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To NAOMI

PREFACE

PERHAPS in the social sciences more than in other fields of learning there is the ever-present danger of a schism between theory and practice—a dichotomy of ideas governing the activities of practitioners on the one hand and the intellectual models of the academicians on the other. The development of such situations—at least temporarily—is no doubt to be expected in any branch of study. But when the lack of intercommunication persists for long there arises the probability of both serious error in practice and unrelieved stagnation in theory. In the study of cost of living index numbers it appeared to me that a situation of this kind was perilously close at hand. My conviction was strengthened by experience during the war and immediate postwar period when I served as an economist in the U. S. Department of Labor and later the U. S. Department of Commerce, while at the same time teaching graduate classes at American University. Circumstances in this period high-lighted the gap between theory and practice—a gap which it is hoped this study will help to close.

To acknowledge one's intellectual debts is a pleasant duty, though likely to be interminable if completeness is required. References in the body of this work fulfill a part of this task. Special acknowledgment may be made here, however, of the more imposing—and more recent—of these obligations.

My serious interest in problems of economic measurement was first stimulated in the seminars of Professors Frederick C. Mills and Wesley C. Mitchell at Columbia University. The keen insights and critical comments of Professor Mills, who read the typescript of this study, and of Professor William Vickrey of Columbia, have done much to improve its clarity and significance. Professors Arthur F. Burns and Leo Wolman of Columbia also read the manuscript and provided numerous helpful suggestions. In Chapter 2, in which I found it necessary to touch upon the philosophical foundations of the approach to index number construction, the advice of Professor Sidney Hook of New York University proved an invaluable guide. It must, of course, be noted that whatever deficiencies remain in this work are the sole responsibility of the author.

M. J. U.

WASHINGTON, JUNE 1948.

TABLE OF CONTENTS

APPENDICES

CHAPTER I

INTRODUCTION

IT is the purpose of this study to explore the problem of measuring changes in living costs over time with a view to determining (1) an effective method for approximating these changes, taking into consideration the limitations of data actually or likely to be available; and (2) a method of estimating the reliability of such measurements. The statement of this objective, however, raises a question. For the subject of index numbers has often been discussed as a fully integrated unit, and the impression has lingered in many quarters that the fundamental principles therein developed are *generally* applicable—regardless of specific purposes.

The basic question raised is one concerning primarily the nature and rôle in scientific research of definitions—a topic discussed at some length in the following chapter. Thus, when we speak of a change in living costs on the one hand, or a change in industrial production on the other, is the economic content of the phenomena so different in the two cases as to require separate and specific analyses, or can objectives be defined in such significantly general terms that specific differences are confined to more or less negligible details? If the latter were true, a consideration of the problem of measuring *cost of living* indices would be unnecessary. The general problem of constructing index numbers would represent a sufficient approach to all particular cases.

It is the object of the following chapter, therefore, to demonstrate that (1) it is the definition of what is to be measured which ultimately limits the practical methods of approximation; and (2) such definitions are meaningful and scientifically useful only when (a) they are stated in operational terms and (b) they are framed in terms of the specific economic phenomena the measurable concept is known to comprehend. To define a concept in specific economic terms does not mean here that generalizations or "abstractions" are ruled out, for neither one is in fact avoidable. It does mean that the economic content of the concept con-

sidered must be faithfully preserved—for a concept which is economically meaningless will remain so, no matter how precise may be its mathematical expression.

But as already noted full discussion of this problem is reserved for the following chapter. It is relevant to state now, however, that one consequence of the present approach is that the results achieved will have only narrow applicability; they will relate specifically to measures of changes in living costs over time. It is true, of course, that much of the analysis will apply with but minor revision to the problem of measuring place to place variations in living costs, for the common attributes of both problems are considerably more numerous than their distinctions. Indeed, the basic theory of the two measures is essentially the same, although their practical settings give rise to both statistical and economic issues which in important respects require separate consideration.

On the other hand, it is anticipated that this study will have but little *direct* bearing on either the theory or construction of index numbers of other types—viz., wholesale prices, industrial production, wage rates, etc. For although underlying all statistical enterprises there are bound to be certain similar activities, such as collecting and classifying data, the objectives of each of these indices in economic terms are basically different.

One other aspect of the approach taken in this study is worthy of comment—viz., that the object is a *practical* one. This is not designed to convey, in the vulgar sense of the term, that "theory" is avoided. On the contrary, the entirety of this work is theoretical. But the purpose of this investigation is to arrive at conclusions which will enhance in the foreseeable future the usefulness, meaning, and accuracy of currently computed index numbers. This implies that consideration is confined to statistical possibilities which—in the light of our current knowledge and of data actually or proximately available—*can* be achieved. Negatively, it implies that much of the work on cost of living index numbers which is most interesting from the standpoint of the pure theory of consumer choice has been given only secondary

consideration. In this category are the pioneer efforts to obtain statistical identification for families experiencing equal real incomes in different situations, for the methods proposed in this connection are uniformly unadaptable to practical use in the construction of current time series on living costs under present conditions. Accordingly, although the practical limitations of these methods have been briefly noted in Chapter III, full description has been confined to Appendix A.

The "practical" approach also imposes an obligation to demonstrate how results achieved may be put to use and how assumptions made in theory may be reconciled with fact. This is the task undertaken in the final chapter, which in general attempts to elucidate the relevance and implications of the principal conclusions of this study which are presented in Chapter IV.

The nature of these conclusions may be indicated in summary. In the first place it is demonstrated that Laspeyre's formula, which is a highly desirable one for reasons of expediency, may also be accorded considerable theoretical significance when employed as an approximation of the "true" (theoretically defined) cost of living index. For it is shown that definite theoretical relationships may be established among the differences, $L - I_0$, $I_1 - P$, $I_0 - I_1$, and $L - P$, where L is Laspeyre's index; I_0 is the true index based on the real income level of the base period; I_1 is the true index based on the real income level of the given period; and P is Paasche's index. Furthermore, two methods are presented for estimating the magnitude of the difference to be expected in practice between Laspeyre's and the true index, and some evidence is presented to indicate that this difference—the "probable error" of Laspeyre's formula—is very small.

It should be noted here that these conclusions may be expected to hold only within certain limitations and only if the use of Laspeyre's formula is coupled with certain other rules of procedure employed in the actual construction of the index. These conditions, along with certain other implications of the results, are detailed in Chapter V. Suffice to note here that the conditions limiting these conclusions are not so restrictive as to vitiate their

importance for the purpose for which they were intended—i.e., for the practical month-to-month task of constructing cost of living index numbers over time.

Finally, for those who are skeptical of the modern mathematical theory of consumer's choice as developed by Pareto[1] and extended by Slutsky,[2] Hicks[3] and Allen,[3] it warrants mention at this point that considerable use is made in succeeding chapters of the notion of an indifference map and the principle of "diminishing marginal rate of substitution." Worth recalling, however, is the view of its proponents that these fundamentals of the theory are grounded directly in experience and are free of the assumption of the measurability of utility and of the comparability of the utilities of different persons.

1 Vilfredo Pareto, *Manuel d'economie politique* (Giard & Briere, Paris, 1909).

2 E. Slutsky, "Sulla teoria del bilancio del consumatore," G. d. E., July 1915.

3 R. G. D. Allen and J. R. Hicks, "A Reconsideration of the Theory of Value," *Economica*, XIV, 1934. J. R. Hicks, *Value and Capital* (Oxford University Press, Oxford, 1939).

CHAPTER II

THE PROBLEM OF DEFINITION

IT is a curious and paradoxical fact that the literature of index numbers is replete with extended discussions of practical methods of measurement, which are at the same time quite innocent of an intelligible circumscription of what is to be measured. This is true in particular of those works which adopt the "general" approach, as previously mentioned. An obvious consequence of the practice is numerous and persistent differences of opinion. For so long as questions remain undefined, answers may compete with impunity and preferences must remain matters of personal predilection. It is relevant and perhaps comforting to recall at this point that "science in general does not begin with precise measurement any more than with clear and exact ideas."[1] Hazy ideas as well as inexact measurements are then to be expected. Progress nonetheless rests on several props, and among them, on the chance that ideas proven crude or mistaken will ultimately be refined or replaced.

The key importance in the study of index numbers of the problem of definition has, of course, been recognized previously. Thus Keynes has complained that many analysts have not been "sufficiently clear" concerning "what they are trying to" measure.[2] Similarly, Frisch censures the frequent omission of statements concerning "the question to which an answer is sought."[3] Yet recognition of the problem serves only to expose further issues. For the authors whose work Keynes and Frisch deplored

1 Morris R. Cohen, *Reason and Nature* (Harcourt, Brace and Company, New York, 1931), p. 95.

2 John Maynard Keynes, *A Treatise on Money* (Harcourt, Brace and Company, New York, 1930), vol. I, p. 100.

3 Ragnar Frisch, "Annual Survey of General Economic Theory: The Problem of Index Numbers," *Econometrica*, IV, no. 1, January 1936, p. 10.

were surely convinced of the relevance of their own investigations. The questions probed, in their view, were at least as lucid as their answers. The core of the difference would appear, then, to turn upon the *content* of a definition required to provide a foundation for measurement.

This latter problem is dealt with at greater length below. For, as noted, the study of index numbers in general—and of cost of living indices in particular—may be approached in several ways, each capable of yielding distinctive conclusions. The choice of an approach in turn hinges primarily upon the problem of definition.

Requisites of a Definition

In general terms, the requirements of a definition designed for measurement offer little problem. None would deny that a definition must be "clear," or "meaningful" in the formal sense. With respect to the problem of economic index numbers, it may also be confidently agreed that a definition ought to be *economically* significant. But the actual implications of these requirements are another matter. When is a definition meaningful?

Fortunately, so long as a definition is destined for measurement there appears to be general agreement on this point, both in the philosophy of science and in scientific practice. Briefly, a concept is said to have meaning when it can be equated, at least in principle, to a unique set of operations.[4] This condition makes it possible for a definition to mean the same thing to all minds. While in practice approximations may differ, this condition provides a standard for appraising their individual relevance and accuracy.

It should be noted, however, that the "unique set of operations" required need not necessarily be physically realizable, for they may comprise what have been termed "mental experi-

4 See esp., Edwin Burtt, "Two Basic Issues in the Problem of Meaning and of Truth," in *Essays in Honor of John Dewey* (Henry Holt and Company, New York, 1929), pp. 71-75; and also P. W. Bridgman, *The Logic of Modern Physics* (Macmillan Company, New York, 1927), pp. 5-28.

ments."[5] Indeed, it may develop that because of practical difficulties it is not possible—given the current status of our knowledge and resources—to measure the concept at all. But so long as the operations have been objectively and uniquely defined, two consequences in particular are worthy of emphasis: (1) there will be no doubt concerning the object of practical approximations; (2) it will be possible to identify only *one* measurement, *in principle*, with a given definition.

For example, the height of a cliff may be defined in terms of the number of times a measuring rod of given size, under given conditions, might be placed end to end along a perpendicular erected from its base to a point level with its peak. In principle, a unique set of operations will thus have been established, although obviously they would qualify more as a "mental experiment" than as a literal program for action. In practice, a simple theodolite and a knowledge of trigonometric functions would provide approximations adequate for most purposes. Indeed, various methods might be adopted, depending in part on the degree of accuracy required for the problem in hand. Nevertheless, in the light of the definition given no doubt could exist concerning the object of approximation, nor would the reliability of any particular estimate be a matter merely for conjecture.

The second requirement of a definition is not so fundamental as the first, for indeed unless the meaning of a concept is clear its economic significance—or usefulness—cannot even be sensibly argued. Furthermore, it is certain that there are no sure stigmata for useful—as opposed to useless—tools, facts, or concepts in science. Nevertheless, there is a sense in which the distinction may be fruitful for present purposes.

Thus, if a concept employed for measurement is already an accepted component of the body of knowledge in the field in which it lies, or if it is *logically related* to such other accepted components, its economic significance—i.e., its usefulness in adding

5 Morris R. Cohen, *A Preface to Logic* (Henry Holt and Company, 1944), p. 58, and John Dewey, *Logic—The Theory of Inquiry* (Henry Holt and Company, New York, 1938), p. 303.

to the store of knowledge or in serving as a pivot for the winning of new knowledge—may *ipso facto* be considered as established. While it may be agreed that this is a sufficient condition, it is not argued here that it is a *necessary* one. Furthermore, it is acknowledged that the existence of the identities and logical relations referred to are not always easily determined. But neglecting the borderlines, application of this standard appears reasonably straightforward at the extremes, and an example should make this clear.

Thus, the level of employment, marginal utility, and a demand curve are all generally recognized concepts in economic theory. Definitions of these ideas designed for measurement would of course need to be stated in explicit operational terms, as indicated above. But the usefulness of these operational statements—for the purposes for which they were presumably devised—would depend on whether the essentials of the meaning of these concepts as found in theory had actually been preserved.[6]

The measurement of demand curves, for example, has been the subject of numerous investigations, perhaps the most ambitious of which are those of Schultz.[7] To define a demand curve in *operational* terms, however, requires what we have termed a mental experiment—as recognized and acted upon by Schultz in the work cited. This operational definition of a demand curve may be stated as follows:

Definition I. If all consumers of commodity A were visited by investigators at a given time of a given day, and confronted with the alternatives of buying this commodity at all possible different prices, then the summations, *at each of these prices*, of the quantities of this commodity which would have been purchased

6 Of course, there is no intention here to deny the possibility that the analysis of theoretical concepts for purposes of quantitative expression may result in their extension and refinement.

7 Henry Schultz, *The Theory and Measurement of Demand* (University of Chicago Press, Chicago, 1938). See also his *Statistical Laws of Demand and Supply with Special Application to Sugar* (University of Chicago Press, Chicago, 1928).

by these consumers would constitute the demand curve for commodity A. This definition may be summarized in the equation:

$$\Sigma x = X = f(p_x, p_{t1}, p_{t2}, p_{t3} \ldots R_t, T_t)$$

where X is the summation of quantities of commodity A purchased at each of the alternative prices assumed by p_x, while p_{t1}, p_{t2} . . . are the prices of all other commodities and R_t the size and distribution of consumers' incomes, at the time (t) of investigation, and T_t represents any other factors which might affect demand existing at that time.[8]

Now this definition is identical with the concept of a demand curve as it is recognized in the major body of economic theory, and it is clear that it refers to the relations between two variables —prices and quantities of commodity A—considered entirely apart from the influence of changes in all other relevant factors such as other prices and incomes. This latter fact, indeed, constitutes the core of the practical problem of estimation faced by Schultz and all other investigators, and numerous methods of approximation—with narrowly limited success at most—have been attempted. But this fact constitutes an integral part of the meaning of this concept as it has been employed in theory.

Now let us suppose that in place of Definition I above another definition is proposed, as follows:

Definition II. The regression obtained between prices and quantities of a commodity, as these prices and quantities are actually

8 It must be emphasized that a demand curve refers to the relation between prices and quantities existing at a *particular moment of time.* As such it denotes the *amount purchased* at alternative prices at that time. However, if Definition I were modified so that: (1) consumers were informed that each alternative price situation was an *enduring* one; (2) consumers were accordingly asked to make purchases for their usual consuming period (say, the week's supply of meat, the season's supply of suits, etc.), thereby eliminating the speculative element; (3) the investigators divided the purchase of each consumer by the usual consuming period noted, obtaining (say) a daily rate of consumption; and (4) these rates of consumption were summated for all consumers; then the modified definition would provide a schedule of instantaneous rates of consumption for the market as a whole at each alternative price.

observed over time, constitutes the demand curve for this commodity.

This definition has several serious defects, but initially attention should be directed to the fact that it has failed to prescribe a unique set of operations. For regressions may be computed by several methods, either of the two variables might be treated as "independent," while the data employed may consist (say) of daily, monthly, or annual observations. Decisions on such issues of method, experience shows, would affect results materially, but nowhere in the definition does a standard appear for appraisal. Let us suppose, however, that this defect is corrected and the definition is revised as follows:

Definition III. The linear least squares regression of quantities on prices of a commodity, as these quantities and prices are observed annually over time, constitutes the demand curve for this commodity.

It may now be concluded that Definition III is operationally meaningful.[9] It may even be granted that the operations therein prescribed will under some circumstances—for example, for some agricultural commodities during periods of fairly stable national income—provide rough approximations of demand curves. The definition is nonetheless a "useless" one as this term has been described above. For it has lost all contact in meaning with the original concept from which it presumes to derive. Indeed, employing this definition one would know, in economic terms, neither what one has found nor what one is seeking—unless Definition I were actually introduced *sub rosa*. Thus, following the prescribed operations literally, relations between prices and quantities for industrial commodities would be obtained, such as those secured

9 Strictly speaking, however, Definition III is not yet entirely intact even in this respect, for it would be necessary still to specify the *kind* of prices (whether spot prices, contract prices, or net realized prices, etc.) to be used, the kind of quantities (whether amounts produced or amounts sold), and certain other details. For simplicity, however, these other details are omitted.

by Moore,[10] which were demonstrably not even approximate demand curves, and indeed contradicted the salient characteristics of demand curves—here because of failure to eliminate the influence of fluctuations in income and other dynamic factors. But such measurements nonetheless would satisfy Definition III. Moreover, it is possible that demand curves might be approximated by methods other than simple linear least squares regressions based on annual observations, but these methods would not satisfy the definition. The important point is that Definition III —*as a definition*—lacks *economic* significance. By analogy this result is of considerable importance for the problem of defining index numbers.

Some Index Number Definitions

Fisher. The definition for index numbers formulated by Fisher,[11] for example, corresponds to Definition II above. In Fisher's view, the problem of constructing index numbers was a general one, entirely divorced from considerations of purpose. Accordingly, he contended that index numbers could be defined in general terms with nevertheless sufficient precision and significance to serve as foundation for measuring the composite of "relative changes in *any* group of magnitudes"—prices, production, wage rates, etc. An index number was defined in general

10 Obtaining positively sloping relations between prices and production of pig iron, Moore actually believed at first that he had discovered a new type of demand curve, contrasting with the "normal" type which he knew prevailed at least for agricultural commodities. He wrote: "Our representative crops and representative producers' good (pig iron) exemplify types of demand curves of contrary character. In the one case, as the product increases or decreases the price falls or rises, while, in the other case, the price rises with an increase of the product and falls with its decrease." See Henry L. Moore, *Economic Cycles, Their Law and Cause* (Macmillan, New York, 1914), p. 114. Moore was the outstanding pioneer in the quantitative study of demand, but his results in this case were obviously the consequence of faulty interpretation.

11 Irving Fisher, *The Making of Index Numbers* (Houghton Mifflin Company, Boston and New York, 1922).

simply as a weighted average of relatives.[12] Specifically, an index number of the "general price level" was a weighted average of price relatives. The only change required in the specific case was to provide a name for the variable of interest.

Now if we consider the latter definition, it may be noted at once that, like Definition II above, it fails to provide a unique set of operations. Prices might be weighted by base year weights, given year weights, or quantities derived in some other manner. Hence, each of an infinite number of different answers—many of them contradictory—would satisfy Fisher's question exactly.[13]

12 *Ibid.*, pp. 2-3.

13 As is well known, Fisher sought to escape indeterminacy through the derivation of "tests" to determine the "best" form of weighting system. But these tests were extraneous to his definition and furthermore their results were inconclusive. The "ideal" formula finally recommended by Fisher was selected from among those which passed both the "time reversal" and "factor reversal" tests primarily on grounds of simplicity.

It should be noted that these two tests, considered most important by Fisher, were essentially derived from analogy with the behavior of prices and quantities of individual commodities. They were, however, not the *only* tests that may have been so derived. The "circular" test advocated by Warren M. Persons—which the "ideal" formula does not pass—would appear to have at least equal claim to authority.

Of the 134 formulas tested by Fisher, 41 met the time reversal test, which may be formulated as follows. If P_{01} is the index number for the year 1 with the year 0 as base, and P_{10} is the index for the year 0 with the year 1 as base, then $P_{01}P_{10} = 1$.

Of the 41 formulas which met the time reversal test, 13 also satisfied Fisher's second chief criterion, the factor reversal test. This latter test requires that if P_{01} and Q_{01} are respectively indices of price and quantity change for the year 1 on the base year 0, then

$$P_{01}Q_{01} = \frac{\Sigma p_1 q_1}{\Sigma p_0 q_0}$$

Although the "ideal" formula passed both the time reversal and factor reversal tests, it failed in the circular test, which requires that $P_{0n} = P_{01}P_{12}P_{23} \ldots P_{n-1,n}$ for any value of n.

It may be noted that the very notion of applying tests in the manner of Fisher implies a "general" approach to index number problems—i. e., a search for a "best" formula regardless of the purposes to which it may be adapted. In the absence of this "general" approach, however, the "tests" may be employed in a different way—i. e., as revelatory of certain character-

That Fisher's definition provides no true guide to practice is related to the fact that it provides no tie to the *economic* concept of the general price level; no effort was made to express the *theoretical* concept in operational terms. To have done so, for one thing, would have required recognition of the fact that prices and quantities are functionally related—a complicating factor for price index numbers of all types.[14] That the task is not so obvious as to be taken for granted is attested by the work of Keynes,[15] who denies that the concept of a "general level of prices"—which Fisher presumably sought to measure—has any meaning at all. Like Definition II above, then, Fisher's definition is devoid of formal (operational) as well as economic meaning.

King. Definitions formulated by King[16] correspond to Definition III above. Thus King has suggested that each formula fulfills a different "purpose," and that in any given case this purpose—or definition—is obtained simply by verbalizing the operations implicit in the formula. We would use a simple average of price relatives if we wished to know the average price change for a group of commodities, assuming that equal amounts of money were spent on each commodity in the first period, and that the same *quantities* of these commodities were purchased in the second period as might have been purchased in the first under this as-

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istics of formulas which may in some cases be of interest even though "success" or "failure" is not taken as a necessary qualification for a particular method of measurement.

14 The fact that the price of any commodity must be properly considered as a function of the demand for that commodity and of the prices of all other commodities was first stated by Léon Walras in 1873 in his *Elements d'economie politique pure* (Lausanne and Paris, 4th ed., 1900), pp. V-Vii. This concept of a system of prices and quantities in complex mutual relationship has been eloquently described by Wesley C. Mitchell in his *Business Cycles. Volume I, The Problem and Its Setting* (National Bureau of Economic Research, New York, 1927), pp. 109-116, and was employed as an explicit frame of reference by Frederick C. Mills in his *Behavior of Prices* (National Bureau of Economic Research, New York, 1927).

15 *Ibid.*, Chapters 4-6.

16 Willford I. King, *Index Numbers Elucidated* (Longmans, Green & Company, New York, 1930).

sumption. Again, we would use Laspeyre's formula if we wished to determine the average price change for a group of commodities, assuming that the same quantities of these commodities had been purchased in the second period as had *actually* been purchased in the first.

Each of these "definitions," of course, is uniquely satisfied by the respective formula to which it refers. Like Definition III above, they are meaningful in the strictly operational sense. But like Definition III as well, they have failed to link operations to an *economic* concept. The possibility can of course be granted that useful results may arise from the operations prescribed in some of King's definitions; after all, King would presumably fit a definition to every conceivable formula. But whatever usefulness may derive from measurements undertaken in response to King's definitions would arise from the fact that they *approximated measurements of concepts defined in some other way*, just as in some cases the operations of Definition III above might provide approximations of the theoretical formulation promulgated in Definition I.

Stated alternatively, King's definitions accurately describe the assumptions implicit in various index number formulas. But any practical investigation involves the question of *which* assumptions are best suited to a particular purpose—say to the measurement of changes in living costs. An intelligible reply further requires a statement of what is meant by a change in the cost of living—which, of course, represents the *actual* problem of definition. For this problem King's formula descriptions are irrelevant.

The Diversity of Definitions

The fact that Fisher contended that there was one "best" formula for all possible purposes and that King responded that each formula had its purpose, has in the past tended to highlight the contrast between their approaches. The foregoing discussion, on the other hand, directs attention to an important similarity. Neither King nor Fisher saw the necessity for denoting—in *economic* terms—the objectives of index number construction.

Moreover, since King and Fisher are by no means alone in their views,[17] it is apparent why both Keynes and Frisch, as previously noted, were struck by what often appeared to be pointless discussion in index number literature—by what even suggested at times the absurd reversal of answers seeking questions.

With respect to the present study, it follows from the foregoing discussion that scientifically fruitful definitions must be framed in terms of the particular economic concept it is desired to measure. Since the concept of a change in living costs is not the same as that of a change in wage rates or in the level of production, the need for different definitions for different types of index numbers is apparent. A *specific* definition of this kind, stated in the following chapter, provides the foundation for this study. That it is framed in operational terms follows as a necessary consequence of the explicit intention to employ it for purposes of measurement.

17 See, for example, John D. Black and Bruce D. Mudget, *Research in Agricultural Index Numbers* (Social Science Research Council, New York, 1938).

CHAPTER III

THE THEORY OF THE COST OF LIVING INDEX

History and Definition

ALTHOUGH wholesale price index numbers had for nearly fifty years[1] enjoyed an accepted place in economic and business literature, it was not until near the close of World War I that there were established regularly published indices purporting to measure changes in the cost of living.[2] Earlier work had apparently been discouraged by the difficulties of securing a representative sample of retail prices and—above all—of securing prices for consumer goods of standard quality over time. But during World War I these obstacles were overcome—or ignored—almost simultan-

1 The first regularly published index of wholesale prices was that of the *London Economist*, begun in 1869. At the outbreak of World War I there were seven regularly maintained wholesale price indices in the United States alone. See Wesley C. Mitchell, *The Making and Using of Index Numbers* (Bulletin No. 656, U. S. Department of Labor, Washington, 1938), p. 8. This bulletin was originally published in 1915 as Part I of Bulletin 284 of the U. S. Bureau of Labor Statistics.

2 It is interesting to note, however, that the first index number known to have been computed in America was *intended* to serve as a measure of changes in the cost of living, but this index was relatively short-lived and did not actually represent an effort to measure changes in living costs *directly*. This was the index begun in Massachusetts in 1780 and designed to provide a basis for equitably adjusting the payment of soldiers in the light of a currency which had been depreciating rapidly throughout the War for Independence. Only four commodities were included, and the index was described as showing changes in the value of "Five Bushels of Corn, Sixty-eight Pounds and four-seventh Parts of a pound of beef, Ten Pounds of Sheep's Wool, and Sixteen Pounds of Sole Leather." This description has led some (*cf.* Irving Fisher's *The Making of Index Numbers*, p. 458) to believe that the index was of the weighted aggregative form. Actually, the quantity "weights" referred to in the description were simply the quantities of these commodities which could have been purchased at base year prices, respectively, for 20 shillings. The formula actually employed was a simple average of price relatives. (See Willard C. Fisher, "The Tabular Standard in Massachusetts History," *Quarterly Journal of Economics*, XXVII, No. 3, May 1913, pp. 417-451).

eously in the major countries of the world in response to a common and powerful stimulus: a sharp and general advance in prices, labor unrest, and industrial conflict threatening in some cases to halt production of munitions and effective prosecution of the war.

Between 1916 and 1918 regularly published indices were developed in Great Britain, Norway, Denmark and the United States.[3] Preliminary work had also been undertaken elsewhere, and by 1921 there were an additional thirteen nations similarly equipped, including Australia, Austria, Belgium, Canada, Finland, France, Germany, India, Italy, New Zealand, Poland, South Africa, and Sweden.[4] Today from Iceland to Panama the construction of cost of living index numbers, usually by Government agencies, has become a virtually universal practice.

Since those responsible for constructing such measures are naturally interested in making known, literally, what their products are, it has become common in official publications to describe cost of living index numbers purely in terms of the formulas by which they are computed. In all important cases the formula is Laspeyre's— $\frac{\Sigma p_1 q_0}{\Sigma p_0 q_0}$ —where p_1 represents prices in the given year and p_0 and q_0 prices and quantities in the base year. Thus, the U. S. Bureau of Labor Statistics index is officially described as showing changes in the cost of a fixed bill of goods composed according to the average purchases of urban wage-earners and lower-salaried workers in the period 1934–1936.

Such descriptions are of course necessary and truly informative. In the light of the discussion of the previous chapter, however, it

3 George E. Barnett, "Index Numbers of the Total Cost of Living," *Quarterly Journal of Economics*, XXXV, no. 2, Feb. 1921. Counted as "cost of living" indices here were only those which were composed of at least three of the following groups of workers' expenditures: food, rent, clothing, heating and lighting, and miscellaneous. Thus, retail prices of foods *alone* had been available in the United States since 1903.

4 International Labour Office, *International Labour Review*, VI, no. 1, July 1922, p. 53.

need not be emphasized here that labels of this kind by no means represent *definitions* from a scientific point of view. For the definition of a cost of living index would indicate the *object* of approximation, not the *particular method* of approximation itself. Indeed, whether an index computed by Laspeyre's formula can even be considered an approximation of a change in the cost of living, has been seriously questioned by previous students of this subject—but this is a question which will be examined at some length in subsequent sections.

It is interesting to note that the underlying idea of a measure of a change in living costs actually preceded by more than two centuries the first systematic attempt to construct one. Writing in 1707, William Fleetwood, Bishop of Ely, set himself the task of determining the relative difference in money income which would provide for a student at the University of Oxford the "same Ease and Favour" in his day and 260 years before.[5] His actual approach to the problem of measurement was of course crude, consisting primarily of a "mental average" of price relatives,[6] but his *definition* nonetheless was in essence the same as that adopted by modern theoreticians. Thus Staehle declares that "the problem of cost of living comparisons may most appropriately be described as that of determining money incomes which yield equivalent satisfaction in two or more situations."[7] Frisch defines the cost of living index as the ratio between money incomes of persons who are equally "well off."[8] Keynes, Bowley,

5 *Chronicon Preciosun: or, an Account of English Money, the Price of Corn, and Other Commodities, for the Last 600 Years—in a Letter to a Student in the University of Oxford* (London, 1707).

6 For a convenient precis of the Bishop's book see Wirth F. Ferger, "Historical Note on the Purchasing Power Concept and Index Numbers," *Journal of the American Statistical Association*, XLI no. 33, March 1946, pp. 53-57.

7 H. Staehle, "A Development of the Economic Theory of Price Index Numbers," *Review of Economic Studies*, XI, no. 3, June 1935, p. 163.

8 Ragnar Frisch, "Annual Survey of General Economic Theory: the Problem of Index Numbers," *Econometrica*, IV, no. 1, January 1936, pp. 11-13. It may be noted that Frisch was apparently of the opinion that this definition

Allen, Haberler, and others present equivalent definitions. From the standpoint of operations the definition calls for either of two (equivalent) procedures: (1) identifying persons (or groups of persons) enjoying equal real incomes in two different situations and comparing their money incomes; or (2) identifying two commodity incomes which yield equal real incomes in two different situations and comparing their money cost. The meaning of terms employed here and in the following sections, however, requires some explanation. Money income is used throughout this work, unless otherwise specified, as synonymous with money expenditures. Commodity income refers to the goods and services purchased with money income. Real income refers to the level of satisfaction—or the income of utility—experienced from these goods and services. Two commodity incomes are considered equivalent when they provide equal real incomes.[9]

Amplification of the Definition

The definition above has thus far been stated in summary form only. Amplification is required before application in either theory or practice is possible. It should be noted, for example, that a change in the cost of living is meaningful only when applied to a *particular* real income level. Indeed, in principle there are as many changes in living costs as there are distinguishable real

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was *generally* applicable. Thus he wrote: "This definition seems, indeed, to be the only plausible one. It is applicable not only to cost of living indices, but equally well to general indices of deferred payments, wholesale prices, etc." In this, however, Frisch appears to be wholly mistaken. The definition above is framed with reference to *consumers*, who enjoy a given real income level, possess roughly the same taste patterns, and are presumably endeavoring to maximize their utility. Wholesalers—those who pay "wholesale" prices—are not consumers (in their capacity as wholesalers), they endeavor to maximize their profits, and in most other respects are highly heterogeneous. It is possible, however, that a definition of a wholesale price index might be *derived* from the one above.

9 The distinctions made here follow those introduced by Gottfried Haberler in his *Der Sinn der Indexzahlen* (Tübingen, 1927), p. 81, and followed also by J. M. Keynes in his *Treatise on Money* (Harcourt, Brace and Company, New York, 1930), p. 97.

incomes. In practice the change in living costs for a given (usually average) real income level may be representative of the change for a considerable positive and negative margin about that average. But at the same time it is obvious that relative changes in living costs would most likely differ substantially for a $100,000-per-year business executive and a $3,000-per-year cutter of coats and suits.

It must also be noted that changes in the cost of living—i.e., in the cost of maintaining a given real income—may be attributable to a variety of factors. The most obvious—and, over the short term, by far the most important—is prices. The other factors may be grouped under the two headings: tastes and environment.

Tastes

Thus, the money cost of a given level of real income in two situations in which prices are precisely the same might vary widely if *tastes* differ materially between the two situations. Indeed, persons may consume precisely the same commodities and yet experience widely different degrees of satisfaction if their attitudes toward these commodities, i.e., the standards of living to which they are accustomed, are at variance. Under such circumstances, obviously, commodities which are physically identical may, in terms of actual social experience, be palpably unlike. The choice luxury in one situation—say the "honey ant" for the aborigines of Australia—may elicit disgust in another.

Indeed, the very meaning of "equivalent commodity incomes" when tastes differ is open to question. On the other hand, given constant tastes, the operational test for this concept is simple. Two commodity incomes are equivalent when there is an indifference of choice between them. But this test implies that the choice be made with reference to a single—i.e., a given—system of wants. When tastes differ the problem becomes indeterminate in principle, for equivalence or lack of equivalence of two commodity incomes might be judged differently, depending upon which system of wants was employed as point of reference.

Environment

The cost of a given level of living may also be influenced by environment—a term used here to embrace especially the variety and quality of goods offered for sale, but also such factors as climate, social services available, and geographical location. Thus the abrupt and enforced limitations on the character of goods marketed in the United States during World War II would have resulted in an appreciable change in the cost of maintaining a given level of living, even if prices had remained unaltered. Similarly, differences in climate necessitate different types of expenditures to maintain a given state of well being.

In the following theoretical discussion it will be assumed that both tastes and environment are constant. For reasons already noted, it is believed by the writer that the former is a necessary assumption if the term "change in living costs" is to be conceded a definite meaning. The latter assumption is made for convenience of analysis only. The practical implications of both assumptions are discussed in the concluding chapter.

The Basic Theory of Cost of Living Index Numbers

In theoretical literature the concept of a cost of living index as defined in the preceding section, has been referred to as the "true" cost of living index. As already noted, all the principal measures of changes in living costs currently published are computed by Laspeyre's formula—and indeed the use of this formula, or one similar to it, would appear to be dictated by the nature of the data now available or likely to be available in the foreseeable future. Expediency alone, however, is a patently deficient justification. The persistent and all-important issue concerns the question of whether any warrant exists in theory for viewing Laspeyre's index as an approximation of the true measure of changes in living costs.

The basic theory bearing on this question—the bulk of which was developed originally by Konüs and Haberler—will be pre-

sented in this section.[10] It will be noted, however, that from the standpoint of the practical problem of providing a current measure of changes in living costs, the net conclusion of these investigations is discouraging. More promising in the light of this practical objective is the further development of the theory presented in the following chapter.

Under the Assumption of Fixed Real Incomes

For ease of exposition, and also to make possible two-dimensional graphic representation, it will be assumed that there are only two goods on the market—X and Y. It will be assumed further that consumers tend to maximize their satisfaction, so that at any time a dollar's worth of any commodity yields the same utility as a dollar's worth of any other commodity, as indicated by the expression:

$$\frac{1}{p_{0x}} \cdot \frac{\partial U}{\partial q_{0x}} = \frac{1}{p_{0y}} \cdot \frac{\partial U}{\partial q_{0y}},$$

where p_{0x}, q_{0x} and p_{0y}, q_{0y} are prices and quantities of the commodities X and Y respectively at time o. And of course

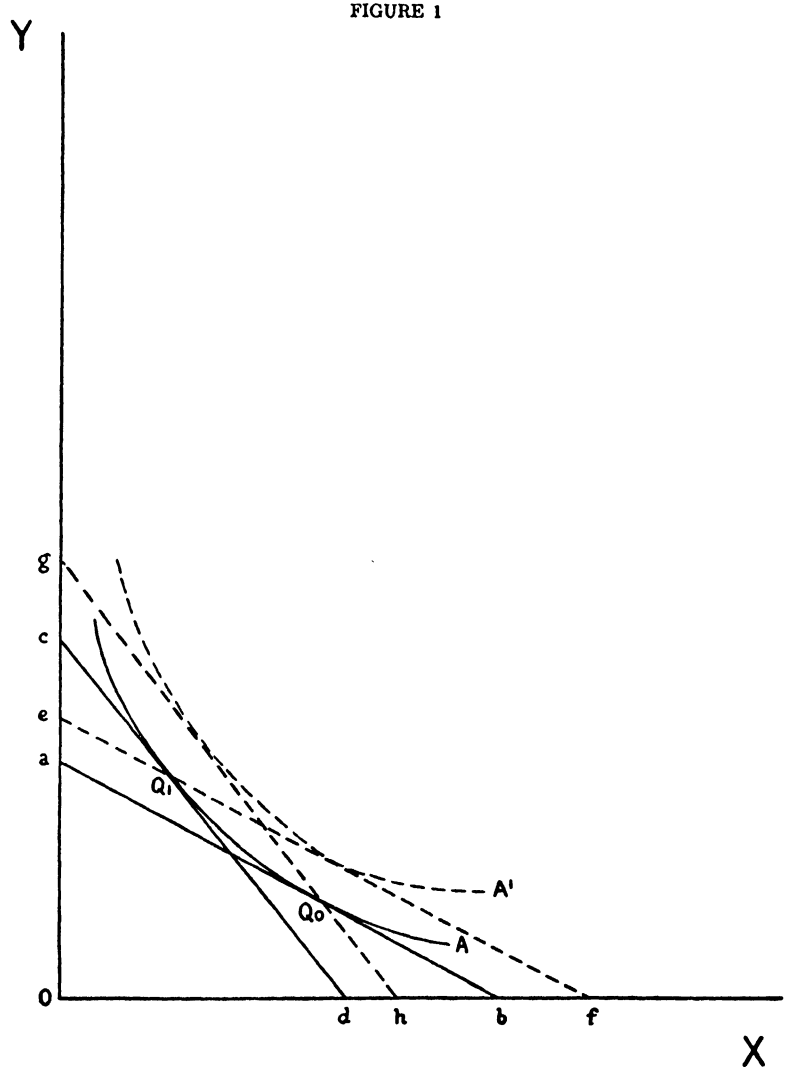
$$p_{0x}q_{0x} + p_{0y}q_{0y} = R_0$$

where R_0 is the total money income (or expenditure) at time o. The latter condition states only that expenditures are equal to the sum of the products of prices and quantities.

10 It has been pointed out by Staehle that at least some aspects of this theory appear to have been worked out initially by Pigou in his *Wealth and Welfare* (Macmillan and Co., London, 1912). See H. Staehle, "International Comparisons of Food Costs," *International Comparison of Cost of Living*, Part 1 (International Labor Office Studies and Reports, Series N, No. 20, Geneva, 1934), p. 75. However, Frisch calls attention to certain material differences between Pigou's approach and the theory subsequently developed by Konüs in "The Problem of the True Index of the Cost of Living," *The Economic Bulletin of the Institute of Economic Conjuncture*, Moscow, No. 9-10, September-October, 1924, pp. 64-71; and by Haberler, *op. cit.* See Ragnar Frisch, "Annual Survey of General Economic Theory: The Problem of Index Numbers," *Econometrica*, IV, no. 1, January 1936, p. 22.

In Figure 1 let the indifference curve A shown represent a part of the indifference map of a suitably defined group of wage-earners sharing the same tastes. The indifference curve represents the preference relations between these two commodities at a given level of utility. Thus by definition every point on this curve

FIGURE 1



represents a bundle of goods of equivalent satisfaction. It is further assumed that the utility function, or preference indicator, increases directly with the quantities of these goods consumed, so that curves relating to superior real income levels could be shown higher on the X-Y scales arranged in ascending order of preference. The "normal" form of the indifference curve requires that they be convex to the origin.

Lines *ab* and *cd* in Figure 1 are expenditure lines of the form $p_x q_x + p_y q_y = R$, tangent to the indifference curve at the points Q_0 and Q_1 respectively. Now the point of tangency of an expenditure line to an indifference curve indicates that combination of purchases which will maximize a consumer's utility at the given level of money expenditure and the given prices. Since indifference curves are convex and non-intersecting, an expenditure line can be tangent to only *one* curve. Since the level of preference rises as we move upwards to the right, the preference level at the point of tangency will be higher than at any other point on the expenditure line.

If commodity Y in Figure 1 is taken as the *numéraire* so that $p_y = 1$ in all price situations, then the numerical value of the slope of an expenditure line will be given by $\frac{p_x}{p_y} = p_x$, and the total income for any line can be obtained from the Y-intercept: i.e., $R = q_y$. Accordingly, it is evident from the figure that if we indicate the true cost of living index by the symbol I, we may write:

$$I = \frac{R_1}{R_0} = \frac{p_{1x}q_{1x} + p_{1y}q_{1y}}{p_{0x}q_{0x} + p_{0y}q_{0y}} = \frac{\Sigma p_1 q_1}{\Sigma p_0 q_0} = \frac{oc}{oa},$$

where R_1 is the total value of the goods consumed at the situation Q_1 and R_0 is the corresponding value for the situation Q_0. This relation simply reiterates the definition of the index as the ratio of the money cost of two equal real incomes.

Now in the same figure the expenditure line *ef* is drawn parallel to *ab*, and through the point Q_1. Since the slope of an expenditure line is given by the ratio of the prices, the equality of slopes shows

that *ab* and *ef* are characterized by the same price system. Moreover, since *ef* passes through Q_1, its intercept, *oe*, indicates the total expenditures required to buy the commodities actually purchased in Q_1 under the price system prevailing in Q_0: that is, $oe = \Sigma p_0 q_1 = R_1{}^0$.

Clearly, however, a rational consumer with the income $R_1{}^0$ would not buy the budget of Q_1, for it is evident that satisfaction would be maximized by moving along the line *ef* to the point at which it touches the higher indifference curve A'. It follows, therefore, that the income $R_1{}^0$ must be higher than the income R_0, for while the prices underlying these two expenditures are the same, the former income can buy a combination of X and Y giving greater satisfaction than does Q_0. In any event, it is evident from the figure that $oe > oa$.

In the same way, if *gh* is drawn parallel to *cd* through the point Q_0, we may write $og = \Sigma p_1 q_0 = R_0{}^1$, representing total expenditures required to buy the commodities actually purchased in Q_0, under the price conditions of Q_1. It is likewise evident in this case that $R_0{}^1$ must be greater than R_1, for at the point of equilibrium the former income can, at the same prices, buy a combination of X and Y that will provide greater satisfaction than Q_1. It is also clear in the figure that $og > oc$. It follows, therefore, that:

$$\frac{og}{oa} > \frac{oc}{oa} \qquad \text{and}$$

$$\frac{oc}{oe} < \frac{oc}{oa} \qquad \text{since } oe > oa.$$

Substituting for these intercepts the expenditures they represent, we obtain:

$$(3.1) \qquad \frac{\Sigma p_1 q_0}{\Sigma p_0 q_0} > I > \frac{\Sigma p_1 q_1}{\Sigma p_0 q_1} \qquad \text{or}$$

$$L > I > P$$

This relationship—that Laspeyre's index is the upper limit and Paasche's index is the lower limit of the true cost of living index—has been accorded considerable weight by Keynes, Bort-

kiewicz, and Allen.[11] It is occasionally referred to in research documents as a guide to practice.[12] Yet, as initially pointed out by Staehle,[13] the principal assumption upon which this relationship is based relieves it of any practical import. For these limits hold only for a very special case—that for which *real incomes are the same* in the two situations compared. But if it were known that real incomes were the same, the construction of "limits" would be superfluous; the true index could be computed exactly from the ratio of the money cost of the two real incomes. Relationship (3.1) in itself, then, is of no practical moment. It does, nevertheless, constitute a step in the derivation of other and more useful relationships.

Allowing for Variation in Real Incomes

It has already been noted that if the situations to be compared are characterized by *different* real incomes, then the definition of a cost of living index implies that there will be more than one true index—indeed, one for each distinguishable real income level. This, of course, is the condition actually encountered in practice and the one which offers serious difficulty. Attention is therefore called to Figure 2, where the problem is posed of comparing the change in living costs between the situation Q_0 on indifference curve A and the situation Q_1 located at a higher preference level. The expenditure lines *ab* and *cd* are drawn tangent to the indifference curves at these points.

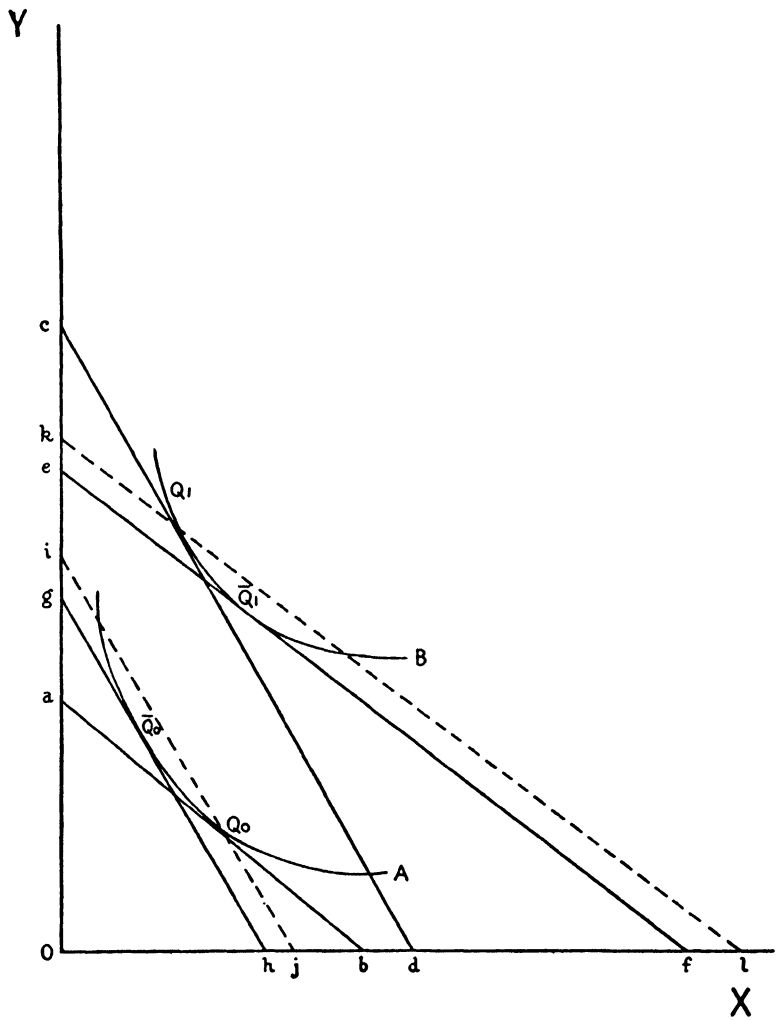
11 John M. Keynes, *op. cit.*, pp. 109-111; L. von Bortkiewicz, "Die Kaufraft des Geldes und ihre Messung," *Nordic Statistical Journal*, IV, 1932, pp. 1-68; R. G. D. Allen, "On the Marginal Utility of Money and its Application," *Economica*, no. 40, May 1933, pp. 186-209.

12 Thus the relationship, $\Sigma p_1q_0/\Sigma p_0q_0 > I > \Sigma p_1q_1/\Sigma p_0q_1$, is explicitly taken as generally applicable in Henry Shavell, "Price Deflators for Consumer Commodities and Capital Equipment, 1929-1942," *Survey of Current Business*, May 1943, p. 14.

13 "A Development of the Economic Theory of Price Index Numbers," *op. cit.*, p. 170.

Employing essentially the same methods as before, the line *gh* may now be drawn parallel to *cd* but tangent to the *indifference curve A* at point $\bar{Q}_0$. Then the point $\bar{Q}_0$ represents those quantities of X and Y which would be purchased at the indifference level A under the price system actually prevailing in Q_1, for *cd* and *gh* are parallel. Hence, the ratio of the total expenditures in $\bar{Q}_0$ to

FIGURE 2



those in Q_0 would indicate one true cost of living index; i.e., the change in money income required to provide under the price conditions of Q_1 a commodity income *equivalent in satisfaction* to that received in Q_0. If commodity Y is again taken as *numéraire*, then this true index may be represented by:

$$I_0 = \frac{\Sigma p_1 \bar{q}_0}{\Sigma p_0 q_0} = \frac{og}{oa}$$

Similarly, *ef* may be drawn parallel to *ab* and tangent to indifference curve B at the point $\bar{Q}_1$. Then the second true cost of living index—the ratio of money expenditures in Q_1 and in $\bar{Q}_1$—will show the change in money income between the price situations Q_0 and Q_1 required to maintain a real income level equal to that actually experienced in Q_1, thus:

$$I_1 = \frac{\Sigma p_1 q_1}{\Sigma p_0 \bar{q}_1} = \frac{oc}{oe}$$

These two true cost of living indices are, of course, quite distinct, and it is easily shown that they do *not* necessarily lie within the limits of Laspeyre's and Paasche's indices.

Thus, if the line *ij* is drawn parallel to *cd* and through the point Q_0, the total income characteristic of this line will represent that required to purchase the commodities actually consumed in Q_0 at the prices prevailing in Q_1: that is, $R_0^1 = \Sigma p_1 q_0 = oi$. It may then be observed that $oi > og$. Similarly, the line *kl* may be drawn through Q_1 parallel to *ab*, and it will be observed that $\Sigma p_0 q_1 = ok > oe$. Accordingly, we may write in summary:

$$\frac{oi}{oa} > \frac{og}{oa} \qquad \text{Since } oi > og, \text{ and}$$

$$\frac{oc}{oe} > \frac{oc}{ok} \qquad \text{since } ok > oe.$$

Substituting for these intercepts the expenditures they represent, we may conclude:

$$(3.2) \qquad \frac{\Sigma p_1 q_0}{\Sigma p_0 q_0} > \frac{\Sigma p_1 \bar{q}_0}{\Sigma p_0 q_0} \quad \text{and} \quad \frac{\Sigma p_1 q_1}{\Sigma p_0 \bar{q}_1} > \frac{\Sigma p_1 q_1}{\Sigma p_0 q_1}, \quad \text{or}$$

$$(3.3) \qquad L > I_0 \quad \text{and} \quad I_1 > P.$$

Considered alone, as other writers have contended, these relationships are of only very limited usefulness. Indeed, the conclusion would appear to be suggested that when account is taken of changes in real income from one situation to another, then Laspeyre's index can in no sense be considered an approximation of a *true* index, nor can Laspeyre's and Paasche's indices be employed *in conjunction* to any advantage. For (3.3) tells us only that L is at some point above I_0, and that P is at some point below I_1; the relationships between L and P and between I_0 and I_1 are entirely unrestricted.

In the following chapter, however, it is shown that more fruitful conclusions may be reached through further exploration of the nature of these relationships—conclusions calculated to illumine both conceptual and the practical quantitative aspects of the problem of constructing cost of living index numbers. Before describing this further development, it will be pertinent to note the nature of an alternative approach taken to the comparison of living costs.

An Alternative Approach

The measurement of changes in living costs by the method of price index numbers of necessity presumes the objective of identifying "equivalent" commodity incomes in two or more situations and comparing their money costs. It will be recalled that two commodity incomes are equivalent when they yield equal real incomes, and that the test for equivalence is the indifference of choice between them for persons of similar tastes. It is on this basis that the preceding analysis was conducted and that the further analysis in Chapter IV is pursued.

As mentioned previously, however, there is an alternative approach—that of identifying persons enjoying equal real incomes in different situations and comparing their money expenditures. That this objective would be fraught with numerous conceptual and statistical obstacles is obvious, but the contributions in this area are of importance aside from possible uses in the construction of cost of living index numbers. Thus, the work of Frisch, Staehle,

and Wald along these lines has substantially expanded the body of knowledge bearing on the theory of consumer choice and—in the case of Frisch—upon the concept and measurement of marginal utility.

Nevertheless, this alternative approach—which is essentially one of deriving cost of living indices from family budget data—is subject to several limitations which, in the present state of statistical and economic knowledge, would appear to preclude its practical application. Details of the various methods proposed which follow the "budget approach," along with the basic theoretical work of Konüs, are described in full in Appendix A. The practical limitations to which, in general, they are subject, however, are noted below.

In the first place, all the methods discussed here require for their computation detailed family budget data providing information on prices paid and quantities purchased by families classified according to income size. Data of this kind are of course now available for certain periods; in the United States the most recent complete compilation is for the years 1934–1936. It is possible also—and surely highly desirable—that such studies will be conducted more frequently in the future. But the budget approach requires these data for each date for which an index is constructed. Aside from the possible frequency of these surveys, it must be noted that the collection and subsequent compilation of family budget statistics comprise a long and arduous task. The lag between actual construction of the index and the period to which it referred, therefore, would inevitably be very substantial. Hence, the maintenance of a *current* measure of changes in living costs by means of the budget approach would, on this consideration alone, be precluded.

More fundamental, however, are the serious limitations upon the reliability of results imposed by assumptions inherent as a matter of statistical expediency in these methods. For it has already been noted that the underlying objective is the exceedingly challenging one of identifying families with equal real incomes in two different situations. Theoretical criteria for this equality are

by no means obvious, while problems involved in their statistical translation have thus far proved overwhelming.

Thus, in the method proposed by Frisch it is held that persons enjoy equal real incomes when the numerical value of their "money flexibilities" is equal.[14] Money flexibility refers to the relative change in the marginal utility of money corresponding to a small relative change in real income. Frisch's efforts to evaluate this criterion in statistical terms have been path-breaking—but still subject to an indeterminate and probably large margin of error. Thus, among the assumptions necessary to his procedure is that one "reference" commodity may be selected which has "independent utility": i.e., its utility is independent of the quantities of all other commodities consumed. Whether it is possible to discover a good answering this description is itself open to question. In any event, experiments thus far have shown that results differ substantially depending upon which commodity is chosen for reference.[15] Until this and related obstacles are surmounted, therefore, cost of living indices computed by this method would be subject to purely arbitrary variation.

14 Ragnar Frisch, *New Methods of Measuring Marginal Utility* (Mohr, Tübingen, 1932) and "Annual Survey of General Economic Theory: The Problem of Index Numbers," *op. cit.*, pp. 31-38.

15 For example, using meat, Frisch obtained the figure of 1.11, and using butter 1.40, for money flexibility. [See Hans Staehle, "Report of the Fifth European Meeting of the Econometric Society," *Econometrica*, V, no. 1, January 1937, p. 95.] Frisch later introduced a modification calling for an independent *subset* of commodities rather than one commodity—but in principle this left the nature of the problem unchanged while raising additional technical problems of measurement, for experiments with subsets have also yielded inconsistent results. See James N. Morgan, "Can We Measure the Marginal Utility of Money," *Econometrica*, XIII, no. 2, April 1945.

Thus, William Vickrey comments: "It is more than likely that the widely varying results obtained with different commodity combinations indicate that at most the independence criterion is satisfied only as between pairs of commodities or groups of commodities. If so, the use of different pairs of commodities would result in different utility functions even with complete and accurate information. Thus the widely divergent results obtained may not be entirely the result of the inadequacy of the data, but may be inherent in the method used." See "Measuring Marginal Utility by Reactions to Risk," *Econometrica*, XIII, no. 4, October 1945, p. 321.

This citation, however, was for illustrative purposes only, for the practical pitfalls in Frisch's as in the other methods of this type are several—a condition characteristic of explorations at the outer frontiers of knowledge in any field. Aside from such distinctive limitations, however, attention should be called to two assumptions common to all the methods proposed for deriving cost of living index numbers from family budget data.

As previously noted, the assumption of fixed tastes for a homogeneous group between situations compared is a necessary one for all cost of living comparisons. In constructing indices from family budget data, however, an additional assumption is necessary. This is the postulate that the families included in the budget statistics for any given point in time share common tastes despite substantial differences in income levels. Thus, in illustrating his own ingenious method Staehle was compelled to assume that German families with incomes of 700 R. M. per person had tastes the same as families enjoying incomes nearly five times as great.[16] Indeed, all these methods—either for purposes of fitting Engel curves or related measures—require comparable assumptions.

The vitiating character of this assumption becomes apparent when it is recalled that *similarity* of income has been frequently appraised as "undoubtedly . . . the best single criterion of similarity of taste."[17] For it is recognized that income itself is a taste-determining factor,[18] while perhaps most of the others—such as social status and cultural background—are closely associated with it.

It should be noted as well that the use of budget data for this

16 *Op. cit.*, pp. 183-187.

17 W. Allen Wallis and Milton Friedman, "The Empirical Derivation of Indifference Functions," in *Studies in Mathematical Economics and Econometrics in Memory of Henry Schultz*, ed. by Lange, McIntyre and Yntema (University of Chicago Press, Chicago, 1942), p. 184. See also James N. Morgan, *op. cit.*, pp. 138-140.

18 But this should not be construed as implying that the tastes of families in a *given* income class will necessarily change if the incomes of these families change *over time*. Reference above is to families in *different* income groups at a *given* time. For further discussion of this point, see below, pp. 61-63.

purpose implies certain other assumptions notably contrary to fact; e.g., that families in different income levels purchase the *same* brands of goods and pay the *same* prices.[19] In the light of these limitations it is obvious that the results of methods relying on family budget data are liable to errors which may be very substantial—and which at the present stage of technical development are not subject even to rough approximation.

19 For further discussion of the limitations of family budget data when employed for this and similar purposes, see R. G. D. Allen, "Expenditure Patterns of Families of Different Types," in *Studies in Mathematical Economics and Econometrics in Memory of Henry Schultz*, ed. by Lange, McIntyre and Yntema (University of Chicago Press, Chicago, 1942); James N. Morgan, *op. cit.*, pp. 131-141, and S. Morris Livingston, "Forecasting Postwar Demand: II," *Econometrica*, XIII, no. 1, January 1945, pp. 19-21.

CHAPTER IV

A PRACTICAL DEVELOPMENT OF THE THEORY

In the previous chapter there were derived the inequalities $L > I_0$ and $I_1 > P$—a relationship which has been generally construed as fatal to conventional approaches to the construction of cost of living index numbers. Nevertheless, it is worthy of emphasis that the evidence borne by this relationship alone must, from any point of view, be judged as inconclusive. For the ultimate key to the significance of either Laspeyre's or Paasche's formula is to be found only in the differences $L - I_0$, $I_1 - P$, $L - P$, and $I_0 - I_1$. If the differences between these indices were established as theoretically indeterminate, or if it were known from experience that they all were capricious or regularly very large, then the practical significance of both formulas would be effectively destroyed. Fortunately, both theoretical and quantitative considerations lead to a contrary conclusion.

The Relationship between the True Cost of Living Indices and the Formulas of Paasche and Laspeyre

Attention may be profitably turned in the first instance to the difference $I_0 - I_1$. Assuming for convenience of exposition the case of only two commodities, as in the previous chapter, these two true indices may be defined as follows:

$$(4.1) \qquad I_0 = \frac{\Sigma p_1 \bar{q}_0}{\Sigma p_0 q_0} = \frac{p_1' \bar{q}_0' + p_1'' \bar{q}_0''}{p_0' q_0' + p_0'' q_0''} \qquad \text{and}$$

$$(4.2) \qquad I_1 = \frac{\Sigma p_1 q_1}{\Sigma p_0 \bar{q}_1} = \frac{p_1' q_1' + p_1'' q_1''}{p_0' \bar{q}_1' + p_0'' \bar{q}_1''},$$

where p_0', p_0'', q_0', q_0'' and p_1', p_1'', q_1', q_1'' are the prices and quantities of the individual commodities purchased in the periods 0 and 1 respectively; and where $\bar{q}_0'$ and $\bar{q}_0''$ are the quantities which *would have been* purchased by consumers experiencing the real income level actually characteristic of the period 0, under the price sys-

tem of period 1, while $\bar{q}_1'$ and $\bar{q}_1''$ similarly are the quantities which would have been purchased at the real income level of the period 1, under the price system of period 0.

Now the quantities purchased at any time are of course functions of the prevailing prices and real income level, and therefore we may write:

$$(4.3) \quad q' = q'(p', p'', U) \qquad \text{and} \qquad q'' = q''(p'', p', U)$$

And since U represents the level of *real* rather than money income, we may further note that:

$$(4.4) \quad q' = f\left(\frac{p''}{p'}, U\right) \qquad \text{and} \qquad q'' = F\left(\frac{p''}{p'}, U\right)$$

Substituting (4.4) in (4.1) and (4.2) we obtain:

$$(4.5) \quad I_0 = \frac{p_1' f\left(\frac{p_1''}{p_1'}, U_0\right) + p_1'' F\left(\frac{p_1''}{p_1'}, U_0\right)}{p_0' f\left(\frac{p_0''}{p_0'}, U_0\right) + p_0'' F\left(\frac{p_0''}{p_0'}, U_0\right)} \qquad \text{and}$$

$$(4.6) \quad I_1 = \frac{p_1' f\left(\frac{p_1''}{p_1'}, U_1\right) + p_1'' F\left(\frac{p_1''}{p_1'}, U_1\right)}{p_0' f\left(\frac{p_0''}{p_0'}, U_1\right) + p_0'' F\left(\frac{p_0''}{p_0'}, U_1\right)}$$

It is then obvious that I_0 and I_1 may differ for *one reason only*. They *may* differ only if real incomes change between the two situations compared, for when $U_0 = U_1$, it is clear that $I_0 = I_1$. *Actual* differences, however, will occur only insofar as the change in real income results in altering the pattern of consumption between the two situations, relatively increasing (or decreasing) consumption of items which have advanced most in price, and relatively decreasing (or increasing) consumption of items which have advanced least. Thus, if the change in real income itself gave rise to *no* change in the pattern of consumption—i.e., if the elasticity of demand with respect to income were the same for *all* goods and services and equal to unity—then we could write:

$$(4.7)\qquad \frac{f\left(\frac{p_0''}{p_0'}, U_1\right)}{f\left(\frac{p_0''}{p_0'}, U_0\right)} = \frac{F\left(\frac{p_0''}{p_0'}, U_1\right)}{F\left(\frac{p_0''}{p_0'}, U_0\right)} = \frac{f\left(\frac{p_1''}{p_1'}, U_1\right)}{f\left(\frac{p_1''}{p_1'}, U_0\right)} = \frac{F\left(\frac{p_1''}{p_1'}, U_1\right)}{F\left(\frac{p_1''}{p_1'}, U_0\right)} = \lambda,$$

and substituting in (4.6), obtain:

$$I_1 = \frac{p_1'\lambda f\left(\frac{p_1''}{p_1'}, U_0\right) + p_1''\lambda F\left(\frac{p_1''}{p_1'}, U_0\right)}{p_0'\lambda f\left(\frac{p_0''}{p_0'}, U_0\right) + p_0''\lambda F\left(\frac{p_0''}{p_0'}, U_0\right)} = I_0$$

Similarly, if prices all changed by equal proportions between the two situations we could write:

$$(4.8)\qquad \frac{p_1'}{p_0'} = \frac{p_1''}{p_0''} = k, \qquad \text{and}$$

$$I_1 = \frac{kp_0'f\left(\frac{kp_0''}{kp_0'}, U_1\right) + kp_0''F\left(\frac{kp_0''}{kp_0'}, U_1\right)}{p_0'f\left(\frac{p_0''}{p_0'}, U_1\right) + p_0''F\left(\frac{p_0''}{p_0'}, U_1\right)} = k = I_0$$

Thus, differences between I_0 and I_1 are possible only if a change in real income has occurred between the two situations. Such differences will actually materialize only (1) if the change in real income succeeds in altering the pattern of consumption, and (2) if relative prices change. Let the letter D_i represent this difference between I_0 and I_1.

If attention is now turned to differences between L and P, it will be noted that these may occur for *two* distinct reasons. The first is the same as that which accounts for any difference occurring between I_0 and I_1. If the difference D_i occurred between I_0 and I_1, then L and P would also differ by D_i, except insofar as this difference might be diminished or enhanced by the play of a second factor noted below. Thus, if relative prices and quantities were independent variables—i.e., if the elasticity of demand with respect to relative prices were zero for all commodities—then L would equal I_0 and P would equal I_1. For under these circum-

stances quantities consumed are functions of the real income level alone, and we obtain:

$$(4.9) \qquad I_0 = \frac{p_1'\phi(U_0) + p_1''\Psi(U_0)}{p_0'\phi(U_0) + p_0''\Psi(U_0)} = L, \qquad \text{and}$$

$$(4.10) \qquad I_1 = \frac{p_1'\phi(U_1) + p_1''\Psi(U_1)}{p_0'\phi(U_1) + p_0''\Psi(U_1)} = P.$$

Hence, differences between L and P and between I_0 and I_1 would on this supposition be precisely the same, and in each case would be due to the possible effects of a change in real income upon the pattern of consumption. We have agreed to call this difference D_i.

As the previous example suggests, the second factor making for differences between L and P is the possible alteration in patterns of consumption attributable to changes in relative prices. It is that factor *alone* which accounts for the fact that L must be greater than I_0; the numerator of L is too large, as shown in Chapter III, because it assumes that consumers do *not* alter their consumption in response to relative price changes, buying relatively more of cheaper items and relatively less of more expensive items. It was also shown that for an analogous reason the denominator of P is too large, and P is therefore smaller than I_1. As already indicated, if the elasticity of demand with respect to relative prices were actually zero for all commodities, the difference between L and P would be D_i, the same as that between I_0 and I_1. Let us represent by the letter D_p the difference which occurs between L and P as the result of alterations in the pattern of consumption attributable to changes in relative prices. Then the *total* difference between L and P may be represented by:

$$(4.11) \qquad L - P = [(L - I_0) + (I_1 - P)] + (I_0 - I_1) = D_p + D_i = D_t$$

The quantity D_p, of course, is necessarily positive, since its effect can be *only* to make L greater than P. The quantity D_i, however, may be positive or negative, and may be numerically greater or less than D_p. Hence D_t may be positive or negative.

Now if the relationship (4.11) is combined with the inequalities $L > I_0$ and $I_1 > P$, it is easily demonstrated that limits may be derived for each of the two true indices, the practical importance of which is indicated below. Since $I_0 - I_1 = D_i$ and $L - P = D_t$, we obtain by substitution:

$$L > I_0 \quad \text{and} \quad I_0 - D_i > L - D_t$$

$$L > I_0 > L - D_t + D_i$$

$$L > I_0 > L - D_p \tag{4.12}$$

Similarly, we obtain for I_1 the limits:

$$P < I_1 < P + D_p \tag{4.13}$$

Application of the Relationship

The limits (4.12) and (4.13) are of practical interest only insofar as they are susceptible to quantitative expression. It is obvious, however, that a direct measurement of D_p is hardly feasible. Indeed, if it were possible to measure D_p directly, it would also be possible to measure I_0 and I_1 directly, and in that case preoccupation with limits, even as a first approach, would be pointless.

Direct observation of D_p would require, first of all, detailed data on prices and quantities relating to the total expenditures of an homogeneous group of families in two or more periods. From these data, of course, it would be possible to compute the Laspeyre and Paasche indices, $\frac{\Sigma p_1 q_0}{\Sigma p_0 q_0}$ and $\frac{\Sigma p_1 q_1}{\Sigma p_0 q_1}$. Then, employing first one of these periods as a base, it would be necessary to isolate those changes in quantities purchased between the two situations which had occurred *only* as the result of changes in relative prices; the effects of any other factors—particularly of a change in real income—upon the pattern of consumption would need to be eliminated. When this task had been completed it would be possible to compute $\frac{\Sigma p_1 \bar{q}_0}{\Sigma p_0 q_0} = I_0$. Employing the second period as a base—i.e., using the real income level of period 1 as a stand-

ard—these derived relationships between prices and quantities would also make possible computation of $\frac{\Sigma p_1 q_1}{\Sigma p_0 \bar{q}_1} = I_0$, and in combination with the other indices, a measurement of D_p.

But derivation of price-quantity relationships of this kind for each of the items in an entire budget is patently far beyond the realm of statistical possibility now and in the foreseeable future. Moreover, even if this enormous task could be accomplished in a particular case, the value of D_p thereby derived for one period would provide no clue to its value at any other time.

Two indirect approaches are available to this problem, however, which are not only much simpler but more useful. These are described below.

Approach I[1]

It follows from definitions previously given that when D_i is positive, then D_t is also positive and greater than D_p. Since D_t is easily measured, this fact suggests the possibility of estimating a probable upper limit for D_p—and therefore a quantitative evaluation of the reliability of both the Laspeyre and Paasche indices when used as approximations for the corresponding two true indices of changes in the cost of living.

Thus it will be recalled from (4.5) and (4.6) that D_i will differ from zero only if real income changes. Furthermore, the *sign* of D_i, in the case of any given observation, depends only upon the particular real income levels which happen to be involved in that comparison and upon which of these levels is chosen as the base. Given a particular choice of base, consecutive observations of D_i over a fairly long period of time would tend to yield an equal number of positive and negative values. This is apparent from Figure 3.

1 The results of this approach are substantially the same as those presented originally in my paper, "On the Economic Theory of Cost of Living Index Numbers," *Journal of the American Statistical Association*, XLI, December 1946, pp. 530-542. The supporting argument has been revised in several important respects, however, and proofs which were initially omitted have now been provided.

FIGURE 3

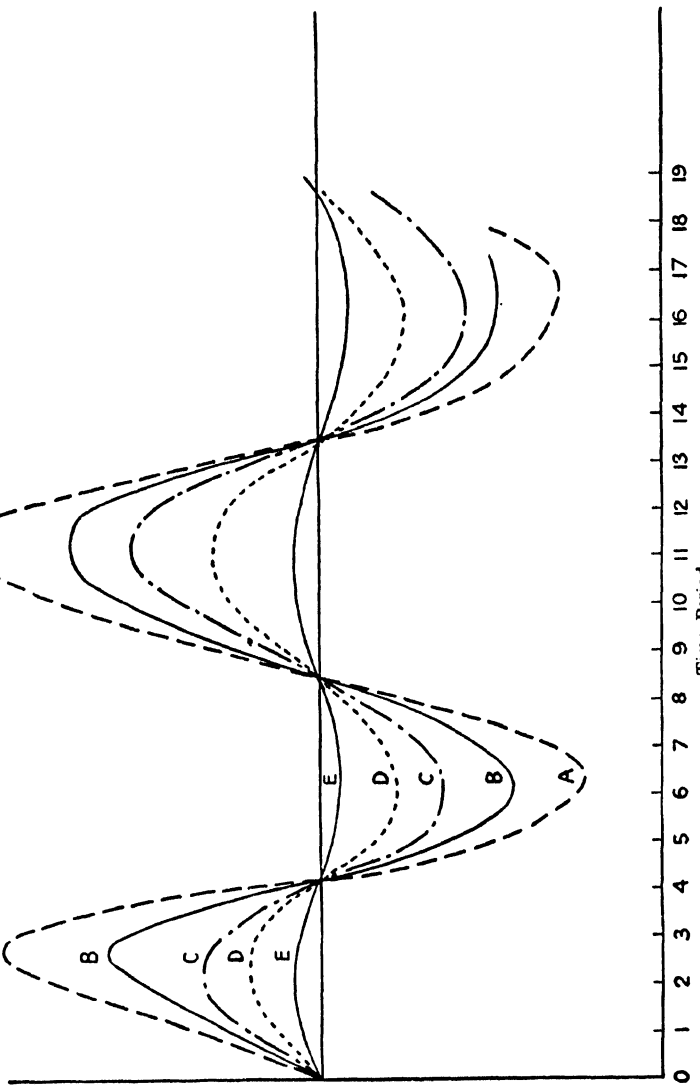

Cost of Living Index (Time 0 = 100)
100
A
B
C
D
E
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19
Time Period

Figure 3 illustrates changes over time in five hypothetical true cost of living indices, each applicable to a given level of real income—A, B, C, D, and E, respectively. It is assumed that these five levels of real income comprise the distinguishable levels which from time to time are actually experienced by the group—say urban wage-earners—for whom a cost of living index is being computed. Thus, in the pit of depression a very low level—say E—would be experienced, while in the best days of prosperity high levels—say B or A—would be the rule. It should be pointed out however, that the height of the curves in the figure do *not* indicate the height of the corresponding real income levels, but rather the magnitude of the true cost of living indices applicable to these levels computed on a base of time o = 100.

For simplicity it was assumed in the figure that the higher the real income level, the more flexible were changes in living costs. Obviously the probable differences in magnitude of true indices applicable to different real incomes was very greatly exaggerated for illustrative purposes—particularly when it is considered that the levels of real income have been restricted to those within the range of experience of urban wage-earners. Time o, taken as 100, is assumed to be an "average" period midway between the peak of prosperity and the trough of depression.

Now, if from time o to time 1 the level of real income for wage-earners rose from C to B, then it is apparent from the figure that $D_i = I_0 - I_1$ would be negative. For in that case I_0 would be the rise in living costs indicated by line C, I_1 the rise indicated by B, and since B is higher than C at time 1, time o = 100, $I_1 > I_0$. Similarly, if the real income level changed from time o to time 3 from C to D, then D_i would be positive. Conversely an alteration in real income level from C to B would result in a positive D_i, and from C to D in a negative D_i, if changes were measured from time o to, say, time 6.

On the assumptions of the previous discussion, then, it is apparent that given any particular base period (such as time o characterized by a real income level C), the chances of obtaining a positive or a negative D_i are clearly even. However, the same

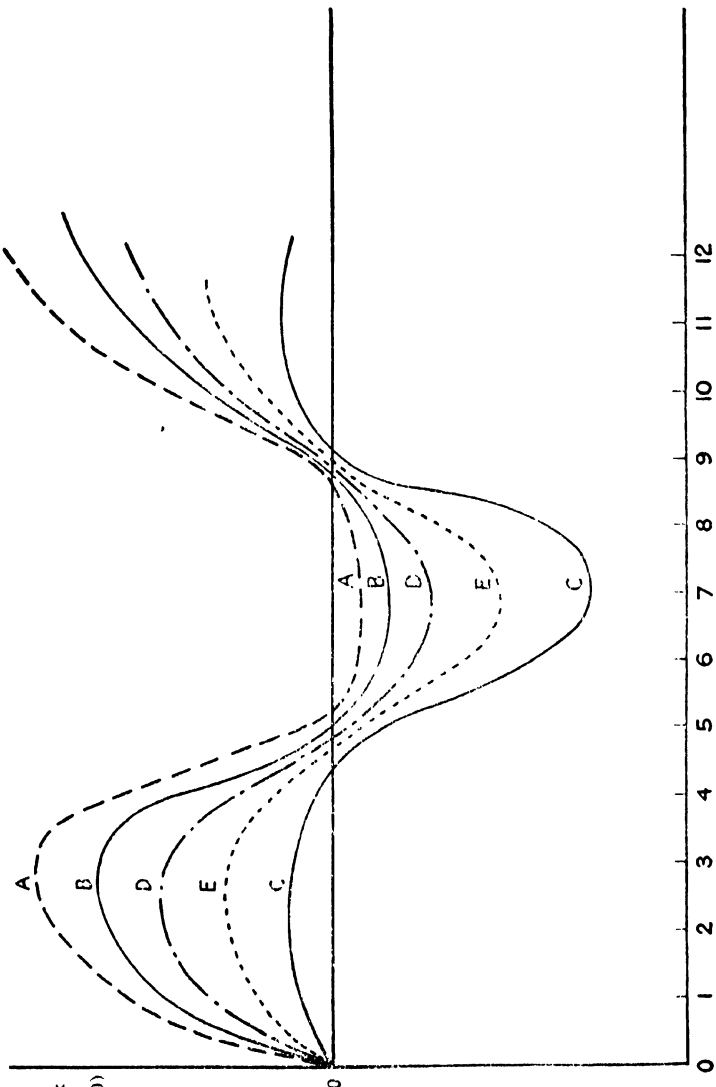
conclusion would follow regardless of special assumptions, with one exception alone. The only circumstance under which negative D_i's (say) would have a greater chance of occurrence than positive D_i's is illustrated in Figure 4.

In Figure 3 it was assumed that true cost of living indices applicable to different levels of real income differed in flexibility in the same way, both during upswings and downswings; whether indices for upper real income levels were more or less flexible was actually immaterial—so long as differences were *consistent* at the various cyclical stages. Figure 4 illustrates an extreme example of another hypothesis—namely, that the order of differences in flexibility of these measures is *reversed* in prosperity and depression.

Thus, it is assumed in the figure that the index applicable to the real income level of the base period—C—is the least flexible of all the indices in prosperity and the most flexible in depression. Under this circumstance D_i would necessarily be negative or zero throughout the entire course of the business cycle, regardless of what changes in real incomes may have occurred. It should be noted at once that there is no justification either in the theory or in the known facts of the behavior of prices to justify the hypothesis of Figure 4, while there is abundant evidence to demonstrate its implausibility. Nevertheless, it is worth noting that even if an extreme case of this kind were to arise, it would be possible to establish a method of observation such that the D_i's obtained would still possess an equal chance of being positive or negative. This could be accomplished by observing D_i throughout the period covered from the vantage point of *each* of the different real income levels successively as described further on pages 58-59 below; i.e., D_i would be measured throughout the period, first using real income C as a base, then real income E as a base, then real income D, etc. Even on this extreme assumption, then, the probability of positive or negative D_i's would be even.

Now as previously noted, although neither D_i nor D_p can be observed directly, D_t can be easily measured. Furthermore, it follows from the previous discussion that if D_t were measured for consecu-

FIGURE 4


Cost of
Living Index
(Time 0 = 100)
100
A
B
D
E
C
0
1
2
3
4
5
6
7
8
9
10
11
12
Time Period

tive periods over a fairly long period of time, the number of D_t's obtained for which D_i's happened to be positive would tend to equal the number of D_t's for which D_i's happened to be negative; in any case, it is virtually certain that D_i would be positive in a substantial number of cases provided the time period involved covered at least the full course of one or more business cycles. If the number of observations were sufficiently large, then, it would follow that (1) the highest value of D_t obtained during the period might be taken as an estimate of the highest value of D_t likely to be encountered in the future—an assumption which can be continuously tested with the passage of time; and (2) that the highest value of D_t to be obtained can also serve as a conservative estimate of the maximum value likely to be obtained for D_p, since $D_t > D_p$ when $D_i > 0$.

Conclusions (1) and (2), of course, follow only if it may be reasonably assumed that the sample of D_t's obtained is unbiased. If, for example, there were some reason to believe that abnormally low values of D_p were regularly associated with positive values of D_i, then the sample as described above would indeed be biased and the results of dubious value. It is obvious from the nature of the measures, of course, that the sign of D_i *in itself* has no influence whatever upon the magnitude of D_p. Nevertheless, there would remain the possibility—which, however, does not appear at all likely—that for a given choice of a base period D_i was regularly positive *only* at *given* periods of price cycles (say throughout depression) and *further* that D_p was regularly small during such periods.

It will be recalled from the discussion of Figures 3 and 4, however, that this would remain a possibility—although a remote one—only if measurements of D_t were obtained on *one* base period alone. If, on the other hand, D_t were computed by obtaining Laspeyre and Paasche indices throughout the period, using first one and then another of the years as base, successively, a representative selection of D_t's (and of D_i's and D_p's) would be assured. Referring to Figure 3, this procedure would be equivalent to observing D_i throughout the period first from the vantage

point of real income A, then B, then C, etc., and positive values of D_t would be distributed equitably in the various stages of the cycle.

Existing Evidence

Thus, from successive observations of the value of D_t it is possible to obtain an estimate of the upper limit of D_p, and since $L > I_0 > L - D_p$ and $P < I_1 < P + D_p$, an estimate of the reliability of the Laspeyre and Paasche indices when employed as approximations of the corresponding true measures of changes in the cost of living. Available evidence indicates that the Laspeyre and Paasche indices provide very close approximations, for it appears that the values of D_p—even at their greatest—are probably very low.

TABLE 1

PER CENT DIFFERENCE BETWEEN OLD AND CURRENT COST OF LIVING INDICES OF THE BUREAU OF LABOR STATISTICS

Date	Indices 1935–39 = 100		Per cent Difference,* Old to Current
	Old Index	Current Index	
	Fixed budget priced: 1917–1918 average purchases	Fixed budget priced: 1934–1936 average purchases	
1935 Mar.	97.8	97.8	0
July	97.6	97.6	0
Oct.	98.0	98.0	0
1936 Jan.	98.7	98.8	.1
Apr.	97.9	97.8	.1
July	99.6	99.4	.2
Sept.	100.0	100 4	.4
Dec.	100.0	99.8	.2
1937 Mar.	101.7	101.8	.1
June	102.6	102.8	.2
Sept.	103.2	104.3	1.1
Dec.	102.6	103.0	.4
1938 Mar.	100.7	100.9	.2
June	101.2	100.9	.3
Sept.	100.4	100.7	.3
Dec.	100.4	100.2	.2
1939 Mar.	99.6	99.1	.5
June	99.2	98.6	.6
Sept.	100.4	100.6	.2
Dec.	99.8	99.6	.2

* Signs ignored.

Source of basic data: U. S. Bureau of Labor Statistics

The existing evidence is shown in Tables 1 and 2. Although Table 1 does not actually present values of D_t, it has supplementary interest. In this table the two measures shown are both fixed base (Laspeyre's) indices. One of these measures of changes in the cost of living, however, is based on average purchases of wage-earners in 1917–1918; the other is based upon purchases in 1934–1936. Most notable is the fact that the percentage difference between these indices during the period covered is at most 1.1 percent.

Now while the percentage differences in Table 1 do not throw a direct light upon the magnitude of D_t or of D_p, they do suggest something concerning the probable order of their values. For the one index shown in this table employs as weights quantities which reflect the real income level prevailing in 1917–1918 and the general pattern of prices prevailing at that time; the second index correspondingly refers to the real income level and price pattern of 1934–1936. The maximum difference of 1.1 percent shows that revisions in these two sets of quantities owing to differences in real income and relative prices were of distinctly minor importance when measured by the differences in the total expenditures they would give rise to under the actual price patterns which prevailed in the months from March 1935 to December 1939. On the assumption that the differential behavior of wage-earners between 1917–1918 and 1934–1936 was typical, the data of Table 1 suggest that the maximum value of D_p may indeed be very small.

More pertinent evidence, however, is presented in Table 2, which shows annual observations of D_t from 1929 to 1940, although not of the kind ideally required for the purpose set forth in this study. Ideally, D_t should be computed from indices measuring changes in the cost of living for some fairly homogeneous group sharing a reasonably common standard of living; the cost of living index in which chief interest lies, of course, is that for urban wage-earners. The retail price indices of Table 2, however, measure price changes only for sales in retail stores, and of course cover *total* sales of this kind in the United States.

TABLE 2

VALUES OF D_t BASED ON RETAIL PRICE INDICES OF DEPARTMENT OF COMMERCE

Date*	Indices 1939 = 100		D_t
	L	P	
	Fixed Weighted Index; Weights based on Sales in 1939	Variable Weighted Index; Weights based on Sales in given year	Expressed as Per cent of L†
1929	131.8	129.9	1.4
1930	124.3	123.6	.6
1931	107.0	106.9	.1
1932	92.7	92.4	.3
1933	90.4	90.2	.2
1934	98.8	98.7	.1
1935	102.3	102.0	.3
1936	102.4	102 1	.3
1937	106 3	105.9	.4
1938	101.7	101.7	.0
1939	100.0	100.0	—
1940	101.2	101.1	.1

* Index numbers for war years subsequent to 1940 are not suited to the purpose of this table and therefore are omitted; see Chapter V.

† The values of D_t, when expressed as percentages of P are the same for every year except 1929. In 1929 the value of D_t, when expressed as a per cent of L, is 1.4; when expressed as a per cent of P, it is 1.46.

Source of basic data: U. S. Department of Commerce, Office of Business Economics.

Nevertheless, until more precisely adapted data are available, the results will remain of considerable interest. They show notably low values of D_t, with some evidence of a cyclical swing in these values.[2] Indeed, taken in conjunction with Table 1, there is the suggestion that cyclical swings in real incomes and relative prices may have greater effects upon the pattern of consumption than the secular changes over a period as long as 17 or 18 years.[3]

2 *Cf.* the remarks of Frederick C. Mills in John D. Black and Bruce D. Mudgett's *Research in Agricultural Index Numbers* (Social Science Research Council, Bulletin No. 10, 1938), p. 53.

3 That this is not a *necessary* conclusion, however, follows from the interpretation of the two Bureau of Labor Statistics indices given above. The particular price patterns prevailing in the years covered also affect the differences between the index numbers, and the time span covered by the Bureau of Labor Statistics indices and the Commerce Department indices are different.

Most of all, however, it is important to note that the maximum value of D_t attained during the entire period is only 1.4 percent. Rounding this figure to 1½ percent and inserting it in the limits (4.12) and (4.13) derived above, we obtain:

$$L > I_0 > L - .015L, \quad \text{or}$$

$$L > I_0 > .985L, \quad \text{and} \tag{4.14}$$

$$P < I_1 < 1.015P. \tag{4.15}$$

Thus, applying these limits, if Laspeyre's index in any given year were 110, then the true index (I_0), based on the base period real income level, would lie between 110 and 108.4. Similarly, if Paasche's index stood at 109, then the true index (I_1), based on the given year's real income level, would lie between 109 and 110.6.

As already noted, the data employed for computing these limits were not ideally suited for the purpose. Limits (4.14) and (4.15) are, therefore, not to be interpreted literally. Nevertheless, they do suggest that the maximum errors for the Laspeyre and Paasche formulas are most likely very small.

Approach II

In table 2 values of D_t were obtained from two series of index numbers computed on the base of 1939 = 100; weights employed for the Laspeyre index were quantities sold in 1939, while weights for the Paasche index of course varied, being derived from the given year. Thus the formulas for these indices were:

$$L = \frac{\Sigma p_i q_{39}}{\Sigma p_{39} q_{39}} \quad \text{and}$$

$$P = \frac{\Sigma p_i q_i}{\Sigma p_{39} q_i},$$

where i = the given year. Using the same data required for these index numbers, however, additional sets of indices might have been obtained simply by shifting the base. Thus the indices might also have been computed by using the formulas:

$$L = \frac{\Sigma p_s q_{40}}{\Sigma p_{40} q_{40}} \qquad \text{and}$$

$$P = \frac{\Sigma p_s q_i}{\Sigma p_{40} q_i}$$

Similarly, additional sets might have been computed by using each of the other years—1929, 1930, 1931, etc.—as base. Furthermore, from each such set of indices additional values of D_t might be obtained. The total number of observations obtained in this way for the 12 years shown in table 2 would be 11 × 11 or 121. It should be noted also that these different observations of D_t would be truly independent (except for the usual limitations which apply to any observations over time), in that both the D_i's and the D_p's underlying the D_t's would in every case be different. The advantages of these additional computations were previously explained above, from the standpoint of avoiding bias and also insuring an adequate representation of positive D_i's. For present purposes, however, it is important to note how extensively the number of observations are increased by this procedure.

Although it was not possible to obtain from the Commerce Department, because of the shortage of clerical assistance available at this writing, these additional observations of D_t which could be computed from the data, the use to which such observations may be put will be briefly indicated. For if a large number of observations of D_t were available, it would be possible to estimate an average value for D_p itself and also to obtain some information concerning the amount of variation to be expected about this average.

This may be demonstrated as follows. Since $D_t = D_p + D_i$, we may write:

$$\bar{D}_t = \bar{D}_p + \bar{D}_i, \tag{4.16}$$

where $\bar{D}_t$, $\bar{D}_p$ and $\bar{D}_i$ are the arithmetic means, respectively, of D_t, D_p and D_i.

Now it should be recalled at this point that the *sign* of D_i, as explained in detail in the discussion of Figures 3 and 4 above, is a

matter of accident, has no relation to its numerical magnitude, and depends only on which real income level is chosen as base for the computation of L and P. Hence, if an unbiased sample of D_t's were obtained, by the procedure described above, the mean of D_i would be zero, and it follows that:[4]

$$\bar{D}_t = \bar{D}_p \tag{4.17}$$

Furthermore, since D_p is statistically independent of the sign of D_i we may note that the covariance between D_i and D_p is zero. Hence the variance of D_t is the sum of the variances of D_i and D_p, and we may conclude:

$$\sigma_p^2 < \sigma_t^2, \qquad \text{and of course} \tag{4.18}$$

$$\sigma_p < \sigma_t, \tag{4.19}$$

where σ_p^2 and σ_t^2 are the variances of D_p and D_t respectively and σ_p and σ_t are their standard deviations.

Thus it is not only possible to measure the average value of D_p with some precision, but it is also possible to obtain an estimate of an upper limit to its standard deviation. This method, therefore, obtains more highly refined results than those secured through Approach I, described above, and would no doubt succeed in narrowing the limits given in (4.14) and (4.15) appreciably.

In concluding this discussion it is relevant to note that data directly suited to the methods of estimation described in this chapter will soon be available. Beginning in 1945 the U. S. Bureau of Labor Statistics inaugurated the practice of collecting detailed data each year on consumer purchases among urban wage-earner families in selected areas. These data, the first of which were made public in 1948, will make possible the construction of a series of Laspeyre and Paasche indices. Measures of D_t relating to the living costs of a reasonably homogeneous group of families may be obtained annually, and estimates of D_p more relevant to the purpose at hand, therefore, may be derived by the methods already described.

4 The argument immediately following was suggested by Professor William Vickrey of Columbia University.

CHAPTER V

IMPLICATIONS

THE popularity of Laspeyre's formula in cost of living indices —as well as for index numbers of other types—has for long rested upon the uncertain ground of expediency. In the foregoing analysis, however, considerable *theoretical* justification for the formula was presented. It was demonstrated that Laspeyre's index may be employed as an approximation of the "true" measure of a change in living costs, and two methods for estimating its maximum error when so employed were presented. Evidence was also introduced which suggests that the size of this maximum error is most likely very small.

In this concluding chapter attention will be devoted to the implications of these results. For it was assumed throughout the preceding analysis that an homogeneous group of persons—persons sharing identical tastes—had in some manner been defined, and that the tastes of these individuals remained unchanged in time. It was further assumed that the environment in which these persons lived was likewise constant. Obviously neither assumption can qualify as a literal transcription of the truth, and their relation to fact and to relevant questions arising in practice must therefore be considered.

THE ASSUMPTION OF CONSTANT TASTES

It is a commonplace that tastes differ among individuals, and that they differ particularly according to income class, cultural background, education, and similar factors. It is equally obvious that because of biological similarities and numerous common elements in the environment of persons living at the same time, tastes of virtually all individuals are at least in *some* respects the same. Moreover, the area of similarity may be expected to broaden materially as the common environmental factors for groups of people are extended. It is on such grounds as these that it is believed that groups of persons with the same general occu-

pational environment—say, wage-earners—living in the same city or in the same type of city, and receiving approximately the same incomes, possess generally similar patterns of tastes.[1] It is obvious that such groups as this can be—and have been—statistically defined.

Attention nevertheless should be directed, especially, to the last criterion of common tastes noted—incomes. This is a particularly important one since income classes are correlated with so many other relevant factors such as cultural background and education. How narrowly must an income class be defined, however, in order to insure that the families it comprehends share, at least roughly, common taste patterns? An examination of this question is obviously beyond the scope of this work, but it is relevant to note that the problem posed is by no means insoluble. An important contribution to the discovery of statistical tests applicable for this purpose may be found in the work of Allen and Bowley,[2] and as the problem receives more attention no doubt further progress may be expected.

Given an adequately circumscribed "homogeneous" group of persons, as defined above, can we reasonably expect that their tastes will remain the same over time, and if so, for how long? In considering this question, attention may be turned initially to the oft-repeated assertion that tastes change as incomes change with the business cycle, since the principal basis for this opinion appears to be a misunderstanding of what is meant by the term "similar tastes." A person's tastes are to be considered unchanged if his indifference map is unchanged—i.e., if he will buy the same quantities of the same commodities when he re-

1 It follows from this that, at least *initially*, separate cost of living indices must be constructed for *each* income group in *each* similar geographical area. The question of whether such distinct cost of living indices may be meaningfully combined into (say) a cost of living index for wage-earners in large cities in the United States, or into a cost of living index for the United States as a whole, is discussed in this chapter below and also in Appendix B.

2 R. G. D. Allen and A. L. Bowley, *Family Expenditures* (P. S. King & Son, Ltd., London, 1935).

ceives the *same income* and faces the same set of prices. Hence the discovery that the distribution of expenditures varies as income changes is no indication that tastes have changed.

This conclusion, however, should not be confused with the preceding suggestion that tastes may be expected to vary with "income class." The term "income class" as employed above may be equated with a *relative* position in the distribution of incomes —rather than with a fixed number of dollars. Persons in the same income class in this sense may regularly receive $1,500 per family in depression on the average and $3,000 in prosperity, and there would be no necessary reason for believing that their intrinsic tastes would change with these cyclical fluctuations. There would be considerable reason for believing, however, that the tastes of these persons would differ at all times from those of individuals regularly earning twice as much on the average—say $3,000 per family in depression and $6,000 in prosperity. For the constant differentials between these two income groups over time would be conducive to constant differentials in consumption habits and in basic attitudes toward commodities, and in addition might reflect, indirectly, those differences in education, cultural background, occupation, etc., referred to above, which in part determine tastes.

Of course it must be conceded that a greatly prolonged and severe depression such as that of the 'thirties may ultimately be expected to influence intrinsic taste patterns—for practices initially adopted out of necessity may in time become an accepted component of the mode of living. Over the long term as well, education, the impact of new products, the stimulation and cultivation of new wants, all result in changes in tastes. Observation suggests, however, that neither cyclical nor secular factors ever result in swift and radical alterations in the general patterns of consumers' wants and preferences. The persistency of common needs, habits, and conventions at all times offers powerful resistance. Tastes indeed change over time—but slowly. The sole exceptions would appear to be periods of great national emergency such as war.

For the maker of cost of living indices, of course, the important question is, when have tastes changed so much as to impair the meaning of an index number series over time? Since such changes are ordinarily gradual, however, it is obvious that there can be no precise answer to this question. The suggestion that it may be possible to name some specific "date" on which tastes have undergone a fundamental change has no connection with reality—except, as already indicated, during abnormal periods such as wartime.[3] It is possible ordinarily to say only that comparisons of changes in the level of living costs have less and less meaning as the time periods compared grow more and more distant.

In the light of these changes in tastes, however, and also because of changes in environment discussed below, it is necessary to replace at relatively frequent intervals—say every five or ten years—the content of the fixed budget priced in a cost of living index. Such replacements may be conceived of as constituting interruptions in the continuity of the index, reflecting the fact that tastes change over time and that comparisons of *levels* of living costs must be confined to fairly nearby periods of time.

Of course, even though the practice of replacing the fixed budget priced every five or ten years were adopted, the present practice of obtaining an overlap and "linking" such index numbers to provide continuous time series is clearly justified and useful. For comparisons of *rates* and the direction of change in living costs may be legitimately made for periods widely distant, and such comparisons are of obvious interest for many purposes.

3 It is perhaps needless to point out that during the period of a major war—such as World War II—the conditions necessary for construction of a cost of living index are in almost all important respects lacking. Fundamental alterations occur in the entire way of life for a large proportion of the people. Tastes change—as well as the motivations for behavior in ordinary economic situations. Products available for consumption vary widely in quality, quantity, and character. The geographical distribution of the population changes, etc. These and other changes occur with a speed and scope possible only in time of war or other serious national emergency.

The Assumption of Constant Environment

It was stated above that a person's tastes may be considered unchanged if he will buy the same quantities of the same commodities when he receives the same income and faces the same set of prices. This is the usual definition, but for complete accuracy an additional condition should be added to it: viz., that the person behave in the way prescribed under the same *environmental* conditions. Thus, an individual whose intrinsic tastes are unchanged may buy *different* quantities of the same commodities under identical price and income conditions—in summer and in winter. In the same way his preferences may differ if he moves from Maine to Florida.

Throughout the preceding analysis it was assumed, however, that environment remained unchanged. From the practical standpoint, insofar as environmental changes are limited to the seasons, this assumption presents no difficulty. For a change in living costs from one month in one year to the same month in the next will still refer to individuals with the same indifference maps, and the fixed budgets employed may be designed to represent expenditures over the course of an entire year. But there are nevertheless, progressive changes in the environment which are enduring rather than seasonal, and some mention must be made of these.

As in the case of changes in tastes, it may be noted at once that, except in time of war, alterations in environment are gradual. Geographical shifts in population assume substantial quantitative significance only over the long term. Social services available to consumers also undergo *gradual* change. The variety and quality of goods and services offered for sale likewise change gradually.

These alterations in the environment, of course, constitute additional grounds for the practice of periodically replacing the fixed budget priced, as mentioned above, and also for the limitations upon the interpretation of cost of living indices previously noted. These observations, however, do not represent sufficient grounds for dismissing the entire subject of environmental

change, for some types of change—as a practical matter—require a continuous, positive policy. This is true of those changes having to do with alterations in the variety and quality of goods offered for sale over time—probably the most important of all variations in environment from the standpoint of index number construction.

Obviously as some goods disappear from retail markets and are replaced by others, sooner or later responsive action of some kind will be required by those who collect retail prices and construct cost of living indices from them. In order to provide a rational basis for these decisions it is necessary to recall that the fixed budget priced under Laspeyre's formula must be regarded as an approximation to a bundle of goods providing a fixed real income of utility rather than a bill of goods of *physically* identical commodities. Indeed the very problem of environmental change with which we are dealing testifies that the goal of a physically identical bundle of goods is literally impossible as well as theoretically incorrect.

Stated alternatively, the objective in introducing new commodities or replacing old ones must be to insure that environmental changes do not distort the theoretical goal of measuring the relative change in money income required to provide equal satisfaction over time. Suppose, for example, that a new product—nylon stockings—ultimately replaces an old product—silk stockings. Suppose further that the new product is more durable and more attractive, but costs $2.00 per pair as against $1.00 per pair for the old product. To link the new product into the index at a price level 100 percent higher than the old would ignore entirely the additional utility afforded by the new product, thus providing an upward bias to the index. On the other hand, to link it in at the same price level as the old product would imply the assumption that the new product actually provided twice as much utility—i.e., that it was worth exactly twice as much to consumers as the old one; of course it might at the time of introduction into the index be worth only one and one-half times as much to most wage-earners, or perhaps three or four times as much and hence either an upward or a downward bias might be present.

There is, to be sure, no direct way of measuring the utility provided by any commodity; even when it is possible to express durability in precise quantitative terms, comfort, appearance and other less tangible characteristics most often play too crucial a rôle to be ignored. Hence policy must be based—insofar as possible—on objective consumer behavior. This policy may be considered in the light of what has been referred to as the "normal" pattern of commodity introduction and growth.[4]

Thus a new commodity usually begins its life in an experimental stage—novel, imperfect, and considerably higher priced than the old commodity which fulfills the same or a comparable function. In this initial stage, the product remains a luxury or novelty, of inconsequential quantitative importance in the economy as a whole and most likely of virtually no importance for the great mass of consumers. As technical problems are solved and mass production is introduced, however, prices are lowered and the new product may compete vigorously with the old one. Eventually the "old" ones—the horse-buggies, gas illumination, and hand-manipulated phonographs—may virtually disappear from the market.

It is during the second, or competitive stage, that action may be profitably taken by the computer of cost of living indices. It appears reasonable to believe, insofar as this normal pattern is followed, that as the product is improved and its price lowered a point may be reached at which it appears that for the mass of consumers a choice between the new product and the old one at the prices then prevailing has reached a condition approaching indifference. If the index were computed for urban wage-earners in a particular income class, then, quantitatively, this condition would be determined when for families in this income class as a group consumption expenditures were approximately equally divided between the new commodity and the old. Annual studies of consumer expenditures will make the determination of such

4 Saul Nelson and Walter G. Keim, *Price Behavior and Business Policy* (Monograph No. 1, Temporary National Economic Committee, Senate Committee Print, 76th Congress, 8th Session, Washington, 1940), pp. 109-110.

points possible. In any event it may be assumed that at this point of indifference a dollar spent on the new commodity provides approximately the same utility as a dollar spent on the old one, for wage-earners as a group. Hence, *at that point in time*, it would be justified to link the new commodity into the index at the same price level as the old one—half the relative weight of the old commodity being given to the new one. Of course, eventually the old commodity may be eliminated from the index entirely—if it is completely displaced in the market—and the weight of the new one increased accordingly.

This procedure is applicable only in one particular—though perhaps the most important—case. Three other cases, for which no objective method of solution appears possible at present, must be mentioned. (1) A commodity may be introduced initially, with quality so high and price so low that the old commodity is displaced from favor without any transitional period of competition. (2) A new commodity may be introduced which satisfies an *entirely* new want. (3) A new commodity may be, in effect, introduced "gradually" through the steady improvement in quality of an old commodity.[5] The first case would obviously require some action on the part of the computer of indices. Since the condition of the case leaves no choice as to *when* the new product is to be introduced into the index, the only question remaining concerns the level at which the link must be made. Chief reliance would clearly need to be placed upon judgment, framed, of course, in the light of the theoretical goal described above. Some assistance could conceivably be obtained from measurement of quality

5 Reference is here to gradual, long term changes in quality in one direction—upward. It may be noted in passing, however, that for some commodities *cyclical* changes in quality occur, periods of improvement being followed by periods of deterioration. For a discussion of these and other forms of indirect price changes see the author's "Indirect Price Increases," *Monthly Labor Review*, November 1942. Insofar as such indirect price changes remain "hidden" from collectors of prices, price index numbers will generally tend to understate the extent of both upswings and downswings.

standards[6] related to durability, in situations in which durability is regarded as a major factor in satisfaction.

The other two cases—(1) commodities satisfying new wants and (2) gradual alterations in quality—represent types of changes which, like gradual changes in tastes, the computer of cost of living index numbers must as a day-to-day matter ignore. In changing the budget priced periodically and in limiting the interpretation of cost of living index numbers as previously described, however, recognition is given to these and to all the other departures from theoretical conditions inherent in a dynamic society.

Other Implications

In the preceding chapter evidence was presented which suggests that the difference between Laspeyre's index and the true index of the cost of living was most likely very small. A word may be in order concerning the "reasonableness" of this finding. It will be recalled that, theoretically, the Laspeyre and Paasche index numbers would exactly equal the corresponding true cost of living indices if (1) there were no changes in *relative* prices from time to time, or if (2) the elasticity of demand with respect to relative prices were zero for all goods and services. The first condition means that all prices change from time to time by equal relative amounts. The second condition means that goods and services are consumed in constant proportions regardless of changes in relative prices (although proportions may change in response to variations in money prices and incomes). Obviously, neither of these conditions is tenable.

However, the tentative conclusion referred to above was that the Laspeyre and Paasche index numbers are *close approximations* to (though they are not identical with) the true cost of living indices. Accordingly, the suggestion follows that the above conditions are in some sense *approximated* rather than realized

6 See Alice L. Edwards, *Product Standards and Labeling for Consumers* (Ronald Press Co., New York, 1940).

exactly. Thus, the aggregate amount of expenditures wage-earners are required to make in order to maintain a given plane of living is affected (A) by changes in money prices. It is also affected by (B) changes in relative prices and the adjustments in consumption patterns made by consumers *in response to these relative price changes*. The effects of factor (A) are measured by the true cost of living indices as well as by the Laspeyre and Paasche index numbers. The effects of factor (B) are measured only by the true indices. The evidence presented in Chapter IV, therefore, indicates that the effects of factor (B) are ordinarily very small, probably most often negligible, when compared with the effects of factor (A).

When the budget of the typical consumer is considered as a whole, this conclusion should not appear surprising. Thus, the force and persistency of common needs, habits, and conventions obviously limit the extent to which consumers may alter their consumption habits in response to relative price changes. Consumption of a large portion of the items for which money is spent, such as housing and electricity, cannot be readily reduced or increased under any circumstances. Interchangeability among commodities is obviously limited.

Furthermore, numerous factors tend to limit the extent of changes which occur in relative prices. Thus, the multitude of interrelationships in both costs and in demand make for considerable similarity in the behavior of most prices. Surely, for example, the bulk of all prices respond to some extent to the major swings of the business cycle. Moreover, though specific turning points differ, there is a pronounced central tendency in timing. Similarly, over periods not too extended there is a marked central tendency in the distribution of price relatives.[7] It is a matter of common observation that while relative prices, in detail, change

7 Unfortunately, because of the paucity of data available, no detailed studies have been made of the behavior of retail price relatives. Numerous studies, however, have been made of wholesale price relatives, the most complete of which is that of Frederick C. Mills, *The Behavior of Prices* (National Bureau of Economic Research, New York, 1927), esp. pp. 76-140 and 223-226.

constantly, the general *order* of relative prices remains always about the same. For example, to the writer's knowledge tenderloin steak is at all times priced considerably higher than beef liver and broadcloth higher than denim, even though the exact differentials may and do change.

The reality of these similarities must not be obscured by the fact that most analyses are designed to highlight *differences* among commodity price and consumption changes, for it is these differences which are of prime importance for the majority of economic investigations and which in general provide insight into economic behavior. For the present investigation the significance of the broad similarities noted outweighs that of the differences.

Additional Proposals

Earlier in this chapter several suggestions were made concerning methods to be employed in constructing meaningful cost of living index numbers—proposals which followed directly from the preceding consideration of the factual and theoretical foundations upon which cost of living index numbers rest. In conclusion, certain other proposals may be made within this framework which emanate not from theoretical "necessity" but from the more flexible base of the apparent requirements of economic analysis.

It has been noted in preceding discussions that a cost of living index must apply to a particular real income level of a particular group of persons sharing common tastes. The more narrowly limited is the definitional group—the persons sharing the same tastes and incomes—the more refined from a theoretical point of view will the index be. The needs of economic analysis as well as of government and business administration, however, may be thwarted as easily by an excess of detail as by the lack of it. Ten thousand different cost of living index numbers loom as useless and bewildering, unless summarized in some way, as the impracticability of their construction from a purely fiscal standpoint.

Of course, neither the foregoing theory nor existing knowledge of the behavior of prices suggests the need for a profusion of

separate cost of living index numbers. It is probable, as noted earlier, that the behavior of living costs for a particular definitional group may represent with accuracy sufficient for all conceivable needs that of "neighboring" definitional groups over a considerable margin.

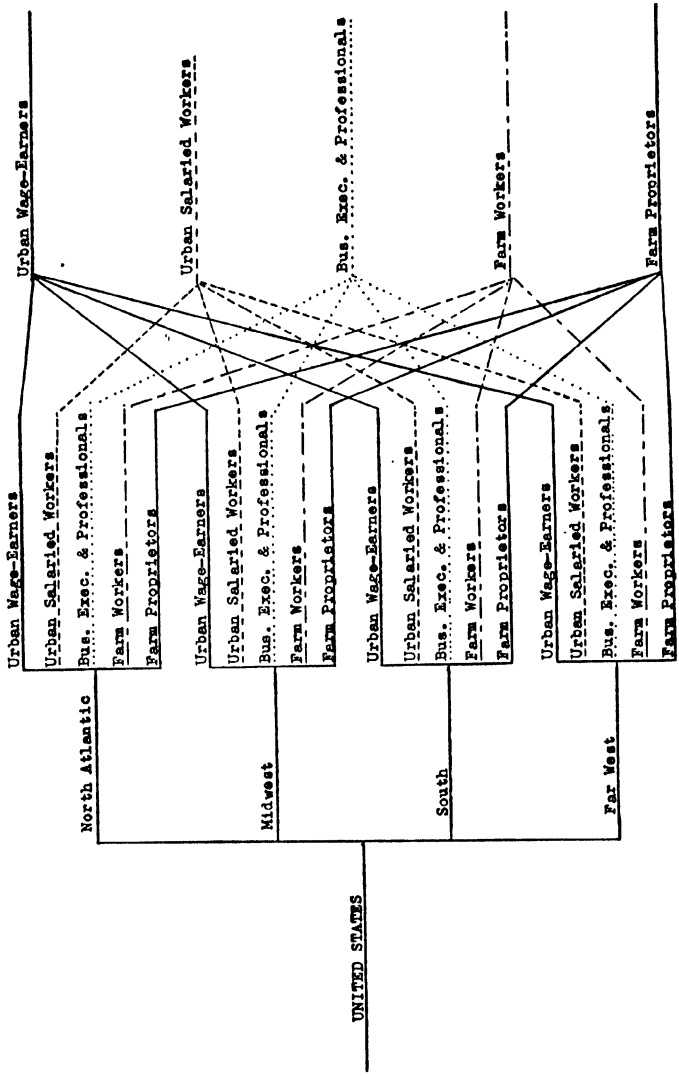
Thus, it has been pointed out above that the question of how narrowly in practice a definitional group must be bounded with respect to income range, geographical location, or other factors bearing on tastes is essentially an empirical problem. Suppose investigation demonstrates that wage-earners in the United States with annual incomes ranging from $2000 to $3000 in a given year represent a relatively homogeneous group and one which differs in significant respects from wage-earners in both lower and higher income classes. Further investigation even so may prove that changes in living costs for the $2000–$3000 group are the same—or very nearly so—as for all other groups of wage-earners. In that case one index number would represent with sufficient accuracy the experience of *all* wage-earners taken as a group.

As suggested, such questions require investigation, and it is to be hoped that eventually a comprehensive study will be undertaken to determine how many separate indices are required to measure the cost of living experience of all the major (i.e. numerically important) groups of persons in the United States. It is the author's guess that perhaps three or four separate vocational index numbers might be required—one each (say) for urban wage-earners and lower-salaried workers, farm workers, farm proprietors, and business executives and professionals. Perhaps for each of these vocational groups separate regional indices may be required. The key to the extent of detail needed in any case would be the actual magnitude of differences found from group to group in changes in living costs. It follows that, at least experimentally, separate index numbers for *more than* three or four vocational or income groups in each of several regions ought to be computed, in order to test these differences.

Now suppose that investigation proves that separate indices are required for five vocational groups in each of four regions, as shown in Figure 5, making 20 separate cost of living index num-

FIGURE 5

SOME TYPES OF COST OF LIVING INDEX NUMBERS



bers in all. It is important to note that nothing in the foregoing theory rules out the construction of *combined* index numbers as shown in the Figure, although care must be employed in interpretation.

Thus, the four regional index numbers for wage earners might be combined in one over-all index number measuring changes in living costs for *all* wage-earners in the United States. The result would not be a "pure" cost of living index as the term has been employed above, but it could nonetheless be defined within the framework of that theory with precision. While the problem of devising and interpreting such combinations is discussed more fully in Appendix B below, it may be noted here that an index of this kind would reflect the average change in the cost of living for urban wage-earners in the United States, assuming that the distribution of wage-earners among the four geographical regions remained the same as it had been in the base period. The qualifying assumption, of course, is a necessary one and would limit the use of the measure.

Similarly, the several vocational index numbers in all regions could be combined to provide a grand cost of living index for the United States as a whole. As detailed in Appendix B, an index of the latter type would be interpreted as measuring the change in living costs for the people of the United States, assuming that the distribution of real incomes, occupations, and geographical location remained the same as they were in the base year. Insofar as the conditions of this assumption are approximated—as they would be, for example, during very short periods of time—the measure will be of interest.

Thus, while "combined" cost of living index numbers are not precluded by the foregoing theory, the limiting assumptions assignable to such measures are given explicit recognition. Of course, the need for these qualifications does not spring—ultimately—from the theory. It arises instead from the nature of the economic events these measures comprehend, to which the theory duly makes reference.

APPENDICES

APPENDIX A

THE DERIVATION OF METHODS FOR CONSTRUCTING COST OF LIVING INDEX NUMBERS FROM FAMILY BUDGET DATA

THE methods discussed in this section all presume the existence of budgetary data for two situations—two periods of time or two places—showing the detailed purchases made, including quantities and prices, by families in each situation classified by income groups. In general, their common objective is to identify two families, one in each of the situations, enjoying equal real incomes. The ratio of the total expenditures of these two families, then, would provide a true measure of the change in the cost of living, at that income level. The limitations common to all these methods, particularly when applied to the measurement of living costs over time, are discussed above in pages 39–43. The details of the derivation of these methods are provided below.

Konüs.[1] As was demonstrated above (pp. 36–39), Konüs proves the following inequalities:

$$L > I_0$$

$$I_1 > P$$

$$I_0 \gtrless I_1$$

$$L \gtrless P$$

1 A. A. Konüs, "The Problem of the True Index of the Cost of Living," *Econometrica*, vol. VII, no. 1, January 1939, pp. 10-29. This article was first published in *The Economic Bulletin of the Institute of Economic Conjuncture*, Moscow, No. 9-10 (36-37), September-October, 1924, pp. 64-71, and was translated from the Russian for publication in *Econometrica*. For criticisms of Konüs' theory see Henry Schultz, "A Misunderstanding in Index-Number Theory: The True Konüs Condition on Cost-of-Living Index Numbers and Its Limitations," *Econometrica*, vol. VII, no. 1, January 1939, pp. 1-9, and Bruce D. Mudgett, "Konüs Condition," *Econometrica*, vol. XIII, no. 2, April 1945, pp. 171-181.

and shows that neither I_0 nor I_1 need lie between L and P. As in previous discussions, L and P represent Laspeyre's and Paasche's formulas respectively, and I_0 and I_1 represent the true cost of living indices on the bases of the real income level of the base period and the real income level of the given period respectively. Assuming that the true index of the cost of living at any time is a continuous and single-valued function of the plane of living, he points out that there exists some plane of living ϵ, such that:

$$I_0 \lesseqgtr I_\epsilon \lesseqgtr I_1.$$

From inequalities previously demonstrated, six possibilities exist with respect to the relations among I_0, I_1, L and P:

$$L > I_0 > I_1 > P$$
$$L > I_1 > I_0 > P$$
$$L > I_1 > P > I_0$$
$$I_1 > L > I_0 > P$$
$$I_1 > L > P > I_0$$
$$I_1 > P > L > I_0$$

It follows from these inequalities that ϵ may be so selected that not only $I_0 \lesseqgtr I_\epsilon \lesseqgtr I_1$ holds, but also $L \gtreqless I_\epsilon \gtreqless P$. This may be done by putting I_ϵ between the middle terms of the inequalities in each case.

Now supposing that he has data from two investigations of consumption, "these data being subdivided into groups according to the quantities of total expenditure of consumers," Konüs sets himself the task of identifying consumers covered in the two studies "having approximately the same standard of living." It follows, as already noted, that if such consumers *were* identified, then the ratio of their average total expenditures would provide a true cost of living index for their standard of living. Accordingly, Konüs seeks to prove that if

$$\frac{\Sigma p_1 q_1}{\Sigma p_0 q_0} = \frac{\Sigma p_1 q_0}{\Sigma p_0 q_1} \tag{1}$$

then

$$L \gtrless \frac{\Sigma p_1 q_1}{\Sigma p_0 q_0} \gtrless P \tag{2}$$

Thus, he considers two possibilities:

I. $$\frac{\Sigma p_1 q_1}{\Sigma p_0 q_0} < \frac{\Sigma p_1 q_0}{\Sigma p_0 q_0}$$

then $$\Sigma p_1 q_1 < \Sigma p_1 q_0$$

and $$\frac{\Sigma p_1 q_1}{\Sigma p_0 q_1} < \frac{\Sigma p_1 q_0}{\Sigma p_0 q_1}$$

But from (1)

$$\frac{\Sigma p_1 q_1}{\Sigma p_0 q_0} = \frac{\Sigma p_1 q_0}{\Sigma p_0 q_1}$$

Therefore

$$\frac{\Sigma p_1 q_1}{\Sigma p_0 q_1} < \frac{\Sigma p_1 q_1}{\Sigma p_0 q_0} < \frac{\Sigma p_1 q_0}{\Sigma p_0 q_0} \quad \text{or}$$

$$P < \frac{\Sigma p_1 q_1}{\Sigma p_0 q_0} < L$$

Now if it is supposed that

II. $$\frac{\Sigma p_1 q_1}{\Sigma p_0 q_0} > \frac{\Sigma p_1 q_0}{\Sigma p_0 q_0}$$

then $$\Sigma p_1 q_1 > \Sigma p_1 q_0$$

and $$\frac{\Sigma p_1 q_1}{\Sigma p_0 q_1} > \frac{\Sigma p_1 q_0}{\Sigma p_0 q_1}$$

Therefore

$$\frac{\Sigma p_1 q_1}{\Sigma p_0 q_1} > \frac{\Sigma p_1 q_1}{\Sigma p_0 q_0} > \frac{\Sigma p_1 q_0}{\Sigma p_0 q_0} \quad \text{or}$$

$$P > \frac{\Sigma p_1 q_1}{\Sigma p_0 q_0} > L$$

Konüs thereupon assumes that since $\frac{\Sigma p_1 q_1}{\Sigma p_0 q_0}$ under the condition

specified falls between the same limits, L and P, as does I_s, the standards of living to which $\Sigma p_1 q_1$ and $\Sigma p_0 q_0$ respectively refer are "approximately equal," and hence the ratio between the two provides a measure of the true cost of living index.

Serious deficiencies in this reasoning have been pointed out by Schultz and Mudgett in the works cited. Nevertheless, because of its historical importance, it may be in order to recall that the basic relationships first developed by Konüs underlie all subsequent work in the theory of cost of living index numbers. However, Konüs' study, which originally appeared in Russian in 1924, did not become available in full translation until 1939. Prior to that his work was known outside of his own country only through a partial summary, which proved to be misleading, incorporated in a review by Bortkiewicz.[2]

As shown above, Konüs attempted to derive a direct approximation of the true cost of living index. On the other hand, Bortkiewicz gave the impression that his objective was to obtain *limits* for the true index, and he placed great emphasis upon the so-called "Konüs condition." But this condition was actually a step rather than the end of Konüs' analysis, and comprised only a partial account of the important relationships he constructed.

Staehle's analysis, given immediately below, was a further development of the work of Konüs, as described by Bortkiewicz. All the relationships employed by Staehle were contained in or were implicit in Konüs' study, but, unlike Staehle, Konüs did not employ them for the purpose of constructing limits.

Staehle I.[3] Using relationships previously demonstrated by Konüs and Haberler,[4] Staehle attempts to show how the limits

2 L. von Bortkiewicz, review of Gottfried Haberler's *Der Sinn der Indexzahlen, Magazin der Wirtschaft*, vol. IV, no. 11, 1928, Berlin, pp. 427-29.

3 H. Staehle, "A Development of the Economic Theory of Price Index Numbers," *Review of Economic Studies*, vol. XI, no. 3, June 1935, pp. 163-188.

4 Some of the results developed by Konüs in 1924, described above, were developed independently three years later by Gottfried Haberler, who had not see Konüs' article. Haberler's work appeared in *Der Sinn der Indexzahlen* (Tubingen, 1927).

to the true cost of living index may be obtained at various real income levels. Thus, he notes that $\frac{\Sigma p_1 q_0}{\Sigma p_0 q_0} > I_0$ and also that if a particular standard of living $e_0 = \Sigma p_0 q_0$ is chosen from the budget data of one period, it is possible, theoretically, to determine a standard of living in another period $e_1 = \Sigma p_1 q_1$, such that $\Sigma p_0 q_0 = \Sigma p_0 q_1$. The bundle of goods, Σq_1, obviously represents a lower standard of living than the bundle Σq_0, since the bundle Σq_1 could have been, but was not, purchased at p_0 prices at the same total expenditure. Hence $\Sigma p_1 q_1 < \Sigma p_1 \bar{q}_0$, where $\bar{q}_0$ represents that bundle of goods providing satisfaction equivalent to Σq_0, which would have been purchased at p_1 prices. Therefore,

$$I_0 = \frac{\Sigma p_1 \bar{q}_0}{\Sigma p_0 q_0} > \frac{\Sigma p_1 q_1}{\Sigma p_0 q_0}$$

and also

$$I_0 > \frac{\Sigma p_1 q_1}{\Sigma p_0 q_1}.$$

In the same way, by noting that $I_1 > \frac{\Sigma p_1 q_1}{\Sigma p_0 q_1}$, by selecting a standard of living, Σq_1, and then determining a standard Σq_0 such that $\Sigma p_1 q_1 = \Sigma p_1 q_0$, it is possible theoretically to determine upper and lower limits of I_1.

Wald.[5] Like Konüs, and also like Frisch in the method described below, Wald seeks to obtain a direct approximation of the true cost of living index. He considers two time periods, 0 and 1, and assumes that C_0 and C_1 are the two corresponding Engel curves "determined from budget data" relating to these two periods.

Given a point $q_0 = (q_0', q_0'' \ldots q^n)$ on C_0, select two points q_1 and $\hat{q}_1$ on C_1, such that

(1) $\qquad \Sigma p_0 q_1 = \Sigma p_0 q_0 \qquad$ and $\qquad \Sigma p_1 \hat{q}_1 = \Sigma p_1 q_0.$

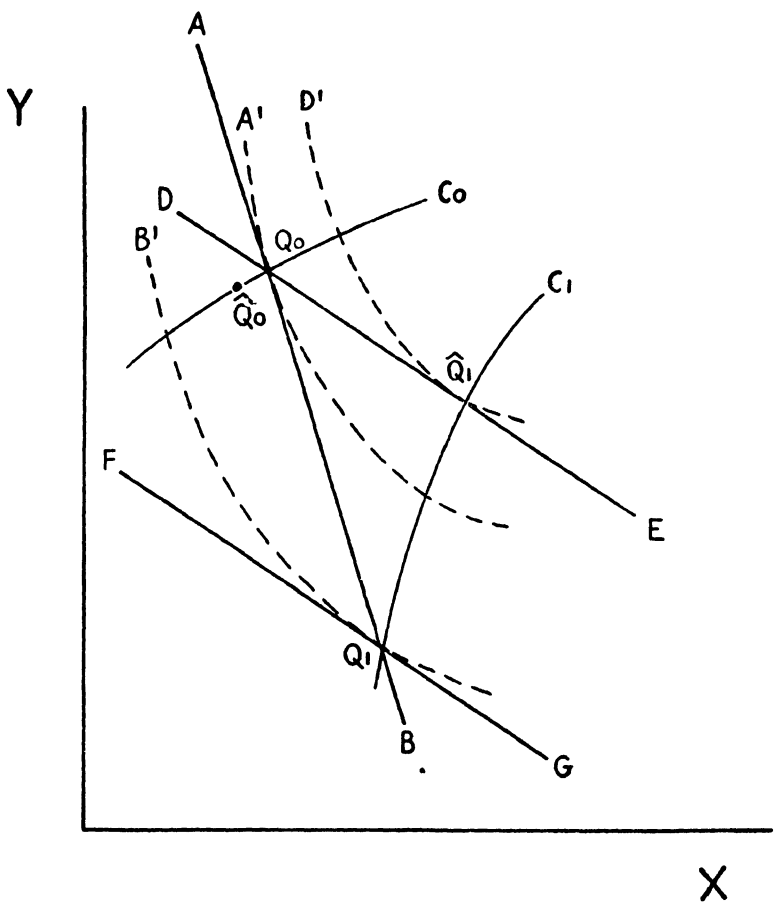
5 A. Wald, "A New Formula for the Index of Cost of Living," *Econometrica*, vol. VII, no. 4, Oct. 1939, pp. 319-331.

The trivial case of $q_1 = \hat{q}_1$ is ruled out, for in that event, subject to the condition (1), the true index sought, I_0, would be given simply by $\Sigma p_1 q_1 / \Sigma p_0 q_0$.

Denote by $\hat{q}_0$ an arbitrary point on C_0 in the neighborhood of q_0 such that $\Sigma p_1 \hat{q}_0 \neq \Sigma p_1 q_0$.

These points, along with the Engel curves on which they lie, are shown in the case of two commodities in Figure 6. The line

FIGURE 6



AB is the expenditure line $\rho(q_0) = \Sigma p_0 q$, tangent to the indifference curve A′ at q_0. The line FG is the expenditure line $\rho(q_1) = \Sigma p_1 q$, tangent to the indifference curve B′ at q_1. The line DE is the expenditure line $\rho(\hat{q}_1) = \Sigma p_1 q$, parallel to FG but passing through the point q_0 and, of course, tangent to the indifference curve D' at $\hat{q}_1$.

Wald assumes that C_0 and C_1 "are not very far from each other"[6] and that the preference indicator $U(q)$ may be approximated by a polynomial of the second degree in the tetrahedron T, determined by the four points q_0, $\hat{q}_0$, q_1, $\hat{q}_1$. Since $\hat{q}_0$ is arbitrary and may be chosen as close to q_0 as desired, this assumption implies primarily that the segment of C_1 between q_1 and $\hat{q}_1$ may be approximated by a straight line.[7]

From this assumption concerning the form of the utility function, the following equation may be deduced with respect to the points q_1 and q_0:

$$(2) \quad U(q_1) - U(q_0) = \tfrac{1}{2}\sum_{i=1}^{n}\left(\frac{\partial U(q_0)}{\partial q^i} + \frac{\partial U(q_1)}{\partial q^i}\right)(q_1{}^i - q_0{}^i)$$

$$= \tfrac{1}{2}\Sigma(\omega_0 p_0 + \omega_1 p_1)(q_1 - q_0),$$

where ω_t denotes the marginal utility of money at $q_t (t = 0, 1)$.

Considering the points q_1 and $\hat{q}_1$ we similarly derive:

$$(3) \quad U(\hat{q}_1) - U(q_1) = \tfrac{1}{2}\Sigma(\omega_1 p_1 + \hat{\omega}_1 p_1)(\hat{q}_1 - q_1)$$

$$= \frac{\omega_1 + \hat{\omega}_1}{2}\Sigma p_1(\hat{q}_1 - q_1)$$

where $\hat{\omega}_t$ denotes the marginal utility of money in $\hat{q}_t (t = 0, 1)$.

Deriving equations similar to (2) and (3) for the pairs of points $\hat{q}_0$ and q_0, q_1 and $\hat{q}_0$ and q_0 and $\hat{q}_1$, we finally obtain:

6 In this connection it should be noted that C_0 will be close to C_1 only if relative prices are much the same. Yet, if it were known that relative prices were similar then there would be no doubt that any conventional fixed weight index would provide a close approximation to the true index.

7 The assumption that the utility function is a polynomial of the second degree implies linearity of the Engel curves.

$$(4)\quad \Sigma(\omega_1 p_1 + \omega_0 p_0)(q_0 - q_1) + (\omega_0 + \hat{\omega}_0)\Sigma p_0(\hat{q}_0 - q_0) + \Sigma(\omega_1 p_1 + \hat{\omega}_0 p_0)(q_1 - \hat{q}_0) = 2\{[U(q_0) - U(q_1)] + [U(\hat{q}_0) - U(q_0)] + [U(q_1) - U(\hat{q}_0)]\} = 0,$$

and

$$(5)\quad \Sigma(\omega_0 p_0 + \omega_1 p_1)(q_1 - q_0) + (\omega_1 + \hat{\omega}_1)\Sigma p_1(\hat{q}_1 - q_1) + \Sigma(\omega_0 p_0 + \hat{\omega}_1 p_1)(q_0 - \hat{q}_1) = 2\{[U(q_1) - U(q_0)] + [U(\hat{q}_1) - U(q_1)] + [U(q_0) - U(\hat{q}_1)]\} = 0.$$

From (1) and (4) we obtain

$$(6)\quad \frac{\omega_1}{\omega_0} = \frac{\Sigma p_0(\hat{q}_0 - q_0)}{\Sigma p_1(\hat{q}_0 - q_0)}$$

and from (1) and (5)

$$(7)\quad \frac{\hat{\omega}_1}{\omega_0} = \frac{\Sigma p_0(\hat{q}_1 - q_1)}{\Sigma p_1(\hat{q}_1 - q_1)}$$

It has previously been noted that because of the assumption concerning the form of the utility function, the Engel curves C_0 and C_1 are linear functions of income in the tetrahedron T. This is likewise true of the marginal utility of money. For the Engel functions $q_t' = f_t'(\rho_t), \ldots, q_t^n = f_t^n(\rho_t)$ and the marginal utility of money $\omega_t(\rho_t)$ are the solutions of the $n + 1$ equations,

$$\frac{\partial U}{\partial q_t'} = \omega_t p_t' = \ldots = \frac{\partial U}{\partial q_t^n} = \omega_t p_t^n, \quad \Sigma p_t q_t = \rho_t,$$

in the unknown quantities $q_t', \ldots, q_t^n$, ω_t, and since $U(q)$ is a polynomial of the second order in the tetrahedron T, $\partial U/\partial q^i$ is linear in T.

Now from (1) and from observation of Figure 7, it is obvious that

$$U(q_1) \leqq U(q_0) \leqq U(\hat{q}_1).$$

Since $U(q)$ is continuous, there exists a point $\bar{q}_1$ which lies on the straight line segment determined by q_1 and $\hat{q}_1$ (all points of which segment lie on C_1) for which

$$U(\bar{q}_1) = U(q_0). \tag{8}$$

If, as previously suggested, $\Sigma p_1 q_1$ be denoted by $\rho(q_1)$, $\Sigma p_1 \hat{q}_1$ by $\rho(\hat{q}_1)$ and $\Sigma p_1 \bar{q}_1$ by $\rho(\bar{q}_1)$, then the marginal utility of money $\omega(\rho_1)$ is a linear function of the income ρ_1 in the interval between $\rho(q_1)$ and $\rho(\hat{q}_1)$. Hence we have within this interval

$$\frac{\omega(\rho_1)}{\omega_0} = \alpha\rho + \beta, \tag{9}$$

where α and β are certain constants. It should be borne in mind that while $\omega(\rho_1)$ in this equation is a variable, ω_0 is a constant indicating the marginal utility of money in the situation q_0. We can calculate the values of α and β since the value of $\omega(\rho_1)/\omega_0$ has already been given above for $\rho_1 = \rho(q_1)$ and $\rho_1 = \rho(\hat{q}_1)$. These values may be written as

$$\frac{\omega[\rho(q_1)]}{\omega_0} = \frac{\omega_1}{\omega_0} = \lambda \qquad \text{and}$$

$$\frac{\omega[\rho(\hat{q}_1)]}{\omega_0} = \frac{\hat{\omega}_1}{\omega_0} = \mu,$$

where λ refers to the right-hand side of the equation (6) and μ to the right-hand side of (7).

For α and β we obtain the values

$$\alpha = \frac{\mu - \lambda}{\rho(\hat{q}_1) - \rho(q_1)} \qquad \text{and} \tag{10}$$

$$\beta = \lambda - \rho(q_1)\frac{\mu - \lambda}{\rho(\hat{q}_1) - \rho(q_1)}$$

Denoting the marginal utility of money in the situation $\bar{q}_1$ by $\bar{\omega}_1$, we get

$$\alpha\rho(q_1) + \beta = \frac{\omega_1}{\omega_0} = \lambda,$$

$$\alpha\rho(\hat{q}_1) + \beta = \frac{\hat{\omega}_1}{\omega_0} = \mu, \tag{11}$$

$$\alpha\rho(\bar{q}_1) + \beta = \frac{\bar{\omega}_1}{\omega_0}.$$

Since (3) depends only on the two points involved being on the same Engel curve, it will still hold if we subststute $\bar{q}_1$ for $\hat{q}_1$ and $\bar{\omega}_1$ for $\hat{\omega}_1$; with the aid of (2) and (8) we have

$$\Sigma(\omega_0 p_0 + \omega_1 p_1)(q_1 - q_0) + (\omega_1 + \bar{\omega}_1)\Sigma p_1(\bar{q}_1 - q_1) = 0$$

and therefore on account of (1)

$$\Sigma p_1(q_1 - q_0) + \left(1 + \frac{\bar{\omega}_1}{\omega_1}\right) \Sigma p_1(\bar{q}_1 - q_1) = 0. \tag{12}$$

From (11) follows

$$\frac{\bar{\omega}_1}{\omega_1} = \frac{\alpha\rho(\bar{q}_1) + \beta}{\alpha\rho(q_1) + \beta} \tag{13}$$

If we substitute in (12) for $\bar{\omega}_1/\omega_1$ the right-hand side of (13) we get

$$\Sigma p_1(q_1 - q_0) + \left(1 + \frac{\alpha\rho(\bar{q}_1) + \beta}{\alpha\rho(q_1) + \beta}\right)\Big(\rho(\bar{q}_1) - \rho(q_1)\Big) = 0.$$

This is an equation of the second degree in $\rho(\bar{q}_1)$, the roots of which are

$$\rho(\bar{q}_1) = \frac{-\beta \pm \sqrt{[\alpha\rho(q_1) + \beta]^2 - \alpha D}}{\alpha} \tag{14}$$

where

$$D = [\alpha\rho(q_1) + \beta]\Sigma p_1(q_1 - q_0).$$

From (14) we obtain

$$\frac{\bar{\omega}_1}{\omega_0} = \alpha\rho(\bar{q}_1) + \beta = \pm\sqrt{[\alpha\rho(q_1) + \beta]^2 - \alpha D}.$$

Since $\bar{\omega}_1/\omega_0$ is a nonnegative number, we have

$$\frac{\bar{\omega}_1}{\omega_0} = +\sqrt{[\alpha\rho(q_1) + \beta]^2 - \alpha D},$$

and hence

$$\frac{\bar{\omega}_1}{\omega_1} = \sqrt{\frac{[\alpha\rho(q_1) + \beta]^2 - \alpha D}{[\alpha\rho(q_1) + \beta]^2}} = \sqrt{1 - \frac{\alpha D}{[\alpha\rho(q_1) + \beta]^2}}$$

$$= \sqrt{1 - \frac{\alpha\Sigma p_1(q_1 - q_0)}{\alpha\rho(q_1) + \beta}}.$$

Since, according to (1), $\Sigma p_1 q_0 = \Sigma p_1 \hat{q}_1 = \rho(\hat{q}_1)$, we get on account of (10) and (11)

$$\frac{\bar{\omega}_1}{\omega_1} = \sqrt{1 - \frac{\alpha[\rho(q_1) - \rho(\hat{q}_1)]}{\lambda}} \tag{15}$$

$$= \sqrt{1 - \frac{\lambda - \mu}{\lambda}} = \sqrt{\frac{\mu}{\lambda}}.$$

If we substitute in (12) for $\bar{\omega}_1/\omega_1$ the value $\sqrt{\mu/\lambda}$, then we get from (12),

$$I_0 = \frac{\Sigma p_1 \bar{q}_1}{\Sigma p_0 q_0} = \frac{\Sigma p_1 \left(q_0 + \sqrt{\frac{\mu}{\lambda}} q_1 \right)}{\Sigma p_0 \left(q_0 + \sqrt{\frac{\mu}{\lambda}} q_1 \right)}, \tag{16}$$

recalling from (1) that $\Sigma p_0 q_0 = \Sigma p_0 q_1$.

This formula is further transformed by Wald as follows.

Consider the Engel function $q_0{}^i = \alpha_0{}^i \rho_0 + \beta_0{}^i$. Multiplying both sides of this equation by $p_0{}^i$ and summating for all commodities we obtain:

$$\Sigma p_0 q_0 = \Sigma p_0 \alpha_0 \rho_0 + \Sigma p_0 \beta_0.$$

Recalling that $\rho_0 = \Sigma p_0 q_0$, it is obvious that $\Sigma p_0 \alpha_0 = 1$ and $\Sigma p_0 \beta_0 = 0$.

Similarly for the Engel function $q_1{}^i = \alpha_1{}^i \rho_1 + \beta_1{}^i$ it may be shown that

$$\Sigma p_1 \alpha_1 = 1 \qquad \text{and} \qquad \Sigma p_1 \beta_1 = 0.$$

Denote $\Sigma \alpha_i p_j$ by a_{ij} and $\Sigma \beta_i p_j$ by b_{ij}. Then $a_{11} = a_{00} = 1$ and $b_{11} = b_{00} = 0$.

We get from (6) and (7)

$$\lambda = \frac{a_{00}}{a_{01}} = \frac{1}{a_{01}} \qquad \text{and} \qquad \mu = \frac{a_{10}}{a_{11}} = a_{10}. \tag{17}$$

Because of (1) we have

$$\Sigma p_0 q_0 = \Sigma p_0 q_1 = \rho(q_0). \tag{18}$$

Since $\Sigma p_1 q_1$ is equal to $\rho(q_1)$ and therefore

$$q_1' = \alpha_1'\rho(q_1) + \beta_1', \ldots, q_1^n = \alpha_1^n\rho(q_1) + \beta_1^n,$$

we get on account of (1)

$$a_{10}\rho(q_1) + b_{10} = \rho(q_0).$$

Hence

$$\rho(q_1) = \frac{\rho(q_0) - b_{10}}{a_{10}} = \Sigma p_1 q_1 \tag{19}$$

Also

$$\Sigma p_1 q_0 = a_{01}\rho(q_0) + b_{01}. \tag{20}$$

If we substitute in (16) for λ, μ, $\Sigma p_1 q_0$, $\Sigma p_1 q_1$, $\Sigma p_0 q_0$, and $\Sigma p_0 q_1$, their expressions from (17), (18), (19), and (20), then we get

$$I_0 = \frac{a_{01} + \frac{1}{a_{10}}\sqrt{a_{10}a_{01}}}{1 + \sqrt{a_{01}a_{10}}} + \frac{1}{\rho(q_0)} \cdot \frac{b_{01} - \frac{b_{10}}{a_{10}}\sqrt{a_{01}a_{10}}}{1 + \sqrt{a_{01}a_{10}}}, \tag{21}$$

or

$$I_0 = \sqrt{\frac{a_{01}}{a_{10}}} + \frac{1}{\rho(q_0)} \cdot \frac{b_{01} - \frac{b_{10}}{a_{10}}\sqrt{a_{01}a_{10}}}{1 + \sqrt{a_{01}a_{10}}}. \tag{22}$$

This is Wald's final formula. The magnitudes in (22) would be known if the α and β coefficients for the Engel curves were statistically determined.

Frisch.[8]

Let $$r(U) = \frac{\rho_t(U)}{P_t(U)},$$

8 The "flexibility method" given below is described in Ragnar Frisch's "Annual Survey of General Economic Theory: The Problem of Index Numbers," *Econometrica*, vol. IV, no. 1, January 1936, pp. 31-38. In this article Frisch also describes the "double expenditure method" which results in the derivation of relationships similar to those obtained by Konüs above. For criticisms of the flexibility method see Abram Burk "Real Income, Expenditure Proportionality, and Frisch's 'New Methods of Measuring Marginal Utility,'" *Review of Economic Studies*, vol. IV, no. 1, 1936, pp. 33-52, and R. G. D. Allen, "On the Marginal Utility of Money and Its Application," *Economica*, vol. XIII, 1933, pp. 186-209.

where r represents real income, ρ_t represents money expenditures, and P_t represents a deflation factor, all variables being a function of the utility level, U, and ρ_t and P_t further depending on the price situation t. Then

$$(1) \qquad r(U) = \frac{\rho_0(U)}{P_0(U)} = \frac{\rho_1(U)}{P_1(U)} = \cdots \frac{\rho_t(U)}{P_t(U)} = \cdots,$$

$$\frac{d \log \rho_t}{d \log r} = 1 + \frac{d \log P_t}{d \log r} = \frac{1}{1 - \dfrac{d \log P_t}{d \log \rho_t}} \qquad \text{and}$$

$$(2) \qquad \frac{d\rho_t}{dr} = P_t\left(1 + \frac{d \log P_t}{d \log r}\right).$$

Frisch then defines *nominal* money utility as

$$\omega_t = \frac{dU}{d\rho_t}$$

and *real* money utility as

$$w(r) = \frac{dU}{dr}$$

Hence

$$(3) \qquad \omega_t = \frac{w(r)}{\dfrac{d\rho_t}{dr}} = \frac{w(r)}{P_t\left(1 + \dfrac{d \log P_t}{d \log r}\right)}$$

Frisch then assumes that "equivalent expenditures," or the receipt of equal real incomes, may be identified by the equality of "money flexibility," which he defines as

$$\breve{w}(r) = \frac{d \log w(r)}{d \log r}.$$

Suppose that an "independent reference set" of commodities may be discovered. This subset of goods, which may in fact consist of one commodity only, has the following characteristics: (1) the utility derived from these commodities must be independent of the quantities of all other commodities; (2) the supply-

price functions for these commodities are independent, so that given the total expenditures for this independent set of commodities, adaptation of prices and quantities within the set is completely determined without reference to prices or quantities of goods outside the set. Denote the aggregate quantity of these independent goods by X, the marginal utility of the aggregate as $\mu(X)$ and the price of this aggregate at H_t. Thus in analogy with (3), he writes:

$$(4)\quad \frac{w(r)}{P_t(r)\left(1+\frac{d\log P_t(r)}{d\log r}\right)} = \frac{\mu(X)}{H_t(X)\left(1+\frac{d\log H_t(X)}{d\log X}\right)}.$$

Since under a given t, X is a function $X = E_t(r)$, Frisch writes:

$$(5)\quad w(r) = \alpha_t(r)\mu(X), \text{ where}$$

$$(6)\quad \alpha_t(r) = \frac{P_t(r)\left[1+\frac{d\log P_t(r)}{d\log r}\right]}{H_t(X)\left[1+\frac{d\log H_t(X)}{d\log X}\right]} = \frac{P_t(r)}{1-\check{P}_t(r)}\cdot\frac{1-\check{H}_t(X)}{H_t(X)}$$

where $\check{H}_t$ and $\check{P}_t$ are the logarithmic derivatives of prices with respect to the corresponding money expenditures, that is

$$\check{H}_t = \frac{d\log H_t(X)}{d\log \xi_t}$$

and

$$\check{P}_t = \frac{d\log P_t}{d\log \rho_t}$$

where

$$\xi_t = X\cdot H_t(X)$$

The price index at the level U between situations o and t can also be written

$$P_{ot}(U) = \frac{P_t(U)}{P_o(U)},$$

and similarly

$$H_{ot}(U) = \frac{H_t(U)}{H_o(U)}.$$

From (5), Frisch says, $\alpha_t(r)$ becomes an observable function provided "the relative price indices, P_{0t} and H_{0t}, are determined by some approximation method . . . and one of the P functions, for instance $P_0(t)$, and one of the H functions, for instance $H_0(t)$, are chosen conventionally . . . "

In any event, if values were obtained for $\alpha_t(r)$ and for r, it would be possible, under the stated assumptions, by choosing r_0 and r_1 in o and 1 close together and such that $X_0 = X_1$, to obtain an approximation of a point measurement of $\check{w}$, since from (5),

$$(7) \quad \check{w}_0 \text{ (approx.)} = \frac{\log w(r_0) - \log w(r_1)}{\log r_0 - \log r_1}$$

$$= \frac{\log \alpha_0(r_0) - \log \alpha_1(r_1)}{\log r_0 - \log r_1} \qquad (\text{if } X_0 = X_1)$$

Similarly, by choosing r_2 and r_3 in 2 and 3 in the same way, an approximation may be determined for $\check{w}_2$. Now from (3) there may be derived

$$(8) \quad \check{w} = \frac{\check{\omega}_t + \check{P}_t}{1 - \check{P}_t} + \frac{\dfrac{d\check{P}_t}{d \log \rho_t}}{(1 - \check{P}_t)^2} \qquad (d \text{ taken along the } t\text{-path})$$

where

$$\check{\omega}_t = \frac{d \log \omega_t}{d \log \rho_t}.$$

By defining $r(U)$ in such a way that P_0 is a constant, $\check{P}_0 = 0$ and $\check{w}_0 = \check{\omega}_0$. Further if $\check{w}(r)$ is a monotonic function of r, then if we find points in o and 2 for which $\check{w}_0 = \check{w}_2$, these two points will have the same real income. Using (8) we may then derive the following relation between the nominal money flexibilities $\check{\omega}$ of pairs of equivalent points on o and 2:

$$1 + \check{\omega}_0 = \frac{1 + \check{\omega}_2}{1 - \check{P}_{02}} + \frac{\dfrac{d \log \check{P}_{02}}{d \log \rho_2}}{(1 - \check{P}_{02})^2} \tag{9}$$

The process of obtaining the cost of living change between 0 and 2—P_{02}—from the above equation is described by Frisch as follows:[9]

"P_{02} in (9) is not known but will be so as soon as equivalence between points on 0 and 2 is determined. And this, in turn, is defined by the fact that the equation is fulfilled. Consequently, it should be possible to determine equivalence by an iteration process.

"If there is expenditure proportionality between 0 and 2, (9) gives

$$\check{\omega}_t = \check{\omega}_2 \tag{10}$$

"The point correspondence between 0 and 2 defined by (10) seems, therefore, a plausible first approximation. Let $P_{02}^{(1)}$ be the index to which it leads. Inserting this in the right member of (9), we get

$$\Omega^{(1)} = \frac{1 + \check{\omega}_2}{1 - \check{P}_{02}^{(1)}} + \frac{\dfrac{d \log \check{P}_{02}^{(1)}}{d \log \rho_2}}{(1 - \check{P}_{02}^{(1)})^2} \tag{11}$$

which may be computed in any point along 2. Next consider the point correspondence defined by

$$1 + \check{\omega}_0 = \Omega_{02}^{(1)}. \tag{12}$$

It leads to an index $P_{02}^{(2)}$ which, inserted in the right member of (9), gives a function of $\Omega_{02}^{(2)}$ that may again be compared with $1 + \check{\omega}_0$, etc. If the process converges, we get the indifference index between 0 and 2."

Staehle II.[10] Let

9 Original references in the following quotation have been altered to conform with the numbering of equations employed here.

10 H. Staehle, "A General Method for the Comparison of the Price of Living," *Review of Economic Studies*, vol. XIV, no. 3, June 1937, pp. 205-214.

$$(1) \qquad \frac{q_1}{q_0} - \frac{\Sigma\left(\frac{q_1}{q_0}\right)q_0p_0}{\Sigma q_0p_0} = \frac{q_1}{q_0} - \frac{\Sigma q_1p_0}{\Sigma q_0p_0}$$

represent the deviation of the quantity ratio (q_1/q_0) from the weighted average of the quantity ratios for all articles. Each such deviation, taken relative to the average $\Sigma q_1p_0/\Sigma q_0p_0$, may be represented by:

$$(2) \qquad \frac{q_1}{q_0}\left(\frac{\Sigma q_0p_0}{\Sigma q_1p_0}\right) - 1$$

Then the weighted average of the absolute values of the relative deviation (2) is, using the q_0p_0's as weights:

$$(3) \qquad D = \Sigma\left|\frac{q_1}{q_0}\frac{\Sigma q_0p_0}{\Sigma q_1p_0} - 1\right| \cdot \frac{q_0p_0}{\Sigma q_0p_0} = \Sigma\left|\frac{q_1p_0}{\Sigma q_1p_0} - \frac{q_0p_0}{\Sigma q_0p_0}\right|$$

Staehle suggests that the value of D be computed for each pair of income groups in the two situations compared. He assumes that the value of D secured for any pair of quantity sets belonging to different situations will arise from two types of factors: (a) differences in tastes, milieu, and prices; and (b) differences in real incomes. Of all the values secured for D, Staehle further assumes, the smallest value will represent an "irreducible minimum dissimilarity" reflecting differences in tastes,[11] environment, habits, prices, etc., and the two income groups for which this minimum value occurs may be considered as enjoying equal real incomes. The ratio of the average money incomes of these groups would therefore provide a true cost of living index.

11 The meaning of "equal real incomes" as between groups with different tastes is not elaborated by Staehle. See p. 30 above.

APPENDIX B

ADDITIONAL PRACTICAL IMPLICATIONS: THE INCOME AND SPACE DIMENSIONS OF COST OF LIVING INDEX NUMBERS

Space

Some of the problems relating to the statistical definition of a homogeneous group—i. e., a group of families sharing the same taste patterns—for cost of living index number construction, were discussed in Chapter V. Suppose that such a group were defined—for example, wage-earners in a particular income class in a particular city. Suppose also that this income class was wide enough to embrace the bulk of all wage-earners, that a cost of living index was computed for this group and that similar indices were computed for corresponding groups defined in a representative selection of other cities in the United States. Could these separate city indices be meaningfully combined into a grand cost of living index for urban wage-earners in the United States?

In approaching this problem attention should be focused initially on the type of question one would expect to answer with this nation-wide index. In analogy with the definition employed in the body of this work, one may expect to determine what change in money-income would be required to maintain the real income levels which had been experienced in the base period. This objective would be satisfied by a weighted average of the individual city indices, provided the indices were weighted by the total monetary expenditures of the wage-earners in each of the cities.[1] In general this is the approach dictated if interest lies in determining the change in the total wage bill required to balance a change in living costs, or in the problem of deflating the monetary value of a bill of goods consumed by wage-earners.

1 Of course, in a practical case if a particular city were presumed to represent within the sample a group of other nearby cities, then the weights employed would be the estimated total monetary expenditures of wage-earners in all the represented cities.

An alternative approach would be suggested by substituting for the question above one asking what is the *average change* in the money incomes required to maintain the real incomes experienced in the base period. Here attention is focused on the average wage-earner family rather than on the total money incomes of wage-earners, and accordingly population weights rather than money expenditure weights for each city are called for.[2]

Whichever of the two methods is employed, however, it is assumed that for each of the cities quantity weights relate to the consumption of wage-earners in the same period in each city—say the year 1940. For it is only in this way that the real income level priced nationally can be given useful meaning—as a composite of the levels actually prevailing in these cities at a particular time. If weights were derived at different time periods in each city, the average of cost of living indices computed in this way might have only nebulous meaning if any, for the real income levels priced in each city would relate to different periods and incidentally might differ rather widely.[3]

Income

Let us assume that a cost of living index has been constructed for urban wage-earners on the basis of the real income level

2 This is the method employed in the Bureau of Labor Statistics Consumers' Price Index.

3 This is especially worth noting because of plans recently proposed for revising weights in the Bureau of Labor Statistics Consumers' Price Index. According to this plan, data will be collected each year on the purchases of consumers; for any given year, however, only from three to seven of the thirty-four large cities included in the index will be covered. It is planned on the basis of these collections to revise weights annually, but only for those cities covered in each year's survey. In the course of time, were this plan actually carried out, the index would reflect a conglomerate of planes of living—relating to that typical in from five to ten different years and differing among the various cities.

In connection with the proposals made in the text above, it should be noted that a simultaneous revision of weights in *all* cities at (say) five year intervals need not necessarily require even a study of consumer purchases every five years simultaneously covering *all* cities; the basis for revising weights throughout the index may be derived from a study of consumer purchases in a carefully selected *sample* of these cities.

experienced in a particular year. It is further assumed that the real income level is that experienced by the *modal group*[4] of wage-earners in each city included, for this would appear to be the most meaningful choice for practical purposes. Since typical real income levels vary with the business cycle, however, it does not follow that the experience of the modal group of any one year will necessarily be of topmost interest in every other year of a business cycle. Even if analysis is confined to wage-earners, therefore, there is the possibility that it may be desirable to price more than one plane of living—one typical of a year of prosperity, perhaps, the other typical of a year in depression.

Whether only one or more than one index is required for wage-earners, however, can be answered only by experiment. If the cost of living index differed very widely for groups separated only by small differences in income, the number of separate measures required might be high. Available evidence, as indicated earlier, suggests that the contrary is true—i. e., that only one cost of living index may be sufficient to represent the experience of the entire group of urban wage-earners throughout the course of the business cycle.

Suppose, however, interests were broadened and in addition to a cost of living index for urban wage-earners separate indices were computed for farmers, salaried workers, and other occupational and income groups. Whether such separate measures were actually needed, of course, would again depend on the magnitude of differences in cost of living experience found by experiment to prevail among the several groups distinguished. Would it be meaningful, in any event, to combine these various index numbers into one grand cost of living index number for the United States as a whole?

As in the problem of combination discussed above, this might be accomplished effectively in either of two ways. If interest were in the change in money income required to provide the people of the United States with the same planes of living as had

4 The question of how *broad* an income range may be included in the modal group was discussed above, on p. 62.

prevailed in the budgetary year, money expenditures would be used as weights in combining the separate cost of living indices into an over-all average. If interest were in the *average* change in money income required for the people of the United States under the same circumstances, population weights would appear to be in order. In either case, however, the grand cost of living index would record the change in the cost of a set of planes of living typical of a particular time—that of the base period—and would reflect (through its weights) the pattern of income and occupational distributions as well as the geographical distribution of persons typical of that time.

Thus, it may be observed that any "combined" cost of living index—as opposed to a "pure" cost of living index which refers to the experience of a single homogeneous group—may be interpreted only in "hypothetical" terms and its usefulness is accordingly limited by the assumptions upon which it is based.

APPENDIX C

A NOTE ON WARTIME EXPERIENCE

In several nations, and particularly in the United States, the accuracy of cost of living index numbers became a matter of public debate[1] during World War II. In substantial part these controversies were political rather than technical and bore at least as much upon the *objectives* of wage control as they did upon the meaning and reliability of the measures concerned. Because of the widespread interest aroused in the questions raised, however, brief attention will be given here to certain underlying theoretical considerations.

At several points previously it has been noted that during the period of a major war the conditions necessary for construction of a meaningful cost of living index are largely lacking. There are numerous reasons for this, though they all converge upon one principle, fundamental to the entire foregoing analysis and indeed to the *meaning* of the measure under discussion: cost of living index numbers rest ultimately on the twin assumptions of constant tastes and constant environment. Insofar as environment changes, the accuracy of a cost of living index diminishes. Insofar as tastes change, the very concept of a cost of living index loses validity. Substantial changes in environment inevitably alter the effective tastes of the community.

1 The most important published reports dealing with or touching upon technical aspects of the subject in the United States were: President's Committee on the Cost of Living, *Report of November 10, 1944* (Washington, 1944). This publication includes the report of June 15, 1944 of the technical committee (Wesley C. Mitchell, Chairman) appointed by the chairman of the President's Committee on the Cost of Living, as well as comments upon this report. American Statistical Association, "An Appraisal of the U. S. Bureau of Labor Statistics Index" (prepared by a special committee of the American Statistical Association, Frederick C. Mills, Chairman), *Journal of the American Statistical Association*, vol. XXXVIII, no. 224, Dec. 1943. Philip Murray and R. J. Thomas, *Living Costs in World War II, 1941-1944* (Congress of Industrial Organizations, Washington, D. C., 1944). It should be noted that none of these reports approached the problem within the theoretical framework of the present study.

At most, over a relatively short span of peacetime years, the assumptions of constant tastes and constant environment are roughly approximated; exceptions at the fringe are always present. During a major conflict as extensive in time and in degree of economic mobilization as World War II there appears little doubt that changes in environment represented *fundamental* revisions rather than exceptions at the fringe, and it is also highly probable that fundamental revisions occurred in tastes as well.

During World War II the variety and quantity of goods sold were profoundly altered, for the most part by Government fiat, with consumers' durable goods, for example, disappearing almost entirely from the market and the supply of numerous other basic commodities strictly limited. Because of material shortages and also as a reaction to price control[2], the *quality* of goods offered for sale changed rapidly and in bewildering variety; nor was there any way of obtaining a quantitative evaluation of these changes. The geographical distribution of the population shifted and for a considerable sector the conditions of living were significantly revised.

There seems little doubt that tastes (as defined above) of the populace were affected, not only by the environmental changes noted but also by the mass psychological impact of the war itself. Workers, in many cases, lived in different places, held new and unaccustomed jobs, faced expenses for transportation, work clothes, housing, etc., often quite different from those experienced before. Families were separated; wives, children, and the retired often worked, and in numerous other ways the fundamental factors affecting the operation of households changed.

Moreover, conditions determining the environment as well as those which might conceivably affect tastes were themselves dynamic. The needs of war and the conditions imposed by them upon civilian life were in an almost constant state of flux.

The important practical difficulties of collecting accurate prices under wartime conditions have been examined at length

2 See my "Indirect Price Increases," *Monthly Labor Review*, November 1942.

elsewhere, and surely in the light of price control the task of obtaining prices at which goods were actually sold must not be underestimated. But even if perfection had been achieved on this score, it is important to note that the foregoing alterations in environment, as well as their probable effects upon tastes, would have vitiated any attempt to measure changes in the cost of living for wage-earners in the sense in which the term has been defined in theory.

At best, then, a "cost of living" index during World War II could have provided only a rough indication of the direction and pace of changes in retail prices. Strictly interpreted, the *magnitude* of changes recorded by these measures could have had only tenuous meaning, if any. That they failed to provide measures of the change in living costs, within the economic meaning of that term, was an inevitable result of the nature of wartime conditions.

These remarks should not be construed as implying that it is impossible to define objective standards for wage control during a war period. In order to examine this latter question it is necessary to know precisely what the goals of wage administration are. Wisely selected goals, however, would be framed in terms of standards which *are* applicable to a war period and by this token the measures analyzed in this study would be ruled out.

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INDEX