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## THE EXCHANGE STABILITY PROBLEM

*“The idea that there exists a natural tendency to equilibrium in international trade is the central principle of the whole theory of our subject. Just for that reason, however, its truth must not be taken for granted too readily”.*

(BARRETT, WHALE: *International Trade*,  
LONDON 1934, p. 86)

# THE EXCHANGE STABILITY PROBLEM

BY

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## PREFACE

The question of the nature of the balance-of-payments equilibrium is an interesting one, from both the theoretical and the practical points of view. The large number of devaluations which took place after 1930 illustrates its practical importance. Is there a 'natural' equilibrium rate of exchange towards which free rates will tend to move, or does no such rate exist? Is it true, as is frequently maintained, that disturbances in the balance of payments can easily be overcome because of the high elasticity of supply and demand of foreign currency? The question of the nature of that equilibrium is also fundamental to our attitude vis-à-vis the dollar shortage problem.

Until recently only a restricted number of students of economic science occupied themselves with a rigorous mathematical formulation of these problems. Interesting contributions were made by such authors as Mr Bickerdike, Mrs Robinson, Mr Brown, Prof. Metzler and a few other investigators. Their results were at first sight partly contradictory. Dr Stuvell has succeeded in showing that these results can also be obtained as special cases of a more general case, which he exposes in this book. Given the complexity of the problem, he has taken care to present the whole matter as clearly and as simply as possible, stating his assumptions one by one, and explaining each relation verbally. The great number of

special cases he studies are listed compendiously. Dr Stuvcl gives not only the general algebraic formulae, but he also tries to calculate numerical values for the final elasticities in which he is interested. Among other things he is able to show that the influence of the import and export elasticity on these final elasticities is only important in certain classes of cases; in other cases it is before all the marginal propensity to spend, and the price elasticity of general home demand that are decisive. I warmly recommend this study to all interested in this field.

J. TINBERGEN

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## CHAPTER I. THE PROBLEM

### I. I

### INTRODUCTION

Economists have always taken a keen interest in the problems of international trade. Therefore one would expect that considerable agreement had been reached at least as to the basic principles of this branch of economic science. But the facts belie this expectation. There appears to be a great deal of controversy as to the fundamental doctrines of foreign trade policy, and as to the theories of international trade and international payments.

As for *foreign trade policy* one need only recall the well-known and, as it seems, everlasting dispute between free-traders and protectionists.

In the field of *the theory of international trade* (in the strict sense of the term) there has been for a long time a good deal of agreement. This was mainly due to the high prestige enjoyed by the classical doctrines of comparative costs and reciprocal demand as developed by RICARDO and MILL. These doctrines were based on the labour-cost theory of value, a theory which unduly emphasized the part played by costs of production in the determination of the value of a product. By the end of the nineteenth century this theory of value had to give way to the so-called marginal utility theory, developed by Menger, Walras, Jevons, Marshall and J. B. Clark. This theory laid more stress on the influence of demand. Notwithstanding this revolution in the theory of value,

the classical doctrines with regard to international trade remained intact long afterwards and even to-day they have their defenders, of whom especially TAUSSIG should be mentioned. His main opponents are OHLIN and HABERLER, who, about 1930, independently of each other, succeeded in providing a well-rounded theory of international trade based on the marginal utility theory of value.

Still different is the situation in the field of *the theory of international payments*, to which the subject-matter of this book belongs. The main doctrine of this theory is that a disturbance of the equilibrium in the balance of payments evokes forces which will automatically restore equilibrium, or, stated in another way, that the balance of payments equilibrium is a stable equilibrium. The authority of this doctrine remained almost unimpaired until the present day, although it was not left completely unchallenged over the course of time. But, as we shall see in what follows, the decisive step in attacking this principle has been taken only comparatively recently.

In 1879 MARSHALL already raised some doubts as to the validity of the principle of stable equilibria in his famous — but to those economists who possess only a limited knowledge of mathematics not easily accessible — essay on the “Pure Theory of Foreign Trade”<sup>1)</sup> later attached as Appendix J to his book “Money, Credit and Commerce”<sup>2)</sup>. MARSHALL himself, in the meantime, did not consider his criticism to be serious as his own words show<sup>3)</sup>.

<sup>1)</sup> A. MARSHALL: *Pure Theory of Foreign Trade and Pure Theory of Domestic Values* (1879), reprinted in *Series of Reprints of Scarce Tracts on Economics and Political Science* (London 1930, re-issued 1935).

<sup>2)</sup> A. MARSHALL: *Money, Credit and Commerce*, first edition (London 1923).

<sup>3)</sup> From the chapter which deals with this matter he says himself that “it has not much bearing on pressing practical problems” (*op. cit.* p. 177), whereas in the Appendix he says, in connection with the very special conditions that should be fulfilled in

The same is true, though to a lesser degree, of WHALE<sup>1)</sup>, who discusses a wide variety of possibilities that the natural tendency to equilibrium does not work because of the presence of counteracting forces of a more or less accidental nature. Such factors as the money illusion, the propensity to spend, and the influence of the volume of production on imports, which will prove to be of strategic importance in our analysis, are however not mentioned.

Reference should also be made to YNTEMA, who undoubtedly made an important contribution to the theory of international trade in his book, which carries the modest title: "A Mathematical Reformulation of the General Theory of International Trade"<sup>2)</sup>. In this book he points out that certain situations may arise in which the equilibrium in the balance of payments is unstable. But, like MARSHALL, he does not think these cases to be of any practical importance and calls them "highly improbable"<sup>3)</sup>.

Apart from a great admiration for YNTEMA's scientific skill I have some objections to his work. For the ordinary economist his work is inaccessible owing to the complicated mathematics, and even more so because of the very condensed connective text. This may be simply a matter of opinion, but I doubt whether his verbal argument is sufficient even for mathematical economists. More serious perhaps is the fact that his equations are of such a nature that they render statistical testing impossible and,

order to render an unstable equilibrium possible: "But all such suggestions derive their origin from the sport of the imagination rather than the observation of facts" (*op. cit.* p. 353) and again: "Nothing approaching to this has ever occurred in the real world: it is not inconceivable, but it is absolutely impossible" (*op. cit.* p. 354).

<sup>1)</sup> B. WHALE: *International Trade* (London 1934).

<sup>2)</sup> T. O. YNTEMA: *A Mathematical Reformulation of the General Theory of International Trade* (Chicago 1932).

<sup>3)</sup> YNTEMA: *op. cit.* p. 87.

therefore, exclude the possibility of confronting his theory with reality (cf. section 2.3). Finally, and this is my most serious objection, his theory is largely based on the simplifying assumption that supply and demand of international traded commodities depend only on the prices of these commodities. This definitely is an over-simplification as will be shown in section 1.6 where the indirect influence of a change in the exchange rate on the demand for imports will be discussed.

It seems TINBERGEN has been the first to stress the strategical importance of this *indirect influence* for the stability problem. In his article on "Unstable and Indifferent Equilibria in Economic Systems"<sup>1)</sup>, published in 1941, he showed — as others had done before him — that unstable and indifferent equilibria in the foreign exchange market are theoretically conceivable and may even materialize in practice, especially if the elasticities<sup>2)</sup> of import and export demand are very low. Apart from that, however, he also made it clear — and this I think was his great achievement — that there is a fairly high probability that such equilibria present themselves in reality, thanks to this indirect influence. In section 1.6 it is demonstrated how essential a part the indirect influence of changes in the rate of exchange plays in the argument which leads to TINBERGEN's important conclusion. In chapter IV a more complete argument is given. It will be shown that there are also certain special circumstances in which unstable and indifferent equilibria emerge quite independently of this indirect influence.

The name of the English economist BROWN should also

<sup>1)</sup> J. TINBERGEN: "Unstable and Indifferent Equilibria in Economic Systems" in *Revue de l'Institut International de Statistique* 1941 (1/2), pp. 36—50.

<sup>2)</sup> The elasticity is the percentage change in volume (quantity demanded or supplied) in response to a one per cent change in price.

be mentioned here, for in 1942 he expressed similar views in an article in the *Oxford Economic Papers* <sup>1)</sup>. It is worth noting that here again, as has so often happened in the history of economics, a new idea emerged more or less simultaneously in the minds of different persons working quite independently of each other.

Before reviewing the still ruling doctrine on our subject (sections 1.4 and 1.5) and some of TINBERGEN's views (section 1.6) it seems desirable to indicate briefly the place of the theory of international payments in the whole of economic theory (section 1.2) and to consider carefully some of the main concepts which will play a part in our argument, in order to avoid misunderstanding as to their definitions (section 1.3).

## I.2

THE SCOPE OF THE THEORY OF  
INTERNATIONAL PAYMENTS

The *subject-matter of economics* is the distribution of the limited resources available for the satisfaction of human wants over various destinations, taking account of the degree of urgency of the various human wants.

There is a large variety of resources, but they may conveniently be classified into a few major groups. To begin with there is the living as well as the dead nature which surrounds us. Furthermore, there is our own capacity for work. From the combination of the two, different kinds of products arise which enable us to satisfy our wants. In order to produce goods and services, however, something more is needed than only the two production factors mentioned so far viz. *nature and labour*. The pro-

<sup>1)</sup> A. J. BROWN: "Trade Balances and Exchange Stability" in *Oxford Economic Papers*, N<sup>o</sup>. 6, April 1942, pp. 57—75.

duction process has to be organised and requires time, and these two aspects account for two more factors of production viz. *enterprise and capital*. It is the entrepreneur in his capacity of organiser of the production process, who decides what products are going to be made and how they will be made. During the period of the process of production he must have command over part of the gifts of nature, part of the labour force and part of the available stock of goods originating from production in previous periods. This means that the owners of the production factors concerned cannot put these factors to any other use during this period. For this they have to be rewarded in terms of goods and services. To be able to give these rewards the entrepreneur has to call upon the owners of the available stock of goods originating from production in previous periods. This is where the production factor capital comes in. By putting the necessary means at the entrepreneurs' disposal they abstain from putting them to some use themselves. Just as the other factors of production (nature, labour, and enterprise) get rewards for their contributions to production (rents, wages, and profits respectively), so the production factor capital gets a reward for its contribution (interest). The time aspect of production, from which the share of capital in production arises, is very important. Production methods which require extra-time, or, stated in another way, more extensive use of the available stock of goods, have proved to be a forceful means of increasing the productivity of labour.

Another division of factors of production is that between imported and home factors of production, which will be used in the course of our argument. What choice is made between the various possible divisions is

mainly a matter of convenience and suitability with regard to the special problem under discussion. The justification of such a broad division lies in the fact that it has proved to be a very useful device in discussing economic problems.

Now let us see where logically the theory of international payments, of which the exchange stability problem forms part, comes in if one examines the structure of economic theory.

It may be true that the resources of nature are almost inexhaustible, nevertheless the means of satisfying wants are limited in each period owing to the fact that the complementary factor of production labour is limited. In this respect one should be aware of the fact that one of the human wants is leisure. One of the most fundamental equilibria in economics, determining the supply of labour, is the one between the satisfaction derived from an additional quantity of goods to be obtained by the supply of one additional unit of labour, and the satisfaction derived from leisure if that additional unit of labour is not supplied. This balancing of marginal utilities in the minds of individuals is the central principle of the subjective or modern *theory of value*.

Such a theory of value, however, is only the bare foundation-stone of economic theory as a whole. In addition the structure of economic theory comprises a theory of prices. Its *raison d'être* is that the abilities of people and the circumstances under which they live differ a great deal, as a consequence of which they have assumed the habit of exchanging the products of their labours so as to attain an optimum in the satisfaction of their wants. In this exchange of products certain quantitative relations, in which the products are exchanged, establish themselves.

These exchange-relations or prices of the units of the various commodities, expressed in terms of units of one or another of the other commodities, form the subject-matter of the *theory of prices*.

The direct exchange of commodities (barter), however, becomes increasingly difficult as the number of different kinds of goods exchanged increases. One of the reasons for this is that those who possess the products one desires, do not want the products one produces oneself. This and other difficulties can be surmounted in a simple way by intermediate exchanges. *A* exchanges his product *a* with *B* against his product *b*, which he does not want, but which he knows *C* wants in exchange for his product *c*, which is the product *A* desires; so *A* exchanges *B*'s product *b* against *C*'s product *c*. In this case the product *b* serves to *A* as a medium of exchange, and the exchange has become an indirect one. It is advisable to confine the number of goods which serve as medium of exchange to preferably one. At the same time this has the advantage that, apart from being a means of exchange, the commodity will become a suitable common denominator for expressing the prices of all other products in units of this commodity. One can take a still further step by freeing the means of exchange of its commodity-function by substituting for it a general assignment to goods and services. It is the *theory of money* which studies the variations in the value of this means of exchange, expressed in terms of goods and services, and the roots of this value.

Unfortunately we have not yet reached the stage where there is one world currency. Consequently one of the trade-partners engaged in importing or exporting goods or services is forced to exchange home currency for foreign currency. This means that there must be a market for

foreign exchange, just as there are markets for goods and services. Now this foreign exchange market is the research-object of the *theory of international payments*. As we have seen this theory does not deal with the exchange of commodities (theory of prices), or with the exchange of money for goods (theory of money), but with the exchange of different kinds of money.

We will refrain from completing the picture of the structure of economic theory. One important aspect of it viz. the distinction between statics and dynamics will be discussed at some length in section 2.5.

## I.3

## SOME OF THE MAIN CONCEPTS

In this section we shall deal briefly with some of the main concepts which enter into the theory of international payments and, as such, will form part of our argument.

The term *balance of payments* does not always have the same meaning in the literature on international trade and payments. As a matter of fact it is used for concepts which are essentially different <sup>1)</sup>.

One version is that the balance of payments is a confrontation *over a certain period* of either:

- (a) quantities of foreign exchange bought and sold, which by definition will be equal to each other, as buying and selling are only two sides of one and the same transaction; or
- (b) payments to and from abroad, which can also be made out of foreign currency holdings, which have been accumulated in previous periods; in the case where the payment-receiving country is on the gold standard gold may be put on one line with foreign currency holdings; or

<sup>1)</sup> See also HABERLER: *Der Internationale Handel* (Berlin 1933), pp. 14 and 15.

- (c) maturing claims on and liabilities to foreigners; or
- (d) new claims on and liabilities to foreigners.

In all cases except the first, the confrontation may refer to all transactions or to certain categories of transactions only (e.g. financial transactions connected with the sale or purchase of goods and services).

Those acquainted with the principles of bookkeeping and social accounting will perhaps express the view that the balance of payments, as defined above, is not a balance sheet in the ordinary sense of the word. They are right. If it were, it should be an account related to the situation at one particular moment, i.e. it should be an inventory account and part of the national balance sheet which shows the amount and composition of the national wealth. In fact the balance of payments should rather be considered as a subsidiary account of the national profit-and-loss account which reflects the development over a certain period and not the situation at one moment.

According to another version the balance of payments should be understood not as an account but in the schedule sense, i.e. as *the quantities of home or foreign currency supplied and demanded or their balance at various rates of exchange*. HABERLER endorses RICARDO's remark that this concept also leaves room for the underlying motives of business transactions to enter the picture. This was not the case with the above-mentioned balance of payments concepts which were all of a merely accounting character. In this study we will make use of the balance of payments concept in the schedule sense. For the determination of the demand and supply schedules for home currency use is made of statistical data, which in principle are based on the concept of the balance of payments as defined under (d).

We have already used several times the term *foreign*

*exchange market*. This should be understood not as an actual market to be found at a certain time at a certain place but as an abstract market embracing *all* exchange of home currency for foreign currency and vice versa.

Furthermore, the *exchange rate* is the price of home currency in terms of foreign currency units. This definition is in accordance with the *continental usage*. So if the exchange rate of the dollar in guilders were four guilders to a dollar and the dollar is considered to be the representative of foreign currency then the exchange rate of the guilder will be  $\frac{1}{4}$ , for f 1.00 = \$ 0.25. In the English usage it would be just the reverse, for there the exchange rate is measured in terms of the number of guilders to the pound, say 10; according to our definition the exchange rate of the guilder, taking the pound sterling as the representative of foreign currency, would be  $1/10$  instead of 10.

Finally, in the course of the argument it will prove to be useful to distinguish the items of the balance of payments into current and capital items. The latter category consists of those items which refer to credits to and from abroad as well as to changes in the country's gold reserves due to international payments. Together they form the *balance of payments on capital account*. All other payments to and from abroad belong to the so-called *balance of payments on income account or current account*. The distinction made is the same as that between the balance of goods and services on the one hand and the balance of gold and capital-flows on the other hand. It is this balance of payments on current account to which our attention will be directed in this monograph. Its balance will be referred to as the *balance on current account*.

#### 1.4 THE DIRECT INFLUENCE OF A CHANGE IN THE RATE OF EXCHANGE

Applying the *supply-and-demand scheme* to a *commodity market* and measuring prices along the vertical axis (ordinate) and quantities along the horizontal axis (abscis), as is usual in economics, the relation between price and quantity demanded will be represented by a downward sloping curve, whereas the relation between price and quantity supplied will be represented by an upward sloping curve. The downward slope of the demand curve is caused by the fact that the consumers attach a lower utility to each successive unit of the commodity put at their disposal. The upward slope of the supply curve has its origin in the increase in marginal costs as the total production of the commodity increases. Increasing marginal costs should be considered as the normal case. Decreasing marginal costs are conceivable mainly in two cases. The first case is that of a pseudo-monopoly, which presents itself if the optimum size of the firm (i.e. the size at which marginal costs reach their lowest level) exceeds the actual size of the industry, because demand falls short and would be unable to absorb the whole of the production of the firm at its optimum size at the then ruling market price (= marginal costs). The second case is that which intermittently arises in each enterprise over small intervals, for instance if a new machine is put to work and at the outset is not fully employed.

Now what happens if we apply the supply-and-demand scheme to the *foreign exchange market*? Should a downward slope of the demand curve and an upward slope of the supply curve here also be considered to be the normal case? The answer requires careful consideration, for in

certain respects the foreign exchange market differs a great deal from commodity markets, and so an answer in the affirmative to the second question should not be given too hurriedly.

Let us first consider what *direct influence* will be exercised on both sides of the market by a change in the rate of exchange. Suppose that for some reason or other the exchange rate of country A falls. This will cause a decrease in the prices of export products in terms of foreign currency and so it will become more attractive for foreigners to buy goods in country A. Foreign demand will become stronger and the *volume* of exports of goods and services will increase. The *value* of exports in terms of home currency will also increase, for *prices* of export products in terms of home currency will become higher as a consequence of the higher demand<sup>1)</sup>. In the case where exports are paid for in home currency foreign importers will demand this currency in exchange for their foreign currency. Where exports are paid for in foreign currency exporters in the home country will demand home currency in exchange for the foreign currency received. So it may be concluded that *ceteris paribus* (i.e. everything else remaining the same) a fall in the rate of exchange will always increase the demand for home currency (cf. however

<sup>1)</sup> In the case of decreasing marginal costs an upward shift in the demand curve may cause a lowering of the price of the product because of the downward slope of the supply curve. But even so this does not necessarily mean that the value of exports will also show a decrease, for where the price goes down the volume of exports goes up. Only in the case where the demand has an elasticity of less than unity will the value in terms of foreign currency decrease. A decrease in the value of exports in terms of home currency will require a still lower elasticity of demand, for a fall in the exchange rate of home currency taken by itself has an effect in the direction of increasing the value of exports in terms of home currency. Now the combination of a downward sloping supply curve (decreasing marginal costs) and a very low elasticity of demand will be extremely rare, and so it seems that there cannot be any serious objections raised against the rule that the value and the price of exports as a whole, expressed in terms of home currency, will show an increase if the exchange rate of the home currency falls.

footnote 1 on page 13). Therefore, there seems to be no reason for a difference of opinion concerning the *downward slope of the demand curve for home currency*.

With regard to the supply curve, matters are not so simple. A depreciation of the home currency raises the prices of imports. *Ceteris paribus* the consequence of this rise in prices is a reduction in the demand for foreign goods and therefore in the *volume* of imports. *Prices* of imported commodities in terms of foreign currency remaining the same (or even being slightly lower as a consequence of the fall in demand) the *demand for foreign currency will be reduced*. But now consider the quantity of home currency required to buy the amount of foreign currency needed to pay for the imports. Will it also be lower than before? This is by no means certain for the exchange rate has fallen, which implies that the price of a unit of foreign currency in terms of home currency has gone up. And this, in its turn, means that the *value* of imports as a whole in terms of home currency, being the *product* of the smaller amount of foreign currency and the higher price of it per unit, might just as well increase as decrease. What in fact will happen depends entirely on the *elasticity of demand for imports* with regard to prices, or, if similar goods are produced at home, on the demand elasticity of substitution of imported goods for home-produced goods with regard to their relative prices. To be brief we will indicate this elasticity by the symbol  $\eta^i$ .

Now if  $\eta^i > 1$ , which means that demand is elastic, a fall in the exchange rate, implying a rise in the prices of imports in terms of home currency, will by definition reduce the value of imported goods and services in terms of home currency, at least if only the direct influence of the fall in the exchange rate is taken into account.

So in this case the supply curve for home currency *ceteris paribus* will have an *upward slope*, for the higher the exchange rate the higher will be the supply of home currency to pay for the imports.

If on the contrary  $\eta^i < 1$  a fall in the rate of exchange will by definition increase the value of imports in terms of home currency, notwithstanding the fact that the increase in prices has reduced the quantity of imported goods. Therefore, we may conclude that *as soon as the demand for imports is inelastic, the supply curve for home currency will ceteris paribus show a downward slope.*

In the "border-line case",  $\eta^i = 1$ , the supply curve for home currency will be vertical, for the supply will be the same whatever the exchange rate may be.

The foremost theorists in the field of international payments have been well aware of the possibility that under certain circumstances the supply curve for home currency may have a downward slope. I here mention only MARSHALL, who, in 1879, dealt with this matter in his essay on the "Pure Theory of Foreign Trade," and YNTEMA, who, in his book "A Mathematical Reformulation of the General Theory of International Trade," paid due attention to the consequences of a downward slope of the supply curve for home currency.

Meanwhile I would like to stress the point that most of these theorists consider an inelastic demand for imports as a merely hypothetical case which will seldom or never present itself in reality. They support their view by two arguments:

- I. Inelasticity in the demand for a commodity, which can also be imported, does not necessarily imply that the elasticity, which reflects the reaction of the imported quantity to a change in price brought about by a change in the exchange rate, will be smaller than unity. This difference

between *the elasticity of demand for imports* and *total elasticity of demand* may be explained as follows. A fall in the rate of exchange will cause a rise in the price in terms of home currency and consequently a fall in the total home demand for the commodity concerned. However, home production will increase as a result of the higher price of the commodity. Therefore, the imported quantity will show a larger fall than home sales as a whole. In other words, the elasticity of import demand is larger than the total demand elasticity, and, if the latter turns out to be smaller than unity, the former may still be higher than unity. This can also be formulated in another way so as to show clearly the part played by the substitution effect. Immediately after the change in the exchange rate there will be a difference between the price at which the imported product is supplied and the price at which the home-produced commodity is marketed. The price of the former has risen, whereas the price of the latter is still unchanged. Consequently the demand for the imported product will decrease and the demand for the home-produced commodity will increase. The process of substitution of home product for imports continues till the price of the home-produced commodity, as a result of the increase in marginal costs which accompanies the increase in production, has again reached the level of the price of the imported product.

2. Even if the import demand elasticities for all imported goods and services prove to be smaller than unity, which is considered to be highly improbable, a demand elasticity for imports as a whole greater than unity may present itself. There are always many goods which are potential import goods. A small rise in the rate of

exchange will induce people to buy these goods abroad which formerly was not done owing to the fact that, at the time, with the lower exchange rate, they were too expensive compared with similar home-produced goods. So here again we see the substitution effect in operation. Yet there is a slight difference. Here the original price difference was such that the outside world could not compete with producers at home, whereas in the first case competition existed already and only tended to equalize prices of home products and foreign products, and to revise the partition of the home market between the two.

#### I.5 EQUILIBRIUM AND EQUILIBRIUM TRANSPOSITION

In the preceding section it was shown that, confining the discussion to the direct influence of a change in the exchange rate, a downward sloping supply curve for home currency is, at least theoretically, conceivable. As our argument proceeds, and the indirect influence of a change in the rate of exchange on the supply of home currency is also taken into consideration (cf. section 1.6), it will become clear that a fall in the exchange rate might just as well be accompanied by a decrease in the supply of home currency as by an increase. What does this mean for the *equilibrium* which establishes itself in the foreign exchange market? In answering this question figure 1, being a schematic representation of the various possibilities which may present themselves, will prove to be rather helpful. Along the vertical axis of the graphs is measured the exchange rate of the home currency in terms of foreign currency ( $k^b$ ) and along the horizontal axis the amount of home currency demanded to pay for exports ( $d$ ) and the amount of home currency supplied to pay for imports

(s). The position of the demand curve is the same in all graphs. The supply curve, however, is turned counter-clockwise; in graph 1 B its slope is already negative.

Now let us consider first what happens if, for some reason or other, the exchange rate drops below the level at which supply and demand are in equilibrium ( $o$ ). In cases A and B the result will be that demand exceeds

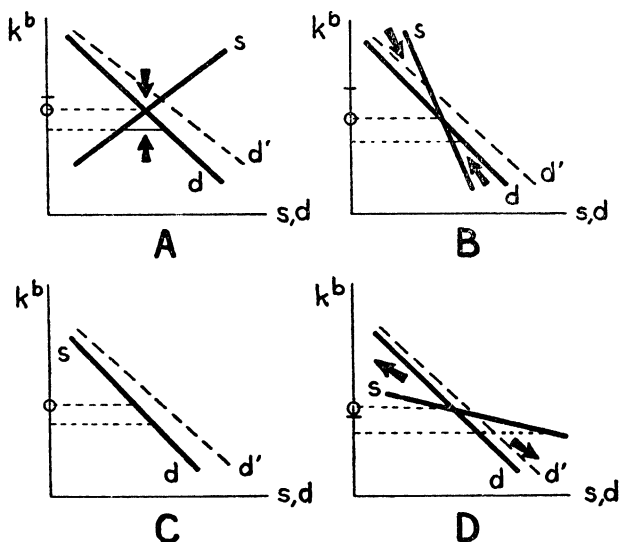


Figure 1

Supply and demand of home currency

supply (—) which leads to an increase in the price, that is to say in the rate of exchange. If, on the contrary, the exchange rate rises above the level mentioned, supply will exceed demand, and consequently the exchange rate will fall again. So in these cases the disturbance itself evokes the forces which automatically restore equilibrium. These equilibria are therefore *stable* equilibria (indicated in the

graphs by arrows pointing towards the equilibrium point). Equilibrium D, on the other hand, is *unstable*, for here supply will exceed demand (.....) if the exchange rate falls below the equilibrium level and such a preponderance of supply over demand will cause the exchange rate to fall still further, which means that it removes itself more and more from the original equilibrium position. The same holds true *mutatis mutandis* (i.e. due alterations made) for a rise in the rate of exchange (the unstable equilibrium is indicated in graph D by arrows pointing away from the equilibrium point). If the supply and the demand curves coincide every exchange rate will balance supply and demand (see graph 1 C). Such an equilibrium is called *indifferent*<sup>1)</sup>.

After having discussed the nature of the equilibrium on the foreign exchange market with regard to the relative position of supply and demand curves, we may now turn to the question of *equilibrium transposition*. What happens if, for one reason or another, either the supply curve or the demand curve shifts? Let us suppose, for instance, that the demand curve shifts towards the right and let this shift be the same in all cases. Such a shift might occur as a result of some autonomous factor, e.g. an increase in incomes abroad, which increases the demand for exports. Now what do we find? *The shift changes the position of the equilibrium point*. In the case represented in graph 1 A, where the supply curve has a positive inclination, a small

<sup>1)</sup> Where at least one of the curves is of a non-linear type, other types of equilibria may present themselves, viz. *mixed* equilibria, being stable in one direction and unstable in the other (cf. N. KARDOR: "A Model of the Trade Cycle" in *The Economic Journal*, L (1940), pp. 79ff), and *metastable* equilibria, being stable for small disturbances and unstable for relatively big disturbances (cf. J. TINBERGEN: "Over verschillende soorten evenwichten en de conjunctuurbeweging" in *De Economist* 92 (1943) pp. 129—147, and J. TINBERGEN: "Legevaegstyper og Konjunkturbewaegelse" in *Nordisk Tidskrift for Teknisk Økonomi* 1944, pp. 45—63, of which a German version has been privately circulated by its author).

increase in the exchange rate suffices to establish the new equilibrium (—). In case B where the slope of the supply curve is a negative one, the transposition of equilibrium requires a much bigger increase in the rate of exchange. TINBERGEN calls the price - here the exchange rate - a good *regulator* in the first case, and a bad one in the second case<sup>1</sup>). In case C the situation is still more serious, for here the exchange rate has no regulating influence at all. Here the original surplus or deficit on current account can only be restored by measures of trade policy, such as changes in tariffs (and bounties), restrictions on exports, and so on. Finally, in case D the rate of exchange will be above its equilibrium level as soon as the demand curve has shifted to the right, and consequently it will continue to rise instead of decrease slightly, which would be sufficient to establish a new equilibrium. In such a case it seems advisable to couple the exchange rate of the home currency with the exchange rate of another currency, or with the value of gold, or with the value of some combination of goods, instead of leaving it to itself. However, where such arrangements exist, a revision of the parity (with a view to changes in the relations of supply and demand) should be considered from time to time.

#### 1.6 THE INDIRECT INFLUENCE OF A CHANGE IN THE RATE OF EXCHANGE

Studying the markets of separate commodities, the usual procedure is to follow MARSHALL's example of ignoring the interdependency which in fact exists between different

<sup>1</sup>) See J. TINBERGEN: "Unstable and Indifferent Equilibria in Economic Systems" in *Revue de l'Institut International de Statistique* 1941 (1/2), p. 38.

markets. This implies that the supply and the demand curves for the commodity concerned are considered as given and are completely independent of each other. Once the structure of the market on the supply-side as well as on the demand-side is known (be it monopoly, or free competition, or something in between these two extremes), the information entailed in these two curves will enable us to tell at which price the forces at each side of the market balance, and also which transposition of equilibrium is called for if one of the two curves changes its shape or its position.

Strictly speaking one is not fully justified in ignoring the interdependency between different markets. Apart from its direct influence, the disturbing factor which causes the equilibrium transposition will also have an *indirect* influence, at least if the equilibrium transposition itself has an influence on one of the factors which determine the shape and position of the supply and demand curves of the market under review. If so, the supply and demand curves will no longer be given quantities, and the equilibrium derived from such curves, which are considered as given and mutually independent, will only be a *partial equilibrium*. An example may serve to illustrate this.

The shape and position of the demand curve are determined by the incomes of the buyers and by the prices of all other products because all these other products compete with the commodity concerned, either directly because they can render similar services as this commodity, or indirectly because the purchasing power of the buyers is limited and the money can only be spent once, so that buying one thing means at the same time forgoing another. Now let us consider the case in which the costs

of production of the commodity concerned decrease, say because of the introduction of a more efficient method of production, or because of a fall in the prices of one or more of the materials required for producing it. This will cause a shift in the supply curve and consequently the price of the product will fall and the sales volume will increase. This represents the direct influence of a lowering of production costs on the equilibrium of the market. Apart from this there may, however, also be an indirect influence.

If the productive apparatus is not fully employed the increase in production of the commodity concerned will probably cause an increase in employment<sup>1)</sup>, which in its turn implies an increase in personal incomes and perhaps also a change in the distribution of incomes. This opens up, at least in theory, the possibility of a change in the position and/or the shape of the demand curve resulting in a further transposition of the equilibrium on the market concerned. If, however, on the other hand, there is full employment, an increase in the consumption of one product seems only possible at the expense of the consumption of the other products<sup>2)</sup>. The demand curves of these other products will, at least in theory, exhibit a shift as a result of change in the price of the commodity concerned, as this price is one of the factors which determine the positions of these curves. Consequently the prices of these other products will also change and, as they form part of the factors which determine the demand curve for the commodity concerned, the result will be a shift of this curve and a further transposition of equilibrium.

<sup>1)</sup> A possible exception is the case of an increase in production as a result of the introduction of labour-saving methods.

<sup>2)</sup> Cf. footnote 1.

We shall not continue this argument any further, for the only point we wanted to prove is that, theoretically at least, there is a possibility of supply and demand curves not being completely independent of each other, so that a shift in one of these curves may also cause a shift in the other. This means that a factor which disturbs the existing equilibrium will have both a direct and an indirect influence as to the transposition of equilibrium. This point, I think, has been proved sufficiently by the argument given above.

The fact that in the theoretical treatment of separate commodity markets supply and demand are considered to be independent of each other is not so surprising. True, the exactness and theoretical consistency of the argument may be slightly impaired by this procedure, but on the other hand there is a considerable gain in simplicity and clarity of argument, and the picture of what happens on the market is not really any poorer than it would be otherwise. As it is, each of these markets constitutes only a very small part of the economy taken as a whole and therefore changes in the total of individual incomes or in the average of the prices of all competing products, brought about by a shift on one such a market, will be so small that there can be no objection to neglecting them altogether. The shifts caused will be hardly noticeable. Therefore it seems quite permissible, in the case of *small markets*, to include all this in the *ceteris paribus* clause. By doing so, however, one should always keep in mind the fact that the total of the partial equilibria determined in this way does not give a proper picture of the new general equilibrium for the economy as a whole, which establishes itself once the disturbances in separate markets have had their full effect.

Now what can be said about the *foreign exchange market* in this connection? In section 1.4 we studied the price-quantity relationship for this market in the same way as does the Marshall school, taking account only of the direct influence of a change in the exchange rate. The result of this investigation was that the necessary condition for a downward sloping supply curve is inelasticity in the demand for imports. However, contrary to the commodity markets, the foreign exchange market cannot be considered as a small market, it is a *big market*. The “price” which establishes itself on the foreign exchange market does not exercise a negligible small influence on the total of all other markets, as was the case with price changes on the markets for separate commodities. In fact a change in the rate of exchange often exercises a decisive influence on numerous sectors of the economy. To realize this one should only think of the importance of the level of the exchange rate with respect to the export industry. Therefore, the picture of the mechanism of the foreign exchange market would be far from complete if we did not take into consideration the *indirect influence* of a change in the rate of exchange. TINBERGEN described this important indirect influence briefly as follows: “The change . . . . in the value of exports causes a *rise in incomes* and therefore in consumers’ expenditure as well as in investments. *For both, as well as for the increased exports, increased imports are required.* So there will be a — probably considerable — shift in the demand curve for imports; a shift which results from the price change from which we started and for that reason cannot be ignored. This shift in itself brings about an increase of the volume of imports” <sup>1)</sup>.

<sup>1)</sup> J. TINBERGEN: “Indifferente en labiele evenwichten in economische stelsels” in *De Economist* (1941), p. 574 (*italics are mine*).

We may conclude from the foregoing that a depreciation of the home currency has, on the one hand, a tendency to *decrease* the volume of imports owing to the resulting increase in prices of import products, and, on the other hand, a tendency to *increase* the volume of imports owing to its increasing effect on the value of exports. So there is not only uncertainty as to the question whether the volume of imports will fall sufficiently to make the value of imports in terms of home currency also decrease, but there is also the question whether the volume of imports will decrease at all. On balance the result may well be an increase in the volume of imports. Taking everything into account there seems no reason whatsoever to exclude a priori a fall in the supply of home currency if this currency depreciates <sup>1)</sup>).

## 1.7

## OUTLINE OF THE STUDY

This first chapter has been devoted mainly to stating in broad outline the problem of exchange stability. In addition to this, attention has been paid to the general structure of economic theory in order to establish the proper place of the exchange stability problem in the whole of economic theory (section 1.2). It has been shown that the body of this theory contains among others the following parts:

1. the theory of value, the central principle of which consists of balancing the marginal utilities obtained and sacrificed;

<sup>1)</sup> One might think that, as regards the value of exports, the indirect influence of a change in the exchange rate on export possibilities should also be taken into account. However, one should not lose sight of the fact that in normal cases imports come from various countries and that for each of these countries the exports to the country concerned form only part of their total exports. Therefore, incomes in those countries will hardly be affected by an increase or decrease of the exports to the country concerned. Furthermore, the import countries may be others than the export countries. So there seem to be good reasons for leaving out of consideration the indirect influence of a change in the rate of exchange on the value of exports.

2. the theory of prices, the subject-matter of which consists of the principles according to which the exchange relations establish themselves, where commodities are exchanged;
3. the theory of money, which studies the value of money in terms of goods and services;
4. the theory of international payments, which deals with the exchange of one currency for another.

Other divisions of economic theory are possible of course, but this one — although definitely incomplete (economic dynamics for one thing has been left out) — seems to be rather useful for establishing the proper place of the exchange stability problem, as part of the theory of international payments, in the whole of economic theory.

In the discussion of the exchange stability problem it has been emphasized that the foreign exchange market is, unlike the markets for separate commodities, a big market. Therefore it is necessary to take account, not only of the direct influence of a change in price — here the exchange rate — (cf. section 1.4), but also of the indirect influence of it (cf. section 1.6). This indirect influence causes shifts in the supply and demand curves for home or foreign currency which may have far-reaching consequences for the stability of the equilibrium on the foreign exchange market (cf. section 1.5). Only in recent years has the practical importance of this been fully perceived (cf. section 1.1).

It is by no means easy to give a fair appraisal of the indirect influence of a change in the rate of exchange, for it turns out that this influence works through various channels. This very much complicates the subject-matter of the theory of international payments. To study it

properly the whole of the economic mechanism of a country's economy must be included in the picture. Verbal treatment of such a complicated matter soon proves to be wholly inadequate. The only feasible approach here seems, therefore, to be mathematical treatment of the subject and simplification of the problem to the barest minimum. The advantages and the implications of the mathematical treatment, in so far as they are of any importance for present purposes, will be set out in chapter II. Due attention will also be given there to the simplification of the picture through handling the matter by means of a macro-economic approach to the problem, through linear approximation of economic relationships, through neglecting "lags" in economic reactions, etc.

In the third chapter a number of definitional and reaction relationships will be described, which, taken together, provide a very concise picture of the working of the economic mechanism in an open economy, and therewith of the forces at work in the foreign exchange market. From this theoretical framework a number of conclusions can be drawn concerning the nature of the equilibrium on the foreign exchange market under various conditions with regard to people's reactions to changes in circumstances. These conclusions, and the discussion thereof, which form the body of the theory on the exchange stability problem to be developed in this monograph, will be dealt with in the fourth and last chapter of this study.

## CHAPTER II. THE METHOD

### 2.1

### MATHEMATICAL TREATMENT

Most authors dealing with the problems of international trade and the foreign exchange market have refrained from making use of mathematics. Nevertheless, the mathematical treatment of economic problems in general, and of balance of payments problems in particular, has much in its favour. I would even go so far as to say that it is the only feasible method of handling such intricate problems as the one dealt with in this monograph.

The fact that the problem of exchange stability will be treated here mathematically should not, however, in the least discourage those readers who consider themselves non-mathematical economists. They will find that the mathematics employed are rather simple and do not surpass, on the whole, the knowledge of algebra acquired in the secondary school. This is already guaranteed, I think, by the fact that the author, who is in the first place an economist, has himself only a limited knowledge of mathematics. Furthermore, constant efforts will be made throughout to express the meaning of mathematical formulae in words. This does not apply only to the basic or elementary formulae from which the argument starts, but also to the conclusions drawn from the combination of these basic formulae.

There are, of course, good reasons which justify the introduction of mathematics into economic treatises; one

being that it seems to do the job better than verbal treatment, and another that it sometimes is the only means by which the job can be done.

MARSHALL, who was against a mathematical exposition of economic theory, pointed out concisely and rightly some of the advantages of the mathematical method by stating that: "The chief use of pure mathematics in economic questions seems to be in helping a person to write down quickly, shortly and exactly some of his thoughts for his own use"<sup>1</sup>), and that it provides the investigator with a simple device "to make sure that he has enough, and only enough, premises for his conclusions (i.e. that his equations are neither more nor less in number than his unknowns)"<sup>2</sup>).

The first advantage pointed out by MARSHALL may add considerably to the preservation of what may be called the *surveyability* of the argument. In economics, as in all human sciences, where problems are often of a rather complicated nature, this is a matter of no little importance.

The second advantage of the mathematical method, mentioned above, provides a safeguard for the *completeness* of a theory (cf. section 2.4). It is interesting to note in this connection TINBERGEN's remark with regard to business cycle theories: "Strictly speaking there are very few, if any, 'literary' theories that are complete and determinate"<sup>3</sup>). Many purely "literary" economists were not careful enough in this respect, and created erroneous theories which cannot stand the test of completeness and therefore are always more or less one-sided. This has caused grave damage to the

<sup>1</sup>) A. MARSHALL: *Principles of economics*, 8th edition (London 1936); Preface to the first edition, p. X.

<sup>2</sup>) MARSHALL, *op. cit.* p. X.

<sup>3</sup>) J. TINBERGEN: *Business Cycles in the United States of America 1919—1932* (Geneva 1939), p. 19.

reputation of the science of economics, and also hampered its progress. The layman interested in economics, if confronted with a number of such one-sided theories which contradict each other completely, will soon lose all interest in economic theory and consider it a mere waste of time to study these theories, and possibly also a waste of money to get expert economic advice. Economists themselves, on the other hand, will be tempted to spend their time on futile debates about the correctness of one theory or another, often forgetting that usually each of these one-sided theories contains part of the truth.

A further advantage of the mathematical method is that it often helps the investigator to get rid of all kinds of obscurities and *to express clearly what are the premises* on which he is going to base his argument.

Usually one will find that "literary" theories of economics confine themselves to explaining that there exist certain relationships between certain phenomena. Very seldom, however, do they give any details about the nature and character of such relationships. At its best the direction of the causal connections is indicated, that is to say if there is assumed to be any such connection and not mutual interdependency. In the language of mathematics this amounts to formulating functions in their most general form, e.g.  $y = f(x, z)$ <sup>1</sup>). Such a formula expresses the fact that with every pair of values of  $x$  and  $z$  there corresponds a certain value of  $y$ . To take full advantage of the opportunities offered by the mathematical approach one should, however, go somewhat further and make also an assumption with regard to the mathematical form of each of the economic relationships involved. Take, for instance,

<sup>1</sup>) Sometimes written as  $y = y(x, z)$ , where the  $y$  on the left-hand side is the symbol for a certain phenomenon and the  $y$  on the right-hand side the function symbol.

the relation between the demand for goods and the price of those goods. Here the investigator will have to make up his mind, if he is going to take full advantage of the mathematical method, whether the volume of demand depends on the first, or second, or even a higher power of the price, or whether there is perhaps an exponential relationship, and so on. His choice may be arbitrary and this, of course, will reduce the generality of his conclusions; yet in many cases this loss of generality turns out to be rather small (cf. section 2.2). This *urge for concreteness*, which usually accompanies the introduction of mathematics into economics, finds its culmination point in econometrics, the science which endeavours to determine numerical values for the coefficients appearing in mathematically formulated economic relationships (cf. section 2.7).

What has so far been said about the merits of the mathematical method does not necessarily call for a mathematical exposition of economic theory. The investigator may use the mathematical method for his research and yet still prefer to present his argument, not in the form of mathematical symbols, but in the written word.

Sometimes, however, the problem under investigation is of such an intricate nature that it renders verbal treatment impossible. So, for instance, some problems of international economics — often considered as “the most difficult and perplexing in economics” — cannot adequately be dealt with verbally. As YNTEMA rightly points out the traditional one-thing-at-a-time method with its *ceteris paribus* assumption cannot be applied here, owing to the fact that “the repercussions of changes in price levels and exchange rates on the interrelations of supply and demand are of primary

importance” <sup>1)</sup> in international trade and payments. This largely accounts for the complicated nature of problems in this field. The same applies, of course, *mutatis mutandis*, to other *big markets*. The fundamental difficulty of verbal treatment with its traditional one-thing-at-a-time method is that it always has to start by explaining changes in one phenomenon by changes in only one other phenomenon. Thereafter it can proceed to explain the changes in a third phenomenon by changes in the two phenomena already dealt with, or one of them. In the explanation of a fourth phenomenon the three phenomena dealt with already may be used if necessary, and so on. This is a serious limitation of the “verbal” method, for where the problem involves a number of mutual relationships matters are not always of such a simple nature. Here the mathematical method is far superior to the “verbal” method, for even if the changes in each of the phenomena are influenced by the changes in all the other phenomena involved in the problem it can reduce the complicated system of mutual relationships to one relationship, which explains the changes in one phenomenon by the changes of only a limited number of other phenomena. This number is equal to the difference between the number of phenomena and the number of elementary relationships connecting them, as will be explained later on in this chapter (cf. section 2.4). A mathematical exposition of economic theory will, therefore, in such cases provide a far *more penetrating analysis* of the subject under investigation than would otherwise have been possible. The mathematical manipulations, by which the conclusions are derived from the premises from which the argument

<sup>1)</sup> T. O. YNTEMA: *A Mathematical Reformulation of the General Theory of International Trade* (Chicago 1932), p. 1.

starts in such a case cannot be given a sensible meaning in the text of a "literary" theory. Also it is difficult to see how the "literary" theorist, who refuses to use the mathematical technique, will ever arrive at certain conclusions which the mathematical economist reaches comparatively easily.

For a theory on which measures of practical policy are to be based the merits of the mathematical method are outstanding. In order to give the politician clear-cut answers the economist will be in need of a *workable apparatus* fit to deal with the practical issues at hand. Such an apparatus can in many cases be provided only by the econometrician, whose work, as stated above, may be considered as the culmination point in the application of the mathematical method in economics.

It is up to the reader to judge how much there is to be gained in the case of the theory of international trade and payments, particularly with regard to the exchange stability problem, by the introduction of mathematics in general, and by the specific assumptions concerning the exact mathematical form of the relationships in particular (which is essentially a mathematically inspired application of the method of diminishing abstraction). It is hoped that, having gone through the argument contained in this monograph, he will be convinced that there is much to be said after all for the mathematical treatment of economic problems.

## 2.2 THE MATHEMATICAL FORM OF THE RELATIONS

In this section we will discuss briefly the question as to what *types of function* should be considered to be the most suitable for describing economic relationships. This question

is of special interest with regard to those relationships the mathematical form of which we do not happen to know beforehand on a priori grounds. Examples of such relationships are the demand and supply functions which play such an important part in economics. Examples of the other group of relationships, the mathematical form of which is given beforehand, are the definitional equations, which also play their part in economic theory.

If there is no indication whatsoever as to the type of function describing a certain relationship, the mathematical economist will, of course, choose the *simplest* type of function conceivable to describe that particular relationship. Why should he make matters more complicated than they already are? Also if there are certain indications, he will still choose, for the sake of convenience, the simplest type of function compatible with the available information.

Now what is the simplest type of function one can think of? Unfortunately there is no single answer to this question in connection with mathematical economics. The economist will prefer the type of function which keeps his argument as simple as possible, and the mathematician will prefer the one that keeps his calculations as simple as possible. However, the two are *not always the same*. Whenever they differ it will be up to the mathematical economist to reconcile the views of the mathematician and the economist to the best of his ability. In reaching a compromise between divergent views he will prefer the type of function which, in his opinion, provides the simplest approach to the problem under investigation.

First let us consider what type of function the *economist* would generally prefer. Let  $y$  be the phenomenon, the variations of which are to be explained, and  $x, z, \dots$  the explanatory phenomena (in mathematical terminology

known as the dependent and independent variables respectively) and  $a, b, c, \dots$  constants, then the economist will, as a rule, prefer a function of the type:

$$(2.2.1) \quad y = a x^b z^c \dots$$

which may also be written according to the definition of a logarithm as:

$$(2.2.2) \quad \log y = \log a + b \log x + c \log z \dots$$

The advantage of this type of function is that it implies *constant elasticities*, which greatly simplifies the economic argument. Let, for instance,  $y$  be the demand for a certain commodity and  $x$  its price, then  $b$  will invariably be the price elasticity of demand for *each* pair of corresponding values of  $x$  and  $y$ . This proposition can be easily proved by differentiating both sides of the equation  $y = a x^b$  with respect to  $x$  (the price)<sup>1</sup>. According to the most elementary rule of differential calculus this results in

$$(2.2.3) \quad \frac{dy}{dx} = abx^{b-1}.$$

Substituting for  $a$  the expression  $y/x^b$ , derived from the original equation, we are left with

$$(2.2.4) \quad \frac{dy}{dx} = b \frac{y}{x} \text{ or } b = \frac{dy}{dx} \cdot \frac{x}{y}$$

which is nothing but the well-known definition of an elasticity (introduced and explained in MARSHALL'S "Principles").

<sup>1</sup>) To reassure the reader who has no knowledge of the principles of differential calculus, it should be noted that the understanding of this proof is not a necessary condition for following the rest of the argument in this and the later chapters of this monograph, for this type of function will not be used.

Now let us turn to the *mathematician*. He will generally prefer a function of the type:

$$(2.2.5) \quad y = a + bx + cz + \dots$$

as this type of function usually, though not always, enables him to solve the problem before him by means of very simple calculation processes. At first sight his choice does not seem to differ very much from the economist's. The only difference between equations (2.2.5) and (2.2.2) is that the former runs in the variables themselves and the latter in their logarithms. On further reflection, however, this difference turns out to be an important one.

As long as only one equation or relationship is involved the mathematician will not object to the economist's choice. As a matter of fact he will be rather indifferent as to the type of function chosen and will accept almost every type of function suggested by the economist. As soon as two or more equations are involved, he will agree with the economist's choice only in so far as it does not stand in his way in the performance of his fundamental task of solving the problem under investigation by combining the various relationships involved (cf. section 2.4). Often, though not always, he will therefore impose on the economist the condition that the type of function chosen is the same for all relationships involved, or at least if different types of functions are chosen that they can all be reduced to the same type by, preferably simple, mathematical processes. Furthermore, the economist, with a view to stating his conclusions in writing, will be prepared to accept all such simplifications as the mathematician might suggest in order to keep the calculations, and the results thereof, as simple as possible, on one condition viz. that the generality of his theory is not unduly impaired.

Now let us again consider the difference between equations (2.2.5) and (2.2.2). In almost every set of relationships, representing the mathematical framework of an economic theory, there are relationships to be found which, on a priori grounds, have to be represented by a certain given type of function. Sometimes, if the relationship concerned runs in the variables themselves, this type of function is such that it can easily be transformed into the type represented by equation (2.2.2). This applies to those functions which contain only products or quotients of the variables, as, for instance, is the case with the relation which defines value ( $U$ ) as the product of quantity ( $u$ ) and price ( $p$ ).

$$(2.2.6) \quad U = up; \text{ therefore, } \log U = \log u + \log p.$$

In other cases, however, where the original function running in the variables themselves contains sums or differences of the variables involved, a transformation into the type of function represented by (2.2.2) is not possible. This is true, for instance, of those relationships which, on a priori grounds, should be described by a function of the type represented by (2.2.5), e.g. the surplus of the balance of payments on current account is equal to the difference between the value of exports and imports of goods and services.

The question now arises as to what the mathematician will do if he is confronted with a set of relationships containing one group of relationships which on a priori grounds should be described by one type of function, and another group which on a priori grounds should be described by a different type of function, which cannot be transformed into the type of function by which the relationships of the first group are to be described. The

only way out apparently is to make use of methods of *approximation* by which the transformation, indispensable for the process of combining the relationships, can be performed. And here the decision as to the type of function to be used passes from the economist to the mathematician.

Now the mathematician will tell us that it is a far more simple procedure to *transform a product of variables into a sum of terms, each containing only one of the variables, measured in deviation from its mean value*, than to do the reverse. We take this for granted and will now show how this approximate transformation should be carried out. Indicating the original or absolute values of the variables, i.e. the values counted as from the point zero on the scale of measurement, with a double bar over the symbols representing the variables; the mean values of the variables by a single bar over those symbols, and the deviations from those means by the symbols without a bar, we get the following. Take for instance equation (2.2.6). It should now be written as

$$(2.2.7) \quad \overline{\overline{U}} = \overline{\overline{up}},$$

and can be transformed into

$$(2.2.8) \quad \overline{\overline{U}} + U = (\overline{u} + u)(\overline{p} + p) = \overline{u}\overline{p} + \overline{u}p + \overline{p}u + pu$$

or, because (2.2.7) will also apply to the mean values of the variables,

$$(2.2.9) \quad U = \overline{u}p + \overline{p}u + pu.$$

This formula only contains one product of deviations from means ( $pu$ ) and as long as these deviations are *small* their product will, of course, be *very* small. Therefore it seems to be quite permissible to neglect this product by way of approximation, but it should be borne in mind

that this approximation can only be justified on the condition that the deviations themselves are small. Omitting the product of deviations we are left with

$$(2.2.10) \quad U = \bar{u}p + \bar{p}u,$$

which proves to be a function of the type of equation (2.2.5),  $\bar{u}$  and  $\bar{p}$  being constants.

In our problem of the stability of the equilibrium in the foreign exchange market we shall have to consider the interconnections in a set of relationships, and will also be confronted with the problem of differences in the types of function which describe these relationships. This forces us to accept the choice of the mathematician, which also implies that in describing the demand and supply relationships we shall have to make use of a function of the type of formula (2.2.5). This type of function is known as the *linear function* because its graphical representation is a straight line.

Once we have made our choice as to the type of function to be used for the description of the relationships which enter into our problem, we have to consider the question to what extent the validity of the conclusions derived from such a set of linear equations is restricted by the choice made. The answer is that the restrictions are in fact less serious than might be thought at first sight. TINBERGEN in his plea for linearity<sup>1)</sup> makes the following points to support this view:

1. *Linearity is not identical with proportionality*, as many non-mathematical economists seem to think. Proportionality is in fact a more stringent condition than linearity; it would require  $a$  to be  $= 0$  in formula

<sup>1)</sup> See, for instance, J. TINBERGEN: "Econometric Business Cycle Research" in *Review of Economic Studies*, VII (1940), p. 73ff.

(2.2.5). That the one is often mistaken for the other is presumably to be explained by the fact that a linear relationship between the original or absolute values of the variables implies proportionality between corresponding changes in the variables (indicated by  $\Delta x$ ,  $\Delta y$ , etc.) and also between deviations from their means. This is easily proved as follows. If

$$(2.2.11) \quad \bar{\bar{y}} = a + b\bar{\bar{x}} + c\bar{\bar{z}} + \dots$$

then we will have, after changes  $\Delta y$ ,  $\Delta x$ , etc. (which correspond with each other) have taken place in the variables,

$$(2.2.12) \quad (\bar{\bar{y}} + \Delta y) = a + b(\bar{\bar{x}} + \Delta x) + c(\bar{\bar{z}} + \Delta z) + \dots$$

and, therefore,

$$(2.2.13) \quad \Delta y = b \Delta x + c \Delta z + \dots$$

and also because (2.2.11) can be written as

$$(2.2.14) \quad (\bar{y} + y) = a + b(\bar{x} + x) + c(\bar{z} + z) + \dots$$

which relation also holds for  $\bar{\bar{y}} = \bar{y}$ ,  $\bar{\bar{x}} = \bar{x}$ , etc.

$$(2.2.15) \quad y = bx + cz + \dots$$

A graphical representation of the difference between linearity and proportionality, together with the still more stringent supposition of equality ( $a = 0$ ,  $b = 1$ ), is given in figure 2.

2. According to a mathematical proposition almost every function can be transformed into a series, which for small intervals in the variables may be reduced to a linear expression. Graphically this means that for

such an interval the tangent may be substituted for the curve itself, which represented the relation. This proposition is very important for our purposes as it implies that for small variations in the variables the conclusions derived from a set of linear equations have the same validity as those derived from the more complicated functions, which give an exact description

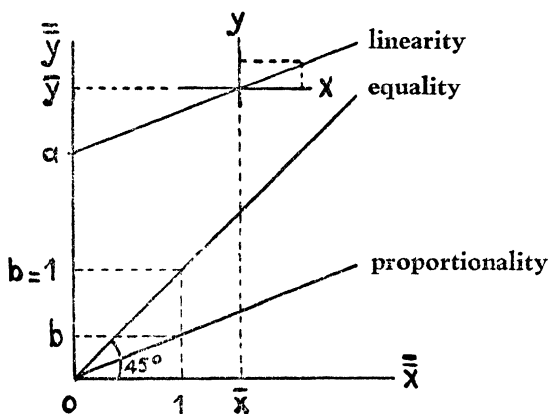


Figure 2

Linearity

of the relationships between certain phenomena and therefore hold true for variations of any size. TINBERGEN puts it neatly as follows: "*In the case of small variations in variables . . . . there is no loss of generality at all*"<sup>1</sup>).

3. Also the fact that in equations of the type of formula (2.2.5) each of the variables is multiplied by a *constant coefficient* (instead of by a varying coefficient) is a less serious restriction than one might think. If one assumes, for instance, that a certain coefficient is a function of

<sup>1</sup>J. TINBERGEN: *Business Cycles in the United States of America 1919—1932* (Geneva 1939), p. 11.

the value of one or more variables, then again we may make use of a *linear approximation* in such a way that we express the coefficient linearly in the variable to which it belongs and the variables on which it is considered to depend. This point will be clarified in the Mathematical Appendix to this chapter. For the rest one is well advised to keep in mind what TINBERGEN says in this respect: "Theory always means reducing variable things to constancy" and "An author who does not bind himself to some 'laws' is able to 'prove' anything at any moment he likes. But then he is telling stories, not making theory"<sup>1</sup>).

From the foregoing it will be clear that as long as only small variations in the variables are involved, linear approximations of supply and demand relations may be considered to be perfectly legitimate. But even if we are studying big variations, there is often a good deal to be said for the use of linear functions, this being one of the simplest forms for functions about the type of which we do not know anything on a priori grounds. And here we may quote once again one of TINBERGEN's remarks: "it is interesting to note that there are astonishing examples of good fits obtained with constant coefficients and linear equations, which suggest that this type of relation is more frequent than is often believed"<sup>2</sup>).

For the problem of stability in the exchange market we shall only have to consider what happens in case of small deviations from the equilibrium position. Therefore, the use of linear equations will be perfectly legitimate and will not in the least restrict the validity of our con-

<sup>1</sup>) J. TINBERGEN: "Econometric Business Cycle Research" in *Review of Economic Studies VII* (1940), p. 80.

<sup>2</sup>) J. TINBERGEN: *Business Cycles in the United States of America 1919—1932* (Geneva 1939), p. 12.

clusions. However, if one deals with such problems as to what change in the terms of trade is required to close the gap in the balance of payments, what the transfer capacity of a country is, etc., it may well be that the solution of such problems involves big variations in the variables and then the linear approximation of some definitional and other equations is no longer to be considered justified. Therefore, in the next chapter we shall not only describe the linear relations, which suffice for the exchange stability problem, but also the curvilinear equations, which we think are most appropriate for those further problems.

### 2.3 MACRO-ECONOMIC RESEARCH

In the beginning of this chapter we have already discussed at some length the advantages of a mathematical formulation of economic problems. The introduction of mathematical symbols, as a kind of shorthand description, for economic phenomena (e.g.  $d$  for demand,  $p$  for price, etc.) and the description of the relations between those phenomena by mathematical functions (e.g.  $d = d(p)$ ) was said to add considerably to the surveyability of the argument. Mainly for this reason the mathematical method may be considered to be of great help to the theorist in detecting economic laws and in proving their general validity, or in ascertaining the conditions subject to which such laws hold true. Furthermore, the mathematical formulation of economic theory has proved to be a useful device in illustrating and describing the mutual interdependence of economic phenomena. It has helped many a scholar in gaining a clear and theoretically correct insight into the interrelations which exist in the economic world. But, so it may be asked, is the mere introduction of mathe-

matics into economics sufficient to forge us the scientific tool which will enable us to deal with the practical problems of the day? The answer to this is in the negative. To deal with practical issues we need a simplified and quantitative picture of the complicated reality.

In this section we shall pay due attention to the *simplification aspect*. Discussion of the quantification aspect will be taken up at a later stage (cf. section 2.7).

Now the first question to be answered, while discussing the need for simplification of the complicated pattern of economic relationships in the real world, is apparently this: why is it that a complete and detailed picture of reality is so complicated? The answer is that what actually happens in economic life is determined by numerous decisions of numerous individuals who are guided by numerous factors in taking their decisions. Take, for instance, the demand for goods and services. The quantity of a particular commodity which an individual wants to buy depends on such factors as the money income he earns, the taxes he has to pay out of this income, the size of his family, the price of this particular commodity, the prices of all other goods and services which compete with this particular commodity for his available income, his thriftiness, his cash position and his mood at the time of purchase, and so on. A complete and detailed picture of demand has to take account of the influence of each of these factors on actual demand. It has to do so separately, not only for every type of commodity, but also for each single individual. The reason for this separate treatment is fairly obvious. Individual wants vary and therefore their economic behaviour in given circumstances, as well as their reactions to changes in those circumstances, will also vary. These differences in wants in their turn can

be explained by differences in age, education, character, and so on.

From the foregoing it will be clear that the so-called *micro-economic* approach to economic problems which, at least in principle, pays attention to every single detail of the problem under investigation, is useless for practical purposes. It makes the total number of relations involved (e.g. in the case of demand, as many for each commodity as there are individuals who may buy this type of commodity), as well as the number of factors entering into these relationships, far too large to cope with. This is the more true if the nature of the problem requires that account be also taken of the interrelationships in the economic system (e.g. the individual's income is not only a determining factor in his demand for goods and services, but this demand itself, together with the demand of others, influences in its turn his income). This, for instance, is the case with the problems concerning big markets. Therefore the purely micro-economic approach must be considered to be of value only to the theorist in clearly stating and discussing economic problems. Usually it will not enable him to do more than formulate the functional relationships which he thinks exist. In most cases he will refrain from indicating the shape of the function or the strength of the influences exercised by the various factors.

Wherever the interrelationships of the economic system enter into the problem, and wherever the economist is expected to give some practical advice to politicians or businessmen, there is need for simplification if we want to avoid the danger of getting lost in the problem by not seeing the wood for the trees. Now the appropriate means for simplification in economics is the so-called

*macro-economic* approach<sup>1)</sup> by which individuals, as well as goods and services (and therefore also the markets where these individuals supply and demand these goods and services), are aggregated into broad *groups*. This procedure reduces tremendously the number of relationships entering into the problem. At the same time it usually enables us to drop some of the explanatory variables in such relationships, namely those which are held responsible for the fact that different individuals faced with the same problem in the same circumstances act in different ways. On the whole the influences of these factors will cancel each other out. In cases where we are only interested in the variations of a certain phenomenon, say demand, and not in the explanation of its absolute volume, we may even go further and omit all those explanatory variables which do not show variations themselves, or only show very slow changes. The average size of the family, for instance, usually changes very slowly and therefore is not of great interest for the explanation of variations in overall demand over time. For the explanation of variations in the demand of an individual family, however, this factor should definitely be taken into account, for the size of the individual family is usually subject to quick, and sometimes even to abrupt, changes (birth and death).

It goes almost without saying that there is no clear distinction between micro-economics and macro-economics. The degree to which the grouping procedure is applied varies a good deal from one case to another. For demand studies of separate commodities, for instance, only individuals are united into one group embracing them all. For deriving a consumption function on the basis of family budgets, on the other hand, all goods and services consumed will

<sup>1)</sup> The terms micro- and macro-economic have been coined by RAGNAR FRISCH.

be brought together into one group, while the individuals are not grouped at all. Sometimes individuals (or goods) are not aggregated into one group but into a small number of broad groups, each of which has its own characteristics, e.g. labourers, entrepreneurs, owners of capital and land owners, or as in this study, natives and foreigners. Similarly, goods and services may be divided into such broad groups as raw materials, semi-finished products and finished products, or import and export commodities and goods for the home market, a distinction which will be used in this monograph. How the grouping procedure is carried out is apparently largely a matter of choice. In making this choice, however, it should be borne in mind that the ultimate aim is "to find the happy medium between the complexity of the real world and the simplicity of an amenable theory"<sup>1</sup>). For each equation groups should therefore be chosen in such a way that they are as *homogeneous* as possible with regard to relevant aspects. What is relevant in this respect also depends on what variables appear in the other equations to be included, as will be explained later.

In order to give an idea of the multitude of relationships in the field of international trade I should like to draw attention to the number of equations in YNTEMA's system. It equals  $m + n$ , namely one balance equation for each of the  $m$  countries stating that expenditure equals receipts, and one balance equation for each of the  $n$  commodities stating that demand equals supply (one of these  $m + n$  equations can be derived from all the others and may not therefore be considered as independent). Add to this the fact that YNTEMA wrongly left out of account certain

<sup>1</sup>) J. TINBERGEN: "Suggestions on Quantitative Business Cycle Theory" in *Econometrica* III (1935), p. 243.

relationships which he should have included (cf. section 1.1) and it will be clear that this type of system consists of such a high number of elements and relations that it cannot be handled for practical purposes. The trouble apparently is that YNTEMA went only half-way on the road which leads from micro-economics to macro-economics. He grouped the individuals according to the countries in which they live, but he refrained from creating country groups, nor did he create any commodity groups. However, he may have had good reasons for this, for closer study of the macro-economic procedure soon reveals the fact that this procedure is not altogether without danger. This applies especially to the procedure of grouping commodities.

There is no objection to the aggregation of individual demand functions and supply functions e.g.

$$(2.3.1) \quad d_1 = d_1(p), d_2 = d_2(p), \dots$$

$$(2.3.2) \quad s_1 = s_1(p), s_2 = s_2(p), \dots$$

as long as the following conditions are fulfilled:

- (i) that demand and supply respectively depend only on the price of the product concerned;
- (ii) that this price is the same for all individuals demanding and supplying the product.

Where these conditions are fulfilled — as happens to be the case in the situation described by the formulae (2.3.1) and (2.3.2) — total demand as well as total supply is also a function of this particular price, for

$$(2.3.3) \quad d_1 + d_2 + \dots = d_1(p) + d_2(p) + \dots = d(p),$$

and

$$(2.3.4) \quad s_1 + s_2 + \dots = s_1(p) + s_2(p) + \dots = s(p).$$

Thanks to the specific assumptions<sup>1)</sup> on which he bases his argument both conditions are fulfilled in YNTEMA's system. Accordingly he does not encounter any difficulties while combining individuals into groups according to the kind of currency they use.

However, as soon as commodity groups are created matters change. Each commodity has its own price, the fluctuations of which over the course of time usually differ from the fluctuations of the prices of other commodities. Now in grouping commodities together the need arises for some sort of average price for the groups as a whole. Let this price be

$$(2.3.5) \quad p = \frac{a_1 p_1 + a_2 p_2 + \dots}{a_1 + a_2 + \dots}$$

$p_1, p_2$ , etc. being the prices of the separate commodities in the group and  $a_1, a_2$ , etc. being the weights attributed to them in the determination of the average price  $p$ . Difficulties may arise out of this procedure of introducing an average price, for a function of  $p_1, p_2$ , etc., can be transformed into a function of  $p$  alone only if a very strict condition is fulfilled. This condition is that each of the prices plays an equally important part in the former function as in the determination of the average price  $p$ . Stated in another way the condition is that the weights are exactly the same in both, i.e.

$$(2.3.6) \quad F(p_1, p_2 \dots) \equiv f(a_1 p_1 + a_2 p_2 + \dots) \equiv \\ \equiv (a_1 + a_2 + \dots) f'(p).$$

Now, as long as there is only one relation under consideration — say the demand function — in which

<sup>1)</sup> YNTEMA, *op. cit.* p. 5.

price appears, care can be taken, of course, that the weights  $a$  are chosen in such a way that the condition is fulfilled, for we are still free in choosing the definition of  $p$  as given in formula (2.3.5). However, once the definition of  $p$  is chosen and a second relation in which the price appears — say the supply function — is introduced, difficulties are encountered, for there is no more room to choose the weights of the separate prices in such a way that the condition formulated in (2.3.6) is fulfilled with regard to this second relation. If we were to choose new weights in order to have the condition fulfilled for the second relation we should have two different  $p$ 's and this would theoretically render it impossible to study the two functions in relation to each other.

Fortunately the practical importance of this objection to the grouping procedure turns out to be very small in most cases. If the prices of all the separate commodities show the same movements, or at least movements which only vary in their respective amplitudes, i.e.

$$(2.3.7) \quad p_1 = f_1(p), \quad p_2 = f_2(p), \text{ etc.}$$

then the weights can be chosen at random and condition (2.3.6) will always be fulfilled. While working with annual figures for prices, which exclude seasonal variations, one will usually find that condition (2.3.7) is approximately fulfilled. Even if the patterns of the separate price movements vary slightly, the influence of the choice of the weights on the pattern of the movement of the average price  $p$  will be comparatively small. Therefore, from a practical point of view it will be perfectly legitimate to introduce, as a first approximation, the same average price  $p$  in the supply function and the demand function of the commodity group as a whole. If the patterns of the

separate price movements vary a good deal it is advisable to aggregate the commodities into different groups according to the shape of their price movements.

It must be admitted that theoretical exactness is sacrificed here, to a certain extent, to simplify the picture of the complicated reality. One has, however, to make one's choice between two evils, and in most cases the application of the macro-economic approach turns out to be the lesser of the two. It all depends on the type of problem dealt with and therefore there may be cases where it is just the reverse.

## 2.4

## SYSTEMS OF EQUATIONS

Having dealt in section 2.2 with the question as to the type of function which should preferably be used for describing economic relationships in mathematical form, we now turn to a discussion of the question of what the economic *content* of such relations is. While discussing this matter we will also have the opportunity of pointing out the importance of the *interrelationships* between different functions which have certain variables in common.

To fix the ideas we will start our story by discussing briefly the content of the *demand function* for a commodity. By such a function we want to express that the volume demanded depends on the price of the commodity concerned and a set of other factors commonly known as demand factors. Let the relation be

$$(2.4.1) \quad u = d(p, d_1, d_2 \dots)$$

In this relation  $u$  stands for turnover or quantity demanded,  $p$  for the price of the commodity and  $d_1, d_2$ , etc. for the demand factors, whereas  $d$  serves as a function symbol for the demand function.

Now let us suppose that one of the demand factors, e.g.  $d_1$ , changes its value. According to formula (2.4.1) this will result in a change in the quantity demanded. Now if, in general, all the factors which are responsible for changes in the volume demanded are independent of each other, i.e. if there is no systematic relationship between them, either directly or indirectly (via  $u$ ), then it may be said that the *unknown* quantity  $u$  in formula (2.4.1) is entirely determined by *known* quantities or data, namely  $p$ ,  $d_1$ ,  $d_2$ , etc. In economics the unknown quantities are often indicated as *economic quantities*, in order to distinguish them from the *data*.

Studying a relation of the given type the economic theorist usually applies what is sometimes called the *variation method*. That means that one considers the effect of a change in one of the data (say  $d_1$ ) on the value of the phenomenon to be explained (here  $u$ ), under the supposition that the other data, figuring as explanatory factors in the relation under consideration (here  $p$ ,  $d_2$ , . . . .) remain constant. This is tantamount to saying that the so-called *ceteris paribus* clause is applied.

The *ceteris paribus* clause may be applied to known, but *not* to unknown quantities. The meaning of this will become clear if one considers the case in which one of the explanatory factors must be looked upon as an unknown quantity. In our example this will be the case if the costs of the additional output exceed the proceeds at the current price which will force producers to increase the price of their product. Their reaction to increased demand, and therefore increased turnover, may be described in the following *supply function*:

$$(2.4.2) \quad p = s(u, s_1, s_2 \dots)$$

in which  $u$  and  $p$  again stand for turnover and price of the commodity, while  $s_1, s_2$ , etc. the so-called supply factors stand for other factors which, from the production side, influence the price; finally  $s$  serves as a function symbol for the supply function. The supply function has here been given the special form of a *price fixation equation* (cf. also section 3.6).

Restoration of equilibrium in the market for the commodity concerned requires the new price to be chosen in such a way that the quantities demanded and supplied (produced) again balance. To find this price we first solve equation (2.4.2) for  $u$ , which gives us:

$$(2.4.2') \quad u = s' (p, s_1, s_2 \dots).$$

Then we put the right-hand sides of equation (2.4.1) and (2.4.2') on a par, which results in an equation from which  $p$  can be solved, the so-called *price equation*:

$$(2.4.3) \quad p = p (s_1, s_2 \dots, d_1, d_2 \dots).$$

This equation could also have been arrived at in another way, namely by substituting straightaway the expression for  $u$ , as given by equation (2.4.1), into equation (2.4.2) and solving the new equation for  $p$ .

Similarly, substitution of (2.4.2) into (2.4.1) leaves us with an equation for  $u$ , which no longer contains  $p$ , the so-called *turnover equation*

$$(2.4.4) \quad u = u (s_1, s_2 \dots, d_1, d_2 \dots).$$

The advantage of equations (2.4.3) and (2.4.4) is that they express the unknown quantities  $p$  and  $u$  as functions of merely data. This implies that the variation method and the *ceteris paribus* clause, which goes with this method,

are again applicable, just as they were in the case of constant prices, discussed earlier.

Now what appears to be the case? In the first case under discussion we dealt with a problem which involved only one unknown (the phenomenon to be explained *u*) and one relation proved to be sufficient to describe the behaviour of that unknown quantity if one of the data which acted as explanatory factors, changed its value. In the second case two unknowns were involved in the problem set and a system of two equations was needed to describe the influence on the unknowns of a change in the data. By the simple algebraic procedure of eliminating one of the unknowns it proved to be possible to reduce the system of two equations to only one equation, the so-called *final equation*. In this equation there appeared only one unknown, apart from the data. The equation thus made it possible to determine *immediately* the influence of changes in the data on the unknown (also called the variable to be explained). That in the second case, as well as in the first, the number of equations proved to be equal to the number of unknowns is not at all accidental. It fits in completely with a well-known lemma of the algebra, according to which the solubility of the unknowns out of a system of equations depends on the condition that the number of independent equations, in which one or more of these unknowns appear, should be equal to the number of unknowns. This rule holds true no matter what the number of unknowns is, and no matter by what types of function the relations between the phenomena entering into the problem are described mathematically. Wherever this condition is fulfilled one calls the system a *complete* one. If the condition is not fulfilled the system will be marked as *incomplete*. Such an incomplete system

only enables us to ascertain a functional relationship between one variable and one or more other variables. The number of explanatory variables in such a function cannot be reduced below the number which corresponds with the difference between the number of variables and the number of equations of the system.

Another equally well-known algebraic rule answers, in broad terms, the question as to what the *number of possible solutions* for an unknown is. According to this algebraic rule the number of solutions is equal to the degree of the final equation i.e. the equation in which, apart from data, only the unknown appears. If, for instance, the system consists of equations which are all of the linear type, then the final equation will also be a linear equation, i.e. an equation of the first degree in the unknown, and only one solution will be possible. If, however, the final equation contains a term with the unknown raised to the second power, then there will be two solutions in general. And so on. This will suffice to bring home to the reader the possibility of more than one solution and herewith the possibility of a *multiple equilibrium* for certain economic systems. This possibility implies that, in certain cases, an equilibrium — stable or unstable, — may establish itself at different levels of employment.

In the preceding section the bearing of two well-known algebraic rules on the solubility of the unknowns in a system of equations and the number of possible solutions for these unknowns has been discussed. Each variable was considered in that context as being constant in time. Nothing, therefore, has been said so far about the *dynamics* of the system.

It often happens that the system shows *endogenous movements*, i.e. movements independent of any change in the data but inherent to the structure of the system. These movements may be either fluctuations or one-sided movements. Fluctuations might have either a decreasing, a constant, or an increasing amplitude (damped, undamped and anti-damped or explosive fluctuations). One-sided movements may be movements either towards an equilibrium or away from it.

These endogenous movements should be well distinguished from exogenous movements, which are caused by changes in the data of the system. Endogenous movements find their explanation in the fact that in the final equation the value of the unknown at a certain moment turns out to depend not only on data, but also on the value of the unknown itself at other moments. This may be due to the appearance in the system either of *time lags*, or of *rates of change* of one or other of the unknowns. Lags may appear in the reaction equations of the system if the reaction is thought not to be instantaneous or almost instantaneous; e.g. changes in wages lag behind changes in the cost of living. Rates of change of an unknown may be represented by the difference between the value of the unknown at one moment and the value at an earlier moment, e.g.  $\Delta p = p_0 - p_{-1}$ . The difference ( $\Delta p$ ) in the values of the variable  $p$  at the moments 0 and  $-1$ , indicated by subscripts, is defined this way.

If none of the elementary equations of the system contains lagged terms in the unknowns in one form or the other, then the final equations for each of the unknowns will yield equilibrium values for the unknowns. If, however, lagged terms appear in the system, then the elimination process will yield a final equation which also contains

lagged terms. This means that the unknown, given its initial values, as a rule does not have one or more equilibrium values but that it is subject to a *development law*. Such a development law depends on the coefficients of the lagged terms in the final equation and on a number of initial values of the unknown. It is these development laws which largely contribute to the explanation of the business cycle phenomenon.

Restraint is required in introducing lags in the elementary equations from which, by means of an elimination process, the final equation is to be derived. The reason is that, in order to find out the type of movement described by the development law and represented by the final equation, one has to solve a so-called characteristic equation, and the degree of this equation is equal to the number of time units of the biggest lag in the final equation. (The time unit should preferably be chosen in such a way that all the lags can be expressed in whole numbers). By introducing different time lags of various lengths one soon faces the difficulty of arriving at a characteristic equation of higher degree which proves to be insoluble. Even the introduction of only one time lag may produce this devastating result.

In the system of equations to be described in the next chapter no lags will be introduced. The reason is that for present purposes we are not especially interested in the development of the economy over the course of time. Our principal aim is to ascertain the characteristic features of certain equilibrium positions.

It should not be overlooked that the procedure of neglecting lags, where they may exist in reality, implies that the picture given by our theory is incomplete. What happens in reality may, therefore, differ from what one would expect on the basis of the theory to be developed

in the following chapters. A formal discussion of the *stability* problem with the help of figure 3 may illustrate this point somewhat further.

If reactions on both sides of the market were *instantaneous*, the equilibrium was said to be stable if a fall in the price caused an excess of demand over supply, for this would cause the price to rise again and would restore the equilibrium in the market (cf. section 1.5). The configuration of the supply and demand curves in cases A, B and C satisfies this condition. Therefore the equilibria A, B and C will be stable if reactions are instantaneous. Equilibrium D, on the other hand, does not satisfy the condition that a fall in price causes an excess of demand over supply. Therefore it should be qualified as unstable. In general it may be said that where reactions are instantaneous equilibria are stable if the slope of the demand curve is lower than the slope of the supply curve, both taken algebraically. (The slope of the demand curve is negative in all graphs, that of the supply curve positive in graphs 3A and 3B, and negative in graphs 3C and 3D). Unstable equilibria are characterised by a supply curve which has a lower slope, taken algebraically, than the demand curve.

Now let us consider what happens if the reactions on the supply side of the market are subject to a certain *time lag*, whereas those on the demand side are still instantaneous. These assumptions on which the well-known cobweb-theorem is based may be represented by the following formulae, if the relationships between changes in the variables concerned viz. volume supplied and demanded ( $s$  and  $d$  respectively) and price ( $p$ ) are linearly approximated.

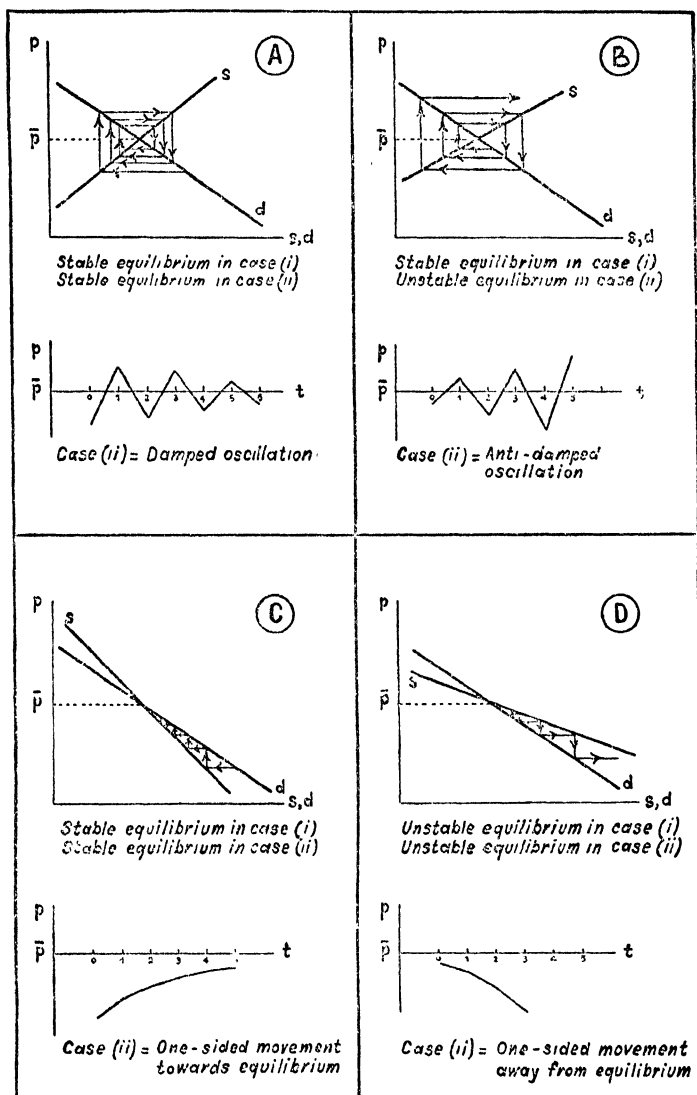


Figure 3

Stability of equilibrium in case of (i) instantaneous reactions on both sides of the market and (ii) delayed action on supply-side and instantaneous reaction on demand-side of the market

$$(2.5.1) \quad s = ap_{-1}$$

$$(2.5.2) \quad d = bp$$

Because

$$(2.5.3) \quad s = d$$

the resulting price movement will be determined by the equation:

$$(2.5.4) \quad p = \frac{a}{b} p_{-1}$$

The following configurations are possible here <sup>1)</sup>:

- (i)  $\frac{a}{b} < 0$  and  $|a| < |b|$  — damped oscillations  
(cf. graph 3A)
- (ii)  $\frac{a}{b} < 0$  and  $|a| > |b|$  — anti-damped oscillations  
(cf. graph 3B)
- (iii)  $\frac{a}{b} > 0$  and  $|a| < |b|$  — one-sided movement towards  
equilibrium (cf. graph 3C)
- (iv)  $\frac{a}{b} > 0$  and  $|a| > |b|$  — one-sided movement away  
from equilibrium  
(cf. graph 3D).

In all graphs  $b$  is negative, whereas  $a$  is positive in graphs 3A and 3B and negative in graphs 3C and 3D. The first and the third case evolve movements towards equilibrium; these equilibria are stable therefore. With the second and fourth case it is just the reverse; they represent unstable equilibria. It is apparently the absolute values of the slopes of supply and demand curve  $\left(\left|\frac{1}{a}\right| \text{ and } \left|\frac{1}{b}\right|\right)$  respectively which determine the question whether the equilibrium is stable or unstable. In cases where

<sup>1)</sup> The border-line cases  $|a| = |b|$  viz. undamped oscillation ( $a = -b$ ) and coinciding supply and demand curve ( $a=b$ ) are left out.

the absolute value of the slope of the demand curve is lower than the absolute value of the slope of the supply curve the equilibrium is stable, whereas in cases where it is higher the equilibrium is unstable. Particularly interesting is the case represented by graph 3B, for the answer to the stability question appears to change if a time lag in the reactions of suppliers to a price change is introduced. This kind of instability due to lags, dealt with here, will *not* be revealed by the system of equations to be developed in the next chapter, because lags will be ignored.

Where the endogenous movements of the system are movements towards equilibrium (cases A and C), the conclusions of a *static theory*, i.e. a theory that does not take account of lags, apply to the equilibrium towards which the endogenous movements carry the system. In reality this equilibrium situation will establish itself only in the long or medium long run, that is to say, if none of the data of the system changes its value in the meantime. Once this equilibrium situation has established itself it will be preserved as long as the data remain unchanged. Whether it will establish itself at all cannot be told by static theory. Only the dynamic theory, which analyses the movements of the system, can answer this question.

Static theory in its simplest form describes and explains only one single equilibrium situation, given the data of the system. However, one can also study how the equilibrium situation is affected by changes in the data, which cause *shifts* in the curves representing certain relationships, e.g. supply and demand curves. Such comparative study of different equilibrium situations is known under the name *comparative statics*. While applying the method of comparative statics one should be well aware

of the fact, already referred to above in connection with static theory, that this method makes sense only if the system, after having been disturbed by outside influences, shows movements towards equilibrium.

## 2.6 LONG-TERM AND SHORT-TERM REACTIONS

Having dealt with the character of the equilibrium (i) in the case of instantaneous reactions on both sides of the market and (ii) in the case of a lagged reaction on one side of the market, and having referred to the study of equilibrium transposition by means of the method of comparative statics (cf. also section 1.5), we now turn to the question of the difference in long-term and short-term reactions. Here we can no longer assume the slope of the curves to remain unchanged during the process of adjustment. Where there is a difference between long-term and short-term reactions it means, apparently, that *the slopes of demand and supply curve* to be employed for the study of the long and the short run equilibrium on one and the same market *are not the same*. Differences in reactions mean differences in structural coefficients, which in their turn imply different slopes of supply and demand curves.

It takes time for changes in data, and also for accidental changes in variables, to work out fully on the variables they directly influence. They only gradually change the values of these variables. This is due to what may be called "imperfections" of the market. For instance, if the price of a certain product falls, a number of people will immediately spot the change, and start to buy more of this product and less of other products. Other people, however, will not be so quick in their reactions. They

will either be unaware for some time that the change has taken place or they will at first look upon it as a temporary change only and feel it not worth while changing their consumption habits. Another example, this time on the supply side of the market, is the introduction of new production methods. These methods may be more profitable than the older ones, but the entrepreneur will only change over to them by the time his old plant or machinery is economically worn out, that is when the initial costs plus the running costs of the new plant and machinery become lower per unit of product than the running costs only of the old plant and machinery.

To take account of this fact the explanatory variables and data should be introduced in the elementary equations of the system with so-called *distributed lags*, that is to say that of each variable or datum the values over a whole range of preceding time units should appear in the equation. Thus, *not*:

$$(2.6.1) \quad d_t =: bp_t,$$

but

$$(2.6.2) \quad d_t = b_0 p_t + b_1 p_{t-1} + b_2 p_{t-2} + \dots$$

It will be fairly obvious that the introduction of all these many lagged terms into a perfectly general system of equations would render such a system far too complicated to handle, although in principle it could be done. This difficulty, however, can be overcome if it is decided beforehand whether the object of study will be (a) the immediate effect, (b) the ultimate effect, or (c) the effect of a change — say  $p_0$  — over a given period of time. If (a) is the case all lagged terms should be disregarded and thus the following equation should be employed:

$$(2.6.3) \quad d_0 = b_0 p_0 (= C_0 p^0).$$

If (c) is the case, and the effect of disturbance  $p_o$  over a period of  $t$  time units is studied in isolation (which means that in formula (2.6.2)  $p^t = p_{t-1} = \dots = p_o$ ), the formula will become:

$$\begin{aligned} (2.6.4) \quad d_t &= b_o p_t + b_1 p_{t-1} + \dots + b_t p_o = \\ &= (b_o + b_1 + \dots + b_t) p_o = \left( \sum_i^t b_i \right) p_o = C_t p_o \end{aligned}$$

whereas for  $t = \infty$  case (c) changes into case (b) and the ultimate effect will be

$$(2.6.5) \quad d_\infty = \left( \sum_i^\infty b_i \right) p_o = C_\infty p_o.$$

So far we have not taken into account the possibility of indirect influences of the disturbance  $p_o$ , which may cause further changes in the price level, through changes in variables affected by changes in the price level (e.g. the wage level), which in their turn again influence the price level. This would mean that  $p_t = p_{t-1} = \dots = p_o$  no longer holds true. Such indirect influences may even cause changes in other explanatory variables appearing in the demand equation (e.g. income), which would result in changes in the volume of demand, which were not a direct effect of changes in the price level. We shall, however, assume for the present that the system under consideration contains sufficient equations to express these changes in changes in the price level. Our model, to be developed in the next chapter, will be such that this condition is fulfilled.

The conclusion from the foregoing must be that the numerical values of the structural coefficients to be entered in the equations depend on the time period for which the adjustment process, following a disturbance, is studied. The answer to the stability problem, studied in this mono-

graph, may accordingly vary. It may even happen that an equilibrium which is stable in the short run will turn out to be unstable in the long run owing to further effects which only make themselves felt later and which counteract the immediate effects. Where our analysis in the fourth chapter will show the coefficients to be such that the equilibrium should be considered to be stable with regard to the period specified, there still remain doubts whether the equilibrium will not be unstable over a shorter period.

For cases (a) and (b) — immediate and ultimate effect of a change respectively — the choice of the proper numerical values of the structural coefficients does not provide any difficulties, at least from a theoretical point of view. However, for the intermediate case (c) — effect of a change over a specified period — the choice is not so easy if there are also indirect influences to account for. An example may illustrate this point. Let the period be three time units, and let the impact of the change in one variable, say investment ( $V$ ), on another variable, say income ( $Y$ ), make itself felt as follows: 10 % of the total effect immediately, 20 % after one time unit, 30 % after two time units, 20 % after three time units and the rest later. Furthermore let this variable ( $Y$ ) exercise a direct influence on another variable, say consumption ( $U'$ ), and let the influence of a change again be distributed over time in the same way. The numerical value of the structural coefficient in the first equation,  $Y = Y(V)$ , should then be equal to 0.8 times the numerical value of the structural coefficient measuring the total direct effect of a change in  $V$  on  $Y$  over the course of time. In the second equation,  $U' = U'(Y)$ , the numerical value of the structural coefficient should, however, be only  $0.31/0.8$  times the numerical value of the structural coefficient measuring the total

direct effect of a change in  $Y$  on  $U'$ , as can easily be seen from the following table:

| Change in $Y$ during time unit: |      | Resulting change in $U'$ during time units: |      |      |      |       |
|---------------------------------|------|---------------------------------------------|------|------|------|-------|
|                                 |      | 0                                           | 1    | 2    | 3    | total |
| 0                               | 0.10 | 0.01                                        | 0.02 | 0.03 | 0.02 | 0.08  |
| 1                               | 0.20 |                                             | 0.02 | 0.04 | 0.06 | 0.12  |
| 2                               | 0.30 |                                             |      | 0.03 | 0.06 | 0.09  |
| 3                               | 0.20 |                                             |      |      | 0.02 | 0.02  |
| total                           | 0.80 | 0.01                                        | 0.04 | 0.10 | 0.16 | 0.31  |

It may seem that there is a contradiction between the case of a definite lag in the reactions on one side of the market, dealt with in section 2.5 where no reference was made to changes in the slopes of the curves describing the market mechanism, and the statement made in this section that the slopes of the curves vary according to the period of adjustment considered on account of the existence of distributed lags. This difference, however, is only seemingly for in fact the slopes of the curves do also change in that case. The case dealt with there is a very special one but, nevertheless, one that is often met in reality. It especially applies to perishable crop products. Once the crop is harvested the volume of supply is fixed. In our graph this would mean that the supply curve is a perpendicular straight line running parallel to the price axis. The price under such circumstances is determined by the demand function,

$$(2.6.6) \quad p = \frac{1}{b} s$$

where  $s$  is a given quantity. For the next period the producer considers the price as given, and adjusts his production in such a way that his marginal costs become equal to this price. He ignores the influence of his action on the price, that is to say he considers the demand curve to be horizontal at the level of the given price. All his colleagues do the same. From this action there will again result a given supply and the price will again be determined by the demand function (2.6.6). This process of adjustment was described in the graphs by horizontal and vertical lines.

## 2.7

## ECONOMETRICS

In the preceding sections of this chapter we discussed various aspects of a method enabling the economist to frame a theory which should at the same time have the merits of being as realistic, as comprehensible, and as simple as circumstances permit. Before discussing the last step to be taken in order to render such a theory, derived by this method, applicable to the economic problems of the real world, it seems desirable to take stock. What were the devices recommended to simplify the picture of the complicated reality, and what were the devices for guaranteeing that the simplified picture of reality still remains realistic in its main features?

In section 2.1 the purely "literary" approach, so often applied in economics, was confronted with the mathematical approach. The *mathematical approach* was thought to be preferable, especially while dealing with complicated economic problems. The reasons given for the introduction of mathematical discipline in economics were: (1) its contribution to the surveyability of the argument, (2) the safeguard it provides for the completeness of a theory, (3) the help

it renders in stating clearly what the premises are on which the argument is based, (4) the urge for concreteness which emerges from its application, and above all (5) the fact that it enables the investigator to give a far more penetrating analysis of his subject than otherwise would have been possible. Comparing the possibilities offered by the two approaches it was found that the "literary" method usually did not carry the theoretical economist any further than the formulation of *functional* or, at its best, causal relationships between economic phenomena.

In section 2.2 consideration was given to the question as to what choice should be made with regard to the mathematical form of the various relations. For relations being part of a system of equations linear functions were recommended in general and more in particular for supply and demand relations, about the mathematical form of which nothing is known on a priori grounds. For definitional equations, however, a linear approximation was thought to be refutable in certain cases where the problem under investigation involves big variations in the variables. With regard to cases of small variations in variables it was argued that linearity usually does not impair the validity of the conclusions.

In section 2.3 the *macro-economic* method was recommended as a device to reduce the *number of equations* by combining individuals, goods, and markets into broad groups. This procedure to simplify the picture of the complicated reality also proved to be quite permissible in most cases. It was said to have the advantage of enabling the investigator to concentrate on the major issues of the problem under investigation without being disturbed by factors which are not really significant.

In section 2.4 attention was paid to the *interrelationships*

between economic phenomena, as they appear in a *system of equations*, or model. Such models are meant to give a complete, although not perfect, picture of an economic system. The test for the completeness of the system was said to be the equality of the number of unknown economic quantities and the number of equations relating the unknown quantities with each other.

In section 2.5 the purposes of *dynamic and static theory*, as well as the connection between the two types of theory, were briefly indicated. Dynamic theory was said to describe and explain the development laws of the economy, particularly in so far as they exhibit themselves in the endogenous movements of the system. It was shown that the lags in people's reactions to changes in certain phenomena are responsible for the existence of these endogenous movements. Static theory, it was argued, is especially fit to deal with an investigation into the properties of the equilibria which establish themselves on different markets. Finally, *comparative statics* was recommended as a method to study equilibrium transposition.

In section 2.6 the difference between *long-term and short-term reactions* to changes in certain phenomena was explained. It was shown that the distributed lags in economic reactions are responsible for this difference, and the point was emphasized that *for different periods of adjustment different numerical values* should be appropriated to the structural coefficients. The choice of these numerical values was illustrated by an example. It was also stated that the answer to the stability problem may vary according to the period of adjustment considered.

Now, there remain two steps to be taken in order to make the much simplified theories, developed by the method described above, applicable to practical issues.

These steps are the *confrontation of theory with reality and its quantification*, i.e. the determination of numerical values for the structural coefficients appearing in the model. These two steps, which usually go hand in hand, will be discussed in the remainder of this section.

For the *confrontation and quantification* of theory a number of natural sciences possess a means which is most suitable, namely the experiment. The tremendous progress made in physics and chemistry since the beginning of the nineteenth century is mainly due to this splendid expedient for quantitative research. The general device in experimenting is to keep constant a number of factors which might be of influence on the size of the phenomenon to be explained. This enables the investigator to ascertain with great accuracy the influence of the one factor that is allowed to vary during the experiment. In the abstract the economist applies the same device when he introduces the *ceteris paribus* clause in his argument. Unfortunately, however, in reality the conditions of this clause are practically never fulfilled, that is to say we very seldom come across a situation in which only the two economic quantities, the causal or functional relationship between which is studied, show variations. Only in the case of vehement changes is this sometimes approximately the situation. However, even in the case of vehement changes usually more than two factors show such vehement changes. From the foregoing it will be clear that economists, because they cannot make use of experiments, must look around for other means of measuring and quantification.

In the course of the twentieth century, economists have succeeded in adopting such other methods of measuring as enable them to quantify the relationships between economic phenomena. This measuring economics is known

by the name of *econometrics*. Since about 1930 it has succeeded in making considerable headway and now has a number of important results on its record.

Without any doubt the most important method of measuring employed is the *multiple (linear) correlation analysis*<sup>1)</sup>. This method is applicable if a series of observations are available for the phenomenon to be explained (e.g. the demand for sugar) and for the phenomena which, according to theory, should be held responsible for the variations in the first phenomenon (examples of explanatory factors may be the price of sugar, national income, etc.). These series of observations are usually so-called time-series, i.e. observations of one phenomenon at a number of consecutive moments or over a number of consecutive time units, normally years, but they may equally well be observations at one time period at various places.

By means of the (linear) correlation analysis the variations in the observations of the phenomenon to be explained can be split up into a number of components, which show variations corresponding with the pattern of variations in each of the explanatory series, but differing from these only in so far as the amplitude of the variations is concerned. In other words, the variations in the explanatory series are each multiplied by a certain number and then added together so as to arrive at the variations of the series to be explained. These numbers are the so-called *regression coefficients*.

The size of these regression coefficients is determined (a) by the importance of the phenomenon concerned with regard to the explanation of the variations of the phenomenon studied and (b) by the respective amplitudes of the

<sup>1)</sup> For a clear and condensed simple exposition of this type of analysis see J. TINBERGEN: *A method and its application to investment activity* (Geneva 1939), chapter II.

phenomena under review. These *amplitudes* depend in their turn on the units in which the phenomena are measured.

Usually the variations in the explanatory series leave part of the variations in the series to be explained unexplained. This is accounted for in the regression formula by an additional term ( $w$ )

$$(2.7.1) \quad x = ay + bz + cu + w$$

This residual ( $w$ ) may be ascribed to (i) a great number of small systematic factors, which influence the phenomenon to be explained partly positively and partly negatively, and (ii) random factors, which sometimes exercise a positive influence on the phenomenon to be explained and sometimes a negative influence.

The correlation analysis does not only have a function in quantifying the relationships which form part of a theory, but it also has the function of controlling and verifying the element of truth contained in the theory put forward. The result of the calculations may be that only part of the variations in the phenomenon to be explained can in fact be explained by the variations in the explanatory factors enumerated by the theory tested, and that the part of the variations which remain unexplained is considerable. In such cases the theory is either wrong or incomplete. It will need revision or completion in order to become applicable to the situation or period for which the relationship is studied. It should be emphasized that this does not imply that the theory is wrong in all cases. For other cases — other periods or situations — it may still give a satisfactory explanation. What is proved is only that the theory is not generally applicable in its given shape. On the other hand, if the theory proves to give a

satisfactory explanation in one case it does not follow that it will do so in another case. So correlation analysis cannot prove a theory to be right in all cases. It cannot even prove the theory to be right at all, because there may always be another explanation which also fits the facts. Another possibility of conflict between theory and reality is that one or more of the explanatory factors in the regression equation appears with a sign which, according to economic theory, is not acceptable on a priori grounds. An example of this is a positive sign for the regression coefficient indicating the relation between quantity demanded and price. In such a case the theory also needs revision, because it contradicts the facts.

#### MATHEMATICAL APPENDIX TO CHAPTER II

##### *Varying regression coefficients*

In section 2.2 it was stated that it is possible, in cases where a coefficient by which one of the variables in an equation is multiplied depends in its turn on one or more variables, to reduce such an equation to one in which only constant coefficients appear. In order to clarify this point an example of such a case will be considered somewhat closer in the following <sup>1)</sup>.

Say that the total of a country's consumption ( $\bar{\bar{U}}'$ ) is a function of the level of its national income ( $\bar{\bar{Y}}$ ) and the price level ( $\bar{p}$ ):

$$(1) \quad \bar{\bar{U}}' = v_1 \bar{\bar{Y}} + v_2 \bar{p} + c + w \quad (w = \text{residual})$$

and that the influence of income on consumption ( $v_1$ )

<sup>1)</sup> This example is, with slight alterations, taken from J. TINBERGEN: "Econometric Business Cycle Research" in *Review of Economic Studies* VII (1940), p. 79.

depends on the prevailing price level:  $v_1 = v_1(\bar{p})$ . Let this relationship be a linear one:

$$(2) \quad v_1 = v_{10} + v_{11} \bar{p}$$

Consequently,

$$(3) \quad \bar{U}' = v_{10} \bar{Y} + v_{11} \bar{p} \bar{Y} + v_2 \bar{p} + c + w.$$

Now let us write this formula as:

$$(4) \quad \bar{U}' + U' = v_{10} \bar{Y} + v_{10} Y + v_{11} (\bar{p} + p) (\bar{Y} + Y) + v_2 (\bar{p} + p) + c + w$$

where  $\bar{U}' + U' = \bar{\bar{U}}'$ , etc. <sup>1)</sup>.

Formula (4) can be transformed as follows:

$$(5) \quad \bar{U}' + U' = (v_{10} \bar{Y} + v_{11} \bar{p} \bar{Y} + v_2 \bar{p} + c) + v_{10} Y + v_{11} \bar{p} Y + v_{11} \bar{Y} p + v_{11} p Y + v_2 p + w,$$

which in its turn can be reduced to:

$$(6) \quad \bar{U}' + U' = c' + v'_{10} Y + v'_2 p + v_{11} p Y + w$$

where  $v'_{10} = v_{10} + v_{11} \bar{p}$  and  $v'_2 = v_{11} \bar{Y} + v_2$ . Further  $\bar{U}' = c'$  and the last term but one in this equation as being a second order term (product of deviations) may be neglected. So we are left with:

$$(7) \quad U' = v'_{10} Y + v'_2 p + w.$$

In this formula  $v'_{10}$  and  $v'_2$  are constants. The difference between  $v'_2$  and  $v_2$  is caused by the influence which the price level exercises on the regression coefficient indicating the relationship between consumers' expenditures and national income.

<sup>1)</sup> As the reader will remember, a double bar over the symbol indicates the absolute value of the variable concerned, a single bar its mean or normal value, and the symbol without a bar the deviation from the average or normal value.

## CHAPTER III. THE MODEL

### 3.1

#### INTRODUCTION

After having stated in the first chapter the problem to be dealt with in this study, and in the second chapter the method by which it will be tackled, we have now come to the stage of setting out what the specific assumptions are on which the theory concerning the foreign exchange market, presented in the fourth chapter, is based. This *theoretical framework*, as put forward in this chapter, consists of a system of equations describing the mechanism which rules the price-formation on the market for foreign exchange. In other words, we are now going to *describe* the elementary laws which connect the various factors that influence, either directly or indirectly, the demand for and supply of home currency in exchange for foreign currency. Perhaps it is even better to substitute the words "assumed relationships" for "elementary laws" for, as long as our theory is not put to the test of reality, we should not pretend to do more than show what the logical consequences are of our theoretical set-up as based on the various assumptions to be laid down in this chapter. This situation is not changed in the least by the fact that we shall not choose our assumptions at random, but will use our common sense in order to arrive at a set of assumptions which, to the best of our knowledge, reflects the actual structure of the economic system and its working, and may therefore be hoped to render the theory applicable to practical problems of economic policy.

Before embarking on the project of outlining a model reflecting the various factors which influence the exchange rate and the balance of payments, and the mutual relationships connecting these factors, we have first to settle in broad terms the question of simplifying the picture of the complex reality, bearing in mind that too micro-economic a model or system of equations contains too many variables to deal with properly. Even with a fairly simple model we soon find ourselves in difficulties, as experience shows.

Several lines of attack are conceivable here. First of all one might choose to deal with a multitude of countries or country-groups and reduce the descriptions of the economic systems of the various countries or country-groups to the barest minimum. In doing so one easily oversimplifies the description of the various economic systems, even if one concentrates on those aspects of the economic mechanism which have a bearing on balance of payments problems. This procedure evokes the danger of not taking sufficiently into account the indirect influence of a change in the rate of exchange, an influence to which attention was drawn in chapter I. It seems to me that J. J. POLAK and T. C. CHANG, who applied this type of approach in a memorandum of the International Monetary Fund<sup>1)</sup>, did not quite succeed in avoiding this danger.

A second line of attack, which is less dangerous, is to put all the countries except the home country into one group called the outside world. This, of course, leaves more room for an adequate description of the economic mechanism at work in both the home country and the outside world. It enables one to take account of the effect

<sup>1)</sup> J. J. POLAK and T. C. CHANG: *An International Economic System* (June 1949, mimeographed draft paper, not published).

which a change in the exchange rate has on the outside world and particularly on the level of incomes and prices abroad. Therefore, there is much to be said for this two-country treatment which J. TINBERGEN applied in a paper published in the *Giornale degli Economisti* <sup>1)</sup>.

In this monograph we will go one step further and confine ourselves to a description of the economic mechanism of the home country only, particularly with a view to the bearing it has on balance of payments problems. This is the procedure which TINBERGEN applied in his earlier writings on the subject <sup>2)</sup> and it seems to me that there is a good deal to be said for it, although from the point of view of theoretical correctness the two-country approach has much in its favour. Usually, I think, the impact of a change in the rate of exchange on incomes and prices abroad is almost negligible, provided the country which changes its rate of exchange is not too big a country, that is to say if its share in international trade is fairly small. Therefore, I am strongly inclined to favour the one-country approach for the more general cases where only one country changes its rate of exchange and where this country has only a moderate share in international trade (the only exceptions here are perhaps Great Britain and the United States). In section 4.3 it will be shown that the exactitude gained in a more ample description of the economic mechanism of the one country concerned is well worth the sacrifice in theoretical correctness of including in the picture some of the effects abroad, which usually are only of a negligible

<sup>1)</sup> J. TINBERGEN: "Modelli di commercio internazionale" in *Giornale degli Economisti e Annali di Economia* VII (new series), Nov./Dec. 1948, pp. 627—648 (a mimeographed English version of this article under the title "Models of International Trade" has been privately circulated by its author).

<sup>2)</sup> See for instance J. TINBERGEN: "Unstable and Indifferent Equilibria in Economic Systems" in *Revue de l'Institut International de Statistique* 1941 (1/2), pp. 36—50, and his book on *International Economic Co-operation* (Amsterdam 1945), pp. 79—100.

size. One should, however, keep in mind the limits set to the applicability of the theory to be developed in this and the following chapter. The devaluations of many currencies in September 1949, for instance, could not be dealt with by our one-country method. However, by first making a calculation as to the changes in world market prices and incomes abroad resulting from such a wholesale devaluation, and by accordingly changing these data of the system, the one-country method would again become applicable.

For the convenience of the reader the main assumptions, on which the theory to be developed here is based, will be numbered and printed in italics. Thus:

*Assumption 1: The effects of a change in the rate of exchange of the home currency on incomes and prices abroad are assumed to be only of negligible size.*

Having made our choice as to the general set-up of our system of equations we still have to bring out one more general feature of the model to be described in this chapter. This model will turn out to be *incomplete*, that is to say the number of unknowns will be greater than the number of equations. This is because the factors governing the balance of the international capital movements will be left out of discussion. Consequently the system will contain one more variable than the number of equations and is therefore not completely soluble. However, it can be reduced to one final equation, describing one of the unknown variables as a function of only one of the other variables of the system and a number of data. The theory, the assumptions of which are laid down in our model, will thus enable us to deduce from the assumed elementary relationships of the system, for

instance, the relationship between the balance on current account and the exchange rate.

For many problems this lack of completeness of the system is by no means a handicap, owing to the fact that the setting of the problem by itself provides one additional assumption with regard to one of the unknowns. The treatment of the well-known *transfer problem*<sup>1)</sup> for example is usually based on the assumption that the condition of stable exchange rates is fulfilled; in other words the exchange rate is treated as a datum instead of as a variable. For the *exchange rate problem*, i.e. the problem of fixing the exchange rate at such a level that the balance of payments on current account will close with a certain given deficit or surplus, the balance on current account is treated as a datum.

What we want to know for the *exchange stability problem* is the relationship which connects the balance on current account and the exchange rate. As explained in chapter I the equilibrium will be a stable one if a fall in the exchange rate causes a favourable change in the balance on current account, for in this case it will become more difficult to obtain home currency, which will tend to increase the rate of exchange of home currency for foreign currency. An unfavourable change in the balance of payments on current account, on the other hand, will make home currency more abundant and will, therefore, tend to make the exchange rate fall still further. Under such circumstances the equilibrium was said to be unstable. This answer to the exchange stability problem refers to *the effect of the forces at work in the field of transactions in goods and services*. It will, however, be a conclusive answer in so

<sup>1)</sup> See, for instance, J. M. KEYNES: "The German Transfer Problem" in *The Economic Journal*, vol. XXXIX (March 1929) pp. 1—7.

far as it may be assumed that a change in the exchange rate will not have repercussions in the sphere of capital transactions — apart from those induced by the resulting change in the balance on current account — which are sufficiently strong to offset the effect in the sphere of commodity transactions. In section 3.7 it will be shown that in practically all circumstances it is quite permissible to assume this to be the case. For reasons given there it appears to be desirable to base our model on an even more stringent assumption, viz:

*Assumption 2: International capital transactions, except those induced by a change in the balance on current account consequent upon a change in the exchange rate, may be treated as data in the discussion of the exchange stability problem.*

To avoid misunderstanding it should be pointed out here that, although this assumption excludes the possibility of the exchange rate influencing the size of international capital transactions, there remains the possibility that an autonomous change in their size will cause a change in the rate of exchange of home currency for foreign currency.

Apart from the commodity items and the type of capital items just dealt with, the balance of payments also contains a number of so-called “balancing items” viz. gold shipments, changes in foreign currency holdings, and certain short-term credit transactions. They serve to overcome short-run discrepancies between supply and demand of currency where the balance on current account and the balance of the type of capital transactions discussed above leave a gap. These balancing items must, of course, be excluded from the concepts of supply and demand of currency, for their inclusion would render the exchange

stability problem meaningless, as the balance of payments defined in this way would always automatically be in equilibrium. It may even be said that it is the object of this study to ascertain whether balancing items (which, under assumption 2, are the counterpart of the change in the balance on current account) are called for by a change in the exchange rate. Their existence is a sure sign of a disequilibrium in the balance of payments.

### 3.2 THE VARIABLES, DATA AND EQUATIONS OF THE SYSTEM

The model referred to in the previous section, which endeavours to give a concise picture of the complicated mechanism ruling the price-formation in the foreign exchange market, consists of a system of equations connecting the various variables and data. Before discussing the actual contents of each of these equations it may be useful, I think, to indicate briefly the different types of equations, variables and data contained in this system, the units of measurement, and the notation of variables and data. The present section is devoted to this purpose.

The unknown *variables* of the system, like all economic quantities, are to be divided into three main groups namely: (a) money-values (b) volumes and (c) prices. The majority of these variables refer to imports, exports, production and consumption of goods and services. Their proper definitions will be given in the context of the following sections, but to avoid misunderstanding it should be made clear right from the start that the production and consumption concepts used in this study are a little unusual. The production concept includes not only the "value added" but also the imported part of the product and could,

therefore, more correctly be referred to as "gross production". Unfortunately, however, the term "gross production" is often used with a different meaning (cf. section 3.4). The consumption concept includes not only the purchases of consumer goods but also those of capital goods. The word "expenditure" would, perhaps, better convey this aspect of our consumption concept, but has also its shortcomings (cf. section 3.4). For the sake of convenience we will, therefore, refer throughout to these concepts as "production" and "consumption" respectively. The variables mentioned so far are indicated by the following symbols:

imports   exports   "production"   "consumption"

|              |       |       |     |      |
|--------------|-------|-------|-----|------|
| money-value  | $U^i$ | $U^e$ | $U$ | $U'$ |
| volume. . .  | $u^i$ | $u^e$ | $u$ | $u'$ |
| price. . . . | $p^i$ | $p^e$ | $p$ | $p'$ |

The remaining variables are:

- (1)  $Y$  = national income (= sum total of remunerations of home factors of production).
- (2)  $S$  = surplus on the balance of payments on current account.
- (3)  $l$  = wage rate.
- (4)  $k^b$  = rate of exchange (= price of home currency expressed in terms of foreign currency units).
- (5)  $p^{eb}$  = export price expressed in foreign currency unit.

One might have expected to find in the list of variables  $p^{eb}$ 's counterpart, the world market price of imported goods and services expressed in foreign currency units, the symbol for which would be  $p^{ib}$ . The omission of this economic quantity from the list of variables is readily explained by assumption I, where it was said that the effect of

changes in the exchange rate on prices abroad would be left out of consideration; therefore  $p^{ib}$  should be treated as a datum.

The choice of the *units* in which the variables of the system are measured is such that the original equilibrium prices are equal to unity. The money-values are expressed in currency units. Consequently volumes are measured in currency units of constant value. Furthermore, it should be noted that in the equations all the variables are measured as *deviations* from their original equilibrium values. To distinguish these deviations from the original equilibrium values the latter are indicated by the appropriate symbol with a single bar over it, whereas the absolute values of the variables, being the sums of original equilibrium values and deviations thereof, are denoted by the appropriate symbol with a double bar over it, e.g.  $\bar{\bar{u}}^i = \bar{u}^i + u^i$ .

The *data* of the system are in general to be classified in three main groups. Firstly, there are the *original equilibrium values* of the variables, referred to above. Secondly, there are the outside or *exogenous influences* working on the system and setting it in motion. They should be indicated by the symbol of the variable, on which they exercise a direct influence, with a subscript  $o$  at the bottom right-hand side. To give just one example, a change in world market prices of imported goods and services, referred to above as  $p^{ib}$ , would have to be denoted as  $u_o^i$  if it were thought to have a direct influence on the volume of imports. For the study of the exchange stability problem no purpose is served by introducing this type of datum. Problems of economic policy, on the other hand, cannot be dealt with without having recourse to this type of datum and it is solely for that reason that it is mentioned here. The third and last category of data consists of what is referred to in the technical language

of the correlation analysis as regression coefficients and in the language of econometricians as *structural coefficients*. The latter name is perhaps not wholly satisfactory, for practically all these coefficients and certainly the more important ones have nothing to do with the structure of the economic system; they are connected more with changes in that structure. The structure of the system may be indicated by the ratios between certain economic quantities, like the average propensities to consume, to invest and to save; the changes in the structure, however, are measured by coefficients which relate the change in one economic quantity to the change in another, like the marginal propensities. Unfortunately there does not seem to be a better word for them in English, unless one would be satisfied with a compromise in calling the former coefficients "average" and the latter ones "marginal" structural coefficients and leave it at that. The structural coefficients will be indicated by Greek symbols to distinguish them clearly from all the other data. The ones that enter into our system of equations are the following:

$v'$  = marginal propensity to spend (with reference to changes in the national income in money terms)

$v^i$  = marginal import quota (with reference to changes in the volume of the national product)

$\eta'$  = price elasticity of home demand for goods and services

$\eta^e$  = substitution elasticity of demand for exports (with reference to prices)

$\eta^i$  = substitution elasticity of home demand for imports (with reference to prices)

$\varepsilon'$  = price elasticity of supply of goods and services for the home market

$\epsilon^e$  = price elasticity of supply of goods and services for export

$\lambda$  = wage coefficient (meaning explained in section 3.6)

$v'_i$  = marginal import quota of goods and services for the home market

$v_i^e$  = marginal import quota of goods and services for export

$v'_l$  = marginal wage quota of goods and services for the home market

$v_l^e$  = marginal wage quota of goods and services for export.

The *equations* of the system are to be divided into three main groups namely:

- a. the definitional equations, each of which relates one variable to one or two others in such a way as follows from the definition of the variable concerned;
- b. the balance equations, describing the fundamental equilibrium conditions of the system;
- c. the behaviouristic or reaction equations, expressing the causal relationships between changes in certain phenomena (variables and data).

The latter group of equations may be said to be of strategic importance for the functioning of the whole system, and this is only natural, for it is after all the intensity of peoples' reactions to changes in certain phenomena which determines the outcome of the economic process. This important group of equations can be further divided into two sub-groups:

- (i) the demand equations, describing the reactions on the demand side of the market;
- (ii) the supply equations, describing the reactions on the supply side of the market.

## 3.3

## THE DEFINITIONAL EQUATIONS

The definitional equations of the system are to be divided into two groups namely:

1. Multiplicative definitional equations, expressing that the absolute value of one variable is equal to the product of the absolute values of two (or more) other variables.
2. Additive definitional equations, expressing that the value of one variable is equal to the sum of the values of two (or more) other variables.

The first group is further to be divided into two sub-groups viz.

- (i) money-value equations, stating that money-value equals volume times price;
- (ii) currency equations, linking prices at home (expressed in home currency) with prices of the same goods and services abroad (expressed in foreign currency).

All these equations are equally true for the absolute values as well as for the original equilibrium values of the variables, but not as far as the equations of the multiplicative type are concerned for the deviations between the two. Take, for instance, the first money-value equation which states that the money-value of imports is equal to the product of volume and price of imports. There cannot be any doubt that this applies to both the absolute values of the variables involved and their original equilibrium values:

$$(3.3.1) \quad \bar{\bar{U}}^i = \bar{\bar{u}}^i \bar{\bar{p}}^i$$

$$(3.3.2) \quad \bar{U}^i = \bar{u}^i \bar{p}^i$$

To find the relation between the deviations of these

variables from their original equilibrium values, equation (3.3.1) should be rewritten as follows:

$$(3.3.3) \quad \bar{U}^i + U^i = (\bar{u}^i + u^i) (\bar{p}^i + p^i) = \bar{u}^i \bar{p}^i + \bar{u}^i p^i + u^i (\bar{p}^i + p^i).$$

Subtracting equation (3.3.2) and putting  $\bar{p}^i = 1$  (cf. section 3.2) gives the following result for the relation between the deviations of the variables from their original equilibrium values:

$$(3.3.4) \quad U^i = u^i (1 + p^i) + \bar{u}^i p^i.$$

Along the same lines can be derived the money-value equations for exports, consumption and production. They turn out to be:

$$(3.3.5) \quad U = u^e (1 + p^e) + \bar{u}^e p^e$$

$$(3.3.6) \quad U' = u' (1 + p') + \bar{u}' p'$$

$$(3.3.7) \quad U = u (1 + p) + \bar{u} p.$$

The derivation of the currency equations, one for imports and one for exports, is equally simple. They state that the price of goods and services expressed in foreign currency is equal to the price of goods and services expressed in home currency multiplied by the rate of exchange which is the price of home currency in terms of foreign currency.

Thus for instance:

$$(3.3.8) \quad \bar{p}^{ib} = \bar{p}^i \bar{k}^b$$

from which results, since  $\bar{p}^{ib} = \bar{p}^i = \bar{k}^b = 1$ ,

$$(3.3.9) \quad p^{ib} = p^i (1 + k^b) + k^b.$$

Likewise the currency equation for the export price level runs as follows:

$$(3.3.10) \quad p^{eb} = p^e (1 + k^b) + k^b.$$

For present purposes equation (3.3.9) may be further simplified on account of the fact that under assumption 1  $p^{ib}$  is to be treated as a datum (cf. section 3.2). Now with regard to the exchange stability problem all one need consider is the functional relationship between  $S$  and  $k^b$ . The influence a change in the data has on the system is a different problem altogether. In order to avoid mixing the two problems we had better assume the data to be constant all the way through. For the problem at hand  $p^{ib}$ , representing a change in the world market price level of imported goods and services, should therefore be put equal to nil, which leaves us with the equation:

$$(3.3.11) \quad p^i (1 + k^b) = -k^b.$$

All these definitional equations can be still further simplified by omitting the products of deviations of the variables from their original equilibrium values. This can be done without any loss of generality on condition that one deals with *small deviations* only, as was shown in section 2.2. With regard to the exchange stability problem this condition may be said to be fulfilled. On account of this simplification we arrive at the following set of *definitional equations* of the multiplicative type:

- (i)  $U^i = u^i + u^i p^i$
- (ii)  $U^e = u^e + \bar{u}^e p^e$
- (iii)  $U' = u' + \bar{u}' p'$
- (iv)  $U = u + \bar{u} p$
- (v)  $p^i = -k^b$
- (vi)  $p^{eb} = p^e + k^b.$

Two more definitional equations have still to be added to the list. They are of the additive type and, therefore,

even more simple than those of the multiplicative type as they are equally true for the absolute and the original equilibrium values of the variables, and for the deviations between the two. They refer to the definition of the money-value of home production, in the sense given to the word in the previous section, and to the definition of the balance on current account and read as follows:

$$(vii) \quad U = Y + U^i$$

$$(viii) \quad S = U^e - U^i.$$

The contents of the various concepts such as national income, home production, etc. will be further elucidated in the next section. There it will also be explained why  $S$  is interpreted as the balance on *current* account which gives equation (viii) the character of a definitional instead of a balance equation.

## 3.4

## THE BALANCE EQUATIONS

First of all it must be made clear in what respect balance equations differ from definitional equations. They were said to describe the fundamental equilibrium conditions of the system. Now equilibrium will be brought about by a process of successive adjustments. Definitions on the other hand are self-evident relationships. Therefore, the relations described by definitional equations will always apply, whereas the relations described by balance equations will apply only if the system is in equilibrium, i.e. when the process of successive adjustments has come to an end. This, it may be said, is the fundamental difference between the two categories of equations. The line of distinction now seems to be a clear one, but as the argument

proceeds it will be seen that it often depends only on subtle differences in the way in which the variables are defined whether a certain equation should be looked upon as a balance equation or a definitional equation.

The balance-equation concept apparently only makes sense if an economic system in an equilibrium position is considered or if two such equilibrium positions are compared with each other, that is to say, if we deal with the pure case of comparative statics (compare section 2.5). This case presents itself if the system of equations, to be developed in this chapter, is used to study the long-run effect of a measure of trade policy, or to ascertain the transfer capacity of a country, and similar problems. For the exchange stability problem the balance-equation concept does not make much sense, for what we are actually concerned with is the departure from equilibrium and its consequences and not the original equilibrium position itself. We characterize the original equilibrium position by what happens if it is disturbed. The situation immediately after the disturbance is no doubt one of disequilibrium in which the fundamental equilibrium conditions formulated in the balance equations are not fulfilled. Supply and demand are not balanced in this situation. Only after a process of successive adjustments has taken place will the balance between the two be restored, and it is to this final or long-run equilibrium that the balance equations refer.

As long as the process of successive adjustments has not worked out completely, there will be at least one of the fundamental equilibrium conditions unfulfilled. This may be the monetary equilibrium, or the price equilibrium, or the volume equilibrium between supply and demand, or the balance of payments equilibrium. The

monetary equilibrium refers to the equality of the demand for and the supply of money; the price equilibrium to the equality of marginal costs and marginal revenue; the volume equilibrium to the equality of the volume of goods and services supplied and demanded; and the balance of payments equilibrium to the equality between the supply of and demand for foreign exchange. The phenomena indicating a state of disequilibrium are the following: hoarding or dishoarding if the monetary equilibrium is upset; marginal profits or losses if the price equilibrium is upset; unwanted stock-piling or stock-depletion if the volume equilibrium in the sphere of goods and services is upset; movements of monetary gold and changes in foreign currency holdings if the balance of payments equilibrium is upset. As soon as the economic system is in equilibrium these phenomena will be absent; the discrepancies will be wiped out completely.

By changing the interpretation of some of the variables of the system the balance equations can still be retained as part of the system of equations describing the economy *in a state of disequilibrium*. All that has to be done is to include the discrepancy in the definition of one of the variables entering into the balance equation concerned. So, for instance, stock-piling should be included in the definition of consumption or home demand, stock-depletion being considered as a negative constituent of home demand. After all, this stock-piling in the hands of producers is part of the country's net capital formation, although it is done unwillingly.

Dealing with the matter in this way we should be well aware of the fact that the balance equations now become self-evident and are no longer dependent on the process of successive adjustments which finally results in a true

equilibrium position. This means that the balance equations, if the variables are defined in the way indicated above, degenerate into *definitional equations*. This reflects the fact stated above that for the exchange stability problem we avail ourselves of a system of equations which describes the economy in a state of disequilibrium and, therefore, leaves no room for balance equations. Furthermore, one should not overlook the fact that now that the unintended stock-piling is included in the consumption concept, the equations explaining the size of home demand should also provide an explanation for this *unintended demand*.

Inter alia it should be remarked that stock-piling is not always done unwillingly. In general there is a certain relation between the volume of production and the stocks of raw materials etc. held by producers. Those stocks are necessary to avoid interruption in the production process. The work in progress also implies a certain amount of stock-holding. Apart from this there is speculative stock-piling. Now if the volume of production increases the work-in-progress stock increases automatically. The normal stocks of raw materials will also have to be increased as increased production requires larger stocks. Finally, stocks may be increased for speculative motives. The stock-piling we referred to in the discussion of the balance-equation concept was only the stock-piling that is done unwillingly.

In our system there are three equations which in the pure case of comparative statics act as balance equations, but for present purposes degenerate into definitional equations. Two of them refer to money-value items, whereas the third (see formula (3.4.4) at end of this section) refers to volume items. Let us first deal with the former two. They run as follows:

$$(3.4.1) \quad S = U^e - U^i$$

$$(3.4.2) \quad Y + U^i = U' + U^e.$$

With a third, which can easily be derived from the other two, and therefore may not be considered as an independent equation, they form a very concise *system of purely national accounts*<sup>1)</sup>. This third equation runs as follows:

$$(3.4.3) \quad Y - S = U'.$$

The respective accounts are:

- I. the foreign or outside world account (= balance of payments on current account);
- II. the product account;
- III. the consolidated income and capital account.

Presented in the form of accounts, the balance equations are the following:

| I     |       | II        |       | III  |     |
|-------|-------|-----------|-------|------|-----|
| $U^e$ | $U^i$ | $Y$       | $U'$  | $U'$ | $Y$ |
|       | $S$   | $U^i$     | $U^e$ | $S$  |     |
|       |       | $\bar{U}$ |       |      |     |

In this system every item has its counter-item and so the whole system balances, the condition for this being, of course, that item and counter-item represent one and the same thing in each case.

In setting up a set of definitions for the variables of the system in such a way that these *definitions* are mutually consistent, this accounting method proves to be rather

<sup>1)</sup> Compare my paper: "Recent Experiences in the Use of Social Accounting in the Netherlands" presented at the 1949-meeting of the International Association for Research in Income and Wealth. The  $U'$  in this monograph corresponds to the sum of  $C$  (consumption) and  $I$  (capital formation).

helpful. It shows, once we have made up our minds regarding the definition of one variable, what the implications of this choice are for the definitions of other variables.

In section 3.2 it has been already mentioned that the commodity items of the system refer to *goods and services*. Now if this were true only for  $U^e$  and  $U^i$ , whereas  $U'$  and  $Y$  would apply to goods only, the  $S$ -s in accounts I and III would not be identical, except in the exceptional case where imports and exports of services would exactly balance.

Another point here is that all money-value items should preferably be expressed in the same type of prices, be it market prices, factor costs<sup>1)</sup> or any other type of price. Personally I would prefer the *market price* concept. A strong point in favour of this concept, according to FRISCH, is not only the fact that market prices are the prices we come across in daily life, but particularly that this concept does not obscure, as the factor-costs concept does, the fact that the government in modern society is supposed to render the community a productive service equal in value to the difference between cost-raising taxes and subsidies<sup>2)</sup>.

It is mainly due to the methods of social accounting, which have been developed in recent years, that the concept of national income at market price is rapidly gaining ground. Like all other market-price concepts of money-value items it includes the item cost-raising taxes minus subsidies. It is this last item which makes it differ from national income at factor costs. In case this factor-costs concept were applied to national income,

<sup>1)</sup> Factor costs are the remunerations of the factors of production.

<sup>2)</sup> See R. FRISCH: *Axiomatic Remarks on Some National Income Concepts* (copy of a letter to Richard Stone; University Institute of Economics, Oslo, October 1949, mimeographed).

while all the other money-value items were expressed in market prices, one would find oneself confronted with a system of equations which was unduly complicated. It would have to render an explanation of the variations in the item cost-raising taxes minus subsidies, an explanation which does not in the least contribute to the solution of the problem studied here.

A further point is that two of the variables of the system, namely  $Y$  and  $U'$ , may be either inclusive or exclusive of *depreciation allowances*. Here again the rule applies that both variables should be treated in the same way. The choice between gross concepts, including depreciation allowances, and net concepts which exclude them is not of great importance. Personally I have a slight preference for the *net concepts* as the *gross concepts* imply a certain amount of double-counting. This may be shown by the following argument. The money-value of goods and services produced over a certain period is equal to the sum of consumption, *gross* capital formation at home, and exports. The market prices at which they are valued consist of the remuneration of home factors of production, the remuneration of foreign factors of production (imports) and depreciation allowances. Out of these depreciation allowances the replacement of capital goods, which have been used up in the production process, is financed. So the only contribution which home and foreign factors of production have rendered on balance in the field of capital formation over a given period is the increase in the stocks of goods at the country's disposal, that is the *net* capital formation. It is for this reason that I prefer to define  $Y$  as net national income and  $U'$  as net home consumption, i.e. home consumption of consumer goods and services plus net capital formation.

A still further point is whether the variables of the system should refer to *commodity flows* or to *financial flows*. The difference between the two types of flow is due to the fact that goods and services are not always paid for on delivery; sometimes they are paid for in advance and sometimes payment takes place only after delivery of the goods or services. In a rapidly growing economy the differences between the commodity flows and the financial flows may be considerable. In a more or less stable economy, however, the discrepancies between the two will usually be rather small, so long as payment habits remain the same, for the value of commodity transactions of a previous period that are paid for in this period will practically be equal to the commodity transactions of this period paid for in a later period. For a static economic system the two types of flows even become identical, because the variables do not change. So if we were to apply the method of comparative statics, this point of commodity or financial flows need not, therefore, worry us. This, however, is not the case with regard to our problem. Differences between commodity flows and financial flows, due to changes in the variables or to changes in payment habits, can exist only so long as the system has not yet fully adapted itself to the new situation. Now for the situation on the exchange market it is certainly the financial flows that are decisive. Notwithstanding this, my preference is for commodity flows, for under assumption 2 we have limited ourselves to a study of the influence of a change of the exchange rate on commodity transactions. Changes in payment habits, however, mean changes in credit facilities and, therefore, belong to the sphere of capital transactions which we have excluded from our analysis. The implications of this limitation for

the applicability of our theory to practical issues should not be overlooked. Take, for instance, the case in which imports are paid for in advance and the receipts from exports only come in some time after delivery has taken place. If, in this case, a devaluation of home currency took place then it would mean that the demand for foreign currency would be affected instantaneously, whereas the supply would only be affected after a time. This may cause a temporary strain on the demand side of the foreign exchange market. Usually, if the economy concerned is a sound economy and confidence exists that the new exchange rate will be retained, this strain will be overcome either by additional credit transactions, or better still by gold shipments which only need to take place once.

So far we have reviewed and further specified the definitions of practically all the money-value items of the system. One, however, the balance of the balance of payments, has been left out. And so we have now to investigate what meaning should be given to the  $S$  in our system. Apparently it can be interpreted either as the balance on current account or as net capital export. If equation (3.4.1) should be considered as a balance equation  $S$  should be interpreted as the net capital export. The same applies to equation (3.4.3), stating that consumption is financed out of net national income minus savings put at the disposal of other countries. If, however, (3.4.1) is considered to be a definitional equation  $S$  should be interpreted as being the balance on current account. Now, I think, assumption 2 forces us to accept the latter interpretation, for by virtue of that assumption the study of the influence of changes in the exchange rate on international capital transactions has been excluded from our system of equations.  *$S$  being interpreted as the balance on*

*current account*, the change of which finds its counterpart in the "balancing items" of the balance of payments, leaves us, therefore, with only one independent balance equation linking money-value items together. This balance equation states that the value of the supply of goods and services ( $Y + U^i = U$ ) should be equal to the value of the demand for those goods and services ( $U' + U^e$ ). By means of the equation (viii), defining the surplus of the balance of payments on current account, this balance equation can easily be transformed into another one stating that the value of home demand for goods and services ( $U'$ ) should be equal to the value of the supply for the home market ( $Y - S$ ).

The second balance equation,

$$(3.4.4) \quad u = u' + u^e$$

(stating that the volumes of the total supply of goods and services ( $u$ ) and total demand for goods and services ( $u' + u^e$ ) should also be equal) will not need any further elucidation after what has been said about the money-value counterpart of this equation.

That we are left with only these two balance equations, which together describe the equilibrium conditions of the system in the sphere of transactions in goods and services, fits in well with assumption 2 which excluded all capital transactions from our field of study.

On the basis of the foregoing discussion the following *balance equations*, which for the present purposes degenerate into *definitional equations*, can now be added to the list:

$$(ix) \quad U = U' + U^e$$

$$(x) \quad u = u' + u^e$$

Our system of equations contains three demand equations, the first explaining the changes in the volume of home demand, the second the changes in the volume of imports, and the third the changes in the volume of exports. Each of these equations has certain characteristics of its own. It seems desirable therefore to deal with each of them separately.

The demand equation explaining the changes in the volume of home demand, which will be dealt with first, will, as the argument proceeds, prove to be of crucial importance. Careful consideration should, therefore, be given to it here. Meanwhile, however, one should be aware of the fact that the whole of the theory of demand is at the back of this equation. Now this subject has been very much in the centre of the economists' interest ever since the marginal utility theory came into being some three-quarters of a century ago. Obviously it cannot be the object of this monograph to review or even summarize the vast amount of literature written on the subject. All that can possibly be expected is a justification of the choice made with regard to the explanatory variables which are allowed to enter into the equation under review here, and a statement as to the simplifying assumptions on which it is based and therewith of the limitations to which its applicability is subject. The inclusion of these equations in our model implies that I do not share the view, expressed recently, that certain demand equations are highly indeterminate because people's consumption habits are largely governed by non-economic and random factors. It seems to me that for the total demand exercised by consumers as a group, the individual irrationalities in their economic behaviour will largely cancel each other out.

The total volume of *home demand* for goods and services is the outcome of a multitude of decisions taken by separate individuals, consumers as well as producers (in their capacity of investors). The demand equation under review can therefore most conveniently be looked upon as a *summation* of all the separate demand equations of separate individuals for separate goods and services. This procedure enables us to take due advantage of the results of econometric research which has been very active in the field of demand. The main finding of this research, I would say, is that almost invariably the individual's income and the price of the product concerned turn out to be among the variables explaining the variations in the volume demanded of the product concerned. Apart from these two, sundry other explanatory variables might crop up in the demand equations for separate commodities, such as the prices of substitutes, the general price level (representing the strength of the competition of all other products for the purchasing power given up in the purchase of this special commodity), and odd factors like the temperature of the weather which plays an important part in the explanation of beer consumption for instance. Furthermore, the individual decisions are governed by the circumstances in which the individual concerned finds himself. People's consumption habits are found to depend on the occupation of the head of the family, the size of the family, the district in which they live, etc.

For the sake of simplicity all factors other than income and price will be left out of account in our demand equation. This means either that the distribution of the population over the country, the frequency distribution of families as to their size, the distribution of the working population as to occupations, the temperature of the

weather, and all other factors apart from individual incomes and prices remain the same, or that the combined influence on total demand of the changes that actually take place in any of these factors is equal to nil. Thus we get:

*Assumption 3: The influences on the total volume of home demand exerted by all factors, other than income and price, cancel each other out.*

This assumption, unavoidable as it is in view of the fact that mathematical economics can only cope with comparatively simple systems of equations, seems to be fairly well in accordance with the facts. This is not so surprising after all, for practically all the factors involved show only gradual or random changes. Their combined influence is usually accounted for in econometric analysis by a so-called trend factor and the so-called residual. The trend factor is a term which is not allowed to show any short-run fluctuations. Usually it is represented by an artificial variable, which shows either a constant increase or a constant percentage increase each year, or by some sort of moving average of the variable concerned (in this case the volume of home demand).

Under assumption 3 we are left with only two explanatory variables, viz. *income and price*. However, different meanings may be attached to the concepts income and price. They may either stand for income and price expressed in terms of money or in terms of goods and services. In the former case they are usually referred to as money-income and money-price, in the latter case as real income and relative price respectively. Now in order to keep the argument as simple as possible there seems everything to be said for leaving money out of the picture at this stage and dealing with the complexities which money intro-

duces in the functioning of the economic system at a later stage. So as a starting-point for our analysis of demand we shall consider the real income of the individual concerned and the relative price of the commodity concerned to be the principal factors governing the individual's decision as to the amount of the commodity he will buy.

The *real income* of an individual is a measure for the quantity of goods and services he has a right to buy on account of his contribution to production. So changes in individual real incomes may be due to either (1) changes in the total of the national income in commodity terms, or (2) changes in the individual's share in this total.

Once the real income of an individual is fixed he is still completely free in the choice of the goods on which he likes to spend this income. He will make this choice on the basis of the *relative prices* of the goods and services that are offered for sale.

In setting up the equation for home demand we will apply the method of diminishing abstraction, starting from the conception of a closed economy in which people spend the whole of their real income, etc. and gradually progressing to a much more realistic picture in which such phenomena as hoarding and the money illusion have their place.

Now let us first consider what happens in a *closed economy* if (1) the total of national income in commodity terms changes, while (2) the price structure (that is the relative prices) and (3) the distribution of income (that is to say the share of each individual in the total income of the country) remains the same. This would apparently mean a proportionate increase in the demand of each individual supposing that each of them would spend the whole of his income. The formulae, explaining the volume of

home demand for the nation as a whole and each individual separately, would in such a case be

$$(3.5.1) \quad u' = \gamma \qquad (3.5.1') \quad u'_i = \gamma_i$$

where  $\gamma$  stands for the change in total national income in commodity terms and  $i$  is a subscript indicating the individual to which the equation refers.

There is no room for price terms in these equations for relative prices were assumed to remain the same, and as all variables are measured as deviations from their original equilibrium values these deviations will be nil for all relative prices.

There is likewise no reason to introduce separate income terms in (3.5.1) — in principle one for each individual — for on the condition that everybody spends the whole of his income, the formula holds true for the demand of each separate individual (cf. (3.5.1')) and so for the sum of all these individual equations (that is equation (3.5.1)).

Even if some individuals do not spend the whole of their available income but transfer part of it to other members of the same community by means of (commodity) *credit transactions*, thereby changing the distribution of available means over the separate individuals, (3.5.1) will still hold true on the condition that each individual spends the whole of his available *means*. The equations (3.5.1'), however, have to be adjusted in such a way that the right-hand side represents the available means instead of the available income of the individuals concerned

$$(3.5.2) \quad u' = \gamma \qquad (3.5.2') \quad u'_i = \gamma_i + t_i$$

where  $t$  stands for the amount of net borrowing expressed in commodity terms (in the case of net lending this term will have a negative sign).

For an *open economy* (that is an economy which has economic relations with the world abroad) there is also the possibility of transfer of income to and from foreigners. This does essentially alter the picture for it means that a change in the total of available means for the community as a whole is drawn into the realm of possibilities and it is available means, not incomes earned and paid out, that decide what the volume of home demand will be on the condition that everybody spends the whole of his available means. To take account of this phenomenon a term  $t$ , representing the amount of *net borrowing from abroad*, should be added to the right-hand side of the demand equation for the community as a whole, for under these circumstances the sum of all  $t_i$ 's is no longer necessarily equal to nil. Thus we get

$$(3.5.3) \quad u' = y + t \qquad (3.5.3') \quad u'_i = y_i + t_i$$

Up to this point the equilibrium condition formulated in equation (x) is automatically fulfilled because by definition  $y = u - u^i$  and  $t = u^i - u^e$ .

Now what happens if the assumption that everybody spends the whole of his available purchasing power is dropped? To answer this question a clear distinction should be made between two sets of phenomena. The first one, which has already been dealt with, is credit transactions which can overcome the contingency that certain people do not like to spend the whole of their available incomes, while others want to spend more. The second phenomenon, which has not yet been referred to, is that certain people do not like to spend the whole of their income but at the same time are unwilling to grant credits to others for that part of their purchasing power, which means that it remains unused. In a barter

economy, like the one we are considering at present, this can only result in *unwanted stock-piling* due to the fact that certain stocks of goods are not claimed. The reverse is also conceivable, namely when certain people overspend their income not by obtaining credits from others but by using claims dating from previous periods in addition to claims accruing to them over the present period. In that case stocks in the hands of producers will be gradually depleted. Neither of these, stock-piling nor stock-depletion, can of course go on for ever. They are not reconcilable with the notion of a long-run equilibrium. This, however, need not concern us for the moment, for all we are trying to explain is human economic behaviour at the demand-side of the market. We are interested in the forces at work. The final outcome of the process of adjustment is quite a different matter, which does not concern us here.

This point of *partial spending of available purchasing power* carries us right into the heart of the matter. People's economic behaviour — so far put in cold storage by means of the assumption that they would spend the whole of their available means — now enters the picture. In our individual demand equations this point can be taken account of by means of a coefficient  $v'_i$ , representing the individual's marginal propensity to spend. These equations will thus run as follows:

$$(3.5.4') \quad u'_i = v'_i y_i$$

Part of their non-spending  $(y_i - v'_i y_i)$  will be taken care of by credit transactions  $(t_i)$  while another part will be taken care of by non-use of claims accruing to them in the present period or by use of claims dating from previous periods, which will result either in stock-piling

in the hands of producers or in the depletion of such stocks.

To find the macro-economic relationships (for the community as a whole) we have to add up all these micro-economic relationships (3.5.4') (for separate individuals). Therefore:

$$(3.5.4) \quad u' = \sum_i u'_i = \sum_i v'_i \gamma_i$$

Now  $\sum_i v'_i \gamma_i$  may be replaced by  $v' \gamma$  only if either (1) the marginal propensity to spend is the same for all individuals or (2) the distribution of available incomes remains the same. In the first case where  $v'_i = v'$  we simply get:

$$(3.5.4.1) \quad \sum_i v'_i \gamma_i = v' \sum_i \gamma_i = v' \gamma$$

In the second case where  $\gamma_i = a \bar{\gamma}_i$  in which  $a$  is a constant and thus  $\gamma = a \bar{\gamma}$  we get:

$$(3.5.4.2) \quad \sum_i v'_i \gamma_i = a \sum_i v'_i \bar{\gamma}_i = \frac{\gamma}{\bar{\gamma}} \sum_i v'_i \bar{\gamma}_i = \gamma \frac{\sum_i v'_i \bar{\gamma}_i}{\sum_i \bar{\gamma}_i} = v' \gamma$$

Apparently the marginal propensity to spend for the community as a whole ( $v'$ ) has in this case to be defined as a weighted average of the individual marginal propensities to spend with the original equilibrium values of the individuals' available incomes in commodity terms as weights.

In order to keep our equation for home demand as simple as possible we will introduce as

*Assumption 4: Either the marginal propensity to spend will be the same for all individuals or the distribution of their available incomes will remain the same.*

This makes our demand equation for the community as a whole run as follows:

$$(3.5.5) \quad u' = v' \gamma$$

Meanwhile it may be said that assumption 4 can hardly be looked upon as an undue simplification of the facts. Econometric research in the field of demand and particularly BOWLEY and ALLEN's work <sup>1)</sup> has revealed the fact that quite often the so-called ENGEL-curves show a distinct linear relationship between spending and income. This finding fits in very well with the first part of assumption 4, for this linear relationship implies that people in all income-brackets have the same marginal propensity to spend. With regard to the alternative in assumption 4 it may be said that, although the income distribution shows fluctuations over the course of time, it is not usually subject to vehement changes. Lately the increased bargaining strength of the labour class has even in many cases acted as a stabilizing factor here, for in several countries the trade unions nowadays keep a watchful eye on the so-called wage quota in the national product, and as soon as any unwelcome developments are spotted speedy action is called for to safeguard labour's share in the total national income.

Introducing *money* into the picture, we find that the phenomenon of partial spending of income no longer necessarily results in stock-piling, neither does the phenomenon of overspending necessarily result in depletion of stocks. Money prices may change in such a way that the goods that are not taken by one or more individuals will be taken by others, leaving stocks where they are. Where this is the case there will be equilibrium between supply and demand of goods and services, but the monetary equilibrium will be upset owing to the *increase in idle balances* (hoarding), for this potential purchasing power will hang over the market and will cause a rise in prices

<sup>1)</sup> BOWLEY and ALLEN: *Family Expenditure* (London 1935).

once it is used. To what extent, in fact, stocks will pile up and to what extent money holdings will increase owing to the phenomenon of partial spending of incomes, depends on the level at which the money-prices of goods and services establish themselves.

In a money economy it is not the changes in real incomes but those in money-incomes and money-prices on which individuals base their decisions with regard to the demand for goods and services. To make our demand equation applicable to such an economy it should be translated into money terms. The phenomenon of partial spending or overspending of available incomes in a money economy, instead of a barter economy, may be represented in the best way, it seems, by an equation, which simply runs as:

$$(3.5.6) \quad U' = v' Y$$

This equation is, however, by no means identical with equation (3.5.5). A proportionate change in all prices would leave the volume of demand under (3.5.5) unaffected, whereas under (3.5.6) the same would be true only if the marginal propensity to spend ( $v'$ ) were equal to  $\bar{U}'/\bar{Y}$ , the average propensity to spend. A somewhat more general discussion of this point will be given later on.

Substituting the definitional equation for  $U'$  in (3.5.6) we get:

$$(3.5.6.1) \quad u' (1 + p') = v' Y - \bar{u}' p'$$

This equation clearly shows that in fact equation (3.5.6) implies that an increase in prices results in an equal percentage decrease in the volume demanded, which is tantamount to saying that the assumed price elasticity of demand in (3.5.6) is equal to unity. In practice it may, and probably

will, differ from unity. To take account of this the demand equation should read:

$$(3.5.7) \quad u' (1 + p') = v' Y - \eta' \bar{u}' p'$$

where  $\eta'$  stands for the price elasticity of demand.

A linear approximation of (3.5.7) finally renders the equation for *home demand* to be introduced in our system of equations

$$(xi) \quad u' = v' Y - \eta' \bar{u}' p'.$$

It should be noted that this equation is still to be considered as a summation of the separate demand equations of all the separate individuals for separate goods and services. This suggests that it is based, not only on an assumption with regard to either the marginal propensity to spend or the distribution of incomes, but also on a similar assumption with regard to prices. This assumption would read as follows:

*Assumption 5: Either the average price elasticity of demand will be the same for all individuals, or the price structure (relative prices of the various goods and services) will remain the same.*

To arrive at this assumption one has to carry out the summation-process starting from the separate demand equations expressed in money terms. The argument is exactly similar to the one given for the summation of the separate equations expressed in commodity terms and has, for the sake of brevity, been left out here.

As shown in section 2.3 this assumption is not a strictly necessary one as long as only one equation, in which the home price level appears, is involved. As, however, the home price level also appears elsewhere in the system

it seems desirable to subject the equation for home demand to the assumption presented here.

This assumption seems not to be too far out of line with the facts. Although the price elasticity of demand varies a good deal from one commodity to another, the weighted averages of the price elasticities of demand for the separate individuals will not differ so much, for the relative quantities of the various goods and services bought by the various individuals do not differ to any great extent. Furthermore, the prices of different goods and services show, in general, changes in the same direction, and so the price structure will be found to be relatively stable. As we are only interested in small changes in the variables with regard to the stability problem, assumption 5 is, therefore, definitely not to be considered as an undue simplification of the facts.

Before bringing the discussion of this equation to an end a few further remarks should be added. First of all there is the point that, if in the original equilibrium situation the balance of payments on current account were exactly balanced so that also  $\bar{U}'$  would be equal to  $\bar{Y}$ , a proportionate change in all prices would leave the volume of home demand unaffected should the marginal propensity to spend ( $\nu'$ ) be equal to the price elasticity of demand ( $\eta'$ ). In this case  $Y$  would be equal to  $\bar{y}p'$ , which in its turn would be equal to  $\bar{u}'p'$  and so  $u'$  would be equal to  $\nu'$  (or  $\eta'$ ) times nil. Equality of  $\nu'$  and  $\eta'$  in this case would mean that people were equally sensitive to changes in money-incomes and to proportionate inverse changes in money-prices. This would be perfectly rational. Usually, however, people's behaviour is less rational because they are subject to the so-called money illusion which makes them act differently in response to changes in their money-

incomes and proportionate inverse changes in money-prices. This is tantamount to saying that usually the structural coefficients  $v'$  and  $\eta'$  show different values. This is not in contradiction with the statement made while discussing equation (3.5.6). There it was said that the volume of home demand would only be left unaffected by a proportionate change in all prices if  $v'$  were equal to  $\bar{U}'/\bar{Y}$ . In our case this would mean equal to unity, for it was shown that equation (3.5.6) actually implied a price elasticity of home demand equal to unity.

Furthermore, it seems desirable to state that our equation is meant to apply to the whole of home demand, that is to say, not only to the demand for consumption goods but also to the demand for *capital goods*. Now one might think that the latter demand is not so much determined by the available money-income and the price level, as by the *profit expectations* of the investors. This is perfectly true, but profit expectations are in their turn mainly determined by three factors: (1) the amount of profits in the present and in the recent past, (2) the prices of the capital goods, and (3) the appropriate rate of interest. The first and most important of these factors — profits — is already included in the first explanatory variable (income), as income was defined in such a way that it embraces all profits distributed and undistributed. The price of capital goods is accounted for in the second explanatory variable (prices). Therefore, of these three factors the rate of interest seems to be the only one that does not have a place in our demand equation, which means that in the set-up of this equation the influence of the rate of interest on the demand for capital goods is considered to be negligible. Thus we get as —

*Assumption 6: Investors' decisions with regard to the purchase of capital goods are taken with a view to their current incomes and the prices of capital goods, but not interest rates.*

This assumption seems to be fairly well in accordance with the facts. TINBERGEN's investigations in the field of investment activity <sup>1)</sup> showed clearly that the influence of the rate of interest on the demand for capital goods could hardly be traced in the cases he studied. Therefore, I am inclined to believe that our equation gives a fair approximation of the facts, even for that part of home demand which consists of the demand for capital goods.

Finally, a word should be said about the way in which the unintended demand, referred to in the previous section, is covered by equation (xi). We do not know very much about the size of this unintended demand due to a misjudgment of the market. However, it seems reasonable to assume that the misjudgments will be in proportion to the changes in the explanatory variables. Therefore, we get as

*Assumption 7: The unintended demand (e.g. unwanted stock-piling) is a function of the changes in the national income and the home price level respectively, both expressed in money terms.*

From a statistical point of view this is a very satisfactory procedure, for it is almost impossible to find out which part of stock-piling was from choice and which was not. One should, however, be aware that on account of this unintended demand the structural coefficients in

<sup>1)</sup> J. TINBERGEN: *A method and its application to investment activity* (Geneva 1939).

equation (xi) no longer strictly reflect how people would react if they could know what the final outcome of their acts would be.

The second demand equation of our system deals with the explanation of changes in the volume of *imports*. The character of this demand equation is different from the one for home demand. It does not deal with the consumers' and investors' demand for the country's products (home demand) but with the producers' demand for foreign products. All demand for imports is considered to be *producers' demand*, as all imported goods and services have still to pass a final stage of production before they reach their final destination (even the handling of the goods by retail traders is here considered as part of the production process).

Producers' demand for imported goods and services will, of course, mainly depend on the volume of production ( $u$ ). But even if this volume is given, the producers are still free to a certain extent to decide what quantity of goods and services they are going to import in order to realize the given volume of production. In a good many cases they have the choice between using home products or foreign products. The demand equation for imports takes account of this fact and studies the forces that cause a *substitution* of the one for the other.

According to CHAMBERLIN<sup>1</sup>), one of the first and foremost writers on the theory of limited competition, there are, in the main, three factors which might cause substitution of the one product for the other, viz. (1) the price of the product, (2) the qualities of the product,

<sup>1</sup>) E. H. CHAMBERLIN: *The Theory of Monopolistic Competition*, 6th edition (Cambridge Mass. 1948).

and (3) the costs incurred in order to promote the sale of the product. It is the relative prices, qualities, and selling costs of the competing products that decide what the imports' *share* in the country's volume of production will be.

For the problem at hand I think it reasonable to refrain from a discussion of the effect of a change in the respective selling efforts of the competitors. Now, as selling costs are made with a view to changing the position and shape of the individual demand curves, and therewith of the total demand curve (in our case for imports), it means that we should introduce here the following assumption:

*Assumption 8: Producers' demand scales for imports are given.*

Furthermore, we shall assume that the differences in quality between home and foreign products are not subject to changes. This means that we introduce as

*Assumption 9: The relative differences in quality between home and foreign products which producers can alternatively use in the production process are not subject to changes.*

In so far as these assumptions are not borne out by the facts, the conclusions of the next chapter will be subject to amendments on this score.

Having excluded from the picture the possible consequences of changes in selling costs and so-called product competition, we are now left with a situation in which, given the price of a home product and the price of a similar foreign product, certain buyers prefer the home product whereas others prefer the foreign product. However, these *preferences* can often be overcome through changes in the relative prices of the products concerned. If, for instance, the home products become relatively cheaper

some buyers will be found to change over from foreign products to home products and so the contribution of foreign factors of production (imports) to total production will be reduced. It is this phenomenon of substitution through *price competition* on which we will first focus our attention in setting up the demand equation for imports.

As a first approximation the ratio between the volume of imports and the volume of home production will be assumed to be a linear function of the ratio between the corresponding price levels. Thus:

$$(3.5.8) \quad \bar{u}^i / \bar{u} = -a \bar{p}^i / \bar{p} + b$$

where  $a$  and  $b$  are constants. This formula can easily be transformed into:

$$(3.5.8.1) \quad \bar{u}^i \bar{p} = -a \bar{u} \bar{p}^i + b \bar{u} \bar{p}$$

Through writing the absolute values of the variables as sums of their respective original equilibrium values and the deviations therefrom, and through making use of the fact that (3.5.8.1) also applies to the original equilibrium values of the variables, this formula may be further transformed into:

$$(3.5.8.2) \quad u^i = (b - a) u - a \frac{(\bar{u} + u)(p^i - p)}{1 + p}$$

in which  $b - a = \frac{\bar{u}^i}{\bar{u}} = \pi_i$ , the average import quota of

the national product, for  $\bar{u}^i = -a\bar{u} + b\bar{u}$ , and furthermore  $a = \eta^i \pi_i$ . This  $\eta^i$  is the price elasticity of substitution for import demand as can easily be shown if the definition of elasticity — viz. the ratio between the percentage

changes in price and volume — is applied to the variables of equation (3.5.8).

$$\eta^i = - \frac{\bar{u}^i/\bar{u} - \bar{u}^i/\bar{u}}{\bar{p}^i/\bar{p} - \bar{p}^i/\bar{p}} = - \frac{-a\bar{p}^i/\bar{p} + b - (b-a)}{\pi_i(\bar{p}^i/\bar{p} - 1)} = \frac{a}{\pi_i}$$

since  $\bar{p}^i = \bar{p} = 1$

Substituting the new symbols for the structural coefficients in (3.5.8.2) we get:

$$(3.5.8.3) \quad u^i = \pi_i u - \eta^i \pi_i \frac{(\bar{u} + u)(p^i - p)}{1 + p}.$$

It should be noted that the structural coefficient of  $u$  is, according to its definition, the average and not the marginal import quota. This is due to the special shape given to equation (3.5.8). It implies that if relative prices of the home and foreign products remain the same as in the original equilibrium situation, that is to say if the ratio between the import price level and the home price level remains the same, namely equal to unity, the ratio between the volume of imports and the volume of production will invariably remain equal to  $b - a = \pi_i$ , no matter what the volume of production will be. The implied assumption that the import quota will remain the same while the volume of production changes seems to be an undue simplification of the facts. Therefore, we should change the equation in such a way that it also provides a true picture of the facts should the volume of production change while the price situation remains unchanged. This means that for  $\pi_i$  should be substituted  $v^i$ , the marginal import quota. Our equation then reads as follows:

$$(3.5.9) \quad u^i = v^i u - \eta^i v^i \frac{(\bar{u} + u)(p^i - p)}{1 + p}$$

and the assumption on which it is based can now be formulated as

*Assumption 10: (A) Given the volume of production, the ratio between the volume of imports and this volume of production will be a linear function of the ratio between the import price level and the home price level, and (B) given the ratio between the import and the home price level the volume of imports will change in proportion to the change in the volume of production.*

The difference between the marginal and the average import quota may be due either to differences in production methods at various levels of production requiring different quantities of imported goods and services per unit of product, or to the fact that the goods and services demanded at different levels of income and production (two closely related variables) do not require the same quantity of imported goods and services per unit of product (an increase in individual income might, for instance, result in an increased demand for luxury products to be imported from abroad).

Linear approximation reduces the demand equation for imports as given in formula (3.5.9) to:

$$(xii) \quad u^i = v^i u - \eta^i \bar{u}^i (p^i - p),$$

as  $\pi_i \bar{u} = \bar{u}^i$  and all products of deviations drop out.

A definite advantage of our demand equation for imports is that, apart from the direct influence of a change in import prices, it also takes into account the *indirect influence* which such a change has through the change in

the volume of production resulting from it. Another advantage is that it deals with international trade as a case of *limited competition*, which certainly is more realistic than to apply to it the assumptions of pure competition. For the latter case the price elasticity of substitution would be infinite. Econometric research so far has revealed only finite values, and even fairly small ones, for this substitution elasticity.<sup>1)</sup>

The third and last demand equation of the system explains the changes in the volume of *exports* by the changes in the prices of the export products expressed in foreign currency. The equation is set up along the same lines as the demand equation for imports, the volume of the country's exports ( $u^e$ ) taking the place of the volume of the country's imports ( $u^i$ ), the total world market demand for this kind of exports ( $u^E$ ) taking the place of the volume of home production ( $u$ ) and the corresponding world market prices of the country's exports ( $p^{eb}$ ) and of all exports of the same kind ( $p^{Eb}$ ), both expressed in terms of foreign currency, taking the place of the price of imports ( $p^i$ ) and the price of home produced goods ( $p$ ), both expressed in home currency. This renders the following basic formula for export demand

$$(3.5.10) \quad u^e = \frac{\bar{u}^e}{\bar{u}^E} u^E - \eta^e \frac{\bar{u}^e}{\bar{u}^E} \frac{\bar{u}^E + u^E}{1 + p^{Eb}} (p^{eb} - p^{Eb})$$

This formula for export demand (comparable to equation (3.5.8.3)) is based on:

<sup>1)</sup> See, for instance, T. C. CHANG: "International Comparison of Demand for Imports" in *The Review of Economic Studies*, vol. XIII (1945-46), pp. 53-67.

*Assumption 11: The country's share in the total world market demand for exports of the same kind is a linear function of the ratio between the price of the country's export and the average price of all exports of the same kind.*

On account of assumption 1 the factors  $u^E$  and  $p^{Eb}$  should be treated as *data*, which means that all the terms with  $u^E$  or  $p^{Eb}$  have to be deleted in equation (3.5.10). This leaves us with the following quite simple *demand equation for exports*.

$$(xiii) \quad u^e = -\eta^e \bar{u}^e p^{eb}$$

The idea behind this treatment of  $u^E$  and  $p^{Eb}$  as data, as has already been stated, is that if the country's share in world trade is relatively small, which it is for most countries, even big changes in the country's foreign trade will have only a negligible effect on price and volume of world trade as a whole. The only exception to this rule is apparently the case where, among the country's export products, there are some the quantities of which are not negligible compared to the total quantities traded in the world market, and where at the same time these products form a considerable part of the total value of the country's exports. This exceptional case might arise for countries cultivating one crop only. If so,  $u^E$  and  $p^{Eb}$  have to be included among the variables of the system, and world supply and demand equations for these goods and services have to be added to the equations of the system. We will, however, pay no more attention to this case.

## 3.6

## THE SUPPLY EQUATIONS

The second set of behaviouristic equations consists of the *supply equations*. Our system contains three such equations, one for exports, one for products sold in the home market, and one for labour. They describe the first part of the economic process, namely the disposal of the means of production at the highest possible reward, and their employment in the production of such goods and services as are supposed to render the highest contribution to the ultimate aim of all economic activity, viz. the satisfaction of human wants. It is the demand equations, dealt with in the previous section, which summarize the main considerations playing a part in the second part of the economic process, viz. the setting up of consumption and investment plans.

Now let us first give a very brief description of the second part of the economic process before we deal with the supply equations themselves. The starting-point for such a description should clearly be the fact that people possess various *means of production*. How far people go with *putting them at the disposal of producers* depends, in principle, on the intensity of the wants the individual can satisfy if he does not part with the use of the means of production concerned, and the wants he can satisfy if he does. He will, at least in theory, make this comparison for each of the successive units of the means of production the use of which he parts with. As soon as the intensity of the respective wants he can satisfy by acting either one way or the other balances he will not give up any more units. In practice the said comparison is slightly falsified by certain institutional factors such as the eight-hour working day, which only

leaves the individual the choice between working eight hours a day and not working at all. But even so he can choose strenuous work or he may prefer to take light work.

With the *destination of the means of production* in the production process it is much the same as with the disposal of the means of production. Each unit can be used only once. Therefore, using it for one purpose means sacrificing the possibility of using it for another. The use which gives the highest yield will be preferred. The result of this process of destination will therefore be that in the end all units of one and the same factor of production obtain the same reward, a reward which tends to be equal to the marginal net product of the factor in question (i.e. the contribution made by the last unit of this factor of production). As long as this is not the case, some units getting a lower reward than others can obviously be more efficiently used elsewhere. The outcome of the destination process will, therefore, be a difference in the *extent* to which the units of one factor of production will be used for different purposes, that is in the production of different products. Furthermore, the extent to which each factor of production (or rather each variety of each of the separate factors of production, e.g. different types of skilled labour) is applied in the production process will be such that the contributions made by the marginal units of each are *proportional* to their prices, i.e. the remunerations per unit of such factors of production.

The picture given above of the process of disposing of the use and allocating the means of production is, in fact, a much simplified one and, therefore, not perfectly realistic. This is mainly due to the fact that the underlying assumption of free choice at any time is not fulfilled in real life. Some decisions taken in the past are

irrevocable in practice and limit people's *freedom of choice* in the present. This applies especially to the volume and composition of the stock of durable goods, capital goods as well as consumers' goods, but is equally true for the composition of the labour force, which may have been trained in the wrong direction. Nevertheless the adjustments which can be made immediately will be made and they will be ruled by the type of consideration indicated above. Further adjustments will be made as time goes on. Therefore, the intensity of people's reactions to certain changes is dependent on the period of adjustment one considers. The structural coefficients in our elementary equations will differ if one deals with a short-run problem in the one case, and with a long-term problem in the other. This point has already been dealt with in section 2.6.

In a rapidly developing economy, with increasing population and increasing stocks of durable goods, this limited freedom of choice in the present, however, is not such a serious matter, owing to the fact that additions to national capital and the labour force can be directed to those parts of the economic system where bottle-necks exist. In a slowly developing economy the process of adjustment will take more time because of the smallness of the annual additions to the capital stock and the labour force. Therefore, we may expect the intensity of the reactions in a rapidly developing economy to be stronger in the short run, although the reactions in the long run will be the same. The final equilibrium will be reached sooner in such an economy than in a slowly developing economy.

In the first two supply equations, viz. those for *home produced goods and services destined for export and the home*

market respectively, the *marginal utility obtained* is represented by the selling price of the product and the *marginal utility sacrificed* by the marginal costs of the product concerned. The whole of these *marginal costs* of a unit of product can be analysed into two parts, viz. one part due to the contribution of foreign factors of production embodied in imported raw materials, semi-finished products and so on, and another part due to the contribution of the home factors of production, mainly consisting of home labour. The quantities of each needed to produce an additional unit of product will be fairly constant, at least in the short run. The same is not true with regard to the prices to be paid for each of these parts, owing to the fact that the changes in the prices of imports are not determined by the same set of factors as those in the prices of the home factors of production. It seems desirable, therefore, to split the marginal costs into their two components: import prices ( $p^i$ ) and wages ( $l$ ), weighed according to their respective shares in the marginal costs of the product ( $v_i$  and  $v_l$ ). The fact that, of the remunerations of home factors of production, only wages are considered to influence marginal costs, implies the following assumption:

*Assumption 12: Of the home factors of production neither nature nor capital will cause changes in the costs of production of marginal units of product if the exchange rate changes.*

It should be noted that this assumption does not exclude rent and interest from the marginal-costs concept. It only excludes them from the variables of the system, which is tantamount to saying that they are treated as data.

Enterprise may also be considered to contribute to the

production of the marginal unit of product, but, just because this is the marginal unit, the remuneration of this factor of production for this contribution will by definition be equal to nil; marginal units are not supposed to carry any profit margin. But as soon as there is any change, either in the price of the product (e.g. because of increased demand), or in one of the constituent parts of the costs of this, up till then, marginal unit (e.g. owing to a change in the import price level), this unit will be produced at either a profit or a loss. This will call for an adjustment in the volume of production. In case of a profit, production will be extended further; in case of a loss, production will shrink until the stage is reached where the then marginal unit does not show either a profit or a loss. The bigger the original discrepancy, the bigger will be the change in the volume of production. The simplest supply equation we can establish on these considerations is the one based on the following assumption:

*Assumption 13: The change in the volume of production is proportional in size to the profit (or loss) margin shown on the up till then marginal unit of product.*

The supply equations for home-produced goods and services sold in foreign markets and the home market respectively, as based on this assumption, run as follows:

$$(3.6.1) \quad u^e = \varepsilon^e \bar{u}^e (p^e - v_l^e l - v_i^e p^i)$$

$$(3.6.2) \quad u' = \varepsilon' \bar{u}' (p' - v_l' l - v_i' p^i)$$

These equations show clearly that it is the margin between price and costs which determines the change in the volume of production.

These equations may also be presented in a slightly different form, namely as *price fixation equations*, which run as follows:

$$(xiv) \quad p^e = v_i^e l + v_i^e p^i + \frac{1}{\varepsilon^e \bar{u}^e} u^e$$

$$(xv) \quad p' = v_i' l + v_i' p^i + \frac{1}{\varepsilon' \bar{u}'} u'$$

The reason for presenting the equations in this form is that usually the seller sets the price of the product, whereas the buyer adjusts his purchases with a view to this set price. If demand falls short of supply, sellers will set a lower price, whereas in the opposite case they will set a higher price. Then the buyers will again adjust their purchases, and so on, until an equilibrium position is reached. Only in the case of pure competition, where sellers have to accept the dictate of the market and have no freedom in choosing the price at which they want to sell their products, does it seem more realistic to use ordinary supply equations.

The obvious reason for giving two supply equations instead of one, explaining the change in the total volume of production, is that the constants in the one supply equation may differ from those in the other.

Finally, and this is more important, one may ask: what phenomena account for a finite, instead of an infinite, *elasticity of supply* ( $\varepsilon$ )? One of the reasons for a finite elasticity of supply is that an increase in production can only be achieved, either by an increase in the production of existing firms, or by an inflow in the field of new entrepreneurs. Now in the one case the efficiency of the management is bound to fall sooner or later owing to

the fact that the capacities of the leaders are limited. In the other case the capacities of the newcomers may be expected to be inferior to the capacities of those already in the field; otherwise they would have been able to compete with them before under less favourable conditions. Likewise the other human factor of production — labour — will show a decrease in efficiency as production increases. Therefore, the marginal labour quota will not remain constant, which if linearly approximated (compare mathematical appendix to chapter II) will result in a finite value for the elasticity of supply of home production. Furthermore, it should be appreciated that the risks involved in the increased outlay on capital goods may become so big that nobody is willing to take them. All this, I think, explains sufficiently the assumed finite elasticity of supply.

The last equation of the system is the *wage rate fixation* equation. It describes how and to what extent changes in certain other variables of the system cause changes in the money-wage rate. As in modern society it is the *trade unions' policy*, rather than the individual actions of the workers, which influences the price formation on the labour market, this equation should be interpreted as describing the strategy of the trade unions in wage deals. This implies that the wage rate fixation equation cannot be considered to be a transformed supply equation, like the price fixation equations dealt with above, for the state of employment is determined by other factors than trade unions' policy. To set up a supply equation for the production factor labour would be fairly difficult, for such an equation would have to take due account of the phenomenon of *involuntary unemployment*, to which atten-

tion was drawn by LORD KEYNES in his famous "General Theory" <sup>1)</sup>. However, for present purposes there is no need to do anything of the sort, for employment does not appear elsewhere in the system. All that is needed is an explanation of changes in the money-wage rate, this being the only variable, apart from the exchange rate, for which changes any explanation by means of changes in other variables has not yet been given.

Trade unions, in their efforts to safeguard the interests of their members to the best of their ability, are, of course, governed in their policy by the same kind of considerations as would, in their absence, underlie the actions of the individual workers. Not being able to influence directly the volume of employment, they bargain for the highest wage. In general "wage" should be interpreted in this context to mean "real wage". So, if the price level of imports, expressed in home currency, rises consequent upon a fall in the rate of exchange, one might expect the trade unions to claim an increase in money-wages to offset the effect of the increase in the home price level, which results from the rise in the import price level. However, where the workers in their capacity of income-earners are subject to the so-called *money illusion*, the trade unions may indulge in a less than proportionate increase in money-wages in the first instance. This phenomenon will be accounted for in our wage rate fixation equation by the wage-coefficient ( $\lambda$ ). This coefficient will be equal to unity if there is no money illusion in the minds of the workers. Where this happens to be the case an increase in the home price level will result in an equal percentage change in the money-wage rate.

<sup>1)</sup> J. M. KEYNES: *The General Theory of Employment Interest and Money* (London 1936).

Furthermore, the trade unions will act according to circumstances. Now, if the employment situation improves, the *bargaining power* of the trade unions will increase. Hence, one might expect an increase in wages to follow the improvement in the employment situation. To simplify matters it will be assumed that changes in employment may be adequately measured by the term  $(u - u')$ . This implies the introduction of

*Assumption 14: The volume of labour needed to produce a certain volume of goods and services is proportionate to this volume of production minus the volume of imports needed for it.*

On the basis of the foregoing, the following (linear) wage rate fixation equation may be added to the list of behaviouristic equations:

$$(xvi) \quad l = \lambda p' + \frac{1}{\epsilon^l (\bar{u} - \bar{u}')} (u - u')$$

in which  $1/\epsilon^l$  stands for what may be called the wage flexibility with regard to changes in the employment situation. This wage flexibility measures the intensity of the reaction of the trade unions to a change in the employment situation. For the sake of convenience, we will refer to  $\epsilon^l$  as the "elasticity of supply of labour" in the next chapter. This has the advantage that the argument given there can run in terms of elasticities. The theoretical objections to identifying the wage rate fixation equation with the (transformed) supply equation for labour, which equally apply to the wording "elasticity of supply of labour", should not be lost sight of however.

The assumptions made with regard to the wage rate fixation equation can be briefly formulated as follows:

*Assumption 15: (A) Given the level of employment the money-wage rate will change in proportion to changes in the home price level, and (B) given the home price level the wage rate will change in proportion to changes in the level of employment.*

To conclude, a word should be said about the possible values of the *wage flexibility*. The change in the bargaining power of the trade unions does not depend only on the percentage change in the volume of employment, but also on the initial state of employment. In a state of *serious unemployment* with many workers on the dole, the wage flexibility may be expected to be almost equal to zero (or  $\epsilon^l = \infty$ ), as in such a situation there will be practically no scope for increases in wages and the bargaining power of the trade unions will be accordingly weak. In a state of *full employment*, however, matters will be just the reverse. The bargaining power of the trade unions will become exceedingly strong in such circumstances, and even the mere threat of a strike may be sufficient to cause employers to give in to the trade unions' claims for higher wages. Employers wanting to extend their production find themselves faced with the fact that the supply of labour has become highly inelastic, owing to the fact that the labour force is already fully employed. Further increases in labour supply will, in such circumstances, be possible only through over-time work and through attracting more female and juvenile labour at relatively high wages. These sources of labour become very quickly exhausted and then a further increase in wages will no longer add

to the supply of labour. The competition of employers to attract more labour may even cause wages to rise above the level necessary to mobilize these potential sources of labour. Involuntary unemployment being absent, the wage flexibility may be identified with the inverse value of the elasticity of the supply of labour. The latter being zero, the former will become *infinite*, at least for small changes in the variables. While discussing the implications of our system of equations in the next chapter, special attention will be paid to the values infinity and zero for  $\epsilon^l$ .

## 3.7

## THE SYSTEM AS A WHOLE

The system of equations developed in this chapter endeavours to give a concise picture of the mechanism ruling the price formation on the foreign exchange market. It consists of the following equations:

| <i>definitional equations</i>    | <i>demand equations</i>                                                       |
|----------------------------------|-------------------------------------------------------------------------------|
| (i) $U^i = u^i + \bar{u}^i p^i$  | (xi) $u' = v' Y - \eta' \bar{u}' p'$                                          |
| (ii) $U^e = u^e + \bar{u}^e p^e$ | (xii) $u^i = v^i u - \eta^i \bar{u}^i (p^i - p)$                              |
| (iii) $U' = u' + \bar{u}' p'$    | (xiii) $u^e = -\eta^e \bar{u}^e p^{eb}$                                       |
| (iv) $U = u + \bar{u} p$         | <i>supply equations</i>                                                       |
| (v) $p^i = -k^b$                 | (xiv) $p^e = v_l^e l - v_i^e p^i + \frac{1}{\epsilon^e \bar{u}^e} u^e$        |
| (vi) $p^{eb} = p^e + k^b$        | (xv) $p' = v_l' l - v_i' p^i + \frac{1}{\epsilon' \bar{u}'} u'$               |
| (vii) $U = Y + U^i$              |                                                                               |
| (viii) $S = U^e - U^i$           |                                                                               |
| (ix) $U = U' + U^e$              |                                                                               |
| (x) $u = u' + u^e$               | (xvi) $l = \lambda p' + \frac{1}{\epsilon^l (\bar{u} - \bar{u}^i)} (u - u^i)$ |

In this system the equations (ix) and (x) are degenerated balance equations (cf. section 3.4). Furthermore, the supply equations have been presented in the form of price fixation equations (cf. section 3.6).

The system consists of 16 equations in which appear altogether 17 variables. It is not completely soluble therefore, but it still allows the expression of all variables but one as a function of the remaining variable (cf. section 2.4). This is exactly what is required for present purposes. An answer to the question whether the equilibrium on the foreign exchange market is stable, unstable, or indifferent, can be given if the relationship between the balance on current account ( $S$ ) and the rate of exchange ( $k^b$ ) is known (cf. section 3.1). If  $S$  is positively influenced by  $k^b$  the equilibrium will be unstable, for this implies that a rise in the price of home currency in terms of foreign currency will result in an excess of the demand for home currency (exports) over the supply of home currency (imports) which will result in a further rise in the rate of exchange. Likewise a fall in the exchange rate will result in a further fall if  $S$  is positively influenced by  $k^b$ . If, on the other hand,  $S$  is negatively influenced by  $k^b$  the equilibrium will be stable, and if it is not influenced at all the equilibrium will be indifferent (cf. chapter I).

The answer to be obtained from the system is, however, subject to three important qualifications, due to the fact that the equations of the system (1) are linearly approximated, (2) do not contain any lagged terms, and (3) are based on certain assumptions, one of which is constancy of data.

*Qualification 1: The answer to the exchange stability problem which emerges from a system of linearly approximated relationships is correct only in the case of small disturbances of the equilibrium.*

For small variations in the variables every relationship can be linearly approximated without loss of generality (cf. section 2.2). The final relationship derived from a system of such linearly approximated relationships will itself also be of the linear type. (cf. section 2.4) and apply to small variations only <sup>1)</sup>. For larger variations, however, the curvilinearity of some of the elementary relationships, if there is any, should be taken into account. Consequently the final equation will also be of the curvilinear type. Where this is the case with the relationship between  $S$  and  $k^b$ ,  $S$  may be negatively influenced by  $k^b$  over certain intervals and positively over others. So if the deviation of  $k^b$  from its original equilibrium value is big enough the system may move from the one interval into the other. This means that, although the original equilibrium may be stable for small disturbances, it will turn out to be unstable for big disturbances. Figure 4 illustrates this point further. It shows that the equilibrium will be stable as long as the disturbance of  $k^b$  is smaller than  $OA$ . If it is bigger, then the system will move into another interval and the original equilibrium will not be restored.  $OA$  is the critical value for changes in the exchange rate. For  $k^b > OA$  the system will move on towards the new equi-

<sup>1)</sup> Only in the exceptional case of a mixed equilibrium (cf. footnote 1 on page 19) will such a linearly approximated final relationship not render the correct answer to the exchange stability problem. The answer obtained will be that the equilibrium is indifferent, whereas in fact it will be stable in one direction and unstable in the other direction.

brium position 1. ( $S = 0$  is an equilibrium condition of the system as long as it is assumed that the international balance on capital account will remain unaltered). This, however, is an unstable equilibrium; the slope of the

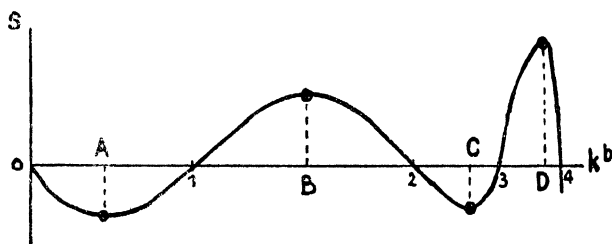


Figure 4

Curvilinear relation between  $S$  and  $kb$

curve is here positive. The slightest disturbance, therefore, might cause the exchange rate to rise further to equilibrium 2, or to fall back to equilibrium 0 both of which are stable equilibria.

*Qualification 2: The answer to the exchange stability problem, which emerges from a system of static equations, only applies to the situation which will have established itself after a certain given period of adjustment, viz. the period of adjustment to which the numerical values of the structural coefficients correspond.*

If, in our system of equations, the lags in people's reactions had been reflected in the elementary equations, the final equation would have read as follows:

$$(3.7.1) \quad S_0 = a_0 k_0^b + a_1 k_{-1}^b + a_2 k_{-2}^b + \dots + a_m k_{-m}^b + \dots + a_n k_{-n}^b$$

whereas the total effect of exchange rate changes over a period of  $n$  time units would be:

$$(3.7.2) \sum_0^n S_t = a_0 \sum_0^n k_t^b + a_1 \sum_{-1}^{n-1} k_t^b + a_2 \sum_{-2}^{n-2} k_t^b + \dots + \\ + a_m \sum_{-m}^{n-m} k_t^b + \dots + a_n \sum_{-n}^0 k_t^b$$

where  $t = 0, 1, 2, \dots, m, \dots, n$ .

Considering only the effect of the disturbance of the equilibrium which occurred at the moment 0, formula (3.7.2) can be reduced to:

$$(3.7.3) \sum_0^n S_t = k_0^b \sum_1^n a_t, \text{ since } k_{-n}^b = \dots = k_{-1}^b = k_{+1}^b = \\ = \dots = k_n^b = 0,$$

and if we study only the effect up till the moment  $m$  the result will be:

$$(3.7.4) \sum_0^m S_t = k_0^b \sum_0^m a_t.$$

For different  $m$ 's the sign of the derived structural coefficient, which connects the change in the balance of payments on current account over a given period of adjustment (0 to  $m$ ) with the change in the rate of exchange at a certain moment (0), may, therefore, be different.

Stating things more concisely one could also say that the full effect of a change in the rate of exchange takes time to materialize because of the (distributed) lags in people's reactions to changed circumstances (cf. section 2.6). Therefore, for different periods of adjustment the answers to the exchange stability problem may well differ. But even the answer obtained on the basis of the numerical values of the structural coefficients which correspond to

the given adjustment period should not be considered as conclusive. If the combined effect up till the end of this period turns out to be a negative influence from  $k^b$  on  $S$  it is still possible that for a shorter period the influence has been a positive one. This would mean that the equilibrium over that period had been unstable, and once an equilibrium shows instability during the process of adjustment it is unstable no matter what the outcome of the calculations for longer periods will be.

*Qualification 3: The answer to the exchange stability problem, which emerges from a system of equations based on certain assumptions, is only correct if these assumptions are in accordance with the facts.*

Particularly important in this connection is the second assumption according to which international capital transactions in response to a change in the rate of exchange may be disregarded. Currency speculation will often give rise to short-run capital transactions, which sometimes have a stabilizing effect and sometimes are detrimental to the stability of the exchange equilibrium. All seems to depend here on the psychological factor confidence. Therefore, even if on account of the factors at work in the commodity sphere the equilibrium would be unstable, currency speculation may prevent the instability from materializing. Likewise currency speculation may sometimes force the exchange rate further and further away from an equilibrium position, which, judged by the forces at work in the commodity sphere, should be considered as a stable equilibrium. A fall in the rate of exchange may, for instance, weaken the confidence of speculators in the currency concerned. They may expect

a further fall, which will make them decide to dispose of their holdings of the currency concerned, thereby increasing its supply and thus counteracting the effect the change in the exchange rate had in the field of transactions in goods and services, if that were a favourable one, or reinforcing that effect if it were an unfavourable one. If, on the other hand, the fall in the exchange rate is considered by them to be a mistake and they expect an increase in the near future, or if the change in the exchange rate has in general restored confidence abroad that in the long run this action will enable the country to pay its way and, therefore, should be considered as a contribution to the long-run equilibrium in the balance of payments of the country concerned, the supply of foreign currency or, what comes to the same thing, the demand for the home currency, may increase. This may either reinforce the favourable effect in the field of transactions in goods and services or counteract this effect if it were unfavourable. In general, I think, the forces at work in the field of capital transactions will reinforce those at work in the field of commodity transactions, for a favourable effect on the balance on current account is most likely to contribute to foreigners' confidence in the sanity of the economy of the country concerned and will therefore encourage them to invest their money in that country, whereas an unfavourable effect on the balance on current account will have the opposite effect. But even if this were not so, our analysis would still have a certain value in so far as it would answer the question to what extent currency speculation can be effective, for the lesser the stability of the equilibrium on account of factors at work in the commodity sphere the more effective currency speculation will be. With this question in view it was decided to impose on the system the

stringent condition, implied in assumption 2, that international capital transactions are not affected by changes in the exchange rate and should be treated as a datum. Furthermore, currency speculation usually has only a short-term effect and if the favourable effect on the balance on current account is strong enough to give the exchange rate a certain power of resistance, this type of speculation will cease to make itself felt after a while and its influence will probably be reversed in due course. I do not, therefore, consider it to be of great importance with regard to the stability problem, that the effect of a change in the exchange rate in the sphere of capital transactions is not taken into account in our picture of the mechanism of the foreign exchange market.

In practice the final outcome of a disturbance in the equilibrium on the foreign exchange market does not depend only on the nature of this equilibrium, but also on the changes in the data which take place during the period of adjustment. They may well counteract the effect of the change in the exchange rate.

## CHAPTER IV. THE THEORY

### 4.1

#### A SIMPLIFIED MODEL

The system of equations, developed in the previous chapter, with its ten definitional equations and only six behaviouristic equations, seems at first sight to be comparatively simple. It is, in fact, simple enough to give a definite answer to the stability problem, once the numerical values of the various structural coefficients and other data appearing in the system are known for a particular case. However, for a general theoretical discussion of the exchange stability problem, to which this chapter is mainly devoted, this system is still too complicated. Such a discussion should aim at disclosing the conditions under which the equilibrium on the foreign exchange market will be either stable, indifferent, or unstable. These conditions can, of course, be formulated only in terms of the numerical values of the structural coefficients and other data of the system that enter into the final equation relating a change in the surplus on current account to a change in the exchange rate. Now, the system, as it stands, contains in all seventeen such structural coefficients and other data. This, no doubt, is too high a number to enable the setting up of a surveyable theory with regard to the exchange stability problem, not to speak of the complete impossibility of giving a comprehensive verbal treatment of the subject.

It will be clear from the foregoing that the only possible

course to take is to further simplify the model. In doing so the argument will proceed along the following lines. Firstly, a number of simplifications will be introduced which do not too seriously impair the general applicability of the model. Their introduction will be based, either on available statistical evidence, or on more general considerations. The simplifications of this type will be discussed in this section, whereas section 4.3 will be devoted to the final equation which can be derived from this simplified model, and to a discussion of the formulæ set up by earlier writers on the same subject. Secondly, closer consideration will be given to some special cases, each of which implies an additional assumption with regard to the numerical values of one or two of the structural coefficients, and therewith simplifies the structure of the model. These special cases referring to the elasticities for home demand, imports and exports, and home supply respectively, will be dealt with in sections 4.4 to 4.6, whereas the possible combination of such special cases will be discussed in section 4.7. This brings the purely theoretical discussion of the exchange stability problem to an end. To illustrate its practical value, section 4.8 will be devoted to a discussion of the exchange stability problem for individual countries. In this discussion use will be made of statistically measured structural coefficients, which have been discussed and commented upon in section 4.2, and partly in this section. Finally, a summary will be given in the last section.

A first simplification, which reduces the number of data contained in the original system of equations by three, is the following:

*Simplification 1: In the original equilibrium situation the value of the imports and the value of the exports of goods and services balance.*

If this is so then we may write

$$(4.1.1) \quad \bar{u}^e = \bar{u}^i = \pi \bar{u}$$

and

$$(4.1.2) \quad \bar{u}' = (1 - \pi) \bar{u} \text{ because } \bar{u}' = \bar{u} - \bar{u}^e$$

In these formulae  $\pi$  stands for the average import quota in the national product in the original equilibrium situation.

As table 1 shows, this simplification is a fairly realistic one, at least in so far as the average values of imports and exports over the period 1924—1938 for the nineteen countries included in the table are concerned (cf. column (4)). The most serious exception appears to be Chile. This country, however, is the only one in the list for which information as regards its balance of payments is available for one year of the inter-war period only. Therefore, I think, we should not put too much weight on this exception. For the rest only Australia, Hungary, New Zealand and Germany appear to have imported a good deal more than they exported during the inter-war period, whereas the United States, Finland and Sweden found themselves in the opposite position, viz. of exporting more than they imported.

For separate years the relative differences between imports and exports have been considerably larger (cf. column (7)). In order to get an idea of the bearing of such differences on the problem of exchange stability, calculations can be made for each individual country, with special reference to the differences between  $\pi^e$  and

TABLE 1

The total import and export quota in the national product and the ratio between them  
 ( $\pi^i$  = total import quota;  $\pi^e$  = total export quota)

| Countries                                         | Average values |         |               | Highest and lowest values |           |               |
|---------------------------------------------------|----------------|---------|---------------|---------------------------|-----------|---------------|
|                                                   | $\pi^e$        | $\pi^i$ | $\pi^e/\pi^i$ | $\pi^e$                   | $\pi^i$   | $\pi^e/\pi^i$ |
| (1)                                               | (2)            | (3)     | (4)           | (5)                       | (6)       | (7)           |
| I. Industrial countries                           |                |         |               |                           |           |               |
| A. Highly industrialized and self-sufficient type |                |         |               |                           |           |               |
| U.S.A. . . . .                                    | .05            | .05     | 1.10          | .07 — .04                 | .06 — .04 | 1.30 — .94    |
| B. Highly industrialized and deficient type       |                |         |               |                           |           |               |
| Germany . . . .                                   | .13            | .14     | .92           | .15 — .07                 | .18 — .07 | 1.11 — .75    |
| U.K. . . . .                                      | .18            | .18     | 1.01          | .24 — .14                 | .23 — .14 | 1.10 — .88    |
| C. Relatively less industrialized type            |                |         |               |                           |           |               |
| Sweden . . . . .                                  | .20            | .19     | 1.07          | .23 — .18                 | .20 — .17 | 1.16 — .93    |
| France . . . . .                                  | .14            | .13     | 1.00          | .24 — .07                 | .21 — .08 | 1.23 — .81    |
| Switzerland . . .                                 | .25            | .24     | 1.05          | .26 — .24                 | .25 — .23 | 1.10 — 1.02   |
| Japan . . . . .                                   | .16            | .17     | .98           | .19 — .14                 | .18 — .14 | 1.09 — .81    |
| Italy . . . . .                                   | .20            | .20     | 1.00          | .23 — .17                 | .21 — .19 | 1.09 — .90    |
| II. Mining and raw material producing countries   |                |         |               |                           |           |               |
| Chile . . . . .                                   | .27            | .34     | .79           |                           |           |               |
| Canada . . . . .                                  | .23            | .23     | 1.02          | .26 — .19                 | .26 — .21 | 1.21 — .79    |
| South Africa . . .                                | .24            | .24     | .98           | .27 — .21                 | .29 — .18 | 1.28 — .86    |
| III. Agricultural countries                       |                |         |               |                           |           |               |
| Estonia . . . . .                                 | .26            | .28     | 1.03          |                           |           | 1.12 — .94    |
| Latvia . . . . .                                  | .20            | .20     | 1.02          | .24 — .15                 | .24 — .13 | 1.19 — .87    |
| New Zealand . . .                                 | .20            | .21     | .94           | .24 — .16                 | .27 — .17 | 1.19 — .77    |
| Finland . . . . .                                 | .26            | .24     | 1.09          | .28 — .23                 | .28 — .20 | 1.28 — .82    |
| Norway . . . . .                                  | .28            | .29     | .99           | .32 — .25                 | .36 — .25 | 1.08 — .88    |
| Hungary . . . . .                                 | .11            | .13     | .92           | .16 — .08                 | .19 — .08 | 1.19 — .64    |
| Denmark . . . . .                                 | .23            | .23     | 1.01          | .27 — .22                 | .27 — .21 | 1.06 — .96    |
| Australia . . . . .                               | .14            | .16     | .88           | .18 — .12                 | .20 — .13 | 1.09 — .61    |

Note: This table summarizes the main findings of calculations, the results of which have been presented in the Statistical Appendix. Statistical sources and periods covered are indicated there. The average values of the coefficients and ratios are the arithmetic means of the values for the separate years for which the coefficients and ratios were calculated. Hence the average ratios do not necessarily coincide with the ratios between the averages of the coefficients. Often the ratios could be calculated for more years than the coefficients as their calculation does not require national income

data  $\frac{\pi^e}{\pi^i} = \left( \frac{\bar{U}^e}{\bar{U}^i} \right)$  The national product figures used refer to the national product at market prices. Furthermore all the figures refer to goods and services (see, however, footnotes to tables in Statistical Appendix).

$\pi^i$  for separate years. The necessary information for such calculations has been compiled in the Statistical Appendix.

Table I also gives the absolute values of the total import and export quota in the national product for the nineteen countries for which calculations were made. With regard to these calculations it should be mentioned that

- (i) the import and export figures used refer to *goods and services*, with the exception of the United Kingdom for which only the balance between receipts and payments for services is known; this balance has been included in the export figure, and the U.K. values for  $\pi^e$  and  $\pi^i$ , as entered in the table, are, therefore, systematically too low, but how much too low is unknown;
- (ii) the national product figures used in the calculations have, whenever possible, been valued at *market prices*<sup>1)</sup>; the only exception here seems to be Chile, for which country only the 1928/29 national product figure could be estimated approximately on the basis of a national income figure for 1928, which most likely is valued at factor costs.

As the table shows, and as has already been stated, the average values of the import and export quota are very much the same for each of the countries concerned taken separately. From one country to another, and from one year to another, (cf. columns (2) and (3), and (5) and (6) respectively), a good deal of variation is shown however.

According to CHANG, who made an investigation into the causes of the variation of the average propensities

<sup>1)</sup> The author's thanks are due to Mr COLIN CLARK — who in his "*The Conditions of Economic Progress*" (London 1940) made a most valuable contribution in the field of national income statistics by putting the national income figures of many countries on a comparable basis — for making available to him the revised figures for national incomes valued at market prices.

to import from country to country<sup>1)</sup>, the following factors are to be held responsible for the differences prevailing in the absolute magnitudes of these average propensities during the inter-war period: (1) the degree of self-sufficiency of the countries concerned, (2) the degree of specialization of their home production, and (3) the average real incomes per capita in these countries, expressed in international units<sup>2)</sup> in order to put them on an internationally comparable basis. However, it does not necessarily follow that the same holds true for  $\pi^i$  for CHANG's findings deal with the ratio of the money-value of retained merchandise imports over the money-value of national income (probably valued at factor costs in most cases), whereas  $\pi^i$  represents the ratio of the money-value of imports of goods and services over the national product (= national income + imports of goods and services) valued at market prices, which is not quite the same.

A second simplification, which further reduces the number of data by three, is to combine the two supply equations — for home market products and export products — into one supply equation for the total of home production. Hence, we get as

*Simplification 2: The prices of home market products and export products change in the same proportion consequent upon a change in the rate of exchange:*

for this is what the amalgamation of the two supply equations amounts to. Expressed in the appropriate symbols this simplification runs as follows:

<sup>1)</sup> T. C. CHANG : "International Comparison of Demand for Imports" in *The Review of Economic Studies*, vol. XIII (1945/46), pp. 53—67.

<sup>2)</sup> COLIN CLARK defines an international unit as the amount of goods and services which one dollar would purchase in the U.S.A. over the average of the period 1925—1934 (cf. *op. cit.*, p. 39/41).

$$(4.1.3) \quad p' = p^e = p$$

whereas the new supply equation becomes

$$(4.1.4) \quad p = v^l l + v^i p^i + \frac{1}{\varepsilon \bar{u}} u$$

where  $v^l$  and  $v^i$  stand for the marginal wage and import quota in the total of home production and  $\varepsilon$  for the price elasticity of supply for home-produced goods. These two equations take the place of equations (xiv) and (xv) in the original model.

It can easily be shown that the price elasticity of supply for the total of home production ( $\varepsilon$ ) will be a weighted average of the supply elasticities for home market products ( $\varepsilon'$ ) and export products ( $\varepsilon^e$ ) if the marginal import and wage quota are the same for both categories of goods and services. If namely

$$(4.1.5) \quad v' = v^e_l = v^l,$$

and

$$(4.1.6) \quad v'_i = v^e_i = v^i$$

then it follows from the formulae for the two separate and the one combined supply equations ((xiv), (xv), and (4.1.4) respectively) and formula (4.1.3) that

$$(4.1.7) \quad u' : \varepsilon' u' = u^e : \varepsilon^e \bar{u}^e = u : \varepsilon \bar{u} \text{ or}$$

$$(u' + u^e) : (\varepsilon' \bar{u}' + \varepsilon^e \bar{u}^e) = u : \varepsilon \bar{u}$$

which by means of equation (x) can be transformed into:

$$(4.1.8) \quad \varepsilon = \frac{\varepsilon' \bar{u}' + \varepsilon^e \bar{u}^e}{\bar{u}} \text{ or } \varepsilon = \varepsilon' (1 - \pi) + \varepsilon^e \pi.$$

On the basis of the foregoing our model, describing the factors which influence the price-formation on the foreign exchange market, may now be rewritten in the following simplified form:

*definitional equations*

$$(1) U^i = u^i - \pi \bar{u} k^b$$

$$(2) U^e = u^e + \pi \bar{u} p$$

$$(3) U' = u' + (1 - \pi) up$$

$$(4) S = U^e - U^i$$

$$(5) Y + U^i = U' + U^e$$

$$(6) u = u' + u^e$$

*demand equations*

$$(7) u' = v' Y - \eta' (1 - \pi) \bar{u} p$$

$$(8) u^i = v^i u + \eta^i \pi \bar{u} (p + k^b)$$

$$(9) u^e = -\eta^e \pi \bar{u} (p + k^b)$$

*supply equations*

$$(10) p = v^l l - v^i k^b + \frac{1}{\varepsilon \bar{u}} u$$

$$(11) l = \lambda p + \frac{1}{\varepsilon^l (1 - \pi) \bar{u}} (u - u^i).$$

(For reference purposes this model has been reproduced on the folding page at the end of the book).

It should be noticed that part of the elimination process has already been carried out in arriving at this simplified model. This has been done in order to bring out as clearly as possible the essential features of the structure of the model. The deleted variables are  $p^e$ ,  $p'$ ,  $p^i$ ,  $p^{eb}$ , and  $U$ . The price variables have been eliminated by means of substituting equation (4.1.3) and the definitional equations (v) and (vi) into the other equations of the system. Furthermore, simplification 2 made equations (ii), (iii), (iv), (ix), and (x) become mutually dependent. Therefore, equation (iv) has been deleted and  $U$  eliminated from equations (vii) and (ix) through substituting the one into the other. As a result of the elimination process and the two simplifications introduced in this section, we are

now left with a model consisting of 11 independent equations, containing 12 variables and 11 structural coefficients and other data.

#### 4.2 THE NUMERICAL VALUES OF SOME STRUCTURAL COEFFICIENTS

One might feel rather tempted to go still a little further with the simplification process by (1) assuming that the marginal costs of a product consist of imports and wages only, that is to say that the marginal import quota ( $v^i$ ) and the marginal wage quota ( $v^l$ ) are together equal to unity, and (2) identifying the marginal import quota ( $v^i$ ) with the total or average import quota ( $\pi$ ).

As regards the first suggested simplification, it should be borne in mind that, although it has been assumed that only the imports and wage components of the price influence marginal costs, these marginal costs may also contain other components (representing the contribution of nature and capital) which remain unaltered. For the numerical illustrations in the following sections, however, it will be assumed that the sum of the marginal import quota and the *marginal wage quota* is equal to unity, as this will simplify our formulae to a considerable extent in a good many cases. In mathematical symbols this simplification reads as:

$$(4.2.1) \quad v^l = 1 - v^i$$

*Assumption A: The sum of the marginal wage quota and the marginal import quota is equal to unity for each of the countries for which numerical illustrations will be provided.*

As table 2 shows, statistical evidence is definitely against the second suggested simplification. With the exception

TABLE 2

*Numerical values of some structural coefficients for nineteen countries*

| Countries                                         | $\pi$ | $\psi^i$ | $\eta^e$ | $\eta^i$ |
|---------------------------------------------------|-------|----------|----------|----------|
| (1)                                               | (2)   | (3)      | (4)      | (5)      |
| I. Industrial countries                           |       |          |          |          |
| A. Highly industrialized and self-sufficient type |       |          |          |          |
| U.S.A. . . . .                                    | .05   | .06      | .43      | .97      |
| B. Highly industrialized and deficient type       |       |          |          |          |
| Germany . . . . .                                 | .14   | .25      | .58      | .37      |
| U.K. . . . .                                      | .18   | .30      | .40      | .28      |
| C. Relatively less industrialized type            |       |          |          |          |
| Sweden . . . . .                                  | .19   | .29      | .36      | .37      |
| France . . . . .                                  | .13   | .18      | .77      | .32      |
| Switzerland . . . . .                             | .24   | .37      | .44      | .26      |
| Japan . . . . .                                   | .17   | .34      | .60      | .47      |
| Italy . . . . .                                   | .20   | .19      | .81      | .27      |
| II. Mining and raw material producing countries   |       |          |          |          |
| Chile . . . . .                                   | .34   | 1.00     | .17      | .32      |
| Canada . . . . .                                  | .23   | .34      | .35      | 1.34     |
| South Africa . . . . .                            | .24   | .42      | .31      | .64      |
| III. Agricultural countries                       |       |          |          |          |
| Estonia . . . . .                                 | .28   | .83      | 1.29     | .34      |
| Latvia . . . . .                                  | .20   | .81      | 1.84     | .48      |
| New Zealand . . . . .                             | .21   | .41      | .52      | .34      |
| Finland . . . . .                                 | .24   | .69      | 1.23     | .25      |
| Norway . . . . .                                  | .29   | .41      | .62      | .86      |
| Hungary . . . . .                                 | .13   | .45      | 1.10     | .54      |
| Denmark . . . . .                                 | .23   | .41      | .45      | .63      |
| Australia . . . . .                               | .16   | .35      | .66      | .67      |
| IV. World as a whole. . . . .                     |       |          | .56      | .56      |

Note:

- (i)  $\pi$  is the average value of  $\pi^i$  in table 1; for sources and periods covered see Statistical Appendix.
- (ii) calculation of  $\psi^i$  has been explained in the text.
- (iii)  $\eta^e$  is taken from T. C. CHANG: "A Statistical Note on World Demand for Exports" in *The Review of Economics and Statistics*, vol. XXX (1948), p. 107.
- (iv)  $\eta^i$  is taken from T. C. CHANG: "International Comparison of Demand for Imports" in *The Review of Economic Studies*, vol. XIII (1945/46), p. 64.
- For further details see text.

of Italy all *marginal import quotas* prove to be higher, and sometimes even considerably higher, than the average import quotas. For the Baltic countries (Estonia, Latvia, and Finland) this marginal import quota proves to be well over 50 per cent, whereas for Chile it seems to be round about 100 per cent, which would imply that for this country every additional unit of product would have to come from imports.

For a proper understanding of the significance of the marginal import quota entered in the table, a few words should be said here about the way in which they were calculated. As a starting-point for our calculations some of the coefficients appearing in CHANG's demand equations for imports were used, viz. the income or employment elasticity and, wherever present, the export elasticity of import demand. The general set-up of his demand equation is as follows:

“Let  $\log x_1$  = quantity of imports;  
 $\log x_2$  = real income or home employment;  
 $\log x_3$  = relative prices; and  
 $\log x_4$  = quantity of exports.

Then, the partial correlation equation, with the trend of all these series eliminated is of the form

$$\log x_1 = b_{12.34} \log x_2 - b_{13.24} \log x_3 + b_{14.23} \log x_4$$

Here,  $b_{12.34}$ ,  $-b_{13.24}$  and  $b_{14.23}$  give respectively the income-elasticity, the relative-price-elasticity and export-elasticity of demand for imports of a country”<sup>1</sup>).

Where  $b_{12.34}$  was calculated by use of a series of home employment, its definition runs as follows:

<sup>1</sup>) T. C. CHANG : “International Comparison of Demand for Imports” in *The Review of Economic Studies*, vol. XIII (1945/46), p. 63.

$$(4.2.2) \quad b_{12.34} = \frac{\Delta u^i}{\Delta u} \cdot \frac{\bar{u}}{\bar{u}^i} = \frac{v^i}{\pi^i},$$

so that  $v^i$  can simply be determined through multiplying  $b_{12.34}$  by  $\pi^i$  (cf. table 1).

Where, as in the majority of cases studied,  $b_{12.34}$  referred to real income its formula is:

$$(4.2.3) \quad b_{12.34} = \frac{\Delta u^i}{\Delta \gamma} \cdot \frac{\gamma}{\bar{u}^i} = \frac{\frac{\bar{\gamma} + \bar{u}^i}{\bar{u}^i} - 1}{\frac{\Delta \gamma + \Delta u^i}{\Delta u^i} - 1} =$$

$$= \frac{1 - \pi^i}{\pi} \cdot \frac{1}{\frac{1}{v^i} - 1}$$

and therefore:

$$(4.2.4) \quad v^i = \frac{\pi^i b_{12.34}}{1 - \pi^i + \pi^i b_{12.34}}$$

In those cases where CHANG's formula also contains a term for exports ( $x_4$ ), the marginal import quota was amended on the consideration that in our system the influence of the volume of exports on the volume of imports is not represented by a separate term, but should be reflected in the influence exerted by the volume of production — of which exports are part — upon the volume of imports. As the formula for the export elasticity is:

$$(4.2.5) \quad b_{14.23} = \frac{\Delta u^i}{\Delta u^e} \cdot \frac{\bar{u}^e}{\bar{u}^i}$$

one should raise the numerical value obtained by either

formula (4.2.2) or formula (4.2.4), with a term of the magnitude:

$$(4.2.6) \quad v^i_0 = \frac{\Delta u^i}{\Delta u^e} \cdot \frac{\Delta u^e}{\Delta u} = b_{14.23} \cdot \frac{\pi^i}{\pi^e} \cdot \frac{\Delta u^e}{\Delta u}$$

to arrive at the proper numerical value of  $v^i$ . By means of a rough-and-ready method the ratio of  $\Delta u^e$  over  $\Delta u$  was estimated to be equal on the average to 0.2 for Germany, 0.4 for the U.K., 0.25 for Japan, and roughly 0.4 for Switzerland.

For Chile, CHANG was forced to use the index of industrial production owing to lack of information about the development of either real income or home employment during the inter-war period. It may be due to this that the actual outcome of the calculation of the marginal import quota for this country turned out to be higher than unity, viz. 1.10. As, however, the highest value conceivable for the marginal import quota is 1.00, it was decided to attribute this value to Chile's marginal import quota. It is only on account of the extremity of this value that Chile was retained in the list of countries studied, as it was thought that it might be useful to demonstrate the consequences of such an extreme value for the marginal import quota by means of an actual example. Apart from this, there seems everything to be said for rejecting altogether the figures calculated for Chile owing to the fact that they are all highly unreliable. (The only balance of payments data available were those for the year 1929, whereas, for the calculation of the average or total import and export quotas the national income figure for 1928 had to be used; this was the only national income for which a conversion from valuation at factor costs to valuation at market prices proved to be impossible by

lack of readily available information concerning the level of indirect taxation).

With regard to the marginal propensity to import, i.e. the part of additional units of income spent on imports (both measured in terms of money), CHANG made three observations:

- (i) this marginal propensity turns out to be higher for agricultural countries than for industrial countries, owing to the fact that a higher proportion of their imports consists of investment goods and "deferred" consumption goods, the demand for which is subject to more violent changes over the trade cycle than the demand for goods for immediate consumption and the raw materials for these goods;
- (ii) the marginal propensities to import of industrial countries cluster around one value, whereas for agricultural countries they vary greatly;
- (iii) in each group the marginal propensities are distributed regularly, namely in such a way that they are higher the lower the country's degree of industrialization.

Furthermore, he found that his first observation applied equally well to the income elasticity of import demand, i.e. the ratio of the percentage changes in the volume of imports and the volume of real income. Likewise he found that the reverse holds true approximately for the income elasticities of export demand<sup>1)</sup>, as one would already expect on a priori grounds, once the observation with regard to the income elasticity of import demand was made. By revealing the fact that for industrial countries the import income elasticity tends to be smaller than the export income elasticity, whereas for agricultural, mining and

<sup>1)</sup> T. C. CHANG : "A Statistical Note on World Demand for Exports" in *The Review of Economics and Statistics*, vol XXX (1948), pp. 106—116.

raw material producing countries it tends to be bigger than the export income elasticity, CHANG was undoubtedly right in claiming that he provided one of the most important clues to the explanation of the cyclical pattern of change in the balance of payments of different types of countries. As he pointed out, however, it should not be overlooked that changes in the terms of trade may, at least partly, offset the changes in the relative quantities exported and imported.

As regards our marginal import quotas, only the first observation made with regard to marginal propensities to import applies, as the table shows. The line of distinction between the two groups of countries proves to be a marginal import quota of about one third.

Thanks to CHANG's pioneering work in the field of econometric research with regard to import and export demand, we are not only in a position to calculate roughly what the countries' marginal import quotas are, but he also supplies us with information about the *price substitution elasticity for export and import demand* ( $\eta^e$  and  $\eta^i$  respectively). The results of his calculations are entered in columns (4) and (5) of table 2. Unfortunately, however, these elasticities refer to merchandise exports and imports only. How far this impediment is a serious one (which also invalidates, though to a considerably lesser extent, the calculated marginal import quotas) is shown by table 3.

The percentage share of merchandise imports in total imports in 1930 proves in general to be well above the 70 per cent level. Notable exceptions are Chile, Australia, Canada and the U.S.A. In the case of Chile this is largely due to the rather high amount of foreign investment in the country. The same applies to Australia and Canada. As for the United States, travel expenditures and personal

TABLE 3

*Percentage share of merchandise imports and exports in total imports and exports of goods and services in 1930*

| Countries                                | Imports | Exports |
|------------------------------------------|---------|---------|
| (1)                                      | (2)     | (3)     |
| 1. U.S.A. . . . .                        | .65     | .70     |
| 2. Germany . . . . .                     | .73     | .87     |
| 3. U.K. <sup>1)</sup> . . . . .          | 1.00    | .61     |
| 4. Sweden . . . . .                      | .89     | .79     |
| 5. France. . . . .                       | .88     | .59     |
| 6. Switzerland <sup>2)</sup> . . . . .   | .90     | .67     |
| 7. Japan . . . . .                       | .82     | .76     |
| 8. Italy. . . . .                        | .90     | .65     |
| 9. Chile <sup>3)</sup> . . . . .         | .56     | .93     |
| 10. Canada . . . . .                     | .60     | .68     |
| 11. South Africa <sup>4)</sup> . . . . . | .75     | .96     |
| 12. Estonia . . . . .                    | .82     | .83     |
| 13. Latvia . . . . .                     | .87     | .81     |
| 14. New Zealand <sup>5)</sup> . . . . .  | .74     | .91     |
| 15. Finland . . . . .                    | .86     | .86     |
| 16. Norway . . . . .                     | .76     | .61     |
| 17. Hungary. . . . .                     | .71     | .85     |
| 18. Denmark . . . . .                    | .89     | .83     |
| 19. Australia <sup>5) 6)</sup> . . . . . | .58     | .87     |

Source: League of Nations' Balances of Payments publications.

<sup>1)</sup> Total imports do not include any invisible imports; total exports only include the balance of invisible exports and imports.

<sup>2)</sup> 1928.

<sup>3)</sup> 1929.

<sup>4)</sup> Merchandise and total exports inclusive of net gold exports.

<sup>5)</sup> 1930/31.

<sup>6)</sup> Exports inclusive of exports of home-produced gold.

remittances play an equally important part in the invisible imports as interest, dividends, shipping and freight. Whether or not it is desirable to enter personal remittances on the income account of the balance of payments may be a matter of dispute. For the sake of comparability, however, the League of Nations' definition of income account has been adhered to throughout in this monograph.

With the exception of the following countries, France, the United Kingdom, Norway, Italy, Switzerland, Canada and the United States, the percentage share of merchandise exports in total exports proves to be above the 70 per cent level. In the case of France, tourists' expenditure plays an equally important part in the invisible exports in 1930 as interest, dividends, and freights taken together. For the United Kingdom, the net income from overseas investments was equally important in 1930 as the net income from shipping, commissions, insurance etc. Both the percentage share of merchandise imports and that of merchandise exports are definitely overestimated here owing to the fact that only the net income from the invisible transactions is known, which made it impossible to establish the correct ratios between merchandise and total imports and exports. For Norway, the invisible exports consist almost completely of shipping services. In the case of Italy, tourists' expenditure and personal remittances were of equal importance in 1930, with income from shipping third on the list. For Switzerland, the income from foreign investment accounted for practically half the income from services, whereas tourists' expenditure played an almost equally important part. For the United States, the invisible exports were mainly made up of interest and dividends, shipping and freight, and travel expenditures. Finally, for Canada, the same items as for the United States formed the main constituents of the invisible exports, but their order of magnitude was just the reverse.

With regard to the numerical values of the price substitution elasticities and the income elasticities of demand for these invisible imports and exports, unfortunately not very much can be said. From the scanty information

available<sup>1)</sup>, however, it seems to follow that the *income elasticities* for invisible imports and exports are not too far out of line with those for visible imports and exports. With regard to the *price substitution elasticities of invisibles*, according to my knowledge no attempts have been made so far to measure them statistically. Nevertheless, some qualitative statements can be made which might serve to partly fill the gap in our statistical knowledge.

For a proper understanding of these qualitative statements it should be realized that a number of invisibles do not have a price in the true sense of the word, e.g. personal remittances. To make the changes in the money-values of such invisibles, consequent upon a change in the exchange rate, fit in properly in our model describing the economic mechanism, fictitious price and volume indices and fictitious price substitution elasticities of demand should be attributed to them. This should be done in such a way that the total money-values of these items show correct changes, that is to say changes in accordance with what they are supposed to be in reality.

As regards *interest and dividends*, it will be clear that a change in the exchange rate can hardly be expected to affect the existing pattern of foreign investments and debts to any appreciable extent, at least in the short run. Only fresh lending and borrowing, as well as the expiration of old contracts, can bring about changes in this pattern, and their amount will be very small compared with the total amount of contracts already in existence. Expressing changes in money-values in terms of home currency, it seems reasonable, therefore, to assume with regard to the

<sup>1)</sup> See, for instance, T. C. CHANG: "The British Balance of Payments 1924—1938" in *The Economic Journal*, vol. LVII (1947), pp. 475—503.

money-values of the invisible imports and exports dealt with here (capital services) that:

- (i) if the interest payments are fixed in the currency of the lender's country (as is usual with loans)
  - (a) inward payments (for exported capital services) will remain constant;
  - (b) outward payments (for imported capital services) will change inversely in proportion to the change in the exchange rate;
- (ii) if the payments are fixed in the currency of the borrower's country (as is usual with dividends)
  - (a) inward payments will change inversely in proportion to the change in the exchange rate;
  - (b) outward payments will remain constant (that is to say if the profitability of home production is left unaffected by the change in the exchange rate).

Now, it should be remembered that the import price level expressed in home currency was assumed to vary inversely in proportion to the rate of exchange, and that for exports the price level expressed in home currency was assumed to vary in proportion to the home price level. With regard to interest payments on existing contracts, being the bulk of all contracts, this home price level will be constant in the short run. Accepting the assumed variations in the import and export price levels as the fictitious price indices for imported and exported capital services, we find that for the cases mentioned under (i) the fictitious volume indices should be constant. For the cases mentioned under (ii) they should be such that they vary inversely in proportion to the changes in the exchange rate in so far as exported capital services are concerned, and in proportion to changes in the exchange rate in so far as imported capital services are concerned. These results

can easily be obtained by attributing price elasticities of substitution equal to nil to the capital services for which payments are fixed in the currency of the lender's country, and price elasticities of substitution equal to unity to the capital services for which payments are fixed in the currency of the borrower's country (compare for this argument equations (8) and (9) on folding page at the end of the book). Consequently the average elasticities of substitution with regard to capital services will be somewhere between nil and unity. Where exactly they will be between these two limits depends on the extent to which contracts are fixed in home and foreign currency respectively.

The argument with regard to *shipping services* runs as follows: Competition in shipping on the supply side of the market is so severe that the prices (freight rates) are always equal for all competitors. Consequently the shares of the various competing countries will remain unaffected in the short run, notwithstanding such changes in the price-costs situation as may occur as a consequence of changes in exchange rates. Such changes in the profitability of shipping make themselves felt only in the long or medium long run and will then lead to relative changes in the merchant shipping tonnage of the various competing countries. Given this state of affairs there cannot be any price substitution effect on the demand side of the market. For small countries this obviously means that freight rates will vary inversely in proportion to changes in their exchange rate, and that, in so far as they are exporting shipping services, the shipping income in terms of home currency will likewise vary inversely in proportion to changes in their exchange rate. In calculating average price elasticities of substitution for import and export demand of all goods and services, this situation may be

conveniently taken account of by putting the price elasticity of substitution for home shipping demand (imports) equal to nil, and that for foreign shipping demand (exports) equal to unity (compare discussion of interest and dividends). The only exception to the foregoing argument is provided by the United Kingdom, whose share in the world's merchant shipping tonnage is such that it is big enough to influence the level of the freight rates.

With regard to *tourists' expenditure* on the debit side of the current account of the balance of payments, it seems reasonable to suppose that real income is the predominant explanatory variable, and that the price elasticity of substitution will be fairly low; usually people decide first whether they will have a holiday abroad and then, in the light of their available means, how long they will stay and what sort of accommodation they will have. The price elasticity of substitution for tourists' expenditure on the credit side of the account, on the other hand, may be fairly high because people spending holidays abroad usually plan carefully to go to the country where they will obtain the highest value for their money.

As for *personal remittances*, it seems reasonable to assume that they are fairly insensitive with regard to changes in the exchange rate of the paying country and, therefore, more or less comparable with the case of capital services, the price of which is fixed in the currency of the borrower's country. Hence, their fictitious price elasticity of substitution may be said to be slightly lower than unity for the paying country, for which these remittances formally may be considered as invisible imports, as well as for the payments-receiving country, for which they constitute invisible exports. "Slightly lower" because the needs of the beneficiaries will also be taken into account in people's

decisions with regard to such personal remittances.

From the foregoing it will be clear that the price elasticities of substitution for invisibles will most probably have values between nil and unity. As statistical evidence shows (cf. table 2) the same applies approximately to the price elasticities of substitution for the merchandise import and export demand of the countries studied.

For lack of more precise information and for the sake of simplicity, it will, therefore, be assumed that

*Assumption B: The income elasticities and the price elasticities of substitution for import and export demand for invisibles are equal to those for the merchandise imports and exports of each of the countries studied.*

Where the price elasticity of substitution for merchandise import or export demand is higher than unity, this assumption seems on a priori grounds not wholly justified. On the imports side, this is the case only for Canada. On the exports side, this is the case for the three Baltic countries and Hungary, but for these countries the percentage share of invisible exports in the total of exports is only small, as table 3 shows, and it therefore seems reasonable to assume that the systematic error made is probably not very serious in these cases.

Another possible objection to CHANG's figures is that in the determination of the equation for import demand he used wholesale and cost of living indices to represent the prices of home-produced substitutes competing with the imported products. Some further investigations to test the degree of correspondence between these two price indices seem desirable.

As can be seen from table 2 the values of the price

elasticities of substitution for import demand are remarkably low and, except in the case of Canada, even below unity. CHANG puts this down to the fact that imports are usually *goods which cannot be produced at home*, at least not cheaply. As for the low values of the price elasticities of substitution for export demand, he seeks the explanation in (1) the *imperfection of competition* in international markets owing to traditional trading connections, the ignorance of home consumers about world supply conditions, special preference for the exports of a particular country, etc. and in (2) the *lack of homogeneity* in the exports of different countries, which means that the exports of one country are hardly ever perfect substitutes for those of another country.

The fact that only for some small agricultural countries like Latvia, Estonia, Finland and Hungary, does the price elasticity of substitution for export demand prove to be higher than unity, may be, at least partly, explained by TINBERGEN's *law of diminishing elasticity*, according to which the price elasticity of substitution for the exports of a country with a large share in the world market may be expected to be smaller than the elasticity for a small country's exports. He illustrates the working of this law with the following example, which refers to the comparison of the market of a single firm with the market of an industry, where the same law manifests itself: "Supposing there to be three competitors, whose prices are  $p_1$ ,  $p_2$  and  $p_3$  and the demand for the products of No. 1 to be:  $u_1 = -a_1p_1 + a_2p_2 + a_3p_3$ , with  $a_1 = a_2 + a_3$ . Assuming the second competitor to be identical in capacity, situation, etc. with the first, the demand for his products will be:  $u_2 = -a_1p_2 + a_2p_1 + a_3p_3$ . Now suppose 1 and 2 to amalgamate and hence their prices to be identical:  $p_1 = p_2$ ;

the total demand for their products will be:

$$u_1 + u_2 = -2 a_1 p + 2 a_2 p + 2 a_3 p_3 = -2(a_1 - a_2) p + 2 a_3 p_3.$$

The elasticity of demand for 1's products was  $\frac{a_1}{\bar{u}_1} \bar{p}_1 = \epsilon_1$ ;

that for the combined demand for 1's and 2's products

$$\epsilon_{12} = \frac{2(a_1 - a_2)}{\bar{u}_1 + \bar{u}_2} \bar{p}; \text{ since } \bar{u}_2 = \bar{u}_1, \text{ and } \bar{p} = \bar{p}_1,$$

$$\epsilon_{12} = \frac{a_1 - a_2}{a_1} \epsilon_1 < \epsilon_1.$$

Since  $a_1 = a_2 + a_3$ ,  $\epsilon_1$  is at the same time the elasticity of substitution; it follows that also  $a_1 - a_2 = a_3$  and  $\epsilon_{12}$  is also the elasticity of substitution." <sup>1)</sup>

After having reviewed for a number of countries the numerical values of some structural coefficients appearing in our foreign trade model, it should be clear that they are not all quite reliable. They should, therefore, be used mainly for *illustrative purposes*, and the conclusions with regard to the exchange stability of different countries, which will be arrived at in section 4.8, should be considered to be *very tentative*.

Apart from this, one should be well aware that the numerical values of some structural coefficients, discussed above, refer essentially to *short-run* reactions, as the calculations were based on yearly figures without the introduction of any lags at all. The qualifications made in section 3.7, and in particular the second one, should therefore be constantly borne in mind while interpreting the results to be obtained in section 4.8.

<sup>1)</sup> J. TINBERGEN: "Unstable and Indifferent Equilibria in Economic Systems" in *Revue de l'Institut International de Statistique* IX (1941), p. 49 footnote.

## 4.3

## THE GENERAL CASE

From the simplified one-country model, presented in section 4.1, the following relation between a change in the exchange rate of home currency ( $k^b$ ) and a corresponding change in the balance on current account ( $S$ ) can be derived by means of eliminating all other variables from the system<sup>1)</sup>:

(4.3.1)

$$S = \frac{\left[ \frac{v^i(1-v^i)}{\varepsilon^i(1-\pi)} + \frac{1}{\varepsilon} \right] \left\{ (1-\eta^i-\eta^e) - v^i \left\{ \frac{v^i}{\pi} + \eta^i \frac{v^i}{\varepsilon^i(1-\pi)} \right\} \right\} (1-\pi)(v'-\eta') - (1-v^i-v^i\lambda) \{ 1-\eta^i-\eta^e(1-v^i) \} (1-v^i)}{\left\{ \frac{v^i(1-v^i)}{\varepsilon^i(1-\pi)} + \frac{1}{\varepsilon} \right\} \left\{ v' \pi \left( 1-\eta^i-\frac{\eta^e}{v'} \right) + (1-\pi)(v'-\eta') \right\} - \left[ (1-v^i-v^i\lambda) + \pi \left\{ \frac{v^i}{\pi} + \eta^i \frac{v^i}{\varepsilon^i(1-\pi)} \right\} \right] \{ (1-v^i)(1-v^i) + v^i \}} \pi \bar{u} k^b$$

$$= \frac{B}{C} \pi \bar{u} k^b = A \pi \bar{u} k^b.$$

This formula is obviously too complicated for any straightforward conclusions to be drawn from it with regard to the exchange stability problem. But once the numerical values of the structural coefficients appearing in it are known for a particular case, an answer can be provided, subject to the various qualifications made in the course of our argument.

<sup>1)</sup> Those who wish to carry out the actual elimination process may be advised to proceed along the following lines: (I) eliminate  $U^i$ ,  $U^e$  and  $U'$  from the system by substituting for them in equations (4) and (5) the right-hand sides of equations (1), (2), and (3); (II) eliminate  $Y$  from the system by substituting for it in equation (7) the right-hand side of the transformed equation (5); (III) eliminate  $u^i$  from equation (11) by substituting for it the right-hand side of equation (8); (IV) eliminate  $l$  from the system by substituting for it in equation (10) the right-hand side of the transformed equation (11); (V) eliminate  $u^e$  and  $u^e$  from the system by substituting for them in equation (6) and the transformed equations (4) and (7) the right-hand sides of equations (8) and (9); (VI) eliminate  $u'$  from the system by substituting for it in the transformed equation (6) the expression for  $u'$  to be derived from equation (7) after the transformation mentioned under (V) has taken place; (VII) eliminate  $u$  from the system by substituting for it in the transformed equations (4) and (10) (solved for  $p$ ) the right-hand side of the transformed equation (6); (VIII) eliminate  $p$  from the system by substituting for it in the equation for  $S$  the expression for  $p$ , established under (VII).

It should be noted that  $\pi\bar{u}$  represents the original equilibrium value of imports and of exports. Hence,  $A$  measures the change in the balance on current account, expressed as a percentage of the original equilibrium value of imports or exports, consequent upon a one per cent change in the exchange rate of home currency.  $A$  may, therefore, be referred to as *the elasticity of the balance on current account* with regard to the exchange rate.

A somewhat closer examination of formula (4.3.1) shows clearly that there is plenty of opportunity for a theoretical discussion of the exchange stability problem, if at least one further simplifying assumption is introduced. Various simplifying assumptions suggest themselves, all of which have the property that they reduce  $A$  to a much simpler form. In section 4.4 to 4.6 we will deal with the following seven special cases:

- (i) for home demand (section 4.4)
  - (a)  $\nu' = \eta'$  which means that there is no money illusion;
  - (b)  $\nu' = 1$  which means that there is no hoarding or dishoarding;
- (ii) for import and export demand (section 4.5)
  - (a)  $\eta^i = \infty$  which means that the competition between imported and home-produced goods on the home market is perfect;
  - (b)  $\eta^e = \infty$  which means that the competition on the country's export markets is perfect;
- (iii) for home supply (section 4.6)
  - (a)  $\varepsilon^l = 0$  which means full employment of labour;
  - (b)  $\varepsilon = 0$  which means that the limit of the country's production capacity has been reached;

(c)  $\varepsilon^l = \varepsilon = \infty$  which means unemployment of labour and underemployment of the country's productive capacity.

Combinations of simplifying assumptions belonging to one and the same group will be dealt with in the appropriate section, whereas combinations of simplifying assumptions belonging to different groups will be dealt with in section 4.7.

The justification for the choice made with regard to additional simplifying assumptions is the following. As explained in section 2.6 and section 3.7 the numerical value of the marginal structural coefficients depends on the period of adjustment taken into consideration. Now in the long run, the pure case of comparative statics, neither permanent hoarding and dishoarding, nor the existence of a permanent money illusion is compatible with the equilibrium concept. This case corresponds to the fundamental hypotheses of the WALRASIAN system. Furthermore, of the special short-run cases, in which either the money illusion or the phenomenon of hoarding and dishoarding is absent, the first corresponds with the fundamental hypothesis of the theoretical economic system of KEYNES. Both KEYNES and WALRAS allowed only price ratios to enter their systems. However, the case of money illusion with absence of the hoarding phenomenon, although not frequently met in economic theory, proves to be equally interesting in our analysis as the KEYNESIAN case and more or less its counterpart. The special cases of perfect competition in international economics are mainly interesting from a theoretical point of view as they reflect the implications of the classical doctrine on the subject of international economics. Finally, the special cases with regard to home

supply find their justification in the fact that the supply elasticities of labour and production change their numerical value over the course of the trade cycle. In this connection it is worth mentioning that, according to modern theory, the trade cycle in all probability is not a fluctuation around an equilibrium, but a movement between two equilibria, viz. a full employment equilibrium and an underemployment equilibrium<sup>1</sup>). The extreme values, nil and infinity, for the supply elasticities, seem, therefore, to be most suitable for further examination of their bearing on the exchange stability problem.

Except for the special cases with regard to import and export demand the argument will, particularly in section 4.8, be illustrated by the numerical values for some of the structural coefficients discussed in sections 4.2 and 4.1. By means of these numerical values and by putting  $v^i$  equal to  $1 - v^i$ , formula (4.3.1) can be reduced to the following comparatively simple form:

(4.3.2)

$$S = \frac{\left(\frac{D}{\varepsilon^I} + \frac{E}{\varepsilon} - F\right)(v' - \eta') - G(1 - \lambda)(1 - v')}{\left(\frac{H}{\varepsilon^I} + \frac{1}{\varepsilon}\right)\{Kv' - L + M(v' - \eta')\} - \left\{N(1 - \lambda) + \frac{Q}{\varepsilon^I} + P\right\}(1 - Nv')} \pi uk^b.$$

The values of the numerical coefficients  $D$  to  $Q$  in formula (4.3.2) are given in table 4. At first sight it can

<sup>1</sup>) See for instance: J. GOUDRIAAN "De berekening van de omvang der werkloosheid als gevolg van prijsdaling in een gesloten economisch milieu" in *De Economist* 83 (1934), p. 849 ff. and in particular J. TINBERGEN: "Legevaegstyper og Konjunkturbævegelse" in *Nordisk Tidsskrift for Technisk Økonomi*, 1944, pp. 45-63 (a mimeographed German version of this article has been circulated privately by its author) and J. TINBERGEN: "Over verschillende soorten evenwichten en de conjunctuurbeweging" in *De Economist* 92 (1943), pp. 129-147. Cf. also the summary of R. M. GOODWIN's paper "The Business Cycle as a Self-Sustaining Oscillation" in *Econometrica*, vol. 17 (1949) p. 184.

be already said that these numerical coefficients vary a good deal from one country to another. It may, therefore, be expected that the answer to the exchange stability problem will distinctly vary from one case to another.

TABLE 4

*The numerical values of the coefficients D to Q in formula (4.3.2) for nineteen countries*

| Countries          | D     | E     | F    | G     | H    | K     | L    | M    | N    | P    | Q    |
|--------------------|-------|-------|------|-------|------|-------|------|------|------|------|------|
| (1)                | (2)   | (3)   | (4)  | (5)   | (6)  | (7)   | (8)  | (9)  | (10) | (11) | (12) |
| 1. U.S.A. . . .    | -0.41 | -0.38 | 0.07 | -0.35 | 0.93 | 0.00  | 0.02 | 0.95 | 0.94 | 0.06 | 0.05 |
| 2. Germany . .     | -0.04 | 0.04  | 0.41 | 0.15  | 0.65 | 0.09  | 0.08 | 0.86 | 0.75 | 0.25 | 0.05 |
| 3. U.K. . . . .    | 0.10  | 0.26  | 0.41 | 0.31  | 0.60 | 0.13  | 0.07 | 0.82 | 0.70 | 0.30 | 0.04 |
| 4. Sweden . . .    | 0.06  | 0.12  | 0.36 | 0.27  | 0.62 | 0.12  | 0.07 | 0.81 | 0.71 | 0.29 | 0.06 |
| 5. France . . . .  | -0.11 | -0.08 | 0.22 | 0.04  | 0.77 | 0.09  | 0.10 | 0.87 | 0.82 | 0.18 | 0.04 |
| 6. Switzerland .   | 0.06  | 0.23  | 0.43 | 0.29  | 0.53 | 0.18  | 0.11 | 0.76 | 0.63 | 0.37 | 0.05 |
| 7. Japan . . . .   | -0.14 | -0.06 | 0.56 | 0.09  | 0.52 | 0.09  | 0.10 | 0.83 | 0.66 | 0.34 | 0.06 |
| 8. Italy . . . . . | -0.10 | -0.06 | 0.14 | 0.06  | 0.82 | 0.15  | 0.16 | 0.80 | 0.81 | 0.19 | 0.06 |
| 9. Chile . . . . . | —     | 0.34  | 1.94 | —     | —    | 0.23  | 0.06 | 0.66 | —    | 1.00 | —    |
| 10. Canada . . .   | -0.60 | -0.53 | 0.39 | -0.38 | 0.57 | -0.08 | 0.08 | 0.77 | 0.66 | 0.34 | 0.27 |
| 11. South Africa.  | -0.14 | 0.04  | 0.56 | 0.10  | 0.45 | 0.09  | 0.07 | 0.76 | 0.58 | 0.42 | 0.12 |
| 12. Estonia . . .  | -0.07 | -0.45 | 1.77 | 0.07  | 0.04 | 0.18  | 0.36 | 0.72 | 0.17 | 0.83 | 0.02 |
| 13. Latvia . . . . | -0.12 | -1.06 | 2.63 | 0.03  | 0.05 | 0.10  | 0.37 | 0.80 | 0.19 | 0.81 | 0.02 |
| 14. New Zealand    | -0.03 | 0.11  | 0.63 | 0.21  | 0.44 | 0.14  | 0.11 | 0.79 | 0.59 | 0.41 | 0.05 |
| 15. Finland . . .  | -0.52 | -0.36 | 1.51 | 0.11  | 1.26 | 0.18  | 0.30 | 0.76 | 0.31 | 0.69 | 0.02 |
| 16. Norway . . .   | -0.38 | -0.34 | 0.41 | -0.13 | 0.49 | 0.04  | 0.18 | 0.71 | 0.59 | 0.41 | 0.21 |
| 17. Hungary . . .  | -0.33 | -0.56 | 0.92 | -0.08 | 0.34 | 0.06  | 0.14 | 0.87 | 0.55 | 0.45 | 0.04 |
| 18. Denmark . .    | -0.18 | -0.06 | 0.56 | 0.06  | 0.45 | 0.09  | 0.10 | 0.77 | 0.59 | 0.41 | 0.11 |
| 19. Australia . .  | -0.30 | -0.28 | 0.64 | -0.06 | 0.50 | 0.05  | 0.11 | 0.84 | 0.65 | 0.35 | 0.08 |

Although it cannot be the purpose of this monograph to describe in full the results of the work of earlier writers on the subject, it seems well worth while to review briefly some of these results in order to show what their relation to our result is. Macro-economic models without prices, and micro-economic models will be left out of consideration as they seem to have very little in common with our simplified one-country model.

As far as one-country models are concerned, the only one comparable with the one presented here seems to be TINBERGEN's first model for international trade<sup>1)</sup>. Owing to the fact that he assumed both  $\varepsilon$  and  $\varepsilon^l$  to be infinite and, furthermore, put  $v^i$  equal to  $\pi$ , the final equation of his model should be:

$$(4.3.3) \quad S = \frac{1 - v'}{\pi + (1 - v')(1 - \pi)} \left\{ (1 - \pi)(1 - \pi')\eta^e - \right. \\ \left. - (1 - \pi')(1 - \eta^i) - (1 - \pi)\pi' \frac{v' - \eta'}{1 - v'} \right\} \pi \bar{u} k^b$$

where  $\pi' = \frac{\pi}{1 - v'\lambda}$ .

It actually turns out to be this formula, if, for the equation  $U^i = U^e$  of the model concerned, equations (4) and (5) of our simplified one-country model are substituted. The former equation seems less suitable for the study of the exchange stability problem, unless the changes in the data necessary to restore a disturbed equilibrium are the object of the investigation. TINBERGEN's model, it appears, corresponds to one of the special cases for home supply (viz.  $\varepsilon = \varepsilon^l = \infty$ ).

Other authors on the subject, and also TINBERGEN in his later work, have been mainly interested in the *two-country-model approach*. Although almost all of them seem to have been quite unaware of the work that had already been done on the subject by earlier writers, the similarity in the results of their work is striking. First in the field, apparently, has been MARSHALL<sup>2)</sup>, who, as early as 1879,

<sup>1)</sup> J. TINBERGEN: "Unstable and Indifferent Equilibria in Economic Systems" in *Revue de l'Institut International de Statistique* IX (1941), p. 43 ff.

<sup>2)</sup> A. MARSHALL: *Pure Theory of Foreign Trade and Pure Theory of Domestic Values* (1879), reprinted as number 1 of Series of Reprints of Scarce Tracts on Economics and Political Science (London 1930, re-issued 1935) and as Appendix J of his *Money Credit and Commerce* (London 1923). Cf. in particular section 9 (pp. 352—354) of this Appendix.

discovered the celebrated case of the "critical elasticities", according to which a country's balance of payments equilibrium is indifferent in cases where the sum of the price elasticities of substitution for export and import demand is equal to unity, and unstable where this sum lies below unity. BICKERDIKE<sup>1)</sup>, who published an article on the subject in 1920, which has gone almost unnoticed, seems to have been the first to state the case in a more general form. JOAN ROBINSON<sup>2)</sup> repeated this performance in 1937, and further completed the picture by an admirably written statement concerning the indirect effects of a change in the balance of payments consequent upon a change in the exchange rate. In 1942 BROWN<sup>3)</sup> published an article in which he introduced a number of refinements in the mathematical version of the theory. Finally, in 1948, TINBERGEN<sup>4)</sup> presented the matter once again, dropping the simplifying assumptions to which his one-country model had been subject, in a very illuminating article, in which he also dealt with *more-country models*. It would carry us too far to compare our simplified one-country model with the latter, but a comparison with the two-country-model approach seems to be in place.

METZLER<sup>5)</sup> summarizes the main findings of the traditional two-country-model approach (BICKERDIKE, JOAN

<sup>1)</sup> C. F. BICKERDIKE: "The Instability of Foreign Exchange" in *The Economic Journal*, vol. XXX (1920), pp. 118—122.

<sup>2)</sup> JOAN ROBINSON: "The Foreign Exchanges" in Part III (pp. 134—155) of her *Essays in the Theory of Employment* (London 1937, second edition 1947), cf. in particular the footnotes on pp. 142 and 143.

<sup>3)</sup> A. J. BROWN: "Trade Balances and Exchange Stability" in *Oxford Economic Papers* No. 6 (April 1942), pp. 56—75.

<sup>4)</sup> J. TINBERGEN: "Modelli di commercio internazionale" in *Giornale degli Economisti e Annali di Economia* VII (new series) Nov./Dec. 1948, pp. 627—648 (a mimeographed English version under the title "*Models of International Trade*" has been circulated privately by its author).

<sup>5)</sup> L. A. METZLER: "The Theory of International Trade" in *A Survey of Contemporary Economics*, edited by H. S. Ellis (Philadelphia, Toronto 1948), p. 226.

ROBINSON) in the following formula:

$$(4.3.4) \quad S = - \frac{\eta_1 \eta_2 (1 + e_1 + e_2) + e_1 e_2 (\eta_1 + \eta_2 - 1)}{(\eta_1 + e_2) (\eta_2 + e_1)} \pi \bar{u} k^b$$

in which  $e_1$  and  $e_2$  are the elasticities of supply of exports and  $\eta_1$  and  $\eta_2$  the elasticities of demand for imports in the two countries.

In our one-country model  $e_2$  has been assumed to be infinite, which reduces the formula to:

$$(4.3.5) \quad S = - \frac{\eta_1 \eta_2 + e_1 (\eta_1 + \eta_2 - 1)}{\eta_2 + e_1} \pi \bar{u} k^b.$$

This formula is not reconcilable with the one derived from our simplified one-country model, because  $\eta_1$  and  $\eta_2$  represent the *price elasticities* of import and export demand, whereas our  $\eta^i$  and  $\eta^e$  stand for the *price elasticities of substitution* for import and export demand. Owing to the fact that in our one-country model the competitors' price for exports has been assumed to be unaffected by changes in the exchange rate of home currency (cf. discussion of assumption 11)  $\eta_2$  and  $\eta^e$  are nevertheless identical and so the demand equation for exports reads the same in both systems. The demand equation for imports, however, reads as follows in the traditional two-country-model approach if simplification 1 (equality of money-values of imports and exports in original equilibrium situation) is imposed on this model:

$$(4.3.6) \quad u^i = \eta_1 \pi \bar{u} k^b \quad \text{since } e_2 = \infty \text{ and thus } p^i = -k^b.$$

Comparing this equation with its equivalent in our one-country model (cf. equation (8) on folding page at end of the book) it appears that the traditional two-country-model approach supposes:

(i) the competing home price level ( $p$ ), and  
 (ii) the change in the volume of production ( $u$ ) consequent upon a change in the exchange rate  
*not* to have any effect on the volume of import demand ( $u^i$ ). The former assumption has been covered by the different meaning attributed to  $\eta_1$  and  $\eta^i$ . The latter assumption implies that the traditional two-country-model approach is based on the assumption that the marginal import quota ( $v^i$ ) is nil.

The supply equation for exports reads as follows in the traditional two-country-model approach if simplifications 1 (equality of money-values of imports and exports in original equilibrium situation) and 2 (proportionate changes in price levels for home-market products and export products) are imposed on this model:

$$(4.3.7) \quad u^e = e_1 \pi \bar{u} p.$$

In this equation  $e_1$  is, apparently, identical with the supply elasticity of exports ( $\epsilon^e$ ), discussed in chapter III. Comparing equation (4.3.7) with its equivalent in our one-country model (cf. equation (10) on folding page) it follows that, where  $v^i = 0$  (see above), the traditional two-country-model approach furthermore proves to be based on the assumption that the home wage rate does not change consequent upon a change in the rate of exchange. This implies that it is assumed that both (i) the wage coefficient ( $\lambda$ ), and (ii) the wage flexibility ( $1/\epsilon^l$ ) are nil (compare equation (11) on folding page).

Equations (4.3.6) and (4.3.7), together with equations (1), (2), (4), and (9) (cf. folding page) form a system of six equations, containing seven unknowns, from which formula (4.3.5) can easily be derived by means of an elimination process.

If also  $e_1$  is assumed to be infinite, formula (4.3.5) reduces to:

$$(4.3.8) \quad S = (1 - \eta_1 - \eta_2) \pi \bar{u} k^b$$

being the formula which represents the well-known *case of the "critical elasticities."* This formula is the same as the one emerging from the final equation of the one-country model (cf. equation (4.3.1) on folding page) for  $\varepsilon = \varepsilon^l = \infty$  and  $v^i = 0$ . The  $\eta_1$  and  $\eta_2$  in formula (4.3.8) are identical with our  $\eta^i$  and  $\eta^e$  respectively. That not only  $\eta_2$  and  $\eta^e$  are identical here (see above) but also  $\eta_1$  and  $\eta^i$ , and that  $\lambda = 0$  is no longer a necessary implication of the classical two-country-model approach in this special case, is due to the fact  $p = 0$  if  $\varepsilon = \varepsilon^l = \infty$  and  $v^i = 0$ . This reduces our equation (8) (cf. folding page) to equation (4.3.6) (see above).

The implied assumptions of the traditional two-country-model approach, as stated above, are (1)  $v^i = 0$ , which means that the marginal unit of product contains no imports (cf. equation (10) on folding page) and that a change in the volume of production for exports, consequent upon a change in the rate of exchange, will be brought about solely by home factors of production (cf. equation (8) on folding page), (2)  $\lambda = 0$ , which means that wages are completely insensitive to changes in the home price level, and (3)  $\varepsilon^l = \infty$ , meaning that wages are equally insensitive to changes in home employment. Although JOAN ROBINSON does not state the assumptions explicitly, it appears that she justifies them by the following general statement: "The actual changes in imports and in exports which will come about as the result of a change in the exchange rate will depend partly upon the reaction on the demand for imports, and on the supply of exports,

of a change in the balance of trade itself..... But these effects influence the magnitude, not the direction, of the change in the balance of trade consequent upon a fall in the exchange rate, for the secondary effects follow from the change in home incomes due to the change in the balance of trade, and if the balance of trade does not alter the secondary effects cannot occur" <sup>1)</sup>). Our formula (4.3.1) (cf. folding page), however, shows clearly that the secondary effects influence not only the magnitude but also the direction of the change in the balance on current account, which follows a change in the exchange rate. This is a result of the fact that with regard to the primary effects of a change in the exchange rate, as reflected in the three behaviouristic equations (8), (9) and (10) (cf. folding page), due account is taken of the phenomenon of imperfect competition in international trade (price-substitution effect and of the import-component of the home price level. The traditional two-country-model approach ignores them both. Apart from  $\eta^i$ ,  $\eta^e$  and  $\varepsilon$  the other structural coefficients prove also to be of importance, and  $v'$  and  $\eta'$  of even strategical importance, in the determination of the direction of the resulting change in the balance on current account.

BROWN, in his version of the two-country model, departed slightly from the traditional approach by taking account of the fact that part of the marginal costs of export products consists of imported raw materials. Again using METZLER's symbols, and denoting the marginal import quota of the export products of the two countries by  $v_1$  and  $v_2$  respectively, BROWN's formula <sup>2)</sup> may be written as:

<sup>1)</sup> JOAN ROBINSON, *op. cit.*, p. 139.

<sup>2)</sup> BROWN, *op. cit.*, p. 60.

$$(4.3.9) \quad S = \left[ - \left\{ \frac{\eta_1 \eta_2 (1 + e_1 + e_2) + e_1 e_2 (\eta_1 + \eta_2 - 1)}{(\eta_1 + e_2) (\eta_2 + e_1)} \right\} + \right. \\ \left. + v_1 \frac{(\eta_2 - 1) e_1}{\eta_2 + e_1} + v_2 \frac{(\eta_1 - 1) e_2}{\eta_1 + e_2} \right] \pi \bar{u} k^b.$$

The first term on the right-hand side is identical with METZLER's formula (4.3.4). The additional two terms imply that the presence of imported raw materials in export products is a stabilizing factor if the foreign demand for such products is inelastic, whereas the reverse is true if this demand is elastic<sup>1</sup>).

*In the one-country model  $e_2$  is assumed to be infinite and  $v_2$  to be equal to nil.* This reduces the formula to:

$$(4.3.10) \quad S = - \frac{\eta_1 \eta_2 + e_1 \{ \eta_1 + (1 - v_1) (\eta_2 - 1) \}}{\eta_2 + e_1} \pi \bar{u} k^b = \\ = - \frac{\eta_1 (\eta_2 + e_1) + (\eta_2 - 1) (1 - v_1) e_1}{\eta_2 + e_1} \pi \bar{u} k^b.$$

The same result can be obtained by means of an elimination process if in the system of equations for the traditional two-country-model approach the following supply equation for exports:

$$(4.3.11) \quad u^e = e_1 \pi \bar{u} (p + v_1 k^b)$$

is substituted for equation (4.3.7).

If also  $e_1$  is assumed to be infinite, formula (4.3.10) reduces to:

$$(4.3.12) \quad S = \{ (1 - v_1) (1 - \eta_2) - \eta_1 \} \pi \bar{u} k^b.$$

<sup>1</sup>) If  $\eta_2 < 1$  then  $v_1 \frac{(\eta_2 - 1) e_1}{\eta_2 + e_1} < 0$ . This contributes to [ ] becoming negative; and where [ ] < 0 the equilibrium will be stable. If  $\eta_2 > 1$  the reverse holds true. For  $\eta_1 < 1$  and  $\eta_1 > 1$  respectively the same argument applies.

For  $v_1 = 0$  formulae (4.3.9) to (4.3.12) become identical with the corresponding formulae for the traditional two-country-model approach.

It should be noted that BROWN, at this stage of his argument, still retained the implied assumption of the traditional approach that a change in the volume of production for exports consequent upon a change in the rate of exchange will be brought about solely by home factors of production. Likewise, he ignores the phenomenon of imperfect competition in international trade and the influence upon the wage level of changes in the home price level and home employment.

It is interesting to see that, if only the assumption of constancy of the wage level (by virtue of  $\lambda = 0$  and  $\varepsilon^l = \infty$ ) is retained, and the other two remaining specific assumptions of BROWN's two-country model are dropped by substituting for equation (4.3.6) the following demand equation for imports:

$$(4.3.13) \quad u^i = v_1 u^e + \eta_1 \pi \bar{u} (p + k^b),$$

the resulting formula for the one-country model appears to be rather simple, viz.:

$$(4.3.14) \quad S = \frac{(1 - v_1) e_1 \{1 - \eta_1 - (1 - v_1) \eta_2\}}{e_1 + \eta_2} \pi \bar{u} k^b.$$

The more general two-country-model formula, from which formula (4.3.14) can be derived by putting  $e_2 = \infty$  and  $v_2 = 0$ , proves to be<sup>1)</sup>

$$(4.3.15) \quad \frac{e_1(1 - v_1)e_2(1 - v_2) \{1 - v_1v_2 - \eta_1(1 - v_2) - \eta_2(1 - v_1)\}}{e_1e_2(1 - v_1v_2)^2 + \eta_{11}\{e_1(1 - v_1) - e_2v_2(1 - v_2)\} + \eta_{22}\{e_2(1 - v_2) - e_1v_1(1 - v_1)\}} \pi \bar{u} k^l$$

<sup>1)</sup> The derivation of this formula will be made available by the author on request

It hardly pays to generalize the two-country model still further by dropping the assumption of constancy of the wage level, and by introducing into the model the indirect effects which a change in the exchange rate has on home production and on home demand in each of the two countries, through changes in the home price level and home incomes. Such a procedure would only burden the formula with a number of coefficients, the numerical values of which tend to be very small in practice in so far as they refer to the "country" which represents the "rest of the world". The impact on the rest of the world of a change in the exchange rate of one country is usually almost negligible. BROWN appreciated this point in the application of his final formula to the British case by putting  $v_2 = 0$  and  $e_2 = \infty$ <sup>1)</sup>. Only in the case of a wholesale devaluation of currencies, as experienced in September 1949, does it make sense to introduce into the model the indirect effects abroad.

Discussing the effect of the multiplier, BROWN<sup>2)</sup> made an attempt to include in his model the indirect effects both at home and abroad. Unfortunately, however, he based this part of his analysis on a set of assumptions which do not seem to be consistent with the earlier part. The alleged assumptions are that for a change in the level of activity (1) the elasticities of supply of all goods and services (including  $e_1$  and  $e_2$ ) may be taken to be infinite, (2) the average real labour costs of production are constant over the relevant range, which implies  $\lambda = 1$  because  $\epsilon^l = \infty$  and a change in the home price level as a direct influence of a change in the exchange rate was taken into account in the earlier part of the analysis. Furthermore, he limits

<sup>1)</sup> BROWN, *op. cit.*, p. 70.

<sup>2)</sup> BROWN, *op. cit.*, pp. 64—66.

himself to an analysis of the case in which there is no money illusion ( $v' = \eta'$ ).

Therefore I think that BROWN and JOAN ROBINSON both failed in their discussion of the indirect effects of a change in the rate of exchange; the former because of inconsistencies in the assumptions made in the two parts of his analysis, the latter because she wrongly excluded from her argument the possibility that indirect effects may influence the direction, as well as the magnitude, of the change in the balance on current account, consequent upon a change in the exchange rate <sup>1</sup>).

As far as I can see, it is a much better policy to analyse the impact of a change in the exchange rate on the home economy in detail (direct as well as indirect effects) by means of a one-country model. To such an analysis the following sections of this chapter are devoted. As stated above, this analysis concerns a number of special cases. The character of these special cases, the order in which they are dealt with and the general structure of the theoretical argument with regard to the exchange stability problem, as given in sections 4.4 to 4.7, may conveniently be indicated by means of a diagram (cf page 178).

#### 4.4 SPECIAL CASES FOR HOME DEMAND <sup>2</sup>)

The first special case for home demand deals with the situation which would arise if the individuals in their

<sup>1</sup>) Doubts as to the validity of the ROBINSON-BROWN approach were also raised by A. C. HARBERGER in a recently published article entitled "Currency Depreciation, Income, and the Balance of Trade" in *The Journal of Political Economy*, vol. LVIII (1950), pp. 47—60 (reprinted as "Cowles Commission Papers", new series, no. 41), which only came to the author's notice after the manuscript of this book had already been handed to the printers.

<sup>2</sup>) This and the following sections, up to 4.7, contain a detailed discussion of the exchange stability problem on the basis of our simplified one-country model. The author felt it his duty to show (i) what the implications of the special cases, indicated in section 4.3, are, (ii) what conclusions can be drawn from them, and (iii) how

capacity of income-spenders, taken on the whole, were *not subject to the so-called money illusion*, that is to say, if they were equally sensitive to changes in real income resulting from changes in money-income and to changes in real income resulting from changes in prices. The only determining factor for the volume of home demand in such a case is the level of *real income*, and the appropriate price level for deflating money-incomes is of course the home price level.

Mathematically expressed the condition imposed on the system in this special case is  $\boxed{v' = \eta'}$ . It reduces the equation for home demand to:

$$(4.4.1) \quad u' = v' \{Y - (1 - \pi) \bar{u}p\} = v' \gamma,$$

where the expression between brackets represents the change in real income ( $\gamma$ ), since (i) in the original equilibrium situation  $\bar{Y} = \bar{U}'$  (due to simplification 1), and likewise  $\bar{\gamma} = \bar{u}' = (1 - \pi) \bar{u}$  (the original equilibrium values of prices being equal to unity, cf. section 3.2), whereas (ii) the linearly approximated definitional equation for  $\gamma$  would read:  $\gamma = Y - \bar{\gamma}p$ .

The final equation of the simplified one-country model (equation (4.3.1)), expressing  $S$  as a function of  $k^b$ , reduces for  $v' = \eta'$  to:

$$\frac{- (1 - v^i - v^l \lambda) \{1 - \eta^i - \eta^e (1 - v^i)\} (1 - v^i)}{-\frac{v^i}{\pi} + \frac{1}{\epsilon}} + \left\{ v^i \pi \left( 1 - \eta^i \frac{\eta^e}{v^i} \right) - \left[ (1 - v^i - v^l \lambda) + \pi \left\{ \frac{v^i}{\pi} + \eta^i \frac{v^l}{\epsilon^l (1 - \pi)} \right\} \right] \right\} \{ (1 - v^i) (1 - v^i) + v^i \} \pi \bar{u} k^l$$

$$\tau \bar{u} k^b = A_1 \pi \bar{u} k^b.$$

these conclusions are obtained. This analysis involves a good deal of mathematics and calls for much patience and endurance on the part of the reader. Those readers who are mainly interested in practical results and in the principal conclusion of our analysis may, therefore, be well-advised to read only the conclusions printed in italics, or to pass on to section 4.8 straightaway.



This formula enables us to formulate a number of conclusions with regard to the exchange stability problem in this special case.

First of all, the equilibrium will be *indifferent* if  $A_1 = 0$ . This will be the case if either  $B_1 = 0$  (and  $C_1 \neq 0$ ) or  $C_1 = \infty$  (and  $B_1 \neq \infty$ ). Now,  $B_1$  will be equal to nil if one of the following conditions is fulfilled:

- (a)  $1 - v^i = v^l \lambda$ . In cases where the marginal costs of production consist of an import and a wage component only (so that  $1 - v^i = v^l$ ) this condition will be fulfilled if individuals in the capacity of wage-negotiators also are free from any money illusion ( $\lambda = 1$ ). Hence, the first conclusion to be drawn from our simplified one-country model appears to be:

*Conclusion 1: When (1) on the whole, people are not subject to the money illusion, either in their capacity of income-spenders or in their capacity of income-earners, and (2) the sum of the cost components which affect marginal costs is equal to unity, the balance of payments equilibrium will be indifferent to changes in the exchange rate.*

This conclusion is not surprising, for one could hardly expect the exchange rate, being an entity referring to the absolute value of a currency, to have any effect on an economy (1) in which all economic decisions are based on changes in the conditions expressed in *real* terms, meaning that only price ratios count, and (2) where prices are perfectly flexible. In such an economy money acts only as a means of exchange, the nominal value of which does not matter. That a change in the exchange rate, in such a case, will

leave the physical sphere of the economy unaffected and will result only in a proportionate change in all price levels can be demonstrated as follows. Assume for a moment that all volume items remain unaltered. If so,

- (i)  $l = p$  for  $\lambda = 1$ , according to equation (11) (cf. folding page at end of the book); hence
- (ii)  $p = -k^b$  for  $1 - v^l = v^i$ , according to equation (10); hence
- (iii)  $u^i = v^i u$  and  $u^e = 0$ , according to equations (8) and (9); hence
- (iv)  $u = u'$ , according to equation (6).

Furthermore, it follows from the foregoing and from equations (5), (1), (2) and (3) that

- (v)  $Y = u + (1 - \pi) \bar{u}p + v^i u$ ; hence
- (vi)  $u = v' u + v' v^i u$ , according to equation (7).

The last result holds true only for  $u = 0$  which, according to the foregoing results, implies constancy of all the volume items of the system. From this it may be concluded that constancy of volume items and inversely proportionate changes in the home price level (and wage rate) on the one hand and the exchange rate on the other hand are not inconsistent with each other in the case under review. To give a complete proof of the fact that changes in the exchange rate have no effect on the volume items of the system would require a preliminary step in the argument, viz. the expression of  $u$  and  $u^i$  in price-terms only. It would not be difficult to include this preliminary step

in the argument, but it would carry us too far to present it here.

It is interesting to note that in cases where the marginal import quota is equal to unity, as it turned out to be for Chile,  $v^i$  which  $= 1 - v^i = 0$  and, therefore, also  $1 - v^i - v^i\lambda = 0$  regardless of the value of  $\lambda$ . In such cases the condition that  $v' = \eta'$  is, therefore, sufficient to render the equilibrium indifferent.

In the remainder of the argument the condition  $1 - v_i = v^i\lambda$  will be referred to as the *special case for labour*.

The second case for which  $A_1 = 0$ , and hence the equilibrium will be indifferent, presents itself if:

- (b)  $\boxed{1 - \eta^i - \eta^e (1 - v^i) = 0}$ . A necessary, though not sufficient, condition here is that the price elasticity of substitution for import demand is smaller than unity, as proves to be the case (taking Mr CHANG's figures for granted) for all countries studied, with the exception of Canada (cf. table 2). Furthermore, this condition appears to be compatible with price elasticities of substitution for export demand larger than unity. The conclusion embodied in the conditions presented here may be formulated as follows:

*Conclusion 2: When (1) the individuals in their capacity of income-spenders are, on the whole, not subject to any money illusion and (2) the sum of the price elasticity of substitution for export demand, multiplied by the share of home factors of production in the marginal costs, and the price elasticity of substitution for import demand is equal to unity, the balance of payments equilibrium will be indifferent to changes in the rate of exchange.*

The third case presents itself if:

- (c)  $\nu' = 1$ . This condition represents the second special case for home demand and will, therefore, be dealt with at the end of this section, where the combination of the two special cases for home demand will be discussed.

For the sake of completeness it may be stated that the conclusions arrived at so far are subject to one limiting condition which is of merely theoretical interest, viz. that when  $B_1 = 0$ ,  $C_1$  should not be at the same time equal to nil, for this would render  $A_1$  indeterminate ( $0 : 0$ ). This, however, would require such a highly special configuration of numerical values for the structural coefficients involved that the fact is hardly worth mentioning here.

$A_1$  will also be nil if  $C_1 = \infty$ . This is possible only if either the price elasticity of substitution for import or for export demand becomes infinite, or the supply elasticity of production or of labour becomes equal to nil. All of these conditions constitute further special cases. The discussion of their combination with the special case for home demand under review will be postponed till later (cf. section 4.7).

To ascertain the conditions which for  $\nu' = \eta'$  render the balance of payments equilibrium *stable and unstable* respectively is a more complicated matter than to establish the conditions which render it indifferent. This is due to the fact that  $C_1$ , the formula of which is highly complicated, plays an equally important part in this problem as  $B_1$ , the formula of which has become comparatively simple as a consequence of the introduction of the condition that  $\nu'$  should equal  $\eta'$ . Nevertheless, it is well worth while to investigate the exchange stability problem by means

of formula (4.4.2), for the analysis (cf. mathematical appendix to this chapter) proves that *in general*

$$A_1 \geq 0 \text{ for } v' \geq 1$$

If  $A_1 > 0$  the equilibrium will be unstable and if  $A_1 < 0$  it will be stable.

Our third conclusion, therefore, reads as follows:

*Conclusion 3: When, on the whole, the individuals in their capacity of income-spenders are not subject to any money illusion the balance of payments equilibrium will, broadly speaking, be stable with regard to changes in the exchange rate if the marginal propensity to spend is smaller than unity, and unstable if it is larger than unity.*

There are, however, *exceptions* to this rule. The reverse, viz.  $A_1 \geq 0$  for  $v' \leq 1$ , will hold true if<sup>1)</sup>:

$$\begin{aligned} 0 &\leq \frac{(1 - v' + v'v^i) \left\{ 1 - v^i\lambda + \eta^i \pi \frac{v^i}{\epsilon^i(1 - \pi)} \right\}}{v' \pi \left\{ \frac{v^i(1 - v^i)}{\epsilon^i(1 - \pi)} + \frac{1}{\epsilon} \right\}} \geq \\ &\geq 1 - \eta^i - \frac{\eta^e}{v'} \leq 1 - \eta^i - \eta^e(1 - v^i) \geq 0. \end{aligned}$$

<sup>1)</sup> To save space the inequalities are printed in a slightly unorthodox way. Rewriting them as  $0 \leq a \leq b \leq c \leq 0$  the top row inequalities  $0 < a > b < c > 0$  imply that  $0 < a$ ,  $b < a$ ,  $b < c$ , and  $0 < c$ . An advantage of our unorthodox way of writing is that it can immediately be seen whether the numerical value of a certain expression should lie between the numerical values of two other expressions or either at one side or the other of *both* the numerical values of the two other expressions. The inequalities under review require the latter condition to be fulfilled.

The exceptional cases represented by the bottom row inequalities require, clearly, that  $-\frac{\eta^e}{v'} > -\eta^e(1 - v^i)$ , and therefore that  $v' > \frac{1}{1 - v^i}$ . Now, as it seems very unlikely that  $v'$  will ever depart from unity to any considerable extent, and certainly not so in the upward direction, this condition can only be expected to be fulfilled in cases where the marginal import quota is very low, as is the case for the United States (0.06). It also appears that for the United States  $1 - \eta^i - \eta^e(1 - v^i)$  is smaller than unity, viz.  $-0.37$ . If, therefore, the marginal propensity to spend and the price elasticity of home demand for the United States were both to the same extent larger than unity, then this would not necessarily imply that the balance of payments equilibrium with regard to changes in the exchange rate would be unstable.

The exceptional cases represented by the top row inequalities require that  $v' < \frac{1}{1 - v^i}$ , which will practically always be the case. The further condition that  $1 - \eta^i - \eta^e(1 - v^i) > 0$  proves to hold good for all countries studied, with the exception of the United States, Canada, Norway, Hungary and Australia. To what extent the other conditions apply remains, however, a matter of doubt, which cannot be answered so long as the numerical values of the other structural coefficients (especially  $v'$ ,  $\epsilon^i$  and  $\epsilon$ ) have not been determined.

It is interesting to note that *the general rule applies where the effect of a change in the rate of exchange on the volume of home demand is "normal"* (that is to say if the volume of home demand increases consequent upon a fall in the rate of

exchange), and that the exceptional cases present themselves if this effect is "abnormal". This can be seen from the following formula:

$$(4.4.3) \quad S = \frac{1 - v'}{v'} u',$$

a formula which can easily be derived from equations (3), (4) and (5) of our simplified one-country model and equation (4.4.1) which takes the place of equation (7) for  $v' = v'_i$ .

The second special case for home demand deals with the situation which arises if, taken on the whole, people spend the whole of their income, which is tantamount to saying that the phenomenon of *hoarding* (either positive or negative) is *absent*. The mathematical expression for this condition is that  $\boxed{v' = 1}$ . This condition reduces the final equation of the simplified one-country model to:

$$\frac{v^l(1-v^i)}{\varepsilon^l(1-\pi)} + \frac{1}{\varepsilon} \left\{ (1-\eta^i-\eta^e) - v^i \left\{ \frac{v^i}{\pi} + \eta^i \frac{v^l}{\varepsilon^l(1-\pi)} \right\} \right\} (1-\pi)(1-\eta')$$


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$$\frac{(1-v^i)}{(1-\pi)} + \frac{1}{\varepsilon} \left\{ \pi(1-\eta^i-\eta^e) + (1-\pi)(1-\eta') \right\} - \left[ (1-v^i-v^l\lambda) + \pi \left\{ \frac{v^i}{\pi} + \eta^i \frac{v^l}{\varepsilon^l(1-\pi)} \right\} \right] v^i$$


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$$\frac{2}{2} \pi \bar{u} k^b = A_2 \pi \bar{u} k^b.$$

According to this formula the balance of payments equilibrium will be *indifferent* to changes in the exchange rate if  $B_2 = 0$  and  $C_2 \neq 0$ . This will be the case if:

$$(a) \quad \boxed{\left\{ \frac{v^l(1-v^i)}{\varepsilon^l(1-\pi)} + \frac{1}{\varepsilon} \right\} (1-\eta^i-\eta^e) = v^i \left\{ \frac{v^i}{\pi} + \eta^i \frac{v^l}{\varepsilon^l(1-\pi)} \right\}},$$

except when, at the same time,

$$\left\{ \frac{v^l (1 - v^i)}{\epsilon^l (1 - \pi)} + \frac{1}{\epsilon} \right\} (1 - \eta^i) \frac{1 - \pi}{v^i} = 1 - v^i - v^l \lambda.$$

The whole of this condition is too complicated to state in words. Nevertheless it enables us to draw a conclusion with regard to the borderline case:  $v^i = 0$ .

*Conclusion 4: When (1) there is neither hoarding nor dishoarding and (2) the marginal unit of product contains no imports, the balance of payments equilibrium will be indifferent to changes in the exchange rate if, at the same time, the sum of the price elasticities of substitution for import and export demand is equal to unity.*

The cases where either (i)  $\epsilon^l = \epsilon = \infty$ , or (ii)  $\epsilon^l = 0$ , or (iii)  $\epsilon = 0$  are left out because they constitute combinations with special cases for home supply, which will be discussed in section 4.7. Notwithstanding its rather limited value, conclusion 4 derives some interest from the fact that it appears to provide a one-country-model counterpart to the celebrated *case of the critical elasticities* of the traditional two-country-model approach, dealt with in the preceding section.

Furthermore, it may be concluded that condition (a) can never be fulfilled if the sum of the price elasticities of substitution for import and export demand is larger than unity, as it proves to be for the majority of the countries studied (cf. table 2).

The equilibrium will also be indifferent if:

- (b)  $\pi = 1$ . This does not make sense, apparently, for it would involve the volume of home demand to be nil

in the original equilibrium situation  $\bar{u}' = (1 - \pi) \bar{u} = 0$ . This condition, therefore, *will never be fulfilled* in reality.

- (c)  $\eta' = 1$ . The discussion of this condition, which constitutes a combination with the first special case of home demand, will be postponed till the end of this section.

The discussion of the cases where  $C_2 = \infty$ , all of which constitute combinations with other special cases (viz.  $\eta^i = \infty$ ,  $\eta^e = \infty$ ,  $\varepsilon^l = 0$  and  $\varepsilon = 0$ ), will be postponed (cf. section 4.7).

With regard to the conditions under which the balance of payments equilibrium is *stable or unstable* respectively in the second special case of home demand, it can be proved by a method similar to the one applied for the first special case for home demand (cf. mathematical appendix to this chapter)<sup>1)</sup> that *in general*

$$A_2 \gtrless 0 \text{ for } \eta' \lesseqgtr 1$$

Stated in words this conclusion reads as follows:

*Conclusion 5: When there is neither hoarding nor dishoarding the balance of payments equilibrium will, broadly speaking, be stable with regard to changes in the exchange rate if the price elasticity of home demand is larger than unity, and unstable if it is smaller than unity.*

The *exceptions* to this rule, for which it can be proved that  $A_2 \gtrless 0$  for  $\eta' \gtrless 1$  holds true, are constituted by the cases in which the following conditions are fulfilled:

<sup>1)</sup> A description of this method will be made available by the author on request.

$$(1 - v^i - v^l \lambda) - \frac{1 - \pi}{v^i} (\alpha + \beta) (1 - \eta') \leq$$

$$\leq \frac{\pi}{v^i} (\alpha + \beta) (1 - \eta^i - \eta^e) - \frac{\pi}{1 - v^i} \alpha \eta^i - v^i \leq 0$$

where  $\alpha = \frac{v^l (1 - v^i)}{\varepsilon^l (1 - \pi)}$  and  $\beta = \frac{1}{\varepsilon}$ .

A necessary, though not sufficient, condition for the cases represented by the bottom row inequalities to materialize appears to be that the sum of the price elasticities of substitution for import and export demand is less than unity, as it proves to be for a minority of the countries studied. Furthermore, where  $1 - v^i - v^l \lambda$  will be a very small positive figure ( $\lambda \approx 1$  and  $1 - v^i \approx v^l$ ), these cases will, in practice, be bound to the condition that  $\eta' > 1$  (for  $\eta' < 1$  the expression in the first part of the inequalities would soon become negative, whereas it should be positive).

A necessary, though not sufficient, condition for the cases represented by the top row inequalities will be that  $\eta' < 1$ , otherwise the expression in the first part of the inequalities cannot become negative as it should. This seems to be about all that can be said of the practical possibility that these exceptional cases will materialize in reality, as long as the numerical values of the other structural coefficients remain unknown.

It can be proved, through combining equations (3), (4), (5) and (7) of our simplified one-country model (with  $v' = 1$  in the last), that for the special case of home demand under review

$$(4.4.5) \quad S = - (1 - \eta') (1 - \pi) \bar{u} p.$$

From this formula it follows that *the general rule will*

apply if the home price level reacts in a normal way to a change in the exchange rate (that is to say if the home price level rises consequent upon a fall in the exchange rate), and that the exceptional cases will present themselves where this happens not to be the case.

An interesting feature of the case under review is that there would still be a definite relationship between  $S$  and  $k^b$  if one of the equations, viz. equation (6), were not included in the system. This equation which, therefore, has a *symptomatic* character, however, cannot be omitted if one wishes to establish a functional relationship between  $u'$ ,  $U'$ , or  $Y$ , and  $k^b$ . From this example it follows that the mere counting of variables and equations is not a wholly satisfactory procedure to answer the question whether the system can be reduced to a relationship between two variables only.

The third and last special case for home demand is a combination of the former two. It deals with the situation which will eventually arise in the *long run*, the so-called *pure case of comparative statics*. Neither permanent hoarding and dishoarding, nor a permanent money illusion is compatible with the notion of a long-run equilibrium. As regards hoarding, it is quite conceivable that because of a change in one of the variables or data of the economic system, one might wish to increase one's stock of available purchasing power, but that such a happening should lead to an ever increasing (or decreasing) stock of available purchasing power in the hands of the public is more difficult to imagine. Such a state of permanent inflation (deflation) would mean that the monetary equilibrium would be permanently upset. As for the money illusion one may expect the individuals sooner or later to adapt

their spending habits to the changes which have taken place in their real incomes, no matter whether these changes are an effect of changes in prices or of changes in money-incomes.

Mathematically formulated the condition imposed on the system in the pure case of comparative statics is  $\boxed{v' = \eta' = 1}$ . It implies, apparently, that people spend the whole of their real income (cf. formula 4.4.1). Furthermore, it reduces the final equation of the one-country model to the following:

$$(4.4.6) \quad S = A_3 \pi \bar{u} k^b = 0.$$

Where  $\boxed{A_3 = 0}$  the equilibrium of the balance of payments with regard to changes in the exchange rate is clearly indifferent. Hence, we get

*Conclusion 6: When, taken as a whole, (1) there is neither hoarding nor dishoarding, and (2) the individuals in their capacity of income-spenders are not subject to any money illusion — which conditions will be fulfilled in the long run — the balance of payments equilibrium will in all circumstances be completely indifferent to changes in the exchange rate.*

This conclusion can easily be derived from the system of equations itself in the following way (compare equations on folding page at end of the book). For  $v' = \eta' = 1$  the equation for home demand may be transformed into:

$$(4.4.7) \quad Y = u' + (1 - \pi) \bar{u} p,$$

the right-hand side of which is identical with the definition of  $U'$ . Hence,

$$(4.4.8) \quad Y = U'.$$

Comparing this equation with the balance equation for the money-value items of the system (equation (5))

$$(4.4.9) \quad Y + U^i = U' + U^e$$

we find the following expression for  $S$ :

$$(4.4.10) \quad S = U^i - U^e = 0.$$

Stated in words the argument would be: where people consume as much as they produce, measured in terms of money, the balance of payments should be in equilibrium no matter what the rate of exchange is.

#### 4.5 SPECIAL CASES FOR IMPORTS AND EXPORTS

The special cases for imports and exports ( $\eta^i = \infty$  and  $\eta^e = \infty$  respectively) are mainly of interest from a theoretical point of view, as it appears that the classical doctrine on the subject has been based (often tacitly) on the assumption of pure competition in both import and export markets. The special cases under review, however, may be given some bearing on practical problems by changing the definitions of imports and exports in such a way that *monetary gold movements* are included among its constituent items. The special case in which both the price elasticity of substitution for import demand and that for export demand are equal to infinity may then be considered to represent the situation which would arise if both the home country and the outside world were on the gold (exchange) standard. Where the central banks in both countries have committed themselves to buy and sell

gold at a fixed price in terms of the home currency, slight changes in the exchange rate will make gold shipments in one direction or the other worth their while. If, for instance, the exchange rate of home currency in terms of foreign currency experienced a fall, it would soon become profitable to obtain foreign currency through buying gold from the home central bank with home currency and re-selling it to the foreign central bank for foreign currency; the opposite would hold true if the exchange rate improved. This is tantamount to saying that the price elasticity of substitution for import and export demand for gold would be equal to infinity in such cases. Now, where the demand for one commodity, in this case gold, is perfectly elastic, the same will apply to the total import and export demand. In such cases, therefore,  $\eta^i = \eta^e = \infty$ . Likewise, the cases where only the price elasticity of substitution for export demand is infinite may be considered to correspond to the cases where the relevant part of the outside world (that is the part with which the home country has established trading connections) is on the gold standard, and the home country is a gold producing country not on the gold standard. Where only the price elasticity of substitution for import demand is equal to infinity it is just the other way round (home country on the gold standard, outside world gold producing).

Now let us see what happens in the first of the special cases under review here, viz. where *the price elasticity of substitution for import demand is infinite*, i.e. where there is pure competition in the market for imported products. Mathematically expressed this condition reads  $\boxed{\eta^i = \infty}$ . Wherever this condition is fulfilled the home price level will vary inversely in proportion to changes in the exchange

rate as long as the data of the system remain unaltered. This conclusion can easily be derived from equation (8) (cf. folding page at end of the book), which for  $\eta^i = \infty$  reads as follows:

$$(4.5.1) \quad p = -k^b.$$

Where this is so, the volume of exports will not change consequent upon a change in the exchange rate; equation (9) now reads:

$$(4.5.2) \quad u^e = 0.$$

On account of equation (6) this means also that the volumes of home production and consumption will vary proportionately whenever the exchange rate changes its value:

$$(4.5.3) \quad u = u'.$$

Furthermore, it can easily be proved that the balance on current account will vary inversely in proportion to the changes in the volume of imports, where in the original equilibrium position imports and exports were assumed to be equal in value. The money-value of the initial volume of exports will change in inverse proportion to the change in the exchange rate, whereas the money-value of the initial volume of imports will do the same. The one effect will, therefore, cancel out the other. The volume of exports remaining the same, the only change in the balance on current account derives, therefore, from the change in the volume of imports. Owing to the omission of products of deviations of variables from their original equilibrium values, this results in:

$$(4.5.4) \quad S = -u^i.$$

The expression for  $S$  in terms of  $k^b$  can be found by substituting  $\eta^i = \infty$  in the final equation of our simplified one-country model (4.3.1)<sup>1</sup>:

$$\begin{aligned}
 (4.5.5) \\
 S &= \frac{\left\{ \frac{v^l}{\varepsilon^l(1-\pi)} + \frac{1}{\varepsilon} \right\} (1-\pi)(v'-\eta') - (1-v^i-v^l\lambda)(1-v')}{\left\{ \frac{v^l}{\varepsilon^l(1-\pi)} + \frac{1}{\varepsilon} \right\} v' \pi + \frac{v^l}{\varepsilon^l(1-\pi)} (1-v') \pi} \pi u k^b = \\
 &= \frac{B_4}{C_4} \pi \bar{u} k^b = A_4 \pi \bar{u} k^b.
 \end{aligned}$$

From this formula it follows that the balance of payments equilibrium will be *indifferent* to changes in the exchange rate ( $A_4 = 0$ ), if ( $B_4 = 0$ ):

$$\eta' = v' - \frac{(1-v^i-v^l\lambda)(1-v')}{\frac{v^l}{\varepsilon^l(1-\pi)} + \frac{1}{\varepsilon}}, \text{ except when, at the}$$

same time, ( $C_4 = 0$ ):  $\frac{v'}{\varepsilon} + \frac{v^l}{\varepsilon^l(1-\pi)} = 0$  which is the case if  $\varepsilon = \varepsilon^l = \infty$ .

The stated condition is too complicated to formulate in words, but it can easily be seen that it will be satisfied if the money illusion is absent in the minds of people in their capacity of income-spenders ( $v' = \eta'$ ) and one of the following conditions is fulfilled:

- (i)  $v' = 1$
- (ii)  $1 - v^i = \frac{v}{v^l} \lambda$
- (iii)  $\varepsilon = 0$
- (iv)  $\varepsilon^l = 0$

<sup>1</sup>Where  $\eta^i = \infty$  all the terms that do not contain  $\eta^i$  may be omitted as they are negligible compared with the terms containing  $\eta^i$ .

The former two cases have already been covered by conclusions 6 and 1 respectively;  $\eta^i = \infty$  appears in both cases not to be a necessary condition to render the equilibrium indifferent. The latter two cases will be discussed in section 4.7, where the combination of special cases belonging to different categories is analysed. If on the contrary there is a money illusion in the minds of people in their capacity of income-spenders then the balance of payments equilibrium can only be indifferent in the special case under review if neither of the four conditions (i) to (iv) is fulfilled. Summing up we get:

*Conclusion 7: When (1) the price elasticity of substitution for import demand is infinite, and (2) the individuals in their capacity of income-spenders are subject to a money illusion, the balance of payments equilibrium cannot be indifferent to changes in the exchange rate if one of the special cases for home demand, home supply, or labour presents itself at the same time (cf., however, end of section 4.7).*

The cases where  $C_4 = \infty$ , viz. (i)  $\varepsilon = 0$  and (ii)  $\varepsilon^I = 0$ , will be dealt with in section 4.7.

The formulation of the conditions which render the balance of payments equilibrium *stable and unstable* respectively in the special case for imports, is greatly simplified by the fact that  $C_4 \geq 0$ . From this it follows that  $A_4 \geq 0$  for  $B_4 \geq 0$ .

$$A_4 \geq 0 \text{ for } \eta' \leq v' - \frac{(1 - v^i - v^I \lambda)(1 - v')}{\frac{v^I}{\varepsilon^I(1 - \pi)} + \frac{1}{\varepsilon}}$$

The mathematical formulation of these conditions enables us to draw the following *approximate* conclusion.

*Conclusion 8: When the price elasticity of substitution for import demand is infinite the balance of payments equilibrium will, broadly speaking, be stable with regard to changes in the exchange rate if the individuals in their capacity of income-spenders are subject to a "negative" money illusion ( $v' < \eta'$ ), and unstable if they are subject to a "positive" money illusion ( $v' > \eta'$ ).*

The *exceptions* to this rule are the cases for which:

$$v' \leq \eta' \leq v' - \frac{(1 - v^i - v^l \lambda)(1 - v')}{v^l} + \frac{1}{\epsilon^l(1 - \pi)} + \frac{1}{\epsilon}$$

A necessary, though not sufficient, condition for the inequalities in the top row appears to be  $v' > 1$ . Therefore, the general conclusion with regard to the appearance of an unstable equilibrium will apply in all circumstances if the marginal propensity to spend is smaller than unity. Likewise a necessary, though not sufficient, condition for the inequalities in the bottom row is  $v' < 1$ . Therefore, the general conclusion with regard to the appearance of a stable equilibrium will hold good in all circumstances if the marginal propensity to spend is larger than unity.

Where  $\boxed{\eta^e = \infty}$ , that is to say where the *price elasticity of substitution for export demand is infinite* — the special case for exports (pure competition in the world market for the country's export products) — the home price level will again prove to vary inversely in proportion to the changes in the exchange rate

$$(4.5.6) \quad p = -k^b.$$

This time, however, this result is brought about by virtue of equation (9) of our simplified one-country model (cf. folding page). As a consequence of (4.5.6) the volume of imports will change only as a result of the changes in the volume of production which follow a change in the exchange rate; equation (8) accordingly reads as follows:

$$(4.5.7) \quad u^i = v^i u.$$

Where the money-values of the initial volumes of imports and exports were, by way of simplification, assumed to be equal, the changes in them will be exactly equal to each other by virtue of (4.5.6) and, therefore, will not affect the balance on current account. Consequently

$$(4.5.8) \quad S = u^e - u^i.$$

The expression for  $S$  in terms of  $k^b$  can be derived from the final equation of our one-country model by substituting  $\eta^e = \infty$  therein:

$$(4.5.9) \quad S = \frac{\left\{ \frac{v^i (1-v^i)}{\varepsilon^I (1-\pi)} + \frac{1}{\varepsilon} \right\} (1-\pi)(v'-\eta') - (1-v^i-v^b \lambda)(1-v^i)(1-v')}{\left\{ \frac{v^i (1-v^i)}{\varepsilon^I (1-\pi)} + \frac{1}{\varepsilon} \right\} \pi} \pi \bar{u} k^b =$$

$$= \frac{B_5}{C_5} \pi \bar{u} k^b = A_5 \pi \bar{u} k^b.$$

This formula is very similar to the formula dealt with in discussing the special case for imports. In order to avoid a repetition of the argument given there we will confine ourselves to only a brief discussion of the implications of the special case for exports.

The equilibrium in the balance of payments will be *indifferent* with regard to changes in the exchange rate if

$$\eta' = v' - \frac{(1 - v^i - v^l \lambda) (1 - v')}{\frac{v^l (1 - v^i)}{\varepsilon^l (1 - \pi)} + \frac{1}{\varepsilon}} \cdot \frac{1 - v^i}{1 - \pi} \quad \text{except when,}$$

at the same time,  $\varepsilon = \varepsilon^l = \infty$ . The negative conclusion to be drawn from this condition is:

*Conclusion 9: When (1) the price elasticity of substitution for export demand is infinite, and (2) the individuals in their capacity of income-spenders are subject to a money illusion, the balance of payments equilibrium cannot be indifferent to changes in the exchange rate if one of the special cases for home demand, home supply, or labour presents itself at the same time, or if the marginal propensity to import is equal to unity (cf. however, end of section 4.7).*

It will be remembered that in the case of Chile the marginal propensity to import was calculated to be unity.

The conditions which render the equilibrium *stable and unstable* respectively may mathematically be formulated as follows:

$$A_5 \geq 0 \text{ for } \eta' \leq v' - \frac{(1 - v^i - v^l \lambda) (1 - v')}{\frac{v^l (1 - v^i)}{\varepsilon^l (1 - \pi)} + \frac{1}{\varepsilon}} \cdot \frac{1 - v^i}{1 - \pi}$$

This leads to the following *approximate* conclusion:

*Conclusion 10: When the price elasticity of substitution for export demand is infinite the balance of payments equilibrium will, broadly speaking, be stable with regard to changes in the exchange rate if the individuals in their capacity of income-spenders are subject to a "negative" money illusion, and unstable if they are subject to a "positive" money illusion.*

The combination of the special case for imports with the special case for exports, on which the classical doctrine was based, and which mathematically can be formulated as  $\eta^i = \eta^e = \infty$ , deprives the system of one independent equation, for both the equations (8) and (9) (cf. folding page) now change into

$$(4.5.10) \quad p = -k^b.$$

Hence it may be concluded that:

*Conclusion 11: When the price elasticities of substitution for import and export demand are both infinite, the relationship between changes in the balance on current account and changes in the exchange rate will become indeterminate.*

Mathematically expressed this amounts to:

$$(4.5.11) \quad S = A_6 \pi \bar{u} k^b \text{ where } A_6 = \text{indeterminate}$$

The same result could also have been obtained by substituting  $\eta^i = \infty$  in the final equation of our simplified one-country model, which would render formula (4.5.5) in which  $\eta^e$  no longer appears. This implies that this formula cannot provide an answer to the question as to what would happen if at the same time  $\eta^e = \infty$ .

*Indeterminateness* should not be confused with *indifference*

( $A = 0$ ). In the latter case the slope of the curve representing the relationship between  $S$  and  $k^b$  is known; the curve coincides with the  $k^b$ -axis if the variables are measured in deviations from their original equilibrium values. In the former case, however, the slope of the curve is completely unknown.

Where the indeterminateness is due to the fact that either the change in the volume of exports or the change in the volume of imports proves to be the product of nil and infinity ( $0 \times \infty$ ), the relationship between  $S$  and  $k^b$  will again become determinate if one of these undetermined variables —  $u^e$  and  $u^i$  — is fixed arbitrarily at a certain level by *measures of economic policy*. This procedure will change the variable concerned into a *datum*. Where trade policy does not belong to the subject-matter of this monograph we will abstain from a further discussion of the implications of such a policy and confine ourselves to the statement that such a policy appears to be highly desirable in the given circumstances.

Finally, it should be remarked that it can be proved that here again the change in the balance on current account depends solely on the changes in the volumes of imports and exports:

$$(4.5.12) \quad S = u^e - u^i$$

This result is fully in accordance with the fact that in the special cases for imports and exports all price levels expressed in home currency vary inversely in proportion to the changes in the exchange rate. This means that *all price competition is excluded* from the picture here, as all price ratios remain the same as long as the data of the system remain unaltered. This should be well distinguished from absence of money illusion, which is a different

matter altogether. In the special cases discussed in this section there is still scope for different reactions to changes in money-incomes and proportionate inverse changes in money-prices, for although changes in price ratios are ruled out by the assumptions on which these special cases are based, changes in the absolute levels of money-prices (and money-incomes) can, and will, still occur.

## 4.6

## SPECIAL CASES FOR HOME SUPPLY

The first special case for home supply deals with the situation which arises if *labour is fully employed*. Any pressure from producers to further increase the supply of labour will in such circumstances merely result in a strengthening of the bargaining position of the trade unions (cf. section 3.6). The wage flexibility in a state of full employment may, therefore, become exceedingly high. With no remaining involuntary unemployment this corresponds with a supply elasticity of the production factor labour equal to nil. Mathematically expressed this condition, on which the first special case for home supply is based, therefore reads  $\boxed{\epsilon^l = 0}$ . It reduces the wage rate fixation equation (cf. folding page) to

$$(4.6.1) \quad u = u^i$$

which means that when labour is fully employed the only possible expedient for changing the volume of production consists of an equal change in the volume of imports. It also means that changes in the volume of consumption and in the volume of exports will fully correspond to changes in the volume of imports:

$$(4.6.2) \quad u^i = u' + u^e$$

A further consequence of full employment of labour is that the supply equation for home products — the only remaining equation containing the wage rate as a variable — becomes symptomatic. In other words it does not affect the final relationship between  $S$  and  $k^b$ . It only serves to determine the change in the wage rate consequent upon a change in the exchange rate. Solved for  $l$  this equation seems to suggest that an increase in the volume of production (or what comes to the same thing, where the labour input in the production process is not liable to change, an increase in the volume of imports) would be accompanied by a fall in the money-wage rate. It should not, however, be overlooked that in the cases studied the exchange rate and the home price level also change.

Where  $\epsilon^l = 0$  the final equation of our simplified one-country model relating changes in the balance on current account to changes in the exchange rate (equation (4.3.1)) reduces to:

$$\begin{aligned}
 (4.6.3) \quad S &= \frac{\left(1 - \eta^e - \frac{\eta^i}{1 - v^i}\right) (1 - \pi) (v' - \eta')}{\left\{\left(1 - \eta^e - \frac{\eta^i}{1 - v^i}\right) - (1 - v')\right\} \pi + (1 - \pi)(v' - \eta')} \pi \bar{u} k^b = \\
 &= \frac{B_7}{C_7} \pi \bar{u} k^b = A_7 \pi \bar{u} k^b.
 \end{aligned}$$

Here the equilibrium will be *indifferent* to changes in the exchange rate for  $B_7 = 0$  (and  $C_7 \neq 0$ ), that is to say if:

$$(a) \quad \boxed{1 - \eta^e - \frac{\eta^i}{1 - v^i} = 0} \quad \text{except when, at the same time,}$$

$$\eta' = v' - \frac{\pi}{1 - \pi}$$

Hence, our first conclusion for this special case appears to be:

*Conclusion 12: When labour is fully employed the balance of payments equilibrium will be indifferent to changes in the rate of exchange if the sum of the price elasticity of substitution for import demand, divided by the share of home factors of production in marginal costs, and the price elasticity of substitution of export demand is equal to unity.*

(b)  $\nu' = \eta'$ , the first special case for home demand; the combination of these special cases is dealt with in section 4.7.

The cases where  $C_7 = \infty$  viz. (i)  $\eta^i = \infty$ , and (ii)  $\eta^e = \infty$  are dealt with in section 4.7.

The conditions which render the balance of payments equilibrium *stable and unstable* respectively can, for the special case under review, mathematically be formulated as:

$A_7 > 0$  for  $B_7 \geq 0$  and  $C_7 \geq 0$ , that is to say for:

$$(i) \nu' > \eta' \text{ and } \frac{\nu' - \eta'(1 - \pi)}{\pi} \geq \eta^e + \frac{\eta^i}{1 - \nu^i} \leq 1$$

$$(ii) \nu' < \eta' \text{ ,, ,, } \geq \text{ ,, } \leq \text{ ,, }$$

$A_7 < 0$  for  $B_7 \geq 0$  and  $C_7 \leq 0$ , that is to say for:

$$(i) \nu' > \eta' \text{ and } \frac{\nu' - \eta'(1 - \pi)}{\pi} \leq \eta^e + \frac{\eta^i}{1 - \nu^i} \leq 1$$

$$(ii) \nu' < \eta' \text{ ,, ,, } \leq \text{ ,, } \geq \text{ ,, }$$

These conditions are too complicated to be stated completely, clearly, and concisely in words. Therefore we will formulate them as follows:

*Conclusion 13: When (1) labour is fully employed and (2) the individuals in their capacity of income-spenders, taken on the whole, are subject to a "positive" money illusion, the balance of payments equilibrium will be stable if  $\eta^e + \frac{\eta^i}{1-\nu^i}$  lies between  $\frac{\nu' - \eta' (1 - \pi)}{\pi}$  and unity, and unstable if the former expression is larger or smaller than both the latter expression and unity. Where there is a "negative" money illusion the conditions for stability will be just the reverse.*

The second special case for home supply describes the situation which arises if the country's productive capacity is used to the full. Wherever this happens to be the case the supply elasticity of home-produced goods and services will be nil. Mathematically expressed, this condition reads  $\boxed{\varepsilon = 0}$ . Consequently the only equation in which this structural coefficient appears (compare equation (10) on folding page) is reduced to:

$$(4.6.4) \quad u = 0$$

meaning that the volume of home production is not liable to change. Accordingly, the sum of the volumes of home demand and exports should also remain constant

$$(4.6.5) \quad u' + u^e = 0.$$

Furthermore, the wage rate fixation equation, being the only equation in which the wage rate appears, will become irrelevant in establishing the final relationship of the

system which connects changes in the balance on current account to changes in the exchange rate. This final relationship can easily be derived from equation (4.3.1) by putting  $\varepsilon = 0$ :

(4.6.7)

$$S = \frac{(1 - \eta^i - \eta^e)(1 - \pi)(\nu' - \eta')}{\nu' \pi \left(1 - \eta^i - \frac{\eta^e}{\nu'}\right) + (1 - \pi)(\nu' - \eta')} \pi \bar{u} k^b =$$

$$= \frac{B_8}{C_8} \pi \bar{u} k^b = A_8 \pi \bar{u} k^b.$$

According to this formula the balance of payments equilibrium will be *indifferent* to changes in the exchange rate if

(a)  $\boxed{1 - \eta^i - \eta^e = 0}$ , except when, at the same time,

$$\frac{\nu' - \eta'}{1 - \nu'} = \eta^e \frac{\pi}{1 - \pi}.$$

Therefore, it may be concluded that:

*Conclusion 14: When the country's productive capacity is fully employed the balance of payments equilibrium will be indifferent to changes in the exchange rate if the sum of the price elasticities of substitution for import and export demand is equal to unity.*

This condition is interesting from the point of view that it constitutes a further one-country-model counterpart to the *case of the critical elasticities* of the traditional two-country-model approach (cf. also section 4.4).

(b)  $\nu' = \eta'$ , the first special case of home demand; the combination of these two special cases is dealt with in section 4.7.

The cases where  $C_8 = \infty$  viz. (i)  $\eta^i = \infty$ , and (ii)  $\eta^e = \infty$  are dealt with in section 4.7.

The conditions which render the balance of payments equilibrium *stable and unstable* respectively with regard to changes in the exchange rate are mathematically formulated as follows:

$A_8 > 0$  (unstable equilibrium) for  $B_8 \begin{smallmatrix} > \\ < \end{smallmatrix} 0$  and  $C_8 \begin{smallmatrix} > \\ < \end{smallmatrix} 0$ , that is to say for:

- |       |                  |     |                                                                                                                                                                                             |
|-------|------------------|-----|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| (i)   | $1 > v' > \eta'$ | and | $0 \begin{smallmatrix} > \\ < \end{smallmatrix} 1 - \eta^i - \eta^e > 1 - \eta^i - \frac{\eta^e}{v'} \begin{smallmatrix} > \\ < \end{smallmatrix} - \frac{(1-\pi)(v' - \eta')}{v' \pi} < 0$ |
| (ii)  | $1 < v' > \eta'$ | " " | " < " > " < "                                                                                                                                                                               |
| (iii) | $1 > v' < \eta'$ | " " | " > " < " > "                                                                                                                                                                               |
| (iv)  | $1 < v' < \eta'$ | " " | " < " < " > "                                                                                                                                                                               |

$A_8 < 0$  (stable equilibrium) for  $B_8 \begin{smallmatrix} > \\ < \end{smallmatrix}$  and  $C_8 \begin{smallmatrix} < \\ > \end{smallmatrix} 0$ , that is to say for:

- |       |                  |     |                                                                                                                                                                                             |
|-------|------------------|-----|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| (i)   | $1 > v' > \eta'$ | and | $0 \begin{smallmatrix} > \\ < \end{smallmatrix} 1 - \eta^i - \eta^e > 1 - \eta^i - \frac{\eta^e}{v'} \begin{smallmatrix} < \\ > \end{smallmatrix} - \frac{(1-\pi)(v' - \eta')}{v' \pi} < 0$ |
| (ii)  | $1 < v' > \eta'$ | " " | " < " < " < "                                                                                                                                                                               |
| (iii) | $1 > v' < \eta'$ | " " | " > " < " > "                                                                                                                                                                               |
| (iv)  | $1 < v' < \eta'$ | " " | " < " < " > "                                                                                                                                                                               |

I will not endeavour to formulate this conclusion in words, for I feel the conditions involved are so complicated that only very little, if anything, can be gained by a verbal statement of the matter.

A combination of the two special cases for home supply dealt with so far will present itself when labour is fully employed and the country's productive capacity is used to the full. Mathematically expressed, this condition reads

$\epsilon^l = \epsilon = 0$ . Wherever this condition is fulfilled the

volume of production will not be liable to changes, if the exchange rate changes its value ( $u = 0$  for  $\varepsilon = 0$ ). Likewise the volume of imports should be constant ( $u^i = u$  for  $\varepsilon^i = 0$ ). Furthermore, the demand equation for imports seems to suggest that the changes in the home price level will be inversely proportional to changes in the exchange rate, for where  $u^i = u = 0$  equation (8) reduces to:

$$(4.6.8) \quad \eta^i \pi \bar{u} (p + k^b) = 0.$$

It should not be overlooked, however, that (4.6.8) also holds true for  $\eta^i = 0$ . At first sight this seems a rather exceptional condition, but on further reflection it appears that where both  $\varepsilon^i$  and  $\varepsilon$  are nil  $\eta^i$  should also be nil. The explanation for this unusual state of affairs, where the numerical values of two structural coefficients determine the value of a third one, is as follows: When the country's productive capacity is used to the full the wage flexibility will only become infinite ( $1/\varepsilon^i = \infty$ ) when every possibility to substitute foreign factors of production (imports) for home factors of production (labour) has been exhausted. If they were not exhausted it would undoubtedly pay to carry on with the process of substituting imports (available at a fixed price) for home labour, which has obtained such a strong bargaining power that it will be able to claim a wage which would deprive entrepreneurs of the monopoly gains otherwise accruing to them in a situation of short supply.

Where this is the case (4.6.8) becomes:  $0 = 0$ . This means a reduction of the system with one equation. Consequently the relationship between  $S$  and  $k^b$  cannot be established.

$$(4.6.9) \quad S = A_9 \pi \bar{u} k^b \quad \text{where} \quad \boxed{A_9 = \text{indeterminate}}$$

It may, therefore, be concluded that:

*Conclusion 15: When (1) labour is fully employed and (2) the country's productive capacity is used to the full, the balance of payments equilibrium with regard to changes in the exchange rate will be indeterminate.*

The last special case for home supply deals with the situation which arises if *both the labour force and the country's productive capacity are seriously underemployed*. It seems permissible to combine the two as they usually go hand in hand. Wherever this situation presents itself the wage flexibility will be practically nil (cf. section 3.6) and the home price level will remain unaltered if the volume of production changes. Mathematically expressed, this condition reads  $\boxed{\epsilon^l = \epsilon = \infty}$ .

It is interesting to note that under these circumstances the wage rate fixation equation reduces to:

$$(4.6.10) \quad l = \lambda p$$

and the price fixation equation for home products to:

$$(4.6.11) \quad p = v^l l - v^i k^b.$$

From the combination of equations (4.6.10) and (4.6.11) it follows that:

$$(4.6.12) \quad p = - \frac{v^i}{1 - v^l \lambda} k^b$$

which implies that if  $1 - v^l \lambda = v^i$  (the special case for labour) the home price level will change inversely in proportion to changes in the exchange rate.

For  $\epsilon^l = \epsilon = \infty$  the final equation of the simplified

one-country model, relating changes in the balance of payments on current account to changes in the exchange rate, reduces to:

(4.6.13)

$$S = \frac{v^i \frac{v^i}{\pi} (1 - \pi) (v' - \eta') + (1 - v^i - v^l \lambda) \{1 - \eta^i - \eta^e (1 - v^i)\} (1 - v')}{(1 - v^l \lambda) \{ (1 - v') (1 - v^i) + v^i \}} \pi \bar{u} k^b =$$

$$= \frac{B_{10}}{C_{10}} \pi \bar{u} k^b = A_{10} \pi \bar{u} k^b.$$

According to this formula the balance of payments equilibrium will be *indifferent* to changes in the exchange rate if:

$$\boxed{\frac{\eta' - v'}{1 - v'} = \frac{(1 - v^i - v^l \lambda) \{1 - \eta^i - \eta^e (1 - v^i)\}}{v^i \frac{v^i}{\pi} (1 - \pi)}}, \text{ except}$$

when, at the same time,  $v' = \frac{1}{1 - v^i}$ .

This condition is again too complicated to be stated in words. Nevertheless it enables us to conclude that it will be satisfied if the individuals in their capacity of income-spenders are not subject to a money illusion ( $v' = \eta'$ ) and one of the following conditions is fulfilled:

- (i)  $v' = 1$
  - (ii)  $1 - v^i = v^l \lambda$
  - (iii)  $1 - \eta^i - \eta^e (1 - v^i) = 0$ .
- } \text{ except when, at the same time, } \eta^i
- } \text{ and/or } \eta^e = \infty

The negative conclusion to be drawn here is the following:

*Conclusion 16: When (1) the labour force and the country's productive capacity are seriously under-employed, and (2) the individuals in their capacity of income-spenders, taken on the whole, are subject to a money illusion, the balance of payments equilibrium cannot be indifferent to changes in the exchange rate if either the hoarding phenomenon is absent, or the special case for labour presents itself, or the sum of the price elasticity of substitution for export demand, multiplied by the share of home factors of production in marginal costs, and the price elasticity of substitution for import demand is equal to unity (cf., however, end of section 4.7).*

The conditions which render the equilibrium *stable and unstable* respectively in this special case may be mathematically formulated as follows:

$A_{10} > 0$  (unstable equilibrium) for  $B_{10} \geq 0$  and  $C_{10} \geq 0$ , that is to say for:

$$\eta' - v' \geq \left\{ 1 - \eta^i - \eta^e (1 - v^i) \right\} \frac{1 - v^i - v^l \lambda}{v^i \frac{v^i}{\pi} (1 - \pi)} (1 - v') \text{ and } v' \geq \frac{1}{1 - v^i}$$

$A_{10} < 0$  (stable equilibrium) for  $B_{10} \geq 0$  and  $C_{10} \leq 0$ , that is to say for:

$$\eta' - v' \geq \left\{ 1 - \eta^i - \eta^e (1 - v^i) \right\} \frac{1 - v^i - v^l \lambda}{v^i \frac{v^i}{\pi} (1 - v^i)} (1 - v') \text{ and } v' \leq \frac{1}{1 - v^i}$$

When the special case for labour presents itself this conclusion is reduced to more simple proportions, viz.

$$A_{10} > 0 \text{ for } \eta' \geq \nu' \geq \frac{1}{1-\nu^i} \text{ and } A_{10} < 0 \text{ for } \eta' \geq \nu' \leq \frac{1}{1-\nu^i}$$

In words this conclusion reads:

*Conclusion 17: When (1) the labour force and the country's productive capacity are seriously under-employed, and (2) the special case for labour presents itself, the balance of payments equilibrium will be unstable with regard to changes in the exchange rate if the numerical value of the marginal propensity to spend lies between the numerical value of the price elasticity of home demand and the reciprocal of the share of the home factors of production in marginal costs. The equilibrium will be stable where this latter condition is not fulfilled.*

## 4.7

## COMBINATIONS OF SPECIAL CASES

So far the combination of special cases belonging to different categories has not been studied in any detail. To complete our analysis of the exchange stability problem these *extra-special cases* will be dealt with in this section.

Systematically reviewing the whole field of possible combinations we have to commence with the first special case for home demand, viz. *absence of money illusion* in the minds of people in their capacity of income-spenders, and to combine it with the special cases for imports and exports, and the special cases for home supply.

Combination with the *special cases for imports and exports* reduces the final equation of the system to:

(4.7.1)

$$S = - \frac{(1-v^i-v^l\lambda)(1-v')}{\left\{ \frac{v^l}{\varepsilon^l(1-\pi)} + \frac{v'}{\varepsilon} \right\} \pi} \pi \bar{u} k^b \quad \text{(compare formulae (4.4.2) and (4.5.5))}$$

and

(4.7.2)

$$S = - \frac{(1-v^i-v^l\lambda)(1-v^i)(1-v')}{\left\{ \frac{v^l}{\varepsilon^l(1-\pi)} + \frac{1}{\varepsilon} \right\} \pi} \pi \bar{u} k^b \quad \text{(compare formulae (4.4.2) and (4.5.9))}$$

respectively. These formulae render the following conclusions:

*Conclusion 18:* When (i) the individuals in their capacity of income-spenders, taken on the whole, are not subject to any money illusion, and (2) the price elasticity of substitution for either import or export demand is infinite, the balance of payments equilibrium will be indifferent to changes in the exchange rate if either (i) the special case for labour presents itself, or (ii) the hoarding phenomenon is absent, or (iii) the labour force is fully employed, or (iv) the country's productive capacity is used to the full.<sup>1)</sup>

With regard to the special case for labour, it should be noted that where the marginal import quota is equal to unity (as in the case of Chile) the condition for this special case is automatically fulfilled, no matter what the numerical value of  $\lambda$  is, that is to say, no matter whether individuals in the capacity of wage-negotiators are subject to a money illusion or not. Furthermore, it should be

<sup>1)</sup> Conclusions, like this one, which do not add to our knowledge, because they are already covered by other conclusions, will not be printed in italics.

pointed out that the only possible *exception* to conclusion 18 is constituted by the case of serious underemployment of the labour force and the country's productive capacity ( $\epsilon^l = \epsilon = \infty$ ), rendering the nature of the balance of payments equilibrium *indeterminate*.

Conditions (i) and (ii) in conclusion 18 confirm conclusions 1 and 6 respectively, showing that the price elasticity of substitution for either import or export demand need not be infinite to render the equilibrium indifferent where these conditions are already fulfilled: (conditions (iii) and (iv) are dealt with further on).

*Conclusion 19: When (1) the individuals in their capacity of income-spenders, taken on the whole, are not subject to any money illusion, and (2) the price elasticity of substitution for either import or export demand is infinite, the balance of payments equilibrium will be stable with regard to changes in the exchange rate if the marginal propensity to spend is smaller than unity, and unstable if the marginal propensity to spend is larger than unity.*

(The other terms of formulae (4.7.1) and (4.7.2) are positive in any case). This conclusion proves to be in accordance with the general rule formulated in conclusion 3.

The combination of the first special case of home demand with the *first two special cases of home supply*, viz. full employment of labour ( $\epsilon^l = 0$ ) and full use of the country's productive capacity ( $\epsilon = 0$ ), reduces the final equation of the system to:

$$(4.7.3) \quad S = 0 \quad (\text{compare formulae (4.4.2), and (4.6.3) and (4.6.7) respectively}).$$

Hence it may be concluded that:

*Conclusion 20: When (1) the individuals in their capacity of income-spenders, taken on the whole, are not subject to any money illusion, and (2) either labour is fully employed, or the country's productive capacity is fully used, the balance of payments equilibrium will be indifferent to changes in the exchange rate.*

Comparing this conclusion with conclusion 18 it appears that where conditions (iii) or (iv) are satisfied the equilibrium will be indifferent no matter what are the numerical values of the price elasticities of substitution for import and export demand. Conclusion 18, therefore, proves to be completely covered by conclusions 1, 6 and 20 (cf. discussion of conditions (i) and (ii) above).

Finally the combination of the first special case of home demand with *the last special case of home supply*, viz. serious underemployment of both the labour force and the country's productive capacity ( $\epsilon^l = \epsilon = \infty$ ), reduces the final equation of the system to:

(4.7.4)

$$S = \frac{(1 - v^i - v^l \lambda) \{ 1 - \eta^i - \eta^e (1 - v^i) \} (1 - v')}{(1 - v^l \lambda) \{ (1 - v') (1 - v^i) + v^i \}} \pi \bar{u} k^b$$

(compare formulae (4.4.2) and (4.6.13)).

From this formula it follows that:

*Conclusion 21: When (1) the individuals in their capacity of income-spenders, taken on the whole, are not subject to any money illusion, and (2) the labour force and the country's productive capacity are both seriously*

underemployed, the balance of payments equilibrium will be indifferent to changes in the rate of exchange if either (i) the special case for labour presents itself, or (ii) the sum of the price elasticity of substitution for export demand, multiplied by the share of the home factors of production in marginal costs, and the price elasticity of substitution for import demand is equal to unity, or (iii) the hoarding phenomenon is absent.

The only possible *exception* to this rule occurs where the marginal import quota is nil, for then the equilibrium will not necessarily be indifferent where the special case for labour presents itself.

Conditions (i), (ii) and (iii) confirm conclusions 1, 2 and 6, which shows that serious underemployment of the labour force and the country's productive capacity is not a necessary condition here to render the equilibrium indifferent. Conclusion 21, therefore, proves to be fully covered by other conclusions.

For the extra-special case under review the conditions which render the balance of payments equilibrium stable and unstable respectively with regard to changes in the exchange rate appear to be the following:

$$A > 0 \text{ for } \eta^i + \eta^e (1 - v^i) < 1 \text{ and } 1 \geq v' \leq \frac{1}{1 - v^i}$$

$$\eta^i + \eta^e (1 - v^i) > 1 \text{ and } 1 \geq v' \geq \frac{1}{1 - v^i}$$

$A < 0$  in the opposite case

*Conclusion 22: When (1) the individuals in their capacity of income-spenders, taken on the whole, are not subject to any money illusion, and (2) the labour force and the country's productive capacity are seriously underemployed, the balance of payments equilibrium will be stable with regard to changes in the exchange rate if (i) the sum of the price elasticity of substitution for export demand, multiplied by the share of home factors of production in marginal costs, and the price elasticity of substitution for import demand is smaller than unity, and, at the same time, (ii) the marginal propensity to spend is smaller or larger than both unity and the reciprocal of the share of home factors of production in marginal costs, and unstable if the marginal propensity to spend lies between those limits. When the sum of the price elasticity of substitution for export demand, multiplied by the share of home factors of production in marginal costs, and the price elasticity of substitution of import demand is larger than unity, the stability conditions will be just the reverse.*

This conclusion can be proved to be in accordance with the stability conditions for the first special case for home demand and the last special case for home supply.

Having dealt with all the possible combinations of the first special case for home demand with other special cases, we may now proceed to discuss the combinations of the second special case for home demand, viz. *absence of the hoarding phenomenon*, with the special cases for imports and exports, and the special cases for home supply.

The combination with the *special cases for imports and exports* reduces the final equation of the system to:

(4.7.5)

$$S = (1 - \eta') (1 - \pi) \bar{u} k^b \quad (\text{compare formulae (4.4.4), and (4.5.5) and (4.5.9) respectively}).$$

From this formula it may be concluded that:

*Conclusion 23: When (1) the hoarding phenomenon is absent, and (2) the price elasticity of substitution for either import or export demand is infinite, the balance of payments equilibrium will be stable, unstable or indifferent to changes in the exchange rate if the price elasticity of home demand is larger, smaller or equal to unity respectively.*

The conditions which render the equilibrium stable and unstable respectively are in accordance with the general rule of conclusion 5. The condition which renders the equilibrium indifferent is the same as the condition formulated in conclusion 6, which shows that the numerical values of the price elasticities of substitution for import and export demand do not matter in this connection.

Combination of the second special case for home demand with the *first special case for home supply*, viz. full employment of labour, reduces the final equation of the system to:

(4.7.6)

$$S = \frac{\left\{ 1 - \eta^e - \frac{\eta^i}{1 - v^i} \right\} (1 - \pi) (1 - \eta')}{\left\{ 1 - \eta^e - \frac{\eta^i}{1 - v^i} \right\} \pi + (1 - \pi) (1 - \eta')} \pi \bar{u} k^b$$

(compare formulae (4.4.4) and (4.6.3))

From this formula it follows that:

*Conclusion 24:* When (1) the phenomenon of hoarding is absent, and (2) labour is fully employed, the balance of payments equilibrium will be indifferent to changes in the exchange rate if either (i) the sum of the price elasticity of substitution for import demand, divided by the share of the home factors of production in marginal costs, and the price elasticity of substitution for export demand is equal to unity, or (ii) the price elasticity of home demand is equal to unity.

In this conclusion condition (i) is already covered by conclusion 12, and condition (ii) by conclusion 6.

The conditions which render the equilibrium stable and unstable respectively are of a rather complicated nature here. They can easily be obtained, however, by substituting  $v' = 1$  in conclusion 13.

Combination of the second special case for home demand with the *second special case for home supply*, viz. full use of the country's productive capacity, reduces the final equation of the system to:

(4.7.7)

$$S = \frac{(1 - \eta^i - \eta^e) (1 - \pi) (1 - \eta')}{(1 - \eta^i - \eta^e) \pi + (1 - \pi) (1 - \eta')} \pi \bar{u} k^b$$

(compare formulae (4.4.4) and (4.6.7) )

From this formula it can be seen that:

*Conclusion 25:* When (1) the hoarding phenomenon is absent, and (2) the country's productive capacity is used to the full, the balance of

payments equilibrium will be indifferent to changes in the exchange rate if either (i) the sum of the price elasticities of substitution for import and export demand, or (ii) the price elasticity of home demand is equal to unity.

Condition (i) of this conclusion is already covered by conclusion 14, and condition (ii) by conclusion 6. This conclusion, therefore, contains nothing new.

Here again the conditions which render the equilibrium stable and unstable respectively are rather complicated. However, they can easily be obtained by substituting  $v' = 1$  in the mathematically expressed stability conditions for the second special case for home demand.

Combination with the last special case for home supply, viz. serious underemployment of both the labour force and the country's productive capacity, reduces the final equation of the system to:

(4.7.8)

$$S = \frac{v^i (1 - \eta')}{1 - v^i \lambda} (1 - \pi) \bar{u} k^b \quad \begin{array}{l} \text{(compare formulae (4.4.4)} \\ \text{and (4.6.13))} \end{array}$$

This formula leads us to the following conclusion:

*Conclusion 26: When (1) the phenomenon of hoarding is absent, and (2) both the labour force and the country's productive capacity are seriously underemployed, the balance of payments equilibrium will be stable with regard to changes in the exchange rate if the price elasticity of home demand is larger than unity, unstable if it is smaller than unity, and indifferent if either*

(i) *this price elasticity of home demand is equal to unity, or (ii) the marginal import quota is nil.*

The conditions which render the equilibrium stable and unstable respectively are in accordance with the general rule of conclusion 5, whereas the first alternative condition for an indifferent equilibrium is already covered by conclusion 6.

Finally, the extra-special cases emerging from a combination of the special cases for home supply and the *special cases for imports and exports* must be dealt with to complete our analysis of the exchange stability problem. For the sake of convenience, they will be discussed in the order of the special cases for home supply.

Combination of the *first special case of home supply*, viz. full employment of labour, with the special cases for imports and exports, reduces the final equation of the system to:

(4.7.9)

$$S = (v' - \eta')(1 - \pi) \bar{u} k^b \text{ (compare formulae (4.5.5) and (4.5.9) respectively, and formula (4.6.3))}$$

Combination of the *second special case for home supply*, viz. full use of the country's productive capacity, with the special cases for imports and exports reduces the final equation of the system to:

(4.7.10)

$$S = \frac{v' - \eta'}{v'} (1 - \pi) \bar{u} k^b \text{ (compare formulae (4.5.5) and (4.6.7))}$$

and

(4.7.11)

$$S = (v' - \eta') (1 - \pi) \bar{u} k^b \quad \begin{array}{l} \text{(compare formulae (4.5.9)} \\ \text{and (4.6.7))} \end{array}$$

respectively. From these formulae the conclusion can be drawn that:

*Conclusion 27: When (1) either labour is fully employed, or the country's productive capacity is used to the full, and (2) the price elasticity of substitution for either import or export demand is infinite, the balance of payments equilibrium will be stable, unstable or indifferent to changes in the exchange rate if the individuals in their capacity of income-spenders, taken on the whole, are subject to a negative ( $v' < \eta'$ ), a positive ( $v' > \eta'$ ), or to no money illusion respectively.*

The conditions which render the equilibrium stable and unstable respectively are in accordance with the general rule of conclusions 8 and 10.

To conclude, the combination of the *last special case for home supply*, viz. serious underemployment of both the labour force and the country's productive capacity, reduces the final equation of the system to:

(4.7.12)

$$S = \frac{(1 - v^i - v^l \lambda) (1 - v')}{(1 - v^l \lambda) \{ (1 - v') (1 - v^i) + v^i \}} \pi \bar{u} k^b$$

(compare formula (4.6.13)).

From this formula the following conclusions can be drawn:

*Conclusion 28: When (1) both the labour force and the country's productive capacity are seriously underemployed, and (2) the price elasticity of substitution for either the import or the export demand is infinite, the balance of payments equilibrium will be indifferent to changes in the exchange rate if either (i) the special case for labour presents itself, or (ii) the phenomenon of hoarding is absent.*

This conclusion constitutes an *exception* to the statements made in conclusions 7, 9 and 16.

*Conclusion 29: When (1) both the labour force and the country's productive capacity are seriously underemployed, and (2) the price elasticity of substitution for either import or export demand is infinite, the balance of payments equilibrium will be stable with regard to changes in the exchange rate if the marginal propensity to spend is either larger or smaller than both unity and the reciprocal of the share of home factors of production in marginal costs, and unstable where the marginal propensity to spend lies between these two limits.*

#### 4.8

#### NATURE OF THE EQUILIBRIUM FOR INDIVIDUAL COUNTRIES

The foregoing analysis does not provide any definite answers with regard to the exchange stability problem for individual countries. In principle such answers can be obtained only if all the structural coefficients entering into the final relation between the balance on current account

and the exchange rate are known. In this section, however, an endeavour will be made to give approximate answers by (1) assuming that in reality both the marginal propensity to spend ( $v'$ ) and the price elasticity of home demand ( $\eta'$ ) lie between the limits 0.8 and 1.2, and (2) by applying our calculations to the three special cases for home supply which, taken together, may be considered to describe, approximately at least, almost the whole of reality.

Starting from formula (4.3.2), which reads

.1)

$$\frac{\left(\frac{D}{\epsilon'} + \frac{E}{\epsilon} - F\right) (v' - \eta') - G (1 - \lambda) (1 - v')}{\left(\frac{H}{\epsilon'} + \frac{1}{\epsilon}\right) \{Kv' - L + M(v' - \eta')\} - \{N(1 - \lambda) + \frac{Q}{\epsilon'} + P\} (1 - Nv')} \pi \bar{u} k^b = A \pi \bar{u} k^b$$

we find that (i) for the first special case for home supply (full employment of labour)

(4.8.2)

$$A = \frac{D (v' - \eta')}{H \{Kv' - L + M (v' - \eta')\} - Q (1 - Nv')}$$

(ii) for the second special case for home supply (full use of the country's productive capacity)

(4.8.3)

$$A = \frac{E (v' - \eta')}{Kv' - L + M (v' - \eta')},$$

and (iii) for the third special case for home supply (serious underemployment of both the labour force and the country's productive capacity)

(4.8.4)

$$A = \frac{F (v' - \eta') + G (1 - \lambda) (1 - v')}{\{N (1 - \lambda) + P\} (1 - Nv')}.$$

In the last case an assumption should also be made with regard to the numerical value of the wage coefficient ( $\lambda$ ). As it is thought that this coefficient will normally be somewhere between 0.8 and 1.0 separate calculations have been made for  $\lambda = 1.0$  and  $\lambda = 0.8$ . For  $\lambda = 1$  formula (4.8.4) reduces to:

$$(4.8.5) \quad A = \frac{F (\nu' - \eta')}{P (1 - N\nu')}.$$

It should be remembered when interpreting the results of the calculations presented in table 5 that (i) coefficients D to Q (cf. table 4) refer to *short-run* reactions and that (ii) formula (4.8.1) has been based on the assumption that only changes in the wage rate and in the import price level affect marginal costs ( $\nu^I = 1 - \nu^I$ ).

From table 5 (cf. page 227) the following *practical conclusions* can be drawn, if Mr CHANG's figures are taken for granted:

1. When there is *full employment* the nature of the balance of payments equilibrium is, as a rule, the same in case I ( $\nu' = 1.2$  and  $\eta' = 0.8$ ) and case II ( $\nu' = 0.8$  and  $\eta' = 1.2$ ). From this it seems to follow that the numerical values of the *marginal propensity to spend and the price elasticity of home demand do not essentially affect the nature of the balance of payments equilibrium*. The only exceptions here seem to be Chile, Estonia and Latvia, but only in so far as the first special case for home supply (full employment of labour) is concerned.
2. When there is full employment the *absolute numerical value of A* (i.e. the numerical value regardless of its sign) proves, as a rule, to be smaller than unity, which means that the degree of stability cannot be very high.

Notable exceptions present themselves in case I (columns (2) and (4)) for Canada and Norway (stable equilibria), and Estonia and Latvia (unstable equilibria when labour is fully employed, and stable equilibria when the country's productive capacity is used to the full).

3. When there is full employment the nature of the balance of payments equilibrium varies a good deal from one country to another. Broadly speaking the equilibrium appears to be:

- (a) *nearly indifferent* for Germany, France, Italy and New Zealand, and also, should their productive capacities be used to the full, for Japan, South Africa and Denmark;

- (b) *fairly stable* for the United States, Canada, Finland, Norway, Hungary and Australia;

- (c) *unstable* for the U.K., Sweden and Switzerland.

The equilibria for Estonia and Latvia are fairly stable, except for the combination of case I and full employment of the labour force, whereas the equilibrium for Chile is unstable, except for the combination of case II and full employment of the labour force.

4. When there is *serious underemployment* of both the labour force and the country's productive capacity *the nature of the balance of payments equilibrium proves to be almost completely determined by the numerical values of the marginal propensity to spend and the price elasticity of home demand*, and, though to a considerably lesser extent, by the numerical value of the wage coefficient. The numerical values of  $\sigma A$  in columns (6) and (8), both referring to case I ( $\nu' = 1.2$  and  $\eta' = 0.8$ ) are all positive, except for the United States; those in columns (7) and (9), referring to case II ( $\nu' = 0.8$  and  $\eta' = 1.2$ ) are all negative. Therefore, *case I* seems almost invariably to

TABLE 5

*Individual countries — The numerical value of the exchange rate elasticity of the balance on current account (A) for the special cases for home supply with (I)  $\nu' = 1.2$  and  $\eta' = 0.8$ , and (II)  $\nu' = 0.8$  and  $\eta' = 1.2$*

| Countries         | full employment of             |      |                                               |      | underemployment ( $\epsilon^l = \epsilon = \infty$ ) |      |                 |      |
|-------------------|--------------------------------|------|-----------------------------------------------|------|------------------------------------------------------|------|-----------------|------|
|                   | labour<br>( $\epsilon^l = 0$ ) |      | country's<br>prod. cap.<br>( $\epsilon = 0$ ) |      | $\lambda = 1$                                        |      | $\lambda = 0.8$ |      |
|                   | I                              | II   | I                                             | II   | I                                                    | II   | I               | II   |
| (1)               | (2)                            | (3)  | (4)                                           | (5)  | (6)                                                  | (7)  | (8)             | (9)  |
| 1. U.S.A. . .     | -0.5                           | -0.4 | -0.4                                          | -0.4 | -35                                                  | -19  | -1.3            | -0.7 |
| 2. Germany. .     | -0.1                           | -0.1 | 0.0                                           | 0.0  | 7                                                    | -1.6 | 4               | -1.0 |
| 3. U.K. . . .     | 0.2                            | 0.2  | 0.3                                           | 0.4  | 3                                                    | -1.2 | 2.2             | -0.8 |
| 4. Sweden. . .    | 0.1                            | 0.1  | 0.2                                           | 0.3  | 3                                                    | -1.2 | 2.0             | -0.7 |
| 5. France . . .   | -0.2                           | -0.1 | -0.1                                          | -0.1 | 22                                                   | -1.4 | 12              | -0.7 |
| 6. Switzerland .  | 0.1                            | 0.1  | 0.2                                           | 0.3  | 1.9                                                  | -0.9 | 1.3             | -0.6 |
| 7. Japan. . . .   | -0.3                           | -0.3 | -0.1                                          | -0.1 | 3                                                    | -1.4 | 2.2             | -1.0 |
| 8. Italy . . . .  | -0.1                           | -0.1 | -0.1                                          | -0.1 | 9                                                    | -0.8 | 5               | -0.4 |
| 9. Chile. . . .   | 0.8                            | -0.8 | 0.3                                           | 1.0  | 0.8                                                  | -0.8 | 0.8             | -0.8 |
| 10. Canada. . .   | -13                            | -0.6 | -1.6                                          | -0.5 | 2.2                                                  | -1.0 | 1.7             | -0.8 |
| 11. South Africa  | -0.5                           | -0.3 | 0.0                                           | 0.1  | 1.8                                                  | -1.0 | 1.4             | -0.8 |
| 12. Estonia. . .  | 3                              | -0.8 | -1.2                                          | -0.4 | 1.1                                                  | -1.0 | 1.0             | -0.9 |
| 13. Latvia . . .  | 4                              | -1.0 | -6.1                                          | -0.7 | 1.7                                                  | -1.5 | 1.6             | -1.5 |
| 14. New Zealand   | -0.1                           | -0.1 | 0.1                                           | 0.1  | 2.1                                                  | -1.2 | 1.6             | -0.9 |
| 15. Finland. . .  | -0.8                           | -0.3 | -0.7                                          | -0.3 | 2.4                                                  | -1.2 | 1.3             | -1.1 |
| 16. Norway . .    | -12                            | -0.5 | -0.9                                          | -0.3 | 1.4                                                  | -0.8 | 1.1             | -0.6 |
| 17. Hungary . .   | -1.6                           | -0.8 | -0.8                                          | -0.5 | 2.4                                                  | -1.5 | 2.0             | -1.2 |
| 18. Denmark. .    | -0.7                           | -0.3 | -0.1                                          | -0.1 | 1.9                                                  | -1.0 | 1.5             | -0.8 |
| 19. Australia . . | -1.0                           | -0.5 | -0.4                                          | -0.3 | 3.3                                                  | -1.5 | 2.4             | -1.1 |

*Explanatory note:* Negative *A* means stability.  
Positive *A* means instability.

render the equilibrium *unstable*, whereas *case II* invariably renders it *stable*.

- When there is serious underemployment the absolute numerical value of *A* appears to be practically always higher than unity, which means that the equilibrium is either stable or unstable but *hardly ever nearly indifferent*. Notable high positive numerical values of *A* are found

for France, Italy and Germany, the currencies of which have all shown definite features of inherent weakness.

The only country which seems to enjoy a fair degree of stability under all circumstances appears to be the United States.

The fourth practical conclusion shows clearly to what extent the celebrated *case of the critical elasticities*, against which serious objections were already raised in section 4.3, suffers from lack of practical value. Running in terms of the price elasticities of import and export demand and implying  $\epsilon = \epsilon^l = \infty$  it obviously ignores the reactions which really matter, namely those for which  $\nu'$  and  $\eta'$  stand in our formulae. Likewise, it may be said that the more general *traditional two-country-model approach* would have gained much in realistic value if it had not been based on the implicit assumption that  $\epsilon^l = \infty$ , which rules out all those cases where labour is fully employed, cases for which the price elasticities (of substitution) of import and export demand really matter.

So far our practical conclusions have been based mainly on Mr CHANG's statistical findings with regard to the numerical values of the price elasticities of substitution for import and export demand. Other authors, however, arrived at somewhat higher numerical values for these structural coefficients. Furthermore, practically all statistically measured price elasticities of substitution refer to *short-run* reactions only. So it seems well worth while to make also a few calculations with higher substitution elasticities. For a number of hypothetical cases, which may be interpreted as referring to the reactions in the *medium long run*, calculations were therefore made. The characteristics of these hypothetical cases and the resulting numerical values of  $A$  have been entered in table 6.

TABLE 6

*Hypothetical cases — The numerical values of the exchange rate elasticity of the balance on current account ( $A$ ) for the special cases for home supply with (I)  $\nu' = 1.2$  and  $\eta' = 0.8$ , and (II)  $\nu' = 0.8$  and  $\eta' = 1.2$*

| Characteristics of hypothetical cases:<br>$\nu_i = 0.50, \pi = 0.25$ |          |          | full employment of             |      |                                               |      | underemployment ( $\epsilon^l = \epsilon = \infty$ ) |      |                 |      |
|----------------------------------------------------------------------|----------|----------|--------------------------------|------|-----------------------------------------------|------|------------------------------------------------------|------|-----------------|------|
|                                                                      |          |          | labour<br>( $\epsilon^l = 0$ ) |      | country's<br>prod. cap.<br>( $\epsilon = 0$ ) |      | $\lambda = 1$                                        |      | $\lambda = 0.8$ |      |
| Nr.                                                                  | $\eta_i$ | $\eta_e$ | I                              | II   | I                                             | II   | I                                                    | II   | I               | II   |
| (I)                                                                  | (2)      | (3)      | (4)                            | (5)  | (6)                                           | (7)  | (8)                                                  | (9)  | (10)            | (11) |
| 1.                                                                   | 1        | 1        | 4                              | —0.7 | —6                                            | —0.5 | 1.5                                                  | —1.0 | 1.3             | —0.9 |
| 2.                                                                   | 1        | 3        | 1.8                            | —0.9 | 2                                             | —0.9 | 1.5                                                  | —1.0 | 1.4             | —0.9 |
| 3.                                                                   | 1        | 5        | 1.6                            | —1.0 | 1.6                                           | —1.0 | 1.5                                                  | —1.0 | 1.5             | —1.0 |
| 4.                                                                   | 3        | 1        | 1.6                            | —1.0 | 1.6                                           | —0.9 | 1.5                                                  | —1.0 | 1.5             | —1.0 |
| 5.                                                                   | 3        | 3        | 1.5                            | —1.0 | 1.4                                           | —1.0 | 1.5                                                  | —1.0 | 1.5             | —1.0 |
| 6.                                                                   | 3        | 5        | 1.4                            | —1.1 | 1.4                                           | —1.1 | 1.5                                                  | —1.0 | 1.6             | —1.1 |
| 7.                                                                   | 5        | 1        | 1.4                            | —1.1 | 1.3                                           | —1.1 | 1.5                                                  | —1.0 | 1.6             | —1.1 |
| 8.                                                                   | 5        | 3        | 1.4                            | —1.1 | 1.3                                           | —1.1 | 1.5                                                  | —1.0 | 1.7             | —1.1 |
| 9.                                                                   | 5        | 5        | 1.3                            | —1.1 | 1.3                                           | —1.1 | 1.5                                                  | —1.0 | 1.8             | —1.2 |

*Explanatory note:* Negative  $A$  means stability.  
Positive  $A$  means instability.

From this table it appears that for higher numerical values of the substitution elasticities the exchange rate elasticity of the balance on current account, and therewith the nature of the balance of payments equilibrium, depends practically entirely on the numerical values of the marginal propensity to spend and the price elasticity of home demand, even in the case of full employment.

That in columns (8) and (9) respectively the numerical values of  $A$  are the same for all hypothetical cases is due to the fact that in the formula from which these coefficients are calculated the price elasticities of substitution for import and export demand do not appear. This formula reads as follows:

$$(4.8.6) \quad A = \frac{\frac{v^i}{\pi} (1 - \pi) (v' - \eta')}{(1 - v') (1 - v^i) + v^i}.$$

## 4.9

## SUMMARY

In this chapter we dealt with the one-country-model version of the exchange stability theory and confronted it with the traditional two-country-model approach. It was found that in order to obtain a more or less workable theory the fairly simple model designed in chapter III had to be further simplified by assuming (i) that in the original equilibrium situation the money-values of imports and exports of goods and services balance, and (ii) that the export and home price level experience equal percentage changes, consequent upon a change in the exchange rate (cf. section 4.1). But even after introducing these simplifications the relation between changes in the balance on current account and the exchange rate proved to be still highly complicated (cf. section 4.3). In order to deal with this complicated relationship in a satisfactory way, a number of alternative simplifying assumptions with regard to (i) the elasticities of home demand, (ii) the price elasticities of substitution for import and export demand, and (iii) the elasticities of home supply had to be introduced (sections 4.4 to 4.6). Furthermore, the consequences of combinations of these further simplifying assumptions were explored (mainly section 4.7). Thanks to this procedure a number of conclusions with regard to the nature of the balance of payments equilibrium could be formulated (conclusions 1 to 29). It should, however, be borne in mind that these conclusions cover only part

of the whole range of possible configurations of numerical values of structural coefficients appearing in the final relationship between changes in the balance on current account and changes in the exchange rate. They all refer to what were called "special" and "extra-special" cases. Furthermore, it should not be overlooked (i) that these conclusions are essentially based on the assumptions set out in chapter III and the simplifying assumptions mentioned above, and (ii) that they are subject to the qualifications discussed in section 3.7. These qualifications emerge largely from the peculiarities of the analytical method applied in this monograph. The implications of this method have been discussed in chapter II.

The complicated character of the results obtained from the one-country-model approach is rather striking if one compares it with the relatively simple conclusions reached by the theorists of the traditional two-country-model school. At first sight one would have expected a one-country model to yield simpler results. As shown in section 4.3 this seeming contradiction can be traced back to a number of rather unrealistic simplifying assumptions implicit in the analysis of the exchange stability problem as provided by the authors of the traditional two-country-model school (MARSHALL, BICKERDIKE, JOAN ROBINSON, BROWN, METZLER). They appear to ignore (1) the phenomenon of imperfect competition (price substitution effect) in international trade, (2) the indirect influence of a change in the exchange rate on the volume of imports through changes in the volume of production (and exports), the importance of which has been stressed particularly in chapter I, (3) the presence of imports in the marginal product (only BROWN's analysis provides an exception here), (4) the sensitiveness of wages with regard to changes in the home price level,

and (5) the sensitiveness of wages with regard to changes in home employment. We may therefore conclude:

*Summary conclusion 1: The theories with regard to the exchange stability problem of MARSHALL, BICKERDIKE, JOAN ROBINSON and BROWN have practical value only in cases where the very specific assumptions on which they are based are fulfilled.*

To provide an approximate answer to the exchange stability problem for individual countries, advantage has been taken of Mr CHANG's calculations as regards the numerical values of the price elasticities of substitution for import and export demand. Although certain objections can be raised against his findings and although they do not perfectly suit our purposes, owing to the fact that they only refer to merchandise trade (cf. section 4.2), we applied them nevertheless to the special cases for home supply, as it was thought interesting to get at least a rough idea of the exchange stability for individual countries. The choice of the special cases to which calculations were applied was made on the consideration that these cases cover, approximately at least, a good number of real situations (cf. section 4.8). The main finding here appeared to be:

*Summary conclusion 2: The nature of the balance of payments equilibrium depends largely on the way in which home demand reacts on changes in incomes and prices. The price substitution elasticities for import and export demand may also influence the nature of this equilibrium, provided*

*they are small (as Mr CHANG's elasticities for individual countries appear to be) and the economy concerned finds itself in a state of full employment.*

This conclusion shows clearly how necessary it is to obtain more information about the marginal propensity to spend and the price elasticity of home demand. Well-designed systems of national accounts, containing price and volume data as well as money-value data, will prove to be of great value in measuring these structural coefficients statistically. The same holds true for the measurement of the price elasticities of substitution for import and export demand which should refer not only to merchandise trade, as Mr CHANG's coefficients do, but also to invisibles (cf. section 4.2). Further research in this field is urgently required in order to provide definite answers to the exchange stability problem for individual countries and will be most helpful in solving, at the same time, a number of other problems in which these structural coefficients play their part. It will be most desirable to make more use than has been done so far of the "distributed lag" concept in this research work so as to enable a clear distinction to be made between short-run and long-run reactions (cf. section 2.6).

Recently some doubts have been raised as regards the legitimacy of using linearly approached structural coefficients for other values of the variables than their averages over the period for which they were calculated. This matter too should be investigated carefully before an attempt is made to give a definite answer to the problem of exchange stability for individual countries.

As soon as this further research has been done, we may hope to be able to determine the nature of the balance of payments equilibrium for individual countries.

#### MATHEMATICAL APPENDIX TO CHAPTER IV

##### *Stability conditions for the first special case for home demand*

The stability conditions for the first special case for home demand ( $v' = \eta'$ ) can be derived from formula

$$\begin{aligned}
 (4.4.2) \quad S &= \frac{-(1-v^i-v^l\lambda)\{1-\eta^i-\eta^e(1-v^i)\}(1-v')}{\left\{\frac{v^l(1-v^i)}{\varepsilon^l(1-\pi)} + \frac{1}{\varepsilon}\right\}v'\pi\left(1-\eta^i-\frac{\eta^e}{v'}\right) - \left[(1-v^i-v^l\lambda) + \pi\left\{\frac{v^i}{\pi} + \eta^i\frac{v^l}{\varepsilon^l(1-\pi)}\right\}\right]\{(1-v')(1-v^i)+v^i\}} \pi \bar{u} k^b = \\
 &= \frac{B_1}{C_1} \pi \bar{u} k^b = A_1 \pi \bar{u} k^b,
 \end{aligned}$$

in the following way. The equilibrium will be unstable if  $A_1 > 0$ , and stable if  $A_1 < 0$ . Now  $A_1 > 0$  if  $B_1 \geq 0$ , and, at the same time,  $C_1 \geq 0$ , and  $A_1 < 0$  if  $B_1 \leq 0$  and, at the same time,  $C_1 \leq 0$ . The conditions for  $B_1$  being positive and negative respectively are

$$\begin{aligned}
 B_1 \geq 0 \text{ for } & \text{(i) } 1-\eta^i-\eta^e(1-v^i) \geq 0 \text{ and } v' > 1; \\
 & \text{(ii) } 1-\eta^i-\eta^e(1-v^i) \leq 0 \text{ and } v' < 1.
 \end{aligned}$$

The conditions for  $C_1$  being positive and negative respectively can be formulated as follows:

$$C_1 \geq 0 \text{ for } 1-\eta^i-\frac{\eta^e}{v'} \geq \frac{(1-v'+v^i v')\left\{1-v^l\lambda+\eta^i\pi\frac{v^l}{\varepsilon^l(1-\pi)}\right\}}{v'\pi\left\{\frac{v^l(1-v^i)}{\varepsilon^l(1-\pi)}+\frac{1}{\varepsilon}\right\}}$$

Furthermore, for  $v' \geq \frac{1}{1-v^i}$  or  $-\frac{1}{v'} \geq -(1-v^i)$ , we find

that  $1 - \eta^i - \frac{\eta^e}{v'} \geq 1 - \eta^i - \eta^e (1-v^i)$ . Likewise, for

$v' \geq \frac{1}{1-v^i}$  or  $0 \geq 1-v' + v^i v'$ , we find that

$$0 \geq \frac{(1-v' + v^i v') \left\{ 1 - v^i \lambda + \eta^i \pi \frac{v^i}{\varepsilon^i (1-\pi)} \right\}}{v' \pi \left\{ \frac{v^i (1-v^i)}{\varepsilon^i (1-\pi)} + \frac{1}{\varepsilon} \right\}}$$

From the combination of these inequalities it may be concluded that:

$A_1 > 0$  for  $B_1 \geq 0$  and  $C_1 \geq 0$ , that is to say for

$$(i) v' > \frac{1}{1-v^i} (> 1) \text{ and } 0 > \frac{(1-v' + v^i v') \left\{ 1 - v^i \lambda + \eta^i \pi \frac{v^i}{\varepsilon^i (1-\pi)} \right\}}{v' \pi \left\{ \frac{v^i (1-v^i)}{\varepsilon^i (1-\pi)} + \frac{1}{\varepsilon} \right\}} \leq 1 - \eta^i - \frac{\eta^e}{v'} > 1 - \eta^i - \eta^e (1-v^i) \geq 0$$

$$(ii) \frac{1}{1-v^i} > v' > 1 \quad ,, \quad ,, < \quad ,, \quad \leq \quad ,, < \quad ,, \quad \geq ,,$$

$$(iii) \left( \frac{1}{1-v^i} > \right) 1 > v' \quad ,, \quad ,, < \quad ,, \quad \leq \quad ,, < \quad ,, \quad \geq ,,$$

$A_1 < 0$  for  $B_1 \geq 0$  and  $C_1 \leq 0$ , that is to say for

$$(i) v' > \frac{1}{1-v^i} (> 1) \text{ and } 0 > \frac{(1-v' + v^i v') \left\{ 1 - v^i \lambda + \eta^i \pi \frac{v^i}{\varepsilon^i (1-\pi)} \right\}}{v' \pi \left\{ \frac{v^i (1-v^i)}{\varepsilon^i (1-\pi)} + \frac{1}{\varepsilon} \right\}} \geq 1 - \eta^i - \frac{\eta^e}{v'} > 1 - \eta^i - \eta^e (1-v^i) \geq 0$$

$$(ii) \frac{1}{1-v^i} > v' > 1 \quad ,, \quad ,, < \quad ,, \quad \geq \quad ,, < \quad ,, \quad \geq ,,$$

$$(iii) \left( \frac{1}{1-v^i} > \right) 1 > v' \quad ,, \quad ,, < \quad ,, \quad \geq \quad ,, < \quad ,, \quad \leq ,,$$

Leaving apart the cases, where the various inequalities are inconsistent with each other (indicated by the sign  $<$  or  $>$ ) the whole of these conditions can be summarized as follows:

$$\boxed{A_1 \geq 0 \text{ for } v' \geq 1}, \text{ except in cases, where}$$

$$0 \leq \frac{(1-v' + v^i v') \left\{ 1 - v^i \lambda + \eta^i \pi \frac{v^i}{\varepsilon^i (1-\pi)} \right\}}{v' \pi \left\{ \frac{v^i (1-v^i)}{\varepsilon^i (1-\pi)} + \frac{1}{\varepsilon} \right\}} \geq 1 - \eta^i - \frac{\eta^e}{v'} \leq 1 - \eta^i - \eta^e (1-v^i) \geq 0$$

## STATISTICAL APPENDIX

### EXPORT AND IMPORT QUOTA

*Explanatory note:*

$\bar{\bar{U}}^e$  = exports of goods and services (excl. monetary gold movements)

$\bar{\bar{U}}^i$  = imports of goods and services (excl. monetary gold movements)

$\bar{\bar{Y}}$  = national income at market prices (i.e. incl. cost-raising taxes minus subsidies)

$a = \bar{\bar{U}}^e / (\bar{\bar{Y}} + \bar{\bar{U}}^i)$  = average export quota in national product

$b = \bar{\bar{U}}^i / (\bar{\bar{Y}} + \bar{\bar{U}}^i)$  = average import quota in national product

$c = a/b$  = ratio between exports and imports

*Sources:*

$\bar{\bar{U}}^e$  and  $\bar{\bar{U}}^i$  — Balances of Payments (League of Nations and United Nations)

$\bar{\bar{Y}}$  — COLIN CLARK: *The Conditions of Economic Progress* (revised edition, not yet published; first edition London 1940).

However, for

U.S.A.: Survey of Current Business, July 1949, p. 11;

U.K.: A. R. PREST: "National Income of the United Kingdom 1870—1946" in *The Economic Journal*, March 1948, table II

(reprinted in University of Cambridge Department of Applied Economics, Reprint Series no. 9) for net national income at factor cost; conversion to market prices based on COLIN CLARK, *op. cit.*;

*Finland:* (1929/38): Statistical Yearbook United Nations 1948 (Lake Success-New York 1949), p. 371;

*Norway:* (1935/38): Nasjonalbudsjettet 1948 (St. meld. no. 1 Handelsdepartementet), p. 6;

*Denmark:* Statistisk Aarbog 1948 (København 1948), p. 241.

TABLE A

*Annual averages of export and import quota (a and b respectively) and their ratio (c) for eight industrial countries*

| Year | U.S.A. |     |      | Germany |     |      | U.K. <sup>1)</sup> |     |      | Sweden |      |      |
|------|--------|-----|------|---------|-----|------|--------------------|-----|------|--------|------|------|
|      | a      | b   | c    | a       | b   | c    | a                  | b   | c    | a      | b    | c    |
| (1)  | (2)    | (3) | (4)  | (5)     | (6) | (7)  | (8)                | (9) | (10) | (11)   | (12) | (13) |
| 1924 |        |     | 1.19 |         |     | .84  | .24                | .23 | 1.06 | .18    | .18  | 1.00 |
| 25   |        |     | 1.12 | .14     | .18 | .78  | .24                | .23 | 1.04 | .19    | .18  | 1.05 |
| 26   |        |     | 1.08 | .15     | .15 | .99  | .22                | .22 | .99  | .20    | .18  | 1.08 |
| 27   |        |     | 1.12 | .14     | .18 | .75  | .22                | .21 | 1.07 | .22    | .19  | 1.15 |
| 28   |        |     | 1.17 | .14     | .18 | .81  | .23                | .21 | 1.10 | .21    | .20  | 1.05 |
| 29   | .07    | .06 | 1.12 | .15     | .18 | .85  | .22                | .21 | 1.08 | .23    | .20  | 1.15 |
| 1930 | .06    | .06 | 1.15 | .15     | .16 | .96  | .20                | .19 | 1.03 | .21    | .20  | 1.05 |
| 31   | .05    | .05 | 1.05 | .15     | .14 | 1.11 | .15                | .17 | .88  | .18    | .20  | .93  |
| 32   | .05    | .04 | 1.07 | .12     | .11 | 1.04 | .14                | .15 | .93  | .19    | .18  | 1.06 |
| 33   | .05    | .05 | 1.05 | .10     | .10 | 1.02 | .14                | .14 | 1.01 | .20    | .17  | 1.16 |
| 34   | .05    | .04 | 1.13 | .08     | .09 | .90  | .14                | .14 | 1.00 | .20    | .18  | 1.13 |
| 35   | .05    | .05 | .95  | .07     | .07 | .98  | .14                | .14 | 1.02 | .19    | .18  | 1.04 |
| 36   | .04    | .05 | .94  |         |     |      | .14                | .15 | .98  | .20    | .18  | 1.06 |
| 37   | .05    | .05 | .99  |         |     |      | .16                | .16 | .96  |        |      |      |
| 38   | .05    | .04 | 1.30 |         |     |      | .14                | .15 | .93  |        |      |      |

TABLE A (continued)

| Year | France <sup>2)</sup> |      |      | Switzerland |      |      | Japan |      |      | Italy |      |      |
|------|----------------------|------|------|-------------|------|------|-------|------|------|-------|------|------|
|      | a                    | b    | c    | a           | b    | c    | a     | b    | c    | a     | b    | c    |
|      | (14)                 | (15) | (16) | (17)        | (18) | (19) | (20)  | (21) | (22) | (23)  | (24) | (25) |
| 1924 | .23                  | .21  | 1.08 | .26         | .25  | 1.02 | .15   | .18  | .81  |       |      | 1.02 |
| 25   |                      |      |      | .26         | .25  | 1.02 | .17   | .18  | .94  | .23   | .21  | 1.09 |
| 26   |                      |      |      | .24         | .23  | 1.04 | .16   | .18  | .90  |       |      | 1.08 |
| 27   | .24                  | .19  | 1.22 |             |      |      | .16   | .17  | .95  | .20   | .19  | 1.01 |
| 28   | .22                  | .18  | 1.23 | .26         | .24  | 1.10 | .16   | .17  | .94  | .17   | .19  | 0.90 |
| 29   | .21                  | .18  | 1.13 |             |      |      | .18   | .18  | 1.01 |       |      | .93  |
| 1930 | .18                  | .17  | 1.09 |             |      |      | .15   | .15  | .99  |       |      | .96  |
| 31   | .13                  | .13  | .96  |             |      |      | .14   | .14  | .97  |       |      |      |
| 32   | .08                  | .10  | .81  |             |      |      | .16   | .16  | 1.02 |       |      |      |
| 33   | .08                  | .09  | .88  |             |      |      | .17   | .17  | 1.01 |       |      |      |
| 34   | .09                  | .09  | .94  |             |      |      | .18   | .18  | 1.02 |       |      |      |
| 35   | .08                  | .08  | .96  |             |      |      | .19   | .17  | 1.09 |       |      |      |
| 36   | .07                  | .08  | .85  |             |      |      | .18   | .17  | 1.04 |       |      |      |
| 37   | .10                  | .12  | .81  |             |      |      |       |      |      |       |      |      |
| 38   | .11                  | .11  | .99  |             |      |      |       |      |      |       |      |      |

<sup>1)</sup> Imports excl. invisible imports; exports incl. net balance of invisibles.

<sup>2)</sup> Trade figures 1936—'38 from frcs 1928-parity converted into frcs of actual parity.

TABLE B

*Annual averages of export and import quota (a and b respectively) and their ratio (c) for three mining and raw material producing countries*

| Year | Chile <sup>1)</sup> |     |     | Canada <sup>2)</sup> |     |      | South Africa <sup>3)</sup> |     |      |
|------|---------------------|-----|-----|----------------------|-----|------|----------------------------|-----|------|
|      | a                   | b   | c   | a                    | b   | c    | a                          | b   | c    |
| (1)  | (2)                 | (3) | (4) | (5)                  | (6) | (7)  | (8)                        | (9) | (10) |
| 1924 |                     |     |     |                      |     | 1.10 |                            |     | .92  |
| 25   |                     |     |     |                      |     | 1.21 |                            |     | .96  |
| 26   |                     |     |     | .26                  | .24 | 1.08 |                            |     | .86  |
| 27   |                     |     |     | .24                  | .24 | .99  |                            |     | .95  |
| 28   |                     |     |     | .24                  | .25 | .98  |                            |     | .91  |
| 29   | .27                 | .34 | .79 | .22                  | .26 | .84  | .27                        | .31 | .88  |
| 1930 |                     |     |     | .20                  | .25 | .79  | .26                        | .28 | .93  |
| 31   |                     |     |     | .19                  | .23 | .85  | .25                        | .26 | .95  |
| 32   |                     |     |     | .20                  | .22 | .89  | .24                        | .19 | 1.28 |
| 33   |                     |     |     | .22                  | .22 | 1.00 | .28                        | .22 | 1.28 |
| 34   |                     |     |     | .23                  | .22 | 1.07 | .22                        | .24 | .88  |
| 35   |                     |     |     | .24                  | .22 | 1.12 | .24                        | .25 | .98  |
| 36   |                     |     |     | .26                  | .22 | 1.21 | .24                        | .25 | .96  |
| 37   |                     |     |     | .26                  | .23 | 1.13 | .25                        | .28 | .91  |
| 38   |                     |     |     | .23                  | .21 | 1.08 |                            |     |      |

<sup>1)</sup> National income presumably valued at factor costs; 1928 national income related to 1929 trade figures.

<sup>2)</sup> National income figures corrected for net balance due outwards of dividends and interest.

<sup>3)</sup> National income at factor costs; conversion to national income at market prices on basis of 1923-figures requires reduction of import and export quota by 4 pCt.

TABLE C

*Annual averages of export and import quota (a and b respectively) and their ratio (c) for eight agricultural countries*

| Year | Estonia |     |      | Latvia <sup>1)</sup> |     |      | New Zealand <sup>2)</sup> |     |      | Finland |      |      |
|------|---------|-----|------|----------------------|-----|------|---------------------------|-----|------|---------|------|------|
|      | a       | b   | c    | a                    | b   | c    | a                         | b   | c    | a       | b    | c    |
| (1)  | (2)     | (3) | (4)  | (5)                  | (6) | (7)  | (8)                       | (9) | (10) | (11)    | (12) | (13) |
| 1924 |         |     | 1.00 |                      |     | .87  |                           |     | .91  | .28     | .26  | 1.06 |
| 25   |         |     | 1.01 | .24 <sup>1</sup>     | .26 | .89  |                           |     | .89  | .27     | .26  | 1.01 |
| 26   |         |     | 1.01 |                      |     | .91  | .21                       | .27 | .77  | .25     | .25  | 1.00 |
| 27   |         |     | 1.05 |                      |     | 1.09 | .24                       | .24 | 1.00 | .25     | .25  | 1.01 |
| 28   | .26     | .28 | .95  | .26                  | .26 | 1.02 | .23                       | .24 | 1.00 | .23     | .28  | .82  |
| 29   |         |     | .94  |                      |     | .93  | .21                       | .25 | .82  | .25     | .27  | .94  |
| 1930 |         |     | .95  | .23                  | .23 | 1.02 | .18                       | .23 | .79  | .24     | .24  | 1.03 |
| 31   |         |     | 1.12 |                      |     | 1.04 | .18                       | .19 | .95  | .25     | .20  | 1.23 |
| 32   |         |     | 1.05 | .16                  | .14 | 1.19 | .19                       | .19 | 1.02 | .26     | .21  | 1.26 |
| 33   |         |     | 1.12 |                      |     | 1.01 | .20                       | .17 | 1.19 | .27     | .21  | 1.28 |
| 34   |         |     | 1.15 |                      |     | 1.01 | .18                       | .18 | .99  | .27     | .21  | 1.28 |
| 35   |         |     | 1.10 |                      |     | 1.03 | .18                       | .18 | 1.01 | .26     | .22  | 1.18 |
| 36   |         |     | .98  | .16                  | .14 | 1.12 | .19                       | .19 | 1.00 | .27     | .22  | 1.18 |
| 37   |         |     | 1.04 | .22                  | .20 | 1.14 | .18                       | .20 | .94  | .27     | .25  | 1.08 |
| 38   |         |     | .96  |                      |     |      | .16                       | .18 | .89  | .24     | .23  | 1.06 |

TABLE C (continued)

| Year | Norway |      |      | Hungary |      |      | Denmark |      |      | Australia <sup>3)</sup> |      |      |
|------|--------|------|------|---------|------|------|---------|------|------|-------------------------|------|------|
|      | a      | b    | c    | a       | b    | c    | a       | b    | c    | a                       | b    | c    |
|      | (14)   | (15) | (16) | (17)    | (18) | (19) | (20)    | (21) | (22) | (23)                    | (24) | (25) |
| 1924 | .32    | .36  | .90  |         |      | .83  |         |      | .98  | .18                     | .19  | .91  |
| 25   | .31    | .33  | .95  |         |      |      |         |      | 1.01 | .16                     | .20  | .79  |
| 26   | .29    | .30  | .99  | .13     | .15  | .87  |         |      | .97  | .14                     | .20  | .68  |
| 27   | .28    | .30  | .95  | .12     | .18  | .64  |         |      | .98  | .14                     | .18  | .77  |
| 28   | .28    | .31  | .91  | .12     | .19  | .66  |         |      | 1.00 | .15                     | .19  | .79  |
| 29   | .30    | .31  | .97  | .16     | .18  | .85  |         |      | 1.02 | .12                     | .19  | .61  |
| 1930 | .29    | .31  | .92  | .16     | .17  | .90  | .27     | .27  | .99  | .13                     | .15  | .85  |
| 31   | .25    | .29  | .88  | .12     | .15  | .78  | .24     | .25  | .96  | .14                     | .13  | 1.09 |
| 32   | .27    | .26  | 1.04 | .08     | .08  | .95  | .23     | .22  | 1.05 | .13                     | .14  | .97  |
| 33   | .27    | .25  | 1.08 | .09     | .08  | 1.09 | .23     | .23  | 1.03 | .14                     | .13  | 1.07 |
| 34   | .28    | .26  | 1.05 | .09     | .09  | 1.02 | .22     | .22  | .98  | .12                     | .14  | .88  |
| 35   | .26    | .26  | 1.02 | .09     | .09  | 1.06 | .22     | .21  | 1.03 | .13                     | .14  | .94  |
| 36   | .27    | .26  | 1.04 | .09     | .08  | 1.14 | .22     | .22  | 1.00 | .15                     | .14  | 1.05 |
| 37   | .29    | .28  | 1.06 |         |      | 1.19 | .25     | .23  | 1.06 |                         |      |      |
| 38   | .28    | .26  | 1.07 |         |      |      | .23     | .22  | 1.06 |                         |      |      |

<sup>1)</sup> National income at factor costs; conversion to valuation at market prices, on the basis of 1925—34 figures, requires a reduction of export and import quota with 7 pCt.

<sup>2)</sup> From 1927 onwards fiscal years.

<sup>3)</sup> Fiscal year figures. Export figures include gold exports out of home production from 1928/29 onwards.

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The above list contains only publications, referred to in the text, which deal with the theoretical and statistical aspects of the exchange stability problem and is, therefore, far from complete. The reader will find further references to the literature on the subject in the above-mentioned publications and also in two papers, which reached the author after the present volume had gone to print viz.:

P. T. ELLSWORTH: "Exchange Rates and Exchange Stability" in *Review of Economics and Statistics*, vol. XXXII (1950), pp. 1—12, and  
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## SIMPLIFIED ONE-COUNTRY MODELS

Simplifications (1)  $\bar{U}^e = \bar{U}^i$ , (2)  $p' = p^e = p$ , and (3)  $e_2 = \infty$  and  $v_2 = 0$  (hence  $p^i = -kb$ ).

## MEANING OF SYMBOLS

## 1. Variables

|                  | imports | exports | "consumption" | "production" |
|------------------|---------|---------|---------------|--------------|
| money-value. . . | $U^i$   | $U^e$   | $U'$          | $(U)$        |
| volume . . . . . | $u^i$   | $u^e$   | $u'$          | $u$          |
| price . . . . .  | $(p^i)$ | $(p^e)$ | $(p')$        | $p$          |

 $S$  = balance on current account $Y$  = national income (at market prices) $l$  = wage rate $kb$  = exchange rate $p^{eb} = (p^e + kb)$  = export price level in terms of foreign currency

## 2. Structural coefficients

 $\pi$  = average import quota $v'$  = marginal propensity to spend $v^i$  = marginal import quota $v^l$  = marginal wage quota $\eta'$  = price elasticity of home demand $\eta^i$  = price elasticity of substitution for import demand $\eta^e$  = price elasticity of substitution for export demand $\varepsilon$  = price elasticity of home supply $1/\varepsilon^l$  = wage flexibility $\lambda$  = wage coefficient $\eta_1$  = price elasticity of home demand for foreign products $\eta_2$  = price elasticity of foreign demand for home products $e_1$  = price elasticity of home supply (of export products) $e_2$  = price elasticity of foreign supply (of import products) $v_1$  = marginal quota of foreign products in home production (for exports) $v_2$  = marginal quota of home products in foreign production (for imports)

| I OUR MODEL                                                        | II MARSHALL *)<br>(case of critical<br>elasticities) | III JOAN ROBINSON *)<br>(traditional<br>approach) | IV BROWN *)                                             |
|--------------------------------------------------------------------|------------------------------------------------------|---------------------------------------------------|---------------------------------------------------------|
| <i>definitional equations</i>                                      |                                                      |                                                   |                                                         |
| (1) $U^i = u^i - \pi \bar{u} kb$                                   | $\times$                                             | $\times$                                          | $\times$                                                |
| (2) $U^e = u^e + \pi \bar{u} p$                                    | $U^e = u^e$                                          | $\times$                                          | $\times$                                                |
| (3) $U' = u' + (1 - \pi) \bar{u} p$                                |                                                      |                                                   |                                                         |
| (4) $S = U^e - U^i$                                                | $\times$                                             | $\times$                                          | $\times$                                                |
| (5) $Y + U^i = U' + U^e$                                           |                                                      |                                                   |                                                         |
| (6) $u = u' + u^e$                                                 |                                                      |                                                   |                                                         |
| <i>demand equations</i>                                            |                                                      |                                                   |                                                         |
| (7) $u' = v' Y - \eta' (1 - \pi) \bar{u} p$                        |                                                      |                                                   |                                                         |
| (8) $u^i = v^i u + \eta^i \pi \bar{u} (p + kb)$                    | $u^i = \eta_1 \pi \bar{u} kb$                        | $u^i = \eta_1 \pi \bar{u} kb$                     | $u^i = \eta_1 \pi \bar{u} kb$                           |
| (9) $u^e = -\eta^e \pi \bar{u} (p + kb)$                           | $u^e = -\eta_2 \pi \bar{u} kb$                       | $u^e = -\eta_2 \pi \bar{u} (p + kb)$              | $u^e = -\eta_2 \pi \bar{u} (p + kb)$                    |
| <i>supply equations</i>                                            |                                                      |                                                   |                                                         |
| (10) $p = v^l l - v^i kb + \frac{1}{\varepsilon} u$                | $p = 0$                                              | $p = \frac{1}{\varepsilon_1 \pi \bar{u}} u^e$     | $p = -v_1 kb + \frac{1}{\varepsilon_1 \pi \bar{u}} u^e$ |
| (11) $l = \lambda p + \frac{1}{\varepsilon^l (1 - \pi)} (u - u^i)$ | $l = 0$                                              | $l = 0$                                           | $l = 0$                                                 |

## FINAL EQUATIONS FOR SIMPLIFIED MODELS

## I OUR MODEL:

(4.3.1)

$$S = \frac{\left\{ \frac{v^l(1-v^i)}{\varepsilon^l(1-\pi)} + \frac{1}{\varepsilon} \right\} (1-\eta^i-\eta^e) - v^i \left\{ \frac{v^i}{\pi} + \eta^i \frac{v^l}{\varepsilon^l(1-\pi)} \right\} (1-\pi)(v'-\eta') - (1-v^i-v^l) \{ 1-\eta^i-\eta^e(1-v^i) \} (1-v')}{\left\{ \frac{v^l(1-v^i)}{\varepsilon^l(1-\pi)} + \frac{1}{\varepsilon} \right\} \left\{ v' \pi (1-\eta^i-\eta^e) + (1-\pi)(v'-\eta') \right\} - [ (1-v^i-v^l) + \pi \left\{ \frac{v^i}{\pi} + \eta^i \frac{v^l}{\varepsilon^l(1-\pi)} \right\} ] (1-v') (1-v^i) + v^i} \pi$$

## II MARSHALL (case of critical elasticities)

$$(4.3.8) \quad S = (1 - \eta_1 - \eta_2) \pi \bar{u} kb$$

## III JOAN ROBINSON (traditional approach)

$$(4.3.5) \quad S = - \frac{\eta_1 \eta_2 + e_1 (\eta_1 + \eta_2 - 1)}{\eta_2 + e_1} \pi \bar{u} kb$$

## IV BROWN (modified traditional approach)

$$(4.3.10) \quad S = - \frac{\eta_1 \eta_2 + e_1 \{ \eta_1 + (1 - v_1)(\eta_2 - 1) \}}{\eta_2 + e_1} \pi \bar{u} kb = - \frac{\eta_1 (\eta_2 + e_1) + (\eta_2 - 1) (1 - v_1) e_1}{\eta_2 + e_1} \pi \bar{u} kb$$

\*) The symbol  $\times$  indicates that the equation concerned is identical to the corresponding equation in our model.











