chromic acid is used instead of potassium bichromate. In doing this the chemical reactions represented by equation (i) are avoided and hence K_2SO_4 is not formed, and instead of chrome-alum we get chromic sulphate ; the chemical reactions are represented by equations (ii), (iii), and (iv). The E.M.F. is about 2 volts, but the current remains constant only for a short time and afterwards falls off. It is useful, however, when a strong current is required for a short time.

So far we have been dealing with single fluid cells only,

i.e. cells in which only one electrolyte is used. But now we shall begin to discuss the two fluid cells.

(*c*) **Grove Cell.**—It is a two-fluid cell and consists of two distinct vessels, an outer glass vessel and an inner porous vessel. The outer porcelain or glass vessel contains the zinc plate immersed in dil. H_2SO_4. The plate is bent round the inner vessel in which is placed a platinum foil immersed in strong nitric acid. The chemical reactions can be represented by the following equations :

$$Zn + H_2SO_4 = ZnSO_4 + 2H,$$

and

$$2H + 2HNO_3 = 2H_2O + 2NO_2.$$

Thus we see that in this case hydrogen is oxidised by the nitric acid, which itself is reduced to nitrogen peroxide. This

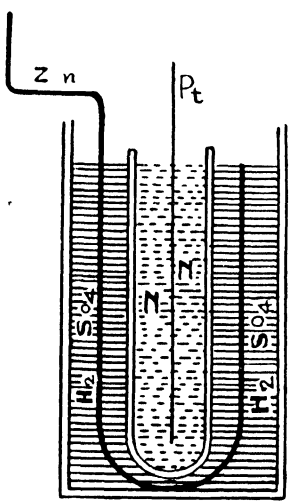


Fig. 6

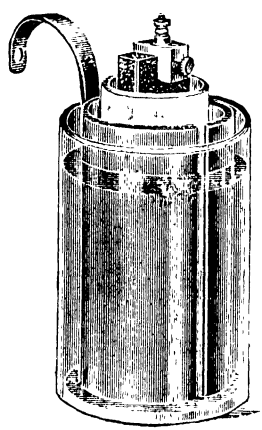


Fig. 7

gas is somewhat soluble in the acid, but if produced in larger quantities, it escapes into the air. The fumes of nitrogen peroxide are, however, very disagreeable. In addition to it the use of platinum makes it very costly.

It is very useful when strong currents are required. Its E.M.F. is fairly high ; it is about 1·9 volts.

(*d*) **Bunsen Cell.**—It is simply a modification of Grove cell. Instead of platinum foil, a plate of hard gas carbon is used.

The zinc plate is formed into a cylindrical form and is immersed in dil. H_2SO_4. Within this is a porous cylinder containing strong nitric acid in which is placed the plate of hard gas carbon. The chemical reactions are similar to those given in the case of Grove cell.

503 (a). III. Replacing Hydrogen.—We have learnt in § 501 that H_2 gets deposited over Cu plate and causes the current to fall down. Now let us suppose that the hydrogen which is liberated is made to pass through the solution of $CuSO_4$; it will form H_2SO_4 with it and leave Cu free, which will be deposited over Cu plate. When Cu is deposited over Cu, no difference in E.M.F. takes place. This is a very useful method, and it is made

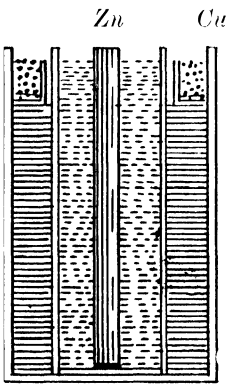


Fig. 8

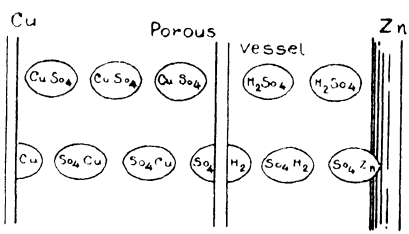


Fig. 9

use of in what is called **Daniell's cell** (Fig. 8). It consists of an outer cylindrical copper vessel which serves also as the positive pole. It contains concentrated solution of $CuSO_4$ which acts as a depolariser in the manner explained above. In this stands an unglazed earthen porous cell containing dil. H_2SO_4, in which is placed a rod of amalgamated zinc.

The action is very simple. In the porous cell hydrogen is liberated by the action of H_2SO_4 on zinc as soon as the circuit is completed or the current begins to pass through it and $ZnSO_4$ is formed. This H_2 attacks the $CuSO_4$ liberating Cu, which gets deposited over the Cu plate. How this process actually takes place can be explained in the following way :

The upper row (Fig. 9) shows the arrangement of molecules of H_2SO_4 and $CuSO_4$, when hydrogen is not liberated. The second row shows the arrangement when H_2 is liberated, *i.e.* when

current begins to flow. Free H_2 ion, which is just next to the porous wall, combines with the SO_4 ion displacing Cu since hydrogen has greater affinity for being oxidised than Cu. This free or liberated Cu ion combines with the next SO_4 ion, and this continues till the last Cu ion is deposited over Cu plate.

Sometimes zinc sulphate is used in place of H_2SO_4. This removes the necessity of amalgamating zinc, and thus the cell becomes more convenient. The chemical reactions are quite simple. Supposing by capital ZN we represent the zinc molecule of the zinc rod and by Zn the zinc already present in the solution, we can express the reactions as

$$ZN + ZnSO_4 = ZNSO_4 + Zn,$$
$$Zn + CuSO_4 = ZnSO_4 + Cu.$$

Thus as before we see that Cu is deposited over Cu. In this case the zinc sulphate has no direct action upon the metallic zinc except when the current is passed through the cell. The chemical action producing the current in this case is the tendency in the presence of free zinc to replace Cu by the more stable compound $ZnSO_4$—an action which evidently can take place where the two liquids meet (in the pores of the porous vessel). It should be noted that the E.M.F. of the cell is greater when the zinc sulphate solution is unsaturated.

But, on the other hand, the E.M.F. of an ordinary Daniell cell is greatest when $CuSO_4$ solution is saturated, and in order to get maximum voltage the $CuSO_4$ solution must be kept saturated. It is done by placing the $CuSO_4$ crystals over the perforated shelf with which all Cu vessels are provided.

The E.M.F. of such a cell is usually about 1 12 volts.

When the cell is not in use, the solutions must not be left in the cell, for otherwise $CuSO_4$ passes through the porous pot on account of diffusion and is decomposed by the zinc, with the result that Cu is deposited on the zinc rod which causes local action.

Its great constancy in E.M.F. and cheapness of its first cost and of its solutions have made it the battery which is most often used in laboratories and telegraph offices.

504. Dry Cells.—The cells which we have explained so far all contain large quantities of electrolytes, and hence they are not easily portable. To facilitate portability we use dry cells. In reality they are by no means dry; *as a matter of fact the very success of all these cells depends upon the contents being kept moist.* Generally the electrolyte disseminates through some solid material through which it can diffuse. Plaster of Paris and

gelatinous compounds are often used as the *solid material.* The usual construction can be explained in a few words as follows :

The outer cup is made up of zinc and acts as the negative pole or positive plate, over this zinc is slipped a cover of some insulating material so that when the cells are touching each other connection may not be set up between them. The positive pole is a plate of carbon. Both the zinc and the carbon carry binding screws. The intervening space is filled in various ways depending upon the make. In order to understand the underlying principle it will be enough to remark that in one make the paste of ammonium chloride solution and of plaster of Paris is used. In another type ammonium chloride, zinc chloride, plaster of Paris and water are used. This shows that more or less the dry cells are modifications of the Leclanche cell already explained.

CHAPTER II

THE ELECTRIC CURRENT

505. We have already spoken of the electric current flowing through a wire which joins the two poles of a voltaic cell. In the cell itself the existence of the current is indicated by the chemical action which is going on there, but how to detect the current in the outer wire or more generally in the outer circuit ? There are certain characteristic properties which a wire carrying a current is found to possess, and it is these properties which enable us to detect the current. The most striking property of the current is the existence of a magnetic field in the space surrounding the current-carrying conductor. Also the wire becomes hot without any apparent application of heat to it. Further, if the two ends of the wire instead of being joined together be dipped in a beaker containing acidulated water, bubbles of the gas are seen rising up at the two ends. We shall consider each of these effects in a somewhat greater detail later on ; at present we shall confine ourselves to the first one.

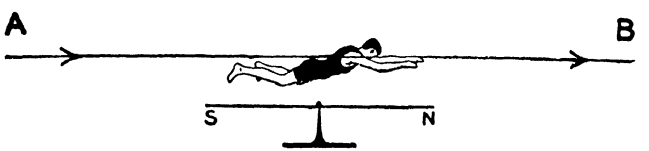


Fig. 10

506. Magnetic Effects of the Current.—It was O'erstead of Copenhagen who first observed that there was a close relationship between magnetic and electrical phenomena. He noticed that when a wire carrying a current was stretched horizontally over a magnetic needle pivoted at the centre, the needle moved to one side as if it tried to set itself at right angles to the wire.

O'erstead's Experiment.—Let *NS* represent the magnetic needle pivoted at the centre, Fig. 10. Hold the wire carrying current above it, the current flowing from *S* to *N*. The north

pole of the needle will be noticed to deflect towards the west. Hold the same wire below, the needle will deflect in the opposite direction, viz. the north pole will deflect now towards the east. Reverse the direction of the current through the wire, so that it now flows from north to south. Hold the wire just above the needle parallel to it, the north pole will move towards the east this time, and when the wire is stretched below the needle, the north pole will move towards the west.

The results need not be committed to memory, they can be easily deduced from **Ampère's rule:**

"*Imagine a man to swim along the wire in the direction of the current, so that it is entering his feet and leaving him at his head, with his face towards the needle, the north-seeking pole of the needle is deflected towards his left hand.*"

Or, remember the word **SNOW.** Interpret it as **S**outh to **N**orth, wire **O**ver the needle, deflection to the **W**est. Remembering this the connection between the deflection, the position of the wire, and the direction of the current can be easily deduced.

If the connection be broken and the same wire be stretched over the needle no deflection is observed, which shows that it was the current which was responsible for the deflection. That there is a magnetic field round a wire carrying current can be shown in a striking manner in the following way. Pass the wire through a piece of cardboard, and hold it vertical. Sprinkle iron filings on the cardboard, the filings arrange themselves in circles round the wire; when the latter carries no current the filings do not arrange themselves in any particular manner, they rather keep on lying as they are thrown. But we have already seen that the filings arrange themselves along the magnetic lines of force; keeping this in mind we come to the conclusion that the wire carrying the current is surrounded by the magnetic lines of force, which are circular. The direction of these lines of force, *i.e.* the direction in which the north pole would be urged, bears a definite relation to the direction of the current. We have already learnt Ampère's rule which gives the direction in which a north pole would be urged. This rule, however, is not always convenient to use. The following rule is sometimes more convenient to use:

"*Imagine an ordinary right-handed screw to be placed along the wire conveying the current with its axis coinciding with the direction of the current, then if the screw is twisted forward, it travels onwards in the direction in which the current flows, and*

the direction in which the thumb rotates is the direction in which the north pole tends to move round the wire." It is called the **corkscrew rule.** When a coil of wire is used instead of a single wire, the lines of force are represented in Fig. 11. Here the two ends of the stout copper wire are passing through two holes in the cardboard. The wire is made into a circular loop, one-half of which is above the cardboard, and the other half below it. When a current is passed through such a loop, and the iron filings are sprinkled on the cardboard, they arrange themselves in circles round each end of the wire. The lines

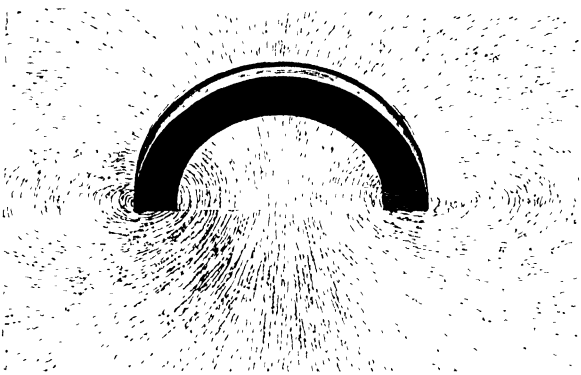


Fig. 11

running near the centre are practically straight and normal to the plane of the circular loop. It is a very important result; it means that near the centre the *magnetic field is almost uniform and is perpendicular to the plane of the coil.* This fact is made use of in the construction of a tangent galvanometer.

507. The Polarity of the Faces of a Coil carrying current.— We have read so far that around the wire carrying current there exists a magnetic field; now we shall see that a coil or a solenoid carrying current behaves just like a bar magnet with its one face as the north pole and the other face as the south pole. This can be shown very easily by what is called *De La Rive's floating battery.* It consists of a flat coil of thin insulated copper wire of 20 to 30 turns. The ends pass through a large cork and are connected below to copper and zinc plates. The cork fits into the mouth of a beaker. Filling the beaker with dilute H_2SO_4, and immersing the plates therein, the current

passes round the coil. If the beaker be floated on water in a trough, on bringing a magnet near it, it will be noticed that one pole of the magnet attracts the battery, whereas the other pole repels it. If the direction of the current be studied, it will be noticed that the face over which the direction of the current is clockwise behaves as the south pole, and the face over which the current is flowing in a counter-clockwise direction behaves as the north pole. What is true of a flat coil is also true of a solenoid. In this case one end behaves as a north pole and the other end as a south pole.

In order to know the kind of polarity of the face of a coil or a solenoid, the following rule is very useful:

Look at the end of a solenoid or the face of a coil, if the direction of the current is ***clockwise, the end or face is a south pole; but if the direction of the current is counter-clockwise, the end or face is a north pole.***

508. Galvanoscope.—When a wire carrying an electric current is stretched horizontally over a magnetic needle, the needle is deflected from the magnetic meridian. We have already learnt that the direction of deflection of the north pole of the needle depends upon the direction of the current in the wire; if the direction be reversed, the direction of the deflection also is reversed. When the wire is placed below the needle the deflection is in the opposite direction to that obtained when it is placed above. But if the direction be reversed in addition to placing it below the needle, the deflection is in the same direction.

Let us study the effect of a rectangular loop carrying an electric current on the needle *NS*. The north pole of the needle *NS* due to the current in *dc* will move towards the west; * *i.e.* the needle will come into the position *N'S'*. Now let us see the effect due to the current in *cb*. Suppose a man to be swimming along *cb*, with his head towards *b* and face towards *N*, the N pole will move towards his left, *i.e.* towards the west. Let us pass on further to the lower wire *ba*, the deflection of the north pole due to the current in this part also will be towards the west. The same will be the direction of the deflection of the north pole of the needle due to the current in *ed*. Thus we see that the deflection due to the current in the loop is much stronger than the deflection produced by a single wire. If a large number of rectangular loops be taken the effect becomes very much stronger. Generally the current is too weak to be detected

* To find the direction in which the north pole will move apply Ampère's rule.

by the deflection of the magnetic needle when a single wire is used, therefore to make the deflection appreciable, the current is made to pass through a large number of coils at the centre of which a magnetic needle is pivoted. Such an instrument is called a **galvanoscope.** As the name implies, it means simply an instrument which serves to show or detect the presence of a current. In order to measure quantitatively the strength of a current we make use of an instrument called a galvanometer. But before we deal with the comparison of the strengths of currents we must know what we mean by the expression "strength of a current." When two points at different potentials are connected together by a conductor, electricity flows from one to the other, which we express by

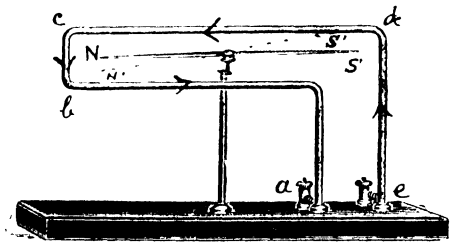


Fig. 12

saying that a current of electricity flows from one to the other. Now either a large number of units of electricity might flow from one point to the other in some definite period, say a second, or a small number of units. In the first case we say that the current is strong, and in the second case that it is weak. Thus we see that the *strength of a current flowing through a wire is measured by the quantity of electricity that flows across any section in unit time.* In other words, it is simply the rate of the flow of quantity of electricity. Supposing that Q units of electricity flow across any section of the wire in t seconds, then the strength of current is $\frac{Q}{t}$ units. When Q is 1 unit and t is one second the strength of the current is also equal to one, and this is taken as the unit of a current. Evidently the magnitude of this unit of current depends upon the magnitude of the unit of Q used. In practice Q is measured in coulombs and not in electrostatic units. When this is so done the unit of current is called an **ampere,** after the name of the celebrated French scientist. *Thus an ampere is the*

strength of a current when 1 *coulomb of electricity flows through the conductor in one second.* If C coulombs pass in one second, then the strength is C units.

509. We have seen that around the wire carrying an electric current there exists a magnetic field, which is proportional to the current; the stronger the current through the wire the stronger the field produced by it at a definite distance. As we know how to measure the strength of a magnetic field, we can compare the two currents by their magnetic fields, keeping other circumstances constant. This gives us a new unit for expressing the strength of an electric current. Let us consider it a little more in detail. One point we have already noticed, that the strength of the magnetic field which accompanies a current is proportional to its strength. But we have not studied so far other factors upon which the magnetic field depends. We shall not go into rigorous mathematics, but try to get simply an idea of the various factors which affect the magnetic field.

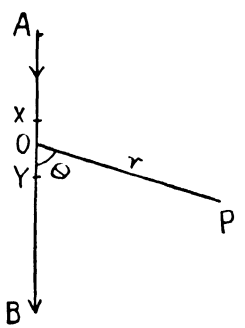


Fig. 13

Suppose AB to be the linear conductor along which the current is flowing, and P to be the point at which we want to determine the intensity of the magnetic field. Let us consider a small element XY of the wire. If it be very small, the direction of OP, where O is the middle point of XY, will be approximately the same as of XP or YP. If the length of XY be denoted by l, the angle between OP and XY or AB by Θ, the current passing through the wire by C, the intensity of the magnetic force at P due to the current in the element XY is represented by the formula

$$H = \frac{Cl}{r^2} \sin \Theta, \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{(i)}$$

where $OP = r$. It should be clearly understood that since Θ is the angle which OP makes with XY, if XY possesses an appreciable size, the angles which AB makes with XP, OP and YP will be different, and our formula will not hold good. It holds good only so long as the angle with AB made by a line joining any point lying on the element XY with P is the same, and that will be true when the element is infinitely small. Now it is clear that in the case of the wire AB each

element into which it can be divided will have different values for r and Θ, and therefore the intensity of the magnetic field produced will be different for each element. If we want to know the total intensity of the magnetic field due to the whole of the wire, we must add up the values of intensities contributed by each element. This is beyond the scope of the present book. But we can, however, consider a special case, in which r is the same, and so is Θ for the various elements. This will be true when the wire is bent into a circular form and the point at which we want to determine the intensity is the centre of the circle. Θ is almost 90° in this case, and r is equal to the radius. (See the two radii drawn in the ⊙ Fig. 14. They make an angle of 90° with the elements, and the distance r also remains the same).

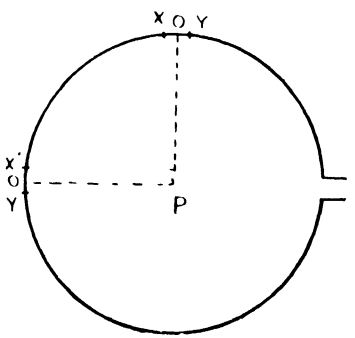


Fig. 14

Let us suppose that we divide the circumference of the circle into m equal elements, each element being indefinitely small. If the length of one element be l, the magnetic field due to it will be given by the formula

$$\frac{Cl}{r^2}\sin\Theta=\frac{Cl}{r^2},\ \text{since}\ \sin\Theta=1,\ \Theta\ \text{being}\ 90°.$$

For all these elements together, the field

$$F=\frac{mlC}{r^2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{(ii)}$$

But ml = circumference $= 2\pi r$

$\therefore$ total intensity of the magnetic field at the centre of a wire bent into a circle

$$=\frac{C\times 2\pi r}{r^2}=\frac{2\pi C}{r}. \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{(iii)}$$

If the current is flowing in a coil of n turns instead of a single loop, then the magnetic field at the centre will be

$$F=\frac{2\pi nC}{r}. \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{(iv)}$$

Let us go back for a moment to formula (ii), that the

magnetic field at the centre $= \frac{Cml}{r^2}$, where ml is the total length of the wire, and r is the distance of the centre. Suppose that the field at the centre is 1 unit, viz. the force on a unit pole is 1 dyne, and that $ml = 1$ cm., and further that $r = 1$ cm., it is clear that C is also equal to 1. Thus we have arrived at another unit of a current. The unit of a current according to this method is "*that much current which flowing in a unit length of an arc of a circle of* 1 *cm. radius exerts a force of* 1 *dyne on a unit pole at the centre.*" This unit is called the absolute or electro-magnetic unit of a current, and it is equal to 10 amperes, the unit which is used actually in practice.

510. Tangent Galvanometer.—Formula (iv) tells us that if there be n turns in a coil, and C be the current in each coil, and r be the radius of the coil, the magnetic field at the centre of the coil will be equal to $\frac{2\pi nC}{r}$; its direction will be perpendicular to the plane of the coil (see § 506, Fig. 11). If at the centre of the coil where the field is uniform there be placed a magnetic needle, it will be acted upon by the magnetic force due to the coil, but at the same time the needle will be acted upon by the horizontal component of the earth's field. If, further, the coil be placed in the magnetic meridian, the two forces will be at right angles to each other, since the force due to the earth will be parallel to the coils, but due to the current in the coils the force will be perpendicular to them. But we know that when two fields at right angles to each other are acting upon a needle it comes to rest according to the formula

$$F = H \tan \Theta,$$

where Θ is the angle which the needle makes with the direction of H; F and H being the two fields. Let us suppose that H denotes the earth's field, and Θ the angle which the needle makes with H, then F will be the force due to the current, and will be equal to $\frac{2\pi nC}{r}$.

Substituting this value of F, the relation $F = H \tan \Theta$ will be reduced to

$$\frac{2\pi nC}{r} = H \tan \Theta,$$

or

$$C = \frac{rH}{2\pi n} \tan \Theta. \quad . \quad . \quad . \quad . \quad . \quad (v)$$

Formula (v) tells us that the current is proportional to the

tangent of deflection produced, and hence the name *tangent galvanometer* given to the instrument.

It consists essentially of a coil of insulated wire wound on a vertical wooden ring with the ends connected to the binding screws at the base, by means of which the coil can be connected to a battery (Fig. 15). Actually there are three distinct sets of coils of insulated wire having their ends connected to terminals marked 2, 50, and 500, which give the number of turns in each set. The first is used when a strong current is in action, the second for a moderate current, and the third for a weak current. The coils can be rotated about a vertical axis to enable us to bring them into the magnetic meridian. At the centre a small magnetic needle is pivoted delicately on a vertical support. The needle is free to turn in a horizontal plane. It carries at right angles to itself a long aluminium pointer, the ends of which move over a graduated scale. The needle, pointer, and the scale are all enclosed in a small cylindrical box. In addition to all these a sheet of mirror glass is fixed below the pointer to remove the error due to parallax. The base of the instrument is provided with three levelling screws.

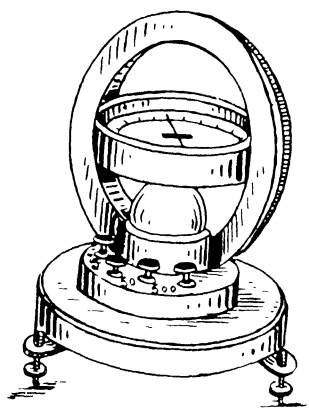


Fig. 15

To use the galvanometer, at first the base is levelled to ensure that the needle is swinging freely, and then the coil is turned into the magnetic meridian so that the magnetic needle lies in the plane of the coil (and hence the magnetic meridian). If the pointer be fixed at right angles to the needle, it will read zero, zero on the scale, and the needle 90°, 90°. For detailed adjustments the student should refer to some book on Practical Physics. When the current is passed round the coil, the field produced is uniform near the centre only, and hence the desirability of having a small magnetic needle, so that it may lie in a uniform field. The needle is deflected from the magnetic meridian due to the magnetic field of the current and comes to rest when

$$\frac{2\pi nC}{r} = H \tan \Theta,$$

as explained already, where Θ is the angle made by the needle with H or by the pointer with 0° and 0° line,

or $$C = \frac{Hr}{2\pi n} \tan \Theta.$$

Writing $\frac{1}{G}$ for $\frac{r}{2\pi n}$, we get

$$C = \frac{H}{G} \tan \Theta.$$

G or $\frac{2\pi n}{r}$ depends simply upon the construction of the instrument, and hence it is called the **galvanometer constant.** If we know r and n we can find out its value.

Writing K for H/G we get

$$C = K \tan \Theta.$$

The constant K is called the **reduction factor** of the galvanometer. If we know the value of H, the horizontal component of earth's field at any place, and further, if we know the values of r and n, we can find out the value of the current by noting simply the deflection and then finding its tangent. The value, however, will be given in absolute units. If we want the value in amperes, we must multiply by 10, because ampere is $\frac{1}{10}$th of the absolute unit.

Thus $C = 10K \tan \Theta$, if the current be measured in amperes.

511. It should be noted here that when there are a good many turns in a coil, say 500, the value of r will not remain the same for all the coils, and if considerable accuracy is required, this fact must be taken into consideration, and the mean value of r should be used.

Precautions.—The same precautions should be taken in making the observations with the tangent galvanometer as are taken in the magnetometer.

(i) Parallax.—It must be avoided by placing the eye vertically above the needle, the eye is moved until the pointer hides its image reflected from the mirror, and in this position the reading is taken.

(ii) The point of suspension (pivoting) may not coincide with the centre of the circular scale.

To avoid the error due to this both ends of the pointer must be read; one end will give too high a deflection and the other too low—the mean being the correct deflection.

(iii) The magnetic axis of the needle may not coincide exactly with the plane of the coil.

This want of symmetry is remedied by taking four readings—two readings corresponding to the two ends of the pointer when

the current is sent in one direction and two when the current is reversed. The mean of these four will give the correct deflection.

(iv) In making observations the deflections should neither be too great nor too small. It is very undesirable to have deflections smaller than 15°, and greater than 75°. It is best if the deflection be somewhere near ***45°—because in that position the galvanometer is most sensitive.***

512. We can find out mathematically the position of the greatest accuracy. Let us suppose that when a current c is sent through the coils of the galvanometer, deflection θ is produced. Now increase c to $c+dc$ where dc is very small, and let the deflection become $\theta+d\theta$. We want that for dc, the change in deflection $d\theta$ should be as great as possible.

We know that $$c = K \tan \theta. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (a)$$

Differentiating (a) with respect to θ we get

$$\frac{dc}{d\theta} = K \sec^2 \theta,$$

or $$dc = K \sec^2 \theta d\theta. \quad . \quad . \quad . \quad . \quad . \quad . \quad (b)$$

Divide (b) by (a) and we get,

$$\frac{dc}{c} = \frac{\sec^2 \theta}{\tan \theta} d\theta,$$

or $$dc = c \frac{1}{\cos^2 \theta} \frac{\cos \theta}{\sin \theta} d\theta$$

$$= c \frac{1}{\cos \theta \sin \theta} d\theta$$

$$= c \frac{2}{\sin 2\theta} d\theta.$$

Now, so that corresponding to dc, $d\theta$ may be as large as possible, the coefficient $2c/\sin 2\theta$ should be as small as possible. The smallest value permissible of the coefficient is when $\sin 2\theta$ is greatest, and it is so when $2\theta = 90°$ or $\theta = 45°$.

Thus we see that, corresponding to a small increment in current, the greatest change in deflection is produced when the deflection is 45°; supposing the accuracy is represented by 100 when the deflection is 45°, and its value is 50 per cent only when the deflection is 15° and 75°. With angles smaller than 15° and greater than 75° the accuracy is less than 50 per cent even. To get satisfactory result the deflection should lie between **30°** and **60°**.

513. To make the Galvanometer sensitive.—Our object is that even for a small current the deflection should be appreciable. The easiest plan is to magnify the scale. This can be done in two ways—one way is to increase the size of the pointer —the longer the pointer, the smaller the angle it can indicate.

But it is not very convenient to have a pointer very long, because the apparatus becomes unmanageable. The second method is to fix a mirror to the magnet and allow a beam of light to fall upon it. The reflected beam is allowed to fall upon a scale at a distance of generally 1 metre. When the mirror moves through a certain angle the reflected ray moves through double that angle. In this way the deflection can be magnified. With this arrangement even a very slight movement of the magnet can be detected.

Note, the beam of light is equivalent to a pointer of length equal to double the distance between the mirror and the scale. To understand it, let us suppose β is the angle through which the spot of light moves on the scale, and α the angle through which the mirror moves. Evidently $\beta = 2\alpha$ (for reflected ray moves through double the angle through which the mirror moves). Further, if β and α be both small, as is usually the case, they can be measured by their tangents. Hence the relation $\beta = 2\alpha$ reduces to

$$\tan \alpha = \tfrac{1}{2} \tan \beta = \frac{\text{deflection of spot of light on scale}}{2 \times \text{distance of the scale from mirror}},$$

which shows that the effect is the same as if a pointer of length equal to double the distance between the mirror and scale were used.

The optical method of measuring the deflection possesses the following advantages :

(*a*) The reflected ray moves through double the angle through which the mirror moves, hence the deflection is doubled.

(*b*) The beam of light is much longer than the mechanical pointer, and hence the deflection is magnified to a greater extent.

(*c*) There is no parallax, as the image directly falls upon the scale.

Consider formula (v)

$$C = \frac{Hr}{2\pi n} \tan \Theta.$$

This shows that, so that $\tan \Theta$ may be as large as possible for a given current C, the coefficient of $\tan \Theta$, *i.e.* $\frac{Hr}{2\pi n}$, should be as small as possible. If this be the case the galvanometer will be very sensitive, because even for a small current it will produce great deflection. Now, so that $\frac{Hr}{2\pi n}$ should be as small as possible, n should be very large, H should be small, and r also should be small. This shows that increasing the number of

coils and decreasing the radius tends to greater sensitiveness. But it should be borne in mind that there are other considerations (resistance, etc.) which prevent us from making n very large, and hence to increase the sensitiveness we cannot depend upon n beyond a certain limit. *The best plan is to make H the smallest possible,* because r also cannot be made very small, otherwise the magnetic needle will not lie in a uniform field and our formula will not hold good. We can make H very small *by placing a magnet near the galvanometer in such a way that it tends to produce the field in the opposite direction to that of the earth.* We shall now consider some other types of galvanometers.

514. The Mirror Galvanometer.—Essentially it consists of a coil of wire wound over a small cylinder, a small magnetic needle, and a mirror. In addition to these, it has an auxiliary magnet to control the field, and which is called, therefore, the controlling magnet. The coil is wound over a small reel which is enclosed in a brass box with a glass front. The magnetic needle is very short and light, and consists of a magnetised strip of steel, attached to the back of a concave mirror. This is suspended by an unspun silk fibre. The deflection is measured by a lamp and scale arrangement. The length of the wire used in the coil depends upon the purpose for which the galvanometer is intended; in some cases the coil consists only of a few yards of stout wire, while in others it consists of several miles of the very finest wire. The wire is carefully insulated by silk covering, and is afterwards soaked in melted paraffin, which, on hardening, forms an excellent insulator.

How to use the Galvanometer.—When the controlling magnet is not used, the needle sets itself in the magnetic meridian, and also the coils must be adjusted in the magnetic meridian. It will simply act as a tangent galvanometer. Its sensitiveness cannot be changed.

But when the controlling magnet is used, let us see what will be the result. If the magnet be placed in the magnetic meridian, so that its south pole is towards the north, the field due to it will be in the same direction as that of the earth, and hence to move the needle out of the magnetic meridian we shall require a greater force than would be necessary in the absence of the controlling magnet. The sensitiveness of the galvanometer will be diminished in this way. But now let the magnet be placed in the opposite way so that its north pole is towards the north, the field due to it is in opposite direction to that of the earth, and by varying the distance the field due to the

earth can be neutralised at the centre of the coil. In this case the galvanometer will be very sensitive, even a very small current will deflect the needle. Fitted with a lamp and scale arrangement this galvanometer is extremely sensitive.

515. Astatic Galvanometer.—In this instrument an astatic pair of needles is used. As explained in § 496, an astatic pair of needles consists of two magnetised needles equal and similar in every respect, with their poles turned in opposite directions. The two needles are rigidly connected together with their axes parallel (Fig. 16). On account of the opposition of the poles, the directive action on the system is very feeble. If the magnetic moments of the needles be exactly equal, the system would remain in any azimuth, and when placed in any uniform magnetic field would have no tendency to take up any definite position; but in practice there is always some outstanding magnetic moment, with the result that the earth's field does produce its effect upon the system, but it is very slight indeed.

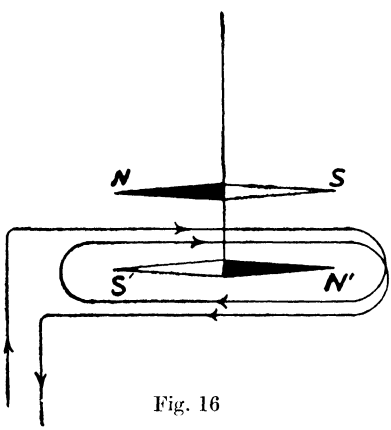


Fig. 16

When such a pair is used for a galvanometer, the coils are made to surround one needle only, the other is placed outside them and is generally placed above (Fig. 16). The magnetic action of the current affects mainly the enclosed needle, but there is some effect on the outside needle as well. The effect of the current in the coil on NS and $N'S'$ will be to move the system in the same direction, which can be seen at once by applying Ampère's rule. To be more exact, the effect of the whole of the coil on the lower needle and of the upper part on the upper needle will be to deflect the system in one direction, but the effect of the lower part of the coil on the upper needle will be to deflect the needle in the opposite direction; the last-mentioned effect is very slight, due to a comparatively large distance between the lower portion of the circuit and the upper needle. Thus on the whole a greater deflection is produced than would be the case if only one needle were used. The actual

advantage of using the astatic pair is, however, not because of this on the whole increased deflection, but, on the other hand, it is due to *the fact that the controlling force or couple due to the earth's field upon the system is very small*, and hence even a feeble current gives a considerable deflection.

How to use an Astatic Galvanometer.—We shall explain very briefly the method of using it. See Fig. 17. The instrument

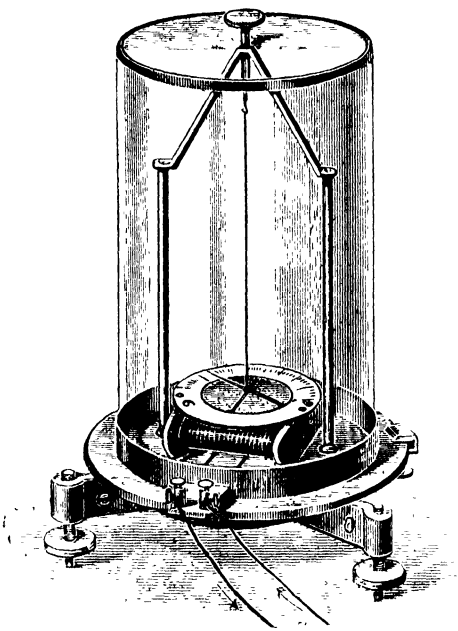


Fig. 17

is provided with three levelling screws. When the galvanometer is levelled the needles swing freely, and are brought to the centre of the scale. The upper needle carries a pointer, which moves over the scale. The coils are capable of rotation, and they are rotated till the diameter* of the coils is in the magnetic meridian. When it is adjusted, the pointer should read zero, zero. The instrument is only meant to detect, and not to measure currents; it is rather a sensitive galvanoscope

* Corresponding to the diameter is a slit, so that it is quite easy to see when the diameter is in the magnetic meridian.

than a galvanometer. In order to measure currents it must be calibrated. We shall describe the construction of a moving coil galvanometer later on in § 563.

515(a). Commercial Instruments.

(i) **Ammeter.**—It is a short name of an ampere-meter. It is a special kind of (*low resistance*) galvanometer which is so calibrated as to read the strength of the current directly in amperes (or milliamperes). It is used when comparatively large currents are to be measured.

(ii) **Volt-meter.**—It is a kind of (*high resistance*) galvanometer which is so calibrated as to read directly the potential difference in volts or millivolts between the points to which the terminals of the instrument (volt-meter) are connected.

CHAPTER III

RESISTANCE

516. We know already that whenever two points having a difference of potential are joined together by a wire, a current flows through it, but we do not know the relation between the potential difference and the strength of a current produced.

It was Ohm in 1827 who first found that the strength of the current of electricity flowing through a wire is directly proportional to the potential difference existing between its two ends; in other words, he found that when the potential difference is doubled the current is doubled, and so on. This is called Ohm's law. It may be stated thus: *The ratio of the potential difference between any two points on a conductor to the current flowing between them is constant, so long as the physical state of the conductor (as temperature, etc.) does not change.*

The same relation can be expressed in other words as

$$\frac{\text{P.D.}}{\text{current}} = \text{constant.}$$

This constant is called the **resistance** of the conductor. This relation is written in a more general form as

$$\left(\frac{E}{C} = R\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where E stands for the electromotive force or the potential difference, C for the current, and R for the resistance of the conductor. The resistance of a conductor does not depend upon the strength of the current which is flowing through it, but only upon its size and shape, on the material of which it is composed, and also upon its physical state as temperature, etc.

Ohm's law can be stated in any one of the five forms:

$$\frac{E}{C} = R, \quad . \quad . \quad . \quad (a) \qquad\qquad C = \frac{E}{R}, \quad . \quad . \quad . \quad (b)$$

$E = CR$, . . (c), $\frac{1}{C} = \frac{R}{E}$, . . (d) $\frac{C}{E} = \frac{1}{R}$. . . (e)

The first three are more often used than the last two.

Let us consider the relation $C = \frac{E}{R}$. This shows that if the difference of potential between two points of a circuit be kept constant, the current through them varies inversely as the resistance of that part of the circuit. If the resistance be decreased, the current increases, but if it be increased, the current diminishes. *The resistance of the wire, so to speak, opposes the flow of electricity.*

Resistance, like other quantities, is measured in terms of a proper unit, called an **ohm.*** It is clear from formula (*a*) $\frac{E}{C} = R$, that if E is one volt and the current flowing through the conductor is one ampere, the resistance is equal to one ohm. It is the resistance of a column of mercury 106 cm. long 1 sq. mm. in cross-section at the temperature of melting ice. Perhaps the student will be able to understand the magnitude of an ohm better if he remembers that it is the resistance of 67·6 metres of No. 18 S.W.G. copper wire (1·22 mm. diameter). If the potential difference be expressed in volts and the current in amperes, resistance is expressed in ohms. For instance, if the E.M.F. acting round a circuit is 10 volts and the current flowing through that circuit is ·5 ampere, the resistance is 20 ohms.

517. Resistance of Wires.—Fit up a cell, and connect it to the low resistance coil of the tangent galvanometer, through say 2 metres long No. 30 S.W.G. iron wire. Note the deflection in the galvanometer. Replace the iron wire by a copper wire having the same length and cross-section, the deflection will be quite different from that in the first case. Use manganin, eureka, or other wires, and it will be noticed that in each case the deflection is different, although the E.M.F. or the potential difference remains the same. But the deflection of the galvanometer is proportional to the current passing through the coils, hence the current in each case is different. As $C = \frac{E}{R}$ by Ohm's law, hence, when E remains the same and C is different, R must be different. Therefore we come to the

* Ohm is the practical unit of resistance. The absolute or C.G.S. unit is very small. Remember 10^9 C.G.S. units = one ohm.

conclusion that the resistance offered by these wires is different. And since their dimensions (cross-section and length) are the same, we see that *the resistance of a wire depends upon the nature of the material.*

Now let us study the effect of changing the length of a particular wire, instead of replacing it by a wire of another material. It is found that the longer the wire used between terminals AB, Fig. 18, the smaller the deflection obtained, and since a small deflection indicates a feeble current—hence we find that *the longer the wire the greater the resistance.*

The current is in like manner weakened by introducing a fine instead of a stout wire. As a matter of fact, the greater the cross-section of the wire, the less the resistance. It is

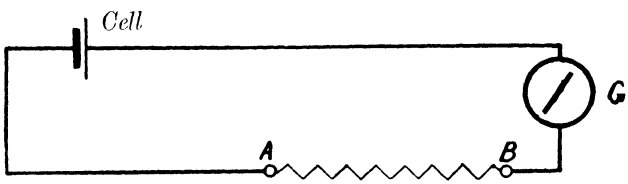


Fig. 18

what is just expected when a flow is taking place through any substance. Consider for simplicity the flow of water through two pipes, supposing that the cross-section of one pipe is double that of the other. Evidently we can regard the bigger pipe to be replaced by two small pipes. This means that we have provided two paths for the current instead of one, and naturally the flow will become double. What is true of the current of water is also true of the current of electricity. In short, it is found that the *resistance varies inversely as the cross-section of the wire.*

To sum up, the resistance of a wire can be written as

$$R = K\frac{l}{a} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

where R is the resistance of the wire, l the length, a the cross-section, and K the constant depending upon the nature of the material. It is called the **specific resistance** of the material. If both l and a are equal to unity, $R = K$. Thus we may define the *specific resistance as the resistance of a wire, one centimetre long and one sq. cm. in cross-section,* or, more generally

as the *resistance offered by a centimetre cube of a substance.* It should be noted that the specific resistance depends upon the physical condition of the substance. The change of temperature usually increases the resistance. It is changed by hardening and annealing. Impurities in a material modify considerably its resistance. This is especially the case with copper; even slight impurities have a very great effect in diminishing its conductivity or, in other words, in increasing its resistance. The reciprocal of resistance, *i.e.* $\frac{1}{R}$, is called the **conductivity** of the material or its **conductance.** The greater the resistance the less the conductivity.

The following table gives the specific resistance of the various substances:

Material.	Sp. Resistance at 0° C.
Silver annealed	1·468 10^{-6} ohm.
Silver hard drawn	1 615 ,, ,,
Copper	1 3 ,, ,,
Platinum	8 2 ,, ,,
Iron	8 6 ,, ,,
Lead	19 0 ,, ,,
German silver (alloy of Cu, Ni, Zn)	20·2 ,, ,,
Manganin	40 0 ,, ,,
Eureka	49 0 ,, ,,
Carbon	4000 ,, ,,

518. Resistance Coils.—In order to measure the resistance of any conductor we must compare it with a certain standard resistance. The resistance of wires, generally, of German silver, Eureka, etc., does not change very much with the temperature and therefore they are used for construction of standard resistances. Each coil is made of carefully insulated wire which is folded in the middle and is then wound on a reel, in this doubled condition. The ends of the coil are soldered to stout brass binding screws, which are fixed to an ebonite plate forming the top of the box enclosing the reel. If the wire used is very long, *i.e.* when the resistance is very high, each layer of the wire is coated with melted paraffin wax, to fix the turns, to ensure good insulation and protection from the moisture.

Generally we require a combination of resistances. Sets of coils, which are so arranged that any desired combination may be used, are called **resistance boxes.** Each coil has its ends

soldered to stout brass pieces fixed to the ebonite plate forming the top of the whole of the box. These pieces are separated from each other by small conical spaces fitted with brass plugs P and Q (Fig. 19). When we do not want any particular resistance to be used, the plug is inserted in the space tightly by generally giving a twist. Then the current passes across the plug, and the coil is not in action. But when the plug is removed the current passes through the coil and thus the desired resistance is introduced into the circuit. These coils

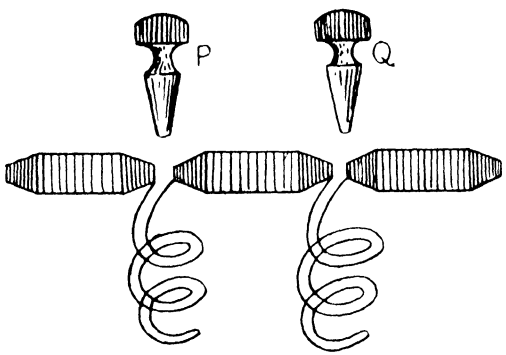


Fig. 19

are so adjusted that their resistances are multiples of the ohm. The usual arrangement of the coil in a resistance box is as follows :

1,	2,	2,	5,	10,	10
20,	50,	100,	100,	200,	500 (ohms).

With these coils any resistance between 0 and 1000 ohms can be inserted in the circuit; as, for instance, when no plug is out, there is 0 resistance, but when all the plugs are out, the resistance is 1000 ohms. By simply taking out different plugs any desired combination can be used.

519. Resistance of a System of Conductors.— There are two methods of joining the conductors together; one is of joining

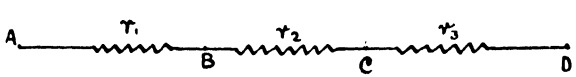


Fig. 20

them end to end, so that the same current traverses each in turn. This method of joining the conductors is called in **series**

(Fig. 20). The second method is that of joining them in **parallels.** This is the case when a good many conductors connect together the same points; in other words, the conductors are connected together at one end and so is the case at the other end. In this case the current is divided between these conductors. Let us study each method in turn.

520. Conductors in Series.—Suppose that the conductors r_1, r_2, and r_3 are joined in series, so that the same current traverses each conductor in turn (Fig. 20). If E be the p.d. between the extreme ends A and D, and i the current passing through each conductor in turn, then by Ohm's law

$$i = \frac{E}{R},$$

or

$$iR = E,$$

where R is the total resistance of the combination. Let e_1, e_2, e_3, etc., be the potential difference between the points A and B, B and C, C and D respectively. Applying Ohm's law to each portion, we get

$$i = \frac{e_1}{r_1} = \frac{e_2}{r_2} = \frac{e_3}{r_3}.$$

But

$$E = e_1 + e_2 + e_3,$$

$$\therefore \quad iR = ir_1 + ir_2 + ir_3 = i(r_1 + r_2 + r_3),$$

$$\therefore \quad R = r_1 + r_2 + r_3 \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Thus we see that *when conductors are joined in series the total resistance is equal to the sum of their resistances.*

521. Conductors in Parallels.—Let us consider the two points A and B connected together by a good many con-

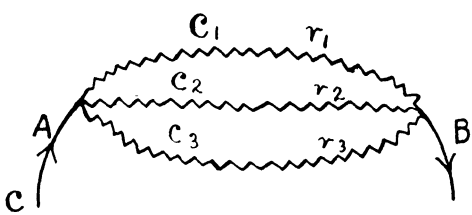


Fig. 20 (*a*)

ductors, say r_1, r_2, and r_3 Fig. 20 (*a*). The current C flowing in at A must be equal to $c_1 + c_2 + c_3$, since there is no accumulation of electricity at the point A. Let, further, the difference of

potential between A and B be E. The difference of potential between the ends of each conductor will obviously be the same.

$$\therefore \quad \text{current in } r_1 = c_1 = \frac{E}{r_1},$$

$$\text{current in } r_2 = c_2 = \frac{E}{r_2},$$

and $$\text{current in } r_3 = c_3 = \frac{E}{r_3}.$$

But $$C = c_1 + c_2 + c_3,$$

$$\therefore \quad \frac{E}{R} = \frac{E}{r_1} + \frac{E}{r_2} + \frac{E}{r_3},$$

where R is the equivalent resistance of r_1, r_2, r_3;

or $$\frac{1}{R} = \frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3} \quad . \quad . \quad . \quad . \quad . \quad (4)$$

Thus we see that *when a number of conductors are joined in parallels, the reciprocal of the equivalent resistance of the system is equal to the sum of the reciprocals of the resistances of separate conductors*; or, in other words, the conductivity of a number of conductors arranged in parallels is the sum of their conductivities. Note, in this case the total resistance of the conductors is less than the resistance of any of the constituent conductors.

We can very easily prove in the light of the formulae established that the resistance of a wire varies inversely as the cross-section and directly as the length. For let us suppose that the wires r_1, r_2, and r_3 are similar and therefore possess the same cross-section and length. Formula (3) tells us that $R = r_1 + r_2 + r_3 = 3r$, when $r_1 + r_2 + r_3 = r$. Thus we see that when a wire is three times as long as another wire, the resistance also is three times. Formula (4) tells us on the other hand that

$$\frac{1}{R} = \frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3} = \frac{3}{r}, \text{ or } R = \frac{r}{3},$$

i.e. when the cross-section becomes three times the resistance becomes $\frac{1}{3}$ times the resistance of a single wire.

522. We have seen that when conductors are joined in parallels,

$$E = c_1 r_1 = c_2 r_2 = c_3 r_3,$$

or $$\frac{c_1}{c_2} = \frac{r_2}{r_1},$$

and $$\frac{c_2}{c_3} = \frac{r_3}{r_2},$$

and $$\frac{c_1}{c_3}=\frac{r_3}{r_1}.$$

Or, in other words, *a current divides itself in two branches of a circuit in the inverse ratio of their resistances.*

Let us consider only two conductors joined in parallels (Fig. 20 (*b*)). Current C is divided into two parts c_1 and c_2 at A. We know from above—

$$\frac{c_1}{c_2}=\frac{r_2}{r_1},$$

or $$c_1=c_2\frac{r_2}{r_1}=(C-c_1)\frac{r_2}{r_1} \text{ (since } C=c_1+c_2),$$

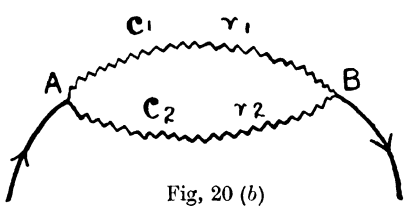
or $$c_1+c_1\frac{r_2}{r_1}=C\frac{r_2}{r_1},$$

or $$c_1\frac{r_1+r_2}{r_1}=C\frac{r_2}{r_1},$$

or $$c_1=C\frac{r_2}{r_1+r_2} \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

Similarly $$c_2=C\frac{r_1}{r_1+r_2} \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

Suppose that we do not want a strong current to pass through r_1, all that we have to do is to make r_2 small in com-



Fig, 20 (*b*)

parison with it. If, for instance, we want that through r_1 only $\frac{1}{10}$ of the main current should pass, then from formula (5) we have

$$c_1=C\frac{r_2}{r_1+r_2},$$

or $$\frac{c_1}{C}=\frac{1}{10}=\frac{r_2}{r_1+r_2},$$

or $$10r_2=r_1+r_2,$$

or $$9r_2=r_1,$$

or $$r_2=\frac{r_1}{9}.$$

In other words, r_2 should be $\frac{1}{9}$ of r_1.

Similarly if we want to pass through r_1 only $\frac{1}{100}$ of the main current, r_2 should be $\frac{1}{99}$ of r_1. This arrangement is used when the galvanometer is very sensitive and we want to pass only a small fraction of the main current through it. To do so we connect to its terminals a wire having a very much smaller resistance than the resistance of the galvanometer coils. This arrangement is called **shunting** the galvanometer or the ammeter. As, for instance, we have seen, to pass $\frac{1}{100}$ of the main current through it, the resistance of the wire connecting its terminals must have resistance $\frac{1}{99}$ of the resistance of the galvanometer coils. Shunting the galvanometer or the ammeter enables us to increase the range of the instrument considerably. For instance, if an ammeter be calibrated to read 1 to 5 amperes by connecting a wire of resistance equal to $\frac{1}{99}$ of the resistance of the instrument across the terminals, we can use it to measure currents 100 to 500 amperes.

Example.—An astatic galvanometer has a resistance of 1000 ohms. What must be the resistance of the shunt so that the current traversing the coils may be $\frac{1}{20}$th of the original amount ?

Let r_1 be the resistance of the galvanometer coils, and r_2 that of the shunt, then

$$\frac{1}{20} = \frac{r_2}{r_1 + r_2}, \text{ where } r_1 = 1000 \text{ ohms.}$$

$$\therefore \quad \frac{1}{20} = \frac{r_2}{1000 + r_2}, \text{ or } 19r_2 = 1000,$$

or

$$r_2 = \frac{1000}{19} = 52{\cdot}63 \text{ ohms.}$$

523. Grouping of similar Cells.—The strength of an electric current flowing round a circuit not only depends upon the total E.M.F., but also upon the total resistance (*i.e.* both external and internal) of the circuit. We have already seen how the conductors can be connected together to vary the resistance of the circuit and thus the current. Now let us see how to connect up the cells and to vary the internal resistance of the circuit. By internal resistance we mean the resistance of the cells. Generally cells are connected in one of the three methods—(i) series, (ii) parallels, or (iii) mixed. What is the best arrangement of connecting them up is determined by the conditions of the experiment. Let us take these arrangements one by one.

524. Cells in Series.—In this arrangement the negative pole of one cell is connected to the positive pole of the other and so

on (Fig. 21). If each cell has an E.M.F. E volts and internal resistance r ohms, the combined E.M.F. and the resistance will be nE * and nr respectively. When the circuit is completed through an external resistance R, the total resistance in the circuit is $R+nr$, and therefore the current

$$C=\frac{nE}{R+nr}. \quad . \quad (7)$$

If r is extremely small when compared with the external resistance R, the formula (7) is reduced to

$$C=\frac{nE}{R}.$$

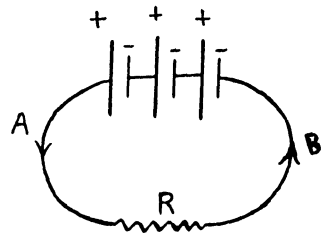


Fig. 21

This is n times the current that one cell would send round the circuit. But if, on the other hand, r is great in comparison with R, then R is negligible as compared with nr and the formula is reduced to

$$C=\frac{nE}{nr}=\frac{E}{r},$$

i.e. the current is the same as given by one cell. Thus we see that *this arrangement is desirable only when the external resistance is very large in comparison with the internal resistance.*

525. Cells in Parallels.—In this arrangement the zincs of all the cells are connected together and so are the coppers. In other words, we simply increase the size of the zinc and copper plates. But we know that the Pot. difference between the two metals placed in an electrolyte depends upon their nature only, and not upon their size. This means that this arrangement has the same E.M.F. as a single cell; but on account of the increase in the cross-section the internal resistance decreases. As a matter of fact, if there are n cells each having a resistance

* For E is the potential difference between the positive and the negative pole of each of the cells 1st, 2nd, and so on. The negative pole of the first cell is joined to the positive pole of the second cell and hence they are at the same potential. But as the positive pole of the second cell is higher by E units than the negative, therefore evidently the negative pole of the first cell is also E units higher in potential than the negative pole of the second cell, and therefore the positive pole of the first cell is $2E$ units higher than the negative pole of the second cell. Repeating the same argument, we can prove that if there be n cells, the p.d. will be nE between the positive pole of the first cell and the negative pole of the last cell.

of r ohms, the resistance of the combination will be $\frac{r}{n}$. If the external resistance be R as before, the current will be equal to

$$C = \frac{E}{R + \frac{r}{n}} \quad . \quad . \quad . \quad . \quad . \quad (8)$$

It is obvious from the formula (8) that when R is very small in comparison with r the formula is reduced to

$$C = \frac{nE}{r};$$

this is n times the current that one cell would give. But if, on the other hand, R is large in comparison with r, then formula (8) is reduced to $\frac{E}{R}$, which is simply equal to the current given by one cell.

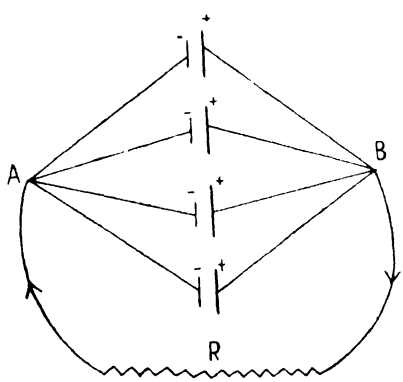


Fig. 22

Thus studying both the arrangements together, we come to the conclusion that **cells should be joined in series when the external resistance is large in comparison with internal resistance, but they should be joined in parallels when the internal resistance is large.**

It will become clear if we take an example. Suppose that the resistance of each cell having 1 volt as E.M.F. is 2 ohms, and we have 4 such cells. The external resistance is 1000 ohms. When the cells are joined in series,

$$C = \frac{4}{1000 + 8} = \frac{4}{1008} = \cdot 004 \text{ ampere (approx.)},$$

and when they are joined in parallels,

$$C = \frac{1}{1000 + \frac{2}{4}} = \frac{1}{1000 + \cdot 50} = \cdot 001 \text{ ampere (approx.).}$$

Evidently in this case it is advantageous to use the cells in series. But let us see what will happen when the external resistance is $\frac{1}{100}$th of an ohm. The current when the cells are joined in series will be

$$C = \frac{4}{\frac{1}{100} + 8} = \frac{4}{8 \cdot 01} = \cdot 5 \text{ ampere (approx.).}$$

And when they are joined in parallels,

$$C = \frac{1}{\frac{1}{100} + \frac{2}{4}} = \frac{1}{\cdot 51} = 2 \text{ amperes (approx.).}$$

Thus we see that in this case it is better to join the cells in parallels.

526. Mixed Grouping.—If we have in all N cells at our disposal to send a current through an external circuit having R ohms resistance, the cells may be arranged all in parallels

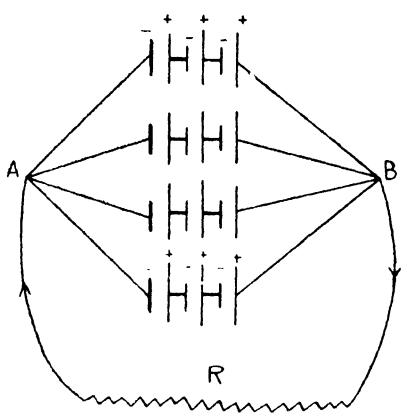


Fig. 23

or all in series; and as we have learnt, if R is very great in comparison with Nr the cells should be arranged in series, and if R is very small in comparison with r the cells should be arranged in parallels. But when R is neither too great nor too small, which arrangement will give maximum current?

Let us suppose that the N cells are divided into m sets, each set having n cells in it, the product mn being equal to N. Join the n cells in series (Fig. 23); the E.M.F. of each row will

be nE, and the internal resistance will be nr. But there are m such batteries joined in parallels, therefore their E.M.F. will remain the same, *i.e.* nE, but the internal resistance will become $\frac{nr}{m}$.

The total resistance will be $R+\frac{nr}{m}$. From this we have

$$C=\frac{nE}{R+\frac{nr}{m}}$$

$$=\frac{mnE}{mR+nr} \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

527. But let us see now when this arrangement will give maximum current. Evidently it will be so when the denominator $mR+nr$ is the smallest, the numerator mnE having a constant value which we cannot alter. But the denominator can be altered, for we can change the number of cells in each row. Now $mR+nr$ may be written as

$$(\sqrt{mR}-\sqrt{nr})^2+2\sqrt{mnRr}.$$

This expression will have the smallest value when

$$(\sqrt{mR}-\sqrt{nr})^2=0,$$

for otherwise it being a square of a quantity will always have positive value.

$\therefore$ for the smallest value

$$\sqrt{mR}-\sqrt{nr}=0,$$

or $$\sqrt{mR}=\sqrt{nr},$$

or $$mR=nr,$$

or $$R=\frac{nr}{m}.$$

But R = external resistance and $\frac{nr}{m}$ is the total internal resistance, hence we see that *the current will be maximum when the external resistance is equal to the internal resistance of the battery.* This arrangement is, however, not the most economical arrangement, for as much energy is utilised inside the battery as outside it, and we are thus utilising only half of the energy.

Example.—Find the maximum current which 24 cells having 1·5 volts E.M.F. and 4 ohms resistance will send round the circuit having 6 ohms resistance.

Here $$mn=24 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (a)$$

and $$C = \frac{mnE}{mR + nr} = \frac{24 \times 1{\cdot}5}{m6 + n4},$$

for C to be maximum,

$$mR = nr \text{ or } 6m = 4n \quad . \quad . \quad . \quad . \quad . \quad . \quad (b)$$

Eliminating n from (a) and (b), we get $m = 4$ and $n = 6$. That is, the arrangement required is 6 cells in series and 4 such rows. The current will be equal to

$$C = \frac{36}{24 + 24} = \frac{36}{48} = \frac{3}{4} = {\cdot}75 \text{ ampere.}$$

When all the cells are joined in series,

the current $$= \frac{36}{6 + 96} = \frac{36}{102} = {\cdot}353 \text{ ampere.}$$

When all the cells are joined in parallels,

the current $$= \frac{1{\cdot}5}{6 + \frac{1}{4}} = \frac{1{\cdot}5}{6{\cdot}17} = {\cdot}243 \text{ ampere.}$$

This example tells us clearly that in the case of mixed grouping when there were 6 cells in series and 4 such rows the current is maximum and very much greater than what we get when we join the cells either all in series or in parallels.

528. Electromotive Force and Potential Difference.—We have been using these words almost as synonyms so far, but now let us see what is the difference between the two. Electromotive force is that which produces or tends to produce motion of electricity. It should be clearly understood that the term electromotive force does not mean force in the dynamical sense of the term. It is called so because of the analogy with the mechanical force which produces motion in matter. Difference of potential between two points will also cause the current to flow between them, and therefore potential difference is also an electromotive force. Now let us consider a cell sending a current round a circuit. The potential diminishes in the direction of the flow of the current, but all the same the potential difference does exist in all parts of the circuit. Further potential difference between two points of a circuit is proportional to the resistance between them ($E = CR$), but electromotive force is independent of the resistance of the circuit, and it exists even when the circuit is not closed. *The term electromotive force or E.M.F. is restricted to the source only; in other words, the E.M.F. is local whereas potential difference is distributed over the whole of the circuit.*

When the external circuit is not closed, the Pot. difference between the two poles of the cell gives the E.M.F., but as

soon as the circuit is closed, the potential difference between the poles falls off on account of the consumption of energy in sending the current through the cell. Suppose, for instance, that we consider a battery having 36 volts as the potential difference between the terminals when the circuit is open; when it will be closed, the current will flow round the circuit, and a part of the energy will be spent in driving the current through the battery which forms part of the circuit. This consumption we do not measure, and hence our recording instrument, if there be any in the circuit, will record less than 36 volts. Thus we see that E.M.F. should be measured by the potential difference at the terminals of a cell on open circuit.

CHAPTER IV

ELECTRICAL MEASUREMENTS

529. We shall deal with only two measurements, that of resistance and electromotive force. Let us take them turn by turn.

530. Measurement of Resistance by the use of Ammeter and Voltmeter.—If we are provided with a coil of unknown resistance, the most direct method of determining its resistance where extreme accuracy is not required is to measure the potential difference between the terminals of the coil by means of a voltmeter and to read the current passing through the coil by means of an ammeter.*

Applying Ohm's law we get $R = \frac{E}{C}$.

Fig. 24 shows the connections. K stands for the key to make or break the circuit, L the resistance (lamp, etc.), A is

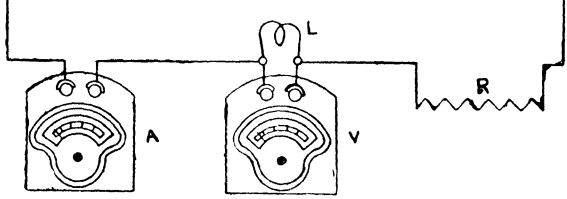


Fig. 24

the ammeter, V the voltmeter, and R the additional resistance to regulate the current.

Note, *the ammeter is to be joined in series with the unknown resistance and the voltmeter in parallels.*

* Instruments of suitable range must be selected for this experiment.

531. Substitution Method.— Suppose R is a coil of unknown resistance. We connect it to a galvanometer, a battery, and a two-way key as shown in Fig. 24*a*. On inserting the plug between D and E the current, after traversing the unknown coil R, will pass through the galvanometer, whose deflection will be proportional to the current. Now remove the plug from the gap between D and E and insert it between D and F, introducing thereby in place of R a resistance box. Insert a resistance either high or low to see how much deflection is produced. For instance, suppose that with the unknown coil the deflection obtained is 30°. And with a high resistance, say 500 ohms, the deflection is 15°. On introducing 400 ohms the deflection is 20°, and so on. Go on continually decreasing the resistance till the deflection produced is just the same as with the unknown coil. It should be remembered that when the deflection is very near the desired deflection, the change in resistance introduced should be very small. Begin a second time from the lower end, *i.e.* with a low resistance, and go on continually increasing it till the deflection produced is equal to that with the unknown coil. The mean of these two readings gives the resistance of the unknown coil ;* since in

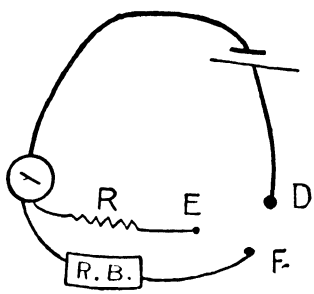


Fig. 24*a*

$$C=\frac{E}{R}=K\tan\Theta,$$

Θ is the same in both the cases and so is E, hence R must be the same.

532. Deflection Method.—Let R be the unknown resistance of the coil, and $b+g$ that of the rest of the circuit, b being the resistance of the battery and g that of the galvanometer.

From Ohm's law we know that current

$$C=\frac{E}{R+b+g} \quad . \quad . \quad . \quad . \quad . \quad . \quad (i)$$

where E is the electromotive force of the cell. At the same

* The degree of accuracy obtainable by this method is not at all large, for the exact equality of deflection is only possible when the unknown resistance happens to be an integral multiple of an ohm or tenth of an ohm.

time we know that the tangent of the deflection in the case of a tangent galvanometer is proportional to C (see § 510);

$$C = K \tan \Theta,$$

where Θ is the deflection of the needle.

Introduce the unknown resistance R into the circuit and note the deflection produced. It should be remembered that both ends of the needle are to be read, and in addition to it the current is to be reversed and both ends read again. The mean of these four readings should be taken as the true deflection. Let the true deflection be Θ_1, and if the current be C_1, then

$$C_1 = K \tan \Theta_1 = \frac{E}{R+b+g} \quad . \quad . \quad . \quad \text{(ii)}$$

Now introduce a known resistance r and suppose the mean deflection (of four readings) is Θ_2. Since E, b, and g are same,

$$C_2 = K \tan \Theta_2 = \frac{E}{r+b+g} \quad . \quad . \quad . \quad \text{(iii)}$$

$$\therefore \quad \frac{E}{R+b+g} \times \frac{r+b+g}{E} = \frac{\tan \Theta_1}{\tan \Theta_2},$$

or

$$\frac{r+b+g}{R+b+g} = \frac{\tan \Theta_1}{\tan \Theta_2} \quad . \quad . \quad . \quad . \quad \text{(iv)}$$

If $b+g$, *i.e.* the resistance of the battery and galvanometer, be very small in comparison with r and R, the formula (iv) is reduced to

$$\frac{r}{R} = \frac{\tan \Theta_1}{\tan \Theta_2} \quad . \quad . \quad . \quad . \quad . \quad \text{(iv}a\text{)}$$

This tells us that while measuring or *comparing resistances always the* ***low resistance coil*** *of the tangent galvanometer should be used* to enable us to neglect g in comparison with the external resistance. However, if $b+g$ be not negligible, then a third observation must be taken with no additional resistance in the circuit. Suppose the mean deflection is Θ when neither R nor any resistance from the box is introduced. Then we know

$$C = K \tan \Theta = \frac{E}{b+g} \quad . \quad . \quad . \quad . \quad \text{(v)}$$

Let us write formulae (ii), (iii) and (v) in a slightly different form.

Formula (ii) can be expressed as

$$R+b+g \propto \cot \Theta_1 \quad . \quad . \quad . \quad . \quad . \quad \text{(vi)}$$

and similarly $$r+b+g \propto \cot \Theta_2 \quad . \quad . \quad . \quad . \quad \text{(vii)}$$

and $$b+g \propto \cot \Theta \quad . \quad . \quad . \quad . \quad \text{(viii)}$$

Equations (vi) and (viii) combined give us the relation

$$R \propto (\cot \Theta_1 - \cot \Theta) \quad . \quad . \quad . \quad . \quad \text{(ix)}$$

and equations (vii) and viii) give us

$$r \propto (\cot \Theta_2 - \cot \Theta),$$

$$\therefore \quad \frac{r}{R} = \frac{\cot \Theta_2 - \cot \Theta}{\cot \Theta_1 - \cot \Theta} \quad . \quad . \quad . \quad . \quad \text{(x)}$$

If we were to express in terms of the tangents of the deflection we would get

$$\frac{r}{R} = \frac{\tan \Theta - \tan \Theta_2}{\tan \Theta - \tan \Theta_1} \times \frac{\tan \Theta_1}{\tan \Theta_2} \quad . \quad . \quad . \quad \text{(x}a\text{)}$$

533. To find out the resistance of a coil of a tangent galvanometer.—Let us take a resistance R_1 out of the resistance box and note the mean deflection produced; say it is Θ_1. Then we know that

$$C_1 = K \tan \Theta_1 = \frac{E}{R_1+b+g} \quad . \quad . \quad . \quad \text{(xi)}$$

Take out another resistance, say R_2, and let the deflection be Θ_2; as before, we have

$$C_2 = K \tan \Theta_2 = \frac{E}{R_2+b+g} \quad . \quad . \quad . \quad \text{(xii)}$$

Dividing (xi) by (xii), we get

$$\frac{\tan \Theta_1}{\tan \Theta_2} = \frac{R_2+b+g}{R_1+b+g} = \frac{R_2+g}{R_1+g} \quad . \quad . \quad . \quad \text{(xiii)}$$

if b, the resistance of the battery, be supposed to be small, as is usually the case

$$g = \frac{R_2 \tan \Theta_2 - R_1 \tan \Theta_1}{\tan \Theta_1 - \tan \Theta_2} \quad . \quad . \quad . \quad \text{(xiv)}$$

534. The Principle of Wheatstone Bridge.—Since this method is one of the most accurate for measuring resistances, we shall first explain its principle before explaining the actual method.

Let the two points A and C be connected together by two conductors, ABC and ADC in parallels (Fig. 25). Wires from a battery are connected to A and C. The current entering at A is divided into two portions c_1 and c_2, which

rejoin at C. Since the current is flowing from A to C there must be a gradual fall of potential along the conductor ABC. Let the potentials of the points A and C be denoted by V_A and V_C. Evidently the potential at the point B will have some intermediate value between V_A and V_C. Call it V. If instead of going along ABC we go from A to C along ADC we shall have the same fall of potential as before, *i.e.* from V_A to V_C. Since the fall of potential is the same along ADC, there must be some point D on this conductor which has the same potential as the point B. The position of the point D can be easily determined experimentally by connecting one terminal of a galvanometer to the point B and by moving the other terminal over ADC till we come to such a point that there is no current through the galvanometer. This point is D. When this is the case the point B and D must have the same potential, for otherwise, since BD is a conductor, a current would flow from the higher to a lower potential. Thus we see that the potential at the point D is also equal to V.

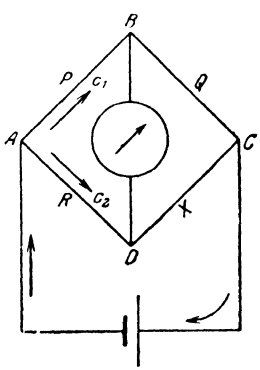


Fig. 25

Let the resistances of the arms AB, BC, AD and DC be called P, Q, R, and X respectively. The current flowing through AB is c_1, and since there is no current through BD the current through BC must also be equal to c_1. Similarly the current through DC is c_2. Let us apply Ohm's law, remembering that $V_B = V_D = V$.

$$V_A - V_B \text{ or } V_A - V = c_1 P,$$

and $$V_A - V_D \text{ or } V_A - V = c_2 R,$$

or $$c_1 P = c_2 R,$$

or $$\frac{P}{R} = \frac{c_2}{c_1} \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{(xv)}$$

Similarly $$V_B - V_C \text{ or } V - V_C = c_1 Q,$$

and $$V_D - V_C \text{ or } V - V_C = c_2 X,$$

or $$c_1 Q = c_2 X,$$

or $$\frac{Q}{X} = \frac{c_2}{c_1} \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{(xvi)}$$

Compare the two equations (xv) and (xvi). Evidently

$$\frac{P}{R} = \frac{Q}{X},$$

or

$$X = R\frac{Q}{P} \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{(xvii)}$$

If the three resistances Q, P, and R be known, the fourth can be easily calculated.

The superiority of this method of measuring resistance lies in the fact that it is far more easy to detect a minute deflection than to read it accurately even when it is considerable.

In order to measure resistances practically with the help of this principle we use either the post-office box or the meter-bridge.

535. Post-office Box.—It is a special kind of resistance box, which was designed originally for the measurement of the

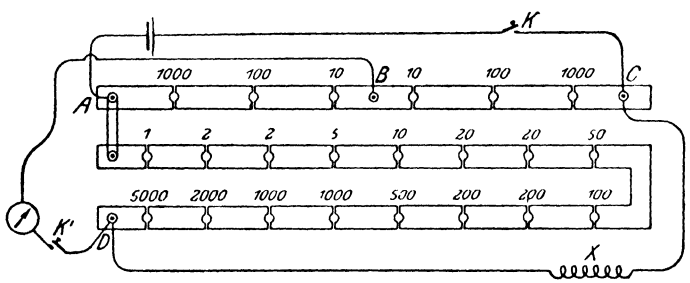


Fig. 26

resistance of telegraph wires and hence the name post-office box. Its plan is shown diagrammatically in Fig. 26. Corresponding points in Figs. 25 and 26 are denoted by the same letters for convenience. The arms AB and BC are called ratio arms, and the ratio $\frac{P}{Q}$ can be made either 1 : 1, 10 : 1, or 100 : 1 as required. The third arm, AD, is so arranged that it gives any whole number of ohms from 1 to 10,000. The unknown resistance is joined to D and C. This is the fourth arm of the bridge. The battery is connected to A and C, and the galvanometer to B and D.

In order to determine the resistance proceed as follows :

Take equal ratios in the arms AB and BC at first, close the battery circuit, and find out which resistance in the third

arm gives no deflection in the galvanometer. This gives the resistance of the unknown coil. But usually it is very difficult to get no deflection with equal ratios, for this means that the unknown resistance is some whole-number of ohms, which may or may not be the case. Suppose, for instance, the unknown resistance is 20·3 ohms. Neither with a resistance of 20 ohms nor with 21, the galvanometer will show zero deflection. But of course with 20 it will give the smallest deflection. Thus we see that with equal arms we get the resistance nearest to an ohm.

Next make the ratio 10 : 1. It will be seen that the galvanometer shows no current when a resistance of 203 ohms is taken out.

Since $$\frac{P}{Q} = 10,$$

therefore $\frac{R}{X}$ must be equal to 10 also;

or $$\frac{203}{X} = 10,$$

or $$X = 20{\cdot}3.$$

Thus we see that with this ratio we can determine the resistance nearest to the first decimal place of an ohm. If we make the ratio 100 : 1 we can find out the resistance nearest to the second place of decimals.

While performing this experiment remember **to close first the battery circuit and then the galvanometer circuit.**

536. The Metre-Bridge.—It consists of a long thin uniform wire of German-silver or manganin, one metre long, stretched

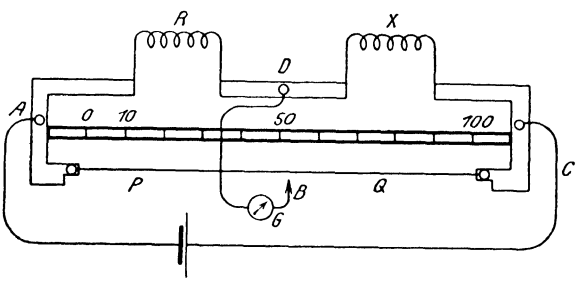


Fig. 27

over a wooden board alongside a metre scale, with its ends soldered to stout copper strips A and C of negligible resistance.

There is a third copper strip fixed on the other side of the board as shown at D in Fig. 27. The unknown resistance is placed at X, and a known resistance R * is placed in the other gap.

The battery is connected to A and C and the galvanometer is connected to D on one side and on the other side to a sliding contact, which moves over the stretched wire. The position of the sliding contact is changed till there is no current flowing through the galvanometer.

Now the wire being uniform, the resistance of any portion of it is proportional to its length. Hence if $AB = d_1$ and $BC = d_2$, the ratio $\frac{P}{Q} = \frac{d_1}{d_2}$. But we know from equation (xvii) that

$$X = R\frac{Q}{P}.$$

Substituting for $\frac{Q}{P}$, $\frac{d_2}{d_1}$ we get

$$X = R\frac{d_2}{d_1} \quad . \quad . \quad . \quad . \quad \text{(xviii)}$$

Thus if we know R, we can at once calculate X.

537. Comparison of Electromotive Forces.—In comparing resistances we have used the low resistance coil of the galvanometer, but in comparing electromotive forces it should be noted at the outset that the **high resistance coil** always is to be used.

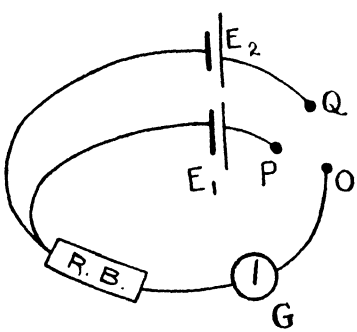


Fig. 28

538. Deflection Method.—We may make use of Ohm's law to compare the electromotive forces of the cells. Suppose we have two cells whose resistances are b_1 and b_2, and the electromotive forces E_1 and E_2.

Let us pass a current from the two batteries in turn through a resistance R (the same in both the cases) and the high resistance

* It should be remembered that the resistance R should be nearly equal to X for good results.

coil of the galvanometer. Let the deflections be Θ_1 and Θ_2. We have as before,

$$\frac{E_1}{R+b_1+g} = K \tan \Theta_1 \quad . \quad . \quad . \quad \text{(xix)}$$

and

$$\frac{E_2}{R+b_2+g} = K \tan \Theta_2 \quad . \quad . \quad . \quad \text{(xx)}$$

Supposing b_1 and b_2 to be almost equal, we get by dividing equation (xix) by (xx)

$$\frac{E_1}{E_2} = \frac{\tan \Theta_1}{\tan \Theta_2} \quad . \quad . \quad . \quad . \quad \text{(xxi)}$$

In this case the cells have sent a current through the same external circuit. For this method we need not know the galvanometer resistance.

539. Equal Deflection Method.—But if the galvanometer resistance be known we can modify our experiment slightly by making the deflection equal in both the cases. *This method is more satisfactory than the first one, because the sensitiveness of the galvanometer does not come in, which plays otherwise a very important part.* If g be known we can make $\Theta_1 = \Theta_2 = \Theta$ by taking R different in the two cases. Suppose that in the first case the resistance is R_1 and in the second case it is R_2.

We shall have

$$\frac{E_1}{R_1+b_1+g} = K \tan \Theta,$$

and

$$\frac{E_2}{R_2+b_2+g} = K \tan \Theta.$$

$$\therefore \quad \frac{E_2}{E_1} = \frac{R_2+b_2+g}{R_1+b_1+g} = \frac{R_2+g}{R_1+g} \quad . \quad . \quad . \quad \text{(xxii)}$$

supposing b_1 and b_2 to be small in comparison with R_1+g or R_2+g.

In the above-mentioned experiment we have neglected the resistance of the battery. If R_1 and R_2 and g be very high, as is usually the case, neglecting of b does not affect the result very much.

540. The Sum and Difference Method.—In this method at first the two cells to be compared are joined in series, *i.e.* the negative pole of the one is joined to the positive pole of the second so as to send the current in the same direction through the circuit consisting of the two cells, a tangent galvanometer, and a resistance box. In this case the total E.M.F. acting

round the circuit is E_1+E_2. Then the cells are joined in opposition, *i.e.* the two negative poles are joined together, so as to send the current in the opposite direction through the same circuit. In the second case the total E.M.F. acting round the circuit is E_1-E_2. The resistance of the circuit is adjusted by changing the resistance in the resistance box, so that suitable deflections are obtained in both the cases. Thus if R be the resistance in the resistance box, b_1, b_2, and g be the resistances of the cells and of the galvanometer respectively, Θ_1 and Θ_2 the deflections in the first and the second case, we have

$$C_1=\frac{E_1+E_2}{R+b_1+b_2+g}=K\tan\Theta_1,$$

and

$$C_2=\frac{E_1-E_2}{R+b_1+b_2+g}=K\tan\Theta_2.$$

$$\therefore\quad \frac{E_1+E_2}{E_1-E_2}=\frac{\tan\Theta_1}{\tan\Theta_2},*$$

or

$$\frac{E_1}{E_2}=\frac{\tan\Theta_1+\tan\Theta_2}{\tan\Theta_1-\tan\Theta_2}.$$

541. Potentiometer Method.—The potentiometer consists of a long thin uniform wire of high resistance, usually of magnanin, stretched between two binding screws, A and B, over a wooden board by the side of a scale divided into millimetres.

To the screw A is connected the positive pole of the battery E of constant E.M.F., say two or three accumulators, and to the screw B is connected the other pole of the battery. Through the wire AB evidently a constant current passes. The potential falls uniformly as we go from A to B. Since the wire is uniform, the resistance therefore is proportional to its length. This means that the potential difference between A and C where C is any point on the wire is proportional to the length of the wire between these two points.

To compare E.M.F.'s proceed as explained below. Join A and C to the ends of a galvanometer; a current will pass through it owing to the potential difference between the points.

---

* $\dfrac{E_1+E_2}{E_1-E_2}+1=\dfrac{E_1+E_2+E_1-E_2}{E_1-E_2}=\dfrac{2E_1}{E_1-E_2}=\dfrac{\tan\theta_1}{\tan\theta_2}+1=\dfrac{\tan\theta_1+\tan\theta_2}{\tan\theta_1}.$

Similarly

$$\frac{E_1+E_2}{E_1-E_2}-1=\frac{E_1+E_2-E_1+E_2}{E_1-E_2}=\frac{2E_2}{E_1-E_2}=\frac{\tan\theta_1-\tan\theta_2}{\tan\theta_1},$$

$$\therefore\quad E_1/E_2=(\tan\theta_1+\tan\theta_2)/(\tan\theta_1-\tan\theta_2).$$

Introduce into this circuit a cell e_1 in such a way that the positive pole of e_1 is connected to A and the other end to the terminal of the galvanometer G, which is very sensitive. Join the second terminal of the galvanometer to a sliding contact through a high resistance. Note that the potential of the positive pole of e_1 is the same as the potential of the point A, since they are joined together and thus form one conductor. A current is sent by e_1 through the galvanometer, but it is opposite in direction to the current sent by the potential difference between the points A and C. If the point C on the wire has the same potential as the sliding contact, which is connected to the negative pole of the cell e_1, no current would pass through the galvanometer. This point can be easily

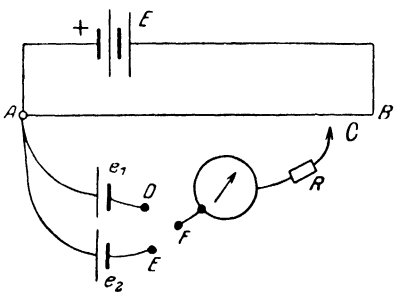


Fig. 29

obtained by moving about the sliding contact and noting the point where no current passes through the galvanometer. Let the length AC be d_1, evidently the E.M.F. of the cell e_1 must be equal and opposite to the potential difference between the points A and C due to the battery E. Let this E.M.F. be denoted by E_1.

Now in place of the cell e_1 introduce the cell e_2 into the circuit, and find out the point on the wire AB for which there is no current in the galvanometer. Let this point be C', the length AC' being d_2. The E.M.F. of the cell e_2 is evidently equal and opposite to the potential difference between A and C'. Let it be called E_2.

But the potential difference between any two points on the wire is proportional to the length between those points as explained above; hence

$$\frac{E_1}{E_2} = \frac{AC}{AC'} = \frac{d_1}{d_2}.$$

There are two points to be remembered in this method of comparing E.M.F.'s. The first is that the potential difference between the points A and B must evidently be greater than the E.M.F. of any of the given cells, for otherwise on no point on the wire the current in the galvanometer will be zero. To ensure this the battery E, which is used to supply the constant current, must have a greater E.M.F. than that of the cells to be compared. The second is that a high resistance should be used in series with the galvanometer to protect the sensitive galvanometer and to avoid the possibility of passing a strong current through the cell.

CHAPTER V

HEATING EFFECT OF THE CURRENT

542. The conversion of electric energy into heat energy is a familiar case. Anybody who has touched an electric bulb emitting light knows that it becomes hot after some time, due to the heat produced along with the light.

The heating of a wire by the passage of a current may conveniently be exhibited by the aid of the apparatus sketched

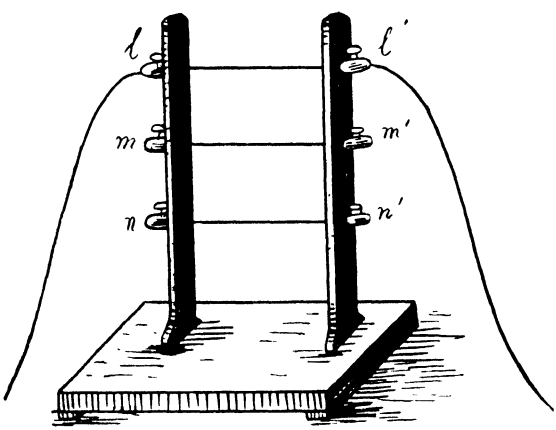


Fig. 30

in Fig. 30. It consists of two uprights mounted on a board which are furnished at different heights with pairs of insulated binding screws l, l' ; m, m' ; and n, n'. Thin wires of different materials, say of Eureka, German silver, etc., are stretched between them, and a current can be sent through any of them by connecting the binding screws to the terminals of a battery. When this is so done with a battery of suitable power, the wire

is first seen to sag in consequence of expansion, then to redden and finally to melt, becoming inflamed if the metal is easily combustible.

By using different wires of the same material we see that the thicker the wire the smaller the rise of temperature, and the greater the resistance that a wire offers the hotter it becomes. Let us study the exact relation between the current, the resistance, and the heat produced.

543. Conversion of Electric Energy into Heat.—The energy of a current is jointly proportional to the quantity of the electricity that passes and the electromotive force that drives it. Let us suppose that in a circuit, E is the E.M.F. acting and C is the current produced. Since E is the potential difference between the ends of the circuit, the work done in moving a unit charge from one point to the other is numerically equal to E (ergs). And when we move Q units of electricity from one point to the other, the work done

$$W = QE.$$

But $Q = Ct$, where C is the current and t is the time for which the current flows,

$$\therefore \quad W = CEt.$$

But from Ohm's law $E = CR$,

$$\therefore \quad W = C^2Rt.$$

Now $W = JH$,

where J is the mechanical equivalent of heat and H is the quantity of heat produced. (See § 240.)

$$\therefore \quad JH = C^2Rt,$$

or

$$H = \frac{C^2Rt}{J},$$

where H is the number of calories produced, C and R are the units of the current and the resistance in C.G.S. system, t is the time in seconds, and $J = 4{\cdot}2 \times 10^7$ ergs.

The formula $H = \frac{C^2Rt}{J}$ is called **Joule's law**, and in words it states that *the heat produced in a wire is proportional to the square of the current, the resistance, and the time for which the current flows.*

We have said that the current and the resistance are to be measured in C.G.S. units, but in practice the current is measured in amperes, which is $\frac{1}{10}$ of the C.G.S. unit, and the resistance in ohms, which is equal to 10^9 C.G.S. units. Let us see when

the current is measured in amperes and the resistance in ohms, what will be the modification in the formula.

When C is measured in amperes, an ampere being $\frac{1}{10}$ of the absolute unit, C becomes ten times the number of the current unit (in C.G.S. system), and hence in order to keep the *number* the same we must divide the number of amperes by 10 and in the case of C^2 by 100. Similarly, as an ohm $= 10^9$ C.G.S. units, when we express the resistance in ohms, we must multiply the number by 10^9, so that it may be equal to the number of C.G.S. units.

Thus we see that

$$H = \frac{C^2 \times \frac{1}{100} \times R \times 10^9 \times t}{4{\cdot}2 \times 10^7},$$

where C and R are measured in the practical units, *i.e.* in amperes and ohms;

$$\therefore \quad H = \frac{C^2 R t}{4{\cdot}2}.$$

This relation, that *the quantity of heat produced is proportional to the square of the current, the resistance and the time* for which the current flows, can be proved experimentally. This has been done by Joule, Lenz, and Becquerel and others with the help of a galvano-thermometer, which consists of a wide-mouthed inverted bottle, stoppered at the lower end. Through the stopper two stout platinum wires pass which end in two platinum cones. To these cones is attached the wire to be experimented upon. The bottle contains water, the temperature of which is recorded by a sensitive thermometer. The resistance of the wire is known, and the strength of the current is measured by a galvanometer, which is included in the circuit. The current is passed for a known time, and the rise in temperature of the water is noted. The mass of water being known, the amount of heat taken up by the water to get heated can be calculated. Thus we know H, we know t, C,

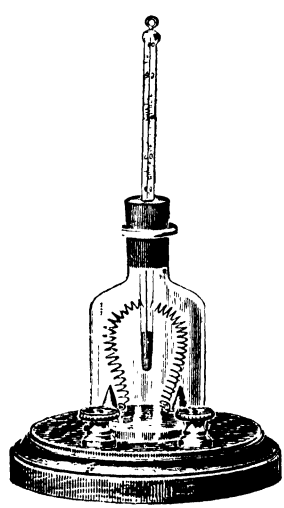


Fig. 31

and R, and hence we can establish the relation between them. Joule found that the relation

$$H = \frac{C^2Rt}{J} \text{ holds good.}$$

544. Closely allied to the heating effect produced by a current is the luminous effect, which has of late become very popular as a method of town-lighting. Before we take it up let us study the unit of electric energy which is consumed in doing the works of various kinds. We have already seen that when the current C flows for the time t seconds against a potential difference E, the work W done is given by the relation

$$W = CEt.$$

In one second the work done by the current is CE. This product CE is called the electrical power. When C is 1 ampere, t is one second and E is one volt,

$$W = 10^7 \text{ ergs,}$$

or is equal to the practical unit of work which is called a **joule.** Note, *joule is the work done when one coulomb is transferred from one point to another against a potential difference of one volt.* The second idea with which we must be familiar is the **electric power.** It is defined generally as the rate at which work is done. When the work done is 1 joule per second the unit of the power is said to be **watt.** If instead of taking 1 second as the unit of time we take 1 hour as the unit, then a current of one ampere flowing against a potential difference of 1 volt for one hour will do 3600 joules of work. This is called **watt-hour.**

The Board of Trade, however, employs a still greater unit than this. They use 1000 watt-hours as their unit, which they call kilowatt-hour. In order to convert joules into watt-hours we must divide by 3600, and then to convert watt-hours into kilowatt-hours we must further divide by 1000. In other words, one kilowatt-hour

$$= 1000 \times 3600 = 3600000 \text{ joules}$$

$$= 36000000000000 \text{ (}i.e.\ 36 \times 10^{12}\text{) ergs.}$$

(This is the unit adopted by the electric supply companies.)

It is useful to remember the relation between the horse-power (the unit of power adopted by the engineers) and the watt. Horse-power is defined as the unit which is equivalent to 33,000 ft. lb. per minute or 550 ft. lb. per second. Now we know that 1 lb. wt.

$=453{\cdot}6$ gram wt. and 1 ft. $=30{\cdot}49$ cm., therefore 1 ft. lb. per second $=(453{\cdot}6 \times 30{\cdot}9)\ g$ ergs per sec.

$=(453{\cdot}6 \times 30{\cdot}49)\ 981$ ergs per second

$=\dfrac{(453{\cdot}6 \times 30{\cdot}49)\ 981}{10^7}$ joules per second or watts

$=1{\cdot}357$ watts.

1 H.P. $=550$ ft. lb. sec. $=550 \times 1{\cdot}357$ watts

$=746$ watts

$={\cdot}746$ kilowatt,

or 1 kilowatt $=1{\cdot}341$ horse-power.

545. Luminous Effect of Electric Current.—Of late, as has been said, this effect has become far more important than the heating effect with which we have dealt already at the beginning of the chapter. The heat produced is $\frac{C^2Rt}{J}$ calories, and if the thermal capacity of the wire which is being heated be known we can find out what will be the temperature of the wire. As the temperature becomes very high, in addition to the heat, luminous effect is produced. We shall survey very briefly arc-lamps and one or two well-known types of the incandescent lamps.

546. Arc Lamps.—When two pieces of carbon are connected to the terminals of a powerful battery, a dynamo, or to the street mains, on bringing the carbon pieces together in contact, the current passes and the ends of the carbons soon become incandescent, the resistance there being very high. They are then separated; the air between gets heated, little particles become detached, are heated to incandescence, and travel from one carbon to the other. This causes one of the carbons to become pointed and the other hollow. *It is these incandescent particles which form the arc between the two carbon pieces.*

The light of the electric arc has a dazzling brilliancy which is painfully and dangerously intense. If a man were to look at it constantly for some time, it would injure his eyes and produce headache. It dazzles rather than illuminates. Its small size also makes it unfit for household use.

What makes the arc-light unfit for household use makes it of the greatest service in the lighthouses. The object of a lighthouse is not to illuminate the surrounding objects but to be itself visible. For this purpose an arc-light which has penetration as its peculiarity is most useful, because in hazy

weather its dazzling brilliancy makes it visible from great distances.

It is also used in projecting pictures over the screens in lecture illustrations or cinemas. The temperature of the arc is generally very high, the negative carbon at about 2000° C. and the positive carbon at 3000° to 4000° C. Many substances which were considered to be refractory were at once melted when they were introduced into the arc.

As has been said, the arc is maintained by the carbon particles which get detached and go from one carbon to the other. It is found that *these particles travel from the positive carbon to the negative. This causes the positive pole to become hollow and the negative pole to become pointed*, on account of the fact that the former is constantly losing particles whereas the latter is constantly gaining them. Both the carbons are consumed constantly during the working of the arc, the positive partly due to the oxidation of the carbon and partly due to the transfer of particles from it to the negative, whereas the negative is consumed only by the oxidation. This explains why the positive carbon is consumed roughly twice as rapidly as the negative.

The light is mostly due to the crater of the positive, and hence if an arc-lamp be used for lighting purposes the positive carbon must be upwards. An ordinary arc-lamp requires only 40 to 60 volts. The candle-power of an ordinary arc-lamp varies from 1000 to 2000.* Roughly, an arc-lamp consumes 1 watt for each candle-power; thus an arc-lamp of 2000 candle-power will take over 9 amperes at 220 volts (street-voltage). As we shall see later, the arc-lamp is much more efficient than the ordinary incandescent lamps, efficient in the sense that for production of equally strong light it requires much less power than the incandescent lamps do.

The *flaming arcs* † which are generally used in streets in big cities like New York, Berlin, etc., consume only 0 27 watt per candle-power.

As has been remarked, the carbons are consumed during the working of the arc. If the carbons be fixed, the length of the arc will go on increasing till the light first becomes unsteady, and then goes out. To remedy it the carbons

* The candle-power of arc-lamps used in lighthouses may be as high as 50,000. Such arc-lamps require a current of about 300 to 400 amperes.

† In these arcs special carbons are used; generally they possess a core of calcium fluoride.

must be brought nearer each other either by hand, in which case the arc is said to be *hand-fed*, or by mechanical devices, in which case the arc is said to possess an *automatic feed*.

The mechanical devices are generally called regulators. They all depend upon the principle that as the distance between the carbons increases, the strength of the current diminishes. Suppose this current passes round an electro-magnet; when it becomes smaller, the electro-magnet loses some of its attracting power, and the carbon which is held fast on account of this attracting power is let loose, falls down a little bit, till the current becomes strong again and the electro-magnet works more efficiently and holds fast the carbon.

547. Incandescent Electric Lamps.—For domestic purposes incandescent lamps are used instead of the arc-lamps, for in the first place their candle-power (which varies from 5 to 100 c.p.) is much less than that of the arc-lamps, and therefore they are not dazzlingly bright. In the second place, the incandescent portion is much longer than is the case with arc-lamps, and therefore they are more useful for illuminating purposes; and thirdly, these lamps are more convenient than the arc-lamps.

All these incandescent lamps are merely a simple application of the principle that a conductor is heated when an electric current passes through it, and if the melting point of the conductor is very high and, in addition to it, possesses a very great resistance, it is rendered incandescent, *i.e.* becomes white hot. The first conductor to be used on a commercial scale was carbon. It was first introduced by Edison and Swan as far back as 1870, but it took some years before it developed into a thoroughly satisfactory type of lamp. But as carbon is readily burnt in air, the filament must be enclosed in vacuum. Carbon is highly refractory, and is therefore capable of withstanding a very high temperature before disintegrating. The resistance of carbon is very high as compared with that of metals, and it is therefore possible to obtain a high-resistance filament of a comparatively short length.

The efficiency of a lamp is the ratio of its candle-power to the power (in watts) consumed by it. A carbon lamp consumes about 4 watts for each candle-power; in other words, the light is 0·25 candle-power per watt. Compare it with the efficiency of an arc-lamp, which takes less than one watt for one candle-power.

Not only are the carbon-lamps inefficient, but there are other serious complaints against them. For instance, carbon-lamps cannot withstand sudden changes in the voltage, and at the same time have very little mechanical strength, and hence cannot withstand even a slight rough handling.

Nowadays the metallic filament lamps are very popular. So long as the metals tantalum, tungsten, etc., were not known, carbon was used in the lamps universally, but with the discovery of these metals the industry of lamp-making was revolutionised. These metals possess a very high melting point, and do not volatilise as carbon does at high temperatures. Moreover, they are less sensitive to the fluctuations of the voltage. But the greatest advantage that these lamps possess over the carbon-lamps is their efficiency. They consume only about 1·5 watts for one candle-power. The latest improved type of lamps is called half-watt lamps. As the name implies, it has a consumption of $\frac{1}{2}$ watt only per candle-power. These lamps are even more efficient than the ordinary arc-lamps. But the difficulty is that only the high candle-power lamps are manufactured so far, and therefore they cannot be used for domestic purposes.

548. Arrangement of Lights and Fans in an Electric Circuit. —The current that is used for lights and fans is generally taken from an electric-supply power-house. The voltage, *i.e.* the difference of potential between the positive and the negative

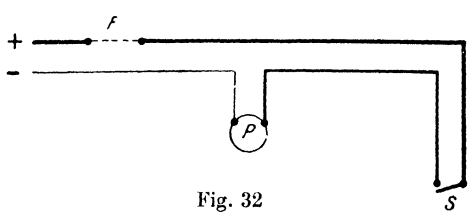


Fig. 32

pole, is usually 220. At some places it is 110. Let us see how electric energy is made available at various *points* for lights and fans. It should be clearly understood that by the word ***point*** *we mean an outlet in a circuit from which connection can be made to an electrical apparatus*, be it a lamp, a fan, or a heater.

The simplest form of the circuit is one which has only one point controlled by a switch. Fig. 32 shows the arrangement of such a point. The switch S is on the wall, somewhere at a convenient position, and P is the ceiling-rose either at the ceiling

or somewhere on the wall where electric energy is wanted. We may connect here either a light or a fan. The same circuit in a more simplified form is shown in Fig. 33.

549. Fuse.—*F* shows the position of the *fuse,* which consists of a short length of wire generally of an alloy of tin and lead enclosed in a porcelain cylinder or some other insulating

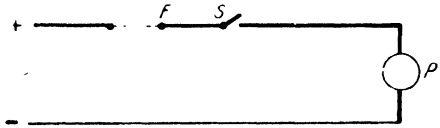
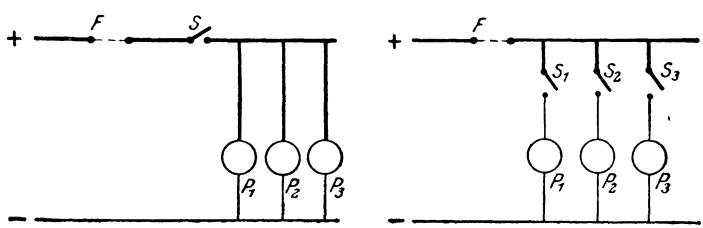


Fig. 33

device. Its object is to prevent too high a current from flowing through the wires which otherwise might melt on account of heat produced (which, as we have seen already, is proportional to the square of the current), and thereby damage the building. Fuse wire is made in different sizes; the size to be used depends upon the maximum current which the circuit is expected to carry. For example, if the maximum current allowable in a circuit is 3 amperes, a three-ampere fuse is used to protect the circuit. If the current passing through the circuit exceeds 3 amperes, the fuse melts and the circuit is broken, and the current is stopped. On replacing the fuse the current begins to flow through the circuit again.

549 (a). In practice one circuit has to feed generally a large number of points. Let us suppose we have three points in



(*a*) Fig. 34 (*b*)

one circuit. These points might be all controlled by one switch, or each point may have a separate switch of its own. The simplified sketch is shown in Fig. 34 (*a*) and (*b*).

The actual arrangement of wires is rather complex.

Note, the lamps or fans are arranged in parallel, which means that each lamp or fan carries the full voltage of the supply, *i.e.* 220 or 110 as the case may be.

If, however, the lamps or fans were designed for 110 volts and the current in our circuit comes at 220 volts, we should use the lamps or fans *in series*, as shown in Fig. 35.

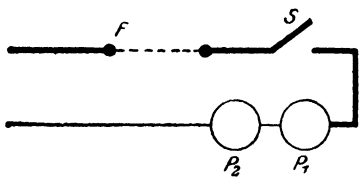


Fig. 35

The reason why we use two 110-volt lamps in series in a circuit carrying current at 220 volts is that the voltage of the current supply being far too great, the lamp would light up very brightly for a second or so, and then the filament would give way because the production of heat will be very great.

If, on the other hand, a lamp requiring 220 volts be used in a circuit carrying current at 110 volts, the lamp would burn very dimly, because the current passing through the filament is insufficient to heat it properly.

550. Thermo-electricity.—We have explained above how a current produces heat when it passes through a conductor having a high resistance. This is not the only relation between thermal and electrical phenomena. There is another, which was discovered by Seebeck, of Germany, in 1821. We shall now consider this relation.

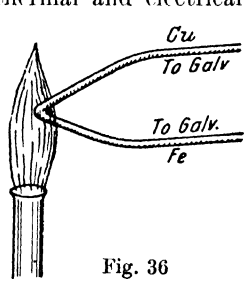


Fig. 36

It is well known that if in an experiment wires of different materials are used to close the circuit, no current flows through the circuit of itself.

Seebeck discovered in 1821 that this is true only if all the junctions are at the same temperature. On changing the temperature at one of the junctions, he found that the current flowed through the circuit. On heating the junction the current flowed in one direction, and on cooling it, the current flowed in the opposite direction. Such a current is called a **thermo-electric current,** and the pair of dissimilar metals used is called a **thermo-couple.** It should be noted that to get a thermo-electric current a pair of dissimilar metals must be used; it is immaterial which metals they are, so long as they are

dissimilar. The effect produced, however, is not equally strong in every case. The greatest effect is produced when a couple of antimony and bismuth is used. The effect in the case of copper and iron is also very great. Suppose two wires, one of copper and the other of iron, are twisted or soldered together at one end, and the free ends are joined to a galvanometer (Fig. 36). It is found that the current at the hot junction goes from copper to iron. If, on the other hand, this junction is placed in ice instead of in the flame of the bunsen burner, the current flows in the opposite direction at this junction, *i.e.* from iron to copper. The effect on the galvanometer is increased if a number of couples is used, as shown in Fig. 37.

If these couples be of antimony and bismuth, the effect on the galvanometer becomes still greater.

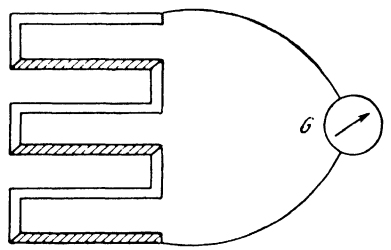


Fig. 37

An arrangement of several thermo-electric pairs grouped in such a manner that alternate junctions can be heated or cooled is called a **thermo-pile.** This instrument is not used as a source of electricity, but, on the other hand, is used to detect minute differences of temperature. All that is to be done is to keep one face at constant temperature and to expose the other face to the radiation, the leads being joined to the low resistance galvanometer. This arrangement can be made quite sensitive. For instance, it is quite easy to detect the heat given out by a match-stick lighted at a distance of 100 feet or more from the thermo-pile.

For measuring high temperatures such as those of molten metals a thermo-couple is generally used. The junction usually of platinum joined to a platinum-rhodium alloy is enclosed in a fireclay cylinder and is immersed in the molten metal. The other end is kept at some constant temperature, say that of steam. The couple is connected in series with a galvanometer. The deflection of the galvanometer is proportional to the temperature. If the instrument be calibrated beforehand, knowing the deflection, we can at once find the temperature of the hot body. This method can be used for measuring even the temperature of furnaces.

CHAPTER VI

CHEMICAL EFFECTS OF A CURRENT

551. The conductors of electricity can be divided into two classes:

1. Pure metals, whether solids or liquids (as mercury); they are not in the least affected so far as their chemical composition is concerned by the flow of an electric current through them.

2. Compounds which are decomposed as an electric current passes through them.

The substances which belong to the second class and suffer decomposition are called ***electrolytes,*** and the *act of decomposition is called* ***electrolysis.*** Electrolytes may be solids, as silver iodide, but generally they are liquids, such as dil. H_2SO_4 or acidulated water. Pure liquids, water, ether, etc., are imperfect conductors of electricity.

Faraday, in 1833, introduced a few technical names which are nowadays universally used. *The metal terminals immersed in the electrolyte are called* ***electrodes;*** *that by which the current enters is called* the ***anode*** or the positive electrode, *and that by which it leaves, the* ***cathode*** or the negative electrode. The products of decomposition are called **ions.** They are given off at the electrodes; *the product appearing at the anode is called the* ***anion,*** and *that which appears at the cathode the* ***cation.*** Sometimes anions and cations are termed as **electro-negative** and **electro-positive** ions respectively, because under the influence of the current the former travel to the positively charged plate, and hence they must themselves be charged negatively; and the latter travel to the cathode, and hence must be charged positively.

552. Electrolysis of Potassium Iodide.—Let us dissolve a few crystals of KI in water, and place the wires from a single cell in the solution at a distance from each other. It will be seen that bubbles of gas begin to rise from the wire connected to the

zinc (and hence the cathode) and that near the end of the second wire (anode, which is connected to the positive pole of the cell), there appears brown coloration due to the liberation of iodine. Actually the reactions are slightly different from those mentioned above. The salt is really decomposed into potassium and iodine as represented by the following equation :

$$2KI = 2K + 2I.$$

The potassium, as soon as it is liberated, begins to act upon the water, and liberates H_2 as

$$2K + 2H_2O = 2KOH + H_2.$$

Thus we find that the liberation of the gas at the cathode is a secondary effect.

This fact supplies us with an excellent test for detecting the kind of polarity of the two wires from a cell or mains. A piece of white paper is first soaked in a solution of starch, and then in the solution of potassium iodide in water. Placing the wires on the paper an inch or two apart, blue colour appears at the anode. The blue colour is due to the action of iodine on starch. This enables us to find out the positive pole or anode at once.

553. Electrolysis of Water.—Potassium iodide can be decomposed by a very weak current, but many substances, water being one of them, can only be decomposed by a strong current. For instance, water cannot be decomposed unless the potential difference between the electrodes amounts to about 1 7 volts. This is due to the presence of a back E.M.F., which we shall study later on.

For exhibiting the decomposition, an apparatus called **voltameter** is used. A common form of it is shown in Fig. 38. It consists of a glass tube bent into the form of a U-tube, calibrated to measure the volumes of the gases given off. Both the extremities of the tube are provided with stop-cocks. There is a platinum foil about half an inch wide and two inches long with a platinum wire welded to it, passing through the cork closing the lower end of each arm of the U-tube. It enables connections to be set up with the battery. To fill the tubes with water, generally a third tube, as shown in the figure, is provided. On passing the current, the gases are rapidly evolved, hydrogen bubbling at the cathode and oxygen at the anode. On collecting both the gases, it is found that the volume of hydrogen is twice that of oxygen, or that the weight of oxygen is eight times that of hydrogen. The chemical decomposition can be represented by the equation

$$2H_2O = 2H_2 + O_2.$$

But in practice the results are always somewhat lower owing to the difference in solubilities of the two gases in water, oxygen being slightly more soluble than H_2, and also due to the fact that a small portion of oxygen is always formed into ozone.

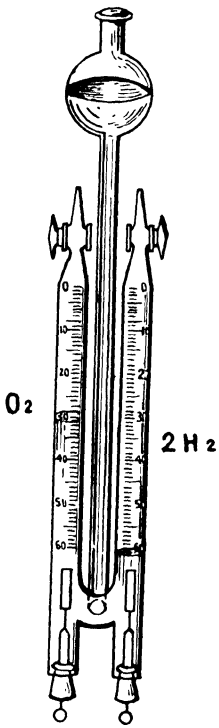


Fig. 38

If instead of water HCl be taken as an electrolyte, we find that hydrogen is liberated on the cathode and chlorine on the anode. In this case the electrodes should be of carbon instead of platinum, because the latter is attacked by nascent chlorine. Further, if the gases be collected, after some time they will be found to be nearly of equal volumes. Theoretically we should get exactly equal volumes according to the equation

$$2HCl = H_2 + Cl_2.$$

But practically we have to wait for some time before the gases can be collected in equal volumes. It is due to the fact that chlorine is very soluble in water, and hence the current must be passed for some time, to allow the hydrogen to escape so long as the liquid does not become saturated with Cl_2. When it is so the gases may be collected for measurement. They will be found to be equal in volumes.

Similarly other electrolytes can be tried. For instance, if $CuSO_4$ solution be tried, it will be seen that it is decomposed into its constituents. At the cathode Cu will be deposited, whereas on the anode oxygen will appear. In this case a secondary reaction is involved, because electrolysis tells us that the solution should be decomposed into Cu and SO_4 ions, according to the equation

$$CuSO_4 = Cu + SO_4.$$

But the SO_4 ions are unstable and hence they attack water, forming H_2SO_4, setting free O_2 as

$$2SO_4 + 2H_2O = 2H_2SO_4 + O_2.$$

554. Potassium and Sodium set free by Electrolysis.—For a long time it was supposed that potassium hydrate and sodium

hydrate were elements. But it was Davy who by using a battery of about 200 zinc-copper cells proved that caustic potash and soda were compounds. His experiment can be repeated with a battery of 5 or 6 grove cells, by making a cavity in the surface of a block of caustic potash and putting in it a small amount of mercury. The potash is placed on a platinum plate which is connected with the positive pole of the battery, while the mercury is touched with the negative pole. Potassium is liberated at the cathode, but forms an amalgam there with mercury, and oxygen is liberated at the anode. Potassium is further separated from mercury by distilling it in the absence of air.

555. Electro-chemical Series.—In all the experiments we have so far discussed, we have seen that H_2 invariably appears on the cathode, and where it does not appear, metals take its place. It is found that with any given substance always the same kind of ion is given off at a particular electrode. But which of the products will appear on the cathode and which on the anode depends upon the nature of the ions. A list can be drawn up which can tell us at once which of the two elements mentioned therein will be electro-positive with respect to the other. For instance, in the following list the element standing last with respect to the elements coming before will be electro-positive:

Oxygen	Platinum
Sulphur	Mercury
Chlorine	Silver
Carbon	Copper
Hydrogen	Zinc

The list shows that H_2 is electro-positive with respect to O_2.

It will be useful for the student to remember a memoria technica that *hydrogen and metals are liberated at the cathode.*

556. Laws of Faraday.—Faraday included in the circuit a good many voltameters in series and filled them with dil. H_2SO_4, having plates of different sizes and at different distances from one another. Hydrogen was collected at all the cathodes, and it was found that the volume was the same in all the cases. Even when the electrodes were made of different materials, say in one case Cu and zinc, in the second case Pt. and zinc, and so on, the volume of H_2 liberated was the same, which showed that *the amount of chemical action was equal at all parts of the circuit.*

He included then in the circuit voltameters containing dil. H_2SO_4 and $CuSO_4$. The amount of ions (H_2 and Cu) liberated at the cathodes were not equal, but if two parts by weight of H_2 were liberated at the cathode of water voltameter, then 63 parts by weight of Cu were liberated at the cathode of Cu voltameter ; these numbers 2 and 63 are to each other as the chemical equivalents.*

This fact was expressed by Faraday, and is called after his name Faraday's first law. *When the same current flows through several electrolytes the amounts of various ions liberated at different electrodes are to each other as their chemical equivalents.* For instance, let us suppose that our circuit is a complex one ; it contains voltameters containing acidulated water, $CuSO_4$, HCl, and silver nitrate ; we shall see that by weight for every gram of H_2 liberated, 31·5 grams of Cu, 35·5 grams of Cl, 8 grams of oxygen, 108 grams of silver are liberated at the corresponding electrodes.

It should be noted that elements which form different sets of salts have different chemical equivalents. For instance, when Cu behaves as a divalent (as in cupric salts) its chemical equivalent is 31·5, but when it behaves as a monovalent (as in cuprous salts) its chemical equivalent is 63.

The amount of an ion liberated at an electrode in a given time is directly proportional to the strength of the current. Thus if a current of 2 amperes liberates a certain amount of an ion, a current of 8 amperes will produce four times that amount. It is clear also that if the same current be allowed to flow for double the time, the amount of an ion liberated will be double, and so on. Combining these two aspects we come to the following law, which is known as Faraday's second law of electrolysis :

The amount of an ion liberated in a given time is directly proportional to the quantity of electricity which passes in that time through the electrolyte.

Both the laws can be combined into one statement as : **The amount of an ion liberated at an electrode is directly proportional to the total amount of electricity which passes and to the chemical equivalent of the substance.**

Faraday's second law of electrolysis can be stated also as that the amount of an ion liberated by a unit quantity of

* It means simply that in chemical reactions 63 parts of Cu can replace 2 parts of H_2. Note, chemical equivalent of an element is obtained by dividing the atomic weight by valency.

electricity (*i.e.* a coulomb) is fixed for any particular substance. It is called the **electro-chemical equivalent** of the substance. So to speak, *this is the mass of the substance liberated at an electrode by one ampere in one second* or two amperes in half a second. The electro-chemical equivalent of silver is known very accurately. The most recent determination gives the value as ·0011183 gram. From this we can calculate the electro-chemical equivalent of hydrogen very easily. It will be

$$\frac{\cdot 0011183}{108} = \cdot 00001036 \text{ gram.}$$

To know the electro-chemical equivalent of any substance we need multiply its chemical equivalent with the electro-chemical equivalent of H_2.

557. Weight of an ion liberated and the strength of Current. —Since the electro-chemical equivalent of a substance is the amount liberated or deposited at an electrode by one coulomb of electricity, naturally we see that when Q units of electricity pass through the electrolyte they will liberate Q times the electro-chemical equivalent. Let us denote by Z the electro-chemical equivalent of the substance, and by W the weight of the ion liberated, then

$$W = QZ$$
$$= CZt,$$

where $Q = Ct$, C being the current strength and t the time for which the current flows.

This enables us to measure the current. If we know the weight of an ion liberated, the time for which the current has flown and the electro-chemical equivalent, we can find out C. It will be equal to $\frac{W}{Zt}$. To determine practically the strength of a current by this method we use voltameters; the one commonly used is shown in Fig. 39. It is Lord Rayleigh's type of silver voltameter. It consists of a platinum basin which serves as the cathode, and an anode made up of pure silver, the electrolyte being a 15 per cent. solution of silver nitrate. Metallic silver is deposited over the platinum dish, when the current is passed for some time. Silver of the anode goes into the solution to form silver nitrate, with the result that anode is constantly consumed. If there be any impurities in silver they will be liberated as silver is dissolved, and they will fall down into the basin if not prevented from doing so in any way. The usual method is to wrap round the disc of the

anode filter paper, which does not allow the impurities to pass through, but allows the current to flow all right. The current employed should not be strong, for otherwise the deposition

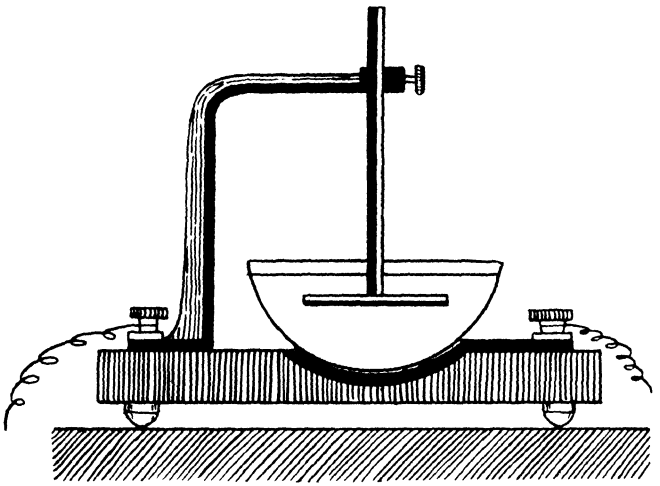


Fig. 39

would be brittle. The platinum dish is weighed before and after passing the current; this gives W; the time for which the current is passed is noted and Z is known from the tables, hence C can be found out very easily.

558. Theory of Electrolysis.—As far back as 1806, Grothüs observed that ions in the free state appear only at the electrodes, and that at the intermediate points they cannot be seen. He explained it as follows :

The molecules of an electrolyte ordinarily are arranged in a haphazard manner shown in the upper row of Fig. 40, each molecule being supposed to be made up of two parts (in the fig. one part is shown as shaded and the other clear), charged respectively with positive and negative electricity. Since these charges are equal, the molecule is neutral on the whole. When a difference of potential is set up between the electrodes A and K the molecules arrange themselves in a row in such a manner that the part charged with negative electricity is towards the anode. The 3rd row shows the stage when the molecule next the anode is decomposed, the electro-negative

part (or anion) goes over to the anode, the other part attacks the next molecule, getting hold of anion and leaving free cation. All the molecules of the electrolyte lying in a row undergo continual decomposition and re-composition. This process extends to the cathode, and the cation portion of the last molecule is set free.

In actual cases the number instead of being 5 only will be many millions.

If acidulated water be the electrolyte, H_2 and O_2 appear at the electrodes.

The modern theory is slightly different. The phenomena of vapour pressure, osmotic pressure, and freezing point of

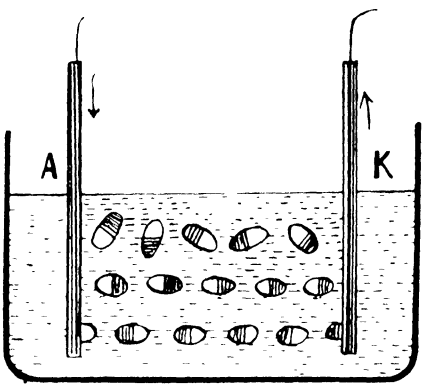


Fig. 40

solutions have changed our point of view. Now it is supposed that in an electrolyte the dissolved substance is either partially or completely dissociated into ions. The more dilute the solution the more complete is the dissociation or ionisation, so that in a very dilute solution practically all the molecules of the dissolved substance dissociate into free ions. Each of these constituent ions carries its own charge of electricity, and since the number of each kind (charged positively and negatively) is equal, hence on the whole the electrolyte is neutral. The explanation of electrolysis is very simple on this basis. When an external E.M.F. is applied, its influence is merely directive. It causes the positively charged ions to go over to the cathode, and the negatively charged ions to go over to the anode. The charges carried by the ions are given over to the electrodes and the ions themselves are liberated.

Let us consider the electrolysis of acidulated water. The electrodes are maintained by the battery at a constant difference of potential, the anode being at a higher potential and the cathode at a lower one. There is thus a field of electric force across the electrolyte. The charged ions begin to move, the positively charged ions to the cathode and the negatively charged ions to the anode. But the steady drift of positively charged ions can be considered as the (positive) current of electricity, and the drift of negatively charged ions in the opposite direction as a (positive) current in the first direction. Hence the total current can be regarded as due to the drift of both the sets of ions. This explains very satisfactorily the electrolytic conduction.

559. Polarisation.—While explaining the action of a voltaic cell, it was remarked that the E.M.F. of a cell rapidly falls off, due to polarisation, *i.e.* to the accumulation of hydrogen on the copper plate. We shall study it now somewhat in greater detail. Take a battery of two grove cells and pass a current for some time by connecting *A* and *C*. (as in Fig. 41)

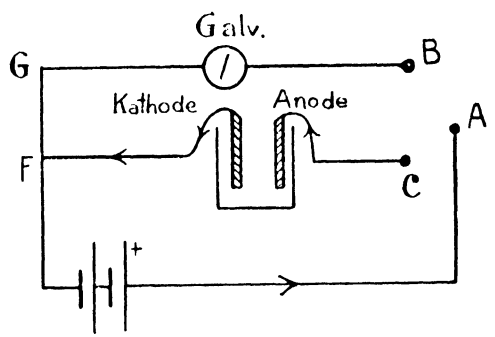


Fig. 41

through a water *voltameter* connected to the galvanometer and the battery. Now take out the plug and connect *C* and *B*. The galvanometer needle will deflect in the opposite direction to that due to the battery current. Where does this current come from when the battery is not in circuit ?

The origin of this current can be explained in the following way. During the process of decomposition each electrode is covered by the layer of liberated ions, in the case of water, with H_2 and O_2. These ions, which have been forced to separate by the primary current, have a strong tendency to

unite again. As a result of this tendency, when there is no external E.M.F., H_2 ions which are positively charged start back from the cathode and the oxygen ions proceed away from the anode. Note that this is the opposite direction, for at first H_2 ions went to the cathode and now they are coming away from it. Formerly the current flowed from *C* to *F*, but now, on the other hand, the current, when the battery is not in circuit, flows from *F* to *C*, *i.e.* in the opposite direction to that of the battery current. *This current is said to be due to polarisation and the plates are said to be* ***polarised.*** Thus we see that due to the deposition of decomposition products there is set up a **back E.M.F.**, so to speak, which opposes the battery current. The strength of this back E.M.F. depends upon the affinity of the liberated ions. In the case of H_2 and O_2 (water) the back E.M.F. is 1·45 volts. It is on account of this opposite E.M.F. that when the applied E.M.F. is smaller, the electrolysis stops after some time. Unless the external E.M.F. is greater than this opposite E.M.F. the action will not be continuous. This is the reason why in the case of water we require about 1·7 volts to produce decomposition.

The measurement of the back E.M.F. is very difficult, for as soon as the oxygen and hydrogen ions are removed, the current stops.

560. Storage Cells or Secondary Batteries.—The arrangement of Cu plates immersed in acidulated water through which an electric current is passed from a battery yields a current in the opposite direction to that of the battery when the plates are joined by a conductor; in other words, it acts as an electric cell. Such an arrangement is called a storage cell. It is sometimes called an accumulator. Generally the plates in the storage cells are not of Cu, but of lead, and the electrolyte is dilute H_2SO_4.

Let us go back once more to our last experiment. When *A* and *C* are joined, the current through the voltameter passes from the anode to the cathode. As a result of decomposition of the electrolyte hydrogen appears at the cathode and oxygen at the anode. The hydrogen has no affinity for lead of the cathode plate, and hence is liberated merely, but on the other hand oxygen combines with the lead of the anode, and forms the dark brown peroxide of lead (PbO_2). When this process has been going on for some time, let the connection between *A* and *C* be broken and that between *B* and *C* be established. It will be seen that the current through the outside circuit, *i.e.* *FGBC*, will be from anode to cathode

in the opposite direction to that of the primary (battery) current. But the deflection in the galvanometer will fall off, showing thereby the decrease of the current strength. The peroxide of lead will disappear, and both the plates, the anode as well as the cathode, will have lead sulphate formed on them, with slight traces of lead monoxide. If the first process is repeated, at the anode oxygen will appear, and it will act upon $PbSO_4$ as well as PbO, converting both into PbO_2, and at the cathode H_2 will act upon $PbSO_4$, reducing it to pure lead. Once more we are in a position to get the current from the voltameter in the external circuit. The first process of sending the current from the battery through the voltameter is termed **charging**, and the second process of taking the current from the voltameter is termed **discharging.**

This, in short, is the outline of a modern accumulator. Actually the plates are very complicated, and can be manufactured with great difficulty. Such a cell when fully charged up gives 2·2 volts as the potential difference between its terminals. The internal resistance is very small. When discharged the voltage should not be less than 1·8. An accumulator is extremely useful in a laboratory, for it gives a constant voltage. But it requires a lot of attention, otherwise it gets spoiled very soon.

561. The chemical changes which occur during the charging and discharging processes may be briefly stated as follows :

Discharging.—We shall start with a fully charged cell, *i.e.* with the anode coated with PbO_2 and the cathode as pure lead in the metallic state. During the discharge the hydrogen will come on to the anode instead of oxygen, and the following changes will take place :

$$PbO_2 + 2H = PbO + H_2O,$$

$$PbO + H_2SO_4 = \mathbf{PbSO_4} + H_2O.$$

Thus the result of discharge is that the anode becomes coated with $PbSO_4$.

Now let us see what takes place on the cathode. The oxygen will come over to it, and the following changes will occur :

$$2Pb + 2O = 2PbO,$$

$$2PbO + 2H_2SO_4 = 2\mathbf{PbSO_4} + 2H_2O.$$

Thus we see that both the anode and the cathode will have lead sulphate formed on them, and along with lead sulphate there will be traces of PbO.

Charging.—In charging, oxygen comes over to the anode, and hydrogen goes over to the cathode. The oxygen at the anode will combine with the monoxide and the sulphate as follows :

$$2PbO + 2O = 2\mathbf{PbO_2},$$

and $$2PbSO_4 + 2O + 2H_2O = 2\mathbf{PbO_2} + 2H_2SO_4.$$

Thus as a result of charging the anode becomes coated with PbO_2.

On the cathode the hydrogen acts upon the sulphate and monoxide as

$$PbSO_4 + 2H = \mathbf{Pb} + H_2SO_4,$$

$$PbO + 2H = \mathbf{Pb} + H_2O.$$

Thus we see that as the result of charging we get the anode coated with PbO_2 and the cathode becomes pure Pb, just the conditions with which we started the discharging process. Hence we learn that by charging, the cell is brought back to the original condition once more, and a discharge can be taken from it.

It should be borne in mind, however, that *it is not electricity which is stored, but it is the chemical potential energy with which an accumulator is charged.*

562. Electro-plating.—The accumulators are merely an application of the polarisation, or what amounts to the same thing, of electrolysis, because without electrolysis there can be no polarisation. Another application of electrolysis is electro-plating. We have learnt that the metal-radical is always deposited on the negative electrode, hence when we want a particular article to be coated with a precious metal, say gold, we must connect it to the negative pole of a battery and thus make it the cathode. In addition to it, the solution (electrolyte) must have gold as the metallic radical. Generally the double cyanide salt (*i.e.* the double cyanide of gold and potassium) is used. It forms an excellent depositing solution. As the electrolysis takes place, the gold of the solution is consumed, and to make up this loss anode is made up of gold, which goes into the solution.

Thus we see that to gild an article with gold we should make it the cathode, the anode should be of pure gold, and the solution should be of some gold salt. The same remarks apply to any other type of electro-plating.

The only point, however, is to make the deposit solid and firmly adherent. As to how it can be best done we cannot explain in detail, but we shall content ourselves with this

much information, that the article must first be thoroughly cleaned before it is made the cathode, and the current must be of suitable strength. Note, the deposit will be brittle if the current strength is high. The current strength will be different for different metals. For gold we require the current produced by, generally, 1 to 3 volts, depending upon the resistance of the circuit; copper requires a current yielded by 5 to 10 volts, and so on.

It is possible to coat even those bodies which are not conductors with these precious metals. Baskets, fruits, leaves, etc., have been gilded. They are, of course, first rendered conductors by depositing some conducting material upon them.

CHAPTER VII

ACTION OF CURRENTS ON CURRENTS AND MAGNETS ON CURRENTS

563. We have already seen that a wire carrying a current possesses a magnetic field around it. Evidently if we bring near the wire carrying current a magnet or, what is the same thing, another current, it will experience a magnetic force. We have studied rather in detail the action of currents on magnets; now we shall study briefly first the action of magnets on currents and then the action of currents on currents. De La Rive's floating battery (see §507) shows that a conductor carrying a current experiences a certain force when a magnet is brought in its neighbourhood. It is owing to this mechanical force that in one case the battery is attracted by the magnet and in the other case repelled. The motion of the conductor carrying a current when placed in a magnetic field can be demonstrated in several interesting ways. A simple experiment, like that of **Barlow's wheel,** very well illustrates this point. Barlow's wheel consists of a star-shaped figure with eight or ten points. It is supported on a horizontal metal axis, and the points of the stars in their lowest position dip into a mercury trough. The current passes from one binding screw up the support to the axis, and from there

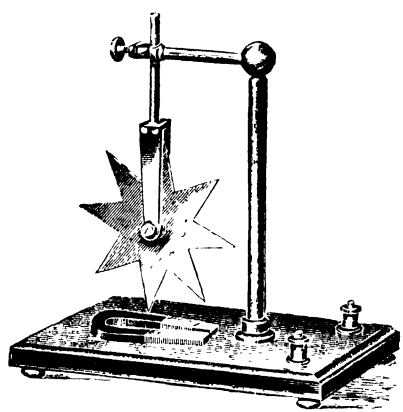


Fig. 42

down the vertical radius of the wheel to the mercury, whence it returns to the battery. If now an electro-magnet be placed with its poles on the opposite sides of the wheel, the latter begins to rotate, one point after another passing out of the mercury. The direction of rotation can be reversed either by changing the direction of the current or by reversing the polarity of the magnet. The effect is the greatest when the current cuts the lines of force at right angles. This shows that a conductor carrying current experiences a mechanical force in a magnetic field. The following is a convenient *memoria technica* for remembering the relations between the three directions—the current, the magnetic lines of force, and the motion of the conductor: *Hold the thuMb and the First finger of the left hand as fully extended as possible, and bend the mIddle finger at right angles to the palm; if the First finger represents the direction of the lines of force and the mIddle finger that of the current (i), the thuMb points in the direction of motion.* It is called **Fleming's left-hand rule.**

A very useful application of this principle is afforded by the **d'Arsonal galvanometer** (Fig. 43), in which the coil carrying the current to be measured moves in a magnetic field due to a permanent cylindrical magnet. The coil is rectangular in shape and is held between two wires, one above and the other below, through which the current comes from and goes back to the battery to complete the circuit. The vertical sides of the coil are perpendicular to the lines of force and hence experience the greatest force. The result is that the coil moves in the field—this motion is checked by the *torsion* of the suspending wire. The movement of the coil is read with the help of a mirror attached to the moving coil.

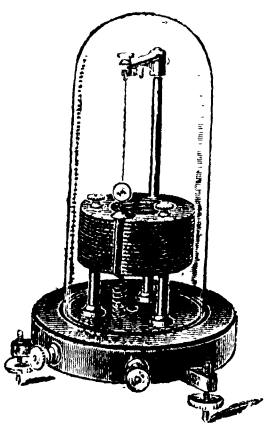


Fig. 43

It is one of the most useful types of galvanometers. The chief advantage arises from the fact that the external magnetic field does not produce any effect, nor is the instrument to be placed in any particular direction, for the zero position of the coil is independent of the direction of the magnetic field in which it is placed.

564. Mutual Action of Currents.—Take two flat spirals and suspend them at a short distance apart parallel to each other. Pass the current through the spirals in such a way that in both of them it flows round in the same direction. The spirals will be noticed to approach each other, showing thereby that the two spirals experience an attraction. Pass next the current in such a way that in one spiral it is in one direction, and in the other it is in the opposite direction. In this case the spirals

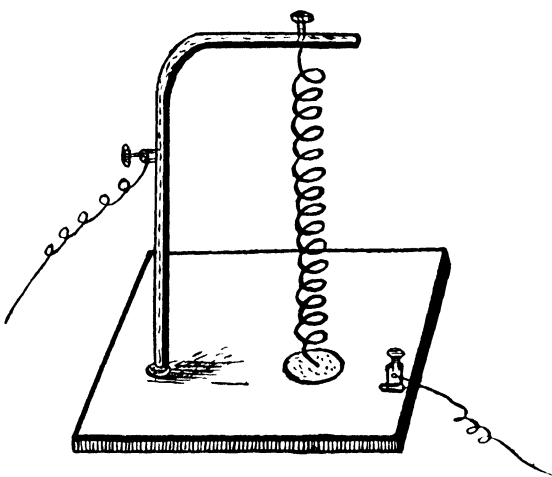


Fig. 44

will be seen to recede from each other, showing thereby that they mutually repel each other. Remembering our fundamental law of electro-magnetic polarity, that looking at the end of the coil, if the direction of the current is counter-clockwise, the end is a north pole, otherwise south, it is easy to see that, when the current is flowing round the spiral nearer to us in the clockwise direction as looked at by us, the face of it opposite to the other (*i.e.* away from us) will have north polarity, whereas the face opposite to us will be a south pole. And similarly the face of the other spiral which is towards us will be a south pole if the current is flowing in the same direction, otherwise a north pole. Thus we see that the attraction and repulsion of two parallel currents is just in accordance with our laws of attraction and repulsion of magnets.

Of course we could infer such a result directly also, knowing

that a circuit, whatever its shape when carrying a curren behaves like a magnet, and that two neighbouring currents wi behave like two magnets in the neighbourhood of each other.

These attractions and repulsions can be more easily demor strated by an apparatus called **Roget's vibrating spiral.** It is mad

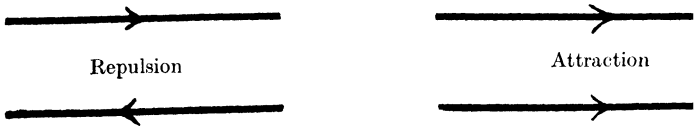
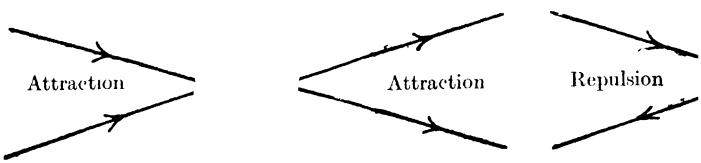


Fig. 45

up of 20 to 30 turns of moderately thick copper wire clampe at the upper end with a screw, the lower end dipping into basin of mercury. On passing the current the successive turı of the spiral attract each other, the result being that the lowe end is drawn out of the mercury and the circuit gets broke There being no current in the spiral there is no attractio



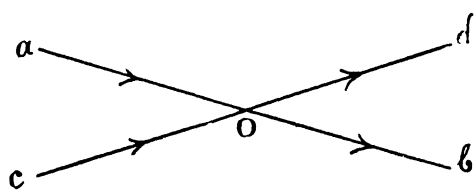


Fig. 45 (*a*)

between the successive turns; and hence the lower end cor sequently falls down again into the mercury cup. The attra tion once more draws out the lower end, and thus a vibratin motion is kept up as long as the battery connection lasts.

Ampère as a result of experiments like these enunciated tl following laws :

1. *Two currents which are parallel but in the opposite dire tions repel each other.*

2. *Two currents which are parallel but in the same direction attract each other.*

3. *When two currents tend to cross at a point they attract each other if they both flow either towards the point or away from it, but they repel each other when one flows towards the point and the other away from it.*

Let us consider next a particular case when two currents, *ab* and *cd*, cross each other in the plane of the paper and are movable about a point *O* (Fig. 45(*a*)). There will be attraction between *a* and *c* and *b* and *d*, while on the other hand there will be repulsion between *a* and *d* and *c* and *b*. The resultant effect will therefore be the tendency to bring the two currents into the same direction.

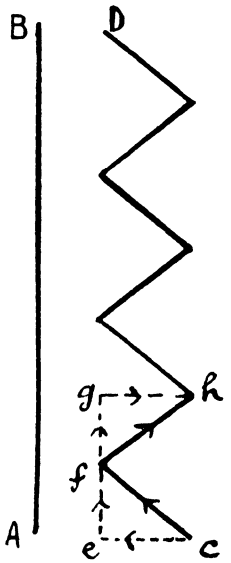


Fig. 45 (*b*)

Suppose further that one of the currents instead of being straight is sinous (Fig. 45 (*b*)). The mutual forces between the two currents *AB* and *CD* will be the same as between two straight currents, *i.e.* the sinous current can be replaced by a straight current without producing any change in the resultant tendency either of attraction or of repulsion

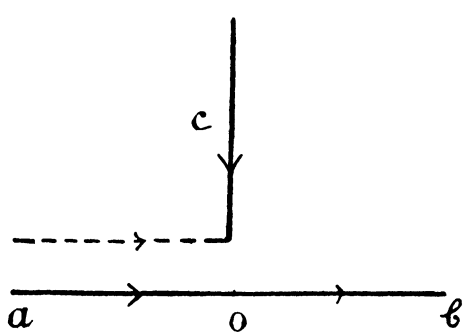


Fig. 45 (*c*)

It will be clearly understood if we remember that each small element of wire *CD* may be resolved into two components

of which that parallel to *AB* (*i.e.* *ef* and *fg*) is effective, whereas the consecutive components perpendicular to *AB* (*ec* and *gh*) neutralise each other, being in the opposite directions.

Further, when the current is wholly on one side of another current, as *c* is on one side of *ab* (Fig. 45 (*c*)), the action is quite simple, for there is attraction between *a* and *c*, and repulsion between *c* and *b*. If *c* be free to move it will tend to move towards *a* on account of attraction and away from *b* on account of repulsion. As a result of this attraction and repulsion *c* will come into the dotted position, *i.e.* will become parallel to *ab*, with the current in the same direction. We can apply Fleming's rule given already to see the direction in which *c* will be urged.

CHAPTER VIII

ELECTRO-MAGNETIC INDUCTION

565. Induction, as we already know, denotes the influence which bodies charged (whether with electricity or magnetism) exert upon bodies at a distance. Hitherto we have dealt, so to speak, with static induction, but now we shall deal with like effects produced by current electricity. The discovery of these

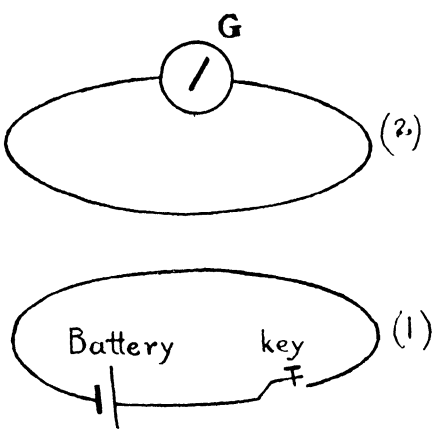


Fig. 46

like effects by Faraday in 1831 constitutes an epoch in the history of electrical science.

The principle of Faraday's original experiment may in a simple way be stated in the following manner. Let us suppose, we have two circuits, 1 and 2 (Fig. 46), of which 1 contains a battery and a key by which the circuit can be closed and broken very suddenly, while circuit 2 remains permanently

closed and contains a galvanometer but no battery. When circuit 1 is closed suddenly the galvanometer in circuit No. 2 shows a momentary deflection. But as soon as the current becomes steady in circuit No. 1 the galvanometer is not affected. Again, when the circuit is broken quickly, a throw in the galvanometer needle is noticed, which is momentary as before but is in the opposite direction.

In his original experiment Faraday used two helices; one of them was connected to the galvanometer and the other to the battery and the key instead of a single coil. He next, in place of making or breaking the circuit, simply changed the distance between the two helices, and he found that *during the time the distance was decreasing there was a current in the galvanometer*, and similarly, *so long as the distance was increasing there was a current.* But when the distance was not changed the galvanometer was not affected in the least. The direction of the deflection was, however, as before, opposite in the two halves of the experiment, viz. when the distance was increasing, it was in one direction; and when the distance was decreasing, it was in the opposite direction. Similar effects could be produced by bringing a magnet near the helix connected to the galvanometer or taking away from it.

As a result of all these experiments he enunciated the following law, which is generally called **Faraday's law:**

When the number of the lines of force through a circuit is changing, an induced E.M.F. is set up; the magnitude of the induced E.M.F. is proportional to the rate at which the number of the lines of force changes.

This induced E.M.F. is *momentary* only.

This law tells us that to induce an E.M.F. all that we have to do is to change the number of the lines of force through a circuit. Let us apply this law to the above-mentioned experiment. At first, when there is no current in circuit No. 1, neglecting the earth's field, there are no lines of force passing through circuit No. 2. But as soon as the circuit No. 1 is closed, the number of the lines of force threading the circuit No. 2 changes from zero to something definite and hence an E.M.F. is induced. Similarly, when the circuit is broken there is a change in the lines of force from something to zero, and hence an induced E.M.F. again. When circuit No. 1 is brought nearer the circuit 2, the number of the lines of force threading the latter increases, and hence an induced E.M.F.; similarly when a magnet is brought nearer, there is a change in the lines of force passing through the circuit containing the

galvanometer, and hence an induced E.M.F. The second part of the same law states, moreover, that the magnitude depends upon the rate at which the number of the lines of force changes, therefore the more suddenly the change is brought about the stronger the E.M.F. induced. This is the reason why all along we have been using the word suddenly with the make or the break of the circuit.

The circuit No. 1, which contains the battery and the key, is called the **primary circuit,** and the circuit No. 2, which contains the galvanometer, is called the **secondary circuit.**

We noticed in the beginning that the direction of the induced current at the make is opposite to that of the induced current at the break. It is found further that the direction of the induced current at the make is opposite to that of the primary current; while at the break the direction is the same as that of the primary current. Faraday's law may be stated more comprehensively as that (1) *when the number of the lines of force through a circuit* ***increases,*** *whether by starting the primary current, by increasing it, or by bringing the primary circuit nearer the secondary, an* ***inverse*** *current is induced in the secondary circuit*; and (2) *when the number of the lines of force* ***decreases,*** *whether by stopping the current or by diminishing it or by moving the primary coil to a greater distance, a* ***direct*** *current is induced.*

566. The number of the lines of force threading a circuit may be changed also by moving a magnet in the vicinity of the circuit containing the galvanometer. Let us study an experiment in which a magnet is moved near a circuit. Suppose we connect a coil to the galvanometer and a battery as shown in Fig. 47. When *A* and *B* are connected, the current from the battery flows through the circuit, but when, on the other hand, *B* and *C* are connected, the circuit is closed through the galvanometer only, the battery being left out. First pass a current from the battery and see whether the direction of the current along the face opposite to you is clockwise or counter-clockwise, and, moreover, which

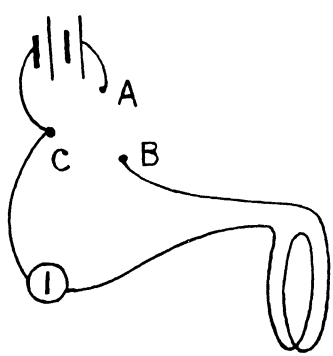


Fig. 47

way the needle of the galvanometer moves. (It should be remembered that by the direction of the deflection of the needle we always mean the deflection of the north pole.) This preliminary experiment connects up the direction of the deflection of the needle with the direction of the current in the coil; say when the deflection is to the right the direction of the current in the coil is clockwise, *i.e.* the face opposite to you possesses south polarity.

Now connect up *B* and *C*. Bring the north pole of the magnet near the coil suddenly. A throw of the needle to the left will be noticed. Take the north pole of the magnet back, the needle will be thrown to the right. Bring the south pole near, the needle will be deflected to the right; and take it back, it will move towards the left. Tabulating the results we see that:

Motion of Magnet	Direction of Deflection.	Polarity of the Face due to induced Current
N. pole moved near.	Left.	North pole.
N. pole withdrawn.	Right.	South pole.
S. pole moved near.	,,	South pole.
S. pole withdrawn.	Left.	North pole.

Just notice, when the north pole is brought near the coil, the polarity induced in the face of the coil opposite to you is also north-seeking. In other words, when we bring the north pole near the coil, the polarity induced tends to repel the north pole of the approaching magnet. Similarly when we withdraw the north pole away from it, the polarity induced is south-seeking; *i.e.* the attraction between the opposite polarity tends to oppose our action. Similar is the case when the south pole is brought near or is withdrawn. Remember, *the polarity induced is such that it tends to oppose our action.*

Compare it to the effect in the case of stopping or diminishing the currents. Whenever the current in the primary coil is stopped or diminished or the coil is removed farther away, the current induced is direct, *i.e.* it is in the same direction as the primary current, which means that we want to stop the current, but the induced current tends to continue it in the same direction in opposition to our attempts. When the current in the primary is started, increased, or the primary coil is brought nearer, the current induced is inverse, *i.e.* in

the opposite direction to that of the primary current, which in simple words means that when we want to start the current, the current induced wants to stop it by flowing in the opposite direction.

All these facts have been generalised into a law by Lenz, a Russian philosopher.

567. Lenz's Law.—*The direction of the induced current (and E.M.F.) is such that it tends to stop the movement which produces it.*

Let us see now if Lenz's law is just in accordance with the *law of conservation of energy.* The induced currents cannot be generated unless an equivalent amount of energy be consumed in some form or other, because otherwise we have produced energy without doing any work. We have already seen that when a north pole is brought near, north polarity is induced in the face of the coil towards the north pole of the magnet. In bringing the north pole nearer we have to work against the repulsion between similar poles. *It is this mechanical work which is converted into electrical energy.* Similarly when we withdraw the north pole, the induced polarity is south, and in the act of withdrawing we have to work against attraction, and as before it is this mechanical work which is converted into electrical energy. When the circuits are approaching, the induced current in the secondary is inverse, the direction of the currents is opposite in the two coils, and hence the repulsion. As before, it is the mechanical work done in overcoming repulsion which is converted into electrical energy. Thus we see that **Lenz's law is in no way an infringement upon the law of the conservation of energy,** but is, on the other hand, just in accordance with it.

We have seen already that when the number of the lines of force is decreasing, a direct current is induced, but when the number is increasing an inverse current is set up. A simple rule which is called Fleming's right-hand rule (clearly distinguish between this right-hand and the already explained left-hand rule) gives us at once the direction of the induced current.

Fleming's Right-hand Rule.—*Extend the thuMb, First finger, mIddle finger, until they are mutually at right angles to one another. Then if the First finger points along the magnetic Field, the thuMb along the Motion, the mIddle finger will give the direction of the induced current (i).*

568. Currents are not only induced in closed circuits, but even in the solid mass of conductors whenever they are made

to cut the magnetic lines of force. These currents that are induced in the solid mass are called **focault** or **eddy currents.**

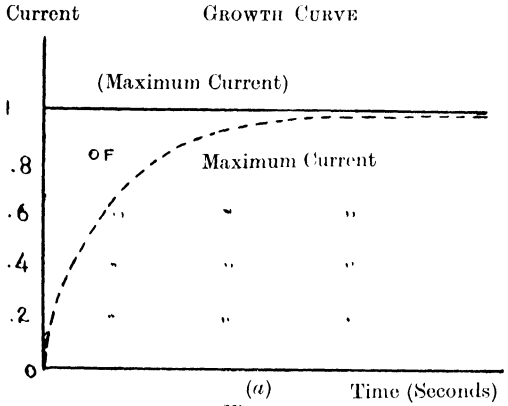


Fig. 48

When we have two circuits, one primary and the other secondary, the induction is said to be mutual, because then the current is

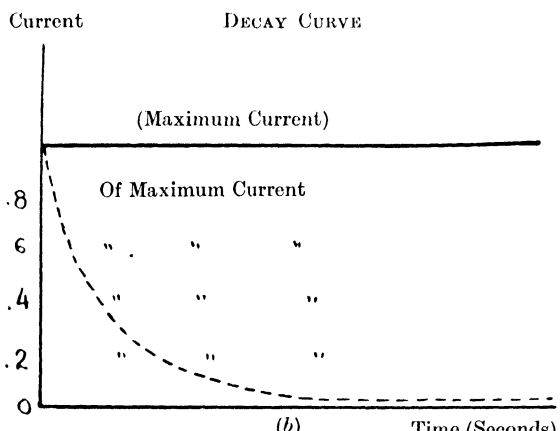


Fig. 48

induced in the secondary by the change in the strength of the current in the primary.

Let us now see what will be the case when we have a single

coil of a good many turns. When we complete the circuit the current in one turn acts on the other turn and induces E.M.F. and so on, with the result that the current takes some time in order to become steady. Similarly when we break the circuit, a direct circuit is induced in the neighbouring turns of the circuit. This induced current at the break is very strong, and is generally called **extra-current,** and it often shows itself as a spark.

When only a single coil is used, the effects are called **self-induction.** This effect is least when the conducting wire is straight, but it is very strong when the coil consists of a large number of turns. *The number of the lines of force threading the circuit when unit current flows in it is called the* **coefficient of self-induction.** It is generally denoted by the letter L.

As has been said, the effect of self-induction is to retard the growth or the decay of the current. In the absence of such an effect the current would take its maximum or zero value at once as the case may be. But on account of this effect the current takes a definite time to attain its final value —see curves in Fig. 48 (*a*) and (*b*).

569. If a graph be drawn representing the growth of the current a curve of the type shown in Fig. 48 is obtained. Unless the circuit contains a great many turns, or it includes iron, L is very small, so that in a very small fraction of a second the current attains its approximately final value. Theoretically the time to attain the final value will be infinite. Supposing L to be the coefficient of self-induction of the circuit and R its resistance, the ratio $\frac{L}{R}$ is called its **time-constant,** which gives roughly the time in which the current attains its $\frac{2}{3}$ of the final value. In the case of decay $\frac{L}{R}$ gives us the time in which $\frac{1}{3}$ of the current is left. The decay curve is always complementary to the growth curve. It should be remembered that this time is very small. One example will suffice to show the magnitude of the quantity we are dealing with. Supposing the resistance of the circuit to be 10 ohms and the coefficient of self-induction to be 1 unit, the time-constant is $\frac{1}{10}$ of a second. In practice the time-constant of the circuits with which we deal is a very small fraction of a second.

One more point should be noted, that the decay curve will be complementary only when the resistance of the circuit remains the same as before breaking it, because otherwise R becomes infinite and therefore $\frac{L}{R}$ infinitely small, *i.e.* zero. Hence it is supposed that after putting the battery out of circuit, the latter is closed.

570. Induction Coil.—A very important use of the electro-

magnetic induction is made in the case of what is called *induction or Ruhmkorff's coil.* It transforms large currents at low potential difference into small currents at high potential difference. The potential difference produced may be as high as that in the frictional electricity, *i.e.* of the order of several thousand volts. The important points in connection with the construction of the induction coil are :

1. The total number of the lines of force threading the primary coil should be large.
2. The secondary coil should consist of a large number of turns.
3. The current in the primary coil should be made and broken very rapidly.

Fig. 49 represents the section of the induction coil in which every essential detail is clear. Suppose PP' represents the

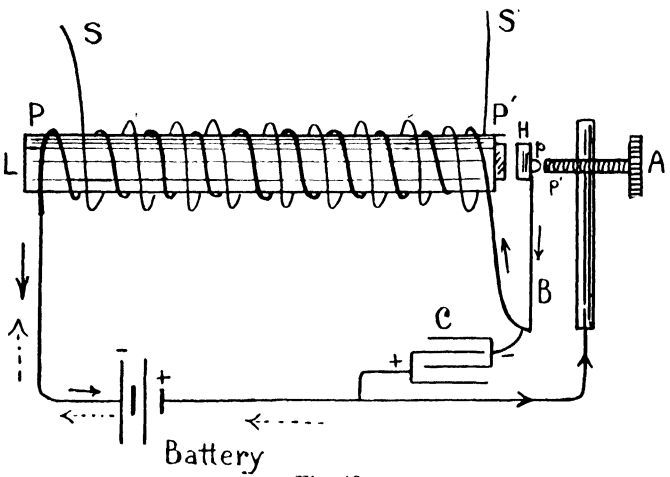


Fig. 49

primary coil, which consists of generally a few turns of thick insulated copper wire. To make the inductive action very strong, an iron core L, made up of generally soft-iron wires, is placed inside the primary coil. Due to the presence of this iron core a large number of the lines of force pass through the primary, and hence, on make and break, the change in the number of the lines of force is very much increased. Since

the E.M.F. induced depends upon the rate at which the number of the lines of force changes, therefore due to the presence of the soft-iron core, the inductive action increases. The secondary coil *SS*, which is wound over an ebonite tube enclosing the primary coil, consists of a very large number of turns of a thin insulated wire. The length of the wire in the secondary depends upon the spark-length required. To give an idea about the length of the wire used, it may be mentioned here that generally for every inch of spark-length about one mile long wire is used. The induction coils of these days give easily 13 to 14 inches spark-length, therefore according to our estimate they should have the wire of the secondary about 20 miles long. It is said that Mr. Spottiswood's induction coil could give a spark 42 inches long, the largest spark-length that any induction coil ever gave—and the length of the wire of the secondary coil was 280 miles! As the potential difference induced in the secondary coil is very great, of the order of several thousand volts, it is desirable that the insulation of each layer of the wire should be very good. The automatic make and break is shown at *A* and *H*. It consists of an iron hammer *H* carried by a brass spring *B*. The hammer is provided with a platinum tip (*p*). Just opposite to it, at the same height and in the same line, is an adjustable screw *A*, having also a platinum tip at the end (*p'*). By screwing in *A*, contact is established between *A* and *H*.

571. The Action of Coil.—Suppose that the current starts from the positive pole of the battery and goes to *A*, which is in contact with *H*. The current after coming to *H* passes through the primary coil *P'P* and thence back to the battery, thereby completing the circuit. The iron core becomes magnetised and consequently attracts *H*, breaking off the contact between *A* and *H*. As soon as this contact is broken, the current stops, and *L* (the iron core) loses its magnetism, the hammer falls back against *A*, due to the action of the spring, thus completing the circuit again. The iron core becomes magnetised and attracts the hammer, thus breaking the circuit once again. As soon as this contact is broken the hammer falls back to complete the circuit once more. This process continues automatically by the movement of the hammer.

Every time the contact is made, E.M.F. is induced in the secondary and so is the case at the break; the direction, however, being opposite in the two cases. Thus an induction coil gives alternating currents, *i.e.* first in one direction, then in the opposite direction, again in the first, and so on. For our

ordinary experiments we want uni-directional current. To do so the induced current at the make is usually suppressed, for this current is very weak in comparison with the induced current at the break.

Why the induced current at the break is stronger than at the make will be clear from the following:

When the current is made, the growth is retarded, due to the self-induction, hence the change in the lines of force is not sudden; and therefore the induced current in the secondary at the make is weakened. When the circuit is broken, the induced current, due to self-induction, retards the decay of the current, hence the induced current at the break is also weakened. We have learnt already that $\frac{L}{R}$ is the time-constant during which the current attains $\frac{2}{3}$ of the final value in the case of growth, and $\frac{1}{3}$ remains in the case of decay. Since, when the circuit is closed, R is very small, being the resistance of the circuit (including primary), $\frac{L}{R}$ is appreciable, but at the break R is infinite, and hence $\frac{L}{R}$ is almost zero. Thus we see that the time taken by the current to decay is very small in comparison with the time taken by the current to grow at the make. *Hence we see that at the break the induced current in the secondary is far stronger than at the make.*

This extra-current at the break, being very strong, causes sparking at the platinum contacts, thus retarding the stoppage of the current. In addition to it, it causes wear and tear of the platinum points. Therefore if we want to make the break sudden we must get rid of the sparking. For this purpose a condenser is put in the primary circuit in such a way that it is in parallels with the spark gap. The action of the condenser may be briefly explained thus. When the current is made, the condenser is at the same time charged and hence the current takes a longer time to attain its final value than it would do in the absence of the condenser. *Its effect is thus to minimise the induced current in the secondary at the make.* But when, on the other hand, the circuit is broken, before the extra-current sparks across the gap it must charge the condenser, which after being charged sends the current in the opposite direction to that of the primary current or the direct current induced. Thus the result is that *it helps to make the break more sudden in addition to stopping the sparking.* Hence we see that the action of the condenser is to make the current at the make very much weaker,

practically nil, and that at the break very strong, and besides this, to minimise the waste by sparking. The result of all this arrangement is that the induction coil gives almost uni-directional currents, viz. only those due to the breaks of the primary current. These currents, however, are intermittent.

572. Discharge through Vacuum Tubes.—Take a glass tube which has at its two ends platinum wires fused in it (Fig. 50). In the middle of this tube a narrow glass tube is sealed on to it, over which is slipped on a rubber-tube connected to an air pump. Connect the two terminals to the secondary coil of an induction coil, and try to pass the discharge through the tube. No effect will be produced, because the air inside the tube offers a very great resistance, and hence discharge cannot pass. Now begin to exhaust the tube; the resistance diminishes until when the pressure is fairly low, the tube is filled with a crimson luminous column, extending from the anode to the cathode. It is called the **positive column.** The colour of this column depends on the gas present inside the tube. With air, for instance, the colour is red; with carbon di-oxide it is white; with hydrogen, blue, and so on. As exhaustion proceeds still further, the positive column is divided into *striae*, and a dark space called **Faraday dark space** appears, which itself becomes covered with a glow called negative glow. At a still lower pressure (*i.e.* at about a pressure of 1 mm. of Hg.) the striae thicken and another dark space called Crookes' dark space (*A* in Fig. 50) makes its appearance. At a still higher vacuum the striations disappear and the second dark space extends over the sides of the tube, which begin to fluoresce. The colour depends upon the composition of glass. For soda-glass it is yellowish-green.

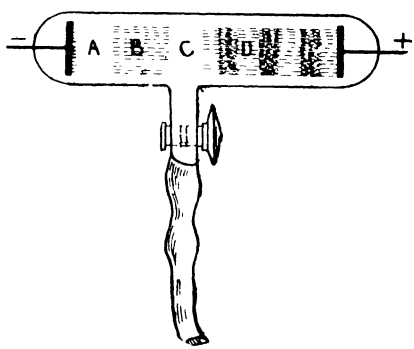


Fig. 50

573. Cathode Rays.—As said above, when the boundary of the Crookes' dark space extends to the walls of the tube, a

bright fluorescence is produced. Crookes while working with a discharge tube in this condition found that if a mica cross be fixed in front of the cathode, a sharp shadow of the cross was formed on the walls of the tube. He found further that by taking a concave cathode considerable heating effect could be produced. From such observations Crookes concluded that a radiation starts from the cathode which travels in straight lines. The name cathode rays was given to this radiation. The cathode rays are found to possess the following properties :

(i) They produce fluorescence in the glass walls of the tube.

(ii) When concentrated on a substance they produce a strong heating effect.

(iii) They cast sharp shadows of objects placed in their path.

(iv) They give a negative charge to the bodies they fall upon.

(v) They are deflected by an electric and a magnetic field.

These properties lead us to think that the cathode rays consist of extremely small negatively charged particles moving with a tremendous speed (it may be as high as 100,000 miles per second). From the magnitude of deflection that a stream of cathode rays undergoes in a known magnetic and electric field the mass and speed of these particles can be calculated. These particles, which possess a mass equal to $\frac{1}{1800}$ of the hydrogen atom, are called **electrons.** Since their discovery it has been found that they are fundamental constituents of all atoms and play a part in phenomena as widely different as conduction of heat, conduction of electricity, emission of spectra, etc.

574. X-Rays.—In 1896 Professor Röntgen of Munich discovered that cathode rays when they strike against a metal of high atomic weight (like platinum) give rise to a new type of invisible rays, which differ from the original cathode rays in that they are deflected neither by a magnetic nor by an electric field. Röntgen called these mysterious rays "X-rays". They are sometimes called Röntgen rays as well.

The interesting property of these rays is their great penetrating power. For instance, they produce a fluorescence after passing through a book of 1000 pages. Thick blocks of wood or ebonite do not prevent this action. A sheet of aluminium an inch thick only reduces this action. Plates of copper, silver, lead, gold, when they are thin, also permit the rays to pass through. The power of penetration varies with the density of the material, the denser the medium the less the penetrating power of the X-rays. It is this difference of penetrating power which has rendered the X-rays of immense value

in surgery. When they are passed through the hand the shadow of the bones on the fluorescent screen is dark, while that of the surrounding flesh is very faint; for the bones being denser stop the X-rays more effectively than the flesh. If a sensitive photographic plate be substituted for the fluorescent screen a photograph can be obtained. The radiograph, as this photograph is generally called, helps the surgeon to detect fractures, foreign bodies, etc. (see Fig. 51).

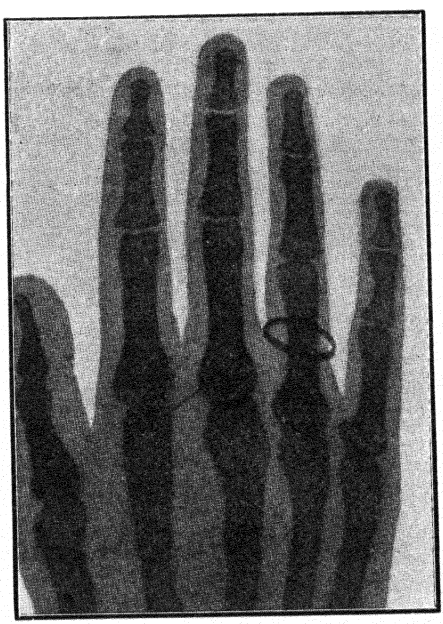


Fig. 51

X-rays have proved of immense value to Physicists as well. They have enabled them to investigate the structure of crystals and the arrangement of the atoms in the molecules, etc.

Nature of the X-rays.—Their nature was a mystery for a long time, but we now know definitely that the X-rays consist of extremely short waves. They are very much shorter than the light waves. It will help the student to form an idea of the shortness of the X-ray waves when he is told that the wave-length of the yellow light is 6×10^{-5} cm. and that of X-rays is

of the order of 10^{-8} cm., *i.e.* a thousand times smaller. They are produced when cathode rays strike against a target made

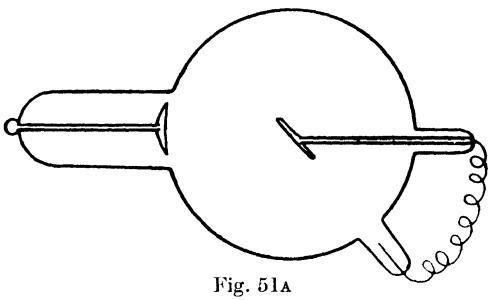


Fig. 51A

of a heavy piece of tungsten or platinum. An X-ray tube is shown in Fig. 51A.

CHAPTER IX

PRACTICAL APPLICATIONS

575. Electric Bell.—The simplest application of electro-magnetism is an electric bell, widely used for domestic and other purposes. One end of the coil of the electro-magnet *EN* (Fig. 52) is connected to one of the terminals *T*, which is a simple binding screw. The other end of the coil is connected to the armature *A*, against which the spring *S* is pressing. By adjusting the screw the spring can be made to touch the armature *A*. Now when the circuit is closed by, say, pressing a key, the current starts from the + pole and goes to one of the terminals, say *T*, and therefrom to the electromagnet, and thence to *A*. From *A* it passes on to *S* and the screw, wherefrom it goes over to *T'*, and finally to the other end of the battery. As soon as the circuit is completed, the electro-magnet attracts the armature *A*; this makes the clapper which is carried by the armature strike against the gong *G*, breaking the contact between the spring and the armature. The current stops and the electro-magnet no longer attracts the armature, which consequently falls back against the spring, thus closing the circuit once more. Again the armature is attracted, producing sound due to the striking of the clapper against the gong. The connection being

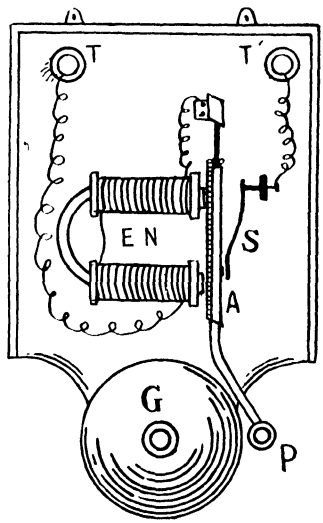


Fig. 52

broken, the armature is no longer attracted and falls back against the spring. This vibratory motion causes the bell to

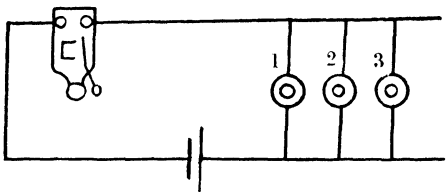


Fig. 53

ring, so long as the key is kept pressed. When one bell is to be rung from several places (1, 2, 3), the circuits are arranged as shown in Fig. 53. At each of these places a simple key called a push is placed. It consists of a small button which when pushed establishes contact between two metal plates in the framework of the button.

576. The Dynamo.—Let (Fig. 54) a plane coil of wire $ABCD$ be so arranged that it can be rotated about the horizontal axis LM, between the poles of two magnets. Suppose to start with, the plane of the coil is vertical. Turn the handle in the anticlockwise direction; the wire AC, if it be supposed to be above first, moves down during the first half turn. By Fleming's right-hand rule we should expect the induced E.M.F.

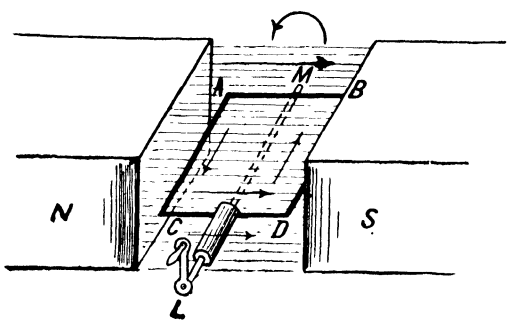


Fig. 54

to send the current round the coil in the direction of A to C. At the same time the wire BD is moving up, hence the induced current will tend to flow from D to B. As a result of this during the first half turn the current tends to flow round the loop in

the direction $ACDB$. Now let us consider the second half turn. The wire DB moves down, hence the induced current tends to flow from B to D and the wire AC moves up, therefore the induced current tends to flow from C to A. In other words, during the second half turn the current flows in the direction $BDCA$, opposite to the former direction. This shows that the induced E.M.F. changes its direction at each half revolution, or, in other words, there are two currents produced in the coil during each revolution. If the coil be

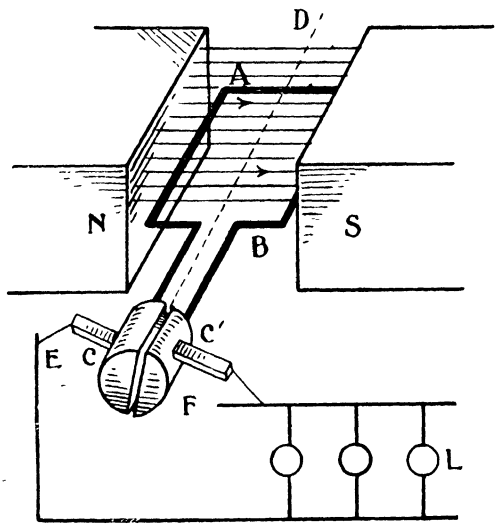


Fig. 55

connected to an outer circuit, it will send round the circuit a current, first in one direction and then in the other, during one revolution.

To get the current in the same direction in the external circuit, a simple device called a **commutator** is used. It consists of a ring split in sectors as shown at C and C' in Fig. 55. One end of the coil is joined to one sector and the other to the other sector. Two brushes, E and F, which lead to the external circuit, are alternately in contact with these sectors as they revolve. The brushes are so arranged in position that when the current in the coil first changes its direction, the brushes change over from one sector to the other. For instance, suppose

the current comes out at C during the first half revolution and at C' during the second half revolution. The current in the external circuit during the first half revolution is in the direction of ELF. During the second half revolution the sector C' (where the current comes out) comes in contact with the brush E, and hence the current in the external circuit flows again in the direction ELF. This shows that with this arrangement we get the current in the external circuit in the same direction. This explains the *principle* of a dynamo (a generator of electricity).

In practice a single coil of wire is too flimsy a thing to be rotated without some support, and moreover, it cuts the magnetic field only twice in every revolution, hence in an actual machine it is replaced by a large number of coils, each coil consisting of a good many turns. The commutator is subdivided into as many segments as the number of coils.

Note carefully the parts of a dynamo. It consists essentially of magnets called **field magnets,** coils of wire called **armature,** and a split-ring called **commutator.**

577. Electric Motor.—In construction the electric motor differs in no essential respect from the dynamo. As a matter of fact, one and the same machine can be used for either purpose.

Suppose in Fig. 55 the external circuit is replaced by a battery or some other source of electricity, as the city mains. A current of electricity will pass through the coils of the armature. The coils being in the magnetic field will experience mechanical force, and hence by applying the left-hand rule we can find out the direction in which the armature will be urged to rotate.

578. The Electric Telegraph.—The object of the telegraph

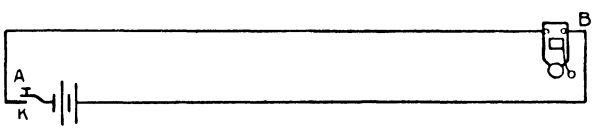


Fig. 56

is to enable two persons at distant stations to communicate with each other. Why it is called telegraph is a matter of wonder. For telegraph means to write at a distance, and throughout the development of telegraphy this idea never swayed the mind of the scientists. Of course nowadays it is possible to write even at the receiving stations, but telegraph was never begun with the idea the name suggests. To

explain the underlying principle, suppose that a man at the station A wants to send a message to another person at the station B (Fig. 56). We shall take it for granted that they have fixed a certain code with the help of which they can understand each other. Suppose they have agreed upon the code that if the electric bell rings for one second continually it is letter A, if for 2 seconds it is B, if for 3 seconds it is C, and so on. Evidently such a code demands that the man at A should be able to ring a bell at B for different durations. The electric bell enables him to do this easily, for if he has at the sending station a battery and a key and at the receiving station an electric bell connected with the sending station as shown in Fig. 56, he can press his key for different durations and thus send a complete message.

As the bell is not a convenient receiver other instruments have been used in practice. The first instrument used was based upon O'ersted's discovery of the deflection of a magnet by a current. It was noticed that by completing a circuit which connected the two distant stations a magnetic needle could be deflected at the receiving station. For each letter of the alphabet a separate magnetic needle was used in the beginning, and with this arrangement it was possible to signal a complete message from one station to the other. Soon after, however, it was realised that by a properly arranged code of signals two magnets were sufficient, one of which moved to the right and the other to the left. Still later, it was found that one needle alone could serve the purpose; by reversing the current the direction of the deflection could be reversed (see the single-needle instrument, Fig. 57). Let us see how, with the help of this, a message can be sent from one station to the other. As soon as the battery circuit is closed, the needle at the receiving station is deflected. (It is generally the deflection of the N. pole of the needle which is called the direction of deflection.)

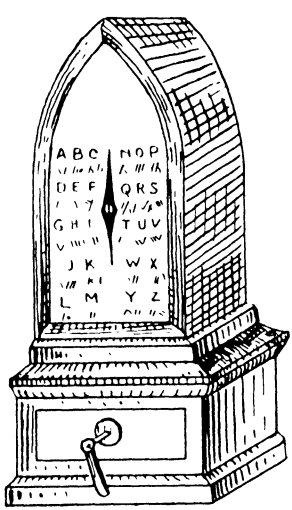


Fig. 57

Suppose that we fix upon a code in the following manner: One deflection to the left and one to the right stand for the letter A. One to the right and three to the left mean B, and so on. By simply fixing a different combination of these deflections all the letters of the alphabet can be denoted very easily. The instrument used is called the **single-needle instrument** (Fig. 57).

579. The recent form of the recorder is called a **sounder.** It consists essentially of an electro-magnet which when the current passes round it attracts an armature at the end of a lever which is pivoted in the middle and whose other end moves between two screws. When the armature is attracted the distant end of the lever moves up and strikes against the screw and produces a sound. The duration of the sound produced may be long or short. By suitably combining the long and short durations the whole of the alphabet can be represented. They correspond to dots and dashes of the

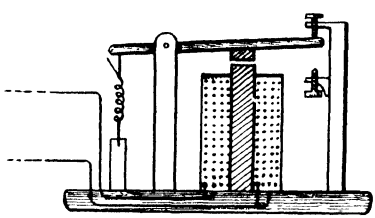


Fig. 58

Morse system. On this system the letters of the alphabet, stop, etc., are represented by the combination of dots and dashes.

A · —	K — · —	U · · —
B — · · ·	L · — · ·	V · · · —
C — · — ·	M — —	W · — —
D — · ·	N — ·	X — · · —
E ·	O — — —	Y — · — —
F · · — ·	P · — — ·	Z — — · ·
G — — ·	Q — — · —	Full stop · · · · · ·
H · · · ·	R · — ·	Comma · — · — · —
I · ·	S · · ·	Interrogative · · — — · ·
J · — — —	T —	Hyphen — · · · ·

In another recent form the end of the lever instead of moving up and down between two screws has a point fixed

to it which presses against a tape of paper wound over a reel worked by a clockwork. The paper passes between two friction rollers. If the key is pressed for a long time a dash is written on the tape, but if it is pressed for a short time a dot is written on that.

This method is far more convenient than that of the sounder, because the latter requires a good deal of training of the ear.

So far we have considered one station as the sending station and the other as the receiving station only, but in actual practice both the stations must be made sending as well as receiving. For this purpose the battery, the key, and the receiver must be present at both the stations. Further, it must be noted that to complete the circuit we require two wires, and actually when we connect two stations at a distance of ten miles from each other the cost of two wires is very great.

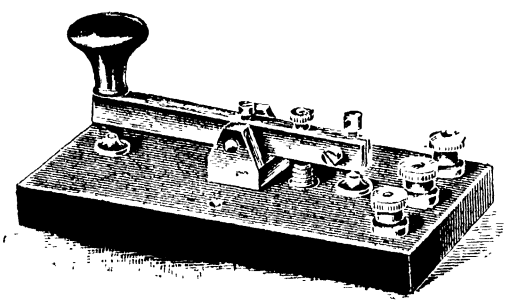


Fig. 59

If somehow or other only one wire could be used the cost would be reduced to half, which means a saving of 50 per cent. in the cost of the wires. Although the rocks and stones are bad conductors of electricity, still the inner layers of the earth, being moist, are fairly good conductors; moreover, the cross-section being so huge the resistance offered by the earth is very small indeed. This fact is taken advantage of and the earth is used to complete the circuit. The ends of the wires at both the stations are connected with metal plates, well buried in permanent damp soil.

Before we take up the actual arrangement for the transmission of the message we shall explain a special instrument (Fig. 59) called the **Morse key,** after the name of its maker. It is a simple contact breaker consisting of a brass lever,

movable about a horizontal axis. It has an ebonite knob over a metal stud at one end and an adjustable screw at the other end of the lever. On the base of the key there are two studs, one just below the adjustable screw and the other below the knob. In the ordinary position when the knob is not pressed, due to the action of a spring the screw remains in contact with the stud below, so that if a wire is joined to the fulcrum the connection is set up through the brass lever between the wire and the stud below the screw. But, on the other hand, when the knob is pressed, connection is set up between the wire and the front stud and is broken between the wire and the back stud.

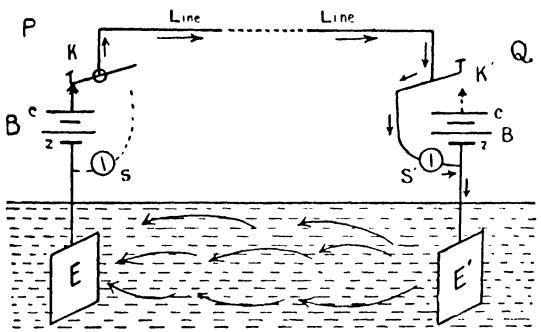


Fig. 60

580. Now we are in a position to understand the arrangement for the transmission and receiving of a message. Each of the two stations P as well as Q is provided with a battery, a Morse key, and a Morse sounder.

We shall first suppose that we want to send the message from P. The zinc electrode of the battery B is joined to the earth, whereas the Cu electrode is joined to the stud under the knob of the Morse key (Fig. 60). The fulcrum is joined to the line. On the other station Q the line is joined to the fulcrum of K'. The key when not pressed connects the lever to the sounder S', which is connected on the other side to the earth at E'. When the key K is pressed the connection between the front stud and the Cu end of the battery is established. The current starts from, say, the Cu plate, goes to the line and thence comes to the screw of the key K', passes through S' to E', the earth plate. Therefrom it comes

back to E and to the zinc end of the battery and the circuit is completed. The sounder produces long or short sound as desired by the sender at P. This explains how the messages can be sent from P to Q. Let us see now how to receive a message at P and send it from Q. On pressing the key K' the connection is established between the battery and K'. The current starts from the copper end of the battery, goes to the line, passes through the back stud of K to S and thence to the earth, wherefrom it comes over to E' and finally to the battery. The circuit is completed, and the sounder S produces sound. Thus we see that with the arrangement sketched in Fig. 60 it is possible to send and receive messages at one and the same station. As a matter of fact, both the stations are sending as well as receiving. It should be noted that it is

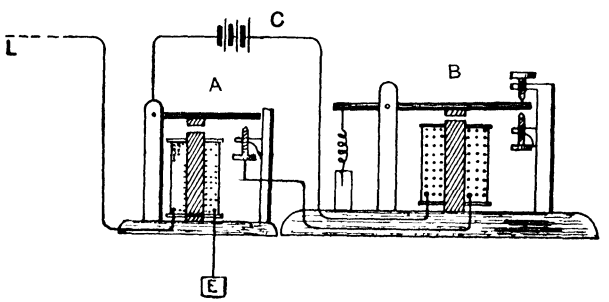


Fig. 61

not possible with these connections to send simultaneously messages from both the stations; for, suppose the man at P is sending a message by pressing his key K and at the same instant the man at Q wants to send a message and presses his key K', the result will be that at no station the current will pass through the sounder.

Simultaneously sending the messages from both the stations can be effected by other systems into which we shall not enter.

581. Relay.—When the distance over which a message is to be sent is very great the current becomes too weak to work the sounder. It then becomes necessary to introduce some contrivance which will enable us to use the local current at the receiving station. The instrument used for this purpose is called a relay. It is an electro-magnet, having a coil of very fine wire A (Fig. 61). The weak current is made to pass round it, and as soon as the current passes round it an arma-

ture of soft iron piece is attracted, which closes the local circuit, including a battery C and a sounder B (see Fig. 61). Thus the weak current brings into action a strong local current which works the sounder.

582. Telephones.—One of the most wonderful applications of Faraday's induced currents in modern times is found in Professor Graham Bell's telephone.

To understand the principle of Bell's instrument, consider the following simple case. Consider a bar magnet NS, surrounded on all sides by its lines of force. Bring near it a diaphragm D (consisting of a thin iron piece). The lines of force try to crowd in this piece and hence undergo a rearrangement. Even when the distance of D from N varies, there is a disturbance in the distribution of the lines of force. Now let us assume that the end N is surrounded by a coil of wire.

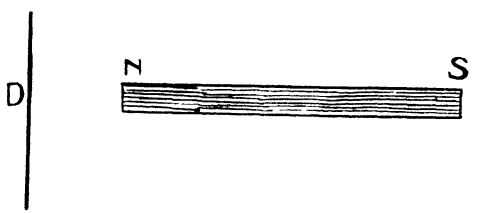


Fig. 62

Due to the magnet a definite number of lines of force will pass through it. But if the number changes in any way, an induced current is set up. When D vibrates in front of N there is a disturbance in the distribution of the lines of force,—hence their number passing through the coil changes, which results in the production of an induced current. The direction of the induced current will depend upon the fact whether the diaphragm moves towards or away from N. This shows that the vibration of the diaphragm in front of the magnet generates alternating currents in the coil. If the terminals of this coil be connected with those of another coil which is similarly wound over the pole of a bar magnet, these alternating currents will flow round the coil surrounding the pole of the second magnet, and will alternately strengthen or weaken the pole. If a diaphragm of iron be placed in front of the second magnet it will move towards or away from the magnet. This is the principle of the telephone.

With such an arrangement it is possible to reproduce the

vibrations of the diaphragm at another station. Remember that every sound is conveyed by the pulses communicated to the air, and that the number of these pulses define the pitch of the note. As the diaphragm at the receiving station is vibrating exactly at the same rate as the diaphragm at the speaker's, hence it is evident that the note given out by the receiver's diaphragm is of the same pitch as that impressed upon the speaker's. Not only is the pitch reproduced exactly, but also the quality or timbre of the note. Thus we see that the induced currents reproduce exactly the same sound at the receiver's instrument, the only difference being that the sound is very much weaker, and that it is affected by the natural vibration of the diaphragm, which gives the sound a peculiar nasal intonation.

Note carefully, *it is* ***electricity and not the sound*** *which goes from one place to another.*

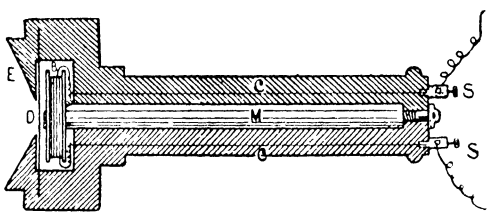


Fig. 63

The well-known Bell's telephone, which was at first used both as a transmitter and a receiver, but which is now used only as a receiver, is shown in Fig. 63. *M* is a permanent steel magnet having a coil of fine silk-covered Cu wire wound round its one end. The magnet is surrounded by a wooden case *CC*. The ends of the coil are connected to the screws *S* and *S*. In front of the magnet, but very near, almost touching it, the diaphragm *D* is placed, which is screwed down by the mouth-piece *E*. At the distant station a similar contrivance is joined to the telephone at the sending station by the two wires. The sender speaks into the mouth-piece of his instrument, while the receiver holds his instrument to his ear.

583. Nowadays there is a separate instrument for speaking. It is called a **microphone.** It depends upon the principle that *the resistance at the point of contact of two conductors varies as they are more or less strongly pressed together.* Carbon

is sensitive especially to such variation. Two carbon plates have a loose packing of granular carbon between them. The carbon plate which is to serve as a diaphragm is protected on the outside by a wire-gauze. The microphone is connected to the receiving telephone as shown in Fig. 64. When a man speaks into the mouthpiece, the diaphragm is set into vibrations. When the diaphragm moves in, the carbon granules are forced closely together; and when it moves out, they are less closely

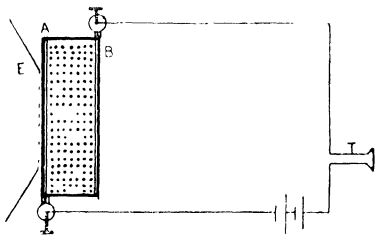


Fig. 64

compressed. This changes the resistance and thereby the strength of the current flowing round the coil. This change in the strength of the current flowing in the coil round the magnet of the receiving telephone causes the diaphragm in front of it to vibrate. The sounds produced with this arrangement are very much louder as compared with those produced by Bell's telephone. Fig. 65 shows the microphone. How both the stations can be used as sending and receiving stations will be clear from Fig. 66. Note, it is important to understand that the change in resistance of the carbon granules is very small. To produce sufficiently great changes in the current, it is necessary that the *total resistance should be very small.* It is in practice impossible to arrange if the microphone be directly connected to the line wires, on account of their considerable length and hence their large resistance.

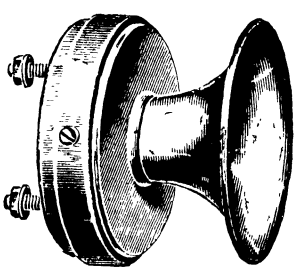


Fig. 65

The difficulty is overcome by inserting an induction coil in the circuit near the microphone. Suppose somebody speaks

into the microphone T (Fig. 66). When the diaphragm moves in, the battery sends a very strong current through the primary P of the induction coil, on account of the low resistance of the primary circuit. These fluctuations of current in the primary set up much stronger fluctuations of E.M.F. in the secondary circuit S of the induction coil. These fluctuations of E.M.F. produce sufficiently * strong changes of current along the line wires to cause vibrations in the diaphragm of the distant receiver R'.

Obviously the diaphragm of the receiver R is also equally affected by the induced current, and hence the speaker will

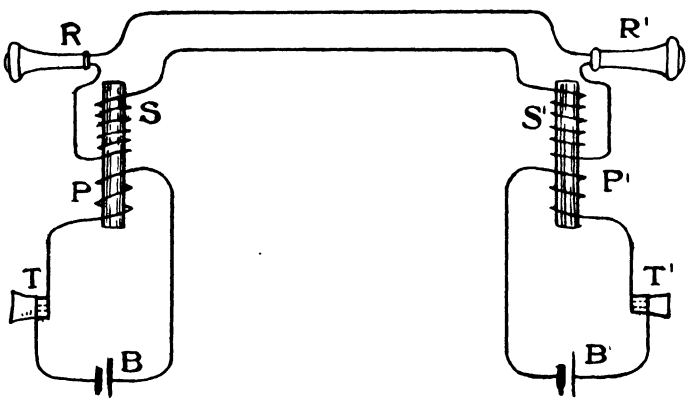


Fig. 66

hear his own voice, but that makes no difference, for we do so even in our daily conversation and without getting confused in any way.

584. Principle of Wireless Telegraphy.— It was in 1888 that Prof. Heinrich Rudolf Hertz of Germany succeeded in producing waves of electricity. It was he who first established definitely that the electric waves are in every way similar to the light waves except that they were very much longer. The light waves are so short that some 50,000 of them go to the inch, whereas the electric waves which he produced were several yards long. He showed experimentally that the electric waves can be reflected and refracted just as light waves.

* It is simply due to a much larger number of turns in the secondary than in the primary.

He could detect them at a distance from the source where they were produced. Instead of describing the apparatus used by Hertz to detect the waves, we shall describe what is called a coherer, the discovery of which was the real beginning of wireless telegraphy.

585. Coherer.—Fig. 67 represents in its simplest form the coherer devised by Branley and improved by Marconi, who

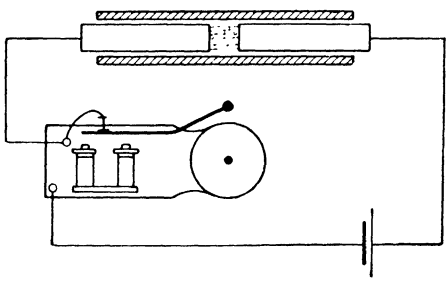


Fig. 67

is the inventor of the first practical system of wireless telegraphy. It consists of a narrow glass tube which is fitted with two silver plugs, separated by a narrow space filled with filings of an alloy of nickel and silver. The filings rest very lightly on one another and offer a considerable resistance to the passage of electricity. When such a coherer is included in the circuit containing a battery and a bell no current passes. But as soon as electric waves fall upon the filings, they become quite good conductors of electricity and allow to flow through the circuit a current sufficiently strong to ring the bell. A slight tap or shake restores the tube to its first badly-conducting state. The coherer as a detector has been replaced in these days by *crystal detectors or thermoionic valves.*

586. How to send Messages.—To produce electric waves we require a battery and an induction coil. The secondary coil has one end connected to the aerial, which terminates at the lower end in a knob (Fig. 68). The other end is connected to a similar knob below. This lower knob is earthed. When the operator closes the switch the current flows through the primary coil and induces a current in the secondary, which produces a spark in the spark-gap. This spark is accompanied by a short train of waves. These waves go out in all direc-

tions. When they fall upon a detector (say a coherer in circuit with a bell at the receiving station) the resistance of the circuit decreases and the bell rings. This is, in a few words, the principle of Marconi's wireless telegraphy.

In actual practice the aerials, which may be as high as 100

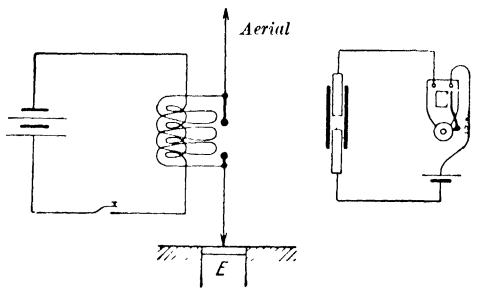


Fig. 68

or 150 feet, are used for signalling as well as receiving messages. In place of a bell a telephone is used. Instead of a battery and an induction coil a dynamo and a transformer along with condensers are used, and in place of a coherer thermoionic valves are used.

The usual wave-length used is 300 to 600 metres. But any wave-length from 20 to 5000 metres can be used.

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