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ASSISTANT MASTER AT CHARTERHOUSE

AND

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ASSISTANT MASTER, MERCERS' SCHOOL

AUTHOR OF 'SCHOOL ALGEBRA', 'ELEMENTARY TRIGONOMETRY'

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PREFACE

THIS work attempts to place the subject-matter of Arithmetic before the average student in a logical sequence, whilst reducing to a minimum the memorizing of rules.

Part I deals primarily with all the calculations and methods (with the exception of Logarithms) which form the machinery of the subject. A student who has worked through this part should be able to solve any ordinary problem in Arithmetic, provided he understands the technical terms introduced into the problem. Consequently the advance to questions dealing with Commercial Arithmetic represents only a very small step, and involves no principle which has not been treated thoroughly in Part I.

Part II is devoted to the applications of arithmetical methods. The commercial side of the subject is kept distinct from the Mensuration necessary for scientific and other purposes.

This arrangement is based upon a system which has been current at Charterhouse for some years. The idea has been that, of the three terms of the school year, the first should be devoted to Mensuration and the second to Commercial Arithmetic, the third being used as a reserve term in which portions of both subjects are revised or completed.

Since logarithmic computation is necessary in both Mensuration and Commercial Arithmetic, a chapter dealing with this is inserted at the beginning of Part II.

The chief features of Part I are:

(i) Decimals have been introduced at the earliest possible stage—in fact, as soon as the ordinary operations with whole numbers have been dealt with. A special advantage of this arrangement is that it is possible to treat 'reduction' and 'decimalization' simultaneously.

(ii) Several rules are discarded, such as the old method of finding L.C.M. and the long method of multiplying sums of money.

(iii) The student is encouraged always to keep in view the degree of accuracy possible. Very long examples in reduction, e.g. of square miles to square inches, have been rigorously excluded; and, throughout the book, four- or five-figure accuracy is regarded as sufficient. Contracted methods have been introduced as more correct than the long methods, rather than as labour-saving devices.

(iv) Great stress is laid upon ratio methods. In the opinion of the authors the use of the 'operator ratio' is the very foundation of sound mathematics.

(v) In most of the side work in the text where division occurs, the Italian method has been used. The continual use of this method effects a considerable lightening of labour and ultimately makes for greater accuracy.

(vi) In written work the student is encouraged to do all his work in a legible form with a minimum of side work.

Some justification is perhaps needed for the insertion of book-work and worked-out examples. It is neither intended

nor expected that students will read the book without some additional instruction; nor is it likely that all teachers will use the methods set out in the text. But, since it is impossible for a teacher of a class to explain separately and continually every point of difficulty that may arise in revision work, it is in practice necessary that pupils should be able to find the help they need in their text-book.

Oral examples (answers to which are given with the other answers) form an important part of the general scheme of the book. Some sets are designed to lead up to new methods; others are intended to provide simple exercises on methods already dealt with.

A remark is necessary on the general arrangement of the examples. In the course of each chapter the examples are mainly manipulative (these are numbered X *a*, X *b*, &c.); whilst a miscellaneous set at the end of each chapter (numbered simply X, &c.) includes more difficult problems involving the use of the methods dealt with in that chapter.

The authors are indebted to Miss Chignell, of Bishop Otter College, Chichester, to Mr. A. S. Paterson, of the Cathedral School, Truro, and to Mr. C. O. Tuckey, of Charterhouse, for much valuable advice and for help in reading the proofs. They also wish to express their appreciation of the care with which the printers and publishers have sought to carry out the recommendations of a Committee of the British Association which has recently issued a report on the Influence of School-books upon Eyesight. This Committee emphasizes the need for clearly printed mathematical symbols, and remarks that

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'sets of examples are the most important part of a book so far as influence upon vision is concerned'

Many examples have been taken from recent examination papers, with the kind permission of the following authorities: The Controller of His Majesty's Stationery Office, the University of Oxford, the University of Cambridge, the University of London, the University of Birmingham, the Intermediate Education Board for Ireland, the Central Welsh Board, the Joint Matriculation Board of the Northern Universities, the Delegacy for Oxford Local Examinations, the Syndicate for Cambridge Local Examinations, the College of Preceptors, and the London Chamber of Commerce.

N. J. C.
W. E. P.

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(1.) BRITISH MONEY.

	4 farthings	= 1 penny (*d.*)
	12 pence	= 1 shilling (*s.*)
	20 shillings	= 1 pound (£)
Also	2 shillings	= 1 florin
	5 shillings	= 1 crown
	21 shillings	= 1 guinea

(2.) FOREIGN MONEY (with approximate English equivalents).

			English Equivalent
France	100 centimes	= 1 franc	9·513*d.*
Italy	100 centesimi	= 1 lire	9·513*d.*
Spain	100 centimo	= 1 peseta	9·513*d.*
Greece	100 leptas	= 1 drachma	9·513*d.*
Germany	100 pfennigs	= 1 mark	11·75*d.*
	3 marks	= 1 thaler	
	10 marks	= 1 krone	
Russia	100 kopecks	= 1 rouble	3*s.* 2*d.*
Turkey	40 paras	= 1 piastre	
	100 piastres	= 1 £T.	18*s.* 0¾*d.*
United States	10 cents	= 1 dime	
and Canada	10 dimes	= 1 dollar ($)	4*s.* 1·32*d.*
	10 dollars	= 1 eagle	
Japan	100 sen	= 1 gold yen	2*s.* 0½*d.*
India	4 pice	= 1 anna	
	16 annas	= 1 rupee	1*s.* 4*d.*
	100,000 rupees	= 1 lac	

(3.) LENGTH.

12 inches	= 1 foot (ft.)	for small measurements
3 feet	= 1 yard (yd.)	
100 links	= 1 chain (ch.)	for surveying
10 chains	= 1 furlong (fur.)	
8 furlongs	= 1 mile (mi.)	
Also		
$5\frac{1}{2}$ yards	= 1 rod, pole, or perch	agricultural measures
40 poles	= 1 furlong	
22 yards	= 1 chain	
6 feet	= 1 fathom (for depth of water)	
6080 feet	= 1 nautical mile [1] (for distance at sea)	

(4.) AREA.

144 square inches	= 1 square foot (sq. ft.)	
9 square feet	= 1 square yard (sq. yd.)	
484 square yards	= 1 square chain (sq. ch.)	
10 square chains	= 1 acre (ac.)	
640 acres	= 1 square mile (sq. mi.)	
Also 40 square poles	= 1 rood (ro.)	
4 roods	= 1 acre	
16 square poles	= 1 square chain	

(5.) VOLUME.

1728 cubic inches (cub. in.)	= 1 cubic foot (cub. ft.)
27 cubic feet	= 1 cubic yard (cub. yd.)

(6.) CAPACITY.

4 gills	= 1 pint (pt.)	for liquids
2 pints	= 1 quart (qt)	
4 quarts	= 1 gallon (gal.)	

[1] A knot is a speed of 1 nautical mile per hour.

2 gallons = 1 peck (pk.)
4 pecks = 1 bushel (bus.) } for grain, &c.
8 bushels = 1 quarter (qr.)

36 bushels = 1 chaldron

(7.) APOTHECARIES' FLUID MEASURE.

60 minims = 1 fluid drachm (fl. dr.)
8 fluid drachms = 1 fluid ounce (fl. oz.)
20 fluid ounces = 1 pint

(8.) WEIGHT (OR MASS).

(a) *Avoirdupois.*

16 drams (dr.) = 1 ounce (oz.)
16 ounces = 1 pound (lb.)
28 pounds = 1 quarter (qr.)
4 quarters = 1 hundredweight (cwt.)
20 hundredweight = 1 ton

Also 7000 grains (gr.) = 1 pound
14 pounds = 1 stone (st.)[1]
480 grains = 1 ounce Troy

(b) *Troy Weight.*[2]

24 grains (gr.) = 1 pennyweight (dwt.)
20 pennyweight = 1 ounce (oz. Troy)
12 ounces = 1 pound (lb. Troy)

(c) *Apothecaries' Weight.*

20 grains = 1 scruple (scr.)
3 scruples = 1 drachm (dr.)
8 drachms = 1 ounce Troy

The grain has the same value in all these measures.
1 gallon of water weighs 10 lb.
1 cubic foot of water weighs 1000 oz. (approximate).

[1] The butcher's stone contains 8 lb.

[2] These measures are extinct with the exception of the ounce Troy. See § 57.

(9.) ANGLES.

60 seconds (60″) = 1 minute (1′)
60 minutes (60′) = 1 degree (1°)
90 degrees = 1 right angle (rt. ∧)

(10.) TIME.

60 seconds (sec.) = 1 minute
60 minutes (min.) = 1 hour (hr.)
24 hours = 1 day
7 days = 1 week

1 sidereal year = approximately 365·2422 days
1 civil year = 365 days
= 52 weeks and 1 day
1 leap year = 366 days

(11.) METRIC SYSTEM (see p. 55).

1 litre = 1 cubic decimetre
1 gramme = weight of 1 cubic centimetre of water
1 are = 1 square dekametre
1 metre = 39·37 inches
1 litre = 1·760 pints
1 gramme = 15·43 grains
1 kilogram = 2·20 lb.
1 yard = ·9144 of a metre
1 acre = ·4047 of a hectare
1 gallon = 4·544 litres
1 pound = 453·6 grammes

CHAPTER I

THIS chapter is intended as a revision chapter for those who have already made some progress in Arithmetic.

NUMERATION AND NOTATION

(**1.**) Arithmetic, as its name shows (it is derived from the Greek *Arithmos*, a number), is the science dealing with numbers.

The word 'number' conveys three ideas which are often confused; these are (i) the number itself, (ii) the name given to that number, and (iii) the symbol denoting the number. Thus there is a definite number of days in a week, and the name given to that number is different in different languages, but the same symbol is used by nearly all civilized nations to denote that number.

(**2**) The first step is to give names to the numbers. This is called **Numeration**. The process of repeating these names in order, beginning with the least, is called **Counting.**

Counting was originally done by means of the fingers; thus there is a different name corresponding to the number of fingers used, viz.:

one, two, three, four, five, six, seven, eight, nine, ten.

(**3.**) When a number greater than ten occurs, these names are used again. It is explained in grammar books that eleven means ten and one, and twelve means ten and two; also twenty, thirty, &c., are abbreviations for two tens, three tens, &c., and the remaining numbers are obvious up to

ninety-nine. Then a new word 'hundred' is introduced, and the system of naming is again obvious up to nine hundred and ninety-nine. Then we have the word 'thousand'. There is no need for a new word till the number 'a thousand thousand' is reached, and this is denoted in English by the word 'million'. No other words are wanted in practice, but occasionally the words 'billion', 'trillion', 'quadrillion', &c., are used. In English, billion means a million million, trillion a million billion, &c.; but on the continent a billion means a thousand million, trillion a thousand billion, &c.

This system is said to be the **decimal system**, because it is built up on the number ten (Latin *decem*).

(**4.**) **Notation** is the portion of Arithmetic that shows how to express a number in symbols. The advantages of the decimal system are better appreciated if we consider first the **Roman System**. In this system the capital letters

I	V	X	L	C	D	M
one	five	ten	fifty	hundred	five hundred	thousand

are used, and they always denote the numbers which are here written underneath them. The rules for using them are simple:

(*a*) If a symbol is repeated, the number is repeated, thus:

II is one and one, i.e. two. XXX is ten and ten and ten, i.e. thirty.

(*b*) If a symbol is placed after one of greater value, the two corresponding numbers are to be added; thus VIII is five and three, i.e. eight.

(*c*) If a symbol is placed before one of greater value, the smaller number is to be taken from the greater; thus XC is a hundred less ten, i.e. ninety. In cases where a symbol comes between two others of greater value, subtraction is performed before addition; thus XIX is X and IX, i.e. ten and nine, or nineteen (it is not XI and X).

(*d*) The symbols are written in descending order of magnitude; e. g. one thousand seven hundred and eighty-seven, which is the same as one thousand, and five hundred, and two hundreds, and fifty, and three tens, a five, and two ones, is represented by MDCCLXXXVII.

It is unusual to use more than three of the same symbol: except on clocks and watches, four is denoted by IV, and not by IIII.

The Roman notation is often used in inscriptions to denote the year, and it is then the custom to use CCCC for four hundred; thus nineteen hundred is more often written MDCCCC than MCM.

When the Roman notation is used for numbering pages of a book, &c., small letters are usually used; thus nine is written as ix.

This system of notation does not agree very clearly with the naming of the numbers, and is obviously quite unfit for purposes of calculation.

(**5.**) In the ordinary, or **Arabic**, system of notation there are ten symbols which are called **digits** (Latin *digitus*, a finger). The numbers from one to nine have each their own symbol, viz.:

1	2	3	4	5	6	7	8	9[1]
one	two	three	four	five	six	seven	eight	nine

The peculiarity of the Arabic system is that the actual number denoted by any one of these digits depends upon the position of the digit with regard to the others. A single digit denotes a number of ones fewer than ten, or, as we generally say, a number of units; if there are two digits, that on the left denotes a number of tens, and the one on

[1] Great care should be taken to write these digits so that no two can be mistaken for one another; badly shaped figures are a common cause of mistakes in calculation.

the right a number of units; thus 72 means seven tens and two, i. e. seventy-two.

The symbol 0 (called zero or nought) is used to denote the absence of any digit in a particular position. Thus 30 means three tens and no units, or thirty. This system is extended indefinitely, the extreme right-hand digit denoting units, the next to it tens, the next hundreds, &c.; so that 3564 means three thousands, five hundreds, six tens, and four units. The sign of the plural is usually omitted, so that the number is read—three thousand, five hundred and sixty-four.

It is seen that, in the Arabic notation, the symbol denoting a number agrees step by step with the name of the number; in fact, the agreement is so exact and simple that the full significance of the Arabic notation is often forgotten.

(6.) Before we can read off the number denoted by a set of figures (i. e. digits), we must determine the **local value** of each digit, and here we have a slight drawback in the convenience of the system.

Consider 7356782.

Before we can say what the 7 represents, we must run our eye to the extreme right and reckon backwards mentally—units, tens, hundreds, thousands, tens of thousands, hundred thousands, millions; the 7 denotes 'seven millions'. Now we can read the number.

To make the reading easier, we mark off the figures in groups of three, beginning at the right; thus: 7,356,782. The number is now seen to be seven million, three hundred and fifty-six thousand, seven hundred and eighty-two.

(7.) The Arabic system of notation is built up on the number ten, and is therefore a 'decimal system'. This is expressed otherwise by saying that ten is the **radix** (i. e. root or base) of the Arabic system. The Roman system is not decimal; it is built up on five quite as much as on ten.

It is quite possible to have any other number as radix; for instance, twelve would give a 'duodecimal system'. This system would have many advantages, but it has never been brought into practical use except in certain questions involving the calculation of areas and volumes.

It is interesting to note that numeration in some foreign languages shows traces of the roots five and twenty as well as ten. In Latin eighteen is *duodeviginti* (two from twenty); in French eighty is *quatre vingt* (four twenties); in Welsh sixteen is *un ar bymtheg* (one on fifteen). In English, too, we have such expressions as threescore (instead of sixty).

Examples I a.

Express the following numbers in words, and write them in the Roman and Arabic systems of notation:

1. The number of lines on this page.
2. ,, ,, sentences on the preceding page.
3. ,, ,, letters in the tenth line of the preceding page.
4. ,, ,, pupils in the room.
5. ,, ,, panes of glass in the windows of the room.

6. Write the following dates in the Roman notation: 871, 1066 1216, 1346, 1509, 1588, 1660, 1703, 1837, 1910.

7. Express in words: XLVII, XCIX, ccxiii, xxxiv, MDCXCIX.

8. Express in figures: two thousand five hundred and two; five million twenty thousand and sixteen; eleven thousand eleven hundred and eleven; forty-nine million one thousand and seven; three hundred and two thousand and ninety-four.

9. Express in words: 5632; 3589036; 75834; 23570002; 509603; 805632; 859306; 22266646; 909099.

10. Two men were knocked over by a motor car. Both were sure that the number of the car had four digits, but one could only recollect 375, and the other only 137. What was the number? Give the answer in words.

11. Three boys, travelling together on a tram, each received

a penny ticket, but the tickets were punctured through the numbers.

The first could only read 2 74 on his ticket, the second 23 5, and the third 237 . State in words the numbers on the tickets.

12. How would the number nineteen be written in a scale whose radix is (i) seven; (ii) eleven; (iii) eighteen?

13. How many numbers are there which, in the Arabic notation, begin with 9 and have four figures?

14. In the ordinary decimal notation, how many numbers (i) less than a hundred, (ii) less than a thousand, have a nought at the end?

Simple Rules.

(**8.**) Accuracy in Arithmetic can only be obtained by accuracy in mental work; the following examples do not provide sufficient practice, but suggest the kind of 'drill' that is needed even by those who have been learning Arithmetic for some years.

\+ is the sign for addition.
− ,, ,, subtraction.
× ,, ,, multiplication.
÷ ,, ,, division.
= ,, ,, denoting equality between what comes before and what follows.

Illustrative Examples. *Fill in the gaps in the following:*

1. $56+4=$. **2.** $8+$ $=12$. **3.** $7+5+8+3+9+6=$
4. $7+5-8+3+9-6=$. **5.** $7\times9+5=$.
6. $7\times9+$ $=71$. **7.** $36\div4=$. **8.** $37\div5=$

The mental work should be as short as possible; it is a waste of time and a cause of inaccuracy even to think unnecessary words.

The mental processes for these examples should be somewhat as follows:

1. 56, **60**.
2. 8 and **4** equal 12.
3. 7, 12, 20, 23, 32, **38**.
4. 7, 12, 4, 7, 16, **10**.
5. 63, **68**.
6. 63 and **8** equal 71.
7. **9**.
8. **7** and 2 over.

Oral Examples. *Fill in the gaps in the following:*

1. $8+7=$.
2. $7+3+9=$.
3. $4+9+7=$.
4. $9+8+4+3+8+3=$.
5. $7+5+6+8+3+1=$.
6. $9+$ $=14$.
7. $4+$ $=11$.
8. $6+$ $=14$.
9. $26+$ $=31$.
10. $28+$ $=35$.
11. $72+$ $=80$.
12. $18-2=$.
13. $17-3=$.
14. $11-6=$.
15. $7+6-4=$.
16. $5+9-8+2=$.
17. $8-3+4-8=$.
18. $3+7-5+8-2+7-9=$.
19. $5+1-3+7-6+8+9-7+4-5=$.
20. $7\times8=$.
21. $7\times8+5=$.
22. $7\times8+$ $=60$.
23. $3\times8=$.
24. $3\times8+4=$.
25. $3\times8+$ $=29$.
26. $6\times7+$ $=41$.
27. $6\times9+$ $=57$.
28. $6\times8+$ $=53$.
29. $42\div6=$.
30. $72\div8=$.
31. $35\div5=$.
32. $48\div7=$.
33. $40\div6=$.
34. $95\div8=$.
35. $23\div$ $=4$ and 3 over.
36. $68\div$ $=9$ and 5 over.
37. $\div2=6$ and 1 over.
38. $6\times$ $+3=33$.
39. $4\times9+$ $=42$.
40. $\times5-11=69$.
41. $82\div2\times3=$.
42. $7\times$ $+8=85$.

ADDITION

(**9.**) The following points should be observed in connexion with addition:

(i) Avoid unnecessary mental steps;

(ii) When the work is arranged in columns, take care that units are placed **directly** under units, tens under tens, &c.;

(iii) When the numbers are arranged on the same line, take care to add units to units, tens to tens, &c.

It is probably simplest to place all the numbers to be added so that their units are all in the same vertical column, but it is not necessary to do so; the student is advised to cultivate the habit of writing down sums of numbers without previously copying them in vertical columns.

Example i. *Add together* 2347, 6251, 237, 4962, 363.

The sum is 14160.

Here the mental work is somewhat as follows:

The sum of the units digits is 20;
put down 0 and carry 2 to be added with the tens.

The sum of the tens digits, with the extra 2, is 26;
put down 6 and carry 2 to be added with the hundreds.

The sum of the hundreds digits is 21;
put down 1 and carry 2 to the thousands.

The sum of the thousands is 14, which is at once put down.

Example ii.

(*a*)	(*b*)	(*c*)		
731	574	693	1998	(i)
562	691	832	2085	(ii)
346	342	561	1249	(iii)
1639	1607	2086	5332	

There are here three addition sums (*a*), (*b*), and (*c*), with vertical columns; the sums are found to be 1639, 1607, and 2086. These when added horizontally give a sum 5332. The same numbers are now added in three horizontal lines (i), (ii), and (iii), giving sums 1998, 2085, 1249 respectively.

When these are added, the final sum should be the same as before.

Oral practice in Addition.

(10.) This exercise can be used in several ways:

(i) Each line can be added from left to right or from right to left;

(ii) Each column can be added from the bottom to the top or vice versa;

(iii) The whole may be treated as one addition sum;

(iv) Any number of lines may be taken to form an addition sum, e. g. the first six lines, the last eight lines, &c.

8	3	7	2	8	6	4	1	0	7	3	5	6	2	7	9	3	8
4	2	0	6	3	2	7	8	4	3	8	7	3	9	0	5	6	3
2	5	3	4	7	5	8	2	0	6	5	7	9	8	3	1	2	0
9	3	5	6	2	9	7	4	1	9	8	3	5	7	2	4	5	6
8	4	2	4	7	5	9	7	2	7	2	4	0	7	1	0	9	6
3	4	5	4	0	4	8	3	5	6	9	1	5	7	1	4	3	7
5	8	3	5	3	6	5	4	7	3	0	8	3	5	6	3	5	4
2	7	7	2	5	0	3	6	8	2	4	3	8	7	0	4	3	1
1	3	5	6	9	4	7	3	5	6	3	2	0	8	3	6	2	0
6	8	9	4	3	2	0	8	2	4	8	6	7	3	5	4	6	9
9	2	3	8	3	5	7	4	2	1	6	3	0	6	2	8	3	8
7	0	5	7	2	1	8	6	3	0	8	5	4	3	9	5	4	1
4	6	7	5	3	7	4	5	2	1	6	3	8	4	0	8	2	5
8	3	8	4	6	3	2	1	5	8	7	4	2	6	4	3	2	1
5	2	9	3	8	9	4	5	6	8	3	2	7	9	5	7	6	9
6	7	2	0	7	3	7	2	1	6	3	5	8	4	6	2	1	7
3	5	0	7	5	8	5	7	8	9	9	1	2	3	7	8	9	5
2	8	4	9	3	4	6	1	2	6	8	2	4	3	2	7	2	8
1	0	7	2	8	3	3	3	5	4	3	8	7	1	3	4	6	7
6	5	8	1	0	2	2	7	8	3	4	5	8	9	0	3	2	1
3	6	6	3	1	7	4	6	5	7	3	2	6	5	4	7	8	4
2	8	5	4	2	4	5	3	0	1	6	5	3	2	7	4	5	0
4	7	8	0	3	5	8	4	7	6	4	3	2	1	8	9	6	2

SUBTRACTION

(**11.**) 'Subtract 7 from 9' may be taken to mean either 'Find what is left when 7 is taken from 9' or 'Find what must be added to 7 to make 9'. Whatever method of subtracting the student has already learnt, he will find it useful to know the '**shop**' method, which is based on the second meaning of subtraction.

Example. *Subtract* 547 *from* 932.

932
547
385

(Mental process:

7 and 5 (put down 5) make 12.
4 and 1 and 8 (put down 8) make 13.
5 and 1 and 3 (put down 3) make 9.)

Note.—This is probably the clearest explanation of the mental process for performing 'shop' subtraction, but it is not exactly correct to say that every one thinks in this way. The 'mental talking' that goes on whilst one is writing down such a subtraction as the example just worked is rather as follows:

Seven — twelve
Five — thirteen
Six — nine,

the figures 5, 8, 3 being put down immediately one says the words 'twelve', 'thirteen', 'nine'.

Here, as elsewhere, the point is to keep the mental processes as short as possible.

COMBINED ADDITION AND SUBTRACTION

(**12.**) The shop method is advisable when it is desired to take the sum of a set of numbers from another number or sum of numbers.

Example. *Find the value of*

$$9456-(3215+2567+1893+348).$$

The numbers enclosed in brackets are to be added, and the resulting sum is to be taken from 9456.

$$\begin{array}{r} 9456 \\ \hline 3215 \\ 2567 \\ 1893 \\ 348 \\ \hline 1433 \\ \hline \end{array}$$

(Mental process:

8, 11, 18, 23 (and **3** make) 26 (carry 2).
2, 6, 15, 21, 22 (and **3** make) 25 (carry 2).
2, 5, 13, 18, 20 (and **4** make) 24 (carry 2).
2, 3, 5, 8 (and **1** make) 9.

The numbers in large type are put down as soon as discovered.)

(**13.**) After a little practice it is easy to simplify any collection of additions and subtractions by this method, without writing the numbers in vertical columns

Thus, $6486+3529-8564+10685-642+386 = 11880.$

The mental process is:

6, 15, 11, 16, 14, 2**0** (the **0** goes down and 2 is carried).
2, 10, 12, 6, 14, 10, **18** (the **8** goes down and 1 is carried).
1, 5, 10, 5, 11, 5, **8** (the **8** goes down).
6, 9, 1, **1** (the **1** goes down).
1.

Examples I b.

Write down, without copying down the figures, the sums of the numbers in each column 1 to 10, and in each row a to m. The results may be checked as in the illustrative example on p. 20.

	1	2	3	4	5
a	37893	65325	73890	78913	38584
b	5627	89456	2516	6317	73256
c	85930	88421	683	18403	89070
d	9090	5382	17594	90584	6489
e	53646	97853	6473	9069	86052
f	31659	75643	87964	60708	43215

	6	7	8	9	10
g	7685	8695	21365	90785	76543
h	86742	35248	86709	76867	32419
i	987	435	5678	34345	9841
k	8641	8608	78051	35793	3052
l	89064	35628	86042	78053	6804
m	8785	32514	23	1000	57490

Find the difference between any number in column A and any number in column B:

	A	B
11.	87953	56532
12.	8605	53689
13.	7583	4437
14.	18059	35086
15.	8751	90807
16.	37593	8405
17.	56372	60539

Find the value of:

18. $36184-(125+476+3859+3910)$
19. $98732-(483+5948+12345+9860)$
20. $59483-(3645+1726+485+38271)$
21. $29584+38469+30295+3847-75610$
22. $38475+56828+3746-85099$
23. $574+396-730+329-852+1201$
24. $2758-786-809+254-1102$
25. $3829+4783-750+375-2258$
26. $2525+5647-1138+5940-11206$
27. $52913+3082-14261-734$
28. $8224-7211+5469-5223$
29. $43292-12621+324-10021+89$
30. $3456-1234+42+307-216$
31. $30752+1223-105+7236-84$
32. $5429+3262-(123+4262)$
33. $89299+32491+107-(2262+32412)$

34. Subtract two million five hundred and seven thousand and eighteen from three million six hundred and seven thousand and sixteen.

35. Find the difference between five million five thousand and eight and ninety-nine thousand and nine.

36. What number must be added to seven hundred and thirty-two thousand two hundred and two to make eight hundred thousand and twenty?

37. Add together ten million three hundred thousand and sixty-five and two million nine hundred and sixty-one thousand and eight, and express the result in words.

38. Subtract fourteen million seven hundred and two thousand and fifteen from one hundred million eighty thousand and twenty-four, and express the result in words.

39. What number must be added to one million ten thousand five hundred and nine to make two million one thousand and eighty-seven?

MULTIPLICATION

(**14.**) Multiplication is an extended form of addition. If we have to find the value of $36+36+36+36+36$, we find it easier, instead of adding, to remember that five sixes added together make 30 and that five threes make 15 (and therefore five thirties make 150), and so obtain the result at once as $150+30$, i.e. 180. The general rule for multiplication follows as soon as we have learnt the multiplication tables.

The number to be multiplied is called the **multiplicand**, the number by which it is multiplied is called the **multiplier**. The result of multiplication is called the **product**: the numbers which are multiplied together are called the **factors** of the product, and the product is said to be a **multiple** of each of its factors. Thus 7 and 8 are factors of 56; and 56 is a multiple of 7 and of 8. More than two numbers may be multiplied together, e.g. $7\times8\times9$ means 'multiply 7 by 8 and the result by 9'.

If all the factors are the same, the result is called a **power** of that factor; thus 9×9, $9\times9\times9$, $9\times9\times9\times9$ are powers of 9, and are denoted respectively by 9^2, 9^3, 9^4.

The small figure which shows the number of factors is called the **index** of the power. For reasons which will appear later, 9^2 is called the **square** of 9, and 9^3 is called the **cube** of 9; 9^4 is called the fourth power of 9, or '9 to the fourth'.

DIVISION

(**15.**) Just as in the case of subtraction, division may be regarded in two ways. 'Divide 324 by 3' may be taken to mean either—

'If 324 is divided into three equal parts, how many will there be in each part?' or—

'What number multiplied by 3 gives 324 as product?'

For purposes of explanation we adopt the second meaning; that is, we treat division as the reverse process to multiplication.

It should be noticed that any multiplication may be verified by reversing the work and performing division, and similarly any division may be verified by multiplication.

The quantity to be divided is called the **dividend**, the dividing quantity the **divisor**, and the result the **quotient**.

There are several methods of denoting division, e.g. $16 \div 2$, $16/2$, $\frac{16}{2}$ all mean that 16 is to be divided by 2.

In most cases division cannot be performed exactly. For instance, if we attempt to divide 17 by 3, we discover that 5 times 3 is 15, and 6 times 3 is 18, so that there is no whole number which, when multiplied by 3, gives 17.

But we see that 17 objects could be arranged in 5 groups each containing 3, and there would remain 2 objects over.

We say therefore that

$17 \div 3$ gives quotient 5 and remainder 2.

This is written more concisely in the form $17 \div 3 = 5\frac{2}{3}$; where $\frac{2}{3}$ means that there remain 2 units which cannot at present be divided by 3.

(**16.**) We can now explain the decimal notation more concisely.

$$73 = 7 \times 10 + 3$$
$$735 = 7 \times 10^2 + 3 \times 10 + 5$$
$$7358 = 7 \times 10^3 + 3 \times 10^2 + 5 \times 10 + 8$$

We notice that each digit, except that on the extreme right, and the one next to it, represents a multiple of some power of ten. We proceed to extend the meaning of 'power' to include these two digits.

(17.) Multiplication and Division of Powers.

Consider $10^3 \times 10^2$.

$$10^3 = 10 \times 10 \times 10;$$
$$10^2 = 10 \times 10;$$
$$\therefore \quad 10^3 \times 10^2 = 10 \times 10 \times 10 \times 10 \times 10$$
$$= 10^5, \text{ by definition of § 14.}$$

This argument could be used for any number other than 10 and for any indices. Hence we have the general rule:

The product of two powers of the same quantity is a power whose index is the sum of the original indices.

Similarly, consider $10^7 \div 10^3$; in other words:

'By what must 10^3 be multiplied to give 10^7 as product?'

The required index added to 3 must equal 7.

$$\therefore \quad \text{new index} = 7 - 3$$
$$= 4.$$
$$\therefore \quad 10^7 \div 10^3 = 10^4.$$

Hence: **The quotient obtained by dividing a power of a number by another power with a smaller index is a power whose index is obtained by subtracting the index of the divisor from the index of the dividend.**

(18.) Meaning of 10^1 and 10^0.

By division rule $10^4 \div 10^3 = 10^{4-3} = 10^1$.

But $10^4 = 10 \times 10 \times 10 \times 10;$

$10^3 = 10 \times 10 \times 10;$

$\therefore \quad 10^4 = 10^3 \times 10,$

and $10^4 \div 10^3 = 10.$

It follows that 10^1 must mean 10.

Again, by division rule

$$10^4 \div 10^4 = 10^{4-4} = 10^0.$$

But by ordinary division

$$10^4 \div 10^4 = 1.$$

It follows that 10^0 must mean 1.

(**19.**) We can now express all the digits in any number as multiples of some power of ten; thus:

$$7358 = 7 + 10^3 + 3 \times 10^2 + 5 \times 10^1 + 8 \times 10^0;$$
$$73582 = 7 \times 10^4 + 3 \times 10^3 + 5 \times 10^2 + 8 \times 10^1 + 2 \times 10^0;$$
$$735 = 7 \times 10^2 + 3 \times 10^1 + 5 \times 10^0.$$

Thus, when any digit is moved one place to the left, the index of the power of ten is increased by one, so that the value denoted by the digit is multiplied by 10. If the digit is moved to the right, its value is divided by 10.

Thus 7 means 7 units or 7×10^0,

but 70 means 7 tens or 7×10^1;

Also 21 means 21 units or 21×10^0,

but 210 means 21 tens or 21×10^1.

It should be noticed that a number may be read in several different ways; thus:

3542 = 3 thousand, 5 hundred and forty two
or, 35 hundred and 42,
or, 354 tens and 2.

If we use indices, these would be written:

$3542 = 3 \times 10^3 + 5 \times 10^2 + 4 \times 10^1 + 2 \times 10^0$,
or, $35 \times 10^2 + 42 \times 10^0$,
or, $354 \times 10^1 + 2 \times 10^0$.

These different methods of regarding a number are sometimes useful in working long multiplications and divisions.

Oral Examples.

State the quotient and remainder in each of the following:

1. $45 \div 9$. **2.** $47 \div 9$. **3.** $74 \div 9$. **4.** $23 \div 4$. **5.** $62 \div 5$.
6. $84 \div 11$. **7.** $63/7$. **8.** $66/7$. **9.** $98/7$. **10.** $42/10$.
11. $39/12$. **12.** $\frac{36}{6}$. **13.** $\frac{38}{6}$. **14.** $\frac{45}{6}$. **15.** $\frac{72}{10}$. **16.** $\frac{83}{11}$.

State the exact value, in terms of powers of 10, denoted by each digit in the following numbers:

17. 3472. **18.** 57352. **19.** 55435. **20.** 809.
21. 1003008. **22.** 937721. **23.** 736000. **24.** 5240000.

Express as powers the following products and quotients:

25. $7^2 \times 7^5$. **26.** $8^5 \times 8^3$. **27.** $10^4 \times 10$. **28.** $9^8 \div 9^2$.

29. $9^6 \div 9^4$. **30.** $10^8 \div 10^3$. **31.** $6^3 \times 6^4 \times 6^5$. **32.** $10^6 \div 10^3$.

33. $10^8 \div 10^5$. **34.** $10^7 \div 10^2 \times 10^3$. **35.** $10^7 \div (10^2 \times 10^3)$.

Express the following products and quotients as multiples of powers of ten:

36. $7 \times 10^2 \times 3 \times 10$. **37.** 700×30. **38.** $5 \times 10^3 \times 2 \times 10^2$.

39. 5000×200. **40.** $6 \times 10^4 \div (3 \times 10)$. **41.** $60000 \div 30$.

42. $2 \times 10^5 \times 5 \times 10^2$. **43.** 200000×500.

44. $56 \times 10^4 \div (28 \times 10^3)$. **45.** $36 \times 10^2 \div (4 \times 10)$.

46. $3600 \div 40$. **47.** $3 \times 10^2 \times 4 \times 10^3 \times 2 \times 10$.

48. What does 68 denote in the following numbers:
3568, 7685, 68007, 49683, 20268?

49. What does 347 denote in the following numbers:
63470, 347, 743473, 347562, 634756?

MULTIPLICATION

(20.) Example. *Multiply* 756893 (i) *by* 7, (ii) *by* 70, (iii) *by* 73.

(i) $756893 \times 7 = 5298251$.

There is no need to write the multiplier underneath the number to be multiplied; but it is advisable to keep corresponding powers of ten in the same column.

(ii) For purposes of explanation we write the indices of the powers above the respective columns:

Indices:	6.	5.	4	3.	2	1.	0
		7	5	6	8	9	3
						7	0
	5	2	9	8	2	5	1	0

Here the 7 in the multiplier stands for 7×10^1, so that, multiplying the 3, we have 21×10^1, i.e. $2 \times 10^2 + 1 \times 10^1$.

The 1 is placed in the column of index 1, and 2 is carried to the next column.

Multiplying the 9, we have $9 \times 10^1 \times 7 \times 10^1 = 63 \times 10^2$; adding the 2 carried over, we have $65 \times 10^2 = 6 \times 10^3 + 5 \times 10^2$; and we place 5 under index 2 and carry 6 to the next column; and so on for the remaining figures.

When the process is thoroughly understood, the sum may be put down as in the first example, viz.:

$$756893 \times 70 = 52982510.$$

(iii) Here we have to multiply by 70 and by 3, and add the two products. It is immaterial which we do first, but it is natural to give precedence to the more important figure, and in some later work it will be found very necessary to do so.

The student is accordingly advised to make a rule of beginning with the left-hand digit.

Indices	7.	6.	5.	4.	3.	2.	1	0.
			7	5	6	8	9	3
							7	3
	5	2	9	8	2	5	1	0
		2	2	7	0	6	7	9
	5	5	2	5	3	1	8	9

The working may also be arranged thus:

$$\begin{aligned} & 756893 \times 73 \\ = \quad & 52982510 \\ + \quad & 2270679 \\ = \quad & 55253189 \end{aligned}$$

(21.) Multiplication by Factors.

If we have to multiply by 72 we might proceed as when multiplying by 73; but, recollecting that $72 = 9 \times 8$, we may first multiply by 9 and the result by 8. This method has many advantages.

$$\begin{aligned} & 756893 \times 72 \\ = \quad & 6812037 \times 8 \\ = \quad & 54496296 \end{aligned}$$

(22.) When the multiplier consists of more than two digits, the method is the same as for two digits.

Example. *Multiply* 348926 *by* 42607.

```
                3 4 8 9 2 6
                  4 2 6 0 7
-----------------------------
1 3 9 5 7 0 4
    6 9 7 8 5 2
    2 0 9 3 5 5 6
        2 4 4 2 4 8 2
-----------------------------
1 4 8 6 6 6 9 0 0 8 2
```

Notice that there should be zeros at the ends of the first three lines of work, but in practice they are omitted.

Oral Exercises on the above sum.

(i) In the first line of work, why is the 4 placed in that particular column? What is the index of that column?

(ii) Why is the 2 in the next line placed where it is?

(iii) Why is the 6 in the next line placed where it is?

(iv) Suggest a mechanical rule for determining where the first digit in a product is to be placed.

(v) Why does the left-hand digit in the first line of work come two places to the left of the extreme left-hand digit of the second line?

(vi) Why does the left-hand digit of the second line come in the same column as the left-hand digit of the third line?

(vii) How could you obtain the second line from the first? Check the accuracy of the work by this method.

(viii) How could you obtain the third line from the second?

(ix) Give the actual value of the figure 2 wherever it occurs in the working.

(**23.**) The student should early accustom himself to carry out multiplication with the multiplier written in different positions relative to the multiplicand. There is no essential reason why it should be written below rather than above or at the side; in practice the multiplier may even be written quite a long way from the actual multiplication, and is sometimes not written at all. For instance

$$756893 \times 72 = 54496296$$

6812037

Here we multiply first by 9 and then by 8. The result of the first multiplication is written in the margin, and this result is then multiplied by 8, and the answer written down where it is wanted, in this case under the original number.

Example. *Find the value of* (i) 3742^2, (ii) 1784^2.

(i)	3742	(ii)	1784
	11226		12488
	26194		14272
	14968		7136
	7484		3182656
	14002564		

In these it would waste time to begin by writing

3742	1784
3742	1784
11226	1784
&c.	&c.

(24.) With a little ingenuity the work may sometimes be shortened, but it is not advisable to spend much time looking for a short method.

Example. *Multiply* 735823 *by* 641288

Notice that $64 = 8 \times 8$
and $128 = 64 \times 2$.

We therefore first multiply by 8, and then that product again by 80000 (thus multiplying by 640000), and lastly multiply the figures of this last line by 2, taking care to get the digits into the right position by considering the fact that we are multiplying actually by 1280.

735823
641288
5886584
47092672
94185344
471874460024

DIVISION

(25.) Example i. *Divide* 53896 *by* 7.

The work is usually written thus:

$$\begin{array}{r} 7\,)\,53896 \\ \hline 7699 \quad \text{Remainder } 3. \end{array}$$

The 53 in the dividend represents 53 thousand. This gives a quotient 7 thousand (and thus 7 is put down in the thousand column) and a remainder 4 thousand.

This is added to the next figure, which in this case represents 8 hundred.

The 48 hundred gives a quotient 6 hundred and a remainder 6 hundred, and we proceed as before.

The work may also be written:

$$53896 \div 7$$
$$= 7699\ \tfrac{3}{7},$$

where $\frac{3}{7}$ indicates that there remains 3 undivided by 7.

Division is the reverse of multiplication, and hence the work can be checked by multiplying the quotient by the divisor and adding in the remainder.

$$7699 \times 7 = 53893$$

Add 3 and we get 53896.

Example ii. *Divide* 74327 *by* 69.

Here we cannot do all the work mentally, and must proceed by taking successive partial divisions, just as we took partial products in multiplication.

$$\begin{array}{rl} 1077 & \\ \hline 69\,)\,74327 & \\ 69 & 69 \times 1000 \\ \hline 5327 & \text{Remainder} \\ 483 & 69 \times 70 \\ \hline 497 & \text{Remainder} \\ 483 & 69 \times 7 \\ \hline 14 & \text{Remainder} \\ \hline\hline \end{array}$$

We have subtracted from the original dividend the result of multiplying 69 by 1000 and 70 and 7, obtaining the final remainder 14.

Hence $74327 \div 69 = 1077\ \frac{14}{69}$.

It is the common practice to bring down from the original dividend at each stage only one digit at a time, but the student is advised to work a few examples by the method just shown.

If subtraction has been learnt by the 'shop' method, it is possible to curtail the work considerably by omitting the lines giving the partial products, performing the multiplication and subtraction in one step.

The work will then be written as follows:

```
      1077
   ---------
69 ) 74327
       532
        497
         14
```

The mental process in the work is:

The first digit is 1.

69 and **5** (put it down) = 74.

The next digit is 7.

$7 \times 9 = 63$ and **9** (put it down) = 72.

$7 \times 6 = 42$ and 7 (carried over) = 49 and **4** (put it down) = 53.

The next digit is 7.

$7 \times 9 = 63$ and **4** (put it down) = 67.

$7 \times 6 = 42$ and 6 (carried over) = 48 and **1** (put it down) = 49.

The quotient is 1077 and remainder 14.

Example iii. *Divide* 4292573 *by* 829.

```
          5178
      ----------
829 ) 4292573
        1475
         6467
          6643
            11
```

The quotient is 5178 and remainder 11.

```
Check.      5178
              829
          -------
          41424
          10356
           46602
              11      (Remainder)
          -------
          4292573
          =======
```

(**26.**) In cases where the divisor factorizes readily, we may use the following method.

Example. *Divide* 496352 *by* 63.

We divide successively by 7 and by 9.

```
7 ) 496352
9 ) 70907   and 3 remainder
    7878    ,,  5      ,,
    ====
```

The quotient is 7878.

The remainder is seen from the fact that the second line gives 'groups of 7, with 3 units over'. The third line gives 'groups of 63, with 5 groups of 7 over'. The remainder is '5 groups of 7, with 3 units', i. e. 35 + 3 or 38.

Examples I c.

Find the products obtained by multiplying each of the numbers 1 to 10 by each of the numbers (*a*) to (*k*). (Results may be verified by division.)

1.	47352.	(*a*) 9.	2.	68947.	(*b*) 72.
3.	573486.	(*c*) 76.	4.	68901.	(*d*) 119.
5.	5328.	(*e*) 243.	6.	76897.	(*f*) 35.
7.	43585.	(*g*) 2003.	8.	63247.	(*h*) 75920.
9.	158324.	(*i*) 8654.	10.	73248.	(*k*) 749196.

Find the quotient and remainder when the numbers 11 to 20 are divided by each of the numbers (*l*) to (*u*). (Results may be verified by multiplication.)

11.	563242.	(*l*) 11.	12.	7358904.	(*m*) 93.
13.	327563.	(*n*) 72.	14.	5873218.	(*o*) 415.
15.	453891.	(*p*) 916.	16.	634285.	(*q*) 358.
17.	27356.	(*r*) 452.	18.	89345239.	(*s*) 6218.
19.	632428934.	(*t*) 7354.	20.	3568921.	(*u*) 98453.

21. Multiply a million and five by a thousand and four, and express the result in words.

22. Express in words the result of dividing 1,007,925,184 by 10,072.

23. Divide 31,885,249 by 779, and express your answer in words.

24. Divide the number seventeen hundred and twelve million fifty-five thousand five hundred and twenty-five by the number fifty-seven thousand and twenty-one. Express the answer in words.

25. Divide 54,076,539 by 63, using the method of factors. Give a reason for the method by which you find the remainder.

26. Find the product of 1235803 and 693.

27. Divide 43769824 by 2569.

What is the least number to be added to the dividend to make it exactly divisible by the divisor?

28. Multiply 4832 by 3735, and divide the product by 12533.

29. The quotient after the division of a certain number by 372 is 273, and the remainder is 237. Find the number.

30. If a man divides five thousand marbles in equal shares among fifty-seven boys, how many marbles will he give to each boy, and how many marbles will remain over?

How many more marbles would the man want in order to be able to give one more marble to each boy?

31. How many times is six hundred and fifty-nine contained in twenty-three million six hundred and eight thousand six hundred and seventy-five? Give the answer in words.

32. Divide 987,654,321 by 693 by Short Division (i.e. by using factors); explain how you obtain the remainder.

33. Divide eight million one hundred and fifty-three by sixty-three, by using factors. Explain the method by which the remainder is found.

34. How many times is it possible to subtract two thousand and fifty-seven from three hundred million three thousand and thirty-nine?

BRACKETS

(**27.**) Very often we have to add two or more numbers and then multiply the result.

To enable us to express these operations we make use of **brackets.** A bracket enclosing a collection of numbers expresses the fact that the operations denoted inside the bracket are to be performed first.

Thus $3 \times (5+2)$ means that 2 is to be added to 5 and the result multiplied by 3.

When there is no risk of ambiguity the multiplication sign is omitted, and we write:

$$3(5+2) = 3 \times 7$$
$$= 21.$$

Example. $4(3 \times 7 - 2 \times 5) - (5 \times 6 - 2) \div 7$

$= 4(21-10) - (30-2) \div 7$

$= 4 \times 11 - 28 \div 7$

$= 44 - 4$

$= 40.$

Note: Great care must be exercised in dealing with brackets. $17\times3+2$ is *not* the same thing as $17\times(3+2)$.

In fact $17\times3+2=51+2$
$=53;$

whilst $17\times(3+2)=17\times5$
$=85.$

Examples I d.

Find the value of each of the following expressions:

1. $4(18-9)-3(12-8)$.
2. $5(2\times5+3\times4)+6(8\times5-16)$.
3. $4\times18-9-3(12-8)$.
4. $(4\times7+8\times9)\div2$.
5. $(3+5\times6)\times(21-6)$.
6. $(7+5-3)\div3$.
7. $(9\times5+6\times7)\div3$.
8. $(9\times7-5\times3)\div(3\times2+2\times5)$.
9. $\dfrac{9\times7-5\times3}{3\times2+2\times5}$.
10. $\dfrac{5\times4-6\times2}{8\times7-6\times8}$.
11. $375(421+779)$.
12. $52(307-224)$.
13. $2(4\times3-2\times2)\div5(4\times3-5\times2)$.

Find the difference between:

14. $5\times4+3\times7$ and $5\times(4+3)\times7$.
15. $7\times9+3\times2$ and $7\times(9+3)\times2$.
16. $27+21\div3+9$ and $(27+21)\div(3+9)$.
17. $27-21\div3+9$ and $(27-21)\div3+9$.
18. $4(5+7-8)$ and $4\times5+4\times7-4\times8$.
19. $2\times(14-6)$ and $2\times14-6$.
20. $(23-9)\times2$ and $23-9\times2$.

(28.) If we wish to speak about numbers in general without specifying particular numbers we use letters. Thus a may stand for *any* number, b for any other number, and so on.

(29.) Addition is the process we use to find how many objects there are in several groups when we know the number in each group. This could be found by counting,

and clearly the number is the same in whatever order we count. Thus, if there are 15 boys in class A and 23 in class B, we shall get the same result whether we first count 15 boys in A and then go on counting in B, or start counting in B and continue in A. It follows that

$$15+23 = 23+15.$$

The argument can be applied to any numbers, and we have the general law

$$a+b = b+a.$$

(**30.**) It should be noted that we can perform addition only when all the objects are of the same kind.

If there are in a field 3 cows and 4 horses, and in another field 5 cows and 6 horses, there are altogether

$$\begin{aligned} &3 \text{ cows} + 4 \text{ horses} + 5 \text{ cows} + 6 \text{ horses} \\ &= 8 \text{ cows} + 10 \text{ horses}. \end{aligned}$$

But, if we regard both cows and horses as being of the same kind, in that they are animals, there are in the field

$$\begin{aligned} &8 \text{ animals} + 10 \text{ animals} \\ &= 18 \text{ animals}. \end{aligned}$$

(**31.**) As with addition, we can subtract only when dealing with quantities of the same kind.

There are two types of question that involve subtraction, viz.: (1) 'Find how many are left when a number b is taken from a number a'; and (2) 'Find what number must be added to a number b to produce a number a'.

If we denote the required number by x, we can state these questions in letters or symbols.

We subtract, when we wish to find a number x such that

$$\begin{aligned} &\text{(i)} \quad x = a-b; \\ &\text{(ii)} \quad b+x = a. \end{aligned}$$

Notice that we should not say 'subtract the two numbers a and b', although we may say 'add the two numbers a and b'. We must say definitely

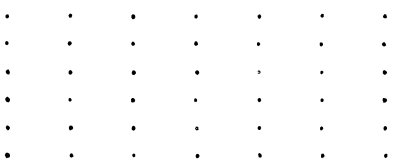
'Subtract b from a'.

Note: The sign $\backsim$ is used to indicate 'the difference between'. Thus $b \backsim a$ means that we are to take the smaller of the two numbers b and a from the larger.

(**32.**) Multiplication is the process by which we determine how many there are altogether in a number of groups when each group contains the same number of units. It is thus a shortened addition, just as addition is shortened counting.

7 multiplied by 6 means $7+7+7+7+7+7$.

If we take a dot to represent a unit, then a horizontal line of 7 dots represents 7, and the total number of dots in six of these lines gives the number obtained when 7 is multiplied by 6.



But in the figure we might take each vertical line to represent 6, and then the number of dots in 7 of these vertical lines gives the number obtained when 6 is multiplied by 7; but there is the same number of dots whether we regard them as in 6 rows of 7 each, or 7 columns of 6 each, and hence

$$7 \times 6 = 6 \times 7.$$

And in general $a \times b = b \times a$.

This is the 'commutative law for multiplication'.

It must be noted that we can multiply only by a number. We can multiply 6 cows by 7 (giving 42 cows) or £7 by 6 (giving £42), but we cannot multiply 6 cows by £7.

(**33.**) Division, as has been stated before, is the reverse of multiplication, but two types of question arise.

Consider the statement 7 cows $\times 6 = 42$ cows.

The reverse question may be either:

(i) If 42 cows are divided into 6 equal groups. how many cows are there in each group?

Or,

(ii) Into how many groups can 42 cows be divided, each group containing 7 cows?

In both cases the actual method of working is the same. In the first we say 'By what must 6 be multiplied to give 42?' and in the second we say 'By what must 7 be multiplied to give 42?'

(**34.**) If we divide a quantity by 2, the result is called a half of the original; if by 3, the result is a third; if by 4, a fourth, &c.

Thus $3765 \div 5$ = one-fifth of 3765.

This may also be written

$$3765 \div 5 = \tfrac{1}{5} \text{ of } 3765.$$

Oral Examples.

1. What is one-fifth of (i) 25, (ii) 125, (iii) 675?
2. „ one-third of (i) 27, (ii) 381, (iii) 492?
3. „ one-tenth of (i) 2740, (ii) 8200, (iii) 16200?
4. „ one-twelfth of (i) 42924, (ii) 54624?
5. What is the value of $a+b$ if

(i) $a = 7$, $b = 2$; (ii) $a = 18$, $b = 3$; (iii) $a = 12$, $b = 4$?

6. What is the value of $a-b$ if a and b have the values given in question 5?
7. What is the value of $a \times b$ if a and b have the values given in question 5?
8. What is the value of $a \div b$ if a and b have the values given in question 5?

9. What is the value of $a \times (b+c)$ if

(i) $a = 3,\ b = 2,\ c = 4$; (ii) $a = 7,\ b = 6,\ c = 9$;
(iii) $a = 2,\ b = 4,\ c = 5$; (iv) $a = 3,\ b = 4,\ c = 5$?

10. What is the value of $a \times b + a \times c$ if a, b, c have the values given in question 9?

11. Find the values of

(i) $x \times (y+z) \times w$; (ii) $x \times y + z \times w$;
(iii) $(x \times y) + z \times w$; (iv) $(x \times y + z) \times w$;
(v) $x \times (y + z \times w)$;

when $x = 2,\ y = 3,\ z = 1,\ w = 4$.

PROBLEMS

(**35.**) In working problems in arithmetic it is most necessary to show how the answer is obtained as well as to obtain the correct answer.

It is also important that the work be arranged in good English, so that it can be read as easily as a novel. Any 'notes' in the way of long multiplications or divisions should always be shown up tidily in the margin.

The pupil should be prepared to show up every piece of paper on which work has been written; this will ensure that work is always written intelligibly. The scrap piece of paper is fatal to good arithmetic.

In working problems it is often useful to think what one would do with numbers smaller than those given in the question, and then to perform the same operations with the given numbers.

Example i. *A plot of ground is 64 yards long and 32 yards wide. How long would the fence round it be? What would be the cost of the fence if each yard costs two shillings?*

Total length of fencing = 64 yards + 32 yards + 64 yards + 32 yards
= 192 yards.

The cost is 192×2 shillings
= 384 shillings
= £19 4*s*.

Example ii. *A man divides six thousand marbles in equal shares among 62 boys. How many marbles will he give to each boy, and how many marbles will remain over? How many more marbles would the man want in order to be able to give one more marble to each boy?*

Each boy gets 6000 marbles ÷ 62
= 96 and 48 over.

$$\begin{array}{r} 96 \\ 62\overline{)6000} \\ \underline{420} \\ 48 \end{array}$$

∴ Each boy gets 96 marbles and there are 48 over.

If he gives 97 to each boy he will want
97 marbles × 62
= 6014.

$$\begin{array}{r} 97 \\ \underline{62} \\ 582 \\ \underline{194} \\ 6014 \end{array}$$

So he will want 14 more marbles.

Or, more shortly:

To give each boy one more, he wants 62 marbles.
He has 48 left.
∴ he wants (62 − 48) more,
i. e. he wants 14 more marbles.

Examples I.

1. Write down in figures ten million nine thousand and ten; and in words 1505005.

The figure 5 occurs three times in the latter number; say exactly what it means in each case.

2. What number must be added to eighty-nine million nine thousand and nine to make one hundred and fifty million?

3. In a division sum the dividend is 4837580, and the quotient 484. What is the divisor?

4. Divide 9876543 by 567 (using factors), and explain how you find the remainder.

5. When the number 1,943,055 is divided by a certain number, the quotient is 2935 and the remainder 85. Find the divisor.

6. The sum of three numbers is one hundred and eleven thousand and eleven. The first two of the numbers are each forty thousand and twenty-four. What is the third number?

7. How many boxes will be required to pack three million two thousand four hundred eggs, if each box holds seventy-two dozen eggs?

8. Find the least number that must be added to twenty-seven million three hundred and one to be exactly divisible by three thousand and nineteen.

9. A reservoir contains 218703 gallons of water. How many cisterns each holding 37 gallons can be filled out of it, and how many gallons will be left in it when they are all full?

10. The average daily supply of water within the area of the Metropolitan Water Board during the month of January, 1906, was two hundred and four million nine hundred and ninety-four thousand gallons. The estimated population within the same area was six million seven hundred and seventy-four thousand eight hundred and thirty-nine. Calculate to the nearest gallon the supply per head per day.

11. The population of England and Wales at the Census of 1906 was thirty-two million five hundred and twenty-six thousand and seventy-five. There were one million eighty-two thousand six hundred and nineteen more females than males. How many were there of each sex?

12. The number of emigrants from the United Kingdom in a given year was five hundred and fifty-seven thousand seven hundred and thirty-seven; fifty-three thousand were Scotch, fifty-two thousand Irish, two hundred and twenty-nine thousand one hundred and forty-two foreigners. How many were English or Welsh?

13. Find, correct to the nearest unit, how many persons, on an average, are represented by each one of 670 Members of Parliament, assuming that the total population represented by them is forty-four million one hundred thousand.

14. The wheat in a field was arranged in 29 rows. Each row contained 27 heaps, and each heap contained 6 sheaves. How many more, or less, than seven thousand sheaves were there in the field?

15. If, in the division of a number by 385, the operation is performed by short division, and the factors 5, 7, 11 are employed in succession, the remainders being 3, 6, 8 respectively, what is the complete remainder?

16. Find three numbers lying between 500,000 and 510,000, each of which is exactly divisible both by 37 and by 73.

17. Find a number between 300,000 and 310,000 which is exactly divisible both by 79 and by 97.

18. A resolution was defeated by a majority of 61 votes out of 501. How many votes were recorded for, and how many against, the resolution?

19. Find two numbers between 900,000 and 1,000,000 which are exactly divisible by 37,259.

20. A mounted troop went into battle, each man riding one horse and leading another. They were defeated and fled on horseback, leaving 17 men and 30 horses dead, while 80 horses were captured by the enemy. Assuming that each man in his flight had only the horse he rode, determine how many men were originally in the troop.

21. The average attendance at 17 football matches was 9000. How many went to them altogether?

22. How many houses each having a frontage of 42 feet can be built on a strip of land which has a frontage of 1500 feet?

23. How many books can be placed in a library, which has 56 shelves, allowing 52 books to each shelf?

24. There are 156 balls to be packed into boxes. If each box is to contain 6 balls, how many boxes will be required?

25. The population of a village is 731, and there are 57 more females than males. Find the number of each sex.

26. Taking the population of the United Kingdom as 42 millions, and the area as 121,380 square miles, how many persons are there to each square mile?

27. In 1909 there were 85,887 emigrants to Canada, 109,700 to the United States, 37,620 to Australia and New Zealand, and 22,017 to South Africa. What was the total number of emigrants in this year?

28. The following table gives the number of members of Parliament and the number of electors in various portions of the United Kingdom. Find the average number of electors to each member in each division.

	Members.	Electors.
England and Wales .	495	6,221,722
Scotland . . .	72	785,208
Ireland	103	698,787

29. In 1909–10 the National Income was £131,697,000 and the National Expenditure £157,945,000. What was the amount of the deficit?

30. The Indian railways have in their employment 515,703 persons, of whom 7,283 are Europeans and 9,698 Eurasians. How many natives are employed?

31. In 1909 the value of the coinage minted in the British Empire was as follows:

Gold minted at London	£14,200,000
Gold minted in Australia and Canada .	£9,742,000
Silver	£1,389,600
Bronze	£121,810

Find the total value of the coinage minted in this year. Find also the difference in value between the gold coinage and the rest of the coinage.

32. By what number must 4009 be multiplied in order that the product may exceed the product of 3065 and 7803 by 125778?

33. A number is divided by the factors 3, 5, and 7 successively. The remainders are 2, 1, and 4 respectively. Find the total remainder, and explain the method of finding it.

34. A village of 1235 inhabitants has 85 more females than males; how many are there of each sex?

35. If 230 is divided by a certain number the remainder is 9, and if 200 is divided by the same number the remainder is 5. Find the number.

36. The following table gives the area, population, and revenue of the British Empire.

	Area in sq. miles.	Population.	Revenue. £
United Kingdom .	121,000	47,000,000	160,000,000
India . . .	1,770,000	300,000,000	73 000,000
Canada . . .	3,750,000	7,200,000	18,000,000
Australia. . .	3,000,000	4,100,000	15,000,000
New Zealand . .	105,000	900,000	9,000,000
South Africa . .	1,200,000	6,000,000	16,000,000

Write down the total area, population, and revenue of the British Empire, and calculate the number of people for each square mile in each country.

CHAPTER II

DECIMALS AND THE METRIC SYSTEM

(36.) In the ordinary method of expressing numbers the digits which compose a number have values which depend not only upon the digits themselves, but also on the positions which they occupy in the written number. Thus, in the number 222, the three digits, although written the same, have very different values; the first represents 'two hundred', the second 'twenty', and the third 'two'.

Thousands.	Hundreds.	Tens.	Units.
	2	2	2

Referring to the diagram, we see that any figure in the tens column has ten times the value it would have if it were in the units column, and one-tenth part of the value it would have if it were in the hundreds column; consequently any movement of a figure from one column into the next column to the left **multiplies** its value by ten, whilst a movement to the right **divides** its value by 10.

Up to the present we have found no meaning for figures placed in any columns to the **right** of the units column, but we can now extend our ideas of numbers to include such figures.

Hundreds.	Tens.	Units.	Tenths.	Hundredths.	Thousandths.
		2			
			2		
				2	
	4	2	6	9	1

In the above diagram we have moved a digit from the units column into the next column to the right, and the previous reasoning shows that we must have divided its value by ten, and that it must represent '**two tenths**'. If we move it again to the right its value is again divided by ten, and it becomes 'two hundredths'.

This process can be carried on indefinitely, and we have a complete system of notation in which numbers placed to the left of the units position represent tens, hundreds, thousands, &c., whilst numbers placed to the right represent tenths, hundredths, thousandths, &c. Thus the number in the fourth row of the above figure represents forty-two, together with six tenths, nine hundredths, and one thousandth.

(**37.**) In practice we naturally leave out the columns, but it then becomes necessary to indicate in some way the position of the unit figure in a number, and we do this by placing a 'point' in the line of figures immediately after the unit figure. This is usually called the 'decimal point'. Thus the number in the figure would be written 42·691.

Note.—The 'decimal point' is merely a convention; any other mark would have done as well, and, in fact, on the Continent, a 'comma' is frequently used. Thus the above number would be written 42,691.

(**38.**) If we wish to express four thousand as a written number we put down the figure 4, and then three zeros to indicate that there are no hundreds, tens, or units, or that the figure 4 **has been moved three places from the units position.**

In like manner, if we wish to express four thousandths, we put zero in the units position to mark that position; then we put the decimal point to mark that position and place zeros in the next two places to indicate 'no tenths or hundredths', and finally we put the 4 in the next position to indicate four thousandths.

Thus this number is written 0·004; but in practice we often leave out the unit zero, and write simply ·004.

Examples. (i) Three tenths and four thousandths is written

0·304,

and is read 'point three nought four'.

(ii) Twenty-one inches, together with three tenths of an inch and five hundredths of an inch, is written

21·35 inches,

and is read as 'twenty-one point three five inches'.

Note.—It must be borne in mind that the magnitude of a number depends upon the position of the figures with regard to the units position or the decimal point. It does not depend upon the number of digits which compose the number. Thus 0·1 is greater than 0·09, and 1 3 is greater than 1 1285.

It will be noticed that the most important digit is that to the extreme left of the number, and that the digits decrease in value from the left to the right. In fact, the digits to the extreme right of a number are relatively so unimportant that for many purposes some of them may be omitted altogether. We shall return to this point later.

Oral Examples.

Point out the actual values of the digits 3, 4, and 7 where they occur in the following numbers:

1. 304·2.	2. 130·074.	3. 73·814.	4. 3064·73.
5. ·07324.	6. 9·067.	7. ·000273.	8. ·347.
9. 103·2317.	10. ·04032.	11. ·0432.	12. ·00967.

How should we write?—

13. Seven tenths.
14. Four hundredths.
15. Six thousandths.
16. Two and one tenth.
17. Five and three thousandths.
18. Sixty-four, with three tenths and nine thousandths.
19. Two hundred-thousandths.
20. Five millionths.
21. Six tenths, four hundredths, and nine ten-thousandths.
22. A hundred and three, with two hundredths.
23. Seventy-one and a tenth.
24. Sixty-two and a thousandth.

In the following pairs of numbers state which is the greater, and why:

25. 2·3, 3·45.	26. 3·4, 2·35.	27. ·796, ·8.
28. ·0036, ·036.	29. ·9212, ·923.	30. ·000378, ·01.
31. 3, 2·87654.	32. ·1064, ·09812.	33. 7·642, 7·66.
34. 8·517, 8·4.	35. ·00028, ·02028.	36. ·222, ·22.

State (i) the unit figure, (ii) the most important figure, (iii) the least important figure, in the following numbers:

37. 32·45.	38. 207·19.	39. 8·67.	40. 10·348.
41. ·456.	42. ·000682.	43. 7·59302.	44. ·0006931.
45. 74·291.	46. 175000.	47. 40800.	48. 100·639.

A farthing is very nearly a thousandth of £1. How many farthings are there in?—

49. £·007.	50. £·024.	51. £·142.	52. £·07.
53. £·15.	54. £·6.	55. £1·042.	56. £·68.
57. £·541.	58. £·022.	59. £·9.	60. £·202.

(**39.**) We have seen that when we multiply a single digit by ten, the process is equivalent to moving the digit into the column to the left of that which it originally occupied; and that division by ten is equivalent to moving the digit into the column to the right.

So if we multiply a number such as 237·41 by ten, each digit has to be moved one place to the left, and the number

becomes 2374·1, since the 4 has now moved into the units position.

Similarly, division of 237·41 by ten will move each digit to the right, and the figure 3 will come into the units position, the number being written 23·741.

In the same way, multiplication or division by any power of ten is equivalent to moving the figures a certain number of places to the left or right of their original positions.

Examples.

(i) $14{\cdot}7218 \times 100$ (or 10^2) $= 1472{\cdot}18$.
(ii) $307{\cdot}4591 \div 1000$ (or 10^3) $= {\cdot}3074591$.
(iii) $\cdot 002416 \times 10000$ (or 10^4) $= 24{\cdot}16$.

(40.) Powers of ten. We have used the expression 10^3 to indicate 1000, and have discovered that 10^0 must be the same thing as 1 (§ 18).

To extend this notation still further, we notice that if we divide 7000 (i.e. 7×10^3) by 10, we obtain 700 (i.e. 7×10^2), the index of the power of ten having been diminished by one.

So we must have

$$(7 \times 10^0) \div 10 = 7 \times 10^{-1}$$
and $$(7 \times 10^{-1}) \div 10 = 7 \times 10^{-2}.$$

But 7×10^0 is simply 7.

$$\therefore \quad 7 \div 10 = 7 \times 10^{-1}.$$

Hence multiplication by 10^{-1} must be the same thing as division by 10, and we must take 10^{-1} to mean 'a tenth'.

So also, multiplication by 10^{-2} must be the same thing as division by 10^2, and hence 10^{-2} means 'a hundredth'. Similarly 10^{-3} means 'a thousandth'; and the notation can be extended indefinitely.

A number such as 73·247 may now be expressed as

$$7 \times 10^1 + 3 \times 10^0 + 2 \times 10^{-1} + 4 \times 10^{-2} + 7 \times 10^{-3},$$

a notation which will be useful when we attempt to perform multiplications with such numbers..

(**41.**) We have seen that 7×10^{-2} must be the same as $7 \div 10^2$.

We now look for a meaning for $7 \div 10^{-2}$.

To answer this, we go back to our first ideas on division, and ask ourselves the question:

'What is the number which will give a result 7 when multiplied by 10^{-2}, i.e. when divided by 10^2?'

The number is 700.

So $7 \div 10^{-2}$ must be the same as 7×10^2.

Similarly, $3 \div 10^{-5}$ must be the same as 3×10^5.

We can now multiply or divide any number by any power of ten.

Examples. (i) $37{\cdot}241 \times 10^{-1}$
$(= 37{\cdot}241 \div 10)$
$= 3{\cdot}7241$

(ii) $106{\cdot}465 \div 10^{-2}$
$= 106{\cdot}465 \times 10^2$
$= 10646{\cdot}5$

(iii) $142{\cdot}31 \times {\cdot}0001$
$= 142{\cdot}31 \times 10^{-4}$
$= 142{\cdot}31 \div 10^4$
$= 0{\cdot}014231$

Oral Examples.

Give the following numbers in the index form.

(Example: $\cdot 004 = 4 \times 10^{-3}$.)

1. 20000.	**2.** 300.	**3.** ·03.	**4.** ·3.	**5.** ·009.
6. 90.	**7.** 800.	**8.** ·00005.	**9.** ·0007.	**10.** 500.
11. 700000.	**12.** ·000006.	**13.** 3000.	**14.** ·07.	**15.** 50000.
16. 2.	**17.** ·0008.	**18.** ·004.	**19.** ·01.	**20.** 7.

Give the results of performing the following operations:

21. 37·212.	(i) $\times 100$.	(ii) $\times 1000$.	(iii) $\div 10$.
22. 406·21.	(i) $\times 10^2$.	(ii) $\div 100$.	(iii) $\div 10^3$.
23. ·0426.	(i) $\times 100$.	(ii) $\times 10^4$.	(iii) $\div 10$.
24. 107·3.	(i) $\div 10^2$.	(ii) $\times 10$.	(iii) $\div 1000$.
25. ·642.	(i) $\times 10$.	(ii) $\times 10^{-2}$.	(iii) $\div 10^2$.
26. ·04293.	(i) $\div 100$.	(ii) $\times 10^{-3}$.	(iii) $\div 10^{-1}$.
27. 1·624.	(i) $\times 10^0$.	(ii) $\div 10^0$.	(iii) $\times 100$.
28. 55·91.	(i) $\times 10^{-3}$.	(ii) $\div 10^{-3}$.	(iii) $\times 10^{-5}$.
29. 742·36.	(i) $\times 10^2$.	(ii) $\div 10^{-2}$.	(iii) $\div 10^3$.
30. ·00741.	(i) $\times 10^4$.	(ii) $\div 10^{-3}$.	(iii) $\div 10^{-6}$.

Read out, or write down, as complete numbers, the following:

31. $4 \times 10^2 + 2 \times 10 + 3 \times 10^0$.	**32.** $5 \times 10 + 7 \times 10^0 + 6 \times 10^{-1}$.
33. $8 \times 10^0 + 8 \times 10^{-1} + 4 \times 10^{-3}$.	**34.** $6 \times 10^2 + 2 \times 10^0 + 9 \times 10^{-1}$.
35. $5 \times 10 + 3 \times 10^{-1} + 8 \times 10^{-2}$.	**36.** $4 \times 10^1 + 3 \times 10 + 9 \times 10^{-2}$.
37. $5 \times 10^4 + 4 \times 10^2 + 3 \times 10^0$.	**38.** $3 \times 10^0 + 7 \times 10^{-2} + 4 \times 10^{-3}$.
39. $5 \times 10^2 + 4 \times 10^{-1} + 6 \times 10^{-2}$	**40.** $4 \times 10^2 + 4 \times 10^{-2} + 5 \times 10^{-4}$.
41. $7 \times 10^0 + 4 \times 10^{-2} + 1 \times 10^{-4}$.	**42.** $2 \times 10^4 + 9 \times 10 + 7 \times 10^0$.

(**42.**) The Decimal or Metric units of measurement are:

The Metre (m.), the unit of length.
The Litre (l.), „ capacity.
The Gramme (g.), „ weight.

These are the chief units, and their multiples or sub-multiples are arranged as follows:

10 Metres = 1 Dekametre (Dm.)
100 „ = 1 Hectometre (Hm.)
1000 „ = 1 Kilometre (Km.)
·1 „ = 1 decimetre (dm.)
·01 „ = 1 centimetre (cm.)
·001 „ = 1 millimetre (mm.)

The three multiples *Deka-*, *Hecto-*, and *Kilo-* are derived

from the Greek, and are written with capital letters in the abbreviated form.

The three sub-multiples *deci-*, *centi-*, and *milli-* are derived from the Latin, and are written with small letters.

These prefixes have the same meanings when attached to the other units; thus:

a centigramme (cg.) is ·01 g.
a Hectogramme (Hg.) is 100 g.
a Kilolitre (Kl.) is 1000 l.

(**43.**) It will be seen that if we prepare a table of columns such as that on p. 50 and write the unit (metre, litre, or gramme) at the head of the unit column, then we write

Deka-	above the	tens column.
Hecto-	„	hundreds column.
Kilo-	„	thousands column.
deci-	„	tenths column.
centi-	„	hundredths column.
milli-	„	thousandths column.

And the complete measure may be written down at once as an ordinary decimal number expressed in terms of the single unit.

	Kilo-	Hecto-	Deka-	Metre Gramme Litre	deci-	centi-	milli-
1		3	2	4	9	4	
2			4		2		
3		3		2		1	

Thus a length of 3 Hm., 2 Dm., 4 m., 9 dm., 4 cm. may be written: 324·94 metres (see line 1 in the figure).

A weight of 4 Dg., 2 dg. may be written:

40·2 grammes (line 2).

A measure of 3 Hl., 2 l., 1 cl. may be written:

302·01 litres (line 3).

In each case we have placed 0 or zero to indicate the absence of certain units.

(**44.**) We have spoken of the metre, gramme, and litre as the units, but for many purposes it is useful to consider one or other of the multiples or sub-multiples as the unit.

For example, we should express a distance on a road in kilometres rather than in metres, whilst for many scientific measurements the centimetre and millimetre are more useful.

If we refer to the length written in the first line of the figure, we see that it could be expressed in terms of the dekametre as the unit, and (remembering that the decimal point is simply an indication of the position of the unit) we should write 32·494 Dm.

Similarly, we might write 3249·4 cm., or in line 2 we might write 4·02 Dg. or ·0402 Kg., and line 3 might be written as 3020·1 dl.

When once the above table is understood we can at once express any metric measures, of the same kind, in terms of any one unit. For example, 3 Dl., 4 l., 5 cl., 1 ml. can be written down as: ·034051 Kl., ·34051 Hl., 3·4051 Dl., 34·051 l., 340·51 dl., 3405·1 cl., or 34051 ml.

(**45.**) The coinage of many countries is arranged on a decimal system. In France and several other countries the chief coin is the franc, which is divided into 100 centimes.

A franc is nearly 10*d.*, so that 10 centimes make nearly a penny.

23 fr. 47 centimes = 23·47 fr. or 2347 centimes.

In the United States and Canada a dollar = 100 cents.

In Germany a mark = 100 pfennig.

The metre, gramme and litre are the chief Metric measures. There are others of minor importance to which we shall refer in Chapter XIII, where we shall also explain the principle upon which the measures are founded.

Oral Examples.

Express the following in terms of the units indicated:

1.	4 Hm.	3 Dm.	7 m.	as	(i) m.	(ii) Dm.	(iii) cm.
2.	3 Km.	4 Dm.	6 m.	,,	(i) Dm.	(ii) Km.	(iii) m.
3.	5 Hm.	7 m.	4 cm.	,,	(i) Dm.	(ii) dm.	(iii) cm.
4.	5 Kg.	4 Hg.	3 Dg.	,,	(i) g.	(ii) Hg.	(iii) Kg.
5.	5 g.	4 dg.	7 cg.	,,	(i) g.	(ii) cg.	(iii) mg.
6.	9 Dg.	4 dg.	3 cg.	,,	(i) dg.	(ii) cg.	(iii) Hg.
7.	4 g.	8 cg.	9 mg.	,,	(i) dg.	(ii) Dg.	(iii) g.
8.	4 Dl.	2 cl.	5 ml.	,,	(i) Dl.	(ii) l.	(iii) ml.
9.	8 Hl.	4 Dl.	5 l.	,,	(i) Dl.	(ii) Kl.	(iii) cl.
10.	4 Kl.	3 l.	2 cl.	,,	(i) cl.	(ii) Kl.	(iii) Dl.

(Further examples of this type can be easily constructed as required.)

11.	Express	37·491 metres	as	(i) cm.	(ii) Dm.	(iii) mm.
12.	,,	·45321 Kg.	,,	(i) g.	(ii) mg.	(iii) Hg.
13.	,,	50·463 Hg.	,,	(i) dg.	(ii) Kg.	(iii) g.
14.	,,	3·742 Kl.	,,	(i) l.	(ii) Hl.	(iii) Dl.
15.	,,	541·2 cg.	,,	(i) g.	(ii) Hg.	(iii) mg.
16.	,,	7·324 Dl.	,,	(i) l.	(ii) Hl.	(iii) Kl.

Express the following (i) in cm., (ii) in Dm., (iii) in Km.

17. 34·21 m. $\times$ 10.
18. 309·6 cm. $\times$ 100.
19. ·0004369 Km. $\times 10^3$.
20. ·542 Km. $\div 10^{-2}$.
21. 2·543 Dm. $\times 10^{-1}$.
22. 43·2 m. $\times 10^{-2}$.

In the following measures, state the unit represented by the most important figure, and express the measure in terms of that unit.

23. 72·34 g. **24.** 853·2 Dg. **25.** 10032 cg.
26. 493 mm. **27.** 341·6 g. **28.** 4391 mg.
29. 54·3 cg. **30.** 4125 cm. **31.** ·894 m.
32. ·00342 Kg. **33.** ·00056 Kl. **34.** ·001764 Hl.
35. 63·921 cl. **36.** 205·9 Dg. **37.** ·000049 Km.
38. 3008 m.

ADDITION AND SUBTRACTION OF DECIMALS

(**46.**) **Example i.** If we have to add 303·41, 27·624, ·3216, 406·03 together, we have to arrange them so that all the unit figures are in the same column, and we can then add in the usual way.

The work would be written:

303·41	(unit figure 3)
27·624	(,, ,, 7)
0·3216	(,, ,, 0)
406·03	(,, ,, 6)
737·3856	

Note.—303·4100 is just the same as 303·41, because the final zeros indicate simply 'no thousandths or ten-thousandths'; and just as we should not write 00423·2 when we mean 423 2, so we do not write 303·4100 when we mean 303 41.

In the above sum any vacant spaces could be filled up with zeros, but they do not look well and are quite unnecessary.

Example ii. *Subtract* 103·241 *from* 705·1962.

705·1962	
103·241	(imagine a zero after the 1)
601·9552	

Example iii. *Subtract* ·065324 *from* 1·86.

1·86	(imagine zeros after the 6)
·065324	
1·794676	

Examples II a.

Add together:

1. 34·329, 7·34, 18·51, 0·0321.
2. 405·2, 7·329, 23·1, 107.
3. 0·00321, 0·0524, 1·326, 0·003.
4. 73·62, 1·547, 0·021, 103·2.
5. 27·65, 1·0429, 0·031, 2·54.
6. 47·32, 0·1249, 2·003, 71·2.
7. 0·0031, 0·205, 0·72, 0·0007.
8. 2·046, 0·24, 10·3, 0·64.
9. 43·5, 0·461, 2·003, 10·4.

Find the value of the following expressions:

10. $62{\cdot}4 \times 10 + 0{\cdot}237 \times 10^2 + 0{\cdot}625 \div 10^{-1}$.
11. $107{\cdot}2 \div 10^2 + 34{\cdot}3 \div 10 + 409{\cdot}4 \times 10^{-2}$.
12. $54{\cdot}3 \div 10 + 20{\cdot}21 \times 10^{-2} + 0{\cdot}044 \times 10$.
13. $604{\cdot}9 \div 10^2 + 2{\cdot}037 \times 10 + 0{\cdot}0725 \times 10^3$.
14. $5300 \times 10 + 0{\cdot}00421 \times 10^6 + 642000 \div 10^3$.
15. $2{\cdot}324 \times 10^3 + 2{\cdot}324 \times 10^2 + 2{\cdot}324 \times 10$.

Find the difference between:

16. 4·516 and 17·243.
17. 0·384 and 1·2.
18. 0·02156 and 0·3426.
19. 17·23 and 104·509.
20. 2·009 and 57·46.
21. 0·00464 and 0·000231.
22. 107·23 and 19·4.
23. 207·84 and 462·5.
24. 1 and 0·00426.
25. 0·02 and 0·006419.

Simplify:

26. $32{\cdot}6 + 24{\cdot}5 - 7{\cdot}43 + 2{\cdot}51 - 18{\cdot}42$.
27. $40{\cdot}21 - 35{\cdot}6 + 2{\cdot}594 - 2{\cdot}021 + 107{\cdot}3$.
28. $212{\cdot}4 \div 10^2 - 9{\cdot}41 \div 10 + 1{\cdot}0412 \times 10 - 389 \times 10^{-2} + 63{\cdot}1 \times 10^{-1}$.
29. $4{\cdot}732 \times 10^2 - 2{\cdot}721 \times 10 + 1{\cdot}751 \times 10 - 1{\cdot}054 \times 10^2$.
30. $4{\cdot}562 \times 10^6 + 3{\cdot}07 \times 10^4 + 5{\cdot}09 \times 10^5 - 4{\cdot}9237 \times 10^6$.

Express the following sums (or differences) as metres, grammes, or litres, as the case may be:

31. 3·4 Hm. + 2·07 Dm. + 54·3 m.
32. 6·03 Kg. + 43·2 Dg. - 307 g. + 5940 dg.
33. 5·41 Kg. + 3·97 Hg. − 54·8 g.

34. 41·62 Hm. + 30·71 Dm. + 4562 cm. − 605·3 dm.
35. 9·42 Dl. + 7·69 l. + 1·006 Hl. + 134 dl.
36. 7·49 Hg. − 40·31 Dg. + 603 dg. − 29·6 g.
37. 3·42 m. + 107·2 cm. − 0·0314 Hm.
38. 45·2 dl. + 3·042 l. − 107 cl. + 0·0062 Hl.
39. 72·43 g. − 701·2 dg. + 74·2 cg.
40. 0·00321 Dm. + 0·0456 m. − 0·000629 Hm.

MULTIPLICATION AND DIVISION IN SIMPLE CASES

(**47.**) Since decimals are merely extensions of whole numbers, the ordinary processes of multiplication and division can be performed in exactly the same way as in the case of whole numbers.

Examples:

(i) *Multiply* 34·216 *by* 3.

$$\begin{array}{r} 34{\cdot}216 \\ \times\quad 3 \\ \hline 102{\cdot}648 \end{array} \qquad [\text{or } 34{\cdot}216 \times 3 = 102{\cdot}648]$$

(ii) *Divide* 307·6128 *by* 9.

$$9\,)\overline{307{\cdot}6128} \qquad [\text{or } 307{\cdot}6128 \div 9 = 34{\cdot}1792]$$
$$34{\cdot}1792$$

(iii) *Multiply* 461 493 *by* 400.

Here we can multiply 461·493 by 400 in one step, if we realize that it is equivalent to multiplying by 100 and then by 4, and that multiplication by 100 is performed by moving the figures two places to the left. We therefore multiply by 4, and, at the same time, move the figures two places to the left.

$$\begin{array}{r} 461{\cdot}493 \\ \times\quad 400 \\ \hline 184597{\cdot}2 \end{array} \qquad [\text{or } 461{\cdot}493 \times 400 = 184597{\cdot}2]$$

(iv) *Multiply* 0·034261 *by* 7000,

$$\begin{array}{r} 0{\cdot}034261 \\ 7000 \\ \hline 239{\cdot}827 \end{array}$$

An examination of these last two examples will show that we can so arrange the work that the multiplication by 100 or 1000 is automatic.

If we place the 7000 so that the unit digit is under the last figure of the number we are multiplying, and start the resulting product under the figure 7, the figures *move themselves* into their correct positions.

Similarly, division by 7000 is equivalent to dividing by 1000 (i. e. moving the figures three places to the right) and then dividing by 7.

(v) *Divide* 1231·23 *by* 7000.

$$\begin{aligned} & 1231{\cdot}23 \div 7000 \\ = \ & 1{\cdot}23123 \div 7 \\ = \ & {\cdot}17589 \end{aligned}$$

(vi) *Divide* 4·23126 *by* 300.

$$3\overline{)\,0{\cdot}0423126}$$
$$0{\cdot}0141042$$

(vii) *Divide* 42·726 *by* 30000.

$$3\overline{)\,0{\cdot}0042726}$$
$$0{\cdot}0014242$$

In the last two examples it will be noticed that division by the power of 10 is performed in writing down the figures.

The student should practise writing the result down without copying down the figures in the usual formal way. Thus:

$$75{\cdot}625 \div 500 = 0{\cdot}15125.$$

(**48.**) Sometimes divisions do not 'come out' exactly. For example, if we divide 10·321 by 3 we obtain.

$3\overline{)\,10{\cdot}321}$ (imagine zeros at the end of the number)

$3{\cdot}440333\ldots\ldots$

and we can evidently go on putting down 3 s for ever.

But it is clear that the further we get towards the right the smaller become the actual values of the figures, and it is usually sufficiently accurate to give the answer approximately by stopping when we have obtained four or five figures in all.

We should write the above result as

$$3{\cdot}4403\ldots,$$

the dots indicating the fact that there are more figures, but that they are so small that we can neglect them in practice.

The reason for neglecting the figures will be quite clear if we think of the number 10·321 as a number of pounds (money).

In this case the division gives us £3·4403 . . .

But the last figure written down is $\frac{3}{10000}$ of £1, and since $\frac{1}{1000}$ of £1 is nearly $\frac{1}{4}d.$, the 3 represents three-tenths of a farthing and is quite negligible, and of course any more figures would be still less important.

When we give a result in this way, obtaining five figures (not including zeros if they begin a number), we say that we have expressed the result CORRECT TO FIVE SIGNIFICANT FIGURES

Examples. (i) *Divide* 372·1 *by* 700.

The result is ·53157 . . .

(ii) *Divide* 2 *by* 9.

The result is ·22222 . . .

(iii) *Divide* 0·074 *by* 70.

The result is 0·0010571 . . .

(**49.**) When we give a result correct to five significant figures, we imply that the number we have written down is the nearest we can get to the required number within that degree of accuracy.

Suppose in Ex. 1 of § 48 we had required four figures only.

The number ·5316... is nearer to ·53157.. than is ·5315..., and so we should write the result as ·5316...

So, in any working of this kind, we must get one more figure than is required, in order to correct, if necessary, the last figure.

Examples. (i) *Divide* 43·2192 *by* 5, *correct to four figures.*

The result is $8{\cdot}6438\ldots$
$= 8{\cdot}644\ldots$ to four figures.

(ii) *Divide* 407·62 *by* 110, *correct to four figures.*

$407{\cdot}62 \div 110$
$[= 40{\cdot}762 \div 11]$
$= 3{\cdot}7056\ldots$
$= 3{\cdot}706\ldots$ correct to four figures.

(**50.**) It will be noticed that, with the help of the negative index notation, we can multiply or divide by a number such as 0·002.

For: $37{\cdot}241 \times 0{\cdot}002$
$= 37{\cdot}241 \times 2 \times 10^{-3}$
$= 0{\cdot}037241 \times 2$
$= 0{\cdot}074482$

An alternative way of writing the working would be:

$$\begin{array}{r} 37{\cdot}241 \\ 0{\cdot}002 \\ \hline 0{\cdot}074482 \\ \hline \end{array}$$

where we have carried out the rule laid down on page 62 of placing the unit digit of the multiplier (in this case a zero) under the last digit of the number we are multiplying.

Similarly: $64{\cdot}46 \div 0{\cdot}02$
$= 64{\cdot}46 \div (2 \times 10^{-2})$
$= 6446 \div 2$
$= 3223$

and
$$\begin{aligned} & 0{\cdot}1073 \div 0{\cdot}03 \\ = \; & 0{\cdot}1073 \div (3 \times 10^{-2}) \\ = \; & 10{\cdot}73 \div 3 \\ = \; & 3{\cdot}5766 \ldots\ldots \end{aligned}$$

Examples II b.

(When divisions do not 'come out' it is sufficient to give five significant figures in the result.)

1.	Multiply	37·24	by (i) 2.	(ii) 500.	(iii) 7000.
2.	,,	103·21	,, (i) 50.	(ii) 700.	(iii) 6.
3.	,,	42·492	,, (i) 30.	(ii) 9.	(iii) 0·08.
4.	,,	307·34	,, (i) 6.	(ii) 0·006.	(iii) 0·05.
5.	,,	64·89	,, (i) 70.	(ii) 0·003.	(iii) 0·6.
6.	,,	57·65	,, (i) 300.	(ii) 0·006.	(iii) 7.
7.	,,	0·0225	,, (i) 4000.	(ii) 500.	(iii) 0·006.
8.	,,	0·006321	,, (i) 3×10^5.	(ii) 4×10^7.	(iii) 2×10^2.
9.	,,	0·008921	,, (i) 2×10^3.	(ii) 5×10^4.	(iii) 9×10^5.
10.	,,	3724	,, (i) 3×10^{-2}.	(ii) 2×10^{-3}.	(iii) 0·005.
11.	,,	49·56	,, (i) 20.	(ii) 3×10^{-2}.	(iii) 4×10^2.
12.	,,	109·9	,, (i) 0·005.	(ii) 0·08.	(iii) 0·7.
13.	Divide	65·4	,, (i) 2.	(ii) 30.	(iii) 500.
14.	,,	87·651	,, (i) 4.	(ii) 300.	(iii) 0·5.
15.	,,	69·82	,, (i) 400.	(ii) 60.	(iii) 700.
16.	,,	305·6	,, (i) 90.	(ii) 0·7.	(iii) 0·08.
17.	,,	0·00342	,, (i) 0·005.	(ii) 0·07.	(iii) 0·006.
18.	,,	64·937	,, (i) 400.	(ii) 60.	(iii) 0·0003.
19.	,,	297·05	,, (i) 6×10^2.	(ii) 8×10^{-1}.	(iii) 5×10^{-2}.
20.	,,	0·0003859	,, (i) 9×10^{-4}.	(ii) 7×10^2.	(iii) 7×10^{-5}.
21.	,,	0·003928	,, (i) 8×10^3.	(ii) 6×10^{-1}.	(iii) 7×10^{-5}.
22.	,,	7004·6	,, (i) 50.	(ii) 0·0007.	(iii) 0·002.
23.	,,	77·9708	,, (i) 9×10^{-1}.	(ii) 4×10^{-2}.	(iii) 8×10^2.
24.	,,	660·06	,, (i) 0·0006.	(ii) 30.	(iii) 8×10^{-3}

Give the results of the following multiplications or divisions as metres, grammes, or litres, as the case may be:

25. 7 Hm. 9 Dm. 5 m.	(*a*) $\times 400$.
26. 9 Kg. 7 Dg. 3 g.	(*b*) $\times 7000$.
27. 9 g. 6 cg. 8 mg.	(*c*) $\times 0{\cdot}005$.
28. 7 Kl. 9 Hl. 5 l.	(*d*) $\div 60$.
29. 7 Dm. 8 m. 4 cm.	(*e*) $\div 0{\cdot}004$.
30. 8·063 Kg.	(*f*) $\div 800$.
31. 7·508 Hl.	(*g*) $\times (6 \times 10^2)$.
32. 73·9 Dm.	(*h*) $\div (3 \times 10^{-1})$.
33. 64·03 Kg.	(*i*) $\times (7 \times 10^{-2})$.

(Numbers **25** to **33** can be combined with operators (*a*) to (*i*) in all possible ways.)

MULTIPLICATION BY ANY NUMBER

(**51.**) We multiply by each digit separately in the same way as for whole numbers, remembering to put down the multiplier in such a way that the figures automatically place themselves in their correct positions.

A little thought will show that this result is obtained by placing the UNIT FIGURE OF THE MULTIPLIER UNDERNEATH THE LAST FIGURE (i. e. the figure to the extreme right) OF THE NUMBER WE ARE MULTIPLYING.

In multiplying by any one figure we START THE LINE IMMEDIATELY UNDER THAT FIGURE, and the figures in the result fall naturally into their proper positions.

Example. *Multiply* 345·61 *by* 239.

345·61	
239	
69122	$= 345{\cdot}61 \times 200$
10368·3	$= 345{\cdot}61 \times 30$
3110·49	$= 345{\cdot}61 \times 9$
82600·79	$= 345{\cdot}61 \times 239$

If the multiplier is itself a decimal, there is no alteration of the method, for we have seen that multiplication by

a number such as 0·002 can be performed by the automatic method just shown, since it is the equivalent of multiplication by 2 and division by 10^3.

We therefore place the multiplier as before, so that the unit figure (i.e. the figure before the decimal point) is under the last figure of the number we are multiplying, and start each line under the figure with which we are operating.

Example i. *Multiply* 472·631 *by* 12·0564.

472·631
12·0564

4726·31	= 472·631 × 10
945·262	= 472·631 × 2
23·63155	= 472·631 × ·05
2·835786	= 472·631 × ·006
·1890524	= 472·631 × ·0004
5698·2283884	= 472·631 × 12·0564

Example ii. *Multiply* 107·645 *by* 0·00324.

107·645
0·00324

·322935	
215290	
430580	
·3487698	(The final 0 is not written down.)

(52.) Note on multiplication of decimals.

We have given the method of multiplication which seems easiest for the beginner to understand. At the same time it must be made clear that accuracy of multiplication should not depend upon any formal system of placing the figures relative to each other, but that each partial product should be obtained by a consideration of the actual value of the digit used to obtain this partial product.

Thus, in multiplying 13·023 by 1·0472 it is possibly more convenient to write the working in the form :

$$
\begin{array}{r}
13{\cdot}023 \\
1{\cdot}0472 \\
\hline
13{\cdot}023 \\
{\cdot}52092 \\
91161 \\
26046 \\
\hline
13{\cdot}6376856
\end{array}
$$

Also, in obtaining the value of

$$(1{\cdot}04843)^2$$

it is not necessary to put down the multiplier, but the work may be written:

$$
\begin{array}{r}
1{\cdot}04843 \\
4194 \\
839 \\
42 \\
3 \\
\hline
1{\cdot}09921
\end{array}
$$

In this last example, it will be noticed that we have left out figures in the partial products which are so small as to be relatively valueless.

We shall return to this point in Chap. VIII.

Examples II c.

Multiply each number in the first column by each number in the second column.

1. 58·04	(*a*) 3·42.
2. 7·906.	(*b*) 5·483.
3. 107·5.	(*c*) 8·402.
4. 596·8.	(*d*) 53·28.
5. 0·08954.	(*e*) 390·7.
6. 69·32.	(*f*) 80·508.
7. 0·000678.	(*g*) 0·0685.
8. 700·1.	(*h*) 0·00748.
9. 0·347.	(*i*) 100·05.
10. 97·41.	(*k*) 70·032.

Multiply each quantity in the first column by each number in the second column, giving the result as metres, grammes, or litres, as the case may be.

11.	3 Dm. 4 m. 7 dm.	(*l*) 41·6.
12.	7 Kg. 9 Hg. 5 g.	(*m*) 0·004.
13.	5 Dg. 7 dg. 8 cg.	(*n*) 50·3.
14.	4 Kl. 2 Hl. 9 Dl.	(*o*) 206·7.
15.	9 Km. 6 Dm. 3 m.	(*p*) 0·059.
16.	5 Hg. 7 Dg. 2 dg.	(*q*) 0·0314.
17.	8 g. 8 dg. 7 cg.	(*r*) 75·9.

DIVISION BY ANY NUMBER

(**53.**) Consider the division sum: Divide 321 2928 by 1·92.

The long division process is applicable, but we have to be careful that we set down the figures in their proper positions.

It is clear that the first digit is 1, but in order to get the 1 of the divisor under the 3 of the dividend for purposes of subtraction we must multiply by 100; in fact, the first figure represents not 'one', but 'one hundred'.

We therefore place the figure 1 in the hundreds column, i.e. over the 3 of the dividend.

The work will then proceed as follows:

167·34	
1·92)321·2928	
192·	(1·92 × 100)
129·2928	(Remainder)
115·2	(1·92 × 60)
14·0928	(Remainder)
13·44	(1·92 × 7)
·6528	(Remainder)
·576	(1·92 × ·3)
·0768	(Remainder)
·0768	(1·92 × ·04)

The quotient is 167·34.

This example has been worked out in detail, but in practice we bring down from the dividend only one figure at a time, as in the case of ordinary numbers; and when the position of the figures relative to the decimal point has been determined, we usually omit the decimal point in the intermediate working, and the zeros between the decimal point and figures to the right. The student is recommended, however, to work one or two examples in the method just shown.

The curtailed working would be as follows:

```
          167·34
        ---------
1·92 ) 321·2928
         192
        ------
         1292
         1152
        ------
          1409
          1344
         ------
           652
           576
          -----
            768
            768
```

(**54.**) If we wish to divide by a number such as 324·6, we see that the number may be written $3{\cdot}246 \times 10^2$, and we divide by 10^2 and then by 3·246.

Example. *Divide* 5622·072 *by* 324·6.

```
          17·32
        ---------
3·246 ) 56·22072   (Dividend divided by 10²)
         32·46
        ------
         23·760
         22·722
        -------
          1·0387
            9738
          ------
            6492
            6492
```

The quotient is 17·32.

(**55.**) A number in which the decimal point comes after the first digit is sometimes said to be in '**standard form**'; and it will be seen that we reduce our divisor to standard form and divide the dividend by the appropriate power of ten in writing it down.

The advantage of the method lies in the fact that when we have got the divisor into this standard form, we can usually see at once an approximate value of the quotient, and can thus avoid the very common error of 'getting the decimal point into the wrong place'.

(**56.**) If the division does not 'come out' exactly, we stop when we have obtained a reasonable number of figures, as in simple cases.

Example. *Divide* 0·107624 *by* 0·04931.

(The divisor is $4{\cdot}931 \times 10^{-2}$, and we divide 0·107624 by 10^{-2} in writing it down, § 41.)

$$
\begin{array}{r}
2{\cdot}1826\ldots\ldots \\
4{\cdot}931\,)\overline{10{\cdot}7624} \\
9{\cdot}862 \\
\hline
9004 \\
4931 \\
\hline
4073 \\
39448 \\
\hline
1282 \\
9862 \\
\hline
2958
\end{array}
$$

To obtain the last figure in the quotient we notice that 29 contains 4·9 just about six times, and we write the 6 in the quotient, but we cannot obtain any more figures with any accuracy, so there is no need to obtain the next remainder.

(57.) *Note*—As in the case of multiplication, it is well to avoid an absolutely formal method of writing the work out.

The method of reducing the divisor to standard form seems to be the easiest for the beginner to understand and work correctly, and it simplifies the checking of the work. In many cases, however, time would be wasted by the method, and the work can be done as well by using the divisor as it stands, and using common sense to see that the figures (and the decimal point) are put into their correct positions.

Thus, to divide 34·83 by ·23 we write:

```
      151·44 . . . . .
    ------------------
23 ) 34·83
     23
     ---
     118
     115
     ---
      33
      23
      --
      10
       92
       --
        8
```

(58.) The 'Italian' or shortened method is, of course, applicable in dividing decimals

Example. *Divide* 37·3242 *by* 103·21.

```
        ·36163 . . . . .
    ------------------
1·0321 ) ·373242
         63612
          1686
           654
            35
             4
```

The quotient is ·36163 . . . . .

It will be noticed that we have omitted several figures that are relatively unimportant.

Examples II d.

Divide each of the numbers 1 to 10 by each of the numbers (*a*) to (*k*), giving the results to a reasonable degree of accuracy.

1. 427·23.	(*a*) 7·624
2. 3041·46.	(*b*) 8·391.
3. 234·71.	(*c*) 23·42.
4. 63·429.	(*d*) 307·63.
5. 2·00321.	(*e*) 4·926.
6. 76·214.	(*f*) ·2347.
7. 803·97.	(*g*) ·7963.
8. 94423·1.	(*h*) ·008232.
9. 706·24.	(*i*) 603·2.
10. ·0042693.	(*k*) 742·631.

Divide each of the quantities 11 to 20 by each of the numbers (*l*) to (*u*), expressing the result as metres, grammes, or litres, as the case may be.

11. 7·324 Kg.	(*l*) 3·21.
12. 3 Hg. 4 Dg. 5 g. 2 dg.	(*m*) 4·69.
13. 4 Kl. 3 Hl. 7 l.	(*n*) 30·73.
14. 6·392 m.	(*o*) 205·2.
15. 53·71 Dm.	(*p*) 9·97.
16. 3 Km. 7 Hm. 8 Dm. 2 m.	(*q*) ·632.
17. 532 Kg. 7 Dg.	(*r*) ·00421.
18. 92·734 g.	(*s*) 73·96.
19. 8 Hl. 72 l.	(*t*) ·00034.
20. 5 Dg. 3 g. 2 cg.	(*u*) 107·6.

Examples II.

(All work must be written out in such a manner that it is immediately intelligible.)

1. Find the value of

$$\frac{7{\cdot}105+2{\cdot}51+1{\cdot}1105}{8{\cdot}23-3{\cdot}714}.$$

2. When unity is divided by a certain integer the quotient is ·0434 . . . . Find the integer, and obtain the next four figures of the quotient.

3. Find the difference between $3 \div 8000$ and $\cdot0123 \times \cdot0123$.

4. How many pieces of cloth each exactly ·00378 Hm. long can be cut from ·1952 Hm. ; and what is the length of the piece left over?

Find the value of

5. $\dfrac{8{\cdot}8-7{\cdot}4}{4{\cdot}5 \div 1{\cdot}8}$; 6. $\dfrac{18{\cdot}2-{\cdot}187+1{\cdot}28}{{\cdot}006}$; 7. $\dfrac{{\cdot}1701 \div 16{\cdot}2}{{\cdot}005 \times {\cdot}07}$.

8. An india-rubber ball rebounds ·46 of the height from which it falls. If it falls from a height of 75 metres, rebounds, and then falls again and rebounds a second time, find how high it will rise on the second rebound.

9. Find, to the nearest millimetre, the value of

$$\frac{{\cdot}1453 \times 3{\cdot}14159}{18} \text{ of } 2{\cdot}51 \text{ metres.}$$

10. If a bushel contains 3·635 Dl., how many Kl., to the nearest tenth of a Kl., are contained in 325·8 bushels?

11. Divide 2·256126 by 24 by short division, using the factors 2, 3, 4 successively. Show that you get the same result by dividing by 24 by long division.

12. Multiply ·00374 by 2·8 ; and divide ·019388 by ·524. Also *write down* the values of $\cdot0374 \times \cdot0028$, and $\cdot0019388 \div 52{\cdot}4$.

13. On a certain day ·025 of a school were absent, ·3125 of the rest went home at midday, and 429 stayed to school dinner. How many pupils were there in the school?

14. Find, to the nearest franc, the cost of 37 Kl. 47 l. of wine at 67·23 fr. per Hl.

Find also, to the nearest litre, the quantity of wine which can be bought at this price for 1000 fr.

15. A book contains 512 pages, and its thickness, without the cover, is 3·2 centimetres.

Find the thickness of the paper it is printed upon, expressed in millimetres.

16. If a chemist buys 2·3 Kg. of a drug for 327 francs, and sells it again at 15 centimes the gram. how much profit does he make on each kilogram?

17. Find the value of

$$1+1\div 9+1\div 9^2+1\div 9^3+1\div 9^4+\ldots$$

correct to five places of decimals.

18. Find, to four places of decimals, the value of

$$(\cdot 3673-\cdot 256)\div(4\cdot 6+2\cdot 527).$$

19. Divide unity by 630, to four significant figures.

20. How much butter at 2 fr. 25 centimes a kilogram would cost 9 fr. 90 centimes?

21. Multiply 1·183 by 2·145, and divide the result by ·0845.

22. Simplify $\dfrac{\cdot 071+\cdot 02341}{1\cdot 313-\cdot 71}$.

23. Find to the nearest centimetre the length of silk at 7 fr. 25 c. per metre which can be bought for 381 fr. 50 c.

24. Divide ·0999 by ·15984.

25. Simplify $(230\cdot 69\times\cdot 0018)\div\cdot 059$.

26. Find the difference between $4\cdot 08\times 0\cdot 37$ and $16\cdot 73\times\cdot 032$.

27. Gold wire of a certain thickness is worth 1 fr. 56 c. per cm. What would be the cost of 2 m. 9 cm. of this wire?

28. Find the sum of $\dfrac{5\cdot 7}{1\cdot 52}$ and $\dfrac{\cdot 0111}{\cdot 74}$.

29. Multiply 678·42 by 13·211.

30. Divide 3·9963 by ·21.

31. Divide 2 by ·0064.

32. How many metres of calico at 1 fr. 5 c. per metre can be bought for 101 fr. 85 c.?

33. What is the cost of 24 m. 8 c of silk at 4 fr. 25 c. per metre ?

34. Divide 8·388655 by ·215.

35. What will be the cost of 40 Kg. 125 g. of tea at 8 fr. 40 c. per Kg. ?

36. Find the cost of 9 Km. 625 m. of silk at 4 fr. 75 c. per metre.

37. If 62 metres 50 cm. of silk cost 293 fr. 75 c., find the price of a metre.

38. When eggs cost 30 centimes each, how many eggs can I buy for 8 fr. 40 c. ?

39. Find the cost of 97 m. 4 cm. at 3 fr. 75 c. per metre.

40. If 77 metres of wire can be bought for 10 centimes, find to the nearest centime the cost of a Km. of wire.

41. The tower of a church is 52·6 metres high, the spire is 43·75 metres high, and the weathercock is 1·25 metres above the top of the spire. How far is the weathercock from the ground ?

42. The contents of 8 equal bottles just fill a 10-litre vessel. How many litres and centilitres does each bottle hold ?

43. On a particular day the sun was above the horizon for 10·55 hours, and was visible for 2·4 hours. How many hours was it hidden ?

44. A kilogram is 2·2 lb. How many lb. are there in 750 Kg. ?

45. A mile is 1·6 kilometres. How many miles are equal to 11·5 Km. ?

46. Fourteen out of every hundred of the 3750 inhabitants of a town were vaccinated in a given year. How many of the inhabitants were vaccinated ?

47. A painter receives 5 fr. 10 c. for painting 34 letters. What is the cost of painting each letter ?

48. Find the cost of 287 m. 7 cm. at 5 fr. 70 c. per metre.

49. A merchant buys 235 m. 50 cm. of silk for 659 fr. 25 c. Find the price per metre.

50. Given that a yard is ·914376 metres, express 4400 yards in kilometres and metres, correct to the nearest metre.

51. Add together 76 Kg., 486 Hg., 39857 Dg., and 4729600 g., giving the answer to the nearest Kg.

52. A kilogram of a certain kind of brass contained 167 grammes of zinc, and the rest was copper. How much copper did it contain?

53. On a certain day the depth of a river was 7·75 feet. After heavy rain the water rose 0·56 ft. one day; 1·83 ft. more the next day; and 1·28 ft. more the day after. How many feet deep was the river then?

54. A cubic centimetre of cast iron weighs 7·2 grammes. How many cubic centimetres are there in a piece of cast iron which weighs 630 grammes?

55. A cubic centimetre of copper weighs 8·8 g., and a cubic centimetre of steel weighs 8 g. How many cubic centimetres of steel weigh as much as 200 cubic centimetres of copper?

56. A litre is 0·22 gallons. How many gallons are there in a jar containing 4·25 litres?

57. What is the value of 3·4 litres of milk at 45 centimes per litre?

58. There are 480 candidates for an examination, and 65 out of every hundred pass; find the number of those who fail.

CHAPTER III

REDUCTION

(**59.**) Most things which we come across in ordinary life are capable of being measured, but before we attempt to measure an object it is necessary to decide upon some standard or unit of comparison

In old days these standards were usually very arbitrary, and, consequently, units of the same name had different values in different parts of the United Kingdom.[1] Nowadays all British measurements are made in terms of units which are fixed by Act of Parliament and are based on the 'Standard Yard' and 'Standard Pound', which are kept in the Standards Department of the Board of Trade

Since quantities of the same kind can be of such very different sizes, it is convenient to have several different units for measuring the same kinds of things. Thus we measure the length of a book in inches; the length of a room in feet; the length of a side of a field in chains or poles; the length of a road in miles; whilst if we wish to think of very much larger distances, such as the distance between the earth and the sun, we use a million miles as our unit.

[1] The *acre*, for example, now defined by Statute as containing 4840 sq. yds., had local values of $2308\frac{3}{4}$ sq yds. (Leicestershire), 6760 sq yds (Westmoreland); 10,240 sq. yds. (Cheshire). The Scottish acre contained $6150\frac{1}{2}$ sq. yds.; the Irish acre 7840 sq yds.; whilst Wales had several land measures which have been called 'acres'.

But whilst it is convenient to have these different units, it is very necessary that they should be connected in some way so that we can compare things of the same kind although they are expressed in terms of different units.

In the metric system of measurements we have seen that these connexions are extremely simple, but unfortunately our own English measures were never carefully thought out and arranged for simplicity of calculation, and so it is necessary to learn by heart the tables of the relations between the various units.

These tables are given on pages 9–12, and they must be learnt very carefully.

(**60.**) For many purposes it is necessary to be able to change the unit by means of which a quantity is expressed, and this process is sometimes called 'reduction'.

For example, when we say that 6 shillings is the same as 72 pence, we have 'reduced' 6 shillings to pence.

When once the tables are learnt, the general process of reduction or change of unit depends only upon multiplication and division, as will be seen from the following examples.

Example i. *Express* £3 (i) *as shillings*, (ii) *as sixpenny pieces.*

(i) There are 20 shillings in £1.
$\therefore$ „ „ 3×20 shillings in £3.
i.e. „ „ 60 shillings in £3.

(ii) There are 2 sixpenny pieces in 1 shilling.
$\therefore$ „ „ 2×60 sixpenny pieces in 60 shillings.
i.e. „ „ 120 sixpenny pieces in 60 shillings.
$\therefore$ £3 = 120 sixpenny pieces.

Example ii. *Express* 720 *pence as pounds.*

Since there are 12 pence in 1 shilling,
there are $720 \div 12 = 60$ shillings in 720 pence,
and $60 \div 20 = 3$ pounds in 60 shillings.
$\therefore$ there are £3 in 720 pence.

Oral Examples.

1. State what operation you would perform on the number representing a quantity when changing the unit of expression from

(i) pounds to shillings; (ii) shillings to pence;
(iii) pence to sixpenny pieces; (iv) shillings to farthings;
(v) half-crowns to sixpenny pieces;
(vi) florins to half-sovereigns;
(vii) threepenny pieces to sovereigns;
(viii) pounds to sixpenny pieces.

2. (i) What is meant by ·5 shillings?
(ii) Express ·5 shillings as pence.
(iii) ,, 6 pence as shillings.

3. (i) Express £·7 as shillings.
(ii) ,, 14*s.* as pounds.
(iii) ,, 16*s.* as pounds.

(**61**.) The decimal notation helps us to change the unit always. Thus at first sight it seems impossible to answer the question: 'How many pounds are there in 6 shillings?' But we remember that to change the unit from shillings to pounds we simply have to divide by 20 the number expressing the quantity.

Dividing 6 by 20 we obtain ·3.

∴ £·3 is the same as 6 shillings.

Similarly, if we wish to express 3 pence in shillings, the result is

$$3 \div 12 \text{ shillings} \quad \text{or} \quad \cdot 25s.$$

Sometimes when we attempt to reduce a number of pence to shillings the result does not 'come out' exactly; but since a hundredth part of a shilling is less than half a farthing, it is quite sufficient if we put down only two figures after the decimal point.

Thus: 2 pence $= 2 \div 12$ shillings
$= \cdot 166 \ldots\ldots$
or $\cdot 17$ shillings correct to two places.

We have taken our examples from measures of money, but the process is identical when any other measures are considered.

Examples of a more complicated type will be shown in the course of the chapter.

MEASURES OF MONEY

(**62.**) The standard unit is the sovereign. Gold is fixed upon as the standard metal, and, strictly speaking, all coins should be of gold, but small coins in gold would be very inconvenient, so we have a silver coinage and a bronze coinage for the smaller values. Since the value of silver and bronze is proportionally smaller than the value of gold, all large sums have to be paid in gold, and any one may refuse to receive more than £2 in silver or more than one shilling in bronze.

It is useful to remember that

8 half-crowns	= £1,
40 sixpences	= £1,
240 pence	= £1,
a florin	= one-tenth of a pound.

Tables of coinages in foreign countries are given in table 2 on page 9. They are mostly constructed on the decimal system and require no explanation. We shall return in Chapter XVI to the question of exchanges from one system of coinage to another.

Oral Examples.

Reduce:

1. £2 (i) to shillings, (ii) to pence.
2. 8 shillings (i) to pence, (ii) to farthings.
3. 10 shillings (i) to pence, (ii) to pounds.
4. 7 five-pound notes (i) to pounds, (ii) to shillings.
5. 108 pence (i) to shillings, (ii) to pounds.
6. 6 pence (i) to shillings, (ii) to pounds.
7. 4 half-crowns (i) to sixpences, (ii) to pounds.
8. 7 florins (i) to threepenny pieces, (ii) to pounds.
9. 9 pence (i) to halfpence, (ii) to shillings.
10. 73 shillings (i) to florins, (ii) to pounds.
11. 50 half-crowns (i) to half-sovereigns, (ii) to florins.
12. 5 threepenny pieces (i) to pence, (ii) to shillings.
13. £2 3*s.* (i) to shillings, (ii) to pounds.
14. 12/6 (i) to pence, (ii) to sixpences.
15. 17/4 (i) to pence, (ii) to halfpence.
16. £3 5*s.* (i) to pounds, (ii) to half-crowns.
17. £2 5*s.* 6*d.* (i) to shillings, (ii) to pounds.
18. £4 3*s.* 6*d.* (i) to threepenny pieces, (ii) to pence.
19. £5 2*s.* 6*d.* (i) to shillings, (ii) to pounds.
20. £3 4*s.* 9*d.* (i) to shillings, (ii) to pounds.

(**63.**) The following examples will make clear the method of reducing any compound quantity.[1]

Example i. *Reduce* £73 4*s.* 8*d.* *to pence.*

£73 4*s.* 8*d.*

= 1464 shillings and 8*d.* (Bringing the pounds to shillings and adding in the 4 shillings.)

= 17576 pence (1464 × 12 + 8).

Example ii. *Reduce* £31 14*s.* 2½*d.* *to pounds.*

£31 14*s.* 2½*d.*

= £31 14*s.* 2·5*d.* (dividing 1 by 2 to express the half-penny in pence)

= £31 14·208*s.* . . . . (reducing the pence to shillings)

= £31·7104 . . . . . (,, ,, shillings to pounds)

[1] **A compound quantity** is one which is expressed in terms of one or more units.

Example iii. *Express* 10423 *farthings in pounds, shillings, and pence.*

10423 farthings
$= 2605\frac{3}{4}d.$ (dividing by 4)
$= 217s.\ 1\frac{3}{4}d.$ (bringing the pence to shillings)
$= £10\ 17s.\ 1\frac{3}{4}d.$ (,, ,, shillings to pounds)

Example iv. *Express* £37·4916 *in pounds, shillings, and pence.*

£37·4916
$= £37\ 9{\cdot}832s.$ (bringing the decimal part to shillings)
$= £37\ 9s.\ 9{\cdot}984d.$ (bringing the decimal part to pence)
$= £37\ 9s.\ 10d.$ nearly.

Examples III a.

(In all reductions that do not 'come out', express the result to a reasonable degree of accuracy.)

Express the following sums of money (i) in pence, (ii) in pounds, (iii) in shillings:

1. £32 4*s.* 2*d.*; **2.** £17 5*s.* 9*d.*; **3.** £105 2*s.* 2*d.*;
4. £721 11*s.* 7*d.*; **5.** £854 6*s.* 8*d.*; **6.** £517 11*s.* $6\frac{1}{2}d.$;
7. £203 4*s.* $8\frac{1}{4}d.$; **8.** £111 6*s.* $3\frac{3}{4}d.$; **9.** £107 14*s.* $8\frac{1}{2}d.$;
10. £354 8*s.* $9\frac{1}{2}d.$; **11.** £872 0*s.* $2\frac{1}{2}d.$; **12.** £521 4*s.* 11*d.*

Express in pounds, shillings, and pence:

13. 4721 pence; **14.** 3092 farthings; **15.** 17234 halfpennies;
16. 207·25 shillings; **17.** 103·75 shillings; **18.** £42·75;
19. £893·125; **20.** £503·725; **21.** 372·651 shillings;
22. 8934 pence; **23.** 49217 farthings; **24.** 273 sixpences;
25. 37·63 shillings; **26.** 47·51 sixpences; **27.** £39·425;
28. 1073·2 pence; **29.** 409·3 shillings; **30.** 9324 pence.

MEASURES OF LENGTH

(**64.**) The chief British units are given in table 3 on page 10. The yard is now the standard unit from which all the other units are derived.

The chain is the surveyors' unit for measuring land, whilst the pole and furlong were originally agricultural units.

The furlong was the average length of a furrow (Anglo-Saxon *furh*); whilst the pole was originally a rod used for measuring shorter distances.

It is useful to remember that

a chain = length of a cricket pitch,
a chain = 4 poles,
a pole = $5\frac{1}{2}$ yards,
a mile = 1760 yards or 5280 feet,
a halfpenny has a diameter of one inch.

The other units of length given in table 3 are sometimes used for special purposes.

Oral Examples.

1. Express in inches: (i) 5 ft., (ii) 2 yds., (iii) 1 yd. 2 ft., (iv) 3 yds. 1 ft. 2 in.

2. Express in feet: (i) 6 yds., (ii) 1 chain, (iii) 2 poles, (iv) 6 in., (v) 3 yds. 1 ft. 3 in.

3. Express in yards: (i) 2 chains, (ii) 4 furlongs, (iii) 8 poles, (iv) 6 feet, (v) 144 in., (vi) $\frac{1}{2}$ mile, (vii) 50 links, (viii) 1 ft. 6 in., (ix) 1 chain 50 links.

4. Express in chains: (i) 2 miles, (ii) 5 furlongs, (iii) 440 yds., (iv) 73 links, (v) 20 poles.

5. Express in furlongs: (i) 5 miles, (ii) 4 chains, (iii) 80 poles, (iv) 8 poles, (v) 300 links.

6. Express in miles: (i) 6 furlongs, (ii) 880 yds., (iii) 200 chains, (iv) 80 poles, (v) 8 poles.

7. How many yds., ft., in. are there in: (i) 70 in., (ii) 12·3 ft., (iii) ·2 yds., (iv) 62 in., (v) 104 in., (vi) 7·5 ft., (vii) ·75 yds., (viii) 609 in.

Examples III b.

(In all reductions that do not 'come out', express the result to a reasonable degree of accuracy.)

Reduce the following (*a*) to inches, (*b*) to feet, (*c*) to yards:

1. 2 yds. 1 ft. 3 in.; **2.** 13 yds. 2 ft. 6 in.; **3.** 10 yds. 0 ft. 6 in.; **4.** 8 yds. 2 ft. 3 in.; **5.** 5 ft. 8 in.; **6.** 17 yds. 2 ft. 5 in.

Reduce the following (*a*) to yards, (*b*) to chains, (*c*) to furlongs:

7. 5 fur. 4 ch. 11 yds.; **8.** 6 ch. 10 yds.; **9.** 3 fur. 5 ch. 6 yds.; **10.** 7 fur. 6 ch.; **11.** 6 fur. 0 ch. 9 yds.; **12.** 2 fur. 2 ch. 2 yds.

Reduce to chains:

13. 2 mi. 3 fur. 5 ch.; **14.** 7 fur. 5 ch. 72 links; **15.** 3 mi. 5 fur. 8 ch.; **16.** 4 mi. 7 fur.

Express in yards, feet, inches:

17. 1073 in.; **18.** 5241 in.; **19.** 307·42 ft.; **20.** 159·6 ft.; **21.** 17·324 yds.; **22.** 8·596 yds.

Express in furlongs, chains, yards:

23. 1562 yds.; **24.** 709 yds.; **25.** 80·32 chains; **26.** 40 ch. 63 links; **27.** 5·632 fur.; **28.** 7·521 fur.

Express in miles, furlongs, chains:

29. 507 ch.; **30.** 641 ch.; **31.** 18·21 fur.; **32.** 107·32 fur.; **33.** 2·435 mi.; **34.** 4·612 mi.

Express in miles, furlongs, poles:

35. 421 po.; **36.** 16·32 fur.; **37.** 2·371 mi.

MEASURES OF WEIGHT

(**65.**) The standard unit is the **pound.** The other units used in general commerce are given in the Avoirdupois table, p. 11. Up to quite recent times most valuable metals

were weighed in terms of a system of measures called Troy measure, but these measures have all gone out of use with the exception of the Troy ounce, which is defined as 480 grains, the grain being one seven-thousandth of the standard pound.

Apothecaries' weight is used for the sale of drugs, and doctors usually use these measures in giving prescriptions.

It should be noticed that the grain has the same value in all measures.

It is useful to remember that

1 ton = 2240 lb.
1 cwt. = 112 lb.
a new penny weighs $\frac{1}{3}$ of an ounce
a new halfpenny ,, $\frac{1}{5}$,,
a new farthing ,, $\frac{1}{10}$,,

Oral Examples.

1. Reduce to ounces:
   (i) 2 lb.; (ii) 5 lb.; (iii) 3 lb. 2 oz.; (iv) 3 oz. 8 dr.
2. Reduce to lb.:
   (i) 2 qrs.; (ii) 3 cwt; (iii) 3 qrs. 3 lb.; (iv) 5 lb. 4 oz.;
   (v) 2 tons; (vi) 5 st.; (vii) 3 st. 7 lb.; (viii) 4900 gr.
3. Reduce to cwt.:
   (i) 5 tons; (ii) 3 tons 2 cwt.; (iii) 2 qrs; (iv) 56 lb.
4. Express in tons, cwt., qrs.:
   (i) 72 cwt.; (ii) 54·5 cwt.; (iii) 4·25 tons;
   (iv) 17·25 cwt.; (v) 100 qrs.; (vi) 1·125 tons.
5. Express in lb. and oz.:
   (i) 325 oz.; (ii) 5·5 lb.; (iii) 5·25 lb.
6. How many grains are there in
   (i) 3 lb.; (ii) 2 oz. troy; (iii) 2·5 lb.?

(**66.**) In working the more complicated types of examples in weights the numbers 16 and 28 are sometimes troublesome. It will usually be found convenient to use the factors of these numbers when multiplying or dividing by them.

Example i. *Thus to reduce* 13 *qrs.* 11 *lb.* 14 *oz. to ounces we proceed as follows:*

13 qrs. 11 lb. 14 oz.
= (13 × 28 + 11) lb. 14 oz.
= (52 × 7 + 11) lb. 14 oz. (multiplying 13 by 4)
= 375 lb. 14 oz.
= 375 × 16 + 14 oz.
= 1400 × 4 + 14 oz. (multiplying 375 by 4)
= 5614 oz.

Example ii. *Reduce* 4·592 *qrs. to qrs., lb., oz.*

4·592 qrs.
= 4 qrs. + ·592 × 28 lb.
= 4 qrs. + 2·368 × 7 lb. (multiplying ·592 by 4)
= 4 qrs. 16·576 lb.
= 4 qrs. 16 lb. + ·576 lb.
= 4 qrs. 16 lb. ·576 × 16 oz.
= 4 qrs. 16 lb. 2·304 × 4 oz.
= 4 qrs. 16 lb. 9 oz.

Example iii. *Reduce* 5739 *oz. to qrs., lb., oz.*

5739 oz.
= 1434 groups of 4 oz + 5 oz. (dividing by 4)
= 358 lb. + 2 groups of 4 oz. + 5 oz. (dividing by 4)
= 358 lb. 13 oz.
= 89 groups of 4 lb. + 2 lb. 13 oz. (dividing by 4)
= 12 qrs. + 5 groups of 4 lb. + 2 lb. 13 oz. (dividing by 7)
= 12 qrs. 22 lb. 13 oz.

After a little practice this work can be considerably shortened, but the student is advised to make each step

perfectly clear in his written work in the manner of the examples just shown.

Examples III c.

(In all reductions that do not 'come out', express the result to a reasonable degree of accuracy.)

Reduce the following weights (*a*) to ounces, (*b*) to pounds, (*c*) to grains:

1. 2 qrs. 6 lb. 9 oz.; **2.** 3 qrs. 5 lb. 11 oz., **3.** 23 lb. 14 oz.; **4.** 2 st. 3 lb.; **5.** 3 lb. 621 grains.

Reduce the following (*a*) to lb., (*b*) to cwt., (*c*) to tons:

6. 2 tons 11 cwt. 12 lb.; **7.** 5 tons 15 cwt. 11 lb.; **8.** 14 tons 2 cwt.; **9.** 1 ton 2 cwt. 3 qrs.

Express as qrs., lb., and oz.:

10. 1732 oz.; **11.** 4561 oz.; **12.** 122·41 lb.; **13.** 73·812 lb.; **14.** 1·234 qrs.; **15.** 3·752 qrs.; **16.** 3·212 stone.

Express as tons, cwt., and qrs.:

17. 471 qrs.; **18.** 543 qrs.; **19.** 37·42 cwt.; **20.** 8·325 tons.

MEASURES OF CAPACITY

(67.) The standard unit is the **gallon**, which is defined as the space occupied by ten pounds mass of water.

The multiples and subdivisions of the gallon are given in the tables. For measuring liquids the gill, pint, quart, and gallon are used, the higher units being used only for measuring seeds, corn, &c.

It is useful to remember that a gallon is approximately equal to 277·3 cub. in.

Oral Examples.

Reduce to pints:

1. 5 qts.; 2. 3 gals.; 3. 2 gals. 3 qts.; 4. 3·5 gals.

Reduce to quarts:

5. 3 gals.; 6. 4 gals. 3 qts.; 7. 2 qts. 1 pt.; 8. 2 pks.

Reduce to gallons:

9. 4 pks.; 10. 1 bus. 2 pks.; 11. 2 qts. 1 pt.; 12. 3·2 qts.

MEASURES OF AREA AND VOLUME

(**68.**) The **acre** is the standard of land measurement, and is divided into four 'roods', or ten square chains.

A rood is the area of a piece of land rectangular in shape, whose length is a furlong (or the length of an average furrow) and whose width is a pole.

The acre was, of course, the old agricultural unit, being roughly the amount of land that could be worked by a single plough in one day.

The square chain is a more modern unit due to surveyors' measurements.

In the lower units it will be noticed that

a square foot is 12^2 square inches,
a square yard is 3^2 square feet,
a square chain is 22^2 square yards.

Volumes are measured by the cubic yard, cubic foot, and cubic inch.

a cubic foot $= 12^3$ cubic inches,
a cubic yard $= 3^3$ cubic feet.

(**69.**) These subdivisions originated with the Babylonians. They were accustomed to work in a sexagesimal system, and, taking the angle of an equilateral triangle as unit, they divided it naturally into 60 parts (giving rise later to the word 'minute', or *pars minuta prima*), and then again into 60 parts (giving our second, or *pars minuta secunda*).

In later times the right angle or quadrant was taken as unit, but the old measures of degree, minute, and second were retained.

(**70.**) Measurements of time are based upon the times of revolution of the earth about its axis, the moon about the earth, and the earth about the sun.

These times, however, are not simple multiples or sub-multiples of each other, so in practice we have had to adopt a somewhat complicated system of averages of these times.

The **day** is the time of rotation of the earth about its axis.[1]

The **week** is defined as seven days.

The **month** (lunar) is the time of revolution of the moon about the earth. It is approximately 29 days. The ordinary or calendar month is approximately $\frac{1}{12}$ of a year, but its length varies from 28 days to 31 days.

The **year** is the time of revolution of the earth about the sun. This time is approximately 365·2422 days. The ordinary year is taken as 365 days, and the slight error is set right by making each fourth year a 'Leap Year' containing 366 days, whilst a further correction provides that the last year of each century (e.g. the years 1800, 1900) is not a leap year unless the number of the year is divisible by 400. Thus 1900 was not a leap year, but 2000 will be.

[1] This is strictly the 'sidereal' day. The 'mean solar day' is the average time of the apparent rotation of the sun about the earth's axis.

Examples III d.

ON CAPACITY; AREAS; ANGLES; TIME.

Reduce to (*a*) pints, (*b*) quarts, (*c*) gallons, (*d*) pecks:

1. 2 pks. 1 gal. 3 qts. 1 pt.; **2.** 3 pks. 3 qts.;
3. 1 bus. 1 gal. 2 qts.; **4.** 1 qr. 3 pks. 1 gal. 2 qts.

Reduce to bushels, pecks, gallons, quarts, pints:

5. 347 pts.; **6.** 143·5 qts.; **7.** 73·125 gals.;
8. 39·625 pks.; **9.** 1·2345 bus.

How many square chains are there in:

10. 5 ac. 4 sq. ch.; **11.** 3 ac. 2 ro.; **12.** 3 ac. 3 ro. 10 sq. po.;
13. ·25 sq. miles.

Express the following angles in degrees:

14. 3 rt. angles; **15.** 32° 30′ 30″; **16.** 54° 12′ 45″;
17. 18° 18′ 48″.

Express in degrees, minutes, and seconds:

18. 1·2345 rt. angles; **19.** 59·3214°; **20.** 107·23′;
21. 38·2417°.

How many hours, minutes, and seconds are there in:

22. 3·4212 days; **23.** 2·34 days; **24.** 307·21 minutes?

Examples III.

1. A straight road is measured with the usual chain, and it is found that it contains 5729 ch. 27·5 links. What is the length of the road in miles and yards?

2. Express 797 yards in chains and links to the nearest link.

3. Express 83 ch. 59 links in feet to the nearest foot.

4. If a penny weigh ·3323 ounces, find the weight of £100 paid in pennies, giving the result in cwt., qrs., lb. to the nearest lb.

5. A man spends 7*d.* every week-day on rail and omnibus fares. How much does that amount to in an ordinary year in which there are 52 Sundays?

6. Find the difference, correct to the nearest second, between 29·5306 days and 27·3217 days.

7. Find the value of

£5·625 + 2·75 guineas − 8·25 half-crowns + 3·33 shillings.

8. A piece of land in London, one-thirtieth of an acre in area, was sold for £20,900. Find the price per square foot to the nearest shilling.

9. A pint of milk weighs 1·288 lb. and a pint of water 1·250 lb. If 3 pints of water be added to 40 pints of milk, what will be the weight of a pint of the mixture?

10. A boy's hoop is 148 inches round. How many miles, &c., will it pass over in 1100 revolutions? (Answer to the nearest chain.)

11. There are 626 pages in a book, 35 lines in each page, and an average of 13 words in each line. How many hours, &c., will it take to read the book through at the rate of 2 words per second?

12. A child was born on Jan. 1, 1890. How many days had he lived up to and including Sept. 30, 1903? (There were only two leap years in that period.)

13. How many ounces are there in ·0125 cwt.?

14. If one hundred and twenty-one millions of marbles be sold in the Empire in the course of a year, at the rate of eleven for a penny, how much money will they bring in?

15. Reduce 3 qrs. 3 lb. 8 oz. to cwt.

16. Reduce 1 year 13 days 4 min. to seconds.

17. Express 8 miles 3 furlongs 24 poles as a decimal of a mile, and reduce £5 6*s.* 8*d.* to pence.

Then find the cost of 8 miles 3 furlongs 24 poles of wire at £5 6*s.* 8*d.* per mile.

18. Express 2 guineas as a decimal of a pound. Hence find the value of ·03125 of 2 guineas.

19. How many inches are there in a mile?

If 1 metre is equal to 39·37 inches, find the number of metres in a mile, correct to the first decimal place.

20. What is the cost of 157 cm. of a material sold at 2 marks 30 pf. per dm.?

21. If a metre is 1·094 yards, find in yards the difference between 8 miles and 14 Km.

22. Find the number of sq. poles in a field of 6 acres, 2 roods, 17 sq. poles.

23. The Cambridge University Library contains 506880 volumes

of an average thickness of 2 inches each. What is the total length, in miles, of the shelves of the Library?

•24. Express 17 cwt. 3 qrs. 21 lb. in tons.

25. Express ·7187 miles in yards, correct to the nearest yard.

26. If a metre contains 39·37 inches, express 5 miles in kilometres, correct to the nearest metre.

27. Find the number of square yards in ·5347 acres.

28. Find the number of pounds in ·0729 tons.

29. Find the value of 1·46875 of £7 4*s.*

30. Find the cost of 29 m. 28 cm. of silk at 7 fr. 75 c. per metre.

31. Find the value of ·03125 of £3 4*s.*

32. How many ounces are there in 1 ton 2 cwt. 3 qrs. 9 lb.?

33. Find, to the nearest centime, the cost of 185 m. 95 cm. of silk at 6 fr. 5 c. per metre.

34. What is the cost of a metre of ribbon, if 303 m. 20 cm. cost 197 fr. 8 c.?

35. Find the value of 2·315 of 5*s.* 4*d.* + 3·28 of 2*s.* 4*d.*

36. A milkman delivers 116 pints of milk in the course of his round. What is that in gallons and quarts?

37. A man smokes 3 oz. of tobacco a week. How many lb. does he smoke in a year?

38. A distance is calculated as 0·673 yards. Express this in inches to the nearest inch.

39. Taking a shilling as 1·24 francs, find the value in English money of 70 francs.

40. How many tons will a mile of rails weigh if they weigh 11 lb. per yard? Give your answer to the nearest tenth of a ton.

41. A deep-sea sounding gave the depth as 10800 feet. Express this depth in miles and yards.

42. In a certain year 1705000 light sovereigns were withdrawn from circulation. On an average each was 2·008*d.* below its proper value. Find the loss to the country.

43. How much will it cost to travel 24563 miles at 1·75*d.* per mile? What is the difference between that cost and 171 guineas?

44. The nineteenth century began on Jan. 1, 1801, and ended on Dec. 31, 1900. Find the number of days in the century.

CHAPTER IV

COMPOUND QUANTITIES: ADDITION

(**71.**) Consider the addition:

$$\begin{array}{r} 4721 \\ 208 \\ 30296 \\ \hline 35225 \\ \hline \end{array}$$

The sum of the figures in the units column gives us 15, i.e. *one ten* and 5

The one ten is carried forward into the tens column, and there added in with the 9, 0, and 2, giving 12 tens, i.e. *one hundred* and 2 tens.

The one hundred is carried forward into the thousands column, and the process continues.

In adding sums of money, or lengths, or weights, or, in fact, any quantity which is expressed in terms of several units, the process is the same, with the modification that the columns do not contain tens, hundreds, &c., but the different units in terms of which we are expressing the quantities.

Example i. *Add:*

£	*s.*	*d.*
3	4	2
	15	7
7	8	$4\frac{1}{2}$
11	8	$1\frac{1}{2}$

There is one odd halfpenny, which we write down at once. The addition of the pence column gives 13 pence, i. e. *one shilling* and 1 penny. The one shilling is carried forward, and added into the shilling column, giving 28 shillings, i. e. *one pound* and 8 shillings. The one pound is carried forward, and the addition of the pound column completes the work.

Example ii. *Add*

tons	cwt.	qrs.	lb.	
3	5	2	7	46
5	8	3	18	28
	7	3	17	18
3	11	0	4	
12	13	1	18	

In a case such as this, the reduction of lb. to qrs. may necessitate some side work.

It is usually easy to see into how many groups of 28 the number of pounds may be divided, and the remainder is obtained by subtraction.

Note.—A difficulty sometimes arises owing to the uncomfortable number $5\frac{1}{2}$ which enters into length measurements.

It is not often that such examples occur, but the following working will show the method of overcoming the difficulty.

Add:

	po.	yds.	ft.	in	
	5	4	2	8	
	7	3	2	7	
	6	3	1	11	
	3	5	1	2	21
		3	2	3	$3 \times 5\frac{1}{2} =$ $16\frac{1}{2}$
Sum =	24	$4\frac{1}{2}$	1	7	$4\frac{1}{2}$
i. e.	24	4	2	13	(Since $\frac{1}{2}$ yd. = 1 ft. 6 in.)
i. e.	24	5	0	1	

(**72.**) Subtraction is worked in a similar way.

Example i. *Subtract* 4 *miles* 3 *fur.* 7 *ch. from* 7 *miles* 5 *fur.* 8 *ch.*

mi.	fur.	ch.
7	5	8
4	3	7
3	2	1

Example ii. *Subtract* £123 14*s.* $10\frac{3}{4}d.$ *from* £304 13*s.* $8\frac{1}{4}d.$

£	*s.*	*d.*
304	13	$8\frac{1}{4}$
123	14	$10\frac{3}{4}$
180	18	$9\frac{1}{2}$

The mental process is somewhat as follows (the figures in thick type being put down as soon as discovered)

$\frac{3}{4}d.$ and $\frac{1}{4}d.$ make 1*d.* and $\frac{1}{4}d.$ more makes $\frac{1}{2}d.$
11*d.* and 1*d.* make 1*s.* and 8*d.* more make **9***d.*
15*s.* and 5*s* make £1 and 13*s.* more make **18***s.*

and the subtraction proceeds as in the case of ordinary numbers.

(**73.**) The 'shop' method of subtraction is especially easy if we have to find the 'change' out of a whole number of pounds or shillings.

A shopkeeper, when giving change for a five-pound note which has been tendered in payment of a bill for £3 2*s.* 2*d.*,

will hand **10***d.* saying £3 3*s.*
and then **17***s.* saying £4
and then **£1** saying £5.

The change is £1 17*s.* 10*d.*

Examples IV a.

Add the following:

1. (*a*)

£	*s.*	*d.*
3724	5	9
203	3	$2\frac{1}{2}$
4212	5	7
33	2	9
107	4	11

(*b*)

£	*s.*	*d.*
365	4	$7\frac{3}{4}$
22	3	$10\frac{1}{2}$
143	5	8
42	4	$2\frac{1}{2}$
307	3	9

2. (*a*)

£	*s.*	*d.*
793	17	$2\frac{1}{2}$
42	3	10
507	0	$3\frac{1}{4}$
23	2	$1\frac{1}{2}$
	4	11
106	3	$9\frac{1}{4}$

(*b*)

£	*s.*	*d.*
727	5	$8\frac{1}{2}$
23	17	11
3421	4	$2\frac{1}{4}$
107	3	$9\frac{1}{2}$
224	2	2
134	6	8

3. (*a*)

£	*s.*	*d.*
4823	8	9
1276	5	11
342	8	10
23	16	$4\frac{1}{2}$
1034	5	11
42	3	$2\frac{1}{4}$
143	5	$8\frac{1}{2}$

(*b*)

£	*s.*	*d.*
429	4	$11\frac{1}{2}$
32	2	$10\frac{3}{4}$
204	5	8
1229	3	$7\frac{1}{4}$
23	8	$4\frac{1}{2}$
107	15	9
123	9	$10\frac{1}{2}$

4. (*a*)

tons	cwt.	qrs.
5	13	3
7	8	2
15	15	1
5	4	2

(*b*)

lb.	oz.	dr.
4	12	13
3	7	9
8	10	7
6	5	15

5. (*a*)

mi.	fur.	ch.
13	3	5
16	7	4
13	5	7
2	6	0

(*b*)

po.	yds.	ft.
3	3	2
5	4	2
8	5	2
1	4	1

6. (*a*)

gal.	qt.	pt.
4	2	1
3	3	0
5	1	1
2	3	1

(*b*)

qrs.	bus.	pk.
2	5	3
5	4	2
3	6	1
4	0	2

The following sums of money, *a* to *w*, are to be used for questions 7 to 33.

	£	s.	d.
a.	1234	12	$2\frac{1}{2}$
b.	349	13	10
c.	272	11	$4\frac{1}{4}$
d.	34	9	$10\frac{1}{2}$
e.	104	14	8
f.	1229	13	$10\frac{1}{2}$
g.	42	1	$7\frac{3}{4}$
h.	1089	4	$9\frac{1}{2}$
i.	34	14	9
k.	107	2	$10\frac{1}{2}$
l.	1293	4	8
m.	142	17	6
n.	359	2	$8\frac{1}{2}$
o.	229	13	7
p.	925	14	$9\frac{1}{2}$
q.	809	9	9
r.	425	8	$8\frac{1}{2}$
s.	9294	19	8
t.	36	18	7
u.	43	4	$7\frac{3}{4}$
v.	889	19	8
w.	1075	7	11

7. Add *a*, *b*, *c*, *g*, *k*.
8. ,, *c*, *d*, *f*, *g*, *k*, *l*.
9. ,, *l*, *t*, *o*, *q*, *v*, *w*.
10. ,, *a*, *d*, *h*, *m*, *r*, *v*, *w*.
11. ,, *m*, *n*, *o*, *p*, *r*, *s*, *t*.
12. ,, *e*, *g*, *i*, *l*, *p*, *r*, *u*, *w*.
13. ,, *b*, *c*, *d*, *f*, *g*, *h*, *k*, *l*, *o*, *r*, *s*, *t*.
14. ,, *e*, *f*, *k*, *m*, *n*, *p*, *r*, *s*, *t*, *v*, *w*.
15. ,, *a* to *l* inclusive.
16. ,, *m* to *w* inclusive.
17. ,, *a* to *t* inclusive.
18. Add *g* to *w* inclusive.
19. Subtract *b* from *a*.
20. ,, *d* ,, *e*.
21. ,, *g* ,, *h*.
22. ,, *i* ,, *k*.
23. ,, *k* ,, *l*.
24. ,, *t* ,, *u*.
25. ,, *v* ,, *p*.
26. ,, *k* ,, *v*.
27. ,, *p* ,, *f*.
28. ,, *c* ,, *w*.
29. Subtract the sum of *d*, *g*, *r*, *v* from the sum of *f*, *h*, *q*, *s*.
30. Subtract the sum of *m*, *o*, *t*, *w* from the sum of *d*, *e*, *a*, *f*.
31. Subtract the sum of *a* to *k* inclusive from the sum of *p* to *w* inclusive.
32. Subtract the sum of *h* to *o* inclusive from the sum of *p* to *u* inclusive.
33. Find the difference between the sums of the quantities indicated by consonants and of those indicated by vowels.
34. Add together:

tons	cwt.	qrs.	lb.
3	5	1	5
2	7	2	15
3	17	1	22

35. Add together: 5 cwt. 3 qrs. 17 lb., 17 cwt. 2 qrs. 16 lb., 11 cwt. 1 qr. 27 lb., 1 qr. 2 lb.

36. Add together: 37 yds. 2 ft., 26 yds. 2 ft., 17 yds. 1 ft., expressing the result in chains, yards, and feet.

37. Add together: 1 mi. 2 fur. 7 ch., 3 mi. 6 fur. 6 ch., 4 mi. 5 fur. 3 ch., 11 mi. 3 fur.

38. Three parcels weighed 1 lb. 13 oz, 5 lb. 2 oz., and 2 lb. respectively. How much did they weigh altogether?

39. A housekeeper's weekly bills are as follows: butcher, 14*s.* 4*d.*; baker, 2*s.* 3*d.*; greengrocer, 3*s.* 9*d.*; fishmonger, 3*s.* 6*d.*; grocer, 3*s.* 10*d.*; milkman, 2*s.* 4*d.* What is the total sum for the week?

40. A week-end ticket to Bournemouth costs 11*s.* 9*d.*, an ordinary return ticket costs 16*s.* 6*d.* What is the saving on the cheaper ticket?

41. Four vessels contain respectively a gallon, 1 gall. 2 qts., 3 galls. 3 qts., 2 qts. 1 pt. If they are all full of water, what must be the capacity of a vessel which will contain all the water?

42. My income for the year 1911 was £175, and my expenditure was £146 16*s.* 8*d.* What had I saved at the end of the year?

43. A farm contains five fields. The areas of these are 2 ac. 3 ro., 5 ac. 2 ro., 7 ac. 1 ro. 23 po., 3 ac. 3 ro. 14 po., 4 ac. 2 ro. 18 po. What is the area of the whole farm?

44. A man's income is £600, and is derived either from land or from investments. If his land brings him in £175 12*s.*, how much does he get from his investments?

45. From a ten-gallon cask of wine three vessels are filled. They hold respectively 3 qts., 2 qts. 1 pt., and 3 galls. 2 qts. How much wine will be left in the cask?

46. A road runs through five parishes, and the lengths of the road in each parish are measured separately. These lengths are 2 mi. 3 fur. 7 ch., 1 mi. 2 fur. 5 ch., 1 mi. 7 fur., 4 mi. 2 fur. 6 ch., and 3 mi. 5 fur. What is the total length of the road?

47. A motor car is racing round a track, and the times taken to cover successive laps are found to be 3 min. 7 sec., 2 min. 58 sec.,

2 min. 54 sec., 3 min. 2 sec., 2 min. 48 sec. How long did the car take to cover the five laps?

48. A mile of wire netting is being used to enclose a field. The sides of the field are 3 ch. 17 yds., 1 fur. 2 ch. 8 yds., 5 ch. 18 yds., and 2 fur. 3 ch. 7 yds. How much netting will be left over?

49. Out of an income of £400 a year a man pays £9 3*s.* income tax, £64 rent, £13 2*s.* 10*d.* rates, £2 3*s.* 7*d.* for water, £7 5*s.* 11*d.* for light, £13 7*s.* for coal. How much has he left for personal expenditure?

50. A rouble is worth 2·11*s.*, and a yen is worth 2·03*s.* Express in pence the difference between these values.

51. A boy has walked 850 yards from a certain milestone. How much nearer is he to that milestone than to the next one along the road?

52. A coal merchant has in stock 300 tons of coal. He sells 6 tons 7 cwt., 5 tons 10 cwt., 3 tons 6 cwt. 2 qrs., 5 tons 7 cwt. 3 qrs., 32 tons, 4 tons 5 cwt. 3 qrs. How much coal has he left?

53. I have to pay wages to four men. They receive £2 2*s.* 10*d.*, £1 18*s.* 6*d.*, 19*s.* 4*d.*, 17*s.* 10*d.* How much change shall I receive from a ten-pound note?

54. Two fields belonging to a farm of 27 acres are let. They contain respectively 3 ac. 2 ro. 15 po. and 5 ac. 3 ro. 18 po. What is the acreage of the remainder?

55. From an area of six square yards of cardboard are cut out five equal areas each of 5 square feet. What area remains?

(**74.**) Small sums of money may be added mentally with a little care.

Example i. *Add* 2*s.* 2½*d.* 4*s.* 9*d.* 3*s.* 7*d.* 2*s.* 11½*d.* 14*s.* 10*d.* 2*s.* 9¼*d.*

The sum = £1 11*s.* 1¼*d.*

The mental work is:

2*s.* 2½*d.* 2*s.* 11½*d* 6*s.* 11½*d.* 7*s.* 6½*d* 10*s* 6½. 10*s.* 7*d.* 11*s.* 6*d.* 13*s.* 6*d.* 14*s.* 4*d.* 28*s.* 4*d.* 29*s* 1¼*d.* 31*s.* 1¼*d.*

(Check by adding the above in the reverse order.)

We have really found the value of:

2*s* 2½*d* + 9*d.* + 4*s* + 7*d.* + 3*s* + ½*d* + 11*d.* + 2*s.*, &c.

Example ii. *Find the value of:*

7*s.* 6*d.* − 2*s.* 8*d.* + 14*s.* 5*d.* + 2*s.* 10*d.* − 3*s.* 6½*d.* − 4*s.* 8½*d.* + 2*s.* 11*d.*

The result is 16*s.* 9*d.*

The mental work is:

7*s.* 6*d.* 6*s.* 10*d.* 4*s.* 10*d.* 5*s.* 3*d.* 19*s.* 3*d* 20*s.* 1*d.*
22*s.* 1*d.* 22*s.* 0½*d.* 21*s.* 6½*d.* 18*s.* 6½*d.* 18*s* 6*d.* 17*s.* 10*d.*
13*s* 10*d.* 14*s* 9*d.* 16*s* 9*d.*

Oral Examples.

Add mentally the rows, columns, and diagonals in the following:

1.	3/1	4/6	5/7	2/9	3/4½
	5/8	7/6	7/9	2/3½	4/1
	3/7	7/11	2/10½	3/8½	3/7½
	19/4	17/6	11/9½	4/7½	5/5½
	16/7¾	2/11¼	3/9¾	2/4½	5/8
2.	2 ft. 6 in.	2 ft. 2 in.	4 ft. 8 in	7 ft. 10 in.	
	1 ft. 8 in.	3 ft. 9 in.	1 yard.	5 ft. 4 in.	
	6 ft. 2 in.	2 ft. 8 in.	7 ft. 3 in.	2 yds.	
	2 ft. 11 in.	13 ft. 9 in.	6 ft. 8 in.	3 yds.	
3.	1 mile 2 fur.	3 mi. 5 fur.	6 mi. 7 fur.	7 fur.	
	4 mi. 5 fur.	5 mi. 6 fur.	7 mi. 1 fur.	8 mi.	
	9 mi. 2 fur.	10 mi. 3 fur.	11 mi. 5 fur.	5 mi. 3 fur.	
	12 mi. 5 fur.	5 mi. 7 fur.	4 mi. 7 fur.	3 mi.	
4.	4 lb. 8 oz.	3 lb. 2 oz.	5 lb. 7 oz.	8 lb. 10 oz.	
	6 lb. 10 oz.	5 lb. 4 oz.	10 lb. 8 oz.	5 lb. 7 oz.	
	3 lb. 12 oz.	12 lb. 11 oz.	11 lb. 12 oz.	5 lb. 1 oz.	
	14 oz.	8 lb.	5 lb. 14 oz.	2 lb. 10 oz.	

Further examples of the above types can be rapidly constructed, and afford excellent mental practice.

MULTIPLICATION AND DIVISION

(**75.**) (*a*) Multiplication and division by simple numbers, and by numbers which can be put into factors.

Example i. *Multiply* £4 5*s.* 10½*d.* *by* 7.
The result is £30 1*s.* 1½*d.*

Mental process.

½*d.* × 7 = 7 halfpennies
= 3 pence and ½*d.*
10*d.* × 7 = 70*d.*
Add in the 3*d.*, obtaining 73*d.* or 6*s.* 1*d.*
5*s.* × 7 = 35*s.*
Add in the 6*s.*, obtaining 41*s.* or £2 1*s.*
£4 × 7 = £28.
Add in £2, obtaining £30.

Example ii. *Multiply* 1 *qr.* 17 *lb.* 5 *oz.* *by* 168.
(168 = 4 × 6 × 7.)

1 qr. 17 lb. 5 oz. × (4 × 6 × 7)

= 1 cwt. 2 qrs. 13 lb. 4 oz. × (6 × 7)

	69
	56
	13

= 9 cwt. 2 qrs. 23 lb. 8 oz. × 7

	79
	56
	23

= 3 tons 8 cwt. 3 qrs. 24 lb. 8 oz.

56	164
48	140
8	24

A certain amount of side work is necessary in performing the reductions. This should always be placed opposite to the piece of work to which it refers.

Note.—In practice, no answer would be of any value to this degree of accuracy. A weight as large as 3 tons could not reasonably be measured correctly to the nearest ounce.

Example iii. *Divide* 17 *mi.* 5 *fur.* 8 *ch.* *by* 8, *giving the result correct to the nearest yard.*

The result is 2 *mi.* 1 *fur.* 7 *ch.* 5 *yds.*

Mental process

17 mi. ÷ 8 = **2 mi.** and 1 mi. remaining.
1 mi. 5 fur. = **13 fur.**
13 fur. ÷ 8 = **1 fur.** and 5 fur. remaining.
5 fur. 8 ch. = **58 ch.**
58 ch. ÷ 8 = **7 ch** and 2 ch. remaining.
2 ch. = **44 yds.**
44 yds. ÷ 8 = **5 yds.** to the nearest yard

Example iv. *Divide* £72 14*s.* 10½*d.* *by* 168.

(168 = 8 × 3 × 7)

£	*s.*	*d.*		
72	14	10½ ÷ (8 × 3 × 7)		
= 9	1	10¼ ÷ (3 × 7)	(to the nearest farthing)	
= 3	0	7½ ÷ 7	,, ,,	,,
=	8	8	,, ,,	,,

Examples IV b.

Multiply the following (using factors if necessary):

1. £7 11*s.* 10*d.* by (i) 3, (ii) 50.
2. £3 4*s.* 9*d.* by (i) 6, (ii) 49.
3. £12 14*s.* 4½*d.* by (i) 9, (ii) 84.
4. £8 19*s.* 7¾*d.* by (i) 8, (ii) 45.
5. £22 4*s.* 11*d.* by (i) 3, (ii) 12.
6. 1 ton 10 cwt. 1 qr. by (i) 8, (ii) 50.
7. 5 cwt. 2 qrs. 17 lb. by (i) 12, (ii) 18.
8. 1 qr. 20 lb. 10 oz. by (i) 9, (ii) 84.
9. 1 mi. 2 fur. 7 ch. by (i) 7, (ii) 72.
10. 3 mi. 3 fur. 6 ch. by (i) 18, (ii) 96.
11. 3 fur. 5 ch. 17 yds. by (i) 8, (ii) 49.
12. 3 ac. 3 ro. 15 sq. po. by (i) 45, (ii) 84.

13. 13 ac. 2 ro. 22 sq. po. by (i) 12, (ii) 300.
14. 1 ac. 5 sq. ch. 128 sq. yds. by (i) 9, (ii) 49.
15. 35 min. 22 secs. by 22.
16. 2° 14′ 15″ by (i) 7, (ii) 10.

Examples IV c

Give the quotient in the following divisions, neglecting any remainder:

1. £23 4*s.* 9*d.*	(i) ÷2.	(ii) ÷5.	(iii) ÷42.
2. £37 5*s.* 10½*d.*	(i) ÷7.	(ii) ÷8.	(iii) ÷36.
3. £203 13*s.* 5¼*d.*	(i) ÷12.	(ii) ÷24.	(iii) ÷54.
4. £5496 12*s.* 11*d.*	(i) ÷9.	(ii) ÷27.	(iii) ÷30.
5. £324 6*s.* 8*d.*	(i) ÷16.	(ii) ÷72.	(iii) ÷50.
6. £7963 14*s.* 10*d.*	(i) ÷24.	(ii) ÷200.	(iii) ÷84.

7. 21 mi. 5 fur. 6 ch. ÷8.
8. 34 mi. 6 fur. 4 ch. ÷12.
9. 60 ch. 16 yds. ÷24.
10. 38 tons 10 cwt. 2 qrs. ÷72.
11. 47 tons 15 cwt. 1 qr. ÷28.
12. 327 ac. 5 sq. ch. ÷25.

(**76.**) (*b*) Multiplication by large numbers.

Example i. *Multiply* 2 *tons* 12 *cwt.* 3 *qrs.* *by* 439.

3 qrs. × 439 = 1317 qrs.
= 329 cwt. **1 qr.**

12 cwt. × 439 + 329 cwt.
= 5268 + 329 cwt.
= 5597 cwt.
= 279 tons. **17 cwt.**

2 tons × 439 + 279 tons
= 878 + 279 tons
= **1157 tons.**

The result is

1157 tons 17 cwt. 1 qr.

Example ii. *Multiply* £23 4*s.* 10½*d.* *by* 579.

		£	*s.*	*d.*
£23 × 579 =		11580		
		+ 1737		
4*s.* × 579	= 2316*s.* =	115	16	
10*d.* × 579	= 5790*d.*			
	= 482*s.* 6*d.* =	24	2	6
½*d.* × 579	= 289½*d.*			
	= 24*s.* 1½*d.* =	1	4	1½
	The total =	13458	2	7½

Example iii. *Multiply* £324 7*s.* 10*d.* *by* 1·723.

£324 × 1·723 =

324
226·8
6·48
·972
£558·252

= £558 5·04*s.*

		£	*s.*	*d.*
=		558	5	0·48
7*s.* × 1·723	= 12·061*s.* =		12	0·732
10*d.* × 1·723	= 17·23*d.* =	·	1	5·23
The total	=	558	18	6·442

The result = £558 18*s.* 6½*d.* to the nearest farthing

Examples IV d.

Multiply:

1. £42 2*s.* 10*d.* by 237.
2. £2 7*s.* 9*d.* by 458.
3. £3 9*s.* 4½*d.* by 684.
4. £5 7*s.* 10¾*d.* by 521.
5. £18 11*s.* 10¼*d.* by 799.
6. £39 4*s.* 5½*d.* by 233.
7. £45 8*s.* 10¼*d.* by 3824.
8. £103 4*s.* 9*d.* by 29.
9. £4 5*s.* 8*d.* by 472.
10. £104 6*s.* 11¼*d.* by 37.
11. 4*s.* 8¼*d.* by 8949.
12. 7*s.* 2½*d.* by 3099.
13. 2 yds. 2 ft. 5 in. by 107 (answer in yards, feet, inches).

14. 3 ch. 4 yds. 2 ft by 223 (answer in miles, furlongs, &c.).
15. 1 fur. 23 po. by 414 (answer in miles, furlongs, &c.).
16. 11 cwt. 1 qr. 17 lb. by 259 (answer in tons, cwt., &c.).
17. 2 qrs. 23 lb. by 241 (answer in tons, cwt., &c.).
18. £1 10*s.* 6*d.* by 3·76.
19. £872 19*s.* 4½*d.* by 133.
20. £2 7*s.* 6*d.* by ·456.
21. £3 12*s.* 6*d.* by ·275.
22. 3 qrs. 10 lb. 8 oz. by 112.
23. £10 8*s.* 7½*d.* by 253.
24. £625 12*s.* 6*d.* by ·375.

(**77.**) (*c*) Division by large numbers.

Divide the highest denomination in the usual way; reduce the remainder to the next lower denomination, and add to the original number of that denomination; divide the resulting number, reduce the remainder, and so on.

Example i. *Divide* £3720 14*s.* 10½*d.* *by* 727.

	£	*s.*	*d.*	farthings
	5	2	4	1
727)	3720	14	10½	
	3635			
	85	1700	3120	
		1714	3130½	
		1454	2908	
		260	222½	890
				727
				163

The result is £5 2*s.* 4¼*d.*, and there is a remainder of 163 farthings, or 3*s.* 4¾*d.*

Explanation.—The remainder of £85 is reduced to shillings and added into the shillings column. This column is then divided in the usual way, and the remainder, after reduction, goes into the pence column. An extra column for farthings is found to be necessary.

In the following example the curtailed, or Italian, method is used, and the working is shortened generally.

Example ii. *Divide* 1723 *tons* 7 *cwt.* 1 *qr.* 18 *lb.* *by* 423, *giving the result correct to the nearest lb.*

	tons	cwt.	qrs.	lb.	
	4	1	1	26	
423)	1723	7	1	18	
	31	627	817	788	} 394 × 28
		204	394	3152	}
				11050	
				2590	
				52	remainder

The result is 4 tons 1 cwt. 1 qr. 26 lb., and there is a remainder of 52 lb.

Example iii. *Divide* £724 14*s.* 2½*d.* *by* 12·34.

	£	*s.*	*d.*
	58	14	6·85
12·34)	724	14	2·5
	617·	179·6	84·58
	107·	123·4	74·04
	98·72	56·2	10·54
	8·28	49·36	9·872
		6·84	·668

The result is £58 14*s.* 6¾*d.* to the nearest farthing.

Examples IV e.

Answers to questions involving money should be given correct to the nearest farthing.

Divide:

1. £14776 15*s.* 8¾*d.* by 199.
2. £453 11*s.* 9¼*d.* by 365.
3. £3259 11*s.* 9*d.* by 39.
4. £7351 16*s.* 6¾*d.* by 71.
5. £815 3*s.* 4½*d.* by 213.
6. £1275 5*s.* 7½*d.* by 305.
7. £2217 14*s.* 5*d.* by 68.
8. £118539 1*s.* 10½*d.* by 693.
9. £726 11*s.* 8*d.* by 249.
10. £543 15*s.* 6½*d.* by 141.
11. £3224 13*s.* 5*d.* by 203.
12. £6242 8*s.* 8*d.* by 173.
13. 253 tons 7 cwt. 1 qr. by 122 (to the nearest lb.).
14. 422 tons 6 cwt. by 421 (to the nearest lb.).

15. 307 tons 3 cwt. 3 qrs. by 73 (to the nearest lb.).
16 2 mi. 3 fur. 7 ch. by 69 (to the nearest foot).
17. 3 mi. 7 fur. 9 ch. by 103 (to the nearest foot).
18. 135 mi. 6 fur. by 424 (to the nearest yard).
19. 372 ac. 2 ro. by 209 (to the nearest square pole).
20. 504 ac. by 69 (to the nearest square pole).
21. £306 4*s.* 5½*d.* by 2·34.
22. £506 7*s.* 11*d.* by 73·2.
23. £8924 15*s.* by 62·1.
24. £8825 17*s.* by 1·42.
25. £3090 14*s.* by 23·73.
26. £625 15*s.* 2*d.* by 307·4.
27. £1020 10*s.* 4*d.* by 63·5.

(**78.**) The kind of division hitherto discussed is sometimes called *Partition*, to distinguish it from another type of division connected with compound quantities called *quotition*. This arises in connexion with problems such as the following.

Example i. *A man has* £233 4*s.* 10*d. out of which to distribute sums of* £1 2*s.* 9*d. each to a number of people. To how many people can he distribute this amount, and how much will he have left?*

(We have to find a number such that when £1 2*s.* 9*d* is multiplied by this number the result will be nearly £233 4*s* 10*d.*)

We cannot divide one sum of money by another in the ordinary way, but we can reduce each sum to the same denomination (pounds, shillings, or pence), and can then divide the number of that denomination in the one quantity by the number of the same denomination in the other.

Now £233 4*s.* 10*d.*
= 4664*s.* 10*d.*
= 55978*d.*

And £1 2*s.* 9*d.*
= 22*s.* 9*d.*
= 273*d.*

$$273 \overline{)\,55978}\ 205$$
1378
13

The division gives 205, and a remainder of 13 pence.

∴ 205 people will get £1 2*s.* 9*d.*, and 1*s.* 1*d.* will remain over.

It is important to notice when performing the division that we are actually dividing *numbers* only. The remainder is a *number* of pence, and the quotient is a *number* of people.

Example ii. *How many parcels, each containing* 2 *lb.* 7 *oz., can be made up out of* 1 *cwt. of goods?*

1 cwt. = 112 lb.
= 112 × 16 oz.
= 448 × 4 oz.
= 1792 oz.
2 lb. 7 oz. = 39 oz.

$$39 \overline{)1792} \quad 45 \quad 232 \quad 37$$

Answer: 45 *parcels may be made up,* and 37 oz. will remain over, i.e. 2 *lb.* 5 *oz. will be left.*

In dealing with examples in the Metric System, difficulties sometimes arise as to the exact unit in terms of which the remainder is expressed.

The following example illustrates this difficulty:

Example iii. *How many lengths of cloth, each* 22 *m.* 3 *dm., can be cut off from a length of* 3 *Hm.* 5 *Dm., and what will remain?*

We express the divisor in terms of the unit which will give 'standard form', and then express the dividend in terms of the same unit:

22 m. 3 dm. = 2·23 Dm.
3 Hm. 5 Dm. = 35 Dm.

$$2{\cdot}23 \overline{)35{\cdot}} \quad 15{\cdot} \quad 12{\cdot}7 \quad 1{\cdot}55$$

The division shows that 15 pieces can be cut off, and 1·55 Dm. will remain.

In performing the division, we stop when we reach the decimal point, i. e. we obtain a whole number as a quotient.

A similar difficulty occurs when dealing with English measures expressed as decimals.

Example IV. *How many pieces of string each* 17·3 *in. long can be cut from a ball containing* 50 *yds.? What is the length of the remainder?*

Express each length as a number of tenths of an inch.

17·3 in. = 173 tenths of an inch.
50 yds. = 18000 tenths of an inch.

$$173 \overline{)\,18000}\ 104,\ 700,\ 8$$

On dividing we find that 104 pieces can be cut off, and there remain 8 tenths of an inch, i.e. the remainder is ·8 in.

Examples IV f.

Find how many times the first of the following quantities is contained in the second, giving the remainder in each case.

1.	£2 2*s.* 3*d.*	£406 5*s.* 9*d.*
2.	£3 3*s.* 3*d.*	£552 10*s.* 11*d.*
3.	£1 5*s.* 9*d.*	£308 5*s.* 7*d.*
4.	£3 3*s.* 6*d.*	£28 7*s.* 6*d.*
5.	£23 2*s.* 2*d.*	£1723 4*s.* 8*d.*
6.	£54 3*s.* 9*d.*	£1200.
7.	3 lb. 2 oz.	2 cwt.
8.	2 lb. 5 oz.	3 cwt. 2 qrs.
9.	2 yds. 2 ft.	$\frac{1}{2}$ mile.
10.	17 yds. 1 ft.	2 fur. 7 ch.
11.	1 ch. 18 yds.	3 miles.
12.	7 ch. 10 yds.	2 mi. 5 fur.
13.	1 qt. 1 pt.	17 gallons.
14.	5 m. 2 dm.	3 Hm. 5 Dm.
15.	11 g. 3 cg.	1 Kg.
16.	5 Dg. 3 dg.	2 Kg. 6 Hg.
17.	3·42 dm.	7·61 Hm.
18.	2 fr. 30 c.	2107 fr.
19.	$6 40 c.	$1000.
20.	5 mk. 20 pf.	323 mk.
21.	8·32 g.	72 Kg.

22.	5 fr. 70 c.	622 fr. 50 c.
23.	\$8 30 c.	\$727.
24.	8 fr. 25 c.	804 fr. 70 c.
25.	7 Kg. 3 Dg.	1043 Kg.
26.	\$5 70 c.	\$349.
27.	£2 18*s.* 6*d.*	£1000.
28.	£3 17*s.* 10*d.*	£2000.
29.	£1 1*s.* 1*d.*	£83 17*s.* 4*d.*

Examples IV.

1. How much will 536 sacks of wheat be worth at 6*s.* 3*d.* per cwt., if a bushel of wheat weighs 63 lb., and each sack contains 4 bushels of wheat?

2. How many tons of oil-cake will be eaten by some bullocks in 144 days, if they eat 105 lb. of oil-cake per day? Also, what will the oil-cake cost at 7*s.* 9½*d.* per cwt.?

3. The ground rent on an estate of 18 acres 2 roods 16 poles is £334 16*s.* How much is this per acre?

4. A man who had 31 Hl. of wine put 17 dl. of the wine into each one of 144 dozen bottles. The remainder of the wine he put into bottles each one of which held 7 dl. How many bottles were filled altogether?

5. What would be the amount of a tax of 4*d.* per lb. on 600,000 tons of tobacco?

6. How many packets, each containing 4 oz. of tea, can be made up from chests containing in all 2 tons 3 qrs. 4½ lb.?

7. A librarian intends to buy 227 books at 2*s.* 8*d.* each, 139 books at 4*s.* 6*d.* each, and the greatest possible number of books at 5*s.* 8*d.* each consistent with not spending more than £100 altogether. What total number of books will he buy, and how much less than £100 will he spend?

8. What sum of money should a booking clerk have charge of after he has sold 9 first, 17 second, and 53 third class tickets at 11*s.* 9*d.*, 8*s.* 3*d.*, and 4*s.* 6*d.* each respectively, if he had £13 14*s.* 9*d.* when he began to sell tickets.

9. Sixteen parcels, each weighing 1 cwt. 3 qrs. 7 lb., and twenty-four others, each weighing 3 cwt. 2 qrs. 14 lb., are taken from a total weight of 15 tons 7 cwt. 3 qrs. 21 lb. What weight is left?

10. If 53 equally valuable shares are now worth £144 4*s.* 1*d.*, what will be the value of 274 similar shares when the price per share has risen 9*d.*?

11. Find the cost of 29 trucks of coal, each truck containing 7 tons 15 cwt. of coal, at 23*s.* 8*d.* per ton.

12. A road is $1\frac{1}{2}$ miles long. I walk it in 22-inch steps. How many steps do I take?

13. A draper's bill includes: 5 pairs of blankets at 12*s.* 9*d.* per pair; 18 yards of muslin at $9\frac{3}{4}d.$ per yard; 72 yards of calico at $2\frac{3}{4}d.$ per yard; 4 dozen yards of flannel at 1*s.* $6\frac{3}{4}d.$ per yard; and 9 yards of tablecloth at 1*s.* $2\frac{1}{4}d.$ per yard. Find the total amount of the bill.

14. My fields measure 9 acres, 8 ac. 2 ro. 22 po., 8 ac. 2 ro. 10 po., 7 ac. 37 po., 15 ac., and 6 ac. 3 ro 27 po. respectively. What is the whole extent of my holding, and what is its rental at 5*s.* 5*d.* per rood?

15. How many coins worth 4*s.* $5\frac{1}{4}d.$ each are equivalent to £409 2*s.* 9*d.*?

16. How many parcels of 16 lb. each can be made up from goods weighing 4 tons 15 cwt. 3 qrs. 25 lb.? If the remainder be sold at $2\frac{1}{2}d.$ an ounce, how much will that bring in?

17. Find the total cost for 93 consecutive days, beginning on a Friday, of 7 papers, 1 at 3*d.*, 5 at 1*d.* each, and 1 at $\frac{1}{2}d.$ on each week-day; and 5 additional papers, at 6*d.* each, on each Saturday.

18. If 5 cwt. of tobacco be made up into packets of $1\frac{3}{4}$ oz., how many packets will there be?

19. Find the total amount of the Poor Rate levied on a population of 3196395 at the rate of 14*s.* $3\frac{1}{2}d.$ per head.

20. A contractor pays in wages for one week £122 3*s.* 6*d.* How many men does he employ if the weekly earnings of each man amount to £1 2*s.* $7\frac{1}{2}d.$?

21. What is the value of five million four hundred thousand and fifty rupees at the rate of 1*s.* 11½*d.* a rupee?

22. What will be the cost of 1453 packages of goods at £5 18*s.* 7*d.* per package?

23. A truck was loaded with the greatest number of packages consistent with the condition that the weight of the loaded truck must not exceed 4½ tons. The weight of the truck was 1 ton 13 cwt 23 lb., and the weight of each package was 1 cwt. 1 qr. 11 lb.

Determine the number of packages and the weight of the loaded truck.

24. Find the total cost of 59 chickens at 2*s* 3*d.* each, 13 geese at 7*s.* 9*d.* each, and 19 turkeys at 13*s.* 10*d.* each.

25. What would be the cost of the coal required to keep an engine continuously at work for 13 weeks 5 days 16 hours if the engine consumes 1·5 cwt. of coal per hour, and the coal costs 24*s.* 6*d.* per ton?

26. The cost of 9 cwt. 2 qrs. 17 lb. of a certain commodity was £26 2*s.* 6*d.* What is the lowest price per lb. expressed in pence at which it can be sold without loss?

27. £840 9*s.* 3¼*d.* is to be divided equally among 65 persons. How much does each receive?

28. What is the greatest number of persons a platform will support if their total weight must not exceed 9½ tons, and their average weight is 12 st. 5 lb.?

29. A number of iron bars, placed in a straight line not quite touching, form a rail 27½ miles long. If the ends of successive bars had been placed in contact, 7 more bars would have been required. The length of each bar being 8 yds., find the distance between successive bars.

(Express the result as a decimal of an inch.)

30. A carpenter takes a plank 10 ft. 4 in. long and saws off as many pieces as he can of length 1 foot 2·5 inches. Allowing ·0625 inches waste for each saw-cut, find the length of the piece of plank left over.

31. On December 1st the sun rose at 7 hrs. 44 min. 20 sec. a.m., and set at 3 hrs. 53 min. 16 sec. p.m.; on each successive day he rose 74 sec. later, and the length of each of these days was 95 sec. shorter than the previous day. Find the times of sunrise and sunset on December 9th.

32. A piece of land is divided into 23 equal plots of 1 acre 2 ro. 25 po. What is the size of the land?

33. What sum is gained by buying 7 cwt. 3 qrs. 24 lb. of tea for £85 and then retailing it at an average price of 1*s.* $11\frac{1}{4}d.$ per lb.?

34. A farmer exchanges 72 lambs, worth a guinea and a half each, for 42 sheep, worth £2 15*s.* 6*d.* each.

Had he to pay money or receive it, and how much?

35. Assuming a metre to be 3·28 ft., find the nearest whole number of metres in one mile.

36. Find the value of ·03125 of £153 16*s.* 8*d.* — 198·6 of $2\frac{1}{2}d.$

37. The 20th of December, 1912, was a Friday. On what day of the week will Christmas Day fall in 1920?

38. From 277,114 tons of ore are obtained 86319·52 ounces of fine gold, which sell for £363,268 15*s.* 7*d.*

Find, to the nearest farthing, the average value of the gold yielded by 1 ton of ore, and the value of 1 oz. of fine gold.

39. A lady buys 16 oranges, weighing 3 lb., for one shilling. She boils them down with 10 pints of water and $11\frac{1}{4}$ lb. of sugar at $2\frac{1}{4}d.$ a pound, and obtains 15 lb. of marmalade.

What weight has evaporated in the boiling, and what has a pound of marmalade cost her in materials?

(A pint of water weighs $1\frac{1}{4}$ lb.)

40. A firm dispatches abroad in a year 1,200 letters under 1 oz., 300 between 1 oz. and 2 oz., and 60 between 2 oz. and 3 oz. Reckoning the postal rate at $2\frac{1}{2}d.$ for the first ounce and $1\frac{1}{2}d.$ for each succeeding ounce or fraction of an ounce, find the total cost of the postage of these letters.

41. A carrier's cart weighs 1 ton 13 cwt. He receives 14 cases, each weighing 149 lb. What is the total weight of the cart and its contents?

42. If I ask for 3 ninepenny stamps, 30 twopence-halfpenny stamps, and 100 penny stamps, and offer a sovereign in payment, what change should I receive?

43. On the 22nd of December the sun rises at 8.6 a.m. and is above the horizon for 465 minutes. At what time does the sun set on that day?

44. From April 1 to October 24, both dates included, the traffic receipts of the London County Council Tramways amounted to £1,053,721. Find, to the nearest shilling, the takings per day during this period.

45. The consumption of sugar in Austria during a single month amounted to 38,700 tons. Taking the average value of the sugar as 23*s.* 4*d.* per cwt., and the population of Austria as 27,000,000, find the expenditure per head for sugar during this month. Give the answer to the nearest penny.

46. A weight of 3 tons 9 cwt. 17 lb. of sugar is divided up into 272 equal packages, and there is 10 lb. left over; find how much sugar there is in each package.

47. If 1 metre = 3·2809 feet, find, correct to the nearest foot, the difference between 161 Km. and 100 miles.

48. A man buys 143·60 metres of ribbon in Paris at 65 centimes per metre, and offers a £5 note in payment, which the shopkeeper agrees to accept at the rate of 25·12 francs to the £. How much change will the purchaser receive?

49. There are 97 leap-years in 400 civil years. Show that the average duration of a civil year exceeds the solar year, which contains 365 days 5 hrs. 48 min. 46 sec., by 26 sec.

50. How many pieces of wire, each 4·13 inches long, can be cut from a rod whose length is 8·24 feet, and what will be the length of the piece left over?

51. A man bought 10 tons 3 cwt. of sugar at £18 a ton and sold it at 2¼*d.* a pound. What profit did he make?

52. Find the value of 3·074 of £2 7*s.* 8*d.*

53. How many pieces, each 3 metres 5 cm. long, can be cut off a rope 35 metres long? What is the length of the remainder?

54. A man buys 3750 tons of coal for £3640; what will he gain or lose by selling 2644 tons at £1 1*s.* 8*d.* per ton and the remainder at 18*s.* 9*d.* per ton?

55. A traveller enters France with £61 10*s.*, which he exchanges for French money at the rate of 25 fr. for £1. He spends 1353 fr. 75 c.; how much French money has he left?

56. A housekeeper takes three half-pints of milk each week-day and one pint on Sunday, and her bill for the week comes to 1*s.* 5½*d.* What is the price of milk per quart?

57. Excursion tickets are issued at the rate of a single fare and a quarter for the double journey. Find the cost of seven and a half such tickets, the single fare being 5*s.* 8*d.*

58. A club employs one boy for 28 weeks at 7*s.* 6*d.* per week, another boy for 5 weeks at 5*s.* 10*d.* per week, and a third for 11 weeks at 4*s.* 4*d.* per week. How much does the club pay its boys altogether?

59. Find the total cost of three dozen pocket handkerchiefs at 1*s.* 10*d.* each, 29 collars at 9½*d.* each, and 13 neckties at 1*s.* 11¾*d.* each.

60. Find the sum of ·0975 of £6 5*s.* and ·3125 of £1 10*s.*, and reduce your result to the decimal of £3.

61. If 5 tons of potatoes are bought for £18 15*s.* and sold at 6*d.* a stone, what is the total gain?

62. Find the total cost of 19½ lb. of nails at 4½*d.* per lb., 300 yards of rope at 7*d.* per dozen yards, 17 iron rods at 1*s.* 3½*d.* each, and 28 hammers at 10½*d.* each.

63. Find the rent of a farm of 1357 acres at £2 7*s.* 8½*d.* per acre.

64. How many times does a wheel, whose circumference is 3 yds. 2 ft. 7 in., turn in rolling 1 mile 186 yds.?

65. A plot of building land containing 3 acres 3 roods 25 poles is sold for £1312 10*s.* What is the price per acre?

(Convert the area to the decimal of an acre.)

66. If 23 Kg. 800 g. of copper are worth 48 marks 80 pf., find to the nearest pfennig the value of a kilogramme of copper.

67. Find to the nearest pfennig the value of £1 in German money, when 2500 mks. are equivalent to £122 7*s.* 6*d.*

(Express the sum of money as a decimal of £1.)

68. A kilogramme is 2·2046 lb. If I order 15½ cwt. of a certain article, and the order is executed in kilogrammes, allowing 51 Kg. to the cwt, shall I get more or less than I ordered, and what will be the difference?

(Give the answer in lb. and oz. to the nearest ounce.)

69. If ribbon costs 1 fr. 8 c. per metre, how much can be bought for 17 fr. 55 c.?

70. A man has £108 13*s.* 4*d.* Find how many things, each costing £5 4*s.* 2½*d.*, he can buy, and how much money will remain over.

71. A man walks at the rate of two steps, of 36 inches each, per second; how long will he take to walk 12 miles? Express the result in hours and minutes.

72. Find the sum of ·058 of a ton, ·166 of 7 cwt., and 5·41 of 24·2 lb. Give the answer in pounds.

73. If a bicycle wheel makes 721 revolutions in a mile, find its circumference to the nearest inch.

74. A consignment of 52 casks of sugar, each containing 7 cwt. 2 qrs., was bought at £26 10*s.* per ton, and sold at 3½*d* per lb. Find the gain or loss.

75. If a metre is approximately 39·37 inches, find the number of kilometres in 375 miles.

76. If 10 metres = 11 yards, express 1 cm. as the decimal of an inch.

77. A business firm has to send 7240 fr. to Paris and 2148 rupees to Calcutta. When the rates of exchange are 25·60 fr. for £1 and 1*s.* 5*d.* for 1 rupee, how much will the two items together cost the firm in English money?

REVISION EXAMPLES

1. Fill in the horizontal and vertical totals in the following table:

VOTERS IN THE UNITED KINGDOM

Country	Electors, 1910						Total for 1910
	Owners	Occupiers	Lodgers	Freemen Freeholders, &c	Total	University Voters	
England—							
Counties	503,571	2,788,052	91,928	—		20,110	
Boroughs	—	2,325,001	118,739	22,365			
Wales—							
Counties	32,678	209,047	5,610	—		—	
Boroughs	—	100,758	8,621	852			
Scotland—							
Counties	63,703	363,404	23,843	—		20,024	
Burghs	30,938	253,882	26,414	—			
Ireland—							
Counties	8,318	566,372	3,774	—		5,020	
Boroughs	—	110,543	2,329	2,431			
Grand Total							

2. If £840 9*s.* $3\frac{1}{4}$*d.* is divided equally among 65 persons, how much will each receive?

3. What is meant by the local value of a digit? Write down the local value of each figure in the number 78·543.

4 Find the values of $\cdot 3 \times \cdot 2 + \cdot 21 \times \cdot 4$ and $\cdot 4 \times \cdot 2 - \cdot 21 \times \cdot 3$; and divide the sum of the squares of the two values by 14·5.

5. A wheel makes 1028 revolutions in passing over 2 mi. 4 fur. 55 yds.; what is the circumference of the wheel to the nearest inch?

6. If 9*d.* is paid as tax on every pound of income above £160, what is the income when the tax is (*a*) 3*s.* 9*d.*, (*b*) £3 16*s.* 6*d.*?

7. Divide eight million one hundred and fifty-three thousand and seventy-eight by sixty-three.

8. In the year 1907, 227 million pounds of tea were exported from India; how many tons was this?

(Give the answer to the nearest ton.)

9. A debtor owes £5476 10*s.*; he is able to pay 12*s.* 4*d.* in the pound. How much money has he?

10. Multiply 34·00376 by ·070605; prove the result to be correct.

11. The season ticket for a year between two places costs £6 10*s.*; the ordinary return ticket costs 8½*d.* How much will a man save by taking a season ticket if he travels the return journey 291 times during the year?

12. How many pieces of string of length 2 ft. 10 in. can be cut from a ball containing 100 yds.? What will be the length of the piece left?

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13. Fill in the figures in the fourth and sixth columns in the following table, giving results in the sixth column to the nearest pound.

Industry	Gross Output Selling Value (1)	Materials used. Cost (2)	Work Given Out. Amount paid to Other Firms (3)	Net Output Excess of Column (1) over Columns (2) and (3) (4)	Persons Employed Total (5)	Net Output per Person Employed (6)
	£	£	£	£		
Mines under the Coal Mines Regulation Acts. . .	123,245,000	16,881,000	—		840,280	
Coke Works	10,141,000	7,148,000	—		10,944	
Shale Oil Works	2,371,000	1,594,000	—		3,391	
Cotton Factories . .	176,940,000	129,095,000	904,000		572,869	
Woollen and Worsted Factories .	70,331,000	49,241,000	1,638,000		257 017	
Tinplate Factories . . .	9,350,000	7,203,000	—		21,222	
Iron and Steel Factories (smelting, founding, and rolling) .	105,597,000	74,049,000	600,000		262,225	
Total .	497,975,000	285,211,000	3,142,000		1,967,948	

14. Divide 298·08 by ·00345.

15. Find the whole cost of 20 dozen boxes of fruit at 14*s.* 7½*d.* per box, 40 dozen at 13*s.* 9¼*d.*, and 60 dozen at 12*s.* 8*d.*

16. A surveyor finds that the length of a road is 5729 chains 27½ links. What is the length in miles, &c. (to the nearest yard)?

17. How many parcels of 16 lb. each can be made up from goods weighing 4 tons 15 cwt. 3 qrs. 25 lb.? What is the value of the remainder if an ounce is worth 2½*d.*?

18. Find, to the nearest millimetre, the circumference of a carriage wheel which makes 59173 turns in travelling 170 kilometres.

---

19. The following table gives the number of unemployed who applied for relief during the year ending March 31, 1910 Calculate the respective numbers for the preceding year.

Applicants	No.	Difference compared with previous year
Total number of applicants (1909–10) .	127,066	+ 69,691
Number of applicants found qualified for assistance under the Act		
i Applicants .	81,749	− 54,840
ii Dependants of applicants . . .	236,094	+ 139,949
Number of applicants provided with work .	58,603	+ 29,587
Number of persons assisted to emigrate, or to remove to other areas in England and Wales . . .	2,201	+ 654

(Notice that the answers show that there is some mistake in the table.)

20. A publisher sells in a year 24516 copies of a book published at 3 fr. 50 c. If the author receives one-twentieth of the published price of each book, how much does he receive in the year?

21. How much money would be required to give every person in London a farthing, if the population of London be estimated at 4913345?

22. What is the cost of 315·75 tons of coal at £1·325 per ton? Give the answer correct to the nearest farthing.

23. A tradesman buys 5760 articles at 1*s.* 3*d.* a dozen and sells them at 2*s.* 6*d.* a score. How much does he gain or lose?

24. A man, who takes seven steps in three seconds, walks $6\frac{1}{2}$ kilometres in an hour. Find the length of his stride, correct to the nearest centimetre.

---

25. Fill in the missing figures in the following table:

IMPORTS IN SEPTEMBER 1911 AND 1912

Class.	Sept. 1911.	Sept. 1912.
	£	£
Food, drink, and tobacco . . .	22,900,000	23,300,000
Raw materials	16,700,000	18,400,000
Manufactures		15,300,000
Miscellaneous	200,000	200,000
Total imports	53,700,000	
Subtract re-exports of imported goods .		8,000,000
Net imports for home consumption .	£45,900,000	

26. A factory employs 215 workmen, each of whom earns on the average 4*s.* 3*d.* each working day. What is the total amount paid in wages for the year? How much extra is paid every leap-year. (No work on Sundays, Bank-holidays, Christmas Day, or Good Friday.)

27. If a pile of 194 sheets of paper is ·958 of an inch thick, what is the thickness of one sheet?

28. The clock in a town-hall strikes the hours, but not the quarters. How many times does the bell sound between 12.5 a.m. Sept. 3 and 12.5 p.m Dec. 20?

29. Reduce 359612 lb. to tons, hundredweights, quarters, pounds.

30. The value of the meat imported during a certain year was £51888314 4*s.* 2*d.* If the meat be taken to be worth 5*d.* per lb., find how many pounds were imported.

---

31. When 1943055 is divided by a certain divisor, the quotient is 2935 and the remainder 85; what is the divisor?

32. A grocer bought 18 cwt. 3 qrs. 24 lb. of sugar; he sold 2 cwt. 3 lb. to one customer, 1 cwt. 2 qrs. 19 lb. to another, 4 cwt. 2 qrs. 15 lb. to a third. How much had he then left?

33. A man earns 3*s.* 6*d.* a day, but is fined 6*d.* every time he is late. He receives five guineas for 31 days' work; how many times was he late?

34. Find the value of $3{\cdot}5\,(2{\cdot}7-1{\cdot}3)+(2{\cdot}6+3{\cdot}7)\,(8{\cdot}9-7{\cdot}8)$.

35. The population of France in a certain year was 39300000; the area of France is 537000 square kilometres. If the land were divided among the people, how many square kilometres would each person receive?

36. A father and son divide £342 2*s.* between them, so that the father receives 9*s.* for every 2*s.* the son receives. How much is the son's share?

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37. Make out an account for the following articles: 34½ yards of ribbon at 7*d.* the yard; 28 yards of muslin at 2*s.* 3½*d.* the yard; one gross of hooks at four for three farthings; 46 feet of curtain-rod at 2*s.* 7½*d.* the yard.

38. How many glasses, each holding half a pint, can be filled from a cask holding 9 gallons. If 2*d.* is charged for each glass, what amount is received for the contents of the cask?

39. How many times can 34·632 be subtracted from 2015?

40. What will it cost to travel 315 miles at an average cost of 4¾*d.* a mile? If the journey starts at 10 a.m. March 5, and if the traveller rests from 7 p.m. to 10 a.m. every day, at what time and on what day will the journey finish, the average speed being 8 miles per hour?

41. Express 1 furlong 4 poles as the decimal of a mile. How many times is the distance contained in 17 miles?

42. What is the difference between ·035 of a ton and ·64 of a cwt.? Express the answer as the decimal of a pound.

43. A grocer buys 146·75 kilogrammes of chocolate at 8 francs 40 centimes per kilogramme; he sells the whole amount at 9 francs 20 centimes per kilogramme. How much does he gain?

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44. Use the following table to find in what year the total trade was greatest and also in what year the imports most exceeded the exports.

SEPTEMBER TRADE, 1900 TO 1912

Year.	Imports. £	Exports of British Goods. £
1900	41,200,000	24,600,000
1901	38,200,000	22,000,000
1902	41,300,000	23,800,000
1903	45,500,000	23,300,000
1904	43,100,000	25,900,000
1905	45,700,000	29,300,000
1906	45,100,000	30,500,000
1907	45,300,000	35,100,000
1908	48,000,000	31,600,000
1909	49,500,000	32,800,000
1910	51,600,000	36,900,000
1911	53,700,000	36,800,000
1912	57,200,000	43,200,000

45. Divide 12 miles 2 furlongs 20 poles 4 yards 2 feet 6 inches by 47.

46. 60 lb. of coffee worth 1*s.* 4*d.* per pound and 12 lb. of chicory worth $5\frac{1}{2}d.$ per pound are mixed together. What is the value of a pound of the mixture?

47. How many pounds of sugar at $2\frac{3}{4}d.$ a lb. are worth as much as 315 lb. of tea at 1*s.* 10*d.* a lb.?

48. If 17·35 metres of cloth cost 89 francs 30 centimes, find, to the nearest centime, the cost of 1 metre.

49. If a confectioner sells 1 ton 2 cwt. 3 qrs. 9 lb. of sweets at 1*d.* an oz., what does he receive?

50. An employer pays each of his workmen a sovereign, a half-crown, a florin, and a shilling; he paid in all £303 9*s.* How many workmen were there?

CHAPTER V

Note —Throughout this chapter the word 'number' is to be understood to mean 'whole number'.

FACTORS

(79.) The number 6 is the product of the two numbers 2 and 3, and each of these two numbers is said to be a **factor** of 6.

In general, one number a is said to be a **factor** of another number b if we can divide b by a without leaving any remainder. b is said to be a **multiple** of a.

We can usually find two or more factors of any number, but there are certain numbers which cannot be so subdivided, and these are called **prime numbers.**

Thus we can find no number which will divide 3, 7, or 11, except unity or the number itself, and these are not factors in the strict sense of the word.

We shall define a prime number as a number which has no factor less than itself, unity being excluded.

(80.) We can express any number as a product of **prime factors** provided we can recognize these factors in the number, and for this purpose it is useful to remember the following **tests of divisibility.**

(i) A number is divisible by 2 if its last or unit digit is divisible by 2.[1]

Thus 306, 472934, 170 are all divisible by 2.

[1] In this connexion it should be noted that 0 is divisible by any number.

A number divisible by 2 is called an **even number**; other numbers are **odd.**

(ii) **A number is divisible by 3 if the sum of its digits is divisible by 3.**

A number such as 17322 can be written in the form

$$10000+7\,(1000)+3\,(100)+2\,(10)+2\,;$$

or in the form

$$(9999+1)+7\,(999+1)+3\,(99+1)+2\,(9+1)+2\,;$$

or $\quad [9999+7\times999+3\times99+2\times9]+(1+7+3+2+2).$

The number in the square bracket is certainly divisible by 3, since each term is so divisible, and the expression in the round bracket is the sum of the digits. If therefore the sum of the digits is divisible by 3, the number itself will be divisible by 3.

Note that the same test is applicable for divisibility by 9.

Thus 47361, 1236, 1002 are each divisible by 3, and 54378 is divisible by 9.

(iii) **A number is divisible by 5 if its last digit is divisible by 5.**

(iv) **A number is divisible by 11 if the sum of alternate digits differs from the sum of the remaining digits by zero or a multiple of 11.**

Consider the number 84238.

This may be written·

$$8\times10000+4\times1000+2\times100+3\times10+8\,;$$

or $\quad 8\,(9999+1)+4\,(1001-1)+2\,(99+1)+3\,(11-1)+8\,;$

or $\quad [8\times9999+4\times1001+2\times99+3\times11]+(8-4+2-3+8).$

The expression in the square bracket is divisible by 11, and therefore the number itself is divisible by 11 if the expression in the round bracket is either zero or divisible by 11.

This expression is obtained by adding together alternate digits, and subtracting from the result the sum of the remaining digits. Thus the numbers 724933, 81939, 872685 are each divisible by 11.

(v) **A number is divisible by 4 if the number formed by the last two digits is divisible by 4; and a number is divisible by 8 if the number formed by the last three digits is divisible by 8.**

(vi) **A number is divisible by 25 if the number formed by the last two digits is divisible by 25 (i.e. if the last two digits are 00, 25, 50, or 75); and by 125 if the number formed by the last three digits is divisible by 125.**

The proof of these statements is left as an exercise.

Thus 4244 is divisible by 4;
4248 „ „ „ 8;
549425 „ „ „ 25.

When expressing a number as the product of its prime factors, we use the index notation (see § 14). Thus we should not say $16 = 2 \times 2 \times 2 \times 2$;
but $16 = 2^4$.

Example i. *Express* 420 *as a product of prime factors.*

$$\begin{aligned} 420 &= 2 \times 210 && \text{(taking out the factor 2)} \\ &= 2^2 \times 105 && (\quad ,, \quad ,, \quad 2) \\ &= 2^2 \times 3 \times 35 && (\quad ,, \quad ,, \quad 3) \\ &= 2^2 \times 3 \times 5 \times 7 \end{aligned}$$

For the sake of neatness, we usually use a full stop instead of the sign $\times$:

Thus $420 = 2^2 . 3 . 5 . 7.$

Example ii. *Express* 6435 *as a product of prime factors.*

$$\begin{aligned} 6435 &= 5 \,.\, 1287 && \text{(5 is clearly a factor)} \\ &= 5 \,.\, 3^2 \,.\, 143 && \text{(after testing for 3 or 9)} \\ &= 5 \,.\, 3^2 \,.\, 11 \,.\, 13 && \text{(after testing for 11)} \end{aligned}$$

(**81.**) If all these tests fail, we have to try other prime numbers such as 7, 13, 17, 19, but the following consideration will save a good deal of time and trouble.

Since $13^2 = 169$, it is no good trying the factor 13 for a number less than 169 if we have tried all the prime numbers less than 13. For if one factor is 13, since the number is less than 13×13, the other factor must be less than 13, and we should have found it before we came to try 13.

Similarly, it is no good trying 17 for a number less than 289.

Example. *Express* 12804 *as a product of prime factors.*

$$\begin{aligned} 12804 &= 2^2 \,.\, 3201 && \text{(since 4 is a factor)} \\ &= 2^2 \,.\, 3 \,.\, 1067 && \text{(after testing for 3)} \\ &= 2^2 \,.\, 3 \,.\, 11 \,.\, 97 && \text{(the tests showing that 11 is a factor)} \end{aligned}$$

Now all the tests show that no one of the numbers $2 \,.\, 3 \,.\, 5 \,.\, 11$ is a factor of 97, and by trial 7 is not a factor. The next prime number is 13, but it is useless to try this, as 97 is less than 13^2. Hence 97 must itself be a prime number, and we have the result as last written.

Oral Examples.

Test the following numbers for divisibility by 2, 3, 5, 11.

1. 304.	**2.** 526.	**3.** 426.	**4.** 375.
5. 1725.	**6.** 2145.	**7.** 3190.	**8.** 123462.
9. 8149.	**10.** 2220.	**11.** 2222.	**12.** 347631.
13. 1492164.	**14.** 17237.	**15.** 42438.	**16.** 2112315.
17. 2010305.	**18.** 121392.	**19.** 89145.	**20.** 90816.
21. 42923.	**22.** 54246171.	**23.** 12423202.	**24.** 149631.

Which of the following are prime numbers?
(If a number is not prime, mention the factor.)

25. 46.	**26.** 47.	**27.** 57.	**28.** 59.
29. 83.	**30.** 101.	**31.** 801.	**32.** 161.
33. 159.	**34.** 89.	**35.** 69.	**36.** 67.
37. 179.	**38.** 407.	**39.** 257.	**40.** 293.

Further examples can be obtained by testing the above numbers 1–24 for divisibility by 4, 8, 9.

It is also a good exercise to construct numbers that will be divisible by 3, 9, or 11.

(**82.**) If we multiply a number by itself we obtain the **square** of the number.

Thus 4 is the square of 2;
25 „ „ 5.

It is sometimes necessary to reverse the process and find a number which, when multiplied by itself, will produce a given number.

Such a result is called the **square root** of the given number.

Thus 2 is the square root of 4 (written $\sqrt{4}$);
5 „ „ 25 („ $\sqrt{25}$).

Similarly $2^3 = 8$, and 2 is called the **cube root**, and the relation is written $2 = \sqrt[3]{8}$.

The method of factors lends itself very easily to the obtaining of square and cube roots in simple cases.

Example 1. *Find the square root of* 853776.

We have
$$\begin{aligned} 853776 &= 2^2 . 213444 \\ &= 2^4 . 53361 \\ &= 2^4 . 3^2 . 5929 \\ &= 2^4 . 3^2 . 11 . 539 \\ &= 2^4 . 3^2 . 11^2 . 49 \\ &= 2^4 . 3^2 . 11^2 . 7^2 \\ &= (2^2 . 3 . 11 . 7)(2^2 . 3 . 11 . 7) \end{aligned}$$

And the square root is
$$\begin{aligned} &2^2 . 3 . 11 . 7 \\ &= 924 \end{aligned}$$

Example ii. *Find the cube root of* 4492125.

$$\begin{aligned} 4492125 &= 5 \,.\, 898425 \\ &= 5^2 \,.\, 179685 \\ &= 5^3 \,.\, 35937 \\ &= 5^1 \,.\, 3^2 \,.\, 3993 \\ &= 5^1 \,.\, 3^1 \,.\, 1331 \\ &= 5^1 \,.\, 3^1 \,.\, 11 \,.\, 121 \\ &= 5^1 \,.\, 3^1 \,.\, 11^3 \end{aligned}$$

The cube root is $5 \,.\, 3 \,.\, 11$ or $\underline{165}$.

Examples V a.

Express the following numbers as products of their prime factors:

1. 90.	2. 420.	3. 495.	4. 168.
5. 2925.	6. 1683.	7. 6292.	8. 798.
9. 18513.	10. 946.	11. 1375.	12. 8073.
13. 219.	14. 57717.	15. 1331.	16. 4444.
17. 2002.	18. 6391.	19. 4557.	20. 17424.
21. 17017.	22. 1881.	23. 11349.	24. 1837.

Find the square roots of the following numbers:

25. 576.	26. 11025.	27. 4356.	28. 148225.
29. 24336.	30. 540225.	31. 23716.	32. 36864.
33. 41409225.	34. 3186225.		35. 213444.

Find the cube roots of the following numbers:

36. 35937.	37. 166375.	38. 1157625.
39. 46656.	40. 1771561.	41. 456533.

L.C.M. AND H.C.F.

(**83.**) Consider the numbers 150 and 63. If we express these numbers in terms of their prime factors we obtain:

$$150 = 2 \,.\, 3 \,.\, 5^2; \qquad 63 = 3^2 \,.\, 7.$$

Now we may set ourselves the problem of finding the smallest number which is exactly divisible by each of these

two numbers, and it is clear that such a number must contain all the factors which occur in the two numbers. It must therefore contain the factors

$$2 . 3^2 . 5^2 . 7,$$

and it must be, in fact, the product of these factors or

$$3150.$$

This number is called the **Least Common Multiple** of the two numbers, and this phrase is sometimes abbreviated into L.C.M.

We might also have set ourselves to find the largest number which would exactly divide each of these numbers, and an inspection of the numbers in factor form shows that 3 is the required number.

This number is the **Highest Common Factor** or H.C.F. of the numbers. It is also sometimes called the **Greatest Common Measure** or G.C.M.

The process can be extended to any collection of numbers.

Example. *Find the L.C.M. and H.C.F. of*

3003, 495, *and* 330.

We first factorize these numbers and obtain

$$\begin{aligned} 3003 &= 3 . 1001 \\ &= 3 . 11 . 91 \\ &= 3 . 11 . 7 . 13 \\ 495 &= 5 . 99 \\ &= 5 . 3^2 . 11 \\ 330 &= 3 . 110 \\ &= 3 . 11 . 2 . 5 \end{aligned}$$

An examination of these factorized forms shows that their L.C.M. is

$$2 . 3^2 . 5 . 7 . 11 . 13.$$

(It is usually convenient to leave an L.C.M. in factor form.)

The H.C.F. is $3 . 11$ or 33.

If there is no factor common to the numbers it is sometimes said that the H.C.F. is 1; but since we do not consider 1 to be a factor in the ordinary sense of the word, it is more correct to say that there is no common factor, or that the numbers are **prime to each other.**

(**84.**) If we have to find the L.C.M. of a large collection of small numbers, it is sometimes easier to write down the result.

Example. *Find the L.C.M. of*

8, 10, 11, 21, 15, 24, 30, 27, 45.

We write down at once

$$\text{L.C.M.} = 2^3 . 5 . 11 \ \ 3 . 7 . 3^2$$
$$= 2^3 . 3^2 . 5 . 7 . 11$$

The mental process is as follows:

$8 = 2^3$. ∴ we must have this factor
$10 = 2 . 5$. We already have the factor 2, and must therefore put in the 5 only.
11 must go in as it stands, as it is a prime number which is not a factor of the L.C.M up to the present.
$21 = 3 \ 7$ We have not got these factors in, so they are put down.
$15 = 3 . 5$. We have both factors in already.
$24 = 2^3 . 3$. We have all factors in already.
$30 = 2 . 3 \ 5$. We have all factors in already.
$27 = 3^3$ We have 3 already in, so add 3^2.
$45 = 3^3 . 5$. And we have all these factors in

The L.C.M may then be written in the neater second form.

Examples V b.

Find the L.C.M. and H.C.F. of the following numbers. (Leave the L.C.M. in factors.)

1. 210, 693.
2. 1287, 780.
3. 2805, 170.
4. 435, 174, 203.
5. 308, 819, 665.
6. 2299, 1045, 5225.
7. 360, 4725, 67375.
8. 23716, 148225, 12100.
9. 5500, 2750, 425.
10. 2783, 12397, 6831.
11. 3038, 5208, 5859.
12. 22385, 10175, 49247.

13. 1404, 3159, 3276
14. 2349, 8613, 347391.
15. 1836, 1989, 2754.
16. 5720, 1584, 396.
17. 462, 1617, 2772.
18. 1836, 1632, 1728.
19. 693, 825, 1764, 1960.
20. 63, 196, 238, 2601.
21. 143, 78, 182, 637, 312.
22. 2646000, 4989600.
23. 3175200, 4158000.

Write down the L.C.M. of the following sets of numbers (leaving the result in factors).

24. 6, 9, 10, 15.
25. 8, 12, 16, 21, 35.
26. 9, 21, 14, 10, 35, 77.
27. 12, 14, 15, 42, 22.
28. 26, 39, 10, 65.
29. 121, 77, 330, 70.
30. 15, 33, 65, 99, 45.
31. 8, 24, 36, 21, 44, 49.
32. 25, 35, 55, 125, 49, 121, 143.
33. 45, 25, 85, 63, 49, 119.
34. 16, 45, 72, 98, 162, 520.
35 16, 63, 84, 112, 182, 624.
36. 8, 18, 28, 48, 98, 108, 168.
37. 6, 16, 26, 36, 56, 96, 156.
38. 33, 117, 135, 143, 195.
39. 24, 77, 126, 231, 44.
40. 12, 18, 28, 32, 60, 63, 90, 120.

(**85.**) Sometimes it is difficult to see the factors if any large prime numbers occur, and in such a case the following method of finding the H.C.F. will be found useful.

The method depends upon the fact that if two numbers have a common factor, and we divide one by the other, then the remainder will also contain that factor; whilst if there is no remainder, the divisor itself is the common factor.

For if we divide a by b, and obtain a quotient Q and a remainder R, then we have $a = bQ + R$,

or $a - bQ = R$,

(on subtracting bQ from each of these equal expressions).

If, then, a and b have a common factor, the left-hand expression is divisible by this factor and therefore R is divisible by this factor; whilst if R is zero (i.e. if there is *no* remainder), b itself is a factor of a, and is therefore the common factor

Example i. *Find the H.C.F. of* 404073 *and* 148903.

1	148903	404073	2
	106267	297806	
2	42636	106267	2
	41990	85272	
	646	20995	3
		1938	
		1615	2
		1292	
		323	

Step 1. Divide 404073 by 148903; the quotient is 2 and remainder 106267.

Step 2. Divide 148903 by 106267; the quotient is 1 and remainder 42636.

Step 3 Divide 106267 by 42636; &c.

And 323 is the common factor, since it is common to 323 and 646.

The figures at the side are for reference only. They are the quotients in the various stages of division.

The work may be shortened by using Italian division.

It is sometimes possible to curtail the work if at any stage the remainders have factors which are obvious

Example ii. *Find the H.C.F. of* 10088 *and* 30749.

10088	30749	3
97	30264	
388	5 | 485	
388	97	

Now if there is a common factor, it is a factor of 485, i.e. it is either 5 or 97.

5 is clearly not a common factor, but 97 is found to be a factor of 10088 by simple division.

Therefore 97 is the Highest Common Factor.

(**86.**) If we have found the H.C.F. of two numbers, it is easy to write down their L.C.M.; for if we multiply the two numbers together, the only factor occurring twice is the H.C.F. we have already found. Rejecting this superfluous

factor by division, we find the Lowest Common Multiple of the two numbers.

We found the H.C.F. of 148903 and 404073 to be 323, and therefore the L.C.M. of these two numbers is

$$148903 \times 404073 \div 323$$
$$= 148903 \times 1251$$

We notice that the product of any two numbers is equal to the product of their H.C.F. and L.C.M.

Examples V c.

Find, by the 'long method', the H.C.F. of the following numbers. Give also the L.C.M. (without multiplying out).

If there is no common factor, say so.

1. 3811, 6623.
2. 6649, 13969.
3. 12126, 47470.
4. 5457, 14338.
5. 10795, 21844.
6. 16541, 22101.
7. 13943, 17009.
8. 3007, 3713.
9. 5662, 7599.
10. 78657, 90275.
11. 149766, 258770.
12. 20711, 18281.
13. 5293, 5561, 7169.
14. 12827, 13837, 16059.

Find the H.C F. only of

15. 1106027, 2439883.
16. 2685592, 2252925.
17. 5348889, 3114043.
18. 161987, 324291.
19. 321321, 654947.

We conclude this chapter with a few notes on rapid calculation.

(**87.**) If we multiply a quantity by 100 and then divide the result by 4, we have, in effect, multiplied the quantity by 25.

So, also, if we multiply by 1000 and divide by 8, we have actually multiplied by 125.

If, then, we wish to multiply a number by 5, 25, or 125, it is easier to multiply by 10, 100, or 1000 (i.e. move the figures with reference to the decimal point), and divide by 2, 4, or 8.

Thus
$$\begin{aligned} 173 \times 25 &= (17300 \div 4) \\ &= 4325 \end{aligned}$$
$$\begin{aligned} 1892 \times 125 &= (1892000 \div 8) \\ &= 236500 \end{aligned}$$

Division by 25 or 125 can be carried out equally easily.

Thus
$$\begin{aligned} &184325 \div 25 \\ &(= 1843{\cdot}25 \times 4) \\ &= 7373 \end{aligned}$$

And
$$\begin{aligned} &1421{\cdot}3 \div 125 \\ &= (1{\cdot}4213 \times 8) \\ &= 11{\cdot}3704 \end{aligned}$$

The steps in brackets may be omitted after practice

(88.) In calculating the value of a difference between two squares, such as $173^2 - 154^2$,
it is usually easiest to use the algebraical fact that

$$a^2 - b^2 = (a-b)(a+b).$$

Thus (i) $173^2 - 154^2 = (173+154)(173-154)$
$= 327 \times 19$
$= 6213$

$$\begin{array}{r} 3270 \\ 2943 \\ \hline 6213 \end{array}$$

(ii) $8432^2 - 8421^2 = 16853 \times 11$
$= 185383$

(89.) Since $(a+b)^2 = a^2 + 2ab + b^2$
and $(a-b)^2 = a^2 - 2ab + b^2$
the squares of some numbers may be calculated by a process which is quicker than actual multiplication.

Thus $301^2 = (300+1)^2$
$= 90000+600+1$
$= 90601$

And $198^2 = (200-2)^2$
$= 40000-800+4$
$= 39204$

Examples V d.

Write down the results of multiplying and dividing the following numbers by (*a*) 25, (*b*) 125.

1. 4725.
2. 89325.
3. 3724.
4. 8949.
5. 7265.
6. 103·2.
7. 8756.
8. 63·498.
9. 474·6.
10. 82·5.
11. 734·9.
12. ·3282.

Calculate, as shortly as possible, the values of the following:

13. 172^2-171^2.
14. 6245^2-6244^2.
15. 192^2-190^2.
16. 4096^2-4092^2.
17. 1717^2-1705^2.
18. $5{\cdot}32^2-4{\cdot}59^2$.
19. $56{\cdot}32^2-43{\cdot}28^2$.
20. $7{\cdot}66^2-6{\cdot}77^2$.
21. 1001^2.
22. 999^2.
23. 1004^2.
24. 896^2.
25. 605^2.
26. 820^2.
27. 1202^2.
28. 1998^2.
29. 3995^2.
30. $500{\cdot}02^2$.

(**90.**) The following examples illustrate types of problems connected with factors.

Example i. *Find the greatest number which, when used to divide* 772, 2932, *or* 547, *leaves a remainder of* 7 *in each case.*

If a remainder of 7 is to be left, the numbers 765, 2925, 540 must be exactly divisible.

$$\begin{aligned} \text{Now} \quad 765 &= 5 \,.\, 153 \\ &= 5 \,.\, 3 \,.\, 51 \\ &= 5 \,.\, 3^2 \,.\, 17 \\ 2925 &= 5^2 \,.\, 117 \\ &= 5^2 \,.\, 3^2 \,.\, 13 \\ 540 &= 5 \,.\, 2^2 \,.\, 27 \\ &= 5 \,.\, 2^2 \,.\, 3^3 \end{aligned}$$

The highest factor common to these numbers is $5 \,.\, 3^2$ or 45.

$\therefore$ if the original numbers are divided by 45, there will be a remainder of 7 in each case.

The answer is 45.

Example ii. *Three bells are tolling at intervals of* 24 *sec.* 30 *sec., and* 42 *sec. They all toll together at a certain moment. When will the first two be tolling together again, and when will they all three toll together again?*

The L.C.M. of 24 and 30 is $2^3 \,.\, 3 \,.\, 5 = 120$.

$\therefore$ after 120 sec., or 2 *minutes*, the first two will be tolling together.

The L.C.M. of 24, 30, and 42 is $2^3 \,.\, 3 \,.\, 5 \,.\, 7 = 840$.

$\therefore$ after 840 sec., or 14 *minutes*, they will all toll together again.

Examples V.

1. Resolve 23256 into prime factors, and also into two factors as nearly as possible equal to one another.

2. If numbers are formed by arranging all the digits 1, 2, 3, 5, 6, 7, 8 so that the odd and even digits occur alternately, show that each number so formed is divisible by 11.

3. Write down in a column all the odd numbers between 290 and 302, and write alongside each one of those numbers the least number, excluding unity, which will divide it exactly, and find the continued product of all those least numbers.

4. Find the even whole numbers of four digits which are perfect squares and which are divisible by 3 and by 7.

5. Find the least square number which is divisible by 7, 8, and 9.

6. Find the least number such that the L.C.M. of 4437, 7163, and that number may be equal to the L.C.M. of 10659, 32799, and that number.

7. Find the value of the missing digit in the number 426*73, such that (i) the number may be divisible by 9, (ii) the number may be divisible by 11.

8. Find the least number which, divided by 33, 51, or 187, in each case leaves a remainder 5.

9. Find the least number which, when divided by 34, 51, or 36, will leave a remainder 7.

10. By what must 693 be multiplied in order that the result may be a square number? Give the square root of the result.

11. Find the three numbers between 2500 and 3000 which are divisible by all the numbers 21, 24, 28.

12. Resolve 1047618 into its prime factors, and write down its factors which lie between 100 and 150.

13. A cogwheel, which has 30 cogs, fits into another which has 56 cogs. How many turns will each have made from the time that two particular cogs are together to the time that they are again together?

14. What is the smallest sum of money of which £3 5*s.* and £3 1*s.* 9*d.* are both exact multiples?

15. What is the smallest bill which could be paid exactly in either francs, half-crowns, or dollars? (A franc = 10*d.*, a dollar = 4*s.* 1*d.*)

16. Find the least number which is a perfect cube and has 2352 as a factor.

17. A number of books will fill an exact number of shelves

with 36, 45, 54, or 60 books on each shelf. What is the least number of books for which this is possible?

18. If the difference between the greatest and least number of shelves that may be used in question 17 is 42, what is the number of books?

19. There are two clocks, one of which beats 238 times and the other 133 times in a minute. They beat exactly together at noon; when do they next beat together, and how often do they do so between 1 p.m. and 11 p.m.?

20. Calculate the value of 372^2, and then write down the values of 373^2 and $372{\cdot}1^2$.

21. Calculate, as shortly as possible, the value of

$$\frac{347{\cdot}2^2 - 53{\cdot}6^2}{42{\cdot}4^2 - 42{\cdot}3^2}.$$

Give the result correct to five significant figures.

22. Verify the fact that the difference of the squares of 11606 and 786 is equal to the difference of the squares of 12073 and 3417.

23. A royal salute is being fired by two guns, each firing 21 times; they fire the first time simultaneously, and then one of them fires at intervals of 36 seconds and the other at intervals of 45 seconds. How many times more will the latter have to fire after the other has finished its salute?

24. Four bells start tolling together, at intervals of 8 sec., 10 sec., 12 sec., and 14 sec. respectively. How long will it be (i) before any two of them are tolling together again, (ii) before they are all tolling together again?

25. Are either of the numbers 437, 457 prime numbers? Give full reasons for your decision.

26. The number 23**67* (where the asterisks stand in the places of unspecified digits) is divisible by 792 without a remainder. What is the quotient?

27. Two clocks sound the first stroke of 12 o'clock at the same instant; one o'clock allows an interval of 20 seconds between each stroke and the next, and the other allows 25 seconds. How many strokes of the slower clock will remain after the quicker

one has finished striking, and what time will elapse between the twelfth stroke of the quicker one and the following stroke of the slower one?

28. There was a question in an arithmetic paper:—To find the L.C.M. of three given numbers. One of them was 3425, another 1829, and the third was an even number of five figures. One candidate, by mistake, copied down 3245 instead of 3425, but obtained the correct answer. What was the third given number?

29. When 983 is divided by a certain number the remainder is 8. When 3166 is divided by the same number the remainder is 16, and when 14626 is divided by the same number the remainder is 1. What is the greatest divisor for which this is possible?

30. Write down the nine numbers which immediately precede 81, and the nine which immediately follow, and show that their sum is exactly divisible by 9. Would the sum still be divisible by 9 if, instead of a group of nine numbers on either side of 81, a group of five, six, or any other amount were chosen? Give your reasons.

CHAPTER VI

FRACTIONS

(**91.**) We have made use of the notations

$$6 \div 3 \qquad 6/3 \qquad \frac{6}{3}$$

to indicate that 6 is to be divided by 3.

We have also found it possible to evaluate an expression such as $\frac{2}{5}$ (i.e. $2 \div 5$) by making use of the extended decimal notation. The result of dividing 2 by 5 was found to be ·4.

But sometimes it is more convenient to leave the expression in the form $\frac{2}{5}$, especially when it is to be combined with other expressions of the same type by the processes of addition, subtraction, &c. Such an expression, when the division is not performed, is called a **fraction**. Fraction means literally a 'broken part' (Latin *frangere*, to break; *fractum*, broken). Expressions such as $\frac{2}{3}$, $\frac{3}{5}$ were called 'vulgar fractions' (i.e. common or ordinary fractions), to distinguish them from 'decimal fractions'. Throughout this book the word 'fraction' will be taken to mean 'ordinary fraction'.

A whole number is often referred to as an integral number or **integer**.

(**92.**) When a number is expressed as a fraction, the dividend is called the **numerator** and the divisor the **denominator.**

Thus in the fraction $\frac{5}{7}$ the numerator is 5 and the denominator 7.

The denominator indicates the kind of fraction we are talking about, e.g. 'thirds,' 'fourths,' 'tenths,' and so on. The term has the same Latin derivation (*nomen*, a name) as the word 'denomination' which we have used in Chapter III.

The **denominator** gives the **name** of the fraction, just as **denomination** gives the **name** of the unit of measurement.

The **numerator** (Latin *numerus*, number) indicates the number of thirds, fourths, tenths, &c.

Thus in the fraction $\frac{5}{7}$ the kind of fraction is indicated by the word 'sevenths', and the number of these sevenths is five. The fraction is read 'five sevenths'.

A fraction in which the numerator is greater than the denominator is called an **improper fraction**, and, by way of distinction, a fraction in which the numerator is less than the denominator is called a **proper fraction**

$\frac{23}{7}$ is an improper fraction, but if we perform the division partially we obtain a **mixed number**, $3\frac{2}{7}$, which is read as three, and two sevenths'.

From the point of view of division, $\frac{23}{7}$ indicates that 23 is to be divided by 7. The result is 3, with 2 remaining still to be divided by 7.

Conversely, a mixed number may be expressed as an improper fraction by performing the multiplication again Thus $5\frac{7}{9}$ is the same as $\frac{52}{9}$. For $5\frac{7}{9}$ indicates that when some number has been divided by 9 the quotient is 5 and the remainder 7.

We obtain the dividend as

$$5 \times 9 + 7 \quad \text{or} \quad 52.$$

The expression is therefore equal to $\frac{52}{9}$.

(**93**.) The fraction $\frac{2}{3}$ means, then, the size of one part when 2 units are divided into 3 equal parts; but there is another and more important point of view.

$\frac{2}{3}$ may mean that one unit is to be divided into three parts, and two of these parts are to be taken.

Clearly the result of dividing two units into three parts is twice as great as the result of dividing one unit into three parts.

Hence these two points of view both give the same result.

The matter may also be illustrated by a figure.

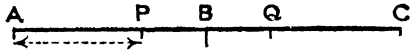


Fig 1

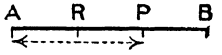


Fig 2.

In Fig. 1 a line AC 2 inches long is divided into three parts. One of these parts is AP. In Fig. 2 a line AB 1 inch long is divided into three parts, and two of these are taken. The result is AP.

We shall usually regard fractions from the second point of view.

Oral Examples.

Express in words:

1. $\frac{3}{5}$;	**2.** $\frac{4}{7}$;	**3.** $\frac{5}{8}$;	**4.** $\frac{2}{15}$;	**5.** $\frac{7}{10}$;
6. $\frac{3}{8}$;	**7.** $\frac{5}{11}$;	**8.** $\frac{15}{26}$;	**9.** $\frac{7}{9}$;	**10.** $\frac{11}{24}$;
11. $\frac{5}{14}$;	**12.** $\frac{9}{19}$;	**13.** $\frac{8}{15}$;	**14.** $\frac{27}{29}$;	**15.** $\frac{2}{9}$;
16. $\frac{5}{17}$;	**17.** $\frac{11}{23}$;	**18.** $\frac{17}{28}$;	**19.** $\frac{19}{20}$;	**20.** $\frac{25}{26}$;
21. $\frac{4}{11}$;	**22.** $\frac{4}{9}$;	**23.** $\frac{7}{100}$;	**24.** $\frac{9}{200}$.	

Express in figures:

1. five-ninths; **2.** seven-elevenths; **3.** fourteen-fifteenths;
4. two-sevenths; **5.** eight-elevenths; **6.** five-sixths;
7. one-third; **8.** nine-tenths; **9.** four-fiftyfifths;
10. sixteen-seventeenths.

Express as mixed numbers:

11. $\frac{17}{4}$; **12.** $\frac{23}{8}$; **13.** $\frac{17}{9}$; **14.** $\frac{52}{7}$; **15.** $\frac{203}{11}$;
16. $\frac{179}{8}$; **17.** $\frac{307}{12}$; **18.** $\frac{444}{11}$; **19.** $\frac{163}{8}$; **20.** $\frac{175}{12}$;
21. $\frac{412}{9}$; **22.** $\frac{18}{13}$; **23.** $\frac{29}{22}$; **24.** $\frac{75}{71}$; **25.** $\frac{163}{82}$;
26. $\frac{1955}{107}$.

Express as improper fractions:

27. $1\frac{2}{9}$; **28.** $3\frac{4}{7}$; **29.** $4\frac{5}{8}$; **30.** $3\frac{1}{10}$; **31.** $2\frac{7}{10}$;
32. $5\frac{6}{11}$; **33.** $8\frac{2}{9}$; **34.** $1\frac{17}{19}$; **35.** $2\frac{5}{38}$; **36.** $1\frac{102}{103}$;
37. $5\frac{1}{5}$; **38.** $2\frac{74}{75}$.

Give the value of:

39. $\frac{2}{3}$ of 1/–; **40.** $\frac{1}{4}$ of 1/–; **41.** $\frac{2}{10}$ of £1;
42. $\frac{3}{10}$ of £2; **43.** $\frac{1}{5}$ of £3; **44.** $\frac{1}{8}$ of 1 lb.;
45. $\frac{2}{3}$ of 1 foot; **46.** $\frac{3}{4}$ of 3 feet;
47 $\frac{1}{5}$ of an acre (in sq. chains); **48.** $\frac{3}{7}$ of 1 qr. (in lb.);
49. $\frac{1}{2}$ a yard (in inches); **50.** $\frac{3}{11}$ of a chain (in yards);
51. $\frac{53}{100}$ of a chain (in links); **52.** $\frac{7}{10}$ of £2 10*s.*;
53. $\frac{5}{6}$ of a foot; **54.** $\frac{1}{7}$ of 1 cwt. (in lb.);
55. $\frac{3}{14}$ of 1 qr. (in lb.); **56.** $\frac{2}{7}$ of a stone (in lb.);
57. $\frac{1}{3}$ of 7/–; **58.** $\frac{2}{5}$ of £3;
59. $\frac{1}{880}$ of a mile (in yards); **60.** $\frac{49}{880}$ of a mile (in yards);
61. $\frac{1}{112}$ of a ton (in lb.); **62.** $\frac{2}{3}$ of 3 m. 3 dm. (in dm.);
63. $\frac{1}{5}$ of 4 m. (in dm.); **64.** $\frac{1}{10}$ of 2 cm. (in mm.);
65. $\frac{3}{10}$ of 4 dm. (in cm.); **66.** $\frac{2}{9}$ of a sq. yard;
67. $\frac{25}{27}$ of a cubic yard; **68.** $\frac{2}{5}$ of an hour (in minutes);
69. $\frac{3}{20}$ of a minute (in sec.);
70. $\frac{1}{5}$ of an ordinary year (in days);
71. $\frac{3}{7}$ of the month of February (in days);
72. $\frac{5}{6}$ of the month of June (in days);
73. $\frac{5}{19}$ of 1 ft. 7 in.

Give the results of performing the following divisions, i.e. express in decimal form:

74. $\frac{1}{2}$; **75.** $\frac{2}{5}$; **76.** $\frac{9}{10}$; **77.** $\frac{3}{5}$; **78.** $\frac{7}{20}$;
79. $1\frac{1}{10}$; **80.** $2\frac{3}{10}$; **81.** $1\frac{7}{10}$; **82.** $3\frac{1}{2}$.

Give the following to one decimal place:

83. $4\frac{1}{3}$; **84.** $5\frac{2}{3}$; **85.** $3\frac{1}{7}$; **86.** $2\frac{2}{7}$; **87.** $4\frac{3}{11}$; **88.** $5\frac{5}{6}$.

Express the following decimals as fractions:

89. ·3;	**90.** ·4;	**91.** 2·3;	**92.** ·5;
93. 3·5;	**94.** 2·5;	**95.** ·25;	**96.** 1·2;
97. 3·7;	**98.** 11·6;	**99.** 10·34;	**100.** 5·65;
101. 8·75;	**102.** 9·35;	**103.** 5·55.	

104. Fig. 3 represents a chess-board of 64 squares.

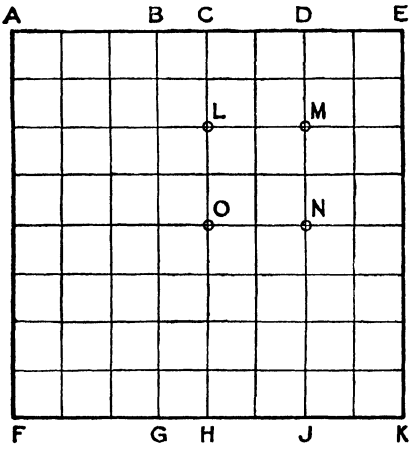


Fig. 3.

(i) How many squares are there in *ABGF*?
(ii) What fraction is *ABGF* of the whole?
(iii) How many squares are there in *BCHG*?
(iv) What fraction is *BCHG* of the whole?
(v) ,, ,, ,, *CDJH* ,, ,, ?
(vi) ,, ,, ,, *CEKH* ,, ,, ?
(vii) ,, ,, ,, *LMNO* ,, ,, ?
(viii) ,, ,, ,, *ONJH* ,, ,, ?

(ix) What fraction is $LMJH$ of the whole?

The square is divided into eight columns.

(x) How many columns are there in $ABGF$?

(xi) What fraction is $ABGF$ of the whole?

(xii) Compare the answers to (ii) and (xi), and draw any deduction you can.

(xiii) Regarding $CDJH$ as made up of columns, find what fraction it is of the whole.

(xiv) Regarding $CDJH$ as made up of squares, find what fraction it is of the whole.

105. AB is a length of 3 inches, each inch being divided into quarters.

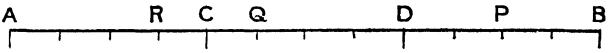


Fig. 4.

(i) How many divisions are there altogether?

(ii) What fraction is AQ of AB?

(iii) ,, ,, ,, AR of AD?

(iv) ,, ,, ,, DP of DB?

(v) ,, ,, ,, AP of AB?

(vi) ,, ,, ,, QB of AB?

(vii) ,, ,, ,, CD of AB?

(viii) ,, ,, ,, RP of AB?

(ix) ,, ,, ,, CP of AB?

(x) ,, ,, ,, DB of CB?

(**94.**) If we have some quantity divided into eight equal parts, we shall have 8 eighths altogether. Now if these eighths be each subdivided into three equal parts, we shall have 24 twenty-fourths altogether.

But suppose we had five of the eighths and then subdivided, we should have fifteen of the twenty-fourths. So

$\frac{15}{24}$ must be the same as $\frac{5}{8}$;

and, conversely, $\frac{5}{8}$ must be the same as $\frac{15}{24}$.

This argument may be applied to any fraction, and we have the general statement that **a fraction is unaltered in value if both the numerator and denominator are multiplied by the same number or are divided by the same number.**

The following geometrical illustration may be found useful. The student is advised to compare it with the results he obtained in Question 104 of the previous oral examples.

Fig. 5 is a rectangle divided into 32 equal squares.

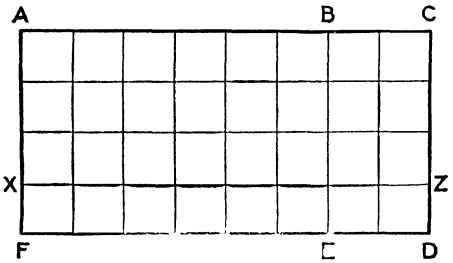


Fig. 5.

The portion $ABEF$ contains 24 of these squares.

$\therefore$ This portion is $\frac{24}{32}$ of the whole.

But the figure may be regarded as being divided into eight equal parts by vertical lines, and of these parts $ABEF$ contains 6.

$\therefore$ $ABEF$ is $\frac{6}{8}$ of the whole.

Again, $ACZX$ contains 24 squares and is therefore $\frac{24}{32}$ of the whole.

But the figure may be regarded as being divided into four equal parts by horizontal lines, and of these parts $ACZX$ contains three.

$\therefore$ $ACZX$ is $\frac{3}{4}$ of the whole.

$\therefore$ $\frac{24}{32}$, $\frac{6}{8}$, $\frac{3}{4}$, all have the same value.

(95.) Hence, if it suits our purposes, we may multiply or divide both the numerator and denominator of a fraction by any number we like, provided we perform the same operation with the same number on both numerator and denominator.

Note.—We *cannot add* the same number to both numerator and denominator of a fraction without altering its value.

$\frac{3+2}{5+2}$ is not equal to $\frac{3}{5}$,

though $\frac{3\times 2}{5\times 2}$ is equal to $\frac{3}{5}$

In particular, if a fraction is such that its numerator and denominator have a common factor, then we may express that fraction in a simpler form by dividing both numerator and denominator by that common factor.

Examples.

(i) $\frac{72}{84} = \frac{6\times 12}{7\times 12} = \frac{6}{7}$.

(ii) $\frac{125}{1000} = \frac{5\times 25}{5\times 200} = \frac{5\times 5\times 5}{5\times 5\times 40} = \frac{5\times 5\times 5}{5\times 5\times 5\times 8} = \frac{1}{8}$.

When a fraction is such that there is no factor common to the numerator and denominator, the fraction is said to be in its **lowest terms.**

(96.) In bringing a fraction to its lowest terms it is best to find all the factors of both numerator and denominator by the methods of Chapter V.

If the factors are not clear, the greatest common factor must be found by the long process explained on p. 134.

Example i. *Reduce* $\frac{420}{882}$ *to its lowest terms.*

$$\frac{420}{882} = \frac{2\times 210}{2\times 441}$$
$$= \frac{3\times 70}{3\times 3\times 49} \quad \text{(after dividing both by 2)}$$
$$= \frac{7\times 10}{3\times 7\times 7} \quad (\text{„ „ „ „ 3})$$
$$= \frac{10}{21} \quad (\text{„ „ „ „ 7})$$

Example ii. *Reduce* $\frac{1164}{10379}$ *to its lowest terms.*

The H.C F. is 97.

$$\text{The fraction} = \frac{12\times 97}{107\times 97}$$
$$= \frac{12}{107}.$$

1	1164	10379	8
	1067	9312	
	97	1067	
		97	
		97	

$$97\overline{)1164}\ 12 \quad 194$$

$$97\overline{)10379}\ 107 \quad 679$$

Note.—Some of this work can be considerably curtailed. At the same time, the student is cautioned against attempting to curtail the work until he thoroughly understands the idea of a fraction. In particular he is warned to avoid the word 'cancel'.

Oral Examples.

Reduce to lowest terms:

1. $\frac{6}{8}$;	2. $\frac{15}{25}$;	3. $\frac{9}{27}$;	4. $\frac{5}{25}$;
5. $\frac{8}{12}$;	6. $\frac{6}{15}$;	7. $\frac{6}{16}$;	8. $\frac{13}{26}$;
9. $\frac{60}{144}$;	10. $\frac{4\times 5}{5\times 6}$;	11. $\frac{3\times 4}{4\times 5}$;	12. $\frac{12\times 13}{13\times 14}$;
13. $\frac{42}{49}$;	14. $\frac{99}{108}$;	15. $\frac{99}{110}$;	16. $\frac{99}{117}$;
17. $\frac{99}{121}$;	18. $\frac{12}{15}$;	19. $\frac{12}{16}$;	20. $\frac{12}{18}$;
21. $\frac{12}{20}$;	22. $\frac{12}{21}$;	23. $\frac{12}{22}$;	24. $\frac{12}{24}$;
25. $\frac{12}{27}$;	26. $\frac{30}{50}$;	27. $\frac{120}{300}$;	28. $\frac{350}{490}$;
29. $\frac{130}{390}$.			

Of the following pairs of fractions, state which is the greater, giving your reasons:

30. $\frac{1}{3}$ or $\frac{1}{5}$;
31. $\frac{2}{5}$ or $\frac{2}{7}$;
32. $\frac{3}{11}$ or $\frac{3}{13}$;
33. $\frac{5}{9}$ or $\frac{5}{10}$;
34. $\frac{5}{9}$ or $\frac{1}{2}$;
35. $\frac{3}{7}$ or $\frac{5}{7}$;
36. $\frac{2}{3}$ or $\frac{5}{3}$;
37. $\frac{7}{9}$ or $\frac{6}{9}$;
38. $\frac{7}{9}$ or $\frac{2}{3}$;
39. $\frac{2}{3}$ or $\frac{3}{6}$;
40. $\frac{2}{3}$ or $\frac{1}{2}$;
41. $\frac{6}{8}$ or $\frac{3}{4}$;
42. $\frac{6}{8}$ or $\frac{5}{4}$;
43. $\frac{7}{5}$ or $\frac{11}{10}$.

Examples VI a.

Reduce to lowest terms:

1. $\frac{300}{1386}$;
2. $\frac{126}{189}$;
3. $\frac{539}{637}$;
4. $\frac{308}{693}$;
5. $\frac{220}{825}$;
6. $\frac{5082}{5445}$;
7. $\frac{420}{616}$;
8. $\frac{504}{4752}$;
9. $\frac{308}{1617}$;
10. $\frac{2254}{3365}$;
11. $\frac{455}{546}$;
2. $\frac{1309}{1496}$;
13. $\frac{306}{595}$;
14. $\frac{441}{539}$;
15. $\frac{303}{717}$;
16. $\frac{927}{1339}$;
17. $\frac{1551}{2256}$;
18. $\frac{11781}{12376}$;
19. $\frac{11704}{15561}$;
20. $\frac{8775}{12025}$;
21. $\frac{1234}{5678}$;
22. $\frac{15029}{19950}$;
23. $\frac{6734}{19943}$;
24. $\frac{39585}{64438}$;
25. $\frac{26059}{44298}$;
26. $\frac{55403}{61081}$;
27. $\frac{7425}{138600}$;
28. $\frac{12053}{27651}$;
29. $\frac{1793}{3687}$;
30. $\frac{13158}{16254}$;
31. $\frac{10625}{36125}$;
32. $\frac{6649}{13969}$;
33. $\frac{24627}{39423}$;
34. $\frac{16541}{22101}$;
35. $\frac{428571}{666666}$;
36. $\frac{78657}{90275}$;
37. $\frac{5369}{18473}$;
38. $\frac{142597}{425179}$;
39. $\frac{186399}{164529}$;
40. $\frac{200475}{475200}$;
41. $\frac{382716049}{876543209}$;
42. $\frac{449298}{1293850}$.

43. Express $\frac{75}{439}$ as a fraction with denominator 1317.
44. ,, $\frac{213}{1341}$,, ,, ,, ,, 22797.
45. ,, $\frac{357}{439}$,, ,, ,, ,, 60143.
46. ,, $\frac{2352}{7893}$,, ,, ,, ,, 1965357.
47. ,, $\frac{14}{989}$,, ,, ,, numerator 84.
48. ,, $\frac{387}{789}$,, ,, ,, ,, 3483.
49. ,, $\frac{2187}{4392}$,, ,, ,, ,, 41553.
50. ,, $\frac{7934}{9999}$,, ,, ,, ,, 2277058.

(97.) The sizes of different fractions can be compared by

(i) dividing the numerator by the denominator, expressing the result as a decimal;

(ii) so arranging the fractions that they each have the same denominator;

(iii) so arranging the fractions that they each have the same numerator.

The last two operations can be performed by making use of the fact that a fraction is unaltered in magnitude if the numerator and denominator are each multiplied by the same number.

Example i. *Compare the fractions* $\frac{3}{5}$, $\frac{5}{9}$, $\frac{11}{15}$, $\frac{23}{30}$ *by arranging them in descending order of magnitude.*

Method (i).

By division
$$\begin{aligned} \tfrac{3}{5} &= \cdot 6 \\ \tfrac{5}{9} &= \cdot 555 \ldots \\ \tfrac{11}{15} &= \overset{2}{\cdot}\overset{2}{7}\overset{}{3} \\ &= \cdot 733 \ldots . \\ \tfrac{23}{30} &= \cdot 766 \ldots . \end{aligned}$$

The order of magnitude is $\frac{23}{30}$, $\frac{11}{15}$, $\frac{3}{5}$, $\frac{5}{9}$.

Method (ii).

The least number of which the denominators are all multiples is
$$5 \times 9 \times 2.$$

$\therefore$ we arrange the fractions such that each has this number 90 for denominator.
$$\begin{aligned} \tfrac{3}{5} &= \tfrac{3\times 18}{5\times 18} = \tfrac{54}{90} \\ \tfrac{5}{9} &= \tfrac{5\times 10}{9\times 10} = \tfrac{50}{90} \\ \tfrac{11}{15} &= \tfrac{11\times 6}{15\times 6} = \tfrac{66}{90} \\ \tfrac{23}{30} &= \tfrac{23\times 3}{30\times 3} = \tfrac{69}{90}. \end{aligned}$$

And again, the order is $\frac{23}{30}$, $\frac{11}{15}$, $\frac{3}{5}$, $\frac{5}{9}$.

Method (iii) is not suitable for this example, but it is easier than method (ii) in the following case:

Example ii. *Arrange the fractions* $\frac{6}{7}$, $\frac{15}{17}$, *and* $\frac{10}{13}$ *in order of magnitude.*

The L.C.M. of 6, 15, 10 is 30.

$$\frac{6}{7} = \frac{6\times5}{7\times5} = \frac{30}{35}$$
$$\frac{15}{17} = \frac{15\times2}{17\times2} = \frac{30}{34}$$
$$\frac{10}{13} = \frac{10\times3}{13\times3} = \frac{30}{39}.$$

Now $30\div35$ is less than $30\div34$ and is greater than $30\div39$.

The order is $\frac{30}{34}$, $\frac{30}{35}$, $\frac{30}{39}$;

or $\frac{15}{17}$, $\frac{6}{7}$, $\frac{10}{13}$.

(**98.**) If we have to add together a number of fractions, or to subtract one fraction from another, we might convert each to a decimal by division.

This method is, however, generally unwieldy, since it involves several divisions, and these divisions will not, as a general rule, 'come out'. But, just as we may add concrete quantities when once we have expressed them in terms of the same denomination, so we can add fractions if we arrange that they shall all have the same denominator; for just as

two men + three men = five men,

so two sevenths of anything + three sevenths of the same thing = five sevenths of the same thing,

i. e. $\frac{2}{7} + \frac{3}{7} = \frac{5}{7}$.

Illustration:

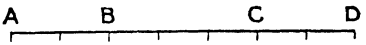


Fig. 6.

$AB = \frac{2}{7}$ of AD

$BC = \frac{3}{7}$ of AD

$AB + BC = AC = \frac{5}{7}$ of AD

Example i. *Add the fractions* $\frac{13}{15}$, $\frac{11}{12}$, $\frac{23}{30}$.

It will be convenient to find the smallest number which will be a multiple of 15, 12, and 30.

The number is $3 \times 5 \times 2^2 = 60$.

In this case all the fractions can be expressed as sixtieths.

The sum $= \frac{52}{60} + \frac{55}{60} + \frac{46}{60}$

(52 sixtieths + 55 sixtieths + 46 sixtieths)

$= \frac{153}{60}$ (153 sixtieths)

$= \frac{51}{20} = 2\frac{11}{20}$.

Example ii. *Simplify* $\frac{13}{14} + \frac{15}{16} - \frac{33}{35}$.

The L.C.M. of 14, 16, 35 is $2 \times 7 \times 8 \times 5 = 560$.

The expression

$= \frac{13 \quad 8 \quad 5}{560} + \frac{15 \quad 7 \quad 5}{560} - \frac{33 \quad 2 \quad 8}{560}$

$= \frac{520}{560} + \frac{525}{560} - \frac{528}{560}$

$= \frac{517}{560}$.

Example iii. *Simplify* $2 - \frac{13}{36}$.

Difference $= 1 + 1 - \frac{13}{36}$

$= 1 + \frac{36}{36} - \frac{13}{36}$

$= 1\frac{23}{36}$.

If the fractions are given as mixed numbers, it is generally convenient to deal with the whole numbers separately.

Example iv. *Simplify* $7\frac{2}{5} - 3\frac{1}{3} + 2\frac{1}{2}$.

The expression $= 7 - 3 + 2 + \frac{12}{30} - \frac{10}{30} + \frac{15}{30}$

$= 6\frac{17}{30}$.

Example v. *Simplify* $8\frac{2}{3} - 5\frac{5}{6} + 1\frac{1}{12} - 2\frac{5}{9}$

The expression $= 2 + \frac{24}{36} - \frac{30}{36} + \frac{3}{36} - \frac{20}{36}$

$= 2 + \frac{27}{36} - \frac{50}{36}$.

Either of the following methods may now be adopted :

$$\begin{aligned} & 2+\tfrac{27}{36}-\tfrac{50}{36} \\ &= 2+\tfrac{27}{36}-\tfrac{27}{36}-\tfrac{23}{36} \\ &= 2-\tfrac{23}{36} \\ &= 1\tfrac{36}{36}-\tfrac{23}{36} \\ &= 1\tfrac{13}{36} \end{aligned}$$

$$\begin{aligned} & 2+\tfrac{27}{36}-\tfrac{50}{36} \\ &= 1\tfrac{36}{36}+\tfrac{27}{36}-\tfrac{50}{36} \\ &= 1\tfrac{13}{36} \end{aligned}$$

Examples in addition and subtraction may usually be curtailed by writing the common denominator once only. The answer should always be given in its lowest terms.

Example vi. *Simplify*

$$3\tfrac{5}{14}-2\tfrac{8}{35}+4\tfrac{7}{30}-1\tfrac{11}{21}.$$

The L.C.M. of the denominators is written down at once in factors.

The expression

$$\begin{aligned} &= 4+\frac{5.15-8.6+7\ \ 7-11.10}{2.7.5.3} \\ &= 4+\frac{75-48+49-110}{2.7.5.3} \\ &= 4-\frac{34}{2\ \ 7.5.3} \\ &= 4-\frac{17}{105} \\ &= 3\frac{88}{105}. \end{aligned}$$

Oral Examples.

State the common denominator in the following, and then simplify the expressions:

1. $\frac{2}{3}-\frac{1}{3}$;
2. $\frac{5}{7}+\frac{1}{7}$;
3. $\frac{3}{5}-\frac{1}{5}+\frac{2}{5}$;
4. $\frac{5}{49}+\frac{17}{49}-\frac{6}{49}$;
5. $\frac{1}{2}+\frac{1}{3}$;
6. $\frac{1}{2}-\frac{1}{5}$;
7. $\frac{2}{3}+\frac{1}{2}$;
8. $\frac{2}{3}-\frac{1}{2}$;
9. $\frac{3}{10}+\frac{5}{15}$;
10. $\frac{3}{10}-\frac{4}{15}$;
11. $\frac{5}{6}-\frac{2}{3}$;
12. $\frac{5}{6}+\frac{1}{2}-\frac{2}{3}$;
13. $\frac{4}{9}+\frac{5}{6}-1\frac{1}{18}$;
14. $\frac{2}{7}+\frac{3}{14}-\frac{1}{2}$;
15. $1\frac{1}{2}-\frac{2}{3}$;
16. $2\frac{1}{2}-1\frac{1}{3}$;
17. $3\frac{5}{9}-2\frac{1}{3}$;
18. $4\frac{4}{15}-2\frac{1}{5}$;

19. $1\frac{1}{2}+2\frac{2}{3}+3\frac{3}{4}$;
20. $1\frac{1}{2}+3\frac{3}{4}-2\frac{2}{3}$;
21. $2\frac{1}{3}+3\frac{1}{6}-4\frac{1}{2}$;
22. $5\frac{1}{7}-2\frac{1}{14}$;
23. $5\frac{1}{14}-2\frac{1}{7}$;
24. $\frac{1}{5}+\frac{2}{5}-\frac{3}{10}$;
25. $\frac{2}{13}-\frac{3}{26}$;
26. $\frac{1}{13}+\frac{1}{26}+\frac{1}{39}$;
27. $2\frac{4}{23}-1\frac{7}{69}$;
28. $\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}$;
29. $\frac{1}{3}-\frac{1}{9}+\frac{1}{27}$;
30. $1\frac{1}{3}-2\frac{2}{9}+3\frac{5}{27}$.

Examples VI b.

Simplify:

1. $\frac{3}{25}-\frac{4}{105}+\frac{4}{35}-\frac{4}{75}$;
2. $\frac{3}{14}+\frac{5}{52}-\frac{17}{91}$;
3. $\frac{2}{3}+\frac{1}{24}+\frac{1}{120}+\frac{1}{720}$;
4. $\frac{3}{4}-\frac{1}{3}+\frac{7}{10}-\frac{11}{15}$;
5. $\frac{2}{5}+\frac{3}{7}-\frac{23}{70}+\frac{4}{15}$;
6. $3\frac{3}{8}+5\frac{5}{6}-6\frac{4}{15}$;
7. $6\frac{3}{28}+\frac{19}{35}-5\frac{19}{30}$;
8. $2\frac{1}{5}+3\frac{1}{15}+1\frac{1}{16}+\frac{8}{45}$;
9. $2\frac{5}{12}-1\frac{1}{3}+\frac{1}{6}$;
10. $3\frac{5}{16}+2\frac{1}{32}-4\frac{19}{64}$;
11. $2\frac{13}{15}-\frac{11}{35}+3\frac{6}{7}-6\frac{2}{5}$;
12. $7\frac{5}{12}-6\frac{14}{15}+1\frac{7}{20}$;
13. $5\frac{1}{14}-2\frac{13}{21}+1\frac{5}{28}$;
14. $2\frac{3}{28}+1\frac{5}{12}+5\frac{2}{21}+\frac{5}{56}$;
15. $\frac{61}{77}+\frac{24}{91}-\frac{8}{143}$;
16. $13\frac{1}{16}+17\frac{16}{21}-26\frac{17}{24}-2\frac{3}{56}$;
17. $4\frac{2}{15}+3\frac{11}{20}-2\frac{1}{12}-3\frac{7}{18}$;
18. $\frac{107}{231}-\frac{43}{44}+\frac{27}{28}$;
19. $\frac{64}{65}-1\frac{91}{117}+1\frac{41}{45}+2\frac{17}{39}$;
20. $19\frac{47}{105}-17\frac{43}{140}+27\frac{23}{60}-28\frac{65}{84}$;
21. $1\frac{23}{35}-2\frac{48}{77}+3\frac{11}{70}$;
22. $4\frac{5}{42}+6\frac{23}{91}+11\frac{51}{52}+1\frac{23}{156}$;
23. $3-\frac{3}{20}-\frac{4}{15}-\frac{7}{12}$;
24. $5+\frac{4}{7}-2\frac{2}{3}+\frac{1}{14}$;
25. $2\frac{2}{11}-5\frac{5}{7}+7\frac{7}{22}-3\frac{3}{14}$;
26. $\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5}+\frac{1}{6}+\frac{1}{7}$;
27. $\frac{2}{3}-\frac{3}{4}+\frac{4}{5}-\frac{5}{6}+\frac{6}{7}$;
28. $\frac{5}{7}-\frac{3}{14}+1\frac{2}{21}-\frac{5}{6}$;
29. $4\frac{2}{13}-2\frac{1}{5}+3\frac{1}{10}-2\frac{1}{39}$;
30. $1\frac{5}{56}-3\frac{2}{63}+8\frac{3}{28}$;
31. $(5\frac{1}{3}+2\frac{1}{4})-(3\frac{1}{7}+4\frac{1}{9})$;
32. $(2\frac{2}{13}-1\frac{1}{26})-(5\frac{4}{39}-4\frac{1}{26})$;
33. $(\frac{1}{3}+\frac{1}{4}+\frac{1}{5})-(\frac{1}{6}+\frac{1}{7}+\frac{1}{8})$;
34. $\frac{1}{2}+\frac{1}{6}+\frac{1}{24}+\frac{1}{120}+\frac{1}{720}$;
35. $\frac{40666}{92002}-\frac{43669}{100023}$;
36. $\frac{5}{6}+\frac{15}{16}+\frac{25}{26}+\frac{35}{36}+\frac{55}{56}$;
37. $3\frac{7}{8}+1\frac{13}{14}+9\frac{31}{42}$;
38. $1\frac{7}{15}-2\frac{1}{24}+1\frac{9}{20}+7\frac{3}{50}$;
39. $(3\frac{1}{10}-1\frac{4}{15})-(2\frac{4}{5}-1\frac{2}{3})$;
40. $2\frac{7}{8}-3\frac{5}{6}+8\frac{5}{24}-5\frac{11}{12}$.

41. A man leaves his property to his three sons, so that the eldest receives $\frac{9}{21}$ of the whole and the second son $\frac{5}{14}$ of the whole; what fraction does the youngest son receive?

42. Three men, *A*, *B*, and *C*, enter into partnership. *A* supplies $\frac{57}{119}$ of the capital and *B* $\frac{29}{136}$; what fraction does *C* supply?

43. A man pays $\frac{3}{80}$ of his income as income tax and $\frac{1}{6}$ as rent;

he spends $\frac{1}{18}$ on rates, $\frac{1}{6}$ on clothes, $\frac{1}{8}$ on insurance. He saves $\frac{1}{10}$ of his income; what fraction does he spend on other expenses?

44. Three robbers found a bag of sovereigns; the first took $\frac{17}{50}$ of the contents, the second $\frac{16}{49}$, and the third $\frac{11}{35}$. If 47 sovereigns were left, how much did each robber get?

45. A man on a journey travelled $\frac{10}{33}$ of the distance by rail, $\frac{19}{55}$ by road, and the rest by river. What fraction did he travel by river?

46. *A* spends £5 in 7 days, *B* £7 in 9 days, and *C* £57 in 63 days. Which of them spends most in 1 day, and which least?

47. A tradesman buys goods at 84*s.* per 100 and sells at £6 per gross. Does he gain or lose?

48. One train travels 149 miles in 3 hrs. 15 min., another travels 316 miles in 7 hrs. Which is the faster train?

MULTIPLICATION OF FRACTIONS

(**99.**) Multiplication of a fraction by a whole number.

Example i. *Multiply* $\frac{3}{5}$ *by* 4.

$$\text{Product} = \frac{3}{5} \times 4$$

$$[= 3 \text{ fifths} \times 4]$$

$$[= (3 \times 4) \text{ fifths}]$$

$$= \frac{3 \times 4}{5}$$

$$= 2\frac{2}{5}.$$

or

$$\text{Product} = \frac{3}{5} + \frac{3}{5} + \frac{3}{5} + \frac{3}{5}$$

$$= \frac{3+3+3+3}{5}$$

$$= \frac{3 \times 4}{5}$$

$$= 2\frac{2}{5}.$$

Example ii. *Multiply* $\frac{3}{8}$ *by* 4.

$$\text{Product} = \tfrac{3}{8} \times 4$$

$$[= 3 \text{ eighths} \times 4]$$

$$[= (3 \times 4) \text{ eighths}]$$

$$= \tfrac{3 \times 4}{8}$$

$$= \tfrac{3}{2} \text{ on reducing to lowest terms.}$$

Division of a fraction by a whole number.

Example i. *Divide* $\frac{8}{9}$ *by* 4.

Quotient $= \frac{8}{9} \div 4$
$[= 8 \text{ ninths} \div 4]$
$[= (8 \div 4) \text{ ninths}]$
$= \frac{2}{9}$

Example ii. *Divide* $\frac{7}{9}$ *by* 4.

Quotient $= \frac{7}{9} \div 4$
$= \frac{7 \times 4}{9 \times 4} \div 4$
$= \frac{7}{9 \times 4}$ (by work as in Example i)
$= \frac{7}{36}$

Oral Examples.

1. Multiply each of the fractions (i) to (viii) by each of the whole numbers (*a*) to (*h*).

(i) $\frac{2}{3}$; (v) $\frac{15}{19}$; (*a*) 2; (*e*) 7;
(ii) $\frac{4}{7}$; (vi) $\frac{8}{5}$; (*b*) 3; (*f*) 9;
(iii) $\frac{8}{15}$; (vii) $2\frac{1}{3}$; (*c*) 5; (*g*) 10;
(iv) $\frac{3}{10}$; (viii) $\frac{16}{27}$; (*d*) 6; (*h*) 12.

2. State two rules for multiplying a fraction by a whole number.

3. Divide each of the fractions (i) to (viii) in Question 1 by each of the whole numbers (*a*) to (*h*).

4. State two rules for dividing a fraction by a whole number.

Find the value of:

5. (*a*) $\frac{3}{7} \times 7 \div 9$; (*b*) $\frac{3}{7} \div 9 \times 7$;
6. (*a*) $\frac{4}{5} \times 3 \div 7$; (*b*) $\frac{4}{5} \div 7 \times 3$;
7. (*a*) $\frac{3}{11} \times 4 \div 5$; (*b*) $\frac{3}{11} \div 5 \times 4$;
8. (*a*) $\frac{5}{18} \times 4 \div 6$; (*b*) $\frac{5}{18} \div 6 \times 4$.

9. State any fact suggested by Examples 5 to 8.

Oral Examples.

Illustrating the use of the word 'of'.

1. If a basket contains 50 apples, how many apples are there in 4 of them?

How many apples are there in $\frac{1}{2}$ of a basket?

How many apples are there in $\frac{1}{5}$ of a basket?

2. If a lb. of tea costs 1*s.* 6*d.*, what is the cost of 2 of the lb.?

What is the cost of $\frac{1}{2}$ of a lb.? What is the cost of $\frac{2}{3}$ of a lb.?

3. If a gallon of water weighs 10 lb., what is the weight of 2 of such gallons?

What is the weight of $\frac{1}{5}$ of a gallon? What is the weight of $\frac{3}{5}$ of a gallon? What is the weight of $\frac{5}{8}$ of a gallon?

4. A cake lasts a family of 8 for 3 days. How much of the cake is eaten each day? What fraction of the cake does each person eat on each day?

If the cake is first cut on Sunday, and two of the family are away on Monday, how much of the cake will be eaten on Monday if they each have a slice of the same size as before?

How big would the slice be if they eat the same total amount on Monday as on Sunday?

5. A boy has 120 marbles. He gives away half of this quantity to another boy, and the next day this boy gives him back $\frac{1}{3}$ of the number he was given. What fraction of the original number did the first boy get back?

6. It is half a mile from my house to the station. If I run $\frac{1}{3}$ of the way and walk the rest, what fraction of a mile do I run?

What fraction of a mile do I walk?

7. A sum of money is divided equally amongst 20 groups of people. What fraction of the whole sum will each group get?

If there are 30 people in each group, what fraction of the whole amount will each person get?

If the original amount was £300, how much will each person get?

8. A man leaves his money to his widow and three sons. His widow gets half of the money and the three sons divide the rest equally.

What fraction of the whole amount will each son get?

If the widow gets $\frac{1}{2}$ of the money and the eldest son $\frac{1}{3}$ of the money, what fraction is left for division between the other two sons?

What fraction will each of these two sons get?

(**100.**) If the above oral examples have been carefully studied, the following points will probably have suggested themselves:

(1) The word 'of' usually implies an instruction to multiply;

(2) Multiplication by $\frac{1}{2}$ is equivalent to division by 2;

(3) Multiplication by $\frac{2}{3}$ is equivalent to the double process of multiplication by 2 and division by 3.

The explanation of the process of multiplication given in Chapter I deals with whole numbers only, and has no meaning when applied to multiplication by a fraction. We must therefore give a new meaning to multiplication, taking care that this new meaning includes the case of multiplication by whole numbers as well as multiplication by fractions.

Careful study of the oral examples in the previous section shows that, in the case of whole numbers, the word 'of' implies a direction to multiply. Let us take this as a new explanation of multiplication, so that, for instance, the direction to multiply a quantity by $\frac{3}{5}$ means that we are to find the value of $\frac{3}{5}$ of that quantity. Notice that the quantity may be either a whole number or fraction or may be a concrete quantity. To find $\frac{3}{5}$ (i.e. 3 fifths) of a quantity we divide by 5, thus obtaining one fifth, and then multiply by 3. Hence it follows from the new meaning of multi-

plication that, in order to multiply by $\frac{3}{5}$, we divide by 5 and multiply by 3.

The definition of multiplication often given is as follows: to multiply by a number, either a whole number or fraction, we do to the multiplicand exactly the same as we do to unity in order to obtain the multiplier. For instance, what do we do to unity to obtain $\frac{3}{5}$? We divide the whole, or unity, into 5 equal parts and take 3 of them, i. e. we divide by 5 and multiply by 3. Therefore in order to multiply any quantity by $\frac{3}{5}$ we divide the quantity by 5 and multiply the result by 3.

Similar arguments can be applied to any fraction.

Example 1. *Multiply*

(a) 4 *by* $\frac{5}{7}$; (b) $\frac{5}{7}$ *by* 4.

(a) Product $= \frac{5}{7}$ of 4
$= 5$ times $\frac{4}{7}$ by definition of a fraction
$= \frac{5\times 4}{7}$ by § 99
$= \frac{20}{7} = 2\frac{6}{7}$.

(b) Product $= \frac{5}{7}\times 4$
$= \frac{5\times 4}{7}$ by § 99
$= 2\frac{6}{7}$.

Example 11. *Multiply*

(a) z *by* $\frac{x}{y}$; (b) $\frac{x}{y}$ *by* z.

(a) Product $= \frac{x}{y}$ of z
$= x$ times $(z \div y)$
$= x$ times $\frac{z}{y}$
$= \frac{x\times z}{y}$.

(b) Product $= \frac{x}{y}\times z$
$= \frac{x\times z}{y}$. by § 99

Example iii. *Multiply*

$$(a)\ \tfrac{2}{3}\ by\ \tfrac{5}{7}; \qquad (b)\ \tfrac{5}{7}\ by\ \tfrac{2}{3}.$$

(a)	(b)	
Product $= \frac{5}{7}$ of $\frac{2}{3}$	Product $= \frac{2}{3}$ of $\frac{5}{7}$	
$= 5$ times $(\frac{2}{3} \div 7)$	$=$ twice $(\frac{5}{7} \div 3)$	
$= 5$ times $\frac{2}{3 \times 7}$	$=$ twice $\frac{5}{7 \times 3}$	by § 99
$= \frac{5 \times 2}{3 \times 7}$	$= \frac{2 \times 5}{7 \times 3}$	by § 99
$= \frac{10}{21}$.	$= \frac{10}{21}$.	

A similar argument can be applied to any fractions, so that

$$\frac{a}{b} \times \frac{x}{y} = \frac{ax}{by} \quad \text{and} \quad \frac{x}{y} \times \frac{a}{b} = \frac{ax}{by}.$$

The same process can be employed with any number of fractions; hence we have the following facts.

(i) **The product of any number of fractions is a fraction whose numerator is the product of the numerators and whose denominator is the product of the denominators. This product should be reduced to its lowest terms.**

(ii) **When several numbers are multiplied together, the product is the same in whatever order they are multiplied, whether the factors are fractions or whole numbers.** (See § 32.)

Example iv. *Find the value of* $\frac{3}{16} \times \frac{4}{15} \times \frac{10}{21}$.

$$\begin{aligned} \text{Product} &= \frac{3 \times 4 \times 10}{16 \times 15 \times 21} \\ &= \frac{3 \times 4 \times 2 \times 5}{4 \times 4 \times 3 \times 5 \times 3 \times 7} \\ &= \frac{1}{2 \times 3 \times 7} \\ &= \frac{1}{42}. \end{aligned}$$

When the reasons underlying the multiplication are quite understood, the work may be arranged as follows:

$$\text{Product} = \frac{\overset{1}{\cancel{3}}}{\underset{4}{\cancel{16}}\ \underset{2}{}} \times \frac{1}{\underset{5}{\cancel{15}}} \times \frac{\overset{1}{\cancel{2}}}{\cancel{10}\ 21}$$

$$= \frac{1}{2 \times 21}$$

$$= \frac{1}{42}.$$

Example v. *Simplify* $3\frac{1}{7} \times 4\frac{2}{3} \times \frac{6}{11}$.

In multiplication a mixed number should usually be reduced to an improper fraction.

$$\text{Product} = \frac{\overset{2}{\cancel{22}}}{\cancel{7}} \times \frac{\overset{2}{\cancel{14}}}{\cancel{3}} \times \frac{\overset{2}{\cancel{6}}}{\cancel{11}}$$

$$= 8.$$

Example vi. *Multiply* $17\frac{2}{3}$ *by* $8\frac{1}{5}$.

The following method may be used:

$$\begin{array}{r} 17\frac{2}{3} \\ 8\frac{1}{5} \\ \hline 141\frac{1}{3} \qquad 17\frac{2}{3} \times 8 \\ 3\frac{8}{15} \qquad 17\frac{2}{3} \div 5 \\ \hline 144\frac{13}{15} \end{array}$$

(101.) We now give simple geometrical illustration of the multiplication of fractions. (Remember that the word 'of' implies multiplication.)

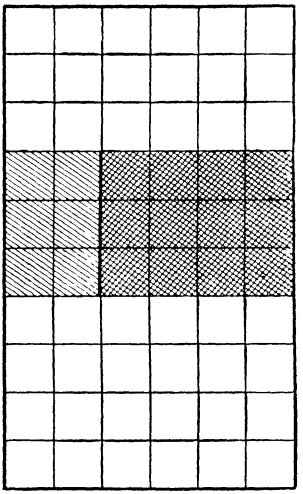


Fig. 7.

In Fig. 7 the shaded portion $= \frac{3}{10}$ of the whole (since it contains 3 rows out of a total of 10 rows). The doubly shaded portion $= \frac{12}{60}$ (or $\frac{1}{5}$) of the whole (since it contains 12 squares out of a total of 60 squares). But the doubly shaded portion is $\frac{4}{6}$ of the whole shaded portion.

$\therefore$ $\frac{4}{6}$ of $\frac{3}{10}$ of the whole $= \frac{12}{60}$ (or $\frac{1}{5}$) of the whole.

Examples.

Draw figures to illustrate the following facts

(i) $\frac{2}{5} \times \frac{3}{7} = \frac{6}{35}$; (ii) $\frac{3}{11} \times \frac{2}{3} = \frac{2}{11}$;

(iii) $\frac{4}{7} \times \frac{7}{9} = \frac{4}{9}$; (iv) $\frac{2}{3} \times \frac{6}{13} = \frac{4}{13}$.

Oral Examples.

Simplify the following:

1. $\frac{2}{3}$ of $\frac{3}{5}$;	**2.** $\frac{1}{2}$ of $\frac{2}{7}$;	**3.** $\frac{3}{5}$ of $\frac{5}{7}$;
4. $\frac{2}{3}$ of $\frac{4}{5}$;	**5.** $\frac{7}{9}$ of $\frac{8}{11}$;	**6.** $\frac{1}{3} \times \frac{5}{8}$;
7. $\frac{1}{2} \times \frac{2}{3} \times \frac{3}{4}$;	**8.** $\frac{1}{2} \times \frac{4}{5} \times \frac{5}{6}$;	**9.** $\frac{2}{5} \times \frac{10}{11} \times \frac{22}{23}$;
10. $\frac{3}{8} \times \frac{1}{3} \times \frac{4}{9}$;	**11.** $\frac{5}{7} \times \frac{14}{15}$;	**12.** $\frac{3}{7} \times \frac{14}{15} \times \frac{5}{8}$;
13. $\frac{1}{2} \times \frac{11}{12} \times \frac{6}{55}$;	**14.** $\frac{2}{13} \times \frac{26}{34} \times \frac{17}{18}$;	**15.** $\frac{3}{4}$ of $\frac{4}{5}$ of $\frac{15}{16}$.
16. $1\frac{1}{2} \times 2\frac{2}{3}$;	**17.** $3\frac{1}{2} \times 1\frac{1}{7}$;	**18.** $2\frac{1}{3} \times 2\frac{1}{7}$;
19. $\frac{1}{2}$ of $3\frac{1}{5}$;	**20.** $\frac{2}{3}$ of $1\frac{1}{2}$;	**21.** $3\frac{1}{7} \times 1\frac{3}{11}$;
22. $4\frac{1}{5} \times 2\frac{1}{7}$;	**23.** $3\frac{5}{6} \times 1\frac{1}{23}$;	**24.** $2\frac{1}{10} \times 3\frac{4}{7}$.

Notice carefully the effect of multiplying a fraction by its denominator or a multiple of the denominator:

25. $\frac{1}{2} \times 2$; **26.** $\frac{3}{7} \times 7$; **27.** $\frac{14}{51} \times 51$;
28. $\frac{3}{5} \times 10$; **29.** $\frac{4}{9} \times 36$; **30.** $\frac{16}{47} \times 94$;
31. $\frac{105}{4783} \times 4783$; **32.** $\frac{31}{55} \times 550$; **33.** $\frac{13}{111} \times 999$.

Examples VI c.

Find the value of:

1. $2\frac{1}{2} \times 15\frac{5}{7}$; **2.** $\frac{12}{13} \times 2\frac{1}{6}$;
3. $2\frac{1}{3} \times \frac{5}{28} \times 1\frac{11}{25}$; **4.** $1\frac{1}{12} \times 1\frac{13}{23} \times 1\frac{10}{13}$;
5. $1\frac{52}{221} \times \frac{425}{875}$; **6.** $1\frac{1}{5} \times 1\frac{1}{6} \times 1\frac{1}{7} \times 1\frac{1}{8} \times 1\frac{1}{9}$;
7. $\frac{35}{66} \times 1\frac{13}{35} \times 2\frac{3}{4}$; **8.** $8\frac{7}{17} \times 6\frac{5}{19}$;
9. $\frac{13}{33} \times \frac{51}{52} \times 4\frac{12}{19}$; **10.** $\frac{124}{327} \times 1\frac{845}{899}$;
11. $6\frac{9}{17} \times \frac{4}{21} \times 10\frac{21}{37}$; **12** $17\frac{1}{7} \times \frac{8}{15} \times 6 \times \frac{5}{48}$;
13. $\frac{3}{4}$ of $(\frac{1}{2}+\frac{1}{3})-\frac{5}{8}$ of $(\frac{1}{2}-\frac{1}{3})$; **14.** $\frac{7}{9}$ of $(4\frac{2}{7}-3\frac{1}{5})$;
15. $\frac{13}{111}$ of $4\frac{1}{9}-\frac{1}{8}$ of $2\frac{2}{3}$; **16.** $1\frac{45}{697} \times \frac{287}{627} \times \frac{741}{742}$;
17. $1\frac{186}{299} \times \frac{207}{679}$; **18.** $2\frac{1}{2} \times \frac{11}{43} \times 3\frac{2}{3} \times 6\frac{5}{11} \times 4\frac{3}{10}$.

19. Justify the statement that $\frac{3}{5} \times \frac{5}{7} = \frac{3}{7}$.

20. Illustrate geometrically the fact that $\frac{2}{3}$ of $\frac{3}{4} = \frac{1}{2}$.

21. A property of 100 acres is divided between A, B, and C, so that A receives $\frac{1}{3}$, and B receives $\frac{2}{5}$ of the remainder. How much does C receive?

22. A property is divided between A, B, and C, so that A receives $\frac{1}{3}$, B receives $\frac{2}{5}$ of the remainder, and C receives 25 acres. How much does B receive?

23. A man gives $\frac{4}{21}$ of his possessions to his eldest son and $\frac{3}{5}$ of this quantity to each of his other two sons; what fraction does he keep for himself?

24. If $\frac{2}{5}$ of a quantity of coal weighs 16 tons, what is the weight of the whole?

25. A post in the middle of a pond shows $\frac{1}{4}$ of its length above the water, $\frac{1}{3}$ of its length is stuck in the mud at the bottom, and 5 feet is immersed in water. What is the total length of the post?

DIVISION BY A FRACTION

(**102.**) Consider the meaning of

$$\tfrac{5}{9} \div \tfrac{3}{7}$$

This gives us instructions to find either

(1) How many times $\frac{3}{7}$ is contained in $\frac{5}{9}$,

or (2) By what $\frac{3}{7}$ must be multiplied to produce $\frac{5}{9}$.

(i) Suppose we had a similar case in dealing with concrete quantities.

How many times is £3 contained in 7 guineas?

We have to bring both quantities to the same *denomination* before we can divide

Thus £3 = 60 shillings.
7 gs. = 147 shillings.

$\therefore$ The number required $= \frac{147}{60} = 2\frac{27}{60} = 2\frac{9}{20}$.

Similarly, to find how many times $\frac{3}{7}$ is contained in $\frac{5}{9}$, we bring them both to the same *denominator*. (This is 7×9.)

$\frac{5}{9} = (5 \times 7)$ sixty thirds.
$\frac{3}{7} = (3 \times 9)$ sixty thirds.

$\therefore$ The result of the division is $\frac{5 \times 7}{3 \times 9}$.

Notice that this is the same as $\frac{5}{9} \times \frac{7}{3}$.

(ii) From the second point of view it is easy to see that

$$\tfrac{3}{7} \times \tfrac{7 \times 5}{3 \times 9} = \tfrac{5}{9}.$$

.. The number we require is, as before,

$$\tfrac{7 \times 5}{3 \times 9}.$$

The student will notice that either of these points of view suggests that **division by a fraction is equivalent to multiplication by a new fraction which is formed by inverting the divisor.**

In algebraical symbols

$$\frac{a}{b} \div \frac{c}{d}$$

$$= \frac{ad}{bd} \div \frac{bc}{bd} \quad \text{(they now have both the same denominator)}$$

$$= \frac{ad}{bc} = \frac{a}{b} \times \frac{d}{c}.$$

Example: $\frac{8}{17} \div \frac{2}{51} = \frac{8}{17} \times \frac{51}{2} = 12$

This means either (1) $\frac{2}{51} \times 12 = \frac{8}{17}$
or (2) $\frac{2}{51}$ of anything is contained in $\frac{8}{17}$ of the same thing twelve times.

(**103.**) **Note.** We have used the word 'of' and the sign of multiplication '$\times$' as meaning the same thing. As a matter of fact there is a distinction which, though slight, leads to occasional difficulty in examination questions.

$$\frac{2}{3} \text{ of } \frac{3}{5} \quad \text{and} \quad \frac{2}{3} \times \frac{3}{5}$$

both produce the same result

The first method of writing gives us instructions to take 'two-thirds of three-fifths', or, in other words, to multiply $\frac{3}{5}$ by $\frac{2}{3}$.

The second method instructs us to multiply $\frac{2}{3}$ by $\frac{3}{5}$. The distinction is, then, that when we write $\frac{2}{3}$ of $\frac{3}{5}$, we regard $\frac{2}{3}$ as the 'operator'; but when we write $\frac{2}{3} \times \frac{3}{5}$ we regard $\frac{3}{5}$ as the 'operator'.

The following example shows the difficulty which may arise:

$$\frac{3}{5} \div \frac{2}{3} \times \frac{7}{6}$$

means divide $\frac{3}{5}$ by $\frac{2}{3}$, and then multiply by $\frac{7}{6}$.

The result is $\frac{3}{5} \times \frac{3}{2} \times \frac{7}{6}$ or $\frac{21}{20}$.

But $\frac{3}{5} \div \frac{2}{3}$ of $\frac{7}{6}$ means $\frac{3}{5}$ is to be divided by $\frac{2}{3}$ of $\frac{7}{6}$ or $\frac{7}{9}$.

$\therefore$ The result is $\frac{3}{5} \times \frac{9}{7}$ or $\frac{27}{35}$.

In these cases the student is advised to read the English sentence expressed by the symbols, and to get at its full meaning before starting to work.

Examples VI d.

1. How many times is $1\frac{2}{3}$ contained in $4\frac{1}{8}$?
2. What must $3\frac{1}{8}$ be multiplied by to give $2\frac{1}{2}$ as the product?
3. Simplify $\frac{4}{5} \times \frac{5}{6} \div 1\frac{1}{6}$.
4. Divide $3\frac{5}{21}$ by $4\frac{1}{4}$.
5. Simplify $4\frac{2}{3} \times 5\frac{4}{7} \div 3\frac{1}{4}$.
6. What must $\frac{3}{5}$ of $7\frac{2}{3}$ be multiplied by in order that the result may be $4\frac{3}{5}$?

Simplify:

7. $\frac{171}{187} \times \frac{153}{209} \div \frac{81}{21}$;
8. $1\frac{6}{13} \times \frac{26}{57} \div 1\frac{1}{2}$;
9. $3\frac{1}{8} \times 2\frac{2}{5} \times 1\frac{5}{9} \div 2\frac{1}{3}$;
10. $1\frac{2}{15} \times 1\frac{11}{34} \div 1\frac{1}{2}$;
11. $7\frac{16}{17} \times 10\frac{1}{5} \div 2\frac{10}{13}$;
12. $4\frac{5}{8} \times 1\frac{11}{13} \div 1\frac{1}{26}$;
13. $\frac{3}{5} \div 1\frac{52}{221}$;
14. $\frac{255}{437} \div \frac{51}{368}$;
15. $2\frac{3}{5} \times 1\frac{3}{26} \div 1\frac{1}{57} \times \frac{4}{19}$;
16. $2\frac{3}{5}$ of $1\frac{3}{26} \div 1\frac{1}{57}$ of $\frac{4}{19}$;
17. $2\frac{7}{10} \times 2\frac{7}{9} \div \frac{5}{8}$;
18. $1\frac{1}{36}$ of $6\frac{1}{4} \div 11\frac{9}{16}$ of $\frac{20}{21}$;
19. $3\frac{3}{4} \div 3\frac{1}{8} \times \frac{15}{16}$;
20. $1\frac{2}{3} \times 3\frac{3}{8} \div \frac{15}{16}$;
21. $3\frac{3}{4} \div 3\frac{1}{8}$ of $\frac{15}{16}$;
22. $10\frac{5}{187} \times 2\frac{11}{20} \div 74\frac{5}{11}$;
23. $6\frac{1}{7}$ of $10\frac{1}{2} \div 2\frac{1}{10}$ of $7\frac{1}{6}$;
24. $\dfrac{2\frac{1}{5} \times 1\frac{3}{23}}{\frac{3}{7} \times 1\frac{2}{17}} \div \dfrac{1\frac{2}{5} \times 1\frac{1}{19}}{\frac{3}{13} \times 1\frac{6}{17}}$;
25. $\dfrac{\frac{4}{5} \text{ of } 1\frac{2}{7} \div 3\frac{7}{15}}{7\frac{1}{4} \div 4\frac{1}{7} \text{ of } \frac{2}{21}}$;
26. $\dfrac{\frac{5}{6} \text{ of } \frac{9}{10} \div \frac{7}{8}}{\frac{3}{4} \div \frac{9}{16} \text{ of } \frac{12}{21}}$.

27. If a piece of work can be finished in 5 days, how much can be done in one day? How much in 3 days?

28. If A can do a piece of work in 5 days and B can do the same work in 6 days, how much will they do in a day when they work together? How long will they take to finish the work?

29. If a man can dig a ditch in 7 days and a boy can dig a ditch of the same length in 12 days, how long would the man and boy take if they work together?

30. If $2\frac{7}{8}$ tons are worth £17 5*s*., what is the value of 1 ton?

31. A man owns $\frac{1}{15}$ of a ship; if his share is worth £215 12*s*., what is the value of the ship?

32. A sum of money is divided between a brother and sister so that the brother's share is $\frac{9}{16}$ of the whole sum. If the sister receives £196, how much will the brother receive?

33. A can give B 100 yards start in a mile race and then finish level. If they run at the same rate throughout, how much does A gain in every yard he runs? How far will B have run when A has run 400 yards?

34. A can beat B by 100 yards in a mile race, and can beat C by 20 yards in 400 yards If they all start level, how far have B and C each run when A has run 400 yards? By how much would C beat B in a mile race?

(**104.**) In dealing with more complicated fractions, the following points should be noted. They are not to be regarded as laws to be learnt, but as indications of the general way in which arithmetical symbols are to be interpreted.

(1) Any expression enclosed in a bracket must be reduced to its simplest form before the bracket is removed.

(2) **The work should always be written as a connected narrative.** Small portions of the expression to be simplified must not be picked out and simplified down in an odd corner of the paper.

(3) It is sometimes useful to multiply both the numerator and denominator of a fraction by some number.

(4) When the signs $+$, $-$, $\times$, $\div$ are mixed, the signs of multiplication and division take precedence, i. e. the operations indicated by these signs must be performed first.

(5) The word 'of' is usually equivalent to the insertion of a bracket round the two fractions which it connects. (See note, p. 167.)

These points are illustrated by the following examples:

Example i. *Simplify* $\dfrac{14\frac{3}{4}+12\frac{1}{3}}{10\frac{17}{18}-9\frac{7}{8}} \div \dfrac{14\frac{2}{7}-2\frac{3}{11}}{8\frac{3}{13}-6\frac{2}{3}}$.

The expression

$$= \frac{26+\frac{9+4}{12}}{1+\frac{68-63}{9\times 8}} \div \frac{12+\frac{22-21}{11\times 7}}{2+\frac{9-26}{13\times 3}}$$

$$= \frac{27\frac{1}{12}}{1\frac{5}{72}} \div \frac{12\frac{1}{77}}{2-\frac{17}{39}}$$

$$= \frac{\frac{325}{12}}{\frac{77}{72}} \div \frac{\frac{925}{77}}{\frac{61}{39}}$$

$$= \frac{325\times 6}{77} \div \left(\frac{925}{77}\times\frac{39}{61}\right)$$

Here we multiply numerator and denominator of the first fraction by 72

It is permissible to cross out factors which divide both numerator and denominator, but great care must be exercised to keep the work neat.

$$= \frac{\cancel{325}^{\,5} \times \cancel{6}^{\,2}}{\cancel{77}} \times \frac{\cancel{77}}{\cancel{925}_{\,185}} \times \frac{61}{\cancel{39}_{\,13}}$$

$$= \frac{122}{37}$$

$$= 3\frac{11}{37}.$$

Example ii. *Simplify* $\dfrac{2\frac{1}{3}+\frac{3}{4}}{15\frac{1}{2}-1\frac{5}{8}}+1\frac{2}{3}$ of $2\frac{4}{15}$.

The expression

$$= \frac{2\frac{13}{12}}{14-\frac{1}{8}} + \frac{5}{3} \text{ of } \frac{34}{15}$$

$$= \frac{\frac{37}{12}}{\frac{111}{8}} + \frac{34}{9}$$

$$= \frac{37\times 2}{111\times 3} + \frac{34}{9}$$

(Multiplying the numerator and denominator of the left-hand fraction by 24—the L C M of 12 and 8.)

$$= \frac{2}{9} + \frac{34}{9}$$

$$= \frac{36}{9}$$

$$= 4.$$

Example iii. *Simplify* $2 - \cfrac{4}{5 - \cfrac{9}{7 - \cfrac{16}{9}}}$.

The expression $= 2 - \cfrac{4}{5 - \cfrac{81}{63 - 16}}$

The numerator and denominator of $\cfrac{9}{7 - \cfrac{16}{9}}$ are each multiplied by 9

$= 2 - \cfrac{4}{5 - \cfrac{81}{47}}$

$= 2 - \cfrac{4 \times 47}{235 - 81}$

The numerator and denominator of the right-hand fraction are each multiplied by 47.

$= 2 - \cfrac{4 \times 47}{154}$

$= 2 - \cfrac{94}{77}$

$= \cfrac{60}{77}$.

Examples VI e

Simplify:

1. $(\frac{7}{8} + \frac{9}{10}) - (\frac{5}{16} + \frac{19}{20}) + \frac{3}{5}$ of $\frac{3}{16}$.
2. $8\frac{2}{7} \times 4\frac{1}{6} \div (2\frac{3}{14} + 1\frac{5}{21})$.
3. $11\frac{1}{2}$ of $3\frac{2}{3} - 2\frac{3}{4}$ of $5\frac{17}{33}$.
4. $(9\frac{1}{5} - 1\frac{2}{3} \times 3\frac{3}{8}) - (2\frac{3}{4} \times 1\frac{1}{6} - \frac{4}{5})$.
5. $(\frac{2}{9}$ of $\frac{3}{14} + \frac{9}{35}) \div (\frac{6}{7} - \frac{2}{3} + \frac{4}{5})$.
6. $11\frac{1}{4} - 1\frac{3}{5}$ of $1\frac{1}{16} + 1\frac{2}{5}$ of $(5\frac{4}{7} - 2\frac{3}{4})$.
7. $\cfrac{\frac{1}{2}\text{ of }\frac{5}{12}\text{ of }1\frac{3}{5}}{\frac{3}{5} + \frac{1}{6} - \frac{1}{10}}$.
8. $\cfrac{\frac{9}{10}\text{ of }\frac{8}{9} - \frac{3}{5}}{\frac{7}{10} \div \frac{14}{15} + \frac{1}{4}}$.
9. $(2\frac{4}{5} + \frac{2}{7}$ of $8\frac{3}{4}) \div (2\frac{3}{4}$ of $2\frac{8}{9} + \frac{3}{10})$.
10. $3\frac{3}{4} \div \frac{5}{24} - 2\frac{1}{3}$ of $2\frac{1}{7} - 1\frac{1}{3}$ of $5\frac{1}{4}$.
11. $\cfrac{3\frac{1}{3} - 2\frac{1}{2} + \frac{5}{12}}{3\frac{1}{3} \div (2\frac{1}{2} - \frac{5}{12})}$.
12. $\cfrac{2\frac{1}{2}}{3\frac{1}{3}} + \cfrac{3\frac{4}{5} - 2\frac{8}{9}}{3\frac{1}{3}\text{ of }4\frac{1}{5}}$.

13. $\dfrac{7\frac{1}{2}+5\frac{1}{6}-10\frac{1}{3}}{5\frac{5}{21}+7\frac{19}{63}+\frac{8}{9}-13}$.

14. $\dfrac{4\frac{1}{7}\times 1\frac{3}{15}\times 3\frac{9}{11}}{5\frac{5}{11}-4\frac{2}{5}}$.

15. $\dfrac{5\frac{9}{14}+1\frac{17}{24}}{17\frac{4}{15}-7\frac{6}{7}} \div (2-\frac{3}{64})$.

16. $7\frac{1}{8} - \dfrac{2\frac{6}{7}-1\frac{3}{5}+\frac{11}{15}}{3\frac{4}{5} \text{ of } 1\frac{5}{7}}$.

17. $(2\frac{1}{5}+4\frac{1}{3}-1\frac{1}{6}) \div (4\frac{1}{10}-2\frac{1}{5}+1\frac{1}{6})$.

18. $\dfrac{3\frac{2}{3}\times 1\frac{1}{6}}{1\frac{2}{9}\times 1\frac{3}{4}} + \dfrac{5\frac{3}{8}+4\frac{1}{4}}{5\frac{1}{4}-4\frac{3}{8}}$.

19. $\dfrac{1\frac{1}{5}+1\frac{5}{6}}{4\frac{1}{8}-3\frac{5}{16}} \div \dfrac{13\frac{1}{9}-1\frac{5}{6}}{4\frac{1}{7}}$

20. $\dfrac{4\frac{7}{12}-2\frac{11}{36}}{5\frac{13}{18}+1\frac{17}{120}} \div \dfrac{7\frac{1}{4}}{18\frac{11}{19}}$.

21. $\dfrac{7\frac{1}{4}\times 3\frac{1}{3}}{6\frac{1}{2} \text{ of } \frac{11}{13}} \div \dfrac{\frac{5}{8} \text{ of } 11}{\frac{1}{7}}$

22. $4 - \dfrac{3\frac{1}{2}-2\frac{1}{3}}{7-4\frac{2}{3}} + \dfrac{16^2-5^2}{23^2-12^2}$.

23. $\dfrac{2\frac{1}{2}-3+1\frac{7}{9}}{(2\frac{1}{2}-1\frac{1}{3})\times(2\frac{1}{4}+1\frac{7}{9})} \times 2\frac{13}{46}$

24. $\dfrac{(1\frac{1}{2}+1\frac{4}{5})\times(3\frac{1}{8}-1\frac{2}{3})}{(4\frac{1}{2}-3\frac{1}{3}) \div (3-\frac{1}{11})}$

25. $\dfrac{3\frac{2}{5}+4\frac{1}{15}}{3\frac{2}{3}-2\frac{8}{9}} \div \dfrac{3\frac{1}{7} \text{ of } 3\frac{1}{5}}{2\frac{1}{4} \text{ of } 3\frac{3}{7}}$.

26. $\dfrac{\frac{5}{7} \div 3\frac{1}{13} - \frac{3}{8} \text{ of } \frac{7}{12}}{(1-\frac{1}{2}+\frac{1}{6}) \div (1\frac{5}{6}-\frac{2}{3})}$.

27. $\dfrac{1\frac{2}{7}+2\frac{2}{3}}{5\frac{1}{2}+4\frac{1}{3}} \div \dfrac{4\frac{4}{5}-3\frac{2}{7}}{2\frac{1}{3}+6\frac{1}{2}}$.

28. $\dfrac{1\frac{5}{11}\times 1\frac{4}{7}-1\frac{4}{5}\times 1\frac{1}{4}}{1\frac{6}{7}\times 1\frac{3}{4}-2\frac{9}{10}\times 1\frac{1}{9}}$.

29. $\dfrac{5\frac{1}{2}-3\frac{3}{8}}{1\frac{1}{2}+1\frac{1}{3}} + \dfrac{1\frac{1}{8}+2\frac{5}{6}}{3\frac{1}{3}-1\frac{3}{4}}$.

30. $2 - \cfrac{3}{3-\cfrac{2\frac{1}{2}+\frac{1}{3}}{3-\frac{1}{4}}}$.

31. $2 - \cfrac{3}{9-\cfrac{7}{1+\cfrac{1}{2-\frac{3}{8}}}}$

32. $\dfrac{\frac{13}{42}\times\frac{1}{6}-\frac{11}{28}\times\frac{1}{9}}{\frac{8}{21}\times\frac{1}{6}-\frac{13}{28}\times\frac{1}{9}}$.

33. $\dfrac{1\frac{2}{3}+4\frac{3}{5}}{2\frac{2}{3}-2\frac{1}{5}} \div (1\frac{5}{8}-\frac{11}{14})$.

34. $\dfrac{\frac{5}{3}+\frac{3}{4}}{1\frac{3}{4}+2\frac{5}{12}} + \dfrac{1}{3\frac{1}{11}}$.

35. $\dfrac{15\frac{3}{4}-1\frac{4}{3}\times 1\frac{5}{6}}{\frac{1}{5}\times 23\frac{1}{3}+\frac{91}{36}}$.

36. $\dfrac{\dfrac{1}{1\frac{1}{2}}+\dfrac{1}{1\frac{1}{3}}+\dfrac{1}{1\frac{1}{4}}}{3-\dfrac{1}{1-\frac{1}{16}}+\dfrac{1}{4-\frac{8}{17}}}$.

37. $5 - \cfrac{2}{4-\cfrac{3\frac{1}{2}+\frac{2}{3}}{4-\frac{2}{3}}}$

38. $\dfrac{5\frac{1}{4}-3\frac{5}{7}-\frac{11}{12}}{4-\frac{1}{23}} \times 5\frac{2}{29} \div 207$.

39. $\dfrac{16\frac{1}{2}-11\frac{3}{4}}{2\frac{2}{3} \text{ of } 3\frac{4}{5}} - \dfrac{2\frac{13}{25}}{9\frac{1}{3} \text{ of } 1\frac{7}{55}}$.

40. $\dfrac{7\frac{51}{56}-3\frac{21}{44}-2\frac{12}{77}}{1\frac{1}{14} \div \frac{8}{15}}$.

41. $\dfrac{4\frac{1}{3} \text{ of } 5-(22\frac{1}{5}-\frac{7}{10})}{3\frac{1}{4}-\frac{4}{5} \text{ of } \frac{5}{6}}$.

42. $\frac{1}{3}+\frac{5}{6}+\frac{1}{8}+\frac{17}{20}+\frac{7}{40}+\frac{41}{60}$.

43. $\dfrac{1}{2+\dfrac{3}{4+\frac{5}{6}}}$.

44. $\dfrac{2\frac{1}{4}-1\frac{1}{2} \text{ of } \frac{5}{6}}{1\frac{3}{8} \text{ of } \frac{4}{11}+\frac{5}{6}} \div \left(1+\dfrac{1}{1+\dfrac{1}{1+\frac{1}{2}}}\right)$.

45. $\dfrac{\frac{4}{9}+\frac{3}{4}-\frac{5}{12}}{\frac{3}{5}+\dfrac{2-\frac{1}{4}}{2\frac{5}{14}+1\frac{1}{7}}}$.

46. $\dfrac{15\frac{1}{4}-\frac{3}{8}}{11\frac{1}{2}-5\frac{5}{6}} \div \left(\dfrac{1}{2+\dfrac{3}{4-\dfrac{5}{6+\frac{7}{8}}}}\right)$.

47. $\dfrac{126}{1+\dfrac{5}{\frac{5}{4}-1}}+\dfrac{124}{1+\dfrac{5}{1-\frac{5}{6}}}$.

48. $\dfrac{5\frac{1}{2} \text{ of } \frac{2}{9} \text{ of } 2\frac{4}{7}-1 \div (\frac{1}{5}+\frac{1}{2})}{1-\frac{3}{14} \text{ of } \left\{\frac{1}{2}+\frac{1}{2} \text{ of } \dfrac{\frac{1}{20}}{\frac{1}{7} \text{ of } \frac{1}{20}}\right\}}$.

49. $\dfrac{7}{9-\frac{11}{13}} \text{ of } \dfrac{1}{3-\dfrac{3}{5-\frac{2}{11}}} \text{ of } \dfrac{1}{2\frac{1}{4}-1\frac{2}{9}}$.

50. $\dfrac{4\frac{1}{4}-3\frac{1}{2}+17}{17-4\frac{1}{4}+3\frac{1}{2}} \times \dfrac{\cdot 04+23\cdot 72-17}{17-\cdot 04+23\cdot 72}$.

(Give the result as a fraction.)

51. $\dfrac{5\frac{1}{2}+2\frac{2}{3}}{4\frac{3}{5}-1\frac{7}{8}} \div \dfrac{4}{5+\dfrac{6}{7-\frac{3}{5}}}$.

52. $\dfrac{7\frac{5}{6}+\frac{3}{4}-\frac{5}{6} \text{ of } 2\frac{1}{3}-2\frac{1}{12}}{2\frac{11}{12}-\frac{2}{3} \text{ of } 2\frac{1}{3}-1\frac{1}{4}+\frac{5}{9} \text{ of } 2\frac{1}{2}}$.

53. $\dfrac{\frac{4}{5}+\frac{17}{20}-\frac{13}{45}}{\frac{7}{8} \text{ of } \frac{4}{5} \text{ of } \frac{15}{28}} \div \dfrac{\frac{8}{9} \text{ of } (\frac{7}{16}-\frac{9}{22})}{\frac{3}{7} \times \frac{8}{15} \times \frac{35}{44}}$.

54. $\left(\dfrac{15\frac{3}{4}-12\frac{1}{3}}{17\frac{1}{2}+4\frac{1}{6}} \div \dfrac{7\frac{1}{5} \text{ of } 4\frac{5}{9}}{9\frac{2}{7} \text{ of } 5\frac{2}{15}}\right) \times 13\frac{1}{11}$.

55. $\left(\dfrac{\frac{1}{10}-\frac{1}{11}}{\frac{1}{13}-\frac{1}{14}}-\dfrac{\frac{1}{11}-\frac{1}{12}}{\frac{1}{14}-\frac{1}{15}}\right) \div \left(\dfrac{\frac{1}{12}-\frac{1}{13}}{\frac{1}{15}-\frac{1}{16}}-\dfrac{\frac{1}{13}-\frac{1}{14}}{\frac{1}{16}-\frac{1}{17}}\right)$.

56. $\dfrac{\frac{1}{50}-\frac{3}{50^2}+\frac{3^2}{50^3}-\frac{1}{53}}{\frac{1}{47}-\frac{3}{50^2}-\frac{3^2}{50^3}-\frac{1}{50}}$.

(**105.**) The following set of examples involves no new principle, but a few worked-out types may be useful.

Example 1. *Express* $\frac{2}{3}+\frac{5}{7}-\frac{11}{14}$ *as a decimal, correct to five significant figures.*

(With larger denominators we should perform the divisions separately, but here it will probably involve less work if we first simplify the expression.)

$$\frac{2}{3}+\frac{5}{7}-\frac{11}{14}=\frac{28+30-33}{3.7.2}$$

$$=\frac{25}{3.7.2}$$

$$=\frac{8{\cdot}333333\ ..}{7.2}$$

$$=\frac{1{\cdot}190476\ ..}{2}$$

$$={\cdot}595238\ ..$$

$$={\cdot}59524...$$

Example 11. *Find the value of*

$$\tfrac{1}{2}+\tfrac{1}{2\ 3}+\tfrac{1}{2\ 3\ 4}+\tfrac{1}{2\ 3\ 4\ 5}+....$$

as a decimal correct to four significant figures.

(Here, since the series of fractions can be continued indefinitely, it is best to evaluate each separately.)

$\frac{1}{2}$ = ·5

$\frac{1}{2\ 3}$ = ·166666.... dividing the previous line by 3

$\frac{1}{2\ 3\ 4}$ = ·041666.... ,, ,, ,, ,, 4

$\frac{1}{2\ 3\ 4\ 5}$ = ·008333.... ,, ,, ,, ,, 5

$\frac{1}{2\ 3\ 4\ 5\ 6}$ = ·001388.... ,, ,, ,, ,, 6

$\frac{1}{2\ 3\ 4\ 5\ 6\ 7}$ = ·000198.... ,, ,, ,, ,, 7

$\frac{1}{2\ 3\ 4\ 5\ 6\ 7\ 8}$ = ·000024.... ,, ,, ,, ,, 8

Total = ·7183....

Example III. *Express* ·3125 *as a fraction in its lowest terms.*
·3125 *is* 3125 *ten-thousandths*

$= \frac{3125}{10000}$
$= \frac{12500}{40000}$ (multiplying numerator and denominator by 4)
$= \frac{50000}{160000}$,, ,, ,, ,, ,,
$= \frac{5}{16}$.

Example IV. *Find the value of*

$(2\frac{1}{3}+3\frac{1}{5})$ of $\frac{5}{7}$ of £10 10*s.*

The expression $= 5\frac{8}{15}$ of $\frac{5}{7}$ of 210 shillings
$= \frac{83}{15}$ of $\frac{5}{7}$ of 210 shillings
$= \frac{83}{21}$ of 210 shillings
= 830 shillings
= £41 10*s.*

Examples VI f.

1. Find, to four significant figures, the value of

$$\frac{1}{6} + \frac{1}{6^2} + \frac{1}{6^3} + \frac{1}{6^4} + \ldots.$$

2. Express as a decimal the sum of the fractions

$$\frac{1}{10}, \quad \frac{1}{5}, \quad \frac{3}{100}, \quad \frac{1}{25}, \quad \frac{7}{100000}, \quad \frac{1}{1250000}.$$

3. Express $\frac{4}{19}$ of $\frac{13}{37}$ as a decimal correct to three figures.

4. Find the value of $\frac{7}{8}$ of a shilling + $\frac{5}{6}$ of a sovereign.

5. Express ·4375 as a fraction.

6. Simplify $\dfrac{3\cdot86-1\cdot6+2\cdot04}{5\cdot22+2\cdot45-1\cdot42}$.

7. Calculate the value of $\dfrac{1}{2}-\dfrac{1}{2.3}+\dfrac{1}{2.3\ 4}-\dfrac{1}{2.3\ 4.5}+\ldots.$
to four significant figures.

8. Divide £6 18*s.* $11\frac{1}{2}$*d.* by $3\frac{2}{7}$.

9. Express as a decimal $\dfrac{2\frac{3}{4} \text{ of } 4\frac{1}{2}}{11\frac{1}{2}+5\frac{1}{4}-1\frac{1}{8}}$.

10. Express as a decimal $2\frac{1}{5}+3\frac{1}{3}$ of $\frac{4\cdot155}{277}$.

11. Simplify $(\cdot437\times4\cdot8)+(\cdot056\times\cdot4)$.

12. Subtract $\frac{1}{2}$ of 3*s.* 11*d.* from $\frac{3}{4}$ of 15*s.*

13. Find the value of

$\frac{6}{7}$ of a guinea $+\frac{4}{5}$ of a sovereign $+\frac{3}{4}$ of a shilling.

14. Find the value of

$1\frac{3}{5}$ of £1 $+1\frac{4}{7}$ of 10*s.* 6*d.* $+3\frac{15}{16}$ of £1 1*s.* 4*d.*

15. Find the value of

$3\frac{1}{5}$ of 5*s.* $2\frac{1}{2}d.+\frac{2}{3}$ of $\frac{1}{7}$ of 4 guineas $+$ £3·625.

16. Find the weight of $\frac{7}{1\,2\,3}$ of 91 tons 4 cwt. 15 lb.

17. Find the value of $3\frac{3}{7}$ of £3 16*s.* 5*d.* $+4\frac{4}{7}$ of £3 15*s.* 10*d.*

18. Find the number of feet in $3\frac{3}{1\,9}$ of 8 mi. 2 fur. 21 yds. 1 ft.

19. Find the value of

$\frac{1}{1\,4}$ of $(\frac{1}{1\,7}+2\frac{1}{9}+1\frac{5}{5\,1}+\frac{6}{2\,7})$ of £2 11*s.*

20. To $\frac{3}{1\,3}$ of the difference between 7*s.* 6*d.* and 2*s.* 1*d.* add $\frac{1}{5}$ of the sum of 4*s.* 8*d.* and 1*s.* 2*d.*

21. Find the value of

$$\frac{1\,0}{1\,1}\text{ of }5s.\ 6d.+\frac{2\frac{1}{3}-1\frac{7}{8}}{2\frac{7}{8}-1\frac{1}{3}}\text{ of }10s.\ 9\tfrac{1}{2}d.-\tfrac{7}{8}\text{ of }1s.$$

22. If $78\frac{3}{4}$ tons are sold for £919, find to the nearest penny the cost of a ton.

23. Find the value of $\dfrac{3\frac{6}{7}}{7\frac{4}{5}\text{ of }\frac{3}{4}}\times\dfrac{2\frac{5}{8}-\frac{1}{3}}{\frac{3}{8}+\frac{3}{7}}$ of £9.

24. Express in shillings and fractions of a shilling the difference between $\frac{1\,7}{5\,6}$ of £1 and $1\frac{1\,6}{2\,1}$ of half a crown.

25. Find the difference between

$\frac{1\,7}{1\,9}$ of £2 15*s.* 5*d.* and 1·03 of £2 12*s.* 1*d.*

26. Find the value of

$$\frac{11\frac{1}{2}-2\frac{1}{5}\text{ of }3\frac{3}{4}}{3\frac{3}{4}\text{ of }1\frac{1}{5}-3\frac{1}{2}}\text{ of }\frac{6\frac{2}{9}}{7\frac{7}{1\,2}}\text{ of £2 7}s.\ 6d.$$

27. Find the value of

$\frac{1}{3}$ of 9*s.* 7*d.* $+\frac{1}{4}$ of £1 6*s.* 9*d.* $-\frac{1}{5}$ of 14*s.* 6*d.*

28. Simplify $\frac{123123}{161007} - \frac{72171}{272646}$.

29. One English mile is $\frac{3}{20}$ of a Swedish mile. How many Swedish miles are there in 54 English miles?

30. The surface of a tidal river first rises $14\frac{1}{2}$ ft. and then falls $15\frac{5}{6}$ ft. How much lower is it than when first observed? (Answer in feet and inches.)

PROBLEMS INVOLVING FRACTIONS

(**106.**) The argument by which the problem is worked must be clearly expressed. The following illustrative examples show the kind of explanation that should be given.

Example i. *An egg merchant sells to one customer* $\frac{1}{5}$ *of his stock, to a second customer* $\frac{1}{4}$ *of what remains, and to a third customer* $\frac{1}{3}$ *of what still remains. He now has ten dozen eggs left. What number had he at first?*

After the 1st customer there remains $\frac{4}{5}$ of the stock.

,, 2nd ,, ,, ,, $\frac{3}{4}$ of $\frac{4}{5} = \frac{3}{5}$ of the stock.

,, 3rd ,, ,, ,, $\frac{2}{3}$ of $\frac{3}{5} = \frac{2}{5}$ of the stock.

Hence $\frac{2}{5}$ of the stock = 120 eggs.

$\therefore$ the whole stock $= 120 \div \frac{2}{5} = 120 \times \frac{5}{2} = 300$ eggs.

Example ii. *A and B can run at the rate of* $12\frac{1}{2}$ *and* $12\frac{7}{16}$ *miles per hour respectively. If A give B ten yards start, in what time will he overtake him? When A has run a mile will he be in front of or behind B?*

In 1 hour A gains $(12\frac{1}{2} - 12\frac{7}{16})$ miles on B,

,, ,, ,, $\frac{1}{16}$ of a mile on B.

How long will A take to gain $\frac{10}{1760} = \frac{1}{176}$ of a mile?

Number of hours $= \frac{1}{176} \div \frac{1}{16} = \frac{1}{176} \times 16 = \frac{1}{11}$.

Hence A will overtake B after $\frac{1}{11}$ hour or $5\frac{5}{11}$ minutes.

A runs $12\frac{1}{2}$ miles in 1 hour.

$\therefore$ A runs 1 mile in $1 \div 12\frac{1}{2} = \frac{2}{25}$ hour.

But in $\frac{2}{25}$ hour B runs $\frac{2}{25} \times 12\frac{7}{16}$ miles;

i. e. when A has run a mile, B has run $\frac{2}{25} \times \frac{199}{16} = \frac{199}{200}$ mile.

But B has 10 yds. start, which equals $\frac{1}{176}$ mile.

$\therefore$ when A is a mile from the start, B is $\frac{1999}{2000} + \frac{1}{176}$ mile from the start.

But $\frac{1}{176}$ is greater than $\frac{1}{2000}$,

$\therefore$ A is behind B.

Examples VI.

Problems involving fractions.

1. If $\frac{7}{11}$ of an estate is worth £4655, what is the value of the remainder?

2. A ship worth £20,000 belonging to four persons A, B, C, and D became a total loss. A owned $\frac{3}{8}$, B $\frac{1}{4}$, C $\frac{1}{8}$, and D the remainder. She was insured for £15,000. What will each receive, and what will each lose?

3. What fraction must be subtracted from the sum of $2\frac{2}{3}$, $3\frac{1}{4}$, $4\frac{5}{6}$, $6\frac{7}{8}$ to leave 18 exactly?

4. An examination paper is done by 2000 candidates, of whom $\frac{2}{5}$ are girls and the rest boys $\frac{1}{10}$ of the boys and $\frac{3}{20}$ of the girls fail. Find what fraction of the whole number succeed in passing.

5. A testator leaves $\frac{3}{7}$ of his estate to his wife, $\frac{2}{9}$ of the remainder to each of his two younger sons, and the rest to his eldest son as residuary legatee. If the estate is worth £12,285, what will the eldest son receive?

6. If $\frac{2}{3}$ of $\frac{4}{5}$ of an estate is worth £23,984, what is the value of the whole estate?

7. The circulation of a newspaper increased in three successive years by $\frac{1}{9}$, $\frac{1}{10}$, and $\frac{2}{11}$ of its amount at the beginning of the respective years. At the end of the third year it was 74,412. What was it at the beginning of the first year?

8. If two-thirds of an estate is worth £70,000, find the value of ·625 of the same estate.

9. An egg merchant sells to one customer one-fourth of his

stock, to a second customer two-fifths of what remains, and to a third customer one-third of what still remains.

He has now 150 eggs left. What number had he at first?

10. A workman saves $\frac{1}{8}$ of his wages. His annual savings amount to £9 15*s.* What are the man's weekly wages, taking a year to be 52 weeks exactly?

11. A man spends $\frac{1}{4}$ of his annual income on rent and rates, $\frac{1}{5}$ on food, and $\frac{1}{12}$ on clothes, and has £119 left for other expenses. Find his annual income.

12. A legacy is divided between three brothers, so that the eldest has $\frac{2}{5}$ of it, the second $\frac{3}{8}$, and the youngest the remainder, which is £35 less than the share of the eldest. Find the value of the legacy.

13. If a person, whose income is £262 a year, spends five-sixths of it, and gives away £21 16*s.* 8*d.*, what fraction of his whole income will he have left?

14. The four floors of a building are to be let for £780, and the rent of each floor above the ground-floor is to be $\frac{2}{3}$ of the rent of the floor below it. Determine the rent of each floor.

15. An estate is let to four tenants, of whom the first holds $\frac{3}{8}$ of the whole, the second $\frac{1}{5}$, the third $\frac{1}{6}$. The fourth holds the remainder, which contains 156 ac. 3 ro. 24 po. What is the extent of the whole estate?

16. A sum of money is divided between *A*, *B*, and *C*. *B* gets $\frac{4}{9}$ of the amount received by *A*, and *C* gets $\frac{3}{5}$ of the sum received by *B*. *C*'s share is £1 18*s.* What is the amount of the sum divided?

17. *A* gave $\frac{2}{3}$ of his money to *B*, $\frac{3}{4}$ of the remainder to *C*, $\frac{4}{5}$ of what he then had left to *D*, and the rest to *E*. If *E* received 5*s.*, how much was given to *B*?

18. I have a bag containing gold, silver, and copper coins. The gold coins form $\frac{1}{7}$ of the whole number, the silver coins form $\frac{1}{4}$ of the whole number, and there are 51 copper coins. What is the whole number of coins in the bag?

19. If one man can do a piece of work in seven days and another can do it in nine days, how many days will they take to do it if they work together?

20. Three-sevenths of the passengers on a steamer are men, nine-elevenths of the remainder are women, and there are forty children. How many passengers are there?

21. The value of a ship and cargo lost at sea was £43,750. If A, B, and C were the owners, and A's share was a third of the whole, and B's share was five-ninths of A's and C's shares taken together, what was C's loss?

22. One pipe can fill a cistern in 3 hours and a second can fill it in 2 hours. If they are both turned on together, how long will they take to fill the cistern?

23. A pipe A can fill a tank in 10 minutes, another pipe B in 16 minutes, and a third C can empty it in 20 minutes. A and B are opened until the tank is one-third full, and then C is opened also. In how long a time from the start will the tank be full?

24. Three men can mow a field in 6 days. At the end of the second day one of them falls ill, and the other two finish the work after mowing 9 days from the time when they began. How long would the third man have taken to mow the whole field by himself?

25. A landlord spends $\frac{1}{6}$ of the rent of a house in repairs, and pays $\frac{11}{240}$ of the remainder as income tax. If he then have £85 17*s.* 6*d.* left, what was the rent?

26. A coal merchant sells to one customer one-eighth of his stock, to a second customer one-third of what remains, and to a third one-tenth of what still remains. He has now 41 tons 6 cwt. 98 lb. left; what had he to begin with?

27. Standard gold consists of $\frac{11}{12}$ of pure gold and $\frac{1}{12}$ of alloy. If a sovereign contains 113 grains of pure gold, find how many will weigh a lb. avoirdupois.

28. How many bottles of wine can be filled from $12\frac{5}{6}$ Hl. of wine, assuming that $2\frac{1}{4}$ litres are sufficient to fill each bottle and that $\frac{1}{55}$ of the whole quantity is wasted during the bottling process?

29. A pipe A can fill a tank in 12 minutes, and another pipe B can fill it in 15 minutes. A third pipe C can empty it in 20 minutes. A and B are opened for 5 minutes, and then C is opened also. In how long a time from the start will the tank be full?

30. I bought a third part of a steamer, and then sold $\frac{3}{8}$ of my share to a friend. Two months afterwards I bought from a fellow shareholder, who owned a half of the steamer, $\frac{8}{15}$ of his share. The steamer shortly afterwards was lost, and £36,000 insurance money was paid to the owners. How much of this sum ought I to receive?

31. Standard gold consists of $\frac{11}{12}$ of pure gold and $\frac{1}{12}$ of alloy. If a sovereign weighs 123 grains, find how many can be coined from a bar of standard gold containing 1 lb. avoirdupois of pure gold.

32. Find the fractions with 49 as denominator which are most nearly equal to ·469 and ·871.

CHAPTER VII

SHORTENED METHODS OF DEALING WITH COMPOUND QUANTITIES

(107.) In Chap. IV we gave methods for performing the usual arithmetical operations upon compound quantities. In this chapter we shall see how some of these operations may be performed in a quicker and neater manner.

Our chief difficulties are due to the fact that the various English measures are complicated by the existence of so many different units. It will help us considerably if we can express sums of money rapidly as decimals of £1.

In order to do this, we notice that

$$\begin{aligned} 2/- &= \tfrac{1}{10} \text{ of } £1 \\ &= £\cdot 1. \end{aligned}$$

Then, by division, we have

$$\begin{aligned} 1/- &= £\cdot 05 \\ 6d. &= £\cdot 025 \\ 3d. &= £\cdot 0125 \\ 1d. &= £\cdot 00416\ldots\ldots \\ \tfrac{1}{4}d. &= £\cdot 00104\ldots\ldots \end{aligned}$$

So that $\frac{1}{4}d.$ is very nearly £·001.

Accordingly any sum of money expressed as a decimal of £1 correct to three places will be correct to the nearest farthing.

Now since 2/– = £·1 and 1/– = £·05, we can at once express any number of shillings as a decimal of £1.

Thus
4/– = £·2
6/– = £·3
9/– = £·45

Oral Examples.

Give the following sums of money as decimals of £1 :

1. 8/–.	**2.** 10/–.	**3.** 14/–.	**4.** 18/–.
5. 19/–.	**6.** 17/–.	**7.** 11/–.	**8.** 7/–.
9. £2 4*s*	**10.** £3 8*s*.	**11.** £5 12*s*.	
12. £10 10*s*.	**13.** £11 17*s*.	**14.** £9 13*s*.	
15. £10 15*s*.	**16.** £12 16*s*.	**17.** £8 2*s*.	
18. £9 1*s*.	**19.** £14 5*s*.	**20.** £5 5*s*.	

Express as pounds and shillings :

21. £3·2.	**22.** £5·25.	**23.** £7·65.
24. £8·35.	**25.** £7·7.	**26.** £9·95.
27. £4·25.	**28.** £·15.	**29.** £6·35.
30. £9·9.	**31.** £10·1.	**32.** £15·05.
33. £21·2.	**34.** £23·3.	**35.** £55·55.

(**108.**) To express a number of pence as a decimal of £1, we use the fact that $\frac{1}{4}d.$ is slightly greater than £·001.

But, since this is not absolutely accurate, there will be some error if we say, for example, that

$7\frac{1}{2}d.$ (= 30 farthings) = £·030.

In order to discover what correction we must make, we notice that 3*d.* is exactly £·0125.

Had we taken 3*d.* as 12 farthings and used the fact that one farthing = £·001, we should have obtained 3*d.* = £·012.

Hence the error in this case is £·0005.

Now it is clear that, if the number of pence is greater than 3*d.*, the error will be greater than ·0005, so that to

obtain a more correct result we must add 1 to the figure in the third decimal place.

Thus $3\frac{1}{2}d.$ (14 farthings) is not £·014...
but £·015 correct to three places.

So $7\frac{1}{2}d.$ is not £·030...
but £·031 correct to three places.

A similar line of argument shows us that, if the number of pence is greater than $9d.$, we must add **2** to the figure in the third decimal place.

Thus $10\frac{1}{4}d.$ (41 farthings)

is £·043 correct to three places.

We can now express any number of pence as a decimal of £1, correct to three decimal places, by reducing the pence to farthings and adding 1 or 2 if the number of pence is over $3d.$ or $9d.$ respectively. It is worth while remembering that $3d.$ = £·0125 and $9d.$ = £·0375 exactly.

Oral Examples, continued.

Express the following as decimals of £1:

36. $2d.$	**37.** $2\frac{1}{2}d.$	**38.** $4\frac{1}{2}d.$	**39.** $5d$
40. $7\frac{3}{4}d.$	**41.** $8\frac{1}{4}d.$	**42.** $9\frac{1}{2}d.$	**43.** $5\frac{1}{4}d.$
44. $11\frac{1}{2}d.$	**45.** $4\frac{3}{4}d.$	**46.** $1\frac{1}{4}d.$	**47.** $7d.$
48. $10\frac{1}{2}d.$	**49.** $10d.$	**50.** $3d.$	**51.** $9\frac{1}{4}d.$
52. $8\frac{3}{4}d.$	**53.** $11d.$		

Give the number of pence in:

54. £·022.	**55.** £·017.	**56.** £·039.
57. £·005.	**58.** £·009.	**59.** £·044.
60. £·033.	**61.** £·023.	**62.** £·032.
63. £·011.	**64.** £·002.	**65.** £·049.
66. £·008.	**67.** £·034.	**68.** £·040.

(**109.**) We can now write down at once the value of any sum of money as a decimal of £1, correct to three decimal places.

Thus £4 4*s.* 7¼*d.*
= £4·230.

(The number of shillings is divided by 2 and written in the tenths column; the pence are reduced to farthings and, with the correction mentioned above, are put in the next two columns.

£5 17*s.* 10½*d.* = £5·894
(i.e. £5·85 + ·044).

The reverse process is equally easy.

£13·247 = £13 4*s.* 11¼*d*

(Here we think of 47 farthings, and make the correction by *deducting* 2 if the number of pence is over 9, or deducting 1 if the number of pence is over 3.)

£27·585 = £27 11*s.* 8½*d.*

(Here the figure in the hundredths column is over 5. We regard the sum as £27·55 + £·035.)

Oral Examples.

Express as decimals of £1 correct to three places of decimals:

1. £2 3*s.* 4*d.*	**2.** £7 5*s.* 10*d.*	**3.** £5 5*s.* 5½*d.*
4. £18 9*s.* 10½*d.*	**5.** £6 4*s.* 8*d.*	**6.** £7 9*s.* 11*d.*
7. £9 19*s.* 4½*d.*	**8.** £2 2*s.* 3*d.*	**9.** £4 14*s.* 7½*d.*
10. £3 4*s.* 9¾*d.*	**11.** £103 7*s.* 10*d.*	**12.** £155 6*s.* 8*d.*
13. £72 15*s.* 2¾*d.*	**14.** £36 8*s.* 9*d.*	**15.** £4 4*s.* 6*d.*
16. £8 8*s.* 8*d.*	**17.** £11 11*s.* 11*d.*	**18.** £14 14*s.* 11½*d.*
19. £2 17*s.* 9*d.*	**20.** £18 13*s.* 3*d.*	

Express in £ *s.* *d.* to the nearest farthing:

21. £3·425	**22.** £2·222 . . . .
23. £2·323	**24.** £3·452 . . . .
25. £11·592	**26.** £12·345 . . . .
27. £23·456	**28.** £34·567 . . . .
29. £45·678	**30.** £82·848 . . . .
31. £97·929	**32.** £30·050 . . . .

33. £36·047 . . . .
34. £107·505 . . . .
35. £58·992 . . . .
36. £365·365 . . .
37. £574·475 . . . .
38. £232·232 . . . .
39. £393·939 . . . .
40. £404·040 . . .

(**110.**) This method will be found of great value later on. For the present it is sufficient to see that it can be used to simplify long division of money.

Example i. *Divide* £3075 14*s.* 11*d.* *by* 176.

(We decimalize the money, put the divisor into 'standard form' and proceed:)

$$
\begin{array}{r}
17{\cdot}4758\ldots \\
\hline
176\,)\,3075{\cdot}75746 \\
17{\cdot}6 \\
\hline
13{\cdot}15 \\
12{\cdot}32 \\
\cdot 837 \\
\cdot 704 \\
\hline
\cdot 1334 \\
\cdot 1232 \\
\hline
1026 \\
880 \\
\hline
146
\end{array}
$$

The answer is £17·476,
or £17 9*s.* $6\frac{1}{4}$*d.*

Example ii. *Divide* £4592 17*s.* $6\frac{1}{2}$*d.* *by* 423 4.

(In this example we have still further shortened the working by using the 'Italian' method of division.)

$$
\begin{array}{r}
10{\cdot}8476 \\
\hline
4{\cdot}234\,)\,45{\cdot}92877 \\
3{\cdot}5887 \\
\cdot 20157 \\
32210 \\
2572
\end{array}
$$

The result is £10 16*s.* $10\frac{1}{2}$*d.*

Notice that this method is especially useful when dividing by any power of 10 or of 5.

Example iii. *Divide* £742 16*s.* 11*d.* *by* 100.

The result is £742·846 . . . . ÷ 100
= £7·428 . . . .
= £7 8*s.* $6\frac{3}{4}$*d.*

Example iv. *Divide* £3429 5*s.* $3\frac{1}{2}$*d.* *by* 125.

The result is £3429·265 . . . $\div \frac{1000}{8}$
= £3·429265 . . . × 8
= £27·43412 . . .
= £27 8*s.* $8\frac{1}{4}$*d.*

Example v. *Find the value of* $\frac{17}{100}$ *of* £32 7*s.* $10\frac{1}{2}$*d.*

The result is $\frac{17}{100}$ of £32·394 . . .
= £5·507 . . . .
= £5 10*s.* $1\frac{3}{4}$*d.*

32·394
17
323·94
226·758
550·698

Examples VII a.

Find the result of dividing each sum of money from 1 to 15 by the numbers (*a*) to (*h*):

1. £374 5*s.* 10*d.*
2. £4654 3*s.* 11*d.*
3. £3593 5*s.* $8\frac{1}{2}$*d.*
4. £2524 7*s.* 2*d.*
5. £397 18*s.* 10*d.*
6. £1549 7*s.* 6*d.*
7. £4242 11*s.* 8*d.*
8. £1397 10*s.*
9. £4243 18*s.*
10. £10240.
11. £9076.
12. £3949 17*s.* $10\frac{1}{2}$*d.*
13. £506 4*s.* 11*d.*
14. £427 8*s.* 9*d.*
15. £3524.

(*a*) 164.
(*b*) 307·2.
(*c*) 56·43.
(*d*) 92·917.
(*e*) 100.
(*f*) 1000.
(*g*) 25.
(*h*) 125.

Find the value of:

16. $\frac{4}{100}$ of £234 5*s.* 9*d.*
17. $\frac{3}{100}$ of £527 6*s.* 10*d.*
18. $\frac{27}{100}$ of £872 3*s.* 9*d.*
19. $\frac{7}{100}$ of £542 9*s.* 2*d.*
20. $\frac{86}{100}$ of £306 5*s.* $1\frac{1}{2}$*d.*

(**111.**) Whilst it is exceedingly useful to decimalize money in this way when divisions are to be performed, it is unfortunately inaccurate if we have to perform multiplications.

The reason for this will be clear if we realize that our decimalization is only approximate, and that there is therefore an error, which, though small, would become considerable if we were to multiply it by any large number.

When dividing, this difficulty does not occur if the divisor is greater than unity, since the error becomes still smaller. Similarly, the difficulty does not arise if we are multiplying by a number less than unity.

If we have to multiply a sum of money, the following method will be found useful. It is a modification of a method which used to be called **Practice.**

The method should generally be used when sums of money have to be multiplied. It may also be used occasionally for the multiplication of other measures.

Example 1. *Find the value of* £32 8*s.* 10*d.* × 3241 (*or find the cost of* 3241 *things at* £32 8*s.* 10*d. each*).

		£3241	= £1 × 3241
		32·4	
		9723	
		6482·	= £32 8*s.* × 3241
		1296·4	
6*d.* = $\frac{1}{40}$ of £1,	∴	81·025	= 6*d.* × 3241
4*d.* = $\frac{1}{60}$ of £1,	∴	54·0166	= 4*d.* × 3241
		£105143·442	= £32 8*s.* 10*d.* × 3241

The result is £105143 8*s.* 10*d.*

Explanation.—The first three lines of the multiplication are simple. Then, since 6*d.* = $\frac{1}{40}$ of £1, the value of 6*d.* × 3241 is $\frac{1}{40}$ of £3241, and we divide the top line of all by 40.

Similarly, to obtain the value of 4*d.* × 3241, we divide the top line by 60.

This method is perfectly general. The pounds and shillings can always be worked off directly, and the pence can be expressed as fractions of £1. A good deal of ingenuity can be shown in finding the best way to treat the pence. In the above example we might equally well have treated 10*d.* as 8*d.* (= $\frac{1}{30}$ of £1) + 2*d.* (= $\frac{1}{4}$ of 8*d.*).

We work two examples to illustrate the method more fully.

Example ii. *Find the cost of* 1234 *things at* £2 17*s.* 7½*d. each.*

	£1234	= cost at £1 each
	2·85	
	2468	= ,, ,, £2 ,,
	987·2	= ,, ,, 16*s.* ,,
	61·70	= ,, ,, 1*s.* ,,
6*d.* = $\frac{1}{40}$ of £1	30·85	= ,, ,, 6*d.* ,,
1½*d.* = $\frac{1}{4}$ of 6*d.*	7·7125	= ,, ,, 1½*d.* ,,
	£3555·4625	

The cost is £3555 9*s.* 3*d.*

Example iii. *Find the value of* £13 5*s.* 9¼*d.* × 924.

	£924	= £1 × 924
	13	
	924	} = £13 × 924
	2772·	
5/– = $\frac{1}{4}$ of £1	231·	= 5/– × 924
6*d.* = $\frac{1}{40}$ of £1 (or $\frac{1}{10}$ of 5/–)	23·1	= 6*d.* × 924
3*d.* = $\frac{1}{2}$ of 6*d.*	11·55	= 3*d.* × 924
¼*d.* = $\frac{1}{12}$ of 3*d.*	·9625	= ¼*d.* × 924
	£12278·6125	

The value = £12278 12*s.* 3*d.*

Labour can often be saved by remembering that

$6s.\ 8d. = \frac{1}{3}$ of £1 ; $3s\ 4d. = \frac{1}{6}$ of £1 ; $13s.\ 4d. = \frac{2}{3}$ of £1.

Example iv. *Find the cost of* 724 *things at* £3 13*s.* 4*d.*

The cost is

$$\begin{aligned} &£3\tfrac{2}{3} \times 724 \\ &= £\tfrac{11}{3} \times 724 \\ &= £\tfrac{7964}{3} \\ &= £2654\tfrac{2}{3} \\ &= £2654\ 13s.\ 4d. \end{aligned}$$

Examples VII b.

Find, by the method of the preceding examples, the value in the following cases :

1. £4 12*s.* $6\frac{1}{2}d. \times 125$.
2. £237 16*s.* $3d. \times 59$.
3. £3 12*s.* $8\frac{1}{2}d. \times 1576$.
4. £14 13*s.* $7\frac{1}{2}d. \times 251$.
5. 4751 articles at £2 9*s.* $2\frac{1}{2}d.$ each.
6. 315 articles at £1 4*s.* 8*d.* each.
7. 170 miles of telegraph wire at £14 16*s.* 8*d.* per mile.
8. £4 15*s.* $6\frac{3}{4}d. \times 905$.
9. 765 tons at £2 17*s.* 8*d.* per ton.
10. 3923 articles at £3 11*s.* $7\frac{1}{2}d.$ each.
11. $103\frac{3}{4}$ acres at £50 12*s.* 6*d.* per acre.
12. 5000 yards of silk at 14*s.* $8\frac{1}{2}d.$ per yard.
13. 1·46875 of £7 4*s.*
14. £13 17*s.* $3\frac{1}{2}d. \times 579$.
15. 777·4 tons of coal at 13*s.* 7*d.* per ton.
16. Find the cost of making a road, 11·65 miles long, at £28 8*s.* 4*d.* per mile.
17. What is the rent of 563 acres of land at £2 12*s.* $6\frac{1}{2}d.$ per acre?

18. What is the value of 98407·714 ounces of fine gold at £4 3*s.* 10¾*d.* per ounce?

19. Find the cost of 23 truck loads of coal, each containing 40 tons, at 17*s.* 8*d.* per ton.

20. Find the value of 1775 cycle-houses at £3 2*s.* 1*d.* apiece.

21. Find the contents of 841 sacks, each containing 6 qrs. 3 bush. 2 pk.

22. If telegraph wire costs £2 8*s.* 9*d.* per mile, what will 621½ miles cost?

23. What is the value of 1684 articles at £1 16*s.* 10½*d.* each?

24. Find the value of £2904 17*s.* 3½*d.* × 805.

25. Convert £317 11*s.* 3*d.* into dollars and cents, reckoning £1 as worth 4 dollars 84 cents, and neglecting fractions of a cent.

26. Find the number of dollars and cents in £13 17*s.* 8*d.* (£1 = 4 dols. 84 cts.)

27. Find the value in francs and centimes of £13 17*s.* 4*d.* at the rate of 25 fr. 16 c. to the £1.

28. Find, correct to the nearest ton, the total weight of 2619 packages, each one of which weighs 3 cwt. 1 qr. 20 lb.

For the following examples take as equivalents of £1: (i) 25 fr. 16 c. (ii) 4 dols. 84 cts. (iii) 20 marks 43 pf.

Find the number of francs and centimes in:

29. £42 5*s.* 10*d.* **30.** £37 16*s.* 8*d.*
31. £25 10*s.* 2*d.* **32.** £2 11*s.* 7½*d.*

Find the number of dollars and cents in:

33. £54 5*s.* 11*d.* **34.** £28 6*s.* 2½*d.*
35. £107 3*s.* 4*d.* **36.** £153 7*s.* 11½*d.*

Find the number of marks and pfennigs in:

37. £33 5*s.* 6½*d.* **38.** £104 9*s.* 10*d.*
39. £342 5*s.* 8*d.* **40.** £27 15*s.* 6*d.*

(**112.**) When we have to find the cost of a quantity of goods such as 3 tons 7 cwt. 1 qr. 18 lb. at a certain price, say **£2** 17*s.* 6*d.* per ton, we can generally manage to express either the goods or the price as a decimal. The following examples will illustrate the method.

Example i. *Find the cost of* 3 *tons* 7 *cwt.* 1 *qr.* 18 *lb. at* £2 17*s.* 6*d. per ton.*

£2 17*s.* 6*d.* = £2·875 exactly.

	£2·875	= cost of 1 ton.
	8·625	= ,, 3 tons.
6 cwt. = ·3 tons	·8625	= ,, 6 cwt.
1 cwt. = $\frac{1}{20}$ of 1 ton	·14375	= ,, 1 cwt.
1 qr. = $\frac{1}{4}$ of 1 cwt.	·03594 . . .	= ,, 1 qr.
14 lb. = $\frac{1}{2}$ of 1 qr.	·01797 . .	= ,, 14 lb.
4 lb. = $\frac{1}{7}$ of 1 qr.	·00513 . . .	= ,, 4 lb.
	£9·690 . . .	

Total cost = £9 13*s.* 9$\frac{1}{2}$*d.*

Example ii. *Find the cost of* 7 *miles* 3 *furlongs* 8 *chains at* £1 17*s* 2$\frac{1}{2}$*d. per mile.*

Here it is not easy to decimalize the money exactly, but 7 mi. 3 fur. 8 ch. = 7 mi. 3·8 fur. = 7·475 mi.

	£7·475	= cost at £1 per mile.
16/– = £·8	5·9800	= ,, 16/– ,,
1/– = $\frac{1}{20}$ of £1	·37375	= ,, 1/– ,,
2*d.* = $\frac{1}{120}$ of £1 (or $\frac{1}{6}$ of 1/–)	·06229 . . .	= ,, 2*d.* ,,
$\frac{1}{2}$*d.* = $\frac{1}{4}$ of 2*d.*	·01557 . . .	= ,, $\frac{1}{2}$*d.* ,,
	£13·907	

Total cost = £13 18*s.* 1$\frac{3}{4}$*d.*

(**113.**) When neither the 'goods' nor the 'price' can be decimalized readily, it is best to use the usual methods to decimalize the 'goods'.

Example i. *Find the cost of* 73 *acres* 3 *roods* 37 *poles of land at* £2 5*s.* 10½*d. per acre.*

73 ac. 3 ro. 37 po.
= 73 ac. 3·925 ro.
= 73·98125 acres.

	£73·98125	= cost at £1 per acre.
	147·96250	= ,, £2 ,,
5/– = ¼ of £1	18·49531 . . .	= ,, 5/– ,,
6*d.* = $\frac{1}{40}$ of £1 (or $\frac{1}{10}$ of 5/–)	1·84953 . . .	= ,, 6*d.* ,,
3*d.* = ½ of 6*d.*	·92476 . . .	= ,, 3*d.* ,,
1½*d.* = ½ of 3*d.*	·46238 . . .	= ,, 1½*d.* ,,
	£169·694	

Total cost = £169 13*s.* 10½*d.*

Example ii. *Find the tax on* £7243 10*s at* 1*s.* 7½*d. in the pound.*

	£7243·5	= tax at £1.
1*s.* = $\frac{1}{20}$ of £1	362·175	= ,, 1*s.* in the pound.
6*d.* = ½ of 1*s.*	181·0875	= ,, 6*d.* ,,
1½*d.* = ¼ of 6*d.*	45·2718	= ,, 1½*d.* ,,
	£588·534	

Tax = £588 10*s.* 8¼d.

Examples VII c.

Find the cost in the following cases:

1. 6 mi. 5 fur. of wire at £1 13*s.* 4*d.* per mile.
2. 3 tons 12 cwt 3 qrs. of copper wire at £50 a ton.
3. 2 cwt. 3 qrs. 8 lb. of flour at £1 3*s.* 4*d.* a cwt.
4. 3 ac. 3 ro. 30 po. of land at £2 10*s.* 8*d.* an acre.
5. 5 tons 16 cwt. 1 qr. 7 lb. of goods at £2 13*s.* 4*d.* per ton.
6. 7 ac. 3 ro. 25 po. of land at £1 12*s.* an acre.
7. 25 ac. 3 ro. 16 po. at £1 16*s.* 8*d.* an acre.

8. 6 tons 7 cwt. 21 lb. at £2 6*s.* 8*d.* per ton.

9. 6 cwt. 3 qrs. 20 lb. at two guineas per cwt.

10. 35 tons 6 cwt. 2 qrs. of coal at 27*s.* 6*d.* per ton.

11. 104 ac. 3 ro. 10 po. of land at £42 16*s.* an acre.

12. 17 tons 11 cwt. 3 qrs. 21 lb. of coal at £1 13*s.* 4*d.* per ton.

13. 13 cwt. 3 qrs. 12 lb. at £15 17*s.* 4*d.* per cwt.

14. 10 tons 2 cwt. 1 qr. 12 lb. at 28*s.* per cwt.

15. 3 tons 6 cwt. 3 qrs. 8 lb. at £31 10*s* a ton.

16. 102 ac. 3 ro. 28 po. at £25 10*s.* per acre.

17. 3 tons 11 cwt. 94 lb. at £4 3*s.* 4*d.* per ton.

18. 21 tons 5 cwt. 3 qrs. 21 lb. at £2 6*s* 8*d.* per ton.

19. 125 qrs. 5 bush. of wheat at £1 8*s.* 6*d.* per qr.

20. 135 ac. 2 ro. 35 po. at £28 6*s.* 8*d* per acre.

21. 46 tons 10 cwt. 2 qrs. of copper at £2 14*s.* 9*d.* per cwt.

22. The duty on 17 cwt. 1 qr. 23 lb. of tobacco at 3*s.* 8½*d.* per lb.

23. 86 tons 4 cwt. of ore at £74 6*s.* 3*d.* a ton

24. 4 tons 5 cwt. 3 qrs. 5 lb. 8 oz. at £32 13*s* 4*d.* per ton.

25. The cost of repairing a road 25 mi 3 fur 170 yds. long at £111 16*s.* 8*d.* per mile

26. The value of 10 oz. 13 dwt. 14 grns. of pure gold at £4 4*s.* 11½*d.* per oz.

27. Find the tax on £5354 13*s.* 4*d.* at 1*s.* 7½*d.* in the pound

28. Find the dividend on £9560 at 15*s.* 10½*d.* in the pound.

29. Find the dividend on £289 13*s.* 8*d* at 14*s.* 7¼*d.* in the pound.

30. Find the dividend on £537 10*s.* at 14*s.* 7½*d* in the pound.

31. Find the dividend on £475 15*s* at 15*s.* 6*d.* in the pound.

32. Find the tax on £4256 4*s.* 8*d.* at 1*s.* 4½*d.* in the pound.

33. Find the tax on £473 8*s.* 10*d.* at 3*s.* 2½*d.* in the pound.

(**114**) The following set of oral examples will illustrate several short methods of mental multiplication and division of compound quantities. One or two of these methods have been mentioned already, but the general guiding principle is: Do not perform the actual multiplications or divisions, even mentally, until you have made a mental note of the sort of expression which has to be simplified.

Thus, (i) to find the cost of ten dozen articles at $2\frac{1}{2}d.$ each, the mental process is:

(The cost is $10 \times 12 \times 2\frac{1}{2}$ pence, but when reducing to shillings the 12 will divide out.)

1 costs $2\frac{1}{2}$ pence.
a dozen cost $2\frac{1}{2}$ shillings.
ten dozen cost 25 shillings.

Or again, (ii) to find the cost of 8 cwt. of coal at 22*s.* per t
(The cost is $\frac{8}{20}$ of 22*s.*)

8 cwt. = ·4 tons
Cost is 8·8 shillings
= 8*s.* 10*d.* to the nearest penny.

(iii) To find the cost of 150 books at 6*s.* 8*d.* each.
The mental process is:

The cost is $150 \times 6s.\ 8d$
but 6*s.* 8*d.* $= \frac{1}{3}$ of £1.
∴ the cost is $150 \times £\frac{1}{3}$
= £50.

Oral Examples.

Find the cost of the following:

1. Twelve dozen eggs at $1\frac{1}{2}d.$ each.
2. Six dozen eggs at $\frac{3}{4}d.$ each.
3. Two dozen books at 1*s.* 2*d.* each.
4. 30 books at 3*s.* 4*d.* each.
5. 10 penholders at 2*s.* per dozen.
6. 140 knives at 17*s.* per score.
7. A dozen firelighters at 3*s.* 6*d.* per gross.
8. 3 dozen hearthstones at 6*s.* per gross.
9. 50 boxes of matches at $3\frac{1}{2}d.$ per dozen.
10. 21 lb. of raisins at 4*s.* per 7 lb.
11. 4 dozen lb. of rice at 4*d.* per 2 lb. bag.

12. A dozen tablets of soap at 1*s.* 3*d.* per box of 3 tablets.
13. A ton of sugar at 18*s.* 6*d.* per cwt.
14. 7 lb. of sugar at 5*s.* 1*d.* per quarter.
15. 8 dozen $\frac{1}{4}$ lb. packets of chocolate at 1*s.* 7*d.* per lb.
16. A gross of eggs at 1*s.* 3*d.* per dozen.
17. 2 bottles of port at 54*s.* per dozen.
18. 54 teaspoons at 3*s.* 6*d.* per dozen.
19. 60 yards of wire netting at 4*s.* 7*d.* per 50 yard roll.
20. 10 dozen pens at 9*d.* per box of 6 dozen.
21. 3 dozen golf balls at 1*s.* 11*d.* each.
22. 8 golf balls at 24*s.* per dozen.
23. 6 tennis balls at 12*s.* 9*d.* per dozen.
24. $1\frac{1}{2}$ dozen tennis balls at 10*d.* each.
25. 600 cartridges at 7*s.* 6*d.* per box of 100.
26. 12 wine-glasses at 3*s.* 4*d.* each.
27. 2 dozen cricket balls at 4*s.* 9*d.* each.
28. 2400 gramophone needles at 1*s* 6*d.* per box of 100.
29. 50 dinners at 2*s.* 6*d.* per head.
30. 2 cwt. of coal at 25*s.* a ton.

Examples VII.

(Decimal methods should be used for these examples.)

1. If the average daily takings of an omnibus are £1 8*s.* $9\frac{1}{2}$*d.*, find the weekly takings of a company owning 729 'buses, each 'bus running 6 days a week.

2. The wages paid by a contractor in one week amounted to £126 8*s.* $7\frac{1}{2}$*d.* ; how many workmen did he employ, if the average weekly earnings of each man were £1 5*s.* $6\frac{1}{2}$*d.* ?

(Express each sum of money as a decimal of £1.)

3. Find the cost of repairing a road, 3 mi. 3 fur. 22 yds. long, at £323 6*s.* 8*d.* per mile.

4. Find the value in English money of 300 bushels of wheat at 1·14 dollars per bushel, assuming that 4·75 dollars = £1.

5. If 1878 tons 6 cwt. 1 qr. of coal are bought for £2504 8*s.* 4*d.*, find the price per ton.

6. If a man pays 4*s.* 7*d.* for every £1 he owes, how much will he pay a tradesman to whom he owes £157 16*s.*?

7 Find the cost of 21 cwt 2 qrs. 10 lb. of cocoa at £3 7*s.* 8*d.* per cwt.

8. What is the total amount of 67 annual payments, each being £26 19*s.* 2*d.*?

9 £368 16*s.* 9*d.* was divided equally amongst a certain number of persons, and the share of each was £3 4*s.* $8\frac{1}{2}d$. How many persons were there?

10. Find the value of 7 ac. 2 ro. 33 po. at £134 11*s.* 8*d* per acre.

11. Find the value of 3·47 of £4 17*s.* 3*d.*

12 Find the value of imports per head, correct to the nearest penny, when the population is 42,793,258 and the total value of the imports is £551,038,628.

13 If the average weekly wage of a railway servant is 25*s.* $5\frac{1}{4}d$, and the number of railway servants employed is 158,000, what sum, correct to the nearest thousand pounds, will pay their wages for 52 weeks?

14. A farm of 625 ac. 1 ro. 10 po. is purchased for £7878 18*s.* 9*d.* Find the price per acre.

15. Find the cost of 89 tons of oilcake at £8 12*s* 9*d.* per ton.

16. On £2340 a profit of 2*s.* $7\frac{1}{2}d$ in the £ is made. Find the total profit.

17. Find the dividend on £4146 12*s.* 6*d.* at 11*s* 8*d.* in the £

18. Find the dividend on £5171 at 5*s* $9\frac{3}{4}d$. in the £.

19. Find the rent of 47 ac. 2 ro. 36 po. $2\frac{3}{4}$ yds. at £20 3*s.* 4*d.* per acre

20. Find the cost of 92 tons 7 cwt 3 qrs. 12 lb. at £1 12*s.* 7*d.* per ton.

REVISION EXAMPLES

1. What fraction must be subtracted from the sum of $2\frac{2}{3}+3\frac{3}{4}+4\frac{5}{6}+6\frac{7}{8}$ to leave 18 exactly?

2. Prove from the definition of a fraction that $\frac{1}{3}-\frac{1}{12}=\frac{1}{4}$.

3. If 13 yds. of cloth cost £2 0*s.* 1*d.*, what must be given for 75 yds?

4. *A* can do a piece of work in 11 days, *B* in 20 days, and *C* in 55 days; how soon can the work be done if *A* is assisted by *B* and *C* on alternate days?

5. Find the cost of 11 tons 12 cwt. $2\frac{1}{2}$ qrs. at £5 6*s.* 8*d.* per ton.

6. Determine the total amount of the Poor Rate levied on a population of 3,196,395 at the rate of 14*s.* $3\frac{1}{2}$*d.* per person.

---

7. Give a definition of multiplication which shall include the multiplication of one fraction by another. Deduce that $\frac{2}{3}\times\frac{3}{4}=\frac{1}{2}$.

8. Simplify $\dfrac{3\frac{1}{2}}{7\frac{1}{4}}\times\dfrac{17\frac{2}{5}}{5\frac{8}{11}}$.

9. Find the value of ·05854 of a guinea; and reduce 16*s.* $3\frac{1}{2}$*d.* to the decimal of £1.

10. Find the value of $\dfrac{23}{24}$ of $\dfrac{2\frac{1}{2}}{85\frac{3}{8}}$ of £1565.

11. A litre is 1·76 pints and 6 ordinary wine bottles hold a gallon. Express the capacity of an ordinary wine bottle as the decimal of a litre, correct to four decimal places.

12. What weight must be added to $\frac{3}{8}$ of $\frac{5}{4}$ of half a cwt. to make it up to $\frac{3}{10}$ of $3\frac{3}{4}$ quarters?

---

13. Find the total amount of duties payable on the following cargo: 500 gallons of brandy at 10*s.* 4*d.* per gallon, 512 gallons of wine at 2*s.* 6*d.* per gallon, 432 gallons of wine at 3*s.* 3*d.* per gallon, 10 gallons of Eau de Cologne at 16*s.* 6*d.* per gallon.

14. Simplify: (i) $1\frac{1}{4}+2\frac{5}{12}+\frac{4}{15}+\frac{7}{25}+\frac{11}{30}$;

(ii) $5\frac{2}{5}+3\frac{1}{5}\div 1\frac{1}{3}-4\div 1\frac{9}{11}$.

15. Find the value of

£1·1875 − 2·25 crowns + 3·75 guineas − 46·125 shillings.

16. If a train runs $78\frac{3}{4}$ miles in 1 hr. 24 min., how many feet will it run in one second?

17. What fraction when multiplied by $3\frac{1}{2}$ of $\frac{4}{5}$ produces $2\frac{2}{5}$ of $\frac{4}{13}$?

18. If numbers are formed by arranging all the digits 2, 3, 4, 5, 6 in any order, show that each of these numbers when divided by 9 will have a remainder 2.

19. Find the cost of 2 tons 14 cwt. 3 qrs. at £16 6*s.* 8*d.* a ton.

20. What fraction added to the sum of $2\frac{2}{3}$, $3\frac{5}{7}$, and $5\frac{7}{12}$ will make a sum total of 12?

21. If $\frac{2}{11}$ of a chicken is worth 4*d.*, and $\frac{3}{4}$ of a chicken is worth $\frac{4}{120}$ of a turkey, how much should be given for 50 turkeys?

22. Simplify $\dfrac{23\frac{4}{15} \div 12}{7\frac{1}{6} + 8\frac{2}{5} - 9\frac{3}{4}}$

23. What weight of copper is worth 723 francs 69 centimes, if 1 kilogram be worth 1 franc 29 centimes?

24. At an election, where only five-sixths of the whole number of voters voted, one of the two candidates received half as many votes again as the other and beat him by 725 votes. What was the whole number of voters, each being supposed to have only one vote?

---

25. Subtract seventeen times three thousand and nine from half a million and seven. How often is the remainder contained in five million twenty thousand and thirty-nine?

26. Simplify: (i) $\dfrac{12053}{27651}$; (ii) $\dfrac{8\frac{2}{11} - 1\frac{8}{9}}{1\frac{1}{3} + 2\frac{10}{11}} - \dfrac{11}{15}$

27. Find, to the nearest penny, the rent of 16 acres 1 rood 34 poles at £5 17*s.* 6*d.* an acre.

28. Multiply 23·625 by ·0034, and prove your answer.

29. A housekeeper takes three half-pints of milk each week-day and one pint on Sunday; her bill for the week comes to 1*s.* $5\frac{1}{2}$*d.* What is the price of milk per quart?

30. Find the smallest number which on division by 21, 24, or 56 will in each case leave remainder 12.

---

31. Divide £45,691 13*s.* $4\frac{1}{2}$*d.* by 546.

32. Add together 3·40765, 537·063, and ·84378. Subtract the result from 601·0473.

33. Find the value of $\frac{1}{14}$ of $(\frac{14}{17}+2\frac{1}{9}+1\frac{5}{51}+\frac{6}{27})$ of £2 11*s.*

34. Express as products of their simplest factors 61710 and 17017; find their G.C.M.

35. Coals at 32*s.* a ton are mixed with coke at 24*s.* a ton in the proportion of 5 tons of coal to 3 tons of coke; find the money saved by using 11 tons of the mixture instead of 12 tons of coal.

36. Two cyclists start abreast at three o'clock to race on a circular track. If one goes round in 3 min. 12 sec. and the other in 3 min. 30 sec., find at what time they will again be riding abreast past the starting-point, and the number of complete circuits they will then have made.

---

37. The nineteenth century began on Jan. 1, 1801, and ended on Dec. 31, 1900. Find the number of days in the century.

38. Simplify $\dfrac{2\frac{2}{3}+4\frac{4}{7}-3\frac{3}{5}}{\frac{1}{3}+\frac{1}{5}+\frac{1}{7}} \div \left(3-\dfrac{1}{3\frac{1}{1\frac{3}{9}}}\right)$.

39. If the rate of exchange is 4 dollars 86 cents for £1, find the value of 317 dollars in English money (to the nearest penny).

40. Find the cost of 20 yds. 2 ft. 3 in. at £1 17*s.* $4\frac{1}{2}$*d.* a yard.

41. If a man, a woman, and a boy can do a piece of work in 3, 4, and 6 days respectively, what fraction of the work would remain unfinished after 1 man, 1 woman, and 2 boys had been working at it for a day?

42. If the average daily takings of a tramcar are £1 5*s.* 10·7*d.*, and the weekly takings of a company are £4621 13*s.* 3*d.*, each car running 6 days a week, how many cars does the company own?

---

43. Find to six places of decimals the difference between $\frac{1}{3.141593}$ and $\frac{7}{22}(1+\frac{4}{10000})$.

44. Find the value of 3 qrs. 13 lb. at £1 10*s.* 8*d.* a quarter.

45. Simplify: (i) $\dfrac{(\frac{1}{5})^3-(\frac{2}{17})^3}{\frac{1}{5}-\frac{2}{17}}$; (ii) $\dfrac{(\frac{8}{11})^2-(\frac{3}{14})^2}{\frac{8}{11}+\frac{3}{14}}$.

46. If a metre equals 39·37 inches, how many metres are there in 4 furlongs? (Give the answer correct to two decimal places.)

47. A and B are to divide 300 cows equally between them. A takes 173, and B 127. If a cow is worth 18 guineas, how much ought A to give B?

48. Find, to four decimal places, the value of $\cdot 61566 \times 1\cdot 62427$.

---

49. Given $u = 25\cdot 24$, and $v = 13\cdot 27$, find the value of

$$1 \div \left(\frac{1}{u} + \frac{1}{v}\right).$$

50. Simplify: (i) $\frac{317}{318} - \frac{197}{198}$; (ii) $\frac{3}{5} \times \frac{4}{7} \times \frac{14}{33} \times \cdot 3674$.

51. From a piece of cloth there were cut in succession pieces whose lengths were respectively 18·35 metres, 7 metres 60 centimetres, 85 centimetres, 12 decimetres; there remained 5 metres 46 centimetres. What was the original length of the piece?

52. A cubic foot of water weighs 1000 oz.; a gallon of water weighs 10 lb. How many pints does a cistern hold, the capacity of the cistern being 1 cubic yard?

53. Find the value of 3 tons 17 cwt. 3 qrs. at £1 17*s.* 3*d.* per ton.

54. Find the value of $\cdot 55678 \div \cdot 83066$ (to four places).

---

55. Find, correct to three figures, the value of $\frac{a^2 + 3b}{b^2 + 3a}$ when $a = 8\cdot 702$, $b = 5\cdot 678$.

56. Simplify $\dfrac{2\frac{1}{5} - 1\frac{2}{3}}{3\frac{1}{3} + 1\frac{2}{5}} \div \dfrac{\frac{2}{7} - \frac{1}{8}}{\frac{2}{7} + \frac{1}{8}}$.

57. A dealer buys 80 pounds of tea at 1*s.* $4\frac{1}{2}d.$ per pound, and mixes them with 63 pounds of tea at 1*s.* $9\frac{1}{2}d.$ per pound. He sells the mixture at 1*s.* 7*d.* per pound; how much does he gain or lose?

58. Find the value of $1\cdot 27994 \times \cdot 78129$ (to five figures).

59. A house was rated at £38. On each £ of this rateable value the tenant had to pay the following amounts: for relief of poor 6·37*d.*, county contributions 6·45*d.*, borough rate 8·0*d.*, police rate 3·95*d.*, expenses of the overseers ·23*d.*, general district rate 1*s.* 6·107*d.*, main drainage rate 1·734*d.*, special drainage rate ·159*d.* What was the total amount of the rate?

60. If $\frac{1}{3}$ of a gallon of brandy costs as much as $\frac{1}{5}$ of a cwt. of sugar, and 9 gallons of brandy costs as much as 8 tons of coal, how many cwt. of sugar cost as much as 10 tons of coal?

CHAPTER VIII

AREAS OF RECTANGLES

Oral Examples.

1. SUGGEST units of measurement for the area of
   (i) A field whose sides are measured in *chains*;
   (ii) A field whose sides are measured in *yards*;
   (iii) A rectangle drawn on paper, whose sides are measured in *inches*;
   (iv) An island whose coast-line is measured in *miles*;
   (v) An island whose coast-line is measured in *Km.*;
   (vi) A window whose sides are measured in *dm.*;
   (vii) A wall whose length and height are measured in *feet.*

2. A plank is 7 ft. long and 2 ft. wide.
   (i) What unit will you use for expressing its area?
   (ii) How many units of area are there in the portion of a floor covered by such a plank?
   (iii) How many of such planks will be wanted for the floor of a room which is 14 ft. long and 6 ft. wide?
   (iv) What will be the area of this room?

3. A lawn-tennis court is 12 yards broad and 26 yards long.
   (i) Suggest a unit of measurement for the area.
   (ii) What is the playing area expressed in terms of this unit?
   (iii) How much grass seed will be wanted to sow such a lawn, allowing a pint of seed to six square yards?

4. Most postage stamps are rectangles of sides 2 cm. and 2·2 cm.
   (i) Suggest a unit of measurement for the area.
   (ii) What is the area, expressed in square centimetres?
   (iii) What is the area, expressed in square millimetres?
   (iv) How many square millimetres are there in a sq. cm.?

5. The average length of the Island of Jersey is about 10 miles, and the average width is about 4 miles. Give a rough idea of the area of the island.

6. A rood is the area of a rectangle whose sides are a furlong and a pole.

(i) How many square poles are there in an acre?

(ii) How many square yards are there in an acre?

(iii) How many acres are there in a rectangular garden whose sides are 20 yards and 1 chain?

(115.) The units by which we measure areas are the square inch, the square foot, the square centimetre, &c. A square inch is defined as a square, each of whose sides is one inch long, and the other measures are similarly defined.

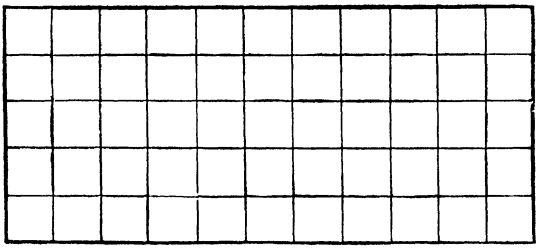


Fig. 8.

Fig. 8 is a reduced plan of a rectangle whose sides are 5 inches and 11 inches respectively. It is divided into a number of square inches by lines drawn parallel to the sides, and it is clear that the number of square inches is 5×11.

This is true of any such figure, and we may say that a rectangle whose sides are a inches and b inches has an area of ab square inches.

Similarly a rectangle whose sides are p chains and q chains has an area of pq square chains.

If the dimensions (i. e. the lengths of the sides) are given in terms of different units, we must express them in terms of the same unit before trying to find the area.

The sum of the sides of a rectangle, or other figure, is called the *circuit* or *perimeter* of the figure.

Oral Examples.

State the areas and perimeters of the rectangles whose sides are as follows:

1. 7 in., 6 in.	**2.** 3 ft., 8 ft.
3. 4 cm., 9 cm.	**4.** 8 yds., 12 yds.
5. 6 ch., 11 ch.	**6.** 5 poles, 14 poles.
7. 8 in., 13 in.	**8.** 2 ft., 4 in.
9. 2 ch., 2 yds.	**10.** 5 yds., 4 ft.
11. 2·3 dm., 6 cm.	**12.** 6·2 dm., 5 dm.
13. 1·2 Km., 4 Hm.	**14.** 3·6 Hm, 2·5 Hm.

How many acres are there in the following rectangular fields, their sides being:

15. 4 ch., 8 ch.	**16.** 12 ch., 7 ch.
17. 2 fur., 3 ch.	**18.** 3 fur., 4 fur.?

How many ares are there in the following rectangular fields, their sides being:

(An are is a square Dekametre.)

19. 2 Dm., 7 Dm.	**20.** 3·2 Hm., 3 Dm.
21. 1·2 Hm., 3 Hm.	**22.** 1·2 Hm., 1·2 Hm.?

23. A square field has a side of length 7 chains. What is its area in acres?

In the following rectangles, the area and the length of one side are given in each case. Give the length of the other side:

24.	Area	24 sq. in.	one side	6 in.
25.	,,	3·6 sq. cm.	,, ,,	4 cm.
26.	,,	12 acres	,, ,,	30 ch.
27.	,,	2 Ha.	,, ,,	100 metres.
28.	,,	3·6 acres	,, ,,	6 ch.
29.	,,	1·44 sq. cm.	,, ,,	1·2 cm.
30.	,,	520 ares	,, ,,	260 metres.
31.	,,	1 sq. ch.	,, ,,	44 yds.
32.	,,	1 ac. 2 ro.	,, ,,	60 yds.

33. A rectangle has sides of ·7 cm. and 1·2 cm.

(i) What is its area in square millimetres?

(ii) What is its area in square centimetres?

(iii) Is it correct to say that its area is $·7 \times 1·2$ sq. cm.?

34. A small square has a side of $\frac{1}{10}$ of an inch. What is its area? (Express as a fraction of a square inch.)

35. A rectangle has sides 1·3 in. and 1·2 in. Express its area in square inches.

36. The sides of a rectangle are given as 2·34 m. and 4·72 m.

(i) If you wish the area expressed as a whole number, what unit would you suggest?

(ii) How would you find the number of square metres in the area?

(iii) How would you find the number of square decimetres in the area?

37. Give in ares the areas of the rectangular fields whose sides have the following lengths:

(i) 3·2 Dm., 2 Dm. (ii) 4·3 Dm., 2 Dm.

(iii) 2 Hm., 1·4 Dm. (iv) ·35 Hm., ·24 Hm.

38. A rectangle has area 4·9 sq. cm., and one side is ·7 cm. What is the length of the other side?

39. One side of a field is 2·3 chains. What is the other side if the field contains ·69 acres?

(116.) When we say that the area of a rectangle of sides a in. and b in. is ab sq. in, we have assumed that a and b contain whole numbers of inches, but the restriction is not necessary.

$$\text{A cm.} = 10 \text{ mm.}$$
$$\therefore \text{ a sq. cm.} = 100 \text{ sq. mm.}$$

If we have a rectangle whose sides are 6·7 cm. and 8·3 cm., we may express these lengths as 67 mm. and 83 mm., and we obtain the area as

$$67 \times 83 \text{ sq. mm.}$$
$$= 5561 \text{ sq. mm.}$$
$$= 55{\cdot}61 \text{ sq. cm.}$$

$$\begin{array}{r} 83 \\ 67 \\ \hline 498 \\ 581 \\ \hline 5561 \end{array}$$

This is identically the same result as we should have obtained by multiplying together 6·7 and 8·3.

Such instances suggest that we can always obtain the number of units of area in a rectangle by multiplying together the numbers representing the lengths of a pair of adjacent sides.

Example. *A rectangle has sides of lengths* 7·32 *in. and* 4·35 *in.*

The area is $7{\cdot}32 \times 4{\cdot}35$ sq. in.

$= 31{\cdot}842$ sq. in.

$$\begin{array}{r} 7{\cdot}32 \\ 4{\cdot}35 \\ \hline 29{\cdot}28 \\ 2{\cdot}196 \\ 3660 \\ \hline 31{\cdot}842 \end{array}$$

Oral Examples.

1. When you measure a line which is about 6 in. long, how accurately do you expect to be able to give the result? (To the nearest tenth? hundredth? thousandth? of an inch.)

2. A boy measures the length of a sheet of paper and tells me that the length is 8·3292 inches. How much of this can I believe?

3. If you measure the length of a book or a sheet of paper, giving the result as a decimal, what degree of accuracy do you expect to get? (3 figures? 4 figures? 5 figures?)

4. When you wish to find the area of a piece of paper, what would you do when you had measured the sides?

5. What degree of accuracy would you expect to get in the area of a rectangle whose sides you had measured?

6. Have you any reason to mistrust the following statement (the multiplication is correct):

A book is 6·34 inches long, and 4·58 inches wide.

$\therefore$ its area is 29·0372 sq. inches.

7. The following were answers given to questions involving areas of rectangles. The actual multiplications and divisions are

correct, but you are asked to express the answers in a more reasonable way.

(i) 2·34 in. by 3·75 in. gives an area of 8·7750 sq. in.

(ii) 4·321 in. by 7·463 in. gives an area of 32·247623 sq. in.

(iii) 7 ft. 6 in. by 8 ft. 4 in. gives an area of 62 sq. ft. 72 sq. in.

(iv) A rectangle has an area of 13·43 sq. cm., and one side is 2·56 cm. ∴ the other side is 5·25391 cm.

(v) The area of the floor of a room is 73·2 sq. ft., and the length of the room is 10·8 sq. ft. ∴ the width is 6·77777 ft.

(**117.**) A very important point arises in connexion with the last example in § 116.

When a length is given as 7·32 inches we have no reason to suppose that the length is *exactly* 732 hundredths of an inch, but that *as far as measurement is possible* the length is 7·32 inches.

The length would be more correctly written as 7·32 . . . . inches, the dots indicating, as usual, that *there may be other figures after the figure* 2, *but that we do not know what they are.*

(**118.**) We will now examine the multiplication in the example of § 116, writing * when we have no information what the digit really is.

$$
\begin{array}{r}
7\cdot 3\,2\,*\,*\,* \\
\quad\quad 4\cdot 3\,5\,* \\
\hline
2\,9\cdot 2\,8\,*\,*\,* \\
2\cdot 1\,9\,6\,*\,* \\
\quad\cdot\, 3\,6\,6\,0\,* \\
\quad\quad\quad *\,*\,*\,* \\
\hline
3\,1\cdot 8\,4\,2\,0
\end{array}
$$

We see that there are unknown figures in the 2nd, 3rd, and 4th decimal columns, therefore the 4, 2, and 0 in these

columns are certainly wrong. Also the missing figure in the 2nd column may affect the number carried to the first column, so that the 8 in that column may be wrong by one. Hence we are not justified in saying that the area is 31·8420 sq. inches, but only that it is 31·8 . . . sq. inches with some doubt as to the accuracy of the last figure.

(**119.**) The point now arises whether it was worth putting down any figure in the thousandths column. The figures we have put there in the partial products are actually correct, but some time may occasionally be saved by omitting them altogether.

Example 1. *Find the area of a rectangle whose sides are* 27·32 *cm. and* 31·24 *cm.*

[A rough answer is $28 \times 30 = 840$ sq. cm. Mental work of this kind provides a useful check on the final position of the decimal point.]

$$
\begin{array}{r}
27{\cdot}32 \quad . \quad . \quad . \\
31\,24 \quad . \quad . \quad . \\
\hline
819{\cdot}6 \quad . \quad . \\
27{\cdot}32 \quad . \quad . \quad . \quad . \quad . \\
5{\cdot}46 \quad . \quad . \quad . \quad . \quad . \\
1{\cdot}09 \quad . \quad . \quad . \quad . \\
\hline
853{\cdot}5 \quad . \quad . \quad . \quad . \quad .
\end{array}
$$

In order to get the 'carry-over' from the 'hundredths column' we put down in that column the figures which we could find accurately, but we omit the figures in all the columns of lower power.

As a matter of fact, in this example, the figure 5 in the tenths column of the final result is possibly inaccurate, since the unknown figure after the 4 in the multiplier would most probably give a figure in this column. Hence we should not venture to say more than that the area of the rectangle is approximately 854 sq. cm.

Example ii. *Find the number of acres in a field whose sides are* 3 *fur.* 4·32 *ch., and* 5 *fur.* 7·64 *ch.*

The sides are 34·32 ch. and 57·64 ch.

The area $= 34{\cdot}32 \times 57{\cdot}64$ sq. ch.

$= 1978$ sq. ch.

$= 197{\cdot}8$ acres.

$$\begin{array}{r} 34{\cdot}32 \\ 57{\cdot}64 \\ \hline 1716\,0 \\ 240\,24 \\ 20\,59 \\ 1{\cdot}37 \\ \hline 1978 \end{array}$$

Example iii. *Find the area of a rectangle whose sides are* 7 *ft.* 4 *in. and* 8 *ft.* 10 *in.*

The sides are $7\frac{1}{3}$ ft. and $8\frac{5}{6}$ ft.

$\therefore$ The area $= \frac{\cancel{22}^{11}}{3} \times \frac{53}{\cancel{6}_{3}}$ sq. ft.

$= \frac{583}{9}$ sq. ft.

$= 64{\cdot}8$ sq. ft. approximately.

In this last example, it is not quite clear how accurate our result can be.

The above is a reasonable answer, but it is sometimes assumed that measurements which do not involve decimals are to be considered as exact.

In this case we should say that the area

$= \frac{583}{9}$ sq. ft.

$= 64\frac{7}{9}$ sq. ft.

$= 64$ sq. ft. $\frac{7}{\cancel{9}} \times \cancel{144}^{16}$ sq. in.

$= 64$ sq. ft. 112 sq. in.

Note.—This result assumes a degree of accuracy which is quite unattainable It is equivalent to the statement that the area is 64·778 square feet; a degree of accuracy not warranted by the data.

It is well to note that, as a general rule, it is useless to try to obtain in any result more figures than were given in the least accurate of the numbers forming the data.

Examples VIII a.

[In the following examples the measurements are supposed to be correct only to the degree of accuracy given. The results must be given to no more than a reasonable degree of accuracy.]

Find the areas of the following rectangles.

1. 3·75 in. by 4·82 in.
2. 53·41 in. by 27·59 in.
3. 2·59 metres by 7·62 metres
4. 5·832 metres by 63·41 dm.
5. 6·43 ch. by 3·79 ch.
6. 2·57 Dm., 5·82 Dm. (Give the answer in ares.)
7. 39·82 cm. by 43·61 cm.
8. 23·25 in. by 72·56 in.
9. 23·82 m. by 17·68 m.
10. 10·43 cm. by 9·82 cm.
11. 8 ft. 4 in. by 6 ft. 3 in.
12. 14 ft. 6 in. by 7 ft. 4 in.
13. 6 ch. 39 lks. by 7 ch. 32 lks. (Give the answer in acres.)
14. 5 ch. 84 lks. by 11 ch. 39 lks.

15. Find the area of a field 49 yds. 2 ft. long by 45 yds. 1 ft. wide. What is the perimeter of the field?

16. A room is 16 ft. long by 12 ft. wide and 11 ft. high. Find the area of (i) the floor, (ii) the ceiling, (iii) the walls.

17. Find the area of the walls of a room 18 ft. long by 15 ft. wide and 12 ft. high. What is the area of wall paper required, if the fireplace, windows, and door have together an area of 150 sq. ft.?

18. What is the area of the floor of a room that can be completely covered by a roll of carpet 27 inches wide and 98 ft. long?

19. A field 6 chains long by 5 chains 17 links wide is surrounded by a fence 6 feet high. What is the total area of the fence?

20. A square carpet, whose side is 12 feet, is placed in a room $17\frac{1}{2}$ ft. by $14\frac{1}{2}$ ft. What area is left uncovered by the carpet?

21. A garden consists of a lawn surrounded by a path 3 ft. wide. If the lawn is 48 ft. by 24 ft., what are the length and

breadth of the garden? What is the area of the garden, and what the area of the path?

22. A garden roller is 27 inches wide, and its circumference is 7 ft. 4 in.; what area does it roll in every complete revolution?

(**120.**) If we are given the area of a rectangle and the length of one side, we can determine the length of the other side by simple division, but we must again be careful not to profess a degree of accuracy which is greater than the measurements will allow.

Example i. *A rectangle has an area of* 38·42 *sq. in., and one side is* 6·34 *in. What is the other side?*

The other side is

$$\frac{38\cdot42\ \ldots}{6\cdot34\ \ldots}\ \text{in.}$$

$$= 6\cdot06\ \text{in.}$$

$$\begin{array}{r} 6\cdot06 \\ \hline 6\cdot34\ \ldots\ )\ 38\ 42\ \ldots \\ 38\ 04\ \ldots \\ \cdot38\ \ldots \\ 38\ \ldots \end{array}$$

In the division, it would be inaccurate to put down more figures than we have done.

Example ii. *A field contains* 43·241 *acres, and one side is* 32·47 *chains. What is the other side?*

The other side is

$$\frac{432\cdot41\ \ldots}{32\cdot47\ \ldots}\ \text{chains.}$$

$$= 13\cdot32\ \ldots\ \text{chains.}$$

$$\begin{array}{r} 13\cdot317\ \ldots \\ \hline 3\ 247\ \ldots\ )\ 13\ 241\ \ldots \\ 10\ 771\ \ldots \\ 1\cdot030\ \ldots \\ \cdot056 \\ 24\ \ldots \end{array}$$

Examples VIII b.

In the following examples, the area and one side of a number of rectangles are given. Find the length of the other side as accurately as the data permit.

1.	Area 37·64 sq. in.	One side	8·49 in.
2.	,, 42·83 sq. cm.	,, ,,	7·82 cm.

3.	Area	103·8 sq. in.	One side	9·21 in.
4.	,,	76·42 sq. metres.	,, ,,	2·34 m.
5.	,,	109·8 ares.	,, ,,	7·64 Dm.
6.	,,	83·92 ares.	,, ,,	6·49 Dm.
7.	,,	372·41 sq. miles.	,, ,,	39·82 miles.
8.	,,	43·89 acres.	,, ,,	11·8 ch.
9.	,,	67·65 acres.	,, ,,	25·7 ch.
10.	,,	43·72 sq. cm.	,, ,,	85·7 mm.
11.	,,	503·9 sq. yds.	,, ,,	27·9 yds.
12.	,,	762·8 sq. yds.	,, ,,	105·3 yds.
13.	,,	643·59 sq. cm.	,, ,,	38·43 cm.
14.	,,	2·426 Ha.	,, ,,	303·7 m.
15.	,,	57·52 Ha.	,, ,,	7·32 Hm.

16. It is required to cut a roll of carpet whose width is 27 inches, so as to cover an area of 200 sq. ft. What length of carpet is needed?

17. What length of carpet, 30 inches wide, is required to cover a room $18\frac{1}{2}$ ft. by 16 ft.?

18. How many yards of wall paper, 21 inches wide, are required to cover an area of 652 sq. ft.?

19. What length of wall paper, 21 inches wide, is needed to paper the walls of a room 24 ft. by 18 ft. and 12 ft. high, allowing 200 sq. ft. of area for fireplace and windows?

20. How far does a man walk when rolling a lawn, 26 yards long by 20 yards wide, with a roller 27 inches wide?

(**121.**) The methods employed in the early part of this chapter are often called 'contracted methods' of multiplication and division.

Formal methods of performing such operations are useless; but the student must always be prepared to omit figures which are of no value.

The following examples will make clear the general method of working.

Example 1. *Find the value of* $27{\cdot}324\ \ldots \times 103{\cdot}242$ *correct to five significant figures.*

[The approximate answer is 2700.]

$$
\begin{array}{r}
27{\cdot}324 \\
103{\cdot}242 \\
\hline
2732{\cdot}4\ \ldots\ldots \\
81{\cdot}97\ \ \\
5{\cdot}46\ \ldots\ \ \\
1{\cdot}09\ \ldots\ \ \\
{\cdot}05\ \ldots\ \ \\
\hline
2821{\cdot}0\ \ldots\ \
\end{array}
$$

In multiplying by any digit, we commence as usual under that digit, but we do not put down, nor even take the trouble to obtain, figures which will be useless For instance, the mental process of obtaining the partial product by the figure 4 is as follows

The multiplication would start under the 4, but the figures there are not wanted. A glance along the columns show that the first product to be of any value will be the result of multiplying the 7 by 4 We take into consideration the carry-over from the previous figure, and say

3×4 12, carry 1
$7 \times 4 = 28$ and $1 = 29$ put down 9)
$2 \times 4 = 8$ and $2 = 10$

In order to obtain accuracy, we keep an extra column so as to get a correct carry-over when performing the final addition.

Example 11. *Find the value of* $3{\cdot}72416 \div {\cdot}049212$, *giving the result correct to four significant figures.*

[Approximate answer is $3\ 7 - 05 = 74$.]

$$
\begin{array}{r}
75{\cdot}675 \\
\hline
4{\cdot}9212\,)\,372{\cdot}416 \\
27{\cdot}932 \\
3{\cdot}526 \\
{\cdot}373 \\
29
\end{array}
$$

The result is $75{\cdot}68\ \ldots$ to four figures.

Before starting the work we notice that if four figures are wanted we shall not have to go beyond the third decimal place : since the rough work shows that the result will be approximately 70.

We actually obtain one figure more than the number required, to enable us to form a more correct estimate of the last figure.

Each partial division is treated separately. E.g. when we have obtained the figures 75 6, the remainder is 373 The divisor is practically 5, and a trial division shows that the next figure in the quotient will be 07. When performing the multiplication of the divisor by ·07, the result of multiplying the figure 4 must come under the 7 of the remainder ·373 ; thus we require only the figures 4·9 of the divisor, though for greater accuracy we consider the carry-over from the next figure (2).

The mental process is then :

$2 \times 7 = 14$, carry 1

$9 \times 7 = 63$, and 1 (carried over) = 64 and 9 makes 73.

(The 9 goes down and 7 is carried.)

$4 \times 7 = 28$, and 7 (carried over) = 35 and 2 makes 37.

The complete remainder is now down, and we proceed as before.

For convenience of reference, the same example is shown with the partial products put down.

$$
\begin{array}{r}
75{\cdot}675 \\
4{\cdot}9212\,)\,372{\cdot}416 \\
344{\cdot}484 \\
\hline
27{\cdot}932 \\
24{\cdot}606 \\
\hline
3{\cdot}326 \\
2{\cdot}953 \\
\hline
{\cdot}373 \\
{\cdot}344 \\
\hline
29
\end{array}
$$

(**122.**) In actual practice it is sometimes useful to employ various devices to assist the work, such as to put some mark against figures in the divisors or multiplicands which are no longer required, or to draw vertical lines showing the limit of accuracy. Such devices are of the greatest use when the student discovers them for himself, but they are

not essential to the method, and are better omitted entirely rather than laid down as universal laws. If it is desired to contract a long multiplication or division, the only rule is, 'Multiply or divide in the usual way, but leave out figures which are useless and whose uselessness is obvious to you.'

For purposes of practice in multiplication and division the following set of examples is inserted.

Examples VIII c.

[In all cases check the result by a rough calculation.]

Find the result of the following multiplications and divisions:

(a) to five significant figures:

1. $72{\cdot}436 \times 105{\cdot}92$.
2. $736{\cdot}89 \times 42\,822$.
3. $834{\cdot}61 \times 107{\cdot}63$.
4. $52{\cdot}4273 \times {\cdot}0082371$
5. $604{\cdot}52 \times 91{\cdot}321$.
6. $8{\cdot}4269 \times {\cdot}0072643$.
7. $(3{\cdot}2462)^2$.
8. $(8{\cdot}59321)^2 + (7{\cdot}4263)^2$.
9. $(5{\cdot}4276)^2 - (3{\cdot}2419)^2$.
10. $(73{\cdot}821)^2 - (48{\cdot}796)^2$.
11. $57{\cdot}623 \div 7{\cdot}8294$.
12. $158{\cdot}726 \div 54{\cdot}391$.
13. ${\cdot}74621 \div {\cdot}0092346$.
14. $1{\cdot}8421 \div {\cdot}024673$.
15. $72{\cdot}324 \div 1083{\cdot}2$.
16. $734{\cdot}42 \div (1{\cdot}035)^2$.
17. $67{\cdot}427 \div 53{\cdot}891$.
18. ${\cdot}012476 \div 4{\cdot}3212$.

(b) to a degree of accuracy justified by the data:

19. $32{\cdot}49\ldots \times 67{\cdot}21\ldots$
20. $43{\cdot}82\ldots \times 65{\cdot}32\ldots$
21. $107{\cdot}63\ldots \times 309{\cdot}21\ldots$
22. $405{\cdot}8\ldots \times {\cdot}007231\ldots$
23. $45{\cdot}82\ldots \times {\cdot}07635\ldots$
24. $909{\cdot}7\ldots \times 1{\cdot}428\ldots$
25. $66{\cdot}82\ldots \times 5{\cdot}848\ldots$
26. $207{\cdot}69\ldots \times 13{\cdot}423\ldots$
27. $759{\cdot}59\ldots \times 9{\cdot}5932\ldots$
28. $(4{\cdot}625\ldots)^2$.
29. $(5{\cdot}927\ldots)^2$.
30. $(103{\cdot}62\ldots)^2 - (99{\cdot}41\ldots)^2$.
31. $(107{\cdot}62\ldots)^3$.
32. $(43{\cdot}45\ldots)^3$.
33. $706{\cdot}4\ldots \div 52{\cdot}76\ldots$
34. $1803{\cdot}5\ldots \div 723{\cdot}4\ldots$
35. $843{\cdot}65\ldots \div (1{\cdot}045)^2$.
36. $742{\cdot}47\ldots \div {\cdot}9231\ldots$
37. ${\cdot}4293\ldots \div {\cdot}8764\ldots$
38. $54{\cdot}727\ldots \div {\cdot}008234\ldots$
39. $\dfrac{72{\cdot}45\ldots \times 63{\cdot}72\ldots}{15{\cdot}54\ldots}$.
40. $\dfrac{107{\cdot}3\ldots \times {\cdot}00824\ldots}{(23{\cdot}24\ldots)^2}$.

Oral Examples.

1. A square has a side of 7 in. What is its area?

2. A square has an area of 49 sq. in. What is the length of its side?

3. Give the lengths of the sides of squares whose areas are:

(i) 121 sq. ft.; (ii) 144 sq. chains;
(iii) 36 sq. cm.; (iv) 1·69 sq. yds.;
(v) 8·1 acres; (vi) 400 ares,
(vii) 1600 sq. mm.; (viii) 324 sq. ft.;
(ix) 3·24 sq. ft.; (x) 32400 sq. yds.

4. Give, as nearly as you can, a rough idea of the lengths of the sides of the squares whose areas are:

(i) 72 sq. cm.; (ii) 153 sq. ft.;
(iii) 69 sq. yds.; (iv) 700 sq. ch.;
(v) 23 acres; (vi) 500 sq. yds.

5. Give the values of:

(i) 1^2; (ii) 10^2; (iii) 100^2;
(iv) $\cdot1^2$; (v) $\cdot01^2$; (vi) $\cdot001^2$

6. If a number is between 1 and 100, between what values will its square root lie?

7. If a number is between 1 and ·01, between what values will its square root lie?

8. If a number is between ·0001 and ·000001, between what values will its square root lie?

(123.) In many problems involving areas it is necessary to find square roots. In Chap. V we have seen how to find the square root of any number which can be readily factorized and is an exact square.

When we come to questions on areas, we find that we are often set to discover the value of expressions such as

$$\sqrt{3}, \quad \sqrt{30}, \quad \sqrt{290}, \quad \sqrt{\cdot007}.$$

These cannot be evaluated exactly, but if we are asked to find $\sqrt{3}$ we can find some number which is such that the result of multiplying it by itself gives very nearly 3.

Thus $(1{\cdot}7321\ldots)^2 = 3{\cdot}0002\ldots$

$= 3{\cdot}000\ldots$

(correct to four significant figures)

$$\begin{array}{r} 1{\cdot}7321 \\ 1{\cdot}7321 \\ \hline 1{\cdot}7321 \\ 1{\cdot}21247 \\ \quad 5196 \\ \quad\quad 346 \\ \quad\quad\ 17 \\ \hline 3{\cdot}0002 \end{array}$$

We have, then, every reason to state that approximately

$$\sqrt{3} = 1{\cdot}732\ldots.$$

(**124.**) We proceed to discover a method for finding such square roots.

The first thing to notice is that we can see at once the approximate size of the square root of any number.

Since $1^2 = 1$ and $10^2 = 100$, we see that a number lies between 1 and 10, its square will lie between 1 and 100. Conversely, a number between 1 and 100 will have a square root between 1 and 10.

We may put this statement into the form · A number with **one** or **two** digits before the decimal point will have a square root with one digit before the decimal point.

Similarly, a number between 100 and 10,000 (i. e. with **three** or **four** digits before the decimal point) will have a square root between 10 and 100 (i. e. with **two** digits before the decimal point).

Now consider the number 6723400.

If we wish to find a square root of this number we space off the digits into pairs : 6,72,34,00

We see now that the number lies between 1000^2 and 10000^2, and hence that there will be four digits before the decimal point in the square root.

Also, since $2000^2 = 4,00,00,00,$

and $3000^2 = 9,00,00,00,$

we see that the square root will lie between 2000 and 3000. In fact, the first digit will be 2.

Similarly in $\sqrt{3425{\cdot}212}$, there will be two digits before the decimal point, and the first of these digits will be 5.

(Since $50^2 = 2500$ and $60^2 = 3600$.)

Oral Examples.

The square roots of the following numbers are required. State (*a*) the number of digits before the decimal point in the result, (*b*) the first of these digits:

1. 34291.	**2** 764230.	**3.** 54291.
4. 7362·2.	**5.** 8492.	**6.** 1·2234.
7. 307 32.	**8.** 23·65.	**9.** 172004.
10. 13000.	**11** 2.	**12.** 63491.
13. 14·221.	**14.** 73441.	**15.** 1892·2.
16. 17344.	**17.** 1734·4.	**18.** 60.
19. 600.	**20.** 6000.	**21.** 123456.
22. 654321.	**23.** 6543·12.	**24.** 6543·21.
25. 80·921.	**26.** 70·324.	**27.** 7·0324.

(**125.**) If a number is less than unity, i.e. is a pure decimal, the same remarks apply.

A number between 10^0 and 10^{-1} has a square between 10^0 and 10^{-2} (i.e. between 1 and ·01).

Conversely, a number between 1 and ·01 has a square root between 1 and ·1.

Thus $\sqrt{{\cdot}34273}$ has a square root between ·5 and ·6.

($·5^2 = ·25$ and $·6^2 = ·36$.)

So also $\sqrt{{\cdot}00341}$ has a square root between ·05 and ·06.

Similarly the first significant figure in the square root of .00000821 is ·002.

Oral Examples, continued.

Give the first significant figure in the square roots of the following numbers:

28. ·54324.	**29.** ·0046321.	**30.** ·0008416.
31. ·000044.	**32.** ·3.	**33.** ·03.
34. ·52763.	**35.** ·0000054.	**36.** ·0000008.
37. ·02.	**38.** ·002.	**39.** ·0002.
40. ·0403.	**41.** ·00403.	**42.** ·000403.
43. ·5008.	**44.** ·000197.	**45.** ·00197.

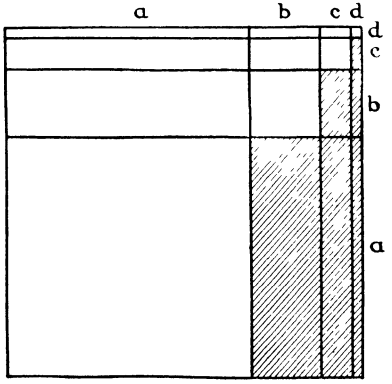


Fig. 9.

(**126.**) Having found the first significant digit of a square root, we use it to help us to find as many more digits as we require.

Fig. 9 represents a square on a side composed of the sum of the four lengths *a*, *b*, *c*, *d*.

The shaded part is removed from its position in the square, and is placed in a new position as shown in fig. 10.

Now the whole square is equal in area to the rather complicated figure which looks like a number of slabs placed upon each other.

The area of the lowest slab is a^2.

The one immediately above it is a rectangle whose sides are $2a+b$ and b;

$$\therefore \text{ its area is } (2a+b)\,b.$$

Similarly the other layers have areas

$$(2a+2b+c)\,c \quad \text{and} \quad (2a+2b+2c+d)\,d$$

Hence we have

$$\begin{aligned}(a+b+c+d)^2 = \quad & a^2 \\ & +b\,(2a+b) \\ & +c\,(2a+2b+c) \\ & +d\,(2a+2b+2c+d).\end{aligned}$$

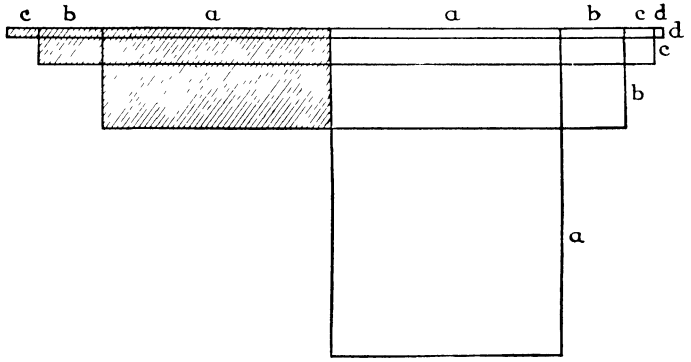


Fig. 10.

The method may be extended to squares whose sides are made up of any number of lengths.

(127.) Now consider the number $(435)^2$.

We have

$$\begin{aligned}(435)^2 &= (400+30+5)^2 \\ &= 400^2+30\,(800+30)+5(860+5) \\ &= 160000+30\times 830+5\times 865 \\ &= 160000+24900+4325 \\ &= 189225.\end{aligned}$$

This is not the quickest way of calculating the square of a number, but the idea is of the greatest importance when performing the reverse operation of calculating square roots.

To illustrate the operation, we will calculate the square root of the above number 189225, and the student is advised to compare very carefully the two workings.

Example. *To find* $\sqrt{189225}$.

We space off the digits into pairs, 18,92,25, and find that the first significant figure is 400.

Subtract 400^2, i. e. 16,00,00.

Remainder is 2,92,25.

We now use 2×400, i. e. 800, as a 'trial divisor' on this remainder. The quotient is 30.

Subtract $30 \times (800 + 30)$ or 24900. The remainder is 43,25.

We now use 2×430, i. e. 860, as a 'trial divisor'. The quotient is 5.

$$5 \times (860 + 5) = 43{,}25.$$

And the square root 'comes out'.

The actual method of performing the work would probably be as follows:

```
      4 3  5
    ---------
    18,92,25
    16
   ---------
83 ) 292
      249
   ---------
865 ) 4325
       4325
      ------
```

Here the zeros are omitted, and several devices for shortening the work are used.

(**128.**) When a square root does not 'come out' we continue the process until we have obtained a reasonable degree of accuracy.

Example i. *Find the square root of* 3079·2.*

```
    30,79·2          | 55·490 . . . .   nearly.
    25
105 ) 579
      525
110·4 ) 54·20
        44·16
110·89 ) 10·0400
           9·9801
110·98 ) 59900
```

The square root is approximately 55·490.

Check.

```
    55·490
      55·49
  ---------
  2774·50
   277·45
    22·20
     4·99
  ---------
  3079 1
  =======
```

We might have obtained greater accuracy by finding a few more figures of the square root, but if we had done so, it would have been better to have curtailed the work as in the following example.

---

* *Note* —The student is advised to refer this example back to the working of art. 126 ·

Let $a = 50$, $b = 5$, $c = \cdot 4$, $d = \cdot 9$

We have

1st partial product	$= a^2$		$= 2500$
2nd ,, ,,	$= (2a + b)\, b$		
	$= 105 \times 5$		$= 525$
3rd ,, ,	$= (2a + 2b + c)\, c$		
	$= 110 \cdot 4 \times \cdot 4$		$= 44 \cdot 16$
4th ,, ,,	$= (2a + 2b + 2c + d)\, d$		
	$= 110 \cdot 89 \times 09$		$= 9 \cdot 9801$
∴	$(a + b + c + d)^2$		$= 3079\ 1$ approx.
i.e.	$(55 \cdot 49)^2$		$= 3079 \cdot 1$ approx.

Example 11. *Find the square root of* 1037·34 . . . . .

```
                10,37·34 . . .      | 32·2077 . . . . .
                 9
              ------------
62            137·
              124·
              ------
64·2           13·34
               12·84
               -----
64·4             ·50
                 45
                  5
```

The last two figures are obtained by division, since the trial divisors have reached such large dimensions as to dwarf completely the extra figures which would usually be added on before performing the divisions. It can be shown that, **when just over half the required figures have been obtained by the square root method, the remaining figures can be found by division.**

Check.

```
      32·2077
          32·2077
      ----------
     966·231
      64·415
       6·442
        ·225
          23
     ---------
     1037·34
     =======
```

The above example illustrates a point similar to that raised in § 117. When we are given the area of a square correct to a certain degree of accuracy, we cannot expect to be able to calculate the length of a side of the square to a greater degree of accuracy than that of the original data.

Thus in this example the original number representing an area was given correct to six significant figures, and we obtained six significant figures in the square root.

Our degree of accuracy was limited by the fact that we had no right to add zeros after the last significant figure 4 of the original number.

Example iii. *Find the length of a side of a square whose area is* 407·3 . . . . *sq. in.*

407·3	| 20·18 . . . .
4	
40) 7·3	
4·0	
3·3	

The side is 20·18 . . . in.

We give an example to show the working still further curtailed by a modification of the 'Italian method' of division.

Example iv. *Find the value of* $\sqrt{20}$ *correct to nine figures.*

(This is an 'abstract' question. Pretence of such accuracy would be unreasonable if we were dealing with concrete areas.)

Here five figures must be found by the square root method, the remaining four figures being obtained by ordinary division.

	20·	| 4·47213596 . . . . .
8·4	1·00	
8·87	6400	
8·942	19100	
8·9441	121600	
	32159	
	5327	
	855	
	50	

The square root is 4·47213596 . . . . .

NOTE.—For many practical purposes it is convenient to use tables of square roots. Those students who have access to such tables are advised to refer to them to check their results in the following examples. In using the tables the decimal point in the square root must be fixed by ordinary common sense.

Thus, if the square root of 407·3 is required, we decide first of all that the result must be about 20. Neglecting all decimal points, and referring to the tables, we find the square roots corresponding to the figures 4073 given as either 2018 or 6382. The square root of 407·3 is then written down as 20·18, in view of our preliminary rough estimate.

Had we been seeking the square root of 407300, we should first have decided that the result was to be something over 600, and the tables would have given us 638·2. Similarly, $\sqrt{4{\cdot}073} = 2{\cdot}018$ and $\sqrt{{\cdot}004073} = {\cdot}06382$.

Examples VIII d.

(Check every third answer by multiplication.)

Find the square roots of the following numbers:

1. 1550025.	2. 547·56.	3. 53·1441.
4. 50·1264.	5. 788544.	6. 8262·81.
7. ·5929.	8. ·427716.	9. ·007569.
10. 25·3009.	11. ·009801.	12. ·00003136.

Find the square roots of the following numbers (which are to be assumed to be exact) to five significant figures.

13. 1032·24.	14. 736·2.	15. 804·91.
16. 176.	17. 3·432.	18. ·00549.
19. 75.	20. 62.	21. 8.
22. ·002.	23. ·5.	24. 7.

The following numbers are correct to the degree of accuracy given. Find their square roots as accurately as possible.

(Do not assume any zeros at the end of the number, and obtain the last few digits by division.)

25. 23·4219.	**26.** 607·22.	**27.** 85·4937.
28. 659·25.	**29.** 574·631.	**30.** 2·5941.
31. 3·14159.	**32.** ·0059621.	**33.** ·00043245.
34. ·0043245.	**35.** ·043245.	**36.** ·000005976.

Find as accurately as possible the lengths of the sides of squares whose areas are as follows:

37. 37·64 sq. metres.	**38.** 273·4 acres.	**39.** 307·6 sq. yds.
40. 142·49 Hectares.	**41.** 347·61 sq. Hm.	**42.** 33·49 sq. in.
43. 95·43 acres.	**44.** 56 acres 7 sq. ch.	**45.** 43·899 Ha

(**129.**) Since $\frac{2}{3} \times \frac{2}{3} = \frac{4}{9}$, the square root of a fraction whose numerator and denominator factorize readily and are perfect squares can be found by taking the square root of *both* numerator and denominator.

If a fraction is given as a 'mixed number' it must be made into an improper fraction before finding the square root. Thus $\sqrt{6\frac{1}{4}} = \sqrt{\frac{25}{4}} = \frac{5}{2} = 2\frac{1}{2}$

Oral Examples.

Give the square roots of the following:

1. $\frac{25}{49}$.	**2.** $\frac{16}{25}$.	**3.** $\frac{121}{144}$.	**4.** $1\frac{23}{121}$.
5. $1\frac{24}{25}$.	**6.** $3\frac{1}{16}$.	**7.** $1\frac{25}{144}$.	**8.** $1\frac{9}{16}$.
9. $2\frac{2}{49}$.	**10.** $11\frac{1}{9}$.	**11.** $30\frac{1}{4}$.	**12.** $3\frac{13}{36}$.
13. $42\frac{1}{4}$.	**14.** $12\frac{1}{4}$.	**15.** $1\frac{40}{81}$.	

VOLUMES

(**130.**) The units of volume are the cubic foot, the cubic inch, the cubic centimetre, &c.

As in the case of units of area, the cubic inch is defined as a cube, each of whose edges is one inch long

Fig. 11 shows a rectangular solid whose edges represent 6 inches, 5 inches, and 4 inches. It may be divided up into six layers similar to the upper figure, and each of these

layers contain 5×4 or 20 cubic inches The total amount is then $5 \times 4 \times 6$ or 120 cubic inches.

We may employ identically the same argument as applied to areas for stating that if the dimensions (i. e. the lengths of

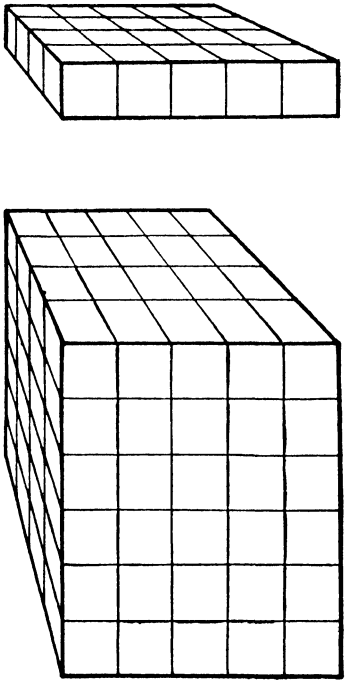


Fig. 11.

the edges) of such a rectangular solid are a, b, and c units respectively, then the volume is $a \times b \times c$ cubic units.

We find also the same difficulties in connexion with volumes as we found in dealing with areas, when the dimensions are not given exactly.

Example i. *A room has length* 23 *ft., breadth* 18 *ft., height* 11 *ft. How many cubic feet of space are there in it?*

The number of cubic feet is:

$$23 \times 18 \times 11$$

$$= 4554.$$

253
1518
4554

Example ii. *A solid rectangular block of wood has edges* 37·32″, 23·41″, 10·34″. *What is its volume?*

The volume is $37{\cdot}32 \times 23{\cdot}41 \times 10{\cdot}34$ cu. in

$$= 9034 \text{ cu. in. approx.}$$

37 32
23·41
746·4
111·96
14·93
·37
873·7
10·34
8737·
262·1
34·9
9034·

Example iii. *A reservoir has a surface of* 43·41 *ares. What is its depth if it contains* 93742 *Kl. of water?*

1 Are = 1 sq. Dm.
∴ Area of section = 43·41 sq. Dm.
A litre = 1 cu. dm.
∴ Volume = 93742000 cu. dm.
= 93742 cu. m.
= 93·742 cu. Dm.

$$\therefore \text{ Depth } = \frac{93{\cdot}742}{43{\cdot}41} \text{ Dm.}$$

$$= 2{\cdot}159 \text{ Dm.}$$

$$= 21{\cdot}59 \text{ metres approx.}$$

2·1594
4 341) 9·3742
·6922
·2581
410
19

Examples VIII e.

Find the volumes of the solids having the following edges (the data are to be considered correct only to the degree given).

1. 32·4 m., 17·5 in., 12·6 in.
2. 407·3 ft., 58·4 ft., 22·1 ft.
3. 64·52 metres, 27·69 m., 18·58 m.
4. 73·68 metres, 39·24 m., 23·25 m.
5. 42·67 cm., 31·82 cm., 12·65 cm.
6. 35·81 cm., 18·73 cm., 15·42 cm.

7. Find the volume in cubic inches of a marble slab 2 ft. 4 in. long, 1 ft. 8 in. wide, and 1 in. thick.

8. How many cubic yards of gravel are required to cover an acre of ground to a depth of 3 inches?

9. The internal dimensions of a tank are: length 5 metres, breadth 350 cm., depth 72 cm. Find the number of litres it will hold.

10. A rectangular tin vessel is 1 m. 30 cm. long, 24 cm. broad, and 7·5 cm. deep; find its capacity in litres.

11. Find, in kilograms, the weight of a rectangular block of wood 75 cm. long, and of section 58 sq. cm., if the weight of a cubic decimetre of the wood is 825 gr.

12. Find the number of litres in a cistern 1·26 m. long, 76 cm. broad, and 60 cm. deep.

13. Find, to the nearest inch, to what depth 100,000 cubic feet of water will fill a reservoir which has an area of an acre.

14. What is the area in square metres of the bottom of a truck which is filled to a depth of 80 cm. by 10 cu. metres of sand.

15. Find to the nearest inch to what depth 400 cu. yds. of gravel will cover an acre of ground.

16. If a cubic cm. of lead weighs 11·3 gr., find, to the nearest Kg., the weight of sheet lead, 2·5 mm. thick, to cover a roof 10·4 m. long by 8·4 m. broad.

Illustrative Problems.

Example i. *If a rectangular pathway which measures* 787·4 *metres in length and* 1·52 *metres in width, is made at a cost of* 1·5 *francs per square metre, find the length, correct to the nearest inch, of a similar pathway* 6 *feet wide, which costs* 1*s.* $1\frac{1}{2}$*d. per square yard, the total cost being the same.* (1 *metre* = 39·47 *inches,* £1 = 25 *francs.*)

Area of 1st pathway $= 787{\cdot}4 \times 1{\cdot}52$ sq. metres.

Cost of 1st pathway $= 787{\cdot}4 \times 1{\cdot}52 \times 1{\cdot}5$ francs.

$\therefore$ cost of 2nd pathway $= \dfrac{787{\cdot}4 \times 1{\cdot}52 \times 1{\cdot}5 \times 20}{25}$ shillings.

$= 787{\cdot}4 \times 1{\cdot}52 \times {\cdot}3 \times 4$ shillings.

But cost of 1 sq. yard $= 1\frac{1}{8}$ shillings.

$\therefore$ area of 2nd pathway $= \dfrac{787{\cdot}4 \times 1{\cdot}52 \times 1{\cdot}2 \times 8}{9}$ sq. yards.

But width of 2nd pathway = 2 yds.

$\therefore$ length of 2nd pathway $= \dfrac{787{\cdot}4 \times 1{\cdot}52 \times 1{\cdot}2 \times 8}{9 \times 2}$ yards.

After some practice most of these steps can be omitted, but they all have to be thought, even though not written. Thus the argument might appear as only one line.

Length of 2nd pathway $= \dfrac{787{\cdot}4 \times 1{\cdot}52 \times \overset{\cdot 3}{1{\cdot}5} \times \overset{4}{20}}{25} \times \frac{8}{9} \times \frac{1}{2}$ yards

$= 787{\cdot}4 \times 1{\cdot}52 \times {\cdot}4 \times \frac{4}{3}$ yds.

$= 638{\cdot}32$ yards

$= 638$ yds. 1 ft.

787·4
393·7
15·75
1196·85
4787·40
1914·96

Example ii. *Determine the weight in pounds of a box, with a lid, made of wood* $\frac{3}{4}$ *of an inch thick, and measuring externally* 4 *ft. by* 3 *ft. by* 3 *ft., the weight of a cubic foot of the wood being* 38·4 *lb.*

The internal dimensions are obtained by subtracting twice the thickness of the wood from each of the external dimensions

Total volume occupied by box
$= 4 \times 3 \times 3 = 36$ cu. ft.

Volume of internal cavity
$= 3\frac{7}{8} \times 2\frac{7}{8} \times 2\frac{7}{8}$ cu. ft.

$\therefore$ volume of wood $= 36 - \dfrac{31 \times 23 \times 23}{8 \times 8 \times 8}$ cu. ft.

$= 3{\cdot}97$ cu. ft.

$\therefore$ weight of wood $= 38{\cdot}4 \times 3{\cdot}97$ lb.
$= 115{\cdot}2$
$+ 34{\cdot}6$
$+ \ 2{\cdot}7$
$= 152{\cdot}5$ lb.
$= 153$ lb. to the nearest lb.

529×31

1587

$8)16399$

$8)2049{\cdot}8$

$8)256{\cdot}23$

$32{\cdot}03$

Example iii. *A gravel walk 6 feet wide runs round a lawn 60 feet long and 40 feet wide. If gravel is 3s. per cubic yard, find the cost of a coat of gravel on the path, 3 inches deep.*

The path and lawn form a rectangle 72 feet long, 52 feet wide; the area of which $= 72 \times 52$ sq. feet.

Area of lawn $= 60 \times 40$ sq. feet.

$\therefore$ area of path $= 72 \times 52 - 60 \times 40$ sq. feet
$= 3744 - 2400$ " "
$= 1344$ sq. feet.

$\therefore$ volume of gravel $= 134\frac{4}{9} \times \frac{1}{4}$ cu. feet.

And cost of gravel $= 336 \times \frac{1}{27} \times 3$ shillings
$= 37\frac{1}{3}$ shillings
$= £1 \ 17s. \ 4d.$

416×9

Examples VIII.

1. Find to the nearest foot the length of a side of a square field of $8\frac{2}{3}$ acres.

2. How many planks 18 feet long and 8 inches wide are required to make a platform 36 yards long and 15 yards wide?

3. How many litres of water are required to fill a cistern 2·3 m. long and 1·6 m. wide, to a depth of ·85 m.?

4. Find the cost of carpeting a room 6·2 metres long and 3·6 metres broad, with carpet 72 cm. wide costing 5 fr. 50 c. per metre.

5. Find the least number of pieces of paper, 12 yds. long by 21 in. wide, required to paper the walls of a room 17 ft. 4 in. long, 14 ft. 2 in. broad, 10 ft. 8 in. high.

6. Find the price of a carpet 6 m. 24 cm. long, and 4 m. 35 cm. broad, at 8 fr. 5 c. per sq. metre.

7. What will be the cost of paper 20 in. wide, at $3\frac{1}{2}d.$ a yard, for a room 21 ft. long, 15 ft. wide, and 10 ft. high?

8. If a cubic foot of iron weighs 7800 oz., find to the nearest ounce the weight of a uniform bar of iron which is 14 ft. 3 in. long, and whose cross-section is one sq. inch.

9. A rectangular room is 6 m. 25 cm. high, 15 m 60 cm. long, and 7 m. 8 cm. broad; find the cost of carpeting the floor at 4*s.* a sq. metre, and of papering the walls at 6*d.* a sq. metre.

10. What length of paper would be needed to paper the walls of a room 16 ft. 3 in. long, 13 ft. 9 in. broad, and 14 ft. 6 in. high, the breadth of the paper being $22\frac{1}{2}$ in.?

11. A rectangular box, without a lid, is to be made of wood half an inch in thickness. Its outside dimensions are to be 14 in. long, 12 in. broad, and $7\frac{1}{2}$ in. deep. What length of wood $3\frac{1}{2}$ in. wide will be required?

12. A piece of land, a quarter of a mile long and 22 yards wide, is required for a railway. How much must be paid for it at £350 per acre?

13. A square field costs 12580 fr. to level and turf at the rate of 85 c. per sq. m.; find the length of a side, correct to the nearest tenth of a metre.

14. Find the total cost incurred in laying down in permanent pasture a field which is a rectangle 430 yards by 223 yards, allowing for four ploughings of the field, each ploughing costing 11*s.* 6*d.* per acre, and taking the cost of the seed and sowing to be 16*s.* 3*d.* per acre. Give your answer to the nearest shilling.

15. What is the cost of turf for a lawn 120 ft. long and 90 ft. wide, at 3*s.* per 100 pieces of turf, each 3 ft. by 1 ft.?

16. It cost £42 to carpet the whole floor of a room 30 ft. long and 28 ft. wide. If a space 1 ft. 3 in. wide be left uncovered all round, what is the saving in cost?

17. How long will it take to roll a field of $3\frac{1}{2}$ acres with a roller 3 ft. $2\frac{1}{2}$ in. wide, working at the rate of 3 miles an hour?

18. A heavy block of concrete, measuring 9 ft. by 9 ft. by 10 ft., is being lowered into a square pit measuring 12 ft. by 12 ft., and 10 ft. deep, which is half full of water. Find the volume of the block of concrete, the volume of water in the pit, and the number of cubic feet of water that will run over when the block is lowered into the pit.

19. Water enters a cistern at the rate of 10 cubic inches in a second. How long will the cistern take to fill, if its base is a square yard, and its depth 2 ft. 6 in.? Give the answer to the nearest minute.

20. On the floor of a room 25 ft. long and 16 ft. wide, there is a cork carpet measuring 22 ft. 6 in. by 13 ft. 6 in., costing 3*s.* 3*d.* a square yard, and a strip of matting 18 inches wide all round the sides of the room, costing 10*d.* a yard. Find the total cost of carpet and matting.

21. How many planks, each 2·5 metres long and 20 cm. broad, will be required for the floor of a room 15 metres long by 10 metres wide?

22. A croquet ground of rectangular shape, 35 yards long and 28 yards wide, is to be surrounded by a rectangular fence of wire netting placed at a distance of 27 inches from the sides of the ground. How many yards of netting will be required, and what is the area enclosed by the wire? (Give the result as the decimal of an acre.)

23. A rectangular field containing 11 acres is twice as long as it is broad. Find, to the nearest penny, the cost of enclosing it by a fence at the rate of 4*s.* 3*d.* per yard.

24. If rain falls to a depth of $\frac{3}{4}$ of an inch, find, to the nearest ton, the weight of the rain that falls on an acre of land. (A cubic foot of water weighs 62·3 lb.)

25. Find the cost of covering a rectangular courtyard 85 ft. 6 in. long, and 52 ft. 6 in. wide, to a depth of 3 in. with gravel costing 5*s.* 8*d.* a cubic yard.

26. Find the area of the walls of a room 23 ft. 8 in. long, 18 ft. 4 in. wide, and 12 ft. high, and the expense of covering them with paper 3 ft. 6 in. wide, costing 4*s.* 6*d.* per dozen yards.

27. Find the cost, at 4*s.* 6*d.* per square yard, of a bordering 2 ft. wide of parquet flooring all round a room, whose length is 18 ft. 7 in., and breadth 15 ft. 9 in.

28. The internal dimensions of a box without a lid are as follows: length 4 ft. 2 in., breadth 2 ft. 4 in., depth 2 ft. 1 in.

Find the cost of lining with a material costing 6*d.* per sq. foot.

29. A square swimming bath with vertical sides contains a million gallons of water when filled to a depth of eight feet. Find, in feet, the length of a side of the bath. (A cu. ft. = $6\frac{1}{4}$ galls.)

30. The rainfall on a roof 20 ft. long and 12 ft. wide is collected in a covered rectangular cistern 5 ft. long, 4 ft. wide, and 3 ft. deep; what depth of water will be found in the cistern after a fall of 2 inches of rain?

31. A square field of grass is surrounded by a path of uniform breadth; the field and path together occupy 8·41 Ha., and the path alone ·04 Ha. Find the breadth of the path.

32. A square field contains ten acres. It has to be fenced all round, and a path 6 ft. wide has to be laid out all round close to the fence inside If the cost of fencing is 5*s.* per yard, and the cost of preparing the path 2*s.* $1\frac{1}{2}d.$ per square yard, find the total expense.

33. A rectangular box, with a lid, is made of wood 15 mm. thick. The external measurements are 60 cm., 55 cm., and 53 cm Find its weight if a cubic decimetre of the wood weighs 850 gr.

34. What will it cost to paper a recess 3 ft. deep, 6 ft. long, and 15 ft. high, with paper sold in rolls, 12 yards long and 21 inches wide, at 1*s.* 10*d.* a roll?

35. A log of timber, 12 ft. 6 in. long, $10\frac{1}{2}$ in. wide, and 9 in. thick, weighs when first cut down 321·3 lb. If it loses one-fifth of its weight in drying, the volume remaining unaltered, what is the weight of a cubic foot when dry?

36. The whole of the square front of a target is to be painted in such a manner that it will show a white square surrounded by a black border of uniform width.

If each side of the front is 1·73 metres long, what must be the width of the black border, correct to the nearest millimetre, in order that one half of the front may be black?

37. Find the cost of papering a room 21 ft. long, $17\frac{1}{2}$ ft. wide, and 11 ft. high, with a 10 in. cornice and 14 in. skirting board, neither of which requires papering. The paper is 21 in. wide, and costs 6*d* a yard, and 9*d*. is charged for hanging each piece of 12 yards.

38. If a piece of iron plate 2·1 metres long, 1·13 metres broad, and 2·3 cm. thick, weighs 425 Kg, find, to the nearest gramme, the weight of a square metre of sheet iron 0·2 mm. thick.

39. The surface of a lake has an area of 3 acres 0 ro. 32 po. After rain, the level of the water rises $\frac{1}{4}$ inch. How many extra gallons of water are there in the lake if $6\frac{1}{4}$ gallons = 1 cu. foot?

40. The perimeter of a square field is a kilometre. Find its area in acres, given that a metre = 1·094 yds.

*

CHAPTER IX

RATIO

(131.) If we wish to compare two quantities, and to obtain a mathematical expression which will make the comparison clear, there are two ways by which we may set to work. For example, suppose that A has £100 and B £200.

For purposes of comparison we may say either

(i) B has £100 more than A, or

(ii) B has twice the amount that A has.

The first result is obtained by subtracting A's money from B's money, and the second result is obtained by dividing the number of pounds that B possesses by the number of pounds that A possesses.

In other words, the second result is obtained by forming a fraction whose numerator is the number of pounds in B's amount, and whose denominator is the number of pounds in A's amount.

A moment's thought will show that this second method of obtaining the comparison shows more clearly the relative wealth of A and B than does the first method; for if we are simply told that B has £100 more than A, we cannot compare their wealth without knowing a good deal more. A might be a pauper whilst B was comparatively wealthy with £100; or A might be a millionaire, with B only very slightly richer. But if we are told that B has just twice the amount that A has, we know at once a good deal more about their relative wealth.

(**132.**) So, in comparing any two sums of money, or two lengths, or two weights, we shall usually form a fraction in which the numerator is the number of units in one of these quantities, and the denominator the number of units in the other. We shall call such a fraction the **ratio** of the two quantities.

If the two quantities, X and Y, contain respectively x and y of the same units, the ratio of X to Y will be the fraction x/y.

It should be noticed that this fraction, when formed, is a mere **number**, and not a number of pounds, or yards, or tons; in fact **a ratio is always an abstract number.**

The fraction x/y may be reduced to its lowest terms If it simplified down to $\frac{2}{3}$, we should say that the ratio of X to Y is 2 to 3, or that X and Y are in the ratio of 2 to 3. This is sometimes written in the form

$$X \,.\, Y \;.\; 2 : 3,$$

but this notation is dying out.

(**133.**) It is most important when constructing ratios to see that both the quantities are expressed in terms of the same unit.

For, just as it is impossible to compare a shilling with a ton, so is it impossible to compare a sovereign (in gold) with a sixpenny piece (in silver) until we have discovered that a sovereign is the equivalent of 240 pennies, and a sixpenny piece is the equivalent of 6 pennies.

Then it is easy enough to see that a sovereign is to a sixpenny piece in the ratio of 240 to 6, i. e. $\frac{240}{6}$ or $\frac{40}{1}$.

When we have constructed this ratio we can say at once that

A sovereign is worth 40 sixpenny pieces,

A sixpenny piece is worth $\frac{1}{40}$ of a sovereign,

and if asked, 'What fraction is sixpence of a sovereign?' we reply at once, 'one-fortieth'.

(**134.**) Clearly the ratio of two quantities is the same in whatever units they are measured.

Thus we might have said that a sovereign = 20 shillings and a sixpenny piece = $\frac{1}{2}$ shilling, and so the ratio of a sovereign to a sixpenny piece is

$$\frac{20}{\frac{1}{2}} = 40 \text{ as before.}$$

It is sometimes useful to express this fraction, when found, as a decimal, in which case we should be answering a question such as: 'What decimal is sixpence of a sovereign?' The answer is ·025.

Oral Examples.

In the following examples, give the ratio of each quantity in the first column to each quantity in the second column.

1.		4.	
(α) 3*s*.	(α) 15*s*.	(α) 3 m. 2 dm.	(α) 2 m. 4 dm.
(β) 7*s*	(b) £1 15*s*	(β) 4 dm. 5 cm.	(b) 3 Dm. 6 m.
(γ) £3 10*s*.	(c) 6*d*.	(γ) 2 Dm. 7 m	(c) 4 m. 8 dm.
(δ) 4*d*.	(d) 5*s*. 6*d*.	(δ) 1 m. 6 dm.	(d) 6 dm. 3 cm.
2.		**5**	
(α) 3 yds.	(α) 2 yds.	(α) 1 ch. 20 lks	(α) 2 ch. 40 lks.
(β) 2 ft. 4 in.	(b) 2 yds. 2 ft.	(β) 4 fur.	(b) 48 lks
(γ) 1 yd. 2 ft.	(c) $4\frac{1}{2}$ in.	(γ) 72 lks.	(c) 2 ch.
(δ) $3\frac{1}{2}$ in.	(d) 1 ft. 6 in.	(δ) 3 ch.	(d) 1 fur.
3.		**6.**	
(α) 2 lb.	(α) 1 qr.	(α) 3 Kg. 5 Hg.	(α) 5 Kg.
(β) 3 lb. 4 oz.	(b) 5 lb.	(β) 4 Kg.	(b) 5 Hg
(γ) 11 oz.,	(c) 7 lb. 2 oz	(γ) 2 Hg. 5 Dg.	(c) 2 Kg. 8 Hg.
(δ) $\frac{1}{2}$ stone.	(d) 15 oz.	(δ) 3 Hg.	(d) 3 Kg.

(135.) The following columns contain several mixed quantities. They may be used for questions of the types:

1. Which quantities in column B can be compared with number 4 in column A?
2. What fraction is A 1 of B 3?
3. How many times is A 4 contained in B 9?

(Express as a decimal if the answer is not a whole number.)

4. Express A 13 as a decimal of B 16.
5. What is the ratio of A 16 to B 6?
6. Give the ratio of A 14 to each quantity in column B with which it can be compared.

Suppose we are comparing £4 with 10*s.*, we first say that £4 = 80 shillings.

Then.

Question.	Answer.
1. What is the ratio of £4 to 10*s.*?	8 to 1.
2. What is the ratio of 10*s.* to £4?	1 to 8.
3. What fraction is 10*s.* of £4?	$\frac{1}{8}$.
4. How many times is 10*s.* contained in £4?	8 times.
5. What decimal is 10*s.* of £4?	·125.
6. How many times is £4 contained in 10*s.*?	·125 times.

A.	B.
1. 2*s.* 6*d.*	1. 3 Dg. 6 g
2. 2 g.	2. 3 chains.
3. 2 tons.	3. 12*s.* 6*d.*
4. 4 dols. 25 cts.	4. 1 quarter (weight).
5. £1.	5. 100 dollars.
6. 2 fr.	6. 400 yds.
7. 12 cwt.	7. £3.
8. 3 dols. 40 cts	8. 21 lb.
9. 7*s.*	9. 8 dols. 50 cts.
10. 1 chain.	10. 14*s.*
11. 1 fr. 60 c.	11. $\frac{1}{4}$ mile.

A.	B.
12. 7*s.* 6*d.*	**12.** 34 dollars.
13. 1 dol. 70 cts.	**13.** 7 stone.
14. 14 lb.	**14.** 1 hr. 30 min.
15 2 fr. 40 c.	**15.** 4*s.* 6*d.*
16. 220 yds.	**16.** 12 dol. 75 cts.
17. 1 hour 20 min.	**17.** 24 g.
18. 17 dollars.	**18.** 1 mile.
19. 1 dg. 2 cg.	**19.** 3 fr. 20 c.
20. 3 stone.	**20.** 6 hours.
21. 11 yds.	**21.** 30 dg.
22. 36 min.	**22.** 4 fr.
23. 1 g. 5 dg.	**23.** 1 hour.
24. 6*s.* 8*d.*	**24.** 5 tons.

(136.) Example. *What is the ratio of* £4 16*s.* $2\frac{3}{4}d.$ to £5 14*s.* 8*d.*?

Both quantities must be expressed in terms of the same unit. This may be done in several ways.

(*a*)
$$\begin{aligned} \text{£4 } 16s.\ 2\tfrac{3}{4}d. &= 96s.\ 2\tfrac{3}{4}d. \\ &= 1154\tfrac{3}{4}d. \\ &= 4619 \text{ farthings.} \\ \text{£5 } 14s.\ 8d. &= 114s.\ 8d. \\ &= 1376d. \\ &= 5504 \text{ farthings.} \end{aligned}$$

The ratio is $\frac{4619}{5504}$.

4619	5504
1079	5 | 885
17	177

(The side work shows that these have no common factor.)

(*b*) The ratio $= \frac{96\frac{11}{48}}{114\frac{2}{3}}$ (Reducing each to shillings.)

$$= \frac{4619}{\cancel{48}\ 16} \times \frac{\cancel{8}}{344}$$

$$= \frac{4619}{5504}.$$

96
576
4608
344
1376
5504

(*c*) The ratio $= \dfrac{£4\ 16s.\ 2\frac{3}{4}d.}{£5\ 14s.\ 8d.}$

$= \dfrac{£19\ 4s.\ 11d.}{£22\ 18s.\ 8d.}$ (Multiplying numerator and denominator by 4.)

$= \dfrac{£230\ 19s.}{£275\ 4s.}$ (Multiplying numerator and denominator by 12.)

$= \dfrac{4619}{5504}$ (Multiplying numerator and denominator by 20.)

(*d*) The following is perhaps the most practical method, where a ratio is to be given as a decimal. The degree of accuracy is, however, small, and the method should be used only with great care.

Ratio $= \dfrac{4{\cdot}811\ldots}{5{\cdot}733\ldots}$ (Reducing each to the decimal of £1.)

$= {\cdot}839\ldots.$

$$
\begin{array}{r}
{\cdot}839\ldots \\
5\,733\,)\,4\,811 \\
{\cdot}2246 \\
526 \\
10
\end{array}
$$

Examples IX a.

[The student is advised to vary his method as much as possible.]

Express as a fraction in its lowest terms the ratio of:

1. £31 17*s.* 2*d.* to £59 8*s.* 4*d.*
2. £437 19*s.* 3*d.* to £342 7*s.* 11*d.*
3. £4 8*s.* $9\frac{1}{4}d.$ to £7 17*s.* $3\frac{1}{2}d.$
4. 3 tons 5 cwt. 2 qrs. to 5 tons 14 cwt. 3 qrs.
5. 61 tons 3 cwt. to 73 tons 2 cwt. 1 qr.
6. 13 cwt. 3 qrs. 16 lb. to 7 cwt. 2 qrs. 14 lb.
7. 14 lb. 5 oz. to 1 qr. 13 lb. 15 oz.
8. 17 fr. 10 c. to 32 fr. 40 c.
9. 122 dol. 10 cts. to 349 dol. 80 cts.
10. 3 Dm. 4 m. 5 dm. to 6 Dm. 5 dm.
11. 4 Kg. 7 Dg. to 3 Hg. 8 g.
12. 72 yds. 2 ft. to 105 yds.

13. 7 fur. 5 ch. to 1 mile 2 fur.
14. £41 2*s.* 9*d.* to £37 3*s.* 6*d.*
15. 62 fr. 70 c. to 105 fr. 60 c.
16. 42 dol. 70 cts. to 38 dol. 50 cts.
17. 24 tons 5 cwt. to 36 tons 11 cwt.
18. 6 Hm. 4 Dm. 2 m. to 7 Km. 3 Hm. 2 Dm.
19. 5 g. 7 cg. to 8 g. 7 cg.
20. 37 dol. 10 cts. to 64 dol. 40 cts.

Express the following ratios as decimals correct to three significant figures:

21. 6*s.* 3*d.* to 11*s.* 8*d.*
22. £22 4*s.* 2*d* to £35 3*s.* 11*d.*
23. £61 3*s.* 8*d.* to £103 4*s.* 9*d.*
24. 82 fr. 30 c. to 72 fr. 50 c.
25. 107 dol. 40 cts. to 370 dol. 30 cts.
26. 83 mks. 30 pf. to 69 mks. 90 pf.
27. 3 ft. 6 in. to 7 yds.
28. 1 mi. 2 fur. 7 ch. to 3 mi. 5 fur. 8 ch.
29. 6 cwt. 2 qrs. 7 lb. to 8 cwt. 3 qrs. 18 lb.
30. £107 9*s.* 9*d.* to £84 11*s.* 10*d.*
31. 32·34 metres to ·07963 Km.
32. 7 Kg. 9 Hg. 4 g. to 8 Kg. 3 Dg. 7 g.
33. 72 fr. 80 c. to 89 fr. 60 c.
34. 2 mi. 7 fur. 70 yds. to 3 mi. 6 fur. 92 yds.
35. 53 mks. 70 pf. to 78 mks. 80 pf.

Give the results of the following to four significant figures:

36. Express £3 7*s.* 2*d.* as a decimal of £72.
37. Express £8 9*s.* 4*d.* as a decimal of £17 5*s.* 10*d.*
38. Express £33 7*s.* 11*d.* as a decimal of £48 2*s.* 9*d.*
39. Express 84 fr. 60 c. as a decimal of 93 fr. 20 c.
40. Express 75 dol. 30 cts. as a decimal of 109 dol. 90 cts.
41. Express 30·75 metres as a decimal of 5·493 Dm.
42. Express 5 tons 3 cwt. 3 qrs. as a decimal of 8 tons 11 cwt.
43. Express 54 mks. 40 pf. as a decimal of 83 mks. 70 pf.

It is sometimes convenient to express the ratios of quantities of very different sizes as decimals in 'standard form'.

Thus the ratio of 3·4 cm. to 2·32 Hm. is:

$$\frac{3{\cdot}4 \times 10^{-4}}{2{\cdot}32} \quad \text{(expressing each quantity in Hm.)}$$

$$= 1{\cdot}465 \times 10^{-4}$$

$$\begin{array}{r} 1\,465\ldots \\ 2\,32\,)\,3\,4 \\ \underline{2\,32} \\ 1\,08 \\ \underline{\cdot 928} \\ \cdot 152 \\ \underline{\cdot 139} \\ 13 \end{array}$$

Express the following ratios as numbers in standard form, multiplied by some power of 10:

44. 3·72 cm. to 8·93 Hm.
45. 4·972 cm. to 9·954 Km.
46. 3·642 g. to 8·957 Kg.
47. 8·929 cg to 9·423 Kg.
48. 5·43 mm. to 3·72 m.
49. 3·176 m. to 3·21 mm.
50. $2{\cdot}432 \times 10^{-2}$ to $3{\cdot}491 \times 10^{1}$.
51. $8{\cdot}793 \times 10^{-3}$ to $9{\cdot}426 \times 10^{2}$.
52. $7{\cdot}694 \times 10^{-8}$ to $9{\cdot}423 \times 10^{-2}$.
53. 1 inch to 1 mile.
54. 6 inches to 1 mile.
55. 1 inch to 6 miles.

The diameter of the earth is 7926 miles.
The diameter of the sun is 866,400 miles.
The diameter of the moon is 2162 miles.
The distance between the earth and moon is 238,840 miles.
The distance between the earth and sun is $93{\cdot}1 \times 10^{6}$ miles.
The distance between the earth and the nearest fixed star is $2{\cdot}55 \times 10^{13}$ miles.

Express in standard form the ratios:

56. Diameter of moon to diameter of earth.
57. Diameter of earth to diameter of sun.
58. Distance between earth and moon to distance between earth and sun.

59. Distance between earth and moon to distance between earth and the nearest fixed star.

60. Diameter of earth to distance between earth and the nearest fixed star.

61. Distance between earth and sun to distance between earth and nearest fixed star.

The following table gives the equivalents of various foreign and colonial coins in English currency:

Canada. Dollar	$= 4s.\ 1\frac{1}{3}d.$
India. Rupee (fixed rating)	$= 1s.\ 4d.$
Belgium. Franc	$= 9{\cdot}513d.$
Denmark. Krone	$= 1s.\ 1\frac{1}{2}d.$
Austria-Hungary. Krone	$= 10d.$
Japan. Gold Yen	$= 2s.\ 0\frac{1}{2}d.$
Egypt Egyptian Pound	$= 20s.\ 3\frac{3}{4}d.$

Express as decimals the ratios:

62. £3 to 4 dollars. **63.** £2 10*s.* to 10·3 Rupees.
64. 4·31 Francs to 7*s.* 2*d.* **65.** 18 Krone (Denmark) to 17*s.* 6*d.*
66. 23 Krone (Austria) to £1. **67.** 53 Yen to £7.
68. £3 Egyptian to £3 English.
69. 4 Francs to 7 Krone (Denmark).
70. 3 Dollars to 8 Rupees. **71.** 1 Gold Yen to £1 Egyptian.
72. 5 Dollars to 40 Krone (Austrian).

(**137.**) **Percentages.** We have seen how two quantities can be compared by constructing the ratio between them, or the fraction that is formed by taking one of them (expressed in terms of a suitable unit) for numerator, and the other (expressed in terms of the same unit) for denominator. But since the easiest way of estimating the size of a fraction is to convert it to a decimal, we usually express our ratios, when obtained, as decimals.

Thus, to compare 6*s.* 6*d.* with £2 4*s.*, we write first the

ratio of these two quantities (expressing each in sixpences) as

$$\frac{13}{88}.$$

Converting this to a decimal, we obtain

$$\frac{13}{88} = \frac{1{\cdot}625}{11}$$
$$= {\cdot}1477\ldots$$

Thus, very approximately 6*s.* 6*d.* is ·15 of £2 4*s.*—or 15 hundredths of £2 4*s.* For convenience of expression we may say that the ratio of 6*s.* 6*d.* to £2 4*s.* is 15 per hundred, or 15 per cent. This is often written 15 %.

When we have obtained the ratio in this form, we have expressed 6*s.* 6*d* as a 'percentage' of £2 4*s.*

We have really converted our fraction-ratio to a fraction with a 'standard denominator' of 100.

When we say that 6*s.* 6*d.* is approximately 15 % of £2 4*s.*, we mean that the ratio of 6*s.* 6*d.* to £2 4*s.* is approximately 15 to 100, or that if £2 4*s.* is divided into 100 parts, then 15 of those parts will be nearly 6*s.* 6*d.*

Example i. *Express* 2 *lb.* 4 *oz. as a percentage of* 1 *stone.*

2 lb. 4 oz. = $2\frac{1}{4}$ lb. = 2·25 lb.
1 stone = 14 lb.

∴ the ratio is $\frac{2{\cdot}25}{14} = \frac{1{\cdot}125}{7} = {\cdot}16\ldots$

∴ 2 lb. 4 oz. is approximately 16 % of one stone.

Example ii. *Express* 1 *m.* 3 *dm.* 4 *cm. as a percentage of* 5 *Dm.* 6 *m.*

Expressing both measures in metres, we have the ratio

$$\frac{1{\cdot}34}{56} = \frac{{\cdot}1914}{8} = {\cdot}024\ldots$$

∴ the result is approximately 2·4 %.

(**138.**) Since 3 % means simply 3 hundredths, it is easy to find a given percentage of any given quantity.

Example i. 4 % *of* £23

$= \frac{4}{100}$ of £23

= £·92 = 18*s.* $4\frac{3}{4}$*d.* to nearest farthing.

Example ii. 3 % *of* £4 17*s.* $2\frac{3}{4}$*d.*

$= \frac{3}{100}$ of £4·861

= £·14583

= 2*s.* 11*d.* to nearest farthing.

Example iii. 11% *of* 3 *g.* 4 *dg.* 7 *cg.*

$= \frac{11}{100}$ of 3·47 g.

= ·3817 g.

= 3 dg. 8 cg. 2 mg approximately.

With big numbers it saves time to employ the contracted methods explained on pages 209–16.

Example iv. *In* 1908 *the imports into the United Kingdom from Germany were of the value* £41,168,648, *those from France were worth* £48,681,160. *Express the value of the German imports as a percentage of the French imports.*

Here our answer will consist of only two, or at most three, figures, and we, therefore, require not more than four significant figures throughout our work.

Compare the following methods of work:

Full method

The German imports $= \frac{41168648}{48681160}$ of the French imports.

```
48681160 ) 41168648        | ·845 . . . .
           22237200
            27647360
```

German imports are slightly more than $84\frac{1}{2}$ % of the French imports.

Contracted method.

The German imports $= \frac{4117}{4868}$ of the French imports.

$$4868 \overline{)4117} \quad \lfloor \cdot 846$$
$$223$$
$$29$$

German imports are slightly more than $84\frac{1}{2}$ % of the French imports.

If we wished to get the percentage correct to the first decimal place, i. e. three significant figures in all, we should keep five figures in our work.

Oral Examples.

Express each quantity in the first column as a percentage of each quantity in the second column:

1.		**4.**	
(α) 2*s.* 6*d.*	(a) 10*s.*	(α) 3 dm.	(a) 2 m.
(β) 4*s.* 6*d.*	(b) £1.	(β) 2 dm. 2 cm.	(b) 1 m.
(γ) 10*s.*	(c) 12*s.* 6*d.*	(γ) 3·2 dm.	(c) 2·5 m.
(δ) 15*s.*	(d) £2.	(δ) 41 cm.	(d) 3 m.
2.		**5.**	
(α) 8 oz.	(a) 1 lb.	(α) £2.	(a) £100.
(β) 12 oz.	(b) 1 lb. 4 oz	(β) £18.	(b) £125.
(γ) 1 lb.	(c) 1 lb. 9 oz.	(γ) £25.	(c) £200.
(δ) 2 oz.	(d) 1 lb. 14 oz.	(δ) £40.	(d) £500.
3.		**6.**	
(α) 2 qts.	(a) 1 gall.	(α) 3 fr.	(a) 50 fr.
(β) 1 qt.	(b) 1 gall. 1 qt.	(β) 10 fr. 50 c.	(b) 100 fr.
(γ) 1 pt.	(c) 2 qts. 1 pt.	(γ) 18 fr.	(c) 150 fr.
(δ) 3 qts. 1 pt.	(d) $2\frac{1}{2}$ galls.	(δ) 48 fr.	(d) 200 fr.

Find (a) 2 %, (b) 4 %, (c) 5 %, (d) 10 %, (e) 12 %, (f) 25 %, (g) 50 %, (h) 75 % of each of the following quantities:

7. £20. **8.** £50. **9.** £200. **10.** £250.
11. £1000. **12.** £800. **13.** 50 lb. **14.** 3 yds. 1 ft.

15. 1 chain (express in links). **16.** 10 fr. **17.** 40 fr.
18. 22 fr. 50 c. **19.** 2 metres. **20.** 4 m. 5 dm.
21. 1 Km. 2 Hm. 5 Dm. **22.** 3 lb. 2 oz.
23. £7 10*s*. **24.** £1 0*s*. 10*d*. **25.** 500 yds.

What fractions are equivalent to the following?

26. $2\frac{1}{2}\%$. **27.** 5 %. **28.** 10 %. **29.** $12\frac{1}{2}\%$. **30.** 20 %.
31. 25 %. **32** 30 %. **33.** 75 %. **34.** 125 %.

Examples IX b.

Express each quantity in the first column as a percentage (to three significant figures) of each quantity in the second column:

1.

(α) £2 14*s*. 5*d*.	(α) £127 5*s*. 9*d*.
(β) £3 18*s*. 10*d*.	(b) £275 4*s*.
(γ) £5 17*s*. 2*d*.	(c) £550.
(δ) £11 8*s*. 9*d*.	(d) £1200.

2.

(α) 2 lb. 14 oz.	(a) 1 ton.
(β) 7 lb. 8 oz	(b) 2 qrs. 27 lb.
(γ) 1 qr. 13 lb.	(c) 3 qrs.
(δ) 2 qrs. 17 lb.	(d) 2 cwt.

3.

(α) 14 fr. 50 c.	(a) 120 fr.
(β) 16 fr. 20 c.	(b) 230 fr. 50 c.
(γ) 29 fr. 30 c.	(c) 370 fr. 40 c.
(δ) 38 fr. 70 c.	(d) 409 fr.

4.

(α) 3 m. 2 dm.	(a) 80 metres.
(β) 272 cm.	(b) 120 metres.
(γ) 54 dm.	(c) 47 Dm.
(δ) 2 dm. 3 cm. 4 mm.	(d) 6 Hm. 4 Dm. 7 m.

5.

(α) 11 mks. 30 pf.	(a) 110 mks.
(β) 17 mks. 40 pf.	(b) 230 mks.
(γ) 35 mks. 50 pf.	(c) 370 mks. 50 pf.
(δ) 100 mks.	(d) 1732 mks.

Find (*a*) $3\frac{1}{2}\%$, (*b*) $7\frac{1}{2}\%$, (*c*) 11%, (*d*) 23%, (*e*) $13\frac{1}{3}\%$, (*f*) $29\frac{1}{4}\%$ of each of the following quantities.

6. £129 14*s.* 10*d.* **7.** £3040 2*s.* 9*d.*

8. £974 5*s.* $2\frac{1}{4}$*d.* **9.** £1724 11*s.* 4*d.* **10.** £394 12*s.* 8*d.*

11. 2 tons 3 cwt. (to the nearest lb.).

12. 3 tons 17 cwt. (to the nearest lb).

13. 18 cwt. 3 qrs. (to the nearest lb.).

14. 1 ton 7 cwt. 2 qrs. (to the nearest lb.).

15. 72 fr. 50 c. (to the nearest centime).

16. 124 fr. 70 c. (to the nearest centime).

17. 339 fr. 40 c. (to the nearest centime).

18. 524 mks. 70 pf. (to the nearest pfennig).

19. 730 marks (to the nearest pfennig).

20. 670 mks. 30 pf. (to the nearest pfennig).

21. \$72.40 c. (to the nearest cent).

22. \$159.30 c. (to the nearest cent).

23. \$669.60 c. (to the nearest cent).

24. 3 Dm 4 m. 7 dm. (to the nearest mm.).

25. 4 Hm. 5 Dm. 8 m. (to the nearest cm.).

26. 5 Kg. 4 Hg. 2 g. (to the nearest dg.).

27. 8 Hl. 7 Dl. 3 l. (to the nearest cl.).

28. 1743 Kg. (to the nearest Dg).

29. In 1906 the total exports of the United Kingdom were worth £465,723,000; in 1909 the value was £477,796,000. What was the percentage increase?

30. In 1908 the cultivated area in Canada increased from 30,065,556 acres to 32,625,482 acres. What was the percentage increase?

31. The area of Europe is 3,860,000 sq. miles, that of Australia 2,974,000 sq. miles. Express the area of Australia as a percentage of that of Europe.

32. In 1909 the population of Ireland was 4,371,365, and the number of emigrants was 28,675. What percentage of the people emigrated?

33. Express each of the following amounts as a percentage (correct to one decimal place) of the total:

	Amount.	Percentage of total.
Public rates	£63,260,940	
Exchequer grants (including local taxation duties)	20,914,877	
Gas, water, electric light, trams and light railways	24,618,992	
Harbours, docks, piers, canals, and quays	6,091,847	
Other sources	12,021,966	
Total	£126,908,622	

34. Calculate, to one decimal place, the percentage in the following tables.

The total amount expended in the year 1906–7 by the local authorities in England and Wales otherwise than out of loans is shown in the following table:

Works and purposes.	Total.	Percentage of total
Administration of justice	£885,185	
Baths, wash-houses, and open bathing places	684,340	
Cemeteries	698,374	
Education:		
i. Elementary education (including industrial schools)	24,692,742	
ii. Higher education	2,970,956	
Electric lighting (other than public lighting), gas-works	6,523,646	
Harbours, docks, piers, canals, and quays	3,896,575	
Highways, bridges, and ferries (excluding lighting, but including Thames tunnels)	13,525,438	
Hospitals, other than poor-law infirmaries	1,895,548	
House refuse—removal and destruction of	2,052,225	
Housing of the working classes	533,344	
Lighting (public)	2,174,073	
Lunatics and lunatic asylums	3,538,826	
Markets	865,531	
Parks, pleasure grounds, and open spaces	1,205,560	
Police and police stations	6,287,796	
Poor relief (excluding cost of maintenance of pauper lunatics in county and borough asylums)	11,598,072	
Private street works and other works of private improvement	1,443,248	
Sewerage and sewerage disposal works	4,488,735	
Tramways and light railways	6,457,472	
Unemployed Workmen Act, 1905—purposes of	217,908	
Vaccination	265,806	
Valuation	191,308	
Waterworks	5,114,264	
Other works and purposes Unapportioned loan charges for purposes numbered 1, 8, 11, 13, 15, 19, and 25	11,927,302	
Total expenditure of local authorities defrayed otherwise than out of loans	£114,134,274	

Statement showing the value of Manufactured and Partly Manufactured Articles (Domestic Produce) exported from the United Kingdom, France, Germany, and the United States to the principal protected foreign countries, to all other destinations, and to all destinations in the year 1908

Country	Value of Manufactured and Partly Manufactured Articles (Domestic Produce) Exported								
	Principal Protected Foreign Countries		All other Destinations						Total to all Destinations
			United Kingdom		Other Countries		Total to all other Destinations		
	Amount	Proportion to Total	Amount	Proportion to Total	Amount	Proportion to Total	Amount	Proportion to Total	
	£ 1,000	Per Cent	£ 1,000	Per Cent	£ 1,000	Per Cent	£ 1,000	Per Cent	£ 1,000
United Kingdom	89,608		—	—	—	—	196,780		286,388
France	49,925		30,584		37,992		68,576		118,501
Germany	118,804		32,731		61 577		94,308		213,112
United States	43 800		30,500		82,000		112,500		156.300

(i) Check the accuracy of the last column
(ii) Calculate, for each country, what percentage each of the first three amounts is of the total in the last column

(**139.**) If two quantities x and y are in the ratio 2 to 3, we have seen that $x/y = 2/3$,

or $x = \frac{2}{3}$ of y (on multiplying each expression by y).

Hence, if we are given a quantity y, and are asked to find another quantity which bears to y the ratio of 2 to 3, we just have to take two-thirds of the quantity y.

For example, we are asked to find a sum of money which is to £30 in the ratio of 2 to 5.

The sum of money is $\frac{2}{5}$ of £30 or £12.

(Verification: the ratio of £12 to £30 is $\frac{12}{30}$ or $\frac{2}{5}$.)

Such a question arises in connexion with numerous problems, of which the following is a type.

Example. *A partner in a business receives* £506 *when the whole receipts of the business in a certain year are* £4927. *The receipts of the firm drop to* £3415 *in a second year. What does this partner get?*

The firm's receipts have fallen in the ratio $\frac{3415}{4927}$.

∴ this partner's receipts must fall in the same ratio, and he gets only $\frac{3415}{4927}$ of £506,

$$= \text{£}350\ 14s.\ 4d.$$

```
                3415
                 506
              17075
               20490
             1727990

              350·718
4 927 ) 1727·990
        1478 1
        249·89
        246·35
          3·540
          3·449
             91
             49
             42
```

(In practice he would probably receive £350, and the balance would be carried to the next year's account.)

(**140.**) In nearly all money problems there are two distinct points of view, both giving the same result.

For example, in the above problem we might have argued thus:

In the first year he gets $\frac{506}{4927}$ of the total profits.

$\therefore$ in the second year he gets $\frac{506}{4927}$ of £3415.

It is immaterial which point of view is chosen, but it is most important to regard the ratio in such problems as a *number* which operates on some concrete quantity.

Oral Examples.

Find quantities which are to the following quantities in the ratio (*a*) 2 to 3, (*b*) 2 to 5, (*c*) 3 to 5, (*d*) 7 to 10, (*e*) 8 to 15, (*f*) 5 to 6, (*g*) 9 to 10:

1. £60.	**2.** £120.	**3.** £7 10*s.*
4. £1 10*s.*	**5.** 2*s.* 6*d.*	**6.** 12*s.* 6*d.*
7. 3 Km. 3 Hm.	**8.** 12 cm.	**9.** 4 fr. 50 c.
10. \$7·50.	**11.** 5 feet.	**12.** 120 yards.
13. 8 fr. 40 c.	**14.** 4 m. 5 dm.	**15.** 9 inches.
16. 27 ft.	**17.** 30 lb.	**18.** \$3.30 cts.
19. 4 fr. 80 c.	**20.** 8 mks. 10 pf.	

21. When income tax is 8*d.* in the £1, a man pays £8 tax; what does he pay when income tax is 1*s.* in the £1?

22. A man has some money invested in a company, and gets therefrom an income of £750 when the company pays a dividend of 3 per cent.

What income will he get if the company pays a 4 per cent. dividend?

23. A contractor has 500 men working for him, and pays in wages £60 per day.

What will he pay per day if he reduces the number of men to 400, if all the men receive the same wage?

24. 50 lb. of sugar cost 12*s.* 6*d*

(i) What will 70 lb. cost?

(ii) What will 120 lb. cost?

(iii) What will 75 lb. cost?

(iv) How many pounds can be bought for 25*s.*?
(v) How many pounds can be bought for 1*s.* 6*d.*?
(vi) How many pounds can be bought for 6*s.* 6*d.*?

25. 20 lb. of tea cost £1 5*s.*
(i) What will be the cost of 4 lb.?
(ii) What will be the cost of 16 lb.?
(iii) How many lb. can be purchased for 15*s.*?
(iv) How many lb. can be purchased for 10*s.*?

26. A mile of wire costs £2 4*s.*
(i) What is the cost of 880 yards?
(ii) What is the cost of 70 yards?
(iii) How many yards can be bought for 6*s.*?
(iv) How many yards can be bought for 19*s.*?

27. 3 men can finish a job in 8 days.
(i) How many days will it take 4 men to finish it?
(ii) How many days will it take 24 men to finish it?
(iii) If the work has to be finished in 2 days, how many men will be wanted?

28. A clock gains 15 seconds per day of 24 hours.
(i) How much will it have gained in 60 hours?
(ii) How long will it take to gain 2 minutes?
(iii) After how many days will it be a quarter of an hour fast?
(iv) How often will it have to be put right if it is never to be more than 10 minutes fast?

29. An investment of £2000 brings in an income of £70.
(i) What income will be produced by the investment of £1600?
(ii) What income will be produced by the investment of £700?
(iii) How much must be invested to obtain an income of £350?
(iv) How much must be invested to obtain an income of £100? (Answer to the nearest £1.)

Examples IX c.

1. What quantity is the same ratio of £315 14*s.* as £5 is of £100?

2. What quantity is the same ratio of £100 as £3 14*s.* 2*d.* is of £110 5*s.*?

3. What quantity is the same ratio of 3 tons 4 cwt. as 6 is of 100?

4. What quantity is the same ratio of 512 fr. 60 c. as $12\frac{1}{2}$ is of 100?

5. What quantity is the same ratio of 14·893 metres as $7\frac{3}{4}$ is of 100?

6. What quantity is the same ratio of 100 as 3 mi. 2 fur. is of 15 mi 7 fur.?

7. What quantity is the same ratio of £5 14*s.* 2*d.* as £3 0*s.* 9*d.* is of £17 2*s* 3*d.*?

8. What quantity is the same ratio of 4 tons 8 cwt. 3 qrs as 2 qrs. 3 lb. is of 1 cwt?

9. What quantity is the same ratio of 42 mks. 30 pf. as £7 is of £60?

10. What quantity is the same ratio of £3 2*s.* 6*d.* as 1 yd. 2 ft. is of 17 yds.?

11. What quantity is the same ratio of 3·429 g. as 4·2 m. is of 7·3 m.?

12. What quantity is the same ratio of 62·3 cm. as 4·51 l. is of 7·32 l.?

13. What quantity is the same ratio of £8 4*s.* 9*d.* as 3 fr. 20 c. is of 7 fr. 80 c.?

14. What quantity is the same ratio of £123 5*s.* as 7 mks. is of 15 mks.?

15. What quantity is the same ratio of 123 fr. 50 c. as 8 fr. is of 25 fr.?

16. If a litre contains 1·76 pints, find the number of litres in $2\frac{1}{2}$ bushels.

17. If a bar of silver, which weighs 593 oz. 19 dwts., is worth £64 14*s.* 5*d.*, find the value of another bar, which weighs 763 oz 13 dwts.

18. Part of the site of Christ's Hospital, 67,680 square feet in area, was sold to St. Bartholomew's Hospital for £238,781. Find the price per acre.

19. If 29 m. 75 cm of cloth cost 190 fr. 40 c., what is the cost of 22 m. 5 cm.?

20. A plot of land measuring 7 ac. 2 ro. 12 sq. yds. is sold for £907 16*s*, what would a field of 8 ac. 16 po fetch if sold at the same price per acre?

21. If 5 qrs. 3 bus. 2 pks. of wheat cost £11 9*s*. 3*d*., what is the value of 3 bus. 2 pks. 1 gal.?

22. If two-thirds of an estate is worth £70,000, find the value of ·625 of the same estate.

23. One hundred and fifty members of a club subscribe 3*s*. 6*d*. each towards a testimonial.

The expenses of collection work out at 2*d*. for every half-crown collected. What is the amount available for the testimonial?

24. Of a regiment, after 1·5 per cent. have been killed, 8·75 per cent. wounded, and 12 per cent. invalided, there still remain 933 men available. What was its original strength?

25. If 3 tons 16 cwt. 77 lb. of salt cost £39 12*s*. $5\frac{1}{4}d$., what will 7 cwt. 7 lb. cost at the same rate?

26. A school of boys and girls contains altogether 449 scholars, and the number of girls is almost exactly ·39 of the number of boys.

Find the number of boys.

(141) [Note.—The following graphical treatment of ratios assumes a knowledge of elementary ideas of co-ordinates, and also of the preliminary geometrical theorems on ratio. It has been thought inadvisable to give any detailed explanation of these theorems here, and consequently this section must be postponed in the case of classes which have not reached that standard. At the same time, the ideas involved are valuable to students who can follow them.]

Suppose two quantities x and y are such that the ratio y/x is always the same, though the quantities themselves may vary.

To fix the ideas, suppose that $y/x = 2/3$.

Now we can get a series of values of x and y satisfying this condition; for example:

$x = 3$	$x = 6$	$x = 9$
$y = 2$,	$y = 4$,	$y = 6$, &c.;

and if we plot these points on an ordinary graphical diagram, they will all lie on a straight line passing through the origin (Fig. 12).

The 'equation' of this straight line is $y = \frac{2}{3}x$.

(**142.**) Now every point on this line is such that its y and x are in the ratio of 2 to 3, and if there is any point whose y and x are in this ratio, it must lie on this line.

Hence we can say that this line represents the ratio $\frac{2}{3}$.

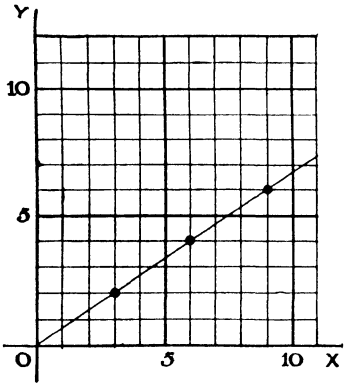


Fig. 12.

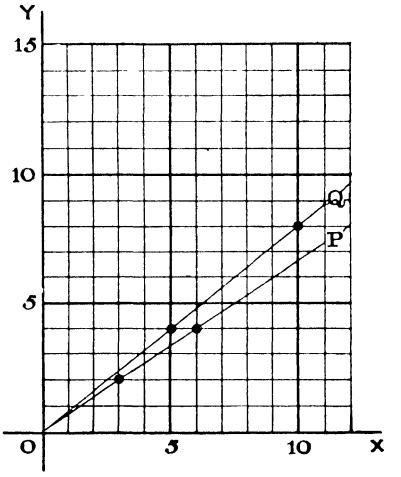


Fig. 13.

Similarly any other ratio can be represented graphically in this way.

It is not necessary to find a *number* of points on the line, but it is sufficient to mark down a point whose x and y represent, on a chosen scale, the quantities whose ratio we are considering, and then draw the line joining the point to the origin O.

In Fig. 13 the line OP represents the ratio $\frac{2}{3}$, and the line OQ represents the ratio $\frac{4}{5}$. A glance at the figure will show that this method of representation indicates very clearly which of these two ratios is the greater.

For on OP when x is 5, y is just over 3, whereas on OQ when x is 5, y is 4.

$\therefore$ the ratio $y : x$ is greater on OQ than on OP. In fact the **slope of the line** gives us at once an idea of the **magnitude of the ratio.**

Example 1. *Place the ratios* $\frac{3}{4}$, $\frac{7}{9}$, $\frac{8}{11}$ *in order of magnitude, by drawing lines representing these ratios.*

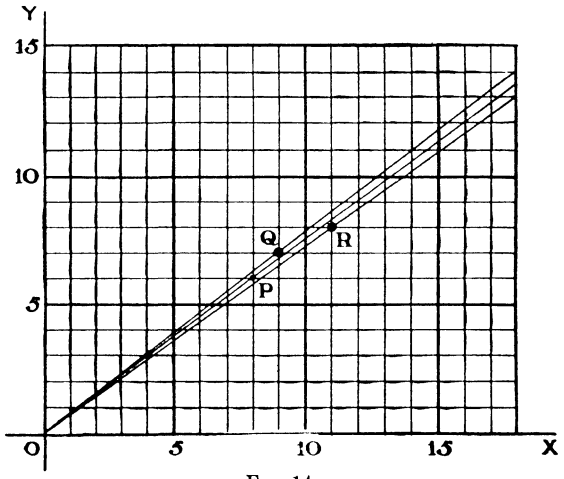


Fig 14.

The point P (Fig. 14) has $x = 4$, $y = 3$;

$\therefore$ OP represents the ratio $\frac{3}{4}$.

The point Q has $x = 9$, $y = 7$;

$\therefore$ OQ represents the ratio $\frac{7}{9}$.

The point R has $x = 11$, $y = 8$;

$\therefore$ OR represents the ratio $\frac{8}{11}$.

Fig. 14 shows that the order of magnitude is $\frac{7}{9}$, $\frac{3}{4}$, $\frac{8}{11}$.

Verification: $\frac{7}{9} = 17777\ldots.$

$\frac{3}{4} = \cdot 75$

$\frac{8}{11} = \cdot 7272\ldots.$

Example ii. *It is required to increase the prices of a number of articles in the ratio* $\frac{107}{100}$. *Draw a graph which will enable you to find the new prices corresponding to any old prices between* £10 *and* £50, *and find the new prices corresponding to* (i) £17 10*s.*, (ii) £23, (iii) £46 15*s.*

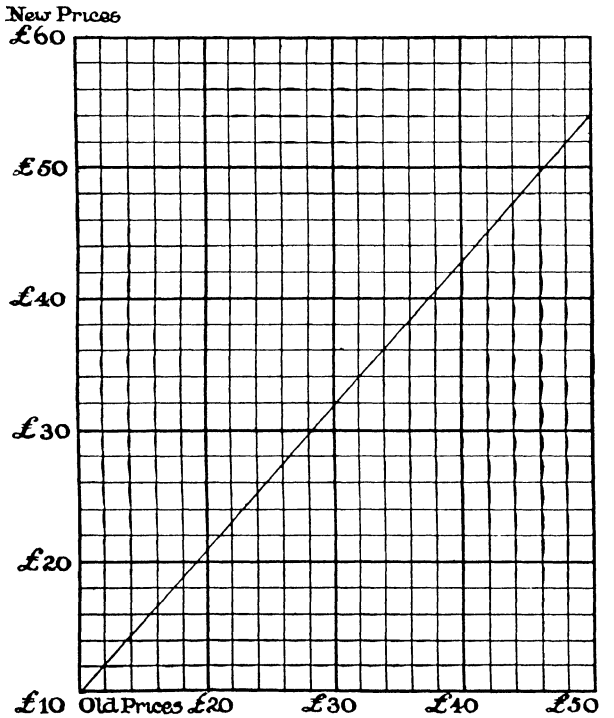


Fig. 15.

In choosing the scale on which the graph is to be drawn, we try to arrange that the whole of the paper at our disposal shall be of use.

Hence it is not advisable to show the origin itself, since we are dealing only with prices between £10 and £50.

In this case we note a pair of corresponding values, namely:

Old Price	£10,	£50;
New Price	£10·7,	£53·5.

Taking 5 spaces to represent £10, and marking new prices vertically, and old prices horizontally, we scale off our paper, and mark the points represented as above.

Joining these points by a straight line, we obtain the required graph (Fig. 15).

From the graph we read off the new prices required:

Old Price.	New Price.
£17·5.	£18·8.
£23.	£24·6.
£46·75.	£49·8.

Example iii. *Given that a metre = 39·3 inches, draw a graph to convert any length up to 50 inches from English measures to metric measures, and obtain from the graph the number of inches in 60 cm., and the number of centimetres in 32 inches.*

We have 100 cm. = 39·3 in.

The graph (Fig. 16) is obtained by joining the point P to the origin, where P shows the correspondence between 39·3 inches and 100 cm.

From the graph we find that

60 cm. = 23·7 in.

32 inches = 81 cm.

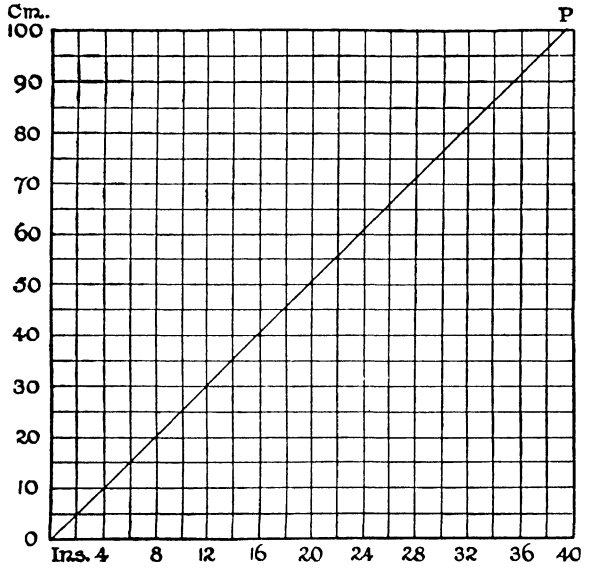


Fig. 16.

Examples IX d.

By drawing lines to represent the ratio, place the ratio in each of the following groups in order of magnitude. Verify by calculation.

1. $\frac{3}{4}, \frac{4}{5}, \frac{5}{6}$.
2. $\frac{4}{7}, \frac{5}{9}, \frac{4}{5}$.
3. $\frac{6}{7}, \frac{2}{5}, \frac{5}{8}$.
4. $\frac{3}{7}, \frac{5}{9}, \frac{2}{3}$.
5. $\frac{1}{8}, \frac{2}{13}, \frac{8}{11}$.
6. $\frac{4}{3}, \frac{5}{7}, \frac{7}{5}$.

7. A number of articles were priced from £10 to £40. The prices are to be increased in the ratio $\frac{115}{100}$.

Draw a graph which will enable you to find the new prices of articles which were originally priced at (i) £13, (ii) £24, (iii) £37.

8. Repeat question 7, increasing the prices in the ratio $\frac{130}{100}$.

9. A Rupee (fixed rating) is equivalent to 1*s.* 4*d.* Draw a graph enabling you to convert any sum of money up to Rs. 50 from Indian to English measure.

Read off the value in English money of (i) Rs. 22, (ii) Rs. 40, (iii) Rs. 37 ; and the value in Indian money of (iv) 18*s.*, (v) 27*s.*, (vi) £2.

10. A franc is equivalent to 9·5 pence. Draw a graph showing the connexion between English and foreign money up to 100 fr., and find the equivalents of (i) 54 fr., (ii) £2, (iii) 80 fr., (iv) £1 17*s*

11. A metre = 39·37 inches.

Draw a graph showing the connexion between yards and metres up to 20 metres, and find the equivalents of (1) 20 yds., (ii) 18 metres, (iii) 11 yards.

12. A litre = ·88 quarts.

Draw a graph showing the connexion between litres and pints, and read off the equivalents of (i) 4 litres, (ii) 3 pints, (iii) 5 pints, (iv) 1 gallon.

Examples IX.

1. Express $\frac{3}{11}$ of £33 13*s.* 9*d.* as a decimal of £26 5*s.*

2. Express 13 st. 1 lb. and 10 st. 3 lb in kilograms, taking a kilogram to be equal to $2\frac{1}{5}$ lb.

3. What will be the rent of 573 acres 1 rood 20 poles of land, if 5 acres 2 roods of similar land are rented for £4 16*s.* 4*d.* ?

4. Express 5 cwt. 3 qrs. 15 lb. as a decimal of 11 cwt. 1 qr. 19 lb., correct to three significant figures.

5. At the uniform rate of $15\frac{3}{4}$ miles in 27 minutes, in what fractional part of an hour will a train travel $22\frac{1}{2}$ miles, and how far will it travel in $1\frac{5}{9}$ hours?

6. How much dress material may be bought for £31 4*s.* $4\frac{1}{2}$*d.* if $303\frac{3}{4}$ yards cost £50 12*s.* 6*d.* ?

7. If 3 tons 18 cwt of lead cost £100 5*s.* 3*d.*, what is the value of 3 cwt. 2 qrs. of the same?

8. A piece of wire measuring 4 Km. in length weighs 47 Kg. Express in grammes, to three significant figures, the weight of 37 cm. of the same wire.

9. If a man takes 11 steps, each 2 ft. 8 in. in length, every 5 seconds, how many yards will he walk in $18\frac{3}{4}$ minutes?

10. A cyclist starts for a ride at 9.15 a.m. At 10.30 a.m. he has ridden 15 miles 150 yards. How far will he have gone altogether by 1.15 p.m., his speed being the same throughout?

11. Assuming that A spends £30,000 in 365 days, and that B spends 24*s.* in 7 days, express B's daily expenditure (i) as a fraction, (ii) as a decimal, of A's average daily expenditure.

12 Find, to three places of decimals, the percentage of salt in the sea if a ton of sea water contains 62 lb. 13 oz. of salt.

13. Express 9*s.* 2*d.* as a decimal of £1 13*s.* 4*d.*

14. The census is taken every ten years. Assuming the increase of the population of England and Wales at each census to be 14 per cent. of the population at the previous census, calculate the population for 1881(to the nearest thousand) if it was 20 millions in 1861.

15. A German 20 mark piece is worth £·979. Find, to the nearest penny, the value of 3725 mks. 39 pf.

16. The circulation of a newspaper increased in three successive years by $\frac{1}{8}$, $\frac{1}{9}$, and $\frac{2}{7}$ of its amount at the beginning of the respective years. At the end of the third year it was 73,125. What was it at the beginning of the first year?

17. In a certain examination 12 per cent. of the candidates are placed in the first class, 24 per cent. in the second, and 36 per cent. in the third. The rest of the candidates, who fail, number 147. How many were there in each of the three classes?

18. The death-rate of London for one week, during which 1259 deaths occurred, was equivalent to 11·8 per thousand per annum of the population. From these data calculate the population of London.

19. What is the value of an ingot of silver which weighs 576 ounces, if 22 ounces of the silver are worth £2 9*s.* $0\frac{1}{2}$*d.*?

20. If it costs £1 10*s.* 3*d.* to run a train 11 miles, how much will it cost to run it 150 miles under the same conditions?

21. If you can buy 7 yards of cloth for 13*s.* 5*d.*, how much can you buy for £1 1*s.* 1*d.*?

22. If a clock gains 5 seconds in 1 hour 10 minutes, how many minutes will it gain in 7 days?

23. If 153 yards of cloth cost £21 19*s.* 10½*d.*, how many yards can be bought for £28 15*s.*?

24. If 125 men can do a piece of work in 120 days, in how many days can 150 men do it?

25. A set of 7 hens lay 1129 eggs, and another set of 11 hens lay 1774 eggs. Which set are, on the average, the better layers?

26. If a field of 15 acres 3 roods 10 poles provides pasture for 46 sheep, for how many sheep will a similar field of 33 acres suffice?

27. If the price of coal is 16*s.* 8*d.* a ton, and if 2½ per cent. is allowed off the bill for cash payment, how much ready money will a man have to pay for 4 tons 8 cwt. of coal?

28. A field of 4 acres 2 roods 16 poles is rented at £13 16*s.*; what is the rent of a similar field of 5 acres 3 roods 12 poles?

29. There were 5,852,276 names of scholars on the registers in 20,513 schools, and the average number of those scholars attending daily was 4,890,237.

Determine (i) what percentage, correct to two places of decimals, the average daily attendance was of the total number on the register; and (ii) at the average rate, what should have been the average daily number of absentees from schools containing 87 and 753 scholars respectively?

30. A property was sold for £8963, and the owner received from the agent £8626 17*s.* 9*d.*

What percentage of the selling price did the agent receive for commission?

31. If gold is worth £3 17*s.* 10½*d.* per oz., and silver is worth 2*s.* 3*d.* per oz., what is the value, to the nearest penny, of 1023 grains of a mixture, 89·4 per cent. of whose weight is gold, and the rest silver?

32. A contractor undertakes to do a certain work in 120 days and employs 85 men for the purpose. At the end of 69 days half the work is done. How many extra men must he employ in order that the work may be finished in time?

33. During a certain year there were 14,471 male and 1971 female offenders, and the total population was 42,371,216.

At the average rate, how many criminal offenders should there have been in a town whose population was 172,598?

What percentage (correct to the nearest tenth) was the number of female of the number of male offenders?

34. When pure silver is worth 2*s.* 7*d.* per Troy ounce of 480 grains, the silver in a Japanese coin, which is composed of silver and alloy, is worth 2*s.* $0\frac{1}{4}d$

The weight of the coin is 416 grains. Find what fraction of the coin is pure silver. (Express as a decimal.)

35. A clock is set right at 9 a.m. on Monday, and gains 3 minutes a day. Find the time indicated by the clock at 5 p.m. on the following Thursday.

36. The fare for 1000 miles on one railway is £5 5*s.*, and on another railway the fare is 11*s.* 8*d.* for 105 miles. By how much per cent. must the second of the two reduce its fares in order that its rate per mile may be identical with that of the first?

CHAPTER X

APPLICATION OF RATIO-METHODS TO PROBLEMS

(**143.**) In the preceding chapter we have shown how to construct ratios between quantities of the same kind, and we have introduced a few simple problems in which ratio-methods are useful

We proceed to give examples of more difficult problems of similar types. In all these problems we shall be either increasing or decreasing a given quantity in a given ratio, and the student is recommended to make up his mind in each case—from principles of common sense alone—whether he is looking for an increase or a decrease in the given quantity; that is to say, whether he is going to multiply by a ratio greater or less than unity

For example, it is given that 6 tons 7 cwt. of coal cost £6 16*s.* 7*d.*, and we are asked to find the cost of 4 tons 11 cwt.

Common sense tells us that the cost would be less; it will in fact be diminished in the ratio of 91 cwt. to 127 cwt.

The cost is:

$\frac{91}{127}$ of £6·829

= £4·893

= £4 17*s.* 10*d.* to the nearest 1*d.*

6·829
91
614·61
6·829
621 439

4 893 . . .
1·27) 6·21439
1·134
·1183
409

The argument might have been worded thus:

6 tons 7 cwt. cost £6 16*s.* 7*d.*

∴ 4 tons 11 cwt. cost £6·829 × $\frac{91}{127}$.

[For clearly 4 tons 11 cwt. cost less than 6 tons 7 cwt., and we therefore multiply by a decreasing fraction.]

In the following pages we include examples of questions which occur in ordinary commercial life. These will be treated more fully in a later chapter, but they serve to illustrate the utility of ratio-methods.

(**144.**) **Rates and Taxes.** The expenses of road making, schools, police, &c., have to be borne by the occupiers of houses in the district

The amount each occupier has to pay depends upon the yearly value of his house, this value being called the *assessment* or *rateable value* of the house. When a *rate* is declared by a district council, it is usually expressed in the form 1*s.* 9*d.* in the £1.

This means that 1*s.* 9*d.* has to be paid on every £1 of rateable value.

Example. *If the rateable value of a house is* £38, *find the amount payable for poor rate at* 1*s.* 9*d. in the* £1.

1*s.* 9*d.* is $\frac{21}{240}$, i.e. $\frac{7}{80}$ of £1.

∴ the rate payable $= \frac{7}{80}$ of £38

$= £\frac{133}{40}$ (Dividing numerator and denominator by 2.)

= £3·325

= £3 6*s.* 6*d.*

(**145.**) **Income Tax** is similarly calculated on a man's yearly income, after certain deductions and allowances have been made.

Example. *A man has an income of* £350 *a year. He is charged income tax at* 9*d. in the* £1, *but is allowed to deduct from his taxable income* (1) £160; (2) *premiums paid for Life Insurance, in this case* £12 14*s.* 10*d. What tax does he pay?*

Total deductions $= £172\ 14s.\ 10d.$
$= £172{\cdot}742.$
$\therefore$ taxable income $= £177{\cdot}258.$
$\therefore$ tax is $\frac{9}{240}$ of £177·258
$= \frac{3}{80}$ of £177·258
$= \frac{1}{80}$ of £531·774
$= £6{\cdot}647$
$= £6\ 12s.\ 11d.$ to the nearest $1d.$

(146.) Ordinary Commercial Dealings. When a man keeps a shop, or carries on any form of commerce, he usually buys something in order to sell it at a higher price and so make a profit. If he wishes to know 'how well he has done', it is natural that he should find the ratio between his profit and the amount he spent, and, for purposes of comparison, it is usual to express this ratio as a percentage.

Example. *A man buys goods for* £1216, *and receives for their sale* £1545; *express his profit as a percentage of his outlay* (*i.e.* '*find his profit per cent.*').

His gain $= £329$
$\therefore$ his profit $= \frac{329}{1216}$ of his outlay
$= \frac{329}{1216} \times 100\ \%$
$=$ rather more than 27 %.

```
      27
12 16)329
      858
       68
```

Alternative method.

Profit on £1216 is £329.
$\therefore$ profit on £100 $= £329 \times \frac{100}{1216}$.

Example II. *A man buys goods for* £315, *and wishes to make a profit of* 8 %. *For what must he sell the goods?*

Profit $= \frac{8}{100}$ of £315
$= £25{\cdot}2.$
$\therefore$ he sells for £340 4*s.*

(**147.**) **Interest.** When a man lends money to another man, he usually expects some annual payment in return for the loan. This payment is called **Interest**, and is calculated as a percentage of the **capital sum** lent. If the money is lent to a company or government, it is said to be **invested**, and the interest payable is called the **dividend** (i. e. the result of **dividing** the profits of the company).

Example i. *Find the yearly interest when* £765 13*s.* 2*d.* *is lent at* $3\frac{1}{2}$ %.

$$\begin{aligned}\text{The interest} &= \frac{3\frac{1}{2}}{100} \text{ of } £765{\cdot}658\\ &= £7{\cdot}65658 \times 3\tfrac{1}{2}\\ &= £22{\cdot}96974\\ &\quad + £3{\cdot}82829\\ &= £26{\cdot}79803\\ &= £26\ 15s.\ 11\tfrac{1}{2}d.\\ &\text{or } £26\ 16s. \qquad \text{to the nearest } 1d.\end{aligned}$$

Example ii. *Find the total interest on* £497 3*s.* 5*d.* *for* 7 *years at* $2\frac{3}{4}$ %.

$$\begin{aligned}\text{Total interest} &= 7 \times \frac{2\frac{3}{4}}{100} \text{ of } £497{\cdot}171\\ &= \frac{7 \times 11}{400} \text{ of } £497{\cdot}171 \qquad \begin{array}{r}5468\,881\\ 38282{\cdot}167\end{array}\\ &= £95{\cdot}705\\ &= £95\ 14s.\ 1d. \qquad \text{to the nearest } 1d.\end{aligned}$$

(**148**) **Trade Discount.** If goods are sold at less than their nominal prices, they are said to be sold at a discount, and the amount deducted from the nominal price is called **discount.** A net price is not subject to discount.

Example. *A tradesman, intending to make* 10 % *profit on his outlay, marks an article at* 5/–. *If he allows* $2\frac{1}{2}$ % *discount for cash payment, what profit does he then make?*

The market price is $\frac{110}{100}$ of the actual cost,

$\therefore$ the actual cost $= \frac{100}{110}$ of $5s. = \frac{600}{11}d. = 54\frac{6}{11}d.$

Also the discount is $2\frac{1}{2}\%$, i. e. $\frac{1}{40}$, of the marked price.

$\therefore$ the actual selling price $= 5s. - \frac{1}{40}$ of $5s.$

$= 4s.\ 10\frac{1}{2}d.$

$\therefore$ the actual gain $= 4s.\ 10\frac{1}{2}d. - 4s.\ 6\frac{6}{11}d.$

$= 3\frac{21}{22}d.,$

and this is the gain on an outlay of $54\frac{6}{11}d.$

Hence the gain $= \frac{3\frac{21}{22}}{54\frac{6}{11}} = \frac{83}{1200}$ of the outlay

$= \frac{83}{1200}$ of $100\,\%$

$= 6\frac{11}{12}\,\%.$

Alternative method.

What is marked at 110 costs 100.

$\therefore$ what is marked at $60d.$ costs $60 \times \frac{100}{110} = \frac{600}{11} = 54\frac{6}{11}d.$

Also what is marked at 100 sells for $97\frac{1}{2}$.

$\therefore$ also what is marked at 60 sells for $97\frac{1}{2} \times \frac{60}{100} = \frac{585}{10} = 58\frac{1}{2}d.$

Hence gain on $54\frac{6}{11}d.$ is $58\frac{1}{2}d. - 54\frac{6}{11}d. = 3\frac{21}{22}d$

$\therefore$ gain on 100 is $\frac{83}{22} \times 100 \div \frac{600}{11} = \frac{83}{22} \times \frac{11}{6} = 6\frac{11}{12}$, i. e. profit is $6\frac{11}{12}\,\%$.

(**149**.) **Bankruptcy.** When a man has run into debt, and his debts (or **liabilities**) are greater than the amount of money or the value of property he possesses (called his **assets**), then, under certain circumstances his affairs are put into the hands of a government official called the 'official receiver', and he becomes a **bankrupt**.

His assets are then compared with his liabilities, and the ratio between them shows what fraction of his debts he is able to pay. His **creditors** (or the people to whom he owes money) then receive just this fraction or ratio of what they ought to receive, and thus his whole assets are divided up

amongst his creditors. Thus, if a man has debts of £3724 10*s.* and assets of £2441 12*s.*, he can pay only

$\frac{2441{\cdot}6}{3724\ 5}$ of what he owes

i.e. ·655 „ „ „ „

```
      ·655
3·7245 ) 2·4416
         20690
         20675
```

This is equivalent to paying £·655, or 11*s.* $1\frac{1}{4}$*d.* approximately, for every £1 that he owes, and it would be said that he pays 11*s.* $1\frac{1}{4}$*d.* in the £1.

Example. *A bankrupt can pay* 12*s.* 6*d.* *in the* £1. *What does a creditor receive to whom he owes* £271 10*s.*?

12*s.* 6*d.* $= (\frac{12\cdot5}{20} =) \frac{5}{8}$ of £1.

∴ this creditor receives $\frac{5}{8}$ of £271·5

$= £\frac{1357\cdot5}{8}$

= £169·687

= £169 13*s.* 9*d.*

ADDITIONAL EXAMPLES

(150.) Example i. *Three partners in a firm have subscribed working capital of* £3000, £5000, *and* £7000.

In a single year the profits amount to £2874. *How should these profits be divided?*

The total capital is £15,000.

The first man supplied $\frac{1}{5}$ of this,

the second supplied $\frac{1}{3}$ of this,

the third supplied $\frac{7}{15}$ of this.

∴ the first man receives $\frac{1}{5}$ of £2874

= £574·8

= £574 16*s.*

The second man receives $\frac{1}{3}$ of £2874

= £958.

The third man receives $\frac{7}{15}$ of £2874
$= \frac{7}{5}$ of £958
$= \frac{1}{5}$ of £6706
$=$ £1351·2
$=$ £1351 4*s.*

(*Check:* by adding the total amount received)

The above is only a special case of a problem such as the following

Example ii. *Divide* £437 18*s.* *into three shares in the ratio of* 7 *to* 5 *to* 8.

The whole must be divided into $7+5+8$ parts $= 20$ parts.

The first share $= \frac{7}{20}$ of £437·9 $= 7 \times$ £21·895
$=$ £153·265
$=$ £153 5*s* $3\frac{1}{2}d.$

The second share $= 5 \times$ £21·895 $=$ £109·475
$=$ £109 9*s.* 6*d.*

The third share $= 8 \times$ £21·895 $=$ £175·160
$=$ £175 3*s.* $2\frac{1}{2}d.$

Example iii. *A contractor agrees to lay down* 12 *miles of railway line in* 30 *days. After* 18 *days he finds that only* $6\frac{1}{2}$ *miles have been completed. He has been employing* 150 *men.*

How many extra men must he take on in order to finish the work in the agreed time?

(*Argument*:—He has done $6\frac{1}{2}$ miles in 18 days with 150 men and he has to do the remaining $5\frac{1}{2}$ miles in 12 days

Since he has to lay only $5\frac{1}{2}$ miles instead of $6\frac{1}{2}$, it will take *fewer* men in the ratio $\frac{5\frac{1}{2}}{6\frac{1}{2}}$ or $\frac{11}{13}$.

Since he has only 12 days instead of 18 in which to do the work, he must employ *more* men in the ratio $\frac{18}{12}$.)

The number of men to be employed during the 12 days

$$= 150 \times \tfrac{11}{13} \times \tfrac{18}{12}$$

$$= \frac{825 \times 3}{13} \quad \text{(on dividing numerators and denominators by 6, and 2.)}$$

$$= \frac{2475}{13}$$

$$13\overline{)2475}\ 190\ (117\ 5)$$

= rather more than 190.

He will have to employ 191 men,

or 41 extra men.

Examples X a.

1. A parish is rated at £5120. The parish is required to raise by rate a sum of £352. What is the rate per £?

2. A grocer buys a cheese weighing 53 lb. for £2 4*s.* 2*d.* At what price per lb. must he sell it so as to gain 15 per cent.?

3. If goods are purchased at £2 16*s.* per cwt., and sold at $7\frac{1}{2}$*d.* per lb, what is the gain per cent.?

4. Find the interest on £245 for $4\frac{1}{2}$ years at $3\frac{1}{3}$ per cent. per annum.

5. Find the interest on £1374 10*s.* 3*d.* for 8 months at $4\frac{1}{4}$ per cent. per annum.

6. If the interest on a certain sum of money at 4 per cent. for 3 years is £18, what is the interest on the same sum at 3 per cent. for 6 years?

7. A debtor pays 14*s.* 9*d.* in the £1. What will a creditor receive to whom he owes £93 10*s.*?

8. An agent receives rents amounting to £862 10*s.*, and deducts $2\frac{1}{2}$ per cent. commission. What sum should he pay over to his employer?

9. If it costs £38 10*s.* to keep a family of 7 persons for 11 weeks, what will be the expense of keeping 15 persons for 9 weeks at the same rate?

10. Find the interest on £850 for 4 months at the rate of $4\frac{1}{4}$ per cent. per annum.

11. If bottles of soda water are bought at 9*d.* a dozen, and retailed at 2*d.* each, what percentage of the receipts is profit?

12. Divide £194 5*s.* among three persons, so that their shares may be in the ratio of 5 to 12 to 13

13. If £1 is equivalent to 25·50 francs, a dollar to 4*s.* 2*d.*, and a rouble to 3·40 francs, find the ratio of a rouble to a dollar.

14. If 70 yards = 64 metres, find how long a man going 2 metres a second will take to walk 18 miles.

15. Find the amount of a rate of 3*s* 8*d.* in the pound on a sum of £47 10*s.*

16. Find the interest on £620 for nine months at the rate of $2\frac{1}{4}$ per cent. per annum.

17. At what rate per cent. per annum will the interest on £250 for 3 years amount to £33 15*s.*?

18. A bankrupt's liabilities amount to £1987 and his assets realize £924 13*s.* $5\frac{1}{2}$*d.* How much in the pound will his creditors receive?

19. If 7 quarters 2 bushels of wheat cost £9 8*s.* 6*d.*, find the cost of 11 quarters 3 bushels 3 pecks.

20. If a train run 1408 yards in 18 seconds, how long will it take to travel a mile and a half?

21. A contractor agreed to complete 500 yards of railway in 40 days, and hired 120 men for the purpose; at the end of 25 days he found only 200 yards finished. How many additional men had he then to engage in order to finish the work in the given time?

22. Find the quarterly interest on £3560 invested at two and a half per cent. per annum, after deducting income tax at 1*s* in the £1.

23. *A* puts £5000 into a business, *B* £7500, and *C* £10,000. The profits amount to £12,000. What should be the share of each?

24. A man dies and leaves £6750 to be divided between his three sons in the ratio of 11 : 9 : 7. How much does each receive?

25. If 12 men working $8\frac{1}{2}$ hours a day can mow 136 acres in 15 days, in how many days can 10 men working 9 hours a day mow 224 acres?

26. If a Kg. = 2·204 lb, and a Ill. = 3·531 cu. feet, what is the weight in grammes of 5·51 litres of a substance of which 10 cu. in. weigh 1 oz.?

27. If the cost of carrying 94 tons for 21 miles is £5 5*s.* 9*d.*, how far can 112 tons be carried for £36?

28. A Russian verst is ·6629 and a French kilometre ·6214 of an English mile. Find how many kilometres are contained in 15535 versts.

29. Find the interest on £643 6*s.* 8*d.* for 3 years 9 months at $2\frac{1}{2}$ per cent.

30. A grocer mixes 28 lb. of tea costing 1*s.* $4\frac{1}{2}d.$ per lb. with 11 lb. costing 2*s.* 2*d.* per lb. What is his gain per cent. if he sells the mixture at 1*s.* $10\frac{1}{2}d.$ per lb.?

31. Find the cost of insuring a cargo valued at £8500 at $12\frac{3}{4}$ per cent.

32. How much will a creditor receive, to whom a bankrupt, paying 8*s.* $10\frac{3}{4}d.$ in the pound, owes £4970?

33. If 1 ton 19 cwt. of lead is worth £50 2*s.* $7\frac{1}{2}d.$, what is the value of 10 cwt. 2 qrs. of the same?

34. Divide £1087 2*s.* 6*d.* among a man's two sons, three daughters, and one grandchild, so that the shares of a son, a daughter, and the grandchild may be in the ratio of 3 : 2 : 1.

35. If 8 men *or* 17 boys can do a piece of work in 91 hours, how long will it take 4 men with 24 boys to do a piece of work 5 times as great?

36. Find the interest on £375 for $2\frac{1}{2}$ years at $2\frac{1}{2}$ per cent. per annum.

37. The cost of the carriage of 1·125 tons for 52 miles is 4*s.* $10\frac{1}{2}d.$; how far can $13\frac{1}{3}$ tons be carried for £2 at the same rate?

38. Assuming that a metre is equivalent to 39·37 inches, and that a kilogram is equivalent to 2·205 lb., find to the nearest gramme the weight of 60 metres of iron wire of which 38 ft. 4 in. weigh 1 lb.

39. A bankrupt owes £9876, and can pay 12*s.* 6*d.* in the £1. What are his assets, and what will a creditor lose on a debt of £250?

40. In a town, the rateable value of which is £95,000, the amount charged on the rates for the purpose of education is £3240 15*s.* Find to the nearest penny the amount paid as education rate by a man whose property has a rateable value of £70.

41. A man pays income tax at 9*d.* in the £1 on his earned income (£675), and at 1*s.* in the £1 on his income from investments (£157). What is his total income tax?

42. A grocer bought 1 cwt. of coffee for £5 19*s.* 6*d.* If he sold three-quarters of it at 1*s.* 7*d.* a lb., and the rest at 1*s.* 4*d.* a lb., what was his gain or loss per cent.?

43. Find the interest for eight months on £1387 10*s.* at $2\frac{3}{4}$ per cent. per annum.

44. What is the interest on £505 2*s.* 6*d.* for 5 years at $4\frac{1}{2}$ per cent. per annum?

45. If 9 tons are carried 220 miles by rail at a cost of £1 13*s.*, what should be charged for the conveyance of 8 tons over a distance of 77 miles?

46. If 7 men working 5 hours a day do a piece of work in 4 days, how long will 10 men take to do the same if they work 4 hours a day?

47. If 100 fr. are worth £3 19*s.* 4*d.*, and 1 Km. = 1093·4 yds., how many Km. can I travel for 35·5 fr., the fare being equivalent to 1*d.* per mile?

48. My house is assessed at £56. What will be the amount of a poor rate payable at 3*s.* 2*d.* in the £1?

49. A merchant buys 72 gallons of whisky at 16*s.* 6*d.* per gallon, and 36 gallons at 19*s.* 6*d.* per gallon; he mixes them and sells the mixture at 20*s.* per gallon. What does he gain per cent. on his outlay?

50. Find the interest on £219 18*s.* 9*d.* for $2\frac{1}{2}$ years at $3\frac{1}{3}$ per cent. per annum.

51. Find the interest on 11250 fr. for 3 years at $2\frac{1}{2}$ per cent. per annum.

52. In journeying from Plymouth to Inverness, a traveller passes over 129 miles of the Great Western Railway, 315 miles

Midland, 1 mile North Eastern, 126 miles North British, 42 miles Caledonian, and 118 miles Highland. If he pays £5 for his ticket, how ought this amount to be distributed between the different railways?

53. In a West Indian Island it is estimated that there are 3 white inhabitants to every 50 blacks. If the total population is 100,000, how many whites are there in the Island?

54. A man, after paying an income tax of 7*d.* in the £1, has £939 15*s.* 4*d.* left; what is his gross annual income?

55. A barrel of beer (36 gallons) is bought for £2 5*s.* and retailed at 6*d.* a quart. Find the gain per cent. on the cost price, if about a gallon and a half is wasted.

56. Find the interest on £4068 15*s.* for $4\frac{2}{3}$ years at $3\frac{3}{7}$ per cent.

57. A bankrupt owes three tradesmen £8 10*s.*, £13 16*s.*, and £16 14*s.* respectively, but has only £20 6*s.* 3*d.* with which to pay them. What does each receive?

58. If 20 men do a piece of work in 14 weeks, working 6 days a week and 8 hours a day, in how many weeks will 24 men do it, working 5 days a week and 7 hours a day?

59. If the cost of carriage of goods in England be 1*d.* per ton per mile, find the equivalent cost of carriage in France in centimes per 1000 Kg. per Km.

(Take 1 Km. = 5 fur.; 1 Kg. = 2·2 lb.; 25 fr. = £1.)

60. A contractor is bound to do a piece of work in 17 days. He engages 20 men, who work 8 hours a day. At the end of 6 days one third of the work is done. How many of the men must, for the rest of the time, work an hour overtime per day, to finish the job in 17 days?

61. A train left King's Cross at 9.45 a.m., passed through Grantham, and reached Doncaster at 12.34 p.m. Assuming that it travelled at a uniform rate without stopping, find, to the nearest minute, the time at which it passed through Grantham, the distances from King's Cross of Grantham and Doncaster being $105\frac{1}{4}$ and 156 miles respectively.

62. In a town, of which the rateable value amounts to £105,000,

the school committee requires £4156 5*s*. Find, to the nearest penny, the amount of school rate paid by a person whose house has a rateable value of £66 13*s*. 4*d*.

63. Find the yearly interest on £600 at $5\frac{1}{2}$ per cent. per annum, after deducting income tax at 9*d*. in the pound.

64. A farmer bought 200 sheep at £2 7*s*. 6*d*. each. If he sold 80 sheep at £2 12*s*. 6*d*. each, at what price per head must he sell the remainder in order to realize a profit of 20 per cent. on the whole of his outlay?

65. I buy eggs at 14 for a shilling, and sell them at 6*s*. 6*d*. a hundred. What is my gain or loss per cent.?

66. Find the interest on £1280 for 5 months at $1\frac{1}{4}$ per cent. per annum.

67. *A*, *B*, *C*, and *D* enter into trade. *A* started business with £500; after 2 months *B* and *C* joined him, *B* contributing £700 and *C* £650. After 2 months more *D* joined with a capital of £800. One year after *A* had started the business, they had gained £932 8*s*.; what was the share of each?

68. A creditor received 16*s*. 3*d*. in the £1; thereby he lost £271. What was the amount due to him originally?

69. If 12 labourers can dig a trench, measuring 30 yards in length and 4 yards in width, in 4 days of 9 hours each, in how many days of 8 hours each will 18 men dig a trench 80 yards long, 3 yards wide, and of the same depth as the other?

70. A man leaves £6400 to be divided among 4 sons, 3 daughters, and 3 nephews. If each daughter receives three times as much as each nephew, and each son five times as much as each nephew, how much does each receive?

71. Taking £1 as equal to 25 fr., find which is the cheaper:—land sold at £35 an acre, or land sold at 2000 fr. a Hectare. Express the cheaper price as a percentage of the dearer.

(An acre = ·4047 Ha.)

72. In 1907 the world's production of gold amounted to £84,825,000, and of silver to £49,232,000. In the same year the British Empire produced gold to the value of £50,909,000, and silver to the value of £7,864,000. Find to the nearest whole

number what percentage of the total amount of (*a*) gold, and (*b*) silver, was produced in the British Empire.

73. Find the fire insurance premium at the rate of $\frac{1}{12}$ per cent. required to insure property of value £4725.

74. A man's income is reduced to £522 after he has paid an income tax of 8*d.* in the £1. Find what would have been his net income if the income tax had been 7*d.* in the £1.

75. A cloth manufacturer makes 1000 metres of cloth. He sells 800 metres at 4 fr. per metre, making a profit of 20 per cent. on the cost, but can only obtain half that price for the remainder. What is his total profit or loss, and what rate per cent. is this on his outlay?

76. A man's net income being £750 *after* paying income tax at 1*s.* 3*d.* in the £1, on what amount is the income tax assessed?

77. A merchant bought 50 cases of wine for £225; he sold 30 cases at a profit of 25 per cent. At what price per case must he sell the other 20 cases so that his total receipts may be 20 per cent. more than his outlay?

78. If timber is purchased at 5*d.* per square foot, how many square feet must be sold for 12*s.* 6*d.* to gain 25 per cent. profit?

79. A bankrupt's debts amount to £8944 18*s.* 9*d*, and his assets to £3570 16*s.* 8*d.* How much will a creditor, to whom he owes £501, receive, supposing that the legal formalities cost a twentieth part of the whole assets?

80. A garrison has supplies of food sufficient to maintain 20,000 men for 48 days, at the rate of 12 oz. per man per day. To what number must the garrison be reduced, in order that the supplies may last for 72 days at the rate of 10 oz. per man per day?

81. A town in a period of five years decreased 5 per cent. in population, but during the next period of five years it increased 3 per cent. How much per cent. has it increased or decreased during these two periods?

82. The rates in a certain town amount to 6*s.* 10*d.* in the £1 on

the rateable value of property. A man living in a house of which the rent is £50 and rateable value £26 moves to another of which the rent is £46 and rateable value £30. By what percentage is his joint expenditure on rent and rates reduced?

83. If goods are marked for sale at 20 per cent. above cost price, and 5 per cent. discount is given for cash, what is the actual profit on an outlay of £700?

84. If 224 equally capable men can do a certain quantity of work in 9 days of 10 hours each, how many of them will be required to do half that quantity of similar work in 6 days of 8 hours each, assuming that each one of the men does $\frac{1}{5}$ more work per hour when he works 8 hours per day than when he works 10 hours per day?

85. Of two steel bars manufactured, one in England and one in France, the one manufactured in England was guaranteed to stand a pressure of six tons per square inch, and the one manufactured in France a pressure of 800 Kg. per sq. cm. Find which of these was the stronger.

(1 lb. = 453·6 gr. 1 yard = ·9144 metres.)

86. If a man with an income of more than £5000 pays an income tax of 1*s.* 2*d.* in the £1 on the first £3000, and of 1*s* 8*d.* in the £1 on the remainder, find what average rate of tax in the £1 a man with an income of £6,800 pays.

87. A person who had bought goods for £500 sold half of them at a profit of 5 per cent., half the remainder with a profit of £18 7*s.* 6*d.*, and the rest for £147 2*s.* 6*d.* What was his whole gain, and the gain per cent. on his outlay?

88. There were 557 inhabited houses in a growing village at the end of the year 1908, and 597 at the end of 1909; what was the percentage increase on the number at the end of 1908? If the percentage increase in the following year on the number at the end of 1909 is the same as the percentage increase that took place during 1909, what will be the number of inhabited houses at the end of 1910?

89. A coal dealer sells some coal at 22*s.* a ton, and thereby loses a penny on every shilling of the cost price. What would he have gained on every shilling by selling his coal at 27*s* a ton?

(**151.**) In all the examples we have been considering, it has been assumed that we are dealing with cases of **direct proportion.**

Such cases arise in questions involving the purchase of anything such as coal, where, if we double the amount of coal, we double the cost; in calculating the amount of work which men can do in a given time, where, if we double the number of men, we double the amount of work they can do (or halve the time they work); in estimating taxes on income, &c.

In all these cases the one quantity not only **depends** upon the other but is *proportional* to it.

Note:—Direct proportion can best be illustrated graphically.

If one quantity y depends on another quantity x (is a function of x), then, if various values are given to x and corresponding values of y are found, the graph obtained will be a straight line when the quantities are directly proportional to one another.

The same idea can be expressed by saying that if x increases steadily, then y will also increase at a steady rate

(**152.**) But there are cases where this is not true.

For example, if we double the side of a square, we do not double the area, but, in fact, we multiply its area by 4 So, also, if two circles have their radii in the ratio of 2 to 3, their areas are in the ratio of the squares of these numbers, or 4 to 9.

Such a result is expressed by saying that *the area of a circle is proportional to the square of its radius*, or *the ratio of the areas of two circles is the same as the ratio of the squares of the measures of their radii.*

Again, the time of oscillation of a pendulum is proportional to the square root of its length, so that the times of oscillation of two pendulums of lengths 6 ft. and 7 ft. would be in the ratio of $\sqrt{6} : \sqrt{7}$.

When we have the law connecting such ratios it is easy enough to work examples.

Example. *The areas of circles are in the ratio of the squares of their radii. A circle of radius 6″ has an area of 113 sq. in. What is the area of a circle of radius 8″?*

The ratio of the areas is $\frac{8^2}{6^2}$.

$\therefore$ the new area $= \frac{64}{36}$ of 113 sq. in.

$= \frac{16}{9}$ " "

$= \frac{1808}{9}$ " "

$= 201$ sq. in. approx.

$$\begin{array}{r} 113 \cdot 0 \\ 678 \\ \hline 1808 \end{array}$$

Examples X b.

1. The area of a circle is proportional to the square of its radii. A circle of radius 5 in. has an area of 78·5 sq. in. What is the area of a circle of radius 7 in?

2. The surface of a ball is proportional to the square of its circumference. A tennis ball 20 cm. in circumference requires 127·3 sq. cm. of cloth to cover it. How much cloth would be required to cover a ball 12 cm. in circumference?

3. The volume of a ball is proportional to the cube of its radius. A ball of 2·3 in. radius has a volume of 50·97 cub. in. What is the volume of a ball of 3 in. radius?

4. The pressure of the wind on the front of a train is approximately proportional to the square of the speed of the train. The pressure on an engine running at 45 m. h. is 12·67 lb. per sq. ft. Find the pressure on an engine running at 60 m. h.

5. The time of swing of a pendulum is proportional to the square root of its length. The length of a pendulum which swings once a second is 39·14 in. Find the time of swing of a pendulum 60 in. long.

6. The time of swing of a pendulum is proportional to the square root of its length. If a pendulum 1 metre in length swings once in a second, find to the nearest millimetre the length of a pendulum which swings 75 times in one minute.

7. The weights of two solids of the same shape and of the same material are in the ratio of the cube of their dimensions.

A model of a gun is made of the same metal as the gun, the dimensions being reduced on the scale of 1·35 inches to the foot. If the model weigh 48 lb., find the weight of the gun to the nearest ton.

8. The strength of a rope is proportional to the square of its diameter. A rope one inch in diameter can stand a strain of a ton.

What is the greatest strain in lb. weight to which a rope of ·84 in. diameter may be subjected?

9. The squares of the times of revolution of the planets of the Solar System are proportional to the cubes of their distances from the Sun. Find, to the nearest day, the time Jupiter takes to go round the Sun, given that his distance from the Sun $= 5{\cdot}2$ times the Earth's distance.

10. The distance which a stone falls in a given time is proportional to the square of the number of seconds it has been falling. A stone dropped from a height of 60 ft takes 1·93 seconds to come to the ground. How far would a stone dropped from a balloon fall in 10 seconds.

11. A stone is dropped into a well, and the time it takes to reach the water is proportional to the square root of the distance through which it has fallen.

A stone takes ·866 sec. to fall 12 ft. How long will a stone take to reach the bottom of an empty well 35 ft. deep?

12. The weight of a bar of iron of circular section is proportional to the product of its length by the square of the diameter.

A bar 30 cm. long and 3 cm. thick weighs 1749 g. What is the weight of a bar 20 cm. long and 4 cm. thick?

13. Two figures of the same shape but of different sizes are cut out of paper. They represent maps of an island on two different scales. On one, the distance between two towns is represented by a line 2″ long, and on the other the same distance is represented by a line 2·7″ long. The area of the smaller piece of paper is 8·34 sq. in. What is the area of the larger?

14. The volume of a cylindrical vessel is proportional to the product of the square of the radius by the depth.

A cylindrical tub 1·25 metres in diameter and 98 cm. high is to be filled with water from a tap by means of a cylindrical vessel 10 cm. high and 12 cm. in diameter. How many times must the measure be filled from the tap?

15. The length of a pendulum is proportional to the square of the time of one oscillation. A pendulum 36 inches long is found to oscillate 61 times in a given interval of time. What change of length will be necessary to make it oscillate 60 times in the same interval?

16. A plate of metal weighs one pound per square foot. Find its weight per square centimetre in grammes, taking one foot equal to 30·479 cm., and one pound equal to 453·593 g.

17. The rate of consumption of coal in a locomotive is proportional to the square of the speed, and a locomotive running at 15 m. h. burns 2 tons an hour. Find the rate of consumption when travelling 60 m. h.

If coal costs 12*s.* a ton, find the cost per mile in each case.

CHAPTER XI

FORMULAE

(**153.**) When we know the length and breadth of a rectangle in terms of some unit of length, we know that we can find the area of that rectangle by multiplying together the two numbers representing the length and breadth.

This may be expressed algebraically thus

If the length is l (in., cm., yd., &c.), the breadth b (in., cm., yd., &c.), and the area A (sq. in., sq. cm., sq. yd., &c.),

then $$A = bl.$$

This is the simplest type of **formula**, and it is true for all rectangles.

If we are given the values of any two of the quantities A, b, l, we can find the third by putting in the values of the two we know, obtaining an 'equation' for the third.

The following examples will give practice in calculating values of certain quantities from such formulae.

Example 1. *The distance (s ft.) described by a falling stone in time (t sec.) is given by the formula*

$$s = 16\,t^2.$$

Find the distance a stone will fall in 5 sec, and the time taken to fall 50 ft.

(1) When $t = 5$.

$$\begin{aligned} s &= 16 \times 5^2 \\ &= 16 \times 25 \\ &= 400. \end{aligned}$$

$\therefore$ a stone falls 400 ft. in 5 sec.

(2) When $s = 50$.

$$50 = 16t^2.$$

$$\therefore \quad t^2 = \frac{50}{16}$$

$$= \frac{12\frac{5}{4}}{4}$$

$$= 3{\cdot}125.$$

$$\therefore \quad t = 1{\cdot}77 \text{ approx.}$$

$$\begin{array}{r|l} 3\,125 & 1{\cdot}77 \\ 27 & 2\,12 \\ 34 & 2350 \end{array}$$

$\therefore$ a stone falls 50 ft. in 1·77 sec.

Example ii. *The Board of Trade Rule for spiral springs is*·

D in. = *diameter of coil, centre to centre,*
d in. = *diameter of spring steel,*
W lb. = *greatest permissible load on spring.*
then $d = {\cdot}05\sqrt[3]{(WD)}$.

If $d = {\cdot}725$ *in. and* $D = 6{\cdot}42$ *in., what load would the spring support?*

Putting in the values we know, we have

$${\cdot}725 = {\cdot}05\sqrt[3]{(W \times 6{\cdot}42)}.$$

$$\therefore \sqrt[3]{(W \times 6{\cdot}42)} = \frac{{\cdot}725}{{\cdot}05} \qquad \text{(dividing by ·05)}$$

$$= 14{\cdot}5.$$

$$\therefore \quad W \times 6{\cdot}42 = (14{\cdot}5)^3. \qquad \text{(cubing each term of this equality)}$$

$$= 3048{\cdot}6.$$

$$\therefore \quad W = \frac{3048{\cdot}6}{6{\cdot}42}$$

$$= 474{\cdot}8 \text{ lb.}$$

$$\begin{array}{r} 14\,5 \\ \hline 145\cdot \\ 58\,0 \\ \hline 7\,25 \\ \hline 210\,25 \\ \hline 2102{\cdot}5 \\ 841\,0 \\ \hline 105{\cdot}1 \\ \hline 3048\,6 \end{array}$$

$$\begin{array}{r} 474{\cdot}8 \\ \hline 6\,42\,)3048{\cdot}6 \\ 480\,6 \\ \hline 31{\cdot}2 \\ 5{\cdot}5 \end{array}$$

$\therefore$ the spring will support just over 474 lb.

Examples XI a.

1. The force (P lb.) required to raise a weight (W lb.) by means of a certain machine is given by the formula

$$P = \cdot 072\ W + 1 \cdot 47.$$

Find the force required to raise 100 lb., and the weight which can be raised by a force of 12 lb.

2. Given $u = 25 \cdot 24$, $v = 13 \cdot 27$, calculate x to four significant figures from the equation

$$\frac{1}{x} = \frac{1}{u} + \frac{1}{v}.$$

3. The weight of cast iron piping per foot can be calculated from the formula

$$w = 2 \cdot 45\ (a^2 - b^2) \text{ lb.},$$

where a is the external, b the internal diameter of the pipe in inches.

Find the weight, to the nearest pound, of 3 yards of piping in which the external diameter is 13·5 in., and the internal diameter 12 in.

4. A ball of diameter d inches has a volume V cubic inches given by the formula

$$V = \cdot 524\ d^3.$$

Calculate the volume of a ball which has a diameter of 2·32 inches.

5. The speed (v ft. per sec.) acquired by a stone falling from a height h ft. is given by

$$v^2 = 64\ h.$$

Find the speed acquired by a stone falling from a height of 70 ft.

6. The time (t sec.) of one swing of a pendulum of length l ft. is given by the formula

$$t = 6 \cdot 283 \sqrt{\left(\frac{l}{32}\right)}$$

Find the number of times a pendulum 6 ft. long will swing in a minute.

7. A company plans an airship, which is stated to be approximately cylindrical, 700 feet long, and 80 feet in diameter. I know that an airship of cylindrical shape, of length l feet and diameter d feet, has a cubic capacity of $\cdot 7854\, d^2 l$ cubic feet, and that the total weight it can support, estimated in ounces, must not exceed the total capacity measured in cubic feet.

Find the greatest weight (in tons) which the airship can support.

8. A hollow circular cylinder, of length l in., whose outside diameter is d in., and thickness t in., has a volume V cub. in., given by the formula

$$V = \tfrac{22}{7}\, l\, t\, (d + t).$$

Calculate the volume of such a cylinder, 2 ft. long, if its outside diameter is 6 in., and its thickness $\frac{1}{2}$ in.

9. With the formula of question 8, calculate the diameter of a lead pipe which has a volume 277·2 cub. in., and thickness ·3 in., if it is 7 ft. long.

10. If a triangle has sides a in., b in., c in., then the area A of the triangle is given by the formula

$$A = \sqrt{[s(s-a)(s-b)(s-c)]},$$

where $s = \frac{1}{2}(a+b+c)$.

Find the area of a triangle whose sides are 5 in., 6 in., 7 in.

11. The weight in pounds of a foot of iron piping is given by the formula

$$W = \frac{3{\cdot}142\, t\, k}{144}\, (b + t),$$

when the thickness is t inches, the bore is b inches, and k is the weight of a cubic foot of iron.

A pipe whose bore is 3 in. and thickness $\frac{3}{8}$ in. weighs 13·3 lb. per ft Find the weight of a cubic foot of iron.

12. The volume of a portion of a cone can be obtained from the formula

$$V = \tfrac{1}{3}\, h\, (a + b + \sqrt{ab}),$$

h in. denoting the height of the portion, and a sq. in. and b sq in. the areas of the two ends. Calculate the volume to the nearest cubic inch when $h = 4{\cdot}5$ in., $a = 28{\cdot}5$ sq. in., $b = 78{\cdot}6$ sq. in.

13. The height H of a mountain in feet is given by

$$H = 49{,}000 \left(\frac{R-r}{R+r}\right)\left(1 + \frac{T+t}{900}\right),$$

where R, r are the observed heights of the barometer in inches, at the bottom and top of the mountain, and T, t are the observed temperatures at the bottom and top.

The following observations were made at the bottom and top of a certain mountain:

	Bar.	Therm.
At bottom	29·60 in.	67°.
At top	25·35 in.	32°.

Find the height of the mountain.

14. The distance s miles visible at sea from a mast of height h ft. is given by the formula

$$s = \sqrt{\frac{Rh}{2640}},$$

where R miles is the radius of the earth (approximately 3960 miles).

Find the distance visible from a conning tower 72 ft. above sea-level.

15. If the line from the eye to the top of a tower some distance away makes an angle $A°$ with the horizon, then the height of the tower is given approximately by the formula

$$h = Ad/57{\cdot}3,$$

where d is the distance to the foot of the tower.

Calculate the height of a tower 170 yards away for which the angle is 9°.

AVERAGES

(**154.**) Suppose a cricketer has made scores of 32, 24, 17, 0, 65, 11, 0, 104, in successive innings. He has had 8 innings and has scored altogether 253 runs.

His **average** is $\frac{253}{8}$ or 31·62 . . ., because, if he scored **31** or **32** in each innings, his total would be very near to the actual total.

Similarly, if we have a collection of quantities we can obtain the average of all of them, by finding their sum, and dividing by the number of the quantities.

(**155.**) In calculating the average of a collection of quantities, such as dates, times, ages, &c., it is often convenient to take a standard somewhere near the probable average, and express all our quantities as numbers above or below that standard.

Example i. *Find the average age of a class of boys whose ages respectively are* (*in years and months*)

13.10,	14.1,	15.1,	12.11,	13.2,
13.6,	14.0,	13.5,	13.2,	13.7,
12.10,	14.2,	14.1,	13.9,	12.8.

We work from the age 13 years as standard, and express all the ages as months above 13 years.

These numbers will be

10.	13.	25.	−1.	2.	6.	12.	5.	2.
7.	−2.	14.	13	9.	−4.			

The sum is 111.

The average of these numbers $= \frac{111}{15} = \frac{37}{5} = 7{\cdot}4$ months.

∴ the average age is 13 years 7 months.

Example ii. *The dates of breaking up for Easter Holidays at a certain school were, during ten consecutive years,*

Apr. 5th,	Apr. 12th,	Mar. 28th,	Mar 27th,	Apr. 10th,
Apr. 9th,	Apr. 8th,	Apr. 14th,	Apr. 13th,	Mar. 29th.

Find the average day of breaking up.

We work from Mar. 31st as standard, and express these dates as days after Mar. 31st.

The numbers are: 5, 12, −3, −4, 10, 9, 8, 14, 13, −2.

The sum is 62.

The average of the numbers is $62 \div 10 = 6{\cdot}2$.

∴ the average day of breaking up is Apr. 6th.

Examples XI b.

1. The following numbers give the population in 1909 of a number of towns. Find the average population of these towns, (*a*) including London, (*b*) excluding London:

London 4,833,938.

Birmingham	563,629.	Manchester	655,435.
Bristol . .	377,642.	Nottingham	263,443.
Hull . . .	275,552.	Sheffield .	470,958.

2. The ages of a number of boys in a class are (in years and months) 16.8, 15 4, 17 2, 17.3, 16.9, 16.11, 17.0, 15.3, 14.8, 16.4, 16.8, 17.1, 15.9, 15.11.

Find the average age of the class.

3. The following numbers give, in thousands of tons, the amount of coal produced in the British Empire during the years 1903 to 1908.

1903,	256,414.	1906,	284,227.
1904,	259,778.	1907,	304,724.
1905,	265,220.	1908,	301,244.

Find the average number of tons produced during these years.

4. The following times are the times of rising and setting of the sun at Greenwich on the first day of each month during the year.

Month .	Jan.	Feb.	March	April	May	June
Rise	8 8	7 42	6 49	5 39	4 35	3 51
Set . .	3 58	4 46	5 37	6 29	7 19	8 21

Month . .	July	Aug	Sept	Oct	Nov	Dec.
Rise . . .	3.48	4 24	5 13	6 0	6.54	7 44
Set . . .	8 18	7 48	6 48	5.40	4 34	3 54

Find (1) the average time of rising;
(2) the average time of setting;
(3) the average duration of daylight.

5. A cricketer has had seven innings with an average of 31·23. He scores in the next four innings 37, 62, 45, 0.

What is his average after these innings?

6. The daily pay of a Second Lieutenant in various branches of the Service is given in the following table:

Household Cavalry	6*s.* 8*d.*
R. H. A.	7*s.* 8*d.*
Line Cavalry	6*s.* 8*d.*
R. Field Artillery, R. G. A., R. E.	5*s.* 7*d.*
Foot Guards, Infantry, A. S. C.	5*s.* 3*d.*

Calculate the average pay of a Second Lieutenant.

7. The times in the Grand Challenge Cup at Henley for the years 1898–1906 were (in minutes and seconds) 7.13, 7.12, 7.6, 7 4, 7 17, 7 9, 7.20, 6.58, 7.9.

Find the average time for the race during these years.

8. The following table gives the rainfall in inches and the number of rainy days during the year 1910.

Month	Jan.	Feb.	March	April	May	June
No. of rainy days	15	24	10	21	20	14
Rainfall	1 53	2 51	0 88	1 65	2·15	1·93

Month	July	Aug.	Sept.	Oct.	Nov.	Dec.
No. of rainy days	17	19	4	15	14	22
Rainfall	2·48	1 83	0·55	1·90	0 79	2 40

Calculate (1) the average rainfall per month;
(2) the average number of rainy days per month;
(3) the average rainfall per rainy day.

9. The following table gives the amount of premium to be paid at the age of 21 for life assurance (with profits) in various offices:

Atlas	£2 2*s.* 5*d.*	Norwich Union	£2 3*s.* 5*d.*
Clergy Mutual	£1 16*s* 0*d.*	Phoenix	£2 0*s.* 11*d.*
Equitable	£2 4*s.* 5*d.*	Prudential	£1 18*s.* 10*d.*
General	£2 0*s.* 0*d.*	Scottish Equitable	£2 1*s.* 7*d.*
		Scottish Widows	£2 3*s.* 1*d.*

Find the average premium payable at this age.

10. The following table gives the dates on which Easter Day falls during the years 1921 to 1930:

1921,	Mar. 27.	1926,	Apr. 4.
1922,	Apr. 16.	1927,	Apr. 17.
1923,	Apr. 1.	1928,	Apr. 8.
1924,	Apr. 20.	1929,	Mar. 31.
1925,	Apr. 12.	1930,	Apr. 20.

Find the average date for Easter Day during these years.

11. On looking up a time-table of trains between two stations, I find that 5 trains take 1 hr., 3 trains take 1 hr. 6 min., 4 trains take 1 hr. 11 min, and 6 trains take 1 hr. 17 min.

What is the average duration of a journey between these stations?

(**156.**) **Graphs of Statistics.** In the examples on averages we have met several 'tables of statistics', such as the table giving the time of sunrise, and the table giving the amount of coal produced in Great Britain. It is often convenient to construct a diagram which will show at a glance how these numbers are increasing or diminishing, and whether the rates of increase or decrease are steady.

The diagram is drawn on squared paper by 'plotting the points' given in the statistics, and drawing a curve as regular as possible through the points.

When the curve is drawn, we can usually make several

discoveries from it, and can sometimes even discover a general relation between the quantities.

We give two illustrations:

Example i. *The times of sunrise (on the first day of the month) are given in the following table:*

Jan	Feb.	March	April	May	June
8 8	7.42	6.49	5 39	4 35	3.51
July	Aug	Sept.	Oct.	Nov.	Dec.
3 48	4 24	5.13	6 0	6 54	7 44

Exhibit these figures graphically, and draw any conclusions you can.

A glance at the figures shows that the latest time is just after 8.0, and the earliest 3.48. The *range* of these values is about 5 hours.

We take distances to the right to represent the successive months, and distances upwards to represent the time of sunrise.

On the paper at our disposal there are 34 spaces in a horizontal direction, and 22 in an upward direction.

We have to spread 12 months over 34 spaces, and can only take 2 spaces to represent a month.

We have to spread 5 hrs. over 22 spaces and can take 4 spaces to represent an hour.

We note these *scales* on the squared paper; we space off the paper along the sides, and plot the points given in the table.

We now draw an even curve through these points and examine the curve. (Fig. 17.)

We notice that when the curve slopes steeply, the time of sunrise is increasing or diminishing rapidly, and when the slope is small, the time of sunrise is increasing or diminishing slowly.

We see that in this case the time of sunrise is getting early at a fairly steady rate between February and May, and at a slower rate in January and between May and the middle of June. Towards the end of June the sun rises earliest, and then the time slowly gets later until the end of July, when it continues

steadily to get later until the end of November, and then slowly reaches its latest time about Jan. 1st.

We can find approximately the time of rising on any date from the graph.

E.g. on April 20th (i.e. $\frac{2}{3}$ of the way through the month of April) the time of rising is approximately 4.54.

We can find also dates on which it rises at a certain time (say 7.0).

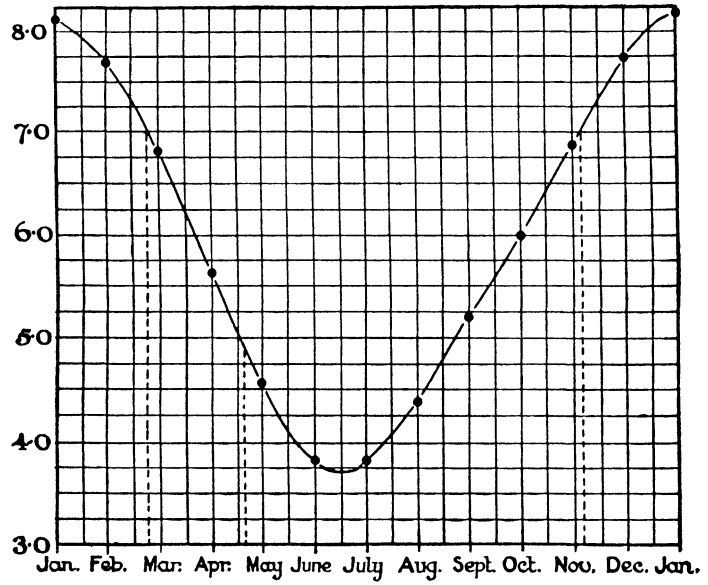


Fig 17.

These dates are about Feb. 22nd and Nov. 5th.

Note.—There is a slight inaccuracy in this graph, owing to the fact that the months are not all of the same length.

Example ii. *The following scheme shows the extension of a spring under various loads.*

Load in lb.	0	1	3	6	9	13
Reading opposite end of spring in inches . .	4·14	4·25	4·48	4·85	5·19	67

Draw a graph to show the connexion between the load and the scale reading, and state what the reading would probably be for a load of 5 lb.

Find also, if possible, a law connecting the reading with the load.

The *ranges* are load 13 lb.
Reading 1·53 in. (i.e. 5·67 − 4·14)

Taking a piece of squared paper with 34 spaces in one direction and 22 spaces in the other, we find that we can take 2 spaces in a horizontal direction to represent 1 lb., and 10 spaces in a vertical direction to represent 1 in.

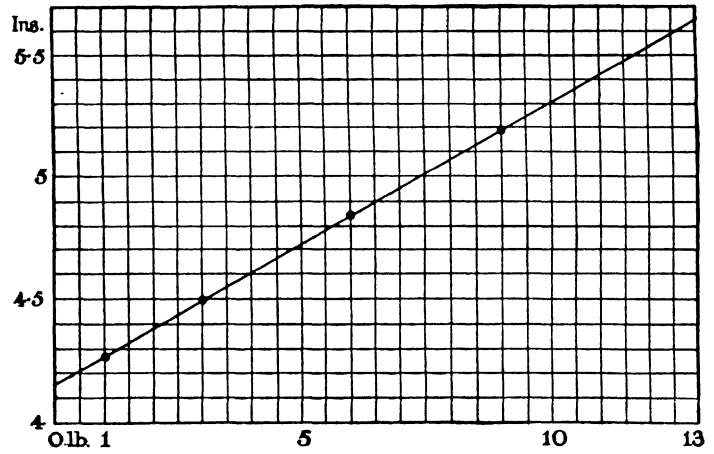


Fig. 18.

When the points are plotted, we see that they appear to be nearly in a straight line.

(This can be discovered by looking along the points with the paper held nearly on a level with the eye.)

We then draw with a ruler the straight line which seems to go most nearly through all the points.

The line shows us that for a load of 5 lb. the reading will be approximately 4·72.

We can obtain the law connecting the load with the reading by realizing that if the graph is a straight line, the ratio between differences of loads and corresponding differences of readings must be constant. (See § 142.)

E.g. the reading is 4·14 in. when the load is 0 lb

,, ,, 5·19 in. , ,, 9 lb.

$\therefore$ the stretching is 1·05 in. for a load of 9 lb.

If the stretching is x in. for a load of P lb., we have

$$\frac{x}{P} = \frac{1{\cdot}05}{9} = {\cdot}116 \text{ approx.}$$

$$\therefore \quad x = {\cdot}116\,P.$$

The reading for a load of P lb. will be $\cdot 116\,P + 4{\cdot}14$ in.

(Verification :—When the load is 5 lb. the reading is

$$\cdot 116 \times 5 + 4{\cdot}14 \text{ in.}$$
$$= {\cdot}58 + 4{\cdot}14$$
$$= 4{\cdot}72 \text{ in.}$$

as discovered above.)

Note.—We seem, in the above work, to have constructed a ratio between quantities of two different kinds, namely lb. and inches.

Actually we have been finding the equation connecting distances on the diagram representing the load, with distances on the diagram representing the readings.

So we have really formed the ratios between *numbers* representing these quantities, and have not really infringed one of the fundamental laws of ratios. (See § 133.)

Examples XI c.

1. During February the time shown by the sundial at Greenwich is slow compared with true Mean Time; the error is given below for certain dates. Represent these graphically and estimate the greatest amount of error.

Find also the error on Feb. 8th and Feb. 28th.

Day	Feb. 1	Feb. 5	Feb. 12
Error (in min. and sec.) . .	13 m. 39 s	14 m. 8 s.	14 m. 25 s.

Day	Feb. 17	Feb. 23	Feb. 27
Error (in min. and sec.) . .	14 m. 14 s.	13 m. 39 s	13 m. 2 s.

2. The following table gives (in millions) the population in England and Wales during the years 1841 to 1911.

Draw as even a curve as you can, showing (*a*) the increase in number of males, (*b*) the increase in number of females, (*c*) the increase in total population. Say whether you think these increases were steady or increasing.

Year.	1841	1851	1861	1871	1881	1891	1901	1911
Males	7·8	8·8	9·8	11·1	12·6	14·1	15·7	17·4
Females	8·1	9·1	10·3	11·7	13·3	14·9	16·8	18·6

Estimate the total population in 1896 and 1916.

3. The following table gives the number of miles (in thousands) of railway open on Dec. 31st in various years.

Draw a graph exhibiting these figures, and find at what period the rate of growth of railways was greatest.

Year.	1845	1855	1865	1875	1885	1895	1905	1909
Length	2·4	8·3	13·3	16·7	19·2	21·2	22·8	23·3

4. An insurance company gives the following quotations for premiums to be paid for £100 insurance at certain ages:

Age.	20	22	24	27	30	33
Premium.	£3 2*s.* 6*d*	£3 7*s.* 2*d.*	£3 12*s.* 4*d.*	£4 1*s.* 11*d.*	£4 16*s.* 6*d.*	£5 17*s*

Estimate the premium payable at age 25.

(You are advised to express these sums of money as decimals of £1.)

5. Two boys, *A* and *B*, had their heights measured on the first days of six consecutive years. The measurements are given in the following table:

	1906	1907	1908	1909	1910	1911
	ft. in.	ft. in.	ft. in.	ft. in.	ft. in.	ft. in.
A	3 0	3 $4\frac{1}{2}$	3 $7\frac{3}{4}$	3 $10\frac{3}{4}$	4 1	4 $2\frac{3}{4}$
B	3 2	3 $3\frac{1}{2}$	3 $5\frac{3}{4}$	3 $8\frac{1}{2}$	3 $11\frac{3}{4}$	4 4

Draw, on one diagram, graphs to illustrate their growth, and find the dates when they were the same height.

6. From the table given below, plot curves showing for the given period (1) the hour of sunrise, (2) the hour of sunset, (3) the duration of sunshine.

Deduce the date of (1) the latest sunrise, (2) the earliest sunset, (3) the shortest day.

Date	Nov. 26	Dec. 3	Dec. 10	Dec 17	Dec 24
Sunrise (hr. and min.)	7 37	7.48	7.58	8.4	8 8
Sunset	3.57	3.51	3 47	3 47	3 50

Date	Dec 31	Jan. 7	Jan. 14	Jan 21
Sunrise (hr. and min.)	8 9	8.7	8.4	7 56
Sunset	3 56	4 4	4 14	4.26

7. Measurements of some spheres resulted as follows:

Diam. in cm.	·50	1 00	1·50	2 00	2 45	2 95	3·10	3·25
Volume in c.cm.	·06	·52	1 77	4 20	7·70	13·45	15 60	18 00

Plot a graph of these values, and deduce the volume of a sphere of diameter 2·60 cm.

If V c.c. is the volume of a sphere and a cm. is the diameter, then $V = ka^3$, where k is some definite number

Find the value of k, and use it to verify the value of the volume found above.

8. In a certain experiment the following observations of volume (v) and pressure (p) of a gas were made:

v	3	3 4	4	5 2	6	7·3	8 5	10
p	107·3	89·8	71·5	49·5	45·0	30·8	24 9	19·8

Draw up a table giving corresponding values of the two variables

$$x = v - 3, \qquad y = \tfrac{1}{20}\, p,$$

and plot a graph of the results.

It is thought that one observation was inaccurate; which do you regard as doubtful?

Correct the table from your graph.

9. For a certain machine and for various loads the table shows what force P kg. was required to move the load W kg.

W	50	100	200	300	400	500	600
P	106	122	152	184	216	246	278

Draw a graph showing these values, and try to discover the law connecting the force with the load.

10. The amount of water in a tank at the end of t seconds from opening a tap is given in the following table:

Time t sec.	0	5	10	15	20	30	40	50	55	60
Gallons of water in tank	64	56·2	49	42·3	36	25	16	9	6·25	4

Estimate the time it takes to empty the tank.

11. The Navy expenditure is given below for certain years. Plot these on a diagram, and draw the straight line which will most nearly pass through the points you obtain. Taking this line to indicate what the expenditure would be in 1905–6, and in 1906–7, if there were no change of plans, estimate the expenditure for each of these years.

1897–8	£20,848,863	1901–2	£30,981,315
1898–9	23,880,875	1902–3	31,003,977
1899–1900	25,731,220	1903–4	35,709,477
1900–1	29,998,529	1904–5	36,889,500

12. A rifle, sighted to 1000 yds., rests on a support 5 ft. from the ground and is fired. The height of the bullet above the support is given by the following table:

Distance in yds. from firing-point	100	200	300	400	500	600	700	800	900	1000
Vertical height above support in ft	7·3	11·2	15·0	18·5	21·0	23·3	25	22·5	16·5	0

Give on squared paper a representation of the path of the bullet, showing also the ground level and the height of the support, and find in what positions a butt of height 20 ft. would stop the bullet.

13. The distances and second class railway fares from Paris to the under-mentioned stations are as follows, the distances being reckoned in miles, and the fares in francs and centimes.

	Distance in miles	Fare. fr.	c
St. Denis	4	0	55
Chantilly	$25\frac{1}{2}$	3	10
Creil	32	3	85
Longueil	45	5	45
Compiègne	53	6	35
Noyon	67	8	15
Chauny	77	9	35
Tergnier	82	9	90
St. Quentin	96	11	65

Draw a diagram showing the relation between distance and railway fare. Examine whether the fare is proportional to the distance travelled.

14. It is known that the power to be applied to a machine and the weight which can be lifted by the power are connected by an equation of the form $P = aW + b$, where P and W represent the power and the weight, and a and b are fixed numbers. Find a and b, if when

$$W = 5{\cdot}5 \text{ lb.}, \qquad P = 4{\cdot}02 \text{ lb.};$$
$$W = 9{\cdot}5 \text{ lb.}, \qquad P = 6{\cdot}66 \text{ lb.}$$

Draw a graph to show the relation between P and W, and read off the value of W when $P = 5$ lb.

15. A sectional plan of a line across certain country is to be drawn, and the following table gives the distances from the starting-point where the 'contour lines' cross the given line

Height above sea level	1000	900	800	700	600	600
Distance in miles	0	·1	21	28	49	·78
Height above sea level	700	800	900	1000	1000	900
Distance in miles	88	98	1 05	1·13	1·22	1·7

Draw, as accurately as you can, a sectional diagram of this line of country.

LIMITS OF ERROR

(157.) The following method should be employed when it is required to estimate the possible error in results obtained from approximate measurement, the possible error in these measurements being known.

Example. *The sides of a rectangle are found to be* 17·56 *inches and* 13·89 *inches respectively, correct to the nearest hundredth of an inch.*

Find the possible error in calculating (i) *the perimeter,* (ii) *the area.*

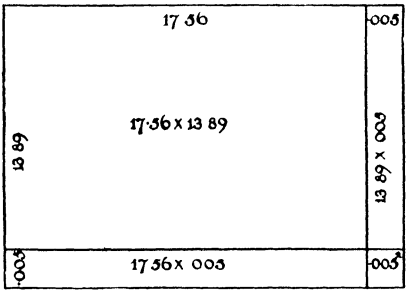


Fig. 19

The real length lies between 17·565 and 17·555 inches, i.e. between (17·56 + ·005) inches and (17·56 − ·005) inches.

The real breadth lies between

(13·89 + ·005) inches and (13·89 − ·005) inches.

Hence the real perimeter lies between

2 (31·45 + ·01) and 2 (31·45 − ·01) inches.

That is, the possible error is ·02 inches.

Similarly, the real area lies between

(17·56 + ·005) (13·89 + ·005) and (17·56 − ·005) (13·89 − ·005) sq. inches.

The calculated area is 17·56 × 13·89 sq. inches.

Hence the possible error is ·005 (17·56 + 13·89) + ·005² sq. inches.

This is shown by the figure above (which is not drawn to scale).

Omitting $\cdot 005^2$ as being relatively unimportant, we find that the possible error is $\cdot 005 \times 31 \cdot 45$, i. e. $\cdot 15725$ sq. inches.

	17 56 × 13 89	
	175 6 . . .	
	52 68 . .	
	14·048 .	
	1 5804	
	. . . .	
Calculated area	243 9084	sq inches.
Possible error	·1572	,, ,,

Hence the real area may be as great as 244·06 sq. inches
or as small as 243·75 sq. inches.

Hence only three figures in the calculated area can be relied on.

PERCENTAGE ERROR

(**158.**) The previous section shows how the greatest possible error may be calculated. In estimating the value of a result it is often more important to know the ratio the possible error bears to the true answer; this ratio is usually expressed as a percentage.

Example 1. *The side of a box is measured as* 23·4 *inches correct to the nearest tenth of an inch. What is the greatest possible percentage error?*

Here the possible error is ·05 inches.

∴ the least value for the true length is 23·35 inches.

∴ possible error $= \frac{\cdot 05}{23 \cdot 35}$ of the true length.

$= \frac{5}{23 \cdot 35}$ per cent.

$= \cdot 21$ per cent.

$$23 \cdot 35 \overline{)5} \begin{matrix} \cdot 21 \\ \cdot 33 \end{matrix}$$

Example ii. *A quantity g is known to be* 981 *when lengths are measured in centimetres and* 32·2 *when lengths are measured in feet. A boy, measuring in centimetres, finds g to be* 975; *another boy, measuring in feet, finds g to be* 31·8. *Which of these results is more accurate?*

In the first case, the error $= \frac{6}{981}$ of the true value.

$= \frac{600}{981}$ per cent.

$= \cdot 6 \ldots$ per cent.

In the second case, the error $= \frac{\cdot 4}{32 \cdot 2}$ of the true value.

$= \frac{40}{32 \cdot 2}$ per cent.

$= 1 \cdot 2 \ldots$ per cent.

Hence the first result is more accurate.

Examples XI d.

Between what limits do the perimeter and area of the following rectangle respectively lie

1. Length 11·32 in., breadth 7·49 in., correct to two decimal places.

2. Length 14·3 cm., breadth 8·3 cm., correct to one decimal place.

3. Length 214 ft., breadth 179 ft., correct to nearest foot.

4. Length 3·16 cm., breadth 2·59 cm., correct to two decimal places.

5. Length 349 yds., breadth 287 yds., correct to nearest yard.

6. What is the possible error in calculating the volume of a cube, its edge being measured as 27·4 cm., correct to the nearest millimetre?

Find the percentage error in the following cases:

	Correct value.	*Calculated value.*
7.	3·14159	3·1428
8.	·86603	·857
9.	1·535	1·378
10.	·65	·59

11. A rectangular piece of paper is measured in inches and in centimetres with the following results.

Length 8·73 inches; 22·17 centimetres.
Breadth 5·57 inches; 14·15 centimetres.

Find the area of the paper in square inches and in square centimetres, and hence find the number of square centimetres in a square inch. If the measurements are correct to the nearest figure in the second place of decimals, to how many places may your results be regarded as accurate?

12. The length, breadth, and thickness of a block of metal to the nearest millimetre are 327, 268, and 132 millimetres respectively. Find the total surface, area, and the volume of the block accurately to as many significant figures as the data permit. What is the greatest possible error in the surface and volume respectively when they are calculated to three significant figures?

13. Find the weight in pounds of a cubic inch of mercury, it being given that a cubic decimetre of mercury weighs 13·596 kilograms, an inch is equal to 2·5400 centimetres, and a pound is equal to ·45359 kilogram. What is the greatest possible error in the result?

14. One metre being equal to 39·3708 inches, find the percentage error made in assuming 1 kilometre to be equal to $\frac{5}{8}$ of a mile.

15. Find to three significant figures the value of $\sqrt{879}$, and find to two significant figures by how much per cent. the square of this approximation differs from 879.

REVISION PAPERS

A.

1. Simplify $\dfrac{1\frac{3}{4}+\frac{2}{3}\text{ of }\frac{7}{8}}{1\frac{3}{4}\div\frac{2}{3}-\frac{7}{8}}$.

2. Find, to the nearest centime, the value of 3·1416 of 5·737 m. at 2·59 fr. per metre.

3. Write down the prime factors of 1155 and 3080, and thence find their Highest Common Factor.

4. Find the exact number of francs which is most nearly equal in value to £62 6*s.* 7*d.*, at the rate of 25 fr. 15 c. for £1.

5. If a cubic dm. of ice weighs 918 g., what is the weight, to the nearest Kg., of a rectangular sheet of ice 53 m. long, 27 m. wide, and 39 mm. thick?

6. A manufacturer paid £52 16*s.* per ton for his raw material, and 5*s.* per cwt. for freight, cost of labour, and other outlay. He sold the manufactured article at 8*d.* per lb. Find his gain on each ton sold, and express it as a percentage, to the nearest whole number, of his outlay.

B.

1. Find the value of $\dfrac{3{\cdot}287\times 19{\cdot}63}{2612{\cdot}3}$

Give the result correct to four significant figures.

2. A bag contains a number of balls. ·4 of them are white; ·3 of them are black; the number of the rest of them is 24. How many of the balls are white?

3. An area is given as 432·42 acres. If it is in the form of a square, find the side of the square as accurately as possible.

4. A publisher produces 10,000 copies of a book at a cost of £1,500, and after giving away 50 presentation copies, sells the rest to booksellers at 4*s.* 6*d.* a copy. What profit does he make, and what percentage is this profit of his outlay?

5. A rectangular court, 44 ft. long, is paved with 1936 tiles, each being 6 in. square. What is the width of the court?

6. The average rainfall in Jamaica in 1901 was:

Division.	Inches.
North-Eastern . . .	107·88.
Northern	64·18.
West-Central . . .	87·31.
Southern	64·50.
Whole Island . . .	80·96.

(i) Express the rainfall in each of the four districts as a percentage of the rainfall over the whole island, giving each percentage to the nearest whole number.

(ii) If the rainfall over the whole island in 1901 was 7 per cent. above the usual rainfall, find the usual rainfall.

C.

1. Simplify (giving the answers as decimals):

(i) $\cdot 3 \times \cdot 1$; (ii) $\cdot 4 \div \cdot 1$; (iii) $3 \cdot 6 \times 2 \cdot 25$; (iv) $3 \cdot 6 \div 2 \cdot 25$; (v) $(\cdot 004)^{\frac{1}{2}} \div (\cdot 02)^{\frac{1}{3}}$.

2. Find the cost of 148 Kl. 8 Dl. of wheat at 15 fr. 20 c. per Hl.

3. Ninety-seven telegraph posts are placed at equal distances along a road. The distance between the first and last posts is 3 miles. What is the distance between two successive posts?

4. In a certain year the total revenue was £144,000,000, and the tax on tea produced £5,790,000. What percentage of the whole revenue, to the nearest whole number, was produced by the tea tax?

5. If £1 is equivalent to 25 fr. 10 c. or $4 85 c., find to the nearest centime the value in French money of 150 dollars.

6. If a water-pipe is L yards long, d inches in diameter, and one end is H feet higher than the other, then $\sqrt{[(3\,d)^5 \times H \div L]}$ gallons of water will flow through the pipe in a minute.

Use this formula to find how many gallons per minute will flow through a pipe a mile long and $4\frac{1}{4}$ inches in diameter, one end being 38 feet higher than the other.

D.

1. Multiply 13·272 by ·046322, giving the result correct to five significant figures.

2. Find the sum of the series

$$\tfrac{1}{2}+\tfrac{2}{3\,0}+\tfrac{3}{4\,0\,0}+\tfrac{4}{5\,0\,0\,0}+\ \ldots\ldots$$

correct to five significant figures.

3. What percentage is £2 13*s.* 7*d.* of £74 10*s.*?

4. Find the interest on £472 12*s.* for 3 years at $4\frac{1}{2}$ %.

5. A metal rod, when heated, expands $1{\cdot}2\times10^{-5}$ of its length for every degree (Centigrade) rise in temperature.

Such a rod is 27·40 cm. long at 0°. What is its length when heated to 50°?

6. A cistern has length 2·49 m., breadth 1·84 m., depth ·82 m. How many litres of water will it hold?

E.

1. Find the value of

·85 of £1 − ·75 of £1 1*s.* + ·25 of 3*s.* − ·35 of 3*s.* 4*d.*

2. Find the prime factors of 1890 and 4928, and write down the smallest number which can be divided exactly by each of them. (Leave the result in factors.)

3. Find, to the nearest pole, the average quantity of land to each person in a county whose area is 505330 acres and population 64411.

4. What will be the cost, in French money, of painting and varnishing the two side walls and one end wall of a hall which is 8 m. 25 cm. long, 5 m. 75 cm. wide, and 4 m. 50 cm. high, at 6 fr. 50 c. a square metre?

5. In Greece in a certain year there were 5,563,100 acres of agricultural land, and of this olives occupied 432,000 acres, yielding 15,000,000 lb.; currants 168,000 acres, yielding 350,000,000 lb.; and figs 52,000 acres, yielding 60,000,000 lb. What was the yield of each per acre, and what percentage of the whole agricultural land was occupied by each?

Give the answers correct to the nearest whole number.

6. Coal is sent to a railway station at 21*s.* 10*d.* per ton, and there is a further charge for delivery of 1*s.* per ton per mile. Find the cost of 6 tons 19 cwt. delivered at a place 3 miles from the railway.

F.

1. The thickness for an unbound book of 272 pages is half an inch; find the thickness of a leaf to the nearest ten-thousandth part of an inch.

2. Find, with as little work as possible, the value of $£11{\cdot}413 \times (1{\cdot}025)^4$ to the nearest shilling.

3. If new-laid eggs are worth 1*s.* 10*d.* a score, how much must be paid for 190 dozen?

4. A gallon of water weighs 10 lb., and a cubic foot of water weighs 1000 oz.; how many gallons are there in 4 cubic yards?

5. A watch gains steadily; at 5 minutes past 9 one morning it pointed to 24 min. past, and at 5.15 in the afternoon it indicated 5.43.

What will it indicate at 9.35 the following morning?

6. During the first six months of a certain year 3182 cars, valued at £1,396,026, and 994 motor-cycles, valued at £32,291, were imported into England. The export of cars numbered 357, of the value of £109,605, and of motor-cycles 356, valued at £11,812. Taking average prices throughout, calculate the ratio of the value of an exported car to that of an imported car, and the ratio of the value of an exported motor-cycle to that of an imported motor-cycle.

Express these ratios as percentages.

G.

1. Multiply 723·06 by 3·02, and verify the result by division.

2. What is the highest factor common to both 12936 and 20328? Also, what numbers between 20 and 50 will exactly divide each of them?

3. Find the cost of 35 tons 13 cwt. 18 lb. at £5 10*s.* a ton.

4. The area of the United Kingdom is 121,377 square miles. Find, to the nearest tenth of a mile, the side of the square which would have this area.

5. In a certain year there were, at a certain place, 196 days on which no rain fell, and the total rainfall recorded during the year was 47·12 inches. Find, to three significant figures, the average fall on the days on which there was rain.

6. Linoleum is laid on a floor whose dimensions are 18 ft. by 14 ft., so as to leave all round a margin of $2\frac{1}{2}$ ft. What will the linoleum cost at 2*s.* 6*d.* per square yard?

(A figure of the floor will help you.)

H.

1. Divide 403·822 by 76·491.

2. Simplify $\frac{2}{3}$ of $(\frac{3}{7} - \frac{5}{11})$ of £47 2*s.* 10*d.*

3. Find all the prime factors of 640332.

What is the least number by which this number must be (*a*) multiplied, (*b*) divided, so that the result may be a perfect square.

Give the square root in each case.

4. A bankrupt has debts amounting to £4739 8*s.*, and assets amounting to £539 12*s.*

What rate in the pound can he pay?

What will a creditor get if he was owed £300?

5. A square field has an area of 43·59 acres. What would be the length of the side of a square field of twice this area?

6. The distance described by a stone falling from a cliff is proportional to the square of the number of seconds it has been in the air.

A stone falling for 2 sec. drops through 64·4 ft. What is the height of a cliff if a stone takes $3\frac{1}{5}$ sec. to fall from top to bottom?

I.

1. Express £7 15*s.* $1\frac{1}{2}$*d.* as a decimal of £10, correct to four significant figures.

2. Reduce $\frac{3\frac{4}{5}-2\frac{1}{4}+9\frac{1}{11}}{5\frac{1}{4}-1\frac{1}{2}+17\frac{5}{33}}$ to a simple fraction in its lowest terms.

3. Find the interest on £4587 14*s.* 2*d.* at 3 per cent. in $4\frac{1}{2}$ years.

4. A farm consists of corn land, pasture land, and fallow land. The areas of these, taken in their order, are 1724 ac. 1 ro 4 po.; 3489 ac. 2 ro. 17 po.; 395 ac. 3 ro. 24 po. Supposing a land tax of 5*d.* an acre to be imposed, what will be the charge upon the whole farm, to the nearest penny?

5. A train going at 55 miles an hour covers a certain distance in $3\frac{1}{2}$ hours. How long will a second train take to cover two-thirds of the distance, its rate of travelling being to the rate of the first train in the ratio 4 to 9?

6. A sum of £1087 2*s.* 6*d.* is divided among two sons, three daughters, and one grandchild, so that the shares of a son, a daughter, and a grandchild are in the ratio of 3 to 2 to 1.

How much does the grandchild receive?

J.

1. Find the smallest number which is exactly divisible by each of the following numbers: 17, 57, 204, 255.

2. Determine which is the greatest, and which the least, of the three quantities

$$\frac{2\frac{1}{2}}{88\frac{1}{4}}, \quad \frac{2\frac{3}{4}}{97\frac{5}{8}}, \quad \frac{3}{105\frac{1}{2}}.$$

3. A quantity of sugar weighing $7\frac{1}{2}$ cwt. is to be made up into parcels each of $10\frac{1}{2}$ lb. How many parcels will there be?

4. If a railway company bought a portion of land 253 miles long, and 15 yards wide, for £50,715, what average price per acre was paid for the land?

5. Through failure of crops, &c. an annual income of £2000 has rapidly diminished. In one year it fell 4 per cent.; in the next year 15 per cent. of the remainder, and in the next year 25 per cent of what then remained.

What was its amount at the close of the third year?

6. How long is the side of a square portion of land which was sold for £43,999 1*s.* at an average price of 15*s.* 8*d.* per square yard?

K.

1. Divide 2789·44 by ·0368, and 2·78944 by 368.

2. On a day in summer there were 15 hours 20 minutes of daylight. What percentage of the whole 24 hours was that? Give your answer to the nearest whole number.

3. Find the cost of making a road 5 miles 6 furlongs 15 poles in length at £68 17*s.* 6*d.* per mile.

4. It was calculated that in boring the Simplon tunnel, the rock yielded a centimetre of its substance to the drill per minute. The whole length of the tunnel is 19·8 Km. How long would it have taken to bore the tunnel if the work had been entirely uninterrupted? Answer to be given to the nearest tenth of a year.

5. An official, after deduction of income tax at 1*s.* in the £1, receives £712 10*s.* What is his gross income (i. e. his income before the tax is deducted)?

6. On a certain map, a length of 1 Km. is represented by a decimetre. How many hectares of land are represented by a square portion of the map, each side of the square being 9 cm. long?

L.

1. Find the value of £84 13*s.* 7*d.* × 84

2. Reduce to its simplest form

$$\frac{(5\frac{1}{7}-2\frac{5}{8})\times(3\frac{1}{2}-\frac{8}{13})}{(2\frac{1}{4}+3\frac{5}{16})-(\frac{11}{12}-\frac{2}{3})}.$$

3. Find the interest for three months on £720 at the rate of $3\frac{3}{4}$ per cent. per annum.

4. How many trees can be planted in $4\frac{1}{2}$ acres, allowing 36 sq. yds. for each tree?

5. Assuming that £1 is equal to 20·35 marks, express in English money a $7\frac{1}{2}$ per cent. dividend on a capital of 585,000 marks.

6. From the following table find, to the nearest shilling, the value per cwt. of coffee in 1903, in 1904, and in 1905. Find also the percentage falling off in 1905 in quantity and in total value as compared with 1904, each percentage to the nearest integer, and reckoned on the 1904 numbers:

Year.	Quantity imported in cwts.	Value in £.
1903.	1,143,526.	3,134,924.
1904.	1,055,866.	3,329,598.
1905.	956,226.	2,707,169.

M.

1. Express in their simplest forms:

(i) $(4\frac{1}{4}-2\frac{1}{8}) \div 3\frac{3}{16}$;

(ii) $\dfrac{1\frac{3}{5}+2\frac{1}{3}}{2\frac{1}{6}-\frac{5}{16}}$.

2. Express £3 4*s.* $10\frac{1}{2}d.$ as the decimal of £4 correct to four significant figures; and £4 7*s.* 2*d.* as a fraction of £9 16*s.* $1\frac{1}{2}d.$

3. In a Post Office, the stamps sold during six months came to the following amounts, taking the months in order: £289 12*s.* 5*d.*, £332 17*s.* 10*d.*, £327 18*s.* 4*d.*, £265 11*s.* 8*d.*, £318 7*s.* 9*d.*, £197 15*s.* 6*d.* What was the average sale per month?

4. Find the cost of 98 tons 17 cwt. 2 qr. of coal at 22*s.* 9*d.* per ton.

5. The incomes of the inhabitants of a group of houses amount to £24,560. They pay rates at 5*s.* $8\frac{1}{4}d.$ in the £1 on rents amounting to £2185. Find what decimal (correct to three figures) their rates are of their total income.

6. A rectangular cistern, which is not full, has in it 100 cubic metres of water. Its length is 7 metres, its width is 3 metres, and its depth is known to be an exact number of metres.

What is the least depth it can have? If this be its depth, how much more water could it contain?

N.

1. Simplify $\dfrac{15 \times 1{\cdot}5 - 25}{{\cdot}5 \times 15 + 2{\cdot}5}$.

2. What is the least number which is divisible by 9, 16, 42, 63, 14, 72?

3. Find the rent of 69 ac. 2 ro. 28 po. at £2 5*s.* per acre.

4. From the first to the fifty-sixth telegraph post on a road I drive in $12\frac{1}{2}$ minutes at the rate of $7\frac{1}{2}$ miles an hour. Regarding the posts as placed at equal distances, how far are each pair of them apart?

5. If a rupee be worth 1*s.* $3\frac{1}{2}$*d.*, and £1 be worth 25 fr. 20 c., how many francs and centimes are 1000 rupees worth?

6. On a certain map a farm of 512 acres is represented by 5 sq. in., and two points on the map are $\frac{5}{11}$ in. apart. How many yards apart are the two objects represented by those points?

O.

1. Find to four significant figures the value of

$$\frac{2{\cdot}301 \times 5{\cdot}11 - 3{\cdot}02}{6{\cdot}32 + {\cdot}002}.$$

2. Find the square root of 250, correct to five significant figures.

3. Find the cost of 5 cwt. 2 qrs. 21 lb. at £4 10*s.* per cwt.

4. The rateable value of a town is £475,000. An assessment has to be made to bring in at least £25,000. Find, in pence and farthings, the smallest rate per pound which will produce this amount.

5. If a man sold his house for £4,500 he would lose £300. For what must he sell it to make a profit of 20 per cent. on the transaction?

6. A surveyor's chain of 100 links should measure 22 yards. A rectangular field is measured by a chain which is 5 inches too short, and is found to be 4·56 ch. long and 3·75 ch. broad.

Assuming that the links in the defective chain are of equal length, find the true area of the field in acres, correct to three significant figures.

P.

1. Simplify $\dfrac{(87\frac{1}{6})^2 - (52\frac{1}{2})^2}{139\frac{2}{3}}$.

2. Find the value of 578 articles at £3 7*s.* $9\frac{1}{2}$*d.* each.

3. In 1903-4 ninety-nine millions nine hundred and ninety-seven thousand telegrams were received in the post offices of Great Britain.

If $\frac{17}{25}$ of these cost 6*d.* each, and the remainder, at an average, $8\frac{1}{2}$*d.* each, what did they bring in to the revenue? (Answer to the nearest pound.)

4. The debts of a bankrupt amount to £4269 1*s.* 0*d.* He has assets amounting to £2846 0*s.* 8*d.* How much in the pound can he pay his creditors?

5. A rectangular cistern to contain 4000 litres is to be fitted into a space 3·47 metres long and 1·32 metres wide. How deep must it be? (Answer to a reasonable degree of accuracy.)

6. The average rainfall per day from Jan. 1st to Jan. 12th was ·115 in., and from Jan. 2nd to Jan. 13th it was ·14 in. (both days included in each case).

On Jan. 1st the rainfall was ·025 in. What was the rainfall on Jan. 13th?

Q.

1. Find which is the greatest and which is the least of the fractions $\frac{6}{11}, \frac{19}{35}, \frac{31}{55}, \frac{43}{77}$.

2. Find all the prime factors of the number 91476. Find the least number by which 91476 may be multiplied so that the product may be a perfect square.

Give also the square root of the product

3. Find the square root of 37 correct to five significant figures.

4. In a certain year there were 37,595 acres of land under wheat in Ireland, which produced 1,175,972 bushels. Find the average yield per acre to the nearest hundredth of a bushel, and the total value of the wheat, to the nearest pound, at 26*s.* 9*d.* per quarter.

5. A landlord's net rental, after paying income tax at the rate of 1*s.* in the £1, is £7101 14*s.* 6*d.* Find his gross rental.

6. The time of oscillation of a pendulum is proportional to the square root of its length, and a pendulum of length 39·1393 in. oscillates 60 times a minute. Find the length of one oscillating 80 times a minute.

R.

1. Find, correct to five significant figures, the value of the series

$$\frac{1}{11} + \frac{1}{11^2} + \frac{1}{11^3} + \frac{1}{11^4} + \ldots\ldots$$

2. Find all the numbers between 2500 and 3000 which are divisible by each of the numbers 12, 18, and 30

3. The tax on tea at the rate of 6*d.* on the lb. produced a revenue for the year ending 31st March, 1904, of £6,559,705; find, to the nearest ton, the weight of tea imported.

4. A train travels 30 miles an hour. Express this speed in feet per second.

5. A contractor undertook to do a certain amount of work in 16 days. He employed 30 men, and they did five-sevenths of the work in 12 days How many extra men did he employ after the twelfth day, if the work was just completed in 16 days, assuming that each man did the same quantity of work per day?

6. A cricketer's average was 47·5 in 14 innings; after his 15th innings his average was 51. How many runs did he make in his last innings?

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ANSWERS

I a (p. 17).

1. XXXIV. 34. **2.** XIV. 14. **3.** XLVI. 46.
6. (i) DCCCLXXI; (ii) MLXVI; (iii) MCCXVI;
(iv) MCCCXLVI; (v) MDIX; (vi) MDLXXXVIII;
(vii) MDCLX; (viii) MDCCIII; (ix) MDCCCXXXVII;
(x) MDCCCCX (or MCMX).
7. (i) 47; (ii) 99; (iii) 213; (iv) 34; (v) 1699.
8. (i) 2000; (ii) 502; (iii) 5,020,016; (iv) 12111;
(v) 49,001,007; (vi) 302,094.
10. 1375. **11.** 2374, 2375, 2376.
12. (i) 25; (ii) 18; (iii) 11.
13. 1000. **14.** (i) 10; (ii) 100.

Oral Examples (p. 19).

1. 15.	**2.** 19.	**3.** 20.	**4.** 35.	**5.** 30.	**6.** 5.
7. 7.	**8.** 8.	**9.** 5.	**10.** 7.	**11.** 8.	**12.** 16.
13. 14.	**14.** 5.	**15.** 9.	**16.** 8.	**17.** 1.	**18.** 9
19. 13.	**20.** 56.	**21.** 61.	**22.** 4.	**23.** 24.	
24. 28.	**25.** 5.	**26.** -1.	**27.** 3.	**28.** 5.	**29.** 7.
30. 9.	**31.** 7.	**32.** 6 and 6 over.		**33.** 6 and 4 over.	
34. 11 and 7 over.		**35.** 5.	**36.** 7.	**37.** 13.	
38. 5.	**39.** 6.	**40.** 16.	**41.** 123.	**42.** 11.	

Oral Examples (p. 21).

Rows.

(1) 89.	(2) 80.	(3) 77.
(4) 95.	(5) 84.	(6) 79.
(7) 83.	(8) 72.	(9) 73.
(10) 94.	(11) 80.	(12) 78.
(13) 80.	(14) 77.	(15) 107
(16) 77.	(17) 101.	(18) 81.
(19) 75.	(20) 74.	(21) 87.
(22) 66.	(23) 87.	

Columns.

(1) 108.	(2) 106.	(3) 118.
(4) 96.	(5) 98.	(6) 104.
(7) 123.	(8) 100.	(9) 88.
(10) 113.	(11) 120.	(12) 94.
(13) 107.	(14) 119.	(15) 85.
(16) 115.	(17) 100.	(18) 102.

Sum of all the digits 1896.

I b (p. 23).

1. 223845.	**2.** 422080.	**3.** 189120.
4. 263994.	**5.** 336666.	**6.** 201904.
7. 121128.	**8.** 277868.	**9.** 316843.
10. 186149.		
(*a*) 294605.	(*b*) 177172.	(*c*) 282507.
(*d*) 129139.	(*e*) 253093.	(*f*) 299189.
(*g*) 205073.	(*h*) 317985.	(*i*) 51286.
(*k*) 134145.	(*l*) 295591.	(*m*) 99812.

A.	11	12	13	14	15	16	17
B. 11	31421	47927	48949	38473	47781	18939	160
12	34264	45084	46106	35630	44938	16096	2683
13	83516	4168	3146	13622	4314	33156	51935
14	52867	26481	27503	17027	26335	2507	21286
15	2854	82202	83224	72748	82056	53214	34435
16	79548	200	822	9654	346	29188	47967
17	27414	51934	52956	42480	51788	22946	4167

18. 27814. **19.** 70096. **20.** 15356.
21. 26585. **22.** 13950. **23.** 918.
24. 315. **25.** 5979. **26.** 1768.
27. 41000. **28.** 1259. **29.** 21063.
30. 2355. **31.** 39022. **32.** 4306.
33. 87223. **34.** 1,099,998. **35.** 4,905,999.
36. 67818. **37.** 13,261,073. **38.** 85,378,009.
39. 10,588,902. **40.** 990,578.

Oral Examples (p. 29).

1. 5. **2.** 5, 2. **3.** 8, 2. **4.** 5, 3.
5. 12, 2. **6.** 7, 7. **7.** 9 **8.** 9, 3.
9. 14. **10.** 4, 2. **11.** 3, 3. **12.** 6.
13. 6, 2. **14.** 7, 3. **15.** 7, 2. **16.** 7, 6.
25. 7^7. **26.** 8^8. **27.** 10^5. **28.** 9^6.
29. 9^2. **30.** 10^5. **31.** 6^{12}. **32.** 10^3.
33. 10^3. **34.** 10^8. **35.** 10^2.
36. 21×10^3. **37.** 21×10^3. **38.** 10^6 or (10×10^5). **39.** 10^6.
40. 2×10^3. **41.** 2×10^3. **42.** 10×10^7, (10^8). **43.** 10^8.
44. 2×10^1. **45.** 9×10^1. **46.** 9×10^1. **47.** 24×10^6.
48. (i) units; (ii) tens; (iii) thousands; (iv) tens; (v) units.
49. (i) tens; (ii) units; (iii) tens; (iv) thousands; (v) hundreds.

I c (p. 37).

	(*a*)	(*b*)	(*c*)	(*d*)	(*e*)
1	426168	3409344	3598752	5634888	11506536
2	620523	4964184	5239972	8204693	16754121
3	5161374	41290992	43584936	68244834	139357098
4	620109	4960872	5236476	8199219	16742943
5	47952	383616	404928	634032	1294704
6	692073	5536584	5844172	9150743	18685971
7	392265	3138120	3312460	5186615	10591155
8	569223	4553784	4806772	7526393	15369021
9	1424916	11399328	12032624	18840556	38472732
10	659232	5273856	5566848	8716512	17799264

	(*f*)	(*g*)	(*h*)	(*i*)	(*k*)
1	1657320	94846056	3594963840	409784208	35475928992
2	2413145	138100841	5234456240	596667338	51654816612
3	20072010	1148692458	43539057120	4962947844	429653417256
4	2411535	138008703	5230963920	596269254	51620353596
5	186480	10671984	404501760	46108512	3991716288
6	2691395	154024691	5838020240	665466638	57610924812
7	1525475	87300755	3308973200	377184590	32653707660
8	2213645	126683741	4801712240	547339538	47384399412
9	5541340	317122972	12019958080	1370135896	118615707504
10	2563680	146715744	5560988160	633888192	54877108608

	(*l*)	(*m*)	(*n*)	(*o*)	(*p*)
11	51203	6056	7822	1357	614
	rem. 9	34	58	87	818
12	668991	79128	102207	17732	8033
	rem. 3	0	0	124	676
13	29778	3522	4549	789	357
	rem. 5	17	35	128	551
14	533928	63152	81572	14152	6411
	rem 10	82	34	138	742
15	41262	4880	6304	1093	495
	rem. 9	51	3	296	471
16	57662	6820	8809	1528	692
	rem. 3	25	37	165	413
17	2486	294	379	65	29
	rem. 10	14	68	381	792
18	8122294	960701	1240906	215289	97538
	rem. 5	46	7	304	431
19	57493539	6800311	8783735	1523925	690424
	rem. 5	11	14	59	550
20	324447	38375	49568	8599	3896
	rem. 4	46	25	336	185

	(*q*)	(*r*)	(*s*)	(*t*)	(*u*)
11	1573	1246	90	76	5
	rem. 108	50	3622	4338	70977
12	20555	16280	1183	1000	74
	rem. 214	344	3010	4904	73382
13	914	724	52	44	3
	rem. 351	315	4227	3987	32204
14	16405	12993	944	798	59
	rem. 228	382	3426	4726	64491
15	1267	1004	72	61	4
	rem. 305	83	6195	5297	60079
16	1771	1403	102	86	6
	rem. 267	129	49	1841	43567
17	76	60	4	3	0
	rem. 148	236	2484	5294	27356
18	249567	197666	14368	12149	907
	rem. 253	207	5015	1493	48368
19	1766560	1399179	101709	85997	6423
	rem. 454	26	2372	6996	65315
20	9969	7895	573	485	36
	rem. 19	381	6007	2231	24613

21. 1004,005,020. **22.** 100,072. **23.** 40,931.
24. 30,025. **25.** 858357, rem. 48. **26.** 856,411,479.
27. 17037, rem. 1771; 798. **28.** 1440. **29.** 101,793.
30. 87; 41; 16. **31.** 35,825. **32.** 1425186, rem. 423.
33. 126986, rem. 35. **34.** 145,844, rem. 1931.

I d (p. 39).

1. 24.	**2.** 254.	**3.** 51.	**4.** 50.
5. 495.	**6.** 3.	**7.** 29.	**8.** 3.
9. 3.	**10.** 1.	**11.** 447600.	**12.** 4316.
13. 1 and $\frac{6}{10}$.	**14.** 204.	**15.** 99.	**16.** 39.
17. 18.	**18.** 0.	**19.** 6.	**20.** 23.

Oral Examples (p. 42).

1. (i) 5; (ii) 25; (iii) 135.
2. (i) 9; (ii) 127; (iii) 164.
3. (i) 274; (ii) 820; (iii) 1620.
4. (i) 3577; (ii) 4552.
5. (i) 9; (ii) 21; (iii) 16.
6. (i) 5; (ii) 15; (iii) 8.
7. (i) 14; (ii) 54; (iii) 48.
8. (i) $3\frac{1}{2}$; (ii) 6; (iii) 3.
9. (i) 18; (ii) 105; (iii) 18, (iv) 27.
10. (i) 18; (ii) 105; (iii) 18; (iv) 27
11. (i) 32; (ii) 10, (iii) 10, (iv) 28; (v) 14.

I (p. 44).

1. 10,009,010, 5 hundred thousand, 5 thousand, 5.
2. 60,990,991. **3.** 9995. **4.** 17418, 537.
5. 662. **6** 30963. **7.** 3475 **8.** 1635.
9. 5910, 33 gals **10.** 30 gals.
11. 15,721,728 males, 16,804,347 females.
12. 223,595. **13.** 65,821. **14.** 2302 less.
15. 313. **16.** 502,386, 505,087, 507,788.
17. 306,520 **18** 220, 281. **19** 931,475, 968,734.
20. 93 men **21.** 153,000. **22** 35 **23.** 2912
24. 26. **25.** 337 males, 394 females. **26.** 346.
27. 255,224. **28.** 12569, 10905, 6784.
29. £26,248,000 **30.** 498,722.
31. £25,453,410, £1,511,410
32. 5997. **33.** 65.
34. 575 males; 660 females. **35.** 13.
36. Area 9,946,000; Pop. 365,200,000; Revenue 291 millions.
U.K. 388; India 169; Canada 2; Australia 1, New Zealand 8; S.A. 5.

Oral Examples (p. 54).

1. 2×10^4. **2.** 3×10^2. **3.** 3×10^{-2}.
4. 3×10^{-1}. **5.** 9×10^{-3}. **6.** 9×10^1.
7. 8×10^1. **8.** 5×10^{-5}. **9.** 7×10^{-4}.
10. 5×10^2. **11.** 7×10^5. **12.** 6×10^{-6}.
13. 3×10^3. **14.** 7×10^{-2}. **15.** 5×10^4.
16. 2×10^0. **17.** 8×10^{-4}. **18.** 4×10^{-3}.
19. 1×10^{-2}. **20.** 7×10^0.

21. (i) 3721 2. (ii) 37212. (iii) 3 7212.
22. (i) 40621. (ii) 4 0621. (iii) 40621.
23. (i) 4 26. (ii) 426. (iii) ·00426.
24. (i) 1·073. (ii) 1073 (iii) ·1073.
25. (i) 6·42. (ii) 00642. (iii) 00642.
26. (i) ·0004293. (ii) ·00004293. (iii) ·4293.
27. (i) 1·624. (ii) 1 624. (iii) 162 4.
28. (i) ·05591. (ii) 55910 (iii) ·0005591.
29. (i) 74236. (ii) 74236. (iii) ·74236.
30. (i) 74 1. (ii) 7 41. (iii) 7410.
31. 423. **32.** 57·6 **33.** 8 804
34. 602 9. **35.** 50·38. **36.** 4030 09.
37. 50403. **38.** 3 074. **39.** 500 46.
40. 400 0405. **41.** 7 0401. **42.** 20097.

Oral Examples (p. 58).

1. (i) 437. (ii) 43·7. (iii) 4370
2. (i) 304 6. (ii) 3 046. (iii) 3046.
3. (i) 50 704. (ii) 5070 4. (iii) 50704.
4. (i) 5430. (ii) 54 3. (iii) 5 43.
5. (i) 5 47. (ii) 547. (iii) 5470.
6 (i) 904 3 (ii) 9043. (iii) ·9043.
7. (i) 40 89 (ii) ·4089. (iii) 4 089.
8. (i) 4 0025. (ii) 40 025. (iii) 40025.
9. (i) 84·5. (ii) 845. (iii) 84500.
10. (i) 400302. (ii) 4 00302. (iii) 400 302.
11. (i) 3749 1. (ii) 3 7491. (iii) 37491.
12. (i) 453·21. (ii) 453210. (iii) 4·5321.
13. (i) 50463. (ii) 5·0463. (iii) 5046·3.
14. (i) 3742 (ii) 37 42. (iii) 374·2.

15. (i) 54·12. (ii) ·5412. (iii) 5412.
16. (i) 73·24. (ii) ·7324. (iii) ·07324.
17. (i) 34210. (ii) 34·21.
18. (i) 30960. (ii) 30·96.
19. (i) 43690. (ii) 43·69.
20. (i) 5420000. (ii) 5420.
21. (i) 254·3. (ii) 2543.
22. (i) 43·2. (ii) 0432.
23. 7·234 Dg. **24.** 8·532 Kg. **25.** 1·0032 Hg.
26. 4·93 dm. **27.** 3·416 Hg. **28.** 4·391 g.
29. 5·43 dg. **30.** 4·125 Dm. **31.** 8·94 dm.
32. 3·42 g. **33.** 5·6 dl. **34.** 1·764 dl.
35. 6·3921 dl. **36.** 2·059 Kg. **37.** 4·9 cm.
38. 3·008 Km.

II a (p. 60).

1. 60·2111. **2.** 542·629. **3.** 1·38461.
4. 178·388. **5.** 31·2639. **6.** 120·6479.
7. ·9288. **8.** 13·226. **9.** 56·364.
10. 653·95. **11.** 8·596. **12.** 6·0721.
13. 98·919. **14.** ·57852. **15.** 2579·64.
16. 12·727. **17.** ·816. **18** ·32104.
19. 87·279. **20.** 55·451. **21.** ·004409.
22. 87·83. **23.** 254·66. **24.** ·99574.
25. ·013581. **26.** 33·76. **27.** 112·483
28. 14·015. **29.** 358·1. **30.** 178000.
31. 415 m. **32.** 6749 g. **33.** 5752·2 g.
34. 4454·19 m. **35.** 215·89 l. **36.** 376·6 g
37. 1·352 m. **38.** 7·112 l. **39.** 3·052 g.
40. ·0148 m.

II b (p. 65).

1. (i) 74·48. (ii) 18620. (iii) 260680.
2. (i) 5160·5. (ii) 722470. (iii) 619·26.
3. (i) 1274·76. (ii) 382·428. (iii) 3·39936.
4. (i) 1844·04. (ii) 1·84404. (iii) 15·367.
5. (i) 4542·3. (ii) ·19467. (iii) 38·934.
6. (i) 17295. (ii) ·3459. (iii) 403·55.

7.	(i) 90.	(ii) 11 25.	(iii) ·000135.
8.	(i) 1896·3.	(ii) 252840.	(iii) 1·2642.
9.	(i) 17·842.	(ii) 446·05.	(iii) 8028·9.
10.	(i) 111 72.	(ii) 744 8.	(iii) 18·62.
11.	(i) 991·2.	(ii) 1·4868.	(iii) 19824.
12.	(i) ·5495.	(ii) 8·792.	(iii) 76·93.
13.	(i) 32 7.	(ii) 2·18.	(iii) ·1308.
14.	(i) 21 91275.	(ii) ·29217.	(iii) 175·302.
15.	(i) ·17455.	(ii) 1·1637 . . .	(iii) ·099743 . . .
16.	(i) 3·3956 . . .	(ii) 436 57 . . .	(iii) 3820.
17.	(i) ·684.	(ii) ·048857 . . .	(iii) ·57.
18.	(i) ·1623425.	(ii) 1·0823 . . .	(iii) 216457 . . .
19.	(i) ·49508 . . .	(ii) 371·3125.	(iii) 5941.
20.	(i) ·42878 . . .	(ii) ·00000055129 . . .	(iii) 5·5129 . . .
21.	(i) ·000000491.	(ii) ·0065467 . . .	(iii) 56·114 . . .
22.	(i) 140·092.	(ii) 10006571 . . .	(iii) 3502300.
23.	(i) 86·634 . . .	(ii) 1949·27 . . .	(iii) ·0974635.
24.	(i) 1100100.	(ii) 22·002.	(iii) 82507·5.

	(*a*)	(*b*)	(*c*)	(*d*)	(*e*)
25	318000	5565000	3·975	13·25	198750 m.
26	3629200	63511000	45·365	151·22 . . .	2268250 g.
27	3627·2	63476	·04534	·15113 . . .	2267 g.
28	3162000	55335000	39·525	131·75	1976250 l.
29	31216	546280	·3902	1·3007 . . .	19510 m.
30	3225200	56441000	40 315	134·38 . . .	2015750 g.
31	300320	5255600	3·754	12·513 . . .	187700 l
32	295600	5173000	3·695	12·317 . . .	184750 m
33	25612000	448210000	320·15	1067·2 . . .	16007500 g.

	(*f*)	(*g*)	(*h*)	(*i*)
25	·99375	477000	2650	55·65 m.
26	11·34125	5443800	30243 . . .	635·11 g.
27	·011335	5440·8	30·227 . . .	·63476 g.
28	9·88125	4743000	26350	553·35 l.
29	·09755	46824	260·13 . . .	5·4628 m.
30	10·07875	4837800	26877 . . .	564·41 g.
31	·9385	450480	2502·7 . . .	52·556 l.
32	·92375	443400	2463·3 . . .	51·73 m.
33	80·0375	38418000	213433 . . .	4482·1 g.

II c (p. 68).

	(a)	(b)	(c)	(d)	(e)
1	198·4968	318·23332	487·65208	3092·3712	22676·228
2	27·03852	43·348598	66·426212	421·23168	3088·8742
3	367·65	589·4225	903·215	5727·6	42000·25
4	2041·056	3272·2544	5014·3136	31797·504	233169·76
5	·3062268	·49094782	·75231508	4·7706912	34·983278
6	237·0744	380·08156	582·42664	3693·3696	27083·324
7	·00231876	·003717474	·005696556	·03612384	·2648946
8	2394·342	3838·6483	5882·2402	37301·328	273529·07
9	1·18674	1·902601	2·915494	18·48816	135·5729
10	333·1422	534·09903	818·43882	5190 0048	38058·087

	(f)	(g)	(h)	(i)	(k)
1	4672·68432	3·97574	·4341392	5806·902	4064·65728
2	636·496248	·541561	·05913688	790·9953	553·672992
3	8654·61	7 36375	·8041	10755·375	7528·44
4	48047·1744	40·8808	4 464064	59709·84	41795·0976
5	7 20868632	·00613349	·0006697592	8·958477	6·27066528
6	5580 81456	4 74842	·5185136	6935·466	4854·61824
7	·054584424	·000046443	·00000507144	·0678339	·047481696
8	56363·6508	47 95685	5·236748	70045·005	49029·4032
9	27·936276	·0237695	·00259556	34·71735	24·301104
10	7842·28428	6 672585	·7286268	9745 8705	6821·81712

	(l)	(m)	(n)	(o)	
11	1443·52	1388	1745·41	7172·49	metres.
12	328848	31·62	397621·5	1633963·5	grammes.
13	2112·448	·20312	2554·234	10496 226	grammes.
14	178464	17 16	215787	886743	litres.
15	377020·8	36·252	455868·9	1873322·1	metres.
16	23720 32	2·2808	28681·06	117860·34	grammes.
17	368 992	·03548	446·161	1833·429	grammes.

	(p)	(q)	(r)	
11	2·0473	1 08958	2633·73	metres.
12	466·395	248·217	599989·5	grammes.
13	2·99602	1·594492	3854·202	grammes.
14	253·11	134·706	325611	litres.
15	534·717	284·5782	687881·7	metres.
16	33·6418	17·90428	43278·18	grammes.
17	·52333	·278518	673·233	grammes.

II d (p. 73).

The results are given correct to five figures.

	(*a*)	(*b*)	(*c*)	(*d*)	(*e*)
1	56 038	50 915	18 242	1·3888	86·730
2	398·93	362·47	129 87	9 8867	617·43
3	30·786	27·972	10 022	·76296	47·647
4	8·3196	7 5592	2 7083	·20619	12·876
5	·26275	·23873	·085534	·0065117	·40666
6	9 9966	9·0828	3 2542	·24775	15 472
7	105·45	95·813	34·328	2 6134	163·21
8	12385	11253	4031 7	306·94	19168
9	92 634	84 166	30·155	2·2957	143 37
10	·00055998	·00050879	·00018229	·000013878	·00086669

	(*f*)	(*g*)	(*h*)	(*i*)	(*k*)
1	1820 3	536 52	51899	·70827	·57529
2	12959	3819·5	369468	5 0422	4·0955
3	1000·0	294 75	28512	·38911	·31605
4	270·26	79·655	7705·2	·10515	·085411
5	8 5352	2 5156	243 34	·0033210	·0026974
6	324·73	95·710	9258 3	·12635	·10263
7	3425·5	1009·6	97664	1 3328	1·0826
8	402314	118577	11470200	156 54	127·15
9	3009·1	886 90	85792	1·1708	·95100
10	·018190	·0053614	·51862	7 0778 $\times 10^{-6}$	5·7489 $\times 10^{-6}$

	(*l*)	(*m*)	(*n*)	(*o*)	(*p*)
11	2281 6	1561·6	238 33	35 692	734 60 g.
12	107·54	73·603	11 233	1·6823	34·624 g.
13	1341·7	918·34	140 16	20·989	432·00 l.
14	1·9913	1 3629	20801	·031150	·64112 m.
15	167 32	114·52	17·478	2·6174	53·872 m.
16	1178·2	806·40	123·07	18·431	379 34 m.
17	165754	113448	17314	2592·9	53367 g.
18	28·889	19 773	3·0177	·45192	9·3013 g.
19	271·65	185·93	28·376	4·2495	87·462 l.
20	16·517	11·305	1·7253	·25838	5·3180 g.

	(*q*)	(*r*)	(*s*)	(*t*)	(*u*)
11	11589	$1{\cdot}7397 \times 10^6$	99·026	$2{\cdot}1541 \times 10^7$	68·067 g.
12	546·20	81995	4·6674	$1{\cdot}0153 \times 10^6$	3·2082 g.
13	6814·9	$1{\cdot}0230 \times 10^6$	58 234	$1{\cdot}2668 \times 10^7$	40·028 l.
14	10·114	1518·3	·086425	18800	·059405 m.
15	849·84	127580	7 2620	$1{\cdot}5797 \times 10^6$	4·9916 m.
16	5984·2	$8{\cdot}9834 \times 10^5$	51·136	$1{\cdot}1123 \times 10^7$	35·149 m.
17	$8{\cdot}4188 \times 10^5$	$1{\cdot}2638 \times 10^8$	7194·0	$1{\cdot}5649 \times 10^9$	4944·9 g.
18	146·73	22027	1·2538	272747	·86184 g.
19	1379·7	207126	11·790	$2{\cdot}5647 \times 10^6$	8 1041 l.
20	83 892	12594	·71687	155941	·49275 g.

II (p. 74).

1. 2 375. **2.** 23; . . . 7826 . . . **3.** ·00022371.
4. 51 pieces, ·00242 Hm. **5.** ·56. **6.** 3215·5.
7. 30. **8.** 15·87 metres. **9.** 64 mm.
10. 11·8. **11.** ·09400525.
12. ·010472, 037, ·00010472, ·000037.
13. 640. **14.** 25191 ft.; 1487 l. **15.** ·125 mm.
16. 7 fr. 83 c. (nearly). **17.** 1·12500. **18.** 0156.
19. ·001587. **20.** 4·4 Kg. **21.** 30·03.
22. ·15656 . . . **23.** 52·62 m. **24.** ·625.
25. 7·038. **26.** ·97424. **27.** 326 fr 4 c.
28. 3 765. **29.** 8962·60662. **30.** 19·03
31. 312·5. **32.** 97. **33.** 102 fr. 34 c.
34. 39·017 **35.** 337 ft. 5 c.
36. 45718 fr 75 c. **37.** 4 fr. 70 c. **38.** 28 eggs.
39. 363 fr. 90 c. **40** 1 fr. 30 c **41.** 97 6 metres.
42. 1 l 25 cl. **43** 8 15 hours. **44.** 1650.
45 7·1875. **46.** 525. **47.** 15 c.
48. 1636 fr. 30 c. **49.** 2 fr. 80 c. **50.** 4 Km. 23 m.
51. 5253 Kg. **52.** 833 g. **53.** 11·42 ft.
54. 87·5 c cm. **55.** 220 c.cm **56.** ·935 gals.
57. 1 ft. 53 c. **58.** 168.

Oral Examples (p. 82).

1. (i) 40. (ii) 480. **2.** (i) 96. (ii) 384.
3. (i) 120. (ii) ·5. **4.** (i) 35. (ii) 700.
5. (i) 9. (ii) ·45. **6.** (i) ·5 (ii) ·025.
7. (i) 20. (ii) ·5 **8.** (i) 56. (ii) ·7

9. (i) 18. (ii) ·75.
10. (i) 36·5. (ii) 3·65.
11. (i) 12·5. (ii) 62·5.
12. (i) 15. (ii) 1·25.
13. (i) 43. (ii) 2·15.
14. (i) 150. (ii) 25.
15. (i) 208. (ii) 416.
16. (i) 3·25. (ii) 26.
17. (i) 45·5. (ii) 2·275.
18. (i) 334. (ii) 1002.
19. (i) 102·5. (ii) 5·125.
20. (i) 64·75. (ii) 3·2375.

III a (p. 83).

1. (i) 7730. (ii) 32·2083 . . . (iii) 644·166 . . . .
2. (i) 4149. (ii) 17·2875. (iii) 345·75.
3. (i) 25226. (ii) 105·1083 . . . (iii) 2102 166 . . .
4. (i) 173179. (ii) 721·5791 . . . (iii) 14431·583 . . .
5. (i) 205040. (ii) 854·3333 . . . . (iii) 17086·666 . . . .
6. (i) 124218·5. (ii) 517·5771 . . . (iii) 10351·542 . . . .
7. (i) 48776·25. (ii) 203·2343. (iii) 4064·687.
8. (i) 26715·75. (ii) 111·3156. (iii) 2226·312.
9. (i) 25856·5. (ii) 107·7354 . . . . (iii) 2154·708 . . . . .
10. (i) 85065·5. (ii) 354·4396 . . . . (iii) 7088·792 . . .
11. (i) 209282·5. (ii) 872·0104 . . . (iii) 17440·208 . . . .
12. (i) 125099. (ii) 521·2458 . . . (iii) 10424·916 . . . .
13. £19 13*s.* 5*d.* **14.** £3 4*s.* 5*d.* **15.** £35 18*s.* 1*d.*
16. £10 7*s.* 3*d.* **17.** £5 3*s.* 9*d.* **18.** £42 15*s.* 0*d.*
19. £893 2*s.* 6*d.* **20.** £503 14*s.* 6*d.* **21.** £18 12*s* $7\frac{3}{4}$*d.*
22. £37 4*s.* 6*d.* **23.** £51 5*s.* $4\frac{1}{4}$*d.* **24.** £6 16*s.* 6*d.*
25. £1 17*s.* $7\frac{1}{2}$*d.* **26.** £1 3*s.* 9*d.* **27.** £39 8*s.* 6*d.*
28. £4 9*s.* 5*d.* **29.** £20 9*s.* $3\frac{1}{2}$*d.* **30.** £38 17*s.* 0*d.*

Oral Examples (p. 84).

1. (i) 60 (ii) 72. (iii) 60. (iv) 122.
2. (i) 18. (ii) 66 (iii) 33. (iv) ·5. (v) 10·25.
3. (i) 44. (ii) 880. (iii) 44. (iv) 2. (v) 4.
(vi) 880. (vii) 11. (viii) ·5. (ix) 33.
4. (i) 160. (ii) 50. (iii) 20. (iv) ·73. (v) 5.
5. (i) 40. (ii) ·4. (iii) 2. (iv) ·2. (v) ·3.
6. (i) ·75. (ii) ·5. (iii) 2·5. (iv) ·25. (v) ·025.
7. (i) 1 yd. 2 ft. 10 in. (ii) 4 yds. 0 ft. 3·6 in.
(iii) 7·2 in. (iv) 1 yd. 2 ft. 2 in.
(v) 2 yds. 2 ft. 8 in. (vi) 2 yds. 1 ft. 6 in.
(vii) 2 ft. 3 in. (viii) 16 yds. 2 ft. 9 in

III b (p. 85).

1. (*a*) 87.	(*b*) 7·25.	(*c*) 2·4166 . . .
2. (*a*) 498.	(*b*) 41·5.	(*c*) 13·8333 . . . .
3. (*a*) 366.	(*b*) 30·5.	(*c*) 10·1666 . . .
4. (*a*) 315.	(*b*) 26 25.	(*c*) 8 75.
5. (*a*) 68.	(*b*) 5·666 . . .	(*c*) 1·8888 . . . .
6. (*a*) 641.	(*b*) 53·4166 . . .	(*c*) 17·8053 . . . . .
7. (*a*) 1199.	(*b*) 54·5.	(*c*) 5·45.
8. (*a*) 142.	(*b*) 6·4545	(*c*) ·64545 . . . . .
9. (*a*) 776.	(*b*) 35·2727 . . .	(*c*) 3·52727 . . . . .
10. (*a*) 1672.	(*b*) 76.	(*c*) 7·6.
11. (*a*) 1329.	(*b*) 60·4091 . . .	(*c*) 6·04091 . . .
12. (*a*) 486.	(*b*) 22·0909 . . .	(*c*) 2·2091 . . . .

13. 195. **14.** 75·72. **15.** 298. **16.** 390.

17. 29 yds. 2 ft. 5 in.
18. 145 yds. 1 ft. 9 in.
19. 102 yds. 1 ft. 5 in.
20. 53 yds. 0 ft. 7·2 in.
21. 17 yds. 0 ft. 11·7 in.
22. 8 yds. 1 ft. 9·5 in.
23. 7 fur. 1 ch.
24. 3 fur. 2 ch. 5 yds.
25. 8 fur. 0 ch. 7 yds.
26. 4 fur. 0 ch. 14 yds.
27. 5 fur. 6 ch. 7 yds.
28. 7 fur. 5 ch. 4·6 yds.
29. 6 mi. 2 fur. 7 ch
30. 8 mi. 0 fur. 1 ch.
31. 2 mi. 2 fur. 2·1 ch.
32. 13 mi. 3 fur. 3 2 ch.
33. 2 mi. 3 fur. 4·8 ch.
34. 4 mi. 7 fur. 3 ch.
35. 1 mi. 2 fur. 21 po.
36. 2 mi. 0 fur. 13 po.
37. 2 mi. 2 fur. 38·7 po.

Oral Examples (p. 86).

1. (i) 32. (ii) 80. (iii) 50. (iv) 3·5.
2. (i) 56. (ii) 336. (iii) 87. (iv) 5·25. (v) 4480.
(vi) 70. (vii) 49. (viii) ·7.
3. (i) 100. (ii) 62. (iii) ·5. (iv) ·5.
4. (i) 3 tons 12 cwt. (ii) 2 tons 14 cwt. 2 qrs.
(iii) 4 tons 5 cwt. (iv) 17 cwt. 1 qr. (v) 1 ton 5 cwt.
(vi) 1 ton 2 cwt. 2 qrs.
5. (i) 20 lb. 5 oz. (ii) 5 lb. 8 oz. (iii) 5 lb. 4 oz.
6. (i) 21000. (ii) 960. (iii) 17500.

III c (p. 88).

1. (*a*) 1001. (*b*) 62·5625. (*c*) 437937.
2. (*a*) 1435. (*b*) 89·6875. (*c*) 627812.
3. (*a*) 382. (*b*) 23·875. (*c*) 167125.
4. (*a*) 496. (*b*) 31. (*c*) 217000.
5. (*a*) 49·42. (*b*) 3·08871. (*c*) 21621.
6. (*a*) 5724. (*b*) 51·1071. (*c*) 2·5554.
7. (*a*) 12891. (*b*) 115·0982. (*c*) 5·7549 . . .
8. (*a*) 31584. (*b*) 282. (*c*) 14·1.
9. (*a*) 2548. (*b*) 22·75. (*c*) 1·1375.
10. 3 qrs. 24 lb. 4 oz. **11.** 10 qrs. 5 lb. 1 oz.
12. 4 qrs. 10 lb. 6·6 oz. **13.** 2 qrs. 17 lb. 13 oz.
14. 1 qr. 6 lb. 9 oz. **15.** 3 qrs. 21 lb. 1 oz.
16. 1 qr. 16 lb. 15·5 oz. **17.** 5 tons 17 cwt. 3 qrs.
18. 6 tons 15 cwt. 3 qrs. **19.** 1 ton 17 cwt. 1·7 qrs.
20. 8 tons 6 cwt. 2 qrs.

Oral Examples (p. 89).

1. 10. **2.** 24. **3.** 22. **4.** 28. **5.** 12. **6.** 19.
7. 2·5. **8.** 16. **9.** 8. **10.** 12. **11.** ·625. **12.** ·8.

III d (p. 91).

1. (*a*) 2·9375. (*b*) 5·875. (*c*) 23·5. (*d*) 47.
2 (*a*) 3·375. (*b*) 6·75. (*c*) 27. (*d*) 54.
3. (*a*) 4·75. (*b*) 9·5. (*c*) 38. (*d*) 76.
4. (*a*) 35·75. (*b*) 71·5. (*c*) 286. (*d*) 572.
5. 5 bus. 1 pk. 1 gal. 1 qt. 1 pt.
6. 4 bus. 1 pk. 1 gal. 3 qts. 1 pt.
7. 9 bus. 1 gal. 1 pt.
8. 9 bus. 3 pks. 1 gal. 1 qt.
9. 1 bus. 1 gal. 3 qts. 1 pt.
10. 54. **11.** 35. **12.** 38·125.
13. 1600. **14.** 270°. **15.** 32·5083 . . .
16. 54·2125. **17.** 18·3133. **18.** 111° 6′ 18″.
19. 59° 19′ 17″. **20.** 1° 47′ 14″. **21.** 38° 14′ 30″.
22. 82 hrs. 6 min. 32 sec. **23.** 56 hrs. 9 min. 36 sec.
24. 5 hrs. 7 min. 13 sec.

III (p. 91).

1. 71 mi. 1084 yds. **2.** 36 ch. 23 l. **3.** 5517 ft.
4. 4 cwt. 1 qr. 22 lb. **5.** £9 2*s.* 7*d.* **6.** 2 days 5 hrs. 49 sec.
7. £7 12*s.* 11½*d.* **8.** £14 8*s.* **9.** 1·285 lb.
10. 2 mi. 4 fur. 6 ch. **11.** 39 hrs. 33 min. 35 sec.
12. 5020. **13.** 22·4. **14.** £45833 6*s.* 8*d.*
15. ·78125. **16.** 32,659,440. **17.** £45 1*s.* 4*d.*
18. 1*s.* 3¾*d.* **19.** 63,360; 1609·3.
20. 36 m. 11 pf. **21.** 1236 yds. **22.** 1057 po.
23. 16 miles. **24.** 896875 tons. **25.** 1265.
26. 8·047 m. **27.** 2588 sq. yds. **28.** 163 lb.
29. £10 11*s.* 6*d.* **30.** 226 fr. 92 c. **31.** 2*s.*
32. 40912 oz. **33.** 1125 fr. **34.** 65 c.
35. £1. **36.** 14 gals. 2 qts.
37. 9·75. **38.** 24 in. **39.** £2 16*s.* 5*d.*
40. 8·6 tons. **41.** 2 mi. 80 yds.
42. £14265 3*s.* 4*d.* **43.** £179 2*s.* 1½*d.*; 8*s.* 10¾*d.*
44. 36524.

IV a (p. 97).

1 (*a*). £8280 2*s.* 2½*d.*
2 (*a*). £1472 12*s.* 1½*d.*
3 (*a*). £7685 14*s.* 8¼*d.*
4 (*a*). 34 tons 2 cwt. 0 qr.
5 (*a*). 46 mi. 6 fur. 6 ch.
6 (*a*). 16 gals. 2 qts. 1 pt.
7. £2006 1*s.* 11*d.*
9. £4334 14*s.* 2*d.*
11. £11414 15*s.* 6½*d.*
13. £14405 3*s.* 5½*d.*
15. £5792 4*s.* 6½*d.*
17. £18016 9*s.* 10*d.*
19. £884 18*s.* 4½*d.*
21. £1047 3*s.* 1¾*d.*
23. £1186 1*s.* 9½*d.*
25. £35 15*s.* 1½*d.*
27. £303 19*s.* 1*d.*
29. £11031 8*s.* 2¼*d.*

1 (*b*). £880 2*s.* 1¾*d.*
2 (*b*). £4638 0*s.* 5¼*d.*
3 (*b*). £2149 11*s.* 1½*d.*
4 (*b*). 23 lb. 4 oz. 12 dr.
5 (*b*). 20 po. 1 yd. 2 ft. 6 in.
6 (*b*). 16 qrs. 1 bus. 0 pk.
8. £2979 4*s.* 3½*d.*
10. £4892 0*s.* 8*d.*
12. £3944 11*s.* 9½*d.*
14. £14592 0*s.* 11½*d.*
16. £14232 17*s.* 6¼*d.*
18. £16799 6*s.* 3*d.*
20. £70 4*s.* 9½*d.*
22. £72 8*s.* 1½*d.*
24. £6 6*s.* 0¾*d.*
26. £782 16*s.* 9½*d.*
28. £802 16*s.* 6¾*d.*
30. £1118 13*s.* 0½*d.*

31. £9002 3*s.* 10¼*d.*
32. £7879 15*s.* 3¼*d.*
33. £16800 11*s.* 10¼*d.*
34. 9 tons 10 cwt. 1 qr. 14 lb.
35. 1 ton 15 cwt. 1 qr. 6 lb.
36. 3 ch. 15 yds. 2 ft.
37. 21 mi. 1 fur. 6 ch.
38. 8 lb. 15 oz.
39. £1 10*s.*
40. 4*s.* 9*d.*
41. 5 gals. 3 qts. 1 pt.
42. £28 3*s.* 4*d.*
43. 24 ac. 0 ro. 15 po.
44. £424 8*s.*
45. 5 gals. 0 qts. 1 pt.
46. 13 mi. 4 fur. 8 ch.
47. 14 min. 49 sec.
48. 3 fur. 4 ch. 16 yds.
49. £290 17*s.* 8*d.*
50. Nearly 1*d.* (·96*d.*).
51. 60 yds.
52. 243 tons 3 cwt.
53. £4 1*s.* 6*d.*
54. 17 ac. 2 ro. 7 po.
55. 29 sq. ft.

Oral Examples (p. 101).

1. *Rows.* (i) 19*s.* 3½*d.* (ii) 27*s.* 3½*d.* (iii) 21*s.* 8½*d.*
(iv) 58*s.* 8½*d.* (v) 31*s.* 5¼*d.*
Columns. (i) 48*s.* 3¾*d.* (ii) 40*s.* 4¼*d.* (iii) 31*s.* 9¾*d.*
(iv) 15*s.* 9*d.* (v) 22*s.* 2½*d.*
Diagonals. (*a*) 23*s.* 9*d.* (*b*) 42*s.* 8¼*d.*

2. *Rows.* (i) 17 ft. 2 in. (ii) 13 ft. 9 in. (iii) 22 ft. 1 in.
(iv) 32 ft. 4 in.
Columns. (i) 13 ft. 3 in. (ii) 22 ft. 4 in. (iii) 21 ft. 7 in.
(iv) 28 ft. 2 in.
Diagonals. (*a*) 22 ft. 6 in. (*b*) 16 ft. 5 in.

3. *Rows.* (i) 12 mi. 5 fur. (ii) 25 mi. 4 fur. (iii) 36 mi. 5 fur.
(iv) 26 mi. 3 fur.
Columns. (i) 27 mi. 6 fur. (ii) 25 mi. 5 fur. (iii) 30 mi. 4 fur.
(iv) 17 mi. 2 fur.
Diagonals. (*a*) 21 mi. 5 fur. (*b*) 31 mi.

4. *Rows* (i) 21 lb. 11 oz. (ii) 27 lb. 13 oz. (iii) 33 lb. 4 oz.
(iv) 17 lb. 6 oz.
Columns. (i) 15 lb. 12 oz. (ii) 29 lb. 1 oz. (iii) 33 lb. 9 oz.
(iv) 21 lb. 12 oz.
Diagonals. (*a*) 24 lb. 2 oz. (*b*) 32 lb. 11 oz.

IV b (p. 103).

1. (i) £22 15*s.* 6*d.* (ii) £379 11*s.* 8*d.*
2. (i) £19 8*s.* 6*d.* (ii) £158 12*s.* 9*d.*

3. (i) £114 9*s.* 4½*d.* (ii) £1068 7*s.* 6*d.*
4. (i) £71 17*s.* 2*d.* (ii) £404 4*s.* 0¾*d.*
5. (i) £66 14*s.* 9*d.* (ii) £266 19*s.* 0*d.*
6. (i) 12 tons 2 cwt. (ii) 75 tons 12 cwt. 2 qrs.
7. (i) 3 tons 7 cwt. 3 qrs. 8 lb. (ii) 5 tons 1 cwt. 2 qrs. 26 lb.
8. (i) 3 cwt. 3 qrs. 17 lb. 10 oz. (ii) 1 ton 16 cwt. 1 qr. 24 lb. 8 oz.
9. (i) 9 mi. 2 fur. 9 ch. (ii) 96 mi. 2 fur. 4 ch.
10. (i) 62 mi. 8 ch. (ii) 331 mi. 1 fur. 6 ch.
11. (i) 3 mi. 4 fur. 6 ch. 4 yds. (ii) 21 mi 7 fur. 2 ch. 19 yds.
12. (i) 172 ac. 3 ro. 35 sq. po. (ii) 322 ac. 3 ro. 20 po.
13. (i) 163 ac. 2 ro. 24 po. (ii) 4091 ac. 1 ro.
14. (i) 13 ac. 7 sq. ch. 184 sq. yds. (ii) 74 ac. 7 sq. ch. 464 sq. yds.
15. 12 hrs. 58 min. 4 sec.
16. (i) 15° 39′ 45″. (ii) 22° 22′ 30″.

IV c (p. 104).

1. (i) £11 12*s* 4½*d.* (ii) £4 12*s.* 11½*d.* (iii) 11*s* 0¾*d.*
2. (i) £5 6*s.* 6¾*d.* (ii) £4 13*s.* 2¾*d.* (iii) £1 0*s.* 8½*d.*
3. (i) £16 19*s.* 5¼*d.* (ii) £8 9*s.* 8½*d.* (iii) £3 15*s.* 5*d.*
4. (i) £610 14*s.* 9*d.* (ii) £203 11*s.* 7*d.* (iii) £183 4*s.* 5*d.*
5. (i) £20 5*s.* 5*d.* (ii) £4 10*s.* 1*d.* (iii) £6 9*s.* 8¾*d.*
6. (i) £331 16*s* 5¼*d.* (ii) £39 16*s.* 4¼*d.* (iii) £94 16*s.* 1½*d.*
7. 2 mi. 5 fur. 7 ch. **8.** 2 mi. 7 fur. 2 ch.
9. 2 ch. 11 yds. 2 ft. **10.** 10 cwt. 2 qrs. 22 lb.
11. 1 ton 14 cwt. 13 lb. **12.** 13 ac. 1 sq. ch.

IV d (p. 105).

1. £9987 11*s.* 6*d.* **2.** £1093 9*s.* 6*d.* **3.** £2372 12*s.* 6*d.*
4. £2810 13*s.* 8¾*d.* **5.** £14855 11*s* 5¼*d.* **6** £9138 18*s.* 9½*d.*
7. £173772 18*s.* 4*d.* **8.** £2993 17*s.* 9*d.* **9.** £2021 14*s.* 8*d.*
10. £3860 16*s.* 8¼*d.* **11.** £2097 8*s.* 5¼*d.* **12.** £1116 18*s.* 7½*d.*
13. 300 yds. 0 ft. 7 in. **14.** 8 mi. 7 fur. 6 ch. 6 yds. 2 ft.
15. 81 mi. 4 fur. 2 po. **16.** 147 tons 13 cwt. 7 lb.
17. 8 tons 9 cwt. 3 qrs. 27 lb. **18.** £5 14*s.* 8*d.*
19. £116104 16*s.* 10½*d.* **20.** £1 1*s.* 7¾*d.*
21. 19*s.* 11¼*d.* **22.** 4 tons 14 cwt. 2 qrs.
23. £2639 2*s.* 1½*d.* **24.** £234 12*s.* 2¼*d.*

IV e (p. 107).

1. £74 5*s.* 1¼*d.* **2.** £1 4*s.* 10¼*d.* **3.** £83 11*s.* 7*d.*
4. £103 10*s.* 11¼*d.* **5.** £3 16*s.* 6½*d.* **6.** £4 3*s.* 7½*d.*
7. £32 12*s.* 3½*d.* **8.** £171 1*s.* 0½*d.* **9.** £2 18*s.* 4¼*d.*
10. £3 17*s.* 1½*d.* **11.** £15 17*s.* 8¼*d.* **12.** £36 1*s.* 8*d.*
13. 2 tons 1 cwt. 2 qrs. 4 lb. **14.** 1 ton 6 lb.
15. 4 tons 4 cwt. 18 lb. **16.** 2 ch. 18 yds. 2 ft.
17. 3 ch. 2 yds. **18.** 2 fur. 5 ch. 13 yds.
19. 1 ac. 3 ro. 5 sq. po. **20.** 7 ac. 1 ro. 8 sq. po.
21. £130 17*s.* 3¼*d.* **22.** £6 18*s* 4¼*d.* **23.** £143 14*s.* 3¾*d.*
24. £6215 7*s.* 8¾*d.* **25.** £130 4*s.* 10½*d.* **26.** £2 0*s.* 8½*d.*
27. £16 1*s.* 5*d.*

IV f (p. 110).

1. 192. Rem. 13*s.* 9*d.* **2.** 174 Rem. £2 5*s.* 5*d.*
3. 239. ,, 11*s.* 4*d.* **4.** 8. ,, £2 19*s* 6*d.*
5. 74. ,, £13 4*s.* 4*d.* **6.** 22. ,, £7 17*s.* 6*d.*
7. 71. ,, 2 lb. 2 oz. **8.** 169. ,, 1 lb. 3 oz.
9. 330 exactly. **10.** 34. ,, 4 yds. 2 ft.
11. 132 exactly. **12.** 28. ,, 1 ch. 6 yds.
13. 45. Rem. 1 pt. **14.** 67. ,, 1·6 m.
15. 90. ,, 7·3 g. **16.** 51. ,, 34·7 g.
17. 2225. ,, 5 cm. **18.** 916. ,, 20 c.
19. 156. ,, $1 60 c. **20.** 62. ,, 60 pf.
21. 8653. ,, 7·04 g. **22.** 109. ,, 1 fr. 20 c.
23. 87 ,, $4 90 c. **24.** 97. ,, 4 fr. 45 c.
25. 148. ,, 2·56 Kg. **26.** 61. ,, $1 30 c.
27. 341. ,, £2 11*s.* 6*d.* **28.** 513. ,, £3 11*s.* 6*d.*
29. 79. ,, 11*s.* 9*d.*

IV (p. 111).

1. £376 17*s.* 6*d.* **2.** 6¾. £52 11*s.* 10½*d.*
3. £18. **4.** 1960.
5. £22,400,000. **6.** 18274.
7. 501. 4*s.* 2*d.* **8.** £37 19*s.* 3*d.*
9. 9 tons 11 cwt. 3 qrs. 21 lb. **10.** £755 15*s.* 8*d.*
11. £265 19*s.* 1*d.* **12.** 4320.
13. £9 0*s.* 11¼*d.* **14.** 55 ac. 1 ro. 16 po. £59 19*s.* 3*d.*
15. 1844. **16.** 671. £2 3*s.* 4*d.*
17. £4 11*s.* 8*d.* **18.** 5120.

19. £2284090 11*s.* 10½*d.*
20. 108.
21. £528754 17*s.* 11*d.*
22. £8615 1*s.* 7*d.*
23. 42. 4 tons 9 cwt. 3 qrs. 9 lb
24. £24 16*s.* 4*d.*
25. £213 3*s.*
26. 6*d.*
27. £12 18*s.* 7¼*d.*
28. 123.
29. ·33 in.
30. 7·5 in.
31. 7 hrs. 54 min. 12 sec. 3 hrs. 50 min. 28 sec.
32. 38 ac. 0 ro. 15 po.
33. £1 8*s* 3*d.*
34. He pays £3 3*s.*
35. 1610.
36. £2 14*s.* 9¼*d.*
37. Saturday.
38. £1 6*s.* 2¾*d.* £4 4*s.* 2*d.*
39. 11¾ lb. Nearly 3*d.*
40. £18 18*s.* 4*d.*
41. 2 tons 11 cwt. 2 qrs. 14 lb
42. 3*s.* 2*d.*
43. 3.51 p m.
44. £5090 9*s*
45. 6*s* 8*d*
46. 1 qr 0 lb. 7 oz.
47. 225 ft.
48. 32·26 fr.
50. 23 pieces. 3·89 in. over.
51. £30 9*s.*
52. £7 6*s.* 6*d.*
53. 11 pieces. 1 m. 45 cm. over.
54. Gains £261 4*s.* 2*d.*
55. 183 fr. 75 c.
56. 3½*d.*
57. £2 13*s.* 1½*d.*
58. £14 6*s.* 10*d.*
59. £5 14*s.* 8¼*d.*
60 £1 1*s.* 6¾*d.* ·359375.
61. £1 5*s.*
62. £3 8*s.* 4¼*d.*
63. £3237 0*s.* 2½*d*
64 504 times.
65. £336.
66. 2 mks. 5 pf.
67. 20 mks. 43 pf.
68. 6 lb. 12 oz. more.
69. 16·25 metres.
70. 20 things. Rem. £4 9*s.* 2*d.*
71. 2 hrs. 56 min.
72. 390·986 lb.
73. 88 inches.
74 £120 5*s* gain
75. 603·505
76. ·396 in.
77. £434 19*s.* 3*d.*

Revision Examples (p. 118).

1.

	Counties.	Boroughs.	Total.	639208	Owners
England	3383551	2466105	5869766	6717059	Occupiers.
Wales	247335	110231	357566	281258	Lodgers.
Scotland	450950	311234	782208	25648	Freemen, &c.
Ireland	578464	115303	698787	45154	University.
		Total	7708327	7708327	Total.

2. £12 18*s.* 7¼*d.*
4. ·144, ·017; ·00145.
5. 4 yds. 1 ft.
6. (*a*) £165; (*b*) £262.

7. 129413, remainder 59. **8.** 101339 tons.
9. £3377 3*s.* 6*d.* **10.** 2·4008354748.
11. £3 16*s.* 1½*d.* **12.** 105; 2 ft. 6 in.
13. *4th Column.* 106364000; 2993000; 777000; 46941000; 19452000; 2147000; 30948000; 209622000.
6th Column. £127; £273; £229; £82; £76; £101; £118.
14. 86400. **15.** £962. **16.** 71 mi. 4 fur. 204 yds.
17. 671; £2 3*s.* 4*d.* **18.** 2873 mm.
19. 57375; 136589; 96145; 29016; 1547. **20.** 4290·3 francs.
21. £5118 1*s.* 4¼*d.* **22.** £418 7*s.* 4½*d.* **23.** £6 gain. **24.** 77 cm.
25. 13900000; 7800000; 57200000; 49200000.
26. £14026 1*s.* 3*d.*; £45 13*s* 9*d.* **27.** ·00494 in.
28. 16926 times. **29.** 160 tons 10 cwt. 3 qrs. 8 lb.
30. 2490639082 lb. **31.** 662. **32.** 10 cwt. 2 qrs. 15 lb.
33. 7 times. **34.** 11·83. **35.** ·0137 sq. km. **36.** £62 4*s.*
37. £6 6*s.* 9½*d.*, total amount of bill. **38.** 144; £1 4*s.*
39. 58 times. **40.** £6 4*s.* 8¼*d.*; 1 hr. 22½ min. p.m March 9th.
41. ·1375 miles; 123 times. **42.** 6·72 lb. **43.** 117 fr. 40 c.
44. 1912; 1903. **45.** 2 fur. 3 po. 4 yds. 2 ft. **46.** 1*s.* 2¼*d.*
47. 2520 lb. **48.** 5·15 francs. **49.** £170 9*s.* 4*d.* **50.** 238.

Oral Examples (p. 128).

(A cross indicates divisibility.)

Divisible by	2.	3	5.	11.	Divisible by	2.	3.	5.	11.
1.	×				**13.**	×	×		
2.	×				**14.**				×
3.	×	×			**15.**	×	×		×
4.		×	×		**16.**		×	×	
5.		×	×		**17.**			×	×
6.		×	×	×	**18.**	×	×		
7.	×		×	×	**19.**		×	×	
8.	×	×			**20.**	×	×		×
9.					**21.**				
10.	×	×	×		**22.**		×		
11.	×			×	**23.**	×			×
12.		×			**24.**		×		

The factors are given. Numbers which are prime are omitted.

25. 2.23. **27.** 3.19. **31.** $3^2.89$. **32.** 7.23.
33. 3.53. **35.** 3.23. **38.** 11.37.

V a (p. 130).

1. $2.3^2.5$.	**2.** $2^2.3.5.7$.	**3.** $3^2.5.11$.
4. $2^3.3.7$.	**5.** $3^2.5^2.13$.	**6.** $3^2.11.17$.
7. $2^2.11^2.13$.	**8.** $2.3.7\ 19$.	**9.** $3^2.11^2.17$.
10. $2.11.43$.	**11.** $5^3.11$.	**12.** $3^3.13.23$.
13. 3.73.	**14.** $3^2.11^2.53$.	**15.** 11^3.
16. $2^2.11.101$.	**17.** $2.7.11.13$.	**18.** $7.11.83$.
19. $3.7^2.31$.	**20.** $2^4.3^2.11^2$.	**21.** $7.11.13.17$.
22. $3^2.11.19$.	**23.** $3^2.13.97$.	**24.** 11.167.

25. 24.	**26.** 105.	**27.** 66.	**28.** 385.
29. 156.	**30.** 735.	**31.** 154.	**32.** 192.
33. 1785.	**34.** 462.	**35.** 33.	**36** 55.
37. 105.	**38.** 36.	**39.** 121.	**40.** 77.

V b (p. 132).

The L.C.M. is given first.

1. $2.3^2.5.7.11$. 21.	**2.** $2^2.3^2.5.11.13$ 39.
3. $2.3.5.11.17$. 85.	**4.** 2 3 5 7 29. 29.
5. $2^2.3^2.5.7.11.13.19$. 7.	**6.** $5^2.11^2.19$. 209.
7. $2^3.3^3.5^3.7^2.11$. 5.	**8.** $2^2.5^2.7^2.11^2$. 121.
9. $2^2.5^3.11.17$. 25.	**10.** $3^3.7^2\ 11^2.23$. 253
11. $2^3.3^3.7^2.31$. 217.	**12.** $5^2\ 11^3.37$ 407.
13. $2^2.3^5.7.13$. 117.	**14.** $3^4\ 11^3$ 29. 261.
15. $2^2.3^4.13.17$. 153.	**16.** $2^4.3^2\ 5.11.13$ 44.
17. $2^2.3^2.7^2.11$. 231.	**18.** $2^5.3^3.17$. 12.

19. $2^3.3^2.5^2.7^2.11$. No common factor.
20. $2^2.3^2.7^2.17^2$,, ,, ,,

21. $2^3.3.7^2.11.13$. 13.	**22.** $2^5.3^4.5^3.7^2.11$. 75600.	
23. $2^5.3^4.5^3.7^2.11$. 75600.	**24.** $2.3^2.5$.	**25.** $2^4.3.5.7$.
26. $2.3^2.5.7.11$.	**27.** $2^2.3.5.7.11$.	**28.** $2.3.5.13$.
29. $2.3.5.7.11^2$.	**30** $3^2.5.11.13$	**31.** $2^3\ 3^2.7^2.11$.
32. $5^3.7^2.11^2.13$.	**33.** $3^2.5^2.7^2.17$.	**34.** $2^4.3^4\ 5.7^2.13$.
35. $2^4.3^2.7.13$.	**36.** $2^4.3^3.7^2$.	**37.** $2^5.3^2.7.13$.
38. $3^3.5.11.13$.	**39.** $2^3.3^2\ 7.11$.	**40.** $2^5.3^2.5.7$.

V c (p. 135).

1. 37. 3811×179 or 103×6623.
2. 61. 6649.229 or 109.13969.
3. 94. 12126.505 or 47470.129.

4. 107. 107.51.134. **5.** 127. 127.85.172.
6. 139. 139.119.159. **7.** 73. 73.191.233.
8. No common factor. 3007.3713.
9. 149. 149.38.51. **10.** 157. 157 501.575.
11. 458. 458.327.565. **12.** No common factor. 20711.18281.
13. 67. 67.79.83.107 **14.** 101. 101.127.137.159.
15. 571. **16.** 527. **17.** 311. **18.** 317. **19.** 107.

V d (p. 137).

1. (*a*) 118125. 189. (*b*) 590625. 37·8.
2. (*a*) 2233125 3573. (*b*) 11165625. 714·6.
3. (*a*) 93100. 148·96. (*b*) 465500. 29 792.
4. (*a*) 223725. 357·96. (*b*) 1118625 71·592.
5. (*a*) 181625. 290·6. (*b*) 908125. 58·12.
6. (*a*) 2580 4·128. (*b*) 12900 ·8256.
7. (*a*) 218900. 350·24. (*b*) 1094500. 70·048.
8 (*a*) 1587·45. 2 53992. (*b*) 7937·25. ·507984.
9. (*a*) 11865. 18·984. (*b*) 59325. 3·7968.
10. (*a*) 2062·5. 3·3. (*b*) 10312·5 ·66.
11. (*a*) 18372·5. 29·396. (*b*) 91862·5. 5·8792.
12. (*a*) 8·205. ·013128. (*b*) 41·025. ·0026256.
13. 343. **14.** 12489. **15.** 764. **16.** 32752.
17. 41064. **18.** 7·2343. **19.** 1298·784. **20.** 12·8427.
21. 1002001. **22.** 998001. **23.** 1008016. **24** 802816.
25. 366025. **26.** 672400. **27.** 1444804. **28.** 3992004.
29. 16040025. **30.** 250020·004.

V (p. 138).

1. $2^3.3^2.17.19$; 152, 153. **3.** 1199835. **4.** 1764 and 7056.
5. 7056. **6** 83259. **7.** 5, 1. **8.** 566.
9. 619. **10** 77. 231. **11.** 2520, 2688, 2856.
12. $2.3^2.11^2.13.37$. 111, 117, 121, 143. **13.** 28 times. 15 times.
14. 3*s* 3*d*. **15.** £6 2*s* 6*d*. **16.** 592704.
17. 540. **18.** 3780. **19.** $\frac{1}{7}$ min. after noon. 4201.
20. 138384, 139131, 138458·71. **21.** 13893. **23.** 4 times more.
24. (i) The first and third are together after 24 sec. (ii) 14 minutes.
25. $437 = 19 \times 23$. 457 is prime. **26.** 3016. **27.** 3 strokes. 5 sec.
28. 75350. **29.** 75. **30.** Yes.

Oral Examples (p. 145).

11. $4\frac{1}{4}$. **12.** $2\frac{7}{8}$. **13.** $1\frac{8}{9}$. **14.** $7\frac{3}{7}$. **15.** $18\frac{5}{11}$.
16. $22\frac{3}{8}$. **17.** $25\frac{7}{12}$. **18.** $40\frac{4}{11}$. **19.** $20\frac{3}{8}$. **20.** $14\frac{7}{12}$.
21. $45\frac{7}{9}$. **22.** $1\frac{5}{13}$. **23.** $1\frac{7}{25}$. **24.** $1\frac{4}{71}$. **25.** $1\frac{81}{82}$.
26. $1\frac{88}{107}$. **27.** $\frac{11}{9}$. **28.** $\frac{25}{7}$. **29.** $\frac{37}{8}$. **30.** $\frac{31}{10}$.
31. $\frac{27}{10}$. **32.** $\frac{61}{11}$. **33.** $\frac{74}{9}$. **34.** $\frac{36}{19}$. **35.** $\frac{81}{38}$.
36. $\frac{205}{103}$. **37.** $\frac{20}{5}$. **38.** $\frac{224}{75}$. **39.** $8d.$ **40.** $3d.$
41. $4s.$ **42.** $12s.$ **43.** $12s.$ **44.** 2 oz. **45.** 8 in.
46 27 ft. **47.** 2 sq. ch. **48.** 12 lb. **49.** 18 in. **50.** 6 yds.
51. 53 ch. **52.** $35s.$ **53.** 10 in. **54.** 16 lb. **55.** 6 lb.
56. 4 lb. **57.** $2s.\ 4d.$ **58.** 24s. **59.** 2 yds **60.** 98 yds.
61. 20 lb. **62.** 22 dm. **63.** 8 dm. **64.** 2 mm. **65.** 12 cm.
66. 2 sq. ft. **67.** 25 cub. ft. **68.** 24 mm. **69.** 9 sec. **70.** 73 days.
71. 12 days. **72.** 25 days. **73.** 5 in. **74.** ·5. **75.** ·4.
76. ·9. **77.** ·6. **78.** ·35. **79.** 1·1. **80** 2·3
81. 1·7. **82.** 3·5. **83.** 4·3. **84.** 5·6 (5·7).
85. 3·1. **86.** 2·2 (2·3). **87.** 4·2 (4·3). **88.** 5·8.
89. $\frac{3}{10}$. **90.** $\frac{4}{10}$ $(\frac{2}{5})$. **91.** $2\frac{3}{10}$. **92.** $\frac{5}{10}$ $(\frac{1}{2})$.
93. $3\frac{5}{10}$ $(3\frac{1}{2})$. **94.** $2\frac{5}{10}$ $(2\frac{1}{2})$. **95.** $\frac{25}{100}$ $(\frac{1}{4})$. **96.** $1\frac{2}{10}$ $(1\frac{1}{5})$.
97. $3\frac{7}{10}$. **98.** $11\frac{6}{10}$ $(11\frac{3}{5})$. **99.** $10\frac{34}{100}$ $(10\frac{17}{50})$. **100.** $5\frac{65}{100}$ $(5\frac{13}{20})$
101. $8\frac{75}{100}$ $(8\frac{3}{4})$. **102.** $9\frac{35}{100}$ $(9\frac{7}{20})$. **103.** $5\frac{55}{100}$ $(5\frac{11}{20})$.
104. (i) 24. (ii) $\frac{24}{64}$. (iii) 8. (iv) $\frac{8}{64}$. (v) $\frac{16}{64}$.
(vi) $\frac{32}{64}$. (vii) $\frac{4}{64}$. (viii) $\frac{8}{64}$. (ix) $\frac{12}{64}$. (x) 3.
(xi) $\frac{3}{8}$. (xii) $\frac{3}{8}$. (xiii) $\frac{2}{8}$. (xiv) $\frac{16}{64}$.
105. (i) 12. (ii) $\frac{5}{12}$. (iii) $\frac{3}{4}$. (iv) $\frac{1}{2}$ or $\frac{2}{4}$. (v) $\frac{10}{12}$ or $\frac{5}{6}$.
(vi) $\frac{7}{12}$. (vii) $\frac{4}{12}$ or $\frac{1}{3}$. (viii) $\frac{7}{12}$. (ix) $\frac{6}{12}$ or $\frac{1}{2}$. (x) $\frac{4}{8}$ or $\frac{1}{2}$.

Oral Examples (p. 150).

1. $\frac{3}{4}$. **2.** $\frac{3}{8}$. **3.** $\frac{1}{3}$. **4.** $\frac{1}{5}$. **5.** $\frac{2}{3}$. **6.** $\frac{2}{6}$. **7.** $\frac{3}{8}$.
8. $\frac{1}{2}$. **9.** $\frac{5}{12}$. **10.** $\frac{2}{8}$. **11.** $\frac{3}{5}$. **12.** $\frac{6}{7}$. **13.** $\frac{6}{9}$. **14.** $\frac{11}{12}$.
15. $\frac{9}{10}$. **16.** $\frac{11}{13}$. **17.** $\frac{9}{11}$. **18.** $\frac{4}{5}$. **19.** $\frac{3}{4}$. **20.** $\frac{8}{9}$. **21.** $\frac{3}{5}$.
22. $\frac{4}{7}$. **23.** $\frac{6}{11}$. **24.** $\frac{1}{2}$. **25.** $\frac{4}{9}$. **26.** $\frac{3}{6}$. **27.** $\frac{7}{8}$. **28.** $\frac{5}{6}$.
29. $\frac{1}{8}$. **30.** $\frac{1}{3}$. **31.** $\frac{2}{5}$. **32.** $\frac{3}{11}$. **33.** $\frac{5}{9}$. **34.** $\frac{-}{9}$. **35.** $\frac{5}{7}$.
36. $\frac{5}{8}$. **37.** $\frac{7}{9}$. **38.** $\frac{7}{9}$. **39.** $\frac{2}{3}$. **40.** $\frac{2}{8}$. **41.** Equal. **42.** $\frac{5}{4}$.
43. $\frac{7}{5}$.

VI a (p. 151).

1. $\frac{50}{231}$. 2. $\frac{2}{3}$. 3. $\frac{11}{13}$. 4. $\frac{4}{9}$.
5. $\frac{4}{15}$. 6. $\frac{14}{18}$. 7. $\frac{16}{22}$. 8. $\frac{7}{66}$.
9. $\frac{4}{21}$. 10. $\frac{22}{35}\frac{54}{65}$. 11. $\frac{5}{8}$. 12. $\frac{7}{8}$.
13. $\frac{18}{35}$. 14. $\frac{9}{11}$. 15. $\frac{101}{259}$. 16. $\frac{8}{13}$.
17. $\frac{11}{16}$. 18. $\frac{99}{104}$. 19. $\frac{88}{117}$. 20. $\frac{27}{37}$.
21. $\frac{617}{2889}$. 22. $\frac{113}{150}$. 23. $\frac{26}{77}$. 24. $\frac{13665}{22222}$.
25. $\frac{11333}{19208}$. 26. $\frac{32559}{35993}$. 27. $\frac{3}{56}$. 28. $\frac{17}{30}$.
29. $\frac{17493}{36887}$. 30. $\frac{17}{21}$. 31. $\frac{5}{17}$. 32. $\frac{109}{229}$.
33. $\frac{84209}{131141}$. 34. $\frac{119}{159}$. 35. $\frac{9}{14}$. 36. $\frac{501}{675}$.
37. $\frac{59}{203}$. 38. $\frac{1425107}{4251179}$. 39. $\frac{207111}{182281}$. 40. $\frac{27}{64}$.
41. $\frac{31}{71}$. 42. $\frac{9981}{28825}$. 43. $\frac{2225}{13117}$. 44. $\frac{36221}{227797}$.
45. $\frac{489908}{601483}$. 46. $\frac{58856048}{196553357}$. 47. $\frac{84}{5934}$. 48. $\frac{3483}{7101}$.
49. $\frac{415553}{834438}$. 50. $\frac{22770758}{28697713}$.

Oral Examples (p. 155).

1. $\frac{1}{3}$. 2. $\frac{6}{7}$. 3. $\frac{4}{5}$. 4. $\frac{16}{49}$. 5. $\frac{5}{6}$.
6. $\frac{3}{10}$. 7. $\frac{7}{6}$. 8. $\frac{1}{6}$. 9. $\frac{17}{30}$. 10. $\frac{1}{30}$.
11. $\frac{1}{6}$. 12. $\frac{2}{3}$ or $\frac{4}{6}$. 13. $\frac{12}{18}$ or $\frac{2}{3}$. 14. 0. 15. $\frac{5}{6}$.
16. $1\frac{1}{6}$. 17. $1\frac{2}{9}$. 18. $2\frac{1}{15}$. 19. $6\frac{23}{12} = 7\frac{11}{12}$.
20. $2\frac{7}{12}$. 21. 1. 22. $3\frac{1}{14}$. 23. $2\frac{13}{14}$. 24. $\frac{3}{10}$.
25. $\frac{1}{26}$. 26. $\frac{11}{78}$. 27. $1\frac{5}{69}$. 28. $\frac{15}{16}$. 29. $\frac{7}{27}$.
30. $2\frac{8}{27}$.

VI b (p. 156).

1. $\frac{1}{7}$. 2. $\frac{181}{364}$. 3. $\frac{517}{720}$. 4. $\frac{23}{30}$. 5. $\frac{33}{50}$.
6. $2\frac{113}{120}$. 7. $1\frac{1}{60}$. 8. $6\frac{1}{2}$. 9. $1\frac{1}{4}$. 10. $1\frac{3}{64}$.
11. $\frac{1}{105}$. 12. $1\frac{5}{6}$. 13. $3\frac{53}{84}$. 14. $8\frac{17}{24}$. 15. 1.
16. $2\frac{1}{16}$. 17. $2\frac{18}{90}$. 18. $\frac{104}{231}$. 19. $3\frac{39}{65}$. 20. $\frac{3}{4}$.
21. $2\frac{71}{110}$. 22. $23\frac{1}{2}$. 23. 2. 24. $2\frac{41}{42}$. 25. $\frac{4}{7}$.
26. $1\frac{83}{140}$. 27. $\frac{311}{420}$. 28. $\frac{16}{21}$. 29. $3\frac{11}{390}$. 30. $6\frac{83}{504}$.
31. $\frac{83}{252}$. 32. $\frac{2}{39}$. 33. $\frac{223}{840}$. 34. $\frac{517}{720}$. 35. $\frac{10}{1877}$.
36. $4\frac{8999}{131104}$. 37. $15\frac{13}{24}$. 38. $7\frac{187}{200}$. 39. $\frac{7}{10}$. 40. $1\frac{1}{5}$.
41. $\frac{9}{42}$. 42. $\frac{223}{952}$. 43. $\frac{251}{720}$. 44. £833, £800, £770.
45. $\frac{68}{165}$. 46. *C* most, *A* least. 47. He loses.
48. The second train.

Oral Examples (p. 158).

		(*a*)	(*b*)	(*c*)	(*d*)	(*e*)	(*f*)	(*g*)	(*h*)
1.	(i)	$1\frac{1}{8}$	2	$3\frac{1}{3}$	4	$4\frac{2}{3}$	6	$6\frac{2}{3}$	8
	(ii)	$1\frac{1}{7}$	$1\frac{5}{7}$	$2\frac{6}{7}$	$3\frac{3}{7}$	4	$5\frac{1}{7}$	$5\frac{5}{7}$	$6\frac{6}{7}$
	(iii)	$1\frac{1}{15}$	$1\frac{3}{5}$	$2\frac{2}{5}$	$3\frac{1}{5}$	$3\frac{11}{15}$	$4\frac{4}{5}$	$5\frac{1}{3}$	$6\frac{2}{5}$
	(iv)	$\frac{3}{5}$	$\frac{9}{10}$	$1\frac{1}{2}$	$1\frac{4}{5}$	$2\frac{1}{10}$	$2\frac{7}{10}$	3	$3\frac{3}{5}$
	(v)	$1\frac{11}{19}$	$2\frac{7}{19}$	$3\frac{18}{19}$	$4\frac{14}{19}$	$5\frac{10}{19}$	$7\frac{2}{19}$	$7\frac{17}{19}$	$9\frac{9}{19}$
	(vi)	$3\frac{1}{5}$	$4\frac{4}{5}$	8	$9\frac{3}{5}$	$11\frac{1}{5}$	$14\frac{2}{5}$	16	$19\frac{1}{5}$
	(vii)	$4\frac{2}{3}$	7	$11\frac{2}{3}$	14	$16\frac{1}{3}$	21	$23\frac{1}{3}$	28
	(viii)	$1\frac{5}{27}$	$1\frac{7}{9}$	$2\frac{26}{27}$	$3\frac{5}{9}$	$4\frac{1}{27}$	$5\frac{1}{3}$	$5\frac{25}{27}$	$7\frac{1}{9}$

2. (i) Multiply the numerator by the whole number, or (ii) divide the denominator by the whole number.

		(*a*)	(*b*)	(*c*)	(*d*)	(*e*)	(*f*)	(*g*)	(*h*)
3.	(i)	$\frac{1}{3}$	$\frac{2}{9}$	$\frac{2}{15}$	$\frac{1}{9}$	$\frac{2}{21}$	$\frac{2}{27}$	$\frac{1}{15}$	$\frac{1}{18}$
	(ii)	$\frac{2}{7}$	$\frac{4}{21}$	$\frac{4}{35}$	$\frac{2}{21}$	$\frac{4}{49}$	$\frac{4}{63}$	$\frac{2}{35}$	$\frac{1}{21}$
	(iii)	$\frac{4}{15}$	$\frac{8}{45}$	$\frac{8}{75}$	$\frac{4}{45}$	$\frac{8}{105}$	$\frac{8}{135}$	$\frac{4}{75}$	$\frac{2}{75}$
	(iv)	$\frac{1}{20}$	$\frac{1}{10}$	$\frac{3}{50}$	$\frac{1}{20}$	$\frac{3}{70}$	$\frac{1}{30}$	$\frac{3}{100}$	$\frac{1}{40}$
	(v)	$\frac{15}{38}$	$\frac{5}{19}$	$\frac{3}{19}$	$\frac{5}{38}$	$\frac{15}{133}$	$\frac{5}{57}$	$\frac{3}{38}$	$\frac{5}{76}$
	(vi)	$\frac{4}{5}$	$\frac{8}{15}$	$\frac{8}{25}$	$\frac{4}{15}$	$\frac{8}{35}$	$\frac{8}{45}$	$\frac{4}{25}$	$\frac{2}{15}$
	(vii)	$1\frac{1}{6}$	$\frac{7}{9}$	$\frac{7}{15}$	$\frac{7}{18}$	$\frac{1}{3}$	$\frac{7}{27}$	$\frac{7}{30}$	$\frac{7}{36}$
	(viii)	$\frac{8}{27}$	$\frac{16}{81}$	$\frac{16}{135}$	$\frac{8}{81}$	$\frac{16}{189}$	$\frac{16}{243}$	$\frac{8}{135}$	$\frac{4}{81}$

4. (i) Divide the numerator by the whole number, or (ii) multiply the denominator by the whole number.

5. (*a*) and (*b*) $\frac{1}{3}$ **6.** (*a*) and (*b*) $\frac{2}{35}$. **7.** (*a*) and (*b*) $\frac{12}{55}$.

8. (*a*) and (*b*) $\frac{5}{27}$.

9. When a fraction has to be multiplied by one number and divided by another, the order of multiplication and division is indifferent.

Oral Examples (p. 159).

1. 200. 25. 10. **2.** 3*s*. 9*d*. 1*s*.

3. 20 lb. 2 lb. 6 lb. $6\frac{1}{4}$ lb. **4.** $\frac{1}{3}$. $\frac{1}{24}$. $\frac{1}{4}$. $\frac{1}{18}$.

5. $\frac{1}{8}$. **6.** $\frac{1}{6}$. $\frac{2}{6}$. **7.** $\frac{1}{20}$. $\frac{1}{600}$. 10*s*. **8.** $\frac{1}{6}$. $\frac{1}{6}$. $\frac{1}{12}$.

Oral Examples (p. 164).

1. $\frac{2}{5}$. **2.** $\frac{1}{4}$. **3.** $\frac{3}{7}$. **4.** $\frac{8}{15}$. **5.** $\frac{56}{99}$. **6.** $\frac{5}{24}$.

7. $\frac{1}{4}$. **8.** $\frac{1}{3}$. **9.** $\frac{8}{23}$. **10.** $\frac{1}{18}$. **11.** $\frac{2}{5}$ **12.** $\frac{1}{4}$.

13. $\frac{1}{20}$. **14.** $\frac{1}{9}$. **15.** $\frac{9}{16}$. **16.** 4. **17.** 4 **18.** 5.

19. $1\frac{3}{8}$. **20.** 1. **21.** 4. **22.** 9. **23.** 4. **24.** $7\frac{1}{2}$.
25. 1. **26.** 3. **27.** 14. **28.** 6. **29.** 16. **30.** 32.
31. 105. **32.** 310. **33.** 117.

VI c (p. 165).

1. $39\frac{2}{7}$. **2.** 2. **3.** $\frac{3}{5}$. **4.** 3. **5.** $\frac{3}{5}$.
6. 1. **7.** 2. **8.** $52\frac{13}{19}$. **9.** $1\frac{15}{19}$. **10.** $\frac{61}{87}$.
11. $13\frac{1}{7}$. **12.** $5\frac{5}{7}$. **13.** $\frac{25}{48}$. **14.** $\frac{38}{45}$. **15.** $\frac{4}{27}$.
16. $\frac{91}{187}$. **17.** $\frac{45}{91}$. **18.** $5\frac{11}{12}$. **21.** 40 acres.
22. $16\frac{2}{3}$ acres. **23.** $\frac{61}{105}$. **24.** 40 tons. **25.** 12 feet.

VI d (p. 168).

1. $2\frac{19}{40}$. **2.** $\frac{4}{5}$. **3.** $\frac{4}{7}$. **4.** $\frac{16}{21}$. **5.** 8. **6.** 1.
7. $\frac{231}{1211}$. **8.** $\frac{4}{9}$. **9.** 5. **10.** 1. **11.** $29\frac{1}{4}$. **12.** $8\frac{2}{9}$.
13. $\frac{17}{35}$. **14.** $4\frac{4}{19}$. **15.** $\frac{3}{5}$. **16.** $13\frac{43}{80}$. **17.** 12. **18.** $\frac{7}{12}$.
19. $1\frac{1}{8}$. **20.** 6. **21.** $1\frac{7}{25}$. **22.** $\frac{125}{361}$. **23.** $4\frac{2}{7}$. **24.** 1.
25. $\frac{40}{1911}$. **26.** $\frac{18}{49}$. **27.** $\frac{1}{5}$, $\frac{3}{5}$. **28.** $\frac{11}{30}$; $2\frac{8}{11}$ days.
29. $4\frac{8}{19}$ days. **30.** £6. **31.** £462. **32.** £252.
33. (i) $\frac{5}{858}$ yds. (ii) $377\frac{3}{11}$ yds.
34. *B* $377\frac{3}{11}$ yds., *C* 380 yds.; $121\frac{2}{19}$ yds.

VI e (p. 172).

1. 1. **2.** 10. **3.** 27. **4.** $1\frac{1}{9}$. **5.** $\frac{4}{11}$. **6.** $131\frac{1}{2}$.
7. $\frac{1}{2}$. **8.** $\frac{1}{5}$. **9.** $\frac{9}{14}$. **10.** 6. **11.** $\frac{25}{32}$. **12.** $\frac{19027}{12865}$.
13. $54\frac{1}{9}$. **14.** 28. **15.** $\frac{2}{5}$. **16** $\frac{155}{2885}$. **17.** $1\frac{3}{4}$. **18.** 13.
19. $1\frac{133}{35}$. **20.** $\frac{5}{7}$. **21.** $\frac{2332}{5741}$. **22.** $4\frac{1}{10}$. **23.** $\frac{18}{29}$. **24.** 12.
25. 9. **26.** $\frac{1}{40}$. **27.** $2\frac{61}{177}$. **28.** $1\frac{2}{7}$. **29.** $3\frac{1}{4}$. **30.** $\frac{31}{85}$.
31. $1\frac{5}{14}$. **32.** $\frac{2}{3}$. **33.** 16. **34.** $\frac{3}{4}$. **35.** 1. **36.** 1.
37. $4\frac{8}{11}$. **38.** $\frac{1}{2261}$. **39.** $\frac{21}{80}$. **40.** $1\frac{1}{9}$. **41.** $\frac{2}{31}$. **42.** 3.
43. $\frac{28}{76}$. **44.** $1\frac{5}{32}$. **45.** $\frac{79}{99}$. **46.** $7\frac{21}{32}$. **47.** 10. **48.** 12.
49. $\frac{13}{37}$. **50.** $\frac{9233}{50835}$. **51.** $4\frac{2341}{2532}$. **52.** $3\frac{1}{27}$. **53.** $26\frac{2}{15}$. **54.** 3.
55. $1\frac{197}{440}$. **56.** $\frac{47}{85}$.

VI f (p. 176).

1. ·2000. **2.** ·3700708. **3.** ·0740.
4. 17*s.* $6\frac{1}{2}$*d.* **5.** $\frac{7}{16}$. **6.** ·688.
7. ·3679. **8.** £2 2*s.* $3\frac{1}{2}$*d.* **9.** ·792.
10. 2·25. **11.** 2·12. **12.** 9*s.* $3\frac{1}{2}$*d.*
13. £1 14*s.* 9*d.* **14.** £5 17*s.* 10*d.* **15.** £4 17*s.* 2*d.*

16. 5 tons 3 cwt. 91 lb. **17.** £31 8*s.* 8*d.*
18. 137760 ft. **19.** 15*s.* 6*d.* **20.** 2*s.* 5*d.*
21. 7*s.* 4*d.* **22.** £11 13*s.* 5*d.* **23.** £16 18*s.* 5½*d.* approx.
24. 1⅜. **25.** 4*s.* 0¾*d.* **26.** £6 6*s.* 8*d.*
27. 7*s.* approx. **28.** ½. **29.** $8\frac{1}{10}$. **30.** 1 ft. 4 in.

VI (p. 179).

1. £2660.

2. *A* receives £5625, *B* £3750, *C* £1875, *D* £3750. *A* loses £1875, *B* £1250, *C* £625, *D* £1250.

3. $\frac{1}{8}$. **4.** $\frac{2}{25}$. **5.** £3900.
6. £44970. **7.** 51516. **8.** £65625.
9. 500. **10.** 30*s.* **11.** £255.
12. £200. **13.** $\frac{1}{1\frac{1}{2}}$. **14.** £324. £216 £144. £96.
15. 607 ac. 1 ro. 16 po. **16.** £12 3*s.* 10*d.*
17. £10. **18.** 84. **19.** Nearly 4 $(3\frac{4}{8})$
20. 385. **21.** £13541 13*s.* 4*d.* **22.** 1 hr. 12 mın.
23. $7\frac{3}{5}\frac{4}{1}\frac{3}{1}$ mın. **24.** 14 days. **25.** £108.
26. 78 tons 15 cwt. **27.** 57 (56 8). **28.** 560.
29. 7½ mın **30.** £17100. **31.** 62 (and a fraction).
32. $\frac{2}{1}\frac{8}{9}$. $\frac{1}{4}\frac{3}{5}$.

Oral Examples (p. 184).

1. ·4. **2.** ·5. **3.** ·7. **4.** ·9.
5. ·95. **6.** ·85. **7.** ·55. **8.** ·35.
9. 2·2. **10.** 3·4. **11.** 5·6. **12.** 10·5.
13. 11·85. **14.** 9 65. **15.** 10·75. **16.** 12·8.
17. 8·1. **18.** 9·05. **19.** 14·25. **20** 5·25.
21. £3 4*s.* **22.** £5 5*s.* **23.** £7 13*s.* **24.** £8 7*s.*
25. £7 14*s.* **26.** £9 19*s.* **27.** £4 5*s.* **28.** 3*s.*
29. £6 7*s.* **30.** £9 18*s.* **31.** £10 2*s.* **32.** £15 1*s.*
33. £21 4*s.* **34.** £23 6*s.* **35.** £55 11*s.* **36.** ·008.
37. ·010. **38.** ·019. **39.** ·021. **40.** ·032.
41. ·034. **42.** ·040. **43.** ·022. **44.** ·048.
45. ·020. **46.** ·005. **47.** ·029. **48.** ·044.
49. ·042. **50.** ·0125. **51.** ·039. **52.** ·036.
53. ·046. **54.** 5¼*d.* **55** 4*d.* **56.** 9¼*d.*
57. 1¼*d.* **58.** 2¼*d.* **59.** 10½*d.* **60.** 8*d.*
61. 5½*d.* **62.** 7¾*d.* **63.** 2¾*d.* **64.** ½*d.*
65. 11¾. **66.** 2*d.* **67.** 8¼*d.* **68.** 9½*d.*

Oral Examples (p. 186).

1. 2·167.	**2.** 7·292.	**3.** 5·273.	**4.** 18·494.
5. 6·233.	**6.** 7·496.	**7.** 9·969.	**8.** 2·1125.
9. 4·731.	**10.** 3·241.	**11.** 103·392.	**12.** 155·333.
13. 72·761.	**14.** 36·4375.	**15.** 4·225.	**16.** 8·433.
17. 11·596.	**18.** 14·748.	**19.** 2·8875.	**20.** 18·6625.
21. £3 8*s.* 6*d.*	**22.** £2 4*s.* $5\frac{1}{4}d.$		**23.** £2 6*s.* $5\frac{1}{2}d.$
24. £3 9*s.* $0\frac{1}{2}d.$	**25.** £11 11*s.* 10*d.*		**26.** £12 6*s.* $10\frac{3}{4}d.$
27. £23 9*s.* $1\frac{1}{2}d.$	**28.** £34 11*s.* 4*d.*		**29.** £45 13*s.* $6\frac{3}{4}d.$
30. £82 16*s.* $11\frac{1}{2}d.$	**31.** £97 18*s.* 7*d.*		**32.** £30 1*s.*
33. £36 0*s.* $11\frac{1}{4}d.$	**34.** £107 10*s.* $1\frac{1}{4}d.$		**35.** £58 19*s.* 10*d.*
36. £365 7*s.* $3\frac{1}{2}d.$	**37.** £574 9*s.* 6*d.*		**38.** £232 4*s.* $7\frac{3}{4}d.$
39. £393 18*s.* $9\frac{1}{4}d.$	**40.** £404 0*s.* $9\frac{1}{2}d.$		

VII a (p. 188).

	(*a*)	(*b*)	(*c*)	(*d*)
1.	£2 5*s.* $7\frac{3}{4}d.$	£1 4*s.* $4\frac{1}{4}d.$	£6 12*s.* 8*d.*	£4 0*s.* $6\frac{3}{4}d.$
2.	£28 7*s.* 7*d.*	£15 3*s.*	£82 9*s.* $6\frac{1}{2}d.$	£50 1*s.* $9\frac{1}{2}d.$
3.	£21 18*s.* $2\frac{1}{2}d.$	£11 13*s.* $11\frac{1}{4}d.$	£63 13*s.* $6\frac{1}{2}d.$	£38 13*s.* $5\frac{1}{4}d.$
4.	£15 7*s.* $10\frac{1}{4}d.$	£8 4*s.* $4\frac{1}{4}d.$	£44 14*s.* $8\frac{1}{4}d.$	£27 3*s.* $4\frac{1}{4}d.$
5.	£2 8*s.* $6\frac{1}{4}d.$	£1 5*s.* $10\frac{3}{4}d.$	£7 1*s.* $0\frac{1}{2}d.$	£4 5*s* 8*d.*
6.	£9 8*s.* $11\frac{1}{4}d.$	£5 0*s.* $10\frac{1}{2}d.$	£27 9*s.* $1\frac{3}{4}d$	£16 13*s.* 6*d.*
7.	£25 17*s.* $4\frac{1}{2}d.$	£13 16*s* $2\frac{1}{2}d.$	£75 3*s.* 8*d.*	£45 13*s.* $2\frac{1}{4}d.$
8.	£8 10*s.* 5*d.*	£4 10*s.* $11\frac{3}{4}d.$	£24 15*s.* $3\frac{1}{2}d.$	£15 0*s.* $9\frac{3}{4}d.$
9.	£25 17*s.* $6\frac{1}{2}d.$	£13 16*s.* $3\frac{1}{2}d.$	£75 4*s.* $1\frac{1}{2}d.$	£45 13*s.* $5\frac{3}{4}d.$
10.	£62 8*s.* $9\frac{1}{4}d.$	£33 6*s* 8*d.*	£181 9*s.* $3\frac{1}{4}d.$	£110 4*s.* $1\frac{1}{4}d.$
11.	£55 6*s.* $9\frac{3}{4}d.$	£29 10*s.* $10\frac{1}{2}d.$	£160 16*s* $8\frac{3}{4}d.$	£97 13*s.* 7*d.*
12.	£24 1*s.* $8\frac{1}{4}d.$	£12 17*s.* 2*d.*	£69 19*s.* 11*d*	£42 10*s.* $2\frac{1}{2}d.$
13.	£3 1*s.* 9*d.*	£1 12*s.* $11\frac{1}{2}d.$	£8 19*s.* 5*d.*	£5 8*s.* $11\frac{1}{2}d.$
14.	£2 12*s.* $1\frac{1}{2}d.$	£1 7*s.* $9\frac{3}{4}d.$	£7 11*s.* 6*d.*	£4 12*s.*
15.	£21 9*s.* 9*d.*	£11 9*s.* 5*d.*	£62 8*s.* $11\frac{3}{4}d.$	£37 18*s.* $6\frac{1}{4}d.$

	(*e*)	(*f*)	(*g*)	(*h*)
1.	£3 14*s.* $10\frac{1}{4}d.$	7*s.* $5\frac{3}{4}d.$	£14 19*s.* $5\frac{1}{4}d.$	£2 19*s.* $10\frac{1}{2}d.$
2.	£46 10*s.* 10*d.*	£4 3*s.* 1*d.*	£186 3*s.* $4\frac{1}{4}d.$	£37 4*s.* 8*d.*
3.	£35 18*s.* 8*d.*	£3 11*s.* $10\frac{1}{4}d.$	£143 14*s.* $7\frac{1}{2}d.$	£28 14*s.* 11*d.*
4.	£25 4*s.* $10\frac{1}{2}d.$	£2 10*s.* $5\frac{3}{4}d.$	£100 19*s.* $5\frac{3}{4}d.$	£20 3*s.* $10\frac{3}{4}d.$
5.	£3 19*s.* 7*d.*	7*s.* $11\frac{1}{2}d.$	£15 18*s.* $4\frac{1}{4}d$	£3 3*s.* $8\frac{1}{4}d.$
6.	£15 9*s.* $10\frac{1}{2}d.$	£1 10*s.* $11\frac{3}{4}d.$	£61 19*s.* 6*d.*	£12 7*s.* $10\frac{3}{4}d.$

	(e)	(f)	(g)	(h)
7.	£42 8*s*. 6¼*d*.	£4 4*s*. 10¼*d*.	£169 14*s*. 0¾*d*	£33 18*s*. 9¾*d*
8.	£13 19*s*. 6*d*.	£1 7*s*. 11½*d*.	£55 18*s*.	£11 3*s*. 7¼*d*.
9.	£42 8*s*. 9¼*d*.	£4 4*s*. 10½*d*.	£169 15*s*. 1½*d*.	£33 19*s*. 0¼*d*
10.	£102 8*s*.	£10 4*s*. 9½*d*.	£409 12*s*.	£81 18*s*. 4¾*d*.
11.	£90 15*s*. 2½*d*.	£9 1*s*. 6¼*d*.	£363 0*s*. 9½*d*.	£72 12*s*. 2*d*.
12.	£39 9*s*. 11¾*d*.	£3 19*s*.	£157 19*s*. 11*d*.	£31 11*s*. 11¾*d*.
13.	£5 1*s*. 3*d*.	10*s*. 1½*d*.	£20 5*s*.	£4 1*s*.
14.	£4 5*s*. 5¾*d*.	8*s*. 6½*d*.	£17 1*s*. 11½*d*.	£3 8*s*. 4½*d*.
15.	£35 4*s*. 9½*d*.	£3 10*s*. 5¾*d*.	£140 19*s*. 2½*d*.	£28 3*s*. 10*d*.
16.	£9 7*s*. 5*d*.	17. £15 16*s*. 4¾*d*.	18. £235 9*s*. 9¾*d*.	
19.	£37 19*s*. 5¼*d*.	20. £263 7*s*. 7¼*d*.		

VII b (p. 191).

1. £578 7*s*. 8½*d*.	2. £14030 18*s*. 9*d*.	
3. £5729 8*s*. 4*d*.	4. £3684 19*s*. 10½*d*.	
5. £11689 8*s*. 9½*d*.	6. £388 10*s*.	
7. £2521 13*s*. 4*d*.	8. £4324 4*s*. 0¾*d*.	
9. £2205 15*s*.	10. £14049 4*s*. 10½*d*.	
11. £5252 6*s* 10½*d*.	12. £3677 1*s*. 8*d*.	
13. £10 11*s*. 6*d*.	14. £8027 11*s*. 10½*d*.	
15. £527 19*s*. 8¼*d*.	16. £331 1*s* 1*d*.	
17. £1479 0*s*. 11½*d*.	18. £412799 17*s* 2*d*.	
19. £812 13*s*. 4*d*.	20. £5509 17*s*. 11*d*.	
21. 5413 qrs. 7 bus. 2 pks.	22. £1514 18*s*. 1½*d*.	
23. £3104 17*s*. 6*d*.	24. £2338415 19*s*. 9½*d*.	
25. $1537.	26. $67 20 c.	27. 348 fr. 89 c.
28. 449 tons.	29. 1064 fr. 6 c	30. 951 fr. 89 c.
31. 641 fr. 79 c.	32. 64 fr. 94 c.	33. $262 79 c.
34. $137 2 c.	35. $518 69 c.	36. $742 45 c.
37. M. 679 85 pf.	38. M. 2134 76 pf.	39. M. 6992 85 pf.
40. M. 567 44 pf.		

VII c (p. 194).

1. £11 0*s*. 10*d*.	2. £181 17*s*. 6*d*	3. £3 5*s* 10*d*.
4. £9 19*s*. 6*d*.	5. £15 10*s* 2*d*.	6. £12 13*s*.
7. £47 7*s*. 10*d*.	8. £14 16*s*. 9¼*d*.	9. £14 11*s*.
10. £48 11*s*. 5¼*d*.	11. £4485 19*s*. 6*d*	12. £29 6*s*. 6¾*d*.
13. £219 17*s*. 4*d*.	14. £283 6*s*.	15. £105 4*s*. 10½*d*.
16. £2624 11*s*. 9*d*.	17. £14 19*s*. 4*d*.	18. £49 13*s*. 10¼*d*.

19. £179 0*s.* $3\frac{3}{4}d.$ **20.** £3845 7*s.* $3\frac{1}{2}d.$ **21.** £2547 4*s.* $10\frac{1}{2}d.$
22. £362 9*s.* $9\frac{1}{2}d.$ **23.** £6405 14*s.* 9*d.* **24.** £140 2*s.* $9\frac{1}{4}d.$
25. £2848 11*s.* $5\frac{1}{2}d.$ **26.** £45 7*s.* $3\frac{1}{2}d.$ **27.** £435 1*s.* 4*d.*
28. £7588 5*s.* **29.** £211 10*s.* 7*d.* **30.** £393 0*s.* $11\frac{1}{4}d.$
31. £366 14*s.* $1\frac{1}{2}d.$ **32.** £292 12*s.* $3\frac{3}{4}d.$ **33.** £75 18*s.* $11\frac{1}{2}d.$

Oral Examples (p 196).

1. 18*s.* **2.** 4*s.* 6*d.* **3.** 28*s.* **4.** £5.
5. 1*s.* 8*d.* **6.** £5 19*s.* **7.** $3\frac{1}{2}d.$ **8.** 1*s.* 6*d.*
9. 1*s.* $3\frac{1}{4}d.$ **10.** 12*s.* **11.** 8*s.* **12.** 5*s.*
13. £18 10*s.* **14.** 1*s.* $3\frac{1}{4}d.$ **15.** £1 18*s.* **16.** 15*s.*
17. 9*s.* **18.** 15*s.* 9*d.* **19.** 5*s.* 6*d.* **20.** 1*s.* 3*d.*
21. £3 9*s.* **22.** 16*s.* **23.** 6*s.* $4\frac{1}{2}d.$ **24.** 15*s.*
25. 45*s.* **26.** £2. **27.** £5 14*s.* **28.** £1 16*s.*
29. £6 5*s.* **30.** 2*s.* 6*d.*

VII (p. 197).

1. £6296 14*s.* 9*d.* **2.** 99. **3.** £1095 5*s.* 10*d.*
4. £72. **5.** £1 6*s.* 8*d* **6.** £36 3*s.* 3*d.*
7. £73 0*s.* $10\frac{1}{2}d.$ **8.** £1806 4*s* 2*d.* **9.** 114.
10. £1037 2*s.* 8*d.* **11.** £16 17*s.* 5*d.* **12.** £12 17*s.* $6\frac{1}{2}d.$
13. £30291000. **14.** £12 12*s.* **15.** £768 14*s.* 9*d.*
16. £307 2*s.* 6*d.* **17.** £2418 17*s.* $3\frac{1}{2}d.$ **18.** £1590 0*s.* $2\frac{1}{4}d.$
19. £962 9*s.* $3\frac{3}{4}d.$ **20.** £150 10*s.* 6*d.* (nearest penny).

Revision Examples (p. 199).

1. $\frac{1}{8}$. **3.** £11 11*s.* 3*d.* **4.** 8 days. **5.** £62 0*s.* 8*d.*
6. £2284090 11*s.* $10\frac{1}{2}d.$ **8.** $1\frac{7}{15}$. **9.** 1*s.* $2\frac{3}{4}d.$; £ 81458 . . .
10. £43 18*s.* 4*d.* **11.** ·7576. **12.** $5\frac{1}{4}$ lb. **13.** £400 15*s.* 8*d.*
14. (i) $4\frac{29}{50}$, (ii) $5\frac{3}{5}$. **15.** £2 5*s.* $1\frac{1}{2}d.$ **16.** $82\frac{1}{2}$ ft. **17.** $\frac{4}{61}$.
19. £44 14*s.* 3*d.* **20.** $\frac{1}{28}$. **21.** £37 10*s.* **22.** $\frac{1}{3}$.
23. 561 kilos. **24.** 4350.
25. (i) 448854, (ii) 11 times, remainder 82645.
26. (i) $\frac{17}{36}$, (ii) $\frac{3}{4}$ **27.** £96 14*s.* 4*d.* **28.** ·080325.
29. $3\frac{1}{2}d.$ **30.** 180. **31.** £83 13*s.* $8\frac{1}{4}d.$ **32.** 59·73287.
33. 15*s.* 6*d.* **34.** $17 \times 13 \times 11 \times 7$; $17 \times 11^2 \times 5 \times 3 \times 2$; 187.
35. £3 5*s.* **36.** 4 hrs. 52 min.; 35 and 32 circuits.
37. 36524. **38.** $1\frac{9}{71}$. **39.** £65 4*s.* 6*d.* **40.** £38 15*s.* $6\frac{1}{2}d.$
41. $\frac{1}{12}$. **42.** 595. **43.** ·000001. **44.** £5 6*s.* 3*d.*

45. (i) $\frac{550}{7225}$, (ii) $\frac{70}{154}$. **46.** 804·67 metres. **47.** £434 14*s*.
48. 1. **49.** 8·697. **50.** (i) $\frac{10}{5247}$; (ii) ·05344.
51. 33·46 metres. **52.** 1350 pints. **53.** £7 4*s*. 9½*d*.
54. ·6703. **55.** 1·59. **56.** $\frac{1}{6354}$. **57.** 3*s*. 6½*d*. gain.
58. 1. **59.** £7 2*s*. 6*d*. **60.** 6¾ cwt.

Oral Examples (p. 205)

1. 42 sq. in.; 26 in. **2.** 24 sq. ft.; 22 ft.
3. 36 sq. cm.; 26 cm. **4.** 96 sq. yds.; 40 yds.
5. 66 sq. ch.; 34 ch. **6.** 70 sq. po.; 38 po.
7. 104 sq. in.; 3 ft. 6 in. **8.** 96 sq. in.; 4 ft 8 in.
9. 88 sq. yds.; 92 yds **10.** 60 sq. ft.; 12 yds. 2 ft.
11. 138 sq. cm.; 58 cm. **12.** 31 sq. dm.; 22·4 dm.
13. 48 sq. hm.; 3·2 km. **14.** 9 sq hm.; 12·2 hm.
15. 3·2 acres. **16.** 8·4 acres. **17** 6 acres. **18.** 120 acres.
19. 14 ares. **20.** 96 ares. **21.** 360 ares. **22.** 144 ares.
23. 4·9 acres. **24.** 4 in. **25.** ·9 cm. **26.** 4 ch.
27. 200 m. **28.** 6 ch. **29.** 1·2 cm. **30.** 200 m.
31. ·5 ch. **32.** 121 yds. **33.** (i) 84 sq. mm; (ii) ·84 sq. cm
34. $\frac{1}{100}$ sq. in. **35.** 1·56 sq. in.
37. (i) 6·4 ares. (ii) 86. (iii) 28. (iv) 8·4. **38.** 7 cm.
39. 3 ch.

VIII a (p. 211).

1. 18·1 sq. in. **2.** 1474 sq. in. **3.** 19·7 sq. m.
4. 36·98 sq. m. **5.** 24·4 sq. ch. **6.** 15·0 ares.
7. 1737 sq. cm. **8.** 1687 sq. in **9.** 421·1 sq. m.
10. 102·4 sq. cm. **11.** 52 sq. ft. 12 sq. in. **12.** 106⅔ sq. ft.
13. 4·68 acres. **14.** 6·65 acres.
15. 2251 sq. yds. 5 sq ft.; 190 yds.
16. (i) 192 sq. ft. (ii) 192 sq. ft. (iii) 616 sq. ft.
17. 792 sq. ft.; 642 sq. ft. **18.** 220½ sq. ft.
19. 983·0 sq. yds. **20.** 109¾ sq. ft.
21. 54 ft. by 30 ft.; 1620 sq. ft.; 468 sq. ft. **22.** 16½ sq. ft.

VIII b (p. 212).

1. 4·43 in. **2.** 5·49 cm. **3.** 11·27 in. (or 11·3).
4. 32·7 m. **5.** 14·4 Dm. **6.** 12·93 Dm.
7. 9·35 miles. **8.** 37·19 ch. **9.** 26·32 ch.
10. 5·10 cm. **11.** 18·1 yds. **12.** 7·244 yds.

13. 16·747 cm. **14.** 79·88 m. **15.** 7·85 Hm.
16. $88\frac{8}{9}$ ft. **17.** 118·4 ft. **18.** 124·2 yds.
19. $461\frac{5}{7}$ ft. **20.** 693 yds. 1 ft.

VIII c (p. 216).

(*a*) **1.** 7672·4. **2.** 31555. **3.** 89829. **4.** ·43185.
5. 55205. **6.** ·61215. **7.** 10·538. **8.** 128·99.
9. 18·949. **10.** 3068·5. **11.** 7·3598. **12.** 2·9182.
13. 80·806. **14.** 74·661. **15.** ·066769. **16.** 685·59.
17. 1·2512. **18.** ·0028871.

(*b*) **19.** 2183·7. **20.** 2862·3. **21.** 33280. **22.** 2·9343.
23. 3·4984. **24.** 1299·1. **25.** 390·76. **26.** 2787·8.
27. 7286·9. **28.** 21·391. **29.** 35 129. **30.** 854·7.
31. 1246500. **32.** 82029. **33.** 13·39. **34.** 2·493.
35. 772·55. **36.** 804·3. **37.** ·490. **38.** 6646.
39. 297·07. **40.** ·001639.

Oral Examples (p. 217).

1. 49 sq. in. **2.** 7 in.
3. (i) 11 ft. (ii) 12 ch. (iii) 6 cm. (iv) 1·3 yd. (v) 9 ch. (vi) 20 Dm. (vii) 40 mm. (viii) 18 ft. (ix) 1·8 ft (x) 180 yds.
4. (i) 8·5 cm. (ii) 12·3 ft. (iii) 8·3 yds. (iv) 27 ch. (v) 15 ch. (vi) 24 yds.
5. (i) 1. (ii) 100. (iii) 10000. (iv) ·01. (v) ·0001. (vi) ·000001.

Oral Examples (p. 219).

1. 3, 1. **2.** 3, 8. **3.** 3, 2. **4.** 2, 8. **5.** 2, 9.
6. 1, 1. **7.** 2, 1. **8.** 1, 4. **9.** 3, 4. **10.** 3, 1.
11. 1, 1. **12.** 3, 2. **13.** 1, 3. **14.** 3, 2. **15.** 2, 4.
16. 3, 1. **17.** 2, 4. **18.** 1, 7. **19.** 2, 2 **20.** 2, 7.
21. 3, 3. **22.** 3, 8. **23.** 2, 8. **24.** 2, 2. **25.** 1, 8.
26. 1, 8. **27.** 1, 2.

(p. 220.)

28. ·7. **29.** ·06. **30.** ·02. **31.** ·006. **32.** ·5.
33. ·1. **34.** ·7. **35.** ·002. **36.** ·0008. **37.** ·1.
38. ·04. **39.** ·01. **40.** ·2. **41.** ·06. **42.** ·02.
43. ·7. **44.** ·01. **45.** ·04.

VIII d (p. 226).

1 1245.	**2.** 23·4.	**3.** 7·29.	**4.** 7·08.
5. 888.	**6.** 90 9.	**7.** ·77.	**8.** ·654.
9. ·087.	**10.** 5·03.	**11.** ·099	**12.** ·0056
13. 32·128.	**14.** 27·133.	**15.** 28 371.	**16.** 13·266.
17. 1·8526.	**18.** ·074095.	**19.** 8·6602.	**20.** 7·8740.
21 1·7321	**22** ·044721	**23.** ·70711.	**24.** 2·6457.
25. 4 8396.	**26.** 24·642.	**27.** 9·2463.	**28.** 25·675.
29. 23 971	**30.** 1·6106.	**31.** 1 7725.	**32.** ·077215.
33. 020797	**34.** ·06576.	**35.** ·20797	**36.** ·002445
37. 6 135 m	**38.** 52 30 ch.	**39.** 17 54 yds.	**40.** 1193 7 m
41. 18·644 Hm	**42.** 5·787 m	**43.** 30 89 ch	**44.** 23·81 ch
45. 662 56 m			

Oral Examples (p. 227)

1 $\frac{5}{7}$.	**2.** $\frac{4}{5}$.	**3** $\frac{11}{12}$	**4.** $1\frac{1}{11}$	**5.** $1\frac{4}{5}$.
6 $1\frac{7}{4}$	**7.** $1\frac{1}{12}$	**8** $1\frac{1}{4}$	**9.** $1\frac{3}{7}$	**10** $3\frac{1}{3}$
11 $5\frac{1}{2}$.	**12.** $1\frac{5}{6}$.	**13.** $6\frac{1}{2}$	**14** $3\frac{1}{2}$	**15** $1\frac{4}{9}$

VIII e (p. 230)

1. 7144 cu in.	**2.** 525700 cu ft.
3. 33195 cu m	**4** 67220 cu m.
5. 17176 cu cm.	**6.** 10342 cu cm.
7. 560 cu in.	**8** 404 cu. yds. (to nearest whole number).
9 1260 litres	**10.** 23·4 litres.
11 3 589.	**12.** 574·6 litres.
13. 2 ft. 4 in.	**14.** 12·5 sq m.
15. 3 in	**16.** 2468 Kg

VIII (p. 232).

1. 614 ft.	**2.** 405
3 3128 litres	**4.** 170 fr. 50 c.
5. 11 pieces.	**6.** 218 fr. 51 c.
7 £2 2*s* 10*d*.	**8.** 772 oz.
9. £22 1*s*. 9*d*., £7 1*s*. 9*d*	**10.** 154 yds 2 ft.
11. 12 ft. 8 in.	**12** £700.
13. 121·7 m.	**14.** £61 12*s*.
15. £5 8*s*.	**16.** £6 18*s* 9*d*.
17. 3 hrs.	**18.** 810 cu. ft. 720 cu. ft. 90 cu. ft.

19. 1 hr. 5 min.
20. £6 10*s* 10*d*.
21. 300
22. 132 yds. ·2224 ac.
23. £208 0*s*. 6*d*
24. 76 tons nearly.
25. £11 15*s*. 6*d*
26. 112 sq. yds. £1 16*s*.
27. £3 0*s*. 8*d*
28. 13*s* $6\frac{1}{2}d$.
29. 141·4 ft
30 2 ft
31 1·7 metres
32. £405 6*s*
33. 22695 gr.
34. 5*s* 6*d*
35. 31·33 lb
36. 253 mm.
37. £3 14*s*. 3*d*
38 1557 gr.
39. 18150 gals
40 15·46 ac

Oral Examples (p 239)

1.		(*a*)	(*b*)	(*c*)	(*d*)
	(*a*)	$\frac{1}{3}$	$\frac{3}{35}$	6	$\frac{6}{11}$
	(*β*)	$\frac{7}{15}$	$\frac{1}{5}$	14	$\frac{14}{11}$
	(*γ*)	$\frac{14}{3}$	2	140	$\frac{140}{11}$
	(*δ*)	$\frac{1}{45}$	$\frac{1}{105}$	$\frac{2}{3}$	$\frac{2}{15}$

2.		(*a*)	(*b*)	(*c*)	(*d*)
	(*a*)	$\frac{2}{3}$	$\frac{2}{8}$	24	6
	(*β*)	$\frac{7}{15}$	$\frac{7}{24}$	$\frac{56}{5}$	$\frac{14}{4}$
	(*γ*)	$\frac{5}{6}$	$\frac{5}{8}$	$\frac{40}{3}$	$\frac{10}{3}$
	(*δ*)	$\frac{7}{114}$	$\frac{7}{152}$	$\frac{7}{8}$	$\frac{7}{56}$

3		(*a*)	(*b*)	(*c*)	(*d*)
	(*a*)	$\frac{1}{14}$	$\frac{2}{5}$	$\frac{10}{57}$	$\frac{2}{15}$
	(*β*)	$\frac{13}{112}$	$\frac{13}{20}$	$\frac{65}{57}$	$\frac{13}{15}$
	(*γ*)	$\frac{11}{148}$	$\frac{11}{20}$	$\frac{11}{114}$	$\frac{11}{15}$
	(*δ*)	$\frac{1}{4}$	$\frac{1}{5}$	$\frac{10}{57}$	$\frac{14}{15}$

4		(*a*)	(*b*)	(*c*)	(*d*)
	(*a*)	$\frac{1}{3}$	$\frac{1}{15}$	$\frac{2}{3}$	$\frac{520}{63}$
	(*β*)	$\frac{3}{16}$	$\frac{1}{80}$	$\frac{3}{32}$	$\frac{5}{4}$
	(*γ*)	$\frac{15}{4}$	$\frac{3}{4}$	$\frac{15}{8}$	$\frac{300}{7}$
	(*δ*)	$\frac{2}{3}$	$\frac{2}{45}$	$\frac{1}{3}$	$\frac{100}{63}$

5.	(*a*)	(*b*)	(*c*)	(*d*)
(*a*)	$\frac{1}{2}$	$\frac{5}{2}$	$\frac{3}{5}$	$\frac{5}{25}$
(*β*)	$\frac{50}{3}$	$\frac{250}{3}$	20	4
(*γ*)	$\frac{3}{16}$	$\frac{3}{2}$	$\frac{9}{25}$	$\frac{9}{125}$
(*δ*)	$\frac{5}{4}$	$\frac{25}{4}$	$\frac{5}{2}$	$\frac{5}{10}$

6.

	(*a*)	(*b*)	(*c*)	(*d*)
(*a*)	$\frac{7}{10}$	7	$\frac{7}{4}$	$\frac{7}{6}$
(*β*)	$\frac{4}{5}$	$\frac{2}{25}$	$\frac{10}{7}$	$\frac{4}{7}$
(*γ*)	$\frac{1}{20}$	$\frac{1}{2}$	$\frac{1}{56}$	$\frac{1}{12}$
(*δ*)	$\frac{1}{50}$	$\frac{1}{5}$	$\frac{3}{28}$	$\frac{1}{10}$

IX a (p. 242).

1. $\frac{3843}{71150}$. **2.** $\frac{105111}{821775}$. **3.** $\frac{42063}{77550}$. **4.** $\frac{2624}{4356}$.
5. $\frac{4892}{5816}$. **6.** $\frac{778}{427}$. **7.** $\frac{239}{671}$. **8.** $\frac{19}{36}$.
9. $\frac{37}{108}$. **10.** $\frac{69}{121}$. **11.** $\frac{1855}{14}$. **12.** $\frac{2158}{315}$.
13. $\frac{3}{4}$. **14.** $\frac{34911}{20674}$. **15.** $\frac{19}{32}$. **16.** $\frac{61}{55}$.
17. $\frac{4857}{7331}$. **18.** $\frac{107}{1220}$ **19** $\frac{169}{289}$ **20.** $\frac{53}{62}$
21. ·536. **22.** ·631. **23.** ·593. **24.** 1·14.
25. ·290. **26.** 1·19 **27.** ·167 **28.** ·359
29. ·736 **30.** 1·27 **31.** ·406. **32.** ·983.
33. ·813. **34.** ·767 **35.** ·682 **36** 0466.
37. ·490. **38.** ·694. **39.** ·908. **40** 685.
41. ·560. **42.** ·607. **43.** ·650

44. $4\cdot17 \times 10^{-5}$ **45.** $4\cdot995 \times 10^{-6}$ **46.** $4\cdot066 \times 10^{-4}$.
47. $9\cdot476 \times 10^{-6}$. **48.** $1\cdot46 \times 10^{-4}$. **49.** $1\cdot083 \times 10^{3}$
50. $6\cdot967 \times 10^{-7}$. **51.** $9\cdot328 \times 10^{-6}$ **52.** $8\cdot165 \times 10^{-7}$.
53. $1\cdot58 \times 10^{-5}$. **54.** $9\cdot47 \times 10^{-5}$ **55.** $2\cdot63 \times 10^{-6}$
56. $2\cdot728 \times 10^{-1}$. **57.** $9\cdot148 \times 10^{-3}$ **58.** $2\cdot56 \times 10^{-3}$.
59. $9\cdot37 \times 10^{-9}$. **60.** $3\cdot11 \times 10^{-10}$. **61** $3\cdot65 \times 10^{-6}$.
62. 3·649. **63** 3 641 **64.** ·4767
65. 1 157. **66.** ·9583 **67.** ·7729
68. 1·016. **69.** ·4027 **70.** 1·156.
71. ·1005 **72** ·6167.

Oral Examples (p. 248).

1.

	(*a*)	(*b*)	(*c*)	(*d*)
(*a*)	25	$12\frac{1}{2}$	20	$6\frac{1}{4}$
(*β*)	45	$22\frac{1}{2}$	36	$11\frac{1}{4}$
(*γ*)	100	50	80	25
(*δ*)	150	75	120	$37\frac{1}{2}$

2.

	(*a*)	(*b*)	(*c*)	(*d*)
(*a*)	50	40	32	$26\frac{2}{3}$
(*β*)	75	60	48	40
(*γ*)	100	80	64	$53\frac{1}{3}$
(*δ*)	$12\frac{1}{2}$	10	8	$6\frac{2}{3}$

3.

	(*a*)	(*b*)	(*c*)	(*d*)
(*α*)	50	40	80	20
(*β*)	25	20	40	10
(*γ*)	$12\frac{1}{2}$	10	20	5
(*δ*)	$87\frac{1}{2}$	70	140	35

4.

	(*a*)	(*b*)	(*c*)	(*d*)
(*α*)	15	30	12	10
(*β*)	11	22	8·8	$7\frac{1}{5}$
(*γ*)	16	32	12·8	$10\frac{2}{5}$
(*δ*)	$20\frac{1}{2}$	41	16 4	$13\frac{2}{5}$

5.

	(*a*)	(*b*)	(*c*)	(*d*)
(*α*)	2	1·6	1	4
(*β*)	18	14·4	9	3·6
(*γ*)	25	20	$12\frac{1}{2}$	5
(*δ*)	40	32	20	8

6

	(*a*)	(*b*)	(*c*)	(*d*)
(*α*)	6	3	2	1·5
(*β*)	21	10·5	7	5 25
(*γ*)	36	18	12	9
(*δ*)	96	48	32	24

	(*a*)	(*b*)	(*c*)	(*d*)
7.	8*s*	16*s*	£1.	£2.
8.	£1.	£2.	£2 10*s*	£5.
9.	£4.	£8	£10	£20.
10.	£5.	£10	£12 10*s*.	£25.
11.	£20	£40	£50.	£100.
12.	£16	£32	£40	£80.
13.	1 lb	2 lb	2 lb. 8 oz	5 lb.
14.	2 4″	4 8″.	6″.	1′.
15.	2 lk.	4 lk.	5 lk	10 lk
16.	20 c.	40 c	50 c.	1 fr.
17.	80 c.	1 fr. 60 c.	2 fr.	4 fr.
18.	45 c.	90 c.	1 fr. 12·5 c.	2 fr. 25 c.
19.	4 cm.	8 cm.	1 dm.	2 dm.
20.	9 cm.	18 cm.	2·25 dm	45 cm.
21.	25 m.	5 Dm.	6·25 Dm.	12·5 Dm.
22.	1 oz.	2 oz.	$2\frac{1}{2}$ oz.	5 oz.

	(*a*)	(*b*)	(*c*)	(*d*)
23.	3*s.*	6*s.*	7*s.* 6*d*	15*s.*
24.	5*d.*	10*d*	1*s.* 0½*d*	2*s.* 1*d.*
25.	10 yds.	20 yds	25 yds	50 yds

	(*e*)	(*f*)	(*g*)	(*h*)
7.	£2 8*s.*	£5	£10.	£15.
8.	£6.	£12 10*s*	£25.	£37 10*s*
9.	£24.	£50.	£100	£150.
10.	£30.	£62 10*s*	£125.	£187 10*s.*
11.	£120.	£250.	£500.	£750
12.	£96.	£200	£400.	£600
13.	6 lb.	12 lb. 8 oz.	25 lb.	37 lb 8 oz
14.	1′ 2·4″.	2′ 6″	5′.	7′ 6″.
15.	12 lk.	25 lk	50 lk	75 lk.
16.	1 fr. 20 c.	2 fr. 50 c	5 fr.	7 fr. 50 c
17	4 fr. 80 c	10 fr	20 fr.	30 fr
18	2 fr. 70 c	5 fr. 62½ c	11 fr. 25 c.	16 fr. 87 5 c.
19	24 cm.	5 dm	1 m.	15 dm.
20.	54 cm.	1 125 m.	2·25 m	3·375 m
21	15 Dm.	3·125 Hm.	6 25 Hm.	9 375 Hm
22.	6 oz.	12½ oz	1 lb. 9 oz	2 lb. 5½ oz
23.	18*s*	£1 17*s.* 6*d.*	£3 15*s*	£5 12*s* 6*d.*
24.	2*s.* 6*d*	5*s.* 2½*d*	10*s.* 5*d*	15*s* 7½*d*
25.	60 yds.	125 yds.	250 yds.	375 yds.

26. $\frac{1}{40}$. **27.** $\frac{1}{20}$ **28.** $\frac{1}{10}$ **29** $\frac{1}{5}$ **30.** $\frac{1}{4}$
31. $\frac{1}{4}$. **32.** $\frac{3}{10}$ **33** $\frac{3}{4}$ **34.** $1\frac{1}{4}$

IX b (p. 249).

1.	(*a*)	(*b*)	(*c*)	(*d*)
(*α*)	2·14	·989	·495	·227
(*β*)	3·10	1·43	·717	·329
(*γ*)	4·60	2 13	1·065	·488
(*δ*)	8 99	4 16	2·08	·953

2.	(*a*)	(*b*)	(*c*)	(*d*)
(*α*)	·128	3 46	3·42	1 28
(*β*)	·335	9·04	8·93	3·35
(*γ*)	1·83	49·4	48·8	18·3
(*δ*)	3·26	87·9	86·9	32·6

3.

	(*a*)	(*b*)	(*c*)	(*d*)
(*a*)	12·1	6·29	3·91	3·55
(*β*)	13·5	7·03	4·37	3·96
(*γ*)	24·4	12 7	7·91	7·16
(*δ*)	32 3	16 8	10·4	9 46

4.

	(*a*)	(*b*)	(*c*)	(*d*)
(*a*)	4	2·67	·681	·495
(*β*)	3·4	2·27	·579	·421
(*γ*)	6·75	4·5	1·15	·835
(*δ*)	·293	195	0498	·0362

5

	(*a*)	(*b*)	(*c*)	(*d*)
(*a*)	10·3	4·91	3 05	·652
(*β*)	15 8	7 57	4 70	1·00
(*γ*)	32·3	15 4	9·55	2·05
(*δ*)	90·9	43·5	27·0	5·77

	(*a*)	(*b*)	(*c*)
6.	£4 10*s.* 9¼*d*	£9 14*s.* 7¼*d.*	£14 5*s.* 5*d.*
7.	£106 8*s.* 1*d*	£228 0*s.* 2½*d*	£334 8*s.* 3½*d*
8.	£34 1*s.* 11¾*d*	£73 1*s.* 4½*d.*	£107 3*s.* 4½*d.*
9.	£60 7*s.* 2½*d*	£129 6*s.* 10*d.*	£189 14*s.* 0½*d*
10.	£13 16*s.* 3*d*	£29 11*s* 11½*d.*	£43 16*s.* 2½*d.*
11.	1 cwt. 2 qrs 1 lb.	3 cwt. 0 qr. 25 lb.	4 cwt. 2 qrs. 26 lb.
12.	2 cwt. 2 qrs 22 lb.	5 cwt. 3 qrs. 3 lb	8 cwt. 1 qr. 25 lb.
13.	2 qrs. 18 lb	1 cwt. 1 qr. 18 lb.	2 cwt. 0 qr. 7 lb.
14.	3 qrs. 24 lb.	2 cwt. 0 qr. 7 lb.	3 cwt. 0 qr. 3 lb.
15.	2·54 fr.	5 44 fr.	7·98 fr.
16.	4·36 fr.	9·35 fr.	13·72 fr.
17.	11·88 fr	25·46 fr.	37·33 fr.
18.	18·36 marks.	39·35 marks	57·72 marks.
19.	25·55 marks.	54·75 marks.	80·3 marks.
20.	23·46 marks.	50·27 marks.	73 73 marks.
21.	$2·53.	$5·43.	$7·96.
22.	$5·58.	$11·95.	$17·52.
23.	$23·44.	$50·22.	$73·66.
24.	1 215 m.	2 603 m.	3·817 m.
25.	16·03 m.	34·35 m.	50·38 m.
26.	189·07 m.	405·15 m.	594·22 m.
27.	30·56 l.	65·48 l.	96·03 l.
28.	61·01 Kg.	130·73 Kg.	191·73 Kg.

	(*d*)	(*e*)	(*f*)
6.	£29 16*s.* 9¼*d.*	£17 5*s.* 11¾*d.*	£37 18*s.* 11¾*d*
7.	£699 4*s.* 7½*d.*	£405 7*s.* 0¼*d.*	£889 4*s.* 9½*d.*
8.	£224 1*s.* 7*d.*	£129 18*s.*	£284 19*s* 4¾*d.*
9.	£396 13*s.*	£229 18*s.* 10*d.*	£504 8*s.* 8½*d.*
10.	£90 15*s.* 3¾*d.*	£52 12*s.* 4¼*d.*	£115 8*s.* 7½*d.*
11.	9 cwt. 3 qrs. 16 lb.	5 cwt. 2 qrs. 26 lb.	12 cwt. 2 qrs. 9 lb.
12.	17 cwt. 2 qrs. 24 lb.	10 cwt. 1 qr. 2 lb	22 cwt. 2 qrs. 2 lb.
13.	4 cwt 1 qr. 7 lb.	2 cwt. 2 qrs.	5 cwt. 1 qr. 26 lb.
14.	6 cwt. 1 qr. 8 lb.	3 cwt. 2 qrs 19 lb.	8 cwt 0 qr 5 lb
15.	16·68 fr.	9·67 fr.	21·21 fr.
16.	28·68 fr.	16·63 fr.	36·47 fr
17.	78·06 fr.	45 25 fr	99 27 fr.
18.	120·68 marks.	69·96 marks	153·47 marks
19	167·9 marks.	97·33 marks.	213·53 marks.
20.	154·17 marks.	89·37 marks.	196·06 marks.
21.	$16·65.	$9 65.	$21·18.
22.	$36·64	$21·24.	$46·60.
23	$154 01.	$89·28.	$195·86
24.	7·981 m.	4·627 m.	10·150 m.
25.	105·34 m	61·07 m.	133·97 m
26.	1242·46 m.	720·27 m.	1580 09 m.
27.	200·79 l.	116·4 l.	255·35 l
28.	400·89 Kg.	232·4 Kg.	509·83 Kg

29. 2·6 %. **30.** 8·5 %. **31** 77 % **32** ·66 %

33 49·8; 16·5, 19·4, 4·8, 9·5.

34. 0·8, 0 6; 0·6, 21·6; 2 6, 5·7, 3·4; 11 8, 1 7, 1·8; 0·5, 1·9, 3·1, 0·8; 1·0, 5·5; 10·2; 1·3, 3 9, 5·7, 0 2; 0 2, 0·2; 4·5; 10·4.

35.

United Kingdom	31·3	—	—	68·7.
France	42·1	25·8	32·1	57·9
Germany	55·7	15·4	28·9	44·3.
United States	28 0	19 5	52·5	72·0.

Oral Examples (p. 254).

	(*a*)	(*b*)	(*c*)	(*d*)
1.	£40.	£24.	£36.	£42.
2.	£80.	£48.	£72.	£84.
3.	£5.	£3.	£4 10*s.*	£5 5*s.*

	(a)	(b)	(c)	(d)
4.	£1.	12*s.*	18*s.*	£1 1*s.*
5.	1*s.* 8*d.*	1*s.*	1*s.* 6*d.*	1*s.* 9*d.*
6.	8*s.* 4*d.*	5*s.*	7*s.* 6*d.*	8*s.* 9*d.*
7.	2·2 Km.	1·32 Km.	1·98 Km.	2·31 Km.
8.	8 cm.	4·8 cm	7 2 cm.	8·4 cm.
9.	3 fr.	1 fr. 80 c.	2 fr. 70 c.	3 fr 15 c.
10.	$5.	$3.	$4 50 c	$5 25 c
11.	3 ft. 4 in.	2 ft	3 ft.	3 ft. 6 in.
12.	80 yds.	48 yds.	72 yds	84 yds
13.	5 fr. 60 c.	3 fr. 36 c.	5 fr. 4 c.	5 fr. 88 c.
14.	3 m.	1·8 m.	2 7 m	3·15 m.
15.	6 in.	3·6 in.	5·4 in	6·3 in.
16.	18 ft.	10 8 ft.	16·2 ft.	18·9 ft.
17.	20 lb.	12 lb	18 lb.	21 lb
18.	$2 20 c.	$1 32 c	$1 98 c	$2 31 c.
19.	3 fr. 20 c.	1 fr. 92 c.	2 fr. 88 c.	3 fr. 36 c
20.	5 m. 40 pf.	3 m. 24 pf.	4 m. 86 pf	5 m. 67 pf.

	(e)	(f)	(g)
1.	£32.	£50.	£54
2.	£64	£100.	£108.
3.	£4.	£6 5*s*	£6 15*s*
4.	16*s.*	£1 5*s.*	£1 7*s*
5.	1*s* 4*d.*	2*s.* 1*d.*	2*s.* 3*d*
6.	6*s.* 8*d.*	10*s* 5*d.*	11*s.* 3*d*
7.	1 76 Km.	2 75 Km.	2·97 Km.
8.	6·4 cm.	10 cm.	10·8 cm
9.	2 fr. 40 c.	3 fr. 75 c	4 fr. 5 c.
10	$4.	$6 25 c	$6 75 c
11.	2 ft. 8 in	4 ft. 2 in.	4 ft. 6 in.
12.	64 yds.	100 yds.	108 yds
13.	4 fr. 48 c.	7 fr.	7 fr. 56 c
14.	2·4 m.	3·75 m.	4·05 m.
15.	4·8 in.	7·5 in.	8·1 in
16.	14·4 ft.	22·5 ft.	24·3 ft.
17.	16 lb.	25 lb.	27 lb
18.	$1 76 c.	$2 75 c.	$2 97 c
19.	2 fr. 56 c.	4 fr.	4 fr. 32 c.
20.	4 m. 32 pf.	6 m. 75 pf.	7 m. 29 pf.

21. £12. **22.** £1000. **23.** £48
24. (i) 17*s.* 6*d.* (ii) £1 10*s.* (iii) 18*s* 9*d.* (iv) 100 lb. (v) 6 lb. (vi) 26 lb
25. (i) 5*s.* (ii) £1. (iii) 12 lb. (iv) 8 lb
26. (i) £1 2*s.* (ii) 1*s.* 9*d.* (iii) 240 yds. (iv) 760 yds.
27. (i) 6 days (ii) 1 day. (iii) 12 men.
28. (i) 37½ sec. (ii) 8 days. (iii) 60 days. (iv) Every 40 days
29. (i) £56. (ii) £24 10*s* (iii) £10000 (iv) £2857.

IX c (p. 255).

1. £15 15*s.* 8½*d* **2.** £3 7*s* 3¼*d* **3** 3 cwt. 3 qrs. 11 lb
4. 64 fr. 7 c. **5** 1 154 m **6.** 20 48
7. £1 0*s.* 3*d.* **8.** 2 tons 6 cwt 3 qrs.
9. 4 mks. 93 pf. **10.** 6*s.* 1½*d* **11.** 1 973 g
12. 38·38 cm. **13.** £3 7*s* 7*d* **14.** £57 10*s.* 4*d*
15. 39 fr. 52 c. **16.** 90·91 **17** £83 4*s.* 3*d*
18. £153,683 10*s.* 4*d* **19** 141 12 ft **20.** £980 2*s*
21. 19*s.* 1¼*d* **22.** £65,625. **23.** £24 10*s*
24. 1200 men **25.** £3 12*s* 11¾*d* **26** 323

IX d (p 262)

1. $\frac{3}{6}$, $\frac{4}{5}$, $\frac{2}{4}$ **2** $\frac{4}{5}$, $\frac{4}{7}$, $\frac{5}{9}$ **3** $\frac{6}{7}$, $\frac{5}{8}$, $\frac{5}{7}$
4. $\frac{2}{3}$, $\frac{5}{9}$, $\frac{3}{7}$. **5** $\frac{8}{11}$, $\frac{2}{13}$, $\frac{1}{8}$ **6** $\frac{7}{9}$, $\frac{4}{5}$, $\frac{5}{7}$.
7 (i) £14 19*s* (ii) £27 12*s* (iii) £42 11*s*
8. (i) £16 18*s.* (ii) £31 4*s.* (iii) £48 2*s*
9. (i) £1 9*s.* 4*d* (ii) £2 13*s.* 4*d* (iii) £2 9*s* 4*d*
(iv) Rs. 13·5. (v) Rs 20·25 (vi) Rs. 30
10. (i) £2 2*s* 9*d.* (ii) 50 fr 53 c (iii) £3 3*s* 4*d.*
(iv) 46 fr. 74 c
11. (i) 18·28 m. (ii) 19·68 yds (or 19 yds. 24·6 in.) (iii) 10·06 m
12. (i) 3·52 qts. (ii) 1·7 litres. (iii) 2·84 litres (iv) 4·54 litres

IX (p. 263)

1. ·35. **2.** 83·18; 65. **3.** £502 2*s.* 9*d*
4. ·515. **5.** $\frac{9}{14}$, 54$\frac{4}{9}$ miles. **6.** 187·3 yds.
7. £4 9*s.* 11¾*d.* **8.** 4·35 gr. **9.** 2200 yds
10. 48 mi. 480 yds **11.** $\frac{73}{35000}$, ·002086. **12.** 2·804 %.
13. ·275. **14.** 25,992,000 **15** £182 7*s.* 2*d.*
16. 45500. **17.** 63, 126, 189
18. 5,548,000 (to nearest thousand) **19.** £64 4*s*
20 £20 12*s.* 6*d.* **21** 11 yds **22** 12 min

23. 200 yds. **24.** 100 days. **25.** The 7 hens.
26. 96. **27.** £3 11*s.* 6*d* **28.** £17 9*s.* 6*d.*
29. 83·56; 14; 124. **30.** $3\frac{3}{4}$. **31.** £7 8*s.* 11*d.*
32. 30 men. **33.** 67, 13·6. **34.** ·90.
35. 5.10 p.m. **36.** $5\frac{1}{2}$ %.

X a (p. 274).

1 1*s* $4\frac{1}{2}$*d.* **2.** $11\frac{1}{2}$*d.* **3.** 25%.
4. £36 15*s.* **5.** £38 18*s.* 11*d* **6.** £27.
7. £68 19*s.* $1\frac{1}{2}$*d* **8.** £840 18*s.* 9*d* **9.** £67 10*s*
10. £12 0*s.* 10*d* **11.** $62\frac{1}{2}$ %.
12. £32 7*s.* 6*d*, £77 14*s.*; £84 3*s* 6*d* **13.** ·64 or 16 to 25.
14. 4 hrs 1 min. $22\frac{2}{7}$ sec **15.** £8 14*s* 2*d* **16.** £12 15*s.* 9*d.*
17. $4\frac{1}{2}$ %. **18.** 3*s.* $8\frac{1}{2}$*d.* **19.** £14 18*s* $2\frac{1}{4}$*d.*
20. $1\frac{1}{2}$ minutes. **21.** 180 men. **22.** £21 2*s* 9*d*
23. £2666 13*s.* 4*d*; £4000; £5333 6*s.* 8*d*
24. £2750, £2250; £1750. **25.** 28 days.
26. 953 37 gr. **27.** 120 miles. **28.** $16572\frac{1}{4}$ K
29. £60 6*s.* 3*d.* **30.** 17 3% gain **31.** £1083 15*s*
32. £2210 12*s* 3*d* **33.** £13 9*s.* $11\frac{1}{4}$*d*
34. Son £250 17*s* 6*d.*, daughter £167 5*s.*; grandchild £83 12*s* 6*d.*
35. 238 hours. **36.** £23 8*s.* 9*d.* **37** 36 miles
38 2329 gr. **39.** £6172 10*s* assets; £93 15*s.* loss.
40. £2 7*s.* 9*d.* **41.** £33 3*s.* 3*d* **42** Gain 42·5 %.
43. £25 8*s.* 9*d* **44.** £113 13*s* 1*d.* **45.** 10*s.* 3*d.*
46. $3\frac{1}{2}$ days. **47.** 564 Km. **48** £8 17*s.* 4*d.*
49. 14·3 % gain. **50.** £18 6*s.* $6\frac{3}{4}$*d.* **51.** 843 fr 75 c.
52. G.W.R. 17*s* $7\frac{1}{4}$*d*; Mid. £2 3*s.* 1*d.*, N E. $1\frac{3}{4}$*d.*; N.B. 17*s.* $2\frac{3}{4}$*d.*, Cal. 5*s.* 9*d*, Highland 16*s.* $1\frac{3}{4}$*d.*
53. 5660 whites. **54.** £968. **55.** $53\frac{1}{3}$ % gain.
56. £651. **57.** £4 8*s.* $6\frac{1}{2}$*d*; £7 3*s.* 9*d.*, £8 13*s.* $11\frac{1}{2}$*d*
58. 16 weeks. **59.** 6·4 c. **60.** 15 men.
61. 11.39 a.m. **62.** £2 12*s.* 9*d* **63.** £31 15*s* 3*d*
64. £3 per head. **65.** 9 % loss. **66.** £6 13*s.* 4*d.*
67. *A* £216; *B* £252; *C* £234; *D* £230 8*s*
68. £1445 6*s.* 8*d.* **69.** 6 days.
70. Son £1000; daughter £600; nephew £200.
71. French is the cheaper. It is 92·56 % of the English price.
72. Gold 60%; silver 16% **73.** £3 18*s.* 9*d.*
74. £524 5*s.* **75.** He gains 8 %. Total profit 266·66 fr.

76. £800. **77.** £5 1*s* 3*d* per case.
78. 24 sq. ft. **79.** £190 **80.** 16,000 men.
81. 2·15 % decrease. **82** 4·48 %. **83.** £98.
84. 175 men
85. The English bar. (The English stands 945 Kg. sq. cm.)
86. 1*s.* 5·4*d.* **87.** £53. 10 6 %. **88** 7·18 %. 640. **89.** 1½*d.*

X b (p 283).

1. 153·9 sq in. **2.** 45·83 sq. cm **3** 113·1 cub. in
4. 22 52 lb. per sq ft **5.** 1·24 sec **6.** 640 mm.
7. 15 tons. **8.** 1580 lb. **9** 4330 days.
10. 1611 ft. **11** 1·48 sec **12** 2073 g.
13. 15 2 sq. m. **14** 1064 times
15. It must be lengthened by 1·2″. **16.** ·488 *g*
17. 32 tons an hour 1*s.* 7·2*d* 6*s* 4 8*d*

XI a (p 288)

1. 8·67 lb , 146 3 lb. **2** 8·697 **3** 843 lb.
4. 6·543 cub m. **5.** 67 ft. per sec **6** 22 times.
7. 98·2 tons **8** 245½ cub m **9.** 3 2″.
10. 14·7 sq m **11** 481 lb. **12** 232 cub. m.
13. 4207 ft. **14** 10·4 miles. **15.** 80·1 ft.

XI b (p 292).

1. (*a*) 1,062,942 (*b*) 434,443. **2.** 16·4. **3.** 278,601.
4. 5 54 ; 6 8 12 hrs 14 min **5.** 32·96 **6.** 5*s.* 10*d.*
7. 7 min. 10 sec **8.** 1·72 m 16 days. ·106 in.
9. £2 1*s.* 2*d.* **10.** Apr 9. **11.** 1 hr. 9 min.

XI c (p 298).

1. 14 min 26 sec. ; 14 min. 20 sec., 12 min 50 sec.
2. 30·6 millions ; 38 millions. **3.** Period 1845–55.
4. £3 15*s.* 6*d.* **5.** July, 1906 ; July, 1910
6. (1) Dec 29th , (2) Dec. 13th ; (3) Dec. 22nd. **7.** 9·2 cub. cm.
8. ($v = 6$, $p = 45$) should probably be $v = 5{\cdot}56$ or $p = 41{\cdot}5$.
9. 3·2 $P = W + 290$. **10.** 70 sec.
11. £39,900,000. £42,120,000.
12. 300 yds. and 923 yds. from support.
13. Fare is 12·2 c. per mile (approx.).
14. $a = {\cdot}66$, $b = {\cdot}39$. $W = 7$.

XI d (p. 305).

	Perimeter.	Area.
1.	37·64 and 37·60 m	84·69 and 84·88 sq. m
2.	45·4 and 45·0 cm	119·8 and 117·6 sq. cm.
3.	788 and 784 ft.	38502 and 38109 sq. ft
4.	11·52 and 11·48 cm	8·213 and 8·156 sq. cm
5.	1274 and 1270 yds	100481 and 99845 sq. yds

6. 112·6 cub. cm **7.** ·039 %. **8** 104°.
9. 10·2 %. **10.** 9·23 %.
11. Correct to units figure, units figure, first decimal place respectively.
12. Area 3300 sq. cm.; error 14·54 sq. cm.
Volume between 11700 and 11400 cub. cm., error 166·176 cub. cm.
13. ·4912 lb.; ·00005 lb **14** ·582 %. **15** 29·6, 0·32 %.

Revision Papers (p. 307).

A. **1.** $1\frac{1}{3}$. **2.** 46 fr. 68 c. **3.** 11 7 5.3 11.7 5 2^3 385
4. 1568 fr. **5** 51233 Kg. **6.** £16 17*s*. 4*d*.; 29 % gain

B. **1.** ·02470. **2.** 32 **3.** 65·759 chains.
4. £738 15*s*., $49\frac{1}{4}$ % gain. **5.** 11 ft
6. (i) 133. 79. 108 80. (ii) 75·66 m

C **1.** (i) ·03. (ii) 4. (iii) 8·1. (iv) 1·6. (v) 2
2. 22508 fr. 16 c. **3.** 55 yds **4** 4′
5. 776 fr. 29 c. **6.** 85·29 gals.

D **1.** ·61479. **2.** ·57506 **3** 3·6 % **4** £63 16*s*.
5. 27·42 cm. **6.** 3756·9

E. **1.** 10*d*. **2** 2.3^3 5 7. $2^6.7.11$. $2^6.3^3$ 5 7 11
3. 7 ac. 3 ro. 15 po. **4.** 650 fr. 81 c.
5. Olives 35 lb. per acre, 8 %. Currants 2083 lb. per acre, 3 %.
Figs 1154 lb. per acre, 1 %. **6.** £8 12*s*. 8*d*.

F. **1.** ·0018. **2.** £12 12*s*. **3** £10 9*s* **4** 675 gals.
5. 10 hrs. 21 min. **6** Car 69·97 %; cycle 102·2 %.

G. **1.** 2183·6412. **2.** 1848. 21. 22. 24. 28. 33. 42. 44.
3. £196 2*s*. 5*d* **4.** 348·4 mi. **5** ·279 m **6** £1 12*s*. 6*d*

H **1.** 5·2793. **2.** £2 4*s*. $10\frac{3}{4}d$.
3. 2^2 $3^3.11^2.7^2$. (*a*) 3 sq. root 1386. (*b*) 3 sq. root 462.
4. 2*s*. $3\frac{1}{4}d$. £34 1*s*. 3*d*. **5.** 29·52 ch. (29 ch. 11·4 yds.).
6 164·9 ft.

I. **1.** ·7756. **2.** $\frac{7928}{137795}$. **3.** £619 6*s.* 10*d.*
4. £116 17*s.* 5*d.* **5.** $5\frac{1}{4}$ hrs. **6** £83 12*s.* 6*d*

J **1** 19380 **2.** $\frac{3}{105\frac{1}{2}}$ is the greatest $\frac{2\frac{1}{4}}{97\frac{5}{8}}$ is the least.
3. 80 parcels **4.** £36 15*s.* **5** £1224. **6** 237 yds.

K. **1** 75800. 00758. **2** 64 %. **3.** £399 5*s* 2*d*
4 3·8 years **5** £750 **6.** 81.

L. **1.** £7113 1*s.* **2.** $1\frac{8}{17}$. **3** £6 15*s.*
4. 605. **5.** £2166 0*s.* 5*d*
6. 1903, £2 15*s* per cwt.; 1904, £3 3*s.* per cwt, 1905, £2 17*s.* per cwt. 1905, quantity 9·4 % decrease, value 19 % decrease.

M **1.** (i) $\frac{7}{9}$. (ii) $3\frac{1}{7}$ **2.** (i) ·8109 (ii) $\frac{1}{9}$.
3. £288 13*s* 11*d.* **4.** £112 9*s.* 5*d.* **5** ·0253
6. 5 metres depth, 5 cub. metres more.

N **1.** 2·225 **2** 1008. **3.** £156 15*s.* $4\frac{1}{2}d$
4 50 yds **5.** 1627 fr 50 c. **6.** 320 yds.

O. **1** 1·382 **2** 15·811. **3.** £25 11*s* $10\frac{1}{2}d$
4. $12\frac{3}{4}d$. **5** £5760. **6.** 1·69 ac

P **1.** $34\frac{2}{3}$. **2.** £1959 3*s.* 7*d* **3** £2,833,248.
4. 13*s.* 4*d* **5** ·87 metre. **6** ·325 m.

Q. **1.** Greatest $\frac{31}{55}$. Least $\frac{17}{35}$.
2. (i) $2^2.3^3.7.11^2$. (ii) 21. (iii) 1386. **3.** 6·0828.
4. 31·28 bus. £196,608. **5.** £7475 10*s.* **6.** 22·0159 m.

R. **1.** ·1. **2.** 2520. 2700. 2880. **3.** 117138 tons.
4. 44 f s. **5.** 6 men **6.** 100.

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CHAPTER XII

LOGARITHMS

(159.) We have seen (§ 54) that any number such as 307·24 can be expressed in the form $3{\cdot}0724 \times 10^2$, i.e. in 'Standard Form', multiplied by some power of ten; similarly ·004623 may be written as $4{\cdot}623 \times 10^{-3}$.

It will be remembered also that, in seeking to obtain a meaning for 10^{-3}, we were guided by the consideration that such an expression had to obey the laws which we had already laid down for powers of ten with positive indices.

From the definition of a power we said that $10^2 \times 10^3 = 10^5$; and we then discovered the law: **The product of powers of ten is a power whose index is the sum of the original indices** (§ 17).

Extending this idea we found that $10^2 \times 10^0 = 10^{2+0} = 10^2$, so that 10^0 must mean 1.

Similarly $10^2 \times 10^{-2} = 10^0 = 1$ (as already found), so that 10^{-2} must represent some number which gives unity when multiplied by 10^2; in other words, 10^{-2} must mean $\frac{1}{10^2}$.

We thus obtained a meaning for any power of ten whose index is a whole number, positive, negative, or zero.

(160.) We are now going to extend the idea of a power still further and try to find a meaning for a power of ten whose index is a decimal.

We notice:

(1) That the numbers

	. . . . .	·001,	·01,	·1,	1,	10,	100, ...
or		10^{-3},	10^{-2},	10^{-1},	10^0,	10^1,	10^2

are increasing from smallness to greatness.

(2) That if x is negative and fairly large (numerically), then 10^x is very small.

(3) That if x is positive and fairly large, then 10^x is very large.

We can sum up these statements by saying that, as x increases from numerically large negative values to large positive values, 10^x increases from very small values to very large values, and is always positive.

It is reasonable, therefore, to suppose that, since $10^1 = 10$, and $10^2 = 100$, $10^{1\frac{1}{2}}$, *if it exists*, must be equal to some number between 10 and 100, and similarly that $10^{\frac{1}{2}}$ must lie between 10^0 and 10^1, i. e. between 1 and 10.

(**161.**) A full discussion of the interpretation and calculation of such fractional or decimal indices is beyond the scope of this work.

For present purposes we shall assume:

(1) That we can (by means of tables) find a value for 10^x, where x is any number, positive or negative, whole number or fractional.

(2) That as x increases from very large (numerically) negative values to very large positive values, 10^x increases gradually from small to large positive values, in accordance with § 160.

(3) That if x lies between 0 and 1, 10^x lies between 1 and 10 (§ 160).

Conversely we shall assume

(1) That if we have any positive number y, we can find a value of x such that $10^x = y$.

(2) That if y lies between 1 and 10 (i. e. is in standard form), then the value of x lies between 0 and 1 (i. e. is a simple decimal less than unity).

Graphical Examples.

1. Tabulate the values of 2^x when x is equal to -3, -2, -1, 0, 1, 2, 3, 4. Hence, draw the graph of $y = 2^x$ for this range of values of x, and obtain approximate values for $2^{\cdot 5}$, $2^{1\cdot 5}$,

$2^{2\cdot5}$, $2^{3\cdot5}$, $2^{-1\cdot5}$. Mention anything you notice about these values. It may help you if you calculate $\sqrt{2}$.

2. Repeat question 1, taking values of 3^x. As before, obtain approximate values of $3^{2\cdot5}$, $3^{1\cdot5}$, &c.

3. You know that $10^2 \times 10^3 = 10^5$. On the assumption that the law $10^a \times 10^b = 10^{(a+b)}$ is always true, state the value of the product $10^{\frac{1}{2}} \times 10^{\frac{1}{2}}$, i.e. $(10^{\frac{1}{2}})^2$.

Now calculate the values of $10^{\frac{1}{2}}$, $10^{\frac{1}{4}}$, $10^{\frac{1}{8}}$, $10^{\frac{1}{16}}$ (i.e. $10^{0\cdot5}$, $10^{0\cdot25}$, $10^{0\cdot125}$, $10^{0\cdot0625}$).

Also obtain $10^{0\cdot75}$ $(= 10^{0\cdot5} \times 10^{0\cdot25})$, $10^{0\cdot375}$, $10^{0\cdot1875}$.

Use these values to plot the graph of $y = 10^x$ for values of x between 0 and 1.

From the graph, estimate the values of $10^{0\cdot3}$, $10^{0\cdot4}$, $10^{0\cdot6}$, $10^{\frac{1}{3}}$.

(162.) Tables have been constructed giving the index of the power of ten corresponding to any number expressed in standard form.

The index is called the **Logarithm of the Number**, and will be found in a **Table of Logarithms.**

These indices or logarithms are given as decimals, and for most practical purposes it is sufficient to have them expressed correct to four figures.

In constructing the tables it is presumed that the number we are looking up is in standard form, and that the index or logarithm lies between 0 and 1 (see § 161); in printing the tables the decimal points are usually omitted.

Suppose the number whose logarithm we require is 6·749. In row 67 (the first two figures) under column 4 (the third figure) we find 8287.

To the right of the tables there is a second set of columns called 'difference columns'. In row 67, under difference column 9 (the fourth figure) we find 6. This is to be added to 8287, and we obtain 8293.

This tells us that the logarithm of 6·749 is ·8293, or that $6\cdot749 = 10^{\cdot8293}$.

'The logarithm of 6·749' is usually written 'log 6·749'.

Example:—*To find the logarithm of* 7·234.

Row 72, column 3, gives 8591.
Diff. column 4 gives 2.
Hence log 7·234 = ·8593.

In practice the addition is done mentally, and the logarithm written down at once.

If we have fewer than four figures given in the number, we may imagine zeros added. Thus the logarithm of 7·2 is found in row 72, column 0, to be ·8573.

Examples XII a.

(These may be taken orally if preferred.)

Write down the logarithms of the following numbers:

1. 4·693.	**2.** 8·397.	**3.** 2·349.	**4.** 5·038.
5. 4·386.	**6.** 4·908.	**7.** 1·212.	**8.** 3·002.
9. 4·69.	**10.** 5·03.	**11.** 7·9.	**12.** 3·4.
13. 9·807.	**14.** 2·101.	**15.** 3·595.	**16.** 9·1.
17. 5.	**18.** 6.	**19.** 9·9.	**20.** 4·444.

(The following should, at first, be done without the use of tables of 'anti-logarithms'.)

Find the numbers which have the following logarithms:

21. ·4839.	**22.** ·3054.	**23.** ·1369.	**24.** ·5049.
25. ·0076.	**26.** ·0853.	**27.** ·4030.	**28.** ·9863.
29. ·0004.	**30.** ·7069.	**31.** ·0435.	**32.** ·4920.
33. ·5911.	**34.** ·5555.	**35.** ·7028.	**36.** ·4069.

Fill in the gaps in the following

37. $4{\cdot}248 = 10^{\square}$. **38.** $6{\cdot}849 = 10^{\square}$.

39. $10^{\cdot 2474} =$. **40.** $10^{\cdot 6231} =$

41. $10^{\cdot 2474} \times 10^{\cdot 6231} = 10^{\square}$.

42. Use the result of **41** to find the product of the two numbers you found in **39** and **40**.

43. $3 = 10^{\square}$. **44.** $2 = 10^{\square}$. **45.** $6 = 10^{\square}$.

46. Can you see any connexion between the three logarithms you obtained in the last three questions?

47. $10^{\cdot 8247} =$ **48.** $10^{\cdot 4249} =$

49. $10^{\cdot 8247} \div 10^{\cdot 4249} = 10^{?}$.

50. Use the result of **49** to find the result of dividing the number you obtained in **47** by that obtained in **48**.

(**163.**) Suppose that we have found from the tables that the logarithm of 7·642 is ·8832.

This is only another way of saying that

$$7{\cdot}612 = 10^{\cdot 8832}.$$

But
$$764{\cdot}2 = 7{\cdot}642 \times 10^{2}$$
$$= 10^{\cdot 8832} \times 10^{2}$$
$$= 10^{2\cdot 8832}. \quad (\text{Just as } 10^{2} \times 10^{3} = 10^{5})$$

Similarly
$$764200 = 7{\cdot}642 \times 10^{5}$$
$$= 10^{5\cdot 8832}.$$

And
$$\cdot 007642 = 7{\cdot}642 \times 10^{-3}$$
$$= 10^{\cdot 8832} \times 10^{-3}$$
$$= 10^{(-3+\cdot 8832)}.$$

(**164.**) Notice that, provided we keep the same sequence of figures in the number, namely 7, 6, 4, 2, the decimal part of the index or logarithm remains unaltered. Hence, in the last case, it is convenient to leave the index in the form just written, without working out the subtraction. In other words, **we always keep the decimal part of the logarithm positive.**

In order to indicate that the whole number alone is negative, we place the minus sign *over* this number. Thus we write

$$\cdot 007642 = 10^{\bar{3}\cdot 8832}.$$

(**165.**) The logarithm, then, contains two distinct parts.

(1) The decimal part, which depends solely upon the sequence of figures in the number.

(2) The whole number, which depends upon the position of the decimal point (i. e. the position of the unit figure) in the original number.

The former is sometimes called the *mantissa* of the logarithm, and the latter the *characteristic*.

We saw that $\log 7{\cdot}642 = {\cdot}8832$ or $7{\cdot}642 = 10^{{\cdot}8832}$.

We can write down at once:

$\log 7642 = \log (7{\cdot}642 \times 10^3) = 3{\cdot}8832$ or $7642 = 10^{3{\cdot}8832}$.

Similarly $\log {\cdot}07642 = \bar{2}{\cdot}8832$ or ${\cdot}07642 = 10^{\bar{2}{\cdot}8832}$.

$\log {\cdot}000007642 = \bar{6}{\cdot}8832$ or ${\cdot}000007642 = 10^{\bar{6}{\cdot}8832}$.

(**166.**) To find a number whose logarithm is given we have to reverse the process of finding the logarithm of a number, remembering that the decimal part of the logarithm gives us the sequence of figures in the number, and the whole number in the logarithm indicates the position of the decimal point (i.e. the position of the unit figure) in the number.

Example. *Find the number whose logarithm is* $2{\cdot}7493$.

Referring to logarithm tables, we find 7490 (the nearest number in the tables less than 7493) under row 56, column 1. The extra 3 is found in the same row under the difference column 4.

The sequence of figures in the number is, therefore, 5614.

The whole number is 2, so we have to multiply the standard form $5{\cdot}614$ by 10^2, and we obtain $561{\cdot}4$ as the required number.

The student is left to discover the use of 'anti-logarithm tables'.

Examples XII b.

(These may be taken orally if preferred.)

Write down the whole numbers (i.e. the characteristics) in the logarithms of:

1. 42·62.	**2.** 703·8.	**3.** 4·897.	**4.** 864·7.
5. ·0492.	**6.** ·00073.	**7.** 39000.	**8.** ·5.
9. ·00836.			

Write down the logarithms of:

10. 36·8.	**11.** 203·9.	**12.** 674·8.	**13.** ·745.
14. ·08392.	**15.** ·9476.	**16.** 1·307.	**17.** ·0004.
18. 506·3.	**19.** 70000.	**20.** 62·7.	**21.** 3·541.

22. $2{\cdot}46 \times 10^5$. **23.** $3{\cdot}124 \times 10^7$. **24.** $5{\cdot}642 \times 10^{-6}$.

Write down the numbers which have the following logarithms. (You may use a table of anti-logarithms if you can discover the method of working with such a table.)

25. 1·4435.	**26.** 2·7947.	**27.** $\bar{1}$·4462.	**28.** ·8383.
29. 6·4231.	**30.** $\bar{4}$·8763.	**31.** 2·8417.	**32.** 3·0049.
33. ·8012.	**34.** ·0901.	**35.** 2·3021.	**36.** 1·3010.
37. ·4972.	**38.** 1·5461.	**39.** $\bar{3}$·8417.	**40.** $\bar{4}$·1020.
41. $\bar{3}$·1763.	**42.** $\bar{1}$·4192.		

Fill in the spaces denoted by an asterisk in the following:

43. $3{\cdot}476 = 10^{*}$.	**44.** $10^{1{\cdot}4221} = {}^{*}$.	**45.** $706{\cdot}9 = 10^{*}$
46. $\cdot00421 = 10^{*}$.	**47.** $10^{2{\cdot}4217} = {}^{*}$.	**48.** $10^{3{\cdot}972} =$

USE OF LOGARITHMS

(**167.**) *Multiplication.*

We wish to evaluate $734{\cdot}2 \times {\cdot}06417$.

From tables we find that $\log 734{\cdot}2 = 2{\cdot}8658$

$\log {\cdot}06417 = \bar{2}{\cdot}8074$

the sum of the logarithms $= \bar{1}{\cdot}6732$

$\therefore$ the expression $= 10^{2{\cdot}8658} \times 10^{\bar{2}{\cdot}8074}$

$= 10^{\bar{1}{\cdot}6732}$ (by addition of indices)

$= 47{\cdot}12$ (by reference to tables).

We have taken a special case, but the method is perfectly general, and can be applied to the continued product of several numbers. Hence

The logarithm of a continued product is the sum of the logarithms of the various factors.

[Algebraical Law:

$$\log(abc\ldots.) = \log a + \log b + \log c + \ldots..]$$

Example. *Find the value of* $637{\cdot}5 \times 72{\cdot}49 \times {\cdot}236 \times 3000$.

	$\log 637{\cdot}5 = 2{\cdot}8044$
	$\log 72{\cdot}49 = 1{\cdot}8602$
	$\log {\cdot}236 = \bar{1}{\cdot}3729$
	$\log 3000 = 3{\cdot}4771$
The expression = 32710000 approximately (or $3{\cdot}271 \times 10^7$).	$\therefore$ log product $= 7{\cdot}5146$

Note.—All logarithmic calculations may be performed by the aid of a mechanical device called a *Slide Rule.*

Students who possess slide rules are advised to check their results in the examples which follow by the use of these instruments.

Examples XII c.

Evaluate:

1. $(2{\cdot}321 \times 10^2) \times (3{\cdot}421 \times 10^{-3}) \times (4{\cdot}625 \times 10^1)$.
2. $(7{\cdot}849 \times 10^{-1}) \times (6{\cdot}483 \times 10^4) \times (7{\cdot}212 \times 10^2)$.
3. $(8{\cdot}512 \times 10^7) \times (5{\cdot}241 \times 10^{-1}) \times (6{\cdot}249 \times 10^{-4})$.
4. $8{\cdot}992 \times 10^6 \times 2{\cdot}417 \times 10^{-4} \times 3{\cdot}842 \times 10^2$.
5. $3{\cdot}246 \times 17{\cdot}49$.
6. ${\cdot}04632 \times 107{\cdot}3$.
7. ${\cdot}764 \times 32{\cdot}4 \times {\cdot}0061$.
8. $214{\cdot}3 \times 72{\cdot}6 \times {\cdot}000342$.
9. $849 \times {\cdot}0723 \times 106{\cdot}4$.
10. $42{\cdot}31 \times 3{\cdot}746 \times {\cdot}08429$.
11. $30700 \times 42{\cdot}3 \times {\cdot}0723$.
12. $53{\cdot}4 \times 207{\cdot}6 \times 1{\cdot}723 \times {\cdot}04261$.

(168.) *Division.*

Suppose we wish to evaluate $\dfrac{64{\cdot}32}{17{\cdot}61}$.

From the tables $64{\cdot}32 = 10^{1{\cdot}8083}$

$17{\cdot}61 = 10^{1{\cdot}2457}$

$\therefore \quad \dfrac{64{\cdot}32}{17{\cdot}61} = \dfrac{10^{1{\cdot}8083}}{10^{1{\cdot}2457}}$

$= 10^{0{\cdot}5626}$ (just as $10^5 \div 10^3 = 10^2$)

$= 3{\cdot}653$ (by reference to the tables).

So **the logarithm of a quotient is obtained by subtracting the logarithm of the denominator (or divisor) from the logarithm of the numerator (or dividend).**

(Algebraical Law: $\log \dfrac{a}{b} = \log a - \log b$.)

(**169.**) *Combined Multiplications and Divisions.*

Example. *Evaluate* $\dfrac{47{\cdot}34 \times 621{\cdot}4 \times {\cdot}00379}{45{\cdot}83 \times 7{\cdot}643}$.

$$\begin{aligned}
\log\ 47{\cdot}34 &= 1{\cdot}6753\\
\log\ 621{\cdot}4 &= 2{\cdot}7934\\
\log\ {\cdot}00379 &= \bar{3}{\cdot}5786\\
\therefore\ \log \text{ numr.} &= \overline{2{\cdot}0473}\\
\log\ 45{\cdot}83 &= 1{\cdot}6612\\
\log\ 7{\cdot}643 &= \ {\cdot}8833\\
\therefore\ \log \text{ denom.} &= \overline{2{\cdot}5445}\\
\therefore\ \log \text{ of expression} &= \bar{1}{\cdot}5028\\
\therefore\ \text{Expression} &= {\cdot}3183.
\end{aligned}$$

(*Note.*—In performing the final subtraction, we were subtracting a number (2·5445) from another smaller number (2·0473). We must *always* subtract the logarithm of the denominator from that of the numerator, since it is essential to keep the decimal part of the logarithm positive. Naturally, in a case such as this, the whole number will be negative.

If the student has any doubt as to the accuracy of the work, he will do well to check by adding together the last two logarithms.)

Examples XII d.

(Some of the simpler examples may be checked by ordinary division.)

Evaluate:

1. $(1{\cdot}027 \times 10^{3}) \div (3{\cdot}024 \times 10^{7})$.
2. $(4{\cdot}521 \times 10^{-2}) \div (7{\cdot}894 \times 10^{3})$.
3. $3{\cdot}725 \div 103{\cdot}4$.
4. $49{\cdot}61 \div 310{\cdot}9$.
5. ${\cdot}00526 \div {\cdot}0003912$.
6. $473{\cdot}2 \div {\cdot}05216$.
7. $\dfrac{3465 \times 403}{7328}$.
8. $\dfrac{{\cdot}0036 \times 579}{3{\cdot}004}$.
9. $\dfrac{54{\cdot}32 \times {\cdot}7291}{154}$.
10. $\dfrac{37{\cdot}45 \times 496{\cdot}2 \times {\cdot}01094}{73{\cdot}41 \times {\cdot}9101}$.
11. $\dfrac{307{\cdot}3 \times {\cdot}529 \times 6{\cdot}823}{54{\cdot}21 \times 3{\cdot}209}$.
12. $\dfrac{{\cdot}094 \times 1{\cdot}673 \times 7{\cdot}592}{3{\cdot}951 \times {\cdot}007}$.

13. $\dfrac{627 \times 443 \times 1{\cdot}22}{107{\cdot}9 \times 3007}$.

14. $\dfrac{1{\cdot}232 \times 4{\cdot}597 \times 1{\cdot}628 \times 10^5}{1{\cdot}443 \times 9{\cdot}392 \times 10^4}$.

15. $\dfrac{82{\cdot}7 \times (43{\cdot}4)^3}{6{\cdot}59 \times (17{\cdot}8)^2}$.

16. $\dfrac{(4{\cdot}84)^2 \times (3{\cdot}91)^3}{(8{\cdot}75)^5}$.

17. $\dfrac{417^2}{293 \times 719}$.

18. $\sqrt{53{\cdot}71} \times \sqrt[3]{44{\cdot}62}$.

(170.) *Powers.*

Remembering that $(43{\cdot}51)^3$ means

$$43{\cdot}51 \times 43{\cdot}51 \times 43{\cdot}51,$$

we see that the logarithm of $(43{\cdot}51)^3$

$= 1{\cdot}6386 + 1{\cdot}6386 + 1{\cdot}6386$ (using the tables)

$= 3 \times 1{\cdot}6386$

$= 4{\cdot}9158.$

$\therefore\ (43{\cdot}51)^3 = 82370$ approx.

Similarly any power with a positive integral index can be calculated.

E. g. $(1{\cdot}824)^5 = 20{\cdot}18.$

log 1 824 = ·2610

5

$\therefore$ log $(1{\cdot}824)^5$ = 1·3050

(Algebraical Law: $\log x^n = n \log x$.)

(171.) *Roots.*

Suppose we wish to calculate

$$\sqrt[5]{17{\cdot}45}.$$

We have to find a number x such that

$$17{\cdot}45 = x^5.$$

$\therefore\ \log 17{\cdot}45 = 5 \log x$ (§ 170).

$\therefore\ \log x = \frac{1}{5} . \log 17{\cdot}45.$ 5) 1·2418 (log 17·45)

$\therefore\ x = 1{\cdot}771.$ ·2483 (log x).

(Algebraical Law: $\log \sqrt[n]{x} = \dfrac{1}{n} . \log x.$

N.B. This is only an extension of Law III, for $x^{\frac{1}{n}}$ has the same meaning as $\sqrt[n]{x}$.)

Note.—We have not discussed the 'index laws' at all fully, but the line of argument we have followed suggests that $x^{\frac{1}{3}}$ is the same as $\sqrt[3]{x}$.

This is proved formally in most works on Algebra,[1] but for present purposes it is sufficient to realize that this is the meaning we must attach to such a fractional index in order to carry out logarithmic calculations correctly.

Similarly $x^{\frac{2}{3}}$ must mean $\sqrt[3]{x^2}$ or $(\sqrt[3]{x})^2$, and $x^{\frac{n}{m}}$ must mean $\sqrt[m]{x^n}$ or $(\sqrt[m]{x})^n$.

This implies that if we wish to calculate the value of a number such as $(62{\cdot}94)^{\frac{3}{5}}$, we must find the logarithm of 62·94, and then multiply by 3 and divide by 5, the order of these last two operations being immaterial.

(**172.**) A difficulty sometimes arises when we are dividing a logarithm with a negative characteristic.

E. g. we have to divide $3{\cdot}2471$ by 5.

The characteristic (whole number) is negative, and the mantissa (decimal part) is positive, so we must divide the two separately.

But in the result the negative part must be a whole number, and this will only be the case if the negative part before division is a multiple of the divisor 5.

In this case we regard $-3+{\cdot}2471$ as $-5+2{\cdot}2471$, and the division is then simple.

The result is $-1+{\cdot}4494$ or $\bar{1}{\cdot}4494$.

Example. *Find the value of* $\sqrt[3]{{\cdot}4796}$.

$$\log {\cdot}4796 = 1{\cdot}6808.$$

Regarding this as $-3+2{\cdot}6808$, and dividing by 3, we obtain, as the logarithm of the result,

$$\bar{1}{\cdot}8936.$$

The value is ·7827.

[1] See *School Algebra* (Clarendon Press), chap. xxi.

Examples XII e. (*Powers and Roots.*)

Evaluate

1. $4{\cdot}75^2 \times 1{\cdot}88^{\frac{1}{3}}$.
2. $\sqrt[3]{32{\cdot}74}$.
3. $\sqrt[4]{17{\cdot}69}$.
4. $\sqrt{{\cdot}2243}$.
5. $\sqrt[5]{{\cdot}02912}$.
6. $\sqrt{{\cdot}0000422}$.
7. $({\cdot}546)^{\frac{1}{3}}$.
8. $(43{\cdot}54)^{\frac{1}{2}} \times (6{\cdot}223)^{\frac{1}{3}}$.
9. $\sqrt{43{\cdot}4 \times 17{\cdot}2 \times 16{\cdot}9}$.
10. $\frac{1}{2}\sqrt{48{\cdot}4 \times 13{\cdot}6}$.
11. $\frac{1}{3}\sqrt{{\cdot}4462}$.
12. $(66{\cdot}71)^{\frac{2}{3}}$.
13. $(23{\cdot}5)^{\cdot 22}$.
14. $£1000 \times (1{\cdot}05)^{20}$. (log 1·05 correct to 7 figures is ·0211893.)
15. $£(1000 \div (1{\cdot}05)^{20})$.

(Give the results of 14 and 15 correct to the nearest £.) Say why it is necessary to give the logarithm to more than 4 figures. Can you suggest any arithmetical question which would give rise to such calculations as these?)

(173.) In working with logarithms it is most important to keep all the working as systematic as possible.

It is advisable to allow plenty of room in a margin, and then to keep all logarithms as side work so as to avoid interfering with the argument of the 'text'.

It is best to keep all logarithms in one vertical column, save when there are many powers or roots to be calculated, when two columns will be useful. E. g.

Example i. *Find the value of* $\dfrac{17{\cdot}6^2 \times {\cdot}3476 \times {\cdot}052^{\frac{1}{3}} \times 720}{\sqrt{84{\cdot}3} \times 5{\cdot}723 \times \sqrt[5]{{\cdot}7628}}$

log 17·6 = 1·2455
. log $(17{\cdot}6)^2$ = 2·4910
log ·3476 = $\bar{1}$·5411
log ·052 = 2·7160
. log $({\cdot}052)^{\frac{1}{3}}$ = $\bar{4}$·1480
log 720 = 2·8573
. log numerator = 1·0374
log 84·3 = 1·9258
.. log $\sqrt{84{\cdot}3}$ = ·9629
log 5·723 = ·7576
log ·7628 = $\bar{1}$·8825
∴ log $\sqrt[5]{{\cdot}7628}$ = $\bar{1}$·9765
∴ log denominator = 1·6970
∴ log of expression = $\bar{1}$·3404

Fraction = ·2190.

After some practice it is unnecessary to 'label' the logarithms in the margin. If the figures are written clearly and systematically, it will always be easy to retrace one's steps in the event of a mistake.

Example ii.

(The student is recommended to follow through the side-work carefully.)

$$\frac{43{\cdot}4^3 \times {\cdot}01702 \times \sqrt{307{\cdot}4} \times 450}{\sqrt[3]{109{\cdot}4} \times 8{\cdot}929 \times \sqrt[4]{{\cdot}892}}$$

1·6375	4 9125
	$\bar{2}$ 2309
2 4877	1 2438
	2 6532
	7 0404
2 0390	·6797
	·9508
$\bar{1}$·9504	1·9876
	1 6181
	5·4223

$$= 2{\cdot}644 \times 10^5$$

or 264400.

Examples XII f.

Evaluate:

1. $\dfrac{34{\cdot}2 \times \sqrt{507{\cdot}3} \times (23{\cdot}4)^2}{306{\cdot}9 \times \sqrt{43{\cdot}5} \times (1{\cdot}62)^3}$.

2. $\dfrac{{\cdot}000426 \times (1{\cdot}89)^4 \times \sqrt[3]{42{\cdot}51}}{63{\cdot}7 \times {\cdot}00587 \times (7{\cdot}62)^2}$.

3. $\dfrac{(5{\cdot}426)^{\frac{1}{5}} \times (108{\cdot}9)^{\frac{1}{3}} \times ({\cdot}0624)^{\frac{1}{2}}}{(58{\cdot}62)^{\frac{1}{4}} \times (5447)^{\frac{1}{5}}}$.

4. $\dfrac{\sqrt{{\cdot}5454} \times \sqrt[3]{{\cdot}07621} \times (4{\cdot}32)^3}{\sqrt[3]{{\cdot}6262} \times \sqrt{{\cdot}084} \times (1{\cdot}76)^2}$.

5. $\dfrac{\sqrt{89} \times \sqrt[4]{33{\cdot}44} \times \sqrt{{\cdot}449}}{\sqrt[3]{75} \times \sqrt{1111} \times \sqrt{111{\cdot}1}}$.

6. $\dfrac{508{\cdot}9 \times (32{\cdot}2)^{\frac{1}{2}} \times \frac{1}{2}(63{\cdot}2)^2}{\frac{1}{4}(44{\cdot}5)^{\frac{1}{4}} \times \frac{1}{3}(72{\cdot}3)^3}$.

7. The time of swing (t seconds) of a pendulum of length l-feet is given by the formula

$$t = 2\pi\sqrt{\frac{l}{g}},$$

where $\pi = 3{\cdot}142$ and $g = 32{\cdot}2$.

Find the time of swing of a pendulum of length 2 ft. 3 in., and the length of a pendulum which swings three times in a second.

8. Find the value of $\sqrt{s(s-a)(s-b)(s-c)}$, where $a = 32{\cdot}6$, $b = 43{\cdot}7$, $c = 65{\cdot}5$, and $s = \frac{1}{2}(a+b+c)$.

9. Find the length of an edge of a cube whose volume is 373 cu. in.

10. If an object of height h feet at a distance of d yards from an observer subtends a small angle $A°$ at the position of the observer, then roughly

$$h = \frac{3Ad}{57{\cdot}3}.$$

Find the height of a tower which subtends an angle of 9° at a point 170 yards away.

11. The volume of a sphere of radius a is given by the formula

$$V = \tfrac{4}{3}\pi a^3,$$

where V is the volume, and $\pi = 3{\cdot}142$.

Find the volume of a sphere whose radius is 3·22 cm.

12. If $CH = D^2_3 V^3$, find D when

$$C = 232{\cdot}4, \quad H = 16150, \quad V = 18{\cdot}38.$$

13. The efficiency of an electric motor is given by the formula

$$\frac{2\pi nT \times 746}{VC \times 12 \times 33{,}000},$$

where $\pi = 3{\cdot}142$, T inch-lbs. is the torque applied to determine the brake horse-power, n the number of revolutions per minute, V the number of volts, and C the current in amperes.

Find the efficiency, correct to 1 per cent., when

$$T = 32{\cdot}8, \quad n = 770, \quad C = 2{\cdot}97, \quad V = 164.$$

14. The distance d in miles of the visible horizon at a point h feet above the surface of the earth is given approximately by the formula

$$d = \sqrt{1{\cdot}5h}.$$

Find the distance visible from the top of a cliff 100 feet high.

15. In Giffard's Injector, if

Q = quantity of water injected in gallons per hour,

P = pressure of steam in atmospheres,

D = diameter of throat in inches,

$$Q = (63{\cdot}4D)^2 \times \sqrt{P}.$$

Find Q to the nearest gallon when $D = {\cdot}15$ and $P = 8{\cdot}5$.

16. Walker's formula for ventilating-fans is $C = 14{\cdot}3\sqrt{HD^4}$, where C = number of cubic feet of air delivered per second,

D = diameter of fan in feet,

H = horse-power required to drive the fan.

If $C = 860$, and $D = 16{\cdot}5$, find H.

17. If H is the horse-power wasted in air-friction when a disk d feet in diameter is revolving at n revolutions per minute, then

$$H = cd^{5\cdot5}n^{3\cdot5}.$$

If $H = 0{\cdot}1$ when $d = 4$, and $n = 500$, find the value of c.

Using this value of c, find the waste of horse-power in the case of a disk of diameter 5 feet, revolving at 580 revolutions per second.

CHAPTER XIII

THE METRIC SYSTEM OF MEASURES

(174.) In nearly all scientific work the units of measurement used are those of the Metric System. These units have been briefly described in an earlier chapter, but the relations between them are so important that we give here some detailed account of their derivations

The *METRE* was originally supposed to be a ten-millionth part of a quadrant of the earth's circumference measured along a meridian (or a 'line of longitude') so that the direct distance from a pole to a point on the equator, measured along the earth's surface, should be ten million metres or ten thousand kilometres.

The measurement was inexact when the unit was fixed upon, so that in practice the metre is now derived from a standard metre kept in the Palais des Archives at Paris.

(175.) The *ARE* (pronounced as 'AIR') is the unit of area as applied to the measurement of land. It is equal to the area of a square whose side is one dekametre.

The usual prefixes (deci-, centi-, &c.) can be applied to the are, but in practice the Hectare (Ha = 100 ares) and the centiare (ca = ·01 are) are alone used.

(176.) The *LITRE* is the measure of capacity for liquids or grain. It is equal to the volume of a cube whose edge is a decimetre.

(177.) The *GRAMME*[1] is defined as the weight of a cubic centimetre of pure water at a certain temperature (4° centigrade).

[1] The official spelling is *GRAMME* for the unit itself, and *KILOGRAM*, *DEKAGRAM*, &c., for the multiples and sub-multiples.

The corrections for variations of temperature and purity are small, and in all questions which are not required to a high degree of accuracy a cubic centimetre of water may be taken to weigh one gramme.

It should be noticed that a litre of water weighs a Kilogram.[1]

For large weights the *quintal* of 100 Kg. and the *tonne* of 1000 Kg. are sometimes used.

(**178.**) The *STERE* is a unit of volume which appears to be used only for the measurement of firewood. It is equal to a cubic metre.

The decastere and decistere are sometimes used.

(**179.**) The important facts to be remembered in connexion with the metric system are here summarized.

(1) The prefixes:

Kilo-	Hecto-	Deka-	deci-	centi-	milli-
1000	100	10	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$.

(2) an are = 1 sq. Dm.
a litre = 1 cu. dm.
a gramme = the weight of 1 cu. cm. of water.

Oral Examples on the relations between the various units in the Metric System.

How many ares are there in?—

1. 40 sq. Dm.
2. 7000 sq. m.
3. 1 sq. Hm.
4. 27·6 sq. Dm.
5. 605·9 sq. Hm.
6. ·4 sq. Km.
7. 403·5 sq. m.
8. 53000 sq. dm.
9. ·032 sq. Km.

Express in ares the areas of rectangles with the following sides:

10. 3 Dm., 5 Dm.
11. 40 m., 50 m.
12. 7 Dm., 2 Hm.
13. 4·3 Dm., 20 m.
14. 23 m., 20 m.
15. ·3 Km., 5 Hm.
16. 5·7 Hm., 10 Hm.
17. 53 Dm., 70 m.
18. 40 m. square.

[1] See note on previous page.

How many litres are there in ?—

19. 4 cu. m.	**20.** 530 cu. dm.	**21.** 3000 cu. cm.
22. 7·3 cu. m.	**23.** 57·6 cu. cm.	**24.** 507·8 cu. cm.

Express in litres the volumes of rectangular solids with the following sides:

25. 3 dm., 2 dm., 2 dm.	**26.** 20 cm., 70 cm., 20 cm.
27. 13 cm., 20 cm., 10 cm.	**28.** ·3 m., ·5 m., ·6 m.
29. 7 m., 8 m., 9 m.	**30.** 2·4 m., 3 m., 10 dm.

Give the weight (in grammes) of water which would fill vessels whose measures are as follows:

31. 532·3 cu. cm.	**32.** 17·62 cu. cm.	**33.** 1 cu. dm.
34. 2·7 litres.	**35.** 1·3 litres.	**36.** 17 litres.
37. 5·6 cu. dm.	**38.** 1 cu. m.	**39.** 1142 cu. cm.

(**180.**) In many questions on areas, approximate square roots have to be calculated. The following examples illustrate methods by which the necessary labour may be shortened.

It will be worth while to remember that

$$\sqrt{2} = 1{\cdot}414\ldots$$
$$\sqrt{3} = 1{\cdot}732\ldots$$

Example i. *Evaluate* (*a*) $\sqrt{32}$, (*b*) $\sqrt{27}$.

(*a*) $\sqrt{32} = \sqrt{16 \times 2}$
$= 4 \times \sqrt{2}$
$= 5{\cdot}656.$

(*b*) $\sqrt{27} = \sqrt{9 \times 3}$
$= 3\sqrt{3}$
$= 5{\cdot}196.$

Example ii. *Evaluate* $\dfrac{3}{\sqrt{2}}$.

Multiply numerator and denominator by $\sqrt{2}$.

We obtain
$$\frac{3}{\sqrt{2}} = \frac{3\sqrt{2}}{2}$$
$$= \frac{4{\cdot}242}{2}$$
$$= 2{\cdot}121.$$

Note.—Square roots which can only be evaluated approximately are called *surds*. For a complete discussion of the theory of surds the student is referred to an Algebra book.[1] The principles underlying the above calculations may be summarized as follows:

(1) When the number whose square root has to be evaluated contains obvious square factors the calculation can sometimes be simplified by expressing the number in terms of its factors.

(2) When a surd occurs in the denominator of a fraction, the denominator can be made *rational* by multiplying both numerator and denominator by that surd.

Oral Examples.

$\sqrt{2} = 1{\cdot}414.$ $\sqrt{3} = 1{\cdot}732.$ $\sqrt{5} = 2{\cdot}236.$ $\sqrt{7} = 2{\cdot}646.$
$\sqrt{10} = 3{\cdot}162.$

Evaluate

1. $\sqrt{50}$.	2. $\sqrt{63}$.	3. $\sqrt{98}$.	4. $\sqrt{147}$.
5. $\sqrt{20}$.	6. $\sqrt{45}$.	7. $\sqrt{162}$.	8. $\sqrt{40}$.
9. $\sqrt{72}$.	10. $\sqrt{108}$.	11. $\sqrt{700}$.	12. $\sqrt{1000}$.
13. $\sqrt{200}$.	14. $\sqrt{75}$.	15. $\sqrt{300}$.	16. $\sqrt{32}$.
17. $\sqrt{48}$.	18. $\sqrt{90}$.	19. $\sqrt{343}$.	20. $\sqrt{125}$.
21. $\sqrt{363}$.	22. $\sqrt{640}$.	23. $\sqrt{288}$.	24. $\sqrt{180}$.
25. $\frac{1}{\sqrt{2}}$.	26. $\frac{1}{\sqrt{3}}$.	27. $\frac{1}{\sqrt{5}}$.	28. $\frac{4}{\sqrt{2}}$.
29. $\frac{6}{\sqrt{3}}$.	30. $\frac{10}{\sqrt{5}}$.	31. $\frac{14}{\sqrt{7}}$.	32. $\frac{110}{\sqrt{10}}$.
33. $\frac{15}{\sqrt{3}}$.	34. $\frac{18}{\sqrt{2}}$.	35. $\frac{18}{\sqrt{3}}$.	36. $\frac{20}{\sqrt{10}}$.
37. $\frac{16}{\sqrt{2}}$.	38. $\frac{14}{\sqrt{2}}$.	39. $\frac{7}{\sqrt{7}}$.	40 $\frac{7}{\sqrt{10}}$.
41. $\frac{6}{\sqrt{10}}$.	42. $\frac{8}{\sqrt{2}}$.	43. $\frac{10}{\sqrt{3}}$.	44. $\frac{10}{\sqrt{7}}$.
45. $\frac{10}{\sqrt{8}}$.	46. $\frac{10}{\sqrt{12}}$.	47. $\frac{12}{\sqrt{10}}$.	48. $\frac{12}{\sqrt{12}}$.

[1] See *School Algebra*, Clarendon Press, § 191

SMALL QUANTITIES

(181.) If we are working a sum with the intention of obtaining an answer correct to a definite number of decimal places, e. g. 5 places, any quantity greater than ·000005 will affect the result when added or subtracted, but any smaller quantity will have no influence on the result and may be neglected.

A number less than unity becomes smaller still when squared or raised to a higher power; so that numbers occur which, while they affect the result themselves, have no effect when squared.

It is easily shown by algebraic division that

$$\frac{1}{1-x} = 1+x+x^2+ \ldots \text{ higher powers of } x,$$

$$\frac{1}{1+x} = 1-x+x^2-x^3+ \ldots \text{ higher powers of } x.$$

These results sometimes enable us to shorten calculation.

Example 1. *Find the value of* $\frac{1}{\cdot 997}$, *correct to* 4 *significant figures.*

There will be one figure before the decimal point, so we require three after it.

$$\begin{aligned}\frac{1}{\cdot 997} &= \frac{1}{1-\cdot 003}\\ &= 1+\cdot 003+\cdot 003^2+\cdot 003^3+ \ldots\\ &= 1+\cdot 003+\cdot 00009+\cdot 000000027+ \ldots\end{aligned}$$

As we require only three decimal places correct, the square and higher powers of ·003 may be neglected.

Hence $$\frac{1}{\cdot 997} = 1\cdot 003.$$

Again,

$(1+x)(1+y) = 1+x+y+xy$ and $(1-x)(1+y) = 1-x+y-xy.$

If x and y are small, it may happen that xy can be neglected, and then $(1+x)(1+y) = 1+x+y$ and $(1-x)(1+y) = 1-x+y$ to the required degree of accuracy.

Example ii. *Find the value of* $\frac{\cdot 994}{1\cdot 007}$ *to 3 decimal places.*

Here we may neglect all decimal numbers beginning with four noughts.

$$\frac{\cdot 994}{1\cdot 007} = (1-\cdot 006)(1-\cdot 007) \quad \text{since} \quad (\cdot 007)^2 = \cdot 000049$$
$$= 1-\cdot 007-\cdot 006 \quad ,, \quad \cdot 006\times\cdot 007 = \cdot 000042$$
$$= \cdot 987.$$

We have

$$(1+x)^2 = 1+2x+x^2; \qquad (1-x)^2 = 1-2x+x^2;$$
$$(1+x)^3 = 1+3x+3x^2+x^3; \qquad (1-x)^3 = 1-3x+3x^2-x^3.$$

If x is so small that x^2 and higher powers may be neglected, then $(1+x)^2 = 1+2x$ and $(1-x)^2 = 1-2x$;

and, if $3x^2$ may be neglected, then

$$(1+x)^3 = 1+3x \quad \text{and} \quad (1-x)^3 = 1-3x.$$

Example iii. *The side of a metal square expands from* 1 *ft. to* 1·000076 *ft., what is now the area of the square, correct to* 5 *decimal places?*

$$\text{Area} = (1+\cdot 000076)^2$$
$$= 1+2\times\cdot 000076 \quad \text{since } \cdot 000076^2 \text{ may be neglected.}$$
$$= 1\cdot 00015.$$

If $(1+x)^2$ may be taken as $1+2x$, then $1+x = \sqrt{1+2x}$. Put y in place of $2x$, and we have

$$\sqrt{1+y} = 1+\tfrac{1}{2}y \quad \text{if } \tfrac{1}{4}y^2 \text{ may be neglected.}$$
Similarly, $\sqrt{1-y} = 1-\tfrac{1}{2}y$,, ,, ,,
and $\sqrt[3]{1+y} = 1+\tfrac{1}{3}y$ if $\tfrac{1}{3}y^2$,, ,,
$\sqrt[3]{1-y} = 1-\tfrac{1}{3}y$,, ,, ,,

Example iv. *Find the cube root of* ·994 *to* 3 *decimal places.*

$$\sqrt[3]{\cdot 994} = \sqrt[3]{1-\cdot 006}$$
$$= 1-\cdot 002 \quad \text{since } (\cdot 006)^2 \text{ may be neglected.}$$
$$= \cdot 998.$$

Oral Examples.

Find the following results correct to 3 decimal places.

1. $\frac{1}{\cdot 993}$. 2. $\frac{1}{1\cdot 006}$. 3. $\frac{1}{\cdot 994}$. 4. $\frac{4}{\cdot 9993}$.

5. $\frac{2}{1\cdot 004}$. 6. $\frac{3}{1\cdot 005}$. 7. $\frac{1\cdot 005}{\cdot 993}$. 8. $\frac{1\cdot 006}{1\cdot 005}$.

9. $\frac{2\cdot 008}{\cdot 996}$. 10. $1\cdot 007 \times \cdot 996$. 11. $1\cdot 004 \times 1\cdot 003$.

12. $\cdot 998 \times \cdot 996$. 13. $(1\cdot 03)^2$. 14. $(1\cdot 02)^3$. 15. $(\cdot 998)^3$.

16. $\sqrt{1\cdot 004}$. 17. $\sqrt{\cdot 992}$. 18. $\sqrt{\cdot 988}$.

19. $\frac{1}{\sqrt{1\cdot 004}}$. 20. $\frac{1}{\sqrt{\cdot 992}}$. 21. $\frac{1}{\sqrt{\cdot 988}}$.

22. $\sqrt[3]{1\cdot 006}$. 23. $\sqrt[3]{\cdot 994}$. 24. $\sqrt[3]{(\cdot 994)^2}$.

(The remaining examples may be postponed until § 182 has been read.)

25. Find the area of a square whose side is 1·06 ft.

26. Find the area of a rectangle whose sides are 1·03 ft. and 1·07 ft.

27. Find the area of a rectangle whose sides are 1·04 ft. and ·97 ft.

28. What is the side of a square whose area is 1·08 sq. yds. ?

29. What is the length of a rectangle whose area is 1·07 sq. ft and breadth ·98 ft. ?

30. What is the volume of a rectangular block whose edges are 1·003 ft., 1·005 ft., ·993 ft. ?

31. What is the side of a cube whose volume is 1·009 cu. ft. ?

32. Find the side of a square whose area equals that of a rectangle 1·05 ft. by ·96 ft.

AREAS

Rectangle and Parallelogram.

(182.) The number of units of area in a rectangle or a parallelogram is the product of the numbers of units of length in the base and the height.

In the case of a rectangle the area can be found by simply counting systematically the number of squares representing units of area.

If the length is a inches and the height h inches, then there will be h rows of square inches in the area, and each row will contain a square inches.[1]

The area of a parallelogram is proved in most Geometry books to be equal to that of a rectangle having the same base and height.

Examples XIII a.

(All results are to be given to a reasonable degree of accuracy.)

1. A book has 325 pages (i.e. 163 sheets), each measuring 5·1 in. by 7·3 in. What area of paper is required for printing the book?

2. What is the total area of the walls of a room of which the dimensions are: height 11 ft., length 15 ft., breadth 12 ft.?

3. A rectangular plot of building land with a frontage of 34 ft. and a depth of 125 ft. was let on a building lease at the rate of £60 per acre. Find the rent of the plot to the nearest penny.

4. The area of a square field is 1 ac. 785 sq. yds. Find the cost of a fence round the field at 2*s.* 7*d.* per yard.

5. Find the cost of papering a room 14 metres long, 6·2 metres broad, and 4·25 metres high, with paper of which the breadth is 70 cm., and the price is 2 fr. per piece of 8 metres, 21 sq. metres being allowed for windows and doors.

(An exact number of pieces of paper must be purchased.)

[1] If h and a are not whole numbers of inches, we take a smaller unit (e.g. $\frac{1}{100}$ in.) in terms of which they may both be expressed as whole numbers. (See § 116.)

6. A building estate is to be let in plots so as to return a ground rent of £65 per acre. What will be the ground rent of a rectangular plot 49 ft. 6 in. wide and 286 ft. deep?

7. A roof 30 metres by 30 metres has to be covered with slates 24 cm. by 24 cm. If each slate overlaps the one immediately below to an extent of 12 cm., find how many slates will be required.

8. The floor of a room 6·8 metres by 4·2 metres is covered with rectangular blocks of wood 4 cm. by 6 cm. How many blocks will be required?

9. What will it cost to paint the inside and outside of an open cistern, fixed to the ground, 6 ft. long, 4 ft. deep, and $5\frac{1}{2}$ ft. wide, at 4*d.* per square foot?

10. A square field contains five acres. Find the length of its side, correct to the nearest foot.

11. A rectangular sheet of paper is $10\frac{1}{4}$ inches long and 8 inches broad. If a uniform border, one inch wide, is removed, what fraction of the original area is left?

12. Find the greatest possible error in the calculated area of a rectangular field whose sides measure 127 yds. 2 ft. and 289 yds. 1 ft. correct to the nearest foot Find also what percentage this error is of the calculated area.

13. A rectangular walled garden 30 yds. by 25 yds. has a border 5 ft. deep just inside the wall, and a path 4 ft. wide next to the border. Find the area of the path and of the border.

14. Calculate the length of a side of a square which shall have the same area as a parallelogram whose base is 17·3 cm. and height 14·2 cm.

15. In an ordnance map of 6 in. to a mile, a field is represented as a rectangle 4·3 in. long and 2·7 in. broad. How many acres does the field contain?

16. A piece of building land has a depth from the road of 310 ft. It is to be divided into a number of two-acre plots. What must be the length of frontage of each plot?

17. A 'tear-off' calendar for 1913 measures 3·97 in. by 2·54 in. What area of paper is required for its printing?

(Other examples on rectangles are given in Part I, p. 232.)

(183.) *Right-angled Triangles.* In many area problems we have to make use of the important fact about right-angled triangles, that **the square on the side opposite to the right angle is equal in area to the sum of the squares on the other two sides.**

(The proof is given in any book on Geometry.)

Since the number of units in the area of a square is obtained by 'squaring' the number of units of length in the side, we have the arithmetical fact that in a right-angled triangle whose sides are a in., b in., and c in. (c being the side opposite to the right angle)

$$c^2 = a^2 + b^2.$$

It is useful to notice that we can also put this fact in the form

$$a^2 = c^2 - b^2.$$

Example 1. *A rectangle has sides 7·3 in. and 4·7 in. What is the length of the diagonal?*

The square on diagonal $= 7{\cdot}3^2 + 4{\cdot}7^2$
$= 53{\cdot}29 + 22{\cdot}09$
$= 75{\cdot}38$ sq. in.

51 1
2 19
53 29

18·8
3 29
22 09

$\therefore$ Length of diagonal $= 8{\cdot}7$ in. approx.

166 | 75 38 | 8 69
11 38
1 42

Example 11. *The diagonal of a rectangular field is 12 ch. 22 links, and one side is 7 ch. 43 links. What is the length of the other side?*

The square on the unknown side
$= 12{\cdot}22^2 - 7{\cdot}43^2$
$= 19{\cdot}65 \times 4{\cdot}79$ sq. ch. (see § 88)
$= 94{\cdot}12$ sq. ch.

19 65
4·79
78 60
13 755
1 7685
94·12

$\therefore$ The length of this side
$= 9{\cdot}70$ ch.
or 9 ch. 70 lks.

187 | 94 12 | 9·70
13 12
3

Examples XIII b.

(Give all results to a reasonable degree of accuracy.)

Find the unknown side in each of the following right-angled triangles:

	Side opposite the right angle.	Other two sides	
1.		3·74 in	4·89 in
2.	14·63 cm	12·42 cm	
3.		6·42 ch	7·53 ch
4.	104·2 yds.	73·6 yds	

5. Find the length of the diagonal of a square whose side is 6·89in.

6. Obtain a formula giving the length of the diagonal (d) of a square in terms of the side (a).

7. Use the formula obtained in question 6 to find the diagonal of a square whose side is 72·3 cm., and the side of a square whose diagonal is 68·5 cm.

8. Find the height of an isosceles triangle in which the equal sides are each 4·64 in., and the base 6·42 in.

(The perpendicular from the vertex bisects the base.)

9. Find the height of an equilateral triangle whose side is 4·25 in.

10. Obtain a formula giving the height (h) of an equilateral triangle whose side is a.

11. Use the formula obtained in question 10 to find (i) the height of an equilateral triangle of side 7·62 ft., and (ii) the side of an equilateral triangle whose height is 23·7 in.

12. A rhombus[1] has its four equal sides each 7·95 in., and one diagonal is 11·32 in. Find the length of the other diagonal.

13. The diagonals of a rhombus are 4·84 cm. and 6·32 cm. Find the length of the side of the rhombus.

14. A circle of radius 6·4 in. is drawn. Calculate the distance from the centre of a chord 7·5 in. in length, and the length of a chord which is distant 1 in. from the centre.

[1] A rhombus is a parallelogram with all its sides equal; its diagonals are at right angles to one another.

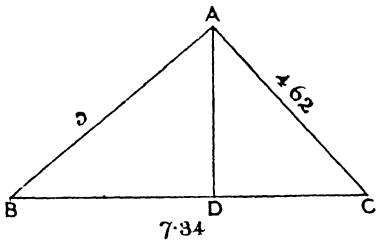
15. A triangle has two sides 3·47 in. and 5·43 in., and the perpendicular to the opposite side from the vertex in which those sides meet is 2·43 in. Calculate the length of the third side.

(184.) *Area of Triangle.* The area of a triangle is half the area of a rectangle with the same base and the same height. (See Geometry book.)

In order to be able to calculate the area of a triangle we must know the length of the perpendicular from one of the vertices to the opposite side.

If we are not given this length, we can at present only obtain it by drawing and measurement, but the following method is useful when we are given the lengths of the three sides.

Example. *Find the area of a triangle whose sides are* 7·34 *in.*, 4·62 *in.*, 5·49 *in.*



The perpendicular AD is drawn to the side BC.

(The perpendicular is drawn to the longest side for preference. Why?)

Let $BD = x$ in., and $DC = y$ in.

Then $AD^2 + BD^2 = AB^2$,

$AD^2 + DC^2 = AC^2$.

Subtracting, we get $BD^2 - DC^2 = AB^2 - AC^2$,

i. e. $x^2 - y^2 = 5{\cdot}49^2 - 4{\cdot}62^2$

$= 10{\cdot}11 \times {\cdot}87$. (See § 88.)

But $x+y = 7{\cdot}34$ in. ——————— (1)

And $\dfrac{x^2-y^2}{x+y} = x-y$

$\therefore \quad x-y = \dfrac{10{\cdot}11 \times {\cdot}87}{7{\cdot}34}$

$= 1{\cdot}198$ ——————— (2)

Logarithms.

1·0047
1 9395
·9442
8657
·0785

Adding (1) and (2) $2x = 8{\cdot}538$

$\therefore \quad x = 4{\cdot}27$ in.

But $AD^2 = AB^2 - BD^2$

$= 5{\cdot}49^2 - 4{\cdot}27^2$

$= 9{\cdot}76 \times 1{\cdot}22.$

9894
0864
2) 1·0758
·5379

$\therefore \quad AD = 3{\cdot}45$ in.

$\therefore$ Area $= \frac{1}{2} . 7{\cdot}34 . 3{\cdot}45$ sq. in.

$= 3{\cdot}67 \times 3{\cdot}45$ sq. in

$= 12{\cdot}67$ sq. in.

5379
·5647
1 1026

(**185.**) By carrying out the work of the previous section in the general case of a triangle whose sides are a, b, and c (expressed in terms of the same unit) we can obtain a formula for the area of a triangle in terms of the lengths of the sides.

With the figure of the previous article, let $AB = c$, $BC = a$, $CA = b$, and let BD and $DC = x$ and y as before.

Now prove the following:

(i) $x^2 - y^2 = c^2 - b^2$.

(ii) $x - y = \dfrac{c^2-b^2}{a}$.

(iii) $x = \dfrac{a^2+c^2-b^2}{2a}$.

(iv) $AD^2 = \dfrac{(a+b+c)(a-b+c)(a+b-c)(b+c-a)}{4a^2}$.

(v) Area $= \sqrt{s(s-a)(s-b)(s-c)}$, where $s = \frac{1}{2}(a+b+c)$.

Example. *Find the area of a triangle whose sides are*

$a = 7{\cdot}34$ *in.*, $b = 4{\cdot}62$ *in.*, $c = 5{\cdot}49$ *in.*

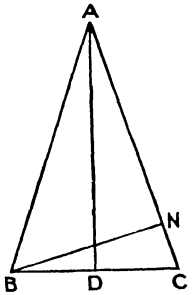
$$2s = a+b+c = 17{\cdot}45.$$
$$\therefore \quad s = 8{\cdot}725$$
$$s-a = 1{\cdot}385$$
$$s-b = 4{\cdot}105$$
$$s-c = 3{\cdot}235.$$

$$\therefore \quad \text{Area} = \sqrt{(8{\cdot}725)(1{\cdot}385)(4{\cdot}105)(3{\cdot}235)}$$
$$= 12{\cdot}67 \text{ sq. in}$$

9407
·1415
·6133
·5099
2) 2 2054
1·1027

(**186.**) *Isosceles and Equilateral Triangles.*

In the case of an isosceles triangle the perpendicular from the vertex to the base bisects the base. This fact will often help us to calculate the altitude.



Example i. *An isosceles triangle has its equal sides of length* 4·62 *in., and a base of* 2·84 *in. Find the area. Find also the length of the perpendicular from either extremity of the base to the opposite side.*

If AD is the altitude, we have in this case

$$BD = DC = 1{\cdot}42 \text{ in.}$$

But $AD^2 = AC^2 - DC^2$
$= 4{\cdot}62^2 - 1{\cdot}42^2$
$= 6{\cdot}04 \times 3{\cdot}20$
$= 19{\cdot}33.$
$\therefore \quad AD = 4{\cdot}40$ in.
$\therefore \quad$ Area $= \frac{1}{2}.\ 2{\cdot}84 \times 4{\cdot}4$ sq. in.
$= 6{\cdot}25$ sq. in.

18·12
1·208
19·33

84 | 19 33 | 4·4
3·33

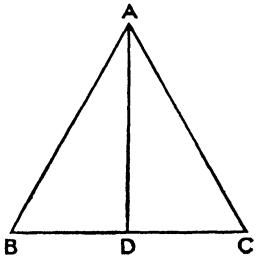
5 68
·568
6 25

If BN is the perpendicular from B to AC, we have

Area $= \frac{1}{2}.\ AC.\ BN$
$= 2{\cdot}31 \times BN.$
$\therefore \quad BN = \dfrac{6{\cdot}25}{2{\cdot}31}$ in.
$= 2{\cdot}70$ in.

2·71
2·31) 6 25
1·63
1

Example 11. *Find the area of an equilateral triangle whose side is* 42·7 *cm.*



$AB = 42{\cdot}7$ cm.
$BD = 21{\cdot}35$ cm.
$\therefore \quad AD^2 = 42{\cdot}7^2 - 21{\cdot}35^2$
$= 64{\cdot}05 \times 21{\cdot}35.$
$\therefore \quad AD = 36{\cdot}98$ cm.
$\therefore \quad$ Area $= \frac{1}{2}.\ 42{\cdot}7 \times 36{\cdot}98$
$= 21{\cdot}35 \times 36{\cdot}98$
$= 789{\cdot}4$ sq. cm.

Logarithms.
1·8065
1·3294
2) 3·1359
1·5679
1·3294
2·8973

(As an exercise the student should find for himself formulae expressing (i) the altitude of an equilateral triangle in terms of

a side (a); (ii) the area of an equilateral triangle in terms of a side (a). These will be useful formulae to remember.)

(187.) A *regular polygon* may be divided up into a number of isosceles triangles by lines drawn from its vertices to the centre of the circumscribing circle. These triangles will have the same height and equal bases, and the sum of the bases will be the perimeter of the polygon.

The area of the polygon is measured by the product —

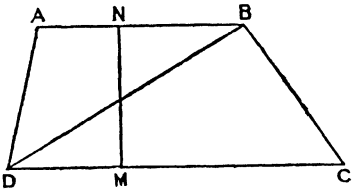
$\frac{1}{2}$ Perimeter $\times$ Perpendicular Distance from the centre to any one side.

In the general case of a regular polygon of any number of sides the calculation of the height of each triangle involves a knowledge of Trigonometry.

(188.) *Area of Quadrilateral* (irregular). The area of an irregular quadrilateral can usually be obtained by drawing a diagonal, and so dividing the figure into two triangles whose areas have to be determined separately.

The altitudes of these two triangles must be part of the data, or they can be obtained by drawing and measurement Where the lengths of the sides and of a diagonal are given, the altitudes may be obtained by the method of § 184.

(189.) *Area of Trapezium.* In the special case when two of the sides of the quadrilateral are parallel (such a quadrilateral is called a Trapezium), we can obtain a simple formula for the area.



$ABCD$ is the trapezium, in which AB and DC are parallel.

Drawing the diagonal BD, we divide the figure into two triangles.

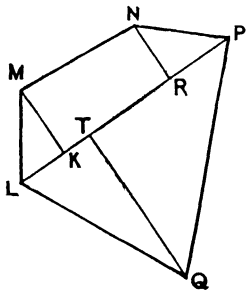
If any line NM is drawn perpendicular to the two parallel sides, this length will be the altitude (i. e. height) of each triangle.

Call this length h, and the sides AB and CD, a and b.

$$\text{The area} = \tfrac{1}{2} . ha + \tfrac{1}{2} hb$$
$$= \tfrac{1}{2} h(a+b).$$

Thus the area of a trapezium is measured by half the product of the height and the sum of the two parallel sides.

(190.) The method of dividing a figure up into triangles can be used for the calculation of the area of any figure bounded by straight lines, but the method tends to become cumbersome, and the following system has been adopted in working surveys of fields and other irregular portions of ground.



$LMNPQ$ is a plan of a figure whose area is required.

A 'base line' LP is taken, and the distances of the 'offsets' MK, QT, NR from this line to the angular points are measured. The distances of the points K, T, R along the line LP are also noted, and the whole is recorded in a 'field book' in the following manner. (The measurements are in chains.)

	L	
	1·34	2·25
4·42	2·61	
	3·28	2·20
	6·39	
	P	

The central column tells us the distances LK, LT, LR, LP, and the figures to the right or left give the lengths of the offsets drawn to one side or the other: thus $MK = 2{\cdot}25$ ch., $QT = 4{\cdot}42$ ch., $NR = 2{\cdot}20$ ch.

The figure is now made up of a series of triangles and trapeziums, and the area can be calculated as follows:

Area of triangle $LMK = \frac{1}{2} LK . MK.$
Area of $MKRN = \frac{1}{2} KR (MK + NR).$
" $NRP = \frac{1}{2} NR . RP.$
" $LQP = \frac{1}{2} LP . TQ.$

With a little care the work can be arranged very concisely as follows:

Area of LMK $= \frac{1}{2} . 1{\cdot}34 \times 2{\cdot}25$
$= {\cdot}67 \times 2{\cdot}25.$ 1·350
Area of $MKRN$ $= \frac{1}{2} . 1{\cdot}94 \times 4{\cdot}45$ ·1575
$= {\cdot}97 \times 4{\cdot}45.$ 4 005
·3115
Area of NRP $= \frac{1}{2} . 2{\cdot}20 \times 3{\cdot}11$ 3·11
$= 1{\cdot}10 \times 3{\cdot}11.$ ·311
Area of LQP $= \frac{1}{2} . 6{\cdot}39 \times 4{\cdot}42$ 12 78
$= 6{\cdot}39 \times 2{\cdot}21.$ 1·278
639
$\therefore$ Whole Area $= 23{\cdot}37$ sq. ch. 23·37
or $2{\cdot}34$ acres.

Examples XIII c.

(Give all results to a reasonable degree of accuracy.)

Find the areas of triangles drawn with the following data:

1. $BC = 7{\cdot}4$ in. Perp. from A to $BC = 4{\cdot}6$ in.
2. $BC = 6{\cdot}8$ in. $CA = 4{\cdot}3$ in. $CD = 3{\cdot}2$ in. (where D is the foot of the perpendicular from A to BC).
3. $AB = 8{\cdot}36$ in. $AC = 8{\cdot}36$ in. $BC = 4{\cdot}22$ in.
4. $\angle ABC = \angle ACB = 45°$. $BC = 6{\cdot}4$ in
5. $AB = BC = CA = 4{\cdot}73$ cm.

---

6. A triangle has an area of 47·6 sq. cm., and one side is 8·4 cm. What is the length of the perpendicular to that side from the opposite vertex?
7. An isosceles triangle has an area of 67·82 sq. in., and the base is 9·32 in. Find the length of the two equal sides.

8. The altitude of an equilateral triangle is 4·72 in. Find the length of a side and the area of the triangle.

Find the areas of triangles with the following data, using either the method of § 184 or the formula of § 185.

9. $a = 47{\cdot}34$ cm. $b = 26{\cdot}25$ cm. $c = 31{\cdot}34$ cm. Find also the lengths of the perpendiculars from the vertices to the opposite sides.

10. $a = 8$ in. $b = 9$ in. $c = 10$ in.

11. $a = 4{\cdot}34$ ch. $b = 2{\cdot}78$ ch. $c = 3{\cdot}92$ ch.

12. $a = 5{\cdot}41$ yds. $b = 4{\cdot}82$ yds. $c = 8{\cdot}87$ yds.

Draw quadrilaterals with the following data, and find in each case the area, making any measurements you think necessary. State what measurements you have made.

13. $AB = 4{\cdot}6$ cm. $BC = 5{\cdot}2$ cm. $CA = 6{\cdot}8$ cm. $CD = 4{\cdot}4$ cm. $DA = 5{\cdot}3$ cm. (B and D are on opposite sides of the line CA.)

14. $AB = 4{\cdot}3$ in. $ABC = 27°$. $BC = 2{\cdot}6$ in. $\angle BCD = 140°$. $CD = 3{\cdot}2$ in.

15. $PQ = QR = 7{\cdot}3$ cm. $\angle PQR = 65°$. $RS = PS = 8{\cdot}5$ cm.

Calculate the areas of the following quadrilaterals without drawing accurate figures.

(Rough figures will help you.)

16. $AB = AD = 8$ in. $BD = 7$ in. $BC = CD = 10$ in.

17. $AB = BC = CD = DA = CA = 6$ in.

18. $\angle PQR = \angle QRS = 90°$. $PQ = 3{\cdot}2$ in. $RS = 4{\cdot}7$ in. $QR = 3{\cdot}4$ in.

19. $\angle PQR = \angle QRS = 90°$. $PQ = 4{\cdot}6$ in. $PS = 7{\cdot}3$ in. $RS = 6{\cdot}5$ in.

20. Diagonal $BC = 6{\cdot}34$ ch. Offsets to the other two vertices are 4·27 ch., 2·86 ch.

(Give the answer in acres.)

Calculate the areas of portions of land represented by the following field-book entries, giving the results in acres, ares, or sq. yds., according to the units given.

21. Chains

	2·41	4·36
	3·88	2·24
2·60	5·40	
2·13	7·42	
	9·44	

22. Metres

	17·3	11·4
8·7	22·1	
	38·6	4·3
	51·4	

23. Yards

	18·4	6·5
17·6	32·2	
	55·8	20·3
	107·1	

24. Metres

18·8	7·6
22·4	8·3
37·6	7·4
48·5	2·3
65 0	

25. Yards

	14·7	2·3
	18·2	12·6
11·2	29·7	
	34·6	

26. Chains

	3·62	2·21
1·64	4·95	
3·41	7·62	
	10·54	

27. Metres

	16·8	7·5
6·2	23·1	
8·4	39·3	
	55·5	

28. Chains

	4·55	4·20
	8 73	2·54
3·50	10·49	
	11·72	

29. Calculate the area of a regular hexagon inscribed in a circle of radius 7·5 in.

30. Calculate the area of a regular hexagon circumscribed to a circle of radius 8·6 cm.

31. Obtain a formula for the area of a regular hexagon of side a.

32. Use the formula of question 31 to calculate the area of regular hexagons of sides (i) 4·3 in., (ii) 6·8 cm., (iii) 32 ft.

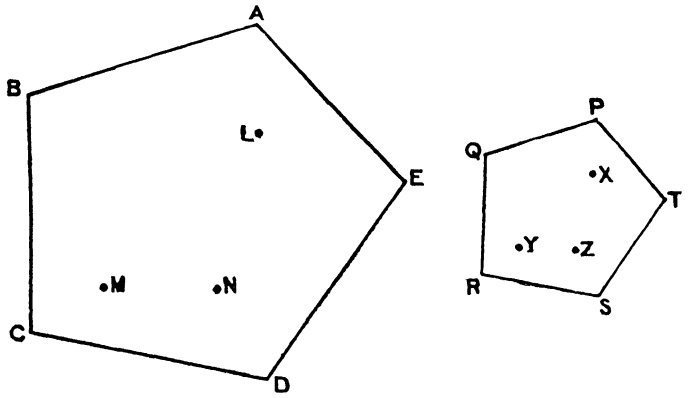
33. Use the formula of question 31 to calculate the lengths of the sides of regular hexagons whose areas are (i) 72·4 sq. in., (ii) 86·9 sq. cm., (iii) 100 sq. metres.

(191.) *Maps and Similar Figures.*[1]

If we have any figure drawn on a piece of paper, we can always draw another figure which will have the same *shape* but a different *size*

This new figure will be such that to every point in the original figure corresponds one point in the new figure, and the distance between two points in one figure will be a certain definite fraction of the distance between corresponding points in the other figure.

Two such figures are said to be similar, and one of them may be called a plan, or map, of the other.



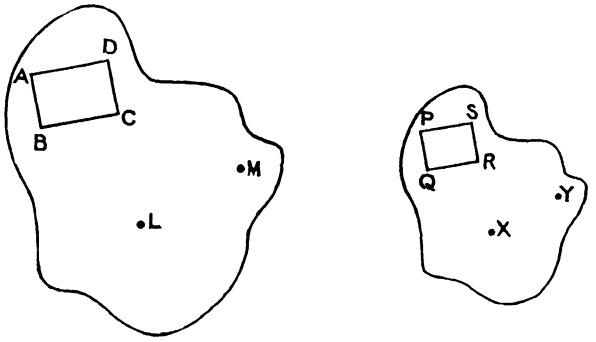
The figures *ABCDE* and *PQRST* are similar, and the points *A*, *B*, *C*, *D*, *E*, *L*, *M*, *N* correspond respectively to the points *P*, *Q*, *R*, *S*, *T*, *X*, *Y*, *Z*. Thus if *PQ* is $\frac{1}{2}$ *AB*, *QR* will be $\frac{1}{2}$ *BC*, *XY* will be $\frac{1}{2}$ *LM*, &c.

In the figures, as drawn, any line in one figure is parallel to the corresponding line in the other figure; but the figures might have been drawn in different positions, in which case we could say that the angle between any two lines in one figure would be equal to the angle between the corresponding lines in the other figure

[1] The student is referred to a Geometry book for a complete theory of similar figures.

(192.) Figures bounded by curved lines may be similar, lengths in one of the figures being a constant fraction of corresponding lengths in the other.

In the two figures drawn, each length in the smaller figure is two-thirds of the corresponding length in the larger.



Thus XY is $\frac{2}{3}$ of LM, PQ is $\frac{2}{3}$ of AB, QR is $\frac{2}{3}$ of BC.

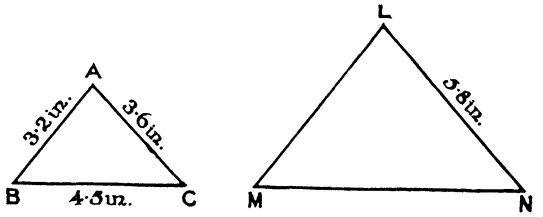
If the sides of the rectangle $ABCD$ are a in. and b in., then the sides of the rectangle $PQRS$ are $\frac{2}{3}a$ in. and $\frac{2}{3}b$ in.

Thus the area of $PQRS$ is $\frac{4}{9}ab$, or $\frac{4}{9}$ of the area of the rectangle $ABCD$.

This is only a special case of the general property which is most important in dealing with maps and similar figures.

If two figures are similar, then the ratio between corresponding areas is the square of the ratio between corresponding lengths.

Example i. *A triangle ABC has* $AB = 3{\cdot}2$ *in.,* $BC = 4{\cdot}5$ *in.,* $CA = 3{\cdot}6$ *in. The triangle LMN is similar to ABC, L and A, M and B, N and C being corresponding points, and* $NL = 5{\cdot}8$ *in. Find the lengths of LM and MN.*



$$\frac{LN}{AC} = \frac{5{\cdot}8}{3{\cdot}6} = \frac{29}{18}.$$

All other corresponding lines are in this same ratio.

$$\therefore \quad LM = \frac{29}{18} \text{ of } AB$$

$$= \frac{29}{18} \times 3{\cdot}2 \text{ in.}$$

$$= \frac{29}{9} \times 1{\cdot}6 \text{ in.}$$

$$= 5{\cdot}16 \text{ in.}$$

$$\begin{array}{r} 29 \\ 17{\cdot}4 \\ \hline 46\,4 \end{array}$$

$$\text{And} \quad MN = \frac{29}{18} \text{ of } BC$$

$$= \frac{29}{18} \times 4{\cdot}5 \text{ in.}$$

$$= \frac{29}{2} \times {\cdot}5 \text{ in.}$$

$$= 7{\cdot}25 \text{ in.}$$

Example 11. *A map is drawn on a scale of 3 in. to a mile. Find the true area (in acres) of an estate represented on the map by an area of 2·31 sq. in.*

A mile $= 1760 \times 3 \times 12$ inches.

$\therefore$ Actual distances are

$$\frac{1760 \times 3 \times 12}{3} \text{ of the map distances.}$$

$\therefore$ Actual area of this estate

$$= (1760 \times 12)^2 \times 2{\cdot}31 \text{ sq. in.}$$

$$= \frac{160 \quad 4 \quad \cdot 77}{1760 \times 1760 \times 12 \times 12 \times 2{\cdot}31} \text{ acres}$$
$$12^2 \cdot 640 \quad 44$$
$$3$$

$$= \frac{640 \times {\cdot}77}{3} \text{ acres}$$

$$= \frac{492{\cdot}8}{3} \text{ acres}$$

$$= 164{\cdot}3 \text{ acres.}$$

$$\begin{array}{r} 448 \\ 44{\cdot}8 \\ \hline 492{\cdot}8 \end{array}$$

Examples XIII d.

ABC, *DEF* are similar triangles, the points *A* and *D*, *B* and *E*, *C* and *F* corresponding. Calculate the lengths of the unknown sides in the following table:

	AB	*BC*	*CA*	*DE*	*EF*	*FD*
1.	5 in.	6 in	7 in.		3 in.	
2.	4 in.	5 in.	6 in.	7 in.		
3.			3 in.	8 in	9 in.	10 in.
4.	5 in.	7 in.		3 in.		10 in.

5. Two figures are similar, and a length of 3·6 cm. in the first is represented by 7·9 cm. in the other. The area of the first is 703 sq. cm. What is the area of the second?

6. The triangles *ABC* and *DEF* are similar, *AB* being 7·2 cm., and *DE* (the side corresponding to *AB*) being 11·6 cm. The area of *DEF* is 85·9 sq. cm. Find the area of *ABC*.

7. One figure has an area of 758 sq. cm., and a similar figure

has an area of 392 sq. cm. Find the length of a line in the second which corresponds to a length of 6·8 cm. in the first.

8. A map is drawn on a scale of $\frac{1}{2}$ in. to a mile. Find the length on the map representing a distance of 3 mi. 4 fur., and find the true area in acres of an estate represented on the map by an area of ·73 sq. in.

9. A rectangular field is represented in a plan by a rectangle whose sides are 1·03 in. and ·84 in respectively. Calculate the area of the field in acres and square yards if 1 in. on the plan represents 220 yds

10. A map is drawn on the scale of 6 in. to the mile. Find the size on the map (in square inches) of a field 12 acres in extent.

11. On a map, the scale of which is 1 2,500,000, the distance between Calais and Marseilles is 35·3 cm. What is the actual distance between the two towns in Km. ?

12. In a map drawn to a scale of 1/5280, a rectangular field is represented by a rectangle $4\frac{1}{2}$ in. by $3\frac{2}{3}$ in. What is the real area of the field in acres?

(193.) The areas of irregular figures can be found approximately, if the figures are drawn to scale on squared paper, by the simple process of counting the squares inside the area.

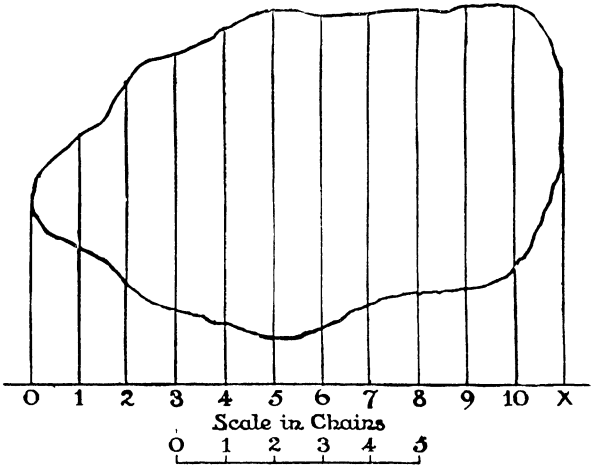
A fair degree of accuracy is obtained by reckoning any portion of a square of which the greater part is within the boundary as a whole square, and neglecting any portion of which the greater part is without the boundary

In estimating areas in this way, great care must be taken that the scales are correctly dealt with For example, if a map is drawn on the scale of an inch to a mile, a square inch on the diagram will represent a square mile of country, but if the scale is 2 inches to a mile, then 4 square inches on the diagram will represent a square mile of country.

Note.—The method can only be applied to maps of small countries or districts. In the case of a large country, the map will not give any idea of the true area, since the map is drawn on a flat surface, and the country itself is a curved surface. For countries whose greatest length does not exceed a few hundred miles, the method gives a result correct to about two significant figures.

(**194.**) An alternative method can be obtained by a modification of the method of § 190.

The figure represents the boundary of an estate, on the scale shown. It is required to find the area of the estate.



The base line OX is taken, and offsets are drawn from this base line at intervals each representing one chain. These offsets are then tabulated (1) for the upper part of the boundary, (2) for the lower part, as shown in the following table.

Distance along OX	0	1	2	3	4	5	6	7	8	9	10	11
Upper offset (1) . .	4	5·2	6·3	7·0	7·5	7·9	7·8	7·8	7·9	8	8	7
Lower offset (2) .	4	2·9	2·1	1·6	1·3	1·0	1·2	1·7	1·9	2	2·5	4·6
Difference of offsets	0	2·3	4·2	5·4	6·2	6·9	6·6	6·1	6·0	6	5·5	2·4

Now each of these portions of the figure is approximately a trapezium, and the height of each trapezium is one chain

$\therefore$ Area of the whole is

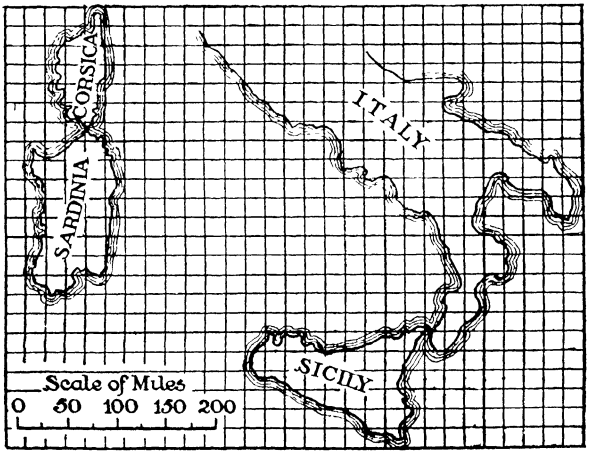
$$\tfrac{1}{2}\{(0+2\cdot3)+(2\cdot3+4\cdot2)+ \ldots + (5\cdot5+2\cdot4)\}$$
$$= \tfrac{1}{2}\{2(2\cdot3+4\cdot2+ \ldots 5\cdot5)+2\cdot4\}$$
$$= 55\cdot2+\tfrac{1}{2}(2\cdot4)$$
$$= 56\cdot4 \text{ sq. ch.}$$

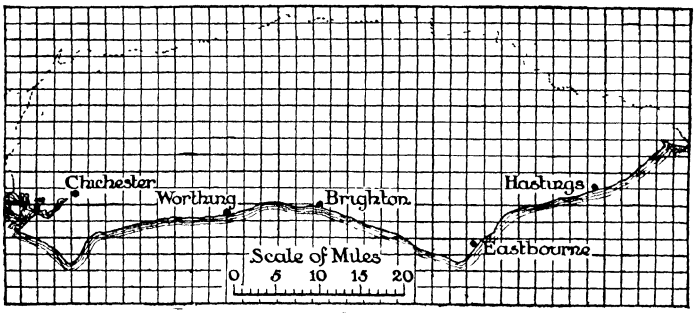
$\therefore$ The area is 56·4 sq. chains or 5·64 acres.

Examples XIII e.

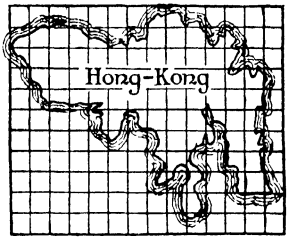
Find the areas of the following countries and counties as accurately as possible. (Do not give more than two significant figures.)

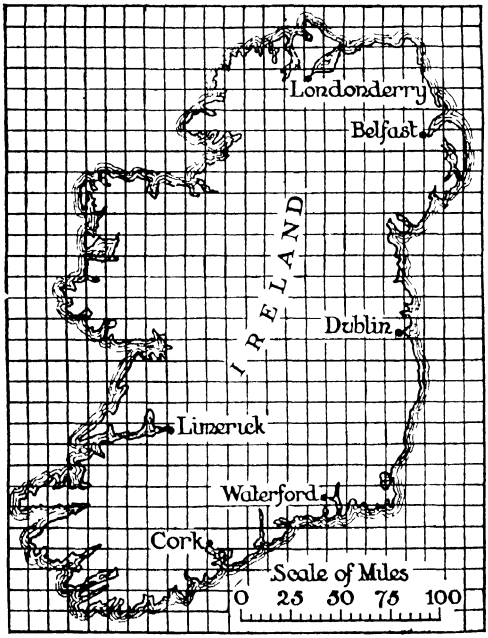
1. Sardinia.
2. Corsica.
3. Sicily.
4. Sussex.
5. Hong Kong.
6. Ireland.
7. The Island of Jersey.
8. The Isle of Wight.

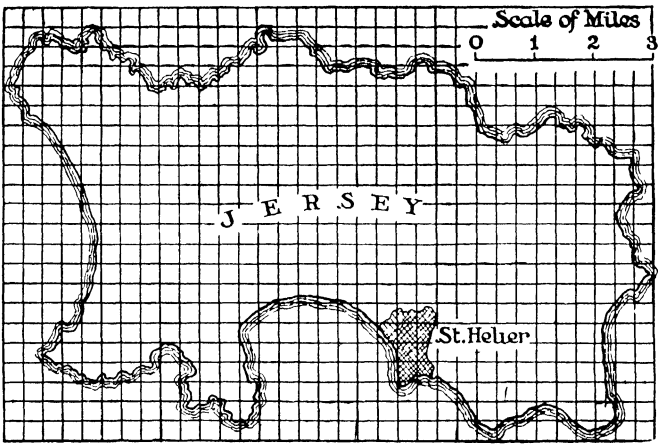


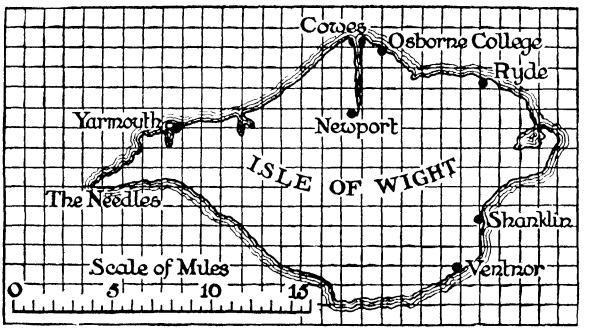


The County of Sussex


Hong-Kong
Scale of Miles
0 1 2 3 4 5 6 7


Londonderry
Belfast
IRELAND
Dublin
Limerick
Waterford
Cork
Scale of Miles
0 25 50 75 100





In the following examples a base line has been taken, and offsets measured at equal distances along the line. The lengths of the offsets to the boundary are on opposite sides of the base line unless otherwise stated.

9. Measurements in *chains*. Offsets 1 ch. apart. Find the area in *acres*

Distance along base line .	0	1	2	3	4	5	6	7	8	9	10	11
Offsets	0	1·2	3·4	4·6	5·7	7·2	4·3	3·1	2·2	2·2	1·7	0
	0	1·3	2·7	3·9	4·6	5·8	7·6	8·2	5·4	3·1	1·8	0

10. Measurements in *metres*. Offsets 10 m. apart Find the area in *ares*.

Distance along base line .	0	10	20	30	40	50	60	70	80	90	100
Offsets	0	7·6	10·4	15·8	18·3	22·4	17·6	14·3	8·7	2·5	0
	0	2·3	4·5	5·7	7·8	9·3	12·5	15·9	6·3	2·2	0

11 Measurements in *yards* Offsets 10 yds. apart, on the *same* side of the base line. Find the area in *acres*.

Distance along base line	0	10	20	30	40	50	60	70	80	90
Offsets	0	3·4	5·7	7·2	7·3	6·1	5·3	4·1	2·2	0
	4	17·2	35·1	49·8	85·1	99·3	102·2	65·5	43·4	3

12. A circle is drawn with radius 10 ft., and a diameter is taken as base line. Calculate the lengths of the offsets to the boundary from points 1 ft., 2 ft., 3 ft . . . 10 ft. along the diameter.

Hence calculate approximately the area of the circle.

The Circle.

(Some preliminary work in measuring the circumferences and diameters of coins, cylinders, &c., is advisable before the work of this section is attempted.)

(**195.**) If the circumference and the diameter of any circle are measured, the ratio between these quantities will be found to be approximately the same.

It is possible to prove theoretically that such a ratio is always exactly the same, but for the present we shall regard this as an experimental fact.

This ratio is usually denoted by the Greek letter π (pronounced 'pi'). Its value has been calculated by theoretical methods correct to many places of decimals; correct to six figures the value is 3·14159 . . . , but the approximation $\frac{22}{7}$ is sufficient for many purposes. It must not be forgotten, however, that when the value $\frac{22}{7}$ is used, the result is only approximate, and in working with decimals it is usually more convenient to use the value 3·14159 , since it shows more clearly the degree of accuracy attainable in the result.

If the circumference is c, and the radius r measured in terms of the same unit, we have then

$$\frac{c}{2r} = \pi,$$

$$\text{or} \quad c = 2\pi r.$$

Using the known value for π, we can now calculate the circumference of a circle whose radius is known, or the radius of a circle whose circumference is known.

Example i. *If a circle has a radius of* 3·45 *in., its circumference is*

$$\begin{aligned} &2\pi \times 3{\cdot}45 \text{ in.} \\ &= 6{\cdot}90 \times 3{\cdot}14159 \ldots . \text{ in.} \\ &= 21{\cdot}68 \text{ in.} \end{aligned}$$

20·70
·690
·276
7
3
21 68

Example ii. *If a circle has a circumference of* 83·49 *cm., its radius*

$$\begin{aligned} &= \frac{83{\cdot}49}{2\pi} \text{ cm.} \\ &= \frac{41{\cdot}745}{3{\cdot}14159 \ldots} \text{ cm.} \\ &= 13{\cdot}29 \text{ cm.} \end{aligned}$$

13 287
3·14159) 41 745
10 329
·903
275
24

(196.) *Length of an arc of a circle.*

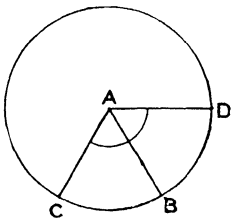
A glance at the figure shows that if the two angles CAB, BAD are equal, then the arcs CB and BD are equal.

(This is strictly a geometrical theorem, but it can be seen to be true by imagining the figure folded about the line AB.)

If the angle CAD is twice the angle CAB, then the length of the arc CD is twice the length of the arc CB.

If, then, we have two arcs in a circle, and if one of them subtends at the centre an angle which is a certain fraction (say $\frac{2}{3}$) of the angle similarly subtended by the other, then the length of the first arc is the same fraction ($\frac{2}{3}$ in this case) of the length of the other.

If, now, we have an arc subtending an angle of 70° at the centre of a circle, this angle is $\frac{70}{360}$ of the angle subtended at the centre by the whole circumference.



Therefore the arc is $\frac{70}{360}$ of the length of the circumference.

This method enables us to effect numerous calculations connected with arcs of circles.

Example 1. *Find the length of an arc which subtends an angle of* 105° *at the centre of a circle of radius* 42·3 *cm.*

The arc is $\frac{105}{360}$ of the whole circumference

$$= \frac{21}{36} \times 3{\cdot}1416 \times 42{\cdot}3 \text{ cm.}$$

$$= 7 \times {\cdot}2618 \times 42{\cdot}3 \text{ cm.}$$

$$= 1{\cdot}833 \times 42{\cdot}3 \text{ cm.}$$

$$= 77{\cdot}5 \text{ cm.}$$

42·3
1·833
42·3
33·84
1·27
·13
77·5

Example 11. *An arc 3·69 in. long is part of a circle of radius 2·83 in. What is the size of the angle subtended by this arc at the centre of the circle?*

The whole circumference $= 2\pi \times 2{\cdot}83$ in.

$\therefore$ The arc $= \dfrac{3{\cdot}69}{2\pi \times 2{\cdot}83}$ of the whole circumference.

$\therefore$ The angle $= \dfrac{3{\cdot}69}{2\pi \times 2{\cdot}83}$ of $360°$

$= 74{\cdot}69°$

$= 74° 41'$.

Logarithms
5670
2 5563
$\overline{3}{\cdot}1233$
·3010
·4972
·4518
$1{\cdot}8733$

Examples XIII f.

Find the lengths of the circumferences of circles whose radii are:

1. 4·34 in.	2. 3·71 in.	3. 93·5 cm.
4. 26·24 yds.	5. 18 yds. 2 ft.	6. 4 ch. 72 lk.

Find the radii of circles whose circumferences have the following lengths:

7. 34·6 in.	8. 43·52 yds.	9. 17·64 metres.
10. 106·5 cm.	11. 53 yds. 2 ft	12. 56 ch

Fill in the missing terms in the following table.

	Radius of circle.	Length of arc.	Angle subtended at centre by the arc.
13.	3·4 in.		40°
14.	5·92 in.		170°
15.	6·84 in		43° 30′
16.	46 cm.	73 cm	
17.	9·84 cm	17·62 cm	
18.		2·57 in.	32°
19.		6·14 ft.	230°
20.	8·54 ch	2·26 ch	

21. A reel contains 100 yards of cotton. If the diameter of the reel is 1 inch, find the number of coils on the reel.

(Neglect the thickness of the cotton.)

22. A bicycle has a 28-inch wheel. If six revolutions of the wheel turn the cog-wheel of a cyclometer through one revolution, find the number of revolutions the cog-wheel of the cyclometer must make to register a mile.

23. A motor bicycle geared to 1 in 7 is travelling at 25 miles an hour. The driving wheel is 28 in. in diameter. Find the number of revolutions the driving pulley makes per minute.

(A gear of 1 in 7 implies that the driving pulley makes 7 revolutions for one of the driving wheel.)

24. What length of hoop-iron is required to put six hoops on a water barrel whose average diameter is 2 ft. 4 in.? Allow 3 in. extra to each hoop for the iron to overlap.

25. In an ordinary bicycle 17 revolutions of the pedals carry the machine 100 yards. What is the gear of the bicycle?

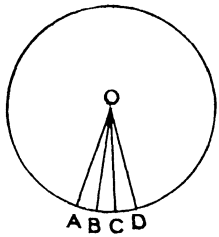
(A gear of 80 means that a single revolution of the pedals will carry the bicycle a distance equal to that traversed in a single revolution by a wheel of 80 in. diameter.)

26. A fly-wheel 3 ft. in diameter is making 50 revolutions a minute. What is the speed (in miles per hour) of any point on the rim?

27. A wire ring of 6 in. diameter is cut, and wrapped partially round a circular disk of 7 in. diameter. What angle will the wire subtend at the centre of the disk?

28. What is the angle subtended at the centre of a circle by an arc equal in length to the radius of the circle?

(**197.**) *Area of a circle and sector.*



Suppose a circle to be divided up into a large number of small sectors OAB, OBC, OCD . . . , &c.

Each of these sectors is very nearly a triangle, of which the base is nearly equal to the small arc of the sector, and the height is nearly equal to the radius.

The area of the circle is equal to the sum of the areas of all these

small sectors, i.e. to the area of a regular polygon with a large number of sides. (See § 187.)

It is then equal to

$$\begin{aligned} &\tfrac{1}{2}r.AB+\tfrac{1}{2}r.BC+\tfrac{1}{2}r.CD+\ldots\ldots\\ &=\tfrac{1}{2}r(AB+BC+CD+\quad..)\\ &=\tfrac{1}{2}r(\text{circumference})\\ &=\tfrac{1}{2}r\times 2\pi r. \end{aligned}$$

$\therefore$ **Area of circle** $= \pi r^2$.

(r is the radius.)

Similarly, *the area of a sector of a circle is* $\frac{1}{2}$ *radius* $\times$ *length of arc.*

Suppose that the arc is a fraction $\frac{x}{y}$ of the whole circumference (i.e. the angle which the arc subtends at the centre is $\frac{x}{y}$ of 360°)

Then the area of the sector is

$$\begin{aligned} &\tfrac{1}{2}r\times\tfrac{x}{y}\text{ of }2\pi r\\ &=\frac{x}{y}\text{ of }\pi r^2\\ &=\frac{x}{y}\text{ of area of circle} \end{aligned}$$

Hence the area of a sector can be calculated by the same 'ratio method' we adopted for the calculation of the length of an arc.

Example 1 *Find* (i) *the area of a circle of radius* 7·64 *cm., and* (ii) *the radius of a circle whose area is* 53·6 *sq. in.*

Logarithms

(i) The area $= \pi.(7{\cdot}64)^2$ — ·8831, 1·7662

$= 183{\cdot}4$ sq. cm. — ·4972[1], 2·2634

(ii) If the radius is r,

then $\pi r^2 = 53{\cdot}6$

$\therefore\ r^2 = \dfrac{53{\cdot}6}{\pi}$ — 1·7292, ·4972, 1·2320

or $r = \sqrt{\dfrac{53{\cdot}6}{\pi}}$ — ·6160

$= 4{\cdot}13$ in.

[1] It is useful to remember the value of log π.

Example ii. *Find the area of a sector of a circle of radius 3·5 ft. if the angle of the sector is 76°.*

The sector $= \frac{76}{360}$ of whole circle
$= \frac{76}{360}$ of $\pi . (3{\cdot}5)^2$
$= 8{\cdot}13$ sq. ft.

5441 | 1 8808 / ·4972 / 1 0882 / 3·4662 / 2·5563 / ·9099

Example iii. *10 sq. ft. of metal are to be cut (as a slice or sector) from a circular disk 10 ft. in diameter. What will be the angle of the slice?*

The area of the disk is $\pi . 5^2$ sq. ft.

$\frac{2}{\cancel{10}\ 5\pi}$ of the whole is cut out.

$\therefore$ Angle $= \frac{2}{5\pi}$ of 360°

$= \frac{144}{\pi}$

$= 45{\cdot}84°$

$= 45°\ 50'$ approx.

3 14159) 144 (45·84 / 18·34 / 2 63 / 12

(**198.**) A *segment of a circle* is a portion of a circle bounded by an arc and the chord joining its extremities. Its area is the difference between the areas of a sector and the triangle bounded by two radii and the chord of the segment.

The necessary calculations involve trigonometry, but measurements can be obtained from an accurate figure.

Examples XIII g.

Fill in the gaps in the following tables

	Radius of circle	Area.
1.	2·34 in.	
2	23 9 cm	
3.		407·6 sq. cm.
4.		28 8 sq. yds.
5.	3·82 ft.	
6.	ch.	4·84 acres
7	200 yds.	acres
8	6 34 metres	
9.		27·45 ares

	Radius of circle.	Angle of sector.	Area of sector.
10.	4·62 cm.	68°	
11	34·4 ft.	84° 30′	
12.	27·6 cm.		42 sq. cm.
13.		17°	232 sq. in.
14	5·44 yds.		10 sq. yds.
15.		123°	300 sq. yds.

16. A circular piece of cardboard of 10 in. diameter has a concentric circle of 5 in. diameter cut out of it. What is the area of the portion of cardboard remaining?

17. Fifty circular slabs each 1 ft. 6 in. in diameter are used to pave the floor of a room. The slabs are in five rows of ten, and are placed touching each other, the slabs in the outside rows just touching the walls of the room. What is the area of the room, and what quantity of cement 3 in. deep is required to fill the spaces between the slabs?

18. A circle has a circumference of 43·52 in. What is its area?

19. The area of a circle is 407·6 sq. cm. What is its circumference?

20. Draw on squared paper a circle of diameter 5 in. (or 10 cm. if centimetre paper is used). By counting squares, or by the method of § 194, find the area of the circle. Also calculate the true area, and express the error as a percentage of the true area.

21. Three equal circular disks, each of 4 in. diameter, are placed touching each other, and an elastic band is passed round them. What is the area enclosed by this band?

22. A regular hexagon is inscribed in a circle of radius 10 in.: (i) find the area of the hexagon; (ii) express the difference between the area and the area of the circle as a percentage of the latter.

23. A circle is circumscribed to a square of side 6 in. What is the area of the circle?

24. Calculate the area of a segment of a circle of radius 8 in., the chord of the segment being equal to the radius of the circle.

(N.B. There are *two* segments satisfying these conditions.)

25. Draw a circle of radius 3·4 in., and place in it a chord 4·2 in. long. Make any necessary measurements and calculate the area of the smaller segment cut off.

26. Find the area of a segment of a circle of radius 6·3 cm., the arc of which subtends an angle of 200° at the centre of the circle.

(Measurements to be obtained from an accurate figure.)

CHAPTER XIV

VOLUMES

(199.) A RECTANGULAR block, or *cuboid*, is a solid whose faces are all rectangles.

The volume of such a solid can be seen by a systematic counting of the number of units of volume of which it is built.

Thus, if the edges of a cuboid are a in., b in., and c in. (where, for the moment, a, b, c are whole numbers of inches[1]), the solid may be built up of c layers, each containing ab inch cubes. The volume is abc cu. in.

(200.) *Specific Gravity.* The Specific Gravity of a substance is defined as the ratio of the weight of any volume of the substance to the weight of an equal volume of water.

The *Density* of a substance is the mass of unit volume of that substance.

Since a cubic centimetre of water weighs one gramme, the number of grammes in a cubic centimetre of the substance is the same as the number giving the specific gravity of the substance[2].

The following table of specific gravities of various substances must be referred to for many examples in this chapter.

Aluminium	2·56	Lead	11·35
Brass (average)	7·61	Mercury	13·58
Copper	8·79	Platinum	21·84
Cork	0·24	Silver	10·50
Gold	19·26	Steel	7·82
Ice	0·93	Sea-water	1·03
Iron (cast)	7·15	Tin	7·29

It will also be necessary to remember that:

A cubic foot of water weighs 1000 oz. (approx.)

A gallon of water weighs 10 lb.

[1] If a, b, c are not whole numbers of inches, we take a smaller unit (e.g. $\frac{1}{100}$ inch) in terms of which they may all be expressed as whole numbers. (See § 130.)

[2] *Specific Gravity* is a ratio, or pure number, *Density* is a weight.

(201.) *Similar Solids.*

If two solids have the same shape, but different sizes, the ratio of their volumes will be the cube of the ratio of corresponding lengths. (See § 191.)

Thus, if we make a model of a solid, reducing all the linear dimensions in the ratio of 2 to 3, we shall obtain a solid whose volume is $\frac{8}{27}$ of the volume of the original solid

If the two solids are made of the same material, the weight of the smaller will be $\frac{8}{27}$ of the weight of the larger.

Example i. *A rectangular bar of iron has dimensions* 7·3 *ft.*, 4·5 *in.*, 4·5 *in. Find its weight.*

The specific gravity of iron is 7·15 (see table).

∴ A cubic foot weighs 7150 oz.

$$\text{The volume} = \frac{7{\cdot}3 \times 4{\cdot}5}{12} \times \frac{4{\cdot}5}{12} \text{ cu. ft.}$$

$$\therefore \text{ The weight} = \frac{7{\cdot}3 \times 4{\cdot}5^2}{144} \times \frac{7150}{16} \text{ lb.}$$

$$= 458{\cdot}6 \text{ lb.}$$

i e. approximately 460 lb.

·6532

·8633
1·3064
3·8543
6·0240
2·1584
1 2041
3 3625
2 6615

Example ii. *A block of metal* 6·3 *cm. by* 5·4 *cm. by* 2·5 *cm. weighs* 647·2 *gm. What is the density of the metal?*

The volume is $6{\cdot}3 \times 5{\cdot}4 \times 2{\cdot}5$ cu. cm.

$$\therefore \text{ The weight of one cu. cm.} = \frac{647{\cdot}2}{6{\cdot}3 \times 5{\cdot}4 \times 2{\cdot}5} \text{ gm}$$

$$= 7{\cdot}61.$$

2 8110
7993
7324
3979
1·9296
8814

Example iii. *A mass of lead is found to weigh* 3·42 *Kg. What is its volume?*

From the table, a cu. cm of lead weighs 11·35 gm.

∴ The volume of this mass of lead is

$$\frac{3420}{11{\cdot}35} \text{ cu. cm.}$$

$$= 301 \text{ cu. cm.}$$

1·135) 342 (301
1·5

Example IV. *Brass is an alloy of copper and tin. Find the percentage of copper in the specimen of brass whose specific gravity is quoted in the table*

Suppose x cu. cm. of copper are combined with y cu. cm. of tin.

Then the weight is $8{\cdot}79\,x + 7{\cdot}29\,y$ gm.

But the weight is also $7{\cdot}61\,(x+y)$.

$$\therefore \quad 7{\cdot}61\,(x+y) = 8{\cdot}79\,x + 7{\cdot}29\,y.$$

$$\therefore \quad 1{\cdot}18\,x = {\cdot}32\,y.$$

$$\therefore \quad x = \frac{32}{118}y$$

$$= \frac{16}{59}y.$$

$\therefore$ There are 16 parts of copper to 59 of tin,

or 16 of copper out of 75 of the brass.

$\therefore$ Percentage of copper $= \frac{16}{75}$ of 100

$= 21{\cdot}3$

Examples XIV a.

Find the weight of the following substances:

1. 50 cu. cm. of gold.
2. 17 cu. cm. of aluminium.
3. 5000 cu. cm. of ice.
4. A litre of sea-water.
5. 62 cu. cm. of cork.
6. A gallon of sea-water.
7. 3 cu. ft. of lead.
8. 75 cu. in. of platinum.
9. 60 cu. ft. of ice.
10. 300 gallons of sea-water.

Find the volume occupied by the following substances:

11. 25 g. of gold.
12. A Kg. of lead.
13. A Kg. of mercury.
14. 60 oz. of silver.
15. 210 oz. of tin.
16. 1 lb. of cork.
17. A ton of ice.
18. 3·5 Kg. of mercury.
19. A ton of cast iron.
20. 40 lb. of steel.

---

21. A bar of steel is a metre in length, and its section is a rectangle 1 cm. by 2 cm. What is its weight?

22. A rectangular pond, 30 yds. by 18 yds., is frozen over with ice to a depth of 3 in. What is the weight of the ice?

23. Aluminium is cast into blocks 8 in. long and 4 in. wide and deep. What is the weight of a block?

24. An alloy is made of platinum and tin, 70% of the alloy being tin. What is the weight of a cu. cm. of this alloy?

25. A volume of 60 cu. cm. of an alloy of copper and tin is found to weigh 480 gm. What weight of tin is there in the alloy?

26. Two liquids of specific gravities ·62 and ·73 are mixed in equal volumes, and there is no shrinkage of volume. What is the specific gravity of the mixture?

27. Equal weights of the two liquids in question 26 are mixed. What is the specific gravity of the mixture?

28. Two liquids of specific gravities ·48 and ·79 are mixed in equal volumes, and there is a 15% shrinkage of volume. What is the specific gravity of the mixture?

29. Equal volumes of two liquids are mixed together. The specific gravities of the two separate liquids are ·73 and ·54, and that of the mixture is found to be ·67. Find the amount per cent. by which the volume has shrunk.

30. An ingot measuring 15 in. by 7 in. by 5 in. is supposed to be of pure gold, but some silver is suspected to have been mixed with the gold. The weight of the ingot is 4720 oz. Find the weight of pure gold in the ingot.

31. What is the specific gravity of a substance, of which a block 3 ft. by 2 ft. 6 in. by 1 ft. 9 in. weighs 3 tons?

32. A hollow cubical box without a lid is made of iron and is 2 in. thick, the external edge being 4 ft. long. Find, in cwt., the weight of the box.

33. A mahogany pattern, made from wood weighing 762 grammes per cubic decimetre, weighed 2·84 Kg.

A gun-metal casting of the same size and shape weighed 30·8 Kg., the gun-metal employed weighing 8·46 Kg. per cubic dm.

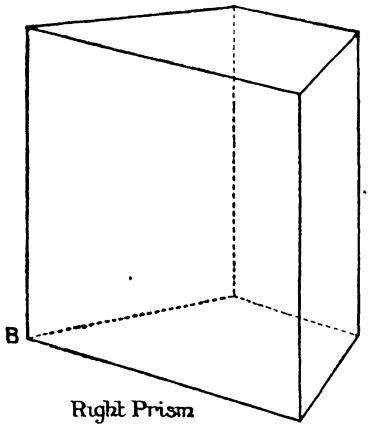
Show that there must be a cavity in the casting, and find what weight of metal would fill it.

34. A tank has a rectangular base 3 ft. by 4 ft., and the sides are vertical. It is filled with water to a depth of 9 in. How high will the water rise (i) if a cube of lead of side 6 in. is placed in the tank, (ii) if a cube of lead of side 12 in. is placed in the tank?

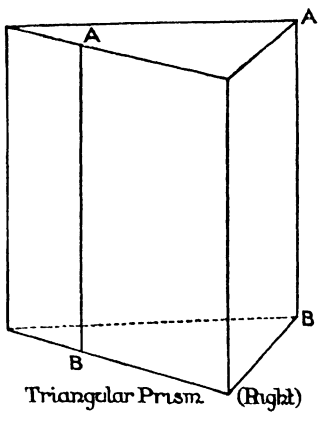
35. If a model made of wood of sp. gr. ·723 weighs 32·3 gm., what will be the weight of a similar solid made of steel, if the linear dimensions are three times the corresponding dimensions of the model?

36. The weight of a railway engine is 80 tons. Find the weight of a model made of the same material on a scale of 1 :30

37. Two solids have their linear dimensions in the ratio 2 to 5. The smaller is made of wood, of sp. gr ·761, and the larger of brass. Find the ratio of their weights.

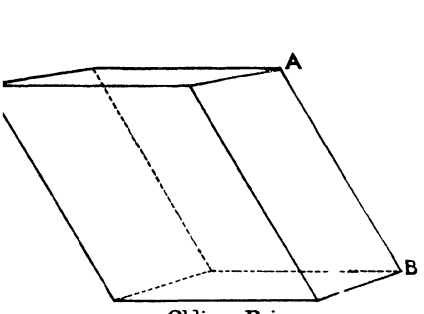


Right Prism

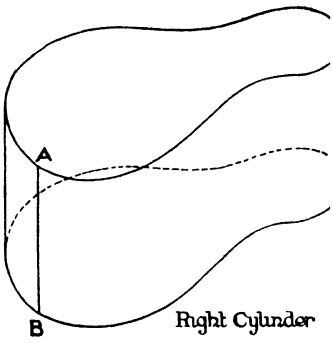


Triangular Prism (Right)

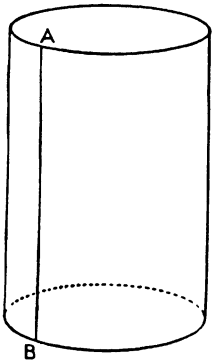
(202.) *Prisms and Cylinders.* If any closed plane figure be moved so that its plane is always parallel to a fixed direction, and all points in its boundary move along parallel straight lines, a solid is traced out called a 'prism' (if the plane figure is formed of straight lines) or a 'cylinder' (if the plane figure is a curve).



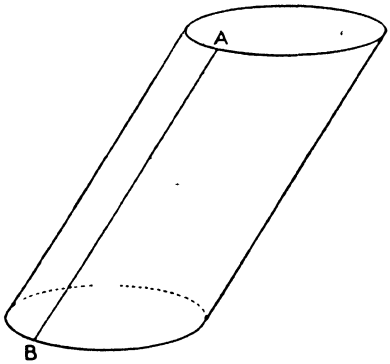
Oblique Prism



Right Cylinder



Right Circular Cylinder



Oblique Circular Cylinder

From another point of view the surface of a prism or cylinder may be considered as traced out by a straight line of given length (the line AB in the drawings) which always has the same direction, and whose end moves round the boundary of a plane figure. Such a line, in any particular position, is called a 'generator' of the prism or cylinder.

If the generators are perpendicular to the plane ends (i. e. if the sides are perpendicular to the ends), the solid is called a 'right prism' or a 'right cylinder'. Otherwise it is called 'oblique'.

The figures illustrate these definitions.

It should be noted that a cuboid is only a particular case of a prism.

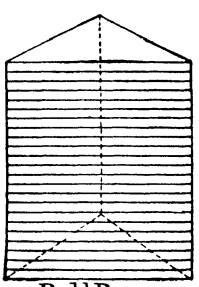
(203.) *The Right Prism.* If the height of a right prism is h inches (i. e. if the length of a generator is h inches), then every square inch in the base contributes h cubic inches to the total volume.

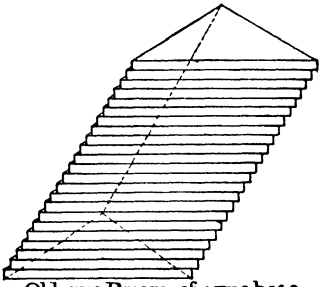
Hence, if the area of the base is A sq inches, and the height h inches,

the volume is Ah cubic inches

Similarly, each inch in the boundary of the base of a right prism contributes h sq. in. to the area of the sides or walls; hence, if the perimeter of the boundary be P inches,

the area of the side surface is Ph sq. inches.





(204.) *The Oblique Prism.* If a right prism be made of very thin layers (see the figure), and these layers be slid to one side (the generators remaining straight lines), an oblique prism will be formed; and since the height and the area of the base remain the same, and since the volume also remains the same, we see that the **volume of an oblique prism is measured by the product of the area of the base and the height** (i. e. the perpendicular distance between the plane ends).

The area of the side surface of an oblique prism is measured by the product of the length of a generator and the perimeter of a section perpendicular to a generator.

(205.) *Cylinders.* A cylinder being only a special case of a prism, its volume and surface area are found in identically the same way as in the case of the prism. Thus **the volume of a cylinder is measured by the product of the number of units of area in the base, and the number of units of length in the height.** In the special case of a right circular cylinder of base-radius r and height h (measured in terms of the same unit), the volume is $\pi r^2 h$.

(206.) *The area of the curved surface of a cylinder is measured by the product of the perimeter of the base and the height.*

In the case of a right circular cylinder, a piece of paper which could be exactly fitted round the cylinder would be a rectangle, one of its sides being the circumference of the base, and the other side being the height of the cylinder.

If the radius of the base is r, and the height is h, then the area of the curved surface will be $2\pi r . h$, and *the total surface area of a solid right circular cylinder* will be

$$2\pi r . h + 2\pi r^2 \quad \text{or} \quad 2\pi r (h + r)$$

Example 1. *A triangular prism is 7·32 in. high, and the base has sides 4·23 in., 3·71 in., and 2·65 in. Find the volume of the prism and the total surface area.*

We first find the area of the base from the formula

$$\sqrt{s(s-a)(s-b)(s-c)}.$$

$2s = 10{\cdot}59$ in.

$s = 5{\cdot}295$ in.

$s - a = 1{\cdot}065$ in.

$s - b = 1{\cdot}585$ in.

$s - c = 2{\cdot}645$ in.

$\therefore$ Area $= 4{\cdot}862$ sq. in.

$\therefore$ Volume $= 4{\cdot}862 \times 7{\cdot}32$ cu. in.

$= 35{\cdot}58$ cu. in.

Logarithms.

·7239
·0273
2001
4224
2)1·3737
6868
·6868
·8645
1·5513

The perimeter = 10·59 in.

∴ The total area of the side surfaces is

$$10{\cdot}59 \times 7{\cdot}32 \text{ sq. in.}$$
$$= 77{\cdot}52 \text{ sq. in.}$$

1 0249
8645
1 8894

But the area of the two ends is

$$2 \times 4{\cdot}86 \text{ sq. in.}$$
$$= 9{\cdot}72 \text{ sq. in.}$$

∴ The total surface area is

$$87{\cdot}24 \text{ sq. in.}$$

Example II. *A reservoir has a surface area of* 16 *acres. The average depth of the water is* 14 *feet. How many gallons of water does it contain?* (*A gallon* = ·1604 *cu. ft.*)

The surface area

$$= 16 \times 4840 \text{ sq. yds.}$$
$$= 16 \times 4840 \times 9 \text{ sq. ft.}$$

∴ The volume of water

$$= 16 \times 4840 \times 9 \times 14 \text{ cu. ft.}$$
$$= \frac{16 \,.\, 4840 \,.\, 9 \;\; 14}{\cdot 1604} \text{ gallons}$$
$$= 6{\cdot}081 \times 10^7,$$

or approximately 60·8 million gallons.

1 2041
3 6848
·9512
1 1461
6 9892
1 2052
7 7840

Example III. *To what depth will a gallon of water* (·1604 *cu. ft.*) *fill a can whose cross-section is a circle of diameter* 8 *inches?*

The area of the base is $\pi \left(\frac{1}{3}\right)^2$ sq. ft.

∴ The depth is $\dfrac{\cdot 1604}{\pi \,.\, \frac{1}{9}}$ ft.

$$= \frac{1{\cdot}4436}{3{\cdot}142} \text{ ft.}$$
$$= {\cdot}4594 \text{ ft.}$$
$$= 5{\cdot}513 \text{ inches,}$$

or $5\frac{1}{2}$ inches, approximately.

·4594
3·142) 1 4436
·1868
297
14

Example IV. *What volume of metal is required to make a pipe 30 ft. long, internal diameter 1 in., thickness $\frac{1}{4}$ in. ?*

The internal radius is ·5 in.
The external radius is ·75 in.

$\therefore$ The area of section is $\pi(\cdot75^2 - \cdot5^2)$
$= \pi \times 1\cdot25 \times \cdot25$ sq. in.
$\therefore$ The volume $= \pi \times 1\cdot25 \times \cdot25 \times 30 \times 12$ cu. in.
$= 353\cdot4$ cu. in.

	·4972
	·0969
	$\bar{1}\cdot3979$
	$1\cdot4771$
	$1\cdot0792$
	$2\cdot\bar{5}483$

Examples XIV b.

Fill in the gaps in the following table, which refers to right circular cylinders.

	Height of cylinder.	Base radius.	Volume.	Area of curved surface.
1.	3·4 cm.	2·4 cm.		
2.	4·62 in.	1·73 in.		
3.	5 ft.	1 in.		
4		2·2 in.	34 cu. in.	
5		3·61 cm.		17·52 sq. cm.
6.	2·64 in.		13·72 cu. in.	
7.	3·5 metres			26·2 sq. m.
8			18·54 cu. cm.	12·82 sq. cm.

9. The base of a right prism is a triangle whose sides are 7 in., 6 in., and 4 in. The height of the prism is 10 in. Calculate the volume.

10. A prism 6·3 cm. long has its base in the form of a rhombus, the equal sides being 7·3 cm., and one diagonal 5·2 cm. Find the volume of the prism, and the total area of its surface.

11. A pencil (unsharpened) is 6 in. long, and its section is a regular hexagon of side ·14 in. Find the volume of the pencil.

If the lead has a circular section, and is ·09 in. thick, find the volume of wood in the pencil.

12. Find the volume of metal in a wire 100 metres long and 3 mm. thick.

13. A volume of metal of 100 cu.cm. is drawn into a wire 2 mm. thick. What will be the length of the wire?

14. 1000 cu. cm. of molten lead are poured into a circular mould of diameter 32 cm. If the lead contracts ·1 per cent. in cooling, what will be the thickness of the leaden disk obtained?

15. A cylindrical can is to be made to hold 10 litres. Its diameter is to be 15 cm. What must be the height?

16. A cylindrical vessel without a lid is made of metal 2 mm. thick. Its external measurements are: diameter 10 cm., height 13 cm. What is the volume of the metal of which it is made?

If the specific gravity of the metal is 7·23, what is the weight of the vessel?

17. A sheet of metal 10 in. by 8 in. is rolled up to form a cylinder: (i) by bringing the long edges together, (ii) by bringing the short edges together. What will be the volume enclosed in each case?

18. A roll of paper is 2 ft. 3 in. in diameter. If the paper is ·02 in. thick, what is the length of paper in the roll?

19. Lead piping has an external diameter of 1·5 in., and is ·1 in. thick. What weight of lead is required for 10 yds. of piping? (See table, p. 377.)

20. Water flows through a pipe of circular section at the rate of 2 ft. per second. If the pipe has an internal diameter of 1 in., find how many gallons are delivered per hour. (A gallon = 277·3 cu. in.)

21. A gasometer is 84 ft. in diameter. How many cubic feet of gas have been used if the gasometer has fallen 2 ft. 6 in.?

22. What is the whole surface area of a cylinder 10 ft. high and 3 ft. across?

23. A motor roller is 3 ft. 6 in. in diameter and 5 ft. wide. What area (in acres) can be rolled per hour if the roller makes 40 revolutions per minute?

At what speed does the roller travel?

24. A solid cylinder whose base radius is 2·34 cm., and whose height is 7·65 cm., weighs 1 Kg. What is the specific gravity of the metal?

25. A copper cylinder is 6 cm. high and has a base radius 4·3 cm. It is thought that there may be a hollow in it. It is found to weigh 3027 gm. Find the volume of the hollow.

26. A hollow copper cylinder, without a lid, is made of metal 2 mm. thick. Its external dimensions are: height 14·5 cm., radius 5·9 cm. Find its weight when full of water.

27. Cylindrical canisters are to be made 12 cm. high (external measurement) without lids, to hold a litre. If the metal is to be 1 mm. thick, find the weight of metal required if its specific gravity is 7·59.

VOLUMES OF IRREGULAR SHAPE

(It may be thought advisable to postpone this section until after the next chapter.)

(**207.**) Figure I represents a mound of earth. We can find approximately the volume of earth in the mound if we can obtain a series of contour lines drawn upon squared paper.

The contour plan for this mound is shown in Figure II, the lines representing sections of the mound at heights 0, 10, 20 .. ft. above the level ground.

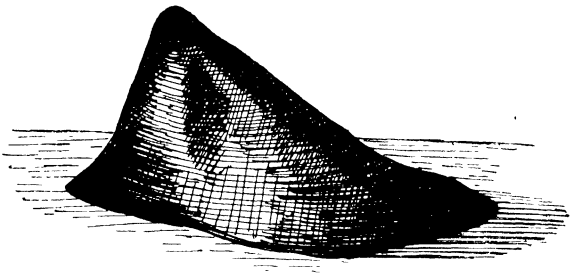


Fig I

The areas of these sections are obtained by counting squares, and a table is obtained as follows.

Height of section in ft. .	0	10	20	30	40	50	60
Area of section in sq. ft	9440	5990	3360	1950	970	400	80

Note.—In the diagram 3 spaces represent 20 ft., so that 9 squares represent 400 sq. ft.

Now, if we regard the mound as being made up of a series of layers, each 10 ft. high (see Fig. III), the lowest layer having

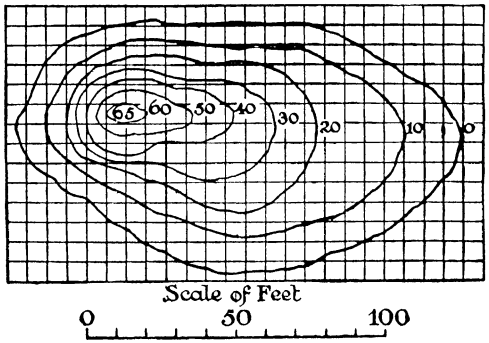


Fig. II

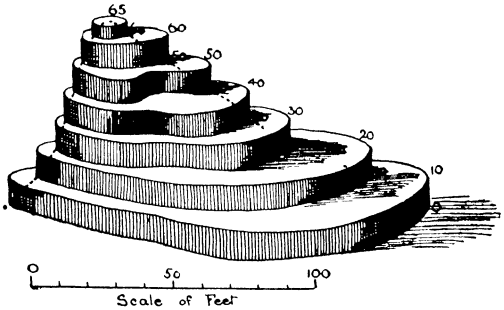


Fig. III

a sectional area of 9440 sq. ft., we shall obtain a volume which we know to be greater than the actual volume of the mound.

This volume is

$$10 \times 9440 + 10 \times 5990 + 10 \times 3360 + \ldots + 10 \times 80 \text{ cu. ft.};$$

i. e. $10\,(9440 + 5990 + 3360 + 1950 + 970 + 400 + 80)$ cu. ft.,

or 221900 cu. ft.

Also, if we regard the layers as in Fig. IV, we shall obtain a volume less than the actual volume of the mound.

This volume is

$$10(5990+3360+1950+970+400+80) \text{ cu. ft.},$$

i. e. 127500 cu. ft.

Now it is not correct to say that the true volume will be the arithmetical average of these two volumes. We have, however, ascertained that the volume lies between 130000 cu. ft. and 220000 cu. ft., and we should probably be justified in saying that the volume is approximately 170000 cu. ft.

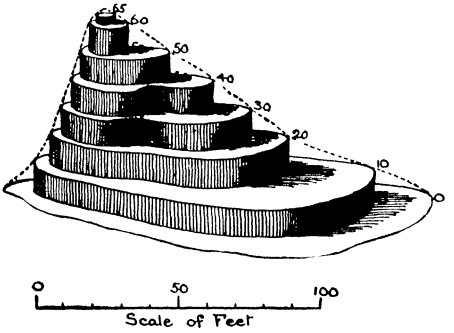


Fig. IV

(208.) We can obtain a more accurate result by drawing a graph connecting the area of the section with its height above the level. Such a graph is shown in Fig. V.

In this graph distances along a horizontal axis represent heights on a scale of 2 spaces to 10 ft., and distances along the vertical axis represent sectional areas on a scale of 1 space to 1000 sq. ft.

Now an area of 1 space will represent a volume of 5000 cu. ft., and the area of paper between the curve and the two axes will represent, on this scale, the volume of the mound. We find the number of squares to be 35, and we obtain the volume of the mound as approximately 175000 cu. ft.

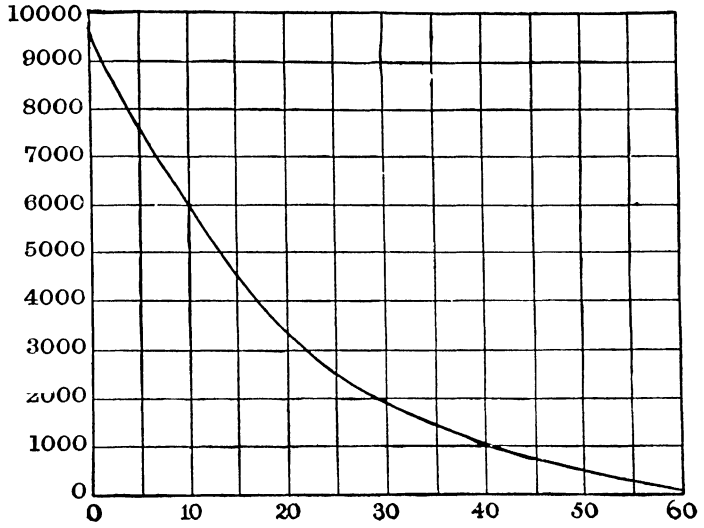


Fig. V.

Examples XIV c.

1. Find the approximate volume (in cubic yards) of the hill shown in the contoured diagram (Fig. I).

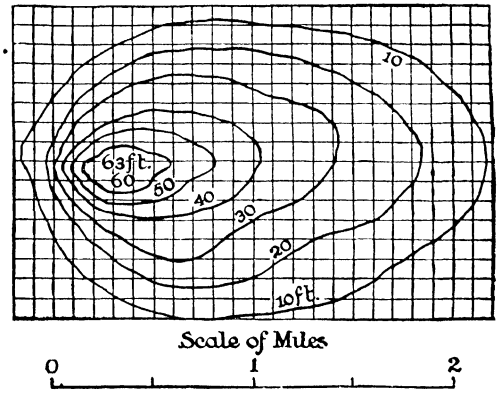


Fig. I.

2. Fig. II shows a contoured plan of a chalk cliff as it exists to-day. The dotted lines indicate the contour lines of the coast some years ago. What volume of chalk has been washed away?

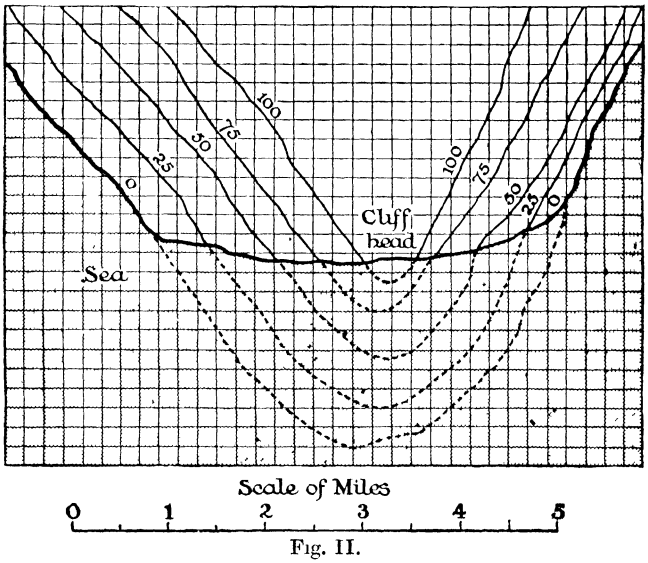


Fig. II.

3. Fig. III shows a lake in the Highlands.

The depth of the lake averages 100 ft. If a dam were to be built across the valley as at AB, and the level of the water raised 500 ft., what would be the total quantity of water (in gallons) contained in the reservoir thus formed?

4. In the grounds of a school in the Thames Valley there is an artificial grass mound, with a flat circular top, and a small perpendicular brick wall round the base.

The wall is 2 ft. high, and the sections of the mound at various heights are circles whose radii are given in the following table:

Height (in feet) .	2	4	8	12	16	20
Radius (in yards) .	7·6	7 2	6 2	5 0	4 0	3·0

Calculate the amount of soil the mound contains.

5. A cone has height 18 in. and base radius 5 in. Tabulate the areas of sections perpendicular to the axis at distances 2 in., 4 in., 6 in. . . . 18 in. from the vertex, and draw a graph com-

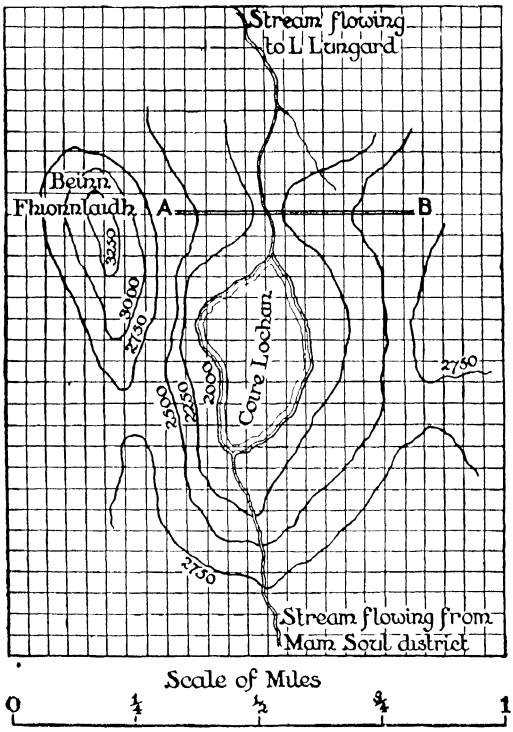


Fig. III.

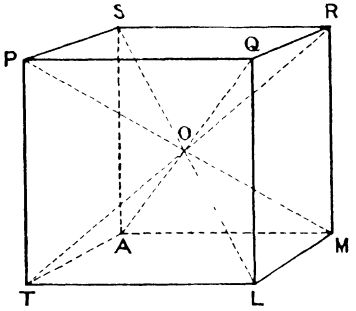
paring the area of a section with its distance from the vertex. From this graph find the volume of the cone.

6. A hemisphere has a radius of 10 in. Calculate the areas of sections at distances 0, 1, 2 . . . 10 in. from the centre, and by drawing a graph, find the volume of the hemisphere.

CHAPTER XV

THE PYRAMID

(209.) Fig I represents a cube cut up into six pyramids. These pyramids each have the same vertex O, and they each have a face of the cube as base.



Cube cut into six pyramids

Fig. I.

Also all the edges OP, OQ, OK, &c., are equal; in fact, the six pyramids are of exactly the same shape and of the same dimensions, and must therefore be equal in volume.

If the side of the cube is a in., the volume of the cube is a^3 cu. in.

Hence the volume of each pyramid

$$= \frac{1}{6} a^3$$

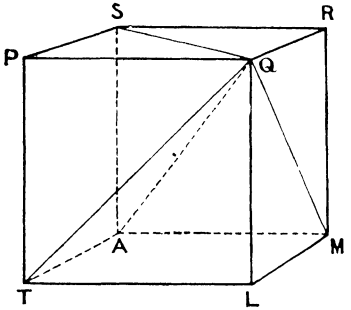
$$= \frac{1}{3} a^2 \times \frac{a}{2}$$

$$= \frac{1}{3} \times \text{area of base} \times \text{height}$$ [1]

[1] We mean, strictly, that the number of units of volume is one-third of the product of the number of units of area in the base and the number of units of length in the height.

(210.) This pyramid is an example of a right pyramid. A *right pyramid* is one in which the base is a regular polygon and the foot of the perpendicular from the vertex to the plane of the base is the centre of the circumscribing circle of the base.

If the foot of the perpendicular is not the centre of the circumscribing circle (it may even fall outside the base), the pyramid is said to be *oblique*.



Cube cut into three pyramids

Fig II.

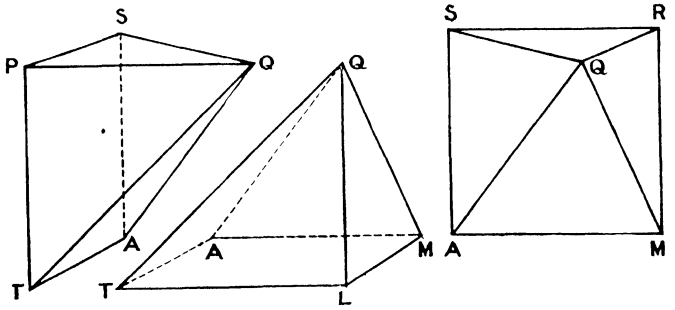


Fig. III

Figs. II and III show a cube divided into three oblique pyramids. These pyramids each have equal bases and equal heights, and we again infer that the volume of each pyramid is measured by a third of the product of the area of the base and the height.

(211.) It can be shown by a method similar to that of § 204 that pyramids having equal bases and equal heights are equal in volume. Also, a prism with a triangular base can be divided into three equal pyramids in much the same way as the cube in Fig. II. Hence, a pyramid with a triangular base is one-third of the prism on the same base and with the same height.

Since any polygon can be divided into a number of triangles, it follows that any pyramid can be divided into a number of pyramids with triangular bases, each of them one-third of a prism

Hence we have the general fact:

Volume of a pyramid $= \frac{1}{3}$ the prism on the same base with the same height

$= \frac{1}{3}$ **area of base** $\times$ **height.**

(212.) A *tetrahedron* (i.e. a 'four-seated' solid) is a pyramid on a triangular base.

A *regular tetrahedron* has all its faces equilateral triangles (and all its edges, in consequence, equal)

If we have sufficient data we can calculate the surface area of a pyramid by the methods applied to triangles, since all the side faces are triangles.

In a right pyramid we can calculate the length of an edge, or the height of a triangular face, if we are given the height of the pyramid and the details of the base, since the perpendicular from the apex to the plane base is at right angles to any diameter of the circumcircle of the plane base.

Example 1. *A pyramid stands on a square base, the side of the square being 3·6 in. The remaining edges of the pyramid are each 4·3 in. Calculate the area of the sloping surface and the volume of the pyramid.*

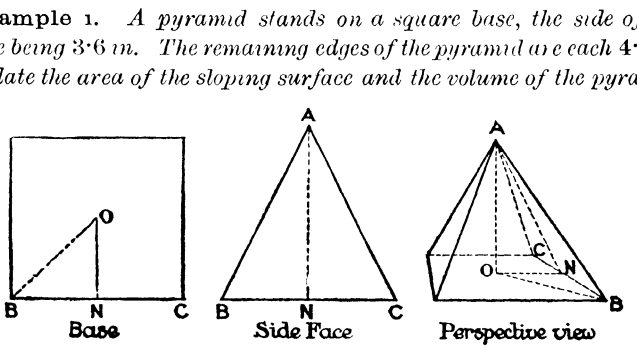


Fig. IV.

Analysis of method—To calculate the area of the face ABC we must find the length of the perp. from A to BC. This is obtained from the right-angled Δ ABN. To calculate the volume we want the height OA. This is obtained from the Δ AOB after a preliminary calculation of the length OB, or it may be obtained from the Δ AON.

The best method will be to calculate AO by both methods independently, and thus obtain a check.

$AB = 4{\cdot}3$ in. $\quad BC = 3{\cdot}6$ in.

$\therefore \quad BN = 1{\cdot}8$ in

Also, $AN^2 = AB^2 - BN^2$

$= 4{\cdot}3^2 - 1{\cdot}8^2$

$= 6{\cdot}1 \times 2{\cdot}5$ (see § 88)

$= 15{\cdot}25.$

$\therefore \quad AN = 3{\cdot}905.$ (using tables of sq. roots or logarithms)

$\therefore$ Area of triangle $ABC = 1{\cdot}8 \times 3{\cdot}9$ sq. in

$= 7{\cdot}02$ sq. in.

3 9
3 12

$\therefore$ Whole side surface $= 28{\cdot}1$ sq. in.

To find the height OA

We have $OB^2 = BN^2 + ON^2$

$= 2 \times 1{\cdot}8^2.$

$\therefore \quad OB = 1{\cdot}8 \times \sqrt{2}$

$= 2{\cdot}55$ in. $(\sqrt{2} = 1{\cdot}414\ldots)$

1 8
72
2

And $OA^2 = AB^2 - OB^2$

$= 4{\cdot}3^2 - 2\ 55^2$

$= 6{\cdot}85 \times 1{\cdot}75.$

$\therefore \quad OA = 3{\cdot}46$ in.

Logarithms
8357
2430
2 | 1 0787
5393

[Check for length of OA.

$OA^2 = AN^2 - ON^2$

$= 3\ 9^2 - 1{\cdot}8^2$

$= 5{\cdot}7 \times 2{\cdot}1$

$= 12\ 0$

11 1
57

$\therefore \quad OA = 3{\cdot}46$ (from tables of sq. roots)]

$\therefore$ Volume $= \frac{1}{3} . 3{\cdot}6^2 . 3{\cdot}46$ cu. in

$= 14{\cdot}95$ cu. in.

or $\quad 15{\cdot}0$ cu. in.

Logarithms
·5563
1·1126
5391
1·6517
4771
1·1746

Example 11. *The base of a pyramid is a regular hexagon inscribed in a circle of radius* 7·4 *cm. Its volume is* 422 *cu. cm. Calculate the length of one of the sloping edges.*

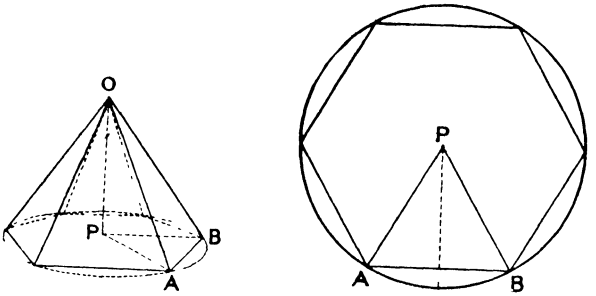


Fig V.

Analysis—We must first find the area of the base; then, knowing the volume, we can find the height OP, and hence the edge OA from the right-angled triangle OPA

The base is composed of six equilateral triangles of side 7·4 cm. The altitude of each is $3{\cdot}7\sqrt{3}$. (See § 185.)

$\therefore$ The area of the base is

$$6 \times \tfrac{1}{2} \times 7{\cdot}4 \times 3{\cdot}7\sqrt{3}$$
$$= 6 . (3{\cdot}7)^2 . \sqrt{3}$$

But the volume $= \frac{1}{3}$ area of base $\times$ height.

$$\therefore \text{ Height } = \frac{3 \times \text{volume}}{\text{area of base}}$$
$$= \frac{\sqrt{3} \times 422}{6(3{\cdot}7)^2}$$
$$= 8{\cdot}90 \text{ cm.}$$

Finally, $OA^2 = 8{\cdot}90^2 + 7{\cdot}4^2$
$= 79{\cdot}21 + 54{\cdot}76$
$= 133{\cdot}97.$
$\therefore \quad OA = 11{\cdot}6$ cm.

Logarithms.

2)	·4771
	·2385
	2 6253
→	2 8638
	·5682
	2
	1·1364
	·7782
→	1·9146
	9492

(The squares and square roots are taken from tables.)

Example III. *Find the volume of a regular tetrahedron whose edge is* 8·42 *in.*

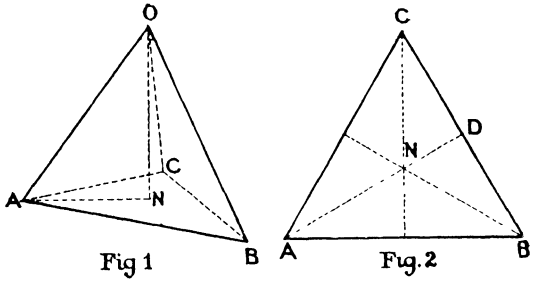


Fig VI.

Analysis of method.—In an equilateral triangle the three altitudes trisect each other Thus $AN = \frac{2}{3} . AD$

We can then calculate the length of AN, and, from the right-angled triangle OAN, we can find the height ON of the tetrahedron. Since we shall be postponing the calculation till the end of the work, it is convenient to use an algebraic symbol for the length of the side, and obtain a formula for the volume

Let the length of edge be $2a$ in. (In this case $a = 4{\cdot}21$.)

Then in Fig. 2 $AD = a\sqrt{3}$. (See § 186)

$\therefore$ Area of base of tetrahedron $= a^2\sqrt{3}$ sq. in.

Again, $AN = \frac{2}{3} . AD = \frac{2}{3} . a\sqrt{3}$.

Then in Fig 1 $ON^2 = OA^2 - AN^2$

$= 4a^2 - \frac{4}{3} a^2$

$= \frac{8}{3} a^2$.

$\therefore \quad ON = \dfrac{2\sqrt{2}}{\sqrt{3}} a.$

$\therefore$ Volume $= \frac{1}{3} a^2\sqrt{3} \times \dfrac{2\sqrt{2}}{\sqrt{3}} a$

$= \dfrac{2\sqrt{2}}{3} a^3$

$= \dfrac{2{\cdot}828}{3} a^3$

$= {\cdot}9427\, a^3$.

In this case, the volume $= 70{\cdot}4$ cu. in

log 4 21 =	·6243
	3
	1 8729
	1·9744
	1·8473

Example IV. *1000 cu. cm. of lead are melted down, and shaped into a regular tetrahedron. What will be the length of an edge of this tetrahedron?*

If $2a$ cm. is the length of an edge, and V cu. cm. the volume, we have found that $V = \cdot 9427 \, . \, a^3$.

$$\therefore \quad a = \sqrt[3]{\frac{V}{\cdot 9427}}$$

$$= 10\cdot 2 \text{ cm.}$$

$\therefore$ The edge is $20\cdot 4$ cm.

	3·
	1·9744
3)	3 0256
	1 0085

Examples XV a.

1. A pyramid has a square base of side 7 in., and it is 10 in. high. Find its volume.

2. The base of a right pyramid is a square of side 6·3 cm., and the sloping edges are each 8·5 cm. Calculate the whole surface area, and the volume.

3. A pyramid is 23 cm high, and its base is an equilateral triangle of side 14 cm Find its volume.

4. Find a formula for the volume of a pyramid whose base is an equilateral triangle of side $2a$ in., and whose remaining edges are each b in long

Use the formula obtained in question 4 to solve the following problems 5 to 8.

5. Find the volume of a tetrahedron (not regular) in which the three base edges are each 5·6 in., and the three remaining edges are each 7·5 in.

6. Find the volume of a regular tetrahedron of edge 12·5 cm.

7. A lump of lead of volume 325 cu. cm. is hammered into the form of a regular tetrahedron. What will be the length of an edge of this tetrahedron?

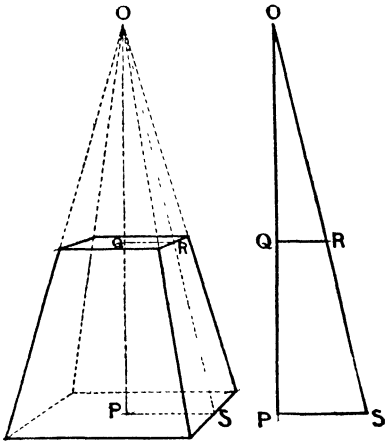
8. 10 Kg. of lead are hammered into the form of a tetrahedron having each sloping edge twice as long as each base edge. Find the lengths of the edges. (Sp. gr. of lead = 11·35.)

9. A pyramid on a square base (side 7 ft) is 10 ft. high. If the top is cut off at a height of 5 ft., what will be the volume of the remainder, and what will be the total area of the sloping surface?

10. The volume of a pyramid is 72·6 cu. in., and its base is a regular hexagon of side 7·3 in. Find the height of the pyramid.

11. Draw a regular pentagon in a circle of 3 in. radius. Make any measurement you think necessary to find the area of this pentagon, and then find the total surface area and the volume of a pyramid whose base is this regular pentagon, and whose height is 6 in.

A portion of a pyramid formed by cutting off the apex by a plane parallel to the base is called a '*frustum*' *of a pyramid.*



Frustum of a Pyramid

Fig. VII

12. A frustum of a right pyramid has its parallel faces squares of sides 7·9 in. and 4·4 in. respectively. Its height is 6·8 in. Calculate its volume.

(Use the fact that the triangles OQR and OPS in the figure are similar.)

13. Find the whole surface area in a frustum of a pyramid, being given that the parallel faces are squares of sides 6·4 yds. and 11·6 yds., and the height is 7·9 yds.

14. A frustum of a pyramid is formed by making a cut parallel

to the base, reducing the height by a third. What fraction is (i) the volume of the frustum of the volume of the original pyramid; (ii) the area of the sloping sides of the frustum of the corresponding area of the original pyramid?

15. One face BCD of a tetrahedron (not regular) is a triangle whose sides are 5 ft., 6 ft., and 7 ft. long respectively, and the face angles at the opposite vertex A are all right angles. Calculate the lengths of the edges meeting in A.

Regarding ABC as the base, calculate the volume of the tetrahedron, and hence find the length of the perpendicular from A to the face BCD.

16. One corner of a rectangular cuboid is chipped off so that the edges meeting in that vertex are reduced to $\frac{1}{2}$, $\frac{2}{3}$, $\frac{6}{7}$ of their former lengths respectively.

Prove that the volume is reduced to $\frac{2}{2}\frac{5}{5}\frac{1}{2}$ of its former value.

THE CONE

(213.) **A cone is of similar construction to a pyramid, but its base is a closed curve instead of a closed rectilineal figure.**

As in the case of the pyramid, **the volume is measured by $\frac{1}{3}$ of the product of the area of the base and the height.** (See note, § 209.)

A *right circular cone* is a cone with a circular base, and such that the perpendicular from the vertex to the base meets the base at the centre of the circle.

Since the area of a circle of radius r is πr^2, **the volume of a circular cone of height h and base-radius r is $\frac{1}{3}\pi r^2 . h$.**

(214.) *Curved surface of a right circular cone.* The curved surface may be regarded as being made up of a large number of small triangles. These all have the same height ($= l$, the length of the slope of the cone), and their total base is equal to the circumference of the circle forming the base of the cone, hence **the area of the curved surface of a right circular cone is measured by $\frac{1}{2}$ the product of the circumference of the base and the slant height, i. e. area of curved surface $= \pi rl$.**

(215.) If a sector of a circle be cut out of paper, and the bounding radii be brought together, the surface of a right circular cone will be formed.

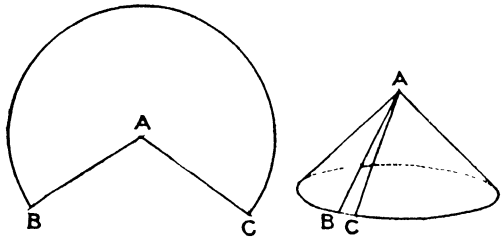


Fig. VIII.

The sloping edge AB of this cone will have the same length as the radius of the original sector, and the circumference of the base of the cone will be the same as the length of the arc of the sector.

In a right circular cone, since the perpendicular from the vertex to the base passes through the centre of the circular base, we can

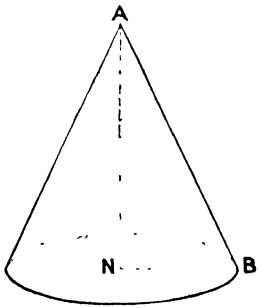


Fig. IX

AN = height. BN = base radius. AB = sloping edge.

calculate any one of the three lengths, (i) height, (ii) base radius, (iii) sloping edge, if we are given the other two, since the three form a right-angled triangle with the sloping edge of the cone as hypotenuse.

Example 1. *A circular cone has base radius* 4·2 *in., and height* 7·3 *in. Find its volume and total surface area.*

$$\text{The volume} = \tfrac{1}{3}\pi r^2.h$$
$$= 134{\cdot}8 \text{ cu. in.}$$

·6232 — ·4972, 1 2464, ·8633, 2·6069, ·4771, 2·1298

If the sloping edge is a in.,

$$a^2 = 4{\cdot}2^2 + 7{\cdot}3^2$$
$$= 17{\cdot}64 + 53{\cdot}29 \text{ (using tables of squares)}$$
$$= 70{\cdot}93 \text{ sq. in.}$$

$$\text{And area of curved surface} = \pi . 4{\cdot}2 . \sqrt{70{\cdot}93}$$
$$= 111{\cdot}1 \text{ sq. in.}$$

2) 1·8508, ·9254, ·6232, ·4972, 2 0458

$$\text{Area of base} = \pi r^2$$
$$= 55{\cdot}42 \text{ sq. in.}$$
$$\therefore \text{ Total surface area} = 166{\cdot}5 \text{ sq. in.}$$

·6232 — 1 2464, ·4972, 1 7436

Example 11. *The volume of a cone is* 764 *cu. cm., and the radius of the base is* 17 *cm. Find the area of the curved surface.*

Argument—We must first find the height of the cone

If the height is h cm.,

we have $\tfrac{1}{3}\pi r^2.h = 764.$

$$\therefore\ h = \frac{3 \times 764}{\pi . 17^2}$$
$$= 2{\cdot}524 \text{ cm.}$$

1·2304 — ·4771, 2·8831, 3·3602, 2·4608, ·4972, 2 9580, 4022

The length of the sloping edge is $\sqrt{17^2 + 2{\cdot}524^2}$

$$= \sqrt{289 + 6{\cdot}370}$$
$$= \sqrt{295{\cdot}4} \text{ cm.}$$

$$\therefore \text{ Area of curved surface} = \pi . 17 . \sqrt{295{\cdot}4}$$
$$= 917{\cdot}9 \text{ sq. cm.}$$

2) 2·4704, 1·2352, 1·2304, ·4972, 2·9628

Example iii. *A sector containing an angle of* 195° *is cut from a circle of radius* 7·5 *in. What is the height of the circular cone which can be formed out of this sector?*

Argument.—The sloping edge of the cone will be 7·5 in. We find the radius of the base from the consideration that its circumference will be the same as the arc of the sector.

The length of the arc of the sector $= \frac{195}{360}$ of $2\pi \times 7{\cdot}5$ (see § 196).

$\therefore$ The radius of the base of the cone $= \frac{195}{360} \times 7{\cdot}5$ in.

$= 4{\cdot}062$ in.

The height of the cone $= \sqrt{7{\cdot}5^2 - 4{\cdot}062^2}$

$= \sqrt{11{\cdot}562 \times 3{\cdot}438}$

$= 6{\cdot}30$ in.

2·2900
·8751
3·1651
2·5563
·6088
1·0629
·5363
2·1·5992
·7996

Example iv. *A paper cone has height* 23·5 *cm., and base radius* 16·4 *cm. Its curved surface is cut along a sloping edge, and it is opened out to form a sector of a circle. Find the angle of this sector.*

Argument.—We must find (i) the length of the arc of this sector, i.e. the circumference of the base of the cone; (ii) the radius of the sector, i.e. the sloping edge of the cone.

The circumference of the base $= 2\pi . 16{\cdot}4$ cm.

The sloping edge $= \sqrt{23{\cdot}5^2 + 16{\cdot}4^2}$

$= \sqrt{552{\cdot}3 + 269{\cdot}0}$

$= \sqrt{821{\cdot}3}$

$= 28{\cdot}65$ in.

$\therefore$ The arc of the sector $= \frac{2\pi . 16{\cdot}4}{2\pi . 28{\cdot}65}$ of the whole circumference of the circle formed.

$\therefore$ The angle of the sector $= \frac{16{\cdot}4}{28{\cdot}65}$ of 360°

$= 206°$.

1·2148
2·5563
3·7711
1·4572
2·3139

Examples XV b.

The following columns refer to right circular cones. Fill in the missing terms

	Radius of base.	Height	Volume.	Area of curved surface
1.	3·4 in.	6·5 in.		
2	2 73 in	5 42 in		
3.	73 6 cm.	85·4 cm		
4.	5 47 in.		104 cu. in.	
5.		7 68 in.	273 cu. in.	
6.	29 4 cm			4320 sq. cm.
7.	18 5 cm			1762 sq. cm

8. A sheet of metal in the form of a semicircle of radius 17 cm. is bent into the form of a cone by soldering together the two bounding radii. Find how many litres of water the hollow cone so formed will hold.

9. A solid cone is 6 in high and has a base radius of 3·2 in. You are asked to cut out a single piece of paper to paste on to the sloping surface, just covering it exactly. State exactly how you would cut the paper.

10. A sheet of zinc has to be cut to form a conical funnel The top of the funnel is to be a circle of diameter 5 in, and the hole at the bottom of the funnel is to be a circle of diameter $\frac{1}{2}$ in The funnel is to be 6 in. deep. Give full particulars as to the method of cutting the zinc.

11. A conical steeple has to be covered with lead The steeple is 23 ft. high, and the base radius is 7 ft. What area of lead will be required? If the lead is ·2 in. thick, and weighs 706 lb. per cu. ft., what weight of lead will be required?

12. An oil can is made in the form of a cylinder (diameter 8 in, height 11 in.) surmounted by a frustum of a cone (height 4 in., diameter of top 4 in). How much oil will the can hold?

(Express the result as a decimal of a gallon.)

13. A conical tent has a height of 10 ft, and its base is a circle of radius 7 ft. Find the area of canvas required for its manufacture.

14. A tent is in the form of a right circular cone mounted on a circular cylinder. The base radius is 8 ft, and the total height 12 ft., the cylindrical portion being 2 ft. high Find the area of canvas required, and the number of cubic feet of air in the tent.

THE SPHERE

(216.) The figure represents a sphere which fits exactly into a circular cylinder. The radius of the base of the cylinder is the same as the radius of the sphere, and the height of the cylinder is the same as the diameter of the sphere.

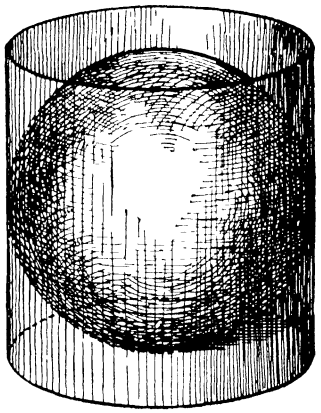


Fig X

It c roved[1] that **the area of the surface of the sphere is exactly equal to the area of the curved surface of the cylinder.**

Also, if any belt of the sphere is cut off by planes parallel to the base of the cylinder, the area of this belt will be equal to the area of the part of the curved surface of the cylinder cut off by the same planes.

[1] We give an outline of the proof, but the average student will probably find it too hard at this stage.

(217.) Figure XII represents a circle inscribed in a square. Prove that the two triangles ONJ and DCK are similar, and hence that $\frac{ON}{NJ} = \frac{DC}{AB}$.

When the cutting planes are very close together, $ON = EA$. Hence, in this case, $NJ \,.\, DC = EA \,.\, AB$.

If the whole figure be revolved about OL as an axis, CD will trace out a portion of a cone, and AB will trace out a portion of a cylinder The areas of the two surfaces traced out will be

$$2\pi \,.\, NJ \,.\, DC, \quad \text{and} \quad 2\pi \,.\, EA \,.\, AB,$$

and the above working shows that these two areas are equal.

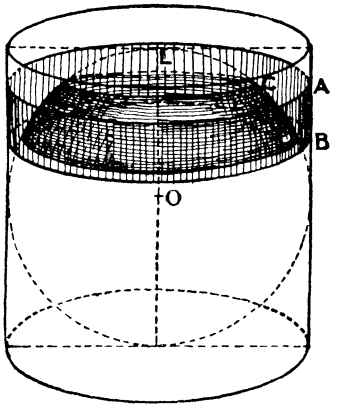


Fig XI.

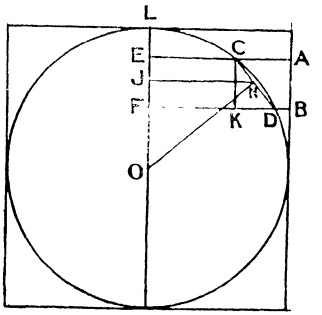


Fig XII.

Considering the surface of the sphere as being made up of a large number of small portions of such cones, we have the final result that **the area of any belt of a sphere cut off by two planes perpendicular to a diameter is equal to the area cut off by the same two planes on a cylinder which just circumscribes the sphere, and whose axis is the diameter of the sphere perpendicular to the planes.** (Fig. XI.)

Since the area of the curved surface of the cylinder is

$$2\pi r \,.\, 2r = 4\pi r^2,$$

the area of the whole surface of the sphere is $4\pi r^2$.

(218.) *Volume of Sphere.*

We regard the sphere as being made up of a number of pyramids each with its vertex at the centre of the sphere, and each having a small portion of the surface of the sphere for base.

The height of each of these pyramids will be practically $= r$, the radius of the sphere; and the total sum of the areas of the bases will be $4\pi r^2$.

$\therefore$ **The whole volume** $= \frac{1}{3} . 4\pi r^2 . r$

$= \frac{4}{3}\pi r^3$.

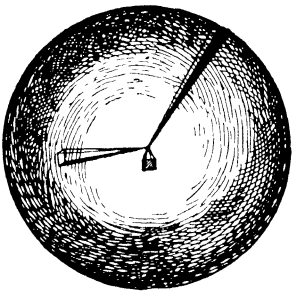


Fig XIII.

Example i. *Find the volume of a sphere whose radius is 3·75 in.*

The volume $= \frac{4\pi . 3{\cdot}75^3}{3}$

$= 220{\cdot}9$

$= 221$ cu. in. (approx.)

6021
·4972
5740 1·7220
2 8213
·4771
2 3442

Example ii. *Find the radius of a sphere whose volume is 736·8 cu. cm.*

If V cu. cm. is the volume, and r cm. the radius,

then $V = \frac{4}{3}\pi r^3$.

$\therefore \quad r = \sqrt[3]{\frac{3V}{4\pi}}$.

In this case, the radius $= 5{\cdot}603$ cm.,

or approximately $5{\cdot}6$ cm.

1771
2 8674
3 3445
·6021
·4972
1 0993
3) 2 2452
·7484

Example III. *A spherical bomb is made of cast iron (sp.gr. 7·150) 1·5 cm. thick, and with an external diameter 13 cm. Find its weight in kilograms.*

The external radius is 6·5 cm.

The internal radius is 5 cm.

$\therefore$ The external volume $= \frac{4\pi \times 6{\cdot}5^3}{3}$ cu. cm. $= 1150$ cu. cm.

The internal volume $= \frac{4\pi \cdot 5^3}{3}$ $= 523{\cdot}7$ cu. cm.

$\therefore$ The volume of metal $= 626{\cdot}3$ cu. cm.

$\therefore$ The weight $= 626{\cdot}3 \times 7{\cdot}150$ grammes $= 1478$ gr. $= 4{\cdot}48$ kg. approx.

·6021
·4972
·8129 2·4387
3·5380
·4771
3 0609

·6021
4972
2 0969
3·1962
·4771
2 7191
2 7968
8543
3 6511

Example IV. *A sphere has a volume of 1000 cu. cm. What is its surface area?*

It is convenient to work in symbols first, obtaining a 'formula' connecting the volume and surface area.

If V cu. cm. is the volume, A sq. cm. the surface area, and r cm. the radius,

then $V = \frac{4}{3}\pi r^3$, (1)

$A = 4\pi r^2$. (2)

(1) Gives $r = \sqrt[3]{\frac{3V}{4\pi}} = \left(\frac{3V}{4\pi}\right)^{\frac{1}{3}}$,

(2) Gives $A = 4\pi \cdot \frac{(3V)^{\frac{2}{3}}}{(4\pi)^{\frac{2}{3}}}$

$= (4\pi)^{\frac{1}{3}} \cdot (3V)^{\frac{2}{3}}$,

or $\sqrt[3]{4\pi \cdot (3V)^2}$,

or $\sqrt[3]{36\pi V^2}$.

In this case the area $= 483{\cdot}7$ sq. cm.

1·5563
·4972
6·
3) 8·0535
2·6845

(219.) A *sector* of a sphere is a figure bounded by a cap of the sphere, and the conical surface formed by radii to the circumference of the circular section forming the edge of the cap. (Fig. XIV.)

Referring to the figure (XV), we see that if we know the radius of the sphere, OA, and the distance, OB, from the centre to a cutting plane, we can find the radius, AB, of the circular section in which the plane cuts the sphere, from the right-angled triangle OAB.

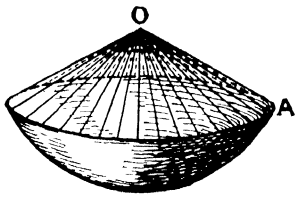


Fig. XIV.

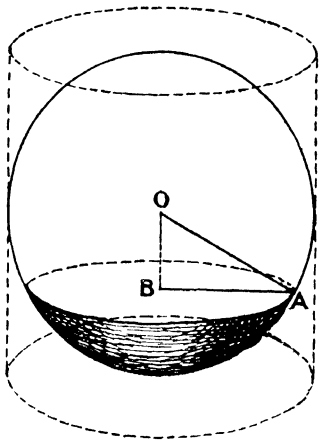


Fig. XV.

Also, the area of the cap is the same as the area of a corresponding portion of the curved surface of the circumscribing cylinder (§ 217), and the volume of the sector is $\frac{1}{3}$ of the product of the area of the surface cut off and the radius of the sphere.

The volume of the conical portion of the sector can be calculated, and hence we can obtain the volume of a *segment of a sphere* (i. e. *a portion cut off by a plane*) by subtracting the volume of the cone from that of the sector.

Example. *A sphere of radius 7 in. is cut into two portions by a plane 3 in. from the centre. Find the volume of the smaller portion.*

(References are to the figure on p. 411.)

We have $OA = 7$ in. and $OB = 3$ in.

$\therefore$ $BA = \sqrt{7^2 - 3^2}$

$= \sqrt{40}$.

$\therefore$ Volume of cone having O for vertex and the circular section for base is

$\frac{1}{3} . \pi . BA^2 . 3$

$= 40\pi$

$= 125{\cdot}7$ cu. in.

Also, area of curved surface of smaller portion = area of curved surface of corresponding portion of the circumscribing cylinder,

$= 2\pi . 7 . 4$ sq. in.

$= 56\pi$ sq. in.

$\therefore$ Volume of sector of sphere, with this portion as surface area,

$= \frac{1}{3} . 56\pi . 7$ cu. in.

$= 410{\cdot}6$ cu. in.

$\therefore$ The volume of the segment

$= 284{\cdot}9$ cu. in.

	1·7482
	·4972
	·8451
	3·0905
	·4771
	2·6134

The segment is the difference between the sector and the cone

Examples XV c.

Find the surface areas of spheres whose radii are :

1. 2·34 in.	2. 3·76 in.	3. 42·3 cm.
4. 15·7 cm.	5. 208 mm.	6. ·0521 m.
7. 6·23 in.	8. 14·3 in.	

Find the volumes of spheres whose radii are :

9. 4·62 in.	10. 5·43 in.	11. 17·9 cm.
12. 16·3 cm.	13. 42·4 mm.	14. ·322 cm.
15. 6·51 in.	16. 2·92 in.	

Find the radii of spheres whose surface areas are:

17. 405 sq. in. **18.** 622 sq. in. **19.** 2394 sq. in.

20. 6050 sq. cm. **21.** 5000 sq. cm. **22.** 3257 sq. cm.

Find the radii of spheres whose volumes are:

23. 522 cu. in. **24.** 1091 cu. in.

25. 3437 cu. in. **26.** 233·5 cu. cm.

27. A million cu. cm. **28.** $2{\cdot}42 \times 10^6$ cu. cm.

29. Find the weight of metal in a spherical bomb whose external diameter is 6 in. and thickness $\frac{1}{2}$ in. if the specific gravity of the metal is 8·78. (Give the answer in lbs.)

30. A sphere of metal of sp. gr. 7·150 weighs 4·02 kg. Its diameter is 10·5 cm. The sphere is supposed to be solid, but it is suspected that there is an air bubble in it. Investigate whether this is so, and find the diameter of the bubble (supposed spherical) if there is one.

31. How many steel ball-bearings (diameter 3·5 mm.) can be made out of a ton of steel?

(Sp. gr. of steel = 7·816. 1 lb. = 453·6 grammes.)

32. An observatory is in the form of a circular cylinder 10 ft. high and 8 ft. radius (external measurement), surmounted by a hemisphere. Find the whole surface area, and the cost of painting the outside at 2*d.* a sq. ft.

33. If lead shot have an average diameter of ·08 in., how many of such shot will go to 10 lb. of lead?

(Lead weighs 709·4 lb. per cubic foot.)

34. Obtain a formula expressing the volume (V) of a sphere in terms of the surface area (A). Use the formula to find the volumes of spheres whose surface areas are:

(i) 522·5 sq. cm. (ii) 68·7 sq. in. (iii) 43·9 sq. in.

35. A solid is in the form of a circular cylinder with hemispherical ends. Obtain formulae for the volume (V) and the surface area (A) in terms of the radius of cross-section r and the total length l.

Use these formulae to find the volumes and surface areas of such solids with the following data:

	Length over all.	Radius of section.
36.	42·4 cm.	7·6 cm.
37.	23·7 cm.	4·5 cm.
38.	54·5 in.	17·4 in.

(Give the results in **38** in cu. ft. and sq. ft.)

39. Find the volume of a sector of a sphere of radius 18 in., the radius of the circular edge of the sector being 7 in.

40. Find the volume of the larger portion cut off from a sphere of radius 14·6 cm. by a plane distant 8·4 cm. from the centre.

41. A bowl is in the form of the larger segment of a sphere of radius 8 in. cut off by a plane which gives a circular rim of radius 7 in. Find the surface area of the bowl.

42. Find the total surface area of a sector of a sphere, being given

Radius of sphere = 16·3 in.

Radius of circular edge of sector = 12·2 in.

43. Assuming the earth to be a sphere of diameter 8100 miles, find the area of the whole surface north of Lat. 30°.

(N.B. The latitude is measured from the equator.)

44. A sphere is cut into six portions of equal surface area by parallel planes. Find the ratios between the volumes of these portions.

MISCELLANEOUS EXAMPLES IN MENSURATION

1. A box with a lid measures externally 32 in. by 27 in. by 10 in., and is made of boards an inch thick. Find the number of cubic inches in the interior of the box.

2. A square reservoir, each side of which is 2 furlongs in length, collects water from an area of 20 sq. miles. It is observed that on a given day the water in the reservoir rises 20 in., and it is assumed that only one-sixteenth of the rainfall reaches the reservoir. How many inches fell on that day?

3. Draw on squared paper a triangle whose sides are 27, 36, and 38 units in length, taking as your unit the side of the smallest square on the paper. Find the area of the triangle by counting squares, and verify by any other means.

4. A kilometre is ·621 of a mile. Find the number of acres in a square field, the perimeter of which is a Km.

5. The floor of a corridor 6 ft. wide and 18 ft. long has a strip of boarding 2 ft. wide down the middle, and the remainder is filled with tiles, each of which is 4 in. by 9 in. How many tiles are there?

6. Find the area of the base of a cubical cistern which contains 300 litres of water when filled to a depth of $2\frac{1}{2}$ metres.

7. Find the number of litres of water in a rectangular trough whose length is 35·3 metres and width 56 cm., the depth being 74·8 mm.

8. A closed wooden rectangular packing case is made of wood 1 in. thick and weighing 48 lb. per cubic foot. If its external dimensions are 26 in., 19 in., and 12 in. respectively, find its cubical contents and its weight.

9. If a man can run round the boundary of a square field whose area is 12 acres in 3 minutes, how long would he take to run at the same rate round another square field whose area is 8 acres?

10. An excavator, used to make a railway tube, cuts a tube 30 ft. in diameter, and moves through 14 ft. per day. How many cubic yards of earth does it remove per day?

11. Two of the angles of a triangle are 65° and 37°, and the length of the side adjacent to them is 2 cm. Construct the triangle, and find its area by any suitable measurement.

12. Draw on squared paper a circle of radius 2 in., with a chord at a distance ·73 in. from the centre. Find by counting squares the area of the segment cut off by the chord.

13. A room is 20 ft. long, 16 ft. broad, and $10\frac{1}{2}$ ft. high. Find the cost of painting the walls at the rate of fourpence per square foot, the area of the windows and fireplace being taken as 25 sq. ft.

14. Find, to the nearest ounce, the weight of water that can be contained in a vessel 2 ft. 6 in. long, 1 ft. $4\frac{1}{2}$ in. wide, and $7\frac{3}{4}$ in. deep.

15. A rectangular cistern is 75 cm. deep, and its breadth is one-half of its length. If the capacity of the cistern is 1265 litres, find the length.

16. In a quadrilateral $ABCD$, AB is 10, BC 9·21, AD 6·45, AC 11·7, DB 10·5. Find the area of the quadrilateral.

17. The area of a rectangular field, whose length is three times its breadth, is 10 acres The field is to be divided into four fields by two hedges planted along the straight lines joining the middle points of the opposite sides. What will be the total cost of planting the hedges at the rate of 1*s.* 6*d.* for every 10 yards?

18. The sides of a triangular field are 2425, 3725, 4000 links. Find the number of roods in the area of the field.

19. The internal measurements of a rectangular cistern are: length 2·40 metres, breadth 1·22 metres, and height 1 metre. How many litres of water will it hold?

20. A map is drawn to a scale of 1 in. to 5 miles. If the map is 7 ft. long and 3 ft. 5 in. wide, what area, in square miles, is represented? What is the greatest distance that can be shown on the map?

21. Brass expands when heated so that a unit of length at the freezing-point of water becomes 1·0019 at the boiling-point of water. Find the increase in volume when a brass cube whose edge is 9 in. at the freezing-point is heated to boiling-point of water.

22. Find, by drawing and measurement, the area of the following polygon

Sides. $AB = 1·9$. $BC = 1$. $CD = 1·85$. $DE = 1·75$. $EA = 1·15$.
Diagonals. $AC = 2·17$. $CE = 2·4$.

(Measurements are in inches.)

23. The area of an equilateral triangle is 100 sq. ft. Find the length of a side, correct to the nearest inch.

24. A square field contains $\frac{2}{5}$ of an acre. What will it cost to put a fence of wire netting round it, leaving an opening of 6 ft. for a gate, wire netting being sold at 11*s.* for a roll of 50 yds., and odd lengths at $3\frac{1}{2}d.$ per yard?

25. What will it cost to line a box without a lid, 3 ft. long, 2 ft. wide, and 8 in. deep, with a material costing 2*s.* 6*d.* a sq. ft.?

26. A square field has an area of $5\frac{1}{2}$ acres. Find the cost of a fence round it at 3*s.* 9*d.* per yard?

27. Two liquids weigh 3·41 g. and 1·24 g. per cu. cm. When they are mixed in the proportion of 1 by weight of the heavier to 4 of the lighter, they contract so that the volume of the mixture is $6\frac{1}{4}$ per cent. less than the total volume before mixture Find the weight per cu. cm. of the mixture.

28. A garden, two-fifths of an acre in extent, is watered by a hose delivering 2 gallons per minute. Find the mean depth of the water supplied by the hose in 1 hour and 50 minutes, assuming that a cubic foot of water contains $6\frac{1}{4}$ gallons.

29. A cistern is 5 ft. long and $2\frac{1}{4}$ ft. wide. How many bricks, each 9 in. by $4\frac{1}{2}$ in. by 3 in., must be placed in the cistern to cause the water to rise 1 ft., supposing that each brick absorbs one-fifth its volume of water?

30. The rise of the tide in Hobson's bay, which is approximately a circle of diameter 45 miles, is 18 in. Find the number of tons of water which flow in.

31. Construct a quadrilateral $ABCD$, in which AB is 8 cm., BC is 5 cm., CD is 3 cm , DA is 5 cm., and the angle BAD is 60°. Find the area of this quadrilateral.

32. A certain map is on a scale of 6 in. to a mile. What area, to the nearest acre, is represented by a rectangle 2·4 in. by 3 in., and what area on the map will represent a farm of 550 acres?

33. A piece of aluminium 1 sq. cm. in area weighs 1·2 dg. How many such pieces can be cut from a plate of the same thickness which weighs 1·5 Kg.?

34. A pond of 8 acres is frozen over with ice, the total weight of which is 2178 tons. Find the average thickness of the ice, if a cubic foot weighs 896 oz.

35. Find the diameter of a circular plot the area of which is $346\frac{1}{2}$ sq. yds.

36. A cylindrical tank, 10 ft. in diameter, contains 12345 gallons of water. Calculate the depth, to the nearest inch, if 11 gallons occupy approximately 3050 cu. in.

37. What will it cost to cover with lead the inside of an open cistern if its depth, length, and breadth are 3 ft. 6 in., 7 ft. 10 in., and 5 ft. 4 in. respectively, and the work costs 10*s.* $1\frac{1}{2}d.$ per square yard?

38. Buttons are made with lead disks ·8 in. across and ·09 in. thick If lead weighs 711 lb. per cu. ft., find the weight of lead required for 1000 buttons.

39. What is the average depth of a rainfall in which 7 million litres of water fall on a square kilometre?

40. Two circular ponds of diameter 20 and 24 ft. contain water to a depth of 4 ft. 6 in. The water is drained off from both into a third circular pond, in which the depth of the water is increased by 3 ft. 9 in. Find the diameter of the third pond.

41. A tank holding 240 litres is filled in 10 min. by a pipe, the internal diameter of which is 1·9 cm. What is the rate of flow of the water in the pipe in metres per second?

42. Pencils are made 7 mm. in diameter and 17 cm. long. A box is made to hold six such pencils, placed side by side in a single layer. What are the smallest internal length, breadth, and depth of the box?

Find also what percentage of the space inside the box is occupied by the pencils.

43. Find the area of a triangle whose sides are 3·9, 3·2, and 4·2 in.

44. The length, breadth, and height of a rectangular room are l, b, and h yards respectively.

Find formulae for (i) V, the volume of the room; (ii) A, the total area of walls, ceiling, and floor; (iii) D, the diagonal of the room. Evaluate these when $l = 8$, $b = 6$, $h = 4$.

45. An irregular six-sided polygon $ABCDEF$ has each of its angles A, B, C, D, 120°, the sides FA, AB, BC, CD, and DE being respectively 1, 1, 3, 2, and $1\frac{1}{2}$ in. long. Draw the polygon and find its area.

46. Find the area of the largest square which could be cut out of a circular field 10 acres in area.

47. Calculate the area of a sector of a circle, radius 17·5 cm., which is bounded by radii making an angle of 70° with each other.

48. Find the weight of a Km. of copper wire of 3 mm. diameter, if the sp. gr. of copper is 8·9.

49. The area of the base of a cistern is 100 sq. ft., and its depth is 10 ft. How many gallons of water will it hold? If the base had been 100 sq m., and the depth 10 dm., how many litres of water would it have held?

50. The sides of a triangular plot are 73·4 ft, 117 ft., and 100 ft. Find the number of acres in the area of the plot.

51. A figure of a certain plot of ground was made on a thin sheet of metal on a linear scale of 1 to 500. The figure was cut out and found to weigh 228 gm. The metal sheet had a mass of ·57 gm. per sq. cm. What was the area of the plot in square metres?

52. Find, to within one yard, the diameter of a circular enclosure whose area is 12 acres.

53. A barometer tube 125 cm. long has an internal diameter of 24 mm.

Find the cost of filling it with mercury at 6*s.* 3*d.* a Kg., taking the sp. gr. of mercury to be 13·6.

54. AB is an arc of a circle which subtends an angle of 60° at the centre of the circle. If the radius of the circle is 14 in., find the difference between the lengths of the chord AB and the arc AB.

55. An exercise ground is in the form of an irregular polygon, A, B, C being consecutive vertices and ABC a re-entering angle. AB is 121 yds., BC 706 yds., CA 725 yds. If the fences AB, BC are removed and one is drawn from A to C, by how many acres will the area of the ground be increased?

56. Find the weight in Kg. of an iron girder which is 5·4 metres long, ·54 metres wide, and ·45 metres thick, if the sp. gr. of iron is 7·76.

57. Out of a circular disk of metal 140 equal circular holes are punched, and the weight of metal thus removed is to the weight of the original disk in the ratio of 45 to 112. Find the ratio of the diameter of the disk to that of each hole.

58. A pillar of stone weighing 5 tons with a square section is to be worked down to one of circular section with a diameter less by one-twentieth than the side of the original square. Find, to the nearest lb., the weight of the stone removed.

59. A diagram in the form of a closed curve was traced out on a sheet of lead of uniform thickness (·5 mm.); the diagram was accurately cut out and weighed; its weight was 3·61 grammes.

Find the area of the figure in sq. cm., assuming that the sp. gr. of lead is 11·35.

60. A pyramid has a square base of 3 in. side, and the length of a sloping edge is 4 in. Find the volume of this pyramid.

61. Draw a triangle with angles 45°, 60°, 75°, and the shortest side of length 75 mm. Find its area by measuring one of the altitudes, and calculate the length of the shortest side of a similar triangle whose area is 10 sq. cm.

62. On a map drawn to the scale of an inch to a mile, a certain estate covers an area of 2·44 sq. in. Find the real area of the estate in acres.

63. The dimensions of four circular disks made of the same material are as follows: the first, diameter 1 in., thickness $\frac{1}{4}$ in.; the second, diameter 2 in., thickness $\frac{1}{2}$ in.; the third, diameter 3 in., thickness $\frac{3}{4}$ in.; the fourth, diameter 4 in., thickness 1 in.

It is required to construct a circular disk, 1 in. thick, of the same material, whose weight shall be equal to the weight of the other four. What must its diameter be?

64. Find the volume and weight of an iron pipe 5 ft. long, its outer diameter being 14·5 in., and the thickness of metal being ·94 in. The sp. gr. of the iron is 7·8.

65. A piece of wire $3\frac{1}{2}$ metres long, with a sectional area of 3 sq. mm., is heated, and then drawn to a length of 18 metres.

Find, to the nearest hundredth of a square mm., the area of a cross-section of the wire thus drawn.

66. A tent, wholly of canvas, is in the form of a cone resting on a cylinder 18 in. high and 12 ft. in diameter, and the vertex of the cone is $8\frac{1}{2}$ ft. above the ground. Find the cost of the canvas at 5*s.* per sq. yd.

67. If in a map x ft. is represented by 1 in., then the side of the square of area 1 acre is represented approximately by $200/x$ in.

Examine the truth of this statement for a map drawn on the scale of 30 ft. to 1 in., and find by how much per cent. the result obtained by the rule differs from the true result.

68. Assuming that the diameters of a penny and of a halfpenny are one-tenth of a foot and one inch respectively, and that their weights are one-third of an ounce and one-fifth of an ounce respectively, compare the thickness of a penny with that of a halfpenny.

69. Two solid leaden cylinders, the first having a diameter of 5 cm and a height of 6 cm., and the second having a diameter of 3 cm. and a height of 4 cm., are melted down and formed into a hollow cylinder 4 mm. thick, of which the inner radius is 4 cm. What will be its height?

70. A litre of a mixture of water and glycerine weighs 1100 gm. If the sp. gr. of pure glycerine is 1·25, what is the proportion of water to glycerine in the mixture?

71. Find the total surface area of a right pyramid 3·75 in. high and with a square base of 2·1 in. side.

72. Find the radius of a spherical globe which will hold 3 gallons.

73. The length of a rectangular field is three times as great as its breadth, and its area is 63·48 Ha. What length of fence is required to enclose it?

74. A cylindrical log is to be sawn into a post whose cross-section is a regular hexagon. Find the percentage waste of wood.

75. If two cylindrical pieces of metal of the same length and of radii 4·32 in. and 5·87 in. are fused and moulded into one cylindrical piece of the same length as before, calculate approximately the radius of this cylinder.

76. How far does a roller, width 5 ft. 3 in., have to be pulled in rolling a rectangular field of area $7\frac{1}{4}$ acres? Allow 10 per cent. extra for turning, and assume that the whole field can be covered without one crease overlapping another.

77. Find how many cu. cm. of liquid whose sp. gr. is ·936 must be mixed with 100 cu. cm. of water to give a liquid of sp. gr. ·954. (Assume that there is no contraction on mixing.)

78. Find the diameter of a hemispherical bowl which will hold 3 gallons of water.

79. There are two liquids which do not contract when mixed, and their specific gravities are ·6 and ·75.

If equal volumes are mixed, what is the specific gravity of the mixture? If equal weights are mixed, what is the specific gravity of the mixture?

80. The driving wheel, 5 ft. in diameter, of a locomotive engine turned 2500 times in going 7 miles. Find what distance was lost owing to the slipping of the wheel on the rail.

81. A certain pint pot is 5 in. high. Of what height must a pot of the same shape be made to hold half a pint?

82. A solid sphere contains 1000 cu. in. Find its radius.

83. The curved surface of a right circular cone, whose height is the same as its base radius, is made by cutting out a sector from a circular sheet of copper, the diameter of the sheet being 56 cm. Find the angle of the sector.

If the metal is 1·4 mm. thick, what will be the weight of the cone, assuming that its base is made of metal of the same thickness, and that the sp. gr. of copper is 8·94?

84. The volume of a cask is $0{\cdot}2618\,H(A^2+AB+B^2)$ cu. ft., where H is the height of the cask, A the bung diameter, and B the head diameter, H, A, B being expressed in feet. How many gallons will a cask hold whose height is 2 ft. 6 in., bung diameter 2 ft. 3 in., and head diameter 1 ft. 9 in.?

(A gallon is 277·3 cu. in.)

85. $ABCD$ is a quadrilateral such that $AB = 5{\cdot}2$ cm., $BC = 3{\cdot}9$ cm., $CD = 2{\cdot}5$ cm., $DA = 6{\cdot}0$ cm., and the angle ABC is

a right angle. Calculate the length of AC, and prove that the angle CDA is a right angle.

Calculate the area of the figure.

86. The volcanic dust from La Soufrière weighed 115 lb. per cubic foot and formed a layer $\frac{3}{4}$ in. thick. Find the number of tons which fell on an acre.

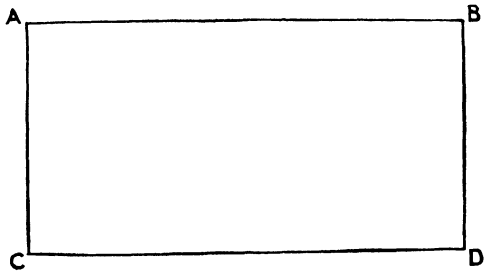
87. A tunnel of diameter 22 ft. is driven $4\frac{1}{2}$ miles. Find in acres what area the earth removed will cover when spread to a depth of $3\frac{1}{4}$ ft.

88. The sides of a triangular field are 100, 135, and 145 yds. long. Find the mean radius of the greatest circular track 30 ft. wide that could be constructed in the field.

89. A tin bucket has its top and bottom diameters 12 in. and 9 in. respectively, and the slant side of the bucket measures 10 in. Draw to scale a pattern for the sheet of tin which can be bent so as to form the sides of the bucket.

Calculate also the area of the tin and the volume of the bucket

90. A bowl is made in the form of part of a hemispherical surface, fitted with a flat bottom. If the diameter of the hemisphere is 10 in. and the depth of the bowl 4 in., find the volume and the surface area of the bowl.



91. The figure $ABDC$ represents, on a scale of 2 in. to the foot, a sheet of metal which is to be bent until AC joins BD, forming a circular cylinder. Calculate the area of the circular bottom that would have to be fitted, and the volume of the vessel so formed.

92. Water is flowing at the rate of 16 Km. per hour through a pipe of cross-section 500 sq. cm. How long will it take to raise the level of a reservoir, 170 metres long and 79 metres wide, by 5 cm.? What volume, in litres, will have passed through the pipe during that time?

93. 1000 yds. of paper, 2 ft. in breadth, are rolled up into a cylinder with internal diameter 4 in. and external diameter 14 in. What is the thickness of the paper?

94. Three equal sticks, 4 ft. long, rest each with one end on the ground, the other ends meeting at a point $3\frac{1}{2}$ ft. above the ground. Find the distance between the ends on the ground.

95. How many shot, each of ·05 in. radius, can be made out of a circular disk of lead 10 in. in diameter and $2\frac{1}{2}$ in. in thickness?

96. The volumes of a cone and a sphere are equal, and the height of the cone is equal to the radius of the base. Find the ratio of the total surface of the cone to that of the sphere.

97. On Aug. 26th, 1912, the rainfall at Yarmouth measured 3·07 in. Calculate the number of tons of water that fell per acre. (1 cu. ft. weighs 1000 oz.)

98. Find the number of acres in a square field, a diagonal of which is 1210 ft.

99. The surface areas of two reservoirs are 2000 sq. ft. and 3500 sq. ft. respectively. If the first discharges into the second at the rate of 500 gallons a minute, how many minutes will it be before the difference in the two levels is diminished by 1 ft.?

100. A book of gold leaf contains 25 leaves $3\frac{1}{3}$ in. square and $\frac{1}{282000}$ of an inch thick. If a cubic inch of gold weighs ·697 lb., and 1 lb. of pure gold is worth £61 19*s.*, find the value of the gold in the book.

101. The external diameter of a hollow sphere is 38·58 cm., and the metal of which it is made is 1 cm. thick. What is the volume of the metal, and what is the internal capacity?

102. A snuff-box is formed by cutting part off a sphere of radius 5 in. by a plane section of radius 3 in. which forms the lid of the box. Find its total surface area and its volume.

103. Obtain a formula for the volume of iron in a pipe of length L, external diameter D, and thickness of metal t.

If 1 cu. ft. of metal weighs 480 lb., calculate the weight of such a pipe of length 10 ft. and external diameter 6 in. for the following thicknesses of metal: $\frac{1}{8}$ in., $\frac{1}{4}$ in., $\frac{3}{8}$ in., $\frac{1}{2}$ in.

104. A flagstaff stands vertically on a horizontal ground. Four ropes, each 56 ft. long, are stretched from a point in the flagstaff 50 ft above the ground to four pegs in the ground, arranged at the corners of a square. Calculate the distance between two adjoining pegs.

105. Find the area cut off from a spherical surface of radius 6 in. by a plane at a distance 4 in. from the centre.

106. Water is discharged at the rate of 500 gallons per minute from a reservoir with a surface of 7200 sq. ft into one with a surface of 5400 sq. ft. Assuming that $6\frac{1}{4}$ gallons occupy one cubic foot, find how many minutes it will take to diminish by 3 in. the difference between the levels of the surfaces.

107. A sphere of radius 6 in. is placed in a hollow cylindrical vessel with a flat base of radius 8 in. Water is poured in until the sphere is half immersed. If the sphere be then removed, how high will the water stand in the vessel?

108. A horse is tethered by a rope 16 yds. long to the corner A of a square hut $ABCD$ whose sides are 12 ft. in length. The horse first walks along BA produced until the rope is fully stretched, and then, keeping it stretched, walks round the hut in the direction indicated by the order of the letters $ADCB$. Draw the path described by the end of the rope on a scale of 12 ft. to an inch, and find the area of the figure traced out by the rope in any way you please.

109. A photograph in the railway station at Lucerne is about 20 metres long and 2 metres high. If six pieces of bromide paper measuring $8\frac{1}{2}$ by $6\frac{1}{2}$ in. and two pieces measuring 6 by $4\frac{1}{4}$ in. can be bought for a shilling, what would be the cost of the bromide paper required for a photograph of this size?

(1 yard = ·9144 metres.)

110. I go to an ironmonger and ask for 18 yds. of a certain wire. The wire is in a coil, 20 in. in diameter. The ironmonger gets the right length by counting off a certain number of rounds from the coil. How many rounds did he count off?

111. A model of a casting is made of wood of sp. gr. ·672, and it weighs 37·4 g. The casting is made of steel of sp. gr. 7·81 and is found to weigh 432·2 g. Show that there is a hollow somewhere in the casting, and find the weight of metal which would just fill it.

Suppose the hollow to be in the form of a spherical bubble, find the diameter of the bubble.

112. A rectangular stone trough is 3 ft. 8 in. long, 2 ft 7 in. broad, and 1 ft. $2\frac{1}{2}$ in. deep externally, and weighs, when empty, 863 lb. It will hold 39 gallons of water. Find the specific gravity of the stone.

(1 cu. ft. of water weighs 62·3 lb., and a gallon of water weighs 10 lb.)

113. During a recent snowfall in London, an analysis of the melted snow is said to have given an average of 30 grains of solid matter to every gallon of water. Taking snow a foot deep as equivalent to ·95 in. of rain, find to the nearest ton the weight of solid matter deposited over an area of 16 sq. miles by a snowfall 8 in. deep.

114. The horizontal cross-sections of the crater of a volcano are circles with their centres all in one vertical straight line. The radii of the circles at different heights are given in the table below. Draw the shape of the vertical section of the crater. Also plot a curve giving the areas of the horizontal sections at different levels, and deduce the volume of the crater in cubic feet.

Height in feet .	0	10	20	30	40	50	60	70	80
Radius in feet .	0	17	24	27	30	35	41	52	65

115. A fine tube of uniform cross-section is 12 cm. long, and contains ·45 g. of mercury. If the sp. gr. of mercury is 13·59 grammes, calculate the radius of cross-section of the tube.

116. The volume of a sphere is 5 cu. in. Find the area of its surface.

117. A space in the form of a rectangle 14 yds. wide with semicircular ends is surrounded with palisading, the length of which is 100 yds. Find the area enclosed.

118. Two circles, radii 32 and 7 in. respectively, have their centres 65 in. apart. Find the lengths of all their common tangents.

119. A cylindrical bolt, of diameter $\frac{3}{4}$ in. and total length $4\frac{1}{2}$ in., has a head in the form of a segment of a sphere. If the diameter of the rim of the head be $1\frac{1}{2}$ in. and its central thickness $\frac{1}{2}$ in., find the weight of the bolt, taking the weight of a cubic inch of wrought iron to be 4·4 oz.

120. A cylindrical jar, whose radius (inside the jar) is 6·25 cm., contains water. How many metal spheres, the radius of each being 3 cm., must be put in to make the surface of the water rise more than 4 and less than 5 cm.?

Find to the nearest mm. how high the surface rises when this number are put in.

121. A wire fence encloses a square field whose area is $2\frac{1}{2}$ acres. Find the length of the fence.

122. An aviation course is in the form of an equilateral triangle, measuring $1\frac{1}{2}$ Km. along each side, and marked out by posts at the vertices. Find the distance that must be traversed in one round by an aeroplane which turns in arcs of 50 metres radius, just grazing the posts. Give a sketch to illustrate your answer.

123. A circular running track is 15 ft broad and $\frac{1}{4}$ mile round, the measurement being taken at 3 ft. from the inner edge of the path. Find the number of cubic yards of cinders required to cover the path to a depth of 4 in.

124. Two men camping out on the river bank have a conical tent 10 ft. high and 12 ft. in diameter at the base. How many square yards of canvas are required to make the tent, allowing 20 % more material than the bare surface of such a cone; and how many cubic feet of air will each person have in the tent when it is closed?

125. A hollow sphere of iron is of uniform thickness. Its external diameter is 9 in. and its weight 72 lb. Find the volume of the internal cavity. The iron weighs 480 lb. per cu. ft.

126. A hemispherical dome has a diameter of 20 ft. and is protected by a thin covering of lead. The lead is removed, and is found to be just sufficient to cover a conical steeple with lead of the same thickness. Find the vertical height of the steeple to the nearest inch if the diameter of its base is 10 ft.

Is it necessary to know the value of π to work this question?

127. Two dogs are chained to posts 25 ft. apart by chains 15 and 20 ft. long respectively. Find, by drawing to scale on squared paper, the area within which a fight can take place (neglecting the dimensions of the dogs).

128. The area of the cross-section of a pipe is 7 sq. in. Assuming the water in the pipe to flow uniformly at 2 miles an hour, how long will it take to fill a cistern 7 ft. long, 4 ft. wide, and 2 ft. 9 in. deep?

129. Find the volume of a spherical shell whose external and internal radii are 9·5 cm. and 8·5 cm. respectively.

130. Three edges of a rectangular cuboid which meet in a point O are bisected at A, B, and C. Prove that the volume of the pyramid $OABC$ is $\frac{1}{48}$ of the volume of the cuboid.

131. Three balls, 5 cm. in diameter, lie on a floor in contact, and a fourth equal ball is placed on them. Find the height of the centre of the fourth ball above the floor.

132. If a diameter of a sphere is divided into four equal parts, and at the points of division planes are drawn perpendicular to the diameter, prove that the areas of the parts into which the surface of the sphere is divided are equal, and the volumes of the portions into which the sphere is cut are in the ratio 5, 11, 11, 5.

CHAPTER XVI

TABLES

(220.) The Arithmetical sums required by any individual are usually of one particular kind, for instance, the man who has to calculate the value of tons, &c., will not as a rule have to calculate the value of yards. In the course of years each trade or business has compiled tables dealing with the calculations it requires, and thus much labour is saved. Occasionally, however, the need for fresh tables arises. We proceed to show how tables may be compiled and how they may be used.

Example 1. *Construct a table to show the value of any number of ounces up to* 5 *lb. at* 8*d.*, 9*d.*, 10*d.*, 11*d.*, *and a shilling a lb. The values are to be given correct to the farthing just greater than the true value.*

1 oz. at 8*d.* a lb. is worth ·5*d.*

1 oz. at 9*d.* a lb. is worth $\frac{1}{4} \times 2{\cdot}25$, i.e. ·5625*d.*

1 oz. at 10*d.* a lb. is worth $\frac{1}{4} \times 2{\cdot}5$, i.e. ·625*d.*

1 oz. at 11*d.* a lb. is worth $\frac{1}{4} \times 2{\cdot}75$, i.e. ·6875*d.*

1 oz. at 12*d.* a lb. is worth $\frac{1}{4} \times 3$, i.e. ·75*d.*

These values ought to differ by $\frac{1}{16}d.$, i.e. ·0625*d.* This provides a check on the accuracy so far. By multiplication the table is now obtained.

Price per lb.		8*d.*	9*d.*	10*d.*	11*d.*	12*d.*
lb.	oz.	*s.* *d.*	*s.* *d.*	*s.* *d.*	*s.* *d.*	*s.* *d.*
	1	$\frac{1}{2}$	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{3}{4}$
	2	1	$1\frac{1}{4}$	$1\frac{1}{4}$	$1\frac{1}{2}$	$1\frac{1}{2}$
	3	$1\frac{1}{2}$	$1\frac{3}{4}$	2	$2\frac{1}{4}$	$2\frac{1}{4}$
	4	2	$2\frac{1}{4}$	$2\frac{1}{2}$	$2\frac{3}{4}$	3
	5	$2\frac{1}{2}$	3	$3\frac{1}{4}$	$3\frac{1}{2}$	$3\frac{3}{4}$
	6	3	$3\frac{1}{2}$	$3\frac{3}{4}$	$4\frac{1}{4}$	$4\frac{1}{2}$
	7	$3\frac{1}{2}$	4	$4\frac{1}{2}$	5	$5\frac{1}{4}$
	8	4	4	$5\frac{1}{2}$	$5\frac{1}{2}$	6
	9	$4\frac{1}{2}$	$5\frac{1}{4}$	$5\frac{3}{4}$	$6\frac{1}{4}$	$6\frac{3}{4}$
	10	5	$5\frac{3}{4}$	$6\frac{1}{4}$	7	$7\frac{1}{2}$
	11	$5\frac{1}{2}$	$6\frac{1}{4}$	7	$7\frac{3}{4}$	$8\frac{1}{4}$
	12	6	$6\frac{3}{4}$	$7\frac{1}{2}$	$8\frac{1}{4}$	9
	13	$6\frac{1}{2}$	$7\frac{1}{2}$	$8\frac{1}{4}$	9	$9\frac{3}{4}$
	14	7	8	$8\frac{3}{4}$	$9\frac{3}{4}$	$10\frac{1}{2}$
	15	$7\frac{1}{2}$	$8\frac{1}{2}$	$9\frac{1}{2}$	$10\frac{1}{2}$	$11\frac{1}{4}$
1	0	8	9	10	11	1 0
2	0	1 4	1 6	1 8	1 10	2 0
3	0	2 0	2 3	2 6	2 9	3 0
4	0	2 8	3 0	3 4	3 8	4 0
5	0	3 4	3 9	4 2	4 7	5 0

With this table we can write down at once the values of any weight less than 5 lb.

3 lb. 10 oz. at 8*d.* a lb. cost 2*s.* 5*d.*
4 lb. 7 oz. at 10*d.* a lb. cost 3*s.* $8\frac{1}{2}$*d.*

Example 11. *Construct a table for expressing cwt., qrs., lb., as a decimal of a ton correct to six places.*

1 cwt. = ·05 of a ton.
14 lb. = ·00625 of a ton.
1 lb. = ·00044642 of a ton.
1 qr. = ·0125 of a ton.
7 lb. = ·003125 of a ton.

lb.	ton.	cwt.	qrs.	ton.
1	·000446		1	·0125
2	·000893		2	·025
3	·001339		3	·0375
4	·001786	1	0	·05
5	·002232	2	0	·1
6	·002679	3	0	·15
7	·003125	4	0	·2
8	·003571	5	0	·25
9	·004018	6	0	·3
10	·004464	7	0	·35
11	·004911	8	0	·4
12	·005357	9	0	·45
13	·005803	10	0	·5
14	·006250	15	0	·75
21	·009375			

7 cwt. 2 qrs. 6 lb. = ·377679 of a ton (i.e. ·35 + ·0250 + ·002679).

13 cwt. 3 qrs. 24 lb. = ·6875 + ·009375 + ·001339 = ·698214 of a ton.

Example III. *In a certain district rates are to be levied at* 2*s.* 1*d. in the £. Construct a table to show the amount of rate on rateable values up to* £100.

Value	Rate.		Value	Rate		
£	*s*	*d.*	£	£	*s*	*d*
·5	1	$0\frac{1}{2}$	10	1	0	10
1	2	1	20	2	1	8
2	4	2	30	3	2	6
3	6	3	40	4	3	4
4	8	4	50	5	4	2
5	10	5	60	6	5	0
6	12	6	70	7	5	10
7	14	7	80	8	6	8
8	16	8	90	9	7	6
9	18	9	100	10	8	4

The rate on £38 is £3 19*s.* 2*d.*,

and on £173 10*s.* is £17 14*s.* 2*d.* + 7*s.* $3\frac{1}{2}d.$ = £18 1*s.* $5\frac{1}{2}d.$

Example iv. *Construct a table for reducing yards, feet, inches to the decimal of a metre correct to three decimal places, being given that* 1 *yd.* = ·91438 *of a metre.*

Hence 1 ft. = ·30479 of a metre, 1 in. = ·025399 of a metre.

ft.	in.	metre.	yds.	metres.
	1	·025	1	·914
	2	·051	2	1·829
	3	·076	3	2·743
	4	·102	4	3·658
	5	·127	5	4·572
	6	·152	6	5·486
	7	·178	7	6·401
	8	·203	8	7·315
	9	·229	9	8·229
	10	·254	10	9·144
	11	·279	11	10·058
1	0	·305	12	10·973
2	0	·610	13	11·887

Example v. *Construct a table for expressing shillings and pence per yard as francs and centimes per metre; being given*

$$1 \textit{ metre} = 1{\cdot}0936 \textit{ yds.}, \quad 25{\cdot}225 \textit{ francs} = £1.$$

1 shilling per yard $= \dfrac{25{\cdot}225}{20}$ francs per $\dfrac{1}{1{\cdot}0936}$ metre

$= \dfrac{25{\cdot}225 \times 1{\cdot}0936}{20}$ francs per metre

$= 1{\cdot}3803$ francs per metre.

1 penny per yard = ·1150 francs per metre.

1 farthing per yard = ·0288 francs per metre.

25·225
·05468
1·26125
·10090
1513
202
1·3803

Per yard. *d.*	Per metre. Francs.	Per yard. *s.*	*d.*	Per metre Francs
$\frac{1}{4}$	·029	1	0	1·379
$\frac{1}{2}$	·058	2	0	2·759
$\frac{3}{4}$	·086	3	0	4·138
1	·115	4	0	5·517
2	·230	5	0	6·897
3	·345	6	0	8·276
4	·460	7	0	9·655
5	·575	8	0	11·034
6	·689	9	0	12·414
7	·804	10	0	13·793
8	·919	12	6	17·241
9	1·034	15	0	20·690
10	1·149	17	6	24·138
11	1·264	20	0	27·586

Example 13*s.* $2\frac{3}{4}$*d.* per yard = 17·931 + ·316 = 18·25 francs per metre.

Examples XVI a.

Use the tables in § 220 to work the following examples:

1. Find the total value of 4 lb. 5 oz. at 8*d.* per lb., 6 lb. 13 oz. at 11*d.* per lb., and 3 lb. 11 oz. at 10*d.* per lb.

2. Find the value of 13 cwt. 2 qrs. 13 lb. at £3 4*s.* 7*d.* per ton.

3. What amount of rates are due on a house rated at £59 10*s.* when the rates are 2*s.* 1*d.* in the £?

4. What will 3·30 metres cost at 5*s.* 3*d.* per yard?

5. What is the value of 13·75 metres if a yard is worth 2*s.* $11\frac{3}{4}$*d.*?

6. By how many metres does a hectometre exceed 100 yds.?

Given 1 metre = 1·09363 yd., 1 mile = 1·60931 kilometre,
kilogram = 2·20462 lb., 1 lb. = ·43593 kilogram.

Construct the following tables:

7. To express shillings and pence as the decimal of £1 correct to five places.

8. To express metres and centimetres up to 20 metres as the decimal of a yard correct to three decimal places.

9. To express miles and furlongs up to 9 miles as the decimal of a kilometre correct to three decimal places.

10. To express kilometres as the decimal of a mile correct to three decimal places.

11. To express kilograms as the decimal of a pound correct to three decimal places.

12. To express the price of a lb. in shillings and pence as the price of a kilogram in francs and centimes.

13. To reduce £ *s.* *d.* to francs (£1 = 25·225 francs).

DECIMALIZATION OF MONEY

(**221.**) The method for mentally reducing £ *s.* *d.* to the decimal of £1 correct to three decimal places has been explained in Part I, § 106, p. 183. If this decimal has to be multiplied by a number greater than one, the result is usually not correct to three places; in such cases the full reduction method (§ 63) should be used or the following mental method

Mental method for expressing £ *s.* *d.* as a decimal of £1 correct to as many places as are desired.

The method of § 106 is accurate as far as shillings are concerned.

If $\frac{1}{4}d.$ were exactly £·001, 6*d.* would be £·024; but 6*d.* is really £·025. Hence, in the case of 6*d.*, we add 1 (i.e. $\frac{1}{6}$ of the number of pence) to the number of farthings in order to obtain the required decimal. The ratio of the correct decimal to the number of farthings is the same in all cases; hence we may always obtain the second and following decimal places by adding to the number of farthings $\frac{1}{6}$ of the number of pence.

Thus $7\frac{3}{4}d.$ = £·0322916 . . . . . the mental process being 31 farthings + 7·75 ÷ 6.

$4\frac{3}{4}d.$ = £·0197916 . . . . , i.e. (19 + 4·75 ÷ 6) ÷ 1000.

3*s.* $9\frac{1}{2}d.$ = £·1895833 . . . . , i.e. ·15 + (38 + 9·5 ÷ 6) ÷ 1000.

This process is easy if continually used, but the reduction method is so short that it is hardly wise to run the risk of error by use of this mental method.

Oral Examples.

Express as a decimal of £1 correct to six decimal places

1. $3\frac{1}{4}d.$	**2.** $6\frac{1}{2}d.$	**3.** $2\frac{1}{4}d.$	**4.** $5s.\ 3\frac{1}{4}d.$
5. $7s.\ 6\frac{1}{2}d.$	**6.** $19s.\ 2\frac{1}{4}d.$	**7.** $15s.\ 11\frac{1}{2}d.$	**8.** $3s.\ 5\frac{1}{2}d.$
9. $13s.\ 10\frac{3}{4}d.$	**10.** $7s.\ 0\frac{1}{4}d.$	**11.** $14s.\ 8\frac{1}{2}d.$	**12.** $17s.\ 5d.$
13. $16s.\ 10d.$	**14.** $3s.\ 8d.$	**15.** $19s.\ 11\frac{3}{4}d.$	**16.** $9s.\ 2\frac{1}{2}d.$
17. $11s.\ 10\frac{1}{2}d.$	**18.** $4s.\ 7d.$	**19.** $1s.\ 9\frac{1}{2}d.$	**20.** $2s.\ 9d.$
21. $8s.\ 4d.$			

MIXTURES

(222.) Example i. *A dealer mixes two kinds of tea, costing respectively* 1*s.* 3*d. per lb. and* 1*s.* 7*d. per lb., in the ratio* 3 *to* 5. *At what price ought he to sell the mixture so as to gain not less than* 2*d. per lb.?*

3 lb. at 1*s.* 3*d.* are mixed with 5 lb. at 1*s.* 7*d.*

Hence 8 lb. of mixture cost $3 \times 1s.\ 3d. + 5 \times 1s.\ 7d.$

i.e. ,, ,, $3s.\ 9d. + 7s.\ 11d. = 11s.\ 8d.$

$\therefore$ 1 lb. of mixture costs $1s.\ 5\frac{1}{2}d.$

Hence, in order to make a profit of 2*d.* per lb., the dealer must sell the mixture at $1s.\ 7\frac{1}{2}d.$ per lb.

Example ii. *In what ratio must two kinds of tea costing respectively* 1*s.* 4*d. per lb. and* 1*s.* 8*d. per lb. be mixed so as to sell, without loss or gain, at* 1*s.* 7*d. per lb.?*

On each pound of cheap tea 3*d.* is gained.

On each pound of dear tea 1*d.* is lost.

But the total loss = total gain.

Therefore there must be 3 times as much dear tea as cheap.

That is, 3 lb. at 1*s.* 8*d.* must be mixed with 1 lb. at 1*s.* 4*d.*

Examples XVI b.

At what price per unit ought the following mixtures to be sold so that there may be neither gain nor loss? (Give prices correct to farthing just above the correct value.)

1. 3 parts at 2*s.* 5*d.* with 2 parts at 1*s.* 7*d.*

2. 5 parts at 6*s.* $7\frac{1}{2}d.$ with 3 parts at 3*s.* 9*d.*

3. 10 parts at $2\frac{3}{4}d.$ with 7 parts at $2\frac{1}{4}d.$

4. 3 parts at £3 5*s.* 10*d.* with 7 parts at 4*s.* 2*d.*

5. 11 parts at 2*s.* 5*d.* with 9 parts at 3*s.* 4*d.*

Find the density of the following mixtures (the parts being in each case by volume); all the densities are supposed to be given in grammes per cu. cm. (see also § 200).

6. 3 parts of density 7 with 4 of density 5·2.

7. 6·3 parts of density 1·9 with 8·6 of density 1·4.

8. 30 parts of density 10·3 with 47 of density 11·4.

9. 24·7 of density 7·8, 58·9 of density 7·1, and 16·4 of density 7·3.

10. 45·9 of density 19·3, 48·7 of density 8·6, and 5·4 of density 20·9.

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11. In what ratio must teas at 1*s.* 2*d.* per lb. and 1*s.* 8*d.* per lb. be mixed so that 3*d.* per lb. may be gained by selling it at 1*s.* 6*d.* per lb.?

12. Two teas were mixed in the ratio 3 to 7; and £1 was gained by selling 50 lb. of the mixture for £4 15*s.* 10*d.* Find the price of the second constituent, it being known that the first constituent was worth 1*s.* 2*d.* per lb.

13. An alloy is formed from two metals of specific gravities 8·6 and 7·0. In what ratio are they mixed if the specific gravity of the alloy is 8·4?

14. A man cycles for 3 hours at 11 miles per hour and then walks for $1\frac{1}{2}$ hours at 4 miles per hour. What was his average speed?

15. A man takes 6 hours to travel 183 miles. Part of the time he walked at $3\frac{1}{2}$ miles per hour, the remainder of the time he travelled by train at the average rate of 44 miles per hour. How far did he walk?

16. A man can walk uphill at the rate of $3\frac{1}{2}$ miles per hour and downhill at the rate of $4\frac{1}{2}$ miles per hour. He takes 5 minutes to walk up and down a hill without stopping. What is the length of the hill?

17. An apple-woman mixes apples at 2 a penny with apples at 3 a penny. Explain why she loses when she sells them at 5 for 2*d.*

18. At a factory there are employed 218 men at £1 5*s.* a week, 147 women at 16*s.* per week, and children at 6*s.* per week. The total wages for the week are £406 12*s.* How many children are employed?

19. An employer has 100 employees. Each man receives £1 12*s.* per week, each woman 18*s.* per week. The total wages for the week is £137 12*s.* Find how many men and women respectively were employed.

20. Income-tax is paid at the rate of 1*s.* 2*d.* in the £ on all income derived from investments, and at the rate of 9*d.* in the £ on all income earned as salary. A man whose total income is £1000 pays £42 14*s.* 2*d.* income-tax. What was his salary?

21. The front seats at a concert cost half a crown each, and the back seats a shilling. If 400 tickets are sold and the receipts are £30 10*s.*, how many front seats are taken?

22. A man walks from *A* to *B* up a hill at $3\frac{3}{4}$ miles per hour, and back again at $4\frac{1}{2}$ miles per hour. If he takes 2 hours 40 minutes for the whole journey, find the distance from *A* to *B*.

23. Strong spirit is mixed with inferior spirit, valued at 5*s.* per gallon, in the ratio of 6 to 1. The mixture is then worth 9*s.* the gallon. Find the value of a gallon of the strong spirit.

24. A cubic centimetre of an alloy of gold and silver weighs $13\frac{1}{3}$ grammes. Assuming that a cubic centimetre of gold weighs 19·6 grammes and a cubic centimetre of silver 10·2 grammes, and that the volume of the alloy is equal to the sum of the separate volumes of the gold and the silver contained in it, find the proportions by weight of gold and silver in the alloy.

PARTNERSHIP

(223.) Example i. *Two men A and B enter into partnership. A is to act as manager and to receive a salary of* £500 *a year for so doing. The remaining profits are to be divided in the ratio of the sums they put into the business. A's share of the capital is* £1000, *B's is* £5000. *If the profits amount to* £1200, *how much does each receive?*

A first takes his salary of £500.

There remains £700 to be divided in the ratio 1000 to 5000, i. e. 1 to 5.

Hence *A* receives $\frac{1}{6}$ of the remaining profits, and *B* $\frac{5}{6}$.

∴ *A* receives £500 + £116 13*s.* 4*d.* = £616 13*s.* 4*d.*, and *B* receives £116 13*s.* 4*d.* × 5 = £583 6*s.* 8*d.*

Example ii. *A, B, and C form a partnership, each putting £1000 into the business. Four months afterwards A withdraws his share and B and C each subscribe an additional £500. Six months later more capital is required, and B puts another £500 into the business. At the end of the year the profits amount to £415. How much ought each to receive?*

A invests £1000 for 4 months, which is equivalent to £4000 for 1 month.

B invests £1000 for 4 months, which is equivalent to £4000 for 1 month.

B invests £1500 for 6 months, which is equivalent to £9000 for 1 month.

B invests £2000 for 2 months, which is equivalent to £4000 for 1 month.

C invests £1000 for 4 months, which is equivalent to £4000 for 1 month.

C invests £1500 for 8 months, which is equivalent to £12000 for 1 month.

Hence the whole profits have to pay for the use of £37000 for 1 month.

$$\therefore A\text{'s share} = \frac{4000}{37000} \times £415 = 4 \times 11{\cdot}2162 = 44{\cdot}8648$$

$$= £44\ 17s.\ 3d.$$

$$B\text{'s share} = \tfrac{17}{37} \times £415 = 17 \times 11{\cdot}2162 = 190{\cdot}6754$$

$$= £190\ 13s.\ 6d.$$

$$C\text{'s share} = \tfrac{16}{37} \times £415 = 4 \times 44{\cdot}8648 = 179{\cdot}4592$$

$$= £179\ 9s.\ 2d.$$

11·2162
415
45
80
60
230
..8

Note.—The sum of these shares is 1*d.* less than £415; the reason for this is that each share has been given to the nearest penny, and each is really a fraction of a penny greater than the amount given.

Example iii. *A and B rent a field for grazing. In the course of* 3 *months A has put in* 50 *sheep for* 70 *days and* 10 *horses for* 15 *days; while B has put in* 30 *sheep for* 40 *days and* 20 *horses for* 25 *days. What fraction of rent ought each to pay, if they agree that a horse eats three times as much as a sheep?*

Here we express the amount eaten in terms of the amount eaten by sheep in one day.

50 sheep in 70 days eat as much as 50×70 sheep in 1 day.

10 horses in 15 days eat as much as $3 \times 10 \times 15$ sheep in 1 day.

The remainder of the work is exactly similar to that in Examples i and ii.

It is found that A's share is equivalent to 3950 sheep for 1 day, and B's share is equivalent to 2700 sheep for 1 day.

Hence A should pay $\frac{79}{133}$ of the rent and B $\frac{54}{133}$.

Examples XVI c.

1. A starts a business at the beginning of the year with £500 capital. Three months afterwards B joins him with £350. They each take £20 a month as salary. At the end of the year the net profits are £450. What income does each of them derive from the business for that year?

2. Two men together buy a motor. The expenses for the year amount to £213. One of them has used the car 115 times during the year, the other 219 times. How ought the expenses to be divided?

3. A, B, and C form a partnership and agree that, after A has received one-third of the net profits for management, the remainder shall be divided proportionally to the amount of capital each contributes. A contributes £530, B £1000, and C £2000; the net profits are £1742 5*s*. How much does each receive?

4. A, B, and C entered into partnership. A advanced £840 for 10 months, B £960 for 8 months, and C £720 for a year. If the profits amounted to £883 4*s*. 6*d*., how much should A, B, and C each receive?

5. A and B enter into partnership; A puts £3500 into the business, but at the end of 3 months withdraws £450, and again 7 months later withdraws £315 of this capital, while B withdraws nothing. If at the end of a year they are entitled to equal shares of the profits, how much capital did B provide?

6. A man leaves £32818 to be divided among his four sons in the proportion of the fractions $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{5}$, and $\frac{5}{6}$. Find the share of each.

7. In 4 days of 6 hours each 5 men and 6 boys can earn £5 8*s.*, a man earning half as much again per hour as a boy. How much can 10 men and 9 boys earn in 6 days of 5 hours each?

8. A garrison of 1140 men is provisioned for 40 days. After 16 days on full rations the garrison is reduced by 152 men, but the existing provisions are increased by one-twelfth by capture. For how many days can the reduced garrison now hold out?

9. Distribute £630 among A, B, C, and D, so that B and C may together have half what A and D have together, that B's money may be $\frac{7}{8}$ of C's, and D's money $\frac{2}{5}$ of A's.

10. A besieged garrison have food which will last for 60 days at the rate of 20 oz. a day for each man. They are reinforced by an addition of 864 men, and the food will now last for only 55 days at 16 oz. a day for each man. How large was the original garrison?

11. If the wages of 45 women amount to £207 in 48 days, how many men must work 16 days to receive £76 13*s.* 4*d.*, the daily wages of a man being double those of a woman?

12. A man having 3 sons left £9656 to be divided among them in proportion to their ages at the time of his death; when he died, their ages were 25, 22, and 21 years respectively; what was the share of each? What difference would it have made to each of the sons if the father had lived a year longer?

13. Two men undertake to do a piece of work for £6. One of them could do it by himself in 16 days, while the other would require 20 days. They employ an assistant; and, all three working together, they complete the work in 8 days. How much of the money has each of them earned?

14. Two people enter into partnership, with £3000 and £4500 respectively. At the end of 8 months the partner with smaller

capital advances £2500 more. Seven months later they close business with a profit of £1040. How should this profit be divided?

15. Three men contribute to a business sums of £1000, £1500, £2700. The profits at the end of the year are £776. Of this sum the first man, as manager, is to receive a commission of £100; what share will each receive out of the profits?

16. Three persons agree to bear a day's expenses in the proportions 2 : 2 : 3. The first pays 7*s.* 6*d.* for luncheon, the second pays 4*s.* in the course of a walk, and the third pays 14*s.* 9*d.* for their dinners. How must they settle up accounts?

COINAGE

(**224.**) The relative values of the gold coins of different countries are determined by the amount of pure gold in them. If we assume that the value of gold is the same in all countries, we can express the value of the gold coins of any one country in terms of the value of the gold coin in any other country.

In England the sovereign is the standard coin, and its value is referred to as one pound or £1. The sovereign is made of an alloy consisting of 22 parts of pure gold to 2 parts of other metals. This alloy is called **standard gold** and is said to have a **fineness** of $\frac{22}{24}$. The legal weight of a sovereign is 123·274478 grains, but a variation of ·2 of a grain is allowed; hence a sovereign is legal tender if its weight is between 123·474 grains and 123·074 grains. The weight of pure gold in a sovereign is legally $\frac{22}{24}$ of the weight of the sovereign, i. e. 113·001605 grains. Hence if we neglect the value of the other metals in the alloy and the cost of manufacture of the coin, we may say that

113·001605 grains of pure gold are worth £1.

If we are now given the weight of pure gold in a foreign coin, we can determine its value.

In England (and other countries in which the value of the coins is based on the value of gold) the value of silver and bronze coins is not the same as the value of the metal of which they are

made, any more than the value of a bank-note or postal order is the value of the paper of which it is made. The value of a shilling is fixed by law as $\frac{1}{20}$ of £1, and of a penny as $\frac{1}{240}$ of £1. At the present time the silver in a shilling is worth considerably less than $\frac{1}{20}$ of £1.

Example. *Given that a ten-dollar (U.S.) gold piece weighs 258 grains and that the fineness of the U.S. standard gold is* $\frac{9}{10}$, *find how many dollars are worth* £1.

Here the mental question is somewhat as follows: 'If $\cdot 9 \times 258$ grains of pure gold are worth 10 dollars, what is the value of 113·0016 grains of pure gold?'

This may be worked by ordinary ratio methods.

$$£1 = \frac{113 \cdot 0016}{\cdot 9 \times 258} \times 10 \text{ dollars}$$
$$= 4 \cdot 866.$$

```
                4·866
232·2 )1130·016
       201·21
        15·456
         1·524
```

The following method of arranging the work is often adopted. It is known as the Chain Rule.

How many dollars are worth £1?

£1 weighs 113·0016 grains.

$258 \times \cdot 9$ grains is the wt. of 10 dollars.

$$\text{Then number of dollars in £1} = \frac{1 \times 113 \cdot 0016 \times 10}{1 \times 258 \times \cdot 9}$$

i. e. £1 is worth 4·8666 dollars.

In using Chain Rule take care that each denomination occurs twice, once on the right and once on the left of the various statements.

(**225.**) The relation between the values of the coins of different countries found by comparing the amount of pure gold they contain is called the **Mint par rate of exchange**. Thus £1 = 25·22 francs is the Mint par rate of exchange. This takes no account of the expenses incurred in sending coins between the two countries. The costs of carriage average about 11 centimes per £. Hence, if a person in London sends coins to Paris, he will receive 25·22 francs for each sovereign, but will have paid ·11 francs for

carriage, so that a sovereign in London is not worth more than 25·11 francs when sent to Paris. It makes no difference whether the English money is changed into French money in London or Paris; the net result is that the sovereign in England can cancel only 25·11 francs of debt in France. Similarly, if a person in Paris wishes to pay a debt in London he must pay 25·22 francs + ·11 francs, i.e. 25·33 francs for each pound of debt he wishes to cancel. The values thus determined are called the **specie points.**

We shall explain in a later chapter other methods of paying foreign debts; the rate of exchange by these methods varies from day to day, but can never go outside the limits imposed by the specie points. Thus the rate of exchange between London and Paris fluctuates between 25·11 francs and 25·33 francs to the £.

Examples XVI d.

(A sovereign contains 113·0016 grains of pure gold.)

Find the values in English money of the following coins:

1. A German mark, which is worth 5·53134 grains of pure gold;

2. An Austrian krona, which is worth 4·70499 grains of pure gold;

3. An Indian rupee, which is worth 7·53344 grains of pure gold;

4. A Belgian franc, which is worth 4·48036 grains of pure gold;

5. An Argentine peso, which is worth 22·40174 grains of pure gold.

6. If a Spanish peseta is worth $8\frac{3}{4}d.$ and 126 francs are worth £5, what will be the value in francs of 4000 pesetas?

7. If the English guinea weighed 8·38 grammes and was of a fineness 0·917, what was its value in francs? It is given that a kilogram of pure gold is worth 3444·44 francs.

8. Given that the British sovereign contains 113·00 grains of pure gold, and that the French 20 franc piece, which is worth 190·3*d.*, contains 5·806 grammes of pure gold, find how many grains are equal to 1 gramme.

9. A merchant in Paris wishes to obtain English money in Paris to send to London to pay a bill of £419. If he has to pay 25·22 francs for each sovereign, and has to pay expenses of carriage averaging ·11 francs per £, how much will he pay in all? Calculate from the data in the first five questions.

10. How many francs are equivalent to 20 marks?

11. How many pesos are equivalent to 20 francs?

12. How many rupees are worth £5?

13. What is the fineness of a French gold coin, being given that a 20 franc piece weighs 6·452 grammes? (See Question 8.)

14. The exchange rate from Madrid on London is 35·94 pesetas to £1. Find the equivalent of 1437 pesetas in English money to the nearest farthing. Show also that if the rate increases by 1 per cent. of its present value the equivalent value is diminished by ·99 . . . per cent.

15. A Spanish dollar is 5 pesetas, and I find that I can get as many pesetas for a sovereign as I can get pence for a Spanish dollar. Find the rate of exchange.

16. If 7 rix dollars are worth 2 ducats and 9 ducats worth 4 moidores and 20 moidores worth £27, how many rix dollars are there in £72?

Examples XVI.

1. The national debt of the United Kingdom is £733,472,610. If this were distributed according to population, find to the nearest million pounds what portion should be assigned to England, Ireland, Scotland, and Wales respectively; their populations being—England 34,043,000; Wales 2,032,000; Scotland 4,559,000; and Ireland 4,382,000.

2. Six men and 4 horses are billeted on one householder for 15 days, 8 men and 3 horses on another for 12 days, and 10 men on a third for 9 days. If the three householders receive £7 10*s.* in all, what, to the nearest halfpenny, is the allowance to each householder? One man's keep is equivalent to that of 2 horses.

3. If 2·45 acres equal 1 hectare, 1 hectare costs 735 francs, 5 francs equal 4 shillings, find the value of 1 acre in English money.

4. A refiner has 10 lb. of gold 20 carats fine, and melts it with 16 lb. of gold 18 carats fine; how much pure gold must he put into it to make the whole 22 carats fine? (24 carats fine is pure gold; 20 carats fine is 20 parts gold and 4 parts alloy.)

5. The railway from Goeschenen to Airolo through the St. Gotthard tunnel rises from Goeschenen (which is 3460 ft. above the sea) at 1 ft. in every 168 up to the highest point in the tunnel, and then descends at 1 in 504 to Airolo (which is 3570 ft. above the sea). The distance between Goeschenen and Airolo by rail being $9\frac{1}{2}$ miles, find how far the highest point of the tunnel is from Goeschenen.

6. Given that 1 metre equals 39·371 in. and 1 kilogram equals 2·2046 lb., and if 1 cubic metre of water weighs 1000 kilograms, find the weight of a cubic foot of water in pounds correct to the nearest tenth of an ounce.

7. In a business there are three partners A, B, and C. A has put £5000 into the business and B has put in £6000. The business is managed by A and B, A receiving an annual salary of £500 and B an annual salary of £350, paid out of the annual profits. At the end of the first year A receives altogether £20 more than B, and C receives £20 more than A. What is the amount of C's capital, and what is the amount of the annual profits?

8. A gallon of water weighs 10 lb.; a cubic foot of water weighs 1000 oz.; a cubic centimetre of water weighs 1 gramme (a centimetre is ·3937 in.). Find the number of grammes in a pound, and the number of cubic centimetres in a gallon, each to two places of decimals. Criticize this question.

9. If 25 francs equal £1, and a gramme equals ·002204 of a pound, find, in French money, the price of a kilogram of a substance which costs a shilling an ounce.

10. A merchant sold 90 cwt. 3 qrs. 14 lb. of madder, which cost him 38*s.* 6*d.* per cwt., to another at the rate of 42*s.* per cwt., the purchase price to be one-quarter in cash and the remaining three-quarters in hemp. If the hemp cost the other merchant 24*s.* 10*d.* per cwt., find how much money he would pay and the

price at which he would reckon the hemp, on the supposition that he is to make the same net profit as the first, so that the exchange is equal.

11. Gunpowder is composed of 2 parts of sulphur, 3 parts of charcoal, and 15 parts of nitre. Given 200 lb. of sulphur, 305 lb. of charcoal, and 10 cwt. 5 lb. of nitre, find the greatest quantity of gunpowder that can be made.

12. Two teas cost a grocer 1*s.* 7*d.* and 1*s.* 1*d.* a lb. He usually makes not less than 3*d.* profit out of each 1*s.* he receives for his wares. If he mixes 9 lb. of the cheaper with 5 lb. of the dearer, at what price per lb., to the nearest penny, must he sell to make his usual profit?

13. Plumbers' solder contains 40 parts of lead to 60 parts of tin. What will be the price of a kilogram of solder if lead costs 0·75 francs per kilogram and tin 2·50 francs per kilogram?

14. The alloy used for making bells is composed of 8 parts of copper to 2 of tin. Copper is worth 4 fr. 75 c. per kilogram and tin 3 fr. 75 c. The expenses of manufacture amount to $\frac{1}{10}$ of the cost of material. What will be the total cost of a bell weighing 1345 kilograms?

15. A workman spends $\frac{1}{3}$ of his wages on food, $\frac{1}{8}$ on lodging and clothing, and $\frac{1}{12}$ on various other expenses; in the course of the year he saves 451 francs. What were his wages for the year?

16. If 25 francs = £1, and a metre = 3·28 feet, what is the cost in francs and centimes of a metre of a ribbon worth 3*s.* 6*d.* a yard?

17. The cost of an article is 60 centimes per gramme. Find in shillings and pence, correct to the nearest penny, the cost of an ounce, given that £1 = 25·12 francs, and a pound = ·454 kilograms.

18. Taking the value of gold as 3 francs 10 centimes per gramme, find its value in English money per ounce Troy, assuming that one gramme is 15·5 grains and £1 is worth 25·25 francs.

CHAPTER XVII

PERCENTAGE

(226.) It is explained in § 137, Part I, that, when a quantity changes, the ratio of the increase or decrease to the original value is often expressed as so many parts in a hundred or so many per cent.

Example. *The population increases from* 4017 *to* 4902, *what is the increase per cent.?*

Increase of population $= 4902 - 4017$
$= 885.$

Ratio of increase to original population $= \frac{885}{4017}$

$$\begin{array}{r|l} & 2203 \\ 4017 & 885\cdot \\ & 81\cdot6 \\ & 1\cdot3 \end{array}$$

$\therefore$ Increase per cent. $= \cdot2203$ of 100
$= 22\cdot03.$

Alternative method:

Increase in a population of 4017 is $4902 - 4017$.

$\therefore$ Increase in a population of 100 is $885 \times \frac{100}{4017}$.

The actual calculation is the same as before.

(227.) If a person is employed to do business for another, he often receives a **commission** on the value of the business he does. This commission is usually expressed as a percentage. In some cases the word **brokerage** is used instead of commission.

The inventor of an article receives from the people who sell the article a *royalty* of so much per cent. on the value of the articles sold. The author of a book may receive a *royalty* on the money received by the publishers for his book; the word *royalty* is also used in connexion with the money received by a land-owner for permission given to a tenant to extract minerals from his land.

Example. *An agent is appointed to sell goods and is to receive a salary of* £100 *together with a commission of* $7\frac{1}{2}$ *per cent. of the money received by his employer for goods sold through his agency. At the end of the year it is found that he has sold goods to the value of* £1536 *; find his total income for the year.*

$$\begin{aligned} \text{Commission} &= \frac{7\frac{1}{2}}{100} \times £1536 \\ &= £107{\cdot}52 \\ &\quad +7{\cdot}68 \\ &= £115{\cdot}2 \\ &= £115\ 4s. \end{aligned}$$

$\therefore$ Total Income = £215 4*s.*

(**223.**) Tradesmen and business firms often allow a deduction of so much per cent. when accounts are paid promptly. This deduction is referred to as a **discount.**

Example. *A tradesman sells goods for* £115 7*s.* 3*d.*, *but allows* $2\frac{1}{2}$ *per cent. discount for cash What does he then receive?*

Amount of bill	£115·362
Discount $= \frac{1}{40}$	£2·884
Cash value	112·478

$\therefore$ He receives £112 9*s.* 7*d.* to the nearest penny.

NOTE.—**All sums of money should be expressed as decimals of £1, as division or multiplication by 100 is thereby rendered easy.**

(**229.**) It is often convenient to express the composition of a substance in terms of percentages.

Example. *A certain salt contains* 23 *parts by weight of sodium to* 14 *of nitrogen and* 48 *of oxygen; what is the percentage composition?*

In 23 + 14 + 48 parts by weight, 23 are sodium,

$\therefore$ In 100 parts by weight, $23 \times \frac{100}{85}$ are sodium,

i. e. $23 \times \frac{20}{17}$ are sodium, 23·528 / 3·529

i. e. $23 \times 1{\cdot}1764$ are sodium, 27·057

i. e. 27·06 are sodium.

Similarly, in 100 parts by weight, $14 \times 1{\cdot}1764$ are nitrogen,

i. e. 16·47 are nitrogen; 47·056 / 9·411

and in 100 parts by weight, $48 \times 1{\cdot}1764$ are oxygen, 56·467

i. e. 56·47 are oxygen.

Thus the salt contains 27·06 % of sodium, 16·47 % nitrogen, and 56·47 % oxygen.

Examples XVII a.

1. The increase in the population of a district of 64000 inhabitants was $2\frac{1}{2}$ per cent. in each year in three consecutive years, the increase for each year being calculated on the population at the beginning of the year. What was the number of inhabitants at the end of the third year?

2. A tradesman saves during each year 20 per cent. of his capital at the beginning of the year, and at the end of the year adds it to his capital. If on Jan. 1st, 1908, his capital was £500, what was it on Jan. 1st, 1912?

3. A bankrupt's debts are £5317 5*s.* and his assets are £4076 11*s.* 2*d.*; how much can he pay in the pound, after defraying the expenses of his bankruptcy, which amount to 10 per cent. of the debts?

4. The population of a certain town increases every year by $7\frac{1}{2}$ per cent. If in April, 1912, the population was 704,000, what will it be in April, 1915?

5. A man borrows £1, agreeing to pay $\frac{1}{2}d.$ per week for the use of the loan. At what rate per cent. per annum is he paying interest?

6. A man bought 1 acre 1 rood 4 perches of land for £6171. He let it at a yearly rental of 1*d.* per sq. ft. What rate per cent. did he obtain on his investment?

7. A tradesman bought 5 cwt. of tea for £37 6*s.* 8*d.* At what price per lb. should he sell it in order to make a profit of 25 per cent.?

8. A traveller sells goods to the value of £3176 in the course of a year and is allowed a commission of $7\frac{1}{2}$ per cent. How much did he earn for that year?

9. In an examination *A* obtains 48 per cent. of full marks and *B* obtains 33 per cent. Together they get 567 marks. What were full marks?

10. Out of 18 parts by weight of water, 2 are hydrogen and 16 oxygen. What is the percentage composition of water?

11. A tailor sends out bills amounting in all to £719 14*s.* 3*d.* He allows 5 per cent. discount for prompt payment; if discount is allowed on one-third of the whole amount, how much does the tailor receive?

12. A man paid 7 per cent. of his income as rates and taxes and then had left £293 4*s.* 7*d.* What was his gross income?

13. A bankrupt's liabilities are £1350. His assets are nominally £970, part of which consists of debts owing to him. He succeeds in recovering only 60 per cent. of these debts, and pays his creditors 12 shillings in the pound. What is the amount of the debts originally owing to him?

14. A gentleman with a capital of £4800 lends £200 free of interest and £1400 at 4 per cent., and the rest at 5 per cent. What rate per cent. does he make on his whole capital?

15. The population of a town is 50,000. If the number of males were increased by 9 per cent. and the number of females by 4 per cent., the population would become 53,200. Find the number of males and females in the town.

16. I take a building lease of 12 acres of ground at £18 an acre ground rent, and build on it 24 similar houses at a cost of £60,000. What must be the rent of each house to give me 6 per cent. on my outlay?

17. The increase in the population of a district of 64,000 inhabitants was $2\frac{1}{2}$ per cent. each year in three consecutive years. What was the number of the inhabitants at the end of the third year?

18. The fare for a journey of 192 Km. is 12·64 fr. Is this cheaper or dearer than the English price of a penny a mile? Express the cheaper price as a percentage of the dearer.

(1 Km. = ·621 mile. £1 = 25·2 fr.)

19. Of a regiment, after 1·5 per cent. have been killed, 8·75 per cent. wounded, and 12 per cent. invalided, there still remain 933 men available. What was its original strength?

20. What is the percentage composition of nitric acid, which contains 1 part of hydrogen to 14 of nitrogen and 48 of oxygen?

21. The percentage of tin in fine solder is 66 per cent., but in coarse solder it is only 25 per cent. Assuming that both kinds of solder are composed entirely of tin and lead, find how much coarse solder can be made by the addition of lead to a hundredweight of fine solder.

22. A man left his property to his three nephews in the proportion 3 : 4 : 7. Each nephew had to pay a legacy duty of 3 per cent. After payment of duty the smallest share amounted to £873. Find the amounts paid to the other nephews.

23. If a metre equals 39·37 in., express a mile as a percentage, correct to one decimal place, of a kilometre.

24. The following table gives the revenue for the last nine months in each of the years 1911 and 1912. Calculate the increase or decrease per cent. (correct to two figures) during 1912 in each source of revenue and in the total revenue.

		1912.	1911.
		£	£
(*a*)	Customs	25,027,000	25,156,000
(*b*)	Excise . . .	28,347,000	29,120,000
(*c*)	Estate Duties .	19,709,000	18,181,000
(*d*)	Stamps . .	7,166,000	6,783,000
(*e*)	Land Tax . .	60,000	120,000
(*f*)	House Duty	400,000	560,000
(*g*)	Income Tax .	12,192,000	13,421,000
(*h*)	Land Value Duties	93,000	150,000
(*i*)	Postal Service . . .	14,090,000	13,325,000
(*k*)	Telegraphs . . .	2,360,000	2,490,000
(*l*)	Telephones .	4,200,000	1,505,000
(*m*)	Crown Lands . .	370,000	370,000
(*n*)	Suez Canal, &c.	843,142	788,936
(*o*)	Miscellaneous . . .	2,160,728	1,947,648

25. In 1910 there were 1,682,153 acres of land in Great Britain on which wheat was grown; in 1911 there were 1,906,038 acres. What was the increase per cent ?

26. In 1911 the value of the manufactured goods exported from Great Britain to Foreign Countries was £239,941,000 and to British Possessions £143,784,000. What percentage of the total was exported to Foreign Countries?

27. The following figures refer to the Federated Malay States:

	1908.	1909.
	$	$
Public Revenue . .	24,623,325	25,246,863
Public Expenditure .	25,854,573	23,633,851
Imports	51,343,592	46,194,598
Exports . .	66,421,978	76,273,438

Express the rate of increase or decrease under each head as a percentage, correct to one decimal place, of the amount in 1908.

28. The following table exhibits the traffic, &c., at Southampton in 1892 and 1911:

	1892.	1911.
Tons of cargo inwards	246,560	707,149
Passengers in and out	122,108	304,045
Length of quays . .	12,054 ft.	23,250 ft.
Shed area . .	554,718 sq. ft	1,064,946 sq ft

Find, to the nearest integer, the percentage increase in each case.

If the shed area had increased in proportion to the length of the quays, find, to three significant figures, what would have been the shed area in 1911.

29. An insurance agent receives 15 per cent. commission on the premiums collected. If he collects £345 in premiums, find the amount of his commission. How much would he have to collect to earn £200?

30. The amount of electricity used annually in London, without counting that used for the traction of the tramcars, is 212,000,000 units. If the cost to produce and distribute this is at the rate of $1\frac{1}{2}d.$ per unit, how much is the annual cost of it? If this cost could by economy be reduced by 20 per cent., how many thousand pounds would be saved each year?

31. The total number of parcels sent to and received from places abroad during the year was 2,075,357, an increase of $10\frac{1}{2}$ per cent. on the number for the previous year. Find the number in the previous year.

32. The total number of foreign and colonial parcels insured was 117,545, as against 98,535 in the previous year; express this increase as a percentage.

33. 55 per cent. of the workers in a factory are men getting an average wage of 35*s.* a week each, and the rest are women getting an average wage of 22*s.* 6*d.* a week each. Find, to the nearest integer in each case, what percentage of the total wages earned is paid to (*a*) men, (*b*) women.

34. In 1900 we imported 23 million quarters of wheat and wheat-flour, 12 per cent. of which came from our colonies. The duty on tea amounted to £6,300,000. Taking these figures, what percentage of decrease in the duty on tea would be equivalent to a duty of 2 shillings a quarter on *foreign* (as distinct from colonial) wheat and wheat-flour? Answer to the nearest whole number.

PROFIT AND LOSS

(230.) If goods are bought for £100 and sold for £110, we may say that £10 has been gained on a cost price of £100 or that £10 has been gained on a selling price of £110; the goods are sold at a profit of 10 % on the cost price or $9\frac{1}{11}$ % on the selling price. If we merely say that goods are sold at a profit of 15 %, there might be a doubt whether we are to reckon profit on the cost price or selling price. **Unless the question distinctly says otherwise, it is always understood that profit and loss are calculated on the cost price.**

Example i. *If a tradesman makes* 20 % *profit by selling goods at* £18, *what will he gain per cent. by selling at* £20?

There is no need to find the cost price. Consider goods costing £100; these sell at £120 to gain 20 %. Thus £120 corresponds to the £18 of the question. We have to find what £120 is raised to when £18 is raised to £20 Hence the written work may be arranged as follows

If the selling price £18 is raised to £20,

then the selling price £120 is raised to £$20 \times \frac{20}{18}$,

i.e. goods costing £100 will now sell for £$133\frac{1}{3}$.

Therefore the goods are now sold at a profit of $33\frac{1}{3}$ %.

Example 11. *A manufacturer sells goods to a wholesale merchant at a profit of* 25 % *on the cost of production. The wholesale merchant sells them to the retail tradesman at a profit of* 15 %, *and the tradesman sells to his customers at a profit of* 20 %. *What will the customers pay for goods which cost the manufacturer* £85 ? *Also, what was the cost to the manufacturer of goods for which the customers pay* £76 10*s.* ?

The first question is a direct calculation of percentages.

	£
Cost to manufacturer . . .	85
Manufacturer's profit at 25% (= $\frac{1}{4}$) .	21·25
Cost to wholesale merchant . .	106·25
Merchant's profit at 10 % (= $\frac{1}{10}$) .	10·625
,, ,, 5 % (= $\frac{1}{20}$) .	5·3125
Cost to tradesman	122·1875
Tradesman's profit at 20 % (= $\frac{1}{5}$) .	24·4375
Cost to customers	146·625

The customers pay £146 12*s.* 6*d.*

The second case involves the inverse process and is more difficult

Goods for which the customer pays £120 cost the tradesman £100.

∴ Goods for which the customer pays £76·5 cost the tradesman

$$£\frac{100\times 76.5}{120}$$

$$= £63{\cdot}75.$$

Goods for which the tradesman pays £115 cost the merchant £100.

∴ Goods for which the tradesman pays £63·75 cost the merchant

$$£\frac{100\times 63.75}{115}$$

$$= £\frac{1275}{23}.$$

Goods for which the merchant pays £125 cost the manufacturer £100.

∴ Goods for which the merchant pays $£\frac{1275}{23}$ cost the manufacturer

$$£\frac{100\times 1275}{125\times 23}$$

$$= £\frac{5100}{5\times 23}$$

$$= £\frac{1020}{23}.$$

Goods cost manufacturer £44 7*s.* to the nearest penny.

After a little experience the student can shorten the work:

$$\text{Manufacturer's cost} = £76{\cdot}5 \times \tfrac{100}{120} \times \tfrac{100}{115} \times \tfrac{100}{125}.$$

Examples XVII b.

1. Find, correct to one decimal place, the profit or loss per cent. obtained by buying at the prices in column A and selling at the prices in column B.

	A.			B.		
	£	*s.*	*d.*	£	*s.*	*d.*
i.	23	4	2	25	0	0
ii.	27	3	6	20	0	0
iii.		3	6		3	$11\frac{3}{4}$
iv.		17	0		15	9
v.	243	6	7	249	18	2

2. Find, correct to one decimal place, the profit or loss per cent. by buying at the prices in column B in previous question and selling at the prices in column A.

3. By selling goods for a certain sum a man gains 5 per cent. If he had sold them for £1 13*s.* 2*d.* more he would have gained 6 per cent. Find their cost price.

4. A man buys some coffee and resells it, losing 3 per cent. on the transaction. If he had obtained £14 more, he would have gained 4 per cent. What was the original sum paid for the coffee?

5. A person bought an estate and subsequently sold it for £625 less than he gave for it, thereby losing $1\frac{1}{4}$ per cent. What did the estate cost him? What should he have received in order to gain $12\frac{1}{2}$ per cent.?

6. A merchant sells an article at an advance of 12 per cent. on the cost price. What is the percentage of profit on the selling price?

7. Eggs are bought at 100 for 5*s.* and sold at 3 for 2*d.* What is the gain or loss per cent.?

8. Eggs are bought at half a crown for 20 and sold at 10 a shilling. What is the gain or loss per cent.?

9. If sugar is bought at 18*s.* 9*d.* per cwt. and sold at $2\frac{1}{4}$*d.* per lb., what is the gain or loss per cent.?

10. Bacon is bought at 77*s.* 6*d.* per cwt. and sold at an average price of 11*d.* per lb. What is the gain per cent.?

11. A tradesman marks his goods so as to get a profit of 25 per cent., but allows 5 per cent. discount for cash payment. What is his net rate of profit?

12. A builder, *A*, sells a house to an agent, *B*, for £4860, by which he loses 19 per cent.; *B* disposes of it to *C* at a price which would have given *A* 17 per cent. profit. Find *B*'s total gain.

13. If a druggist were to buy by avoirdupois weight and to sell by troy weight, what would he gain per cent. by buying at 3*d.* per oz. and selling at 6*d.* per oz.?

14. A man added 10 gallons of water to 60 gallons of wine, which he bought at £1 1*s.* per gallon. At what price per gallon must he sell the mixture to gain 30 per cent. on his outlay?

15. A retail butcher buys a consignment of 50 tons of American beef at 4*d.* per lb. and sells it at 7*d.* per lb. Find his total gain and his gain per cent. Find also his gain per cent. if he uses dishonest scales which make 14 oz. appear to be 1 lb.

16. If a grocer gains 10 per cent. by selling tea at 2*s.* 3*d.* per lb., what would he gain per cent. if he were to sell at 2*s.* 9*d.* per lb.?

17. In what ratio must two teas, costing £15 and £10 per cwt. respectively, be mixed in order that a profit of 10 per cent. may be made by selling the mixture at 2*s.* 9*d.* a pound?

18. A dealer by selling goods at 7*s.* 6*d.* per lb. gains 10 per cent. How much will he gain per cent. should the price rise to 8*s.* 9*d.*, and how much will he lose if it falls to 6*s.*?

19. A dealer buys two pictures from an artist at £135 and £90, and sells them again at profits of 20 per cent. and 25 per cent. respectively on the purchase price. Find his percentage of profit on the whole transaction.

20. *A* buys a pipe of wine and sells it to *B* at a profit of 5 per cent.; *B* sells it to *C* at a profit of 5 per cent.; and *C* sells it to *D* for £49 12*s.* 3*d.*, making a profit of 12½ per cent. What did the wine cost *A*?

21. If 4 per cent. be lost by selling silk at 10*s.* per yard, at what price should it be sold to gain 5 per cent.?

22. An article passes through the hands of three dealers, each of whom makes a profit of 10 per cent. If the third sells it for £33 5*s.* 6*d.*, what did the first pay for it?

23. A bankrupt owes £676; his property realizes £31 5*s.*, and he has debts owing to him to the amount of £480, of which he recovers 14*s.* 6½*d.* in the £1. What dividend can he pay?

24. I offer a watch for £5 15*s.*, but if I reduce the price by 5 per cent. I still make a profit of 9¼ per cent. How much did the watch cost me?

25. A cattle dealer buys sheep at 35*s* each, and twice as many pigs at 16*s.* each; he sells the sheep at 37*s.* 6*d.* each, and the pigs at 17*s.* 4*d.* each. Find his profit per cent. on his outlay.

26. *A*, who bought certain goods for £8915 16*s.* 8*d*, sold them to *B*, and made a profit at the rate of 6½ per cent. For what sum, to the nearest penny, must *B* sell the goods in order that his rate of profit may be 3 per cent.?

27. A kilogram is 2·2046 lb. *A* contracts to supply *B* with 10 cwt. of a certain article. He finds himself obliged to deliver the article in kilograms, and allows 51 Kg. to the cwt. Find how many lb. *B* receives, and by how much per cent he gains or loses.

28. Two teas, costing £12 and £10 per cwt. respectively, are mixed in the ratio 2 to 5. What profit per cent. is made by selling the mixture at 2*s.* 2*d.* a pound?

29. If a profit of 12 per cent. is made by selling eggs at 15 a shilling, what profit per cent. would be made by selling them at 15*d* a dozen?

30. If by selling a horse for £153 I lose 10 per cent., for what sum must I sell it in order to gain 15 per cent.?

31. A manufacturer marks his goods at an advance of 80 per cent. on cost price; but he allows 15 articles to the dozen, and also allows 10 per cent. discount for cash. What rate of profit on his outlay does he obtain from a customer who pays cash?

32. Assuming that *A* states his profit as a percentage of his *outlay*, and *B* as a percentage of his selling price, find for what

sums respectively A and B must sell their houses, which cost £2125 apiece, in order that each may be able to state that he makes 15 per cent. profit.

33. How much in the pound is income-tax (paid on the whole income) when 14 per cent. rise of the tax causes 1 per cent. diminution of the net income?

34. A merchant took £481 5*s.* for coal of a certain quality on one day, and £444 13*s.* 6*d.* on the next day, when his price per ton was 10 per cent. higher. How much per cent. was the weight of coal sold on the second day less than that sold on the first day?

35. A man paid £53 2*s.* 6*d* income-tax last year at 1*s.* 3*d.* in the £; this year, on an increased salary, he paid £41 5*s.* at 11*d.* in the £. If he obtain the same rise in salary, and income-tax be reduced to 10*d.*, what income-tax will he pay next year?

36. A house in Germany is bought for 37,500 marks. It is liable to 60 marks a year for ground rent, and 75 marks a year for taxes. What must be the rent paid by a tenant in order that the owner may receive $5\frac{1}{2}$ per cent. on his outlay?

37. If goods are bought at £61 7*s.* 1*d.* per ton, at what rate per lb. must they be retailed in order that, allowing for a waste of 5 per cent. of the goods, a profit at the rate of 12 per cent. may be made?

38. A dealer bought some pears at 9*s.* per hundred, and sold them for £2 5*s.* 10*d.* If he obtained an average price of 1*s.* $4\frac{1}{2}$*d.* per dozen, what whole profit, and what rate per cent. of profit, did he make?

39. Find the cost in cash of $83\frac{1}{2}$ tons of a commodity advertised at £9 10*s.* per ton, and how much of such cost will be profit to the seller when the cash price of the commodity is 5 per cent. less than the advertised price, and yields the seller a profit at the rate of 14 per cent. on his outlay.

40. A merchant, having bought some pieces of cloth, found a quarter of it worthless. At what percentage advance on the cost price did he sell the remainder if, on the whole, he lost 10 per cent.?

41. A manufacturer sells chocolate at 8 fr. 40 c. per Kg., and 15 per cent. of that price is profit. If he raises the price to

8 fr. 75 c. per Kg., what percentage of that price will be profit?

42. Goods are bought at 70*s.* per cwt. and sold at $8\frac{1}{4}d.$ per lb. What is the profit per cent.?

43. A chemist sells a patent medicine in two forms—i. e. in bottles and in tabloids—and, reckoned on the prices at which he sells them, he gets 30 and 20 per cent. profit respectively. Last year the profit on the total sale of the medicine was $23\frac{1}{2}$ per cent. This year the number of bottles sold decreases 10 per cent, but the number of tabloids sold increases 13 per cent. What will now be his percentage profit on the whole sale of the medicine?

44. A tradesman, who bought 13 cwt. 1 qr. 10 lb of plums for £20 15*s.*, sold two-thirds of the plums at 12 lb. for 3*s* 8*d.*, and the remainder at 4*d.* per lb. What rate per cent. of profit did he make?

45. A sovereign weighs 123·274 grains, but of this only 113·001 grains is gold. Neglecting the value of the other metal, find, correct to *three* decimal places, the rate per cent. of profit when sovereigns are exchanged for their weight in gold.

46. A dealer purchases 56 dozen of wine at 32*s.* per dozen. He sells 32 dozen at 42*s.* per dozen. At what price per dozen must he sell the remainder so that his gain by the whole transaction may be 25 per cent.?

Examples XVII.

1. I blend 600 gallons of malt whisky, worth 3*s.* a gallon, with 360 gallons of grain spirit at 1*s.* 2*d.* a gallon, and after 3 years sell the blend at 4*s.* a gallon. What is my profit, after charging 5 per cent. per annum interest on my outlay?

2. A tradesman, who is selling off, states that he will make a reduction of 20 per cent. from the marked price. At what price will he mark goods for which he wishes to receive 11*s.* 8*d.*?

3. A woman bought two parcels of eggs, the same number in each parcel. The first she bought at 1*d.* each, but the second at 3 for 2*d.* She then mixed them all together, and sold them at 13*d.* a dozen. Find her gain per cent.

4. By reducing the selling price of wine from £4 16*s.* 6*d.* to £4 15*s.* per dozen, a wine merchant reduces his profit by $2\frac{1}{2}$ per cent. What was the cost price of a single bottle?

5. A dealer purchases a liquid at 8*s.* a gallon and dilutes it with so much water that when he sells the compound at 6*s.* a gallon he gains 20 per cent. on his outlay. How much water is there in every gallon of the compound sold?

6. A man buys a house for £3400. At what rent should the house be let so that, after deducting £12 yearly from the gross rent for repairs, the man may obtain 5 per cent. on his capital?

7. A merchant bought a certain quantity of coal and sold one-half of it at a price which was 10 per cent. above the cost price, the remainder being sold at 19*s.* 2*d.* per ton. If on the whole transaction he gained $12\frac{1}{2}$ per cent., find what was the cost price per ton.

8. If I buy articles at the rate of $7\frac{1}{2}d.$ a dozen and retail them at 7 for 6*d.*, what will be my percentage gain or loss?

9. A dealer buys and mixes together 40 chests of one kind of tea at 1*s.* 1*d.* per lb., and 20 chests of another kind at 1*s.* 2*d.* per lb., each of the chests containing the same quantity of tea. At what price must he sell the mixture in order to gain, as nearly as possible, 10 per cent. on his outlay?

10. If 15 per cent would be lost by selling an estate for £27,625, for what must it be sold so as to produce a gain of 20 per cent.?

11. A greengrocer buys 1500 bananas at 4*d.* a dozen, and sells 900 of them at the rate of two for three halfpence and the remainder at three for twopence. Find his percentage gain.

12. A tradesman sells an article for 1*s.* $2\frac{1}{2}d.$, making a profit of 45 per cent. When the cost price is reduced one penny he reduces the selling price by a penny. What percentage profit is he now making?

13. A man goes from Oxford to London to buy a certain article, the price of which in London is 10 per cent. less than its price in Oxford. If the expenses of the journey are 15*s.*, and he makes a clear saving of 24*s.*, what does he pay for the article?

14. A man sold an article for 5*s.* 6*d.*, and by so doing gained three-eighths of what it had originally cost him. What would he have gained per cent. if he had sold it for 6*s.* 6*d.*?

15. A merchant who sold his goods at a profit of 10 per cent. found that, when he allowed $2\frac{1}{2}$ per cent. discount off his selling price, his business increased by one-third. Find whether his total profits were increased or diminished by adopting this plan, and in what proportion.

16. 33 per cent. of the candidates in an examination pass; 42 per cent. fail in mathematics, and 52 per cent. fail in classics. What per cent. of the whole of the candidates fail in both mathematics and classics?

17. Of the children attending school in Switzerland, 71·7 per cent. speak German only, 22·4 speak French only, 4·5 speak Italian only, and the remainder consists of 5,838 children who speak only Romansch; find the total number of children attending school.

18. A manufacturer reduces the price of his goods by $2\frac{1}{2}$ per cent.; what percentage increase in sales after the reduction will produce an increase of 1 per cent. in gross receipts?

19. A builder has two houses which cost him equal amounts; he sells the first to *A* at a profit of 30 per cent. and the second to *B* at a loss of 20 per cent. *A* then buys *B*'s house from him, giving the same price as he did for the first house. What profit per cent. did *B* make?

20. The total amount of tea consumed in the United Kingdom in 1905 was 258,779,000 lb., and in 1910 it was 286,892,000 lb. The population was approximately 42,970,000 in 1905, and 44,900,000 in 1910. Find, in each case correct to two decimal places, (*a*) the number of lb. of tea consumed per head of population in 1905 and in 1910, (*b*) the percentage increase of population during this period. Find also, as accurately as your results allow, the percentage increase in consumption of tea per head of population.

21. An alloy of copper and zinc contains 73 per cent. of copper; a second alloy of zinc and tin contains 54 per cent. of zinc; a third

alloy of tin and copper contains 12 per cent. of tin. If these three alloys are mixed in quantities proportional to 1, 2, and 3, what are the percentages of copper, zinc, and tin in the mixture?

22. *A* sells goods to *B* at a profit of 10 per cent. *B* sells the same goods to *C* for £2 7*s.* 8*d.*, making a profit of 4 per cent. What did *A* pay for the goods in the first case?

23. *A* left £15,325 between *B*, *C*, and *D*. To *B* he left such a sum that, after paying legacy duty at the rate of 10 per cent., *B* actually received £600. *C*'s legacy was to *D*'s legacy as 3 to 2. What net sums did *C* and *D* receive if they paid legacy duty at the rate of 3 per cent.?

24. If 500,000 rabbits bought in Belgium, in any year, at the rate of 75 francs per 100 are sold in England at an average price of 10*d.* each, how much do the importers gain after paying 2 per cent. on their outlay for freight, &c.? (25 francs = £1.)

25. A tradesman bought 84 baskets of fruit at 7*s.* 3*d.* a basket. Seven baskets proved bad and were thrown away. At what price must he sell the remainder in order, nevertheless, to gain £2 5*s.* 6*d.* by the transaction?

26. A tax is imposed on a certain commodity, and the amount of the tax is the same on every pound of the commodity, irrespective of quality. This tax increases the price per lb. of one quality of the commodity by 20 per cent. and the price per lb. of another quality of the commodity by 50 per cent. Compare the prices of the two qualities when the tax is included.

27. The table below gives information as to the quantity of general traffic carried by a Brazilian railway, and the receipts from it, in the years 1909 and 1910:

	Quantity.	Receipts.
1909	328,154 tons	5,471,136 milreis.
1910	377,324 ,,	5,944,524 ,,

Express the increase for 1910 as a percentage of the figures for 1909 in each case to the nearest integer; and find, to the nearest penny, the average price paid per ton in each year, taking a milreis as 16·28*d.*

28. The price of bread is 0 fr. 35 c. per kilogram. A certain baker allows his poor customers a reduction of 0 fr. 10 c. on every 4 kilograms. How much per cent. is this reduction?

29. A wood merchant buys a pile of wood 18·75 metres long, 1·14 metres wide, and 2·10 metres high at 6·75 francs per half-stere. He gives in payment a 500 franc note. What change should he receive, or how much does he still owe, if there is an allowance of $\frac{1}{2}$ per cent. on the bill? (A stere = 1 cubic metre.)

30. A person sold a horse at a loss of 20 per cent.; if he had received £10 more for it he would have gained 10 per cent. Find the cost of the horse.

31. A legacy of £2420 is left to three persons in such proportions that after the payment of legacy duty of 15 per cent. the first receives twice as much as the second, the second three times as much as the third. What are their respective shares?

32. If a kilometre is ·62137 of a mile, show that the percentage of error involved in taking a hectare equal to $2\frac{1}{2}$ acres is less than $1\frac{1}{4}$.

33. At an election where there were two candidates and 6732 voters on the register, approximately 58 per cent. voted, and the successful candidate received a majority of 332. How many votes did he receive?

34. A grocer wishes to make a mixture of three kinds of tea; he intends to sell the mixture at 2*s.* 6*d.* a pound and to make a profit of 25 per cent. He takes 300 lb. of the cheapest tea, for which he has paid 1*s.* 6*d.* a pound. Of the remaining two kinds he takes twice as much of one as he does of the other, the cost to him of these being 2*s.* 3*d.* and 3*s.* 6*d.* a pound respectively. How many pounds of the mixture does he make, and what is his total profit?

35. A quantity of tea was sold in London to a merchant in Paris at 16 lb. for 15*s.* 7*d.* At what price, expressed in francs and centimes per kilogram, must the buyer sell the tea in order to gain 10 per cent.? (1 kilogram = 2·204 lb., £1 = 25·26 francs.)

REVISION EXAMPLES

1. Simplify the fraction $\dfrac{\frac{1}{7}+\frac{1}{13}+\frac{7}{11}-\frac{1}{143}}{1-\frac{1}{91}-\frac{1}{11}-\frac{7}{143}}$.

2. Two pendulums start swinging together; one swings ten times in 3 seconds, the other fifteen times in 8 seconds. After what period will they, for the first time, swing together again?

3. The hot-water tap of a bath can fill it in 15 minutes, the cold-water tap in 3 minutes; and the waste tap can empty it, when full, in 2 minutes. The bath being full, all three taps are opened; in how many minutes will it be emptied?

4. An iron rod is known to weigh 17·343 lb. per foot run. What are the corresponding figures in grammes per centimetre?

5. In a certain chemical compound, potassium, chlorine, and oxygen are present. The proportions (by weight) are 78, 71, and 96 respectively. What percentage of each is present, and what weight of oxygen does a kilogram of the substance contain?

6. *A* and *B* are partners in a trading venture, *A* contributing £2000 and *B* £3000. *A*, however, acts as manager, the understanding being that of the profits *A* shall receive 25 per cent. for his services as manager, the remainder to be divided in the ratio of their contributions. *B*'s profits are £120; what do *A*'s amount to?

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7. Two men start from the same point to walk round a circular course in the same direction. One takes 10 minutes, and the other 18 minutes for each round. After how short a time will they be side by side again?

8. Without actual extraction of the root, prove that the square root of 6 lies between 2·449 and 2·450.

9. Simplify $\dfrac{4\frac{7}{8}-3\frac{1}{5}+5\frac{1}{10}}{\frac{7}{6}\text{ of }\frac{9}{8}\div\frac{15}{2}}$.

10. If a pound of tea costs 2*s.* 8*d.* and a ton of coal costs 18*s.* 6*d.*, how many ounces of tea should be exchanged for a hundredweight of coal?

11. Find the cost of repairing a road 29 mi. 7 fur. 200 yds. long at £34 16*s.* 8*d.* per mile.

12. A chemist, in making a mixture which should contain 9 grains of a drug to the ounce of water, puts by mistake 36 grains only into an 8 oz. bottle ; the mistake is discovered when 2 oz. of the solution have been used. How much of the drug should now be added to make it of correct strength ?

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13. Calculate, correct to two decimal places, the value of ·003089 of 817·34.

14. The voters of two parties were conveyed in carriages to the polling-booths, those of one party accommodating 5 and those of the other 4 passengers In all 120 journeys were made. Find the majority by which the election was decided, the total poll being 500.

15. On the assumption that 35 yds. are equal to 32 metres, find the difference in sq. inches between 5 sq. yds. and 4 sq. metres.

16. By selling goods at £240 a merchant gains 25 per cent. How much would he gain per cent. by selling them at £216 ?

17. A tradesman makes a profit of 30 per cent. on half his stock of a certain article and 25 per cent. on another third. What percentage of profit does he make on the remainder if he makes a total profit of 26 per cent. ?

18. The garrison of a fortress numbering 1000 men fight an action in which 40 men are killed and 120 wounded. Before the action it was calculated that their food would last for three weeks. After the action the wounded are allowed to remain on full rations, but the unwounded are put on three-quarter rations. How long will the food last under this arrangement ?

---

19. Find to the nearest penny the cost of 387 tons 5 cwt. 15 lb. of coal at 17*s.* 9*d.* per ton.

20. Show without actual multiplication or division that the following statement is incorrect :

$$65721 \times 7134 = 468823614.$$

21. What fraction of an acre is 5 po. 13 sq. yds. 6 sq. ft. 108 sq. in.?

22. A tradesman pays 16*s.* 8*d.* for an article which he marks for sale at £1 6*s.* 8*d.* He allows 25 per cent. discount for cash. What is his profit per cent.?

23. A dairyman obtained 41 lb. of butter from 100 gallons of milk, and a gallon of milk weighs 10·3 lb. What percentage of the total weight of milk was the weight of butter produced? Find the price realized per quart of milk used, if the butter is sold at 1*s.* 6*d.* a pound. Give your answer to the nearest farthing.

24. The price of certain goods in France is 13 fr. 16 c. per kilogram, and the carriage to England costs 5*s.* 3*d.* per cwt. Find to the nearest penny the price per lb. at which the goods must be sold in England so as to make a profit of 10 per cent. (1 kilo. = 2·204 lb. £1 = 25·22 francs.)

---

25. If 3 cwt. 1 qr. 4 lb. cost £3 1*s.* 4*d.*, what will be the cost of 7 cwt. 3 qrs. 11 lb.?

26. If a certain metal be worth 1 fr. 18 c. per kilogram, what is its equivalent price in £ per ton? (1 kilo. = 2·204 lb. £1 = 25·22 francs.)

27. A man pays away one-tenth of his income in life insurance premiums, and pays an income-tax of 9*d.* in the pound on the remainder. After making these payments he has £606 7*s.* 6*d.* left. Find his gross income.

28. A tradesman sells goods at a price which gives him a profit of 10 per cent. on the wholesale price. When the wholesale price goes up 30*s.* a cwt. he raises the retail price 4*d.* a lb. and then makes 12 per cent. profit. What was the original wholesale price per cwt.?

29. A train performs a distance of 27 miles in 1 hour 17 minutes; 15 miles of the distance are level and the remaining 12 miles are run on an uphill gradient, where the average speed is only two-thirds of the average speed on the level. Find the average speed on the level in miles per hour.

30. If 96 lb. of tea at 1*s.* 8*d.* per lb. be mixed with 128 lb. at 1*s.* 3*d.* per lb., at what rate per lb. should the mixture be sold so as to gain 40 per cent. upon the original outlay?

---

31. Evaluate to three decimal places:

$$\frac{27{\cdot}4605 \times 3{\cdot}76482}{{\cdot}43789 \times 15{\cdot}999}$$

32. Gunpowder is composed of 33 parts by weight of potassium nitrate, 7 of charcoal, and 5 of sulphur. Find how many pounds and ounces of charcoal are in $4\frac{1}{5}$ cwt. of powder.

33. The import duty on wine is increased from 40 per cent. to 50 per cent. In consequence the retail price is increased from 4*d.* to 5*d.* a glass. The retail seller originally made 20 per cent. on his outlay; find what he now makes.

34. How many yards of fencing will be required to enclose a square field containing 18·225 acres?

35. A contractor has 20 men working 8 hours a day engaged on a piece of work to be finished in 25 days. At the end of 14 days only one-third of the work is done. He engages more men, and all his men now work 9 hours a day. What is the least number of additional men that he must engage if the work is to be finished by the specified time?

36. *A*, *B*, and *C* have respectively £5000, £3000, and £2000 invested in a business. *A* and *B* receive respectively 20 per cent. and 10 per cent. of the annual profits in payment for managing the business. The remainder of the profits is divided among *A*, *B*, and *C* in proportion to their capital. If at the end of the year *A* receives altogether £600 more than *B*, what does each of them receive?

---

37. A man weighs 11 st. $5\frac{1}{2}$ lb. Express this in kilograms and grammes, given that a kilogram is equal to 2·204 lb.

38. Find the least number which, when divided by 63, 119, and 153, leaves a remainder of 7 in each case.

39. From the following data find the percentage of pensioners (correct to two significant figures) for each country and for Great Britain and Ireland:

	Population.	Pensioners.
England and Wales . . .	35,756,615	421,432
Scotland	4,877,618	75,134
Ireland	4,374,158	186,202

40. Average limestone consists of 112 parts of carbonate of lime to 9 of clay and 29 of sand. Find its percentage composition.

41. If a cubic yard of coal weighs a ton, find to the nearest whole number how many kilograms a cubic metre will weigh. Assume that a metre is 39·37 in. and that a kilogram is 2·205 lb.

42. Brass expands when heated so that a unit of length at the freezing-point of water becomes 1·0019 at the boiling-point of water. Express in cubic inches, correct to three significant figures, the increase in bulk when a brass cube whose edge is 9 in. at the freezing-point is heated to boiling-point of water.

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43. Explain our decimal system of arithmetic, and show how it is that by means of a few digits and a 0 we are able to express any number, however great. If we had a duodecimal (12) system, how should we write down the product of nine by five, in figures?

44. Find the product of 186·43876 and 2·3742, true to five places of decimals.

45. Simplify (i) $\dfrac{1\frac{1}{2}+2\frac{2}{3}+3\frac{5}{6}}{\frac{4}{5}\text{ of }\frac{3}{4}\text{ of }2\frac{1}{3}}$; (ii) $\dfrac{1\frac{1}{9}\text{ of }\frac{11}{20}}{10-\dfrac{10}{2-\dfrac{3}{4\frac{5}{6}}}}$

46. If a centimetre is ·0328 ft. and a cubic foot of water weighs 1000 oz., how many pounds are there in a kilogram?

47. At the end of a year the population of a town is 16,000; during the year the population increases by 560. What is the percentage increase during the year?

48. Air is a mixture of oxygen and nitrogen. The oxygen forms 20·8 per cent. of the volume of the air. If the weights of equal volumes of oxygen and nitrogen are as 16 to 14, find what percentage of the weight of air is oxygen.

---

49. Each member of a society subscribes as many pence as there are members in the society; the total subscription is £29 8*s.*; how many members are there?

50. Simplify $\frac{17}{151}$ of $\dfrac{3\frac{5}{11}-2\frac{3}{7}+1\frac{1}{2}}{2\frac{1}{3}\text{ of }1\frac{1}{7}-1\frac{1}{2}\text{ of }\frac{3}{7}}$.

51. It pays a tradesman, when a customer is purchasing 4 articles at $2\frac{1}{2}d.$ each, to induce him to take 7 for $1s.\ 3d.$; but it does not pay him to do so when he can sell 5 at $2\frac{1}{2}d.$ each. Show that his profit lies between 50 per cent. and 100 per cent.

52. The adult population of a town at two successive censuses was, respectively, 16,125 and 16,899. At the former census the men were 60 per cent. of the total; and the actual increase in the number of each sex was the same. What was the rate per cent. of the increase in the number (1) of men, (2) of women, (3) of the total?

53. The following table gives the record of three football teams, A, B, C:

Team	Games played.	Games won.	Games drawn	Games lost	Goals for.	Goals against.
A	16	10	3	3	27	16
B	15	9	4	2	23	14
C	16	8	7	1	25	15

A victory counts 2 points, a draw 1 point, and a defeat 0. Arrange the teams in order, the order being determined by the percentage of points obtained out of the maximum obtainable on the number of games played. If two teams have the same percentage, their order is determined by the ratios of the number of goals scored for each team to the number of goals scored against it.

54. Find from the following table the average value, to the nearest penny, per ton for each metal:

	Value in £.	Weight in Tons.
Copper . .	6,093,815	71,975
Lead	3,519,653	208,253
Tin	7,820,627	43,608
Zinc	2,399,315	93,704

---

55. I have to cycle to a meeting at a distance of 5 miles, 4 miles of which is uphill and the remainder on the level, my cycling pace uphill, on the level, and downhill being respectively 8, 10, and 12 miles an hour. The meeting will last an hour. How long shall I be away from home?

56. The total area of three estates is 1768 acres. If the areas of the two smaller estates be respectively $\frac{3}{5}$ and $\frac{2}{3}$ that of the largest, find the area of each.

57. Air contains 23 per cent. of its weight of oxygen. If 10 cu. ft. of air weigh 12·3 oz., find, to the nearest ounce, the weight of oxygen in an empty room 12 ft. high, measuring 27 ft. 4 in. by 18 ft.

58. The following table deals with the yield of potatoes in England:

	Yield.	Acreage on which grown.
For 1906 . .	2,439,063 tons	396,516 acres
For 1907 . . .	2,098,239 tons	381,891 acres

Find the yield per acre in tons, to two decimal places, for each of the two years; and give the difference of the yield per acre for the two years as a percentage, to the nearest whole number, of the yield per acre for 1906.

59. If a kilogram is equal to 2·204 lb. and 25 francs equal £1, find the cost in francs and centimes of half a kilogram of an article that costs 2*s.* 3*d.* per oz.

60. A man saves annually one-third of his income. When his income is increased by £100 a year, he saves only one-fourth, but finds that he is saving annually £12 10*s.* more than before. What was his original income?

---

61. Find, correct to the nearest penny, the cost of collecting 243,519 lb. of rubber at 16·87 cents per lb., 100 cents being equivalent to 2*s.* 4*d.*

62. Prove from the definition of a fraction that $\frac{1}{3} - \frac{1}{12} = \frac{1}{4}$.

63. Find the value of 22 cwt. 3 qrs. 21 lb. at £2 9*s.* 6*d.* per cwt.

64. An express train travels between two stations in a certain time. If the average speed of the train were increased by 20 per cent. the time would be shortened by one hour. How would the time be altered if the average speed were reduced by 10 per cent.?

65. The receipts of a railway during a half-year include £83,077 9*s.* 6*d.* from 2,233,564 second-class passengers and £553,186 2*s.* 6*d.* from 23,717,341 third-class, the average length of journey being the same for both classes. If the second-class carriages were made more attractive so as to induce 5 per cent. of the passengers who now travel third class to travel second class instead, by how much would the receipts of the railway be increased?

66. In consequence of the inaccurate length of his chain, a surveyor records the dimensions of a rectangular field as 275 yds. and 188 yds. If the true area is 10·4 acres, give the percentage error in the area and also in the length of the chain.

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67. If a yard is $\cdot\frac{2}{3}\frac{2}{5}$ of a metre, find, to two places of decimals, the difference, in cubic inches, between a cubic yard and $\frac{4}{5}$ of a stere. (A stere is a cubic metre.)

68. Find the dividend on £9648 8*s.* at 16*s.* $0\frac{1}{2}d.$ in the £.

69. What does a person gain or lose per cent. by selling butter at $8\frac{1}{2}d.$ per pound which cost £5 5*s.* per cwt.?

70. IMPORTS TO THE UNITED KINGDOM.

	1890	1900	1910
	£	£	£
Channel Islands	1,212,158	1,586,243	1,599,596
Canada	12,506,642	22,621,164	26,827,668
Newfoundland	401,004	516,518	332,342
West Indies	1,938,022	1,895,212	2,123,315
British Guiana	853,564	487,874	633,770
Australia	23,546,368	23,568,918	29,074,112
New Zealand	8,285,662	12,741,510	14,664,331
India	27,648,857	36,472,636	29,615,570
Straits Settlements	4,584,783	6,283,820	7,942,940
Ceylon	4,101,275	4,135,027	5,127,938
Hong Kong	630,818	466,811	547,339
Mauritius	1,224,530	1,264,554	1,270,146
Aden	204,419	137,668	128,917
Brazil	3,940,069	6,237,746	6,927,368
Chile	3,711,544	5,422,941	7,381,156
Cape Colony	4,301,521	4,933,489	6,097,134
Peru	1,070,949	2,371,692	2,887,438
Argentina	6,168,624	23,035,202	35,727,547
United States	89,607,392	99,227,802	99,037,603

Find the total imports for each year and calculate, correct to one decimal place, the percentage increase or decrease in the total during each period of ten years.

71. At 8 a.m. three electric cars leave the same dépôt, and make three different journeys. The first is timed to take 1 hr. 45 min. over its journey, and to remain 15 min. at the dépôt before setting out on the same journey again, and so on. The second and third cars take 1 hr. 5 min. and 1 hr. 18 min. respectively over their journeys, and remain 7 min. and 12 min. respectively at the dépôt. Find when the cars next leave together.

72. A lady bought two kinds of material for a dress, 20 yds. in all, for £5; one kind cost 4*s.* a yard, the other 9*s.* a yard. How many yards of each kind did she buy?

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73. (i) Express $\frac{3}{2}+\frac{4}{3}+\frac{5}{4}+\frac{6}{5}+\frac{7}{6}+\frac{8}{7}+\frac{9}{8}$ as an improper vulgar fraction.

(ii) Find two numbers whose sum is 138·752 and difference 84·53.

74. Taking a gallon to contain 277 cubic inches and an inch to be 2·5 centimetres, find the number of gallons in 1567 litres.

75. By selling an article at 6*s.* $1\frac{1}{2}d.$ a tradesman gains 5 per cent. What would he gain if he sold it at 6*s.* 5*d.*?

76. Fill in the gaps in the following statement.

During the year 83,029,999 telegrams of all descriptions were sent, showing an increase over the previous year of 3,606,443, or per cent.

The ordinary inland telegrams increased by 4·95 per cent. from 65,561,817 to . The total value of inland telegrams was £2,130,973 as compared with £2,070,045 for the previous year, making the average value per telegram as compared with

The number of foreign telegrams transmitted by the Post Office was 6,482,806, an increase of 174,819, or of per cent. over the previous year's figures.

77. There are two clocks in a building, one of which gains three minutes in four days and the other loses four minutes in five days. They are set right at noon on Sunday. Determine

the time indicated on each clock, to nearest second, when they differ by five minutes.

78. A joiner takes 47 hours to make a cupboard. If his rate of pay is $11\frac{1}{2}d.$ per hour, and the material costs $9d.$ per cubic foot, what charge will be made if the amount of material used is $6\frac{2}{3}$ cu. ft., the cost of sundries is $5s.$ $9d.$, and the charge is 10 per cent. above the actual cost of labour and material?

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79. Add the difference between ·035 of a ton and ·064 of a cwt. to the difference between ·27 of a qr. and ·78 of a lb., giving the answer in pounds to three decimal places.

80. What is the least sum of money that contains $14s.$ $7d.$, £1 $11s.$ $6d.$, and £3 $15s.$ each an exact number of times?

81. With what two quantities of spirits that cost me $14s.$ $6d.$ and $17s$ $6d.$ respectively did I make a blend of 12 gallons, if by selling the mixture at $16s.$ $3d.$ a gallon I should have incurred a loss of $2\frac{1}{2}$ per cent.?

82. The value per acre of some uncultivated land was given as 3·29 dollars in 1902, 5·84 dollars in 1906, and 14·84 dollars in 1910. Find, to the nearest whole number, the increase in the value in 1906 over that in 1902 as a percentage of the value in 1902, and the increase in the value for 1910 over that in 1906 as a percentage of the value in 1906. Also, give the value in 1910 in English money, taking a sovereign as equal to 4·87 dollars.

83. A steamer A is in distress and stationary, and fires a gun which is heard on another steamer B coming direct towards her. A fires another gun 10 min. after the first, and this is heard on B 9 min. 48 sec. after the first. At this moment B is 4 miles from A. How soon will B reach A? Give the answer to the nearest second, supposing sound to travel at the rate of 1130 ft. per second.

84. A farm of 36 acres 3 roods is rented at £1 $2s.$ $6d.$ per acre. Express the rent in francs per square kilometre. (1 kilometre = ·6214 miles. £1 = 25·22 francs.)

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85. Find, correct to three places of decimals, the value of:

(i) $(\frac{2}{3}\frac{7}{7} - \frac{8}{1}\frac{}{1}\frac{}{3}) \div (\frac{1}{2}\frac{5}{1}\frac{3}{1} + \frac{1}{1}\frac{7}{9})$.

(ii) The square root of ·03651.

86. A rubber plantation has 49,201 rubber trees planted on 311 acres. On 28 of these acres the trees are planted so that each tree has an allowance of 300 sq. ft. Find the average allowance per tree on the remainder of the plantation.

87. Find, to the nearest shilling, the cost of 12 acres 22 sq. poles at £56 14*s.* an acre.

88. The total receipts of a railway for a year amounted to £4,601,933 18*s.* 4*d.* The total expenditure for the same year was as follows:

		£	*s.*	*d.*
(A)	Permanent way and works . .	422,069	17	10
(B)	Rolling stock maintenance	376,062	17	5
(C)	Running and traffic expenses . .	1,297,764	12	5
(D)	Tax	54,540	13	3
(E)	Other expenses	366,313	18	11

Express each of these items, to two decimal places, as a percentage of the total receipts; and express, to one decimal place, the balance surplus of receipts over expenditure as a percentage of the total receipts.

89. A man buys oranges at 6*s.* 8*d.* a gross and sells them at 1*s.* 3*d.* a score. What is his gain per cent.?

90. If A can beat B by 10 yds. in 100 yds. race, and B can beat C by 15 yds., by how much will A beat C?

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91. Simplify (i) $5\frac{3}{1}\frac{}{0} + 4\frac{1}{7}\frac{9}{5} + 1\frac{5}{8} - 6\frac{2}{3}$;

(ii) $1\frac{6}{1}\frac{}{9} \times 2\frac{1}{2}\frac{7}{0} + 4\frac{4}{1}\frac{}{1} \div 1\frac{5}{1}\frac{}{1}$.

92. A train 210 yds. long, going 25 miles an hour, overtakes another train 318 yds. long, going 17 miles an hour in the same direction on a parallel line; how long will the first train take in passing the second train?

93. It was said at the end of June, 1909, that in the previous 52 weeks 38,542,650 cubic yards had been excavated for the Panama Canal; and that there remained to be excavated a

hundred million cubic yards. If the work were continued at the same rate, find when the excavation would be finished.

94. If A, B, and C could reap a field in 18 days; B, C, and D in 29 days; C, D, and A in 24 days; and D, A, and B in 27 days, in what time would it be reaped when they all work together?

95. What length of carpet, 30 in. wide, will be required for a room 17 ft. 4 in. long and 13 ft. 9 in. wide? If the cost of the carpet is £5 19*s.* 2*d.*, what is the price of it per yard?

96. A merchant in New York buys goods in Geneva of the value of 4004 francs; find the value of the goods in dollars, when we see by the newspapers that the exchange between London and New York is 4·8650 dollars to £1, and between London and Geneva is 25·48 francs to £1.

CHAPTER XVIII

SIMPLE INTEREST

Revise § 147, Chap. X, Part I.

(231.) If money is lent, the borrower has to pay for the use of it, and also has at some time to pay back the whole sum borrowed. The original sum borrowed is called the *Principal*, the additional sum is called *Interest*, while the total amount paid back, i. e. the Principal and Interest together, is called the *Amount*. The interest paid for the use of £100 for one year is referred to as the rate per cent. per annum. The rate of interest and the periods when the interest is to be paid are settled at the time of borrowing; also a date may be fixed for repayment of the loan.

Sometimes the interest is not paid until the principal is repaid. If the interest for each year is calculated on the original sum lent, the money is said to be lent at *Simple Interest*. If, however, the interest owing at the end of each year is added to the principal, so that for the second year, and each year following, interest has to be paid on the total amount owing at the beginning of the year, the money is said to be lent at *Compound Interest*. For the present we consider only Simple Interest.

(232.) The interest for one year is the given percentage of the principal; since the interest for each year is the same, the interest for any period is found by multiplying the interest for one year by the number of years. If the period involves days or months, they must be expressed as the fraction of a year; it is always understood that a month is $\frac{1}{12}$ of a year unless the name of the month is given, also a day is always reckoned as $\frac{1}{365}$ of a year even though it is known that the particular year is a leap year.[1]

[1] In business interest is usually reckoned by means of tables, and these tables have been calculated on the understanding that there are 365 days in the year.

Example i. *Find the simple interest on* £372 10*s.* *for* 3 *years at* 4 *per cent.*

(Mental. Interest for 1 year = $\frac{4}{100}$ of the Principal.)

$$\begin{aligned}\text{Interest} &= 3 \times \tfrac{4}{100} \text{ of } £372{\cdot}5 \\ &= 12 \times £3{\cdot}725\,^{1} \\ &= £44{\cdot}7 \\ &= £44\ 14s.\end{aligned}$$

Be careful not to divide through by any of the factors of 100. It is far easier, when we use decimals, to divide by 100 than by 25; consequently we do not reduce $\frac{4}{100}$ to $\frac{1}{25}$.

Example ii. *Find the simple interest on* £1732 11*s.* 5*d.* *for* 3 *years* 4 *months at* $3\frac{1}{5}$ *per cent.*

(Mental 3 years 4 months = $3\frac{1}{3}$ years.

Interest for 1 year = $\frac{3\frac{1}{5}}{100}$ of the Principal.)

$$\begin{aligned}\text{Interest} &= \frac{10}{3} \times \frac{16}{500} \times £1732{\cdot}571 \text{ (§ 221).} \\ &= \tfrac{32}{300} \times £1732{\cdot}571\,^{1} \\ &= \tfrac{4}{3} \times £138{\cdot}60568 \\ &= 4 \times £46{\cdot}2019 \\ &= £184{\cdot}8076 \\ &= £184\ 16s.\ 2d. \text{ to nearest penny.}\end{aligned}$$

When dealing with fractional rates per cent. it is often best to arrange the work as in the next example.

Example iii. *Find the simple interest on* £842 10*s.* 4*d.* *at* $4\frac{1}{2}$ *per cent. for* 6 *years and* 4 *months.*

	£	
$4\% = \frac{4}{100}$	842·517	Principal
$\frac{1}{2}\% = \frac{1}{8}$ of 4%	33·70068	Interest at 4 % for 1 year
	4·21258	,, $\frac{1}{2}$ % ,,
	37·91326	,, $4\frac{1}{2}$ % ,,
	227·47956	,, ,, 6 years
	12·63775	,, ,, $\frac{1}{3}$ year
	240·117	,, ,, $6\frac{1}{3}$ years.

Interest = £240 2*s.* 4*d.* to the nearest penny.

[1] After some practice the steps marked [1] may be omitted.

Example iv. *Find the interest on* £491 7*s.* 2*d.* *at* $3\frac{1}{4}$ *per cent. from Jan.* 5*th*, 1913, *to May* 12*th*, 1913.

$$\text{Number of days} = 26+28+31+30+12$$
$$= 127.$$

Note that Jan. 5th is not counted, but May 12th is counted.

$$\text{Interest} = \frac{127}{365} \times \frac{13}{400} \times £491{\cdot}358$$
$$= \frac{127 \times 13 \times £245{\cdot}679}{73000}$$
$$= £5{\cdot}556$$
$$= £5\ 11s.\ 1\tfrac{1}{2}d.$$

245·679 × 13
2456·79
737·037÷
3193·827 × 127
383259·24
22356 789
73) 405616·029 (5556
406
411
466
28

(Rough Check. Interest on £500 at $3\frac{1}{4}$ for 4 months
$= £\frac{1}{3} \times 5 \times 3\frac{1}{4}$.
$\therefore = £5\frac{5}{12}$)

Third, tenth, and tenth rule.

$\frac{1}{73}$ = ·0137, very nearly.

$$1+\tfrac{1}{3}+\tfrac{1}{10} \text{ of } \tfrac{1}{3}+\tfrac{1}{10} \text{ of } (\tfrac{1}{10} \text{ of } \tfrac{1}{3})$$
$$= 1 + \frac{100+10+1}{300}$$
$$= 1{\cdot}37.$$

·0137
73) 1·00
270
510

Hence $\frac{1}{73000}$ = ·0000137, nearly,

$= \frac{1}{10000}(1+\frac{1}{3}+\frac{1}{30}+\frac{1}{300})$, nearly.

This gives a compact method for dividing by 73,000.

In the last example

$$\text{Interest} = \frac{127 \times 13 \times £245{\cdot}679}{73000}.$$

Multiply out the numerator, obtaining the product correct to nearest whole number.

In this case seven figures must be kept:

	£
	245·679
	13
	3193·827
	127
	383259·2
	22356 8
	405616·
add $\frac{1}{3}$ of line above	135205·
add $\frac{1}{10}$ of line above	13520
add $\frac{1}{10}$ of line above	1352
	555693·

Divide by 100,000
Interest = £5 11s. 2*d*. to nearest penny.
This method gives a result which is too great by $\frac{1}{4}d$. for every £10 in the answer.

Example v. *A man has a balance at the bank on Jan. 1st of £472. At the beginning of February and each alternate month in the ensuing half-year he deposits £25. In the middle of April he withdrew £215. If interest is allowed at the rate of 2 per cent. per annum, what interest should be added to his account at the end of the half-year?*

Interest on £472 for January = Int. on £472 for 1 month.
(On Feb. 1st there is £497 in the bank.)
Interest on £497 for 2 months[1] = Int. on £497 × 2 for 1 month
= ,, £994 ,,
(On April 1st there is £522 in the bank.)
Interest on £522 for $\frac{1}{2}$ month = Int. on £522 × $\frac{1}{2}$ for 1 month
= ,, £261 ,,
(There is now £522 − £215 = £307 in the bank.)
Interest on £307 for $1\frac{1}{2}$ months = Int. on £307 × $\frac{3}{2}$ for 1 month
= ,, £410$\frac{1}{2}$,,
(On June 1st there is £332 in the bank.)
Interest on £332 for 1 month = Int. on £332 for 1 month.
Hence by addition
the total interest on June 30th = Int. on £2469$\frac{1}{2}$ for 1 month.
$= \frac{1}{12}{}^{1} \times \frac{2}{100} \times$ £2469·5
= £4·116
= £4 2*s*. 4*d*.

[1] It is seen that the method of working is similar to that of Partnership sums. In order to make the principle clearer, all months in the above working have been reckoned as $\frac{1}{12}$ of a year; the time ought really to have been reckoned in days.

Examples XVIII a.

Find the Interest and the Amount on the following sums of money:

1. £3200 for 2 years at 4 per cent.
2. £1740 for 3 years at 5 per cent.
3. £160 10*s.* for 3 years at 4 per cent.
4. £1730 5*s.* for $3\frac{1}{2}$ years at $2\frac{1}{2}$ per cent.
5. £8471 6*s.* 6*d.* for 4 years at $3\frac{1}{2}$ per cent.
6. £1929 11*s.* for $4\frac{1}{2}$ years at $3\frac{1}{2}$ per cent.
7. £8193 3*s.* 6*d.* for 6 years at 4 per cent.
8. £462 5*s.* 10*d.* for 7 years at $3\frac{1}{2}$ per cent.
9. £316 6*s.* 8*d.* for $3\frac{1}{3}$ years at $2\frac{1}{5}$ per cent
10. £643 13*s.* 10*d.* for 2 years 3 months at $7\frac{1}{2}$ per cent.
11. £754 14*s.* 11*d.* for 8 months at $6\frac{1}{4}$ per cent.
12. £392 5*s.* 8*d.* for 10 months at 5 per cent.
13. £147 18*s.* 4*d.* for 11 months at $4\frac{1}{2}$ per cent.
14. £3027 for $6\frac{1}{2}$ years at $3\frac{1}{2}$ per cent.
15. £4123 4*s.* 8*d.* for 6 years at $5\frac{1}{2}$ per cent.

Find the Interest on the following sums of money:

16. £3120 from Jan. 1st to Mar. 12th at 5 per cent.
17. £4000 from Feb. 10th to April 10th at $2\frac{1}{2}$ per cent.
18. £3070 from Mar. 11th to June 12th at 2 per cent.
19. £400 from Mar. 10th to Aug. 4th at 3 per cent.
20. £6700 from June 12th to Jan. 4th at $2\frac{3}{4}$ per cent.
21. £9800 from July 12th, 1911, to Mar. 4th, 1912, at 4 per cent.
22. What is the interest on £P at r per cent. for t years?

Use the result to work any of examples 1 to 21.

23. Prove that the third, tenth, and tenth rule gives an error of about $\frac{1}{4}d.$ in every £10 of the correct answer.

24. Make a table, correct to six decimal places, to show the interest on £1 for any number of days less than a year at 2 per cent., 3 per cent., and 5 per cent. (See Chap. XVI for construction of tables. The table need only show the interest for 1, 2, . . . 10, 20, 30, ... 100, 200, 300 days. Show also 3, 6, 9, 12 months.)

Use the table to find the interest on

a. £57 at 2 per cent. from Mar. 3rd to June 21st.

b. £163 5*s.* 4*d.* at 3 per cent. from Jan. 14th to Sept. 20th.

c. £117 4*s.* 2*d.* at $2\frac{1}{2}$ per cent. ($\frac{1}{2}$ of 5 per cent.) from April 3rd to Dec. 4th.

25. At the beginning of the year a man has £493 on deposit in a bank. Two months later he withdraws £110, but replaces £75 a month and a half afterwards. At the end of May he withdraws £52 What interest is due on June 30th, at $2\frac{1}{4}$ per cent. per annum? (Each month is to be reckoned as $\frac{1}{12}$ of a year.)

26. The entries in a bank pass book are shown below. Find the interest due at the end of the year at 2 per cent. per annum.

	Deposit.			Withdraw.		
	£	*s.*	*d.*	£	*s.*	*d.*
Jan. 1st, Balance.	74	7	6			
Feb. 3rd . . .	45	0	0			
Mar 15th . .				52	3	4
May 30th . . .	47	5	0			
June 19th . . .				24	10	0
Aug. 24th . .	20	0	0			
Sept. 12th . .				35	4	0
Oct. 19th . . .	35	0	0			
Dec. 20th . . .				40	13	2

27. Extract from regulations of the Post Office Savings Bank:

'Interest at the rate of £2 10*s.* per annum is allowed on every complete pound deposited, and commences on the first day of the month next following the deposit, and is calculated to the 31st of December, when such interest is added to the principal and begins to carry interest as an ordinary deposit.'

Calculate the interest due on the following accounts, each month to be reckoned as $\frac{1}{12}$ of a year:

(*a*)

Date.	Amount of Deposit.			Amount of Withdrawal.		
	£	*s.*	*d.*	£	*s.*	*d.*
Jan. 1st, Balance .	48	7	6			
Jan. 15th . . .				20	0	0
Feb. 5th	23	8	0			
April 15th . .				15	0	0
Sept. 19th . .	12	14	0			
Oct. 8th				11	10	0

(*b*)

1911.	£	s.	d.	£	s.	d.
Jan. 1st, Balance .	115	12	0	.	.	.
Mar. 25th . . .	.	.		13	6	0
July 4th	24	3	0	.	.	.
Dec. 20th . .	.	.	.	20	0	0
1912.						
Mar. 3rd . . .	16	17	0	.	.	
April 17th . . .	.	.	.	12	10	0
June 7th . . .	25	9	0	.	.	.
Sept. 20th . . .	.	.	.	13	0	0
Dec. 19th . .	10	10	0	.	.	.

(*c*) On Jan. 1st the balance was £23 12*s.* and 15*s.* was paid in on the first day of every succeeding month.

28. What is the interest on 1865 francs for 6 years at $3\frac{1}{2}$ per cent.?[1]

29. Find the interest on 34,500 francs at $5\frac{1}{4}$ per cent. for 2 years 82 days.

30. What is the interest on 53,765 francs from Jan. 25th to May 11th at 3 per cent.?

31. Calculate the interest on 7568·35 dollars at 6 per cent from Feb. 17th to Sept. 12th.

32. What is the interest on 4957·72 dollars for 3 years at $3\frac{3}{4}$ per cent.?

33. Which is the greater, and by how much: the interest on 4975·8 marks at $3\frac{1}{2}$ per cent. for 7 months or the interest on 5852·75 marks at $2\frac{3}{4}$ per cent. for 8 months?

(233.) Since the rate per cent. per annum gives the interest on £100 for one year, all sums in Simple Interest can be worded as examples of ratio methods, with which the student is now fully acquainted. We proceed to work a few examples by different methods, but the student is urged to devise his own method.

[1] In France, Germany, and the United States it is customary to calculate simple interest on the understanding that there are 360 days in the year and that each month is $\frac{1}{12}$ of a year; this considerably simplifies the calculation. The answers to these examples are worked out on this assumption. In English examinations, however, the student is advised to reckon 365 days to the year, even though the question deals with foreign money.

Example i. *Find a sum of money which in* 3 *years at* 4 *per cent. amounts to* £132 4*s.* 10*d.*

£100 gains £12 interest and amounts to £112.

$\therefore$ Principal $= \frac{100}{112}$ of amount

$= \frac{100}{112}$ of £132 4*s.* 10*d.*

$= £\frac{13200242}{112}$

$= £118{\cdot}074$

$=$ £118 1*s.* 6*d.* to nearest penny.

$$1{\cdot}12 \overline{)\,132{\cdot}242}\quad 118{\cdot}074$$

20 2

9 04

82

4

Example ii. *Find a sum of money which gives* £32 5*s.* 4*d. interest in* 4 *years at* $3\frac{1}{2}$ *per cent.*

£100 gains £$(4 \times 3\frac{1}{2})$, i.e. £14 interest.

$\therefore$ Principal $= \frac{100}{14}$ of interest.

Here we have to multiply by a ratio greater than unity, and we require an answer correct to three decimal places. We must, therefore, have more than three decimal places before multiplying by this ratio, since the carry from the fourth place will affect the third place in the answer. We must either use the full method for reducing to a decimal of a pound or the mental method of § 221.

$\therefore$ Principal $= \frac{100}{14}$ of £32 5*s.* 4*d.*

$= \frac{100}{14}$ of £32·26667

$= £\frac{1613338}{7}$

$= £230{\cdot}477$

$=$ £230 9*s.* 6*d.* to the nearest penny.

$$20 \overline{)\,5{\cdot}33333\ s.}\quad 26667\ £$$

Example iii. *At what rate per cent. per annum will* £1885 15*s.* *amount to* £2569 6*s.* $8\frac{1}{4}$*d. in* 7 *years and* 3 *months at simple interest?*

The interest is £683 11*s.* $8\frac{1}{4}$*d.*

Rough answer. Interest on £1900 for 7 years is about £700

$\therefore$ interest on £100 for 1 year is about $\frac{£700}{19 \times 7} = \frac{100}{19}$.

Required rate is about 5 per cent.

The interest on £1885·75 for $7\frac{1}{4}$ years is £683·585;

$\therefore$ the interest on £100 for 1 year is £683·585 $\times \frac{100}{1885{\cdot}75} \times \frac{4}{29}$.

Hence rate per cent. $= 68358{\cdot}5 \times \frac{4\times4}{7543\times29}$

$= \frac{1093736}{218747}$

$= 5$ per cent.

273434×4
1093736

7543×29
15086
67887
218747

If the interest, £683 11*s.* 8¼*d.*, had been expressed as a vulgar fraction, the answer would have come exactly 5 per cent. Interest is worked to the nearest penny or farthing, consequently when the sum is reversed the rate per cent. will usually not come exactly a whole number or simple fraction, but very nearly to such a value. It is better to work in decimals, as above, and to give the answer correct to the nearest shilling. It is only in examination questions, and not always then, that exact answers are obtained.

Example iv. *In how many months will* £1550 12*s.* 6*d. amount to* £1580 15*s.* 6¼*d. at* 3⅓ *per cent. per annum, simple interest?*

The student should be careful to notice that, while simple interest is doubled when the time (or rate) is doubled, the amount is not doubled. To prevent error it is advisable to subtract at once and so find the interest.

Rough answer. Interest on £1550 for 1 year is $£15\frac{1}{2} \times 3\frac{1}{3}$, i.e. £52 nearly.
∴ the time is about $\frac{30}{52}$ of a year, i.e. about 7 months.

Interest on £1550·625 for 1 year is $£1550{\cdot}625 \times \frac{3\frac{1}{3}}{100}$.

$= £516{\cdot}875 \left(\text{since } \frac{3\frac{1}{3}}{100} = \frac{1}{30}\right)$.

Interest for required time is £30 3*s.* 0¼*d.*

∴ required time $= \frac{30{\cdot}151}{51{\cdot}6875}$ of a year

$= \frac{241{\cdot}208}{413{\cdot}5} \times 12$ months (multiplying numerator and denominator by 8)

$= \frac{2894{\cdot}5}{413{\cdot}5}$ months

$= 7$ months.

Example v. *A dealer offers to sell an article for* 10*s. cash or for* 2*s. cash and five additional payments of* 2*s. at intervals of* 1 *month. Reckoning simple interest, calculate the rate of interest charged per annum.*

We suppose that the dealer invests each payment as it is received. He ought to have the same amount 5 months after selling the article whether he is paid cash down or by instalments.

If the dealer receives 10*s.* cash, he has at the end of 5 months

10*s.* + interest on 10*s.* for 5 months
= 10*s.* + interest on 50*s* for 1 month.

If the dealer receives 2*s.* cash and the instalments, he has at the end of 5 months

2*s.* + interest on 2*s.* for 5 months (cash payment)
+2*s.* + ,, ,, 4 ,, (1st instalment)
+2*s.* + ,, ,, 3 ,, (2nd ,,)
+2*s.* + ,, ,, 2 ,, (3rd ,,)
+2*s.* + ,, ,, 1 month (4th ,,)
+2*s.* (5th ,,)

i. e. he has 12*s.* + ,, ,, 15 months
= 12*s.* + ,, 30*s.* for 1 month.

Hence 10*s.* + interest on 50*s.* for 1 month = 12*s.* + interest on 30*s.* for 1 month.

∴ interest on 50*s.* − interest on 30*s.* for 1 month = 12*s.* − 10*s.*
i. e. , 20*s.* for 1 month = 2*s.*
,, 100*s.* for 12 months = 120*s.*

Required rate = 120 per cent. per annum.

Examples XVIII b.

Find the principal which will amount to the following sums after the given time at the given rate per cent. :

1. £3245 after 3 years at 4 per cent.
2. £1543 5*s.* 8*d.* after $3\frac{1}{2}$ years at 5 per cent.
3. £1492 11*s.* 6*d.* after 6 years at $3\frac{1}{4}$ per cent.
4. £698 7*s.* 6*d.* after 4 years at $5\frac{1}{4}$ per cent.
5. £493 8*s.* 3*d.* after 2 years 8 months at $7\frac{1}{2}$ per cent.
6. £529 5*s.* 5*d.* after 7 months at 2 per cent.

7. Find the sum of money on which the simple interest for 4 years at $2\frac{3}{4}$ per cent. per annum is £312 8*s.*

8. On what sum will the simple interest for 5 months at $2\frac{1}{4}$ per cent. be £35 14*s.* 2*d.*?

9. At what rate per cent. will the simple interest on £3064 11*s.* 8*d.* amount in $2\frac{1}{2}$ years to £263 3*s* $0\frac{1}{4}$*d.*?

10. At what rate per cent. will the simple interest on £415 7*s.* 2*d.* amount to £9 4*s.* 7*d.* in 1 year 4 months?

11. The amount of £420 put out at simple interest for 8 months is £429 16*s.* At what rate is the interest calculated?

12. If £341 amounts to £356 at the end of $2\frac{1}{2}$ years, what will £427 3*s.* amount to in 14 months, the rate of interest being the same in both cases?

13. In how many years will £523 7*s.* 2*d.* amount to £586 3*s.* 3*d.* at 3 per cent. per annum?

14. In what time will the simple interest on £55 12*s.* 6*d.* amount to £5 11*s.* 3*d.* at $2\frac{1}{2}$ per cent. per annum?

15. In how many days will the interest on £493 12*s.* amount to £14 4*s.* 11*d.* at $3\frac{1}{2}$ per cent.

16. Find the rate per cent. at which £525 will produce £183 15*s.* in 7 years at simple interest.

17. In what time will £2396 13*s.* 4*d.* amount to £2825 at $3\frac{1}{4}$ per cent. simple interest?

18. Find the sum of money which will amount to £2050 in 3 years time at 4 per cent., reckoning simple interest.

19. If the simple interest on £3000 amounts to £810 in 6 years, at what rate per cent. was the money borrowed?

20. What sum of money put out to interest at 5 per cent. will produce a yearly income of £341 5*s.*?

21. At what rate per cent. simple interest will £910 amount to £1109 1*s.* 3*d.* in $6\frac{1}{4}$ years?

22. In what time will £850 amount to £1011 10*s.* at 4 per cent. simple interest?

23. The simple interest on £7894 in $2\frac{1}{2}$ years amounts to £690 14*s.* 6*d.* What is the rate per cent.?

24. In what time will the simple interest on £312 10*s.* at 15 per cent. per annum amount to £390 12*s.* 6*d.*?

25. A billiard table may be bought for £4 15*s.* cash, or 10*s.* cash may be paid and nine monthly payments of 10*s.* What is the rate of interest charged per annum?

26. A man borrows £10 on Jan. 1st; he pays the loan by paying £1 on the first of each month for the rest of the year. If each month be reckoned as $\frac{1}{12}$ of a year, what rate of interest per annum does he pay?

COMMERCIAL DISCOUNT

(234.) In wholesale business transactions it is unusual to pay cash or even to pay by cheque. Suppose Mr. John Brown buys goods from Messrs. Jones, Robinson & Co. to the value of £512 on May 17th; it may be arranged that he shall pay in 3 months' time. The document showing Mr. Brown's liability to pay may be either a Promissory Note or a Bill of Exchange.

PROMISSORY NOTE.

May 17th, 1913.

Three months after date I promise to pay to the order of Messrs Jones, Robinson & Co. the sum of Five hundred and twelve pounds for value received.

JOHN BROWN.

Since this is payable to the *order* of Messrs. Jones, Robinson & Co., the note must be *endorsed* by them, that is, their business signature must be written across the back of the note.

BILL OF EXCHANGE.

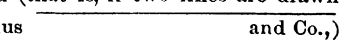
May 17th, 1913.

Three months after date, pay to our order the sum of Five hundred and twelve pounds for value received.

JONES, ROBINSON & CO.

To MR. JOHN BROWN.

A Bill of Exchange is of no value until it has been *accepted* by the person to whom it is addressed; this he does by writing across the front of the bill 'Accepted John Brown'. The acceptor usually states where the payment of the bill will be made. The custom has grown up, and is recognized by law, to allow three *days of grace*; this means, in this case, that payment of the note or bill cannot be enforced till three days after August 17th. The bill is legally due on August 20th.

(235.) Depositors in a bank can pay money by means of a cheque. This is a direction to the banker to pay a specified person a definite stated sum of money; the cheque must, of course, be signed by the depositor. If the cheque reads 'Pay Mr. C. R. Editor or bearer', the bank-cashier will pay the money to any one who presents the cheque for payment. If, however, it reads 'Pay Mr. C. R. Editor or order', the cashier will not pay unless Mr. Editor has endorsed the cheque (i.e. has signed his name on the back of the cheque). If the cheque is crossed (that is, if two lines are drawn across the cheque, from bottom to top, thus ) the cashier will not pay the money across the counter. The holder of a crossed cheque can obtain its cash value only by paying it into his own bank account or to a friend's account. Thus a crossed cheque is of no value to any one who steals or finds it; it is usual to cross all cheques that are sent by post.

(236.) Very likely Messrs. Jones, Robinson & Co. may need money before August 20th. They will then take the bill to a Bill Broker, who will buy the bill from them. Naturally the broker does not intend to lose over the transaction, so instead of paying the full amount for the bill he will pay less. The amount which he deducts is called *Discount*; and the bill is said to be discounted.

Suppose the bill is discounted on July 2nd. If the broker instead of parting with cash on July 2nd had kept it on deposit in his bank, it would have earned interest by August 20th. In order that the broker may not lose, he should pay for the bill such a sum of money as together with the interest on that sum would amount to £512 on August 20th. The rate of interest is

fixed from time to time by the Bank of England. Let us suppose it is 2 per cent. Then, in theory, the broker ought to make the following calculation:—What sum of money will in 49 days (29 days in July and 20 in August) amount to £512 at 2 per cent. per annum?

This would give the *True Present Value* of the bill; the difference between this and the amount of the bill is called the *True Discount.*

(**237.**) As we have seen, it is not so simple to find the Principal when the Amount is given as it is to find the Amount when the Principal is given.. But it is the custom to deduct from the amount of the bill, not the True Discount, but the interest on that amount; this is called *Commercial* or *Bankers'* Discount. Hence the broker would use a book of interest tables to find the interest on £512 for 49 days at 2 per cent. and deduct that amount from the £512.

Discount questions are generally set in some such form as the following:

Example. *A bill for £512 is drawn on May 17th at 3 months. What is the discount if it is discounted on July 2nd, the rate of interest being 2 per cent.?*

The bill is legally due on August 20th.

Number of days to run = 29 + 20 = 49.

Discount = Interest on £512 for 49 days at 2 per cent.

$$= £512 \times \frac{2}{100} \times \frac{49}{365}$$

$$= £\frac{512 \times 49 \times 4}{73000}$$

= £1 7*s.* 6*d.* (§ 232, p. 478).

3584 × 7
25088 × 4
100352
3340
3345
334
137481

(238.) We have then the following statements:

True Present Value + Int. on True Present Value = Amount . *A.*
Int. on True Present Value = True Discount on Amount . *B.*
Banker's Discount = Int. on Amount . . . . *C.*
∴ Banker's Discount = Int. on True Present Value
+ Int. on True Discount
= True Discount + Int. on True Discount *D.*

Note carefully that Banker's Discount is the *real* discount of commercial transactions and that True Discount is entirely theoretical.

Example. *The True Discount on a bill, discounted 173 days before it is legally due, is £5 4s. 4d.; the Banker's Discount is £5 6s. 9d. Find the amount of the bill.*

The difference between the discounts is 2*s.* 5*d.*; and this is the interest on the True Discount, £5 4*s.* 4*d.*

The Banker's Discount, £5 6*s.* 9*d.*, is the interest on the amount of the bill for the same time and at the same rate.

2*s.* 5*d.* is interest on £5 4*s.* 4*d.*
∴ £5 6*s.* 9*d.* is interest on £5·21666 × $\frac{53375}{1208}$
= £230·495
Amount of bill = £230 9*s.* 11*d.*

```
53375
    5 21666
------------
266875
 10675
   533
   320
    32
     8
------------
1208 ) 278438·
       3683
        598
        115
          6
```

It should be noted that the multiplying fraction is nearly 50, hence £5 4*s.* 4*d.* must be expressed as a decimal to 5 places in order that the final result may be correct in the third decimal place. Also in the side multiplication no decimal places are needed, as the subsequent division by 1208, which is more than 1000, causes the first decimal place in the product to have no effect till the fourth place in the quotient is reached.

(239.) The following example illustrates a different point of view of the difference between Banker's Discount and True Discount.

Example. *Find the True Discount on a bill of* £3470 *due in* 3 *month's time, reckoning interest at* $2\frac{1}{4}$ *per cent. Find also at what rate per cent. Banker's Discount would be deducted to give the same discount.*

Interest on £100 in 3 months at $2\frac{1}{4}$ per cent. $= £\frac{1}{4} \times 2\frac{1}{4} = £\frac{9}{16}$.

$\therefore$ True discount on £100 $\frac{9}{16}$ is $£\frac{9}{16}$

True discount on £3470 is $£\frac{9}{16} \times 3470 \times \frac{16}{1609}$

$= £\frac{31230}{1609}$

$= £19\ 8s.\ 2d.$

$$1609 \overline{)\ 31230\ (19{\cdot}409}$$
$$15140$$
$$659$$
$$15$$

We now have to find at what rate per cent. £19·409 will be the Banker's Discount (i.e simple interest) on £3470 for 3 months.

Interest on £3470 for 3 months is £19·409

$\therefore$ Interest on £100 for 12 months is $£19{\cdot}409 \times \frac{100}{3470} \times 4$

$= £\frac{77.63}{34.7}$

$$3{\cdot}47 \overline{)\ 7{\cdot}7636\ (2{\cdot}237}$$
$$823$$
$$1296$$
$$255$$

Required rate = £2 4*s.* 9*d.* per cent.

Hence the True Discount at £2 5*s.* per cent. is the same as Banker's Discount at £2 4*s.* 9*d.* per cent.

EQUATION OF PAYMENTS

(240.) **Example** 1. A *purchases goods from* B *and in payment gives three bills, one for* £500 *due on Oct.* 3*rd, another for* £300 *due on Nov.* 10*th, and a third for* £400 *due on Oct.* 15*th; on what day will a single payment of* £1200 *cancel all three bills, so that neither* A *nor* B *loses by the transaction, the rate of interest being the same in all cases?*

This is calculated on the assumption that the sum of the present values of the three bills on any date not later than Oct. 3rd (the earliest of the dates of payment) is equal to the present value of the single payment.

Take Oct. 3rd as the date of the present values.

Present value of the £500 bill = £500

Present value of the £400 bill = £400 − int. on £400 for 12 days
= £400 − int. on £400 × 12 for 1 day

Present value of the £300 bill = £300 − int. on £300 × 38 for 1 day

∴ Sum of the present values = £1200 − int. on £(4800 + 11400) for 1 day
= £1200 − int. on £16200 for 1 day
= £1200 − int. on £1200 for $13\frac{1}{2}$ days.

Hence £1200 is due 14 days after Oct. 3rd.
i.e. a single payment is made on Oct. 17th.

The time of payment is called the *equated time.* The process of finding it is usually referred to as the *equation of payments.* The work may be arranged shortly thus:

$$\text{No. of days after Oct. 3rd} = \frac{500 \times 0 + 300 \times 38 + 400 \times 12}{500 + 300 + 400}$$
$$= \frac{11400 + 4800}{1200}$$
$$= \frac{162}{12} = 13\tfrac{1}{2} \text{ days.}$$

Required date is Oct. 17th.

Any interim payment cancels an equal amount of debt; the corresponding product is therefore subtracted.

Example ii. *If in the previous example* A *paid* £200 *on Oct. 9th, on what day should he pay the balance to close the account?*

Reckon from Oct. 3rd.

$$\text{Number of days} = \frac{500 \times 0 + 300 \times 38 + 400 \times 12 - 200 \times 6}{500 + 300 + 400 - 200}$$
$$= \frac{11400 + 4800 - 1200}{1000}$$
$$= \frac{150}{10}$$
$$= 15.$$

The balance should be paid on Oct. 18th.

The equation of payments may also be worked backwards from any assigned date.

Suppose no additional payment had been made before Nov. 10th

A would then owe £500 + int. on £500 × 38 for 1 day
+ £300 + no interest
+ £400 + int. on £400 × 26 for 1 day
i.e. £1200 + int on £29,400 for 1 day
and would have paid £200 + int. on £200 × 32 for 1 day
i.e. he would still owe £1000 + int. on £23,000 for 1 day
i.e. £1000 + int. on £1000 for 23 days.

Hence he ought to have paid £1000 23 days before Nov. 10th.
∴ balance ought to have been paid on Oct. 18th.

FOREIGN BILLS OF EXCHANGE

(241.) If A in London owes money to X in Paris, he can discharge the debt by sending actual coin or gold, as has been explained in Chap. XVI. In ordinary commercial transactions it is usual to pay by bills of exchange. Suppose

A (in London) is owed 10,000 francs by X (in Paris), and B (in London) owes 10,000 francs to Y (in Paris).

These accounts may both be settled in the following manner:

A draws a bill on X for 10,000 francs.

A sells the bill to B for its equivalent value in £ *s.* *d.*, so that A is now paid the equivalent of 10,000 francs.

B has bought the bill, so that he has now paid the equivalent of 10,000 francs

B sends the bill to Y.

Y takes the bill to X, who pays the money to Y;
so that Y receives 10,000 francs,
and X pays 10,000 francs.

In actual practice A and B would not deal with one another directly; A would sell to a Bill Broker and B would buy from the Bill Broker.

(242.) If a large number of merchants in London owe money to merchants in Paris, there will be a great demand in London for bills on Paris; consequently the price of such bills will go up.

This rise shows itself in the rate of exchange; the broker will reckon that £1 is worth less than the par rate, i.e. £1 will be less than 25·22 francs. Similarly, if there is little demand for bills on Paris, the rate of exchange will go up. The rate of exchange changes every day and may be different for different towns; for instance, the rate of exchange between Manchester and Lyons may be different from that between London and Paris on the same day.

Foreign Bills of Exchange may be discounted in the same way as ordinary bills.

Example. *What should be paid in London for a bill of* 10,000 *marks due* 80 *days hence, interest being reckoned at* 4 *per cent.? When the bill is discounted the rate of exchange is* £1 = 20·47 *marks.*

$$\text{Discount} = \tfrac{80}{365} \times 4 \times 100 \text{ marks}$$
$$= \tfrac{6400}{73} \text{ marks}$$
$$= 87{\cdot}67 \text{ marks}$$

$$\begin{array}{r|l} & 87{\cdot}67 \\ \hline 73 & 6400 \\ & 560 \\ & 490 \\ & 520 \end{array}$$

$$\text{Present value} = 9912{\cdot}33 \text{ marks}$$
$$= £\tfrac{9912\,33}{20\,47}$$
$$= £484\ 4s.\ 9d.$$

$$\begin{array}{r|l} & £484\ 237 \\ \hline 20{\cdot}47 & 9912{\cdot}38 \\ & 1724{\cdot}3 \\ & 86{\cdot}73 \\ & 4{\cdot}85 \\ & 76 \\ & 15 \end{array}$$

Examples XVIII c.

Find the Banker's Discount and the Present Value in the following cases:

1. A bill for £763 5*s.* drawn Oct. 1st at 3 months, discounted Nov. 3rd at 3 per cent.

2. A bill for £517 10*s.* 3*d* drawn May 7th at 4 months, discounted May 18th at $2\frac{1}{2}$ per cent.

3. A bill for £257 13*s.* 9*d.* drawn Dec. 15th at 6 months, discounted Jan. 29th at $3\frac{1}{2}$ per cent.

4. A bill for £812 15*s.* 2*d.* drawn Mar. 29th at 2 months, discounted May 8th at 4 per cent.

5. A bill for £543 7*s.* 5*d.* drawn July 3rd at 5 months, discounted Sept. 12th at 3 per cent.

6. A bill for £975 17*s.* 4*d.* drawn Jan. 15th, 1912, at 4 months, discounted Feb. 13th at $3\frac{1}{4}$ per cent.

7. Explain the difference between true discount and banker's discount. Find the difference between the two for a bill for £610 discounted 4 months before it is due at 5 per cent.

8. A bill due 3 months hence is discounted at 4 per cent., and its true present value is £1225. What is the amount of the bill?

9. Find the true present worth, correct to the nearest sixpence, of £620 due in $1\frac{3}{4}$ years at 3 per cent. simple interest.

10. Find the discounted value of a bill for £1825 drawn Oct. 30th at 2 months and discounted by a banker on Dec. 21st at $4\frac{1}{2}$ per cent.

11. A bill for 6900 francs legally due on Oct. 4th was discounted on May 4th at 6 per cent. How much was paid for the bill? (See note on p. 482.)

12. Find the discount on a bill for 9372 francs discounted on Mar. 3rd at 4 per cent. if it was legally due on April 7th.

13. At what rate per cent. was a bill for £578 discounted if the holder received £572 11*s.* 8*d.* for it 70 days before it was due?

14. A bill for £789 10*s.* was discounted on April 19th at 4 per cent. for £770 15*s.*; when was it nominally due?

15. At what rate per cent. will the true discount on £956 for 3 months be equal to the banker's discount at $4\frac{1}{2}$ per cent.?

16. A bill for £463 12*s.* legally due on Aug. 7th was discounted at $3\frac{1}{2}$ per cent. for £456 7*s.*; on what day was it discounted?

17. If the true discount on a bill is £5 3*s.* 2*d.* and the commercial discount £5 14*s.* 6*d.*, what is the amount of the bill?

18. A bill is discounted 43 days before it is due at 4 per cent.; if the difference between the true discount and banker's discount is 17*s.* 3*d.*, find the amount of the bill.

19. A man holds a bill for £527 15*s.* 3*d.* which is legally due on Aug. 23rd. He discounts it on July 15th. Will it pay him better to receive the true present value at 5 per cent. or the commercial present value at $4\frac{3}{4}$ per cent.?

20. In return for goods A has given B the following bills; £417 at 3 months, £1426 at 2 months, £542 at 4 months. If they are all dated Feb. 14th, when will a single payment of £2385 close the account?

21. A sum of £650 is due from A to B on a certain day and £495 12*s.* 6*d.* is also due 7 months later. If A pay both debts at 3 months after the first bill was due, will it be to B's advantage or not, interest being allowed at 5 per cent. per annum?

22. B holds the following bills from A:—£417 10*s.* dated Mar. 3rd at 6 months, £525 13*s.* dated Mar. 29th at 4 months, and £715 18*s.* dated April 14th at 2 months. If A pays £1659 1*s.* on July 31st, does he gain or lose by the transaction?

What balance must A pay, and when, to close the following accounts?

			£	*s.*	*d.*		£	*s.*	*d.*
23.	Mar. 2nd.	To goods at 3 months .	627	13	6				
	April 4th.	,, 4 ,,	715	8	0				
	April 20th.					By Cash	950	8	0
	May 10th.	,, 3 ,,	890	0	0				
24.	June 5th.	,, 6 ,,	873	12	0				
	July 3rd	,, 2 ,,	1015	0	0				
	July 15th.					By Cash	546	0	0
	Aug. 3rd.	,, 4 ,,	756	17	6	,,	245	12	0
	Aug 8th.	,, 3 ,,	450	10	0				

25. A has to pay £100 per annum half yearly for two years; when should he make a single payment of £400 to cancel the debt?

Find the present value in English money of the following bills:

26. 95,613 francs due in 15 days at 4 per cent., £1 = 25·26 francs.

27. 112,318 marks due in 29 days at $3\frac{1}{2}$ per cent., £1 = 20·41 marks.

28. 5767 dollars due in 34 days at $3\frac{1}{2}$ per cent., £1 = 4·77 dollars.

29. 48,759 francs due in 3 months at $4\frac{1}{2}$ per cent., £1 = 25·42 francs.

30. 6981 dollars due in 30 days at 4 per cent., £1 = 4·86 dollars.

Examples XVIII.

1. I borrow £560 for the term of 5 months at the rate of $2\frac{1}{2}$ per cent. simple interest. I repay principal with interest at the end of the term. What shall I pay in all?

2. Find the simple interest for 5 years at $2\frac{1}{2}$ per cent. on £4572 3*s.* 6*d.* and also on 60,000 francs.

3. What sum of money will amount to £425 19*s.* 6*d.* in 10 years at $3\frac{1}{2}$ per cent. simple interest; and in how many years more will it amount to £453 11*s.* 9*d.*?

4. At what rate per cent. will the simple interest on 18,320 francs amount to 2473 francs 20 centimes in 3 years?

5. A sum is placed out at interest so that after 11 months the capital and interest amount to 6302 francs 50 centimes and after $2\frac{1}{2}$ years to 6825 francs. What was the original sum?

6. In what time will the interest on 25,000 francs amount to 1775 francs at $3\frac{1}{2}$ per cent.?

7. *A* invests £15,600 and *B* £18,000 at the same rate of interest. *B* receives every year £144 more interest than *A*; what is the rate per cent.?

8. Find, to the nearest penny, the true present value of a debt of £150 due a year and 4 months hence at 5 per cent. per annum.

9. Find the true discount on £130 due a year and 9 months hence, reckoning simple interest at $2\frac{1}{2}$ per cent. per annum.

10. What difference in amount of simple interest shall I obtain by lending £5250 for 2 years at $3\frac{1}{2}$ or $4\frac{1}{2}$ per cent. per annum?

11. If £718 15*s.* is the present worth of £869 13*s.* 9*d.* due in 7 years, what rate per cent. is reckoned?

12. Find the simple interest for an ordinary year, correct to the nearest penny, on a deposit of £5257, the rate per cent. being 3 for the first 6 calendar months, and $3\frac{1}{2}$ for the remainder of the year.

13. The simple interest for 12 months on a deposit of £1680 was £62 6*s.* For 7 months at the beginning of the year the rate per cent. was $3\frac{1}{2}$. What was it for the rest of the time?

14. At what rate per cent. per annum will the simple interest on £8647 10*s.* for $3\frac{5}{9}$ years be £1153?

15. A man borrowed £1200 on July 12th at 4 per cent., and a further £800 on the following Sept. 15th. How much interest was due on Dec. 31st of the same year?

16. In what time will £795 10*s.* amount to £921 at $3\frac{1}{4}$ per cent. simple interest? (Answer to the nearest day.)

17. On what sum will the simple interest for 11 months at $3\frac{1}{4}$ per cent. per annum be £47 1*s.* 5*d.*?

18. For what time must £1329 be lent, at 5 per cent. per annum simple interest, in order that its amount may be equal to that of £1415 lent for the same time at 3 per cent.?

19. Find the simple interest on £17,520 for 151 days at $2\frac{3}{8}$ per cent. per annum.

20. If £100 7*s.* 6*d.* was allowed by a banker as simple interest on £2555, deposited with him for $1\frac{1}{4}$ years, what average rate per cent. per annum of interest was obtained by the investor?

21. Find the cost of discounting on Sept. 1st, 1904, a bill of £580 12*s.*, maturing on Oct. 18th, 1904. (Mercantile discount at the rate of 4 per cent. per annum.)

22. A person puts £15 in the Post Office Savings Bank on January 20th, £10 on April 25th, £15 on September 20th. What interest will be due on December 31st, if interest, at the rate of $2\frac{1}{2}$ per cent. per annum, is allowed on every complete pound deposited, and commences on the first day of the month next following the deposit?

23. Find the true present value and the commercial present value of £2432 due four months hence, the rate of interest being 4 per cent. per annum. Express the difference between the two present values as a percentage of the true present value.

24. The simple interest on £242 13*s.* 4*d.* for 5 years amounted to £30 6*s.* 8*d.* What was the rate per cent. per annum?

25. The purchaser of a farm lets it for £175 a year and spends 12 per cent. of the rent on repairs, &c.; the remainder yields him 4 per cent. per annum on the purchase money. What sum did he give for the farm?

If he now reduces the rent by 30 per cent., what percentage of the new rent may be spent on repairs, &c., so that the residue yields 3 per cent per annum on the purchase money?

26. A row of 36 leasehold houses is to be sold by auction. They let at £35 a year each, but the tenants often change, so that the average number vacant at any time is 3 Repairs and decoration amount to 12 per cent. of the rental, whether the house is empty or occupied. At what price should the whole row be bought so as to give the buyer a clear 5 per cent. on his outlay?

27. Two sums of money amounting to £1946 5*s.* were invested, the smaller at 4 per cent. and the larger at $4\frac{1}{2}$ per cent. per annum. At the end of 18 months the simple interest on the two sums amounted together to £125 4*s.* $10\frac{1}{2}d.$ What were the two sums?

28. A loan of £7000 is repaid in half-yearly instalments of £1000 together with simple interest at 5 per cent. paid at the time of each repayment. If the first instalment is paid at the end of a year from the date of the loan, how much will be paid altogether?

29. For what time must £1329 be lent at 5 per cent. per annum simple interest, in order that its amount may be equal to that of £1415 lent for the same time at 3 per cent. per annum simple interest?

30. Find the true discount and the banker's discount on £1062 10*s.* due 16 months hence at $3\frac{3}{4}$ per cent.

What is the rate per cent. for the aforesaid time when the banker's discount exceeds the true by $\frac{1}{15}$ of the latter?

31. A man borrowed some money from a bank for 162 days; for 82 days the rate of interest was $7\frac{1}{2}$ per cent per annum, for 80 days the rate was 60 per cent. per annum. Show that he paid altogether exactly 3 per cent. on his loan.

32. A man lends a sum of money, part at 5 per cent. and part at 4 per cent. The whole amount lent is £3652, and his total income from it is £158 12*s.* Find how much is lent at 5 per cent. and how much at 4 per cent.

33. Draw a graph to show the sum to which £50 will amount in various times at 5 per cent. simple interest. Over the negative

portion of the time axis, draw a graph to indicate the sums which will in various times amount to £50.

34. Explain the difference between true and commercial discounts. Find the ratio of these discounts on £100 payable 4 years hence, interest being reckoned at $6\frac{1}{4}$ per cent. What would be the ratio in the case of a bill for £519 7*s.* 2*d.* due 4 years hence at $6\frac{1}{4}$ per cent.?

35. In how many months will the interest on £575 14*s.* at $\frac{1}{2}$ per cent. per month be equal to the interest on £436 10*s.* for 3 years at 5 per cent. per annum?

36. The difference between the banker's discount and the true discount on a bill due 3 months hence is 2*s.* 1*d.*, interest being reckoned at 3 per cent. per annum; find the amount of the bill.

37. Find the commercial discount at $3\frac{3}{4}$ per cent. on £787 13*s.* 5*d.* due 6 months hence. By how much does it differ from the true discount?

38. A bill for £671 13*s.* 4*d.* is due on Sept. 29th. The market rate of interest being $3\frac{3}{4}$ per cent. per annum, what would a banker give for the bill on July 18th? What is its real value on that day?

CHAPTER XIX

STOCKS AND SHARES

(**243.**) If two or more people carry on a business as partners, they usually divide the profits in the ratio of the money they put into the business. This money is called *Capital.* If a large capital is required, the public in general may be asked to subscribe and a *Company* is formed. Each contributing person then becomes a kind of partner, and is entitled to his proportional share of the profits. He must therefore know what the total capital is, in order that he may know exactly what fraction of the profits he is entitled to. The *Prospectus* of the company states how many shares there will be and how much money must be paid for each share; these may be £5 shares, £1 shares, or even 2*s.* 6*d.* or 1*s.* shares; but purchasers must buy an exact number of shares and then become *Shareholders.* The Shareholders elect representatives to manage or direct the business; these are called *Directors*; apart from this the Shareholders take no part in the business whatever.

Profits vary from year to year, so that the amount to be divided (the Dividend[1]) is unknown, and the percentage to be paid on the shares is not always stated. Some people would rather have an assured dividend, so *Preference* shares are issued which receive a definite percentage before the other, or *Ordinary,* shares receive any dividend. There may even be a third series of shares which receive no dividend until all other shares have received an assigned percentage; these are called *Deferred* shares.

After the company has started, more capital may be required. This may be obtained either by issuing more shares or by borrowing.

[1] The word dividend is also applied to any portion of the profits and to the rate per cent. of profit, thus shares are said to pay a dividend of 6 per cent.

Money borrowed in this way is called *Debenture Stock*, and a definite rate of interest is paid just as in ordinary cases of borrowing. Interest on Debentures must be paid before a dividend is paid on any stock; or, in other words, debts must be paid before the profits are divided.

Note—In the case of a partnership, each partner is liable for all the debts incurred by the business. A partner may have provided £1000 of the total £10,000 capital; and may have other property to the value of £50,000. If now the partnership is a failure and debts are incurred to the amount of £20,000, the creditors can compel that partner to pay the whole of the £20,000 debt, even though he is entitled to only one-tenth of the profits, had there been any; his liability is unlimited. In the case of a *limited liability* company the liability of each shareholder is limited to the amount of his shares. If the company does badly, he may lose that amount but he cannot be called upon to pay any more, however much other property he may have. Sometimes shareholders are not required to pay up the whole share value, they may, for instance, be required to pay up only £5 when taking a £20 share. In such a case the shareholder is liable to pay the remaining £15 when called upon, but the dividend is calculated on the paid up capital.

(**244.**) Governments, county councils, and other public bodies borrow money in the same way as companies. They promise a fixed rate of interest, and sometimes fix a date when the money will be repaid. The money thus borrowed is called Stock, and is named by reference to the borrowing government or body and to the rate of interest payable. Thus a man who buys £100 of Canada $3\frac{1}{2}$ per cent. stock has bought the right to receive from the Canadian Government an income of £$3\frac{1}{2}$ every year. If the stock is redeemable, he will receive £100 cash on the date mentioned; if it is not redeemable, the only way of getting his money back is by selling the stock (i.e. the right to receive $3\frac{1}{2}$ per cent. income) to somebody else. Stock may be bought and sold in any amount; it is not divided into definite portions as shares are.

Both stock and shares are often referred to as 'stock'.

It must be carefully noticed that £ *s. d.* stock is quite different from £ *s. d.* money. For example, a man who has £3500 stock has either a definite share in a business or a right

to receive a definite income; but it does not follow that this share, or right, is worth £3500 cash or money

Example. *Extract from a Company's report: 'After making allowances for depreciation and provision for doubtful debts, the net profit for the year is £33,154, which with £5658 brought forward gives £38,812. The directors recommend that £10,000 be set aside for writing down goodwill, investments, &c., and that a dividend of 6 per cent. be paid on the Ordinary shares, leaving £6012 to be carried forward'. From this find the amount of the Ordinary shares.*

£	
33154	net profit in year
5658	balance of profit from previous year
38812	gives amount available for dividend
10000	set aside to cover loss in value of the Company's property
6012	balance carried forward to next year
22800	amount distributed as dividend on Ordinary shares.

Hence 6 per cent. of Ordinary shares = £22,800.

$\therefore$ 100 " " " = £22,800 $\times \frac{100}{6}$.

Amount of Ordinary shares = £380,000.

Oral Examples.

What yearly dividend will a man receive who holds the following amounts of stocks or shares?

1. £2000 of $2\frac{1}{2}$ per cent. stock.
2. £350 of 6 per cent. Preference shares.
3. £1500 of $2\frac{1}{2}$ per cent. Consols.[1]
4. £750 of New Zealand 4 per cent. stock.
5. £350 of Copenhagen 4 per cent. stock.
6. £2300 of 3 per cent. stock.
7. £4950 of 5 per cent. debentures.

[1] If several different issues of stock are 'consolidated' into one stock, the stock so formed is called 'Consols'. Unless otherwise stated, the word Consols used alone is always to be taken to refer to the English Government stock of that name.

8. £900 of Uraguay 5 per cent. stock.
9. £450 of London County Council 3 per cent. stock.
10. £2300 of $2\frac{3}{4}$ per cent. stock.
11. £70 of 7 per cent. Preference shares.
12. £3500 of $3\frac{1}{4}$ per cent. stock.
13. £2700 of Port of London $3\frac{1}{2}$ per cent. stock.
14. £1250 of 4 per cent. stock.
15. £650 of 6 per cent. debentures.
16. £875 of 2 per cent. stock.
17. £4500 of Natal 3 per cent. stock.
18. £2300 of Canadian $3\frac{1}{2}$ per cent. stock.
19. £4900 of 7 per cent. debentures.
20. £3825 of 4 per cent. stock.

Examples XIX a.

Find the yearly income obtained from

1. £898 L. C. C. 3 per cent. stock.
2. £2498 7*s.* 6*d.* $2\frac{1}{2}$ per cent. Consols.
3. £763 14*s.* Canadian $3\frac{1}{2}$ per cent. stock.
4. £7642 Natal 3 per cent. stock.
5. £795 10*s.* $2\frac{3}{4}$ per cent. stock.
6. £8975 $4\frac{1}{2}$ per cent. stock.
7. 795 6 per cent. Preference shares of £5 each.
8. £343 Copenhagen 4 per cent. stock.
9. 343 Ordinary £1 shares paying $8\frac{3}{4}$ per cent.
10. £7532 Railway $3\frac{1}{2}$ per cent. stock.
11. £8947 10*s.* New Zealand 4 per cent. stock.
12. £649 Port of London $3\frac{1}{2}$ per cent. stock.

13. If the total capital of a company is £8000 and the net yearly profits, after paying all claims, are £756 13*s.*, find the dividend, correct to the nearest penny, on £452 Ordinary stock.

14. A company issues £10,000 6 per cent. Preference shares and £50,000 Ordinary shares; after paying all other claims there remains a profit of £3718; what is the greatest rate per cent. that can be paid on the Ordinary shares?

15. The paid-up capital of a certain bank was £3,500,000 in 1912 and a total dividend of $20\frac{1}{4}$ per cent. was paid. What was the total amount paid to the shareholders as dividend?

The capital was divided into £20 shares, on each of which only £5 was paid. How much additional capital could the shareholders be called upon to pay?

16. The capital of the U—— U—— Company consists of $1,000,000 7 per cent. Preference stock and $5,000,000 Ordinary stock. The company have issued $1,250,000 6 per cent. Debenture bonds. What total sums will be paid as dividends on the respective stocks when the total profits are (i) $75,000; (ii) $145,000, (iii) $500,000.

17. The capital of the B—— and P—— Company, Limited, is made up of the following items:

SHARE CAPITAL	
5 per cent. First Preference Stock	£1,200,000
5 per cent. Second Preference Stock	1,000,000
Ordinary Stock	10,000,000
DEBENTURE CAPITAL	
4 per cent. First Debenture Stock	£2,925,000
$4\frac{1}{2}$ per cent. Second Debenture Stock	2,075,000
5 per cent Debenture Stock	307,327
$4\frac{1}{2}$ per cent. Consolidated Debenture Stock	7,942,673

The net profits for 1911 amounted to £1,823,653; if all this were paid away as dividend, how much of it was available for the Ordinary stock, and at what rate could the dividend on that stock be paid?

18. The capital of a company is £117,000. Its working expenses are 35 per cent. of the gross receipts, and 10 per cent. of the net profits are transferred to a reserve fund. What must be the yearly gross receipts of the company, in order that the shareholders may receive a dividend of 8 per cent.?

19. What is the quarterly income from £1241 6*s.* 8*d.* $2\frac{1}{2}$ per cent. Consols, after deducting income-tax at 1*s.* in the pound?

20. A man holds £11,356 14*s.* 8*d.* in G. N. R. 4 per cent. debentures. Find his half-yearly income; find also the amount which will be deducted for income-tax at 1*s.* 3*d.* in the pound.

(**245.**) If a shareholder wishes to have money in place of his shares, he cannot demand the money from the Company; he must find some one who will buy his shares. Also if any one wishes to buy shares, he cannot buy them from the Company except when the shares are first issued. The same remarks apply to any form of stock except those stocks which are redeemable on a fixed date. The reasons which cause the cash value of stock to vary do not concern us, but, speaking roughly, we may say that the value of shares and stock is subject to the usual law of supply and demand. If the stock is paying a good dividend, and if the credit of the Company or Government is good, then people will be anxious to buy, while shareholders will not be anxious to sell. In such a case those who want the shares must pay a high price for the stock. But if the stock is paying a low rate of dividend, or if there is any doubt as to whether the dividend will be paid, then shareholders will be anxious to sell, while others will not wish to buy. In such a case the shareholders will be willing to accept a low price for the stock.

The price of stock is usually quoted as so much per £100 stock; thus, on a certain date, Mexican Ordinary were quoted at $54\frac{1}{4}$, Mexican 8 per cent. 1st Preference at 139. This means that £100 of the Ordinary stock could be bought for £$54\frac{1}{4}$ money while £100 of 8 per cent. 1st Preference stock was worth £139 money. The price of shares is given as so much per share; thus Aerated Bread $3\frac{7}{8}$, Drury Lane $\frac{31}{32}$, means that the shares were worth £$3\frac{7}{8}$ money and £$\frac{31}{32}$ money respectively.

If the value of stock or shares is the same as the nominal value, the stock is said to be *at par*. New stock need not be issued at par, but may be issued *at a premium* (i. e. at more than the nominal value) or *at a discount* (i. e. at less than the nominal value). Thus stock issued at 95 per cent. is issued at a discount of 5 per cent.

Oral Examples.

What amount of cash must be paid for

1. £500 $2\frac{1}{2}$ per cent. Consols at 75?
2. £400 Transvaal 3 per cent. stock at 92?
3. £1000 Manchester 3 per cent. stock at $82\frac{1}{2}$?
4. £2000 Great Western stock at 118?
5. £1200 United States stock at 67?
6. £1500 Water 'B' stock at 80?
7. £700 Bombay-Baroda stock at 105?
8. £800 Burma $2\frac{1}{2}$ per cent. guaranteed stock at 109?
9. 1000 Premier Pipe shares at 16*s.* 6*d.*?
10. 500 Harrods £1 shares at $4\frac{3}{4}$?
11. £750 Debenture stock at a premium of 5 per cent.?
12. £1200 5 per cent. Preference stock at a discount of 3 per cent.?

How much is gained or lost

13. by buying £100 Consols at 82 and selling at $79\frac{1}{2}$?
14. by buying £300 Great Northern stock at $86\frac{1}{2}$ and selling at $88\frac{1}{4}$?
15. by buying £1200 Grand Trunk stock at 90 and selling at $94\frac{1}{4}$?
16. by buying £600 Bank Stock at 230 and selling at 215?
17. £95 money is invested in 3 per cent. stock at 95, and the stock is afterwards sold at $97\frac{1}{2}$. How much money is gained?
18. £214 money is invested in 5 per cent. stock at 107, and the stock is afterwards sold at 79. How much money is lost?
19. How much money will a Government receive by issuing £3,000,000 4 per cent. Debentures at 98?
20. What must a man pay for £1000 stock issued at a premium of 3 per cent.?

What income is obtained by investing

21. £75 in $2\frac{1}{2}$ per cent. Consols at 75?
22. £150 in $2\frac{1}{2}$ per cent. Consols at 75?
23. £82 10*s.* in Manchester 3 per cent. stock at $82\frac{1}{2}$?
24. £165 in Manchester 3 per cent. stock at $82\frac{1}{2}$?

25. £327 in Burma $2\frac{1}{2}$ per cent. stock at 109?

26. £515 in 4 per cent. stock at 103?

27. £1000 in 5 per cent. stock at par?

28. £45 5*s.* in 2 per cent. stock at $90\frac{1}{2}$?

29. £105 in $2\frac{1}{2}$ per cent. stock at 70?

30. £412 in $3\frac{3}{4}$ per cent. stock at 103?

How much money must be invested in

31. Manchester 3 per cent. stock at $82\frac{1}{2}$ to obtain an income of £3?

32. $2\frac{1}{2}$ per cent. Consols at 75 to obtain an income of £5?

33. 4 per cent. stock at 101 to obtain an income of £12?

34. Burma $2\frac{1}{2}$ per cent. stock at 109 to obtain an income of £10?

35. Natal 3 per cent. stock at 81 to obtain an income of £21?

36. Canada $3\frac{1}{2}$ per cent. stock at $94\frac{1}{4}$ to obtain an income of £7?

37. Chinese 5 per cent. stock at 99 to obtain an income of £25?

38. Mexican 5 per cent. stock at 98 to obtain an income of £30?

39. Chilian $4\frac{1}{2}$ per cent. stock at 91 to obtain an income of £18?

40. Bulgarian 6 per cent. stock at $102\frac{1}{2}$ to obtain an income of £18?

Examples XIX b.

1. What must be paid for £1250 Argentine 5 per cent. stock at $102\frac{1}{2}$?

2. How much Argentine 5 per cent. stock at $102\frac{1}{2}$ can be bought for £1250?

3. A man sells £2315 Servian 4 per cent. stock at 78. How much money does he receive?

4. A man sells £5672 India 3 per cent. stock for £4367 8*s.* 10*d.*; what was the price of the stock?

5. What is the value of £456 10*s* Consols when they stand at 75?

6. How much must be paid for £737 Irish land $2\frac{3}{4}$ per cent. stock at 87?

7. What is received for £3452 Great Northern Preferred stock when they are sold at $87\frac{1}{2}$?

8. How much 5 per cent. stock at $103\frac{1}{2}$ can be bought for £2000?

9. If £735 stock can be bought for £690 18*s.*, how much of the same stock can be bought for £390 11*s.* 5*d.*?

10. How much is gained by buying £770 London County Council $3\frac{1}{2}$ per cent. stock at $96\frac{1}{2}$ and selling the same stock at $98\frac{3}{4}$?

11. How much is gained by investing £770 in London County Council $3\frac{1}{2}$ per cent. stock at $96\frac{1}{2}$ and afterwards selling at $98\frac{3}{4}$?

12. If £5375 of 4 per cent. stock are bought at 98 and sold at £$101\frac{1}{2}$, how much money is gained?

13. If £5375 money is invested in 4 per cent. stock at 98 and the stock is afterwards sold at $101\frac{1}{2}$. how much money is gained?

14. What is gained by buying £437 Consols at $78\frac{1}{2}$ and selling them at $81\frac{3}{4}$?

15. What is gained by investing £437 cash in Consols at $78\frac{1}{2}$, and selling the stock at $81\frac{3}{4}$?

16. How much does a man lose by investing £1535 in $4\frac{1}{2}$ per cent. stock at 101 and afterwards selling at $98\frac{3}{4}$?

17. By buying stock at $94\frac{1}{4}$ and selling at $93\frac{1}{2}$ a man lost £67 17*s.* 6*d.* How much stock did he buy?

18. A man invested £1947 10*s.* in 3 per cent. stock at 95 and lost £56 7*s.* 6*d.* when he sold out. At what price did he sell?

19. How much stock is obtained by investing £5012 in 5 per cent. stock at $89\frac{1}{2}$?

20. A man buys Canadian 3 per cent. stock at 97 and by so doing obtains an income of £49 5*s.* How much money did he invest?

21. What income would be obtained from an investment of £1271 12*s.* in $2\frac{3}{4}$ per cent. stock at $93\frac{1}{2}$?

22. A man receives an income of £76 10*s.* from 3 per cent. stock. He sells out at 97; how much money does he receive?

23. What sum must be invested in $2\frac{1}{2}$ per cent. Consols at $89\frac{1}{2}$ in order to secure an income of £150 a year?

24. What income is obtained by investing £222 1*s.* 3*d.* in a 3 per cent. stock at $93\frac{1}{2}$?

25. A man has an income of £415 derived from capital invested in 4 per cent. stock. He sells out this stock at 119. How much money does he receive?

26. A man invests £12,346 5*s.* in 5 per cent. stock and so secures an income of £425. What was the price of the stock?

27. A man invests £1995 in a $4\frac{1}{2}$ per cent. stock at 84. What income does he receive?

28. A man receives an income of £12 11*s.* 5*d.* from money invested in $3\frac{1}{2}$ per cent. stock at 90. How much money does he invest?

(**246.**) Stock may be bought either with a view to securing a yearly income or with a view to selling the stock at a profit. The investor who seeks an income has to consider the rate of interest he receives on his money; this is usually quite different from the nominal rate of interest of the stock. To take an extreme case, if a man buys £100 of 8 per cent. stock at 200, he receives a yearly income of £8 on an investment of £200 money. Hence he gets 4 per cent. on his money, although he invests in 8 per cent. stock.

The following examples illustrate the point of view of a person who has money to invest and is considering what stock to invest it in.

Example i. *Which is the better investment, a* 5 *per cent. stock at* 109 *or a* 3 *per cent. stock at* 92?

First Method. Consider the income on £1 money invested in each.

Income obtained by investing £109 money in 5 % stock = £5.

∴ Income obtained by investing £1 money in 5 % stock = £$\frac{5}{109}$.

Income obtained by investing £92 money in 3 % stock = £3.

Income obtained by investing £1 money in 3 % stock = £$\frac{3}{92}$.

$\frac{5}{109} \sim \frac{3}{92} = \frac{460 \sim 327}{109 \times 92}$, ∴ $\frac{5}{109}$ is greater than $\frac{3}{92}$.

The 5 % stock at 109 is the better investment.

Second Method. Consider the income obtained by investing a common multiple of the two prices in each.

£109 money invested in 5 % stock gives an income of £5.

∴ £109 × 92 money invested in 5 % stock gives an income of £5 × 92 = £460.

£92 money invested in 3 % stock gives an income of £3.

∴ £109 × 92 money invested in 3 % stock gives an income of £3 × 109 = £327.

Hence the 5 % stock at 109 is the better investment.

Example ii. *What rate of interest is earned by investing in* $2\frac{1}{2}$ *per cent. Consols at* 75?

Here we have to find the income obtained by investing £100 money.

£75 money secures an income of $£2\frac{1}{2}$.

$\therefore$ £100 money secures an income of $£2\frac{1}{2} \times \frac{100}{75} = £3\frac{1}{3}$.

Hence the rate of interest is $3\frac{1}{3}$ per cent.

Example iii. *How much money must be invested in Manchester* 3 *per cent. stock at* $82\frac{1}{2}$ *to secure an income of* £50 *per annum?*

£3 is the income obtained by investing $£82\frac{1}{2}$ money.

$\therefore$ £50 is the income obtained by investing $£82\frac{1}{2} \times \frac{50}{3}$

$= £27\frac{1}{2} \times 50$

$= £1375$.

£1375 must be invested

Alternative method.

$$\text{Income} = \frac{3}{82\frac{1}{2}} = \frac{3 \times 2}{165} = \frac{2}{55} \text{ of money invested.}$$

$\therefore$ Money invested $= \frac{55}{2}$ of income.

$= £\frac{55}{2} \times 50$

$= £1375$.

Example iv. *A man holds* £2500 *Transvaal* 3 *per cent. stock. He sells out at* 92 *and invests the proceeds in* $2\frac{1}{2}$ *per cent. Consols at* 80. *What is the change in his income?*

Original income = 3 per cent. on £2500 = £75.

When he sells he receives £92 × 25 cash.

On every £80 cash invested in Consols he gets an income of $£2\frac{1}{2}$.

$\therefore$ On £92 × 25 cash invested in Consols he gets an income of

$$£2\frac{1}{2} \times \frac{92 \times 25}{80}$$

$$= £\frac{46 \times 25}{16}$$

$$= £\frac{575}{8}.$$

Second income = £71 17*s.* 6*d.*

His income is decreased by £3 2*s.* 6*d.* per annum.

Examples XIX c.

What rate of interest on the money invested is earned by the following stocks? Give answers to one decimal place.

1. South Australian $3\frac{1}{2}$ per cent. stock at $90\frac{1}{4}$.
2. Metropolitan Water Board 3 per cent. stock at $78\frac{7}{8}$.
3. India $3\frac{1}{2}$ per cent. stock at $90\frac{7}{8}$.
4. New South Wales 4 per cent. stock at 101.
5. French 4 per cent. stock at 88.
6. Japan $4\frac{1}{2}$ per cent. stock at $95\frac{1}{4}$.
7. Salvador 6 per cent. stock at 101.
8. German 3 per cent. stock at $76\frac{1}{2}$.
9. Greek 5 per cent. stock at 59.
10. 4 per cent. Debentures at $93\frac{3}{4}$.

11. A man holds £550 South Australian $3\frac{1}{2}$ per cent. stock. He sells out at $90\frac{1}{4}$ and invests the proceeds in Salvador 6 per cent. stock at 101. What is the increase in his income? Does he lose in any way?

12. What is the change in income caused by selling £2350 Metropolitan Water Board 3 per cent. stock at $78\frac{7}{8}$ and investing the proceeds in French 4 per cent. stock at 88?

13. A man finds that by investing in 4 per cent. stock he receives $4\frac{1}{2}$ per cent. on his money. What is the price of the stock?

14. Which is the better investment, South Australian $3\frac{1}{2}$ per cent. stock at $90\frac{1}{4}$ or New South Wales 4 per cent. stock at 101?

15. Find the total income obtained by investing £2527 in $3\frac{1}{2}$ per cent. stock at $90\frac{1}{4}$ and £3165 in 3 per cent. stock at $78\frac{7}{8}$.

16. A man derived a yearly income of £43 10*s.* from some Japan $4\frac{1}{2}$ per cent. stock. He sold out at $95\frac{1}{4}$ and invested the proceeds in India $3\frac{1}{2}$ per cent. stock at $90\frac{7}{8}$. What was his new income?

17. How much 3 per cent. stock does a man hold if, by selling out at 80 and investing the proceeds in a 5 per cent. stock at 85, he increases his income by £203 per annum?

18. What is the difference in income obtained by investing £2250 (1) in a 4 per cent. stock at 90, (2) in a 3 per cent. stock at 75?

What will be the respective rates per cent. obtained upon the money invested?

19. £5000 was invested in a 3 per cent. stock at par, subject to an income-tax of 1*s.* 3*d.* in the £. Another £5000 was invested in $3\frac{1}{4}$ per cent. stock at 104, free from income-tax. Which investment was the more advantageous, and by how much?

20. A man buys a certain stock at $112\frac{1}{2}$; for the first half-year the dividend was at the rate of 4 per cent. per annum per £100 stock, and for the next half-year at the rate of 3 per cent. per annum; what is the total interest per cent., correct to the nearest penny, on his outlay?

21. A man invests one-third of his capital in $2\frac{1}{2}$ per cent. stock at 96, one quarter in 4 per cent. stock at 110, and the remainder in 5 per cent. stock at 120; find, correct to the nearest farthing, the average rate per cent. returned on the capital.

22. A person invests £1730 by placing £900 of it in $2\frac{3}{4}$ per cent. stock at $112\frac{3}{8}$, and the remainder in 5 per cent. stock at $103\frac{5}{8}$. Find, to the nearest penny, the average rate of interest he receives on his money.

23. A man invests equal sums in 3 per cent. stock at 96 and in 4 per cent. railway debentures at 127. What percentage will he get for his money on the whole? (Find the result to the nearest penny.)

24. A man buys £6000 $2\frac{1}{2}$ per cent. stock at $91\frac{1}{2}$ and £4900 New South Wales 3 per cent. stock at 90 What rate of interest does he get on the total sum invested?

25. A man invests a sum of money in 3 per cent. stock at 99. If he had invested in the same stock when the price was 98, he would have received an income of £2 10*s.* greater. How much did he invest?

26. If money invested in the $2\frac{1}{2}$ per cent. Consols yielded exactly $2\frac{1}{2}$ per cent. interest, after paying one shilling in the pound income-tax, what would be the price of Consols?

27. A man invests £1404 16*s.* in $2\frac{3}{4}$ per cents. at 99; how much must he invest in $3\frac{1}{2}$ per cents. at 105 that the rate of interest on his whole investment may be 3 per cent.?

(**247.**) *Brokerages.* If Mr. A. wishes to sell some stock, he might have difficulty in finding some one to buy it; he therefore employs a stockbroker, who, as it were, finds him a customer and sees that all the documents are prepared that are necessary for the transfer of the stock. Similarly, if Mr. A. wishes to buy stock, the stockbroker finds some one who wishes to sell. In return for these services the stockbroker charges a percentage on the amount of stock transferred; this is called *brokerage.* Brokerage is charged both on selling and buying. The amount of brokerage varies somewhat with the kind of stock, but in working examples it is usual to take the brokerage to be $\frac{1}{8}$ per cent., unless some other brokerage is mentioned. If stock is at 95, any one buying £100 of the stock will pay £95 money for the stock and £$\frac{1}{8}$ brokerage; thus the actual price to a buyer is $95\frac{1}{8}$. But any one selling £100 stock will receive £95 money for the stock, of which £$\frac{1}{8}$ is paid as brokerage, so that the stockbroker will pay him only £95 − £$\frac{1}{8}$, i.e. £$94\frac{7}{8}$.

The main facts to be remembered are

Brokerage is charged on the amount of stock and not on the cash value. Brokerage increases the price when buying and diminishes the price when selling.

Oral Examples.

Reckon brokerage at $\frac{1}{8}$ per cent.

1–10. What is the actual price (*a*) to a seller, (*b*) to a buyer, of the stocks mentioned in Examples XIX c, 1–10?

11. What does a broker receive for selling £5600 of $3\frac{1}{2}$ per cent. stock at 97?

12. What does a broker receive for buying £5600 of $3\frac{1}{2}$ per cent. stock at 97?

13. What does a broker receive for selling £5600 of $3\frac{1}{2}$ per cent. stock at $97\frac{1}{8}$?

14. What does a broker receive for buying £5600 of $3\frac{1}{2}$ per cent. stock at $96\frac{7}{8}$?

15–18. How does the brokerage affect the broker's client in each of the cases 11–14?

How much must a man pay so as to secure an income

19. of £3 by investing in 3 per cent. stock at 94?

20. of £7 10*s.* by investing in $3\frac{3}{4}$ per cent. stock at $102\frac{1}{4}$?

21. of £9 by investing in $2\frac{1}{4}$ per cent. stock at $85\frac{3}{8}$?

22. of £12 10*s.* by investing in $2\frac{1}{2}$ per cent. stock at $75\frac{3}{4}$?

23. of £15 by investing in 5 per cent. stock at $103\frac{1}{4}$?

Examples XIX d.

In the following examples brokerage is to be reckoned at $\frac{1}{8}$ per cent., unless some other brokerage is mentioned.

1. A man invests £1200 in $2\frac{1}{2}$ Consols at 81. How much stock will he buy and what will the broker charge?

2. A stockbroker sells £2500 4 per cent. stock at $109\frac{1}{2}$ for a client and buys £3700 3 per cent. stock at 96 for him. How much will the broker receive for his services?

3. For selling stock a stockbroker received £23 7*s.* 6*d.*; what was the amount of the stock sold?

4. If £3470 17*s.* 6*d.* 3 per cent. stock was bought at 92 and sold at $92\frac{1}{4}$, what was gained on the transaction (*a*) by the stockbroker, (*b*) by the client?

5. What per cent. does a man receive for his money by investing in $2\frac{3}{4}$ per cent. stock at $97\frac{1}{2}$?

6. £5460 is invested in $3\frac{1}{2}$ per cent. stock at $90\frac{7}{8}$ The stock is afterwards sold at 91. How much money is gained or lost?

7. I transfer £4950 stock from 3 per cents. at $97\frac{5}{8}$ to railway 4 per cent. debentures at $89\frac{7}{8}$; find the change in my income.

8. What is the cost of 1000 shares at $\frac{11}{16}$ a share, brokerage being charged at 6*d.* a share?

9. How many shares at $2\frac{5}{16}$ can be bought for £100, brokerage being reckoned at 9*d.* a share?

10. What sum must be invested in India $3\frac{1}{2}$ per cents. at $89\frac{3}{4}$ to obtain a yearly income of £50?

(**248.**) *The Stock Exchange.* In London and many other large towns there is a place where stockbrokers meet and transact business; this is known as the Stock Exchange. The general public are not allowed in the Stock Exchange; those who use it are of two classes. There are the *jobbers* who actually buy and sell stock, and the stockbroker (§ 247) who acts as the connecting link between the outside public and the jobber. The jobbers usually deal in one kind of stock only, and the jobbers dealing in the same kind of stock gather in the same portion of the building; so that the stockbroker knows exactly where to go when he wishes to buy mining shares, government stock, or any other kind of stock. The jobber makes his profit by selling at a higher price than he buys. In the daily list of stock and share prices two prices are always given for each stock (not one price as in all the previous examples and sections); e. g. Brighton Ordinary 105–107. This means that the jobber will buy the stock at 105, but will sell at 107. If, therefore, a member of the public wishes to buy this stock, he must pay $107\frac{1}{8}$; if he wishes to sell, he will receive only $104\frac{7}{8}$. This difference should always be kept in mind by those who hope to make money by selling stock at a profit. In addition to this money paid to the stockbroker and jobber, there are also duties to be paid to the Government, by means of stamps, on every transaction.

Examples XIX e.

The following table shows the prices of certain stocks on two different dates:

Stock	Oct. 8th, 1912		Feb. 5th, 1913.	
Consols $2\frac{1}{2}$ %	$73\frac{15}{16}$	$74\frac{1}{16}$	$74\frac{13}{16}$	$74\frac{15}{16}$
India $3\frac{1}{2}$ %	$91\frac{1}{8}$	$91\frac{3}{8}$	$90\frac{7}{8}$	$91\frac{1}{8}$
London County Council 3 %	$80\frac{7}{8}$	$81\frac{1}{8}$	$80\frac{1}{8}$	$80\frac{3}{8}$
French 3 %	90	91	89	90
Turkish 4 %	$86\frac{1}{2}$	$87\frac{1}{2}$	86	87
Great Central Preference	$30\frac{3}{8}$	$30\frac{5}{8}$	$32\frac{3}{4}$	$33\frac{1}{4}$
South Eastern	90	92	93	95
Bombay-Baroda	100	102	106	107

Use the table above and reckon brokerage at $\frac{1}{8}$ per cent. :

1. What does a man gain or lose by investing £515 in Consols on Oct. 8th, 1912, and selling them on Feb. 5th, 1913?

2. How much must be invested in French 3 per cents. on Oct. 8th, 1912, to secure an income of £97 4*s.*?

3. A man invested £1000 on Oct. 8th, 1912, in India $3\frac{1}{2}$ per cents., received a quarter's dividend, and sold out on Feb. 5th, 1913. What was his gain or loss?

4. How much is gained by investing £2000 in Great Central Preference stock on Oct. 8th, 1912, and selling on Feb. 5th, 1913?

5. A man gained £155 by buying Great Central Preference stock on Oct. 8th, 1912, and selling on Feb. 5th, 1913; how much stock did he buy?

6. What was the difference in the rate of interest earned by investing £790 in London County Council 3 per cent. stock on Oct. 8th and Feb. 5th respectively?

7. What income is obtained by investing £500 in Turkish 4 per cents. on Feb. 5th, 1913?

8. On Oct. 8th, 1912, £2000 of $2\frac{1}{2}$ per cent. Consols are sold and the proceeds invested in French 3 per cents. What is the change in annual income?

9. How much is gained by buying £1625 South Eastern stock on Oct. 8th, 1912, and selling the same on Feb. 5th, 1913?

10. A person who held £3500 Turkish 4 per cents. sold them on Oct. 8th and invested half the proceeds in Bombay-Baroda and half in London County Council 3 per cents. He sold these on Feb. 5th and reinvested the proceeds in Turkish 4 per cents. How much Turkish stock to the nearest £ does he now hold?

(**249.**) In examinations problems on stock should be found quite simple if the various transactions are followed through stage by stage, and if the distinction between money and stock is kept clearly in mind at each stage.

Example i. *A man possesses* £10,000 *of* $3\frac{1}{2}$ *per cent. stock and* £7500 *of* $4\frac{1}{2}$ *per cent. stock. He sells out when the stocks are at* 126 *and* 168 *respectively, and invests the proceeds in* $2\frac{3}{4}$ *per cent. stock. What price must he pay in order that his income may be the same as before?*

Income on £10,000 of $3\frac{1}{2}$ per cent. stock $= £100 \times 3\frac{1}{2} = £350$

Income on £7500 of $4\frac{1}{2}$ per cent. stock $= £75 \times 4\frac{1}{2} = £337\frac{1}{2}$

Total income $= £687\frac{1}{2}$.

He sells £10,000 of $3\frac{1}{2}$ per cent. stock for $£100 \times 126 = £12,600$.

He sells £7500 of $4\frac{1}{2}$ per cent. stock for $£75 \times 168 = £12,600$.

He thus invests £25,200 money and receives an income of $£687\frac{1}{2}$.

What must he invest so as to receive an income of $£2\frac{3}{4}$ (being the income on £100 of the $2\frac{3}{4}$ per cent. stock)?

Price required $= £25,200 \times 2\frac{3}{4} \div 687\frac{1}{2}$.

$$= £25,2\cancel{00} \times \frac{\cancel{11}}{\cancel{4}} \times \frac{2}{\cancel{1375}\ \cancel{55}\cancel{00}}$$

$$= £100\tfrac{4}{5}. \qquad 5$$

Example ii. *A person invests* £6030 *in India* 3 *per cents. at* $100\frac{1}{2}$ *and pays income-tax at* 1*s.* 2*d. in the pound; on the stock rising to* 102 *he sells out and invests the proceeds in railway stock at* 180 *paying* 5 *per cent. free of income-tax. Find the change in his net income*

Tax on £3 = 3*s.* 6*d.*

Net income obtained by investing $£100\frac{1}{2} = £2\ 16s.\ 6d.$

$\therefore$ Net income obtained by investing $£6030 = £2\ 16s.\ 6d. \times \frac{6030}{100\frac{1}{2}}$.

$= £2\ 16s.\ 6d. \times 60.$

$= £33\ 18s. \times 5.$

$= £169\ 10s.$

The India stock bought for $£100\frac{1}{2}$ sells for £102.

$\therefore$ The India stock bought for £6030 sells for $£102 \times 60 = £6120$.

Income obtained by investing £180 in second stock = £5.

$\therefore$ Income obtained by investing £6120 in second stock $= £5 \times \frac{612}{18}$.

$= £5 \times 34.$

$= £170.$

$\therefore$ Net income is increased by 10*s.*

Examples XIX.

1. Find the income produced by the investment of £36,500 in $2\frac{3}{4}$ per cent. Consols at $91\frac{1}{4}$. What will be the change in the income when the interest is reduced to $2\frac{1}{2}$ per cent.?

2. A person sold out £12,500 Canada $2\frac{1}{2}$ per cent. stock at 90, and invested the proceeds in Victoria 4 per cent. stock at 108. What will be the change in his income?

3. The yearly dividend on a 3 per cent. stock is £237. If the stock is sold at $79\frac{7}{8}$ per cent., what sum does the sale produce?

4. What is the price of a $2\frac{1}{2}$ per cent. stock when the yield to the investor is $3\frac{1}{4}$ per cent.?

5. A man invested £869 5*s.* in a 3 per cent. stock, and obtained therefrom an income of £28 10*s.* At what price did he purchase the stock?

6. If I derive an annual income of £45 7*s.* 6*d.* from a 3 per cent. stock, how much stock do I hold? If I sell out at 91, and invest the proceeds in $3\frac{3}{4}$ per cent. stock at $112\frac{1}{2}$, what is my new income?

7. I invest £480 in railway shares, and find that if I were to sell out at $109\frac{1}{2}$, I should realize £486 13*s.* 4*d.* Find the price at which I bought the shares.

Also, if I derive the same income from these shares as I would by investing £768 in $2\frac{1}{2}$ per cent. Consols at the same price, find what dividend the railway is paying.

8. If I sell £4000 $2\frac{1}{2}$ per cent. Consols at $88\frac{1}{16}$, and invest the proceeds in Great Central Railway 5 per cent. Preference stock at 125, how much of the latter stock do I obtain, and what difference does it make in my income? The cost of each transaction is included in the price.

9. Find the income obtained by investing £525 16*s.* 8*d.* in Bank of England stock at $315\frac{1}{2}$ paying $9\frac{1}{2}$ per cent. How much money would have to be invested in $2\frac{1}{2}$ per cent. Consols at 90 in order to obtain the same income?

10. A man has £7925 in a 3 per cent. stock. He sells out at 96 as much of this stock as will realize £4200, and, keeping the

remainder in the 3 per cents., he invests the proceeds of the sale in a 5 per cent. stock at 105. Find the change in his income.

11. A person received a legacy of £1000 which he invested in a railway stock at 80. When the stock rose to 138 he sold out, and invested the proceeds in 3 per cent. debentures at 75. Find his income, and the market value of his capital, when, subsequent to his purchase, the debentures rose to par.

12. What sum must be invested in 3 per cent. stock at 98 in order to produce a net income of £229 after deducting income-tax at 11*d.* in the £?

13. The difference between the incomes derived from investing a certain sum of money in $2\frac{1}{4}$ per cent. stock at 75 and in 3 per cent. stock at 96 was £7 10*s.* What was the sum invested?

14. The difference between the incomes derived from investing a certain sum of money in 5 per cent. stock at 160 and in $3\frac{1}{2}$ per cent. stock at 125 was £13. What was the sum invested?

15. What sum must be invested in 4 per cent. stock at 105 in order to produce an income of £156 after deducting income-tax at 6*d.* in the £?

16. A man invests £970 in Manchester Corporation 3 per cent. stock at 85, £820 in Cape $3\frac{1}{2}$ per cent. stock at $97\frac{3}{4}$, and £1710 in Argentine 5 per cent. stock at $102\frac{1}{4}$. Find the average rate of interest on the capital invested.

17. Find the quarterly dividend on a sum of £27,215 6*s.* 3*d.* invested in $2\frac{3}{4}$ per cent. Consols at $88\frac{3}{4}$, after deducting income-tax at 1*s.* in the £. Brokerage is to be reckoned at $\frac{1}{8}$ per cent.

18. A company at the end of one half-year puts £1000 of its profits into its reserve fund and pays a dividend of 8 per cent. on its capital. During the next half-year its profits increase 40 per cent., and it puts £1500 into the reserve fund and pays a dividend of 11 per cent. on its capital. What are its profits for the year, and what is its capital?

19. A man invests his money in a $5\frac{1}{2}$ per cent. stock at 130 and a $3\frac{3}{4}$ per cent. stock at 95. Which will be the more profitable manner of investing it—to invest it equally in the two stocks, or to purchase equal amounts of the two stocks?

20. A man invests £8190 in a 3 per cent. stock at 91. He sells out £6000 stock when the price has risen to $93\frac{1}{2}$ and the remainder when it has fallen to 85. How much does he gain or lose by the transaction?

21. The sum of £25,200 has been lying for a year on deposit in a bank at $2\frac{1}{2}$ per cent. interest. What will be the change in income if the money is withdrawn and invested in Japanese 4 per cent. bonds at 84?

22. A man invests £33,000 in a 3 per cent. stock at 88, and subsequently, when the stock has risen to 90, sells out sufficient to purchase a house. If his income is diminished by £120 a year, find (1) the price which he paid for the house, (2) the amount of stock remaining in his possession.

23. I have 20 £100 shares in a company paying a dividend of $4\frac{1}{2}$ per cent. When the shares are quoted at $103\frac{1}{2}$ I sell out, and lend £2000 of the proceeds on a mortgage at 5 per cent., leaving the balance at my bankers on deposit at $2\frac{1}{2}$ per cent. What alteration do I effect in my income?

24. Find the amount that must be invested in a 4 per cent. stock at 114 to produce a net income of £100 after income-tax at a shilling in the pound has been deducted.

25. If a man transfers his money from a 4 per cent. stock at 103 to a $3\frac{1}{2}$ per cent. stock at $97\frac{1}{2}$, will he increase or diminish his income, and by how much per cent.?

26. How much 5 per cent. stock must be held in order that an income of £285 may be derived therefrom after paying an income-tax of 1*s.* in the £ on the whole income received? If the income-tax is raised to 1*s.* 3*d.*, how much more stock must be bought in order that the net income may be the same as before?

27. Which is the more profitable investment—a 5 per cent. stock which can be bought at 101, or a 4 per cent. stock which stands at 93?

28. What yield per cent. does a man receive for his money if he invests in a $5\frac{1}{2}$ per cent. stock which he buys at $137\frac{1}{2}$?

29. A sum of £4004 was invested in a 3 per cent. stock at $89\frac{3}{8}$, and after a year's dividend had been received the stock was sold out. The whole gain, including the year's dividend, was £302 8*s.* At what price was the stock sold?

30. A person holds 35 shares of $100 each on which a yearly dividend of 8 per cent. is made. If a dollar is worth 4*s.* 3*d.*, find the amount of the dividend in English money.

If the shares are sold at $130 each, and the proceeds invested in a 6 per cent. stock at 91, what is the change in the yearly income?

31. A man buys 20 railway shares at £65 and sells them when they have risen £7 each, buying $3\frac{1}{4}$ per cent. stock at 90. The shares paid £3 5*s.* dividend. Find the change in his income.

32. A man invests £7878 in $2\frac{3}{4}$ per cent. Consols at 101, and pays income-tax at 8*d.* in the £; on the stock rising to 103 he sells out, and invests the proceeds in stock at 156, yielding 5 per cent., free of income-tax. Find the change in his net income.

33. What will be the income derived from the investment of £3000 in the 4 per cents. at $71\frac{1}{4}$?

34. A holder of railway debentures receives one year a dividend of 5 per cent. on his stock, and pays an income-tax of 4*d.* in the £. The next year he receives a dividend of 6 per cent., and pays an income-tax of 3*d.* in the £, and finds that he is £352 18*s.* 4*d.* richer. How much railway stock does he hold?

35. A sum of £1057 10*s.* is invested in 3 per cent. stock at $93\frac{7}{8}$, is sold out when the stock reaches $95\frac{1}{8}$, and reinvested in $3\frac{1}{2}$ per cent. stock at $99\frac{5}{8}$, brokerage of $\frac{1}{8}$ per cent. being charged on each transaction. Find the alteration in income.

36. The $2\frac{1}{2}$ per cents. are now at $104\frac{1}{2}$, and a certain railway 4 per cent. stock is at 85. What would be the gain in income by selling out £6810 stock from the first and buying the second, brokerage $\frac{1}{8}$ per cent. being charged on each transaction?

37. What amount of $2\frac{3}{4}$ per cent. Consols at $97\frac{1}{8}$ (this price including all charges) can be bought for £1750, and what annual income will be derived from the investment? (Calculations to be made to the nearest shilling)

38. An investor finds that he will obtain £25 a year more income by investing in the $2\frac{3}{4}$ per cents. at $93\frac{1}{4}$ than in the $2\frac{1}{2}$ per cents. at 85. Find how much money he has to invest, assuming that the prices named include brokerage.

39. How much money must be invested in the $3\frac{1}{2}$ per cents. at 105 to obtain the same income as that derived from £2000 of 4 per cent. stock?

40. If I invest my money equally in two stocks, a $5\frac{1}{2}$ per cent stock at 132 and a $2\frac{3}{4}$ per cent. stock at $82\frac{1}{2}$, what rate of interest do I get on my investment?

41. What income do I receive from £925 of a 4 per cent. stock? If I sell out this stock at 91, and invest the proceeds in a $5\frac{1}{2}$ per cent. stock at $129\frac{1}{2}$, what will be the change in my income?

42. What income do I receive from £1000 of a 3 per cent. stock? If I sell out this stock at $91\frac{3}{4}$ and invest the proceeds in a 5 per cent. railway stock at 125, what will be the change in my income?

43. Find the income derived from investing £1628 9*s.* 2*d.* in a $4\frac{1}{2}$ per cent. stock at $93\frac{1}{2}$.

What net income would be received from this investment by an investor who has to pay income-tax at the rate of a shilling in the pound?

44. What annual income is derived from £14,800 $2\frac{3}{4}$ per cent. Consols? A lady who possesses that amount of Consols is aware that the rate of interest will soon be changed to $2\frac{1}{2}$ per cent. Calculate for her what quantity of Consols she must sell at 95, and with the proceeds buy an annuity at the price of £100 for £7 10*s.* annuity, in order that, when the change in rate takes place, her annuity and her income from the remaining Consols may be equal to her present income from Consols. (No brokerage to be reckoned.)

45. A man who spent £603 15*s.* in buying $3\frac{1}{2}$ per cent. stock receives £20 2*s.* 6*d.* per annum dividend. Find the price at which he bought the stock.

46. What must be the price of a 6 per cent. stock in order that an investment in it may return an income of $3\frac{3}{4}$ per cent. on the money invested?

47. If a certain sum is invested in $2\frac{1}{2}$ per cent. stock, an income of £712 10*s.* is obtained. If two-thirds of the sum is invested in $2\frac{1}{2}$ per cent. stock and the remainder put out to simple interest at 4 per cent. per annum, an income of £801 16*s.* is obtained. What is the price of the $2\frac{1}{2}$ per cent. stock?

48. Which investment will bring in the higher rate of interest, one in a $2\frac{1}{2}$ per cent. stock at 95, or one in a 4 per cent. stock at 145? Find, to the nearest pound, the difference of annual income when the sum invested is £10,000.

49. A man owns £5950 $2\frac{1}{2}$ per cent. Consols, for which he paid £6217 15*s.* How much additional money must he now invest in the same stock at $91\frac{1}{2}$ in order that he may get $2\frac{1}{2}$ per cent. interest on the whole sum invested? (No brokerage to be reckoned.)

50. If I buy £2750 Indian railway $3\frac{1}{2}$ per cent. stock at £107, how much money do I invest and what is my annual income? (Disregard brokerage.)

51. The capital of a company is 125,000 shares of £5 each. The profit in the year is £48,853. Find the greatest dividend, which must be an exact number of pence, which they can pay on each share without drawing on their reserve.

52. I buy a house, leasehold, for £1220, and, to pay for it, borrow £450 on mortgage at $4\frac{1}{2}$ per cent. and sell out enough $2\frac{1}{2}$ per cent. Consols at $87\frac{1}{2}$ to make up the balance. If the repairs which I have to do amount to $1\frac{3}{4}$ per cent. annually on the purchase price, and the ground rent I have to pay is £15 a year, and if I let the house at £90 a year, find how much my yearly income is increased.

53. By buying 3 per cent. stock at a certain price I find I obtain $3\frac{3}{5}$ per cent. for my money and derive a net income therefrom, after paying an income-tax of 6*d.* in the pound, of £421 4*s.* Find the amount of stock and the price at which I bought it.

54. A man has part of his money invested at $3\frac{1}{2}$ per cent. and part at $2\frac{1}{2}$ per cent. He sells out half of each and reinvests the proceeds at 6 per cent. If his original and final incomes are £469 and £684 10*s.* respectively, find his total capital and the amounts of his original investments.

55. The shares of a rubber company were issued at par at 2*s.* a share and are now quoted at 52*s.* 3*d.* Last year the company declared a dividend of 225 per cent. One shareholder secured an income of £315; what would he get for his shares if he sold out now?

56. A man buys railway stock at 118. At the end of a year he receives a dividend of $4\frac{1}{4}$ per cent. on his stock and sells out at $117\frac{1}{2}$. Find what rate of interest per cent. he makes on the year's investment. At each transaction he pays brokerage at the rate of 2*s.* 6*d.* per £100 of stock.

57. A man invests £14,560 in the 4 per cents. at 104. Six months later he sells £800 of his stock at the same price and invests the proceeds in the 5 per cents. at 116. Find the rate of interest he makes in the first year on his investment; dividends are paid each half-year.

58. A speculator buys 100 rubber shares at 128 free from brokerage. They rise to 132. He sells out 56 shares and reinvests the proceeds in railway stock at 154. He sells out both stocks when the railway stock has fallen to 153 and the rubber stock to 118. Find his total gain or loss.

59. A man invests his money equally in two stocks, a 3 per cent. stock at 88 and a 4 per cent. stock at 96. Had he with the same money purchased equal amounts of the two stocks, his annual income would have been 25 shillings more. How much did he invest?

60. If a man transfers his money from Great Northern 4 per cents. at $115\frac{7}{8}$ to London and South Western $3\frac{1}{2}$ per cents. at $99\frac{5}{8}$, will he increase or diminish his income, and by how much per cent.? Reckon brokerage at $\frac{1}{8}$ per cent. in each case.

CHAPTER XX

COMPOUND INTEREST

(250.) A SUM of money is said to be invested at Compound Interest when the interest is added to the principal and thus itself earns interest. To calculate Compound Interest we proceed as follows:

Example. *Find the amount of* £1724 5*s.* 6*d.* *for* 3 *years at* 4 *per cent.*

Principal	= £1724·275
4 %	= 68·97100
Amount after 1 year	= £1793·24600
4 %	= 71·72984
Amount after 2 years	= £1864·97584
4 %	= 74·59903
Final Amount	= £1939·57487

The amount = £1939 11*s.* 6*d.*
The increase = £215 6*s.* 0*d.*

In working an example such as this, the following points must be carefully noted.

1. Keep five figures after the decimal point, so as to ensure accuracy.

2. In taking 4 per cent. of any sum of money, we are multiplying the sum by 4, and dividing by 100, and since the latter process is done by moving the figures two places to the right, the two steps can be done at once.

3. Be particularly careful to label each line.

4. *Do no side work whatever.*

(**251.**) In cases where the interest for any year cannot be calculated at once, it is better to use a practice method, splitting the rate per cent. into two or more parts.

Example 1. *Find the amount of* £2745 4*s.* 8*d. after* 2 *years at* $3\frac{1}{4}$ *per cent.*

Principal =	£2745·233
3 % =	82·35699
$\frac{1}{4}$ % =	6·86308
	£2834·45307
3 % =	85·03359
$\frac{1}{4}$ % =	7·08613
	£2926·573

The amount is £2926 11*s.* 5*d.* to the nearest penny.

In taking $\frac{1}{4}$ % we are simply dividing the Principal by 400, and we may check the result by dividing the 3 % line by 12

A check on the work should be constantly maintained by noticing that any line representing interest is slightly greater than the corresponding line for the previous year. In this way any error in the position of the figures can be rectified.

In the following example the time is not a whole number of years.

Example ii. *Find the amount of* £474 8*s. for* $2\frac{1}{2}$ *years at* $4\frac{1}{2}$ *per cent.*

For the first two years we proceed in the ordinary way, and for the next half-year we take half the rate per cent., and work as before; for the interest at $4\frac{1}{2}$ % for $\frac{1}{2}$ a year equals the interest at $\frac{1}{2}$ of $4\frac{1}{2}$ % for 1 year.

Principal	= £474·4
4 % =	18·976
$\frac{1}{2}$ % =	2·372
Amount after 1 year	= £495·748
4 % =	19·82992
$\frac{1}{2}$ % =	2·47874
Amount after 2 years	= £518·05666
2 % =	10·36113
$\frac{1}{4}$ % =	1·29514
Amount after $2\frac{1}{2}$ years	= £529·71293

Total amount = £529 14*s.* 3*d.*

The interest or increase is £55 6*s.* 3*d.*

(252.) The interest may be added at periods other than whole years.

Example. *Find the amount of* £4725 *after* $1\frac{1}{2}$ *years at* 7 *per cent., interest being reckoned every half-year.*

(For each period of half a year we take half the rate of interest)

Principal		£4725
Interest for 1st half-year	3 % =	141·75
	$\frac{1}{2}$ % =	23·625
Amount after 1st half-year		£4890·375
Interest for 2nd half-year	3 % =	146·71125
	$\frac{1}{2}$ % =	24·45187
Amount after 1st year		£5061·53812
Interest for 3rd half-year	3 % =	151·84614
	$\frac{1}{2}$ % =	25·30769
Amount after $1\frac{1}{2}$ years		£5238·69195

The amount = £5238 13*s.* 10*d.*

When the interest is added every half-year, the interest is greater than if it were added every year. For instance, if interest is payable half-yearly, £100 at 5 per cent. amounts at the end of the year to £105·0625. This is the same amount as if the rate of interest had been 5·0625 per cent. payable yearly. Hence 5·0625 per cent. is called the *effective rate of interest*, when interest at 5 per cent. is payable half-yearly.

$$\begin{array}{r} 102{\cdot}5 \\ 2{\cdot}5625 \\ \hline 105{\cdot}0625 \end{array}$$

(253.) The following method is very useful for long periods and for working inverse sums in Compound Interest.

Example. *Find the amount of* £1724 5*s.* 6*d. for* 3 *years at* 4 *per cent.*

At 4 per cent., £1 amounts to £1·04 at the end of a year.

Hence in each year the principal at the beginning of the year is multiplied by 1·04.

Hence £1 becomes £1·04 at the end of 1 year.

„ „ £(1·04) × (1·04)

= £(1·04)² at the end of 2 years.

„ „ £(1·04)³ at the end of 3 years,

and so on for any number of years.

∴ £1724·275 amounts to £1724·275 × (1·04)³ at the end of 3 years.

The actual calculation proceeds as in § 250, Example i.

If, however, the number of years is large, the calculation is tedious; the work is much lightened by using logarithms. There must be at least as many figures in the logarithms as we require in the final amount; four-figure logarithms are rarely sufficiently accurate. (Use the logarithms on p. 589.)

Thus the amount of £1724 in 20 years at 4 per cent.

= £1724 × (1·04)²⁰

= £3777.

log (1·04)²⁰ = 20 × ·01703

= ·3406

log (1724) = 3·2365

∴ log (amount) = 3·5771

If the interest is payable yearly and the amount is required at the end of a fraction of a year, care is necessary in applying the method. For instance, the amount of £930 at 4 per cent. at the end of $7\frac{1}{2}$ years is £930 × (1·04)⁷ × (1·02) (see Example ii, § 251), not £930 × (1·04)^{7·5}.

Examples XX a.

Find the amount and the increase to the nearest penny in the following cases, reckoning Compound Interest:

1. £345 2*s.* for 2 years at 3 per cent.
2. £407 8*s.* for 2 years at 4 per cent.
3. £7452 3*s.* 6*d.* for 3 years at 5 per cent.
4. £1763 9*s.* 4*d.* for 2 years at $4\frac{1}{2}$ per cent.
5. £845 4*s* for 3 years at $3\frac{1}{2}$ per cent.
6. £592 3*s.* 6*d.* for 3 years at $2\frac{1}{2}$ per cent.
7. £4921 9*s.* for $3\frac{1}{2}$ years at $2\frac{1}{2}$ per cent.
8. £454 3*s.* 8*d.* for 3 years at $2\frac{2}{3}$ per cent.

9. £516 4*s.* 10*d.* for $2\frac{1}{2}$ years at $2\frac{2}{3}$ per cent.
10. £1923 4*s.* for $2\frac{1}{2}$ years at $4\frac{1}{4}$ per cent.
11. £35 2*s.* 8*d.* for 4 years at $3\frac{1}{2}$ per cent.
12. £492 1*s.* 2*d.* for 3 years at $3\frac{3}{4}$ per cent.

Find the difference between the Simple Interest and the Compound Interest in the following cases:

13. £491 4*s.* 11*d.* for 2 years at $3\frac{1}{2}$ per cent.
14. £507 8*s.* 5*d.* for $3\frac{1}{2}$ years at $3\frac{1}{2}$ per cent.
15. £8524 5*s.* for 3 years at $4\frac{1}{3}$ per cent.
16. £519 8*s.* 8*d.* for $2\frac{1}{2}$ years at $4\frac{1}{2}$ per cent.
17. £1023 4*s.* 4*d.* for $3\frac{1}{2}$ years at 5 per cent
18. £1936 for 2 years at $5\frac{1}{4}$ per cent.
19. £514 8*s.* 4*d.* for 3 years at $4\frac{1}{6}$ per cent.

Find the amount in the following cases:

20. £422 for $1\frac{1}{2}$ years at 10 per cent. per annum reckoned half-yearly.
21. £1049 for 1 year at 12 per cent. per annum reckoned quarterly.
22. £543 for $1\frac{1}{4}$ years at 8 per cent. per annum reckoned half-yearly during the first year.
23. £4821 for 8 months at 20 per cent. per annum reckoned quarterly.
24. Find, by logarithms, the amount of £5500 in 15 years at 5 per cent. per annum compound interest.
25. A person borrowed £11,000 for 2 months at 5 per cent. per annum. At the end of the time the interest was added on and the debt renewed for another two months. This was continually repeated till at the end of 2 years the debt and interest were paid. Find the total amount paid.
26. If the annual increase in the population of a state is 25 per thousand and the present number of inhabitants is 2,624,000, what will be the population in 3 years' time? What was it a year ago?

27. When land is treated with bone manure, the manure loses 40 per cent. of its original value during the first year, 30 per cent. during the second year, 20 per cent. during the third year, and the remaining 10 per cent. is exhausted in the fourth year. If a farmer treats his land each year with 4 tons of bone manure costing £5 8*s.* a ton, what will be the value of the manure left in the ground at the end of 4 years, that is, just before he treats with bone manure for the fifth time?

28. The population of a town has increased from 160,000 to 212,000 in 10 years. What will be its population 5 years afterwards if it goes on increasing at the same rate?

29. For a certain substance, through which light is passing, the intensity of the light at the end of any inch (measured from the source of light) is ·97 of what it was at the beginning of the inch; what fraction is the intensity at 100 inches of the intensity at 50 inches?

30. Find the sum of money such that, when put out at compound interest, the interest for the first year is £15 12*s.* 6*d.* and for the second year £16 1*s.* 10½*d.*

31. If the present population of a county is two millions, what will it be at the end of 3 years if it increases in the interval at the rate of 70 per 1000?

32. *A* put out £30 on his 16th birthday to accumulate at 2½ per cent. compound interest, interest being added yearly. What will be the amount on his 20th birthday? If he had invested £30 in the same way on his 17th, 18th, and 19th birthdays, what would have been the amount on his 20th birthday?

33. What is the effective rate of interest when money accumulates at 3½ per cent. compound interest, interest being paid (i) half-yearly, (ii) quarterly?

INVERSE COMPOUND INTEREST

(254.) Example i. *Find the principal which will amount to £3742 4s. 8d. in $2\frac{1}{2}$ years at $4\frac{1}{4}$ per cent. compound interest.*

The answer will have 4 figures before the decimal; if we require the answer correct to nearest penny we must have 7 significant figures

$\therefore$ Amount $=$ Principal $\times (1{\cdot}0425)^2 \times (1{\cdot}02125)$

$=$ Principal $\times\ 1{\cdot}10990088.$

$\therefore$ Principal $= \dfrac{£3742{\cdot}23333}{1{\cdot}10990088}$

$=$ £3371 13*s.* 5*d.*

1 0425 × 1 0425
417
260625
1 0868062$_{5}$ × 1·02125
2173612
135851
1 10990088

1 10990083) 3742 23333 (3371·6824
412·53069
79 56043
1 86737
75747
9153
274
52

Just as in the case of simple interest, this principal is called the Present Value of the Amount.

Example ii. *In how many years will £735 amount to £803 3s. 1d. at 3 per cent. compound interest?*

Here the principal is multiplied by 1·03 every year, so that the answer can be found by continually multiplying £735 by 1·03 till the amount is £803 3*s.* 1*d.*

£735·
22·05 × ·03
£757·05 Amount at end of 1 year.
22·7115
779·7615 Amount at end of 2 years.
23·3928
£803·1543 Amount at end of 3 years.

Required time is 3 years.

This method may be very long, and it is far better to use logarithms. If n be the number of years, we have

$735 \times (1{\cdot}03)^n = 803{\cdot}15$

$\therefore\ n \log 1{\cdot}03 = \log 803{\cdot}15 - \log 735$

$\therefore$ number of years $= \dfrac{{\cdot}0385}{{\cdot}0128} = 3.$

2·9048
2·8663
2·0385

Example iii. *At what rate per cent. will* £735 *amount to* £803 3*s.* 1*d. in* 3 *years at compound interest?*

Here there is an unknown ratio which, operating three times on £735, gives an amount £803·154, i.e. the cube of the ratio $= 803{\cdot}154 \div 735$.

If we denote the ratio by R, we have

$$803{\cdot}154 = 735 \times R^3$$
$$\therefore \quad R^3 = 803{\cdot}154 \div 735$$
$$3 \log R = {\cdot}0385$$
$$\log R = {\cdot}0128$$
$$\therefore \quad R = 1{\cdot}03.$$

2·9048
2·8663
·0385

Hence the required rate is 3 per cent.

Examples XX b.

1. Find the present value of £828 14*s.* 3*d.* due $2\frac{1}{2}$ years hence at 5 per cent. per annum compound interest.

2. What principal will amount to £498 18*s.* $5\frac{1}{2}d.$ in 3 years at $3\frac{1}{2}$ per cent. per annum compound interest?

3. What sum would in 3 years amount to £2811 18*s.* at compound interest, if the rate were 3 per cent. per annum for the first year, 4 per cent. for the second year, and 5 per cent. for the third?

4. The compound interest on a certain sum, reckoned annually at 3 per cent. per annum, in 3 years exceeds the simple interest for the same period by £68 3*s.* 6*d.*, find the sum.

5. If the compound interest on £2371 for 4 years is £349 15*s.* 3*d.*, find the rate per cent.

6. What principal amounts to £477 15*s.* in 2 years at 5 per cent. compound interest?

7. What is the present value of £308 14*s* due 3 years hence at 5 per cent. compound interest?

8. On what sum of money would the compound interest for 3 years at 5 per cent. per annum exceed the simple interest by £38 2*s.* 6*d.*?

9. A person has a legacy of £2000 left him, payable at the end of 3 years. He wishes to obtain cash for it now; for how much

can he sell his claim if money at compound interest yields 5 per cent. per annum, paid yearly. (Answer to nearest pound.)

10. Find to the nearest shilling the present worth of £450 due 5 years hence at $4\frac{1}{2}$ per cent. compound interest.

11. What sum of money must be put out to compound interest at $3\frac{1}{4}$ per cent. per annum to amount in 3 years to £1000?

12. Calculate the number of years that a capital takes to double itself at compound interest at 4, 5, and 6 per cent. per annum?

(255.) It often happens that equal sums of money are invested every year and allowed to accumulate at compound interest. The following examples show how we may deal with such cases

Example i. *A man invests* £10 *for his son every year, beginning on the day of the boy's birth. If the money has increased at* 3 *per cent. compound interest, how much will the son receive when he is* 21 *years old?*

The first £10 amounts to $£10 \times (1{\cdot}03)^{21}$.
The second £10 amounts to $£10 \times (1{\cdot}03)^{20}$.
The twenty-first £10 amounts to $£40 \times (1{\cdot}03)^{1}$.
The final £10 amounts to $£10 \times 1$.

$$\text{Amount} = £10\,(1{\cdot}03^{21} + 1{\cdot}03^{20} + \ldots + 1{\cdot}03 + 1).$$

Multiply by 1·03.

$$\text{Amount} \times 1{\cdot}03 = £10\,(1{\cdot}03^{22} + 1{\cdot}03^{21} + \ldots + 1{\cdot}03^{2} + 1{\cdot}03).$$

Subtract the previous line.

$$\text{Amount} \times {\cdot}03 = £10\,(1{\cdot}03^{22} - 1)$$
$$= £10 \times {\cdot}916.$$
$$\text{Amount} = £9{\cdot}16 \div {\cdot}03 = £305{\cdot}3$$
$$= £305 \text{ to the nearest pound.}$$

$\log 1{\cdot}03$	·012837
	11
	·141207
	2
$\log (1{\cdot}03)^{22}$	·282414

$\therefore (1{\cdot}03)^{22} = 1{\cdot}916$

Such investments as this are usually made by paying a yearly sum or *Premium* to an insurance company.

Example ii. *A man wishes to pay a yearly sum so that his payments with compound interest at 3 per cent. may amount to at least £500 in ten years' time; find the amount of the yearly payment.*

Consider a yearly payment of £1.

If he pays £1 now, and £1 at the end of the 10th year, there will be 11 payments in all.

As in the previous example, we see that

Final amount $= £1\ (1{\cdot}03^{10} + 1{\cdot}03^{9} + \ldots + 1{\cdot}03 + 1)$.

$\therefore$ Amount $\times\ 1{\cdot}03 = £1\ (1{\cdot}03^{11} + 1{\cdot}03^{10} + \ldots + 1{\cdot}03^{2} + 1{\cdot}03)$

and Amount $\times\ {\cdot}03 = £1\ (1{\cdot}03^{11} - 1)$.

$\therefore$ Amount $= £{\cdot}384 \div {\cdot}03$

$= £12{\cdot}8$.

$\log 1\ 03^{11} = 11 \times {\cdot}012837$

$= 141207$

$1{\cdot}03^{11} = 1{\cdot}384$

Hence to produce £12·8, yearly payment must be £1.

$\therefore$ to produce £500, yearly payment must be $\frac{£500}{12{\cdot}8}$

$= £39{\cdot}1$.

$$12{\cdot}8 \overline{)\ 500\ \ 39{\cdot}1}\ \ 116\ \ 1$$

A yearly payment of £40 will therefore produce just over £500.

Example iii. *What sum should a man pay on Jan. 1st, 1913, for the right of receiving a yearly income of £50 in each year from 1920 to 1930 inclusive, payable on Jan. 1st; it being supposed that the money can be used so as to increase at 4 per cent. compound interest?*

[Such questions as this occur in connexion with house-property, annuities, and pensions. The calculations are complicated by the uncertainty as to how long the various people concerned are going to live; we shall suppose here that either the man or his heirs will receive the sums of £50, so that their payment is certain]

We may calculate on the understanding that a separate payment is made for each of the future payments of £50. What sum invested on Jan. 1st, 1913, will amount to £50 on Jan. 1st, 1920? The required sum multiplied by $(1{\cdot}04)^{7}$ equals £50.

Hence Present Value of £50 due in 1920 $= £50 \div (1{\cdot}04)^{7}$

$$= \frac{£50}{(1{\cdot}04)^{7}}$$

and Present Value of £50 due in 1921 $= \dfrac{£50}{(1{\cdot}04)^{8}}$

and Present Value of £50 due in 1930 $= \dfrac{£50}{(1{\cdot}04)^{17}}$,

$\therefore$ Total Present Value

$$= £50\left\{\frac{1}{(1{\cdot}04)^{7}} + \frac{1}{(1{\cdot}04)^{8}} + \ldots + \frac{1}{(1{\cdot}04)^{16}} + \frac{1}{(1{\cdot}04)^{17}}\right\}.$$

Divide by 1·04.

Present Value $\times \dfrac{1}{1{\cdot}04}$

$$= £50\left\{\frac{1}{(1{\cdot}04)^{8}} + \frac{1}{(1{\cdot}04)^{9}} + \ldots + \frac{1}{(1{\cdot}04)^{17}} + \frac{1}{(1{\cdot}04)^{18}}\right\}.$$

Subtract.

Present Value $\times \left(1 - \dfrac{1}{1{\cdot}04}\right)$

$$= £50\left\{\frac{1}{(1{\cdot}04)^{7}} - \frac{1}{(1{\cdot}04)^{18}}\right.$$

$\therefore$ Present Value $= \dfrac{\dfrac{1}{(1{\cdot}04)^{7}} - \dfrac{1}{(1{\cdot}04)^{18}}}{1 - \dfrac{1}{1{\cdot}04}} \times £50$

$$= \frac{(1{\cdot}04)^{11} - 1}{(1{\cdot}04)^{17}(1{\cdot}04 - 1)} \times £50$$

$$= \frac{{\cdot}539}{(1{\cdot}04)^{17}} \times £1250$$

$$= £345{\cdot}9$$

$$= £345\ 18s.$$

Logarithms.

$\overline{{\cdot}017033} \times 11$
$\overline{187363}$
$= \log(1{\cdot}539)$

$\overline{017033} \times 17$
$\overline{{\cdot}289561}$

$\bar{1}{\cdot}7316$
$3\ 0969$
$\overline{2{\cdot}8285}$
$-\overline{{\cdot}2896}$
$\overline{2{\cdot}5389}$

Example iv. *Find the sum that should be paid for the right to receive a yearly payment, beginning one year hence, of* £1 *for* (a) 10 *years,* (b) 100 *years,* (c) 1000 *years,* (d) *for ever.*

(*a*) The required sum equals the sum of the present values of the successive yearly payments.

$$\text{Required sum} = £1\left(\frac{1}{1{\cdot}05} + \frac{1}{1{\cdot}05^2} + \ldots + \frac{1}{1{\cdot}05^{10}}\right).$$

Multiplying by $\frac{1}{1{\cdot}05}$ and subtracting, we obtain, as in the previous examples,

$$\text{Required sum} = £\frac{\frac{1}{1{\cdot}05} - \frac{1}{1{\cdot}05^{11}}}{1 - \frac{1}{1{\cdot}05}}$$

$$= £\frac{1 - \frac{1}{1{\cdot}05^{10}}}{{\cdot}05}$$

$$= £20\left(1 - \frac{1}{1{\cdot}05^{10}}\right)$$

$$= £20\,(1 - {\cdot}6139)$$

$$= £20 - £12{\cdot}278$$

$$= £7{\cdot}722$$

$$= £7\ 14s.\ 5d.$$

$\log 1{\cdot}05 = 02119$
$\log 1\ 05^{10} = {\cdot}2119$
$\log \frac{1}{1{\cdot}05^{10}} = \bar{1}\ 7881$

(*b*) As in (*a*) we obtain

$$\text{Required sum} = £20\left(1 - \frac{1}{1{\cdot}05^{100}}\right)$$

$$= £20\,(1 - {\cdot}00760)$$

$$= £20 - £{\cdot}152$$

$$= £20 - 3s.$$

$$= £19\ 17s.$$

$\log 1\ 05^{100} = 2\ 119$
$\log \frac{1}{1\ 05^{100}} = \bar{3}{\cdot}881$

(*c*) As in (*a*), we obtain

$$\text{Required sum} = £20\left(1 - \frac{1}{1{\cdot}05^{1000}}\right)$$

$$= £20\,(1 - 10^{-22} \times 6{\cdot}5).$$

$\log 1\ 05^{1000} = 21{\cdot}19$
$\log \frac{1}{1\ 05^{1000}} = \bar{22}{\cdot}81$

This differs from £20 by a decimal of a pound beginning with 21 noughts. This is of no practical value, and may therefore be neglected.

Hence required sum = £20.

(*d*) We see from (*a*), (*b*), and (*c*) that the present value of the yearly payment is less than £20 and that the amount to be subtracted decreases as the number of years increases. In the case of 1000 years the amount to be subtracted is negligible; still more may it be neglected when the number of years is unlimited. Hence, when the payment continues for ever, the amount to be paid is £20.

Such a payment is called a *perpetual annuity*, and occurs when freehold property is let. Clearly a perpetual yearly payment of £1 can be obtained by investing £20 at 5 per cent.; so that £20 is the sum which will purchase a yearly income of £1. Hence the present value obtained by the above argument is verified by this more elementary method.

Examples XX c.

1. If £5 is invested at the beginning of every year at 3 per cent. compound interest, what will be the amount at the end of 20 years?

2. What sum should be paid for an annuity of £52 a year, payable yearly, the first payment to be made a year hence and payments to continue for 15 years; compound interest to be reckoned at $3\frac{1}{2}$ per cent.?

3. A man buys the lease of a house for 25 years for £1250. If interest is reckoned at 4 per cent. per annum, how much of the rent ought he to lay aside to form a sinking fund which will just amount to his purchase money when the lease expires?

4. The yearly output of a gold-mine decreases every year by 13 per cent. of its amount during the previous year. Given that the first year's output is £260,000, find (*a*) the total output for the first 10 years, (*b*) the total output for all time.

5. A bank gives interest on deposits at the rate of $2\frac{1}{2}$ per cent. per annum, the interest being credited half-yearly. If neither principal nor interest were withdrawn, how long a time would be required for a deposit of £500 to increase to £650?

6. A boy in a swing works up the movement until he swings through a distance of 18 ft. in his forward movement. While the motion is dying away, the distance covered in any movement, either backward or forward, is 5 per cent. less than that covered in the preceding movement in the reverse direction. Find the total distance (including the first 18 ft.) that the boy goes while the swing is coming to rest.

7. A man receives a pension starting with £100 the first year, but each year he receives 90 per cent. of what he received the previous year. Find the total amount he receives in the first 6 years; find also the greatest amount he could possibly receive, even if he were to live for ever

8. A person pays the principal of a debt by annual instalments, paying £120 the first year, and each succeeding year 10 per cent. more than the year before; how much will he have repaid in 10 years?

9. At the beginning of each of 20 consecutive years a man invests £100 at 4 per cent. per annum compound interest; what single sum invested at the beginning of this period would give the same amount at the end of 20 years?

10. The holder of 80 4 per cent. debenture shares of £20 in a company which pays interest on Jan. 1st and July 1st invests the interest regularly in a bank which allows $2\frac{1}{2}$ per cent. interest and adds the interest to the sum invested at the end of each year. Find the total amount at the bank at the end of 6 years.

11. A man borrows £2000 to be paid back in 10 yearly instalments, the first instalment being paid at the end of a year and compound interest being reckoned at 4 per cent. per annum. How much did he repay each year?

12. If at the age of 65 the expectation of life is 11 years, show that the present value of a life annuity of £1 per annum is £9·254, reckoning compound interest at 3 per cent.

13. It is expected that a company will ultimately pay 50 annual dividends of 25 per cent. on its stock, the first dividend being expected in 10 years from the present time. What is the present worth of £100 of stock, calculated on a basis of 5 per cent. compound interest?

14. Show that by payment of £1 a year, beginning on his 21st birthday and ending on his 60th, a man should become entitled to an old-age pension of about £8 11*s.* a year, paid from his 61st to his 75th birthdays, compound interest being reckoned at 4 per cent.

15. A tree is $3\frac{1}{2}$ ft. high when planted and grows in height in each year by $4\frac{1}{2}$ per cent. of its height at the beginning of the year. Find its height at the end of 12 years.

RECURRING DECIMALS

(256.) A vulgar fraction is reduced to a decimal fraction by dividing the numerator by the denominator, the numerator being reduced to tenths, the first remainder to hundredths, &c.; the process is continued till the required number of decimals have been obtained, or until there is no remainder. In this latter case the decimal is said to terminate. If we perform this process with the fraction $\frac{1}{3}$, we find that

$$\tfrac{1}{3} = \cdot 3333333 \ldots\ldots,$$

and we see that the succession of threes is unending. This is denoted thus:

$$\tfrac{1}{3} = \cdot\dot{3}.$$

Such a decimal is called a *recurring* or *circulating* decimal.

Hence $\cdot\dot{3} = \cdot 333 \ldots\ldots$

$$= \frac{3}{10} + \frac{3}{10^2} + \frac{3}{10^3} + \ldots\ldots$$

$$= \frac{3}{10}\left(1 + \frac{1}{10} + \frac{1}{10^2} + \frac{1}{10^3} + \ldots \text{ without end}\right).$$

This is exactly similar to the series of fractions we had in Example iv, § 255, and may be treated in the same way.

Suppose we stop at $\frac{1}{10^{1000}}$; this gives an approximate value for $\cdot\dot{3}$.

$$\cdot 33\ldots\ldots = \frac{3}{10}\left(1 + \frac{1}{10} + \frac{1}{10^2} + \ldots + \frac{1}{10^{1000}}\right).$$

Multiply by $\frac{1}{10}$, and subtract.

$$\frac{9}{10} \times \cdot 333\ldots = \frac{3}{10}\left(1 - \frac{1}{10^{1000}}\right).$$

$$\therefore \cdot 333\ldots = \frac{3}{9}\left(1 - \frac{1}{10^{1001}}\right)$$

$$= \frac{1}{3} - \cdot 0000\ldots\ldots 3 \text{ (1000 noughts before the 3)}.$$

If we go on for ever, we must subtract from $\frac{1}{3}$ a quantity smaller than any quantity we can think of, however small. That is, $\cdot\dot{3}$ does not differ from $\frac{1}{3}$ at all.

The work is more compactly arranged thus:

$$\cdot\dot{3} = \cdot 333\ldots\ldots \ldots\ldots$$

$$10 \times \cdot\dot{3} = 3\cdot 333\ldots \ldots\ldots\ldots$$

Subtract the previous line.

$$9 \times \cdot\dot{3} = 3.$$

$$\therefore \cdot 3 = \tfrac{1}{3}.$$

Example i. *Reduce* $\cdot\dot{7}1\dot{2}$ *to a vulgar fraction.*

$\cdot\dot{7}1\dot{2}$ means $\cdot 712712712\ldots\ldots\ldots\ldots$

$$\therefore 1000 \times \cdot\dot{7}1\dot{2} = 712\cdot 712712712\ldots\ldots\ldots\ldots$$

Subtract the previous line.

$$999 \times \cdot\dot{7}1\dot{2} = 712.$$

$$\therefore \cdot\dot{7}1\dot{2} = \frac{712}{999}.$$

Example ii. *Reduce* $\cdot 23\dot{4}87\dot{3}$ *to a vulgar fraction.*

$\cdot 23\dot{4}87\dot{3}$ means $\cdot 23487348734873$.

Denote the decimal by F.

$$F = \cdot 23487348734873 \ldots\ldots$$

Multiply by 1000000 (this moves the decimal point to the end of the first set of recurring figures).

$$1000000\,F = 234873 \cdot 48734873 \ldots\ldots$$

Multiply by 100 (this moves the decimal point to the beginning of the first set of recurring figures).

$$100\,F = 23 \cdot 48734873 \ldots\ldots$$

Subtract.

$$999900\,F = 234873 - 23.$$

$$\therefore \quad F = \frac{234873 - 23}{999900}\text{; which can be simplified.}$$

Examples XX d.

Reduce to decimals

1. $\frac{7}{12}$.	**2.** $\frac{3}{11}$.	**3.** $\frac{2}{7}$.	**4.** $\frac{5}{13}$.
5. $\frac{9}{16}$.	**6.** $\frac{5}{7}$.	**7.** $\frac{3}{32}$.	**8.** $\frac{8}{13}$.
9. $\frac{5}{48}$.	**10.** $\frac{4}{15}$	**11.** $\frac{7}{90}$.	**12.** $\frac{1}{81}$.

13. By considering the factors of the denominator, show how to determine when the corresponding decimal will terminate and when it will not terminate.

14. Show that if the decimal derived from a vulgar fraction does not terminate it must recur. (Consider the possible remainders.)

Reduce to vulgar fractions in their simplest form:

15. $\cdot\dot{5}$.	**16.** $\cdot\dot{0}\dot{9}$.	**17.** $\cdot\dot{4}28157\dot{}$.
18. $\cdot 3\dot{5}$.	**19.** $\cdot 57\dot{0}\dot{9}$.	**20.** $\cdot 73\dot{4}28157\dot{}$.
21. $\cdot 2478\dot{9}$.	**22.** $\cdot 247\dot{8}\dot{9}$.	**23.** $\cdot 2\dot{4}78\dot{9}$.

24. State rules for reducing circulating decimals to vulgar fractions.

Examples XX.

1. Find the amount, to the nearest £, of £12,000 in 4 years at $3\frac{1}{2}$ per cent. per annum, compound interest.

2. Find, correct to the nearest penny, the compound interest on £1250 for 3 years at $3\frac{3}{4}$ per cent. per annum.

3. The difference between the simple and compound interest, at the same rate per cent. per annum, on a certain sum is for 2 years £3 15*s.*, and for the third year £7 13*s.* What is the sum?

4. Find, to the nearest penny, the difference between the simple and compound interest on £576 14*s.* for 3 years at $3\frac{1}{3}$ per cent.

5. Sixty-four years after the bankruptcy, a creditor, to whom £1000 was owed, received a dividend at the rate of $2\frac{3}{4}d.$ in the £. If compound interest be reckoned at 4 per cent, find, to the nearest penny, the worth at the time of the bankruptcy of the sum now received by the creditor.

6. The value of certain land was 9 per cent. less at the end of each one of 4 consecutive years than it was at the beginning of that year; and at the end of each one of the next 2 years the value was a certain percentage greater than at the beginning of that year; also, at the end of the 6 years the value was ·932 of the value at the beginning of the 6 years. Determine, correct to *one* decimal place, the rate per cent. of the increase in value during the fifth and sixth years.

7. A debt of £1000 bearing compound interest at 4 per cent., payable annually, is to be repaid by four equal annual instalments, the first payable 1 year after the contraction of the debt. Find the amount of each instalment, so that the creditor may receive the equivalent of the whole sum with compound interest.

8. If a sum of money has to be put out at interest for 3 years, find whether it is more profitable to invest it at $3\frac{1}{2}$ per cent. simple interest or at 3 per cent. compound interest.

9. The difference between the simple interest and the compound interest on a certain sum of money for 2 years at 3 per cent. per annum was £1. Find the sum of money.

10. A man borrows £1000 at 3 per cent. Show that three

annual payments of £353 10*s.* 7*d.*, the first payment being made at the end of one year, will pay off the loan.

11. A piece of property, if sold now, will realize £14,400 ; but, if retained for 4 years, it will bring in an annual rent of £300, and will at the end of the 4 years bring in a price of £15,000. Find which way of disposing of it is more profitable, assuming that money can be invested at 4 per cent. compound interest.

12. A debtor owes £2000, on which he is charged 4 per cent. interest at the end of each year. Each year he pays into the bank £200 to meet the interest and reduce the principal debt. Find, to the nearest pound, how much he owes after 5 years.

13. Two offers were made for some property : one of £4500 payable at once, the other of £5200 payable at the end of 3 years. Express the less offer as a percentage, correct to the nearest $\frac{1}{4}$, of the other offer, taking money to be worth 4 per cent. per annum and reckoning compound interest.

14. A speaker in Parliament recently asserted that, in a certain district where rents are fixed by a court of arbitration, on each of three consecutive appeals a reduction was granted of 20 per cent. of the rent at the time of appeal, making in all a reduction of about 50 per cent. of the original rent. Examine the arithmetical accuracy of this last statement.

15. Taking the present worth of £1 due in 5 years at 3 per cent. per annum compound interest to be £·862609, find, correct to the nearest pound, the present worth of £2474 due in 10 years at 3 per cent. per annum compound interest.

16. The present value of a sum of money due 3 years hence is £150, and in 2 years' time it will be £160 3*s.* 4*d.* ; find the rate of compound interest.

17. Draw up a table showing the sums of money (expressed in decimals) to which £1 would amount in 1, 2, and 3 years respectively at 4 per cent. per annum compound interest, payable half-yearly.

18. Express the compound interest on £318 7*s.* 6*d.* for 4 years at $3\frac{1}{2}$ per cent. as a decimal of the original amount, correct to 4 significant figures.

19. Find the compound interest on £3368 13*s.* for 3 years at $4\frac{1}{2}$ per cent.

20. What sum to the nearest shilling paid *now*, together with £300, £250, and £200 paid respectively at the ends of 1, 2, and 3 years from the present time, will be equivalent to three sums of £350 due at the ends of 1, 2, and 3 years from the present time if money is worth 3 per cent. per annum and compound interest is reckoned?

21. In a certain country the birth-rate per annum is 41, and the death-rate 25, for every 1000 inhabitants at the beginning of the year. If the population at the beginning of 1913 was four millions, find to the nearest thousand what it was at the beginning of 1910, assuming no immigration or emigration.

22. If in a certain town the birth-rate per annum is 35 and the death-rate 20 per thousand inhabitants, find to the nearest hundred what its population must have been three years ago if it be now 235,000

23. If the population of a town increases every year by 1·6 per cent. of the population at the beginning of that year, in how many years will the total increase of the population of the town be 40 per cent.?

24. What should be the rate of interest if capital is to treble itself in 19 years?

25. A debt of £200,000 is to be paid off as follows: At the end of every year £13,000 is to be paid, part of which is in payment of 5 per cent. interest on the part of the debt not paid off, and the remainder in reduction of the debt. Find how soon the debt will be liquidated.

26. A man having borrowed a sum of money undertakes to pay back £4 a month. At his twelfth payment he settles his debt by an additional payment of £56 15*s.* What was the sum borrowed, if the lender receives compound interest at 6 per cent. per annum, interest being converted to principal once a month?

MISCELLANEOUS EXAMPLES

For many of these examples reference must be made to the tables on pp. 9–12 for the relations between various units.

A knowledge of logarithms is essential for some of the later examples.

The examples are grouped in batches of six, and may be used, in this form, as test papers.

1. Find the value, correct to five figures, of the sum of the series:

$$1 + \frac{1}{3} + \frac{1}{3\ 6} + \frac{1}{3.6\ 9} + \frac{1}{3.6\ 9\ 12} + \ldots$$

2. Find, in square yards, the area of the smallest square courtyard that can be exactly paved with flagstones each 5 ft. 3 in. long and 4 ft. 8 in. broad, all laid the same way.

3. A croquet lawn is 35 yds. long by 28 yds. broad. How long will it take a man to mow it, if he pushes a 14-in. mowing machine at 3 miles an hour?

4. Find the length and breadth of a rectangular table whose area is 5·5 square metres and which is twice as long as it is wide.

5. A number is divided first by 4 and then by 7, the respective remainders being 2 and 3. Find the remainders when the same number is divided first by 7 and then by 4.

6. What volume of wood is used in the construction of a closed chest of a uniform thickness of 3 in., whose external measurements are 10 ft., 8 ft., and 6 ft. respectively?

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7. During a certain financial year the total expenditure for a village school was £438 8*s.* 11*d.*, and the number of children attending the school was 113. The corresponding items for another school were £543 9*s.* 3*d.* and 197. Find, correct to the nearest penny, the difference between the costs per head in the two schools.

8. A courtyard 20 ft. square is paved with black and white tiles, all 5 in. square, in the following manner: all round the court there is a band $2\frac{1}{2}$ ft. wide of black tiles, and the rest are white. How many tiles are there of each colour?

9. Express 68,796 as a product of prime factors. Hence find the smallest whole number which, multiplied by 68,796, will give a perfect square, and the smallest whole number which will give a perfect cube.

10. A steam-engine employed in pumping water from a coal-mine is found to consume an amount of coal proportional to the product of the amount in gallons of water pumped up and the depth in feet from which the water is raised. If 112 lb. of coal are burnt in pumping 9,600 gallons of water from a depth of 600 ft., how much coal will be burnt when 12,000 gallons are raised from a depth of 750 ft.?

11. It is required to raise £1500 by means of a rate levied on a district assessed at £12,500. If the expenses of collection are 3 per cent. of the money collected, find, to the nearest farthing, how much in the pound the rate must be.

12. If the price of an article in England is 3*s.*, find, to the nearest 5 cents, what the price of it would be in American money, assuming that 10 dollars exchanges for 41*s.* $1\frac{1}{4}d.$

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13. The water-supply of a district containing 750,000 inhabitants is drawn from a lake whose area is 10,000 acres. Allowing 26 gallons per day for each person, find how many days' supply are represented by a depth of 1 ft. of water in the lake, taking a gallon to contain 277 cu. in.

14. The map of a county is drawn on the scale of 6 in. to the mile. What area on the map will represent a lake 8 acres in extent?

15. The value of the meat imported during a certain year was £51,888,000. Taking the population to be forty-two millions and the meat to be worth 5*d.* per lb., determine the average weight, correct to the nearest lb., of the meat imported for each person.

16. Find, as accurately as the data permit, the weight in lb. of a cubic inch of mercury, being given that a litre of mercury weighs 13·596 Kg., 1 in. = 2·5400 cm., 1 lb. = ·45359 Kg.

17. A contractor undertakes to finish a piece of work in 30 days and engages 15 men. At the end of 16 days only three-eighths of the work is done, and now the men are prevented from working for three days. What is the least number of extra men he must engage in order to finish the work in the specified time?

18. An uncovered cistern is made of a material which is $7\frac{1}{2}$ times as heavy as water. It is one-tenth of an inch thick, and its external dimensions are: length 3 ft., width and height each 2 ft. What is the weight of the cistern?

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19. Find the dimensions of a tank which is 1·28 metres deep, and which holds 3000 litres, the length of the tank being three times the width.

20. A vacant plot of land in a town is in the form of a rectangle with two sides parallel to the street, the other two sides being 80 ft. long If the plot was sold at the rate of £2420 per acre, what was the price per foot frontage to the street?

21. A man's income is half 'earned' and half 'unearned'. On the former he pays an income-tax of 9*d.* in the pound, and on the latter 1*s.* 2*d.* in the pound. His net income after payment of the tax is £1218 13*s.* 4*d.* What is his gross income?

22. From the following table make a similar one showing all these measures of length as decimals of a kilometre

	English miles.
English mile	1·000
English geographical mile	1 150
Kilometre	0 621
Russian verst	0 663
Austrian mile	4·714
Dutch ure	3 458
Norwegian mile	7·021

23. An open rectangular tank is made of cement, walls and base being 1 foot thick. Externally the tank is 13 ft. long, 8 ft. broad, and 5 ft. deep. Find the volume of cement required, and the weight of water which the tank will contain when full. (1 cu. ft. of water weighs 1000 oz.)

24. In a mile race, when A and B run together, A beats B by 16 yds.; when B and C run together, B beats C by 110 yds. When A and C run together, by how much ought A to beat C?

---

25. If income-tax is levied on excess of salary over £160 and the tax on a salary of £370 is £7 17*s.* 6*d*, find the tax on a salary of £380.

26. Find the weight to the nearest lb. of a cubic foot of iron which weighs 7·8 gr. per c.c., taking a metre to be 39·37 in. and a Kg. to be 2·204 lb.

27. If an inch depth of snow lying on the ground gives a pressure of 0·4325 lb. on every square foot, and if a cubic foot of water weighs 62·3 lb., how many inches of snow are equivalent to a fall of 1 inch of rain?

28. Calculate the numerical factor (to three figures) that will reduce the price of a substance in francs per Kg. to shillings per lb., having given that £1 = 25·18 fr. and 1 Kg. = 2·205 lb.

29. A, B, and C have respectively £8000, £4000, and £2000 invested in a business. A and B receive respectively 20 per cent. and 10 per cent. of the total profits for managing the business and the remainder of the profits is divided among the three in proportion to their capital. Out of a profit of £3400 what does each receive?

30. During a year a light railway carried 37,000 first-class passengers at an average fare of 5·6*d*, 220,000 second-class passengers at an average fare of 4·4*d*., and 1,000,000 third-class passengers at an average fare of 3 7*d*. The following year the different classes were abolished, and the fares reduced, with the result that the railway carried 1,500,000 passengers at an average fare of 3·4*d*. Find the percentage increase or decrease in the total amount received.

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31. Find, to the nearest shilling, the cost of filling a tank 37 cm. by 15 cm. by 29 cm. with liquid costing 17*s.* 4*d.* per gallon.

32. An English sovereign weighs 123·3 grains, and contains 91·67 per cent. of pure gold. Find the value of this gold at the rate of £3 17*s.* 9*d.* for 480 grains.

33. A man pays annually in house rent and rates £67 10*s.* The rent is 20 per cent. more than the rateable value of the house, and the rate is 3*s.* in the pound. What rent does he pay?

34. The cyclometer attached to a bicycle registers 1 mile when the machine has gone 1 mile 23 yards. Find what it will mark, correct to the nearest quarter of a mile, when the machine has been ridden 4529 miles.

35. A man whose assets are £1764 owes *A* £250, *B* £5820, *C* £365, and *D* £180. How much in the pound can he pay, and how much ought *A*, *B*, *C*, and *D* respectively to receive?

36. A man who takes seven steps in three seconds walks $6\frac{1}{2}$ Km. in an hour. Find the length of his stride.

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37. What is the average depth of a rainfall in which 7 million litres of water fall on a square Km.?

38. You are given that one metre = 39·371 in.; 1 Kg. = 2·2046 lb. Use these data to find the weight of a cubic foot of water in pounds, correct to the nearest ounce.

39. An open tank whose external measurements are 4 ft., 3 ft., and 2 ft. 9 in. is made of wood 1 in. thick. Calculate its weight if a cubic foot of the wood weighs 36 lb.

40. A builder has two houses which cost him equal amounts; he sells the first to *A* at a profit of 30 per cent., and the second to *B* at a loss of 20 per cent. *A* then buys *B*'s house from him, giving him the same price as he paid for the first house. What profit per cent. did *B* make?

41. A man invests £13,050 in a 3 per cent. stock which stands at 87, and has to pay income-tax at the rate of 1*s.* 2*d.* in the pound on the income derived therefrom. What is the net income

therefrom, and what percentage does he actually earn on the investment?

42. A man living in a town A travels to a town B to buy goods which he can obtain 10 per cent. cheaper in B than in A. If the expenses of the journey are 13*s.* 6*d.*, and he saves 15*s.* altogether, what does he pay for the goods?

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43. Taking a metre to be 3·28 ft., find, to the nearest cm., the distance travelled in one second by a train running at 45 miles an hour.

44. A cistern is 3 ft. long and 2 ft. 8 in. broad. How much will the level of the water sink when 10 gallons are drawn off? (1 gallon = $277\frac{1}{4}$ cu. in.)

45. A stone trough consists of a rectangular block of granite roughly hollowed out. The external dimensions are 3 ft. 2 in. long, 2 ft. 3 in. broad, and 1 ft. 5 in. deep. When filled to the brim, it holds 31 gallons of water. The empty stone trough weighs 866 lb. Given that 1 gallon of water weighs 10 lb. and 1 cu. ft. of water weighs 1000 oz., find how many times granite is as heavy as water. (Answer to two significant figures.)

46. A greengrocer sells half his stock of strawberries at a gain of 60 per cent., a third at a gain of 25 per cent., and the rest go bad. What percentage of profit does he make altogether?

47. The rainfall in a certain district is 5·76 cm. How many litres per hectare is this?

Taking an inch as $2\frac{1}{2}$ cm., and a cubic foot of water as weighing $62\frac{1}{2}$ lb., how many tons per acre is this? Give your answer to the nearest ten tons.

48. Answer the following questions with as little work as possible, stating your method: (*a*) One train goes 30 miles in $52\frac{1}{2}$ min., a second goes 58 miles in 96 min.; which is the faster? (A graphical solution may be given.) (*b*) Between what limits is the area of a rectangle which is from 12 to 14 cm. long, and from 31 to 32 cm. broad? (*c*) Show that the square root of 156 is less than $12\frac{1}{2}$.

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49. Given that 1 ft. = 30·48 cm., find, correct to the nearest foot, the side of a square field of area 15,600 square metres.

50. A tank measuring 8 in. by 11 in. by 17 in. is to be filled with a liquid which is sold in France at 13·7 fr. per litre. Find the cost of the liquid to the nearest shilling. (£1 = 25 fr. ; 1 in. = 2·54 cm.)

51. The average depth of a pond of area 3 acres is 7 in. If a cubic foot of water weighs $62\frac{1}{2}$ lb., find the weight (in tons) of water contained in the pond.

52. On a rectangular plot of ground three tennis courts are marked out each 78 ft. long and 36 ft. wide. The courts are side by side with their longer sides parallel, and there is a distance of 10 ft. between adjacent pairs of courts. The long sides of the two outer courts are 10 ft. from the sides of the ground, and the short sides of the courts are 20 ft. from the ends of the ground. Find the dimensions of the ground and give its area to the nearest square pole. Draw a sketch of the ground showing the courts, and mark the dimensions on your sketch.

53. A tradesman sells three-quarters of his stock of a certain article at a gain of 30 per cent., and sells off the remainder at a loss of 10 per cent. What percentage of profit does he make altogether?

54. A money-changer will give me 25 francs 5 centimes for £1 and make a small profit on the transaction. He makes exactly the same amount of profit if I give him 20 francs and he gives me 15*s.* 9*d.* in exchange. Find, to the nearest centime, how many francs and centimes are equivalent to £1.

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55. One metre being equal to 39·3708 in., find the percentage error made in assuming 1 Km. to be equal to $\frac{5}{8}$ mile.

56. The resistance of a sample of Leicestershire granite to crushing is found to be 5·740 tons per square inch. The resistance of a Welsh granite is 1·121 tonnes per square centimetre. Find which is the harder, and express as a decimal the ratio of its resisting power to that of the other to as many significant figures as the data justify.

57. A rectangular table top is measured to the nearest tenth of an inch and is found to be 47·4 in. by 72·6 in. Show that its area may be anything between 3435·2 and 3447·2 sq. in.

58. An alloy of gold and silver weighs 60 oz., of which 60 per cent. is gold. If 10 oz. of gold be added, how much silver must also be added so that the gold in the resulting alloy may be reduced to 50 per cent.?

59. A dealer buys two horses and sells one of them for £45, making on this one a profit of 35 per cent. He sells the second horse at a loss of 20 per cent. On the whole he neither gains nor loses. How much did each horse cost?

60. Write down an expression for the amount of £x at r per cent. compound interest for t years.

Find, to the nearest penny, the amount of £453 for 3 years at $2\frac{1}{2}$ per cent. compound interest.

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61. A rectangular tank contains 350 gallons. Its length is 6 ft., and breadth 3 ft. 6 in.; find its depth, correct to within one-tenth of an inch, being given that 1 gallon = 277·174 cu. in.

62. An acre of land in England costs £92 15*s.*, and a hectare of land in France costs 2150 fr. Express the cost of land in France as a percentage of the cost in England.

(£1 = 25 fr.; 1 metre = 1·0936 yds.; 1 Ha. = 10^4 sq. metres.)

63. In one year we imported 23 million quarters of wheat and wheat-flour, 12 per cent of which came from our colonies. The duty on tea amounted to £6,300,000. Taking these figures, what percentage of decrease in the duty on tea would be equivalent to a duty of 2*s.* a quarter on *foreign* (as distinct from colonial) wheat and wheat-flour? Answer to the nearest whole number.

64. Find, correct to the nearest ton, what weight of soil will be required to cover a circular field, whose diameter is 87 yds., to a depth of $2\frac{1}{2}$ ft. The soil weighs 97 lb. per cu. ft.

65. Find the volume of a cube which shall have the same total surface as a rectangular block whose dimensions are 7 in. by 8 in. by 9 in.

66. The owner of a taxi-cab can make 8*d.* per mile run in fares, but estimates that the expenses of the cab per mile run are proportional to the rate at which the cab is running, and are $2\frac{1}{2}d.$ per mile when running at 10 miles per hour. At what rate should the cab be run so as to make a profit at the rate of 4*s.* per hour whilst running?

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67. A town has a population of 400,000, and a water-supply of 30 gallons per head per day is required. A reservoir is provided to hold six months' supply with a mean depth of 20 ft. What must the area of the reservoir be?

Find the area of the catchment basin, if 20 in. of rainfall in the year will yield sufficient water to supply the town.

68. The earth's circumference, measured along a meridian, is 40 million metres, or 21,600 nautical miles. Assuming that a metre = 39·370 in., find the number of feet in a nautical mile.

69. A German workman earning £2 a week has to pay per annum, income-tax 36*s.*, church tax 9*s.*, and town tax 45*s.* How much, to the nearest farthing, does the total of these taxes amount to per pound of his income? The year may be taken as 52 weeks.

70. A tradesman has been selling an article at a profit of 45 per cent. The cost of the article to him is reduced 1*d.* and he reduces the price at which he sells it by 1*d.*, and now finds that his profit is increased to 50 per cent. At what price is he now selling it?

71. A cubic foot of water weighs approximately 1000 oz. An inch = 2·54 cm. From these data calculate the number of grammes in 1 lb.

72. A cyclist starts from A at noon on a journey of 60 miles to B; he travels steadily at 12 miles an hour. At 2 o'clock a motor starts from B to meet him and travels at 20 miles an hour. When and where do they meet?

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73. If the manufacturer makes a profit of 10 per cent., the middleman of 8 per cent., and the shopkeeper of 10 per cent.,

find the cost to the manufacturer of goods which the shopkeeper sells for £54 9*s*.

74. Equal masses of two alloys each consisting of the same four metals in the proportions 1 : 3 : 5 : 7 and 2 : 3 : 4 : 9 respectively are mixed together so as to form a single alloy. Find the proportions of the metals in the new alloy.

75. A sheet of metal weighs $4\frac{1}{2}$ lb. to the square foot. How many Kg. to the square metre is this? (1 metre = 39·370 in.; 1 gramme = 0·0353 oz.)

76. Assuming that £100 will accumulate in 7 years at 5 per cent. compound interest to £140 14*s*. 2*d*., find the amount to which it will accumulate in 14 years.

77. The population of a town is 22,491. Two years ago the population was 20,400. Assuming that the annual increase in any year is proportional to the population at the beginning of that year, calculate the estimated population 3 years hence.

78. A and B travel from a town X to a town Y, a distance of $10\frac{1}{4}$ miles. They have a bicycle between them, which they can ride at 7 and 10 miles an hour respectively, their walking speeds being $3\frac{1}{2}$ and 4 miles an hour respectively. They start together from X, B walking, A on the bicycle. On arriving at a point P, A leaves his bicycle and walks on. As he is walking into Y he is overtaken by B, who has picked up the bicycle and ridden from P. Find the distances of P from X and Y.

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79. A wine-merchant buys in Bordeaux a cask of claret, and the freight to London is £1. It contains 208 litres of wine at 75 c. per litre, and for the cask itself he has to pay 12 fr. He has to pay duty 1*s*. 3*d*. per gallon and bottles the wine at a cost of 2*s*. a dozen bottles. If he sells it at 18*s*. a dozen, what percentage of this price is profit? (£1 = 25 fr.; 6 bottles = 1 gallon; 1 litre = ·220 of a gallon.)

80. A train 88 yds. long enters a tunnel 6 fur. long. It is noticed that exactly one minute elapses from the moment the engine enters the tunnel to the time when the last carriage leaves the tunnel. What is the speed of the train in miles per hour?

81. The $2\frac{1}{2}$ per cent. Consols stand at $82\frac{1}{4}$, and a certain $4\frac{1}{2}$ per cent. stock stands at $91\frac{1}{2}$. A man invests £1000 in Consols, and buys £1000 of the other stock. What is his total income from the two investments? Express his total income as a percentage of the total sum invested.

82. The population of a country increases by 14·5 per cent., while the expenditure to be met by taxation increases 23·2 per cent. Find, to the nearest integer, the increase per cent. in the average taxation per head of the population.

83. Brass is made by melting together copper and zinc, and the mixture shrinks 3 per cent. in the process. One cubic centimetre of copper weighs 8·6 grammes, and of zinc 7·1 grammes. 64 per cent., by weight, of the brass is copper. Find (i) the weight of 1 cubic centimetre of brass, (ii) what percentage of the volume is copper.

84. What sum of money must be invested at 4 per cent. per annum compound interest (interest payable yearly), so as to gain £37 5*s.* 6*d.* as interest in three years?

85. In 1904 the weight and value of the fish landed on the coasts of the United Kingdom (excluding shellfish) were as follows:

	Weight in tons.	Value in £.
England and Wales . .	367,271	6,481,605
Scotland	397,339	2,230,484
Ireland	46,263	311,675

Find what percentage of the total weight and what percentage of the total value of fish were landed in Scotland. Give each percentage to the nearest integer.

86. A pedestrian and a car started together from A to go to B, $5\frac{1}{2}$ miles distant. On reaching B, the car immediately returned to A. On reaching A, the car immediately returned to B. It *overtook* the pedestrian $\frac{1}{2}\frac{1}{2}$ mile from B. Assuming that the pedestrian and the car travelled at uniform rates and along the same road, find how far the pedestrian was from B when the car *met* him.

87. In consequence of migration into towns the population of a rural district is diminished each year by 4 per cent. of its number at the beginning of the year. To what number is such a population of 945,126 reduced in 5 years?

88. The sp. gr. of an alloy of gold and silver is 13·33; assuming that the sp. gr. of gold is 19·6, and that of silver 10·2, and that the volume of the alloy is equal to the sum of the separate volumes of the gold and the silver contained in it, find the percentages by weight of gold and silver in the alloy.

89. At first it was calculated that a bankrupt's assets were 77 per cent. of his liabilities, but later and more correct calculations showed that the percentage was $62\frac{1}{2}$. By how much per cent were his assets over-estimated at first, if his debts were under-estimated then by $10\frac{5}{7}$ per cent.?

90. A tenant has a lease of a house which has three more years to run. The rent is £150, paid at the end of the year, but in consideration of a certain sum of money down the landlord reduces the rent to £125 for the remainder of the term. Assuming that the landlord can obtain 3 per cent. interest on his money, what sum should the tenant pay?

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91. Certain new records for a motor-cycle were recently established, as shown below:

	New Record.	Old Record.
50 miles . .	45 min. $31\frac{1}{5}$ sec	47 min. $24\frac{3}{5}$ sec
One hour	65 miles 803 yds.	63 miles 194 yds
100 miles . .	1 hour 34 min. 8 sec.	1 hour 37 min. $52\frac{2}{5}$ sec
Two hours	122 miles 210 yds.	118 miles 1367 yds

Calculate, to two significant figures, the percentage by which the old record was in each case increased or diminished.

92. An express train 332 ft. long, and going at 62 miles an hour, overtakes a goods train 526 ft. long running in the same direction on a parallel line at 17 miles an hour. Find how many seconds the trains take to pass each other.

93. A tradesman marks certain goods 30 per cent. above cost price; he allows cash customers a discount of 12 per cent., and credit customers a discount of 5 per cent.; five-eighths of his sales are for cash and the rest on credit. When the whole stock has been sold and paid for, what percentage is his return over the cost price?

94. A journey of 100 miles is divided into five equal stages of 20 miles each; the first stage is covered at 8 miles an hour, the second at 12, the third at 15, the fourth at 16, and the fifth at 18 miles per hour. Find the average speed for the whole journey.

95. How much money must be invested so that, with compound interest at 5 per cent. per annum, it may amount to £1000 at the end of three years?

96. A man holds £1600 of a 3 per cent stock at 93. He sells out, the expenses of the sale amounting to 3*s.* 6*d.* on every £100 of stock, lends £1100 of the proceeds on a mortgage at 4 per cent., and leaves the remainder on deposit with his bankers at $2\frac{1}{2}$ per cent. What is the alteration in his income?

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97. In the year 1905–6 the amounts of revenue derived by local authorities from rates, from exchequer grants, and from other sources were £58.256,000, £19,850,000, and £3,561,200 respectively. Find, to the nearest tenth, the percentage each is of the total revenue.

98. A man buys a house for £1150. There is a ground rent of £25 a year, and annual repairs come, on an average, to $1\frac{1}{2}$ per cent on the purchase price of the house. At what rent must he let the house to clear 8 per cent. per annum on the purchase price?

99. Find the rate per cent. per annum at which a sum of money will double itself in 16 years if lent at compound interest.

100. A goods train passes a railway bridge at 2 o'clock, at a speed of 31 miles an hour. Twenty-five minutes later a passenger train passes on parallel rails in the same direction at a speed of 56 miles an hour. At what time will the goods train be overtaken?

101. The 11.20 train from Paddington is due at Newport, a distance of 133½ miles, at 1 53 p.m. If the average rate of the train during the first two-thirds of the distance be 58 miles an hour, by how much must the engine-driver reduce the speed during the remaining part of the journey if the train is to arrive precisely to time?

102. The area of the surface of a lake is 3 acres. Assuming that the volume of a gallon is ·16 cu. ft., find the number of gallons of water required to raise the level of the surface of the lake a quarter of an inch.

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103. Two persons, *A* and *B*, carry on business in partnership, in which *A* provides the entire capital, £8750 The profits are divided in the following manner: *B*, first of all, receives £200 a year for his services as manager; *A* then recives 4 per cent. on his capital; and finally the residue of the profit is divided equally. If in one year *A* receives in all 10 per cent. on his capital, find the total amount of the profits.

104. In a certain year Italy produced 62,185,000 hectolitres of wheat on 5,137,000 hectares; in the same year Great Britain produced 59,092,000 bushels of wheat on 1,755,696 acres. Give the yield in bushels per acre, to the nearest integer, for each country, and express the total yield of wheat of Great Britain as a percentage of that of Italy.

105. Some boys, having rested, returned to school, distant 4 miles, at the rate of 3 miles an hour, with the exception of one of them who, 15 minutes after they had started, went back to the resting-place to look for his knife. At what average rate did he travel alone if his successful search at the resting-place occupied 8 minutes and he just reached school with the others?

106. A policeman stated that a motor-car took 31 seconds to travel a quarter-mile. Assuming that he may have been 2 seconds wrong either way, calculate the limits between which the velocity of the car must lie, in miles per hour, correct to the nearest mile.

107. If a cubic foot of iron weighs 480 lb., find the weight of a spherical bomb, of external diameter 8 in. and thickness $\frac{1}{2}$ in.

108. If, at a certain rate of interest, £1 amounts at the end of one year to £a, find the amount of £P at the end of x years, reckoning compound interest at the same rate and payable annually. Use this result to calculate the compound interest on £750 for 9 years at $3\frac{3}{4}$ per cent. per annum. (See table, p. 589.)

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109. The gross receipts of a company are divided as follows: 49 per cent. to provide a dividend of $3\frac{1}{2}$ per cent., 44 per cent. to pay working expenses, and the remainder, £4000, is reserved. Find the capital of the company.

110. The perimeter of a rectangular table is 200 in.; its area is halved when a strip 6 in. wide is cut off all round. Find the dimensions of the original table.

111. n fish weigh m lb. between them. Of these b average c lb. each. What is the average weight of the remainder?

112. Assuming that the square of the speed of a falling body is proportional to the height fallen through, and that after falling through a height of 1 ft. the speed is 8·025 ft. per second, find what the speed will be after falling through 873·4 ft

113. A man has part of his money invested at $3\frac{1}{2}$ per cent., and part at $2\frac{1}{2}$ per cent. He sells out half of each and reinvests the proceeds at 6 per cent.

If his original and final incomes are £469 and £684 10s., find his total capital and the amount of his original investments.

114. Taking the present worth of £1, due in 5 years at 3 per cent. per annum compound interest, to be £·862609, find, correct to the nearest pound, the present worth of £2474 due in 10 years at 3 per cent. per annum compound interest.

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115. From the following data draw a graph representing the time of sunrise, and estimate the time of rising on Feb. 15th, May 16th, and on the day of earliest rising:

Date :	Jan. 1.	Jan. 30.	Mar 2.	Apr. 1	May 1.	May 31.	June 30.	July 30.
Time .	8h. 8m.	7h. 44m.	6h. 47m.	5h. 39m.	4h. 35m.	3h. 51m.	3h. 48m.	5h. 8m.

116. A dealer buys 80 lb. of tea at 1*s.* 6*d.* per lb.; he mixes it with 20 lb. of another sort, and sells the mixture at 2*s.* 2*d.* per lb., making 30 per cent. profit. How much did he pay for the second sort of tea?

117. A cyclist and a motor-car start at the same time from two places 30 miles apart; the car travels twice as fast as the cyclist. After meeting and resting, each starts back, travelling one mile per hour faster than it did on the outward journey. The cyclist reaches home 5 minutes earlier than the car does. Find the pace of each on the way out.

118. A country is made up of two districts, one having q inhabitants per square mile and the other r inhabitants per square mile. Given that the total population of the country is p, and its area is m square miles, find the areas of the two districts.

119. A owes B £1500, but it is not due for payment till the end of three years from now. How much ought B to be willing to accept now in order to clear off the debt, (*a*) taking money to be worth 4 per cent. per annum simple interest, (*b*) taking it to be worth 4 per cent. per annum compound interest?

120. Find to the nearest Kg. the weight of a solid block of ice in the form of a right circular cone whose height is 45 cm., and the diameter of whose base is 75 cm. The specific gravity of ice is ·92. If by the melting of some of the ice the block is reduced to a smaller cone of half the given dimensions, what part of the original weight of ice will remain?

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121. If the same kind of tobacco is sold in England at $6\frac{1}{2}d.$ per oz. and in France for 13 fr. 50 per Kg., find, to the nearest farthing, by how much the English duty on tobacco exceeds that paid in France. (£1 = 25 fr ; 1 Kg. = 2·205 lb.)

122. A man rows down a river from a place A to a place B and back again without stopping in 2 hrs. 36 min If the speed of the current be $1\frac{1}{4}$ miles per hour and the distance from A to B be 3 miles, find the speed of the man in still water and the times of his two journeys.

123. A piece of gold in the form of a square, of which each side is 3 cm. and the thickness of which is 1 mm., is worth £4 10*s*. What would be the cost of a circular medal of radius 4 cm. and thickness $2\frac{1}{2}$ mm., composed of a material of which 90 per cent. is gold and 10 per cent. an inferior metal whose value is one-eightieth of that of pure gold?

124. The capital of a company consists of 500,000 £1 shares, divided into 200,000 5 per cent. preference shares. 200,000 ordinary shares, and 100,000 deferred shares The profits in a certain year were sufficient, after placing £25,000 to the reserve fund, to pay the preference dividend and to pay $7\frac{1}{2}$ per cent. and 8 per cent. respectively on the ordinary and on the deferred shares. What was the amount of the profits?

125. A railway train is timed to run between two stations in 3 hrs. 18 min. On a certain journey the average speed of the train over half the distance between the stations was 10 per cent. more than the time-table speed, and over the remainder of the distance it was 10 per cent. less. How long did the train take to perform the journey?

126. Each of one pair of opposite sides of a square field is shortened by 77 yds., and each of the other pair of sides is lengthened by 11 yds. If the area of the rectangular field thus obtained be 1·2 acres, find the side of the original square.

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127. Assuming that 1 metre = 39·37 in., 1 gallon = 277·27 cu. in., find, correct to three figures, the number of pints in a litre.

128. Six circular flower beds, each 12 ft. in diameter, are cut in a lawn, and the turf removed is formed into another lawn in the form of a circle, in the centre of which is a circular flower-bed, 6 ft. in diameter. What is the diameter of this lawn?

129. A man who has £3100 3 per cent. stock sells out at 92, and with the proceeds purchases some 5 per cent. stock, thereby increasing his income by £31. Find the price which he paid for the second stock.

130. Two cyclists, *A* and *B*, ride a race. The wheels of *A*'s bicycle are 30 in. in diameter and those of *B*'s bicycle are 27 in.

in diameter. A gives B 100 yds. start, and both arrive at the winning post at the same time, but A's wheel has made exactly 37 turns less than B's wheel. Find the length of the course ridden by each, to the nearest foot.

131. On Jan 1st, 1912, a man had a balance of £137 at his bank, and an income arising from £12,400 stock paying $3\frac{1}{2}$ per cent. During that year he spent £355, and at the end of the year bought another £100 of the stock at the price 115 How much can he afford to spend this year, and yet have as large a balance at the bank on Jan. 1st, 1914, as he had on Jan. 1st, 1912?

132. A pair of tantalum lamps, each equivalent to 25 standard candles, cost 25 per cent. more for electric current per hour than a single ordinary lamp equivalent to 16 such candles. Tantalum lamps cost 2*s.* 6*d.* each, ordinary lamps 10*s.* 3*d.* per dozen, and both have to be replaced after burning for 1000 hours, during which time the cost of current for an ordinary lamp is 16*s.*

In a certain room four ordinary lamps are replaced by two pairs of tantalums. Find, to the nearest integer in each case, the percentage increase or decrease effected by the change:

(*a*) In light;

(*b*) In total cost of lamps and current for 1000 hours;

(*c*) In cost per standard candle.

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133. A rectangular oil tank is 2 metres long, 1 m. 60 cm. wide, and 1 m. 30 cm. deep. How many litres of oil can it hold, and how many cylindrical oil drums each 40 cm. in diameter and 60 cm. high could be completely filled from the full tank?

134. How much must a man invest in a 3 per cent. stock at $92\frac{1}{2}$ so as to secure a net annual income of £150, after paying income-tax at a shilling in the pound, brokerage being reckoned at $\frac{1}{8}$ per cent.?

135. A man walks at x miles an hour, and the length of his step is y inches Find an expression for the number of steps which he takes in a minute, and evaluate the expression when $x = 4$ and $y = 31$.

136. Calculate to within about one-thousandth of an inch the difference in length between two chords of a circle whose radius is 12 in., one chord being at a distance 6 in. and the other 7 in. from the centre.

137. A person who holds £4865 $3\frac{1}{2}$ per cent. stock sells it at $93\frac{3}{4}$, and invests the proceeds in the purchase of a 3 per cent. stock at $87\frac{1}{2}$ Find the change in his income

138. On a tramway route cars are running both ways every five minutes, at the rate of ten miles an hour. A man travels two miles in one tram-car. How many cars, travelling in the opposite direction, may he meet?

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139. The following table gives the scheduled times of a certain train:

Station *A*	. . . . .	depart 5 35
„ *B* (48 miles from *A*)	. . .	arrive 6 30
„ *B*	. . . .	depart 6 32
„ *C* (90 miles from *A*)	. .	arrive 7 28

Find the average value of the scheduled speed of running for each section of the journey and for the whole journey.

In an actual journey the train leaves *A* 5 min. behind time, arrives at *B* 10 min. behind time, stops 3 min. at *B*, and arrives at *C* 4 min. behind time. Find the average value of the speed of running for each section and for the whole journey, as before.

140. A clock which is right at 10 a.m. on Monday gains 2 seconds an hour from 10 a.m. to 10 p.m. and loses 3 seconds an hour from 10 p.m. to 10 a.m. every day. What is the correct time when the clock indicates 12 noon on the following Friday?

141. The rim of a circle whose diameter is 55 cm. is graduated to intervals of 5 minutes of arc; find the distance, measured along the rim, between consecutive divisions.

142. Assuming that a gallon of water weighs 10 lb. at a temperature of 4° C., find the weight of a gallon of water at boiling-point, given that 1000 cu. in. of water expand to 1043 cu. in. when heated to boiling-point.

143. A man has a certain sum in the 3 per cents., and another in the 5 per cents., the amount of stock held by him in the latter being three times as great as that held by him in the former. His total income is £1500. How much stock has he in each, and what would his holdings realize in all if the selling price of the 3 per cent. stock be 95, and that of the 5 per cent. stock 120?

144. A tax is imposed on a certain commodity, and the amount of the tax is the same on every pound of the commodity, irrespective of quality. This tax increases the price per lb. of one quality of the commodity by 20 per cent., and the price per lb. of another quality of the commodity by 50 per cent. Compare the prices of the two qualities when the tax is included?

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145. By selling articles at the rate of a articles for b pence, a dealer makes a profit of c per cent. What would be his percentage profit if he sold them at the rate of b articles for a pence?

146. The claims against a bankrupt company are £5000 claimed by the bank for an over-draft, £4000 claimed by A, and £11000 by other creditors. Moreover, A has guaranteed the bank against loss. The available assets are £4000. The case is taken into court, where A's claim against the company is reduced to £2000, and the other claims are allowed in full. How much more is A out of pocket after discharging his guarantee than he would have been if the court had not reduced his claim?

147. Given that a quadrant of the earth is 10 million metres, and that the earth's radius is 3956 miles, find the value of the metre in English measure.

148. Construct a quadrilateral $ABCD$ in which $AB = 2$ in., $BC = 3{\cdot}5$ in., $CD = 3$ in., $DA = 2{\cdot}8$ in., and the diagonal $AC = 4$ in. Find the area of the quadrilateral.

149. On a circle railway $16\frac{1}{2}$ miles long the stations are $\frac{3}{4}$ mile apart. Each train stops 20 seconds at a station and runs at an average speed of 20 miles per hour between the stations. If the distances between the leading carriages of two successive trains is never to be less than $\frac{1}{2}$ mile, find the greatest

number of trains that can pass through a station in an hour, and the number of trains required to maintain this continuous maximum service round the circle.

150. A steamer leaves New York for Liverpool at noon on Saturday (New York time), and travels at an average speed of 28 miles per hour. Another steamer leaves Liverpool for New York at 3 p.m. (Greenwich time) on Saturday, and travels at an average speed of 24 miles per hour. The distance is 3045 miles, and it is 4.45 p.m. at Liverpool when it is mid-day at New York. When and where do the steamers meet?

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151. I wish to fix the size of a circular enclosure which I may not enter; I therefore note the following points: It is surrounded by a footpath 10 ft. wide, and four lamp-posts stand along the outer boundary of the footpath at the corners of a square. Each side of this square cuts from the enclosure a strip whose greatest width is equal to the width of the path. Find the diameter of the enclosure.

152. A man sold for £2 one hundred pounds of a substance which he bought at 1 fr. per Kg. Taking £1 = 25 fr. 30 c., and 1 Kg. = 2·205 lb., find his gain per cent.

153. The sides of a cube all increase from 3 in. to 3·002 in. Show that the increase of volume is very nearly 2 per cent.

154. The distance between two places is 67 miles by rail and 76 miles by road. A train which leaves one place at 1.7 p.m. arrives at the other at 3.41 p.m., having spent 12 minutes on stoppages during the journey. Find, to the nearest mile per hour, the average speed of the train whilst running, and the average speed at which a motor-car must travel to go from one place to the other in as short a time as the train.

155. A bankrupt has $m+n$ creditors. To m of these he owes £p each and to the remaining n he owes £q each. If all his creditors had an equal claim, they would each receive £r. How much does each of the m, and how much does each of the n actually receive?

156. At noon on Monday, one clock, which loses 10 minutes a day, is an hour fast, and a second clock, which gains a quarter of an hour a day, is 20 minutes slow. When will they both indicate the same time?

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157. In order to determine the volume of a cube and the area of one face, the edge of the cube is measured and found to be 5·6 cm. The measurement may be as much as 0·1 cm. too great or too small. Find the greatest error that might arise in (*a*) the area of a face, (*b*) the volume. In view of this possible error, to what degree of accuracy should (*a*) the area, (*b*) the volume, be given?

158. A rail weighs 84 lb. per yard length. Find the cross-sectional area of the rail if a cubic foot of the metal weighs 470 lb. Also find the cost per mile of the rails for a single track (2 rails) at £6 6*s.* 6*d.* per ton.

159. If p dozen oranges worth b pence per score be selected from a case containing q gross oranges of average value 23 for a shilling, what is the average value per dozen of the oranges which are left in the case?

160. The weight of a length of $10\frac{1}{2}$ 'nauts' of round copper wire, 0·025 in. in diameter, is 119·7 lb. The weight of $4\frac{1}{4}$ miles of round copper wire, 0·035 in. in diameter, is 83·41 lb. From this, calculate the length of the 'naut' in terms of the mile.

161. A circular plate of metal of uniform thickness, and 2 ft. in diameter, is pierced by 20 circular rivet holes, all of the same diameter. If by the piercing of the holes the weight of the plate is reduced by $1\frac{1}{4}$ per cent., find the diameter of each hole.

162. A, B, and C eat a quantity of nuts, each eating at a uniform rate; A eats 15 more than B and 27 more than C. If B and C alone had eaten the same number of nuts, B would have eaten 19 more than C. How many nuts did A eat?

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163. Below are tabulated the total values of certain commodities imported during corresponding months of the years 1907 and 1908,

together with the percentage increase or decrease in the quantities imported.

	1907.	1908.	Percentage increase or decrease in quantity as compared with 1907.
Bacon . . .	£1,164,121	£1,236,807	11·0 increase.
Butter . . .	2,384,943	1,997,546	19·9 decrease.
Wheat . . .	2,268,151	3,145,264	5·8 increase.
Oats	273,891	277,755	3·0 decrease.

Calculate to two significant figures for each commodity:

(*a*) The percentage increase or decrease in the total value imported.

(*b*) The total value for 1908, reckoned at 1907 prices.

164. The following formula gives G the number of gallons of water delivered per hour by a pipe of diameter D in. and of length L yds., under a head of water H ft.

$$G = \sqrt{\frac{(15D)^5 \times H}{L}}$$

Use it to find the diameter of a pipe two miles long which will deliver 140,000 gallons of water an hour under a head of 30 ft.

165. If a hollow metal cylinder, whose outer radius is 4·3 cm. and inner radius is 1·1 cm., and whose length is 4 cm., is cast into a solid cylinder 12 cm. long, what is the radius of the solid cylinder?

166. From a vessel containing 100 litres of wine 5000 cu. cm. are removed and replaced by water. This operation is repeated three more times. Find, correct to the nearest integer, the number of cu. cm. of wine in the resulting mixture.

167. A ship, 40 miles from the shore, springs a leak which admits $3\frac{3}{4}$ tons of water in 12 minutes. 60 tons would suffice to sink her, but the ship's pumps can throw out 12 tons of water in an hour. Find the average rate of sailing so that she may reach the shore just as she begins to sink.

168. A car is timed to run over a motor track half a mile long in 43 seconds. What is its speed in feet per second? Suppose

there is an error of 10 yds. in the distance, and of $1\frac{1}{2}$ seconds in the time, between what limits must the real speed lie?

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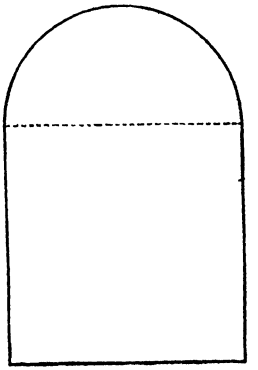
169. The capital of a company is £150,000. In a certain year the expenses were 40 per cent. of the receipts, and out of the profits £2000 was placed to the reserve fund, the remainder of the profits being sufficient to pay a dividend of 5 per cent. on the capital. In the next year the receipts were increased by 20 per cent., and the expenses were 45 per cent. of the receipts How much can be placed to the reserve fund in this year if the dividend for this year is declared at 6 per cent. on the capital?

170. There are two clocks in a building, one of which gains 3 minutes in 4 days, and the other loses 4 minutes in 5 days. They are set right at noon on Sunday. Find the time indicated on each clock, to the nearest second, when they differ by 5 minutes.

171. A steamer takes 18 hours to do the journey up the Danube from Vienna to Linz, and only 8 hours to do the journey down from Linz to Vienna. How long ought a raft floating with the stream to take?

172. An alloy, whose value is £a per lb., is composed of two metals. One of the metals, worth £b per lb., is extracted; this metal was contained in the alloy in the ratio of x to 1. What is the value of the other metal per lb.?

173. Draw on squared paper two circles with radii 3 in. and 2·4 in., and their centres 3·5 in. apart. Find, by counting the squares, the area of the overlapping part of the circles.



Section of Arched passage
Scale 1 to 50

174. The figure is the section across an arched passage with vertical sides and a semicircular roof, drawn to a scale of 1 to 50. The passage is 20 metres long. Measure up the figure and find (1) the cost of

paving the floor at 8*s.* a square metre, and (2) the cost of painting the walls and roof, at a cost of 4*d.* per square metre.

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175. In consequence of the inaccurate length of his chain, a surveyor records the dimensions of a rectangular field as 275 yds. and 188 yds. If the true area is 10·4 acres, give the percentage error in the area, and also in the length of the chain.

176. The coal bunkers of a steamer, when filled with a certain quality of coal, will hold 375 tons; how many tons will they hold when filled with a lighter and inferior coal, if the space required for the better coal is to that required for the other in the ratio 10 to $11\frac{1}{4}$?

Suppose the steamer, when using the better coal, can steam 1,650 miles; how many miles will it steam when the bunkers are filled with the inferior coal, assuming that the steaming properties of the latter are to the former in the ratio 1 to 1·2?

177. In a certain year the world's production of silver would have served to make silver coins of the face value of £59,654,000, at the rate of an ounce troy to 5*s.* $11\frac{1}{2}d.$ An ounce troy being equal to ·0686 lb. avoir., find to the nearest ton the weight of silver produced in that year. The market value of silver in that year having been $24\frac{3}{8}d.$ per ounce troy, find the market value of the year's production to the nearest £1000.

178. The diameter of a circular brass disk, cut out of a sheet 5 mm. thick, is known to lie between 3·48 and 3·52 cm. If its volume is calculated on the assumption that the diameter is 3·5 cm., what is the greatest possible error in the result?

179. The perimeter of the base of a right circular cone measures 301·6 in., and its slant side is 5 ft. Find its volume.

180. A motor-car passing through a town meets every 2 minutes a tram-car going in the opposite direction, and overtakes every 8 minutes one going in the same direction. Assuming that the tram-cars run at uniform rates and at equal intervals of half a

mile from each other, find the speed of the motor-car and of the tram-cars.

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181. A rectangular plot intended to be 62 ft. long by 44 ft. wide was measured out with a yard measure which had half an inch broken off the end, so that the graduations only read up to $35\frac{1}{2}$ in. The error was not noticed. Find the deficiency in the area.

182. An article which costs 7*s.* 6*d.* in London sells for 4 dollars in New York. Find, to the nearest penny, the profit made by a man who buys 1000 of these articles in London and sells them in New York after paying 5*d.* per article for freight, &c., and a customs duty of 20 per cent. on the New York selling price. (£1 = $4.85.)

183. A man takes a boat and rows upstream. When he has gone $1\frac{1}{2}$ miles upstream he passes a bottle floating down with the stream. He goes on up the river for another 18 minutes and then turns. When he gets back to the starting-place he finds that the bottle has just arrived there. Assuming that his speed through the water and the speed of the stream are both uniform, find the speed of the stream in miles per hour.

184. If the price of gold be £140 a Kg., find the approximate value of a hollow cylinder of gold, with open ends, whose height is 1 in., and outer and inner diameters $\frac{3}{5}$ in. and $\frac{1}{2}$ in. respectively. The sp. gr. of gold is 19·25, and 1 metre is equal to 39·37 in.

185. A sphere of wax 6 in. in diameter is melted, and part is poured into a cylindrical jar of 3 in. diameter until it is filled to the height of 10 in. The remainder is poured into a cylindrical jar of 4 in. diameter. Find the height to which the latter jar will be filled.

186. An electric light company has the following tariff for private consumers: 6*d.* per Board of Trade unit for the first 26 such units consumed in any quarter, and 2*d.* per unit for all in excess of that number.

Draw a diagram from which may be read the cost per quarter for any number of units up to 250.

The readings of a consumer's meter on certain dates are tabulated below:

Date.	Meter Reading (B of T units).
31st December, 1911	757
31st March, 1912	989
30th June, 1912	1087
30th September, 1912	1158
31st December, 1912	1407

Find from your diagram the cost to this consumer for each of the quarters for which data are given

187. The following table exhibits the cotton yarn spun, and cotton woven piece goods produced in British India and Native States in April, 1907, 1908, and 1909:

		April, 1907	April, 1908	April, 1909.
Cotton yarn spun	Weight in lb	51,347,484	52,863,095	58,533,364
Grey and bleached piece goods.	Weight in lb	12,898,781	13,216,937	13,912,840
	Length in yds.	56,507,157	59,002,972	60,703,100
Coloured piece goods.	Weight in lb	1,836,484	1,815,468	2,383,538
	Length in yds	7,610,211	7,384,852	9,618,173

Estimate (*a*) the total weight of woven piece goods produced in April, 1909, as a percentage (to the nearest tenth) of the weight of yarn spun in April, 1909. (*b*) The average number of yards to the pound in (i) grey and bleached piece goods, (ii) coloured piece goods, each calculated to the nearest tenth from the above data for April, 1909. (*c*) The increase per cent. (to the nearest tenth) in the total weight of cotton goods produced in April, 1908, as compared with April, 1907, and of those produced in April, 1909, as compared with April, 1908

188. A rectangular tank with a sloping bottom is 56 in. long and 44 in. broad. The ends are rectangular, one of them 60 in. deep and the other 34 in. deep. The tank is made of sheet iron. Find the area of sheet iron used, and the capacity of the tank in gallons.

189. A sum of £1000 is lent on condition that it is repaid in two equal instalments at the ends of the second and third years, together with compound interest at 3 per cent. per annum. What is the amount of each instalment?

190. A yard stick is 0·14 in. too short. If it is used to obtain the area of a floor, how much per cent. will the area thus obtained exceed the true area?

191. Tram-cars run at uniform speed and at equal intervals along a certain route. As I walk in the same direction, a car passes me every 5 minutes. If I walk $\frac{1}{2}$ mile an hour faster, 5 cars pass me in 27 minutes. If I stand still, 8 cars pass me in 37 minutes. Find the speed of the cars in miles per hour, and the intervals between them in feet.

192. If an object h ft. above the sea-level is just visible above the sea horizon to a person whose eye is H ft. above the sea-level, the distance of the object from the observer is $\frac{3}{4}(\sqrt{h}+\sqrt{H})$ miles. Hence find the rate at which a steamer is moving directly from a person whose eye is 10 ft. above the sea, it being noted that the top of the mast, 50 ft. high, disappears 15 minutes after the hull, the top of which is 12 ft. above the sea-level. Also find the length of time after leaving land before the steamer disappears.

193. A lends B a certain sum, at the same time he insures B's life for £737 12*s.* 6*d.*, paying annual premiums of £20. At the end of 3 years, and just before the fourth premium is to be paid, B dies, having repaid nothing. What must A have lent B in order that he may have just enough to recoup himself together with 5 per cent. per annum compound interest on the sum lent and on the premiums?

194. A cubic foot of copper weighs 556 lb., and a cubic foot of zinc 442 lb.; and 64 parts, by weight, of copper melted with 36 parts of zinc form brass, a cubic foot of which weighs 517 lb. Find how much per cent. the volume has shrunk in making the alloy.

195. The greatest weight which can safely be hung on a steel rope of diameter d in. is $2{\cdot}63d^2$ tons. Find the greatest weights which can be hung on ropes of diameters 0·5 in., 1 in., 1·5 in., 2 in., 2·5 in., 3·5 in., 4 in. respectively; and from your results draw an accurate graph showing the weight in terms of the diameter.

Use your graph to find

(i) Whether 6·5 tons can safely be hung from a rope of 17 in. diameter.

(ii) The greatest weight which a rope of diameter 2·9 in. can carry.

(iii) The smallest rope which can carry a load of 25 tons.

196. Find the cost of the canvas required to make a conical tent 12 ft. in diameter (at the base), and 8 ft. high, at 2*s.* per square yard.

197. A man at the age of 40 pays a lump sum to an insurance company to secure an annuity of £100 for his old age, the first £100 to be paid when he is 60, the next when he is 61, and so on. How much should he pay if the calculation is made at 3 per cent. per annum compound interest, added half-yearly, and on the assumption that he will probably receive six payments of £100?

198. A cylinder, graduated to show cu. cm., contains some water, and the reading is 125 cu. cm. On a spherical metal ball being immersed in the water, the reading is 255·8 cu. cm. Find the diameter of the ball.

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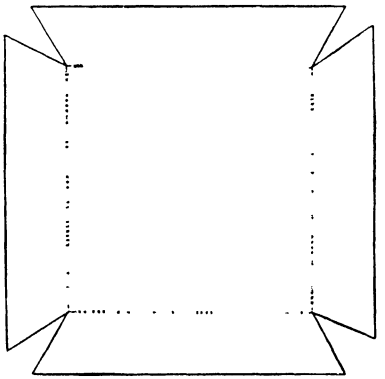
199. There is a row of 21 trees planted at intervals of one year. The youngest tree is 10 ft. high, and the middle tree is 25·9 ft. high. Assuming that during each year of growth a tree adds to its height a constant fraction of its height at the beginning of that year, find the heights of the oldest tree and of the third tree. (Use logarithms.)

200. Plot on squared paper the points whose coordinates are given in the following table, and draw a curve through the points:

x	1	1·5	2	2·5	3	4
y	4	2·66	2	1·6	1·33	1

By counting squares estimate the area contained between the curve, the x-axis, and two perpendiculars to the x-axis through the points (1, 4) and (4, 1).

201. A cardboard tray, with a square base and sloping sides, is to be made by cutting from a sheet of cardboard a piece shaped somewhat as in the figure, and folding along the dotted lines to make the sides of the tray. The dimensions are to be: edge of base 4 in., edge of top of tray $5\frac{1}{4}$ in., vertical depth 1 in. Find the breadth of the strip that forms a side of the tray. Draw an accurate full-size figure of the piece of cardboard cut out ready for folding, and explain your method.



202. A man owns a house whose rent is £100 a year. He lets it for a period of 10 years, arranging to receive a lump sum at once instead of the yearly rent. How much ought he to receive, money being worth 3 per cent. compound interest?

203. A spherical segment is 1 metre high, and the radius of its base is 2 metres. This segment is cut in two by a plane parallel to its base and 50 cm. from it. Find the radius of the sphere and the area of the section.

204. It is desired to make with the least amount of sheet metal a closed cylindrical tin to contain exactly 1000 cu. cm. of water. Plot a curve which will show the relation between the amount of metal in sq. cm. and the diameter of the tin, and use this curve to find the required diameter.

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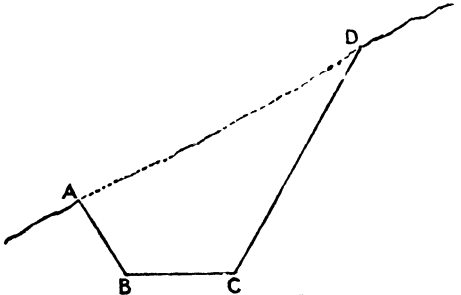
205. The figure shows, on a scale of 1 to 900, a section of the balloon of an airship in the shape of a cylinder with hemispherical



Section of Airship
Scale 1 to 900

ends. The weight it can lift is the difference between the weight of the hydrogen with which it is filled and the weight of an equal volume of air. Find the weight it can lift, if 1 cu. metre of air weighs 1229 gm. and 1 cu. metre of hydrogen weighs 85 gm.

206. Find the volume of a sphere inscribed in a right circular cone whose height is 5 in., and the diameter of whose base is 10 in.



Section of Railway Cutting
Scale 1 in 400

207. The figure shows the section of a railway cutting on the scale of 1 in 400, the dotted line indicating the original outline of the

ground. Find the area of the section $ABCD$ by pricking it off on to squared paper.

Hence find the number of tons of earth that have to be moved to make such a cutting a quarter of a mile long, taking the weight of a cubic yard of earth to be 38 cwt.

208. A lightship is 3 miles from a straight shore, and it is desired to lay a telegraph line from it to a place on the shore 15 miles distant from the ship. Submarine cable costs £60 per mile, and a wire on land £40 per mile. Find the cost of the line if x miles consists of cable and the rest is laid along the shore. Hence, by drawing a graph, find the cheapest way of laying the line.

209. Find to within 1 mm. the length and the breadth of a rectangular table whose area is 5·5 square metres and which is twice as long as it is wide.

210. The following table gives the acreage under wheat in Great Britain in the four years 1905–8, and the estimated total produce in bushels for each year:

	1905.	1906.	1907.	1908.
Acreage under wheat . .	1,796,995	1,755,696	1,625,445	1,626,733
Estimated total produce .	58,902,499	59,091,772	55,206,195	52,535,139

Find, to two decimal places, the average yield in bushels per acre for each year, and for the whole period.

211. Assuming that $976547 \times 979397 = 956427202159$, deduce the value of 976547×984397, and hence that of 979577×984397.

212. The cost of travelling third class on the Swiss railways is 5·2 centimes per kilometre, and luggage is charged for at the rate of 5 centimes per 100 kilograms per kilometre. Find the greatest number of lbs. of luggage a traveller can take with him so that his total expenditure shall not exceed 1*d* per mile.

1 kilometre = 6214 mile. 1 kilogram = 2·205 lb. 25 francs = £1.

213. Fill up the blanks in the following table:

£	s	d	£	s.	d.	£	s.	d.	Total. £	s.	d.
64	6	9	311	3	7	719	15	2			
34	7	4	560	7	7	2671	13	3			
26	2	9	986	2	11	26	18	10			
73	19	10	619	17	11	4377	11	11			
4	3	4	675	5	4	3612	7	6			
87	9	5	598	13	5	2320	9	6			
26	18	11	240	14	8	590	18	11			
24	8	10	207	6	11	3703	5	8			
32	4	2	281	1	10	5984	6	8			
64	2	1	804	2	3	176	6	1			
47	1	11	160	11	6	228	10	9			
82	17	4	339	11	6	273	3	3			
69	15	7	716	9	8	649	14	5			
33	9	1	81	12	3	9184	14	2			
24	0	3	970	1	4	3984	13	11			
79	11	4	278	15	8	7490	8	5			
22	14	8	584	2	2	4948	8	11			
5	12	2	241	13	10	3621	7	5			
9	13	5	630	5	2	25	16	6			
77	8	10	381	3	6	1057	14	4			
40	3	2	472	6	2	471	12	9			
28	2	8	488	2	1	803	11	6			
13	9	2	657	8	10	583	3	5			
85	15	11	125	9	9	7175	17	7			
58	0	1	614	3	7	1234	15	6			
30	8	5	647	6	2	2935	8	7			
21	17	3	232	1	11	9023	8	5			
40	5	8	226	13	7	72	2	5			
54	18	9	472	10	2	488	6	6			
33	9	5	295	18	4	7124	5	9			

214. The table below gives the times of morning high tide on various days of June, 1912, at London Bridge:

Day . .	13	15	17	19	21	23	25
	h. m.	h. m.	h. m.	h. m.	h. m.	h. m.	h. m.
Time . .	12 0	1 22	2 59	4 43	6 31	8 24	10 30

Draw a graph to show the times of high tide, taking three small divisions to represent a day on one scale, and one small division to represent 20 minutes on the other. Read off from your graph and record the times of morning high tide on 14th, 16th, 18th, 20th, 22nd, and 24th of June.

215. When a circular sheet of lead of radius 15 in. is allowed to sink in a tank of water which is 6 ft. long and 3 ft. 6 in. wide the level of the water rises 0·2 in. Find the thickness of the lead to the nearest hundredth of an inch.

216. At a certain colliery the earnings per day of the miners in a particular week were as follows:

63	earned	3s. 9d.
255	,,	5s. 3d.
192	,,	6s. 7d.
135	,,	7s. 8d.
18	,,	8s. 2d.

Find the average earnings per day of the whole number.

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217. The total amount of wheat imported into the United Kingdom in the year 1910 was 105,222,638 cwt., and of this amount 48,116,138 cwt. were received from countries of the British Empire. What percentage, to the nearest integer, of the total amount imported was obtained from countries of the British Empire? Find the value, to the nearest million pounds, of the total imports of wheat in the year mentioned at the rate of 35*s.* 6*d.* per quarter, assuming that on the average the wheat weighed 62 lb. per bushel.

218. Find the number of feet of wire 0·04 of an inch in diameter that can be made from a cubic foot of metal. Also find the diameter of a wire of which a length of one mile can be made from a cubic foot of metal.

219. State why the answers to the following questions are obviously wrong:

(1) The cost of carpeting a room 16 ft. long and 15 ft. 3 in. wide is £3 1*s.* 2*d.* Find the cost of carpeting a room 32 ft. long and 30 ft. 6 in. wide with similar carpet. Ans. £6 2*s.* 4*d.*

(2) Find H.C.F. of 8883 and 1117. Ans. 329.

(3) Simplify $\frac{5}{12} \times \frac{7}{8}$ of $3\frac{1}{3} + \frac{1}{5} \times \frac{15}{16}$. Ans. $1\frac{49}{70}$.

(4) Evaluate £3·12476 to the nearest penny. Ans. £3 2*s.* 7*d.*

220. Find the present value of an annuity of £300 payable every year for 3 years, beginning at the end of the first year, at 5 per cent. per annum compound interest.

221. Some land bought at £180 an acre is sold several years later at 3*s.* a square yard. If the land has an area of 6 acres 16 sq. po., find the actual buying and selling prices, and express the latter as a percentage of the former. If the purchase money could have been put out at 20 per cent. compound interest, the interest being added yearly, in how many years would it have yielded so much profit? Give a complete number of years as your answer.

222. Fill up the blanks in the following table:

£	*s.*	*d.*	£	*s.*	*d.*	£	*s.*	*d.*	Total. £	*s*	*d.*
53	12	9	261	6	9	501	17	2	.		
43	7	2	560	7	7	2671	13	3	..		
27	4	11	986	9	6	26	12	10	. ...		
73	13	10	619	17	11	4377	11	9	. .		
8	5	4	675	5	4	3612	5	8	.		
78	9	3	589	16	2	2320	9	6	.		
63	18	7	240	18	8	950	14	10	. ...		
42	8	10	207	8	11	3703	7	7	.. ..		
32	2	2	218	4	10	5894	6	8	...		
46	2	1	804	2	5	176	3	5	.. ...		
47	4	9	106	16	6	228	10	7	. ...		
28	16	4	393	13	2	237	3	3	. .		
59	15	7	716	9	8	469	14	5	. ..		
33	6	4	81	12	3	9184	15	2	.. ..		
27	0	3	790	1	10	3198	13	11	.. .		
79	11	4	278	15	8	4790	8	5	.		
22	15	5	584	2	3	4948	8	11	. ..		
5	14	6	241	13	10	3621	7	5	.. ...		
4	13	5	360	5	2	52	12	8	. . .		
77	10	10	471	12	5	1057	15	5	..		

223. A train that leaves Bristol at 2.20 is due in London at 4.30. It was 52 miles from London at 3.46 and 38 miles distant at 4.4. If this average speed was maintained throughout the whole journey, find (1) how many minutes late the train will arrive in London, and (2) how far Bristol is from London.

224. A racing track for motor-cars is 2,010 metres round. Two cars which were racing on it were observed to pass a particular point at 47 min. 12·4 sec. and 47 min. 48·2 sec. past two, and to pass it next at 48 min. 7·6 sec. and 48 min. 37 sec. past two. Find, to the nearest tenth of a kilometre per hour, the difference between the speeds of the two cars, and say which is the faster.

225. In 1910 the United States of America produced 56,889,000 tons of iron ore, valued at 140,735,000 dollars, and 27,303,000 tons of pig iron, valued at 425,115,000 dollars. A dollar = 100 cents. Find, to the nearest cent, the value of a ton of iron ore and of a ton of pig iron. The State of Minnesota was the chief producer of iron ore, and produced 31,966,000 tons, valued at 78,462,000 dollars. Calculate what percentage by weight of the total production of iron ore came from this state giving the percentage to the nearest whole number.

226. In a newspaper it was recently stated that on a certain area the rainfall of last December worked out at 1640 million tons of water with a volume of about $58\frac{3}{4}$ million cu. ft. Taking the weight of a cubic foot of water as roughly equal to 1000 oz., show that these two numbers cannot both be right. Supposing the area in question to be about 3800 sq. miles, and the rainfall about 6·7 in., determine which of the former numbers was wrong and suggest a simple correction.

227. The number of cubic feet in a parcel of uniform circular section may be calculated from the formula $0{\cdot}0796 \times g \times g \times l$, where g is the number of feet in the girth and l the number of feet in the length of the parcel. The dimensions of three parcels of this shape were as follows:

(1) girth, 43 in.; length, 29 in.;
(2) girth, 41 in.; length, 24 in.;
(3) girth, 53 in.; length, 19 in.

Find which of the three parcels contained the greatest number of cubic feet, and find what this number was to the nearest tenth.

228. A rectangular hop-field is planted in rows. The rows are $7\frac{1}{2}$ ft. apart, and there are 95 rows. The plants in each row are $5\frac{1}{2}$ ft. apart, and there are 120 plants in each row. If the distance between each outside row of plants and the adjacent boundary of the field is 3 ft., and the distance between the end plant of any row and the boundary is 8 ft., find, to the nearest tenth of an acre, the acreage of the field. If every plant requires 3 hop poles, how many poles to the acre are required?

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229. Make a list or table of the integral powers of 2 from index -2 to index $+4$. Use your values to draw a curve the ordinates of which show the logarithms of numbers between 0·3 and 16·0 to base 2, as unit for the numbers 1 cm., and for the logarithms 5 cm. From your curve determine as accurately as you can $\log_2 1{\cdot}7$ and $\log_2 5{\cdot}4$.

230. A person invests £1842 15*s.* in 4 per cents. at $102\frac{3}{8}$; he afterwards sells out at 105 and re-invests at 126 in 5 per cents.; find the change of income.

231. A hollow closed cubical box is made of iron 2 in. thick, the external edge being 4 ft. long; it being given that iron weighs 7·112 times as much as water, and that a cubic foot of water weighs 1000 oz., find in tons the weight of the box.

232. A train travelled via the Severn Tunnel from Newport to Paddington, a distance of $133\frac{3}{4}$ miles, in 2 hrs. 30 min. The time of passing through the tunnel, which is 4 miles 636 yds. long, was 6 min. 35 sec. Taking the journey as a whole, what was the average number of feet the train passed over in one second? What was the average number of feet the train passed over in one second in its passage through the tunnel? Give your answer in each case to the nearest whole number.

233. It is required to find how much per cent. of the cost of copper wire will be the cost of aluminium wire for telegraphic purposes; and the following are the data:

The aluminium wire must have 1·28 times the diameter of the copper wire; and the price of aluminium is £90 per ton against £70 per ton for copper wire.

On the other hand, the weight of a certain length of aluminium wire is

$$D^2 \times 2{\cdot}71 \times T \text{ tons};$$

and the weight of the same length of copper wire is

$$D^2 \times 8{\cdot}95 \times T \text{ tons};$$

where D is the diameter of the wire, and T depends on the length.

234. Find, in British money, to the nearest halfpenny, the value of a yard of cloth worth 31 francs per metre, assuming that a metre contains 39·37 in., and that a sovereign is worth 25 francs 20 centimes.

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235. Find the value of

$$\frac{\sqrt{13}-\sqrt{11}}{\sqrt{13}+\sqrt{11}-6}$$

correct to five significant figures.

236. A can tie up 15 faggots, or cut 10 faggots, in the same time that B can tie up 18 faggots, or cut 9 faggots. If, working together, they have to cut and tie up 684 faggots and wish to do it in the least possible time, how many faggots should be cut by A and B respectively?

237. I buy a house, leasehold, for £1260, and, to pay for it, borrow £500 on mortgage at 5 per cent. and sell out enough 3 per cent. stock at 95 to make up the balance. If the repairs which I have to do amount to $2\frac{1}{4}$ per cent. annually on the purchase price, and the ground rent I have to pay is £18 a year, and I let the house at £100 a year, find how much my yearly income is increased.

238. The height of a mountain is given on a map in metres and I want to turn it into feet. If I reckon 11 metres as equal to 12 yds., I get the answer too small by 20 ft. If I reckon 8 kilometres as equal to 5 miles, I get the answer too big by 46 ft. From these data calculate, to two places of decimals, the number of inches in a metre.

239. The selling price of each one of certain machines is calculated by increasing the cost (of manufacture) at the rate of 15 per cent. and adding thereto a certain sum. When the cost of one is £55 the selling price is £65. What is the selling price of one when the cost is £95; and what is the cost of one when the selling price is £88?

240. How many gallons of tar per mile will be required to give a road 50 ft. wide a coating of tar of an average thickness of 0·07 in.? (A gallon = $277\frac{1}{4}$ cu. in.)

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241. Find the dividend, at the rate of 12*s.* $10\frac{1}{2}$*d.* in the £, on

(*a*) £475 12*s.* 6*d.*;

(*b*) 45 francs 37 centimes;

(*c*) 453 dollars 70 cents.

(1 franc = 100 centimes; 1 dollar = 100 cents.)

242. Genuine milk contains 88·75 per cent. by bulk of water, 2·75 of fat, and 8·5 of non-fatty solids. A purchaser buys 7 gallons of milk at 3*d.* a quart; the milk on being analysed is found to contain 90·84 per cent. of water, 2·24 of fat, the residue being non-fatty solids. Find whether anything besides water has been added to it; and find the sum of which the purchaser has been defrauded.

243. A man sells out £5833 6*s.* 8*d.* $2\frac{1}{2}$ per cent. Consols at 96, and invests the proceeds partly in $4\frac{1}{2}$ per cent. railway debentures at 135, and partly in 3 per cent. Colonial stock at 96; his income is increased by £35 19*s.* 2*d.* Find how much of each kind of stock he bought on re-investment.

244. Compute the value of

$$\frac{1}{\sqrt{3}+1}+\frac{1}{\sqrt{2}-1}$$

correct to two places of decimals.

Compute the sum of the infinite series of terms

$$1+\frac{1}{1}+\frac{1}{1.2}+\frac{1}{1.2.3}+\frac{1}{1.2.3.4}+\ldots$$

to six places of decimals.

245. A business was paying regularly 7 per cent per annum; it was extended by increasing the capital by one-half, and after that it paid regularly 6 per cent. per annum. What was the return per cent. on the new capital?

246. A man invests one-third of his capital in $2\frac{3}{4}$ per cent. stock at 98, one quarter in $3\frac{1}{2}$ per cent. stock at 105, and the remainder in 4 per cent. stock at 110; find the average rate per cent. return on his capital.

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247. A watch loses 3 minutes in 24 hours; it is set right at noon on Monday; what is the correct time when it indicates $1\frac{1}{2}$ minutes past one o'clock on the following Wednesday afternoon?

248. A box, with lid, has to be made out of 5 sq. ft. 62 sq. in. of plank $\frac{1}{2}$ in. thick. If the internal dimensions be as 1 : 3 : 9, find the external dimensions.

249. A person sells out $2\frac{3}{4}$ per cent. Consols at $94\frac{1}{8}$, and by investing the proceeds in shares which pay an annual dividend of £4 per share, raises his income $4\frac{1}{2}$ per cent.; determine the price of each share to the nearest sixty-fourth of a pound.

250. A rectangular piece of paper is measured in inches and in centimetres, with the following results:

Length, 8·73 in.; 22·17 cm.

Breadth, 5·57 in.; 14·15 cm.

Find the area of the paper in square inches and in square centimetres, and hence find the number of square centimetres in a square inch. If the measurements are correct to the nearest figure in the second place of decimals, to how many places may your results be regarded as accurate?

251. A railway engine running at 47 miles an hour picks up water from a trough a quarter-mile long between the rails, at the rate of 177 gallons per minute. Find, using the data below, to the nearest integer the number of (i) pounds, (ii) cubic feet, of water it picks up in the quarter-mile.

(A gallon of water weighs 10 lb., and occupies 0·16057 cu. ft.)

252. An electric tram-line with a single line of rails is $3\frac{1}{4}$ miles long. There are 13 trams available, and it is desired to arrange a 3-minute service of trams. How many crossing-places must be provided, and what must be the speed of the trams?

253. Simplify $\sqrt{\dfrac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}}$,

and calculate it to 3 places of decimals.

254. How far does a roller, width 5 ft. 3 in., have to be pulled in rolling a rectangular field of area $7\frac{1}{4}$ acres? Allow 10 per cent. extra for turning, and assume that the whole field can be covered without one crease overlapping another.

255. Two men were walking, each at 3 miles per hour, in opposite directions at the side of a railway; a train passed one of the men in 5 seconds and the other in 6 seconds. Find the length of the train.

256. In the year 1907 the United States produced 27,781,361 tons of pig iron of the value of 529,958,000 dollars, and 868,996,491 lb. of copper of the value of 173,799,300 dollars. Taking a dollar as 49·32*d.*, find the value per ton of each of the above commodities in English money, to the nearest penny.

257. A grocer calculated that, if he mixed 1 lb. of Orange Pekoe with 2 lb of Souchong, he would get 20 per cent. profit by selling the mixture at 2*s.* a lb.; and if he mixed 1 lb. of Orange Pekoe with 4 lb. of Souchong he would get 25 per cent. profit by selling this mixture at 2*s.* a lb. What was the price of the Orange Pekoe?

258. A piece of string and a sheet of note-paper weigh the same. It is found that the length of the string is 389 cm. and that six sheets of the paper weigh 17 grams. What length of string, to the nearest metre, is there in a ball weighing a quarter of a kilogram?

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259. A man's gross income in 1902 was 20 per cent. greater than in 1901, and in both years £150 of his income was exempt from income-tax, the tax on the rest of his income being 14*d.* in the £ for 1901 and 15*d.* in the £ for 1902. Also the amount of income-tax paid for 1902 exceeded the amount for 1901 by £6 4*s.* 2*d.* What was the man's gross income in 1901?

260. A person dies worth £31,000, leaving his property to his son, his widow, and a cousin, so that the legacy, after payment of duty, to the son shall be twice that to the widow, and 20 times that to the cousin. How much does he leave to each, the duty payable by a son being 1 per cent. on the gross legacy, that by a cousin 10 per cent., while the legacy to the widow is free?

261. A person derives his income from 3 per cent. stock at 76 when the income-tax is 1*s.* in the £; he sells out and re-invests in 4 per cent. stock and finds that with income-tax at 1*s.* 2*d.* in the £ his net income is unaltered. What was the price of the 4 per cent. stock?

262. A table is drawn up showing the various sums of money to which £1 would amount in different times (one year, two years, three years, &c.) at a certain rate per cent. per annum, compound interest, payable yearly. The following amounts appear opposite successive years: £1·124864, £1·169858, £1·216652, £1·266318. Show that there must be a mistake in the table. State what you think the rate of interest is, and why.

263. The exposure of a photograph is proportional to the square of the stop number (n) and inversely proportional to the speed number (p) of the plate. For a certain fast plate $p = 350$, and with a stop $n = 6{\cdot}4$ the exposure is found to be $\frac{1}{50}$ of a second; what would be the proper exposure with a plate $p = 100$, and with a stop $n = 45$?

264. Find to the nearest £ the present value of an annuity of £10 for 15 years to commence 20 years hence, interest being reckoned at 4 per cent. per annum, the annuity and interest being paid quarterly.

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265. What must be the market price of 7 per cent. stock in order that, after deducting the income-tax of 14*d.* in the £ it may yield 7 per cent. interest?

266. Twenty men can make a number of articles in one hour which a machine, tended by one man, can make in ten minutes. Reckoning interest and depreciation at 12 per cent. per annum,

and a man's wages at 33*s.* per week, find the capital value of such a machine to an employer.

267. The geographical mile being a minute of latitude on the surface of the Earth, supposed spherical, prove that the circumference of the Earth is 21,600 geographical miles, and that the surface is over 148 million square geographical miles.

268. One month's working of a mine yielded 5,623 oz. of gold; the estimated net profit was £11,100. Find the percentage of gross return absorbed by working expenses, the market value of gold being £3 17*s.* 9*d.* per ounce.

269. Find, correct to 3 decimal places, the value of

(*a*) $7{\cdot}2351 \times 121{\cdot}0234$, (*b*) $6{\cdot}0251 \div 2{\cdot}0143$.

270. A distillation was made in four operations, and the percentage of alcohol in the portions which came over were 72, 69, 62, and 52, while the weights of these portions were in the ratios 4 : 3 : 2 : 1; what is the percentage of alcohol in the whole liquid that came over?

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271. A man who gives 10 per cent. discount on his listed price and wishes to make 35 per cent. on his outlay buys a motor-car for £500. At what price should he list the car?

272. What sum ought a man to pay for an annuity of £100 a year for 10 years, to commence 5 years hence, the annuity to be paid yearly, and the rate of interest being 4 per cent.?

273. Calculate to 4 significant figures $\dfrac{(\cdot 03972)^{\frac{1}{3}}}{(57{\cdot}14)^{2}} \times 441{\cdot}2$.

274. A square field has an area of 5 acres. A circular running track 2 yds. wide is to be made in the field. Find the greatest length the track can have, if it is measured along the centre of the track. Express the area covered by the track as a decimal of an acre.

275. (*a*) State and prove the rule for the divisibility of any number by 11.

(*b*) Express the number 187 in the scale in which the radix is 5.

276. The whole capital stock of a railway is £5,000,000. There is besides £1,000,000 of debentures which have to receive interest at $4\frac{1}{2}$ per cent. The working expenses come to 0·71 of the gross receipts, which are then found to return a dividend of 9 per cent. on the capital stock. An extension of the line is made, the whole capital stock being raised to £6,000,000, and the gross receipts are found to have increased by 16 per cent., while the working expenses have increased by 12 per cent. Find what dividend can be paid on the stock

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Logarithms of R for Compound Interest

R	log R	R	log R
1·0025	·00108438	1·0325	·01389006
1·005	·00216606	1·035	·01494035
1·0075	·00324505	1·0375	·0159881 1
1·01	·00432137	1·04	·01703334
1·0125	·00539503	1·0425	·01807606
1·015	·00646604	1·045	·01911629
1·0175	·00753442	1·0475	·02015403
1·02	·00860017	1·05	·02118930
1·0225	·00966332	1·0525	·02222210
1·025	·01072387	1·055	·02325246
1·0275	·01178183	1·0575	·02428038
1·03	·01283722	1·06	·02530587

Constants used in Mensuration and their Logarithms

	logarithm		logarithm
$\pi = 3{\cdot}14159265$	0·497150	$1 \div \pi = 0{\cdot}31830989$	$\bar{1}{\cdot}502850$
$\frac{1}{2}\pi = 1{\cdot}57079633$	0 196120	$1 \div 4\pi = 0\ 07957747$	$\bar{2}\ 900790$
$\frac{1}{6}\pi = 0\ 52359878$	$\bar{1}{\cdot}718999$	$\sqrt{6 \div \pi} = 1{\cdot}24070098$	0·093667
$\frac{4}{3}\pi = 4{\cdot}18879020$	0·622089	$\sqrt[3]{3 \div 4\pi} = 0{\cdot}62035049$	$\bar{1}\ 792637$
$\sqrt{\pi} = 1{\cdot}77245385$	0·248575	$\sqrt{1 \div \pi} = 0{\cdot}56418958$	$\bar{1}{\cdot}751425$
$\pi^2 = 9{\cdot}86960440$	0 994300	$1 \div \pi^2 = 0{\cdot}10132118$	$\bar{1}\ 005700$
$\sqrt[3]{\pi} = 1{\cdot}46459189$	0·165717	$\sqrt[3]{\pi^2} = 2{\cdot}14502940$	0·331433
$\pi/180 = 0{\cdot}01745329$	$\bar{2}{\cdot}241877$	$180/\pi = 57{\cdot}29577951$	1·758123

	0	1	2	3	4	5	6	7	8	9	1 2 3 4	5	6 7 8 9
10	0000	0043	0086	0128	0170	0212	0253	0294	0334	0374	4 9 13 17 4 8 12 16	21 20	26 30 34 38 24 28 32 37
11	0414	0453	0492	0531	0569	0607	0645	0682	0719	0755	4 8 12 15 4 7 11 15	19 19	23 27 31 35 22 26 30 33
12	0792	0828	0864	0899	0934	0969	1004	1038	1072	1106	3 7 11 14 3 7 10 14	18 17	21 25 28 32 20 24 27 31
13	1139	1173	1206	1239	1271	1303	1335	1367	1399	1430	3 7 10 13 3 7 10 12	16 16	20 23 26 30 19 22 25 29
14	1461	1492	1523	1553	1584	1614	1644	1673	1703	1732	3 6 9 12 3 6 9 12	15 15	18 21 24 28 17 20 23 26
15	1761	1790	1818	1847	1875	1903	1931	1959	1987	2014	3 6 9 11 3 5 8 11	14 14	17 20 23 26 16 19 22 25
16	2041	2068	2095	2122	2148	2175	2201	2227	2253	2279	3 5 8 11 3 5 8 10	14 13	16 19 22 24 15 18 21 23
17	2304	2330	2355	2380	2405	2430	2455	2480	2504	2529	3 5 8 10 2 5 7 10	13 12	15 18 20 23 15 17 19 22
18	2553	2577	2601	2625	2648	2672	2695	2718	2742	2765	2 5 7 9 2 5 7 9	12 11	14 16 19 21 14 16 18 21
19	2788	2810	2833	2856	2878	2900	2923	2945	2967	2989	2 4 7 9 2 4 6 8	11 11	13 16 18 20 13 15 17 19
20	3010	3032	3054	3075	3096	3118	3139	3160	3181	3201	2 4 6 8	11	13 15 17 19
21	3222	3243	3263	3284	3304	3324	3345	3365	3385	3404	2 4 6 8	10	12 14 16 18
22	3424	3444	3464	3483	3502	3522	3541	3560	3579	3598	2 4 6 8	10	12 14 15 17
23	3617	3636	3655	3674	3692	3711	3729	3747	3766	3784	2 4 6 7	9	11 13 15 17
24	3802	3820	3838	3856	3874	3892	3909	3927	3945	3962	2 4 5 7	9	11 12 14 16
25	3979	3997	4014	4031	4048	4065	4082	4099	4116	4133	2 3 5 7	9	10 12 14 15
26	4150	4166	4183	4200	4216	4232	4249	4265	4281	4298	2 3 5 7	8	10 11 13 15
27	4314	4330	4346	4362	4378	4393	4409	4425	4440	4456	2 3 5 6	8	9 11 13 14
28	4472	4487	4502	4518	4533	4548	4564	4579	4594	4609	2 3 5 6	8	9 11 12 14
29	4624	4639	4654	4669	4683	4698	4713	4728	4742	4757	1 3 4 6	7	9 10 12 13
30	4771	4786	4800	4814	4829	4843	4857	4871	4886	4900	1 3 4 6	7	9 10 11 13
31	4914	4928	4942	4955	4969	4983	4997	5011	5024	5038	1 3 4 6	7	8 10 11 12
32	5051	5065	5079	5092	5105	5119	5132	5145	5159	5172	1 3 4 5	7	8 9 11 12
33	5185	5198	5211	5224	5237	5250	5263	5276	5289	5302	1 3 4 5	6	8 9 10 12
34	5315	5328	5340	5353	5366	5378	5391	5403	5416	5428	1 3 4 5	6	8 9 10 11
35	5441	5453	5465	5478	5490	5502	5514	5527	5539	5551	1 2 4 5	6	7 9 10 11
36	5563	5575	5587	5599	5611	5623	5635	5647	5658	5670	1 2 4 5	6	7 8 10 11
37	5682	5694	5705	5717	5729	5740	5752	5763	5775	5786	1 2 3 5	6	7 8 9 10
38	5798	5809	5821	5832	5843	5855	5866	5877	5888	5899	1 2 3 5	6	7 8 9 10
39	5911	5922	5933	5944	5955	5966	5977	5988	5999	6010	1 2 3 4	5	7 8 9 10
40	6021	6031	6042	6053	6064	6075	6085	6096	6107	6117	1 2 3 4	5	6 8 9 10
41	6128	6138	6149	6160	6170	6180	6191	6201	6212	6222	1 2 3 4	5	6 7 8 9
42	6232	6243	6253	6263	6274	6284	6294	6304	6314	6325	1 2 3 4	5	6 7 8 9
43	6335	6345	6355	6365	6375	6385	6395	6405	6415	6425	1 2 3 4	5	6 7 8 9
44	6435	6444	6454	6464	6474	6484	6493	6503	6513	6522	1 2 3 4	5	6 7 8 9
45	6532	6542	6551	6561	6571	6580	6590	6599	6609	6618	1 2 3 4	5	6 7 8 9
46	6628	6637	6646	6656	6665	6675	6684	6693	6702	6712	1 2 3 4	5	6 7 7 8
47	6721	6730	6739	6749	6758	6767	6776	6785	6794	6803	1 2 3 4	5	5 6 7 8
48	6812	6821	6830	6839	6848	6857	6866	6875	6884	6893	1 2 3 4	4	5 6 7 8
49	6902	6911	6920	6928	6937	6946	6955	6964	6972	6981	1 2 3 4	4	5 6 7 8
50	6990	6998	7007	7016	7024	7033	7042	7050	7059	7067	1 2 3 3	4	5 6 7 8

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	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
51	7076	7084	7093	7101	7110	7118	7126	7135	7143	7152	1	2	3	3	4	5	6	7	8
52	7160	7168	7177	7185	7193	7202	7210	7218	7226	7235	1	2	2	3	4	5	6	7	7
53	7243	7251	7259	7267	7275	7284	7292	7300	7308	7316	1	2	2	3	4	5	6	6	7
54	7324	7332	7340	7348	7356	7364	7372	7380	7388	7396	1	2	2	3	4	5	6	6	7
55	7404	7412	7419	7427	7435	7443	7451	7459	7466	7474	1	2	2	3	4	5	5	6	7
56	7482	7490	7497	7505	7513	7520	7528	7536	7543	7551	1	2	2	3	4	5	5	6	7
57	7559	7566	7574	7582	7589	7597	7604	7612	7619	7627	1	2	2	3	4	5	5	6	7
58	7634	7642	7649	7657	7664	7672	7679	7686	7694	7701	1	1	2	3	4	4	5	6	7
59	7709	7716	7723	7731	7738	7745	7752	7760	7767	7774	1	1	2	3	4	4	5	6	7
60	7782	7789	7796	7803	7810	7818	7825	7832	7839	7846	1	1	2	3	4	4	5	6	6
61	7853	7860	7868	7875	7882	7889	7896	7903	7910	7917	1	1	2	3	4	4	5	6	6
62	7924	7931	7938	7945	7952	7959	7966	7973	7980	7987	1	1	2	3	3	4	5	6	6
63	7993	8000	8007	8014	8021	8028	8035	8041	8048	8055	1	1	2	3	3	4	5	5	6
64	8062	8069	8075	8082	8089	8096	8102	8109	8116	8122	1	1	2	3	3	4	5	5	6
65	8129	8136	8142	8149	8156	8162	8169	8176	8182	8189	1	1	2	3	3	4	5	5	6
66	8195	8202	8209	8215	8222	8228	8235	8241	8248	8254	1	1	2	3	3	4	5	5	6
67	8261	8267	8274	8280	8287	8293	8299	8306	8312	8319	1	1	2	3	3	4	5	5	6
68	8325	8331	8338	8344	8351	8357	8363	8370	8376	8382	1	1	2	3	3	4	4	5	6
69	8388	8395	8401	8407	8414	8420	8426	8432	8439	8445	1	1	2	2	3	4	4	5	6
70	8451	8457	8463	8470	8476	8482	8488	8494	8500	8506	1	1	2	2	3	4	4	5	6
71	8513	8519	8525	8531	8537	8543	8549	8555	8561	8567	1	1	2	2	3	4	4	5	5
72	8573	8579	8585	8591	8597	8603	8609	8615	8621	8627	1	1	2	2	3	4	4	5	5
73	8633	8639	8645	8651	8657	8663	8669	8675	8681	8686	1	1	2	2	3	4	4	5	5
74	8692	8698	8704	8710	8716	8722	8727	8733	8739	8745	1	1	2	2	3	4	4	5	5
75	8751	8756	8762	8768	8774	8779	8785	8791	8797	8802	1	1	2	2	3	3	4	5	5
76	8808	8814	8820	8825	8831	8837	8842	8848	8854	8859	1	1	2	2	3	3	4	5	5
77	8865	8871	8876	8882	8887	8893	8899	8904	8910	8915	1	1	2	2	3	3	4	4	5
78	8921	8927	8932	8938	8943	8949	8954	8960	8965	8971	1	1	2	2	3	3	4	4	5
79	8976	8982	8987	8993	8998	9004	9009	9015	9020	9025	1	1	2	2	3	3	4	4	5
80	9031	9036	9042	9047	9053	9058	9063	9069	9074	9079	1	1	2	2	3	3	4	4	5
81	9085	9090	9096	9101	9106	9112	9117	9122	9128	9133	1	1	2	2	3	3	4	4	5
82	9138	9143	9149	9154	9159	9165	9170	9175	9180	9186	1	1	2	2	3	3	4	4	5
83	9191	9196	9201	9206	9212	9217	9222	9227	9232	9238	1	1	2	2	3	3	4	4	5
84	9243	9248	9253	9258	9263	9269	9274	9279	9284	9289	1	1	2	2	3	3	4	4	5
85	9294	9299	9304	9309	9315	9320	9325	9330	9335	9340	1	1	2	2	3	3	4	4	5
86	9345	9350	9355	9360	9365	9370	9375	9380	9385	9390	1	1	2	2	3	3	4	4	5
87	9395	9400	9405	9410	9415	9420	9425	9430	9435	9440	0	1	1	2	2	3	3	4	4
88	9445	9450	9455	9460	9465	9469	9474	9479	9484	9489	0	1	1	2	2	3	3	4	4
89	9494	9499	9504	9509	9513	9518	9523	9528	9533	9538	0	1	1	2	2	3	3	4	4
90	9542	9547	9552	9557	9562	9566	9571	9576	9581	9586	0	1	1	2	2	3	3	4	4
91	9590	9595	9600	9605	9609	9614	9619	9624	9628	9633	0	1	1	2	2	3	3	4	4
92	9638	9643	9647	9652	9657	9661	9666	9671	9675	9680	0	1	1	2	2	3	3	4	4
93	9685	9689	9694	9699	9703	9708	9713	9717	9722	9727	0	1	1	2	2	3	3	4	4
94	9731	9736	9741	9745	9750	9754	9759	9763	9768	9773	0	1	1	2	2	3	3	4	4
95	9777	9782	9786	9791	9795	9800	9805	9809	9814	9818	0	1	1	2	2	3	3	4	4
96	9823	9827	9832	9836	9841	9845	9850	9854	9859	9863	0	1	1	2	2	3	3	4	4
97	9868	9872	9877	9881	9886	9890	9894	9899	9903	9908	0	1	1	2	2	3	3	4	4
98	9912	9917	9921	9926	9930	9934	9939	9943	9948	9952	0	1	1	2	2	3	3	4	4
99	9956	9961	9965	9969	9974	9978	9983	9987	9991	9996	0	1	1	2	2	3	3	3	4

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
00	1000	1002	1005	1007	1009	1012	1014	1016	1019	1021	0	0	1	1	1	1	2	2	2
01	1023	1026	1028	1030	1033	1035	1038	1040	1042	1045	0	0	1	1	1	1	2	2	2
02	1047	1050	1052	1054	1057	1059	1062	1064	1067	1069	0	0	1	1	1	1	2	2	2
03	1072	1074	1076	1079	1081	1084	1086	1089	1091	1094	0	0	1	1	1	1	2	2	2
04	1096	1099	1102	1104	1107	1109	1112	1114	1117	1119	0	1	1	1	1	2	2	2	2
05	1122	1125	1127	1130	1132	1135	1138	1140	1143	1146	0	1		1	1	2	2	2	2
06	1148	1151	1153	1156	1159	1161	1164	1167	1169	1172	0	1	1	1	1	2	2	2	2
07	1175	1178	1180	1183	1186	1189	1191	1194	1197	1199	0	1	1	1	1	2	2	2	2
08	1202	1205	1208	1211	1213	1216	1219	1222	1225	1227	0	1	1	1	1	2	2	2	3
09	1230	1233	1236	1239	1242	1245	1247	1250	1253	1256	0	1	1	1	1	2	2	2	3
10	1259	1262	1265	1268	1271	1274	1276	1279	1282	1285	0	1	1	1	1	2	2	2	3
11	1288	1291	1294	1297	1300	1303	1306	1309	1312	1315	0	1	1	1	2	2	2	2	3
12	1318	1321	1324	1327	1330	1334	1337	1340	1343	1346	0	1	1	1	2	2	2	2	3
13	1349	1352	1355	1358	1361	1365	1368	1371	1374	1377	0	1	1	1	2	2	2	3	3
14	1380	1384	1387	1390	1393	1396	1400	1403	1406	1409	0	1	1	1	2	2	2	3	3
15	1413	1416	1419	1422	1426	1429	1432	1435	1439	1442	0	1	1	1	2	2	2	3	3
16	1445	1449	1452	1455	1459	1462	1466	1469	1472	1476	0	1	1	1	2	2	2	3	3
17	1479	1483	1486	1489	1493	1496	1500	1503	1507	1510	0	1	1	1	2	2	2	3	3
18	1514	1517	1521	1524	1528	1531	1535	1538	1542	1545	0	1	1	1	2	2	2	3	3
19	1549	1552	1556	1560	1563	1567	1570	1574	1578	1581	0	1	1	1	2	2	2	3	3
20	1585	1589	1592	1596	1600	1603	1607	1611	1614	1618	0	1	1	1	2	2	2	3	3
21	1622	1626	1629	1633	1637	1641	1644	1648	1652	1656	0	1	1	2	2	2	2	3	3
22	1660	1663	1667	1671	1675	1679	1683	1687	1690	1694	0	1	1	2	2	2	2	3	3
23	1698	1702	1706	1710	1714	1718	1722	1726	1730	1734	0	1	1	2	2	2	2	3	3
24	1738	1742	1746	1750	1754	1758	1762	1766	1770	1774	0	1	1	2	2	2	2	3	3
25	1778	1782	1786	1791	1795	1799	1803	1807	1811	1816	0	1	1	2	2	2	2	3	3
26	1820	1824	1828	1832	1837	1841	1845	1849	1854	1858	0	1	1	2	2	3	3	3	4
27	1862	1866	1871	1875	1879	1884	1888	1892	1897	1901	0	1	1	2	2	3	3	3	4
28	1905	1910	1914	1919	1923	1928	1932	1936	1941	1945	0	1	1	2	2	3	3	4	4
29	1950	1954	1959	1963	1968	1972	1977	1982	1986	1991	0	1	1	2	2	3	3	4	4
30	1995	2000	2004	2009	2014	2018	2023	2028	2032	2037	0	1	1	2	2	3	3	4	4
31	2042	2046	2051	2056	2061	2065	2070	2075	2080	2084	0	1	1	2	2	3	3	4	4
32	2089	2094	2099	2104	2109	2113	2118	2123	2128	2133	0	1	1	2	2	3	3	4	4
33	2138	2143	2148	2153	2158	2163	2168	2173	2178	2183	0	1	1	2	2	3	3	4	4
34	2188	2193	2198	2203	2208	2213	2218	2223	2228	2234	1	1	2	2	3	3	4	4	5
35	2239	2244	2249	2254	2259	2265	2270	2275	2280	2286	1	1	2	2	3	3	4	4	5
36	2291	2296	2301	2307	2312	2317	2323	2328	2333	2339	1	1	2	2	3	3	4	4	5
37	2344	2350	2355	2360	2366	2371	2377	2382	2388	2393	1	1	2	2	3	3	4	4	5
38	2399	2404	2410	2415	2421	2427	2432	2438	2443	2449	1	1	2	2	3	3	4	4	5
39	2455	2460	2466	2472	2477	2483	2489	2495	2500	2506	1	1	2	2	3	3	4	5	5
40	2512	2518	2523	2529	2535	2541	2547	2553	2559	2564	1	1	2	2	3	4	4	5	5
41	2570	2576	2582	2588	2594	2600	2606	2612	2618	2624	1	1	2	2	3	4	4	5	6
42	2630	2636	2642	2649	2655	2661	2667	2673	2679	2685	1	1	2	2	3	4	4	5	6
43	2692	2698	2704	2710	2716	2723	2729	2735	2742	2748	1	1	2	3	3	4	4	5	6
44	2754	2761	2767	2773	2780	2786	2793	2799	2805	2812	1	1	2	3	3	4	4	5	6
45	2818	2825	2831	2838	2844	2851	2858	2864	2871	2877	1	1	2	3	3	4	5	5	6
46	2884	2891	2897	2904	2911	2917	2924	2931	2938	2944	1	1	2	3	3	4	5	5	6
47	2951	2958	2965	2972	2979	2985	2992	2999	3006	3013	1	1	2	3	3	4	5	5	6
48	3020	3027	3034	3041	3048	3055	3062	3069	3076	3083	1	1	2	3	4	4	5	6	6
49	3090	3097	3105	3112	3119	3126	3133	3141	3148	3155	1	1	2	3	4	4	5	6	6

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
50	3162	3170	3177	3184	3192	3199	3206	3214	3221	3228	1	1	2	3	4	4	5	6	7
51	3236	3243	3251	3258	3266	3273	3281	3289	3296	3304	1	2	2	3	4	5	5	6	7
52	3311	3319	3327	3334	3342	3350	3357	3365	3373	3381	1	2	2	3	4	5	5	6	7
53	3388	3396	3404	3412	3420	3428	3436	3443	3451	3459	1	2	2	3	4	5	6	6	7
54	3467	3475	3483	3491	3499	3508	3516	3524	3532	3540	1	2	2	3	4	5	6	6	7
55	3548	3556	3565	3573	3581	3589	3597	3606	3614	3622	1	2	2	3	4	5	6	7	7
56	3631	3639	3648	3656	3664	3673	3681	3690	3698	3707	1	2	3	3	4	5	6	7	8
57	3715	3724	3733	3741	3750	3758	3767	3776	3784	3793	1	2	3	3	4	5	6	7	8
58	3802	3811	3819	3828	3837	3846	3855	3864	3873	3882	1	2	3	4	4	5	6	7	8
59	3890	3899	3908	3917	3926	3936	3945	3954	3963	3972	1	2	3	4	5	5	6	7	8
60	3981	3990	3999	4009	4018	4027	4036	4046	4055	4064	1	2	3	4	5	6	6	7	8
61	4074	4083	4093	4102	4111	4121	4130	4140	4150	4159	1	2	3	4	5	6	7	8	9
62	4169	4178	4188	4198	4207	4217	4227	4236	4246	4256	1	2	3	4	5	6	7	8	9
63	4266	4276	4285	4295	4305	4315	4325	4335	4345	4355	1	2	3	4	5	6	7	8	9
64	4365	4375	4385	4395	4406	4416	4426	4436	4446	4457	1	2	3	4	5	6	7	8	9
65	4467	4477	4487	4498	4508	4519	4529	4539	4550	4560	1	2	3	4	5	6	7	8	9
66	4571	4581	4592	4603	4613	4624	4634	4645	4656	4667	1	2	3	4	5	6	7	9	10
67	4677	4688	4699	4710	4721	4732	4742	4753	4764	4775	1	2	3	4	5	7	8	9	10
68	4786	4797	4808	4819	4831	4842	4853	4864	4875	4887	1	2	3	4	6	7	8	9	10
69	4898	4909	4920	4932	4943	4955	4966	4977	4989	5000	1	2	3	5	6	7	8	9	10
70	5012	5023	5035	5047	5058	5070	5082	5093	5105	5117	1	2	4	5	6	7	8	9	11
71	5129	5140	5152	5164	5176	5188	5200	5212	5224	5236	1	2	4	5	6	7	8	10	11
72	5248	5260	5272	5284	5297	5309	5321	5333	5346	5358	1	2	4	5	6	7	9	10	11
73	5370	5383	5395	5408	5420	5433	5445	5458	5470	5483	1	3	4	5	6	8	9	10	11
74	5495	5508	5521	5534	5546	5559	5572	5585	5598	5610	1	3	4	5	6	8	9	10	12
75	5623	5636	5649	5662	5675	5689	5702	5715	5728	5741	1	3	4	5	7	8	9	10	12
76	5754	5768	5781	5794	5808	5821	5834	5848	5861	5875	1	3	4	5	7	8	9	11	12
77	5888	5902	5916	5929	5943	5957	5970	5984	5998	6012	1	3	4	5	7	8	10	11	12
78	6026	6039	6053	6067	6081	6095	6109	6124	6138	6152	1	3	4	6	7	8	10	11	13
79	6166	6180	6194	6209	6223	6237	6252	6266	6281	6295	1	3	4	6	7	9	10	11	13
80	6310	6324	6339	6353	6368	6383	6397	6412	6427	6442	1	3	4	6	7	9	10	12	13
81	6457	6471	6486	6501	6516	6531	6546	6561	6577	6592	2	3	5	6	8	9	11	12	14
82	6607	6622	6637	6653	6668	6683	6699	6714	6730	6745	2	3	5	6	8	9	11	12	14
83	6761	6776	6792	6808	6823	6839	6855	6871	6887	6902	2	3	5	6	8	9	11	13	14
84	6918	6934	6950	6966	6982	6998	7015	7031	7047	7063	2	3	5	6	8	10	11	13	15
85	7079	7096	7112	7129	7145	7161	7178	7194	7211	7228	2	3	5	7	8	10	12	13	15
86	7244	7261	7278	7295	7311	7328	7345	7362	7379	7396	2	3	5	7	8	10	12	13	15
87	7413	7430	7447	7464	7482	7499	7516	7534	7551	7568	2	3	5	7	9	10	12	14	16
88	7586	7603	7621	7638	7656	7674	7691	7709	7727	7745	2	4	5	7	9	11	12	14	16
89	7762	7780	7798	7816	7834	7852	7870	7889	7907	7925	2	4	5	7	9	11	13	14	16
90	7943	7962	7980	7998	8017	8035	8054	8072	8091	8110	2	4	6	7	9	11	13	15	17
91	8128	8147	8166	8185	8204	8222	8241	8260	8279	8299	2	4	6	8	9	11	13	15	17
92	8318	8337	8356	8375	8395	8414	8433	8453	8472	8492	2	4	6	8	10	12	14	15	17
93	8511	8531	8551	8570	8590	8610	8630	8650	8670	8690	2	4	6	8	10	12	14	16	18
94	8710	8730	8750	8770	8790	8810	8831	8851	8872	8892	2	4	6	8	10	12	14	16	18
95	8913	8933	8954	8974	8995	9016	9036	9057	9078	9099	2	4	6	8	10	12	15	17	19
96	9120	9141	9162	9183	9204	9226	9247	9268	9290	9311	2	4	6	8	11	13	15	17	19
97	9333	9354	9376	9397	9419	9441	9462	9484	9506	9528	2	4	7	9	11	13	15	17	20
98	9550	9572	9594	9616	9638	9661	9683	9705	9727	9750	2	4	7	9	11	13	16	18	20
99	9772	9795	9817	9840	9863	9886	9908	9931	9954	9977	2	5	7	9	11	14	16	18	20

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ANSWERS

PART II

XII a (p. 328).

1. ·6715.	**2.** ·9242.	**3.** ·3709	**4.** ·7023.	**5.** ·6421.
6. ·6909.	**7.** ·0835.	**8.** ·4774.	**9.** ·6712.	**10.** ·7016.
11. ·8976.	**12.** 5315.	**13.** ·9915.	**14.** ·3224.	**15.** ·5557.
16. ·9590.	**17.** ·6990	**18.** ·7782.	**19.** ·9956.	**20.** ·6478.
21. 3·047.	**22.** 2·020.	**23.** 1·371	**24.** 3·199.	**25.** 1·017.
26. 1·217.	**27.** 2 529.	**28.** 9·690.	**29.** 1·001.	**30.** 5·093.
31. 1·105.	**32.** 3·105.	**33** 3·900.	**34** 3·593.	**35.** 5·044.
36. 2·552.	**37.** ·6282.	**38** ·8357.	**39.** 1·768.	**40.** 4·199.
41. ·8705.	**42.** 7·422.	**43.** ·4771.	**44.** ·3010.	**45.** ·7782.

46. ·4771 + ·3010 = ·7782 approx.

47. 6·679.	**48.** 2·661	**49.** ·3998	**50.** 2·511

XII b (p. 330).

1. 1.	**2.** 2.	**3.** 0.	**4.** 2.
5. −2.	**6.** −4.	**7.** 4.	**8.** −1.
9. −3.	**10.** 1·5658.	**11.** 2·3094.	**12.** 2·8292.
13. $1\bar{1}.8722$.	**14.** $2\bar{2}.9239$.	**15.** $\bar{1}.9766$	**16.** ·1162.
17. $\bar{4}.6021$.	**18.** 2·7045.	**19.** 4 8451.	**20.** 1·7973.
21. ·5491.	**22.** 3·3909.	**23.** 7·4948.	**24.** $\bar{6}.7515$.
25. 27·76.	**26.** 623·3	**27.** ·2794.	**28.** 6·892.
29. $2{\cdot}65 \times 10^6$.	**30.** ·0007521.	**31.** 694·5.	**32.** 1011.
33. 6·327.	**34.** 1·230.	**35.** 200·4.	**36.** 20.
37. 3·142.	**38.** 35·17.	**39.** ·006945.	**40** ·0001265.
41. ·001501.	**42** ·2625.	**43.** ·5411.	**44.** ·2643.
45. 2·8494.	**46.** $\bar{3}.6243$.	**47.** 265·3.	**48.** 2·496.

XII c (p. 332).

1. $3{\cdot}672 \times 10^4$. **2.** $3{\cdot}669 \times 10^7$. **3.** $2{\cdot}787 \times 10^4$.
4. $8{\cdot}349 \times 10^5$. **5.** 56·75. **6.** 4·971.
7. ·1510. **8.** 5·320. **9.** 6530.
10. 13·36. **11.** 93870. **12.** 813·7.

XII d (p. 333).

1. $3{\cdot}396 \times 10^{-5}$. **2.** $5{\cdot}726 \times 10^{-6}$. **3.** ·03602.
4. ·1596. **5.** 13·45. **6.** 9074.
7. 190·5. **8.** ·6939. **9.** ·2572.
10. 3·043. **11.** 6·377. **12.** 43·16.
13. 1·044. **14.** 6·803. **15.** 3238.
16. ·02730. **17.** ·8252. **18.** 2·066.

XII e (p. 336).

1. 150. **2.** 3·199. **3.** 2·050. **4.** ·4736.
5. ·3077. **6.** ·006495. **7.** ·8174. **8.** 12·13.
9. 112·3. **10.** 12·82. **11.** ·2226. **12.** 16·44.
13. 2·003. **14.** £2654. **15.** £377.

XII f (p. 337).

1. 49·01. **2.** ·0004624. **3.** ·1082.
4. 32·88. **5.** ·01026. **6.** ·7092.
7. 1·66 sec. 1·09 inches. **8.** 631·6.
9. 7·197 cu. in. **10.** 80·1 ft. **11.** 140 cu. cm.
12. 14860. **13** ·614 or 61 %. **14.** $12\frac{1}{4}$ miles.
15. 264 gallons. **16.** 2·935. **17.** $1{\cdot}746 \times 10^{-14}$. ·5667.

Oral Examples (p. 341).

1. 40. **2.** 70. **3.** 100. **4.** 27·6. **5.** 60590.
6. 4000. **7.** 4·035. **8.** 5·3. **9.** 320. **10.** 15.
11. 20. **12.** 140. **13.** 8·6. **14.** 4·6. **15.** 1500.
16. 5700. **17.** 371. **18.** 16. **19.** 4000. **20.** 530.
21. 3. **22.** 7300. **23.** ·0576. **24.** ·5078. **25.** 12.
26. 28. **27.** 2·6. **28.** 90. **29.** 504000. **30.** 72000.
31. 532·3. **32.** 17·62. **33.** 1000. **34.** 2700. **35.** 1300.
36. 17000. **37.** 5600. **38.** A million. **39.** 1142.

Oral Examples (p. 343).

1. 7·070.	**2.** 7·938.	**3.** 9·898.	**4.** 12·12.	**5.** 4·472.
6. 6·708.	**7.** 12·73.	**8.** 6·324.	**9.** 8·484.	**10.** 10·39
11. 26 16.	**12.** 31·62.	**13.** 14 14.	**14.** 8 660	**15.** 17·32.
16 5·656.	**17.** 6·928.	**18.** 9 486.	**19.** 18·52.	**20.** 11·18
21. 19·05.	**22.** 25·30.	**23.** 16·97.	**24.** 13·42	**25** ·7070.
26. ·5773.	**27.** ·4472.	**28.** 2 828.	**29.** 3 464.	**30.** 4 472.
31. 5·292.	**32.** 34·78.	**33.** 8·660	**34.** 12·73.	**35.** 10·39.
36 6·324.	**37.** 11·31.	**38.** 9·898.	**39.** 2·646.	**40.** 2·213.
41. 1·897.	**42.** 5 656.	**43.** 5 773.	**44** 3·780.	**45.** 3 535.
46. 2·887.	**47.** 3·794.	**48.** 3·464.		

Oral Examples (p 346).

1. 1·007.	**2** ·994.	**3.** 1·006.	**4.** 4·003
5. 1·992.	**6.** 2 985.	**7** 1·012	**8.** 1·001.
9. 2 016.	**10.** 1 003.	**11.** 1·007.	**12.** ·992.
13. 1·061.	**14.** 1·060	**15** ·994.	**16.** 1 002
17. ·996.	**18.** ·994	**19.** ·998	**20.** 1·004
21. 1·006	**22.** 1 002.	**23.** 998.	**24.** 996.
25. 1·12 sq ft.	**26.** 1 10 sq ft	**27.** 1·01 sq ft.	**28.** 1·04 yd.
29. 1·09 ft	**30.** 1 001 cu ft	**31.** 1·003 ft.	**32.** 1·005 ft

XIII a (p 347).

1. 6068 sq. in or 42 sq. ft. 20 sq. in. **2.** 594 sq. ft.
3. £5 17*s* 1*d*. **4.** £38 15*s*. **5.** 54 fr. **6.** £21 2*s*. 6*d*.
7. 31125. **8.** 11900. **9** £3 1*s*. 4*d*. **10.** 155 yds. 2 ft.
11. $\frac{99}{1864}$ in. approx. **12.** 626 sq. ft. ·19 %.
13. Path, 1096 sq. ft. Border, 1550 sq. ft. **14.** 15·7 cm.
15 206 acres. **16.** 281 ft. **17.** 3681 sq. in. (or 25·6 sq ft)

XIII b (p. 350).

1. 6·16 in.	**2.** 7·73 cm.	**3.** 9·90 ch.	**4.** 73·8 yds.
5. 9·74 in.	**6.** $d = a\sqrt{2}$.	**7.** (i) 102·2 cm.	(ii) 48·4 cm.
8. 3·35 in.	**9.** 3·68 in.	**10.** $h = a\sqrt{3}$.	
11. (i) 6·60 ft.	(ii) 27·4 in.	**12.** 11·17 in.	**13.** 3·98 cm.
14. (i) 5·19 in.	(ii) 12·65 in.	**15.** 7·33 in.	

XIII c (p. 357).

1. 17·0 sq. in. **2.** 9·76 sq. in. **3.** 17·1 sq. in.
4. 10·2 sq. in. **5.** 9·69 sq. cm. **6.** 11·3 in
7. 15·28 in. **8.** (i) 5·45 in. (ii) 12·86 sq. in.
9. (i) 385·6 sq cm. (ii) 16·29 cm. 29·38 cm. 24·61 cm.
10. 34·2 sq. in. **11.** 5·34 sq. ch. **12.** 11·3 sq yds.
13. 23·6 sq. cm. **14.** 3·42 sq. in. **15.** 53·7 sq. cm.
16 58·0 sq. in. **17.** 31·2 sq. in. **18.** 13·4 sq. in.
19. 39·1 sq. in. **20.** 2·26 acres. **21.** 3·03 acres.
22. 5·17 ares. **23** 2024 sq. yds **24.** 2·01 ares.
25. 340 sq. yds. **26.** 2 73 acres. **27.** 4·66 ares.
28. 4·80 acres. **29.** 146·1 sq in. **30.** 256 sq. cm.
31. $\frac{3}{2}a^2\sqrt{3}$. **32.** (i) 48·0 sq. in. (ii) 120 sq. cm. (iii) 2660 sq. ft.
33. (i) 5·27 in. (ii) 5·78 cm. (iii) 6·20 metres.

XIII d (p 363).

1. 2·5 in. 3·5 in **2.** 8·75 in. 10·5 in **3.** 2·4 in. 2·7 in.
4. $CA = 16\ 7$ in. $EF = 4{\cdot}2$ in. **5.** 3386 sq. cm.
6. 33·1 sq. cm. **7.** 4·89 cm. **8.** 1·75 in. 1869 acres.
9. 8 ac. 3156 sq. yds. **10.** ·675 sq in. **11.** 882·5 Km.
12. $73\frac{1}{3}$ acres.

XIII e (p. 366).

1. 9300 sq. miles. **2.** 3400 sq miles. **3.** 9900 sq. miles.
4. 1500 sq. miles. **5.** 30 sq. miles. **6.** 33000 sq miles.
7. 45 sq. miles. **8.** 150 sq. miles. **9.** 8 acres.
10. 18·4 ares. **11.** Nearly 1 acre (·95 ac.). **12.** 310 sq. ft.

XIII f (p. 372).

1. 27·27 in. **2.** 23·3 in. **3.** 587·5 cm.
4. 164·9 yds. **5.** 117 3 yds. **6.** 29 ch. 66 lks.
7. 5·507 in. **8.** 6·93 yds. **9.** 2·807 m.
10. 16·95 cm. **11.** 8 yds. 2 ft. **12.** 8·9 ch.
13. 2·374 in. **14.** 17·56 in. **15.** 5·19 in.
16. 90° 54′. **17.** 102° 36′. **18.** 4·60 in.
19. 1·60 ft. **20.** 15° 10′. **21.** 1145
22. 120. **23.** 2100. **24.** $45\frac{1}{2}$ ft.
25. 67·4. **26.** 5·355 m. p. h. **27.** 308° 34′.
28. 57° 18′.

XIII g (p. 375).

1. 17·2 sq. in. **2.** 1795 sq. cm. **3.** 11·4 cm.
4. 3·03 yds. **5.** 45·85 sq. ft. **6.** 3·92 ch.
7. 26·0 acres. **8.** 126·3 sq. m. **9.** 29·6 m.
10. 12·67 sq. cm. **11.** 873 sq. ft. **12.** 6° 19′.
13. 39·55 in. **14.** 154° 54′. **15.** 16·7 yds.
16. 58·9 sq. in. **17.** (i) 112·5 sq. ft. (ii) 6 cu. ft.
18. 150·7 sq. in. **19.** 71·6 cm. **20.** Area = 19·63 sq. in.
21. 43·5 sq. in. **22.** (i) 259·8 sq. in. (ii) 17·3 %.
23. 56·56 sq. in. **24.** (i) 5·80 sq. in. or 195·3 sq. in.
25. 2·08 sq. in. **26.** 76·1 sq. cm.

XIV a (p. 379).

1. 963 gm. **2.** 43·5 gm. **3.** 4650 gm.
4. 1030 gm. **5.** 14·9 gm. **6.** 10·3 lb.
7. 2128 lb. or 19 cwt. **8.** 59·3 lb.
9. 1·557 tons or 1 ton 11 cwt. **10.** 27·6 cwt. or 3090 lb.
11. 1·30 cu. cm. **12.** 88·1 cu. cm. **13.** 73·6 cu. cm.
14. 9·87 cu. in. **15.** 49·8 cu. in. **16.** 115·2 cu. in.
17. 38·5 cu. ft. **18.** 257·7 cu. cm. **19.** 5·01 cu. ft.
20. 141·4 cu. in. **21.** 1564 gm. **22.** 31·5 tons.
23. 189·6 oz. **24.** 11·65 gm. **25.** 230 gm.
26. ·675. **27.** 670. **28.** ·747.
29. 5·2 %. **30.** 3360 oz. **31.** 8·19.
32. ·796 cwt. **33.** 700 gm. **34.** (i) $\frac{1}{8}$ in. (ii) $\frac{9}{10}$ in.
35. 9·434 Kg. **36.** 6·64 lb. **37.** ·0064.

XIV b (p. 386).

	Height.	Radius.	Volume.	Area
1.			61·5 c.c.	51·3 sq. cm.
2.			43·4 cu. in.	50·2 sq. in.
3.			188·5 cu. in.	377 sq. in.
4.	2·24 in.			31 sq. in.
5.	·77 cm.		31·6 cu. cm.	
6.		1·29 in.		21·4 sq. in.
7.		1·19 in.	15·6 cu. in.	
8.	·71 cm.	2·89 cm.		
9.	120 cu. in.			

10. 223·5 cu. cm. 255 sq. cm.
11. ·305 cu. in. ·267 cu. in.
12. 707 cu. cm. **13.** 31·83 metres **14.** 1·24 cm.
15. 56·57 cm. **16.** (i) 95 cu. cm. (ii) 684 gm.
17. (i) 50·9 cu. in (ii) 63·6 cu. in. **18.** 2335 ft.
19. 65 lb. **20.** 245 gallons. **21.** 13860 cu. ft
22. 108·4 cu. ft. **23.** 3·03 acres. 5 m. p h. **24.** 7 61.
25. 2·89 cu. cm. **26.** 2568 gm. **27.** 357 gm.

XIV c (p. 391)

1. $4{\cdot}6 \times 10^7$ cu. yds. **2.** $1{\cdot}9 \times 10^8$ cu. yds. **3.** 20 million galls
4. 700 cu. yds. **5.** 500 cu. in. **6.** 2100 cu. in.

XV a (p. 400).

1. 163·3 cu. in. **2.** (i) 139·2 sq. cm. (ii) 95·8 cu. cm.
3. 651 cu. cm. **4.** Vol. $= \frac{1}{3} a^2 \sqrt{3b^2 - 4a^2}$.
5. 30·6 cu. in. **6.** 230 cu in. **7.** 14·0 cm.
8. 14·72 cm.; 29·44 cm. **9.** 143 cu. ft. 111 sq ft **10.** 1·57 in
11. 42·80 cu. in. 61·42 sq in. **12.** 264·1 cu. in.
13. 475·4 sq. yds. **14.** $\frac{26}{27}$ $\frac{8}{9}$.
15. Edges are 2·45 in., 4·36 in., 5·48 in.
Volume = 19·5 cu. in.
Perp. = 3·98 in.

XV b (p. 406).

	Radius.	Height.	Volume.	Curved surface.
1.			78·7 cu. in	78·4 sq. in.
2.			42·32 cu. in.	52·1 sq. in.
3.			48460 cu. cm.	26060 sq cm.
4.		3 32 in.		110 sq. in.
5.	5·825 in.			176·4 sq. in.
6.		36·4 cm.	32930 cu. cm.	
7.		24·0 cm.	8608 cu cm.	

8. 1·114 litres. **9.** Radius of circle = 6·8 in. Angle of sector = 169°
10. Concentric circles. Radii 7·12″, ·71″. Angle of sector 126·4°.
11. 528 sq. ft. 622 lb. **12.** 2·42 gallons.
13. 268·4 sq. ft. **14** 422 sq ft. 1072 cu. ft.

XV c (p. 412).

1. 68·82 sq. in. **2.** 177·7 sq. in. **3.** 22490 sq. cm.
4. 3098 sq. cm. **5.** 5439 sq. cm. **6.** 341·1 sq. cm.
7. 487·8 sq. in. **8.** 2569 sq. in. **9.** 413·0 cu. in.
10. 670·8 cu. in. **11.** 24·04 cu dm. **12.** 18·14 cu. dm.
13. 319·5 cu. cm. **14.** 140 cu. mm. **15.** 1156 cu. in.
16. 104·3 cu. in. **17.** 5·68 in. **18.** 7·03 in.
19. 13·8 in. **20.** 21·9 cm. **21.** 20·0 cm.
22. 16·1 cm. **23.** 5·00 in. **24.** 6·39 in.
25. 9·36 in. **26.** 3·82 cm. **27.** 62·03 cm.
28. 83·28 cm. **29.** 15·15 lb. **30.** Diam. = 4·4 cm.
31. 5739000. **32.** 904·8 sq. ft. £7 10*s* 10*d*. **33.** 90860.

34. $V = \frac{1}{3}\sqrt{\frac{A^3}{4\pi}}$. (i) 74·92 cu. cm. (ii) 53·56 cu. in. (iii) 27·35 cu. in.

35. $V = \pi r^2 (l - \frac{1}{3}r)$. $A = 2\pi rl$.
36. $V = 6770$ cu. cm. $A = 2025$ sq. cm.
37. $V = 1320$ cu. cm. $A = 670$ sq. cm.
38. $V = 23{\cdot}6$ cu. ft. $A = 41{\cdot}4$ sq. ft.
39. 62·5 cu. in. **40.** 11520 cu cm. **41.** 597 sq in.
42. 1178 sq. in. **43.** $5{\cdot}15 \times 10^7$ sq. miles.
44. 4 to 10 to 13 (in pairs).

Miscellaneous Examples (p. 414).

1. 6000 cu. in. **2.** 1 in. **3.** 464 sq. units.
4. 15·43 acres. **5.** 288. **6.** 12 sq. dm.
7. 1478·6 litres. **8.** 4080 cu. in. $51\frac{1}{3}$ lb.
9. 2·45 min. **10.** 366·5 cu. yds. **11.** 1·115 sq. cm.
12. ·342 sq. in. **13.** £12 3*s*. 8*d*. **14.** 2220 oz.
15. 183·7 cm. **16.** 61 sq. units. **17.** £3 16*s*. 3*d*.
18. 176·7 roods. **19.** 2928.
20. 86100 sq. mi. 467 miles. **21.** 4·16 cu. in.
22. 3·81 sq. in. **23.** 15 ft. 2 in. **24.** £2.
25. £1 11*s*. 8*d*. **26.** £122 7*s*. 4*d*. **27.** 1·515 gm.
28. ·024 in. **29.** 200 bricks.
30. 1857 million tons. **31.** 23·8 sq. cm.
32. 128 ac. $30\frac{15}{16}$ sq. in. **33.** 12500.
34 1·5 in. **35.** 63 ft. **36.** 25 ft. 3 in.
37. £7 10*s*. 8*d*. **38.** 18·6 lb. **39.** ·7 cm.
40. 34 ft. 3 in. **41.** 1·41 m. per sec.

42. Length, 17 cm. Breadth, 4·2 cm. Depth, ·7 cm. 78·6 %.
43. 5·93 sq. in.
44. $V = lbh$. $A = 2(bl + lb + bh)$.
$D = \sqrt{h^2 + b^2 + l^2}$.
192 cu. yds. 208 sq. yds. 10·8 yds.
45. 9·1 sq. in.
46. 30812 sq yds. **47.** 187·1 sq. cm **48.** 62·92 Kg.
49. 6234 gallons. 100,000 litres. **50.** ·08 acres.
51. 10,000 sq. m. **52.** 272 yds. **53.** £2 8*s.* 0½*d.*
54. ·66 in. **55.** 8·80 acres. **56.** 10,182 Kg.
57. 3 to 56. **58.** 1 ton 9 cwt. 13 lb.
59. 6·36 sq. cm. **60.** 10·17 cu in.
61. 33·3 sq. cm. 41 mm. **62.** 1562 acres.
63. 5 in. **64.** 1·39 cu ft. 678 lb.
65. ·58 sq. mm. **66.** £6 8*s.* **67** 4·17 % too small
68. 125 to 108 **69** 13·8 cm. **70.** 3 to 2.
71. 20·77 sq. in. **72.** 5·83 in. **73.** 3680 metres.
74. 17·3 %. **75** 7·29 in.
76. 12 mi. 936 yds. or 12·53 mi. **77.** 255·5 cu. cm.
78 14·70 in. **79** ·675. ·667. **80** 2310 ft.
81. 3·97 in. **82.** 6·203 in.
83. 254° 34′. 3·72 Kg **84.** 49 gallons
85. $AC = 6{\cdot}5$ cm. Area = 17·64 sq. cm.
86. 139·8 tons **87.** 63·8 acres. **88.** 29·2 yds
89. Radius of concentric circles, 40 in. and 30 in. Angle of sector, 54°
Area = 330 sq. in. Volume = 862 cu. in.
90. Volume = 247·2 cu. in Whole surface area = 153·9 sq. in.
91. 3·56 sq. in. 24 8 cu. in.
92. 50 min. 22 sec. 671500 litres. **93.** ·004 in.
94. 3·35 ft. **95.** 375000.
96. 1·521. **97.** 310·9 tons. **98.** 16·8 acres.
99. 15·86 min. (15 min. 52 sec.). **100** 10*d.*
101. 4430 cu. cm. 25640 cu. cm.
102. Surface area = 311·1 sq. in. Volume = 509 cu. in.
103 $\pi Lt(D - t)$. 76·9 lb. 150·5 lb 221·0 lb 288·0 lb.
104. 35 7 ft. **105.** 75·41 sq. in. **106.** 9·64 mm.
107. 3¾ in. **108.** 3393 sq. ft. **109.** £8 2*s.* 1*d.*
110. 10⅕. **111.** Weight = 2·5 gm. 8·4 mm.
112. 2·7. **113.** 281 tons. **114.** 300,000 cu. ft.
115. ·30 mm. **116.** 14·14 sq. in. **117.** 546·1 sq. yds.

118. 60 in. 52 in. **119.** 10 oz. **120.** 4. 4·5 cm.
121 $\frac{1}{4}$ mile. **122.** 4·554 Km. **123.** 249·7 cu. yds.
124. 29·31 sq. yds 188·5 cu. ft. **125.** 122·5 cu in.
126. 39 ft 8 in. **127.** 112 sq. ft. **128.** $\frac{3}{4}$ min.
129. 1019 cu. cm. **131.** 6·58 cm.

XVI a (p 433).

1. 12*s.* 2$\frac{1}{2}$*d.* **2.** £2 3*s.* 11$\frac{1}{2}$*d.* **3.** £6 3*s.* 11$\frac{1}{2}$*d.*
4. 23·90 francs. **5.** 56 50 francs. **6.** 8·562 metres.

7.

s.	*d.*	£	*s.*	£
	0$\frac{1}{4}$	·00104	4	·2
	0$\frac{1}{2}$	·00208	5	·25
	0$\frac{3}{4}$	·00313	6	·3
	1	·00417	7	·35
	2	·00833	8	·4
	3	·0125	9	·45
	4	01667	10	5
	5	·02083	11	·55
	6	025	12	6
	7	·02917	13	·65
	8	·03333	14	·7
	9	·0375	15	·75
	10	·04167	16	·8
	11	·04583	17	·85
1	0	·05	18	·9
2	0	·10	19	·95
3	0	·15	20	1·0

8.

Metre.	Yard.	Metres	Yards
·01	·011	2	2 187
·02	·022	3	3·281
·03	·033	4	4·375
·04	·044	5	5·468
·05	·055	6	6·562
·06	·066	7	7·655
·07	077	8	8·749
·08	·087	9	9·843
·09	·098	10	10·936
·1	·109	11	12·030
·2	·219	12	13·124
·3	·328	13	14·217

Metre.	Yard.	Metres.	Yards.
·4	·437	14	15·311
·5	·547	15	16·404
·6	·656	16	17·498
·7	·766	17	18·592
·8	·875	18	19·685
·9	·984	19	20·779
1·0	1·094	20	21·873

9.

Mile.	Fur.	Kilometre.	Miles.	Kilometres.
	1	·201	2	3·219
	2	·402	3	4·828
	3	·603	4	6·437
	4	·805	5	8·047
	5	1·006	6	9·656
	6	1·207	7	11·265
	7	1·408	8	12·874
1	0	1·609	9	14·484

10.

Kilometre.	Mile.	Kilometres.	Miles.
·1	·062	1	·621
·2	·124	2	1·243
·3	·186	3	1·864
·4	·249	4	2·486
·5	·311	5	3·107
·6	·373	6	3·728
·7	·435	7	4·350
·8	·497	8	4·971
·9	·559	9	5·592
1·0	·621	10	6·214

11.

Kilogram.	Lb.	Kilogram.	Lb.
·1	·220	1·0	2·205
·2	·441	2·	4·409
·3	·661	3·	6·614
·4	·882	4·	8·818
·5	1·102	5·	11·023
·6	1·323	6·	13·228
·7	1·543	7·	15·432
·8	1·764	8·	17·637
·9	1·984	9·	19·842
1·0	2·205	10·	22·046

12.

Per lb. *d.*	Per kilogram. Francs.	Per lb. *s.* *d.*	Per kilogram. Francs.
¼	·06	1 0	2·89
½	·12	2 0	5·79
¾	·18	3 0	8·68
1	·24	4 0	11·57
2	·48	5 0	14·47
3	·72	6 0	17·36
4	·96	7 0	20·25
5	1·21	8 0	23·15
6	1·45	9 0	26·04
7	1·69	10 0	28·93
8	1 93	12 6	36 17
9	2·17	15 0	43·40
10	2·41	17 6	50·63
11	2 65	20 0	57·86

13.

d.	Francs.	*s.* *d.*	Francs.
¼	·026	1 0	1·261
½	·053	2 0	2·523
¾	·079	3 0	3·784
1	·105	4 0	5·045
2	·210	5 0	6 306
3	·315	6 0	7·568
4	·420	7 0	8 829
5	·526	8 0	10 090
6	·631	9 0	11·351
7	·736	10 0	12·613
8	841	12 6	15·766
9	·946	15 0	18·919
10	1·051	17 6	22·072
11	1·156	20 0	25·225

Oral Examples (p. 435).

£	£	£
1. ·0125416 .	**2.** ·0250833 . . .	**3.** ·009375.
4. ·2525416 . .	**5.** ·3770833 . . .	**6.** ·959375.

£	£	£
7. ·7979166 . .	**8.** ·1729166	**9.** ·6947916 . . .
10. ·35ɔ125.	**11** ·7354166 . .	**12.** ·8708333 . .
13. ·8416666 .	**14** ·1833333	**15** ·9989583 . . .
16. ·4604166 . . .	**17** 59375	**18.** ·2291666 .
19. ·0895833 . . .	**20.** ·1375.	**21.** ·4166666 .

XVI b (p. 435).

1. 2*s.* 1*d.* **2** 5*s.* $6\frac{3}{4}d.$ **3** $2\frac{3}{4}d.$
4. £1 2*s.* 8*d.* **5.** 2*s.* 10*d.* **6.** 5·97.
7. 1·61. **8.** 10·97. **9.** 7 31.
10. 14·18. **11.** 5 to 1. **12.** 1*s.* 8*d.*
13. 7 to 1. **14.** $8\frac{1}{3}$ miles per hour. **15.** 7 miles.
16. $288\frac{1}{4}$ yds. **17.** Average cost is $1\frac{5}{9}d.$ **18.** 55 children.
19. 68 men, 32 women. **20.** £750. **21.** 140.
22 $5\frac{5}{11}$ miles **23.** 9*s.* 8*d.* **24.** 49 gold to 51 silver.

XVI c (p 439).

1. *A* £535 1*s.* 8*d*, *B* £334 18*s* 4*d.*
2. £139 13*s.* 3*d.*, £73 6*s* 9*d.*
3. *A* £755 2*s.* 9*d*, *B* £329 0*s* 9*d.*, *C* £658 1*s* 6*d.*
4. £300 2*s* 6*d* , £274 8*s.*; £308 14*s* **5.** £3110.
6. £7173 6*s* 8*d.*; £8070, £8608, £8966 13*s.* 4*d.*
7. £12. **8.** 30 days.
9. *A* £300, *B* £98, *C* £112, *D* £120.
10. 2376 men. **11.** 25 men.
12. £3550, £3124, £2982, £3536, £3128, £2992.
13. £3, £2 8*s*, 12*s*. **14.** £500, £540.
15. £230, £195, £351. **16** 2nd pays 3*s.* 6*d.* to the 3rd person.

XVI d (p. 443).

1. 11·748*d.* **2.** 9·993*d.* **3.** 1*s.* 4*d.*
4. 9·516*d.* **5.** 3*s.* 11·6*d* **6.** 3675 francs.
7. 26·45 francs. **8.** 15·43 grains. **9.** 10,613 francs.
10. 24·69 francs. **11.** 4 pesos. **12.** 75 rupees.
13. ·9. **14.** £39 19*s.* 8*d.* **15.** £1 = 34·64 pesetas.
16. 420.

XVI (p. 444).

1. England 555 million, Wales 33 million; Scotland 74 million; Ireland 71 million.
2. £2 15*s.* 7*d.*; £2 12*s.* 9*d*; £2 1*s.* 8*d.*
3. £12 **4.** 42 lb. **5.** 5 miles. **6** 62 lb. 6·8 oz.
7. £9000, £3450. **8.** 453·0 grains; 4530 c.c.
9. 44·08 francs. **10** £47 14*s.* $2\frac{1}{4}d.$; 27*s.* $11\frac{1}{4}d.$
11. 1500 lb. **12.** 1*s.* 9*d* (1*s.* 8·18*d.*). **13.** 1·80 francs.
14. 6731 fr. 73 c. **15.** 984 francs **16.** 4 fr. 78 c.
17. 13*s.* 7*d.* **18.** £3 16*s.* $0\frac{1}{2}d$

XVII a (p. 449).

1. 68921. **2.** £936 16*s.* **3** 13*s.* 4*d.*
4. 874577. **5.** $10\frac{5}{6}$ %. **6.** $3\frac{3}{4}$ %.
7. 1*s.* 8*d.* **8.** £238 4*s* **9.** 700.
10. $11\frac{1}{9}$ hydrogen; $88\frac{8}{9}$ oxygen **11** £707 14*s.* 4*d.*
12. £315 6*s.* **13.** £400. **14.** $4\frac{1}{2}$ %.
15. 24000 males, 26000 females. **16.** £159.
17. 68921. **18.** Dearer; 99·05 % **19** 1200.
20. 1·6; 22·2; 76·2. **21.** 2 64 cwt.
22. £1164, £2037. **23.** 160 9 %.
24. (*a*) −·51, (*b*) −2 7, (*c*) 8 4, (*d*) 10·1, (*e*) −50, (*f*) −28 6, (*g*) −9·2, (*h*) −38, (*i*) 5·7, (*k*) −5·2, (*l*) 179, (*m*) 0, (*n*) 6·9, (*o*) 11·0. (1912) £117317870, (1911) £113917584. Increase 3 %.
25. 13·31 %. **26** 62·5 %.
27. 2·6 %; −8·6 %; −10 %, 14·8 %.
28. 187 %; 149 %; 93 %; 92; 1070000.
29. £51 15*s.*; £1333 6*s.* 8*d.* **30.** £1060000; £212000.
31. 1878151. **32.** 19·3 %. **33.** 66 %; 34 %.
34. 32 %.

XVII b (p. 455).

1. (i) 7·7 gain. (ii) 26·4 loss. (iii) 13·7 gain. (iv) 7 4 loss (v) 2·7 gain.
2. (i) 7·2 loss. (ii) 35·9 gain. (iii) 12·0 loss. (iv) 7·9 gain. (v) 2·6 loss.
3. £165 16*s.* 8*d.* **4.** £200. **5.** £50000, £56250.
6. $10\frac{5}{7}$ %. **7.** Gain of $11\frac{1}{9}$ %. **8.** Loss of 20 %.
9. Gain 12 %. **10.** Gain 32·5 %. **11.** $18\frac{3}{4}$ %.
12. £2160. **13.** $82\frac{7}{24}$ %. **14.** £1 3*s.* 5*d.* per gallon.

15. £1400; 75 %; 100 %. **16.** $34\frac{4}{9}$ %.
17. 4 to 1. **18.** $28\frac{1}{3}$ %; 12 %. **19.** 22 %.
20. £40 **21.** 10*s.* $11\frac{1}{4}$*d.* **22.** £25.
23. 11*s.* 3*d.* in the £. **24.** £5. **25.** 7·71 %.
26. £9780 4*s.* 6*d* **27.** 1124 346 lb ·39 % gain.
28. 14·8 %. **29.** 75 %. **30** £195 10*s.*
31. $29\frac{3}{5}$ %. **32.** £2443 15*s.*; £2500.
33. 1*s.* 4*d.* **34.** 16 %. **35.** £39 11*s.* 8*d*
36. 2197·5 marks. **37.** $7\frac{3}{4}$*d.* **38.** 9*s.* 10*d.* 26·4 %.
39. £793 5*s.*; £92 10*s.* 11*d.* **40.** 20 %.
41. 18·4 %. **42.** 10 %. **43.** 23·0 %.
44. $13\frac{1}{3}$ %. **45.** 9·091 %. **46.** 37*s.* 4*d.*

XVII (p. 459).

1. £64 7*s.* **2.** 14*s.* 7*d.* **3.** 30 %.
4. 5*s.* a bottle. **5.** 3 pints. **6.** £182.
7. 16*s.* 8*d.* **8.** $37\frac{1}{7}$ % gain. **9.** 1*s.* $2\frac{3}{4}$*d.*
10. £39000. **11.** 115 %. **12.** 50 %.
13. £17 11*s.* **14.** $62\frac{1}{2}$ %. **15.** Diminished; 30 to 29.
16. 27 %. **17.** 417000. **18.** $3\frac{23}{39}$ %.
19. $62\frac{1}{2}$ % **20.** (*a*) 6·02 lb., 6·39 lb.; (*b*) 4·49 %; 6 %.
21. $56\frac{1}{8}$ copper, $22\frac{1}{2}$ zinc, $21\frac{1}{8}$ tin. **22.** £2 1*s* 8*d.*
23. *C* £8531 3*s.*; *D* £5687 8*s.* 8*d.* **24.** £5533 6*s* 8*d.*
25. 8*s.* 6*d.* **26.** 2 to 1.
27. 15 %; 9 %. £1 2*s.* 7*d.*; £1 1*s.* 4*d.* **28.** 7·14 %.
29. He still owes 102 fr. 95 c. **30.** £33 6*s.* 8*d.*
31. £1234 4*s.*; £617 2*s.*; £205 14*s.* **33.** 2118.
34. 525 lb. £13 2*s.* 6*d.* **35.** 2 fr. 98 c. per kilo.

Revision Examples (p. 464).

1. 1. **2.** $4\frac{4}{5}$ sec. **3.** 10 min.
4. 258·7 grammes per centimetre.
5. Potassium 31·84 %, Chlorine 28·98 %, Oxygen 39·18 %. 391·8 grammes.
6. £146 13*s.* 4*d.* **7.** $22\frac{1}{2}$ minutes. **9.** $38\frac{4}{7}$.
10. 5·55. **11.** £1044 12*s.* 1*d.* **12.** 27 grains.
13. 2·52. **14.** 300. **15.** $278\frac{7}{16}$ sq. in.
16. $12\frac{1}{2}$ %. **17.** 16 %. **18.** 28 days.
19. £343 13*s.* 10*d.* **21.** $\frac{3}{88}$. **22.** 20 %.
23. 3·98 %. $1\frac{3}{4}$*d.* **24.** 5*s.* 4*d.* per lb. **25.** £7 6*s.* 6*d.*

26. £47·55 per ton. **27.** £700. **28.** £9 6*s.* 8*d.*
29. $25\frac{5}{7}$ miles per hour. **30.** 2*s* per lb.
31. 14·757. **32.** 73 lb. 2·77 oz. **33.** 40 %.
34. 1188 yds. **35** 26 men.
36. *A* £1375, *B* £775, *C* £350. **37.** 72·37 kilograms
38. 1078. **39.** E. 1·2 %; S 1·5 %, I. 4·3 %; G.B 1·5 %.
40. $74\frac{2}{3}$ %, 6 %, $19\frac{1}{3}$ %. **41.** 1329 kilos.
42. 4·16 cu. m. **43.** 39. **44.** 442·64290.
45. (i) $5\frac{5}{7}$. (ii) $\frac{5}{9}$. **46.** 2·205. **47.** 3·6 %.
48. 23·1 %. **49.** 84. **50.** $\frac{3}{20}$.
52. (i) 4 %. (ii) 6 %. (iii) 4·8 %. **53.** *B*, *A*, *C*.
54. Copper £84 13*s.* 6*d.*, Lead £16 18*s.*, Tin £179 6*s.* 9*d.*, Zinc £25 12*s.* 1*d.*
55. 2 hrs. 2 min. **56.** 780 acres, 468 acres, 520 acres.
57. 1670 oz. **58.** 1906, 6·15, 1907, 5·49; 11 %.
59. 49·59 francs. **60.** £150. **61.** £4792 17*s.* 2*d.*
63. £56 15*s.* 5*d.* **64.** Increased by 40 minutes.
65. £16449 1*s.* **66.** 2·71 %; 1·35 %. **67.** 2181·30 cu. m.
68. £7738 16*s.* 5*d.* **69.** $24\frac{1}{9}$ % loss.
70. 1890, £195,938,201; 1900, £253,406,827; 1910, £277,946,230 1890-1900, 29·3 % increase, 1900–1910, 9·7 % increase.
71. 2 p.m. **72.** 16 at 4*s*, 4 at 9*s*
73. (i) $\frac{2441}{280}$. (ii) 111·641 and 27·111.
74. 382 gallons. **75.** 10 %.
76. 4·54 %; 68807127; 7·43*d*; 7·58*d.*; 2·77 %.
77. 5 hrs. 21 min. 35 sec.; 5 hrs. 22 min. 35 sec.
78. £3 1*s.* $4\frac{1}{2}d.$ **79.** 78·012 lb. **80.** £78 15*s.*
81. $3\frac{1}{3}$ gallons and $8\frac{2}{3}$ gallons. **82.** 78 %; 154 %; £3 0*s.* 11*d*
83. 15 min. 16 sec. **84.** 7012 francs per square kilometre.
85. (i) ·407. (ii) ·191. **86.** 273 sq. ft. **87.** £688 4*s.*
88. *A* 9·17 %, *B* 8·17 %, *C* 28·20 %, *D* 1·19 %, *E* 7·96 %, excess 45·31 %
89. 35 %. **90.** 23·5 yds. **91.** (i) $4\frac{307}{800}$; (ii) $6\frac{3}{4}$
92. $2\frac{1}{4}$ minutes. **93.** 2 years 31 weeks nearly.
94. $17\frac{823}{1057}$ days. **95.** 31 yds. 2 ft. 4 in.; 3*s* 9*d.*
96. 764·5 dollars.

XVIII a (p. 480).

1. £256; £3456. **2.** £261; £2001.
3. £19 5*s.* $2\frac{1}{2}d.$; £160 15*s.* $2\frac{1}{2}d.$
4. £151 7*s.* 11*d.*; £1881 12*s.* 11*d.*

5. £1185 19*s.* 9*d.*; £9657 6*s.* 3*d*
6. £303 18*s.* 1*d*; £2233 9*s.* 1*d.*
7. £1966 7*s.* 3*d.*; £10159 10*s.* 9*d.*
8. £113 5*s.* 3*d.*; £575 11*s.* 1*d.*
9. £23 3*s.* 11*d.*, £339 10*s.* 7*d.*
10. £108 12*s.* 6*d.*; £752 6*s* 4*d*
11. £31 8*s.* 11*d.*; £786 3*s.* 10*d.*
12. £16 6*s.* 11*d.*; £408 12*s.* 7*d.*
13. £6 2*s.*; £154 0*s.* 4*d.*
14. £688 12*s.* 10*d.*; £3715 12*s.* 10*d*
15. £1360 13*s.* 4*d.*; £5483 18*s.*
16. £29 18*s.* 4*d.*
17. £16 3*s.* 3*d.*
18. £15 12*s* 10*d.*
19. £4 16*s.* 8*d.*
20. £103 19*s.* 9*d.*
21. £253 9*s.* 2*d.*
22. *P rt* / 100.

24.

Days.	2 per cent.	3 per cent.	5 per cent.
1	·0000548	·0000822	·0001370
2	·0001096	·0001644	·0002740
3	·0001644	·0002466	·0004110
4	·0002192	·0003288	·0005479
5	·0002740	·0004110	·0006849
6	·0003288	·0004932	·0008219
7	·0003836	·0005753	·0009589
8	·0004384	·0006575	·0010959
9	·0004932	·0007397	·0012388
10	·0005479	·0008219	·0013699
20	·0010959	·0016438	·0027397
30	·0016438	·0024658	·0041096
40	·0021918	·0032877	·0054795
50	·0027397	·0041096	·0068493
60	·0032877	·0049315	·0082192
70	·0038356	·0057534	·0095890
80	·0043836	·0065753	·0109589
90	·0049315	·0073973	·0123288
100	·0054795	·0082192	·0136987
200	·0109589	·0164384	·0273973
300	·0164384	·0246575	0410959

24. (*a*) 6*s.* 10½*d.* (*b*) £3 2*s.* 5*d* (*c*) £1 19*s.* 4*d.*
25. £4 19*s.* 6*d.*
26. £1 16*s.* 2*d.*
27. (*a*) 18*s.* 1*d*
27. (*b*) 1911, £2 16*s.* 3*d.*; 1912, £3 0*s.* 4*d.*
27. (*c*) 13*s.* 3*d.*
28. 391·65 francs.
29. 4035·06 francs.
30. 474·92 francs.
31. $261·11.
32. $557·74.
33. The second by 5·71 marks.

XVIII b (p. 485).

1. £2897 6*s* 5*d*
2. £1313 8*s*. 8*d*
3. £1249 0*s*. 4*d*.
4. £577 3*s*. 5*d*.
5. £411 3*s*. 6½*d*.
6. £523 3*s*. 4*d*
7. £2840.
8. £3808 17*s*. 9*d*.
9. 3½ %
10. 1⅔ %.
11. 3½ %.
12. £435 18*s*. 4*d*.
13. 4 years.
14. 4 years.
15. 301 days.
16. 5 %.
17. 5½ years.
18. £1830 7*s*. 2*d*.
19. 4½ %.
20. £6825
21. 3½ %.
22. 4¾ years.
23. 3½ %.
24. 8⅗ years
25. 14·8 %
26. 22⅔ %.

XVIII c (p. 494).

1. £3 17*s* 9*d*.; £759 7*s*. 3*d*.
2. £4 1*s* 6*d*.; £513 8*s*. 9*d*
3. £3 9*s*. 3*d* ; £254 4*s* 6*d*.
4. £2 2*s* 9*d* , £810 12*s*. 5*d*
5. £3 15*s*. 11*d* , £539 11*s*. 6*d*.
6. £8 3*s* 4*d*., £967 14*s*.
7. 3*s* 4*d*.
8. £1237 5*s*.
9. £589 1*s* 6*d*.
10. £1822 6*s*.
11. 6727·5 francs
12. 36 45 francs
13. 4½ %.
14. Nov. 19th.
15. 4·71 %
16. Feb. 25th.
17. £52 1*s* 7*d*.
18. £39,023 16*s*.
19. Commercial Discount at 4¾ %.
20. May 4th.
21. To *B*'s advantage by about 1 day.
22. *A* pays 9 days late, i. e. he gains.
23. £1282 13*s*. 6*d* on Oct 1st
24. £2304 7*s*. 6*d*. on Dec 10th
25. After 1¼ years.
26. £3778 18*s*. 8*d*.
27. £5487 11*s*. 9*d*.
28. £1205 1*s*. 6*d*.
29. £1896 11*s*. 2*d*.
30. £1431 14*s*

XVIII (p. 497).

1. £565 16*s* 8*d*
2. £571 10*s*. 5*d* , 7500 francs
3. £315 10*s*. 9*d*.; 2½ years.
4. 4½ %.
5. 6000 francs.
6. 2 years 11 days.
7. 6 %.
8. £140 12*s* 6*d*.
9. £5 9*s*.
10. £105.
11. 3 %.
12. £170 19*s*. 3*d*.
13. 4 %.
14. 3¾ %.
15. £32.
16. 4 years 312 days.
17. £1580.
18. 3 years 7 months.
19. £172 2*s*. 10*d*.
20. 3½.
21. £2 19*s*. 10*d*.
22. 12*s*. 1*d*.
23. £2400; £2399 11*s*. 6*d*., ·018 %.
24. 2½ %.
25. £3850; 5⅘ %.
26. £20076.

27. £817 1*s.* 8*d.*; £1129 3*s.* 4*d.* **28.** £7700.
29. 3 years 7 months **30.** £50 11*s* 11*d.*, £53 2*s.* 6*d.*; 5 %.
32. £2400; £125.
34. Commercial to true discount as 5 to 4 in both cases
35. 22·7 months **36.** £1865 14*s.* 10*d*
37. £14 15*s.* $4\frac{1}{2}d.$; 5*s.* $5\frac{1}{4}d$ **38.** £666 12*s.* 7*d.*, £666 13*s* 4*d*

Oral Examples (p. 503).

1. £50.	**2.** £21	**3.** £37 10*s.*	**4.** £30
5. £14.	**6.** £69	**7.** £247 10*s*	**8.** £45.
9. £13 10*s.*	**10.** £63 5*s.*	**11.** £4 18*s.*	**12.** £131 5*s*
13. £94 10*s.*	**14.** £50	**15.** £39	**16.** £17 10*s*
17. £135.	**18.** £80 10*s*	**19** £343.	**20.** £153

XIX a (p 504).

1. £26 18*s* 10*d* **2.** £62 9*s* 2*d* **3.** £26 14*s.* 7*d*
4. £229 5*s.* 2*d.* **5.** £21 17*s.* 6*d* **6.** £403 17*s* 6*d*
7. £238 10*s.* **8.** £13 14*s* 5*d.* **9.** £30 0*s* 3*d.*
10. £263 12*s.* 5*d.* **11** £357 18*s.* **12.** £22 14*s.* 4*d.*
13. £42 15*s.* **14.** 6·23 %. **15.** £708,750; £10,500,000.
16. Preference, (i) 0. (ii) $70,000. (iii) $70,000
Ordinary, (i) 0 (ii) 0 (iii) $355,000
Debentures, (i) $75,000 (ii) $75,000 (iii) $75,000
17. £1,130,491 7*s* 4*d* ; 11·30 % **18** £16,000.
19. £7 7*s* 5*d.* **20.** £227 2*s* 8*d.*, £14 3*s.* 11*d*

Oral Examples (p. 507).

1. £375.	**2.** £368.	**3.** £825	**4.** £2360.
5. £804.	**6.** £1200.	**7.** £735	**8.** £872.
9. £825.	**10.** £2375.	**11.** £787 10*s.*	**12.** £1164.
13. $£2\frac{1}{2}$ loss.	**14.** $£5\frac{1}{4}$ gain.	**15.** £51 gain.	**16.** £90 loss.
17. $£2\frac{1}{2}$ gain.	**18.** £56 loss.	**19.** £2,940,000.	**20.** £1030.
21. $£2\frac{1}{2}$.	**22.** £5.	**23.** £3.	**24.** £6.
25. £7 10*s.*	**26.** £20.	**27.** £50.	**28.** £1.
29. £3 15*s.*	**30.** £15.	**31.** £82 10*s.*	**32.** £150.
33. £303.	**34.** £436.	**35.** £567.	**36.** £188 10*s*
37. £495.	**38.** £588.	**39.** £364.	**40.** £307 10*s*

XIX b (p. 508).

1. £1281 5*s*.
2. £1219 10*s* stock
3. £1805 14*s*
4. 77.
5. £342 7*s*. 6*d*
6. £641 3*s* 10*d*.
7. £5020 10*s*.
8. £1932 7*s*. 4*d*. stock
9. £415 10*s*.
10. £17 6*s*. 6*d*
11. £17 19*s* 1*d*
12. £188 2*s*. 6*d*.
13. £191 19*s*. 3*d*.
14. £14 4*s*. 1*d*.
15. £18 1*s* 10*d*.
16. £34 3*s*. 11*d*.
17. £9050.
18. $92\frac{1}{4}$
19. £5600.
20. £1592 8*s* 4*d*
21. £37 8*s*
22. £2473 10*s*.
23. £5370
24. £7 2*s*. 6*d*
25. £12346 5*s*.
26. $145\frac{1}{4}$.
27. £106 17*s* 6*d*.
28. £323 5*s*.

XIX c (p. 512)

1. 3·9 %.
2. 3·8 %.
3. 3·9 %.
4. 3·96 %.
5. 4·6 %
6. 4·7 %.
7. 5·94 %.
8. 3·9 %.
9. 8·5 %.
10. 4 3 %.
11. £10 4*s*. 9*d*. His investment is not so safe.
12. £13 15*s*.
13. $88\frac{8}{9}$.
14. New South Wales stock at 101.
15. £218.
16. £35 9*s* 3*d*.
17. £11,900.
18. £10; $4\frac{4}{9}$ %, 4 %.
19. The second income was greater by £15 12*s* 6*d*.
20. £3 2*s*. 3*d*.
21. £3 10*s* $3\frac{1}{4}d$.
22. £3 11*s* 9*d*.
23. £3 2*s*. 10*d*.
24. 3 %.
25. £8085
26. 95.
27. £936 10*s*. 8*d*

Oral Examples (p. 514).

	(*a*)	(*b*)		(*a*)	(*b*)		(*a*)	(*b*)
1.	$90\frac{1}{8}$	$90\frac{3}{8}$.	2.	$78\frac{3}{4}$	79.	3.	$90\frac{3}{4}$	91.
4.	$100\frac{7}{8}$	$101\frac{1}{8}$.	5.	$87\frac{7}{8}$	$88\frac{1}{8}$	6.	$95\frac{1}{8}$	$95\frac{3}{8}$.
7.	$100\frac{7}{8}$	$101\frac{1}{8}$.	8.	$76\frac{3}{8}$	$76\frac{5}{8}$.	9.	$58\frac{7}{8}$	$59\frac{1}{8}$.
10.	$93\frac{5}{8}$	$93\frac{7}{8}$.						

11. £7.
12. £7.
13. £7
14. £7.
15. Client receives £7 less.
16. Client pays £7 more.
17. Client receives £7 less.
18. Client pays £7 more.
19. £94 2*s*. 6*d*.
20. £204 15*s*.
21. £342.
22. £379 7*s*. 6*d*
23. £310 2*s*. 6*d*

XIX d (p. 515).

1. £1479 4*s.*; £1 17*s.* **2.** £7 15*s.* **3.** £18,700.
4 (*a*) £8 13*s.* 7*d.* (*b*) £8 13*s.* 7*d.* **5.** 2·8 %
6. £7 10*s.* loss. **7.** £66 increase. **8.** £712 10*s.*
9. 33. **10.** £1283 18*s.* 7*d.*

XIX e (p. 516).

1. £3 9*s* 5*d.* gain. **2.** £2952 9*s.*
3. £1 7*s.* 4*d.* gain. **4.** £121 19*s.* **5.** £8266 13*s.* 4*d*
6. ·0344 % rise. **7.** £22 19*s.* 1*d.* **8.** £1 8*s.* fall.
9 £12 3*s* 9*d* **10** £3490.

XIX (p. 519).

1. £1100; £100. **2.** £104 3*s.* 4*d.* gain.
3. £6310 2*s.* 6*d.* **4.** $76\frac{1}{13}$. **5.** $91\frac{1}{2}$.
6. £1512 10*s.*; £45 17*s.* 7*d* **7.** 108; 4 %.
8. £2818; £40 18*s.* gain. **9.** £15 16*s.* 8*d.*; £570.
10. £68 15*s.* gain. **11.** £69; £2300. **12.** £7840.
13. £6000. **14.** £4000. **15.** £4200. **16.** 4 21 %.
17. £200. **18.** £12,000; £50,000.
19. To purchase equal amounts. **20.** £30 loss.
21. £570 gain. **22.** £3600; £33,500. **23** £11 15*s.* increase.
24. £3000. **25.** Diminished by $7\frac{2}{59}$ % (7·56 %).
26. £6000; £80. **27.** The 5 % stock. **28.** 4 %.
29. $93\frac{1}{8}$. **30.** £59 10*s.*; £4 5*s.* gain. **31.** £13 loss.
32. £50 3*s.* increase. **33.** £168 8*s.* 5*d.* **34.** £35,000.
35. £3 15*s.* increase. **36.** £163 15*s.* gain.
37. £1801 16*s.*; £49 11*s.* **38.** £317,050. **39.** £2400.
40. $3\frac{3}{4}$ %. **41.** £37; £1 5*s* loss. **42.** £30; £6 14*s.* gain.
43. £78 7*s.* 6*d.*; £74 9*s.* $1\frac{1}{2}$*d.* **44.** £407; £800. **45.** 105.
46. 160. **47.** 86. **48.** The 4 % stock; £13.
49. £2882 5*s.* **50.** £2942 10*s.*; £96 5*s.*
51. 7*s.* 9*d.* **52.** £11 8*s.* **53.** £14,400; $83\frac{1}{3}$.
54. £15,000; £9400, £5600. **55.** £3657 10*s.* **56.** 2·96 %.
57. 3·86 %. **58.** He loses £264 money. **59.** £7590.
60. 1·5 per cent., approximate gain.

XX a (p. 529).

1. £366 2*s*. 4*d*.; £21 0*s* 4*d*.
2. £440 12*s*. 11*d*.; £33 4*s*. 11*d*
3. £8626 16*s*. 6*d*.; £1174 13*s*.
4. £1925 15*s*.; £162 5*s*. 8*d*
5. £337 1*s*. 9*d*.; £91 17*s*. 9*d*.
6. £637 14*s*. 2*d* ; £45 10*s*. 8*d*.
7. £5366 2*s*. 3*d*.; £444 13*s*. 3*d*.
8. £491 9*s*. 11*d*.; £37 6*s*. 3*d*.
9. £551 7*s*. 11*d*.; £35 3*s*. 1*d*.
10. £2134 11*s*. 3*d*.; £211 7*s*. 3*d*.
11. £40 6*s*. 4*d*.; £5 3*s*. 8*d*.
12. £549 10*s*. 4*d*.; £58 9*s*. 2*d*.
13. 12*s*.
14. £2 17*s*.
15. £48 14*s*. 3*d*.
16. £4 2*s*. 6*d*.
17. £11 16*s*. 8*d*.
18. £5 6*s*. 9*d*
19. £2 14*s*. 4*d*.
20. £488 10*s*. 4*d*.
21. £1180 13*s*. 2*d*
22. £599 1*s*. 1*d*.
23. £5492 6*s*. 6*d*
24. £11,430
25. £12,151 16*s*. 9*d*.
26. 2,825,761; 2,560,000.
27. £20 15*s*. 8*d*.
28. 244,029.
29. ·2180.
30. £520 16*s*. 8*d*.
31. 2,450,086
32. £33 2*s*. 3*d*.; £127 13*s*. 10*d*.
33. (i) 3·530 %. (ii) 3·546 %.

XX b (p. 533).

1. £733 6*s*. 8*d*.
2. £450.
3. £2500.
4. £25,000.
5. $3\frac{1}{2}$ %.
6. £433 6*s*. 8*d*.
7. £266 13*s*. 4*d*.
8. £5000.
9. £1727 12*s*.
10. £361 2*s*.
11. £908 10*s*. $2\frac{1}{2}$*d*.
12. 17·7 years; 14·2 years, 11·9 years.

XX c (p. 538).

1. £143 6*s*.
2. £598 18*s*.
3. £30.
4. £1,503,160; £2,000,000.
5. $10\frac{1}{2}$ years nearly.
6. 360 feet.
7. £468 11*s*. 2*d*.; £1000.
8. £1912 9*s*. 9*d*.
9. £1413 4*s*. (nearly).
10. £416 5*s*. (nearly).
11. £246 10*s*.
13. £294 4*s*.
15. 5·936 feet.

XX d (p 542).

1. $\cdot 58\dot{3}$ **2.** $\cdot 2\dot{7}$. **3.** $\cdot \dot{2}85714\dot{2}$. **4.** $\cdot \dot{3}84615\dot{3}$.
5. ·5625. **6.** $\cdot \dot{7}14285\dot{7}$. **7.** ·09375. **8.** $\cdot \dot{6}15384\dot{6}$.
9. $\cdot 10416\dot{6}$. **10.** $\cdot 2\dot{6}$. **11.** $\cdot 0\dot{7}$. **12.** $\cdot \dot{0}12345679\dot{0}$
13. If denominator contains no factors other than 2 or 5, the decimal terminates; if it contains any other factor it does not terminate.
15. $\frac{5}{6}$ **16.** $\frac{1}{11}$. **17.** $\frac{3}{7}$ **18.** $\frac{16}{45}$. **19.** $\frac{137}{175}$
20. $\frac{257}{350}$. **21.** $\frac{123711}{499950}$. **22.** $\frac{1851}{6660}$. **23.** $\frac{247787}{999990}$.

XX (p 543).

1 £13,770. **2.** £145 19*s.* 3*d* **3.** £2343 15*s*
4. £1 18*s* 10*d.* **5.** 18*s* 7*d* **6.** 16·6 %.
7. £275 9*s* 10*d* **8.** Simple Interest. **9.** £1111 2*s* 3*d.*
11. To sell now **12** £1350 **13.** $97\frac{1}{4}$.
14. Actually 48·8 % **15.** £1841 **16.** $3\frac{1}{4}$ %.
17. 1 year £1 04040, 2 years £1 08243, 3 years £1·12616
18. ·1475. **19** £475 10*s.* 9*d.* **20.** £280 1*s*
21. 3,815,000 **22.** 224,700 **23** 22 years
24. 6 %. **25** 30 years **26.** £100 (£99·93)

Miscellaneous Examples (p. 546).

1 1·3956. **2.** 196 sq yds. **3.** About 28 minutes
4 1658 mm 3317 mm **5.** 0, 2 **6.** $88\frac{1}{8}$ cu. ft
7 £1 2*s.* 5*d.* **8** 1296 white 1008 black.
9. $2^2.3^3.7^2$ 13 3.13 2.7 13^2. **10** 175 lb.
11. 2*s.* $5\frac{3}{4}$*d.* **12** 75 cents **13.** 139. **14.** i.e. (45) 45 sq in
15. 59 lb. **16.** ·4912 lb. **17.** 22 **18** 100·97 lb.
19. 1·28, 2·652, ·884 metres **20.** £4 8*s.* 11*d.* **21.** £1280.
22. English mile 1·610 Km. Austrian mile 7·591 Km.
English geog mile 1·852 Km Dutch ure 5·568 Km.
Kilometre 1·000 Km. Norwegian mile 11·306 Km
Russian verst 1 068 Km.
23. 256 cu. ft. 7·36 tons. **24.** 125 yds. **25.** £8 5*s.*

26. 487,lb. **27.** 12 in **28** ·360.
29. *A* £2040 *B* £1020 *C* £340 **30.** 4·61 % increase
31. £3 1*s* **32.** 18*s* 4*d*. **33** £60 **34** 44,01 miles
35. 5*s* 4*d* in the £.
A £66 13*s*. 4*d* *B* £1552 *C* £97 6*s* 8*d*. *D* £48.
36 77·4 cm **37.** 7 cm **38** 62 lb 7 oz
39 145 3 lb **40** 62½ %. **41** £423 15*s* 3·2
42. £12 16*s* 6*d* **43** 20 12 metres **44.** 2 4 in
45. 2·7. **46.** 21⅝ % **47** 576,000 l per Ha 230 tons per acre.
48. (*a*) The second (*b*) 372 sq cm and 448 sq cm.
49. ·410 ft. **50** £13 9*s* **51** 2127 tons
52. ·148 ft. by 118 ft 1 10 24 sq po. **53.** 20 %
54. 25 ft. 20 c **55.** ·582 %
56. The latter. Ratio = 1 240 **58** 22 oz
59. £33 6*s*. 8*d* £58 6*s*. 8*d*. **60.** £$a\left(1+\frac{r}{100}\right)^t$ £487 16*s*. 8*d*
61. 2 ft. 8 1 in **62** 37 5 %. **63** 32 % **64.** 5792 tons
65. 508 cub. in **66.** 8 or 24 m. p h
67. 404 acres 15 sq mi 80 acres **68** 6076 ft
69 10½*d* per £ **70** 1*s* 1½*d* **71** 453·1 g.
72. 3 hr 7½ min p m 22½ miles from *B* **73** £41 13*s*. 4*d*
74. 25 51 77 135 **75** 22 Kg. per sq m.
76. £197 19*s*. 9*d* **77** 26,034 **78.** 6½ mi. and 3½ mi
79. 44 % (percentage reckoned on selling price) **80** 48 m p. h.
81. £75 7*s* 11*d* 3·94 % **82** 8 % **83** 8 24 gr 59 5 %
84. £298 10*s*. 6*d* **85.** 49 % weight 25 % value
86. 3 miles. **87** 770,630 **88.** 49 % gold. 51 % silver
89. 10 % **90.** £70 14*s* 4*d*
91 −3·9 % 3 7 −3·8 % 2·8 %. **92.** 13 sec.
93. 17 8 %. **94** 12·72 m p. h. **95.** £863 16*s* 9*d*
96. Increase of £5 12*s*. 7*d*. **97** 71 3 %. 24 3 % 4·4 %
98. £135 (£134 5*s*.) **99.** 4·4 %. **100** 2 h. 56 m.
101. 14·18 m p. h **102** 17,016 **103** £1600.
104. 13, 31. 34·6 %. **105** 5 m p h. **106** 27 3 m. p h and 31 0 m. p. h.
107. 24½ lb. **108.** £*Pa*'. £295. **109** £800,000.
110. 69 7 m., 30·3 m. **111.** $\frac{m-bc}{n-b}$ lb **112.** 237·2 f. s.
113. Capital £15,000. Investments £9400 and £5600
114. £1841. **115.** 7 h. 17 m., 4 h. 10 m, 3 h 44 m.
116. 2*s*. 4*d*. per lb. **117.** 7 m. p. h. 14 m. p. h.

118. $\frac{p-mr}{q-r}$. $\frac{p-mq}{r-q}$ sq. miles. **119.** £1339 5*s.* 9*d.* £1333 9*s.* 11*d.*
120. 61 Kg. $\frac{1}{8}$ or 8 Kg. **121.** 3*s.* 9$\frac{1}{4}$*d.* per lb. or 2$\frac{3}{4}$*d.* per oz.
122. 3$\frac{1}{4}$ m. p. h. 2 hrs. $\frac{1}{3}$ hr. **123.** £56 12*s.* 6*d.* **124.** £58,000
125. 3 hrs. 20 min. **126.** 121 yds. **127.** 1·76. **128.** 30 ft.
129. 115. **130.** 5615 ft. 5315 ft. **131.** £401 10*s.*
132. 56 % increase. 23 % decrease. 53 % decrease.
133. 4160 l. 55. **134.** £4875. **135.** $\frac{1056x}{y}$. $136\frac{8}{31}$.
136. ·645 m. **137.** £13 18*s.* loss. **138.** 5 cars.
139. 52·4, 45, and 47 8 m. p. h , 48, 51·4, and 48·2 m. p. h.
140. 12 hrs. 0 min. 44 sec. (nearly). **141.** ·4 mm **142.** 3 59 lb
143. £8333 6*s.* 8*d.* £25,000. £37,916 13*s.* 4*d.* **144.** 2 to 1.
145. $\frac{a^2(100+c)}{b^2}-100$. **146.** £355 11*s.* 1$\frac{1}{2}$*d.* **147.** 39·37 in
148. 7·7 sq. m.
149. 32 trains and sometimes 33 in an hour. 31 trains.
150. 1428 m. from Liverpool, Tuesday, 2 30 a m. (Greenwich time), or 1617 m. from New York, Monday, 9 45 p.m. (New York time).
151. 116·6 ft. **152.** 11·57 %. **154.** 28·3 m. p h. 29 6 m. p h
155. £$\frac{(m+n)rp}{pm+qn}$. £$\frac{(m+n)rq}{pm+qn}$. **156.** 4 48 p.m. on Thursday
157. (*a*) 1·1 sq. cm. (*b*) 10 cu. cm. **158.** 8·56 sq in. £834 18*s*
(*a*) One sig. fig. (*b*) Two sig figs
159. $\frac{2880q-23bp}{460(12q-p)}$ shillings **160.** 1·14 miles.
161. $\frac{3}{5}$ in. **162.** 147 nuts.
163. (*a*) 6·24 % inc. 16·2 % dec. 38·7 % inc. 1·41 % inc.
(*b*) 1·29, 1·91, 2·40, ·266 millions.
164. 19·8 in. **165.** 2·4 cm. **166.** 81,451 cu. cm. **167.** 4·5 m. p. h.
168. 61·4 f. s. 58·7 f s. and 64·3 f. s. **169.** £1450.
170. Weds.: 5 hrs. 27 min 35 sec. 5 hrs. 22 min. 35 sec.
171. 28$\frac{4}{5}$ hrs. **172.** £$(a-bx)\div(1-x)$ per lb.
173. 5·3 sq. in. **174.** £12 8*s.*; £1 16*s.* 10$\frac{3}{4}$*d.*
175. 2·7 % in area. 1·3 % in length of chain.
176. 333$\frac{1}{3}$ tons. 1222 miles. **177.** 6132 tons. £2,034,000.
178. 55·1 cu. mm. **179.** 50·27 cu. ft. **180.** 9$\frac{3}{8}$ m. p.h. 5$\frac{5}{8}$ m. p. h
181. 75 sq. ft. 36 sq. in. **182.** £264 19*s.* 3*d.*
183. 2$\frac{1}{2}$ m. p. h. **184.** £3 16*s.* 3*d.* **185.** 3·375 in.
186. £2 7*s.* 4*d.* £1 5*s.* £1 0*s.* 6*d.* £2 10*s.* 2*d.*

187 (*a*) 27·8 %. (*b*) (i) 4·4. (ii) 4·0 (*c*) (i) 2·0 %., (ii) 8·4 %.
188. 417·8 gallons. 12,116 sq in. (9 sq. yds. 3 sq. ft. 20 sq. in.).
189. £538 6*s*. **190.** ·7 %
191. 10 m. p. h. 2970 ft. (my speed is $3\frac{1}{4}$ m p. h.)
192. 22·05 m. p. h. 37·5 min. **193.** £580. **194.** 1·6 %.
195. (i) Yes; (ii) 22·1 tons; (iii) 3·08 in. **196.** £2 1*s*. 11*d*.
197. £307 (to the nearest £). **198.** 6·298 cm.
199 67·08 ft. 12·1 ft. **200.** 11·4 sq. units.
201. 1·18 in. **202.** £853.
203. $2\frac{1}{2}$ metres. 7·069 sq. m **204.** 10·8 cm.
205. 6826 kilograms. **206.** 37·2 cu. in.
207. 2,787,000 tons.
208 £722 to the nearest £ ($x = 4{\cdot}024$).
209. 1658 metres; 3317 metres
210. 32·78; 33·66; 33·97; 32·29; 33·18.
211. 961,309,937,159. 964,292,660,069. **212.** 56 lb.

213. Rows:

(i)	£1095 5*s*. 6*d*.	(xvi)	£7848 15*s*. 5*d*.
(ii)	£3266 8*s*. 2*d*.	(xvii)	£5555 5*s*. 9*d*
(iii)	£1039 4*s*. 6*d*.	(xviii)	£3868 13*s*. 5*d*.
(iv)	£5071 9*s* 8*d*.	(xix)	£665 15*s*. 1*d*.
(v)	£4291 16*s*. 2*d*.	(xx)	£1516 6*s*. 8*d*.
(vi)	£3006 12*s*. 4*d*	(xxi)	£984 2*s*. 1*d*.
(vii)	£858 12*s*. 6*d*.	(xxii)	£1314 16*s*. 3*d*.
(viii)	£3935 1*s*. 5*d*.	(xxiii)	£1254 1*s*. 5*d*.
(ix)	£6297 12*s*. 8*d*.	(xxiv)	£7387 3*s* 3*d*.
(x)	£1044 10*s*. 5*d*.	(xxv)	£1906 19*s*. 2*d*.
(xi)	£436 4*s*. 2*d*.	(xxvi)	£3613 3*s*. 2*d*.
(xii)	£695 12*s* 1*d*.	(xxvii)	£9277 7*s*. 7*d*.
(xiii)	£1435 19*s*. 8*d*.	(xxviii)	£339 1*s*. 8*d*.
(xiv)	£9299 15*s* 6*d*.	(xxix)	£965 15*s*. 5*d*.
(xv)	£4978 15*s*. 6*d*.	(xxx)	£7453 13*s*. 6*d*.

Columns: £1296 18*s*. 6*d*.; £13,896 3*s*. 7*d*.; £85,510 18*s*.
Total: £100,704 0*s*. 1*d*.

214. 12 h. 38 m. 2 h. 8 m. 3 h. 49 m. 5 h. 34 m. 7 h. 24 m. 9 h. 26 m.

215. ·86 in. **216.** 6*s.* 3½*d.*

217. 46 %. £42,000,000 **218.** 114,591 ft.; ·19 in.

220. £816 19*s.* 6*d.* **221.** £1098. £4428 12*s.* 403·3 %. 8 yrs.

222. Rows:

(i)	£816 16*s.* 8*d.*	(xi)	£382 11*s.* 10*d.*
(ii)	£3275 8*s.*	(xii)	£659 12*s* 9*d.*
(iii)	£1040 7*s* 3*d*	(xiii)	£1245 19*s.* 8*d.*
(iv)	£5071 3*s.* 6*d.*	(xiv)	£9299 13*s.* 9*d.*
(v)	£4295 16*s.* 4*d.*	(xv)	£4315 16*s.* 0*d.*
(vi)	£2988 14*s.* 11*d.*	(xvi)	£5148 15*s.* 5*d.*
(vii)	£1255 12*s.* 1*d.*	(xvii)	£5555 6*s.* 7*d.*
(viii)	£3953 5*s.* 4*d.*	(xviii)	£3868 15*s.* 9*d.*
(ix)	£6144 13*s.* 8*d.*	(xix)	£417 11*s.* 3*d.*
(x)	£1026 7*s.* 11*d.*	(xx)	£1606 18*s.* 8*d.*

Columns: £855 13*s.* 8*d.* £9189 0*s.* 10*d.* £52,324 12*s.* 10*d.*

Total: £62,369 7*s.* 4*d.*

223. Nearly 23 min. late; 119 miles (nearly).

224. The latter by 17·2 Km. per hour.

225. $2·47; $15·57; 56 %. **226.** $58\frac{3}{1}$ should read 58750.

227. The first; 2·5 cu. ft. **228.** 10·9 acres; 3126 poles.

229. ·765; 2·433. **230.** £3 gain.

231. 2·918 tons. **232.** 78 ft.; 58 ft.

233 63·8 %. **234.** £1 2*s.* 6*d.*

235. ·31331. **236.** *A* 540, *B* 144.

237. £4 13*s.* **238.** 39·37

239. £111; £75 **240.** 9598.

241. (*a*) £306 3*s.* 8*d.*; (*b*) 29·21 francs; (*c*) 292·07 dollars.

242. Nothing but water; 1*s* 3½*d.*

243. £2414$\frac{2}{21}$; £2437½. **244.** 2·78; 2·718282.

245. 4 %. **246.** 3·284 %. **247.** 7 min. 38 sec. past one.

248. 4 in., 10 in., 28 in. **249.** 131$\frac{1}{61}$.

250. 48·6261 sq. in., 313·7055 sq. cm., 6 451. Correct to (i) units, (ii) units, (iii) first decimal place.

251. (i) 565 lb.; (ii) 9·1 cu. ft **252** 12; 10 miles per hour

253 $\sqrt{3}-\sqrt{2}$; ·318 **254** 12 m. 936$\frac{4}{7}$ yds.

255. 264 ft. **256.** £3 18*s.* 5*d.*; £92 1*s.* 3*d.* **257.** 2*s.*

258. 343 metres. **259.** £530 **260.** £20,000, £9900; £1100.

261. 100$\frac{1}{9}$. **262** Last number should read £1·265318 4 %.

263. 3$\frac{1}{2}$ sec. **264.** £51 **265** £94$\frac{1}{6}$ **266.** £85,085.

268. 49·22 %. **269.** 875 616, 2 991 **270.** 67·1 %.

271. £750. **272.** £693 2*s.* **273.** 0·07089

274. 482·4 yds., ·1993 acre. **275** (*b*) 1222 **276.** 9·63 %.

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PRINTER TO THE UNIVERSITY