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APPLIED MECHANICS

APPLIED MECHANICS

BY

J. BOOTHROYD, B.Sc., A.M.I.C.E.

HEAD OF THE MECHANICAL ENGINEERING DEPARTMENT,
COLLEGE OF TECHNOLOGY AND ARTS, RUGBY

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PREFACE

This book is written primarily for students in the third year of a Mechanical Engineering Course, but it should also be useful for students who have some elementary knowledge of Mechanics and require a general knowledge of Applied Mechanics.

The scope of the book has been made sufficiently large to allow for variations in the syllabuses of work taken in different Technical Institutions.

In order to obtain facility in the application of the principles of Applied Mechanics, and to impress those principles on his mind, it is necessary for a student to work a number of examples. At the end of each chapter will be found a large number of examples chosen for these purposes, many of them also amplifying the text. A neat and orderly method of laying out the working of an example is a great help to develop logical sequence of thought, and accuracy of working.

By the permission of the Union of Lancashire and Cheshire Institutes, the Union of Educational Institutions, and the Northern Counties Technical Examination Council, many of the examples are taken from past examination papers of these bodies.

J. B.

Feb. 23, 1931.

CONTENTS

CHAP.		PAGE
I	~~SIMPLE STRESSES AND STRAINS~~	1
II	~~FORCE DIAGRAMS FOR FRAMED STRUCTURES~~	28
III	~~BENDING MOMENTS AND SHEARING FORCES~~	40
IV	~~BEAMS AND BENDING~~	53
V	~~TORSION~~	66
VI	~~DYNAMICS~~	76
VII	BALANCING	94
VIII	PISTON VELOCITY AND ACCELERATION	101
IX	PISTON AND CRANK EFFORT	113
X	FRICTION	126
XI	HYDROSTATICS	137
XII	HYDRAULICS. FLOW OF WATER	144
XIII	WATER TURBINES	164
	ANSWERS TO EXAMPLES	175
	INDEX	181

APPLIED MECHANICS

CHAPTER I

SIMPLE STRESSES AND STRAINS

1. Nature of Forces. In designing any part of a machine, or structure, it is necessary to know (*a*) the nature and amount of the forces which that part will have to resist; (*b*) the strength of the material used and its behaviour when acted on by a force. In (*a*) are included one or more of the following—(1) the useful load carried; (2) the weight of the part itself; (3) forces due to friction; (4) forces due to the inertia of parts moving with varying velocity including centrifugal forces due to rotating masses; (5) forces due to expansion or contraction caused by change of temperature. It is also necessary to know whether the forces are applied slowly, rapidly, or in a repeated or alternating manner. In (*b*) are included the composition and physical properties of the material as found by analysis, and examination and testing of sample pieces in special testing machines. It is also necessary in many cases, e.g. high-pressure steam plant, to test the material at high temperatures.

2. Strain and Stress. When a force is applied to a piece of material the shape of that piece of material is changed; the proportional change in the dimension of the piece is called the *strain*. The resistance of the material to deformation causes internal forces, called *stresses*, in the material. If an ideal surface in the material be considered across which a force is being transmitted, the *intensity of stress* across this surface will be the force

transmitted per unit area if the force is transmitted uniformly over the area. When the force is not uniformly distributed over the area the intensity of stress at any point is the ratio of force to area over an indefinitely small area surrounding the point. The intensity of stress is very commonly referred to as stress, and the word stress will be used with this meaning in the following paragraphs.

3. Tensile Strain and Stress. If a piece of material be pulled in opposite directions by equal forces P, P at the ends the bar will be stretched from its original length l by an amount x to a length $l + x$.

The *tensile strain*, therefore, will be

$$x/l; \text{ or } e = x/l,$$

where e is the strain in the material.

Across any plane surface perpendicular to the axis of the force P internal forces are induced equal to P , and the *tensile stress* will be P/A , where A is the area of the cross-section; or $f = P/A$ where f is the stress in the material. This is assuming that the surface is taken some distance from the end where the intensity of stress may be considered uniform over the surface.

4. Compressive Strain and Stress. If the forces P, P in Article 3 act towards each other, the piece of material will be shortened an amount x . The *compressive strain*, therefore, will be x/l , and the *compressive stress* will be P/A .

5. Shear, Strain and Stress. If a piece of material be acted on by two equal parallel forces P, P , acting in opposite directions, but not in the same straight line, and the material is prevented from turning, there will be a tendency for the material to slide, or *shear*, along surfaces such as AB (Fig. 1), parallel to the axes of PP . The effect of this will be to distort the piece of material, a plane CD perpendicular to the axes of PP being distorted through an angle of ϕ radians. The *shear strain* will be ϕ . Since ϕ is always a very small angle under

ordinary working conditions, the shear strain may be taken as x/l (see Fig. 1) without appreciable error.

The shearing resistance of the material acts tangentially on surfaces such as AB, and the *shear stress* will be P/A , where A is the area of the surface.

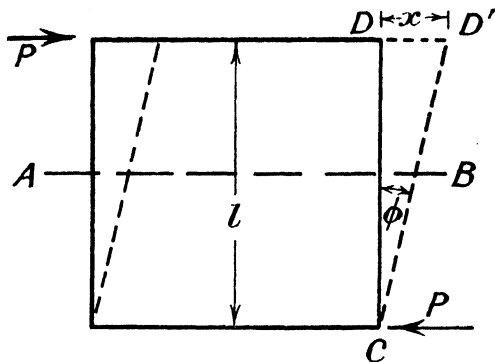


FIG. 1.

6. Elasticity. When a load is applied to a piece of material that piece is deformed; if the deformation disappears when the load is removed the material is said to be *elastic* up to that load, or to the stress produced by the load. The greatest stress, after which a given material can regain its original shape, is called the *elastic limit* of the material. If the material is loaded beyond the elastic limit, part of the strain produced will not disappear on removal of the load; that part of the strain which does not disappear is called the *permanent set*, and that part which disappears is often called *elastic strain*, though the term *elastic strain* is more correctly applied to strains produced by stresses within the elastic limit.

7. Plasticity. If none of the strain disappears on removal of the stress the material is said to be perfectly *plastic*. Soft clay, putty, wax are almost perfectly plastic. Some metals possess plastic properties at high

temperatures or under high stresses ; the former property is needed for forging, the latter for pressing and stamping.

8. Ductility. If a material is capable of being drawn out by tension to a smaller section, that material is said to be *ductile*. This property enables a material to be drawn out into wire. If a material can be hammered or rolled into sheets, that material is said to be *malleable*. Malleability is a very similar property to ductility.

9. Brittleness. When a material has no ductile or malleable properties it is said to be *brittle*.

10. Behaviour of Materials subjected to Tension. If a bar of ductile material be subjected to a gradually increasing tensile force in a testing machine, and a stress-strain graph be drawn, the shape of the graph will be

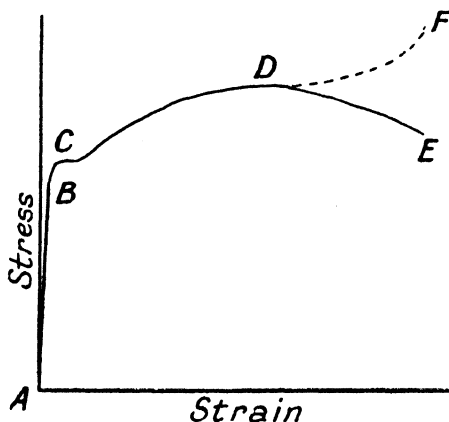


FIG. 2.

generally as shown in Fig. 2, which is the graph of a mild steel bar. The first part of the graph, A to B, is a straight line, the material obeying Hooke's Law up to the stress at B, or strain is proportional to stress. The point B is called the *limit of proportionality* of the material. With some metals, such as wrought iron and steel, the *elastic*

limit coincides with the limit of proportionality, but with other metals, such as rolled aluminium, the limit of proportionality is much lower than the elastic limit.

The extensions during this period are extremely small and can only be measured by an instrument called an *extensometer*, which magnifies the extensions. The slope of the graph decreases from B to a point C, where the strain suddenly increases, and is sometimes accompanied by a drop in the stress; this point is called the *yield point*. At this point the material appears to undergo some change, yielding suddenly and then standing up to the load again, for after this point the graph is a smooth curve becoming gradually flatter until a point, D, is reached, after which the curve drops to a point E at which the bar breaks. Up to the point D the bar has increased uniformly in length and decreased uniformly in diameter, the lateral strain being about one quarter the longitudinal strain, but the bar now begins to draw out at some point in its length, generally about the middle, and the area reduces rapidly here, producing *local stricture*. The curve from D to E shows the *nominal stress*, or stress calculated on the original area, which is less than the *real stress*, as the area is reducing rapidly. The curve of real stress would be as shown from D to F.

In commercial testing nominal stresses are usually given, that is, ultimate strength is equal to greatest load carried divided by original area; also the yield point is often taken as the commercial elastic limit.

The quantities *percentage reduction in area* and *percentage elongation* should be measured, as these give a measure of the ductility of the material. In stating the percentage elongation, the length on which it is measured must be stated, as this influences the percentage elongation. If a bar 8 inches long between gauge points has the inch lengths marked off before testing and after testing these are found to be 1.21, 1.22, 1.22, 1.25, 1.45, 1.57, 1.27, and 1.23 inches measured in order, fracture occurring between the 4- and 6-inch marks (Fig. 3); the

percentage elongation measured on 8 inches will be $\frac{2.42 \times 100}{8} = 30\%$ (2.42 inches being the total elongation on 8 inches), while the percentage elongation on 2 inches at fracture will be $\frac{1.02 \times 100}{2} = 51\%$.

A test piece of metal should be turned to the shape shown in Fig. 3 ; the centre part, on which the observations are

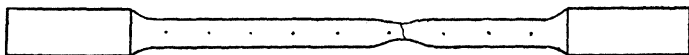


FIG. 3.

made, should be turned truly parallel, smooth, and co-axial with the enlarged ends left for the testing-machine grips, and there should be no internal angle where the bar changes section. The *gauge length* is a length measured, and marked, on the central section ; the ends of this length should be far enough away from the enlarged ends of the test piece for the stress to become uniform over the section. For comparison of results sizes of test-pieces are specified by the British Engineering Standards Association. Many firms use a test length of four times the square root of the cross-section area.

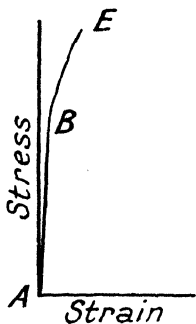


FIG. 4.

The graph described above can be obtained most readily by using an Autographic Recorder fixed to the testing machine.

When testing a bar of hard material, such as heat-treated alloy steel, there is often no marked yield point or local stricture, the elastic limit falling not far short of the breaking stress or ultimate stress. The stress-strain graph is of the form shown in Fig. 4.

A series of stress-strain graphs for different materials

are shown in Fig. 5. For the sake of clearness the curves are drawn from different points on the strain axis.

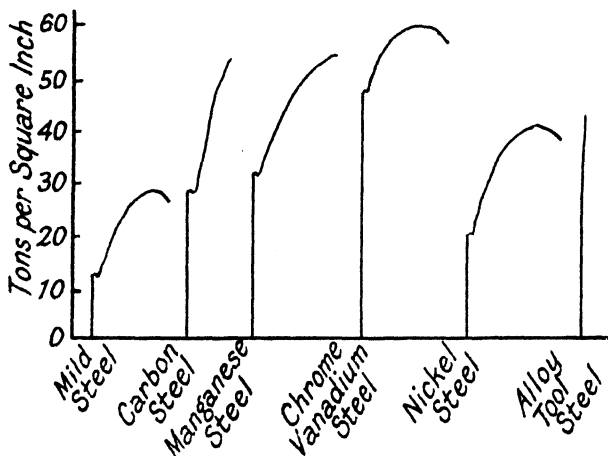


FIG. 5.

11. Raising the Elastic Limit. If a piece of ductile material be loaded beyond the elastic limit and then released, the next time the material is loaded the elastic limit will be raised approximately to the previous load. This process may be repeated several times, the elastic limit approaching the ultimate strength of the material, and the ultimate strength itself being raised. The material is then said to be *hard*, and possesses little, or no, ductility. If the material be now annealed, the hardness and high elastic limit and ultimate strength will disappear, and the material will revert to its original properties.

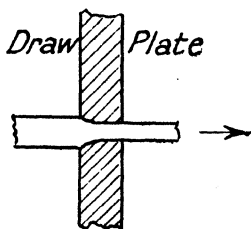


FIG. 6.

These phenomena occur in wire drawing, where the process of drawing (Fig. 6) is equivalent to producing an

elongated stricture. After several passes the wire becomes hard, the ultimate strength is raised considerably, and the ductility is reduced. If further passes are required the wire must be annealed.

12. Behaviour of Materials subjected to Compression. To get the true strength in compression short specimens must be used, of a length about two and a half times the diameter, to prevent bending. Ductile materials behave in much the same way as in tension

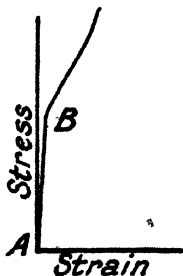


FIG. 7.

up to the elastic limit. At the yield point, however, the strain does not increase suddenly, and when the plastic stage is reached the material spreads with increase of sectional area. Very soft materials will squeeze out to a flat disc without fracture. The form of the stress-strain graph is shown in Fig. 7. Brittle materials have no marked elastic limit; there is little or no spread, and fracture occurs. When fracture occurs one part of the specimen slides over

the other at an angle of about 45° , or the specimen breaks up into prisms or pyramids, as shown in Figs. 8 and 9.

13. Behaviour of Materials subjected to Shear. Ductile materials in shear pass through an elastic stage similar to that of materials in tension, with an elastic limit, but there is no sudden yield, or alteration of area. The shearing strength of ductile materials, both at the elastic limit and ultimate strength, is about $4/5$ of their tensile strength.

Brittle materials in shear are elastic up to the point of fracture.

14. Strength of Timber. Wood is not a homogeneous material and its strength is not uniform; being different along and across the grain; and depending on the part of the tree from which the sample is taken, i.e. heartwood or sapwood, and also on the seasoning. When testing in tension, large specimens should be used if possible to get a more average result. When wood is used in

SIMPLE STRESSES AND STRAINS

tension the pull should be along the grain, as the strength is greater in that direction than across the grain. The compressive strength is also greater along the grain than across it.

The commonest test for wood is the Bending Test, rectangular test beam being supported at the ends and loaded at the centre, the *modulus of rupture* being given by

$$f = \frac{3Wl}{2bd^2}$$

where W is the central breaking load, l the length of span

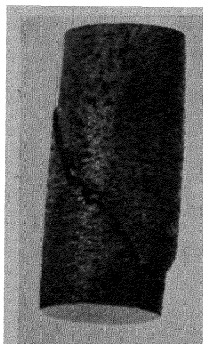


FIG. 8.—Cast Iron.



FIG. 9.—Concrete.

b the breadth and d the depth of the section, the dimensions being in inches. (See Art. 34.)

15. Modulus of Elasticity. Since the working stress in any material should never reach the limit of proportionality, the relation

$$\text{stress} \div \text{strain} = \text{modulus}$$

may be used in calculations on stresses in loaded members. The equations (Art. 3) $e = x/l$, $f = P/A$, and $E = f/e$

may be combined into the equation $E = \frac{Pl}{Ax}$.

EXAMPLE. A steel rod 2 inches diameter 12 feet long is stretched 0.058 inch. Find the load on the rod and the stress induced. $E = 13,000$ tons per square inch.

$$P = \frac{EAx}{l} = \frac{13,000 \times 3.14 \times 0.058}{144} = 16.5 \text{ tons.}$$

$$f = P/A = 16.5/3.14 = 5.25 \text{ tons per square inch}$$

16. Resilience. The work done in elastic stressing of a bar is called the *resilience* of the bar. If the bar is stressed up to the elastic limit this is then called the *proof resilience* of the bar. If a bar is stretched or compressed an amount x by a gradually applied load, the work done will be $\frac{1}{2}Px$, where P is the load corresponding to the stretch or compression x and $\frac{1}{2}P$ is the average load during the straining period. Since $P = Af$, $x = fl/E$, and $Al = V$ where V is the volume of the bar,

$$\text{resilience} = \frac{Afl^2}{2E} = \frac{Vf^2}{2E}.$$

The resilience of a bar is a measure of its power to resist a blow or shock without taking a permanent set.

EXAMPLE. Find the resilience of a steel bar 1 inch diameter, 3 feet long when carrying a tensile load of 4 tons. $E = 13,000$ tons per square inch.

$$f = 4/.7854 = 5.1 \text{ tons per square inch.}$$

$$\text{Resilience} = \frac{Afl^2}{2E} = \frac{.7854 \times 36 \times 5.1^2}{2 \times 13,000} = 0.028 \text{ inch-tons.}$$

17. Suddenly Applied Load. Let a bar AB of

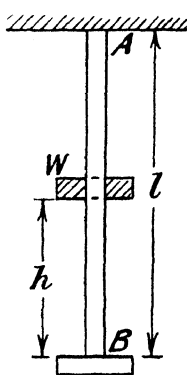


FIG. 10.

length l and area of cross-section A be rigidly suspended from one end, and have a ring weight W threaded on it, and a head formed on the lower end. If the weight be allowed to fall through a height h on to the head, the bar will be instantaneously stretched an amount x , and the maximum stress, f , in the bar will be given by $x = \frac{fl}{E}$. If the bar is not stressed

beyond the elastic limit, the work done on the bar will be $\frac{1}{2}Afx$, which will be equal to the loss of potential energy of the weight, $W(h + x)$.

Therefore $W(h + x) = \frac{1}{2}Afx$

and $W\left(h + \frac{fl}{E}\right) = \frac{Af^2l}{2E}$ since $x = \frac{fl}{E}$,

which reduces to the form

$$f = \frac{W}{A} + \frac{\sqrt{W^2 l^2 + 2WEAlh}}{Al}$$

If $h = 0$, that is, the weight is not dropped from a height but only applied suddenly, then $f = \frac{2W}{A}$. Therefore the stress induced by a suddenly applied load is twice as great as the stress induced by the same load if applied gradually.

18. Repeated and Reversed Stresses. Loads on engineering members may be applied (a) gradually, (b) rapidly, (c) repeatedly, or (d) repeatedly reversed in kind. Wöhler was the first to investigate the last two cases; his results, which were published in 1870, some of which are given in the table below, are still in use, though much work has been done since by other investigators. The results of many experiments show that materials will break under much lower repeated stresses than static stresses, the factors controlling the breaking stress being the maximum stress reached, and the *range of stress*, that is, the algebraic difference between the maximum and minimum stresses.

In the table below are given some of Wöhler's results for axle steel, the stresses being in tons per square inch.

Maximum Stress (Tension).	Minimum Stress (Compression).	Range of Stress.	Number of Repetitions before Fracture.
+ 15.3	— 15.3	30.6	56,430
12.4	12.4	24.8	479,490
9.6	9.6	19.2	41,917,992
7.6	7.6	15.2	132,250,000 (not broken)
21.01	0	21.01	106,910
17.19	0	17.19	409,481
15.28	0	15.28	10,141,645
	(tension)		
+ 21.01	+ 9.55	11.46	2,373,424
+ 21.01	+ 11.46	9.55	4,000,000 (not broken)

Thus the limiting maximum stress for the three types of load are as given in the following table.

Kind of Repeated Load.	Limiting Maximum Stress.	Limiting Range.
Completely reversed . . .	7.6	15.2
From maximum to zero . .	15.28	15.28
From maximum to half . .	21.01	9.55

Later investigation confirms the general results given above. The speed of reversal is found to have an effect on the limiting range of stress; up to 1,000 reversals per minute there is no noticeable change, at speeds from 1,400 to 2,400 per minute the range is considerably less, while at high speeds (about 7,000) the range is greater.

From the experimental data obtained several formulæ have been constructed for the use of engineers. One of these of general application, known as the Launhardt-Weyrauch formula, is

$$f_{\max.} = f_1 + (f - f_1) \frac{f_{\min.}}{f_{\max.}}$$

Where $f_{\max.}$ is the required limiting stress which the material can withstand for an unlimited number of repetitions or reversals, f_1 is the value of $f_{\max.}$ when the lower stress $f_{\min.}$ is zero, and f is the ultimate strength in tension under a steadily applied load. When $f_{\min.}$ is compressive, its sign changes to minus. It is also assumed that $f_1 = \frac{1}{2}(f + f_2)$ where f_2 is the limiting stress for complete reversal.

19. Factor of Safety. The load which a member of a structure carries is called the *working load*, and the stress induced the *working stress*. The working stress must obviously be less than the ultimate strength of the material. In practice it is usual to divide the ultimate stress by a suitable *factor of safety* to get the allowable working stress. In choosing a suitable factor of safety the following points must be considered—(1) the maximum load and the certainty with which it can be stated; (2) the

nature of the load, gradually or suddenly applied, repeated or reversed ; (3) the reliability of the material and the certainty with which its strength can be stated ; (4) the effect of corrosion or wear on the dimensions of the member ; (5) errors in workmanship ; (6) the consequences of a breakdown.

Unwin gives the following table of Factors of Safety :

Material.	Factors of Safety for			
	Dead Load.	Live or Varying Load.		Structure subject to Shock.
		Stress of One Kind only.	Reversed Stresses.	
Cast iron	4	6	10	15
Wrought iron and steel	3	5	8	12
Timber	7	10	15	20

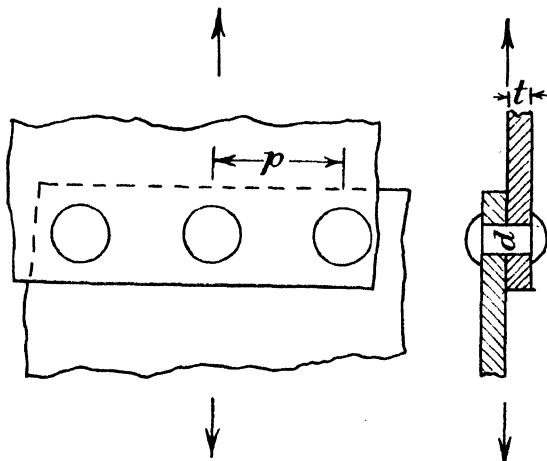


FIG. 11.

20. Riveted Joints. A simple lap-riveted joint (Fig. 11) when under tension may fail (1) by tearing the

plate between the rivets; (2) by shearing the rivets; (3) by crushing the rivet or plate; (4) by tearing the plate between the rivet-hole and the edge of the plate.

Let

p = pitch of rivets. f_t = tensile stress in plates.
 d = diameter of rivets. f_s = shear stress in rivets.
 t = thickness of plates. f_c = crushing stress in rivets
 or plates.

Consider a strip of plate equal in width to the pitch. If the distance from the rivet-hole to the edge of the plate is made equal to the diameter of the rivet the joint will not fail here.

$$\text{Resistance of strip to tearing} = (p - d)tf_t \quad . \quad (1)$$

$$,, \quad ,, \quad \text{shearing} = \frac{\pi}{4}d^2f_s \quad . \quad . \quad (2)$$

$$,, \quad ,, \quad \text{crushing} = dtf_c \quad . \quad . \quad (3)$$

Since the values of the working stresses can be obtained for any given case, these give two equations to find p and d .

In many cases d is fixed by an empirical rule (Unwin, $d = 1.2\sqrt{t}$) when (1) and (2) are equated to find p .

The ratio of the strength of the riveted joint to that of the solid plate is called the *efficiency of the joint*. There are three efficiencies, the resistance of the solid plate being ptf_t .

$$\text{Tearing efficiency} = \frac{(p - d)tf_t}{ptf_t} = \frac{p - d}{p}.$$

$$\text{Shearing efficiency} = \frac{\pi d^2 f_s}{4ptf_t}.$$

$$\text{Crushing efficiency} = \frac{dtf_c}{ptf_t} = \frac{df_c}{pf_t}.$$

If there are n rows of rivets the shearing and crushing resistances must be multiplied by n , the tearing resistance remaining as above.

EXAMPLE. Find the pitch of rivets for a double riveted lap joint, $t = \frac{1}{2}$ inch; $d = \frac{1}{8}$ inch; $f_t = 6$, $f_s = 5$, and $f_c = 12$ tons per square inch. Calculate the efficiencies of the joint.

$$(p - d)tf_t = 2 \times \frac{\pi}{4} d^2 f_s$$

$$(p - 0.9375) \times \frac{1}{2} \times 6 = \frac{2 \times \pi \times 0.9375^2 \times 5}{4}$$

$$p = 3.24 \text{ inches.}$$

$$\text{Tearing efficiency} = \frac{p - d}{p} = \frac{3.24 - 0.9375}{3.24} = .71$$

$$\text{Shearing efficiency} = \frac{2\pi d^2 f_s}{4ptf_t} = \frac{2 \times \pi \times 0.9375^2 \times 5}{4 \times 3.24 \times 0.5 \times 6} = .71$$

$$\text{Crushing efficiency} = \frac{2df_c}{pft} = \frac{2 \times 0.9375 \times 12}{3.24 \times 6} = 1.15.$$

The solid plate would tear before the rivet crushes.

A butt joint with single cover plate (Fig. 12) is similar as regards calculations of strength to a lap joint. A

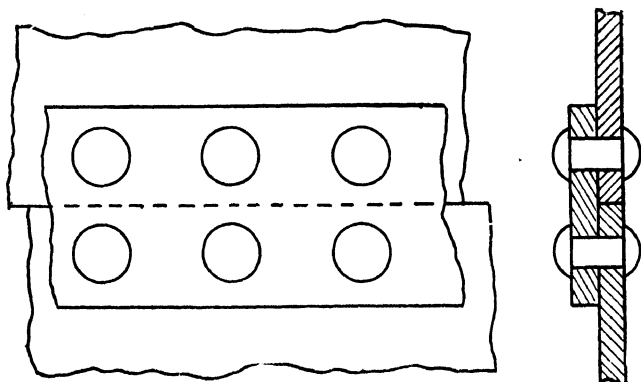


FIG. 12.

butt joint with double cover plates (Fig. 13) has the rivets in double shear. Although the strength of a rivet in double shear is twice that in single shear the Board of Trade only allow the load on a rivet in double shear to be 1.75 times that allowed on the same rivet in single shear to allow for errors in workmanship. Each cover plate should only carry half the load on the plate, but it is usual to make the thickness of the cover plates equal to $\frac{5}{8}t$ for double cover plate joints.

When the pitch of the rivets in the second or subsequent rows is a multiple of the pitch in the first row, the

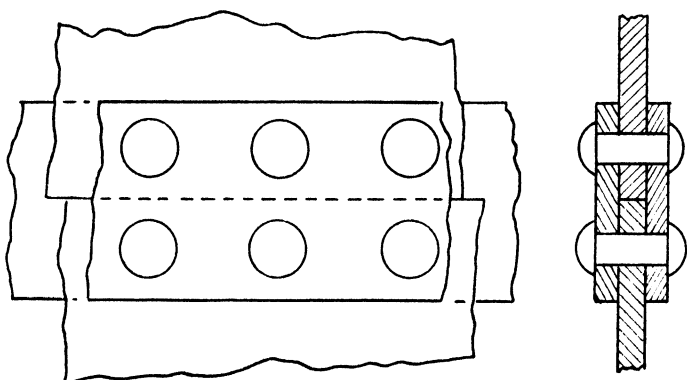


FIG. 13.

width of strip to be taken when calculating the resistances to tearing, shearing and crushing is equal to the larger

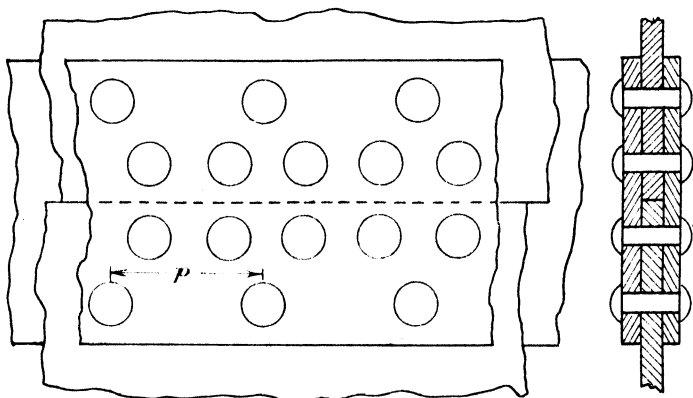


FIG. 14.

pitch. In a joint of this type, failure may also occur by the last row of rivets being sheared and the plate torn along the next row of rivets simultaneously.

EXAMPLE. Find the pitch of rivet required for the double cover plate butt joint shown in Fig. 14. $t = 1$ inch, $d = 1.125$ inches. $f_t = 6$, $f_s = 5$ and $f_c = 12$ tons per square inch.

$$\text{Tearing resistance} = (p - d)tf_t.$$

$$\text{Shearing resistance} = \frac{3\pi d^2 f_s \times 1.75}{4}$$

$$(p - 1.125) \times 1 \times 6 = \frac{3 \times \pi \times 1.125^2 \times 5 \times 1.75}{4}$$

$$\therefore p = 5.47 \text{ inches.}$$

$$\text{Tearing resistance} = (5.47 - 1.125) \times 1 \times 6 = 25.47 \text{ tons.}$$

Tearing and shearing resistance

$$= (5.47 - 2.25) \times 1 \times 6 + \frac{\pi \times 1.125^2 \times 5 \times 1.75}{4} = 27 \text{ tons.}$$

The plate will not fail by tearing and shearing.

In practice diameters of rivets are taken as the nearest sixteenth of an inch, and the pitch as the nearest eighth of an inch.

21. Stresses in Thin Cylinders. When a cylindrical vessel is subjected to an internal fluid pressure there

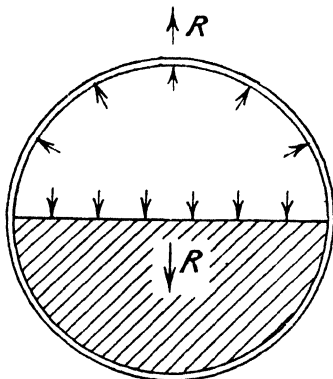


FIG. 15.

is a tendency to burst the vessel either longitudinally or transversely, which is resisted by the tensile stresses induced in the plates forming the cylinder.

Consider a cross-section of a thin cylinder (Fig. 15)

the following assumptions are made:—(1) That the structure is made up of bars, or members, connected by frictionless pin points, and therefore the forces in the members must act along their axes; (2) that the loads are concentrated at the joints. If a load acts at a point in a member, or is distributed along a member, it must be divided proportionally between the two ends.

Two kinds of forces have to be considered: (a) the forces due to the loads and the reactions of the supports, which must be in equilibrium; (b) the internal forces induced in the members of the structure by the external forces.

Since the forces in the members act along their axes, and the directions of the loads and reactions of the supports are known, a polygon of forces can be drawn for each point if a sufficient number of the forces are known. There are generally sufficient known forces at the supports to draw this polygon, and this should give sufficient known forces to draw the polygon at the next joint; the process being repeated until the forces in all the members have been found.

Instead of drawing a separate polygon of forces for each joint it is much simpler, and neater, to draw a combined force polygon. The method to be adopted to draw the combined force diagram is illustrated in the next two articles.

26. Roof Truss Simply Supported. Reactions Vertical. Draw the outline of the structure to a linear scale, showing the loads, and letter according to Bow's Notation (Fig. 24). Draw the forces AB, BC, CD to a force scale (Fig. 25); take a pole O and join Oa, Ob, Oc, Od. In Fig. 24 produce the lines of the forces EA, AB, BC, CD and DE downwards. Starting at any point in the line EA draw the lines 12 in the A space parallel to Oa, 23 in the B space parallel to Ob, 34 in the C space parallel to Oc, and 45 in the D space parallel to Od. Join 15 in the E space and draw Oe in Fig. 25 parallel to 15. Then *de* and *ea* close the force polygon for the external forces and give the magnitudes and directions of the two

the plates in a longitudinal direction is only half the stress induced in a circumferential direction; consequently, if the shell is built up of plates riveted together the longitudinal joints need to be twice as strong as the circumferential joints.

If there are riveted joints in the shell, then the stress induced must be multiplied by $1/e$, where e is the efficiency of the joint.

In the foregoing two assumptions are made: (1) that the stress is uniform across the plates; this is very nearly true if d is great compared with t ; (2) that the ends of the cylinder give no support to the sides; this is very nearly true if the length is much greater than the diameter.

EXAMPLE. Find the thickness of plate required for a boiler 6 feet diameter whose working pressure is 300 lb. per square inch gauge if the working stress in the plates is not to exceed 5 tons per square inch and the efficiency of the riveted joint is 70 per cent.

$$f_t = \frac{pd}{2te}.$$

$$t = \frac{pd}{2ef_t} = \frac{300 \times 72}{2 \times 0.7 \times 5 \times 2240}$$

$$= 1.37 \text{ inches.}$$

Required thickness = $1\frac{3}{8}$ inches.

22. Stress in Revolving Hoop. When a circular hoop, such as a flywheel rim, is revolving every part tends to fly outwards due to centrifugal force (Fig. 17), and the effect on the hoop is the same as if an internal fluid pressure were acting on it.

Consider a hoop whose cross-sectional area is A square inches, diameter d inches, linear velocity V feet per second, and density of material w lb. per cubic inch. The weight of 1 inch length of the rim is Aw lb., and the centrifugal force on this is

$$\frac{AwV^2}{gR} \text{ (see Article 47) or } \frac{24AwV^2}{gd}$$

since R (the radius) in feet is equal to $d/24$. This centrifugal force is equivalent to the internal fluid pressure p

in the last article, and therefore the resultant force tending to burst one half of the flywheel from the other is

$$\frac{24AwV^2 \times d}{gd} = \frac{24AwV^2}{g}.$$

This is resisted by the force induced in the rim, which is

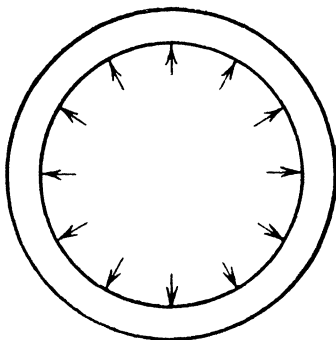


FIG. 17.

equal to $2Af_t$, where f_t is the tensile stress induced in the material.

Therefore
$$2Af_t = \frac{24AwV^2}{g},$$

or
$$f_t = \frac{12wV^2}{g} = \frac{12w\omega^2 R^2}{g}$$

where ω is the speed in radians per second.

EXAMPLE. What is the bursting speed of a cast-iron flywheel if the ultimate strength of cast iron in tension is 9 tons per square inch, and density is 0.26 lb. per cubic inch.

$$f_t = \frac{12wV^2}{g}$$

$$9 \times 2240 = \frac{12 \times 0.26 \times V^2}{32.2}$$

$$V = 456 \text{ feet per sec.}$$

STRENGTH OF MATERIALS IN TONS PER SQUARE INCH

Material.	Elastic Limit.			Ultimate Strength.			Modulus of Elasticity.	
	Ten-sion.	Comp.	Shear.	Ten-sion.	Comp.	Shear.	E.	G.
Cast iron .	—	—	—	7-11	35-60	8-12	7,000	4,000
Wrought iron	12-15	12-14	10-12	20-24	—	17-19	12,000	6,000
Mild steel .	13-14	—	10-11	26-29	—	16-18	{ 13,000 14,000	{ 5,000 6,000
1 per cent. carbon steel	28	23	22	55	—	45	14,000	6,000
Copper . .	2·5	—	—	9-14	—	—	7,000	—
Brass . .	5·5-13	—	—	13-25	—	—	{ 4,000 7,000	{ 2,000
Gun-metal .	5-11	—	—	10-22	30-50	—	6,000	—

EXAMPLES I

1. A test piece of copper is 0·73 inches diameter, and the distance between the gauge points is 3 inches.

It is subjected to an ordinary tensile test, the maximum load on the material being 5·82 tons and the load at fracture 4·7 tons. The ultimate length of the test piece between gauge points is 4·84 inches, and the diameter at fracture 0·41 inches. Find :

- (i) The load per square inch (a) when the maximum load is applied, and (b) when fracture takes place ;
- (ii) The elongation per cent., and
- (iii) The reduction of area per cent.

Draw the probable load extension diagram for the test piece.

U.L.C.I.

2. A rolled bar of Monel metal $\frac{3}{4}$ inch diameter, and having a gauge length of 2·66 inches, gave the following results when tested in tension in a testing machine :—

Load at yield point, 9·8 tons. Maximum breaking load, 18·78 tons. Maximum elongation, 1·22 inches. Diameter of bar at section of fracture, 0·424 inches. Determine :—

- (a) The yield point in tons per square inch ;
- (b) The breaking strength in tons per square inch ;
- (c) The percentage elongation ;
- (d) The percentage reduction of area.

By what indication is the yield point detected during a commercial test of the material ?

U.L.C.I.

3. A specimen, 0·564 inch diameter and 2 inches long, of a nickel chrome steel connecting rod when tested gave the following

results. Load at elastic limit, 11.25 tons. Ultimate load, 14.5 tons. Total elongation, 0.4 inch. Diameter at fracture, 0.37 inch. Find :—

- (a) The elastic strength in tons per square inch ;
- (b) The breaking strength in tons per square inch ;
- (c) The elongation per cent. ;
- (d) The reduction of area per cent.

U.L.C.I.

4. A steel test piece was turned to a diameter of 1.128 inches and the distance between the gauge points was 8 inches. On being tested in tension, the breaking load was 26.47 tons and the load at the elastic limit was 15 tons. The elongation for a load of 12 tons was 0.0071 inch and the dimensions after fracture were : Smallest diameter, 0.74 inch and total length, 10.1 inches. Determine :—

- (a) The modulus of elasticity of the steel ;
- (b) The tenacity of the steel in tons per square inch ;
- (c) The elongation per cent.
- (d) The reduction of area per cent.

Sketch a curve showing the relation between the extension and the load up to the breaking-point naming important points. U.L.C.I.

5. What do you understand by the "elastic limit" of a material ? A bar of metal $1\frac{1}{4}$ inches diameter and 8 inches long was gradually loaded until it broke under a load of 36 tons. During the test it was noted that the maximum elastic load was 16 tons. When a load of 12 tons was applied the extension of the bar was 0.0059 inch. Find :—

- (a) The tenacity of the material ;
- (b) The modulus of elasticity, and

- (c) The work stored in the material when the 12-ton load was applied.

U.L.C.I.

6. What do you understand by "resilience" ? Show that the maximum amount of energy that can be stored up in an elastic body per unit volume = $\frac{f^2}{2E}$ where f is the stress at elastic limit, and E the modulus of elasticity of the material. U.L.C.I.

7. A rod of steel $1\frac{1}{2}$ inches diameter and 10 feet long is subjected to a tensile load of 21 tons. Find the amount of energy stored in the rod in inch-pounds, assuming it not to be stressed beyond its elastic limit. $E = 30,000,000$ lb. per square inch. U.L.C.I.

8. For a given maximum stress compare the strain energy stored in a bar 12 inches long and 1 inch diameter throughout with that stored in a bar of the same length which is $1\frac{1}{4}$ inches diameter for 5 inches of its length and 1 inch diameter for the remainder. If you use a formula, prove it. N.C.T.E.C.

9. A bar of steel, 30 inches long and $\frac{7}{8}$ inch diameter, is subjected to a pull of 4.5 tons. Determine the extension of the bar due to the load, and calculate the amount of energy stored in the bar so extended. $E = 30,000,000$ lb. per square inch. U.L.C.I.

10. A mild steel bar $1\frac{1}{4}$ inches diameter and 8 feet long is subjected to a tensile load of 6 tons. Determine the energy stored up in the strained bar. $E = 30,000,000$ lb. per square inch. If you use a formula, prove it. U.L.C.I.

11. Compare the work done when a load of 5 tons is gradually applied to (a) a steel bar 1 inch diameter, 3 feet long, (b) a steel bar $\frac{3}{4}$ inch diameter, 3 feet long. $E = 13,000,000$ tons per square inch.

12. Find the work done when a load of 10 tons is gradually applied to each of the following steel bars :—(a) 6 inches long, 1 inch diameter ; (b) 6 inches long, 1 inch diameter for 3 inches length and 2 inches diameter for 3 inches length.

13. A steel bar 1 inch diameter and 4 feet long is subjected to stress by a weight of 150 lb. which falls freely through 3 inches. Calculate the maximum stress induced in the material if $E = 30,000,000$ lb. per square inch. U.L.C.I.

14. A bar of mild steel is $\frac{1}{2}$ inch diameter and 5 feet long. Determine the instantaneous stress induced in the bar when a weight of 30 lb. falls freely from rest through a height of 5 inches and is then suddenly pulled up by the bar. $E = 30,000,000$ lb. per square inch. U.L.C.I.

15. A bar of steel 10 feet long and $1\frac{1}{4}$ inches diameter is subjected to an axial load of 6 tons. Determine the maximum stress induced in the bar and the extension produced when the load is a dead load. If the load were applied suddenly, determine the instantaneous stress and extension produced. Explain your reasoning in arriving at the results for the live load. Would the permanent stress and strain be different in the two cases ? $E = 30,000,000$ lb. per square inch. U.L.C.I.

16. A steel bar is 10 feet long and 2 inches diameter. Find the work done when a load of 20 tons is applied gradually. Take $E = 13,500$ tons per square inch. U.E.I.

17. A vertical bar 4 inches by 4 inches in cross-section and 13 feet long, is intended to carry a weight of 25 tons. The weight is lowered slowly until the bar just commences to take the load ; the weight is then released and the bar takes the weight suddenly. Find the maximum instantaneous tensile stress, and the work stored in the bar at the instant of its maximum elongation. $E = 12,500$ tons per square inch. N.C.T.E.C.

18. A weight of 60 lb. falls through a height of 4 inches and then commences to stretch a steel bar of diameter 0.6 inches and length 2 feet 6 inches. Calculate the maximum stress induced. If you use a formula show how it is derived. $E = 30,000,000$ lb. per square inch. U.E.I.

19. A steel rod 6 feet long and 2 inches diameter hangs vertically from a rigid support and has a stop at the lower end. A ring weight of 100 lb. placed on the bar drops 2 feet before striking the stop. Find the maximum stress produced in the rod.

20. A ring weight of 50 lb. falls through a distance h feet before being checked by a stop fixed to the bottom of a vertical steel bar hung from a rigid support. If the bar is 1 inch diameter and 10 feet long, what must be the value of h so that the maximum extension of the bar may be 0.025 inch?

21. Assuming pure tensile and compressive stresses only, find the diameter of a mild steel connecting rod transmitting a maximum force of 8,000 lb. Ultimate strength of steel 30 tons per square inch. State a suitable factor of safety. What other stresses are being neglected?

22. An engine draws a train weighing 300 tons along the level. At a certain instant the acceleration is 0.15 feet per second per second and the frictional resistance is 15 lb. per ton. If the diameter of the drawbar is 1.25 inches, what will then be the stress in it?

23. A steel bar $1\frac{1}{2}$ inches diameter and 12 feet long is raised in temperature through 60°C. , after which its ends are firmly secured. After cooling to normal temperature again, the length of the bar is found to be 0.05 inch less than when at its highest temperature. Determine the total pull in tons exerted by the cold bar and the intensity of stress in it.

Coefficient of linear expansion of steel per degree C = 0.000011, $E = 13,000$ tons per square inch. U.L.C.I.

24. A steel rod 4 feet long and 0.5 inches diameter is cooled from 150°F. to 60°F. Find what forces are required in order to prevent any change in length. Take $E = 13,500$ tons per square inch and the coefficient of linear expansion as 0.000007. U.E.I.

25. A steel bar is 10 feet long and $1\frac{1}{4}$ inches diameter. Its temperature is raised from 14°C. to 74°C. Its ends are then firmly gripped so that it cannot contract as it cools to 14°C. What is the pull exerted by the cold bar? Take $E = 13,500$ tons per square inch. Coefficient of linear expansion of steel = 0.000011 per degree C. U.E.I.

26. A steel bar is heated to 300°F. A 3-foot length is then clamped to perfectly rigid supports and the bar allowed to cool. Find the stress in the bar when it has cooled to 20°F. Coefficient of linear expansion per degree F = 0.000006, $E = 13,000$ tons per square inch.

27. A piece of steel 6 inches long and 3 inches diameter is placed between rigid supports 6 inches apart. If the temperature of the steel is raised 150°C. , what force will then be exerted on the supports? Coefficient of linear expansion per degree C. = 0.000011, $E = 13,000$ tons per square inch.

28. Determine the least diameter of hole that can be punched through a metal plate $\frac{3}{8}$ inch thick, if the ultimate shearing stress of the metal is 45,000 lb. per square inch and the punch can withstand a total force of 120,000 lb. per square inch. U.L.C.I.

29. Find the work done in punching a hole 2 inches diameter in a plate 1 inch thick. Ultimate strength of plate in shear 25 tons per square inch. Assume that shearing is completed when the punch has penetrated one-third the thickness of the plate.

30. What is the maximum compressive stress in a punch when punching a hole 1 inch diameter through a steel plate $\frac{1}{2}$ inch thick? Ultimate shear strength of plate, 22 tons per square inch.

31. Find the pitch and diameter of rivets and efficiency of joint for a single lap riveted joint in plates of the following thicknesses:—(a) $\frac{1}{2}$ inch, (b) $\frac{5}{8}$ inch, (c) $\frac{3}{4}$ inch, (d) $\frac{7}{8}$ inch, (e) 1 inch. Ultimate strength of plates in tension, 30 tons per square inch. Ultimate strength of rivets in shear, 24 tons, and in crushing, 40 tons per square inch.

32. The same as question 29 for double riveted lap joints.

33. The same as question 29 for single riveted butt joints with double cover plates. Strength of rivets in double shear to be taken as 1.75 times that in single shear.

34. The same as question 31 for double riveted butt joints with double cover plates.

35. Two steel boiler plates $\frac{3}{8}$ inch thick are to be united by a single riveted lap joint with rivets of $\frac{3}{4}$ inch diameter. Determine the correct pitch of the rivets if the tensile stress in the plate is to be 6 tons per square inch and the shear stress in the rivets is to be 4 tons per square inch. U.E.I.

36. Two lengths of a flat steel bar, $\frac{3}{4}$ inch thick, are to be connected by a double butt joint for carrying a load of 41 tons. Find the number of 1-inch diameter rivets and the necessary width of bar. Rivets in double shear = $1\frac{1}{2}$ rivets in single shear. $f_t = 6$; $f_s = 5$; $f_b = 10$ tons per square inch respectively. U.E.I.

37. Determine the stress in the shell plate of a torpedo 20 inches diameter (assuming it a thin cylinder) when subjected to an air-pressure of 2,000 lb. per square inch, the thickness of the plate being $\frac{3}{8}$ inch. U.L.C.I.

38. What should be the thickness of shell plate of a cylindrical drum 5 feet diameter subjected to an internal (gauge) pressure of 200 lb. per square inch, the efficiency of the joints of the plates being 70 per cent.? Assume a safe stress of 10,000 lb. per square inch for the material. U.L.C.I.

39. Determine the thickness of the shell plates of a thin cylinder, 7 feet diameter, subjected to an internal fluid pressure of 140 lb. per square inch by gauge, if the allowable stress in the material is 5 tons per square inch and the efficiency of the longitudinal joints is 70 per cent. Prove the formula you use. U.L.C.I.

40. What thickness of shell plate would be required to make a cylinder 6 feet in diameter, the internal gauge pressure being 120 lb. per square inch and the efficiency of the riveted joint 0.7?

The plate is to be of mild steel, having a tensile strength of 28 tons per square inch and the factor of safety is to be 4. U.L.C.I.

41. A cylindrical boiler shell is 8 feet in diameter and the plates are $\frac{7}{8}$ inch thick. Find the average tensile stress in tons per square inch, in the plates due to a steam pressure of 140 lb. per square inch, the efficiency of the longitudinal joints being 70 per cent.

(a) On a longitudinal section of the plates along the line of rivets ;

(b) On a longitudinal section of the plates at some distance from the joint ;

(c) On a transverse section of the plates at some distance from the joint. U.E.I.

42. A boiler is to be made of internal diameter 4 feet for raising steam of 200 lb. per square inch gauge pressure. What must be the thickness of the steel plates used if the efficiency of the riveted joints is 76 per cent. and the tensile strength of the steel plates is 30 tons per square inch ? Assume a safety factor of 4. U.E.I.

43. A cylindrical shell, 3 feet in diameter, is built up of plates $\frac{9}{16}$ inch thick. The plates are connected longitudinally by double riveted butt joints, with double straps, the rivets being $\frac{3}{4}$ inch diameter with a pitch of 3 inches. Assuming that a rivet in double shear has $1\frac{1}{2}$ times the strength of the same rivet in single shear, and that the working shear stress in the rivets and the tensile stress in the plates are to be 4 and 5 tons per square inch respectively, find the plate and rivet efficiencies. What is the joint efficiency ? Determine the strength of the joint along the inner line of rivets. What should be the working pressure of the fluid in the shell in lb. per square inch ? U.E.I.

44. Find the maximum speed in revolutions per minute of a cast-iron flywheel 20 feet diameter if the hoop stress is not to exceed 4,250 lb. per square inch. Also find the bursting speed if the ultimate strength of cast iron in tension is 9 tons per square inch. 1 cubic inch of cast iron weighs 0.26 lb.

45. Deduce an expression for determining the stress induced in the rim of a wheel by centrifugal action.

The flywheel of a rolling mill engine is 16 feet in diameter and runs at 100 revolutions per minute. Find the stress induced in the rim of the wheel. Weight of material, 0.262 lb. per cubic inch. U.L.C.I.

46. At what speed in revolutions per minute would a thin cylinder or rim of steel 3 feet in diameter be rotating if the stress in the rim due to rotation is 1 ton per square inch ? Weight of steel, 0.28 lb. per cubic inch. Prove any formula you use. U.L.C.I.

47. Assuming that the maximum tensile stress allowed is 1,000 lb. per square inch and that the weight of 1 cubic inch of cast iron is 0.261 lb., find the greatest permissible number of revolutions per minute of a cast-iron flywheel of outside diameter 15 inches.

If you use a formula to obtain the speed, you should show how it is deduced.

U.E.I.

48. A flat bar of steel of cross-section 3 inches by $\frac{1}{4}$ inch is placed over a flat bar of copper of cross-section 3 inches by $\frac{1}{8}$ inch and of the same length as the steel bar. The two pairs of ends are rigidly clamped together and the combined bar is subjected to an axial pull of 6 tons. Find the pull sustained by each bar. E for steel = 13,000 tons per square inch and E for copper = 4,200 tons per square inch.

U.E.I.

49. A flat steel bar, 3 inches by $\frac{3}{4}$ inch in cross-section, is rigidly attached to two flat bars of brass, one on each side. If the bars are of equal length and the cross-section of each of the brass bars is 3 inches by $\frac{1}{4}$ inch, determine the load carried by each of the three bars when a pull of 25 tons is applied to the composite bar.

E for steel = 12,500 tons per square inch.

E for brass = 4,000 tons per square inch. N.C.T.E.C.

50. A steel tube 6 inches long, 3 inches external diameter and 2 inches internal diameter is bushed with a bronze tube of the same length 2 inches external diameter and 1 inch internal diameter. Find the stress in each material when the compound tube is subjected to an axial compressive load of 5 tons.

E for steel = 13,000 tons per square inch.

E for bronze = 6,000 tons per square inch.

CHAPTER II

FORCE DIAGRAMS FOR FRAMED STRUCTURES

23. Graphical Representation of the Moment of a Force. (*Note.* Bow's Notation for forces is used throughout this and the following articles.)

To find the moment of the force AB about the point P (Fig. 18). Let m feet to an inch be the linear scale to which Fig. 18 is drawn. Draw ab (Fig. 19) to represent the force AB in magnitude and direction, and let n lb. to an inch be the force scale. Take a pole O , distance Oh inches perpendicularly from ab , and join Oa and Ob . Through P (Fig. 18) draw a line parallel to

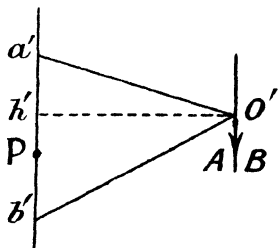


FIG. 18.

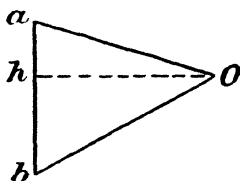


FIG. 19.

AB , and from any point O' in AB draw $O'a'$ and $O'b'$ parallel to Oa and Ob respectively, cutting the line through P in a' and b' . The triangles Oab and $O'a'b'$ are similar; therefore $ab : a'b' :: Oh : O'h'$, and $ab \times O'h' = a'b' \times Oh$.

$ab \times O'h'$ (both measured to scale) is equal to the moment of the force AB about P .

Therefore $a'b'$ (to the scale 1 inch = $m \times n$ lb.-feet) multiplied by the distance Oh in inches is equal to the moment of the force AB about P .

Instead of multiplying by the polar distance Oh each time it is simpler, and more convenient, to use a *moment scale* where 1 inch is equal to $m \times n \times (Oh \text{ measured in inches})$ lb.-feet. Other units, such as tons and inches, may be used in a similar manner.

24. Resultant, and Resultant Moment of a System

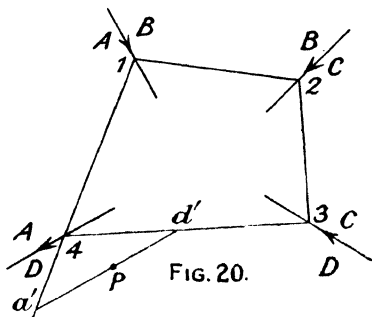


FIG. 20.

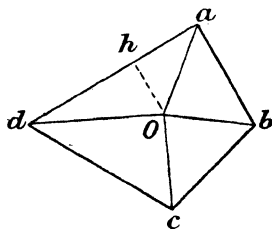


FIG. 22.

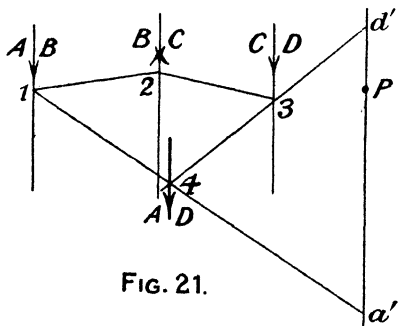


FIG. 21.

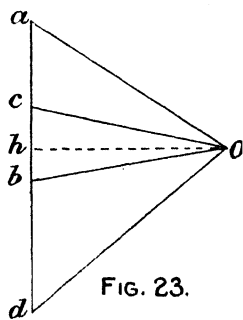


FIG. 23.

of Forces. Let AB, BC and CD (Figs. 20 and 21) be the given forces, and P the point about which the Resultant Moment is to be found. Draw the Force Polygon $abcd$ (Figs. 22 and 23) and the closing line, da , will give the equilibrant force in magnitude and direction. The

reverse of the equilibrant, ad , will give the *Resultant Force*. In the second case (Fig. 23), as the forces are parallel the polygon appears as a straight line.

To find the position of the Resultant Force.

In the Force Polygon take a pole O and draw the lines Oa , Ob , Oc and Od . From any point in AB (Figs. 20 and 21) draw a line, 12 , to BC (in the B space) parallel to Ob . In the C space draw the line 23 parallel to Oc . From 1 on AB draw a line in the A space parallel to Oa , and from 3 on CD draw a line in the D space parallel to Od . Where these lines meet, 4 , is a point on the line of action of the Resultant AD . The polygon 1234 is called the *Funicular* or *Link Polygon*.

Consider the lines 12 , 23 , 34 and 41 , to represent rigid links pin jointed together with the forces AB , BC , CD and DA acting at the joints ; these links taking the place of the body on which the forces are acting. Forces would be induced in the links and, if the system is in equilibrium when acting on the body, it would be in equilibrium when acting on the links, and therefore a triangle of forces can be drawn for each joint. Referring to Figs. 20 to 23, Oab is the triangle of forces for joint 1 , Obc is the triangle of forces for joint 2 , Ocd is the triangle of forces for joint 3 , and Oda is the triangle of forces for joint 4 . Therefore the forces are in equilibrium when the line of action of the force AD passes through the point 4 .

Through P (Figs. 20 and 21) draw a line parallel to AD , and produce the lines 14 and 43 to cut this line in a' and d' . Then $a'd'$, to the moment scale, gives the Resultant Moment of the forces about the point P , for if the triangles Oad and $4a'd'$ are compared with the triangles Oab and $O'a'b'$ in Article 23, the construction will be seen to be similar. The perpendicular distance Oh (Figs. 22 and 23) is the polar distance to be used in computing the moment scale.

25. Forces in Roof Trusses and Built-up Girders.
In drawing the Force Diagrams for *Framed Structures*

the following assumptions are made:—(1) That the structure is made up of bars, or members, connected by frictionless pin points, and therefore the forces in the members must act along their axes; (2) that the loads are concentrated at the joints. If a load acts at a point in a member, or is distributed along a member, it must be divided proportionally between the two ends.

Two kinds of forces have to be considered: (*a*) the forces due to the loads and the reactions of the supports, which must be in equilibrium; (*b*) the internal forces induced in the members of the structure by the external forces.

Since the forces in the members act along their axes, and the directions of the loads and reactions of the supports are known, a polygon of forces can be drawn for each point if a sufficient number of the forces are known. There are generally sufficient known forces at the supports to draw this polygon, and this should give sufficient known forces to draw the polygon at the next joint; the process being repeated until the forces in all the members have been found.

Instead of drawing a separate polygon of forces for each joint it is much simpler, and neater, to draw a combined force polygon. The method to be adopted to draw the combined force diagram is illustrated in the next two articles.

26. Roof Truss Simply Supported. Reactions Vertical. Draw the outline of the structure to a linear scale, showing the loads, and letter according to Bow's Notation (Fig. 24). Draw the forces AB, BC, CD to a force scale (Fig. 25); take a pole O and join *Oa*, *Ob*, *Oc*, *Od*. In Fig. 24 produce the lines of the forces EA, AB, BC, CD and DE downwards. Starting at any point in the line EA draw the lines *12* in the A space parallel to *Oa*, *23* in the B space parallel to *Ob*, *34* in the C space parallel to *Oc*, and *45* in the D space parallel to *Od*. Join *15* in the E space and draw *Oe* in Fig. 25 parallel to *15*. Then *de* and *ea* close the force polygon for the external forces and give the magnitudes and directions of the two

reactions of the supports, for *12345* is the Link Polygon for the loads and reactions.

Through *a* (Fig. 25) draw a line parallel to *AF*, and through *e* draw a line parallel to *EF* cutting the line through *a* at *f*. Then *caf* is the triangle of forces for the joint *EAF*. To find the directions of the forces, note that

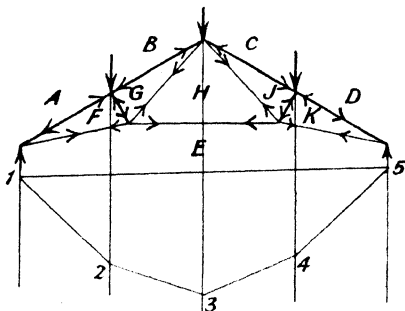


FIG. 24.

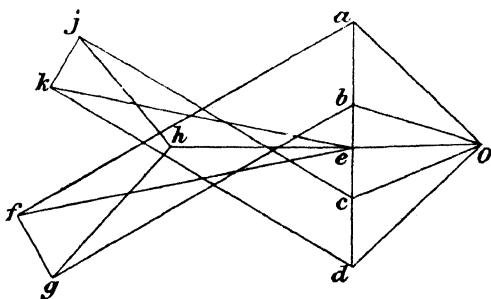


FIG. 25.

ea is upwards, and continue round the triangle obtaining the directions of the forces in the members *at the joint* as shown by arrows in Fig. 24. Since these forces are internal forces they will be reversed at the opposite ends of the members.

The force polygon for the joint *ABGF* can now be drawn,

starting with fa , ab , and continuing with bg and gf parallel to BG and GF in Fig. 24; the directions of the forces *at the joint* being found by following round the polygon from a force whose direction is known; e.g., ab downwards, bg , gf and fa .

In a similar manner, and on the same diagram, draw the force polygons for the joints $FGHE$, $BCJHG$, $CDKJ$, and $JKEH$. Note that the last line, ke , if the working is correct, will pass through e when drawn parallel to KE .

The members where the arrows point inwards are tension members, or ties, and those where the arrows point outwards are compression members, or struts. It is usual, when the force diagram is completed, to thicken the lines representing struts in the structure outline to show clearly which members are struts and which members are ties.

Complete the work by making a table of the members, with the forces in each member, and state the nature of such forces.

When there are loads acting on the joints at the supports these loads will increase the reactions of the supports by an equal amount, but will have no effect on the forces in the members.

27. Roof Truss with Fixed Support and Inclined Loads. Should the loading not be vertical, one support at least must be fixed, and the reaction there will not be vertical. To find the reactions of the supports, first find the line of action of the resultant, R , of the loads, and continue the line of action of the resultant until it meets the vertical through the free support at P (Fig. 26). Then, by the triangle of forces, the line of action of the other support must pass through this point. The force diagram (Fig. 27) can now be drawn as shown in the last article.

EXAMPLES II

Note.—The diagrams for the examples below are given on pages 36 and 38.

1. Draw the force diagram for the roof truss with the loading as shown in Fig. 28, and find the forces in the members A , B and C .

2. Draw the force diagram for the roof truss with the loading as shown in Fig. 29, and tabulate the forces in all the members.

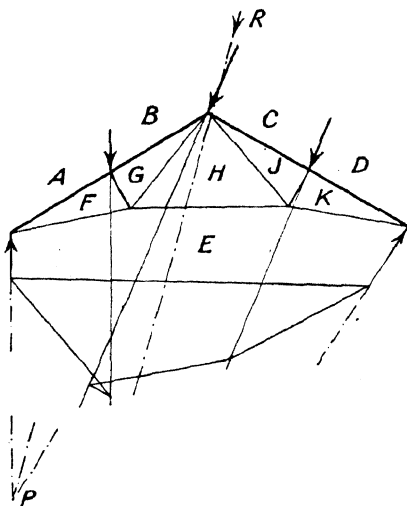


FIG. 26.

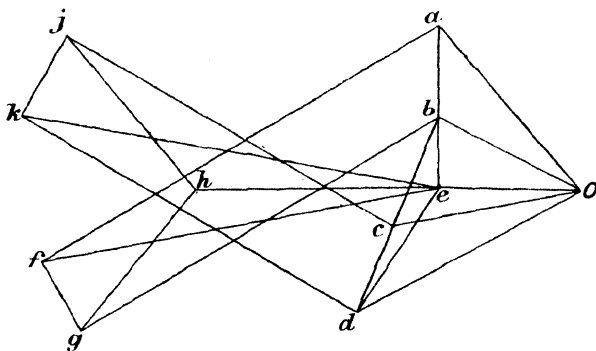


FIG. 27.

3. Draw the force diagram for the roof truss with the loading as shown in Fig. 30. Find the reactions of the supports and tabulate the forces in all the members. $AB = BC = CD = DE$.

4. Find the forces in the members of the roof truss shown in

truss shown in Fig. 53, the end A being hinged and the reaction at the end B being vertical. U.L.C.I.

28. The roof truss shown in Fig. 54 is supported on a hinge at the end B, and on rollers at the end A. The inclined loads at A and C are perpendicular to AC, and that at D is perpendicular to CD. Find the magnitude and direction of the reactions at the supports.

29. Two equal rods AB and BC are in a vertical plane and are pin-jointed together at B so as to permit of relative motion of AB and BC in the vertical plane. A is hinged to a fixed abutment and C is hinged to a slider which is constrained by guides to move in a horizontal plane passing through A.

A constant vertical force of 300 lb. is applied to the joint B. Find the magnitudes of the horizontal forces that act on the slider when the apex angles are respectively 60° , 120° and 170° .

Draw a graph of these forces on a base line of angles. U.L.C.I.

30. A weight of 100 lb. hangs from a tripod, the three legs of which are 8 feet long, and when set up on the ground are placed at points of an equilateral triangle of 6 feet sides. Find the thrust in each leg. U.E.I.

CHAPTER III

BENDING MOMENTS AND SHEARING FORCES

28. Bending Moment and Shearing Force. When a beam or cantilever is simply loaded the external forces are perpendicular to the length of the beam, they are all in one plane, and they are also in equilibrium, otherwise there would be movement of the beam. The plane in which the forces act will be taken to be the plane of the paper.

Consider a cantilever (Fig. 55) acted on by a number of loads, or forces, P , Q , S , T . Take any cross-section

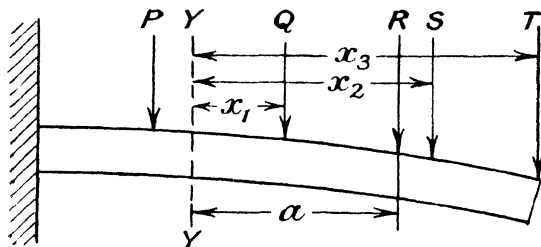


FIG. 55.

YY and consider the equilibrium of the part to the right of the section. This part is acted on by the forces Q , S , T , and internal forces at YY. The forces Q , S , T may be replaced by a resultant force $R = Q + S + T$, at a distance a from YY such that $Ra = Qx_1 + Sx_2 + Tx_3$.

The force R tends to turn the part about a horizontal axis at YY, and also to slide the part vertically down the section YY. If two opposite vertical forces R_1 and R_2 (Fig. 56), each equal to R , are applied at YY the equilibrium will not be altered. R and R_2 form a couple

of moment Ra giving a turning or *bending moment* about YY , and R_1 gives a sliding or *shearing force* at YY .

In the case of a section YY in a loaded beam (Fig. 57), the conditions on either side of the section are similar to those of the cantilever, noting that the force due to the reaction is of opposite sign to the forces due to the loads.

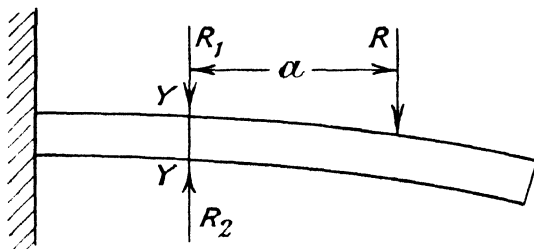


FIG. 56.

The bending moment at any section in a beam or cantilever is equal to the algebraic sum of the moments of all the forces on either side of the section.

The shearing force at any section in a beam or cantilever is equal to the algebraic sum of all the forces on either side of the section.

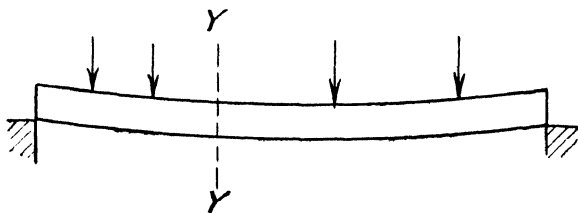


FIG. 57.

To calculate the bending moment at any section in a beam, first calculate the reactions of the supports, and then take moments on either side of the given section. If the section is taken at a point where a load occurs, that load will have no moment at the section, but the load is not neglected, as it has been taken into account when

AB = 6 feet, BC = 28 feet, and CD = 6 feet. Loads of 5 tons each are suspended at A and D, and a load of 6 tons in the centre of the length. Neglecting the weight of the girder, draw diagrams of the B.M. and S.F. for the girder. At what points in the girder will there be no B.M. ?

U.L.C.I.

13. A beam, ABC, 34 feet long, is supported at A and B. AB = 25 feet, and BC = 9 feet. Loads of 3 tons, 2 tons, 4 tons and 3 tons act respectively at points which are 5 feet, 15 feet, 20 feet and 30 feet distant from A. Draw to scale the B.M. and S.F. diagrams for the beam, and write on the diagrams the magnitudes of the maximum B.M. and S.F.

U.L.C.I.

14. A horizontal beam AB, 30 feet long, is supported at A and at a point C, 24 feet from A. Vertical loads of 5, 8, 4 and 2 tons are carried by the beam at sections respectively distant 4, 10, 16 and 30 feet from A. Draw to scale the B.M. and S.F. diagrams, and write on the diagrams the values of the maximum B.M. and S.F.

U.L.C.I.

15. Draw to scale the B.M. diagram for a beam 16 feet long, freely supported at points 10 feet apart so that the beam overhangs 3 feet at each end. Loads 1, 2, 3 and 4 tons are placed at distances of 0, 4, 8 and 16 feet respectively along the beam.

U.E.I.

16. A steel beam 24 feet long is supported at one end and at a point 8 feet from the other end. It carries a distributed load of 2 tons per foot run over its whole length, and one concentrated load of 6 tons midway between the supports and another of 4 tons at the free end. Draw the B.M. and S.F. diagrams and give the positions where the B.M. and S.F. respectively have their maximum values.

U.E.I.

17. A beam AC 25 feet long is supported at A and at an intermediate point B. AB = 20 feet. A uniformly distributed load of $\frac{1}{2}$ ton per foot run rests on the beam from A to D, the middle point of AB, and concentrated loads of 2 tons and 1 ton are supported respectively at E and C. AE = 15 feet. Draw the B.M. and S.F. diagrams for the beam and state the values of the maximum B.M. and S.F.

U.L.C.I.

18. AB is a horizontal beam, 24 feet in length, supported in the following manner:—The end A is inserted underneath a firmly fixed joist; at C, 8 feet from A, the beam is supported whilst for the rest of its length it is free. The beam carries a uniformly distributed load of $\frac{1}{2}$ ton per foot run, together with loads of 6, 4 and 2 tons at points 4, 12 and 20 feet from A respectively. Calculate the B.M. at sections 8 and 12 feet respectively from A. Draw to scale a diagram showing the variation in S.F. along the length of the beam. What is the area of this diagram between the sections A and C, and between C and a section 12 feet from A ? What do these results suggest ? What do you know about the

concavity on the upper surface *negative bending*. Also it is convenient to call a shearing force which would cause the part of the beam to the left of the section to rise a *positive shearing force*, and that which would cause the part of the beam to the right of the section to rise a *negative shearing force*.

29. Bending Moment and Shearing Force Diagrams. If the bending moments and shearing forces are calculated at a sufficient number of sections, and the results plotted on a base parallel to the beam, curves are obtained which are called *Bending Moment Diagrams*, and *Shearing Force Diagrams*. Positive bending moments and shearing forces are generally plotted above the base-line, and negative bending moments and shearing forces below the base-line.

Below are given some examples of bending moment and shearing force diagrams.

1. *Cantilever with Concentrated Loads* (Fig. 59).

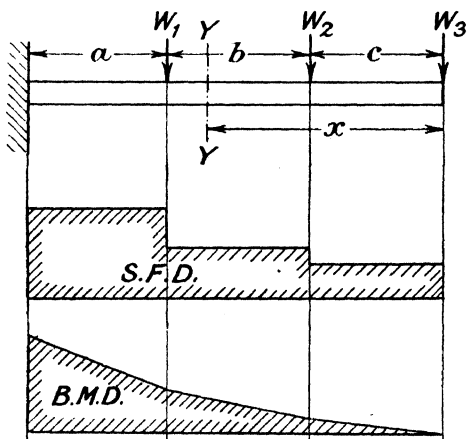


FIG. 59.

$$\text{B.M. at YY} = W_2(x - c) + W_3x.$$

$$\text{S.F. at YY} = W_2 + W_3.$$

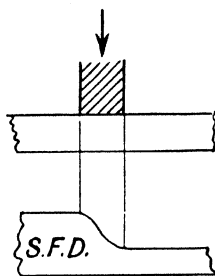


FIG. 60.

In actual practice a load is not applied at a point, but over a length short in comparison with the length of the beam, and which appears as a point on a small scale diagram. Drawn more correctly, to a larger scale, the shearing force diagram would appear as shown in Fig. 60. It is, of course, impossible for there to be two values of the shearing force at one section.

2. *Cantilever with Distributed Load, w per unit length.* (Fig. 61.)

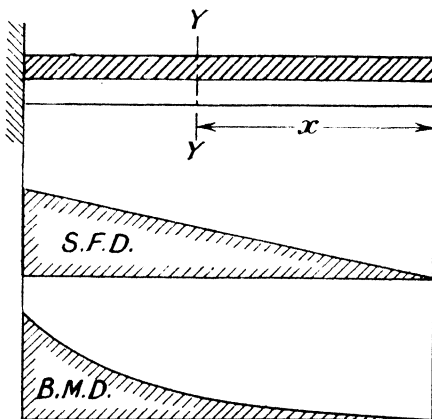


FIG. 61.

$$\text{B.M. at YY} = wx \times \frac{x}{2} = \frac{wx^2}{2}$$

$$\text{S.F. at YY} = wx.$$

3. *Beam with Concentrated Load at Centre.* (Fig. 62.)

$$\text{B.M. at YY} = \frac{1}{2}Wx.$$

$$\text{B.M. at centre (maximum)} = \frac{1}{4}Wl.$$

$$\text{S.F. at YY} = \frac{1}{2}W.$$

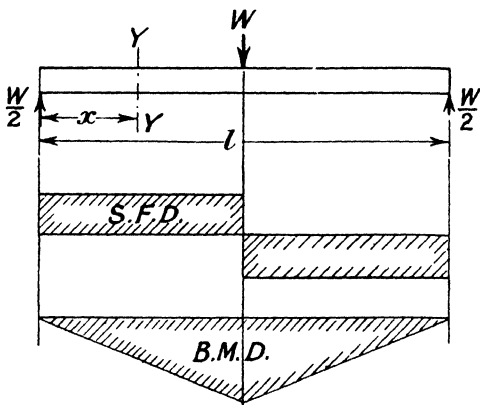


FIG. 62.

4. *Beam with Distributed Load, w per unit length.*
(Fig. 63.)

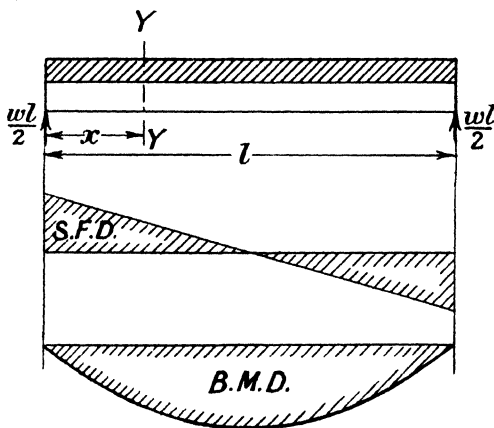


FIG. 63.

$$\text{B.M. at YY} = \frac{1}{2}wlx - wx \times \frac{x}{2}$$

$$\text{B.M. at centre (maximum)} = \frac{1}{4}wl^2 - \frac{1}{8}wl^2 = \frac{wl^2}{8}$$

$$\text{S.F. at YY} = \frac{1}{2}wl - wx.$$

The bending moment diagram curve for a uniformly distributed load over the whole span is a parabola whose axis is vertical and passes through the centre of the span.

Where there are both concentrated and distributed loads the bending moments and shearing forces may be calculated directly, or they may be calculated separately for each type of loading and then added together.

30. Relation between Bending Moment and Shearing Force Diagrams. Let a beam of span l (Fig. 64)

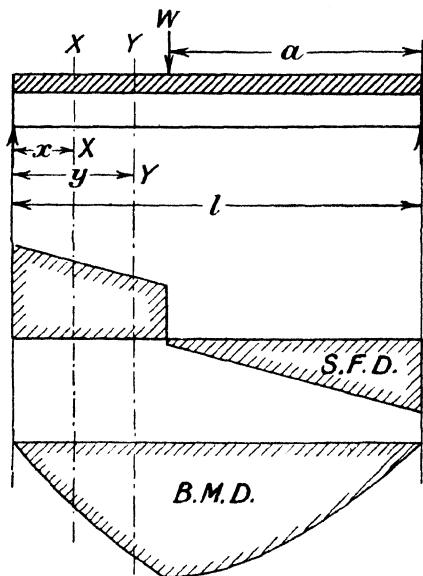


FIG. 64.

carry a distributed load of w per unit length and a concentrated load W at a distance a from the right-hand support. The reaction of the left-hand support will be $\frac{wl}{2} + \frac{Wa}{l}$. Consider the bending moments and shearing forces at two sections XX and YY at distances x and y respectively from the left-hand support.

$$\text{S.F. at XX} = \frac{wl}{2} + \frac{Wa}{l} - wx.$$

$$\text{S.F. at YY} = \frac{wl}{2} + \frac{Wa}{l} - wy.$$

Area of S.F. diagram between XX and YY

$$\begin{aligned} &= \frac{1}{2} \left\{ \frac{wl}{2} + \frac{Wa}{l} - wx + \frac{wl}{2} + \frac{Wa}{l} - wy \right\} (y-x) \\ &= \left\{ \frac{wl}{2} + \frac{Wa}{l} \right\} (y-x) - \frac{wy^2}{2} + \frac{wx^2}{2}. \end{aligned}$$

$$\text{B.M. at XX} = \left\{ \frac{wl}{2} + \frac{Wa}{l} \right\} x - \frac{wx^2}{2}$$

$$\text{B.M. at YY} = \left\{ \frac{wl}{2} + \frac{Wa}{l} \right\} y - \frac{wy^2}{2}.$$

Difference between the bending moments at XX and YY

$$= \left\{ \frac{wl}{2} + \frac{Wa}{l} \right\} (y-x) - \frac{wy^2}{2} + \frac{wx^2}{2}.$$

Therefore the area of the shearing force diagram between the two sections is equal to the difference of the bending moments at those sections. This result would be obtained whatever the type of loading.

As the distance y (Fig. 64) is increased, the area of the portion of the shearing force diagram will increase until the point where the shearing force becomes zero is reached, after which, as the shearing force diagram is now negative, the area will decrease. Consequently the bending moment will increase with increase of y up to this point, and then it will decrease. Therefore the Bending Moment will have a (mathematical) maximum value at the point where the shearing force is zero, or changes from a positive to a negative value. This property of the shearing force diagram is useful to find the point of maximum bending moment. In the case of an overhung beam there will be more than one such maximum value; the greatest of such values must be taken as the actual maximum bending moment on the beam.

To obtain the shearing force diagram carry the points a, b, c, d and e in the force polygon horizontally to the corresponding spaces A, B, C, D and E between the forces on the beam. The shearing force at any section YY will be the vertical height, 67 , of the diagram vertically under the section. The scale is the force scale for the distance 67 is equal to $ed - dc$, which is equal to the reaction ED minus the load CD, which is the shearing force at YY.

A distributed load may be split up into short lengths, the weight of each part acting at its centre of gravity, and these parts then treated as a number of concentrated loads. The bending moment diagram should then be corrected to a smooth curve on the inside of the diagram obtained by the above method. The shearing force diagram should also be corrected to a straight line.

If the distributed load is uniform over the whole span the bending moment at the centre of the span can be calculated and plotted, and the parabolic curve then drawn.

EXAMPLES III

1. A beam simply supported over a span of 20 feet carries loads of 3 and 4 tons at points 5 and 12 feet respectively from one support. Draw the B.M. and S.F. diagrams and find the B.M. under each load.

2. A beam simply supported over a span of 20 feet carries loads of 2, 3, 4 and 5 tons at points 4, 8, 12 and 16 feet from one support. Find the B.M. and S.F. at each load.

3. A beam simply supported over a span of 20 feet carries a uniformly distributed load of $\frac{3}{4}$ ton per foot run over its whole length. Draw the B.M. and S.F. diagrams and write down the value of the B.M. and S.F. at quarter and half span.

4. The axles of a locomotive are 7, 5, 6 and 7 feet apart, and the loads on the axles are 10, 10, 20, 20 and 20 tons respectively. Draw the B.M. and S.F. diagrams for a bridge of 50 feet span supported at each end, on which the engine is standing with the leading axle (10-ton load) 10 feet from the left support.

Determine the magnitude of the maximum B.M. and state where it acts.

U.L.C.I.

5. A beam simply supported over a span of 30 feet carries loads of 4, 8 and 10 tons at points 4, 14 and 20 feet respectively from the

left support, and a distributed load of 1 ton per foot over its whole length. Draw the B.M. and S.F. diagrams, and find the position and magnitude of the maximum B.M.

6. A shaft weighing 400 lb. per foot run is supported in bearings 5 feet apart and carries a flywheel weighing 15 tons 2 feet from one bearing. If the C.G. of the flywheel is 1 inch from the axis of the shaft, find the maximum B.M. in the shaft at the centre of the flywheel when the speed of the shaft is 120 r.p.m.

7. A girder supported at the ends is 30 feet long and supports a wall over 20 feet of its length from the left end and a column 5 feet from the right end. The weight of the wall is 2,400 lb. per foot run, and the column supports a load of 6,000 lb. Draw to scale the B.M. and S.F. diagrams.

What is the magnitude of the maximum B.M. and where does it act? How could the B.M. at any section be determined from the S.F. diagram?

Use scales as follows :—

For B.M. diagram 1 inch equals 50,000 lb.-ft.

For S.F. diagram 1 inch equals 10,000 lb.

U.L.C.I.

8. A horizontal girder which is freely supported at its ends and has a span of 30 feet supports a uniformly distributed load of $\frac{1}{2}$ ton per foot run over the whole span, and also two concentrated loads of 2 and 3 tons at points 20 and 24 feet respectively from the left support. Draw to scale the B.M. and S.F. diagrams and state the values of the maximum B.M. and S.F.

U.L.C.I.

9. A girder 32 feet long is supported at both ends. It carries a distributed load of 2,500 lb. per foot run on the left hand 16 feet and a single load of 6,000 lb. at 8 feet from the other end. Draw the S.F. and B.M. diagrams to scale. Find where the B.M. is a maximum and calculate its amount.

Scales—B.M. 1 inch = 50,000 lb.-ft.

S.F. 1 inch = 10,000 lb.

U.E.I.

10. A beam AD 30 feet long is supported at two points B and C such that AB = 5 feet, and AC = 20 feet. Loads of 5 tons and 4 tons act on the beam at A and D respectively, and a load of 10 tons acts at a point midway between B and C. Find the maximum B.M. and S.F. acting on the beam, and draw the B.M. and S.F. diagrams.

U.L.C.I.

11. A girder AB 25 feet long is supported at the end A and at a point C 5 feet from B. The girder is loaded with a uniformly distributed load of 1 ton per foot run over its entire length, and in addition two loads are suspended from it, namely, one of 4 tons 15 feet from A, and one of 2 tons 2 feet from B.

Draw the B.M. and S.F. diagrams for the girder and determine the magnitudes of the maximum B.M. and S.F. and state the positions where they occur.

U.L.C.I.

12. A girder ABCD of I section rests on supports at B and C.

AB = 6 feet, BC = 28 feet, and CD = 6 feet. Loads of 5 tons each are suspended at A and D, and a load of 6 tons in the centre of the length. Neglecting the weight of the girder, draw diagrams of the B.M. and S.F. for the girder. At what points in the girder will there be no B.M. ?
U.L.C.I.

13. A beam, ABC, 34 feet long, is supported at A and B. AB = 25 feet, and BC = 9 feet. Loads of 3 tons, 2 tons, 4 tons and 3 tons act respectively at points which are 5 feet, 15 feet, 20 feet and 30 feet distant from A. Draw to scale the B.M. and S.F. diagrams for the beam, and write on the diagrams the magnitudes of the maximum B.M. and S.F.
U.L.C.I.

14. A horizontal beam AB, 30 feet long, is supported at A and at a point C, 24 feet from A. Vertical loads of 5, 8, 4 and 2 tons are carried by the beam at sections respectively distant 4, 10, 16 and 30 feet from A. Draw to scale the B.M. and S.F. diagrams, and write on the diagrams the values of the maximum B.M. and S.F.
U.L.C.I.

15. Draw to scale the B.M. diagram for a beam 16 feet long, freely supported at points 10 feet apart so that the beam overhangs 3 feet at each end. Loads 1, 2, 3 and 4 tons are placed at distances of 0, 4, 8 and 16 feet respectively along the beam.
U.E.I.

16. A steel beam 24 feet long is supported at one end and at a point 8 feet from the other end. It carries a distributed load of 2 tons per foot run over its whole length, and one concentrated load of 6 tons midway between the supports and another of 4 tons at the free end. Draw the B.M. and S.F. diagrams and give the positions where the B.M. and S.F. respectively have their maximum values.
U.E.I.

17. A beam AC 25 feet long is supported at A and at an intermediate point B. AB = 20 feet. A uniformly distributed load of $\frac{1}{2}$ ton per foot run rests on the beam from A to D, the middle point of AB, and concentrated loads of 2 tons and 1 ton are supported respectively at E and C. AE = 15 feet. Draw the B.M. and S.F. diagrams for the beam and state the values of the maximum B.M. and S.F.
U.L.C.I.

18. AB is a horizontal beam, 24 feet in length, supported in the following manner:—The end A is inserted underneath a firmly fixed joist; at C, 8 feet from A, the beam is supported whilst for the rest of its length it is free. The beam carries a uniformly distributed load of $\frac{1}{2}$ ton per foot run, together with loads of 6, 4 and 2 tons at points 4, 12 and 20 feet from A respectively. Calculate the B.M. at sections 8 and 12 feet respectively from A. Draw to scale a diagram showing the variation in S.F. along the length of the beam. What is the area of this diagram between the sections A and C, and between C and a section 12 feet from A ? What do these results suggest ? What do you know about the

B.M. at a section of the beam where the S.F. passes through a zero value ?

U.E.I.

19. A horizontal girder AB, 24 feet long, is hinged at the end A and rests on a support at B. It carries loads of 2, 3 and 1 tons at distances of 4, 12 and 18 feet from A respectively. The 3 ton load is inclined at 30° to the vertical on the side towards A. Determine the reactions of the supports and draw to scale the B.M. and S.F. diagrams. State the values of the maximum B.M. and S.F.

U.L.C.I.

20. A girder over a 20 feet span weighs $\frac{1}{2}$ ton per foot run. A load of 5 tons travels across the girder. Draw a diagram showing the maximum B.M. at each section of the girder.

CHAPTER IV

BEAMS AND BENDING

32. Stresses Induced by Bending of Beams.

When a beam is loaded the loads and reactions impress a bending moment on the beam, causing the beam to become curved, one part of the beam being stretched, and another part being compressed. These deformations set up resisting forces (tensile and compressive) in the material of the beam. The resultants of these forces at any section, called the *resisting moment* of the beam, will oppose and balance the bending moment on the beam at that section. The method by which the resisting moment, or strength, of a beam can be calculated, is shown below.

In the theory of the strength of beams, the following assumptions are made. First, that originally plane sections of a beam remain plane after bending. Second, that the modulus of elasticity in tension is equal to the modulus of elasticity in compression. Neither of the above assumptions are perfectly true, but they are so near the truth that for practical purposes they may be considered to be true, but *only so long as the stress in the material does not exceed the elastic limit*.

ABCD (Fig. 66) is the elevation of a short length of an unloaded beam. A'B'C'D' (Fig. 67) is an elevation of the same length of the beam when it is bent in a plane represented by the plane of the paper. The upper part of the beam is stretched, and the lower part compressed, by bending, and there must be a surface, EF, called the *Neutral Surface*, between these two portions where the fibres of the beam are not strained, and consequently where there is no stress in the material of the beam. The line where the neutral surface cuts a transverse section is called the *neutral axis* of the section.

The transverse parallel sections AB and CD will be inclined to one another after bending, and their planes will intersect at O , which is the *centre of curvature* of that part of the beam. The *Radius of Curvature* of that part of

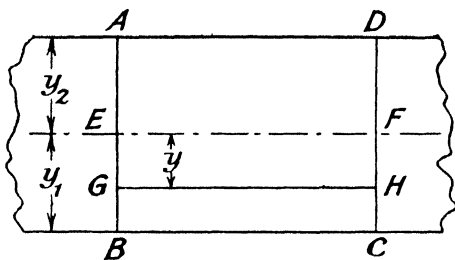


FIG. 66.

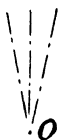
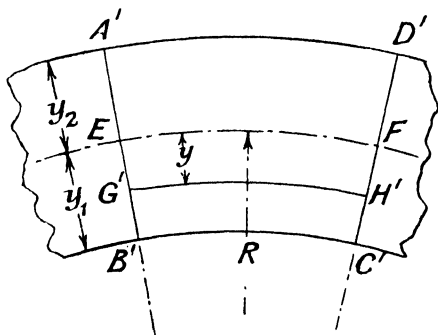


FIG. 67.

the beam is the distance from the centre of curvature to the neutral surface of the beam.

An indefinitely thin layer, GH , parallel to the axis of the beam, and distant y from the neutral surface, will

have its length altered to $G'H'$ after bending. The amount of strain induced in this layer by the bending of the beam will be

$$\begin{aligned}\frac{GH - G'H'}{GH} &= \frac{EF - G'H'}{EF} = 1 - \frac{G'H'}{EF} \\ &= 1 - \frac{EF(R - y)}{EF \times R} = \frac{y}{R}.\end{aligned}$$

But $E = \frac{\text{stress}}{\text{strain}}$, and if f is the stress induced in the layer GH by bending, then

$$E = f \div \frac{y}{R}, \text{ or } \frac{f}{y} = \frac{E}{R}.$$

Therefore the stress in any layer parallel to the axis due to bending is proportional to the distance of the layer from the neutral surface, since E and R are constant for that section. This gives the equation

$$\frac{f}{y} = \frac{f_1}{y_1}$$

where f is the stress in a fibre at a distance y from the neutral

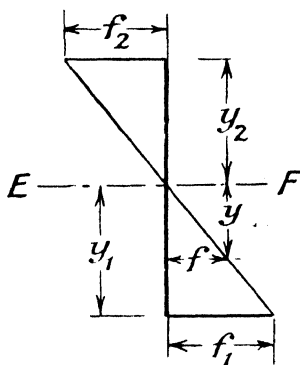


FIG. 68.

axis, and f_1 is the stress in a fibre at a distance y_1 from the neutral axis. The maximum stress will be the stress on the outermost layers, and the distribution of stress on a cross-section will be as shown in Fig. 68.

33. Position of the Neutral Axis. Because it is a moment which produces bending, the resultant of the external forces which produce pure bending is a couple; therefore the resultant of the internal forces which resist bending must be a couple, since a couple can only be balanced by a couple. The internal forces are due to the tensile and compressive stresses induced by bending, and the resultants of these forces must therefore be equal and parallel to form the two forces of the couple.

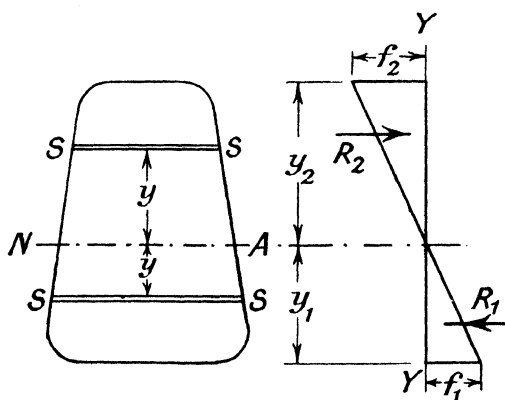


FIG. 69.

Consider a transverse section of a beam (Fig. 69), NA being the neutral axis, and YY an edge view of the section. Let y_1 and y_2 be the distances from the neutral axis to the extreme fibres, and f_1 and f_2 the stresses in these fibres. On an indefinitely narrow strip, ss , of the section parallel to NA, of area a and at a distance y from NA, and below NA, the stress, f , will be given by

$$\frac{f}{y} = \frac{f_1}{y_1}, \text{ and therefore } f = \frac{f_1 y}{y_1}.$$

The force on ss is fa , and

$$fa = \frac{f_1 a y}{y_1}.$$

The resultant, R_1 , of the force on the part of the section below NA will be equal to the sum of the forces on all the strips of area below NA, that is

$$\Sigma fa = \frac{f_1}{y_1} \Sigma ay.$$

Similarly, the resultant, R_2 , of the force on the part of the section above NA will be

$$\Sigma fa = \frac{f_2}{y_2} \Sigma ay.$$

Therefore, since $R_1 = R_2$, and $\frac{f_1}{y_1} = \frac{f_2}{y_2}$, Σay for the part of the section below NA must be equal to Σay for the part of the section above NA.

But Σay is the moment of the area about NA, and since the moments of area above and below NA are equal, there is no resultant moment of area about NA. This can only be so if NA passes through the centre of gravity of the section. Therefore *the neutral axis must pass through the centre of gravity of the section*, whatever the shape of the section.

34. Moment of Resistance to Bending. Consider a cross-section of a beam, Fig. 70, and let the lettering have the same meaning as in Fig. 69. The force on the strip ss below NA is

$$fa = \frac{f_1}{y_1} ay,$$

due to the stresses set up in the beam in resistance to the Bending Moment applied at the section. The moment of

this force is $\frac{f_1}{y_1} ay^2$, and the total Moment of Resistance of

the section is $\frac{f_1}{y_1} \Sigma ay^2$ on the part below the Neutral Axis,

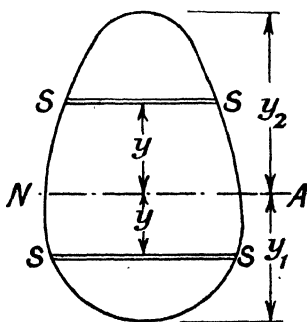


FIG. 70.

plus $\frac{f_2}{y_2} \Sigma ay^2$ on the part above the Neutral Axis. But Σay^2 is equal to the second moment of area of the section, therefore the sum of these two quantities is equal to

$$\frac{f_1 I}{y_1}, \text{ or } \frac{f_2 I}{y_2},$$

where I is the Second Moment of Area of the whole section about the Neutral Axis.

For equilibrium the Moment of Resistance of the section must be equal to the Bending Moment, M , at the section, therefore

$$M = \frac{f}{y} I,$$

where f is the stress at a distance y from the Neutral Axis. In general it is only the maximum, or skin stress, f_1 or f_2 , which is required, but there are cases where the magnitudes of the stresses at different points in the section are required to be found. Since $\frac{M}{I} = \frac{f}{y}$ we get the further relation

$$\frac{M}{I} = \frac{f}{y} = \frac{E}{R}.$$

Let $z_1 = \frac{I}{y_1}$, and $z_2 = \frac{I}{y_2}$,

then

$$M = f_1 z_1, \text{ or } M = f_2 z_2,$$

where z_1 and z_2 are called the *moduli* of the section. If the section is symmetrical about the Neutral Axis, $y_1 = y_2$ and $f_1 = f_2$, and there is only one Modulus of Section. Unsymmetrical beams are found where angles, tees, or similar sections are used, or where the material is cast iron, as in parts of machines. In the latter case the beam is designed to bring the Neutral Axis near to the tension edge, as cast iron will withstand safely a much greater compressive than tensile stress.

EXAMPLE. At a section in a 12 inch by 6 inch beam there is a Bending Moment of 288 ton-inches. Find the maximum

stress due to bending, and the stress 2 inches from the neutral axis.

$$I = \frac{BD^3}{12} = \frac{6 \times 12^3}{12} = 864 \text{ inches}^4.$$

$$f_1 = \frac{My}{I} = \frac{288 \times 6}{864} = 2 \text{ tons per square inch. Maximum.}$$

$$f = \frac{288 \times 2}{864} = \frac{2}{3} \text{ tons per square inch at 2 inches from N.A.}$$

Owing to the similarity between Second Moment of Area and Moment of Inertia the Second Moment of Area is very often called the Moment of Inertia of the Section.

Because the resisting Moment of a beam depends on the Moment of Inertia of Section about the Neutral Axis, a beam should, if possible, be fixed in such a manner that the greatest Moment of Inertia of Section is obtained, e.g. : an I beam should be fixed with the web vertical. If an I beam is fixed with the web horizontal, the Moment of Inertia of Section will be the least value of any about an axis through the centre of gravity of the section, and the resisting moment therefore the least possible.

35. Second Moments of Area, and Moduli of Typical Sections.

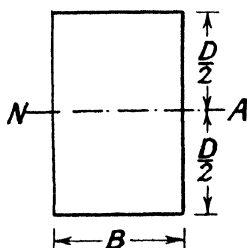


FIG. 71.

$$I = \frac{BD^3}{12}.$$

$$Z = \frac{BD^2}{6}.$$

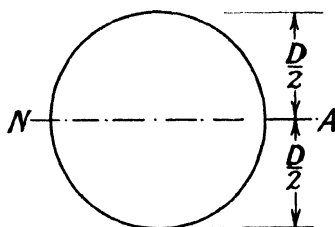


FIG. 72.

$$I = \frac{\pi D^4}{64}$$

$$Z = \frac{\pi D^3}{32}$$

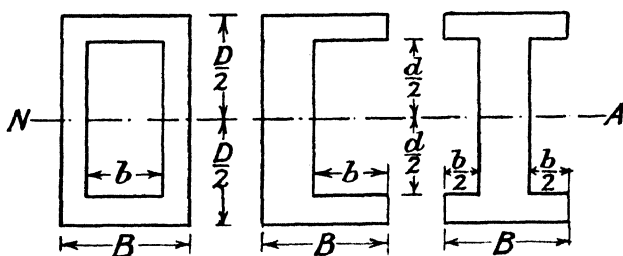


FIG. 73.

$$I = \frac{1}{12}(BD^3 - bd^3).$$

$$Z = \frac{BD^3 - bd^3}{6D}.$$

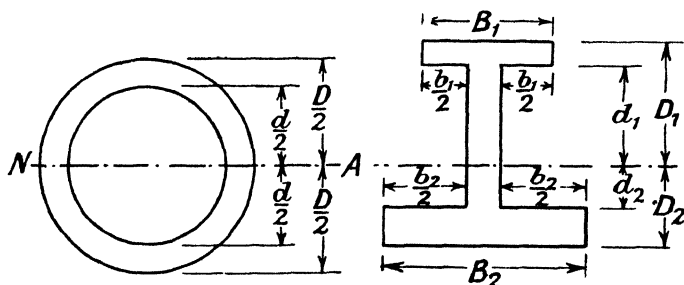


FIG. 74.

FIG. 75.

$$I = \frac{\pi}{64}(D^4 - d^4)$$

$$= \frac{\pi(D^4 - d^4)}{32D}$$

$$I = \frac{1}{12}(B_1 D_1^3 - b_1 d_1^3)$$

$$+ \frac{1}{12}(B_2 D_2^3 - b_2 d_2^3)$$

$$Z_1 = \frac{I}{D_1},$$

$$Z_2 = \frac{I}{D_2}.$$

EXAMPLES IV

1. Calculate the M.I. of the section for the following beams :—
 (a) 24 inches by 7½ inches I joist, flanges 1 inch and web 0.6 inch thick ; (b) 18 inches by 6 inches I joist, flange 0.7 inch and web 0.5

inch thick ; (c) 12 inches by 6 inches I joist, flanges 0.7 inch and web 0.5 inch thick ; (d) 9 inches by 4 inches I joist, flanges 0.5 inch and web 0.3 inch thick ; (e) 6 inches by 3 inches I joist, flanges 0.35 inch and web 0.25 inch thick ; (f) 4 inches by 3 inches I joist, flanges 0.34 inch and web 0.22 inch thick.

2. A 10 inch by 6 inch steel joist, whose web is 0.4 inch thick, has flanges which may be taken 0.75 inch thick. Find the maximum and minimum M.I. of the cross section. U.E.I.

3. A 12 inches by 6 inches rectangular beam carries a load of 1 ton at the centre of a 12-foot span. Find the greatest tensile and compressive stresses.

4. If, at a section of joist (a) question 1 there is a B.M. of 300 ton-inches, find the stress due to bending at a point 4 inches from the neutral axis.

5. A girder of I Section has a total depth of 12 inches, flanges 6 inches wide and $\frac{3}{8}$ inch thick, and web $\frac{1}{2}$ inch thick. Determine the greatest B.M. to which such a section may be subjected in order that the maximum tensile or compressive stress induced in the material may not exceed $7\frac{1}{2}$ tons per square inch. U.L.C.I.

6. State the assumptions that are made in the ordinary theory of beams.

A horizontal girder 20 feet long and supported at the ends is loaded with a uniformly distributed load of 1.5 tons per foot run, which includes the weight of the girder. The depth of the girder is 18 inches and the M.I. of a section is 1.150 in inch units. Find the maximum stress acting in the girder. U.L.C.I.

7. A rolled joist, 20 feet long, rests on supports at the ends and carries a uniformly distributed load of $\frac{1}{2}$ ton per foot run together with a concentrated load of 5 tons at 6 feet from a support. Calculate the maximum B.M. and the maximum stress due to bending. The M.I. of the joist is 600 inches⁴, and the depth of the section is 20 inches. N.C.T.E.C.

8. A simply supported steel joist, 10 inches by 5 inches in section, has a span of 16 feet. It carries a uniformly distributed load of 5 tons, and a concentrated load of 2 tons acting at 6 feet from one end. The M.I. of the section is 150 (inch)⁴. Determine the maximum B.M. and the skin stress. N.C.T.E.C.

9. A simply supported beam of 18 feet span carries loads of 1, 5 and 2 tons at points 5, 8 and 11 feet from one support. Find the modulus of section of the beam if the greatest stress due to bending is not to exceed 5 tons per square inch. Neglect the weight of the beam.

10. A temporary bridge of 10 feet span is made of two steel tubes, 6 inches internal diameter and 7 inches external diameter, laid parallel and 3 feet apart and covered with flooring. If the distributed load, including the weight of the bridge, is 2 cwt. per

foot run and there is a concentrated load at the centre of 20 cwt., find the maximum stress in the tubes due to bending.

11. A steel pipe, 12 inches internal diameter and 13 inches external diameter, is used as a beam. Find the central load on a span of 8 feet if the stress due to bending is not to exceed 5 tons per square inch. You may neglect the weight of the pipe.

12. At a section of a cast-iron girder, 9 inches total depth, there is a B.M. of 60 ton-inches. The bottom flange is 6 inches wide and 2 inches deep; the top flange is 3 inches wide and 1 inch deep; and the web is 1 inch thick. Find the skin stresses due to bending.

13. What is the greatest load a 6 inch by 6 inch by $\frac{1}{2}$ inch Tee, with the leg vertical, can carry on a span of 5 feet without exceeding a stress due to bending of 6 tons per square inch?

14. A cast-iron beam of Tee section, total depth 12 inches, is supported at the ends of a 10-foot span and carries a load of 2.75 tons at the centre of the space. The flange, $7\frac{1}{2}$ inches wide and 2 inches thick, is underneath the leg, which is vertical and $\frac{3}{4}$ inch thick. Find the distance of the Centre of Gravity of the Section, from the lower side of the tension flange, the M.I. of the Section, and the maximum tensile and compressive stresses induced. U.L.C.I.

15. A tilt hammer weighing 5 cwt. on the end of a lever 5 feet long is to be raised with a maximum acceleration of 5 feet per second per second. Find the size of rectangular lever required if the depth is to be twice the breadth and the stress is not to exceed $\frac{3}{4}$ ton per square inch. Neglect the weight of the lever.

16. A girder, 30 feet long, is supported at both ends. It carries a uniformly distributed load of 1 ton per foot run over a length of 16 feet, commencing at 2 feet from the left-hand support, and a concentrated load of 2 tons at a point 5 feet from the right-hand support. Draw to scale the curve of shearing force, and find the position of the section of the girder at which the B.M. is a maximum. What is the magnitude of this maximum B.M.?

If the girder is of I section 24 inches deep, with a second moment of the area (M.I. of the cross-section) about the neutral axis of 1,670 inch⁴ units, find the maximum stress intensity set up in the material, at the section at which this maximum B.M. occurs.

U.E.I.

17. Calculate the maximum stresses induced in a shaft 9 inches diameter and 30 feet long—

(a) When it is supported at each end;

(b) When it rests on two supports which are respectively 6 feet from the ends of the shaft.

Draw to scale the S.F. and B.M. diagrams for each case. 1 cubic inch of the material weighs 0.28 lb.

U.L.C.I.

18. A girder AC 50 feet long is supported at the end A and at a point B 40 feet from A. It is loaded with a load of 10 tons at C and a uniformly distributed load of 1 ton per foot run between

A and B. Draw to scale the B.M. and S.F. diagrams and determine the magnitude of the maximum B.M.

Allowing a stress of $7\frac{1}{2}$ tons per square inch, what should be the modulus of section of the girder used ? U.L.C.I.

19. A rolled steel joist 16 inches deep, with a M.I. of 726 inch-units is used to support a uniformly distributed load. If the span is 13 feet and the maximum stress in the flanges 4.5 tons per square inch, what is the distributed load per foot run ? U.E.I.

20. A girder 20 feet long and 10 inches deep is supported at the ends and carries a load W at 7 feet from one end. Find the maximum permissible value of W if the maximum stress in the material due to bending is not to exceed $7\frac{1}{2}$ tons per square inch, and the M.I. of the section is 146 (inches)⁴. U.L.C.I.

21. A horizontal I section girder 15 inches by 5 inches is 20 feet long and is freely supported at its ends. The weight of the girder is 42 lb. per foot run and the M.I. of its section is 428, in inch units. Determine the uniformly distributed load, in addition to its own weight, that the beam can carry, the maximum direct stress allowed being $7\frac{1}{2}$ tons per square inch. U.L.C.I.

22. A warehouse floor is to be designed to sustain a load of 560 lb. per square foot and the floor is to be carried on wooden joists of 4 inches breadth and 12 inches depth and span 12 feet. If the maximum stress to be permitted in the wood is 1,120 lb. per square inch, calculate the distance apart of the joists. U.E.I.

23. What are the usual assumptions made in discussing the theory of the bending of beams ?

The maximum B.M. acting in a loaded beam of I section is 384 inch-tons. The total depth of the section is 9 inches and the M.I. is 230 (inches)⁴. Find the maximum stress in the section and the radius of curvature of the beam at the section where the maximum B.M. acts.

$E = 30,000,000$ lb. per square inch. U.L.C.I.

24. A flat steel bar of thickness 0.2 inch has a Young's Modulus of 13,000 tons per square inch. What will be the radius of curvature to which it may be bent if the greatest stress is to be 20 tons per square inch ? Prove the formula you use.

If the width of the strip is 1.5 inches, and the length 6 feet, what two equal loads applied one at each end would produce this stress if the bar were supported at the centre ? U.E.I.

25. A girder of I section has a depth of 10 inches and the M.I. of its section is 345 (inch)⁴ units. It has a length of 20 feet and is supported at both ends ; its weight is 70 lb. per foot, and it carries a uniformly distributed load of 15 tons in addition. Find the maximum B.M. in the girder and at that section calculate the maximum stress and the radius of curvature of the girder. $E = 30,000,000$ lb. per square inch. U.E.I.

26. A rolled girder of I section has a depth of 10 inches with a

M.I. of 300 inch⁴ units. It is 20 feet long, supported at both ends, and has a uniformly distributed load of 1 ton per foot run. Calculate—

- (1) The maximum B.M. ;
- (2) Maximum stress in the girder ;
- (3) Radius of curvature where the greatest B.M. occurs.

$E = 13,500$ tons per square inch.

U.E.I.

27. A steel bar, 1 inch by $\frac{1}{4}$ inch section, 6 feet long, is laid symmetrically on two knife edges at the same level 4 feet apart, with the 1 inch edge horizontal. Loads of 10 lb. hung at each end cause an upward deflection of 0.9 inch at the centre. Calculate E for the material.

28. A strip of spring steel 20 inches long, 0.3 inch wide and 0.1 inch thick is bent to the form of a circular arc 72 inches diameter. Determine the B.M. acting and draw the B.M. diagram for the strip. Find also the maximum stress induced in the steel. $E = 30,000,000$ lb. per square inch.

U.L.C.I.

29. A piece of flat steel has to be bent round a drum 6 feet diameter. What is the maximum thickness the strip can be made so that there shall be no permanent deformation when it is removed from the drum ? The elastic limit of the steel is 14 tons per square inch, and E is 14,000 tons per square inch.

U.E.I.

30. Fig. 76 shows an arrangement in which a pin is used to turn a spindle. The spindle is to be subjected to a torque of $22\frac{1}{2}$ lb.-

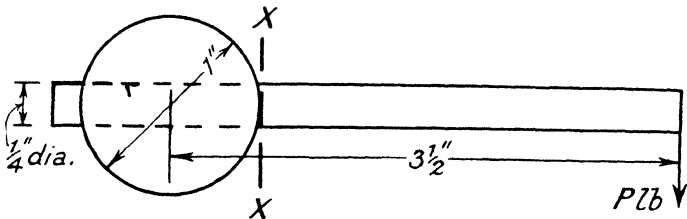


FIG. 76.

inches by a force P which acts at right-angles to the axis of the pin as shown in the sketch. Find the maximum stress in the material of the pin at the point where it enters the spindle (or at the Section XX).

U.L.C.I.

31. The axle of a railway truck is carried in bearings 5 feet 6 inches apart. The centres of the wheels are 4 feet 10 inches apart and are placed symmetrically between the bearings. Find the diameter of the axle if the ultimate strength of the material is 32 tons per square inch, and the total load on the axle is 10 tons.

32. A gear wheel with six spokes has a pitch circle diameter of

4 feet, and the tangential force at the pitch circle is 1,200 lb. Assuming that the driving effort is taken by three spokes, find the size of rectangular spoke required if the stress due to bending is not to exceed 2,000 lb. per square inch. The depth of the spoke section is to be twice the depth. The wheel hub is 12 inches diameter.

33. At a section of an I girder, $7\frac{1}{2}$ inches by 24 inches, with flanges 1 inch and web 0.6 inch thick, there is a B.M. of 1,052 ton-inches. Draw a graph showing the variation of tensile and compressive stresses with distance from the Neutral Axis, and hence find the proportion of the resisting moment due to the flanges, and that due to the web.

34. A girder 130 feet long, supported at one end and at a point 30 feet from the other end, carries a uniformly distributed load of 1 ton per foot run. The girder is in two parts, connected together at the section where there is no B.M. by a pin joint. Find the necessary diameter of pin if the shear stress is not to exceed 3 tons per square inch.

CHAPTER V

TORSION

36. Transmission of Power by Shafting. The relation between the horse-power, H , being transmitted by a shaft and the turning moment or torque, T , on the shaft in pound-inches is given by the equation

$$T = \frac{2\pi TN}{33000 \times 12}$$

where N is the speed of the shaft in revolutions per minute.

Since the power is being transmitted by a torque of one sense at one end of the shaft overcoming a resisting torque of the opposite sense at the other end of the shaft, one end of the shaft will be made to twist in relation to the other end, or the shaft will be subjected to torsion. The deformation will set up resisting shear stresses in the material of the shaft in planes perpendicular to the axis of the shaft, and at any section the moment of the shear force on the area of the section will oppose and balance the torque at that section.

Consider a thin square layer $ABCD$ (Fig. 77) on the surface of a shaft. When a torque is applied to the shaft this will become distorted to $AB'C'D$, and the shear strain will be $\frac{BB'}{AB} = \frac{x'}{l}$. There will obviously be no shear stress on the outer square face, and assuming that radial lines will remain radial, which is very nearly true, there can be no shear stress on the inner square face. If a length l of the shaft is considered an axial line AE will, after strain, become part of a helix, AE' , and the strain will be $\frac{x}{l}$. But $x = r\theta$, where r is the radius of the

shaft, and θ is the angle of twist in radians in the length l ; also

$$\text{Modulus of Rigidity, } G = \frac{\text{shear stress}}{\text{shear strain}}.$$

Therefore
$$G = f \div \frac{x}{l},$$

and
$$\frac{fl}{G} = x = \theta r = \frac{1}{2}d\theta,$$

where d is the diameter of the shaft in inches.

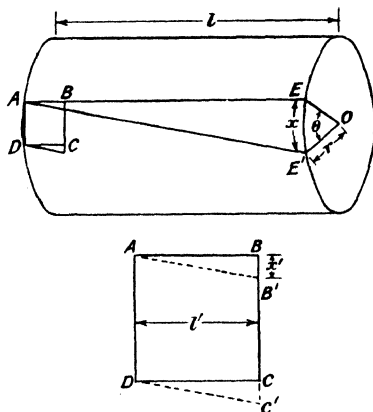


FIG. 77.

This equation is more conveniently written

$$\theta = \frac{fl}{Gr} = \frac{2fl}{GD}, \text{ or } n = \frac{360fl}{\pi Gd}$$

where n is the angle of twist in degrees.

If the stress and modulus are expressed in inch units, the length, diameter and torque must also be expressed in inch units.

37. Moment of Resistance of a Shaft to Torsion.

Since $\theta = \frac{fl}{Gr}$; and for all points on a given cross-

section of a shaft subjected to a torque θ , l and G are the same. Therefore the stress at any point in the cross-section is proportional to the distance of that point from the centre of the shaft, and

$$\frac{f_1}{r_1} = \frac{f}{r}$$

where f_1 is the stress at a radius r_1 , and f is the stress at

a radius r . Also, if a solid shaft were made up of a number of thin concentric tubes rigidly fixed together at their ends, the strength of the shaft to resist torsion would be the same as that of a similar shaft made out of one piece of material since there is no shear stress induced between layers which would form adjacent concentric rings (Article 36).

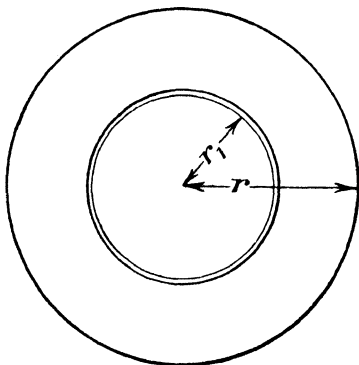


FIG. 78.

Consider a cross-section of a solid shaft of radius r made up of such thin concentric rings (Fig. 78). The shear stress on a tube whose radius is r_1 , and area of cross-section a_1 , will be f_1 where $f_1 = \frac{f}{r} r_1$, f being the stress at the external radius r . The total shear resistance of the tube will be $\frac{f}{r} a_1 r_1$, and the moment of resistance of the tube to torque will be $\frac{f}{r} a_1 r_1^2$. The total moment of resistance of the whole shaft to torque will be the sum of the moments of resistance of all the tubes making up the shaft, that is $\frac{f}{r} \sum a_1 r_1^2$, or $\frac{f}{r} J$, where J is the polar Moment of Inertia of the circular cross-section of the shaft about

the axis of the shaft. J for a circle is equal to $\frac{\pi r^4}{2}$, therefore the resisting moment of the shaft is equal to

$$\frac{\pi r^3 f}{2} = \frac{\pi d^3 f}{16} = T,$$

where T is the torque applied to the shaft in pound-inches, and f is the maximum stress, on the outermost layer of the shaft, in pounds per square inch, because for equilibrium the resisting moment must be equal to the torque.

Since $\theta = \frac{2fl}{Gd}$, by substituting the above value for f we get

$$\theta = \frac{32Tl}{\pi G d^4} = \frac{Tl}{GJ}$$

38. Hollow Shafts. The method of deducing the moment of resistance of a hollow shaft (Fig. 79) is the

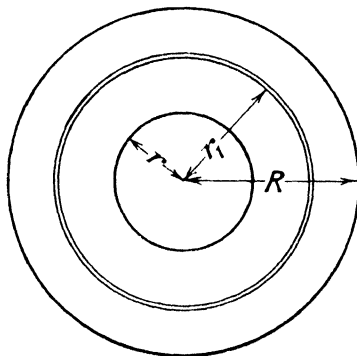


FIG. 79.

same as that for a solid shaft, but the summation $\Sigma a_1 r_1^2$ will be from the internal radius, r , to the external radius R , and $T = \frac{f}{R}J$, where J for a circular ring is equal to $\frac{\pi(R^4 - r^4)}{2}$, and therefore

$$T = \frac{\pi(R^4 - r^4)f}{2R} = \frac{\pi(D^4 - d^4)f}{16D}.$$

Also $\theta = \frac{32Tl}{\pi G(D^4 - d^4)}.$

The same result may be obtained by subtracting the resisting moment of a shaft of radius r from the resisting moment of a shaft of radius R .

For the same weight of material per unit length a hollow shaft is stronger than a solid shaft. This is because material may be considered to have been moved from the centre where the stress is small, or the material is doing little work, to the outside where the stress is greater and the material is doing more work.

EXAMPLE. If a torque of 26 ton-inches is applied to a shaft 3 inches in diameter, find the maximum shear stress induced, and the angle of twist on a length equal to 20 diameters.

$$G = 12.5 \times 10^6 \text{ lb. per square inch.}$$

$$f = \frac{16T}{\pi d^3} = \frac{16 \times 26}{\pi \times 27} = 4.9 \text{ tons per square inch.}$$

$$\begin{aligned} \theta &= \frac{32Tl}{\pi Gd^4} = \frac{32 \times 26 \times 2240 \times 20 \times 3}{\pi \times 12.5 \times 10^6 \times 3^4} \\ &= 0.035 \text{ radians} \\ &= 2^\circ. \end{aligned}$$

39. Helical Springs. The material of a *close coiled* helical spring carrying an axial load may be regarded as subject to torsion only, the shear stress due to the axial load being in most cases negligibly small compared with the stress due to twisting of the wire forming the spring.

Consider a close coiled helical spring (Fig. 80), and let R be the mean radius of the coil in inches, d be the diameter of the round wire in inches, W be the axial load in pounds, δ be the axial deflection in inches, n be the number of complete coils, and l the length of wire in the coils in inches. Since the coils are close $l = 2\pi Rn$ very nearly.

The whole of the wire is subjected to a twisting moment equal to $WR = T$ (Fig. 81), and if one end of the wire is held fast, the other end will twist through an angle θ , where

$$\theta = \frac{Tl}{GJ} = \frac{32Tl}{\pi Gd^4} = \frac{32WRl}{\pi Gd^4} \text{ radians.}$$

The free end of the wire will have an axial movement of $R\theta$, therefore

$$\delta = R\theta = \frac{32WR^2l}{\pi Gd^4} = \frac{64WR^3n}{Gd^4} \text{ inches.}$$

In the case of an *open coiled* helical spring, the material is subjected to both torsion and bending, but if the obliquity of the coils is not greater than 10° , the error due to neglecting the bending effect is less than 1 per cent.

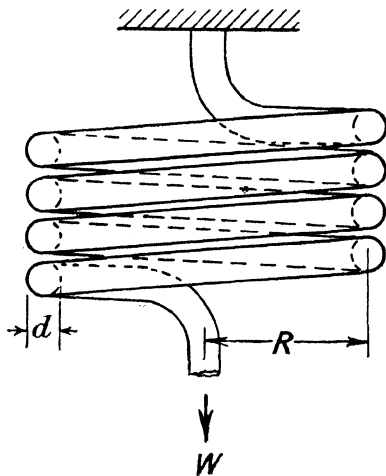


FIG. 80.

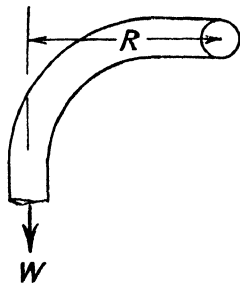


FIG. 81.

EXAMPLE. A close coiled helical spring made of $\frac{1}{4}$ inch diameter steel wire has a mean diameter of 4 inches. (a) What is the greatest axial load it can carry if the stress in the material is not to exceed 20,000 lb. per square inch? (b) If there are 10 coils, what will be the axial deflection? $G = 12 \times 10^6$ lb. per square inch.

$$(a) \quad T = WR = \frac{\pi d^3 f}{16}.$$

$$W = \frac{\pi \times 0.25^3 \times 20,000}{16 \times 2} = 30.6 \text{ lb.}$$

$$(b) \quad \delta = \frac{64WR^3n}{Gd^4} = \frac{64 \times 30.6 \times 2^3 \times 10}{12 \times 10^6 \times 0.25^4} \\ = 3.35 \text{ inches.}$$

EXAMPLES V

1. A shaft is subjected to simple torsion. Deduce an expression which connects the torque with the diameter of the shaft and the maximum stress induced.

A shaft subjected to pure torsion is to transmit 60 H.P. at 100 r.p.m. What diameter of shaft would be required if the maximum stress in the material is not to exceed 8,000 lb. per square inch?

U.L.C.I.

2. Find the maximum stress induced in a shaft transmitting 7,500 H.P. at a speed of 300 r.p.m., the diameter of the shaft being 12 inches. Derive the formula used.

U.L.C.I.

3. A shaft $6\frac{1}{2}$ inches diameter transmits 1,500 H.P. at 350 r.p.m. Find the maximum stress induced in the shaft, and also the relative angle of twist in degrees between two cross-sections of the shaft 10 feet apart. $G = 12,000,000$ lb. per square inch.

U.L.C.I.

4. A shaft 8 inches diameter transmits 1,400 H.P. at a speed of 160 r.p.m. Determine the maximum stress induced in the shaft. Derive the formula.

U.L.C.I.

5. A solid shaft 9.5 inches diameter transmits 2,500 H.P. at a speed of 450 r.p.m. The ratio of the maximum to the mean torque acting on the shaft is 1.6. Determine the maximum shear stress acting in the material.

U.L.C.I.

6. A shaft 3 inches diameter running at 300 r.p.m. transmits 60 H.P. Find the maximum stress in the shaft and the twist in degrees in a length of 100 feet. $G = 12,000,000$ lb. per square inch. (Assume the torque is uniform and neglect all bending actions.)

U.E.I.

7. Show that the intensity of shear stress at a point in a transverse section of a circular shaft subjected to twisting is proportional to the distance of the point from the centre. Find the diameter of a shaft to transmit 300 H.P. at 180 r.p.m. assuming a maximum stress of 7,500 lb. per square inch.

U.L.C.I.

8. A circular shaft is subject to torsion. Prove that the intensity of shear stress at any point is proportional to its distance from the axis. Find the diameter of the shaft which will transmit 100 H.P. at 200 r.p.m. if the maximum stress allowed is 7,500 lb. per square inch, and the maximum torque is 1.4 times the mean torque.

U.E.I.

9. A solid steel shaft is to transmit 30 H.P. at 120 r.p.m. Find the necessary shaft diameter, if the shear stress is not to exceed 7,500 lb. per square inch, and the maximum torque is 1.5 times the mean torque.

U.E.I.

10. Find the diameter of shaft necessary to transmit 4,000 H.P.

at a speed of 220 r.p.m. if the angle of twist is not to exceed 1° in a length of 20 diameters, and determine the maximum stress induced in the shaft. $G = 12,000,000$ lb. per square inch.

U.L.C.I.

11. Calculate the maximum torque which can be applied to a steel bar 2 inches diameter, so that the shear stress shall not exceed 6 tons per square inch.

If a length of 3 feet of the bar were stressed to this limit, determine the amount of twist in degrees.

$G = 12,000,000$ lb. per square inch.

N.C.T.E.C.

12. A solid steel shaft, 3 inches in diameter, twists through 16 degrees on a length of 40 feet when transmitting power. Determine (a) the twisting moment in inch-tons, (b) the H.P. transmitted if the shaft is revolving at 90 r.p.m.

$G = 5,500$ tons per square inch.

N.C.T.E.C.

13. Determine the maximum torque to which a 4-inch diameter shaft should be subjected, given that the maximum allowable stress is 5,000 lb. per square inch and that the twist is not to exceed 1 degree in a length of 20 diameters.

$G = 12,500,000$ lb. per square inch.

You ought to work from first principles, and not merely write down a formula.

U.L.C.I.

14. A shaft 9 inches diameter revolves 150 times per minute. Find the H.P. transmitted by the shaft when it has an angle of twist of 1° for each length of 20 diameters.

$G = 12,000,000$ lb. per square inch.

U.L.C.I.

15. A rod of elastic material $\frac{1}{2}$ inch diameter is subjected to a twisting moment of 162 inch-lb. and in a length of 20 inches it is observed that the angle of twist of the rod is 1° . Determine the modulus of rigidity of the material.

U.L.C.I.

16. 10,000 H.P. is transmitted by a hollow shaft, 15 inches external diameter and 9 inches internal diameter, at a speed of 100 r.p.m. Find the maximum shear stress induced in the shaft.

17. What H.P. can be safely transmitted by a hollow shaft of external diameter 6 inches and internal diameter 3 inches at a speed of 200 r.p.m. ? The shaft is assumed to be subjected to pure torsion and the angle of twist is not to exceed 1 degree in a length of 10 feet. What will be the maximum intensity of stress induced in the material of the shaft ?

$G = 12,000,000$ lb. per square inch.

U.E.I.

18. A shaft 9 inches diameter transmits a given H.P. at a certain speed. What diameter of shaft of the same material with a hole through its centre would be required to replace the 9-inch shaft if the diameter of the hole is to be 0.4 of the external diameter of the shaft ? What would be the ratio of the weights of the two shafts ?

U.L.C.I.

19. Determine the external diameter of a hollow shaft to transmit 10,000 H.P. at 180 r.p.m., the maximum shear stress allowed being 3 tons per square inch, and the internal diameter being 0.6 of the external. U.L.C.I.

20. What H.P. can be safely transmitted by a hollow shaft of external diameter 12 inches and internal diameter 6 inches at a speed of 100 r.p.m. ? The shaft is assumed to be subjected to pure torsion, and the angle of twist is not to exceed 1° in a length of 10 feet. What will be the maximum intensity of shear stress induced in the material of the shaft ?

$G = 12,000,000$ lb. per square inch.

21. A closely coiled helical spring, 6 inches in mean diameter, is made of $\frac{3}{8}$ inch diameter wire. What axial pull can this spring maintain if the shear stress is not to exceed 20,000 lb. per square inch ? N.C.T.E.C.

22. A closely coiled helical spring is made from wire one quarter inch diameter, so that it will elongate 1 inch with an axial pull of 30 lb. How many coils of 4 inches mean diameter are required ? $G = 12,000,000$ lb. per square inch. N.C.T.E.C.

23. A spiral spring is made from round steel closely coiled in 10 turns whose outside diameter is 5 times that of the rod. If the spring is to stretch 0.5 inches under an axial load of 300 lb., find the diameter of the rod. $G = 12,000,000$ lb. per square inch. The case may be taken as one of pure torsion. U.E.I.

24. A spiral compression spring is made of steel rod of diameter 0.3 inches, it has 10 coils of external diameter 1.7 inches, and its unloaded length is 10 inches. The results of a test on this spring are given :

Axial load in cwt. .	1	2	3	4	5	6	7	10
Compression of spring in inches . . .	.26	.51	.77	1.04	1.29	1.55	1.81	2.59

Plot a graph of these results. Select a convenient pair of values from the graph, and, considering the case as one of pure torsion, calculate the twist per unit length for your own case and the modulus of rigidity of the material. U.E.I.

25. A solid steel shaft is 3 inches diameter. Compare the maximum permissible twisting moment and bending moment which may be applied to the shaft if the shear stress is not to exceed 10,000 lb. per square inch, and the tensile stress is not to exceed 12,000 lb. per square inch.

26. A flange coupling connects two lengths of 3-inch shafting. The 6 coupling bolts are arranged in a circle 8 inches diameter. The shaft transmits 40 H.P. at 250 r.p.m. Find the diameter of the bolts if their average stress is to be the same as the maximum stress in the shaft.

U.E.I.

CHAPTER VI

DYNAMICS

40. Force, and Inertia and other Resistances.

On the inertia of a body depends its resistance to change of motion (increase or decrease in velocity, or change of direction), and the inertia depends directly on the quantity of matter in the body. The engineers' unit quantity of matter is that amount which is contained in a mass whose weight is 1 lb.

When a force acts on a body there may be more than one resistance to overcome, such as gravity, friction, and inertia. The two former may be called static resistances, and the third a dynamic resistance, for the two former can be balanced without change of motion taking place, and the inertia is a resistance to change of motion due to any force in excess of the force required to overcome the static resistances. In solving problems in which these different resistances occur, it is often helpful to split up the force applied to the body into components (all in the same direction) each of which is equal to one of the resistances.

EXAMPLE. A mass of 100 lb. weight is being pulled up an incline of 5° to the horizontal by a force of 20 lb. If the coefficient of friction is 0.1, find the acceleration produced.

$$\text{Friction Force} = 100 \times 0.1 \times 0.996 = 9.96 \text{ lb.}$$

$$\text{Gravity Force} = 100 \times 0.087 = 8.7$$

$$\text{Accelerating Force} = 20 - (9.96 + 8.7) = 1.34 \text{ lb.}$$

$$\text{Acceleration} = \frac{Pg}{W} = \frac{1.34 \times 32.2}{100} = 0.431 \text{ feet per sec. per sec.}$$

41. Relation between Linear and Angular Motion.

In linear motion displacements are linear distances and

velocities are linear displacements divided by time. In angular, or rotational, motion displacements are reckoned as the angular movements of a radius, and velocities as angular displacements divided by time, the angles being measured in radians. There is therefore a direct analogy between linear and angular motion, and similar equations can be deduced for both. In the case of angular inertia, momentum and energy since the change in motion is due to the *turning moment* of the force applied, and not to the force alone irrespective of its position with relation to the axis of rotation, the moment of inertia, momentum and energy must be taken instead of the linear values to form the respective equations.

A list of equations for linear motion is given below for revision purposes, and the methods of obtaining the angular equations are shown in the following articles.

ELEMENTARY DYNAMICAL EQUATIONS

$$S = Vt. \qquad S = \frac{(V_1 + V_2)t}{2}$$

$$V_2 = V_1 + at. \quad S = V_1t + \frac{1}{2}at^2. \quad V_2^2 = V_1^2 + 2as.$$

$$P = \frac{Wa}{g}, \quad Pt = \frac{W(V_2 - V_1)}{g}, \quad \text{Work done} = Ps$$

$$\text{Potential Energy} = Wh. \quad \text{Kinetic Energy} = \frac{WV^2}{2g}.$$

Where—

V_1 = initial linear velocity. V_2 = final linear velocity.

a = acceleration. S = distance travelled.

t = time taken. V = uniform velocity, or velocity at any instant.

P = Force in gravitation units (e.g., lb. weight).

W = Weight of body.

Note. In all equations similar quantities must be expressed in the same units ; e.g., if the value 32.2 is used for g , all linear quantities must be expressed in feet and

time in seconds. Also distances, velocities and accelerations measured in reversed directions must be given negative values.

42. Angular Motion. If a body is travelling in a circular path, of radius R feet, with a uniform speed of V feet per second the radius

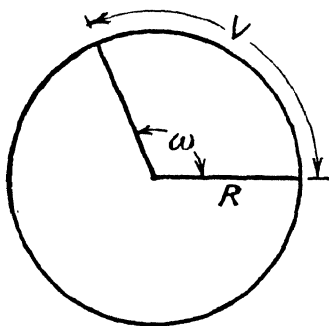


FIG. 82.

joining the body to the centre of curvature will be moving with a uniform angular velocity of ω radians per second (Fig. 82). Since angle in radians equals arc divided by radius, $\omega = V/R$. If the velocities are varying the relationship holds good for the velocities at any instant. Similarly, if the body is moving with a linear acceleration of a feet per

second per second, the angular acceleration, α radians per second per second, will be given by $\alpha = a/R$.

EXAMPLE. If a point on the rim of a flywheel 6 feet diameter is moving with a speed of 24 feet per second, what is the angular velocity of the flywheel?

$$\omega = V/R = 24/3 = 8 \text{ radians per sec.}$$

If the speed of the point increases to 30 feet per second in two seconds, what is the angular acceleration of the flywheel?

$$a = (V_2 - V_1)/t = (30 - 24)/2 = 3 \text{ feet per sec.}^2$$

$$\alpha = a/R = 3/3 = 1 \text{ radian per sec.}^2$$

If a radius, or body, is moving with a uniform angular velocity, ω , for a time t , the angle moved through, θ radians, will be given by $\theta = \omega t$. When the velocity changes at a uniform rate from ω_1 to ω_2 , the angle turned through will be given by—

$$\theta = \frac{(\omega_1 + \omega_2)t}{2}$$

Also $\omega_2 = \omega_1 + \alpha t$, and the angle moved through can be written as

$$\theta = \frac{(\omega_1 + \omega_1 + \alpha t)}{2} t,$$

or $\theta = \omega_1 t + \frac{1}{2} \alpha t^2$.

Since $2\theta = (\omega_1 + \omega_2)t$, and $\omega_2 = \omega_1 + \alpha t$, by substituting for t in the first equation the value of t in the second one we get $\omega_2^2 = \omega_1^2 + 2\alpha\theta$.

EXAMPLE. Find the angular acceleration of, and the angle turned through by a flywheel when it changes its speed from 120 r.p.m. to 360 r.p.m. in $2\frac{1}{2}$ minutes.

$$120 \text{ r.p.m.} = \frac{120 \times 2\pi}{60} = 12.56 \text{ rad. per sec.}$$

$$360 \text{ r.p.m.} = \frac{360 \times 2\pi}{60} = 37.68 \text{ rad. per sec.}$$

$$\alpha = \frac{\omega_2 - \omega_1}{t} = \frac{37.68 - 12.56}{150} = .167 \text{ rad. per sec.}^2$$

$$\theta = \frac{(\omega_1 + \omega_2)}{2} t = \frac{37.68 + 12.56}{2} \times 150 = 3768 \text{ radians.}$$

= 600 revolutions.

43. Rotational Inertia or Moment of Inertia.

If a small body A, of weight w , which is constrained to move in a circular path of radius R about an axis O, is acted on by a force P whose line of action is always tangent to the path of the body (Fig. 83), the body will be given a linear

acceleration a such that $P = \frac{wa}{g}$; or, since $a = \alpha R$, where α equals the angular acceleration,

$$P = \frac{wR\alpha}{g}.$$

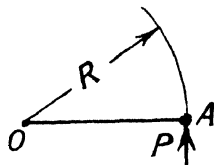


FIG. 83.

The torque, T , causing rotation, is equal to PR , and therefore

$$PR = T = \frac{wR^2\alpha}{g}.$$

In linear motion we have simply a force P acting on a body of mass W , the mass being the measure of the inertia

of the body ; but in angular, or rotational motion, it is not sufficient to specify the force P only, but the radius R at which the force acts must be specified also. It is therefore the turning moment, T , of the force which has to be considered, and not the individual values of P and R singly or separately. Also, since we are taking the moment of the force it is necessary to equate this to the *moment of inertia* of the body, coupled with the angular acceleration instead of the linear acceleration of the body. Above it is shown that in this quantity the radius is brought in twice, once when changing from linear to angular motion, and again when taking moments. We thus get mass multiplied by radius squared, wR^2 , as a measure of the moment of the inertia of the body to change of angular velocity.

Therefore, if $I = wR^2$, the moment of inertia of the body about the axis O , then $T = \frac{I\alpha}{g}$.

The above refers to a body whose mass is so small that the whole mass may be considered to be at the radius R , but in a body of definite size this is not so, all the small elements, $w_1, w_2, w_3, \dots$, being at different radii, $r_1, r_2, r_3, \dots$, from the centre of rotation. In the case of a body of definite size, therefore, the moment of inertia is equal to the sum of the products $w_1r_1^2, w_2r_2^2, w_3r_3^2, \dots$. The moment of inertia of a body of simple regular shape may be calculated by integrating the quantity r^2 between given limits ; $w_1, w_2, w_3, \dots$ being of equal magnitude, and together equal to the weight, W , of the body. If a distance k from the axis be taken such that $k^2 = (r_1^2 + r_2^2 + r_3^2 + \dots)$ the moment of inertia of the body would not be altered if the whole mass were considered concentrated at this distance from the axis, for $Wk^2 = W(r_1^2 + r_2^2 + r_3^2 \dots)$. This distance k is called the *Radius of Gyration* of the body. For rotational inertia this is analogous to concentrating the mass of the body at its centre of gravity for linear inertia, but the distinction between the two must be noted ; the case

of a body rotating about an axis through its centre of gravity illustrates this.

The distance r is measured perpendicular to the axis of rotation, therefore any displacement of the body, or part of the body, *parallel* to the axis will not cause any change in the moment of inertia of the body.

To calculate the moment of inertia of a solid which is symmetrical about three mutually perpendicular axes through the centre of gravity, Routh's Rule is often used. It is stated as follows:—The M.I. of a solid about any one of three mutually perpendicular axes through the C.G. is equal to the weight of the body multiplied by the sum of the squares of the other two *semi-axes*, divided by three for a cube or rectangular solid, by four for a circle or ellipse, and by five for a sphere.

The moment of inertia, I_x , of a body of weight W about an axis parallel to one through the centre of gravity of the body, and distant h from it, is given by the equation $I_x = I_G + Wh^2$, where I_G is the moment of inertia about the axis through the centre of gravity of the body.

EXAMPLE. Find the moment of inertia of a rectangular plate 24 inches by 12 inches by 2 inches about an axis along the centre of the 12 inch side. Weight of 1 cubic inch of material, 0.28 lb.

$$W = 24 \times 12 \times 2 \times 0.28 = 161 \text{ lb.}$$

$$I_G = 161 \times \frac{12^2 + 1^2}{3} = 7,782 \text{ lb.-ins.}^2$$

$$\begin{aligned} I_x &= I_G + Wh^2 \\ &= 7,782 + 161 \times 12^2 \\ &= 30,966 \text{ lb.-ins.}^2 \end{aligned}$$

EXAMPLE. A torque of 600 lb.-feet acts on a flywheel whose weight is 2,000 lb., and radius of gyration 3 feet. Find the angular acceleration produced, and the time required to reach a speed of 240 r.p.m. from rest.

$$T = \frac{Wk^2\alpha}{g}.$$

$$\alpha = \frac{600 \times 32.2}{2000 \times 3^2} = 1.07 \text{ rad. per sec.}^2$$

$$240 \text{ rev. per min.} = \frac{240 \times 2\pi}{60} = 25.12 \text{ rad. per sec.}$$

$$t = \frac{\omega}{\alpha} = \frac{25.12}{1.07} = 23.5 \text{ secs.}$$

44. Angular Momentum. Since $T = \frac{I\alpha}{g}$, if the torque, T , acts on the body for a time, t , then

$$Tt = \frac{I\alpha t}{g} = \frac{I(\omega_2 - \omega_1)}{g}.$$

The quantity $\frac{I\omega}{g}$ is called the *Angular Momentum*, or *Moment of Momentum*, of the body.

EXAMPLE. If a torque of 0.2 lb.-feet acts on a body weighing 60 lb., whose radius of gyration is 0.75 feet for 2 minutes; find the increase of angular velocity.

$$Tt = \frac{Wk^2(\omega_2 - \omega_1)}{g}$$

$$0.2 \times 120 = \frac{60 \times 0.75^2(\omega_2 - 0)}{32.2}$$

$$\omega_2 = 34 \text{ rad. per sec.}$$

45. Work Done by a Torque. The work done by a torque, T , which turns through an angle θ against a resistance is equal to $T\theta$; for the torque, T , may be replaced by a tangential force, P , at a radius R , where $T = PR$. The distance, s , moved through by the force P in its own direction is equal to $R\theta$. Therefore

$$Ps = PR\theta = T\theta.$$

EXAMPLE. How many foot-pounds of work are done when a 3 feet-diameter pulley is turned through 40 revolutions by an effective belt pull of 120 lb.?

$$\text{Work done} = T\theta = 120 \times 1.5 \times 40 \times 2\pi = 45,216 \text{ ft.-lb.}$$

46. Kinetic Energy of a Rotating Body. If a small body of weight w be moving with a linear velocity v in a circle of radius R , its

$$\text{Kinetic Energy} = \frac{wv^2}{2g} = \frac{wR^2\omega^2}{2g}, \text{ since } v = \omega R.$$

A large body of weight W can be divided up into a number of small parts, w_1, w_2, w_3 , etc., whose distances

from the axis are r_1, r_2, r_3 , etc. respectively : then the kinetic energy of the whole body is

$$\frac{(w_1 r_1^2 + w_2 r_2^2 + w_3 r_3^2 + \text{etc.}) \omega^2}{2g} = \frac{W k^2 \omega^2}{2g} = \frac{I \omega^2}{2g},$$

since each part of the body will have the same angular velocity.

EXAMPLE. A flywheel weighing 200 lb. has a radius of gyration of 9 inches. Find the work which must be done to increase its speed from 10 radians per second to 20 radians per second.

$$\text{K.E. at 10 rad. per sec.} = \frac{W k^2 \omega^2}{2g} = \frac{200 \times 0.75^2 \times 10^2}{2 \times 32.2} = 174.7 \text{ ft.-lb.}$$

$$\text{K.E. at 20 rad. per sec.} = \frac{200 \times 0.75^2 \times 20^2}{2 \times 32.2} = 698.8 \text{ ft.-lb.}$$

$$\text{Work done} = 698.8 - 174.7 = 524.1 \text{ ft.-lb.}$$

47. Centrifugal Force. When a body is moving in a circular path with uniform *speed* the direction of motion at any instant is tangential to the path of the body, or perpendicular to the radius ; but this direction is changing continuously, and therefore the *velocity* is changing continuously. To cause this change of velocity there must be a force impressed on the body in the direction of the change of velocity, or acceleration, to overcome the inertia of the body. Consider a body moving round a circle of radius R and centre O , with uniform speed V . Let A be its position at any instant and B be its position t seconds later, t being very small, and therefore the distance AB very small (Fig. 84). Draw a vector, oa , to represent the body's velocity when it is at A , and a vector ob to represent its velocity when it is at B , then ab will represent the change of velocity from A to B . If a third position of the body, c , be taken t seconds later, another velocity triangle, obc , can be drawn, and if the process be repeated until the body reaches the position A again, the total change of velocity will be equal to the circumference of a circle of radius V , that is, $2\pi V$. If equal intervals of time are taken, it will be seen that the changes in velocity are equal, or that the acceleration is constant, and that

it is always along the radius towards the centre. The time taken for A to make a complete revolution is $\frac{2\pi R}{V}$, and therefore the change of velocity per unit time or *radial acceleration* is equal to

$$\frac{2\pi V}{2\pi R/V} = \frac{V^2}{R}.$$

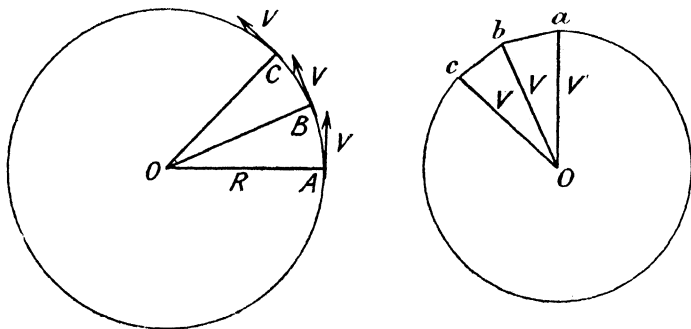


FIG. 84.

If the body's speed is given as ω radians per second, since $V = \omega R$, by substitution the radial acceleration is equal to $\frac{\omega^2 R}{g}$.

Since $P = \frac{Wa}{g}$, by substituting the above values for a the force towards the centre, or *centripetal force* required to keep a body of weight W moving with uniform speed V , or ω , in a circle of radius R will be equal to $\frac{WV^2}{gR}$, or $\frac{W\omega^2 R}{g}$. The inertia of the body, resisting change of motion, will exert an equal outward or *centrifugal force*.

EXAMPLE. A stone weighing 2 lb. is whirled round in a vertical circle at the end of a string 3 feet long at a speed of 60 r.p.m. Find

the tension in the string when the stone is at the highest and lowest points.

$$P = \frac{W\omega^2 R}{g} = \frac{2 \times (2\pi)^2 \times 3}{32 \cdot 2} = 7 \cdot 4 \text{ lb.}$$

$$\text{Tension at the top} = 7 \cdot 4 - 2 = 5 \cdot 4 \text{ lb.}$$

$$\text{Tension at the bottom} = 7 \cdot 4 + 2 = 9 \cdot 4 \text{ lb.}$$

In the case of a large body the body may be considered as split up into a number of small parts of weights w_1 , w_2 , w_3 , etc. at radii r_1 , r_2 , r_3 , etc.: the centrifugal force of the body would then be equal to $\frac{\omega^2}{g}(w_1 r_1 + w_2 r_2 + w_3 r_3 + \text{etc.})$, but $(w_1 r_1 + w_2 r_2 + w_3 r_3 + \text{etc.}) = WR$, therefore the whole body may be considered as concentrated at its centre of gravity, which is at a radius R from the centre of rotation.

48. Simple Harmonic Motion. When a body oscillates about a point in such a manner that it has an acceleration always towards that point, the magnitude of the acceleration being proportional to the distance from the point, the body is said to be moving with *Simple Harmonic Motion*.

If a weight is hung on the end of a spiral spring, equilibrium is obtained when the upward force due to the extension of the spring is equal to the downward force due to the weight. If the weight is pulled down below the equilibrium position, and released, it will oscillate about this position, which is called the *centre of oscillation*. During the period when the weight is below the centre the resultant force on it will be an upward force, because the downward force due to the weight is constant, while the upward force due to the spring increases, proportional to the distance from the centre, and the weight's acceleration will be towards the centre and proportional to the distance from it. Similarly, during the period when the weight is above the centre, the acceleration will be downwards and proportional to the distance from the centre. The weight will, therefore, oscillate with *Simple Harmonic Motion*.

Simple Harmonic Motion may take place when a body is moving in a straight path, a curved path, or rotating about a centre. The distance from the centre to the extremity of the oscillation is called the *amplitude*; and the time taken from passing any point to the next time of passing in the same direction the *period* of the oscillation.

Let a point, P (Fig. 85), be moving round a circle of radius r with uniform angular velocity ω . Then Q, the projection of P on a diameter, will make a complete oscillation along the diameter while P makes a complete

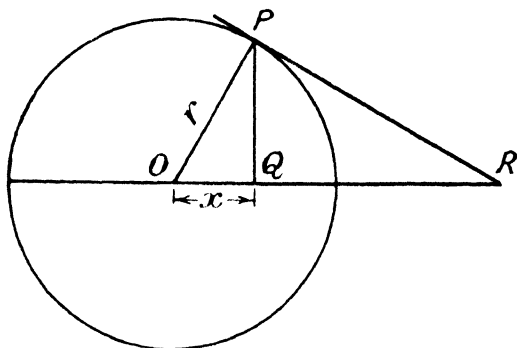


FIG. 85.

revolution. At any instant, when the distance of Q from the centre, O, is x , the acceleration of P will be $\omega^2 r$ along PO; and, since Q is moving along the diameter, Q's acceleration, a , will be the component of this along the diameter. Therefore $a = \omega^2 r \cos \text{POQ} = \omega^2 r \times \frac{x}{r} = \omega^2 x$. The point Q is therefore moving with Simple Harmonic Motion because its acceleration is always towards the centre, and proportional to the distance from the centre, since ω^2 is constant. This property of the point Q gives a simple method of obtaining equations to solve problems on Simple Harmonic Motion.

The velocity of P is ωr in the direction RP at right-angles to PO. The velocity v of Q is the component of this along the diameter.

Therefore $v = \omega r \cdot \cos PRO = \omega r \cos OPQ$

$$= \omega r \times \frac{\sqrt{r^2 - x^2}}{r} = \omega \sqrt{r^2 - x^2}.$$

From the above two equations we see that the acceleration is zero at the centre, and a maximum at the extremity, while the velocity is zero at the extremity and a maximum at the centre.

The time of a complete oscillation, t , is given by $t = \frac{2\pi}{\omega}$, the time taken for P to make a complete revolution. Since the force acting on the body, giving it Simple Harmonic Motion, is proportional to the distance from the centre, the force, P, at any distance (x) is equal to λx , where λ is the force towards the centre, or restoring force, at unit distance from the centre. Using the inertia equation, we have

$$a = \frac{Pg}{W} = \frac{\lambda xg}{W} = \omega^2 x.$$

$$\text{Therefore } \omega = \sqrt{\frac{\lambda g}{W}},$$

which gives an expression in terms of the weight of the body and the force towards the centre to substitute for ω in the above equations, which then become—

$$\text{Acceleration } a = \frac{\lambda g}{W} \cdot x, \text{ at displacement } x.$$

$$\text{Velocity } v = \sqrt{\frac{\lambda g}{W}} \times \sqrt{r^2 - x^2}, \text{ at displacement } x.$$

$$\text{Periodic Time } t = 2\pi \sqrt{\frac{W}{\lambda g}}$$

EXAMPLE. If a force of 2 lb. causes the end of a flat spring to deflect 3 inches, find the number of oscillations per minute and the acceleration and velocity at a displacement of 6 inches,

when a weight of 5 lb. is fixed to the end of the spring and made to oscillate with an amplitude of 12 inches.

$$\lambda = \frac{P}{x} = \frac{2}{.25} = 8 \text{ lb. for a displacement of 1 foot.}$$

$$t = 2\pi\sqrt{\frac{W}{\lambda g}} = 2\pi\sqrt{\frac{5}{8 \times 32.2}} = .875 \text{ sec.}$$

$$N = \frac{60}{.875} = 68.6 \text{ oscillations per minute.}$$

$$a = \frac{\lambda g}{W} \cdot x = \frac{8 \times 32.2}{5} \times 0.5 = 25.76 \text{ ft. per sec. per sec.}$$

$$v = \sqrt{\frac{\lambda g}{W} \cdot \sqrt{r^2 - x^2}} = \sqrt{\frac{8 \times 32.2}{5} \cdot \sqrt{1^2 - 0.5^2}} \\ = 6.2 \text{ ft. per sec.}$$

EXAMPLES VI

1. A velocity-time diagram has an area of 27.75 square inches. The time-base is 7.5 inches long and represents 20 seconds. The velocity scale is 1 inch = 5 feet per second. If the initial displacement of the body is 30 feet, what is the displacement of the body at the end of the 20 seconds?

If the tangent at a certain point of this velocity time curve is inclined at 42° to the time base, what is the acceleration at the time considered?

U.E.I.

2. Establish formulæ for the velocity and displacement from starting-point in time t in the case of a body moving in a straight line, with an initial velocity u and a constant acceleration f .

A body moving in a straight line with uniform acceleration travelled 150 feet in 3 seconds, and 200 feet in the next two seconds. Calculate the initial and final velocities and the acceleration.

N.C.T.E.C.

3. A stone is dropped down a shaft and its impact against the bottom is heard after 11 seconds. Taking the velocity of sound to be 1,100 feet per second, find the depth of the shaft.

N.C.T.E.C.

4. A wheel starting from rest receives a uniform angular acceleration of 2 radians per second per second. Find the speed in r.p.m. at the end of 1 minute.

If the effort is then taken off, and if the frictional resistance of the bearings is equivalent to a uniform retardation of $\frac{3}{4}$ radian per second per second, in how many minutes will the wheel come to rest?

N.C.T.E.C.

5. Explain what the letters represent in the formula $\frac{WV^2}{2g}$, which is used for determining the kinetic energy of a moving body,

At a certain instant a body weighing 20 lb. has stored in it 360 ft.-lb. of kinetic energy. Determine the speed with which it is moving, and also the reduction of speed that takes place when its kinetic energy is reduced to 30 ft.-lb. U.L.C.I.

6. The total load on the axles of the coupled wheels of a locomotive is 50 tons, and the coefficient of friction between the wheels and the rails is 0.19. Determine the greatest pull the engine can exert, and the maximum weight it can draw up an incline of 1 in 80, if the resistance to motion is 12 lb. per ton on the level. U.L.C.I.

7. A train weighing 450 tons is drawn up an incline of 1 in 48 at a speed of 12 miles per hour by a locomotive of 90 tons weight. What H.P. is required, the frictional resistance of the engine and train being 10 lb. per ton? What speed could be attained by this train on the level with the same frictional resistance and utilising the same H.P.? U.L.C.I.

8. A certain car can be regarded as having fixed parts weighing 15 cwt. and four wheels each weighing 1 cwt. These wheels have an external diameter of 4 feet, and a radius of gyration of $1\frac{1}{2}$ feet. What horse-power will be required for this car to attain a speed of 60 miles per hour in half a mile from rest? The average wind resistance may be assumed to be 80 lb., but all internal losses may be neglected. U.E.I.

9. A train weighing 200 tons, travelling at 60 miles per hour down an incline of 1 in 100, with steam shut off, has the brakes applied and stops in 500 yards. Find the space-average retarding force, in tons, of the brakes. U.E.I.

10. A train having a total weight of 480 tons moved from rest over a distance of 5 miles in 7 minutes and attained a maximum speed of 60 miles per hour after it had run 2 miles, this speed being afterwards maintained. Assuming the acceleration uniform while the speed was being increased, determine—

- (a) The duration of the acceleration period;
- (b) The magnitude of the acceleration, and
- (c) The magnitude of the accelerating force acting on the train.

U.L.C.I.

11. A railway truck weighing 6 tons and moving at 3 miles per hour is brought to rest by the compression of two springs of equal stiffness. The resisting force of each spring at the beginning of compression is 600 lb., and it increases by 400 lb. per inch of compression. Find the distance through which each spring is compressed in bringing the truck to rest. U.L.C.I.

12. A half-ounce bullet, travelling horizontally, at 2,000 feet per second, becomes imbedded in a stationary block, of mass 8 lb., which is free to move horizontally. Find the velocity of the block after impact, and the ratio of the original kinetic energy of the bullet to the kinetic energy of the block and embedded bullet, N.C.T.E.C.

13. A swing bridge weighs 500 tons, and its radius of gyration is 30 feet. It is moved by a pinion working on a circular rack whose radius is 15 feet from the centre of rotation. If the maximum force which the pinion can exert on the rack is 0.75 ton, find the maximum velocity which the bridge can attain after turning through 45° . You may neglect friction.

14. A swing bridge weighs 798 tons and swings about a pivot, the radius of gyration of the bridge about the axis of rotation being 65.4 feet. The bridge has to be turned through 90° in 60 seconds, the first 24 seconds being a uniform acceleration period, the next 12 seconds a uniform speed period, and the remaining 24 seconds a uniform retardation period. Determine—

- (a) The maximum angular velocity of the bridge during a swing;
- (b) The angular acceleration;
- (c) The magnitude of the torque necessary to produce the acceleration, and
- (d) The magnitude of the force which must act at a radius of 18 feet $7\frac{1}{2}$ inches to produce this torque. U.L.C.I.

15. A flywheel of weight 120 lb., and radius of gyration 1.4 feet, is mounted on a horizontal axle, of radius 1.75 inches, so that it may be rotated by a falling weight attached to a cord wrapped round the axle. Neglect friction, and find what will be the angular acceleration of the flywheel, and the linear acceleration of the weight, if a weight of 12 lb. is attached to the cord.

16. A balanced wheel and axle is made to rotate by a weight suspended from a cord wound round the axle. The diameter of the axle is $3\frac{1}{16}$ inches, and the weight on the cord necessary to overcome friction is $3\frac{3}{4}$ oz. An additional weight of 6 oz. is attached to the cord (making $9\frac{3}{4}$ oz. altogether), and this weight is allowed to fall vertically through 12 feet. The time of descent is 14.8 seconds. Find the moment of inertia of the wheel and axle about the axis of rotation. U.L.C.I.

17. What do you understand by "radius of gyration" and "angular momentum"?

A flywheel weighing 2,240 lb. loses 10,000 ft.-lb. of energy when its speed changes from 120 to 90 r.p.m. What is its angular momentum when rotating at 120 r.p.m.? What is its radius of gyration? If the change in speed takes place in 1.5 seconds, find the mean effective torque exerted on the flywheel in lb.-feet.

U.E.I.

18. A wheel having a moment of inertia of 9,500 ft.-lb. units is brought from rest to 200 r.p.m. in 30 seconds. Calculate the couple that must have acted on it. U.E.I.

19. Find the moment of inertia about its axis of rotation of a grindstone, 4 feet diameter, weighing 322 lb.

A man takes 12 seconds to set the grindstone rotating at a rate

of 120 r.p.m. If the handle has a radius of 3 feet, find the average force he exerts.

N.C.T.E.C.

20. A disc flywheel 6 feet diameter and weighing 2 tons is keyed to a machine shaft. Determine the torque necessary to reduce the speed of the shaft from 150 r.p.m. to 90 r.p.m. in 5 seconds. How many ft.-lb. of energy would the wheel give out during the 5 seconds?

U.L.C.I.

21. Give a clear description of the function of a flywheel. A flywheel weighing 1.75 tons has a radius of gyration of 4 feet and revolves at 150 r.p.m. How much energy is stored in the wheel? By how much will the speed be reduced when 90,000 ft.-lb. of energy have been withdrawn from the wheel?

U.L.C.I.

22. Determine from first principles the weight of a flywheel which has a radius of gyration of 4.5 feet, if its kinetic energy is 420,000 ft.-lb. when running at a speed of 260 r.p.m.

U.L.C.I.

23. The flywheel of a shearing machine weighing 15 tons, radius of gyration 5 feet, revolves at a speed of 100 r.p.m. Find the torque required to get up speed in 2 minutes. If the work done in shearing a plate is 70 ft.-tons, what will be the reduction in speed of the flywheel, assuming all the energy required to come from it?

24. A flywheel has a rim of weight 1.2 tons and radius of gyration of 3 feet. Calculate the kinetic energy released when the wheel slows from 250 to 240 r.p.m. If this change in speed takes place in 2 seconds, what is the H.P. momentarily exerted?

U.E.I.

25. A flywheel of a steam engine has an internal trough turned in it to contain cooling water for a brake. If the diameter of the trough be 5 feet, determine the least speed of the flywheel in r.p.m. that will permit the retention of the water.

26. If the coefficient of friction between the wheels and road is 0.5, what is the greatest speed in miles per hour at which a car can travel round a curve of radius 30 feet without skidding? If the wheel base of the car is $4\frac{1}{2}$ feet wide, and the centre of gravity lies midway between the wheels and is $2\frac{1}{2}$ feet above the ground, what is the greatest speed at which the car can go round the curve without overturning, assuming friction sufficient to prevent skidding?

27. A four-wheeled truck is loaded to a total weight of 20 tons. The distance between two wheels on the same axle is 4 feet, and the height of the centre of gravity is 6 feet above the rails. If the truck runs round a horizontal curve of radius 121 feet at a speed of 15 miles per hour, what will be the vertical reactions on the inner and outer wheels respectively? Through what angle should the track be tilted for the resultant force to be perpendicular to the line joining the rails?

U.E.I.

28. A motor-car of weight 2 tons is approaching a curve of radius 100 feet. The distance between the two back wheels is $4\frac{1}{2}$ feet and the height of the centre of gravity 4 feet from the ground, the coefficient of friction between the wheel and road is 0.3. Find (a) the greatest speed at which the corner can be taken without skidding; (b) neglecting skidding the speed at which the road would exert no reaction on the inner wheels. The road is horizontal everywhere. U.E.I.

29. On a railway curve of radius R the outer rail is raised to prevent side thrust. Make sketches showing (a) the forces acting on the engine (weight, w), and (b) the mass-acceleration produced; the speed of the engine being v . Hence determine the correct super-elevation if the gauge of the railway is l .

Calculate the super-elevation of the outer rail of a railway on a curve having a radius of 500 yards if there is to be no side thrust at a speed of 30 miles per hour; the gauge is 4 feet $8\frac{1}{2}$ inches. N.C.T.E.C.

30. A flywheel weighs 2 tons, and its centre of gravity is $\frac{3}{8}$ inch from the centre of the shaft. Find the pull on the shaft bearings at 250 r.p.m. If the shaft is 6 inches diameter and the coefficient of friction is 0.01, find the resisting torque in lb.-feet due to the wheel being out of balance. U.E.I.

31. A 10-lb. weight is hung on the end of a spring and the time of a complete vibration is found to be $\frac{1}{4}$ second. What is the stiffness of the spring in lb. per inch, and what is the greatest tension in the spring if the amplitude of vibration is 3 inches?

32. The lift of a valve which moves with simple harmonic motion is $\frac{3}{16}$ inch, and the total time occupied by the opening and closing of the valve is $\frac{1}{10}$ second. Determine the accelerating forces at the ends of the stroke of the valve per lb. weight of valve. U.L.C.I.

33. A cylindrical spiral spring which supports a weight of 20 lb. is set vibrating. The strength of the spring is such that a force of 15 lb. is necessary to stretch it 1 inch. Determine the period of the vibration. U.L.C.I.

34. Show that in a S.H.M. the acceleration is proportional to the displacement from mid-position.

A cylindrical spiral spring supports a weight of 35 lb. and is made to vibrate vertically. The stiffness of the spring is such that a weight of 10 lb. would stretch it 1 inch. Show that the motion is S.H.M. and find the periodic time of a vibration. U.L.C.I.

35. What is meant by simple harmonic motion? Deduce a formula connecting the amplitude and time of one vibration with the greatest acceleration towards the position of rest.

A spring is found to increase 1 inch in length when a load of 21 lb. is attached. If a weight of 56 lb. is attached to this spring,

drawn below its position of rest and then released, how many vibrations per minute will it make ? U.E.I.

36. In a simple harmonic motion along a straight line, prove that the acceleration at any point is directly proportional to the distance of the point from the mid-point of the motion.

Assuming that a piston moves with simple harmonic motion and that its stroke is 2 feet, find the force to overcome the inertia of the reciprocating parts at the ends of the stroke and also when the crank has turned through 45° from either dead centre. The reciprocating parts weigh 500 lb. and the crank shaft makes 120 r.p.m. U.E.I.

37. A pit cage weighs 5 tons loaded ; the winding rope weighs 3 lb. per foot ; the winding drum 20 feet diameter weighs 18 tons, and its radius of gyration is 8 feet ; and the depth of the shaft is 300 yards. Find the maximum torque on the drum shaft if the starting acceleration of the cage is 1.5 foot per second per second.

CHAPTER VII

BALANCING

49. Balancing one Revolving Mass. If a mass A (Fig. 86), of weight W , be attached to an arm, XOY, revolving round O, then the Centrifugal Force exerted by A is $\frac{W\omega^2 r}{g}$, where ω is the angular velocity of the arm, and r the distance from O to the centre of gravity of A. This force will be transmitted through the bearings to the framework of the machine, and, as it is always changing in direction, will tend to set up vibration in the machine.

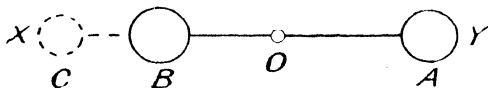


FIG. 86.

If, however, another mass B, of weight W_1 , be attached to the arm at a distance r_1 on the other side of O so that

$$\frac{W_1\omega^2 r_1}{g} = \frac{W\omega^2 r}{g},$$

then the Centrifugal Force due to B will balance that due to A, in all positions, and there will be no tendency to set up vibration. The centres of gravity of A and B must lie in the same plane at right-angles to the axis. Either W_1 or r_1 may be assigned any given value, the value of the other quantity being then obtained from the equation.

Note. Since, in any given problem, $\frac{\omega^2}{g}$ is the same for all the masses, it may be omitted from the calculations on balancing, as it cancels out in each equation, but it

must be used when obtaining the values of any unbalanced forces.

50. Balancing Two or More Revolving Masses in one Plane. If a number of masses, X, Y, Z, of weights W_1 , W_2 , W_3 , and radii r_1 , r_2 , r_3 respectively, be revolving in one plane in positions as shown, then each mass will be exerting a force equivalent to W_1r_1 , W_2r_2 , W_3r_3 , respect-

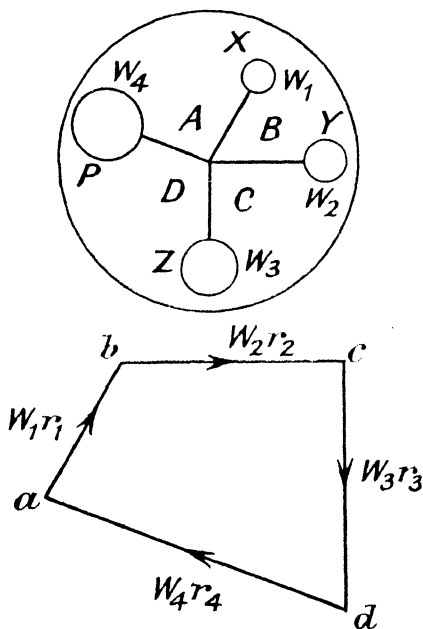


FIG. 87.

ively along its radius line (see *Note*, Article 49). The equilibrant of these forces may be obtained by drawing a Polygon of Forces, as shown in Fig. 87, the closing side, da , of the Polygon giving the *Weight-arm* product, W_4r_4 , in magnitude and direction. A mass P placed in the plane so that its radius is parallel to da , and its weight-arm product is equal to W_4r_4 , will give complete balance.

51. Balancing one Revolving Mass by two others in different Planes of Revolution. In Article 49 is shown the method of balancing one rotating mass by another in the same plane of revolution, but many cases occur where this is not practicable, as in the case of the revolving masses at the crank pin of an engine. The revolving mass has then to be balanced by two other masses in different planes of revolution.

It is obvious that the centres of gravity of all the masses must lie in one plane through the axis of revolution. Let A be the mass to be balanced by the masses B and C (Figs. 88 and 89), the weights of the masses being, respect-

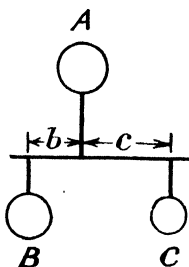


FIG. 88.

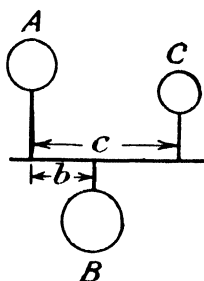


FIG. 89.

ively, W_1 , W_2 , and W_3 ; the distances of their centres of gravity from the axis of revolution r_1 , r_2 , and r_3 , and the distances from the plane of revolution of A to those of B and C, b and c respectively.

The centrifugal forces of the three masses, A, B and C, are parallel, and two conditions are necessary for equilibrium.

(1) The algebraic sum of the forces must be zero.

$$W_1 r_1 = W_2 r_2 + W_3 r_3 \quad . \quad . \quad (\text{Fig. 88})$$

$$W_1 r_1 + W_3 r_3 = W_2 r_2 \quad . \quad . \quad (\text{Fig. 89})$$

(see *Note*, Article 49).

This will give *Standing Balance*. That is, if the shaft carrying the masses is placed on two horizontal parallel straight edges there will be no tendency to roll.

(2) The algebraic sum of the moments of the forces about any point must be zero. (To reduce the calculation choose the point in the place of revolution of B.)

$$W_1 r_1 \cdot b = W_3 r_3 (b + c) \quad . \quad . \quad (\text{Fig. 88})$$

$$W_1 r_1 \cdot b = W_3 r_3 (c - b) \quad . \quad . \quad (\text{Fig. 89})$$

(see *Note*, Article 49).

This will give *Running, or Dynamic Balance*, that is, when the shaft is revolving there will be no unbalanced moment tending to rock the shaft.

Static Balance may be obtained without obtaining Running Balance, but Running Balance cannot be obtained without Static Balance.

EXAMPLE. The rotating parts of a gas engine weigh 250 lb., and may be considered as rotating at a radius of 18 inches. Find the balance masses required in the two flywheels whose planes are respectively 2 feet 6 inches and 3 feet 6 inches from the centre line of the engine. The balance masses are to be placed at a radius of 2 feet.

Referring to Fig. 88.

$$W_1 = 250, r_1 = 1.5 \text{ feet. } r_2 = r_3 = 2 \text{ feet.}$$

$$b = 2.5 \text{ feet. } c = 3.5 \text{ feet.}$$

$$(1) 250 \times 1.5 = W_2 \times 2 + W_3 \times 2.$$

$$(2) 250 \times 1.5 \times 2.5 = W_3 \times 2 \times 6.$$

$$W_3 = 78.125 \text{ lb. } W_2 = 109.375 \text{ lb.}$$

52. Balancing Two or More Revolving Masses in different Planes of Revolution by two Masses in two given Planes. Let A and B be two masses which have to be balanced by masses C and D in the planes of revolution 1 and 2 respectively (Fig. 90). Consider Reference Planes in the planes 1 and 2, keyed to, and revolving with the shaft. The true projections of these planes, looking in the direction of the arrow, are shown in Figs. 91 and 92.

By the method of Article 51 find the weight-arm products, $W_2 r_2$ and $W_3 r_3$, in these planes to balance the mass A. By the same method find the weight-arm products, $W_4 r_4$ and $W_5 r_5$, in these planes to balance the mass B. The resultant weight-arm product in each plane $W_6 r_6$ and $W_7 r_7$ may now be obtained by drawing parallelo-

has its centre of gravity $\frac{1}{16}$ inch out of true. What diameter hole must be bored through the disc at a radius of $2\frac{1}{2}$ feet to give balance?

6. A balanced disc 4 feet diameter has two holes drilled out, 2 inches and 3 inches diameter at radii of 15 inches and 12 inches respectively, the angle between the radii being 135° . Where must a hole 4 inches diameter be drilled to restore balance?

7. A circular wheel carries three weights, A, B and C, attached to its spokes. A weighs 60 lb., and is 3 feet from the centre D of the wheel. B weighs 100 lb. and the angle ADB is 90° and $BD = 4$ feet. C weighs 120 lb. and the angle ADC is 120° (in the same direction as ADB) and DC is $3\frac{1}{2}$ feet. What weight must be placed at E 5 feet from the centre to balance the centrifugal forces set up by A, B and C when the wheel rotates? What will be the value of the angle ADE? U.E.I.

8. The two flywheels of a gas engine are 30 inches apart, and the centre of the crank is 18 inches from one wheel. What masses must be placed in the flywheels at a radius of 18 inches to balance the rotating parts of the crank, etc., which are equivalent to a mass of 150 lb. at a radius of 9 inches?

9. What is meant by "static" and "dynamic" balancing of rotating masses?

The rotating parts of a gas engine which require balancing are equivalent to 200 lb. at a radius of 9 inches. The distance between the flywheels is 5 feet, and the centre line of the cylinder is 2 feet from one wheel. Determine the masses which must be placed in the flywheels at a radius of 20 inches to balance these parts.

10. A single-cylinder engine, whose stroke is 24 inches, has rotating masses equivalent to 150 lb. at the crank-pin. The centre line of the cylinder lies between two flywheels which are 6 feet apart and is 2 feet from one of them. Find the weights which must be placed in the wheels at a radius of $2\frac{1}{2}$ feet to give balance.

11. A machine has a rotating mass of 180 lb. whose centre of gravity is 12 inches from the axis of rotation. What masses must be placed at radii of 18 inches, and in planes of revolution 3 feet and $4\frac{1}{2}$ feet from, and on the same side of, the plane of revolution of the mass?

12. A shaft rests in bearings $2\frac{1}{2}$ feet apart and overhangs 12 inches. A wheel weighing 12 lb. is fixed on the overhanging end of the shaft. If the centre of gravity of the wheel is 0.12 inches out of true, find the forces on the bearings due to the wheel being out of balance when the shaft revolves at 240 r.p.m.

13. A locomotive, whose stroke is 20 inches, has two cranks at right-angles, the rotating masses being equivalent to 250 lb. at each crank-pin. The wheel centres are 4 feet 10 inches apart, and the cylinder centres, which are 2 feet apart, are placed symmetrically between them. What weights must be placed in the

wheels at 3 feet radius to give balance ? If the left-hand crank is leading, find the angles of the balance weights in advance of this crank.

14. A two-cylinder engine of 3-feet stroke has cranks at right-angles and the rotating masses are equivalent to 295 lb. and 245 lb. at the crank-pins. The rotating parts are to be balanced by masses placed at 2 feet radius in flywheels on each side of the engine ; one flywheel is $2\frac{1}{2}$ feet from the crank carrying the 295 lb. mass, and the other flywheel 2 feet from the crank carrying the 245 lb. mass. Find the masses, and the positions in which they must be placed if the centre lines of the cylinders are $2\frac{1}{2}$ feet apart.

15. A three-cylinder engine with cranks at 120° has rotating masses equivalent to 700 lb., 800 lb. and 1,000 lb. at the crank-pins, the 700-lb. mass leading. The cranks are 2 feet radius, and the planes of revolution of the 700-lb. and 800-lb. masses are 3 feet apart, and of the 800-lb. and 1,000-lb. masses $4\frac{1}{2}$ feet apart. The revolving masses are to be balanced by weights placed at $2\frac{1}{2}$ feet radius in flywheels on each side of the engine, one being $3\frac{1}{2}$ feet from the 700-lb. mass, and the other $4\frac{1}{2}$ feet from the 1,000-lb. mass. Find the balance weights in magnitude and position.

CHAPTER VIII

PISTON VELOCITY AND ACCELERATION

53. Relative Velocity. The velocity of a body is its rate of change of position, but this must be considered in relation to some other body. In engineering problems the earth is considered as a fixed body, and velocities relative to the earth as absolute velocities; i.e., a train travelling north at 40 miles per hour. If there is another train travelling south at 30 miles per hour, then, to a man in the second train the first train appears to be travelling north at 70 miles per hour, and this would be the *Relative Velocity* of the first train to the second one. The Relative Velocity, therefore, of two bodies moving in the same, or parallel lines, is the difference of their velocities, taking sign into account. If the bodies are not moving in the same or parallel lines, the subtraction must be done by vectors, taking direction into account.

Suppose two points, A and B (Fig. 93) are moving over a plane, P, with velocities V_A and V_B in the given directions; each point has a velocity relative to the other of the same magnitude, but of opposite sense. To find the velocity of B relative to A give to the plane, and hence to both A and B, a velocity equal to that of A, but in the opposite direction. A will now be at rest relative to the earth, and B will have a velocity compounded of its own velocity over P and P's velocity relative to the earth; add these velocities by vectors (Fig. 94), and the resultant, V_R , will be B's velocity relative to the earth, and therefore its velocity relative to A, which is now at rest.

A simpler, and better, construction is to draw a vector pa (Fig. 95) to represent the velocity of A relative to P

in magnitude and direction, and a vector pb to represent the velocity of B relative to P, then the closing vector, ab , will represent the velocity of B relative to A, for the triangle pab is equal to the triangle XYZ in Fig. 94 if the same velocity scales are used in both diagrams. The diagram pab is called the *Velocity Image*.

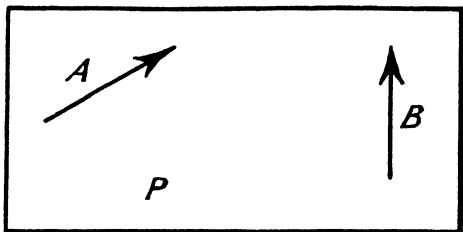


FIG. 93.

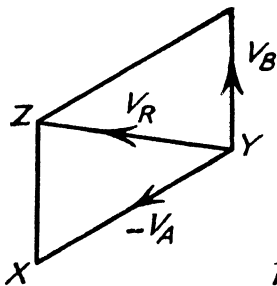


FIG. 94.

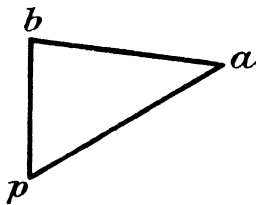


FIG. 95.

Note.—Special care must be taken with the lettering of the vectors in the Velocity Image; the first letter of the vector represents the plane, or body, of reference, the second letter the moving body, and the direction of motion is from the first to the second letter; thus the vector pa represents the velocity of A relative to P and is in the direction from p to a . The velocity of B relative to A is represented by ab .

PISTON VELOCITY AND ACCELERATION 103

EXAMPLE. What will be the relative velocity of a ship, A, travelling N.E. at 15 knots to another ship, B, travelling W. at 10 knots?

Draw pa to represent the velocity of A relative to the sea (Fig. 96), and pb that of B. Then $ba = 23$ knots, 27° N. of E. will be the velocity of A relative to B.

One point in a rigid link may have a velocity relative to another point in the link, e.g. the centre of the crank-pin to the centre of the shaft, but this relative velocity can have no component along the line joining the points, but must be at right-angles to this line, otherwise the link would alter in length, which is impossible. This property is of use in solving certain types of problems.

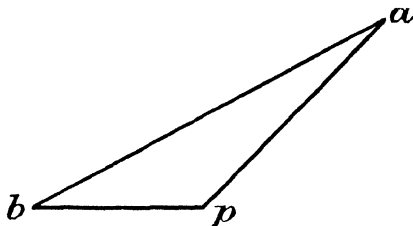


FIG. 96.

EXAMPLE. The connecting rod of an engine is 36 inches long and the crank 12 inches long. If the velocity of the crank-pin is 10 feet per sec., find the velocity of the cross-head when the crank is 60° from the inner dead centre.

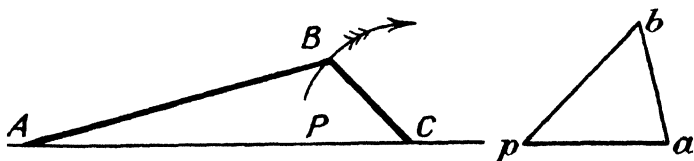


FIG. 97.

Draw pb to represent the velocity of the crank-pin, B, relative to the engine-frame P (Fig. 97); its direction will be at right-angles to the crank BC.

The velocity of A relative to P will be along AC; draw a line through p parallel to AC, then a will be on this line.

The velocity of A relative to B will be at right-angles to AB; draw a line through b at right-angles to AB, the point a will be on this line also, and will be where this line cuts the line through p parallel to AC.

The velocity of A, pa , is 10.02 feet per sec.

54. Instantaneous or Virtual Centre. Let a rigid body AB (Fig. 98) be moving in any manner, and let the directions of motion of two points, A and B, at any instant, be given by the arrows *aa* and *bb*. The motion of A would not be altered *at that instant* if it were fixed to a rigid link at right-angles to *aa* rotating about any point in its length since at any instant the path of a point in a rotating body is perpendicular to the radius drawn to that point.

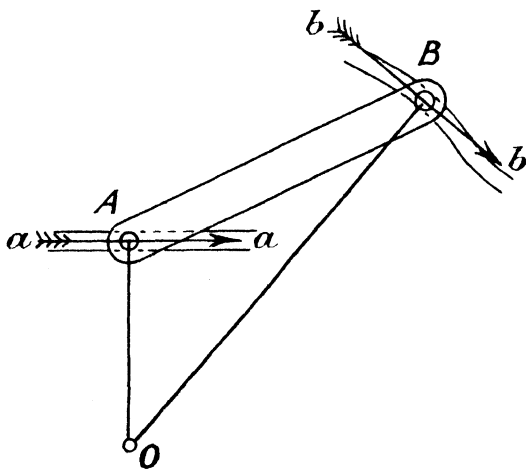


FIG. 98.

Similarly, the motion of B would not be altered *at that instant* if it were fixed to a rigid link at right-angles to *bb* rotating about any point in its length. These links would cross at some point O, and the motion of the body AB would not be altered *at that instant* if it were rotating about the point O. This point is called the *Instantaneous or Virtual Centre*.

Unless the motion of the body is one of pure rotation the position of O is constantly changing. Since the body at that instant is rotating about O, every point in the body will have the same angular velocity about O, and the

ratio of the velocities of any two points in the body will be equal to the ratio of their distances from O, for

$$V_A = \omega \times AO \text{ and } V_B = \omega \times BO,$$

$$\therefore \frac{V_A}{V_B} = \frac{OA}{OB},$$

where ω is the angular velocity about O.

55. Piston Velocity Diagrams. In the methods usually adopted it is the velocity of the cross-head which is studied, but since the piston and cross-head are rigidly connected and move in the same straight line their velocities are always the same. The velocity of the crank-pin is considered to be uniform, the small cyclic variations being neglected. The piston velocity curve may be obtained by finding the velocity at a number of points using either the velocity-image or the instantaneous centre method, but the work is considerably shortened by the following method.

In Fig. 99 A is the axis of the crosshead pin, AB the axis of the connecting rod, B the axis of the crank-pin, BC the axis of the crank, C the axis of the crank-shaft, and the line of stroke passes through the axis of the crank-shaft. Let O be the Instantaneous Centre of the connecting rod for the position shown, V the velocity of A, and v the velocity of B. Produce AB to cut the vertical through C at D.

$$\text{Then } \frac{V}{v} = \frac{OA}{OB} = \frac{CD}{CB},$$

since the triangles OAB and CBD are similar. Therefore, since v is constant, if the velocity scale is so chosen that the length CB represents v , the intercept CD of AB produced on the perpendicular through C will represent the velocity of A. Draw DE parallel to AC to meet the perpendicular through A at E. By repeating the construction for a number of positions and drawing a curve through these points, a piston-velocity diagram on a stroke base is obtained. If with centre C and radius CD a curve be drawn cutting CB at e , and the construction repeated for

a number of positions of the crank, a polar piston-velocity diagram is obtained.

Note. The motion of that part of the simple engine mechanism known as the *Slider Crank Chain*, including frame, cross-head, connecting rod and crank, is studied in detail here as this is an example of the chain in its simplest form. This chain, in various modified forms, is

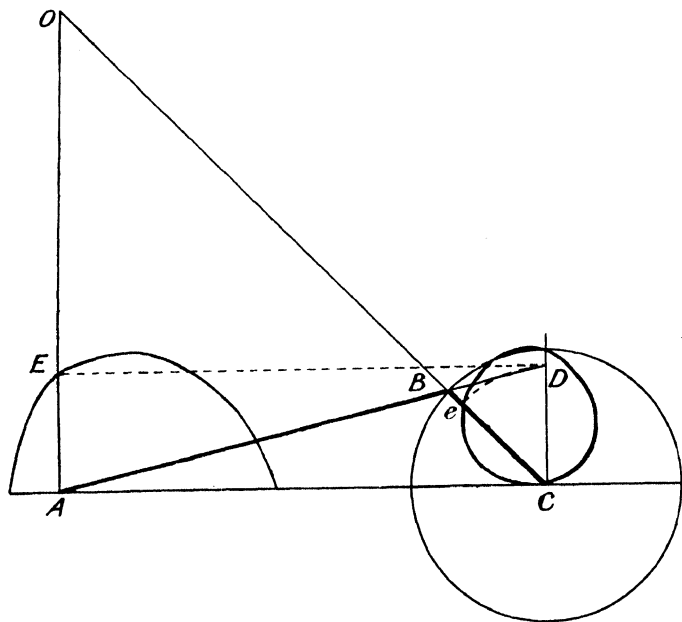


FIG. 99.

an essential part in many other mechanisms, such as a quick-return motion, and the principles studied here will apply to these other machines in a modified or extended form; also these principles form the basis for the study of the motion of other mechanisms.

56. Centripetal and Tangential Accelerations. The direction of the velocity of a point is always in the

direction of motion, but the direction of the acceleration of a point is not in the direction of motion unless the point moves in a straight line. If a rod PA, of length r (Fig. 100), rotates about P with uniform angular velocity ω , the velocity of A is $\omega r = V$, perpendicular to PA, and the acceleration of A is $\omega^2 r = \frac{V^2}{r}$ in the direction A to P, that is, in a direction perpendicular to that of motion. If the rod is rotating about P with varying angular velocity then the point A at any instant has, in addition to its centripetal acceleration due to the instantaneous angular

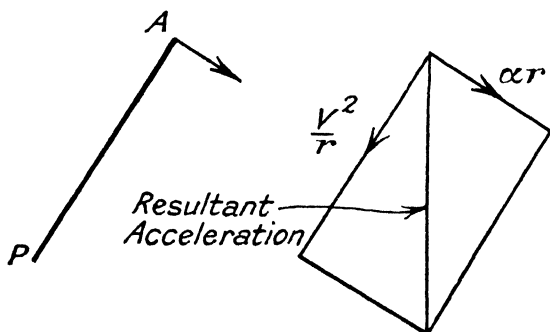


FIG. 100.

velocity, a tangential acceleration in a direction perpendicular to PA due to the angular acceleration of the rod. The magnitude of this acceleration will be αr , where α is the angular acceleration of the rod. The resultant acceleration of A will be obtained by adding vectorially the centripetal and tangential accelerations.

57. Piston Acceleration. The motion of the connecting rod is compounded of linear motion and rotational motion, both of which are constantly changing. The relative acceleration of one end of the connecting rod to the other end is therefore compounded of centripetal and tangential relative accelerations.

To obtain the acceleration of the piston, or cross-head,

A, in the position shown (Fig. 101), first draw the velocity-image pba , where pb represents the crank pin velocity, v , and pa the piston velocity, V . An acceleration image must now be drawn, beginning with the known acceleration of B, $\frac{v^2}{R}$, where R is the length of the crank, represented by $p'b'$ parallel to BC and in the direction B to C. Since A moves in a straight line, AC, its acceleration, will be along this line, therefore through p' draw a line $p'a'$ parallel to AC, a' will lie on this line. A's acceleration relative to the frame can also be obtained by adding, vectorially, A's acceleration relative to B to B's accelera-

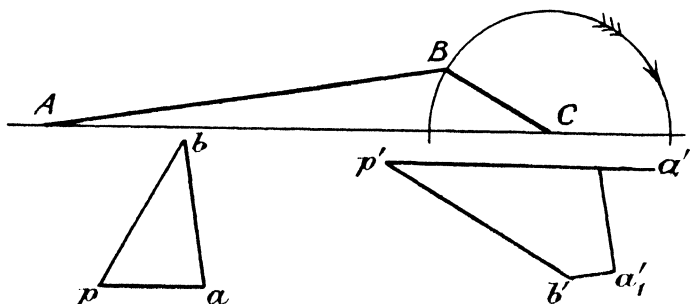


FIG. 101.

tion relative to the frame. A's centripetal acceleration relative to B will be $\frac{(ab)^2}{AB}$, where ab is A's velocity relative to B, and AB the length of the connecting rod, and its direction will be from A to B; draw $b'a_1'$ to represent this. A's tangential acceleration relative to B will be perpendicular to AB ; through a_1' draw a line parallel to this direction, cutting $p'a'$ in a' . Then $p'a'$ will represent A's acceleration.

58. Piston Acceleration Diagram, Klein's Construction. The construction of a piston-acceleration diagram by the method of the last paragraph is somewhat

tedious, but the work can be considerably shortened by using Klein's construction. On the connecting-rod, AB (Fig. 102), as diameter, draw a circle. Produce the line of connecting rod to cut the perpendicular through C at D; with B as centre and BD as radius draw a circle cutting the former circle in E and F. Through EF draw a line cutting AB at G, and AC at H. If the figures CBGH and $p'b'a_1'a'$ of Article 57 be compared they will be found to be similar. Therefore CH will represent the acceleration of A to a

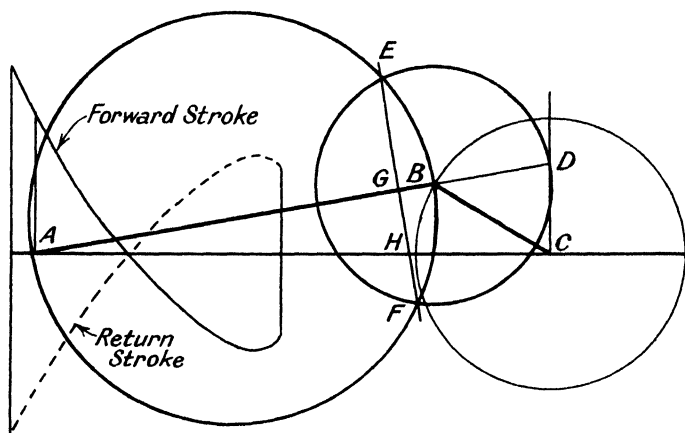


FIG. 102.

scale where BC represents the acceleration of B, that is, the length BC on the drawing represents an acceleration of $\frac{v^2}{R}$ units, where v is the crank-pin velocity, and R the crank radius.

EXAMPLE. A connecting rod is 48 inches long, crank 18 inches long, and crank-pin velocity 25 feet per sec. If the scale of the drawing is 1 inch to 1 foot, find the acceleration scale.

$$\begin{aligned} \text{Crank-pin acceleration} &= \frac{v^2}{R} = \frac{25^2}{1.5} \\ &= 416.7 \text{ feet per sec.}^2 \end{aligned}$$

Length of BC on drawing, 1.5 inches.

$$\text{Acceleration scale } \frac{416.7}{1.5} = 277.8$$

$$\therefore 1 \text{ inch} = 277.8 \text{ feet per sec.}^2$$

By repeating the above construction for a number of positions and plotting the value of CH opposite A, a piston acceleration diagram on a stroke base is obtained. The curve for the return stroke is the same shape as that for the forward stroke, but reversed.

59. Inertia Force on Reciprocating Parts. Since the reciprocating parts of an engine have mass, force must be exerted in the direction of motion during acceleration to overcome their inertia, and in the opposite direction during retardation. It follows from this that the inertia of the reciprocating parts exerts a force in a backward direction during acceleration, and a force in a forward direction during retardation. This force will be changing during the stroke as the acceleration, or retardation, changes; its magnitude, P , at any instant being given by the equation $P = \frac{Wa}{g}$, where W is the weight of the reciprocating parts and a is their acceleration at that instant.

Since, for any engine, W and g are constant, P varies as a , and the height of the ordinate of an inertia force curve on a stroke base at any point will always be the same proportion of the height of the acceleration curve ordinate at that point; the force and acceleration scales being constant for the stroke. If, therefore, for any point in the stroke the inertia force be calculated and plotted to a force scale on the same base as the acceleration curve the ordinate reduction factor can be obtained, and then the force curve obtained by reduction of acceleration ordinates.

EXAMPLE. If, at a point in the stroke of an engine, the acceleration of the reciprocating parts is 120 feet per second per second, and the reciprocating parts weigh 50 lb., find the ordinate reduction

PISTON VELOCITY AND ACCELERATION 111

factor; the acceleration scale being 1 inch = 50 feet per second per second, and the force scale 1 inch = 100 lb.

$$\text{Accelerating force, } P = \frac{50 \times 120}{32 \cdot 2} = 186 \text{ lbs.}$$

$$\text{Height of acceleration ordinate} = \frac{120}{50} = 2 \cdot 4 \text{ inches.}$$

$$\text{Height of force ordinate} = \frac{186}{100} = 1 \cdot 86 \text{ inches.}$$

$$\text{Reduction factor} = \frac{1 \cdot 86}{2 \cdot 4} = 0 \cdot 77.$$

EXAMPLES VIII

1. Define Relative Velocity. What is the relative velocity of an aeroplane travelling N.W. at 80 miles per hour to a motor-car travelling E. at 40 miles per hour?

2. A man in a train travelling due N. at 40 miles per hour sees an aeroplane which appears to him to be travelling N.E. at 30 miles per hour. Determine the true velocity of the aeroplane.

3. A ship, A, is sailing due E. at the rate of 15 knots. Another ship, B, is sailing at the rate of 20 knots. From A the ship B appears to be moving due N. Find the direction in which B is actually sailing, and the velocity of B relative to A. N.C.T.E.C.

4. At noon a ship A, sailing due E. at a rate of 12 knots, is 12 sea miles due N. of a ship B sailing N.E. at a rate of 12 knots. What is the relative velocity of B to A, and how far are the ships apart at 1 p.m.?

5. Two trains, A and B, are travelling S. to N. and W. to E. respectively at speeds of 60 miles per hour, along tracks crossing each other at C. A passes C 2 minutes before B arrives there. Find the relative velocity of A to B, and the shortest distance between A and B.

6. A railway line, A, crosses another, B, by means of a bridge, the angle of intersection being 30 degrees. A train, A, is approaching the point of intersection with a velocity of 40 miles per hour, and another train, B, is receding from the intersection, on the same side of it, with a velocity of 20 miles per hour. Find the relative velocity of the trains. U.E.I.

7. A mechanism consists of four links, AB, BC, CD and DA, pin-jointed together, the link DA being fixed. AB = 3 inches, BC = 6 inches, CD = 4 inches, and DA = 5 inches. The link AB is revolving about A at a speed of 120 r.p.m. Find the velocity of C when the angle BAD is equal to 90°.

8. A steam engine has a crank 2 feet long, and a connecting rod 8 feet long, and runs at a speed of 100 r.p.m. Draw the piston-velocity diagram for the out stroke, and find the piston velocity

when the crank has turned through 45° from inner dead centre. Find also the velocity of the mid-point of the connecting rod for this position.

9. An engine running at 600 r.p.m. has a crank 3 inches long and a connecting rod 9 inches long. Draw the piston-velocity diagram and state the piston velocities at a quarter and three-quarters of the in-stroke and out-stroke.

10. A steam engine has a crank 15 inches long and connecting rod 60 inches long. The eccentric has a radius of 3 inches and is 135° in advance of the crank; the eccentric rod is 48 inches long. Find the relative velocity of piston and valve when the crank has turned through an angle of 140° from inner dead centre. Engine speed 120 r.p.m.

11. The crank of an engine is 18 inches and the connecting rod 42 inches long. The speed of the engine is 180 r.p.m. Draw the piston-acceleration diagram, and write on it the magnitude of the acceleration when the piston has turned through 45° from inner dead centre.

12. An engine is running at 80 r.p.m.; the crank is 18 inches and the connecting rod 45 inches long. Draw the inertia force diagram; the reciprocating parts weigh 160 lb. Find the force required to overcome the inertia of the reciprocating parts at quarter and three-quarter stroke in each direction.

13. An engine has a crank $1\frac{1}{2}$ feet long, connecting rod $4\frac{1}{2}$ feet long; the reciprocating parts weigh 175 lb., and the engine speed is 240 r.p.m. Find the force required to overcome the inertia of the reciprocating parts when the angle between the centre line and the crank is 45° .

14. A stamp weighing 56 lb., sliding in vertical grooves, is worked by an eccentric of 3 inches radius whose centre is vertically above the stamp. The connecting rod is 18 inches long. Find the tension in the connecting rod when the eccentric has turned through an angle of 45° from the bottom of the up-stroke. Speed of eccentric, 60 r.p.m.

15. A twin-cylinder engine, with the cylinders opposed, has a stroke of 6 inches and connecting rods 9 inches long. The reciprocating parts in each cylinder weigh 5 lb., and the engine speed is 1,000 r.p.m. Draw the inertia force curve for one cylinder for one stroke. Find the net inertia force on the two sets of reciprocating parts when the crank has turned through 30° from dead centre (a) when there is one crank and a common crank-pin, (b) when there are two cranks at 180° .

CHAPTER IX

PISTON AND CRANK EFFORT

60. Piston Effort Diagrams. The effort on the piston at any instant is due to the difference between the steam, or gas, pressure on the working side of the piston

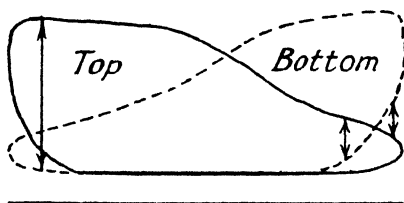


FIG. 103.

and that on the exhaust side. Referring to the Indicator Diagrams (Fig. 103), the piston effort per square inch will

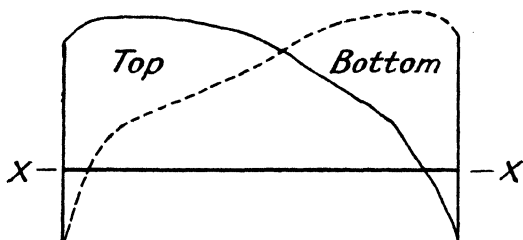


FIG. 104.

be equal to the vertical distance between the top of one diagram and the bottom of the other. This distance is plotted on a straight base, XX (Fig. 104), and shows more

clearly the piston effort during the stroke. Where the curve lies below XX the effort is negative.

Note. As the force scale of the indicator diagram is in lb. per square inch of piston area, the subsequent working is simplified if the weights of reciprocating parts are also expressed in lb. per square inch of piston area. The crank-pin and crank efforts obtained will then be per square inch of piston area, and the total effort will be obtained by multiplying by the piston area.

61. Effect of Piston-rod. In most engines there is a piston-rod on one side of the piston only. Let p_1 and p_2 be the intensities of pressure on the front and back of the piston, and A and a the areas of the piston and

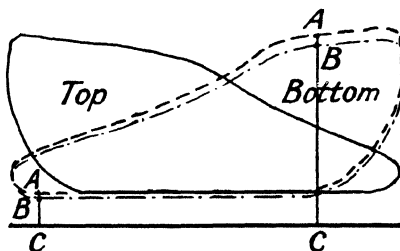


FIG. 105.

piston-rod respectively, then the total piston effort will be $p_1A - p_2(A - a)$. Subsequent working, however, can be simplified by reducing the pressure on the back of the piston in the ratio of the areas, the difference of areas then being neglected. To do this the height of the diagram above the zero line must be reduced in the ratio of the areas; thus in Fig. 105

$$BC = AC \times \frac{A - a}{A}.$$

The new diagram is now used to obtain the piston effort.

62. Weight of Reciprocating Parts in Vertical Engine. In a vertical engine the weight of the reciprocating parts is assisting the steam pressure during the

downstroke, and detracting from it during the upstroke. These effects are most conveniently allowed for by lowering the base-line of the piston-effort diagram for the downstroke by an amount equal to the weight of the reciprocating parts per square inch of piston (Fig. 106), and raising the base-line of the diagram for the upstroke by the same amount, as the effective pressure will be increased, or diminished, by this amount throughout the whole stroke.

63. Inertia Effect of Reciprocating Parts.

Since reciprocating parts are being accelerated during the early part, and retarded during the latter part of the stroke, force will be required to overcome the inertia of these parts. During the early part of the stroke this force must come from the steam pressure, and thus diminishes the available piston effort: during the latter part of the stroke the force is in the reverse direction, and the inertia of these parts is now increasing the available piston effort. The net effect of inertia on the work done during a stroke is nil, but the intensity of effort at different parts of the stroke is altered.

The necessary correction of the indicator diagrams for the inertia forces is made as shown in Figs. 107 and 108. The diagram ABDC is the piston-effort diagram due to steam pressure on a straight base, AB, the curve EF is the piston acceleration curve on the stroke base, AB, and the curve GH is the inertia force curve. The force curve is obtained from the acceleration curve by means of the

equation $P = \frac{Wa}{g}$, where P is the inertia force, W is the weight of reciprocating parts per square inch of piston,

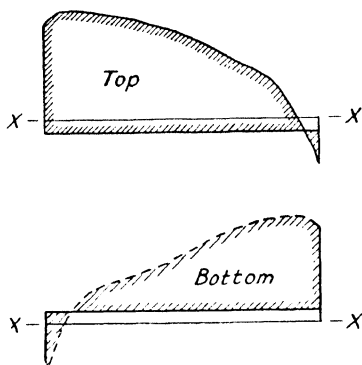


FIG. 106.

and a is the piston acceleration at that part of the stroke. The force P is plotted to the same scale as that of the indicator diagram. The net piston effort at any point of the stroke is the vertical distance between the curves CD and GH , which is plotted on a straight line base in Fig. 108.

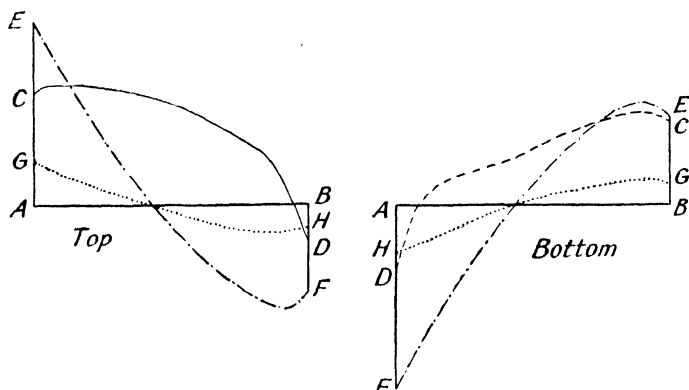


FIG. 107.

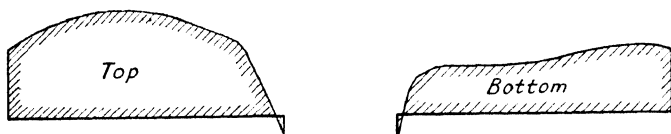


FIG. 108.

64. Distribution of Weight of Connecting Rod.

As the motion of the connecting rod is partly reciprocating and partly rotating, its weight must be divided between the reciprocating and rotating masses. If the connecting rod be supported on knife edges, supported on weighing machines, at the centres of the cross-head pin and the crank-pin, the weight at the cross-head end should be added to the reciprocating masses, and that at the crank-pin end to the rotating masses. If this data is not available half the weight is usually apportioned to each of the masses.

65. Crank Effort Diagrams. The component of

the force in the connecting rod perpendicular to the crank is called the *crank-pin effort*, and the crank-pin effort multiplied by the length of the crank is called the *crank effort*, or *turning effort*. Since the length of the crank is constant, the crank effort is proportional to the crank-pin effort. If P (Fig. 109) is the piston effort when the crank, BC , is in the position shown, the force in the connecting rod, Q , is equal to $P/\cos \phi$, and the crank-pin effort, T , is equal to $Q \sin (\theta + \phi)$. Therefore

$$T = \frac{P \sin (\theta + \phi)}{\cos \phi}.$$

If a length, CD , be marked off along the crank, or the crank produced, to represent P , and a line, DE , be drawn

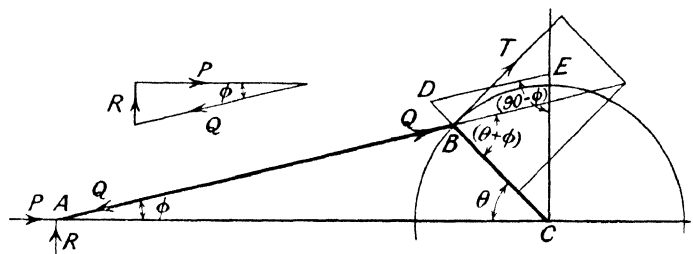


FIG. 109.

parallel to the connecting rod to meet a line through C perpendicular to the stroke in E , then CE will represent T ; for

$$\frac{CE}{CD} = \frac{\sin (\theta + \phi)}{\sin (90 - \phi)} = \frac{\sin (\theta + \phi)}{\cos \phi}.$$

$$\therefore \frac{CE}{CD} = \frac{T}{P}.$$

If this construction be repeated for a number of positions of the crank, and T be plotted on a straight base (e.g. crank-pin circle base), a *Crank-pin Effort Diagram* is obtained. If T multiplied by the Crank Radius be plotted on a straight base (e.g. crank angle base), a *Crank Effort Diagram* is obtained. Neglecting friction, the area

of either of these diagrams, for one revolution, expressed in work units, is equal to the sum of the areas of the piston-effort diagrams. It should be noticed that, while the piston-effort diagrams, developed from the indicator diagrams, are forward and reverse in direction, the crank-pin effort, and crank-effort diagrams are continuous in direction.

The crank-effort should now be changed from per square inch of piston area to total effort. This can be done by multiplying the scale by the piston area. (Art. 60.)

If the crank-effort diagram (Fig. 110) is drawn to the scales one inch equals T pounds-feet (crank effort), and one inch equals θ radians (crank angle), then one square inch of the diagram equals $T\theta$ foot-pounds.

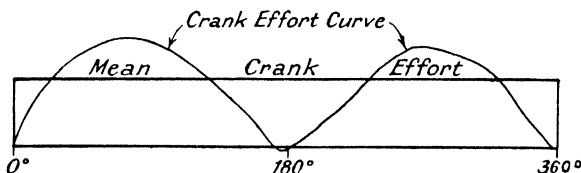


FIG. 110.

The variation in the crank effort is clearly shown in this diagram, and the maximum crank effort, which is needed when designing the shaft, can be obtained from it.

The mean crank effort is obtained by drawing a line parallel to the base enclosing an area equal to that under the crank-effort curve. When there are more than one crank the total crank effort at any instant is equal to the sum of the separate crank efforts at that instant. A combined crank-effort diagram may be obtained by drawing the crank-effort diagrams for the separate cranks on the same base, and to the same crank-effort scale, and adding the ordinates

$$AE = AB + AC + AD.$$

As the cranks are usually set at different angles, the

diagram for each crank must begin at the correct crank angle, as shown in Fig. 111.

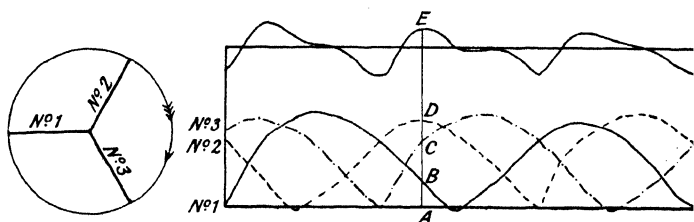


FIG. 111.

66. Fluctuation of Energy. Consideration of the diagrams, Figs. 110 and 111, shows that, while the resistance overcome remains sensibly constant, the crank effort fluctuates, and consequently the speed of the engine will fluctuate, increasing during the periods when the crank effort is greater than the resistance, and decreasing when it is less.

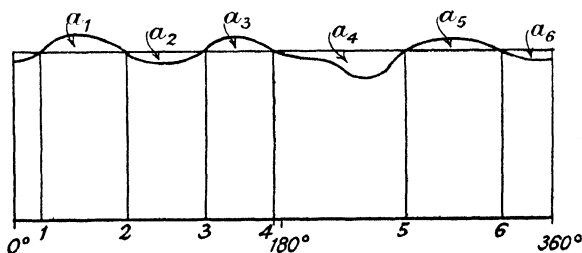


FIG. 112.

In Fig. 112 let A , in work units, equal the area under the mean resistance line, and a_1, a_2 , etc. the areas of the parts of the crank-effort diagram above or below the mean line. Let E equal the kinetic energy in the moving parts when the crank-pin is at 1 , then $E + a_1$ will equal the kinetic energy in the moving parts at 2 as the engine has given out a_1 units of work greater than the resistance

overcome. At 3 the kinetic energy will be equal to $E + a_1 - a_2$; at 4 to $E + a_1 - a_2 + a_3$; at 5, to $E + a_1 - a_2 + a_3 - a_4$; at 6, to $E + a_1 - a_2 + a_3 - a_4 + a_5$; and at 1 to $E + a_1 - a_2 + a_3 - a_4 + a_5 - a_6$ since $a_1 - a_2 + a_3 - a_4 + a_5 - a_6 = 0$. At those points in the cycle where the kinetic energy has maximum and minimum values, the speed of the engine will have maximum and minimum values. Suppose the points of maximum and minimum speed are 2 and 5, then the difference between the kinetic energy of the moving parts at the points of maximum and minimum speed, $a_2 - a_3 + a_4$, is called the *fluctuation of energy*. The ratio which the fluctuation of energy bears to the work done per cycle, $\frac{a_2 - a_3 + a_4}{A}$, is called the *coefficient of fluctuation of energy*.

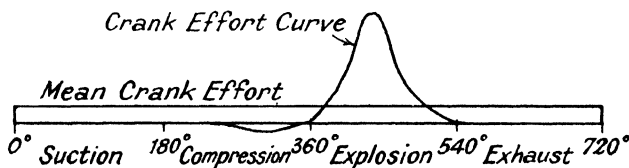


FIG. 113.

In a steam engine, or two-stroke internal combustion engine, the cycle takes place in one revolution of the crank-shaft, while in an Otto cycle internal combustion engine the cycle takes place in two revolutions of the crank-shaft. In the latter case the crank-effort diagram would be of the form shown in Fig. 113. If an internal combustion engine is governed on the "hit and miss" principle, the complete cycle may take a number of revolutions.

EXAMPLE. In Fig. 112, if $A = 140$, $a_1 = 2.3$, $a_2 = 2.9$, $a_3 = 5.1$, $a_4 = 3.7$, $a_5 = 4.2$, and $a_6 = 5.0$ units, find the points of maximum and minimum speed, and the coefficient of fluctuation of energy.

At 1, K.E. = E units.

At 2, K.E. = $E + 2.3$ „

At 3, K.E. = $E - 0.6$ „

At 4, K.E. = $E + 4.5$ units.

At 5, K.E. = $E + 0.8$,,

At 6, K.E. = $E + 5.0$,,

At 1, K.E. = $E + 0$,,

Therefore points of maximum and minimum speed are at 6 and 3 respectively.

$$\begin{aligned}\text{Coefficient of Fluctuation of Speed} &= \frac{5.1 - 3.7 + 4.2}{140} \\ &= 0.04.\end{aligned}$$

67. Flywheels. As most machines, and especially electrical machines such as generators, are required to run at a steady speed, it is necessary to damp out the fluctuations in speed of the engine due to the variations in the crank effort. To do this it is necessary to absorb and store up the surplus energy during the periods when the crank effort is greater than the resistance, and to give out this stored-up energy when the crank effort is less than the resistance. The function of the flywheel is to absorb the surplus energy during the former periods and give it out during the latter periods. As the flywheel can only absorb energy by increasing its speed, and give it out by decreasing its speed, it is not possible entirely to eliminate fluctuation of speed, but the fluctuation of speed can be reduced to any desired figure by suitably designing the flywheel.

Let ω be the mean speed of an engine, ω_1 and ω_2 the permissible maximum and minimum speeds respectively in radians per second, W and K the weight and radius of gyration, respectively, of the flywheel, and E the fluctuation of energy. The change in the kinetic energy of the flywheel between maximum and minimum speeds is

$$\frac{WK^2\omega_1^2}{2g} - \frac{WK^2\omega_2^2}{2g},$$

and this must be equal to the fluctuation of energy. Therefore

$$E = \frac{WK^2(\omega_1^2 - \omega_2^2)}{2g}$$

which gives an equation for finding the required moment of inertia of the flywheel.

It should be noted that a flywheel can absorb a greater amount of energy for a given fluctuation of speed at a higher than at a lower mean speed.

When an engine is driving heavy rotating masses, such as the armature of an electrical generator, these masses have considerable flywheel effects, and their moments of inertia should be added to that of the flywheel. In the case of moving engines, such as locomotives and motor-cars, the whole moving mass forms what may be called a "linear" flywheel, and little, or no rotating flywheel is required.

EXAMPLE. Find the moment of inertia of the flywheel required to keep the variation in speed of an engine within 1 per cent. of the mean speed of 300 revs. per min. if the fluctuation of energy is 12.1 foot-tons.

$$\omega = \frac{300 \times 2\pi}{60} = 31.42 \text{ radians per sec.}$$

$$\omega_1 = 31.42 \times 1.01 = 31.73 \quad \text{,,} \quad \text{,,}$$

$$\omega_2 = 31.42 \times 0.99 = 31.11 \quad \text{,,} \quad \text{,,}$$

$$E = \frac{WK^2(\omega_1^2 - \omega_2^2)}{2g}$$

$$12.1 = \frac{WK^2(31.73^2 - 31.11^2)}{2 \times 32.2}$$

$$WK^2 = 20 \text{ tons-feet.}^2$$

EXAMPLES IX

1. The crank of an engine is 18 inches and the connecting rod 42 inches long, and the engine is running at 180 r.p.m. If the total effective thrust on the piston is 2,500 lb. when the crank is at an angle of 45° from inner dead centre, find the turning effort on the crank for this position. Assuming this value to be equal to the mean turning effort, find the H.P. of the engine. The engine is double-acting.

2. The piston stroke of an engine is 24 inches, and the forces transmitted through the piston-rod to the cross-head, during the in-stroke, when the piston is at $\frac{1}{4}$, $\frac{1}{2}$ and $\frac{3}{4}$ stroke are 14,000 lb., 8,400 lb., and 5,600 lb. respectively. Find the torques acting on the crank-shaft for the above piston positions, and draw a torque diagram for half a revolution of the crank on a base which represents angular motion of the shaft. The length of the connecting rod is 5 feet.

Plot to scales—1 inch = angular motion of crank of 45° ; and 1 inch = 5,000 lb.-feet units of torque. U.L.C.I.

3. In a single crank steam engine the diameter of the cylinder is 10 inches, stroke 1 foot, and connecting rod length 2 feet 3 inches. Assuming a uniform effective pressure of 25 lb. per square inch throughout the stroke, draw the crank-effort diagram on a rectangular base which is equal to the circumference of the crank-pin path and determine the mean torque acting on the crank-shaft. Determine the crank angles when the speed of the crank-shaft is either a maximum or a minimum. Use scales 1 inch equals 6 inches for the base-line and 1 inch equals 300 lb.-feet for the torque.

U.L.C.I.

4. The crank-shaft of an engine has a mean speed of 120 r.p.m., but during a revolution its speed varies from a maximum to a minimum, which are respectively at rates of 123 and 117 r.p.m. Explain briefly the cause of the variation in speed during a revolution.

A flywheel weighing 12 cwt. and having a radius of gyration of 4 feet is keyed to the crank-shaft. Find the number of foot-pounds of energy the flywheel must store up during one part of a revolution and give out again during another part. U.L.C.I.

5. In a single-cylinder double-acting engine the lengths of the crank and connecting rod are 10 inches and 45 inches respectively, and the diameter of the cylinder is 18 inches. Assuming the net steam pressure to be constant throughout the stroke and to be equal to 35 lb. per square inch, draw on a base representing crank angles a diagram of the torque or turning moment acting on the crankshaft for a revolution, and from it find the value of the maximum torque.

Scales to be used—For base-line $1\frac{1}{2}$ inches = 90° , for ordinates 1 inch = 2,000 lb.-feet. U.L.C.I.

6. A horizontal steam engine running at 180 r.p.m. has a cylinder 12 inches diameter, stroke 18 inches, length of connecting rod 48 inches, and weight of reciprocating parts 275 lb. The net effective pressures on the piston at each eighth of the outstroke are 110, 104, 80, 68, 56, 46, 35, 25 and 0 lb. per square inch respectively. Draw the crank-effort diagram on a crank-angle base, find the work done during the stroke, and the ratio of maximum to mean torque.

7. In a four-stroke single-cylinder gas engine the work done during the explosion stroke is 12,000 ft.-lb., and the work done during the compression stroke is 800 ft.-lb. The speed of the engine is 250 r.p.m. Find the mean turning effort on the crank-shaft, and the H.P. of the engine. Neglect work done in suction and exhaust strokes. If the fluctuation of speed is not to be more than 2 per cent. on either side of the mean, find the moment of inertia of the flywheel required.

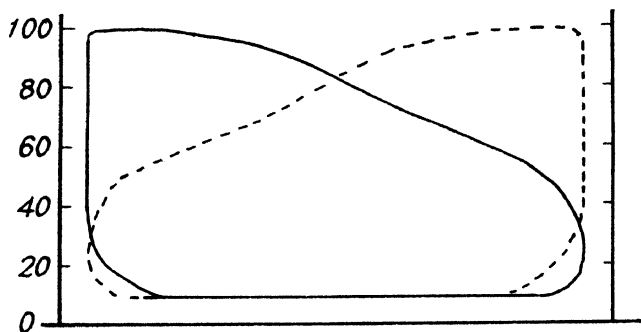


FIG. 114.

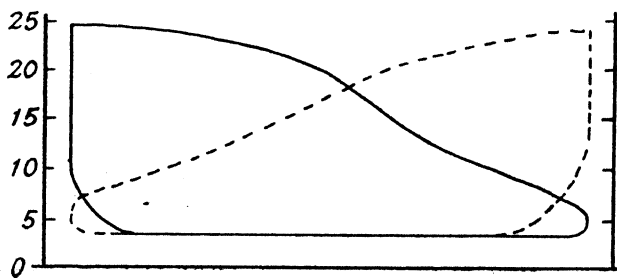
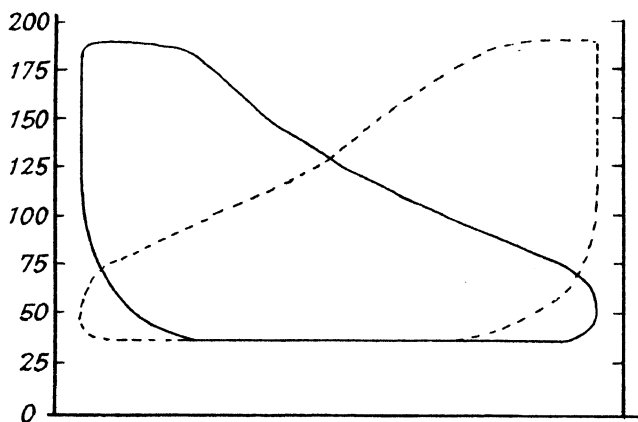


FIG. 115.

8. In a crank-effort diagram for a three-cylinder engine (Fig. 112), the total area of the diagram is 7.68 square inches, and the areas cut off by the mean resistance line are + 0.18, - 0.17, + 0.14, - 0.16, + 0.18, - 0.16, + 0.13, and - 0.14 square inches, taken in sequence, the positive areas being above and the negative areas below the line respectively. The turning effort scale is 1 inch = 150 lb.-ft. per square inch of piston, and the crank-angle scale is 1 inch = 60° . The piston is 18 inches diameter, and the engine is running at 180 r.p.m. Find the average torque, the work done per revolution, the H.P. of the engine, and the coefficient of fluctuation of energy.

9. The indicator diagrams (Fig. 114) are taken from a single-cylinder double-acting horizontal steam engine. The stroke is 12 inches, cylinder diameter 8 inches, piston-rod $1\frac{1}{2}$ inches diameter, and weight of reciprocating parts 170 lb. The speed of the engine is 150 r.p.m. The full line diagram is for the out-stroke. The scale is in lb. per square inch. Draw the crank-effort diagram and find the coefficient of fluctuation of energy.

10. A two-cylinder vertical compound engine has the following particulars. High-pressure cylinder 9 inches diameter and weight of reciprocating parts 400 lb. Low-pressure cylinder 18 inches diameter and weight of reciprocating parts 620 lb. The cranks are 9 inches long, connecting rods 30 inches long, and piston-rods $2\frac{1}{4}$ inches diameter. The angle between the cranks is 90° , the high pressure leading. The speed is 200 r.p.m. In the indicator diagrams (Fig. 115) the full-line diagram is for the top of the cylinder, and the scale is in lb. per square inch.

Draw the combined crank-effort diagram on a crank-angle base; find the magnitude of the mean and maximum torques on the crank-shaft, the work done per cycle, and the coefficient of fluctuation of energy.

CHAPTER X

FRICTION

68. Angle of Friction. If a body of weight W be placed on a rough inclined plane (Fig. 116), and the angle

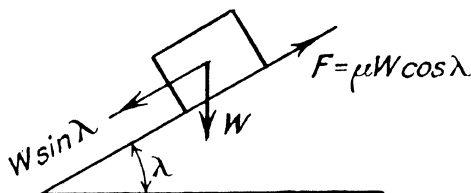


FIG. 116.

of the plane be increased slowly, the body will slide down the plane when the force down the plane, $W \sin \lambda$, is

equal to the friction between the body and the plane, $\mu W \cos \lambda$, where μ is the coefficient of friction.

Then $W \sin \lambda = \mu W \cos \lambda$ and $\tan \lambda = \mu$. λ is called the angle of friction.

If a body of weight W be dragged along a plane surface by a force P (Fig. 117), the total reaction of the plane, R , will make an angle λ with the normal to

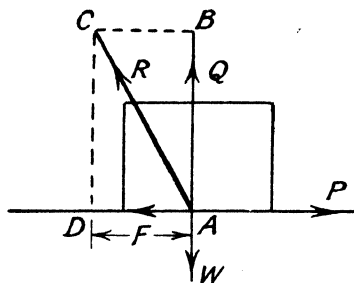


FIG. 117.

the surface. Let AB represent Q , the normal reaction of the plane, and AD represent $F = \mu Q$, the force of friction, then $F/Q = \lambda$.

69. Inclined Plane. When a body of weight W is dragged up a rough inclined plane of inclination α by a force P , there are three forces acting on the body, W , P and R , the reaction of the plane. The force P may be found by drawing the triangle of forces.

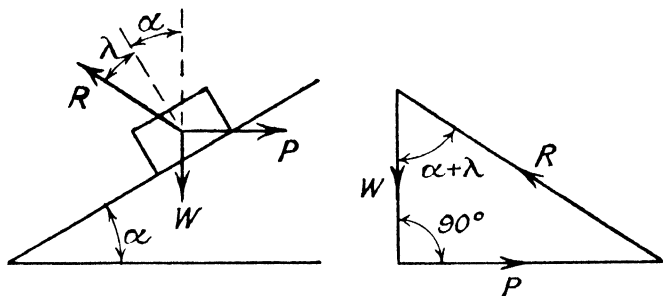


FIG. 118.

Case 1. Force horizontal (Fig. 118). From the triangle of forces

$$P = W \tan (\alpha + \lambda).$$

Case 2. Force parallel to the plane (Fig. 119). From the triangle of forces

$$\begin{aligned} \frac{P}{\sin (\alpha + \lambda)} &= \frac{W}{\sin (90 - \lambda)} = \frac{W}{\cos \lambda} \\ \therefore P &= \frac{W \sin (\alpha + \lambda)}{\cos \lambda} = \frac{W (\sin \alpha \cos \lambda + \cos \alpha \sin \lambda)}{\cos \lambda} \\ &= W (\sin \alpha + \mu \cos \alpha). \end{aligned}$$

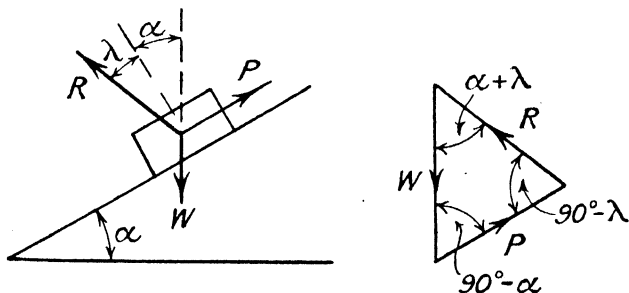


FIG. 119.

Case 3. Force inclined at an angle to the plane (Fig. 120).
From the triangle of forces

$$\frac{P}{\sin(\alpha + \lambda)} = \frac{W}{\sin(90 + (\beta - \lambda))} = \frac{W}{\cos(\beta - \lambda)}$$

$$P = \frac{W \sin(\alpha + \lambda)}{\cos(\beta - \lambda)}.$$

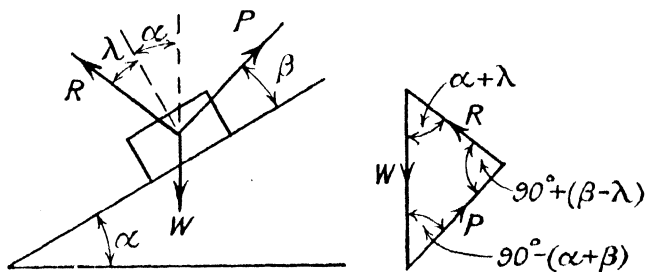


FIG. 120.

70. Friction of Screw. A square-threaded screw may be considered as an inclined plane wrapped round a

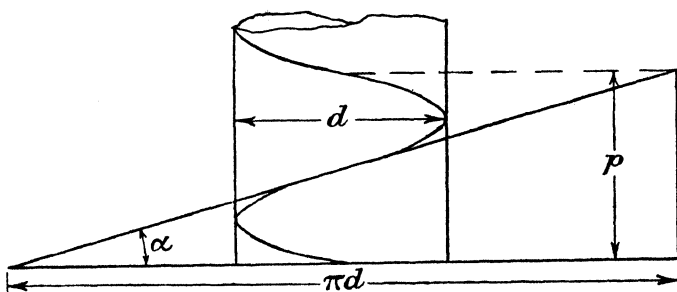


FIG. 121.

cylinder (Fig. 121), where $\tan \alpha = P/\pi d$, the force being generally applied parallel to the base. The screw then becomes a special application of *Case 1*, Article 69. Let

the force required at the mean radius of the screw to raise a weight W be P . Then

$$P = W \tan (\alpha + \lambda) = \frac{W(\tan \alpha + \tan \lambda)}{1 - \tan \alpha \tan \lambda}$$

$$= \frac{W(p + \mu \pi d)}{\pi d - \mu p}.$$

If the screw is rotated by a force Q at a radius R then $QR = Pd/2$.

In the case of a triangular screw thread the force, R ,

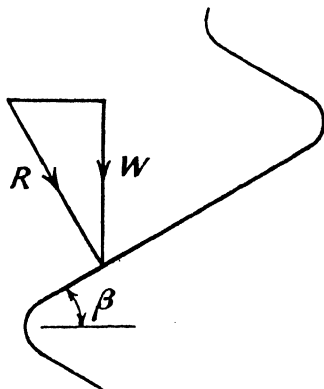


FIG. 122.

normal to the thread is equal to $W/\cos \beta$, where β is the semi-angle of the thread (Fig. 122). Since friction is proportional to R , $\tan \lambda$ may be increased to $n \tan \lambda$ where $n = 1/\cos \beta$, and

$$P = \frac{W(\tan \alpha + n \tan \lambda)}{1 - n \tan \alpha \tan \lambda} = \frac{W(p + n \mu \pi d)}{\pi d - n \mu p}.$$

EXAMPLE. In a screw jack the mean diameter of the screw is 3 inches, and the pitch of the screw is 1 inch. Neglecting the friction of the collar, what force at a radius of 2 feet is required to lift a weight of 2 tons if the angle of friction is 15° ?

$$\mu = \tan \lambda = \tan 15^\circ = 0.268$$

$$\begin{aligned} Q &= \frac{W(p + \mu\pi d)}{\pi d - \mu p} \times \frac{d}{2R} \\ &= \frac{2 \times 2,240(1 + 0.268 \times \pi \times 3)}{\pi \times 3 - 0.268 \times 1} \times \frac{3}{2 \times 24} \\ &= 114 \text{ lb.} \end{aligned}$$

71. Friction of Flat Pivot. If the load, or thrust, W , be evenly distributed over the whole surface of the pivot, the intensity of pressure is $p = W/\pi R^2$ (Fig. 123).

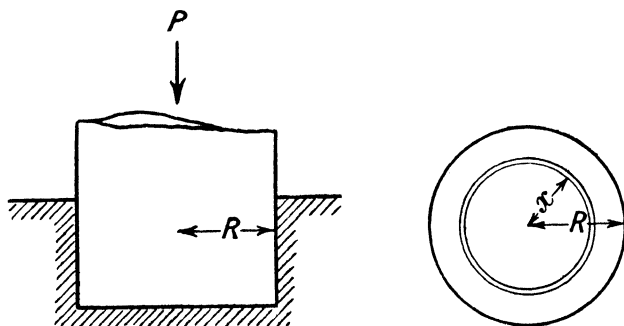


FIG. 123.

The pressure on an elementary ring of radius x and width dx is $2\pi x p dx$.

The moment of friction on an elementary ring

$$= 2\pi \mu p x^2 dx.$$

The moment of friction, T , on the whole surface

$$= 2\pi \mu p \int_0^R x^2 \cdot dx.$$

Therefore
$$T = \frac{2}{3} \pi \mu p R^3.$$

Substituting the value of p from above,

$$T = \frac{2}{3} \mu W R.$$

72. Friction of Collar Bearing. In this case (Fig. 124)

$$p = \frac{W}{\pi(R^2 - r^2)},$$

$$\begin{aligned} \text{and } T &= 2\pi\mu p \int_r^R x^2 dx, \\ &= \frac{2}{3}\pi\mu p(R^3 - r^3). \end{aligned}$$

Substituting the value of p ,

$$T = \frac{2\mu W(R^3 - r^3)}{3(R^2 - r^2)}.$$

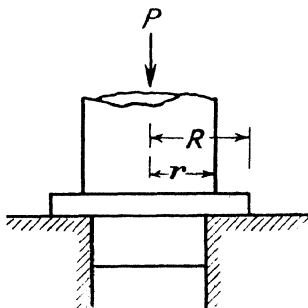


FIG. 124.

EXAMPLE. The total load on a vertical shaft carried by a collar bearing is $\frac{1}{3}$ ton. If the inside and outside radii of the bearing are 3 and 4½ inches respectively, the coefficient of friction is 0.08 and the speed of the shaft 120 r.p.m., find the horse-power absorbed in friction.

$$\begin{aligned} T &= \frac{2\mu W(R^3 - r^3)}{3(R^2 - r^2)} = \frac{2 \times 0.08 \times 2,240(4.5^3 - 3^3)}{3(4.5^2 - 3^2) \times 3} \\ &= 227.5 \text{ lb.-ins.} \end{aligned}$$

$$\begin{aligned} H &= \frac{2\pi TN}{33,000 \times 12} = \frac{2 \times \pi \times 227.5 \times 120}{33,000 \times 12} \\ &= 0.43. \end{aligned}$$

73. Friction in Journal Bearings. Let W = the load on the bearing in lb., R = the radius of the bearing in inches, and μ = the coefficient of friction; the torque required to overcome friction will be μWR , and the work done against friction in any given time will be $\mu WR\theta$, where θ is the angle turned through by the shaft in radians in that time.

EXAMPLE. A flywheel weighing 5 tons, radius of gyration 4 feet, is carried on two bearings by a shaft 9 inches diameter. If the flywheel is running at 100 r.p.m. when the power is cut off, find the number of revolutions it makes before coming to rest. Coefficient of friction = 0.08.

(a) *By energy method.*

$$\text{K.E. of flywheel} = \frac{Wk^2\omega^2}{2g} = \frac{5 \times 4^2 \times 10.47^2}{2 \times 32.2} = 136 \text{ ft.-tons.}$$

Work done against friction = $\mu WR2\pi N$, where N = number of revolutions.

$$\therefore \frac{0.08 \times 5 \times 4.5 \times 2\pi \times N}{12} = \frac{5 \times 4^2 \times 10.47^2}{2 \times 32.2}$$

$$N = 144$$

(b) By momentum method.

$$Tt = \frac{Wk^2\omega}{g}$$

$$\frac{0.08 \times 5 \times 4.5}{12} \times t = \frac{5 \times 4^2 \times 10.47}{32.2}$$

$$t = 173 \text{ secs.}$$

$$\text{Average speed} = \omega/2, \text{ and } N = \frac{\omega t}{2 \times 2\pi},$$

$$= 144$$

74. Friction of a Belt on a Pulley. Let a belt, ABCD (Fig. 125), be partly wrapped round a fixed pulley

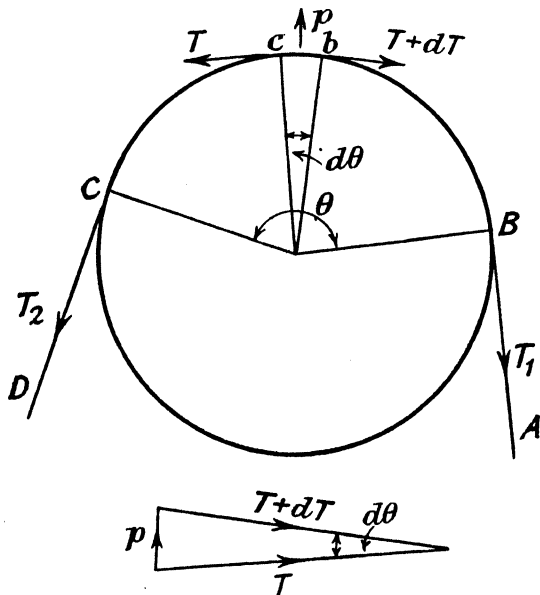


FIG. 125.

so that the angle of lap is θ , and let the tension at one

end, T_1 , be greater than that at the other, T_2 , so that the belt is just on the point of slipping, and let the coefficient of friction between the belt and the pulley be μ . The tension in the belt will be increasing from C to B, and therefore the intensity of pressure of the belt on the pulley will be increasing, and also the friction per unit length between the belt and the pulley.

Consider a short length, bc , of the belt. Let the tensions at b and c be $T + dT$, and T respectively, and the resultant pressure of the pulley on the belt be p . Then, since the belt is not slipping, the difference between the tensions in the belt at b and c must be equal to the friction on that length, therefore

$$T + dT - T = dT = \mu p.$$

From the triangle of forces for bc , $p = Td\theta$; therefore

$$dT = \mu T d\theta, \text{ and } \frac{dT}{T} = \mu d\theta.$$

Integrating
$$\int_{T_2}^{T_1} \frac{dT}{T} = \mu \int_0^\theta d\theta,$$

therefore
$$\log_e \frac{T_1}{T_2} = \mu\theta, \text{ or } \frac{T_1}{T_2} = e^{\mu\theta}.$$

θ is measured in radians, and the logarithms are the Napierian logarithms.

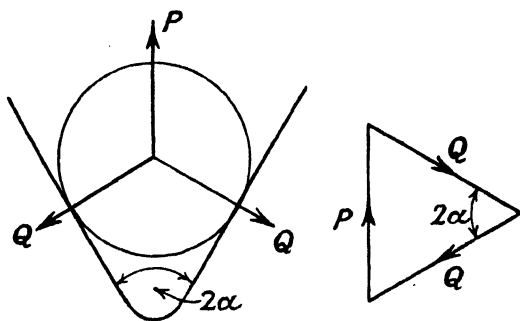


FIG. 126.

In the case of a rope in a V-grooved pulley (Fig. 126) the normal pressure on each surface is Q , and

$2Q = p \operatorname{cosec} \alpha$ where 2α is the angle of the groove. The resistance to slipping is $2\mu Q = \mu p \operatorname{cosec} \alpha$. Comparing this with the resistance to slipping of a flat belt, μp , it will be seen that the above equations apply to this case if μ is increased to μ_1 , where $\mu_1 = \mu \operatorname{cosec} \alpha$.

75. Horse-Power transmitted by a Belt. If the tensions on the tight and slack sides of a belt over a pulley be T_1 and T_2 respectively, and the speed of the belt be V feet per minute, the horse-power transmitted, H , is given by

$$H = \frac{(T_1 - T_2)V}{33,000} = \frac{T_1 \left(1 - \frac{1}{e^{\mu\theta}}\right)V}{33,000}.$$

When $\mu = 0.4$ and $\theta = 180^\circ$, this becomes

$$H = \frac{.72T_1V}{33,000}.$$

EXAMPLE. If the safe tension in a single ply belt is 100 lb. per inch width, find the width of belt required to transmit 30 H.P. to a 36-inch diameter pulley running at 300 r.p.m. Coefficient of friction = 0.4, and angle of lap = 180° .

$$V = \pi DN = \pi \times 3 \times 300 = 2,826 \text{ feet per min.}$$

$$H = \frac{.72T_1V}{33,000}$$

$$T_1 = \frac{30 \times 33,000}{0.72 \times 2,826} = 486 \text{ lb.}$$

$$\text{Width of belt} = \frac{486}{100}$$

$$= 4.86 \text{ inches, say 5 inches.}$$

EXAMPLES X

1. Find the work done when a body weighing 112 lb. is drawn a distance of 100 feet up a slope of inclination 10° , the coefficient of friction being 0.3. Find also the work done when the body is drawn the same distance down the plane.

2. If a body of weight 1 ton is being lowered down a plane of inclination 30° , the coefficient of friction being 0.25, by a rope, find the pull in the rope to maintain uniform velocity (a) when the rope is parallel to the plane, (b) when the rope is horizontal.

3. A sliding body weighing 100 lb. is hauled up a plane inclined at 30° to the horizontal by a pull parallel to the plane. If the coefficient of friction is 0.2, calculate the work done in moving the body through a distance of 20 feet, (a) if the velocity is uniform, (b) if the acceleration is 2 feet per second per second. N.C.T.E.C.

4. The handle of a screw jack has a radius of 18 inches, the screw is 2 inches diameter and $\frac{1}{2}$ inch pitch. What force at the end of the handle is required to raise a load of 10 cwt. ? The coefficient of friction is 0.15, and the load rotates with the screw.

5. The saddle of a lathe weighs 5 cwt., and the coefficient of friction between saddle and bed is 0.1. The saddle is moved by a leading screw $1\frac{1}{2}$ inches diameter and $\frac{1}{2}$ inch pitch driven by a 12-inch P.C. diameter gear wheel. If the coefficient of friction of the screw is 0.1, find the tangential force required on the pitch circle of the gear wheel to move the saddle. Neglect collar friction.

6. Find the turning moment necessary to raise a weight of 1 ton by a vertical screw having a square thread of 1 inch pitch and a mean diameter of thread of 3 inches. The coefficient of friction of the screw is 0.2. U.E.I.

7. The mean diameter of the threads of a $\frac{1}{2}$ -inch bolt is 0.45 inches, and the angle of the thread is 55° . Find the tension in the bolt when the nut is being tightened up by a force of 30 lb. at the end of a spanner 9 inches long. There are 12 threads per inch and the coefficient of friction of the screw is 0.2. Neglect friction between nut and bearing.

8. What width of belt is required to transmit 30 H.P. to a pulley 24 inches diameter running at 250 r.p.m., if the angle of lap is 180° , and the coefficient of friction 0.4 ? Allowable tension per inch width of belt is 100 lb.

9. An overhead crane drum is 12 inches diameter ; the brake drum is 18 inches diameter, the brake band laps round the bottom half of the drum, and the ends are carried up vertically to points A and B of a horizontal lever ABC. $BC = 3$ feet. If the coefficient of friction between band and brake drum is 0.3, what vertical force at C will be required to support a load of 1 ton on the winding rope when the brake lever is hinged (a) at A, (b) at B ? The winding rope is on the A side of the pulley.

10. Two equal pulleys, 2 feet diameter, are connected by a tight belt ; the coefficient of friction between the belt and pulley is 0.2. What must be the tensions in the belt if 20 H.P. is transmitted at 200 r.p.m. ? U.E.I.

11. A truck is being pulled up an incline by a rope wrapped round a hydraulic capstan. If the pull on the truck is to be 2 tons, how many laps must the rope make round the capstan so that the pull on the free end of the rope may not exceed 20 lb. ? Coefficient of friction between rope and capstan 0.25.

12. How many ropes $2\frac{1}{2}$ inches diameter will be required to transmit 6000 H.P. to a pulley 12 feet diameter running at 120 r.p.m. ? Angle of lap, 200° ; angle of grooves, 45° ; and coefficient of friction of rope, 0.3. The maximum pull in a rope is not to exceed 5,500 lb.

13. A thrust bearing on a shaft has a collar 15 inches outside and 9 inches inside diameters. The thrust on the bearing is 5,000 lb., and the coefficient of friction is 0.08. Find the H.P. absorbed in the bearing when the shaft is running at 120 r.p.m.

14. What H.P. can be transmitted by a multiple plate disc clutch running at 1,200 r.p.m. ? There are eight rubbing surfaces of 8 inches internal and 10 inches external diameters, and the coefficient of friction is 0.35. The axial pressure on the clutch is 150 lb.

CHAPTER XI
HYDROSTATICS

76. Pressure on Plane Surface Immersed in Water. Let ABCD (Fig. 127) be a plane surface immersed vertically in water, whose area is A , and let AB be a narrow horizontal strip of area a_1 , at a depth below the surface of x_1 . Since the pressure of a liquid at rest

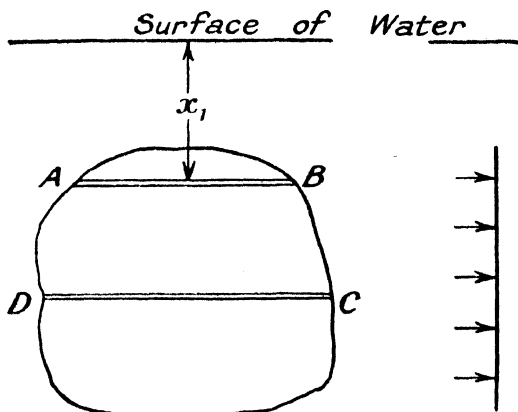


FIG. 127.

is always perpendicular to the surface with which it is in contact, and the intensity of pressure is proportional to the depth, the pressure on AB will be horizontal and uniform. Also the total pressure on any plane surface may be considered as made up of a number of differing parallel forces which are the pressures on each element of the area.

The pressure intensity on AB will be $w x_1$, where w is the density of water and x_1 the depth of the strip below the surface. The total pressure, or thrust, on the strip will be $w x_1 a_1$ where a_1 is the area of the strip. If the surface is divided into a number of thin horizontal strips of areas a_1, a_2, a_3 etc., at depths x_1, x_2, x_3 etc., below the surface, the thrust on the whole surface, T, will be equal to the sum of the thrusts on each of the strips, or

$$\begin{aligned} T &= (w x_1 a_1 + w x_2 a_2 + w x_3 a_3 + \dots) \\ &= w (x_1 a_1 + x_2 a_2 + x_3 a_3 + \dots). \end{aligned}$$

The total thrust is also equal to the area multiplied by the average pressure on the area. Let d be the depth below the surface at which the pressure is equal to the average pressure on the area. Then

$$T = w A d.$$

Therefore $w A d = w (x_1 a_1 + x_2 a_2 + x_3 a_3 + \dots)$

or $A d = (x_1 a_1 + x_2 a_2 + x_3 a_3 + \dots).$

But $(x_1 a_1 + x_2 a_2 + x_3 a_3 + \dots)$ is the sum of the moments of each element of area about the surface of the liquid, and is equal to the moment of the whole area about this surface, that is, is equal to $A \bar{x}$, where $\bar{x}$ is the depth of the centroid of the area below the surface of the water. Therefore $d = \bar{x}$, and *the average pressure is equal to the pressure at the centroid of the immersed plane surface.*

If the surface is not vertical the above proof will hold good if modified to include the angle of inclination to the vertical.

EXAMPLE. Find the total pressure on a rectangular door in a dock wall whose edges are 4 feet and 2 feet; the 4 feet edge being horizontal and 10 feet below the surface of the water.

Depth to centroid = $10 + 1 = 11$ feet.

Average pressure = 11×62.3 lb. per square foot.

Total pressure = $11 \times 62.3 \times 4 \times 2 = 5,482$ lb.

77. Centre of Pressure on Plane Surface Immersed in Water. Since the pressure of water on any plane surface may be considered as made up of a number

of parallel forces there will be a point through which the resultant of these parallel forces acts, this point is called the *Centre of Pressure* of the surface. The method to be used to find the position of the centre of pressure is therefore similar to that used for finding the position of the resultant of a number of parallel forces, or the total pressure on the surface (resultant) multiplied by the depth of the centre of pressure below the surface is equal to the sum of the products of the pressure on each element of the surface multiplied by the depth of that element below the surface of the water.

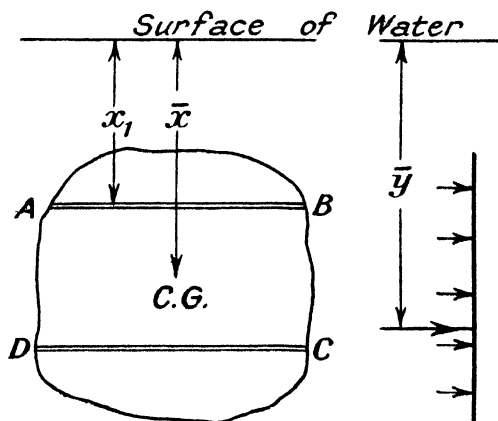


FIG. 128.

Let ABCD (Fig. 128) be a plane surface whose area is A , immersed vertically in water, and let $\bar{x}$ be the depth of the centroid and $\bar{y}$ the depth of the centre of pressure below the surface of the water. Divide the surface into a number of thin horizontal strips of area a_1, a_2, a_3 etc., at depths x_1, x_2, x_3 etc., below the surface of the water. The total pressure on the surface is $wA\bar{x}$, and the moment of this about a line XY in the surface of the water is $wA\bar{x}\bar{y}$. The total pressure on the strip AB is wa_1x_1 ; the moment of this pressure about XY is $wa_1x_1^2$; and the

moment of the total pressure on the surface is equal to the sum of the moments of all the strips about XX, that is, $w(a_1x_1^2 + a_2x_2^2 + a_3x_3^2 + \dots)$. But $(a_1x_1^2 + a_2x_2^2 + a_3x_3^2 + \dots)$ is equal to AK_1^2 (the moment of inertia, or second moment, of the surface about XY) where K_1 is the radius of gyration of the surface about XY. By the Theorem of Parallel Axes $AK_1^2 = AK^2 + A\bar{x}^2$, where K is the radius of gyration about an axis through the centroid parallel to XX.

Therefore $wA\bar{x}\bar{y} = wAK^2 + wA\bar{x}^2$, and

$$\bar{y} = \frac{AK^2 + A\bar{x}^2}{A\bar{x}} = \frac{\text{Second moment about surface of water}}{\text{First moment about surface of water}}.$$

If the surface is not vertical the above proof will hold good if modified to include the angle of inclination to the vertical. In this case it is usually more convenient to measure $\bar{x}$ and $\bar{y}$ along the slope.

In certain cases the above equation can be reduced to a much simpler form.

Case 1. Rectangle with One Edge in Surface of Water.

Let B be the breadth and H the depth of the rectangle.

$$\bar{x} = \frac{H}{2}, \text{ and } K^2 = \frac{H^2}{12}.$$

$$\text{Therefore } \bar{y} = \frac{A\frac{H^2}{12} + A\left(\frac{H}{2}\right)^2}{A\frac{H}{2}} = \frac{2}{3}H.$$

Case 2. Triangle with Base in Surface of Water. Let

B be the breadth and H the depth of the triangle. $\bar{x} = \frac{H}{3}$,

$$\text{and } K^2 = \frac{H^2}{18}.$$

$$\text{Therefore } \bar{y} = \frac{A\frac{H^2}{18} + A\left(\frac{H}{3}\right)^2}{A\frac{H}{3}} = \frac{1}{2}H.$$

Case 3. Triangle with Apex in Surface of Water and Base horizontal. Let B be the breadth and H the depth of the triangle. $\bar{x} = \frac{2H}{3}$, and $K^2 = \frac{H^2}{18}$.

$$\text{Therefore } \bar{y} = \frac{A \frac{H^2}{18} + A \left(\frac{2H}{3} \right)^2}{A \frac{2H}{3}} = \frac{3}{4}H.$$

Case 4. Circle with Edge touching the Surface. Let D be the diameter of the circle. $\bar{x} = \frac{D}{2}$, and $K^2 = \frac{D^2}{16}$.

$$\text{Therefore } \bar{y} = \frac{A \frac{D^2}{16} + A \left(\frac{D}{2} \right)^2}{A \frac{D}{2}} = \frac{5}{8}D.$$

EXAMPLE. A circular opening 2 feet diameter in a vertical dam is closed with a flap which is hinged at the top. Find the horizontal pull required at the bottom to open the flap when the surface of the water is 10 feet above the top of the opening.

Thrust on flap, $T = \pi \times 1^2 \times 11 \times 62.3 = 2,154$ lb.

Depth of Centre of Pressure from

$$\text{Surface} = \frac{A \frac{2^2}{16} + A 11^2}{A 11} = 11.02 \text{ feet.}$$

Distance of C.P. from hinge = $11.02 - 10 = 1.02$ feet.

Moment of Thrust about hinge = $2,154 \times 1.02$ lb.-ft.

Moment of Pull, P, about hinge = $P \times 2$ lb.-ft.

$$P \times 2 = 2,154 \times 1.02.$$

$$P = 1,098 \text{ lb.}$$

78. Pressure Energy of Water. If V cubic feet of water under a pressure of P lb. per square foot are introduced into a cylinder fitted with a piston 1 square foot in area, neglecting friction, the water would be able to force the piston through a distance of V feet against a force of P lb.; that is, the water could do PV ft.-lb. of work. This work would be done at the expense of the pressure energy in the water, which is, therefore, equal to PV ft.-lb.

The pressure energy is usually put into the water by a power-operated force pump, the water flowing along mains from the pump to the hydraulic machinery. As the demand for power will fluctuate the working of the pump would be irregular, or some water must go to waste. To steady the work of the pump a hydraulic accumulator is usually connected to the main. The accumulator is a vertical cylinder with a loaded ram, the load being equal to the total pressure of water on the end of the ram. The ram will rise or fall as the demand for power is less, or greater, than the output of the pump.

EXAMPLE. If the water pressure is 700 lb. per square inch, find the energy which can be stored in a hydraulic accumulator whose ram is 12 inches diameter and has a stroke of 6 feet.

$$\text{Stroke volume} = \frac{\pi \times 6^2 \times 6}{144} = 4.7 \text{ cubic feet.}$$

$$\text{Energy} = PV = 700 \times 144 \times 4.7 = 475,000 \text{ ft.-lb.}$$

EXAMPLES XI

1. A lock gate, 10 feet wide, has water on both sides, the depth of water on one side being 10 feet, and on the other side 5 feet above the bottom. Determine the magnitude of the resultant thrust on the gate, and the position at which it acts.

2. Find the magnitude of the resultant thrust and the position of the centre of pressure on a vertical circular plate, 4 feet diameter, whose top is 4 feet below the surface of water.

3. A vertical sluice gate 2 feet wide, 4 feet deep, has its top 12 feet below the surface of the water. Assuming the gate to be held in position by one bolt at the top and another at the bottom, find the force in each bolt.

4. In the vertical side of a tank there is an opening 3 feet square, one side being horizontal. The opening is closed by a door held in place by two bolts on the centre line of the door, one 2 feet above and the other 2 feet below, the centre of the door. If the surface of the water is 3 feet above the top of the opening, find the tension in each bolt.

5. A vertical rectangular sluice gate is 4 feet by 2 feet, and is submerged with the longer sides horizontal; the top side being 4 feet below the surface of the water. Find the position of the centre of pressure.

U.E.I.

6. A horizontal drain, whose cross-section is a square of side 1 foot, is closed by a heavy iron plate 1 foot broad, so that when

the drain is closed the lid is inclined at 45° to the horizontal. The plate can turn about its upper edge, which is horizontal. Find the weight of the lid if it is on the point of opening when the drain is half-full of water.

7. A force pump plunger is 2 inches diameter, with a stroke of 6 inches. If the force on the plunger is 2,000 lb., and the plunger makes 90 strokes a minute, find the H.P. supplied.

8. A hydraulic crane lifts a weight of 2 tons at a speed of 3 feet per second. If the efficiency of the crane is 70 per cent., how many cubic feet of water per minute at a pressure of 1,000 lb. per square inch must be supplied while the crane is working?

9. A hydraulic hoist lifts a gross load of 30 tons at a speed of 180 feet per minute. Assuming the hoist efficiency to be 80 per cent., and the working pressure 700 lb. per square inch, find the number of cubic feet of water supplied per minute. U.L.C.I.

10. A hydraulic accumulator has a ram 9 inches diameter with a stroke of 6 feet. The water pressure in the accumulator is 1,200 lb. per square inch. What is the total energy which can be stored in the accumulator? If the water can be led from the accumulator to a hydraulic machine through a 2-inch diameter pipe, flowing with a velocity of 5 feet per second, what H.P. can be supplied to the machine, and for how long can the accumulator supply this H.P.? Neglect friction in the pipe.

11. A hydraulic motor develops 50 H.P., with an efficiency of 72 per cent. How many gallons of water will be required per minute if the water supplied is under a pressure of 700 lb. per square inch?

U.E.I.

CHAPTER XII

HYDRAULICS. FLOW OF WATER

79. Energy of Water. A given quantity of water may possess energy in one, two, or three different forms simultaneously; these forms are—(1) *potential energy*, or the energy due to its height above *datum level*; (2) *pressure energy*, or the energy due to the pressure exerted by the water; (3) *kinetic energy*, or the energy due to motion.

For simplicity the energy in 1 lb. of water will be considered in this and subsequent work, unless otherwise stated; the energy in W lb. of water can be obtained by multiplying the following expressions by W .

(1) The potential energy of 1 lb. of water at a height of h feet above datum is h foot-pounds;

(2) The pressure energy of 1 lb. of water, which exerts a pressure of P lb. per square foot, is P/w , where w is the weight of 1 cubic foot of water. The energy in V cubic feet is PV foot-pounds, for if V cubic feet of water at a pressure of P lb. per square foot be admitted into a cylinder whose ram is 1 square foot in area, the ram will be moved a distance of V feet against a force of P lb. The volume of 1 lb. of water is $1/w$;

(3) The kinetic energy of 1 lb. of water which is moving with a velocity of v feet per second, is $v^2/2g$.

In many cases the energy contained in one form is so small compared to the energy in another form that the former quantity may be neglected.

The total energy, H , in 1 lb. of water is therefore given by the equation

$$H = h + \frac{P}{w} + \frac{v^2}{2g}.$$

If the water moves without doing external work, such

as against friction or in producing turbulence, or without having external work done on it, H is a constant quantity; but h , P and v may vary.

This result is known as *Bernoulli's Theorem*. This theorem assumes frictionless streamline flow, but it can be modified to include friction and turbulence.

80. Velocity of Water due to Head. When water flows out of a vessel through an orifice at a depth h below the free surface (Fig. 129), the velocity of the issuing water may be calculated by consideration of energy.

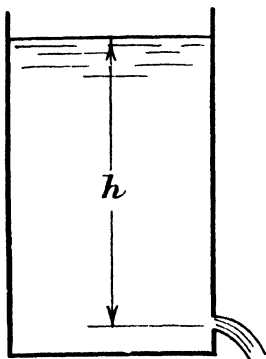


FIG. 129.

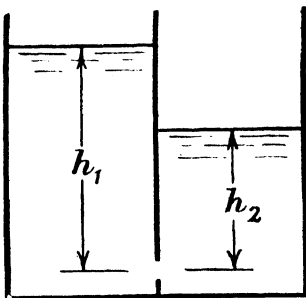


FIG. 130.

Consider the energy in 1 lb. of water. At the free surface the potential energy is h , and the kinetic energy is zero; the datum being taken at the orifice level. At the orifice the potential energy is zero, and the kinetic energy is $\frac{v^2}{2g}$.

Therefore
$$h + 0 = 0 + \frac{v^2}{2g},$$

or $v = \sqrt{2gh}$.

That is, the water issues from the orifice with a velocity equal to that gained by a body falling freely through a height equal to the head of water above the orifice.

If there is friction in the orifice the velocity will be reduced as some of the potential energy will be used in overcoming this friction.

When water flows through an orifice under a head h_1 , from one vessel into another in which the head is h_2 (Fig. 130), the velocity of the water will be given by $v^2 = \sqrt{2g(h_1 - h_2)}$, where $h_1 - h_2$ is the head producing flow.

81. Flow of Water through Sharp-edged Orifices.
When water flows through a sharp-edged orifice the quan-

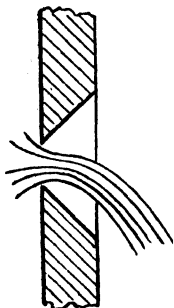


FIG. 131.

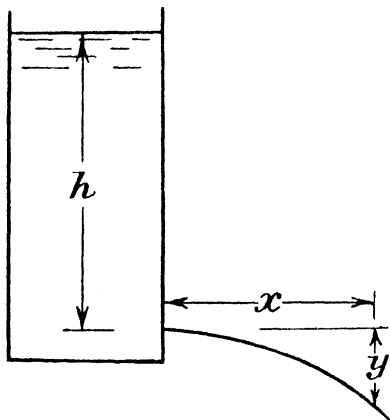


FIG. 132.

tity flowing per second, Q , in cubic feet, is given by $Q = Av$, where A is the cross-sectional area of the jet in square feet, and v the velocity of the water at this section in feet per second. As the particles of water are not approaching the orifice in parallel lines, but as shown in Fig. 131, the jet contracts, and has a minimum section, or *vena contracta*, a short distance in front of the orifice. The ratio of the area at the vena contracta to the area in the orifice is called the *coefficient of contraction*, C_c . The value of C_c for sharp-edged orifices is about 0.62, but varies slightly with the shape of orifice and head of water. Since there is friction at the orifice, the velocity

of the water will be less than the theoretical value due to the head.

The coefficient of velocity, C_v , may be found as follows. In a time, t , a particle of water will have to be moved horizontally a distance, x (Fig. 132), given by $x = vt$, where v is the actual velocity of the jet, and vertically through a distance y , where $y = \frac{1}{2}gt^2$.

$$\text{Therefore} \quad \frac{x^2}{y} = \frac{2v^2}{g} = 4C_v^2h,$$

$$\text{or} \quad C_v = \frac{x}{2\sqrt{hy}}.$$

$C_v = 0.96$ to 0.994 for a sharp-edged orifice.

These two coefficients are generally combined in one *coefficient of discharge*, $C = C_c \times C_v$;

$$Q = C_c A C_v \sqrt{2gh} = CA\sqrt{2gh}.$$

C is usually found by experiment, and its average value is 0.61 .

Measuring columns for gauging small quantities of water are based on this principle.

82. Flow of Water over Rectangular Notches or Weirs. When water flows over a rectangular notch the velocity of the water in any thin horizontal strip varies as $\sqrt{h}$ where h is the depth of the strip below the free surface of the water.

Let B be the breadth of the notch, and H the height of still water above the sill (Fig. 133). Consider a strip of thickness dh at a depth h below the surface; the quantity of water, q , flowing past this strip per second will be given by $q = Av = Bdh\sqrt{2gh} = B\sqrt{2g}h^{\frac{3}{2}}dh$. The total quantity of water, Q , flowing over the weir will be given by $Q = B\sqrt{2g} \int_0^H h^{\frac{3}{2}}dh = \frac{2}{3}B\sqrt{2g}H^{\frac{3}{2}}$. As in the case of the orifice there will be a contraction of area, due to the sides, and also a loss due to friction, therefore $Q = \frac{2}{3}CB\sqrt{2g}H^{\frac{3}{2}}$, where C is the coefficient of discharge. The value of C varies from 0.59 to 0.62 .

It has been assumed that the water has no velocity as it approaches the notch. If the stream is wide compared with the notch this is approximately true. If the water approaches the notch with an average velocity v_o , there is an additional kinetic head $h_o = \frac{v_o^2}{2g}$, and the head must be increased to allow for this, from H to $(H + h_o)$.

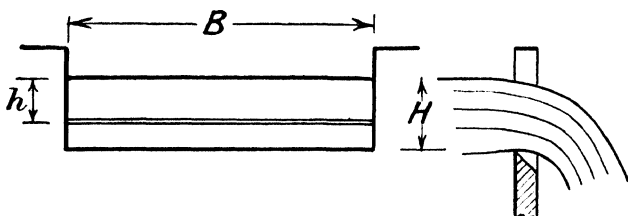


FIG. 133.

As the water curves and drops over the notch, the head must be measured at the still water a short distance above the notch.

EXAMPLE. Water is flowing from a reservoir over a weir 12 feet wide, with a head of 9 inches. Calculate the discharge in gallons per day, assuming $C = 0.6$.

$$\begin{aligned} Q &= \frac{2}{3}CB\sqrt{2g}H^{\frac{3}{2}} = \frac{2}{3} \times 0.6 \times 12 \times \sqrt{64.4} \times 0.75^{\frac{3}{2}} \\ &= 25.13 \text{ cubic feet per second} \\ &= 25.13 \times 6.25 \times 60 \times 60 \times 24 \\ &= 13,560,000 \text{ gallons per day.} \end{aligned}$$

83. Flow of Water over Triangular Notch. Let the angle of the notch be 2θ , and the head of water be H (Fig. 134). The width of the notch at the surface of the water will be $2H \tan \theta$. Consider a horizontal strip of thickness dh at a depth h below the free surface. The area of this strip will be $2(H - h) \tan \theta dh$, and the quantity of water, q , flowing past this strip per second will be given by

$$\begin{aligned} q &= 2(H - h) \tan \theta dh \sqrt{2gh} \\ &= 2 \tan \theta \sqrt{2g} (Hh^{\frac{1}{2}} - h^{\frac{3}{2}}) dh. \end{aligned}$$

The total quantity of water, Q , flowing over the notch will be given by

$$Q = 2 \tan \theta \sqrt{2g} \int_0^H (Hh^{\frac{1}{2}} - h^{\frac{3}{2}}) dh$$

$$= \frac{8}{15} \tan \theta \sqrt{2g} H^{\frac{5}{2}}.$$

Taking the coefficient of discharge into consideration $Q = \frac{8}{15} C \tan \theta \sqrt{2g} H^{\frac{5}{2}}$. The value of C is about 0.61.

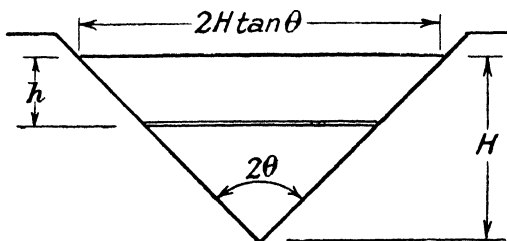


FIG. 134.

EXAMPLE. Calculate the quantity of water flowing over a rectangular notch when the head is 5 inches.

$$\tan \theta = \tan 45^\circ = 1$$

$$Q = \frac{8}{15} C \tan \theta \sqrt{2g} H^{\frac{5}{2}} = \frac{8}{15} \times 0.61 \times 1 \times \sqrt{64.4} \times 0.417^{\frac{5}{2}}$$

$$= 0.143 \text{ cubic feet per second.}$$

84. Loss of Head due to Sudden Enlargement of Pipe. If water is flowing in a steady stream along a pipe which suddenly enlarges (Fig. 135) there will be eddy motion set up in the corner of the larger pipe which will cause a loss of energy, or head.

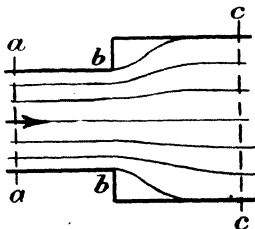


FIG. 135.

Consider two sections, aa and cc , each at such a distance from bb that the flow is steady. If h is the loss of head due to sudden enlargement

$$\frac{P_a}{w} + \frac{v_a^2}{2g} = \frac{P_c}{w} + \frac{v_c^2}{2g} + h.$$

Let A_a and A_c be the areas of the pipes at aa and cc respectively. Then $v_a A_a = v_c A_c$.

The momentum of the quantity of water which passes aa in unit time is equal to $\frac{w A_a v_a^2}{g}$, and the momentum of the water which passes cc is $\frac{w A_c v_c^2}{g}$. The change of momentum is therefore

$$\frac{w A_a v_a (v_a - v_c)}{g}.$$

The forces on the water between aa and cc to produce this change of momentum are $P_a A_a$ acting on aa , $P_c A_c$ acting on cc , and $P(A_c - A_a)$ acting on the annular ring at bb . Experiments have shown that P may be taken as equal to P_a .

$$\text{Therefore } P_c A_c - P_a (A_c - A_a) - P_a A_a = \frac{w A_a v_a (v_a - v_c)}{g},$$

$$\text{and } (P_c - P_a) A_c = \frac{w A_a v_a (v_a - v_c)}{g}.$$

Since $A_a v_a = A_c v_c$, therefore

$$\frac{P_a}{w} = \frac{P_c}{w} - \frac{v_a v_c}{g} + \frac{v_c^2}{g}.$$

By adding $\frac{v_a^2}{2g}$ to both sides of the equation, and putting $\frac{v_c^2}{2g} + \frac{v_c^2}{2g}$ for $\frac{v_c^2}{g}$ the equation becomes

$$\frac{P_a}{w} + \frac{v_a^2}{2g} = \frac{P_c}{w} + \frac{v_c^2}{2g} + \frac{(v_a - v_c)^2}{2g}.$$

Therefore the loss of head, h , due to sudden enlargement, is equal to $\frac{(v_a - v_c)^2}{2g}$ per lb. of water flowing through the pipe. The loss of head is the same whether there is steady or turbulent flow in the pipe.

85. Loss of Head due to Sudden Contraction of

Pipe. When water flows from the larger into the smaller pipe (Fig. 136), there will be a contraction of area in the smaller pipe similar to the contraction of area when water flows through an orifice. The only loss of head up to the contracted area, aa , is that due to friction, which is small and may be neglected. After passing aa the water expands and fills the pipe, as at cc , and there is a loss of head here due to the sudden enlargement.

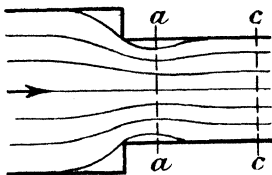


FIG. 136.

Using the same notation as in the last paragraph, this loss of head is equal to

$$\frac{(v_a - v_c)^2}{2g} = \frac{v_c^2}{2g} \left(\frac{A_c}{A_a} - 1 \right)^2.$$

If C is the coefficient of contraction at aa , then $\frac{A_c}{A_a} = \frac{1}{C}$.

The value of the coefficient C in this case varies with the ratio of the diameters of the pipes. Where the ratio is very large (as in the case of water flowing out of a reservoir into a pipe), if $C = 0.6$, then $\left(\frac{1}{C} - 1 \right)^2 = 0.45$. An experimental value which is usually taken is 0.5.

86. Flow through Frictionless Pipe. When water flows from a Reservoir A through a pipe of varying section to a Reservoir B at a lower level (Fig. 137) with the pipe running full, the same quantity of water will pass every section per second; and the velocity of the water will vary inversely as the area of the section. Since there is no friction, the total energy in 1 lb. of water will be constant, but the kinetic energy will vary, the potential energy will vary as the water reaches different levels, and generally the pressure energy will vary also. The energy of 1 lb. of water at each point can be expressed as in the table below :

Position.	Potential Energy.	Pressure Energy.	Kinetic Energy.	Total Energy.
A . .	H	O	O	H
C . .	h_1	$\frac{P_1}{w}$	$\frac{v_1^2}{2g}$	$H = h_1 + \frac{P_1}{w} + \frac{v_1^2}{2g}$
D . .	h_2	$\frac{P_2}{w}$	$\frac{v_2^2}{2g}$	$H = h_2 + \frac{P_2}{w} + \frac{v_2^2}{2g}$
B . .	O	O	$\frac{v^2}{2g}$	$H = O + O + \frac{v^2}{2g}$

If water is flowing full bore along a horizontal water main of varying sectional area, the pressure at a small

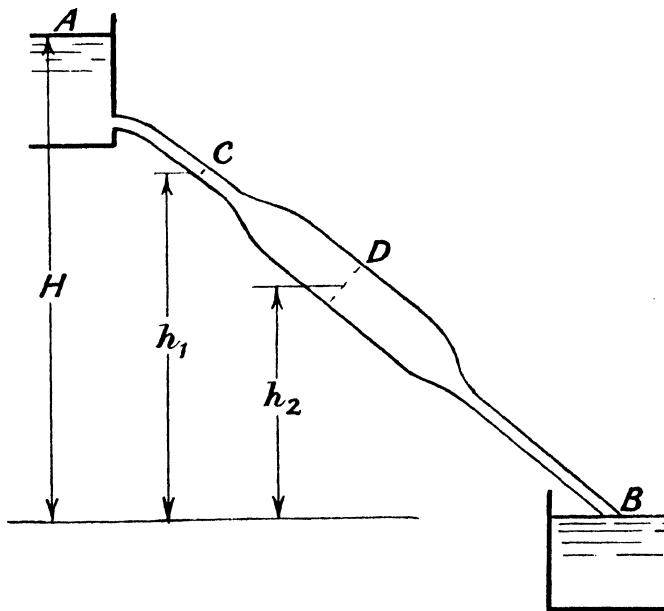


FIG. 137.

section will be smaller than at a larger section as the velocity of the water, and therefore the kinetic energy

of the water, will be greater at the small section; this increase of kinetic energy can only be obtained at the expense of the pressure energy as the potential energy remains constant.

87. Venturi Water Meter. Bernoulli's Theorem is applied in the Venturi Water Meter to the measurement of the flow of water in a pipe. The meter consists of a double conical pipe inserted in a horizontal length of the water pipe, or main, with two pressure pipes, one from the full section of the main and one from the smallest section or *throat* of the meter, to a manometer. The manometer shows the difference of pressure at these two sections from which the flow of water in the main can be calculated.

Conditions up to the throat only are considered, where

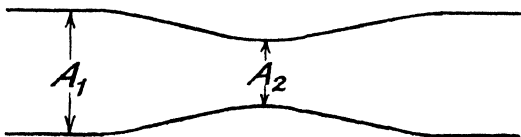


FIG. 138.

the flow is streamline and Bernoulli's Theorem is applicable.

Let A_1 and A_2 be the cross-sectional areas of the main and the throat respectively (Fig. 138), also P_1 and v_1 , P_2 and v_2 be the pressures and velocities of the water at the same points respectively. Neglecting friction in the meter the total energy in 1 lb. of water will be the same at both points.

$$\text{Therefore } h_1 + \frac{P_1}{w} + \frac{v_1^2}{2g} = h_2 + \frac{P_2}{w} + \frac{v_2^2}{2g}.$$

Or, since $h_1 = h_2$,

$$\frac{v_2^2 - v_1^2}{2g} = \frac{P_1 - P_2}{w} = h,$$

where h is the manometer height measured in feet of water.

As the quantity of water passing both points per second is the same, $A_2 v_2 = A_1 v_1$, and

$$v_2 = \frac{A_1}{A_2} v_1 = n v_1, \text{ where } n = \frac{A_1}{A_2}.$$

Therefore
$$h = \frac{v_1^2 (n^2 - 1)}{2g}$$

or
$$v_1 = \sqrt{\frac{2g}{n^2 - 1}} \sqrt{h}.$$

The quantity of water, Q , flowing per second is given by

$$Q = A_1 v_1 = A_1 \sqrt{\frac{2g}{n^2 - 1}} \sqrt{h} = C \sqrt{h},$$

where $C = A_1 \sqrt{\frac{2g}{n^2 - 1}}$, a constant depending on the dimensions of the meter.

For all but small differences in pressure a water manometer becomes impracticably large, and a mercury manometer must be used. If the height of the mercury manometer is read in inches the constant must be multiplied by $\frac{13.59}{12}$, where 13.59 is the specific gravity of mercury.

The friction factor for a Venturi Meter is 0.97 or 0.98, according to the condition of the meter.

EXAMPLE. A Venturi meter on a water main 12 inches diameter has a throat 6 inches diameter. Find the discharge in gallons per minute when the head in the mercury manometer is 8.5 inches.

$$A_1 = \frac{\pi}{4} \times 12^2 = .7854 \text{ square feet.}$$

$$n = \frac{12^2}{6^2} = 4$$

$$Q = A_1 \sqrt{\frac{2g}{n^2 - 1}} \sqrt{h} = .7854 \sqrt{\frac{2 \times 32.2}{4^2 - 1}} \sqrt{\frac{8.5 \times 13.56}{12}}$$

$$= 5.04 \text{ cubic feet per sec.}$$

$$= 5.04 \times 6.25 \times 60$$

$$= 1,890 \text{ gallons per minute.}$$

88. Fluid Friction. When a body moves in still water, or water flows steadily over a surface at very low

velocities only a thin film of water is in contact with the surface and the remainder of the water slides past. As the velocity increases local eddying is caused for some distance from the body ; in pipes this would cause turbulence. Professor Osborne Reynolds has demonstrated this effect with the apparatus shown in Fig. 139. Water from a vessel, A, is made to flow through a horizontal glass tube, BC, by varying the head h . Deeply coloured water is led by a fine tube, D, to the axis of BC. When the velocity of the water in BC is low the coloured water flows through in an unbroken filament, but when the velocity is increased beyond a certain amount, the fila-

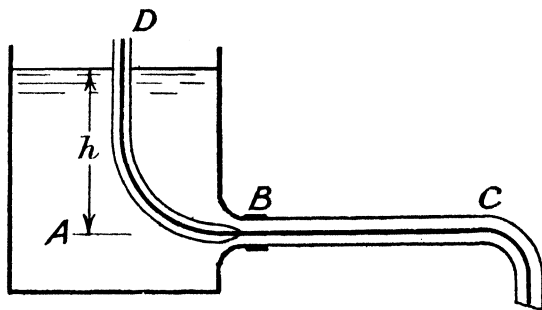


FIG. 139.

ment is diffused throughout the water, showing that the motion is now turbulent. This velocity is called the *Critical Velocity*.

Experiments by Froude and others have established the following laws of fluid friction :—

1. The friction varies directly as the area of wetted surface.
2. The friction varies directly as the roughness of the surface.
3. The friction varies as the n^{th} power of the velocities. At low velocities $n = 1$; at velocities common in water-mains $n = 1.7$ to 2.2 , an average value of 2 being generally used.
4. The friction is independent of the pressure.

5. The friction varies directly as the density and viscosity of the fluid.

89. Loss of Head due to Friction in a Pipe. Experiments have shown that the loss of energy in a pipe due to friction is directly proportional to the wetted surface, the square of the velocity, the coefficient of friction, and inversely proportional to the area of the cross-section at normal velocities in water-pipes. Therefore $h' \propto f \frac{sv^2}{A}$, where h' is the loss of energy expressed as loss of head, s is the wetted area, v the velocity of the water, A the area of cross-section of the pipe, and f the coefficient of friction. As the head or energy due to velocity is $\frac{v^2}{2g}$, the loss of head may be written

$$h' = f \frac{s}{A} \cdot \frac{v^2}{2g}, \text{ or } h' = f \cdot \frac{4L}{D} \cdot \frac{v^2}{2g},$$

since $s = \pi DL$ and $A = \frac{\pi}{4} D^2$.

Consider the energy in 1 lb. of water at two points, A and B, in a uniform pipe flowing full, where h_1 and h_2 are the respective heights in feet of A and B above datum, P_1 and P_2 are the respective pressures in lb. per square foot at A and B. By Bernouilli's Theorem the energy in the 1 lb. of water at B is less than that at A by the amount of the friction head, and the energy equation becomes

$$h_1 + \frac{P_1}{w} + \frac{v^2}{2g} = h_2 + \frac{P_2}{w} + \frac{v^2}{2g} + h',$$

where h' is the loss of head due to friction, or

$$h' = \frac{P_1 - P_2}{w} + h_1 - h_2.$$

Since all the quantities in the last expression can be measured experimentally, we have here a method of finding the value of f . From such experiments D'Arcy

deduced the formula $f = C\left(1 + \frac{1}{12D}\right)$, where D is the diameter of the pipe in feet, and C varies from 0.005 for clean new pipes to 0.01 for old encrusted pipes.

If pressure tubes, or piezometers, are inserted in a water pipe running full (Fig. 140) at points A and B , a distance l apart, the water will rise in these tubes to heights $\frac{P_1}{w}$ and $\frac{P_2}{w}$ where P_1 and P_2 are the pressures at A and B respectively. The line CD joining these points is called the *hydraulic gradient* or *virtual slope* of the pipe. h'' , the difference between the levels at C and D , is called

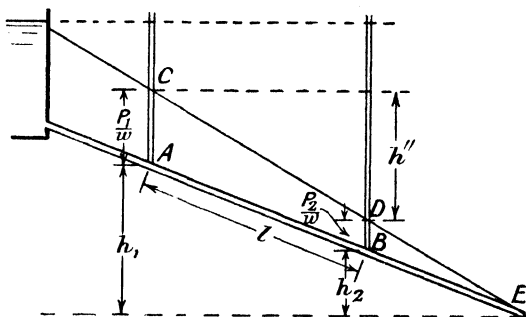


FIG. 140.

the *virtual fall* of the pipe AB , and is given by the equation $h'' = h_1 + \frac{P_1}{w} - h_2 - \frac{P_2}{w}$, which is equal to the loss of head in the length of pipe AB . The angle i which the hydraulic gradient makes with the horizontal is generally small, and $\sin i$ may be taken as equal to i , so that $i = h''/l$. The pressure at the hydraulic gradient is equal to that of the atmosphere.

When water enters a pipe from a reservoir the loss at entrance may be taken as $0.5v^2/2g$, and the kinetic energy of the water in the pipe is $v^2/2g$, where v is the velocity of the water in the pipe. If, therefore, a piezo-

meter were inserted in the pipe close to the entrance (Fig. 141), the water would rise in it to a height $1.5v^2/2g$ less than that of the free surface in the reservoir. If the pipe is uniform the loss of head will be proportional to the length of the pipe, and the hydraulic gradient is a straight line. Where there are sudden changes of section in the pipe line there will be a virtual fall at each change of section, due to the loss of head there, and a change in the hydraulic gradient.

If the pipe rises above the hydraulic gradient, as at C, the pressure in the pipe there will be less than that of the

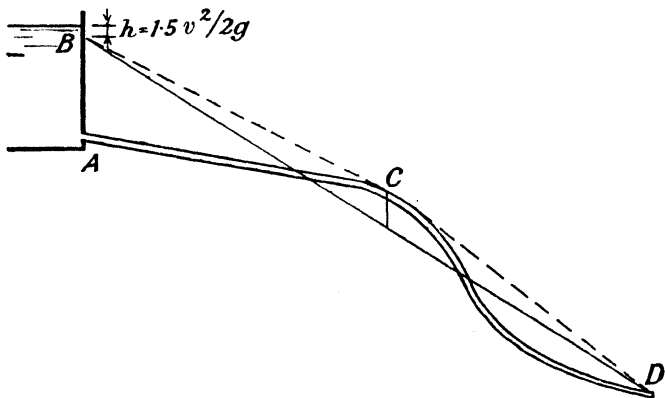


FIG. 141.

atmosphere. As long as the pipe remains air-tight this part of the pipe will act as a syphon, and the discharge will not be altered unless air accumulates at the point of low pressure or the pipe rises to the height of the water barometer above the hydraulic gradient. Water at the free surface in the reservoir dissolves air at atmospheric pressure; when the pressure falls below that of the atmosphere part of this air is liberated, air valves should therefore be placed at such points in a pipe line.

When there is a leak at the point C air enters and the pressure will then be equal to that of the atmosphere; the

hydraulic gradient will now be BC, and the flow will be diminished, as the available head to overcome friction between A and C will be reduced. The part of the pipe CD will not be kept full.

90. Flow of Water in Pipes. When water from a reservoir flows through a pipe and discharges at a lower level there is a loss of potential energy in the water. Part of this potential energy has been used in overcoming the friction in the pipe, and part is converted into kinetic energy in the issuing water. Let the height of the water surface in the reservoir be H feet above the pipe outlet, the diameter of the pipe be D feet, and its length L feet, and the velocity of the water in the pipe be v feet per second. Consider the energy changes in 1 lb. of water; at the surface there is H foot-pounds of energy, at the entrance to the pipe there is a loss of energy due to sudden contraction which may be taken as $0.5\frac{v^2}{2g}$, due to friction in the pipe there is a loss $h' = f\frac{4L}{D} \cdot \frac{v^2}{2g}$, and at the outlet the kinetic energy in the water is $\frac{v^2}{2g}$. The energy equation may therefore be written,

$$H = 0.5\frac{v^2}{2g} + f\frac{4L}{D} \cdot \frac{v^2}{2g} + \frac{v^2}{2g},$$

from which the velocity of the water, and consequently the discharge, may be calculated in any given case.

That part of the boundary of the cross-section of a channel or pipe which is in contact with the water is called the *wetted perimeter*, and the area of the cross-section of the water in the channel or pipe divided by the wetted perimeter is called the *hydraulic mean depth*, or *hydraulic mean radius*. In a circular pipe of diameter D , running full, the hydraulic mean depth, m , is equal to $D/4$, and the above equation may be written

$$H = 0.5\frac{v^2}{2g} + f\frac{L}{m} \cdot \frac{v^2}{2g} + \frac{v^2}{2g}.$$

EXAMPLE. Find the discharge in gallons per day through a pipe 12 inches diameter, 7 miles long, if the surface of the reservoir is 200 feet above the outlet. Take $f = 0.005$.

$$H = \frac{v^2}{2g} \left(0.5 + f \frac{4L}{D} + 1 \right)$$

$$200 = \frac{v^2}{2 \times 32.2} \left(0.5 + 0.005 \times \frac{4 \times 7 \times 5,280}{1} + 1 \right)$$

$$v = 4.47 \text{ feet per sec.}$$

$$Q = Av = \frac{\pi}{4} \times 1^2 \times 4.47 = 3.51 \text{ cubic feet per sec.}$$

$$= 3.51 \times 60 \times 60 \times 24 \times 6.23$$

$$= 1,890,000 \text{ gallons per day.}$$

If the pipe is not of the same diameter throughout its whole length, but consists of sections of different diameters, the loss of head due to friction must be calculated separately for each section, since the velocity of the water will vary inversely as the areas of the pipes in each section. Also, if there are sudden enlargements or contractions in the pipe line, the loss of head due to these must be added to the loss of head due to friction in the pipe.

EXAMPLES XII.

1. In an experiment to determine the coefficients of contraction, velocity and discharge of water through an orifice, the following results were obtained:—Diameter of orifice, 0.3 inch; head of water above centre of orifice, 4 feet; discharge of water, 93.5 lb. in 5 minutes. The jet issues horizontally, and in a horizontal distance of 4 feet it falls 12.4 inches. Determine the values of the coefficients.

U.E.I.

2. Water flowing through a right-angled triangular notch is passed into a tank and weighed. Write down the expression for determining the flow in cubic feet per second for such a notch, and determine the coefficient from the following data:—Quantity of water weighed in 5 minutes, 5,540 lb.; head of still water level over the apex of the notch, 5 inches. Explain how the head is measured.

U.L.C.I.

3. In 10 minutes 635 gallons of water passed over a right-angled V notch, the head of water being 4 inches. Determine the value of the numerical coefficient in the expression used for calculating the flow over the notch.

U.L.C.I.

4. A right-angled V notch is used to measure the discharge of water from a machine. If the mean head of the water flowing over

the notch be 11.4 inches, and the coefficient of discharge of the notch be 0.622, find the number of gallons of water flowing per hour. U.L.C.I.

5. Show how to obtain the expression for the quantity of water flowing over a rectangular notch.

In an experiment on the flow of water over a rectangular notch, 6 inches wide, it was found that the flow was 12.4 cubic feet per minute when the depth was 3 inches. Determine the coefficient of discharge for the notch.

6. A pipe 3 inches diameter is suddenly enlarged to 6 inches diameter. Water is flowing through the pipe at the rate of 10 cubic feet per minute. Find the loss of energy per minute at the change of section (a) when the water is flowing from the 3-inch to the 6-inch diameter pipe, (b) when the water is flowing in the reverse direction.

7. Establish a formula giving the loss of head of water due to a sudden enlargement in the pipe in which it is flowing. The diameter of a straight horizontal pipe at one point is changed abruptly from 6 inches to 1 foot. If 1,875 gallons of water are being delivered per minute, find the loss of head due to the enlargement. U.E.I.

8. State Bernoulli's theorem for the motion of liquids in pipes. A pump sometimes used for pumping waste acid from a tank is constructed in the following manner:—By the condensation of steam in a chamber, a pressure of 8 lb. per square inch below that of the atmosphere is maintained at the top of a pipe of 3 inches internal diameter, which dips down into the acid. If it is to be raised an average height of 3 feet before being discharged, how many gallons of acid of specific gravity 1.2 will be raised per minute? Friction in the pipe may be neglected. U.E.I.

9. A short portion of a horizontal pipe is uniformly tapered in diameter from a maximum of 4 inches to a minimum of $1\frac{1}{2}$ inches. The pressure of the water at the maximum diameter is 10 lb. per square inch gauge, and its velocity there is 6 feet per second. Determine the difference in the water pressure at the maximum and minimum diameters. U.L.C.I.

10. Water flows through a pipe which has a converging portion in which its diameter is gradually reduced, and from the smallest section a diverging portion in which its diameter is again gradually increased to the original diameter. The largest and smallest diameters of the pipe, the axis of which is horizontal, are 9 feet and 3 feet respectively, and the water pressures at the largest and smallest diameters are represented by heads of 26 feet and 20 feet of water respectively, when the pipe is running full. Find the velocity of the water through the smallest section, and also the number of cubic feet of water flowing through the pipe per second. U.L.C.I.

11. A Venturi meter has an inlet of 6 inches and a throat diameter of 3 inches. What will be the difference of head in feet between the inlet and throat if the water passing through the meter is 500 gallons per minute? U.E.I.

12. A Venturi meter has an inlet of 6 inches and a throat of 2 inches diameter. If the difference in pressure head between the water at the inlet and throat is 5 feet when flow takes place, find the discharge in cubic feet per second.

13. A Venturi meter has an inlet of 6 inches and a throat of 4 inches diameter. If the water pressures at the inlet and throat are 120 and 115 lb. per square inch respectively, find the quantity of water flowing through the pipe in cubic feet per minute.

14. State the chief facts concerning the friction of water flowing in a pipe. What do we mean by (1) hydraulic gradient, (2) virtual slope of a pipe, (3) hydraulic mean depth?

A pipe 18 inches in diameter and 4 miles long connects two reservoirs having a difference of level of 200 feet. If 3,000 gallons per minute are delivered, find the coefficient of resistance to the flow. U.E.I.

15. During an experiment 70 gallons of water flowed through a horizontal pipe $\frac{3}{4}$ inch diameter in 12.45 minutes, and the measured difference of pressure head in a length of 10.4 feet was 1.95 feet. Determine the coefficient of resistance to the flow. U.L.C.I.

16. A pump is delivering 190 gallons of water per minute through a hose pipe 500 feet long of 3 inches internal diameter. The pressure as given by a gauge at the pump end of the hose is 150 lb. per square inch, and at the end near the nozzle is 50 lb. per square inch above atmospheric pressure. Assuming that the whole loss is due to friction in the pipe, calculate the value of the coefficient of friction for this hose-pipe. U.E.I.

17. Determine the head of water lost in friction in a pipe 5,000 feet long and $2\frac{1}{2}$ feet diameter when it discharges $9\frac{1}{2}$ millions of gallons in 24 hours; the coefficient of resistance to flow is 0.007. U.L.C.I.

18. If the value of f in the formula for loss of head due to friction in pipes is 0.01, find the head required to deliver 1,000 gallons of water per minute through a horizontal pipe 2 miles long and 1 foot in diameter. U.E.I.

19. Describe experiments by which the loss of head incurred by water passing along a pipe can be measured. What is the effect of (a) varying the velocity, (b) using pipes of different diameters?

3 cubic feet of water per second are admitted to a pipe of 9 inches internal diameter at a pressure of 800 lb. per square inch. (a) If the pipe is horizontal and 1,500 yards long, what is the loss of head? (b) If the water issuing from the pipe enters a hydraulic motor, what is the H.P. supplied to the motor? Take the coefficient of friction, $f = 0.01$. U.E.I.

20. What do you understand by Bernoulli's theorem? A pipe AB is 250 feet long and has an inclination with the horizontal of $\sin^{-1} \frac{1}{5}$. When the pipe is running full the pressure head is 35 feet at the upper end A, where the pipe is 2 feet in diameter, and the velocity of flow 3.5 feet per second. Determine the pressure head at the end B where the pipe has been reduced to 1.5 feet diameter.

U.L.C.I.

21. Water flows from a reservoir which is open to the air to a jet which is 15 feet below the surface of the water in the reservoir. The jet issues into a condenser in which the pressure is 4 lb. per square inch absolute. Find the velocity of the water issuing from the jet.

U.E.I.

22. At a section A in a pipe running full of water the diameter of the pipe is 2 feet, and the velocity of the water is 4 feet per second. At another section B, which is 30 feet below the level of A, the diameter of the pipe is $1\frac{1}{2}$ feet. Determine the difference between the water pressures at A and B.

U.L.C.I.

23. A certain quantity of water flows through a pipe 1,000 feet long and 6 inches in diameter with a loss of head of 150 feet. Determine the loss of head if the same quantity of water passed through a pipe 1,400 feet long and 8 inches diameter, assuming both pipes running full and the same coefficient of resistance in each case.

U.L.C.I.

24. A pipe, one mile long, 6 inches diameter, conveys water flowing at 3 feet per second to a reservoir. Assuming a coefficient $f = 0.01$, find the frictional loss in head along the pipe. What H.P. is lost in forcing the water through the pipe?

U.E.I.

25. A nozzle at the end of a 3-inch internal diameter water pipe has an internal diameter at its open end of $1\frac{1}{8}$ inches. When water flowed through the pipe a pressure gauge at the entrance to the nozzle registered a pressure of 75 lb. per square inch gauge. Calculate the velocity of the water leaving the nozzle and the number of gallons of water issuing from the nozzle per minute.

U.L.C.I.

26. A centrifugal pump draws 500 gallons of water per minute from a sump 12 feet below, and delivers to a higher level. To prevent the liquid from breaking up, the pressure head in the suction pipe must not fall below $6\frac{1}{2}$ feet. What must be the least diameter of the suction pipe to ensure no breaking up?

U.E.I.

CHAPTER XIII

WATER TURBINES

91. Water Wheels. In a water wheel the water does work on the wheel by virtue of its original potential energy. On the rim of the wheel are fixed a number of buckets

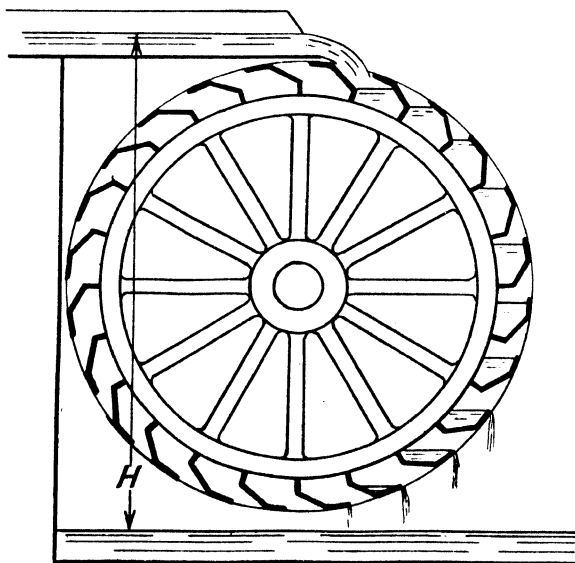


FIG. 142.

into which the water flows as near the top of the wheel as possible, and empties out as near the bottom as possible. If the water is carried over the top of the wheel, as in Fig. 142, the wheel is called an *Overshot Wheel*; but if

the available fall is not great enough for this, and the water is led straight to the wheel below its centre as in Fig. 143, the wheel is called an *Undershot Wheel*.

The energy supplied to a water wheel in a given time is equal to the loss of potential energy of the water in that time, or to the weight of water flowing over the wheel multiplied by the difference in height—between the *headrace* and the *tailrace*.

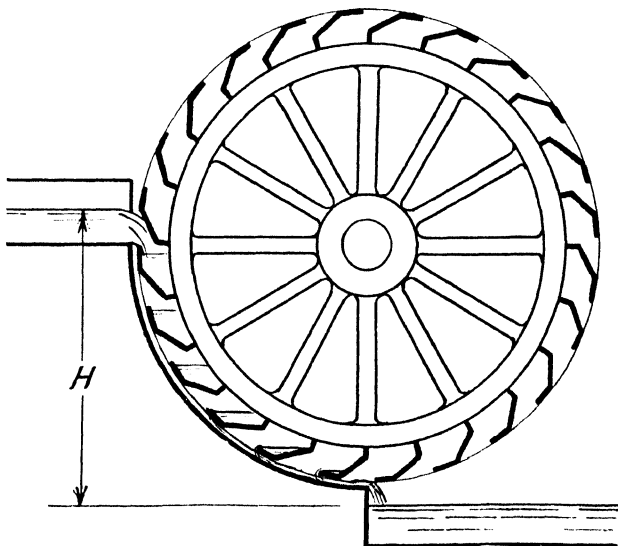


FIG. 143.

Generally the velocity of the water as it approaches the wheel is small, and its kinetic energy may be neglected.

92. Water Turbines. In a water turbine the water is led to the turbine through pipes from a source of supply at a higher level and has considerable pressure energy when it reaches the turbine; the kinetic energy of the water may be neglected as the velocity must be small on account of friction in the pipe. In the turbine this pressure

energy is converted into velocity energy and thus does work on the turbine rotor. There are two main types of turbines, *Impulse Turbines* and *Reaction Turbines*.

In an impulse turbine the water enters the turbine casing through a nozzle in which all the pressure energy is converted into velocity energy ; the water then impinges on the rotor vanes, or buckets, and by continuous loss of momentum causes a continuous pressure on the vanes, thus driving the rotor.

In a reaction turbine the water enters the vanes under pressure, and in the vanes the pressure energy is converted into velocity energy, the water gaining momentum and by reaction imparting an equal momentum to the vanes of the rotor.

93. Pressure of a Jet of Water on Vanes. When considering the pressure of a jet of water on vanes, there are several different cases to be studied, but one principle is involved in all of them. Since the velocity of the water is changed in magnitude, or direction, or in both magnitude and direction, there is a change of momentum in the direction of motion of the vane, and consequently force is exerted in that direction. This force is equal to the rate of change of momentum of the water, or the

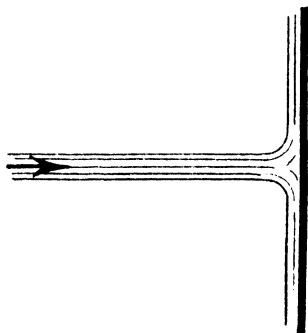


FIG. 144.

change of momentum per second.

Case I. Stationary Flat Vane normal to Jet. In this case (Fig. 144), the velocity of the water in the direction of the jet is destroyed after impinging on the vane. Let A be the area of cross-section of the jet in square feet ; v be the velocity of the water in feet per second ; w the weight of water per cubic foot in pounds ; and P the force in pounds exerted on the vane. The weight of water

impinging on the vane per second will be wAv , and the change of momentum per second will be

$$\frac{wAv \times v}{g}.$$

Therefore
$$P = \frac{wAv^2}{g}.$$

In this case no work is done.

Case II. Flat Vane normal to, and moving in the direction of the Jet. In this case the water does not have the whole of its velocity in the direction of the jet destroyed after impinging on the vane, nor does the whole of the water strike the vane per second. Let v_1 be the velocity of the vane in feet per second, and the same notation as in Case I hold good for the other quantities. The weight of water impinging on the vane per second will be $wA(v - v_1)$; the change of velocity of the water will be $(v - v_1)$; and the change of momentum per second will be

$$\frac{wA(v - v_1)^2}{g}.$$

Therefore
$$P = \frac{wA(v - v_1)^2}{g}.$$

The work done per second will be
$$\frac{wA(v - v_1)^2 \times v_1}{g}.$$

Case III. Succession of Flat Vanes normal to Jet and moving in the direction of the Jet. In this case the water does not have the whole of its velocity in the direction of the jet destroyed after impinging on the vane, but the whole of the water will strike the vanes per second. With the same notation as above, the weight of water impinging on the vanes per second will be wAv ; the change of velocity of the water will be $(v - v_1)$; and the change of momentum per second will be

$$\frac{wAv(v - v_1)}{g}.$$

Therefore
$$P = \frac{wAv(v - v_1)}{g}.$$

The work done per second will be

$$\frac{wAvv_1(v - v_1)}{g},$$

and since the kinetic energy supplied per second in the water is

$$\frac{wAv \times v^2}{2g},$$

the efficiency will be

$$\frac{wAvv_1(v - v_1)}{g} \div \frac{wAv^3}{2g} = \frac{2v_1(v - v_1)}{v^2}.$$

Differentiating this expression shows that the efficiency is greatest when $v = 2v_1$, and the efficiency is then 50 per cent.

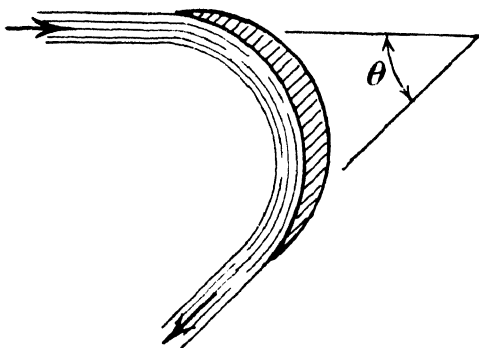


FIG. 145.

Case IV. Stationary Curved Vane. The jet of water should enter the vane tangentially, and the direction of the jet on leaving the vane will be inclined at an angle θ to the original direction as shown in Fig. 145. With the same notation as above, the weight of water impinging on the vane per second will be wAv ; the change in velocity of the water in the direction of the jet (neglecting friction

in the vane) will be $v(1 + \cos \theta)$; and the change of momentum per second will be

$$\frac{wAv^2(1 + \cos \theta)}{g}.$$

Therefore
$$P = \frac{wAv^2(1 + \cos \theta)}{g}.$$

There is no work done in this case.

Case V. Succession of Curved Vanes moving in the direction of the Jet. This case is similar to Case III. Again with the same notation as above, the weight of water impinging on the vanes per second will be wAv ; the change in velocity of the water in the direction of the jet (neglecting friction in the vane) will be $(v - v_1)(1 + \cos \theta)$; and the change of momentum per second will be

$$\frac{wAv(v - v_1)(1 + \cos \theta)}{g}.$$

Therefore
$$P = \frac{wAv(v - v_1)(1 + \cos \theta)}{g}.$$

The work done per second will be

$$\frac{wAvv_1(v - v_1)(1 + \cos \theta)}{g},$$

and the efficiency (see Case III) will be

$$\frac{wAvv_1(v - v_1)(1 + \cos \theta)}{g} \div \frac{wAv^3}{2g} = \frac{2v_1(v - v_1)(1 + \cos \theta)}{v^2}.$$

Differentiating this expression shows that the efficiency is greatest when $v = 2v_1$, and the efficiency is then $0.5(1 + \cos \theta)$. If $\theta = 0^\circ$, that is the direction of the jet is reversed, the efficiency (neglecting friction) is 100 per cent.

EXAMPLE. Water under a head of 500 feet flows through a nozzle 1 square inch sectional area and impinges on a fixed flat vane normal to the direction of the jet. (a) Find the pressure exerted on the vane. (b) If the jet impinges on a succession of flat vanes moving with a velocity of 70 feet per second fixed to a turbine rotor, find the Horse Power of the turbine, and its efficiency.

(a) Velocity of jet, $v = \sqrt{2gh} = \sqrt{2 \times 32.2 \times 500} = 179.4$ feet per sec.

$$P = \frac{wAv^2}{g} = \frac{62.3 \times 1 \times 179.4^2}{144 \times 32.2} = 432.5 \text{ lb.}$$

$$\begin{aligned} \text{(b) Work per sec.} &= \frac{wAvv_1(v - v_1)}{g} \\ &= \frac{62.3 \times 1 \times 179.4 \times 70(179.4 - 70)}{144 \times 32.2} \\ &= 18,460 \text{ ft.-lb.} \end{aligned}$$

$$\text{Horse Power} = \frac{18,460}{550} = 33.4.$$

$$\begin{aligned} \text{Efficiency} &= \frac{2v_1(v - v_1)}{v^2} = \frac{2 \times 70(179.4 - 70)}{179.4^2} \\ &= 0.476. \end{aligned}$$

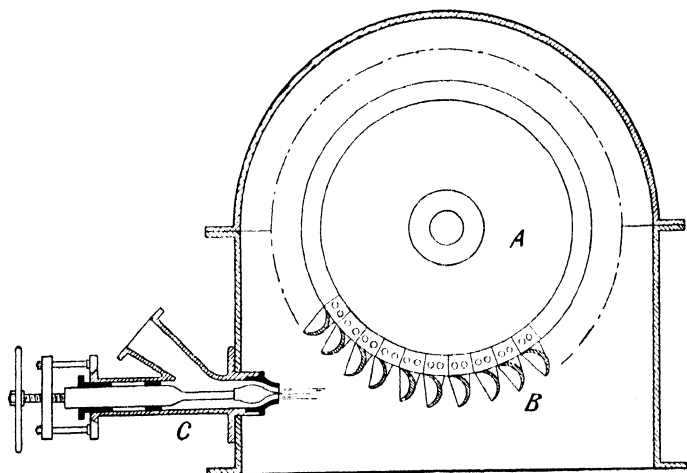


FIG. 146.

94. Pelton Wheel. The Pelton Wheel (Fig. 146) is one of the simplest forms of impulse water turbine. The rotor, A, is a vertical wheel, or disc, carried on a horizontal shaft, with vanes or brackets, B, fixed radially on the rim. The water enters through a nozzle, C, fixed to the casing, so that the jet strikes the centre of the buckets

in a direction tangential to the wheel. The present form of bucket, known as the *Dobble Bucket*, is of double ellip-

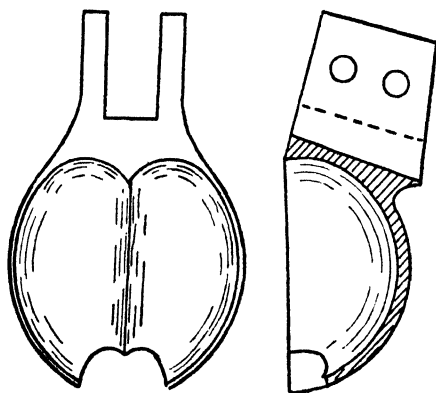


FIG. 147.

soidal form (Fig. 147), the jet striking the ridge in the centre and half the jet being deflected on each side to avoid end thrust on the shaft. The outer lip of the bucket is cut away to clear the jet as the bucket comes into action. The angle through which the water is deflected is less than 180° , so that the returning stream may clear the following bucket. The quantity of water flowing through the nozzle is regulated by a *spear head-* or *needle-valve*, moved along the axis of the nozzle by means of a screw; the object of this type of valve being to give a steady solid jet of water whatever quantity is flowing through the nozzle.

In large wheels two or more nozzles are sometimes fitted to give increased power.

Neglecting friction, the greatest efficiency is obtained when the bucket speed is 0.5 the jet speed if the water is deflected through 180° . In practice, taking bucket friction into account, and deflecting the water through 160° , the bucket speed should be about 0.46 of the jet

speed. The efficiency of a Pelton Wheel is from 70 to 80 per cent.

EXAMPLE. If 10 cubic feet of water per second with a velocity of 100 feet per sec. are supplied to a Pelton Wheel, find the Water Horse Power of the wheel, neglecting friction. Bucket angle, 160° and bucket speed 55 feet per sec.

$$\begin{aligned}\text{Efficiency} &= \frac{2v_1(v - v_1)(1 + \cos \theta)}{v^2} \\ &= \frac{2 \times 55 \times 45 \times 1.94}{10,000} = .96.\end{aligned}$$

$$\text{K.E. of feet per second} = \frac{10 \times 62.3 \times 100^2}{64.4} = 96,730 \text{ ft.-lb.}$$

$$\text{H.P. of feet} = \frac{96,730}{550} = 176.$$

$$\text{W.H.P. of wheel} = 176 \times .96 = 169.$$

EXAMPLES XIII

1. 2 cubic feet of water per minute flow through a nozzle $\frac{1}{4}$ inch diameter and impinge perpendicularly on a plate. Find the pressure on the plate (a) when the plate is stationary, (b) when the plate is moving in the direction of the jet with a velocity of 20 feet per second.

2. A jet of water impinges on a flat plate. If the velocity of the jet is 100 feet per second, and the quantity of water is 2 cubic feet per second, find the pressure on the plate (a) when it is fixed, (b) when it is moving in the direction of the jet with a velocity of 30 feet per second.

3. Water under a head of 98 feet issues from a nozzle 2 inches diameter and impinges normally on a series of flat vanes moving in the direction of the jet with a velocity of 35 feet per second. Find the H.P. and the efficiency of the machine. Neglect nozzle friction.

4. A jet of water under an effective head of 120 feet is to strike a fixed vane and give a pressure of 200 lb. What must be the diameter of the jet? If the jet is required to maintain a pressure of 100 lb. on a series of flat vanes moving in its own direction with a velocity of 40 feet per second, find the diameter.

5. A jet of water issues from a nozzle $\frac{1}{2}$ inch diameter, with a velocity of 250 feet per second, and impinges on a series of flat vanes moving in the direction of the jet. Plot a graph showing the pressure of the jet on the vanes as the velocity of the vanes increases from 170 to 250 feet per second. State the magnitude of the maximum pressure.

6. A jet of water from the nozzle 2 inches diameter with a velocity

of 200 feet per second strikes a fixed cup and has its direction completely reversed. Find the pressure on the cup (*a*) when it is stationary, (*b*) when it is moving in the direction of the jet with a velocity of 70 feet per second.

7. A jet of water, 1 inch diameter, coming from a reservoir whose surface is 200 feet above the nozzle, strikes a fixed hemispherical cup so that its direction is reversed. If the jet has 90 per cent. of the full velocity due to its head, find the force it exerts on the cup. If the jet acts on a turbine whose efficiency is 80 per cent., find the H.P. developed.

8. A jet of water issuing from a nozzle 1 inch diameter with a velocity of 400 feet per second, strikes a series of cups attached to a wheel. The radius of the wheel is 2 feet, and the angle of the cups is 160° . Find the speed at which the wheel should run for maximum efficiency, and the H.P. developed at that speed.

9. A jet of water impinges on a series of hemispherical cups moving in the same direction, and is deflected through 180° . Obtain an expression for the pressure on the cups. If the velocity of the jet is 100 feet per second, and that of the cups 40 feet per second, find the work done per lb. of water striking the cups. U.E.I.

10. 20 cubic feet of water are supplied per second to a Pelton Wheel through a nozzle the area of which is 40 square inches. The velocity of the cups is 38 feet per second. Find the H.P. of the wheel, assuming an efficiency of 80 per cent.

11. A Pelton wheel of 3,200 H.P. is driven by a jet of water $5\frac{1}{2}$ inches diameter, which issues from a nozzle under an effective head of 850 feet. Determine the velocity of the jet, the weight of water issuing from the nozzle per minute, and the efficiency of the wheel. U.L.C.I.

12. One hundred cubic feet of water per minute under a head of 462 feet are supplied to a Pelton wheel 2 feet diameter. Determine the H.P. of the wheel and its efficiency when running at 600 r.p.m. U.L.C.I.

13. A test of a 12-inch Pelton wheel gave the following results:—Weight of water passing through nozzle per second, 6.15 lb.; velocity of water issuing from nozzle, 72.5 feet per second; revolutions of wheel per minute, 310; brake horse power, 0.46. Find the H.P. given to the wheel by the water, and the efficiency of the wheel. U.L.C.I.

14. A Pelton water wheel is supplied with 350 lb. of water per minute. If the diameter of the nozzle is 0.4 inch, determine (*a*) the velocity of the water issuing from the nozzle, (*b*) the number of foot-pounds of energy possessed by the water leaving the nozzle per minute. The H.P. developed is measured by a rope brake whose effective diameter is 12 inches, and the effective load on which is 9.5 lb. at 550 r.p.m. Calculate the B.H.P. and the efficiency of the wheel under these conditions. U.E.I.

15. A series of tests were made on a Pelton wheel fitted with a Prony brake. The pressure head of water and nozzle area were kept constant during a test, while the speed of the wheel and brake readings were varied by altering the pressure of the brake drum. The mean radius of the bucket wheel was $6\frac{7}{8}$ inches, and the arm of the Prony brake was 18 inches. The quantity of water used was measured over a 55° V notch whose coefficient of discharge was 0.6. For each of the series of readings given below plot the curves of Brake Horse Power, and efficiency, against ratio of bucket speed to jet velocity, and state for what speed ratio the B.H.P. and efficiency are greatest.

"N" is the wheel speed in r.p.m., and "P" is the brake spring balance reading in lb.

(a) Pressure head of water at nozzle, 95 feet; head of water over notch, 0.365 feet.

N	730	680	650	612	580	548	503	470	420	390	320
P	2.2	3.4	3.8	4.4	4.7	4.9	5.25	5.5	6.0	6.2	6.7

(b) Pressure head of water at nozzle, 110 feet; head of water over notch, 0.401 feet.

N	800	773	748	725	705	680	660	628	590	545	510	450	375
P	2.7	3.8	4.75	5.4	5.8	6.0	6.25	6.4	6.9	7.5	8.2	8.6	9.0

(c) Pressure head of water at nozzle, 80 feet; head of water over notch, 0.505 feet.

N	700	675	650	605	575	550	520	470	350
P	5.05	6.04	7.5	8.5	9.8	10.0	10.8	11.7	13.0

16. A 3-inch water pipe has a right-angled bend. Determine the vector change of velocity and momentum per second at the bend when 90 gallons of water flow through the pipe per minute. Of what value is the determination of this vector change of momentum in practice?

U.L.C.I.

17. A certain type of ram for pumping water is constructed on the following principle:—Waste water flows through a pipe, 20 feet long and of 6 inch bore, at a speed of 16 feet per second. Its flow is stopped and its momentum destroyed in 0.1 second by operating against a piston of diameter 4 inches. This raises some other water through a pipe 200 feet long and of 3-inch bore to a reservoir 50 feet vertically above the ram. Calculate (a) the force acting on the piston, (b) the number of cubic feet of water obtained per hour if the piston is operated 60 times per minute. Friction in pipe, etc., may be neglected.

U.E.I.

ANSWERS TO EXAMPLES

EXAMPLES I

1. (ia) 13.9 tons; (ib) 35.6 tons; (ii) 61.3; (iii) 68.4.
2. (a) 22.7; (b) 42.5 nom.; (c) 45.9; (d) 68. 3. (a) 43; (b) 58 nom.; (c) 20; (d) 57. 4. (a) 13,530 tons/sq. in.; (b) 26.5 nom.; (c) 26.25; (d) 56.9. 5. (a) 29.3 tons/sq. in.; (b) 13,258 tons/sq. in.; (c) 0.035 inch-tons. 7. 2,505.
8. 1:1.56. 9. 0.017 ins.; 84.5 inch-lb. 10. 241 inch-lb.
11. 1:1.78. 12. (a) 5.86 ft.-lb.; (b) 3.46 ft.-lb.
13. 26,970 lb./sq. in. 14. 27,820 lb./sq. in.
15. 4.9 tons/sq. in.; 0.44 ins.; 9.8 tons; 0.88 ins.
16. 0.57 inch-tons. 17. 3.125 tons/sq. in.; 0.325 inch-tons.
18. 41,400 lb./sq. in. 19. 25,300 lb./sq. in. 20. 1.2 ins.
21. $1\frac{1}{8}$ ins.; 8. 22. 2.8 tons/sq. in. 23. 7.2; 4.1 tons/sq. in.
24. 1.67 tons. 25. 10.9 tons. 26. 20.98 tons/sq. in.
27. 151.4 tons. 28. 0.937 ins. 29. 52.4 inch-tons.
30. 44 tons/sq. in. 31. (a) $1\frac{1}{16}$ ins.; $2\frac{1}{2}$ ins.; 56.5%; (b) $1\frac{5}{16}$ ins.; 3 ins.; 57.7%; (c) $1\frac{3}{8}$ ins.; $3\frac{5}{8}$ ins.; 56.5%; (d) $1\frac{7}{8}$ ins.; $4\frac{3}{8}$ ins.; 57.2%; (e) $2\frac{1}{8}$ ins.; 5 ins.; 56.8%.
32. (a) $1\frac{1}{16}$ ins.; $3\frac{7}{8}$ ins.; 72.5%; (b) $1\frac{5}{16}$ ins.; $4\frac{3}{4}$ ins.; 72.8%; (c) $1\frac{3}{8}$ ins.; $5\frac{3}{4}$ ins.; 71.4%; (d) $1\frac{7}{8}$ ins.; $6\frac{7}{8}$ ins.; 72.7%; (e) $2\frac{1}{8}$ ins.; $7\frac{7}{8}$ ins.; 72%. 33. (a) $\frac{5}{8}$ in.; $1\frac{1}{2}$ ins.; 55.6%; (b) $\frac{3}{4}$ in.; $1\frac{3}{4}$ ins.; 56.5%; (c) $\frac{15}{16}$ ins.; $2\frac{1}{4}$ ins.; 55.7%; (d) $1\frac{9}{16}$ ins.; $2\frac{1}{2}$ ins.; 56.7%; (e) $1\frac{1}{4}$ ins.; 3 ins.; 55.6%.
34. (a) $\frac{5}{8}$ in.; $2\frac{1}{4}$ ins.; 72.4%; (b) $\frac{3}{4}$ in.; $2\frac{3}{4}$ ins.; 72%; (c) $\frac{15}{16}$ ins.; $3\frac{1}{2}$ ins.; 71.7%; (d) $1\frac{1}{16}$ ins.; 4 ins.; 71%; (e) $1\frac{1}{4}$ ins.; $4\frac{5}{8}$ ins.; 72%.
35. 1.53 ins. 36. 12; 10.1 ins. 37. 53,300 lb./sq. in. 38. $\frac{7}{8}$ in. 39. $\frac{3}{4}$ in. 40. $\frac{1}{16}$ in.
41. (a) 10,970 lb./sq. in.; (b) 7,680 lb./sq. in.; (c) 3,840 lb./sq. in. 42. $\frac{3}{8}$ in. 43. 75%; 78.5%; 75%; 3.4 tons per in.; 262.5 lb./sq. in. 44. 200 r.p.m.; 436 r.p.m.
45. 685 lb./sq. in. 46. 933 r.p.m. 47. 1,549 r.p.m.
48. 5.17 tons; 0.83 tons. 49. 20.6 tons; 2.2 tons; 2.2 tons.
50. 0.997 tons/sq. in.; 0.46 tons/sq. in.

EXAMPLES II

Note. Compression members indicated by heavy type, tension members by ordinary type.

1. A, 1 ton; B, 2 tons; C, 1·7 tons. 2. AB, **900 lb.**; BC, **900 lb.**; CD, **1,380 lb.**; DE, 1,080 lb.; EA, 700 lb.; BE, 250 lb.; EC, **250 lb.** 3. AB, **3,350 lb.**; BC, **2,235 lb.**; CD, **2,235 lb.**; DE, **3,350 lb.**; EF, 3,000 lb.; FA, 3,000 lb.; BF, **1,110 lb.**; CF, 1,000 lb.; DF, **1,110 lb.**; L.H., **2,000 lb.**; R.H., **2,000 lb.** 4. AB, **4·6 tons**; BC, **4·6 tons**; CD, **6 tons**; DE, **6 tons**; EG, 5·6 tons; GF, 3 tons; FA, 4·3 tons; BF, 1 ton; CF, 1·6 tons; CG, 3·2 tons; DG, **2 tons**.
5. Legs **19·5 tons**; rope 12·8 tons. 6. AB, **10,100 lb.**; BC, **6,740 lb.**; CD, **6,740 lb.**; DE, **10,100 lb.**; EF, 9,050 lb.; FA, 9,050 lb.; BF, **3,380 lb.**; CF, 3,000 lb.; DF, **3,380 lb.**
7. AB, **4,625 lb.**; BD, **4,150 lb.**; DC, **7,625 lb.**; CF, 7,000 lb.; FE, 7,000 lb.; EA, 2,300 lb.; BE, 3,000 lb.; DE, **4,100 lb.**; DF, nil. 8. A, **24·7 tons**; B, 3·9 tons; C, 21·3 tons. 9. BC, 7·85 tons; CD, 7·07 tons; DA, **5 tons**; CA, **2·2 tons**; A, hor. 6·8 tons, vert. 1·1 tons; B, hor. 6·8 tons, vert. 3·9 tons. 10. AB, **2·9 tons**; BC, **1·75 tons**; CD, **4 tons**; DE, 2·25 tons; EA, 1·45 tons; EB, 0·6 tons; EC, **0·6 tons**.
11. AB, **2·25 tons**; BC, **1·46 tons**; CD, **3·6 tons**; DE, 1·9 tons; EA, 1 ton; BE, 0·88 tons; CE, **0·88 tons**.
12. AB, **2,530 lb.**; BC, **1,950 lb.**; CD, **3,000 lb.**; DE = EF, 1,500 lb.; FG = GE, 1,270 lb.; BG, 1,000 lb.; BF, 1,370 lb.; CF, 940 lb.; CE, 1,800 lb. 13. AB, 9·625 tons; BD, 7·8 tons; DC, **12·625 tons**; BC, **3 tons**; P, 10 tons; Q, 7 tons.
14. 12·5 tons; AC, **37 tons**; BC, 27·7 tons; AB, 12·5 tons; BD, 17·7 tons; DA, 12·5 tons. 15. AB, **4 tons**; AD, 5·6 tons; BD, **1·7 tons**; CD, 6·3 tons; C, hor. 5·5 tons, vert. 3·2 tons; B, hor. 5·5 tons, vert. 0·8 tons. 16. EA, **3·9 tons**; AF, **1·5 tons**; FD, **2·85 tons**; DB and BE, 1·4 tons; AB and FB, nil. 17. 78 lb. 18. 47·5 lb. 19. 63 lb.
20. AB, 1·1 tons; BC, **2·1 tons**; CD, **0·4 tons**; DA, **1·75 tons**; BD, 1 ton. 21. AB, **4·2 tons**; BC, **2·8 tons**; AF, 3·75 tons; BF, **1·3 tons**; CF, 1·6 tons. 22. A, **55·5 tons**; B, 27·75 tons; C, 41·5 tons. 23. BG, **2·85 cwt.**; CH, **2·25 cwt.**; DJ, **4·5 cwt.**; GH, 1 cwt.; GF, 2 cwt.; HI, **2·25 cwt.**; IJ, nil; IF = JF, 4 cwt. 24. AD, **4·5 tons**; DC, **3·5 tons**; CB, **4·3 tons**; DE, **1·75 tons**; CE, 1·75 tons; AE, 3·9 tons; EB, 2·15 tons. 25. L.H., 333 lb.; R.H., 914 lb. 36° to vert. 26. L.H., 4,975 lb. 12° 40' to vert.; R.H., 4,017 lb. 26. A, 4,800 lb. 13° 20' to vert.; B, 4,200 lb. 27. A, 6,900 lb. 24° 24' to vert.; B, 4,380 lb. 28. A, 1,335 lb.; B, 1,060 lb., 15° 40' to vert. 29. 86·5 lb.; 260 lb.; 1,710 lb. 30. 37 lb.

EXAMPLES III

1. 19.25; 25.2 tons-ft. 2. 24; 40; 44; 32 tons-ft.
 3. 28.125 tons-ft.; 3.75 tons; 37.5 tons-ft.; 0 tons.
 4. 720.8 tons-ft. at 28 ft. from L.H. 5. 227.2 tons-ft. at 14.07 ft. from L.H. 6. 26.8 tons-ft. 7. 226,860 lb.-ft.: 13.75 ft. from L.H. 8. 76.85 tons-ft.; 11.23 tons. 9. 12.6 ft. from L.H.; 198,450 lb.-ft. 10. 40 tons-ft.; 6 tons.
 11. 50.7 tons-ft. 10.075 ft. from A; 13.925 tons at C. 12. 16 ft. from ends. 13. 36 tons-ft.; 5.6 tons. 14. 66.7 tons-ft.; $9\frac{2}{3}$ tons. (16.) 8 ft. from free end. (17.) 16 tons-ft.; 4 tons. 18.) 104 tons-ft.; 52 tons-ft. (19.) A, 3.55 tons; B, 2.38 tons; 22.6 tons-ft.; 3.216 tons.

EXAMPLES IV

1. (a) 2,520 ins.⁴; (b) 821 ins.⁴; (c) 318 ins.⁴; (d) 85 ins.⁴; (e) 20 ins.⁴; (f) 7.5 ins.⁴. 2. 213.4 ins.⁴; 27 ins.⁴.
 3. 0.25 tons/sq. in.; 15.5 lb./sq. in. 4. 1.43 tons/sq. in.
 5. 463 ton-ins. 6. 7.05 tons/sq. in. 7. 494 ton-ins.; 8.23 tons/sq. in. 8. 202.5 ton-ins.; 6.75 tons/sq. in.
 9. 76.8 ins.³. 10. 5,320 lb./sq. in. 11. 12.3 tons.
 12. 1.05 tons/sq. in.; 1.9 tons/sq. in. 13. 0.09 tons.
 14. 3 ins.; 315 ins.⁴; tensile 0.8 tons/sq. in.; comp. 2.36 tons/sq. in. 15. $3\frac{1}{4}$ ins. and $6\frac{1}{2}$ ins. 16. 13 ft. from L.H.; 990 ton-ins.; 7.1 tons/sq. in.
 17. (a) 4,032 lb./sq. in.; (b) 805 lb./sq. in. 18. 153 tons-ft.; 245 ins.³. 19. 1.6 tons. 20. 3.7 tons. 21. 13.89 tons.
 22. 10.67 ins. 23. 7.5 tons/sq. in.; 8,022 ins. 24. 65 ins. 12.44 lb. 25. 1,050,000 lb.-ins.; 15,217 lb./sq. in.; 9,857 ins.
 26. (a) 600 ton-ins.; (b) 10 tons/sq. in.; 6,750 ins.
 27. 29,600,000 lb./sq. in. 28. 20.8 lb.-ins.; 41,667 lb./sq. in.
 29. 0.072 ins. 30. 12,570 lb./sq. in. 31. 3.7 ins.
 32. $1\frac{1}{2}$ ins. $\times$ $3\frac{1}{2}$ ins. 33. 0.79; 0.21. 34. 4.4 ins.

EXAMPLES V

1. 2.88 ins. 2. 4,632 lb./sq. in. 3. 5,000 lb./sq. in.; 0.88°. 4. 5,485 lb./sq. in. 5. 3,328 lb./sq. in.
 6. 2,378 lb./sq. in.; 9°. 7. 4.15 ins. 8. 3.1 ins. 9. 2.52 ins.
 10. 10.8 ins.; 5,235 lb./sq. in. 11. 21,150 lb.-ins.; 2.3°. 12. 25.45 ton-ins.; 81.4. 13. 62,840 lb.-ins. 14. 1,784.
 15. 12,400,000 lb./sq. in. 16. 10,900 lb./sq. in. 17. 660; 5,234 lb./sq. in. 18. 9.08 ins.; 1.17. 19. 14.45 ins.
 20. 1,050; 10,400 lb./sq. in. 21. 69 lb. 22. 3. 23. $\frac{1}{4}$ in.
 25. 5:3. 26. 0.53 in.

EXAMPLES VI

1. 400 ft.; 16.88 ft./sec.². 2. 20 ft./sec.; 120 ft./sec.; 20 ft./sec.². 3. 1,491 ft. 4. 1,145.8; $2\frac{2}{3}$. 5. 34 ft./sec.; 24.2 ft./sec. 6. 9.5 tons; 532 tons. 7. 979; 68 m.p.h. 8. 30.2. 9. 18 tons. 10. (a) 4 min.; (b) 0.367 ft./sec.²; (c) 5.47 tons. 11. 10.285 ins. 12. 7.78 ft./sec.; 256:1. 13. 0.0354 rad./sec. 14. (a) 0.044 rad./sec.; (b) 0.0018 rad./sec.²; (c) 193 tons-ft.; (d) 10.36 tons. 15. 0.24 rad./sec.²; 0.35 ft./sec.². 16. 1.79 lb.-ft.². 17. 3,636 f.p.s. units; 2.04 ft.; 606. 18. 206 lb.-ft. 19. 644 lb.-ft.²; 7 lb. 20. 787.5 lb.-ft.; 49,482 ft.-lb. 21. 31 r.p.m. 22. 1,800 lb. 23. 5.7 r.p.m. 24. 20,186 ft.-lb.; 18.3. 25. 34.2 r.p.m. 26. 15 m.p.h.; 20.1 m.p.h. 27. 3.14 tons; 6.86 tons; $7^{\circ} 6'$. 28. (a) 45.5 m.p.h.; (b) 62.5 m.p.h. 29. $1v^2/\sqrt{g^2R^2 + v^4}$; 2.27 ins. 30. 159 lb.; 0.4 lb.-ft. 31. 16.3 lb./in.; 59 lb. 32. 3.8 lb. 33. 0.35 sec. 34. 0.6 sec. 35. 115. 36. 2,452 lb.; 1,734 lb. 37. 70.3 tons-ft.

EXAMPLES VII

1. 861.5 lb.; $108^{\circ} 26'$ to first radius; 25.3 lb. 2. 135° ; $124\frac{1}{2}^{\circ}$. 3. 44 lb.; 9 in. radius, $131\frac{1}{2}^{\circ}$ from 8 lb. 4. 1:1.8:2.35. 5. 3.28 ins. 6. 4.87 ins. 78° to 15 ins. 7. 153 lb.; $87\frac{1}{2}^{\circ}$. 8. 45 lb.; 30 lb. 9. 54 lb.; 36 lb. 10. 40 lb.; 20 lb. 11. 360 lb.; 240 lb. 12. 33 lb.; 9.5 lb. 13. 53.3 lb.; $112\frac{1}{2}^{\circ}$; $157\frac{1}{2}^{\circ}$. 14. 150 lb., $159\frac{1}{2}^{\circ}$ from 295 lb.; $152\frac{1}{2}^{\circ}$ lb. 149° from 245 lb. 15. 180 lb. 138° from 700 lb.; 390 lb. $198\frac{1}{2}^{\circ}$ from 1,000 lb.

EXAMPLES VIII

1. 112 m.p.h. 2. 64.8 m.p.h. 3. $41^{\circ} 24' N$ of E; 13.2 knots. 4. 9.2 knots; 5 sea miles. 5. 84.6 m.p.h.; 1.41 miles. 6. 52.9 m.p.h. 7. 3.36 ft./sec. 8. 17.4 ft./sec.; 17.7 ft./sec. 9. 15.1 ft./sec.; 12.9 ft./sec. 10. 2.6:1. 11. 170 ft./sec.². 12. 295 lb.; 336 lb. 13. 3,780 lb. 14. 69 lb. 15. (a) 746 lb.; 0.

EXAMPLES IX

1. 3,525 lb.-ft.; 121. 2. 13,000; 8,600; 4,600 lb.-ft. 3. 636 lb.-ft.; 132° ; 232° . 4. 7,000 lb.-ft. 5. 7,630 lb.-ft. 6. 9,720 ft.-lb.; 1.75. 7. 890 lb.-ft.; 42.4; 10,800 lb.-ft.². 8. 48,800 lb.-ft.; 307,200 ft.-lb.; 1,670; 0.026. 9. 0.16. 10. 4,120 lb.-ft.; 6,880 lb.-ft.; 25,800 ft.-lb.; 0.13.

EXAMPLES X

1. 5,260 lb.; 1,360 lb. 2. (a) 0.716 tons; (b) 0.966 tons.
3. (a) 1,349 lb.; (b) 1,473 lb. 4. 14.4 lb. 5. 1.5 lb.
6. 1,035 lb.-ins. 7. 3,900 lb. 8. $8\frac{3}{4}$ ins. 9. (a) 318 lb.;
(b) 1,225 lb. 10. 1,125 lb.; 603 lb. 11. $3\frac{1}{2}$ laps. 12. 9.
13. 4.64. 14. 54.2.

EXAMPLES XI

1. 23,360 lb.; 3.89 ft. above bottom. 2. 4,697 lb.;
0.17 ft. below centre. 3. 3,323 lb.; 3,653 lb. 4. 1,177 lb.;
1,346 lb. 5. 0.067 ft. below centre. 6. 25.9 lb. 7. 2.73.
8. 8. 9. 150. 10. 458,050 ft.-lb.; 34.3; 24.3 sec. 11. 141.5.

EXAMPLES XII

1. $C = 0.635$; $C_v = 0.984$; $C_c = 0.646$. 2. 0.618.
3. 0.619. 4. 52,520. 5. 0.618. 6. (a) 62.7 ft.-lb.;
(b) 35.3 ft.-lb. 7. 5.7 ft. 8. 433. 9. 12 lb./sq. in.
10. 19.78 ft./sec.; 140 c. ft./sec. 11. 10.8. 12. 0.39.
13. 159. 14. 0.011. 15. 0.0079. 16. 0.017. 17. 106.5 ft.
18. 76 ft. 19. (a) 172 ft.; (b) 570. 20. 84.6.
21. 50.6 ft./sec.². 22. 12.75 lb./sq. in. 23. 49.8 ft. 24. 59 ft.;
3.9. 25. 105.5 ft./sec. 2,720. 26. 2.77 ins.

EXAMPLES XIII

1. (a) 6.3 lb.; (b) 5.02 lb. 2. (a) 387 lb.; (b) 271 lb.
3. 9.48; 0.49. 4. 1.57 ins.; 2.12 ins. 5. 98.9 lb.
6. (a) 3,377 lb.; (b) 1,427 lb. 7. (a) 220 lb.; (b) 8.2.
8. 929 r.p.m.; 595. 9. 149 ft.-lb. 10. 145.9. 11. 234 ft./sec.;
144, 300 lb.; 86.1%. 12. 80.8; 92.6%. 13. 0.91; 50.4%.
14. (a) 107.4 ft./sec.; (b) 62,647; 0.5; 26.2%.
16. 6.9 ft./sec.; 3.2 f.p.s. units. 17. (a) 1,221 lb.; (b) 1,165.

INDEX

- Acceleration, angular, 76
 - of piston, 107
 - radial, 84
 - tangential, 106
- Accumulator, hydraulic, 142
- Angle of friction, 126
 - of twist of shaft, 67
- Angular acceleration, 76
 - momentum, 82
 - velocity, 76
- Axis, neutral, 56

- Balancing, 94
- Beam, bending moment on, 40
 - moment of resistance of, 57
- Bearings, friction in, 131
- Behaviour of materials in Testing Machine, 4
- Belt, friction of, 132
 - horse-power transmitted by, 134
- Bending moment diagram, 43
 - — on beams, 40
 - of beams, 53
 - — resistance to, 57
- Bernouilli's Theorem, 145
- Bucket, Dobbie, 171

- Cantilever, 43
- Centre, instantaneous, 104
 - of pressure, 138
- Centrifugal force, 83
- Coefficient of contraction, 146
 - of discharge, 147
 - of fluctuation of energy, 119
 - of friction, 127
- Coefficient of velocity, 147
- Collar bearing, friction of, 131
- Compressive strain, 2
 - stress, 2
- Connecting rod, weight of, 116
- Crank effort diagram, 116
- Critical velocity of water in pipes, 155

- Diagram, bending moment, 43
 - crank effort, 116
 - indicator, 113
 - piston effort, 113
 - shearing force, 43
 - stress-strain, 4
- Discharge, coefficient of, 146
- Dobbie bucket, 171
- Ductility, 4
- Dynamical equations, 77

- Efficiency, riveted joint, 14
- Effort, crank, 116
 - piston, 113
- Elasticity, 3
 - modulus of, 9
- Elastic limit, 3
- Elongation of test piece, 5
- Energy, fluctuation of, 119
 - of water, 144
 - of water due to pressure, 141

- Factor of safety, 12
- Fatigue of metals, 11
- Flow of water in pipes, 159
- Fluctuation of energy, 119
- Fluid friction, 88
- Flywheels, 121

- Force, 76
 — centrifugal, 83
 — diagrams, 30
 Forces, resultant moment of, 28
 Friction, angle of, 126
 — fluid, 88
 — inclined plane, 127
 — in journal bearings, 131
 — of belt on pulley, 132
 — of collar bearing, 131
 — of flat pivot, 130
 — of screw, 128
 — of water in pipes, 156
 Funicular polygon, 30
 Gas engine, fluctuation of energy in, 120
 Girders, moment of inertia of, 59
 — resisting moment of, 57
 Gradient, hydraulic, 157
 Harmonic Motion, Simple, 85
 Head, loss of, due to sudden contraction, 150
 — loss of, due to sudden enlargement, 149
 — lost through friction in pipes, 156
 — of water, pressure of, 137
 — of water, velocity due to, 145
 Helical springs, 70
 Hollow shaft, 69
 Hooke's Law, 4
 Hoop tension, 19
 Hydraulic accumulator, 142
 — gradient, 157
 — mean depth, 159
 — mean radius, 159
 Impact of jet on vanes, 166
 Impulse turbines, 166
 Inclined plane, 127
 Indicator diagrams, 113
 Inertia, 76
 Inertia curves, 116
 — force on reciprocating mass, 110
 — moment of, 79
 — rotational, 79
 Instantaneous centre, 104
 Jet of water, pressure of, 166
 Joint, riveted, 13
 Kinetic energy of a rotating body, 82
 — — of water, 144
 Klein's construction, piston acceleration diagram, 108
 Link polygon, 30
 Loss of head due to friction in pipes, 156
 — of head due to sudden contraction, 150
 — of head due to sudden enlargement, 149
 Materials, strength of, 21
 Metals, fatigue of, 11
 Modulus of elasticity, 9
 — of rigidity, 67
 — of rupture, 9
 — of section, 58
 Moment of a force, 28
 — of inertia, 59, 79
 — of inertia. Routh's rule, 81
 — of resistance to bending, 57
 — of resistance to torsion, 67
 Momentum, angular, 82
 Motion, simple harmonic, 85
 Neutral axis of a beam, 56
 Nominal stress, 5
 Notch, rectangular, 147
 — triangular, 148
 Orifice, flow of water through, 146
 Overshot water-wheel, 164

- Pelton wheel, 170
- Periodic time, 85
- Permanent set, 3
- Pipes, flow of water in, 151, 159
 - friction in, 156
- Piston acceleration, 107
 - effort diagrams, 113
 - velocity, 105
- Pivot, friction of, 130
- Plasticity, 3
- Polygon, funicular, 30
 - of forces, 29
- Potential energy of water, 144
- Power transmitted by belt, 132
- Pressure, centre of, 138
 - energy of water, 141
 - on plane immersed surface, 137
- Proof resilience, 10
- Pulley, belt friction on, 132

- Radius of curvature of a beam, 54
 - of gyration, 80
- Reaction turbine, 166
- Reciprocating parts, inertia of, 110
- Rectangular notch, 147
- Relation between Shearing Force and Bending Moment diagrams, 46
- Relative velocity, 101
- Repeated and reversed stresses, 11
- Resilience, 10
- Resultant moment, 29
 - pressure, 138
- Revolving hoop stress, 19
 - vane, 167
- Rigidity, modulus of, 67
- Riveted joints, 13
- Roof truss, 30
- Rope pulley, 133
- Rotational inertia, 79

- Routh's rule, 81
- Rupture, modulus of, 9

- Safety, factor of, 12
- Screw, friction of, 128
- Second moment of area, 58
- Shaft, angle of twist in, 67
 - hollow, 69
 - moment of resistance to torsion, 67
- Shafting, 66
- Shear strain, 2
 - strength of materials, 21
 - stress, 2
- Shearing force in beam, 40
 - — diagram, 43
- Shock, 13
- Simple harmonic motion, 85
 - stress and strain, 1
- Slider crank chain, 106
- Spring, helical, 70
- Strain, 1
 - compressive, 2
 - shear, 2
 - tensile, 2
- Strength, elastic, 3
 - of materials, 21
 - ultimate, 5
- Stress, 1
 - compressive, 2
 - due to bending, 53
 - in revolving hoop, 19
 - in thin cylinders, 17
 - repeated and reversed, 11
 - shear, 2
- Stress-strain diagram, 4
- Stress, tensile, 2
- Suddenly applied load, 10

- Tailrace, 165
- Tensile strain, 2
 - stress, 2
 - strength of materials, 21
- Tension test bars, 4
- Testing, compression, 8
 - shear, 8

- Testing tension, 4
— timber, 8
Thin cylindrical shells, 17
Torque, work done by, 82
Torsion of shaft, 67
Triangular notch, 148
Truss, roof, 30
Turbine, impulse, 166
— reaction, 166
Turning moment, work done
by, 82
Ultimate strength, 5
— — of materials, 21
Undershot water-wheel, 165
Vanes, pressure of jet of water
on, 166
Velocity, coefficient of, 147
Velocity of piston, 105
— of water due to head, 145
— of water in pipes, 159
— relative, 101
Vena contracta, 146
Venturi water meter, 153
Virtual centre, 104
— fall, 157
— slope, 157
Water, energy of, 144
— meter, Venturi, 153
— turbines, 165
— wheels, 164
Weir, flow of water over, 147
Wetted perimeter, 159
Working load, 12
Young's Modulus, 9

