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MAGNETIC PROPERTIES

OF

MATTER

BY

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PREFACE TO THE ORIGINAL JAPANESE EDITION.

The science of magnetism has in recent years made such rapid progress that its aspect has been entirely altered. About ten years ago the study of this branch of science was largely confined to iron, nickel and cobalt—the so-called ferromagnetic substances—but recently the study of the magnetic properties of matter in general, in addition to these particular substances, has become so significant that it is now recognised that magnetism forms an important element in the general properties of matter. And, just as we are aided in investigating the constitution of atoms and molecules by our knowledge of various phenomena of heat, light, electricity and radioactivity, so also our knowledge of magnetism is of essential importance in elucidating the structure of matter. In this connexion, it is worthy of notice that inasmuch as the phenomena of magnetism are considered to be caused by the rotation of the magnetic atoms constituting matter, we can treat of the relation of the characteristic properties of substances resulting from the various orientations of the axes of [illegible]oms. Most of the phenomena of heat, light, electr[illegible]u others, however, undergo no change even when the orientation of the atoms is in any way altered, and hence these phenomena cannot be made use of in determining the characteristic properties which are due to the differences in their orientation. But from what has been said it is not difficult to infer that the study of magnetism will furnish us with very important knowledge regarding the arrangement and orientation of atoms in crystalline bodies.

The investigations into the problems of magnetism made in the last ten years by Japanese and Western scientists

are very numerous, but they have been chiefly published in Western journals and we can find no book either in Japan or in Western countries, in which these investigations are unified and combined in one volume. The only special treatises on magnetism which we have are those of Ewing in England, written ten years ago and of Du Bois and Gans in Germany. Ewing's classical treatise does not contain the new investigations made in recent years, while the work of Du Bois and Gans is one written for special purposes, giving general theories and informations necessary for technical engineers. Some results of recent investigations find their places in English and German encyclopædias and handbooks of physics, but they are by no means complete, which fact the author has always considered to be a matter of no small regret. Fortunately, the author has been in close contact with this branch of science during the period of its greatest advance and expansion, and has for years been engaged in the investigation of magnetism, so that he considers himself to be slightly better acquainted than others with the order of its development and its present state and to possess an advantage in unifying these numerous facts. This is one of the reasons which has made him take up his pen to write this book. Moreover, for the last five years he has been giving special lectures on the subject of "The Magnetic Properties of Matter" in the Tohoku Imperial University, Sendai, which has resulted in a satisfactory arrangement and treatment of the subject in a manuscript form. This circumstance is the second of the reasons which have led to the publication of this book. Lastly the fact that the ardent efforts of Japanese and Western scientists have led to the establishment of acceptable theories of magnetism, have encouraged him to send this volume to the press.

The present work is divided into eight chapters. The first forms an introduction to the matter which is found in the subsequent chapters. As to the methods of explanation the author has followed Professor Du Bois' work. In the

following chapters, from the second to the seventh, numerous phenomena of magnetism are systematically arranged. But since the author's object is the unification of a large number of magnetic phenomena, not a few of those, even though well-known, have been omitted, because of their being not so closely related to the fundamental theories of magnetism. In the eighth chapter various theories regarding magnetism are arranged and criticised according to the author's view. It is believed that these theories cover, on the whole, the various phenomena described in the preceding chapters. If his readers can get some general idea of the magnetic properties of matter through this book, the author will be well satisfied.

In compiling this volume the author wishes to acknowledge his great indebtedness to the books of Ewing and of Du Bois, and also to Messrs. Mitsuo Yamada and Yoshiyuki Suzuki for their useful suggestion and valuable assistance during its preparation.

February, 1917.

KOTARŌ HONDA.

PREFACE TO THE ENGLISH EDITION.

This book is the author's translation of his Japanese text entitled "Magnetism and Matter," written ten years ago. During these years the science of magnetism has made such great progress both, in theory and in experimental results, that it has been thought advisable to enlarge the contents and to undertake the necessary revision. In this translation, therefore, this has been done. Thus Chapters V and X have been added, though their contents are not all new, and Chapter IX has been entirely rewritten. A new theory regarding the origin of magnetism has recently been proposed by the author, and in this edition, many observed facts have been unified and explained on the basis of this theory.

It was Emeritus Professor H. Nagaoka, of the Tokio Imperial University, who initiated me into this interesting branch of Physics, and I owe much to the results of his work in the preparation of this book. I also wish to take this opportunity of expressing my deepest feeling of gratitude for his ever willing guidance in the course of my investigations.

In conclusion, the author wishes to express his heartfelt thanks to Dr. C. Noss and Assistant Professor K. Someya for giving aid in the course of the translation. The same gratitude is also due to Mr. G. S. Carey, M. A., Professor J. Okubo and Professor M. Yamada for reading and correcting the proof-sheets. Thanks are also due to Messrs. Syokwabo & Co. for the generous willingness shown in furthering the publication of the book.

March, 1928.

KOTARO HONDA.

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CHAPTER I.

Introduction: Preliminary Considerations.

1. Magnetic Field.

Experimental facts show that the space surrounding a conductor, through which an electric current passes, is in a particular state. This particular state is called an electro-magnetic field or simply a magnetic field. This magnetic field exerts a force on any other conductor through which the electric current passes, and generates an induced electro-motive force in a conductor moving in the field. If a change of the magnetic field in the neighbourhood of a circuit of a conductor takes place, an induced current is generated in that conductor.

The magnetic field produced by an electric current is a vector, and its magnitude and direction vary generally from point to point in the field.

As mentioned above, when a change of the magnetic field takes place in the neighbourhood of an electric circuit, an induced current is produced in the circuit. The relation between the induced electro-motive force and the change of magnetic field is expressed by Faraday's law, namely,

$$e = -\frac{dN}{dt}, \qquad N = \int H\cos\theta\, dS,$$

where e is the induced electro-motive force, H the intensity of the magnetic field, S the area enclosed by the circuit, θ an angle between H and the normal to dS, and N a quantity called the magnetic flux.

Let i denote the strength of an electric current in the conductor induced by the change of magnetic field, and t the time. Then, the total electric quantity Q is given by

$$\begin{aligned} Q &= \int_0^{\tau} i\,dt = -\int_0^{\tau} \frac{1}{R}\frac{dN}{dt}\,dt \\ &= -\frac{1}{R}(N_{\tau} - N_0) = \frac{S}{R}(H_0 - H_{\tau}). \end{aligned}$$

In this case, the magnetic field at each point in the small area S is considered to be uniform and perpendicular to S. If then $H_\tau=0$,

$$Q = \frac{S}{R} H_0 .$$

If instead of changing the magnetic field, we make the circuit revolve through a certain angle, we obtain a similar result. That is, if we suppose that the circuit is at first perpendicular to the magnetic field H_0, and then made parallel to the magnetic field by a rotation through $\pi/2$, the electric quantity Q is given by the same expression as before. We can, therefore, determine the magnetic field H_0 by rotating the circuit $S=1$ and $R=1$ through $\pi/2$ and measuring Q induced in the circuit. In this chapter, it is assumed that a circuit of this kind is always used to determine the magnetic field.

2. Paramagnetic and Diamagnetic Substances.

In the foregoing paragraphs, the magnetic field is considered to be in a vacuum. If the magnetic field is present in air or other substances, the intensity of the field, and consequently, the quantity of electricity Q, differ more or less from the above values. In most cases, however, this difference is very small and not greater than some ten-thousandths as compared with the value for a vacuum. Thus for different media, the values of Q are more or less different from each other for the same currents producing the magnetic field, its value increasing in some media and decreasing in others. We call the former class of media or substances paramagnetic substances, and the latter diamagnetic substances. Air, oxygen, aluminium, sodium, manganese, chromium, platinum, thorium, etc., belong to the former class; hydrogen, chlorine, carbon, bismuth, antimony, water, etc. to the latter.

If a magnetic field acts on a substance, the latter undergoes a change in its physical state, which is revealed by the difference in the values of Q in the substance and in a vacuum; in this case, the medium is said to be magnetically polarised or induced. Experiments show that Q is proportional to the intensity of the magnetic field both for paramagnetic and diamagnetic substances; that is, if H_0 be the intensity of the magnetic field, then

$$Q \propto H_0 .$$

If μ be the proportional constant, then

$$Q = \mu H_0 .$$

The coefficient μ is called the magnetic permeability; it is greater or less than unity for paramagnetic or diamagnetic substances respectively. The product μH_0 is called the magnetic induction and is usually denoted by B; that is,

$$B = \mu H_0 .$$

In a vacuum, where $\mu = 1$, the intensity of the magnetic field is equal to the magnetic induction.

If a line is drawn in a magnetic field, such that its tangent at any point coincides in direction with the magnetic field at the same point, it is called a line of magnetic force. Similarly, if the direction of the tangent at any point on a curve coincides with that of the magnetic induction, this curve is called a line of magnetic induction. In the case of isotropic substances, the direction of the lines of magnetic induction coincides with that of the lines of magnetic forces, while in the case of anisotropic substances, the directions of these two generally differ from each other. The density of the lines of magnetic induction at a point is so drawn that the number of lines passing through a unit area perpendicular to these lines is equal to the magnetic induction at the point; hence, if we denote an elementary area perpendicular to the lines of magnetic induction by dS, and the number of them passing through this area by dn, we then get

$$\frac{dn}{dS} = B \quad \text{or} \quad dn = BdS.$$

3. Ferromagnetic Substances.

Besides the substances mentioned above, there are others whose intensity of magnetization or magnetic induction is very great and not proportional to the magnetic field; not only does their magnetization depend on the intensity of the magnetic field, but also, more or less, on the treatment they have undergone or their previous history. This phenomenon is called the magnetic hysteresis. These substances preserve a portion of the magnetism after the action of the magnetic field has ceased; ordinary magnets, which are sometimes called permanent magnets, are made by utilizing this property. The substances which have the above-mentioned properties, are called ferromagnetic substances. Iron is the most remarkable among them; besides iron, nickel, cobalt, alloys composed of these elements, and some compounds of manganese and chromium, etc., belong to this class.

In a strict sense, ferromagnetic and paramagnetic substances cannot be distinguished from each other. Usually a ferromagnetic substance is understood to be one which is easily magnetisable, and whose permeability varies with the intensity of the magnetic field, showing the phenomenon of magnetic hysteresis. On the other hand, a paramagnetic substance is understood to be one which is not easily magnetisable, and whose permeability is independent of the magnetic field, showing no phenomena of magnetic hysteresis. When however, these properties are examined in detail, it is easily found that the above criteria are not conclusive. Namely, the phenomenon of hysteresis last mentioned is nothing but a result necessarily associated with strong magnetization in the case of ferromagnetic substances, as compared with paramagnetic substances, and therefore cannot be considered an independent criterion. This will be clearly understood, if we compare the case of magnetization with that of elasticity. That is, within the limit of elasticity, the deformation is a definite function of the stress; if, however, the deformation exceeds the limit, the substance can no longer recover its original form, and shows elastic hysteresis. Similarly, the second criterion, namely, whether the magnetization is proportional to the magnetic field or not, is reduced to the question whether by the application of a given field the substance is strongly or weakly magnetized; even in the case of paramagnetic substances, if the magnetic field is extremely strong their permeability would vary with it. We can also find a similar phenomenon occurring in an elastic substance; in the case of a slight tension, the elongation is proportional to it, but in that of a great tension, the elongation increases more rapidly than the tension. Hence this criterion also cannot be independent of the first. Hence the actual criterion is the first only, that is, whether the substance is easily magnetized or not. This also is not a very legitimate criterion; because theoretically, it is not correct to fix the critical value of magnetization, beyond which the substance is to be called ferromagnetic, and below it paramagnetic. Actually, we are able to prepare some alloys whose permeability has any value lying between that of iron and that of a paramagnetic substance; as an example, we may quote an alloy of iron and a non-magnetic metal which makes with it a solid solution. Thus, we cannot distinguish the one class of substances from the other by the above criteria; in fact the ferromagnetic and the paramagnetic substances are two extreme members of a series, whose magnetization varies continuously from the former to the latter.

4. Intensity of Magnetization.

As we have already shown, when a ferromagnetic or paramagnetic substance is placed in a magnetic field, the quantity of electricity Q induced in the exploring coil within the substance is μ times as great as that in a vacuum. This fact is due to a particular state of the substance affected by the action of the magnetic field. We are accustomed to express this particular state by the term "intensity of magnetization." Now, if B be the magnetic induction within the substance when acted on by a magnetic field H, then the difference $B-H$ is due to the particular or polarised state in the substance; consequently, the intensity of magnetization can be measured by this difference. But for the sake of convenience, it is usual to measure the same quantity by the difference divided by 4π. That is,

$$\frac{B-H}{4\pi} = \text{intensity of magnetization} = I,$$

whence

$$B = 4\pi I + H.$$

Dividing both sides of this equation by H, we obtain

$$\frac{B}{H} = \frac{4\pi I}{H} + 1.$$

If we put

$$\frac{I}{H} = \kappa,$$

then

$$\mu = 4\pi\kappa + 1.$$

This is an important relation existing between μ and κ; κ is called the magnetic susceptibility.

Let us suppose that within a magnetized substance we make a very flat cavity perpendicular to the direction of magnetization. As this cavity is very thin, the magnetic state in the substance remains unchanged by the presence of the cavity. If we insert the above-mentioned coil into the cavity and withdraw it rapidly, an electric current is induced in the coil, its quantity Q being

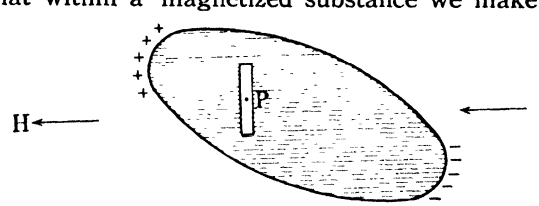


Fig. 1.

$$Q = B = 4\pi I + H,$$

where H is the sum of the force of the magnetic field applied from outside and of the magnetic field due to the magnetism induced on

the surface of the substance. On one side of the cavity, a positive magnetism and on the other side, a negative magnetism appears; hence if their surface density be σ, the total magnetic force at a point within the cavity is $4\pi\sigma+H$, which must be equal to B. Hence σ is equal to I, and the walls of the cavity have a surface density $+I$ on one side and $-I$ on the other. Thus we see that the magnetic induction B at a point in a magnetic substance is equal to the sum of the forces of the magnetic field acting on the substance at the same point and of the magnetic field $4\pi I$ due to the magnetism appearing on the walls of a very thin cavity, which is formed round about the point perpendicular to the direction of magnetization. We see also that if a magnetized substance is cut by a plane perpendicular to the direction of the magnetization, the magnetism of surface density I appears on the outer surface.

5. Magnetic Moment.

If a long rod made of a magnetic substance, length l and sectional area S, is placed in a uniform magnetic field of intensity H parallel to its direction, it is magnetized by induction, $+m$ appearing at one end and $-m$ at the other end of the bar; m is equal to IS. The product of m and l is then called the magnetic moment of this substance; for a couple of magnitude ml acts on a substance placed perpendicular to the direction of a magnetic field of unit intensity. The magnetic moment is usually denoted by M; hence,

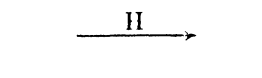


Fig. 2.

$$ml = M.$$

Therefore,

$$M = ISl = Iv,$$

where v denotes the volume of the magnetized rod. From this expression, it follows the relation

$$I = \frac{M}{v}.$$

The intensity of magnetization is, therefore, equal to the magnetic moment per unit volume.

We shall next consider the relation between the intensity of magnetization and the magnetic field; in the case of paramagnetic and diamagnetic substances, this relation is very simple, I being proportional to H, while in that of ferromagnetic substances, it is not so simple. Hence, the quantities μ and κ are constant in relation to the former class of substances, but in relation to the latter, they are com-

plicated functions of the magnetic field. Though the relation between I and H is not the same for different substances, it can generally be expressed by a curve as shown in Fig. 3. Namely, if the field H is weak, I increases proportionally to H, the curve being straight. When H gradually increases beyond a certain value, I goes up rapidly, and when H increases still further, I approaches asymptotically to a definite value I. This curve representing the relation between I and H is called the curve of magnetization. As the magnetic susceptibility κ is equal to I/H, the κ,H curve has a maximum as represented by the dotted line in Fig. 3. The μ,H curve has a form similar to the κ,H curve; because μ is equal to $4\pi\kappa+1$.

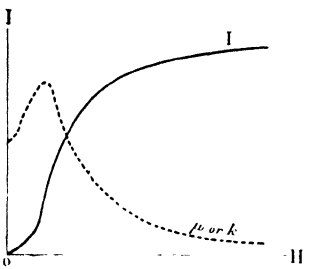


Fig. 3.

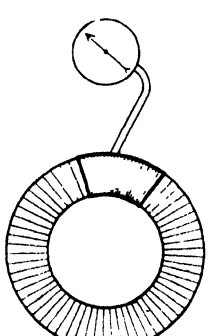
Hitherto many scientists have tried to get an expression for the relation between I and H, but we have not yet any satisfactory expression for it. An empirical formula convenient for calculation s as follows:

$$\frac{I}{I_0} = \frac{aH^n}{1+aH^n},$$

where I_0 denotes the value of saturated magnetization, a and n are two constants, which are to be determined suitably.

6. Demagnetizing Force.

If we wind a coil uniformly on a long iron bar bent into the form of a ring and pass an electric current through the coil, the iron ring inside the coil will be uniformly magnetized by the magnetic field due to the coil. In this case, however, the magnetic poles do not appear and hence there is no magnetic action outside the coil. For the purpose of measuring the intensity of magnetization in such an iron ring, we make a secondary coil by winding another coil on the former one, and connect this secondary coil with a ballistic galvanometer; then


Fig. 4.

by passing an electric current through the primary coil, we measure the current induced in the secondary coil. If now we denote the sectional area of the ring by S and the number of windings of the secondary coil by n, the quantity of electricity Q passing through this circuit is expressed as follows—

$$Q = \frac{N}{R},$$

where R is the resistance of the secondary circuit. As N is equal to nSB,

$$Q = \frac{nS}{R}(4\pi I + H).$$

Q can be measured by a ballistic galvanometer and, as H is the intensity of the magnetic field due to the primary coil, this is a known quantity. From this expression, therefore, we can determine the value of I. Thus, the relation between I and H is experimentally determined and graphically shown by a curve as oA in Fig. 6. This curve depends upon the nature of the substance and is characteristic of each substance; it is called the normal curve of magnetization.

Next let it be supposed that a gap be made perpendicular to the axis of the ring as shown in Fig. 5, and let this gap be extremely thin. Measuring again the intensities of magnetization for different magnetic fields, we notice that, although the result is very similar to that in the former case, there is a small difference between them. That is, in this case, the curve has

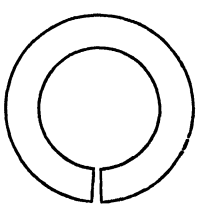


Fig. 5.

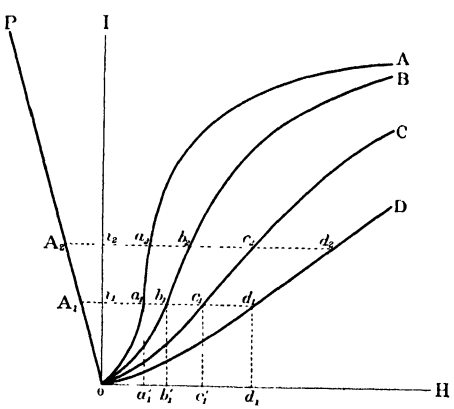


Fig. 6.

a form as shown by curve oB in Fig. 6. If we make the gap wider and wider, we get curves such as oC, oD, etc. Thus, even for the same substance the curves of magnetization have different forms according to the widths of the gap, becoming straighter as the gap is wider. Consequently, the curves except oA are not characteristic of the substance, and in order

to obtain the normal curve of magnetization from other curves, we must make the following construction:—

Let the intersecting points of a straight line $I=i_1$ with the curves oA, oB, oC and oD be a_1, b_1, c_1 and d_1 respectively. From these points draw perpendiculars to oH and let the intersecting points of these lines with the axis oH be a_1', b_1', c_1' and d_1'. Then it is evident from the figure that the magnetic field necessary to produce the same value of I, is different for different curves; that is, $H=Oa_1'$ for oA, $H=Ob_1'$ for oB, $H=Oc_1'$ for oC and $H=Od_1'$ for oD. Similarly, let the intersection of these curves with a line $I=i_2$ be a_2, b_2, c_2 and d_2 respectively. On the line $I=i_1$, take a point A_1 such that A_1i_1 is equal to a_1b_1; and take a point A_2 on the line $I=i_2$ such that A_2i_2 is equal to a_2b_2, and so on. If we consider a case where the gap is not very wide, the points A_1, A_2, etc. lie nearly on a straight line oP. Similar remarks apply for other curves. Denoting by ΔH the difference of the magnetic fields necessary for inducing the same value of I, we get

$$\Delta H \propto I, \quad \text{or} \quad \Delta H = NI,$$

where N is a proportional constant. Hence in order to get the normal curve of magnetization oA from the curve oB, it is only necessary to give a shear corresponding to an angle PoI to the figure. ΔH is usally called the demagnetizing force; for, with a gap in the iron ring, an excess ΔH of the magnetic field must be applied to the ring in order to get the same intensity of magnetization as in the normal curve, and hence the matter is the same as if a magnetic field ΔH acts in the opposite direction to the magnetizing field H. The factor N is called the demagnetizing factor; it increases with the width of the gap.

If a long magnetic bar is placed parallel to a uniform magnetic field, the magnetic force at a point P inside the substance is the resultant of the applied magnetic field and the field due to the induced magnetism near the ends of the bar. Since the latter field acts in the direction opposite to the former, it is a demagnetizing force. This force has different values for different portions of the bar; it is minimum at the middle point and increase toward the ends of the bar. The magnitude of the force at every point is, however, proportional to the intensity of magnetization I, hence the mean value of these forces throughout the bar is also proportional to I. Denoting it by $\bar{H}_d$, we have

$$\bar{H}_d = \bar{N}I.$$

$\bar{H}_d$ is called the mean demagnetizing force. This force diminishes as the bar becomes longer. The ratio of the length of a bar to its diameter, that is, $l:2r = \mathfrak{m}$, is called the dimension ratio. For a rod of a definite length l, N becomes greater as $\mathfrak{m}$ diminishes. Also $\bar{H}_d$ is proportional to the product of I and the sectional area S; hence

$$\bar{H}_d \propto IS,$$

consequently,
$$H_d \propto Ir^2, \text{ or } \bar{H}_d \propto \frac{I}{\mathfrak{m}^2};$$

whence
$$\frac{\bar{H}_d}{I} \propto \frac{1}{\mathfrak{m}^2} \propto N,$$

$$\therefore \quad \bar{N}\mathfrak{m}^2 = const. = C.$$

That is, the product of the coefficient of the mean demagnetizing force and $\mathfrak{m}^2$ is constant. The result of experiments shows that the above relation nearly holds for bars of small section. Namely, for $\mathfrak{m} > 80$, C has nearly a constant value of 50 or 40 in the case of the magnetometric or ballistic method respectively; for $\mathfrak{m} < 100$, C decreases with $\mathfrak{m}$. This is probably due to the fact that for a short bar, the magnetic poles do not locate on the ends of the bar, but are a little shifted inwards, their positions depending on the strength of the magnetic field.

7. Substances to be Uniformly Magnetized.

If a ferromagnetic substance of an arbitrary form is placed in a uniform magnetic field, it is not uniformly magnetized; that is, the magnetic induction at every point is not the same, depending on the position of the points. The form of the substance, in which it is uniformly magnetized, is an ellipsoid. The problem of magnetization of an ellipsoid placed in a uniform field was first solved by Poisson[1].

Consider a point $P(x, y, z)$ in the neighbourhood of a body of density ρ; if the gravitational potentional at the point due to the body be V, then it can be shown that $\partial V/\partial x$ is equal to the magnetic potential due to a body which

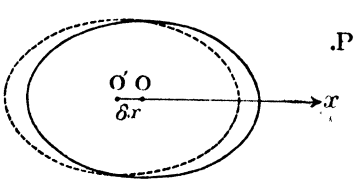


Fig. 7.

(1) Maxwell's Electricity and Magnetism, 2, 66.

has the same form as the above and is uniformly magnetized in the direction of x with the intensity of magnetization $I=\rho$.

In order to prove this, we consider that the body is slightly displaced to the left by ∂x and its density changed to $-\rho$; the center O is then displaced to O', being $oO'=\partial x$. The gravitational potential V' due to the body at P after the displacement is given by $-(V+\partial V/\partial x \,.\, \partial x)$, and the gravitational potential at the same point due to these two bodies before and after the displacement is equal to

$$V + V' = -\frac{\partial V}{\partial x}\,\partial x .$$

Next consider two elementary volumes as shown in Fig. 8 corresponding to these two bodies, and let their sectional area and length be denoted by S and ∂l respectively. The common parts of these volumes, which are shaded in the figure, have equal and opposite masses, their effects canceling each other, and hence there remains only an effect of positive mass on the right side and that of negative mass on the left. Consequently, if we consider the effects at P due to these two small volumes, it is exactly the same as the action of a magnet which has magnetic poles $\rho S\partial x$ and $-\rho S\partial x$ at a distance of ∂l, ∂x being assumed to be still very small as compared to ∂l. The magnetic moment of this magnet is

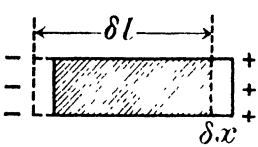


Fig. 8.

$$M = \rho S\partial x \times \partial l = \rho\partial x \times S\partial l .$$

Since $S\partial l$ in this expression denotes the volume of the magnet, the magnetic moment per unit volume, that is, the intensity of magnetization I is

$$I = \rho\partial x .$$

Thus we see that the action at P due to two equal elementary volumes having densities $+\rho$ and $-\rho$ is equivalent to that at the same point due to a small magnet, which has the same form as the elementary volumes and the intensity of magnetization $\rho\partial x$. This relation holds good for all corresponding portions of these bodies, or for the whole bodies; hence the magnetic potential at P due to a magnetized body whose intensity of magnetization is $\rho\partial x$, is equal to the gravitational potential $-\partial V/\partial x \,.\, \partial x$ due to two

bodies which have the same form and densities $+\rho$ and $-\rho$ respectively. Consequently the magnetic potential at P due to a magnetized body of the intensity of magnetization ρ is equal to $-\partial V/\partial x$. From this we conclude that if a gravitational potential V at a point due to a body is known, we can obtain an expression for the magnetic potential at the same point due to the same body uniformly magnetized in the direction of x by differentiating V with respect to x and changing its sign.

In the above magnetized body, we observe that positive magnetism appears on one side of the body and negative magnetism on the other side. If we draw a normal n at a point on the surface and denote the angle between n and x by θ, its surface density is given by $\rho\partial x\cos\theta$. As this is the surface density for the case where the intensity of magnetization is equal to $\rho\partial x$, the surface density in the case of the intensity of magnetization $I=\rho$, is $\rho\cos\theta$.

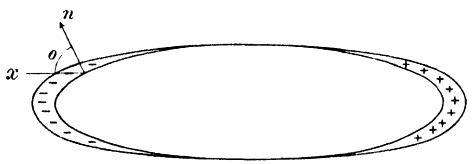


Fig. 9.

In order that a body may be uniformly magnetized, the magnetic field at every point in that body must be uniform. The magnetic field consists of two parts; the one is external, and the other due to an induced magnetism on the surface of the body. If the external field be uniform, the resultant field can only be uniform in the case when the field due to the induced magnetism is uniform.

Now, in order that a magnetic field due to an induced magnetism on the surface of a body is uniform at every point within it, its magnetic potential must be a function of the first order with respect to the co-ordinates; that is, $\partial V/\partial x$ is a function of the first order with respect to x, y, z, and hence V must be a function of the second order with respect to x, y, z. According to the theory of potential, in order that V may be a function of the second order with respect to x, y, z, the body must have a surface of the second degree. The body having such a property and a finite volume is an ellipsoid. Thus we conclude that a body to be uniformly magnetized in a uniform magnetic field is an ellipsoid.

Let an ellipsoid be expressed by the following equation

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1;$$

denoting by φ, L, M and N respectively the following expressions

$$\varphi = \int_0^\infty \frac{du^2}{\sqrt{(a^2+u^2)(b^2+u^2)(c^2+u^2)}},$$

$$L = -4\pi abc \frac{\partial \varphi}{\partial a^2},$$

$$M = -4\pi abc \frac{\partial \varphi}{\partial b^2},$$

$$N = -4\pi abc \frac{\partial \varphi}{\partial c^2},$$

the gravitational potential at a point in the ellipsoid is given by

$$V = -\frac{\rho}{2}(Lx^2 + My^2 + Nz^2) + const.$$

If we denote the direction cosines of the intensity of magnetization I by l, m, n, and its components by A, B, C, we have

$$A = Il, \quad B = Im, \quad C = In.$$

Consequently, if the magnetic potential is expressed by Ω,

$$\Omega = ALx + BMy + CNz,$$

or

$$\Omega = I(Llx + Mmy + Nnz).$$

This is the magnetic potential due to the induced magnetism on the surface of the body. There is also a magnetic potential due to a magnetic field applied from outside. Let the components of the external magnetic field be denoted by X, Y and Z, then

$$\Omega_0 = -(Xx + Yy + Zz).$$

Hence, the components of the resultant magnetic force at a point in the body is

$$X - AL, \quad Y - BM, \quad Z - CN,$$

where AL, BM and CN are the components of the demagnetizing force and L, M and N their coefficients, since they act in the directions opposite to X, Y, Z. If the magnetic susceptibility of the substance constituting the ellipsoid be denoted by κ, then

$$A = \kappa(X - AL),$$

$$B = \kappa(Y - BM),$$

$$C = \kappa(Z - CN).$$

Or,
$$A = \frac{\kappa X}{1+\kappa L},$$
$$B = \frac{\kappa Y}{1+\kappa M},$$
$$C = \frac{\kappa Z}{1+\kappa N}.$$

This is the expressions for the relation between the intensity of magnetization at a point within a uniformly magnetized ellipsoid and the magnetic field.

It is worth while to consider the case where the magnetic susceptibility κ is very large or very small. In the case where κ or L, M and N are very small, the second term in the denominator of the expressions for A, B, C is negligibly small as compared to 1; hence A, B, C are independent of L, M, N. Accordingly, in this case, the intensity of magnetization is independent of the form of the body. On the other hand, if κ or L, M and N are very large, 1 can be neglected as compared with the second term; we then get,
$$A = \frac{X}{L}, \quad B = \frac{Y}{M}, \quad C = \frac{Z}{N}.$$

That is, the intensity of magnetization is proportional to the applied magnetic field. Thus in the case of a ferromagnetic substance in the form of a long bar placed parallel to the magnetic field, the coefficient of demagnetizing force is very small, and the intensity of magnetization is nearly unaffected by a further variation of the length of the substance; the curve of magnetization is very similar to the normal curve, as shown by *oa* in Fig. 10. If, however, the substance is short and thick, the coefficient of demagnetizing force is very large, and the intensity of magnetization is proportional to the magnetic field up to a moderate field, and the curve of magnetization is straight, as shown by curve *ob* in Fig. 10. Since, however, κ decreases with an increasing magnetic field, the rate of increase in magnetization gradually decreases in very strong fields. Even in such a case, the curve of magnetization will take the form of the normal curve, provided we take the demagnetizing force into account and plot

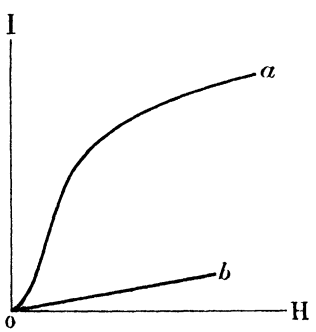


Fig. 10.

the intensity of magnetization against $H-NI$.

We shall next find the coefficient of demagnetizing factor in some special cases.

(i) If the ellipsoid of rotation is flat in the direction of a-axis, we have

$$b=c=\frac{a}{\sqrt{1-e^2}},$$

where e is the eccentricity. Consequently

$$L=4\pi\left(\frac{1}{e^2}-\frac{\sqrt{1-e^2}}{e^3}\sin^{-1}e\right),$$

$$M=N=2\pi\left(\frac{\sqrt{1-e^2}}{e^3}\sin^{-1}e-\frac{1-e^2}{e^2}\right).$$

(ii) If the ellipsoid of rotation is long in the direction of c-axis, we have

$$a=b=c\sqrt{1-e^2};$$

hence

$$L=M=2\pi\left(\frac{1}{e^2}-\frac{1-e^2}{2e^3}\log\frac{1+e}{1-e}\right),$$

$$N=4\pi\left(\frac{1}{e^2}-1\right)\left(\frac{1}{2e}\log\frac{1+e}{1-e}-1\right).$$

(iii) If the ellipsoid becomes a sphere, we have

$$L=M=N=\frac{4\pi}{3}.$$

(iv) A cylinder may be considered as a special case of an ellipsoid, extremely elongated, and therefore the coefficient of demagnetizing force, when it is magnetized perpendicularly to the axis, can be derived from the value in the case (ii). That is, putting $e=1$, we get

$$L=M=2\pi.$$

(v) A thin plate may be considered as a special case of an ellipsoid, extremely flat, and hence the coefficient of demagnetizing force, when it is magnetized perpendicularly to the surface, may be derived from the value in the case (i). That is, putting $e=1$, we get

$$L=4\pi.$$

Thus, in the case of an ellipsoid, the coefficients of demagnetizing force, L, M, N, can be theoretically calculated; these coefficients are very important in discussing the intensity of

magnetization of different substances; because the magnetic field which it is necessary to know, is not the magnetic field applied from the outside, but the effective field, which is the difference between the external field and the demagnetizing force, the latter being only obtainable from the knowledge of the above-mentioned coefficients.

The following table contains the values of the coefficients of demagnetizing force for ellipsoids of rotation and of cylinders, magnetized in the direction of the major axis.

m	Ellipsoid of rotation N	Cylindrical rod(1) $\bar{N}$ (magnetom.)	Cylindrical rod(2) N (ballistic)
5	0.7015	0.6800	—
10	0.2549	0.2550	0.204
15	0.1350	0.1400	0.106
20	0.0848	0.0898	0.0672
25	0.0587	0.0628	0.0467
30	0.0432	0.0460	0.0344
40	0.0266	0.0274	0.0211
50	0.0181	0.0183	0.0144
60	0.0132	0.0131	0.0104
70	0.0101	0.0099	0.0079
80	0.0080	0.0078	0.0062
90	0.0065	0.0063	0.0051
100	0.0054	0.0052	0.0042
150	0.0026	0.0025	0.0020
200	0.0016	0.0015	0.0012
300	0.0008	0.0008	—

Since the demagnetizing factors for cylinders cannot be theoretically calculated, the above values of them were obtained by comparing the value of magnetization in different fields for an ellipsoid with those of magnetization for cylinders made of the same material as the ellipsoid. In the case of the magnetometric method, the observed intensity of magnetization for a rod is always its mean value; while in the case of the ballistic method, the secondary coil is usually wound on the middle portion of the rod, where the intensity of magnetization is maximum, so that the observed value is always the maximum of magnetization along the rod. Hence in the former case, the mean value $\bar{N}$ of the demagnetizing factor is to be taken, and in the latter, the minimum value N corresponding to the middle portion of the rod.

(1) R. Mann, Dissert., Berlin, (1895). (2) Shuddemagen, Phys. Rev. **31**, (1910), 165.

CHAPTER II.

Magnetization of Ferromagnetic Substances.

8. Method of Measuring Magnetization, I.

There are two methods of measuring the intensity of magnetization, one of which is by using a magnetometer and the other by using a ballistic galvanometer. We shall first describe the former method.

The essential apparatus for this method is a magnetometer, which consists of a small horse-shoe magnet suspended by an extremely fine silk or quartz fibre, a small mirror being fixed to the upper part of it. If now a magnet is brought near this magnetometer, the horse-shoe magnet turns by a certain angle, which is read on a scale with a telescope. Besides the magnetometer, a magnetizing and a compensating coil are necessary for the measurement. Three different cases arise, these depending on the relative positions of the coils and the magnetometer.



Fig. 11.

First position: Usually the compensating coil is made of the same shape as the magnetizing coil, and these two coils are placed in an east and west direction, their axes being situated horizontally along the same line, and the magnetometer at the middle point between these coils as shown in Fig. 12. Before we insert a ferromagnetic bar in the magnetizing coil, the action of each coil upon the magnetometer is made to cancel that of the other, by passing a strong electric current in series through them in opposite directions and adjusting the distances from the coils to the magnetometer. The test-bar is then inserted inside the magnetizing coil, and a current is passed through both coils. While the action of the coils by themselves would balance each other, the magnetized rod influences the magnetometer, causing the deflection of the horse-shoe magnet carrying the mirror. The rela-

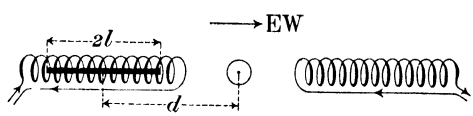


Fig. 12.

tion between this angle of deflection and the intensity of magnetization of the rod can be obtained as follows:—

When no current passes through the magnetizing coil, the axis of the magnet of the magnetometer is parallel to the direction of the horizontal component of terrestrial magnetism. When, however, a current passes through magnetizing coil, the bar acts on the magnet of the magnetometer with a magnetic force F_1 perpendicular to the horizontal component of terrestrial magnetism. Consequently, this magnet, being influenced by these two forces perpendicular to each other, will come to rest pointing in the direction of the resultant force. Let this angle of deflection be θ, and m and m' the pole-strengths of the bar and the magnet respectively. Then,

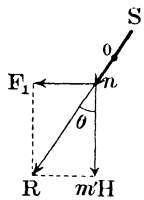


Fig. 13.

$$F_1 = m' H \tan\theta.$$

If we denote the length of the bar by $2l$ and the distance between the centres of the bar and the magnet of the magnetometer by d,

$$F_1 = \frac{mm'}{(d-l)^2} - \frac{mm'}{(d+l)^2} = \frac{4mm'ld}{(d^2-l^2)^2}.$$

Since the magnetic moment is $M=2lm=Iv$,

$$F_1 = \frac{2m'Ivd}{(d^2-l^2)^2} = m'H\tan\theta.$$

$$\therefore \quad I = \frac{(d^2-l^2)^2}{2vd} H\tan\theta.$$

Experimental results show that the distance between the poles of a bar magnet is not equal to its full length, but shorter by $\frac{1}{6}$ of its length. Hence, for the purpose of obtaining an accurate result, $\frac{5}{6}l$ must be substituted for l in the above expression. Since also the bar cannot be uniformly magnetized, the value of magnetization obtained from the above-mentioned measurement is a mean value.

The action of an ellipsoid of revolution on a small magnet situated at a great distance is similar to that of a bar magnet of definite length. Let the major axis of the ellipsoid be $2c$, the minor axis $2a$, and the intensity of magnetization I, then it is easily proved that the action of the ellipsoid at a distant point along c-axis is

equivalent to that of a bar magnet, which has a pole-strength equal to $\pi a^2 I$ and a pole-distance of $^4/_3\, c$.

The above result holds good only for the case where the distance d of the point from the ellipsoid is large. However, when this condition is not satisfied, the problem can be treated more generally. The results[1] of calculation show that, putting

$$e=\sqrt{1-\frac{a^2}{c^2}} \text{ and } n=\frac{d}{c},$$

$$F_1=\frac{2Mm'}{d^3}\left\{1+\frac{6}{5}\left(\frac{e}{n}\right)^2+\frac{9}{7}\left(\frac{e}{n}\right)^4+\frac{4}{3}\left(\frac{e}{n}\right)^6+\frac{15}{11}\left(\frac{e}{n}\right)^8+\cdots\right\}.$$

When accuracy in the result is desired, the terms in this expression are taken up to a certain suitable member and substituted in the place of F_1 in the expression on the preceding page.

Second position: The axes of the magnetizing and compensating coils are placed horizontally in the east-west direction, the line connecting the centres of these coils coinciding with the magnetic meridian. The magnetometer is placed at the middle point of this line, as shown in Fig. 14. An electric current is passed through these coils in opposite directions, and the position of the compensating coil is so adjusted that the magnetic effects of these coils on the magnetometer cancel each other. The test bar is next placed in the magnetizing coil. In this position, the component forces in the magnetometer in the north-south direction due to the bar magnet cancel each other, and as a consequence, the component force in the east-west direction only remains. Hence,

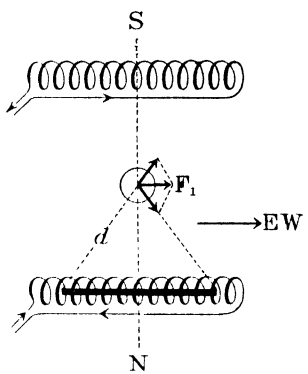


Fig. 14.

$$F_1=\frac{2mm'l}{(d^2+l^2)^{\frac{3}{2}}}=\frac{Ivm'}{(d^2+l^2)^{\frac{3}{2}}}=m'H\tan\theta.$$

Thus we obtain the following expression for the intensity of magnetization

(1) Roessler, Untersuchung über die Magnetisierung des Eisens, Inaug. Dissert., Zürich, 1892.

$$I = \frac{(d^2+l^2)^{\frac{3}{2}}}{v} H\tan\theta.$$

The calculation for an ellipsoidal magnet is the same as in the former case.

Third position: The magnetizing and compensating coils are vertically placed in such a position that the line connecting the centres of these coils lies in the east-west direction. The magnetometer is placed midway between these coils (Fig. 15). Denoting the vertical distance from the centre of the magnetized bar to that of the magnetometer magnet by h and the horizontal distance between the centres of the bar and the magnetometer by d, we get the following expression for the horizontal component F_1 acting in the east-west direction,

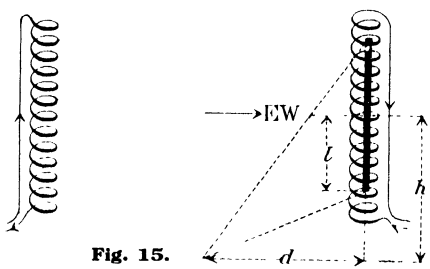


Fig. 15.

$$F_1 = mm'\left\{\frac{d}{[d^2+(h-l)^2]^{\frac{3}{2}}} - \frac{d}{[d^2+(h+l)^2]^{\frac{3}{2}}}\right\}$$

$$= \frac{Ivm'd}{2l}\left\{\frac{1}{[d^2+(h-l)^2]^{\frac{3}{2}}} - \frac{1}{[d^2+(h+l)^2]^{\frac{3}{2}}}\right\}$$

$$= m'H\tan\theta.$$

$$\therefore \quad I = \frac{2lH\tan\theta}{vd\left\{\frac{1}{[d^2+(h-l)^2]^{\frac{3}{2}}} - \frac{1}{[d^2+(h+l)^2]^{\frac{3}{2}}}\right\}}.$$

It is here to be remarked that the action of the bar magnet on the magnetometer varies according to its position in the magnetizing coil. Suppose the magnet to be placed sufficiently high in the coil, then since the distance from the magnet to the magnetometer is large, its effect on the magnetometer will be small. On the other hand, if the magnet be so placed that its centre is in the same horizontal plane as the magnet of the magnetometer, the poles of the magnet in the coil are at the same distances from the magnetometer, and hence their deflecting action on the magnetometer is balanced. Accordingly, the effect of the magnet on the magnetometer is at a maximum at some position between the above two.

In this position, the deflection of the magnetometer is not only a maximum, but a small error due to the measurement of the height of the magnet in the coil does not affect the final result. The position of the maximum[1] can be theoretically calculated in the case of an ellipsoid of rotation. When the horizontal distance d of the ellipsoid from the magnet in the magnetometer is large, the vertical distance h for the maximum effect is equal to

$$h = \frac{d}{\sqrt{3}} .$$

Among these three positions, the first position is the best adapted for observation, although any one of the other two can be used according to circumstances.

To obtain the correct value of magnetization, it is absolutely necessary that before each magnetization, the residual magnetism should be completely removed. For this purpose, an alternate current of a gradually diminishing strength is passed through the coils. In this case, the specimen in the magnetizing coil is magnetized in opposite directions in rapid succession, its intensity of magnetization gradually diminishing to zero, and hence the specimen will finally be found to be in a magnetically neutral state. In the first and second positions, the test-bar is placed horizontally in the east-west direction, and hence the horizontal component of the earth field is perpendicular to this direction and does not affect the magnetization in the longitudinal direction of the bar. In the third position, however, the bar is vertically placed, and therefore affected by the vertical component of the earth field. In this case, it is not possible to demagnetize the specimen completely, even though the above-mentioned process is carried out by applying an alternating current of a gradually diminishing intensity. The vertical component of the earth field can be eliminated by an opposite magnetic field caused by a constant current of suitable strength flowing through a long coil, which is co-axially placed within the magnetizing coil.

In the calculation of the intensity of magnetization by the foregoing formula, the horizontal component of the earth field H must be known. The value of H in different districts is generally known, but inside buildings it is affected by the iron in the neighbourhood and is considerably different from that of the outside.

(1) H. Nagaoka, Ann. der Phys. u. Chem. **57**, (1896), 275.

Hence the value of H in the position of the magnetometer must be measured. For this purpose, the comparison method is the most suitable; a horizontally suspended magnet is vibrated in a required position, and also in the outside field, where the earth field is known. Let the horizontal components of the field in these two positions be H and H_0, and their periods T and T_0, respectively; then

$$T_0 = 2\pi\sqrt{\frac{K}{MH_0}}, \quad \text{and} \quad T = 2\pi\sqrt{\frac{K}{MH}},$$

where K denotes the moment of inertia of the magnet and M the magnetic moment of the vibrating magnet. Hence we have

$$\left(\frac{T_0}{T}\right)^2 = \frac{H}{H_0}.$$

Or

$$H = H_0\left(\frac{T_0}{T}\right)^2.$$

Hence if H_0 is known, H can be determined.

The following direct method is also adapted for the determination of the earth field. A circular coil of a diameter a having n number of turns is placed vertically with its axis in the east-west direction. If an electric current i is passed through this coil, its magnetic force at a point on the axis distant b from the centre of the coil acts in the direction of the axis with a magnitude F

$$F = \frac{2\pi n a^2 i}{(a^2+b^2)^{\frac{3}{2}}}.$$

Hence the magnetometer placed at the point is acted on by two forces F and H perpendicular to each other, and the magnet of the magnetometer will be in equilibrium taking the direction of their resultant force. If θ be the angle of deflection of the mag netometer, then we have

$$H\tan\theta = F = \frac{2\pi n a^2 i}{(a^2+b^2)^{\frac{3}{2}}},$$

whence we can find the value of H.

9. Method of Measuring Magnetization, II.

In applying this method, a secondary coil is wound round the test-bar, and the total quantity of electricity induced in the

secondary coil during magnetization is measured by a ballistic galvanometer placed in the circuit of the secondary coil; whence the magnetic induction or the intensity of magnetization is calculated.

In order to determine the constant of the galvanometer, we make use of the so-called standard coil, which consists of primary and secondary coils possessing each a known number of turns, and the latter coil is always placed in the circuit of the ballistic galvanometer. If an electric current is passed through the primary coil, a magnetic field is induced in the coil, this giving rise to an induced current in the secondary coil. The total quantity of electricity flowing through the secondary coil can be found from the current strength, the dimensions, and the number of turns of these coils. This combined with the deflection of the galvanometer gives the required constant of the instrument. If we pass an electric current of intensity i through the primary coil of a sectional area S_0, the quantity of electricity flowing through the secondary circuit having a number of turns m_0 is given by:—

$$Q_0 = \frac{N_0}{R} = \frac{4\pi n i m_0 S_0}{R}$$

where R is the resistance of the circuit. Let us denote the maximum deflection of the galvanometer by θ_0; then

$$Q_0 \propto \theta_0 .$$

$$\therefore \quad Q_0 = \kappa \vartheta_0 \quad \text{or} \quad \kappa = \frac{Q_0}{\theta_0} ,$$

where κ is the galvanometer constant. Next, an electric current is passed through the magnetizing coil and the maximum deflection θ of the ballistic galvanometer is observed; the electric quantity Q flowing through the secondary coil is then given by the following expression:—

$$Q = \kappa\theta = Q_0 \frac{\theta}{\theta_0}$$

$$= \frac{N}{R} = \frac{mSB}{R} ,$$

where m is the number of turns of the secondary coil wound round the test-bar and S its sectional area. Consequently,

$$B = 4\pi I + H = \frac{4\pi n i m_0 S_0 \vartheta}{m S \vartheta_0} .$$

In this equation, H can be found from the intensity of the magnetizing current. Hence, by measuring the values of ϑ for different currents, we can obtain the values of magnetic induction B or the intensity of magnetization for various magnetic fields.

The period of the ballistic galvanometer should be sufficiently long, at least, longer than six seconds. The vibration of the galvanometer needle should damp as rapidly as possible; because, if the damping be slow, the observation requires a long time. If, however, the damping is too great, Q may not be proportional to θ; it is therefore desirable that the damping is as great as possible within the range, in which the above proportionality exists. To examine whether this proportionality holds good or not, it is sufficient to compare the deflections of the ballistic galvanometer caused by currents through the primary of the standard coil with the current intensity.

The ballistic method can be applied not only to test a substance in the form of a bar or an ellipsoid, but is equally applicable to a substance in the form of a ring. In the latter case, the diameter should, for uniform magnetization, be sufficiently large compared with its thickness. Otherwise, the magnetic field inside the ring cannot be entirely uniform, even though the magnetizing coil is uniformly wound over the whole ring. That is, the magnetic field is the strongest at the innermost part of the ring and the weakest at the outermost part. As shown in Fig. 16*a*, this is caused by the fact that, since the diameter of the inner side is a little smaller than that of the outer, the number of turns per unit length is a little greater in the inner side than in the outer. We can therefore diminish this defect in a greater or less degree by making the section of the ring flat as shown in Fig. 16*b*.

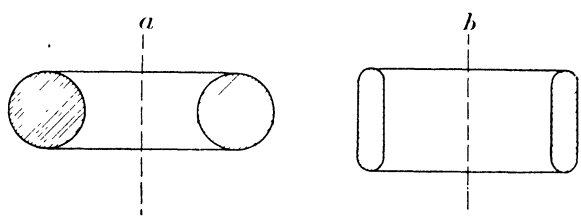


Fig. 16.

Besides this method, there is another one called the yoke method, which is a variation of the ballistic method. A test-bar is

inserted inside the framework of a large yoke made of iron, and the primary and secondary coils are wound on the bar. It is desirable to have the sectional area of the frame very large as compared with that of the bar. Since the frame absorbs most of the lines of force due to the magnetized bar, the free magnetism appearing at both ends of the bar or on the frame is inappreciable, so that the demagnetizing action vanishes. The bar is therefore magnetized uniformly as in the case of a ring.

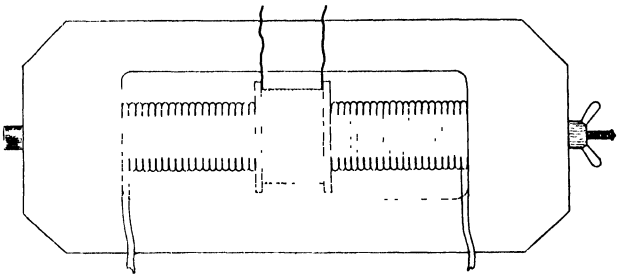


Fig. 17.

10. Electromagnets.

In order to get a very strong magnetic field by means of a coil, a very strong current must be used, which heats the coil, and hence the use of too strong a current becomes inadmissible. Thus a limit is put to the magnitude of the magnetic field produced by a coil. The strongest magnetic field hitherto obtained by means of a coil does not exceed three thousand gausses in strength. To obtain a still stronger field, we must have recourse to other methods, one of which lies in the use of an electromagnet.

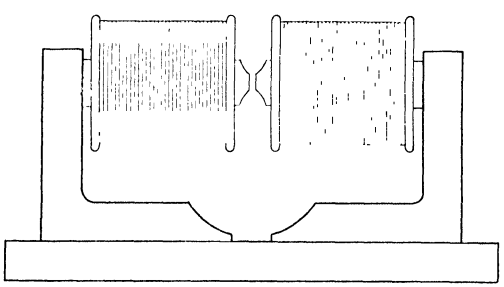


Fig. 18.

There are various types of electromagnets, the first type being that of Rumkorf. It consists of an electromagnet whose two poles are placed horizontally and face to face in the same line. The pole distance can be adjusted at will. If we pass an electric current through the coil, the iron cores are strongly magnetized and produce a strong field in the interspace between the two poles.

P. Weiss modified this electromagnet, and obtained a very powerful magnetic field. Essential parts of his improvements are:— (i) The iron cores as well as the iron frame connecting them are made sufficiently thick to minimize leakage of magnetic flux into the surrounding space. (ii) A cooling device prevents the strong heating of the coils. (iii) The central portion of the pole-pieces is replaced by ferro-cobalt (33% Co). By this improved electromagnet, he obtained a magnetic field amounting to a strength of 54,000 gausses for a distance of 1 mm between the pole-pieces, the diameter of its end surface being one centimetre.

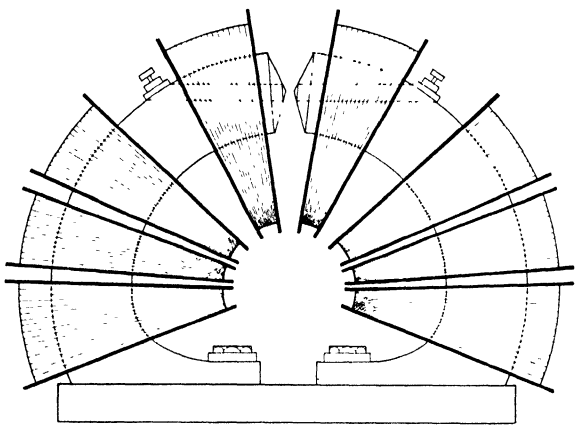


Fig. 19.

The second type of electromagnet is due to H. du Bois. The iron cores are of a semi-circular form instead of a straight one, as shown in Fig. 19. With cores of this form the leakage of the flux is comparatively small, and hence this type also gives a strong field.

In order to obtain, further, a much stronger field, such as one hundred thousand gausses, will it be necessary to magnify only the dimensions of the electromagnet? As the following explanation will show, this is not the case.

A magnetic field produced at a point P in the interspace between the pole-pieces is the sum of the fields produced, one by the coils and the other by the distribution of free magnetism appearing on the surface of the pole-pieces, the former being usually very small as compared with the latter. We shall consider first the field due to the free magnetism, which may be regarded as consisting of two parts:— One is the field due to the free magne-

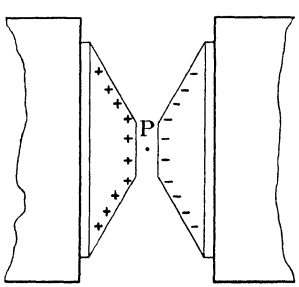


Fig. 20.

tism appearing on the flat surfaces of the pole-pieces and nearly equal to $4\pi I$ for a small pole-distance, and the other is due to the free magnetism appearing on their slant surfaces. Suppose the electromagnet to become greater in dimensions, its form remaining always similar; then the area and therefore the free magnetism on the slant surfaces gradually increase, but the distance of the increased area of the slant surfaces from point P also increases; hence the increase of the field at the same point is comparatively small. Since the intensity of magnetization has a saturation value I_0 and for pole-pieces made of iron, I_0 is 1700, $4\pi I_0$ cannot exceed 21,300. Hence we see the reason why by making the electromagnet sufficiently large, we cannot obtain a stronger magnetic field beyond a certain limit, this limit being probably one of sixty to seventy thousand gausses.

It is, however, very important that for the investigation of the nature of paramagnetic and diamagnetic substances, we should be able to avail ourselves of a much stronger field than that hitherto obtained. For this purpose, we again have recourse to the coil method in place of the electromagnet, this time, however, furnished with an effective cooling device for the coil.

H. Deslandres and A. Perrot[1] used an additional copper band coil, the thickness and the width of the band being respectively 0.5 mm and 1 mm, as shown in Fig. 21, and a current of cold oil, cooled to -20^0, was constantly passed in the interspace round the coil by pumping. In this way, they were able to pass a current of 1000 ampères through the coil, and the field due to this alone was one of nearly ten thousand gausses; they remark edthat with a Weiss electromagnet of the middle type, a field of fifty thousand gausses was obtained.

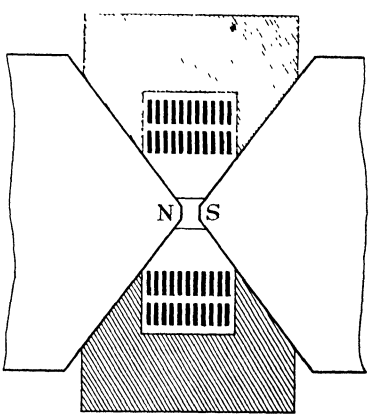


Fig. 21.

Let us now suppose that the cooling device of the coil is sufficiently effective and that a strong electric current can be passed

(1) H. Deslandres et A. Perrot: Contribution à la réalisation de champs magnétiques élevés. Concentration des ampères-tours dans un très petit volume. C.R. **158**, (1914), 226.

through the coil without sensible heating; the magnetic field inside the coil is then,

$$H = 4\pi ni.$$

If the number of layers of copper band, 0.5 mm thick and 1 cm wide, be 100, then

$$H = 1257i.$$

In the case where i=1000, that is, 100 e. m. unit,

$$H = 125{,}700.$$

Thus, it appears that we should be able to get a stronger field than one of one hundred thousand gausses; hence, though great difficulty accompanies the practical realization, it would not be impossible to get a constant field of far beyond one hundred thousand gausses by means of the coil method. In such a strong field, by inserting an iron core into the coil, the field strength can only be increased by nearly twenty thousand gausses, that is, by less than 20 percent; hence the effect of the iron core is here relatively small. This small increase of the field can be obtained more easily by increasing the strength of the current.

Special methods are required for the measurement of the strong field due to an electromagnet. We shall describe them below:—

(a) *The Bismuth Spiral Method.* Bismuth has the property of changing its electric resistance when it is placed in a strong magnetic field. Hence if we find once the relation between the field and the change of resistance, the intensity of magnetic field can be found from the measurement of the resistance. A thin bismuth wire is wound in a spiral form as shown in Fig. 22, each turn being well insulated, and covered on both faces by thin mica plates for protection. The two ends of the spiral wire are connected with thick copper wires. To measure a magnetic field by means of this apparatus, we first measure the resistance of the bismuth wire in the case where there is no magnetic field, and then measure its resistance by placing the spiral face perpendicular to the field. From these two measurements, we obtain the percentage change of resistance due to the magnetic field, and from this change and the given

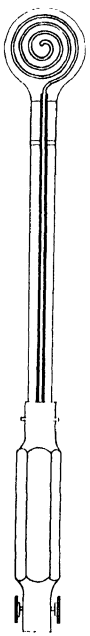


Fig. 22.

curve expressing the relation between the change of resistance and the magnetic field, the strength of the field can be found, a small correction due to the variation of room temperature being of course made. Though this method, due to P. Lenard, is very simple, it is not a very accurate one; for not only does the resistance depend on the temperature, but its temperature coefficient varies with the strength of the magnetic field, so that the correction due to the variation of room temperature cannot be made accurately. Fig. 23 shows a curve representing the relation between the change of electric resistance and the magnetic field.

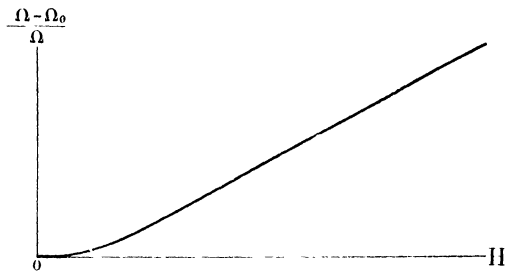


Fig. 23.

(b) *The Method of Plane-polarised Light.* When a plane-polarized light passes through the plate of a transparent substance placed in a strong field, the plane of polarization is rotated. If we denote the angle of rotation by θ, θ is proportional to the intensity of magnetic field H, and to the length of the path through which the light travels, that is,

$$\theta = cHl,$$

where c is a proportional constant called Verdet's constant. This c is a well-known constant depending not only on the property of the substance, but also on the wave-length of light. If θ and l be directly measured, H can be known from the above expression. Quartz is commonly used as a substance giving rise to a rotation of the plane of polarization in a magnetic field; but as it rotates the plane of polarization without the magnetic field, to eliminate this rotation, a plate consisting of dextro and levo quartz plates of equal thickness is used. The following table[1] shows the values of c in quartz for given wave-lengths of light:—

(1) A. Borel, Arch. des Sci. **16**, (1903), 24, 157.

Wave length ($\mu\mu$)	c (min)
643.9	0.01385
589.0	0.01684
508.6	0.02285
480.0	0.02605
467.9	0.02785
360.9	0.04684
257.3	0.10725
219.4	0.16032

(c) *The Coil Method.* This method is based on the measurement of the total quantity of electricity induced in a small flat coil placed perpendicularly to a magnetic field. Usually, the magnetic field between the pole-pieces of an electromagnet is confined to a small space, so that the exploring coil must be sufficiently small. We usually make use of a coil, 5 mm in diameter and 1 mm in thickness, wound closely of fine insulated copper wire. In order to find an effective area of the coil for induction, we compare it with another coil consisting of one layer and having a known diameter. We place these coils in series in the circuit of a ballistic galvanometer, and observe the deflections caused by drawing each of the coils out of a uniform field; the ratio of these deflections is equal to that of the areas of the coils. We can thus find the area of the exploring coil from that of the known coil. The area so found is called the equivalent area of the coil. Among the three methods abovementioned, this affords the most accurate result.

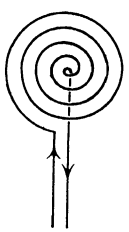


Fig. 24.

11. Method of Measuring the Intensity of Magnetization in the Field of an Electromagnet.

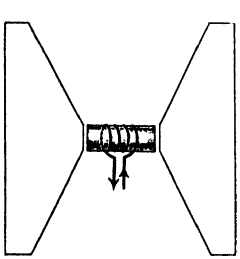


Fig. 25.

There are two methods for the measurement of the intensity of magnetization in a strong field due to an electromagnet. The first is the method making use of a ballistic galvanometer, in which a test specimen in a cylindrical form is brought across the pole-pieces of the electromagnet, wound round by a secondary coil of several turns and connected with a ballistic galvanometer placed in the circuit of the standard coil. A cur-

rent is first passed through the electromagnet, and the total flux due to the specimen is measured by withdrawing it suddenly from the field, and thus the value of B is obtained. A defect of this method is that the residual magnetism does not enter into the measurement; this defect may, however, be removed by measuring the flux, which is due to the reversal of the current passing through the electromagnet, instead of taking the specimen out of the field. In this case, we obtain a deflection of the galvanometer corresponding to $2B$. This method furnishes a good result for a strong field, but does not give an accurate result in the case of a weak field, on account of the phenomenon of hysteresis.

In using the ballistic method, the magnetic induction B is directly measured, and hence the magnetic field must be known in order that the intensity of magnetization can be known. Nevertheless, the measurement of the magnetic field is not an easy matter; for the field in the interspace between the pole-pieces, when there is no specimen, may differ from the field in the same space with the specimen within it, and hence the actual field can only be obtained by measuring it in the latter case. This may be effected in the following way:—

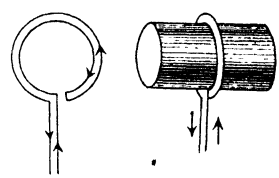


Fig. 26.

A fine insulated wire is wound on the specimen in a ring-formed coil, several turns in all, as shown in Fig. 26, the distance between the inner and outer wires being about 1 mm. As the magnetic field varies continuously from the inside to the outside of the specimen in the direction normal to the field, the magnetic field just outside the specimen in its middle portion must be the same as that inside, and hence the magnetic field inside the annular space may be taken as an actual field within the specimen. We may therefore obtain the required magnetic field by measuring the magnetic flux induced in the annular coil.

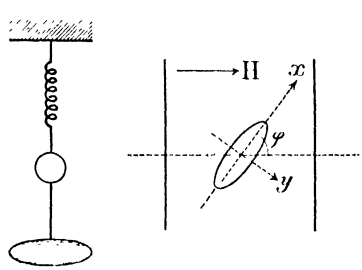


Fig. 27.

The second method for measuring the intensity of magnetization in a strong field is to make use of the torsional couple; by this method, P. Weiss obtained several important results. An ellipsoid of rotation is suspended by a spring in a uniform field, and if its principal axis shows an inclination in relation to the field, a couple

is exerted on the ellipsoid. This couple may be known from the angle of twist of the spring, which is measured by means of a mirror fixed to the ellipsoid, a scale and a telescope being placed in front of it.

The relation between the magnetization I and the couple C can be obtained as follows:—Let the axes of an ellipsoid horizontally suspended be taken as x and y axes, the components of I be denoted by A and B, and those of H by X and Y, then we get

$$A=\frac{\kappa X}{1+\kappa L} \quad \text{and} \quad B=\frac{\kappa Y}{1+\kappa M},$$

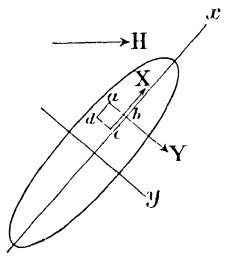


Fig. 28.

L and M being the demagnetizing factors in the direction of x and y. Consider now an elementary volume $a\,b\,c\,d$ in the ellipsoid as shown in Fig. 28, and calculate the couple about the vertical axis z. On the surfaces ab and dc, free magnetism $+A\,dy\,dz$ and $-A\,dy\,dz$ and on the surfaces ad and bc, $+B\,dx\,dz$ and $-B\,dx\,dz$ appear. Hence, the couple ∂C acting on this elementary volume is equal to $(AY-BX)\,dx\,dy\,dz$. But

$$\partial C=(AY-BX)\,dx\,dy\,dz=\kappa XY\left\{\frac{1}{1+\kappa L}-\frac{1}{1+\kappa M}\right\}dx\,dy\,dz,$$

and if φ be the angle between the field and the x axis, we have

$$X=H\cos\varphi, \qquad Y=H\sin\varphi.$$

$$\therefore\ \partial C=\kappa H^2\sin\varphi\cos\varphi\left\{\frac{1}{1+\kappa L}-\frac{1}{1+\kappa M}\right\}dx\,dy\,dz$$

$$=\kappa H^2\sin\varphi\cos\varphi\,\frac{M-L}{(1+\kappa L)(1+\kappa M)}\,dx\,dy\,dz.$$

On the other hand, if ψ be the angle between the magnetization and the x axis, we have

$$A=I\cos\psi=\frac{\kappa H\cos\varphi}{1+\kappa L}, \qquad B=I\sin\psi=\frac{\kappa H\sin\varphi}{1+\kappa M}.$$

Consequently,

$$I^2\cos\psi\sin\psi=\frac{\kappa^2H^2\cos\varphi\sin\varphi}{(1+\kappa L)(1+\kappa M)}.$$

Substituting this expression in the expression for ∂C, we get

$$\delta C=(M-L)I^2\sin\psi\cos\psi dx\,dy\,dz.$$

Since the ellipsoid is magnetized uniformly, the resultant couple C is,

$$C = (M - L) I^2 \sin\psi \cos\psi \int dx\, dy\, dz$$

$$= \frac{1}{2} v (M - L) I^2 \sin 2\psi,$$

where v is the whole volume of the ellipsoid. This couple becomes a maximum for $\psi = 45^0$, and if this maximum value be denoted by C_m, then

$$C_m = \frac{1}{2} v (M - L) I^2.$$

Hence, we can find I from the above expression by measuring the maximum couple C_m.

It is the special advantage of this method that the intensity of magnetization can be obtained without the knowledge of the magnetic field; this is therefore the most suitable method for determining the saturation value of the intensity of magnetization. It is, however, to be noted that in using this method, the field should be uniform over a wide range, otherwise the specimen is easily attracted by the pole-piece.

12. Intensity of Magnetization of Ferromagnetic Substances.

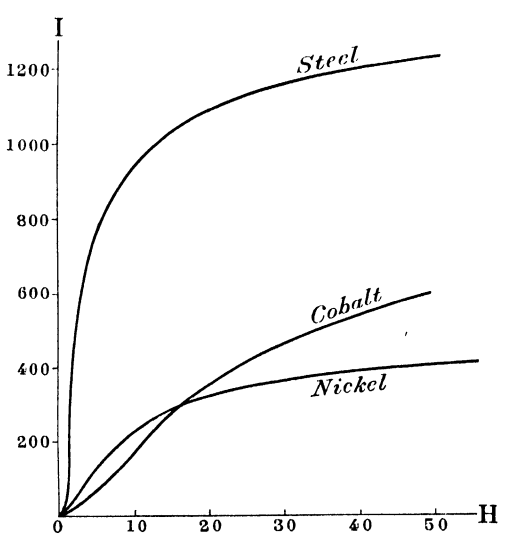


Fig. 29. (Nagaoka, Honda)

Iron, nickel and cobalt are three elements belonging to ferromagnetic substances, and the alloys of these elements are also ferromagnetic. For an accurate determination of the intensity of magnetization of ferromagnetic substances, we must use a long ellipsoid of rotation with a small demagnetizing factor. The intensity of magnetization of the ellipsoid of rotation of iron, nickel and cobalt has been determined by

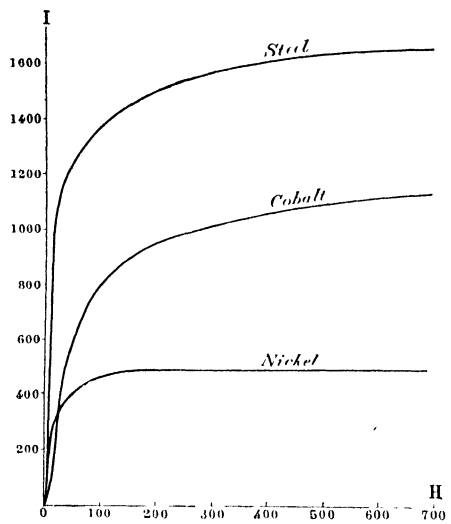


Fig. 30. (Nagaoka, Honda)

H. du Bois[1], H. Nagaoka and K. Honda[2], K. Honda and S. Shimidzu[3], P. Weiss[4], etc. Among them, du Bois and Weiss have determined the saturation value of magnetization in strong fields. Fig. 29 shows the intensity of magnetization in weak fields, and Fig. 30 that of magnetization in strong fields. We see that the intensity of magnetization increases gradually at first, then very rapidly, and afterwards again slowly with the increase of the field; while in very strong fields, the intensity of magnetization gradually approaches its saturation value. P. Weiss used small ellipsoids of rotation made of iron, nickel and cobalt, and measured the saturation value of magnetization in strong fields of 4 to 5 thousand gausses by the couple method; the following table contains his results:—

Substances	Major axis *2c*	Minor axis *2a*	Temperature °C	Intensity of magnetization
Iron(0.09%C)	8.95mm	2.58mm	19°	1706
nickel	9.02	3.64	18°	479
cobalt	7.93	3.26	17°	1412

13. Magnetic Hysteresis.

When during the process of the magnetization of a piece of iron, the magnetic field is diminished at a point *b* in the curve of magnetization, the *I,H* curve does not take the same path *ba*,

(1) H. du Bois, Phil. Mag. **29**, (1890), 293. (2) Nagaoka and Honda, Jour. Coll. Sci. **16**, Art. 8, (1900). (3) Honda and Shimidzu, Jour. Coll. Sci. **20**, Art. 6, (1904). (4) P. Weiss, Jour. de Phys., **9**, (1910) 373.

but takes another course, such as curve *bc* in Fig. 31, which is quite different from the curve *ab*; hence if the field be reduced to zero, the magnetization does not vanish. This magnetization is called residual magnetism. Next we apply a weak field in the opposite direction and make it gradually increase, then the intensity of magnetization gradually diminishes and vanishes finally at a certain value of the field. The magnetic field which it is necessary to apply in the opposite direction in order to make the magnetization vanish, is called the coercive force. A substance having a large residual magnetism and a large coercive force is adapted for a permanent magnet.

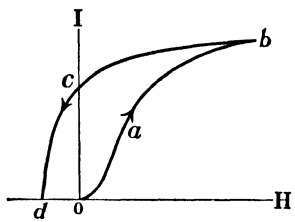


Fig. 31.

When the magnetic field is cyclically varied between certain values, the curve of magnetization forms a loop as shown in Fig. 32. This is called a hysteresis-loop. As will be explained in Art. 17, the area enclosed by this curve expresses the loss of energy during the cyclic process of magnetization. The area of the loop varies from substance to substance; for example, it is small in soft iron, but large in steel. Especially, for electrolytic iron obtained in a magnetic field, the area of the hysteresis-loop is considerably large as shown in Fig. 33.

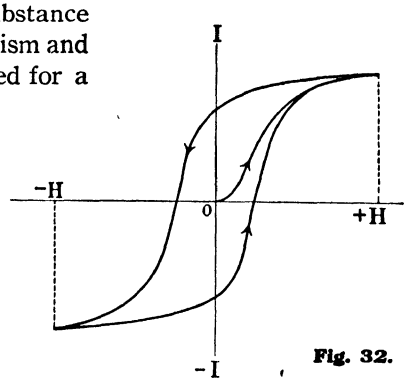


Fig. 32.

Beetz[1] and Maurain[2] showed by experiments that electrically deposited iron has a strong residual magnetism and coercive force and has a hysteresis-loop similar to a rectangle. Maurain explained this fact by assuming that the molecules of electrically deposited iron are regularly arranged under

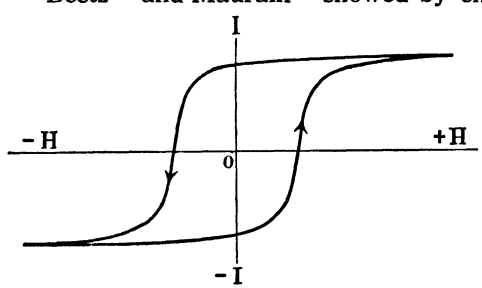


Fig. 33.

(1) W. Beetz, Pogg. Ann., **111**, (1860), 107. (2) M. C. Maurain, C.R., **131**, (1900), 410, 880.

the action of the magnetic field. Kaufmann and Meier[1] further investigated the phenomenon, and concluded that the phenomenon is due to occluded hydrogen rather than to the action of the magnetic field. Now a single crystal of iron shows a very small hysteresis, and hence Maurain's view that a regular arrangement of iron atoms is the cause of the large hysteresis, is not valid. On the other hand, since electrolytic iron absorbs hydrogen to a considerable extent, the hydrogen atoms in the interspace of the iron lattice prevent more or less the rotation of the atomic magnets and make the magnetization of the iron difficult, thus giving rise to the large hysteresis as suggested by Kaufmann and Meier.

14. Magnetization in Very Weak Fields.

We shall next consider the magnetization in very weak fields. Bauer[2] and Lord Rayleigh[3] first made an accurate investigation of the matter and found proportionality between the magnetization and the field; especially, the latter succeeded by an ingenious method in proving the above law of proportionality. Thus, he placed a soft iron wire in a magnetizing coil and magnetized it by passing a weak current through the coil; the total deflection of the magnetometer, due to the wire and the magnetizing coil, was then compensated by means of the magnetic action of a compensating coil in the usual way. In a field of 0.04, the compensation was made perfect; then he decreased the field by reducing the magnetizing current gradually and found that the compensation remained perfect down to a field of 0.00004. This shows that the proportionality between I and H holds good within the range from 0.04 to 0.00004. His experiments also show that in a range of fields from 0.04 to 1.2, the following relation is satisfied:

$$I = 6.4H + 5.1H^2,$$

or

$$\kappa = 6.4 + 5.1H.$$

The constants in this expression vary with the samples of iron. Afterwards, P. Weiss[4], Holborn[5], Pierce[6], Gumlich and Ragowski[7], &c., also investigated the same subject and confirmed the above-mentioned relation.

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(1) W. Kaufmann u. W. Meier, Phys. Zeitschr. **12**, (1911), 513. (2) C. Bauer, Wied. Ann. **11**, (1880), 394. (3) Lord Rayleigh, Phil. Mag., **23**, (1887), 225. (4) P. Weiss (1896). (5) Holborn, (1897). (6) Pierce, (1910). (7) Gumlich u. Ragowski, (1911).

In the case of magnetization in a very weak field, there is a phenomenon called magnetic viscosity; that is, in the magnetization of a thick bar in a very weak field, the magnetization does not attain its final value at the instant of the application of the magnetizing field, but some interval of time is required for its increase. The thicker the bar and the weaker the field, the more remarkable is this phenomenon. According to Ewing's experiments, in which a bar, 4 mm thick and 40 cm long, was magnetized in a field of 0.044, the relation between the time which elapsed after passing the magnetizing current and the increase of magnetization was as follows:—

t″ (seconds)	0	5	60
Deflection of magnetometer	44	58	67

The relation between the time which elapsed after breaking the current and the decrease of magnetization was as follows:

t″ (seconds)	0	5	60
Deflection of magnetometer	24	9	0

Thus, we see that an interval of time is required both for the increase and the decrease of magnetization.

Y. Homma[1] also thoroughly investigated the above phenomenon. According to him, this phenomenon is due to the fact, that the state of magnetization is propagated from the outside towards the inside under the action of the magnetic field, as in the case of the conduction of heat, and that the thicker the bar, the longer is the time required to reach its final magnetization.

15. Distribution of Free Magnetism on a Magnetized Substance.

In order to ascertain the distribution of free magnetism on a bar or an ellipsoid made of a ferromagnetic substance placed in a uniform magnetic field, it is convenient to move a flat secondary coil a definite distance on the substance and measure the induction thus produced by means of a ballistic galvanometer. The usual process of measurement is to shift the secondary coil fitted to

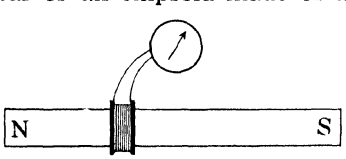


Fig. 34.

(1) Y. Homma, Jour. Coll. Sci. **9**, (1896) 295.

the substance by 5 or 10 cm along the length, and to obtain the distribution of free magnetism from the corresponding deflection of the galvanometer. By a similar arrangement, C. Benedicks[1], Yoshida and Kadooka[2] made accurate investigations of the distribution of free magnetism on magnetized bars. Figs. 35 and 36 are the results given by Yoshida and Kadooka. The curves in Fig. 35 show the distributions of free magnetism along the length of the bar under the action of various fields. As the field increases, the centre of mass of the free magnetism, that is, the magnetic poles gradually approach the ends of the bar. We know also that in the case of an ellipsoid of rotation, the distribution of free magnetism increases nearly proportionally to the distance from the centre of the ellipsoid, excepting its end portion.

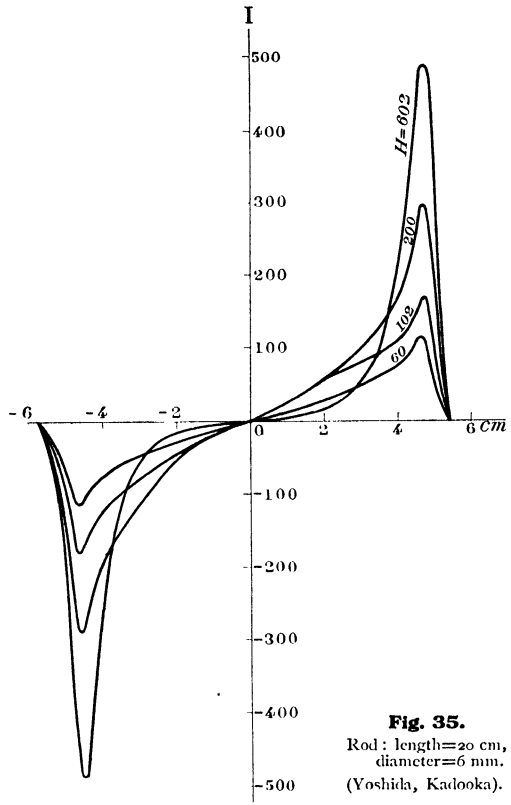


Fig. 35.
Rod: length=20 cm, diameter=6 mm.
(Yoshida, Kadooka).

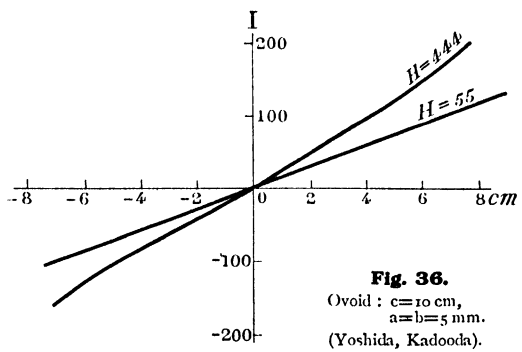


Fig. 36.
Ovoid: c=10 cm, a=b=5 mm.
(Yoshida, Kadooda).

(1) C. Benedicks, Ueber die Polabstand magnetisiert. Cylinder, Acad. Handl. **27**, Anfl. **1**, No. 5. (2) Yoshida and Kadooka, Tokyo Sug.-Butu. Kiji **3**, (1906), 150.

Fig. 37 shows the distribution of free magnetism on a bar magnet made of Hadfield's nickel steel (29% Ni, 3% Mn). If this steel is once heated to a high temperature of about 800° and then cooled, it is very weakly magnetic; if, however, it is immersed in liquid air, it becomes strongly magnetic. In the case where one-half of the bar made of the steel is immersed in liquid air, that half of the bar becomes strongly magnetic, while the other half is only weakly magnetic. A permanent magnet made in this way has four poles instead of two.

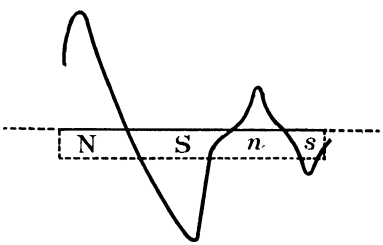


Fig. 37. (Honda)

16. Effect of Vibration on Magnetization.

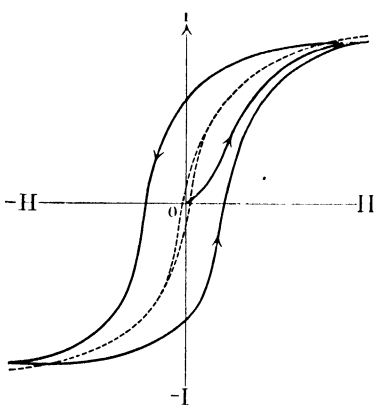


Fig. 38.

If during magnetization, we give an iron bar a constant vibration by repeatedly striking it with a hammer, it is strongly magnetized even in a weak field, and the hysteresis phenomenon is considerably reduced. This fact is shown in Fig. 38; that is, the full line shows the usual hysteresis curve, and the dotted one, the same curve taken under a constant vibration. The effect of vibration is considerable in a weak field, but small in a strong field. In an increasing field, vibration increases the magnetization, but in a decreasing field, diminishes it. The fact that in work-shops, drills are sometimes strongly magnetized, is explained by the magnetization of drills by the earth magnetic field under repeated shocks. If a permanent magnet undergoes a vibration, its intensity of magnetization slightly diminishes.

Instead of giving a mechanical vibration, J. R. Ashworth[1] passed an alternating current through an iron wire and obtained

(1) R. Ashworth, Phil. Mag. **27**, (1914), 357.

a result similar to the case of a mechanical vibration. He also investigated the same effect on magnetization at various high temperatures. The effect due to an alternating current flowing through an iron wire is not so simple as that due to meachnical vibration; because, by the passing of an alternating current through an iron wire, a circular field perpendicular to the wire is produced inside it and hence an alternating screw magnetization is produced by the combined effect of longitudinal and circular fields. As the heating of a substance results in a molecular agitation, it gives the same result as the mechanical vibration.

17. Hysteresis-Loop.

When a ferromagnetic substance is magnetized and the field varied cyclically between certain ranges, the curve showing the relation between the magnetization and the magnetic field forms a hysteresis-loop enclosing an area. This area expresses the energy consumed during the cyclic magnetization; this fact was discovered independently by Warburg[1] and Ewing[2].

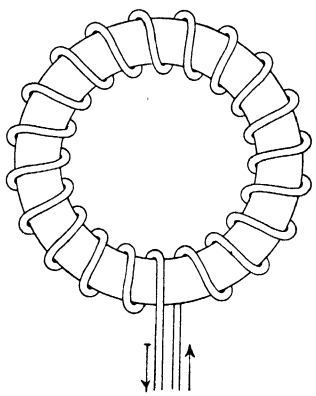


Fig. 39.

Let us suppose that a magnetizing coil is uniformly wound on a ring-shaped ferromagnetic substance and magnetized by passing an electric current through it. If we denote the length of the coil by l, the number of turns of the coil per unit length by n, and the current flowing through the coil by i, then the field inside the coil is

$$H = 4\pi ni.$$

Let us suppose that during time dt the current i changes its magnitude by di; then at the same time, the magnetic induction B changes by dB. If the change of lines of induction passing through the coil be dN and the area of a section of the ring be S, then

(1) E. Warburg, Wied. Ann. **13**, (1881), 141. (2) J. A. Ewing, Proc. Roy. Soc. **34**, (1882), 39.

$$dN = lnSdB.$$

Consequently, the induced electromotive force e due to the change of the current is

$$e = -\frac{dN}{dt} = -lnS\frac{dB}{dt},$$

and its direction is opposite to that of the magnetizing current. Since the magnetizing current is to flow against this induced electromotive force, an excess of work must be applied for the change of magnetization. In other words, the current must perform an excess of work $-eidt$ on the substance.

Hence $$-eidt = lniSdB.$$

Denoting the work per unit volume of the magnetic substance by dW, we get

$$dW = nidB = \frac{H}{4\pi}dB.$$

Accordingly the work necessary to magnetize the substance from B_1 to B_2 is

$$W = \frac{1}{4\pi}\int_{B_1}^{B_2} HdB.$$

Substituting the relation $B = 4\pi I + H$ in the above expression, we get

$$W = \int_{I_1}^{I_2} HdI + \frac{1}{4\pi}\int_{H_1}^{H_2} HdH.$$

For cyclic magnetization, the second term in the above expression vanishes, and hence we obtain

$$W = \int HdI.$$

That is, W denotes the area of the hysteresis-loop. Hence during the process of cyclic magnetization the energy equal to this work is given to the magnetic substance, and converted into heat, causing a small rise of temperature. In the following table, actual examples of the rise of temperature due to the hysteresis-loss is given for the case of soft iron.

H	B	I	W	Temperature rise
1.5	1974	167	410 erg	0.000012°
2.56	5950	473	2190	0.000062
6.62	11480	913	6160	0.000175
26.5	13720	1090	8690	0.000247

The relation between the loss of energy and the induction is given by Steinmetz's empirical formula, that is,

$$W = \eta B^n,$$

where n has usually a value 1.6 and η is a constant depending on the property of the substance. Although this formula applies satisfactorily to the results of experiments in moderate fields, it does not hold good so well in weak fields or strong fields. The following are the values of n and η for iron, steel and cobalt:—

Soft iron	$n = 1.6$	$\eta = 0.0020$
Carbon steel	$n = 1.6$	$\eta = 0.0025$
Cobalt	$n = 1.6$	$\eta = 0.0010$.

CHAPTER III.

Magnetism and Stress.

18. General Theory.

In attempting to find the relation between magnetism and stress, or between magnetism and strain, in a magnetic body, let it be assumed that the conditions of a body placed in a magnetic field are determined by the temperature T, the stress p and the magnetic field H, and the principles of thermodynamics be applied to the body.

By the law of conservation of energy, the energy dU which is acquired by a body is given by

$$dU = dQ + dW,$$

where dQ is the quantity of heat received by the body and dW the work done on the body from the outside. The relation between dQ, dW and the change of the independent variables dT, dp and dH may be written

$$dQ = Q_1 dT + Q_2 dp + Q_3 dH,$$

$$dW = W_1 dT + W_2 dp + W_3 dH.$$

When these values are substituted in the above expression, we have

$$dU = (Q_1 + W_1)\, dT + (Q_2 + W_2)\, dp + (Q_3 + W_3)\, dH.$$

Hence

$$\frac{\partial U}{\partial T} = Q_1 + W_1,$$

$$\frac{\partial U}{\partial p} = Q_2 + W_2,$$

$$\frac{\partial U}{\partial H} = Q_3 + W_3.$$

Consequently,
$$\left.\begin{aligned}\frac{\partial}{\partial p}(Q_1+W_1) &= \frac{\partial}{\partial T}(Q_2+W_2),\\ \frac{\partial}{\partial H}(Q_2+W_2) &= \frac{\partial}{\partial p}(Q_3+W_3).\end{aligned}\right\} \quad (1)$$

Again since the entropy of the body is dQ/T, we have

$$\frac{dQ}{T} = \frac{Q_1}{T}dT + \frac{Q_2}{T}dp + \frac{Q_3}{T}dH;$$

by proceeding as before, the following relations are obtained

$$\left.\begin{aligned}\frac{\partial}{\partial p}\left(\frac{Q_1}{T}\right) &= \frac{\partial}{\partial T}\left(\frac{Q_2}{T}\right),\\ \frac{\partial}{\partial H}\left(\frac{Q_2}{T}\right) &= \frac{\partial}{\partial p}\left(\frac{Q_3}{T}\right).\end{aligned}\right\} \quad (2)$$

If the differentiation of the first equation be conducted, remembering that p and T are variables which are independent of each other, we have from the first of the above equations

$$\frac{1}{T}\frac{\partial Q_1}{\partial p} = \frac{1}{T}\frac{\partial Q_2}{\partial T} - \frac{Q_2}{T^2}.$$

Therefore

Similarly,
$$\left.\begin{aligned}\frac{\partial Q_1}{\partial p} &= \frac{\partial Q_2}{\partial T} - \frac{Q_2}{T}.\\ \frac{\partial Q_2}{\partial H} &= \frac{\partial Q_3}{\partial p}.\end{aligned}\right\} \quad (3)$$

Transforming the first of equation (3), we get

$$Q_2 = T\left\{\frac{\partial Q_2}{\partial T} - \frac{\partial Q_1}{\partial p}\right\}. \quad (4)$$

Hence from the first of equations (1),

$$Q_2 = T\left\{\frac{\partial W_1}{\partial p} - \frac{\partial W_2}{\partial T}\right\}. \quad (I)$$

Also, from the second of equations (1) and (3), we get

$$\frac{\partial W_2}{\partial H} = \frac{\partial W_3}{\partial p}. \quad (II)$$

The above relations (I) and (II) hold good generally and are independent of the form of the body and the kind of the stress.

19. Special Cases.

To apply the relation obtained in the foregoing article to special cases, let a rod of length l, be subjected to a tension P in a magnetic field H. If the rod has suffered an increase ∂l in length and dB in the magnetic induction on account of a heat quantity introduced into the body, then the work done dW on the body is given by

$$dW = Pdl + \frac{Hv}{4\pi}\, dB,$$

where v is the volume of the body. The first term of this expression is the work done for the change of length and the second term that for the magnetization. Since l and B are both functions of T, P and H, we have

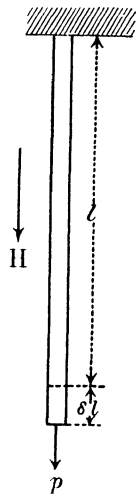


Fig. 40.

$$\begin{aligned} dW &= P\left(\frac{\partial l}{\partial T}\, dT + \frac{\partial l}{\partial P}\, dP + \frac{\partial l}{\partial H}\, dH\right) \\ &\quad + \frac{vH}{4\pi}\left(\frac{\partial B}{\partial T}\, dT + \frac{\partial B}{\partial P}\, dP + \frac{\partial B}{\partial H}\, dH\right) \\ &= \left(P\frac{\partial l}{\partial T} + \frac{vH}{4\pi}\frac{\partial B}{\partial T}\right) dT \\ &\quad + \left(P\frac{\partial l}{\partial P} + \frac{vH}{4\pi}\frac{\partial B}{P\partial}\right) dP \\ &\quad + \left(P\frac{\partial l}{\partial H} + \frac{vH}{4\pi}\frac{\partial B}{\partial H}\right) dH. \end{aligned}$$

This expression corresponds to the relation mentioned before, that is,

$$dW = W_1 dT + W_2 dP + W_3 dH;$$

hence from the relation (II), we have

$$\frac{\partial}{\partial H}\left(P\frac{\partial l}{\partial P} + \frac{vH}{4\pi}\frac{\partial B}{\partial P}\right) = \frac{\partial}{\partial P}\left(P\frac{\partial l}{\partial H} + \frac{vH}{4\pi}\frac{\partial B}{\partial H}\right).$$

Differentiating and simplifying the above relation,

$$\frac{H}{4\pi}\frac{\partial v}{\partial H}\frac{\partial B}{\partial P} + \frac{v}{4\pi}\frac{\partial B}{\partial P} = \frac{\partial l}{\partial H} + \frac{H}{4\pi}\frac{\partial B}{\partial H}\frac{\partial v}{\partial P}.$$

If now the small terms $\frac{H}{4\pi}\frac{\partial v}{\partial H}\frac{\partial B}{\partial P}$ and $\frac{H}{4\pi}\frac{\partial v}{\partial P}\frac{\partial B}{\partial H}$ be neglected, the following simple relation is obtained,

$$\frac{v}{4\pi}\frac{\partial B}{\partial P}=\frac{\partial l}{\partial H}.$$

But since $B=4\pi I+H$, if S be the section of the rod,

$$S\frac{\partial I}{\partial P}=\frac{1}{l}\frac{\partial l}{\partial H}.$$

If $\partial l/l$ be denoted by ∂e, the elongation per unit length, and $\partial P/S$, the change of tension per unit area, by ∂p, we obtain the following relation

$$\frac{\partial I}{\partial p}=\frac{\partial e}{\partial H}, \tag{III}$$

where $\partial l/\partial p$ and $\partial e/\partial H$ are respectively differential coefficients, when H and p are kept constant. It follows from this relation that in the case of a body, which increases its length when under a constant tension, subjected to a magnetic field, the intensity of magnetization under a constant field is increased by the application of tension.

If pressure p, instead of tension, be applied, the results will be

$$\frac{\partial I}{\partial p}=-\frac{\partial e}{\partial H}. \tag{III'}$$

If the equation (III) be differentiated with respect to p, the following relation is obtained,

$$\frac{\partial}{\partial p}\left(\frac{\partial e}{\partial H}\right)=\frac{\partial^2 I}{\partial p^2}=\frac{\partial}{\partial H}\left(\frac{\partial e}{\partial p}\right).$$

If E denotes the modulus of elasticity, $\frac{\partial e}{\partial p}=\frac{1}{E}$, hence we have

$$\frac{\partial}{\partial H}\left(\frac{\partial e}{\partial p}\right)=-\frac{1}{E^2}\frac{\partial E}{\partial H},$$

or,

$$\frac{\partial E}{\partial H}=-E^2\frac{\partial^2 I}{\partial p^2}. \tag{IV}$$

Thus we see that if the intensity of magnetization of a body be expressed as a function of the second or higher degree of stress, the modulus of elasticity will suffer a change by the action of a magnetic field.

Also by using the relation (I), we obtain from the expression for dW the following relation

$$Q_2 = T\left\{\frac{\partial}{\partial P}\left(P\frac{\partial l}{\partial T} + \frac{vH}{4\pi}\frac{\partial B}{\partial T}\right) - \frac{\partial}{\partial T}\left(P\frac{\partial l}{\partial P} + \frac{vH}{4\pi}\frac{\partial B}{\partial P}\right)\right\}.$$

$$\therefore \quad Q_2 = T\left\{\frac{\partial l}{\partial T} + \frac{H}{4\pi}\left(\frac{\partial v}{\partial P}\frac{\partial B}{\partial T} - \frac{\partial v}{\partial T}\frac{\partial B}{\partial P}\right)\right\}$$

$$= T\left\{l\frac{\partial e}{\partial T} + \frac{Hv}{4\pi}\left(\frac{\partial \omega}{\partial P}\frac{\partial B}{\partial T} - \frac{\partial \omega}{\partial T}\frac{\partial B}{\partial P}\right)\right\},$$

where $\partial\omega$ is the change of volume per unit volume. If now the coefficient of expansion be α,

$$\alpha = \frac{\partial e}{\partial T} = \frac{1}{3}\frac{\partial \omega}{\partial T}.$$

Also

$$Q_2 = \frac{\partial Q}{\partial P} = \frac{1}{S}\frac{\partial Q}{\partial p}.$$

Hence

$$\frac{\partial Q}{\partial p} = Tv\alpha + \frac{THv}{4\pi}\left(\frac{\partial \omega}{\partial p}\frac{\partial B}{\partial T} - 3\alpha\frac{\partial B}{\partial p}\right).$$

$\partial Q/\partial p$ is the quantity of heat to be added from outside to maintani the temperature constant, when the body is subjected to tension. So if the density of the body be ρ and its specific heat c,

$$\frac{\partial Q}{\partial p} = -\rho c v\frac{\partial T}{\partial p}.$$

Therefore

$$-\frac{\partial T}{\partial p} = \frac{T\alpha}{\rho c} + \frac{TH}{4\pi\rho c}\left(\frac{\partial \omega}{\partial p}\frac{\partial B}{\partial T} - 3\alpha\frac{\partial B}{\partial p}\right). \qquad \text{(V)}$$

The above is the relation giving the temperature variation, when a magnetic substance is subjected to tension in a magnetic

field. The first term on the right-hand side is already well known in the thermodynamics of elasticity, and the second is the term which represents the influence of the magnetic field. Since α is positive in ferromagnetic substances, tension causes a diminution of their temperature, and pressure an increase.

Next let us consider the case where the substance is subjected to hydrostatic pressure or tension. The work done on the substance from the outside is, in this case, given by

$$dW = pdv + \frac{vH}{4\pi} dB.$$

Hence proceeding in the same way as before, we obtain

$$\frac{\partial I}{\partial p} = \frac{\partial \omega}{\partial H}, \qquad \text{(VI)}$$

where p denotes the intensity of hydrostatic pressure or tension. $\partial I/\partial p$ and $\partial \omega/\partial H$ are respectively the differential coefficients, when the magnetic field and the hydrostatic pressure or tension is kept constant. From the above relation it follows that in a substance for which the intensity of magnetization increases by the application of hydrostatic tension, its volume is increased by magnetization. If hydrostatic pressure, instead of hydrostatic tension, be applied, the results will be reversed.

Lastly let us consider the case of twisting a cylindrical wire made of a ferromagnetic substance placed in a magnetic field. If the twist of the wire per unit length be τ, and the couple n, the work done on the wire from the outside is

$$dW = nld\tau + \frac{vH}{4\pi} dB.$$

Hence, by a calculation similar to the one before, we obtain,

$$S\frac{\partial I}{\partial n} = \frac{\partial \tau}{\partial H}. \qquad \text{(VII)}$$

where $\partial I/\partial n$ and $\partial \tau/\partial H$ are respectively the differential coefficients when the magnetic field and the couple are kept constant. From this relation it follows that in the case of a substance for which the intensity of magnetization increases by the application of a couple, its twist is likewise increased by magnetization. If K be the rigidity and r the radius of the cylinder, then

$$n = \frac{\pi}{2} Kr^4\tau,$$

Hence

$$dn = \frac{\pi}{2} Kr^4 d\tau.$$

Consequently the relation (VII) may be written as follows,

$$\frac{\partial\tau}{\partial H} = \frac{2}{Kr^2} \frac{\partial I}{\partial\tau}. \qquad \text{(VII')}$$

If two different phenomena are so related to each other that the one follows as a necessary consequence of the other, then these two are said to be reciprocally related to each other.

The above results are the general summary of the investigations made by Koláček[1], Heydweiller[2], Gans[3], Terada and the author[4]. As in these calculations some factors were disregarded, the results cannot claim a high degree of accuracy. Moreover in the above calculations the hysteresis phenomena are entirely left out of consideration, so that we should have here an important cause of the disagreement between the theory and the experiment.

20. Effect of Tension or Pressure on Magnetization.

In the foregoing article, the theory of the effect of stress on magnetization was expounded; we shall now proceed to describe the results of experiments. The apparatus for measuring the effect of tension[5] will be first described. In the following figure, *CC* is a magnetizing coil placed vertically, within which is hung a wire *W* to be tested. Around this wire is wound a secondary coil, its axis being brought into coincidence with that of the magnetizing coil. To eliminate the induced current due to the magnetizing coil alone, another secondary coil is connected in series with the above-mentioned secondary coil, and inserted into another primary coil connected in series with the magnetizing coil, its position being so adjusted, that, in the case of the absence of the test-wire, the currents induced in these secondary coils just cancel each other. In order to measure the current induced in the secondary coil during the magnetization of the wire, a ballistic galvanometer is used, which is connected with the secondary circuit, its deflection

(1) Koláček, Ann. der Phys. **14**, (1904), 177. (2) A. Heydweiller, Ann. der Phys. **12**, (1903), 602. (3) R. Gans, Ann. der Phys. **13**, (1904), 634. (4) K. Honda and T. Terada, Jour. Coll. Sci. **21**, Art. 7, (1906). (5) K. Honda and T. Terada, ditto.

being read on a scale through a telescope. To both ends of the wire are fixed brass rods, *A* and *B*, the upper rod *A* to the frame above the coil, and to the end of the lower rod a hook is attached from which hangs a string passing over the pulleys *pp* and subjected to tension by the weight *P*. Further, an arm *aa* is fixed to the lower rod and perpendicular to it. This arm can slide freely along the vertical groove inside the cylinder *D*, which can be rotated round its axis on its stand. Thus the vertical groove is to allow the arm to slide freely downwards, when the wire is subjected to tension, and the rotating device of the cylinder is to give the wire a twist. This angle of twist is read on a graduated disk *bb* by making use of the index *E* attached to the brass rod *B*.

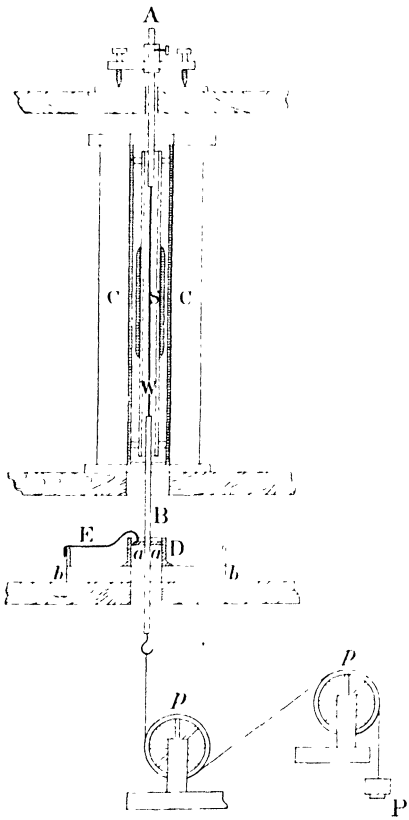


Fig. 41.

There are two different methods of measuring the effect of tension on magnetization. The first is to keep the magnetic field constant and measure the change in the intensity of magnetization ∂I due to tension, these processes being repeated under various magnetic fields and tensions. The second is to keep the tension constant and measure the intensity of magnetization under varying fields. Since the hysteresis phenomenon is very pronounced in ferromagnetic substances, the value of the magnetization for the same strengths of the magnetic field and the tension differs somewhat according to the order of their application. Thus the relation

$$\frac{\partial^2 I}{\partial H \partial p} = \frac{\partial^2 I}{\partial p \partial H}$$

does not hold good exactly, and hence the results of measurement by these two methods should not exactly agree with each other.

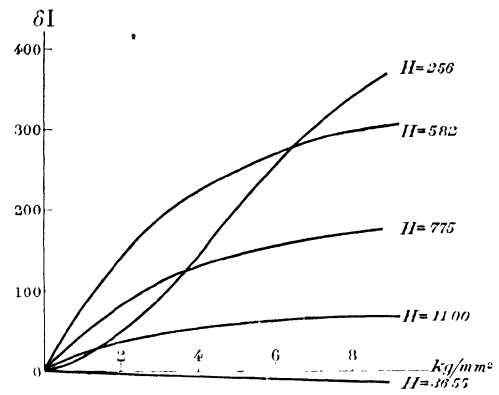


Fig. 42.
Iron (Honda, Terada).

The result of the measurement for the variation of magnetization under a constant field and varying tension, in the case of iron, is given in Fig. 42. Thus the intensity of magnetization I increases with the tension below a certain value of the magnetic field, but on the other hand, decreases when the field exceeds this value. Under a constant magnetic field, the magnetization increases with the tension.

Next examining the case of constant tension and varying field, we also observe a similar effect, that is, the intensity of magnetization increases with the tension below a certain value of magnetic field, but if the field increases beyond this value, the tension causes a decrease of magnetization.

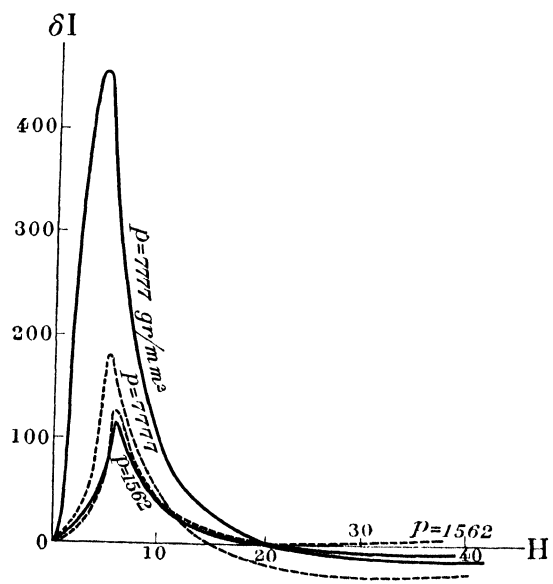


Fig. 43.
Iron (Honda, Terada).

From the above two results, we obtain the relation between δI and H, as shown in Fig. 43. The full lines represent the result obtained by the first method, and the dotted lines that obtained

by the second. These results show that at a certain magnetic field, the variation of magnetization with tension becomes zero. This value of magnetic field is called the Villari critical point.

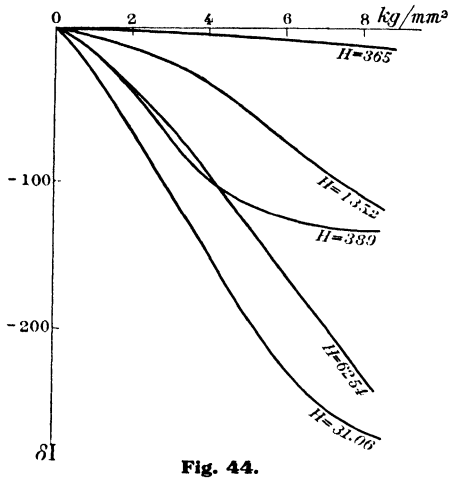


Fig. 44.
Nickel (Honda, Terada)

We shall next mention the result of experiments made with nickel. The results obtained under a constant field and varying tension are as shown in Fig. 44. Thus, the magnetization decreases with the increase of tension at an increasing rate, and its variation increases to a certain value of the magnetic field, but when that value is passed it begins to decrease.

In the case of a constant tension and a varying magnetic field, a similar change is observable, that is, the intensity of magnetization always decreases with tension. The rate of the variation of magnetization gradually increases with an increasing field and, after passing a maximum, gradually decreases.

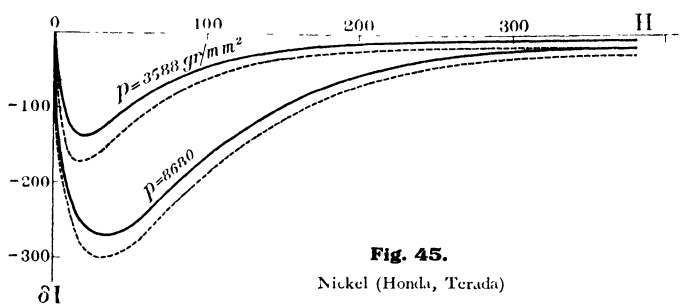


Fig. 45.
Nickel (Honda, Terada)

From the above two sets of experiments the relation between δI and H is obtained. The full lines show the results of the first experiment and the dotted ones those of the second, these two sets of curves coinciding approximately with each other.

Heydweiller[1] discovered that nickel also increases in magne-

(1) A. Heydweiller, Wied. Ann. **52**, (1894), 462.

tization, if it is subjected to tension under a very weak magnetic field, and hence the difference of the variation of magnetization between iron and nickel is only quantitative, so that the Villari point also exists in nickel. The author studied the same phenomenon in co-operation with S. Shimidzu[1], and found that what was discovered by Heydweiller is not a true effect of tension, but a secondary effect coming into play by virtue of the undesirable strain to which the wire under investigation was subjected.

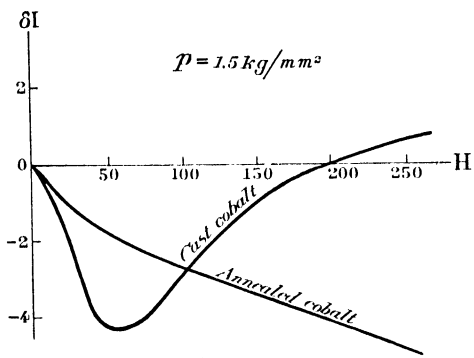


Fig. 46.

The measurement of the effect of pressure on magnetization is not so easy as that of the effect of tension. Ewing[2] carried out this experiment by a yoke method. He took a thick, short nickel bar, which was placed in a thick iron yoke, and the pressure was exerted by means of a lever, the change in magnetization thereby caused being measured by a ballistic galvanometer. The effect of pressure is opposite of that of tension, showing a notable increase in magnetization. Fig. 48 is the graphical representation of Ewing's results.

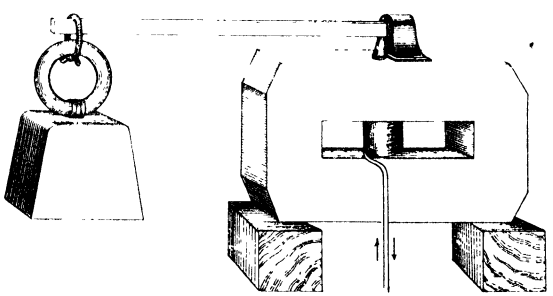


Fig. 47.

21. Change of Length by Magnetization.

Many investigations have hitherto been made with regard to the

(1) K. Honda and S. Shimidzu, Drud. Ann. 14, (1904), 791. (2) Ewing, Phil. Trans. A, (1888), 333.

reciprocal effect of tension on magnetization by a number of workers. The most important work has been done by Bidwell,[1] Nagaoka[2], Shimidzu[3], Terada and the author[4].

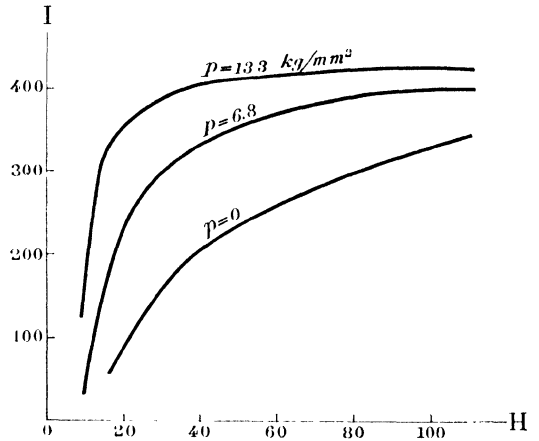


Fig. 48.

Nickel (Ewing)

To measure a small variation of length Bidwell used an optical lever. In the following figure, *S* is an iron rod to be tested, both ends being soldered to brass rods *FF*. The lower rod is fixed to the stand *E*, and the pointed end of the upper rod touches a point near the end of a long lever, *B*, to act as the point of application of force in the lever. *M* is a small mirror which can turn round the axis *D*, a small arm *G* projecting perpendicularly from the back of the mirror and touching the other end

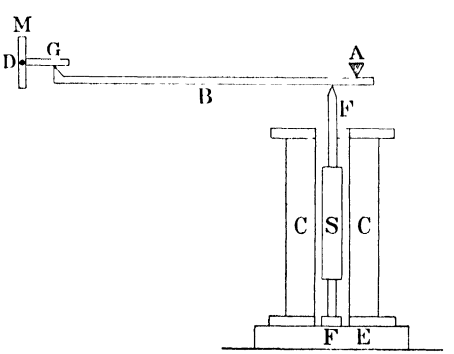


Fig. 49.

(1) S. Bidwell, Proc. Roy. Soc. **40**, (1886), 109, 257; **43**, (1888), 406. (2) H. Nagaoka, Phil. Mag., **37**, (1894), 131; Rapport au Congres International de Physique, Paris, 1900. (3) K. Honda and S. Shimidzu, Phil. Mag. **4**, (1902), 338. (4) K. Honda and T. Terada, Jour. Coll. Sci. **21**, Art. 4, (1906).

of the lever. If an electric current is passed through a magnetizing coil *C* to magnetize the iron rod, a change in its length will take place, which makes the lever rotate round the point of support *A*. The consequence is that the other end of the lever will move the arm *G* and rotate the small mirror. This angle of rotation can be measured on a scale through telescope, which method will enable us to detect a change in length of one in ten million. Professor H. Nagaoka also used an optical lever, and was successful in eliminating the effect of the temperature change, due to the electric current in the magnetizing coil, by using a compensation system.

Lastly, the apparatus used by T. Terada and the author will be described. Since the change in the length produced by magnetization is extremely small, the measurement is best made by a so-called differential method. In Fig. 50 *A*, *C* is a magnetizing coil, inside which vertically hang an iron and a copper wire parallel to each other. At both ends of the two wires are soldered brass bars. The upper bars hang separately from two horizontal beams *aa*. Each of these beams rests on two pillars, the distance between them being so adjustable, that the elongations of the two wires, when no magnetic field acts, are made exactly equal. A frame which supports the small mirror *M* is attached to the lower end of the copper wire, and the small springs *ss* attached to the two sides of the frame keep the axis of rotation of the mirror horizontal. A band *f* made of a bundle of very thin copper wires is connected with the lower bar and lightly presses the axis of rotation of the small mirror. A silk thread fastened to the lower

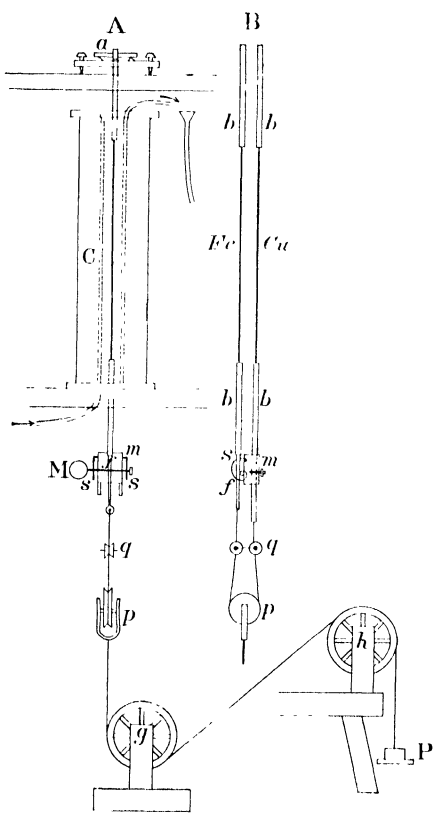


Fig. 50.

ends of the bar and to the copper wire bundle, is connected with a weight P through the pulleys q, p, g, h. In this way the weight exerts the same tension on both wires without giving rise to any disturbing vibration in the small mirror.

If now we suppose that the elongation of the iron and copper wires is so adjusted that it is the same for both, the axis of the small mirror will not be rotated at all by applying or increasing the tension. But if a magnetic field is applied by passing an electric current to the magnetizing coil, the iron wire will be magnetized and there will be a change in its elasticity, so that the axis of the mirror will be rotated by a small amount, which can be read on a scale through a telescope. If the length of the iron wire be l, the change in length by magnetization ∂l, the radius of the axis of the mirror r and the angle of rotation $\partial\varphi$, we have

$$\partial l = 2r\partial\varphi.$$

If the distance between the small mirror and the scale be d, and the scale reading n, then

$$\partial\varphi = \frac{n}{2d}.$$

Hence

$$\frac{\partial l}{l} = \frac{rn}{ld}.$$

In our case, $2r = 1.61$ mm, $d = 7.32$ m, $= 72$ cm; then the change of length for 1 mm of the scale is

$$\frac{\partial l}{l} = 4.5\times10^{-8}.$$

Thus, by means of this apparatus, we can measure the elongation of the wire under a constant tension and in a varying magnetic field; from this, we are able to find by calculation the change in the modulus of elasticity in a magnetic field. On the other hand, if we measure the deflection of the mirror by varying the tension under a constant field, after adjusting the pillars in such a way that when no magnetic field is acting, the application of tension produces no rotation of the mirror, we obtain directly the change in the modulus of elasticity caused by magnetization. In the absence of hysteresis, these two methods should give the same result. For, if δe be the elongation per unit length and if the hysteresis phenomenon be absent, we have

$$\frac{\partial^2 e}{\partial H \partial p} = \frac{\partial^2 e}{\partial p \partial H} .$$

Hence if this relation be integrated with respect to H,

$$\left(\frac{\partial e}{\partial p}\right)_{II} - \left(\frac{\partial e}{\partial p}\right)_0 = \frac{\partial (e_{II} - e_0)}{\partial p} .$$

The left-hand side of this equation shows the change in the reciprocal of the modulus of elasticity E, and the right-hand side the change in magnetic expansion caused by tension. If the section of the wire be S,

$$\frac{\partial e}{\partial p} = \frac{1}{SE} ,$$

$$\therefore \quad \left(\frac{\partial e}{\partial p}\right)_{II} - \left(\frac{\partial e}{\partial p}\right)_0 = \frac{1}{S}\left(\frac{1}{E_{II}} - \frac{1}{E_0}\right) = -\frac{1}{S}\frac{\partial E}{E^2} ,$$

or

$$\frac{\partial E}{E} = -SE \frac{\partial (e_{II} - e_0)}{\partial p} .$$

But in the case of iron and nickel, this relation does not strictly apply, by reason of the hysteresis.

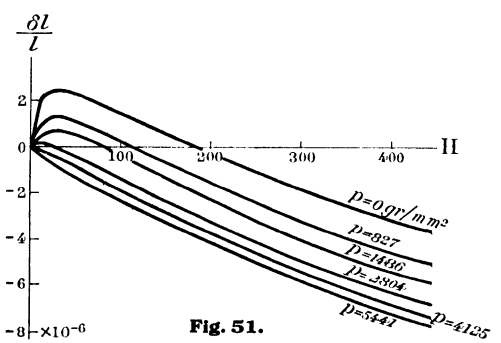


Fig. 51.
Iron (Honda, Shimidzu).

We shall now describe the result of experiments. The change in length caused by a varying field under a constant tension[1] is given by the curves in Fig. 51. In the case of iron, the elongation gradually increases with the magnetic field, when the tension is small, but after reaching a maximum, decreases gradually till it becomes zero. If the magnetic field is further increased, iron contracts on being subjected to magnetization.

As the tension increases, the elongation in weak fields gradually decreases, while the contraction in strong fields markedly increases. At 5.5 kg/mm, iron contracts for all fields. From the

(1) K. Honda and S. Shimidzu, Jour. Coll. Sci. **16**, Art. 9, (1902).

above result, $\partial(e_{\mu}-e_0)/\partial p$ and, consequently, the change in elasticity can be calculated.

In the case of nickel, we have only the contraction of the wire with an increasing magnetic field, and as the tension increases, the contraction gradually decreases in a weak field, while in a strong field, it increases.

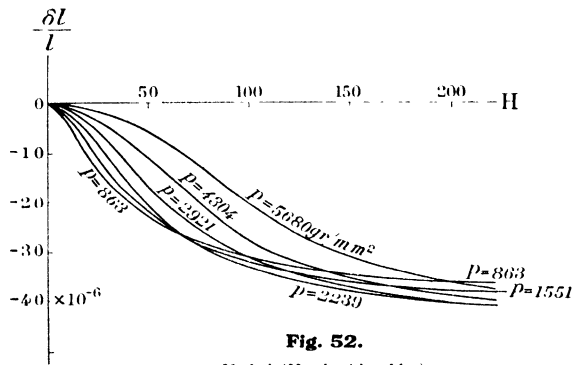


Fig. 52.
Nickel (Honda, Shimidzu)

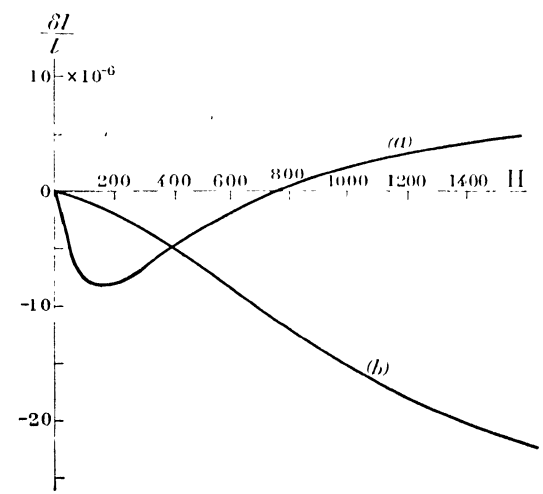


Fig. 53.
Cobalt (Nagaoka, Honda).

The magnetic expansion of cast cobalt[1] is the reverse of that of iron, i.e., the length decreases in a weak field, and increases in a strong field, as shown in Fig. 53*a*, but that of annealed cobalt always decreases with an increasing field as seen from Fig. 53*b*. As will be explained later on, this remarkable difference in magnetic properties is due to the allotropy of cobalt. The magnetic expansion of cobalt under tension has not yet been studied.

The phenomena connected with the change of dimensions in ferromagnetic substances caused by a magnetic

(1) H. Nagaoka and K. Honda, Jour. Coll. Sci. **16**, Art. 8, (1902); K. Kanda, and S. Shimidzu, **19**, Art. 10, (1903). Paul McCorkle, Phys. Rev. **22**, (1923), 271.

field as mentioned above, is called magnetostriction.

Next we shall describe the result of the change in the modulus of elasticity measured by the second method. In Fig. 54, the curve shows a small variation in the modulus of the elasticity in iron, which gradually increases with the magnetic field.

Fig. 55 shows graphically the same variation in the case of nickel.

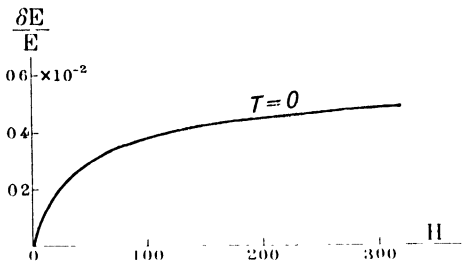


Fig. 54.
Iron (Honda, Tanaka)

The rate of decrease of elasticity increases with the magnetic field, after reaching its maximum gradually diminishes, and above a certain value of the magnetic field, the modulus of elasticity increases. As the tension increases, the change of elasticity decreases in weak fields, but increases in strong fields, and the strength of the magnetic field in which this change vanishes, becomes greater. Generally, the variation of the modulus of elasticity in nickel is markedly larger than that in iron.

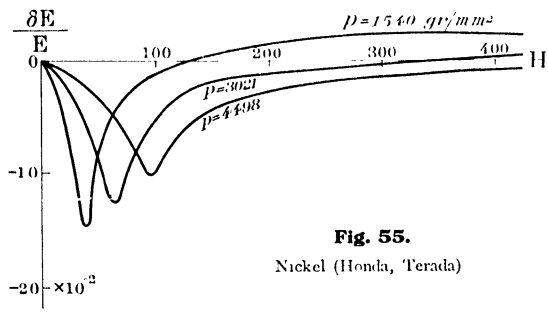


Fig. 55.
Nickel (Honda, Terada)

Finally in the following tables, it is intended to show to what degree the reciprocal relations

$$\frac{\partial I}{\partial p} = \frac{\partial e}{\partial H}$$

and

$$-\frac{1}{E^2}\frac{\partial E}{\partial H} = \frac{\partial^2 I}{\partial p^2}$$

hold good. Here the data obtained by the second method were availed of.

Iron; $p = 1.63$ kg/mm^2.

H	$\frac{\partial e}{\partial H}$	$\frac{\partial I}{\partial p}$
11.9	$+14.0 \times 10^{-8}$	$+1.3 \times 10^{-8}$
24.2	+ 0.8	+0.6
36.6	− 0.7	−1.7
97.1	− 1.8	−2.4
207.0	− 2.0	−2.2
367.0	− 1.4	−1.7

Iron; $p = 3.19$ kg/mm^2.

H	$-\frac{1}{E^2}\frac{\partial E}{\partial H}$	$\frac{\partial^2 I}{\partial p^2}$
11.9	-21.0×10^{-17}	-29×10^{-17}
24.2	− 6.4	− 4.5
36.6	− 3.3	+ 1.8
97.1	− 0.53	+ 1.6
207.1	+ 0.03	+ 0.28
367.0	+ 0.21	+ 0.28

Nickel; $p = 1.5$ kg/mm^2.

H	$\frac{\partial e}{\partial H}$	$\frac{\partial I}{\partial p}$
31.1	-7.7×10^{-17}	-3.7×10^{-17}
62.5	−2.2	−2.2
135.0	−0.68	−0.71
206.0	−0.32	−0.32
365.0	−0.05	−0.05

Nickel; $p = 3.02$ kg/mm^2.

H	$-\frac{1}{E^2}\frac{\partial E}{\partial H}$	$\frac{\partial^2 I}{\partial p^2}$
31.7	$10. \times 40^{-16}$	
62.5	12.2	-4.0×10^{-16}
135.0	− 2.7	−2.1
206.0	− 0.67	−1.6
365.0	− 0.67	−1.6

If we consider the circumstances that in the above calculation, the hysteresis phenomenon was not taken into account, it is to be concluded that these reciprocal relations are confirmed by the above investigation. Especially in nickel these relations are very well satisfied, since in this element, its hysteresis phenomenon is of minor importance.

22. Effect of Hydrostatic Pressure on Magnetization and its Reciprocal Relation.

The apparatus used by Professor H. Nagaoka and the present author[1] to test the effect of hydrostatic pressure on magnetization is as follows:—*CC* is a magnetizing coil, *T* a stout brass tube, in the interior of which rests vertically an ellipsoid of rotation of iron or nickel to be tested, and the space round it is filled with water. The upper end of the tube is connected with a Cailletet hydrostatic pressure pump through a narrow copper tube. About 300 atmospheres of pressure can be exerted in the interior of the tube.

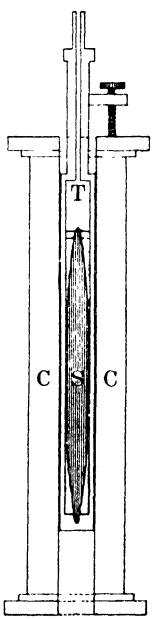


Fig. 56.

At first an electric current is passed through the magnetizing coil to magnetize the ellipsoid *S*. After eliminating its effect on the magnetometer by an ellipsoid of the same shape placed in another coil of the same dimensions, hydrostatic pressure of various intensities is applied to the first ellipsoid, and the deflection produced in the magnetometer is observed, the variation of magnetization being thus calculated. Fig. 57 shows the effect of the hydrostatic pressure, which is extremely small. In the case of iron, hydrostatic pressure diminishes the intensity of magnetization, and in that of nickel, increases it. This increase or decrease is approximately proportional to the hydrostatic pressure. It is here to

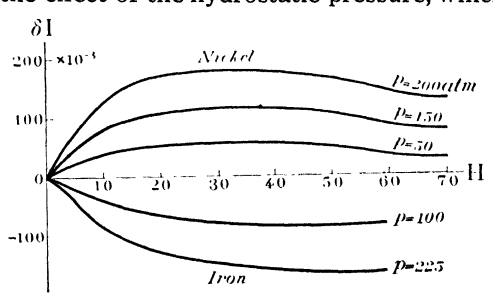


Fig. 57. (Nagaoka, Honda).

(1) H. Nagaoka and K. Honda, Jour. Coll. Sci. **9**, (1898), 353.

be remembered that the nickel used was made by forging a thin plate into a bar, so that the internal structure was not uniform, but was in a highly strained condition.

The volume change produced by magnetization can be measured by a dilatometer. The ellipsoid of rotation made of iron or nickel is placed in a glass tube T with a capillary neck as shown in Fig. 58, the ellipsoid being fixed by brass rings *aa*. The inside of the tube is filled with water, and the surface of the water m is found in the capillary. The tube T is placed in a magnetizing coil CC and the position of the meniscus is read before each magnetization on an ocular in a microscope; then a current of short duration is passed through the magnetizing coil. The ellipsoid undergoes a change of volume, which will cause the meniscus of the water in the capillary to move. The result of the measurement for $\delta v/v$ in various magnetic fields is shown graphically in Fig. 59.

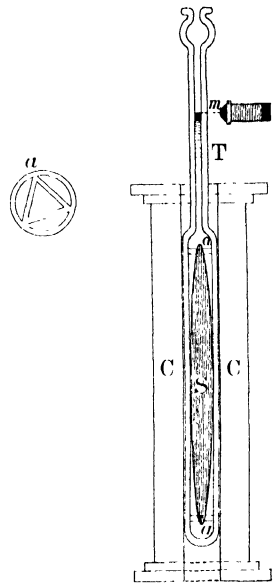


Fig. 58.

From this figure, we see that iron is expanded by magnetization while nickel is contracted. According to the reciprocal relation (VII), a substance, whose intensity of magnetization is increased by hydrostatic pressure, contracts on magnetization, and a substance, whose magnetization diminishes by hydrostatic pressure, expands on magnetization. These conclusions are in good agreement with the above results.

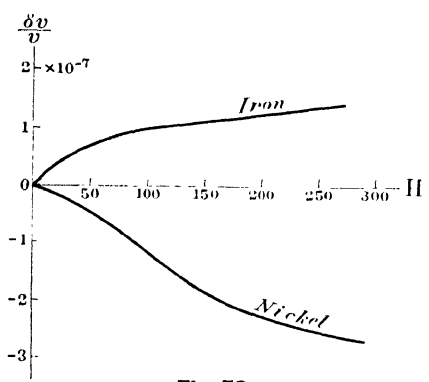


Fig. 59.
Iron (Nagaoka, Honda).

Fig. 60 is the result of measurement made by Professor H. Nagaoka and the author[1], of the variation of volume for different kinds of

(1) H. Nagaoka and K. Honda, Jour. Coll. Sci. **13**, (1900) 57.

steel and nickel in strong magnetic fields. We see that the volume of these iron, steel and nickel is increased by magnetization, the tendency becoming more pronounced with the increase of the magnetic field. The nickel used here was made from a thick bar into an ellipsoid of rotation and was not heterogeneous as was the above-mentioned nickel. Hence from the point of view of the reciprocal relation, it may be concluded that the effect of hydrostatic pressure in the case of homogeneous nickel will be an increase of magnetization, as in the case of iron.

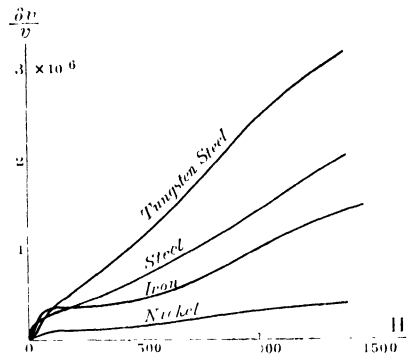


Fig. 60. (Nagaoka, Honda)

23. Effect of Twist on Magnetization and its Reciprocal Relation.

The effect of twist on magnetization can be studied by the apparatus described in Art. 20. Here we have also two ways of measuring this effect. The first is to give a twist of varying magnitude under a constant field, and measure the corresponding variation of magnetization. The second is to magnetize a wire which is subjected to a constant twist and find its magnetization curve, the same process being repeated for different angles of twist. These two ways of observation do not give exactly the same results for the same magnetic field and the same twist, on account of hysteresis. The dotted line in Fig. 61 is the result of the second way of observation in iron and shows the decrease of magnetization; its amount increases at first rapidly, attains a maximum and then gradually decreases. In the case

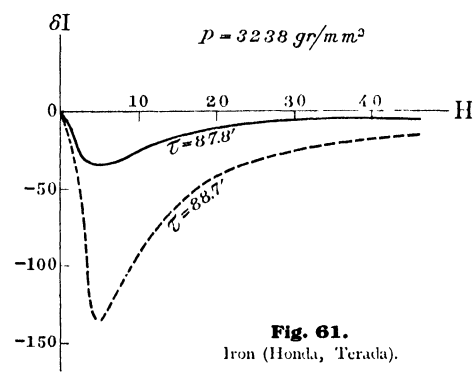


Fig. 61.
Iron (Honda, Terada).

of the first way of observation, the initial effect produced when the first twist is given to a magnetized wire, is to increase the magnetization. This effect is, however, not reversible; thus when the same amount of twist is given to the same wire a second time, the magnetization does not increase, but decreases. The initial effect may therefore be regarded as a disturbing effect of the vibration produced during twisting. The decreasing effect being reversible, is the true effect of twist, and is shown in Fig. 61 by a full line.

Fig. 62 shows the results of similar experiments made with nickel, the full lines representing the result of the first way of observation and the dotted ones that of the second way. The twist increases the magnetization of nickel in a weak field and decreases it in a strong one. The disturbing effect of vibration due to the twist, is, in the case of nickel, very small, in comparison with that of iron.

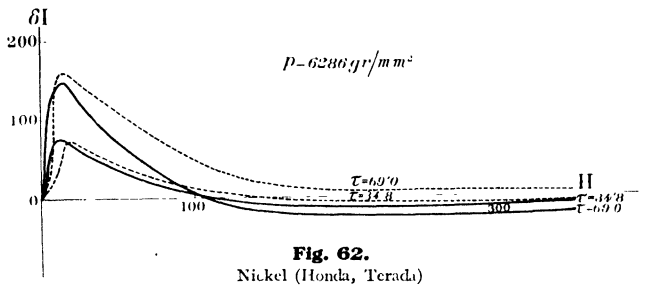


Fig. 62.
Nickel (Honda, Terada)

Let us next examine the reciprocal effect of the above phenomenon, i.e., the change in the angle of twist in a twisted wire due to the action of a magnetic field. By using Barus' differential method[1], the author, in co-operation with T. Terada[2] made an experiment on this subject. In the next figure, *W* is an iron or nickel wire to be tested, to both ends of which are soldered brass rods *a* and *b*. To the rod *a* is attached a small mirror *M* and to the rod *b* is fixed at right angles to it a brass arm *F*, the ends of which slide in the vertical grooves cut in the inside of a hollow cylinder *E*, so that it may be rotated inside the round ring *G*. The wire can thus be twisted under a constant tension applied on the arm *F*. The rod *a* is connected with the brass rod *c* by a copper wire which can be twisted by rotating the disk *B*.

At first the disk *B* is rotated in a certain sense to twist the copper

(1) C. Barus, Amer. Jour. Sci. **34**, (1887), 175; Phy. Rev. **13**, (1901), 283.
(2) Honda and Terada, Jour. Coll. Sci. **21**, loc. cit.

wire, which will also rotate, more or less, the mirror M. Next the lower cylinder E is so rotated in the opposite sense that the mirror returns to its initial position. The copper and iron wires are thus in equilibrium under the same couple. Then an electric current is passed through the coil C to magnetize the wire W. If the rigidity of the wire suffers a change by virtue of the magnetic field, it gives rise to a new rotation of the mirror, so that by observing the angle of rotation with a scale and telescope, we can find the change in rigidity.

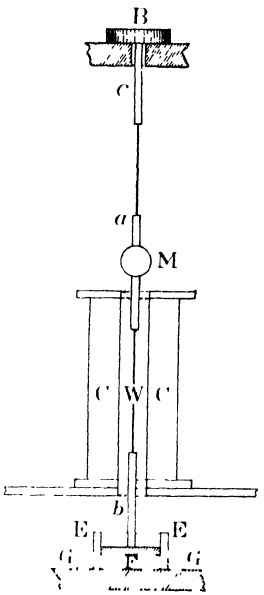


Fig. 63.

If now the angles of twist of the copper and iron wires, when no magnetic field is in action, be θ' and θ respectively, and the couples per unit angle of twist of these wires be n' and n respectively, we then have

$$\theta' n' = \theta n.$$

If the magnetic field is brought into action, the rigidity of the iron wire changes, but that of the copper wire does not; hence, denoting the change of twist caused by magnetization by $\delta\theta$, we obtain the following relation

$$(n + \delta n)(\theta + \delta\theta) = n'(\theta' - \delta\theta),$$

or

$$\delta n(\theta + \delta\theta) = -\delta\theta(n + n').$$

Hence

$$\frac{\delta n}{n} = -\frac{\delta\theta}{\theta + \delta\theta}\left(1 + \frac{n}{n'}\right)$$

$$= -\frac{\delta\theta}{\theta}\left(1 + \frac{n}{n'}\right)\left(1 - \frac{\delta\theta}{\theta}\right).$$

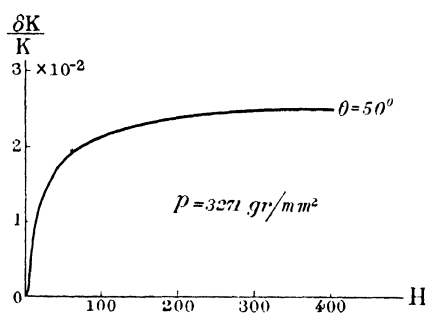


Fig. 64.
Iron (Honda, Terada).

If K be the rigidity of the iron wire,

$$\frac{\delta K}{K} = \frac{\delta n}{n} = -\frac{\delta\theta}{\theta}\left(1 + \frac{n'}{n}\right)\left(1 - \frac{\delta\theta}{\theta}\right),$$

so that by measuring θ, $\partial\theta$, n and n', we can find the change of rigidity $\partial K/K$. Figs. 64 and 65 show the variation of rigidity of iron and nickel in a magnetic field. From these figures, we see that as the magnetic field increases, the rigidity of the iron wire increases, at first rapidly and then gradually. The rigidity of the nickel wire decreases in weak fields and after reaching a minimum, begins to increase as the field increases; when the tension applied to the test-wire is increased, the amount of the diminution of rigidity increases as seen from Fig. 65.

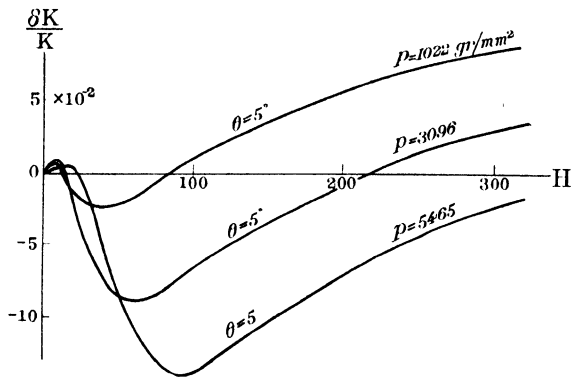


Fig. 65. Nickel (Honda, Terada).

In the following tables we shall show to what extent the reciprocal relation (VII) before referred to is quantitatively satisfied.

Iron.

H	$\frac{1}{\tau}\frac{\partial\tau}{\partial H}$	$\frac{S}{\tau}\frac{\partial I}{\partial n}$
8	-11×10^{-4}	-8.6×10^{-4}
14	-3.6	-6.2
22	-2.7	-4.8
50	-1.1	-0.96
110	-0.33	-0.48
200	-0.11	-0.48
300	-0.07	-0.24

Nickel.

H	$\frac{1}{\tau}\frac{\partial\tau}{\partial H}$	$\frac{S}{\tau}\frac{\partial I}{\partial n}$
12	$+2.3\times10^{-3}$	$+2.3\times10^{-3}$
24	$+0.84$	$+0.52$
50	-0.37	-0.57
97	-0.60	-0.70
168	-0.40	-0.70
358	-0.17	-0.52

Thus the agreement between the theory and the experiment may be said to be very satisfactory, if we consider that in the theoretical deduction of the relationship, the hysteresis is not taken into account.

24. Kirchhoff's Constants.

The general theory of magnetostriction was first developed by Kirchhoff.[1] He assumed the intensity of magnetization to be a function of strain and took as the co-ordinate axes the principal axes of strain; denoting the component magnetization along these directions by I_x, I_y, I_z, he gives:—

$$I_x = \left\{\kappa - \kappa'\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right) - \kappa''\frac{\partial u}{\partial x}\right\} H_x,$$

$$I_y = \left\{\kappa - \kappa'\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right) - \kappa''\frac{\partial v}{\partial y}\right\} H_y,$$

$$I_z = \left\{\kappa - \kappa'\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right) - \kappa''\frac{\partial w}{\partial z}\right\} H_z.$$

where u, v, w are the component displacements at a point (x, y, z) and H_x, H_y, H_z the component magnetic fields at the same point. Of the constants κ, κ' and κ'', κ is the ratio between the magnetization and the field when the strain is zero, that is, the coefficient of magnetization. κ' and κ'' are the coefficients which denote the influence of the volume and length changes on magnetization, and are called Kirchhoff's constants. But, as a matter of fact, κ' and κ'' are not really constant, but are functions of the magnetic field. Kirchhoff calculated from the above relations how much the volume of a sphere changes on magnetization, when placed in a homogeneous magnetic field. Cantone[2] afterwards calculated the changes of volume and length in the case of an ellipsoid of rotation. But as the theories are very complex, we shall not enter into them here, but try to solve the inverse problem.

Suppose a ferromagnetic rod to be placed parallel to a homogeneous magnetic field[3], and if the changes per unit of volume and of length when the rod is subjected to hydrostatic pressure, be respectively ω and e, then we have by Kirchhoff's relation,

(1) G. Kirchhoff, Wied. Ann. **24**, (1885), 52. (2) Cantone, Memoria d. Acad. Lincei, (1890), Rendiconte d. Acad. d. Lincei. (3) H. Nagaoka and K. Honda, Jour. Coll. Sci. **9**, (1898), 353.

$$I = (\kappa - \kappa'\omega - \kappa''e)\, H.$$

The change of magnetization produced by the hydrostatic pressure is

$$\delta I = I - \kappa H = -(\kappa'\omega + \kappa''e)\, H.$$

But since we can put $\omega = 3e$,

$$\delta I = -\left(\kappa' + \frac{1}{3}\,\kappa''\right)\omega\, H. \tag{1}$$

On the other hand, if we apply a tension to the rod, the latter expands lengthwise, but contracts laterally, so that, if the contraction per unit length be f,

$$\omega = e - 2f.$$

If now Poisson's ratio be σ, then $f = \sigma e$.

$$\therefore \quad \omega = (1 - 2\sigma)\, e.$$

If the modulus of elasticity be E and the rigidity K, we have the following relation between E, K and σ:—

$$\frac{E}{K} = 2\,(1 + \sigma).$$

Hence

$$\omega = \left\{1 - \left(\frac{E}{K} - 2\right)\right\} e = \left(3 - \frac{E}{K}\right) e.$$

If this is put in the expression for δI,

$$\delta I = \left\{\kappa' \frac{E}{K} - 3\left(\kappa' + \frac{1}{3}\,\kappa''\right)\right\} eH. \tag{2}$$

Hence if we measure the change of magnetization due to hydrostatic pressure and tension, we can calculate κ' and κ'' from equations (1) and (2).

The following table contains the values of κ' and κ'' for iron and nickel[1]; Figs. 66 and 67 show the relation between these constants and the magnetic field.

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(1) H. Nagaoka and K. Honda, Jour. Coll. Sci. **13**, (1900), 57.

Iron.

H	κ'	κ''
5	+ 21900	— 22610
10	+ 23520	— 28450
20	+ 13280	— 16420
30	+ 7302	— 8650
60	+ 2139	— 2220
80	+ 1207	— 1100
100	+ 750	— 550
120	+ 500	— 255
160	+ 239	+ 18
200	+ 122	+ 131
300	+ 25	+ 175
500	— 9	+ 130
800	— 9	+ 70

Nickel.

H	κ'	κ''
5	— 229100	+ 712800
10	— 188900	+ 578900
20	— 71000	+ 216700
30	— 36370	+ 111200
60	— 8163	+ 34540
80	— 6906	+ 20960
100	— 4653	+ 14120
120	— 3373	+ 10260
160	— 1297	+ 3968
200	— 1283	+ 3888
300	— 591	+ 1790
500	— 216	+ 655
800	— 86	+ 259

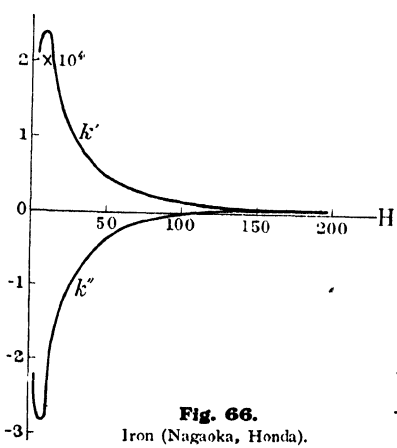


Fig. 66.
Iron (Nagaoka, Honda).

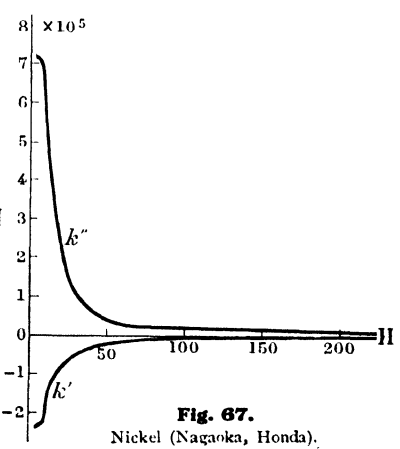


Fig. 67.
Nickel (Nagaoka, Honda).

The fact that κ' and κ'' generally assume very large values is to be expected from the extreme smallness of the strain; in fact, if the magnetization depends on the strain, the coefficients κ' and κ'' must necessarily be very large.

25. Some Phenomena Relating to the Constants κ' and κ''

(a) *Effect of Twist on Longitudinal Magnetization.* The tangential strain produced when a wire is twisted, can be decomposed into elongation and contraction each acting in a direction making 45° with that of the axis. If these two directions be taken as co-ordinates, and r be the distance to any arbitrary point (x, y) from the centre of the wire, R its radius, and τ the twist per unit of length, we have

$$\frac{\partial u}{\partial x} = +\frac{1}{2}\tau r.$$

$$\frac{\partial v}{\partial y} = -\frac{1}{2}\tau r,$$

$$\frac{\partial w}{\partial z} = 0.$$

If these values are put in the Kirchhoff's equations, we get

$$I_x = \left(\kappa - \frac{1}{2}\tau r\kappa''\right)\frac{H}{\sqrt{2}},$$

$$I_y = \left(\kappa + \frac{1}{2}\tau r\kappa''\right)\frac{H}{\sqrt{2}}.$$

If I_c be the intensity of circular magnetization,

$$I_c = I_x\frac{1}{\sqrt{2}} - I_y\frac{1}{\sqrt{2}} = -\frac{1}{2}\tau r\kappa''H.$$

Since I_c varies with r, its mean circular magnetization being $\bar{I}_c$, we have

$$\bar{I}_c = \frac{1}{\pi R^2}\int_0^R 2\pi r dr I_c$$

$$= -\frac{1}{3}\tau\kappa''RH \qquad (3)$$

Thus it follows that the intensity of circular magnetization is proportional to $\kappa''H$, and as κ'' differs in sign for iron and nickel,

the circular magnetization must also differ in sign for these metals. The manner in which this magnetization varies with the magnetic field, may be inferred from κ'' curves in Figs. 66 and 67.

(b) *Effect of Twist on Circular Magnetization.* Consider the effect of a twist in a wire through which is passed an electric current to magnetize it circularly. If the strength of the electric current be C, the magnetic field at a point distant r from the centre of the wire is

$$H = \frac{2Cr}{R^2}.$$

If we replace, as in the preceding case, the twist by the elongation and contraction perpendicular to each other, and find the intensity of the component magnetization in each direction, we have

$$I_x = \left(\kappa - \frac{1}{2}\tau r\kappa''\right)\frac{H}{\sqrt{2}},$$

$$I_y = -\left(\kappa + \frac{1}{2}\tau r\kappa''\right)\frac{H}{\sqrt{2}}.$$

Hence if the longitudinal component of magnetization be I_l,

$$I_l = I_x\frac{1}{\sqrt{2}} + I_y\frac{1}{\sqrt{2}} = -\tau r\kappa''\frac{H}{2} = -\frac{C\tau\kappa'' r^2}{R^2}.$$

Since I_l varies with r, if its mean value be denoted by I_l,

$$I_l = \frac{1}{\pi R^2}\int_0^R 2\pi r dr I_l = -\frac{1}{2}C\tau\kappa''.$$

Here though κ'' is really a function of r, in the above integration, it is regarded as constant. From the above we see that I_l is proportional to $C\kappa''$. In iron and nickel, κ'' differs in sign, and hence the longitudinal magnetization produced when the circularly magnetized wire is twisted has also a different sign.

(c) *Twist Produced by Longitudinal and Circular Magnetizations.* When an electric current is passed through a wire which is magnetized longitudinally, a twist is produced in the wire. The relation of this twist τ, the magnetic field H and the electric current C can be easily found. The wire suffers at the same time a change of length, but as its magnitude is very small in comparison with the change of twist, it may be neglected.

If as in the previous case, the directions making an angle of 45° with the axis of the wire be taken as x and y axes, we have at a point distant r from the axis

$$\frac{\partial u}{\partial x} = \frac{1}{2}\tau r, \quad \frac{\partial v}{\partial y} = -\frac{1}{2}\tau r, \quad \frac{\partial w}{\partial z} = 0;$$

$$H_x = \frac{1}{\sqrt{2}}(H+h), \quad H_y = \frac{1}{\sqrt{2}}(H-h), \quad H_z = 0,$$

where $h=2Cr/R^2$ and represents the intensity of the circular field. Hence

$$I_x = \frac{1}{\sqrt{2}}\left(\kappa - \frac{1}{2}\tau r\kappa''\right)(H+h),$$

$$I_y = \frac{1}{\sqrt{2}}\left(\kappa + \frac{1}{2}\tau r\kappa''\right)(H-h),$$

$$I_z = 0.$$

Let the length of the wire be taken as unity and consider a virtual twist $\partial\tau$, then the work done is $N\partial\tau$, N being the resisting couple. Since the magnetization depends on τ, the magnetic energy also changes in virtue of $\partial\tau$, and must necessarily be equal to $N\partial\tau$.

In general, the magnetic energy is

$$W = -\frac{1}{2}\int (I_x H_x + I_y H_y)\, dxdydz.$$

But

$$\partial I_x = -\frac{1}{2\sqrt{2}} r\kappa''(H+h)\,\partial\tau,$$

$$\partial I_y = +\frac{1}{2\sqrt{2}} r\kappa''(H-h)\,\partial\tau.$$

Hence

$$N\partial\tau = -\frac{\partial W}{\partial\tau}\partial\tau = \frac{1}{2}\int (H_x\partial I_x + H_y\partial I_y)\, 2\pi r dr,$$

$$= \frac{\pi}{4}\int \kappa'' r \left\{-(H+h)^2 + (H-h)^2\right\} \partial\tau r dr.$$

But as h is usually very small in comparison with H, κ'' may be regarded as independent of r. Therefore

$$N = -\frac{2\pi\kappa''HC}{K^2}\int_0^c r^3 dr = -\frac{\pi}{2}\kappa''R^2HC$$

$$= -\frac{1}{2}\kappa''HC \times (\text{section of wire}).$$

Thus we see that the couple N is proportional to $\kappa''CH$. Consequently, for a given electric current, the twisting of the wire is determined by the value of $\kappa''H$, so that we may infer from the κ'' curve how the twist varies with the magnetic field. κ'' in iron differs in sign from that in nickel, and hence they are twisted in opposite directions. Since the couple resulting from the twist is proportional to R^4, the angle of twist due to the mutual action of the longitudinal and circular fields is inversely proportional to R^2 for the same current and the same magnetic field.

The above-mentioned phenomenon, i.e., that a twist is produced by the mutual action of longitudinal magnetic field and electric current passing through the wire, is called the Wiedemann effect.

26. Comparison of Theory and Experiment.

(a) *Circular Magnetization Produced by Twisting a Longitudinally Magnetized Wire.*[1] The apparatus for the experiment is nearly the same as that mentioned in Art. 20. Two brass rods B and b, which are soldered on both ends of the test-wire, are connected with a ballistic galvanometer, by which the current induced in this circuit is measured. If an electric current is passed through the magnetizing coil to magnetize the wire W longitudinally, and the wire is then twisted, circular magnetization is produced in the wire, the consequence of which is to produce an induced current in the above-mentioned circuit. Hence if we measure this current by the ballistic galvanometer, we can find the intensity of the circular magnetization.

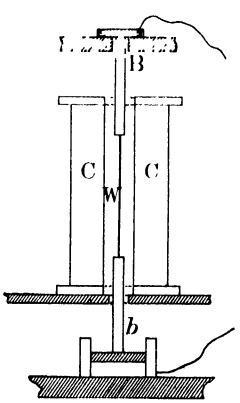


Fig. 68.

Thus, if the length of the wire be l, and its radius R, the quantity of the induced electric current produced by the twist is

(1) H. Nagaoka and K. Honda, Jour. Coll. Sci. **13**, (1900), 263.

$$Q = \frac{N}{w},$$

where w is the total resistance of the circuit. If the mean intensity of circular magnetization be B_c, we have

$$N = \int_0^R B_c \, l dr = 4\pi l \int_0^R I_c dr.$$

Hence if the value I_c obtained before be substituted in the above expression, we get

$$Q = -\frac{\pi\tau\kappa'' l R^2 H}{w}.$$

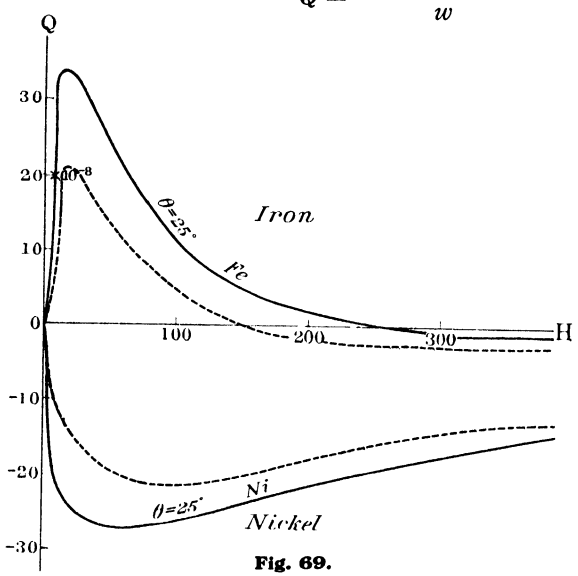


Fig. 69.
Nickel (Nagaoka, Honda).

As l and R are constants, Q is proportional to $\kappa'' H\tau$.

The full lines in Fig. 69 shows the results of experiments with iron and nickel wires, the angle of twist being 25°. Thus, in iron, Q first increases with the magnetic field, after reaching a maximum begins to decrease, and becomes zero at a certain magnetic field. If the field is further increased, Q changes its sign. In nickel, Q is always negative, and as the field is increased, its magnitude reaches a maximum and then gradually decreases. The dotted lines in the figure show the theoretical results calculated from the value of κ''; a fair agreement of the results of the experiment with the theory is noticeable.

(b) *Longitudinal Magnetization Produced by Twisting a Circularly Magnetized Wire.*[1] The apparatus for this experiment is

(1) H. Nagaoka and K. Honda, Jour. Coll. Sci., loc. cit.

nearly the same as in the previous case. A secondary coil which is connected with a ballistic galvanometer, is co-axially fixed round the wire, and an electric current is passed through it; the wire is then suddenly twisted and the induction thereby caused is measured by the galvanometer. To produce a strong circular magnetization, the current must be sufficiently intense, and hence the wire is strongly heated by the current. This heating is, however, partly prevented by a cooling device, which constantly circulates cold oil round the wire.

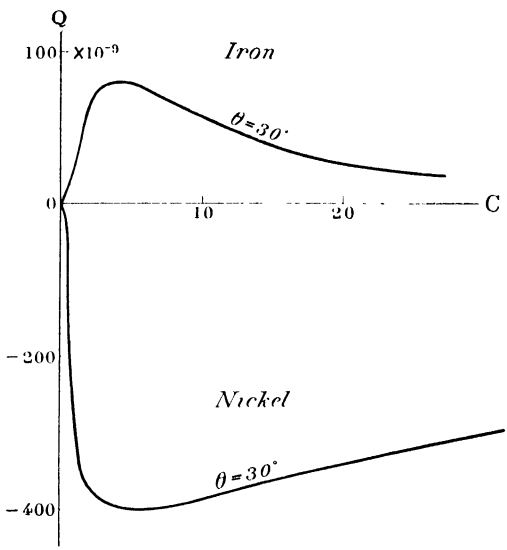


Fig. 70.
(Nagaoka, Honda).

The results of experiments with iron and nickel wires are shown in Fig. 70. Here Q is the quantity of the induced current due to the longitudinal magnetization produced by twisting the wires, and is a quantity proportional to the longitudinal magnetization. In the case of iron, Q increases with the current C, and gradually decreases after reaching a maximum. According to the theoretical consideration, Q will change its sign in a strong circular field, but the corresponding current is very much greater than that used in this experiment. In the case of nickel, Q is always negative, and with the increase of the current, its amount reaches a maximum and then gradually decreases.

The circular magnetization is the greatest at the surface of the wire and zero at the centre, so that the value of κ'' varies considerably from the centre to the surface, and, therefore, the theoretical result which in its calculation involves the assumption of the constancy of κ'', cannot be compared with that of the above experiment. But qualitatively, we find a good agreement between the theory and the experiment.

(c) *Wiedemann Effect.*[1] Fig. 71 shows an apparatus for the measurement of the Wiedemann effect. *CC* is a coil, *S* the test-wire, *a* and *b* brass rods soldered to both ends of it. To the lower brass rod *b* is attached a small mirror *M*, and from its lower end is hung a brass wire, its end are dipped in a mercury pool *g*; the upper rod *a* and the pool are connected with a battery. If we produce a longitudinal magnetic field by passing an electric current through the coil and also a current through the wire, this latter will be twisted and the small mirror will be rotated through a small angle, the amount of which can be read by means of a scale and telescope in the usual manner.

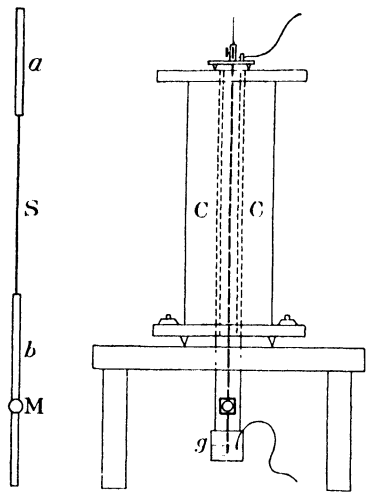


Fig. 71.

In Fig. 72, the full lines show the result of experiments, τ representing the angle of twist per unit of length. If the longitudinal and circular fields are in the relation of clockwise rotation, and if the lower end of the wire is twisted in the direction of the circular field, τ is taken as positive, and if twisted in the opposite direction, it is taken as negative.

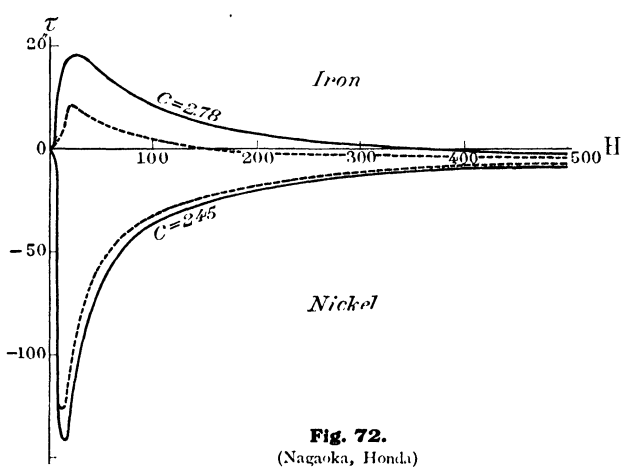


Fig. 72.
(Nagaoka, Honda)

In the case of iron, τ increases for a

(1) H. Nagaoka and K. Honda, Jour. Coll. Sci., loc. cit. K. Honda and S. Shimidzu, Jour. Coll. Sci. **16**, (1902), Art. 14.

constant current, with an increasing magnetic field, and after attaining a maximum value, gradually decreases to zero at a certain field. When the field is further increased, τ changes its sign. In the case of nickel, τ is always negative, and after reaching a maximum, gradually decreases with the increase of magnetic field. If the current passing through the wire is increased, the above variation also increases nearly proportionally. An experiment with wires of different thickness shows in accordance with the theory, that under a given electric current and in a given magnetic field, the angle of twist is inversely proportional to the square of the radius. The dotted lines in the above figure shows the result of the theoretical calculation from the value of κ'', and in the case of nickel, for which the hysteresis is comparatively small, we observe a good coincidence between the experiment and the theory. In the case of iron, the hysteresis is particularly conspicuous, and hence the divergence of the experiment from the theory is large, but on the whole the two curves resemble each other.

The numerous phenomena concerning the relation between magnetization, stress and strain mentioned in the present chapter may be classified as follows:

I. Effect of stress on magnetization:
   (a) Effect of tension and compression.
   (b) Effect of hydrostatic pressure.
   (c) Effect of twist.

II. Change in the dimensions in ferromagnetic substances by magnetization:
   (a) Change in length.
   (b) Change in volume.
   (c) Change in twist.
   (d) Change in the modulus of elasticity and rigidity.

III. Mutual effect between longitudinal and circular magnetizations.
   (a) Effect of twisting a longitudinally magnetized wire.
   (b) Effect of twisting a circularly magnetized wire.
   (c) Wiedemann effect.

Amongst those, I and II are phenomena reciprocally related; one may be inferred from the existence of the other. Thus, (a), (b) and (c) of I and II are corresponding reciprocal relations; also (d) in II is the same change as (a) and (c) in II and nothing but a change as seen from the different points of view. That III stands in a reciprocal relation with I is clear from the explanation in Art. 24, so that all the above phenomena are, as shown by

Kirchhoff's theory, contained in the fact that the intensity of magnetization is a function, not only of the magnetic field, but also of the change in length and volume.

The complete theory of magnetism must be one which not only explains the important phenomena relating to the magnetization curve, magnetic hysteresis, the magnetization of crystalline bodies, the effect of temperature on magnetization, but also the complex phenomena relating to magnetostriction. This last requirement is apparently very difficult to fulfil, but it is not really so. For, as explained above, in order to arrive at the constitution of matter fulfilling all these requirements, it is only necessary to conceive of a model of the atomic system in which the intensity of magnetization is influenced by the change in volume and length. Hence it will not be long before such a complete theory of magnetism is put forward.

CHAPTER IV.

Magnetism and Temperature.

27. Historical.

It has long been known that when magnetized iron is heated to a red heat, it loses its magnetism, but this is restored on cooling. Not only iron, but nickel and cobalt also possess a similar property. The temperature at which cobalt loses its magnetism is, however, far higher than in the case of iron, while on the contrary, the corresponding temperature in nickel is considerably lower. Hopkinson named the temperature at which magnetism disappears on heating, and reappears on cooling, the critical temperature. This disappearance of magnetism is due to some particular change taking place within the substance, and consequently, it is accompanied by a change in other physical properties. For instance, in 1869 Gore measured the change in length of a steel wire during cooling from a high temperature and found that at first the wire slowly contracted, but at a temperature of dull red heat, it underwent a sudden slight elongation, again contracting afterwards. In 1875, Tait[1] discovered that in iron an abrupt change in the thermo-electric property takes place at the temperature of red heat, and, in nickel, a similar change also occurs at a much lower temperature, and remarked that these changes are closely related to the disappearance of magnetism. Kohlrausch[2] and Hopkinson[3] found that the temperature coefficient of electric resistance changes abruptly at the critical temperature.

In 1874, Barrett[4] found that if a steel wire was allowed to cool from a very high temperature, not only was the cooling stopped temporarily at a certain temperature, but the temperature of the wire was raised by 10° to 20°, whereby the redness of the wire was increased for a moment. He called

(1) P. G. Tait, Sci. Papers, **1**, 218. (2) Kohlrausch, Wied. Ann. **33**, (1888), 42. (3) Hopkinson, Phil. Trans. Series A, **180**, (1889), 443. (4) Barrett, Phil. Mag., Jan. and May, (1874).

this phenomenon recalescence. Recalescence is especially remarkable in the case of high carbon steel, and is intimately related to the critical temperature.

28. Change of Magnetization at High Temperatures.

The ballistic and magnetometric methods can be used for the investigation of the magnetization at high temperatures. In using the former method, the test-specimen in the form of a ring is wrapped in asbestos, and over it are wound the primary and secondary coils; the whole is then put into a furnace of uniform temperature. An electric current is passed through the primary coil to magnetize the ring, and the induced current thereby caused in the secondary coil, is measured with a ballistic galvanometer. For the measurement of temperature, it is advantageous to use Le Chatelier's therm-ocouple, that is, a thermo-couple consisting of platinum and platinum-rhodium wires.

The ballistic method is, however, not very suitable for a measurement to be carried out at high temperatures, and the magnetometric method is usually preferred. In the latter method, in order to protect the test-specimen in the form of a bar or an ellipsoid of rotation from oxidation, the specimen is placed in a silica tube at its closed end and inserted in a heating coil, this coil being in turn placed in a magnetizing coil. The other end of the silica tube is connected with a vacuum pump, so that the heating of the specimen can take place in vacuo. The intensity of magnetization at different temperatures is then measured with a magnetometer in the usual way.

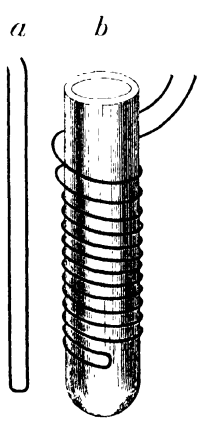


Fig. 73.

The heating coil consists of a porcelain tube wound round with platinum wire, this being pasted over with kaolin or magnesia powder, and then covered with asbestos paper. The powder protects the platinum wire against the action of asbestos at high temperatures. The platinum wire, 0.6 mm thick, is bent at its middle point so as to form parallel wires (Fig. 73*a*) and wound on the tube at a rate of three or four turns per centimetre (Fig. 73*b*). Since the platinum wire is wound anti-inductively, an electric current passing through the heating coil does not give rise to a magnetic field inside the coil. In order to prevent the rise of

temperature in the magnetizing coil during the heating of the specimen, this coil is provided with a water-jacket.

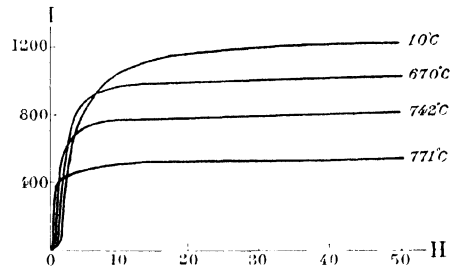


Fig. 74.
Iron (Hopkinson).

We have a number of investigations concerning the change in magnetization at high temperatures, of which those by Hopkinson[1], Curie[2], Morris[3], Nagaoka and Kusakabe[4], Shimidzu[5], Takagi[6] and the author[7] are to be mentioned. The graphs in Fig. 74 show the results for the curve of magnetization of pure iron at different temperatures. Generally the rise of temperature increases the magnetization in a weak field, but decreases it in a strong field. The magnetization does not considerably change its value in the interval from 10° up to 600°, but above it, rapidly decreases and becomes zero at 790°. The relation between the intensity of magnetization of iron under a constant field and the temperature is graphically shown in Fig. 75. We see that in a field of several hundred gausses, the magnetization gradually decreases from the lowest temperature upwards, its rate of decrease becoming large from ca. 600°, the magnetization vanishing at 790°C. In a field of some fifty gausses, the initial decrease of

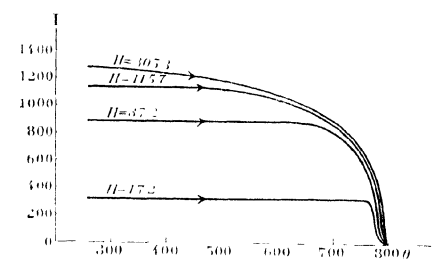


Fig. 75.
Iron (Honda)

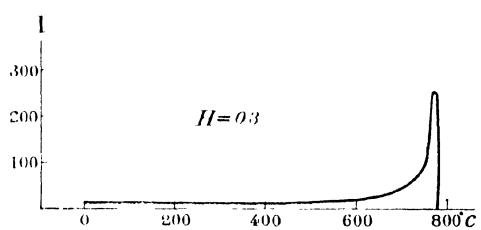


Fig. 76.
Iron (Hopkinson)

(1) Hopkinson, Phil. Trans., Series A, **180**, (1889) **443**; Proc. Roy. Soc. **45**, (1889), 318. (2) P. Curie, Ann. de Chem. et phys. (7) **5**, (1895), 289; Oeuvres, (1908), 232. (3) Morris, Phil. Mag. **44**, (1897), 213. (4) Nagaoka and Kusakabe, Jour. Coll. Sci. **19**, (1904) Art. 9. (5) Honda and Shimidzu, Jour. Coll. Sci. **20**, (1904), Art. 6. (6) Honda and Takagi, Sci. Rep. **1**, (1912), 207; **2**, (1913), 203. (7) Honda, Sci. Rep. **2**, (1913), 69.

magnetization is very slow. If the field is weaker than 10 gausses, the value of magnetization is nearly constant for the rise of temperature and at 760° begins rapidly to decrease, vanishing finally at 790°C. If the field is less than one gauss, the magnetization gradually increases with the rise of temperature, and at 700°, suddenly begins to increase, and after reaching a maximum, rapidly decreases (Fig. 76). We see from the above results that the critical temperature for pure iron is 790° and independent of the strength of the field.(1)

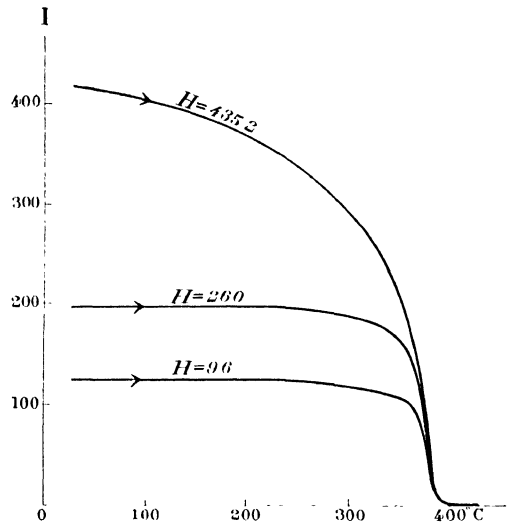


Fig. 77.
Nickel (Honda)

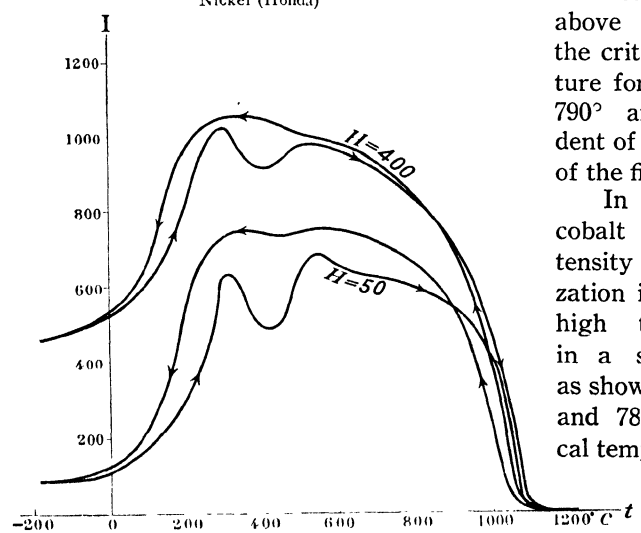


Fig. 78.
Annealed Cobalt (Honda, Shimidzu).

In nickel and cobalt also, the intensity of magnetization is changed at high temperatures in a similar way, as shown in Figs. 77 and 78; their critical temperatures are respectively 390° and 1115°. The abnormal course of the curve for cobalt below 500° is due to the allotropic change of cobalt, as will be explained later on.

(1) K. Honda, Sci. Rep. **10**, (1921), 433.

of heat begins to occur at the critical temperature and extends to some lower temperature; during heating, a reverse change takes place. In cobalt, another evolution of heat is observable at 470°, i. e. at its allotropic transformation point.

(b) *Electric Resistance.* We have a considerable number of investigations concerning the change of electric resistance at high temperatures, amongst which the most prominent are those of Le Chatelier,[1] Hopkinson,[2] Harrison,[3] Meyer[4], Burgess and Kelberg[5] and others. It is, however, to be remarked that these investigations do not give us an exact knowledge of the relation between electric resistance and magnetization. Hence the present author in co-operation with Y. Ogura[6] measured, simultaneously, magnetization and electric resistance, at high temperatures, and ascertained an important relation existing between them. An iron wire to be tested was bent in the form of a narrow lattice, each portion of which was insulated with a fine porcelain tube, as shown in Fig. 94. The whole was then tied up in a bundle and placed in a heating coil, which was in turn placed in a magnetizing coil. The magnetization of this bundle of wire was measured with a magnetometer, and at the same time the electric resistance was measured with a Wheatstone bridge. The result of experiments is shown in Fig. 95. Here, R is the electric

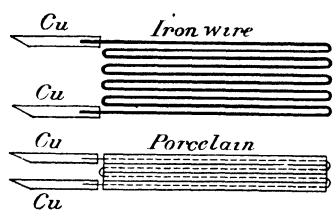


Fig. 94.

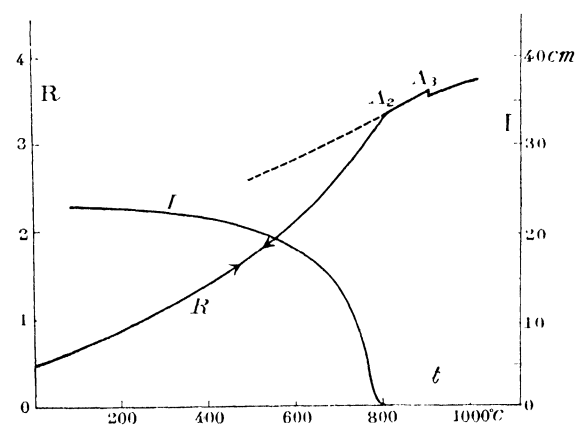


Fig. 95. Iron (Honda, Ogura)

(1) H. Le Chatelier, C. R. **110**, (1890), 283. (2) J. Hopkinson, Trans. Roy. Soc., (A), **180**, (1889), 462. (3) E. P. Harrison, Phil. Mag., **3**, (1902), 177. (4) Meyer, Inaug. Dissert., Greifswalds, (1911). (5) Burgess and Kelberg, Jour. of Washington Acad. **5**, No. 15, (1914), 435. (6) K. Honda and Y. Ogura, Sci. Rep., **3**, (1914), 113.

resistance and I the deflection of the magnetometer, which is proportional to the intensity of magnetization. In iron, the resistance increases gradually with the rise of temperature; the R,t curve shows the relation between the resistance and the temperature; it gradually bends upwards, then changes its direction at the A_2 point to become straight, and afterwards suffers a small break at the A_3 point.

The same curve for nickel shows a similar change (Fig. 96), that is, the curve bends upwards below the A_2 point, and becomes straight above it. It is very interesting to observe that if the straight portion of the curve for iron or nickel is produced in the direction of lower temperatures, it intersects the axis of temperature at nearly absolute zero. This agrees with the general observation that the electric resistance of non-magnetic metals is approximately proportional to the absolute temperature. Hence the fact that the resistance of iron and nickel at high temperatures below the A_2 point is much less than might be expected from the linear relation observed at higher temperatures, is attributed to their ferromagnetic property. In the same figures, we also see that the amount of difference between the resistance to be expected from the linear relation and that actually observed, varies in the same way as the intensity of magnetization considered as a function of temperature. In studying the change of resistance of ferromagnetic substances at high temperatures, previous investigators directed their attention only to the point, where the R,t curve changes its direction; but this point is merely the end of the A_2 transformation, having in itself no important significance, and the bending of the curve below this point furnishes us an important knowledge of the electric resistance of ferromagnetic substances.

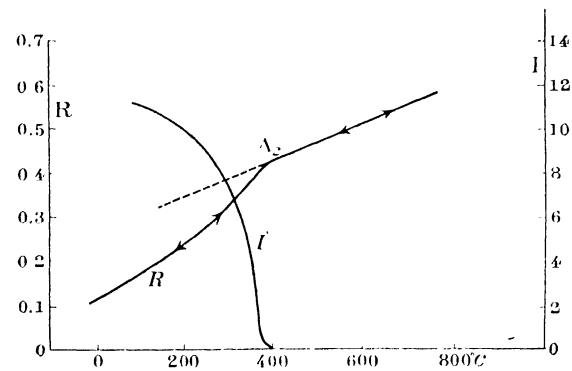


Fig. 96.

Nickel (Honda, Ogura)

(c) *Thermal Expansion.* The first accurate determination of the expansion of iron and steel at high temperatures was made by

Charpy and Grenet,[1] the range of temperature being from the ordinary to 1000°.

The author[2] also measured the expansion of ferromagnetic metals at high temperatures with the following apparatus. In Fig. 97, S is a specimen in the form of a bar, 20 cm long and 5 mm thick, having a small hole at its left end to receive one junction of a thermocouple. A quartz tube b, 1.2 cm thick and 60 cm long, is closed at its right end and fixed to a brass piece d at its left end. A quartz tube a is 6 mm thick and 45 cm long; it is made to press the specimen to

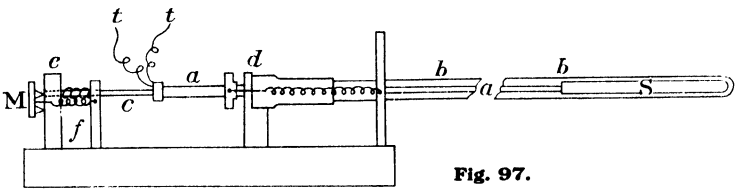


Fig. 97.

the closed end of the tube b by means of two springs. A brass bar c transmits the motion of the left end of the quartz tube to a small reflecting mirror M. A platform carrying this mirror has three legs, one of which rests on the left end of the bar c and the other two on small stands with conical holes fixed on a support e, and the mirror

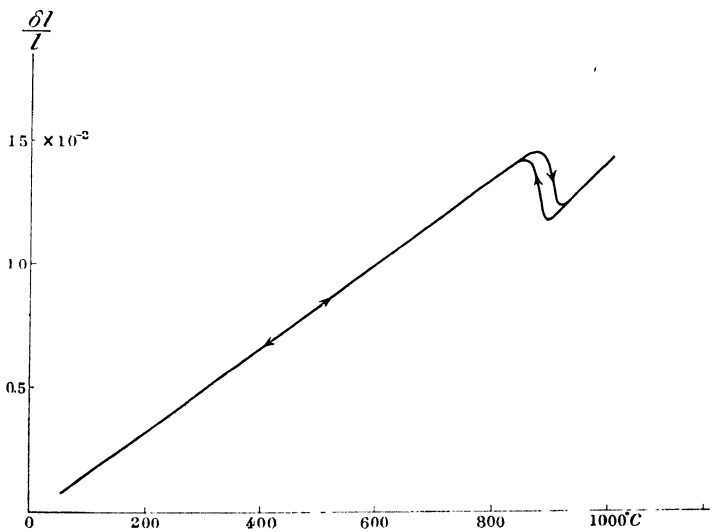


Fig. 98. Pure Iron (Honda)

(1) G. Charpy et L. Grenet, Bulletin de la Société d'Encouragement, **104**, (1903), 464, 883. (2) K. Honda, Sci. Rep., **6**, (1917), 203.

is pulled to the right by means of a spring *f*. In this way, a small change in length of the specimen gives rise to a small rotation of the mirror, which can be read by means of a vertical scale through a telescope. The thermo-couple is inserted into the tube *a* at its left end and is in contact with the specimen. The specimen is heated by an electric resistance furnace 50 cm long, into which the tube *b* is inserted.

The correction for the small expansion of the quartz tubes can be easily made. In order to avoid the oxidation of the specimen at high temperatures, the left half of the apparatus is enclosed in a bell-jar so as to make observations in vacuo.

The result of observation is graphically shown in Fig. 98; the expansion curve of pure iron rises almost linearly with the increase of temperature, and abruptly falls at the A_3 point, after which a linear expansion again takes place. During cooling the A_3 transformation takes place at a temperature lower by 25° than in the case of heating. The abnormal change of length accompanying the A_2 transformation is extremely small and cannot be distinctly detected, unless we use an apparatus of much higher sensibility.

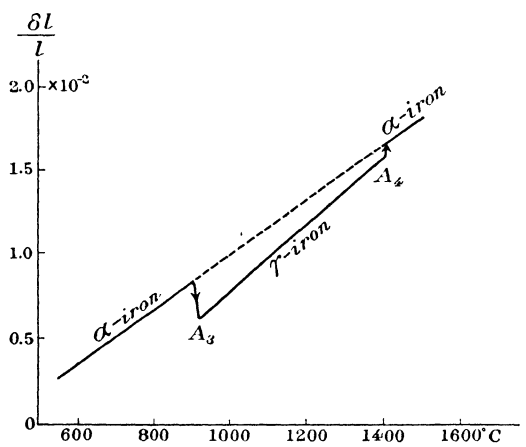


Fig. 99. Pure Iron (S. Sato).

Fig. 99 is an expansion-temperature curve obtained by S. Sato[1] at very high temperatures, in which the abrupt contraction and expansion at the A_3 and A_4 points are clearly seen. When the same curve of α-iron is produced into the region of γ-iron, it forms a continuation of the curve above the A_4 point, again showing the fact that above this point iron has the same lattice as α-iron.

Nickel undergoes only one magnetic transformation, and hence the expansion-temperature curve is almost straight. Cobalt has one allotropic transformation; during heating, the same curve shows an abrupt expansion at 470° and during cooling an abrupt contraction at 403°, as shown in Fig. 100.

(1) Sci. Rep., **14**, (1925), 513.

33. Steel.

Steel is an iron alloy containing a small amount of carbon, usually less than 1.6 percent. The carbon contained in steel sometimes exists as graphite, but in most cases in the form of the compound Fe_3C called cementite. Although the amount of carbon is small, it combines with iron to produce a considerable amount of cementite; hence even with a very small amount of carbon content, steel shows a remarkable difference in its properties, as compared with pure iron. Cementite is ferromagnetic and its critical point lies at 215°. Fig. 101 is that part of the equilibrium diagram for the iron-carbon system which has an important relation to the transformations in steel. In this figure, the ordinate represents the temperature and the abscissa the percentage of carbon contained in iron. The A_3 point of pure iron is 910°, but that of steel falls with the increase of carbon, becoming 726° at 0.85 percent of carbon (Line A_3B). The steel existing in the upper field of the line A_3B is called austenite, which consists of a solid solution of carbon in γ-iron. When austenitic iron is cooled down to the temperature of the A_3 point, a minute portion of the austenite begins to undergo the A_3 transformation and sets free iron, which is usually called ferrite. As

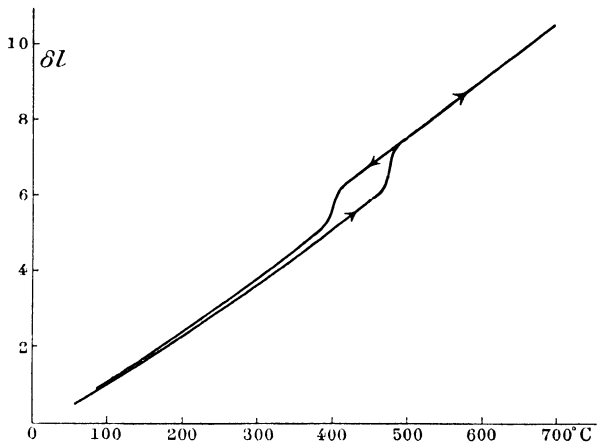


Fig. 100. Cobalt (H. Masumoto).

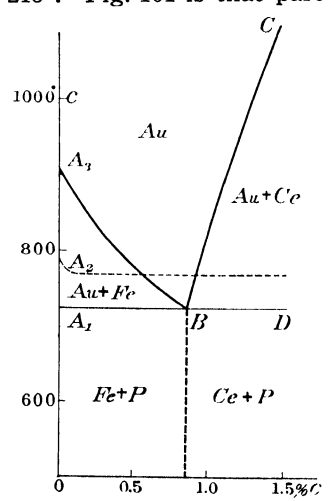


Fig. 101.
A=austenite, *Fe*=ferrite,
Ce=cementite, *P*=pearlite

the temperature gradually falls, the same process of transformation and precipitation of ferrite goes on continuously, and the composition of the remaining solid solution continuously approaches that of the point B. At this point, that is, 726°, the remaining austenite changes into a mixture of ferrite and cementite called a eutectoid mixture, or pearlite. This precipitation at the constant temperature is called the A_1 transformation. Though the carbon content increases from 0 to 0.85 or to the higher percentage, the A_1 transformation always takes place at the same temperature (line A_1D), and the amount of pearlite increases proportionally with the content of carbon, the whole becoming pearlite at the point B. Steel having a concentration of 0.85 percent of carbon changes wholly from austenite to pearlite at the same temperature, that is, at the A_1 point. In steels containing a higher carbon content than 0.85 percent, cementite first begins to separate along the line BC during cooling, and the composition of the remaining solid solution gradually approaches that of the point B, where the residual solid solution changes into pearlite at 726°. The amount of pearlite decreases gradually as the content of carbon increases from 0.85 to a higher percent.

The A_2 point for pure iron is 790°, but with the addition of carbon to iron, it decreases slightly at first and afterwards remains nearly constant at 770° ; hence the A_2 line is nearly parallel to the abscissa. Since this line does not express any change of phase as explained before, it is shown by a dotted line in Fig. 101.

In the same figure the area above the line A_3BC is a field of austenite, the area A_3BA_1 a field in which ferrite and austenite coexist, and the area BCD that in which cementite and austenite coexist. The area below the line A_1B is a field in which ferrite and pearlite coexist, and that below the line BD a field in which cementite and pearlite coexist.

The transformation points are, in general, different for heating and cooling; for example, during heating the A_3 point is higher than that during cooling by 10° to 20°. The A_2 point is the same for heating and cooling; but the A_1 point during heating is considerably higher than that during cooling, by as much as 30° to 60°. The lines in Fig. 101 represent their mean positions. The fact that there is thus a difference between the transformation points for heating and cooling, is attributed to the following causes:—The first is the rapidity of heating and cooling, and if this is done at a sufficiently slow rate, the difference between these points is considerably reduced. The second is the presence of impurities such as, manganese, silicon, etc. When the impurities are present as a solid solution, they

reduce the velocity of transformation, and hence, at the ordinary rate of cooling, the transformation point for heating occurs at a higher temperature and that for cooling at a lower temperature, than in the case of an extremely slow heating and cooling, so that the difference between these two may be considerable. Accordingly, when pure carbon steel is heated or cooled at an extremely slow rate, this difference will disappear.

34. Thermal Phenomena and Changes of Physical Properties Accompanying Transformation in Steel.

(a) *Thermal Phenomena.* The evolution or absorption of heat during transformations in steel is measured by means of the thermal analysis before mentioned.

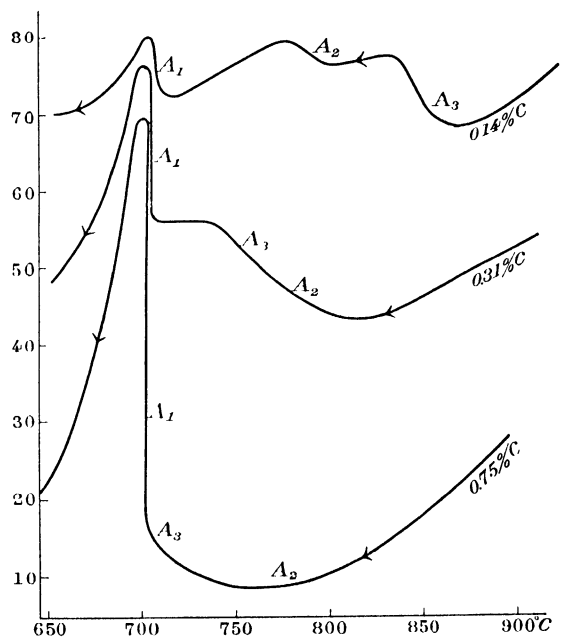


Fig. 102.
Carbon steels (Honda).

In the case of steels, the evolution of heat takes place at three temperatures, viz. the A_1, A_2 and A_3 points. Fig. 102 shows the cooling curves for three steels containing 0.14, 0.31, and 0.75 percent of carbon, these curves being drawn displaced at suitable distances to avoid overlapping. As we have already explained in the last section, we see by these curves that the A_3 point falls gradually as the carbon content is increased, and that at the same time the evolution of heat at the A_3 point becomes gradual. The A_2 point remains nearly constant for all steels. As the carbon content is increased, the heat effect at the A_2 point becomes so small that it is difficult to note this effect on the thermal curve.

From the same figure, we see also that the A_1 point remains perfectly constant for all steels, and that the heat evolution during cooling at the A_1 point is very conspicuous, and increases with the carbon content, giving rise to the phenomenon of recalescence observable in high carbon steels. Thus the recalescence is the evolution of heat accompanying the decomposition of austenite into a eutectoid mixture.

(b) *Magnetization.* Formerly in investigating the magnetization of steels many physicists did not pay due attention to their constitution or structure, and therefore, very little was known about the relation between the magnetization and the structure. The present author[1] in co-operation with H. Takagi investigated this relation for various kinds of steel, containing from 0.14 to 1.5 percent of carbon.

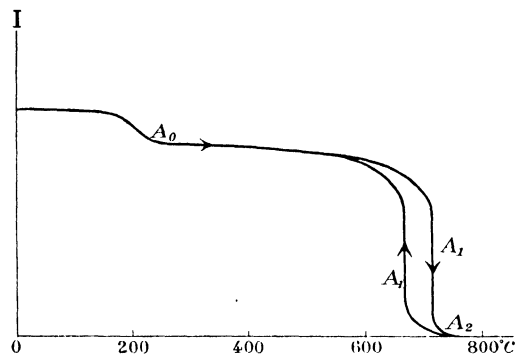


Fig. 103. Steel (Honda).

As seen in Fig. 103, the curve showing the relation between the magnetization of steel (0.5%C) and the temperature shows a change consisting of three steps. The first step, that is the first decrease of magnetization, takes place from 150° to 215°; at higher temperatures, the decrease of magnetization is slow, but the second discontinuous decrease takes place at 740°, and at the third step, the change becomes slow again and the magnetization diminishes to zero at 770°. The temperature 215° is the critical point of cementite called the A_0 point; at higher temperatures, cementite is not ferromagnetic. 740° is the A_1 point at which the eutectoid mixture changes into a solid solution; it is, however, considerably higher than the A_1 point during cooling, that is, 700°. The magnitude of the change of magnetization at the first two temperatures depends on the amount of cementite present, being zero in pure iron and increasing with the content of carbon. Hence in high carbon steels, most of the magnetization disappears at the A_1 point. The gradual diminution of magnetization above the A_1 point is the final phase of the A_2 transformation.

(1) K. Honda and H. Takagi, Sci. Rep., **1**, (1912), 207; **2**, (1913), 203.

(c) *Thermal Expansion.* The present author measured the thermal expansion of different kinds of steel with the dilatometer before mentioned. The results of observation are graphically shown in Fig. 104, the curves being drawn displaced at suitable distances to avoid overlapping. The expansion curve of a Swedish low carbon steel, which contains only 0.1 percent of carbon, rises almost linearly with the increase of temperature, and abruptly contracts at the A_3 point, after which a linear expansion again takes place. The abnormal change of length accompanying the A_2 transformation, being very small, is not observable in the curve. In steels of higher car-

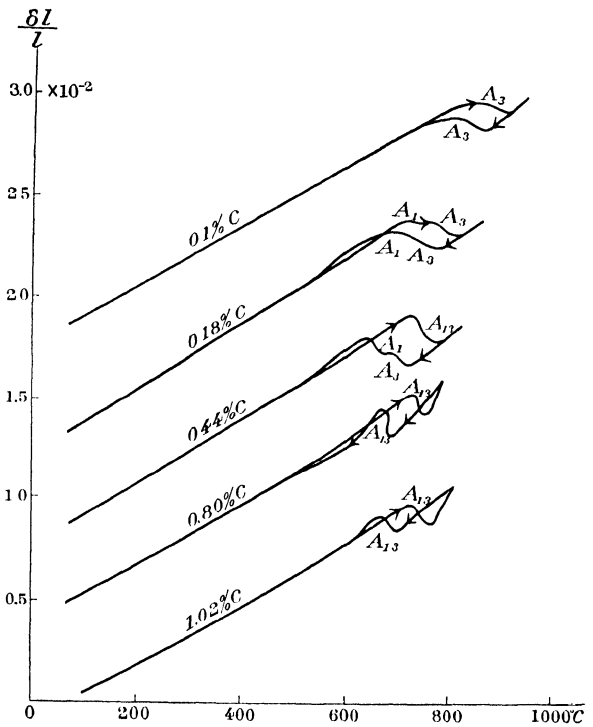


Fig. 104.
Carbon steels (Honda).

bon content, another abnormal change is also seen at the A_1 point. For example, in steel containing 0.18 percent of carbon, an abnormality at the A_1 point is already observable, and as the carbon content increases, this change becomes conspicuous. On the other hand, the abnormal change due to the A_3 transformation, whose

temperature falls with the carbon content, becomes less conspicuous. In the second and third curves, we can distinguish the two steps due to the A_1 and A_3 transformations, but in the other curves, these two changes coincide and unite into a single step. In all cases, the transformation points during cooling are decidedly lower than those during heating.

(d) *Electric Resistance.* Using the same method as in the case of iron and nickel, Y. Ogura and the present author[1] measured simultaneously the magnetization and the electric resistance in steels differing in carbon content at various high temperatures. Fig 105 shows an example of our observation. It is to be noted that in this case, the *R,t* curve shows also a hysteresis in the A_1 transformation during heating and cooling, just as in the case of the *I,t* curve, and that the change of electric resistance during the A_0 transformation is too small to be seen on the *R,t* curve.

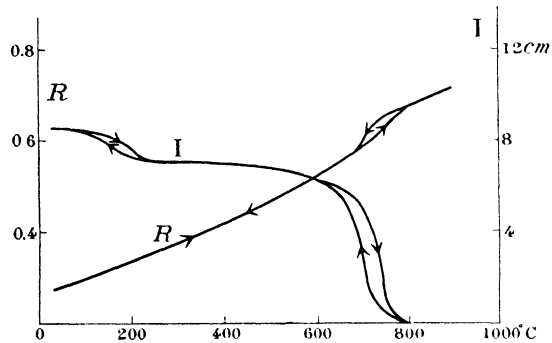


Fig. 105.
Steel 1.0%C (Honda, Ogura).

The electric resistance of a bar, several millimeters thick, at high temperatures can be conveniently measured in the following way[2]:— A constant electric current of 10—15 amperes is passed through the test-bar by means of two wires connected to both ends of it, and the difference of potentials at two points situated at a distance of 10—15 cm in the middle portion of the bar is measured by means of a potentiometer. Then the difference of potentials divided by the current gives the resistance of the bar between the two points. If the same measurement be conducted at different high temperatures by placing the bar in an electric resistance furnace of a uniform temperature, the resistance at such high temperatures is easily obtained.

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(1) loc. cit. (2) I. Iitaka, Sci. Rep. **7**, (1918), 167; T. Matsuda, Sci. Rep. **11**, (1922), 223; H. Imai, Sci. Rep. **11**, (1922), 313; T. Isihara, Sci. Rep. **13**, (1924), 75.

CHAPTER VI.

Ferromagnetic Alloys and Compounds.

35. Iron-Nickel Alloys.

According to the recent investigations carried out in our Institute, the equilibrium diagram of the iron-nickel alloys consists mainly of the fields of solid solutions α and γ, and a narrow heterogeneous field corresponding to the mixture of these two solid solutions, as shown in Fig. 106. The dotted curve in the diagram represents the boundary between the ferromagnetic and paramagnetic alloys. A thorough study of the alloys was made by T. Kase[1], who fully confirmed this diagram. He also explained the heterogeneous structure of meteorite (iron-nickel alloys) consisting of camasite and taenite as due to the presence of the α and γ solid solutions formed during a rapid cooling, their compositions naturally varying with the rapidity with which they were formed. The above diagram also agrees very satisfactorily with the result of the X-ray analysis.[2]

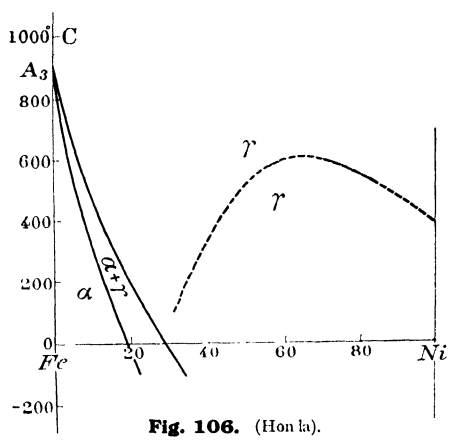


Fig. 106. (Hon la).

The magnetic properties of the iron-nickel alloys were first investigated by J. Hopkinson[3]. Afterwards, Guillaume[4], Osmond[5],

(1) Sci. Rep., **14**, (1925), 173; **14**, (1925), 537. (2) A. Osawa, Sci. Rep., **15**, (1926), 387. (3) J. Hopkinson, Proc. Roy. Soc. **47**, (1889) 23; **47**, (1890), 138; **48**, (1890), 1. (4) Guillaume, Les applications des aciers au nickel, Paris, (1898); Jour. de Phys. (3), **7**, (1898),262. (5) Osmond, C. R. **118**, (1894), 532; **128**, (1899), 304, 1395.

Dumont[1], L. Dumat[2], Nagaoka[3], Shimidzu[4], Takagi and the author[5] made various investigations of the same alloys, covering various aspects, the reults of which will be described below.

(a) *Magnetization at Ordinary Temperature.* Fig. 107 shows the magnetization curves[6] of nickel steels (0.3%C) of different concentrations, the form of the specimens being an ovoid, 20 cm in the longer diameter and 10 mm in the shorter. They are similar in form to the magnetization curve for iron or nickel, but are different from the

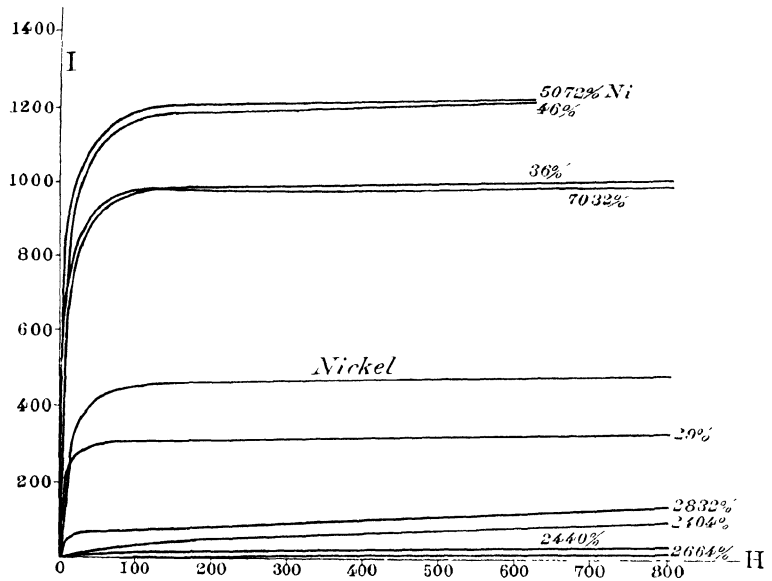


Fig. 107. Nickel steels (Nagaoka, Honda).

curve of the latter in that the magnetization reaches its saturation-value in a comparatively weak field. In the case of nickel steels containing less than 30 percent of nickel, the magnetization at a given temperature as well as the critical point, differs considerably according as the alloy is being heated or cooled, especially if it is cooled down to the temperature of liquid air. In the case of nickel steels of higher concentrations, the intensity of magnetization is almost reversible with regard to the temperature, and the critical temperatures during heating and cooling differ very little from

(1) Dumont, C. R. **126**, (1898), 741. (2) L. Dumas, C. R. **130**, (1900), 1311. (3) Nagaoka and Honda, Jour. Coll. Sci. **16**, Art. 8, (1902); **19**, Art. 11, (1903). (4) Honda and Shimidzu, Jour. Coll. Sci. **20**, Art. 6, (1905). (5) Honda and Takagi, Sci. Rep. **6**, (1917), 321. (6) Nagaoka and Honda, Jour. Coll. Sci., **19**, Art. 11, (1903).

each other. Consequently the alloys containing less than 30 percent of nickel are called irreversible alloys, and those containing more than that percentage reversible alloys.

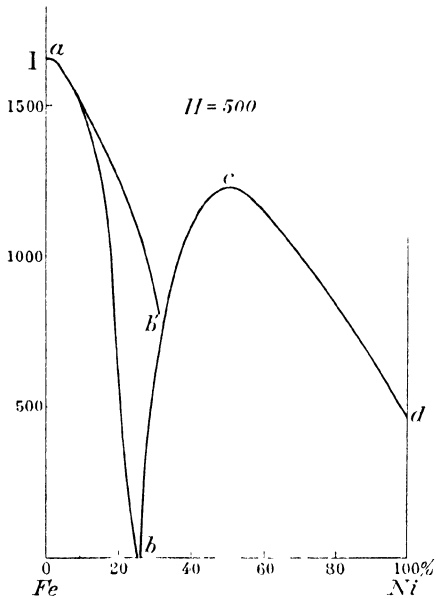


Fig. 108. Nickel-iron (Honda).

Fig. 108 shows the relation between the intensity of magnetization I in a very strong field and the content of nickel in the case of iron-nickel alloys of a very low carbon content; *a b c d* is the curve for the annealed specimens. Thus, as the content of nickel increases from zero, the intensity of magnetization shows an inconspicuous maximum at 1 percent of nickel, then rapidly decreases and vanishes at 26 percent. If the content of nickel further increases, the magnetization reappears and afterwards rapidly increases. At about 50 percent of nickel,

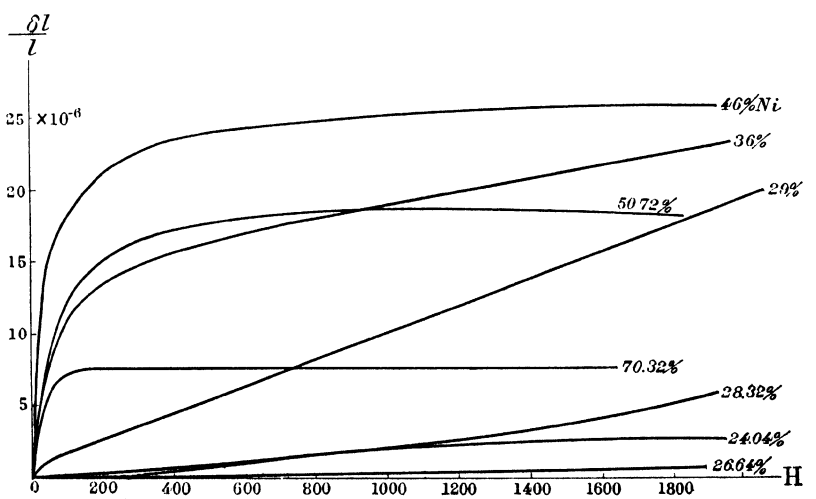


Fig. 109. Nickel steels (Nagaoka, Honda).

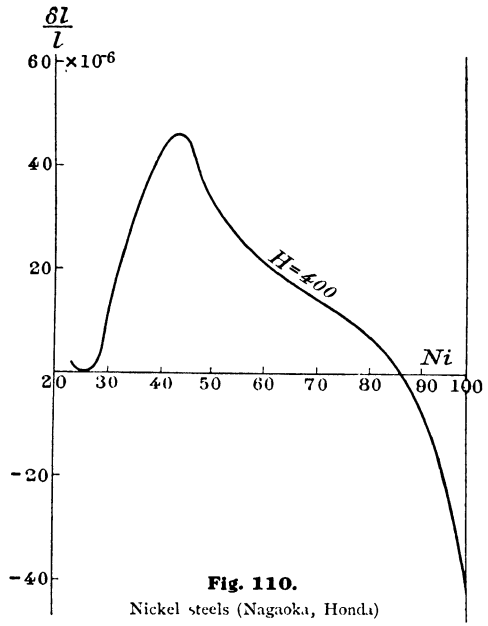


Fig. 110.
Nickel steels (Nagaoka, Honda)

the intensity of magnetization reaches a maximum value and then gradually decreases. The curve *a b' c d* is the result for specimens, which have been immersed in liquid air, and shows a large hysteresis with respect to the former curve.

(b) *Change of Length by Magnetization.* The change in length in the case of iron-nickel alloys, caused by magnetization is remarkably large as compared with that of iron. In the case of alloys containing from 25 to 80 percent of nickel, their length is increased by magnetization. According to the investigation of H. Nagaoka and the author, the change of length in ovoids of iron-nickel alloys containing 0.3 percent of carbon in different magnetizing fields is given by the curves in Fig. 109. From this figure, the relation between the change in length and the content of nickel is obtained, as shown in Fig. 110.

In the case of iron-nickel alloys[1] of a very low carbon content, the

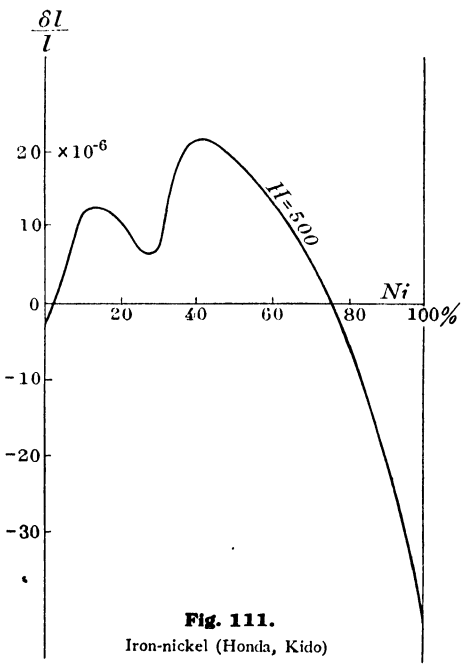


Fig. 111.
Iron-nickel (Honda, Kido)

(1) Honda and Kido, Sci. Rep., **9**, (1920), 221.

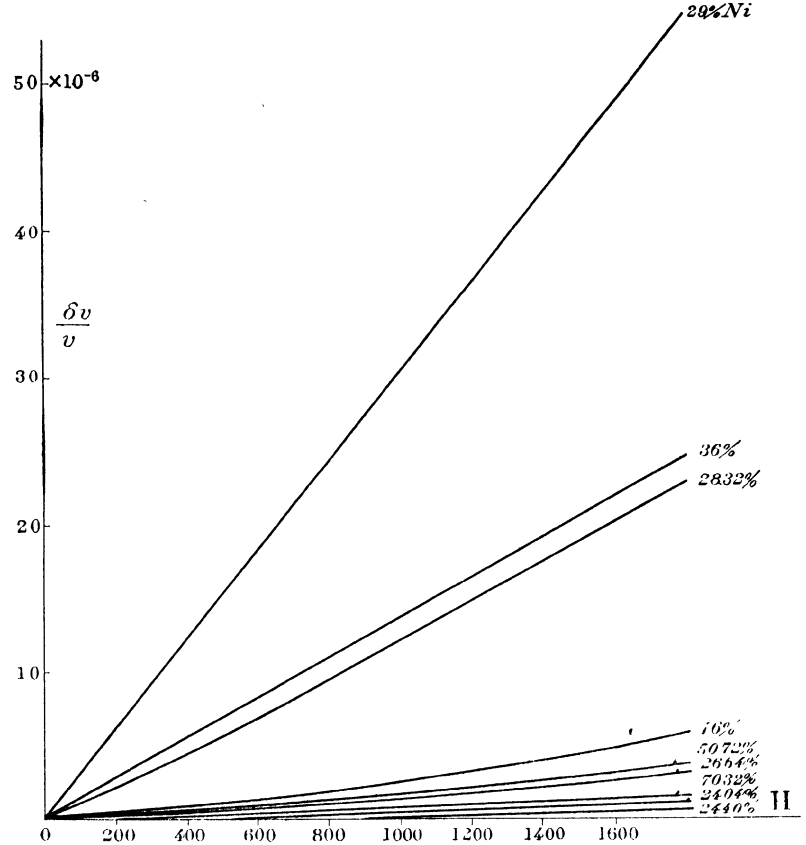
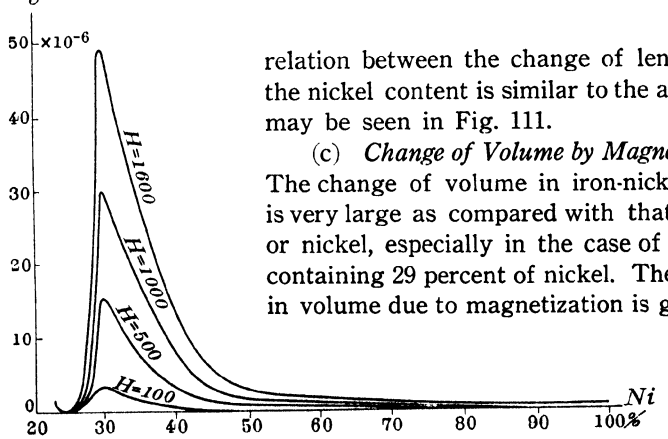


Fig. 112. Nickel steels (Nagaoka, Honda)



relation between the change of length and the nickel content is similar to the above, as may be seen in Fig. 111.

(c) *Change of Volume by Magnetization.* The change of volume in iron-nickel alloys is very large as compared with that in iron or nickel, especially in the case of an alloy containing 29 percent of nickel. The change in volume due to magnetization is generally

small as compared with that in length, since it is a differential effect of the longitudinal elongation and lateral contraction. For several kinds of iron-nickel alloys investigated by Professor H. Nagaoka and the author, the expansion of volume is rather greater than the elongation, and hence it may be concluded that, contrary to the other cases, these alloys are elongated both longitudinally and laterally, the most conspicuous example being furnished by the 29 percent nickel alloy. It is a remarkable fact that in the case of these alloys the change of volume increases almost proportionally to the magnetic field, as shown in Fig. 112. The relation between the change of volume and the content of nickel is given in Fig. 113.

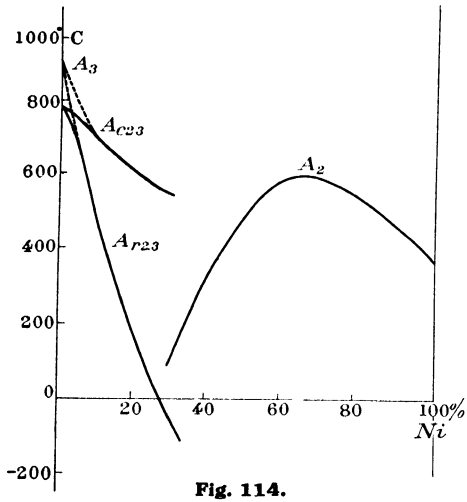


Fig. 114.
Critical point-concentration curve (Honda).

(d) *Magnetization at Different Temperatures.* As we have already remarked, the change of magnetization due to temperature in the reversible as compared with the irreversible alloys differs entirely. The relation between the critical temperatures during heating (Ac) and cooling (Ar), and the concentration of nickel is given in Fig. 114. Below 30 percent of nickel content, the critical temperature-concentration curve during cooling lies considerably below that obtained during heating; the difference between the critical temperature during heating and that during cooling increases gradually with the content of nickel. Above 30 percent of nickel, the curve obtained

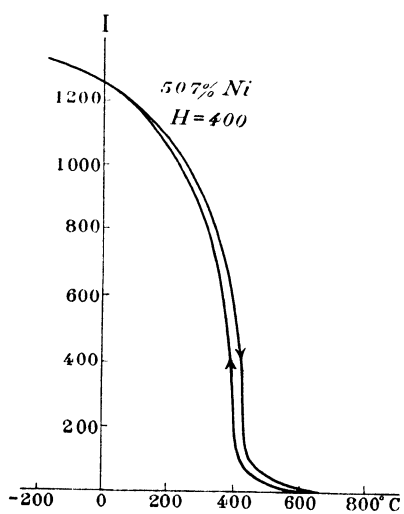


Fig. 115.
(Honda, Shimidzu).

during heating and that obtained during cooling coincide with each other.

The experiment of the magnetization of iron-nickel alloys at different high and low temperatures was made by S. Shimidzu and the author. The samples were tested in the form of an ovoid; Fig. 115 shows an example of the change of magnetization in reversible nickel steels, the temperature-hysteresis being very small. On the other hand, for irreversible nickel steels, a conspicuous hysteresis appears in heating and cooling; examples are given in Figs. 116, 117, 118. In a 14.0 percent nickel alloy the temperature hysteresis is already large and still more so in 26.6 and 29.2 percent nickel alloys; the latter alloys may be in a state of a strong magnetization or in a state of little or no magnetization at ordinary temperature. If we cool the above-mentioned alloys from a suitable temperature in the range of transformation, they may be brought to a state, which has at ordinary temperature any value of magnetization lying between

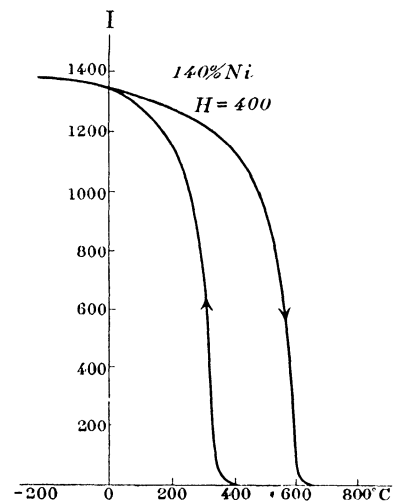


Fig. 116. (Honda).

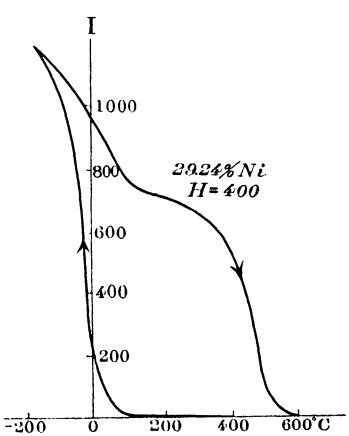


Fig. 117. (Honda, Shimidzu).

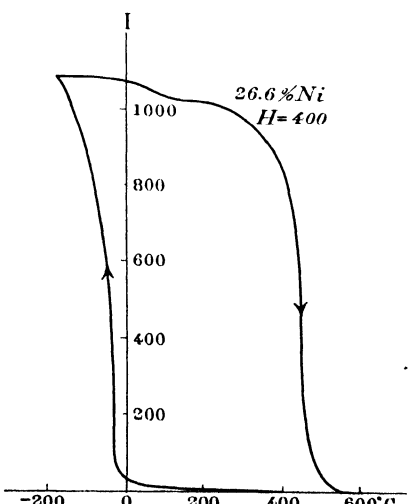


Fig. 118. (Honda, Shimidzu).

zero and the maximum. An abnormal bend of the curve observable in the two alloys in the vicinity of 120° is due to the fact that these alloys consist of a mixture of two solid solutions, that is, iron dissolving nickel and nickel dissolving iron, the critical point of the latter being about 120°.

(e) *Permalloy.* Permalloy is an iron-nickel alloy containing 78 percent of nickel; when properly heat-treated, it has a very high permeability and a very small hysteresis-loss in weak fields. The heat-treatment which gives the best result is to anneal it at 900° for one hour, then cool it slowly to 600° and finally, rapidly to room temperature. Fig. 119 is given for the sake of comparison of the hysteresis curves for permalloy and Swedish iron in very weak fields.

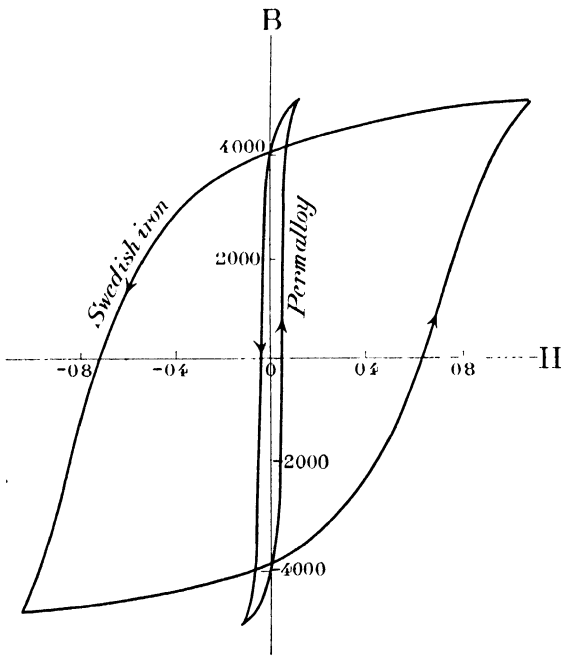


Fig. 119.

The following table shows also a comparison of the permeability and the hysteresis-loss of permalloy with those of a transformer sheet iron:—

Specimens	Permeability Max.	Coercive force $B_{max.} = 15000$	Hysteresis-loss $B_{max.} = 10000$
Sheet iron (4% Si)	3400	0.88	2260
Electrolyt. iron	13000	0.34	1060
Permalloy	120000	0.05	180

The most important use of permalloy is found in its application in submarine cables to increase their self-induction.

36. Iron-Cobalt Alloys.

The equilibrium diagram of the iron-cobalt system was investigated by Guertler and Tammann[1], Ruer and Kaneko[2]. Quite recently H. Masumoto[3] completed the diagram by introducing the new transformation lines found by himself. His diagram is given in Fig. 120. In the solid phase, it consists of three fields, each being a homogeneous solid solution, with two narrow ranges of heterogeneous mixtures. The critical or A_2 point of pure iron rises rapidly as the content of cobalt increases, and at 14 percent of cobalt content coincides with the A_3 point, which also rises with the increase of cobalt. As the content of cobalt further increases, the $A_{2,3}$ point rises and reaches its maximum at about 45 percent of cobalt, and afterwards gradually falls, up to 70 percent. With a further increase of cobalt, the A_3 point rapidly falls and attains room temperature at 78 percent of cobalt, while the A_2 point steadily rises from 70 percent to pure cobalt.

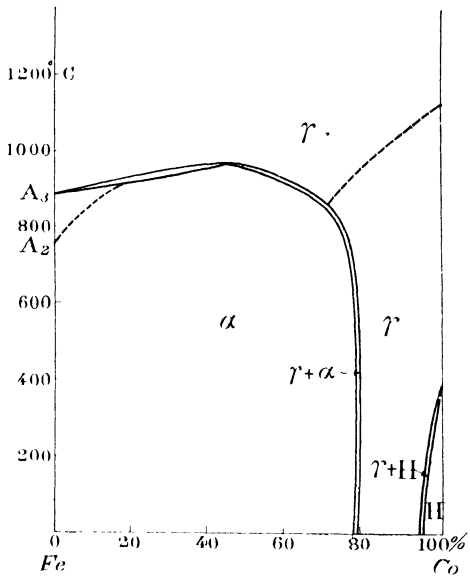


Fig. 120.
Iron-Cobalt (Masumoto).

The new transformation point of cobalt rapidly falls with the addition of iron to cobalt and attains room temperature at 95 percent of cobalt.

The relation between the magnetization and the concentration was investigated by Preuss[4] and the present author[5]; the result is shown in Fig. 121. In a field of 100 gausses, the curve shows two maxima, and between them a minimum at 22 percent

(1) W. Guertler and Tammann, Zeits. anorg. Chem., **45**, (1905), 205. (2) R. Ruer and K. Kaneko, Ferrum, **11** Jahrg., (1913), 33. (3) H. Masumoto, Sci. Rep. **15**, (1926), 449. (4) A. Preuss, Dissert., Zürich, (1912). (5) K. Honda, Sci. Rep. **8**, (1919), 51.

of iron, where the structure of the alloys changes from γ to α solid solution; but in stronger fields, this minimum becomes less conspicuous, and in a field of 1200 gausses no longer appears. As may be expected from the standpoint of the molecular theory of magnetism, the connection between the magnetization-concentration curve and the structure of an alloy system, which is at first very conspicuous, becomes generally less so, as the magnetic field increases.

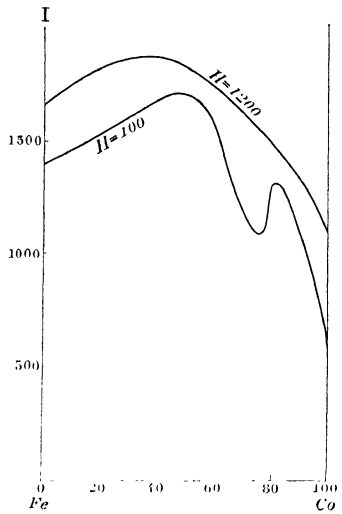


Fig. 121. Iron-Cobalt (Honda).

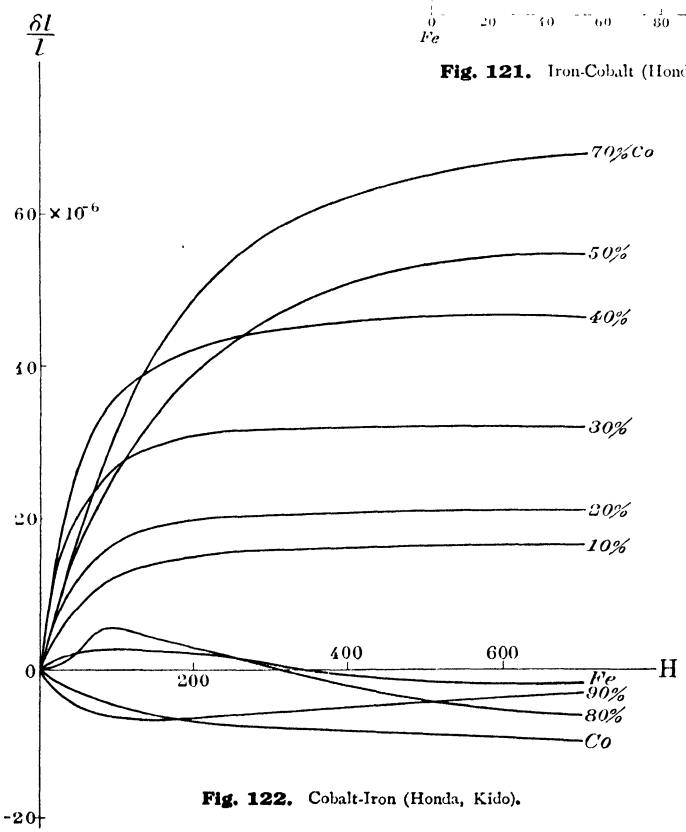


Fig. 122. Cobalt-Iron (Honda, Kido).

The change in length of the iron-cobalt alloys resulting from magnetization was investigated by K. Kido and the present author[1]. The specimens were first annealed and then subjected to experiments. The relation between the change of length and the magnetic field is given by the curves shown in Fig. 122 and that of the change of length to cobalt content is given in Fig. 123. In this figure it is to be observed that the concentration at which magnetic expansion changes into contraction lies at the point where the structure of the alloy system changes from the α to the γ solid solution.

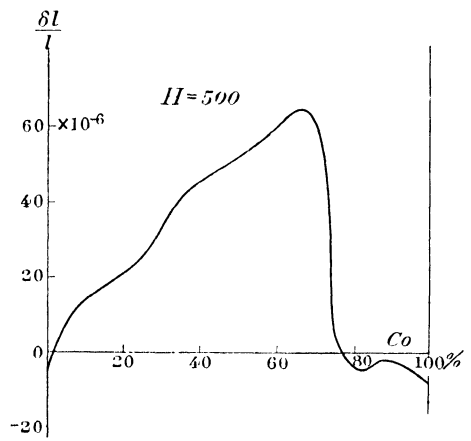


Fig. 123. Cobalt-Iron (Honda, Kido)

The K. S. magnet steel invented in our Institute is the best magnet steel known to us, having a very high coercive force and a large residual magnetism. It is a cobalt-iron alloy of the following composition:—

0.6—0.8%C, 30—35%Co,
6—8%W, 1.5—2.0%Cr.

Ordinary tungsten magnet steel containing 5—7 percent of tungsten and 0.5—0.7 percent of carbon has, when quenched, a coercive force of 80 gausses, while that of quenched K. S. steel is 235 gausses, that is, 4 times greater than that of the tungsten steel. In the case of a long magnet, the demagnetizing action of which is very small, the intensity of the permanent magnetism of both the K. S. and the tungsten steel does not much differ; but in the case of a short magnet, owing to its large coercive force, the permanent magnetism of the former steel is much greater than that of the latter, its amount depending on the dimension ratio of these magnets. Thus:

Dimension ratio	5	10	15	20	30
M_p (K. S.): M_p (T. S.)	2.36	2.33	1.77	1.40	1.08

The most important property of K. S. magnet steel is its permanency. By virtue of its large coercive force, its permanent magnetism is hardly affected by a mechanical shock or ageing for years.

(1) Sci. Rep. **9**, (1920), 221.

37. Cobalt-Nickel Alloys.

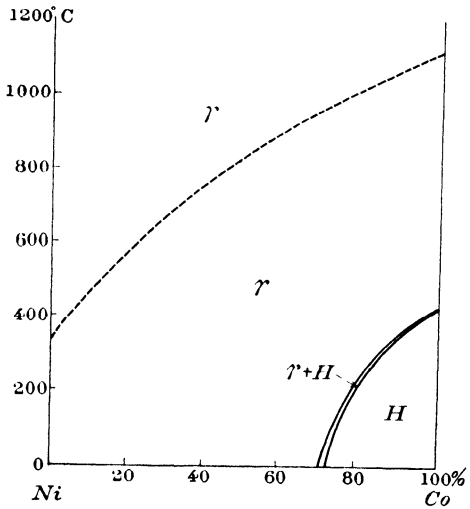


Fig. 124. Cobalt-Nickel (Masumoto).

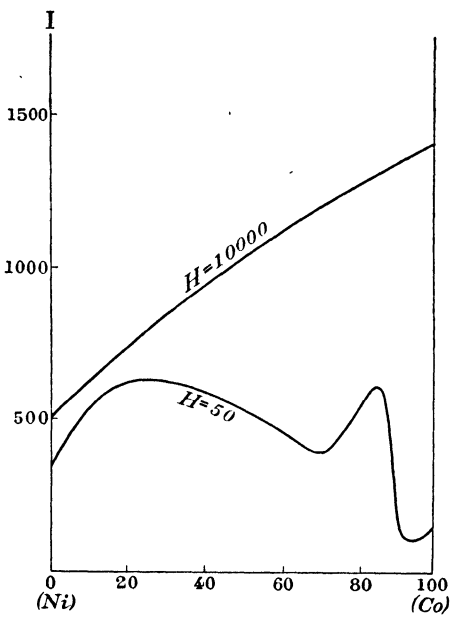


Fig. 125. Cobalt-Nickel (Bloch, Masumoto).

The equilibrium diagram of the cobalt-nickel system consists, in its solid phase, of two fields of homogeneous solid solutions. Fig. 124 is the diagram given by H. Masumoto[1]. The transformation point of cobalt at 470° falls rapidly as nickel is added to cobalt, and attains room temperature at 28 percent of nickel. The left side of this line consists everywhere of the γ solid solution having a face-centred lattice, and the right side of it of another solid solution, H, having a hexagonal close-packed lattice; between these two there is a narrow field of a heterogeneous mixture of these solid solutions. The broken line in the diagram extending from one side to the other is the critical or A_2 line, which shows the boundary between the ferromagnetic and paramagnetic fields.

O. Bloch[2] and H. Masumoto[3] investigated the magnetization of the alloys in different magnetizing fields, the result being shown in Fig.

(1) Sci. Rep. **15**, (1926), 449; R. Ruer and K. Kaneko, Ferrum, **11**, l.c. (2) Arch. des Sci. **33**, (1912), 293. (3) Sci. Rep. **16**, (1927), 321.

125. In weak fields, we observe an intimate connection between the magnetization and the structure of the alloys, but as the field increases, this connection becomes less conspicuous. In a very strong field of 10000 gausses, it almost disappears and regardless of the variation of the structure of the alloys, the intensity of magnetization of the alloys increases with a slight curvature from that of nickel to cobalt.

In comparing the magnetization-concentration curves for the three ferromagnetic systems, viz. the iron-nickel, iron-cobalt and cobalt-nickel with each other, a very interesting fact is observable. In a moderate field, these curves have each a minimum at the concentration which forms the boundary between two homogeneous solid solutions, and also a maximum at a certain concentration in each solid solution. The following table gives the concentrations of these maxima and minima:—

Max. or Min.	Iron-Nickel	Iron-Cobalt	Cobalt-Nickel
Min. (boundary of α, γ or α, H)	25 % Ni	22 % Fe	28 % Ni
1 Max.	1	19	15
2 „	50	54	76

The change of length resulting from magnetization in cobalt-nickel alloys was investigated by H. Masumoto[1]; the relation of the change of length to the magnetic field and that to the concentration are given in Figs. 126 and 127. From the latter figure, it is evident that the maximum expansion lies at the concentration, where the *H* solid solution changes into the γ solid solution.

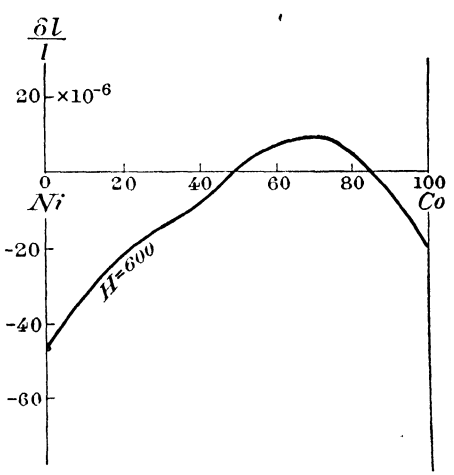


Fig. 126. Cobalt-Nickel (Masumoto).

(1) O. Bloch, Arch. des. Sci. **33**, (1912), 293.

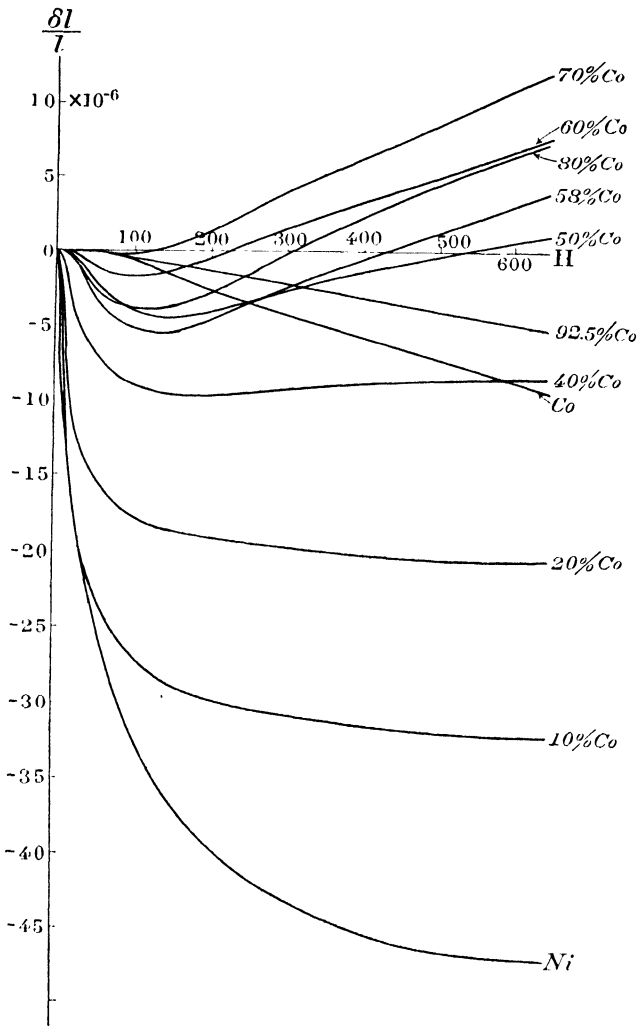


Fig. 127. Cobalt-Nickel (Masumoto).

38. Alloys of Manganese.

In 1892 Hogg[1] made an alloy consisting of 11 % Fe, 55 % Mn, 25 % Al, with a small amount of carbon, silicon, phosphorus and sulphur. He found that this alloy showed nearly as great an inten-

(1) Hogg, Chem. News, **66**, (1892), 140.

sity of magnetization as iron. In 1900, Heusler[1] made an alloy by mixing aluminium, tin, antimony or bismuth with a commercial bronze (30 % Mn, 70 % Cu), and found that notwithstanding that the component metals are all non-magnetic, the alloy is ferromagnetic. This discovery attracted the special attention of many physicists, since it is a remarkable fact which cannot be explained by the ordinary theory of molecular magnetism. Starck, Haupt, Richarz, Take[2], and others of the University of Marburg in Germany, at once began to investigate this kind of alloys. Their results show that if manganese, copper and aluminium are contained in the proportion of one atomic weight each, the alloy shows a strong magnetization. For instance, the intensity of magnetization is 430 in a field of 100 gausses. Quite recently T. Ishiwara[3] made a similar investigation and found that the strongest magnetic alloy contains these elements in the atomic proportion of about Mn:Cu:Al=3:1:3, and that the intensity of magnetization of this alloy is about 350 in a field of 550 gausses and its critical point is 170°.

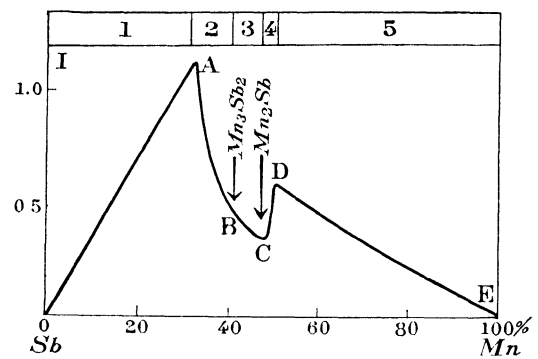
Heusler's alloys show a complicated phenomenon of magnetic hysteresis with regard to temperature, their critical points lying in a range from 660° to 350° according to their composition. Austin[4] and Grondahl[5] investigated the magnetostriction of these alloys, and found that their volume and length expand on magnetization.

Guillaume explains the magnetic property of Heusler's alloy in the following way:—Manganese is similar to iron in its properties and belongs to the ferromagnetic substances, its critical temperature being very low, so that it loses its magnetic property at ordinary temperature. Alloying other elements, such as copper or aluminium, with manganese raises the critical temperature of the alloy so that it becomes magnetic at room temperature. This explanation does not, however, agree with the facts. For then there must be a close relation between the ferromagnetic property of the alloy and the solid solution of the components; but this is not the case. On the contrary, it is evident from the investigation of Richarz and other physicists that an intimate relation exists between the ferro-

(1) F. Heusler, Verhandl. d. Deuts. Phys. Gesell. **5**, (1903), 219; Zeits. angew. Chem., **17**, (1904), 260. (2) Starck and Haupt, Verhandl. d. Deuts. Phys. Gesell. **5**, (1903), 224; Heusler, Richarz, Starck and Haupt, Marburger Schriften **13**, (1904), 234; Take, Marburger Schriften **14**, (1905), 35, Richarz, Marburger Schriften **19**, (1910) 67. (3) Kinzoku no Kenkyu, **3**, (1926), 13. (4) L. Austin, Verhandl. d. Deuts. Phys. Gesell. **6**, (1904), 211. (5) L. O. Grondahl, Phys. Rev. **4**, (1914), 325.

magnetic property of the alloys and the concentration of the compounds. Hence, the ferromagnetic property of the alloys, consisting of non-magnetic elements, owes its origin to the compound formation.

The results of magnetization with the alloys of manganese and antimony investigated by the author[1] are graphically shown in Fig. 128. It is seen that the variation of magnetization with the concentration completely agrees with the structure of these alloys. In the uppermost portion of the figure, the structure of the antimony-manganese system as investigated by Williams[2] is given, the concentrations of 40.7 and 47.8 percent of antimony corresponding to ferromagnetic compounds Mn_3Sb_2 and Mn_2Sb respectively. The alloys of intermediate concentration consist of a mixture of these two compounds, and their intensity of magnetization varies linearly. In the alloys of the ranges AB and CD, the structure consists of unsaturated solid solutions of Sb to Mn_3Sb_2, and Mn to Mn_2Sb respectively, and hence their magnetization will vary from B to A or C to D and not in a linear way. In the alloys of the ranges OA and DE, the structure consists each of a mixture of antimony and the antimony-saturated solid solution and that of manganese and the manganese-saturated solid solution, respectively, and hence the magnetization in these ranges ought to vary linearly. These conclusions agree completely with the results of experiments, as shown in Fig. 128. The critical temperature of the compound Mn_3Sb_2 is 315° and that of Mn_2Sb 270°.



1. *Sb*+Saturated solid solution.
2. Unsaturated solid solution.
3. Two compounds.
4. Unsaturated solid solution.
5. *Mn*+Saturated solid solution.

Fig. 128. Antimony-Manganese (Honda).

Alloys of manganese and tin[3], and those of manganese and bismuth[4] are also ferromagnetic, but their magnetic property is

(1) K. Honda, Ann. der Phys. **32**, (1910), 1003. (2) R. S. Williams, Zeits. anorg. Chem., **55**, (1907), 1. (3) K. Honda, Ann. Phys. **32**, (1910), 1003; R. S. Williams, Zeits. anorg. Chem., **55**, (1907), 1. (4) Hilpert und Dieckmann; Ber. der Deuts. Chem. Gesell. **44**, (1911), 2831.

considerably weaker as compared with the above-mentioned alloys. The critical temperature of Mn_4Sn is 117° and that of $MnBi$ 370°.

39. Ferromagnetic Compounds.

According to G. Tammann's investigation[1], a compound consisting of a ferromagnetic element and a non-magnetic one is generally non-magnetic. But, to this rule there are exceptions, examples being Fe_3O_4 and Fe_nS_m[2], etc. Hilpert[3] concluded from his investigation that the formation of a ferromagnetic compound by the combination of a ferromagnetic element with a non-magnetic one is confined to the case, in which the compound has such a molecular constitution as $MeO.Fe_2O_3$, where *Me* denotes a metallic element. For instance, magnetite Fe_3O_4 can be written $FeO.Fe_2O_3$. Compounds which have *K*, *Na*, *Ca*, *Mg*, *Zn*, *Pb*, etc. as metallic elements are each ferromagnetic. These compounds are not ferromagnetic when they have just been separated from a solution; but, if they are once heated to a high temperature and cooled, they become ferromagnetic. But the compounds $CoO.Fe_2O_3$ and $CuO.Fe_2O_3$ are ferromagnetic even when just separated from solutions. The critical temperature is 280° for the former and 270° for the latter. Besides, the compounds $Co_2O_3.2Fe_2O_3$, *NiB*, *CoB*, *FeB*, etc. are ferromagnetic.

As regards the magnetic properties of the amalgams of iron and cobalt, we have an investigation by H. Nagaoka[4]. Wünsche[5] also investigated nickel amalgam. From their results we see that amalgams of iron and cobalt in a field of 3200 gausses have specific magnetizations of 180 and 112 respectively, these values being nearly equal to those for pure iron and pure cobalt. Hence it is to be concluded that in these amalgams, iron and cobalt are suspended as fine particles. It is characteristic of these amalgams that their coercive forces are remarkably large. On the other hand, the specific susceptibility of nickel is only 49.3×10^{-6}; hence the nickel in the amalgam is either present as a solution or as a compound.

Lastly, we shall give some instances of ferromagnetic compounds consisting of paramagnetic elements. In 1859, Wöhler[6] discovered that an oxide Cr_5O_9 is ferromagnetic and easily attracted by a magnet. The molecular formula of this compound can be written

(1) G. Tammann, Zeits. Phys. Chem. **65**, (1909), 73. (2) Pyrrhotite. (3) Hilpert, Verhandl. der Deuts. Phys. Gesell. **11**, (1909), 293. (4) H. Nagaoka, Wied. Ann. **59**, (1896), 66. (5) Wünsche, Ann. der Phys. **7**, (1902), 116. (6) F. Wöhler, Lieb. Ann. **111**, (1859), 117.

$2Cr_2O_3.CrO_3$. Two years later, Geuther[1] investigated this compound and confirmed the fact of its being ferromagnetic. Shukow[2] further investigated it and showed that the most strongly ferromagnetic compound is Cr_4O_9 and its specific susceptibility is 6500×10^{-6}. According to T. Soné and T. Ishiwara's investigations[3], the results obtained by Shukow are not confirmed, but the most strongly ferromagnetic compound is found to be Cr_5O_9. They showed that the intensity of its specific magnetization reaches 20 in a field of 360 gausses, and also that its critical point is 150°.

The compounds[4] MnB and Mn_5P_2 are both ferromagnetic, and the magnetization of Mn_5P_2 is weaker than that of MnB. The critical point of Mn_5P_2 is 24°, and hence its magnetization may be made to vanish by warming it with the hands. Besides, the compounds[5] Mn_2As and $MnAs$ are also ferromagnetic, and the critical point of the latter is from 40° to 50°. According to the investigation by Wedekind[6] and by Ishiwara[7], a compound of manganese and nitrogen, Mn_5N_2, is ferromagnetic and has an intensity of specific magnetization of 4.4 in a field of 294 gausses, and its critical point is 500°.

Manganese is often reported to have a large susceptibility of an order of 500×10^{-6}, but this is obviously due to the presence of a small amount of the above compound mixed in the metallic manganese introduced during its preparation.

The above examples afford us important data for the framing of a theory concerning the nature of magnetization, which will be described in the ninth chapter.

(1) Geuther, Lieb. Ann. **118**, (1861), 61. (2) J. Shukow, C. R. **146**, (1908), 1396. (3) T. Soné and T. Ishiwara, Sci. Rep. **3**, (1914), 271. (4) Binet du Jassoneix, C. R. **139**, (1904), 1209; C. R. **142**, (1906), 1336; Wedekind, Ber. der Deuts. Chem. Gesell. **38**, (1905), 1228; **40**, (1907), 1259, 3851. (5) Wedekind und Veit, Ber. der Deuts. Chem. Gesell. **44**, (1911), 2663. Hilpert und Th. Dieckmann, ibid. **44**, (1911), 2378. (6) Wedekind, Zeits. Phys. Chem. **66**, (1909), 614. (7) T. Ishiwara, Sci. Rep. **5**, (1916), 53.

CHAPTER VII.

Magnetization of Paramagnetic and Diamagnetic Substances.

40. Methods of Observation.

(a) *Faraday's Method.* When a small substance is placed in a heterogeneous magnetic field, it is, if diamagnetic, displaced along the gradient from the strong field towards the weak. But, if the substance is paramagnetic, the contrary is the case. Hence, if we measure the force acting on the substance, we can obtain its magnetic susceptibility in the following way:—

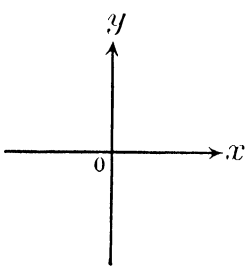


Fig. 129.

Let us suppose that the magnetic field H is uniform in the direction of x, but varies in the direction of y. Then under the action of the magnetic field, a small substance placed at the origin o tends to move in the direction of the y-axis. If we denote the force acting on the substance by f, the work done by the magnetic field during the displacement δy is equal to $f\,\delta y$. If the volume of the substance be v and the intensity of magnetization I, the magnetic energy W[1] is equal to

$$W = -\frac{1}{2} IHv = -\frac{1}{2} \kappa H^2 v,$$

where κ is the magnetic susceptibility.

Since $f\delta y$ is equal to the decrease of the magnetic energy, we have

$$f\delta y = -dW = \kappa v H \frac{\partial H}{\partial y} \delta y.$$

$$\therefore \quad f = \kappa v H \frac{\partial H}{\partial y},$$

(1) Maxwell's Electricity and Magnetism **2**, third edition 74.

whence $$\kappa = \frac{f}{vH\frac{\partial H}{\partial y}},$$

or $$\chi = \frac{f}{mH\frac{\partial H}{\partial y}},$$

where m is the mass of the substance and is equal to ρv. Hence χ or κ can be obtained by measuring H, $\partial H/\partial y$ and f acting on the substance.

A torsion balance[1] is usually employed for the measurement of the force f. In Fig. 130, s is a substance to be tested and is placed at a point midway between the pole-pieces of an electromagnet, but deviating a little from the straight line joining their centres. The position of the substance can be read on a scale by means of an index attached to the other end a of the beam of the torsion balance. If an electric current is passed through the electromagnet to excite a magnetic field, the substance will be displaced along the gradient of the magnetic field or in the opposite direction, the wire suspending the beam being thus twisted. Then by twisting the torsion head so much in the opposite direction that the beam takes its original position, the force f is just balanced by the twisting couple of the suspending wire, and hence its value can be found from the angle of twist of the wire.

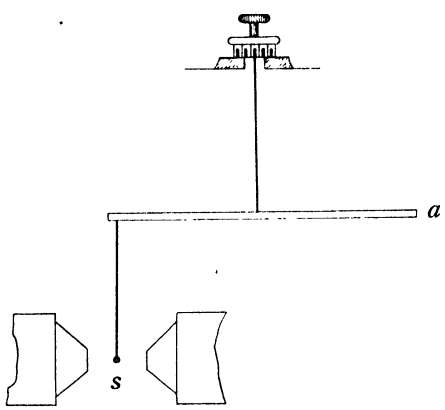


Fig. 130.

In order to find the magnetic field and its gradient, an exploring coil with its plane perpendicular to the magnetic induction is placed in various positions in the magnetic field. Then, it is suddenly removed from the field, and the quantity of electricity thus induced in the circuit of the coil is measured by a ballistic galvanometer. In this way, the intensity of the magnetic field at various points

(1) Sci. Rep. **1**, (1911), 1; Ann. d. Phys. **32**, (1910), 1027.

can be measured, and hence its gradient can also be found from these values of the field. The value of $\partial H/\partial y$ is zero in the straight line joining the centres of the pole-pieces, increases with the distance from the line, and then decreases after reaching a maximum. It is convenient to place the substance to be tested at the point, where the value of $H\partial H/\partial y$ is a maximum; for, in this position, there will be a minimum of error due to the small difference in the position of the substance.

If the susceptibility at high temperatures is to be measured, an electric furnace as previously described, is placed vertically between the pole-pieces of the electromagnet.

(b) *Weiss' Method.* Instead of a torsion balance, P. Weiss makes use of the mutual action of two horizontal coils c_1 c_2 as shown

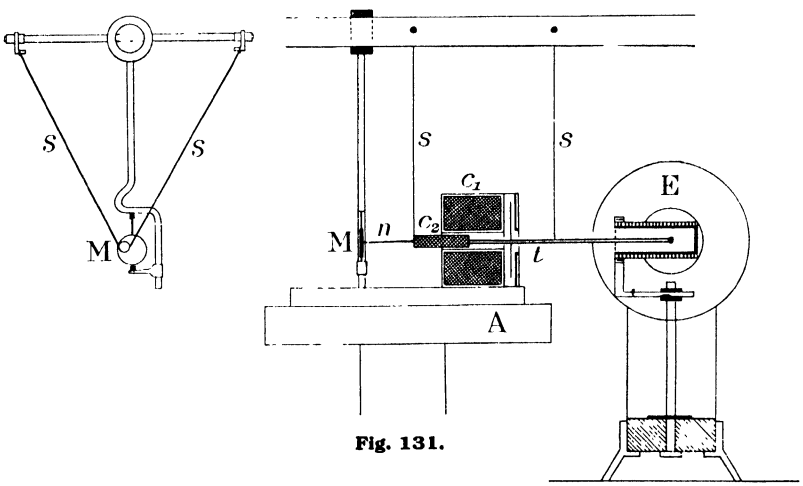


Fig. 131.

in Fig. 131, one of which is wound on a movable silica tube *t* horizontally suspended by two pairs of strings *ss*, while the other is placed co-axially with the first coil, but quite free from it, and is fixed to a stout adjustable stand A. One end of the tube has a small vessel containing the test-specimen, also made of silica, while a magnetized steel needle *n* is fixed to the other end. Near the free end of the needle, a small mirror M is pivoted and can freely be rotated about a vertical axis. On the back of this mirror, there is a small iron disc having a small shallow conical hole to receive the pointed end of the needle. The silica tube *t* being in its natural position of equilibrium, the stand of the mirror is so adjusted that the pointed end of the needle is just in magnetic contact with

the hole in the iron disc, and the plane of the mirror is perpendicular to the silica tube. In this way a good and inseparable contact between the needle and the iron disc is obtained; this magnetic contact, a device of the author's, has been proved to work excellently.

The specimen is placed in the proper position between the pole-pieces of an electromagnet, and a constant current is passed through the fixed horizontal coil c_1. When the electromagnet is excited, the specimen is slightly displaced and causes a rotation of the mirror, which may be read on a scale with a telescope. A suitable current is then sent through the movable coil c_2 in such a direction that the specimen is brought exactly back to its original position. This current i is then proportional to the magnetic force f acting on the specimen. In a given exciting field, κ or χ is proportional to f or to i, and the constant of proportionality is conveniently determined from an experiment with a substance of known susceptibility. Thus κ or χ can be found from the values of this constant and i.

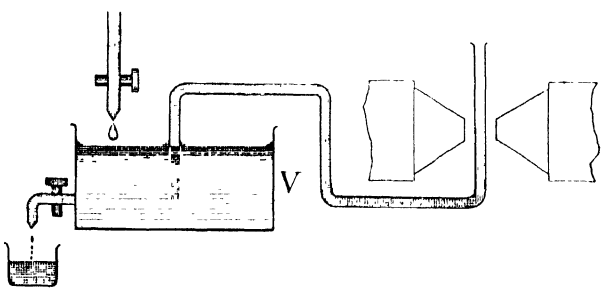


Fig. 132.

(c) *Hydrostatic Pressure Method.* The magnetic susceptibility of liquids is usually measured by the hydrostatic pressure method. The liquid to be tested is contained in a vessel V and led by means of a capillary tube to a point midway between the pole-pieces of an electromagnet as shown in Fig. 132, the meniscus of the liquid lying on the line joining the centres of the pole-pieces. When the magnetic field is excited, the meniscus of the liquid in the capillary tube rises in the case of a paramagnetic liquid, and falls in the case of a diamagnetic liquid. By observing the amount of rise or fall of the meniscus with a microscope, the susceptibility of the liquid can be found from the following calculation:—

Let the rise or fall of the meniscus due to the magnetic field be a, the density of the liquid ρ, the force of gravity g,

and the sectional area of the tube S. Then the force acting on the liquid is

$$f = aS\rho g.$$

Suppose a virtual displacement ∂a be given to the meniscus, then the work done is $f\partial a$. But the change in volume of the liquid column being $S\partial a$, the change of the magnetic energy is

$$dW = -\frac{1}{2}\kappa H^2 S\partial a.$$

Consequently,

$$f\partial a = \frac{1}{2}\kappa H^2 S\partial a,$$

or

$$f = \frac{1}{2}\kappa H^2 S = aS\rho g,$$

whence

$$\kappa = \frac{2a\rho g}{H^2},$$

or

$$\chi = \frac{2ag}{H^2}.$$

Hence from the measurement of a, we can find the value of or χ. The value of a can be found from the difference in the positions of the meniscus in the capillary tube with and without the magnetic field. For an accurate measurement, it is convenient to find the value of a from the pressure required to bring back the meniscus to its original position. For instance, in the case of a diamagnetic substance such as water, the meniscus in the capillary tube falls as the result of the action of the magnetic field. From a burette filled with water and placed above the vessel V, the water is made to fall into the vessel drop by drop, until the meniscus returns to its original position; then the value of a can be found from the mass of the water poured in, and the section of the vessel. In the case of a paramagnetic liquid, the meniscus rises, and hence, in order to bring back the meniscus to its original position, a certain quantity of water must be removed from the vessel and this quantity of water gives the value of a, as referred to above.

(d) *Cylinder Method.* This method is also applied to the measurement of the susceptibility of liquids. As shown in Fig. 133, a cylindrical glass tube is half filled with a liquid and suspended between the pole-pieces of an electromagnet, the height of the tube being so adjusted that the meniscus of the liquid lies on the line joining the centres of the pole-pieces. If the electromagnet is excit-

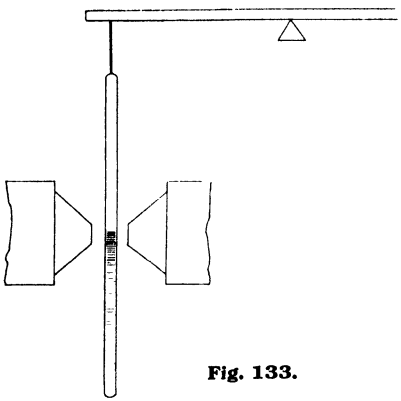


Fig. 133.

ed, the liquid meniscus will be affected by the magntic field, causing an inclination of the balance-beam. Consequently, the weight required to bring the beam back to the horizontal position is equal to the magnetic force f, and the susceptibility of the liquid can be found from the following formula deduced above:—

$$\kappa = \frac{2f}{SH^2} \quad \text{or} \quad \chi = \frac{2f}{\rho SH^2}.$$

41. Magnetic Susceptibility of Elements.

There has been a number of investigations concerning the magnetic susceptibility of elements; but the results of different investigators do not generally coincide with each other, and in some cases, the divergence is so great that for the same element, the susceptibility, as given by different observers, not only differs in numerical value, but also in sign. This discrepancy is caused by the impurities contained in the elements, among which iron has the most striking effect. Iron is a substance which has the widest distribution in nature, and a trace of it is liable to be found in any substance. Since iron is strongly magnetic, its presence in an element, even in an extremely small amount, greatly affects the apparent susceptibility of that element, especially when iron is present in the free state. We have three methods for detecting iron present in an element as an impurity. The first is the usual method of chemical analysis, by which the total content of iron can be found, but there is no telling whether it is contained as a compound or in the free state. Hence, by this method we cannot decide how much the iron present as an impurity actually affects the susceptibility of the element to be tested. The second method of detection consists in the measurement of susceptibility in different fields. If a substance does not contain iron at all, its susceptibility will be independent of the strength of the magnetic field, but if it does contain iron, the susceptibility varies with the strength of the magnetic field. Consequently, by measuring the susceptibility in different fields, we may judge whether a substance contains iron or not. The third method consists in the measurement of the susceptibility at different high

temperatures. If the substance contains free iron as an impurity, the susceptibility-temperature curve is concave towards the temperature axis and rapidly falls in the vicinity of the critical point of iron; otherwise the curve falls, rapidly at first and then gradually, with the rise of temperature. Hence from the form of the curve, we may conclude the presence or absence of free iron.

In 1909, the author[1] measured the magnetic susceptibility of different elements, 43 in number, at temperatures ranging from room temperature up to 1000°. Afterwards Owen[2] measured the magnetic susceptibility of these elements at different low temperatures down to that of liquid air. In addition, he also measured the susceptibility of fifteen other elements. In these measurements, special attention was paid to the effect of iron or other ferromagnetic substances, which may be contained as impurities.

Atomic number		Atomic weight	$\chi \times 10^6$	Atomic number		Atomic weight	$\chi \times 10^6$
1	H	1.008	−1.98	40	Zr	91.2	−0.45
2	He	4.00	−0.47	41	Nb	93.5	1.29
3	Li	6.94	0.50	42	Mo	96.0	0.04
4	Be	9.02	−1.03	44	Ru	101.7	0.43
5	B	10.82	−0.71	45	Rh	102.9	1.10
6	C[3]	12.00	−0.49	46	Pd	106.7	5.20
7	N	14.008	−0.26	47	Ag	107.88	−0.20
8	O	16.00	104.1	48	Cd	112.4	−0.18
10	Ne	20.2	−0.33	49	In	114.8	−0.11
11	Na	23.00	0.51	50	Sn[4]	118.7	0.027
12	Mg	24.32	0.55		Sn[5]		−0.35
13	Al	26.97	0.58	51	Sb	121.8	−0.82
14	Si	28.06	0.13	53	I	126.92	−0.36
15	P	31.04	−0.90	54	Xe	—	—
16	S[3]	32.07	−0.48	55	Cs	132.8	−0.10
17	Cl	35.46	−0.57	56	Ba	137.4	0.93
18	Ar	39.88	−0.45	57	La	138.9	1.04
19	K	39.10	0.40	58	Ce	140.2	15.3
20	Ca	40.07	1.10	59	Pr	140.9	25.2
22	Ti	48.1	1.25	60	Nd	144.3	36.2
23	V	51.0	1.50	68	Er	167.7	22.3
24	Cr	52.01	3.75	73	Ta	181.5	0.87
25	Mn	54.93	9.66	74	W	184.0	0.22
29	Cu	63.57	−0.086	76	Os	190.9	0.039
30	Zn	65.37	−0.016	77	Ir	193.1	0.13
31	Ga	69.72	−0.24	78	Pt	195.2	1.10
32	Ge	72.60	−0.12	79	Au	197.2	−0.15
33	As	74.96	−0.31	80	Hg	200.6	−0.19
34	Se	79.2	−0.32	81	Tl	204.4	−0.24
35	Br	79.92	−0.39	82	Pb	207.2	−0.12
37	Rb	85.5	0.09	83	Bi	209.0	−1.39
38	Sr	87.6	−0.24	90	Th	232.1	0.08
39	Y	89.0	5.30	92	U	238.2	2.60

(1) K. Honda, Ann. der Phys. **32**, (1910), 1027; Sci. Rep. **1**, (1912), 1. (2) M. Owen, Ann. der Phys. **37**, (1912), 657. (2) Diamond. (3) Rhombic sulphur. (4) White tin. (5) Grey tin.

The foregoing table contains the result of accurate measurements of the specific susceptibilities of different elements at 18°—20° made by the author, M. Owen, T. Soné[1], A. P. Wills and L. G. Hector[2], and others.

The above values of the susceptibilities are plotted in Fig. 134 against the atomic number of the elements. In the curve we notice that the magnetic susceptibility of different elements varies periodically with the increase of the atomic number, and the relation between them has a close connection with Mendeleff's periodic system.

At first sight the curve seems to be a very complicated one, but a closer examination reveals the fact that there is a regular periodic variation. Thus, in Fig. 134, A_1, A_2, A_3; B_1, B_2, B_3; C_1, C_2, C_3; D_1, D_2, D_3; etc. are corresponding points in the periodic

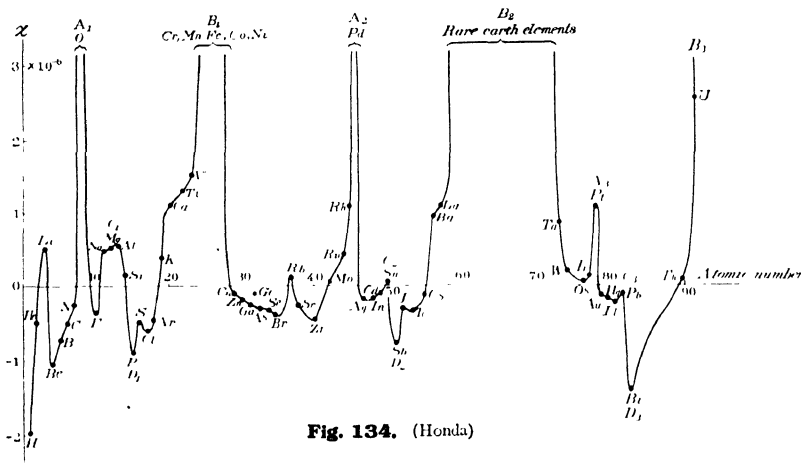


Fig. 134. (Honda)

variation, the whole curve having three great periods. We also see that elements which belong to the same group in the periodic system and have similar properties, are found in a similar position on the curve. For example, phosphorus, antimony and bismuth of the fifth group are all found at minimum points on the curve; ruthenium, rhodium, palladium, and osmium, iridium, platinum of the eighth group are all situated in similar positions. Formerly, the relation between the magnetic susceptibilities of elements and their atomic weights was quali-

(1), (2) See the next section.

tatively considered, i.e. only in connection with their sign, and hence oxygen, being a strong paramagnetic element and situated amidst diamagnetic elements in the periodic system, has been considered to show a remarkable abnormality. If, however, the susceptibility is considered quantitatively in relation to the atomic number, this element, is in no way an exception to the law of periodic variation.

It is worthy of notice that the susceptibility-atomic number curve crosses the zero line quite continuously, as if the paramagnetic and diamagnetic properties were merely relative, but not essentially different in their nature.

42. Effect of Temperature on Magnetic Susceptibility.

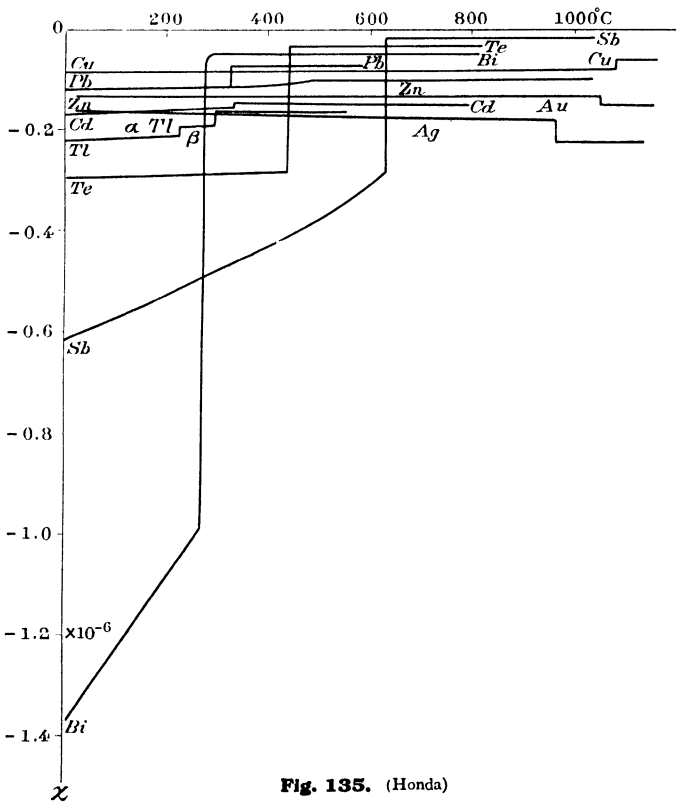


Fig. 135. (Honda)

The effect of high and low temperatures on the magnetic susceptibilities of elements has been investigated by P. Curie, the

present author, M. Owen and H. Endo[1]. Some of the results of investigation are graphically given in Figs. 135—137. Based on his investigation with several elements and compounds, P. Curie deduced two laws known by his name; that is,

(i) The magnetic susceptibility of diamagnetic substances is independent of the temperature and the state of the substance.

(ii) The magnetic susceptibility of a paramagnetic substance is inversely proportional to the absolute temperature.

Strictly speaking, the first law is only applicable to a certain number of cases and fails in most cases as regards diamagnetic substances. This will also be seen in Fig. 135. According to the investigations of Owen and of the author, the susceptibility of

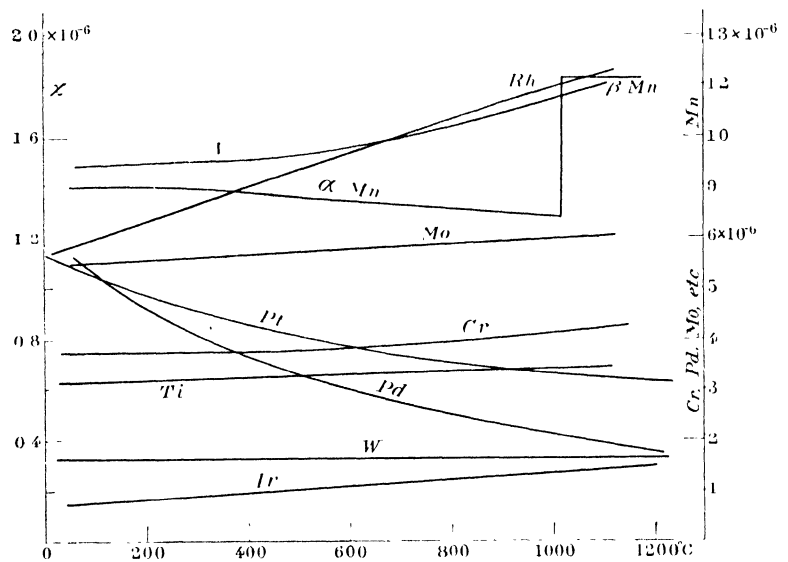


Fig. 136. (Honda)

diamagnetic substances generally decreases with the rise of temperature, graphite, cadmium, antimony, tellurium, bismuth and others being good examples. Gallium, germanium, silver, tin, antimony, gold, mercury, bismuth and others undergo an abrupt decrease of susceptibility during fusion. Carbon, sulphur and thallium change their susceptibility as the result of polymorphic transformation.

Generally speaking, the susceptibility of solid diamagnetic elements decreases with the rise of temperature and makes an abrupt change on being subjected to polymorphic transformation; it

(1) Sci. Rep., **16**, (1927), 201.

also suffers an abrupt diminution during fusion. In the liquid state, the susceptibility of diamagnetic elements is independent of temperature.

As to the second law, it is to be concluded that within a wide range of temperature extending over 800°, almost none of the paramagnetic elements exactly follows this law, though, for many elements, it seems to be applicable in a narrow range of temperature viz. 300°—500°, as in the case of Curie's experiments. For example, as we see in Fig. 137, paramagnetic elements such as cerium, neodymium, praseodymium, erbium and others follow fairly well Curie's second law in a range from room temperature down to

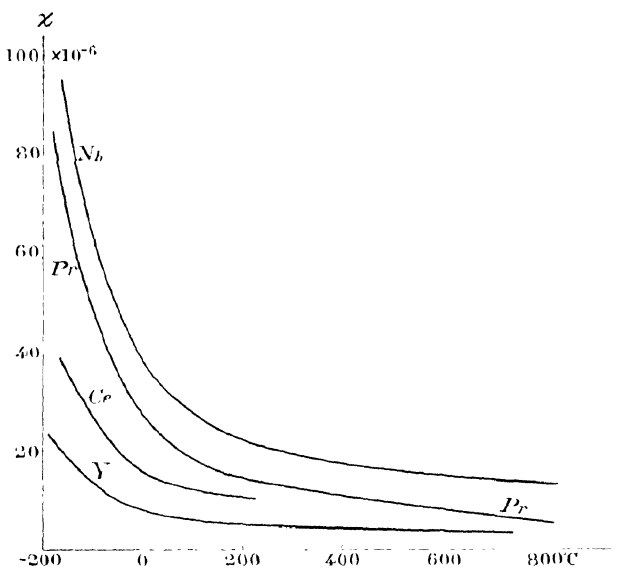


Fig. 137. (Owen)

that of liquid air, but fail to do so at higher temperatures. The susceptibility of many paramagnetic elements decreases with a rise of temperature, but there are others, the susceptibility of which is independent of it or increases with the rise of temperature, as shown in Fig. 136. In fact, Curie's two laws are only applicable to a small number of elements.

The above complicated variations of susceptibility may be expressed with reference to the susceptibility-temperature curve. Thus the change of magnetic susceptibility in both the paramagnetic and diamagnetic elements, takes place in such a direction that with a rise of temperature, their atomic weight apparently increases. For

example, the susceptibility of the elements situated at the maximum and minimum points of the curve, such as bismuth, antimony, platinum, palladium, etc., numerically decreases as the temperature rises, and that of the elements situated at the ascending portion of the curve, such as rhodium, ruthenium, iridium, etc., numerically increases with a rise of temperature. Those elements, which lie in the portion of the very flat minima in the curve, suffer little change of susceptibility as the effect of temperature.

Considering the thermomagnetic properties of the elements, it is very important to observe that, as shown in Fig. 138, at a very

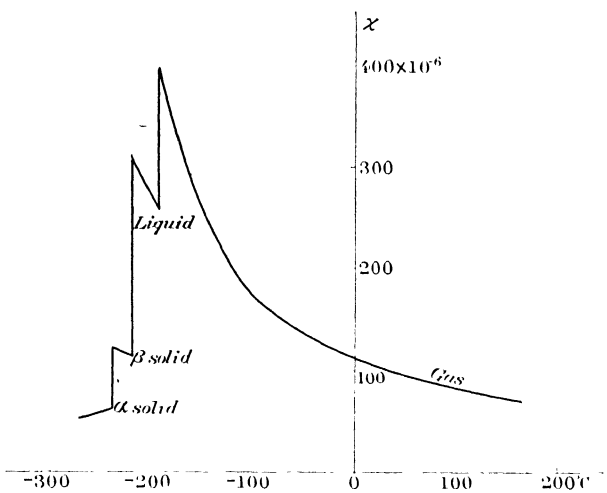


Fig. 138.

Oxygen (Onnes, Perrier)

low temperature, solid oxygen[1] has two modifications α and β, its transformation point being -240°, that the β modification, stable at higher temperatures, is more magnetic than the α modification, stable at lower temperatures, and that liquid oxygen is more magnetic than solid oxygen and gaseous oxygen still more magnetic than liquid oxygen. The curve belonging to the gaseous oxygen is the one calculated from the result obtained by T. Soné, assuming Curie's law, and hence its course may differ from the actual curve to some extent.

The magnetic susceptibility of tin[2] changes its sign twice as the temperature rises. Tin has two modifications, that is, ordinary

(1) A. Perrier and K. Onnes, Proc. Akad. Amsterdam, **16**, (1914), 894. (2) K. Honda, Sci. Rep. **1**, loc. cit.

white tin and grey tin. Grey tin is obtained when white tin is exposed to very low temperatures for a long period of time; it changes into ordinary tin at 18°, its volume thereby decreasing by 20 %. The grey tin, which is stable below 18°, is a strong diamagnetic substance. On slow heating, it changes into ordinary tin at 18°, and its susceptibility abruptly increases so as to become paramagnetic, afterwards remaining constant up to 232°, at which point it melts, to become diamagentic again. This very interesting phenomenon suggests that paramagnetism and diamagnetism may have some connection with each other.

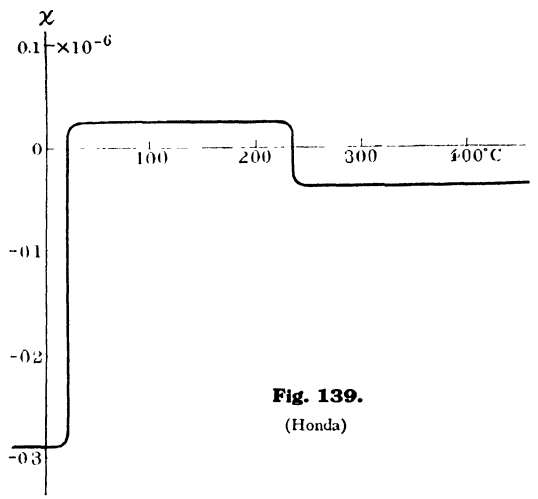


Fig. 139.
(Honda)

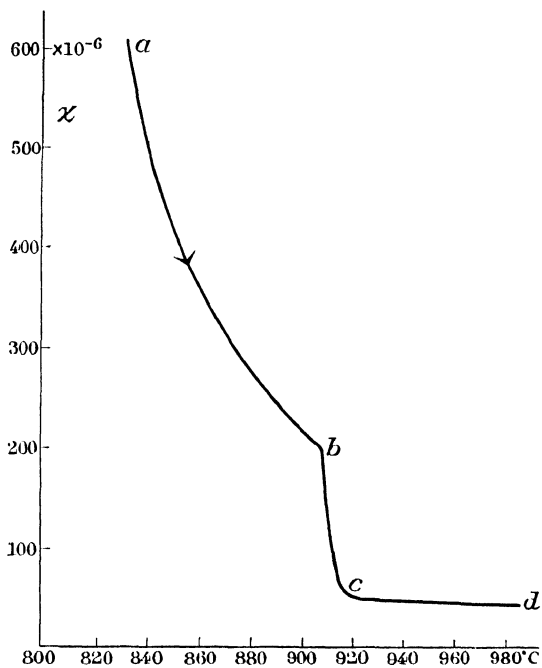


Fig. 140.
(Honda, Takagi)

It is also to be remarked that manganese has a transformation point at 1020°, at which the magnetic susceptibility changes abruptly.

We shall lastly consider the change of magnetic suscep-

tibility in iron, nickel and cobalt above their critical points. The specimens of iron examined by Curie[1], Weiss[2] and Terry[3] contained some carbon, all being low carbon steels. H. Takagi and the author[4] obtained pure iron from the Bureau of Standards, Washington, and investigated the relation between the susceptibility and the temperature with the result shown in Fig. 140. Above the critical point, as the temperature rises, the susceptibility first decreases rapidly, shows an abrupt diminution at the A_3 transformation point, that is at 910°, and afterwards only slightly decreases.

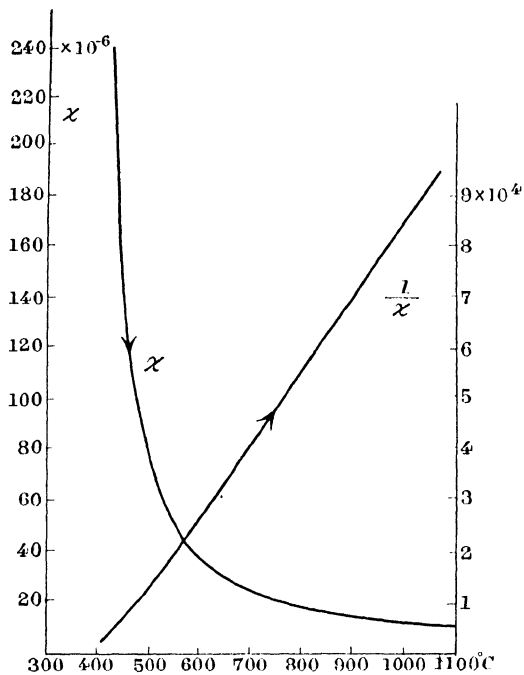


Fig. 141.

Nickel (Honda, Takagi)

As regards nickel and cobalt, there are the investigations made by Weiss[5], Bloch[6], Takagi, the author[7] and E. M. Terry[8]. The curves in Figs. 141 and 142 show the results obtained by Takagi and the author.

P. Weiss adopts the following formula to express the relation between the susceptibility χ and the temperature T above the critical point θ:—

$$\chi\,(T - \theta) = \text{const.}$$

As will be described later, P. Weiss applied this formula to his magneton theory and calculated the number of magnetons con-

(1) P. Curie, Oeuvres, loc. cit. (2) P. Weiss et G. Foëx, Arch. des Sci. **31**, (1911), 89. (3) E. M. Terry, Phys. Rev. **9**, (1917), 394. (4) K. Honda and H. Takagi, Sci. Rep. **4**, (1915), 261. (5) Weiss et Foëx, Arch. des Sci. **31**, loc. cit. (6) O. Bloch, Arch. des Sci. **33**, (1912), 293. (7) K. Honda and H. Takagi, Sci. Rep. **1**, (1912), 229. (8) E. M. Terry, Phys. Rev., loc. cit.

tained in one molecule; but this law being only approximately valid, his conclusion with regard to the number of magnetons is not quite correct. This formula applies fairly well in the case of nickel, except in the vicinity of the critical point; but in the case of cobalt, it is not applicable. As we have already explained, iron possesses two transformations A_3 and A_4 above the critical point, and the above-mentioned formula cannot of course be applied in the transformation ranges. Weiss considers the susceptibility-temperature curve in iron to consist of two portions corresponding to different phases, named by him β_1 and β_2 (*ab* and *bc* in Fig. 140); but β_2 being a transformation range of the A_3 change, Weiss' view cannot be correct.

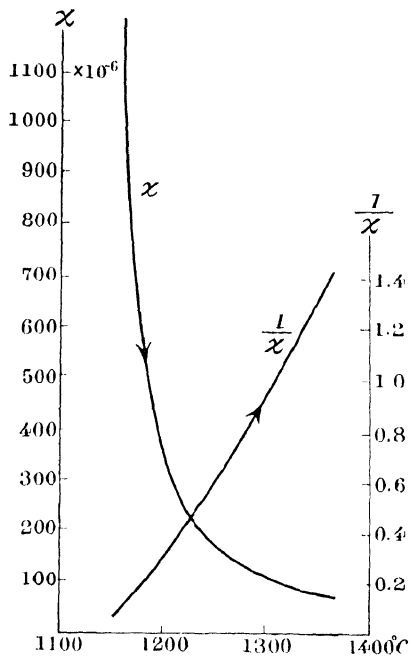


Fig. 142.
Cobalt (Honda, Takagi).

43. Magnetic Susceptibility of Different Gases.

The magnetic susceptibility of gases was formerly measured by the hydrostatic method[1], the upper part of the tube above the meniscus of a liquid of a known susceptibility being filled with the gas to be tested. Since the density of a gas is generally very small, this method of measuring the susceptibility cannot give a very accurate result, unless its specific susceptibility is very large. T. Soné[2] improved the cylinder method and made an accurate determination of the susceptibilities of different gases.

(1) H. du Bois, Wied. Ann. **35**, (1888), 137. R. Hennig, Wied. Ann. **50**, (1893), 485. A. Piccard, Arch. des Sci. **35**, (1913), 458. W. P. Roop, Phys. Rev. **7**, (1916), 529. E. Bauer, P. Weiss and A. Piccard, C. R. **167**, (1918), 484. (2) T. Soné, Sci. Rep. **8**, (1919), 115; **11**, (1922), 139.

The details of the magnetic balance used in his measurement are shown in Fig. 143. *AB* is the aluminium arm of the balance, *C* an agate knife-edge resting on a smooth plane of steel, and *HD* an aluminium pointer. *DE* shows the side view of the bifilar system consisting of two thin Wollaston wires, *M* is a mirror attached to the lower end of the bifilar system, and facing the plane of the figure at right angles, its deflection being observed on a scale with a telescope in the usual way. *F* is a copper vane-damper dipped in a vessel containing oil, and *I* is a pillar with a slide arm

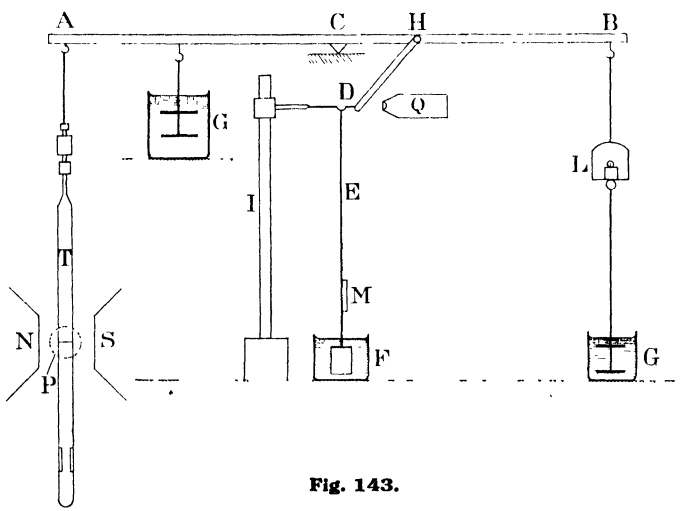


Fig. 143.

carrying the fixed suspension of the bifilar system. *T* is the measuring tube suspended between the pole-pieces of a Weiss electromagnet. *P* is a microscope with which the measuring tube can be adjusted to the correct position. *L* is a pan in which a balancing weight is to be placed, and *GG* metal dampers dipped in oil.

The measuring tube is made of glass and has a glass partition. The upper half of the tube is filled with the compressed gas under examination, and the air in its lower half is evacuated, the lower end of it being sealed. The tube is vertically suspended from the arm of the magnetic balance between the pole-pieces of the Weiss electromagnet, and the upper surface of the partition is placed on the axial line of the pole-pieces. The field is then applied and the corresponding inclination of the balance, which causes a deflection of the mirror in the bifilar suspension, is ob-

served on a scale with a telescope. Since the tube alone produces some deflection of the scale, it is necessary to eliminate the effect by making further two similar observations, firstly after evacuating the upper half, and secondly after filling it with distilled water.

The gas to be tested is compressed into the measuring tube with a Cailletet hydraulic compressor, its pressure being from 25 to 50 atmospheres. The susceptibilities of different gases at 20° and 760 mm, as determined by T. Soné, are given below:—

Gases	χ	κ
Air	23.85×10^{-6}	0.03084×10^{-6}
Oxygen, O_2	104.1	0.0488
Carbon dioxide, CO_2	−0.423	−0.000836
Nitrogen, N_2 (chemical)	−0.265	−0.000331
„ (atmospheric)	−0.360	−0.000452
Hydrogen, H_2	−1.982	−0.0001781
Nitric oxide, NO	48.80	—
Nitrous oxide, N_2O	−0.429	17° liquid and solid
Nitrogen peroxide, NO_2	4.54	
Nitrogen tetroxide, N_2O_4	−0.276	−16°, liquid
Nitrogen trixide, N_2O_3	−0.206	18°, liquid
Nitrogen pentoxide, N_2O_5	−0.332	16°, solid

Recently, A. P. Wills and L. G. Hector[1] used a modified form of the hydrostatic method and measured the magnetic susceptibilities of different gases. Their method consists in balancing magnetically the gas to be tested against an aqueous solution of nickel chloride. By varying the concentration of the solution its susceptibility is made roughly equal to that of the gas; then by varying the pressure of the gas, or the temperature of the gas and the solution, the two susceptibilities are made exactly equal to each other. From the susceptibility of the solution, that of the gas under a pressure of one atmosphere and at 20° is deduced. The following table contains the results of their observation, those of hydrogen, nitrogen and oxygen being here omitted:—

Gases	χ	κ
Argon	-0.452×10^{-6}	-0.000752×10^{-6}
Helium	−0.473	−0.0000780
Neon	−0.334	−0.000277

(1) Wills and Hector, Phys. Rev. **23**, (1924), 209; Hector, Phys. Rev. **24**, (1924), 418.

From the theoretical point of view, the above values for the susceptibility of the gases are very important; because in the case of gases, the molecules do not exert any mutual action upon each other, and hence their magnetization can be theoretically treated. That is, if we assume a model for a gas molecule, its susceptibility may be calculated from known data and compared with the observed value.

44. Magnetic Susceptibility of Compounds.

In determining the susceptibility of a substance, it is usual to make the relative measurement, water being taken as the standard substance and hence the susceptibility of water is a very important constant. The magnetic susceptibility of water is very accurately known, a number of reliable experiments having been made; the values are:—

Sève[1] (1912)	-0.720×10^{-6}	(24°)
de Haas and Drapier[2] (1912)	-0.721	(21°)
Weiss and Piccard[3] (1912)	-0.719	(26°)
Piccard and Devaud[4] (1920)	-0.719	(20°)

The mean of these values is -0.720×10^{-6}, and the probable error certainly does not exceed one half percent. Moreover, the susceptibility of water is almost independent of temperature.

(a) *Inorganic Compounds.* The important investigations concerning the magnetic susceptibilities of inorganic compounds at room temperature were made by Meyer[5], Feytis[6], Wedekind[7], Urbain, Jantsch[8], Soné, the author[9], Pascal[10], and others[11]. Generally speaking, the magnetic susceptibility of a compound is different from the sum of the susceptibilities of its components; in other words, the magnetic susceptibility is not generally subject to the law of addition. The compounds MgO and Al_2O_3

(1) Séve, Jour. de Phys. (5), **3**, (1913), 8. (2) de Haas and Drapier, Ann. der Phys. **42**, (1913), 673. (3) Weiss et Piccard, C. R. **155**, (1912), 1234. (4) Piccard and Devaud, Arch. des Sci. (5), **2**, (1920), 410. (5) St. Meyer, Sitz. Ber. der Wiener Akad. **107**, (1899), 108; (1901), 110; (1902), 111; Wied. Ann. **68**, (1899), 325. (6) E. Feytis, C. R. **152**, (1911), 708; **153**, (1911), 668; **156**, (1913), 886. (7) E. Wedekind, Magnetochemie, (1911). Ber. d. Deutsch. Chem. Gesell. **46**. (1913), 3763; **45**, (1912), 262; **48**, (1915), 105; **54**, (1921), 253; Zeits. angew. Chem. **37**, (1924), 87. (8) B. Urbain et G. Jantsch, C. R. **147**, (1908), 1286. (9) K. Honda and T. Soné, Sci. Rep. **2**, (1913), 1. (10) P. Pascal, C. R. **156**, (1913), 323, 1906; **158**, (1914), 37; **173**, (1921), 144, 712; **174**, (1922), 457, 1698; **175**, (1922), 1063. (11) R. H. Weber, Jahrbuch der Radio. u. Electronick **12**, (1915), 74.

are remarkable examples, both being diamagnetic, while each element constituting the compounds, such as manganese, oxygen and aluminium, is paramagnetic. On the other hand, the compounds $CuBr_2$ and $CuCl_2$ are both strongly paramagnetic, while their component elements, copper, bromine and chlorine are all diamagnetic. But it is also to be noted that the additive law holds good for certain groups of paramagnetic or diamagnetic compounds of similar constitution as found by P. Pascal.

The magnetic susceptibility of inorganic compounds at different temperatures from ordinary temperatures down to the low temperature of liquid air or liquid hydrogen was investigated by Kamerlingh Onnes and his co-workers[1], and also by T. Ishiwara[2]. According to their results a small number of the compounds obey Curie's law, but a large number of them follow a more extended law:

$$\chi\,(T+\Delta) = \text{const.},$$

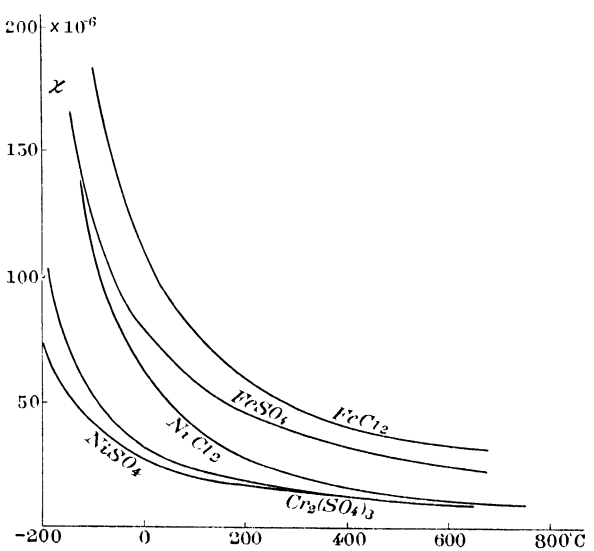


Fig. 144.
(Ishiwara, Honda)

where Δ is a positive or negative constant. Since it was thought necessary to ascertain definitely whether the above law is applicable to a wider range of temperatures, the author[3] in co-operation with Soné and Ishiwara made further the measurements of the susceptibility of these compounds at high and low temperatures and found

(1) K. Onnes, A. Perrier and E. Oosterhuis, Leiden Comm. No. 116, 122a, 124a, 129b 132e, 134d, 139, 140. Jackson, Dissert., Leiden, (1923), 47; Jackson and Onnes, C. R. **177**, (1923), 154; Proc. Roy. Soc. London, **104** A, (1923), 671. (2) T. Ishiwara, Sci. Rep. **3**, (1914), 303. (3) K. Honda and T. Soné, Sci. Rep. **3**, (1914), 139. K. Honda and T. Ishiwara, Sci. Rep. **4**, (1915), 215.

that in many cases the above law holds good satisfactorily (Fig. 144). We have, however, found some cases in which this law is valid in a range of temperature, say 200°, below room temperature, but is not applicable at high temperatures.

It is an interesting fact to observe that during the measurement of the susceptibility of a compound at high temperatures, a discontinuous change in it takes place when the compound is decomposed or oxidized. As a compound has its own susceptibility, it is quite conceivable that this will be changed by decomposition or oxidation. Consequently, if the change of the magnetic susceptibility of a compound is continuously observed during heating or cooling, we can detect, from the susceptibility-temperature curve, the internal change taking place in

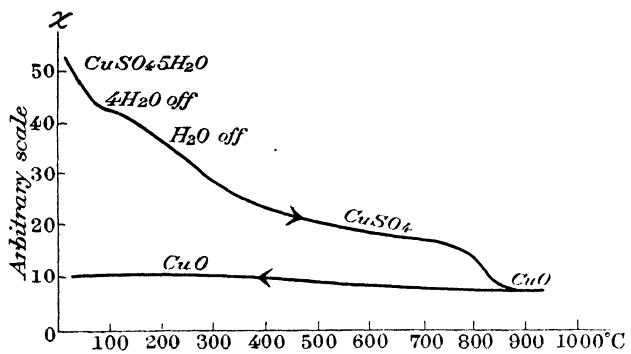


Fig. 145.

$CuSO_4 \cdot 5H_2O$ (Honda, Ishiwara).

the compound, if the change be an abrupt one. As an example, the change of magnetization in $CuSO_4.5H_2O$ is given in Fig. 145; thus we may notice on the χ,t curve the change of susceptibility corresponding to the two-stepped separation of the water of crystallization, and also the change corresponding to the formation of CuO from $CuSO_4.5H_2O$. Fig. 146 gives another example; when the compound $KMnO_4$ is heated to 230°C, it decomposes into K_2MnO_4 and MnO_2, the susceptibility abruptly increasing during the decomposition. As the temperature further rises the susceptibility decreases logarithmically, and at 600°, the substance changes again into $K_2O.2MnO_2$, its susceptibility slightly increasing. With a further rise of temperature, the compound does not change in composition, and hence its susceptibility slowly decreases. During

cooling, no change takes place in its composition, and hence the χ,t curve increases logarithmically.

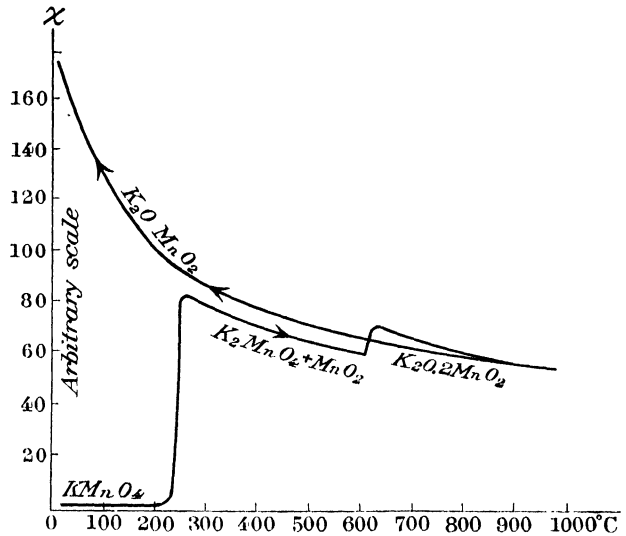


Fig. 146.

$KMnO_4$ (Honda, Ishiwara).

(b) *Organic Compounds.* Organic compounds are generally diamagnetic. From investigations extending over many years, Pascal[1] concludes that in a series of organic compounds of a similar constitution, the molecular susceptibility of a compound can be given by addition from the susceptibilities of the constituent atoms or radicals. For example, a compound $A_aB_bC_c$ which is composed of a, b, c atoms of the elements A, B, C, has the molecular susceptibility given by the following expression:—

$$\chi_m = \Sigma a\chi_a + \lambda,$$

where λ is a constant characteristic of the molecular constitution and χ_a, χ_b, χ_c are respectively the atomic susceptibilities for the elements A, B, C. These atomic susceptibilities are generally different from those as measured directly, and have the same values for the compounds of similar constitution, but may have

(1) P. Pascal, C. R. **147**, (1908), 57, 242, 742; **148**, (1909), 413; **150**, (1910), 1167; **152**, (1911), 862, 1010; Ann. de Chim. et Phys. (8), **19**, (1910), 5, 62; C. R. **173**, (1921), 145.

different values for those of another constitution. For instance, if we calculate the atomic susceptibility for the hydrocarbons and halogen compounds given below, the following values are obtained:—

Carbon	-6.25×16^{-6}	Bromine	-3.19×10^{-6}	$\lambda = -1.50\times10^{-4}$	(1)
Hydrogen	-3.05	Iodine	-4.65	$= 11.0$	(2)
Nitrogen	-5.30	Sulphur	-15.60	$= 5.70$	(3)
Chlorine	-20.95				

Using these values of atomic susceptibility, the molecular susceptibilities are calculated and compared with the observed values, as shown in the table below:—

Hydrocarbons	Cal. values	Obs. values
$CH_3(CH_2)_4CH_3$	-80.2×10^{-6}	-79.6×10^{-6}
$CH_3(CH_2)_8CH_3$	-129.6	-129.7
$(C_6H_5)_2$	-57.0	-57.4
C_6H_6	-57.3	-57.4
$C_6H_5 . CH_3$	-69.65	-69.9
$C_6H_5CH(CH_3)_2$	-70.2	-70.0
Halogen compounds	**Calc. values**	**Obs. values**
C_6H_5Cl	$-75.2\ 10^{-6}$	$-74.9\ 10^{-6}$
C_6H_5Br	-86.15	-85.6
C_6H_5I	-100.75	-100.0
$C_6H_5CH_2Cl$	-87.55	-87.7.

From these tables, we see that the agreement between the calculated and the observed values is very close. It must, however, be noted that λ has different values for different constitutions of molecules; this shows that in general the additive law does not strictly hold.

According to the investigations made by Oxley[4] and by Ishiwara[5], diamagnetic susceptibilities of organic compounds in a liquid state are nearly constant in the temperature range from room temperature down to that of liquid air. When the compounds solidify at low temperatures, their susceptibility varies discontinuously in a greater or less degree.

(1) Benzene ring. ⬡. (2) One ethylene bond. (3) Two or more ethylene bonds. (4) A. E. Oxley, Phil. Trans. Roy. Soc. London, Ser. A, **214**, (1914) 109. (5) T. Ishiwara, Sci. Rep. **3**, (1914), 303.

CHAPTER VIII.

Magnetization of Crystals.

45. Magnetization of Paramagnetic and Diamagnetic Crystals.

According to experimental data, there is the same symmetry in paramagnetic and diamagnetic crystals in relation to magnetization, as there is in relation to light. For instance, crystals belonging to the regular system are isotropic both to light and as regards magnetization. Crystals belonging to tetragonal and hexagonal systems are optically uniaxial, and there is a symmetry of rotation as regards magnetization. Crystals belonging to other systems are optically biaxial and magnetically their symmetry is that of an ellipsoid having three different axes. The directions of the axes of symmetry are called the principal axes of magnetization, and the magnetic susceptibility in these directions, the principal susceptibilities. Generally speaking, the direction of a magnetic field does not coincide with that of magnetization, but when the magnetic field acts in the direction of the principal axes, the directions of these two coincide with one another.

The principal susceptibility can be measured in the following way:—One of the principal axes of a small crystal is placed in the direction of a magnetic field, and its susceptibility measured by the Faraday or Weiss method. Assuming that the surrounding medium is a vacuum, the magnetic energy W will be

$$W = -\frac{1}{2}\kappa_s H^2 v,$$

where $s=1, 2, 3$ denotes the directions of the principal axes. Consequently, as before, the principal susceptibility κ_s or χ_s is given by

$$\kappa_s = \frac{f}{vH\frac{\partial H}{\partial y}},$$

or $$\chi_s = \frac{f}{mH\frac{\partial H}{\partial y}}.$$

As regards the magnetic susceptibilities of crystals, an accurate measurement was first made by W. Voigt and S. Kinoshita[1]. Faraday's method was used in their measurement, in which a small circular disc of a crystal, cut perpendicularly to one of its principal axes, was suspended between the pole-pieces of an electromagnet from one arm of a torsion balance by means of a bifilar suspension, so that the face of the disc was perpendicular to the field, in other words, so that the direction of the principal axes coincided with that of the field. In the case of uniaxial crystals two such discs, and for biaxial crystals three such, were prepared, and the magnetic susceptibility in each direction was measured. The following table contains the results of their experiment:—

Regular System.

Rock salt	$\chi = -0.376 \times 10^{-5}$
Alum	„ -0.579
Fluorite	„ -0.627
Pyrite	„ 10.666

Hexagonal System.

Beryl	$\chi_1 = -0.827 \times 10^{-6}$,	$\chi_2 = +0.386 \times 10^{-6}$
Apatite	„ -0.264	„ -0.264
Calcite	„ -0.364	„ -0.406
Dolomite	„ -0.788	„ -0.121
Quartz	„ -0.461	„ -0.466
Tourmaline	„ -1.12	„ -0.748

Tetragonal System.

Rutile	$\chi_1 = 1.96 \times 10^{-6}$,	$\chi_2 = 2.09 \times 10^{-6}$
Zircon	0.170	„ 0.732

Rhombic System.

Azurite	$\chi_1 = -0.342 \times 10^{-6}$,	$\chi_2 = -0.314 = 10^{-6}$,	$\chi_3 = -0.359 \times 10^{-6}$
Aragonite	„ -0.392	„ -0.387	„ -0.444

Finke[2] investigated the susceptibilities of crystals belonging to the monoclinic system. In the case of these crystals, one of the principal axes of magnetization may be found from the crystal-

(1) W. Voigt u. S. Kinoshita, Gött. Nachr., (1907), 123; Ann. der Phys. **24**, (1907), 492. (2) Finke, Gött. Dissert., (1909); Ann. der Phys. **31**, (1910), 149.

lographic form, but the other two axes are not evident, so that their positions must be experimentally determined at the same time as the principal susceptibilities. If a, b, c be the axes of a monoclinic crystal, axes a, b and b, c are both perpendicular to each other, while c and a make a certain angle other than a right angle, and if the direction perpendicular to the plane containing a and c axes, be taken as the z-axis, this axis will be one of the principal axes of magnetization. The susceptibility in this direction may be obtained experimentally by making use of a disc cut perpendicularly to this axis. On the other hand, though the other two principal axes lie evidently in the ac-plane, their directions are not known, so that they must be experimentally determined together with the principal susceptibilities χ_1 and χ_2. Let us suppose that x', y', z' be the principal axes of magnetization and x, y, z another system of axes taken in the crystal, the z-axis coinciding with z', and that the magnetic field H acts in the xy-plane. If the intensities of magnetization in the direction of the principal axes be denoted by σ_1, σ_2 and σ_3, then we have

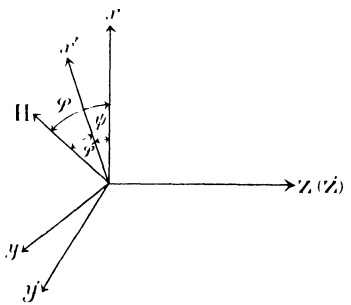


Fig. 147.

$$\sigma_1 = \chi_1 H_{x'} \quad \text{and} \quad \sigma_2 = \chi_2 H_{y'}.$$

If the intentsity of magnetization in the direction of the field be σ, and the angle between H and x' be φ', then

$$\begin{aligned}\sigma &= \sigma_1 \cos\varphi' + \sigma_2 \sin\varphi' \\ &= \chi_1 H_{x'} \cos\varphi' + \chi_2 H_{y'} \sin\varphi' \\ &= (\chi_1 \cos^2\varphi' + \chi_2 \sin^2\varphi')\, H.\end{aligned}$$

Consequently, the value of χ in the direction of the field is

$$\begin{aligned}\chi = \frac{\sigma}{H} &= \chi_1 \cos^2\varphi' + \chi_2 \sin^2\varphi' \\ &= \frac{\chi_1+\chi_2}{2} + \frac{\chi_1-\chi_2}{2}\cos^2\varphi'.\end{aligned}$$

Let us denote the angle between x and H by φ, and that between x and x' by ψ, then $\varphi'=\varphi-\psi$. Hence

$$\chi = \frac{\chi_1+\chi_2}{2} + \frac{\chi_1-\chi_2}{2}\{\cos 2\varphi \cos 2\psi + \sin 2\varphi \sin 2\psi\},$$

where χ_1, χ_2 and ψ are the quantities to be determined. If instead of these unknown quantities, we introduce another set of the unknown quantities:—

$$\frac{\chi_1+\chi_2}{2} = A, \qquad \frac{\chi_1-\chi_2}{2}\cos 2\psi = B, \qquad \frac{\chi_1-\chi_2}{2}\sin 2\psi = C,$$

the above expression becomes

$$A + B\cos 2\varphi + C\sin 2\varphi = \chi,$$

where χ is a quantity to be determined by experiment. The unknown quantities A, B and C can be determined as follows:— Three discs are made by cutting a crystal by three planes passing through its z-axis, and their susceptibilities are measured by placing each of these discs perpendicular to the field. Then the above relation holds good for each disc, and the following three equations are obtained,

$$A + B\cos 2\varphi_1 + C\sin 2\varphi_1 = \chi',$$

$$A + B\cos 2\varphi_2 + C\sin 2\varphi_2 = \chi'',$$

$$A + B\cos 2\varphi_3 + C\sin 2\varphi_3 = \chi'''.$$

Solving these equations for A, B, C and then for χ_1, χ_2 and χ_3, we obtain

$$\chi_1 = A + \frac{B}{\cos 2\psi}, \qquad \chi_2 = A - \frac{B}{\cos 2\psi}, \qquad \tan 2\psi = \frac{C}{B}.$$

By means of this method, Finke measured three principal susceptibilities and ψ for some crystals belonging to the monoclinic system, and obtained the following results:—

Crystals	$\chi_1 \times 10^6$	$\chi_2 \times 10^6$	$\chi_3 \times 10^6$	ψ
Augite	26.6	12.9	22.7	-7°
Hornblende	24.0	16.7	18.0	$-21^\circ\ 55'$
Adular	−1.09	−0.98	−0.81	$-13^\circ\ 20'$
Cane sugar	−0.60	−0.55	−0.57	$-1^\circ\ 50'$
$Fe\,SO_4.(NH_4)_2\,SO_4.6H_2O$	85.4	75.7	74.9	$-18^\circ\ 58'$
$Co\,SO_4.(NH_4)_2\,SO_4.6H_2O$	55.9	43.5	45.5	$-27^\circ\ 31'$
$Ni\,SO_4.(NH_4)_2\,SO_4.6H_2O$	19.6	15.7	18.0	$-16^\circ\ 17'$
$Fe\,SO_4.7H_2O$	84.3	78.7	76.2	$30^\circ\ 10'$
$Co\,SO_4.7H_2O$	70.9	64.0	68.5	$-39^\circ\ 40$

Thus we see that the three principal susceptibilities differ considerably from each other, and that neither of the two principal axes of magnetization, χ_2 and χ_3, coincide with the a- or c-axis.

Recently Foëx and L. C. Jackson investigated the change of the principal susceptibilities at different low temperatures down to 14°K for siderose[1], cobalt ammonium sulphate hexahydrate and nickel sulphate heptahydrate[2]. It is found that Weiss' law

$$\chi\,(T+\Delta) = \text{const.}$$

is only applicable down to 80°K.

Jackson used the Weiss method[3], in which the maximum couple exerted on a small cylinder of the crystal suspended in a homogeneous field was measured. The results of observations are given below:—

Nickel sulphate heptahydrate $NiSO_4.7H_2O$ (rhombic system).

T	$\chi_1 \times 10^6$	$\chi_2 \times 10^6$	$\chi_3 \times 10^6$
288°K	16.7	16.5	16.6
169.5	35.1	34.5	34.8
77.3	78.1	76.4	77.3
64.5	92.6	90.3	91.5
20.3	228.2	215.9	219.9
16.6	278.8	255.5	261.3
14.6	304.9	283.3	290.4

Cobalt ammonium sulphate hexahydrate $CoSO_4.(NH_4)_2SO_4.6H_2O$ (monoclinic system).

T	$\chi_1 \times 10^6$	$\chi_2 \times 10^6$	$\chi_3 \times 10^6$	ψ
290°K	26.1	21.9	25.5	31° 18′
77.2	91.2	58.6	85.6	28° 17′
20.3	286.5	165.5	277.0	27° 8′
16.7	346.0	192.0	325.0	27° 41′
14.5	388.0	213.0	365.0	27° 40′

In the last table, ψ is the angle in the symmetry plane, which the axis of the greatest susceptibility makes with the c-axis.

---

(1) Foëx, Thèse, Strassburg, 1921; Ann. de Phys. **16**, (1921), 174. (2) L. C. Jackson, Dissert., Leiden, (1923), 47. (3) See section 11.

Quite recently, I. I. Rabi[1] devised a useful method for measuring the principal susceptibility of crystals. The crystal to be tested is immersed in a solution, its orientation being first so adjusted that one of its principal axes coincides with that of the field, in which case the application of the magnetic field does not cause any rotation of the crystal; then the concentration of the solution is so varied that there is no movement of the crystal due to the field. In this case, the susceptibility of the solution is just equal to that of the crystal; hence by measuring the susceptibility of the solution, the principal susceptibilities can be obtained. It is an advantage of this method that it does not require any preparation of a crystal section.

Rabi measured the principal susceptibilities of 14 crystals, of which 11 were paramagnetic and belong to the isomorphous series of the monoclinic double sulphate hexahydrate $MeSO_4.R_2SO_4.6H_2O$, the rest being the diamagnetic crystals of $NaNO_3$ and KNO_3. In the series containing copper, the greatest difference in the maximum and minimum susceptibilities of a crystal is 28 percent, in the nickel series 4 percent, in the cobalt series 32 percent, in the iron series 16 percent and in the manganese series 1 percent. The diamagnetic crystals show also similar magnetic properties.

46. Method of Measuring the Magnetization in Ferromagnetic Crystals.

As we have already seen, crystals belonging to the regular system are isotropic in relation to the magnetization; in these crystals, the change of orientation of the atoms due to the magnetic field is extremely small. In ferromagnetic crystals, the same change increases to a considerable amount, in which case the crystals of the regular system are no longer isotropic in relation to the magnetization.

The magnetization of ferromagnetic crystals was investigated by P. Weiss, J. Kunz, Quittner, K. Beck and others. P. Weiss[2] made use of two methods; the first one is the torsion method based on the measurement of a couple exerted on a crystal by a magnetic field, and the other is the ballistic method.

In Fig. 148, D is a thin circular plate of crystal cut parallel to a principal axis, and is suspended horizontally between the pole-pieces of an electromagnet. M is a small mirror fixed on a

(1) Phys. Rev. **29**, (1927), 174. (2) Thèse prés. à la Faculté des Sci. de Paris, (1896).

thick vertical wire, and its angle of rotation can be read on a scale with a telescope. The upper end of the wire is fixed to the lower end of a spring suspended from the accessorial bar of a torsion disc B. The lower end of the wire, carrying a weight, is immersed in water to damp the vibrations of the suspended system. If the electromagnet be excited, a couple will generally act on the disc, provided the direction of the principal axis of the crystal does not coincide with that of the field; but in the case of such coincidence, no rotation is produced.

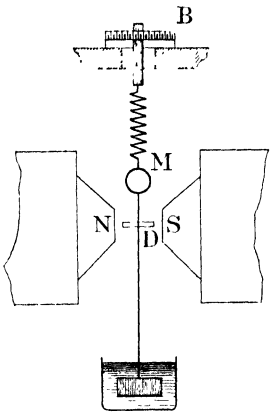


Fig. 148.

Fig. 149 shows the plan of the disc. If the intensity of magnetization I makes an angle φ with the field H, the couple acting on a unit volume is $HI\sin\varphi$. When the field is uniform, the resultant couple will be equal to the couple multiplied by the volume v, that is, $vHI\sin\varphi$. If we denote the component of magnetization perpendicular to the field by $I_\perp$, then

$$I_\perp = I\sin\varphi,$$

and the resultant couple N is expressed by

$$N = vHI_\perp.$$

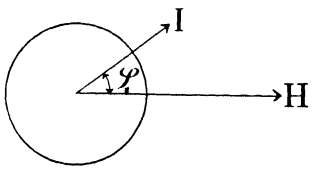


Fig. 149.

This couple will be in equilibrium with the couple produced by the twisting of the suspending spring, that is, $K\theta$. Accordingly,

$$I_\perp = \frac{K\theta}{vH}.$$

Hence for the determination of $I_\perp$, the principal axis of the crystal is placed at an angle with the direction of the field, and then a magnetic field is applied, by which a rotation of the disc is produced, thus causing a corresponding deflection of the mirror. We can thus find $I_\perp$ from the above expression by measuring the angle of rotation θ. If we keep the magnetic field constant, and vary the direction of the principal axis, the relation between $I_\perp$ and the angle which the principal axis makes with the field can be obtained.

As regards the measurement of the component of magnetization

parallel to the magnetic field, the circular disc is vertically suspended, and placed in such a way that its face makes an angle of 3° or 4° with the direction of the field; then under the action of the magnetic field, the disc will rotate about a vertical axis through a small angle. Fig. 150 is a side view of the disc. If the horizontal component of magnetization be I_h, we have

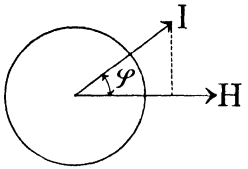


Fig. 150.

$$I_h = I\cos\varphi.$$

Since I_h plays the same rôle as I in the former case, the couple acting on the disc will be

$$N = vHI_h \sin\alpha = K\theta,$$

where α is the angle between I_h and H. But the component of magnetization $I_\parallel$ parallel to the field is

$$I_\parallel = I_h \cos\alpha \div I_h;$$

that is,

$$I_\parallel = \frac{K\theta}{vH\sin\alpha}.$$

If we keep H constant and vary the angle between the principal axis and H, the relation between $I_\parallel$ and the angle which the principal axis makes with the field, can be obtained.

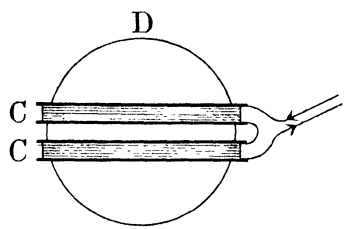


Fig. 151.

We shall next describe the ballistic method. Two fine secondary coils CC are placed on both sides of the centre of the circular disc D to be tested and a little apart from each other (Fig. 151), and they are in turn connected to a ballistic galvanometer. The disc is horizontally fixed to a support A with a circular stand F, as shown in Fig. 152, and the secondary coils CC are fixed to a support B set up on a stand E; the disc and the coils can be rotated independently of each other. The whole apparatus is

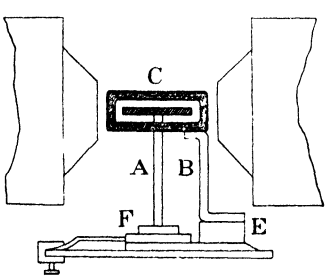


Fig. 152.

placed between the pole-pieces of an electromagnet, and the field is applied parallel to the face of the disc.

In order to determine $I_{\parallel}$, the axis of the coil is placed parallel to the field, and a magnetic field is excited by passing an electric current through the electromagnet. Then, the stand E is suddenly rotated by 180°, and the quantity of the current thus induced is measured with the galvanometer; this gives the value of $2B$. For the measurement of the magnetic field, the test-disc is removed and a similar observation is made once more, with the coils only. $I_{\parallel}$ can thus be obtained from B and H.

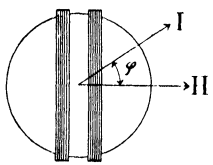


Fig. 153.

As regards the measurement of $I_{\perp}$, the axis of the coil is placed perpendicular to the field, and then, after exciting the magnetic field, the coil is suddenly rotated by 180°. By observing the quantity of the current thus induced, the value of $I_{\perp}$ can then be obtained. In this case, the field in the direction of $I_{\perp}$ is of course zero.

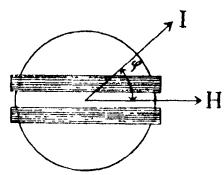


Fig. 154.

If we make the field constant and vary the orientation of the disc, the relation between $I_{\parallel}$ or $I_{\perp}$ and the angle which the principal axis makes with the field, can be obtained.

47. Magnetization and Magnetostriction of Pure Iron.

Formerly the preparation of single crystals of iron was made by Goldschmidt's method, by which K. Beck(1) obtained large crystals of iron, which however contained silicon, and made a thorough study of the magnetic properties of these crystals. His results are very important from the point of view of crystallomagnetism. In recent years, a new method of producing large crystals of iron has been developed, and those ranging from 10–20 cm in linear dimensions can be obtained. The method is as follows:—

From pure iron containing no trace of carbon, a test-piece in the form of a plate or a rod is made by forging, and then an elongation of 2.8–3.0 percent is given by a tensile test machine. Then, the specimen is heated continuously at 880°, i. e. just below the A_3 point,

(1) K. Beck, Dissert., Zürich (1918).

for 3—4 days, during which period the test-piece changes into a single crystal, or an aggregate of a few large crystals, its external form being of course unchanged.

S. Kaya, Y. Mashiyama and the author[1] prepared large crystals of pure iron by the above method from which three oblate ellipsoids were made and then subjected to magnetic measurements. The longer axis of the ovoids was 20 mm and their shorter axis 0.4

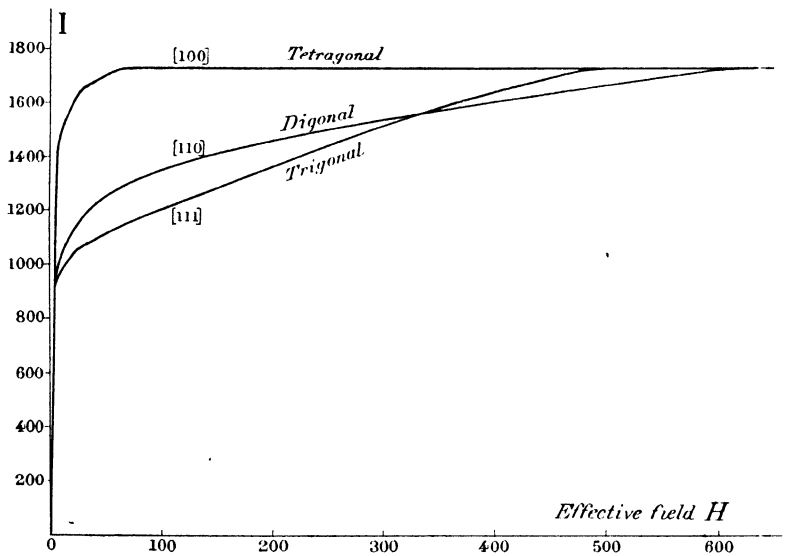


Fig. 155.

(Honda, Kaya

mm, the flat plane of the ellipsoids coinciding with one of the three principal planes (100), (110), (111).

The magnetization in the directions of the principal axes of the crystal was measured by the ballistic method; the results are graphically given in Fig. 155. As seen in this figure, the magnetization curves in the directions of the tetragonal [100], diagonal [110] and trigonal [111] axes are almost straight up to the intensities of magnetization 1300, 900 and 860, respectively. Then the curves show sharp breaks or bendings, twice or thrice. The saturation of magnetization is much more easily attained than in the

(1) Sci. Rep. **15**, (1926), 721, 755. See also W. L. Webster, Proc. Roy. Soc. **107**, (1925), 496; **109**, (1925), 570. W. Gerlach, Phys. Zeitschr. **26**, (1925), 914.

case of ordinary iron, its value amounting to 1717. In the direction of the tetragonal axis, the saturation is reached at 70 gausses only, and in those of the diagonal and trigonal axes at 590 and 470 gausses, respectively The initial and maximum susceptibilities of the crystal are very large, but on the other hand, its residual magnetism is very small. In plane (100), the direction of the tetragonal axis is more easily magnetizable than that of the diagonal. In plane (110), the directions of the tetragonal, diagonal and trigonal axes are in a decreasing order of magnetizability.

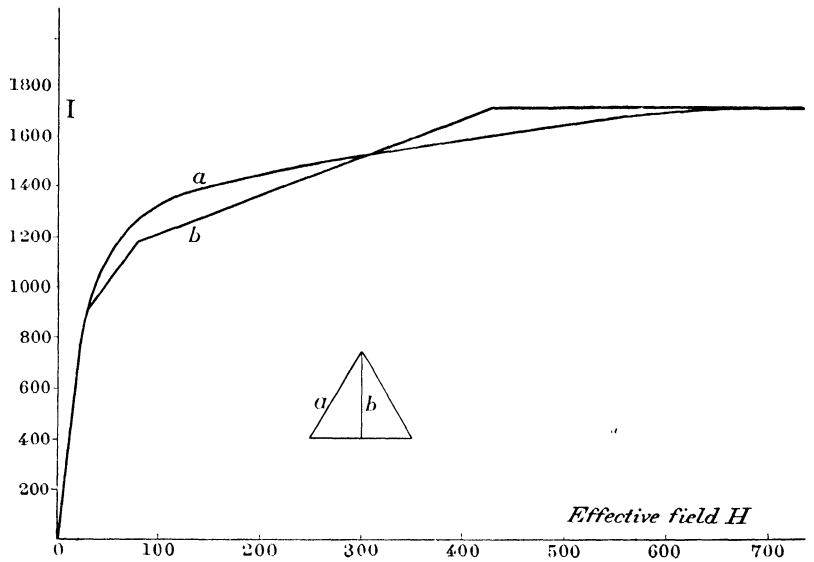


Fig. 156.

(Honda, Kaya)

The magnetization in plane (111) is given in Fig. 156; the space-lattice in this plane consists of a series of equilateral triangles. The form of the magnetization curves are similar to those in the above cases. Below a field of 300 gausses, the direction of the sides of the triangle (*a*-axis) is more magnetizable than that of the bisector of the triangle (*b*-axis); but above the field, the contrary is the case.

The results of observation for the components of magnetization parallel and perpendicular to the magnetizing field are given in Figs. 157-162. Here the applied field being kept constant, the orientation of the ovoid is varied and the components of magnetization measured. In these figures, the abscissa represents the

orientation of the ovoid measured from the tetragonal axis in planes (100) and (110) and from the *b*-axis in plane (111), and the ordinate the parallel or perpendicular components of magnetization.

In plane (100), the space-lattice consists of a series of squares, and hence both the parallel and perpendicular components of magnetization vary with a period of 90°, the form of the curves being

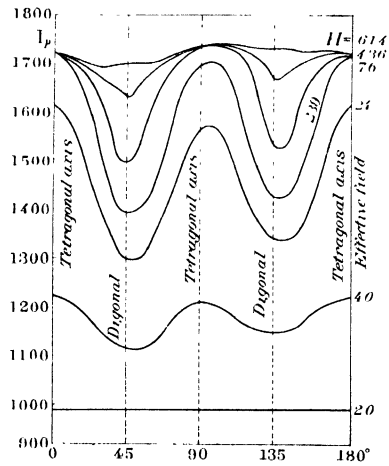


Fig. 157.

Intensity of magnetization in plane (100)
Parallel component. (Honda, Kaya)

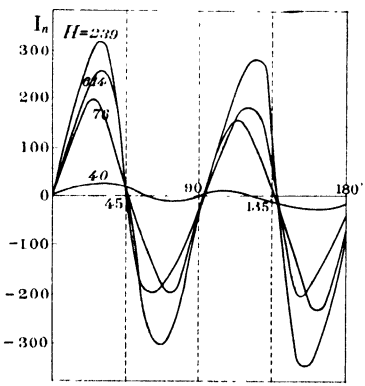


Fig. 158.

Intensity of magnetization in plane (100).
Perpendicular component. (Honda, Kaya)

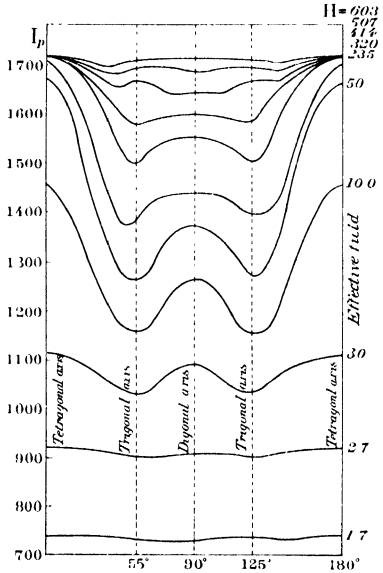


Fig. 159.

Intensity of magnetization in plane (110).
Parallel component (Honda, Kaya)

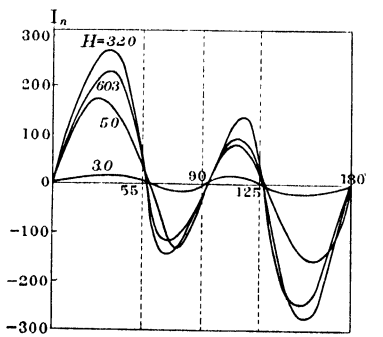


Fig. 160.

Intensity of magnetization in plane (110)
Perpendicular component. (Honda, Kaya)

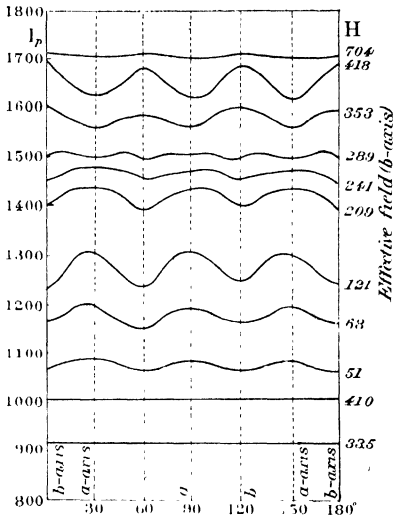


Fig. 161.
Intensity of magnetization in plane (111).
Parallel component. (Honda, Kaya)

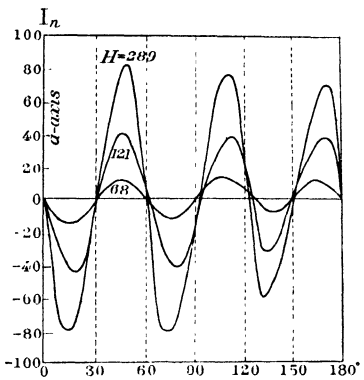


Fig. 162.
Intensity of magnetization in plane (111)
Perpendicular component. (Honda, Kaya)

sinusoidal. For the parallel component, the direction of the tetragonal axis has the maximum magnetization, and that of the digonal the minimum; but for the perpendicular component, the magnetization vanishes in the direction both of the tetragonal and digonal axes and attains a maximum or minimum between them. Below a field of 3 and above one of 650 gausses, the parallel component of magnetization in plane (100) is almost independent of the direction of the field. As the field increases beyond 3 gausses, the amplitude of the periodic change becomes greater, attains a maximum and afterwards gradually decreases. In the case of the perpendicular component, the magnetization appears in a weaker field and is observable up to a stronger field, than in the case where the periodic variation appears in the parallel component.

In plane (110), the space-lattice consisting of a series of rectangles, the two components of magnetization vary with a period of 180°. For the parallel component, the principal and secondary maxima occur respectively in the directions of the tetragonal and digonal axes, and the minimum in the directions of 55° and 125° from the tetragonal axis. For the perpendicular component, the magnetization vanishes in the direction of the tetragonal, trigonal and digonal axes, and attains a maximum or a minimum midway

between them. The amplitude of this periodic change varies in a similar way to that in the case of plane (100).

In plane (111), the space-lattice being a series of equilateral triangles, the two components of magnetization vary with a period of 60°. In weak fields, the parallel component of magnetization in

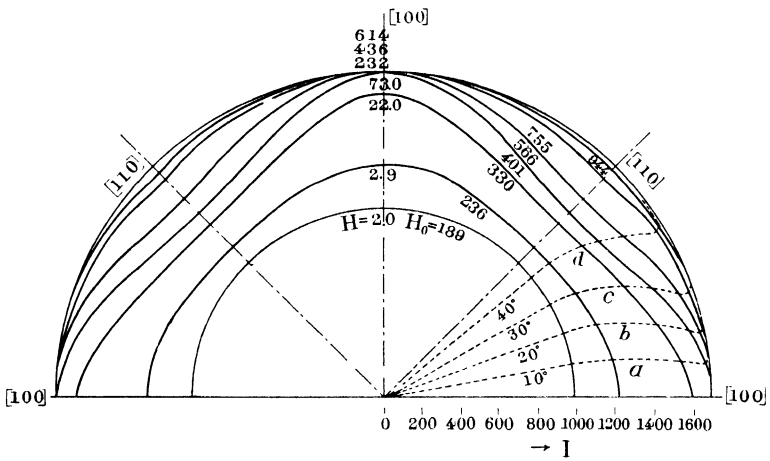


Fig. 163.

Distribution of magnetization in plane (100). H_0=external field, H=effective field. (Honda, Kaya)

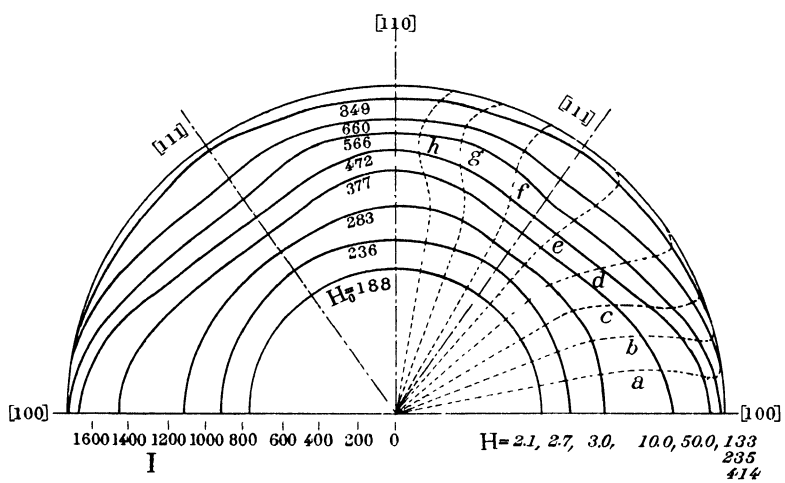


Fig. 164.

Distribution of magnetization in plane (110). (Honda, Kaya)

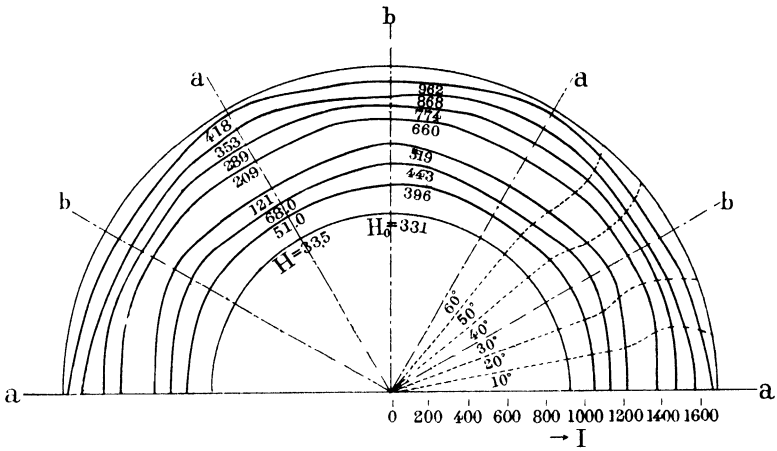


Fig. 165.

Distribution of magnetization in plane (111). H=effective field for *a*-axis.
(Honda, Kaya)

the direction of the sides of the equilateral triangles forming the space-lattice is a maximum, and that in the direction of the bisectors of the vertical angle of the triangle is a minimum; but in a stronger field above 300 gausses, the opposite is the case. The perpendicular component of magnetization vanishes in these two directions and attains a maximum or a minimum between them.

Figs. 163—165 show the distribution of the resultant intensity of magnetization for different directions of the magnetizing field in the three planes (100), (110) and (111). H_0 is the applied field and H the effective field in the direction of the tetragonal or *a*-axis; the dotted curves show the deviation of magnetization from the direction of the field. Below a certain strength of the field, and also above a field which gives nearly the saturation value of magnetization, the resultant intensity of magnetization is independent of the direction of the field; but in an intermediate field, it depends considerably on its direction.

The hysteresis-loss of the single crystal of iron is very small as compared with that of ordinary iron, amounting only to one-tenth of that of ordinary sheet iron containing a few percent of silicon. In the case of polycrystals, the hysteresis as well as the initial and maximum permeabilities increases very rapidly with the number of grains or crystals in the specimen, as is shown in the table below:—

Specimen No.	Grain number per mm²	Initial permeability	Maximum permeability	Hysteresis-loss in ergs
1	92.0	600	3400	3730
2	12.1	900	3740	2070
3	1.9	980	3700	1840
4	0.15	1000	—	1325
5	0.092	1500	4430	1170
6	0.0067	1700	4300	460

It is therefore to be concluded that these quantities are mainly affected by the irregular distribution of elementary magnets on the grain boundaries.

Lastly the results of observation of the magnetostriction of the single crystal of iron by Y. Mashiyama and the author[1] will be described. In Figs. 166 and 167, the longitudinal and transverse effects of magnetic expansion, that is, the components of the magnetic expansion in the direction of the field and that perpendicular to it, are given. In the direction of the tetragonal axes [100], the longitudinal effect of magnetic expansion is positive for all fields; it increases at first rapidly, and afterwards slowly. In the direction of the digonal axis [110], a small expansion is observable below 120 gausses, above which a contraction takes place. The magnetic expansion in the direction of the trigonal

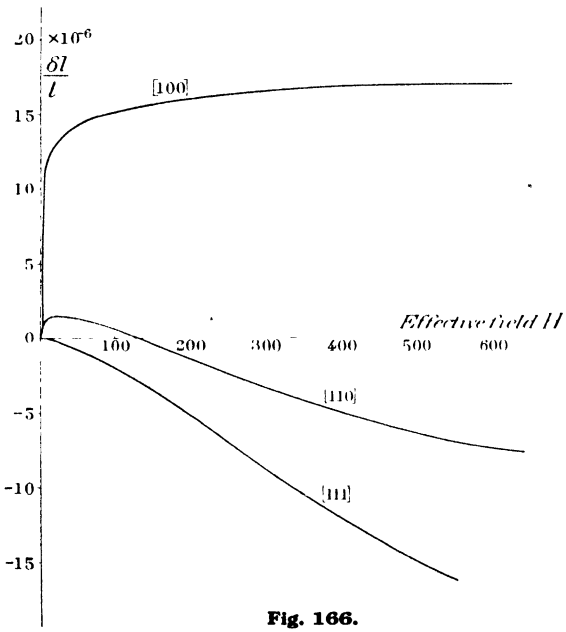


Fig. 166.
Longitudinal effect of magnetic expansion.
(Honda, Mashiyama)

(1) Sci. Rep. **15**, (1926), 755.

axis [111] is negative for all fields. The transverse effect is just the reverse of the longitudinal effect, except in the direction [110] in which a small initial contraction is not observable. From these results, it is to be concluded that the magnetic expansion in ordinary iron consisting of an immense number of micro-crystals, is a differential effect of the expansion and the contraction above referred to, the crystal axes being orientated at random. The smallness of the magnetic expansion of ordinary iron, as compared with that of the single crystal, also confirms this view.

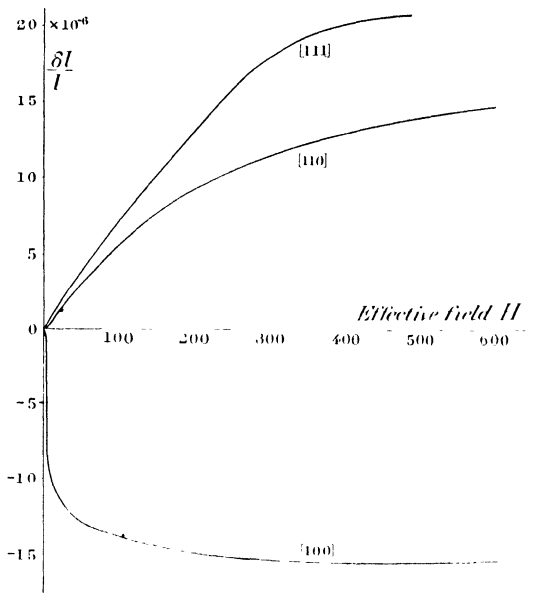


Fig. 167.

Transverse effect of magnetic expansion (Honda, Mashiyama)

The magnetic expansion-orientation curves under a constant field in three planes (100), (110), (111) are all sinusoidal, having periods of 90°, 180° and 60° respectively, and are similar to the case of the magnetization.

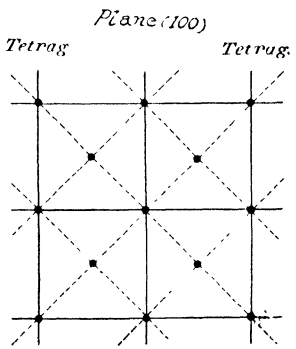


Fig. 168.

48. Magnetization of Magnetite and Pyrrohotite.

(a) *Magnetite.* The Magnetization of magnetite was investigated by P. Weiss[1] and Quittner[2] with three discs cut parallel to planes (100), (110),

(1) Thèses prés. à la Faculté des Sci. de Paris, (1896). (2) Arch. des Sci. **26,** (1908), 358, 455, 585.

(111). Magnetite belongs to the regular system and has a double face-centred cubic lattice as regards divalent iron atoms which may have a close connection with the ferromagnetism. In fields from 40 to 500 gausses, the magnetization is strongest in the direction of the trigonal axis, weaker in the direction of the digonal axis and weakest in that of the tetragonal axis.

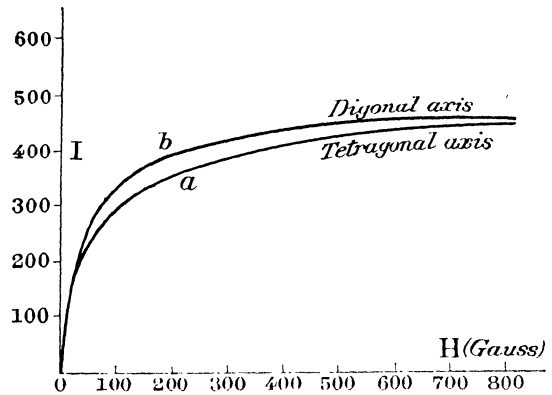


Fig. 169.
(Quittner)

In plane (100), the space-lattice consists of a series of squares, the sides of which make an angle of 45° with the tetragonal axis (Fig. 168); hence this axis, in the case of iron, corresponds to the digonal axis in the case of magnetite, and vice versa. So the magnetization of magnetite in the direction of the digonal axis is stronger than that in the direction of the tetragonal axis,

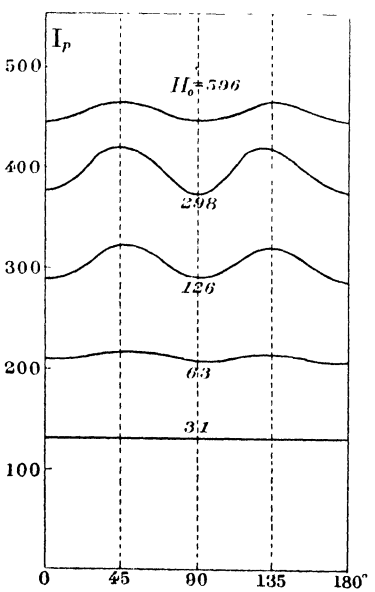


Fig. 170.
Intensity of magnetization in plane (100).
Parallel component (Quittner).

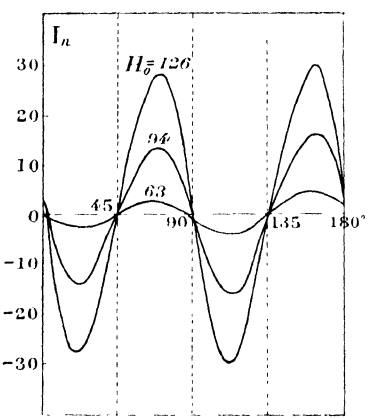


Fig. 171.
Intensity of magnetization in plane (100).
Perpendicular component (Quittner).

as shown by Quittner. In Fig. 169, curves *a* and *b* are respectively the magnetizations in the tetragonal and digonal axes. As seen in Figs. 170 and 171, the parallel and perpendicular components of magnetization show a period of 90°, and the maximum or

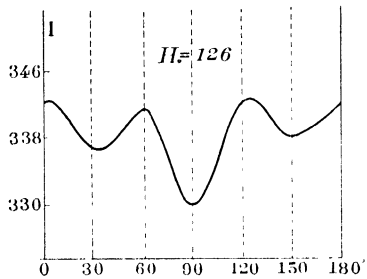


Fig. 172.
Intensity of magnetization in plane (111). Parallel component (Quittner).

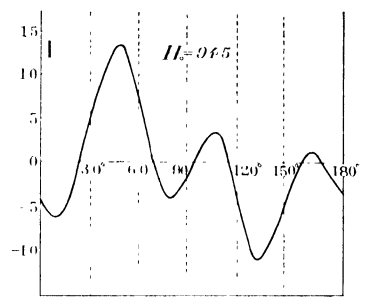


Fig. 173.
Intensity of magnetization in plane (111) Perpendicular component (Quittner)

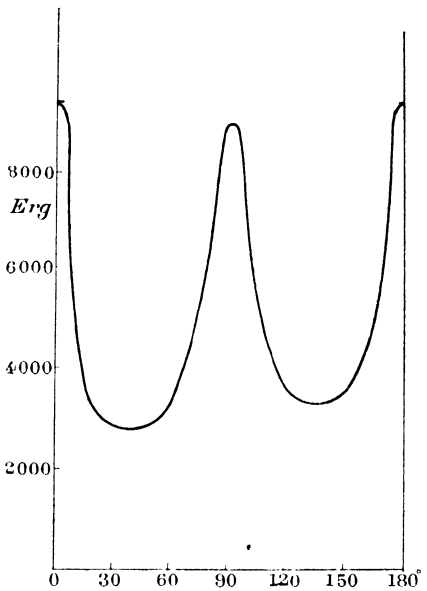


Fig. 174.
Hysteresis-loss in plane (100) (Quittner).

minimum point of the parallel component coincides with the zero points of the perpendicular component.

In plane (110), the space-lattice consists of a series of rectangles, so that the two components of magnetization repeat themselves with a period of 180°. In plane (111), the space-lattice is formed by a series of equilateral triangles; hence as shown in Figs. 172 and 173, the two components of magnetization show a variation of a period of 60°.

As regards the hysteresis-loss during cyclic magnetization, the results in the case of different orientations of the discs are very important. As is to be expected, the curve (Fig. 174) in plane (100) possesses a period of 90°, and that (Fig. 175) in plane (111) shows a period of 60°.

(b) *Pyrrhotite.* The magnetization of pyrrhotite ($Fe_{II}S_{III}$) was investigated by P. Weiss[1] and J. Kunz[2]. This crystal belongs to the hexagonal system and has a prismatic form. It is a remarkable fact that the crystal is paramagnetic in the direction of its axis, but ferromagnetic in the plane parallel to the base.

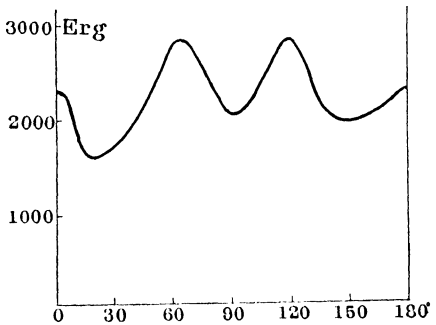


Fig. 175.
Hysteresis-loss in plane (111) (Quttner).

When a circular disc of pyrrhotite cut parallel to its base is placed between the poles of an electromagnet, and its intensity of magnetization is measured for different orientations, there appear two particular directions perpendicular to each other; one is the direction in which the crystal is easily magnetizable and the other the direction in which it is not. The former is called the direction of easy magnetization, and the latter that of difficult magnetization. The relation between the intensity of magnetization and the field in these directions is given in Fig. 176

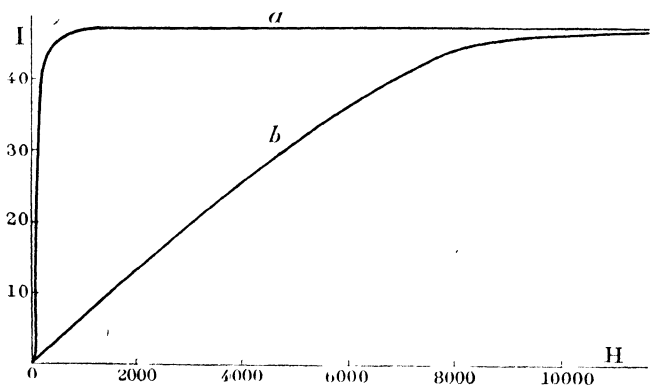


Fig. 176. (Weiss)

in these cases, the direction of magnetization coincides with that of the field, but does not do so for intermediate directions.

(1) P. Weiss, Jour. de Phys. **4**, (1905), 469, 829. (2) P. Weiss et J. Kunz, Arch. des Sci. **20**, (1905), 621; Jour. de Phys. **4**, (1905), 847.

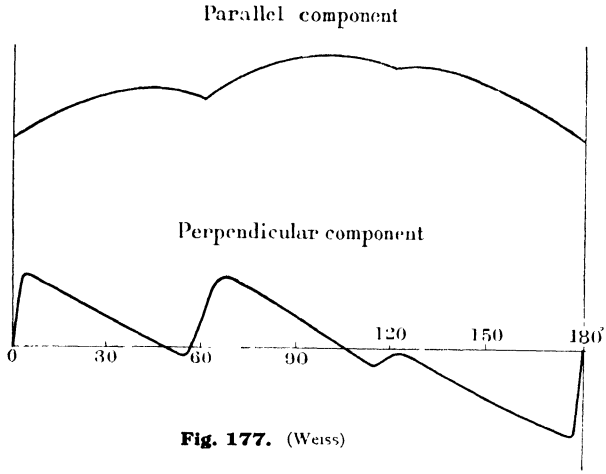


Fig. 177. (Weiss)

The parallel and perpendicular components of magnetization, for different orientations of the disc, form curves having a short period of 60° superposed on a long period of 180°, as shown in Fig. 177. Weiss considers that the normal period of pyrrhotite in the magnetization-orientation curve is 180°; but the fact that the crystal actually has a period of 60°, is explained by him by supposing that the crystal is not of a simple structure, but is composed of three crystals having a period of 180°, in which

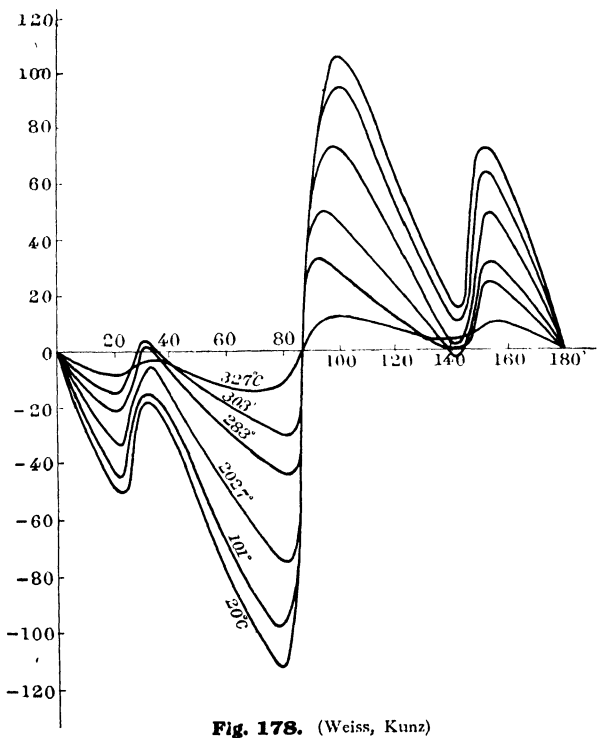


Fig. 178. (Weiss, Kunz)

their long axes are parallel and other axes have a phase-difference of 60°. This assumption is too complicated and probably does not correspond with the actual fact. Since the crystal of pyrrhotite belongs to the hexagonal system, the present author considers its normal period to be 60°, and explains the presence of the longer period as caused by the distortion[1] of the space-lattice of the crystal.

When pyrrhotite is heated to high temperatures, there is a marked difference between the normal and abnormal crystals in the change of their magnetization. In the case of the normal crystals, the periods of the parallel and perpendicular components at high temperatures, do not change as regards their orientations, and also the ratio of the amplitudes for the same orientation remains unchanged. Fig. 178 shows the variation of the perpendicular component at different temperatures. In the case of the abnormal crystals, not only are the periods of the parallel and perpendicular components changed by heating, but also their ratio of amplitudes in the same orientation.

(1) K. Honda and J. Okubo, Sci. Rep. **5**, (1916), 203.

CHAPTER IX.

Theories of Magnetism.

49. Nature of the Elementary Magnet.

So far the foregoing descriptions have to do with the experimental facts concerning magnetism. A complete theory of magnetism must be one that explains all these phenomena and at the same time does not contradict the results of investigation in other branches of physics. At present, the most trustworthy theory of the structure of atoms is that which regards them as composed of a nucleus with electrons revolving round it, and hence it will be most appropriate to apply this theory also to the explanation of the nature of magnetism. The nucleus itself is not a simple centre of force, but has a very complicated structure, consisting of a certain number of protons and electrons, each of which has a positive and a negative charge of an equal amount, respectively. The difference between the numbers of protons and electrons in the nucleus is nearly equal to the atomic number of the atom. These protons and electrons are also believed to make a very rapid orbital motion round the common centre. A proton has a mass of one hydrogen atom, while an electron has a mass of nearly one-eighteen hundredth of one such atom. Also, from electrical and thermal conductivities of solid metals the conclusion is drawn that besides these revolving electrons or "bound electrons," the metals contain so-called "free electrons" moving like gas molecules in the interspace of the atoms. The electrons that have an intimate relation with the problems of magnetism, are chiefly those described as bound electrons.

Experiments show that an electric circuit is equivalent in its action to a flat magnet. As the electrons making a rapid orbital motion may be regarded as an electric circuit, they exert an action similar to that of a small magnet. If e be the charge of an electron and T the period of its circular motion, then the electric current in the case of a single electron is

$$i = \frac{e}{T}.$$

If M be the corresponding magnetic moment and r the radius of the orbit, then

$$M = i \times (\text{area of orbit}) = \frac{\pi e r^2}{T}.$$

But $e = 4.77 \times 10^{-10}$ E.S.U., $r = 10^{-8}$ cm, and $T = 10^{-15}$ sec.; hence we have

$$M = 14.8 \times 10^{-11} \text{ E.S.U.}$$
$$= 0.49 \times 10^{-20} \text{ E.M.U.}$$

This value is nearly equal to the magnetic moment possessed by one nickel atom. Thus if we consider a single electron as making a circular motion, it gives rise to a magnetic moment of a reasonable magnitude.

In the case of an atom having several electrons moving in different orbits, its resultant magnetic moment will be the geometrical sum of the magnetic moments due to these electrons. As these moments have generally different values and directions, the resultant moment has different values for different kinds of atoms. Thus each atom has its own magnetic moment, and consequently it may at first sight appear that if we consider a substance to be composed of such atoms, we shall be able to explain its magnetic property; but as is shown below, this is not the case.

50. Effect of a Magnetic Field on a System of Revolving Electrons.

Let us first suppose that an electron is making under a central force an elliptic motion with a certain period, and consider the effect of a magnetic field on the orbital motion of the electron. If the magnetic field H acts in the direction of z-axis, then the equation of motion will be

$$\left.\begin{aligned} m\left(\frac{d^2x}{dt^2} - \kappa\frac{dy}{dt}\right) &= X \\ m\left(\frac{d^2y}{dt^2} + \kappa\frac{dx}{dt}\right) &= Y \\ m\frac{d^2z}{dt^2} &= Z \end{aligned}\right\} \quad (1)$$

where X, Y, Z are the components of the central force, and $\kappa=eH/m$, e and m having the same meaning as before.

If we refer the original motion in the absence of the magnetic field to the co-ordinate axes x', y', z', which have the origin and z-axis common to the axes x, y, z, and are rotating about the z-axis with a constant angular velocity ω, then the component velocities referred to the moving axes will be

$$u = \frac{dx'}{dt} - \omega y', \qquad v = \frac{dy'}{dt} + \omega x', \qquad w = \frac{dz'}{dt}.$$

Similarly the component accelerations[1] are

$$\frac{du}{dt} - \omega v, \qquad \frac{dv}{dt} + \omega u, \qquad \frac{dw}{dt};$$

that is,

$$\frac{d^2x'}{dt^2} - 2\omega\frac{dy'}{dt} - \omega^2 x',$$

$$\frac{d^2y'}{dt^2} + 2\omega\frac{dx'}{dt} - \omega^2 y',$$

$$\frac{d^2z'}{dt^2}.$$

Now the dimensions of the atom are very small and the velocity of the electron is very large, and hence the third term in the above expressions may be neglected in comparison with other terms. Hence the equations of motion referred to the rotating axes become

---

(1) To prove this, let us suppose that at the time $t=0$, the rotating axes x', y', z' coincide with the fixed axes x, y, z; then at the time t, we have

$$x = x' \cos\omega t - y' \sin\omega t,$$

$$y = x' \sin\omega t + y' \cos\omega t.$$

Hence

$$\frac{dx}{dt} = \frac{dx'}{dt}\cos\omega t - \frac{dy'}{dt}\sin\omega t - \omega x' \sin\omega t - \omega y' \cos\omega t,$$

$$\frac{dy}{dt} = \frac{dx'}{dt}\sin\omega t + \frac{dy'}{dt}\cos\omega t + \omega x' \cos\omega t - \omega y' \sin\omega t,$$

$$\frac{dz}{dt} = \frac{dz'}{dt}$$

At the instant at which the axes coincide with each other at $t=0$, we have

$$\left.\begin{aligned} m\left(\frac{d^2x'}{dt^2} - 2\omega\frac{dy'}{dt}\right) &= X', \\ m\left(\frac{d^2y'}{dt^2} + 2\omega\frac{dx'}{dt}\right) &= Y', \\ m\frac{d^2z'}{dt^2} &= Z', \end{aligned}\right\} \qquad (2)$$

where X', Y', Z' are the components of the central force referred to the same axes. These equations become identical with the equations (1), if 2ω be put equal to κ. Hence the effect of a magnetic field H on an electron which is revolving round a centre under the influence of a central force, is simply to impose on the undisturbed motion of the electron a rotation of an angular velocity $\omega = eH/2m$ round the direction of the field as the axis.

The conclusion can be extended to the case where a certain number of electrons are revolving on a common orbit. The above motion of the orbital plane is called the Larmor precession, and is shown diagrammatically in Fig. 179, where the upper circle represents the path of aphelion and the lower that of perihelion.

As we have already remarked, revolving electrons, like an electric circuit, possess a magnetic moment, and hence, it may at first sight appear very probable that such a system of electrons, when acted on by a magnetic field, will have a component of the magnetic moment in the direction of the field; but from what has been said above, we see that the angle between the axis of the magnetic moment due to the system of revolving electrons and the direction of the field remains unchanged by the application of the field. Hence we arrive at an important conclusion that a system of revolving electrons cannot have any component of its magnetic moment in the direction of the field; that is, such a system has a definite magnetic moment, but cannot be mag-

---

$$\frac{dx}{dt} = \frac{dx'}{dt} - \omega y', \qquad \frac{dy}{dt} = \frac{dy'}{dt} + \omega x', \qquad \frac{dz}{dt} = \frac{dz'}{dt}.$$

The same reasoning may also be applied to the case of acceleration; that is,

$$\frac{dx}{dt} = u\cos\omega t - v\sin\omega t, \qquad \frac{dy}{dt} = v\sin\omega + v\cos\omega t, \qquad \frac{dz}{dt} = \frac{dz'}{dt},$$

and so at $t = 0$, we have

$$\frac{d^2x}{dt^2} = \frac{du}{dt} - \omega v, \qquad \frac{d^2y}{dt^2} = \frac{dv}{dt} + \omega u, \qquad \frac{d^2z}{dt^2} = \frac{d\omega}{dt}$$

netised by the magnetic field. This rather unexpected result was shown by J. J. Thomson[1] and W. Voigt[2] 26 years ago.

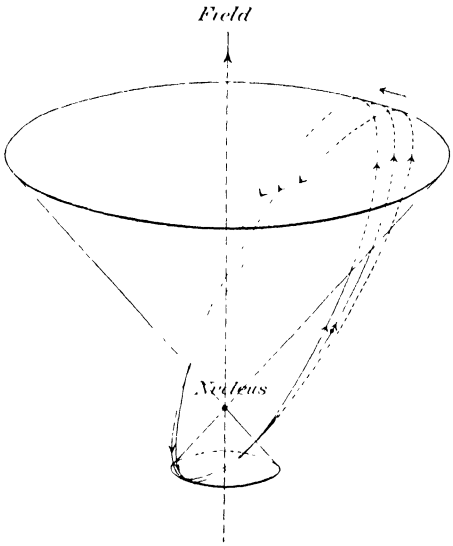


Fig. 179.

Considering next the precessional motion of the electrons, it is easily realized that this motion gives rise to a diamagnetic effect, that is, to a very weak magnetization in a direction opposite to that of the field. For the sake of simplicity, take the simple case of a circular orbit of radius r. If θ be the angle between the magnetic field and the axis of the revolving electrons, the components of the magnetic field parallel and perpendicular to the axis will be $H\cos\theta$ and $H\sin\theta$ respectively. The diamagnetism due to the precessional motion corresponding to the former component is then: –

$$\Delta M_1 = -\frac{1}{2} e r^2 \omega ;$$

but since

$$\omega = \frac{eH\cos\theta}{2m},$$

$$\Delta M_1 = -\frac{e^2 r^2 H\cos\theta}{4m},$$

the component of which in the direction of the field is therefore

$$\Delta M_1' = -\frac{e^2 r^2 H\cos^2\theta}{4m}.$$

Similarly the diamagnetism in the direction of the field corresponding to the latter component is

$$\Delta M_2' = -\frac{e^2 r^2 H\sin^2\theta}{4m},$$

(1) Phil. Mag. **6**, (1903), 673. (2) Ann. d. Phys. **9**, (1902), 115.

where $\bar{r}$ is the mean radius of the orbit of the electron, which in virtue of the precessional motion due to the component field, is assumed to make a circular motion about the field, and is equal to $r/\sqrt{2}$. Hence the resultant diamagnetism will be:—

$$\Delta M = \Delta M_1' + \Delta M_2'$$

$$= -\frac{e^2 r^2 H}{4m}\left(\cos^2\theta + \frac{1}{2}\sin^2\theta\right). \qquad \text{(I)}$$

From the above it is to be concluded that the ferro- and paramagnetism cannot be explained by a system of revolving electrons, but the diamagnetism can be. It is well known that the atoms constituting matter each contain a system of revolving electrons, and it is usual to assume that the magnetic properties of matter are due to these revolving electrons; but this assumption has now been shown to be untenable.

Consider next the case of the magnetization of a molecule consisting of two or more atoms; for the sake of simplicity, take the model known as a hydrogen molecule, in which two electrons are revolving on a common circular orbit in diametrically opposite positions, and two protons symmetrically situated on the axis of the orbit on opposite sides. This system may be considered as a gyroscope with a constant magnetic moment. When an external field acts on the molecule in a direction making an angle with the axis of the revolving electrons, this molecule will make a precessional motion combined with a nutation. The limits of this nutational motion are restricted to a range between the initial angle and a certain angle less than this, and therefore this motion contributes to the magnetization to a certain extent. The precessional motion is of such a nature that it gives rise to a magnetic moment opposite to the field. Hence the resultant magnetic moment in the direction of the field will be the difference of the above two, that is, the magnetizations due to the nutational and precessional motions.

Now the magnetic moment and the moment of momentum of the system of revolving electrons n in number are respectively given by

$$M = \frac{\pi n e r^2}{T} \quad \text{and} \quad p = \frac{2\pi m n r^2}{T};$$

hence

$$p = \frac{2m}{e} M.$$

But according to the theory of the gyroscope, the moment of momentum in the direction of the field remains unchanged by the application of the field, and therefore from the above relation it is to be concluded that the magnetic moment of the molecule in the same direction is not changed by the action of the magnetic field, that is, the magnetization due to the nutation is just equal and opposite to that due to the precession.

In the above consideration, it has been assumed that the orbital motion of the electrons is not in the least affected by the action of the field, but as a matter of fact it undergoes a slight change in their velocity, so that this change results in the diamagnetism of the molecule. Hence, it is again to be concluded that the above model of a hydrogen molecule cannot be magnetized by the magnetic field, but that it gives rise to the diamagnetic phenomenon.

The model of a hydrogen molecule is usually considered to undergo a paramagnetic effect under the action of a magnetic field; but this conclusion is not based on any dynamical theory of electrons, and follows naturally as the result of the application of the distribution law to the case of the gas under consideration.

The above conclusions can be extended to the case of more complicated molecules; hence we arrive at the important conclusion that an atom or a molecule consisting of a system of revolving electrons, which has a definite magnetic moment, cannot be magnetized by a magnetic field, but gives rise to the phenomenon of diamagnetism.

51. The Origin of the Ferromagnetism and the Paramagnetism.

Having thus far confirmed the fact that a system of revolving electrons in atoms or molecules has a definite magnetic moment, but that this fact cannot explain the origin of the ferro- and paramagnetism, the next step is to find the true explanation for these phenomena.

The atoms consist each of a nucleus and a system of revolving electrons, and as we have already remarked, the nucleus itself also contains a certain number of revolving electrons, which is nearly equal to the atomic weight diminished by the atomic number. The existence of these innermost electrons is confirmed by the radioactive phenomena; thus, the β-rays accompanying the radioactive disintegration are nothing but the electrons pro-

jected from the nucleus. The velocity of these electrons is extremely great, approaching the velocity of light. These swiftest electrons, according to Rutherford, are also revolving in an extremely small circular orbit within the nucleus, and are constrained by a number of protons to revolve in a fixed orbit round the centre. It is here to be noted that if these electrons are rapidly revolving with a high velocity approaching that of light, the angular momentum of the nucleus is so great that a very strong magnetic field can only produce an extremely small turning towards the field, as the theory of the gyroscope shows. To avoid this difficulty, it is assumed with Rutherford that just outside these revolving electrons, but still within the nucleus, a certain number of protons are revolving in a circular orbit, but in the opposite direction, and hence the angular momenta due to the protons and to the electrons nearly or completely annul each other, while the magnetic moment is slightly increased by the revolving protons. Under the action of the magnetic field, such an atom will tend to turn towards the field, this being analogous to the motion of a circuit made by a metallic wire carrying an electric current, that is, the atom can be magnetized by a magnetic field. Hence if the magnitude of magnetic moment due to these revolving electrons can account for that of the magnetic moment of an atom of a magnetic substance, then the origin of the magnetism may be safely attributed to the nuclear electrons.

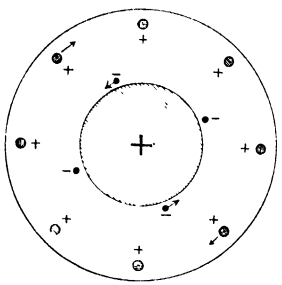


Fig. 180.

Now the magnetic moment M due to the innermost electrons may be calculated as follows:—

$$M = \frac{1}{2} nerv;$$

taking $r = 5 \times 10^{-12}$ cm, $v = 2 \times 10^{-10}$ cm/sec.,

$e = 1.59 \times 10^{-20}$ E. M. U. and $n = 30$

for iron atoms, we have

$$M = 2.4 \times 10^{-20},$$

which agrees roughly with the observed value 2.05×10^{-20} for one iron atom. Thus the above calculation shows that the nuclear electrons are sufficient to account for the ferro- and paramagnetic properties of matter.

Solids are generally crystals or crystalline substances, and the atoms constituting them are distributed along their space-lattice, and are executing thermal motions of translational and rotational vibrations, a free continuous rotation being here excluded; the period of these vibrations is probably common to them, and their amplitude increases with the rise of temperature. Though the translational vibration does not appreciably affect the magnetization, the rotational vibration is intimately connected with it.

In the case of ferromagnetic substances, the thermal agitation at ordinary temperature is probably very small and hence the atoms constituting these substances form an immense number of elementary groups, in each of which the magnetic axes of these atoms, by virtue of their mutual action, take the same orientation, along the axes of the space-lattice in accordance with the principle of the minimum energy; but the directions of the magnetic axes differ from one group to the other, its distribution as a whole being uniform in all directions of the lattice. Hence when no external field acts on the substance, the magnetic moment vanishes on the whole. In the case of para- and diamagnetic substances, the effect of thermal agitation is assumed to be so great that the magnetic axes of the nuclei cannot form elementary groups as in the case of ferromagnetic substances, and at any instant they are distributed uniformly in all directions.

In ferromagnetic substances, both the resultant angular momentum of the nucleus and the thermal effect are assumed to be very small, and hence, under the action of the field, the atoms will turn in the direction of the field against their mutual magnetic action, thus giving rise to the ferromagnetic phenomenon. In paramagnetic substances, the resultant angular momentum of the nucleus has a certain magnitude, and hence under the action of the magnetic field, its gyrostatic effect combined with the thermal agitation and the mutual action will resist the turning of the atoms in the direction of the field. In diamagnetic substances, the annihilation of the angular momentum within the nucleus is very incomplete, and hence its gyrostatic effect is so great that the nucleus cannot be deflected towards the field in any appreciable degree by the application of the field. On the other hand, the outer or the optical electrons will undergo the Larmor precession and thus give rise to the diamagnetism, as remarked in the foregoing section. Hence in this theory, even the diamagnetic atoms have a definite magnetic moment.

52. Langevin's Theory of Paramagnetic Gases.

In the above, we have explained the paramagnetism as caused by the revolving electrons in the nucleus, its gyrostatic as well as the temperature effect acting as the resistance to magnetization. The question how much the magnetization is affected by temperature, was first solved statistically by P. Langevin in the case of diatomic gases. Starting from the assumption that all the molecules of the gas have each a constant magnetic moment, he developed his theory[1] in the following way:—

Construct a unit sphere with its centre at the point o, and place the south poles of all the magnetic molecules at the same point o; then the lines drawn in the direction of the north pole will each meet the sphere at a point. If the magnetic field is not applied, these points will be uniformly distributed all over the sphere, but if applied, each molecule tends to point in the direction of the field, and the uniform distribution is slightly disturbed, the points on the sphere becoming a little denser in the direction of the field. Langevin assumes this distribution of the points on the sphere to be the same as that of the density of a gas subjected to the force of gravity, that is, to be the Maxwell-Boltzmann distribution. If we denote the potential energy of a gas molecule at a certain height by W and the gas constant for one molecule by r, the number of gas molecules at the given height will be proportional to $e^{-W/rT}$, T being the absolute temperature.

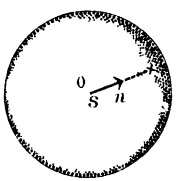


Fig. 181.

In the case of a molecular magnet placed in a magnetic field H, its potential energy is equal to $-MH\cos\alpha$, where M is the moment of the molecular magnet and α the angle between the field and the magnetic axis. If we denote by dn the number of molecules whose magnetic axis falls between two cones having vertical angles α and $\alpha+d\alpha$, we have

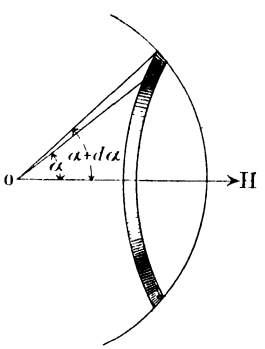


Fig. 182.

$$dn = ke^{\frac{HM\cos\alpha}{rT}}\, d\omega,$$

where $$d\omega = 2\pi \sin\alpha\, d\alpha,$$

(1) P. Langevin, Ann. de Chim. et Phys. **5**, (1905), 70.

where k is a constant and $d\omega$ the area of the zone on the unit sphere intercepted between the above two cones. If we put

$$\frac{MH}{rT} = a,$$

then

$$dn = 2\pi k\, e^{a\cos\alpha} \sin\alpha\, d\alpha.$$

Integrating this expression with respect to α between the limits 0 and π, we get the total number of molecules n. That is,

$$n = 2\pi k \int_0^{\pi} e^{a\cos\alpha} \sin\alpha\, d\alpha = \frac{4\pi k}{a} \sinh a;$$

consequently,

$$k = \frac{an}{4\pi \sinh a}.$$

If n be the number of molecules in one gram molecule, the intensity of magnetization will be

$$\sigma = \int_0^{\pi} M\cos\alpha\, dn = nM\left(\coth a - \frac{1}{a}\right) = \sigma_0 L(a), \qquad \text{(II)}$$

where σ_0 is the saturation value of magnetization and $L(a)$ a function of a. $L(a)$ is zero for $a=0$ and increases gradually as the value of a increases, tending to an asymptotic value 1, as shown in Fig. 183. Now if R be the gas-constant,

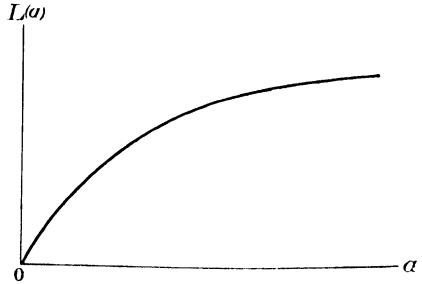


Fig. 183.

$$a = \frac{MH}{rT} = \frac{\sigma_0 H}{RT}.$$

Here a is very small, unless the temperature is very low. Hence if we expand $L(a)$ into a power series of a and neglect the terms higher than the third, we get

$$L(a) = \frac{1}{3} a.$$

Therefore

$$\frac{\sigma}{\sigma_0} = \frac{1}{3} a = \frac{\sigma_0 H}{3RT},$$

or

$$\sigma = \frac{\sigma_0^2 H}{3RT}; \qquad \text{(III)}$$

since $\sigma/H = \chi_m$, we obtain Curie's law

$$\chi_m T = \frac{\sigma_0^2}{3R} = const. \qquad (IV)$$

Thus we see that for paramagnetic gases, Curie's law stands on a theoretical basis. But for liquids and solids, in which the molecules exert a mutual action on each other, the above theory is not applicable without some modification. As a matter of fact, experiments show that this law does not hold good for solids and liquids.

In the above theory, it is assumed that for each of the elementary magnets every orientation is possible, whereas, according to the quantum theory, only certain angles with reference to the direction of the magnetic field are permissible, since the presence of the field gives a reference line for quantising the directions of the magnets in space. But it should be remarked that the quantising of the orientation in gas molecules is only allowable at an extremely low temperature or pressure, where the effect of thermal impacts on the orientation of the molecules is very small. At an ordinary temperature and pressure, gas molecules are undergoing an immense number of impacts every second, and their orientations are changed by each impact, so that before each molecule has time to assumes its quantised orientation, it may be deflected in other directions by impacts, and hence the possibility of each molecule assuming the quantised orientation is very small. Thus, at an ordinary temperature and pressure, the quantised orientation of molecules is hardly possible.

The fact that the specific heat of different gases actually determined agrees with the result of the theoretical deduction from the classical dynamics also confirms the above view; for these gases are always subjected to the earth magnetic field, and hence if the quantised orientation of the gas molecules does actually take place irrespective of the thermal impacts and the strength of the magnetic field, their specific heat should be more or less different from the result given by the classical dynamics; but actually such is not the case.

53. Deduction of Langevin's Formula from the Theory of Gyroscope.

Let us suppose[1] that a magnetic field acts on a gas molecule which is rotating about its magnetic axis, its south pole 0 being fixed, and let the magnetic moment of the molecule be $M=mr$,

(1) K. Honda and J. Okubo, Sci. Rep. **7**, (1918), 141; W. Weaver, Phys. Rev. **16**, (1920), 438.

m and r being its pole-strength and pole-distance respectively. Again let α be the angle between the field H and the magnetic axis of the molecule. We assume for the sake of simplicity the form of the molecule to deviate little from a sphere, so that as a first approximation, the three principal moments of inertia may be taken as equal to each other.

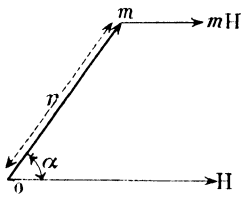


Fig. 184.

We know from the theory of gyroscope,

(i) the angular momentum about oH remains constant,

(ii) the sum of the kinetic and potential energies of the spinning molecule is constant.

Hence using the following notations,

K = moment of inertia about the magnetic axis,
ω = angular velocity about the same axis,
$\dot{\psi}$ = precessional velocity,
$\dot{\theta}$ = nutational velocity,

we have,

$$K\omega \cos\theta + K\dot{\psi} \sin^2\theta = K\omega \cos\alpha,$$

$$\frac{1}{2} K\omega^2 + \frac{1}{2} K\dot{\psi}^2 \sin^2\theta + \frac{1}{2} K\dot{\theta}^2 - MH\cos\theta = \frac{1}{2} K\omega^2 - MH\cos\alpha;$$

$$\therefore \qquad \dot{\psi} = \frac{\omega(\cos\alpha - \cos\theta)}{\sin^2\theta},$$

$$\omega^2(\cos\theta - \cos\alpha)^2 + \dot{\theta}^2 \sin^2\theta = \frac{2MH}{K}(\cos\theta - \cos\alpha)\sin^2\theta.$$

Hence the limits of the nutational angle, θ and θ', are given by

$$\theta = \alpha,$$

and

$$\cos\theta' = -\frac{1}{a} + \sqrt{1 + \frac{2}{a}\cos\alpha + \frac{1}{a^2}},$$

where

$$a = \frac{4HM}{K\omega^2}.$$

The positive value of the radical is taken as the possible root. For small values of a, $\cos\theta'$ may be written thus:

$$\cos\theta' = \cos\alpha + \frac{1}{2} a(1 - \cos^2\alpha) + \frac{1}{2} a^2(\cos^3\alpha - \cos\alpha) + \cdots$$

Thus the spinning molecule makes a nutational motion between α and θ',

Since the angular velocity is very large and the applied couple very small, the precession is very slow and the nutation rapid. In this case, the motion may be described as a steady precession combined with a nutation in a circle about a point which advances with the precession. This precession takes place with an angle of inclination to the direction of the field lying halfway between α and θ'. Since this circular motion is harmonic, the time mean will be the same as if the axis precessed steadily at an angle equal to the mean of α and θ'. Hence the magnetic moment for a gram molecule of the gas is given by

$$\sigma = \frac{Mn}{2}\int_0^{\pi} \cos\left(\frac{\theta'+\alpha}{2}\right)\sin\alpha\, d\alpha,$$

or

$$\frac{\sigma}{\sigma_0} = \frac{1}{2}\int_0^{\pi}\left(\cos\frac{\theta'}{2}\cos\frac{\alpha}{2} - \sin\frac{\theta'}{2}\sin\frac{\alpha}{2}\right)\sin\alpha\, d\alpha.$$

For small values of a, we have

$$\cos\frac{\theta'}{2} = \cos\frac{\alpha}{2} + \frac{a}{2}\cos\frac{\alpha}{2}\sin^2\frac{\alpha}{2} + \cdots,$$

$$\sin\frac{\theta'}{2} = \sin\frac{\alpha}{2} - \frac{a}{2}\sin\frac{\alpha}{2}\cos^2\frac{\alpha}{2} + \cdots;$$

hence

$$\frac{\sigma}{\sigma_0} = \frac{1}{2}\int_0^{\pi}\left(\cos\alpha + \frac{a}{4}\sin^2\alpha\right)\sin\alpha\, d\alpha$$

$$= \frac{a}{6}$$

$$= \frac{2MH}{3K\omega^2} = \frac{1}{3}\,\frac{MH}{\frac{K\omega^2}{2}}. \qquad \text{(V)}$$

Since $MH/\frac{1}{2}K\omega^2$ corresponds to a in the Langevin theory, we have arrived at the same result. It is here to be noted that Langevin considers the case of diatomic gases and therefore takes into account the rotational energy of a molecule for two degrees of freedom, that is, rT, but in our case, only the energy for the rotation about the magnetic axis of a molecule comes into consideration. Thus to assume Langevin's distribution function for the gas molecules is equivalent to giving each molecule a rotation round its magnetic axis with an energy rT.

54. Paramagnetic Solids.

In solid substances, atoms are distributed along the space-lattice belonging to their crystallographic system and exert a mutual action upon each other, and when acted on by an external field, their mutual action prevents them from turning in the direction of the field. This mutual action affects the orientation of the atoms in the same way as the thermal agitation does. Hence to take this action into account statistically is equivalent to giving these atoms an increase of the kinetic energy denoted by φ. This φ is a quantity which is constant, or diminishes slowly with the rise of temperature.

As we have already remarked, the rotational vibration of the atoms has an important connection with magnetization; the energy of this vibration is generally small as compared to that of free continuous rotation, that is, rT, and hence it may be denoted by srT, s being a small fraction, which may be assumed to depend on the shape of the atoms. Again if there remains any resultant angular momentum as the difference between the two opposite rotations of electrons and protons in the nucleus, the energy due to the nuclear rotation Q has to be added to that of the rotational vibration. Thus in the case of solid substances, we assume

$$dn = k\, exp.\left(\frac{nMH\cos\alpha}{sRT+Q+\varphi}\right)d\omega;$$

putting

$$a' = \frac{nMH}{sRT+Q+\varphi} = \frac{\sigma_0 H}{sRT+Q+\varphi},$$

we obtain as in the former case,

$$\sigma = \sigma_0 L(a').$$

For a small value of a',

$$\frac{\sigma}{\sigma_0} = \frac{1}{3}a' = \frac{\sigma_0 H}{3(sRT+Q+\varphi)}.$$

If we put $\sigma/H = \chi_m$, then

$$\chi_m = \frac{\sigma_0^2}{3(sRT+Q+\varphi)}; \qquad \text{(VI)}$$

or, putting

$$\Delta = \frac{1}{sR}(Q+\varphi),$$

$$\chi_m(T+\Delta) = \frac{\sigma_0^2}{3sR} = const. \qquad \text{(VII)}$$

If the shape of the atoms deviates considerably from a sphere, the first term sRT in the denominator of the expression of $\chi_{\prime\prime\prime}$ becomes important as compared with the second and third terms. Since Q is independent of temperature and φ diminishes only slightly with the rise of temperature, Δ may be considered to be constant. In this case expression (VII) shows the relation which has been experimentally found by K. Onnes and A. Perrier.

If the shape of the atoms is nearly spherical, the first term sRT is very small and may be neglected as compared to $Q+\varphi$, then we have

$$\chi_{\prime\prime\prime} = \frac{1}{3}\frac{\sigma_0^2}{Q+\varphi}. \qquad \text{(VIII)}$$

Hence the susceptibility remains constant or slowly increases with the rise of temperature, according as Q is very large or comparable with φ. As we have already seen, such variation of susceptibility with temperature is actually found in the case of chemical elements and compounds.

R. Gans[1] made several attempts to improve the theory of dia- and paramagnetism; but, strictly speaking, his theory is only applicable to the case of gas molecules, where the mutual action does not exist. In a later paper, Gans[2] developed a theory of paramagnetism by taking the mutual action into account in terms of a molecular field, which made the theory applicable to solids. Though his theory gives relations which in some cases agree well with experimental data, the scope of the present work does not permit us to enter into the details of the papers.

From the determination of the specific heat at very low temperatures, the view has been confirmed that in the case of molecules at low temperatures, the law of the equipartition of energy does not apply, but instead, a quantum distribution of energy has to be assumed. Hence, at very low temperatures, the formulas so far deduced require some modification. Oosterhuis[3] was the first to propose a modification of Langevin's equipartition formula by replacing the kinetic energy rT by

$$\frac{h\nu}{e^{\frac{h\nu}{rT}}-1} + \frac{1}{2}h\nu,$$

and for very low temperatures he arrived at the relation (VII) experi-

(1) R. Gans, Gött. Nachrich. (1910), 197; Ditto (1911), 118; Ann. Phys. **49**, (1916), 149. (2) R. Gans, Ann. Phys., **50**, (1916), 163. (3) E. Oosterhuis; Phys. Zeits. **14**, (1913), 862.

mentally found by K. Onnes and A. Perrier.

Instead of assuming a common angular velocity for all molecules as in the above, Keesom[1] considered the molecular rotation to be resolved into a system of standing elastic waves, as was done by Debye in his theory of specific heat. His theory shows a fairly good agreement with experiment, in fact to about the same degree as that of Oosterhuis.

In his paper "Ueber Paramagnetismus[2]," R. Gans proposed a quantum modification in order to arrive at a theory which would be applicable to paramagnetic substances at low temperatures, and obtained an expression for the susceptibility, which is in some cases in a good agreement with experimental results.

Von Weyssenhoff[3], F. Reiche[4], S. Rotszajn[5] and A. Smekal[6] also made investigations of the same subject. To enter into the details of their theories would carry us too far beyond the scope of the present work and the reader who is interested in the matter is referred to the original papers. But a brief remark will not be out of place. Of these theories, the most satisfactory is probably that of Reiche, which, so far as the fundamental assumptions and the final results are concerned, is the same as that of Rotszajn. From the standpoint of theoretical development, von Weyssenhoff's theory is also satisfactory, but his fundamental assmptions are somewhat artificial. The theory of Oosterhuis, which is founded upon far simpler assumptions, nevertheless showing a fairly good agreement with experiments, may be taken as the representative of the quantum theories. Except Gans' theory, none of them takes the mutual action of the molecules into account and in this respect they are defective.

55. Diamagnetic Substances.

It has already been stated in section 50 that the origin of the diamagnetism is attributable to the outer or optical electrons, which are rapidly revolving round the nucleus of atoms, the Larmor precession giving rise to the said effect. P. Langevin was the first to develope the theory of diamagnetism from the electron theory of matter,

(1) W. H. Keesom, Phy. Zeits. **15**, (1914), 8. (2) R. Gans, Ann. Phys. **50**, (1916), 163. (3) J. von Weyssenhoff, Ann. Phys. **51**, (1916), 285. (4) F. Reiche, Ann. Phys. **54**, (1917), 401. (5) S. Rotszajn, Ann. Phys. **57**, (1918), 81. (6) A. Smekal, Ann. Phys. **57**, (1918), 376.

which was published in the same paper[1] as that of paramagnetism.

Without following his method of calculation, let us first suppose that an electron is rapidly revolving in a circular orbit of radius r and that a magnetic field H acts in the direction making an angle θ with the axes of the orbit; then the diamagnetism in the direction of the field is, by (I) in section 50, given by

$$\Delta M = -\frac{e^2r^2H}{4m}\left(\cos^2\theta + \frac{1}{2}\sin^2\theta\right).$$

Let the number of revolving electrons contained in a unit volume, whose magnetic axes make with the direction of the field an angle lying between θ and $\theta+d\theta$ be dn, n being the total number of electrons, then the intensity of diamagnetism will be

$$I = -\frac{e^2r^2H}{4m}\int_0^{\pi}\left(\cos^2\theta + \frac{1}{2}\sin^2\theta\right)dn.$$

Since the directions of the axes of the orbital motion of atoms are uniformly distributed, we have

$$dn = \frac{n}{2}\sin\theta\, d\theta.$$

Hence, integrating, we get

$$I = -\frac{ne^2r^2H}{6m} = -\frac{nm}{6}\left(\frac{e}{m}\right)^2 r^2H.$$

$$\therefore \qquad \kappa = -\frac{nm}{6}\left(\frac{e}{m}\right)^2 r^2.$$

If in each atom several electrons with different radii be present, we have

$$\kappa = -\frac{nm}{6}\left(\frac{e}{m}\right)^2 \Sigma r^2, \tag{IX}$$

where the summation is to be extended to all the orbits in an atom.

As we have shown above, the diamagnetism is caused by the outer or optical electrons and therefore the same effect must

(1) P. Langevin, Ann. de Chim. et Phys. **5**, (1905), 70.

also exist in the case of the magnetization of paramagnetic substances, the origin of which is to be attributed to the nuclear electrons. Hence as a general case, we have for the intensity of magnetization I

$$I = \frac{I_0^2 H}{3(sNT+Q+\varphi)} - \frac{nm}{6}\left(\frac{e}{m}\right)^2 \Sigma r^2 H, \qquad \text{(X)}$$

where I_0 is the intensity of saturation magnetization, and N the gas constant per unit volume. If κ_p and κ_d denote respectively the para- and diamagnetic susceptibilities, we have

$$\kappa = \kappa_p - \kappa_d.$$

Thus the susceptibility κ actually observable is the difference between κ_p and κ_d, which may be positive or negative, according as $\kappa_p \gtrless \kappa_d$. In either case, the atoms have each a definite magnetic moment.

It is well known that the Zeeman effect in optics is due to the change in the orbital motion of the optical electrons caused by the magnetic field, which is also the origin of the diamagnetism as explained above. In other words, the Zeeman effect and the diamagnetism are different aspects of one and the same process going on in atoms.

The expression (X) above given for the magnetization explains many observed facts quite satisfactorily. As we have already remarked, many thermomagnetic properties of paramagnetic substances, such as the empirical law $\chi(T+\Delta) = const.$, the constant susceptibility, the increasing susceptibility, etc., can be explained quite naturally by the above expression.

Since the Langevin diamagnetism is caused by the orbital motion of the outer electrons, it is hardly affected by temperature, while the diamagnetism actually observable, being the difference between the para- and diamagnetisms, may sometimes vary with temperature in virtue of the variation of the paramagnetic term. Thus the decrease of diamagnetism with the rise of temperature is probably due to the increasing paramagnetism, which is caused by a gradual decrease of the mutual action accompanying the rise of temperature.

The observed susceptibility in diamagnetic substances, is given by

$$\kappa = \kappa_p - \kappa_d = -(\kappa_d - \kappa_p);$$

hence the paramagnetic term in the expression (X) must be small, and therefore the quantity Q which is independent of temperature, will be very large as compared with sNT and φ, so that the paramagnetic term and therefore the diamagnetism, hardly varies with temperature, which fact is confirmed by actual experiments.

Consider next the change in magnetic susceptibility during allotropic change or melting. In the paramagnetic term κ_p, I_0 and Q are both constants characteristic of the nucleus of the atoms and therefore do not vary at all during these changes. In the case of the allotropic change, sNT and φ may vary more or less, both increasing when a contraction of volume occurs, and decreasing in the opposite case; in the case of melting, sNT probably increases to become NT and φ vanishes, so that $sNT+\varphi$ may increase or decrease.

As regards the diamagnetic term κ_d, none of the quantities except Σr^2 changes with the allotropic transformation or the melting. In solids, the diameter of the orbital motion of electrons is comparable with the spacing of the lattice, and hence when this diameter approaches or is greater than the spacing, some of the electrons in the outermost rings will be deflected by neighbouring atoms from their own orbit and stray off as free electrons among atoms, the latter electrons thus playing the principal rôle in the conduction of electricity. This ingenious idea is due to K. Höjendahl[1] and J. Frenkel[2]. Accordingly, when a contraction of volume takes place during an allotropic change or melting, the free electrons increase at the cost of the bound electrons; but when an expansion of volume takes place, the reverse is the case. In the former case Σr^2 or the Langevin diamagnetism decreases and the electric conductivity increases, and in the latter, the diamagnetism increases and the conductivity decreases.

From the above considerations the following qualitative relations between the change of magnetic susceptibility $\partial\kappa$, the change of volume ∂V and the change of electric resistance ∂R during transformation and melting may be expected:—

Phase change	∂V	$\partial\kappa_p$	$\partial\kappa_d$	∂R
Allotropic change	$\begin{matrix}+\\-\end{matrix}$	$\begin{matrix}+\\-\end{matrix}$	$\begin{matrix}+\\-\end{matrix}$	$\begin{matrix}+\\-\end{matrix}$
Melting	$\begin{matrix}+\\-\end{matrix}$	$\begin{matrix}\pm\\\pm\end{matrix}$	$\begin{matrix}+\\-\end{matrix}$	$\begin{matrix}+\\-\end{matrix}$

To compare these conclusions with the result of observations, take the case of the allotropic changes A_3 and A_4 in iron; in this case, the diamagnetic susceptibility is very small in comparison

(1) Kristian Höjendahl, Phil. Mag. **48**, (1924), 349. (2) J. Frenkel, Zeit. f. Phys. **29**, (1924), 214.

with the paramagnetic one, and therefore can be neglected. As we have shown in sections 31 and 32, during heating α iron is transformed into γ iron at 910°, which is again transformed into α iron at 1400°. With the first transformation, there is a volume contraction and a decrease of the paramagnetic susceptibility and with the second a volume expansion and an increase of susceptibility take place. With regard to the electric resistance in the vicinity of the A_3 point, Burgess and Kelberg[1] found a decrease of resistance during this transformation; the change of resistance at the A_4 point has not yet been determined. So far the agreement between the theory and the experiment is complete.

Consider next the case of thallium; this metal is diamagnetic and undergoes an allotropic change at 225° from the α to the β form. Since in diamagnetic substances, κ_p should be very small as compared with κ_d, the sign of $\partial\kappa$ generally depends on that of $\partial\kappa_d$. Now in thallium ∂V is negative and hence $\partial\kappa$ and ∂R should be negative; this conclusion agrees with the results of experiment[2]. Grey tin is also transformed into white tin at 18°, and a large contraction and a marked diminution of diamagnetism take place in it; in fact during this transformation, the diamagnetism changes into a paramagnetism (Fig. 139).

Lastly consider the case of melting. Generally, during melting the volume of an element with the exception of bismuth and antimony increases. As has already been remarked, the paramagnetic term κ_p may increase or decrease on melting. In the diamagnetic term, during melting, Σr^2 varies, increasing in the case of volume expansion and decreasing in the case of contraction, so that the Langevin diamagnetic term generally increases with the exception of the cases of bismuth and antimony, for which the susceptibility decreases. Hence the resultant change of susceptibility may be positive or negative.

In the case of tin, during melting the paramagnetic term probably decreases and the diamagnetic term numerically increases, so that the resultant decrease must be considerable; in fact its susceptibility actually changes from the positive to the negative value. In most of the diamagnetic elements, the diamagnetic susceptibility slightly decreases during melting, as the result of a similar change in κ_p and κ_d, but in a few cases, such as gold and silver, it increases

(1) Bureau of Standards, **11**, (1915), 457. (2) M. Werner, Zeits. anorg. Chem. **83**, (1913), 275.

during melting. In these elements, the electrical resistance[1] is increased by melting, as the result of the diminution of free electrons due to the volume expansion during melting. In the case of bismuth and antimony, the volume contraction takes place during melting, and therefore the electric resistance must considerably decrease as the result of the increase of free electrons. On account of the same cause, the diamagnetic susceptibility must decrease during melting; this conclusion is actually confirmed by experiment.

As we have already remarked, in the susceptibility-atomic number curve given in section 40, the para- and diamagnetic susceptibilities of elements form a continuous curve in crossing the zero line, in spit of the fact that these two kinds of magnetism are quite different from each other in origin. This peculiar fact is easily realised from the expression $\kappa=\kappa_p-\kappa_d$, no further explanation being needed.

Pascal found that in the case of compounds of a similar constitution, the molecular diamagnetic susceptibility is the sum of the atomic susceptibilities. This law can be easily explained by the present theory; because in this case the first paramagnetic term must be very small as compared with the second term, and the condition of a similarity of constitution signifies that for each component an expression of the form (IX) holds, and hence the resultant is given as the sum of such expressions.

The fact that a diamagnetic compound may be formed from two paramagnetic atoms, and a paramagnetic compound from two diamagnetic atoms, can also be explained by the same theory. For, according to the theory, their angular momentum in the case of diamagnetic atoms, is assumed to be very large, but in the formation of the molecule from these atoms, it may happen that the same quantity as a whole greatly diminishes in magnitude, so that the molecule now behaves as paramagnetic. On the other hand, the angular momentum in the case of paramagnetic atoms is not so large as in that of diamagnetic atoms, but the formation of the molecule from these atoms may increase the same quantity and result in a diamagnetic molecule.

As has already been remarked, three diamagnetic elements, aluminium, copper and manganese, form a ferromagnetic com pounds called the Heusler alloy; this curious fact is also explain ed from our theory by assuming that the atoms of these elements have each a large angular momentum, but, when they form a

(1) H. Tsutsumi, Sci. Rep. **7**, (1918), 94.

molecule, the same quantity as a whole nearly concel each other.

So far the susceptibility refers to crystalline substances, which consist of an aggregate of a large number of minute crystals orientated uniformly in all directions. But in the case of a single crystal, the susceptibility can be obtained by assuming φ to be a periodic function of the inclination θ of the magnetic field with respect to a crystallographic axis of the crystal, that is, $\varphi=\varphi(\theta)$; hence

$$\kappa = \frac{I_0^2}{3\{sNT+Q+\varphi(\theta)\}} - \frac{mn}{6}\left(\frac{e}{m}\right)^2 \Sigma r^2.$$

In the case of a square-lattice, $\varphi(\theta)$ has a form $A+B\sin 4\theta$, and in a hexagonal lattice a form $A+B\sin 6\theta$, etc.

56. Weiss' Molecular Field.

According to Langevin's theory, the intensity of magnetization per gram molecule of a paramagnetic substance is given by

$$\sigma = \sigma_0\left(\coth a - \frac{1}{a}\right),$$

where $a=\sigma_0 H/RT$. Since R is very large, a and consequently σ are extremely small even in a very strong field which may be obtained by ordinary means. Hence if we try to explain also ferromagnetism from his theory, it becomes necessary to introduce some assumption. P. Weiss[1] assumed that each molecule of a substance under an external field is simultaneously subjected to the action of another extremely strong field in the same direction as the external field. He considered that this field, which is called by him the molecular field, is due to the mutual action of the molecules and that it is proportional to I. Accordingly, the actual field H is the sum of the external field H_0 and the molecular field H_m. Thus if N is the proportional constant, we have

$$H = H_0 + H_m = H_0 + NI.$$

In order to explain ferromagnetism from the above conception, the molecular field must at least be of a magnitude of several millions of gausses; otherwise, a and σ become very small in virtue of the large value of R, and the substance cannot pass out of the paramagnetic domain. In short, according to Weiss' theory, the distinction between ferro- and paramagnetic substances is based

(1) P. Weiss, Arch. des Sci. **31**, (1911), 401; **37**, (1914), 105, 201.

on the presence or absence of the molecular field. He also considers that even in the case where the ferromagnetic substance is not subjected to an external field, its interior is magnetized almost to saturation by the molecular field. The fact that we cannot recognize such magnetization, is explained in the following way: A ferromagnetic substance, such as iron, is composed of an immense number of fine crystals, and each of these crystals is strongly magnetized by a molecular field, but the directions of these crystals are quite arbitrary, and therefore the magnetization as a whole vanishes. The process of magnetization then consists in directing, to a greater or less degree, these groups of molecular magnets which are in equilibrium under the action of a molecular field, in the direction of an external field.

It is, however, to be noted that the above idea of the molecular field involves several difficulties. For example, it is very difficult to explain the origin of the molecular field of such an enormous magnitude of an order 10^7 gausses by means of the electron theory. To get out of this difficulty, Weiss has recently come to assume that this field is of a non-magnetic nature. If so, it cannot be treated as a magnetic force and added to a magnetic field, as was done by Weiss; for, while the magnetic force is such that it exerts a couple on elementary magnets and thus plays an important part in the magnetization of ferromagnetic substances, other kinds of forces are not. Hence the author concludes that Weiss' attempt to explain the ferromagnetism by introducing the molecular field into the Langevin paramagnetism is not appropriate. A few remarks will, however, be made on the thermomagnetic properties of ferromagnetic substances, which are given by Weiss as a confirmation of his theory.

(a) *Spontaneous Magnetization.* According to Weiss' theory, fine crystals constituting a ferromagnetic substance are magnetized almost to saturation by its own molecular field, though they are not acted on by any external field. This magnetization is called spontaneous magnetization. The relation between the spontaneous magnetization of a substance and the temperature was investigated by Weiss in the following way:—

Since below the critical point θ the substance is ferromagnetic, we have

$$H_m = NI \quad \text{and} \quad a = \frac{MH}{rT} = \frac{\sigma_0 H_m}{RT}.$$

If ρ denotes the density and W the atomic weight of the substance, then

$$H_m = \frac{N\rho\sigma}{w}$$

and

$$a = \frac{N\rho\sigma\sigma_0}{wRT}. \qquad (1)$$

At the critical point θ, a_θ is very small; hence

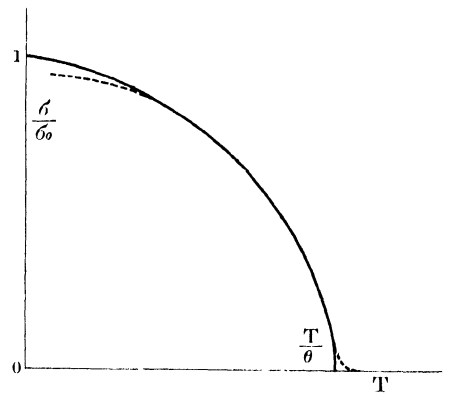


Fig. 185.

$$a_\theta = \frac{N\rho\sigma_\theta\sigma_0}{wR\theta},$$

$$\frac{\sigma_\theta}{\sigma_0} = \frac{1}{3}a_\theta = \frac{N\rho\sigma_\theta\sigma_0}{3wR\theta};$$

or

$$\theta = \frac{N\rho\sigma_0^2}{3wR}. \qquad (2)$$

From equations (1) and (2), we get the relation

$$\frac{T}{\theta} = \frac{3}{a}\frac{\sigma}{\sigma_0}. \qquad (3)$$

Below the critical point, we have

$$\frac{\sigma}{\sigma_0} = coth\, a - \frac{1}{a}. \qquad (4)$$

In equations (3) and (4), a only depends on the nature of the substance, and therefore if a be eliminated from them, a relation between T/θ and σ/σ_0 is obtained. This relation being independent of the nature of the substances is called by Weiss the equation of corresponding states. The full line in Fig. 185 is the curve expressing this relation, and the dotted line in the same figure is the experimental result for the magnetization-temperature curve of magnetite in a very strong field. We see that except at very low temperatures and also in the neighbourhood of the critical point, both curves coincide. In the case of a 34% nickel-iron, a similar coincidence is also observable. But according to the results obtained by the author[1] for iron, nickel and some nickel steels, the divergence between the theoretical and the experimental curves is very large.

(b) *Relation between Susceptibility and Temperature above the Critical Point.* Above the critical point, I is very small and therefore H_m is also very small, though it exists.

(1) Sci. Rep. **5**, (1916), 210.

In this case, the external field H_0 cannot be neglected, so that

$$T = \frac{\sigma_0(H_0+NI)}{aR}.$$

Combining this with equation (2), we have

$$\frac{T}{\theta} = \frac{3w(H_0+NI)}{a\sigma_0 N_\rho}.$$

Since a is very small above the critical point θ, we have

$$\frac{\sigma}{\sigma_0} = \frac{1}{3}a.$$

Hence

$$\frac{T}{\theta} = \frac{w(H_0+NI)}{\sigma N_\rho}.$$

Since $I=\sigma_\rho/w$, we get

$$\frac{T}{\theta} = \frac{\left(H_0+\frac{N_\rho\sigma}{w}\right)w}{\sigma N_\rho} = \frac{w}{\chi N_\rho}+1,$$

or

$$\frac{T-\theta}{\theta} = \frac{w}{\chi N_\rho},$$

or

$$\chi(T-\theta) = \frac{w\theta}{N_\rho} = \frac{\sigma_0^2}{3R}.$$

Thus the theory of the molecular field leads to a relation, the form of which is the same as that experimentally found by Weiss. But as will be shown later on, the same relation may be obtained in a very simple way without introducing the conception of the molecular field.

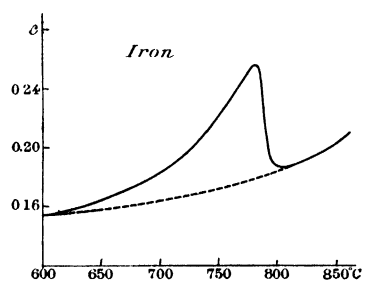


Fig. 186. (S.Umino)

(c) *Change of Specific Heat during the Magnetic Transformation.*

Weiss and Beck[1] measured the specific heat of iron, nickel and magnetite at various temperatures, ranging from the ordinary to those above the critical point, and found that the specific heat increases with the temperature, at first slowly and then rapidly, afterwards decreasing abruptly

(1) Weiss et Beck, Journ. Phys. **7**, (1908), 249; Ferrum **10** Jahrg. (1912), 1.

when approaching very near the critical point, and again increasing gradually. Recently S. Umino[1] made an accurate determination of the specific heat of iron at high temperatures in the range of the A_2 transformation and obtained a result similar to that above mentioned. Fig. 186 shows the result of his observation. Weiss explained this change of specific heat as the change of magnetic energy and found a good agreement between the theory and experiment. But, as the following considerations show, his explanation cannot be correct.

Thus consider the case of the determination of the specific heat of a ferromagnetic substance with and without a magnetic field. According to Weiss' view, the internal energy per unit of volume in these two cases are given by

$$u + \frac{1}{2}(H_m + H_0)I \quad \text{and} \quad u - \frac{1}{2}H_m I,$$

where u is the internal energy independent of magnetization; since H_0 is very small as compared with H_m, the magnetic energy is practically the same in these two cases. Hence if it is assumed, as was done by P. Weiss[2], that in the case of demagnetization by heating, the magnetic energy is wholly converted into heat, then the respective specific heats will be given by

$$c_1 = \frac{1}{\rho}\frac{du}{dt} + \frac{NI}{\rho}\frac{dI}{dt}$$

and

$$c_2 = \frac{1}{\rho}\frac{du}{dt} - \frac{NI}{\rho}\frac{dI}{dt},$$

where ρ is the density of the substance.

Since during heating dI/dt is always negative, c_1 is less than c_2 by a considerable amount, the difference increasing with the rise of temperature; during cooling, the opposite is the case. Hence if we conduct the thermal analysis with and without the magnetic field, the heat evolution or absorption during heating or cooling through the critical range must be considerably different from each other in these two cases. But according to the author's experiment[3], the heat absorption or the evolution during heating or cooling respectively is not materially changed by the action of the magnetic field. Hence the second term in the expression

(1) Kinzoku no Kenkyu **3**, (1926), 527. (2) Weiss et Beck, Journ. Phys. **7**, (1908), 249. (3) Sci. Rep. **2**, (1913), 69.

for c must be very small as compared with the first term; that is, there can exist no molecular field of such an enormous magnitude as that conceived by Weiss.

We may therefore conclude that the good coincidence between the theoretical and observed values obtained in the case of Weiss' calculation, for the change of specific heat in the critical range is merely accidental, and that his theory of the molecular field is not consistent with the observed facts.

57. Relation between Paramagnetic and Ferromagnetic Substances.

Let us first compare the intensities of magnetization in ferro- and paramagnetic substances. As a concrete example, consider the case of iron and manganese. The intensity of magnetization of iron increases, at first rapidly, with the magnetic field, and soon reaches an asymptotic value of 220 per unit mass. In the case of manganese, the intensity of magnetization is extremely small in ordinary fields, but increases proportionally with the strength of the field, its proportional constant being 10^{-5}. Hence the ratio of these two magnetizations is very large in ordinary fields; but with an increasing field, it becomes steadily less. For example, at H=100 the ratio is about 180000; but at H=20000 it is only 1100. Though the susceptibility will not remain constant in a still higher field, it is highly probable that in such a strong field, as has not been attained up to the present time, the above ratio will be far less than 100. Hence it is to be concluded that the magnetic moments of iron and manganese atoms do not differ so much from each other as is generally believed to be the case, and that, from the point of view of atomic magnetism, ferromagnetic substances do not take up any special position in relation to other substances.

Admitting that there is only a small difference in the atomic magnetism of ferro- and paramagnetic substances, what causes then the great difference of magnetizability in an ordinary field? As we have already remarked, this is attributable to the great resistance to magnetization offered by the combined effect of the thermal agitation and the angular momentum of the nucleus of the atoms. The author has already expressed his view that in the case of ferromagnetic substances the thermal agitation and the angular momentum of the nucleus are at ordinary temperature negligibly small in comparison with the mutual action, while in

the case of paramagnetic substances, the first two effects predominate over the last, the cause of the difference being attributable to that in the shape of the atoms. He also considers that the thermal agitation affecting the magnetization consists of the rotational vibration of the atoms, the amplitude of which increases with the rise of temperature; because the magnetization in the direction of the field must obviously decrease in a greater degree with increasing amplitude of the rotational vibration, becoming zero, when this vibration becomes a continuous uniform rotation.

The well known fact that at high temperatures, the ferromagnetic substance changes into a paramagnetic in consequence of the magnetic or A_2 transformation, is understood to be caused by the continuous increase of the amptitude of the rotational vibration about the magnetic axes of the atoms, which is caused by the rise of temperature. The steady increase of the rate of diminution of magnetization up to the critical temperature, is probably due to a gradual change in the shape of the atoms from a sphere-like form to an ellipsoidal one, and the heat of the transformation consists of the energies required for the deformation and the change of the rotational vibration of the atoms.

Lastly, we shall deduce Weiss' formula regarding the relation between the magnetic susceptibility of a ferromagnetic substance and the temperature above its critical point. Strictly speaking, as we have already remarked, the transformation of a ferromagnetic substance into a paramagnetic is continuous, there being no distinct point to be called the critical point; but for practical purposes, the existence of such a temperature θ may be assumed. Thus, let us assume that the substance is paramagnetic above θ and ferromagnetic below it. In the case of paramagnetic solids, which become ferromagnetic at the critical point θ, Q must be negligibly small and φ is also very small as compared with sRT. At $T=\theta$ the susceptibility will become very large and therefore in the distribution function, we have to put $sR(T-\theta)$ in place of sRT. Hence neglecting a small diamagnetic term, we obtain for the molecular susceptibility

$$\chi_m = \frac{\sigma_0^2}{3\{sR(T-\theta)+\varphi\}}, \qquad \text{(XI)}$$

or neglecting the small quantity φ in the denominator, we have

$$\chi_m (T-\theta) = \frac{\sigma_0^2}{3sR} = const. \qquad \text{(XII)}$$

This is the same relation as obtained by Weiss with the exception of the factor s, which is less than 1, and is fairly well confirmed by experiments in the case of iron, nickel, etc., except at the temperature near θ.

This relation may also be applied to the substances, which are paramagnetic at ordinary temperatures, and become ferromagnetic at very low temperatures.

58. Theory of Magnetization in Ferromagnetic Substances, I.

The theory of ferromagnetic substances was first published by Poisson. He considered that the molecules of the magnetic substance contain two kinds of magnetic fluids, positive and negative, and that by the action of a magnetic field, they seperate from each other, giving rise to the phenomenon of magnetization. Weber assumed that the molecules of the magnetic substance have a constant magnetic moment, but in the ordinary state, these magnetic axes point uniformly in all directions, so that their magnetic effect vanishes as a whole, and that if acted on by an external field, the magnetic axes tend to turn in the direction of the field against the mutual action of the molecules, resulting in the magnetization of the substance. He also assumed the mutual action of molecules to be an elastic force, and obtained the curve of magnetization as shown in Fig. 188.

Let the direction of the magnetic axis of a molecule be sn and that of a magnetic field sp; then the magnetic axis will point in the direction of the resultant R of the field H and the elastic force h produced by the rotation of the molecule, which we assume to act in the direction of the initial position of the magnetic axis. Denoting the angle between R and H by θ and the magnetic moment of the molecule by M, the component of the moment in the direction of H is $M\cos\theta$. Hence by integrating this component for all the molecules, we get the resultant moment or the intensity of magnetization of the substance. If we denote by dn the number of molecules, whose magnetic axes make with the field an angle lying between α and $\alpha+d\alpha$, then

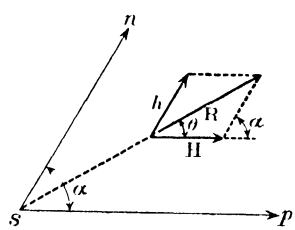


Fig. 187.

$$dn = \frac{n}{2} \sin \alpha \, d\alpha.$$

Hence the intensity of magnetization I is given by

$$I = \int_0^\pi M \cos\theta dn = \frac{I_0}{2} \int_0^\pi \cos\theta \sin\alpha \, d\alpha. \tag{1}$$

But $$R^2 = h^2 + H^2 + 2hH\cos\alpha.$$

$$\therefore \quad RdR = -hH \sin\alpha \, d\alpha \tag{2}$$

From the condition of equilibrium, we have

$$H \sin\theta = h \sin(\alpha - \theta)$$

$$= h(\sin\alpha\cos\theta - \cos\alpha\sin\theta).$$

Hence, $$\tan\theta = \frac{h\sin\alpha}{H + h\cos\alpha},$$

$$\therefore \quad \frac{1}{\cos\theta} = \frac{R}{H + h\cos\alpha}. \tag{3}$$

By substituting these values of α and θ in the expression for I, we get

$$I = -\frac{I_0}{2hH} \int_0^\pi (H + h\cos\alpha) \, dR.$$

But

$$h\cos\alpha = \frac{R^2 - H^2 - h^2}{2H}.$$

$$\therefore \quad I = -\frac{I_0}{2hH} \int \left(H + \frac{R^2 - H^2 - h^2}{2H} \right) dR$$

$$= -\frac{I_0}{4hH^2} \left(H^2 R + \frac{1}{3} R^3 - h^2 R \right).$$

Two cases are to be distinguished:—

Case i. $$H < h,$$

we have

$$R = h - H \quad \text{for} \quad \alpha = \pi,$$

$$R = h + H \quad \text{for} \quad \alpha = o.$$

Hence,
$$I = \frac{2I_0}{3h} H.$$

Thus so long as H is less than h, I increases proportionally to H.

Case ii. $H > h$,

we have
$$R = H - h \quad \text{for} \quad \alpha = \pi,$$

$$R = H + h \quad \text{for} \quad \alpha = o.$$

Therefore,
$$I = I_0 \left(1 - \frac{1}{3} \frac{h^2}{H^2}\right).$$

That is, as H increases from h, I gradually approaches a constant value I. The form of the curve is shown in Fig. 188; it represents roughly the observed curve of magnetization, but differs from the latter in not having the point of inflexion. Moreover, the above theory does not explain the magnetic hysteresis. Afterwards, Maxwell tried to modify Weber's theory to explain the hysteresis phenomenon, but he could not arrive at any satisfactory result.

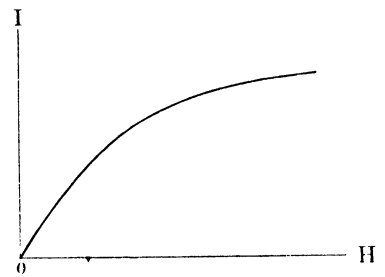


Fig. 188.

In 1890, an important advance on Weber's theory was made by Ewing[1] by introducing a very simple assumption that the mutual action of the molecules above referred to is of a magnetic nature. That is to say, in the case of ferromagnetic substances belonging to the regular system, such as iron and nickel, the atoms in each microcrystal composing such a substance are all situated in the space-lattice of the cubic system, and when no field is in action, the directions of the magnetic axes of these atoms are parallel to one of the sides of the space-lattice; but even in the same microcrystal a number of such groups may be formed.

(1) J. A. Ewing, Phil. Mag. (5), **30**, (1891), 205. See also "Magnetic Induction in Iron and other Metals."

In this orientation of the atoms, the distance between the magnetic poles of the neighbouring atoms is the smallest, so that the potential energy is the minimum, and hence the equilibrium is most stable. A group of such elementary magnets is called an elementary complex. Since the orientations of the minute crystals are uniformly distributed in all directions, the magnetic effect of a large number of such groups of the elementary magnet vanishes as a whole. When, however, an external field is applied, these magnets in each group tend to point in the direction of the field, but the deflection from the orientation of the minimum potential energy causes a quasi-elastic force tending to restore these atoms to their initial orientation, thus resisting the action of the field. Hence with the increase of the magnetic field, the magnetization becomes gradually strong. Ewing experimentally obtained the curve of magnetization and that of hysteresis by making use of a model consisting of a large number of pivoted magnets, and showed that his theory agrees well with the observed facts.

Peddie[1] assumed that magnetic atoms are of a spherical form situated in contact with each other, and that their magnetic axes all point in a certain direction. He then calculated the mutual action of these magnetic atoms and attempted to explain the result of Weiss' investigation concerning ferromagnetic crystals. Gans[2] also published a paper concerning the theory of magnetism. He assumed that an elementary magnet or "magneton" is an electrified body of revolution rotating rapidly about its axis. An elementary complex consists of a group of such magnetons distributed according to the law of probability in a space in the form of an ellipsoid, the magnetons being free to move about within the complex like a gas molecule. A ferromagnetic crystal is assumed to consist of such complexes situated at the intersections of a space-lattice, with their corresponding axes parallel. By applying the laws of statistical mechanics to a system of such complexes, he obtained a relation between the magnetic and thermal quantities similar in form to those of Weiss. From considerations of stability, he also deduced the hysteresis curve; but from his theory, the curve of magnetization possessing an inflexion point cannot be obtained.

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(1) Proc. Roy. Soc. Edinburgh, **25**, (1905), 1025; **28**, (1908), 643; **32**, (1912), 216.
(2) R. Gans, Gött. Nachr., (1910), 197; (1911), 118.

In short, all the above theories are not fully satisfactory in explaining the observed facts.

59. Theory of Magnetization in Ferromagnetic Substances, II.

Ewing's molecular theory of magnetism has been developed by J. Okubo and the author[1] for the simple case of the two-dimensional problem. As the magnetization of a ferromagnetic substance is the integral effect of the magnetization of elementary complexes, we shall first consider the magnetization of a single complex.

(A) Magnetization of a Single Complex.

Let us take one side of the square lattice as x-axis and the other side as y-axis, the initial direction of the elementary magnets being supposed to coincide with the direction of y-axis, as shown in Fig. 189. If an external field H acts on the system of magnets in a direction making an angle α with the y-axis, each of these magnets is turned through a common angle θ and is in equilibrium under the action of the external and internal forces. This internal force may be calculated in the following way: If we assume that the force acting on a magnetic pole of an atom is mainly due to eight atomic magnets situated round the atom, the action due to the remaining atoms being assumed to be negligibly small, the number of magnetic poles acting on the pole is sixteen, of which the action of eight poles cancel each other on account of their symmetrical position, and the other eight act on the same pole according to Coulomb's law.

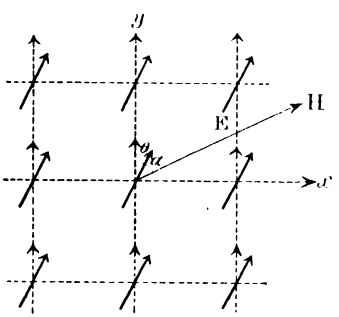


Fig. 189.

Let the resultant force be R and the component forces in the directions of x and y be respectively X and Y; also let m be the pole-strength of the elementary magnet, $2r$ its length, and $2a$ the spacing of the lattice. Then we have the following expression for the condition of equilibrium:—

(1) K. Honda and J. Okubo, Sci. Rep. **5**, (1916), 153.

$$Hm\sin(\alpha-\theta) = -Y\sin\theta + X\cos\theta$$

$$= F(\theta).$$

The values of X and Y and therefore that of $F(\theta)$ may be directly calculated. Thus

$$F(\theta) = -\frac{m}{2r^2}\left[\frac{k}{(1+k^2)^{\frac{1}{2}}}\frac{\sin\theta}{(1-p^2\cos^2\theta)^{\frac{3}{2}}}\left\{(1-p\cos\theta)^{\frac{3}{2}} - (1+p\cos\theta)^{\frac{3}{2}}\right\}\right.$$

$$+\frac{k}{(1+k^2)^{\frac{3}{2}}}\frac{\cos\theta}{(1-p^2\sin^2\theta)^{\frac{3}{2}}}\left\{(1+p\sin\theta)^{\frac{3}{2}} - (1-p\sin\theta)^{\frac{3}{2}}\right\}$$

$$+\frac{k}{(1+2k^2)^{\frac{3}{2}}}\frac{\cos\theta-\sin\theta}{[1-q^2(\cos\theta+\sin\theta)^2]^{\frac{3}{2}}}\left\{[1+q(\cos\theta+\sin\theta)]^{\frac{3}{2}}\right.$$

$$\left.-[1-q(\cos\theta+\sin\theta)]^{\frac{3}{2}}\right\}$$

$$+\frac{k}{(1+2k^2)^{\frac{3}{2}}}\frac{\cos\theta+\sin\theta}{[1-q^2(\cos\theta-\sin\theta)^2]^{\frac{3}{2}}}\left\{[1+q(\cos\theta-\sin\theta)]^{\frac{3}{2}}\right.$$

$$\left.\left.-[1-q(\cos\theta-\sin\theta)]^{\frac{3}{2}}\right\}\right],$$

where $k = \frac{a}{r}$, $p = \frac{2k}{1+k^2}$, $q = \frac{2k}{1+2k^2}$,

and $a > r$, $0 < p < 1$, $0 < q < \frac{2}{3}$.

Since p and q are both less than 1, the above expression may be expanded in their power series, that is,

$$F(\theta) = \frac{mk}{r^2}\cdot\frac{3.5.7}{2.2.2}\sin 4\theta\left\{\frac{p^3}{4(1+k^2)^{\frac{1}{2}}}\left(\frac{1}{3!}+\frac{9}{2}\frac{p^2}{5!}+\frac{9}{2}\frac{11}{2}\frac{p^4}{7!}+\cdots\right)\right.$$

$$-\frac{q^3}{(1+2k^2)^{\frac{1}{2}}}\left(\frac{1}{3!}+\frac{9}{2}\frac{q^2}{5!}+\frac{9}{2}\frac{11}{2}\frac{q^4}{7!}+\cdots\right)$$

$$-\frac{9}{2}\frac{11}{2}\left[\frac{p^7}{4(1+k^2)^{\frac{3}{2}}}\left(\frac{1}{7!}+\frac{1}{2}\frac{13}{2}\frac{p^2}{9!}+\cdots\right)\right.$$

$$\left.\left.+\frac{q^7}{(1+2k^2)^{\frac{3}{2}}}\left(\frac{1}{7!}+\frac{1}{2}\frac{13}{2}\frac{q^2}{9!}+\cdots\right)\right]\sin^2 2\theta+\cdots\right\} \quad (1)$$

For a given substance, k, m and r are all constants and the above expression is a function of θ alone. If we write

$$F(\theta) = \frac{m}{r^2} f(\theta),$$

$f(\theta)$ contains, besides θ, only k. In the above expression, $sin^2 2\theta$ is a function of $sin 4\theta$, and therefore, $F(\theta)$ is a function of $sin 4\theta$, its period being $\pi/2$. This is also to be expected from the property of the space-lattice; because, in a space-lattice consisting of squares, the directions of its two sides are both the orientations of equally stable equilibrium for the elementary magnets.

In the above calculation, the action of eight surrounding atoms only was considered; but in the second row, we have sixteen atoms, in the third twenty-four, in the fourth thirty-two, and so on. Although the number of surrounding atoms is thus increased, their distances from the pole under consideration are also increased, and hence $F(\theta)$ rapidly converges to a constant value. For example, if we compare the coefficient of $sin 4\theta$ for the cases of eight and twenty-four elementary magnets, there is only a difference of 4.5 percent between them for $k=2$.

In the case of ferromagnetic substances such as iron, nickel and cobalt, the spacing $2a$ may be considered to be much less than $2r$, and therefore k may be assumed to be at least greater than 2. In this case, the term containing $sin^2 2\theta$ is very small and may be neglected; viz. $F(\theta)$ is simplified into the following expression

$$F(\theta) = A \sin 4\theta.$$

In this expression, A is a constant containing k, m and r, and may be written as follows:

$$A = \frac{m}{r^2} \varphi(k).$$

Consequently, the condition of equilibrium is

$$H \sin(\alpha - \theta) = A \sin 4\theta.$$

If we put $H/A = h$, the above expression becomes

$$h \sin(\alpha - \theta) = \sin 4\theta. \qquad (2)$$

Now h is called the reduced field and A a function of k; $\varphi(k)$ becomes infinite for $k=1$, and rapidly decreases with the increase of k, the form of the φ, k curve resembling a hyperbola or a logarithmic curve, as shown in Fig. 190.

Having thus found the internal force, we shall next consider the magnetization of an elementary complex. The intensity of magnetization of a complex is understood to be the resultant moment of the elementary magnets constituting the complex in the direction of an applied field. If we denote the number of the magnets in a complex by n, then we have for the intensity of magnetization I

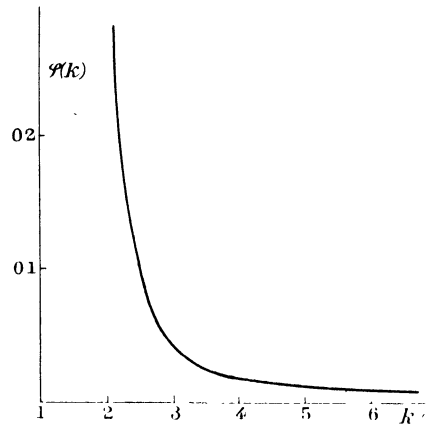


Fig. 190.

$$I = 2rnm \cos(\alpha-\theta).$$

Since $2rnm$ is the intensity of saturation magnetization I_0, we have

$$I_0 = 2rmn.$$

$$\therefore \quad I = I_0 \cos(\alpha-\theta).$$

Putting $I/I_0=i$, which we call the reduced intensity of magnetization, we have

$$i = \cos(\alpha-\theta). \tag{3}$$

This is an expression for the magnetization of a complex, the relation between α and θ being given by equation (2). If h and α are given, the intensity of magnetization i can be calculated from the above two equations (2) and (3). Since these equations do not contain any quantity depending on the nature of the substance, they apply to all ferromagnetic substances belonging to the cubic system. Hence they may be called the equation of magnetization for the corresponding states.

In order to find θ by means of equation (2), it is convenient to use the graphical method. Thus, if we draw two curves corresponding to given values of h and α, that is,

$$y = h \sin(\alpha-\theta)$$

and

$$y = \sin 4\theta,$$

θ will be given as the intersection of these curves. Substituting

this value in equation (3), the value of i is obtained. Fig. 191 shows an example; here the curve I represents $y=\sin 4\theta$ and curves a, b, c, etc. represent $y=0.6\ \sin(\alpha-\theta)$ corresponding to $\alpha=30°$, 70°, 120° and 170°. In this way, the relation between i and h for different values of α, that is, the curve of magnetization may be obtained. We shall discuss below the different cases of magnetization.

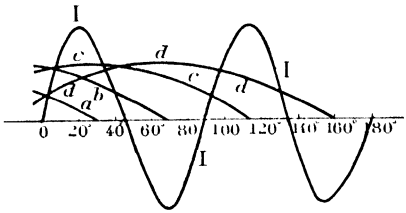


Fig. 191.

In Fig. 192, four curves representing the relation between i and h are given, taking for the angle α, 30°, 70°, 120° and 170° respectively. They give the intensity of magnetization in the direction of the respective fields, which is obtained when the magnitude of the latter is so varied that it is always in equilibrium with the internal resisting force $\sin 4\theta$. In the curve for $\alpha=30°$ the initial point a corresponds to the value of $\cos 30°$; as h increases θ becomes greater, but is always less than α, and therefore $i=\cos(30°-\theta)$ steadily increases, tending asymptotically to the value of $i=1$ with $h=\infty$. In the curve for $\alpha=70°$ the point b corresponds to the value of $\cos 70°$; as h increases from 0, θ and therefore $\sin 4\theta$ also increases. Since, however, the latter quantity reaches a maximum at $\theta=\pi/8$, h must diminish from a certain value of θ upwards,

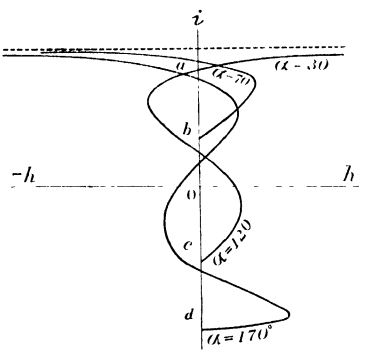


Fig. 192.

if the magnetization is to be effected statically or reversibly. At $\theta=\pi/4$, the resisting force $\sin 4\theta$ vanishes and therefore h must be diminished to zero; with a further increase of θ, $\sin 4\theta$ will change sign, and therefore h must be applied in the opposite direction, if the magnetization is to be effected reversibly. If θ approaches 70°, h becomes $-\infty$ in the limit and the magnetization tends asymptotically to unity. The curve for $\alpha=120°$, which begins at the point c on the negative side of i, passes through a maximum and a minimum of h, and coincides with the curve for $\alpha=30°$, as the value of i increases. Thc curve for $\alpha=170°$, beginning at a point d on the negative side of i, passes through two

maxima and one minimum of h with the increase of i, and approaches asymptotically to the line $i=1$.

In the ordinary case of magnetization the field is continuously increased, and therefore the magnetization is only partly reversible. But it is easy to see in what way the magnetization in the direction of the field is increased by applying a continuously increasing field.

Case i, $0<\alpha<\pi/4$: The component magnetization in the direction of the field increases with h and becomes 1 for $h=\infty$. If the field is gradually reduced, i assumes its original value, and there is no hysteresis.

Case ii, $\pi/4<\alpha<\pi/2$: In this instance, i increases with h continuously up to the maximum resisting force; here it undergoes an abrupt change, and assumes a value corresponding to a change of $\pi/2$ in the initial orientation of the elementary magnets. With a further increase of the field, i continuously increases as though the initial orientation were $\alpha-\pi/2$. If the field is reduced, i assumes a value quite different from its initial one as shown in Fig. 193, DCE; that is, a hysteresis occurs.

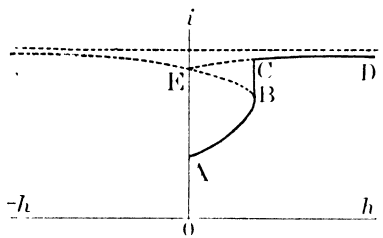


Fig. 193.

Case iii, $\pi/2<\alpha<3\pi/4$: Here i increases with h, at first continuously and then abruptly, when the resisting force reaches a maximum. After this, the curve of magnetization follows a course corresponding to the case of the initial orientation of $\alpha-\pi/2$ (Fig. 194). With the removal of the field a hysteresis is also observed.

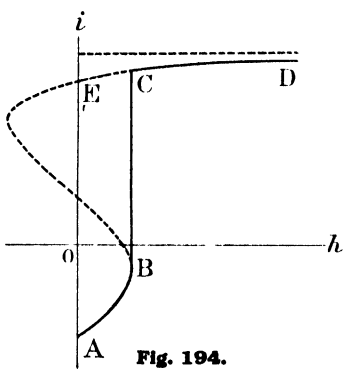


Fig. 194.

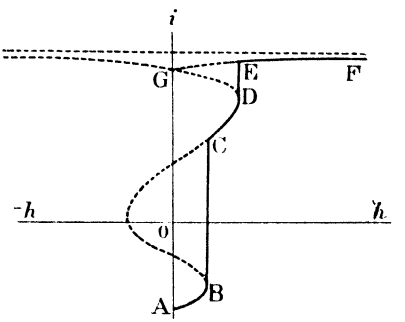


Fig. 195.

Case iv, $3\pi/4 < \alpha < \pi$: The curve of initial magnetization is the same as in the above cases. If the first maximum of the resisting force is less than the second, its subsequent magnetization undergoes twice a discontinuous increase, as shown by the curve *ABCDE* in Fig. 195; if the first maximum is greater than the second, the magnetization follows a curve *ABEF* in Fig. 196. The subsequent magnetization takes place continuously. By reducing the field, a hysteresis phenomenon (Figs. 195, 196, FEG) is observed.

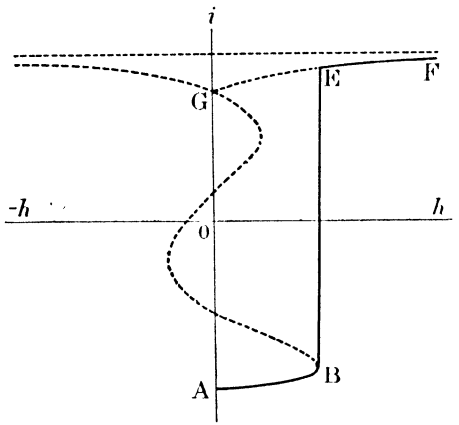


Fig. 196.

The relation between the initial orientation α and the maximum resisting force h_m can be found in the following way:—

From the equation

$$h = \frac{\sin 4\theta}{\sin(\alpha - \theta)},$$

we have

$$\frac{dh}{d\theta} = \frac{5\sin(\alpha + 3\theta) + 3\sin(\alpha - 5\theta)}{2\sin^2(\alpha - \theta)}.$$

If the value of θ corresponding to the maximum force be denoted by θ_0, we have

$$5\sin(\alpha + 3\theta_0) = 3\sin(5\theta_0 - \alpha) \tag{4}$$

and

$$h_m = \frac{\sin 4\theta_0}{\sin(\alpha - \theta_0)}. \tag{5}$$

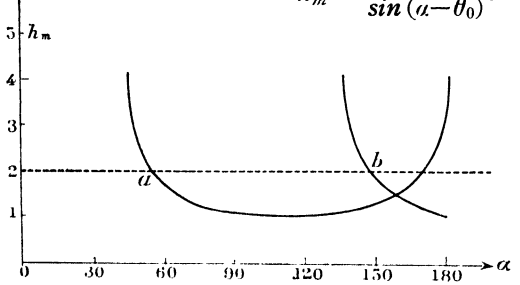


Fig. 197.

The existence of such values of θ may be understood from Fig. 192. The calculated values of h_m for different values of α are given in the following table and in Fig. 197.

α	h_m	α	h_m
45°	4.000	100°	1.025
50°	2.625	120°	1.008
60°	1.750	140°	1.137
70°	1.405	160°	1.541
80°	1.205	170°	2.018
90°	1.088	180°	4.000

Curve a in Fig. 197 refers to the first maximum; in the interval between 135° and 180° a second maximum is also possible. However, as h_m corresponding to α for the first maximum is equal to that corresponding to $\alpha+\pi/2$ for the second, curve b for the second maximum has the same form as curve a, only being displaced through $\pi/2$.

(B) Magnetization of Ferromagnetic Substances.

Hitherto, we have considered exclusively the magnetization of a single complex. We are now able to study the magnetization of a mass of ferromagnetic substance, such as iron, which consists of a large number of such elementary complexes with their magnetic axes uniformly distributed in all directions. Now, the elementary complexes are in actual cases disposed in all directions; but for the sake of simplicity of calculation, it will here be assumed that the complexes have one of their faces all parallel to a common plane, the other faces being distributed quite arbitrarily, and that the magnetic field acts parallel to this plane. The problem is then reduced to a two dimensional one. The magnetization obtained in this simple case obviously does not differ in character from that of an actual case.

Let n be the number of elementary complexes per unit volume; if there is no magnetic force acting on them, the number of complexes whose magnetic axes make with the direction of the field, an angle lying between α and $\alpha+d\alpha$, is equal to dn. Thus

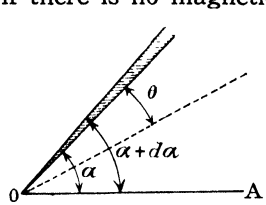


Fig. 198.

$$dn = \frac{n}{2\pi}\, d\alpha.$$

If M be the magnetic moment of a complex of an initial angle α, then its component in the direction of the field is $M\cos(\alpha-\theta)$.

Assuming M to be the same for all complexes, the total magnetization due to these complexes is

$$I = \frac{1}{2\pi}\int_{-\pi}^{+\pi} Mn\cos(\alpha-\theta)\,d\alpha = \frac{I_0}{\pi}\int_0^{\pi}\cos(\alpha-\theta)\,d\alpha,$$

where $I_0=Mn$ is the saturation value of magnetization. Hence we have

$$i = \frac{1}{\pi}\int_0^{\pi}\cos(\alpha-\theta)\,d\alpha. \qquad (6)$$

The relation connecting α and θ must, however, be different from that for a single complex. In this case, besides $A\sin 4\theta$, which measures the force due to the sixteen neighbouring poles, we must also consider the magnetic force due to the surrounding complexes. If no field acts on the substance, the resultant effect of the surrounding complexes is obviously zero; but in its magnetised state this is not the case. To calculate this force exactly is hardly possible; but it is not difficult to estimate approximately its mean effect. Since the total action of a complex on a magnet within it is the same as the sum of the effects of neighbouring magnets, those of the distant ones being very small, we may consider the external form of the complex under consideration to be a sphere, without causing sensible error in the value of $A\sin 4\theta$. The magnetic effect of other complexes on the magnet under consideration may approximately be replaced by one due to a uniform distribution of magnetization with a mean intensity in the space in which other complexes are found. As the boundary of the above complex is assumed to be a sphere, this force is $4\pi/3I$, acting in the direction of the external field, and not generally coinciding in direction with that of the axis of the magnet under consideration, and hence it exerts a couple, tending to turn the magnet in the direction of the field. Hence instead of the relation (2) we must use the following one :—

$$\left(H+\frac{4}{3}\pi I\right)\sin(\alpha-\theta) = R\sin(\alpha-\theta) = A\sin 4\theta.$$

But for a given value of H, I is a constant, so that for a while we may regard R as an external field and proceed to calculate I for different values assigned to R. After finding I, the actual field may be found by simply subtracting $4\pi/3I$ from the assigned field. Hence the same relation as (2), that is,

$$h = \frac{\sin 4\theta}{\sin(\alpha - \theta)}$$

may also be used in the present case.

If h be given, this equation gives θ in terms of h, and if this value of θ be substituted in equation (6), it gives the intensity of magnetization i in terms of h, and thus the problem is formally solved. But in actual calculation, some complications are involved, and we must separately consider the cases corresponding to several graded values of h.

(i) *External Field Small.*—First let us consider the case where h is very small; then θ is also small, and therefore $\sin 4\theta = 4\theta$. From the above equation we get

$$h(\sin\alpha - \theta\cos\alpha) = 4\theta$$

$$\therefore \quad \theta = \frac{h\sin\alpha}{4 + h\cos\alpha}.$$

Equation (6) becomes

$$\begin{aligned} i &= \frac{1}{\pi}\int_0^{\pi} (\cos\alpha + \theta\sin\alpha)\, d\alpha \\ &= \frac{1}{\pi}\int_0^{\pi} \cos\alpha\, d\alpha + \frac{h}{4\pi}\int_0^{\pi} \sin^2\alpha \left(1 + \frac{h}{4}\cos\alpha\right)^{-1} d\alpha \\ &= \frac{h}{4}\left(\frac{1}{2} + \frac{h^2}{8.4^2} + \frac{\pi}{16}\frac{h^4}{4^4} + \cdots\right) \\ &= 0.125h + 0.00195h^3 + 0.00007h^5 + \cdots \end{aligned} \tag{7}$$

As it ought to be, i is an odd function of h. If h be sufficiently small, the terms of a higher order of h than the third may be neglected, and i and h are linearly related to each other. This fact was confirmed by the experiments of Baur[1], Lord Rayleigh[2] and others. In this case the magnetization is perfectly reversible, that is, there is no hysteresis.

(ii) *External Field Large.* — Next we shall consider the case where h is large. In order to change the integration variable from α to θ, we differentiate equation (2) and obtain

(1) Baur, Inaug. Diss. Zürich, 1879; Wied. Ann. **11**, (1880), 394. (2) Rayleigh, Phil. Mag. **23**, (1887), 225; see also Ewing's Magnetic Induction, p. 124.

$$\frac{d\alpha}{d\theta} = \frac{4\cos 4\theta}{h\cos(\alpha-\theta)} + 1$$

and also

$$\cos(\alpha-\theta) = \pm\frac{1}{h}\sqrt{h^2-\sin^2 4\theta},$$

$$\therefore\quad i = \frac{1}{\pi}\int\left\{\frac{4\cos 4\theta}{h} \pm \frac{1}{h}\sqrt{h^2-\sin^2 4\theta}\right\}d\theta. \qquad (8)$$

Now during magnetization, all the complexes do not necessarily change their angle of deflection continuously; in fact, some of these complexes make an abrupt rotation. Hence in evaluating the above integral it is necessary to divide the limits of integration into several parts. If h be given, we can find from Fig. 197 the value of α for which h is equal to h_{III}; the value of θ for these values of α may then be found from equation (2). We have generally three values of α and θ; let us call them α_1, α_2, α_3 and θ_1, θ_2, θ_3. Then we have

$$\int_0^{\pi} = \int_0^{\alpha_1} + \int_{\alpha_1}^{\alpha_2} + \int_{\alpha_2}^{\alpha_3} + \int_{\alpha_3}^{\pi}.$$

In the first and fourth integrals the elementary magnets in the complexes belonging to these integrals remain stable, since in these cases the field is less than the critical value. The magnets in the complexes belonging to the second integral all lie beyond the position of stable equilibrium, and therefore the magnetization is the same as if the initial orientation of these complexes were $\alpha-\pi/2$. Hence the limit of the second integral must be changed from α_1 and α_2 to $\alpha_1-\pi/2$ and $\alpha_2-\pi/2$. In the third integral the magnets in the complexes lie beyond the first and second positions of stable equilibrium, and therefore the magnetization is the same as if the initial orientation were $\alpha-\pi$. Hence the limits of the third integral are to be changed from α_2 and α_3 to $\alpha_2-\pi$ and $\alpha_3-\pi$. If the integration variable be then changed from α to θ, we have

$$\int_0^{\pi} = \int_{\theta_1}^{\theta_1'} + \int_{\theta_2}^{\theta_2'} + \int_{\theta_3}^{\theta_3'} + \int_{\theta_4}^{\theta_4'}.$$

Now from equation (8) we have

$$i = \frac{1}{\pi h}(\sin 4\theta' - \sin 4\theta) \pm \frac{1}{\pi}\left\{\int_0^{\theta'}\sqrt{1-k^2\sin^2 4\theta}\,d\theta - \int_0^{\theta}\sqrt{1-k^2\sin^2 4\theta}\,d\theta\right\},$$

where $k^2=1/h^2$. Hence if E be an elliptic integral of the second kind, we have

$$i=\frac{1}{\pi h}(\sin 4\theta'-\sin 4\theta)\pm\frac{1}{4\pi}\left\{E(k_1,4\theta')-E(k_1,4\theta)\right\}. \quad (9)$$

The double sign of the second term must be so chosen that the upper and lower signs correspond to $\alpha-\theta<\pi/2$ and $\alpha-\theta>\pi/2$ respectively, with the condition implied that if an abrupt turning of the magnets through $\pi/2$ takes place, α and θ are measured from the new position of equilibrium.

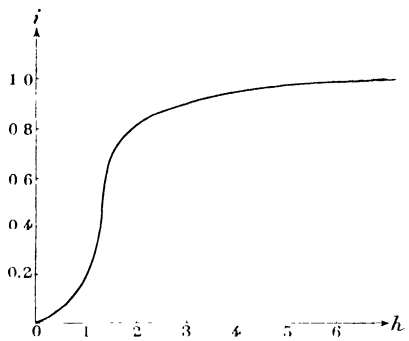


Fig. 198.

In the following tables and in Fig. 198, the results of the calculation based on the above relations are given. Up to $h=0.5$, i was calculated from equation (7), and for higher fields by means of equation (9), taking the sum of the four integrals having different limits of integration.

h	i	h	i
0.1	0.012	1.5	0.677
0.3	0.037	2.0	0.816
0.5	0.062	2.5	0.875
1.0	0.183	3.0	0.909

Thus the form of the curve of magnetization agrees precisely with that experimentally found. This curve starts from the origin at a definite angle, and increases at first linearly with the field. With a further increase of the field, the magnetization increases more and more rapidly, and in a certain field its rate of increase attains a maximum and then gradually decreases. The curve of magnetization passes therefore through an inflexion point, and gradually approaches an asymptotic value 1 as the field is increased. This curve is the normal curve of magnetization with the reduced intensity of magnetization and field; it is common for all the ferromagnetic substances belonging to the regular system.

The curve of magnetization belonging to a particular substance can be obtained from i and h by multiplying by I_0 and A, the characteristic constants of the substance, respectively. Thus we have

$$I = i I_0,$$

and

$$R = hA = H + \frac{4\pi}{3} I,$$

or

$$H = hA - \frac{4\pi}{3} I$$

$$= hA - \frac{4\pi}{3} iI_0.$$

Hence from different pairs of i and h, the corresponding pairs of I and H can be obtained. The I,H curve thus obtained does not materially differ in form from the curve of magnetization shown in Fig. 198.

Recently J. A. Ewing[1] published a paper in which he modified his old model of the elementary magnet in an important point. In his new model, the atom itself does not rotate on the application of a field, but what rotates is only a small part of the atom. The idea by which Ewing was led to modify his theory, is based on the fact that the actual limit of the reversible magnetizing field is very small as compared with that required by his original model. But it must be remarked that he considers only a single complex, and does not take into account the effect of a large number of surrounding complexes[2]. If this be done, it will be seen that his objection to his former model loses its foundation. Moreover the periodic variation of the magnetization for different orientations of ferromagnetic crystals actually observable cannot be explained by his new model of the elementary magnet.

60. Residual Magnetism and Hysteresis Curve.

If a mass of iron is once magnetized to saturation and the field is then reduced to zero, there remains a residual magnetism. The amount of this residual magnetism can easily be found in the following way: The complexes whose magnetic directions has initially lain between 0 and $\pi/4$ will return to their original positions when $h=0$; the complexes whose magnetic directions have been initially

(1) Pro. Roy. Soc. Edinb **42**, (1922), 97; Phil. Mag. **43**, (1922), 493. (2) K. Honda and J. Okubo, Sci. Rep. **12**, (1923), 27.

$\pi/4 > \alpha > \pi/2$ or $\pi/2 > \alpha > 3\pi/4$, will assume a new position of equilibrium differing from the initial one by $\pi/2$ when $h=0$. Lastly, the complexes whose magnetic directions have initially been $3\pi/4 > \alpha > \pi$, will come to a new position differing by π from the initial position when $h=0$. Hence, if the field be reduced to zero, the magnetic axes of all these complexes are distributed uniformly within an angle making $\pi/4$ on both sides of the field. The residual magnetism may therefore be found thus:—

$$R = 2\int_0^{\frac{\pi}{4}} M\cos\theta\, dn, \qquad dn = \frac{2n}{\pi}\, d\theta,$$

$$= \frac{4I_0}{\pi\sqrt{2}};$$

hence the reduced residual magnetism is

$$r = \frac{R}{I_0} = 0.8927. \tag{10}$$

Thus there remains a residual magnetism of about 90 percent. Experiments with very long soft iron wires confirm the correctness of this conclusion.

According to the above consideration, the process of reducing the field from ∞ to 0 is reversible, that is, the magnetization during the reduction of the field from ∞ to 0 exactly coincides with the new magnetization from 0 to ∞, the initial magnetization being r. This curve of magnetization can easily be found, since the initial orientation of the complexes is known to be uniformly distributed within an angle subtended by the lines inclined at $\pi/4$ to the field. If h be small,

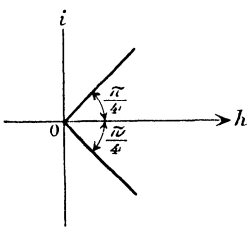


Fig. 199.

$$i = \frac{4}{\pi}\int_0^{\frac{\pi}{4}} \cos(\alpha-\theta)\, d\alpha \quad \text{and} \quad \theta = \frac{h\sin\alpha}{4+h\cos\alpha},$$

$$\therefore\ i = \frac{1}{\pi}\int_0^{\frac{\pi}{4}} (h + 4\cos\alpha)\left(1 + \frac{h}{4}\cos\alpha\right)^{-1} d\alpha$$

$$= 0.9003 + 0.0452\,h - 0.0094\,h^2 + \cdots \tag{11}$$

For a large value of h we find the value of i from equation (9) by simple substitution of the limits of integration.

Starting from the residual magnetism, the magnetization by a gradually increasing negative field may be calculated in a similar way. This case is equivalent to that of the magnetization by a posi-

tive field of a group of complexes whose initial magnetic axes are uniformly distributed within an angle given by $\pm(3\pi/4) > \alpha > \pi$. For small values of h, we have the same expression for i as in equation (11). For large values of h we find i from equation (9), as in the former cases. In this way we can obtain a well-known

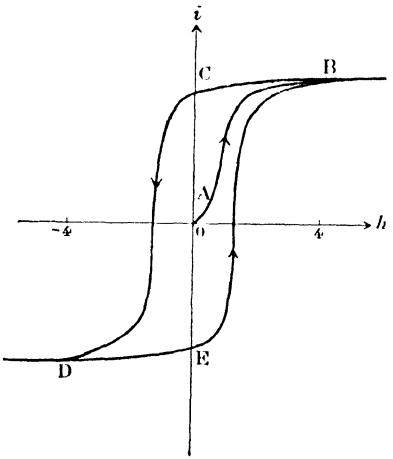


Fig. 200.

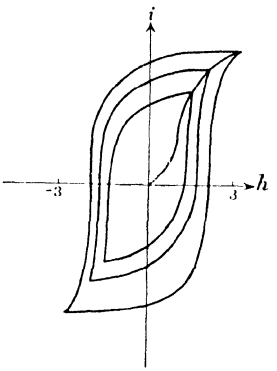


Fig. 201.

hysteresis-loop, when the field is varied between $+\infty$ and $-\infty$, as shown in Fig. 200. It possesses all the characteristics shown by iron, nickel and cobalt[1].

The hysteresis-loop accompanying a cyclic change of magnetic field between $+h$ and $-h$ can also be calculated in a similar manner. For this purpose, the residual magnetism obtained by reducing the field from h to 0 is first calculated. Then the curve of magnetization having this residual magnetism as the initial state can be obtained; it must coincide with the curve of demagnetization obtained by reducing the field from h to 0. Next, the curve of magnetization from 0 to $-h$ having the same state of residual magnetism as the initial one, can be obtained, and so on. In this way we obtain a complete cycle of magnetization as shown in Fig. 201.

61. Effect of Temperature on Magnetization.

As we have already seen, in the case of iron and nickel, the magnetization in very weak fields increases with the rise of tem-

(1) K. Honda and J. Okubo, Sci. Rep. **6**, (1917), 183.

perature at first very slowly; but from 600° upwards its rate of increase becomes steadily greater. At about 750°, the magnetization reaches a sharp maximum, and then falls abruptly to zero. As the field is increased, the increase at high temperatures becomes less, and in a field of about ten units the magnetization remains almost constant up to about 730°, and then rapidly falls. With a further increase of the field, the temperature at which the magnetization begins to decrease becomes steadily lower, and in a strong field of several hundred gausses, it begins to decrease from a temperature far below room temperature, the rate of decrease becoming steadily greater as the temperature rises.

From these facts, we may conclude that the temperature affects the magnetization in two opposite ways[1]. The first effect, which exists in all fields, is to diminish the magnetization, and the second, which is noticeable only in weak fields, is to increase it. As we have already remarked, the first effect is probably due to the rapid rotational vibration of the atoms; the amplitude of this vibration is assumed to be small at room temperature, but to become greater as the temperature is raised. It is evident that the component of magnetization in the direction of the field, when the elementary magnets make a rotational vibration about their mean orientation, is less than that in the case when the magnets make no such vibration. Thus the magnetization diminishes with an increasing amplitude of the rotational vibration, or, with the rise of temperature. Thus, the first effect of temperature is explained. Here it is to be assumed that when the field ceases to act, the direction of the magnetic axes of the atoms is again distributed uniformly in all directions owing to the thermal agitation.

The second effect is due to the abrupt turning of the atoms towards the field caused by their rotational vibration. If the thermal motion be absent, the elementary magnets in each complex will assume a common direction determined by the external field and the internal resisting force. Suppose this direction to make an angle θ with the field; in virtue of the thermal energy, these magnets will in actual cases execute rotational vibrations about their mean orientation. The amplitude of this vibration will actually differ from one complex to another according to Maxwell's law; but at a given temperature, we may assume, as a first approximation, their mean value to be 2β. Since, in each complex, the

(1) K. Honda and J. Okubo, Sci. Rep. **5**, (1916), 325.

atoms exert a mutual action on each other, the rotational vibration of atoms with a common phase will take place more easily than in the case of the vibration with arbitrary phases. Hence in a stationary state, we may assume that all the magnets in each complex oscillate with a common phase, but that the phase of the oscillation in one complex differs from that in the other. If for a complex (α), $\theta+\beta$ is less than the critical angle, the elementary magnets in the complex will oscillate about its mean orientation θ; on the other hand, if it is greater than the critical angle, they will undergo an abrupt turning and assume a position, as if the initial orientation were $\alpha-\pi/2$, or $\alpha-\pi$, causing thereby an increase of magnetization. Hence, even in weak fields, where in the case of the absence of thermal agitation, there can be no complex in which elementary magnets abruptly turn towards the field, an increasing number of the elementary complexes will begin to make an abrupt turning with the rise of temperature, thus causing an increase of magnetization. The second effect of temperature is thus explained.

If the field becomes greater, an increasing number of complexes turns abruptly towards the field, even if there is no thermal motion, and consequently the increase of magnetization due to the thermal agitation becomes less and less. In a sufficiently strong field, where all the complexes have finished their possible abrupt turning, the effect of temperature in increasing magnetization must vanish, and there exists only the effect of the diminishing magnetization due to the rotational vibration of the atoms. Thus the effect of temperature on magnetization is explained by our theory, at least qualitatively.

62. Magnetization of Ferromagnetic Crystals.

So far we have considered the case of the magnetization of ferromagnetic substances consisting of an immense number of microcrystals. In this section, we shall consider the case of a single ferromagnetic crystal such as iron or nickel[1]. It is here to be noticed that the theory does not take the thermal motion of atoms into account, and hence strictly speaking, applies only to the case of magnetization at the absolute zero of temperature.

(a) *Iron Crystal.* The iron crystal has a body-centred cubic

(1) K. Honda and J. Okubo, Sci. Rep. **5**, loc. cit.

lattice. Let us consider first the magnetization in a plane parallel to the face (100). Taking two sides of the space-lattice as the x and y axes, we may assume that two pairs of one-sixth of the whole number of complexes are directed in the positive and negative directions of the x and y axes and the remaining pair in the direction perpendicular to the xy plane, the total magnetic effect being thus cancelled. For the calculation of the magnetization, we may, however, suppose the last pair to lie in the xy plane, their magnetic axes being perpendicular to the direction of the field. If the magnetic field be applied in a direction making an angle α with the x axis, the intensity of magnetization is giyen by

$$\begin{aligned} i &= \frac{1}{6}\left\{\cos(\alpha-\theta_1)+\cos(\pi-\alpha-\theta_2)+\cos\left(\frac{\pi}{2}-\alpha-\theta_3\right)\right. \\ &\qquad \left.+\cos\left(\frac{\pi}{2}+\alpha-\theta_4\right)+2\cos\left(\frac{\pi}{2}-\theta_5\right)\right\} \\ &= \frac{1}{6}\left\{\cos(\alpha-\theta_1)-\cos(\alpha+\theta_2)+\sin(\alpha+\theta_3)\right. \\ &\qquad \left.+\sin(\theta_4-\alpha)+2\sin\theta_5\right\}, \end{aligned} \tag{12}$$

where α and θ are connected by the relations:

$$\begin{aligned} h &= \frac{\sin 4\theta_1}{\sin(\alpha-\theta_1)} = \frac{\sin 4\theta_2}{\sin(\alpha+\theta_2)} \\ &= \frac{\sin 4\theta_3}{\cos(\alpha+\theta_3)} = \frac{\sin 4\theta_4}{\cos(\theta_4-\alpha)} = \frac{\sin 4\theta_5}{\cos\theta_5}. \end{aligned} \tag{13}$$

In this calculation, we must take into account the abrupt change of rotation of the magnets in passing through the critical value of θ. If h be very large or very small, it may readily be shown that i is independent of α, and that the crystal behaves like an isotropic substance.

If h be very small, θ is also very small; hence from equation (13), we get

$$\theta_1 = \frac{h\sin\alpha}{4+h\cos\alpha}, \qquad \theta_2 = \frac{h\sin\alpha}{4-h\cos\alpha}, \qquad \theta_3 = \frac{h\cos\alpha}{4+h\sin\alpha},$$

$$\theta_4 = \frac{h\cos\alpha}{4-h\sin\alpha}, \qquad \theta_5 = \frac{h}{4}.$$

Substituting these values of θ in equation (12) and simplifying, we have

$$i = \frac{4h}{3}\left\{\frac{\sin^2\alpha}{4^2-h^2\cos^2\alpha} + \frac{\cos^2\alpha}{4^2-h^2\sin^2\alpha} + \frac{1}{16}\right\}.$$

If we neglect h^2 in comparison with 16, we have

$$i = \frac{1}{6}h.$$

Thus i is independent of α, that is, the crystal behaves as an isotropic substance. This conclusion is confirmed by actual experiment.

If the field is moderate and acts in the direction of x-axis, $\alpha=0$,

$$\therefore \quad i = \frac{1}{6}\left\{1-\cos\theta_2+\sin\theta_3+\sin\theta_4+2\sin\theta_5\right\}$$

and

$$h = \frac{\sin 4\theta_2}{\sin\theta_2} = \frac{\sin 4\theta_3}{\cos\theta_3} = \frac{\sin 4\theta_4}{\cos\theta_4} = \frac{\sin 4\theta_5}{\cos\theta_5}.$$

The value of i for different values of h is calculated by means of the above equations and the result is graphically given in Fig. 203. The initial ascending portion of the magnetization curve is due to

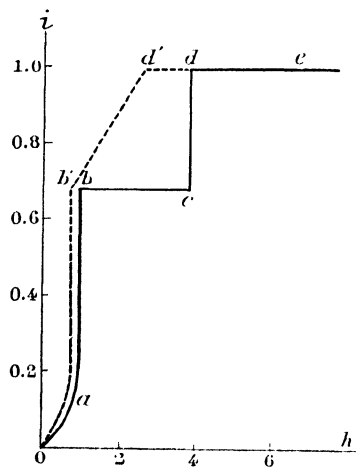


Fig. 203.

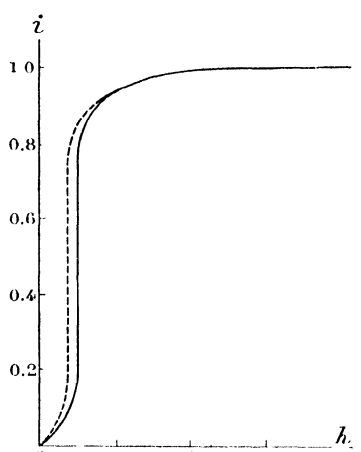


Fig. 204.

the effect of the continuous turning of the elementary complexes caused by an increasing field, and the rapid ascending portion *ab*

to the effect of the abrupt turning of the complexes, the axes of which were initially perpendicular to the field. Above this field, the complexes which contribute to the increase of magnetization, are only those the magnetic axes of which lie in the direction opposite to the magnetic field. Under the action of the magnetic field, the elementary magnets in these complexes do not turn to any appreciable extent, until the field increases by a certain high value *bc*, but when the field attains this value, all of them turn over at once in the direction of the field, thus causing an abrupt increase of magnetization *cd*. A further increase of the field cannot cause any increase of magnetization.

However, as we have already remarked, the elementary magnets are actually undergoing thermal agitation; hence even in the case where in the absence of thermal agitation, no abrupt turning of the elementary magnets can take place, some complexes begin to turn abruptly and the magnetization commences to increase below the critical field h_m. Consequently, the magnetization curve, instead of taking the course *o a b c d e*, will follow another course *o b′d′e* with two breaks, as confirmed by the actual experiment.

Similarly, if the field acts along the diagonal axis of the space-lattice, we have $\alpha=\pi/4$, and the curve of magnetization takes the course which is shown in Fig. 204.

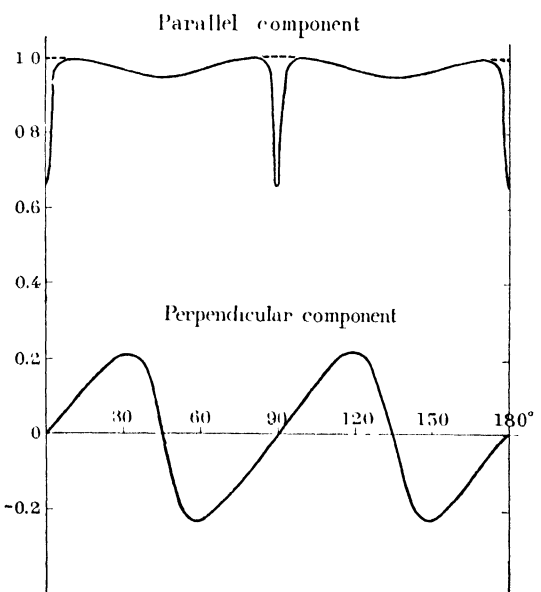


Fig. 205.

Let us next suppose the field to be kept constant, and its direction α to be changed continuously from 0 to π. The calculation gives us different values of the parallel and perpendicular components of magnetization corresponding to different directions of the field. The results for $h=3$ are shown

in Fig. 205. At $\alpha=0$, $\pi/4$, $\pi/2$, $3\pi/4$, π, the direction of the magnetization coincides with that of the field, so that there is no component perpendicular to the field. The sharp minima seen at $\alpha=0$, $\pi/2$, π are to be attributed to the complex whose magnetic axis points in the direction opposite to the field; but in actual cases, the atoms are in the state of thermal agitation, so that the orientation of a complex $\alpha=0$ is only its mean position, and therefore these minima do not actually appear. Hence in these directions, the parallel component of magnetization must be a maximum, as shown by the dotted lines. Both parallel and perpendicular components vary periodically with a period of $\pi/2$. These theoretical conclusions agree with the experimental results very satisfactorily.

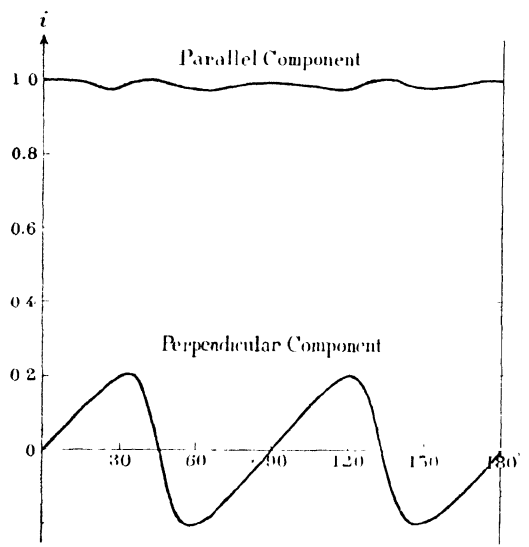


Fig. 206.

Consider next the case where the field is so strong as to cause an approximate saturation of magnetization; then all magnets are initially directed nearly in the positive direction of x. The parallel and perpendicular components of magnetization, i and i', are then given by

$$i = \cos(\alpha-\theta) \text{ and } i' = \sin(\alpha-\theta),$$

together with the relation

$$h = \frac{\sin 4\theta}{\sin(\alpha-\theta)}.$$

In Fig. 206, the results of calculation for $h=7$ are given. Here the variation of the parallel component with the orientation is very small, while that of the perpendicular component does not much differ from the variation in the former case. The parallel

component shows maxima at $\alpha=0$, $\pi/2$, π, two minor maxima newly appearing at $\alpha=\pi/4$ and $3\pi/4$. These conclusions are borne out in our experiment under strong fields (Figs. 157, 158).

If the magnetizing field is very strong, $\theta \doteqdot \alpha$ and consequently $i=1$, $i'=0$; thus the magnetization is constant irrespective of the direction of the field.

The magnetization in a plane parallel to the face (110) can be studied in the same way as before. The space-lattice in this plane consists of a system of rectangles, whose two sides are respectively $2a$ and $2a_1/\sqrt{2}$, and the internal resisting force $f(\theta)$ is somewhat different from that for the square lattice, having a period of π. Hence both parallel and perpendicular components of magnetization will vary with a period of π; this result is actually confirmed by experiment.

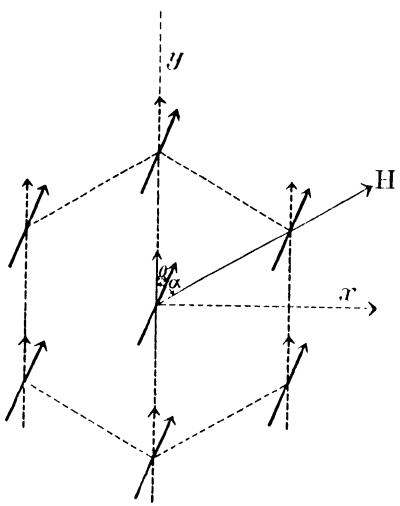


Fig. 207.

Lastly the case of magnetization in the plane parallel to the face (111) will be considered; the space-lattice in this plane consists of a system, of equilateral triangles with the side $2a_1/\sqrt{2}$. Assuming that every atom is acted on by its six neighbours situated at the corners of the regular hexagon, as shown in Fig. 207, the internal resisting force may be calculated in the same way as in the former cases. Thus

$$F(\theta) = \frac{3.5.7.9}{2.2.2}\frac{mk}{4r^2(1+k^2)^{\frac{3}{2}}}\sin 6\theta\left\{\left(\frac{16}{5!}+\frac{36}{7!}\frac{11}{12}p^2+\cdots\right)p^5 - \left(\frac{11.13}{2.2}\frac{486}{11!}p^9+\cdots\right)\sin^2 3\theta+\cdots\right\}, \quad (14)$$

where $k=a/r$, $p=k/(1+k^2)$. Obviously $F(\theta)$ is a periodic function of θ with a period of $\pi/3$. If k is greater than 2, the second term in

$F(\theta)$ is negligibly small, and the form of the function is reduced to a simple sine function, that is,

$$F(\theta) = A \sin 6\theta.$$

Hence as the condition of equilibrium, we have

$$H \sin(\alpha - \theta) = A \sin 6\theta,$$

or

$$h \sin(\alpha - \theta) = \sin 6\theta.$$

The parallel and perpendicular components of magnetization for a field $h=3$ are calculated and shown in Fig. 208; as we have remarked before, the sharp minima existing in the directions of

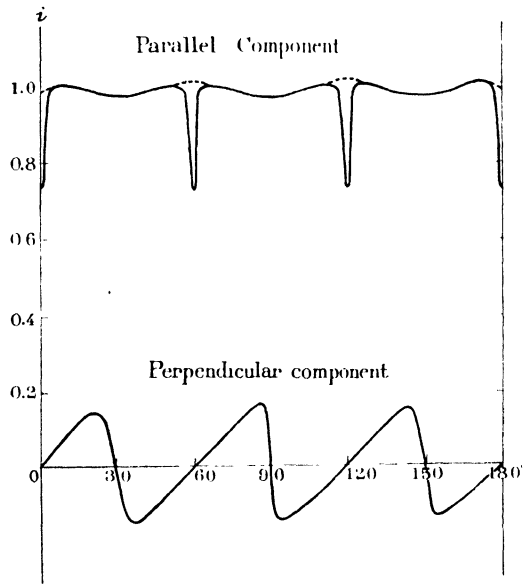


Fig. 208.

the lattice will vanish in the actual case owing to the thermal agitation. These results agree with the experimental one obtained by N. Beck, S. Kaya and the author. They are also applicable to the case of the magnetization of pyrrhotite in planes parallel to the base of the hexagonal prism.

(b) *Nickel Crystal.* Nickel has a face-centred cubic lattice, and hence, the plane parallel to the face (100) contains iron atoms which are arranged in the corners of new squares formed from the

diagonals of the original square lattice. The sides of the new squares are, in magnitude, $1/\sqrt{2}$ of those of the original squares and are inclined at an angle of 45° to them. The problem of magnetization is therefore the same as in the case of iron, except that in the case of nickel, the orientation of the maximum and the minimum of the parallel and perpendicular components of magnetization is displaced by $\pi/4$ with respect to the direction of its lattice. This conclusion is borne out by actual experiment.[1]

In the plane parallel to the face (110), the atoms are distributed in the corners of the rectangular lattice in the same manner as in the case of iron. The distribution of atoms in the face (111) is also the same as in the corresponding case of iron, the space-lattice in this face consisting of equilateral triangles. These conclusion are in a good agreement with the results of recent experiments[2]. They are also applicable to the case of magnetite, in which the divalent iron atoms assume a face-centred cubic lattice.

63. Magnetic Hysteresis in Ferromagnetic Crystals.

Considered from the theoretical point of view, when an iron crystal is magnetized in the direction of the principal axes and the field is then reduced to zero, the whole or a greater part of the magnetization must remain as the residual magnetism, provided that there is no thermal agitation. In actual cases, however, the residual magnetism in iron crystals is almost zero or very small; this discrepancy between theory and observation may be explained as follows:—If all or most of the elementary magnets lie in the same direction, the external field being zero, a strong demagnetising force acts in the interior of the magnetized mass. Thus all the elementary magnets in the complexes are exposed to thermal agitation under the action of this demagnetising force, and consequently as soon as the external field is withdrawn, the magnetization diminishes very rapidly with time, till it almost vanishes.

S. Kaya and the present author[3] found that the hysteresis-loss of single crystals of iron is very small, and in the case of polycrystals, it increases very rapidly at first and then more slowly, with the increase in the number of grains or crystals in a unit volume. From this fact it is to be concluded that the hysteresis

(1), (2) W. Sucksmith, H. H. Potter and L. Broadway, Proc. Roy. Soc. A, **117**, (1928), 471. S. Kaya. Sci. Rep. **17**, (1928). (3) Sci. Rep. **15**, (1926), loc. cit.

in ordinary iron is mainly affected by an irregular distribution of elementary magnets on the grain boundary. For, in boundary regions, the distribution of these magnets being irregular, they are denser in one place and rarer in other place than in the interior of grains. Where the magnets are denser, the magnetization is more dfficult and so after the field is withdrawn more residual magnetism remains. With an increase in the grain number, the number of elementary magnets in the closely packed state also increases and hence the residual magnetism or hysteresis increases. The fact that the best kind of magnet steel, especially K. S. magnet steel, possesses a high coercive force and also an extremely fine structure, confirms the correctness of the above view. Silicon steel used in dynamo machines has a small hysteresis-loss, and this smallness of hysteresis is explained by the comparative largeness of the grains, and not as a direct effect of silicon, as is usually assumed to be the case. The same explanation will probably apply to the case of the permalloy which has a very high susceptibility; because if we measure the grain number for the series of iron-nickel alloys, which have been thermally treated in the same manner, it attains a very conspicuous maximum at the concentration of permalloy.

Next we shall consider the mechanism of the hysteresis-loss. According to our theory, the hysteresis-loss occurs only when the rotation of the elementary magnets caused by an external field becomes discontinuous. That is, if the field is very small, there is no sudden rotation of atoms, and hence no hysteresis by magnetization. If, however, the field increases to a certain value, some of the atoms make an abrupt rotation and give rise to the hysteresis phenomenon. The number of such atoms increases with the strength of the field and reaches a maximum value, so that the hysteresis-loss increases also in a similar way.

In the hysteresis curve shown in Fig. 200, the hysteresis-loss takes place in portion AB of the magnetization curve. During the demagnetization from $h=\infty$ to 0, no abrupt rotation of atoms occurs and therefore we have no hysteresis. But the magnetization in the opposite direction from $h=0$ to 4 involves a loss of energy. Similarly, in portion DE of the magnetization curve, there is no loss of energy, but in portion EB, we have a loss of energy equal in amount to that in portion CD.

According to the general theory of magnetization, the total loss of energy during a complete cycle of magnetization is equal to the area of the hysteresis-loop. By our theory, the hysteresis-

loss is the kinetic energy obtained by the atoms during their abrupt rotation, and hence it is very interesting to investigate whether in a cyclic process of magnetization, the kinetic energy thus obtained is equivalent to the area enclosed by the hysteresis-loop. As the following calculation will show, the result completely agrees with the above theory; moreover, in the process of magnetization, we can distinguish the energy dissipated during the magnetization from the total energy.

We shall first consider the energy loss of a single complex during magnetization. Let α be the angle between the direction of the axis of the elementary magnets and the field. If h increases from 0 to h_m, which is the critical field for the abrupt turning, the magnets in the complex will turn reversibly towards the field; and at $h=h_m$, an abrupt turning of the atoms occurs, and their axis assumes a new orientation corresponding to the initial position differing by $\pi/2$ or π from the original. During the abrupt turning, the atoms will acquire a kinetic energy which is transformed into heat energy; this energy for one cycle of magnetization must be equal to the sum of the work done on the atoms.

The couple N acting on an elementary magnet, whose magnetic moment is M, is given by

$$N = M\left\{H_m \sin(\alpha-\theta) - A \sin 4\theta\right\},$$

where $M=2mr$. If θ_0 and θ_1 be the angles of deflection of an atom from its initial position, which correspond to the positions just before and after the abrupt turning, we have

$$W = \Sigma \int_{\theta_0}^{\theta_1} N d\theta = \Sigma M \int_{\theta_0}^{\theta_1} \{H_m \sin(\alpha-\theta) - A \sin 4\theta\} d\theta,$$

where the summation Σ is to be extended to all the atoms n in the complex. Since M, α, θ are the same for all of them, the above equation may be written as

$$W = nM \int_{\theta_0}^{\theta_1} \{H_m \sin(\alpha-\theta) - A \sin 4\theta\} d\theta,$$

or

$$w_s = \frac{W}{nMA} = \int_{\theta_0}^{\theta_1} \{h_m \sin(\alpha-\theta) - \sin 4\theta\} d\theta,$$

where w_s is the reduced hysteresis-loss for a single complex caused by magnetization. The latter does not involve any quantity depend-

ing on the nature of the substance; it is therefore applicable to all substances belonging to the cubic system.

Now the reduced intensity of magnetization for a single complex is

$$i = \cos(\alpha-\theta);$$

we have therefore

$$di = \sin(\alpha-\theta)\, d\theta.$$

Hence
$$w_s = \int_{I_0}^{I_1} h_m\, di - \int_{\theta_0}^{\theta_1} \sin 4\theta\, d\theta$$

$$= \int_{I_0}^{I_1} h_m\, di + \int_{0}^{\theta_0} \sin 4\theta\, d\theta - \int_{0}^{\theta_1} \sin 4\theta\, d\theta,$$

where I_0 and I_1 are referred to Fig. 209. But the last two terms are the integrals along the reversible courses of magnetization, in which case we have the relation

$$h \sin(\alpha-\theta) = \sin 4\theta;$$

$$\therefore\ h \sin(\alpha-\theta)\, d\theta = h\, di = \sin 4\theta\, d\theta.$$

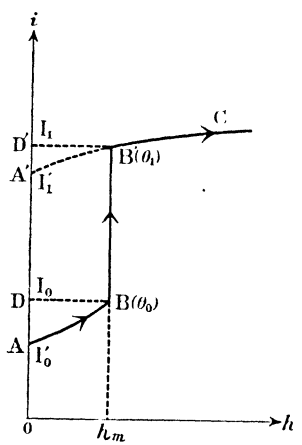


Fig. 209.

Moreover, the elementary magnets at θ_1 have the same potential energy with regard to the direction $\theta=0$ or that perpendicular to it. Hence

$$\int_0^{\theta_1} \sin 4\theta\, d\theta = \int_{\frac{\pi}{2}}^{\frac{\pi}{2}+\theta_1} \sin 4\theta\, d\theta,$$

and therefore we get finally

$$w_s = \int_{I_0}^{I_1} h_m\, di + \int_{I'_0}^{I_0} h\, di - \int_{I'_1}^{I_1} h\, di.$$

Referring to the annexed figure, in which $ABB'C$ is the curve of magnetization and $B'A'$ the course taken by the demagnetization curve, when the field is reduced to zero, we see that the first integral represents the area $DBB'D'$, the second the area ABD and the third the area $A'B'D'$, so that w_s is equal to the area $ABB'A'$.

Next, consider the case of the mass of a ferromagnetic substance consisting of an immense number of minute complexes, whose magnetic axes are uniformly distributed in all directions. From the above result, we see that if I_1 and I_2 be the intensities

of magnetization of a complex corresponding to the magnetizing and demagnetizing stages for the same strength of the field, we have

$$w_s = \int_0^{h_m} (I_2 - I_1)\, dh = \int_0^{h} (I_2 - I_1)\, dh,$$

where h may take any value whatever, as w_s vanishes for larger values of h than h_m. Hence, for the hysteresis-loss w of a mass of the ferromagnetic substance, we must summarise the above expression for all the complexes constituting the substance. Thus

$$w = \Sigma w_s = \int_0^{h} \Sigma\, (I_2 - I_1)\, dh\,;$$

but ΣI_1 and ΣI_2 are respectively the resultant intensities of magnetization corresponding to the ascending and descending branches of the magnetization curve. Hence putting

$$\Sigma I_1 = i_1 \qquad \text{and} \qquad \Sigma I_2 = i_2,$$

$$w = \int_0^{h} (i_2 - i_1)\, dh.$$

Referring to Fig. 210, w represents the area $OABC$ enclosed by the magnetizing and demagnetizing branches (1) and (2), of the magnetization curve. The area $OABD$ is known to be the total energy of magnetization, and therefore the area CBD, which is the difference between the areas OBD and OBC, is the net energy of magnetization.

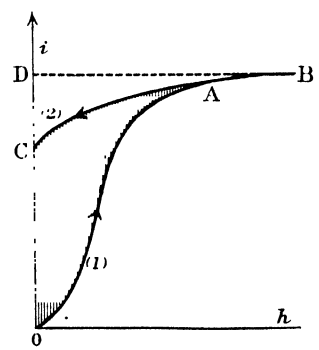


Fig. 210.

In the same way, it can be shown that in a cyclic process of magnetization, as shown in Fig. 200, the hysteresis-loss during the magnetization EB is given by the area EBC and the loss during the demagnetization CD by the area CDE, and that the total loss during the cyclic magnetization is equal to the area $BCDEB$ enclosed by the hysteresis-loop. Thus the important fact that the loss of energy during a complete cycle of magnetization is equal to the area of the hysteresis-loop, is explained atomistically, and the mechanism of the hysteresis phenomenon is made more comprehensive.

In concluding this section, one important question which has a close connection with the magneton theory to be described in the next chapter, will here be discussed. This is the question whether the saturation value of magnetization obtained by the usual method of observation is absolute or merely relative. It is often asserted by some physicists that in ferromagnetic crystals, a crystalline force acting in the interior of the magnetized substance prevents, more or less, the elementary magnets from turning in the direction of the magnetic force, however strong it may be, and hence the observed value of saturation magnetization is merely apparent. If this were true, the saturation value of magnetization must have a different value for certain orientations of the crystal in relation to the magnetizing force from those for other orientations; but as we have seen in section 47, it is entirely independent of the orientation of the crystal, and consequently the following important conclusion can be drawn:—

The saturation value of magnetization in ferromagnetic substances as observed by the usual method is the true value of saturation.

CHAPTER X.

Magnetic Moment of Atoms.

64. Magnetic Moment of Atoms and Molecules.

(a) *Ferromagnetic substances.* In the case of ferromagnetic substances, the magnetic moment of the atoms can be calculated from the saturation value of magnetization by merely dividing it by the number of atoms, which contribute to the magnetization, that is,

$$M = \frac{I_0 w}{\rho N},$$

where w and ρ are the atomic weight and the density respectively and N the Loschmidt number 6.06×10^{23}. The following table shows the results of calculation :—

Metals	$\rho_{20°C}$	ρ_{0K}	$I_{20°C}$	I_{0K}	σ_0	$M \times 10^{20}$
Fe (Gumlich) [1]	7.876	7.961	1721	1775	12460	2.06
Co (Preuss) [2]	8.920	9.020	1477	1515	9908	1.63
Ni (Gumlich) [3]	8.912	9.018	509	543	3535	0.58

(b) *Paramagnetic and Diamagnetic Gases.* In the case of strong paramagnetic gases, the small diamagnetic term may be neglected. For diatomic gases, Langevins' formula can be applied to calculate the magnetic moment of atoms, that is,

$$\sigma_0 = \sqrt{3RT\chi_m}.$$

$$\therefore \quad M = \frac{\sigma_0}{N} = \frac{1}{N}\sqrt{3RT\chi_m}.$$

For gases at ordinary pressure and temperature, the quantum

(1) E. Gumlich, Elektrot. Zeitschr. **30**, (1909), 1065. (2) A. Preuss, Dissert., Zürich, (1912). (3) E. Gumlich, Wiss. Abh. d. Phys. Techn. Reichsanstalt, **4**, H.3.

distribution of molecular magnets may be disregarded. The following table contains our results of calculation:—

Gases[1]	T	$\chi_m \times 10^6$	σ_0	$M \times 10^{20}$
O_2	293°	3331	15610	2.57
NO	293	1464	10350	2.20

The hydrogen atom has no nuclear electrons and so no magnetizable magnetic moment, but it has a non-magnetizable moment due to the outer electron, so that the hydrogen atom or molecule should be purely diamagnetic. The molecular susceptibility may be calculated by the formula given before by assuming Bohr's model, that is,

$$\chi_m = -\frac{Nm}{6}\left(\frac{e}{m}\right)^2 \Sigma r^2,$$

and compared with the value obtained experimentally.

Taking $m = 8.99 \times 10^{-28}$, $\frac{e}{m} = 1.769 \times 10^7$,

$$\Sigma r^2 = 2 \times 0.2575 \times 10^{-16},$$

we get

for the first orbit $n = 1$, $\chi_m = -1.47 \times 10^{-6}$,

for the second orbit $n = 2$, $\chi_m = -23.40 \times 10^{-6}$.

The observed value is -3.96×10^{-6}, compared with which the first value is too small and the second too large. In actual cases, a certain fraction of the whole number of molecules[2] may have the first orbit, and the other fraction the second orbit, etc. As the orbit becomes greater, there is a greater chance that it will collapse into a smaller one; so the number of molecules with an orbit n at any instant rapidly decreases with the increase of n. Hence, if we assume that the hydrogen gas at an ordinary temperature and pressure contains only the first and second kinds of molecules in the proportions of 83.6 and 16.4 percent respectively, then the calculated value may be made to coincide exactly with that observed.

In the case of the inert monatomic gases, helium, neon and argon, they have nuclear electrons 2, 10, 22 in number respectively, and therefore their nucleus has each a magnetizable magnetic moment. The atomic susceptibility of these gases is then given by

(1) Soné's values were taken. (2) Sci. Rep. **12**, (1923), 123.

$$\chi_a = \frac{\sigma_0^2}{3(RT+Q)} - \frac{Nm}{6}\left(\frac{e}{m}\right)^2 \Sigma r^2 ;$$

but owing to the incomplete annihilation of the angular momentum, it is probable that Q is very large and consequently the first paramagnetic term is negligibly small. Their susceptibilities are then diamagnetic. A similar calculation of the diamagnetic susceptibility of these gases, assuming Bohr's models, as in the case of hydrogen gas, gives the following results:—

Gases	$\chi_a \times 10^6$ ($n=1$)	$\chi_a \times 10^6$ ($n=2$)	$\chi_a \times 10^6$ (obs.)	Percentages ($n=1:2$)
He	−0.538	−6.81	−1.88	78.8 : 21.2
Ne	−3.35	−53.6	−6.67	93.4 : 6.6
Ar	−7.58	−121.3	−17.95	91.3 : 8.7

Thus to account for the observed diamagnetic susceptibility of these gases, it is sufficient to assume that each of them consists of two kinds of molecules ($n=1$, 2) of the above percentages.

(c) *Dilute solution of salts.* In a dilute solution, the solute molecules may be considered to behave like gaseous ones, so that Langevin's formula is also applicable to the dilute solution. For the following salts, for which the diamagnetic term may be neglected, the calculation gives the following values for the molecular moment:—

Substances	T	$\chi_m \times 10^6$	σ_0	$M \times 10^{20}$	Experimenter
$FeSO_4$	293°	12110	29750	4.91	Weiss, Frankamp[1]
$FeCl_3$	,,	12640	30400	5.02	Cabrera, Moles[2]
$Fe(NO_3)_3$	,,	12700	30470	5.03	,, ,,
$CoSO_4$	293°	9903	26910	4.44	Cabrera, Marquina, Jimeno[3]
$CoCl_2$	,,	10140	27240	4.49	
$Co(NO_3)_2$	,,	10410	27580	4.55	Quartarolli[4]
$NiSO_4$	293°	4418	17970	2.96	Weiss, Bruins[5]
$NiCl_2$	,,	4409	17950	2.96	,, ,,
$Ni(NO_3)_2$	,,	4418	17970	2.96	,, ,,
$MnSO_4$	293°	14830	32930	5.43	Cabrera, Moles, Marquina[6]
$MnCl_2$	,,	14830	32930	5.43	
$Mn(NO_3)_2$	,,	14840	32940	5.44	,, ,, ,,
$CrSO_4$	293°	9893	26890	4.44	Cabrera, Pina[7]
$CrCl_2$	,,	9750	26700	4.41	,, ,,
$CrCl_3$	,,	6010	20960	3.46	J. B. Elias[8]
$Cr(NO_3)_2$	,,	6100	21110	3.48	Cabrera, Marquina[9]

(1) Weiss and Frankamp, Proc. K. Akad. Wet. Amst. **18**, (1915), 254.

From the above table it is to be concluded that in the same metal, the magnetic moment of an atom having the same valency is the same for different radicals or components, but is different for different valencies.

(d) *Paramagnetic Solids.* In the case of strong paramagnetic solids, the small diamagnetic term may be neglected and we have for the molecular susceptibility the expression,

$$\chi_m = \frac{\sigma_0^2}{3(sRT+Q+\varphi)},$$

or

$$\sigma_0 = \sqrt{3\chi_m(sRT+Q+\varphi)}.$$

If s be previously known and χ_m be determined at different temperatures, then σ_0 can be calculated. The value of s depends probably upon the form of atoms or molecules and slowly increases with the rise of temperature. Since, however, we have little information regarding s at hand, σ_0 cannot be known; hence the numerous results of calculation made by P. Weiss and his followers for obtaining the number of magnetons in solid salts to be described later on lacks a sound basis, for in their calculation, s was simply taken as unity.

In the case of ferromagnetic substances above their critical points, they are paramagnetic and the formula

$$\chi_m = \frac{\sigma_0^2}{3sR(T-\theta)}$$

is applicable. From the values of susceptibility at various temperatures, θ can be evaluated, and since σ_0 is known for three ferromagnetic metals, s can be calculated by the formula

$$s = \frac{\sigma_0^2}{3R\chi_m(T-\theta)}$$

in the case of these metals. The following table contains the

(2) Cabrera et Moles, Anales de la Sociedad española de Fisica y Quimica, **10**, (1912), 216, 394. (3) Cabrera, Marquina et Jimeno, *ibid.*, **14**, (1916) 367. (4) Quartarolli, Gazz. Chim. **46**, (1916), 371. (5) Weiss and Bruins, Proc. K. Akad. Wet. Amst, **18**, (1915), 246. (6) Cabrera, Moles et Marquina, Anales de la Sociedad española de Fisica y Quimica, **13**, (1915), 256. (7) Cabrera et Pina, *ibid.*, **17**, (1919), 149. (8) J. B. Elias, *ibid.*, **16**, (1918), 467. (9) Cabrera et Marquina, *ibid.*, **15**, (1917), 199.

results of our calculation[1]:—

Metals	s
Fe	0.529
Co	0.496
Ni	0.123

From the above table, we see that in the case of iron and cobalt the value of s at very high temperatures, is nearly $1/2$, that is, sRT is roughly equal to the rotational energy of one degree of freedom, and that in nickel, it is far less than this value. This remark will probably apply to the case of other paramagnetic substances, though it is not confirmed, as σ_0 is unknown.

The theoretical value of the specific heat of solid substances at high temperatures is

$$C = 3R = 5.96$$

per gram atom. Above the critical point of a ferromagnetic substance, atoms are assumed to make a violent rotational vibration about their magnetic axes with an energy sRT, this motion being not completely free, but constrained in some way by the mutual action of neighbouring atoms. Hence at a sufficiently high temperature, the atomic heat of the substance may increase beyond the above theoretical value by an amount 1—3. The fact that the atomic heat of ferromagnetic and many other substances at very high temperatures is decidedly greater than the theoretical and has a value ranging from 7—8.5 cal., is for the most part attributable to the energy of rotational vibration.

65. Theory of Magnetons.

This theory was first put forward by P. Weiss[2] in 1911. According to him, the magnetism of an atom always consists of a whole multiple of a single elementary magnet called the "magneton," just as electricity is composed of electrons; he also considers the

(1) In the case of cobalt, the above relation does not hold good, unless the temperature is very high above the critical point, and hence 0 was determined at a range above 1300°. (2) Phys. Zeitschr., **12**, (1911), 935.

magneton to be an important constituent forming matter. But in the opinion of the present author, magnetism being a vector quantity differs from electricity in the important respect that the latter is essentially a scalar quantity, and hence it is not very likely that in the former the additive law is always valid.

According to the present author, the origin of ferro- and para-magnetism is in the innermost nuclear electrons rapidly revolving about the centre of the atom. To assume the additivity to hold in the case of atomic magnetism is equivalent to considering that all these electrons are revolving in a common orbit within the nucleus, or orbits of the same form and parallel orientation; but it is not very likely that this condition is always fulfilled. Certain electrons may circulate in orbits of different radii and may give rise to a fractional resultant moment. Again, even if we admit that the resultant moment of an atom is a certain integral multiple of an elementary unit, this is not always the case for a molecule; for, in forming a molecule, the constituent atoms may be bound together in such a way that their orientations are generally different from each other and consequently their magnetic axes are not necessarily parallel.

Thus it appears to the present author that the theory of magnetons is untenable, unless the possibility of the occurrence of fractional values of magnetons is precluded by assuming the quantum orientation of nuclear magnets in atoms or molecules. At any rate, the validity of the theory must be founded upon experimental evidence.

In 1910, K. Onnes and P. Weiss[1] measured the saturation value of magnetization in iron, nickel, cobalt and magnetite at the low temperature of liquid hydrogen, and obtained the values shown below:—

Fe	$\sigma_0 = 12360$
Co	$= 9650$
Ni	$= 3370$
Fe in Fe_3O_4[2]	$= 7417$

If magnetism consists of an assemblage of magnetons with their axes parallel, these saturation values of magnetization should have a common divisor. Weiss first found the greatest common measure between the values of magnetization for iron and nickel

(1) K. Onnes and P. Weiss, Communication from Phys. Lab. of Leiden, No. 114, (1910), 3. (2) Weiss assumed that the magnetization of magnetite is due to the iron atoms, that due to oxygen being negligibly small.

and obtained 1123.5 as a number very near to this; thus

Fe	12360 ÷ 11 =	1123.6
Ni	3370 ÷ 3 =	1123.3
Mean		1123.5[1]

This number is called the magnetic moment of one gram magneton; if the value is divided by the Loschmidt number 6.06×10^{23}, we get the magnetic moment of one magneton that is 1.85×10^{-21}. Thus he concluded that an iron atom contains eleven magnetons and that of nickel three magnetons. If the magneton theory be the correct one, the number of magnetons contained in an atom of cobalt or magnetite must also be an integer. Dividing the saturation values of magnetizaion for cobalt and magnetite by 1123.5, we get 8.59 and 6.61 respectively. Weiss explains this discrepancy by assuming that in the case of these metals, the complete saturation of magnetization has not yet been attained.

In the case of paramagnetic substances, it is impossible to obtain directly the saturation value of magnetization, because our magnetic field available at present is far less than that corresponding to the saturation of magnetization. There are, however, ways of estimating the magnetic moment of molecules without obtaining the saturation value, such as are given in the last section; that is, in the case of gases and dilute solutions, we can estimate the saturation value by means of Langevin's formula from the measurement of susceptibility at a known temperature and thence the number of magnetons. Thus in the case of oxygen and nitrogen monoxide gases, this number was found to be 13.93 and 9.23 respectively.

In his experiment[2], Pascal measured the susceptibilities of iron compounds relative to water, taking its susceptiblity to be -0.79×10^{-6}; but as these values have later been found to be too large, Weiss recalculated Pascal's results by taking the susceptibility of water to be -0.75×10^{-6} and tested his theory of magnetons. Since, however, this value is again too large, its correct value being -0.720×10^{-6}, Weiss' results were recalculated by the present author. Again, what was observed by Pascal was the susceptibility of iron compounds, not of the iron atom, and hence Weiss applied the so-called "diamagnetic correction" to the observed results by assuming the additive law and calculated the susceptibility of the iron atom; but we cannot be sure that this method of calcula-

(1) Weiss has recently corrected this value to 1126. (2) Ann. d. Chim. et Phys. (8), **16**, (1909), 520.

tion is always valid. For, as has been confirmed by several experimenters, the magnetic moment of a compound differs sometimes considerably from the sum of those of its components.

Take a simple case of ferrous chloride $FeCl_2$. The atomic susceptibility of iron was calculated by Weiss from the observed susceptibility of the chloride by subtracting the diamagnetic susceptibility of the component Cl_2, which was obtained by Pascal from the measurement of the susceptibility of organic compounds. But we are not certain as to whether this method of calculation is not erroneous; chlorine itself is diamagnetic, but, in the combined state with iron, it is quite possible that the chlorine has a magnetic moment which contributes a measurable amount to the paramagnetism of the molecule. If the additive law were valid, the diamagnetic correction due to chlorine would be in fact a small number, but we have no theoretical or experimental ground for assuming this law to be correct.

Assuming for a while the additive law and the diamagnetic correction to be applicable and taking the susceptibility of water to be -0.720×10^{-6}, the present author calculated the number of magnetons in iron atoms from Pascal's results for eleven solutions of iron salts. The results of calculation are given in the following table:—

Substances	$\chi_m\times10^6$	σ_0	n
$K_3(NH_4)_3[Fe(CN)_6]_2$	1660	11460	10.21
$Fe(NH_4)P_2O_7$	7670	24100	21.45
$Fe(NH_4)C_3H_4(OH)(CO_2)_3$	7720	24180	21.53
$FeNaP_2O_7$	9270	26550	23.63
$FeNa(PO_3)_4$	12480	30870	27.47
$FeCl_3$	12570	30760	27.37
$Fe_2(SO_4)_3$	14590	33120	29.46
$FeK(PO_3)_3$	10840	28620	25.46
$FeNa_2(C_2O_4)_2$	11810	29870	26.57
$FeNa(PO_3)_3$	12480	30740	27.36
$FeSO_4$	12380	30480	27.13

From the above table, it is to be concluded that the Weiss theory is not confirmed, though in the case of Weiss' calculation in which -0.75×10^{-6} was adopted for the susceptibility of water, the results were much nearer to the whole number than in the above case.

Weiss also made use of the results for solid compounds obtained by Feytis, Pascal, Quincke, etc. and calculated the num-

ber of magnetons in iron, chromium and manganese atoms; but as we have shown before, in the case of solids, Weiss' relation

$$\chi_m(T+\Delta) = \frac{\sigma_0^2}{3R}$$

must be replaced by

$$\chi_m(T+\Delta) = \frac{\sigma_0^2}{3sR},$$

and hence unless s is known from other experiments, it is not possible to calculate the saturation value of magnetization, so that Weiss' calculation for solid substances is not quite correct.

The same remark applies also to the case of the number of magnetons in the atom of ferromagnetic metals above their critical points, as calculated by Weiss; but leaving this matter for a while out of consideration, the following fact may also deserve notice. In nickel, the $1/\chi, T$ curve (Fig. 141) above the critical point is not quite straight, but slightly bends below 414°; Weiss considered the curve to consist of two broken lines intersecting at 414°, which he assumed to represent two different phases of nickel, and obtained for the number of magnetons in these phases 8 and 9 respectively. But the existence of a phase change at 414° is very improbable and has never been confirmed. In the case of cobalt, the linear relation between $1/\chi$ and T is only roughly satisfied, so that the number of magnetons in cobalt atoms at high temperatures is very uncertain.

Iron is known to undergo two allotropic transformations at 910° and 1400°, the first change during heating being $\alpha \rightarrow \gamma$ phase and the second the reverse change $\gamma \rightarrow \alpha$. Weiss calculated the number of magnetons for the β and γ phases of iron, but the non-existence of the former phase is in the present day already a completely established fact. As, however, he could not find a linear relation between $1/\chi$ and T in the β region, he divided this curve into two broken lines and considered each of them to correspond to different phases β_1 and β_2; but as we have shown before, β_2 is in reality the transitional stage of the A_3 transformation, and is not a single phase, to which only Weiss' law is applicable. According to his calculation, the number of magnetons for the β_1 and γ phases are respectively 12 and 20.

Subsequently several investigators measured the susceptibility of different salts of iron, nickel, cobalt, manganese and chromium in a dilute solution or in a solid state. Assuming the additive

law for these salts, and applying diamagnetic correction, the number of magnetons contained in the atoms of these metals was calculated by means of Langevin's formula. The general results are as follows:—In many cases, the number of magnetons is very near to integers ranging from 10 to 30, but in other cases, the deviation from integers is considerable. Small deviations from integers are always explained as due to experimental errors; for example Kamerlingh Onnes obtained 39.2 for the number of magnetons in the gadolinium atom[1], which is, according to him, 39 and this difference is considered to lie within experimental error. The recent experiments made by Théodridès[2] and Cabrera[3] showed sometimes a large deviation from the whole number law. Thus in some cases, half magnetons or one-third magnetons were found.

From the experimental facts so far described, it is to be concluded that the existence of Weiss' magneton as a fundamental unit is far from being confirmed.

It may not be out of place here to remark upon the accuracy of the whole number law of magneton. Taking a concrete example, consider the case of the number of magnetons $n=20$; then the most favorable case for the observed number of magnetons is $n=20.0$, and the most unfavorable case $n=20.5$, the neutral case being $n=20.25$. Hence, an error of only one percent in the value of n alters the most favorable case to the neutral and the neutral case to the most unfavorable. So, to test the whole number law experimentally, great accuracy is required in the measurement of susceptibility; but in most of the measurements hitherto made, the experimental error is estimated to be one percent. Moreover the uncertain diamagnetic correction may sometimes becomes as large as one percent. Considering these circumstances, it is to be concluded that the experimental evidence for the whole number law as given by Weiss and his followers is far from being satisfactory.

Another point also deserves notice. A fact which is unfavorable to the theory which postulates the magneton as the fundamental unit of magnetism, is the multiplicity of the magneton number obtained for the same element; for example, iron contains so many different numbers of magnetons according to circumstances; thus 10, 11, 17, 20, 21, 23, 25, 26, 27, 29 have already been

(1) K. Onnes, Solvay Conference Report, Gautier, Villars, Paris, (1923). (2) Théodoridès, Journ. d. Phys. **3**, (1922), 1. (3) Cabrera, Journ. d. Phys. **3**, (1922), 443.

found. It is probable that as similar experiments are further repeated, every integer extending over a wide range will be obtained, thus greatly detracting the value of the theory. According to the present author, magnetism is due to the nuclear electrons, and hence each element has its own magnetic moment which does not, however, vary with any physical or chemical changes. The number of magnetons in the atom of all elements, if such a unit actually exists, must remain unaltered throughout these changes.

In the above no account regarding the quantum distribution of atoms is given; because all the measurements of susceptibility have hitherto been made under normal pressure and temperature where thermal impacts are extremely frequent, and hence the quantum distribution is hardly realised.

Lastly, we shall consider Bohr's magneton based on the quantum theory. As we have already shown, if an electron of mass m and charge e moves in a circular orbit of radius r, then the relation of the moment of momentum p to the magnetic moment M is expressed by

$$M = \frac{e}{2m} p;$$

now from the quantum relation for optical electrons, we have

$$p = \frac{nh}{2\pi},$$

where h is Planck's constant and n an integer. Hence we get

$$M = \frac{nhe}{4\pi m}.$$

Taking $n=1$ and $h=6.55\times10^{-27}$ *erg, sec.*, we have

$$M = 9.22\times10^{-21}$$

and $$\sigma = MN = 5589 \text{ per gram atom.}$$

Thus according to the quantum condition, the magnetic moment of a revolving electron is always an integral multiple of the above unit. This unit is called the Bohr magneton, which is nearly five times as large as the Weiss magneton. But it must be remembered that, although Bohr's magneton has a magnetic moment, it cannot be magnetised at all by the magnetic field, and hence it does not constitute units of magnetic moments in ferro- and paramagnetic substances.

Leaving this important point out of our consideration for a moment, it is to be noted that as the fundamental unit of magnetism Bohr's magneton is too large; because we have a large number of substances, for which the saturation value of magnetization is less than the magnetic moment of a Bohr magneton. For example, the saturation values of magnetization for nickel, Mn_3Sb and Mn_2Sb[1] are respectively 3370, 1513 and 610 per gram atom or molecule, which are obviously far less than one Bohr magneton 5589; the same values for ferromagnetic compounds Mn_4Sn, $MnBi$, NiB, CoB, FeB, Mn_5P_2, Mn_2As, $AsMn$, etc. are also less than the Bohr unit.

66. Magnetic Deflection of Atomic Rays.

In our theory the nucleus of an atom is the seat of the magnetizable magnetic moment and the outer or optical electrons are the carriers of non-magnetizable moment. Gerlach and Stern[2] first succeeded in determining the latter kind of magnetic moment; their experiment not only yielded a direct determination of the magnetic moment of the Bohr magneton, but they also afforded a direct proof of the quantization of the direction of the outer revolving electrons in the atom in relation to the magnetic field. But what Gerlach and Stern have found is not the magnetizable magnetic moment of the nucleus, which has a large mass and the orientation of which it is consequently very difficult to change in a very short interval of time.

The metal to be investigated is heated in a small furnace, and the atoms evaporated escape through a narrow slit with a velocity depending on the temperature of the furnace. The stream of atoms passes through a heterogeneous field at right angles to it, the field and its gradient being in the same direction, and impinges on a glass plate, forming a very thin deposit, which is only visible after a special developing. The traces of the deposit with and without the magnetic field give the deflection of the atomic rays; the amount of deflection y depends on the magnetic moment M of the optical electrons, the field H and its gradient $\partial H/\partial y$ and the velocity v of the atoms. If the flying atom has a mass m and traverses a path of length l, then we have from elementary considerations

(1) K. Honda and T. Ishiwara, Sci. Rep. **6**, (1917), 9. (2) Ann. d. Phys. **74**, (1924), 673; **76**, (1925) 163.

$$y = \pm \frac{1}{2} \frac{M}{m} \frac{\partial H}{\partial y} \left(\frac{l}{v}\right)^2,$$

according as M has the same direction or the opposite direction to that of the field.

According to the quantum theory, an atom which possesses a resultant moment of momentum, must so orientate itself in a magnetic field that the component of the moment in the direction of the field is an integral multiple of the unit $he/4\pi m$. Hence, in the case of the unit Bohr magneton, its direction must be either parallel to or opposite to the field. In a heterogeneous field, the magneton in the former orientation will be attracted towards the stronger field, while the magneton in the latter orientation will be repelled in the opposite direction. Thus, if an atomic ray be sent through the heterogeneous field in the direction perpendicular to it, it will be split up into two discrete beams. On the other hand, if every direction of the magneton in relation to the magnetic field is possible as follows from the classical theory, the atomic beam will merely be broadened. Hence, the experiment of the atomic beam will decide for or against the theory of the space-quantization of the Bohr magnetons by the manner of the splitting of the beam.

In general, the number of parts into which the beam is split up will be $2j + 1$ in terms of the spectral theory, where j is Sommerfeld's j for the basic term of the spectrum of the neutral atom.

According to the theory of the present author, the nucleus has a magnetic moment and exerts a magnetic field on the outer electrons; but an external field of 20000 gausses, as used by Gerlach and Stern, being very large as compared with the internal field due

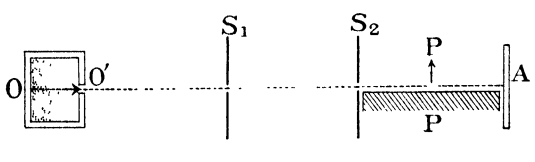


Fig. 211a.

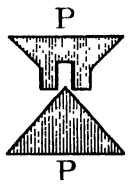


Fig. 211b.

to the nucleus, the resultant field acting on the Bohr magneton is very near in direction to the external field.

Fig. 211 *a, b* shows diagramatically the arrangement of experiments made by Gerlach and Stern. O is a small furnace with an aperture O', in which the metal is vaporised. The vaporised

atoms are continuously making thermal motion within the furnace, some of which escape through the aperture O'. Two narrow slits S_1, S_2 limit the atomic rays to a very narrow beam. This beam passes through the slit S_2 into a heterogeneous field between the pole-pieces PP of an electromagnet, the field and its gradient being both perpendicular to the atomic beam. A is a plate, which can be cooled, if necessary, and on which the atoms are deposited; this deposit is generally too faint to be seen, but may be made visible by means of a special technique. PP in Fig. 211*b* is the side-view of the electromagnet. The whole arrangement including the furnace, the slits and the magnetic poles, is enclosed in an evacuated vessel.

The gradient of the field is largest along the knife-edge of the electromagnet and falls very rapidly on both sides of it, and hence if the space-quantization does actually take place, an atomic beam of a unit moment will split into two in traversing the heterogeneous field. The separation is a maximum in the direction of the knife-edge and diminishes on either side, until a point is reached where the gradient is not strong enough to cause the resolution of the beam. In many cases, splitting into two beams was observed, but the case of triple or multiple splitting seldom occurred.

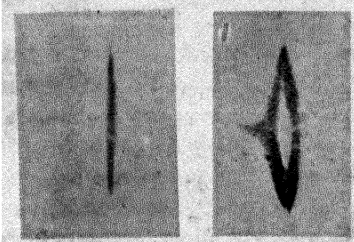
Silver, copper, gold, thallium, bismuth, tin, lead, antimony, iron and nickel were studied by Gerlach and Stern[1], hydrogen by T. E. Phipps, J. B. Taylor and E. Wrede[2]. Zinc, cadmium, thallium, potassium and sodium by Alfred Leu[3]. Pl. I shows the traces of the atomic rays of these metals impinging on the plates. The atomic rays of silver, copper, gold, hydrogen, thallium, potassium and sodium show the double splitting of the beam, while nickel and bismuth give deflected as well as undeflected beams. Other metals do not show any splitting of the beam. The approximate number of the Bohr magnetons deduced from these deflections is tabulated below:

Ag	Cu	Au	H	K	Na	Ni	Tl
1	1	1	1	1	1	$1/2$	$1/3$

Thus, the first six elements have each a magnetic moment of one Bohr magneton, which orientates in the magnetic field in accordance with the space-quantization. Although the Bohr magneton cannot be magnetized, it may take the quantum orientation in relation to the magnetic field. Thus Gerlach and Stern's method

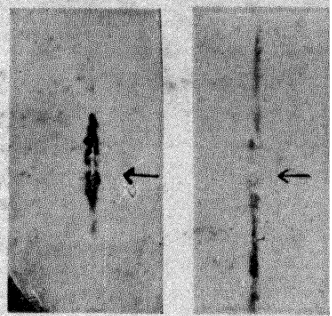
(1) Gerlach and Stern, loc. cit. (2) T. E. Phipps and J. B. Taylor, Phys. Rev. **29**, (1927), 309; Erwin Wrede, Zeit. für Phys. **41**, (1927), 569. (3) Alfred Leu, Zeit. für Phys. **41**, (1927), 551.

PLATE I.



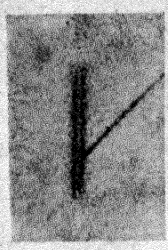
Silver

Without field. With field.

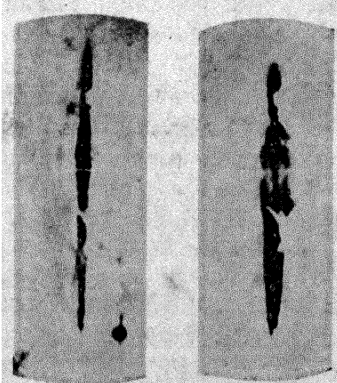


Copper Gold

With field.

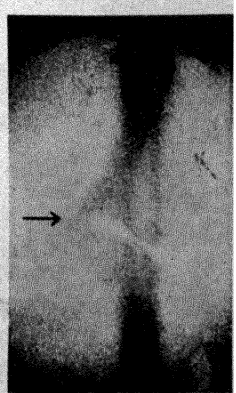


Thallium with field.



Nickel

Without field. With field.



Bismuth with field.

furnishes us with a direct proof of the space-quantization of the outer revolving electrons. On the other hand, zinc, tin, lead, cadmium, antimony and mercury have no Bohr magneton; this shows that in the case of these metals, the actions of the Bohr magneton having magnetic moments of opposite signs in the same atom just balances each other. Thallium has only a small magnetic moment of one-third of a Bohr magneton. Nickel shows two deflected beams and one undeflected beam; that is, some nickel atoms have no Bohr magneton and others two magnetons. Iron atoms seem to have no magneton, but this requires further confirmation.

Lastly, we shall consider the above result in connection with the structure of the atoms. Hydrogen atoms have no nuclear electron and therefore no magnetizable moment, but they have one Bohr magneton due to one revolving electron, as confirmed by experiment. Atoms of higher atomic weights have each a nuclear moment, which is, however, small in light atoms and gradually increases in heavy atoms. Sodium and potassium are paramagnetic, and their atoms have each one outer electron. In these metals, the nuclear moment is probably small as compared with that due to the outer electron, and therefore the splitting of the atomic beam is to be attributed chiefly to this electron or to one Bohr magneton. Similarly, atoms of copper, silver and gold have all one outer electron, but the susceptibility of these metals is diamagnetic not only in the solid state, but also in the liquid state. From the ordinary theory, it is very difficult to explain why the gaseous atoms of these metals have a magnetic moment, while the liquid atoms are diamagnetic. But the author's theory stands in good harmony with this apparently anomalous fact; because the splitting of the atomic beam is due to the outer non-magnetizable electron or to the Bohr magneton. The fact that the magnetic moment of the nickel atom contains two units of Bohr's magneton and also that iron atoms have no magnetic moment, does not agree with the results deduced from the saturation value of these ferromagnetic metals, but this discrepancy may be explained in the same way as shown above. The fact that the magnetic moment of thallium atoms has a value of $1/3$ Bohr magneton, is in good agreement with the result of the Zeeman effect of the element. In the case of zinc, tin, lead, cadmium, antimony and mercury the action of the Bohr magnetons having magnetic moment of opposite signs just balances each other, so that no deflection of the beam is observable.

The well-known relationship between the spectroscopic evid-

ence and the number of Bohr magnetons, as detected by the atomic rays, is naturally to be expected from our theory; because by this theory also the origin of the magnetic moment of the atomic ray is attributed to the optical electrons or to the Bohr magnetons, which, independent of the nucleus, may orientate themselves in certain discrete directions with respect to the magnetic field in accordance with the quantum mechanism.

67. Gyro-magnetic Effect.

If an electron of charge e and mass m is revolving in a circle of radius r round the positive nucleus of an atom, its magnetic moment and its angular momentum are shown to be

$$M = \pi ner^2 \qquad \text{and} \qquad p = 2\pi nmr^2$$

respectively, where n is the number of revolutions per second. Hence we have the relation

$$\frac{p}{M} = \frac{2m}{e} = 1.13 \times 10^{-7}.$$

Here the relation is obtained by assuming a circular orbit for the motion of the electron, but even in the general case, where the electron describes an elliptic orbit, the magnetic moment and the angular momentum are connected by the same relation.

Let us next consider the case of magnetization of a magnetic substance, which contains a large number of such orbital electrons. If it be assumed that a system of revolving electrons in atoms behaves, when acted on by an external field, like a rigid magnet and its magnetic axis turns towards the direction of the field against the mutual action of the atoms,—an assumption, which is, as shown before, not plausible from the dynamical theory of electrons, the resultant magnetic moment σ and the moment of momentum P in the direction of the field are then obtained by simply summing up the components of each of these two vectors taken in the same direction. Thus we have

$$P = \Sigma p_H = \frac{2m}{e} \Sigma M_H = \frac{2m}{e} \sigma,$$

or

$$\frac{P}{\sigma} = \frac{2m}{e}.$$

Hence any change in the magnetization must be accompanied by a change in the angular momentum.

According to the law of conservation of the angular momentum in mechanics, the angular momentum of an isolated system always remains constant. Hence if a magnetic substance is magnetized or demagnetized by an electric current or by a permanent magnet, a mechanical angular momentum is produced in the opposite direction to compensate for the change in the intrinsic angular momentum P, that is, a reactional mechanical force appears so as to cause a rotational motion of the body as a whole, provided that atoms or parts of the atoms are not free to turn owing to the presence of the neighbouring atoms.

According to the theory of the present author, the origin of the magnetism is in the nuclear electrons and their angular momentum is nearly balanced by that of the protons, and hence the magnetization is not accompanied by any appreciable increase of the angular momentum; that is,

$$\frac{P}{\sigma} = 0.$$

Consequently, the magnetization cannot give rise to any reactional couple in a magnetized substance.

The above rotational reaction accompanying magnetization was suggested early in 1908 by Richardson[1]. He failed, however, to detect this effect experimentally. Einstein and de Haas[2] first succeeded in confirming the existence of this effect, making use of the resonance method. A cylinder of soft iron was vertically suspended by a thin glass fibre in a magnetizing coil, and the period of torsional oscillation was so adjusted as to equal that of an alternating current passing through the coil. When the resonance was set up, the amplitude of the torsional oscillation became a maximum; and from the maximum amplitude, the damping coefficient and the saturation value of magnetization, the ratio m/e was calculated. Their experiment was repeated by E. Beck[3] and Arvidsson[4].

J. Q. Stewart[5] carried out the same investigation with a similar arrangement of suspending a ferromagnetic rod in a coil. The rod was first magnetized by a magnetizing current and then its direction was suddenly reversed, the corresponding rotation of the rod about its axis being measured by the ballistic throw of a

(1) Phys. Rev. **26**, (1908), 248. (2) Verhand. d. D. Phys. Ges. **17**, (1915), 152; **18**, (1916), 173. (3) Phys. Zeit. **20**, (1919), 490; Ann. d. Phys. **60**, (1919), 109. (4) Phys. Zeit. **21**, (1920), 88. (5) Phys. Rev. **11**, (1918), 100.

light spot reflected from a mirror attached to the rod. Very accurate experiments with iron and nickel were also made by Chattock and Bates[1] who made use of a method similar to that of Stewart. While Einstein and de Haas found the ratio of angular momentum to magnetic moment agreeing with that of the theory within 2 percent, later experimenters obtained for the same ratio a value of one-half the theoretical value.

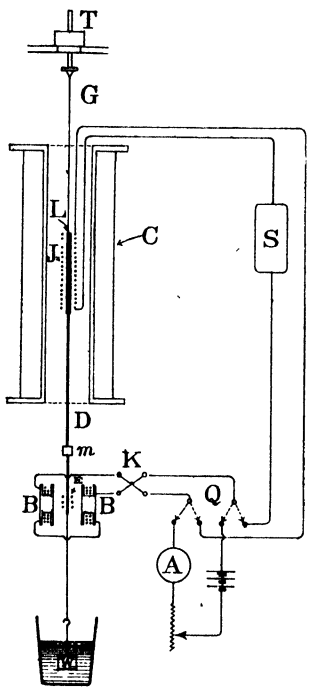


Fig. 212.

Sucksmith and Bates[2] repeated the same experiment by making use of the null method. Fig. 212 shows their arrangement. A rod *L* to be tested is suspended from the torsion head *T* by means of a thin German silver wire G, so that the rod lies symmetrically along the axis of the vertical magnetizing coil *C*, through which a current from an A. C. generator is passed. To the lower end of the rod is attached an aluminium wire *D* carrying a small mirror *m* and a number of short magnetized needles *E*. These needles are so adjusted as to be situated midway between a pair of small Helmholtz coils *BB*. A small coil *J*, co-axial with the specimen *L*, is fixed inside the magnetizing coil. The coils *BB* are placed in series with a resistance *S* and the induction coil *J*, a commutator *K* being included to reverse the current when necessary. The switch *Q* allows a known current to be passed through Helmholz's coils if desired. *W* is the weight immersed in oil and stretches the suspended system downwards.

When an alternating current flows through the magnetizing coil, the magnetization of the specimen is periodically reversed, and the gyro-magnetic effect makes the system oscillate about its vertical axis, the amplitude of which will be a maximum when the frequency

(1) Phil. Trans. A, **223**, (1923), 257. (2) Proc. Roy. Soc. A, **104**, (1923), 499; A, **108**, (1925), 638.

of the alternating current is equal to the natural frequency of oscillation of the suspended system. The induced current in the coil J flowing through the coils BB and the resistance S, is so adjusted by means of the resistance that the gyro-magnetic effect is just balanced by the momentum delivered to the system by the action of the current in the coils BB on the magnets E; then the ratio in question can be obtained in the following way:—

Let N be the number of lines of induction through the small coil J, where a specimen of length l and cross section S is magnetized; then if n be the number of turns per centimetre of the coil,

$$N = (4\pi I + H)\, Sln.$$

Let Γ be the couple which acts on the magnets E by virtue of a unit current flowing through the coils BB, and i the current required just to compensate for the gyro-magnetic effect due to magnetization; then if R be the electric resistance in the circuit, we have

$$Ri = \frac{dN}{dt} = \left(4\pi \frac{dI}{dt} + \frac{dH}{dt}\right) Sln,$$

and the angular momentum P of the gyro-magnetic effect is related to Γ in such a way that

$$\frac{dP}{dt} = \Gamma i\ ;$$

hence

$$P = \frac{\Gamma Sln}{R}(4\pi I + H).$$

If σ is the total magnetic moment of the specimen, then

$$\sigma = ISl.$$

Neglecting the small quantity H in the expression for P, we get

$$P = \frac{4\pi\sigma\Gamma n}{R}.$$

If a steady deflection θ is produced when a known current i flows through the coils BB, then

$$\Gamma i = k\theta,$$

where k is the torsional constant of the suspension. Eliminating Γ from the last two equations, we get

$$\frac{P}{\sigma} = \frac{4\pi kn\theta}{iR}.$$

In this ratio all the quantities involved are capable of accurate measurement, and hence this method affords us a very accurate means of determining the above important ratio.

The results of determination obtained by Sucksmith and Bates are given below:—

Specimen	Number of exp.	$\frac{P}{\sigma}\frac{1}{1.13\times10^{-7}}$
Iron	16	0.503 ± 0.006
Nickel	6	0.501 ± 0.002
Cobalt	9	0.515 ± 0.032
Magnetite	10	0.495 ± 0.024
Heusler's alloy	14	0.501 ± 0.003

Thus the ratio actually obtained is, within the limits of experimental error, half that expected from the ordinary theory, and is equal to the mean of the values obtained from the above and the author's theory. At present, we cannot explain the above discrepancy between experiment and theory. Apparently many more experiments under different conditions, especially in comparatively weak fields and also at high temperatures, are needed before we are sufficiently clear about this point.

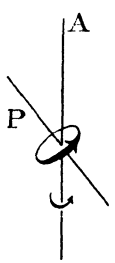


Fig. 213.

A phenomenon, which is usually considered to be the inverse effect of the above, has also been observed. A system of revolving electrons may be considered as a gyroscope. It is a well-known fact that a gyroscope rotating with a large angular velocity shows a peculiar property with regard to the change in the direction of its axis. If we take the gyroscope, which is spinning round its axis P, and put it under the action of a couple C round another axis A, then the resultant motion is not only to rotate P round A, but also is to make P rotate in the PA plane and finally to set it in the direction of the axis A. A corresponding effect may be expected to occur in a magnetic substance, whose atoms are, so to speak, all small gyroscopes.

Thus the elementary magnets consisting of rapidly revolving electrons must have a tendency to set their axes in the direction of the rotational axis, so that the substance will be magnetized by rotation.

The first experiment based on this idea was made by J. Perry[1], who tried, but without success, to detect a change in the magnetization of an iron rod produced by rotating it about its axis. S. J. Barnett[2] in collaboration with L. J. H. Barnett first obtained a positive result in his experiment in 1914. The substance under investigation was in the form of a cylindrical rod; it was mounted horizontally and perpendicularly to the magnetic meridian in a space where the terrestrial magentic field was annulled, and was rapidly rotated about its axis. Two methods of measuring magnetization, that is the method of electromagnetic induction and the magnetometric method, were used. In the former method the change of magnetization was measured ballistically by means of a coil surrounding the rod, while in the latter, an astatic magnetometer was mounted with the centre of its lower magnet in the polar axis, or preferably in the equatorial plane of the rod under investigation. The magnetization produced by rotation was compared with that under the action of an external field. In this way the ratio of the angular momentum in ferromagnetic substances to the magnetic moment was determined. S. J. Barnett and L. J. H. Barnett found that for iron, steel, nickel and cobalt, the ratio in question is the same in each case, their value multiplied by $1/1.13\times10^{-7}$ being very nearly equal to $1/2$.

Thus we again find the same discrepancy between theory and experiment; but no matter how this discrepancy may be explained in future, it may safely be concluded that the origin of magnetism is to be sought in the revolving electrons within the atoms.

(1) Spinning Tops, Oct., (1890), 112. (2) Phys. Rev. **6**, (1915), 239; **10**, (1917), 7; **17**, (1921), 404; **20**, (1922), 90; Phys. Zeit. **24**, (1923), 14.

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