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ARITHMETIC FOR ENGINEERS

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WITH AN APPENDIX ON
VERNIERS AND MICROMETERS

BY
CHARLES B. CLAPHAM
B.Sc. (HONS.) ENG., A.M.I.MECH.E.
Lecturer and Demonstrator in the Mechanical Engineering Dept., Finsbury Technical College; Author of "Metric System for Engineers"

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EDITORIAL NOTE

THE DIRECTLY-USEFUL TECHNICAL SERIES requires a few words by way of introduction. Technical books of the past have arranged themselves largely under two sections: the Theoretical and the Practical. Theoretical books have been written more for the training of college students than for the supply of information to men in practice, and have been greatly filled with problems of an academic character. Practical books have often sought the other extreme, omitting the scientific basis upon which all good practice is built, whether discernible or not. The present series is intended to occupy a midway position. The information, the problems, and the exercises are to be of a directly useful character, but must at the same time be wedded to that proper amount of scientific explanation which alone will satisfy the inquiring mind. We shall thus appeal to all technical people throughout the land, either students or those in actual practice.

THE EDITOR.

AUTHOR'S PREFACE

THE following work is an endeavour to treat the elementary portions of what is usually called "Practical Mathematics" in a thorough and practical manner, suitable for elementary students of technical schools and for home study. Although a great many books on the same subject already exist, the Author has been unable to find one dealing with the necessary matter in sufficient detail and with a sufficient amount of engineering application to meet the needs of his own students. For private study, too, he is of the opinion that the existing text-books treat the elementary matter in a manner too cursory to really fulfil their object. The treatment in the following pages, therefore, has been developed from his own lecture notes and class instruction on the subject, and numerous diagrams have been introduced to assist in making the work clear.

Many will think, no doubt, that the title "Arithmetic for Engineers" is not sufficiently comprehensive for the matter contained, but it was found impossible to frame a short original title to adequately describe the contents. An examination of the examples and exercises should show that the whole of the matter is *directly useful;* all purely academic work such as Highest Common Factor, Recurring Decimals and the like being discarded. Wherever possible the examples are *truly* practical, *i. e.*, are problems *actually met with* in the Drawing Office, Workshop and Laboratory, while the data are of correct dimensions. Such examples should stimulate the reader's interest in the mathematical work and show the applications of the principles to practice; at the same time a little general engineering knowledge will be gained. Even at the risk of becoming verbose, all matter is treated at length, *every principle being followed by worked examples.* Where slight variations of the problem may cause difficulty to the beginner, or where special precautions have to be observed on certain points, *several* illustrative examples are given. A case in point is that of the extraction of a square root on pp. 102 *et seq.* For the private student this is particularly necessary, as he is often unable to obtain guidance in working a new problem which presents features slightly different from those in the example worked to illustrate the principle.

Stress has been laid, in Chap. II, on limiting the number of figures to be given in a result, as class-room experience shows that many students will persist in using, and stating in results, far more figures than are necessary from the practical point of view, where

an accuracy of 1 per cent. is often as much as is desired. The method of "approximating for a result" shown on pp. 58 *et seq.* is, it is believed, not generally known. In Chap. III positive and negative quantities, which are of importance in logarithms and higher work, are given special attention, as they often present difficulty to the beginner. The simple equation is treated very methodically in Chap. IV, with concrete illustrations of the earlier examples, and is followed immediately by a similar treatment of the literal simple equation, which to a beginner often presents great difficulty, even when he is well able to work a similar numerical example. The use, only, of logarithms is taken in Chap. VI; it was hoped to add something of the theory, but space would not permit. The Mensuration in Chaps. VII and VIII may appear rather extensive, but calls for little knowledge beyond the evaluation of a formula or the solution of a simple equation. This division of the subject was framed to be of use for reference in the Drawing Office, in which direction the tables on pp. 312, 313, 345 and 346 should prove useful. The "more exact" formula for the circumference of an ellipse on p. 260 (due to Boussinesq) is not often seen, but is very accurate. It is to be feared that many writers repeat the formula $\pi\sqrt{2(a^2 + b^2)}$ as being more exact than $\pi(a + b)$, when really it is but little better and has only a very limited range of application. Chap. IX takes Graphs in an elementary degree, considerable attention being paid to the details of setting out and finishing off, in which direction many students fail. The last chapter is devoted to the Slide Rule, with illustrations of readings and settings, and the method of instruction was tested while in manuscript form. The chapter was carefully worked through by a novice, who, finally, and without other instruction, could use the rule with ease and certainty: these pages should be helpful to the private student.

It is hoped that teachers of Practical Mathematics will be saved much research work by the numerous classified practical exercises throughout the book: answers are given to all these exercises.

To MR. W. J. LINEHAM, B.Sc., M.I.C.E., the Author tenders his sincere thanks for much kindness throughout his career, and also for generous assistance and useful criticism in the production of the book.

The Author's gratitude is also tendered to MR. JOHN L. BALE for his generous help with the book in all its stages, from the earliest research, throughout the manuscript and proof forms to the final production.

The notification of errors, clerical and otherwise, will be gratefully appreciated.

CHARLES B. CLAPHAM

Goldsmiths' College,
New Cross, S.E.,
May, 1916.

PREFACE TO THE THIRD EDITION

In preparing a third edition of "Arithmetic for Engineers" a chapter on Elementary Trigonometry has been added. It is hoped that this addition, by filling a gap in the original treatment, will render the book of greater value to Technical Schools, in many of which a first course of Practical Mathematics includes at least the elementary conceptions of this branch of the subject. The new chapter covers little more than the meaning of the important ratios sine, cosine, and tangent, but is believed to include sufficient to show how even this small amount can be of great utility to the practical man in drawing office and workshop. The treatment is similar to that followed throughout the rest of the book, *viz.*, detailed explanations followed by really practical illustrative examples and exercises at every step.

The matter on Verniers and Micrometers, originally added as an Appendix to the Second Edition, has been retained as an Appendix, as it is rather in the nature of a special application.

It is a matter of deep regret to the Author that, by the death in 1919 of Mr. W. J. Lineham, founder of the D.U. Series, he has been deprived of much valuable assistance and criticism.

The Author wishes to acknowledge with many thanks the assistance of the various readers who have kindly notified him as to errors in the first two editions, and he hopes that this useful assistance will continue in the future.

CHARLES B. CLAPHAM.

Goldsmiths' College,
New Cross, S.E.
October, 1922.

PREFACE TO THE FOURTH EDITION

THE opportunity has been taken to revise one or two sections and to add a short account of the finding of logarithms and anti-logarithms on the slide rule. About fifty new exercises have been added, covering a variety of details including some simple applications to wireless and aircraft.

The notification of errors is gratefully acknowledged; the Author will always be pleased to receive further information in this direction.

CHARLES B. CLAPHAM.

Finsbury Technical College,
Leonard St., E.C. 2.
September, 1925.

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ARITHMETIC FOR ENGINEERS

INTRODUCTORY

Measurement and Units.—From the practical standpoint our calculations are never concerned with numbers alone, but always with some properties of materials or machines: simple, such as length and weight; or complicated, such as horse-power and electrical resistance. We have always, therefore, to deal with quantities which have been measured and which, besides a number, will have a name, such as feet, pounds, etc.

In some of the examples to follow, numbers alone will be used when the calculations are only for the explanation of a certain mathematical process. A measured quantity is described as being so many times larger than some particular "standard" of reference. This "standard" is called a **unit,** a word denoting the number 1, and which for each particular kind of measurement is given a distinctive name. Thus the unit in the table of weights is the **pound,** and in the table of length the **yard,** and so on. These units are fixed by law in the case of the more common measurements, such as length, weight, etc., and official samples are preserved in Government offices. With more special measurements, such as velocity, horse-power, etc., the units are fixed by general consent.

Multiples and Sub-multiples.—It is very seldom that we find any particular unit suitable for all measurements of a certain kind. Thus the yard would be very inconvenient when measuring long distances on the earth's surface, for our measurements would come to millions of yards. Such numbers, besides being difficult to imagine, would render calculation very laborious. Therefore to make our measurements of long distances consist of a *smaller number* we employ a *larger unit* which contains an exact number of standard units. Such "large" units are called **multiples** of

the standard. The mile (1760 yards) and the ton (2240 lbs.) are examples. On the other hand, when measuring very small lengths (such as the thickness of sheet metal) the yard would be too large a unit, and therefore small units called **sub-multiples** are adopted. Common examples are the **inch**, of which 36 make 1 yard, and the **ounce**, of which 16 make 1 pound. Taking the more special measurements used by the engineer, the number of multiples and sub-multiples is small, while in some cases, as with horse-power, only the standard unit is employed.

Abbreviations, Mathematical Signs and Terms.—In order to save time and space, and to make our calculations more easily read, it is customary to use certain signs for phrases which are constantly occurring in mathematical work. These must be committed to memory. The more frequent ones are given below; others will be introduced as required.

$=$ stands for "equals" or "is equal to";
$+$ „ „ "plus," meaning "added to";
$-$ „ „ "minus," meaning "subtract or take away";
$\times$ „ „ "multiplied by";
$\div$ „ „ "divided by";
$\therefore$ „ „ "therefore."

Thus the statement $3 - 2 + 5 = 6$ would be read as "three minus two plus five equals six," meaning that if 2 is subtracted from 3, and 5 added to what is left, the result equals 6.

Very often a division is not stated with the sign $\div$, but in the form $\frac{24}{8}$, meaning 24 divided by 8. This form is similar to the sign $\div$, the dot representing the numbers, and is then often read as "twenty-four over eight."

The following words have mathematical meanings—

Sum is the result of an addition.
Difference is the result of a subtraction.
Product „ „ „ „ multiplication.
Quotient „ „ „ „ division.
Dividend is a number to be divided.

An **expression** is any mathematical statement containing numbers, signs, etc.

If several numbers be multiplied together they are said to be **factors** of the product.

Thus since $3 \times 4 \times 5 = 60$, the numbers 3, 4 and 5 are "factors of 60." Any number may be split up into factors by knowledge of the multiplication tables, but it is seldom necessary to find the factors of large numbers.

Several different factors may be found of the same number. Thus besides the above factors of 60, we may also have 20 and 3, 30 and 2, 2, 10 and 3, etc. This is because some of these factors can themselves be split up into factors. Thus 30×2 can be written as $6 \times 5 \times 2$, and so on.

The simplest factors of all are those which cannot themselves be split up, *e. g.*, 2, 3, 5, 7, 11, 13, 17, 19, etc. These are called **"prime factors."**

If a number contains another number exactly (*i. e.*, can be divided exactly by that number), then it must contain the prime factors of that number. Thus 36 contains 12, *i. e.*, 36 can be divided exactly by 12; then 36 contains the prime factors of 12, *i. e.*, 2, 2 and 3, as may be proved by simple division.

Also the following abbreviations will be met with—

i. e.	meaning	"that is";
e. g.	,,	"for example";
viz.	,,	"namely";
approx.	,,	"approximately" or "very nearly";
revs. per min. or r.p.m.	,,	"revolutions per minute";
m.p.h.	,,	"miles per hour";
h.p. (H.P. also used)	,,	"horse-power."

Previous Knowledge.—In the following pages it is assumed that the reader is acquainted with the four simple rules of arithmetic —addition, subtraction, multiplication, and division when applied to whole numbers; and with the simple kinds of measurements, such as money, length, weight, time, etc.

The tables of these measurements are given below for reference—

British Table of Length

12 inches (ins.) = 1 foot;
3 feet (ft.) = 1 yard;
5½ yards (yds.) = 1 rod, pole, or perch;
40 poles (po.) = 1 furlong;
8 furlongs (fur.) = 1 mile.

Weight (Avoirdupois)

16 drams	= 1 ounce (oz.);
16 ounces	= 1 pound;
28 pounds (lbs.)	= 1 quarter;
4 quarters (qrs.)	= 1 hundredweight (cwt.)
20 cwt.	= 1 ton.
also 112 lbs.	= 1 cwt.;
and 2240 lbs.	= 1 ton.

Square Measure

144 square inches (sq. ins.)	= 1 square foot;
9 square feet (sq. ft.)	= 1 square yard;
30¼ square yards (sq. yds.)	= 1 square pole;
40 square poles	= 1 rood;
4 roods	= 1 acre;
640 acres	= 1 square mile.

Cubic Measure

1728 cubic inches (cu. ins.)	= 1 cubic foot;
27 cubic feet (cu. ft.)	= 1 cubic yard.

Measure of Capacity

4 gills	= 1 pint;
2 pints	= 1 quart;
4 quarts	= 1 gallon;
2 gallons	= 1 peck;
4 pecks	= 1 bushel;
8 bushels	= 1 quarter.

CHAPTER I

VULGAR FRACTIONS

Fractions generally.—As the quantities with which we have to deal seldom contain an exact number of units, we have to consider how to deal with parts, or **fractions,** of a unit. Since 1 is the smallest whole number in our written figures, and we have to deal with quantities less than 1, it is evident that our ordinary system of notation cannot be employed to represent the magnitude of such quantities.

Two systems of representation are in general use, both employing the following idea: the unit is divided into *a certain number of equal parts*, and any particular fraction is said to be equal to *so many of those parts*. The number of parts into which the unit is divided may be chosen within wide limits, but in practice only the most convenient are employed.

In the **Vulgar System** this number may be any convenient whole number whatever, for example: 8, 13, 100, 346, etc.

In the **Decimal System** the number is always 1 with one or more noughts after it, *i. e.*, 10, or 100, or 1000, etc.

The decimal system is thus a special case of the vulgar system, but appears rather different in practice on account of the different method of statement adopted. In Great Britain the vulgar system is employed chiefly in connection with money and the everyday use of the ordinary weights and measures. The decimal system is certainly more valuable for scientific purposes and for the more accurate measurements demanded in the workshop, while on the Continent it has been adapted, with few exceptions, to all measurements.

Vulgar Fractions.—In the vulgar system the following notation is adopted: below a short horizontal line is placed the number which shows exactly into how many equal parts the unit has been divided; above the line is placed the number which indicates to how many of these parts our fraction is equivalent. Thus in the case of $\frac{3}{4}$ (read as "three-fourths" or "three-quarters") the unit

has been divided into 4 equal parts, and 3 of them taken. This is shown graphically at *a*, Fig. 1, where the length AE represents our unit (say 1 foot, for example) divided into 4 equal parts, AB, BC, CD, and DE. The fraction $\frac{3}{4}$ is then represented by the length AD.

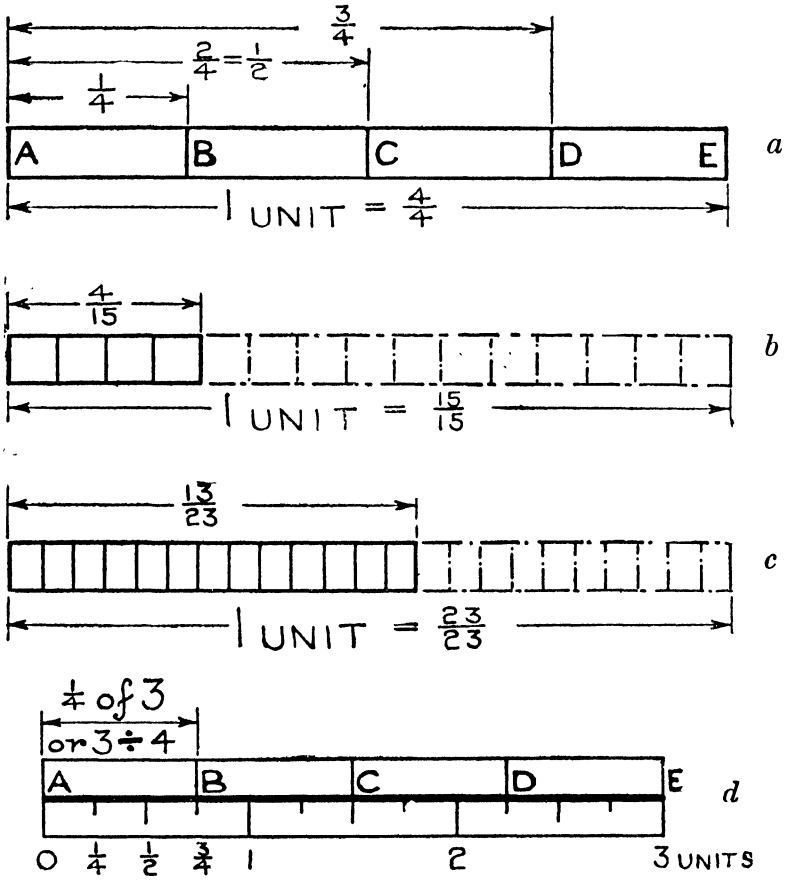


Fig. 1.—Illustrating Vulgar Fractions.

The number underneath the line, which indicates the size of the equal parts, is called the **Denominator**; while that above the line, which shows the number of equal parts taken, is called the **Numerator.** Thus in the above example, 3 is the numerator and 4 the denominator.

Similarly *b* and *c*, Fig. 1, indicate respectively the fractions $\frac{4}{15}$ (*four fifteenths*) and $\frac{13}{23}$ (*thirteen twenty-thirds*), in which 4 and 13 are numerators and 15 and 23 are denominators. We shall frequently refer to the numerator and denominator as the "top" and "bottom" of the fraction.

From the foregoing, the line between the numerator and denominator is merely a means of distinguishing between them. On p. 2 it is stated that this mid-line indicates the division of the upper number by the lower one. This is quite in accordance with our fractional notation, as may be seen from *d*, Fig. 1. Here the distance AE, which represents 3 units, is shown divided into 4 equal parts on the top side of the heavy line. Beneath the line each of the 3 units is shown divided into 4 equal parts, and it is easily seen that AB = $\frac{3}{4}$ of 1 unit. Therefore the fraction $\frac{3}{4}$ unit is the result of dividing 3 units into 4 equal parts, and hence we may consider any vulgar fraction as indicating the result of dividing the numerator by the denominator.

Such of the simpler denominators as quarters (or fourths), eighths, and sixteenths, will be familiar to most readers as being the usual divisions on the common 12-inch rule, while thirty-secondths and sixty-fourths are occasionally marked. In the monetary system —where the sovereign (£) is the unit—twentieths and two hundred-and-fortieths are employed, but are better known by the names of shillings (*s.*) and pence (*d.*) respectively.

Other peculiar denominators such as those at *b* and *c*, Fig. 1, are not often met with in practice.

Forms of the Vulgar Fraction.—A true fraction is less than a whole unit, and therefore its numerator must always be less than its denominator. When this is so the fraction is called a **proper fraction.** Thus $\frac{2}{3}$, $\frac{5}{8}$, $\frac{15}{16}$, $\frac{3}{10}$, are proper fractions.

When a measurement consists of an addition of whole units to a fraction of a unit, as, for instance, two inches and three-quarters of an inch (commonly "two and three-quarter inches"), it could be expressed as $2'' + \frac{3}{4}''$. It is customary, however, to omit the + sign and write the whole number immediately in front of the fraction, thus, $2\frac{3}{4}''$. Care should be taken, when writing in this form, that the figure representing the whole number is written larger than the figures composing the fraction; otherwise the 2 may be confused with the 3 or the 4, and the quantity read as $\frac{23}{4}$ or $\frac{3}{24}$. Numbers written in the form $2\frac{3}{4}$ are termed **mixed numbers.**

In the course of calculation we sometimes arrive at fractions in which the numerator is greater than the denominator, a con-

dition which cannot denote a true fraction, since we have more parts than are contained in our unit. This type of quantity, which is known as an **improper fraction,** is only a statement of a mixed number in fractional form, and may be treated in all calculations *as a proper fraction.* Thus $\frac{15}{8}$, $\frac{17}{16}$, $\frac{111}{5}$, etc., are improper fractions.

To convert a mixed number into an improper fraction proceed thus :—**Multiply the whole number by the denominator and add the numerator. The result of this is the numerator of the improper fraction. The denominator remains the same.** The method consists in finding the total number of equal parts contained in the mixed number and stating this in fractional form. Thus in (*a*), Ex. 1, below, the denominator is 4, or each equal part is a quarter. We therefore convert the 2 into quarters, and as 1 unit = 4 quarters, 2 units = 8 quarters. The total number of quarters is then 8 + 3 = 11, and therefore $2\frac{3}{4} = 11$ quarters, *i. e.*, $\frac{11}{4}$.

Example 1.—Convert into improper fractions (*a*) $2\frac{3}{4}$; (*b*) $11\frac{7}{16}$

(*a*) Whole number × Denominator

$$2 \times 4 = 8$$

add numerator = 3

Sum = 11

$$\therefore 2\tfrac{3}{4} = \underline{\tfrac{11}{4}}$$

This may be stated more mathematically thus,

$$2\tfrac{3}{4} = \frac{(2 \times 4) + 3}{4} = \frac{8 + 3}{4} = \frac{11}{4}$$

(*b*) $$11\tfrac{7}{16} = \frac{(11 \times 16) + 7}{16} = \frac{176 + 7}{16} = \frac{183}{16}$$

With a little practice the operation may be performed mentally.

The reverse operation of converting an improper fraction into a mixed number is, naturally, the opposite of the foregoing, thus :—**Divide the numerator by the denominator, giving the whole number. The remainder of the division is the numerator of the true fractional piece.**

Example 2.—Convert into mixed numbers (*a*) $\frac{11}{4}$; (*b*) $\frac{101}{12}$.

(*a*) Numerator ÷ denominator = whole number.

$$11 \div 4 = 2 \text{ with remainder } 3.$$

$$\therefore \tfrac{11}{4} = \underline{2\tfrac{3}{4}}$$

(*b*) $101 \div 12 = 8$ with 5 remainder.

$$\therefore \tfrac{101}{12} = \underline{8\tfrac{5}{12}}$$

---

* The use of the bracket signs ( ) is given later. Here it is adopted to show definitely that the multiplication of 2 and 4 must be done before adding the 3.

This also may be done mentally after some practice. A special case occurs when the remainder of the division is 0: that is, when the denominator divides into the numerator an exact number of times; then the improper fraction is equal to an exact whole number, *e. g.*, $\frac{32}{16} = 2$ and 0 remaining, $\therefore \frac{32}{16} = 2$.

Exercises 1. On Forms of Vulgar Fractions.

Write out the following numbers, describing each one as a Proper Fraction, an Improper Fraction, or a Mixed Number—

1. $\frac{1}{8}$, $\frac{101}{10}$, $\frac{5}{9}$, $\frac{4}{5}$, $\frac{22}{7}$.
2. $\frac{41}{12}$, $\frac{13}{13}$, $\frac{7}{6}$, $\frac{31}{32}$, $\frac{33}{32}$.
3. $5\frac{2}{3}$, $30\frac{1}{4}$, $\frac{16}{15}$, $\frac{17}{16}$, $1\frac{1}{16}$.
4. $\frac{28}{7}$, $6\frac{1}{4}$, $\frac{9}{8}$, $\frac{3}{17}$, $7\frac{1}{50}$.

Convert the following mixed numbers into improper fractions—

5. $1\frac{1}{2}$, $2\frac{1}{4}$, $1\frac{1}{10}$, $3\frac{1}{8}$, $2\frac{3}{7}$.
6. $5\frac{1}{9}$, $1\frac{7}{16}$, $1\frac{15}{16}$, $2\frac{1}{12}$, $3\frac{3}{11}$.
7. $10\frac{1}{2}$, $15\frac{1}{4}$, $11\frac{3}{8}$, $19\frac{1}{2}$, $7\frac{1}{32}$.
8. $10\frac{9}{10}$, $11\frac{1}{4}$, $17\frac{3}{4}$, $21\frac{3}{8}$, $13\frac{3}{7}$.
9. $14\frac{3}{8}$, $15\frac{11}{12}$, $17\frac{3}{16}$, $38\frac{15}{32}$.
10. $19\frac{11}{16}$, $41\frac{5}{12}$, $16\frac{3}{11}$, $31\frac{9}{16}$.

Convert the following improper fractions into mixed numbers—

11. $\frac{5}{4}$, $\frac{7}{2}$, $\frac{8}{3}$, $\frac{10}{7}$, $\frac{12}{5}$.
12. $\frac{17}{16}$, $\frac{19}{2}$, $\frac{25}{4}$, $\frac{22}{7}$, $\frac{29}{3}$.
13. $\frac{37}{12}$, $\frac{52}{6}$, $\frac{27}{11}$, $\frac{32}{32}$, $\frac{99}{6}$.
14. $\frac{21}{16}$, $\frac{61}{15}$, $\frac{75}{16}$, $\frac{97}{8}$, $\frac{113}{12}$.
15. $\frac{96}{16}$, $\frac{71}{18}$, $\frac{66}{25}$, $\frac{131}{11}$, $\frac{265}{12}$.
16. $\frac{179}{16}$, $\frac{253}{40}$, $\frac{413}{128}$, $\frac{355}{113}$.

Cancelling: Reduction to Lowest Terms.—As a general rule it is advisable to keep the magnitude of the figures in any calculation as small as possible. By this means we are less liable to error, while space, time, and patience are economised. An examination of (*a*), Fig. 1, will show that the length AC is dimenmensioned as $\frac{2}{4} = \frac{1}{2}$, a statement which is easily seen to be true. Also from Fig. 2 it can be seen that—

$$\frac{3}{4} = \frac{6}{8} = \frac{9}{12} = \frac{15}{20}$$

These fractions may also be written as—

$$\frac{3}{4} = \frac{3 \times 2}{4 \times 2} = \frac{3 \times 3}{4 \times 3} = \frac{3 \times 5}{4 \times 5}$$

Thus we may *multiply* the numerator and denominator of a fraction by the *same* number without altering its value, and similarly we may *divide* both numerator and denominator by the *same* number without altering the value of the fraction. This division is termed **cancelling,** and is of great use in reducing fractions to their **lowest terms,** *i. e.*, making the numerator and denominator as small as possible while keeping the value of the fraction the same.

Cancelling is usually shown by drawing a line through the existing numbers, and writing the new numbers immediately above and below the old ones. The operation should be performed very

neatly, so that it is possible to read the original figures afterwards. As a simple illustration take the $\frac{9}{12}$ previously used—

$$\frac{\overset{3}{\cancel{9}}}{\underset{4}{\cancel{12}}} = \frac{3}{4}$$

The reduction may be performed in a single operation by looking for the highest number that will divide into the numerator and denominator exactly; but as much time may be spent in such a search, it may be advisable to divide in steps, using those numbers which can be seen at a glance to be suitable; *c*, Example 3, will

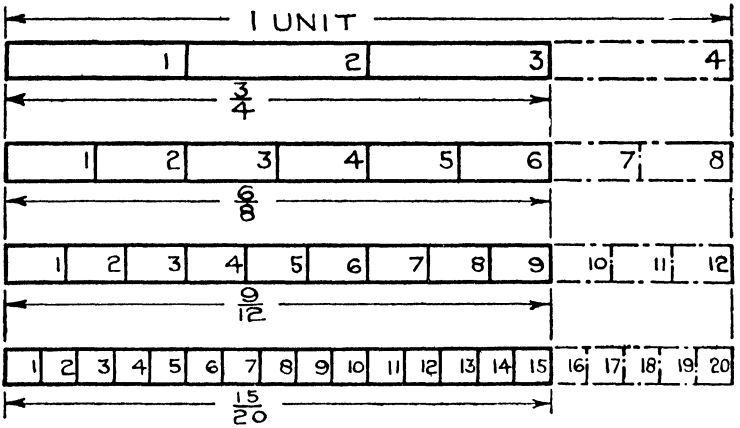


Fig. 2.—Illustrating Cancelling.

illustrate. In connection with this it is useful to remember the following points—

A number is divisible by 2 when the last figure is even, *i. e.* is one of the figures 0, 2, 4, 6 or 8. Thus 32, 248, 500 are divisible by 2.

A number is divisible by 5 when the last figure is 0 or 5; and is divisible by 10 when the last figure is an 0. Thus 25, 40, 2005 are divisible by 5, and 40, 2000, 530 are divisible by 10. The division by 10 is very easily performed by striking off the last 0. Thus $2000 \div 10 = 200$.

A number is divisible by 3 if the sum of its separate figures (or digits) is divisible by 3; and is divisible by 9 if the sum of the digits is divisible by 9.

Thus, taking 51, $5 + 1 = 6$, which may be divided exactly by 3. Therefore 51 is divisible by 3, giving 17 as a result. Taking 957, we have $9 + 5 + 7$ (done mentally) $= 21$, and this is divisible

by 3. Therefore 957 is divisible by 3, giving 319 as a result. Taking 738, the sum of the digits is 18; therefore the number is divisible either by 3 or 9.

A number is divisible by 11 when the sum of the odd digits (*i.e.*, the first, third, and so on) equals the sum of the even digits; or, when the difference between these sums is exactly divisible by 11.

Thus in the number 3575 the odd digits (the 3 and the 7) add up to 10, while the even digits (the two 5's) also add up to 10. The number is then divisible by 11, giving 325 as result. The totalling of the odd and even digits can be done mentally.

In the case of the number 9196 the sum of the odd digits is $9 + 9 = 18$, while that of the even digits is $1 + 6 = 7$. The difference between 18 and 7 is 11, which, of course, is divisible by 11. Then the number 9196 can be divided by 11 without remainder, giving 836 as result.

There are tests for divisibility by other numbers, but they are somewhat complicated and of doubtful value, the test taking longer than cancelling by steps. The rule for 11 is given, as, being a prime factor, cancelling by steps is impossible.

Example 3.—Reduce the following fractions to their lowest terms: (*a*) $\frac{15}{18}$; (*b*) $\frac{51}{63}$; (*c*) $\frac{42}{54}$; (*d*) $\frac{19}{38}$; (*e*) $\frac{141}{120}$.

(*a*) Cancelling by 3, $\frac{\overset{5}{\cancel{15}}}{\underset{6}{\cancel{18}}} = \frac{5}{6}$ (*b*) Cancelling by 3, $\frac{\overset{17}{\cancel{51}}}{\underset{21}{\cancel{63}}} = \frac{17}{21}$

(*c*) Cancelling by 6 in one operation, $\frac{\overset{7}{\cancel{42}}}{\underset{9}{\cancel{54}}} = \frac{7}{9}$

(or by 3 and 2, in two operations).

(*d*) Cancelling by 19, the only possible number, $\frac{\overset{1}{\cancel{19}}}{\underset{2}{\cancel{38}}} = \frac{1}{2}$

(*e*) This is an improper fraction. It is usually better to reduce to a mixed number before cancelling.

$$\text{Thus } \frac{141}{120} = 1\frac{\overset{7}{\cancel{21}}}{\underset{40}{\cancel{120}}} = 1\frac{7}{40}$$

More complicated examples are seldom required.

Exercises 2. On Cancelling.

Reduce the following proper fractions to their lowest terms—

1. $\frac{4}{6}, \frac{3}{9}, \frac{8}{12}, \frac{4}{10}, \frac{6}{16}$.
2. $\frac{12}{16}, \frac{22}{28}, \frac{9}{36}, \frac{7}{21}, \frac{14}{28}$.
3. $\frac{24}{30}, \frac{44}{84}, \frac{11}{33}, \frac{35}{80}, \frac{15}{75}$.
4. $\frac{14}{100}, \frac{108}{144}, \frac{510}{540}, \frac{450}{855}$.
5. $\frac{17}{34}, \frac{13}{39}, \frac{18}{84}, \frac{19}{38}, \frac{57}{81}$.
6. $\frac{156}{224}, \frac{436}{608}, \frac{120}{405}, \frac{132}{165}$.

Simplify the following numbers, reducing them to their lowest terms—

7. $\frac{18}{8}, \frac{28}{6}, \frac{12}{8}, \frac{22}{16}, \frac{36}{32}$.
8. $2\frac{9}{8}, 3\frac{14}{7}, 1\frac{17}{16}, 4\frac{23}{8}, 1\frac{12}{14}$.
9. $2\frac{41}{12}, 5\frac{20}{16}, \frac{122}{12}, \frac{160}{15}, \frac{108}{104}$.
10. $7\frac{37}{12}, 3\frac{50}{88}, \frac{405}{120}, \frac{143}{132}$.
11. $\frac{152}{10}, \frac{5280}{60}, \frac{212}{36}, \frac{48}{16}$,
12. $\frac{33000}{60}, \frac{98}{72}, 11\frac{39}{13}$.

Addition of Vulgar Fractions.—Addition is only possible between quantities of the same kind of measurement. Pounds sterling (£) can only be added to pounds sterling, and feet to feet, etc. No one would attempt to add pounds to feet or tons to shillings. Quantities of the same kind of measurement, but of different units *can* be added, *provided that the sum of the numbers is not found.* Thus £5 and 3 shillings cannot be added and called 8 something, but *can* be stated as £5 3*s.*; to obtain this result there has been no addition of the numbers 5 and 3.

Similarly when dealing with fractions, the denominators (*names*) may be regarded as units; and no addition of numerators (*numbers*) can be performed until the denominators or units are the same. This is usually stated by saying that the fractions to be added must have a *common denominator.* To take a very simple example, let it be required to add $\frac{1}{8}$ to $\frac{3}{16}$. Either the eighths must be changed into sixteenths, or the sixteenths into eighths. Now it is seldom, if ever, convenient to convert any fraction into one of smaller denominator (*i. e.* larger equal parts). To illustrate this let us convert the sixteenths into eighths.

Then, since $\frac{2}{16} = \frac{1}{8}$, $\frac{3}{16} = 1\frac{1}{2}$ eighths, and we may therefore write—

$$\frac{1}{8} + \frac{3}{16} = \frac{1}{8} + \frac{1\frac{1}{2}}{8} = \frac{2\frac{1}{2}}{8},$$

a form which, with any but the simplest numbers, would become hopelessly muddled.

But let us convert into *smaller parts, i. e.*, convert the eighths into sixteenths.

Then we may write—

$$\frac{1}{8} + \frac{3}{16} = \frac{2}{16} + \frac{3}{16} = \frac{5}{16},$$

a practical proof of this being easily obtained by an inspection of the common 12-inch rule. It will be noticed that the numbers which compose the fraction $\frac{5}{16}$ (*i. e.*, 5 and 16) are entirely *whole* numbers.

Example 4.—The tapping size (size of a hole before being screwed) of a certain Whitworth bolt is given as $\frac{5}{16}'' + \frac{3}{64}''$. Express this as a single fraction.

$$\frac{5}{16} + \frac{3}{64}$$
$$= \frac{20}{64} + \frac{3}{64} \text{ converting the } \frac{1}{16}^{\text{ths}} \text{ into } \frac{1}{64}^{\text{ths}}, \frac{1}{16} \text{ being } \frac{4}{64}$$
$$= \frac{23}{64}''.$$

Exercises 3. On Addition of Fractions.

Simple Cases

1. Drilled bolt holes in pipe flanges are $\frac{1}{8}''$ larger than the bolts for bolts over $\frac{5}{8}''$ diameter. What sized holes are required for the following bolts: (*a*) $\frac{7}{8}''$, (*b*) $\frac{3}{4}''$, (*c*) $1\frac{1}{8}''$?

2. For bolts up to $\frac{1}{8}''$ diameter the drilled holes are $\frac{1}{16}''$ larger than the bolt. What sized holes are required for the following bolts: (*a*) $\frac{9}{16}''$, (*b*) $\frac{5}{8}''$, (*c*) $\frac{1}{2}''$?

3. The diameter of the tapping hole for certain Whitworth bolts and nuts is given as follows: (*a*) $\frac{1}{2}''$ bolt, diameter of tapping hole $= \frac{3}{8} + \frac{1}{32}$; (*b*) $\frac{9}{16}''$ bolt, diameter of tapping hole $= \frac{7}{16} + \frac{1}{32}$. Express each of these diameters as a single fraction.

4. The thickness of the head in certain Whitworth standard bolts and nuts is given as follows: (*a*) $\frac{1}{4}''$ bolt, thickness of nut $= \frac{3}{16} + \frac{1}{32}$; (*b*) $\frac{3}{8}''$ bolt, thickness of nut $= \frac{5}{16} + \frac{1}{64}$. Express each of these sizes as a single fraction.

Express the following Whitworth bolt-head thicknesses as a single fraction in each case:—

5. (*a*) $\frac{3}{4}''$ bolt. Head is $\frac{5}{8} + \frac{1}{32}$. (*b*) $1\frac{1}{8}''$ bolt. Head is $\frac{9}{16} + \frac{1}{32}$.

6. (*a*) $\frac{5}{8}''$,, ,, ,, $\frac{1}{2} + \frac{3}{64}$. (*b*) $1\frac{3}{16}''$,, ,, ,, $1\frac{1}{16} + \frac{3}{64}$.

7. (*a*) $\frac{15}{16}''$,, ,, ,, $\frac{1}{4} + \frac{1}{64}$. (*b*) $1\frac{1}{8}''$,, ,, ,, $1\frac{5}{16} + \frac{3}{64}$.

Express the following Whitworth tapping sizes as a single fraction in each case—

8. (*a*) 1″ bolt, tapping size is $\frac{13}{16} + \frac{1}{32}$. (*b*) $1\frac{1}{16}''$ bolt, tapping size is $\frac{9}{16} + \frac{1}{64}$.

9. (*a*) $\frac{5}{8}''$,, ,, ,, ,, $\frac{1}{2} + \frac{1}{64}$. (*b*) $\frac{3}{8}''$,, ,, ,, ,, $\frac{1}{4} + \frac{3}{64}$.

10. (*a*) $\frac{7}{8}''$,, ,, ,, ,, $\frac{11}{16} + \frac{3}{64}$. (*b*) $1\frac{5}{16}''$,, ,, ,, ,, $\frac{3}{4} + \frac{3}{64}$.

A rather different appearance is presented by the case $\frac{1}{4} + \frac{2}{5}$. We cannot convert the quarters into fifths or the fifths into quarters without complicating the figures; but we can obviously give each fraction the same denominator by multiplying the 4 by 5, and the 5 by 4, *i. e.* multiplying the denominators together, giving 20 in each case. Then, remembering that the value of a fraction is unaltered if both top and bottom be multiplied by the *same number*, the example may be written thus—

$$\frac{1}{4} + \frac{2}{5} = \frac{1 \times 5}{4 \times 5} + \frac{2 \times 4}{5 \times 4}$$
$$= \frac{5}{20} + \frac{8}{20} = \frac{13}{20}$$

In Fig. 3 this is shown graphically.

The foregoing method may be applied to *any* example, but in the more difficult cases the labour and figuring may be reduced by employing the *Least Common Denominator* (abbreviated to L.C.D.). As an illustration, consider the addition of the fractions $\frac{5}{12}$ and $\frac{3}{8}$. Working on the previous method, the common denominator would be $12 \times 8 = 96$. Now a little thought will show that the numbers 48 and 24 *also* contain 12 and 8 an exact number of times, so that for the purpose of addition the fractions may equally well be converted into forty-eighths or into twenty-fourths. By using 24 as

the common denominator instead of 96, smaller numbers will be produced, and the calculation simplified in consequence. This 24 is then the *smallest possible* common denominator, and is the **Least Common Multiple** (abbreviated to L.C.M.) of the given denominators. Now the L.C.M. of a set of numbers is defined as the *least number which will contain each of the given ones an exact number of times*, so, keeping this in view, we may deduce the 24 from the given numbers 12 and 8.

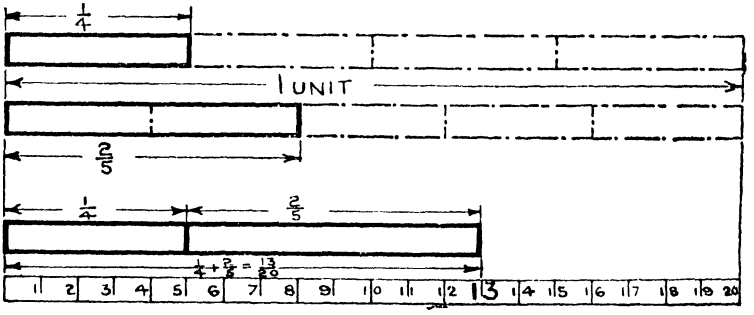


Fig. 3.—Showing that $\frac{1}{4} + \frac{2}{5} = \frac{13}{20}$.

The required L.C.M. is to contain the numbers 12 and 8 exactly, and must therefore contain the factors of these numbers (*see* p. 3). Splitting into prime factors, we have—

$$12 = 2 \times 2 \times 3 \qquad 8 = 2 \times 2 \times 2$$

Now *all* the factors of 12 will be required, since the 12 itself must divide into the L.C.M. But in the case of the 8, two of the factors *already appear* in the 12, viz. 2×2, and need not, therefore, be used again. The remaining 2 in the 8 does not appear in the 12, and must therefore be counted. Then the L.C.M. of 12 and $8 = 2 \times 2 \times 3 \times 2 = 24$.

The detailed method of finding an L.C.M. is laid out in the following example:—

Example 5.—To find the L.C.M. of 8, 15, 4 and 12.

$8 \quad 15 \quad 4 \quad 12$

$8 = 2 \times 2 \times 2$

$15 = 3 \times 5$

$12 = 2 \times 2 \times 3$

L.C.M. $= 2 \times 2 \times 2 \times 3 \times 5$

$= 8 \times 3 \times 5$

$= 120$

Explanation of the Method.—Examine the given numbers to see if any one will divide exactly into any other one; if so, cross it

out. Thus 4, crossed out, will divide exactly into 8 and 12, so that if the L.C.M. contains 8 it *must* contain 4; we may therefore neglect such numbers. Then split each of the remaining numbers into its *prime factors* (*see* p. 3) as shown. The L.C.M. is to contain all the numbers, and so must contain the 8. The factors of the first number are then left alone. Now examine the prime factors of the second number (15) and see if any of them already exist in the first number. If so, cross these out.* In this case neither the 3 nor the 5 appears in the factors of 8, and so are left in. Next examine the prime factors in the third number and cross out any that still appear in the first and second numbers. Thus the 3 already appears in the factors of 15 and the two 2's already appear in the factors of 8. All these three factors are then crossed out. The process is repeated until all the prime factors have been examined, when the L.C.M. is obtained by multiplying up all the factors which are left (in this case $2 \times 2 \times 2 \times 3 \times 5$), giving 120.

Example 6.—Find the L.C.M. of 8, 12, 16, 6 and 18.

~~8~~ 12 16 ~~6~~ 18

$12 = 2 \times 3 \times 2$ L.C.M. $= 2 \times 3 \times 2 \times 2 \times 2 \times 3$

$16 = \not{2} \times \not{2} \times 2 \times 2$ $= \underline{144}$

$18 = \not{2} \times \not{3} \times 3$.

Note.—The numbers 8 and 6 are crossed out, being contained in 16 and 18 (or 12) respectively. The factors of 12 include two 2's; then two of the 2's forming the factors of 16 are crossed out. The other pair *must be left in*, as the L.C.M. must contain four 2's if it is to contain 16, and in the first line only two 2's appear. In the third line the 2 and *one* 3 are cut out. Only one 3 appears in the first two lines, and so only one may be cut out. Finally, the L.C.M. is the product of the remaining factors = 144 as shown.

Exercises 3 (contd.). On L.C.M.

Find the L.C.M. in each of the following cases—

11. 4, 8, 6. **12.** 4, 3, 6, 2. **13.** 9, 3, 4, 8.
14. 16, 9, 8. **15.** 12, 10, 5. **16.** 14, 21, 7, 2.
17. 16, 48, 18. **18.** 55, 11, 15. **19.** 100, 150, 75.
20. 3, 51, 9, 17. **21.** 34, 17, 3. **22.** 33, 15, 3.
23. 125, 25, 150. **24.** 280, 14, 150, 50.
25. 30, 180, 45, 9, 5. **26.** 3, 17, 51, 34, 9.

Applying the method to addition problems, the L.C.M. of the denominators is first obtained, and each fraction is converted so that it shall have the L.C.M. for its denominator, *i. e.* the common denominator is the *Least* Common Denominator.

* See Note to Example 6.

Example 7.—Add together $\frac{3}{8}$, $1\frac{2}{15}$, $\frac{1}{4}$, and $1\frac{5}{12}$.

The L.C.D. has been shown in Example 5 to be 120. Dividing this L.C.D. 120 by each denominator in turn, gives the number 15, which is required to convert the fraction.

Thus, $\frac{120}{8} = 15$, and therefore $\frac{3}{8} = \frac{3 \times 15}{8 \times 15} = \frac{45}{120}$

and similarly for the others.

$$\therefore \quad \tfrac{3}{8} + 1\tfrac{2}{15} + \tfrac{1}{4} + 1\tfrac{5}{12}$$

$$= \frac{3 \times 15}{8 \times 15} + \frac{2 \times 8}{15 \times 8} + \frac{1 \times 30}{4 \times 30} + \frac{5 \times 10}{12 \times 10}$$

$$= \frac{45}{120} + \frac{16}{120} + \frac{30}{120} + \frac{50}{120}$$

$$= \tfrac{141}{120} = 1\tfrac{21}{120}, \text{ or cancelling by 3, } = 1\tfrac{7}{40}$$

These steps may be omitted or shortened. See a few lines further on.

In an actual example the working would not be shown as above, in which there is a considerable amount of repetition for explanatory purposes. The common denominator need only be written once if a long line be placed above it instead of the several short lines. Also the actual multiplication of numerators may, in most practical cases, be performed mentally. The working would then appear in the following form:—

$$\tfrac{3}{8} + 1\tfrac{2}{15} + \tfrac{1}{4} + 1\tfrac{5}{12}$$

$$= \frac{45 + 16 + 30 + 50}{120} = \frac{141}{120}$$

$$= 1\tfrac{21}{120} = 1\tfrac{7}{40}$$

Example 8.—In connection with an electrical resistance problem it was found necessary to evaluate (*i. e.*, find the value of) the expression $\frac{1}{4} + \frac{1}{2} + 1\frac{1}{18}$. Calculate the required value.

$$\tfrac{1}{4} + \tfrac{1}{2} + 1\tfrac{1}{18}$$

$$= \frac{9 + 18 + 2}{36} = \frac{29}{36}$$

4, 2, 18

$4 = 2 \times 2$

$18 = 2 \times 3 \times 3$

$\therefore$ L.C.M. $= 2 \times 2 \times 3 \times 3$

$= 36.$

Explanation.—The L.C.M. is quite simple. Taking the first fraction, 4 into 36 goes 9. Then $9 \times 1 = 9$, which is written above the long line. Then 3 into 36 goes 12, and $12 \times 1 = 12$, which appears as shown. Lastly, 18 into 36 goes 2, and $2 \times 1 = 2$. Addition of the numerators completes the example.

Where the given quantities include whole numbers, then the easier and safer method is to add the whole numbers independently of the fractions. Usually the whole numbers may be

added mentally. *It is unwise to convert the mixed numbers into improper fractions*, as unwieldy figures result, and the subsequent labour is greatly increased. Where improper fractions *are* given (an unlikely condition), they should be converted into mixed numbers.

Example 9.—Find the value of $3\frac{7}{8} + 4 + 7\frac{1}{2} + 6\frac{1}{3}$.

Add the whole numbers mentally and write the result in front of the addition of the fractions, thus—

$3 + 4 + 7 + 6 = 20$

The expression then $= 20\frac{21 + 12 + 8}{24}$

$= 20\frac{41}{24}$

$= 20 + 1\frac{17}{24}$ May often be done mentally.

$= 21\frac{17}{24}$

8, 2, 3
L.C.M. $= 8 \times 3$
$= 24$

Example 10.—The dimensions of the spindle for a sluice valve are shown in Fig. 4. Determine the overall (*i. e.*, total) length of the spindle.

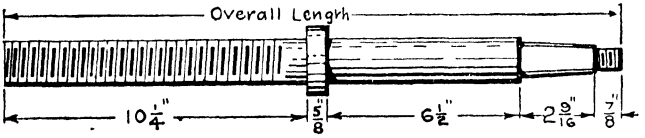


Fig. 4.—Spindle for a Sluice Valve.

Writing down the dimensions in order we have—

$10\frac{1}{4} + \frac{5}{8} + 6\frac{1}{2} + 2\frac{9}{16} + \frac{7}{8}$

$= 18\frac{4 + 10 + 8 + 9 + 14}{16}$

$= 18\frac{45}{16} = 18 + 2\frac{13}{16} = 20\frac{13}{16}$"

¼, ⅝, ½, 16, ⅞
L.C.M. = 16

Examples like this are of frequent occurrence when dealing with drawings, and with practice they can be done mentally. It is then necessary to be thoroughly familiar with the ordinary measurements of $\frac{1}{8}$, $\frac{1}{16}$, etc., and to know at once the simpler conversions such as $\frac{5}{8} = \frac{10}{16} = \frac{20}{32}$, etc. Such knowledge is obtained by doing a considerable amount of mechanical drawing. Then the above example would be done mentally as follows: $10\frac{1}{4} + \frac{5}{8}$ is $10\frac{7}{8}$, add 6 gives $16\frac{7}{8}$, add $\frac{1}{2}$ gives $17\frac{3}{8}$, add 2 gives $19\frac{3}{8}$. Leave the $\frac{9}{16}$ till the last as being the most troublesome figure. Then $19\frac{3}{8}$ and $\frac{7}{8}$ gives $20\frac{1}{4}$; and $20\frac{1}{4}$ and $\frac{9}{16}$ is $20\frac{4}{16} + \frac{9}{16} = 20\frac{13}{16}$.

Exercises 3 (contd.). Addition of Fractions with L.C.M.

Find the value of the following :—

27. $\frac{1}{4} + \frac{2}{5} + \frac{3}{10}$. **28.** $\frac{3}{8} + 1\frac{5}{12} + \frac{1}{24}$. **29.** $\frac{2}{15} + \frac{3}{10} + \frac{1}{8}$.

30. $\frac{3}{4} + \frac{3}{8} + 3\frac{5}{12}$. **31.** $1\frac{3}{16} + \frac{25}{48} + \frac{5}{18}$. **32.** $9\frac{5}{8} + \frac{13}{16} + 3\frac{1}{2}$.

33. $1\frac{1}{12} + \frac{3}{8} + \frac{1}{10}$. **34.** $1\frac{3}{8} + 5\frac{1}{16} + \frac{3}{20} + \frac{1}{4}$. **35.** $\frac{9}{16} + 1\frac{1}{8} + \frac{1}{96} + 5\frac{3}{8}$.

36. In finding the current required for some electric glow lamps fed in parallel, it was necessary to obtain the value of the expression $\frac{1}{100} + \frac{1}{110} + \frac{7}{75}$. Find this value.

37. Find the overall length in inches of the lathe mandrel shown at *a*, Fig. 5. Convert the result into feet and inches.

38. *b*, Fig. 5, shows an eccentric strap bolt. Find its overall length in inches.

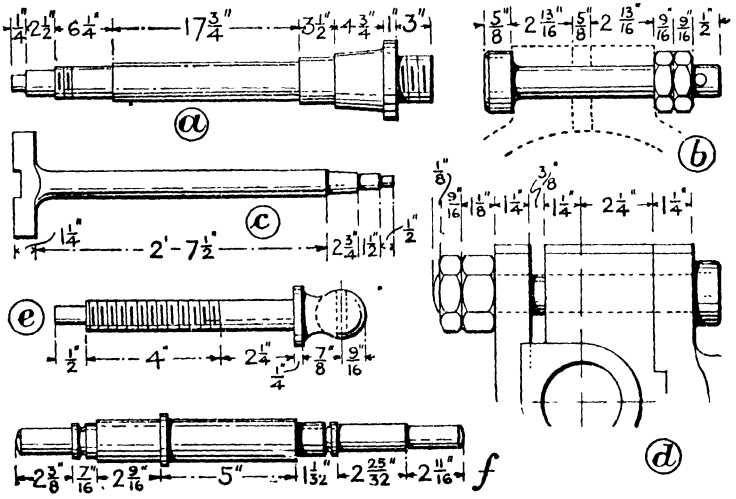


Fig. 5.—Exercises on Addition of Vulgar Fractions.

39. Find the overall length in feet and inches of the piston rod shown at *c*, Fig. 5.

40. Find the length of the cross-head bolt, shown at *d*, Fig. 5, from the underside of the head to the point.

41. The vice screw shown at *e*, Fig. 5, is to be cut from the solid bar. Allowing an extra inch of length for holding, etc., what length of bar must be cut off?

42. Find the overall length of the armature shaft for an electric motor, shown at *f*, Fig. 5.

Subtraction of Vulgar Fractions.—This involves no new principle, the method of finding the L.C.D., and the separate treatment of whole numbers, being adopted as in addition. An example is the best explanation.

Example 11.—Find the value of $5\frac{7}{12} - 2\frac{3}{8}$.

$$5\tfrac{7}{12} - 2\tfrac{3}{8}$$
$$= 3\frac{14 - 9}{24} = 3\tfrac{5}{24}$$

$$12 = 2 \times 2 \times 3$$
$$8 = 2 \times 2 \times 2$$
$$\therefore \text{L.C.M.} = 2 \times 2 \times 3 \times 2$$
$$= 24$$

Explanation.—Taking the whole numbers first, $5 - 2 = 3$. L.C.M. $= 24$. Then 12 into 24 goes 2, and $7 \times 2 = 14$. Similarly 8 into 24 goes 3 and $3 \times 3 = 9$. Then 9 from $14 = 5$.

Example 12.—From $20\frac{1}{30}$ take $13\frac{7}{12}$.

$$20\tfrac{1}{30} - 13\tfrac{7}{12}$$
$$= 7\frac{22 - 35}{60}$$
$$= 6\frac{82 - 35}{60} = 6\tfrac{47}{60}$$

$$30 = 2 \times 3 \times 5$$
$$12 = 2 \times 2 \times 3 \times 2$$
$$\text{L.C.M.} = 30 \times 2$$
$$= 60$$

Explanation.—As before, the whole numbers are dealt with first, followed by the finding of the L.C.M., and the conversions as shown. A special point arises in the second line. We cannot take 35 from 22, but it must be remembered that the statement means $7\frac{22}{60} - \frac{35}{60}$, which of course is possible. The subtraction may be performed by converting a unit into sixtieths, and adding this unit to the fraction $\frac{22}{60}$, as shown in the last line, giving $\frac{82}{60}$, from which $\frac{35}{60}$ can be subtracted.

A short combination of addition and subtraction presents but little difficulty, and is illustrated by the following example—

Example 13.—Evaluate $15\frac{3}{4} - 3\frac{1}{9} + 1\frac{1}{3}$.

Whole numbers $15 - 3 + 1 = 13$

$$\text{Expression} = 13\frac{27 - 4 + 12}{36}$$
$$= 13\frac{39 - 4}{36} = 13\tfrac{35}{36}$$

$$4 \quad 9 \quad 3$$
$$\text{L.C.M.} = 4 \times 9$$
$$= 36$$

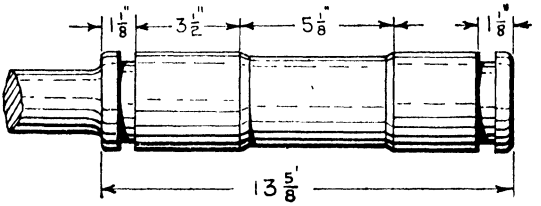


Fig. 6.—Rock Drill Piston.

Example 14.—Part of a rock drill piston is shown in Fig. 6, and by a draughtsman's oversight a dimension has been omitted. Assuming that all the other figures are correct find the value of the missing dimension.

It is evident from the picture that if the sum of the small lengths

be found, and subtracted from the "overall" distance, this will give the value of the missing quantity.

Then $1\frac{1}{8} + 3\frac{1}{2} + 5\frac{1}{8} + 1\frac{1}{8} = 10\frac{1+4+1+1}{8} = 10\frac{7}{8}$.

Then missing dimension $= 13\frac{5}{8} - 10\frac{7}{8}$

$= 3\frac{5-7}{8} = 2\frac{13-7}{8} = 2\frac{6}{8}$,

$\therefore$ Missing dimension $= 2\frac{3}{4}''$

Note.—With practice this type of example can also be done mentally.

With a more lengthy combination it may not be easy for a beginner to deal with the whole numbers mentally. Then it is best to find the sum of the whole numbers to be added, and the sum of the whole numbers to be subtracted; the final subtraction being simple. The numerators of the converted fractions may be dealt with in the same way.

An example will illustrate—

Example 15.—Find the value of $\frac{5}{6} + 2\frac{3}{5} - 5\frac{7}{12} - 3\frac{3}{20} + 9\frac{11}{15}$.

Whole numbers: $2 - 5 - 3 + 9 = 11 - 8 = 3$.

Expression $= 3\frac{50 + 36 - 35 - 9 + 44}{60}$

$= 3\frac{130 - 44}{60} = 3\frac{86}{60}$

$= 4\frac{26}{60}$ or $4\frac{13}{30}$

6	5 12 30 15
$12 = 2 \times 2 \times 3$	
$30 = 2 \times 3 \times 5$	
L.C.M. $= 12 \times 5 = 60$	

Note.—Taking the whole numbers first, 2 and 9 are to be added, and 5 and 3 to be subtracted. Combining these pairs gives $11 - 8 = 3$, as shown. After converting to the L.C.M., all the plus numerators are added, giving 130, and all the minus numerators, giving 44. Then the final subtraction gives 86.

Exercises 4. On Subtraction of Vulgar Fractions.

1. The correct size of tapping hole for a certain screw is $\frac{13}{32}''$. If a $\frac{7}{16}''$ drill is the nearest size available, by how much will the hole be too large?

2. By how much will a hole be too large if a $\frac{13}{16}''$ hole be drilled when the correct size is $\frac{51}{64}''$?

3. A $\frac{3}{8}''$ hole is drilled for tapping when the size should be $\frac{23}{64}''$. What is the difference in size, and is the hole larger or smaller than it should be?

Find the value of the following:—

4. $\frac{5}{6} - \frac{1}{3}$. **5.** $\frac{5}{8} - \frac{1}{8}$. **6.** $\frac{7}{16} - \frac{1}{8}$. **7.** $\frac{3}{10} - \frac{2}{15}$.

8. $1\frac{5}{8} - \frac{3}{8}$. **9.** $2\frac{1}{8} - 1\frac{9}{16}$. **10.** $\frac{14}{25} - \frac{3}{10}$. **11.** $4\frac{3}{5} - 2\frac{7}{9}$.

12. $2\frac{3}{4} - 1\frac{7}{8}$. **13.** $1\frac{9}{10} - \frac{7}{8}$. **14.** $9\frac{3}{14} - 6\frac{13}{15}$.

15. From $3\frac{3}{7}$ take $2\frac{3}{4}$.

16. From $1\frac{15}{16}$ take $\frac{21}{32}$.

17. From $1\frac{1}{8}$ take $\frac{7}{20}$.

18. Subtract $23\frac{1}{4}$ from $30\frac{1}{2}$.

19. Subtract $4\frac{11}{16}$ from $5\frac{35}{64}$.

20. Subtract $\frac{13}{16}$ from $1\frac{7}{8}$.

Find the value of the following:—

21. $3\frac{1}{4} - \frac{5}{8} + 1\frac{1}{16}$. **22.** $\frac{13}{32} - \frac{5}{8} + 30\frac{7}{4}$. **23.** $\frac{10}{11} + \frac{1}{3} - \frac{14}{33}$.
24. $\frac{4}{15} - \frac{2}{3} + 1\frac{1}{5}$. **25.** $2\frac{5}{8} - 1\frac{1}{4} - 1\frac{1}{12}$. **26.** $4\frac{7}{12} + 5\frac{1}{8} - 7\frac{7}{16}$.
27. $12 - 3\frac{1}{4} + \frac{11}{32} - \frac{3}{8}$. **28.** $\frac{13}{20} - 2\frac{3}{10} - \frac{4}{5} + 5\frac{1}{4}$.
29. $3\frac{7}{12} - \frac{3}{16} + 1\frac{5}{8} + 2\frac{3}{4} - \frac{5}{6}$. **30.** $1\frac{9}{40} + \frac{3}{8} - 2\frac{17}{20} + 3\frac{3}{5} - \frac{7}{10}$.
31. Find the value of $3\frac{1}{16} - 1\frac{1}{4}$ and take the result from $2\frac{1}{32}$.
32. ,, ,, ,, $\frac{3}{8} - \frac{1}{5}$,, ,, ,, $\frac{9}{20}$.
33. Take the result of $1\frac{3}{16} + \frac{2}{5}$ from $3\frac{3}{20}$.
34. ,, ,, ,, $9\frac{3}{4} + \frac{5}{12}$,, $13\frac{5}{8}$.

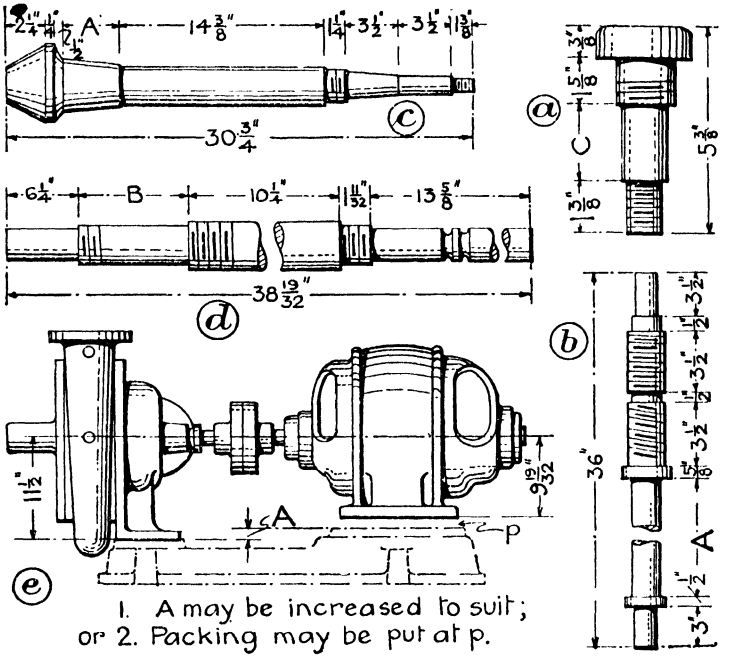


Fig. 7.—Exercises on Subtraction of Vulgar Fractions.

35. From $2\frac{7}{12}$ take the result of $\frac{2}{3} + \frac{5}{8}$.
36. ,, $21\frac{3}{4}$,, ,, ,, $3 + 4\frac{1}{4} + 2\frac{5}{8}$.
37. ,, $37\frac{5}{8}$,, ,, ,, $3\frac{1}{2} + 21\frac{5}{8} + \frac{3}{16}$.
38. From the result of $2\frac{1}{16} - \frac{3}{32}$ take $1\frac{11}{16}$.
39. ,, ,, ,, $\frac{7}{10} + \frac{3}{4}$,, $\frac{3}{20}$.
40. ,, ,, ,, $\frac{1}{3} - \frac{3}{11}$,, $\frac{9}{33}$.

41. *a*, Fig. 7, shows a screw from a lathe head-stock. Find the length of the part C.

42. The valve spindle for a stop valve is shown at *b*, Fig. 7. Find the length of the plain portion marked A.

43. A milling machine spindle is shown at *c*, Fig. 7, where by a

draughtsman's oversight the length A of the front bearing has been omitted. Find this value from the other dimensions.

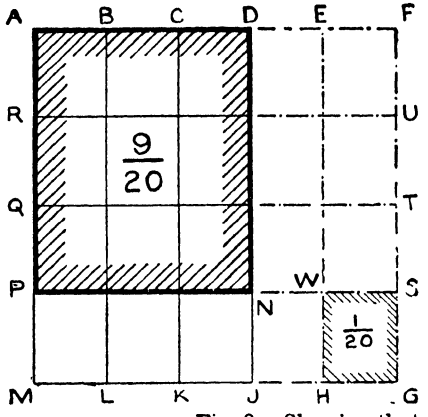
44. Find the value of the missing dimension B from the drawing of the electric motor shaft shown at *d* in Fig. 7.

45. An electric motor and a centrifugal pump, built by different makers, are to be assembled on a single bedplate as shown at *e*, Fig. 7, which also gives dimensions from centre line to base.

(*a*) Supposing a special bedplate is to be made to suit, find what the dimension A must be.

(*b*) Supposing that a standard bedplate, in which the dimension A is $1\frac{3}{4}$", be used, find what packing thickness is required under the motor feet.

Multiplication of Vulgar Fractions.—Multiplication is a convenient method of performing a long series of additions of equal quantities. Thus, 5×4 is a shorter way of stating both $5 + 5 + 5 + 5$ (*i. e.*, 5 added 4 times) and $4 + 4 + 4 + 4 + 4$



Large Area A F G M represents 1 UNIT. Then:—
Area A D J M = $\frac{3}{5}$,
and shaded area
A D N P = $\frac{3}{4}$ of A D J M
= $\frac{3}{4}$ of $\frac{3}{5}$
= $\frac{9}{20}$

Fig. 8.—Showing that $\frac{3}{4} \times \frac{3}{5} = \frac{9}{20}$.

(*i. e.*, 4 added 5 times). The multiplication of a fraction by a whole number may be shown the same way.

Thus, $\frac{3}{4} \times 5 = \frac{3}{4} + \frac{3}{4} + \frac{3}{4} + \frac{3}{4} + \frac{3}{4}$

$$= \frac{3 + 3 + 3 + 3 + 3}{4} = \frac{3 \times 5}{4}$$

$$= \frac{15}{4} = 3\frac{3}{4}$$

So that the result is obtained by multiplying the numerator by the whole number.

The multiplication of two fractions, *e. g.*, $\frac{3}{4} \times \frac{3}{5}$, is very conveniently shown graphically, as in Fig. 8. The multiplication sign ($\times$) may be replaced by the word "of," for $\frac{3}{4} \times \frac{3}{5}$ is $\frac{3}{5}$ taken $\frac{3}{4}$ of a "time"; the result will naturally be less than $\frac{3}{5}$. In Fig. 8,

the square A F G M represents 1 unit, which is divided by vertical lines B L, C K, D J, and E H, into fifths. Then the area A D J M represents the fraction $\frac{3}{5}$. Similarly the horizontal lines R U, Q T, and P S, divide both the unit and the area A D J M into 4 equal parts. Therefore area A D N P is $\frac{3}{4}$ of area A D J M, *i. e.*, $\frac{3}{4}$ of $\frac{3}{5}$ of a unit. The two sets of lines cut the unit square into 20 small figures such as H G S W, and in the area A D N P there are 9 such figures—

$$\therefore \frac{3}{4} \text{ of } \frac{3}{5} = \frac{9}{20}$$

But the 9 is the product of the numerators 3 and 3, and the 20 is the product of the denominators 4 and 5.

Hence the rule for the multiplication of vulgar fractions—

Multiply the numerators together to give the new numerator, and the denominators together to give the new denominator.

Reduction to lowest terms should follow, but in many cases the cancelling may be more conveniently performed *before* multiplying up. Thus in the following we first cancel by 3:—

$$\frac{\overset{3}{\cancel{9}}}{16} \times \frac{7}{\underset{4}{\cancel{12}}} = \frac{3 \times 7}{16 \times 4} = \frac{21}{64}$$

The method is applicable to any number of fractions. When mixed numbers occur, it is usually advisable to convert them into improper fractions.

Example 16.—Find the product of $\frac{2}{3}$, $1\frac{1}{7}$, $2\frac{1}{4}$ and $\frac{4}{11}$.

$$\frac{2}{3} \times 1\frac{1}{7} \times 2\frac{1}{4} \times \frac{4}{11}$$
$$= \frac{2}{3} \times \frac{\overset{1}{\cancel{8}}}{7} \times \frac{\overset{3}{\cancel{9}}}{\underset{1}{\cancel{4}}} \times \frac{\cancel{4}}{\cancel{11}} = \frac{2 \times 3}{7} = \frac{6}{7}$$

Cancelling being performed with 11, 4 and 3.

Example 17.—If 1 kilogramme is $2\frac{1}{5}$ lbs., find the number of lbs. in 235 kilogrammes.

Evidently 235 kilogrammes = 235 times as much as 1 kilogramme.
Then as 1 kilogramme = $2\frac{1}{5}$ lbs.
235 kilogrammes = $2\frac{1}{5} \times 235$ lbs.

$$= \frac{11}{\cancel{5}} \times \overset{47}{\cancel{235}}$$
$$= \frac{11}{1} \times 47 = \underline{517 \text{ lbs.}}$$

The cancelling should always be shown *neatly*, so that the original figures may be easily read. Care in this direction enables the working to be checked through, for possible errors, by any one. Examples involving heavy cancelling are met with when calculating

the speeds of various combinations of gear wheels, such as occur in the feed motions of machine tools, variable speed gears, etc. In examples of this kind, special care must be taken and plenty of space allowed above and below the expression so that the cancelling may not be too confined.

Example 18.—The following figures were obtained when calculating the feed in inches of a drilling machine, for one revolution of the drill. Complete the calculation.

$$\frac{33}{20} \times \frac{15}{22} \times \frac{1}{16} \times \frac{1}{30} \times 16 \times \frac{5}{8}$$

$$\frac{\overset{1}{\cancel{3}}}{\underset{4}{\cancel{20}}} \times \frac{\overset{1}{\cancel{15}}}{\underset{1}{\cancel{22}}} \times \frac{1}{\underset{1}{\cancel{16}}} \times \frac{1}{\underset{10}{\cancel{30}}} \times \frac{\overset{1}{\cancel{16}}}{\underset{1}{\cancel{1}}} \times \frac{\overset{1}{\cancel{5}}}{\underset{1}{\cancel{8}}}$$

$$= \frac{1}{20} \times \frac{1}{10} = \underline{\frac{1}{200} \text{ inch per revolution.}}$$

It must be noted that "multiplication" *does not necessarily imply an "increase."* If the multiplier is itself a fraction, then the resulting product is only a piece of the quantity multiplied, in which case a *decrease* results.

Exercises 5. On Multiplication of Vulgar Fractions.

Find the value of—

1. $\frac{2}{5} \times 7$. **2.** $\frac{7}{16} \times 5$. **3.** $\frac{5}{33} \times 3$. **4.** $\frac{17}{20} \times 4$.

5. $\frac{2}{3} \times \frac{5}{8}$. **6.** $\frac{4}{5} \times \frac{7}{12}$. **7.** $\frac{25}{27} \times \frac{18}{55}$. **8.** $\frac{13}{11} \times \frac{7}{39}$.

9. $1\frac{2}{3} \times \frac{14}{15}$. **10.** $\frac{5}{16} \times 2\frac{3}{10}$. **11.** $3\frac{3}{8} \times 1\frac{5}{9}$. **12.** $5\frac{1}{5} \times 1\frac{2}{13}$.

13. $\frac{19}{5} \times 1\frac{7}{8} \times \frac{4}{3}$. **14.** $\frac{1}{16} \times \frac{33}{25} \times \frac{4}{11}$.

15. $2\frac{1}{4} \times 1\frac{1}{15} \times \frac{3}{8} \times \frac{1}{4}$. **16.** $\frac{5}{8} \times \frac{3}{16} \times 5\frac{1}{3} \times 1\frac{1}{4}$.

17. Find the value of $1\frac{1}{2} \times 1\frac{1}{3} \times 1\frac{1}{5} \times 1\frac{1}{6} \times 1\frac{1}{7}$.

The following multiplications refer to certain spur wheel trains. Complete the calculations.

18. $\frac{120}{11} \times \frac{132}{10} \times \frac{48}{8}$. **19.** $500 \times \frac{1}{55} \times \frac{22}{75}$.

20. $1\frac{1}{45} \times \frac{20}{33} \times \frac{27}{35}$. **21.** $\frac{35}{100} \times \frac{20}{80} \times \frac{48}{70}$.

The following multiplications give the feed in inches per revolution of the mandrel in an 8″ self-acting lathe. Complete the calculation.

22. Greatest surfacing feed $= \frac{64}{32} \times \frac{1}{36} \times \frac{54}{18} \times \frac{1}{2}$.

23. Lowest surfacing feed $= \frac{32}{64} \times \frac{1}{36} \times \frac{54}{18} \times \frac{1}{2}$.

24. Greatest traversing feed $= \frac{64}{32} \times \frac{1}{36} \times \frac{20}{70} \times 14 \times \frac{3}{8}$.

25. Lowest traversing feed $= \frac{32}{64} \times \frac{1}{36} \times \frac{20}{70} \times 14 \times \frac{3}{8}$.

26. The speed of a certain lathe spindle, when driving through the back gear, is given by the value of the expression—

$$140 \times \frac{6}{16} \times \frac{18}{54} \times \frac{18}{54} \text{ r.p.m.}$$

Find the value required.

27. The feed of a drilling machine, in inches, for one revolution of the drill spindle, is given by the figures : $\frac{30}{17} \times \frac{17}{12} \times \frac{1}{75} \times \frac{1}{40} \times 16 \times \frac{5}{8}$. Complete the calculation.

28. (*a*) The finest measurement that can be made with a certain micrometer is $\frac{1}{25}$ of the "smallest scale division"; if the latter is $\frac{1}{40}$ of an inch what is the finest measurement?

(*b*) When the instrument is fitted with a "vernier" we can measure $\frac{1}{10}$ of the result of (*a*). What is then the smallest measurement?

29. The ordinary mile contains 5280 ft., and the nautical mile is equal to $1\frac{5}{53}$ ordinary miles. How many feet are there in a nautical mile?

30. If 1 nautical mile be considered equal to $1\frac{1}{7}$ ordinary miles, how many ordinary miles are there in a distance of $10\frac{1}{2}$ nautical miles?

31. One mile contains 1760 yds. If a "kilometre" is $\frac{5}{8}$ of a mile how many yards are there in 1 kilometre?

32. If 1 cubic metre $= 1\frac{3}{10}$ cubic yds., find how many cubic yards there are in 260 cubic metres.

Division of Vulgar Fractions. — The division of one quantity by another is the determination of the number of times that the second quantity is contained in the first. The result

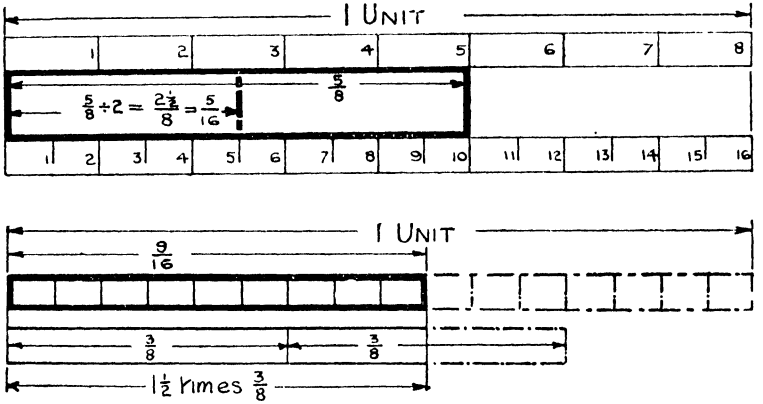


Fig. 9.—Illustrating Division of Vulgar Fractions.

could be obtained by continued subtraction, but such a process would be extremely laborious.

First take the case of dividing a fraction by a whole number, *e. g.*, $\frac{15}{16} \div 3$. The number of parts in the fraction is 15 and these are to be divided into 3 equal groups; then obviously each group must contain $15 \div 3 = 5$ parts, and each part being $\frac{1}{16}$, the result is then $\frac{5}{16}$, so that the numerator is divided by the whole number. But this method becomes rather inconvenient when the numerator is not *exactly* divisible by the whole number, *e. g.*, $\frac{5}{8} \div 2$. According to the foregoing the result would be $\frac{2\frac{1}{2}}{8}$, obviously an inconvenient form. A better statement of the result is obtained if the

denominator be multiplied by the whole number, giving $\frac{5}{16}$. The truth of the method is evident from the upper diagram in Fig. 9.

The second, and more general, case is the division of a fraction by a fraction. As an example take $\frac{9}{16} \div \frac{3}{8}$, *i. e.*, "how many times is $\frac{3}{8}$ contained in $\frac{9}{16}$?" First divide $\frac{9}{16}$ by 3, *i. e.*, the numerator only, giving $\frac{9}{16 \times 3}$. But the actual *divisor* is 8 times *smaller* than 3, and therefore the *result* must be 8 times *larger* than the previous one of $\frac{9}{48}$, that is—

$$\frac{9}{16} \div \frac{3}{8} = \frac{9}{16 \times 3} \times 8$$

$$= \frac{\overset{3}{\cancel{9}} \times \overset{1}{\cancel{8}}}{\underset{2}{\cancel{16}} \times \underset{1}{\cancel{3}}} = \frac{3}{2} = 1\tfrac{1}{2}$$

The proof is seen in the lower diagram in Fig. 9.

It can be seen that the $\frac{3}{8}$ has been turned upside down and now multiplies the $\frac{9}{16}$. Hence the rule for dividing one fraction by another: **Invert the divisor, and proceed as in multiplication.** Where mixed numbers occur, it is advisable, as when multiplying, to convert these into improper fractions.

Cancelling should be used where possible.

Example 19.—Divide $1\frac{5}{12}$ by $1\frac{8}{9}$.

$$1\tfrac{5}{12} \div 1\tfrac{8}{9}$$

$$= \underbrace{\frac{17}{12} \div \frac{17}{9}} = \frac{\cancel{17}}{\underset{4}{\cancel{12}}} \times \frac{\overset{3}{\cancel{9}}}{\cancel{17}} = \frac{3}{4}$$

This step usually omitted.

Example 20.—Find the value of $1\frac{1}{2} \times \frac{7}{8} \div 2\frac{2}{5}$.

$$1\tfrac{1}{2} \times \tfrac{7}{8} \div 2\tfrac{2}{5}$$

$$= \frac{\overset{1}{\cancel{3}}}{\underset{1}{\cancel{2}}} \times \frac{7}{8} \times \frac{5}{\underset{4}{\cancel{12}}} = \frac{35}{64}$$

Example 21.—A large pipe-joint is to have a number of bolts evenly spaced on a circle whose length right round is $75\frac{1}{2}''$. The bolts are to be spaced $3\frac{3}{4}''$ from centre to centre. How many bolts must be used?

A glance at Fig. 10 will show that there are as many bolts as spaces.

Then if we find how many times the $3\frac{3}{4}$ is contained in the $75\frac{1}{2}$ we shall have the number of spaces and therefore the number of bolts.

Then, $75\frac{1}{2} \div 3\frac{3}{4}$

$$= \frac{151}{2} \div \frac{15}{4} = \frac{151}{2} \times \frac{4}{15}^{2}$$

$$= \frac{302}{15} = 20\,\frac{2}{15}$$

or, as we cannot use a fraction of a bolt, say 20 bolts.

Example 22.—An engineering drawing is made to a scale of "$\frac{3}{4}$" to 1 ft." (*i. e.*, every 1 ft. on the job is drawn as $\frac{3}{4}$" on the drawing). Find what fraction of full size this represents (*i. e.*, find what fraction of an inch on the drawing represents 1" on the job).

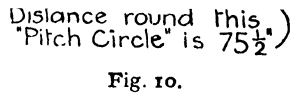


Fig. 10.

The quantities must be changed into the same units before dividing. Now "$\frac{3}{4}$" to 1 ft." means that 1 ft. is shown as $\frac{3}{4}$", also, changing to inches, 12" are shown as $\frac{3}{4}$".

$$\therefore 1'' \text{ will be } \frac{3}{4}'' \div 12$$

$$= \frac{3}{4} \times \frac{1}{12} = \frac{1}{16}''.$$

$$4$$

Thus 1" on the job appears as $\frac{1}{16}$" on the drawing, or a scale of $\frac{3}{4}$" to 1 ft. = $\frac{1}{16}$ full size.

Example 23.—It is desired to make measurements from a certain drawing of which the "scale" is not known. A dimension figured as $12\frac{3}{4}$" measures $2\frac{1}{8}$". What is the scale?

$12\frac{3}{4}$" is shown as $2\frac{1}{8}$".

Then 1" will be shown by $2\frac{1}{8}'' \div 12\frac{3}{4}''$

$$= \frac{17}{8} \div \frac{51}{4}$$

$$= \frac{17}{8} \times \frac{4}{51} = \frac{1''}{6}$$

$$2 \qquad 3$$

Then the drawing is $\frac{1}{6}$ full size or 2" to 1 ft.

It should be noted that division does not necessarily produce a decrease, as might at first be inferred from the word "divide." When we *divide by a fraction*, i. e., *by something less than* 1, then the result of the division may be considerably *greater* than the number divided. Thus, if 3 be divided by $\frac{1}{4}$ the result is 12, the reason being that $\frac{1}{4}$ is contained 4 times in 1, and therefore 12 times in 3; and as a proof $12 \times \frac{1}{4} = 3$.

Exercises 6. On Division of Vulgar Fractions.

Find the value of the following—

1. $\frac{3}{4} \div \frac{1}{16}$. **2.** $1\frac{5}{12} \div \frac{3}{16}$. **3.** $\frac{3}{32} \div \frac{15}{64}$. **4.** $\frac{18}{35} \div \frac{3}{10}$.
5. $3\frac{11}{12} \div \frac{17}{24}$. **6.** $1\frac{5}{12} \div 3\frac{7}{9}$. **7.** $4\frac{7}{8} \div 1\frac{9}{7}$. **8.** $1\frac{7}{15} \div 1\frac{1}{45}$.
9. $\frac{7}{16} \div 14$. **10.** $5\frac{5}{8} \div 9$.

11. The top joint of an oil-separator shell is to have 40 bolts evenly spaced round the pitch circle, and the length of this circle is $123\frac{1}{8}''$. Find the "pitch" of the bolts (*i. e.*, the distance from the centre of any bolt to the centre of the next. Fig. 10 will make this clear).

12. The length round a boiler ring on the pitch line of the rivets is 18′–9″, and the rivets are to be $2\frac{1}{4}''$ pitch. (Fig. 10 will show what is meant.) How many rivets will be required? (*Note.*—Work in inches.)

13. The size of the teeth in a spur wheel are often given by the "diametral pitch," *i. e.*, the number of teeth that the wheel has for every inch of its diameter (*e. g.*, a wheel of 50 teeth and 5″ diameter has 10 teeth per inch of diameter). Find the diameter of a wheel having 35 teeth and $2\frac{1}{2}$ teeth per inch of diameter.

Find the diameters of the following spur wheels—

14. To have 162 teeth, and 12 teeth per inch of diameter.
15. „ 55 „ „ $2\frac{1}{2}$ „ „ „
16. „ 22 „ „ $2\frac{3}{4}$ „ „ „

The following figures are taken from tests on toothed wheels. Find in each case the "pressure per inch of width," *i. e.*, pressure ÷ width.

17. Pressure on teeth 270 lb.; width $1\frac{1}{4}''$.
18. „ „ „ 434 „ „ $1\frac{3}{4}''$.
19. „ „ „ 416 „ „ $1\frac{5}{8}''$.
20. „ „ „ 1638 „ „ $2\frac{5}{8}''$.

What fractions of full size do the following scales represent :—

21. $\frac{1}{8}''$ to 1 ft. **22.** $\frac{1}{10}''$ to 1 ft. **23.** $\frac{1}{4}''$ to 1 ft.
24. $\frac{3}{8}''$ to 1 ft. **25.** $1\frac{1}{2}''$ to 1 ft. **26.** $4\frac{1}{2}''$ to 1 ft.
27. 4″ to 1 ft. **28.** $\frac{3}{16}''$ to 1 ft. **29.** $\frac{3}{4}''$ to 1 ft.

On referring to certain drawings it is found that the "scales" to which they are made are not stated. Find what fraction of full size the drawings are made in each of the following cases :—

30. A dimension figured as 8″ measures 2″.
31. „ „ „ $10\frac{1}{2}''$ „ $\frac{7}{8}''$.
32. „ „ „ 6″ „ $2\frac{1}{4}''$.
33. „ „ „ $9\frac{3}{4}''$ „ $1\frac{3}{16}''$.
34. „ „ „ $7\frac{1}{2}''$ „ $2\frac{13}{16}''$.
35. „ „ „ $45\frac{3}{8}''$ „ $3\frac{5}{32}''$.
36. „ „ „ $10\frac{3}{4}''$ „ $1\frac{11}{32}''$.

Various Examples. Brackets.—The study of the four simple rules applied to Vulgar Fractions is usually followed by the working of *compound examples*, i. e., those in which addition, multiplication, etc., are mixed up together in various ways. Such examples, however, seldom occur in practice, as decimal work is much more convenient, but one or two cases of a simple character will be taken as they serve to revise the foregoing principles of

vulgar fractions, and these principles are of some importance in certain parts of algebraic work.

They also provide a convenient opportunity for introducing another sign, the **bracket,** of which there are three common forms, thus : (the round), the {curly}, and [the square]. These are used to denote that the numbers and quantities contained within the two pieces of the bracket are to be treated *as one quantity only.* They may not be removed or inserted anywhere without obeying certain laws, which are given in the algebraic section, where brackets are of considerable importance.

For the present it may be taken that the operations within any bracket must be carried out and a result obtained, *before* the remainder of the problem can be proceeded with.

As a simple illustration of their use, let it be required to take the *difference* between $\frac{1}{2}$ and $\frac{1}{8}$, from $\frac{3}{4}$. Using brackets, this may be concisely stated thus : $\frac{3}{4} - (\frac{1}{2} - \frac{1}{8})$.

The statement is by no means the same as $\frac{3}{4} - \frac{1}{2} - \frac{1}{8}$.

In the first case—

$$\tfrac{3}{4} - (\tfrac{1}{2} - \tfrac{1}{8})$$
$$= \tfrac{3}{4} - \tfrac{3}{8} = \tfrac{3}{8}$$

In the second case—

$$\tfrac{3}{4} - \tfrac{1}{2} - \tfrac{1}{8}$$
$$= \frac{6 - 4 - 1}{8} = \tfrac{1}{8},$$

which shows a considerable difference.

It should be noted that the bar of division also acts as a bracket. Thus, in the case of $\frac{6 + 8}{16}$, the 6 and the 8 must be considered as one quantity. **Cancelling is not permissible where the terms of the division contain + and − signs, unless all the numbers on either side of the line be divided at the same time.** For instance, if it be desired to cancel the above expression by 2, both the 6 and 8 must be divided by 2, thus—

$$\frac{\overset{3}{\cancel{6}} + \overset{4}{\cancel{8}}}{\underset{8}{\cancel{16}}} = \frac{3 + 4}{8} = \frac{7}{8}$$

and *not* either of the following—

(i) $\dfrac{\overset{3}{\cancel{6}} + 8}{\underset{8}{\cancel{16}}}$ or $\dfrac{6 + \overset{1}{\cancel{8}}}{\underset{2}{\cancel{16}}}$

Thus (i) $= \frac{11}{8} = 1\frac{3}{8}$ and (ii) $= \frac{7}{2} = 3\frac{1}{2}$

Now $\frac{6 + 8}{16}$ evidently $= \frac{14}{16}$, which can be cancelled to $\frac{7}{8}$, thus proving the first system of cancelling to be correct.

With compound examples, like the following, it will be found that the working presents a much better appearance, and can easily be re-read at any time, if the various operations are carried out in their proper places in the given expression. The steps in the next examples should be easily followed with but little explanation.

Example 24.—Find the value of $\frac{1\frac{7}{15}}{1\frac{1}{3}+\frac{3}{10}}$.

$$\frac{1\frac{7}{15}}{1\frac{1}{3}+\frac{3}{10}} = \frac{1\frac{7}{15}}{1\frac{10+9}{30}} = \frac{1\frac{7}{15}}{1\frac{19}{30}}$$

$$= \frac{7}{15} \div \frac{49}{30} = \frac{\overset{1}{\cancel{7}}}{\underset{1}{\cancel{15}}} \times \frac{\overset{2}{30}}{\underset{7}{\cancel{49}}} = \underline{\frac{2}{7}}$$

Explanation.—The first operation must be to evaluate the bottom line, then the $1\frac{7}{15}$ can be divided by the result. The addition *may* be done in a separate place and the result collected.

Example 25.—Simplify the expression $\frac{\frac{3}{8}}{1\frac{1}{2}\div\frac{3}{14}}$.

Expression $= \frac{\frac{3}{8}}{\underset{6}{\cancel{12}} \times \frac{14}{3}}7$ inverting the $\frac{3}{14}$ and cancelling by 2

$$= \frac{\frac{3}{8}}{1\frac{7}{8}} = \frac{3}{\underset{4}{\cancel{8}}} \times \frac{\overset{9}{\cancel{18}}}{7} = \underline{\frac{27}{28}}$$

Where + and − signs are found mixed up with × and ÷ signs, **the operations of multiplication and division must be performed before those of addition and subtraction,** *unless the existence of brackets indicates some other order.* The order of the signs is quite immaterial. The reason for the rule is as follows: A sign of multiplication (and division is merely multiplication inverted) only stands between the components or factors of a number, and does not denote the existence of two separate numbers. As an illustration consider the expression $5 + 3 \times 4$. Now it must be remembered that the sign × is only an abbreviation for a long addition, and **3 × 4** in the above **means 4 + 4 + 4** (or $3 + 3 + 3 + 3$). We may not then follow any method which neglects this meaning.

Then— $5 + 3 \times 4$

$= 5 + 4 + 4 + 4 = 17$

To obtain this result without replacing the multiplication by its lengthy meaning, the multiplication must be done first, as follows—

$$5 + 3 \times 4$$
$$= 5 + 12 = 17$$

Following the *wrong scheme* of taking the signs in order (which may appear the natural way at first sight) we should have—

$$5 + 3 \times 4$$
$$= 8 \times 4 = 32$$

But in so doing *we have neglected the meaning of* 3×4 *as given above.* The signs of $\times$ and $\div$ may be looked upon as **strong signs,** and those of $+$ and $-$ as **weak signs,** so that the former have to be obeyed first.

Should it be desired to give the expression $5 + 3 \times 4$ the meaning of 8×4, then brackets must be used thus $(5 + 3) \times 4$, thus indicating that both the 5 and the 3 are to be multiplied by 4. Of course, the existence of a fraction, in any form, makes no difference to the above principles.

Example 26.—Find the value of $\frac{1}{2} + \frac{2}{3} \times \frac{3}{14} + \frac{9}{16}$.

$$\text{The expression} = \frac{1}{2} + \frac{\overset{1}{\cancel{2}}}{\underset{1}{\cancel{3}}} \times \frac{\overset{1}{\cancel{3}}}{\underset{7}{\cancel{14}}} + \frac{9}{16}$$
$$= \frac{1}{2} + \frac{1}{7} + \frac{9}{16}$$
$$= \frac{56 + 16 + 63}{112} = \frac{135}{112} = 1\frac{23}{112}$$

Note.—The multiplication must be done first.

Example 27.—Simplify the expression $(\frac{1}{2} + \frac{2}{3}) \times \frac{3}{14} + \frac{9}{16}$.

$$\text{The expression} = \frac{3 + 4}{6} \times \frac{3}{14} + \frac{9}{16}$$

First do the addition in bracket.

$$= \frac{\overset{1}{\cancel{7}}}{\underset{2}{\cancel{6}}} \times \frac{\overset{1}{\cancel{3}}}{\underset{2}{\cancel{14}}} + \frac{9}{16}$$

Then multiply.

$$= \frac{1}{4} + \frac{9}{16} = \frac{13}{16}$$

Example 28.—Find the value of $1 - \frac{\overset{3}{\cancel{10}}}{\frac{1}{2} + \frac{1}{5}}$.

$$\text{The expression} = 1 - \frac{\overset{3}{\cancel{10}}}{\frac{5 + 2}{10}}$$
$$= 1 - \frac{\overset{3}{\cancel{10}}}{\underset{7}{\cancel{10}}} = 1 - \frac{3}{\underset{1}{\cancel{10}}} \times \frac{\overset{1}{\cancel{10}}}{7}$$
$$= 1 - \frac{3}{7} = \frac{4}{7}$$

Example 29.—Cast-iron piston rings, $\frac{9}{16}$″ wide, are to be turned from a rough casting. The width of the cutting-off tool is $\frac{5}{16}$″, and 3″ is required at one end for holding in the chuck. Allowing $\frac{1}{4}$″ for facing up the end of the pipe, find the least possible length of pipe from which to cut 1 dozen rings (see Fig. 11).

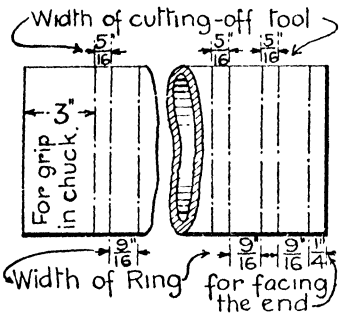


Fig. 11.

After facing, which will finish one side of the first ring cut off, each parting will produce one ring. Therefore there will be 12 partings.

Total length required = Chuck grip + total width of partings + total width of rings + allowance for finishing end.

$$= 3 + \overset{3}{\cancel{12}} \times \frac{5}{\cancel{16}_{4}} + \overset{3}{\cancel{12}} \times \frac{9}{\cancel{16}_{4}} + \frac{1}{4}$$

$$= 3 + \frac{15}{4} + \frac{27}{4} + \frac{1}{4}$$

$$= 13\tfrac{3}{4}, \text{ say } 14 \text{ in.}$$

Example 30.—A mortar is to be composed of 1 part of cement to 3 parts of sand, by weight. Find how much sand and cement are needed to make 3 cwt. of this mortar.

Using 1 lb. of cement, say, we must use 3 lbs. of sand, as there are to be 3 parts of sand to 1 of cement; the total weight of the mortar is then 4 lbs. Thus the weight of cement is $\frac{1}{4}$ of the total weight (*i. e.*, 1 lb. cement in 4 lbs. of mortar), and the weight of the sand is $\frac{3}{4}$ of the total weight (*i. e.*, 3 lbs. sand in 4 lbs. of mortar).

The actual working may be as follows—

Cement = 1 part
Sand = 3 parts
$\therefore$ Mortar = 4 parts

Then cement = $\frac{1}{4}$ of mortar = $\frac{1}{4} \times 3$ cwt. = $\frac{3}{4}$ cwt.
and sand = $\frac{3}{4}$ „ „ = $\frac{3}{4} \times 3$ „ = $2\frac{1}{4}$ „

Total (as proof) = 3 cwt.

Note.—The method of working is always the same, no matter how many constituents there may be in the mixture.

Exercises 7. Compound Examples.

Find the value of the following expressions—

1. $\dfrac{\frac{2}{3} - \frac{1}{8}}{8\frac{2}{5}}$.
2. $\dfrac{1\frac{1}{5} + 2\frac{1}{4}}{3\frac{3}{10}}$.
3. $\dfrac{2\frac{3}{4}}{\frac{5}{5} + \frac{1}{5}}$.
4. $\dfrac{\frac{9}{10}}{\frac{3}{5} - \frac{9}{20}}$.
5. $\dfrac{2\frac{3}{4}}{\frac{5}{8} \div 3\frac{3}{4}}$.
6. $\dfrac{\frac{2}{5} \times \frac{1}{8}}{\frac{3}{5}}$.
7. $\dfrac{1\frac{2}{15} \times 1\frac{1}{4}}{13 \div 13}$.
8. $\dfrac{\frac{2}{3} \times 1\frac{1}{5}}{\frac{2}{5} \times 1\frac{1}{3}}$.
9. $\dfrac{\frac{2}{3} + 1\frac{1}{5}}{\frac{2}{5} + 1\frac{1}{3}}$.
10. $\dfrac{2\frac{1}{3} + 1\frac{1}{2}}{\frac{1}{4} + \frac{1}{3}}$.
11. $\dfrac{2\frac{1}{5} - 1\frac{1}{3}}{3\frac{1}{3} + 1\frac{1}{5}}$.

Simplify the following expressions:—

12. $2\frac{2}{5} - 1\frac{1}{4} \times 1\frac{1}{15}$.

13. $(2\frac{2}{5} - 1\frac{1}{4}) \times 1\frac{1}{15}$.

14. $1\frac{1}{2} \times \frac{3}{4} + \frac{3}{8} \times \frac{5}{6}$.

15. $1\frac{1}{2} \times (\frac{3}{4} + \frac{3}{8}) \times \frac{5}{6}$.

16. $\frac{3}{8}$ of $\frac{2}{3} \div \frac{4}{5} + \frac{2}{3}$.

17. $\frac{3}{8}$ of $\frac{2}{3} \div (\frac{4}{5} + \frac{2}{3})$.

18. $3\frac{1}{8} + 2\frac{1}{2} \div \frac{3}{4} - 1\frac{3}{16}$.

19. $(3\frac{1}{8} + 2\frac{1}{2}) \div (\frac{3}{4} - \frac{3}{16})$.

20. $\dfrac{1\frac{1}{16}}{\frac{1}{2} \times \frac{3}{8}} - 1\frac{1}{4}$.

21. $\dfrac{1\frac{3}{8}}{\frac{5}{16} \div 1\frac{7}{8}} + 2\frac{4}{15}$.

22. $\dfrac{\frac{3}{4} + \frac{2}{5}}{\frac{3}{4} - \frac{2}{5}} \times \dfrac{2\frac{2}{3}}{\frac{2}{3} + \frac{2}{7}}$.

23. Special set screws are to be cut from bar stock in an automatic machine (see *a*, Fig. 12). The overall length of the screw is $1\frac{3}{16}$". The width of the cutting-off tool is $\frac{1}{8}$", and the bar stock is in 5'-0"

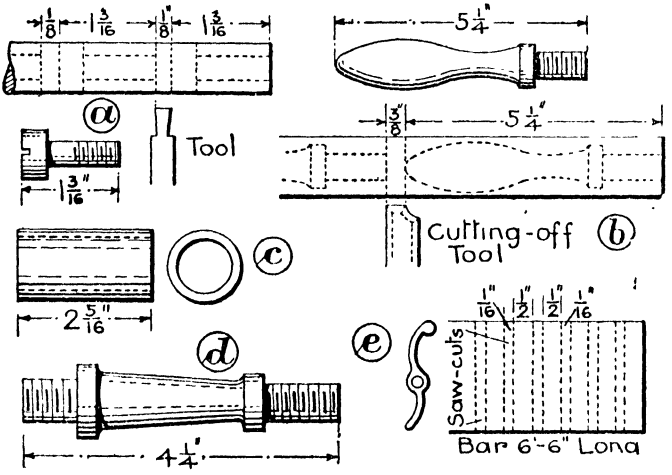


Fig. 12.—Illustrating Exercises 7.

lengths. How many screws are obtainable from each bar, and how many bars will be required for an order of 12 gross of screws? (*Note.*—1 gross = 12 dozen = 144.) (*Hint.*—Remember that if, say, $20\frac{1}{2}$ screws can be cut from 1 bar, only 20 screws are really obtainable; but if, say, $55\frac{1}{4}$ bars are required, 56 must be bought.)

24. It is required to turn 9 handles each $5\frac{1}{4}$" long, as shown at *b*, Fig. 12, from a mild steel bar. Allowing $\frac{3}{8}$" extra on each handle for cutting-off, etc., find what length of bar must be cut off for the 9 handles.

25. Some sleeves, as at *c*, Fig. 12, are to be cut from a piece of wrought-iron pipe, in the lathe. If we allow $\frac{3}{16}$" extra on each sleeve for parting off and $1\frac{1}{2}$" extra at the end for holding when cutting off the last one, what length of pipe is required for 16 such sleeves?

26. Pillars for stop valves, as shown at *d*, Fig. 12, are to be turned from bars. Allowing an extra $\frac{1}{4}$" on each one for cutting off, how many pillars can be cut from a bar 5'-6" long?

27. Electric contact fingers, $\frac{1}{2}$" wide, are to be sawn in a power hack-saw, from extruded bars each 6'-6" long. (See *e*, Fig. 12.) If the saw-cut is $\frac{1}{16}$" wide, how many fingers can be cut from each bar?

28. Special nuts are to be turned from the bar as follows : Seven $\frac{5}{8}''$ thick, and fifteen $\frac{3}{4}''$ thick. Allowing an extra $\frac{3}{16}''$ on each nut for parting off, find the total length of bar required.

29. Forty-eight angle stiffeners for a plate girder are required $40\frac{3}{4}''$ long, and are sheared from the long angle bar without waste. What length of angle will be required? If the stock angle bar is in 45 ft. lengths, how many lengths will be needed?

30. A concrete is to be made of cement 1 part and coke breeze 4 parts by weight. How much of each constituent is required for 15 cwt. of concrete?

31. What weight of each metal must be used to make 150 lbs. of each of the following solders, assuming nothing lost in the melting?

(*a*) Lead 3 parts; tin 2 parts.

(*b*) Tin 4 parts; lead 1 part; bismuth 1 part.

32. Black gunpowder is composed of nitre 15 parts, charcoal 3 parts, and sulphur 2 parts, by weight. Find the quantities of each material required to make 60 lbs. of gunpowder.

33. The white metal for fusible plugs is made of varying proportions, according to the temperature at which it is required to melt. Find what weight of each metal is required to make 1 cwt. (112 lbs.) of each of the following alloys : (*a*) tin 2 parts, bismuth 1 part; (*b*) lead 1 part, tin 4 parts, bismuth 5 parts; (*c*) lead 5 parts, tin 3 parts, bismuth 8 parts.

34. Find what weight of each material is required to make the following quantities of foundry sand : (*a*) 2 cwt. of facing sand consisting of 6 parts old sand, 4 parts new sand, and 1 part coal dust; (*b*) 10 cwt. of floor sand consisting of 6 parts old sand, 2 parts new sand, and $\frac{1}{2}$ part coal dust. (Give results in lbs.)

35. A filling material, for cast iron, which will expand on solidifying, is composed of lead 9 parts, antimony 2 parts, and bismuth 1 part. How much of each metal must be used to make 30 lbs. of filling?

36. Find what weight of each material is necessary to make 75 tons of concrete, to consist of cement 1 part, sand 3 parts, and aggregate (broken brick, etc.) 6 parts.

37. When helical springs are used in compression the last three-quarters of the coil at each end is flattened to provide a flat seating; these flattened portions do not form part of the effective coils. Eighteen such springs are required, each having 7 effective coils, the average length of a coil being $3\frac{3}{4}''$. Calculate the total length of wire in a complete spring and also the total length of wire required for the 18 springs, allowing $2''$ per spring as wastage in cutting off.

Find the values of the following expressions.

38. $\dfrac{1}{3 - \dfrac{1}{2 - \frac{1}{3}}}$.

39. $\dfrac{1}{\frac{1}{2} + \frac{1}{3} + \dfrac{\frac{1}{8}}{\frac{3}{4} - \frac{1}{3}}}$.

40. $\dfrac{\frac{3}{4} \times (\frac{6}{7} + \frac{5}{6})}{\frac{3}{14} + \frac{5}{7} \times \frac{3}{16}}$.

41. $3\frac{1}{2} + \dfrac{\frac{162}{\frac{3}{4}} - 140}{20}$.

42. $(\frac{5}{6} \times \frac{7}{12}) \div (\frac{1}{7} + \frac{19}{28})$.

43. $\dfrac{1\frac{1}{8} \div 1\frac{1}{8}}{\frac{2}{3} \times \frac{5}{9} \div 10\frac{2}{3}}$.

CHAPTER II

DECIMAL FRACTIONS

Decimal Notation.—The ordinary method of numbering is built upon a scale which uses only ten signs, and is therefore called **decimal notation.** The ten different signs employed, which are called digits or figures, are 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9; and all numbers are expressed with these only, by employing multiples of the unit in which 10 of each kind make 1 of the next larger variety. As examples of these multiples we have the **ten,** which is a set of 10 units; the **hundred,** a group of 10 tens, *i. e.*, $10 \times 10 = 100$; the **thousand,** a group of 10 hundreds, *i. e.*, $100 \times 10 = 1000$; and so on.

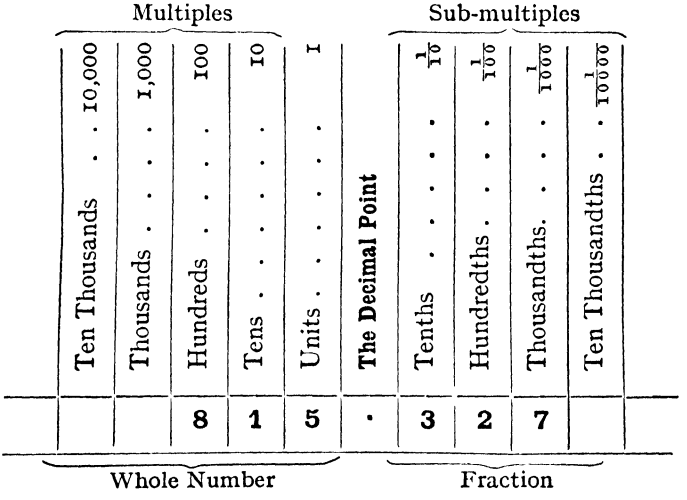
The value of each digit in a number (*i. e.*, the multiple to which that figure refers) is indicated by its position in the number. Thus the extreme right-hand digit is the "units" figure, the next to the left the "tens" figure, the next the "hundreds" figure, and so on.

For example, the number 15 means 1 set of 10 units + 5 single units (*i. e.*, $1 \times 10 + 5$)
the number 746 means $(7 \times 100) + (4 \times 10) + 6$, and
" " 5280 " $(5 \times 1000) + (2 \times 100) + (8 \times 10) + 0$

From the above examples it can be seen that, starting from the extreme left with the most important figure (*i. e.*, the one of the largest multiple) and proceeding to the right, every multiple is $\frac{1}{10}$ of its left-hand neighbour, until finally we arrive at the simplest unit. Now proceeding to divide the unit, in order to deal with fractional quantities, it is only natural that we should make the first sub-multiple $\frac{1}{10}$ of the unit, and the next $\frac{1}{10}$ of that (*i. e.*, $\frac{1}{10}$ of $\frac{1}{10} = \frac{1}{100}$), and so on. Then, if the vulgar system of writing down such fractions were employed, we should have for denominator a number such as 10, 100, 1000, etc., *i. e.*, a number composed

of 1 with one or more noughts following it.* Fractions having such denominators are called "**decimal fractions,**" but the denominator is omitted in writing down the fraction. The numerator only is written, the denominator being indicated by the position of the figures in the numerator. A dot (·), called a **decimal point,** is used to distinguish between whole numbers and the numerators of decimal fractions; figures to the **left** of the point are **whole numbers,** and those to the **right** are **fractions.** The first place of figures to the right of the point is the $\frac{1}{10}$ place, the second the $\frac{1}{100}$ place, the third the $\frac{1}{1000}$ place, and so on.

All the foregoing points are illustrated in the diagram below, which shows the composition of a mixed number when expressed decimally.



The number 815·327 means $815 + \frac{3}{10} + \frac{2}{100} + \frac{7}{1000}$

But, by the rules of Chap. I, $\frac{2}{100} = \frac{20}{1000}$, and $\frac{3}{10} = \frac{300}{1000}$ so that the number $815{\cdot}327 = 815\frac{327}{1000}$.

Besides omitting the denominator in writing, it is also omitted in reading a decimal fraction: the figures following the point are simply read in order. Thus the above quantity 815·327 is read as *eight hundred and fifteen point three two seven.* As further examples—

39·37 reads thirty-nine point three seven and $= 39 + \frac{3}{10} + \frac{7}{100}$.

* Numbers such as these (*i.e.*, those composed of 1 followed by one or more noughts) are called "*powers*" of 10: see Chap. III.

But as $\frac{3}{10} = \frac{30}{100}$, a shorter statement in the vulgar form is $39\frac{37}{100}$. Similarly, 62·425 reads sixty-two point four two five, and $= 62 + \frac{4}{10} + \frac{2}{100} + \frac{5}{1000} = 62\frac{425}{1000}$.

When a whole number alone is written down, the decimal point is omitted, as there is no fraction to consider. If a number such as 15·00 is obtained as the result of any calculation, then the dot and the noughts may be omitted, and the figure written simply as 15, since ·00 is, of course, 0. But when a proper fraction has to be written down, *then the dot is absolutely necessary*, as its omission will convert the fraction into a whole number. Thus ·305 is the fraction $\frac{305}{1000}$, and if the dot were omitted the figures would read 305! For this reason a nought is sometimes written in front of the point thus, 0·305, so that, should the dot be lost, the appearance of the figures 0 305 would indicate its intended existence. The number ·305 may be read as either "nought point three nought five" or simply "point three nought five."

It should also be noted that noughts may be added to the *right of the figures of the fraction*, and to the *left of the figures of the whole number*, without affecting the value of the number. Thus 3·1400 is only 3·14, the 0's merely stating that no $\frac{1}{1000}$ths or $\frac{1}{10000}$ths exist, which is equally well shown by writing nothing whatever beyond the 4. For similar reasons 0078·54 = 78·54.

Movement of the Decimal Point.—It is a feature of the decimal system that any number may be multiplied or divided by 10, or 100, or 1000, etc., without any actual operation on the figures themselves beyond moving the decimal point. As an example take the number 39·37. This $= 39\frac{37}{100}$, which may be expressed in the improper form $\frac{3937}{100}$. If the decimal point be moved one place to the right the figures will appear as 393·7, which $= 393\frac{7}{10} = \frac{3937}{10}$. Converting to $\frac{1}{100}$ths we have $\frac{39370}{1000}$, which $= \frac{3937}{100} \times 10$, so that 393·7 is 10 times 39·37. The converse is, of course, equally true. Thus 39·37 is $\frac{1}{10}$ of 393·7. Hence the rule: **to multiply or divide a number by 10 move the decimal point 1 place to the right or left respectively.**

Since $100 = 10 \times 10$, then to multiply or divide by 100 the above rule should be obeyed *twice*. Similarly, to operate by 1000, which $= 10 \times 10 \times 10$, the decimal point must be moved *three* times. Hence we have a more general statement of the above rule: **To multiply or divide a number by any power of 10,* move the decimal point to right or left respectively, 1 place for each 0 in the multiplier.**

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* See foot-note on p. 36.

As examples: $2{\cdot}5 \times 10 = 25$ (the dot is not needed finally).
$2{\cdot}5 \div 10 = {\cdot}25$
${\cdot}7854 \times 100 = 78{\cdot}54$
$3{\cdot}1416 \times 1000 = 3141{\cdot}6$.

When a whole number is to be operated upon in this manner, and apparently has no decimal point, *e. g.*, 1760, then it must be remembered that the dot really exists immediately to the right of the units figure, *i. e.*, after the 0. Then places may easily be counted off when moving the dot to the left. Thus $1760 \div 100 = 17{\cdot}60$. When moving the dot to the right the operation is that of adding a 0 for every time that the number is multiplied by 10. Thus $1760 \times 100 = 176{,}000$.

Similarly, when moving the decimal point to the left and no more existing figures are available, noughts may be introduced as necessary. Thus $3{\cdot}14 \div 100 = {\cdot}0314$, for we may consider 3·14 as 03·14.

Also $\cdot0625 \div 10 = \cdot00625$.

Exercises 8. On Movement of Decimal Point.

What is the result of—

1. (*a*) $5{\cdot}5 \times 10$. (*b*) $5{\cdot}5 \div 10$. (*c*) $1{\cdot}34 \times 100$.
(*d*) $1{\cdot}34 \div 100$. (*e*) $453{\cdot}6 \times 1000$. (*f*) $453{\cdot}6 \div 1000$.

2. (*a*) $\cdot001118 \times 1000$. (*b*) $\cdot0000087 \times 10{,}000$. (*c*) $\frac{\cdot68}{100000}$.
(*d*) $350 \times 1{,}000{,}000$.

3. If 1 kilowatt = 1000 watts, convert the following into watts—
(*a*) 35 kw. (*b*) 7·3 kw. (*c*) ·08 kw.
(*Hint.*—Multiply by 1000.)

4. Convert the following into kilowatts—
(*a*) 29,500 watts. (*b*) 231 watts. (*c*) 305,000 watts.
(*Hint.*—Divide by 1000.)

5. If 1 volt = 100,000,000 absolute units, convert the following into absolute units—
(*a*) 2·7 volts; (*b*) ·005 volts; (*c*) ·000003 volts.
(*Hint.*—Multiply by 100,000,000.)

6. If 1 Joule = 10,000,000 ergs, convert the following into Joules—
(*a*) 42,600,000 ergs. (*b*) 550,000 ergs.

Conversion from Decimals to Vulgar Fractions.—This has already appeared in the section on decimal notation, and only a few points remain to be noted. It will be seen, from the examples given in the section referred to, that the denominator consists of 1 followed by as many o's as there are figures in the numerator; for example, in the case of 39·37 the denominator (100) contains two noughts, one for each of the figures behind the decimal point. Similarly with ·425 there are three noughts, *i. e.*, the denominator is 1000. Hence we have the following rule: **For the numerator**

write the figure or figures behind the decimal point. For the denominator write 1 for the decimal point and a 0 for every figure following it. Finally reduce the fraction to its lowest terms.

Example 31.—Convert the following decimals into vulgar fractions—

(*a*) ·2; (*b*) ·5; (*c*) 0·3; (*d*) ·45; (*e*) ·125.

(*a*) Numerator = 2. Denominator = 1 followed by one nought, *i. e.*, 10, as there is one figure behind the decimal point.

$\therefore$ ·2 $= \frac{2}{10}$; cancelling by 2 gives $\frac{1}{5}$.

(*b*) ·5 $= \frac{5}{10}$; " " 5 " $\frac{1}{2}$.

(*c*) 0·3 $= \frac{3}{10}$; no reduction possible.

(*d*) ·45 $= \frac{45}{100}$; cancelling $\frac{\cancel{45}^{9}}{\cancel{100}_{20}} = \frac{9}{20}$.

(*e*) ·125 $= \frac{125}{1000}$; cancelling by 125, or 3 times by 5, gives $\frac{1}{8}$.

In cases where the decimal contains a 0, this must be counted as a " figure behind the decimal point."

Thus ·305 $= \frac{305}{1000} = \frac{61}{200}$ cancelling by 5.

This rule must be noticed particularly when a 0 or 0's follow immediately after the decimal point.

Thus ·025 $= \frac{025}{1000} = \frac{25}{1000} = \frac{1}{40}$ cancelling by 25 or by 5 and 5 again.

Similarly ·0036 $= \frac{0036}{10000} = \frac{36}{10000} = \frac{9}{2500}$ cancelling by 4.

(The steps $\frac{025}{1000}$ and $\frac{0036}{10000}$ above are only introduced for explanation and need not be put in the working.)

Since many of the simpler fractions are frequently used in the vulgar form, it is desirable to remember the decimal equivalents of the more common ones. The following table gives the decimal equivalents for every $\frac{1}{32}$ up to 1. Certainly the equivalent for every $\frac{1}{8}$ should be memorised, and it is advisable to learn also the equivalents for the intermediate sixteenths.

DECIMAL EQUIVALENTS.

Fraction.	Decimal Equivalent.	Fraction.	Decimal Equivalent.	Fraction.	Decimal Equivalent.
$\frac{1}{32}$	·03125	$\frac{3}{8}$	**·375**	$\frac{23}{32}$	·71875
$\frac{1}{16}$	·0625	$\frac{13}{32}$	·40625	$\frac{3}{4}$	**·75**
$\frac{3}{32}$	·09375	$\frac{7}{16}$	·4375	$\frac{25}{32}$	·78125
$\frac{1}{8}$	**·125**	$\frac{15}{32}$	·46875	$\frac{13}{16}$	·8125
$\frac{5}{32}$	·15625	$\frac{1}{2}$	**·5**	$\frac{27}{32}$	·84375
$\frac{3}{16}$	·1875	$\frac{17}{32}$	·53125	$\frac{7}{8}$	**·875**
$\frac{7}{32}$	·21875	$\frac{9}{16}$	·5625	$\frac{29}{32}$	·90625
$\frac{1}{4}$	**·25**	$\frac{19}{32}$	·59375	$\frac{15}{16}$	·9375
$\frac{9}{32}$	·28125	$\frac{5}{8}$	**·625**	$\frac{31}{32}$	·96875
$\frac{5}{16}$	·3125	$\frac{21}{32}$	·65625	1	1·00
$\frac{11}{32}$	·34375	$\frac{11}{16}$	·6875		

Exercises 9. On Conversion of Decimals to Vulgar Fractions.

Convert the following decimal fractions into the vulgar form—

1. (*a*) ·65; (*b*) ·45; (*c*) ·425.
2. (*a*) ·028; (*b*) ·875; (*c*) ·3264.
3. (*a*) 2·205; (*b*) 1·34; (*c*) ·305.
4. (*a*) ·964; (*b*) 3·142; (*c*) ·00256.
5. Convert into vulgar fractions : (*a*) ·515625; (*b*) ·390625.

Degree of Accuracy. Significant Figures.—Mathematically there is no limit to the number of figures which may be written on either side of the decimal point, but a few only are required for practical problems. As some operations with decimals produce large numbers of digits in the results, it is desirable to consider how many figures are really necessary in any mixed number. All the quantities with which we have to deal are connected with measurement; the accuracy with which a quantity can be stated depends upon the nature of the measurements relating to that quantity. Thus we might say with truth that the size of a steel ball, taken from a ball bearing, was 1·2498″; machines and instruments which can measure $\frac{1}{10000}$″ (·0001″) in a distance of 1¼″ are fairly common, and it is also very desirable to know the size of the ball accurately in order to secure smooth and safe running of the bearing. But to say that the length of a liner's voyage is 2536·4978 miles would be utterly absurd, for the actual distance travelled would depend so much on the state of the weather, ocean currents, etc., etc., that we could not be sure that even the last figure in the whole number (the 6) was correct, much less any of the succeeding figures. We could not give, with confidence, the distance any more accurately than 2540 miles. Evidently, then, there is no use in stating the figures 6·4978, and still less in using them in any calculation in which the above distance is required. A quantity such as this 2540 miles is, then, only an *approximate*, or nearly correct, measurement. When describing the accuracy to which this distance has been measured we say that it is "*correct to* 3 *significant figures*," the digits 2, 5 and 4 being the "significant figures"; that is to say, it is as near as possible to the truth while only containing 3 significant figures. The 0 only helps to denote the various multiples of the figures 2, 5 and 4, *i. e.*, it fixes the position of the decimal point and does not itself signify any number.

All the digits 1, 2, 3, 4, 5, 6, 7, 8, and 9 are significant figures in all cases. The digit 0 is only a significant figure when it has a higher digit on both sides of it, as in the case of the number ·305. Then the 0 is significant, distinguishing the number ·305 from ·035 or ·350. Thus, taking the following numbers:—

In ·305 the sig. figs. are	305.	In 606·5	the sig. figs. are 6065.
,, ·035 ,, ,, ,, ,,	35.	,, 6080	,, ,, ,, ,, 608.
,, ·035 ,, ,, ,, ,,	35.	,, ·000341	,, ,, ,, ,, 341.
,, 2938 ,, ,, ,, ,,	2938.	,, 30,000,000	,, ,, fig. is 3.

Most of our measurements fall under this "approximate" heading, and consequently are only stated "correct to so many significant figures." The actual number of significant figures depends upon the nature of the measurement. Thus in giving the horse-power of a ship's engines 2 significant figures are usually all that can be relied upon, *e. g.*, 12,000 h.-p.; but when stating the length of a base-line used for the large surveys of Great Britain, 7 or more significant figures are given, *e. g.*, 26405·78 ft.; and in accurate chemical work it is quite possible to work to 7 significant figures. No definite rules can be given for the number of significant figures desirable in any measurement or result. It is a matter of experience, and the reader must be guided by the style of answer given to the various examples, both worked and set, throughout this book. It may be taken, as a *general* rule, that **3 or 4 significant figures** are quite sufficient for most engineering calculations.

As a matter of interest it may be noted that calculations on the ordinary slide rule (Chap. X) are performed with the "significant figures" only, the position of the decimal point being fixed later.

The adjustment of a quantity to any required number of significant figures is governed by the idea that any error introduced thereby shall be as small as possible. Thus if the 2536 be required correct to 3 significant figures it should be written 2540, and not 2530. The difference between 2536 and 2540 is 4, whereas between 2536 and 2530 it is 6. Hence less error is introduced by using 2540 rather than 2530. But the number 2533 correct to 3 significant figures is 2530 and not 2540, the difference between 2533 and 2530 being 3, whereas between 2533 and 2540 it is 7.

Hence we have the rule: **increase the last figure retained by 1 when the rejected figures are over 5000 ..... but leave it unaltered when the rejected figures are less than 5000 .....** When one figure only is to be rejected and is 5 exactly, either of the above may be applied.

Example 32.—Express the following numbers correct to 2 significant figures:—(*a*) ·333; (*b*) 1·666; (*c*) ·01734; (*d*) ·7071; (*e*) 10·01, (*f*) 235·5; (*g*) 1·5708; (*h*) 0·9951.

(*a*) ·33 as 3 is less than 5; (*b*) 1·7 as 66 is greater than 50;
(*c*) ·017 as 34 ,, ,, ,, 50; (*d*) ·71 ,, 71 ,, ,, ,, 50;
(*e*) 10 as 01 ,, ,, ,, 05; (*f*) 240 ,, 55 ,, ,, ,, 50;
(*g*) 1·6 as 708 is greater than 500; (*h*) 1·0 as 51 is greater than 50.
Note.—In case (*h*) only one sig. figure can be given.

Example 33.—Express the following numbers correct to 3 significant figures:—(*a*) 3744; (*b*) 39·37; (*c*) 2·7183; (*d*) 2·3026; (*e*) 1·897; (*f*) 4875.

(*a*) 3740; (*b*) 39·4; (*c*) 2·72; (*d*) 2·30 (*i.e.*, 2·3);
(*e*) 1·90 (*i.e.*, 1·9); (*f*) 4880.

Note.—In (*d*), since only 3 figures are required, all figures to the right of the 0 (the third significant figure) must be eliminated, leaving 2·30. The last 0 has no use, so that our result is 2·3. This contains only 2 significant figures but cannot be avoided since the third figure is a 0.

In (*e*), since 3 figures are required, the digit to the right of the 9 must go out, but being a 7, *i. e.*, over 5, must be allowed for by adding 1 to the 9. This gives 1·90, *i. e.*, 1·9, or only 2 figures, which, however, cannot be avoided.

Example 34.—Express the number 3·1415928* correct to
(*a*) 5 significant figures; (*b*) 4 significant figures;
(*c*) 3 significant figures.

(*a*) 3·1416; (*b*) 3·142; (*c*) 3·14.

It should be noted that, after adjusting, the last significant figure is not reliable. Where several numbers which have been thus adjusted are used in a calculation there is a possibility of introducing errors beyond those of measurement. To avoid this it is usually advisable to carry our calculations to 1 figure more than is required, and to adjust to the required number finally. Thus a problem requiring an answer to 2 significant figures should be worked to 3, and the answer reduced to 2 finally.

Exercises 10. On Significant Figures.

What are the significant figures in the following numbers:—

1. 39·37; 5280; 2·205; 30·45; ·0365.
2. 29,850,000; 10,500,000; ·0807; 1083; 454.
3. ·00000066; 606·5; ·001118; 32·09; 1·5708.
4. 10·9; ·0000087; ·0000105; 600·9; 2000.

* This particular number, which is very important in connection with circles, has been *calculated* to 700 *decimal places !*

Express the following numbers correct to 3 significant figures —

5. 453·6; 30·48; ·4343; 62·425; ·001118.

6. 14380; 1393; 32·182; 600·9; 3749·9.

Express the following numbers correct to 2 significant figures:—

7. ·0807; ·282; 14·7; ·0646; 5·37.

8. 2·205; 606·5; ·01604; 29,850,000; ·1145.

9. The Imperial Gallon contains 277·274 cubic inches. Express this (*a*) correct to 4 significant figures; (*b*) correct to 3 significant figures.

10. The British pound weight contains 453·5924 grammes. Express this (*a*) correct to 5 significant figures; (*b*) correct to 3 significant figures.

Express the following numbers correct to (*a*) 5 significant figures; (*b*) 4 significant figures; (*c*) 2 significant figures—

11. ·142857. **12.** 2·71828.

Addition and Subtraction of Decimals.—Since ten is the "rate of exchange" on each side of the decimal point, addition and subtraction with decimal fractions are carried out in exactly the same way as with whole numbers, provided that all the decimal points are kept in a vertical line, and the various decimal figures representing the same decimal place are directly beneath each other. That is to say, tens may only be added to tens and units to units, and similarly tenths may only be added to tenths and hundredths to hundredths, etc.

Thus, finding the total of 25·3, 12·57 and 0·62,

$$\begin{array}{r} 25{\cdot}3 \\ 12{\cdot}57 \\ 0{\cdot}62 \\ \hline 38{\cdot}49 \\ \hline d\ c\ b\ a \end{array}$$

Explanation—

Column *a*, 2nd place of decimals,	2 + 7	= 9, write 9.	Nothing to carry.
,, *b*, 1st ,, ,,	6 + 5 + 3	= 14, ,, 4.	Carry 1.
,, *c*, units	1 + 2 + 5	= 8, ,, 8.	Nothing to carry.
,, *d*, tens	1 + 2	= 3, ,, 3.	

Result = 38·49.

Apart from mistakes in common addition, the only blunders likely to arise are from not writing the figures and decimal points in distinct separate columns, when figures of the wrong columns may get added together. Therefore great care should be given to the writing down of the figures.

Example 35.—When considering the strength of a certain steel column it was necessary to add together the quantities 6·48, 0·312, and 2·37. Find the value required.

6·48
0·312
2·37
9·162

The 2 may be neglected.
Then result = 9·16.

In subtraction, the number to be taken away is written below the number from which it is to be taken, the subtraction being performed in the ordinary way. The previous remarks on the decimal point and the neat spacing of the figures should be noted.

Thus in taking 1·875 from 2·327—

2·327
1·875
·452 = Result.

Example 36.—When measuring the difference of pressure between two points of a pipe in which water is flowing, a mercury U gauge (see Fig. 13) is sometimes employed. The top surface of the mercury stands opposite ·817 ft. on the scale, and the bottom surface opposite 199 ft. What is the "difference of level," *i. e.*, the distance marked "?"

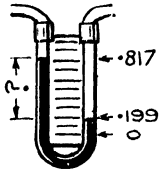


Fig. 13.

Top reading = ·817
Bottom „ = ·199
∴ Difference of level = ·618 ft.

Example 37.—In connection with the strength of a cast-iron rail section certain areas had to be measured with a planimeter.* The following three pairs of readings were taken, the area in each case being the difference of the pair of figures. Find these areas (sq. ins.).

(*a*) 19·45, 12·25; (*b*) 57·63, 56·52; (*c*) 70·76, 69·09.

(*a*) 19·45 — 12·25 = 7·20 sq. ins.
(*b*) 57·63 — 56·52 = 1·11 sq. in.
(*c*) 70·76 — 69·09 = 1·67 sq. in.

* An instrument for measuring accurately, and with very little calculation, the area of an irregular figure.

Example 38.—In finding the bending moment at a certain section of a girder it was necessary to take 62·2 ft. tons from the sum of 3·4, 45·6 and 37·6 ft. tons. Find this value.

		3·4 ft. tons	
		45·6 ,,	
		37·6 ,,	
Sum	=	86·6 ,,	
		62·2 ,,	
Difference	=	24·4 ft. tons	Result = 24·4 ft. tons.

Example 39.—A portion of a surveyor's levelling book is shown. The height of the place called G above the place called A is found by subtracting the sum of column 3 from the sum of column 2. Find this height, all the figures being in feet.

1	2	3
Point.	Back Sight.	Fore Sight.
A	7·52	
B	4·09	4·41
C	9·75	5·16
D	4·14	1·12
E	2·01	2·73
F	1·99	4·69
G		8·78
Totals .	29·50	26·89

Deduct Total Fore Sights (*i. e.*, Col. 3) 26·89

2·61

Result : Place G is 2·61 ft. above place A.

The engineer has seldom to add up long columns of figures. A good deal of the decimal addition will be only with two or three numbers at a time, and hence the method of writing down in vertical columns, shown in the foregoing examples, is often omitted as being laborious. With a little practice one is soon able to sort out the last decimal place (say the fourth) in each number to be added, and to perform the addition of those particular figures mentally; then pass on to the next larger decimal place (say the third) and repeat the process; and so on until the example is complete; similarly with subtraction.

The following examples will serve as illustrations—

Example 40.—In a problem on a retaining wall it was necessary to find the sum of the distances 1·19 ft. and 2·33 ft., and then to subtract 2·83 ft. from this sum. Find the resulting measurement.

$$\begin{aligned} \text{We have } & 1{\cdot}19 + 2{\cdot}33 - 2{\cdot}83 \\ &= 3{\cdot}52 - 2{\cdot}83 \\ &= \underline{{\cdot}69 \text{ ft.}} \end{aligned}$$

Example 41.—The expression $92{\cdot}1 + {\cdot}56 - (25 + 17{\cdot}9)$ relates to the flow of water through a pipe. Find the value required.

$$\begin{aligned} & 92{\cdot}1 + {\cdot}56 - (25 + 17{\cdot}9) \\ &= 92{\cdot}66 - 42{\cdot}9 \\ &= 49{\cdot}76, \text{ say } \underline{49{\cdot}8}. \end{aligned}$$

Exercises 11. On Addition and Subtraction of Decimals.

1. Find the sum of 10·5, 63·58 and 25·2.

2. Add together 67·35, 85·62, 105·14 and 5·7.

3. Add together 125·5, 67·825, 1·5625, 13·125 and 12·03125.

4. The *total* head against which a centrifugal pump has to work is made up of three heads : loss of head in pump 5·5 ft., loss by friction in the pipe 93·6 ft., and height through which water has to be lifted 250·5 ft. Find the total head.

5. In a laboratory experiment the following weights (lbs.) were placed on a certain hook : ·7, 2·9, 3, 3, 2·9, 2·9, 3, 3, 2, 2, 1·5, 1·5, 1·1, 2, 3, 1·5. Find the total weight on the hook.

6. The lengths of the lines in a traverse survey were as follows: 13·63, 3·8, 6·97, 9·41, 3·48, 6·11, 9·66, 5·08, 6·84, 2·07 chains. Find the total length of line surveyed.

7. In calculating the bending moment on a continuous girder the expression $5 + 3{\cdot}75 + 1{\cdot}06 + 1{\cdot}42$ was obtained. Finish the calculation.

8. The following table gives the chemical analysis of some gaseous fuels. The combustible constituents are hydrogen, methane, and carbon monoxide. Determine the total number of parts of combustible matter in each gas.

Gas.	Carbon Monoxide.	Hydrogen.	Carbon Dioxide.	Methane.	Nitrogen.
	parts	parts	parts	parts	parts
Dowson Gas .	25·07	18·73	6·57	·62	49·1
Mond Gas . .	10·00	23·00	15·00	3·00	49·00
Natural Gas .	·41	1·64	·25	93·70	3·41

9. Find the difference between 68·75 and 59·85.

10. Find the result of 125·3 — 68·45 — 26·9.

11. In an experiment on linear expansion, the following were some figures (millimetres) taken for brass and steel—

(*a*) Brass. Initial reading ·5 Final reading 2·3

(*b*) Steel. ,, ,, 3·215 ,, ,, 4·225

Find the actual expansion in each case (*i. e.*, the difference between each pair of figures).

12. The following figures were taken in a delicate experiment on the electrolysis of copper sulphate—

Weight of plate before depositing . . . 63·8203 grammes
,, ,, ,, after ,, . . . 64·5091 ,,

Find the weight of copper deposited on the plate.

13. In an experiment on the latent heat of steam the following weights were measured—

Weight of beaker (empty) . . . . . 264·9 grammes
,, ,, ,, + cold water . . . . 1428·6 ,,
,, ,, ,, + cold water + steam condensed 1480·5 ,,

Find (*a*) weight of cold water used; (*b*) weight of steam condensed.

14. Find the difference (*a*) between ·3575 and ·5463, and (*b*) between ·3575 and ·6453, figures which occurred in measuring the quantity of water flowing over a V-notch.

15. Extreme sizes of a shaft and hole, nominally 3″ diameter, when made to limit gauges are given below—

Maximum diameter of hole = 3·0035 in.
Minimum ,, ,, ,, = 3·0000 ,,
Maximum ,, ,, shaft = 2·9965 ,,
Minimum ,, ,, ,, = 2·9930 ,,

Find the extreme clearances which can occur, *i. e.*, find the difference in diameter between the largest shaft and the smallest hole; and between the smallest shaft and the largest hole.

16. and **17.** The following tables are from a surveyor's level book. Find the difference between the sums of columns FS and BS in each case (which gives the difference of level between the first and last points).

16.

Pt.	B.S.	F.S.
A	·23	
B	3·41	13·96
C	4·18	9·57
D		4·95

17.

Pt.	B.S.	F.S.
A	2·475	
B	3·37	3·94
C	2·15	4·59
D	1·15	6·21
E	2·13	5·38
F		2·15

18. Figures from a surveyor's level book are given. If the levelling is correct the total of column R will equal the total of column F. Any difference is an error. Find the totals of columns R and F and find what error exists, if any. (All measurements in feet.)

Pt.	B.S.	F.S.	R.	F.
A	7·336	—		
B	10·265	9·455	—	2·119
C	3·441	5·055	5·210	—
D	4·60	3·365	·076	—
E	6·61	9·885	—	5·285
A	—	4·49	2·12	—

19. Referring to the last exercise, the sum of column B.S should equal the sum of column F.S. Any difference is an error. Find what error, if any, exists.

20. Find the value of 3·75 − 1·42 + 1·5, figures obtained in calculating the bending moment on a beam.

21. One of the reactions of a continuous girder was calculated to be 2·5 + 1·5 + 2·5 − ·27, all in tons. Complete the calculation.

22. Find the value of the expression (connected with a bending moment) 12·7 − 4·7 − 15·55 + 12·3 − 1·75.

Multiplication of Decimals.—Let us consider the multiplication of two decimals, *e. g.*, ·8 × ·2. Convert each decimal into a vulgar fraction, but omit the cancelling. Then ·8 becomes $\frac{8}{10}$ and ·2 becomes $\frac{2}{10}$. The expression ·8 × ·2 may now be written as $\frac{8}{10} \times \frac{2}{10}$.

Multiplying (without cancelling) we have—

$$\frac{8}{10} \times \frac{2}{10} = \frac{16}{100}, \textit{ i. e.,}\ 1 \text{ tenth} + 6 \text{ hundredths},$$

or ·16 in the decimal form. It should be noticed that the figures in ·16 are simply the product of the figures in ·8 and ·2; also that *the total number of decimal places* in the quantities multiplied is **2** (1 in ·8 and 1 in ·2) and there are **2** *decimal places* in their result ·16.

Again, let us consider 2·45 × 1·3:

$$2{\cdot}45 = 2\tfrac{45}{100} = \tfrac{245}{100},$$

$$\text{also } 1{\cdot}3 = 1\tfrac{3}{10} = \tfrac{13}{10}.$$

Hence we have $\frac{245}{100} \times \frac{13}{10} = \frac{3185}{1000} = 3{\cdot}185$.

Again notice that the digits in 3·185 are the product of the digits in 2·45 and 1·3; also *the total number of decimal places* in the numbers multiplied is **3** (2·45 has 2 decimal places and 1·3 has 1), and there are **3** *decimal places* in the result 3·185.

Any example may be similarly treated and the same points will be seen. Hence we have the rule for the multiplication of decimals—

Multiply the figures together without considering the decimal point. Count up the total number of decimal places in the quantities multiplied, and starting from the right, point off this number of figures in the result obtained.

To illustrate this rule, and also the method of writing down the work, consider the second example just taken, *i. e.*, 2·45 × 1·3. Either number may be taken as the multiplier; but usually it will be found easier to take the one with the fewer digits by which to multiply, in this case 1·3. Set down the quantities as in ordinary multiplication; the decimal point *need not* be placed in any particular position. Multiply, *disregarding the decimal point* (*i. e.*, as in ordinary multiplication).

Thus— Mental work to fix the decimal point

2·45	. . . . .	2 decimal places
1·3	. . . . .	1 decimal place
735		
245		
3·185	. . . . .	3 decimal places.

This gives the figures 3185.

Now fix the decimal point thus: Count the number of decimal places in 2·45 (two) and add this to the number of decimal places in 1·3 (one), making 3 decimal places in all. Starting from the right in the product 3185 count off 3 digits and place the decimal point: thus point off the 5, the 8 and the 1, and then write the dot. Thus the product of 2·45 and 1·3 is 3·185.

Example 42.—If 1 cubic foot of water weighs 62·5 lbs. and cast iron is 7·2 times heavier than water, what is the weight of 1 cubic foot of cast iron?

1 cu. ft. of cast iron will, of course, weigh 62·5 × 7·2 lbs.

62·5	. . . . .	1 decimal place
7·2	. . . . .	1 ,, ,,
1250		
4375		
450·00	. . . . .	2 decimal places.

The total number of decimal places in 62·5 and 7·2 being two, two digits are pointed off from the right in the product 450·00.

∴ 1 cu. ft. of cast iron weighs 450 lbs.

Occasionally, when a large number of decimal places are required to be pointed off in the result, it will be found that there are not sufficient figures available. Then a nought must be supplied for every required figure which does not exist in the product obtained. The following example will illustrate.

Example 43.—In finding the area of a small circle it was necessary to multiply together the numbers ·785 and ·0039. Find this value.

Again the number with the fewer multiplying figures is taken as the multiplier, *i. e.*, ·0039.

·785	. .	3 decimal places
·0039	. .	4 ,, ,,
7065		
2355		
·0030615	. .	7 decimal places. ∴ Result = ·00306

The multiplication of 785 by 39 gives the figures 30615 as result. Seven decimal places being required and five figures only existing, two noughts must be inserted to the left of the 3, in order to complete the seven decimal places. The answer then becomes ·0030615. The last two figures may be neglected and the result given as ·00306.

The foregoing method can be used for the multiplication of more than two numbers. In these cases a large number of figures will be obtained in the result, and many will have to be discarded as useless. *Much labour can be saved, and many useless figures avoided, by reducing the result each time to* 3 *significant figures before using the next multiplier.* In the following example the multiplication is worked in ordinary "long hand" manner, and also by the method just suggested. It will be seen that there is a big saving in labour, and the difference in the results is not worth considering. *To* 3 *significant figures the results are the same.*

Example 44.—Evaluate 1670 × 2·75 × 3·73 × 6·78.

Long hand working.

1670
2·75
8350
11690
3340
4592·50
3·73
137775
321475
137775
17130·025
6·78
137040200
119910175
102780150
116141·56950

to 3 sig. figs. = 116000

Working when each result is reduced to 3 sig. figs.

1670
2·75
8350
11690
3340
4592·50

say 4590 to 3 sig. figs.

3·73
13770
32130
13770
17120·70

say 17100
6·78
136800
119700
102600
115938·00

to 3 sig. figs. = 116000

It can be seen from the above that the multiplication of decimals becomes very long and tedious when more than two numbers are multiplied together. There are "contracted" methods of multiplying which are sometimes advocated, but their value is very doubtful, and they are probably never used in practice. The majority of engineering multiplication (and division) can be carried out with sufficient accuracy by logarithms (see Chap. VI), which are very simply and quickly used and certainly should be known by any one attempting calculation seriously.

Exercises 12. On Multiplication of Decimals.

(*Note.*—Nos. 1 to 10 are set to test the student's accuracy of multiplying; the answers should be given both fully and to 3 sig. figs.)

1. Multiply 29·5 by 1·38.
2. Multiply 235 by 1·72.
3. ,, ·057 ,, 13·9.
4. ,, 15·95 ,, ·785.
5. ,, 3·14 ,, 19·6.
6. ,, ·0087 ,, ·054.
7. ,, 13400 ,, ·0036.
8. ,, 191·5 ,, ·0986.
9. ,, ·0046 ,, 21·2.
10. ,, 1114 ,, 19·25.

11. If a cruiser's speed is 27 knots, what is this in miles per hour? (1 knot = 1·15 miles per hour.)

12. The speed of a cargo boat is 10½ knots. The speed of a tramcar is 12 miles per hour. Which vehicle is the faster?

13. The calculation of the "moment of inertia" of a cast-iron rail section (a figure used in connection with the strength of the section) gave the figures 2 × 1·78 × 2·7 × 2·7. Calculate the required value.

14. In calculating the weight per foot of a certain angle bar the following figures appeared: 7·11 × 12 × ·28 lb. Finish the calculation.

15. The following calculation was necessary to convert the readings of a water meter into lbs. per min.: 60 × 62·5 × ·026. Find the required value.

16. The calculation for the weight per foot of a large wrought-iron pipe produced 10·25 × ·785 × 3·4 lbs. Find the required value.

17. The working volume of an engine cylinder was given by the expression 36 × 3·14 × 24 cu. ins. Find the required value.

18. Find the value of the following figures, which give the amount a certain rail will expand under certain conditions: 360 × 25 × ·0000087 inches.

19. Pressures in connection with compressed air are frequently spoken of as "atmospheres," 1 atmosphere being 14·7 lbs. per sq. in. Find the number of lbs. per sq. in. in 35 atmospheres.

20. Find the number of lbs. per sq. in. in 32½ atmospheres.

21. The area of the steam cylinder in a steam-driven air compressor

is to be 1·38 times the area of the air cylinder. Find the steam-cylinder area when the air-cylinder area is 113 sq. ins.

22. Find the total resistance of 2 miles of electric cable whose resistance per yard is ·008 ohms (1 mile = 1760 yards).

Division of Decimals.—Let us first consider the case where the divisor is a whole number, *e. g.*, 11·75 ÷ 5. Express 11·75 as an improper vulgar fraction without cancelling, and it becomes $\frac{1175}{100}$. Divide by 5 according to the method stated in Chap. I, for the division of vulgar fractions, *i. e.*, divide the numerator. In this case divide 1175 by 5 = 235.

$$\text{Then } 11{\cdot}75 \div 5 = \tfrac{1175}{100} \div 5 = \tfrac{235}{100}.$$

Expressed decimally the result is 2·35. Now it will be seen that the digits in 2·35 have been obtained by dividing the digits in 11·75 by 5 in the ordinary manner; also that the whole number of the quotient is complete when the whole number in the 11·75 is finished with, *i. e.*, 2 is obtained when 11 has been divided. Or, when the decimal point is reached in 11·75 it is also reached in the 2·35.

Hence to divide a decimal by a whole number—

Divide the digits in the ordinary manner, and place the decimal point in the result when the decimal point is reached in the number being divided.

Then, the working of the above appears thus—

$$\begin{array}{r|l} 5 & 11{\cdot}75 \\ \hline & 2{\cdot}35 \end{array}$$

The decimal point is reached when 11 has been divided by 5, and therefore comes after the 2 in the result.

Now take the general case of a mixed number divided by a mixed number, *e. g.*, 8·672 ÷ 2·71. By a simple adjustment the example can be converted into one of the foregoing nature and worked in a similar manner.

Let us write the example in the form of a fraction, thus $\frac{8{\cdot}672}{2{\cdot}71}$. Now on p. 9, when considering cancelling, it was shown that a fraction may be multiplied top and bottom by the same

number without altering the value of the fraction. Applying this here let us multiply our fraction, both top and bottom, by 100.

Then we have $\frac{8{\cdot}672 \times 100}{2{\cdot}71 \times 100} = \frac{867{\cdot}2}{271}$ (See p. 37, on movement of decimal point.)

The example is now similar to the first one taken in this section, with the exception that "long" division is more convenient.

$$
\begin{array}{r}
271)867{\cdot}2(3{\cdot}2 \\
813 \\
\hline
542 \\
542 \\
\hline
\cdots
\end{array}
$$

Explanation.—Divide in ordinary way. Place decimal point in quotient when it is reached in number divided, *i.e.*, 271 into 867 goes 3, and since decimal point is then reached in 867·2, it is placed after the 3 in quotient.

Thus, $\frac{8{\cdot}672}{2{\cdot}71} = \frac{867{\cdot}2}{271} = 3{\cdot}2.$

A rough test is available: 8·672 is nearly 9, and 2·71 is nearly 3; dividing 9 by 3 gives 3, which is in agreement with our result 3·2.

If the divisor contains three decimal places then multiplication by 1000 would be necessary, instead of 100, to convert to a whole number. That is, the decimal point would have to be shifted three places to the right in both numbers, and so on.

From this we have the following rule for the division of decimals:—**Convert the divisor into a whole number by shifting the decimal point to the right. Move the decimal point in the dividend the same number of places, also to the right, and then perform the division in the ordinary way. The decimal point must be placed in the quotient when it is reached in the number divided.** All cases may be dealt with by this rule.

The following examples and remarks serve to illustrate certain special points that arise. When the divisor is already a whole number (a case of frequent occurrence) then no shifting of the decimal point is necessary.

It will frequently be found, when doing the actual dividing, that there are not sufficient figures in the dividend to "bring down." In this case a nought may be brought down each time, since a number like 34·5 may, of course, be written as 34·500. The following example illustrates.

Example 45.—Divide 3·45 by 1·2.

Move the decimal point in 1·2 one place, making 12
" " " 3·45 " " 34·5
Then divide—

```
12)34·5(2·875
   24
   ---
   105
    96
   ----
     90
     84
     --
      60
      60
      --
```

Note.—12 into 34 goes 2. Decimal point now being reached in 34·5 place it after the 2 in quotient. Continue as in ordinary division.

Hence 3·45 ÷ 1·2 = 2·875.

It will soon be found when dividing, that very few examples will work out "exactly," as have those taken up to the present. This is especially so with examples of a practical nature, and in these cases **the division should not be carried too far.** Thus if 23·75 be divided by 3·14, the answer is found to be 7·5636942, while yet leaving a remainder! It would be ridiculous to give a result of anything like this accuracy. In the first number (23·75) there are only 4 significant figures, while the second (3·14) only contains 3.

We may therefore reckon with safety that **3** *significant figures* would be sufficient in the answer; but the division should be carried to **4** *significant figures* in order that the third may be as correct as possible. Thus the division can be stopped when the quotient reads 7·563. The answer is then stated as 7·56.

The general rule of **3** or **4** *significant figures in the final answer* may be followed in all cases, *unless other instructions are given.*

When shifting the decimal point in the dividend it will sometimes be found that sufficient figures do not exist, in which case noughts may be added as desired. This is shown in the following case—

Example 46.—Divide 231·4 by 1·938.

The dot has to be moved three decimal places in 1·938, but only one place is available in 231·4. Hence two noughts must be added to

make up the three places, and the example appears as $\frac{231400}{1938}$, the decimal point in the numerator standing after the second 0.

```
1938)231400(119·40
     1938
     ----
     3760
     1938
     ----
     18220
     17442
     -----      <- When this point is reached the decimal
      7780          point is placed in quotient.
      7752
      ----
       280
```

Result = 119·4.

Note.—Four sig. figs. are given in result as there are four in each of the given numbers.

When the number divided is a small fraction it is possible that even after shifting the decimal point there will be no whole number. The procedure in this case is shown by the following example—

Example 47.—Divide ·01193 by 2·3.

Making the divisor 23 the dividend becomes ·1193.

```
23)·1193(·005186
    115
    ---
     43
     23
     --
     200
     184
     ---
      160
      138
      ---
```

Result = ·00519.

(This can be checked roughly by multiplying ·005 by 2·3.)

Explanation.—The first thing met with in the dividend is the decimal point. Therefore write the dot in the quotient, for if a fraction be divided by a whole number the result must be a fraction. Try if the first figure in the dividend can be divided by the divisor. If so divide, if not put a nought in the quotient and then try the first two figures, and so on. Thus 23 into 1 won't go; put a nought after the decimal point. 23 into 11 won't go; put a second nought. 23 into 119 goes 5 times. Put 5 in the quotient and continue as in ordinary division.

Exercises 13. On Division of Decimals.

1. Divide 219·5 by 5.
2. Divide 32·73 by 7.
3. „ 413·8 „ 22·75.
4. „ 13500 „ 4·36.
5. „ 1·897 „ 21·35.
6. „ ·158 „ ·0065.
7. „ ·0076 „ ·016.
8. „ ·098 „ 11·57.
9. „ 621 „ ·31.
10. „ 33000 „ 117.

11. If 14·7 lbs. per sq. in. = 1 atmosphere, find how many atmospheres in the following: (*a*) 165 lbs. per sq. in.; (*b*) 90 lbs. per sq. in.

12. The following figures are extracts from the results of an experiment to determine the coefficient of discharge of a rectangular notch. The coefficient is the value $\frac{\text{Actual discharge}}{\text{Theoretical discharge}}$. Calculate the coefficient in each case.

Actual discharge. Cu. ft. per sec. .	·0372	·0531	·1085
Theoretical discharge. Cu. ft. per sec.	·0571	·0829	·1750
Coefficient			

13. If 1 gallon of water = 10 lbs., and 2·205 lbs. = 1 kilogramme, find how many kilogrammes there are in 1 gallon of water.

14. If 1 kilogramme = 1000 grammes, and 453·6 grammes = 1 lb., find how many lbs. there are in 1 kilogramme.

15. If 1 kilogramme = 2·205 lbs., and 1 cwt. = 112 lbs., find how many kilogrammes go to 1 cwt.

16. If 746 watts = 1 horse-power, find how many horse-power = 1 kilowatt (*i. e.*, 1000 watts).

17. The reaction of a plate girder (*i. e.*, the upward force at the support) is 97 tons, and 1 square foot of surface is required for every 20 tons of load. Find how many square feet are required.

18 to 20. In certain experiments the quantity of water delivered by a pipe in a certain time was measured with the results below. Find in each case the quantity of water passing through in 1 minute.

18. 100 lbs. of water passed in 4·55 minutes.
19. 100 „ „ „ 2·37 „
20. 94 „ „ „ 3·02 „

21. Calculate the resistance per foot (*i. e.*, ohms for 1 foot) of an electric cable which is 520 yards long, and has a resistance of 20·7 ohms.

22. If 62·5 lbs. of water occupy a volume of 1 cubic foot, find the volume occupied by 1 lb. of water.

Conversion of Vulgar Fractions to Decimals.—In Chap. I, p. 7, it is shown that a vulgar fraction is the result of dividing one number by another. If now this division be performed as just described for decimals, a decimal answer will be obtained, and hence our vulgar fraction will have been converted into a

decimal fraction. Therefore to convert a vulgar into a decimal fraction : **Divide the numerator by the denominator.**

Example 48.—Convert the following vulgar into decimal fractions :—
(*a*) $\frac{1}{4}$; (*b*) $\frac{1}{8}$; (*c*) $\frac{7}{16}$

(*a*) 4)1·00
·25 ∴ $\frac{1}{4}$ = ·25.

(*b*) 8)1·00
·125 ∴ $\frac{1}{8}$ = ·125
or $\frac{1}{8}$ = $\frac{1}{2}$ of $\frac{1}{4}$
= $\frac{1}{2}$ of ·25 = ·125

(*c*) 16)7·0(0·4375
64
60
48
120
112
80
80
∴ $\frac{7}{16}$ = ·4375.

With the more uncommon vulgar fractions that appear, the division may not work right out without remainder. Then the rule of 3 or 4 significant figures may be followed.

Example 49.—The value of π,* a constant of great use in calculations with circles, may be given by the fraction $3\frac{16}{113}$. Express this decimally.

113)16·0(0·1415
113
470
452
180
113
070
505

∴ $3\frac{16}{113}$ = 3·14 to 3 sig. figs.

Example 50.—In a certain experiment the following intervals of time were read with a stop-watch—
(*a*) 1 min. 42 sec.; (*b*) 2 min. 35 sec.; (*c*) 1 min. 13 sec.
For the purposes of calculation it is desired to state these times as minutes only. Find the necessary values (*i. e.*, convert the seconds to decimals of a minute).

1 min. = 60 sec.

(*a*) 1 min. 42 sec. = $1\frac{42}{60}$ min. = $1\frac{7}{10}$ cancelling by 6 = 1·7 mins.

(*b*) 2 min. 35 sec. = $2\frac{35}{60}$ min. = $2\frac{7}{12}$ cancelling by 5
$2\frac{7}{12}$ = 2·583 min. or 2·58 to 3 sig. figs.

(*c*) 1 min. 13 sec. = $1\frac{13}{60}$ min. = 1·217 min. or 1·22 to 3 sig. figs.

* Greek letter "pi."

Exercises 14. On Conversion of Vulgar Fractions into Decimals.

Convert the following into decimal fractions—

1. $\frac{5}{8}$. **2.** $\frac{3}{32}$. **3.** $\frac{9}{16}$. **4.** $\frac{19}{21}$. **5.** $\frac{11}{64}$.
6. $\frac{5}{33}$. **7.** $\frac{9}{20}$. **8.** $\frac{5}{9}$. **9.** $\frac{6}{12}$. **10.** $3\frac{16}{113}$.

11 to **15.** In certain experiments measurement of time had to be taken in minutes and seconds. Convert the seconds into decimals of a minute in the following cases (*i.e.*, convert the fraction $\frac{\text{seconds}}{60}$ into a decimal, since 1 min. = 60 sec.)—

11. 18 sec. **12.** 34 sec. **13.** 11 sec. **14.** 54 sec. **15.** 13 sec.

16 to **20.** The "pitch" of a screw thread is $\frac{1}{\text{no. of threads}}$ per inch. Find the pitch as a decimal for the following cases:—

16. 24 threads per inch. **17.** 18 threads per inch.
18. 9 threads per inch. **19.** 4·5 threads per inch.
20. 3·5 threads per inch.

21 to **24.** Find the pitch, in decimals, of the following Whitworth Gas Threads:—

21. 11 threads per inch. **22.** 14 threads per inch.
23. 19 threads per inch. **24.** 28 threads per inch.

25. The "go in" end of an internal limit gauge is ·0004″ smaller than the nominal size, and the "not go in" end is ·0012″ larger than the nominal size, the latter being $1\frac{5}{8}$″. What are the actual sizes of the ends?

Compound Examples. Approximation for Result.—Some of the calculations that occur most frequently in engineering problems are the evaluation of expressions containing multiplication and division, such as $\frac{3{\cdot}28 \times 58{\cdot}7 \times 58{\cdot}7}{32 \times 600}$. These may be worked out on the lines of the previous paragraphs. Cancel where possible. Then the product of the numbers above the line may be obtained; also the product of the numbers below the line. Finally the division is performed. The number of figures in the calculations is often very large, and, in consequence, there are plenty of opportunities for making blunders and slips in the working. The most common mistake is that of wrongly placing the decimal point, and in many cases the blunder may pass unnoticed. Sometimes the nature of the problem gives a check on the accuracy of the result, *e. g.*, if on calculating the thickness of a boiler plate a result of 7·5″ was obtained, then there is evidently a mistake in the working; $7\frac{1}{2}$″ boiler plate is obviously absurd, and an examination of the calculations would no doubt show that the result should be ·75″, which is reasonable. But in cases where there is no such guide, and, in fact, *in all problems, it is advisable to get a*

rough idea of the result before doing the actual detailed working. To those who use a slide rule, or intend to do so, this "approximating for a result" is of *great use*, for the slide rule really works with the significant figures only, independently of the position of the decimal point.

The general method of approximating is to replace the actual figures by some near convenient round numbers, so that, using these, the approximate calculation becomes a very simple one, and may often be done mentally. The following example will illustrate:—

Example 51.—Find the approximate value of $\frac{293 \times 4{\cdot}9}{63}$

The 293 may be called 300
" 4·9 " " " 5
" 63 " " " 60

} This is mental work and is only written here as an explanation.

Then our approximate expression would be $\frac{300 \times 5}{60}$ and cancelling by 60, $\frac{\overset{5}{\cancel{300}} \times 5}{\underset{1}{\cancel{60}}} = \underline{25}$.

Hence our result must be about 25; it may be anything from, say, 20 to 30, but must have two figures in the whole number.

The actual result is 22·8 to 3 significant figures. Now supposing our detailed working had been bungled, and a result of, say, 2·28 or 228 had been obtained: the mistake would have been shown by our approximation of 25.

While the method just shown is suitable for simple examples, it is not accurate enough for more complicated ones. Also, for those readers who are not accustomed to much mental calculation, a more systematic method is desirable.

Considering the expression $\frac{19{\cdot}5 \times 529 \times {\cdot}67}{31 \times 1{\cdot}8}$

let us shift the decimal point in each of the compo... ... until a units figure *only* stands in front of the point (as in the 1·8); at the same time multiplying or dividing by the necessary number of tens to preserve the value of each number. Thus in the top line—

19·5 becomes $1{\cdot}95 \times 10$
529 " $5{\cdot}29 \times 10 \times 10$
and ·67 " $\frac{6{\cdot}7}{10}$

while in the bottom line—

31 becomes $3{\cdot}1 \times 10$

and the 1·8 remains unchanged, being already in the form required.

Using these "converted numbers," we may rewrite the given expression as—

$$\frac{(1{\cdot}95 \times 10) \times (5{\cdot}29 \times 10 \times 10)}{(3{\cdot}1 \times 10) \times 1{\cdot}8} \times \left(\frac{6{\cdot}7}{10}\right)$$

For convenience, let us separate the tens into one expression (*b*), and the remaining numbers into another (*a*), thus—

$$\underbrace{\frac{1{\cdot}95 \times 5{\cdot}29 \times 6{\cdot}7}{3{\cdot}1 \times 1{\cdot}8}}_{(a)} \times \underbrace{\frac{10 \times 10 \times 10}{10 \times 10}}_{(b)}$$

Expression (*b*), by cancelling, evidently reduces to 10.

Now replace each number in (*a*) by its *nearest whole number*, *i. e.*, write 2 in place of 1·95 and 3 in place of 3·1, and so on.

Then we have—

$$\underbrace{\frac{2 \times 5 \times 7}{3 \times 2}}_{(a)} \times \underbrace{10}_{(b)}$$

$$= \frac{35}{3} \times 10$$

$$= 12 \text{ (very nearly)} \times 10$$

$$= 120$$

which is then an *approximate answer* to the given expression. The actual answer, it may be noted, is 123·9.

In practice this method may be considerably abridged. Thus in expression (*a*) the "nearest whole numbers" may be obtained at one operation from the values in the given expression, the intermediate step being a mental one. For instance, taking the 19·5, the step 1·95 is performed mentally, the 2 only being written down; again, taking the ·67, the step 6·7 is mental, 7 only being written. Also, the result of expression (*a*) is not required with great accuracy; thus in the foregoing $\frac{35}{3}$ is called "12 very nearly," since the expression $\frac{36}{3}$ would give 12. A certain amount of caution in this direction is necessary, however, otherwise some of the "approximate" results may be *too far* from the truth. In expression (*b*) a stroke (1) may very conveniently be used to represent each 10, while the × signs may be omitted, since the only operations involved are those of multiplication and division.* These strokes may be cancelled in the usual way, and every stroke remaining

* It must be clearly understood that this method of approximating cannot be applied to expressions involving + and − signs.

after the cancelling will, of course, signify a 10. Instead of expression (*b*) in the foregoing we should then have $\frac{\text{X}\text{X}\text{I}}{\text{X}\text{X}}$, the remaining stroke on the top line indicating that the result of expression (*a*) is to be multiplied by 10; similarly a stroke *beneath* the line would indicate *division* by 10. These "strokes" are perhaps best placed at the right-hand side of the sheet, and will be shown in this position in the succeeding examples, under the heading of "Tens."

In connection with these "strokes" (*i. e.*, tens), it is seen from the detailed working already shown, that when a "converted number" contains a multiplying 10 the stroke must appear on the same side of the division bar as the number itself; but when containing a dividing 10, the stroke must appear on the *other* side of the bar. Thus, in the case of the ·67 $\left(\text{which} = \frac{6{\cdot}7}{10}\right)$ the stroke must be *under* the bar, while the number itself is on top, *i. e.*, the stroke is on the *other* side. Note also that in a number containing a multiplying 10 the decimal point is shifted to the *left* when obtaining the "nearest whole number," and in one containing a dividing 10, to the *right*. Considering all these points, we obtain the following **rules for approximation**—

> Taking each number in turn, shift the decimal point mentally until there is a units figure only in front of the point. Find the nearest whole number to the quantity so obtained, and write it in a new expression (called (*a*) in the foregoing working); at the same time place a stroke in an auxiliary expression (called (*b*) in the foregoing working) for every place that the decimal point has been shifted, according to the following rules—
>
> When the point is shifted to the *left* place the stroke (or strokes) on the *same side of the division bar* as the number occupies in the original expression; when shifted to the *right* place the stroke (or strokes) *on the opposite side* of the line.
>
> Work out the simple expression (*a*) and cancel the "strokes" so far as possible. For every remaining stroke, multiply or divide the result of (*a*) by 10, according to whether the strokes are above or below the line. This gives an approximate result.

It is important to notice that this method, although lengthy in description, is very quick and easy in actual use.

Example 52.—The expression $\frac{\cdot 39}{\cdot 000328 \times 1200}$ amperes had to be evaluated in connection with an experiment on electrolysis. Find the required value.

Approximation—

$$\frac{4}{3 \times 1{\cdot}2} = \frac{4}{3{\cdot}6} = 1 \text{ very nearly.}$$

Tens—

$$\frac{\text{tttt}}{\text{tttt}}$$

no strokes remaining.

∴ no multiplication or divisions by 10.

Therefore the result is about 1 ampere.

Explanation of Approximation.

Top line.—Convert ·39 to 3·9 and write 4. Decimal point having been moved one place to right, place a stoke *below* the line.

Bottom line.—Convert ·000328 to 3·28 and write 3. Decimal point moved 4 places to right, place 4 strokes *above* the line.

Convert 1200 to 1·2 *and write* 1.

The strokes all cancel out, so that the approximate answer is the result of the first expression, *i. e.*, about 1.

Working in full—

·000328
1200
65600
328
·393600

$$\frac{\cdot 39}{\cdot 394} = \frac{390}{394}$$

394)3900(0·9898
3546
3540
3152
3880
3546
3340

∴ Result is ·99 amperes, which agrees with the approximate answer.

Example 53.—Find an approximate answer to each of the following calculations. Given also the significant figures in the result (obtained by a slide rule, for example), state, from an inspection of the approximate answer, what the actual results must be.

(*a*) $\frac{32{\cdot}2 \times {\cdot}09}{4 \times 3{\cdot}14}$ Sig. figs. are 23.

Approximation—

$$\frac{8 \times 1}{4 \times 8} = \frac{1}{4} = {\cdot}25$$

·25 = approximate result

Tens.

$$\frac{1}{1}$$

∴ Actual result must be ·23, this being the nearest number to ·25 that can be made from the sig. figs. 23.

(*b*) $\frac{5 \times 33000 \times 12}{35 \times 144 \times 3{\cdot}14 \times 6}$ Sig. figs. are 209.

Approximation—

$\frac{5 \times \cancel{3} \times 1}{\cancel{3} \times 1{\cdot}5 \times 3 \times 6} = \frac{5}{27}$

= about $\frac{1}{5}$ = ·2

∴ an approx. result = ·2 × 100 = 20

∴ Actual result must be 20·9.

Tens—

$\frac{11111}{111}$

Two strokes above.

∴ Multiply by 10 × 10 = 100.

(*c*) $\frac{\cdot 695}{99{\cdot}7 \times 1{\cdot}605}$ Sig. figs. are 434.

Approximation—

$\frac{7}{1 \times 1{\cdot}5}$ = about 5

∴ an approx. result = $\frac{5}{1000}$ = ·005

∴ Actual result = ·00434.

Tens—

$\frac{}{111}$

∴ divide by 10 × 10 × 10 = 1000.

Note.—When a number, after being "converted," is nearly 1·5, as with the 144 at (*b*) above, it is best to call it 1·5 rather than 1 or 2, to avoid an answer which is too approximate (see also *c* above).

Example 54.—Determine, without actually working out the expressions, whether the answers given below are reasonably correct.

(*a*) $\frac{\cdot 000267}{\cdot 00852} = \cdot 0314$

Approximating— $\frac{3}{9} = \frac{1}{3} = \cdot 3$

∴ Result is about $\frac{\cdot 3}{10}$ = ·03, and answer given is reasonably correct.

Tens—

$\frac{111}{1111}$

∴ divide by 10.

(*b*) $\frac{4 \times 7{\cdot}14 \times 60}{8{\cdot}41 \times 6{\cdot}28 \times 2{\cdot}14} = 15{\cdot}1$

Approximating—

$\frac{\cancel{4} \times 7 \times \cancel{6}}{\cancel{8} \times \cancel{6} \times 2} = \frac{7}{2}$ = 2 about.

∴ Result is approximately 2 × 10 = 20, and answer given is reasonably correct.

Tens—

$\frac{1}{}$

∴ multiply by 10.

(*c*) 1·41 × 182 × ·00034 × 1·08 × 187 × 187 = 33200.

Approximating—

1 × 2 × 3 × 1 × 2 × 2 = about 24.

∴ Result is about 24 × 100 = 2400.

Tens—

$\frac{111111}{1111}$

∴ multiply by 100.

∴ Given result is incorrect, being 10 times too large, and should be 3320.

(*d*) $\dfrac{\cdot00307 \times \cdot0123 \times 8\cdot02 \times 1\cdot02}{\cdot0119} = \cdot26$

Approximating—

$$\frac{3 \times 1 \times 8 \times 1}{1} = 24 \text{ about}$$

∴ Result is about $\frac{24}{1000} = \cdot024$.

Tens—

11
11111

∴ divide by 1000.

∴ Given result is incorrect, and should be ·026.

Example 55.—A calculation for the indicated horse-power (I.H.P.) of a petrol engine reduced to the figures

$$\frac{74\cdot4 \times \cdot785 \times 3\cdot5 \times 3\cdot5 \times 800}{33000 \times 3}.$$ Find the I.H.P.

Cancel two noughts in 800 and 33000, making 8 and 330.

Approximation—

$$\frac{7 \times 8 \times \cancel{3} \times \cancel{3} \times 8}{\cancel{3} \times \cancel{3}} = 448$$

Tens—

1
111

∴ divide by 100.

∴ Approximate result is $\frac{448}{100} = 4\cdot48$, *i. e.*, about $4\frac{1}{2}$ H.P.

Working in full, expression $= \frac{5723}{990} = 5\cdot78$ I.H.P.

Example 56.—On testing a certain coal gas for its calorific value (*i. e.*, heating value) the figures $\frac{2250 \times 17\cdot7}{454 \times \cdot146}$ were obtained. Approximate for the answer, and, given that the significant figures are 6009, state the calorific value (in B.T.U. per cu. ft.) correct to the units figure.

Approximation—

$$\frac{\cancel{2} \times \cancel{2}}{\cancel{4} \times 1\cdot5} = \frac{1}{1\cdot5} = \frac{2}{3} = \cdot6$$

Tens—

11111
11

∴ multiply by 1000.

∴ Approximate result = ·6 × 1000 = 600 and actual result = 600·9.

∴ Calorific value to units fig. = 601 B.T.U. per cu. ft.

Exercises 15. On Approximating.

1 to **8.** Determine, without actually working out the expressions, whether the answers given in the following are reasonably correct.

1. $\frac{\cdot016}{\cdot00136} = 11\cdot8$.

2. $\frac{144\cdot6}{\cdot00341 \times \cdot7} = 60600$.

3. $\frac{1\cdot31 \times 33000 \times 150}{6400 \times 32\cdot2} = 315$.

4. $\frac{4 \times 9 \times 38 \times 3\cdot14}{32 \times 144 \times 6} = 1\cdot55$.

5. $\frac{\cdot000267}{\cdot0054} = \cdot494$.

6. $\frac{9\cdot87}{3\cdot22 \times 900} = \cdot00341$.

7. $2 \times \cdot785 \times 6\cdot25 \times 140 = 1375$.

8. $\frac{1\cdot075 \times 1\cdot075 \times 80\cdot4}{428} = \cdot217$.

9 to **18.** Find an approximate answer to each of the following expressions. Given also the significant figures in the results (obtained, say, from a slide rule), state, from an inspection of your approximate answers, what the actual results are (*i. e.*, put the decimal point in its correct place among the given figures).

9. $\frac{8 \times 9}{3{\cdot}14 \times 48}$	Sig. figs. in answer are	478.
10. $\frac{10000 \times {\cdot}5 \times {\cdot}5}{3{\cdot}25 \times 12}$	,, ,, ,,	64.
11. $\frac{100 \times 110 \times 2640}{33000}$	,, ,, ,,	88.
12. $\frac{24000}{30 \times 1{\cdot}06 \times 10{\cdot}56}$	,, ,, ,,	714.
13. $\frac{25 \times 3{\cdot}14 \times 1{\cdot}5 \times 24}{4 \times 33}$	,, ,, ,,	214.
14. $\frac{3{\cdot}14 \times 24 \times 1000}{1728 \times {\cdot}16}$	,, ,, ,,	272.
15. $\frac{87 \times 900 \times 32{\cdot}2}{9{\cdot}66 \times 9{\cdot}87 \times {\cdot}6}$	,, ,, ,,	44.
16. $\frac{4 \times 3{\cdot}14 \times {\cdot}18 \times 72{\cdot}5 \times 72{\cdot}5}{{\cdot}342}$	,, ,, ,,	347.
17. $\frac{6{\cdot}85 \times {\cdot}35 \times {\cdot}35 \times {\cdot}35}{12}$	,, ,, ,,	245.
18. $\frac{7 \times {\cdot}0625 \times 2240}{6 \times 232}$	,, ,, ,,	705.

In some examples expressions appear with the four simple arithmetical rules in various combinations. Then the best method of working is to preserve the general form of the expression throughout the various steps, as was adopted in Chap. I, p. 30. The following example illustrates:—

Example 57.—In calculating the safe eccentric load on a stanchion, the following expression was arrived at:—

$$\frac{238}{1 + \frac{6{\cdot}5 \times 6{\cdot}5}{5{\cdot}36 \times 5{\cdot}36}} \text{ tons.}$$ Complete the calculation.

The first step necessary is to evaluate the right-hand term under the line.

$$\frac{238}{1 + \frac{6{\cdot}5 \times 6{\cdot}5}{5{\cdot}36 \times 5{\cdot}36}} \text{ tons}$$

(*a*) $6{\cdot}5 \times 6{\cdot}5 = 42{\cdot}3$

(*b*) $5{\cdot}36 \times 5{\cdot}36 = 28{\cdot}7$

$\frac{a}{b} = 1{\cdot}47$

$$= \frac{238}{1 + 1{\cdot}47} \text{ tons}$$

$$= \frac{238}{2{\cdot}47} \text{ tons} = \underline{96{\cdot}4 \text{ tons}}$$

Exercises 16. Compound Examples.

1. The height of a mercury barometer is 30″. The height of the water barometer in feet is found by multiplying this figure by $\frac{13{\cdot}6}{12}$. Find the required height.

2 and **3.** In finding the centre of gravity of two rolled sections the following expressions had to be evaluated. Find the values required.

2. $\frac{3{\cdot}18 \times 5{\cdot}7}{7{\cdot}19}$ inches. **3.** $\frac{1{\cdot}67 \times 5{\cdot}5}{3{\cdot}6}$ inches.

4. The current to pass through an electro-magnet was calculated to be $\frac{5{\cdot}6 \times {\cdot}997}{28}$ amperes. Complete the calculation.

5. The resistances in an electric circuit, which is to carry a current of 4·5 amperes, are 3·5 ohms, ·75 ohms, and 2·38 ohms. The voltage (volts) required is equal to "current (amperes) × total resistance (ohms)." Find (*a*) total resistances; (*b*) voltage required.

6. The bending moment at a point on a continuous girder was given by the expression $\frac{{\cdot}5 \times 11380}{4 \times 70}$. Find the required value.

7. A stream is found to deliver 48 cu. ft. of water per sec. with a total fall of 120 ft. Find the horse-power of the stream, which is given thus: $\frac{48 \times 62{\cdot}5 \times 120}{550}$.

8. Find the thickness of plate for a steam boiler 6′-6″ diameter, to work at 100 lbs. per sq. in. pressure, from the following expression: $\frac{100 \times 78}{2 \times 9000 \times {\cdot}7}$ inches.

9. A dynamo is to supply 600 lamps, each of 16 candle-power. Each lamp takes 1·2 watts for every candle-power. (*a*) How many watts must the dynamo supply? If the voltage is 110, the current (amperes) supplied is $\frac{\text{watts}}{110}$ (*b*), what current is supplied?

10. The tensile test of an iron wire showed that the "Modulus of Elasticity" $= \frac{32 \times 84}{{\cdot}105 \times {\cdot}00104}$ lbs. per sq. in. Complete the calculation (3 sig. figs. will do).

11. The following figures were obtained in a torsional test on wrought iron: $\frac{10{\cdot}2 \times 57{\cdot}8 \times 57{\cdot}3 \times 51{\cdot}8}{{\cdot}388 \times {\cdot}388}$ lbs. per sq. in. Find the required value.

12. A coal gas was tested for its "calorific" or "heating" value. When ·146 of a cubic foot of gas was burnt, the temperature of 4·9 lbs. of water rose from 51° F. to 69° F. Calculate the calorific value (in B.T.U.) which equals $\frac{\text{lbs. of water} \times \text{rise in temperature}}{\text{cu. ft. of gas burnt}}$

13. Find the value of the expression 6·48 + ·3125 + 4·68 × ·506, figures obtained when calculating the strength of a stanchion.

14. Calculation in connection with a column gave the expression—

$$\frac{36}{1 + \frac{225}{2000}} \text{ tons.}$$ Complete the calculation.

15. Evaluate the expression $2 \times (21{\cdot}6 + 246{\cdot}5) + 427$, which refers to the strength of a built-up girder.

16. The bending moment on a girder, at a certain point, was found to be given by the expression $\frac{150 + 20(14 + 20 + 26 + 32)}{40}$ tons ft. Complete the working.

17. Find the value of the expression $5 \times (9{\cdot}5 + 8{\cdot}8 + 5{\cdot}6) + 15 \times (7{\cdot}8 + 6{\cdot}7)$ tons ft., which is the bending moment on a bridge truss.

18. One of the reactions of a continuous girder was found to be $\frac{{\cdot}5 \times 20}{2} + \frac{{\cdot}5 \times 10}{2} + \frac{16{\cdot}45}{20} + \frac{16{\cdot}45 - 13{\cdot}75}{10}$ tons. Finish the working.

19. The weight of a proposed plate girder was estimated to be $\frac{80 \times 70 \times 14}{1400 \times 7 - 70 \times 14}$ tons. Find its amount.

20. In calculating the safe eccentric load on a column the following expression was arrived at:—

$$\frac{64}{1 + \frac{3{\cdot}25 \times 5}{2{\cdot}82 \times 2{\cdot}87}} \text{ tons.}$$ Complete the calculation.

21. Find the value of the expression $130 + {\cdot}85\,(606{\cdot}5 - {\cdot}7 \times 170)$.

22. Evaluate $\frac{120 \times (1 + {\cdot}917)}{2{\cdot}5} - 4$ lbs. per sq. in., which occurred in a steam engine calculation.

23. The following figures were obtained when calculating the theoretical amount of air required to burn 1 lb. of oil—

$4{\cdot}35(2{\cdot}26 + 1{\cdot}2)$ lbs. Find the required value.

24. Complete the following calculation, which refers to the mean effective pressure in a steam engine—

$75 \times {\cdot}67 \times 1{\cdot}405 - 17$ lbs. per sq. in.

25. Find the value of the expression $\frac{37{\cdot}5 \times 80}{4} - \frac{37{\cdot}5 \times 18}{4}$ ft. tons, figures relating to a bending moment.

26. Find the weight of a roof truss, given by the following figures: $\frac{3 \times 10 \times 45}{4} \times \left(1 + \frac{45}{12}\right)$ lbs.

Averages.—When a quantity has various values at different times it is often necessary to state a value that shall give a *general idea* of the size of the quantity. Let us suppose that a boiler in the heating apparatus of a building had consumed the following quantities of coal during a certain month: 1st week 11 tons, 2nd week $6\frac{1}{2}$ tons, 3rd week $12\frac{1}{2}$ tons, 4th week 8 tons. If it is desired to state how much was generally burnt per week at that season of the year, it is hardly fair to quote any one of the actual numbers, since each number only applied to one particular week. Thus, the $12\frac{1}{2}$ tons was burnt in a cold week, while the $6\frac{1}{2}$ was used when the weather was mild; generally, the amount burnt was more than the $6\frac{1}{2}$ and less than the $12\frac{1}{2}$. Now let us imagine

that all the four weeks had been equally cold, so that an equal amount of coal was burnt each week; also that the *total amount burnt under the supposed conditions is equal to the total amount actually burnt.* Then we consider the total amount to have been spread equally over four weeks.

The total amount burnt $= 11 + 6\frac{1}{2} + 12\frac{1}{2} + 8 = 38$ tons, and, dividing this equally among four weeks, the consumption for one week would be $\frac{38}{4} = 9\frac{1}{2}$ tons. This supposed constant consumption is called the **average value** or **mean value.** A common way of making the statement would be, "On the average the consumption is $9\frac{1}{2}$ tons per week," meaning that the consumption per week is in the neighbourhood of $9\frac{1}{2}$ tons. From the above we obtain the rule:—

To find the average of a series of quantities, find the sum of all the quantities and divide by the number of quantities.

There are many applications of the use of the average, as will be seen from the examples and exercises in this section. Particularly, when testing or experimenting to find the value of a certain quantity, the experiment is repeated three or four or more times and slightly different results will be obtained owing to errors of instruments, etc. Then the average of all the measured results is found, and we may safely assume that this average is nearer the actual truth than any of the measured values, as the errors have been spread over the whole set of numbers.

Example 58.—A wrought-iron shaft under a torsion (twisting) test had its diameter measured in several places, the values being ·625″, ·621″, ·626″, ·623″, ·624″, ·622″, ·623″, ·621″. Find the average diameter of the shaft.

8 values.

·625″
·621
·626
·623
·624
·622
·623
·621

8)4·985

·6231

Average diameter = ·623″

It should be noticed that the accuracy of the average should not exceed the accuracy of the given figures.

A case of common occurrence to the engineer is the "averaging" of an indicator diagram from a steam or gas engine for the purpose of finding the indicated horse-power. Roughly an indicator diagram is of the form shown on the left of Fig. 14, and as will be seen in Chap. VII, the idea of "averaging," *i. e.*, finding the average height, is to get a figure equal in area to the diagram, but with a constant or unvarying height as on the right of Fig. 14. Certain

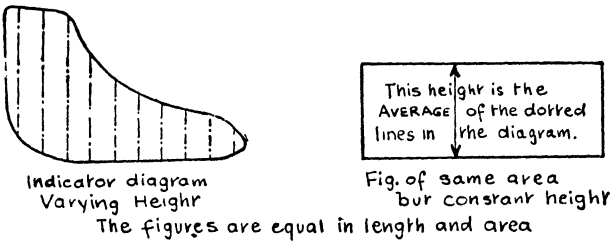


Fig. 14.—"Averaging" an Indicator Diagram.

lines (called ordinates) are drawn, as indicated by the dotted lines, and measured, and the average of these heights is found.

Example 59.—The ordinates, in inches, of an indicator diagram from a steam engine, were as follows: ·47, ·72, ·82, ·83, ·76, ·60, ·44, ·28, ·20, and ·08. Find (*a*) the average ordinate. One inch of ordinate represents 70 lbs. per sq. in. Find (*b*) the average pressure in lbs. per sq. in. (called the mean effective pressure, or M.E.P.).

10 ordinates.

·47
·72
·82
·83
·76
·60
·44
·28
·20
·08

10)5·20

·52

(*a*) Average ordinate = ·52″

1″ of ordinate = 70 lbs. per sq. in.

∴ ·52″ of ordinate = 70 × ·52 = 36·4 lbs. per sq. in.

(*b*) M.E.P. = 36·4 lbs. per sq. in.

Example 60.—The readings of temperature in the table on p. 70 were made in an experiment on the calorific value of coal gas. Find (*a*) the average inlet temperature, (*b*) the average outlet temperature, and,

(*c*) the rise in temperature (*i. e.*, average outlet temperature, — average inlet temperature).

Inlet temp. °C.	Outlet temp. °C.
9·47	30·62
	31·32
	31·63
	31·63
	31·80
9·46	31·90
	32·20
	32·61
	32·82
	33·00
	33·13
9·42	33·22
3)28·35	12)385·88
9·45	32·157

Result—

(*a*) 9·45° C.

(*b*) 32·16° C.

(*c*) 22·71° C.

Exercises 17. On Averages.

1. Three separate experiments were made to determine the "refractive index" of a certain kind of glass, the results being 1·52, 1·505 and 1·49. Find the average value.

2. Experiments to determine the "coefficient of discharge" of a V-notch gave the following results: ·575, ·574, ·580, ·581, ·581, ·581. Find the average value.

3. In measuring the internal resistance of an accumulator the experiment was performed five times with the following results: ·025, ·028, ·03, ·0286, ·0275 ohms. Find the average resistance.

4. The diameter of an iron wire measured in several places for a tensile test was as follows: ·0365, ·0346, ·0365, ·0364, ·0358, ·0368, ·0365, ·0348 in. Find the average diameter.

5. The following values of the coefficient of friction between two wood surfaces were obtained in an experiment: ·417, ·409, ·367, ·395, ·37, ·389, ·403 and ·415. Find the average value.

6. During a boiler test the following readings were taken on the water gauge measuring the supply of feed water to the boiler: 31·6, 31·7, 31·7, 31·6, 33, 34·5, 32·4, 26·5, 20·0, 23·8, 24·0, 24·0, 24·5, 24·7, 24·5 lbs. per minute. Find the average value of the supply.

7 to **9.** The following figures were taken during the test of a steam electric plant; the steam pressure, voltage, and current being kept as steady as possible.

Steam pressure (lbs. per sq. in.): 96, 100, 99, 100, 100, 98, 102, 102, 102, 103. Voltage: 110, 111, 111·5, 109, 109·5, 109·5, 111, 112, 111, 110. Current (amperes): 220, 223, 223, 230, 229, 224, 225, 217, 220, 200.

7. Find the average steam pressure.

8. Find the average voltage.

9. Find the average current.

10. The following readings were taken on an anemometer at different parts of the ashpit, in measuring the air supplied to a furnace: 570, 580, 620, 650, 660, 600, 600, 460 cu. ft. per min. Find the average value.

11. In an experiment to find the specific resistance of a piece of wire the following readings of diameter were taken: ·465, ·463, ·460, ·460, ·461, ·460, ·462, ·464 millimetres. Find the average diameter.

12. Tests on a number of materials, to determine the ratio of shear to tensile strength gave the following results: ·67, ·64, ·66, ·72, ·81, ·82, ·88, ·85, ·85, ·87, ·83, ·73, ·91, ·59, ·90, ·98. Find the average value.

Percentage.—The words "Percentage" and "so much per cent." are frequently met with, a percentage being another way of expressing a fraction (or division) whose denominator is 100. Thus the statement "5 per cent." (written as 5%) means 5 parts out of 100 parts, *i. e.*, $\frac{5}{100}$ in the vulgar form. Cancelling reduces this to $\frac{1}{20}$; or it may be expressed in the decimal form ·05, shifting the point two places to the left to divide by 100. Both forms are useful. From the above we obtain the following rules:—

1. To convert a percentage into a decimal fraction, shift the decimal point two places to the left.

2. To convert a percentage into a vulgar fraction, write the percentage as numerator, and write 100 as denominator; then reduce to the lowest terms.

Example 61.—Convert the following percentages into decimal fractions: (*a*) 30%, (*b*) 37·5%, (*c*) 1·13%, (*d*) ·028%.

(*a*) Shifting decimal point two places to left (to divide by 100), 30% = ·3

(*b*) Similarly 37·5% = ·375

(*c*) ,, 1·13% = ·0113

(*d*) ,, ·028% = ·00028

Example 62.—Convert the following percentages into vulgar fractions: (*a*) 45%, (*b*) 12½%, (*c*) 33⅓%, (*d*) 26·25%, (*e*) ·025%.

(*a*) $45\% = \frac{45}{100}$. Cancelling by 5 $= \frac{9}{20}$

(*b*) $12\frac{1}{2}\% = \frac{12\frac{1}{2}}{100} = 12\frac{1}{2} \div 100 = \frac{25}{2} \times \frac{1}{100} = \frac{1}{8}$

(*c*) $33\frac{1}{3}\% = \frac{33\frac{1}{3}}{100} = 33\frac{1}{3} \div 100 = \frac{100}{3} \times \frac{1}{100} = \frac{1}{3}$

(*d*) $26{\cdot}25\% = \frac{26{\cdot}25}{100} = \frac{2625}{10000}$. Cancelling by 25 and 5 $= \frac{21}{80}$

(*e*) $\cdot025\% = \frac{\cdot025}{100} = \frac{25}{100000}$. Cancelling by 25 $= \frac{1}{4000}$

It is often required to find a certain percentage of a quantity, meaning that we are to find a certain fraction of the quantity. **To do this, express the percentage as a fraction and multiply by the quantity.**

Example 63.—An electric motor is to work at 30 h.-p. under ordinary full load conditions. It is required, however, to deliver 25% overload (*i. e.*, 25% extra) for a certain period. Find the greatest h.-p. the motor must be capable of giving.

Full load = 30 h.-p.

$$\text{Overload} = 25\% = \frac{\overset{5}{\cancel{25}}}{\underset{2}{\cancel{100}}} \times \cancel{30} = 7{\cdot}5$$

$\therefore$ Greatest h.-p. = 30 + 7·5 = 37·5 h.-p.

The vulgar form is the more useful when the percentage is equivalent to a simple fraction with 1 as numerator; *e. g.*, 20% = $\frac{1}{5}$, 12½% = $\frac{1}{8}$, etc. In these cases, to multiply as per rule we merely have to divide by the denominator of the fraction; thus the above can be done mentally: 25% = $\frac{1}{4}$, therefore divide the 30 h.-p. by denominator 4. For other cases the decimal form is better, as multiplying by a decimal is easier than by a complicated vulgar fraction. It is useful to memorise the equivalents of the simpler percentages, which are given below.

1%	= $\frac{1}{100}$ = ·01	25%	= $\frac{1}{4}$ = ·25
5%	= $\frac{1}{20}$ = ·05	33⅓%	= $\frac{1}{3}$ = ·3333
10%	= $\frac{1}{10}$ = ·1	50%	= $\frac{1}{2}$ = ·5
12½%	= $\frac{1}{8}$ = ·125	66⅔%	= $\frac{2}{3}$ = ·6667
20%	= $\frac{1}{5}$ = ·2	75%	= $\frac{3}{4}$ = ·75

100% of course = $\frac{100}{100}$ = 1

Example 64.—The catalogue price of a certain machine is £7 10s. The makers announce that "owing to the increased cost of raw material, all prices are advanced 10%." An agent who buys the machine under these new conditions is allowed 20% discount. Find the actual price he pays.

Note.—The figures in brackets thus [ ] are only to assist the beginner and do not appear on the actual bill.

		£	s.	d.
Catalogue price	=	7	10	0
10% advance [= $\frac{1}{10}$ of £7 10s.]	=		15	0
New price	=	8	5	0
20% discount [= $\frac{1}{5}$ of £8 5s.]	=	1	13	0
Price paid [= difference]	=	£6	12	0

Explanation.—10% of £7 10*s.* = $\frac{1}{10}$ of 150*s.* = 15*s.* 20% of £8 5*s.* = $\frac{1}{5}$ of £8 5*s.* Dividing £8 by 5 gives 1 and £3 over. £3 = 60*s.* and adding in the 5 gives 65*s.* This divided by 5 = 13*s.* Thus 20% of £8 5*s.* = £1 13*s.*

A common error in such examples as the last is to say that "10% advance with 20% discount is equal to 10% discount." This is not so, since the 10% and the 20% *refer to different quantities:* the 10% to the old price, and the 20% to the new price: the 20% cannot be used until the old price has been found.

Example 65.—In steel-makers' catalogues it is stated that the actual weights of rolled steel section bars may vary 2½% on either side of the listed weights. Calculate the greatest and least weights of a channel bar listed at 19·3 lbs. per foot.

Listed weight = 19·3 lbs. per ft.

2½% allowance = $\frac{2{\cdot}5}{100} \times 19{\cdot}3$

= ·482 lbs. per ft.

∴ Greatest weight = 19·3 + ·48 = 19·78 lbs. per ft.

and least ,, = 19·3 − ·48 = 18·82 lbs. per ft.

Conversion to a Percentage.—It is frequently necessary to express some quantity as a fraction of another, and the most usual way is to express it as a percentage. Suppose that the nominal full load of an alternator is 60 kilowatts, but at some particular time it is called upon to deliver 75 kw., an increase of 15. Now this in itself is not much guide as to how much the machine has been overloaded, as the overload allowable depends on the size of the machine. Thus an alternator designed for 6000 kw. could easily deliver another 15 kw.; but it would be impossible to get another 15 kw. out of a machine designed for, say, 20 kw. In each case the *actual increase* is the same, but *as a fraction of the full load* is very different; this fraction is the important figure. Taking the big machine 15 kw. expressed as a fraction of 6000 kw. is $\frac{15}{6000}$; with the small machine 15 as a fraction of 20 is $\frac{15}{20}$. Now to avoid the confusion of having vulgar fractions with different denominators, the fractions are expressed as a percentage, *i. e.*, they are converted so that the denominator is always 100.

From Rule 2 on p. 71, it follows that to convert a vulgar fraction into a percentage we must multiply by 100. Converting the above fractions to percentages we have—

$$\frac{\overset{1}{15}}{\underset{4}{6000}} \times \overset{1}{100} = \tfrac{1}{4}\% \text{ or } {\cdot}25\% \text{ overload, and } \frac{15}{\underset{1}{20}} \times \frac{\overset{5}{100}}{1} = 75\% \text{ overload.}$$

Then 15 kw. increase on 20 kw. is an overload of 75% but an increase of 15 on 6000 kw. is only ·25% overload. Thus we have the rule: **To express one quantity (A) as a percentage of another quantity (B) write (A) over (B) as a vulgar fraction, multiply by 100, and evaluate.**

Thus A expressed as a percentage of B is $\frac{A}{B} \times 100$.

Examples where this occurs are many and include: efficiencies of all kinds, errors of instruments and experiments, changes of speed and load, extensions of test bars, the composition of alloys, etc., etc.

Example 66.—An ammeter, when tested, was found to read 4·63 amperes, the true value being 4·5 amperes. Find the percentage error (*i. e.*, express the error as a percentage of the true value).

Apparent value = 4·63
True value = 4·50

∴ Error = ·13 too high

$$\therefore \%\ \text{error} = \frac{\cdot 13 \times 100}{4 \cdot 5} = 2 \cdot 89\%\ \text{too high,}$$

i. e., for every 100 amperes indicated the error is 2·9 amperes.

Example 67.—The following figures were obtained in a tensile test of a piece of steel—

Original length 5″. Original area of cross section ·994 sq. in.
Final length 6·05″. Final area of cross section ·407 sq. in.

(*a*) Express the total extension as a percentage of the original length. (*b*) Express the contraction in area as a percentage of the original area.

(*a*) Total extension = Final length − original length.
= 6·05 − 5 = 1·05″

$$\therefore \text{Extension as \% of original length} = \frac{1 \cdot 05 \times 100}{5} = 21\%$$

(*b*) Contraction in area = Original − Final area = ·994 − ·407 = ·587 sq. in.

$$\therefore \text{Contraction as \% of original area} = \frac{\cdot 587 \times 100}{\cdot 994} = 59 \cdot 05\%,$$

say, 59%

Actually, the statements made are not as complete as in the above two examples. Thus in Example 66, the usual statement is "find the percentage error," and the figure of which the error is to be a percentage would not be stated. Then the student may be in doubt as to whether he ought to express the ·13 amperes as a

percentage of the true value 4·5, or the apparent value 4·63. A slight knowledge of the particular work with which the example is connected is useful in this respect, but the following will help—

Instrument and experimental errors are stated as a percentage of the **true value.**

With changes of load and speed, the error is stated as a percentage of the **normal** or **full load, or speed.**

Extensions and contractions of test bars are stated as percentages of the **original length or area.**

With composition of alloys, etc., the quantity of each material is given as a percentage of the **total quantity.**

Example 68.—A sample of coal weighing 3·35 lbs. was found on analysis to contain: Carbon 2·52 lbs., hydrogen ·234 lb., oxygen ·197 lb., nitrogen ·133 lb., and the remainder ash. Find the percentage composition of the coal (*i. e.*, express the weight of each constituent as a percentage of the total weight).

Carbon	2·52 lbs.	Total coal . . .	3·35 lbs.
Hydrogen	·234 lb.	Total (less ash) .	3·084 ,,
Oxygen	·197 ,,	∴ Ash = difference =	·266 lb.
Nitrogen	·133 ,,		
Total (without ash) .	3·084 lbs.		

Carbon	$\frac{2\cdot52 \times 100}{3\cdot35} = \frac{252}{3\cdot35}$	=	75·2%
Hydrogen	$\frac{\cdot234 \times 100}{3\cdot35} = \frac{23\cdot4}{3\cdot35}$	=	6·99%
Oxygen	$\frac{\cdot197 \times 100}{3\cdot35} = \frac{19\cdot7}{3\cdot35}$	=	5·88%
Nitrogen	$\frac{\cdot133 \times 100}{3\cdot35} = \frac{13\cdot3}{3\cdot35}$	=	3·97%
Ash	$\frac{\cdot266 \times 100}{3\cdot35} = \frac{26\cdot6}{3\cdot35}$	=	7·95%
		Total	99·99%

The sum of the percentages of *all* the constituents should be 100%. Any serious difference would indicate a mistake somewhere. Due to slight approximations, etc., small differences may arise, but if, as in the above, the percentages total 99·99%, the figures can quite well be accepted. The exact difference allowable depends on the delicacy of the experiments, etc.

Example 69.—The chemical analysis of a "Self-hardening" steel gave the following results: Carbon ·63%, chromium 1·04%, manganese ·05%, silicon ·15%. Find the percentage of iron "by difference" (*i. e.*, subtract the total % of matter other than iron, from 100%).

This example does not involve any finding of percentages, but is merely an addition dealing with percentages.

Carbon . . .	·63%	100·00
Chromium . .	1·04 „	1·87
Manganese . .	·05 „	
Silicon . . .	·15 „	98·13% Iron
	1·87%	∴ Iron " by difference " = 98·13

Exercises 18. On Percentages.

1. A firm engaged in the manufacture of valves, makes the following extra charges for drilling holes in the flanges of cast-iron bodies:—
1½″ holes 4*s.* per dozen holes; 1″ holes 2*s.* 6*d.* per dozen.
For cast-steel bodies they charge 50% extra. Find the rates for the above holes in cast steel.

2. If 5% extra is charged for packing a job for export, find the charge for packing a job costing £50.

3. If the estimated cost of a certain job is 26*s.* and a profit of 12½% is to be added, find the selling price of the job.

4. The ordinary price of a certain clutch is £4. Due to an increase in cost of material the price is increased 5%. Find (*a*) the new charge. A customer is now allowed a discount of 25%. Find (*b*) the actual amount he is to pay.

5. A wrought-iron shaft will only transmit 70% of the h.-p. that the same-sized steel shaft will transmit at the same speed. If the h.-p. of a certain steel shaft is 256, what is that of the same-sized wrought-iron shaft?

6. A square foot of wrought-iron plate 1″ thick weighs 40 lbs. If steel is 2% heavier than wrought iron, find the weight of the same-sized sheet of steel.

7. The " working volume " of a steam engine cylinder is 5·96 cu. ft. The " clearance " volume is 8% of the working volume. Find (*a*) this clearance volume in cubic feet; (*b*) the total volume in cubic feet (*i. e.*, clearance + working).

8. A variation in weight of 2% either side of the standard weight is allowed in electric conductors. The standard weight of 1000 yds. of 37/13 conductor is 2900 lbs. Find the maximum and minimum weights allowable.

9. The Admiralty test load on a ½″ crane chain is 3 tons. At the Elswick Works the test load is 10% higher. Find the Elswick test load.

10. It is found that an engine governor keeps the speed between 486 and 474 revolutions per minute. Find the change of speed and express it as a percentage of the average speed.

11. At 40° F., 1 cu. ft. of water weighs 62·43 lbs., and at 400° F. weighs 53·63 lbs. Find the decrease in weight, and express it as a percentage of the greatest weight.

12. The correct value of " *g* " (the acceleration due to gravity) in the metric system at London is 981 units. An experiment gave the value 992. Calculate the percentage error (*i. e.*, express difference as a percentage of the true value).

13. A pump was rated by its makers at 300 gallons per minute. On test it filled a tank, holding 750 gallons, 4 times in 9 min. 27 sec.

Find (*a*) the test rating of the pump (*i. e.*, the gallons delivered in 1 min. on test), and (*b*) the difference between the makers' and the test ratings. Express this as a percentage of the makers' rating, saying whether it is above or below.

14 to **16.** The composition of some alloys is given below. Find in each case the percentage composition.

14. Bell metal : 16 parts copper, 5 parts tin.

15. Soft gun metal : 16 parts copper, 1 part tin.

16. Ajax plastic bronze : 13 parts copper, 1 part tin, 6 parts lead.

17. A solution of electrolyte for copper plating is composed of 1 lb. copper sulphate, 1 lb. sulphuric acid, and 10 lbs. water. Calculate the percentage composition of the solution.

18. A quantity of exhaust gas is known to consist of the following : Water ·18 cu. ft., carbon dioxide ·24 cu. ft., nitrogen ·852 cu ft., oxygen ·08 cu. ft. Express the quantity of each constituent as a percentage of the total quantity.

19. The following figures were calculated from an analysis of a sample of flue gas : Carbon dioxide 1·54, oxygen 1·81, nitrogen 11·43. Express these figures as percentages of the total.

20 to **23.** The chemical analyses of various irons and steels are given in the accompanying table. Find the percentage of iron in each case, by difference.

No.	Material	Carbon %	Silicon %	Sulphur %	Phosphorus %	Manganese %	Iron %
20	Locomotive Boiler Plate (Mild Steel)	·148	·024	·039	·022	·562	
21	Tram Rail (Mild Steel)	·739	·347	·030	·028	·720	
22	Cast Iron . . .	3·23	1·46	·14	·638	·56	
23	Wrought Iron . .	·04	·17	·013	·31	·09	

24 to **26.** Engine governors are designed to prevent the engine speed from changing more than a certain amount when the load is removed or applied suddenly. The variation allowed depends upon the kind of work—

For cotton spinning the change is not to be more than 2% above and below normal speed.

For electric traction the change is not to be more than 1·5% above and below normal speed.

For machine shop work the change is not to be more than 3% above and below normal speed.

Find the greatest and least speeds in the following cases—

24. Normal speed 200 revs. per min., for cotton spinning.

25. Normal speed 180 revs. per min., for electric traction.

26. Normal speed 85 revs. per min., for machine shop drive.

27. A salt solution is made by adding 65 grammes of salt to 250 grammes of water. Find the percentage of salt in the solution (*i. e.*, salt as a % of the total).

28. A mixture for " blueing " steel is composed of water 10 lbs., hypo 2 oz., sugar of lead 2 oz. Calculate the percentage composition by weight of the mixture. (*Note.*—Work in ounces.)

Ratio.—It is often necessary to state the number of times that one quantity contains another, and this is usually given in the form of a **ratio**. Thus, if the diameter of a steam engine cylinder is 12″ and the stroke 18″, the stroke is evidently $1\frac{1}{2}$ times the diameter, since $18'' = 1{\cdot}5 \times 12''$. Now this fact of the stroke being equal to $1\frac{1}{2}$ diameters is expressed by the words "*the Ratio of stroke to diameter is* **1·5** *to* **1**." The 1·5 is evidently the value of $\frac{18}{12} = \frac{\text{stroke}}{\text{dia.}}$ *i. e.*, $\frac{\text{1st quantity}}{\text{2nd quantity}}$. Thus, the ratio of one quantity (A) to another (B) is the number of times that (A) contains (B).

It should be clear that we cannot give a ratio between two quantities of different measures. Thus, we cannot say that the ratio of 27 tons to 9 miles is 3 to 1; certainly 27 tons is not 3 times 9 miles. Hence *the measures must be alike.* Further, the value of the ratio cannot be worked out without bringing both quantities to the **same unit.** Thus, consider the ratio of 2 ft. to 6 in. Obviously this is *not* $\frac{2}{6}$ or $\frac{1}{3}$ or 1 to 3, for 2 ft. is not one-third of 6 in. But, converting the 2 ft. into inches (24″) the ratio is $\frac{24}{6}$ or $\frac{4}{1}$ or 4 to 1, meaning that 2 ft. is 4 times 6 in., which is certainly true.

Thus we obtain the rule: **To find the ratio of one quantity (A) to another quantity (B), reduce both to the same units and divide the first quantity by the second,** *i. e.* $\frac{A}{B}$.

Example 70.—The "grate area" of a boiler is 22 sq. ft., and the heating surface is 1365 sq. ft. Find the ratio of heating surface to grate area.

The first-named quantity in the ratio required is heating surface, so that heating surface is to be divided by grate area.

Then, ratio of heating surface to grate area $= \frac{1365}{22} = 62{\cdot}1$

or, say, 62 to 1.

Example 71.—A locomotive crank is 13″ long and the connecting rod is 6 ft. 2 in. long. Find the ratio of the length of connecting rod to length of crank.

Convert to the same units, preferably inches. Then the rod is 74″ long. The required ratio is "connecting rod to crank," so that connecting rod is to be divided by crank.

Then ratio of connecting rod to crank $= \frac{74}{13} = 5{\cdot}69$, or, say, 5·7 to 1.

Slight variations in the manner of stating a ratio are met with. The word "to" is frequently replaced by two dots, thus (:), so that

the statement "4 : 1" is read as "4 to 1." Sometimes only the result of the division is stated, the words "to 1" being understood. Thus a ratio 5·7 means a ratio of 5·7 to 1.

It should be noticed that two ratios always exist between two quantities. Thus, considering our original case of stroke and diameter we can have—

(*a*) The ratio of stroke to diameter, which $= \frac{\text{stroke}}{\text{dia.}} = \frac{18}{12}$ or 1·5 to 1

i. e., stroke = 1·5 times diameter.

(*b*) The ratio of diameter to stroke, which $= \frac{\text{dia.}}{\text{stroke}} = \frac{12}{18}$ or ·667 to 1

i. e., diameter is ·667 times stroke.

In order to avoid any doubt as to which is required the form of statement with the words "of" and "to" should be carefully kept; a statement such as "the ratio between stroke and diameter" is not sufficiently clear and should be avoided. Then the quantity first mentioned will be the numerator of the ratio fraction, *e. g.*, in (*a*) the stroke is first mentioned and therefore stroke value is above diameter value in the fraction.

Also the two ratios are "reciprocal," *i. e.*, either one is the other inverted.

Thus stroke to diameter $= \frac{18}{12} = \frac{1·5}{1}$ and diameter to stroke $= \frac{12}{18} = \frac{1}{1·5} = ·667$; one can always be obtained from the other.

Example 72.—The following figures refer to a steam-driven air compressor: Area of steam cylinder = 113 sq. ins.; area of air cylinder = 93·5 sq. ins. Calculate the ratio of air cylinder area to steam cylinder area and give also the reverse ratio of steam cylinder to air cylinder.

Ratio air cylinder area to steam cylinder area $= \frac{93·5}{113}$

$= ·827$, *i. e.*, ·827 to 1

Then the ratio steam cylinder to air cylinder = 1 to ·827

$= \frac{1}{·827} = 1·21$, *i. e.*, 1·21 to 1.

Example 73.—The efficiency of any machine is the ratio of the output to the input. A generator gives an output of 27·6 kilowatts for an input of 51 h.-p. Find the efficiency.

(*Note*—1 kilowatt = 1·34 h.-p.)

Reduce to same units, preferably horse-power.

Then, since 1 kw. = 1·34 h.-p.

27·6 kw. = 27·6 × 1·34 = 37 h.-p.

Efficiency = Ratio of output to input = $\frac{27}{37}$ = ·725

Note.—In the case of efficiency it is usual to state only the decimal result as shown, omitting the words "to 1." In many cases also this result is converted to a percentage, which for the above would be 72·5%.

Ratios are used when speaking of the relative sizes of the cylinders in a compound or multi-stage expansion steam engine. The case of the compound engine, with only 2 cylinders, may be similarly treated, but with triple and quadruple expansion 3 and 4 cylinders have to be dealt with, and 2 or 3 ratios are combined into one short statement.

Example 74.—The triple-expansion engines of the White Star liner *Olympic* have the following cylinder areas: High Pressure (H.P.) 2290 sq. ins.; Intermediate Pressure (I.P.) 5542 sq. ins., and Low Pressure (L.P.) 7390 sq. ins. Find the "cylinder ratios" (viz. ratios of the cylinder areas), taking H.P. as 1.

There will be two ratios; and H.P. area will be the 2nd quantity in each case.

The area ratios are: **1.** I.P. to H.P. **2.** L.P. to H.P.

1. Ratio of I.P. to H.P. = $\frac{5542}{2290}$ = 2·42 : 1

2. Ratio of L.P. to H.P. = $\frac{7390}{2290}$ = 3·23 : 1

Instead of writing these results as two, they are combined into one by giving "the ratio of L.P. to I.P. to H.P."; thus 3·23 : 2·42 : 1, meaning that if the H.P. area be 1, then the I.P. area is 2·42 and the L.P. area 3·23. Then the ratio of the L.P. to the I.P. should be according to this, $\frac{3·23}{2·42}$ = 1·335. This is proved by taking the actual areas given. Then ratio of L.P. to I.P. = $\frac{7390}{5542}$ = 1·335 as before. The dots (:) are invariably used when giving cylinder ratios.

Ratios connected with certain triangles are very important, and are the foundation of a special branch of mathematics known as "Trigonometry." (*See* Chap. XI.)

Exercises 19. On Ratios.

Find the ratio of connecting rod length to crank length in the following engines:—

1. (*a*) A petrol motor; crank $2\frac{1}{4}''$, connecting rod $11''$.
(*b*) A marine engine; crank $30''$, connecting rod $5'-1''$.

2. (*a*) A high-speed electric-lighting engine; crank $4''$, connecting rod $1'-8''$.
(*b*) A power-driven air-compressor; crank $2\frac{3}{8}''$, connecting rod $14''$.

The "diagram factor" of a steam engine is the ratio of the actual mean pressure to the theoretical mean pressure. Find the diagram factor for the following cases:—

3. A marine engine; theor. mean press. 36 lbs. per sq. in.; actual mean press. 23 lbs. per sq. in.

4. Corliss engine; theor. mean press. 29 lbs. per sq. in.; actual mean press. 25 lbs. per sq. in.

5. The "buckling factor" of a column is the ratio of its length (inches) to a measurement known as its "least radius of gyration" (inches). Find the buckling factor: (*a*) when length = 14 ft., and least radius of gyration = $4{\cdot}13''$; (*b*) when length = $38'-6''$, and least radius of gyration $4{\cdot}1.''$

6. Find the ratio of length to diameter of a Lancashire boiler: (*a*) when length = 24 ft. and diameter $7'-6''$; (*b*) when length = 21 ft. and diameter = $6'-6''$. *Note*—work in feet.

7. Find in each of the following boilers the ratio of heating surface to grate area:—

(*a*) Heating surface 720 sq. ft.; grate area 24·75 sq. ft.
(*b*) Heating surface 1616 sq. ft.; grate area 53·6 sq. ft.

8. Values for the tensile and shearing strength of two metals are given below. Find in each case the ratio of shear strength to tensile stress:—

(*a*) Copper: Tensile 14 tons per sq. in.; shear 11·6 tons per sq. in.
(*b*) Hard rolled bronze: Tensile 26·9 tons per sq. in.; shear 16·06 tons per sq. in.

9. The results of tests giving the elastic limit stress and breaking stress (in tons per sq. in.) of various metals are given below. Find for the two materials ratio of the elastic stress to breaking stress in each case.

(*a*) Gun steel. Elastic stress 24·8. Breaking stress 46·9.
(*b*) Lowmoor iron. Elastic stress 12·5. Breaking stress 22·1.

10. At atmospheric pressure ·016 cu. ft. of water, when evaporated, becomes 26·37 cu. ft. of steam. Find the "relative volume" (*i. e.*, the ratio of steam volume to water volume).

11. A two-stage air-compressor has the following dimensions:—

Area of H.P. steam cylinder, 314 sq. ins.; Area of H.P. air cylinder, 380 sq. ins.

Area of L.P. steam cylinder, 1385 sq. ins. Area of L.P. air cylinder, 1018 sq. ins.

Find the ratio of L.P. to H.P. (*a*) for steam end; (*b*) for air end.

12. The areas of the cylinders in a triple-expansion marine engine are as follows: H.P. 962 sq. ins; I.P. 2290 sq. ins.; L.P. 6082 sq. ins. Find the ratio of L.P. : I.P : H.P.

13. The quadruple expansion engines of an Atlantic liner have the

following areas: H.P. 1097; I.P. No. 1, 1905; I.P. No. 2, 4390; and L.P. 9852 sq. ins. Find the cylinder ratios (*i.e.*, L.P. : 2nd I.P. : 1st I.P. : H.P.).

14. The following figures refer to two American locomotives:—

(*a*) Firebox heating surface 231 sq. ft.; tubes heating surface 3193 sq. ft.

(*b*) Firebox heating surface 294·5 sq. ft.; tubes heating surface 3625 sq. ft.

Find for each case the ratio of total heating surface to the firebox heating surface. (*Note.*—Total heating surface covers tubes and firebox.)

15. The efficiency of any simple lifting machine is the ratio of the theoretical effort to the actual effort. Find the efficiency if the theoretical effort is ·59 lb. and the actual 1·43 lbs.

16. and **17.** The "velocity ratio" of any machine is the ratio of the movement of the effort to the movement of the load. Find this ratio in each of the following cases:—

16. Weston pulley block. Effort moves 30"; load moves 2·5".

17. Geared capstan. Effort moves 31'-5"; load moves 18".

18. The "expansion ratio" in a simple steam engine is approximately the ratio of the initial pressure to the final pressure. Find the expansion ratio in the following cases:—

(*a*) Initial pressure 72 lbs. per sq. in.; final pressure 18 lbs. per sq. in.

(*b*) Initial pressure 43 lbs. per sq. in; final pressure 17·5 lbs. per sq. in.

19. If a cast-iron propeller costs £24, then the same sized propeller would cost: (*a*) in steel, £38, (*b*) in Delta metal £115, (*c*) in gun metal £130, (*d*) in manganese bronze £135, (*e*) in aluminium bronze £145, (*f*) in phosphor bronze £170. Find the ratio of the cost of each of these materials to the cost of the cast-iron propeller.

Proportion.—When two ratios have the same value, the four quantities composing the ratios are said to be "proportionals," or "in proportion." Thus the ratio of 3 to 2 is $\frac{3}{2}$ or 1·5. Similarly the ratio of 12 to 8 is $\frac{12}{8}$ which also equals 1·5. Then evidently the ratio of 3 : 2 = the ratio of 12 : 8 and the numbers 3, 2, 12 and 8 are said to be in proportion. Thus 4 quantities are in proportion when the ratio of the 1st to the 2nd is the same as the ratio of the 3rd to the 4th. The equals sign is commonly replaced by four dots, thus (: :), and the above result is stated as 3 : 2 : :12 : 8, and is read shortly as "3 is to 2 as 12 is to 8." A more useful form is obtained by stating the ratios as vulgar fractions, thus $\frac{3}{2} = \frac{12}{8}$. Although each ratio must have the same units and measures, they need not be the same in the two ratios, since each ratio is only a number.

Proportion has its use when, knowing the ratio between two quantities, and the value of *one* of them, we wish to find the value of the other quantity; that is, knowing three of the quantities in the proportion we wish to find the fourth.

Thus, supposing the ratio of the H.P. diameter to the L.P. diameter in a compound steam engine is to be 3 : 5, and the L.P. diameter is 32″; and it is desired to find the H.P. diameter.

Then the statement H.P. : L.P. : : 3 : 5 is a proportion, meaning that if the H.P. is, say, 3″ dia. the L.P. is 5″ dia., and if the H.P. is, say, 6″ dia. the L.P. is 10″ dia., or, whatever the actual size the H.P. is $\frac{3}{5}$ of the L.P.

Then since in this case L.P. = 32″

$$\text{H.P.} = \tfrac{3}{5} \text{ of } 32 = 19{\cdot}2'' \text{ or, say, } 19\tfrac{1}{4}'' \text{ dia.}$$

Now if our answer is correct the ratio of the calculated H.P. to the L.P. should be $3 : 5 = \frac{3}{5}$ or ·6 to 1.

Ratio of $19\frac{1}{4}$ (H.P.) to 32 (L.P.) $= \dfrac{19{\cdot}2}{32} = {\cdot}6$ to 1, which proves the work.

Proportion is best treated by the simple equation (see p. 164), when any one of the quantities can be easily obtained. But from the foregoing calculation we can deduce a rule which will suit all ordinary cases. It is seen in the above that H.P. = L.P. $\times \frac{3}{5}$. Now numbering the terms in the proportion H.P. : L.P. : : 3 : 5, from left to right as No. 1, 2, 3, and 4, then No. 1 $= \dfrac{\text{No. 2} \times \text{No. 3}}{\text{No. 4}}$.

Should the given ratio be written so that the required quantity is No. 2, then the ratio should be reversed as in Ex. 77.

Example 75.—Brass is an alloy of copper and zinc; how much copper must be mixed with 80 lbs. of zinc when the ratio of copper to zinc is 7 : 3?

$$\text{Copper} : \text{Zinc} : : 7 : 3$$

$$\text{Then copper} = \frac{\text{Zinc} \times 7}{3}$$

$$= \frac{80 \times 7}{3} = \underline{186{\cdot}7 \text{ lbs.}}$$

When the given ratio is stated as "so much to 1," then the 1 will have no effect on the result, so that only one operation, instead of two as above, is required.

Example 76.—The ratio of the lengths of connecting rod to crank in a steam engine is to be 4·5 (*i. e.*, 4·5 : 1). Find the length of the connecting rod if the crank is 8″.

$$\text{Connecting rod} : \text{Crank} : : 4{\cdot}5 : 1$$

$$\text{Then connecting rod} = \frac{\text{Crank} \times 4{\cdot}5}{1}$$

$$= \frac{8 \times 4{\cdot}5}{1} = \underline{36''}$$

Example 77.—The efficiency (*i. e.*, the ratio of the output to the input) of a transmission gear is ·87. Find the input if the output is required to be 35 horse-power.

$$\text{output} : \text{input} :: \cdot 87 : 1$$

Input (which is here the 2nd quantity) is required; then the statement must be changed to—

$$\text{input} : \text{output} :: 1 : \cdot 87.$$

$$\text{Then input} = \frac{\text{output} \times 1}{\cdot 87} = \frac{35}{\cdot 87} = \underline{40 \cdot 3 \text{ h.-p.}}$$

Example 78.—In a triple expansion engine the ratios of the cylinder areas (H.P. : I.P. : L.P.) are to be as 2 : 5 : 13. The L.P. area is 2463 sq. ins. Find the areas of the other two cylinders.

Considering first the I.P., we have the proportion—

$$\text{I.P.} : \text{L.P.} :: 5 : 13$$

$$\text{Then I.P.} = \frac{\text{L.P.} \times 5}{13} = \frac{2463 \times 5}{13} = 946 \text{ sq. ins.}$$

Now take the H.P.; we have another proportion—

$$\text{H.P.} : \text{L.P.} :: 2 : 13$$

$$\therefore \text{H.P.} = \frac{\text{L.P.} \times 2}{13} = \frac{2463 \times 2}{13} = \underline{379 \text{ sq. ins.}}$$

Exercises 20. Proportion.

1. Pewter consists of tin and lead, the ratio of tin to lead by weight being 4 : 1. If a founder only has in stock 1½ cwt. (168 lbs.) of tin and desires to use the whole of it in making pewter, how much lead will be required?

2. A quantity of sulphuric acid is to be diluted so that the final solution contains water and acid in the ratio of 9 : 1. How much water must be mixed with ½ gallon of acid?

3. The ratio of the L.P. cylinder area to the H.P. cylinder area in a compound air compressor is to be 2·25 : 1. If the L.P. area is 113 sq. ins. find the area of the H.P. cylinder.

4. If gun metal weighs heavier than cast iron in the ratio of 505 to 450, find the weight of a casting in gun metal to replace one in cast iron of the same size weighing 14·5 lbs. What is the increase in weight?

5. The cylinder diameters of a triple expansion steam engine are to be in the ratio 3 : 5 : 8 (*i. e.*, H.P. : I.P. : L.P.) and the L.P. is to be 42½″ diameter. Find the diameter of the H.P. and I.P. cylinders.

6. The areas of the cylinders of a quadruple expansion marine engine are to be in the ratio 1 : 2·2 : 4·6 : 10 (*i.e.*, H.P. : 1st I.P. : 2nd I.P. : L.P.). If the area required in the L.P. is 7543 sq. ins. find the areas of the other three cylinders.

7. The ratio of the H.P. area to the L.P. area in a hydraulic intensifier

is to be 35 : 800. If the H.P. cylinder is required to be 154 sq. ins. area, find the area of the L.P. cylinder.

8. A solution for pickling castings is to be made of sulphuric acid and water in the ratio of 4·5 parts water to 1 part acid. How much water is required for 30 gallons of acid?

9. Another pickling solution is composed of hydrofluoric acid and water in the ratio of 1 : 35. What quantity of water must be mixed with 5 gallons of acid?

10. If the ratio of copper to tin in bell metal is 16 : 5, how much tin must be alloyed with 12 cwt. of copper, assuming no loss in the melting?

Exercises 20a. Miscellaneous Examples involving Decimals.

1. Find the value of the expression $\frac{14\cdot14 - 7\cdot65}{3} + \frac{10.}{2\cdot8}$

2. Find the value of $27(17\cdot4 + 2\cdot5) + 30 \times 6\cdot5 \times 11\cdot75$.

3. An Avro aeroplane has a total wing area of 330 square feet, an engine of 110 horsepower, and when fully loaded weighs 1820 pounds. Calculate (*a*) the loading per square foot of wing area, (*b*) the loading per H.P.

4. A variable electrical condenser has 22 fixed plates, each 0·032″ thick, spaced uniformly 0·1″ apart. The two supporting end plates are 0·104″ thick and are each $\frac{3}{8}$″ beyond the outside fixed plates. Calculate the overall height of the condenser.

5. The loss of liquid by evaporation in a liquid air container is required to be not greater than 10% of the full charge in a period of 24 hours. An actual container on test lost 1¼ lb. in 6½ hours. If the full charge is 50 lb. of liquid, calculate the percentage loss in this case.

6. The wing span of a twin-engined aeroplane is 67 ft. 2 ins., and the chord of the wings is 10 ft. 6 ins. Calculate the "aspect ratio," *i.e.*, the ratio of span to chord.

7. A batch of compressed gas cylinders, all of the same size, was weighed after manufacture, the weights being as follows: 10 lb. 6 oz., 10 lb. 2 oz., 10 lb. 7 oz., 10 lb. 8 oz., 10 lb. 4 oz., 10 lb. 7 oz., 10 lb. 5 oz., 10 lb. 7 oz. Calculate (*a*) the average weight of a cylinder, (*b*) the variation in weight (*i.e.*, the difference between the heaviest and lightest cylinders) as a percentage of the average weight.

8. Calculate the total weight of the instrument equipment of an aeroplane, which requires the following: 2 Air Speed Indicators; 1 Pressure Head; 56 feet of Air Speed Tubing; 2 Altimeters; 2 Cross Levels; 1 Compass Type A; 1 Compass Type B; 1 Revolution Indicator; 1 Flexible Drive for latter, 10 feet long; 1 Radiator Thermometer; 2 Pressure Gauges; 2 Watches. The weights of the various components are as follows:—

Air Speed Indicator	1·45 lb.	Compass Type A	4·5 lb.
Pressure Head	·25 lb.	Compass Type B	6·7 lb.
Air Speed Tubing	·035 lb. per foot.	Revolution Indicator	2·7 lb.
		Flexible Drive	·26 lb. per foot.
Altimeter	·94 lb.		
Cross Level	·5 lb.	Radiator Thermometer	1·25 lb.
Watch	·375 lb.	Pressure Gauge	·625 lb.

CHAPTER III

SYMBOLS AND THEIR USES

Symbols and Formulæ.—To avoid repetition in connection with the weights and measures in every day use, it is customary to use some letter or sign instead of the full name of a unit. Thus "pounds sterling" is replaced by the sign £, "dollars" by $, "pounds" (in weight) by lbs., etc., while every one understands that £5 means 5 pounds sterling, or that 15 lbs. means 15 pounds in weight. This system is extended to other uses, and signs are employed to denote particular measured quantities. The branch of calculation dealing with these signs, or **symbols,** as they are called, is usually known as Algebra.

The engineer frequently has to indicate that certain quantities or measurements must be added or multiplied, or generally operated upon in various definite ways; and often this has to be done without giving any actual figures. If words be employed, then cumbersome sentences are obtained, and calculation would be very slow and laborious. But if some sign, say a letter, be used to stand for each quantity dealt with, then any relationship, however complicated, may be easily expressed. For example, taking the statement—

$$\text{Voltage} = \text{Current} \times \text{Total resistance}$$

Instead of writing this out every time it is required, a shorter statement may be obtained by using letters to stand for the three quantities involved. Let us represent "Voltage" by the letter V, "Current" by C, and "Total resistance" by R, using the initial letters in each case, a method usually adopted where possible. We may then write—

$$V = C \times R$$

Knowing what V, C and R indicate, then this statement conveys as much information as the former written sentence. The new statement is known as a **Formula** (plural: Formulæ) and the various letters in it are called **Symbols.** If the value of current and resistance be given for some particular case, we can calculate

the corresponding value of V by "substituting" the given values for C and R, and working out the expression so obtained.

Thus, if C = 4·5 amperes, and R = 6·63 ohms
then $V = C \times R$
$= 4{\cdot}5 \times 6{\cdot}63 = 29{\cdot}8$ volts

Letters Used.—Many kinds of signs may be used, but the simplest are the letters of the alphabet, both small and capital. The latter form should be printed and not written, as they are then clearer, and the formulae are more easily read. Letters of the Greek alphabet, α (alpha), β (beta), π (pi), etc., are also used. Further symbols may be obtained by attaching a "suffix" to a letter. Thus a_1 (read as "a one") and T_s may be used to denote different quantities from that represented by a and T respectively; but suffixes are best avoided, as being small they are liable to be forgotten, when confusion would result. Of course, **the same symbol should never be used to denote two different things in the same example.**

It is unfortunate that there is no universal system of symbols for use in engineering work. Thus one man will use f for stress, another q, and so on, while in the same book t will be adopted to denote temperature, as well as thickness. This is confusing, as the same formula has to be considered in two or three different forms. Usually each person keeps some particular letter for each quantity in all his *own* calculations, and it is necessary to say in each example what the various letters stand for. If this precaution is not adopted considerable confusion may arise, wrong values being substituted and the like. It is noteworthy that an attempt has been made to standardise the symbols for use in Reinforced Concrete calculations, and in Electrical work; and in most engineering calculations there are certain letters employed by every one for certain quantities, *e. g.*, π (pi) for a constant used in circle calculations, "g" for the "acceleration due to gravity," etc.

Signs of × and ÷.—When dealing with letters, the multiplication sign × is almost invariably omitted, the letters being written close to one another, so that C × R would appear as CR. This is perfectly understood for letters, but it must be remembered that the same scheme cannot be employed for numbers. Thus 46 cannot mean 4 × 6. The original intention was that a dot (.) should be placed between the symbols multiplied; thus C × R would appear as C.R, but the dot being very small, it is not a difficult matter to forget to write it, especially as its omission does not alter the value of the expression. The dot is, however, sometimes used with

numbers; thus 4.6 means 4×6, but its use in such a manner is not advisable.

The algebraic method just mentioned may be adopted whenever a letter and a figure are to be multiplied together.

Thus $3a$ (read as "three a") means $3 \times a$, *i. e.*, 3 times a,

$= a + a + a$

or $3 + 3 + 3 + 3 + \ldots\ldots$ until there are "a" threes.

Of course in this last case we cannot say how many threes there are unless we know the value of a.

Similarly $wl = w \times l$

i. e., $l + l + l + l$ . . . w times

or $w + w + w + w$ . . . l ,,

A common example of this nature occurs in connection with a beam supporting a "uniformly distributed load," *i. e.*, a load spread evenly over some distance. At *a*, Fig. 15, the load is spread evenly

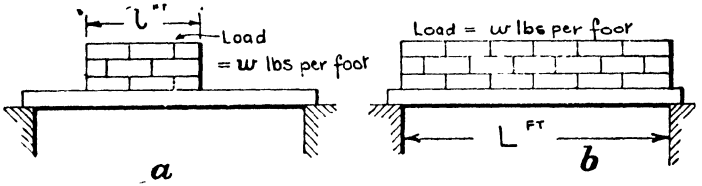


Fig. 15.—Uniform loading of Beams.

over a portion of the beam. If the intensity of the loading, *i. e.*, the load upon 1 ft. length, be w lbs per ft., then—

on 1 ft. the load will be w lbs.
,, 2 ,, ,, ,, $2w$,,
,, 3 ,, ,, ,, $3w$,,
,, 4 ,, ,, ,, $4w$,,
,, l ft. the total load will be lw lbs. or wl lbs.

Then if w were $1\frac{1}{2}$ tons per ft., and l were 18 ft., the total load would be $1\frac{1}{2} \times 18 = 27$ tons.

At *b*, Fig. 15, the total load $= w$L, where L = span. This statement is true for any kind of beam, with any span and any intensity of loading.

The system may be extended to three or more quantities. Thus $2\pi rn$ means $2 \times \pi \times r \times n$.

The symbols need not be written in any particular order, since the order in which a multiplication is carried out does not affect the result. Thus $2\pi rn$ is the same thing as $\pi n2r$ or $r2n\pi$. It is

usual, however, to retain a particular order in any formula, as it is then more easily remembered. It is also customary to write the figures before the symbols. In expressions of this type the number in front of the symbols is called a **coefficient,** and is, of course, a constant quantity. When an expression contains no coefficient, then a "1" must be understood and *not* nought. Thus—

ab means $1ab$
wly ,, $1wly$ and so on.

The division sign $\div$ is seldom used, the fractional method of writing being preferred. This gives a more compact form of statement than the sign $\div$, as can be seen from a comparison of the following expressions:—

$\frac{\cdot 66 \times 4 \times 36}{100 \times 11\cdot 3}$ which equals $\cdot 66 \times 4 \times 36 \div (100 \times 11\cdot 3)$

and $\frac{T\pi dN}{33000}$ which equals $T\pi dN \div 33000$.

Similarly $\frac{WL}{a\Delta^*}$ denotes that the result of $W \times L$ is to be divided by the result of $a \times \Delta$.

Signs of + and − : Brackets.—These signs are used in algebra in exactly the same manner as in arithmetic. Thus $s + L$ means the addition of the quantities represented by s and L. Similarly $W - w + b$ means that w is to be subtracted from W and then b added to the result. In any "expression" (*i. e.*, mathematical statement) the signs of + and − separate the various quantities into groups of symbols called "terms." Thus in the expression $H + \frac{P}{G} + \frac{v^2}{2g}$, the terms are H, $\frac{P}{G}$ and $\frac{v^2}{2g}$.

The remarks made in Chap. I on brackets (p. 29) and on the order in which multiplication and addition operations are to be performed (p. 30) must be observed when dealing with symbols. When brackets appear thus $(d - 1)(r + 2)$, then the *results* of the brackets are to be multiplied together.

Simple Evaluation.—The evaluation of formulæ by substitution, *i. e.*, finding the value of the expression when certain values are given to the symbols, is an important piece of work, and some simple examples will be given here. In all cases the reader is advised, when working through the exercises, to copy closely the method of stating the work shown in these examples.

* Δ, Greek letter "delta."

The first step should merely be the *substitution* of the given values in place of their symbols.

Example 79.—If E is the voltage supplied to an electric circuit, and C is the current in amperes, then the power supplied is $\frac{EC}{1000}$ kilowatts. Find the power supplied when E = 110 volts and C = 25 amperes.

$$\frac{EC}{1000} = \frac{110 \times 25}{1000} \text{ with values substituted}$$

$$= \underline{2{\cdot}75 \text{ kilowatts}}$$

$$\frac{\overset{2}{\cancel{110}} \times \overset{5}{\cancel{25}}}{\underset{4}{\cancel{1000}}} = 11$$

Example 80.—The "Modulus of Elasticity" in tons per sq. in. of a bar of metal is given by the formula $\frac{WL}{a\Delta}$, where W = load on bar in tons : L = length of bar in inches : a = area of cross section in sq. ins. : and Δ = extension in inches. Find the Modulus when W = 5 tons, L = 8″, a = ·44 sq. in. and Δ = ·0067 in.

$$\frac{WL}{a\Delta} = \frac{5 \times 8}{{\cdot}44 \times {\cdot}0067}$$

$$= \frac{40}{{\cdot}00295}$$

$$= \underline{13600 \text{ tons per sq. in.}}$$

Approximation.

$$\frac{\overset{2}{\cancel{5}} \times \overset{1}{\cancel{8}}}{\underset{1}{\cancel{4}} \times 7} \qquad \frac{1111}{\quad\quad}$$

$$= \frac{10}{7} = 1{\cdot}4$$

about 14000.

Example 81.—The voltage required to maintain an electric arc is $a + bL + \frac{d + eL}{C}$ where a, b, d and e are constants found by experiment : L = length of arc in millimetres : and C = current in amperes. If a = 39, b = 2·1, d = 11·7 and e = 10·5, find the volts necessary for an arc 5 millimetres long with a current of 10 amperes.

$$\text{Voltage required} = a + bL + \frac{d + eL}{C}$$

$$= 39 + 2{\cdot}1 \times 5 + \frac{11{\cdot}7 + 10{\cdot}5 \times 5}{10}$$

$$= 39 + 10{\cdot}5 + \frac{11{\cdot}7 + 52{\cdot}5}{10}$$

$$= 49{\cdot}5 + 6{\cdot}42$$

$$= 55{\cdot}92, \text{ say } \underline{55{\cdot}9 \text{ volts.}}$$

Note.—The × operations must be done before the + operations. Also the division line is a bracket; hence the addition above it must be carried out before dividing by 10.

Example 82.—The horse-power (h.-p.) per cylinder of a petrol engine can be calculated from the formula : h.-p. $= \cdot 45\ (d + s)(d - 1 \cdot 18)$ where d = dia. of cylinder in inches, and s = stroke. Find the h.-p. for an engine of 3·23″ dia. and 5″ stroke.

$$\begin{aligned} \text{h.-p.} = \cdot 45\ (d + s)(d - 1 \cdot 18) &= \cdot 45\ (3 \cdot 23 + 5)(3 \cdot 23 - 1 \cdot 18) \\ &= \cdot 45 \times 8 \cdot 23 \times 2 \cdot 05 \\ &= \underline{7 \cdot 6 \text{ h.-p.}} \end{aligned}$$

Exercises 21. On Simple Evaluation: × and ÷.

1. If C is the current in amperes in an electric circuit, and R the resistance in ohms, then the voltage necessary = CR. Find the voltage if C = 35 amps. and R = 2·48 ohms.

2. The weight (grammes) of metal deposited in electro-plating = eCt where C = current in amperes, t = time in seconds that current flows, and e is a number depending on the metal deposited. Find the weight of copper deposited when C = 22·5 amps., t = 1800, and e = ·000328 for copper.

3. The formula Dwr refers to the " power " of a lathe. Find the " power " in each of the following cases:—

(*a*) Small lathe : D = $8\frac{1}{2}$, w = $1\frac{3}{4}$ and r = 7;

(*b*) Large lathe : D = 38, w = 3 and r = 14.

4. The speed of a belt in feet per minute is given by the formula πDN, where π = 3·14, D = dia. of pulley in ft., and N = revs. per min. of the pulley. Find the speed when D = 3·5 ft. and N = 140 revs.

5. The formula $Eaat$ refers to the force exerted by a metal bar when heated. Find the force exerted (lbs.) by a wrought-iron bar if E = 25,000,000, a = 3·1, α = ·0000066 and t = 300.

6. Find the value of the expression $2\pi nLI$ (a formula used in connection with alternating electric currents) when π = 3·14, n = 87, L = ·02, and I = 65.

7. If a load of W lb. stretches a bar a distance of Δ inches, then the work done in inch-pounds is equal to $\frac{F\Delta}{2}$. Find the work done if F = 24000 and Δ = ·0075″.

8. The area of a certain geometrical figure is $\frac{2}{3}lh$ sq. ins. when l = length in inches and h = height in inches. Find the area when l = 9 and h = 4·8.

9. Find the value of the formula Iw_1w_2, which relates to a gyroscope, when I = 1·5, w_1 = ·0785, and w_2 = 210.

10. If W tons is the total distributed load on a girder and l is the length of the girder in feet, then the " Bending Moment " in tons feet = $\frac{Wl}{8}$. Find the bending moment for a girder where W = 75 tons and l = 22·5 ft.

11. The number of poles required in an alternator is $\frac{2f}{n}$ where f = frequency of current and n = revs. per sec. Find the number when f = 60 and n = 2·5.

12. The formula $\frac{A_1 h}{A}$ occurs in connection with the strength of beams. Find its value when A = 3·6, A_1 = 1·67, h = 5·5.

13. The resistance of any electrical conductor is $\frac{sl}{a}$ ohms, where l = length, a = its area and s a figure depending on the material. Find the resistance where l = 36000, a = ·00181, and s = ·00000066.

14. The expression $\frac{pr}{f}$ gives the minimum thickness of a hydraulic pipe, where p = pressure in lbs. per sq. in., r = radius in ins., and f = safe stress in material (lbs. per sq. in.). Find thickness t when p = 750, r = 2·5″, f = 2500.

15. Find the value of the expression $\frac{gH}{Vv}$, which relates to a centrifugal pump, when g = 32, H = 12·5, V = 27·5, and v = 23.

16. The formula $K = \frac{kA}{4\pi t}$ refers to electrical condensers. Find K when k = 2·2, A = 9·75, π = 3·14 and t = ·15.

17. When measuring resistances by a Wheatstone Bridge we have the formula $x = \frac{RR_1}{R_2}$. Find the value of x when R = 287, R_1 = 1000, and R_2 = 10.

18. The H.P. of a steam engine is given by the formula $\frac{pLaN}{33000}$ when p = mean effective pressure, a = cylinder area, L = length of stroke and N = number of working strokes per min. Find the H.P. when p = 22·5, L = 1·75, a = 113 and N = 220.

19. When measuring the B.H.P. of an engine we have the formula B.H.P. = $\frac{2\pi RNW}{33000}$ where π = 3·14, R = radius of brake arm (ft.), N = revs. per min., and W = load supported (lbs). Find the B.H.P. for a petrol engine when R = 2·5, N = 1250, and W = 5·3.

20. The B.H.P. of a three-phase motor is given by the formula $\frac{1{\cdot}732p\eta EC}{746}$, where p = power factor, η = efficiency, E = voltage, C = current. Find the B.H.P. of a motor when p = ·8, η = ·93, E = 107, C = 35.

Exercises 22. On Simple Evaluation. × and ÷ with + and −.

1. If the thickness of plate in which a hole is to be punched is t in. and d is the diameter of the punch, then the diameter of the hole required in the bolster is $d + \frac{t}{6}$. Find what size of hole is required if a $\frac{7}{8}$″ dia. hole is to be punched in a $\frac{3}{4}$″ plate. (*N.B.*—Work in decimals.)

2. The diameter of tapping hole for a Whitworth bolt is $d - 1{\cdot}28p$, where d = bolt dia., and p = pitch of screw. Find this value when d = ·75″ and p = ·1″.

3. The total heat of steam (H) at t° F. is very nearly $1082 + {\cdot}3t$. Find H when t = 345°.

4. The latent heat of steam at t° F. is very nearly $1114 - {\cdot}7t$. Find the latent heat when t = 330.

5. The total heat in 1 lb. of wet steam is $S + xL$, where S = sensible heat, x = dryness, and L = latent heat. Find the total heat when S = 310, x = ·93 and L = 896.

6. The maximum speed allowable (ft. per min.) when cutting cast iron with high-speed tools is given approximately by the following formulæ: Soft cast iron $115-1300a$; hard cast iron $40-400a$, where a = area of cut. Calculate the speed in the following cases: (*a*) cutting soft cast iron, area of cut = ·0124 sq. in.; (*b*) hard cast iron, area of cut = ·046 sq. in.

7. The circumference of an ellipse may be obtained approximately from the formula $1{\cdot}82D + 1{\cdot}315d$, where D = long axis, and d = short axis. Find circumference when D = 2·5 and d = 1·5.

8. The dimensions of American countersunk-head screws are calculated from the following formulæ (see Fig. 16): $D = 2d - {\cdot}008$; $H = \dfrac{d - {\cdot}008}{1{\cdot}74}$; $w = {\cdot}173d + {\cdot}015$; $h = \dfrac{H}{3}$ where d = diameter of screw. Calculate the values of D, H, w and h for a screw ¾" dia.

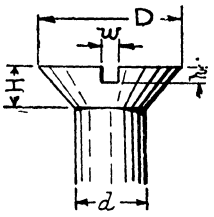


Fig. 16.

9. The piston speed of a petrol motor is given as $600(r + 1)$ ft. per min., where r = ratio of stroke to diameter. Find the speed if r = 1·25.

10. If E is the E.M.F. of a battery, r its own resistance, and R the resistance through which it is supplying current, then the current C is given by the formula $C = \dfrac{E}{R + r}$. Find C when E = 2·5, R = 5·7, and r = ·02.

11. When measuring the flow of water in a stream by a notch the following formula is met with: $C = {\cdot}57 + \dfrac{B}{10W}$. Find the value of C if B = 3 and W = 15.

12. The "velocity ratio" in a Weston pulley block is $\dfrac{2R}{R - r}$ where R is the large radius, and r the small radius, of the compound pulley. Find the velocity ratio if R = 3·5 and r = 3·25.

13. If C is the temperature on the Centigrade scale, the corresponding Fahrenheit temperature is $\frac{9}{5}$ C + 32. Find this value if C = 85.

14. The current delivered by a number of cells connected in series is $\dfrac{nE}{R + nr}$ amperes, where n = number of cells, E = E.M.F. of 1 cell, r = internal resistance of 1 cell, and R = outside resistance. Find the current delivered by 57 cells if E = 2·7, R = 2·3 and r = ·03.

15. The coefficient of friction in a pipe carrying water is $·0075\left(1 + \dfrac{1}{d}\right)$ where d is diameter in inches. Find this value when d = 9".

16. The expression $h\left(\dfrac{d - d_1}{d}\right)$ gives the cross-section of a chimney under certain conditions. Find the value of the expression when h = 100, d = ·0761 and d_1 = ·0414.

17. A formula for the allowable working stress under live loads is $\dfrac{f}{1 + \dfrac{w}{W}}$. Calculate the value of the expression when f = 6, W = 7, and w = 2.

18. The expression $\frac{h}{3} \cdot \frac{a + 2b}{a + b}$ occurs in certain investigations on the strength of materials.* Find its value when $h = 15$, $a = 2{\cdot}5$, and $b = 7$.

19. The weight of a girder before designing may be obtained from the expression $\frac{wlr}{cf - lr}$ tons. For a particular plate-web girder $w = 50$, $r = 12$, $c = 1300$, $l = 45$, $f = 6{\cdot}5$. Find its weight in tons.

20. If a wire rope contains n wires, each d ins. dia., then the diameter of the rope is approximately $\frac{nd}{13} + 7d$. Calculate this diameter if $n = 72$ and $d = {\cdot}064$.

21. The expression $(p + {\cdot}38)(v - {\cdot}41)$ relates to the expansion of steam. Find its value when $p = 165$ and $v = 2{\cdot}7$.

22. The rating formula used by the Society of Motor Manufacturers for a petrol engine is $\cdot 197d(d - 1)(r + 2)$ H.P. per cylinder, where d = cylinder dia. in ins., and r = ratio of stroke to diameter. Calculate this H.P. when $d = 3{\cdot}23$ and $r = 1{\cdot}55$.

23. The expression $\frac{W[F - (s + c)]}{Lhb}$ is used in connection with a drop test for materials. Find its value when $W = 103$, $F = 5$, $s = 2{\cdot}8$, $c = {\cdot}57$, $L = 36$, $h = 2$ and $b = 1$.

24. The expression $f\ (1 - ec)$ tons is a formula relating to long columns. Find its value when $f = 6$, $e = {\cdot}0053$ and $c = 95$.

25. Cochran's evaporation formula is $E = 26(G - 2)\left(5 + \frac{\frac{H}{G} - 13}{7{\cdot}5}\right)$ where E = lbs. of water evaporated per hour, G = grate area in sq. ft., and H = heating surface in sq. ft. Find E for a boiler where $G = 14$ and $H = 250$.

Powers and Indices.—When we wish to indicate that a quantity is to be multiplied by itself, as for example in certain calculations with circles where the diameter has to be multiplied by the diameter, we could write "$d\ d$" if d stood for the diameter, meaning of course $d \times d$. But in such cases, when the quantities multiplied together (*i. e.*, the factors) are *the same*, a shorter method of statement is used. The symbol for the quantity is only written down *once*, and a small figure is placed at its right-hand top corner to indicate the number of symbols which are to be multiplied together. Thus the foregoing statement "$d\ d$" would be written as d^2, to indicate that two d's are multiplied together. This small figure is called an **index,** or "exponent," while the quantity represented by a symbol together with an index, is said to be a **"power"** of the symbol; thus d^2 is a power of d. The index $(^2)$ is used when *two* of the same quantities are to be multiplied together, 3 when *three* of the same quantities are to be multiplied together, and so on.

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* See p. 87 for use of dot as a multiplication sign.

Thus we have—

$\mathbf{d^2} = \mathbf{d} \times \mathbf{d}$, *i. e.*, two *d*'s multiplied together,
(Read as "*d* squared" or "*d* to the second power.")

If $d = 3$, then $d^2 = 3 \times 3 = 9$.
Similarly $x^2 = x \times x$, and so on.

$\mathbf{d^3} = \mathbf{d} \times \mathbf{d} \times \mathbf{d}$, *i. e.*, three *d*'s multiplied together.
(read as "*d* cubed" or "*d* to the third power.")

If $d = 3$, then $d^3 = 3 \times 3 \times 3 = 27$.

$\mathbf{R^4} = \mathbf{R} \times \mathbf{R} \times \mathbf{R} \times \mathbf{R}$ (read as "R to the fourth," meaning "R raised to the fourth power").

The index must not be confounded with a suffix, mentioned on p. 87. The index is placed at the *top*, but the suffix at the *bottom* of a symbol; also the actual figure of an index has a definite *mathematical* meaning, while that of a suffix has none.

It must be clearly understood that an index is *not* the same as a coefficient. Thus x^3 *is not the same as* $3x$.

For $x^3 = x \times x \times x$, *i. e.*, three *x*'s multiplied together, but $3x = x + x + x$, *i. e.*, three *x*'s added together.

Let $x = 4$,

then $\mathbf{x^3} = 4^3 = 4 \times 4 \times 4 = \mathbf{64}$
but $\mathbf{3x} = 3 \times 4 = \mathbf{12}$

showing a considerable difference.

Similarly d^5 is not $5d$.

For let $d = 3$, then $d^5 = 3^5 = 3 \times 3 \times 3 \times 3 \times 3$ (*i. e.*, the product of five 3's)

$= 27 \times 9 = 243$
whereas $5d = 5 \times 3 = 15$.

The indices met with in practical problems are neither numerous nor large. The index 2, representing the "square," is the most common. It occurs in all calculations dealing with areas of circles, and in many others besides. It is useful to remember the "squares" of the simpler numbers as given in the table on p. 101.

The indices 3 and 4 are less common, but still find a prominent place in connection with the strength of shafts and beams, among other things.

The 5th power is found in hydraulic work. Indices beyond 5 are not common, and are usually only introduced for convenience in calculation. Mathematically an index may be of any form and

size whatever, but only the whole number form will be considered in this book.

Certain peculiarities met with are worth noting. Thus 1 raised to any power whatever = 1.

For example $1^2 = 1 \times 1 = 1$

$1^5 = 1 \times 1 \times 1 \times 1 \times 1 = 1$, etc.

Similarly 0 raised to any power = 0.

Also a number raised to the first power is not altered, since the first power of any quantity is just the single quantity. Thus $x^1 = x$ and $3^1 = 3$. For this reason the index 1 is not written.

If 10 be raised to any power such as we have been dealing with, the result is always a number composed of a 1 and some noughts, *the number of noughts being the same as the index.*

Thus	$10^1 =$	$=$ 10, *i. e.*, 1 nought	
	$10^2 = 10 \times 10$	$=$ 100 ,, 2 noughts	
	$10^3 = 10 \times 10 \times 10$	$=$ 1000 ,, 3 ,,	
	$10^4 = 10 \times 10 \times 10 \times 10$	$=$ 10000 ,, 4 ,,	

Where large numbers such as 10,000,000 are required in formulae, it is customary to replace them by their corresponding power of 10, to simplify the appearance of the number. Thus the formula for the Electromotive Force (E.M.F.) of a dynamo is stated as $\frac{PFNZ,}{10^8}$ volts, meaning $\frac{PFNZ}{100,000,000}$ volts. All that is necessary is to count the number of 0's after the 1, which gives the index to be written above the 10.

Expressions of the forms $3r^2$, a^2b, etc., are of frequent occurrence, and it must be noted that **the index applies only to that symbol against which it is placed, unless brackets be used.**

Thus $3r^2$ (read as " 3 r squared ") means 3 times the square of $r = 3 \times r \times r$.

It does *not* equal $3r \times 3r$, which would be the square of $3r$, and would have to be written as $(3r)^2$ and read as " $3r$ in brackets, squared," or " $3r$, all squared." The bracket indicates that *everything inside* it must be considered as one quantity. Similarly a^2b (read as " a squared b ") means the " square of a " multiplied by b, and therefore $= a \times a \times b$.

It does not mean $ab \times ab$, which would be the " square of ab," *i. e.*, $(ab)^2$.

Similarly $5r^3 = 5 \times r \times r \times r$;

But $(5r)^3 = 5r \times 5r \times 5r$

The difference in meaning may be further shown if we use some figures. Thus, let $r = 2$.

Then $5r^3 = 5 \times 2^3$
$= 5 \times 8 = 40.$

But $(5r)^3 = (5 \times 2)^3$
$= 10^3 = 1000.$

Taking the statement $(5r)^3 = 5r \times 5r \times 5r$, this, of course, $= 5 \times r \times 5 \times r \times 5 \times r$. Since the multiplication may be carried out in *any* order, we may write $5 \times 5 \times 5 \times r \times r \times r$,

i. e., $5^3 \times r^3 = 125r^3$.

Hence to simplify an expression such as $(5r)^3$ **raise each quantity inside the bracket to the power outside. Then evaluate the powers of the numbers.**

$$\therefore (5r)^3 = 5^3r^3 = 125r^3.$$

When all the quantities inside the brackets are symbols, then only the first sentence of the above rule can be obeyed. Thus $(ab)^2 = a^2b^2$ and nothing further can be done. The expression a^2b^2 is no simpler than the one $(ab)^2$ when it stands alone; but when it forms part of a larger expression then the form a^2b^2 is to be preferred, as other simplifications may then be carried out.

Collecting, for comparison, some of the foregoing statements showing the factors in full—

$a^2b = a \times a \times b$
$ab^2 = a \times b \times b$
$(ab)^2 = ab \times ab = a \times a \times b \times b = a^2b^2.$

Then if $a = 3$ and $b = 4$

$a^2b = 3^2 \times 4 = 9 \times 4 = 36$
$ab^2 = 3 \times 4^2 = 3 \times 16 = 48$
$(ab)^2 = a^2b^2 = 3^2 \times 4^2 = 9 \times 16 = 144.$

Example 83.—Find the value of the following expressions: (*a*) r^4, (*b*) $4r$, (*c*) $4a^2$, (*d*) $(4a)^2$, (*e*) $2{\cdot}5p^2$, (*f*) $(2{\cdot}5p)^2$, when the symbols have the following values: $a = 3$, $r = 4$, $p = 2{\cdot}5$.

(*a*) $r^4 = 4^4 = 4 \times 4 \times 4 \times 4 = 256.$
(*b*) $4r = 4 \times 4 = 16.$
(*c*) $4a^2 = 4 \times 3^2 = 4 \times 9 = 36.$
(*d*) $(4a)^2 = (4 \times 3)^2 = 12^2 = 144.$
or $= 4^2a^2 = 16a^2 = 16 \times 9 = 144.$
(*e*) $2{\cdot}5p^2 = 2{\cdot}5 \times 2{\cdot}5^2 = 2{\cdot}5 \times 6{\cdot}25 = 15{\cdot}625.$
(*f*) $(2{\cdot}5p)^2 = (2{\cdot}5 \times 2{\cdot}5)^2 = 6{\cdot}25^2 = 39{\cdot}0625.$

Example 84.—Find the value of the expression $\frac{bd^3}{12}$, a formula relating to the strength of beams, when $b = 3$, $d = 5$.

$$\frac{bd^3}{12} = \frac{3 \times 5^3}{12} = \frac{3 \times 125}{12} = \frac{375}{4} = 31{\cdot}25.$$

Example 85.—The formula $\frac{\pi^2 E}{5C^2}$ gives the safe stress in tons per sq. in. in a column, under certain conditions. Find the value of the expression when $\pi = 3{\cdot}14$, $E = 13400$ and $C = 95$.

$$\frac{\pi^2 E}{5C^2} = \frac{3{\cdot}14^2 \times 13400}{5 \times 95^2}$$

$$= \frac{9{\cdot}86 \times 13400}{5 \times 9025} = \frac{132100}{45100}$$

$$= 2{\cdot}93 \text{ tons per sq. in.}$$

Approximation.

$$\frac{3 \times 3 \times 1}{5 \times 1 \times 1} = \frac{9}{5} = 2 \text{ approx.}$$

Top line = 132124
say 132100
Bottom line = 45125
say 45100.

Example 86.—The expression $H + \frac{P}{G} + \frac{v^2}{2g}$ is very important in hydraulics. Find its value when $H = 15$, $P = 2200$, $G = 62{\cdot}5$, $v = 4$, and $g = 32$.

$$H + \frac{P}{G} + \frac{v^2}{2g} = 15 + \frac{2200}{62{\cdot}5} + \frac{4^2}{2 \times 32}$$

$$= 15 + 35{\cdot}2 + \frac{16}{64}$$

$$= 15 + 35{\cdot}2 + {\cdot}25 = 50{\cdot}45.$$

Example 87.—The strength of copper alloys at high temperatures is given by Unwin's formula : $f = a - b(t - 60)^2$. Find the strength (tons per sq. in.) of rolled brass, when $t = 500$, $a = 24{\cdot}1$, and $b = {\cdot}000028$

$$f = a - b(t - 60)^2 = 24{\cdot}1 - {\cdot}000028(500 - 60)^2$$

$$= 24{\cdot}1 - {\cdot}000028 \times 440^2$$

$$= 24{\cdot}1 - {\cdot}000028 \times 193600$$

$$= 24{\cdot}1 - 5{\cdot}42$$

$$= 18{\cdot}68, \text{ say, } 18{\cdot}7 \text{ tons per sq. in.}$$

Exercises 23. On Evaluation with Powers.

Write out the factors of the following:—

1. 4^3, 10^2, 3^4, $\cdot 2^3$, $\cdot 5^2$, 10^5. **2.** r^2, D^4, a^3, h^5.

Find the values of the following expressions when $a = 3$, $b = 2$, $R = 4$, and $p = 1{\cdot}5$.

3. (i) a^2; (ii) $2a$. **4.** (i) R^3; (ii) $3R$. **5.** (i) $2p$; (ii) p^2.

6. (i) a^4; (ii) $4a$. **7.** (i) $5b$; (ii) b^5.

Write out the factors of the following expressions:—

8. (i) c^2r; (ii) cr^2; (iii) c^2r^2; (iv) $(cr)^2$; (v) What do you notice about (iii) and (iv)?

9. (i) $2M^2$; (ii) $3M^2$; (iii) $(2M)^2$; (iv) $(3M)^2$.

10. (i) a^3r; (ii) ar^3; (iii) a^2r^3; (iv) a^3r^2.

11. (i) $3n^2p$; (ii) $3np^2$; (iii) $3n^2p^2$.

12. (i) $3(np)^2$; (ii) $(3n)^2p$; (iii) $(3np)^2$.

Find the values of the following expressions when T = 3, $e = 2$, $x = 3$, V = 10, C = ·5, $w = 12$, $l = ·1$.

13. (i) T^2C; (ii) TC^2; (iii) T^2C^2; (iv) $(TC)^2$.

14. (i) $4C^2$; (ii) $2C^2$; (iii) $(4C)^2$; (iv) $(2C)^2$.

15. (i) e^3x; (ii) ex^3; (iii) e^2x^3; (iv) e^3x^2.

16. (i) $3w^2l$; (ii) $3wl^2$; (iii) $3w^2l^2$; (iv) $3(wl)^2$.

17. (i) $(2V)^3$ C; (ii) $(2VC)^3$; (iii) $3V^5l$; (iv) $3l^5V$.

18. The area of a circle is given by the formula ·785 D^2 if D is its diameter. Find the area (sq. ins.) if D = 2·5″.

19. If r is the wave resistance of a ship's model when towed in an experimental tank and k is the number of times the ship is bigger than its model, then the wave resistance of the ship itself is k^3r. Find this value if $k = 40$ and $r = 1·76$ lbs.

20. Find the value of the expression A_2h^2, which relates to the strength of a beam, when $A_2 = 2·22$ and $h = 5·7$.

21. Find the value of the expression $\frac{1}{2} I \omega^2$, which refers to flywheels, when $I = 47·2$ and $\omega = 35·7$.

22. The safe load in tons on a white manilla rope is $\frac{C^2}{30}$ where C = girth in inches. Find the safe load for a rope of $1\frac{3}{4}$″ girth.

23. The H.P. of a petrol motor (1 cylinder) may be roughly estimated from the formula $\frac{d^2s}{12}$, where d = diameter in ins., and s = stroke in ins. What is the H.P. when $d = 3·3$ and $s = 3·74$?

24. Find the value of the expression $\frac{bh^3}{3}$, which refers to the strength of beams, when $b = 7·5$ and $h = 1·1$.

25. The "kinetic energy" (*i. e.*, energy of motion) of a body weighing w lbs. and moving at v ft. per sec. is $\frac{wv^2}{2g}$ where $g = 32$. Modern rifle ammunition carries a bullet weighing ·0249 lb., which leaves the muzzle with a velocity of 2440 ft. per sec. Find the kinetic energy of the bullet (ft. lbs.).

26. The H.P. of a low-pressure fan for ventilation is given by the expression ·0000115 $\frac{Q^3}{D^4}$ where Q = cu. ft. of air per sec., and D = dia. of fan in ft. Find the H.P. required for a fan 4·5 ft. dia., delivering 250 cu. ft. per sec.

27. Henderson's formula for the rating of a petrol engine is $·2d^2(r + 1)$ where d = dia. in ins., and r = stroke ratio. Find the rating when $d = 4·5$, and $r = 1·15$.

28. The copper loss in a transformer is $C_1^2 R_1 + C_2^2 R_2$ where

C_1 and R_1 are current and resistance of primary winding, and C_2 and R_2 current and resistance of the secondary. Find the loss when $C_1 = 20$, $C_2 = 1$, $R_1 = \cdot 08$, and $R_2 = 1 \cdot 8$.

29. White and Poppe's formula for the weight of a piston and connecting rod in a petrol engine is $\cdot 037\ (d + 1 \cdot 9)^3$ lbs. Calculate the weight for a case where $d = 4 \cdot 02''$.

30. Burls' formula for a purpose similar to that in Ex. 29 is $\cdot 08d^3(1 + \cdot 15r) + 1 \cdot 5$ lbs. Calculate the weight for a car engine when $d = 3 \cdot 15''$ and $r = 1 \cdot 5$.

31. The greatest stress in a thick cylinder under internal pressure is $p \cdot \frac{R^2 + r^2}{R^2 - r^2}$. A cylinder has $R = 9$ and $r = 6$ while $p = 750$. Calculate the greatest stress.

32. When measuring the power in an alternating current circuit by the 3-voltmeter method we have

$$\text{Power} = \frac{C}{2V_1}\left(V^2 - V_1^2 - V_2^2\right) \text{ watts.}$$

Calculate the value of the power when $C = 6 \cdot 5$ amperes, $V = 218$ volts, $V_1 = 68$ volts, $V_2 = 156$ volts.

33. The formula $\frac{1}{2}\left(a - \frac{b^2}{a}\right)$ refers to radial-arm bogies on locomotives. Find the value (ft.) when $a = 12$ ft. and $b = 6 \cdot 75$ ft.

Square Root.—Consider the statement $d \times d = d^2$. We have already seen that d^2 is called the "square" of d, and that if the value of d be known then the value of d^2 can be obtained. Thus if $d = 4$, then $d^2 = 4 \times 4 = 16$.

Now in many cases our calculations will give us the value of an expression like d^2, from which we have to obtain the value of d. Thus we may require the value of d when we know that $d^2 = 16$, that is to say we require the value of some number which if multiplied by itself will give 16 as the product. Now we know that $4 \times 4 = 16$, hence 4 is the number required in this case. Then 4 is said to be the **"square root"** of 16. The square root of any quantity A is that quantity which, if multiplied by itself, gives A as product.

Thus 1 is the square root of 1 because $1 \times 1 = 1$
" 2 " " " " 4 " $2 \times 2 = 4$
" 3 " " " " 9 " $3 \times 3 = 9$
" 4 " " " " 16 " $4 \times 4 = 16$

Similarly, from our previous work on powers, we know that—

$d \times d = d^2$, therefore d is the square root of d^2
$ab \times ab = (ab)^2$ " ab " " " $(ab)^2$
or a^2b^2.

In formulæ the words "square root of" are replaced by the sign $\sqrt{\ }$. The horizontal line extends over *all* the quantities of which the square root has to be taken.

Thus we write $\sqrt{25} = 5$ because $5 \times 5 = 25$

$\sqrt{10000} = 100$ because $100 \times 100 = 10000$

$\sqrt{a^2b^2} = ab$,, $ab \times ab = a^2b^2$

The square roots of certain numbers, such as those of 4, 9, 16, etc., are found in the common multiplication tables, and should therefore be known after a moment's thought. Some others, not usually remembered, such as $\sqrt{225} = 15$, $\sqrt{324} = 18$, etc., are useful, and are included in the following table:—

TABLE OF CERTAIN SQUARES AND SQUARE ROOTS.

Number.	Square Root.	Number.	Square Root.	Number.	Square Root.
1	1	25	5	169	13
2·25	1·5	36	6	196	14
4	2	49	7	225	15
6·25	2·5	64	8	256	16
9	3	81	9	289	17
12·25	3·5	100	10	324	18
16	4	121	11	361	19
20·25	4·5	144	12	400	20
Square.	Number.	Square.	Number.	Square.	Number.

The squares are in the left-hand columns reading from the headings at the bottom; square roots are in the right-hand columns, reading from the headings at the top.

The square root of a number such as 56 is not given above, as it is not a whole number.

Now $7 \times 7 = 49$, which is less than 56

$8 \times 8 = 64$,, ,, greater ,, 56

Hence $\sqrt{56}$ is between 7 and 8, *i. e.*, it is 7 and a decimal. Such square roots as these can be calculated or "extracted" by a kind of division as given below. The method can only be applied to decimal numbers. If it be required to find the square root of a number containing a vulgar fraction, then usually this must be first converted into a decimal. To illustrate the method consider the following example. (The mathematical explanation of the method is omitted as being somewhat lengthy.)

Example 88.—Extract the square root of 22·09.

Write the number down with a bracket to its right-hand side; the answer is to be placed to the right of this bracket. Mark off the figures (or digits) *in pairs* (called periods) by dashes, starting from the decimal point and working to the right and left.

Thus— $'22\cdot09'($

Consider the extreme left-hand period, the 22. Find mentally or from the table the highest whole number whose square is not greater than 22; in this case it is 4. Place this number (4), in the answer and its square (16) under the period 22. Subtract the square 16 from the period 22, giving 6 as a remainder. The next thing in the number is the decimal point. Therefore place the decimal point in the answer, which, as in ordinary division of decimals, is done when all the figures in the whole number have been dealt with. Bring down the next period or *pair of figures, i. e.*, 09, to the side of the remainder 6, giving 609 as a new number to be divided.

Thus—

$'22\cdot09'(4$
16
—
609

Place a vertical line to the left of the working. Double the partial answer 4, making it 8, and write this 8 opposite the remainder 609, on the left of the vertical line.

Thus—

	$'22\cdot09'(4\cdot$
	16
8	609

Divide the 8 into the first two figures of the remainder, 60. It will go 7 times. Write this 7 in the answer, and also at the side of the 8 to the left of the vertical line, giving 87.

Thus—

	$'22\cdot09'(4\cdot7$
	16
87	609

Now multiply this new number 87 by the last figure put in the answer, 7, and write the product under the remainder 609. Now $87 \times 7 = 609$. No subtraction can be made, and the example has

worked right out. The complete working should now appear thus:—

$$\begin{array}{r|l} & {}'22{\cdot}09'(4{\cdot}7 \\ & 16 \\ \hline 87 & 609 \\ & 609 \\ \hline & \cdots \end{array}$$

Then, $\sqrt{22{\cdot}09} = \underline{4{\cdot}7}$.

This and the following examples should be proved by actual multiplication, *i. e.*, squaring.

The above method, if followed *strictly*, will give the square root of any number whatever. Certain points arise which require a little explanation.

When the example does not work out without a remainder, as does the above, then it is usually sufficient to work to 4 or 5 significant figures and to give the result correct to 3 or 4.

The extreme left-hand or starting period will often contain only one figure, as in the case of 583·75, which, when marked off in pairs, will appear as 5'83·75'. Here proceed exactly as before, treating the figure 5 as a period; *i. e.*, finding the highest number whose square is not greater than 5, in this case 2.

When it is found that there are not sufficient figures in the number to "bring down," then noughts may be supplied to the right of the decimal point. Thus, 597 may be marked off as 5'97·00'00', the number not being affected by such adding of noughts. It is not absolutely necessary to write the noughts, but care must be taken that they are "brought down" *in pairs.* To avoid the very probable error of adding only a single 0 to a remainder, it is safer to write the additional noughts, and point them off as shown.

Should the last or right-hand period only contain one figure, then a single nought should be added to complete the period. Thus 692·5 should be marked off as 6'92·50'. It is advisable to write a few additional noughts as required.

When dividing the first figure of a divisor into the first two figures of a dividend, the former may go exactly into the latter, or very nearly so. Thus, in Example 89, which follows, taking the first division, 4 into 29 goes 7 times almost exactly. In such cases it will usually be found that this result is not suitable on account of the resulting product being too large, and a smaller number will have to be taken. This is shown in the explanations to Examples

89 and 92. To avoid writing any figures which may have to be discarded, it is advisable to jot down each product in a place reserved for scribbling and to test its correctness before writing it in the complete working of the example.

Example 89.—Find the square root of 692·5.

```
        6'92·50'00'(26·31
        4
       ---
46  |   292
    |   276 .... After obtaining this product
    |  ----          place the decimal point.
523 |    1650
    |    1569
    |   -----
5261|     8100
    |     5261
    |    -----
```

Proof by Squaring.

```
  26·3
  26·3
 -----
   789
  1578
   526
 -----
 691·69
 ------
```

$$\therefore \sqrt{692{\cdot}5} = \underline{26{\cdot}3}.$$

Explanation and Notes.—The highest number whose square is not greater than 6 is 2. Hence 2 is placed in answer and $2^2 = 4$, under the 6. Subtraction gives 2. The next period to bring down is the 92, which is placed at the side of the 2; the new dividend is thus 292. Double the partial answer 2 making 4, and write the 4 on the left opposite the 292. Dividing 4 into 29 gives 7, but this is found unsuitable, thus: placing the 7 in the answer and in the new divisor to the right of the 4, we should have $47 \times 7 = 329$, which is bigger than 292. Hence the next smaller number must be taken, which is 6. Then 6 is written to the right of the 2 in the answer, and also to the right of the 4 on the left, so that the new divisor is 46; then $46 \times 6 = 276$, which appears beneath the 292. Subtraction gives 16. The next thing in our given number is the decimal point, which is now written in the answer. The third period is the 50, which is brought down to the right of the 16, making 1650. Double the partial answer 26, making it 52, which appears on the left of the 1650. Dividing 5 into the 16 gives 3. The 3 is written after the 52 (making the new divisor 523), and also in the answer after the decimal point. Multiplying 523×3 gives 1569. Subtracting gives 81. The next period is 00. Doubling the 263 gives 526, which is placed on the left. Dividing 5 into 8 gives 1. Then 1 is put after the 526 and after the 263; and then $5261 \times 1 = 5261$. The answer to three significant figures is 26·3. As a proof, 26·3 is squared, giving 691·69 (shown at the side). The error due to using only three significant figures is ·81, or about ·1% of the original quantity, which error is so small as to be completely ignored.

Great care must be taken when a divisor will not go *at all* into

one of the dividends. Then a nought must be placed in the answer, as in the ordinary division of decimals.

Example 90.—To find $\sqrt{940}$.

$$
\begin{array}{rl}
 & 9'40\cdot00'(30\cdot659 \\
 & 9 \\
606 & 4000 \\
 & 3636 \\
6125 & 36400 \\
 & 30625 \\
61309 & 577500 \\
 & 551781
\end{array}
\qquad \therefore \sqrt{940} = \underline{30\cdot66}
$$

Notes.—The first period gives 3 in the answer, and leaves no remainder. Bringing down the second period, the new dividend is 40. Doubling the 3, the new divisor is 6. Now 6 into 40 will go 6. But placing this with the first 6, giving a divisor of 66, the product would be 66 × 6 which is evidently much larger than 40. Even 1 is too large, 61 × 1 giving 61. Therefore the result of the division in this case is 0, *and this* 0 *must be placed in the answer.* Bringing down the next period 00, proceed as before.

When the number of which the square root is to be extracted is a decimal fraction, then the decimal point is the first thing met with, and is then the first thing placed in the answer. After this the decimal point may be disregarded, the example being worked out as for a whole number, remembering that the "pointing off," *must* commence from the decimal point.

When dealing with a decimal fraction which commences with one or more noughts, care must be given to the number of noughts which are to appear in the answer, as in the next example.

Example 91.—To find $\sqrt{\cdot000972}$.

$$
\begin{array}{rl}
 & \cdot00'09'72'00'(\cdot03117 \\
 & 9 \\
61 & 72 \\
 & 61 \\
621 & 1100 \\
 & 621 \\
6227 & 47900 \\
 & 43589
\end{array}
\qquad \therefore \sqrt{\cdot000972} = \underline{\cdot0312}
$$

Note.—Here the decimal point is first met with; hence it is the first thing put in the answer. The first period is 00; hence a 0 is put in the answer. Next period 09, and from there the example is similar to previous ones.

A common mistake, in cases such as Example 91, is to obtain the wrong number of noughts following the decimal point in the result. The number of noughts may be checked by the following rule:—

For every complete period of noughts following the decimal point, there must be a nought after the decimal point in the result.

Thus, in the above cases, there is 1 *complete* period of noughts in ·00'09'72, and therefore there must be *one* nought in the result, as shown.

The square root of a whole or mixed number is always smaller than the number itself (except with 1, for $\sqrt{1} = 1$). But with fractions the square root is always *larger* than the fraction itself. Thus, in Example 91, $\sqrt{\cdot 000972} = \cdot 0312$, the root being much larger than the number. Squaring the result will prove the truth of the statement.

It may be found that after subtraction, a remainder is obtained which is larger than the divisor. This would indicate a blunder in an ordinary division, yet it is quite possible in square root. The following example illustrates:—

Example 92.—Extract the square root of 284·5.

```
                2'84·50'(16·867
                1
              ---
26   |        184
              156 ..... Decimal point is placed when this
              ---             product is obtained.
328  |         2850
              2624
              ----
3366 |        22600
              20196
              -----
33727|        240400
              236089
```

$\sqrt{284\cdot 5} = \underline{16\cdot 87}$

Notes.—On bringing down the second period the new dividend is 184. Dividing 2 into the 18 the result is 9 exactly, which is discarded as being too large. Also both 8 and 7 give products too high. Then 6 must be taken, and we have $26 \times 6 = 156$; 156 from 184 leaves 28, which is larger than the divisor 26. Now, in ordinary division this

would indicate that the figure 6 in the answer was too small, but here it must be accepted, as 7 is too large. This peculiarity is due to the last figure in the answer at each step being placed in the corresponding divisor; thus we have 26 and 6, using the 6, but would have 27 and 7 if the 7 were used. The twofold increase is sufficient to give a product larger than our dividend.

When dealing with large numbers which end in several noughts, whose roots are whole numbers ending in noughts, care must be taken towards the end of the working that the decimal point is placed correctly.

Example 93.—Extract the square root of 7,290,000.

$$\begin{array}{r|l} & 7'29'00'00(2700 \\ & 4 \\ \hline 47 & 329 \\ & 329 \\ \hline & \ldots \end{array} \qquad \therefore \sqrt{7290000} = \underline{2700}$$

In this case the decimal point, although not seen, is on the extreme right of the number, hence mark off in pairs from the *right*. The first and second periods present no difficulty. The subtraction leaves *no remainder* and the figures in the answer up to this stage are 27. The third period is 00, hence a 0 is placed in the answer. The fourth period is also 00, hence *another* 0 is placed in the answer. This brings us to the end of the number, *i.e.*, to the decimal point (not placed) and the point comes after the 2700. The result, 2700, can be proved by actual squaring.

The three following square roots should be memorised for future use; and, as an exercise, the reader should verify their accuracy—

$\sqrt{2} = 1{\cdot}414$ $\qquad$ $\sqrt{3} = 1{\cdot}732$ $\qquad$ $\sqrt{5} = {\cdot}707$.

Exercises 24. On Square Root.

Extract the square roots of the following numbers. (As these exercises from 1 to 17 are set as a test of the student's grasp of the method, they should be worked right out.)

1. 11·56.	**2.** 240·25.	**3.** 4·41.	**4.** 841.
5. 84·2724.	**6.** 110·25.	**7.** 1624·09.	**8.** 11881.
9. ·6084.	**10.** ·0729.	**11.** ·009025.	**12.** 1·0201.
13. ·00000961.	**14.** 252·81.	**15.** 2490·01.	**16.** 102,400.
17. 6,760,000.			

Work out the following square roots to 4 decimal places and state the answer to 3:—

18. 22·7.	**19.** 2·61.	**20.** 2558.	**21.** 3682.
22. 763·1.	**23.** 583·7.	**24.** ·09075.	**25.** ·1267.

Evaluation, including Square Root.—When square roots appear in a formula the algebraic method of expressing multiplication and division is adopted.

Thus— $\cdot 18\sqrt{D}$ means $\cdot 18 \times$ Square Root of D,

$\frac{850}{\sqrt{P}}$ means $850 \div$ Square Root of P.

When the square root of a single quantity only is required in an expression (as in the two cases shown), the root is first found and can then be combined with the other quantities. But when the root sign covers a complicated expression as in $\sqrt{\frac{D^3 l}{120R}}$ we have the rule:—

The value of any expression under a root sign must first be obtained before attempting to extract the root.

When writing out an expression containing a square root, care should be taken that only the proper quantities are covered, as the inclusion or omission of any quantities will alter the value of the expression considerably. Thus taking $\sqrt{\frac{a}{b}}$ and $\frac{\sqrt{a}}{b}$, the first means that a must be divided by b and the square root of the quotient *then* extracted; but the second indicates that the square root of a must first be found, and then divided by b.

Similarly, $\sqrt{a} + b$ is by no means the same as $\sqrt{a + b}$, but carelessness in writing the sign might turn the first expression into the second. If possible, it would be better to write $b + \sqrt{a}$ instead of $\sqrt{a} + b$. Also $\sqrt{a}\,b$ (meaning $\sqrt{a} \times b$) is best written as $b\sqrt{a}$, as the latter form, unlike the former, cannot be mistaken for $\sqrt{ab}$.

Example 94.—The permissible current in an electric cable may be calculated from Kennelly's rule, $C = 560\sqrt{d^3}$ where C = current in amperes and d = diameter of conductor core in inches. Calculate the current which may be carried by a conductor having a core ·16″ dia.

$C = 560\sqrt{d^3}$
$= 560\sqrt{\cdot 16^3}$
$= 560\sqrt{\cdot 004096}$
$= 560 \times \cdot 064 = \underline{35 \cdot 84 \text{ amps.}}$

$\cdot 16^3 = \cdot 004096$

Extraction of $\sqrt{\ }$

$\cdot 00\,|\,40\,|\,96 (\cdot 064$

36

$124 \mid 496$

496

Note.—The value of $\cdot 16^3$ is first found. Then the square root of the result is extracted as shown. Finally, the root is multiplied by 560.

Example 95.—The velocity of steam in the nozzle of a steam turbine is given by the formula $V = \sqrt{2gJ(H_1 - H_2)}$. Find the velocity when $H_1 = 1194$, $H_2 = 1130$, and J and g are constants, being 778 and 32·2 respectively.

$$V = \sqrt{2 \times 32{\cdot}2 \times 778(1194 - 1130)}$$
$$= \sqrt{64{\cdot}4 \times 778 \times 64}$$
$$= \sqrt{3{,}207{,}000}$$
$$= \underline{1791 \text{ ft. per sec.}}$$

$$\begin{array}{r|l} & 3'20'70'00(1790\cdot \\ & 1 \\ \hline 27 & 220 \\ & 189 \\ \hline 349 & 3170 \\ & 3141 \\ \hline 358 & 2900 \end{array}$$

Next figure would be 7 or 8.
No need to work out.

Example 96.—The outside diameter of a hydraulic pipe is given by the formula—

$$D = d\sqrt{\frac{f + p}{f - p}}$$

when D = outside diameter in inches.
d = inside ,, ,,
f = safe stress in the metal, in lbs. per sq. in.
p = pressure of the water, ,, ,,

Find the value of D when $d = 4''$, $f = 2800$ lbs. per sq. in., and $p = 750$ lbs. per sq. in.

$$D = d\sqrt{\frac{f + p}{f - p}}$$
$$= 4\sqrt{\frac{2800 + 750}{2800 - 750}}$$
$$= 4\sqrt{\frac{3550}{2050}}$$
$$= 4\sqrt{1{\cdot}73}$$
$$= 4 \times 1{\cdot}315 = 5{\cdot}26''$$

say, $\underline{5\frac{3}{8}''}$ to the next largest $\frac{1}{8}''$

$$\begin{array}{r|l} & 1{\cdot}73'00'(1{\cdot}315 \\ & 1 \\ \hline 23 & 73 \\ & 69 \\ \hline 261 & 400 \\ & 261 \\ \hline 2625 & 13900. \end{array}$$

Exercises 24 (contd.). On Evaluation with Square Root.

26. If an observer is situated with his eye h ft. above sea-level the distance of the apparent horizon in nautical miles is $1{\cdot}23\sqrt{h}$. Calculate this value when $h = 38$ ft.

27. When calculating the size of steam pipe required for a steam engine the velocity of the steam (ft. per sec.) is given by the expression $\frac{850}{\sqrt{P}}$ where P = pressure in lbs. per sq. in. Calculate the velocity if P = 130 lbs. per sq. in.

28. The expression $\frac{D}{60}\sqrt{P}$ gives the diameter (ins.) of a steam engine piston rod where D = cylinder diameter and P = boiler pressure. Calculate the rod diameter when D = 33″ and P = 150 lbs. per sq. in.

29. The Lanchester formula for petrol engine rating is $\cdot 4d^2\sqrt{r}$ where d = cylinder diameter in inches and r = stroke ratio. Find the rated H.P. of an engine when $d = 3{\cdot}5''$ and $r = 1{\cdot}285$.

30. Find the value of the expression $A - \cdot 06\sqrt{A}$, which relates to chimney stacks, when A = 12·6.

31. In locomotive boilers the thickness of the firebox plates is calculated from the formula $T = \sqrt{\frac{P}{2}} - 1$, where P = working pressure in lbs. per sq. in. and T = thickness in *sixteenths of an inch* (*i. e.*, if T is 9, plate is $\frac{9}{16}''$ thick). Calculate the thickness required for a boiler to work at 175 lbs. per sq. in.

32. The thickness of a steam engine cylinder may be calculated from the formula $\frac{\sqrt{D}}{5} + \cdot 015D$ when D = cylinder diameter in ins. Calculate the thickness of a cylinder 33″ dia. Give your actual result and also the result to the next $\frac{1}{16}''$.

33. Find the value of the expression $\sqrt{d_1 d_2}$, which relates to the inertia of links in machines, when $d_1 = 12$ and $d_2 = 21$.

34. If l is the length of a pendulum in ft. then the time of a double swing in seconds is $2\pi\sqrt{\frac{l}{32{\cdot}2}}$ where $\pi = 3{\cdot}14$. Calculate the time when $l = 3{\cdot}26$ ft.

35. The permissible current in an electric conductor may be obtained from the formula $138\sqrt{d^3}$ amps. where d = dia. of core in centimetres. Calculate the current allowable when $d = \cdot 64$ cms.

36. The expression $\sqrt{l^2 - r^2}$ refers to an engine governor. Find its value when $l = 1{\cdot}25$ and $r = \cdot 85$.

37. The expression $B + \sqrt{B^2 + T^2}$ occurs in shaft calculations. Calculate its value when B = 4·55 and T = 5·73.

38. If a trolley wire l ft. long be hung between two poles L ft. apart, then the dip at the centre of the span in feet is $\cdot 612\sqrt{Ll - L^2}$. Calculate the dip for a wire 120·1 ft. long hung between two standards 120 ft. apart.

39. The expression $\frac{T}{2}\sqrt{\frac{L - S}{L}} - \frac{l}{2}$ relates to the movement of a slide valve. Find its value when T = 5·5, L = 1, S = ·75 and $l = \cdot 25$.

Other Roots.—Besides square root we have to deal with Cube Root, Fourth Root, Fifth Root, etc., all of which involve an idea similar to that of square root.

Thus, 2 is the cube root of 8 because $2 \times 2 \times 2 = 8$,

i. e., because 8 is the cube of 2;

3 is the cube root of 27 because $3 \times 3 \times 3 = 27$.

Similarly 3 is the fourth root of 81 because $3 \times 3 \times 3 \times 3 = 81$,

i. e., because 81 is the fourth power of 3.

Thus, the cube root (or fourth or fifth root, etc.), of any number A, is such that when 3 (or 4 or 5) roots are multiplied together the result is A.

In formulæ the root sign $\sqrt{\ }$ is used in conjunction with a small figure placed immediately outside, near the top, to denote whether the root is cube, fourth, etc.

Thus— $\sqrt[3]{\ }$ signifies cube or third root

$\sqrt[4]{\ }$,, fourth root

$\sqrt[5]{\ }$,, fifth root

When writing these signs the little figure must be small and *close to the root sign.* If this is not done there is a possibility of it being taken for a coefficient.

Then, $\sqrt[3]{125} = 5$ because $5 \times 5 \times 5 = 125$

$\sqrt[3]{a^3b^3} = ab$ because $ab \times ab \times ab = (ab)^3 = a^3b^3$

$\sqrt[4]{10000} = 10$ because $10 \times 10 \times 10 \times 10 = 10000$

$\sqrt[5]{D^5} = D$ because $D \times D \times D \times D \times D = D^5$

Therefore $\sqrt[3]{x} = a$, if $a^3 = x$

i. e., $a \times a \times a = x$

and— $\sqrt[4]{y} = n$, if $n^4 = y$

i. e., $n \times n \times n \times n = y$

Roots beyond the fifth are not common in practical problems. It is possible to extract these "higher" roots by arithmetical methods somewhat similar to that followed for a square root, but the methods are so much more complicated and laborious that they are never used.

Most engineering pocket-books give tables of square and cube roots for numbers 1 to 1000, in sufficient detail for practical work. *Any root* can be quickly and easily extracted by Logarithms. The actual method is dealt with in Chap. VI. Examples involving cube or other roots cannot be evaluated until the reader can use logarithms, and will be left until then.

Powers of Fractional Expressions.—Consider the fraction $\frac{l}{3}$. If this is to be squared then it may be indicated in the form $\left(\frac{l}{3}\right)^2$. It is *not correct* to write $\frac{l^2}{3}$, as the index only refers to the symbol against which it is placed. If the value of l be known the easier method of working is to find the value of $\frac{l}{3}$ and then square it. But often the value of l is not known, and a different form is required from $\left(\frac{l}{3}\right)^2$, to enable further working to be carried out.

$$\text{Now } \left(\frac{l}{3}\right)^2 = \frac{l}{3} \times \frac{l}{3}$$

Carrying out the multiplication of these two vulgar fractions in the ordinary way, we should have for the numerator $l \times l = l^2$, and for the denominator $3 \times 3 = 9$.

$$\therefore \left(\frac{l}{3}\right)^2 = \frac{l^2}{3^2} = \frac{l^2}{9}$$

Therefore to square a fractional expression, the numerator is squared to give a new numerator and the denominator is squared to give a new denominator.

A similar treatment can be applied to other powers.

Thus— $$\left(\frac{m}{5}\right)^3 = \frac{m}{5} \times \frac{m}{5} \times \frac{m}{5} = \frac{m^3}{5^3} = \frac{m^3}{125}$$

All this is contained in the following:—

To raise any fractional expression to any power, raise both numerator and denominator to that power, to give a new numerator and a new denominator. Then simplify where possible.

Example 97.—Remove the brackets from the following expressions and simplify where possible.

(*a*) $\left(\frac{E}{R}\right)^2$ (*b*) $\left(\frac{5}{x}\right)^3$ (*c*) $\left(\frac{D}{2}\right)^5$ (*d*) $\left(\frac{2r}{3}\right)^3$ (*e*) $\left(\frac{\pi DN}{60}\right)^2$

(*a*) $\left(\frac{E}{R}\right)^2 = \underline{\frac{E^2}{R^2}}$ (No simplification possible)

(*b*) $\left(\frac{5}{x}\right)^3 = \frac{5^3}{x^3} = \underline{\frac{125}{x^3}}$

(*c*) $\left(\frac{D}{2}\right)^5 = \frac{D^5}{2^5} = \underline{\frac{D^5}{32}}$

(*d*) $\left(\frac{2r}{3}\right)^3 = \frac{2^3r^3}{3^3} = \frac{8r^3}{27} = \underline{\cdot 297r^3}$

or $\left(\frac{2r}{3}\right)^3 = (\cdot 667r)^3 = \cdot 667^3\, r^3 = \underline{\cdot 297r^3}$

Note here that in certain cases the form $\frac{8r^3}{27}$ would be retained as being useful for cancelling purposes.

$$(e)\ \left(\frac{\pi DN}{60}\right)^2 = \frac{\pi^2 D^2 N^2}{60^2} = \frac{\pi^2 D^2 N^2}{3600}$$

Exercises 25. On Powers of Fractional Expressions.
Remove the brackets from the following expressions, and simplify where possible:—

1. $\left(\frac{l}{r}\right)^2$ 2. $\left(\frac{d}{2}\right)^2$ 3. $\left(\frac{D}{2}\right)^3$ 4. $\left(\frac{D}{2}\right)^4$ 5. $\left(\frac{2}{3}h\right)^3$
6. $\frac{(2x)^2}{8}$ 7. $\left(\frac{wr}{2}\right)^2$ 8. $\left(\frac{x}{3}\right)^5$ 9. $\left(\frac{C}{3r}\right)^2$ 10. $\left(\frac{T}{2\pi}\right)^2$
11. $\left(\frac{\pi d}{3}\right)^3$ 12. $\left(\frac{22V}{15}\right)^2$ 13. $\left(\frac{2\pi N}{60}\right)^2$ 14. $\left(\frac{R}{1{\cdot}5}\right)^3$ 15. $\left(\frac{r}{2\pi}\right)^2$
16. $\left(\frac{S_2}{S_1}\right)^2$ 17. $\left(\frac{y}{L}\right)^3$ 18. $(1{\cdot}3rn)^2$ 19. $\left(\frac{wl}{4}\right)^2$ 20. $\left(\frac{{\cdot}95l}{x}\right)^3$

Roots of Fractional Expressions.—With regard to roots, exactly similar laws apply. When it is required to denote the square root of, say, $\frac{D^2}{4}$ this must be written as $\sqrt{\frac{D^2}{4}}$, care being taken that the *whole of the fraction comes under the root sign*

$$\text{Now } \sqrt{\frac{D^2}{4}} = \frac{D}{2} \text{ because } \frac{D}{2} \times \frac{D}{2} = \frac{D^2}{4}$$

$$\text{Note that } 2 = \sqrt{4} \text{ and } D = \sqrt{D^2}$$

We might write $\frac{\sqrt{D^2}}{\sqrt{4}}$, although this form has no value here, yet in many cases it would be employed.

Hence we have the law:—

To extract a root of a fractional expression find the root of both numerator and denominator to give a new numerator and a new denominator. Then simplify where possible.

$$\text{Thus, } \sqrt{\frac{D}{9}} = \frac{\sqrt{D}}{\sqrt{9}} = \frac{\sqrt{D}}{3}$$

Similar remarks to the foregoing apply to other roots.

$$\text{Thus, } \sqrt[3]{\frac{L^3}{12}} = \frac{\sqrt[3]{L^3}}{\sqrt[3]{12}} = \frac{L}{2{\cdot}29}$$

Where a root sign covers a product, the expression may be written as the product of the roots of the factors.

$$\text{Thus } \sqrt{abc} = \sqrt{a} \times \sqrt{b} \times \sqrt{c}$$

This point is of use when some of the factors have an index the same as the root number.

Thus $\sqrt{a^2b} = \sqrt{a^2} \times \sqrt{b}$

$= a\sqrt{b}$ since $\sqrt{a^2} = a$

The form $a\sqrt{b}$ is much simpler than $\sqrt{a^2b}$

Similarly $\sqrt[3]{27S^3} = \sqrt[3]{27} \times \sqrt[3]{S} \times \sqrt[3]{r^3}$

$= 3 \times \sqrt[3]{S} \times r = 3r\sqrt[3]{S}$

This may also occur in fractional forms.

Thus— $\sqrt[4]{\frac{p^4S}{16}} = \frac{\sqrt[4]{p^4S}}{\sqrt[4]{16}} = \frac{p\sqrt[4]{S}}{2}$

Example 98.—The expression $\sqrt{\frac{D^2S}{4000}}$ is given for the diameter of the exhaust pipe of a steam engine, where D = dia. of cylinder and S = piston speed. Reduce to a simpler form.

$$\sqrt{\frac{D^2S}{4000}} = \frac{\sqrt{D^2S}}{\sqrt{4000}} = \underline{\frac{D\sqrt{S}}{63{\cdot}2}}$$

The result can either be left in this form or, as multiplication is usually more direct than division, it can be slightly modified thus—

$$\frac{D\sqrt{S}}{63{\cdot}2} = \frac{1}{63{\cdot}2} \times D\sqrt{S} = \underline{{\cdot}0158\, D\sqrt{S}}$$

Example 99.—The expression $\sqrt{\frac{2g}{\mu}}$ relates to the flow of water in pipes. In all cases g is a constant quantity and is 32. Reduce to a simpler form.

$$\sqrt{\frac{2g}{\mu}} = \sqrt{\frac{2 \times 32}{\mu}} = \sqrt{\frac{64}{\mu}}$$

$$= \frac{\sqrt{64}}{\sqrt{\mu}} = \underline{\frac{8}{\sqrt{\mu}}}$$

Exercises 26. On Roots of Fractional Expressions.

In each of the examples Nos. 1 to 5 the expressions are to be simplified so far as possible.

1. $\sqrt{\frac{2938}{H}}$ which gives the speed in revs. per min. of an engine governor of height H ft.

2. $\sqrt{\frac{2}{3}RT}$ relating to the wheel base on a tramway track.

3. $\sqrt{\frac{d^2P}{\cdot 75p}}$ giving the steam cylinder diameter in a direct-acting steam pump.

4. $\sqrt{\frac{D^2P}{3500}}$ which gives the diameter of a piston rod.

5. $\sqrt{\cdot 12r}$ relating to masonry arches.

6. The velocity of a body falling h ft. is $\sqrt{2gh}$ where g = a constant due to gravity = 32·2. Substituting this value, simplify the expression.

7. In metric units the value of g in the above formula is 981. Reduce the formula by using this value.

8. In connection with water flowing from orifices we have the expression $\sqrt{\frac{gx^2}{2y}}$. Simplify this if $g = 32$.

9. The diameter of a gas engine valve is $\sqrt{\frac{D^2S}{V}}$ where S = piston speed, V = gas velocity through valve, and D = cylinder diameter. Taking S as 300 and V as 4000 reduce to a simpler form.

10. The velocity of steam issuing from a turbine nozzle is $\sqrt{2gJH}$. Simplify this if J = 778 and g = 32·2.

11. If $g = 32$, simplify the expression $\sqrt{2g \times 2 \cdot 3p}$ which refers to the velocity of water under pressure p.

12. The formula $\sqrt{\frac{8\pi Mg}{S}}$ refers to the measurement of voltage by an electrometer. π and g are constants, π being 3·14, while g is 981. Substituting these values, reduce to a simpler form.

13. The diameter of a firebox stay bolt is given by the expression $\sqrt{\frac{ps}{\cdot 785f}}$ In locomotive boilers with copper stays $f = 3500$ and $s = 16$. Substituting values, simplify the expression.

14. The formula $\sqrt{\frac{P(S-6)}{20}} - 1$ refers to the thickness of firebox plates. For locomotive boilers S = 16. Substitute this value and simplify.

Laws of Indices: Multiplication.—When multiplying expressions containing symbols, it will often be found that the same kind of quantities have to be multiplied together, as for example $a^2 \times a^3$. Now by the ordinary system they would appear as a^2a^3; but a shorter expression may be employed as shown below.

$$a^2 = a \times a \text{ and } a^3 = a \times a \times a$$

Hence $a^2 \times a^3$ means $a \times a \times a \times a \times a$

i.e., 5 a's multiplied together.

Now this may be written as a^5

Hence $a^2 \times a^3 = a^5$

Note that the $5 = 2 + 3$, *i. e.*, the *sum* of the indices. Therefore the indices are *not multiplied* as might be expected for a multiplication, but are **added.**

Thus $a^2 \times a^3 = a^{2+3} = a^5$

A further proof may be obtained if figures be used. Thus, let $a = 3$, then $a^2 = 9$ and $a^3 = 27$.

Then $a^2 \times a^3 = 9 \times 27 = 243$

But if $a = 3$, $a^5 = 3^5 = 243$

Similarly $r^4 \times r^3 = r^{3+4} = r^7$

Hence the rule : **To multiply together two or more powers of the same quantity, write down the quantity and add the indices.**

This does not, of course, apply to cases of different quantities. Thus in $a^2 \times b^3$ the indices cannot be added or interfered with in any way. The product $a^2 \times b^3$ can only be written as a^2b^3.

When one of the quantities is of the first power, then it must be remembered that, although not written, its index is 1.

Thus— $p \times p^3 = p^{1+3} = p^4$

Similarly $x^2 \times x = x^{2+1} = x^3$

and $n \times n = n^{1+1} = n^2$

The general rule for multiplication of indices applies to any number of powers of the same quantity.

Thus $r^2 \times r \times r^3 = r^6$ since the sum of the indices 2, 1, and 3 = 6.

(Remember the index 1 for r).

The addition of the indices may usually be done mentally.

Example 100.—Simplify the following expressions:—

(*a*) $h \times h^3$; (*b*) $x^2 \times x^4 \times x$; (*c*) $c \times c^4$; (*d*) $10^3 \times 10^5$

(*a*) $h \times h^3 = h^{1+3} = \underline{h^4}$

(*b*) $x^2 \times x^4 \times x = x^{2+4+1} = \underline{x^7}$

(*c*) $c \times c^4 = c^{1+4} = \underline{c^5}$

(*e*) $10^3 \times 10^5 = 10^{3+5} = \underline{10^8}$

Example 101.—Simplify the following:—

(*a*) $r^2 \times f \times f^2$; (*b*) $p^2\mathrm{H} \times \mathrm{H}p$; (*c*) $ab^2 \times a^3c$.

(*a*) $r^2 \times f \times f^2 = r^2f^{1+2} = \underline{r^2f^3}$

(*b*) $p^2\mathrm{H} \times \mathrm{H}p = p^2 \times p \times \mathrm{H} \times \mathrm{H} = p^{2+1} \times \mathrm{H}^{1+1} = \underline{p^3\mathrm{H}^2}$

(*c*) $ab^2 \times a^3c = a \times a^3 \times b^2 \times c = \underline{a^4b^2c}$

Exercises 27. On Laws of Indices: Multiplication.

Multiply:—

1. a^3 *by* a^2. **2.** f by f^2. **3.** N^2 by N.
4. d by d^4. **5.** r^3 by r. **6.** a^4 by a^3.
7. H^5 by H. **8.** 10 by 10^3. **9.** 10^2 by 10^5.

Simplify the following expressions:—

10. $wr \times w$. **11.** $px \times x$. **12.** $b^2 \times b \times b^2$.
13. $D \times D^4 \times D$. **14.** $R^2 \times R \times R$. **15.** $10 \times 10^2 \times 10$.
16. $10^3 \times 10 \times 10^2$. **17.** $f^2T \times f$. **18.** $RE^3 \times R^2E$.
19. $\pi n \times \pi n^2$. **20.** $pd^2 \times p \times d^3$. **21.** $x^4 \times xy \times yv$.

Laws of Indices: Division.—In a similar manner the division of powers of the same quantity may be treated.

Thus, take $\frac{b^5}{b^2}$. Now b^5 means 5 b's multiplied together, and b^2 means 2 b's multiplied together.

Hence $$\frac{b^5}{b^2} = \frac{b \times b \times b \times \not{b} \times \not{b}}{\not{b} \times \not{b}}$$

Cancelling leaves only 3 b's on top, *i.e.*, $\frac{b^5}{b^2} = b^3$

But the index 3 is the result of taking the index 2 from the index 5,

$$\therefore \frac{b^5}{b^2} = b^{5-2} = b^3.$$

Hence we have the rule: **To divide a power by another power of the same quantity, write the quantity and subtract the index of the divisor from the other index.**

Remember that the indices are *not divided* but are subtracted.

Similarly $$\frac{p^4}{p^3} = \frac{\not{p} \times \not{p} \times \not{p} \times p}{\not{p} \times \not{p} \times \not{p}} = p$$

or $\frac{p^4}{p^3} = p^{4-3} = p^1$, written as p.

Example 102.—Simplify the following:—

$$(a)\ \frac{H^3}{H^2} \quad (b)\ \frac{10^3}{10^2} \quad (c)\ \frac{a^5}{a} \quad (d)\ \frac{a^2R^6}{aR^3}$$

(a) $\frac{H^3}{H^2} = H^{3-2} = H^1 = \underline{H}$ (b) $\frac{10^3}{10^2} = 10^{3-2} = \underline{10^1}$

(c) $\frac{a^5}{a} = a^{5-1} = \underline{a^4}$ (d) $\frac{a^2R^6}{aR^3} = \frac{a^2}{a} \times \frac{R^6}{R^3} = a^{2-1} \times R^{6-3} = \underline{aR^3}$

Exercises 28. On Laws of Indices: Division.

Simplify the following:—

1. $\frac{m^4}{m^3}$	**2.** $\frac{S^5}{S}$	**3.** $\frac{t^2}{t}$	**4.** $\frac{R^3}{R^2}$
5. $\frac{10^7}{10^3}$	**6.** $\frac{10^3a^3}{10a^2}$	**7.** $\frac{a^3n^2}{a^2n}$	**8.** $\frac{\pi r^2}{r}$
9. $\frac{p^2d^4}{pd}$	**10.** $\frac{r^2w^2}{rw}$	**11.** $\frac{10^3R^2}{10^2R}$	**12.** $\frac{10^7 \times 10^4}{10^3 \times 10^2}$

Laws of Indices: Powers.—Occasionally we require to find the value of an existing power raised to a power of *itself*, *e. g.* the square of a^3 or $(a^3)^2$. Now just as $x^2 = x \times x$ so $(a^3)^2 = a^3 \times a^3$, the a^3 *as a whole* being like the x.

Now $a^3 \times a^3 = a^{3+3} = a^6$. Thus $(a^3)^2 = a^6$ where, of course, $6 = 3 \times 2$. Similarly $(a^2)^4 = a^2 \times a^2 \times a^2 \times a^2 = a^8$; and $8 = 2 \times 4$.

The actual operation on the indices of the quantity in the brackets is addition, but at the same time it is essentially a multiplication. Hence the rule: **To raise a power of a quantity to some power of itself write the quantity and multiply the indices.**

This must not be confounded with the rule for the adding of indices.

Example 103.—Simplify the following expressions:—

(*a*) $(n^2)^3$; (*b*) $(10^4)^2$; (*c*) $(a^2r^2)^3$; (*d*) $(np^2)^2$; (*e*) $(a^2b^3)^3$

(*a*) $(n^2)^3$ *i.e.*, $n^2 \times n^2 \times n^2 = n^{2\times3} = \underline{n^6}$

(*b*) $(10^4)^2 = 10^{4\times2} = \underline{10^8}$

(*c*) $(a^2r^2)^3 = a^{2\times3}\, r^{2\times3} = \underline{a^6r^6}$

(*d*) $(np^2)^2 = n^{2\times1}\, p^{2\times2} = \underline{n^2p^4}$

(*e*) $(a^2b^3)^3 = a^{2\times3}\, b^{3\times3} = \underline{a^6b^9}$

Exercises 29. On Laws of Indices: Powers.

1. Find the 4th power of a^3. **2.** Find the 3rd power of a^3.
3. ,, ,, 3rd ,, ,, 10^2. **4.** ,, ,, square of N^4.
5. ,, ,, 5th ,, ,, D^2. **6.** ,, ,, cube of R^4.

Simplify the following:—

7. $(10^3)^2$	**8.** $(10^2)^6$	**9.** $(R^3)^3$	**10.** $(m^5)^2$
11. $(r^2)^2$	**12.** $(cd^2)^3$	**13.** $(c^2d)^3$	**14.** $(c^2d^2)^3$
15. $(a^2p^3)^2$	**16.** $(ap^3)^3$	**17.** $(10^2 \times 10^3)^2$	**18.** $(m^3p)^4$

Substitution of Symbols.—It is often necessary to evaluate a formula when the given values are themselves symbols; the result obtained is another formula satisfying certain conditions. Much of

this work requires a knowledge of the rules in the previous sections. Many of the expressions obtained will be of the vulgar fraction form, and cancelling is of great assistance. The method is best seen from the following examples. The student should, first of all, *merely substitute.* It is only with considerable practice that the processes of substituting and evaluating can be combined.

Example 104.—In connection with the bending of beams the expression $\frac{2}{3}Lh$ occurs. Under certain conditions L has the value $\frac{l}{2}$ and $h = \frac{wl^2}{8}$. Find by substituting and simplifying, the value of the given expression under these conditions.

Given expression $= \frac{2}{3}Lh$

$= \frac{1}{\cancel{3}} \cdot \frac{l}{\cancel{2}} \cdot \frac{wl^2}{8}$ substituting $\frac{l}{2}$ for L

$= \underline{\frac{wl^3}{24}}$ and $\frac{wl^2}{8}$ for h

Note.—After substituting the new values and cancelling, the fractions can be multiplied up in the ordinary way.

Example 105.—The Marine Motor Association formula for the H.P. rating of a petrol engine is 3·25 A where A = area of the exhaust valve. Now $A = \cdot 785\delta^2$ * where δ = dia. of exhaust valve; and, on the average, $\delta = \cdot 5d$, where d = dia. of the motor cylinder. By substituting first for A and then for δ find the rated H.P. in terms of d, the dia. of the cylinder.

H.P. $= 3 \cdot 25A$

$= 3 \cdot 25 \times \cdot 785\delta^2$ substituting $\cdot 785\delta^2$ for A

$= 3 \cdot 25 \times \cdot 785 \times (\cdot 5d)^2$,, ,, $\cdot 5d$ for δ

$= 3 \cdot 25 \times \cdot 785 \times \cdot 25d^2$

$= \underline{\cdot 637d^2}$

Example 106.—The expression $\frac{v^2}{r}$ relates to the inertia of moving engine parts. Find its value when $v = \omega r$.†

Given expression $= \frac{v^2}{r}$

$= \frac{(\omega r)^2}{r}$ substituting ωr for v

$= \frac{\omega^2 \cancel{r^2}}{\cancel{r}} = \underline{\omega^2 r}$

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* δ is a Greek small letter " delta."

† ω is a Greek small letter " omega."

Example 107.—Find the value of the expression $\frac{p}{r^2}$ when $r = \frac{p}{3}$

The given expression $= \frac{p}{r^2} = \frac{p}{\left(\frac{p}{3}\right)^2}$ (writing $\frac{p}{3}$ in place of r)

$= \frac{p}{\frac{p^2}{9}}$

$= \frac{p \times 9}{p^2} = \frac{9}{p}$ (inverting the divisor and cancelling by p)

Exercises 30. On Substitution of Symbols.

Find the values of the following expressions, Nos. 1 to 8, when $a = 2n$ and $x = \frac{n^2}{2}$.

1. ax^2 **2.** a^2x **3.** a^2x^2 **4.** $\frac{x}{a}$

5. $\frac{a^2}{x}$ **6.** $\frac{a^2}{x^2}$ **7.** $\frac{a}{x^2}$ **8.** $\frac{a^3}{x}$

9. If E is the voltage and C the current in an electric circuit, the power supplied is EC. Find another expression for the power: (*a*) when $E = CR$; (*b*) when $C = \frac{E}{R}$.

10. In connection with beams the expression ax^2 appears. In a particular case $a = bh$ and $x = \frac{h}{2}$. Find the value of the given expression under these conditions.

11. Evaluate ax^2 for the case where $a = \frac{2}{3}BH$ and $x = \frac{3}{5}B$.

12. If $\pi = 3{\cdot}14$, find the value of πr^2 when $r = \frac{d}{2}$.

13. Find the value of the expression PV when $V = \frac{vP}{p}$.

14. The formula $\frac{bh^2}{6}$ relates to beams, b being the width and h the depth. Find its value for a beam whose depth is twice the width (*i. e.*, $h = 2b$).

15. The expression $\frac{pr}{f}$ gives the thickness of a pipe under internal pressure. When used for water supply it is more convenient to replace p by $\frac{H}{2{\cdot}3}$ and r by $\frac{d}{2}$; if $f = 2800$, convert the formula to give the thickness in terms of H and d.

16. Simplify the expression $\frac{px^2}{2}$ when $x = \frac{3l}{8}$.

17. The expression $\frac{Sy}{l}$ is of importance in connection with continuous girders. Find its value for the special case where $S = \frac{l^3}{12}$ and $y = \frac{l}{2}$.

18. If Q is the charge of electricity supplied to an electric condenser and V = the potential, the energy = $\frac{1}{2}QV$. Find the value of this when $V = \frac{Q}{K}$, K being the capacity.

19. The formula $\frac{\pi^2 EI}{l^2 A}$ relates to long columns. A more useful form is obtained by substituting Ak^2 for I. Perform this substitution.

20. The fraction $\frac{AB}{2}$ was met with in a problem on beams, where $A = \frac{l}{2}$ and $B = \frac{ab}{l}$. Substitute these values and simplify.

21. The expression $\frac{1}{2}I\omega^2$ relates to fly-wheels. Find its value if $I = \frac{W}{g}k^2$ and $g = 32$.

22. The volume of a ball is $\frac{4}{3}\pi R^3$ where R = radius. Using diameter, $R = \frac{D}{2}$. Substitute this value and simplify the formula if $\pi = 3{\cdot}142$.

23. The expression $\frac{I}{y}$ is connected with beams. For a particular case $I = \frac{\pi d^4}{64}$ and $y = \frac{d}{2}$. Substituting these values reduce the given expression.

24. The volume of a ring is πDa where D = mean diameter and a = area of cross-section. If the ring is of circular section wire, $a = \frac{\pi}{4}d^2$ if d is diameter of wire. If $\pi = 3{\cdot}14$ find a formula for the volume in terms of D and d.

25. In connection with masonry structures we have the expression $\frac{K^2}{d}$. For a certain case $K = \frac{R}{2}$ and $d = R$. Find the value of the given expression under these conditions.

26. Find the value of the expression in the last example for the case where $K = \frac{D}{4}$ and $d = \frac{D}{2}$.

27. Connected with the bending of a beam we have the expression $\frac{M}{A}$. For a certain case $M = \frac{2}{3}B^2H$ and $A = \frac{2}{3}BH$. Substitute these values and simplify.

28. A formula giving the deflection of a beam is $\frac{Py}{H}$. Find the value of this expression when $P = \frac{Wl^2}{16}$, $y = \frac{l}{3}$, and $H = EI$.

Positive and Negative Quantities.—The signs + and − are also used in Algebra to denote "positive" and "negative"

quantities. Up to the present we have only considered quantities greater than 0. While in the ordinary weights and measures of everyday life, quantities "less than nothing" do not occur, yet in engineering science they are often to be found. As an example let us consider the Centigrade or Continental Thermometer, which is used in all scientific measurements. The Zero or 0 on this is the temperature of freezing of water, and temperatures above this are numbered 1, 2, 3, 4, . . . and so on, as far as may be desired. But we often have to consider temperatures **colder than that of freezing**; for example, the atmospheric temperature in winter, when the thermometer mercury will stand at some point **below the zero,** say 8° (read as "8 degrees") as indicated at A in Fig. 17. Now the temperature cannot be said to be "8 degrees" since this is used to denote 8 divisions *above zero*. We can say "8 degrees of frost" or "8 degrees below zero," but neither of these methods is sufficiently compact for use in calculation. But if we call the temperatures **above zero, positive** or **plus quantities,** and denote them by the addition sign +, then we may call the temperatures **below zero, negative** or **minus quantities,** and denote them by the subtraction sign —. Thus a temperature "above zero," as at B in Fig. 17, is 0 + 8 degrees. The 0 itself not having any value, is omitted, the reading appearing as + 8 degrees (read as "plus 8 degrees"). Similarly a temperature "below zero," as at A, is 0 — 8 degrees. As before, the 0 is omitted and the reading is called — 8 degrees (read as "minus 8 degrees"). These forms are now quite suitable for calculation. Although not apparent when we write + 8 or — 8, the meaning of add and subtract still holds good, the numbers really meaning 0 + 8 and 0 — 8. Similarly if we consider *changes* of temperature, a rise may be called + and therefore a fall will be —.

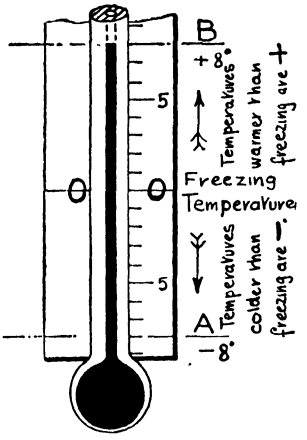


FIG. 17.

Again, suppose we take the case of direct stress in a bar, which may be tension or the effect of pulling apart; or compression which is the effect of pushing together. The two forms are quite opposite in character, as a bent bar subjected to a tension will straighten, but if subjected to a compression will become more bent. Now if,

when measuring stresses, we call the tension +, the compression must be called —. Supposing now that we have to find the stress in some part of a machine or structure, and we cannot say from an inspection what kind of stress exists there. If the answer to our calculation comes out +, we know it is a tension; if — it is a compression.

Further, positive or + quantities may be regarded as "income," and negative or — quantities as "expenditure." Many other examples may be found: thus with alternating electric currents the voltage is + when in one direction, and — when in the other: if the extension of a helical spring is +, the compression is —: if the height of a mountain is +, taking the sea-level as the zero of heights, then the depth of the sea is —.

It is customary when a positive number or letter stands alone, or is the first thing met with in an expression, to omit the + sign. Thus 5 means + 5 and x means + x. Similarly with a compound expression, $\frac{wl^3}{8}$ means + $\frac{wl^3}{8}$. In the middle of any expression the + sign must appear. The minus sign, however, *must always be included*, no matter what the position of the quantity.

Addition of + and — Quantities.—This may be illustrated by reference to changes of temperature on a thermometer, with the aid of Fig. 18. Actual readings of temperature will be

\+ when above the zero
and ∴ — „ below „ „

Considering changes of temperature (*i. e.*, distances through which the mercury moves)—

a Rise of temperature will be +
and ∴ a Fall „ „ „ —

Now supposing the temperature is 8° and it rises 5° (*i. e.*, changes + 5°), then obviously the final temperature is 13°. Thus we start at 8, change 5, and finish at 13. Then,

Start + Change = Final,

a statement true for all temperatures and for rises as well as falls.

1.—*Addition of Quantities with the Same Sign.*

(i) Both quantities +. This has already been taken as 8 + 5 = 13.

(ii) Both quantities —. Let a temperature be — 2° (*i. e.*, 2° below zero) and let it *fall* 6° (*i. e.*, the change is — 6). From *a*,

Fig. 18, it can be seen that the final temperature is 8° below zero, *i. e.*, − 8. Then, since

$$\text{Start} + \text{Change} = \text{Final}$$
$$-2 + (-6) = -8$$

Note that in both (i) and (ii) the result is the *sum* of the actual figures, and the sign of the result is the same as the sign of the numbers.

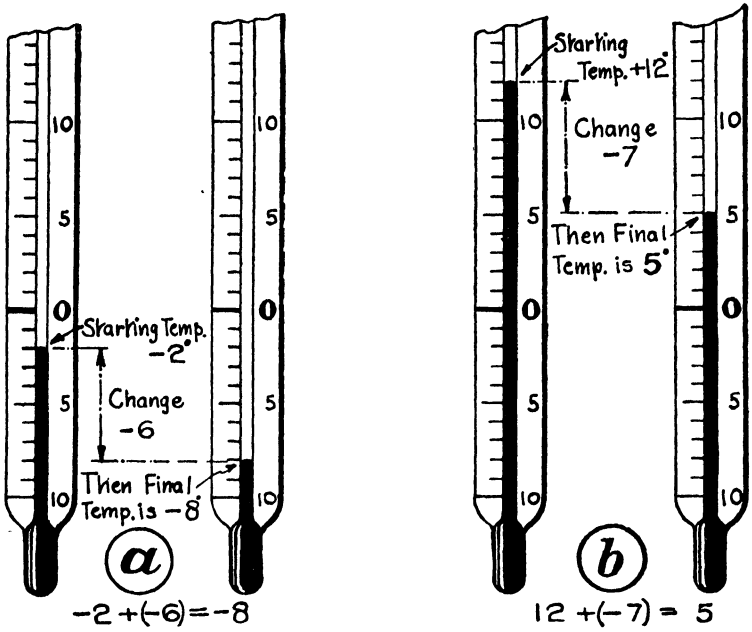


FIG. 18.—Illustrating the Addition of + and − Quantities.

2.—Addition of Quantities with Unlike Signs.

(i) Let the temperature be 12°, and let it fall 7° (*i. e.*, the change is − 7°); then it can be seen from *b*, Fig. 18, that the final temperature is 5°. Then, since

$$\text{Start} + \text{Change} = \text{Final}$$
$$12 + (-7) = 5$$

(ii) Let the temperature be 10° and let it fall 16° (*i. e.*, the change

is -16). Then from Fig. 18, it can be seen that the final temperature is 6° below zero, *i. e.*, $-6°$. Then, since

$$\text{Start} + \text{Change} = \text{Final}$$
$$10 + (-16) = -6$$

Note that in both (i) and (ii) the result is the *difference* of the actual figures, *e. g.*, in (*i*), $5 = 12 - 7$; and that the sign of the result is the sign of the larger number, *e. g.*, in (i) sign of result 5 is the sign of the 12. Hence we have the rules:

To add two quantities—

1. When the signs are the same, add the numbers, and write the sign in front of the sum.

2. When the signs are different, take the difference of the numbers, and write the sign of the larger number in front of the difference.

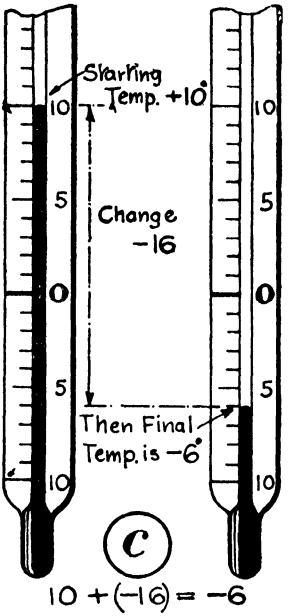


FIG. 18c.

Example 108.—Find the value of the expression $p + q$ for the following cases: (*a*) $p = 10$, $q = 5$; (*b*) $p = 4$, $q = -2$; (*c*) $p = -7$, $q = -3$; (*d*) $p = -3$, $q = 5$; (*e*) $p = 6$, $q = -15$.

(*a*) $p + q = 10 + 5 = \underline{+15}$ Ordinary case.

(*b*) $p + q = 4 + (-2) = \underline{+2}$ Signs are different, and thus the difference of 4 and 2 is found. The sign is that of the larger number (4).

(*c*) $p + q = -7 + (-3) = \underline{-10}$ Signs are the same, so that the numbers are added, and the same sign (−) given to the sum.

(*d*) $p + q = -3 + 5 = \underline{+2}$ Signs are different, therefore the difference of the numbers is taken, and the sign of the larger number (7) is prefixed.

(*e*) $p + q = 6 + (-15) = \underline{-9}$ Signs are different, therefore the difference is found, and the sign is that of the larger number.

The whole *idea* throughout this section has been *addition*, although in some cases subtraction has been performed between the numbers. This is due to the *signs* of the quantities, and although

not used in ordinary arithmetic, is very important in scientific work. The result of an "addition" carried out by this consideration of the signs or nature of the quantities is called an **Algebraic Sum,** and does not necessarily produce an increase in the size of the quantities. An *arithmetical sum* is simply an addition of the *figures,* disregarding the signs.

Example 109.—(**a**) Add together $5ab$, $6ab$, $13ab$, and ab. (**b**) Prove the result to be correct when $a = 2$ and $b = 3$.

(a)		(b) Proof		
$5ab$	=	$5 \times 2 \times 3$	=	30
$6ab$	=	$6 \times 2 \times 3$	=	36
$13ab$	=	$13 \times 2 \times 3$	=	78
ab	=	$1 \times 2 \times 3$	=	6
Sum = $25ab$				150

$25\ ab = 25 \times 2 \times 3 = 150.$

Notes.—The beginner had better write the terms in a vertical column, as indicated; but with a little practice this may be dispensed with. The letters ab may be regarded as a unit like feet, etc., and thus the unit of the answer will be the same as that of each term, *i. e.*, ab. (*Note* that ab is really 1 ab.) *Proof.*—Substitute the value of $a \times b$ in each term and evaluate, giving 150 as the sum of the quantities. Substituting the values of a and b in $25\ ab$ we obtain 150, thus proving the algebraic addition to be correct.

With positive and negative terms it is usually advisable to sort out all the positives and all the negatives, add separately, and finally make an addition of the positive and negative results.

Example 110.—(*a*) Add together $5r^2f$, r^2f, $-9r^2f$, $2r^2f$, $-2r^2f$, and $-r^2f$.

(*b*) Prove the result correct when $r = 2$ and $f = 1$.

Positive Quantities.	Negative Quantities.
$5r^2f$	$-9r^2f$
r^2f	$-2r^2f$
$2r^2f$	$-r^2f$
$8r^2f$ = Sum of + Quantities	$-12r^2f$
$-12r^2f$ = " " − "	
$-4r^2f$ = Result of Addition.	

Note.—Adding $8r^2f$ and $-12r^2f$ gives $-4r^2f$ by Rule 2, p. 125.

Proof.—The sum of the + and − quantities need not be questioned, the method having been proved in the previous example.

$$8r^2f = 8 \times 2^2 \times 1 = 32$$
$$-12r^2f = -(12 \times 2^2 \times 1) = -48$$
$$\text{Sum} = -16$$

Now $-4r^2f = -(4 \times 2^2 \times 1) = -16$, thus proving the foregoing result.

This addition and subtraction of positive and negative numbers is very important in connection with calculation by Logarithms (Chap. VI).

Exercises 31.—On Addition of + and − Quantities.

Find the value of the expression T + t in the following cases:—

1. T = 5, t = 3. **2.** T = 7, t = − 4. **3.** T = − 9, t = − 6.
4. T = − 14, t = 4. **5.** T = − 2, t = 19.

Find the value of the expression l + m in Exs. Nos. 6 to 10.

6. l = ·7, m = 1·2. **7.** l = − ·4, m = ·6. **8.** l = − 1·9, m = ·3.
9. l = − ·3, m = − ·8. **10.** l = − ·15, m = − 1·4.

11. Add + ·3570 to + ·5160. **12.** Add + ·2180 to − ·3560.
13. ,, − ·2250 ,, − ·7910. **14.** ,, − ·7750 ,, + ·1050.
15. ,, − ·0050 ,, + 1·05. **16.** ,, − 2 ,, + ·305.
17. ,, + 1 ,, − ·046. **18.** ,, + 3 ,, − 2·05.
19. ,, − 1·505 ,, − ·61. **20.** ,, + 2 ,, − 1·65.

Add together:—

21. 3, − 5, − 7, + 1, − 2, + 12. **22.** − 5, + 6, + 2.
23. 2, − 7, − 1. **24.** − 4, + 6, − 5.
25. − 3, − 2, + 5. **26.** − 1, + 1, + 7.
27. 7, − 2, − 5, + 1. **28.** − 3, − 1, + 2, − 6.
29. 14, − 7, − 6, + 3, − 7. **30.** − 1, + 2, + 6, − 5, − 9.

Complete the following additions:—

31.	**32.**	**33.**	**34.**
$4ab$	$-x^2y$	$-9r^2$	$14pm$
$-3ab$	$4x^2y$	$-6r^2$	$-9pm$
ab	$3x^2y$	$+3r^2$	pm
$-2ab$	$-2x^2y$	$+11r^2$	$-2pm$

Add together the following:—

35. $4a^2c$, $-3a^2c$, $-a^2c$, $+2a^2c$.
36. $-5wl$, $-3wl$, $+wl$, $+14wl$, $+11wl$.
37. $-7p^2$, $-3p^2$, p^2, $7p^2$, $10p^2$.
38. $+5d$, $-3d$, $-d$, $-6d$, $+2d$.
39. $-2{\cdot}5n$, $3n$, $1{\cdot}5n$, $-7{\cdot}5n$.
40. $2{\cdot}8w$, $-3w$, $-5{\cdot}75w$, $-3{\cdot}22w + 0{\cdot}5w$.

Subtraction of + and − Quantities.—This we will also examine by considering changes of temperature, with the aid of Fig. 19. In this case, knowing the starting and final temperatures,

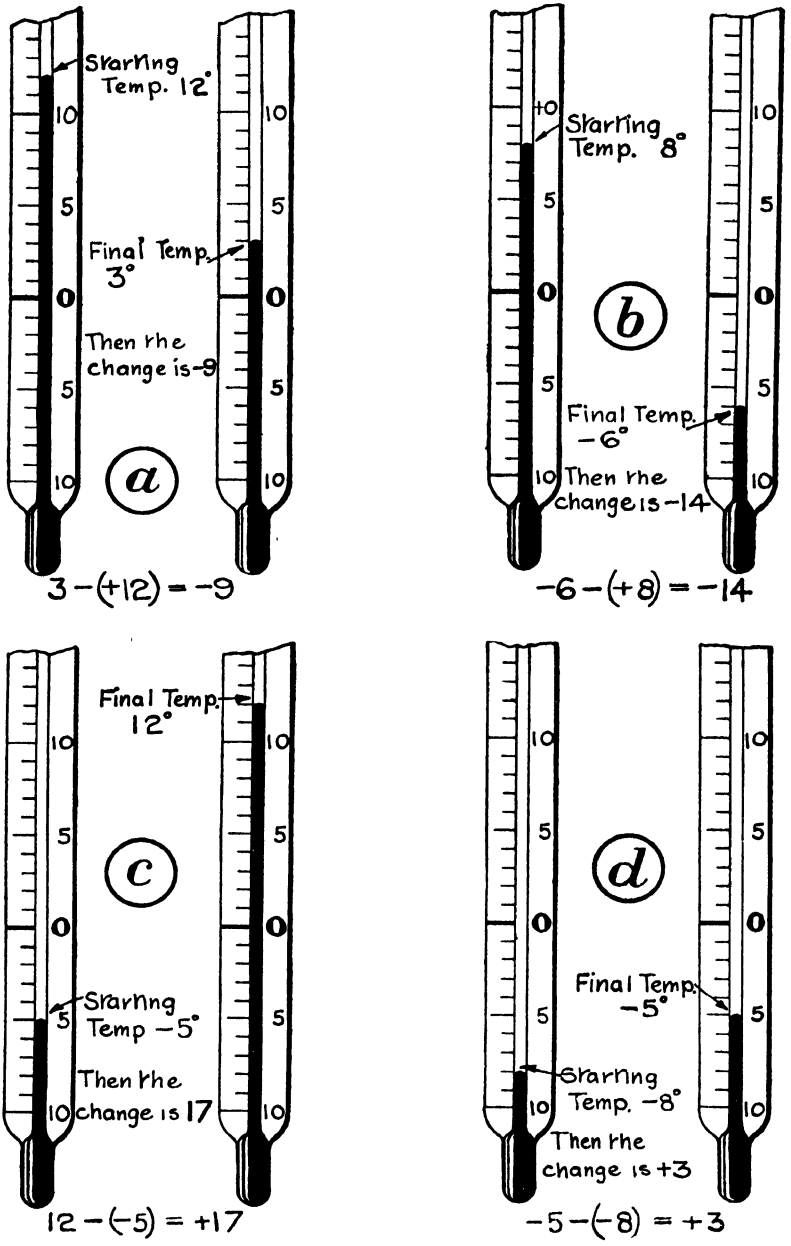


Fig. 19.—Illustrating the Subtraction of + and − Quantities.

we shall find the size and nature of the change. The ordinary arithmetical subtraction is a case of both quantities being +, and in all cases the quantity to be subtracted is less than the other. Algebraically these conditions need not exist.

Let the temperature on our thermometer be 7°, and suppose it rises to 18°, what is the change in temperature? Evidently the temperature has *risen 11°*; then the change algebraically is + 11.

Thus, $18 - 7 = 11$, or in words,

Final — Start = Change,

a statement true for all temperatures, no matter whether they rise or fall.

1.—*Subtraction of a + Quantity.*

Consider the case where the first number is smaller than the second.

(i) Let the temperature be 12°, and let it change to 3° (final temperature). From *a*, Fig. 19, it is seen that the change is *a fall of 9°, i. e.,* — 9°. Then since—

$$\begin{array}{rrrr} \text{Final} & - & \text{Start} & = \text{Change} \\ 3 & - & (+12) & = -9 \end{array}$$

The + sign may be omitted before the 12; hence the above statement may be written as $3 - 12 = -9$, and when the idea of opposite nature or opposite direction is considered the statement is evidently true, since to pass from 12° to 3° the mercury must *fall* 9°. To perform the operation algebraically we notice that the result — 9 is also the result of $3 + (-12)$, reversing the two signs before the 12. Evidently $3 - (+12) = 3 + (-12)$, from which we conclude that "*the subtraction of + 12 is equivalent to the addition of — 12.*" We will examine this for another case.

(ii) Let the temperature be 8° above zero, and let it change to 6° below zero. Then from *b*, Fig. 19, the change is a *fall of 14°* (*i. e.,* — 14°). Then as—

$$\begin{array}{rrrr} \text{Final} & - & \text{Start} & = \text{Change} \\ -6 & - & (+8) & = -14 \end{array}$$

But — 14 is also the result of $-6 + (-8)$, or *the subtraction of + 8 is equivalent to the addition of — 8.*

Both cases show us that **the subtraction of a + quantity is equivalent to the addition of a — quantity.**

Denoting the first number by *a* and the second by *b* we can write:—

$$\mathbf{a - (+b) = a + (-b) = a - b}$$

2.—Subtraction of a — Quantity.

(i) Let our temperature be 5° below zero and let it change to 12° above zero. Then from *c*, Fig. 19, the change is evidently *a rise of 17°* (or change = + 17°). Then as—

Final — Start = Change
12 — (— 5) = + 17

Notice that + 17 is also the result of 12 + 5, which we may write as 12 + (+ 5). Then 12 — (— 5) = 12 + (+ 5) whence we conclude that *the subtraction of — 5 is equivalent to adding + 5*. The truth of this is evident from *c*, Fig. 19 for certainly the mercury has *risen 17°*.

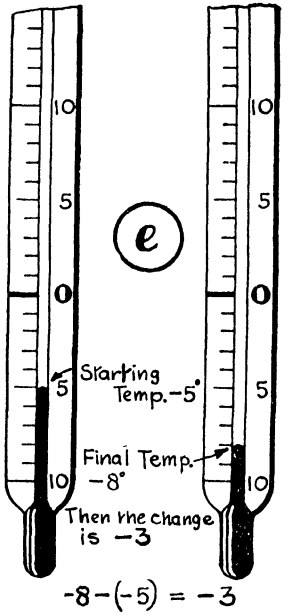


Fig. 19*e*.

As a further illustration : If we call a "possible event" a + event, then an "impossible event" must be a — event. Now suppose we say that an event is "not impossible," which corresponds to — (—), then we mean that it *is* possible, *i. e.*, it is +.

We will examine the above for two other cases.

(ii) Let the temperature be 8° below zero, and let it become 5° below zero. Then from *d*, Fig. 19, the change is evidently 3° rise. Then as—

Final — Start = Change
— 5 — (— 8) = + 3

But + 3 is also the result of — 5 + 8, *i. e.*, =5 + (+ 8); hence, as before, the subtraction of — 8 is equivalent to adding + 8. The "addition" is, of course, algebraic.

(iii) Let the temperature be — 5, and let it change to — 8. Then the change should be — 8 — (— 5) = — 8 + (+ 5) = — 3, or a fall of 3°. From *e*, Fig. 19, this is evidently true. Hence we conclude that **the subtraction of a — quantity is equivalent to the addition of a + quantity,** or, if *a* is the first number and — *b* the one to be subtracted—

$$\mathbf{a - (-b) = a + (+b) = a + b}$$

In both our conclusions we notice that a change of operation (*i. e.*, addition replacing subtraction) accompanies a change of sign. Hence we have the rule : **To take one quantity from another, change the sign of the quantity to be subtracted, and proceed to add algebraically.**

Example 111.—(*a*) Take $5wl^2$ from $8wl^2$; (*b*) $7wl^2$ from $2wl^2$; (*c*) $-3wl^2$ from $4wl^2$; (*d*) $-6wl^2$ from $-9wl^2$.

(*a*)	$8wl^2$	Change sign of $5wl^2$ to $-$ and add.
	$5wl^2$	Sum of 8 and -5 is 3.
Difference =	$3wl^2$	The " unit " wl^2 remains.
(*b*)	$2wl^2$	Change sign of $7wl^2$ to $-$ and add.
	$7wl^2$	Sum of 2 and -7 is -5.
Difference =	$-5wl^2$	
(*c*)	$4wl^2$	Change sign of $-3wl^2$ to $+$ and add.
	$-3wl^2$	Sum of 4 and 3 is 7.
Difference =	$7wl^2$	
(*d*)	$-9wl^2$	Change sign of $-6wl^2$ to $+$ and add.
	$-6wl^2$	Sum of -9 and $+6 = -3$.
Difference =	$-3wl^2$	

Exercises 32.—On Subtraction of + and − Quantities.

Find the value of the expression W − w in the following cases:—

1. W = 11, w = 5. **2.** W = − 4, w = 7. **3.** W = − 7, w = − 3.
4. W = 1, w = − 10. **5.** W = − 3, w = − 2. **6.** W = − 15, w = + 3.
7. W = − 2, w = + 12. **8.** W = − 4, w = − 3. **9.** W = − 3, w = − 3.

10. Subtract 1·5 from 3·7. **11.** Subtract 1·2 from − 2·4.
12. ,, − ·7 ,, 1·6. **13.** ,, − 1·9 ,, − 4·6.
14. ,, 2·6 ,, − ·3. **15.** ,, − 2·18 ,, − 1·28.
16. ,, − 3·13 ,, 3·59. **17.** ,, 2·29 ,, − 1·16.
18. ,, 5·35 ,, − 1·56. **19.** ,, ·358 ,, − ·759.
20. ,, 7·2 ,, 5·5. **21.** ,, − 1·355 ,, 2·746.
22. ,, 2·38 ,, 2·05. **23.** ,, − 1·357 ,, − 3·359.
24. ,, − 3 ,, − 2·155. **25.** ,, 2·35 ,, − 5.
26. ,, $7wl$,, $14wl$. **27.** ,, $19rf$,, $7rf$.
28. ,, $-12ad^2$,, $13ad^2$. **29.** ,, $-c^2p$,, $-3c^2p$.
30. ,, $4xy$,, $-xy$. **31.** ,, $5{\cdot}5xy$,, $4{\cdot}25xy$.

Addition and Subtraction of Several Terms.

The addition and subtraction of expressions containing several terms is subject to the same general rule that applies to vulgar and decimal fractions, and in fact to everything: *i. e.*, that only things of the same name or kind may be directly added together. Thus, in adding the expressions $a^3 - 2a^2b + ab^2$ and $- a^2b + 2ab^2 - b^3$ care must be exercised in distinguishing the terms containing a^2b from those containing ab^2. The distinction between these has already been given on p. 96.

In the earlier examples, as with decimals, the expressions may be written in separate lines, separate columns being arranged for terms of the same nature; but, unlike decimals, each column is complete in itself, there being no carrying to think of. Thus we shall have a column for a^3, another for a^2b, another for b^3, and so on. It should be remembered that the order of the terms in any expression may be altered at will, provided that the *correct signs be retained:* thus $a^3 - 2a^2b + ab^2$ is equivalent to $- 2a^2b + ab^2 + a^3$, etc.

Example 112.—Add together the expressions $a^3 - 2a^2b + ab^2$, and $- a^2b + 2ab^2 - b^3$.

$$\begin{array}{rl} & a^3 - 2a^2b + ab^2 \\ & \quad - a^2b + 2ab^2 - b^3 \\ \hline \text{Sum} = & a^3 - 3a^2b + 3ab^2 - b^3 \\ \hline & \text{(1)} \quad \text{(2)} \quad \text{(3)} \quad \text{(4)} \end{array}$$

Explanation.—Write the first expression down (the order given will usually be suitable), keeping the terms well spaced. Write the terms of the second expression beneath this, placing them beneath those of the same kind in the first line. Thus $- a^2b$ in the second expression is written beneath the second term in the top line $- 2a^2b$; also $+ 2ab^2$ is placed beneath $+ ab^2$. The term $- b^3$ must make a new column, there being no similar term in the top line. When all are arranged the addition may be proceeded with. It must be "algebraic," *i.e.*, regarding sign.

Column (1) a^3 is the only term: write a^3 in lower line.
(2) $- 2a^2b$ and $- 1a^2b$, added give $- 3a^2b$.
(3) $+ 1ab^2$ and $+ 2ab^2$, added give $+ 3a\,b^2$.
(4) $- b^3$ is the only term: write $- b^3$.
$\therefore$ Final result is $a^3 - 3a^2b + 3ab^2 - b^3$.

When several terms, including + and − signs, have to be added in a column, the addition may be performed continuously after a little practice, mentally retaining the sum from step to step without separating the + from the − quantities.

Example 113.—Add together the expressions $3p^2q + 5q^2 - p$, $3p - 2q^2$, $-2pq + 2p^2q^2 - p$, $-6p^2q + 2pq - 7q^2 + 3p$.

$$\begin{array}{rl} & 3p^2q + 5q^2 - p \\ & \quad - 2q^2 + 3p \\ & \qquad - p - 2pq + 2p^2q^2 \\ & - 6p^2q - 7q^2 + 3p + 2pq \\ \hline \text{Sum} = & - 3p^2q - 4q^2 + 4p \qquad + 2p^2q^2 \end{array}$$

Explanation.—The first expression is written down as given. With the other expressions, like terms are placed beneath each other as shown; and if there is no like term, a new column is made.

Column of p^2q : -6 and $+3 = -3$ $\therefore$ write $-3p^2q$ in result.
,, ,, q^2 : -7, -2 and $+5 = -4$ $\therefore$,, $-4q^2$,, ,,
,, ,, p : $+3 + 3 - 1 - 1 = +4$ $\therefore$,, $+4p$,, ,,
,, ,, pq : $+2$ and $-2 = 0$ $\therefore$,, nothing ,, ,,
,, ,, p^2q^2 : only $+2p^2q^2$ exists, $\therefore$ write $+2p^2q^2$ in result.

$\therefore$ Result of addition $= -3p^2q - 4q^2 + 4p + 2p^2q^2$ or any other arrangement of these terms *with their proper signs.*

Example 114.—From $4s - pr$ take $3s + 2pr - 3r$.

$$\begin{array}{rl} & 4s - pr \\ & 3s + 2pr - 3r \\ \hline \text{Difference} = & s - 3pr + 3r \end{array}$$

Explanation.—The expressions are arranged in columns of s, pr, and r.

Column of s : $4s - 3s = 1s$ written as s.
,, ,, pr : change sign of $2pr$ and add,
i.e., $-pr - 2pr = -3pr$.
,, ,, r : change sign of $-3r$ and add,
i.e., $0 + 3r = +3r$.
$\therefore$ Result of subtraction $= s - 3pr + 3r$.

At a later stage, when dealing with the removal of brackets, it will be necessary to perform addition without arranging the terms in columns. Then the various like terms are picked out by looking along the expression. The following example will illustrate:—

Example 115.—Simplify the expression—

$$3a^2 - 10 + 4a^2 + 10 - 2a^2 + 3$$

$$\overset{\checkmark}{3a^2} - \overset{\checkmark}{10} + \overset{\checkmark}{4a^2} + \overset{\checkmark}{10} - \overset{\checkmark}{2a^2} + \overset{\checkmark}{3}$$

$$= 5a^2 + 3.$$

Start from one end, say the left, and take all the terms with a^2 and add mentally. Thus, $3a^2$ and $4a^2 = 7a^2$, $7a^2$ and $-2a^2 = 5a^2$, which completes the terms in a^2. Then take all the terms of another kind, here numbers only, and repeat: -10 and $+10 = 0$. Then $+3$ alone remains.

When a term has been added in, it is advisable to tick it off before proceeding to another. This serves to show what terms have been dealt with, and indicates when all terms are included.

This method is sometimes known as **"collecting-up like terms."**

Example 116.—Collect up like terms in the expression $750 - 3t + 5t^2 - t + 24 - 3{\cdot}5t^2 + 4{\cdot}7t$.

$$\overset{\surd}{750} - \overset{\surd}{3t} + \overset{\surd}{5t^2} - \overset{\surd}{t} + \overset{\surd}{24} - \overset{\surd}{3{\cdot}5t^2} + \overset{\surd}{4{\cdot}7t} = 774 + {\cdot}7t + 1{\cdot}5t^2.$$

Taking the numbers first, $750 + 24 = 774$. Next taking terms in t: $-3t$ and $-t = -4t$, then $-4t$ and $+4{\cdot}7t = +{\cdot}7t$. Now taking terms in t^2 (quite distinct from those in t): $+5t^2$ and $-3{\cdot}5t^2 = +1{\cdot}5t^2$.

Exercises 33. On Addition and Subtraction of Several Terms.

(The first few examples are arranged in columns to accustom the student to the kind of example before setting out his own work.)

Find the sums of the following:—

1.
$$\begin{array}{r} 2p^2 - 3pq + q^2 \\ + pq - 2q^2 \\ -3p^2 \qquad - q^2 \\ \hline \\ \hline \end{array}$$

2.
$$\begin{array}{r} -ar^2 - ar - r^3 \\ 2ar^2 + 4ar \\ 3ar^2 - 5ar - 3r^3 \\ \hline \\ \hline \end{array}$$

3.
$$\begin{array}{r} -l + w^2l^2 \\ 3w + 3l - 2w^2l^2 \\ -4w \\ \hline \\ \hline \end{array}$$

4.
$$\begin{array}{r} 3m^3 - 2m^2 + m \\ -m^3 \qquad - 4m \\ 2m^3 + m^2 + 2m \\ \hline \\ \hline \end{array}$$

5. Add $a^3 + a^2b + ab^2$ to $-a^2b - ab^2 - b^3$.

6. Add $p^3 - p^2q + pq^2$ to $p^2q - pq^2 + q^3$.

7. Add together the following expressions:—

$x^6 + 3x^5y + y^2$; $-2x^3y^3 + y^2$; $-x^4 - 4x^5y - 3x^6$; $3y^2 - x^3y^3$.

8. Add together:—

$4ml - 17 + l^2$; $3l^2 + 4$; $-m^2 + l^2 - ml$; $-3m^2 - 5$.

Collect up like terms in the following—

9. $-3r^2 - 10 + 12 + 2r^2 - r^2$.

10. $11 - t^2 + 2t - 15 - 11t^2 - t$.

11. $\cdot75l + 3x - 1{\cdot}3l - x$.

12. $2{\cdot}75y^2 + 1{\cdot}45y - 3y^2 - 2y + 7$.

13. $n^2 - n - 5 + 17 - 3n + 4n + 6n^2$.

14. $\frac{1}{2}a + a - \frac{1}{4}a + d.$
15. $c^2 - \frac{1}{2}c - 1\frac{1}{2}c^2 + 1\frac{1}{4}c.$
16. $\frac{3}{8}x^2 - 10 + \frac{1}{8}x^2 + x.$
17. $\frac{3}{8}d - x + 3x^2 - 2x - \frac{1}{8}d.$
18. $\frac{1}{4}n + \frac{1}{8}t - \frac{4}{3}t + \frac{3}{8}n + t.$

Add together the following expressions:—

19. $bd^2 - 2b^2d - b + 3d$; $-5b^2d + 3b - 2d$; $2bd^2 - 7b - d$; $5b^2d + 2b$.

20. $7e - 3h + f$; $-e - 4h - 3f$; $+4e + h - f$; $-3e - f$; $+2e + 6h$.

Multiplication and Division of + and − Quantities.—This may be investigated by numbers and then applied to symbols.

1. *Multiplication of two Plus Quantities, e. g.,* $(+3) \times (+7)$.

This is the common case already known and used, and written as 3×7.

Now 3×7 means 3 sevens added together.

For use in other cases we will consider that these 3 sevens are added to 0, thus $0 + 7 + 7 + 7 = +21$

Hence $\underline{(+3) \times (+7) = +21}$

Similarly with letters $(+a) \times (+b)$ means a times $+b$, or $+b$ added a times to 0,

i. e., $0 + b + b + b + b + \ldots$ until the letter b is written a times $= ab$.

2. *Multiplication of a Positive and a Negative Quantity.*

e. g., $(+3) \times (-7)$

Now in a similar manner to 1, $(+3) \times (-7)$ means 3 times -7, or -7 added 3 times to 0, *i. e.,* $0 + (-7) + (-7) + (-7)$

$= -7 - 7 - 7$ (See case (ii), p. 123.)
$= -21$

Hence $\underline{(+3) \times (-7) = -21.}$

Similarly with letters, $(+a) \times (-b)$ means $-b$ added a times to 0, *i. e.,* $0 + (-b) + (-b) + (-b) + \ldots$ until the quantity $-b$ is added a times to 0,

$= 0 - b - b - b \ldots$ until the number of "b's" is a
$= -ab.$

3. *Multiplication of Two Negative Quantities.*

e. g., $(-3) \times (-7)$.

Now if $(+3) \times (-7)$ means -7 added 3 times to 0, then $(-3) \times (-7)$ must mean -7 *subtracted* 3 times from 0, since the sign of $+$ and $-$ have directly opposite meanings.

Thus $(-3) \times (-7) = 0 - (-7) - (-7) - (-7)$
$= +7 + 7 + 7$ (by case (iii), p. 130)
$= +21$

Hence $(-3) \times (-7) = +21$.

Similarly with symbols: $(-a) \times (-b)$ means $-b$ subtracted a times from 0.

That is, $0 - (-b) - (-b) - (-b)$ . . . . until the symbol $(-b)$ is written "a" times.

$= +b + b + b + b +$ . . . until the number of "b's" is a
$= ab$.

Collecting our results we have the following:—

From 1: $(+3) \times (+7) = +21$ or $(+a) \times (+b) = +ab$
„ 3: $(-3) \times (-7) = +21$ or $(-a) \times (-b) = +ab$
„ 2: $(+3) \times (-7) = -21$ or $(+a) \times (-b) = -ab$

whence it is seen that the *same* signs (*i. e.*, two positives or two negatives) give $+$, and that two different signs (*i. e.*, a $+$ and a $-$) give $-$; or, as usually given by the **"Rule of Signs"**:—

Like signs give plus.
Unlike signs give minus.

This rule is equally true for division, as may be seen from the following:—

(i) The division of a positive quantity by a positive quantity needs no explanation, being the ordinary arithmetical case. This is a question of "like" signs.

(ii) *Division with unlike signs.*

From case 2, p. 135, we have the statement $(+3) \times (-7) = -21$. Now if we divide -21 by $+3$, the result must be -7 to agree with our multiplication.

$$\therefore \frac{-21}{+3} = -7, \text{ or } \frac{\text{Minus}}{\text{Plus}} = \text{Minus.}$$

From case 3, p. 136, we have the statement $(-3) \times (-7) = +21$. Now dividing $+21$ by -3, the result must be -7 to agree with the multiplication.

$$\therefore \frac{+21}{-3} = -7, \text{ or } \frac{\text{Plus}}{\text{Minus}} = \text{Minus.}$$

Evidently then unlike signs, no matter in what order, give minus.

(iii) *Division with two minus signs, i. e.,* a case of " like " signs.

From case 2, p. 135, we have as before $(+3) \times (-7) = -21$. Now if we divide the -21 by the -7, then our result must be $+3$ to ensure agreement with the multiplication.

$$\therefore \frac{-21}{-7} = +3, \text{ or } \frac{\text{Minus}}{\text{Minus}} = \text{Plus.}$$

Now combining (i) and (iii), since the result is of the same sign, we have as for multiplication—

Like signs give plus.
Unlike signs give minus.

When more than two quantities, with varying signs, have to be multiplied together the product of them all is found in the usual way, after which a consideration of the signs will readily show the sign of the final product. Thus, in the case of $-6 \times 5 \times -3 \times 4$, the first sign is $-$ and the next $+$, giving $-$; then this $-$ multiplied by the $-$ belonging to the 3 gives a $+$; and finally this $+$ multiplied by the $+$ of the 4 gives $+$ as result; thus the final product is $+360$.

A similar effect to the Rule of Signs is obtained from the action of two suspended bar magnets. Thus, two unlike poles, *i. e.,* a North and a South, will always attract each other; but two North poles brought together will have the same effect as two South poles brought together, for they will repel each other. This action is usually summed up in the phrase: **Unlike poles attract and like poles repel.**

Exercises 34. On Multiplication and Division of + and − Quantities.

Find the value of the expression $a\mathrm{R}$ for the following cases:—

1. $a = 4$, $\mathrm{R} = 5$.
2. $a = -4$, $\mathrm{R} = -5$.
3. $a = 4$, $\mathrm{R} = -5$.
4. $a = -4$, $\mathrm{R} = 5$.
5. $a = -3{\cdot}5$, $\mathrm{R} = {\cdot}6$.
6. $a = 4{\cdot}9$, $\mathrm{R} = -1{\cdot}1$.
7. $a = -2$, $\mathrm{R} = 1{\cdot}563$.
8. $a = -1{\cdot}5$, $\mathrm{R} = -{\cdot}65$

Find the value of the expression alx for the following cases:—

9. $a = 2, l = -3, x = 4$. **10.** $a = -2, l = 3, x = 4$.
11. $a = -2, l = -3, x = 4$. **12.** $a = -2, l = -2, x = -4$.
13. $a = -1, l = 2, x = -5$. **14.** $a = 1{\cdot}5, l = -3, x = -1{\cdot}5$.
15. $a = -1{\cdot}5, l = -6, x = -1$.

Find the value of the expression $\frac{p}{q}$ for the following conditions:—

16. $p = 10, q = 2{\cdot}5$. **17.** $p = -10, q = 2{\cdot}5$.
18. $p = 10, q = -2{\cdot}5$. **19.** $p = -10, q = -2{\cdot}5$.
20. $p = -7{\cdot}5, q = 3$. **21.** $p = -7{\cdot}5, q = -4$.

Powers of Minus Quantities.—We may now consider the value of expressions such as $(-3)^2$, $(-r)^3$, *i. e.*, the powers of negative quantities. A power applied to a minus quantity has the same meaning as when applied to a plus quantity, thus $(-3)^2$ means $(-3) \times (-3)$. Its value is, then, by the preceding paragraph, $+9$.

$$\therefore \underline{(-3)^2 = +9.}$$

Also, $(-3)^3 = (-3) \times (-3) \times (-3)$
$= (+9) \times (-3)$
$= -27$ by preceding paragraph
$$\therefore \underline{(-3)^3 = -27.}$$

Again, $(-3)^4 = (-3)^{3+1} = (-3)^3 \times (-3)$
$= (-27) \times (-3)$
$= +81$
$$\therefore \underline{(-3)^4 = +81.}$$

Continuing in this way we should find that all the **even powers** (*i. e.* "squares," 4th powers, 6th powers, etc.) are **positive,** while all the **odd powers** (*i. e.*, "cubes," 5th powers, etc.) are **negative.**

This is equally true with symbols.

Thus $(-r)^2 = (-r) \times (-r) = +r^2$.

It must be remembered that $(-r)^2$ is *not* the same thing as $-r^2$. The first means the "square of $-r$," and the second, "$-$ the square of r."

Thus $(-r)^2$ means $(-r) \times (-r) = +r^2$, whilst $-r^2$ means $-(r \times r)$, or just the subtraction of r^2.

Example 117.—Find the value of x^2 for the following cases:—(*a*) $x = -6$; (*b*) $x = -3r$; (*c*) $x = -2r^2$.

(*a*) $x^2 = (-6)^2 = (-6) \times (-6) = \underline{+36}$
(*b*) $x^2 = (-3r)^2 = (-3r) \times (-3r) = \underline{+9r^2}$
(*c*) $x^2 = (-2r^2)^2 = (-2r^2) \times (-2r^2) = \underline{+4r^4}$

Exercises 35. Powers of − Quantities.

If $n = -2$ find the values of the following expressions:—

1. n^2. **2.** n^3. **3.** n^4. **4.** n^5. **5.** n^6.
6. $-n^2$ **7.** $-n^3$. **8.** $-n^4$. **9.** $-n^5$. **10.** $2n^3$.

Find the value of x^3 in the following cases:—

11. $x = 4$. **12.** $x = -4$. **13.** $x = 4r$. **14.** $x = -4r$.

Evaluation, including Positive and Negative Quantities. We will here take a few actual examples where positive and negative quantities are used.

Example 118.—The width of the expansion valve in a Meyer valve gear is given by the expression $a + S - R$ where a, S and R represent certain sizes of the gear. Find the width (inches) for the following cases:—

(1) $a = 1{\cdot}7$, $S = 3{\cdot}27$, $R = {\cdot}5$; (2) $a = 1{\cdot}5$, $S = 3$, $R = -{\cdot}75$.

(1) $a + S - R = 1{\cdot}7 + 3{\cdot}27 - {\cdot}5 = \underline{4{\cdot}47''}$

(2) $a + S - R = 1{\cdot}5 + 3 - (-{\cdot}75)$
$= 1{\cdot}5 + 3 + {\cdot}75 = \underline{5{\cdot}25''}$

Example 119.—The latent heat of Ammonia as used in refrigeration is $566 - {\cdot}8t$, where t = temperature F. Calculate the latent heat when the temperature is − 10° F.

$566 - {\cdot}8t = 566 - {\cdot}8 \times -10$
$= 566 - (-8) = 566 + 8 = \underline{574}$

Note.—$-{\cdot}8t$ is to be subtracted, and since t is − 10, then ·8 of t is ·8 of − 10 or − 8. Now subtracting − 8 is adding + 8, hence the result.

Example 120.—In connection with an apparatus for drawing large circular arcs we have the expression $\frac{r(r + b + R)}{R - r}$. Find the value of this expression when $r = 5$, $b = 4{\cdot}5$, and $R = -25$.

$$\frac{r(r + b + R)}{R - r} = \frac{5\{5 + 4{\cdot}5 + (-25)\}}{-25 - 5}$$
$$= \frac{5 \times \{-15{\cdot}5\}}{-30} = \frac{-77{\cdot}5}{-30} = \underline{2{\cdot}58}$$

Note.—After substituting, the expression $5 + 4{\cdot}5 + (-25) = 9{\cdot}5 + (-25)$, or, $9{\cdot}5 - 25 = -15{\cdot}5$, Finally the division with like signs gives a + result.

Exercises 36. On Evaluation with + and − Quantities.

1 to **4.** In heat engine work it is frequently necessary to know the "absolute temperature," which $= t + 461$, where t = Fahrenheit temperature. Find the absolute temperature when t has the following values:—

1. 250. **2.** − 10. **3.** − 25. **4.** 0.

5 to **8.** The absolute temperature, if t is Centigrade temperature, is $t + 273$. Find the absolute temperature when t has the following values:—

5. 10. **6.** − 5. **7.** 0. **8.** − 241.

9 and **10.** Find the value of the expression $a + S - R$ (which refers to a certain steam engine expansion valve) for the following cases:—

9. $a = 1{\cdot}25$, $S = 2{\cdot}75$, $R = 0$. **10.** $a = 1{\cdot}06$, $S = 3{\cdot}5$, $R = -1{\cdot}8$.

11 to **14.** If F. is the Fahrenheit temperature, the corresponding Centigrade temperature is $\frac{5}{9}$ (F − 32). Calculate the temperature °C when F has the following values:—

11. 15. **12.** 0. **13.** − 2. **14.** 32.

15 to **17.** If C is the Centigrade temperature, the corresponding Fahrenheit temperature is $\frac{9}{5}C + 32$. Calculate the temperature °F when C has the following values:—

15. − 15. **16.** 0. **17.** − 37·5.

18 to **22.** The expression $\frac{n-a}{m-a}$ is important in connection with epicyclic gearing. Calculate its value for the following cases:—

18. $n = 5$, $a = 3$, $m = 1$. **19.** $n = -4$, $a = 1$, $m = 0$.

20. $n = 4$, $a = -2$, $m = 0$. **21.** $n = -2{\cdot}5$, $a = 0$, $m = 1{\cdot}5$.

22. $n = -5{\cdot}6$, $a = -2{\cdot}8$, $m = 1$.

23 and **24.** The latent heat of sulphur dioxide, a refrigerating agent, is $175 - {\cdot}27t$ when t = Fahrenheit temperature. Calculate the latent heat when t has the following values:—

23. 0. **24.** − 20.

25 to **28.** The expression $A - \frac{B}{r^2}$ relates to thick cylinders under pressure. Find its value for the following cases:—

25. $A = -8{\cdot}48$, $B = 212$, $r = 4{\cdot}5$. **26.** $A = -8{\cdot}48$, $B = 212$, $r = 5$.

27. $A = 3{\cdot}6$, $B = -57{\cdot}5$, $r = 4{\cdot}5$. **28.** $A = 3{\cdot}6$, $B = -57{\cdot}5$, $r = 5{\cdot}5$.

29 and **30.** Find the value of the expression $\frac{r(r+b+R)}{R-r}$ for the following cases:—

29. $r = 6$, $b = 6$, $R = -55$. **30.** $r = 7{\cdot}5$, $b = 7$, $R = 20$.

31. The formula $v = u + at$ is important in questions on velocity. Find v when $u = 5$, $t = 3{\cdot}5$, and $a = -2$.

32 to **35.** The expression $\frac{v-u}{t}$ refers to the acceleration of a moving body. Find its value for the following cases:—

32. $v = 25$, $u = 12$, $t = 2{\cdot}5$. **33.** $v = 6$, $u = 22{\cdot}5$, $t = 3{\cdot}5$.

34. $v = -10$, $u = 5$, $t = 1{\cdot}5$. **35.** $v = -12$, $u = 15$, $t = 1{\cdot}5$.

36 to **39.** In solving "quadratic" equations an expression of the form $b^2 - 4ac$ has to be evaluated. Find the value of this expression in the following cases:—

36. $a = 3, b = 5{\cdot}5, c = -2{\cdot}5$. **37.** $a = -2, b = 1, c = 1{\cdot}6$.
38. $a = 1, b = -4{\cdot}5, c = -2$. **39.** $a = -1, b = 35, c = -1{\cdot}8$.

40 to **44.** An expression of the form $\frac{-b+R}{2a}$ also occurs when solving quadratics. Find its values for the following cases:—

40. $a = 1, b = 5, R = -1$. **41.** $a = -2, b = 1, R = 3{\cdot}7$.
42. $a = -2, b = 1, R = -3{\cdot}7$. **43.** $a = 15, b = -6, R = 13{\cdot}5$.
44. $a = 15, b = -6, R = -13{\cdot}5$.

The Removal of Brackets.—The meaning and use of a bracket have been introduced in Chap. I, p. 29. They are employed in a similar manner when dealing with symbols, and then require some further consideration. In arithmetic a bracket can always be removed, and an expression simplified, by completing the operations within the bracket, as in Example No. 27, etc. But with an algebraic expression very frequently the quantities within the bracket cannot be worked out, and yet it is required to simplify the expression by removing the brackets. The removal may easily be carried out if certain adjustments be made, depending on two things: (1) the sign immediately in front of the bracket, and (2) any quantity by which the bracketed expression is to be multiplied. To illustrate and prove the "adjustments" we will take expressions with numbers only, and work them out in two ways: (1) as in ordinary arithmetic, keeping strictly to the meaning of the bracket, and (2) by omitting the brackets and making "adjustments."

First, consider the sign in front of the bracket:

1.—*Plus sign in front, e.g.*, (i) $9 + (7 + 4)$.

Evaluating the expression within the bracket first, we have—

$$\begin{aligned} & 9 + (7 + 4) \\ = \ & 9 + \quad 11 \quad = 20 \end{aligned}$$

Now let us write the given expression and simply omit the brackets. Then we have—

$$\begin{aligned} & 9 + (7 + 4) \\ = \ & 9 + \ 7 + 4 \\ = \ & \ 16 \quad + 4 = 20 \end{aligned}$$

which agrees with the former.

Now take the expression (ii) $9 + (7 - 4)$.

Evaluating the expression within the bracket first, we have—

$$\begin{aligned} & 9 + (7 - 4) \\ = \ & 9 + \quad 3 \quad = 12. \end{aligned}$$

Now do as before, *i. e.*, write the given expression without the brackets. Then we have—

$$\begin{aligned} & 9 + 7 - 4 \\ = \quad & 16 \quad - 4 = 12 \end{aligned}$$

which also is in agreement with the first value.

Hence we see that when the sign immediately in front of the bracket is +, we may "remove the brackets" by simply omitting them.

2.—Minus sign in front, e. g., (i) $9 - (4 + 3)$.

Evaluating the expression within the bracket first, we have—

$$\begin{aligned} & 9 - (4 + 3) \\ = 9 - \quad & 7 \quad = 2 \end{aligned}$$

Now evidently the meaning of the given expression is that both the 4 and the 3 are to be taken away from the 9, so that if we wish to omit the brackets the expression should be written as—

$$\begin{aligned} & 9 - 4 - 3 \\ = \quad & 5 \quad - 3 = 2 \end{aligned}$$

which agrees with the former result.

$$\therefore 9 - (4 + 3) = 9 - 4 - 3$$

Note that in the given expression the sign of the 4 is + (not written, but understood), but in the altered expression is —. Also the sign of the 3 is + originally, and becomes — when the brackets are removed. Thus the sign of the quantities inside the brackets have been changed.

Now take the expression (ii) $9 - (4 - 3)$

Evaluating the portion within the brackets first, we have—

$$\begin{aligned} & 9 - (4 - 3) \\ = 9 - \quad & 1 \quad = 8 \end{aligned}$$

Now, following the method of the preceding paragraph, let us omit the brackets and change the signs of the quantities inside, *i. e.*, change the 4 into -4 and the -3 into $+3$. Then we have—

$$\begin{aligned} & 9 - (4 - 3) \\ = 9 - \ & 4 + 3 \\ = \quad & 5 \quad + 3 \ = 8 \end{aligned}$$

which again agrees with the former answer.

$$\therefore 9 - (4 - 3) = 9 - 4 + 3$$

Hence the rule : **To remove brackets from an expression whilst retaining its original value ; if the sign immediately in front of the bracket is +, simply remove the brackets : if it is −, change the signs of all quantities within the brackets when removing them.**

Representing these results with symbols—

$$\left.\begin{aligned} a + (b + c) &= a + b + c \\ a + (b - c) &= a + b - c \end{aligned}\right\} \text{ + sign in front, no change.}$$

$$\left.\begin{aligned} a - (b + c) &= a - b - c \\ a - (b - c) &= a - b + c \end{aligned}\right\} \text{ − sign in front, signs changed.}$$

Secondly, consider the quantity multiplying the bracketed expression, and let us take the case of $8 + 5(6 + 3)$. It should be noted that the algebraic method of indicating multiplication is adopted here. This is usually followed with brackets, the given expression of course meaning $8 + 5 \times (6 + 3)$.

Also it should be remembered that the multiplication of the quantities in the brackets by 5 must be done before the 8 is added (*see* p. 30). Working on the ordinary method of evaluating the quantities in brackets first, we have—

$$\begin{aligned} & 8 + 5(6 + 3) \\ = {} & 8 + 5 \times 9 \\ = {} & 8 + 45 \qquad = 53 \end{aligned}$$

Now the *sum* of the 6 and 3, *i. e.*, *both* the 6 and the 3, have to be multiplied by 5, so that multiplying by 5 *before* evaluating the bracket we have—

$$\begin{aligned} & 8 + 5(6 + 3) \\ = {} & 8 + 5 \times 6 + 5 \times 3 \quad \text{(this step may be done mentally)} \\ = {} & 8 + 30 + 15 \quad = 53 \text{ which agrees with the former result.} \end{aligned}$$

Similarly with the expression $8 + 5(6 - 3)$

Ordinary Method

$$\begin{aligned} & 8 + 5\,(6 - 3) \\ = {} & 8 + 5 \times 3 \\ = {} & 8 + 15 \quad = 23 \end{aligned}$$

Removing Brackets

$$\begin{aligned} & 8 + 5(6 - 3) \\ = {} & 8 + 5 \times 6 - 5 \times 3 \\ = {} & 8 + 30 - 15 \\ = {} & 38 - 15 = 23 \end{aligned}$$

Hence we see that when there is a multiplier with a plus sign immediately outside a bracketed expression the brackets may be removed if **all the quantities inside the bracket be multiplied.**

When a minus sign stands in front of the bracket and its multiplier, then both the multiplication and the change of sign must be performed when removing the brackets.

Thus—

$$\begin{aligned} & 15 - 4(3 - 2) \\ = {} & 15 - 4 \times 3 + 4 \times 2 \\ = {} & 15 - 12 + 8 \\ = {} & 3 + 8 = 11 \end{aligned}$$

Proof, working on ordinary method

$$\begin{aligned} & 15 - 4(3 - 2) \\ = {} & 15 - 4 \times 1 \\ = {} & 15 - 4 = 11 \end{aligned}$$

Combining all the foregoing rules we may represent them with symbols, thus—

$$\begin{aligned} a + b(c + d) &= a + bc + bd \\ a + b(c - d) &= a + bc - bd \\ a - b(c + d) &= a - bc - bd \\ a - b(c - d) &= a - bc + bd \end{aligned}$$

Example 121.—The " latent heat of steam " is found by experiment to be $966 - \cdot 7\,(t - 212)$. Reduce the expression to a simpler form.

$$\begin{aligned} & 966 - \cdot 7\,(t - 212) \\ = {} & 966 - \cdot 7t + \cdot 7 \times 212 \\ = {} & 966 - \cdot 7t + 148 \cdot 4 \\ = {} & \underline{1114 \cdot 4 - \cdot 7t} \end{aligned}$$

Note.—Remove brackets, changing sign inside, and multiply all terms inside by $\cdot 7$. " Collect up " the two numbers.

Example 122.—Simplify the expression $r + \frac{2}{3}(1 - r)$, which occurs in connection with the strength of materials.

$$\begin{aligned} & r + \tfrac{2}{3}(1 - r) \\ = {} & r + \tfrac{2}{3} - \tfrac{2}{3}r \\ = {} & \tfrac{1}{3}r + \tfrac{2}{3}, \text{ or } = \frac{r}{3} + \frac{2}{3} = \underline{\frac{r + 2}{3}} \end{aligned}$$

Example 123.—Simplify the expression $2(x + 3) - (x - 5) + 3(1 + 2x) - 7$.

$$2(x + 3) - (x - 5) + 3(1 + 2x) - 7$$
$$= 2x + 6 - x + 5 + 3 + 6x - 7 = \underline{7x + 7}.$$

When an expression contains brackets which enclose other brackets, *start by removing the inside brackets*, then remove the next pair, and so on until all have been removed. At each stage the quantities within the *innermost brackets* should be reduced as much as possible by collecting.

Example 124.—Simplify the expression $3\{2 + 2(r - 1) - 3(r - 6)\}$.

$$\begin{aligned} & 3\{2 + 2(r - 1) - 3(r - 6)\} \\ & \quad \checkmark \quad \checkmark \quad \checkmark \quad \checkmark \\ = {} & 3\{2 + 2r - 2 - 3r + 18\} \\ = {} & 3\{18 - r\} \quad = \underline{54 - 3r} \end{aligned}$$

Example 125.—The E.M.F. of the Clark Standard Cell is given as $1{\cdot}434\ [1 - {\cdot}0008(t - 15)]$ where t = temperature °C. Simplify the expression.

$$\begin{aligned} & 1{\cdot}434\ [1 - {\cdot}0008(t - 15)] \\ = & 1{\cdot}434\ [1 - {\cdot}0008t + {\cdot}012] \\ = & 1{\cdot}434\ [1{\cdot}012 - {\cdot}0008t] \\ = & \underline{1{\cdot}451 - {\cdot}001147t} \end{aligned}$$

Exercises 37.—On Removal of Brackets.

Remove the brackets from the following expressions and simplify as far as possible :—

1. $H - (H - M)$.
2. $(H + C) - (H - M)$.
3. $2x + (3 - x)$.
4. $4p - (2 - p)$.
5. $C + 4(C - 1)$.
6. $25 - 3(d + 1)$.
7. $2w^2 - w(1 - w)$.
8. $2(a - 1) + 3(1 - a)$.
9. $r(2 - r) - 2(r^2 - r)$.
10. $4(H + 3) - 2(2 - H) + H$.
11. $A + (m - 1)\,A$.
12. $-f - (p^3 + f) + 3p^3$.
13. $v_2 - (v_1 - v_2)$, which relates to the efficiency of a Pelton wheel.
14. $3(n - 1) - 4$, referring to balancing of reciprocating engines.
15. ${\cdot}133(1 - {\cdot}0021t)$, in connection with the effect of temperature upon fluid friction.
16. $14500(C + 4{\cdot}28H)$, giving the " heating value " of oil.
17. $99\left(1 - \frac{1}{d}\right)$, which refers to the mean pressure in a petrol engine.
18. $\left(P - \frac{W}{R}\right)R$, an expression in connection with a worm-geared pulley block.
19. $a + \frac{1}{2}(b - a)$, relating to the strength of a dam.
20. $\frac{R}{l_2}(l_1 - l_2)$, which appeared in a problem on measuring the internal resistance of a cell.
21. $W - W\left(1 - \frac{x}{l}\right)$, a formula appearing in a problem on continuous girders.
22. $x + \frac{1}{2}\left(\frac{h}{2} - x\right)$, which refers to the shear stress in beams.
23. $c^2 + \frac{1}{2}(1 - c^2)$, an expression occurring in a problem on the strength of materials.
24. $EC - C(E - e)$, in connection with an electric motor.
25. $1 + {\cdot}0002\ (t - 15)$, giving the E.M.F. of the Hibbert 1 volt cell at temperature $t°$ Centigrade.
26. $100\left(1 + \frac{v}{s}\right) + 461$, which gives the absolute initial temperature of the charge in a scavenging gas engine.
27. $167 - {\cdot}27(t - 32)$, relating to the latent heat of sulphur dioxide, a refrigerating agent.
28. $s + 966 + {\cdot}48(p - 32)$, an expression referring to the heat lost in the exhaust gases of an oil engine.

29. $1114 - \cdot 7t + (t - 32)$, which gives the total heat of 1 lb. of steam at temperature $t°$ Fahrenheit.

30. $\cdot 44x + (1 \cdot 2x - \cdot 5x^2)$, an expression which occurred in a problem on the strength of materials.

31. $1 \cdot 019\{1 + \cdot 00003(t - 20)\}$ volts, which is the E.M.F. of the Weston cadmium cell at temperature $t°$ Centigrade.

32. $\cdot 72\{1 - \cdot 0007(t - 60)\}$, giving the specific gravity of petrol at temperature $t°$ Fahrenheit.

33. $14500\{C + 4 \cdot 28(H + \frac{1}{8}O)\}$, a formula giving the heating value of coal when the chemical composition is known.

34. $\frac{1}{100}[(212 - t) + 966 + \cdot 48(T - 212)]$, which refers to the loss of heat due to moisture in coal.

Insertion of Brackets.—It is sometimes necessary to insert a bracket in a given expression for the purpose of further calculation. Thus, suppose we have the expression $CR + Cr$ and it is desired to use the symbol C in some particular way. For ease of later calculation the C, now appearing in two terms, should only appear once. The C is a factor of *each* term in the expression; it is therefore said to be " common to the two terms " or it is a **" common factor."** When we adjust the expression (without, of course, altering its value) so that this symbol C occurs only in one place, we are said to " take out " the common factor C from the two terms. When doing this, brackets must be inserted, and exactly similar rules must be obeyed as when removing brackets; the operation is, in fact, exactly opposite to that of removal.

The expression $CR + Cr$ would appear as $C(R + r)$ if we take out the common factor C. Removing the brackets from this expression according to the rules given on p. 143, we obtain the original expression.

Taking the formulae given on p. 144 and reversing the order of statement, we have

$$1.\ a + bc + bd = a + b(c + d)$$
$$2.\ a + bc - bd = a + b(c - d)$$
$$3.\ a - bc - bd = a - b(c + d)$$
$$4.\ a - bc + bd = a - b(c - d)$$

Thus, when inserting brackets, we must conform with the methods given on p. 143, for their removal. Hence we have the rule: **When brackets are put in after a + sign, the signs then coming within the bracket remain unchanged; when brackets are put in after a − sign, the signs then coming within the bracket are changed.**

An inspection of the four cases shown will make this clear. Thus, in 1 and 2, where the bracketed quantity follows the + sign in

front of bc, the sign between c and d is unchanged; but in 3 and 4 the bracket is inserted after the — sign in front of bc, and the sign between c and d is changed.

Taking out a Common Factor.—The terms to be put inside the bracket are obtained by dividing the terms of the given expression by the common factor. Where a term in the given expression does not contain the common factor under consideration, then, of course, this term is not included in the operation of inserting brackets, but is merely repeated.

Take case 2 in the previous section—

$$a + bc - bd = a + b(c - d)$$

The only factor which is common to two terms (or more) is b; hence b is the factor to be taken out. The first term a does not contain b, and therefore can only be repeated. Then the second term bc will supply the first term inside the bracket. Now bc divided by b, *i.e.*, $\frac{bc}{b}$, equals c, which is then put inside. Then the sign — is written (not changed as the sign in front is +) and the third term bd is considered. Similarly, $\frac{bd}{b} = d$, which is then placed after the — sign; the bracket is then closed. The actual division can usually be done mentally.

Example 126.—Take out the common factor in the expression $PV + PVk$.

$$PV + PVk = PV(1 + k)$$

Note.—The common factor in this case consists of a product of two symbols P and V; it is however treated in exactly the same manner as a single symbol.

Remember that $\frac{PV}{PV} = 1$

Unless a certain common factor is specified to be taken out, then the factor taken should be the largest possible. Thus, in Example 127, which follows, R is a common factor, and might perhaps be needed. But R^2 is also common, and being larger than R, is therefore taken.

Example 127.—Take out the common factor from the expression $fR^2 - pR^2$. (This expression occurs in connection with thick cylinders, such as guns and hydraulic pipes.)

$$fR^2 - pR^2$$
$$= R^2(f - p)$$

Example 128.—Take out the common factor from the expression $2vV - 2V^2$, relating to water turbines.

Here 2 appears in each term and also V. Hence 2V is the common factor.

$$2vV - 2V^2$$

$$= 2V(v - V)$$

$$\frac{\cancel{2}vV}{\cancel{2}V} = v$$

$$\frac{2V^2}{2V} = V$$

Example 129.—Take out the common factor from the expression $\frac{v^2A^2}{a^2 2g} - \frac{v^2}{2g}$ (This occurs in connection with the Venturi water meter.)

Note here that v^2 is common on the top line, and $2g$ on the bottom. Hence $\frac{v^2}{2g}$ is the common factor.

$$\frac{v^2A^2}{a^2 2g} - \frac{v^2}{2g}$$

$$= \frac{v^2}{2g}\left(\frac{A^2}{a^2} - 1\right)$$

Note that $\frac{v^2A^2}{a^2 2g} \div \frac{v^2}{2g}$

$$= \frac{v^2A^2}{a^2\cancel{2g}} \times \frac{\cancel{2g}}{v^2}$$

$$= \frac{A^2}{a^2}$$

Example 130.—Take out the common factor x in the expression $4l^3 - 5xl^2 + xy^2$, which refers to moving loads on continuous girders.

$$4l^3 - 5xl^2 + xy^2$$

$$= 4l^3 - x(5l^2 - y^2)$$

$$\frac{5xl^2}{x} = 5l^2$$

$$\frac{xy^2}{x} = y^2$$

In this case the signs are changed, as the brackets have been put in after a — sign. The term $4l^3$ does not contain the common factor, and is therefore merely repeated. When there are two common factors in the expression (*e. g.*, x and l^2 in the last example), the factor to be taken out will be mentioned in the exercise work here. In ordinary calculation where the taking out of factors is merely a step in the work, the nature of the calculation must decide the actual factor to be taken out.

Exercises 38. On Taking out a Common Factor.

Rewrite the expressions given in Nos. 1–11, in each case taking out the common factor.

1. $a - ea$, which occurs in a problem on epicyclic gearing.
2. $aR - ar$, referring to a machine for drawing large arcs.
3. $cA + mca$, a formula relating to reinforced concrete.
4. $M + M\frac{x}{y}$, in connection with the bending of beams.

5. $K - \frac{Kl}{L}$, relating to a coil spring.

6. $\frac{nd}{13} + 7d$, giving the outside diameter of a wire rope.

7. $2Ml_1 + 2Ml_2$, a formula referring to continuous girders.

8. $a^2 + 2ab$, which relates to a machine for drawing large arcs.

9. $8hr - 4h^2$, referring to the area of a geometrical figure.

10. $w^2r - \frac{w^2r^2}{l}$, a formula in connection with the inertia of engine parts.

11. $x - \frac{x^2}{L}$, which occurred in a problem on beams.

In Nos. 12–21 the given expressions are to be rewritten, taking out the common factor mentioned in each case.

12. $\frac{pl_1^3}{4} + \frac{pl_2^3}{4}$, relating to continuous girders. $\left(\text{Common factor } \frac{p}{4}.\right)$

13. $5l^2 - \frac{4l^3}{x}$, another continuous girder expression. (Common factor l^2.)

14. $\frac{bd^3}{12} + bdx^2$, a formula in connection with beams. (Common factor bd.)

15. $\frac{nPV - npv}{n - 1}$, giving the work done in air compression. (Common factor $\frac{n}{n-1}$.)

16. $Wy - \frac{5Wxy}{4l} + \frac{Wxy^3}{4l^3}$, an expression occurring in connection with rolling loads on continuous girders. (Common factor Wy.)

17. $a^2b + ab^2$. (Common factor ab.)

18. $3p^3r - 2p^2r$. (Common factor p^2r.)

19. $\frac{hA}{A - a} + \frac{h\sqrt{aA}}{A - a}$, an expression relating to the volume of a certain solid. $\left(\text{Common factor } \frac{h}{A - a}.\right)$

20. $\frac{l^2x^2}{2} - \frac{lx^3}{3} + \frac{x^4}{12}$, referring to the bending of a cantilever. (Common factor lx^2.)

21. $(x + a)l + (x + a)\frac{l^2}{2}$. (Common factor $(x + a)$.)

22. Take out the common factor $\frac{x^3}{3}$ in the expression given in No. 20.

23. In the expression $x^2 + ax + cx + ac$, take out the common factor in the first two terms, and also that in the last two terms. Then take out the common factor in the resulting expression.

24. Simplify the expression $\frac{ny^2}{by - y}$ which refers to bridge girders. (*Hint.*—Take out common factor in bottom line. Then cancel.)

25. The expression $\frac{2}{7}HB^3 - \frac{6}{25}HB^3 + \frac{8}{75}HB^3$ relates to the bending of beams. Take out the common factor HB^3 and simplify.

26. Simplify the expression $bh + \frac{h}{2}(a - b)$, which refers to the area of a certain geometrical figure. (*Note.*—First remove the brackets; then take out the common factor h and reduce.)

Multiplication with Expressions of Two Terms.—It is sometimes necessary to multiply together two quantities, each of which is represented by an expression containing 2 terms, *e. g.*, $a - b$ multiplied by $a + c$. The operation would be indicated thus: $(a - b)(a + c)$. It must be noted that *it is incorrect to write the statement as* $a - b \times a + c$; in this case only the a and b in the middle of the expression are to be multiplied. But when we say "multiply the expression $a - b$ by the expression $a + c$," it is intended that the *whole* of the expression $a - b$ is to be multiplied by the *whole* of the expression $a + c$. Hence brackets must be used to denote that each expression is taken as a whole. These symbolic expressions with several terms and brackets are exactly similar to the compound expressions containing vulgar fractions, as given on p. 33, etc.

The method is to take one expression, say the $a - b$, and multiply it, first by one term, say a of the other expression; then by the other term c and add the results together; just as in multiplying 33×24 we multiply the 33 first by 4 giving 132, and then by 20 giving 660, and add the two results together, giving 792.

The usual method of laying out the work is as follows:—

$$\begin{array}{l} a - b \\ a + c \\ \hline a^2 - ab \\ \qquad + ac - bc \\ \hline a^2 - ab + ac - bc = \text{Product.} \end{array}$$

Explanation.—Multiply the whole of the top line (*i. e.*, the expression $a - b$), by the symbol a in the expression $a + c$: a times $a = a^2$, which is written beneath the line under $a + c$. Now $+ a$ multiplied by $- b = - ab$ which is written after the a^2. This completes the multiplication by the a. Now multiply the whole

of the top line by the c: $(+c) \times (+a) = ac$. This must be written in another column, as there is no existing column for ac. Then $(+c) \times (-b) = -bc$. Another column has to be made for this, as shown. This completes the multiplication, and now the products are added up. In this particular case there is no actual addition to do, as each column contains only one term. Then the result is $a^2 - ab + ac - bc$; the order of the terms does not matter so long as the signs are correct.

Example 131.—Simplify the expression $(b - x\mathrm{R})(a - 2x) + ax\mathrm{R}$, which occurred in an investigation on retaining walls.

$$(b - x\mathrm{R})(a - 2x) + ax\mathrm{R}$$
$$= \checkmark ab - \checkmark ax\mathrm{R} - \checkmark 2xb + \checkmark 2x^2\mathrm{R} + \checkmark ax\mathrm{R}$$
$$= ab - 2xb + 2x^2\mathrm{R}$$

$$\begin{array}{r} b - x\mathrm{R} \\ a - 2x \\ \hline ab - ax\mathrm{R} \\ \qquad\qquad - 2xb - 2x^2\mathrm{R} \\ \hline ab - ax\mathrm{R} - 2xb - 2x^2\mathrm{R} \end{array}$$

The multiplication is exactly similar to the previous one. When "collecting up the like terms" in the second line, note that $-ax\mathrm{R}$ and $+ax\mathrm{R} = 0$; tick off the terms when collected so that all are included.

Complicated examples in this kind of work are not very common. The most important application is given in the following section:—

Evaluating such Expressions as $(a \pm b)^2$.*

1. $(a + b)^2$, *i. e.*, "the sum of two quantities, squared," or, "the square of the sum of two quantities."

2. $(a - b)^2$, *i. e.*, "the difference of two quantities, squared," or, "the square of the difference of two quantities."

3. $(a + b)(a - b)$, *i. e.*, "the sum of two quantities, multiplied by the difference of the *same* two quantities," or, "the product of the sum and difference of two quantities."

Such expressions often occur, and the reader should be able to write them down at once when required. This operation is frequently called "expanding the expression."

It must be noticed that $(a + b)^2$ is *not* equal to $a^2 + b^2$, and that $(a - b)^2$ is *not* equal to $a^2 - b^2$, as might be imagined at first sight.

* The statement $(a \pm b)^2$ is a short way of saying "$(a + b)^2$ and $(a - b)^2$."

1. $(a + b)^2 = (a + b)(a + b)$.

$$\begin{array}{l} a + b \\ a + b \\ \hline a^2 + ab \\ \quad + ab + b^2 \\ \hline \therefore (a + b)^2 = a^2 + 2ab + b^2 \end{array}$$

Multiply by $+ a$
$a \times a = a^2$; $b \times a = ab$
Multiply by $+ b$
$a \times b = + ab$; written beneath the previous one
$b \times b = b^2$

Let us call "a" the first term and "b" the second term. Then it is seen that our result contains—

$+ a^2$ which is the square of the first term.
$+ b^2$ „ „ „ „ second term.
$+ 2ab$ „ „ twice the product of the two terms.

Then the result is best summed up in the following sentence:—

The square of the sum of two quantities = the square of the first term + the square of the second term + twice the product of the two terms.

Thus, in writing out the value of $(p + x)^2$,

$+$ Square of first term $=$ Square of $p = + p^2$
$+$ „ „ second term $=$ „ „ $x = + x^2$
$+$ Twice product of the two terms $= + 2px$
$\therefore (p + x)^2 = p^2 + 2px + x^2$

2. Difference of two quantities, squared.

$$(a - b)^2 = (a - b)(a - b)$$

$$\begin{array}{l} a - b \\ a - b \\ \hline a^2 - ab \\ \quad - ab + b^2 \\ \hline a^2 - 2ab + b^2 \end{array}$$

Multiply by $+ a$
$a \times a = a^2$; $- b \times + a = - ab$
Multiply by $- b$
$a \times - b = - ab$; written beneath the previous one
$- b \times - b = + b^2$

Note that in this case we have—

$+ a^2$ which is the square of the first term.
$+ b^2$ „ „ „ „ second term.
$- 2ab$ which is minus twice the product of the two terms.

Therefore—

The square of the difference of two quantities = the square of the first term + the square of the second term − twice the product of the two terms.

Thus, in writing out the value of $(n - y)^2$

$+$ Square of first term $=$ Square of $n = n^2$
$+$,, ,, second term $=$,, ,, $y = y^2$
$-$ Twice product of the two terms $= -2ny$
$\therefore (n - y)^2 = n^2 - 2ny + y^2$

3. Sum of two quantities multiplied by the difference of the same two quantities.

$(a + b)(a - b)$

$$\begin{array}{r} a + b \\ a - b \\ \hline a^2 + ab \\ \quad - ab - b^2 \\ \hline a^2 \qquad - b^2 \end{array}$$

Multiply as before
It will be seen that the sum of $+ ab$ and $- ab = 0$, since $+ 1$ neutralizes $- 1$

Note that here there are only two terms, thus—

$+ a^2$ which is the square of the first quantity
$- b^2$ which is minus the square of the second quantity

Therefore—

The sum of two quantities multiplied by the difference of the same two quantities = the square of the first term − the square of the second term.

Hence in writing out the value of $(N + n)(N - n)$

$+$ Square of first term $=$ Square of $N = N^2$
$-$,, ,, second term $=$ $-$ Square of $n = -n^2$
$\therefore (N + n)(N - n) = N^2 - n^2$

It should be noted that the order of the terms in the two brackets should preferably be the same, *e. g.*, $(N + n)(N - n)$ and not $(n + N)(N - n)$. The latter is equally correct, but might lead to confusion as to which is the "first term." Actually the *first term* is the one which comes *first in the subtraction.*

The advantage of remembering the rules in the above manner is that *any* two-term expression may be dealt with. Should one of the terms be a number, then some reduction may be made after having written out the result according to the above rules.

Thus in evaluating the expression $(x - 3)^2$, an example of case 2,

$+$ Square of first term $= x^2$
$+$ Square of second term $= 3^2 = 9$
$-$ Twice product of the two terms $= -2 \times 3 \times x = -6x$
$\therefore (x - 3)^2 = x^2 - 6x + 9$

With a little practice the result may be written down almost directly, the squaring being done mentally.

When the terms contain more than a single symbol it is advisable to work in two steps. First write out the "squares of terms," etc., without any reduction. Then take each term of the result and simplify where possible.

Example 132.—Expand $(3m + ny)^2$.

Result = Square of first term + Twice product + Square of second term

$$\therefore (3m + ny)^2 = (3m)^2 + 2 \times 3m \times ny + (ny)^2$$

Now reduce terms where possible.

Thus, $(3m)^2 = 3^2m^2 = 9m^2$
$2 \times 3m \times ny = 6mny$
$(ny)^2 = n^2y^2$
Hence result $= \underline{9m^2 + 6mny + n^2y^2}$

Working directly without the intermediate step, there is a greater possibility of error. With practice, of course, it is quite possible to obtain the answer directly.

Example 133.—Find the value of $\left(f - \frac{P}{2}\right)^2$

$$\left(f - \frac{P}{2}\right)^2 = f^2 - 2 \times f \times \frac{P}{2} + \left(\frac{P}{2}\right)^2$$
$$= \underline{f^2 - fP + \frac{P^2}{4}}$$

Example 134.—Simplify the expressions—

$$(a)\ (n - 1)^2;\ (b)\ \left(\frac{q}{3} + 2r\right)\left(\frac{q}{3} - 2r\right)$$

$$(a)\ (n - 1)^2 = n^2 - 2 \times 1 \times n + 1^2$$
$$= \underline{n^2 - 2n + 1}$$

$$(b)\ \left(\frac{q}{3} + 2r\right)\left(\frac{q}{3} - 2r\right) = \left(\frac{q}{3}\right)^2 - (2r)^2$$
$$= \underline{\frac{q^2}{9} - 4r^2}$$

Example 135.—Simplify the expression—

$v^2 - V^2 - (v - V)^2$, which relates to water turbines.

After expanding $(v - V)^2$, the result must be enclosed in a bracket to indicate that it is *all* to be subtracted.

$$\begin{aligned} & v^2 - V^2 - (v - V)^2 \\ &= v^2 - V^2 - (v^2 - 2vV + V^2), \text{ expanding the square} \\ &= v^2 - V^2 - v^2 + 2vV - V^2, \text{ removing the bracket} \\ &= 2vV - 2V^2, \text{ collecting like terms.} \end{aligned}$$

The expression $2vV - 2V^2$ may also be stated in the form $2V(v - V)$.

The case of $(a + b)(a - b) = a^2 - b^2$ is often of use when reversed, or stated in words—

The difference of the squares of two quantities may be expressed as the product of the sum and difference of the two quantities.

Or, $M^2 - N^2 = \text{sum} \times \text{difference} = (M + N)(M - N)$

This form is useful, as it may enable further calculation to be done in a simpler manner, or it may make cancelling possible. For example, in certain calculations connected with fly-wheels, expressions like $N^2 - n^2$ occur when N and n have values which are nearly equal. Then the form $(N + n)(N - n)$ makes the evaluation of the expression an easy matter. The following example will illustrate.

Example 136.—Find the value of $N^2 - n^2$, which relates to the energy in a fly-wheel, when $N = 250$ and $n = 247$.

$$\begin{aligned} N^2 - n^2 &= (N + n)(N - n) \\ &= (250 + 247)(250 - 247) \\ &= 497 \times 3 \\ &= 1491 \end{aligned}$$

Proof.

$250^2 = 62500$

$247^2 = 61009$

1491

Note.—If we had taken $250^2 - 247^2$ the working would be less easy, as squaring these numbers is comparatively a long process.

As an example of its use to effect cancelling and further simplification of a problem, the following example may be considered:—

Example 137.—The expression $\frac{2V(v - V)}{v^2 - V^2}$ occurs in connection with the efficiency of hydraulic turbines. Reduce to a simpler form.

$$\begin{aligned} \frac{2V(v - V)}{v^2 - V^2} &= \frac{2V(v - V)}{(v + V)(v - V)} \\ &= \frac{2V}{v + V} \quad \text{cancelling } (v - V) \text{ in top and bottom} \end{aligned}$$

Note.—By converting the $v^2 - V^2$ into the form $(v + V)(v - V)$, the term $(v - V)$ may be cancelled.

Exercises 39. On Expansion of Squares of Two Terms, etc.

Multiply :—

1. $x - 3$ by $2x + 1$.
2. $x^2 - 4$ by $x^2 + 7$.
3. $l + x$ by $2l - x$.
4. $2a + r$ by $a - 2r$.
5. $2s + 3p$ by $s - p$.
6. $b^3 - 3b^2$ by $b + 2$.
7. $a^2 - ab + b^2$ by $a + b$.
8. $a^2 + ab + b^2$ by $a - b$.

Expand the following expressions :—

9. $(2x + y)^2$.
10. $(x - 2y)^2$.
11. $(3n - 2p)^2$.
12. $(p - 4n)^2$.
13. $(l + px)^2$.
14. $\left(\frac{p}{2} + y\right)^2$.
15. $\left(x - \frac{l}{2}\right)^2$.
16. $\left(\frac{a}{2} - \frac{b}{3}\right)^2$.
17. $(1{\cdot}5x - 2)^2$.
18. $(3m + 1{\cdot}2p)^2$.
19. $\left(\frac{x}{3} + l\right)^2$.
20. $(2n - {\cdot}5)^2$.

Simplify the following expressions :—

21. $(2 - l)(2 + l)$.
22. $(n + 3)(n - 3)$.
23. $\left(\frac{l}{2} - x\right)\left(\frac{l}{2} + x\right)$
24. $\left(r + \frac{x}{2}\right)\left(r - \frac{x}{2}\right)$.
25. $\left(xy - \frac{y}{x}\right)\left(xy + \frac{y}{x}\right)$.
26. $\left(\frac{l}{2} - \frac{1}{x}\right)\left(\frac{l}{2} + \frac{1}{x}\right)$

27. Simplify the expression $(l + z)(l^2 - 3z^2) - (l^2 - z^2)z$, which occurs in a problem on moving loads over suspension bridges.

By using factors find the value of $N^2 - n^2$ in the following cases:—

28. N = 100, n = 98.
29. N = 450, n = 445.
30. N = 130, n = 127.
31. N = 144, n = 141.

32. In connection with a differential hydraulic accumulator it was necessary to evaluate $D^2 - d^2$ when D = 14 and d = 12. Find the required value by using factors.

33. With reference to the lengths of the sides of right-angled triangles we have the statement $b^2 = a^2 - c^2$. Find by the "expansion method" the value of b^2 when $a = 9{\cdot}625$ and $c = 3{\cdot}125$. (Note the shortness of this method compared with squaring.)

34. Simplify the expression $PV + \frac{PV - pv}{n - 1} - pv$ which relates to the power required for air compression. (*Hint.*—Reduce all terms to the common denominator $n - 1$, remove brackets from the resulting numerator and simplify, then take out common factor.)

35. For the area of a certain geometrical figure which has certain dimensions, a, c, A and h, there is an approximate formula $\frac{2}{3}ch$; there is also another approximate formula $\frac{2}{3}Ah$. By adding $\frac{7}{10}$ of the first expression to $\frac{3}{10}$ of the second expression, a third formula is obtained which is found to be very accurate. Obtain this formula and in it substitute for A the expression $\frac{8a - c}{3}$. Simplify to give a formula for the area in terms of the dimensions a, c and h.

CHAPTER IV

SIMPLE EQUATIONS

Equations Generally.—Speaking generally an equation is a short statement that two things are equal, the sign = being used, *e. g.*, $2'\text{-}6'' = 30''$. But the word equation is usually applied to a statement of equality containing various symbols, from which something has to be calculated.

The statement on p. 86, V = CR is, of course, an equation, but it is also called " a formula for V in terms of C and R," because the value of V can be obtained directly by simple arithmetic, after substituting values for C and R. Now supposing that we know the values of C and V, and desire to find the corresponding value of R. Let V = 110 volts and C = 250 amperes; then substituting these values in the above formula we can write—

$$110 = 250R$$

Now this statement is called an **equation in one unknown quantity,** since it contains only one symbol whose value is not yet known. From it the value of R can be calculated, but other operations than those of simple arithmetic will have to be performed. The object of this chapter is to show how to proceed in various cases. The quantities contained in any equation are divided into two classes: **known quantities,** *i. e.*, those whose numerical values are known, as the V and C above; and **unknown quantities,** *i. e.*, those whose values have to be calculated, as R above. The process of finding the value of the unknown quantity is known as **" solving the equation,"** and the final result is called the **solution.** When describing the operations to be performed we shall speak of the " left-hand side " of the equation, meaning all written to the left of the equals sign, and the " right-hand side," meaning all on the right of the sign.

The all-important thing that must be kept in view when dealing with equations of any type is that *the two sides must always be equal.* To suit our own convenience the sides may be considerably changed

in appearance, but the *values* must remain the same. As soon as the equality is disturbed the statement is no longer true.

An equation can be looked upon as a balance or pair of scales, when just in equilibrium (see Fig. 20). The actual things in the scale pan may be very different, but the *weight* of each side must be the same. The weight in the pair of scales corresponds to the value in our equation, and the two scale pans correspond to the two "sides" of the equation. We can operate upon the weights in the scale pans in various ways without destroying the balance, but whatever is done to one side must be done to the other. Thus if we add 2 lbs. to one side we must add 2 lbs. to the other; if 3 lbs. be taken from one side, 3 lbs. must also be taken from the other side. Also if the weight on one side is multiplied or divided by 4 exactly, the same must be done to the other side. *Exactly the same idea applies to equations.*

Simple Equations.—Equations are divided according to the *highest power* of the unknown quantity which they contain. When only the *first power* is present then the equation is a *Simple Equation*, *e. g.*, $966 = 1114 - \cdot 7t$, where t is the unknown quantity, and is only of the first power. When the second power, or square, is the highest power of the unknown present then the equation is said to be a **Quadratic Equation,** *e. g.*, $x^2 + 5x + 18 = 12$ where x is the unknown and x^2, *i. e.* the second power, is the highest power present. Only simple equations will be taken in this book, but in all equations the principles of the simple equation apply.

The solving of an equation requires manipulation of the quantities involved, and therefore a knowledge of the preceding chapters. The most common operations which have to be performed are those of multiplication, division, addition, and subtraction. These will be taken singly and in combination.

Equations requiring Division.—As a simple example consider the equation

$$3w = 168$$

To give this a real meaning let w represent the weight in pounds of a small bag of cement. Then the equation can be represented as in Fig. 20, where on one scale pan we have 3 bags of cement (all the same size), and 168 lbs. on the other pan just balancing. The 168 lbs. has been made up of three 56-lb. weights.

Then evidently if three sacks weigh three 56-lb. weights, one sack will weigh one 56-lb. weight.

$$\text{That is, } w = 56 = \tfrac{1}{3} \text{ of } 168$$

Writing this in mathematical form—

$$3w = 168$$

$$\therefore \quad w = \frac{1}{3} \text{ of } 168 = \frac{168}{3} = 56 \text{ lbs.}$$

Notice that on the left-hand side, to obtain the w above we have divided the $3w$ by 3. Notice also that the right-hand side has *also* been divided by 3.

Thus in detail: $\frac{3w}{3} = \frac{168}{3}$ *i. e.*, $w = 56$ lbs.

Hence we see that if in the given equation we find the unknown multiplied by a coefficient (as the 3 above), then we can isolate the

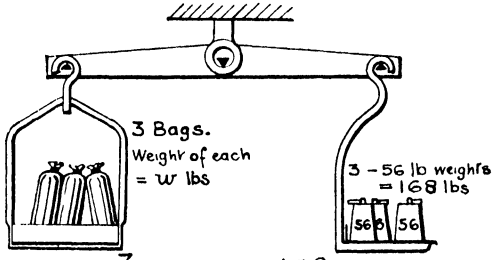


Fig. 20.

unknown and still keep the equality, by dividing *each side* by this coefficient. This is usually stated as " dividing across by 3."

Example 138.—If h = height of water in feet in a vertical pipe, then p the pressure in pounds per square inch at the bottom is given by the equation $2{\cdot}3\,p = h$. Find the pressure when the height is 25 ft.

$$2{\cdot}3p = h$$

Substituting for h $\quad 2{\cdot}3p = 25$

$\div 2{\cdot}3$ $\quad \frac{2{\cdot}3p}{2{\cdot}3} = \frac{25}{2{\cdot}3}$

$$\therefore \quad p = \underline{10{\cdot}9 \text{ lbs. per sq. in.}}$$

Having obtained a solution, a check on the correctness of the working is always available by substituting in the given equation the calculated value for the unknown, and then simplifying each side. If correct, the two sides of the equation will be found equal;

otherwise some wrong operation has been performed. Beginners should certainly check all their equations. For the earlier examples of each type in this chapter the check will be shown.

To prove the truth of the result in Ex. 138, we substitute the value 10·9 for p in the equation.

Thus—	$2{\cdot}3p = 25$
Substituting—	$2{\cdot}3 \times 10{\cdot}9 = 25$
Simplifying—	$25{\cdot}07 = 25$

which is quite satisfactory, as the slight difference is due to the value 10·9 being calculated to only 3 significant figures.

With a simple equation as above, the operation performed (dividing by 2·3) is easily seen. With more difficult types the operations may involve many steps. It is therefore advisable to state at the left-hand side what operation has been performed on each line as shown above. Then when returning to the equation for checking or reference, it is easy to pick out any operation. The steps can be very briefly stated : thus " $\div$ 2·3 " in the previous Example, No. 138, means " dividing both sides by 2·3 " and similarly " $\times$ 25 " would mean " multiplying both sides by 25." The reader can also use abbreviations such as " simpli." for " simplifying." Adopted right from the commencement of equations, the scheme becomes a habit and is of distinct advantage in more difficult work. The sign $\therefore$ (therefore) is placed before the last step in the working to emphasize the conclusion arrived at.

The following example shows that the method is just the same if the coefficient is a decimal.

Example 139.—Find the value of x from the equation $\cdot 2x = 6$.

	$\cdot 2x = 6$	*Proof.*
$\div \cdot 2$	$\frac{\cdot 2x}{\cdot 2} = \frac{6}{\cdot 2}$	Substituting 30 for x in given equation—
	$\therefore x = \underline{30}$	$\cdot 2 \times 30 = 6$
		$\therefore \quad 6 \quad = 6$
		$\therefore$ Result is correct.

With practice the number of steps may be reduced. Thus, in the above example, the step $\frac{\cdot 2x}{\cdot 2} = \frac{6}{\cdot 2}$ may be omitted, the left-hand side being understood. The working would then appear thus :—

$$\cdot 2x = 6$$
$$\div \cdot 2 \qquad x = \frac{6}{\cdot 2} = 30.$$

Exercises 40. On Equations requiring Division.

Solve the following equations:—

1. $5a = 16{\cdot}5$.	**2.** $12d = 134$.	**3.** $\cdot 75\mathrm{D} = 16$.
4. $3{\cdot}7x = 3{\cdot}5$.	**5.** $\cdot 105s = 11{\cdot}05$.	**6.** $\cdot 004\mathrm{M} = 17$.
7. $251x = 1{\cdot}35$.	**8.** $44c = 11{\cdot}5$.	**9.** $\mathrm{R} \times 1{\cdot}34 = 37$.

10. If $la = \mathrm{V}$ calculate the value of a when $l = \cdot 667$ and $\mathrm{V} = 15$.

11. The formula $\mathrm{P}v = p\mathrm{V}$ refers to the expansion of gases. Find the value of v when $\mathrm{P} = 165$, $p = 20$, and $\mathrm{V} = 13{\cdot}4$.

12. The speed of a belt is given by the formula $\mathrm{V} = \pi\mathrm{DN}$ where V = speed in ft. per min., D = dia. of pulley in ft., and N = revs. per min., and $\pi = 3{\cdot}14$. A planing machine belt is to run at 1500 ft. per min., over a pulley running at 240 revs. per min. What must be the diameter of the pulley?

13. The formula $pr = ft\eta$* relates to steam boilers. Calculate the plate thickness t (inches) for a large marine boiler when $r = 114$, p is to be 210, $f = 18000$, and η is to be $\cdot 9$. Give the actual result and also the thickness to the next $\frac{1}{16}$".

14. In a hydraulic press $\mathrm{P} = \cdot 785d^2p$ where P = total load in pounds, d = dia. of ram in inches, and p = pressure of water in pounds per square inch. Calculate the pressure p when the ram is 12" dia. and is to exert a total pressure of 150 tons (*i.e.*, $\mathrm{P} = 150 \times 2240$ lbs.).

15. Find the value of h from the formula $v^2 = 2gh$, which relates to hydraulics; when $v = 30$ and $g = 32$.

16. Using the formula in Ex. 13, calculate p the pressure allowable (pounds per square inch) in a compressed air receiver where $r = 9$", $t = \cdot 25$, $\eta = \cdot 66$, and $f = 10{,}000$.

17. For a direct-acting steam pump we have the formula $\cdot 75\mathrm{D}^2\mathrm{P} = d^2p$. A certain pump has $\mathrm{D} = 8$, $\mathrm{P} = 60$, $d = 3{\cdot}5$. Calculate p using these values.

18. If $\mathrm{V} = 2{\cdot}47dt^2$ calculate the value of d when $\mathrm{V} = 257$ and $t = 1{\cdot}56$.

Equations requiring Multiplication.—Sometimes the coefficient multiplying the unknown appears as a vulgar fraction, *e.g.*, $\frac{1}{5}\mathrm{P}$, or in the form $\frac{\mathrm{P}}{5}$. To isolate the unknown here we must *multiply* by the denominator, since $\frac{\mathrm{P}}{5} \times 5 = \mathrm{P}$, the 5's cancelling; we have therefore to "multiply across by 5."

Example 140.—Solve the equation $\frac{\mathrm{P}}{5} = 8{\cdot}3$.

$$\frac{\mathrm{P}}{5} = 8{\cdot}3$$

$$\times 5 \quad \frac{5\mathrm{P}}{5} = 5 \times 8{\cdot}3$$

$$\therefore \quad \mathrm{P} = \underline{41{\cdot}5}$$

Proof.

Substitute for P in equation

$$\therefore \quad \frac{41{\cdot}5}{5} = 8{\cdot}3$$

$$\therefore \quad 8{\cdot}3 = 8{\cdot}3$$

Result is correct.

* η is the Greek small letter "eta."

Example 141.—Solve the equation $\frac{m}{30{\cdot}5} = 13$.

$$\frac{m}{30{\cdot}5} = 13$$

$$\times\ 30{\cdot}5 \qquad \frac{30{\cdot}5m}{30{\cdot}5} = 13 \times 30{\cdot}5$$

$$\therefore \quad m = \underline{396{\cdot}5}$$

After the first few examples it is possible to dispense with part of the second step, and to set down as in the next example.

Example 142.—Find the value of l from the equation $\frac{l}{{\cdot}43} = 2{\cdot}31$.

$$\frac{l}{{\cdot}43} = 2{\cdot}31$$

$$\times\ {\cdot}43 \qquad l = 2{\cdot}31 \times {\cdot}43 = \underline{{\cdot}993}.$$

The above two sections may be summed up into one general rule :—

Either side of an equation may be multiplied, or divided, by any number provided that the other side is also multiplied, or divided, by the same number.

Thus if $A = B$
then $xA = xB$
and $\frac{A}{x} = \frac{B}{x}$ } where x is any number

Exercises 41. On Equations requiring Multiplication.

Solve the following equations :—

1. $\frac{f}{5} = 6.$
2. $\frac{s}{1{\cdot}15} = 4{\cdot}33$
3. $\frac{D}{1{\cdot}1} = 12.$
4. $\frac{x}{{\cdot}5} = 5.$
5. $\frac{E}{{\cdot}955} = 115.$
6. $\frac{H}{2{\cdot}3} = 45.$
7. $\frac{r}{2 \times 3{\cdot}5} = 3\ 15.$
8. $\frac{a}{3{\cdot}14 \times 6{\cdot}25} = 1.$
9. $\frac{m}{3{\cdot}85} = {\cdot}015.$

10. For an electric battery supplying current we have the formula $c = \frac{e}{r}$ where c = current, e = volts lost in cell, and r = resistance of cell. If $c = {\cdot}56$, and $r = 2{\cdot}5$, find e.

11. The formula $\frac{l}{k} = c$ refers to long columns. Calculate the value of l when $c = 160$ and $k = 1{\cdot}075$.

12. If $\frac{V}{\pi D} = N$, a formula relating to belts, calculate the value of V when $\pi = 3{\cdot}14$, $D = 1{\cdot}25$, and $N = 165$.

13. The formula $\frac{R_1}{R_2} = \frac{R_3}{R_4}$ is employed when measuring electric resistance by a Wheatstone bridge. If $R_3 = 10$, $R_4 = 1000$, and $R_2 = 247$, calculate the value of R_1.

14. The formula $c = \frac{v}{r - 1}$ relates to the clearance volume in a gas engine. If $c = 25$, $r = 6{\cdot}5$, calculate v.

15. The formula $\frac{P}{p} = \frac{v}{V}$ applies to the expansion of gases. Calculate the value of v when $P = 115$, $p = 20$ and $V = {\cdot}25$.

16. In a transformer if E_1 = primary voltage, E_2 = secondary voltage, S_1 = turns in primary coil, and S_2 = turns in secondary coil, then $\frac{E_1}{E_2} = \frac{S_1}{S_2}$. Find S_1 if $E_1 = 6000$, $E_2 = 500$, and $S_2 = 12$.

Combination of Multiplication and Division.—Both operations are required in equations like the following. They may be combined or performed in one step, but first will be shown separated for clearness.

Example 143.—Solve the equation $\frac{22}{7}d = 13$, giving the diameter of a circle whose circumference is 13".

$$\frac{22}{7}d = 13$$

$\div 22$ $\qquad \frac{1}{7}d = \frac{13}{22}$

$\times 7$ $\qquad d = \frac{13}{22} \times 7 = \underline{4{\cdot}14}.$

The first step is to divide across by 22 since this is a multiplying coefficient. Secondly, multiply by 7, since this is a dividing number.

Combining the two operations we multiply by 7 and divide by 22, *i. e.*, multiply by $\frac{7}{22}$.

Then— $\frac{22}{7}d = 13$

$\times \frac{7}{22}$ $\qquad d = 13 \times \frac{7}{22}$

$= \underline{4{\cdot}14}$

Proof.

$\frac{22}{7} \times 4{\cdot}14 = 13$

$\therefore \quad 13{\cdot}01 = 13$

$\therefore$ Result is correct.

Example 144.—When a gas expands under the application of heat, the formula $\frac{V}{v} = \frac{T}{t}$ is of importance. Find the value of t if $V = 2{\cdot}5$, $v = 7{\cdot}8$, and $T = 550$

$$\frac{V}{v} = \frac{T}{t}$$

Substituting— $\quad \frac{2{\cdot}5}{7{\cdot}8} = \frac{550}{t} \quad . \ . \ . \ . \ . \ . \ 1$

$\times\ t \quad \frac{2{\cdot}5t}{7{\cdot}8} = 550 \quad . \ . \ . \ . \ . \ 2$

$\times\ 7{\cdot}8 \quad 2{\cdot}5t = 550 \times 7{\cdot}8 \quad . \ . \ . \ 3$

$$\therefore \quad t = \frac{550 \times 7{\cdot}8}{2{\cdot}5} = \underline{1716}$$

The double multiplication performed in the above example in passing from line 1 to line 3 can be usefully combined into a step called **Cross Multiplication.** Notice that in line 3 we have on the left-hand side $2{\cdot}5 \times t$; *i.e.*, the top of the left-hand fraction (in line 1) $\times$ the bottom of the right-hand fraction. Similarly on the right-hand side we have the top of the right-hand fraction $\times$ bottom of left-hand fraction. Thus two multiplications have been performed, as indicated here, $\frac{2{\cdot}5}{7{\cdot}8} \times \frac{550}{t}$, and the two products equated. By means of this we are enabled to at once straighten out any equation in which the whole of each side is separated into numerator and denominator by a division line; the unknown can then be readily found. Exercises on proportion are best treated when worked on the above lines, since A : B : : C : D can be stated as $\frac{A}{B} = \frac{C}{D}$.

Example 145.—The horse-power of a steam engine is given by the formula H.P. $= \frac{pLaN}{33000}$ where H.P. = horse-power, p = mean effective pressure in lbs. per sq. ins., L = stroke in ft., a = area of piston in sq. ins., and N = number of strokes per min. Find the stroke L when H.P. = 50, $a = 143$, $p = 27$ and N = 300.

Substituting the given values in the formula, the following equation is obtained (reversed for ease of reading).

$$\frac{27 \times L \times 143 \times 300}{33000} = 50$$

$\times \frac{330}{27 \times 143 \times 3}$, $\quad L = \frac{50 \times 330}{27 \times 143 \times 3}$

$= 1{\cdot}42'$

say $\underline{17''}$

Approximation.

$$\frac{5 \times 3}{3 \times 1 \times 3} \qquad \frac{111}{111}$$

$= \frac{5}{3}$ = about 2'.

In equations of this form it is best to perform all the algebraic operations before doing any actual simplifying. Then one expression is obtained for the result, and is easily dealt with. Where a slide rule is used this is a decided advantage.

Thus in the above example it is best to work in the form shown rather than to simplify some of the figures on the left-hand side first.

Exercises 42. Multiplication and Division combined.

1. For a direct current generator $\frac{EC}{1000} = K$ where E = volts, C = current in amperes, and K = kilowatts. A generator to develop 150 kilowatts is to work at 220 volts. What current will it give?

2. The formula $r = \frac{2\pi R}{p}$ refers to a screw-jack. If $p = \cdot 5$, $\pi = 3 \cdot 14$, and $r = 250$, find the value of R.

3. The formula $K = \frac{kA}{4\pi t}$ gives the capacity of a parallel plate condenser. Calculate the value of A when $K = 18$, $k = 3 \cdot 5$, $\pi = 3 \cdot 14$ and $t = \cdot 15$.

4. A rough rule for petrol motors is $H = \frac{d^2 s}{12}$ where H = horse-power, d = dia. of cylinder in inches, and s = stroke in inches. A motor with a cylinder 3·25″ dia. is to develop 3·5 H.P. Calculate its stroke.

5. The formula $d^2 l = \frac{cK}{N}$ refers to dynamos. Calculate the value of c for a small multi-polar machine where $d = 18$, $l = 26$, $K = 37 \cdot 5$, and $N = 475$.

6. In connection with the strength of beams $Z = \frac{bh^2}{6}$. Calculate the value of b if $Z = 24$ and $h = 6$.

7. The formula $h = f\frac{Lv^2}{d}$ refers to pipes conveying water. In a certain test $L = 12$, $v = 6 \cdot 5$, $d = 4$ and $h = \cdot 85$. Calculate the value of f.

8. For hydraulic pipes $t = \frac{Hd}{12500}$ where t = thickness of pipe in inches, d = dia. in inches, and H = head of water in feet. A pipe is 5″ dia. and $\frac{1}{2}$″ thick. Calculate the value of H.

9. The pull of a lifting magnet is $P = \frac{B^2 A}{11,180,000}$ lbs. Calculate the value of A if $P = 3490$ and $B = 16000$.

10. The formula $H = \frac{1 \cdot 26CT}{l}$ refers to the magnetic field produced by a coil of wire. In a certain coil $l = 20$, $T = 250$ and $H = 75$. Calculate the value of C.

11. The formula $S = \frac{PD}{.2t}$ relates to the strength of boilers when metric units are used. Calculate the value of P when S = 7·5, D = 1·35, and t = 12.

12. If H = horse-power obtained from a waterfall, Q = quantity of water in cubic feet per second, and h = height of fall, then $H = \frac{GQh}{550}$ where G = 62·4. Calculate the value of Q when H = 40 and h = 10·5.

13. The formula $f = \frac{2}{3}\frac{R^2}{t^2}p$ relates to flat circular plates loaded uniformly. If R = 18, t = ·625, and f = 9000, calculate the value of p.

14. Find the value of N from the formula $\frac{2\pi N}{60} = \omega$ when π = 3·14 and ω = 35·6.

15. The formula $\theta = \frac{2fl}{Cd}$ relates to the twisting of shafts. Calculate the value of l, if θ = ·35, f = 5·5, C = 5400, and d = ·375.

(In the following few examples the unknown is in the denominator. They are best solved by *cross-multiplication*.)

16. The formula $R = \frac{sl}{a}$ relates to the resistance of an electric conductor. In a pair of feeders supplying current to a number of lamps l = 21600, R = ·286, and s = ·00000066. Calculate the value of a.

17. The formula $\frac{2\pi R}{p} = r$ relates to a screwjack. Find p if r = 200, R = 20, and π = 3·14.

18. The formula $\frac{P_1V_1}{\tau_1} = \frac{P_2V_2}{\tau_2}$ relates to the mixture in a gas engine. If P_1 = 15, V_1 = 1·3, τ_1 = 480, P_2 = 102, and V_2 = 25, calculate τ_2.

19. The capacity of a plate condenser is given by the formula $K = \frac{kA}{4\pi t}$. Calculate the value of t if k = 3·6, K = 150, A = 650, and π = 3·14.

20. The formula $d = \frac{wL^2}{8T}$ refers to overhead electric conductors. Calculate the value of T when d = 12, L = 1320 and w = ·5.

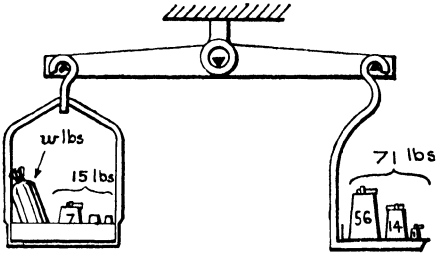
Equations requiring Subtraction.—As an example consider the equation $w + 15 = 71$. Let w represent the weight of a bag of cement, as before; then this equation is represented by Fig. 21. On the left-hand scale pan we have the bag of cement (weight unknown) together with a weight of 15 lbs. All this is balanced by 71 lbs. on the right-hand pan. To obtain the bag of cement by itself on the left, we must remove the 15 lbs. from the left-hand pan, and *to preserve the balance we must also remove 15 lbs. from the right hand.*

Then, $w + 15 = 71$

$-15 \quad w + 15 - 15 = 71 - 15$

$\therefore \quad w = 56$ lbs., *i. e.*, the sack weighs 56 lbs.

Hence when there is a known quantity added to the unknown, the latter is isolated by subtracting the known quantity from each side of the equation.



$w + 15 = 71$

$\therefore \quad w = 71 - 15$

$= 56$

Fig. 21.

Example 146.—Find the value of r from the formula $R + r = s$ when $R = 199$ and $s = 256$.

Substituting given values, $199 + r = 256$

$-199 \quad 199 + r - 199 = 256 - 199$

$\therefore \quad r = \underline{57}$

Proof.

$199 + 57 = 256$

$\therefore$ Result is correct.

Example 147.—If $w + w_1 = w_2$, find the value of w_1 when $w = 21{\cdot}7$ and $w_2 = 22{\cdot}63$.

Substituting, $21{\cdot}7 + w_1 = 22{\cdot}63$

$-21{\cdot}7 \quad 21{\cdot}7 + w_1 - 21{\cdot}7 = 22{\cdot}63 - 21{\cdot}7$

$\therefore \quad w_1 = \underline{{\cdot}93}$

Proof.—$21{\cdot}7 + {\cdot}93 = 22{\cdot}63$, $\therefore$ Result is correct.

In practice the second step may be shortened thus—

$21{\cdot}7 + w_1 = 22{\cdot}63$

$-21{\cdot}7 \quad w_1 = 22{\cdot}63 - 21{\cdot}7$

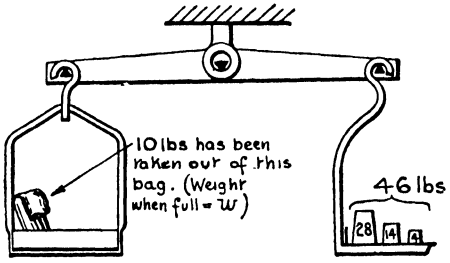
$= {\cdot}93$

Equations requiring Addition.—Consider the equation $w - 10 = 46$. If w again represents the weight of our bag of cement, this equation is seen in Fig. 22. On the left-hand scale pan is a bag of cement (weight w when full) from which 10 lbs. of cement

have been taken out. This partly filled bag is found to weigh 46 lbs. Now let us supply 10 lbs. to the left-hand pan to make up the weight of a full sack; then 10 lbs. must be added to the right-hand pan to keep the balance. So that—

$$w - 10 = 46$$
$$+ 10 \qquad w - 10 + 10 = 46 + 10$$
$$\therefore \quad w = 56 \text{ lbs.} = \text{weight of sack.}$$

Hence when there is a known quantity subtracted from the unknown, the latter may be isolated by adding this known quantity to each side.



$$W - 10 = 46$$
$$\therefore \quad W = 46 + 10$$
$$= 56$$

Fig. 22.

Example 148.—Solve the equation $T - 35 = 81$.

	$T - 35 = 81$	*Proof.*
$+ 35$	$T - 35 + 35 = 81 + 35$	$116 - 35 = 81$
$\therefore$	$T = 116$	$\therefore$ Result is correct.

Example 149.—Find the value of T from the equation $T - 273 = 131$.

	$T - 273 = 131$	*Proof.*
$+ 273$	$T - 273 + 273 = 131 + 273$	$404 - 273 = 131$
$\therefore$	$T = 404$	$\therefore$ Result is correct.

As before, the work may be shortened by combining steps 2 and 3.

Thus, $T - 273 = 131$
$+ 273 \quad T = 131 + 273 = 404.$

The operations in the two previous sections may be summed up in the following rule :—

Any number may be added to or subtracted from either side of an equation provided that the same number is also added to or subtracted from the other side.

Thus if $A = B$
then $A + x = B + x$
and $A - x = B - x$
} where x is any number

The operations may be conveniently dealt with in a more mechanical manner. Consider the shorter form of working in the four previous examples—

In Ex. **146,** the 199 was + when on the left, and — when on the right
„ **147,** „ 21·7 „ + „ „ — „ „
„ **148,** „ 35 „ — „ „ + „ „
„ **149,** „ 273 „ — „ „ + „ „

It is noticeable that in each case the sign becomes changed. Hence : **Any term (*i. e.*, set of symbols or numbers between a pair of + and − signs) in an equation may be written on the other side if its sign be changed.** This is called **"transposing."** Thus in Ex. 148, the 35 is *transposed, i. e.*, it is written on the other side with an altered sign.

Exercises 43. On Equations requiring Addition and Subtraction.

Solve the equations—

1. $x + 5 = 12$. **2.** $p + 14{\cdot}7 = 96$. **3.** $h + {\cdot}015 = 37$.
4. $275 + L = 1236$. **5.** $f + 1200 = 6250$.
6. $11{\cdot}5 + D = 23{\cdot}85$. **7.** $r + 1 = 7{\cdot}7$.
8. $n + {\cdot}333 = 1{\cdot}667$. **9.** $\cdot004 + C = \cdot00415$.

10. The formula $H = S + xL$ refers to wet steam. Calculate the value of S when $H = 1115$, $x = {\cdot}895$, and $L = 889$.

11. The formula $W = H + Cr$ refers to the energy given out by a cell. If $W = 2{\cdot}7$, $r = {\cdot}028$ and $C = {\cdot}5$, calculate the value of H.

12. The formula $D = Cs + k$ is used when measuring distances with a telescope. Find the value of k when $D = 166{\cdot}2$, $C = 100$, and $s = 1{\cdot}65$.

Solve the equations—

13. $m - 5 = 1{\cdot}1$. **14.** $x - 21 = -27{\cdot}85$.
15. $H - 34 = 2{\cdot}5$. **16.** $r - {\cdot}15 = {\cdot}16$.
17. $y - {\cdot}006 = -{\cdot}001$. **18.** $D - {\cdot}05 = 1{\cdot}13$.
19. $P - 14{\cdot}7 = 21{\cdot}3$. **20.** $L - {\cdot}105 = 125{\cdot}605$.
21. $l - {\cdot}55 = -{\cdot}01$.

22. The formula $u = v - at$ is used in mechanics. Find the value of v when $u = 25$, $a = 3{\cdot}8$, and $t = 7$.

23. From the formula $L + {\cdot}7t = 1114$, find L, the latent heat of steam if $t = 250$.

Equations combining the Four Rules.—The equation usually met with requires two or more of the foregoing operations.

Example 150.—Find the value of a from the equation $v = u - at$, when $v = 6{\cdot}5$, $u = 33$, and $t = 3{\cdot}5$.

Substituting values— $6{\cdot}5 = 33 - 3{\cdot}5a$

Transposing— $3{\cdot}5a = 33 - 6{\cdot}5$

$\div\ 3{\cdot}5$ $\qquad a = \dfrac{26{\cdot}5}{3{\cdot}5} = \underline{7{\cdot}57}$

Proof.—Left-hand side $= 33 - 3{\cdot}5 \times 7{\cdot}57$

$= 33 - 26{\cdot}5 = 6{\cdot}5$

$\therefore$ Result is correct.

Note.—Before the value of a can be obtained the $3{\cdot}5\,a$ must be isolated, which is done by transposing. This also gives a plus sign to the term containing the a.

Example 151.—Solve the equation $27{\cdot}6 + {\cdot}45a = 13$

$27{\cdot}6 + {\cdot}45a = 13$

$-\ 27{\cdot}6$ $\qquad {\cdot}45a = 13 - 27{\cdot}6 = -\ 14{\cdot}6$

$\div\ {\cdot}45$ $\qquad a = \dfrac{-\ 14{\cdot}6}{{\cdot}45} = -\ \underline{32{\cdot}4}$

Note.—After transposing, the right-hand side, when evaluated, becomes minus. Then, dividing across, we have a minus divided by a plus quantity. Remembering the points in Chap. III, the result is minus.

Example 152.—If r is the internal resistance of a battery, n the number of batteries connected in parallel, and R the external resistance, then the total resistance R_1 is given by the formula—

$$R_1 = R + \frac{r}{n}$$

Find the value of r if $R_1 = 1{\cdot}07$, $R = {\cdot}45$ and $n = 4$

Substituting the given values

$1{\cdot}07 = {\cdot}45 + \dfrac{r}{4}$

$-\ {\cdot}45$ $\qquad 1{\cdot}07 - {\cdot}45 = \dfrac{r}{4}$

$\times\ 4$ $\qquad {\cdot}62 \times 4 = r = \underline{2{\cdot}48}$

Note.—The term $\dfrac{r}{4}$ is first isolated by subtracting the term ·45. Then $\dfrac{r}{4}$ is converted into r by multiplying across by 4.

Example 153.—If F is a temperature on the Fahrenheit (English) Thermometer, and C the corresponding temperature on the Centigrade (French) Thermometer, then $F = \dfrac{9}{5}C + 32$. Find the Centigrade reading when $F = 212$.

Substituting— $212 = \frac{9}{5}C + 32$

-32 $212 - 32 = \frac{9}{5}C$

$\times \frac{5}{9}$ $180 \times \frac{5}{9} = C$

$\therefore$ $C = \underline{100}$

Example 154.—If p is the pitch (in inches) of a riveted joint, d the rivet diameter, and η the efficiency of the joint, then $\frac{p-d}{p} = \eta$. Find the value of p when $d = \cdot75$ and $\eta = \cdot7$.

Substituting given values, $\frac{p - \cdot75}{p} = \cdot7$

$\times p$ $p - \cdot75 = \cdot7 \times p.$

Transposing— $p - \cdot7p = \cdot75$

Simplifying— $\cdot3p = \cdot75$

$\div \cdot3$ $p = \frac{\cdot75}{\cdot3} = \underline{2\cdot5 \text{ inches}}$

Note.—Since p is the only denominator we multiply through by p, the fact that there is also a p in the numerator making no difference.

Exercises 44. On Equations combining the Four Rules.

1. If F is the Fahrenheit temperature, and C the corresponding Centigrade temperature, then $F = \frac{9}{5}C + 32$. Calculate the temperature °C when F = 86°.

2. If t is the thickness of a boiler flue and L the maximum unsupported length, then according to the Board of Trade rule:— $L = 120t - 12$. Find the value of t when L = 3'-6". (*Note.*—Work in inches.)

3. The formula $H = S + xL$ refers to saturated steam. Calculate x, the "dryness fraction" when H = 1113, S = 338, and L = 876.

4. The formula $B = H + 4\pi I$ refers to magnetism. Calculate the value of I when B = 18300, H = 22·5 and $\pi = 3\cdot14$.

5. From the formula $v = u + at$, important in mechanics, calculate a, if $v = 47$, $u = 5$, and $t = 28$.

6. The formula $R + \frac{r}{n} = S$ relates to electric batteries in parallel. Calculate the value of r when R = 15, $n = 10$, S = 17·5.

7. If $25 - \frac{l}{2} = 14\cdot6$, calculate the value of l.

8. In connection with wire ropes we have the formula $D = \frac{nd}{13} + 7d$. If $D = 1\frac{1}{2}$ and $d = \cdot064$, find the value of n.

9. When turning soft cast iron the formula $v = 115 - 1300a$ may be used, where v = cutting speed in ft. per min. and a = area of cut in sq. ins. If v is to be 70 ft. per min., what will be the value of a?

10. The expression $p = A + \frac{B}{r^2}$ relates to thick cylinders. Calculate the value of B when $p = 750$, $A = -700$, and $r = 6{\cdot}5$.

11. The formula $c^2 = 8hr - 4h^2$ refers to a circular arc. Calculate the value of r when $c = 15$ and $h = 4{\cdot}7$.

12. The tapping size of a Whitworth bolt is given approximately by the formula $d = {\cdot}9D - {\cdot}05$, where d is tapping size and D = outside diameter. A certain bolt must be $1{\cdot}16''$ at the bottom of the thread (*i. e.*, $d = 1{\cdot}16$). Calculate the outside diameter of the bolt to the next largest sixteenth.

13. The formula $v^2 = u^2 + 2as$ refers to moving bodies changing their speed. Calculate the value of a when $v = 3500$, $u = 3750$ and $s = 30$.

14. The latent heat of ammonia is $L = 566 - {\cdot}8t$, where t = Fahrenheit temperature. Calculate the temperature at which $L = 502$.

15. Using the formula in Ex. 14, above, calculate t when $L = 595$.

16. The formula $L = 176 - {\cdot}27t$ gives the latent heat of sulphur dioxide, where t = temperature °F. Calculate the value of t when $L = 160$.

17. The equation $nr - mR = 0$ gives the condition for the maximum current from a certain arrangement of cells. Calculate R when $r = 2{\cdot}5$, $n = 5$, and $m = 7$.

18. Using the formula in Ex. 1, calculate the temperature C when $F = 25$.

19. The formula $ad = \frac{b}{e} - c$ refers to the relative cost of gas and electricity as lighting agents, where a = cost of electricity in pence per B.O.T. unit, and b = cost of gas in pence, per 1000 cu. ft. If $a = 4$, $d = 3\frac{1}{2}$, $c = 1\frac{1}{2}$, and $e = 2$, find b.

20. The formula $t = {\cdot}0001PD + {\cdot}1$ refers to steam boilers. Calculate the value of P when $t = {\cdot}625$ and $D = 42$.

21. The formula $s = 1 + \frac{n}{150}$ is given in connection with helical springs, to allow for reversing loads. Calculate the value of n when $s = 3{\cdot}7$.

22. The formula $\frac{v - u}{t} = a$, relates to bodies in motion. If $v = 15{\cdot}5$, $t = {\cdot}75$ and $a = 76$, calculate the value of u.

23. The formula $\frac{T - t}{T} = n$ relates to the efficiency of heat engines. If n is to be ·33 and t is 520, what must be the value of T?

24. Calculate the value of E from the formula $C = \frac{E - V}{r}$, which refers to a dynamo supplying current, when $V = 108$, $C = 215$, and $r = {\cdot}028$.

25. Using the formula in Ex. 6, above, find n when $S = 31{\cdot}3$, $R = 30$, and $r = 6{\cdot}5$.

26. The formula $\frac{HN^2}{2938} = \frac{W + w}{w}$ refers to a "Porter" governor. Calculate w when $H = {\cdot}33$, $N = 270$, and $W = 54$. (*Hint.*—First evaluate the left-hand side.)

Equations with Several Terms.—In some calculations equations are obtained with several terms on each side, with the unknown quantity appearing in more than one term. In these cases all the terms containing the unknown are collected to one side, and all those containing the known to the other side. On simplifying, the resulting equation is quite simple.

Example 155.—Solve the equation $3x = 255 - 4x$

	$3x = 255 - 4x$
Transposing,	$3x + 4x = 255$
Collecting,	$7x = 255$
$\div 7$	$x = \frac{255}{7} = \underline{36{\cdot}4}$

Explanation.—In this case the unknown x occurs on two sides of the equation, and hence either the $3x$ or the $-4x$ must be transposed to the opposite side. By transposing the latter, the sign of the unknown is kept positive. After transposing, the " like terms " $3x$ and $4x$ are " collected " as in Chap. III, p. 134, reducing the equation to a simple form.

Proof—

$$3x = 255 - 4x$$
$$\therefore 3 \times 36{\cdot}4 = 255 - 4 \times 36{\cdot}4$$
$$\therefore 109{\cdot}2 = 255 - 145{\cdot}6$$
$$\therefore 109{\cdot}2 = 109{\cdot}4, \text{ and result is satisfactory.}$$

In this and other cases the two sides do not prove exactly equal. This is due to limiting our calculations to three significant figures but the agreement is sufficiently close to prove our method correct.

Example 156.—In an experiment to find the specific heat of copper the following equation was obtained : $43000s = 3960 + 1510s$. Find the value of s.

	$43000s = 3960 + 1510s$
Transposing,	$43000s - 1510s = 3960$
Collecting,	$41490s = 3960$
$\div 41490$	$s = \frac{3960}{41490} = \underline{{\cdot}0955}$

Example 157.—Solve the equation $5 + R = 15 - 3R$.

	$5 + R = 15 - 3R$
Transposing,	$R + 3R = 15 - 5$
Collecting,	$4R = 10$
$\div 4$	$R = \frac{10}{4} = \underline{2{\cdot}5}$

Note.—Two changes are necessary here to collect the two terms with the R on the left and the numbers upon the right.

Example 158.—The equation $300 + 883x = 1148 + 1{\cdot}4$ occurred in a boiler test, "x" representing the dryness of the steam. Calculate the value of x.

$$300 + 883x = 1148 + 1{\cdot}4$$

Transposing— $$883x = 1148 + 1{\cdot}4 - 300$$

$\div 883$ $$x = \frac{849{\cdot}4}{883} = \underline{{\cdot}962}$$

Example 159.—Find the value of n from the formula $e(m - a) = n - a$, when $e = 1{\cdot}33$, $m = 0$, and $a = 1$.

Substituting the given values—

$$1{\cdot}33(0 - 1) = n - 1$$

Simplifying— $$1{\cdot}33 \times -1 = n - 1$$

Simplifying— $$-1{\cdot}33 = n - 1$$

Transposing— $$1 - 1{\cdot}33 = n$$

$$\therefore \quad n = \underline{-{\cdot}33}$$

Note.—After substituting, the bracket $(0 - 1)$ is reduced to -1. The rest is simple if the multiplication of unlike signs be remembered.

Equations containing Brackets.—These may be solved with the help of the laws in Chap. III, p. 143, for removing brackets from expressions. After any necessary removal or insertion of brackets, the equation can usually be dealt with by some of the foregoing methods. Any bracket which contains *figures only* may be removed by simplifying these figures; removal by algebraic means is then unnecessary.

Example 160.—Solve the equation $65 - 4s = 7(3 + s)$.

$$65 - 4s = 7(3 + s)$$

Removing brackets— $$65 - 4s = 21 + 7s$$

Transposing— $$65 - 21 = 7s + 4s$$

Simplifying— $$44 = 11s$$

$\div 11$ $$s = \frac{44}{11} = \underline{4}$$

Explanation.—First remove brackets on right-hand side, by multiplying every term inside the bracket by the 7. Then transpose the known values to the left, and the unknowns to the right, thus keeping the sign of the latter plus. Then proceed as usual.

Example 161.—The following formula occurs in heat experiments: $W(T - t) = w(t - t_1)$. Find the value of t when $w = 71{\cdot}5$, $T = 95$, $W = 8{\cdot}3$, and $t_1 = 15$.

	$W(T - t) = w(t - t_1)$
Substituting given values—	$8{\cdot}3(95 - t) = 71{\cdot}5(t - 15)$
Removing brackets—	$788{\cdot}5 - 8{\cdot}3t = 71{\cdot}5t - 1072{\cdot}5$
Transposing—	$788{\cdot}5 + 1072{\cdot}5 = 71{\cdot}5t + 8{\cdot}3t$
Collecting—	$1861 = 79{\cdot}8t$
$\div 79{\cdot}8$	$t = \frac{1861}{79{\cdot}8} = \underline{23{\cdot}3}$

Note.—In removing brackets care must be taken to multiply *all* the inside quantities by the outside number.

Example 162.—The formula $V = \frac{2R}{R - r}$ gives the " velocity ratio " of a Weston pulley block, when R = radius of large pulley and r = radius of small pulley. Find the value of R when V = 25 and $r = 2$.

Substituting given values—	$25 = \frac{2R}{R - 2}$
$\times (R - 2)$	$25(R - 2) = 2R$
Removing brackets—	$25R - 50 = 2R$
Transposing and simplifying—	$23R = 50$
	$\therefore \quad R = \frac{50}{23} = \underline{2{\cdot}17}$

Note.—Multiply through by the denominator R − 2, inserting brackets to ensure correct multiplication.

Example 163.—Find the value of a from the formula $e = \frac{n - a}{m - a}$ (used in toothed gearing), when $e = \frac{16}{5}$, $n = 55$, and $m = 0$.

Substituting given values,	$\frac{16}{5} = \frac{55 - a}{0 - a} = \frac{55 - a}{-a}$
Cross-multiplying—	$-16a = 5(55 - a)$
Removing brackets—	$-16a = 275 - 5a$
Transposing—	$-275 = 16a - 5a = 11a$
$\div 11$	$a = \frac{-275}{11} = \underline{-25}$

Example 164.—The equation $450 - 7{\cdot}5w - 7{\cdot}6(45 + w) = 0$ refers to a problem on a loaded beam. Determine the value of w.

	$450 - 7{\cdot}5w - 7{\cdot}6(45 + w) = 0$
Removing brackets—	$450 - 7{\cdot}5w - 342 - 7{\cdot}6w = 0$
Collecting—	$108 - 15{\cdot}1w = 0$
Transposing—	$108 = 15{\cdot}1w$
$\div 15{\cdot}1$	$w = \frac{108}{15{\cdot}1} = \underline{7{\cdot}15}$

Note.—Change the sign when removing the bracket.

Example 165.—Solve the equation $15(x - 1) - 4(x - 3) = 2(7{\cdot}5 + x)$.

	$15(x - 1) - 4(x - 3)$	$= 2(7{\cdot}5 + x)$
Removing brackets—	$15x - 15 - 4x + 12$	$= 15 + 2x$
Transposing—	$15x - 4x - 2x$	$= 15 + 15 - 12$
Collecting—	$9x$	$= 18$
$\div 9$	x	$= 2$

Exercises 45. Equations requiring Removal or Insertion of Brackets.

1. Solve the equation $5(3x - 4) = 10$.

2. The equation $n = {\cdot}65(15 - n)$ occurred in a calculation on a reinforced concrete beam. Find the value of n.

3. Find the value of R from the equation $20(R - 2{\cdot}5) = 2R$, which refers to a Weston pulley block.

4. Find the value of a in the equation $22(1{\cdot}5 - a) = 3(4 - a)$, which occurred in a problem on epicyclic gearing.

5. Find the value of a in the equation $-{\cdot}85(1 - a) = -2 - a$, referring to toothed gearing.

6. The formula $L + T - t = W(t - t_1)$ occurs in connection with jet condensers. Calculate the value of t when $L = 1015$, $T = 142$, $W = 28{\cdot}5$, and $t_1 = 54$.

Solve the equations—

7. $3x - 2(5 - 2x) = -24$.

8. $n - 2(n + 1) = 3n - 5$.

9. $3t - (t + 3) = 1 - 4(7 - 2t)$.

10. The equation $81(f - 3000) = 20{\cdot}25(f + 3000)$ occurred when finding the stress in the metal of a hydraulic cylinder. Find the value of the stress f.

11. The formula $\frac{F\Delta}{2} = W(H + \Delta)$ relates to forces caused by shock. Calculate the value of Δ when $F = 11200$, $W = 56$, and $H = 18$.

12. Find the value of r from the equation $2(r - 5) = 3 + 5(2 - r)$.

13. The formula $w(L + t) = W(T - t)$ deals with a mixture of ice and water. Calculate the value of t when $w = 28{\cdot}5$, $W = 397$, $L = 80$, and $T = 18$.

14. The equation $130c = 300(19{\cdot}5 - c) + 560(7{\cdot}5 - c)$ occurred in a calculation to find the voltage drop in an electrical conductor supplied at both ends. Solve for c.

15. The formula $3G(a + b) = h(a + 2b)$ relates to the strength of masonry dams. Calculate the value of a when $G = 15{\cdot}6$, $h = 33$, and $b = 15$.

16. The formula $\frac{G + S}{S} = m$ is used in connection with galvanometer shunts. Calculate the value of S when $G = 54$ and $m = 20$.

17. The formula $g = \frac{T - t}{H - t}$ refers to an economiser. If $g = {\cdot}15$, $T = 250$, and $H = 1185$, calculate the value of t.

18. The formula $e = \frac{n-a}{m-a}$ applies to epicyclic trains of gear wheels. Calculate the value of n when $e = \frac{8}{15}$, $a = 30$ and $m = 0$.

19. Using the formula in Ex. 18, calculate the value of n when $e = -\cdot25$, $m = 5$, and $a = -1\cdot4$.

Solve the following equations:—

20. $\frac{1-p}{p+1} = -\cdot5$. **21.** $\frac{2T-1}{T-2} = 3\cdot5$.

22. Calculate the value of f from the equation $\frac{f+950}{f-950} = \frac{16}{9}$, which relates to the strength of a hydraulic cylinder.

23. The formula $R = \frac{r+rC}{1+rC}$ refers to clearance in a steam engine. If $R = 2\cdot53$, and $r = 2\cdot67$, calculate the value of C.

24. The formula $C = \frac{nE}{R+nr}$ relates to the electric current supplied by a number of cells connected in series. Calculate the value of n when $C = \cdot8$, $E = 1\cdot08$, $R = 5$, and $r = 1\cdot23$. (Give the calculated result and the result to the next whole number.)

25. The formula $\frac{n}{d-n} = \frac{mc}{t}$ relates to the strength of reinforced concrete beams. Find the value of n when $d = 11$, $m = 15$, $c = 600$, and $t = 16000$.

26. The formula $w = \frac{Wlr}{cf - lr}$ relates to the weight of built-up girders. If $w = 13\cdot3$, $W = 95$, $l = 76$, $f = 6\cdot5$ and $r = 14$, calculate the value of the ratio c.

Equations containing Brackets: Cases where Removal can be avoided.—Any equation containing brackets can, of course, be dealt with by their removal as in the previous section. In certain cases, however, by retaining the brackets, the working is easier. This is true for those equations where the unknown only occurs *once*, that once being *inside the bracket.* The following examples will illustrate.

Example 166.—An experiment to determine the latent heat of steam (in metric units) produced the following equation: $51\cdot9(L + 60) = 1190 \times 26$. Calculate the value of L.

$$51\cdot9(L + 60) = 1190 \times 26$$

$\div 51\cdot9$ $\qquad L + 60 = \frac{1190 \times 26}{51\cdot9} = 596$

Transposing— $\qquad L = 596 - 60 = \underline{536}$

Explanation.—Note that L only occurs once, and is inside the bracket. Dividing across by the coefficient 51·9 the bracketed quantity (L + 60) is isolated, and since there is a + sign outside, the bracket is not necessary. Continue as usual.

For comparison, the same example is shown worked by removing the brackets in the ordinary way—

	$51{\cdot}9(L + 60)$	$= 1190 \times 26$
Remove brackets—	$51{\cdot}9L + 3114$	$= 30900$
Transpose—	$51{\cdot}9L$	$= 30900 - 3114$
		$= 27786$ (say 27790)
$\div 51{\cdot}9$	$\therefore L$	$= \frac{27790}{51{\cdot}9} = \underline{536}$

Note that by the first method the working is simpler, and the operation of multiplying the 60 by the 51·9 is saved.

Example 167.—The Fahrenheit temperature F and its corresponding Centigrade temperature C, are connected by the equation $C = \frac{5}{9}(F - 32)$. A refrigerating machine keeps a cold store at a temperature of − 8°C; what is the equivalent F reading?

Substituting given value—	$-22 = \frac{5}{9}(F - 32)$
$\times \frac{9}{5}$	$-\frac{22 \times 9}{5} = F - 32$
Simplifying—	$-39{\cdot}6 = F - 32$
Transposing—	$-39{\cdot}6 + 32 = F$
	$\therefore \quad F = \underline{-7{\cdot}6}$

Note.—Multiplying across by $\frac{9}{5}$ isolates the bracketed quantity.

Example 168.—The formula $w\{L + (t - 32)\} = W(T - t)$ occurs when finding the latent heat of water in English units. Solve for L when $w = {\cdot}125$, T = 54, W = 1·1, and $t = 37$.

Substituting values,	$\cdot125\{L + (37 - 32)\}$	$= 1{\cdot}1(54 - 37)$
Simplifying—	$\cdot125\{L + 5\}$	$= 1{\cdot}1 \times 17$
$\div 125$	$L + 5$	$= \frac{18{\cdot}7}{{\cdot}125} = 149{\cdot}5$
Transposing—	L	$= 149{\cdot}5 - 5 = \underline{144{\cdot}5}$.

Example 169.—Find the value of s from the formula—

$$W(T - t) = (w_1 + w\,s)(t - t_1)$$

when,

$W = 25 \quad w_1 = 68{\cdot}3 \quad w = 15{\cdot}5 \quad T = 91 \quad t = 36 \quad t_1 = 16{\cdot}3$

Substituting values—	$25(91 - 36)$	$= (68{\cdot}3 + 15{\cdot}5\,s)(36 - 16{\cdot}3)$
Simplifying—	25×55	$= (68{\cdot}3 + 15{\cdot}5\,s) \times 19{\cdot}7$
$\div 19{\cdot}7$		$\frac{1375}{19{\cdot}7} = 68{\cdot}3 + 15{\cdot}5\,s$
Simplifying and transposing :	$69{\cdot}8 - 68{\cdot}3$	$= 15{\cdot}5\,s$
	$1{\cdot}5$	$= 15{\cdot}5\,s$
$\div 15{\cdot}5$		$s = \frac{1{\cdot}5}{15{\cdot}5} = \underline{{\cdot}0968}.$

Exercises 46. Equations with Brackets where Removal is not needed.

1. A formula for the piston speed of a petrol engine is $333(r + 2) = V$, where r = stroke-ratio and V = speed in ft. per min. If V is to be 1200, calculate the value of r.

2. The expression $w(L + t) = W(T - t)$ refers to a expression on the latent heat of water. Calculate the value of L when $w = 120$, $W = 452$, $T = 30{\cdot}5$ and $t = 7{\cdot}5$.

3 The formula $\frac{F\Delta}{2} = W(H + \Delta)$ relates to the strength of bodies under impact. Calculate the value of H when $F = 12{\cdot}6$, $\Delta = {\cdot}27$, $W = {\cdot}5$.

4. Find the value of the Fahrenheit temperature F from the formula $R = \frac{4}{9}(F - 32)$ when $R = 57{\cdot}5°$. (R is Rèaumur temperature.)

5. Using the formula in Ex. 4, above, calculate the value of F when $R = -10$.

6. The formula $c^2 = h(D - h)$ refers to circular arcs. Calculate the value of D when $c = 3{\cdot}78$ and $h = {\cdot}77$.

7. In heat experiments we have the formula $H = (W + w)(T - t)$. If $T = 60$, $t = 23$, $W = 435$, and $H = 18650$, find w.

8. The formula $\frac{c^2}{4} = h(2r - h)$ applies to the measurement of flat circular arcs. Calculate the value of r when $c = 3{\cdot}5$ and $h = {\cdot}43$.

9. The formula $Z = h\left(A + \frac{a}{6}\right)$ is connected with the strength of built-up beams. If $Z = 176$, $h = 27$, and $a = 13{\cdot}5$, calculate the value of A.

10. Using the formula given in Ex. 9, above, calculate the value of a when $Z = 255$, $A = 7{\cdot}5$, and $h = 33$.

11. The formula $R = R_0(1 + \alpha t)$ refers to the increase in resistance of an electric conductor with a rise of temperature. In a test on a sample of aluminium wire $R_0 = {\cdot}327$, $R = {\cdot}42$ and $t = 65$. Calculate the value of α.

12. The formula $H = {\cdot}2d^2(r + 1)$ gives the horse-power of a petrol engine when d = cylinder diameter in inches, and r = stroke-ratio. Calculate the value of r when $H = 5$ and $d = 3{\cdot}3$.

13. The expression $w(T - t) = (W + e)(t - t_1)$ occurs in heat experiments. In a particular case $w = 96{\cdot}8$, $W = 278{\cdot}5$, $T = 96$, $t = 35$, $t_1 = 15{\cdot}5$. Calculate the value of e.

14. $p = f(1 - gc^2)$ is Johnson's formula for columns and struts. Calculate the value of g when $p = 4{\cdot}73$, $f = 7$, and $c = 45$.

15. In a test to find the latent heat of water we have the statement $w(t - T) = w_1\{L + (T - 32)\}$. Find the value of L when $w = 20$, $t = 60$, $T = 45$, $w_1 = 2$.

16. The formula $c = \frac{E}{r + R}$ relates to electric batteries. If $c = 2$, $E = 2{\cdot}7$, and $r = {\cdot}032$, calculate the value of R.

17. The formula $p = \frac{45000}{60v + 20}$ relates to the allowable pressure on slides. If p is to be 300 what is the value of v?

18. The formula $\frac{D_1}{D_2} = \frac{R}{R_1} \cdot \frac{G + r}{G + x}$ is used when measuring high resistances. In a particular test $D_1 = 135$, $D_2 = 207$, $R = 100{,}000$, $R_1 = 30{,}000$, $G = 4150$, and $r = 7000$. Calculate the value of x.

Fractional Equations.—Certain equations contain terms in the form of vulgar fractions, more or less complicated, in which the unknown may occur as part of the numerator, denominator, or of both.

To explain the method let us solve the equation—

$$\frac{x}{2} + \frac{2x}{5} = \frac{3{\cdot}6}{2}$$

Considering the left-hand side we have to add two terms which *in form* are exactly like vulgar fractions: the fact that they contain an unknown quantity need not create any difficulty, and in fact these terms can be treated exactly as fractions. It is best, however, to work according to the method already treated in this chapter when clearing an equation of quantities below the line. Thus, if we multiply both sides of the equation by the product of all numbers below the line, in this case $2 \times 5 \times 2 = 20$, we can, by cancelling, clear the equation of fractions.

Thus— $$\frac{x}{2} + \frac{2x}{5} = \frac{3{\cdot}6}{2}$$

$\times 20$ $$\frac{\overset{10}{\cancel{20}}x}{\underset{1}{\cancel{2}}} + \frac{\overset{4}{\cancel{20}} \times 2x}{\underset{1}{\cancel{5}}} = \frac{3{\cdot}6 \times \overset{10}{\cancel{20}}}{\underset{1}{\cancel{2}}}$$

$$10x + 8x = 3{\cdot}6 \times 10$$

Collecting— $$18x = 36$$

$\div 18$ $$x = \frac{36}{18} = \underline{2}$$

Proof.—Substitute the value 2 for x in the given equation—

Then— $$\frac{2}{2} + \frac{2 \times 2}{5} = \frac{3{\cdot}6}{2}$$

$\therefore$ $1 + {\cdot}8 = 1{\cdot}8$, which is true, and

$\therefore$ the result is correct.

The second step, with a little practice, may be omitted. Thus, when multiplying by 20, the process is carried out mentally in the following way:—

First term, 20 times $\frac{x}{2}$	2 into 20 goes 10 times
	10 times $x = 10x$. *Write* $10x$.
Second term, 20 times $\frac{2x}{5}$	5 into 20 goes 4 times.
	4 times $2x = 8x$. *Write* $8x$.
Other side, 20 times $\frac{3{\cdot}6}{2}$	2 into 20 goes 10 times
10 times 3·6	*Write* $10 \times 3{\cdot}6$.

In the examples worked, this multiplying step will be shown in detail, but will be enclosed in brackets, thus [ ], to indicate that it is for explanation only.

It will be seen that in the next example we multiply by 12 instead of by the product of 6 and 4, although the latter would give the same result. Multiplying by a lower number that will still cancel the 6 and the 4, we get figures that are easier to work with. Obviously, then, we multiply by the L.C.M. of the numbers below the line.

Example 170.—Solve the equation $\frac{7a}{6} - \frac{a}{4} = 11$.

$$\frac{7a}{6} - \frac{a}{4} = 11$$

× L.C.M. of 6 and 4 = 12

$$\left[\frac{\overset{2}{\cancel{12}} \times 7a}{\underset{1}{\cancel{6}}} - \frac{\overset{3}{\cancel{12}}a}{\underset{1}{\cancel{4}}} = \frac{11 \times 12}{1}\right]$$

$$\therefore \quad 14a - 3a = 11 \times 12$$

Collecting— $\qquad 11a = 11 \times 12$

÷ 11 $\qquad a = \frac{\cancel{11} \times 12}{\cancel{11}} = \underline{12}$

Note.—When multiplying across, the 11 must be included, although it has apparently no denominator. Its denominator is really 1, as shown in the second step, but, of course, has no effect on the figures and so may be omitted in the working. When the term or terms on the other side are also fractional, then the L.C.M. taken must be that of *all* the denominators, as in the next example.

Also in dealing with a term in which the numerator is composed of two parts, *e.g.*, $\frac{3x+5}{7}$, it must be carefully remembered that the line of division performs the same office as a bracket, the expression

merely being a shorter statement of $\frac{3x}{7} + \frac{5}{7}$, as used in ordinary vulgar fractions; and when multiplying, *both* the quantities on top of the line must be multiplied.

Example 171.—Solve the equation $\frac{3x + 5}{7} = \frac{x}{2} + \frac{1}{3}$

$$\frac{3x + 5}{7} = \frac{x}{2} + \frac{1}{3}$$

× L.C.M. of 7, 2, and 3 = 42

$$\left[\overset{6}{\cancel{42}} \times \frac{(3x + 5)}{\cancel{7}} = \overset{21}{\cancel{42}x} + \overset{14}{\cancel{42}}\right]$$

$$\therefore 18x + 30 = 21x + 14$$

Transposing— $30 - 14 = 21x - 18x$

Simplifying— $16 = 3x$

÷ 3 $x = \underline{5\tfrac{1}{3}}$

Note.—After cancelling the left-hand side, remember that *both* the terms $3x$ and 5 are to be multiplied by the 6, giving $18x + 30$. It may assist beginners if a bracket is put round such terms as the $3x + 5$. Then, on multiplying, the form $6(3x + 5)$ is obtained, which shows more clearly that both the terms have to be multiplied. This becomes very important when minus signs occur; see Ex. 173.

Example 172.—Solve the equation $\frac{6p + 5}{9} + \frac{5}{3} = p + \frac{5 - p}{18}$.

$$\frac{6p + 5}{9} + \frac{5}{3} = p + \frac{5 - p}{18}$$

× L.C.M. of 9, 3, and 18 = 18

$$\left[\overset{2}{\cancel{18}}\frac{(6p + 5)}{\cancel{9}} + \overset{6}{\cancel{18}} \times 5 = 18p + \frac{\cancel{18}(5 - p)}{\cancel{18}}\right]$$

$$\therefore 12p + 10 + 30 = 18p + 5 - p$$

Transposing— $10 + 30 - 5 = 18p - p - 12p$

Collecting— $35 = 5p$

÷ 5 $p = \underline{7}$

Explanation.—The L.C.M. of the denominators being 18, we multiply throughout by 18. In the second step the compound numerators are shown closed in brackets to show that they must both be operated on. Remember to multiply the first p on the right-hand side.

When the signs between the terms are minus, great care must be exercised, as a *change of sign occurs.* The following example will illustrate:—

Example 173.—Solve the equation $\frac{5}{6} - \frac{3x+3}{5} = \frac{x}{3}$

$$\frac{5}{6} - \frac{3x+3}{5} = \frac{x}{3}$$

× L.C.M. of 6, 5, and 3 = 30 $\quad \left[\overset{5}{\cancel{30}} \times 5 - \frac{\overset{6}{\cancel{30}}(3x+3)}{\cancel{5}} = \overset{10}{\cancel{30}} \times x\right]$

$$\therefore 25 - 18x - 18 = 10x$$

Transposing— $\quad 25 - 18 = 10x + 18x$

Collecting— $\quad 7 = 28x$

÷ 28 $\quad x = \frac{7}{28} = \underline{\frac{1}{4} \text{ or } \cdot 25}$

Explanation.—Only the second term on the left side calls for any comment. On multiplying across by the 30 this term becomes $-\frac{30(3x+3)}{5}$, and on cancelling reduces to $-6(3x+3)$. On removing the bracket, as the sign outside is minus, the + sign is changed, and we have $-18x - 18$, as shown in the working.

An unknown in the denominator should create no difficulty; the equation is cleared of fractions in the same way.

Example 174.—If r and s are two electrical resistances connected in parallel, and R is the equivalent resistance, then these quantities are connected by the formula $\frac{1}{r} + \frac{1}{s} = \frac{1}{R}$. Find the value of r when $s = 6$ and $R = 2$.

Substituting given values— $\quad \frac{1}{r} + \frac{1}{6} = \frac{1}{2}$

× L.C.M. of 6, 2, and $r = 6r$ $\quad \left[\frac{6\cancel{r}}{\cancel{r}} + \frac{\overset{3}{\cancel{6}}r}{\cancel{6}} = \frac{\cancel{6}r}{\cancel{2}}\right]$

$$\therefore 6 + r = 3r$$

Transposing— $\quad 6 = 3r - r = 2r$

÷ 2 $\quad r = \frac{6}{2} = \underline{3}$

Proof.

$$\frac{1}{r} + \frac{1}{6}$$

$$= \frac{1}{3} + \frac{1}{6} = \frac{1}{2}$$

Example 175.—Find the value of m from the equation $1 - \frac{2}{m} = \frac{2}{5}$

$$1 - \frac{2}{m} = \frac{2}{5}$$

× L.C.M. = $5m$, $\quad \left[5m - \cancel{5m} \times \frac{2}{\cancel{m}} = \cancel{5}m \times \frac{2}{\cancel{5}}\right]$

$$\therefore \quad 5m - 10 = 2m$$

Transposing— $\quad 5m - 2m = 10$

$$3m = 10$$

÷ 3 $\quad m = \frac{10}{3} = \underline{3{\cdot}33}$

Exercises 47.—On Fractional Equations.

Solve the following equations:—

1. $\frac{2}{3}a + \frac{a}{5} = 6{\cdot}5$
2. $\frac{1-n}{2} = \frac{n}{3}$
3. $\frac{x-1}{2} - \frac{x-2}{3} = \frac{x-3}{4}$
4. $\frac{2-c}{5} = c - \frac{1}{2}$
5. $r - 1 = \frac{r-7}{3}$
6. $\frac{d}{2} - \frac{2+3d}{5} = 1 + \frac{d}{4}$
7. $\frac{2}{3} - y = 2 - \frac{y}{3}$
8. $\frac{q}{4} - 2 = \frac{3q-2}{4}$
9. $\frac{\theta}{2} - 2\theta = \frac{1-\theta}{3}$*
10. $\frac{\Delta+2}{3} = 2\Delta$†
11. $\frac{2y-1}{5} - \frac{3y-1}{2} = \frac{2}{5}$
12. $\frac{x-2}{x} = \frac{2}{3}$

13. The formula $\frac{V}{R} = C - \frac{V}{r}$ is used when measuring electrical resistances. Find R if V = 35, C = 7, and r = 590.

14. Using the equation in Ex. 13, find V if R = 6·7, r = 535, and C = 5·4.

15. Calculate the value of a from the formula $g = \frac{h}{3} \cdot \frac{a+2b}{a+b}$, referring to the strength of dams; when g = 7·3, h = 18, and b = 7.

16. The formula $\frac{\mu}{v} - \frac{1}{u} = \frac{\mu - 1}{r}$‡ refers to the refraction of light at spherical surfaces. If v = 15, u = 20, and r = 10, find the value of μ.

Equations requiring Square Root.—Although quadratic equations are not taken in this book, there are some simple equations containing squares which may be solved without any further knowledge. Such equations are those in which the square is the *only power* of the unknown which is present, as in the following examples. Briefly the method is to solve the equation as though x^2(say) were the unknown quantity. Arriving at some *numerical* value for x^2, the value of x is then easily obtained. The equations may be of any of the forms in the foregoing sections, and exactly the same methods will be adopted to determine the value of the square of the unknown.

Example 176.—The formula F = ·00034 WRN² is used in connection with engine governors. Find N when W = 3, F = 69, and R = ·6.

Substituting values	$69 = {\cdot}00034 \times 3 \times {\cdot}6 \times N^2$
$\div$ ·00034 × 1·8	$N^2 = \frac{69}{{\cdot}00034 \times 1{\cdot}8} = 112700$
Taking $\sqrt{\ }$	$N = \sqrt{112700} = 336$ revs. per min.

* θ is a Greek small letter "theta." † Δ is a Greek capital "delta."
‡ μ is a Greek small letter "mu."

Explanation.—Treating N^2 as the unknown, the equation is divided across by the coefficient of N^2, leaving N^2 alone. Taking the square root of the value of N^2 gives the value of N.

Example 177.—If h ft. is the "height" of an engine governor and N its speed in revs. per min., then $h = \frac{2938}{N^2}$. Find the speed when the height is 0·35 ft.

Substituting—	$\cdot 35 = \frac{2938}{N^2}$
$\times N^2$	$\cdot 35N^2 = 2938$
$\div \cdot 35$	$N^2 = \frac{2938}{\cdot 35} = 8400$
Taking $\sqrt{\ }$	$N = \sqrt{8400} = \underline{91 \cdot 6 \text{ revs. per min.}}$

Example 178.—The formula $q = A - \frac{B}{r^2}$ is used in connection with thick cylinders. Find the value of r when $q = -1120$, $A = -560$, $B = 5040$.

Substituting given values	$-1120 = -560 - \frac{5040}{r^2}$
Transposing—	$\frac{5040}{r^2} = -560 + 1120 = 560$
$\times r^2$	$5040 = 560r^2$
$\div 560$	$\frac{5040}{560} = r^2 = 9$
Taking $\sqrt{\ }$	$r = \sqrt{9} = \underline{3}$

Note.—Here it is advisable to transpose first, thereby reducing the size of the figures.

Example 179.—In connection with the strength of materials we find the formula $\frac{m^2}{m^2 - 1} = \frac{E_2}{E_1}$. Find the value of m when $E_1 = 13400$, $E_2 = 14400$.

Substituting—	$\frac{m^2}{m^2 - 1} = \frac{14400}{13400} = 1 \cdot 074$
$\times (m^2 - 1)$	$m^2 = 1 \cdot 074(m^2 - 1)$
Removing brackets—	$m^2 = 1 \cdot 074m^2 - 1 \cdot 074$
Transposing—	$1 \cdot 074 = 1 \cdot 074m^2 - m^2 = \cdot 074m^2$
$\div \cdot 074$	$m^2 = \frac{1 \cdot 074}{\cdot 074} = 14 \cdot 5$
Taking $\sqrt{\ }$	$m = \sqrt{14 \cdot 5} = \underline{3 \cdot 81}$

Exercises 48.—On Equations requiring Square Root.

1. The formula $s = \frac{c^2}{30}$ relates to manilla ropes, where c = circumference in inches and s = safe load in tons. Calculate the value of c for a rope to support a load of ·45 ton.

2. The formula $V = 2{\cdot}47dt^2$ relates to the volume of an anchor ring of round section. If $V = 18{\cdot}7$, and $d = 6$, find t.

3. The formula $f = \frac{wv^2}{ga}$ refers to the stress caused by centrifugal force. If $f = 200$, $w = {\cdot}83$, $g = 32$ and $a = 2{\cdot}4$, find v.

4. The formula $H = \frac{{\cdot}816}{n^2}$ relates to engine governors. Calculate the value of n when $H = {\cdot}375$.

5. The pull on an electric magnet is $P = \frac{B^2A}{11{,}180{,}000}$ in lbs. In a magnet where P is to be 600 the value of A is 7·2. Calculate the value of B.

6. For a direct-acting steam pump we have the formula $\cdot75D^2P = d^2p$. If $p = 150$, $P = 110$, and $d = 8\frac{1}{2}$, find the value of D.

7. The formula $f = \frac{pa^2}{t^2}$ relates to the strength of a flat square plate. If $f = 15000$, $p = 15$, and $a = 21$, find the value of t.

8. In the formula given in Ex. 7, calculate the value of a when $f = 2800$, $p = 150$, and $t = 1{\cdot}125$.

9. The formula $f = \frac{2}{3}\frac{pR^2}{t^2}$ refers to the strength of a flat circular plate. Calculate the value of R if $f = 9000$, $p = 60$, and $t = {\cdot}625$.

10. If a timber beam be loaded at the centre of the span, the breaking load in lbs. is $W = \frac{abd^2}{l}$, where b, d and l are the width, depth and span in inches, and a = a constant depending on the material. Find depth d for a beam 3″ wide to support 6000 lbs. on an 18″ span, when $a = 19200$.

11. The formula $h = \frac{g^2L}{730d^5}$ relates to water pipes. Calculate g (galls. per min.) for a pipe 4″ dia. (= d), and 450 ft. long (= L) when the friction head $h = 30$ ft.

12. The formula $\frac{v^2}{2g} + h = H$ occurs in hydraulics. If $h = 3{\cdot}35$, $H = 3{\cdot}53$, and $g = 32$, calculate the value of v.

13. The formula $\frac{V^2}{2gJ} = H_1 - H_2$ relates to the expansion of steam in a turbine nozzle. If $H_1 = 1200$, $H_2 = 1090$, $g = 32{\cdot}2$, and $J = 778$, find the value of V.

14. The formula $q = A - \frac{B}{r^2}$ refers to the strength of thick cylinders. If $A = 7200$, $B = -64800$, and $q = 14{,}400$, find r.

15. The formula $\frac{1}{c^2} - 1 = x$ occurs in some problems in hydraulics. Calculate the value of c if $x = {\cdot}0628$.

16. The formula $P = \frac{c(t+1)^2}{s-6}$ is used in reference to the strength of flat stayed surfaces in boilers. Calculate t when $P = 180$, $c = 125$, and $s = 36$.

17. Calculate d, the diameter of cylinder (ins.) required for a petrol motor to develop a horse-power (H) of 7·5, using Henderson's formula $H = \cdot 2d^2(r + 1)$, when $r = 1 \cdot 2$.

18. Work Ex. 17, using Poppe's formula $H = \cdot 81(d - \cdot 79)^2$, *i. e.*, find d when $H = 7 \cdot 5$.

Equations requiring Squaring.—Often it is necessary to solve an equation containing the square root of the unknown quantity, as in the remaining examples in this chapter. These are easily solved by previous methods, with only one extra operation. Thus, in the formula, $V = 8 \cdot 02\sqrt{h}$, when V is known, the value of $\sqrt{h}$ can be easily found, and squaring gives the value of h, since $\sqrt{h} \times \sqrt{h} = h$. Sometimes the operation of squaring must be performed at an early stage in the working, and sometimes right at the end, but in all cases *the whole of one side must be under the root sign before any squaring is performed.*

Example 180.—If water falls from a height h ft. it acquires a velocity of V ft. per sec. as given by the formula $V = 8 \cdot 02\sqrt{h}$. Find the height to give a velocity of 66 ft. per sec.

Substituting—	$66 = 8 \cdot 02\sqrt{h}$
$\div 8 \cdot 02$	$\frac{66}{8 \cdot 02} = \sqrt{h}$
Simplifying—	$\sqrt{h} = 8 \cdot 23$
Squaring—	$h = (8 \cdot 23)^2 = \underline{67 \cdot 7 \text{ ft.}}$

Explanation.—Here the $\sqrt{h}$ must be isolated before h can be found. Hence the equation must be divided by the coefficient 8·02. Then squaring the value of $\sqrt{h}$ gives the value of h. As a proof, the reader should substitute the result in the given formula, and see whether V comes to 66, as it should do.

Example 181.—If T is the time in secs. of the double swing of a pendulum and l is its length in ft., then $T = 2\pi\sqrt{\frac{l}{32 \cdot 2}}$ where $\pi = 3 \cdot 142$. What length of pendulum is required to give a double swing of 2 secs.?

Substituting—	$2 = 2\pi\sqrt{\frac{l}{32 \cdot 2}}$
$\div 2\pi$	$\frac{2}{2\pi} = \sqrt{\frac{l}{32 \cdot 2}}$
Simplifying—	$\sqrt{\frac{l}{32 \cdot 2}} = \cdot 3183$
Squaring—	$\frac{l}{32 \cdot 2} = (\cdot 3183)^2 = \cdot 1013$
$\times 32 \cdot 2$	$l = \cdot 1013 \times 32 \cdot 2 = 3 \cdot 262$ ft.
	say, $\underline{3 \text{ ft. } 3\frac{1}{8}''}$.

Example 182.—In connection with water flowing in channels and pipes, we have the formula $c = \sqrt{\frac{2g}{\mu}}$ where $g = 32$. By experiment we find that $c = 110$. What is the corresponding value of μ?

Substituting—	110	$= \sqrt{\frac{2 \times 32}{\mu}}$
Squaring—	110^2	$= \frac{2 \times 32}{\mu}$
Simplifying—	12100	$= \frac{64}{\mu}$
$\times \mu$	$12100\,\mu$	$= 64$
$\div 12100$	μ	$= \frac{64}{12100} = \underline{\cdot00529}$

Note.—Squaring is the first operation that must be performed as the whole of the right hand side is under the root sign.

Example 183.—Find the value of r from the formula $h = \sqrt{2{\cdot}25 - r^2}$ (which relates to an engine governor) when $h = {\cdot}833$.

Substituting—	${\cdot}833$	$= \sqrt{2{\cdot}25 - r^2}$
Squaring	${\cdot}694$	$= 2{\cdot}25 - r^2$
Transposing—	r^2	$= 2{\cdot}25 - {\cdot}694 = {\cdot}556$
Taking $\sqrt{\ }$	r	$= \sqrt{{\cdot}556} = \underline{{\cdot}746}$.

Note.—This example involves both squaring and square root. To obtain r we must know r^2; and to obtain this we must know the value of $2{\cdot}25 - r^2$. Therefore the first operation must be to square each side. Then r^2 is obtained by transposition, and r follows from a square root.

Exercises 49. On Equations requiring Squaring.

1. The formula $w = 1000\sqrt{d}$ refers to roller bearings for long bridges. Calculate the value of d if $w = 2500$.

2. The formula $d = 1{\cdot}2\sqrt{t}$ relates to riveted work. Find the value of t if $d = \frac{13}{16}$.

3. Calculate the value of r from the Lanchester motor rating formula $H = {\cdot}4d^2\sqrt{r}$ when $H = 5{\cdot}5$, and $d = 3{\cdot}75$.

4. If $T = 2\pi\sqrt{\frac{l}{g}}$ calculate the value of g when $T = 1{\cdot}82$, $l = 2{\cdot}73$ and $\pi = 3{\cdot}14$.

5. The formula $B = \sqrt{\frac{2Rt}{3}}$ applies to curves on tramway tracks. If $B = 6$, and $t = 1{\cdot}375$, calculate the value of R.

6. Calculate the value of i from the formula $v = c\sqrt{mi}$, which relates to the flow of water, when $v = 3{\cdot}8$, $c = 95$, and $m = 1{\cdot}7$.

7. In the formula given in Ex. 6, calculate the value of m if $v = 6$, $c = 120$ and $i = \cdot00045$.

8. The expression $f = \sqrt{\frac{B}{\mu b}}$ refers to reinforced concrete beams. Calculate the value of μ when $B = 190{,}000$, $b = 10$, and $f = 15$.

9. The formula $p = \sqrt{\frac{2 \cdot 5P}{R \times k}}$ relates to helical tooth gears. Find R if $p = 2\frac{3}{4}$, $P = 5450$, and $k = 1075$.

10. The formula $T = \sqrt{\frac{P}{2}} - 1$ refers to the strength of locomotive fireboxes. If $T = 8$, calculate the value of P.

11. In connection with slide valves we have the formula $\frac{\sqrt{x}}{1 - \sqrt{x}} = n$. If $n = 2 \cdot 41$ calculate the value of x.

12. In connection with the strength of thick cylinders we have the formula $\frac{D}{d} = \sqrt{\frac{f + p}{f - p}}$. Calculate f when $D = 4$, $d = 3$, and $p = 1500$.

13. Calculate the value of p in the foregoing formula when $D = 7$, $d = 4$, and $f = 12000$.

14. A formula connecting the cylinder diameter of a steam engine with the indicated horse-power is $d = 205\sqrt{\frac{I.H.P}{pLN}}$. Calculate L if $d = 12$, I.H.P. $= 37$, $p = 40$, and $N = 180$.

15. The formula $A = \frac{4}{3} h^2 \sqrt{\frac{D}{h}} - \cdot61$ gives the area of a segment of a circle. Calculate D if $A = 8 \cdot 83$ and $h = 2 \cdot 21$.

Exercises 49a. Miscellaneous Examples on Equations.

1. For a compressed gas cylinder the formula $p(R - t) = ft$ is applicable, where p = internal pressure, R = outside radius, f = stress in the material, and t = thickness. Calculate the necessary value of t for a case where $p = 1 \cdot 5$, $R = 3 \cdot 5$, and $f = 8$.

2. The wave length of an oscillatory electric circuit is given by the formula $\lambda = k\sqrt{LC}$ where L = inductance, C = capacity, and k = a constant. Calculate the value of C when $k = 1884$, $L = 130$ and λ is to be 500. (λ is the Greek small letter "lambda".)

3. For the Pitot tube used for measuring the velocity of a fluid the fundamental formula is $p = \frac{1}{2}\rho v^2$. For a particular case $p = 2490\,h$, $\rho = 0 \cdot 73$, and $v = \cdot0268\,\frac{W}{a}$. Substitute these values in the fundamental formula and reduce all the constants to give a new formula, giving h in terms of W and a. (ρ is the Greek small letter "rho".)

4. The inductance of a circular solenoid of circular section is given by the formula $L = \cdot0126\,n^2\,[R - \sqrt{R^2 - a^2}]$, where n = number of turns of wire. Calculate the value of n required if L is to be 100, and $R = 4 \cdot 5$ and $a = 1 \cdot 5$.

5. Referring to the formula in Ex. 4, calculate the value of a when $L = 150$, $n = 220$ and $R = 4 \cdot 8$.

CHAPTER V

TRANSPOSITION OF FORMULÆ

The General Solution.—The process of solving an equation is not always confined to cases like those in Chap. IV, where a numerical answer is obtained. In many cases an equation is composed entirely of symbols, and the solution is a more or less complicated algebraic expression. Such an equation is called a *literal equation*, and the solution is a **general solution**, *i. e.*, one that will suit *for all numerical values* of the quantities involved.

The applications of this are as follows: Consider a formula such as $H = 1082 + \cdot 3t$. Frequently several values of H are known, and it is required to find the corresponding values of t. If each value of H be substituted in turn in the given formula, and the resulting equations worked out as in Chap. IV, a considerable amount of time and space will be occupied in repeating a number of similar statements. This can be saved if, in calculating the various values of t, all the *algebraic* operations be performed before any substitution. Thus from the above formula it can be calculated that $t = \frac{H - 1082}{\cdot 3}$, and all that is necessary is to substitute each value of H in turn, and to evaluate the right-hand side in each case (see Ex. 186, p. 192). Often in the process of finding the general solution some of the symbols and figures may cancel (see Ex. 187, p. 192), thus reducing the amount of arithmetic necessary in the final substitutions.

Again, when studying problems in some branch of science, it is constantly necessary to find a formula by which a particular quantity can be calculated. An examination will show some connection between the various quantities involved, as, for example, in the case of a boiler, where we can determine the relation $prl = ftl\eta$. If it is necessary to calculate t for any particular conditions a formula for t can be obtained from the above relation, *i. e.*, $t = \frac{pr}{f\eta}$. The

operation of finding this formula from the known conditions is purely an algebraic one, and is independent of the values involved.

The name **"transposition"** will be given to the process of finding a new formula for some particular quantity contained in a given formula. In the majority of cases it will be found better to transpose the given formula before substituting numerical values.

The Unknown Quantity.—Considering an equation such as $ap + r = f$, we cannot merely say "solve this equation," as there are four symbols whose values are unknown. Hence it is necessary to specify the particular symbol whose "value" is desired. Strictly speaking a "value" means a numerical value, but it is quite evident what is meant. Thus suppose a formula for p is required from the statement $ap + r = f$, we can express ourselves in several ways, *e. g.*: "solve for p in the equation"; "find the value of p in the equation"; "find a formula for p"; or "transpose for p."

Some difficulty may be experienced at first in distinguishing the unknown quantity from the remaining symbols, especially if, in the course of working, the unknown occupies different positions. To assist in this direction the unknown will be shown in heavy type throughout the working of most of the examples in each section; the student, in his first few cases, can put a ring round the unknown quantity.

The methods of working in this chapter are practically identical with those in Chap. IV if the unknown be regarded as a symbol, and all the other symbols as numbers, and treated accordingly: also the order of treatment is similar. For the first example of each type a numerical equation will be shown as an illustration: and proving these will readily show that the method is correct.

Equations requiring Division.—As with similar cases in Chap. IV, the equation must be divided across by the quantity which multiplies the unknown.

Example 184.—If P represents the pressure of water in lbs. per sq. ft., and p the pressure in lbs. per sq. in., then $144p = \mathrm{P}$. Find a formula for p.

			Similar Numerical Example.		
	$144p$	$= \mathrm{P}$		$3x = 15$	
$\div 144$	p	$= \frac{\mathrm{P}}{144}$	$\div 3$	$x = \frac{15}{3}$	

Note.—In the numerical case one other step is possible, *i. e.*, the evaluation of $\frac{15}{3}$. This, of course, cannot be done in the left-hand case until the value of P be known.

Example 185.—If E = primary voltage, e = secondary voltage, C = primary current, and c = secondary current in a transformer, then neglecting losses ec = EC. Transpose for c.

$$ec = EC$$

$$\div e \qquad c = \frac{EC}{e}$$

Example 186.—If D = the diameter of a belt pulley in feet, N = revs. per min., and V = velocity of belt in ft. per min., then $V = 3{\cdot}14DN$. Find a formula for D, and calculate the values of D when V has the values 800, 1000 and 1200, N being 200.

$$3{\cdot}14DN = V$$

$$\div 3{\cdot}14N \qquad D = \frac{V}{3{\cdot}14N}$$

(1) $V = 800 \quad \therefore D = \frac{800}{3{\cdot}14 \times 200} = 1{\cdot}273$ ft

(2) $V = 1000 \quad \therefore D = \frac{1000}{3{\cdot}14 \times 200} = 1{\cdot}592$ ft.

(3) $V = 1200 \quad \therefore D = \frac{1200}{3{\cdot}14 \times 200} = 1{\cdot}91$ ft.

Note.—It should be easily seen that the multiplier of the unknown D is $3{\cdot}14N$. Hence the equation is divided throughout by this quantity.

Example 187.—The relation $\frac{\pi}{4}d^2 = \pi dl$ occurs when finding the lift of a mushroom valve, d being the valve diameter and l its lift. Find a formula for l. What should be the lift of a 3″ valve?

$$\pi dl = \frac{\pi}{4}d^2$$

$$\div \pi d \qquad l = \frac{\pi d^2}{4\pi d}$$

Cancelling, $\qquad l = \frac{d}{4}$

When $d = 3''$ $\qquad l = \frac{3''}{4}$

Explanation.—Divide across by πd to get l on left-hand side by itself. The expression is then cancelled by πd, leaving a very simple result. It is evidently better to calculate the lift from the final expression than to substitute for π and d in the given equation and then work out.

Equations requiring Multiplication. — Equations where the unknown is divided by a number, or symbol, require to be multiplied by this number or symbol to isolate the unknown.

Example 188.—The formula $\frac{H}{2\cdot3} = p$ connects the pressure p of water in lbs. per sq. in. with the head H in ft. Find a formula for H.

		Similar Numerical Example.	
	$\frac{H}{2\cdot3} = p$		$\frac{x}{2} = 4$
$\times 2\cdot3$	$H = 2\cdot3p$	$\times 2$	$x = 4 \times 2$

Example 189.—If Q = the quantity of electricity in coulombs, and t the time in secs in which it flows, then C the current in amperes is given by $C = \frac{Q}{t}$. Transpose for Q.

$$\frac{Q}{t} = C$$

$$\times t \qquad Q = ct$$

Example 190.—With an alternating electric current $p = \frac{W}{EC}$ where p = power factor, W = true power (watts), E = volts, and C = current. Find a formula for W, and find the values of W: (*a*) when E = 110, C = 35, and p = ·81; (*b*) when E = 6000, C = 23, and p = ·739.

$$\frac{W}{EC} = p$$

$$\times EC \qquad W = pEC$$

(*a*) $W = \cdot81 \times 110 \times 35 = 3120$ watts

(*b*) $W = \cdot739 \times 6000 \times 23 = 102000$ watts.

Exercises 50. On Equations requiring (a) Division, (b) Multiplication.

(*a*)—Division.

1. Transpose for a in the formula $F = Ma$.

2. The formula $w = eCt$ relates to electrolysis. Transpose for e.

3. In connection with the expansion of gases we have the formula $Pv = pV$. Transpose for P.

4. In Exs. 40, No. 12, we have the formula $V = \pi DN$. Find a formula for N.

5. Transpose for R_4 in the equation $R_1R_4 = R_2R_3$, which relates to the measurement of electrical resistances.

6. In Exs. 40, No. 14, we have the formula $P = \cdot785d^2p$. Transpose for p.

7. Transpose for t in the formula $pr = ft\eta$, which relates to steam boilers.

8. With reference to an alternating current the formula $W = ECp$ occurs. Transpose for C.

9. Solve the equation $v^2 = 2gh$, for h, this occurring in hydraulics.

10. In Exs. 40, No. 17, we have the formula $\cdot75D^2P = d^2p$. Solve for P.

11. Transpose for s in the formula $H = ws\ (T - t)$, which relates to problems on heat.

12. Lewis's formula for the strength of wheel teeth is $L = c \times p \times w \times F$. Find a formula for w.

(*b*)—MULTIPLICATION.

13. If $R = \frac{E}{C}$ transpose for E.

14. The statement $f = \frac{M}{Z}$ refers to stress in beams. Find a formula for Z by cross-multiplying.

15. If Q is the actual discharge through a nozzle, and Q_1 is the theoretical discharge, then the "coefficient of discharge" $c = \frac{Q}{Q_1}$. Find a formula for Q.

16. The formula $\frac{E}{W} = \frac{d}{D}$ refers to levers. Solve for E.

17. The formula $V = \frac{Q}{kr}$ relates to an electrically charged sphere. Transpose for Q.

18. Transpose for v in the formula $c = \frac{v}{r - 1}$ which relates to the clearance in gas-engine cylinders.

19. With reference to gear wheels: Pitch dia. $= \frac{\text{No. of teeth}}{\text{Real pitch}}$. Find a statement for "No. of teeth."

20. The equation $\frac{H}{R} = \frac{W}{F}$ occurs in connection with engine governors, and $F = \cdot 00034WRN^2$. Substitute this value of F and transpose for H.

Equations requiring Multiplication and Division.—To obtain a formula for the unknown the equation must be multiplied by the quantities dividing the unknown, and divided by those multiplying it. The operation may be done separately if so desired, but very little practice should enable the student to combine the two operations.

Example 191.—If p = the number of pairs of poles in an alternator, n = the number of revs. per sec. and f = the frequency of the current, then $\frac{pn}{2} = f$. Transpose for p.

		Similar Numerical Example	
	$\frac{pn}{2} = f$		$\frac{3x}{2} = 9$
$\times \frac{2}{n}$	$p = \frac{2f}{n}$	$\times \frac{2}{3}$	$x = \frac{9 \times 2}{3}$

Example 192.—The formula $S = \frac{PD}{\cdot 2t}$ occurs in connection with boilers, using metric units. Transpose for P.

$$\frac{PD}{\cdot 2t} = S$$

$$\times \frac{\cdot 2t}{D} \qquad P = \frac{\cdot 2tS}{D}$$

Example 193.—If $c^2r = \left(\frac{c}{2}\right) R$, find a formula for R.

$$c^2r = \left(\frac{c}{2}\right)^2 R$$

Simplifying $$c^2r = \frac{c^2}{4} R$$

$$\times \frac{4}{c^2} \qquad \frac{4c^2r}{c^2} = R$$

Cancelling by c^2 $\therefore$ $R = 4r$.

Note.—The resulting formula is far easier to use than the original expression.

Example 194.—In an investigation concerning the arrangement of the cells in an electric battery the following equation appeared: $\frac{E^2p}{4Rr} = \frac{w}{R}$; and it was desired to find the value of p. Transpose for p.

$$\frac{E^2p}{4Rr} = \frac{w}{R}$$

$$\times \frac{4Rr}{E^2} \qquad p = \frac{w4Rr}{RE^2} = \frac{4rw}{E^2}$$

Explanation.—The unknown being p, the equation must be *divided by* E^2 *and multiplied by* $4Rr$; combining the two operations the equation is multiplied by $\frac{4Rr}{E^2}$. R is then cancelled. When the result contains both numbers and symbols it is usual to place the numbers in front.

Example 195.—The Board of Trade formula for steel tube-plates in boilers is $p = \frac{20000T(D - d)}{WD}$, p being working pressure, T thickness, W width of combustion chamber, D pitch of tubes, and d inside diameter of tubes. Transpose the formula for the thickness T.

$$\frac{20000T(D - d)}{WD} = p$$

$$\times WD \qquad 20000T(D - d) = pWD$$

$$\div 20000(D - d) \qquad T = \frac{pWD}{20000(D - d)}$$

Note.—The bracketed quantity (D − *d*) should be looked upon as a single symbol or number; there is no need for removing the bracket. The operations of multiplying and dividing are done separately in this example as the expression has rather a heavy appearance.

In the following example the unknown is below the line, but exactly the same principles apply.

Example 196.—The capacity of a plate electric condenser is given by the formula $K = \frac{kA}{4\pi t}$. Transpose for t.

$$K = \frac{kA}{4\pi t}$$

Cross-multiplying, $4\pi tK = kA$

$\div 4\pi K$ $\qquad t = \frac{kA}{4\pi K}$

Note. — Clear equation of fractional appearance by cross-multiplying, which brings unknown to left-hand side. Then proceed as usual.

Exercises 51. Multiplication and Division combined.

1. The Board of Trade Rule for Morrison Furnaces is $P = \frac{13500T}{D}$, where P is working pressure, T is thickness, D is diameter. Find a formula for T.

2. In Exs. 42, No. 1, the formula $\frac{EC}{1000} = K$ appears. Transpose for C.

3. The horse-power taken by a D.C. motor is given by the formula $\frac{EC}{746} = H$, where E = volts, C = current in amperes, and H = horse-power. Transpose for C.

4. The expression $W = 2{\cdot}71\frac{p}{\tau}$ refers to the weight of air.* Solve for p.

5. In Exs. 42, No. 8, we have the formula $t = \frac{Hd.}{12500}$. Solve for d.

6. In connection with the horse-power transmitted by belts the formula $H = \frac{SV}{33000}$ occurs. Transpose for S.

7. The formula $Z = \frac{bh^2}{6}$ occurs in connection with beams. Transpose for b.

8. Solve for S in the formula $H = \frac{d^2Sn}{12720}$, which refers to the rating of petrol engines.

9. Transpose for d in the formula $W = C\frac{dh^2}{b}$, which refers to the weight of chimneys.

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* τ is a small Greek letter "tau."

10. In Exs. 42, No. 10, the formula $H = \frac{1{\cdot}26CT}{l}$ occurs. Transpose for CT.

11. In a law applying to wedges we have $\frac{R}{P} = \frac{L}{t}$. Obtain a formula for P (*i. e.*, cross-multiply).

12. The formula $V = \frac{\tau}{2{\cdot}71p}$ refers to volume of air. Solve for p.

13. Find a formula for k from: $K = \frac{kA}{4\pi t'}$ which gives the capacity of a parallel plate condenser.

14. Transpose for d in the formula $h = f\frac{Lv^2}{d}$, which refers to pipes conveying water.

15. In Exs. 42, No. 5, the formula $d^2l = \frac{cK}{N}$. Transpose for c.

16. Transpose for w in the formula $W = w \times \frac{H - h + h_1}{966}$.

17. In Exs. 42, No. 13, we have the formula $f = \frac{2}{3}\frac{R^2}{t^2}p$. Solve for p.

18. Transpose for h in the formula $H = \frac{h(d - d_1)}{d}$, which relates to the height of chimneys.

19. The formula $C = \frac{nE}{R + nr}$ relates to electric batteries in series. Find a formula for E.

20. Transpose for f in the equation $\frac{wl}{4} = f\frac{bh^2}{6}$, which relates to beams.

Equations requiring Addition and Subtraction.—When an equation contains terms separated by + and − signs, then these terms may be transposed as desired provided that the signs are changed.

Example 197.—If t = Fahrenheit temperature and T = absolute Fahrenheit temperature, then $t + 460 = T$. Transpose for t.

	Similar Numerical Example.
$t + 460 = T$	$x + 2 = 5$
Transposing the 460, $t = T - 460$	$\therefore x = 5 - 2$

Example 198.—If p = the pitch of a riveted joint, d = dia. of rivet, and w = effective length between rivets, then $p - d = w$ Transpose for p.

$$p - d = w$$

Transposing d $$p = w + d$$

Example 199.—The formula $v = u + at$ occurs in connection with velocity calculations. Find a formula for u.

$$u + at = v$$

$$-at \qquad u = v - at$$

Note.—The term "at" is considered as one symbol and taken to the other side with a change of sign.

Example 200.—If $H - h = \frac{P}{G}$, transpose for h.

$$H - h = \frac{P}{G}$$

$$+h \qquad H = \frac{P}{G} + h$$

$$-\frac{P}{G} \qquad H - \frac{P}{G} = h$$

Explanation.—The unknown has a $-$ sign, and if the H be taken across a formula would be found for $-h$. But since transposing a term changes its sign, the unknown can be made $+$ by taking it to the other side. Then transpose the term $\frac{P}{G}$.

Exercises 52. Equations involving Addition and Subtraction.

1. Transpose for p in the formula $P = p + 14{\cdot}7$.

2. The formula $H = S + xL$ refers to wet steam. Transpose for S.

3. Solve for t in the formula $T = t + 273$.

4. In the formula $B = H + 4\pi I$ transpose for H.

5. The formula $M_2 = M_1 + mM$ refers to properties of sections. Transpose for M_1.

6. Solve for R in the formula $R + \frac{r}{n} = S$.

7. Transpose for c in the formula $ad = \frac{b}{e} - c$ (see Exs. 44, No. 19).

8. If E = the E.M.F. of a battery, e = volts lost internally and V = "potential difference," then $E - e = V$. Transpose for e.

9. Transpose for H in the formula $W = H + Cr$.

10. The formula $u = v - at$ is used in mechanics. Transpose for v.

11. Transpose for C in the formula $\frac{V}{R} = C - \frac{V}{r}$.

12. Solve for A in the formula $\frac{Z}{h} = A + \frac{a}{6}$.

Equations requiring the Four Rules.—Where various combinations of multiplication, addition, etc., occur, then two main operations are necessary:

1. Transpose as in the last section to obtain the value of the *term containing the unknown.*

2. Divide or multiply this result by the quantity which respectively multiplies or divides the unknown, as in the first section. This may be seen by the working of a simple numerical example—

$$\begin{array}{lll} & 2x + 1 = 7 & \\ -1 & 2x = 7 - 1 & \ldots \text{ transposing to find the value of } 2x \\ \div 2 & x = \dfrac{7-1}{2} & \ldots \text{ dividing by 2 to obtain the value of } x \end{array}$$

The method will be exactly the same if the 2, the 1, and the 7 in the given equation be replaced by symbols, x being considered as the unknown throughout.

Thus, let the 7 be replaced by a symbol, say r,

$$\begin{array}{ll} \text{Then—} & 2x + 1 = r \\ -1 & 2x = r - 1 \\ \div 2 & x = \dfrac{r-1}{2} \end{array}$$

Now let the 1 be replaced by a symbol, say d,

$$\begin{array}{ll} \text{Then—} & 2x + d = r \\ -d & 2x = r - d \\ \div 2 & x = \dfrac{r-d}{2} \end{array}$$

Finally replace the 2 by a symbol, say a,

$$\begin{array}{ll} \text{Then—} & ax + d = r \\ -d & ax = r - d \\ \div a & x = \dfrac{r-d}{a} \end{array}$$

Example 201.—If D = dia. of a Whitworth bolt, and d = dia. at the bottom of the screw thread, then the following relation is approximately true : $d = \cdot 9\mathrm{D} - \cdot 05$. Transpose for D.

$$\begin{array}{ll} & \cdot 9\mathrm{D} - \cdot 05 = d \\ + \cdot 05 & \cdot 9\mathrm{D} = d + \cdot 05 \\ \div \cdot 9 & \mathrm{D} = \dfrac{d + \cdot 05}{\cdot 9} \end{array}$$

Example 202.—The Fahrenheit temperature F, and the Centigrade temperature C, are connected by the formula $F = \frac{9}{5}C + 32$. Transpose for C.

$$\frac{9}{5}C + 32 = F$$

$$-32 \qquad \frac{9}{5}C = F - 32$$

$$\times \frac{5}{9} \qquad C = \frac{5}{9}(F - 32)$$

Explanation.—The value of $\frac{9}{5}C$ is obtained by transposing the 32. Now the equation has to be multiplied by 5 and divided by 9 to get C; and the *whole* of the right-hand side must be so treated. Therefore a bracket is first placed round the F − 32 and then the $\frac{5}{9}$ written outside to conform to this.

Example 203.—Let d = dia. of a punch, t = thickness of plate, and D = dia. of hole in the bolster; then $D = d + \frac{t}{6}$. Transpose for t.

$$d + \frac{t}{6} = D$$

$$-d \qquad \frac{t}{6} = D - d$$

$$\times 6 \qquad t = 6(D - d)$$

Note.—After transposing the d, the *whole* must be multiplied by 6; a bracket must therefore be placed round the $D - d$ to indicate this.

Example 204.—The formula $1 + 4\pi K = \mu$ is used in connection with magnetism. Transpose for K.

$$1 + 4\pi K = \mu$$

$$-1 \qquad 4\pi K = \mu - 1$$

$$\div 4\pi \qquad K = \frac{\mu - 1}{4\pi}$$

Example 205.—The formula $c^2 = 8hr - 4h^2$ relates to circular arcs. Transpose for r.

$$8hr - 4h^2 = c^2$$

$$+4h^2 \qquad 8hr = c^2 + 4h^2$$

$$\div 8h \qquad r = \frac{c^2 + 4h^2}{8h}$$

Exercises 53. On Equations combining the Four Rules.

1. Transpose for t in the formula $L = 120t - 12$.

2. The formula $W = \cdot 362H + 7$ was obtained from tests on a Diesel engine. Transpose for H.

3. Find a formula for q when $n = 1 \cdot 035 + \cdot 1q$, a statement relating to the expansion of steam.

4. The formula $K = 48 + \cdot 45Y$ relates to the consumption of power in a textile factory. Transpose for Y.

5. The formula $t = 1\cdot 54PD + 2\cdot 6$ relates to the strength of steam boilers. Transpose for P.

6. The formula $F = \frac{9}{4}R + 32$ connects Fahrenheit and Réaumur temperatures. Find a formula for R.

7. Find a formula for n from the equation $R + nr = S$.

8. In Exs. 44, No. 9, the formula $v = 115 - 1300a$ occurred. Find a formula for a.

9. In the formula $L = 566 - \cdot 8t$, relating to the latent heat of ammonia, transpose for t.

10. Transpose for l in the formula $25x - \frac{l}{2} = 14\cdot 6$.

11. The formula $D = CS + K$ is used when measuring distances with a tacheometer. Transpose for C.

12. Find the value of a from the formula $v = u + at$.

13. In Exs. 44, No. 17, we have the equation $nr - mR = 0$. Solve for r.

14. Solve the equation in Ex. 13 above, for R.

15. Find a formula for I from the equation $B = H + 4\pi I$, which is used in magnetism.

16. Find a formula for a from the equation $v^2 = u^2 + 2as$, which relates to velocity problems.

17. Transpose for r in the formula $-l^2 = -6\cdot 5pr + 1\cdot 8p^2$.

18. Find a formula for n from the equation $s = 1 + \frac{n}{150}$ which relates to helical springs.

19. Transpose for r in the formula $R + \frac{r}{n} = S$, relating to electric batteries in parallel.

20. The formula $D = \frac{nd}{13} + 7d$ occurs in connection with wire ropes. Transpose for n.

21. Transpose for B in the formula $p = A + \frac{B}{r^2}$ which relates to thick cylinders.

22. In Exs. 44, No. 19, we have the formula $ad = \frac{b}{e} - c$. Solve for b.

23. The formula $1 - \frac{A}{L} = n$, relates to epicyclic gearing. Transpose for A.

24. Transpose for E in the equation $c = \frac{E - e}{r}$.

Equations with Brackets.—Where removal is necessary, or desirable, the removing will generally reduce the example to an equation similar to those in the last section. When the unknown is multiplied by some symbol (as by c in the following example), then it is usually advisable to remove the brackets.

Example 206.—The formula $p = f(1 - ec)$ is used in calculating the sizes of struts and stancheons. Transpose for e.

	$p = f(1 - ec)$
Removing brackets,	$p = f - fec$
Transposing,	$fec = f - p$
$\div fc$	$e = \frac{f - p}{fc}$

Where the unknown stands alone as a term in the bracket then it is usually better *not* to remove the brackets. By multiplying or dividing, the value of the *bracket* is obtained. The brackets can then be left out, without altering the value of the expression, and the example then usually appears as those in the last section. The following two examples illustrate:—

Example 207.—If R is the temperature on the Réaumur thermometer, and F that on the Fahrenheit, then $R = \frac{4}{9}(F - 32)$. Find a formula for **F**.

	$\frac{4}{9}(\mathbf{F} - 32) = \mathrm{R}$
$\times \frac{9}{4}$	$\mathbf{F} - 32 = \frac{9}{4}\mathrm{R}$
$+ 32$	$\mathbf{F} = \frac{9}{4}\mathrm{R} + 32$

Example 208.—The following formula is met with in connection with plate-web girders $Z = h\left(A + \frac{a}{6}\right)$. Transpose for **A**.

	$h\left(A + \frac{a}{6}\right) = Z$
$\div h$	$A + \frac{a}{6} = \frac{Z}{h}$
$- \frac{a}{6}$	$\mathbf{A} = \frac{Z}{h} - \frac{a}{6}$

Exercises 54. On Equations involving Use of Brackets.

[*Note.*—In many cases the removal of the brackets will not be necessary. See above.]

1. Transpose for T in the formula $H = ws(T - t)$, which relates to heat experiments.

2. The formula $P = C(A_c + mA_s)$ refers to reinforced concrete columns. Transpose for A_c.

3. Transpose for D in the formula $c^2 = h(D - h)$, which relates to circular arcs.

4. In Exs. 46, No. 1, the formula $333(r + 2) = V$ occurs. Transpose for r.

5. The formula $V(R - 2{\cdot}5) = 2R$ refers to Weston pulley blocks. Find a formula for R.

6. Transpose for r from the equation $\frac{c^2}{4} = h(2r - h)$.

7. Solve for w in the formula $H = (W + w)(T - t)$, which refers to heat experiments.

8. The formula $C = \frac{5}{9}d(W + w)$ relates to experiments on the "calorific value" of fuels. Transpose for w.

9. Transpose for F in the temperature conversion formula:

$$C = \frac{5}{9}(F - 32).$$

10. In Ex. 22, No. 9, the formula $S = 600(r + 1)$ occurs. Transpose for r.

11. Lloyd's Rule for Fox Furnaces is $P = \frac{1259(16T - 2)}{D}$, where P = working pressure, D = greatest diameter, and T = thickness. Transpose for T.

12. The formula $H.P. = \frac{(T_1 - T_2)S}{33000}$ refers to transmission of power by ropes. Transpose for T_1.

13. In Ex. 46, No. 11, we have the formula $R = R_0(1 + \alpha t)$. Find a formula for α.

14. In Ex. 46, No. 12, we have the formula $H = {\cdot}2d^2(r + 1)$. Transpose for r.

15. Transpose for g in the formula $p = f(1 - gc^2)$.

16. In the formula $P = \frac{Wh}{6(d + 1)}$ transpose for d.

17. Find a formula for a from the equation $Z = h\left(A + \frac{a}{6}\right)$, which refers to the strength of plate girders.

18. Find a formula for t from the equation in Ex. 7 above.

19. In connection with the "wire test" of Whitworth screw threads we have the equation $\frac{d}{1{\cdot}6p - d} = {\cdot}4617$. Solve for d.

Cases requiring the Insertion of Brackets.—When the unknown symbol occurs in more than one term in the given formula, then usually brackets will have to be *inserted*, the unknown being taken out as a common factor, as on p. 147.

Example 209.—The formula $W = cA + mca$ refers to a reinforced concrete column. Transpose for c.

$$cA + mca = W$$

Taking out common factor c, $\quad c(A + ma) = W$

$\div (A + ma)$ $\quad c = \frac{W}{A + ma}$

Explanation.—The unknown occurs in two terms, hence its value cannot be found directly. But if the common factor c (the unknown)

be taken out as shown, the bracketed quantity may be regarded as a single symbol multiplying the unknown, and by which we can divide across.

Example 210.—Transpose for n in the equation $d - n = nf$.

	$d - n = nf$	
$+ n$	$d = nf + n$	
Inserting brackets,	$= n(f + 1)$	
$\div (f + 1)$	$n = \frac{d}{f + 1}$	

Note.—Transpose unknown from the left so that all terms containing it are on the right. Take out the unknown as a common factor and divide across by the bracketed quantity.

Example 211.—If $em - ea = n - a$, transpose for a.

	$em - ea = n - a$
Transpose,	$em - n = ea - a$
Taking out common factor a,	$= a(e - 1)$
$\div (e - 1)$	$\frac{em - n}{e - 1} = a$

There are other equations, of fractional form, which usually involve the removal or the insertion of brackets. The unknown may be in the denominator or in the numerator, or in both, and the equation is usually first cleared of fractions by multiplying through by the denominator (or the L.C.M. of the denominators, if there are several of these).

Example 212.—When two electrical resistances, r ohms and s ohms, are connected in parallel, then the equivalent single-resistance R ohms is given by the equation $\frac{1}{R} = \frac{1}{r} + \frac{1}{s}$. Find a formula for R.

	$\frac{1}{R} = \frac{1}{r} + \frac{1}{s}$	*Note.*—
	$\frac{1}{R} = \frac{s + r}{rs}$	$\frac{Rrs}{R} = rs$
Cross-multiplying,	$rs = R(s + r)$	$\frac{Rrs}{rs} = R.$
$\div (s + r)$	$\frac{rs}{s + r} = R$	

Explanation.—The right-hand side of the equation is the addition of two vulgar fractions. Add these by the rules of Chap. I. The L.C.M. of r and s can only be rs, and therefore $\frac{1}{r} = \frac{s}{rs}$ and $\frac{1}{s} = \frac{r}{rs}$. Cross-multiply, put brackets round s and r to take the place of the horizontal dividing line, and divide by $(s + r)$, giving the formula for R.

Example 213.—Transpose for n in the formula $\frac{n}{n-1} = x$.

$$\frac{n}{n-1} = x$$

$\times (n-1)$ $\qquad n = x(n-1)$

$\qquad = xn - x$

Transposing, $\qquad x = xn - n$

$\qquad = n(x-1)$

$\div (x-1)$ $\qquad n = \frac{x}{x-1}$

Example 214.—Transpose for p in the formula $\frac{p-q}{p+q} = s$, a formula in connection with compound stresses.

$$\frac{p-q}{p+q} = s$$

$\times (p+q)$ $\qquad p - q = s(p+q)$

$\qquad = sp + sq$

Transposing, $\qquad p - sp = sq + q$

Inserting brackets, $\qquad p(1-s) = q(s+1)$

$\div (1-s)$ $\qquad p = \frac{q(s+1)}{1-s}$

Explanation.—Multiply across by the denominator $p+q$, placing brackets round the $p+q$, since the whole of this is to be multiplied by s. Then remove the brackets algebraically, so that all the terms containing p can be brought to the left, and the others to the right. Then insert brackets and divide across by $1-s$.

Example 215.—When a number of secondary cells are connected in series we have the formula $C = \frac{nE}{R + nr}$, where C = current in circuit, n = number of cells, E = electro-motive force of one cell, r = internal resistance of one cell and R = external resistance. Find a formula for n.

$$C = \frac{nE}{R + nr}$$

$\times (R + nr)$ $\qquad C(R + nr) = nE$

$\qquad CR + Cnr = nE$

$- Cnr$ $\qquad CR = nE - Cnr$

$\qquad = n(E - Cr)$

$\div (E - Cr)$ $\qquad \frac{CR}{E - Cr} = n$

Exercises 55. Equations requiring Insertion of Brackets.

1. Transpose for e in the formula $em + a = n + ea$.
2. Find a formula for Δ (delta) from the equation $\frac{F\Delta}{2} = W(H + \Delta)$ which relates to the strength of bodies under shock.
3. Transpose for n in the formula $C(nr + R) = nE$.

4. Find a formula for T from the equation $T - t = \eta T$.

5. Transpose for d in the formula $d + k(t - d) = N$, which relates to plate condensers.

6. Transpose for t in the expression $w(L + t) = W(T - t)$, which relates to the latent heat of water.

7. In connection with wire ropes the formula $D = \frac{nd}{13} + 7d$ occurs. Solve for d.

8. Transpose for S in the formula $G + S = Sm$, which relates to a galvanometer.

9. Find a formula for n from the equation $n = k(d - n)$.

10. Transpose for t in the formula $g(H - t) = T - t$, which relates to economisers.

11. The formula $S = \frac{Cdh^2}{2} - \frac{Wb}{2}$ refers to the strain on a chimney due to wind pressure. Transpose for C.

12. Transpose for m_1 in the formula $v_1 = \frac{mv}{m + m_1}$ which relates to momentum (or quantity of motion).

13. Find a formula for a if $e = \frac{n - a}{m - a}$: a gear-wheel formula.

14. Transpose for a if $g = \frac{h}{3} \cdot \frac{a + 2b}{a + b}$.

15. Solve for p in the formula $\frac{b}{p} = \frac{R^2r^2}{R^2 - r^2}$, which refers to the stress in thick pipes. (*Hint.*—Cross-multiply.)

16. The equation $h - e = \frac{1}{2}e$ occurs in a problem on hydraulics. Find the value of e.

Equations requiring Square Root.—Many cases occur where the unknown appears in an equation as a square, and the equation is solvable by using the methods of the true simple equation with the one addition of taking a square root. Generally the equation should be worked as though the *square of the unknown* were the unknown quantity. When a value is obtained for the *square*, then by taking the square root of each side of the equation the value of the actual unknown is obtained. In no case should the root be taken until the *whole of one side* of the equation is contained under the index.

Example 216.—If V is the velocity of the wind in miles per hour, and P the pressure per sq. ft. caused by it on a flat surface, then the following formula (found by experiment) connects P and V: $P = KV^2$. Find a formula for V.

$$P = KV^2$$

$\div K$ $$\frac{P}{K} = V^2$$

Taking $\sqrt{\ }$ $$\sqrt{\frac{P}{K}} = V$$

Explanation.—Consider (V^2) as the unknown, and divide the equation by K. Then take square root of each side, obtaining a formula for V. The *whole* of the left-hand side must be written under the root sign.

Example 217.—If C is the circumference of a white manilla rope in inches, and S the safe load in tons that it can carry, then $S = \frac{C^2}{30}$. Transpose for C.

$$S = \frac{C^2}{30}$$

$\times 30$ $\quad 30S = C^2$

Taking $\sqrt{\ }$ $\quad \sqrt{30S} = C$

$\therefore \quad C = \sqrt{30}\ \sqrt{S}$

$\quad = 5{\cdot}48\ \sqrt{S}$

Note.—Multiply by 30 to find value of C^2. Then take $\sqrt{\ }$ of whole, giving value of C. By separating the 30 and S as shown, $\sqrt{30}$ is reduced to a definite figure.

Example 218.—In connection with rope-driving we have the formula $f = \frac{wv^2}{ga}$. Find the formula for v.

$$f = \frac{wv^2}{ga}$$

$\times \frac{ga}{w}$ $\quad \frac{fga}{w} = v^2$

Taking $\sqrt{\ }$ $\quad \sqrt{\frac{fga}{w}} = v$

Example 219.—If H is the "height" of an engine governor in feet, and N its speed in revolutions per minute, then $H = \frac{2938}{N^2}$. Transpose for N.

$$H = \frac{2938}{N^2}$$

$\times N^2$ $\quad HN^2 = 2938$

$\div H$ $\quad N^2 = \frac{2938}{H}$

Taking $\sqrt{\ }$ $\quad N = \sqrt{\frac{2938}{H}}$

$$= \frac{\sqrt{2938}}{\sqrt{H}} = \frac{54{\cdot}1}{\sqrt{H}}$$

Note.—After taking the root, the expression is simplified by taking roots of both numerator and denominator. The numerator being a number, its root may be evaluated, as shown.

Example 220.—If a circular plate be supported all round its circumference and be uniformly loaded, we have the following formula:

$f = \frac{2}{3}\frac{R^2}{t^2}p$, when f = stress in plate, R = radius of plate, t = thickness, and p = pressure. Find a formula to give thickness t directly.

$$f = \frac{2}{3}\frac{R^2}{t^2}p$$

$\times t^2$ $$t^2 f = \frac{2}{3}R^2 p$$

$\div f$ $$t^2 = \frac{2R^2 p}{3f}$$

Taking $\sqrt{\ }$ $$t = \sqrt{\frac{2R^2 p}{3f}}$$

$$= \sqrt{\frac{2}{3}} \cdot \sqrt{R^2} \cdot \sqrt{\frac{p}{f}}$$

$$\therefore \quad t = \cdot 816\, R\sqrt{\frac{p}{f}}$$

Note.—In clearing of fractions here, it is only necessary to multiply by t^2. Using the whole denominator $3t^2$ would necessitate dividing by $3f$ to find the value of t^2, bringing the 3 back to its original place. The final expression is much simplified by taking separate roots in the last steps, as shown.

Example 221.—The formula $\frac{I}{C^2} - 1 = x$ relates to the flow of water through orifices. Transpose for C.

$$\frac{I}{C^2} - 1 = x$$

$+ 1$ $$\frac{I}{C^2} = x + 1$$

$\times C^2$ $$I = C^2(x + 1)$$

$\div (x + 1)$ $$\frac{I}{x + 1} = C^2$$

Taking $\sqrt{\ }$ $$\sqrt{\frac{I}{x + 1}} = C$$

Explanation.—As the left-hand side contains two separate terms, first transfer the 1 to the other side, so as to find the value of the term containing the unknown. Then proceed as usual, taking care to insert the brackets in the third step.

Example 222.—Transpose for x in the formula $(x + 1)^2 = c$.

$$(x + 1)^2 = c$$

Taking $\sqrt{\ }$ $$x + 1 = \sqrt{c}$$

$- 1$ $$x = \sqrt{c} - 1$$

Note.—As the whole of the left-hand side containing the unknown is squared, the square root is taken first of all.

Example 223.—The Board of Trade formula for flat surfaces with screwed stays (over 6″ pitch) is $P = \frac{C(t+1)^2}{S-6}$: where P = working pressure; C = a constant; t = thickness of plate in sixteenths of an inch; and S = surface supported in sq. ins. Find a formula for t.

$$P = \frac{C(t+1)^2}{S-6}$$

$\times (S-6)$ $\qquad P(S-6) = C(t+1)^2$

$\div C$ $\qquad \frac{P}{C}(S-6) = (t+1)^2$

Taking $\sqrt{\ }$ $\qquad \sqrt{\frac{P}{C}(S-6)} = t+1$

-1 $\qquad \sqrt{\frac{P}{C}(S-6)} - 1 = t$

Note.—The first two steps can be combined by multiplying by $\frac{(S-6)}{C}$; they are separated here, as the expression is rather heavy. Note in last line that the -1 must *not* be included under the $\sqrt{\ }$ sign.

Equations requiring Squaring.—Certain cases, where the unknown appears under a square root sign, may be solved by considering the square root of the unknown as the unknown, solving for this by previous methods, and finally squaring to give the true unknown. The squaring must only be performed when the *whole* of one side of the equation is under the square root sign.

Example 224.—The formula $w = 1000\sqrt{d}$ relates to the rollers for expansion bearings of large bridge girders, d being the diameter in ins., and w the load in lbs. per in. of length. Transpose for the diameter d.

$$w = 1000\sqrt{d}$$

$\div 1000$ $\qquad \frac{w}{1000} = \sqrt{d}$

Squaring— $\qquad \left(\frac{w}{1000}\right)^2 = d$

$$\therefore \quad d = \frac{w^2}{1{,}000{,}000}$$

Explanation.—Divide across by 1000, as the value of $\sqrt{d}$ is first required. Square each side to give value of d. The formula may be simplified by squaring the top and bottom separately.

Example 225.—If B = the greatest permissible wheel base in ft., R = radius of quickest curve in ft., and T = width of rail slot in ins., on an electric tramway track, then $B = \sqrt{\frac{2RT}{3}}$. Find a formula to

give the radius of the quickest curve for a given wheel base and width of slot (*i. e.* find a formula for R).

$$B = \sqrt{\frac{2RT}{3}}$$

Squaring— $$B^2 = \frac{2RT}{3}$$

$\times \frac{3}{2T}$ $$\frac{B^2 \times 3}{2T} = R$$

$$\therefore \quad R = \frac{1{\cdot}5B^2}{T}$$

Note.—As the whole of the right-hand side is under a root sign, squaring is the first operation.

Example 226.—With the usual pitch of 4″ for the screwed stays in locomotive fire-boxes, $T = \sqrt{\frac{P}{2}} - 1$, where T = thickness of plate in sixteenths of an inch, and P = working pressure in lbs. per sq. in. Transpose for P.

$$T = \sqrt{\frac{P}{2}} - 1$$

$+ 1$ $$T + 1 = \sqrt{\frac{P}{2}}$$

Squaring— $$(T + 1)^2 = \frac{P}{2}$$

$\times 2$ $$2(T + 1)^2 = P$$

Note.—The 1 is first transposed to get the value of the term containing the unknown.

Example 227.—If T is the time in seconds of one swing of a pendulum and l is its length in feet, then $T = 2\pi\sqrt{\frac{l}{g}}$, where g is a constant. Find a formula for l.

$$T = 2\pi\sqrt{\frac{l}{g}}$$

$\div 2\pi$ $$\frac{T}{2\pi} = \sqrt{\frac{l}{g}}$$

Squaring— $$\left(\frac{T}{2\pi}\right)^2 = \frac{l}{g}$$

Simplifying— $$\frac{T^2}{4\pi^2} = \frac{l}{g}$$

$\times g$ $$\frac{T^2 g}{4\pi^2} = l$$

Example 228.—The equation $C = \sqrt{\frac{2g}{\mu}}$ relates to problems on water supply. Transpose for μ.

$$C = \sqrt{\frac{2g}{\mu}}$$

Squaring $$C^2 = \frac{2g}{\mu}$$

$\times \mu$ $$C^2\mu = 2g$$

$\div C^2$ $$\mu = \frac{2g}{C^2}$$

Example 229.—Transpose for x in the formula $C = \sqrt{\frac{1}{1+x}}$.

$$C = \sqrt{\frac{1}{1+x}}$$

Squaring— $\quad C^2 = \frac{1}{1+x}$

$\times(1+x)$ $\quad C^2(1+x) = 1$

$\div C^2$ $\quad 1 + x = \frac{1}{C^2}$

-1 $\quad x = \frac{1}{C^2} - 1$

Note.—Comparing with Example 221, it will be seen that the operations are in exactly the reverse order.

Exercises 56. Equations involving Square Root and Squaring.

1. Transpose for C in the equation $C^2R = x$.

2. If $\frac{E^2}{R} = x$, find a formula for E.

3. From the formula $E = \frac{1}{2}I\omega^2$ deduce a formula for ω.

4. The formula $A = \cdot 525K^2$ refers to egg-shaped sewers. Transpose for K.

5. Transpose for B in the formula $P = \frac{B^2A}{11{,}180{,}000}$, which relates to the pull on an electric magnet.

6. Solve for D in the equation $\cdot 75D^2P = d^2p$.

7. Transpose for a in the formula $f = \frac{pa^2}{t^2}$, which relates to the strength of a flat square plate.

8. The formula $W = \frac{abd^2}{l}$ occurs in Exs. 48, No. 10. Transpose for d.

9. The formula $T = \frac{wL^2}{8d}$ relates to trolley wires. Solve for L.

10. Solve for n in the formula $H = \frac{\cdot 816}{n^2}$, which relates to engine governors.

11. Transpose for t in the formula in Ex. 7 above.

12. If b = depth and h = length of winding space, and d = dia. of wire outside insulation, and S = no. of turns, then in a field coil $S = \frac{bh}{d^2}$. Transpose for d.

13. The formula $I = \frac{EK}{w^2}$ refers to fly-wheels. Solve for w.

14. In Exs. 48, No. 13, the formula $\frac{V^2}{2g}J = H_1 - H_2$ occurs. Transpose for V.

15. Transpose for v in the formula $\frac{v^2}{2g} + h = H$.

16. Solve for r in the formula $q = A - \frac{B}{r^2}$, which relates to the strength of thick cylinders.

17. Transpose for d in the formula $H = \cdot 2d^2(r + 1)$.

18. Transpose for x in the equation $(x + 1)^2 = 81l^2$.

19. Cooper's formula for the spacing of stiffeners in plate girders is $f = \dfrac{5}{1 + \dfrac{1500d^2}{t^2}}$. Solve for d.

EQUATIONS INVOLVING SQUARING

20. Solve for x in the equation $\sqrt{x - 1} = y$.

21. Transpose for m in the formula $v = C\sqrt{mi}$, which relates to the flow of water.

22. The formula $d = 1 \cdot 2\sqrt{t}$ relates to riveted work. Transpose for t.

23. Transpose for B in the formula $f = \sqrt{\dfrac{B}{\mu b}}$ which relates to reinforced concrete beams.

24. Transpose for μ in the previous formula (No. 23).

25. The Lanchester motor rating formula is $H = \cdot 4d^2\sqrt{r}$. Find a formula for r.

26. Transpose for N in the formula $D = p\sqrt{\dfrac{4N}{\pi}}$, which refers to condenser plates.

27. The formula $d = \sqrt{\dfrac{D^2}{R}}$ relates to cone pulleys where d = smallest diameter, D = large diameter, and R = ratio of cone. Transpose for D.

28. Solve for l in the formula $x = \sqrt{\dfrac{a(l + b)}{3}}$, which refers to deflection of beams.

29. Transpose for d, in the formula $\dfrac{f}{f_1} = \sqrt{\dfrac{d}{d_1}}$.

30. The formula $t = 2\pi\sqrt{\dfrac{W}{gF}}$ refers to spring calculations. Transpose for F.

31. If $\dfrac{D}{d} = \sqrt{\dfrac{f + p}{f - p}}$, transpose for f. (*Hint.*—Cross-multiply after squaring.)

32. Transpose for p in the previous formula (Ex. 31).

33. Transpose for C in the formula $\lambda = k\sqrt{LC}$, which relates to wireless telegraphy.

34. The equation $a = \sqrt{b^2 - c^2}$ occurs in connection with right-angled triangles. Transpose for c.

35. In the formula in Ex. 31 above, which relates to thick cylinders subject to internal pressure, D always equals $d + 2t$, where t is the thickness of the cylinder. For a particular case $d = 10t$. Substitute for D and d in terms of t in the equation which is the result of Ex. 31, and simplify to give an equation like $f = ap$, where a is a number. (*Hint.*—If $D = d + 2t$, and $d = 10t$, then $D = 12t$.)

CHAPTER VI

USE OF LOGARITHMS

Introductory.—Much arithmetical calculation can be considerably shortened and simplified by using **logarithms,** which are certain numbers calculated by mathematicians and entered into tables for reference. By their use, the lengthy and laborious operations of multiplication and division (which are of very frequent occurrence), are replaced by the simpler operations of addition and subtraction; while, in higher branches of calculation, there are certain operations which would be practically impossible without logarithms.

It is not possible, in this book, to explain fully the principles underlying logarithms and their uses. The reader must accept the statements made until a later stage, but he can, with the aid of Chap. III, learn the meaning of a logarithm. Briefly, a logarithm is an "index." It has already been seen on p. 94, that in statements such as $10^2 = 100$, $10^3 = 1000$, and $10^4 = 10000$, the numbers 2, 3, and 4 are called "indices," while the quantities 10^2, 10^3, and 10^4 are "powers of 10." Now, considering the *values* of these powers as **numbers,** the indices are said to be **logarithms** of the numbers, while the number 10, which appears in all the given statements, is called the **base** of the logarithms. Describing the indices 2, 3, and 4 in greater detail, they are "logarithms of the numbers 100, 1000 and 10000, respectively, to the base 10." As many statements similar to $10^2 = 100$ are possible, we can use symbols to give a more general form and write

$$a^x = N$$

where N is the number, x is the logarithm and a is the base, and x is then "the logarithm of N to the base a."

Any positive number may be chosen as base. Similarly, the symbols x and N may have a variety of values, but *with any one base,* each value of N has a corresponding value of x, or, every number has its own particular logarithm. A base having been decided upon, it is possible, with the aid of higher mathematics, to calculate to

any accuracy desired the logarithms of all numbers to this base, and so a " system " of logarithms is constructed. The only system considered in this book will be that in which 10 is the base, this being the one of greatest practical importance. To keep the size of the tables within reasonable limits, and to shorten the actual working, only a certain number of significant figures is considered. Thus we have **four-figure logarithms,** which are logarithms calculated to four decimal places *for numbers containing only four significant figures.*

For the great majority of engineering calculations these are quite suitable. The actual logarithm table for four-figure logarithms is quite small, only occupying two pages. For more accurate calculation there are 7-figure logarithms which are logarithms given to 7 decimal places, having been calculated for numbers containing 7 significant figures. The logarithm table in this case is about 100 times larger than the four-figure table, and is consequently more tedious to handle, while the working is correspondingly longer.

Of course, when it is desired to find the logarithm of a number which is given to, say, 5 or 6 significant figures, then the number must first be reduced to 4 significant figures if the 4-figure tables are to be used.

A logarithm (or " log " as it is commonly called) is usually a mixed number, such as 3·4512, but it is convenient to consider it as consisting of two parts (*a*) the whole number, (*b*) the decimal fraction.* It is only the decimal portion that is actually found from the tables, the whole number being supplied from an examination of the number. The whole number may be either + or −, as will be shown, *but the decimal part,* as found from the tables, *is always positive.*

Finding Logarithms. (1) Whole Number.—The whole number of a logarithm is found by observing the following rules:—

1. When the number is 1 or over, then the whole number of the logarithm is 1 less than the number of figures (or digits) in front of the decimal point, and is plus.

It has no connection with the actual figures in the number. Thus, in 235·9 the whole number of the log is 2, because there are 3 figures in front of the decimal point and 3 − 1 = 2.

---

* In mathematical language the whole number is called the " characteristic " and the decimal portion the " mantissa."

Similarly—

13500 . . .	5 figs.	in	front	∴	whole	number	is	4
29,500,000. .	8 ,,	,,	,,	,,	,,	,,	,,	7
2240 . . .	4 ,,	,,	,,	,,	,,	,,	,,	3
14·7. . . .	2 ,,	,,	,,	,,	,,	,,	,,	1
3·142 . . .	1 ,,	,,	,,	,,	,,	,,	,,	0

(because 1 − 1 = 0).

2. When the number is less than 1, then the whole number of the logarithm is 1 more than the number of noughts between the first figure and the decimal point, and is minus. Again, it does not depend upon the actual figures in the number.

Thus in ·0952 the whole number is − 2 because there is one nought between the decimal point and the first figure 9, and 1 + 1 = 2.

Similarly—

·0807 . . . . . 1 nought between dec. point and first figure
∴ whole number is − 2

·0000087 . . . 5 noughts between dec. point and first figure
∴ whole number is − 6

·000341 . . . . 3 noughts between dec. point and first figure
∴ whole number is − 4

·3937 . . . . . 0 nought between dec. point and first figure
∴ whole number is − 1

(because 1 more than 0 is 1).

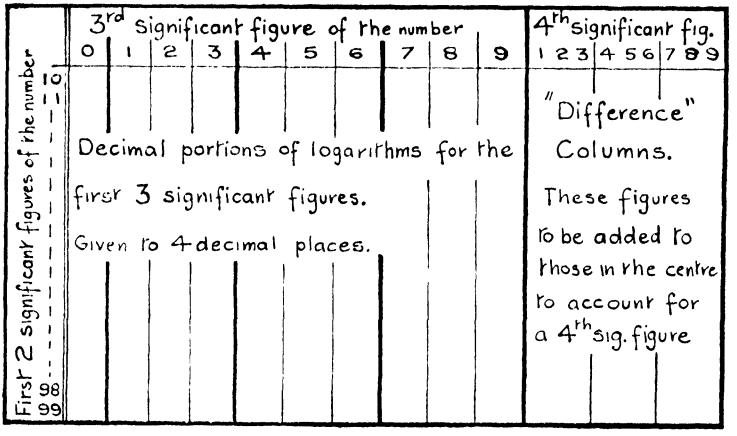
When the whole number is + , the complete log is written in the ordinary way, *e. g.*, 3·2396, since both the whole number and decimal are here positive. But since the whole number of the log is sometimes −, while the decimal is always +, then the − sign cannot be written in front of the log in the ordinary way. Thus with the number ·095, the whole number is − 2, and the decimal part is + ·9777. Now we cannot write − 2·9777, because this would mean that the whole of the number 2·9777 was negative instead of only the 2. Hence, to show this distinction, **the minus sign is written above the whole number.** Thus the log of ·095 is written as $\bar{2}$·9777, meaning − 2 + ·9777.

Finding Logarithms. (2) The Decimal Part.—It must first be noted that the decimal part of a log is independent of the position of the decimal point in the number; it depends only on the significant figures. Thus the decimal part of the logs of the numbers: 39370, 39·37, ·003937 are all the same, being ·5952 in each case. Hence when finding logs from the tables, only the significant

figures in the number need be considered, no decimal point appearing anywhere in a logarithm table.

The logarithm table is arranged in the following way: the extreme left-hand vertical column contains the first two significant figures of the number, while the extreme top horizontal column contains the third and fourth figures. In the body of the table the four-figure numbers are the decimal parts of the logs of the first three significant figures. The one or two figures on the right of the table are "differences" which are to be *added* to the four-figure numbers, to account for the fourth and last significant figure. By using these "difference" columns, the table is made as small as possible. For convenience the table is split horizontally at about the figures

DIAGRAM ILLUSTRATING LOGARITHM TABLE.



54 in the left-hand column, and the lower half is placed on the right-hand side on the opposite page. The above diagram should be compared with the complete table at the end of the book.

It must be remembered that the figures in the left-hand and top columns are not numbers but only significant figures. Thus, on the first line in the left-hand column we have 10. This should not be read as "ten" but as "one nought," and similarly with the other numbers.

The decimal part is found in the following way: Required the decimal of the log of 39·37. The significant figures are 3937. Look down the extreme left-hand column until the figures 39 are reached Pass horizontally across the table until the column is reached which is headed by the 3rd significant figure 3. Here we find the figures

5944. Keeping this number marked with the finger or a pencil, pass across to the " difference " columns until the column headed by the 4th significant figure 7 is reached. Here we find a figure 8. These two numbers 5944 and 8 are shown in heavy type and underlined in the following extract from the logarithm table.

EXTRACT FROM LOGARITHM TABLE.

	0	1	2	3	4	5	6	7	8	9	1 2 3	4 5 6	7 8 9
10	0000	0043	0086	0128	0170	0212	0253	0294	0334	0374	4 9 13 4 8 12	17 21 26 16 20 24	30 34 38 28 32 37
38	5798	5809	5821	5832	5843	5855	5866	5877	5888	5899	1 2 3	5 6 7	8 9 10
39	5911	5922	5933	**5944**	5955	5966	5977	5988	5999	6010	1 2 3	4 5 7	**8** 9 10
40	6021	6031	6042	6053	6064	6075	6085	6096	6107	6117	1 2 3	4 5 6	8 9 10

Then the " difference " 8 is added to the extreme right-hand figure of the four-figure number 5944. Thus—

Opposite 39 at side and under 3 at top	5944
„ „ „ „ „ 7 in difference column	8
Sum	5952

Then the decimal portion of the log of 39·37 is ·5952. The addition can with a little practice be done mentally. The whole number being 1, the complete log is 1·5952.

As another example take the log of 501·9. Look down the left-hand column until the figures 50 are reached. Pass across to the column headed 1, when the figures 6998 are reached. In the " difference " column under 9 and the same horizontal line we find 8.

Then opposite 50 at side and under 1 at top	6998
„ „ „ „ „ „ 9 in difference column	8
Sum	7006

The whole number being 2 the logarithm of 501·9 = 2·7006.

The logs of numbers containing only 2 significant figures, *e. g.*, 93; ·067, etc., are found in the column headed 0, *i. e.*, the one next to the extreme left-hand column, the " difference " column not being required in such cases.

With those numbers containing only 1 significant figure, *e. g.* 700; ·005, etc., a nought should be added mentally to the single significant figure to make two figures, as a single significant figure is not found in the left-hand column of the table. Then in the case of the 700, look opposite 70, and in the case of the ·005 look opposite 50.

It will be noticed from the given logarithm table (or from the extract just given), that in the earlier part of the table there are, for each number in the first column, two lines in the " difference " column. This arrangement (the copyright of Messrs. Macmillan & Co., Ltd.) gives greater accuracy. The logarithm is looked out as explained, and the " difference " figure is found in the same line as the 3rd significant figure of the number. Thus, for log 1048, following the 4 along horizontally, 34 is found in the " difference " column. For log 1078, the " difference " is in the lower line and is 32.

Example 230.—Find the decimal parts of the logarithms of the following numbers : (*a*) 606·5, (*b*) 6080, (*c*) 3·009, (*d*) ·001016.

(*a*) Opposite 60 at side and under 6 at top we find . 7825
On same line under 5 in " difference " column . 4

Then decimal part is . . . . . ·7829

(*b*) Opposite 60 at side and under 8 at top, we find . . 7839 .
No 4th significant figure ∴ Decimal is . . . . ·7839

(*c*) Opposite 30 at side and under 0 at top we find . 4771
On same line under 9 in " difference " column . 13

∴ Decimal part is . . . . . ·4784

Note.—The " difference " column has 2 figures which are added to the 3rd and 4th figures as shown.

(*d*) Opposite 10 at side and under 1 at top we find . 0043
On same line under 6 in " difference " column . 26

∴ Decimal part is . . . . . ·0069

Note.—In the " difference " figures the 6 of 26 goes under the 3.

Example 231.—Find the complete logarithms of the following numbers : (*a*) 5·728; (*b*) ·0001652; (*c*) 5; (*d*) 7004; (*e*) ·1183; (*f*) ·00000066; (*g*) 40400.

(*a*) 1 figure in front of decimal point ∴ whole number is 0.
Opposite 57 and under 2 we find 7574
" " " " 8 in " difference " column . 6

∴ Decimal is . . . . . ·7580

∴ Logarithm = 0·7580

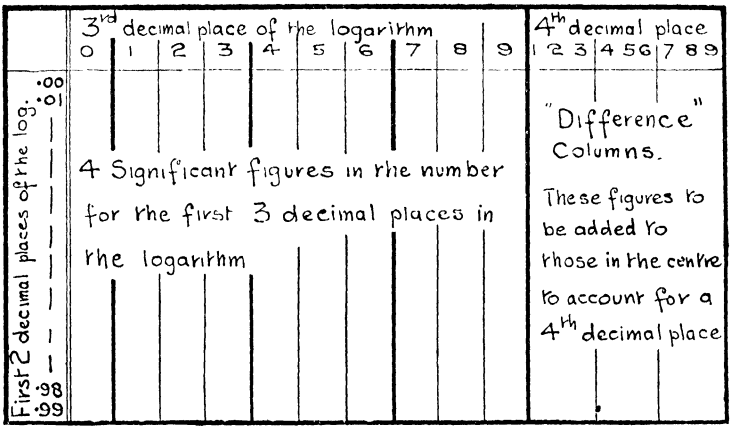
(*b*) 3 noughts between 1st figure and decimal point ∴ whole number is — 4.
Opposite 16 and under 5 we find 2175
" " " " 2 in " difference " column . 5

2180

∴ Logarithm = $\bar{4}$·2180.

Finding Antilogarithms. (1) Significant Figures.—*Only the decimal part of the log is considered.* The antilogarithm table is arranged thus :—The first two decimal places of the log are contained in the extreme left-hand column, while the third and fourth places are found in the extreme top horizontal line. The four figure numbers in the body of the table are the significant figures for the logarithms up to the third decimal place. The numbers of 1 or 2 figures to the right of the table are "differences" which are to be added to the four-figure numbers to account for the fourth decimal place.

DIAGRAM ILLUSTRATING ANTILOGARITHM TABLE.



It will be seen that, in the full tables, the decimal point appears in front of each pair of figures in the left-hand column. This is to distinguish such numbers as ·01 from ·1, etc. The operation of finding the significant figures is carried out in a similar manner to that for finding a logarithm. Thus, let us find the significant figures for the antilog of 2·7829. Look down the left-hand column until the first two decimal places ·78 are found. Move across the table until the column headed by the third decimal place 2 is reached. Here we find the figures 6053. Keeping this marked with the finger, or a pencil, move across to the "difference" columns until the column headed by the 4th decimal place 9 is found. Here we find the figure 13 which is to be added to the four-figure number previously found.

Thus opposite 78 and under 2 we find	6053
" " " " 9 in the "difference" column	13
∴ Significant figures in antilog are . .	6066

These numbers are shown in bold type and underlined in the following extract from the antilogarithm table:—

EXTRACT FROM ANTILOGARITHM TABLE.

	0	1	2	3	4	5	6	7	8	9	1 2 3	4 5 6	7 8 9
·00	1000	1002	1005	1007	1009	1012	1014	1016	1019	1021	0 0 1	1 1 1	2 2 2
·77	5888	5902	5916	5929	5943	5957	5970	5984	5998	6012	1 3 4	5 7 8	10 11 12
·78	6026	6039	**6053**	6067	6081	6095	6109	6124	6138	6152	1 3 4	6 7 8	10 11 **13**
·79	6166	6180	6194	6209	6223	6237	6252	6266	6281	6295	1 3 4	6 7 9	10 11 13

As another example, let us find the significant figures in the antilog of $\bar{3}\cdot2104$.

Looking opposite ·21 in the left-hand column and under the column headed 0 we find 1622
On the same line and under 4 in the "difference" column 2
∴ Significant figures in antilog are 1624

Example 232.—Find the significant figures in the antilogarithms of the following: (*a*) 0·7103; (*b*) $\bar{3}\cdot2997$; (*c*) 2·0019; (*d*) $\bar{1}\cdot4779$.

(*a*) Opposite ·71 and under 0 we find 5129
" " " " 3 in "difference" column . 4
∴ Significant figures in antilog are . . 5133

(*b*) Opposite ·29 and under 9 we find 1991
" " " " 7 in "difference" column . 3
∴ Significant figures in antilog are . . 1994

(*c*) Opposite ·00 and under 1 we find 1002
" " " " 9 in "difference" column . 2
∴ Significant figures in antilog are . . 1004

(*d*) Opposite ·47 and under 7 we find 2999
" " " " 9 in "difference" column . 6
∴ Significant figures in antilog are . . 3005

Finding Antilogarithms. (2) Placing the Decimal Point. Having obtained the significant figures in the antilog the decimal point is fixed according to the following rules:—

1. If the whole number of the logarithm is plus, then the number of figures before the decimal point must be one more than the whole number.

Taking (*c*), Example 232, the log is 2·0019 and the significant figures are 1004. Then the antilog will be 100·4, *i.e.*, the whole

number is 2 and therefore there will be three figures in front of the decimal point.

2. If the whole number is minus, then the number of noughts between the first figure and the decimal point must be made 1 less than the whole number.

Taking (*b*) Example 232, the log is $\bar{3}\cdot2997$, and the significant figures are 1994. Then the antilog will be ·001994, *i. e.*, the whole number is $\bar{3}$, and therefore there will be two noughts between the first figure and the decimal point.

Example 233.—Find the antilogarithms of the following (*i. e.*, find the numbers whose logs are) : (*a*) ·7103; (*b*) $\bar{1}\cdot4779$; (*c*) $\bar{2}\cdot9786$; (*d*) 5·3001.

(*a*) Significant figures are 5133; see (*a*) Ex. 232. Whole number is 0, ∴ there must be 1 figure before the decimal point.

∴ Antilog is 5·133.

(*b*) Significant figures are 3005; see (*d*) Ex. 232. Whole number is $\bar{1}$, ∴ there must be no noughts between the first figure and the decimal point.

∴ Antilog is ·3005.

(*c*) Opposite ·97 and under 8 we find	9506
,, ,, ,, ,, 6 in "difference" column .	13
∴ Significant figures are	9519

Whole number is $\bar{2}$, ∴ there must be 1 nought between the first figure and the decimal point.

∴ Antilog is ·09519.

(*d*) Opposite ·30 and under 0 we find	1995
,, ,, ,, ,, 1 in "difference" column .	0
∴ Significant figures are	1995

Whole number is 5, ∴ there must be 6 figures in front of the decimal point.

∴ Antilog is 199500.

A mistake sometimes made when using logarithms is to look up the antilog table in place of the log table, and *vice versâ*. To avoid this, it should be remembered that *there is no decimal point anywhere in the logarithm table;* the decimal point only appears in the left-hand column of the antilogarithm table.

When working examples logarithmically, it is usual to abbreviate the words "the logarithm of" to "Log," so that the statement—

$$\text{Log } \cdot09519 = \bar{2}\cdot9786$$

means "the logarithm of ·09519 = $\bar{2}\cdot9786$."

Exercises 58. On Finding Antilogarithms.

THE SIGNIFICANT FIGURES

Find from the antilogarithm table the significant figures in the antilogs of the following:—

1. 1·245, 2·659, 4·50, 3·301.
2. 0·005, 3·0004, 0·227, 1·707.
3. $\bar{1}$·7495, $\bar{2}$·3989, $\bar{1}$·7307, 2·1486.
4. ·9989, $\bar{5}$·6739, $\bar{1}$·8055, 3·9499.
5. 0·2699, 3·4928, $\bar{1}$·8407, $\bar{2}$·0002.
6. $\bar{4}$·9599, ·7956, $\bar{3}$·9039, 1·9009.

PLACING THE DECIMAL POINT

7. to 12. Place the decimal point in the significant figures found from Questions 1 to 6 above, according to the whole number in each case.

COMPLETE ANTILOGARITHMS.

Find the antilogs of the following:—

13. $\bar{3}$·0095, 2·193, 1·301, ·3069.
14. 4·3286, 1·3595, $\bar{1}$·301, ·4972.
15. $\bar{5}$·1001, 3·3511, ·5956, 6·0969.
16. ·7575, $\bar{1}$·2091, $\bar{6}$·2201, 7·4771.
17. 3·3502, ·9896, $\bar{1}$·0021, 1·9795, $\bar{1}$·8951.
18. 1·0001, 3·2375, $\bar{3}$·1115, ·1009, $\bar{1}$·8751.

Multiplication by Logarithms.—This is effected according to the following rule:—

Find the logarithms of the numbers to be multiplied. Add them together, and then find the antilogarithm of the sum.

Expressed in symbols—

If $X = A \times B$, then $\log X = \log A + \log B$.

Note that *the multiplication of numbers* is performed by *the addition of their logarithms.*

The following examples illustrate the best method of setting down this work, and should be closely followed. Care should be taken that the decimal points and the various decimal places are arranged in straight vertical columns, as emphasised for decimal addition in Chap II.

Example 234.—Find the result of 7·2 × 62·5 by logarithms.

Log 7·2	= 0·8573
Log 62·5	= 1·7959
Log of product	= 2·6532 = Sum

Product = Antilog = 450

The addition of the decimal portions presents no difficulty, as these are always +, but when adding the whole numbers care must be taken that the *algebraic sum* is obtained, as both + and − numbers will be met with.

Example 235.—Perform the calculation of Ex. 43, p. 49 (·785 × ·0039), by logarithms.

Log ·785	= $\bar{1}$·8949
Log ·0039	= $\bar{3}$·5911
Log of product	= $\bar{3}$·4860 = Sum

Product = Antilog = ·003062

Note.—When adding the logs, the carrying figure from the first decimal place is + 1. Then we have to add + 1, − 1, and − 3, which gives − 3 for the whole number.

When several numbers have to be multiplied then all the logs may be added up and the antilog found only at the end.

Example 236.—Calculate by logarithms the value of 3·14 × 3·14 × 13000 × ·021 tons.

Log 3·14	= 0·4969
Log 3·14	= 0·4969
Log 13000	= 4·1139
Log ·021	= $\bar{2}$·3222
Log of product	= 3·4299
Antilog	= 2691
∴ Result	= 2691 tons.

Note.—Carrying figure from the first decimal place is + 1. Then the sum of + 1, − 2 and + 4 = − 1 + 4 = 3.

If the student should work this example by the ordinary method the amount of labour saved will at once be seen.

Example 237.—The expression ·667 × ·475 × 2·023 × 4·093 occurred in connection with the strength of a channel section. Complete the working, using logarithms.

Log ·667	= $\bar{1}$·8241
Log ·475	= $\bar{1}$·6767
Log 2·023	= 0·3060
Log 4·093	= 0·6120
Log of product	= 0·4188

Product = Antilog = 2·623

Note.—Carrying figure from first decimal place is + 2. The sum of + 2, − 1, and − 1 is 0.

Exercises 59. On Multiplication by Logarithms.

Work the following by logarithms:—

1. $3{\cdot}56 \times 21{\cdot}95$ **2.** $\cdot095 \times 316.$ **3.** $\cdot00032 \times 60 \times 13.$
4. $1{\cdot}414 \times \cdot5 \times \cdot7071.$ **5.** $\cdot7 \times 384{\cdot}5 \times \cdot93 \times 12.$

For further practice work a number of the following exercises by logarithms:—

Chap. II: Exs. 12; Chap. III: Exs. 21, Nos. 1 to 6.

Division by Logarithms.—The division of one number by another, using logs, is carried out according to the following rule:—

Find the logarithm of each number. Subtract the logarithm of the divisor from the logarithm of the number to be divided. Find the antilogarithm of the difference.

Expressed in symbols—

If $X = \frac{A}{B}$ **then** $\log X = \log A - \log B.$

Note that *the division of numbers* is performed by *the subtraction of their logarithms.*

Example 238.—Find the value of $\frac{238}{2{\cdot}47}$ (last piece of calculation in Ex. 57, p. 65).

Log 238	$= 2{\cdot}3766$
Log $2{\cdot}47$	$= 0{\cdot}3927$
Log of quotient	$= \underline{1{\cdot}9839}$ = Difference

Quotient = Antilog = $\underline{96{\cdot}36}$

The subtraction of the decimal part presents no difficulty as it is always positive. But since the whole numbers may be both positive and negative, the subtraction must be performed in the *algebraic* way.

Example 239.—Find the value of $\frac{\cdot695}{160}$ by logarithms.

Log $\cdot695$	$= \bar{1}{\cdot}8420$
Log 160	$= 2{\cdot}2041$
Log of quotient	$= \underline{\bar{3}{\cdot}6379}$ = Difference

Quotient = Antilog = $\underline{\cdot004344}$

Note.—When taking the 2 from the -1 remember the algebraic method. Changing the sign of the quantity subtracted, the 2 becomes -2. Adding -2 to -1 gives -3.

When the decimal part to be subtracted is larger than the other decimal part, then the scheme of "borrowing one" must be very carefully watched on account of the presence of both + and − numbers that may occur. The following example gives a case.

Example 240.—Find the value of $\frac{\cdot 01193}{2\cdot 3}$ (see Ex. 47, p. 55) by logarithms.

Log ·01193	$= \bar{2}\cdot 0766$
Log 2·3	$= 0\cdot 3617$
Log of quotient	$= \bar{3}\cdot 7149$

Quotient = Antilog = ·005187.

Note.—Coming to the first decimal place the 3 cannot be taken from 0. Therefore borrow 1 from the $\bar{2}$; then 3 from 10 is 7. Now borrowing 1 from the $\bar{2}$ is *taking* + 1 from $\bar{2}$ which therefore becomes $\bar{3}$. Now proceeding to the whole numbers 0 from $\bar{3}$ leaves $\bar{3}$

Hence the difference is $\bar{3}\cdot 7149$

The following alternative explanation of the above will apply to those who are accustomed to "paying back" after borrowing. After subtracting the 3 from the 10, consisting of the 0 and the borrowed 1, then 1 must be "paid back" to the 0 in 0·3617, making + 1 from $\bar{2}$. Working algebraically this gives $\bar{3}$ as before.

Example 241.—Find the value of the expression $\frac{67\cdot 8}{\cdot 0007502}$: figures relating to an engine governor.

Log 67·8	$= 1\cdot 8312$
Log ·0007502	$= \bar{4}\cdot 8752$
Log of quotient	$= 4\cdot 9560$

Quotient = Antilog = 90360

Note.—Here again a 1 has to be borrowed to enable the subtraction of the first decimal place to be performed. Then in the top line, when "paying back," 1 taken from 1 leaves 0. Now 4 has to be taken from 0. Changing sign and adding gives + 4 for result.

Exercises 60. On Division by Logarithms.

Work the following by logarithms:—

1. 219·5 ÷ 31·83.
2. ·00963 ÷ ·785.
3. 29580 ÷ 1·986.
4. ·1352 ÷ 23·66
5. ·00058 ÷ ·000415.
6. 2382 ÷ 299·8.

Work by logarithms Exercises 13, Chap. II.

Compound Examples.—Examples involving both multiplication and division are very easily and quickly treated by means of logarithms. The following examples show the most convenient method of laying out the work. It is usually best to find the log of the whole of the top line, and the log of the whole of the bottom line, and then subtract.

Example 242.—Find the value of the expression $\frac{2250 \times 17{\cdot}7}{454 \times {\cdot}146}$

B.T.U.: figures relating to the heating value of a gas.

Top line		Bottom line	
Log 2250	= 3·3522	Log 454	= 2·6571
Log 17·7	= 1·2480	Log ·146	= $\bar{1}$·1644
Log of product	= 4·6002	Log of product	= 1·8215
Log of bottom line	= 1·8215		
Log of quotient	= 2·7787 = Difference		

Quotient = Antilog = 600·8 B.T.U.

Explanation.—First obtain the log of the top line and the log of the bottom line. Now, unless it is desired to know the two products there is no need to look up the antilogs of the two logs obtained. The log of the bottom line may be brought under the log of the top line, and the log of the quotient obtained directly.

The student is warned against the practice of looking up the various logarithms and working the example *without stating to which numbers the logarithms refer.* The time necessary for the writing of statements such as "log 454" is not worth considering, and when completed in this manner the example is understandable by any one at any time, and can always be readily referred to.

Example 243.—Work Ex. 55, p. 64, by means of logarithms.

$$\frac{74{\cdot}4 \times {\cdot}785 \times 3{\cdot}5 \times 3{\cdot}5 \times 800}{33000 \times 3}$$

Top line		Bottom line	
Log 74·4	= 1·8716	Log 33000	= 4·5185
Log ·785	= $\bar{1}$·8949	Log 3	= 0·4771
Log 3·5	= 0·5441	Log of product	= 4·9956
Log 3·5	= 0·5441		
Log 800	= 2·9031		
Log of product	= 5·7578		
Log bottom line	= 4·9956		
Log of quotient	= 0·7622		

Quotient = Antilog = 5·784; say, 5·78

Comparing this with the "longhand" working, it will be seen that the saving of labour is considerable.

Some simple mental work will often save labour with logarithms, as in the above example, where $33000 \times 3 = 99000$. By looking up the logarithm of 99000 the working out of a log is saved.

Example 244.—Find the value of the expression $\frac{51{\cdot}9}{{\cdot}000341 \times 4 \times {\cdot}45}$; figures relating to an engine governor.

Log 51·9	$= 1{\cdot}7152$	Log ·000341	$= \bar{4}{\cdot}5328$
Log bottom line	$= \bar{4}{\cdot}7881$	Log 4	$= 0{\cdot}6021$
Log of quotient	$= 4{\cdot}9271$	Log ·45	$= \bar{1}{\cdot}6532$
		Log of product	$= \bar{4}{\cdot}7881$

Quotient = Antilog = 84550

Exercises 61. Compound Examples.

(In order that the student may check his calculations at various steps, log top line, log bottom line, and results are given in the answers for Nos. 1 to 4.)

Find the value of—

1. $\frac{7{\cdot}11 \times 29{\cdot}38}{{\cdot}093 \times 11{\cdot}36}$

2. $\frac{1{\cdot}003 \times 7 \times 6{\cdot}28}{{\cdot}00051 \times {\cdot}957}$

3. $\frac{78{\cdot}5 \times 113 \times 2{\cdot}167 \times 194}{33000}$

4. $\frac{87{\cdot}5}{{\cdot}000341 \times 5{\cdot}6 \times {\cdot}416}$

5. $\frac{6{\cdot}345 \times 0{\cdot}1075}{{\cdot}00374 \times 15250 \times 1{\cdot}021}$

6. $\frac{{\cdot}0129 \times {\cdot}785 \times 525}{33000 \times 10{\cdot}5}$

7. The bending moment on a girder was given by the figures $\frac{62 \times 24 \times 28}{2240 \times 8}$. Complete the working.

8. Find the value of the expression $\frac{35 \times 1080}{2 \times 3 \times 1{\cdot}5 \times 1{\cdot}5}$ lbs. per sq. in. which was obtained in calculating the stress in the teeth of a gear wheel.

9. A calculation for the area of the conductors to supply a certain house with electric light produced the figures $\frac{{\cdot}66 \times 4 \times 36}{100 \times 10{\cdot}3}$ sq. ins. Find the actual area required.

10. Evaluate the expression $\frac{9{\cdot}87 \times 30{,}000{,}000 \times {\cdot}021}{62 \times 62}$ lbs. which refers to the buckling load on a steel column.

Work by logarithms Nos. 1 to 11 of Exs. 16, Chap. II, and Nos. 12 to 20 of Exs. 21, Chap. III.

Examples involving + and −. There are plenty of examples in which the operations of addition and subtraction are mixed

up with those of multiplication and division. If it is desired to evaluate these with the aid of logs, then two things must be remembered—

1. That addition and subtraction must be performed in the ordinary manner, viz., without the use of logarithms;

2. That the addition of two logarithms is connected with the multiplication of their numbers.

Hence any addition or subtraction of logs must be confined to those *portions* of the expression involving multiplication or division only. Then the antilogs or results of these portions must be found, and any addition or subtraction desired must be performed upon these partial results.

In this type of example it is best to keep the form of the expression as far as possible, and to do the logarithmic work at the side of the sheet. The student should state carefully to what numbers each log refers; it is very desirable in this class of example to be able to trace all the steps in the working.

The following will illustrate:—

Example 245.—Find the value of the expression $7 \times \cdot575 \times 1\cdot75 + 9\cdot85 \times \cdot475 \times \cdot2375$, figures relating to the strength of a channel section bar.

$$= \underbrace{7 \times \cdot575 \times 1\cdot75}_{7\cdot044} + \underbrace{9\cdot85 \times \cdot475 \times \cdot2375}_{1\cdot111}$$

$$= \underline{8\cdot155}$$

N.B.—The brackets are merely used to show the necessary steps.

Log 7	$= 0\cdot8451$
Log ·575	$= \bar{1}\cdot7597$
Log 1·75	$= 0\cdot2430$
Log of product	$= 0\cdot8478$
Antilog	$= \underline{7\cdot044}$

Log 9·85	$= 0\cdot9934$
Log ·475	$= \bar{1}\cdot6767$
Log ·2375	$= \bar{1}\cdot3756$
Log of product	$= 0\cdot0457$
Antilog	$= \underline{1\cdot111}$

Explanation.—The first part of the expression is evaluated by logs, care being taken that the antilog 7·044 is found and written down under the main expression. Similarly the second part of the expression is evaluated by logs, and the antilog 1·111 written next to the 7·044. Great care must be taken that these antilogs are found before the + sign is obeyed. Note also that the final addition produces the actual result without the use of log tables.

Example 246.—Evaluate the expression $\dfrac{64}{1 + \dfrac{6{\cdot}5 \times 5}{2{\cdot}82 \times 2{\cdot}82}}$ tons, figures relating to the eccentric load on a column.

$$= \frac{64}{1 + \dfrac{6{\cdot}5 \times 5}{2{\cdot}82 \times 2{\cdot}82}}$$

$$= \frac{64}{1 + 4{\cdot}088}$$

$$= \frac{64}{5{\cdot}088}$$

$$= \underline{12{\cdot}58 \text{ tons}}$$

Log 6·5	= 0·8129	Log 2·82	= 0·4502
Log 5	= 0·6990	Log 2·82	= 0·4502
Log of product	= 1·5119	Log of product	= 0·9004
Log of lower product	= ·9004		
	·6115		
Antilog	= 4·088		

Log 64	= 1·8062
Log 5·088	= 0·7066
Log quotient	= 1·0996
Antilog	= 12·58

Explanation.—First evaluate by logs the right-hand term in the bottom line, and insert its antilog (4·088) in the second line of the main working. Then add the 1 to the 4·088, when the expression becomes a simple division, logs being used. The final antilog in this case is the answer.

Example 247.—Find the value of the expression—

$$\frac{6{\cdot}395}{32{\cdot}2}\left(\frac{{\cdot}626 \times {\cdot}626}{4} + \frac{10 \times 10}{12}\right)$$

figures relating to an experiment on the twisting of wires.

Expression =

$$\frac{6{\cdot}395}{32{\cdot}2}({\cdot}098 + 8{\cdot}333)$$

$$= \frac{6{\cdot}395 \times 8{\cdot}431}{32{\cdot}2}$$

$$= \underline{1{\cdot}674}$$

Log ·626	= $\bar{1}$·7966	Log 6·395	= 0·8058
Log ·626	= $\bar{1}$·7966	Log 8·431	= 0·9259
Log of product	= $\bar{1}$·5932	Log of product	= 1·7317
Log 4	= 0·6021	Log 32·2	= 1·5079
	$\bar{2}$·9911		0·2238
Antilog	= ·09797	Antilog	= 1·674
say,	·098		

Explanation.—First evaluate the bracketed terms; the second term can be found mentally, being $\frac{100}{12} = 8\frac{1}{3}$, *i.e.*, 8·333. Then the two results must be added, and their sum operated upon by the two numbers outside the bracket, logs being used.

Exercises 62. Examples involving + and −

Evaluate by logarithms—

1. $9{\cdot}52 \times {\cdot}459 + 2{\cdot}35 \div {\cdot}958$. **2.** $2{\cdot}7 + 3{\cdot}58 \times 2{\cdot}895 \times 3{\cdot}142$.

3. $\dfrac{9{\cdot}75 - 8{\cdot}25}{10{\cdot}385 + 5{\cdot}5} \times \dfrac{6{\cdot}875 \times 9}{{\cdot}758}$. **4.** $\dfrac{17{\cdot}25 \times 2{\cdot}56 + 2{\cdot}894 \times {\cdot}015}{6{\cdot}785 \times {\cdot}0125}$.

5. $\dfrac{17{\cdot}5 \times {\cdot}75 \times {\cdot}5625}{12} + 17{\cdot}5 \times {\cdot}75 \times 4{\cdot}59 \times 4{\cdot}59$.

6. The expression $7 \times {\cdot}57 \times 1{\cdot}75 + 9{\cdot}85 \times {\cdot}475 \times \dfrac{{\cdot}475}{2}$ occurred in connection with finding the centre of gravity of a channel section. Complete the working.

7. The weight of a proposed bridge truss is estimated to be $\dfrac{60 \times 125 \times 10}{1800 \times 7{\cdot}5 - 125 \times 10}$ tons. Complete the working.

Work by logarithms, Nos. 13 to 25 of Exs. 16, Chap. II.

Powers by Logarithms.—Logarithms afford a very simple and rapid method of raising numbers to all kinds of powers. Only whole number powers, such as squares and cubes, will be taken in this book.

A number can be raised to a power by logarithms according to the following rule: **Find the logarithm of the number and multiply it by the index denoting the power. This product is the logarithm of the result, and its antilogarithm is the power required.**

Expressed symbolically—

$$\textbf{If } X = a^n \textbf{ then } \log X = n \times \log a$$

A slightly different method of laying out the work is advisable in these cases.

Example 248.—Find the value of $(6{\cdot}25)^3$

$$\begin{aligned} \text{Log } (6{\cdot}25)^3 &= 3 \times \log 6{\cdot}25 \\ &= 3 \times 0{\cdot}7959 \\ &= 2{\cdot}3877 \\ \text{Antilog} &= 244{\cdot}2 = 6{\cdot}25^3 \end{aligned}$$

Note.—Brackets used as in the 6·25 above are not a necessity, but may help to keep the statement more clear, especially in written work.

These examples present no difficulty when the whole number of the log is positive, as the multiplication is easily performed. But when the whole number is negative, then the log really consists of two parts, one + and the other −, and care must be taken in dealing with it.

Example 249.—Find the value of $\cdot 596^2$.

$$\begin{aligned} \text{Log } \cdot 596^2 &= 2 \times \log \cdot 596 \\ &= 2 \times \bar{1}\cdot 7752 \\ &= \bar{1}\cdot 5504 \\ \text{Antilog} &= \cdot 3551 = \cdot 596^2. \end{aligned}$$

To explain: When carrying out the multiplication by 2 it must be remembered that, of the log, only the 1 is negative, while the ·7752 is positive. Multiplying $\cdot 7752 \times 2$ we obtain 1·5504, the ·5504 being the decimal portion (positive) of the new log., and the 1 (also positive) being a carrying figure. Now multiplying the $\bar{1}$ by 2 gives $\bar{2}$. Adding the carried 1 to the $\bar{2}$ gives $\bar{1}$. Hence the new log is $\bar{1}\cdot 5504$, and its antilog is the required number.

Example 250.—Find the value of $\cdot 0365^2$

$$\begin{aligned} \text{Log } \cdot 0365^2 &= 2 \times \log \cdot 0365 \\ &= 2 \times \bar{2}\cdot 5623 \\ &= \bar{3}\cdot 1246 \\ \text{Antilog} &= \cdot 001332 = \cdot 0365^2 \end{aligned}$$

Explanation.—Multiplying the decimal portion of the log by 2, the carrying figure is 1, which is, of course, positive. Then $\bar{2} \times 2$ is $\bar{4}$, and adding the carried $+ 1$ gives $\bar{3}$.

Example 251.—The area of a circle of diameter d'' is given by the formula $\cdot 7854d^2$. Find the area of the section of a wire No. 000 S.W.G. ($\cdot 372''$ diameter).

$$\begin{aligned} \text{Area} &= \cdot 7854d^2 \\ &= \cdot 7854 \times \cdot 372^2 \\ \text{Then log area} &= \log \cdot 7854 + 2 \times \log \cdot 372 \\ &= \bar{1}\cdot 8951 + 2 \times \bar{1}\cdot 5705 \\ &= \bar{1}\cdot 8951 + \bar{1}\cdot 1410 \\ &= \bar{1}\cdot 0361 \\ \therefore \text{Area} &= \text{Antilog} = \cdot 1086 \text{ sq. in.} \end{aligned}$$

Note.—In this case the log of the square must be obtained by multiplying by the 2 before adding to the log of ·7854.

Exercises 63. On Powers by Logarithms.

Find the value of the following by logs:—

1. $(3\cdot 142)^2$. **2.** $(\cdot 7854)^2$. **3.** $2\cdot 5^5$. **4.** $\cdot 0875^4$.
5. $27\cdot 85^2$. **6.** $7\cdot 325^3 \times \cdot 875$. **7.** $(8\cdot 75 \times 5\cdot 265)^2$.

Work by logarithms, Nos. 18 to 30 of Exs. 23, Chap. III.

Roots by Logarithms.—Similarly any root may be easily and quickly extracted by means of logarithms. Only square roots, cube roots, and the like will be considered in this book.

To extract any root of a number: **Find the logarithm of the number and divide it by the figure denoting the particular root. The quotient obtained is the logarithm of the result, and its antilogarithm is the root.**

Expressed in symbols—

$$\text{If } X = \sqrt[n]{a} \text{ then } \log X = \frac{\log a}{n} \text{ or } \frac{1}{n} \times \log a$$

Example 252.—Extract the square root of 22·09.

$$\text{Log } \sqrt{22{\cdot}09} = \frac{1}{2}\log 22{\cdot}09$$
$$= \frac{1}{2} \text{ of } 1{\cdot}3441$$
$$= {\cdot}6721$$
$$\text{Antilog} = 4{\cdot}7 = \sqrt{22{\cdot}09}$$

Explanation.—In the case of square root 2 is the figure denoting the root. Hence log 22·09 has to be divided by 2; the actual division can be done mentally.

Referring to the ordinary working of this example on p. 103, the saving in time and labour is seen at once to be considerable.

Roots by Logarithms. Adjustment of the logarithm when its whole number is negative. As when dealing with powers, these examples present no difficulty while the whole number of the log is positive, but an adjustment has to be made when it is negative. Two ways of making this adjustment will be shown. As an example let us find the value of $\sqrt[3]{{\cdot}2512}$. In this case 3 is the number denoting the root.

$$\text{Then log } \sqrt[3]{{\cdot}2512} = \frac{1}{3}\log {\cdot}2512$$
$$= \frac{1}{3} \text{ of } \bar{1}{\cdot}4000$$

METHOD I

Now at first it does not seem easy to divide a number having + and − parts, but the difficulty is got over by changing the number, so that it only possesses one sign. Thus $\bar{1}{\cdot}4000$ means $-1 + {\cdot}4000$, which if evaluated according to the algebraic idea equals $-{\cdot}6000$.

Now $-{\cdot}6000$ can be divided by 3 quite easily, giving $-{\cdot}2000$, which is then the log of the result. But it must be remembered that the logs in the tables (the decimal parts) are *always positive,*

so the logarithm $-\cdot2000$ must first be converted (without, of course, altering its value) so that it will have a *positive decimal part.*

Now let us add $+1$ and -1 to the logarithm $-\cdot2000$. The value will remain the same, since the addition of $+1-1$ is the addition of 0. Then we may write $-\cdot2000+1-1$, which, combining the first two numbers, $=+\cdot8000-1=\bar{1}\cdot8000$, and the decimal part is now positive, whilst the log is of the ordinary form.

The complete working would appear as follows:—

$$\begin{aligned}\text{Log } \sqrt[3]{\cdot2512} &= \frac{1}{3}\ \log \cdot2512\\ &= \frac{1}{3}\text{ of } \bar{1}\cdot4000\\ &= \frac{1}{3}\text{ of } -\cdot6000\\ &= -\cdot2000\\ &= \bar{1}\cdot8000\\ \sqrt[3]{\cdot2512} &= \text{Antilog} = \underline{\cdot6310}\end{aligned}$$

The chief new points in the above method are—

1. Convert the log to a single negative number, which really consists of taking the decimal from the negative whole number and reducing the digit of the latter by 1.

2. Find the required fraction of this.

3. Add $+1$ and -1 mentally, which is actually done by taking the decimal from 1 and increasing the digit of the whole number by 1.

Method II

An alternative method of "making the adjustment" is as follows: add a sufficient negative number to the whole number to make it exactly divisible by the divisor representing the root, *i. e.*, in this case add $\bar{2}$ to the $\bar{1}$, making it $\bar{3}$, which is then exactly divisible by the divisor 3. Now to the decimal part add the same number, only *positive*, in order to keep the same value for the log, *i. e.*, in this case add $+2$ to the decimal (making it $2\cdot4000$), the $+2$ and the -2 neutralizing each other. Then the original logarithm $\bar{1}\cdot4000$ becomes $\bar{3}+2\cdot4000$, in which the negative part is exactly divisible by the divisor 3. Dividing the $\bar{3}$ by 3 gives $\bar{1}$, and dividing $+2\cdot4000$ by 3 gives $+\cdot8000$. Hence our new log is $\bar{1}\cdot8000$, in which the decimal is $+$, so that the log is of the ordinary form. If the whole number is already exactly divisible this method is

shorter than Method I, as division can be directly proceeded with. Performed mentally, Method II is in all cases the quicker. The complete working in this case is as follows:—

$$\begin{aligned}\text{Log } \sqrt[3]{\cdot2512} &= \tfrac{1}{3}\log \cdot2512\\ &= \tfrac{1}{3}\text{ of } \bar{1}\cdot4000\\ &= \tfrac{1}{3}(\bar{3} + 2\cdot4000)\\ &= \bar{1} + \cdot8000\\ &= \bar{1}\cdot8000\\ \sqrt[3]{\cdot2512} = \text{Antilog} &= \underline{\cdot6310}\end{aligned}$$

Note.—Brackets are needed in the third line to denote that both the $\bar{3}$ and the 2·4 are to be divided by 3.

Example 253.—Evaluate $\sqrt[5]{\cdot000247}$, figures which occurred in a problem on water supply.

Working by Method I—

$$\begin{aligned}\text{Log } \sqrt[5]{\cdot000247} &= \tfrac{1}{5}\text{ of log } \cdot000247\\ &= \tfrac{1}{5}\text{ of } \bar{4}\cdot3927\\ &= \tfrac{1}{5}\text{ of } -3\cdot6073\\ &= -\cdot7215\\ &= \bar{1}\cdot2785\\ \text{Antilog} &= \underline{\cdot1899}\end{aligned}$$

Explanation.—Converting the logarithm $\bar{4}\cdot3927$ into a number of single sign, the $+\cdot3927$ is taken from the $-4\cdot0000$, giving $-3\cdot6073$. Dividing by 5, $-\cdot7215$ is obtained. To convert we add $+1$ and -1. The -1 is written as $\bar{1}$, and the $+1$ added to the $-\cdot7215$, giving $+\cdot2785$. The final log is then $\bar{1}\cdot2785$.

Working by Method II—

$$\begin{aligned}\text{Log } \sqrt[5]{\cdot000247} &= \tfrac{1}{5}\text{ of log } \cdot000247\\ &= \tfrac{1}{5}\text{of } \bar{4}\cdot3927\\ &= \tfrac{1}{5}(\bar{5} + 1\cdot3927)\\ &= \bar{1}\cdot2785\\ \text{Antilog} &= \underline{\cdot1899}\end{aligned}$$

Explanation.—To make the whole number of the log $\bar{4}\cdot3927$ exactly divisible by 5, a -1 must be added, when it becomes $\bar{5}$. $+1$ must therefore be added to the ·3927, making it 1·3927. Dividing $\bar{5}$ by 5 gives $\bar{1}$ and dividing 1·3927 by 5 gives ·2785, so that our final log is $\bar{1}\cdot2785$, as before.

Exercises 64. On Roots by Logarithms.

Find the value of the following by logs:—

1. $\sqrt{56{\cdot}5}$.	2. $\sqrt{217{\cdot}8}$.	3. $\sqrt[3]{{\cdot}19652}$.
4. $\sqrt{{\cdot}00165}$.	5. $\sqrt[3]{97{\cdot}8}$.	6. $\sqrt[6]{1{,}135{,}000}$.
7. $\sqrt[4]{19{\cdot}06}$.	8. $\sqrt[5]{179{\cdot}3}$.	9. $\sqrt[4]{1{\cdot}212}$.
10. $\sqrt[3]{{\cdot}969}$.	11. $\sqrt[7]{{\cdot}0565}$.	12. $\sqrt[5]{{\cdot}0005573}$.
13. $\sqrt[4]{{\cdot}00043}$.	14. $\sqrt[7]{{\cdot}0000087}$.	15. $\sqrt[4]{10{\cdot}05}$.

16. Rankine's rule for the thickness (ins.) of a masonry arch is $12\sqrt{{\cdot}12r}$. Calculate the thickness when radius $r = 10{\cdot}5$.

17. The formula $\frac{1{\cdot}49G}{\sqrt{H}}$ gives the area of cross-section (sq. ft.) of a chimney H ft. high, when G = total grate area in sq. ft. of the boilers. Calculate the value of the expression when G = 30 and H = 70.

18. The formula $P = \frac{Q\delta}{\sqrt{r}}$ refers to live rollers. Calculate the value of P when Q = 500, $r = 2{\cdot}5$, and $\delta = {\cdot}0073$.

19. The expression $\frac{\sqrt{x}}{1 - \sqrt{x}}$ relates to the motion of a steam engine slide valve. Find its value when $x = {\cdot}625$.

20. Calculate the diameter of a piston-rod from the formula $D = 1{\cdot}2\sqrt{dL}$, when $d = 13$ and $L = {\cdot}667$.

21. The expression $1{\cdot}73\sqrt{\frac{d^3}{t}}$ relates to the collapsing of short tubes under external pressure. Calculate its value when $d = 2{\cdot}5$ and $t = {\cdot}5$.

22. The greatest allowable wheel base on a tramway track of R ft. radius, where the width of rail groove is w'', is $\sqrt{\frac{2Rw}{3}}$ ft. Calculate this length when R = 70 ft. and $w = 1{\cdot}25''$.

23. The expression $\sqrt{\frac{gx^2}{2y}}$ occurred in a certain hydraulic problem. Find its value when $g = 32$, $x = 6$, and $y = 1{\cdot}25$.

24. In calculating the sizes of motor and dynamo field coils Esson's Formula may be used, which is $C = \sqrt{\frac{tA}{355R}}$. Calculate the value of C when $t = 50$, $A = 1150$, and $R = 10$.

25. The current allowed in bare overhead electric conductors is given as $\sqrt{\frac{D^3t}{320R}}$ amps. Calculate this value when $D = 9{\cdot}45$, $t = 20$, and $R = {\cdot}00000177$.

26. The "impedance" of an A.C. circuit is $\sqrt{R^2 + p^2L^2}$. Calculate its value when $R = 2{\cdot}3$, $p = 546$, and $L = {\cdot}015$.

27. The formula $P = {\cdot}24\sqrt{D} + {\cdot}625 - {\cdot}175$ ins. gives the pitch of a U.S. standard thread where D = diameter of bolt in inches. Calculate P for a $1\frac{1}{2}''$ bolt (*i. e.*, $D = 1{\cdot}5$).

28. The expression $\sqrt{\frac{1{\cdot}25CT(D + d)}{E}}$ gives the diameter of wire required for the field coils of a shunt-wound dynamo. Calculate the size of wire required when E = 110, D = 15, d = 10, and CT = 11000.

29. The horse-power formula given by Messrs. Rolls Royce for petrol engines is H.P. = $\cdot 25\left(d - \frac{1}{2}\right)^2\sqrt{S}$ per cylinder, where d = cylinder diameter and S = stroke. Calculate the H.P. if d = 4·33 and S = 5·12.

30. The "number of threads per inch" on bolts over 1" diameter having British standard fine threads is given by the expression $\frac{10}{\sqrt[5]{d^5}}$, where d = diameter of bolt in inches. Calculate the number of threads per inch on bolts of the following diameters: (*a*) 2", (*b*) 1¼". (*Note.*—In each case give the actual answer, and also the nearest *whole number* of threads.)

Further examples will be found in Exercises 24 (p. 109), Chap. III.

Various Examples.—We will close this chapter with some examples of a more difficult nature involving all the foregoing points.

Example 254.—The diameter of a turbine nozzle is given by the figures $\sqrt{\frac{\cdot 1675}{\cdot 785}}$ inches. Complete the working.

Log ·1675	=	$\bar{1}{\cdot}2240$	
Log ·785	=	$\bar{1}{\cdot}8949$	
Log of quotient	=	$\bar{1}{\cdot}3291$	
	=	$-\cdot 6709$	(By Method I, p. 234)
Log $\sqrt{\text{quotient}}$	=	$-\cdot 3354$	(Dividing by 2)
	=	$\bar{1}{\cdot}6646$	

Antilog = ·4619"; say ·46".

Example 255.—In a problem on water supply the figures $\sqrt[5]{73{\cdot}6^2}$ ft. per sec. appeared. Find the required value.

$$\text{Log. } \sqrt[5]{73{\cdot}6^2} = \frac{1}{5} \text{ of log } (73{\cdot}6^2)$$

$$= \frac{1}{5} \text{ of } (2 \times \log 73{\cdot}6)$$

$$= \frac{1}{5} \text{ of } (2 \times 1{\cdot}8669)$$

$$= \frac{1}{5} \text{ of } 3{\cdot}7338$$

$$= \cdot 7468$$

$\therefore$ Antilog = 5·582, say, 5·58 ft. per sec.

Example 256.—It was calculated that under certain conditions the speed of an engine governor would be $\sqrt{\frac{69{\cdot}2}{{\cdot}000341 \times 4 \times {\cdot}55}}$ R.P.M. Complete the calculation.

Bottom line

Log ·000341	$= \bar{4}{\cdot}5328$	Log 69·2	$= 1{\cdot}8401$
Log 4	$= 0{\cdot}6021$	Log bottom line	$= \bar{4}{\cdot}8753$
Log ·55	$= \bar{1}{\cdot}7404$	Log of quotient	$= 4{\cdot}9648$
Log of product	$= \bar{4}{\cdot}8753$	Log $\sqrt{\text{quotient}}$	$= 2{\cdot}4824$

Antilog = 303·7 R.P.M.; say, 304.

Example 257.—The deflection of a beam loaded centrally is given by the formula $\frac{WL^3}{48EI}$ ins. Find the deflection of such a beam where W = 11·5 tons, L = 24 ft., E = 12500 tons per sq. in., I = 725·7.

Converting the 24 ft. into ins., L = 24 × 12 = 288 ins. Deflection $= \frac{WL^3}{48EI} = \frac{11{\cdot}5 \times 288^3}{48 \times 12500 \times 725{\cdot}7}$ (substituting given values).

Top line		Bottom line	
Log (288^3)		Log 48	$= 1{\cdot}6812$
$= 3 \times 2{\cdot}4594$	$= 7{\cdot}3782$	Log 12500	$= 4{\cdot}0969$
Log 11·5	$= 1{\cdot}0607$	Log 725·7	$= 2{\cdot}8607$
Log top line	$= 8{\cdot}4389$	Log of product	$= 8{\cdot}6388$

Log top line = 8·4389
Log bottom line = 8·6388
Log of quotient = $\bar{1}{\cdot}8001$

Antilog = ·6311
Deflection = ·631″

Example 258.—A number giving the inclination of a water supply pipe is given by the expression $\left(\frac{2{\cdot}5}{110\sqrt{1{\cdot}817}}\right)^2$. Complete the working.

Bottom line

Log $\sqrt{1{\cdot}817}$		Log 2·5	$= 0{\cdot}3979$
$= \frac{1}{2} \times 0{\cdot}2593$	$= 0{\cdot}1297$	Log bottom line	$= 2{\cdot}1711$
Log 110	$= 2{\cdot}0414$	Log of quotient	$= \bar{2}{\cdot}2268$
Log bottom line	$= 2{\cdot}1711$	Log $(\text{quotient})^2$	$= \bar{4}{\cdot}4536$
		Antilog	$= {\cdot}0002843$

Required number = ·000284, say.

Example 259.—Dr. Pole's formula, giving the quantity of gas flowing through a pipe in cu. ft. per hr., is $1350d^2\sqrt{\frac{hd}{ls}}$, where d = diameter of pipe in ins., h = effective gas pressure in ins. of water, l = length of pipe in yds., s = specific gravity of gas.* Find the discharge when $d = 6''$, $h = 1{\cdot}6''$, $l = 1250$ yds., and $s = {\cdot}425$.

Discharge = $1350d^2\sqrt{\frac{hd}{ls}}$ cu. ft. per hr.

$= 1350 \times 6^2\sqrt{\frac{1{\cdot}6 \times 6}{1250 \times {\cdot}425}}$ (substituting the given figures)

$= 1350 \times 36\sqrt{\frac{9{\cdot}6}{1250 \times {\cdot}425}}$ (doing a little mental calculation)

Log 1250	= 3·0969	Log 9·6	= 0·9823
Log ·425	= $\bar{1}$·6284	Log bottom line	= 2·7253
Log of product	= 2·7253	Log of quotient	= $\bar{2}$·2570
		Log $\sqrt{\text{quotient}}$	= $\bar{1}$·1285
Log of the sq. root	= $\bar{1}$·1285		
Log 36	= 1·5563		
Log 1350	= 3·1303		
Log of product	= 3·8151		
Antilog	= 6533		

Discharge = 6533 cu. ft. per hr.

Exercises 65. Various Examples.

1. The formula $d = \sqrt[3]{\frac{T}{\cdot 2f}}$ relates to shafts under torsion. If T = 55800 and f = 9000, calculate the value of d.

2. The Board of Trade rule for safety-valve springs of square section is $d = \sqrt[3]{\frac{SD}{11000}}$. Find d when S = 3350 and D = 4.

3. The diameter in inches of a pipe to transmit g gallons of water per min. a distance of L ft. with a loss of head of h ft. is $d = \sqrt[5]{\frac{gL}{730h}}$. Calculate d if g = 150, L = 1470, and h = 25.

4. The following expression relates to helical springs: $\sqrt[4]{\frac{64wnr^3}{C\Delta}}$. Calculate its value when w = 250, n = 12, r = 1·75, C = 12,000,000, and Δ = 1·5.

5. The distance apart (in ft.) at which the bearings of shafting should be placed is $5\sqrt[3]{d^2}$, when d = diameter of shaft in inches. Calculate the distance for a shaft $2\frac{1}{2}''$ diameter.

---

* A figure connected with the weight of the gas.

6. The formula $K = \sqrt[4]{\frac{d}{d_4}}$ relates to a certain kind of galvanometer. If $d = 117$ and $d_4 = 219$, calculate the value of K.

7. The diameter of a propeller fan to deliver Q cu. ft. of air per sec. with a consumption of H horse-power is $\sqrt[4]{\frac{\cdot 0000115 Q^3}{H}}$. If $H = \cdot 85$ and $Q = 230$, find the diameter.

8. Find the value of the expression $\left(\frac{3\cdot 7}{90\sqrt{1\cdot 57}}\right)^2$, which relates to a water-supply pipe.

9. Using Dr. Pole's formula (Ex. 259), $Q = 1350 d^2 \sqrt{\frac{hd}{ls}}$, calculate the quantity of gas Q if $d = 10$, $h = 2\cdot 5$, $l = 2355$, and $s = \cdot 43$.

10. The formula $D = \sqrt[3]{\frac{320RC^2}{t}}$ relates to bare overhead conductors. Calculate D when $R = \cdot 0000018$, $t = 20$, and $C = 270$.

11. With reference to steam temperature and pressure we have $\theta = \sqrt[5]{\frac{85}{3\cdot 176 \times 10^{10}}} - 35\cdot 16$. Find the value of θ.

12. With reference to the Venturi meter for measuring water we have the formula $Q = \frac{8\cdot 02 \times C a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \sqrt{h}$. Calculate Q if $C = \cdot 975$, $h = 6\cdot 3$, $a_1 = \cdot 785$, $a_2 = \cdot 136$.

13. The equation $\triangle = \frac{16W}{\lambda G d^4}(R^4 - r^4)$ gives the deflection of a conical spiral spring. Calculate the deflection $\triangle$ when $W = 500$, $\lambda = \cdot 15$, $G = 12 \times 10^6$, $d = \cdot 192$, $R = \cdot 8$, $r = \cdot 3$.

14. The true air speed v of an aeroplane flying with an indicated speed of V m.p.h. in air of density ρ is given by the expression $v = V\sqrt{\frac{1221}{\rho}}$. At an altitude of 10,000 ft., $\rho = 910$. Calculate the true air speed represented by an indicated speed of 100 m p.h.

15. For a variable electrical condenser of the "square law" type, $R = \sqrt{4a\theta + r^2}$. Calculate the value of R when $a = 6$, $\theta = 3\cdot 142$, $r = \cdot 375$.

16. The pressure of the atmosphere, p inches of mercury, at an altitude of H feet above sea-level is given approximately by the formula $H = 62580 \log \frac{29\cdot 92}{p}$. Calculate the altitude H corresponding to a pressure of 16 inches of mercury. (*Hint.*—$\log \frac{29\cdot 92}{p} = (\log 29\cdot 92 - \log p)$, and when the two logarithms have been looked out they become *ordinary numbers* for the purpose of the subsequent calculation.)

As soon as the student, by sufficient practice, has made himself reasonably proficient in the use of logarithms, he should use logarithms for the numerical work in subsequent calculation, and if possible, should take up the use of a slide rule.

CHAPTER VII

MENSURATION

LENGTHS AND AREAS

In this chapter it is assumed that the reader can use, or is learning to use, the ordinary drawing instruments such as T-square, set-squares, and compasses; also that he has some knowledge of the method of showing solid objects on paper by "plan, elevations, and sections."

Measurement of Length.—In the introductory chapter it is stated that the **Yard** is the British standard of length or **Unit**; and that multiples and sub-multiples of the yard are employed for larger and smaller distances. The smallest sub-multiple unit employed is the **Inch,** which is $\frac{1}{36}$ of the yard; below this, fractions of an inch are used, in the vulgar form for general use, and in the decimal form for the finer measurements. The largest multiple in ordinary use is the **Mile,** which is 1760 yards. Between these extremes there are various other units as given in the table on p. 3.

In addition, the following relations are very useful:—

36 inches = 1 yard
1760 yards = 1 mile
5280 feet = 1 mile

For certain classes of work special units are employed. Thus, in nautical work the "nautical mile" is used for long distances, while the speeds of ships are stated in "knots."

NAUTICAL MEASURE

1 nautical mile = 6080 feet = 1·15 ordinary miles
1 fathom = 6 feet = 2 yards
[Depths of sea bottom are given in fathoms]
1 knot = 1 nautical mile per hour
= 6080 feet per hour

In Land Surveying, measurement is made with a chain 22 yards or 66 feet long, called Gunter's Chain. For certain special work a chain 100 feet long is used. Unless otherwise stated, the "chain" is taken to mean the Gunter Chain.

SURVEYORS' MEASURE

100 links = 1 chain (Gunter)
10 chains = 1 furlong
80 chains = 1 mile
[1 link = 7·92 inches]

Conversion or Reduction.—When stating lengths or distances, it is customary to use two or more units for sizes above 1 ft. Thus on drawings we find such dimensions as 22′-6″, 3′-4¾″, or the length of a road may be given as 3 miles, 5 furlongs, and 35 yards. But for many calculations it is desirable that such a length should be stated with a *single* number, *i. e.*, it should be *in terms of one unit only*. The 22′-6″ would then be "converted" into (say) *feet*, becoming 22·5 ft., while the 3′-4¾″ might all be converted into inches, becoming 40·75″. Again, a calculated dimension is usually obtained with a single unit, *e. g.*, 4·833 ft.; on a drawing this would be given as 4′-10″, the decimal of a foot having been converted into inches. The amount of this "conversion" or "reduction" required in connection with length measurement is not very great.

The method of working is best illustrated by examples.

Reduction to Smaller Units.

Example 260.—Reduce the distance 4m. 3 fur. 25 po. 3 yds. to yards.

m.	fur.	po.	yds.
4	3	25	3
8			
35 fur.			
40			
1425 poles			
5½			
7128			
712½			

Result = 7840½ yds.

Explanation.—Reduce 4 miles to furlongs by multiplying by 8, since 1 m. = 8 fur.; while multiplying, add in the 3 fur.; similarly convert the 35 fur. to poles by multiplying by 40, adding in the 25 poles. The method of multiplying by 5½ to convert to yards should be noted as being quick and easy. First multiply the 1425 by 5 and add in the extra 3. Then multiply by ½ and add the two results.

Example 261.—(*a*) Convert $5'\text{-}7\frac{1}{2}''$ into inches; (*b*) convert 4·74 ft. into feet and inches.

(*a*) $5'\text{-}7\frac{1}{2}''$

12

$60 + 7\frac{1}{2} = 67\frac{1}{2}$

$\therefore \quad 5'\text{-}7\frac{1}{2}'' = \underline{67\frac{1}{2}''}$

(*b*) 4·74 ft. ....only the decimal of a ft.

12 is multiplied by 12.

8·88 ins.

= 4′-8″ + ·88 of an inch

·88″ = $\frac{7}{8}$″ full (see table p. 39).

∴ Dimension = $\underline{4'\text{-}8\frac{7}{8}''}$

Note.—Conversion of this type is largely done mentally.

Example 262.—Convert 35 miles per hour to feet per second.

1 mile = 5280 ft.

∴ 35 m. per hr. = 35 × 5280 ft. in 1 hr.

1 hr. = 60 mins. = 60 × 60 secs. = 3600 secs.

$\therefore$ 35 × 5280 ft. in 1 hr. $= \frac{35 \times 5280}{3600}$ ft. in 1 sec.

$= \underline{51{\cdot}3 \text{ ft. per sec.}}$

Conversion to Larger Units.

Example 263.—A warship opens fire at a range of 12,000 yds. How far away is the target: (*a*) in miles and yards; (*b*) in miles only?

(*a*) 1760)12000(6 m.

10560

1440 yds. Distance = $\underline{6 \text{ m. } 1440 \text{ yds.}}$

(*b*) 1440 yds. $= \frac{1440}{1760}$ m. = ·818 m.

∴ Distance is also 6·818 m., $\underline{\text{say } 6{\cdot}82 \text{ m.}}$

Example 264.—Reduce 16950 ft. to miles, etc.

3)16950

5650 yds.

2

11)11300 half yds.

4,0) 1027 + 3 half yds.

8) 25 + 27 po.

3 + 1 fur.

Result = $\underline{3 \text{ m. } 1 \text{ fur. } 27 \text{ po. } 1 \text{ yd. } 1 \text{ ft. } 6 \text{ ins.}}$

Explanation.—The feet are converted into yards by dividing by 3. As it is not easy to divide by $5\frac{1}{2}$ direct, the yards are multiplied by 2 (*i. e.*, converted to $\frac{1}{2}$ yds.) and then divided by 11. The 3 remainder

is the 3 half yds., *i.e.*, 1½ yds. or 1 yd. 1 ft. 6 ins. To divide by 40, point off the 0 in the 40 and the last figure in the 1027 as shown (*i.e.*, dividing by 10). Then divide the 102 by 4 and place the remainder 2 before the 7.

Addition and Subtraction in Length Units.—It is occasionally necessary to add and subtract various lengths when given in two or more units. The method is exactly the same as when dealing with money. In addition, add up the figures of the smallest unit first; convert to the next higher unit and write the remainder. Repeat, carrying forward the converted figure from the previous column.

Example 265.—Find the sum of the following dimensions from the wheel base of a 4-coupled locomotive, which gives the overall length of the engine : 6′-10½″, 10′-2½″, 8′-7″, 4′-7½″.

6′	10½″
10	2½
8	7
4	7½
30′	3½″

Note.—Sum of inch column is 27½ = 2′-3½″. Write 3½ carrying 2 to feet column.

Subtraction is performed similarly. Occasionally a 1 from the feet column has to be "borrowed" and converted into inches in order to carry out a subtraction in the inch column.

Example 266.—A locomotive is 13′-1⅝″ from rail to top of chimney and 7′-9″ from rail to centre line of boiler. Find the distance from centre line of boiler to top of chimney.

	13′-1⅝″
	7′-9″
Difference =	5′-4⅝″

In subtracting the inches it is necessary to "borrow" a 1 from the feet column.

Exercises 66. On the Conversion, etc., of Length.

1. Reduce 2 m. 1 fur. 20 po. 5 yds. to yards.
2. Reduce 7 fur. 3 yds. to yards.
3. Reduce 1 m. 28 po. 3 yds. to feet.

Convert :—

4. 115·5 yds. to inches.
5. 27·85 ft. to inches.
6. 5 yds. 2·5 ft. to inches.
7. 5′-6½″ to inches.
8. 11′-10¾″ to feet.
9. 1′-9⅝″ to yards.
10. 2785 ft. to a decimal of a mile.
11. A machine shop requires the following lengths of a certain width belting :—14′-6″, 28′-0″, 7′-4″, 19′-10″. What total length must be ordered ?

12. A plate girder requires the following lengths of plate for its flanges :—16′-9″, 29′-6″, 45′-4″, 45′-4″, 29′-6″, 16′-9″. Find the total length of plate required.

13. In connection with boiler flues it was necessary to find the difference between 3′-8½″ and 2′-10½″. What is this difference?

14. The overall length of a lathe bed is 6′-6″. The fast head-stock occupies 1′-2½″ and the loose headstock 11″. What is the greatest length available between the centres?

Simple Geometrical Terms.—The well-known "Square" and "Circle" are examples of "plane geometrical figures," *i. e.*, flat figures which have something definite about them, such as straight sides, or sides made up of smooth curves; such figures are constantly occurring in engineering work. Before proceeding to any

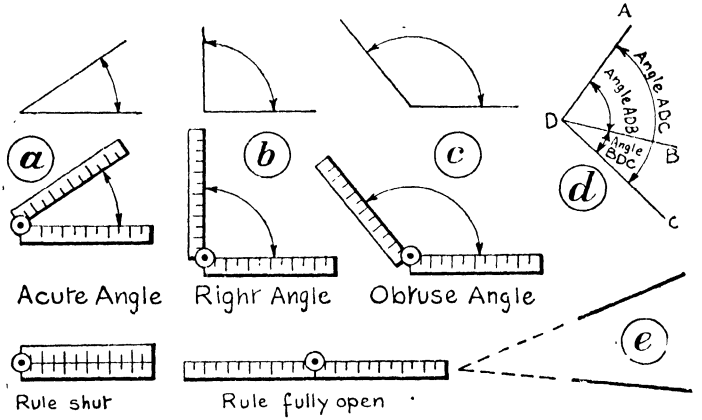


Fig. 23.—Illustrating Various Angles.

actual calculation some descriptions and explanation of terms are necessary.

Angle is the geometrical name for "corner." More particularly, it is the opening between the two lines making a corner. Thus, when a folding rule is opened the two legs enclose or include an angle, as shown in Fig. 23. At *a*, in this figure, the opening is small and the angle is said to be "acute" meaning *sharp*. At *c* the opening is very large and then the angle is said to be "obtuse," meaning *blunt*. At *b*, the opening is exactly halfway between the acute and obtuse positions, or the rule is just halfway between being quite shut and being full open. This angle is a **Right Angle,** or a square corner, and is the most important of all. In ordinary language we should say that the two legs of the rule are "square with each other."

It should be noticed that the angle between two lines *is not the distance between them;* the distance is different at different places. Two lines may include an angle although they do not meet and make a proper corner, as at *c*, Fig. 23; here the two full black lines are inclined to each other and therefore enclose an angle. The appearance of a corner is obtained if each line be continued to the left until the two meet as shown dotted. In geometrical language this continuing of a line is called "producing," and the line is said to be "produced." To indicate any particular angle on a figure the corners are lettered. When only two lines meet at a corner a single letter is sufficient. Thus at *d* in Fig. 23, we have the angle D. When more than two lines meet at a point several letters are necessary. Thus, the angle BDC is the one between the lines BD

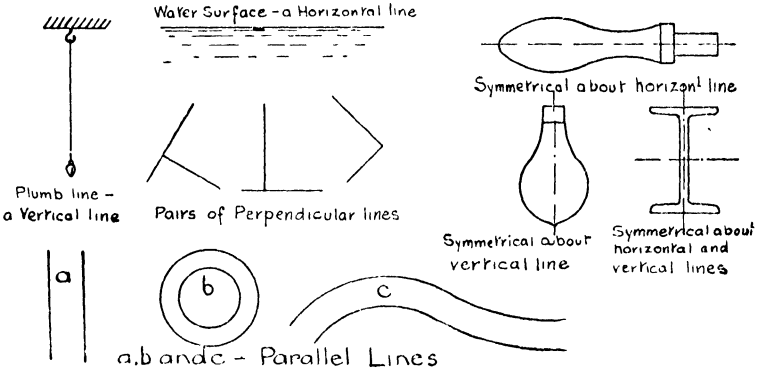


Fig. 24.—Illustrating Various Lines.

and CD. The angle ADB is that between the lines AD and BD; while the angle ADC is the whole angle between the lines AD and CD, *i. e.*, the sum of BDC and ADB.

Parallel lines are those which are the same distance apart everywhere; they would never meet however far produced in each direction. Parallel lines may be either straight or curved, as in Fig. 24, or like the rails on a railway or tramway track.

A **Vertical line** is one which stands straight up, and is at right angles to the earth's surface, as with a plumb line (see Fig. 24).

A **Horizontal line** is one which is quite level, like the surface of a pond, as shown in Fig. 24. It is at right angles to a vertical line.

Lines which are at right angles to each other are said to be **perpendicular** to each other. Neither line need necessarily be vertical

or horizontal, as shown in Fig. 24. The vertical and horizontal directions are of course perpendicular to each other.

A figure is said to be **symmetrical about a certain line** when the portions on either side of that line are exactly the same shape, but reversed (see Fig. 24).

A **plane figure** is one that is quite flat and has no thickness.

Simple Plane Figures

(Letters refer to Fig. 25)

All figures having *four straight sides* are called **Quadrilaterals.** Forms shown from A to F are examples.

When opposite pairs of sides in quadrilaterals are parallel the figure is called a **Parallelogram,** and the opposite sides are then equal. Examples are shown from A to D.

A **Rectangle** is a parallelogram in which all angles are *right angles.* Strictly the term applies to both A and B, but it is customary to restrict it to the form in which adjacent sides are unequal as at A, which is sometimes known as an "oblong." When all four sides are equal as at B, the figure is called a **Square.**

C. A **Rhomboid** is a parallelogram in which the angles are *not* right angles and in which adjacent sides are *not* equal. This is the general form of parallelogram.

D. A **Rhombus** is a parallelogram in which *all four sides are equal,* but the angles are *not* right angles. When placed so that the longest dimension is vertical, we recognise the "diamond" or "lozenge" shape.

E. A **Trapezoid** is a quadrilateral having only *one pair of parallel sides.* There are various forms as shown.

F. A **Trapezium** is a quadrilateral in which *all sides are unequal.* A name which better describes this figure is **Irregular Quadrilateral.**

G. A **Triangle** is a *three-sided figure,* and has therefore *three angles.* At *a,* all three sides are equal in length, and the triangle is said to be "equilateral"; the angles are then all equal. At *b* only two sides are equal and the triangle is "isosceles." At *c* and *g* all three sides are unequal.

The **Right-angled Triangle** (shown at *g*) is one in which one angle is a right-angle. This triangle is the most important in higher work.

H. A **Hexagon** is a *six-sided* figure, in which all sides are equal and all angles are equal; the opposite sides are then parallel. This is the shape of the common nut and bolt head.

J. An **Octagon** is an *eight-sided* figure, in which all sides are

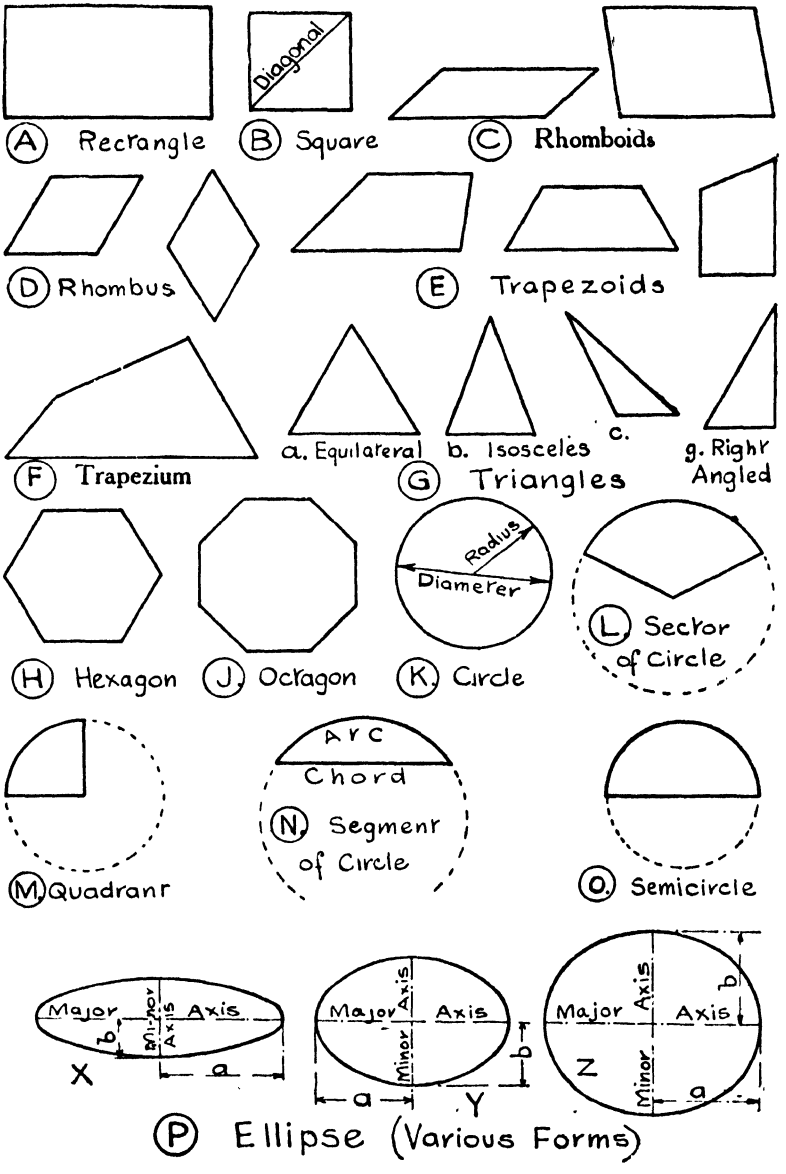


Fig. 25.—Simple Geometrical Figures of Importance to the Engineer.

equal and all angles are equal; the opposite sides are then parallel. Chipping chisels are frequently made from "octagon bar steel."

The remaining straight-sided figures are of no importance here.

K. The **Circle** is a figure with only one *continuous* outline which is always the same distance from some point called the centre. This distance is the "Radius" (plural, radii). The dimension right across from one side to the other when taken *through the centre* is the diameter of the circle, and is, of course, equal to twice the radius. The actual curved outline is called the **Circumference.**

L. A **Sector** of a circle is a piece of a circle with three sides; two sides being radii (and therefore straight) and the third side a piece of the circumference. This curved piece is called an **arc,** and is said to "*subtend*" the angle at the centre (*i. e.*, it is opposite this angle).

M. A **Quadrant** is a particular sector in which the angle between the two radii is a right angle. Since there are four such sectors in the whole circle a quadrant is a quarter of a circle.

N. A **Segment** is a piece of a circle with only two sides; one an arc and the other a straight line from side to side, which is called a **Chord.** When this chord passes through the centre of the circle the segment becomes exactly half a circle and is then called a **Semicircle** (shown at O). It should be noted that the distance along the arc is considerably longer than that along the chord.

P. The **Ellipse** (or Oval) is a figure with one continuous outline and is longer one way than the other, being in fact a stretched or flattened circle. It is symmetrical about two perpendicular lines passing through its centre, the longer being called the "major axis," and the shorter the "minor axis." At X, the major axis is much longer than the minor axis. This is approximately the shape of the cross-section of the tube in a steam-pressure gauge. At Y the shape is a medium one, as in a boiler manhole. At Z the major and minor axes are very nearly equal; when equal, our ellipse becomes a circle. Figures like the above drawn with circular arcs are not necessarily ellipses, as certain laws exist as to the shape of the curves. These laws cannot be dealt with until a much later stage.

The majority of the shapes we deal with in engineering work are these regular figures or combinations of them, since the regular figures are easily produced by machine processes, *e. g.*, the circle in the lathe, and rectangular shapes on the planer. Irregular shapes require either expensive machines to produce, or much hand labour, which is both costly and slow.

Perimeter or Circumference.—The actual distance round the outline (or the length of the boundary) of any figure is called its **Perimeter** or **Circumference.** With the straight-sided figures the calculation of the perimeter is an easy matter when the length of the sides are known. If all the sides are not known, but sufficient information is given to draw out the figure, then it should be drawn, and the sides actually measured.

In all the straight-sided figures **the perimeter is the sum of the lengths of all the sides,** particular cases being formulated as follows :—

1. **Rectangle :** Let a = length of long side and b = length of short side. Then Perimeter = $2a + 2b$ or $2(a + b)$.
2. **Square :** Let s = length of side. Perimeter = $4s$.
3. **Equilateral Triangle :** Let s = side. Perimeter = $3s$.
4. **Hexagon :** Let s = length of side. Perimeter = $6s$.
5. **Octagon :** Let s = length of side. Perimeter = $8s$.

Circumference of a Circle. Meaning of π. The length of the circumference of a circle cannot be so easily reckoned as that of a straight-sided figure. Roughly the circumference is a little more than three times the diameter. If several circles be drawn and their diameters and circumferences measured in some way, then it will be found that the ratio $\frac{\text{Circumference}}{\text{Diameter}}$ is always very nearly 3·14. This figure or "constant" is a very important one, and occurs in a great many calculations. In formulæ it is usually denoted by the Greek letter π (pi), its value not being known with *absolute* exactness, although it has been calculated to 700 decimal places ! Approximate values of varying accuracy may be used depending on the class of work in hand, so that for practical calculations we can consider it to be known exactly.

To 8 significant figures $\pi = 3{\cdot}1415928$
„ 4 „ „ $\pi = 3{\cdot}142$

The latter is quite suitable for the great majority of engineering calculations. Then we have

Circumference of a Circle = $\pi \times$ diameter

or in symbols, if c = circumference and d = diameter

$$c = \pi d.$$

Since the diameter of a circle is twice the radius, then if r = radius, $d = 2r$

Then $c = 2\pi r$, a useful form.

Occasionally π is given in the form of a vulgar fraction; the commonest value being $\frac{22}{7}$, *i. e.*, $3\frac{1}{7}$. The decimal equivalent of this is 3·142857 . . . . so that the difference is not great. Another, and *very accurate*, form is $\frac{355}{113}$, *i. e.*, $3\frac{16}{113}$, of which the decimal equivalent is 3·1415929. . . .

Determination of π by Measurement.—Students should find the value of π by actual measurement of the circumference of a circle. There are several methods possible, of which one follows :—

Draw a circle with a diameter of 3″ or more and make a mark somewhere on the circle as at A, Fig. 26. Open a pair of dividers

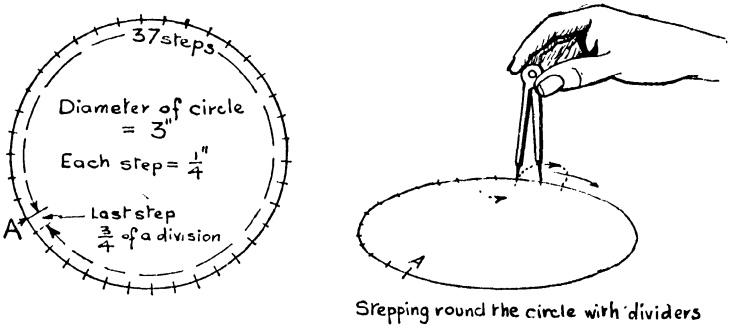


Fig. 26.—Determination of π by Measurement.

to some small even distance, say $\frac{1}{4}$″, and starting from A, step carefully round the circle as indicated. The points of the dividers must be very carefully placed on the circumference each time. Count the number of steps taken from the start until the mark A is again reached. The last step may be less than $\frac{1}{4}$″, in which case it should be estimated as $\frac{1}{2}$ or $\frac{3}{4}$ of a step. If the step is small and the circle fairly large, the length along the arc from one point to the next is almost exactly equal to the length of the chord, *i. e.*, the straight distance between the points.

Then if N be the number of steps and l the length of the step the circumference = Nl.

Thus, with a 3″ dia. circle, and $\frac{1}{4}$″ step, $37\frac{3}{4}$ steps were made round the circle.

$$\text{Then circumference} = 37\tfrac{3}{4} \times \tfrac{1}{4}''$$

$$= \frac{151}{16} = 9{\cdot}438''$$

$$\therefore \pi = \frac{\text{circum.}}{\text{dia.}} = \frac{9{\cdot}438}{3} = 3{\cdot}146''$$

which is very near to the actual value of 3·142.

If (in a class) several students do this, each having a different sized circle, all the ratios should come in the neighbourhood of 3·14, and the average value should be very near to 3·142.

Examples involving Circumferences of Circles.

Example 267.—A steam engine cylinder is lagged with wood strips held in place by two brass bands as shown at *a*, Fig. 27. Determine what length of brass strip is necessary for each band.

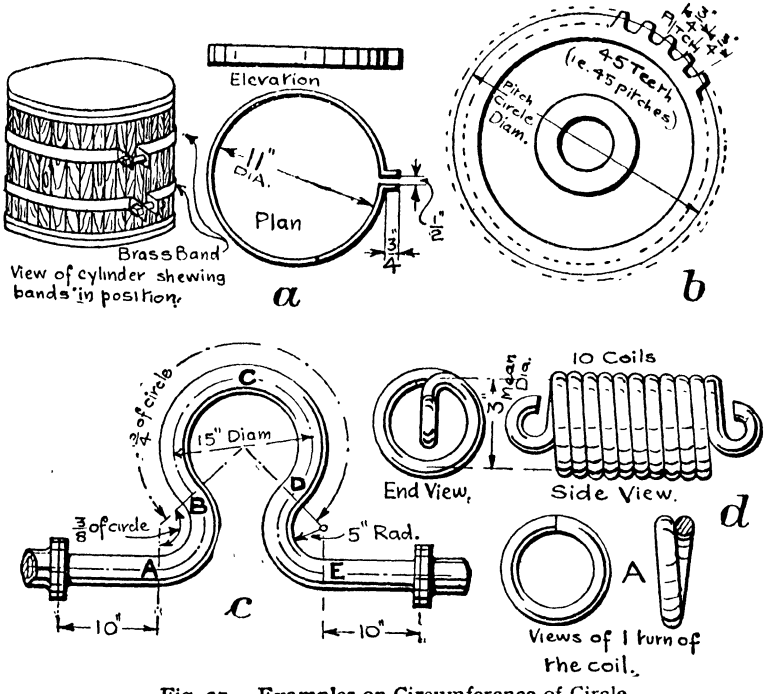


Fig. 27.—Examples on Circumference of Circle.

The portion round the cylinder is less than a full circumference by the $\frac{1}{2}$" between the ends. In addition to this there are the two projecting pieces each $\frac{3}{4}$" long, for the bolt.

$$\text{Circum. of circle} = \pi d$$
$$= 3{\cdot}142 \times 11 = 34{\cdot}56"$$

Add for two lugs each $\frac{3}{4}$" long, $1\frac{1}{2}$" and deduct $\frac{1}{2}$" piece for space, therefore add 1".

$$\therefore \text{Length required} = 35{\cdot}56, \text{ say } 35\tfrac{5}{8}"$$
$$\text{or, } \underline{2'\text{-}11\tfrac{5}{8}"}$$

Example 268.—A spur wheel is to have 45 teeth of $\frac{3}{4}''$ pitch. What will be the "pitch diameter" of the wheel (*i. e.*, the diameter of the circle on which the pitch is measured—see *b*, Fig. 27)?

$$\text{Circum.} = 45 \times \tfrac{3}{4}'' = 33{\cdot}75$$
$$\therefore \text{Dia.} = \frac{\text{circum.}}{\pi} = \frac{33{\cdot}75}{3{\cdot}142} = \underline{10{\cdot}73''}$$

In certain cases, given the number of revolutions per unit of time, we need to know the *surface speed* of a revolving wheel, *i. e.*, the speed of a point on the circumference. Obviously in 1 revolution a point on the circumference will travel a distance equal to the circumference, in 2 revolutions 2 × circumference, or for N revolutions N × circumference; or—

$$\text{Surface speed (ft. per min.)} = \text{Revolutions (per min.)} \times \text{circumference (ft.)}$$

(The surface speed is also called the "circumferential" or the "peripheral" speed—periphery being another name for circumference.)

The following examples will illustrate :—

Example 269.—The commutator of a dynamo is 19″ dia., and makes 470 revolutions per minute. Find the surface speed in feet per minute.

$$\text{Surface speed} = \text{circum.} \times \text{revs.}$$
$$\text{Circum. of commutator} = \frac{19}{12} \times \pi = 4{\cdot}975 \text{ ft.}$$
$$\therefore \text{Surface speed} = 4{\cdot}975 \times 470$$
$$= \underline{2338 \text{ ft. per min.}}$$

Note.—In first line 19 is divided by 12 to bring inches to feet.

Example 270.—A milling cutter is 3″ dia. and is to be used with a cutting speed of 50 ft. per min. At what speed (revs. per min.) must it run?

$$\text{Revs. per min.} = \frac{\text{cutting speed}}{\text{circumference}}$$
$$\text{Cutting speed} = 50 \text{ ft. per min.} = 50 \times 12 \text{ ins. per min.}$$
$$\text{Circum. of cutter} = 3 \times \pi = 9{\cdot}426''$$
$$\therefore \text{Revs. per min.} = \frac{50 \times 12}{9{\cdot}43} = \underline{63{\cdot}6 \text{ revs. per min.}}$$

Example 271.—A copper expansion bend in a long length of steam pipe is shown at *c*, Fig. 27. The portion BCD is $\frac{1}{4}$ of a complete circle, and the portions AB and DE are each $\frac{3}{8}$ of a circle. Find

the total length of pipe from flange to flange necessary to make the bend.

Portion BCD $= \frac{3}{4}$ of circum. of circle
$= \frac{3}{4} \times \pi \times 15$ $= 35{\cdot}4''$
Portion AB $= \frac{3}{8}$ of circum. of circle
Portion DE $= \frac{3}{8}$ " "
$\therefore$ AB + DE $= \frac{3}{4}$ " "
$= \frac{3}{4} \times \pi \times 5$ $= 11{\cdot}8''$
Two straight portions 10" each $= 20{\cdot}0''$
$\therefore$ Total length required $= 67{\cdot}2''$
say, $5'\text{-}7\frac{1}{4}''$

Example 272.—A close-coiled helical (or coil) spring is 3" average dia. (*i.e.*, dia. measured at centre of coils) and contains 10 complete coils. What length of wire is necessary to make the spring, allowing an extra 8" for hooks at the ends? (See *d*, Fig. 27.)

Note.—Considering one coil or turn it is seen from A, Fig. 27, that, like a turn of a screw thread, it is not a true circle, its length being a little more than the circumference of a circle, but the difference is so small in a *closely coiled* spring that for practical purposes it can be neglected. Then the length of 1 turn is calculated as for a true circle, and is multiplied by the number of coils.

Length of 1 turn $= \pi \times 3 = 9{\cdot}42''$
$\therefore$ Length of 10 turns $= 9{\cdot}42 \times 10 = 94{\cdot}2''$
Add for ends 8
$\therefore$ Total length required per spring $= 102{\cdot}2''$
or, $102\frac{1}{4}''$

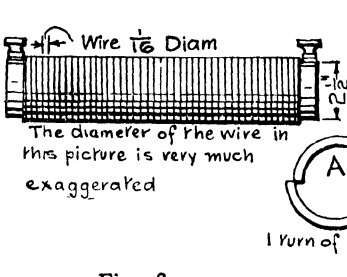


Fig. 28.

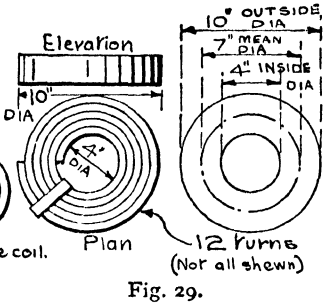


Fig. 29.

Example 273.—A resistance for an arc lamp is to be in the form of a close wound coil as in Fig. 28. The total length of wire is to be 44 yds., and the coil is to be wound on a cylinder of $2\frac{1}{2}''$ dia. How many turns will there be? If the wire is $\frac{1}{16}''$ dia., what length of cylinder will the wire occupy?

$$\text{Circum. of } 2\tfrac{1}{2}'' \text{ circle} = \pi \times 2{\cdot}5 = 7{\cdot}85''$$

$$\text{Total length} = 44 \text{ yds.} = 44 \times 36 \text{ ins.} = 1584''$$

$$\text{No. of turns} = \frac{\text{Total length}}{\text{Length of turn}} = \frac{1584}{7{\cdot}85} = \underline{202 \text{ turns}}$$

$$\therefore \text{ Length of coil} = 202 \times \tfrac{1}{16} = 12{\cdot}625'' = \underline{12\tfrac{5}{8}''}$$

Example 274.—A coil of leather belting (see Fig. 29) has an outside dia. of 10″ and an inside dia. of 4″, and there are approximately 12 turns. What is the approximate length of belting in the coil?

Note.—Any single turn of the coil is not a true circle but a turn of a spiral as shown at A, Fig. 29. For practical purposes its length is that of a circle. Since all turns are of different diameters it would be very tedious to calculate the length of each turn separately. Therefore we take the average or mean (or centre) turn and calculate on that. For every turn smaller than the mean diameter there is one larger than the mean.

$$\text{Mean dia.} = \frac{\text{outside dia.} + \text{inside dia.}}{2}$$

$$= \frac{10 + 4}{2} = 7''$$

$$\therefore \text{ Mean circum.} = \pi \times 7 = 22''$$

$$\therefore \text{ Total length in 12 turns} = 12 \times 22'' = \frac{12 \times 22}{12} \text{ ft.}$$

$$= \underline{22 \text{ ft.}}$$

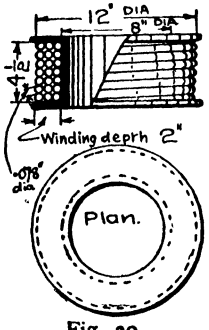


Fig. 30.

Example 275.—Fig. 30 shows a field coil for a dynamo, with dimensions. Find the length of the wire in the coil, which is required in calculating the resistance, etc., of the field windings.

Considering 1 vertical layer of the wire.

$$\text{No. of turns in 1 layer} = \frac{4{\cdot}5}{{\cdot}078} = 57{\cdot}7, \text{ say } 57 \text{ turns.}*$$

Considering the horizontal layers.

$$\text{No. of layers} = \frac{2}{{\cdot}078} = 25{\cdot}6, \text{ say } 25 \text{ layers.}$$

Thus we have 25 layers with 57 turns in each layer.

$\therefore$ Total number of turns in the coil $= 25 \times 57 = 1425$ turns. Now the turns vary in diameter; the smallest being in the inner-

* In examples of this nature it is best to discard any decimal of a turn rather than work to the nearest whole number; it is preferable to have the calculated answer a little *small*.

most layer and the largest in the outermost. Then we may calculate on the length of an *average* turn,

$$\text{Outside dia.} = 12'' \quad \text{Inside dia.} = 8''$$

$$\therefore \text{Dia. of average turn} = \frac{12 + 8}{2} = 10''$$

$$\therefore \text{Length of average turn} = \pi d = \frac{3{\cdot}14 \times 10}{12} = 2{\cdot}62 \text{ ft.}$$

$$\therefore \text{Total length of wire in coil} = \text{Average turn} \times \text{No. of turns}$$
$$= 2{\cdot}62 \times 1425 = \underline{3730 \text{ ft.}}$$

Example 276.—An electrically driven hoist is to run at a speed of 200 ft. per min., and the winding drum is to revolve at 26 revs. per min. What must be the diameter of the drum?

Note.—In 26 revs. the drum must wind up 200 ft. of rope; hence we can find the length wound on in 1 rev., which gives the circum. of the drum.

$$\text{Circum. of drum} = \frac{200}{26} = 7{\cdot}7 \text{ ft.}$$

$$\therefore \text{Dia.} = \frac{7{\cdot}7}{\pi} = 2{\cdot}45 \text{ ft.}$$

$$\text{say, } \underline{2'\text{-}5\tfrac{1}{2}''}$$

Exercises 67. On Circumferences of Circles.

(*Letters refer to Fig.* 31.)

1. The rings of a locomotive boiler, 4'-3" dia., are butt-jointed as shown at A. Calculate the length of plate necessary for each ring.

2. The rings in a large marine boiler are built up of three equal pieces, butt-jointed as shown for the single joint at A. If the boiler is 14'-7½" dia., find the length of the pieces.

3. An angle ring for the top of a cylindrical tank is to be 3'-2½" dia. (measured on the centre line). What length of angle bar must be cut?

4. Find the length of plate required for a lap-riveted boiler ring 6'-0" dia., if the lap-joint requires an extra 4½ ins.

5. A wrought-iron pulley is 3'-0" dia. Find the length of plate required for the rim. There are 10 arms evenly spaced. Find the distance apart of the holes which must be drilled in the rim to receive the ends of the arms.

6. A concrete culvert is reinforced with two sheets of expanded steel, as shown at B, with a lap of 8". What is the width of each sheet?

7. In a steam engine cylinder lagged as shown at *a*, Fig. 27, the lagging is to be composed of wood strips 1½" wide and is to be 11" dia. as nearly as possible. Find the number of strips required per cylinder.

8. An electric cable is to be "armoured" with a ring of steel wire, No. 8, S.W.G. (*i. e.*, ·16") diameter, as at C. The diameter of the circle through the centre of the wires is to be 2¼". How many wires are required? (Give the nearest whole number.)

9. An alternator armature is 60″ inside diameter and is to have 60 slots, as at D. What will be the "pitch" of the slot (*i.e.*, the distance from the edge of one slot to the same edge of the next slot)? If the slot is 1·2″ wide, what will be the width of the "tooth"?

10. A spur wheel is to have a pitch circle diameter of 3′-9½″ and a pitch of 2¾″. How many teeth will it have? (*Note.*—Work in inches and give nearest whole number.)

11. A boiler-maker's wheel tool for measuring curved lines on plates is exactly 2 ft. circumference. What is its diameter in inches?

12. A worm wheel in an electric lift drive is to have 43 teeth of 1¾″ pitch. What will be its pitch circle diameter?

13. A spur wheel in a lathe back gear is to have 79 teeth, and 6 teeth for every 1″ of diameter. Find its diameter; also calculate the pitch of the teeth.

14. An emery wheel 18″ dia. runs at 1200 revs. per min. Find its circumferential speed in feet per min.

15. A wood-planing machine with a 5″ dia. cutter is to run at 2000 ft. per min. Determine the revs. per min.

16. An electric motor has a belt pulley 6″ dia., and runs at 550 revs. per min. Calculate the speed of the belt in feet per min. If this belt drives a machine shop shaft by a pulley 24″ dia., find the revs. per min. of the shop shafting.

17. Find the length of pipe required from flange to flange to make the expansion bend shown at E.

18. A band saw is to run over two equal pulleys 3 ft. dia., their centres being 5 ft. apart as shown at G. Calculate the length of saw-blade required.

19. At F is shown a bent copper steam pipe. What length is required from flange to flange?

20. A pipe hanger is required to the dimensions shown at H. What length of bar is necessary?

21. Dimensions are given at J of the curved portion of the splasher guard over the driving wheel of a locomotive. What length of plate is required?

22. Dimensions of a sheet metal guard over the back gear of a milling machine are given at K. Find what length of strip is required.

23. Dimensions are given at L of a strap brake on a traction engine. Determine what length of strap is required.

24. A closely coiled spring for a coil clutch is shown at M. Find what length of bar is required, there being 4½ complete coils and 4″ extra for the ends.

25. An inductance coil, as shown at N, is required for a wireless installation. If it has 60 turns and is wound on a cylinder of 3½″ dia., what length of wire is required?

26. A flat coiled spring in an indicator reducing gear is 1½″ inside dia. and 3″ outside dia., as shown at O, and there are 22 coils. Find the length of strip in the spring.

27. A volute spring for the buffer between engine and tender on a locomotive has 11 complete coils, and is 6″ outside and 2″ inside dia., as shown at P. Calculate the length of material required. (*Note.*—This may be treated as a flat coil spring.)

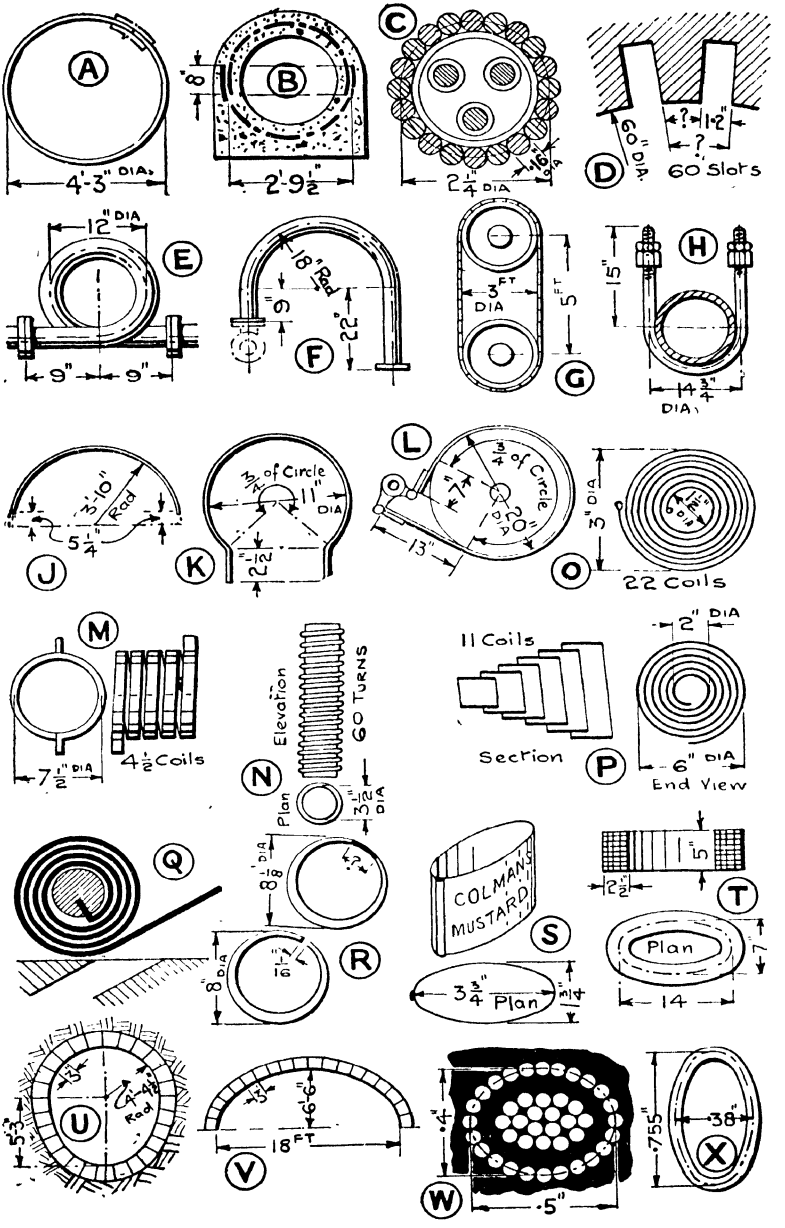


Fig. 31.--Exercises on Circumferences of Circles and Ellipses.

28. A coil of leather belt as in Fig. 29 is 8″ outside and $2\frac{1}{2}$″ inside dia., and has 11 coils. What length of belt does it contain?

29. A roll of sheet zinc is found to have 24 coils and is $7\frac{1}{2}$″ outside and $3\frac{1}{2}$″ inside dia. What length does it contain?

30. At Q is shown a "Gutermuth Valve," used in certain pumps. The outside diameter is 4″, and the inside $1\frac{1}{2}$″, and there are 4 complete coils. Find the length of strip required, allowing $4\frac{1}{2}$″ for the straight flap and $\frac{3}{4}$″ for attaching to the central spindle.

31. Calculate the length of wire in the field coil of an electric motor as in Fig. 30, the dimensions being outside dia. 8″, inside dia. 5″, length 3″, winding depth $1\frac{1}{2}$″, and dia. of wire (outside insulation) ·07″.

32. A cylinder is 8″ dia. and the spring rings are required to be open $\frac{1}{16}$″ when in the cylinder (see R). If the rings are turned $8\frac{1}{8}$″ dia. how much must be cut out?

Circumference of Ellipse.—The exact formula for this is very complicated and far too cumbersome for practical use. Hence various approximate formulæ have been proposed. Of these two are given, the first being the simpler and the second the more exact. The accuracy desired in the particular example should decide which formula is to be used. The more exact formula gives less than 1% error if major axis is less than 5 times minor.

Simpler form: **Circumference of Ellipse** $= \pi\ (a + b)$ where $a = \frac{1}{2}$ major axis and $b = \frac{1}{2}$ minor axis (see Fig. 25). The error in using this formula is 2·7% when $a = 2b$ and as much as 10% when $a = 5b$.

Note.—When our ellipse becomes a circle, $a = b = r$, then circumference $= \pi(r + r) = \pi \times 2r = 2\pi r$ as already known.

More exact form: **Circumference of Ellipse** $= \pi\{1{\cdot}5(a + b) - \sqrt{ab}\}$

Example 277.—An oval or elliptic chain link is shown at *a*, Fig. 32. Find the length of bar in the link.

This will be worked by the more exact formula. The student should work by the simpler one to see the difference.

$$\begin{aligned} a = \tfrac{1}{2}\text{ major axis} &= \tfrac{1}{2} \times 2 = 1'' \\ b = \tfrac{1}{2}\text{ minor axis} &= \tfrac{1}{2} \times 1\tfrac{1}{4} = \tfrac{5}{8}'' = {\cdot}625'' \\ \therefore \text{Circum. of ellipse} &= \pi\{1{\cdot}5(a + b) - \sqrt{ab}\} \\ &= \pi\{1{\cdot}5(1 + {\cdot}625) - \sqrt{1 \times {\cdot}625}\} \\ &= \pi\{2{\cdot}44 - {\cdot}79\} \\ &= \pi \times 1{\cdot}65 \end{aligned}$$

$\therefore$ Length of rod per link = $\underline{5{\cdot}18''}$

Example 278.—Fig. 32 shows at *c* an elliptic gear wheel. The major axis is 9″ long and the minor axis 4·7″ long, and there are 30 teeth. What is the "pitch" (*i.e.*, distance from centre of one tooth to centre of next)?

In this case the more exact form is desirable.

$$a = \tfrac{1}{2} \times 9 = 4{\cdot}5 \qquad \therefore\ a + b = 6{\cdot}85$$
$$b = \tfrac{1}{2} \times 4{\cdot}7 = 2{\cdot}35 \qquad \text{and } \sqrt{ab} = \sqrt{4{\cdot}5 \times 2{\cdot}35}$$
$$= \sqrt{10{\cdot}6} = 3{\cdot}25$$
$$\text{Circum.} = \pi\{1{\cdot}5(a + b) - \sqrt{ab}\} = \pi\{10{\cdot}3 - 3{\cdot}25\}$$
$$= 3{\cdot}14 \times 7{\cdot}05 = 22{\cdot}1''$$
$$\therefore \text{Pitch} = \frac{\text{Circum.}}{\text{No. of teeth}} = \frac{22{\cdot}1}{30}$$
$$= {\cdot}737'', \text{ say, } \tfrac{3}{4}''$$

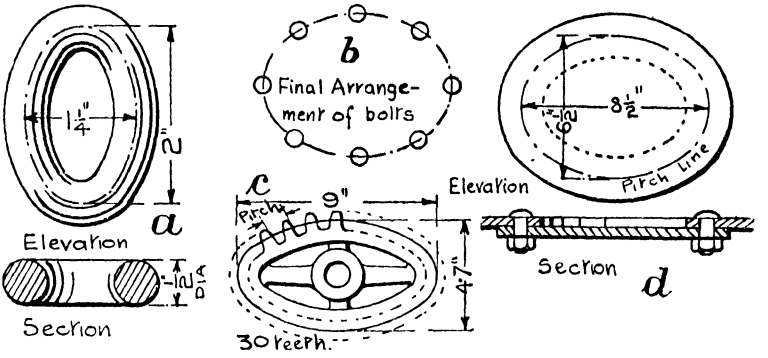


FIG. 32.—Examples on Circumference of Ellipse.

Example 279.—At *d*, Fig. 32, are given dimensions of an oval hand hole on a cylindrical tank, under a vacuum. For staunchness the pitch of the bolts must not exceed 6 times the bolt diameter. Using $\frac{5}{8}''$-bolts how many must be used to satisfy this rule? (*Note.*—It is preferable to use an *even* number of bolts. If a mixed number be obtained then use the *next larger even number*.)

In this case the simpler formula may be used—

Major axis $= 8\frac{1}{2}''$ Minor axis $= 6\frac{1}{2}$

$\therefore\ a = 4{\cdot}25;\ b = 3{\cdot}25;$ and $a + b = 7{\cdot}5$

$\therefore$ Circum of ellipse $= \pi \times 7{\cdot}5 = 23{\cdot}52''$

Bolt dia. $= \frac{5}{8}'' = {\cdot}625''$

$\therefore$ Largest allowable pitch $= 6 \times {\cdot}625 = 3{\cdot}75''$

$\therefore$ Least possible number of bolts $= \dfrac{23{\cdot}52}{3{\cdot}75} = 6{\cdot}27$

Next even number $= 8$ $\therefore$ use eight $\frac{5}{8}''$-bolts.

The arrangement would then be as at *b*, Fig. 32.

A circumference formula often used is $\pi\sqrt{2(a^2 + b^2)}$, but this is almost as approximate as $\pi(a + b)$, and will in certain cases give an error of 7 %.

Exercises 68. On Circumferences of Ellipses.

(*Letters refer to Fig.* 31.)

1 to 3. Find the circumferences of each of the following ellipses, using both the simple and the more exact formulæ:—

1. Major axis 6″. Minor axis $1\frac{1}{2}$″.

2. Major axis 15″. Minor axis 10″.

3. Major axis 12″. Minor axis 10″.

4. An elliptic gear as at *c*, Fig. 32, has major axis 2″ and minor axis 1·414″, and 38 teeth. Calculate the pitch.

5. Calculate the length of rod for an elliptic chain link (see *a*, Fig. 32), if the centre-line dimensions are major axis $3\frac{1}{8}$″, minor axis $2\frac{1}{8}$″.

6. Mustard tins of elliptic section are to be made of dimensions shown at S. What length of sheet tin is required for the walls of each tin if an extra $\frac{3}{8}$″ be allowed for the joint?

7. A boiler mud-hole is elliptical, similar to *d*, Fig. 32, and the pitch line of the rivets measures 18″ × $13\frac{1}{2}$″. If the rivets are pitched 2″ apart how many rivets will be required? (Give nearest even number.)

8. At T is shown, with dimensions, a magnet coil, elliptical in section. If the wire is ·078″ dia., calculate the length of wire required for the coil. (*Note.*—Work this exactly as Example 275, but use the circumference of the average ellipse.)

9. A semi-elliptic arch is shown at V. How many 3″ bricks are required for the inner ring?

10. The conveyor tunnel shown at U is semi-circular at the top and semi-elliptic at the bottom. Find how many 3″ bricks are required to line the inside.

11. Part of the cross section of a split concentric electric cable is shown at W. The outer ring of wires is elliptical, the wire being No. 19, S.W.G. (·04″) dia. Determine how many wires there are in this ring.

12. At X are given dimensions of a $\frac{1}{8}$″ "Simplex" oval conduit for carrying electric wiring. It is made out of one strip of steel, rolled and brazed. Calculate the width of strip required.

Measurement of Angles. Degrees.—When measuring the size of an angle, two systems may be employed. The one in general use is that in which the unit is the **degree.**

The degree is $\frac{1}{360}$ part of a complete revolution.

At X, Fig. 33, is shown a circle divided up into 36 equal parts, each division indicating 10 degrees. The divisions are numbered off in a left-handed or anti-clockwise direction, *i. e.*, in a direction opposite to that in which the hands of a clock move. The first division is split into 10 parts, so that each of the smallest divisions represents 1 degree. The unit is a very small one, as can be seen from Y, Fig. 33, which shows an angle of 5 degrees. When writing down angles, the word "degree" is replaced by a small o (°) at the right-hand top corner of the figure. Thus 90° reads as 90 degrees.

The **right angle** being $\frac{1}{4}$ of a revolution, is $\frac{1}{4}$ of 360°, *i. e.*, 90°. Similarly two right angles, *i. e.*, $\frac{1}{2}$ a revolution, is 180°, and three right angles 270°.

For finer measurements, such as are required in surveying work, the degree is sub-divided into **minutes** and **seconds,** as shown in the following table.

ANGULAR MEASURE

60 seconds (secs.)	= 1 minute (min.)
60 minutes	= 1 degree (° or deg.)
360 degrees	= 1 revolution (rev.)

The minute is indicated by a single tick, and the second by two ticks. Thus, 35° 15′ 28″ would be read as 35 degrees, 15 minutes, 28 seconds.

The minute and second are far too small to be shown in a picture, and their use in instruments is only possible by means of special apparatus. Another system of angle measurement adopts as its unit the Radian (= 57·3°), but this will not be dealt with here.

The Protractor.

To measure and set out angles in degrees a portable divided circle is used, known as a protractor. It may be either semi-

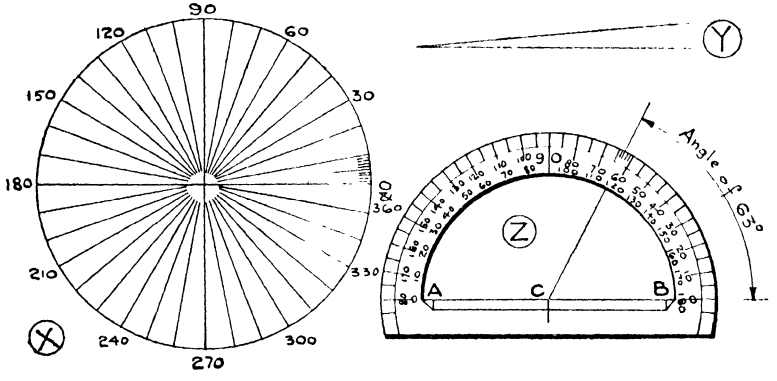


Fig. 33.—Measurement of Angles.

circular (the most common form), circular or rectangular, and may be of celluloid, brass, wood, or ivory. Z, Fig. 33, shows a semi-circular one; other forms are shown in instrument-makers' catalogues. They are usually marked in both directions of reading, *i. e.*, left and

right handed, so as to be equally useful either way. A convenient size is 6″ diam.; below this, reading is limited to an accuracy of about 1°.

In measuring angles the bottom line AB opposite 0° and 180° is placed carefully along one line of the angle, so that the central mark C is at the point of the angle. Then the angle is read off on the scale, taking care to read in the correct direction, since the divisions are marked for both directions. Thus, the protractor in Fig. 33 is shown reading an angle of 63°. Full degrees are obtained exactly from direct markings; parts of a degree may be estimated by eye, although half-degrees are sometimes indicated.

In setting out an angle, the instrument is placed as for reading, and then the required angle is marked by a fine dot or tick at the proper figure on the scale. Removing the protractor this tick is joined to the point at which C was placed.

Important Angles.—Besides the right angle there are certain angles of frequent occurrence which deserve special mention. The **angle of 45°** or half a right angle, shown in Fig. 34, appears on the 45° set-square, *i. e.*, the one with two equal sides.

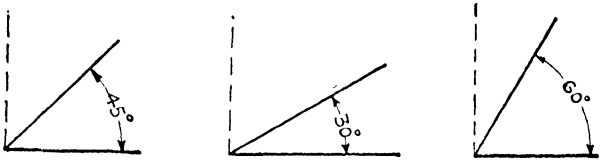


Fig. 34.—Important Angles.

The **angle of 30°**, *i. e.*, $\frac{1}{3}$ of a right angle, and the **angle of 60°**, *i. e.*, $\frac{2}{3}$ of a right angle, appear on the 30° — 60° set-square, and are also shown in Fig. 34.

In any triangle *the sum of the three angles is always* 180°. Thus if two of the angles are known and their sum is subtracted from 180°, the third angle is found.

Addition and Subtraction of Angles are carried out in columns, in much the same manner that money is dealt with. These operations appear very considerably in surveying work where the measurement of angles is very important.

Example 280.—The angles of a four-sided paddock were measured, with the results given below. If the work has been exact then the sum of the angles should be 360°. Find this total, and the error of measurement, if any.

Station (or corner).	Measured Angle.
A	33° 42′
B	111° 51′
C	49° 12′
D	164° 12′
Total =	358° 57′

360 00
358 57
Error = 1° 3′

Note.—The sum of the minutes column is 117 = 1°-57′. Carry 1° to degree column. Difference between total and 360° gives error. Remember that a 1 has been borrowed from the 360° when taking the 57′ away.

Example 281.—In a traverse survey, with a theodolite, of a five-sided figure, the angles in the following table were obtained. Find the total, which should be 1260° exactly. Find the error if any.

Station (or corner).	Angle.
A	277° 15′ 00″
B	215° 8′ 20″
C	229° 29′ 20″
D	246° 11′ 40″
E	291° 52′ 40″

Total 1259° 57′ 00″ ..... *Note.*—Top line to be
1260 00 00 subtracted from bottom.

Error = 3′ 00″ too small.

Reduction of Angles.—It is sometimes easier when calculating, to use angles in degrees and decimals of a degree, rather than degrees, minutes, etc. Hence it is necessary to convert from one style to the other.

Example 282.—Convert 39·38° into degrees, minutes, and seconds.

The degrees remain : decimal to be converted.

·38° × 60 = 22·8′
·8′ × 60 = 48″
∴ Angle = 39° 22′ 48″

Example 283.—Reduce the angle 73° 19′ 12″ to degrees and decimals.

$$12'' = \frac{12}{60} \text{ mins.} = \frac{1}{5} = \cdot 2'$$

$$\therefore 19' 12'' = 19 \cdot 2'$$

$$= \frac{19 \cdot 2}{60} \text{ degs.}$$

$$= \cdot 32 \text{ degs.}$$

$$\therefore \text{Angle} = 73 \cdot 32°$$

Exercises 69. Addition, Subtraction, and Reduction of Angles.

1. The angles measured in a traverse survey are given in the table. Find the total of these measured angles. This should be exactly 540°; any difference is an error. Find the error if any.

Station.	Angle.
A	115° 12′
B	119° 3′
C	110° 0′
D	121° 11′
E	74° 54′

2. A series of measured angles have been adjusted to account for slight errors in reading, etc. These angles are: 277° 15′ 36″; 215° 8′ 56″; 229° 29′ 56″; 246° 12′ 16″ and 291° 53′ 16″. The total should be exactly 1260°. Try if this is so.

3. The three angles of a triangle should total up to 180°. In a certain triangle two angles, when measured, were found to be 35° 14′ and 98° 47′. Find the third angle.

Convert the following angles into degrees, minutes and seconds :—

4. 19·35°. **5.** 17·28°. **6.** 75·06°. **7.** 0·785°.

Convert the following angles into degrees and decimals of a degree :—

8. 25° 15′. **9.** 30° 18′ 50″. **10.** 65° 22′ 30″.

11. 11° 5′ 50″. **12.** 49° 13′ 49″.

The Right-angled Triangle.—In any right-angled triangle we have the following very important relation :—

Square of Hypotenuse = Sum of the Squares of the other two sides ;

or in symbols, if l = hypotenuse (a word used for the side opposite the right angle, in a right-angled triangle, *i. e.*, the longest side), and h and b the other two sides, then

$$l^2 = h^2 + b^2$$

Thus, if the two sides of a triangle enclosing a right angle be made 3″ and 4″ long, then on measurement the other side will be found to be 5″ long exactly.

Thus, $3^2 + 4^2 = 9 + 16 = 25 = 5^2$

This particular triangle is shown in Fig. 35, where the squares on each side are drawn in detail. It can be seen that the number of square inches in the big square is equal to the number in the other two squares. The student should draw several right-angled triangles at random and measure the sides of each; square all the values obtained for each triangle and add together the two smaller figures, which will be found to equal the third and largest square.

From the above formula we obtain by simple transposing $h^2 = l^2 - b^2$ and $b^2 = l^2 - h^2$. Hence, when dealing with right-angled triangles : if the length of two of the sides be known, the length of the third can always be calculated.

The fact that a right-angled triangle with shorter sides of 3″ and 4″ has a long side of 5″ is useful for setting out a right angle by measurement only. A triangle is drawn with sides 3, 4, and 5 long (in inches, feet, etc., or in chains for surveying work) thus producing a right angle between the 3 and 4 sides.

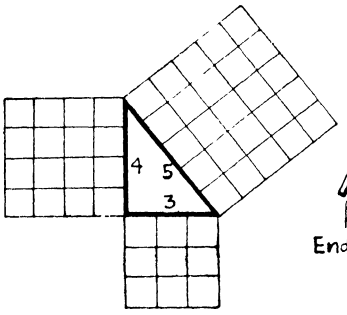


Fig. 35.

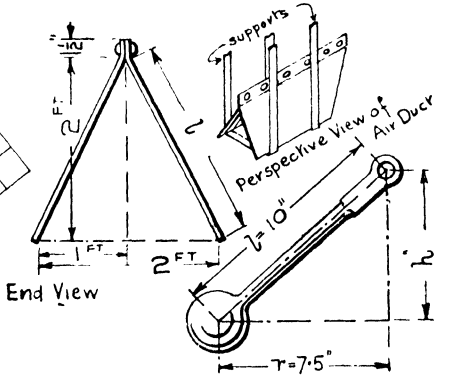


Fig. 36. Fig. 37.

Example 284.—The length of a governor arm is 10″. If the radius is 7·5″, what is the height? See Fig. 37.

Let l = length of arm, h = height, and r = radius.

$$h^2 = l^2 - r^2$$
$$= 10^2 - 7{\cdot}5^2$$
$$= 100 - 56{\cdot}25 = 43{\cdot}75$$
$$\therefore\ h = \sqrt{43{\cdot}75} = \underline{6{\cdot}61''}$$

Example 285.—A triangular air duct for use in a cooling tower is made up as shown in Fig. 36, which also gives dimensions. What will be the width of plate necessary for each side?

The portion on either side of the centre line forms a right-angled triangle, short side 1′; vertical side 2′.

Then $l^2 = 1^2 + 2^2 = 5$

$\therefore\ l = \sqrt{5} = 2{\cdot}236'$

$= 2'\text{-}2\frac{7}{8}''$, say

Lap for riveting at top = $1\frac{1}{2}''$

$2'\text{-}4\frac{3}{8}''$

$\therefore$ Plate must be $2'\text{-}4\frac{3}{8}''$ wide

Example 286.—An open-coiled spring has an average diameter of $2\frac{1}{2}$", and has 8 complete turns or coils in a length of 16". What total length of wire is necessary to form the spring? (See Fig. 38.)

In this case, unlike that taken on p. 255, the length of one complete turn is appreciably more than the circumference of the coil, since the forward movement is so large. Fig. 38 shows that a screw thread or helix is merely a straight inclined line wrapped round a cylinder. Thus, at (*a*) is shown a sheet of paper with an inclined line drawn upon it, resting flat against a cylinder. At (*b*) the paper is partly wrapped round and at (*c*) it is completely

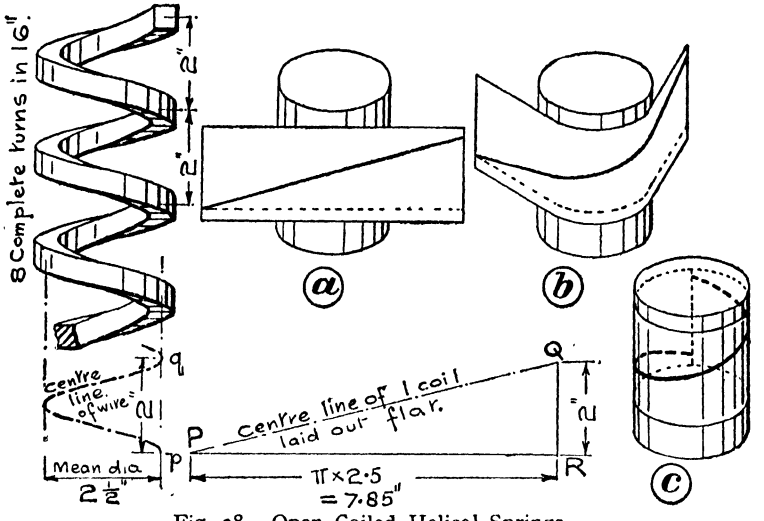


Fig. 38.—Open Coiled Helical Springs.

wrapped round. The straight line now presents the appearance of a screw thread. If therefore we unfold one turn of a screw thread or helix we can find the length of the inclined or screw surface.

In Fig. 38, the triangle PQR represents one turn of the centre line of our spring unwound. P is the starting point *p*, and Q is the finish of the complete turn as at *q*. Then QR is the distance advanced in one turn, *i. e.*, the "pitch," and PR is the distance round the cylinder from *p* to *p* again, *i. e.*, the circumference of the cylinder. Then PQR is a right-angled triangle and PQ, the actual length of the centre line, may be calculated.

$$\text{Circum. of cylinder} = \pi d = \pi \times 2{\cdot}5 = 7{\cdot}85''$$

$$\text{Pitch} = \frac{\text{Length}}{\text{No. of Coils}} = \frac{16}{8} = 2''$$

Then, if l = length of one turn = PQ,

$$l^2 = 7{\cdot}85^2 + 2^2 = 61{\cdot}6 + 4 = 65{\cdot}6$$

$$\therefore\ l = \sqrt{65{\cdot}6} = 8{\cdot}1'' = \text{length per single turn}$$

$$\therefore\ \text{Length for 8 turns} = 8{\cdot}1 \times 8 = 64{\cdot}8''$$

say, 65″ or 5′-5″

Proportions of 45° and 60°— 30° Right-angled Triangles. With the aid of the foregoing relation we can examine certain useful proportions in the two right-angled triangles most often met with.

The 45° Right-angled Triangle.

See A, Fig. 39. Let the short side be 1.

i. e., $b = h = 1$ ($b = h$ is only true for a 45° right-angled triangle)

$$\text{Then } l^2 = b^2 + h^2$$

$$= 1^2 + 1^2 = 2$$

$$\therefore\ l = \sqrt{2} = 1{\cdot}414$$

i. e., if short side is 1, hypotenuse is 1·414

or **Hypotenuse = 1·414 × Short side.**

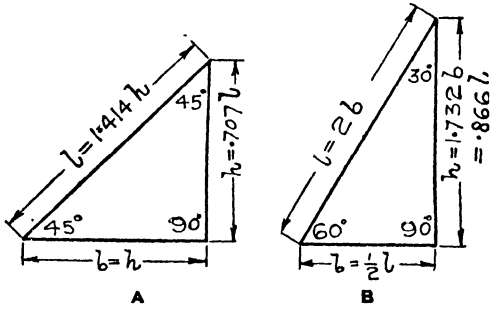


Fig. 39.

From this we get—

$$\textbf{Short side} = \frac{\text{Hypotenuse}}{1{\cdot}414} = {\cdot}707 \times \textbf{Hypotenuse}$$

If a square is divided into two parts by a diagonal each triangle is a 45°—90° triangle. Hence the diagonal is 1·414 × side.

The 60°—30° Right-angled Triangle. See B, Fig. 39.

In this case we cannot deduce the relations by use of the right-angled triangle only : some higher work is needed. But by proving certain proportions found by measurement they will be better remembered. If the student draws a 60°—30° right-angled triangle carefully he will find on measurement that the shortest side is half the longest side (or hypotenuse).

Thus if $b = 1$, then $l = 2$

Then $h^2 = l^2 - b^2$

$= 4 - 1 = 3$

$\therefore\ l = \sqrt{3} = 1{\cdot}732$ as may be verified by measurement.

Thus if shortest side = 1, longest side = 2, and other or intermediate side = 1·732 ; or—

Longest side = 2 × shortest side
Intermediate side = 1·732 × shortest side

From the above we also obtain—

Shortest side = ·5 × longest side
Intermediate side = 1·732 × shortest
= 1·732 × (·5 × longest)
= **·866 of longest side**

which again may be verified by measurement.

Exercises 70. On Right-angled and 60° – 30° Triangles.

(*Letters refer to Fig.* 40.)

1. A telegraph pole is strengthened by a wire rope stay as shown at A. Determine the length of the stay between the points of attachment.

2. A crane jib is to operate at a radius of 18 ft., when the top is 20 ft. above the bottom, as at B. What is the length of the jib?

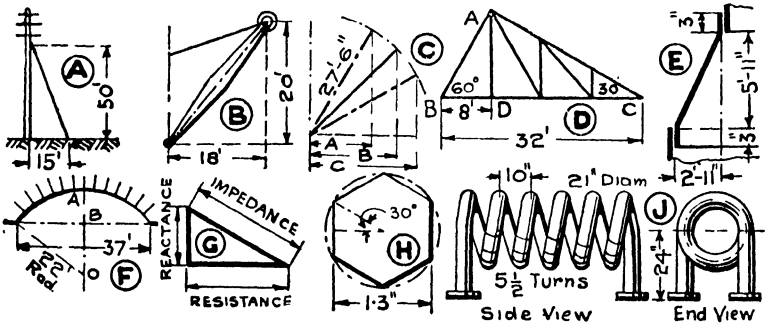


Fig. 40.—Exercises on Sides of Triangles.

3 to **5.** A crane jib is 27′-6″ long. At what radius does the head lie when the jib makes the following angles with the horizontal? (See C.)

3. 60° **4.** 45°. **5.** 30°.

(*i. e.*, what are the distances A, B, and C.)?

6. At D are given dimensions of a North light roof truss. Calculate the length of the two rafters AB and AC.

7. In the roof truss in Ex. 6, calculate the length of the strut AD, using the result of AB found in Ex. 6.

8. At E are given dimensions of the conical portion of a cooling tower shell. Calculate the total width of plate required for the inclined portion.

9. In connection with the flow of water in open channels of the form shown at M, Fig. 60, it is necessary to find the length of the inclined side AB. Find this length for the case shown.

10. A segmental arch has a span of 37 ft. and a radius of 22 ft. as shown at F. Calculate the " rise," *i.e.* the height AB. (*Hint.*—First find the distance BO, and subtract from the radius.)

11. In connection with alternating electric currents, we have the right-angled triangle shown at G. If resistance = ·5 ohm and reactance = ·01, calculate the value of the impedance.

12. Referring to G, if impedance = ·83, and resistance = ·47, find the reactance.

13. Bright hexagon-head screws are to be cut from round stock as indicated at H, and are required to be 1·3″ across the flats of the head. What must be the least diameter of the round stock? (*Hint.*—The least size is that across the corners. The method is indicated by the small triangle.)

14. Calculate the length of piping required for the evaporating coil shown at J, if the average diameter is 21″.

15. Find the length of hand-railing required for a spiral staircase, the diameter being 10 ft. and the pitch 9 ft. 6 ins., if there are 12 complete turns.

16. An open coiled helical spring (see Fig. 38) has an average diameter of $4\frac{1}{4}$″ and has 15 complete turns in a length of 2′-6″. Find the length of wire necessary to form the spring.

17. A spiral shoot in a warehouse (similar to a spiral staircase) is made of steel plate edged on the outside with angle bar. The diameter is 6 ft. and the pitch 12 ft. Calculate the total length of angle bar required for 11 complete turns.

Length of Arc of Circle.

1. *Given Radius and Angle.*—The length of the arc of a circle can be calculated exactly if its radius and the angle at its centre be known. If the ends of the arc are joined to the centre from which it was drawn a sector is formed. If the central angle is $n°$ then the sector is $\frac{n}{360}$ of a complete circle. Since the whole circumference of a circle is $2\pi r$, the circumference of the sector is only $\frac{n}{360}$ of $2\pi r$.

$$\therefore \text{Arc of Sector} = \frac{n}{360} \times 2\pi r$$

$$= \frac{2 \times 3{\cdot}14}{360} \times nr = \frac{\mathbf{n°r}}{\mathbf{57{\cdot}3}}$$

The length is in the same units as r.

Example 287.—Fig. 41 shows an end view of a 5/5″ corrugated sheet with dimensions (*i.e.*, it has 5 complete waves from A to B, each of 5″ pitch). Find the width of flat sheet necessary to make this corrugated sheet.

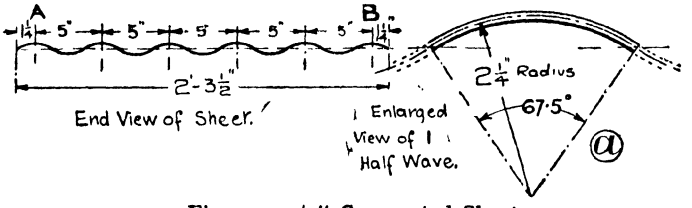


Fig. 41.—5/5″ Corrugated Sheet.

Consider 1 half wave as shown at (*a*).

It is a sector of a circle, radius $2\frac{1}{4}$″ and angle at centre $67\frac{1}{2}°$.

$$\text{Length of arc} = \frac{rn}{57{\cdot}3}$$

$$= \frac{2{\cdot}25 \times 67{\cdot}5}{57{\cdot}3} = 2{\cdot}65''$$

In all there are 11 of these half waves, 6 on top of, and 5 below the centre line, as may be seen from the figure.

$$\therefore \text{ width of sheet required} = 11 \times 2{\cdot}65 = 29{\cdot}2''$$

$$\text{say, } \underline{2'\text{-}5\tfrac{1}{4}''}$$

2. *Given Chord and Height.*—In many cases the angle n is not known, the dimensions c (*i.e.*, chord) and h (height) being given. The sector may be drawn and n and r measured, or the following approximate formula may be used, with an error of less than 1%.

$$\text{Length of arc} = \frac{8a - c}{3}$$

where a = chord of half the arc = AC in Fig. 78, and c = chord of whole arc = AB.

The value of a may be calculated from the right-angled triangle ACD. Thus $a^2 = \left(\frac{c}{2}\right)^2 + h^2$ and $a = \sqrt{\frac{c^2}{4} + h}$.

3. *Given Chord and Radius* (*i.e.*, c and r).—Height h must be calculated from the right-angled triangle OAD (Fig. 78), where $r^2 = \left(\frac{c}{2}\right)^2 + \text{OD}^2$ or $\text{OD} = \sqrt{r^2 - \frac{c^2}{4}}$ and h then $= r - \text{OD}$. Then proceed as in 2.

Exercises 71. On Arc of Sector.

(*Letters refer to Fig.* 42.)

1. A curved channel-section bar in a vaulted roof is to be 11′-6″ radius and is to subtend an angle of 75°, as shown at A. Find the length of bar required.

2. A pipe bend is required to be of 39″ radius, and to subtend an angle of 55°. What length is required from flange to flange?

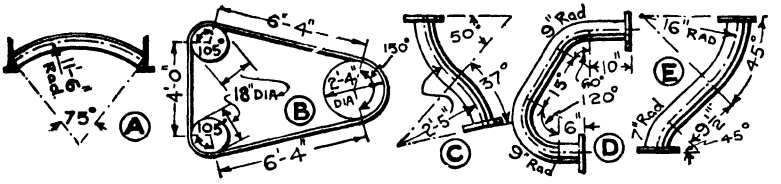


Fig. 42.—Exercises on Arc of Sector.

3. At B are given the arrangement and dimensions of a band-sawing machine. Calculate the length of the saw blade.

4. Calculate the length of the bent pipe shown at C.

5. At D are given dimensions of a bent copper pipe on a steam engine. Calculate its length from flange to flange.

6. Calculate the length of the curved pipe shown at E.

Area

Measurement of Area.—The area of any figure is the amount of surface it contains. It is made up of two measurements commonly known as length and breadth (or width), and consequently

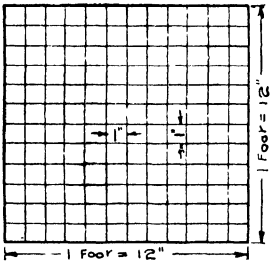


Fig. 43.

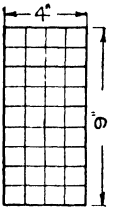


Fig. 44.

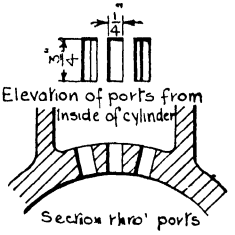


Fig. 45.

its units are derived from those in the ordinary table of length measure. The **Unit** of area or surface measure is the **Square Foot**, *i. e.*, the area of a square whose side is 1 Foot. The smallest sub-multiple in use is the Square Inch, *i. e.*, the area of a square of 1″

side. Larger multiples include the square yard, the square pole, etc., as set out in the table on p. 4. The latter half of the table is only used in the measurement of land where very large areas have to be dealt with. The reason for the 144 in converting square inches to square feet is seen in Fig. 43. Here a square of 1 foot side is divided up into squares of 1″ side. Each side being 12″ long, it is seen that the big square contains 12 rows of small squares, each row itself containing 12 small squares.

$$\therefore \text{ 1 sq. ft. } = 12 \times 12 = 144 \text{ sq. ins.}$$

It is also seen from this that the area of a square is found by "squaring" the side, or multiplying the length by the breadth.

Also a square of 1 yard side has a side 3 ft. long, and therefore contains $3 \times 3 = 9$ sq. ft.,

$$\textit{i. e.}, \text{ 1 yd. } = 3 \text{ ft.} \qquad \therefore \text{ 1 sq. yd. } = 3^2 = 9 \text{ sq. ft.}$$

$$\text{Similarly 1 pole } = 5\tfrac{1}{2} \text{ yds.} \qquad \therefore \text{ 1 sq. po. } = 5\tfrac{1}{2}{}^2 = 30\tfrac{1}{4} \text{ sq. yds.}$$

Area units = length units × length units = (length units)^2 or square units.

The Surveyors' Measure was designed to enable areas to be easily calculated in acres.

Since 1 chain = 22 yds., 1 sq. ch. = 22^2 = 484 sq. yds.

$$\therefore \textbf{ 10 sq. ch. = 4840 sq. yds. = 1 acre}$$

Now **1 acre** = 4 roods
= 4 × 40 sq. po. = 160 sq. po.
= 160 × $30\frac{1}{4}$ sq. yds. = **4840 sq. yds.**

Reduction.—The reduction from one unit to another used by the average engineer is practically confined to the use of the 144 and the 9.

Thus, **to convert square inches to square feet divide by 144,** and **to convert square feet to square inches multiply by 144,**

and similarly in conversion with the square yard.

Conversion among the other units is necessary when any land measurement occurs, the method being exactly similar to that of length.

Example 288.—The survey of a plot of land gives the area as 115·2 square chains. Convert this into acres, etc.

10 square chains = 1 acre
∴ 115·2 sq. ch. = 11·52 acres

4
2·08 rds.
40
3·20 sq. po.
30
6·00
¼ of ·2 ..... ·05
6·05 sq. yds.
9
·45 sq. ft.

Area = 11 ac. 2 rds. 3 sq. po. 6 sq. yds. 0·45 sq. ft.

Example 289.—A plot of building land measured with the 100 ft. chain gave an area of 82,728 sq. ft. State this in acres, etc.

9)82728 sq. ft.
9192 sq. yds.
4
121)36768(303 sq. po.
363
468
363
105 quarter sq. yds. over
= 26¼ sq. yds.

4,0)30,3 sq. po.
4) 7 rds. + 23 sq. po.
1 ac. + 3 rds.

∴ Area = 1 ac. 3 rds. 23 sq. po. 26¼ sq. yds.

Note.—Instead of dividing by 30¼ as 30·25, the sq. yds. are multiplied by 4, and then divided by 121, since $30\frac{1}{4} = \frac{121}{4}$. By this means the decimal point is not introduced. The remainder being ¼ sq. yds. is divided by 4 to bring to sq. yds.

Areas of the Simple Figures.

In calculating areas, *dimensions must be in the same units, i. e.*, feet and feet, or inches and inches, etc. A length in feet must never be multiplied by a width, say, in inches.

Square and Rectangle.

Area = length × breadth

Fig. 44 shows in detail the truth of the method. A rectangle is

9″ by 4″ (*i. e.*, 9″ long by 4″ wide), and evidently there are four rows of little squares, each row containing 9 sq. ins.

$$\therefore \text{ in all there are } 4 \times 9 = 36 \text{ sq. ins.}$$

i. e., length × breadth.

A square is merely a rectangle with equal sides.

Example 290.—In a certain two-stroke petrol motor the exhaust port is in the form of three rectangular holes each $\frac{3}{4}'' \times \frac{1}{4}''$ as shown in Fig. 45. Determine the area of the port.

$$\text{Area of 1 rectangular hole} = \tfrac{3}{4} \times \tfrac{1}{4} = \tfrac{3}{16} = \cdot 1875 \text{ sq. in.}$$

$$\therefore \text{ Area of 3 rectangular holes} = 3 \times \cdot 1875 = \cdot 5625 \text{ sq. in.}$$

$$\therefore \text{ Exhaust port area} = \cdot 562 \text{ sq. in.}$$

Example 291.—A steam port in an engine cylinder is 16″ wide, and the area required through the port (which is rectangular) is ·165 sq. ft. What must be the length of the port?

$$\text{Area} = \text{Length} \times \text{width}$$

$$\therefore \text{ Length} = \frac{\text{Area}}{\text{Width}}$$

$$= \frac{\cdot 165 \times 144}{16} = 1 \cdot 485''$$

$$\therefore \text{ Length of port} = 1\tfrac{1}{2}'' \text{ (to the nearest } \tfrac{1}{16}'')$$

Note.—The area is given in sq. ft. and length in ins. For calculation both must be in the same units. Hence the 144 is introduced to convert sq. ft. to sq. ins. The result is in inches.

Example 292.—When calculating the discharge of water through channels it is necessary to find the "hydraulic mean depth" which is "area of cross section of channel ÷ wetted perimeter." Find the

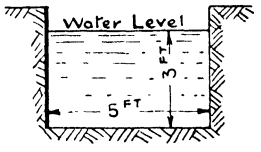


Fig. 46.

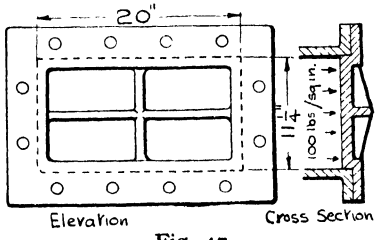


Fig. 47.

hydraulic mean depth for the channel shown in Fig. 46 when the water runs 3′-0″ deep.

Area of cross section = area of rectangle 5′ × 3′ = 15 sq. ft. Wetted perimeter is that portion of the channel perimeter touched by the water.

$$\therefore \textit{Wetted} \text{ perimeter} = 5 + 3 + 3 = 11 \text{ ft}$$

$$\therefore \text{Hydraulic Mean Depth} = \frac{\text{Area}}{\text{Wetted perimeter}}$$

$$= \frac{15}{11} = \underline{1{\cdot}363}$$

When dealing with areas we frequently meet with expressions such as "lbs. per square inch," "amperes per square inch," etc. meaning the number of lbs. (or amperes, etc.) on every 1 square inch of area. Thus if the total load (number of lbs.) on a piston is 5000 lbs. and the area is 100 sq. ins., the pressure is $\frac{5000}{100} = 50$ lbs. on 1 sq. in., or "50 lbs. per sq. in." Similarly if a conductor is carrying 1000 amperes per sq. in., and has an area of ·25 sq. in. the total current is $1000 \times {\cdot}25 = 250$ amperes.

Example 293.—A valve chest cover is 20″ × 11¼″ (inside dimensions) as shown in Fig. 47, and the steam pressure is 100 lbs. per sq. in. What is the total force tending to blow the cover off? If the cover is held down by 12 bolts what is the force on each bolt?

Area of rectangle under steam pressure $= 20 \times 11\frac{1}{4}$
$= 225$ sq. ins.

At 100 lbs. on each sq. in., total force on cover
$= 225 \times 100 = 22500$ lbs.

Then force on each bolt $= \frac{22500}{12} = \underline{1875 \text{ lbs.}}$

The areas of other figures can be obtained by dividing the surface up into squares, by two perpendicular sets of parallel lines The figure should be drawn either full size or to scale upon "squared paper" (see p. 349) where the perpendicular lines are already provided. The complete squares and fractions of squares enclosed by the outline of the figure are then counted up, and any piece which is less than half a square is neglected, whilst any piece more than half a square is counted as one. Thus, taking the quarter of a circle shown in Fig. 48 (only a quarter need be set down), there can be counted 71 complete squares; and around the outline there are 8 pieces greater than half a square

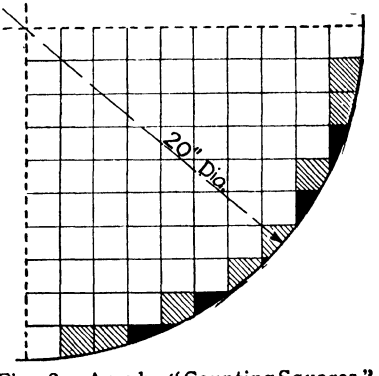


Fig. 48.—Area by "Counting Squares."

The area of the quadrant is then 79 sq. ins., and the total area of the circle $4 \times 79 = 316$ sq. ins. The pieces less than half a square (which are to be neglected) are shown black in Fig. 48; while those pieces greater than half a square are shown shaded. The correct area as obtained by the formula on p. 287, is 314·2 sq. ins., so that the result obtained from "counting squares" is very close.

Any figure may be treated in the foregoing manner, but the method is laborious, and areas can usually be found more readily by formulæ or simpler graphical methods, as will be shown.

Rhomboid.—This figure may be regarded as a distorted rectangle. In Fig. 49, at (*a*) we have a rectangle divided into a number of narrow horizontal strips. If the lowest strip be fixed, and each of the others shifted a small distance to the right of the one just below it, then the rectangle will appear as at (*b*), *i. e.*, as a rhomboid.

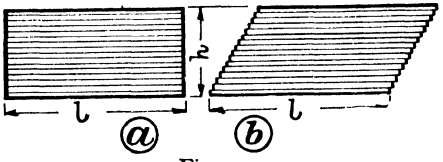


Fig. 49.

Now the number and size of the strips have not been altered in any way, and therefore the area of the rhomboid must equal the area of the rectangle. If the thickness of the strips be made very small then the slant sides of the rhomboid will become practically straight lines.

Then, **Area of Rhomboid** = Area of Rectangle
= **Base** × **Perpendicular Height**

and this statement is true for all parallelograms.

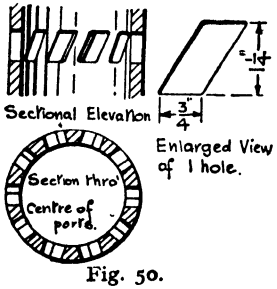


Fig. 50.

It is very important that the *perpendicular height* be used when calculating this area, and not the length of the slant side. If the height is not actually given, the figure should be drawn out and the perpendicular height measured.

Example 294.—The steam port for the piston valve in a high-speed engine has 10 holes with dimensions as shown in Fig. 50. Determine the total area through the port.

Area of 1 hole = base × perpendicular height
= ·75″ × 1·25″ = ·938 sq. ins.
∴ Area of 10 holes = 9·38 sq. ins. = total port area.

Example 295.—The steam port for a piston valve of a marine engine is to have an area of 97 sq. ins. There are 12 holes, as shown in Fig. 51. Calculate the height of the holes.

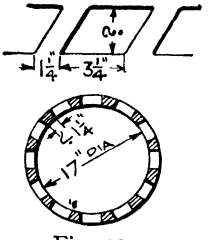


Fig. 51.

$$\text{Area per hole} = \frac{97}{12} = 8{\cdot}083 \text{ sq. ins.}$$

$$\therefore \text{ Height} = \frac{\text{Area}}{\text{Length}} = \frac{8{\cdot}083}{3{\cdot}25}$$

$$= 2{\cdot}485, \text{ say } 2\tfrac{1}{2}''$$

Triangle.—Consider any triangle, as ABC, Fig. 52. From two of the corners, say A and C, draw lines parallel to the other sides as AD and CD. Then the figure ABCD is a rhomboid and its area = lh, where l = base and h = perpendicular height. It can be seen, and measurement will prove, that the added triangle ADC is exactly the same size and shape as the original one ABC. Hence the area of triangle ABC is *half* the area of the parallelogram ABCD.

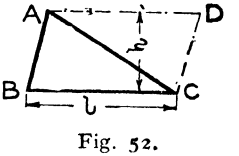


Fig. 52.

$\therefore$ **Area of triangle** = ½ area of parallelogram = ½ lh

= ½ **Base** × **Perpendicular Height**

It does not matter which side of the triangle is called the *base*, provided that the *corresponding* perpendicular height is taken. Thus consider the triangle in Fig. 53

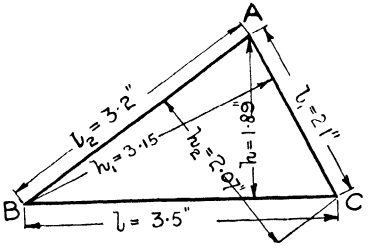


Fig. 53.

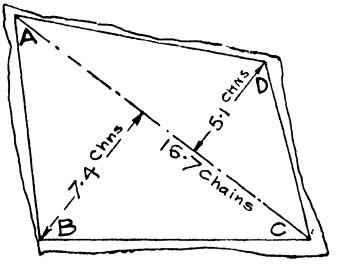


Fig. 54.

Base.	Perpendicular Height.	Area.
BC = l = 3·5	h = 1·89	½(3·5 × 1·89) = 3·307
AC = l_1 = 2·1	h_1 = 3·15	½(2·1 × 3·15) = 3·307
AB = l_2 = 3·2	h_2 = 2·07	½(3·2 × 2·07) = 3·307

Whence the area is the same in each case.

Should the perpendicular height not be given, then the triangle should be drawn out, and the height measured. In a right-angled triangle if either perpendicular side be taken as base, then the other perpendicular side is the perpendicular height. (An additional rule for area of triangles is given in the table at the end of these sections on Area.)

Example 296.—The area of the plot of ground shown in Fig. 54 is measured by dividing the shape into triangles, finding the areas of these and calculating approximately the surplus outside the triangles. If this surplus area is 18·6 square chains, find the total area of the plot.

Triangle ABC. Area $= \frac{1}{2}$ base $\times$ height
$= \frac{1}{2} \times 16{\cdot}7 \times 7{\cdot}4$
$= 61{\cdot}75$ sq. ch.

Triangle ADC. Area $= \frac{1}{2} \times 16{\cdot}7 \times 5{\cdot}1$
$= 42{\cdot}5$ sq. ch.

$61{\cdot}75$
$42{\cdot}5$
Surplus area $18{\cdot}6$

Total area $= 122{\cdot}85$ sq. ch. $= 12{\cdot}285$ acres.

Example 297.—A cooling tower has two air ducts in the form of triangles as shown at *a*, Fig. 55. What is the total cross sectional area in square feet? It is desired to replace them by a single triangular duct with base and height equal, which shall have the same area as the two combined. What must be the size of the new duct?

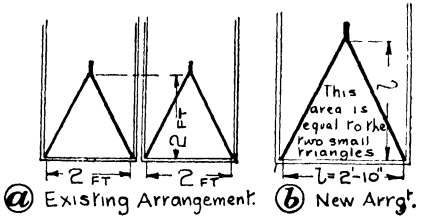


Fig. 55.—Air Ducts for Cooling Tower.

Area of 1 duct $= \frac{1}{2}$ base $\times$ height
$= \frac{1}{2} \times 2 \times 2 = 2$ sq. ft.
$\therefore$ Total area of two $= 4$ sq. ft.

Let l = base of new triangle; then since base is to equal perpendicular height, base also $= l$.

$\therefore$ Area of new triangle $= \frac{1}{2}$ base $\times$ height
$= \frac{1}{2}l \times l = \frac{1}{2}l^2$

$\therefore \frac{1}{2}l^2 = 4$
$\therefore \quad l^2 = 8$
$\therefore \quad l = \sqrt{8} = 2{\cdot}83'$ or $2'-10''$

The new duct will then appear as at *b*, Fig. 55.

Trapezium.—This may be divided up into two triangles by joining a pair of opposite corners with a straight line, measuring the bases and perpendicular heights of each, and then calculating the areas of the separate triangles, and finally adding the results together. Ex. 296, just dealt with, illustrates this.

A briefer statement in symbols is as follows : Refer to Fig. 56.

Take the diagonal l.

Measure extreme height of figure perpendicular to this $= h$.

$$\text{Then, Area of Trapezium} = \frac{lh_1}{2} + \frac{lh_2}{2} = \frac{lh}{2}$$

This last applies to any *quadrilateral*, but h must always be taken perpendicularly to l. (The student should work the above taking diagonal L, Fig. 56.)

Thus, given the diagonals of a square instead of the sides :

Area $= \frac{1}{2}$ diagonal $\times$ diagonal.

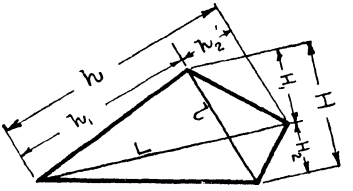


Fig. 56.—Area of Trapezium.

Trapezoid.—Consider the trapezoid ABCD in Fig. 57. Draw a line EF exactly half way between the two parallel lines. From points E and F draw lines perpendicular to the parallel lines as GH and KL. Continue the top line as shown dotted. Then it can be seen by both eye and measurement that the triangle AHE is

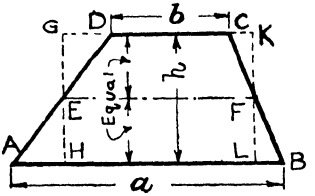


Fig. 57.

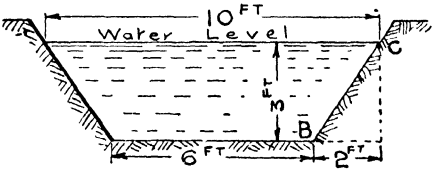


Fig. 58.

exactly the same size and shape as triangle EGD. Similarly triangle FKC is the same size and shape as FLB and replaces it. Then the area of the rectangle HLKG is the same as the area of the trapezoid.

But area of rectangle = length $\times$ height

The length is the length EF, halfway between the top and bottom, which is thus the average between the top and bottom.

$$\text{The average length} = \frac{\text{bottom} + \text{top}}{2} = \frac{a + b}{2}$$

Then, Area of Trapezoid = Average length × height.

$$= \left(\frac{a + b}{2}\right) \times h$$

Example 298.—Find the hydraulic mean depth (*i. e.*, area ÷ wetted perimeter, see Ex. 292) for the channel shown in Fig. 58.

$$\text{Average width} = \frac{10 + 6}{2} = 8 \text{ ft.}$$

$$\therefore \text{Area} = \text{average width} \times \text{height}$$
$$= 8 \times 3 = 24 \text{ sq. ft.}$$

Length of slant sides of trapezoid may be measured after carefully drawing out the section, or may be calculated with the aid of the right-angled triangle relation, thus—

$$\text{Slant side BC} = \sqrt{2^2 + 3^2}$$
$$= \sqrt{13} = 3{\cdot}61 \text{ ft.}$$
$$\text{Then, wetted perimeter} = \text{base} + 2 \text{ slant sides}$$
$$= 6 + 2 \times 3{\cdot}61$$
$$= 13{\cdot}22 \text{ ft.}$$
$$\therefore \text{Hydraulic mean depth} = \frac{\text{area}}{\text{wetted perimeter}}.$$
$$= \frac{24}{13{\cdot}22} = \underline{1{\cdot}815}$$

Example 299.—Fig. 59 gives dimensions of a positive conductor rail employed on an electrified railway. Find the area of the section in square inches, neglecting any small radii.

Cross section is difference between large and small trapezoids.

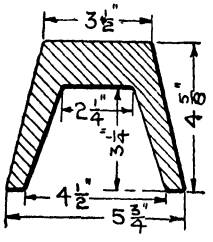


Fig. 59.

LARGE TRAPEZOID—

$$\text{Mean width} = \frac{3\frac{1}{2} + 5\frac{3}{4}}{2} = 4{\cdot}625$$

$$\therefore \text{Area} = \text{mean width} \times \text{height}$$
$$= 4{\cdot}625 \times 4{\cdot}625 = 21{\cdot}4 \text{ sq. ins.}$$

SMALL TRAPEZOID—

$$\text{Mean width} = \frac{2\frac{1}{4} + 4\frac{1}{2}}{2} = 3{\cdot}375$$

$$\therefore \text{Area} = 3{\cdot}375 \times 3{\cdot}25 = 10{\cdot}96 \text{ sq. ins.}$$

$$\therefore \text{Net area of section} = 21{\cdot}4 - 10{\cdot}96$$
$$= \underline{10{\cdot}44 \text{ sq. ins.}}$$

Exercises 72. On Areas of Triangles and Quadrilaterals.

(*Letters refer to Fig.* 60.)

1. The grate of an American locomotive is 9′-6″ long and 6′-3¼″ wide. Determine the grate area in square feet.

2. The heating surface in a locomotive fire-box is as follows: Top, 5′-4″ long, 3′-3″ wide; 2 sides each 5′-6″ long, 5′-4″ high; 2 ends each 3′-3″ wide, 5′-6″ high. Calculate the total heating surface in square feet.

3. When calculating the strength of masonry dams it is necessary to know the total pressure of the water on the face for a length of 1 ft. The total pressure on such a rectangular area is 62·4 HA lbs., where H = depth to the centre of the rectangle and A = the area of the rectangle. Find the total pressure when the total depth of water is 21 ft.

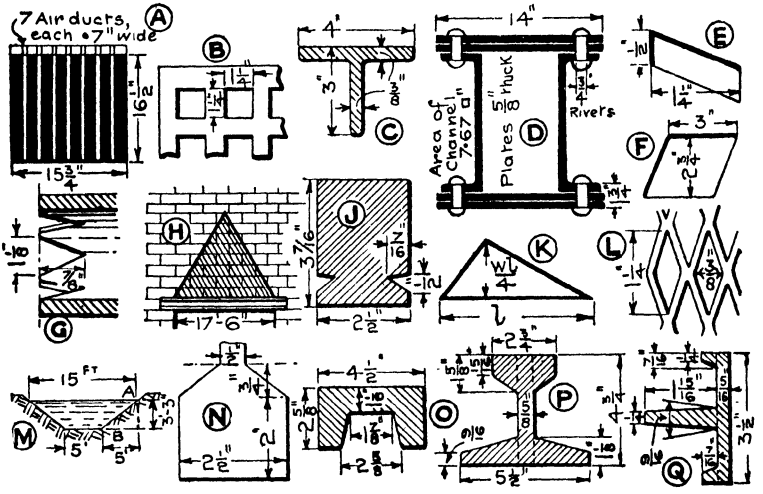


Fig. 60.—Exercises on Areas of Triangles and Quadrilaterals.

4. A column base plate is 2′-3″ square and rests on a York stone block. Allowing a pressure of 20 tons per sq. ft. on the block, what load can the column carry?

5. In a test to determine the bearing pressure permissible on a certain soil a platform having 4 legs each 8″ square was loaded with 31 cwts. Calculate the bearing pressure in lbs. per sq. ft.

6. In an Edwards' air pump the inlet ports consist of 16 rectangular holes each 4⅝″ × 4¼″. Find the total port area.

7. At A is shown the cross section of the rim of a dynamo armature. Calculate the *net* cross section (*i. e.*, the shaded area).

8. A rectangular screening plate 3′-2″ by 2′-2″ has 1¼″ square holes as shown at B, there being 18 rows of holes and each row containing 12 holes. Determine the total area through the holes. What percentage is this of the plate area?

9. In a tensile test on a piece of boiler plate the cross section of the specimen was $1\frac{3}{4}'' \times \frac{1}{2}''$. The breaking load was 25·8 tons. Calculate the breaking stress (*i. e.* load per sq. in.).

10. In a tensile test on a piece of mild steel plate the following results were obtained: original section 1·753″ wide, ·64″ thick; final section 1·472″ wide, ·482″ thick. Calculate the reduction in area, and express it as a percentage of the original area.

11. Calculate the area of the T-section shown at C, neglecting any radii at the corners.

12. At D is shown the cross section of a compound girder built of plates and channels. Find the net area of the cross section shown shaded. (*Note.*—Do not include space occupied by rivets.)

13. When charging a certain make of lead accumulators, a current of 5 amperes per sq. ft. of positive plate may be allowed. An accumulator has three positive plates each 9″ × 7″. Calculate the charging current in amperes. (*Note.*—Each plate has two sides.)

14. The rafters in a roof truss are 28 ft. long and are spaced 12 ft. apart. If the wind pressure is 25 lbs. per sq. ft., what is the total load upon the roof between a pair of rafters, due to the wind?

15. An ammeter shunt for switchboard use has four manganin strips each 3″ long and $1\frac{3}{8}''$ wide. Find the total amount of radiating surface. (*Note.*—Each strip has two sides.)

16. If the strips in Ex. 15 above are ·02″ thick, calculate the total cross section (*i. e.*, width × thickness). The resistance is $\frac{sl}{a}$ ohms where s = specific resistance, l = length of strips, and a = total area. Calculate the resistance if s = ·000043.

17. A parallel screen has 96 holes with dimensions as at E. Find the total area through the holes.

18. Find the area through the holes forming the inlet port of an Edwards' air pump, which has 12 holes as shown at F.

19. The steam ports in a pressure regulator on a turbine-driven centrifugal feed pump, are triangular, as shown at G. If there are 6 holes in all, find the total port area.

20. When designing girders to carry a wall over an opening we assume that the load on the girder consists only of the brickwork enclosed within an *equilateral* triangle as shown shaded at H. Calculate the area of the face of the wall supported by a 17′-6″ span girder, and the total load (per half-brick thick), if 1 sq. ft. of brickwork (per half-brick thick) weighs 125 lbs.

21. At J are given dimensions of the "keystone section bar," employed as third rail by the Underground Electric Railway. Calculate the area of the section, neglecting the radii. (*Hint.*—Deduct the triangles from the rectangle.)

22. In connection with deflection of beams it is necessary to calculate the area of certain triangular diagrams. Find a statement for the area in the case shown at K.

23. Expanded steel of diamond mesh, as shown at L, is used in an oil separator. Calculate the total area through the holes in a sheet 6′-0″ wide by 4′-0″ high, which contains 144 holes across and 32 holes vertically.

24. A flat screening plate has 20 diamond-shaped holes, the long diagonal being 2″ and the short one $\frac{3}{4}''$. Find the total area through the holes.

25. Find the area enclosed (in square chains) between the straight lines on the plan of a plot of ground, given in Fig. 61. (Any dimensions required beyond those given on the picture are to be measured on the scale of chains shown.)

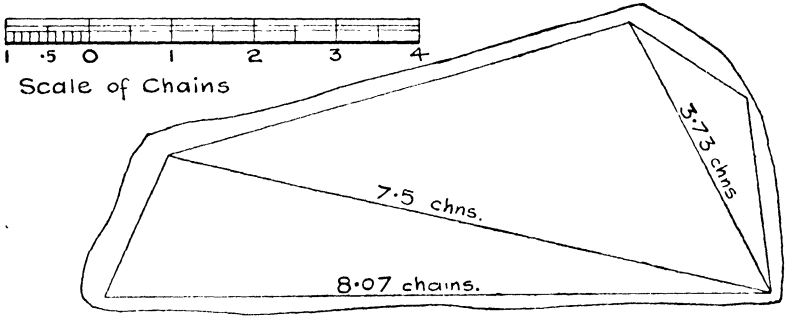


Fig. 61.—Area of Triangles.

26. A section of a supply channel is shown at M. Calculate the cross-sectional area.

27. The "hydraulic mean depth" = area of section ÷ wetted perimeter (see Ex. 292). Calculate this for the channel shown at M.

28. The kathode in an electrolysis experiment is shown at N. It is immersed until the level of the liquid reaches the narrow shank. Determine the area in the liquid. Allowing a current of ·09 amps. per sq. in., what total current may be applied?

29. At O are given dimensions of the section of the third rail employed on the Central London Railway. Determine the area of the cross section, neglecting any rounding of corners. (*Note.*—The section is a rectangle minus a trapezoid.)

30. The third rail section used on the Metropolitan District Railway is shown at P. Determine the area in square inches. (*Note.*—For calculation divide the figure up as suggested by dotted lines.)

31. At Q is shown the section of the conductor rail on the L.C.C conduit tramway system. Calculate the cross-sectional area. (The mode of division is suggested.)

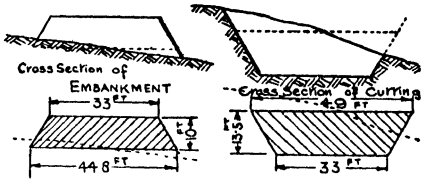


Fig. 62.

32. When calculating the volume of earth to be dealt with in constructing cuttings and embankments, sections occur such as in Fig. 62. In finding the area of these sections the sloping ground line is replaced by a straight horizontal line, converting the section into a trapezoid of approximately the same area. Find in square feet the two adjusted areas (cross-hatched), shown dimensioned in Fig. 62.

Hexagon.—It is a property of the hexagon that the length of its side is the same as that of the radius of the circle through the

points, so that a hexagon drawn in a circle of $\frac{1}{2}$" rad. has a $\frac{1}{2}$" side. If the opposite corners be joined by straight lines then all will pass through the centre of the outside circle as at O in Fig. 63. This will divide the hexagon into six equal triangles like OAB, which are equilateral, *i. e.*, OA = AB = OB

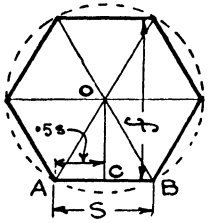


Fig. 63.

Then the area of the hexagon = 6 times area of triangle OAB.

Let S = side of hexagon
then AC = $\frac{1}{2}$ side = ·5 S

The triangle OAC has a right angle at C and a 60° angle at A.

Then referring to p. 270,

the intermediate side CO = 1·732 × short side
= 1·732 × ·5 S
= ·866 S

Area of triangle = $\frac{1}{2}$ base × height
= $\frac{1}{2}$ S × ·866 S = ·433 S^2

∴ **Area of hexagon** = 6 × ·433 S^2
= 2·598 S^2
say, **2·6 S^2**

also if f = distance across the flats

Area = ·866 f^2

Octagon.—Let S = side of octagon (see Fig. 64).

AB = S and in an octagon $\theta = 45°$
∴ AC = ·707 S (see p. 269)

Side of outer square = S + 1·414 S
= 2·414 S

∴ Area of outer square = 5·827 S^2

Deduct for four corners as ABC = 2 squares of side ·707
= 2 × ·707 × ·707 = 1·00 S^2

Deducting gives 4·827 S^2

∴ **Area of octagon = 4·83 S^2**

also if f = distance across the flats

Area = ·829 f^2

Circle.—The area of a circle may be examined by the aid of Fig. 65. At (*a*) a circle is divided up into 16 equal sectors by means of 8 diameters. These sectors are numbered from 1 to 16. If the

circle is cut along these diameters the sectors may be arranged as at (*b*), forming a parallelogram if the slight curvature at the ends of the sectors be neglected. The more sectors that the circle is divided into, the straighter will the top and bottom of this parallelogram become. It can be seen that the length of the parallelogram is only that occupied by sectors numbers 1 to 8: *i. e.*, half a circumference.

Then, length of parallelogram = half circum. $= \frac{2\pi r}{2} = \pi r$

Also height ,, ,, = length of sector = radius = r

Then, **area of circle** = area of the parallelogram

= length × perpendicular height

$= \pi r \times r = \pi r^2$

or **Area** $= \pi \times$ **Square of Radius.**

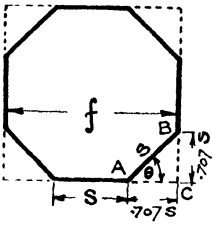


Fig. 64.

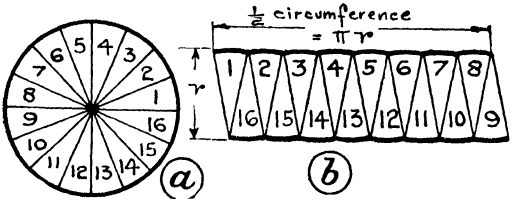


Fig. 65—Area of Circle.

Another form which is often useful is in terms of the diameter, and may be obtained as above—

$$\text{Half circum. of circle} = \frac{\pi d}{2} \text{ and radius} = \frac{d}{2}$$

$$\therefore \text{Area} = \frac{\pi d}{2} \times \frac{d}{2} = \frac{\pi}{4} d^2$$

$$\text{Taking } \pi = 3{\cdot}142, \frac{\pi}{4} = {\cdot}7854$$

$$\therefore \textbf{Area of circle} = {\cdot}7854\, d^2.$$

Note.—3·14 and ·785 are sufficiently accurate for most practical work.

This may also be obtained from the first form, by substituting for r the value $\frac{d}{2}$.

$$\text{Thus } \pi r^2 = \pi \left(\frac{d}{2}\right)^2 = \frac{\pi d^2}{4}$$

Example 300.—A water-pipe is 14″ diameter. What is its cross-sectional area?

$$\text{Radius} = 7''$$
$$\therefore \text{Area} = \pi r^2 = \pi \times 7^2$$
$$= 3{\cdot}14 \times 49 = 154 \text{ sq. ins.}$$

Example 301.—A 3/20 electric cable is composed of three wires each of No. 20, S.W.G. (*i. e.*, ·036″ dia.) as shown in Fig. 66. Find the total area of the section and the current allowable at 1000 amperes per sq. in.

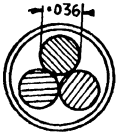


Fig. 66.

Area of 1 strand $= {\cdot}785d^2$
$= {\cdot}785 \times ({\cdot}036)^2$ *
$= {\cdot}785 \times {\cdot}001296$
$= {\cdot}001017$ sq. in.

$\therefore$ Total area of section $= 3 \times {\cdot}001017$
$= {\cdot}003051$ sq. in.

At 1000 amps. per sq. in.
Current allowable $= {\cdot}003051 \times 1000$
$= 3{\cdot}05$ amps.

[* If the student finds it of any assistance he can put a bracket round the quantity to be squared.]

Example 302.—A reinforced concrete beam is to have a total area of steel of 1·07 sq. ins. How many bars must be used if they are to be $\frac{7}{8}$″ dia.?

Area of $\frac{7}{8}$″ dia. circle $= {\cdot}785 \times {\cdot}875^2$
$= {\cdot}785 \times {\cdot}766 = {\cdot}601$ sq. in.

$$\therefore \text{No. of bars} = \frac{\text{total area}}{\text{area of 1 bar}} = \frac{1{\cdot}07}{{\cdot}601} = 1{\cdot}78$$

We must, therefore, use 2 bars.

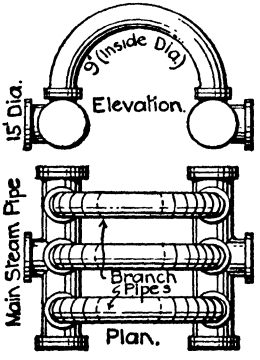


Fig. 67.

Example 303.—A set of expansion bends for a main steam-pipe is arranged as in Fig. 67. The combined areas of the small bends is slightly greater than the area of the main steam-pipe. This pipe is 15″ dia., and there are three small bends each 9″ dia. Determine by how much the total area of the bends is greater than that of the main pipe. Also express this as a percentage of the main steam-pipe area.

Area of main steam-pipe $= \cdot 785d^2$
$= \cdot 785 \times 15^2$
$= \cdot 785 \times 225 = 176 \cdot 6$ sq. ins.
Area of 1 bend $= \cdot 785d^2$
$= \cdot 785 \times 9^2$
$= \cdot 785 \times 81 = 63 \cdot 6$
$\therefore$ Total area of bends $= 3 \times 63 \cdot 6 = 190 \cdot 8$
Excess area of bends $= 190 \cdot 8 - 176 \cdot 6 = \underline{14 \cdot 2 \text{ sq. ins.}}$

$\therefore$ As a percentage of the main steam-pipe area this

$$= \frac{14 \cdot 2 \times 100}{176 \cdot 6} = \underline{8\% \text{ approx.}}$$

Example 304.—A cottered joint for pump rods is shown at *a*, Fig. 68, and an enlarged cross section through the cotter hole is given at *b*,

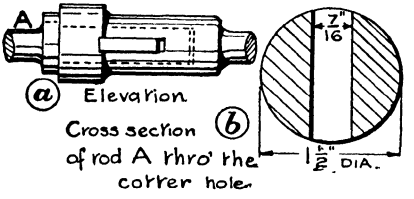


Fig. 68.

with dimensions. Determine the effective or net area (shown shaded) at this section. If the load on the rod is 7500 lbs., what is the stress (*i. e.*, force per sq. in.)?

Area of $1\frac{1}{2}''$ dia. circle $= \cdot 785 \times 1 \cdot 5^2$
$= \cdot 785 \times 2 \cdot 25 = 1 \cdot 766$ sq. ins.
Deduct plain rectangular hole $1\frac{1}{2}'' \times \frac{7}{16}'' = \cdot 656$
(neglect rounded ends)
$\therefore$ net area $= \underline{1 \cdot 110 \text{ sq. ins.}}$

$$\therefore \text{Stress} = \frac{\text{load}}{\text{area}} = \frac{7500}{1 \cdot 11} = \underline{6750 \text{ lbs. per sq. in.}}$$

Determination of Diameter of Circle from Area.—When the area of a circle is known the diameter can be obtained by solving an equation. Either the radius or the diameter may be obtained directly—

Thus $\pi r^2 = \text{area}$

$\div \pi$ $r^2 = \dfrac{\text{area}}{\pi}$

Taking $\sqrt{\ }$ $r = \sqrt{\dfrac{\text{area}}{\pi}}$

Hence **to determine the radius of a circle, knowing its area, divide the area by π, and extract the square root of the result.**

$$\text{Similarly} \qquad \frac{\pi}{4}d^2 = \text{area}$$

$$i.\,e., \qquad \cdot 785d^2 = \text{area}$$

$$\div \cdot 785 \quad \therefore \quad d^2 = \frac{\text{area}}{\cdot 785}$$

$$\text{Taking } \sqrt{\ } \qquad d = \sqrt{\frac{\text{area}}{\cdot 785}}$$

Hence **to obtain the diameter of a circle, when its area is known, divide the area by ·785, and extract the square root of the result.**

Example 305.—The area of a steam-pipe is to be 12·3 sq. ins. What will be its inside diameter?

$$\cdot 785d^2 = 12 \cdot 3$$

$$d = \sqrt{\frac{12 \cdot 3}{\cdot 785}}$$

$$= \sqrt{15 \cdot 7} = 3 \cdot 96''$$

say, $4''$ dia.

Example 306.—The area of the low pressure cylinder of a compound condensing engine is to be $3\frac{1}{4}$ times the area of the high-pressure cylinder, and is 30″ dia. Find the diameter of the H.P. cylinder.

$$\text{Area of L.P. cylinder} = \pi r^2$$

$$= 3 \cdot 14 \times 15^2$$

$$= 3 \cdot 14 \times 225 = 706 \text{ sq. ins.}$$

$$\text{Area of L.P. cylinder} = 3\tfrac{1}{4} \times \text{area of H.P.}$$

$$\therefore \text{Area of H.P.} = \frac{\text{Area of L.P.}}{3 \cdot 25}$$

$$= \frac{706}{3 \cdot 25} = 217 \cdot 5 \text{ sq. ins.}$$

$$\therefore d = \sqrt{\frac{217 \cdot 5}{\cdot 785}} = \sqrt{277} = 16 \cdot 64$$

say, $16\frac{5}{8}''$ dia.

The above may be extended to three or four cylinders when the ratio of the area of each cylinder to the H.P. (or L.P.) is known.

Example 307.—In an expansion bend of the form shown in Fig. 67, the main steam-pipe is 18″ dia. There are to be four bends, and their combined area is to be about 8% greater than the steam-pipe area. Determine the diameter of the bends.

Area of main steam-pipe $= \pi r^2$
$= 3{\cdot}14 \times 9^2$
$= 3{\cdot}14 \times 81 = 254$ sq. ins.

Taking 8% of this $= \frac{8}{100} \times 254 = 20{\cdot}32$ sq. ins.

$\therefore$ Total area of bends $= 254 + 20{\cdot}3 = 274{\cdot}3$ sq. ins.

There being four bends the area of each

$$= \frac{274{\cdot}3}{4} = 68{\cdot}6 \text{ sq. ins.}$$

$\therefore {\cdot}785d^2 = 68{\cdot}6$ if d = dia. of branch

$$d = \sqrt{\frac{68{\cdot}6}{{\cdot}785}}$$

$$= \sqrt{87{\cdot}4} = 9{\cdot}35''$$

say, $9\frac{3}{8}''$ dia.

Example 308.—The total force on a valve spindle is 8400 lbs. and the stress in the spindle (*i. e.*, the force per sq. in. of area) is not to be more than 2500 lbs. per sq. in. What must be the diameter of the spindle?

Area required in spindle $= \frac{8400}{2500} = 3{\cdot}36$ sq. ins.

$$\therefore d = \sqrt{\frac{3{\cdot}36}{{\cdot}785}}$$

$$= \sqrt{4{\cdot}28} = 2{\cdot}07''$$

$\therefore$ Spindle diameter $= 2\frac{1}{16}''$

Exercises 73. On Areas of Circles.

(*Letters refer to Fig.* 69.)

1. The outlet from the turbine wheel in a hydro-electric installation is circular and 18 ft. dia. Determine the cross-sectional area.

2. Find the cross-sectional area of a wire the diameter of which is ·109″.

3. A tray in a contact feed water heater has three rows of holes $\frac{3}{8}''$ dia., there being 56 in each row. Determine the total area through the holes.

4. At W, Fig. 31, is shown the arrangement of a split concentric electric cable. Determine the cross-sectional area in the central portion (which contains 18 wires, No. 16 S.W.G., *i. e.*, ·064″ dia.); in the outer ring (which contains 34 wires, No. 19, S.W.G., *i. e.*, ·04″ dia.); and the total area.

5. A concrete column is reinforced with 20 steel rods $\frac{7}{8}''$ dia. Find the total area of steel.

6. A motor piston is $3\frac{1}{4}''$ dia. Find the total force upon it if the pressure is 300 lbs. per sq. in.

7. Calculate the area through the boiler flue shown at A.

8. Calculate the area through the discharge flume for a water turbine, as shown at B.

9. A wire rope consists of six strands, each having 24 wires of No. 15, S.W.G. (·072″ dia.). Calculate the total cross-sectional area of the rope. Assuming that every square inch requires a breaking load of 100 tons, what is the breaking load of this rope?

10. A $1\frac{1}{8}$″ eye-bolt is ·942″ dia. at the bottom of the screw thread (*i. e.*, at the weakest place). Find its cross-sectional area, and allowing a stress of 9000 lbs. per sq. in., calculate the load it will carry, in tons.

11. A locomotive safety valve has two valves each $2\frac{1}{2}$″ dia., held down by a central spring as shown at C. Find the total force on the spring when the steam is just blowing off at 140 lbs. per sq. in.

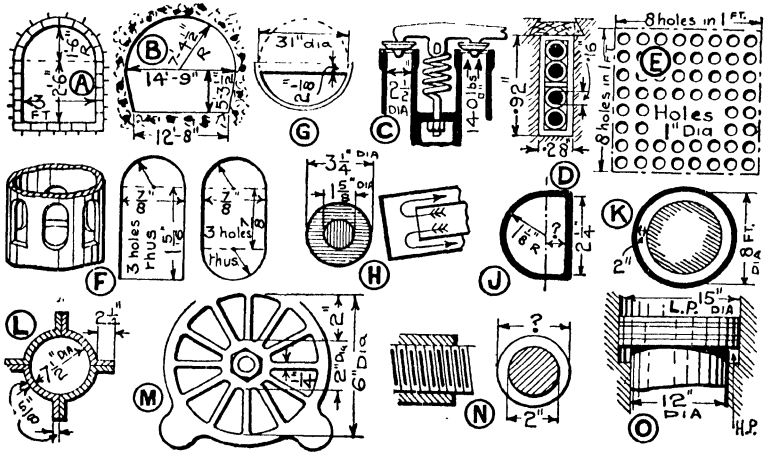


Fig. 69 —**Exercises on Areas of Circles and Hollow Circles.**

12. A two-stage air compressor has a low-pressure cylinder 10″ dia. and a high-pressure cylinder $6\frac{1}{2}$″ dia. Find the ratio of piston area of low pressure to high pressure.

13. At D is shown an armature slot with 4 round conductors ·16″ dia. Owing to waste space between wires and thickness of insulation, the slot is not completely filled with conductors. Find the "space factor," which is "area of conductors ÷ area of slot."

14. A steam-collecting pipe in a boiler is 7″ dia., and is to be perforated with rectangular slots 2″ × $\frac{3}{4}$″. How many such slots are required so that the total area through the holes is $1\frac{1}{2}$ times the pipe area?

15. A boiler manhole is 16″ dia. and the steam pressure is 60 lbs. per sq. in. The cover is held on by sixteen $\frac{7}{8}$″-bolts. Find the stress in the bolts (*i. e.*, the force per sq. in. of bolt area) if the diameter of a $\frac{7}{8}$″-bolt at the bottom of the thread is ·733″.

16. A perforated screening plate is made with holes at $1\frac{1}{2}$″ pitch as at E. Determine the area of perforation per square foot of plate. If the area of perforation is to be not more than two-fifths of the whole plate area, find if this rule is satisfied here.

17. The low-pressure exhaust port in a Willans' central valve engine is composed of six holes in a hollow tube, as shown dimensioned at F. Determine the total area through the port.

18. At G is shown the front of an ash-pit in a Lancashire boiler. In a boiler test it was necessary to know the area through the ash-pit. Find the required area. (*Note.*—The area is very nearly a semicircle less a rectangle.)

19. H gives dimensions of the Field tube in a Niclausse boiler. Find the ratio of the outer area to the inner area (the two areas are indicated by shade lines).

20. It is calculated that a reinforced concrete beam requires a total area of steel of 1·56 sq. ins. If round rods $\frac{5}{8}$″ dia. are to be used, how many will be required? (Take the nearest whole number.)

DIAMETER FROM AREA.

21. An exhaust main is to take the exhaust steam from three engines. Two of the exhaust pipes are $5\frac{1}{2}$″ dia. and the third is $2\frac{3}{4}$″ dia. Calculate the diameter of the exhaust main, the area of which is to equal the total area of the other three exhaust pipes.

22. A 64/38 flexible wire for electric work is made up of 64 strands of No. 38 gauge (*i. e.*, ·006″ dia.). Find what diameter of solid wire is equivalent in cross-section to this flexible wire.

23. A copper conductor, No. 6/0, S.W.G. (·464″ dia.) is to be replaced by an aluminium conductor. If the area of the latter is to be 1·7 times the area of the former (being a worse conductor), what must be the diameter of the aluminium conductor?

24. A hydraulic riveter is required to give a total force of 100 tons and the pressure water is supplied at 1500 lbs. per sq. in. Find the area and the diameter of the ram. (*Hint.*—Bring total force to lbs.)

25. A Venturi meter in a pipe 30″ dia. is to have a throat area one-eighth the pipe area. What will be the diameter of the throat?

26. The exhaust pipe of a petrol motor is of the cross-section shown at J. The circular part is to be $1\frac{1}{8}$″ radius and the whole area required is 2·4 sq. ins. What must be the depth of the rectangular piece? (*Hint.*—Area of half-circle subtracted from given area gives area of rectangular piece.)

27. In a hydraulic accumulator pump the piston is 4″ dia. and the piston rod area is required to be one-half of the piston area. Calculate the diameter of the piston rod.

28. The Board of Trade Rule for safety valves is $A = \frac{37{\cdot}5G}{P}$, where A = area of safety valve in sq. ins., G = total grate area in sq. ft., and P = working pressure + 15 lbs. A Lancashire boiler to work at 120 lbs. per sq. in. has two grates each 6 ft. long and 2′-6″ wide. Calculate the diameter of the safety valve.

29. In an expansion bend of the form shown in Fig. 67, the main steam-pipe is $12\frac{1}{2}$″ dia. The total area of the bends is to be 10% more than the steam-pipe area. Calculate the diameter of each branch if there are to be three branch pipes of equal area.

30. The areas of the cylinders in a triple expansion engine are in the ratios of 1 : 2·8 : 7·1. If the low-pressure cylinder diameter is 7′-4″, what will be the diameters of the high and intermediate pressure cylinders? (*Hint.*—Area of L. P. cylinder is 7·1 times H. P. area. Work as in Example 306. Also area of I. P. cylinder is 2·8 times H. P. area.)

Hollow Circle or Ring (also called an Annulus).—The area of a ring, *i. e.*, the area enclosed between two circles, one inside the other (as in Fig. 70) is evidently *the larger area minus the smaller.* If the diameters are known, then by calculating each area separately, and subtracting, the value can be found. But it is usually more convenient to have a single formula for this area, thus—

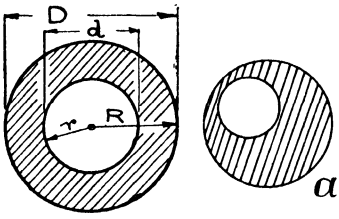


Fig. 70.

Area of large circle $= \pi R^2$ if R = rad. of large circle
Area of small circle $= \pi r^2$,, r = ,, ,, small ,,
Then **Area of Ring** = large circle − small circle
$= \pi R^2 - \pi r^2$
$= \pi(\mathbf{R^2} - r^2)$ taking out the common factor π.

By using this form only one multiplication by π has to be made.

A similar form can be obtained in terms of the diameters, thus

Area of large circle $= \cdot 785D^2$ if D = dia. of large circle
Area of small circle $= \cdot 785d^2$,, d = ,, ,, small ,,
Then **Area of Ring** $= \cdot 785D^2 - \cdot 785d^2$
$= \mathbf{\cdot 785(D^2 - d^2)}$ taking out the common factor ·785.

This may also be stated as $\cdot 785(D + d)(D - d)$, as dealt with in Chap. III.

The above also applies if the inside circle is not in the centre of the outside one (*i. e.*, is eccentric) provided that it is completely inside. (See *a*, Fig. 70.)

Example 309.—A hollow shaft is 15″ outside diameter and 11″ inside diameter. Find the area of its cross-section.

Area $= \cdot 785(D^2 - d^2)$
$= \cdot 785(15^2 - 11^2)$
$= \cdot 785(225 - 121)$
$= \cdot 785 \times 104 = \underline{81 \cdot 6 \text{ sq. ins.}}$

Working on the radius form—

$R = 7 \cdot 5''$ and $r = 5 \cdot 5''$
Then area $= 3 \cdot 14(7 \cdot 5^2 - 5 \cdot 5^2)$
$= 3 \cdot 14(56 \cdot 25 - 30 \cdot 25)$
$= 3 \cdot 14 \times 26 = \underline{81 \cdot 6 \text{ sq. ins.}}$

Example 310.—A thrust bearing in a 5-ton testing machine has 4 collars each 2″ inside and $2\frac{3}{4}$″ outside diameter as in Fig. 71. Find the bearing pressure per sq. in. when the full load of 5 tons is on the machine.

Area of 1 collar $= \cdot 785(D^2 - d^2)$
$= \cdot 785(2 \cdot 75^2 - 2^2)$
$= \cdot 785(7 \cdot 56 - 4)$
$= \cdot 785 \times 3 \cdot 56 = 2 \cdot 795$ sq. ins.

$\therefore$ Total area of 4 collars $= 4 \times 2 \cdot 795$
$= 11 \cdot 18$ sq. ins.

$\therefore$ Bearing pressure in lbs. per sq. in. $= \frac{5 \times 2240}{11 \cdot 18}$
$= 1002$ lbs. per sq. in.

Example 311.—In a double-beat valve of a steam engine (Fig. 72) the upper valve is 7″ dia. and the lower one $5\frac{1}{2}$″ dia. If the steam pressure between the discs is 120 lbs. and the weight of the valve and its

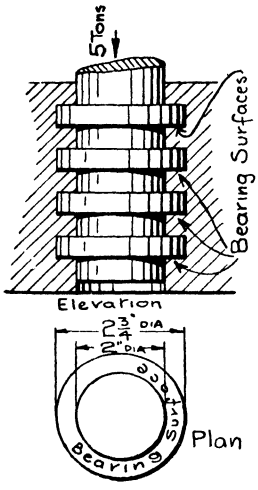


Fig. 71.

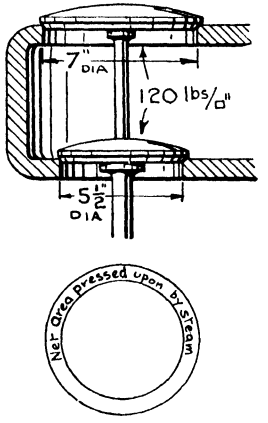


Fig. 72.

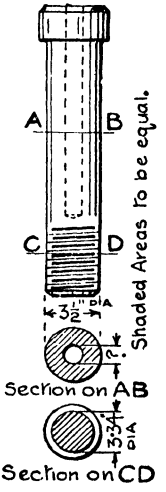


Fig. 73.

Practical Examples involving Area of Hollow Circle.

fitting is 25 lbs., what force must be exerted by a spring (not shown) which holds the valve down on its seat?

Note.—The top disc being the larger, and pressure being inside, there is a force acting upwards trying to lift the valve. This is partly neutralized by the weight of the valve, etc., which acts downwards.

Effective area = area of upper valve minus area of lower
$= \cdot 785 \times 7^2 - \cdot 785 \times 5 \cdot 5^2$
$= \cdot 785(7^2 - 5 \cdot 5^2)$
$= \cdot 785(49 - 30 \cdot 25)$
$= \cdot 785 \times 18 \cdot 75 = 14 \cdot 7$ sq. ins.
Then upward force of steam $= 120 \times 14 \cdot 7$
$= 1764$ lbs.
Then final upward force = force on spring
= steam force minus weight of valve
$= 1764 - 25 = \underline{1739 \text{ lbs.}}$

Example 312.—Bolts subjected to heavy shocks are sometimes made with a hole drilled up the unscrewed portion as in Fig. 73, so that the area of the bolt at any section as AB is equal to the area at the bottom of the thread. A particular bolt in a connecting-rod end is $3\frac{1}{2}''$ dia., and the diameter at the bottom of the thread is $3 \cdot 34''$. What must be the diameter of the central hole?

Area at bottom of screw thread $= \cdot 785 \times 3 \cdot 34^2$
$= \cdot 785 \times 11 \cdot 15 = 8 \cdot 75$ sq. ins.
Then net area as at AB $= \cdot 785(D^2 - d^2)$
$= \cdot 785(3 \cdot 5^2 - d^2)$
$= \cdot 785(12 \cdot 25 - d^2)$

and this is to equal 8·75 sq. ins.

$\therefore \cdot 785(12 \cdot 25 - d^2) = 8 \cdot 75$
$\div \cdot 785 \quad \therefore \quad 12 \cdot 25 - d^2 = \frac{8 \cdot 75}{\cdot 785}$
Transpose $\therefore \quad 12 \cdot 25 - 11 \cdot 15 = d^2$
$\therefore \quad d^2 = 1 \cdot 1$
Take $\sqrt{\ }$ $\therefore \quad d = \sqrt{1 \cdot 1} = 1 \cdot 05''$, say, $\underline{1\frac{1}{16}'' \text{ dia.}}$

Note.—This was the actual size of hole used in the particular bolt mentioned.

The area of a hollow circle may also be obtained in another way which is sometimes preferable. Suppose we imagine the ring to be made of a long thin rectangle bent round into a circle, the length of the rectangle being the *average circumference* of the circle, and the width, of course, the width of the ring.

Then area = length of rectangle × width
Now length of rectangle = average circumference of ring
$= \pi \times$ average diameter
and width of rectangle = width of ring
$= \frac{1}{2}$ difference of diameters

Then **area** $= \pi \times$ **average diameter** $\times \frac{1}{2}$ **difference of diameters.** If this be put into symbols and evaluated we obtain a proof of its accuracy.

$$\text{Average diameter} = \frac{D + d}{2}$$

$$\text{Width} = \tfrac{1}{2}(D - d)$$

$$\text{Area} = \pi \times \frac{D + d}{2} \times \tfrac{1}{2}(D - d)$$

$$= \frac{\pi}{4}(D + d)(D - d)$$

$$= \frac{\pi}{4}(D^2 - d^2)$$

Refer to p. 153.

The foregoing method by average diameter, etc., is somewhat easier when the ring is very thin, *i. e.*, the inside and outside diameters are nearly equal. Then it is usually easier to find an average diameter, and multiply by the thickness, than to find the difference of the two squares. The method can only be applied to cases of concentric circles.

Example 313.—Find the cross-sectional area of the metal in the lap-welded wrought-iron pipe shown in Fig. 74.

Inside diameter is 15″, and thickness is $\frac{1}{4}$″. Then outside diameter is $15\frac{1}{2}$″, and average diameter $= 15\frac{1}{4}$″.

$\therefore$ Average circum. $= \pi \times 15\frac{1}{4}$

and area $= \pi \times 15{\cdot}25 \times {\cdot}25 = 11{\cdot}98$ sq. ins.

say, 12 sq. ins.

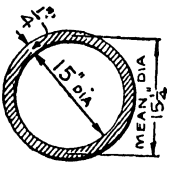


Fig. 74

As a comparison the student should work this example by the other method.

Exercises 74. On Areas of Hollow Circles.

(*Letters refer to Fig.* 69.)

1. A hollow shaft has an outside diameter of $6\frac{1}{4}$″ and an inside diameter of $3\frac{1}{8}$″. Find its cross-sectional area.

2. A hollow cast-iron column is 10″ inside diameter and $12\frac{1}{2}$″ outside. Find the area of its cross-section, and the load it can carry at 3·5 tons per sq. in.

3. In a bolt such as is shown in Fig. 73, the bolt diameter is $2\frac{1}{2}$″ and the diameter at the bottom of the thread is 2·18″. What size of hole is required?

4. A chimney stack is 8 ft. inside diameter. If the effect of friction, etc., be considered equivalent to the loss of a layer of air 2″ thick round the inner surface, as shown by the plain area at K, calculate this area in square feet. (*Hint.*—Use "average diameter" method.)

5. A wrought-steel riveted pipe is 16″ internal diameter and is $\frac{1}{4}$″ thick. Find its cross-sectional area. (Use "average diameter" method.)

6. Calculate the area of the section of a column shown at L.

7. At M is shown the plan view of the grid in a flexible disc valve. Calculate the area through the holes. (*Hint.*—Deduct the area occupied by the bars from the area of the annulus.)

8. The plunger in a hydraulic press consists of a hollow rod moving over a screw as shown at N, the area producing the pressure being that of the annular area (the plain area in end view). If the area required is 2·75 sq. ins. and the inside diameter is 2″, what must be the outside diameter?

9. The arrangement of a compound air compressor with differential pistons is shown at O. The low-pressure piston is the large piston at the top and the high-pressure piston is the annular area between the large and small pistons. Calculate the L. P. and H. P. piston areas and find the ratio of L. P. area to H. P. area.

10. In a hydro-electric plant the total weight of the turbine wheel, generator rotor, etc., is 550,000 lbs., and is to be supported by oil pressure acting on an annular ring 6′-0″ outside and 2′-1″ inside diameters. What pressure of oil (lbs. per sq. in.) is necessary?

Sector of Circle.—Suppose a sector of a circle contains an angle of 60°. Since a full circle contains 360°, then the sector is $\frac{60}{360}$, *i. e.*, $\frac{1}{6}$ of a full circle. Hence its area is $\frac{1}{6}$ that of the circle, *i. e.*, $\frac{1}{6}\pi r^2$.

Similarly for any other angle, say 27°. Then this sector is $\frac{27}{360}$ of a circle and its area is $\frac{27}{360} \times \pi r^2$.

Hence, if n be the angle of a sector in degrees, as in Fig. 75,

$$\textbf{Area of sector} = \frac{\pi n r^2}{360}$$

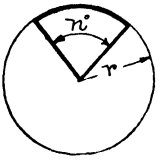


Fig. 75.

Thus, when $n = 90°$, *i. e.*, the angle is a right angle,

Area $= \frac{90}{360} \times \pi r^2 = \frac{1}{4}\pi r^2$, as is already known, since a sector of 90° angle, *i. e.*, a **quadrant,** is a quarter of a circle.

Splitting the formula into factors we get $\frac{nr}{57{\cdot}3} \times \frac{\pi r}{6{\cdot}28}$; the first factor = length of arc (l), as on p. 271, whence area $= \frac{lr}{2}$, since π cancels twice into 6·28.

Example 314.—A rotary valve of a petrol engine has a port shaped as in Fig. 76, which also gives dimensions. Determine the area through the port, in sq. ins.

Notice here that the port is a sector of a hollow circle. The same principle applies as laid out above.

$$R = 1\tfrac{5}{8}'' = 1{\cdot}625;\ r = \frac{15}{16} = {\cdot}9375$$

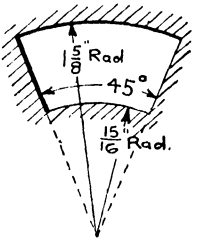


Fig. 76.

Then area of complete ring $= \pi(1{\cdot}625^2 - {\cdot}9375^2)$

$$\therefore \text{Area of Port} = \frac{45\pi}{360}(1{\cdot}625^2 - {\cdot}9375^2)$$

$$= \frac{45\pi}{360}(2{\cdot}64 - {\cdot}879)$$

$$= \frac{\overset{1}{\cancel{45}} \times \pi \times 1{\cdot}76}{\underset{8}{\cancel{360}}} = \underline{{\cdot}691 \text{ sq. in.}}$$

Area of Fillet.—The fillet or quarter-circle is very common in the corners of various shapes in engineering work, and it is occasionally necessary to find the area of the shaded portion shown in Fig. 77, when the radius is a big one. In most cases the rounding of corners, etc., may be neglected.

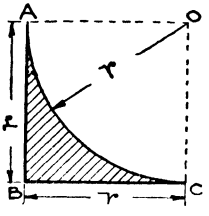


Fig. 77.

Let r = radius of the fillet

Then area of the square OABC $= r^2$

and area of quarter-circle $= \frac{1}{4} \times \pi r^2 = {\cdot}785r^2$

$\therefore$ **Area of fillet** = area of shaded portion

= the square − the quarter-circle

$= r^2 - {\cdot}785r^2$

$= r^2(1 - {\cdot}785)$ taking out the coefficient r^2

$= {\cdot}215r^2$ (or approximately $\frac{1}{5}r^2$)

Segment of a Circle.—Referring to Fig. 78, it can be seen that the area of the segment ACB can be obtained by taking the area of the triangle OAB from the area of the sector OACB

Then, if r = radius of arc; C = length of chord

h = height of segment; n = angle at centre in degrees

Then, area of sector $= \frac{n \times \pi r^2}{360}$. . . . . i.

Height of triangle OAB $= r - h$

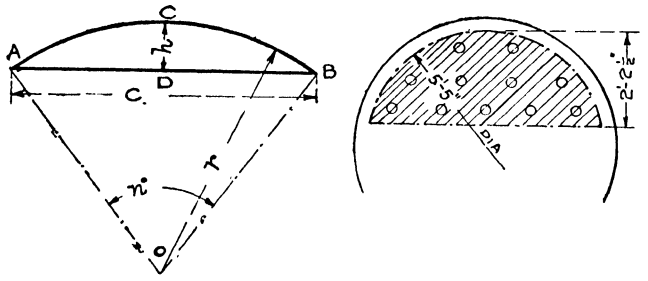


Fig. 78.—Area of Segment.

$\therefore$ Area of triangle OAB $= \frac{C}{2}(r - h)$ . . . . . ii.

Taking (ii) from (i) gives **area of segment.**

To use these formulæ the segment must be drawn out to scale and any necessary dimensions not given must be measured off. Thus, if two of the quantities C, h and r are known, the segment can be drawn, and the remaining dimension and n measured.

The area of a segment can be calculated directly by the use of higher work, but several approximate formulæ can be used. Thus, given diameter (D) of complete circle, and height (h) of segment, which are the two measurements usually known,

$$\textbf{Area of segment} = \frac{4}{3}h^2\sqrt{\frac{D}{h} - \cdot 608}$$

or given length of chord (C) and height of segment (h)

$$\textbf{Area of segment} = \frac{h^3}{2C} + \frac{2}{3}Ch.$$

For segments greater than a semicircle find the area of the remaining portion or segment of the circle, and subtract this from the area of the whole circle.

When the segment is a *flat* one, *i.e.*, h is less than $\frac{1}{9}$C, then the area is given approximately by the simple formula $\frac{2}{3}Ch$, the error being less than 1%. The error increases as the segment becomes less flat, and is 4·5% when $h = \frac{1}{4}$C.

Example 315.—In calculating the number of stays necessary for a boiler head, as in Fig. 78, which gives dimensions, it is required to find the area (shaded) which they will support. Find this area.

The area is a segment of a circle—

$$D = 5'\text{-}5'' = 5{\cdot}42'; \quad h = 2'\text{-}2\tfrac{1}{2}'' = 2{\cdot}21'; \quad \therefore h^2 = 4{\cdot}88$$

$$\text{Area of segment} = \frac{4}{3}h^2\sqrt{\frac{D}{h}} - {\cdot}608$$

$$= \frac{4}{3} \times 4{\cdot}88\sqrt{\frac{5{\cdot}42}{2{\cdot}21}} - {\cdot}608$$

$$= \frac{4}{3} \times 4{\cdot}88\sqrt{1{\cdot}842}$$

$$= \frac{4}{3} \times 4{\cdot}88 \times 1{\cdot}358$$

$$= \underline{8{\cdot}83 \text{ sq. ft.}}$$

Ellipse.—The area of an ellipse cannot be examined in as simple a manner as the area of a circle. The following formula is obtained by the use of higher mathematics :—

$$\textbf{Area of Ellipse} = \pi ab$$

where $a = \frac{1}{2}$ major axis and $b = \frac{1}{2}$ minor axis (see p. 249, Fig. 25).

This is very similar to the formula for the area of a circle. Thus, when an ellipse becomes a circle $a = b = r$.

Then area, $= \pi ab$

$= \pi rr = \pi r^2$, the well-known formula.

Example 316.—The mud-hole door of a boiler is elliptical, and is $15'' \times 11''$. What is the total force acting on it when the steam pressure is 60 lbs. per sq. in. ?

$$a = \tfrac{1}{2} \text{ major axis} = \frac{15}{2} = 7{\cdot}5''$$

$$b = \tfrac{1}{2} \text{ minor axis} = \frac{11}{2} = 5{\cdot}5''$$

Then, area $= \pi ab$

$= \pi \times 7{\cdot}5 \times 5{\cdot}5 = 129{\cdot}5$ sq. ins.

$\therefore$ Total force at 60 lbs. per sq. in. $= 60 \times 129{\cdot}5$

$= \underline{7770 \text{ lbs.}}$

Example 317.—Fig. 79 shows the standard proportions of egg-shaped sewers for drainage. The lower portion is practically half an ellipse. Find an expression for the total area if h = the depth.

Area of ellipse $= \pi ab$ $\therefore$ Area of half ellipse $= \frac{\pi}{2}ab$

$a = \frac{1}{2}$ major axis $= \frac{2}{3}k$ $\quad b = \frac{1}{2}$ minor axis $= \frac{1}{2} \times \frac{2k}{3} = \frac{k}{3}$

$\therefore$ Area of half ellipse $= \frac{\pi}{2} \times \frac{2}{3}k \times \frac{k}{3}$

$= \frac{\pi}{9}k^2 = \cdot 349k^2$

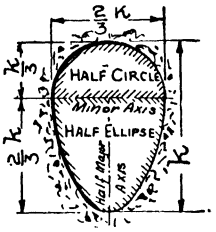


Fig. 79.

Area of half circle $= \frac{\cdot 785d^2}{2}$.

Here $d = \frac{2k}{3}$

$\therefore$ Area of half circle $= \frac{\cdot 785}{2} \times \left(\frac{2k}{3}\right)^2$

$= \frac{\cdot 785 \times 4k^2}{2 \times 9}$

$= \cdot 1745k^2$

$\therefore$ Total area = half circle + half ellipse

$= \cdot 1745k^2 + \cdot 349k^2$

$= \underline{\cdot 5235k^2}$

Exercises 75. On Areas of Sectors and Segments of Circles and Ellipses.

(*Letters refer to Fig.* 80.)

Sector of a Circle

1. Find the area in sq. ins. of a sector of a circle $6\frac{1}{2}''$ radius, if the angle is 55°.

2. The sector of a circle has a radius of $1'\text{-}1\frac{3}{8}''$ and the arc subtends an angle of 215°. Find the area of the sector in sq. ft.

3. Calculate the area of the port of a rotary valve shaped as in Fig. 76, if the large radius is $1\frac{5}{16}''$, the short radius $\frac{3}{16}''$, and the angle 45°. (*Hint.*—This is a sector of a hollow circle.)

4. The throttle regulator in a petrol engine is shown at A. Neglecting the rounded corners, find the total area through the ports, in sq. ins.

Segment of a Circle

5 to **7.** B shows the segment of a circle with necessary dimensions.

5. Calculate the area exactly by subtracting the triangle from the sector.

6. Calculate the area using the approximate formula $\frac{4}{3}h^2\sqrt{\frac{D}{h}} - \cdot 608$. Find the difference between this result and the true one (as found from No. 5), and express it as a percentage of the true area.

7. Calculate the area, using the approximate formula $\frac{h^3}{2C} + \frac{2}{3}Ch$, and find the percentage error, as in Ex. 6.

8. At C is given all necessary dimensions of a segment where h is less than $\frac{C}{4}$. Calculate the area exactly by subtracting the triangle from the sector.

9. Calculate the area using the approximate formula $\frac{2}{3}Ch$. Find how much this differs from the true result of Ex. 8, and express this difference as a percentage of the true area.

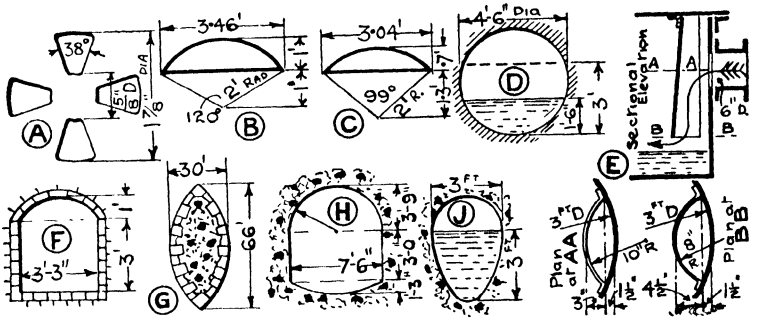


Fig. 80.—Exercises on Areas of Segments, etc.

10. At D is shown a circular culvert running one-third full. Calculate the area occupied by the water in sq. ft.

11. Calculate the wetted perimeter, and find the hydraulic mean depth, which is Area ÷ Wetted Perimeter, for the above case. (*Hint.*—Obtain chord by drawing; or by a right-angled triangle with a vertical to the centre.)

12. Calculate (*a*) the area of the cross-section; (*b*) the wetted perimeter; and (*c*) the hydraulic mean depth (area ÷ wetted perimeter) for the circular channel in Ex. 10, above, when running two-thirds full (*i. e.*, 3 ft. deep as at D).

13. A furnace flue is shown at F. Calculate the area of the cross-section.

14. The plan of a bridge pier is shown at G, and the pier consists of two segments. Find its cross-sectional area.

15. At H is given dimensions of the filling culvert to a dry dock. Calculate the area of the section.

16. Particulars of the steam inlet to an oil separator are given at E. Calculate the cross-sectional area enclosed between the two arcs at section BB. The inlet pipe being 6″ dia., find how many times the area at BB is greater than the pipe area.

17. Repeat the calculation in Ex. 16 above, for section AA.

ELLIPSE

18. Find the area of an ellipse whose major axis is $14\frac{3}{4}''$ and minor axis $10\frac{1}{2}''$.

19. In finding the capacity of an elliptic petrol tank for the back of a motor-car the area of the cross-section was required. Find this area if the two axes were $1'\text{-}1''$ and $8\frac{1}{2}''$.

20. Find the cross-sectional area of an elliptical magnet core $18'' \times 9''$.

21. Find the cross-sectional area of an elliptical funnel $13'' \times 10''$.

22. The egg-shaped culvert used for drainage is very nearly a half ellipse in the lower two-thirds usually occupied by the water. Calculate the area of the water section in the culvert shown at J.

23. A manhole door in the low-pressure end of a turbine casing is elliptical, $16'' \times 12''$. What is the total force on the door in lbs., if the net pressure of the atmosphere is 14·4 lbs. per sq. in.?

[Further examples involving areas of ellipses will be found in Exercises 80 on "The Volumes of Various Prisms."]

Area of Irregular Figures.—The area of a figure with an irregular outline cannot be obtained by the simple application of a formula; but can be found by drawing and measurement on the actual figure.

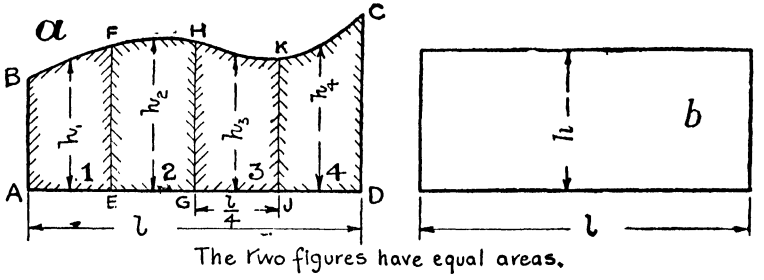


Fig. 81.—Area of Irregular Figure.

At *a*, Fig. 81, is shown a figure with one irregular side BC; the sides AB and CD are parallel and at right angles to the side AD. The figure has been divided into four strips of equal width by the parallel lines EF, GH, and JK. Taking any one of these strips, say No. 3, it can be seen that the side HK, although strictly curved, is very near indeed to a straight line. Suppose we assume for the present that it *is* a straight line: then the figure GJKH is a trapezoid

$$\begin{aligned} \text{and area of GJKH} &= \text{area of a trapezoid} \\ &= \text{average height} \times \text{width} \\ &= h_3 \times \frac{l}{4} \end{aligned}$$

where h_3 = the height at the centre of the width and l = length of figure. Assuming the side HK as straight, then the height at the centre is the average height. If h_1, h_2, h_3 and h_4 are the heights at the centres of the 4 strips, then the areas of the various strips are—

Strip No. 1 . . . . . $h_1 \times \frac{l}{4}$

2 . . . . . $h_2 \times \frac{l}{4}$

3 . . . . . $h_3 \times \frac{l}{4}$

4 . . . . . $h_4 \times \frac{l}{4}$, the width of all the strips being $\frac{l}{4}$.

Then the total area is very nearly

$$h_1\frac{l}{4} + h_2\frac{l}{4} + h_3\frac{l}{4} + h_4\frac{l}{4}$$

which $= l \times \left(\frac{h_1 + h_2 + h_3 + h_4}{4}\right)$ taking out the common factor l.

Note that the bracketed factor is the average of the four central heights. At *b*, Fig. 81, is shown a rectangle of the same length as the given figure, with height equal to this calculated average height. The area of the given irregular figure is, then, the area of this rectangle. Now if the length is divided into a larger number of strips, *i. e.*, the width of each strip is made less, then the curved side of each strip is practically a straight line, and its area for all practical purposes is given exactly by " width × central height " : therefore the area of the whole figure is given by *length* × *average of these central heights.* The heights of a curve are usually called " ordinates," and thus these heights at the centre of strips have been called " mid-ordinates " and the foregoing method of finding irregular areas is called the **Mid-Ordinate Method.** This method may be summed up in the following rule :—

Divide the figure up into a number of narrow strips and measure the height at the centre of each strip. Find the average of these heights, and multiply by the length of the figure, measured perpendicularly to the heights.

The number of strips into which the figure should be divided depends upon the state of the curve; when fairly straight a few would suffice and when very irregular many should be used. For

general purposes **10 strips** is a very convenient number to adopt, as the average is then very easily obtained from the total by merely shifting the decimal point. Using the mid-ordinate method, areas can be measured with an accuracy of within 1 %.

Method of Dividing-up.—Where the length is an even amount, as 6″ or 8″, and the number of strips is 10 or 20 or some round number, then the widths of the strips can be set off with an ordinary rule divided into tenths. But with uneven lengths a mechanical method of dividing-up is necessary, and that shown in Fig. 82 is both rapid and successful. The two end lines of the figure are produced for some distance. If 10 strips be required, a rule is laid diagonally across the figure and adjusted so that the two end lines

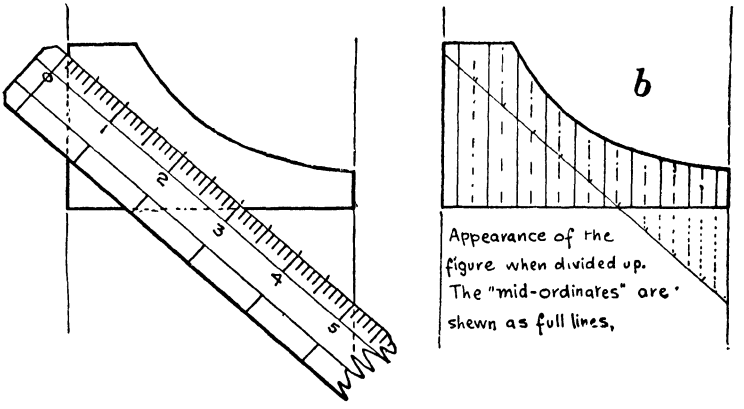


Fig. 82.—Method of Dividing for Mid-Ordinate Rule.

just enclose a distance easily divisible by 10 : in this case 5″, *i. e.*, 10 half inches. Then a line is drawn along the edges of the rule, and the 10 spaces marked off together with the centre of each space. Removing the rule, lines are drawn through these centre points parallel to the end lines as indicated at *b*, Fig. 82. The drawing and setting off should be done very carefully with a finely pointed pencil. It is not really necessary to show the actual strips, as only the centre or mid-ordinates are required to be measured.

In many cases, the figure has no parallel end lines, and perhaps no straight base line, being completely irregular. In such cases two parallel lines are drawn one at each extreme of the figure as shown in the following examples; the "length" is then the perpendicular distance between these lines, and the mid-ordinates are drawn parallel to these end lines.

Example 318.—In order to find the volume of material held in an elevator bucket it is necessary to find the area of its end (shown in Fig. 83). Find this area in sq. ins.

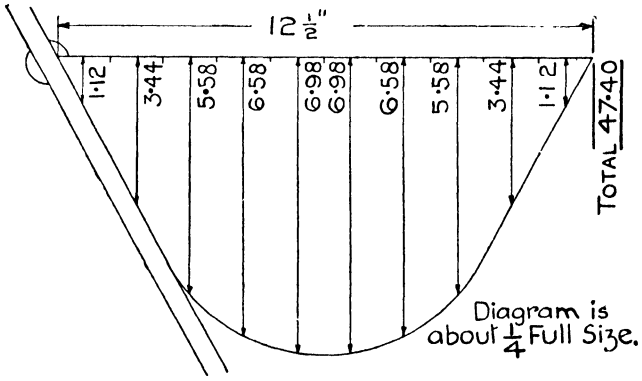


Fig. 83.—Use of Mid-Ordinate Rule.

The top has been taken as the base or length, and 10 mid-ordinates set up perpendicular to this by the method described. The measured lengths are shown.

$$\text{Sum} = 47{\cdot}4$$
$$\therefore \text{Mean height} = 4{\cdot}74$$
$$\text{Area} = \text{Mean height} \times \text{length}$$
$$= 4{\cdot}74 \times 12{\cdot}5 = \underline{59{\cdot}25 \text{ sq. ins.}}$$

Example 319.—In calculating the volume of water impounded in a reservoir it was necessary to find the area of a number of irregular

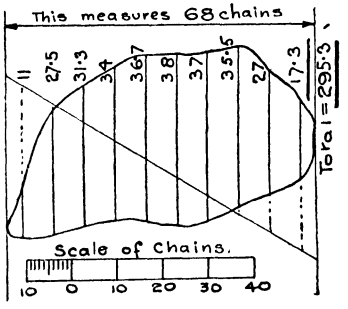


Fig. 84.

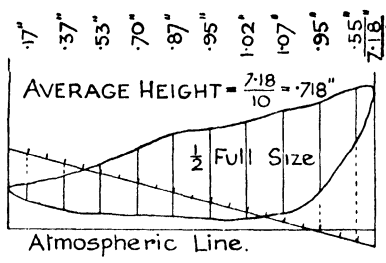


Fig. 85.

Use of Mid-Ordinate Rule.

figures. Find the area in square chains and acres of one of these shown in Fig. 84. (Measurements to be made on the scale attached.)

The figure has no straight side, so that any over-all dimension may

be taken as the length. Two parallel lines have been drawn touching the extreme ends, and on measurement they are found to be 68 chains apart. Ten mid-ordinates have been drawn, and their lengths are shown on the figure.

Then sum of 10 mid-ordinates = 295·3

$\therefore$ Mean height = 29·53 chains

Area = length × mean height

= 68 × 29·53 = 2008 sq. chains

or 200·8 acres.

Example 320.—Fig. 85 shows an indicator diagram from a non-condensing steam engine. Find the average height. If 1″ of height represents 24 lbs. per sq. in., find the average, or mean effective pressure. (*Note.*—Only the outline and the atmospheric line appear on the diagram as given.)

In this case there is no straight base, but the "atmospheric" line must be the length as pressures are measured vertically. Two lines are set up perpendicular to the atmospheric line and touching the extreme ends of the figure. Ten mid-ordinates are drawn and measured with the results shown, and the average obtained.

Average or mean ordinate = ·718″

1″ of ordinate represents 24 lbs. per sq. in.

$\therefore$ Mean pressure = 24 × ·718

= 17·2 lbs. per sq. in.

Note.—When dealing with the indicator diagram it is usually only the mean height that is required and not the area.

Exercises 76. On Area of Irregular Figures.

[*Note.*—In many cases the figures to the following exercises are shown much smaller than they would be drawn in practice; all drawing and measurement should therefore be done with the utmost care. For ease of working the student is advised to make a careful tracing with a fine pencil of the figures and to work on the tracing. Where drawings are not full size a scale of inches, etc., is given and all measurements should be made on this scale.]

1. Fig. 86 shows at A the cross-section of a six-pole dynamo magnet frame. Find the area of the section.

2. C, Fig. 86, shows the cross-section of an aqueduct for water supply. Find the area below the water level in sq. ft.

3. The water line section (half only) of a reinforced concrete barge is given at A, Fig. 87. Find the area of the whole section in sq. ft.

4. Find the cross-sectional area of the magnet frame shown at B, Fig. 86.

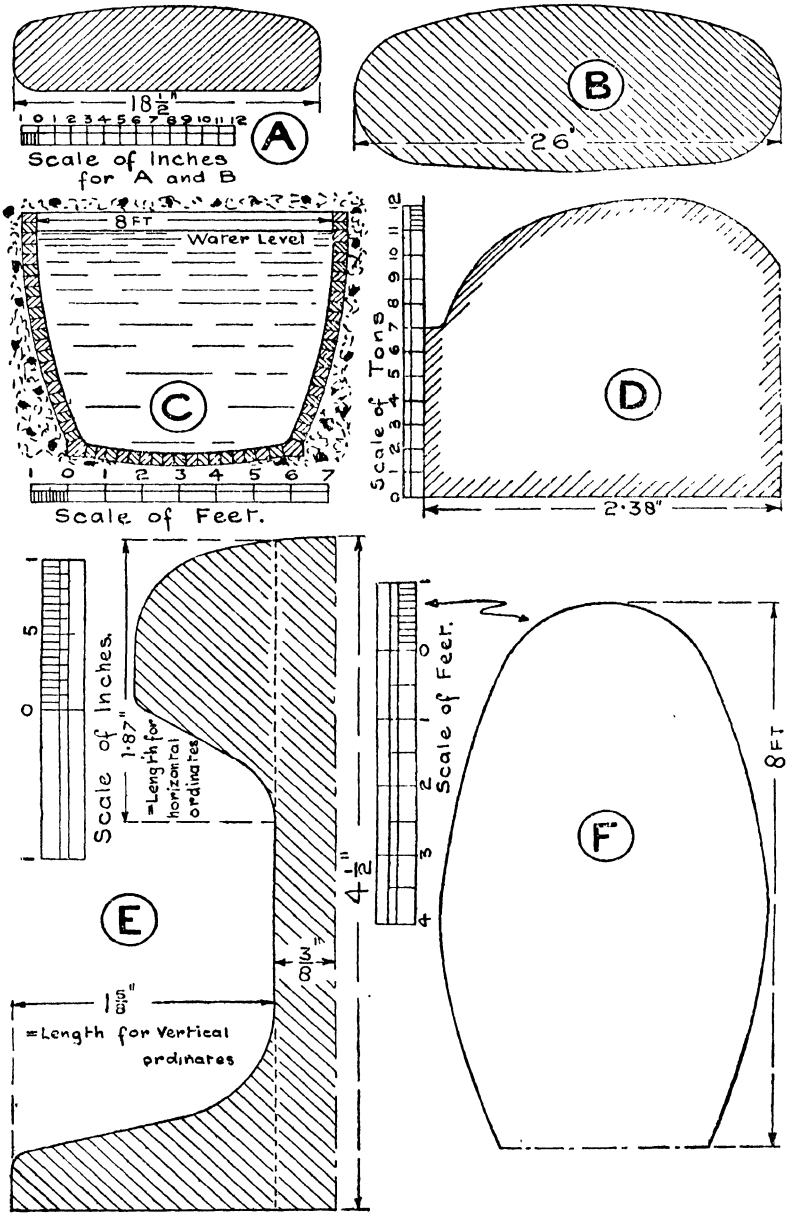


Fig. 86.—Exercises on Area of Irregular Figures.

5. Fig. 86 at F shows the development of one blade of a screw propeller. Find its area in sq. ft.

6. The curve at D, Fig. 86, was obtained from the tensile test of a steel bar. The area beneath the curve is the work done in breaking the bar, in inch tons. Find the work done. (*Hint.*—Measure vertical distances on scale of tons. Then "height in tons × length in ins." gives work in inch tons.)

7. B, Fig. 87, shows a "contour" line obtained when surveying in connection with a proposed reservoir. Find the enclosed area in square chains.

8. Find the cross-sectional area of the cast-iron beam section, half of which is shown at E, Fig. 86. (*Hint.*—The quickest method is to

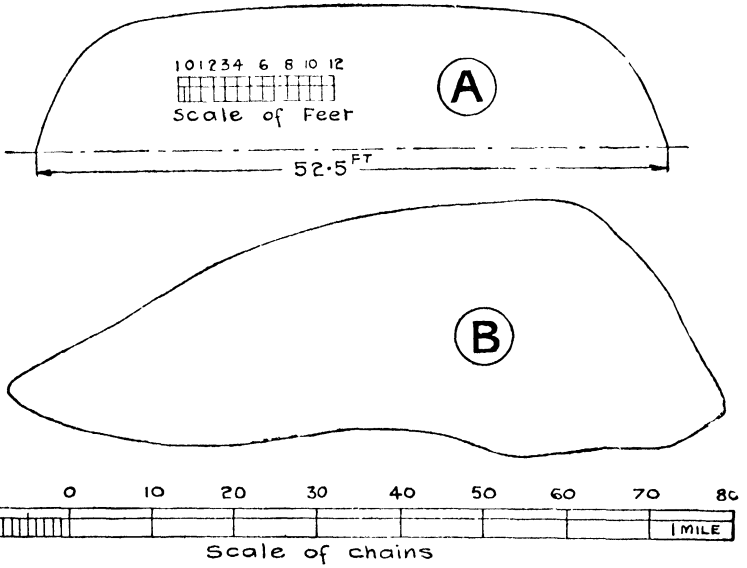


Fig. 87.—Area of Irregular Figures.

calculate as a rectangle the area of the half web from top to bottom, as shown by dotted lines. Then only the projecting portions of the flanges need be averaged. The bottom flange projections are best done by *vertical* mid-ordinates, and those of the top flange by *horizontal* ordinates. Multiplying by 2 will give the area of the whole beam.)

Questions **9** to **12** give examples of indicator diagrams. In each case the average height and the mean effective pressure are to be found.

9. A, Fig. 88. Diagram from the H.P. air cylinder of a two-stage air compressor. 1″ = 50 lbs. per sq. in.

10. B, Fig. 88. Full load diagram from scavenging pump of a two-cycle Marine Diesel Engine. 1″ = 10 lbs. per sq. in.

11. C, Fig. 88. Gas Engine Diagram. 1″ = 243 lbs. per sq. in.

12. D, Fig. 88. Diagram from four-cycle Diesel Engine. 1″ = 300 lbs. per sq. in.

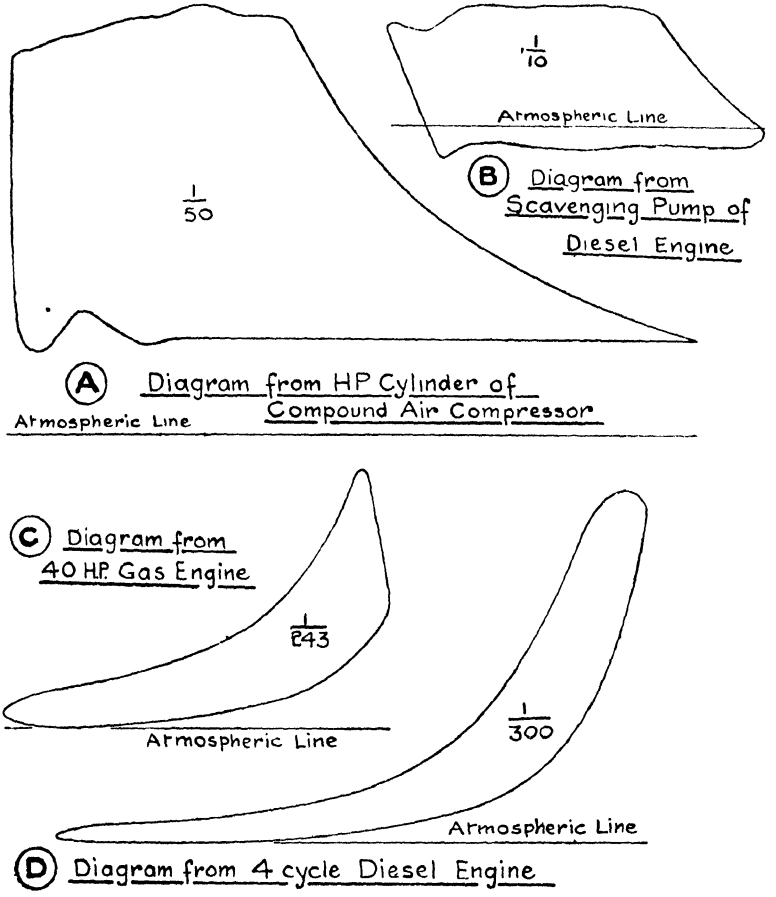


Fig. 88.—Indicator Diagrams.

TABLE.—For convenience of reference, all the formulæ, etc., relating to areas and circumferences, mentioned in the previous pages, have been collected into the following table :—

TABLE OF AREAS AND CIRCUMFERENCES OF PLANE FIGURES.

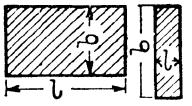
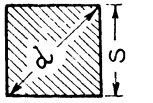
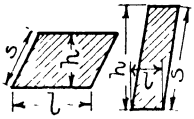
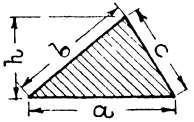
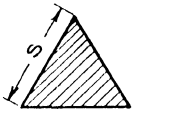
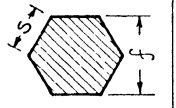
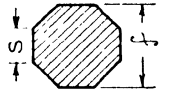
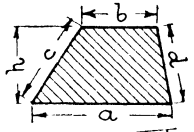
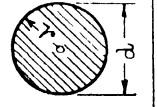
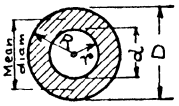
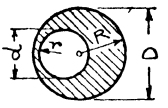
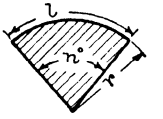
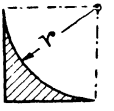
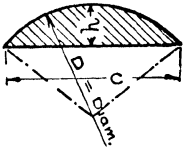
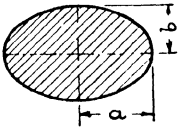
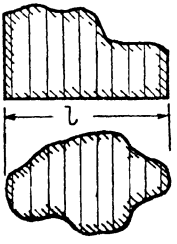
Title	Figure.	Circumference or Perimeter.	Area.
Rectangle .		$2(l+b)$	lb
Square . .		$4s$	s^2 or $\frac{d^2}{2}$
Rhomboid .		$2(l+s)$	lh
Triangle . .		$a+b+c$ $s=\frac{1}{2}$ perimeter	$\frac{ah}{2}$ or $\sqrt{s(s-a)(s-b)(s-c)}$
Equilateral triangle .		$3s$	$\cdot 433s^2$
Hexagon . .		$6s$ or $3\cdot 46f$	$2\cdot 6s^2$ or $\cdot 866f^2$
Octagon . .		$8s$ or $3\cdot 32f$	$4\cdot 83s^2$ or $829f^2$
Trapezoid .		$a+b+c+d$	$h\left(\frac{a+b}{2}\right)$
Irregular quadrilateral or trapezium. .		Sum of all four sides.	Divide into two triangles by either diagonal. Find area of each triangle and add. Or area $=\frac{lh}{2}$
Circle . . .		πd or $2\pi r$	$\frac{\pi}{4}d^2=\cdot 7854d^2$ or $\pi r^2=3\cdot 142r^2$

TABLE OF AREAS AND CIRCUMFERENCES OF PLANE FIGURES (*continued*).

Title.	Figure.	Circumference or Perimeter.	Area.
Hollow circle (annulus) .			$\frac{\pi}{4}(D^2-d^2)=\cdot7854(D^2-d^2)$ or $\pi(R^2-r^2)$ or $\pi \times$ mean dia. $\times$ thickness
Hollow circle (eccentric)			$\cdot7854(D^2-d^2)$ or $\pi(R^2-r^2)$
Sector of circle . .		$l=\frac{rn}{57\cdot3}$	$\frac{\pi nr^2}{360}$ or $\frac{lr}{2}$
Sector of hollow circle .			$\frac{\pi n(R^2-r^2)}{360}$
Fillet . . .			$\cdot215r^2$ or approx. $\frac{1}{5}r^2$
Segment of circle . .			Area = sector − triangle Various approx. formulæ on p. 300.
Ellipse . .		$\pi(a+b)$ approx. or $\pi\{1\cdot5(a+b)-\sqrt{ab}\}$ more nearly	πab
Irregular figures . .		Step round curved portions in small steps, with dividers; add in any straight pieces.	Divide into narrow strips; measure their mid-ordinates. Then— Area = aver. mid-ordinate $\times$ length l

CHAPTER VIII

MENSURATION—continued

VOLUMES AND SURFACE AREAS

Volume is the amount of space occupied by a body, or is its bulkiness, and includes measurement in three main directions, usually known as length, breadth, and depth or thickness. The unit of volume is the **cubic foot** (cu. ft.), which is the volume of a solid body, with six faces, each face being a square of 1 ft. side, as shown in Fig. 89. This solid is called a **cube.**

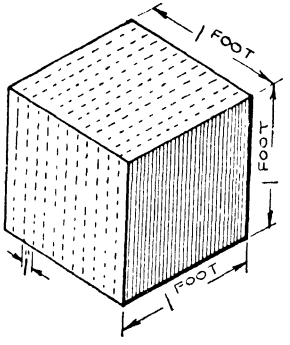


Fig. 89.—The Cube.

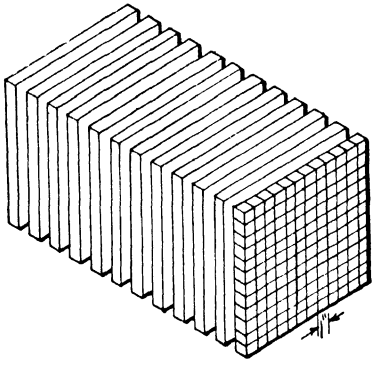


Fig. 90.—Relation between the Cubic Inch and Cubic Foot.

For smaller measurements the **cubic inch** (cu. in.) is used, which is the volume of a cube of 1″ side. The relation between the cubic inch and the cubic foot may be obtained by reference to Fig. 90. Here the big cube represents a cube of 1 ft. side, and is therefore a cubic foot. Since 1 ft. = 12 ins. then the cube may be sawn into 12 slabs, each 1″ thick, by cutting along the dotted lines. The cube, when sliced, is shown in Fig. 90, the slabs being slightly separated from one another. Now each slab is 1 in. thick and 1 ft. side, *i. e.*, 12 ins. each side, and may therefore be divided up

into a number of little cubes each 1 in. side as indicated on the foremost slab. Evidently the first slab contains $12 \times 12 = 144$ cubes of 1 in. side, *i. e.*, 144 cu. ins. There being 12 slabs in all each of which may be divided like the first one, the whole cubic foot is obviously 144×12 cu. ins., *i. e.*, 1728 cu. ins.

Then 1 cu. ft. $= 12 \times 12 \times 12$ cu. ins. $= 1728$ cu. ins.
Similarly for the cubic yard—

$$1 \text{ yard} = 3 \text{ feet}$$
$$\therefore 1 \text{ cubic yard} = 3 \times 3 \times 3 = 27 \text{ cubic feet}$$

These three units are the only ones of their type employed, and our table of volume measure is then—

Smallest unit	= 1 cubic inch (cu. in.)
1728 cubic inches	= 1 cubic foot (cu. ft.)
27 cubic feet	= 1 cubic yard (cu. yd.)

The only conversions which are required may be summed up in the following :—

Cubic inches	= cubic feet $\times$ 1728
Cubic feet	= cubic inches $\div$ 1728
Cubic feet	= cubic yards $\times$ 27
Cubic yards	= cubic feet $\div$ 27

There is another unit often employed when dealing with the capacities of various vessels, etc., and which relates to the weight of water. This is the **gallon** (volume of 10 lbs. of water).*

* 1 gallon (gall.)	= 277·4 cubic inches
	= ·1605 cubic feet
or 6·24 gallons	= 1 cubic foot *

Occasionally when dealing with the storage of grain, etc., the following are required :—

8 gallons = 1 bushel
8 bushels = 1 quarter.

The weight of an article depends on its volume, and it is very often necessary to find the weight, knowing the volume and the weight per cubic foot or per cubic inch of the material of which it is made. This merely requires a simple multiplication.

---

* This is when measured at 62° F; the weight of a cubic foot of water depends upon the temperature, and becomes less as the temperature rises. The greatest weight is 62·425 lbs., at 39° F.

Conversion.—The amount of conversion and reduction wanted is very small. The following examples should suffice :—

Example 321.—The volume of an engine cylinder is 1830 cu. ins. Convert this into cubic feet.

$$\text{Volume of cylinder} = 1830 \text{ cu. ins.}$$
$$= \frac{1830}{1728} \text{ cu. ft.} = \underline{1{\cdot}06 \text{ cu. ft.}}$$

Example 322.—A pipe is discharging ·28 cu. ft. of water per second. Find the discharge in gallons per minute.

$$\cdot 28 \text{ cu. ft. per sec.} = \cdot 28 \times 6{\cdot}24 \text{ galls. per sec.}$$
$$= \cdot 28 \times 6{\cdot}24 \times 60 \text{ galls. per min.}$$
$$= \underline{105 \text{ galls. per min.}}$$

Example 323.—A pump is rated at 4800 gallons per hour. How many cubic feet of water can it discharge per second ?

$$6{\cdot}24 \text{ galls.} = 1 \text{ cu. ft.}$$
$$\therefore \ 4800 \text{ galls. per hr.} = \frac{4800}{6{\cdot}24} \text{ cu. ft. per hr.}$$
$$= \frac{4800}{6{\cdot}24 \times 3600} \text{ cu. ft. per sec.}$$
$$= \underline{\cdot 214 \text{ cu. ft. per sec.}}$$

Exercises 77. On Volume Conversion.

1. The volume of a steam engine cylinder is 12·56 cu. ft. The clearance volume is 8% of this. Find the clearance volume in cubic inches.

2. A storage tank is to hold 5000 gallons of water. What must be its volume in cubic feet ?

3. A pump for an experimental hydraulic plant is required to deliver 1·5 cu. ft. of water per second. What must it deliver in gallons per hour ?

4. A feed pump delivers 160 cu. ins. of water per stroke, and makes 25 strokes per minute. How many gallons of water does it deliver per minute ?

5. A " Navvy " or steam shovel had an average output of 63 cu. yds. per hour. What is this in cubic feet per minute ?

6. If a casting has a volume of 3250 cu. ins. and 1 cu. ft. of the metal of which it is cast weighs 390 lbs., find the weight of the casting.

Volumes of Regular Solids.

Prisms.—The great majority of solid bodies met with in engineering work are either regular solids, or are made up of a number of regular solids. A " regular solid " is one which has some definite

geometrical shape. There are two main classes: "Prisms" and "Pyramids." A **prism** is a solid with two parallel ends, both of the same shape and size, joined by a number of sides which are plane rectangles. The ends are generally perpendicular to the sides. The shape of the end may be that of any of the regular geometrical figures, *e. g.*, square, hexagonal, circular, etc., and the prism is named according to the shape of the end. Thus Fig. 92 shows a rectangular prism. The "circular prism" is given the special name of **cylinder.** A square prism in which the length of the side is the same as the side of the square end is called a **cube**; every face is then a square, so that any pair of opposite faces may be called the ends. A rectangular prism, *i. e.*, one whose ends are rectangles, is sometimes known as a **cuboid.**

Any prism may be regarded as the volume swept out by a plane figure when moved perpendicularly to its plane. Thus in

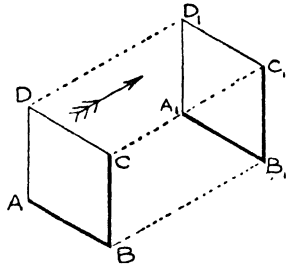


Fig. 91.

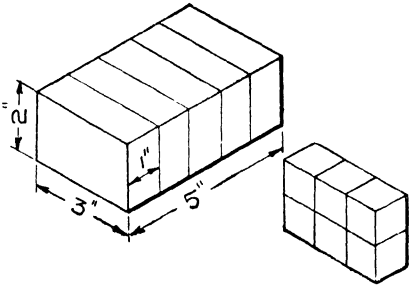


Fig. 92.—Volume of Rectangular Prism or Cuboid.

Fig. 91, the square ABCD is moved in the direction of the arrow to the position marked $A_1B_1C_1D_1$ and in doing so sweeps out a square prism.

Volume of a Rectangular Prism.—Fig. 92 shows a cuboid whose end is $2'' \times 3''$, and whose length is $5''$. Now let the solid be cut into five pieces each $1''$ long as indicated by the faint lines. Taking the end section, this may be cut up as indicated (Fig. 92) into a number of small cubes each of $1''$ side; these small cubes are then cubic inches. Evidently in the end section there are $2 \times 3 = 6$ cu. ins.

Now every other section is like this one and therefore contains 6 cu. ins. There being five sections in all, the total number of cu. ins. in the solid is $6 \times 5 = 30$. Thus the volume of the prism

$= 2 \times 3 \times 5 = 30$ cu. ins. Supposing we call the 5″ measurement the "length," the 3″ measurement the "breadth," and the 2″ measurement the "thickness": then evidently—

Volume of rectangular prism = length × breadth × depth

All three dimensions must be in the same one unit, *i. e.*, they must all be in inches, or all in feet, etc. If the measurements are given in various units, say one in feet and the others in inches, then all must be converted to some one unit, say feet or inches. Again, if some of the dimensions are in two or more units, *e. g.*, length = 3′-2″, then before any multiplying is carried out this dimension must be converted into inches (38″) or feet (3·167′). See Ex. 326.

Example 324.—A rectangular tank for measuring purposes is 5 ft. long and 2 ft. broad. Find how many gallons of water it holds when the water is 4 ft. deep. See Fig. 93.

(All measurements are in feet, and may therefore be multiplied up directly, giving the result in cubic feet.)

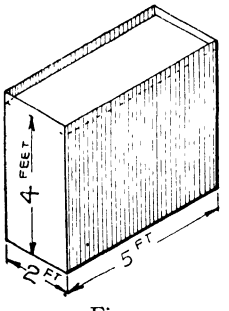


Fig. 93.

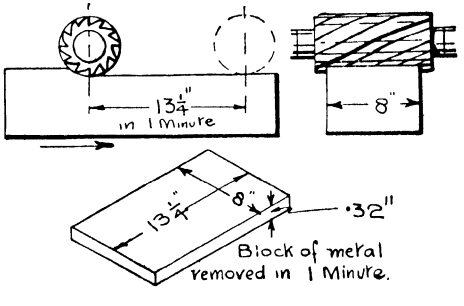


Fig. 94.

Volume of water in tank $= 5 \times 2 \times 4$
$= 40$ cu. ft.

1 cu. ft. $= 6{\cdot}24$ gallons
$\therefore$ Gallons in tank $= 40 \times 6{\cdot}24$
$= 249{\cdot}6$, say, 250 gallons.

Example 325.—In some milling tests on cast iron, a block 8″ wide was milled with a cut ·32″ deep, the feed being 13¼″ per min. (See Fig. 94.) Determine the volume of metal removed per minute.

In 1 min. the cutter will pass over a length of 13¼″. Then the metal removed is a rectangular prism 13¼″ × 8″ × ·32″.

$\therefore$ Volume removed $= 13{\cdot}25 \times 8 \times {\cdot}32$
$= 33{\cdot}92$ cu. ins.

Example 326.—A concrete foundation for an engine is to be 9′-6″ long, 3′-9″ wide, and 1′-6″ deep. Find what volume of concrete will be required.

(All measurements must be converted to some one unit: in this case feet will be the most suitable.)

Length 9′-6″ = 9·5 ft. Breadth 3′-9″ = 3·75 ft. Depth 1′-6″ = 1·5 ft.

$$\therefore \text{ Volume required} = 9{\cdot}5 \times 3{\cdot}75 \times 1{\cdot}5$$
$$= \underline{53{\cdot}4 \text{ cu. ft.}}$$

Exercises 78. On Volume of Rectangular Prisms.

1. What volume of concrete will be required for an engine foundation 15′ long, 5′-6″ wide, and 4′-6″ deep?

2. A railway wagon is 7′-0″ wide, 4′-6″ deep, and 13′-0″ long. What weight of coal will it carry when just full, if 1 cu. ft. of coal weighs ½ cwt.?

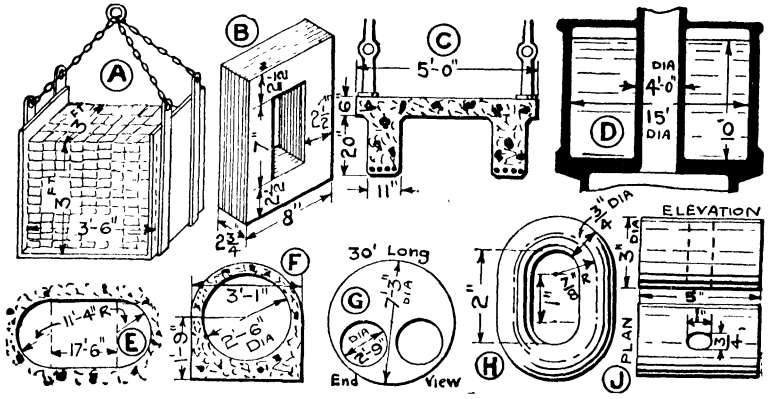


Fig. 95.—Exercises on Volumes of Prisms.

3. A storage tank is 40 ft. long and 12 ft. wide. If the water be 8′-6″ deep, how many gallons of water does the tank hold?

4. A storage battery contains 60 cells each 12″ × 15″. How many gallons of acid will be required to fill the cells to a depth of 10″?

5. A hod used for lifting bricks by crane is shown at A, Fig. 95. Taking 1 cu. ft. of brick as weighing 112 lbs., calculate the weight to be lifted. (Neglect weight of hod.)

6. Calculate the weight of a railway sleeper, 9 ft. long, 10″ wide and 5″ deep, if 1 cu. ft. of the wood weighs 52 lbs.

7. A rectangular acid tank is 2′-0″ long, 1′-3″ wide, and 1′-10″ deep. Calculate the number of gallons of sulphuric acid it will hold when filled to within 2″ of the top.

8. A steel ingot containing 11 cu. ft. of metal is rolled into a plate 1″ thick and 9 ft. wide. Find the approximate length of the plate. (Give the answer to the nearest foot.)

9. The cross-sectional area of the Standard Bull-Head Rail used on British railways is 8·81 sq. ins. If 1 cu. ft. of the steel weighs 490 lbs., calculate the "weight per yard" of the rail (*i. e.*, the weight of a piece 1 yd. long).

10. A rectangular bin is required to have a volume of 150 cu. ft. The length is to be 9 ft., and the height is limited to 4′-3″. What width must it be made?

11. An overhead storage tank, 13 ft. square, is required to hold 5000 gallons of water. How deep must the water be?

12. A small transformer core is shown at B, Fig. 95. In a test it was necessary to know the volume of metal in the core. Find this volume.

13. The cross-section of a reinforced concrete foot-bridge is shown at C, Fig. 95. The total length is 42′-7″. If 1 cu. ft. of concrete be taken as 120 lbs., calculate the weight of the concrete.

14. A rectangular channel is required to discharge 10 cu. ft. of water per second. If the water travels 4 ft. in 1 sec., what must be the area of the cross-section of the channel? If the channel be made 2′-3″ wide, how deep will the water run?

Volume of Other Prisms.—When the shape of the end is other than rectangular we cannot take length × breadth × depth to give volume. Notice that in the rectangle in Fig. 92, 3″ × 2″ = 6″ = area of the end, *i. e.*, breadth × thickness.

So that we can also say volume = area of end × length.

Now whatever the shape of the end, provided that it be one of the regular shapes, we can calculate its area. Hence we have for *any* prism

Volume of prism = area of end × length

The same units must be used throughout, as emphasised in the previous section.

Taking some particular cases—

Hexagonal prism: Let s = side of hexagon, l = length

Area of end = $2{\cdot}6s^2$

$\therefore$ Volume = $\mathbf{2{\cdot}6s^2l}$

Cylinder: d = diameter, h = length, r = radius

Area of end = $\frac{\pi}{4}d^2$ or πr^2

$\therefore$ Volume = $\mathbf{\frac{\pi}{4}d^2h}$ or $\mathbf{\pi r^2h}$

The remaining cases are given in the table at the end of the chapter, as they are all of the same nature.

Example 327.—A steam engine cylinder is 13″ dia. and the stroke is 16″. Determine the "working volume" in cubic feet (*i. e.*, the volume swept out by the piston).

This is a case of a cylinder 13″ dia. and 16″ long.

$$\text{Area of end} = \pi r^2 \text{ and } r = 6\tfrac{1}{2}''$$
$$= \pi \times 6{\cdot}5^2 = 133 \text{ sq. ins.}$$
$$= \frac{133}{144} \text{ sq. ft.}$$
$$\text{Length} = 16'' = 1'\text{-}4'' = 1{\cdot}33 \text{ ft.}$$
$$\therefore \text{Volume} = \text{area of end} \times \text{length} = \frac{133 \times 1{\cdot}33}{144} \text{ cu. ft.}$$
$$= \underline{1{\cdot}23 \text{ cu. ft.}}$$

Example 328.—A boiler feed pump is 5″ dia. and has an 8″ stroke. Find the discharge in gallons per minute when making 30 strokes per minute.

$$\text{Area of cylinder} = \frac{\pi}{4}d^2 = {\cdot}785 \times 5^2 = 19{\cdot}6 \text{ sq. ins.}$$
$$= \frac{19{\cdot}6}{144} \text{ sq. ft.}$$
$$\text{Stroke } 8'' = \frac{2}{3} \text{ ft.} = {\cdot}667 \text{ ft.}$$
$$\therefore \text{Volume per stroke} = \frac{19{\cdot}6}{144} \times {\cdot}667 = {\cdot}0908 \text{ cu. ft.}$$

At 30 strokes per min., volume per min. $= {\cdot}0908 \times 30$ cu. ft. per min.

$$1 \text{ cu. ft.} = 6{\cdot}24 \text{ galls.}$$
$$\therefore \text{Galls. per min.} = {\cdot}0908 \times 30 \times 6{\cdot}24$$
$$= \underline{17 \text{ galls. per min.}}$$

In many cases it is necessary to invert the operations when dealing with volume, *i. e.*, knowing the volume and some other dimensions to find, say, the length of the prism.

$$\text{Since volume} = \text{area of end} \times \text{length}$$
$$\text{then } \textbf{length} = \frac{\textbf{volume}}{\textbf{area of end}}$$
$$\text{and also } \textbf{area of end} = \frac{\textbf{volume}}{\textbf{length}}$$

With the particular case of the cuboid

$$\text{volume} = \text{length} \times \text{breadth} \times \text{depth}$$
$$\therefore \text{Breadth} = \frac{\text{volume}}{\text{length} \times \text{thickness}}. \quad \text{Etc.}$$

Example 329.—A tank is 5′-0″ long, and 3′-6″ wide. How deep will the water be when the tank contains 120 galls.?

$$\text{Volume of water} = 120 \text{ galls.}$$
$$= \frac{120}{6{\cdot}24} \text{ cu. ft.} = 19{\cdot}23 \text{ cu. ft.}$$
$$\text{Then } 19{\cdot}23 = 5 \times 3{\cdot}5 \times \text{depth}$$
$$\therefore \text{Depth} = \frac{19{\cdot}23}{5 \times 3{\cdot}5} = \underline{1{\cdot}1 \text{ ft.}}$$

Example 330.—A cylindrical receiver for an air compressor is to have a capacity of 25 cu. ft. What diameter must it have if its length is 5′-0″?

$$\text{Volume} = \cdot 785 d^2 l$$
$$\therefore \cdot 785 d^2 \times 5 = 25$$
$$\therefore \quad d^2 = \frac{25}{\cdot 785 \times 5} = 6 \cdot 37$$
$$\therefore \quad d = \sqrt{6 \cdot 37} = 2 \cdot 52 \text{ ft.}$$
$$= 2 \text{ ft. } 6\tfrac{1}{4} \text{ ins.}$$

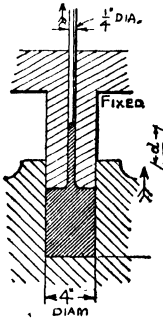


Fig. 96.

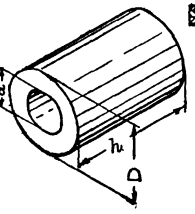


Fig. 97.

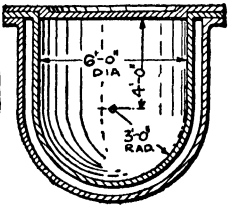


Fig. 98.

Example 331.—In making bullets, lead wire is produced by squeezing molten lead from a cylindrical chamber through a hole in the top, as shown in Fig. 96. If the chamber is 4″ dia. and 8″ long, what length of $\frac{1}{4}$″ dia. wire can be produced?

The 4″ dia. cylinder, 8″ long, is made into a long $\frac{1}{4}$″ dia. cylinder, whose length is to be found.

$$\text{Volume of lead in chamber} = \frac{\pi}{4} d^2 h$$
$$= \cdot 785 \times 4^2 \times 8 = 100 \cdot 5 \text{ cu. ins.}$$
$$\text{Cross-sectional area of wire} = \cdot 785 \times \cdot 25^2 = \cdot 0491 \text{ sq. in.}$$
$$\therefore 100 \cdot 5 = \cdot 0491 \times \text{length in inches}$$
$$\therefore \text{Length} = \frac{100 \cdot 5}{\cdot 0491} = 2047'' \text{ or } 56 \cdot 9 \text{ yds.}$$

Hollow Cylinders and Tubes.—Take the hollow cylinder in Fig. 97, and let it be required to find the volume of the solid portion.

$$\text{Area of the end} = \cdot 785(D^2 - d^2)$$
$$\text{Then volume} = \text{area of end} \times \text{height}$$
$$= \cdot 785(D^2 - d^2)h$$

Alternatively, the volume of the solid "walls" of the tube is evidently the volume of the big cylinder of diameter D, less the volume of the small cylinder of diameter *d*.

Then volume of large cylinder $= \cdot 785D^2h$
and volume of small cylinder $= \cdot 785d^2h$

$\therefore$ **Volume of tube** $= \cdot 785D^2h - \cdot 785d^2h$
$= \cdot 785h(D^2 - d^2)$
taking out the common factor $\cdot 785h$

Example 332.—Fig. 99 shows the sump or reservoir of a circular cooling tower. How many gallons of water does it hold when the water is 3′-6″ deep?

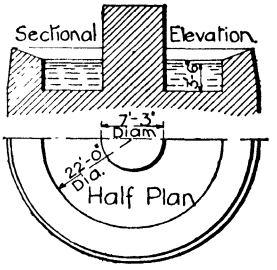


Fig. 99.

Volume of water $= \cdot 785(D^2 - d^2)h$
$= \cdot 785(22^2 - 7{\cdot}25^2) \times 3{\cdot}5$ cu. ft.
(reducing all measurements to feet)
$= \cdot 785 \times 431{\cdot}4 \times 3{\cdot}5$
$= 1185$ cu. ft.

$\therefore$ Gallons of water $= 1185 \times 6{\cdot}24 = \underline{7396}$

Example 333.—In making lead pipes by extrusion, what would be the least volume of metal required to produce 60 ft. of pipe $\frac{1}{2}$″ inside diameter and $\frac{11}{16}$″ outside diameter? What weight of metal would be required if the lead weighed ·41 lb. per cu. in.?

This is the case of a very long hollow cylinder.

Area of end $= \cdot 785(D^2 - d^2)$
$= \cdot 785(\cdot 6875^2 - \cdot 5^2)$
$= \cdot 785 \times \cdot 2228$
$= \cdot 1749$ sq. in.
Length $= 60$ ft. $= 720$ ins.

$\therefore$ Volume of metal = area of end × height
$= \cdot 1749 \times 720 = 125{\cdot}9$ cu. ins.
say, 126 cu. ins.

$\therefore$ Weight of metal at ·41 lb. per cu. in. $= 126 \times \cdot 41 = 51{\cdot}66$ lbs.
say, 52 lbs.

Exercises 79. On Volumes of Cylinders.

1. A cylindrical gas-holder is 150 ft. dia. and 115 ft. high. How many cubic feet of gas does it contain?

2. An oxygen cylinder is 5″ dia. and 3′-0″ long. It is filled with gas under pressure so that every cubic foot of the cylinder contains 120 cu. ft. of gas. How much oxygen does the cylinder hold?

3. The H.P. cylinder of a compound Corliss engine is 15″ dia. and has a 36″ stroke. Calculate the "working volume" in cubic feet (*i. e.*, the volume swept out by the piston).

4. A separator drain tank is 1′-6″ dia. and 2′-0″ long. Calculate how many gallons of water it will hold. (1 cu. ft. = 6·24 galls.)

5. The governor ball on a high speed engine is a cast-iron cylinder 3″ dia. and $2\frac{1}{2}$″ thick. Calculate its weight if 1 cu. in. of cast iron weighs ·26 lb.

6. A three-throw force pump (*i. e.*, one with three cylinders) has cylinders $14\frac{3}{4}$″ dia. and 24″ stroke, and makes 100 r.p.m. Calculate the gallons of water it delivers in 1 min. (*Hint.*—In 1 rev., pump delivers the volume of the 3 cylinders.)

7. The clearance volume in a gas-engine cylinder is 209 cu. ins. It is desired to reduce this volume to 165 cu. ins. by attaching a plate $9\frac{1}{2}$″ dia. to the back of the piston. What thickness of plate is required?

8. In a cement testing machine the load is applied by running water into a tall cylindrical tank. In a certain machine the greatest load required ·962 cu. ft. of water to be in the tank. If the length of the tank is limited to 3′-9″, what must be its diameter?

9. The cylinder of an air compressor is required to have a working volume of 5 cu. ft. If the diameter is to be 20″, what must be the stroke?

10. The cross-section of the discharge flume from a hydro-electric installation is given at E, Fig. 95. Calculate the cubic feet of water discharged per second if the water leaves at 4 ft. per sec.

11. Determine the volume of concrete (cubic feet) required to build the culvert whose section is given at F, Fig. 95, if it is 650 yds. long.

12. A condenser tube plate of rolled Muntz metal is 3′-6″ dia. and $1\frac{1}{8}$″ thick. Calculate its weight if the metal weighs ·31 lbs. per cu. in.

13. Dimensions of a Lancashire boiler are given at G, Fig. 95. How many gallons of water will be required to fill the boiler for its hydraulic test? (*Hint.*—The water volume is the large cylinder less the two flue cylinders.)

14. A Diesel engine cylinder is $19\frac{1}{2}$″ dia. and has a 37″ stroke. When at the top of its stroke the piston is $2\frac{1}{2}$″ away from the cylinder cover. The resulting space enclosed is the "clearance volume." Express this volume as a percentage of the "working volume" (*i. e.*, the volume swept out by piston) and also as a percentage of the total volume.

15. In a test on a pump driven by producer gas the quantity of gas used was measured by noting the height fallen by the gas-holder top during the run. The holder was 20 ft. dia. and dropped 2′-$7\frac{1}{2}$″ in the test. How many cubic feet of gas were used?

16. H, Fig. 95, gives dimensions of a link for a crane chain. Calculate the weight in pounds of one link, if wrought iron weighs ·28 lb. per cu. in., and also the weight per fathom (*i. e.*, number of lbs. for 6 ft.). (*Hint.*—When finding the number of links in the 6 ft., the length of

one link is the inside dimension 2″. Work to the nearest complete link.)

17. A lead hammer head is shown at J, Fig. 95. If the metal weighs ·41 lb. per cu in., calculate the weight of the head.

HOLLOW CYLINDERS

18. A hollow steel shaft has an outside diameter of 7″ and an inside diameter of 3½″. Calculate the weight of a 12 ft. length if the steel weighs 491 lbs. per cu. ft.

19. Calculate the weight per foot of a cast-iron pipe 9¼″ inside dia., and ⅜″ thick, if cast iron weighs 460 lbs. per cu. ft. (*Hint.*—Use " average diameter " method.)

20. A circular brick chimney is 100 ft. high, 8 ft. average dia., and 13½″ average thickness. If 1 cu. ft. of brickwork weighs 120 lbs., estimate the weight of the chimney.

21. An elevated storage tank is shown at D, Fig. 95. Calculate the gallons of water it will hold.

22. The body of a gas-storage cylinder is made by " cold drawing " from a flat plate. If the body is produced from an annular plate as indicated at F, Fig. 102, and the volume of the annular plate is equal to the volume of the cylinder body, calculate the thickness of plate required for dimensions shown.

We may also have prisms whose ends are segments of circles or combinations of any of the regular figures, or even irregular shapes. In all these cases find the area of the end and multiply by the length. See Exercises 80.

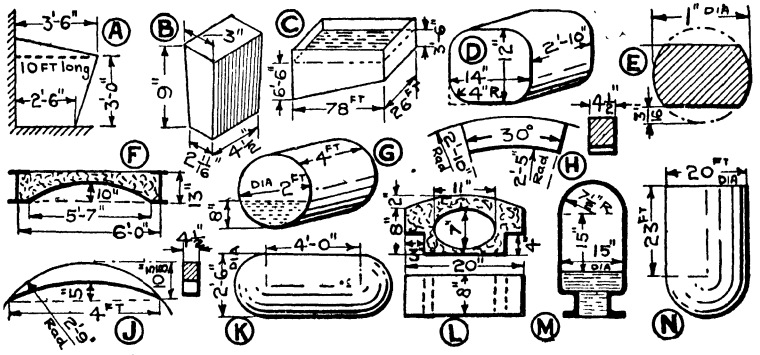


Fig. 100.—Exercises on Volume of Prisms and Spheres.

Exercises 80. On Volumes of Various other Prisms.

(*Letters refer to Fig.* 100.)

1. A sandbin built against a wall is shown in end view at A. Calculate the cubic yards of sand it contains when filled to the dotted line. Also find the total weight of the sand if 1 cu. ft. weighs 105 lbs.

2. Taper fire-bricks for use in the furnace arches of mechanical stokers are shown at B. Calculate the weight per 1000 bricks if 1 cu. ft. of fire-brick weighs 137 lbs.

3. A swimming bath is shown at C. Find how many gallons of water are required to fill it.

4. Find (*a*) the weight of a foot of hexagonal bar $\frac{3}{4}''$ across the flats and (*b*) the weight of a foot of hexagonal bar of $\frac{1}{2}''$ side, both to be used for making bolts and both of steel weighing ·28 lb. per cu. in.

5. An octagonal bar $\frac{3}{4}''$ across flats is to be used for chisels. Find its weight per foot if the steel weighs 489·3 lbs. per cu. ft.

6. At D, Fig. 102, is shown a V-block for shop use. Find its weight if of metal weighing 489 lbs. per cu. ft.

7. Calculate the capacity in gallons of an elliptic milk-can $10'' \times 6''$ and $15''$ deep, if filled to within $1\frac{1}{2}''$ of the top.

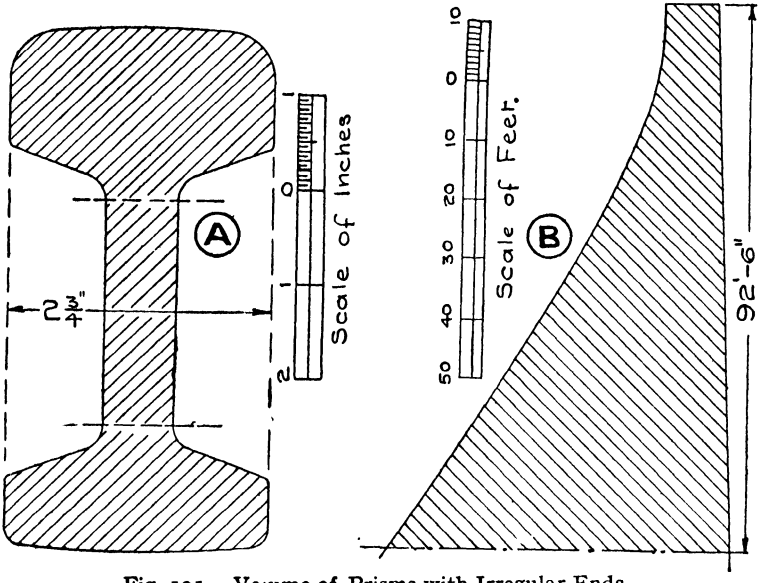


Fig. 101.—Volume of Prisms with Irregular Ends.

8. Calculate the capacity in gallons of the elliptical petrol tank for the back of a motor-car, shown at B, Fig. 102.

9. The dimensions of the cross-section of a $\frac{3}{4}''$ "Simplex" oval conduit similar to that at X, Fig. 31, are as follows :—Mean axes ·95" and ·42", and thickness ·042". Find the weight of a 100 ft. length if steel weighs ·28 lb. per cu. in. (*Hint.*—This is a hollow elliptical cylinder, and the "average" sizes of the section are given.)

10. Calculate the capacity in gallons of the petrol tank for a car, as shown at D. (*Hint.*—As the corners have a large radius, allow for them by deducting the "fillets" when finding the area, see p. 299.)

11. At H are given dimensions of a locomotive wheel balance weight. Calculate the weight if of steel weighing ·28 lb. per cu. in.

12. F shows the cross-section of an arched reinforced concrete floor. If the length is 18 ft., find the total volume of concrete between two

girders. If the concrete weighs 120 lbs. per cu. ft., what is the weight of the floor between the two girders? (*Hint.*—In finding the cross-sectional area subtract the segment from the rectangle.)

13. A section of chisel-steel bar is shown at E. Calculate the weight per foot if 1 cu. in. of cast steel weighs ·28 lbs. Find the cost of 8 ft. at 6½*d.* per lb.

14. J gives dimensions of a crescent-shaped balance weight for the driving wheel of a locomotive. Calculate the weight if 1 cu. in. of steel weighs ·28 lbs.

15. An oil-storage tank partly filled with oil is shown at G. Find the number of gallons it contains.

16. Dimensions of a hollow concrete slab for the fire-proof construction of flooring are given at L. Calculate the volume of concrete required for one slab and also its weight if 1 cu. ft. of concrete weighs 120 lbs. (*Hint.*—In finding the area of the end of the prism deduct the ellipse and the pieces out of the corners from the sum of the segment and the rectangle below it.)

17. When designing dams and retaining walls it is necessary to know the weight of 1 ft. length of the wall. B, Fig. 101, gives the section of a certain dam. Calculate the weight of 1 ft. if the masonry weighs 156 lbs. per cu. ft. (*Hint.*—Use horizontal mid-ordinates.)

18. A, Fig. 101, shows the section of a British Standard Bull-Head Rail. Find the weight per yard in lbs. (*Hint.*—Calculate the area of the web between a pair of lines indicated; work by *vertical* mid-ordinates on the two flanges.) 1 cu. ft. of the steel weighs 490 lbs.

Volume of Sphere.—The volume of a sphere or round ball cannot be examined in any simple way. At this stage, then, the following rule must be accepted.

$$\textbf{Volume of sphere} = \frac{4}{3}\pi \mathbf{R^3} \text{ where R = radius of sphere}$$

or in terms of diameter $R = \frac{D}{2}$

$$\therefore \frac{4}{3}\pi R^3 = \frac{4}{3}\pi\left(\frac{D}{2}\right)^3$$

$$= \frac{4}{3}\pi\frac{D^3}{8} = \frac{\pi \mathbf{D^3}}{6} \text{ or } \cdot 5236\mathbf{D^3}$$

By a similar deduction to that for the hollow cylinder (p. 323),

$$\textbf{Volume of hollow sphere} = \frac{4}{3}\pi(\mathbf{R^3} - \mathbf{r^3})$$

Example 334.—A spherical float for an automatic valve is 5″ dia. How much water will it displace when half submerged? The upward force that the water exerts on a floating body is equal to the weight of water displaced. What will be the force in this case if 1 cu. in. of water = ·0361 lb.?

$$\text{Volume of sphere} = \frac{\pi D^3}{6}$$

$$= \frac{\pi \times 5^3}{6} = \frac{\pi \times 125}{6} = 65{\cdot}5 \text{ cu. ins.}$$

$\therefore$ Volume of water displaced when half submerged

$$= \tfrac{1}{2} \times 65{\cdot}5 = \underline{32{\cdot}75 \text{ cu. ins.}}$$

Upward force = weight of displaced water

$$= 32{\cdot}75 \times {\cdot}0361 = \underline{1{\cdot}18 \text{ lbs.}}$$

Example 335.—A cauldron for melting down fat is cylindrical, with a hemispherical bottom, as shown in Fig. 98; determine the capacity of the tank.

The volume required is composed of a cylinder 6′-0″ dia. and 4′-0″ long, and half a sphere of 3′-0″ rad.

$$\text{Volume of cylinder} = \pi r^2 h$$

$$= \pi \times 3^2 \times 4 = 113 \text{ cu. ft.}$$

$$\text{Volume of hemisphere} = \tfrac{1}{2} \text{ volume of sphere}$$

$$= \tfrac{1}{2} \times \tfrac{4}{3}\pi r^3$$

$$= \tfrac{2}{3} \times \pi \times 3^3$$

$$= 2\pi \times 3^2 = 2\pi \times 9 = 56{\cdot}5 \text{ cu. ft.}$$

$\therefore$ Capacity of tank $= 113 + 56{\cdot}5 = \underline{169{\cdot}5 \text{ cu. ft.}}$

$$\textbf{Volume of segment of sphere} = \frac{\pi h}{6}(3r^2 + h^2)$$

where r = radius of the base (not radius of the sphere)
h = height of segment. (See Table on p. 346.)

Example 336.—An evaporating pan is segmental and is 6 ft. dia. and 2′-9″ deep. Calculate its capacity in cubic feet.

Radius of base = 3 ft.; height = 2·75 ft.

$$\text{Volume} = \frac{\pi h}{6}(3r^2 + h^2)$$

$$= \frac{\pi \times 1{\cdot}25}{6}(27 + 7{\cdot}56)$$

$$= \frac{\pi \times 1{\cdot}25 \times 34{\cdot}56}{6} = \underline{22{\cdot}6 \text{ cu. ft.}}$$

Exercises 81. On Volume of Sphere, etc.

1. Calculate the weight of a cast-iron governor ball $2\frac{3}{4}$″ dia., weighing ·26 lb. per cubic inch.

2. A lead melting pot is hemispherical and is 7″ dia. What volume of lead can it hold? How many vice clamps, each requiring $6\frac{1}{2}$ cu. ins., can be cast from one charge of the above pot?

3. An air vessel for a pump is shown at M, Fig. 100. Calculate the air volume in cubic feet when the average position of the water is as shown.

4. A storage tank has hemispherical ends as shown at K, Fig. 100. Calculate the quantity of water held when half full (in gallons).

5. An elevated storage tank, with a hemispherical base as shown at N, Fig. 100, is supported on a trestle tower. Calculate the gallons of water it will hold.

6. An evaporating pan is segmental and is 7′-6″ dia. and 3 ft. deep. Calculate its capacity in cubic feet.

7. Fig. 102 shows at E a rivet with a segmental head. Calculate the weight of 100 such rivets if made of steel weighing 490 lbs. per cu. ft.

8. Shrapnel bullets are of lead, spherical and of $\frac{1}{2}$″ dia. Calculate the weight per 1000 if 1 cu. in. of lead weighs ·41 lb.

9. At C, Fig. 102, is shown a balance weight used on a testing machine. Find its weight if of cast-iron weighing 450 lbs. per cu. ft.

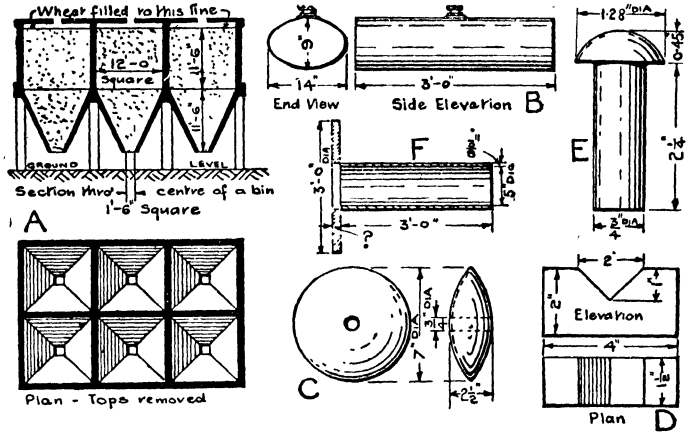


Fig. 102.—Exercises on Volumes.

10. Find the weight of a hollow sphere of 3″ inside and $3\frac{1}{4}$″ outside radius which was for part of an experimental boiler. (Take the weight of the material as ·26 lb. per cu. in.)

Pyramids.—A pyramid is a solid having a base of any geometrical shape, from which the sides taper uniformly to a point or apex. As with prisms, pyramids are designated by the shape of their ends, in this case called the base. Thus *a*, Fig. 103, shows a square pyramid. The circular pyramid *b*, Fig. 103, is given the special name of **cone.** From the engineering standpoint the cone and square pyramid are the most important solids of this class.

The volume of a pyramid cannot be examined in any simple manner, and must therefore be accepted as truth until a later stage. It is, however, one-third of the volume of the circumscribed prism, this latter being a prism whose end is the same shape and

size as the pyramid, and whose height is the same. Thus at *a*, Fig. 103, the full lines show a square pyramid and the chain-dotted lines indicate the circumscribing prism.

Since the volume of a prism is "area of base × height"

Volume of any pyramid = $\frac{1}{3}$ area of base × height

Taking the two important cases: with the square pyramid

Let s = side of square base; h = height of pyramid

Then volume of circumscribing prism = s^2h

and **Volume of square pyramid = $\frac{1}{3}s^2h$**

Similarly for a cone. The circumscribing prism is a cylinder of radius r and height h, as at *b*, Fig. 103.

Volume of circumscribing cylinder = πr^2h

∴ **Volume of cone = $\frac{1}{3}\pi r^2h$**

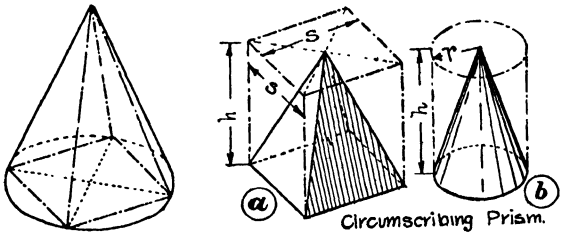


Fig. 103.—Cone and Pyramid.

It must be carefully observed that by the "height" is meant the "*height perpendicular to the base,*" and not the length of the sloping side.

Example 337.—A wooden cone for demonstration purposes is 8″ dia. and 18″ high. Determine its volume and also its weight, if made in pine weighing ·0318 lb. per cu. in.

$$\text{Volume of cone} = \tfrac{1}{3}\pi r^2h$$
$$= \tfrac{1}{3}\pi \times 4^2 \times 18$$
$$= \frac{\pi \times 16 \times 18}{3} = 301{\cdot}4 \text{ cu. ins.}$$

∴ Weight = 301·4 × ·0318 = 9·6 lbs.

Example 338.—It is desired to cut down the cone mentioned in the previous example, into a square pyramid as indicated on the left of Fig. 103, keeping the diameter of the cone as the diagonal of the square base. What will be the volume of the square pyramid?

It is not necessary here to find the side of the square. Referring to

p. 281 area of square = half product of diagonals. In this case the diagonals are 8″.

$$\text{Area of base} = \tfrac{1}{2} \text{ product of diagonals}$$
$$= \tfrac{1}{2} \times 8 \times 8 = 32 \text{ sq. ins.}$$
$$\text{Volume} = \tfrac{1}{3} \text{ base} \times \text{height}$$
$$= \tfrac{1}{3} \times 32 \times 18 = \underline{192 \text{ cu. ins.}}$$

From the formulæ given either the height or the size of the base can be determined, when the volume, etc., are known.

Example 339.—A conical measure as in Fig. 104 is required to hold one pint (34·66 cu. ins.) when filled to a depth of $5\frac{1}{2}$″. What must be the diameter at the surface of the liquid?

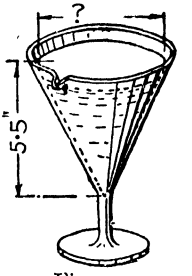


Fig. 104.

$$\text{Volume of cone} = \tfrac{1}{3}\pi r^2 h$$
$$34{\cdot}66 = \frac{3{\cdot}14r^2 \times 5{\cdot}5}{3}$$
$$r^2 = \frac{3 \times 34{\cdot}66}{3{\cdot}14 \times 5{\cdot}5} = 6{\cdot}03$$
$$\therefore\ r = \sqrt{6{\cdot}03} = 2{\cdot}455''$$
$$\therefore \text{ Diameter at surface} = 2 \times 2{\cdot}455 = \underline{4{\cdot}91''}$$

Frusta of Cone and Pyramid.—If a cone or pyramid is cut into two parts by a plane parallel to the base, the lower portion is termed a **Frustum** (plural "frusta"). Thus Fig. 105 shows a frustum of a square pyramid. Evidently the volume of the frustum = Volume of large cone minus volume of small cone cut off. (See Fig. 105.) The usual dimensions known will be the sizes of the small and large ends and the height of the frustum, *i. e.*, the perpendicular distance between the ends. If the height of the complete cone is known (or is obtained by drawing out and measuring) then the volume of the frustum can be obtained as indicated above.

But it may often be inconvenient to find the height of the complete cone, and in any case the finding of two volumes is a long job. A formula for the volume is then useful.

$$\textbf{Volume of any frustum} = \frac{h}{3}(A + a + \sqrt{Aa})$$

where h = height of frustum
A = area of large end, and
a = area of small end

(This formula cannot be proved here, as it involves a considerable amount of algebraical work, including a quadratic equation.)

The formula is applicable to any frustum, whatever the shape of the end. As the most important cases are those of the square pyramid and cone we will derive a particular formula for each.

FRUSTUM OF SQUARE PYRAMID.—Let S = side of base
s = side of top

Then A = area of base = S^2
and a = area of top = s^2

$$\therefore \text{Volume of frustum of square pyramid} = \frac{h}{3}(A + a + \sqrt{Aa})$$

$$= \frac{h}{3}(S^2 + s^2 + \sqrt{S^2 s^2}) \quad \text{substituting for A and } a$$

$$= \frac{h}{3}(S^2 + s^2 + Ss)$$

FRUSTUM OF CONE.—Let R = radius of base
r = radius of top

Then A = area of base = πR^2
a = area of top = πr^2

$$\text{Volume of frustum of cone} = \frac{h}{3}(A + a + \sqrt{Aa})$$

$$= \frac{h}{3}(\pi R^2 + \pi r^2 + \sqrt{\pi R^2 . \pi r^2})$$

$$= \frac{h}{3}(\pi R^2 + \pi r^2 + \pi Rr) \quad \text{reducing the square root}$$

$$= \frac{\pi h}{3}(R^2 + r^2 + Rr) \quad \text{taking out the common factor } \pi$$

If diameters be used instead of radii—

$$\text{the volume} = \frac{\pi h}{12}(D^2 + d^2 + Dd)$$

The reader is cautioned against calculating the volume of a frustum by the expression "height × average of the two end areas." This approximate formula can only be used in cases of *slight taper* and where the height of the frustum is a *very small fraction* of the height of the complete pyramid. Large errors may be introduced by the indiscriminate use of this formula; and examination for each case will take as long as working by the exact formula.

Example 340.—Fig. 106 gives dimensions of a coal bunker, consisting of a square prism and the frustum of a square pyramid. Find

the capacity of the bunker in cubic feet, and also the weight of coal it holds, if 1 cu. ft. of coal = ½ cwt.

Volume of the square prism $= 10{\cdot}5 \times 10{\cdot}5 \times 7{\cdot}5$
$= 826{\cdot}9$ cu. ft.

Volume of lower portion (frustum of sq. pyramid) $= \frac{h}{3}(S^2 + s^2 + Ss)$
$= \frac{5}{3}(10{\cdot}5^2 + 1{\cdot}5^2 + 10{\cdot}5 \times 1{\cdot}5)$
$= 1{\cdot}667(110{\cdot}2 + 2{\cdot}25 + 15{\cdot}75)$
$= 1{\cdot}667 \times 128{\cdot}2 = 213{\cdot}7$ cu. ft.

$\therefore$ Total capacity of bunker $= 826{\cdot}9 + 213{\cdot}7 = 1040{\cdot}6$ cu. ft.

$\therefore$ Weight of coal held $= 1040{\cdot}6 \times \frac{1}{2}$ cwts. $= 520{\cdot}3$ cwts.

$\therefore$ Capacity (coal level with top of bunker) = say, 520 cwts. or 26 tons.

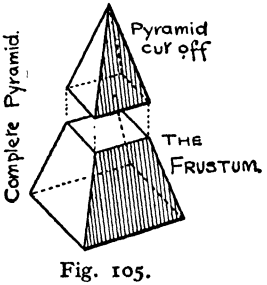


Fig. 105.

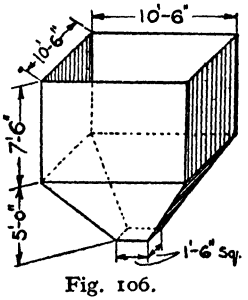


Fig. 106.

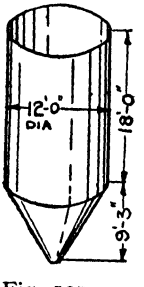


Fig. 107.

Example 341.—An ash receiver in a suction ash plant, with a 16″ dia. hole for discharging, is cylindrical, with a taper bottom, as shown with dimensions in Fig. 107. Find its capacity.

Volume of cylindrical portion $= \frac{\pi}{4}d^2h$
$= {\cdot}785 \times 12^2 \times 18$
$= {\cdot}785 \times 144 \times 18$
$= 2035$ cu. ft.

Conical frustum, radius of top $= 6$ ft.
radius of bottom $= 8'' = {\cdot}667$ ft.

$\therefore$ Volume $= \frac{\pi h}{3}(R^2 + r^2 + Rr)$
$= \frac{\pi \times 9{\cdot}25}{3}(6^2 + {\cdot}667^2 + 6 \times {\cdot}667)$
$= \frac{\pi \times 9{\cdot}25}{3}(36 + {\cdot}445 + 4)$
$= \frac{\pi \times 9{\cdot}25 \times 40{\cdot}44}{3} = 392$ cu. ft.

$\therefore$ Total capacity of receiver $= 2035 + 392 = 2427$ cu. ft.

Example 342.—Fig. 108 gives dimensions of a gas-holder tank. Determine how many gallons of water are required to fill it to a depth of 35 ft.

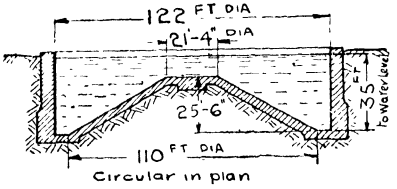


Fig. 108.

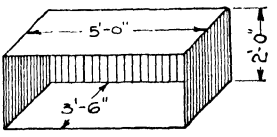


Fig. 109.

The volume of water here is the volume of a cylinder, minus the volume of the conical frustum in the centre.

Volume of cylindrical portion $= \pi R^2 h$
$= \pi \times 61^2 \times 35 = 408{,}900$ cu. ft.

Frustum to be deducted—

Radius of large end	$= 55'$
Radius of small end	$= 10'\text{-}8'' = 10{\cdot}67$ ft.
Height	$= 25{\cdot}5'$

$\therefore$ Volume $= \frac{\pi h}{3}(R^2 + r^2 + Rr)$

$= \frac{\pi \times 25{\cdot}5}{3}(55^2 + 10{\cdot}67^2 + 55 \times 10{\cdot}67)$

$= \frac{\pi \times 25{\cdot}5}{3}(3025 + 114 + 587)$

$= \frac{\pi \times 25{\cdot}5 \times 3726}{3} = 99420$ cu. ft.

$\therefore$ Volume of water $= 408900 - 99420$
$= 309480$ cu. ft.

$\therefore$ Gallons of water $= 309480 \times 6{\cdot}24$
$= 1{,}932{,}000$ galls.

Exercises 82. On Volumes of Pyramids, Cones, and Frusta.

1. Find the volume of a square pyramid if the side of the base is 5″ and the perpendicular height is $12\frac{1}{2}$″. Find what fraction this volume is of the volume of a square prism, of the same height and same base.

2. A square pyramid is to have a volume of 160 cu. ins. and a base of $7\frac{1}{2}$″ side. What will be its height?

3. A cone is $11\frac{1}{2}$″ high and $5\frac{3}{4}$″ in diameter at the base. Find its volume. What relation can you find between this volume and the volume of a cylinder of the same height and on the same base ($5\frac{3}{4}$″ dia.)?

4. A, Fig. 110, shows a supply bucket (for holding sand) as used in a special testing machine. Find its capacity in cubic inches. The

sand is allowed to run into a cylindrical tin 10″ dia. If this tin is to hold all the contents of the supply bucket, what must be its least depth?

5. A conical part of a measuring glass, as shown in Fig. 104, is to have a length of 4″ and is to hold half a pint. What will be its diameter at the surface of the liquid (1 pint = 34·66 cu. ins.)?

6. A bucket for a coal conveyor consists of a frustum of a square pyramid, and is 26″ square at the top, 18″ square at the bottom, and 12″ deep. Find how many tons of coal 20 such buckets will carry, if the coal weighs ½ cwt. per cu. ft. (*Hint.*—Reduce all dimensions to feet.)

7. Calculate the capacity in cubic feet of a coal bunker such as is shown in Fig. 106, with the following dimensions. Upper rectangular portion 12 ft. square, 10 ft. deep. Lower tapered portion 1′-2″ square at the bottom opening and 9 ft. deep. If 1 cu. ft. of loose coal weighs 56 lbs., what weight in tons does the bunker hold?

8. At A, Fig. 102, is shown a group of grain silos or bins filled with wheat. Find the total capacity in cubic feet and also the weight of the wheat if 1 cu. ft. weighs 48 lbs.

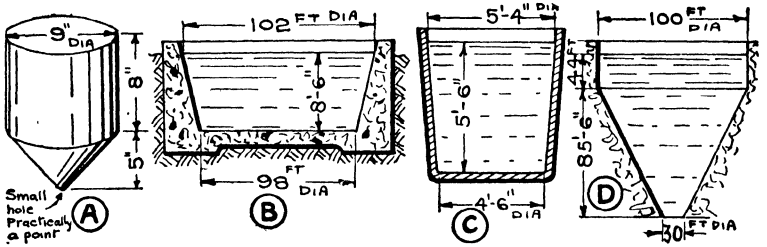


Fig. 110.—Exercises on Volumes of Cones and Frusta.

9. B, Fig. 110, gives dimensions of a gas-holder tank. Calculate the quantity of water it contains in gallons.

10. A milk churn is 12″ dia. at the top and 18″ dia. at the bottom, and 3′-6″ high. How many gallons of milk will it hold? (The churn is a frustum of a cone.)

11. The approximate dimensions of a steel-works ladle are given at C, Fig. 110. Calculate the weight of steel (in tons) it holds at 490 lbs. per cu. ft.

12. A surge tank at a hydro-electric station is shown at D, Fig. 110. Calculate the gallons of water it contains when the water reaches the high-water level.

The Calculation of Weights.—In the drawing office the calculation of the weights of proposed machine parts is of common occurrence. In all cases the *volume* of the object must first be found, and then be multiplied by the "density" of the material used. This **density** is the weight of a unit volume of material. The usual forms are "lbs. per cubic inch" and "lbs. per cubic foot." Thus the density of cast iron is ·26 lb. per cu. in.

Since 1728 cu. ins. equal 1 cu. ft., then—

lbs. per cubic foot = lbs. per cubic inch × 1728

The densities of the many materials employed vary considerably with the composition, method of manufacture, etc., and will be found in engineering pocket-books. The following densities of the more common materials should be memorised :—

Material.	Weight in lbs. per cu. in.	Weight in lbs. per cu. ft.
Cast iron	·26	450
Wrought iron	·278	480
Mild steel	·284	490
Brass and gun-metal . .	·307	530
Lead	·41	708
Water	·0361	62·4

The majority of engineering structures or parts are combinations of the simple solids already taken, so that the total volume can usually be obtained by dividing the object up into its component parts, and applying the formulæ already given. Certain shapes appear, however, which do not lend themselves, either easily or at all, to the application of a formula. Such figures, by a "give-and-take" process, can usually be reduced to a form permitting of easy

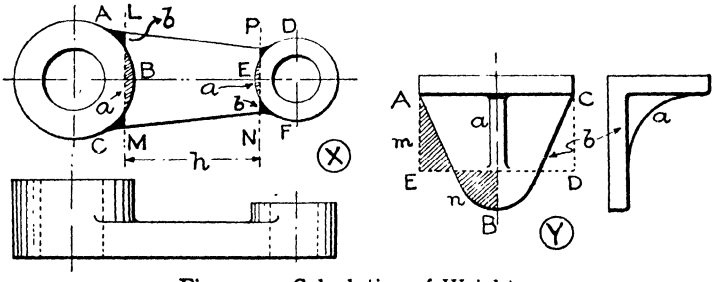


Fig. 111.—Calculation of Weights.

calculation. With practice the process of "give-and-take" can become very reasonably accurate. When calculating weights, an *exact* answer is seldom required, as the weight of the actual article is affected by so many conditions not considered in our calculations, such as the blow-holes in casting, lack of uniformity in material, and the like.

Space does not permit of a number of examples being taken here, but Fig. 111 will illustrate the principle. At X is shown a

forging for an engine crank, which, for calculation, divides naturally into the two bosses and the crank arm. The bosses are easily dealt with, but when finding the volume of the crank arm the area of the figure ABCFED is required, which is not easily found by formula. A straight line is then drawn across each curved end as at LM and PN, so that at each end the areas *b*, cut off, balance the areas *a*, put on. The balance is obtained by eye only and its accuracy is dependent upon judgment and experience. The area of the irregular figure is thus replaced by that of the trapezoid LMNP, which can be dealt with by the formula given on p. 282.

Similarly with the bracket casting at Y, Fig. 111, the complex figure ABC, forming the back *b*, can be replaced by a rectangle AEDC if the line ED be drawn so that the area *m* is judged to balance the area *n*.

Fillets, if small in comparison with the rest of the object, can usually be neglected, but when large, as in the rib *a*, at Y, should be treated by the formula on p. 299.

Surface Areas.

In many cases we are concerned with the *area of the surface* of some of the solids previously mentioned. In the case of the **cuboid and square prism** the calculation of the area of the surface is simple, given the necessary dimensions. An example will therefore be sufficient; frequently only part of the area is required.

Example 343.—The inside of a lift cage is to be lined with sheet zinc, on top and bottom, sides and back. The dimensions are given in Fig. 109. Find how many square feet of zinc are required.

Back 5′-0″ × 2′-0″; area = 2 × 5 = 10 sq. ft.
Two sides each 3′-6″ × 2′-0″; area = 2 × 3·5 × 2 = 14 sq. ft.
Top and bottom each 5′-0″ × 3′-6″; area = 2 × 5 × 3·5 = 35 sq. ft.

∴ Total area required = 59 sq. ft.

Cylinder.—In all prisms the surface can be divided into two parts: the **lateral surface,** which is the surface of the sides; and the two ends. Suppose that a cylinder is covered with paper. Then if it be slit along a straight line lengthwise and opened out, as in Fig. 112, the area of the rectangular sheet is evidently the area of the curved or lateral surface of the cylinder. Now the length of the sheet is the length of the cylinder, and the width of the sheet is the circumference.

Then if d = diameter of cylinder and l = length

$$\left.\begin{matrix}\text{Area of curved surface} \\ \text{or lateral surface}\end{matrix}\right\} = \text{length} \times \text{circumference}$$

$$= l \times \pi d$$

$$= \pi dl \text{ or } 2\pi rl$$

All measurements must, of course, be in the same units.

The two ends each have in area πr^2

$\therefore$ Area of ends $= 2\pi r^2$

$\therefore$ **Total surface area** = lateral + ends

$$= 2\pi rl + 2\pi r^2$$

$$= 2\pi r(l + r) \text{ taking out the common factor } 2\pi r$$

In many cases, however, only the lateral surface is required.

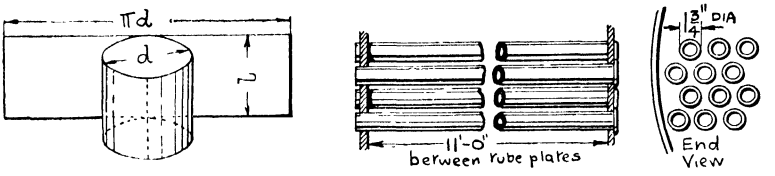


Fig. 112.—Illustrating Surface of Cylinders.

Example 344.—The smoke tubes in a locomotive boiler are shown in Fig. 112. Find the heating surface of 1 tube (*i.e.*, the lateral surface). If the boiler has 220 such tubes in all, calculate the total heating surface of the tubes.

Lateral surface is that of a cylinder $1\frac{3}{4}$" dia. and 11 ft. (=132") long.

$\therefore$ Heating surface of 1 tube $= \pi dl$

$$= 3{\cdot}14 \times 1{\cdot}75 \times 132 \text{ sq. ins.}$$

$$= \frac{3{\cdot}14 \times 1{\cdot}75 \times 132}{144} = 5{\cdot}036 \text{ sq. ft.}$$

$\therefore$ Total heating surfaces of tubes $= 5{\cdot}036 \times 220$

$$= \underline{1108 \text{ sq. ft.}}$$

Example 345.—A surface condenser is to have 12000 sq. ft. of cooling surface (lateral). How many tubes $\frac{3}{4}$" dia. and 14 ft. long will be required?

Lateral surface of 1 tube $= \pi dl$

$$= \frac{3{\cdot}14 \times {\cdot}75 \times 14 \times 12}{144_{12}} \text{ sq. ft.}$$

$$= 2{\cdot}75 \text{ sq. ft.}$$

$\therefore$ Number of tubes required $= \dfrac{\text{Total surface}}{\text{Surface of 1}}$

$$= \frac{12000}{2{\cdot}75} = \underline{4368}$$

Example 346.—The field coil of an electric motor is 4″ dia. and 4½″ long. Determine the watts radiated per sq. in. of lateral surface when the voltage is 100 and the current is ·48 ampere.

(*Note.*—Watts = volts × amperes.)

$$\begin{aligned} \text{Surface of coil} &= \pi dl \\ &= 3{\cdot}14 \times 4 \times 4{\cdot}5 \text{ sq. ins.} \\ &= 56{\cdot}5 \text{ sq. ins.} \\ \text{Watts radiated} &= \text{volts} \times \text{amperes} \\ &= 100 \times {\cdot}48 = 48 \text{ watts} \end{aligned}$$

$$\therefore \text{ Watts radiated per sq. in.} = \frac{48}{56{\cdot}5} = \underline{{\cdot}85}$$

Example 347.—A dynamo core is 24″ long, and it is to have at least 2000 sq. ins. of curved surface for radiation. Determine a suitable diameter.

$$\text{Surface} = \pi dl$$

$$\therefore \quad 2000 = 3{\cdot}14 \times d \times 24$$

$$\text{whence} \quad d = \frac{2000}{3{\cdot}14 \times 24} = 26{\cdot}54'', \text{ say, } \underline{26\tfrac{1}{2}'' \text{ dia.}}$$

Surface of Sphere.—This cannot be examined in any simple manner, and the following formula must, therefore, be accepted until a later stage.

Surface of sphere = $4\pi R^2$ where R = radius of sphere.

Example 347*a*.—What is the surface of a sphere 3½″ dia.?

$$\begin{aligned} \text{Radius} &= 1\tfrac{3}{4}'' = 1{\cdot}75'' \\ \text{Surface} &= 4\pi R^2 \\ &= 4\pi \times (1{\cdot}75)^2 = \underline{38{\cdot}5 \text{ sq. ins.}} \end{aligned}$$

Curved surface of segment of sphere = $2\pi Rh$

where R = radius of sphere; h = height of segment

In cases where h is not given the following formula can be used where r = rad. of base of segment.

$$\text{Curved surface} = 2\pi R(R - \sqrt{R^2 - r^2})$$

Example 348.—Find the area of the dished end of a water-tube boiler drum, as shown in Fig. 113.

$$\begin{aligned} \text{Surface} &= 2\pi Rh \\ &= 2\pi \times 3 \times \frac{5}{12} \text{ sq. ft.} \\ &= \underline{7{\cdot}85 \text{ sq. ft.}} \end{aligned}$$

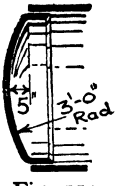


Fig. 113.

Example 349.—The dished ends of boiler drums are flanged from a circular plate as indicated in Fig. 114. Assuming that the thickness

of the plate and the volume of the metal are not altered, find what diameter of plate is required to produce the end shown in Fig. 114.

Thickness of plate is very small in comparison with other dimensions, and so we may calculate the volume of metal as "surface area × thickness." If volume and thickness are to be the same in each case, surface areas must also be equal.

$\therefore$ Surface of circular plate = surface of dished end
(1 face only) = surface of segment of sphere + surface of cylindrical flange.

Surface of segment $= 2\pi Rh = 2 \times 3{\cdot}14 \times 30 \times 6 = 1130$ sq. ins.
Surface of cylinder $= \pi dl = 3{\cdot}14 \times 36 \times 5 = 565$ " "
$\therefore$ Total surface of dished end $= 1695$ " "

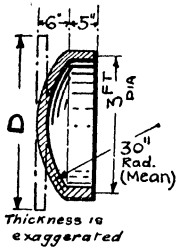


Fig. 114.

Surface of circular plate $= {\cdot}785D^2$
$\therefore \quad {\cdot}785D^2 = 1695$
$\therefore \quad D = \sqrt{\frac{1695}{{\cdot}785}} = 46{\cdot}5''$

i. e. a plate 3′-10½″ dia. is required.

In cases of "cold drawing" of tubes, etc., where the thickness alters, the volumes may be equated instead of surface areas.

Surface of Cone.—Make a mark on the base of a cone, and lay the cone on its side with this mark to the ground, as at *a*, Fig. 115. Roll the cone quite freely until the mark goes just once round, and arrives at the ground again as at *c*. It will be found that a sector OAB has been marked out, which is the curved surface of the cone. The radius of the sector is evidently the "slant height" of the cone, *i. e.*, the length *l* of the sloping side. Also the length of the arc of the sector is evidently the circumference of the base of the cone, *i. e.*, πd if d = diameter of cone base.

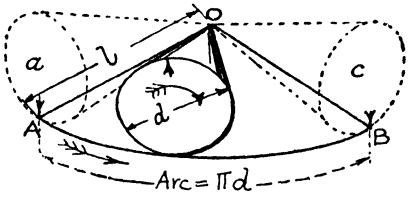


Fig. 115.—Surface of Cone.

Then, **Curved surface of cone** = Area of Sector
$= \frac{1}{2}$ arc × radius (see p. 313)
$= \frac{1}{2}\pi d \times l$
$= \pi rl$

l may be obtained by drawing, or by calculation of a right-angled

triangle, as indicated in Ex. 350. The total surface is evidently the curved surface plus the base.

$$i.\,e.,\ \pi rl + \pi r^2 = \pi r(l + r)$$

Example 350.—Find the curved surface of a cone 8″ dia. and 10″ high.

The slant height is required and will be calculated here. It is the hypotenuse of a right-angled triangle whose perpendicular sides are 12″ and 4″.

$$\therefore \text{ Slant height } l = \sqrt{10^2 + 4^2}$$
$$= \sqrt{116} = 10{\cdot}77''$$
$$\therefore \text{ Curved surface } = \pi rl$$
$$= \pi \times 4 \times 10{\cdot}77 = \underline{135{\cdot}3 \text{ sq. ins.}}$$

Surface of Square Pyramid.—The lateral surface consists of four triangles. Let s = side of base and l = slant height, *i. e.*, the distance from point to centre of side of base, and *not the slant edge;* the distinction is shown at Fig. 116.

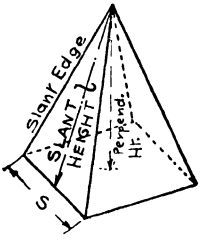


Fig. 116.

$$\text{The area of one triangle } = \tfrac{1}{2}sl$$
$$\therefore \textbf{ Lateral surface of pyramid } = 4 \times \tfrac{1}{2}sl$$
$$= \mathbf{2}sl$$
$$\text{The base area } = s^2$$
$$\therefore \textbf{ Total surface } = 2sl + s^2$$
$$= s(2l + s)$$

Example 351.—Find the area of the lateral surface of a square pyramid, if perpendicular height is 24″ and side of base 9″.

$$\text{Slant height } = \sqrt{24^2 + 9^2}$$
$$= \sqrt{576 + 81}$$
$$= \sqrt{657} = 25{\cdot}64''$$
$$\therefore \text{ Area of lateral surface } = 2sl$$
$$= 2 \times 9 \times 25{\cdot}64$$
$$= \underline{461 \text{ sq. ins.}}$$

Surface of Frustum of Square Pyramid.—The lateral surface consists of four faces each of which is a trapezoid. Let S = side of base, s = side of top, and l = slant height as shown at Fig. 117.

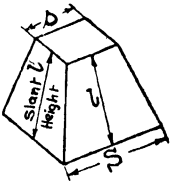


Fig. 117.

Then—

$$\text{Area of one trapezoid } = \frac{S + s}{2} \times l \quad \text{(as on p. 282)}$$

$$\therefore \text{ Lateral surface } = 4 \times \frac{S + s}{2} \times l$$
$$= \mathbf{2}l(S + s)$$

The total surface easily follows, being: lateral surface + S^2 + s^2.

Where l, the slant height, is not given it may be obtained by drawing, or by calculation, using the right-angled triangle property.

Example 352.—Find how many square feet of plating are required to make the tapering portion, shown in Fig. 118, of a coal bunker.

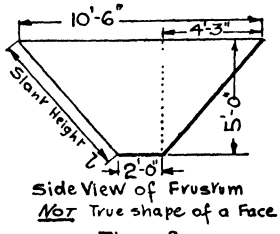


Fig. 118.

$$\text{Slant height} = \sqrt{5^2 + 4{\cdot}25^2}$$
$$= \sqrt{25 + 18{\cdot}06}$$
$$= \sqrt{43{\cdot}06} = 6{\cdot}56'$$
$$\therefore \text{Lateral surface} = 2l(S + s)$$
$$= 2 \times 6{\cdot}56(10{\cdot}5 + 2)$$
$$= 2 \times 6{\cdot}56 \times 12{\cdot}5$$
$$= 164 \text{ sq. ft.}$$

Surface of Frustum of Cone.—This is somewhat similar to the square pyramid.

$$\text{Lateral surface} = \frac{\pi l}{2}(D + d)$$
$$= \pi l(R + r)$$

where l = *slant height* as before

Example 353.—Fig. 119 gives dimensions of a "Galloway" tube, placed in the flue of a boiler to provide extra heating surface. Determine the heating surface (lateral) in the case shown.

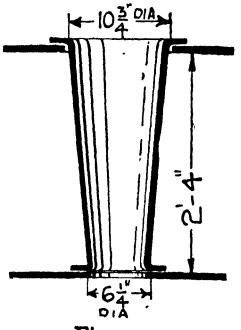


Fig. 119.

The heating surface is the lateral or curved surface of the frustum.

$$\text{Slant height} = \sqrt{28^2 + (5{\cdot}375 - 3{\cdot}125)^2}$$
$$= \sqrt{28^2 + 2{\cdot}25^2}$$
$$= \sqrt{789{\cdot}1} = 28{\cdot}09$$
$$\text{Curved surface of frustum} = \frac{\pi}{2}(D + d)l$$
$$= \frac{\pi}{2}(10\tfrac{3}{4} + 6\tfrac{1}{4}) \times 28{\cdot}1$$
$$= \frac{\pi \times 17 \times 28{\cdot}1}{2} = 750 \text{ sq. ins.}$$
or, 5·2 sq. ft.

Exercises 83. On Surface Areas.

(*Letters refer to Fig.* 120.)

1. Calculate the total heating surface (= tubes + firebox) of a large American locomotive having 446 tubes, each 2¼" dia. and 24 ft. long, and a firebox of 353 sq. ft. heating surface.

2. An English locomotive boiler has 157 tubes 2¼" dia. and 24 tubes 5¼" dia., all being 11'-6" long. Calculate the total heating surface in sq. ft. (*i.e.*, lateral surface).

3. A surface condenser has 2680 tubes $\frac{3}{4}$" outside dia., and 14'-2$\frac{1}{2}$" long. Calculate the cooling surface in sq. ft. If the condenser handles 52,000 lbs. of steam per hour, calculate the weight of steam condensed per sq. ft. of surface.

4. In tests on the adhesion of round steel bars embedded in concrete, a bar 1$\frac{1}{8}$" dia. and 3 ft. long was pulled length-wise from the concrete with a force of 18 tons. Calculate the adhesive stress (*i.e.*, the force in lbs. per sq. in. of lateral surface).

5. In a test to determine the loss of heat from a bare steam pipe the pipe was 6" outside diameter and 3'-6" long. In 1 hour, 2210 units of heat were lost. Calculate the loss per sq. ft.

6. A Leyden jar is 4" mean dia. and is coated with tinfoil for a depth of 5" up the sides and on the base as shown at A. Calculate the area of the coating. (One side only.)

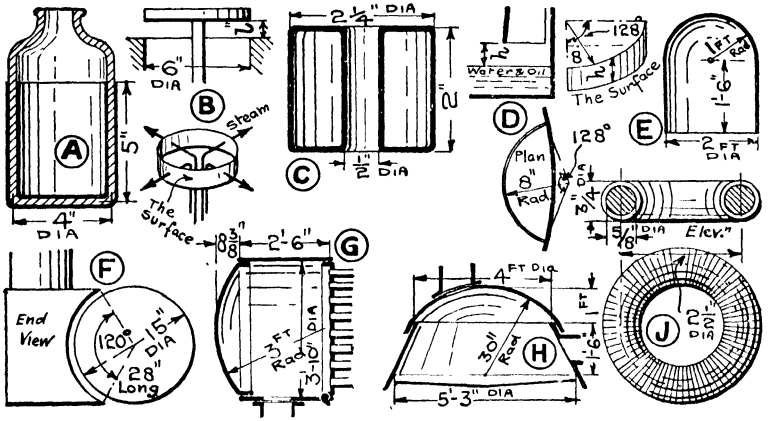


Fig. 120.—Exercises on Surface Areas.

7. If the bevel on a mushroom valve be neglected, and the valve considered as merely a circular plate closing the pipe, as shown at B, the area through which steam passes is the surface of the cylinder as shown. When the valve is full open this area should equal the area of the pipe. Find what must be the "lift" of a 6" dia. valve for full opening.

8. Refer to Ex. 7 above: if d = dia. of valve, and l = lift, find what the value of l must be for full opening. (*Hint.*—Equate formula for area of pipe to formula for surface of cylinder and solve for l.)

9. A tank for use in boiler tests is to be 2'-6" dia. and 3'-6" high with an open top. Calculate its weight if 1 sq. ft. of the sheet metal of which it is made weighs 2·86 lbs.

10. A float for a carburettor is shown at C. Calculate its weight if of sheet brass weighing ·87 lb. per sq. ft. (*Hint.*—There are four surfaces to consider; the outside and inside lateral surfaces, and the two annular ends.)

11. A dynamo armature is 10$\frac{1}{2}$" dia. and 23" long, and the power lost by radiation is 680 watts. Calculate the watts lost per square inch.

12. The pole piece in a two-pole dynamo embraces an angle of 120° at the centre as shown at F. The armature is 15″ dia. and 28″ long. Calculate the area of the lateral surface of the armature opposite the pole piece. (This area is wanted when calculating the field winding, etc.)

13. The area beneath the edge of the inlet tube in the oil separator (see E, Fig. 80) is to equal 1½ times the area of the steam pipe. Further dimensions are given at D (Fig. 120). Calculate the height h if the steam-pipe is 6″ dia.

14. In a fat melting tank as shown in Fig. 98, the space between the inner and outer vessels is a steam jacket. If the metal is ½″ thick calculate the heating surface in square feet. (*Hint.*—The heating surface is the outside surface of the inner vessel.)

15. A pressed sheet metal dome is shown at E. Calculate its weight if the metal weighs 2·6 lbs. per sq. ft.

16. A dished boiler end plate, as shown in Fig. 114, has the following dimensions: dia. 3′-6″, mean rad. of spherical part 3′-0″, height of segment 6¾″, length of flange 5″. Calculate the diameter of the flat plate required to make it, assuming that the thickness does not alter during flanging.

17. At G is shown the combustion chamber in a vertical boiler. Calculate the heating surface (*i. e.*, lateral surface of cylinder plus area of spherical end; neglect the entering flue and the tube plate).

18. A taper flue in a Lancashire boiler is 2′-8″ dia. at one end and 3′-3″ dia. at the other, and is 3′-3″ long. Calculate its heating surface (lateral surface).

19. The furnace of a vertical boiler is shown at H. Calculate the heating surface (*i. e.*, lateral surface of conical frustum plus curved surface of segment of sphere. Neglect the two openings).

20. Calculate the weight of the plating necessary for the coal bunker in Fig. 106, if the plate of which it is made weighs 20 lbs. per sq. ft.

21. At J is shown an iron ring of circular section (called an "Anchor Ring"—see end of Volume Table, p. 346) wound with a coil of wire. Calculate the watts radiated per square inch of surface if the watts wasted in a test were 10·5. Also find the volume of the iron ring within the coil.

22. In the "Brinell" hardness test for metals, a hard steel ball is pressed into the test piece under a certain load, and the diameter of the spherical depression produced is measured. Then the "hardness number" is "Load ÷ Curved Surface of Depression." In a certain test the load was 3000 kilogrammes and the diameter of the depression was 3·8 millimetres, the steel ball being 10 millimetres dia. Calculate (*a*) the curved area of the depression; (*b*) the hardness number.

23. In a certain aeroplane instrument a pivoted tube is kept head to wind by a trailing conical tube of sheet metal. This conical tube is the frustum of a cone; diameter at large end 4″, at small end ⅛″, length on centre line 7½″. It is necessary to know its weight for the purpose of balancing the instrument. Calculate the weight if the metal weighs ·53 lb. per sq. ft.

Exercises 83a. Miscellaneous Examples on Mensuration.

(*Letters refer to Fig.* 120*a*.)

1. The plates of a variable electrical condenser are semicircular, $2\frac{1}{2}''$ diameter. If the portion round the axis for a radius of $\frac{1}{4}''$ be ineffective, find the effective area of the two sides of a single plate.

2. A flywheel casting has a rim of rectangular section, 6′–2″ outside and 5′–4″ inside diameter, 8″ broad. The hub is 7″ dia. by 9″ long with a 3″ dia. cored hole. There are 6 arms connecting rim to hub, of elliptical cross section $4\frac{1}{2}'' \times 2\frac{1}{2}''$. Calculate the weight in cast iron.

3. A frame aerial consists of a square pancake coil as indicated at A. If there are 14 complete turns, spaced $\frac{1}{4}''$ apart, the side of the outside square being 4 ft., estimate the total length of wire required. (*Hint.*—14 turns include 13 spaces of $\frac{1}{4}''$ wide. Calculate the average length of a turn.)

4. At B are given dimensions of a cast iron segment used for lining an underground railway tunnel. Calculate its weight. (*Hint.*—The cylindrical part is a sector of a tube; use average diameter method. The side flanges can be treated similarly and the end flanges are rectangular. The material at corners must not be included twice.)

5. A conical spring for the overhead valves of an aero engine is shown at C. Estimate the total length of wire required per spring, allowing $1\frac{1}{2}$ coils extra to the working coils to allow for the flattened ends. (*Hint.*—Find the length of a turn of average diameter.)

6. A hardness test consists in pressing a hard steel cone into the material under a certain load and measuring the diameter of the conical depression so produced. The apex angle of the cone is 90° so that the depth of the conical depression, measured along the axis of the cone, is one-half of its diameter. The "hardness number" is then "Load ÷ Curved Surface of Depression." Calculate (1) the curved surface of the depression, (2) the hardness number, when a load of 1000 kilogrammes produces a depression 3 millimetres diameter.

7. A special milling cutter is required having a diameter over the teeth of $4\frac{5}{8}''$. The teeth are to be cut with a 45° angular cutter as indicated at D. The width of the gap is required to be $\frac{1}{2}''$ and the width of the top of the tooth is to be about ·05″. Determine how many teeth must be cut and the actual width of the top of the tooth. (*Note.*—An exact number of teeth must be cut.)

8. Big end bolts for marine engine connecting rods are frequently reduced in area in the shank as a provision against shock. One way of doing this is as shown at E, where three semicircular flutes are milled in the shank. If the diameter of such a bolt is 3″ and the diameter at the bottom of the threads is 2·634″, find what diameter of flute is needed to make the area at the section AB equal that at the section CD. (*Hint.*—The area at AB is that of a circle less the area of three flutes, and each of the latter may be taken as a complete semicircle.)

9. Earthenware draw-in conduits for carrying electric cables underground are made of octagonal cross section with a circular hole. If the octagon be 4″ across the flats and the hole 3″ diameter, estimate the weight of a section 15″ long, the material weighing 150 lbs. per cubic foot.

10. A motorcycle petrol tank is a cylinder 5″ diameter and 8″ long, with conical ends, the length of cone, measured on the axis, being 3″. Estimate the capacity of the tank in gallons, when quite full.

11. A cast iron balance weight is spherical, with a hole cored through the centre as shown at F. If the ball is 10″, and the hole 5″, diameter, determine the weight of the balance weight. (*Hint.*—From the volume of the sphere subtract the volume of the cylinder forming the hole and also the volume of the two segments of the sphere cut off beyond the ends of the hole. The length of the hole must first be calculated from the right-angled triangle OAB.)

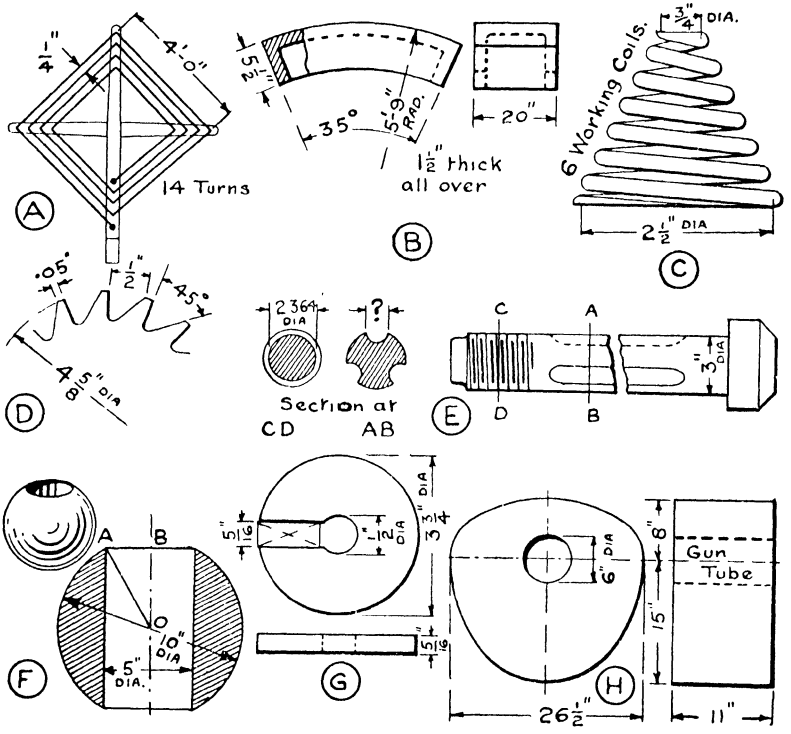
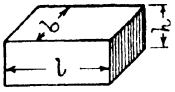
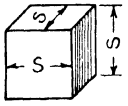
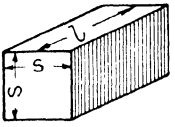
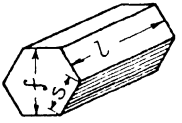
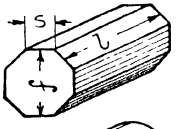
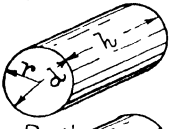
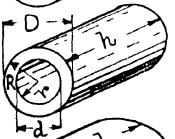
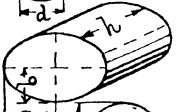


Fig. 120a.—Diagram for Exercises 83a on Mensuration.

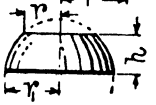
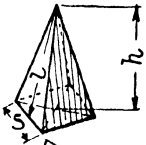
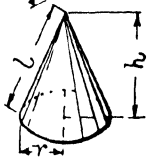
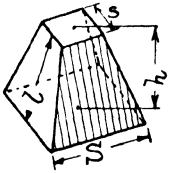
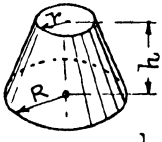
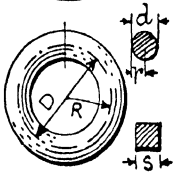
12. A number of weights for testing purposes are required of the form shown at G, to be of brass and to weigh 1 lb. each. If the outside diameter be $3\frac{3}{4}$″, the central hole be $\frac{1}{2}$″ diameter, the thickness be $\frac{5}{16}$″, determine whether a slot about $\frac{5}{16}$″ wide will give enough volume to permit of adjustment to the exact weight, *i.e.*, the weight when calculated from the tentative dimensions should be a little more than 1 lb. (*Note.*—For practical purposes, the area of the top of the weight may be obtained by subtracting the area of the rectangle shown crossed from that of the hollow circle.)

13. An aeroplane petrol tank is shown at H, with dimensions. The shape of the end may be considered as consisting of two semi-ellipses. Calculate the capacity in gallons when quite full.

TABLE OF VOLUMES AND SURFACE AREAS OF SOLIDS.

Title.	Figure.	Volume.	Surface Area.
Any prism .		Area of base × height	Circumference of base × height
Rectangular prism or cuboid . .		lbh	Whole area $= 2(lb+lh+bh)$
Cube . . .		S^3	Whole area $= 6S^2$
Square prism		S^2l	Lateral surface $= 4Sl$ Ends $= 2S^2$ Whole surface $= 2S(2l+S)$
Hexagonal prism . .		$2{\cdot}6S^2l$ or $\cdot866f^2l$	Lateral $= 6Sl$ or $3{\cdot}46fl$ (For ends see Table in Chap. VII.)
Octagonal prism . .		$4{\cdot}83S^2l$ or $\cdot829f^2l$	Lateral $= 8Sl$ or $3{\cdot}32fl$
Cylinder . .		πr^2h or $\cdot7854d^2h$	Lateral $= 2\pi rh$ Two ends $= 2\pi r^2$ Whole area $= 2\pi r(h+r)$
Hollow cylinder . . .		$\pi(R^2-r^2)h$	Outer lateral surface $= 2\pi Rh$ Inner lateral surface $= 2\pi rh$
Elliptical prism . .		πabh	Lateral $= \pi h\{1{\cdot}5(a+b)-\sqrt{ab}\}$ or $\pi(a+b)h$ (less accurate)
Sphere . .		$\frac{4}{3}\pi R^3$ or $\frac{\pi}{6}D^3$ or $\cdot5236D^3$	$4\pi R^2$
Hollow sphere . .		$\frac{4}{3}\pi(R^3-r^3)$	$4\pi(R^2+r^2)$

TABLES OF VOLUMES AND SURFACE AREAS OF SOLIDS (*continued*).

Title.	Figure.	Volume.	Surface Area.
Segment of sphere . .		$\frac{\pi h}{6}(3r^2 + h^2)$ or $\cdot 5236h(3r^2+h^2)$	Curved surface $= 2\pi Rh$ or $2\pi R(R - \sqrt{R^2 - r^2})$ where R=rad. of sphere
Zone of sphere . .		$\frac{\pi h}{6}\left\{3(r^2+r_1^2)+h^2\right\}$	
Any pyramid		$\frac{1}{3}$ area of base $\times$ height	Lateral $= \frac{1}{2}$ circum. of base $\times$ slant height
Square pyramid . . .		$\frac{1}{3}S^2h$	Lateral $= 2Sl$
Cone . . .		$\frac{1}{3}\pi r^2 h$	Lateral $= \pi rl$
Frustum of any pyramid . . .	h=height of frustum A=area of large end B=area of small end	$\frac{h}{3}(A + B + \sqrt{AB})$	Lateral=$\frac{1}{2}$ mean circum. $\times$ slant height
Frustum of square pyramid .		$\frac{h}{3}(S^2 + s^2 + Ss)$	Lateral $= 2l(S + s)$ (l = slant height)
Frustum of cone. . .		$\frac{\pi h}{3}(R^2 + r^2 + Rr)$	Lateral $= \pi l(R + r)$ (l = slant height)
Anchor ring .		Round section $2\pi^2 Rr^2$ Square section πDS^2	$4\pi^2 Rr$ $4\pi DS$

CHAPTER IX

CURVES OR GRAPHS

[*All the worked examples should be drawn out by the student.*]

Curves and their Uses.—After a machine has been built, especially if it is one of a new type, it is usual to test its performance in order to see if it satisfies the conditions for which it was designed, to determine any defects, or to obtain information for later designs. Thus, if the machine is an electric motor, it is necessary to find its "efficiency" (*i. e.*, what fraction of the electric energy supplied, is given out usefully) at various loads. The motor is worked at a number of different loads ("brake horse powers") varying from nothing to rather more than the rated horse power, and certain measurements are made at each load from which the horse power and the efficiency are calculated. Thus two sets of figures are obtained, and for each horse power there is one particular efficiency. Such a set of figures is given below, calculated from the results of tests on a direct current motor rated at 90 H.P., and each efficiency is written beneath its particular horse power.

B.H.P. . . .	10	20	42	60	74	92	102	111
Efficiency . .	·60	·74	·83	·86	·89	·90	·905	·905

Looking at these figures alone does not supply much useful information. Thus we cannot say what is the efficiency at horse powers *other than those in the table;* again, although it can be seen that the efficiency increases as the B.H.P. increases, it is not an easy matter to see *how* it increases, *i. e.*, whether faster or slower than the H.P.

For these reasons the results of the test are shown *graphically* by drawing a "**graph**" or "**curve**" as shown in Fig. 123 *et seq.* The two quantities are then said to be *correlated* by the graph, and from it the efficiency at *any* H.P. between the lowest and the highest

stated can be found, and from the shape of the curve we can say *how* the efficiency changes.

Besides being useful for testing purposes, curves are of great value in science and calculations. Many equations may be easily solved by their aid, and in some cases a graph is the *only* means of solving an equation. Again, when designing machines it is frequently necessary to calculate from a certain formula, and when the class of work demands its frequent use it is a great advantage to draw a curve for the formula; then when any value is required it is merely read from the curve, no calculation being necessary. In this case a graph is often called a **chart.**

Since a graph gives to the eye a much better understanding of the changes in a varying quantity than a table of figures, it is customary with quantities constantly changing to have graphs drawn automatically. Thus "recording ammeters" and "recording voltmeters" in electric stations show for any period of the day how much energy has been given out, and how the generators have been regulated. Waterworks are also provided with "recorders" which register the quantity of water being delivered at any time. The height of the barometer and the temperature, together with many other physical measurements, are automatically recorded in observatories. A further advantage in these cases is that, being automatic, practically no attention is required, whereas if figures were read at certain times there is a danger of certain readings being omitted or wrongly read. The common indicator diagram is an example of an automatically drawn graph giving the pressure in an engine cylinder at *any* point of the stroke.

Rectangular Co-ordinates.—The principle of the graph is as follows. **Each pair of figures** is represented by the **position of a point** upon a sheet of paper, the distance of the point being measured from two fixed lines at right angles to each other, such as the left-hand edge and the bottom of the sheet. These two lines are "lines of reference" and are known as the **axes**; the point at which they cross is called the **origin.** For each set of figures a "scale" has to be arranged in order that a quantity like horse power can be shown by a distance. Taking our case of the electric motor we might say that 1″ should represent 20 H.P., and that 1″ should represent 0·2 efficiency. The method of choosing these scales will be dealt with later.

Let us now take one particular pair of figures as an example, say the pair H.P. = 60, efficiency = ·86. Then if every 20 H.P. is to be shown by 1″, 60 H.P. will be shown by 3″. Also if every

·2 efficiency is to be shown by 1″, ·86 efficiency will be shown by 4·3″, since $\frac{\cdot 86}{\cdot 2} = 4{\cdot}3$. Thus this pair of figures will be shown by a point 3″ from one axis and 4·3″ from the other. These two dimensions are known as the **rectangular co-ordinates** of the point, and the position of the point on the paper is shown in Fig. 121. Continuing this process for each point, they can be "plotted" in the same manner as the one taken, and are shown as dots in Fig. 121. Evidently they are points on a regular curve.

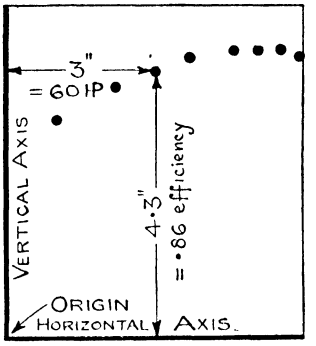


Fig. 121.—Rectangular Co-ordinates.

Either quantity may be measured horizontally or vertically, but certain arrangements are kept to in various kinds of work in order to obtain uniform plottings. In the particular case just taken the H.P. would be measured horizontally and the efficiency vertically, as shown. It is then said that "*the efficiency is plotted to a base of H.P.*," the "base" quantity being the one plotted horizontally.

Squared Paper.—It would obviously be very inconvenient if the rectangular co-ordinates of each point had to be calculated and

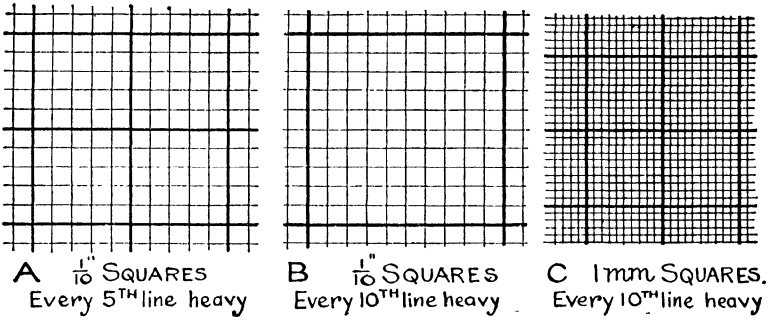


Fig. 122.—Types of Square Paper.

measured, as shown above. This is entirely avoided by employing **squared paper.** This paper is ruled all over both horizontally and vertically, with equi-distant parallel lines, thus dividing the surface up into a number of small squares. Several varieties may be obtained, but the two most useful ones are those where the lines

are $\frac{1}{10}''$ apart, and 1 millimetre apart. The lines are printed in blue or gray, these colours being easily seen without tiring the eye, but at the same time being fainter than black, and therefore not so prominent as to interfere with the appearance of the "curve" when drawn in black. Usually every fifth or tenth line is printed heavier than the others so that numbers of 10, 15, 20, etc., can be quickly and easily counted. Fig. 122 shows three different types of paper :—

A. lines $\frac{1}{10}''$ apart, every 5th line heavy
B. „ „ „ „ 10th „ „
C. „ 1 millimetre apart, every 10th line heavy

Type A will generally be found most useful, although type C is in considerable use. Squared paper can be obtained in various sized sheets or in exercise books, at slightly more than the cost of ordinary paper. Its use in exercise books is not recommended, as the fold in the book renders the drawing of the curves rather difficult; it is easier to draw on a flat sheet of paper and to stick this in the book finally.

The most common size of sheet is "foolscap" ($13'' \times 8''$), and the examples taken here will be arranged, so far as possible, to suit such a sheet. Allowing a small margin all round, the largest working size is about $12'' \times 7''$. The curves shown will not appear so distinctly as on the actual paper, as the division lines are of the same colour as the curves. Also, since the plottings cannot be shown full size, all the $\frac{1}{10}''$ lines cannot be drawn. The first example (Fig. 123) is shown half size in order that the style of work may be clearly seen. The heavy lines $1''$ apart are shown, together with every other $\frac{1}{10}''$ line, while the heavy $\frac{1}{2}''$ lines are indicated by the short lines standing out from the axes. The remaining examples are shown about one-fourth full size; only the heavy lines $\frac{1}{2}''$ apart being drawn.

Method of Plotting.—The method of plotting a curve from a table of given values, and the various points to be observed, will now be dealt with in the order in which the work should be done. The example will be the curve of efficiency and B.H.P., of which the figures are given on p. 347. The B.H.P. will be plotted horizontally and the efficiency vertically. The finished curve is shown half size in Fig. 123, and should be referred to continually in reading the following description. In all cases a student should endeavour to produce a neat and complete job; a graph always looking more

important and workmanlike when finished with neatly printed titles, figures, etc.

1. Choice of Scales.—This is decided by the condition that where possible the curve should fill the sheet; but it is very important that the scales should be such that the decimal system can be used with ease. Thus 1″ (*i. e.*, 10 squares) should equal 1, or 10, or 20, or 50 units, etc., and should not be used to represent 3 or 6 or 8 units. This condition should be obeyed even if it requires drawing a smaller curve or using a larger sheet of paper.

Taking our particular case, the maximum value of the H.P. to be plotted is 111, let us say 110. Taking 1″ to represent 10 H.P. we require 11″, which is just within the length of our working page and therefore quite satisfactory. Then, **scale of H.P. is 1″ = 10 H.P.** For the efficiency, the maximum value is ·905, say 1. Space will not permit of using 10″ to represent 1, so that we must employ 5″ = 1, *i. e.*, 1″ = ·2. There is nothing between which will give a decimal system. Then, **scale of efficiency is 1″ = ·2.**

2. Setting-out Scales.—The axes should now be drawn in ink, *each upon one of the heavy lines, and not less than* ½″ *from the edge of the paper.* This margin is necessary for titles, etc. If the curve is not likely to fill the sheet with a convenient scale, the margin may be increased to say 1½″. The lines may be fairly thick, and may, with advantage, be put in with the ruling pen and Indian ink. Both horizontal and vertical scales should be marked as follows:—Taking the horizontal scale, first mark the origin 0, and then mark every second heavy line (*i. e.*, every 1″) as 10, 20, 30 and so on to 110. The figures should be *printed,* and placed centrally under their particular heavy line, a little beneath the axis. Parallel and near to the axis the title of the scale should be neatly printed, describing the **kind of measurement,** and the **unit** in which it is measured (see Fig. 123, *et seq.*). Taking the vertical scale, the origin is numbered 0, and then each inch is marked ·1, ·2, ·3, and so on to 1·0, as shown. The title in this case is merely "efficiency."

3. Plotting the Points.—The points should first be plotted *in pencil* as follows. From the table of figures on p. 347, the first pair of figures is H.P. 10, efficiency ·6. Taking the base figure first, *i. e.*, the 10 H.P., look along the H.P. scale for the line marked 10. Follow this line upwards with the pencil until, glancing at the efficiency scale on the left-hand side, the heavy line marked ·6 is reached. Make a small dot at the crossing point of the two heavy lines, and draw a small ring round it in order that it may easily be found again.

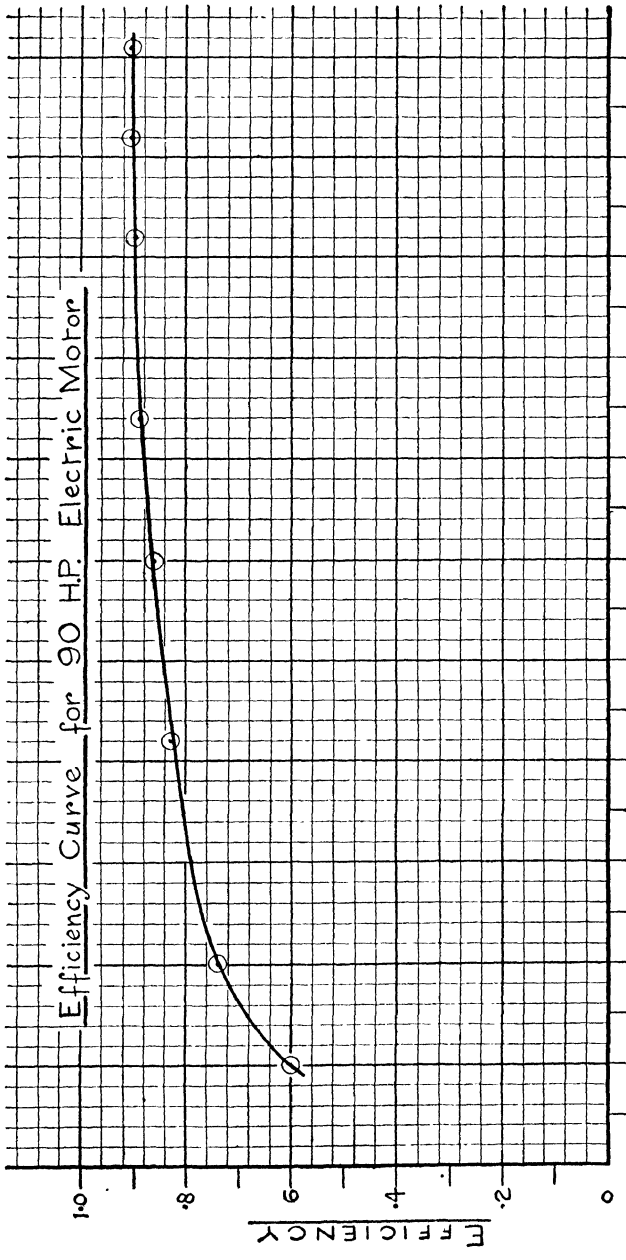


Fig. 123.—Example of a Finished Curve.

The next pair of figures is H.P. = 20, efficiency = ·74. Find the heavy line at 20 on the base scale and follow upwards with the pencil until the heavy line between ·6 and ·8 on the efficiency scale is reached, *i. e.*, ·7. Then since the distance between ·7 and ·8 is divided into five parts, each of the small squares represents ·2, and standing from the ·7 the light lines read ·72, ·74, ·76, and ·78 (this is shown full size at A, Fig. 124). The required point is then at the intersection of the second of these light lines (*i. e.*, ·74) and the vertical heavy line at 20. The third pair of figures is H.P. 42, efficiency ·83. Along the base, 42 is at the second fine line past the 40 (since 1″ = 10 H.P., then from 40 to 50 the lines read 41, 42, 43, and so on to 50). Following this upwards with the pencil, the figure ·83 is required on the efficiency scale. Now the first fine

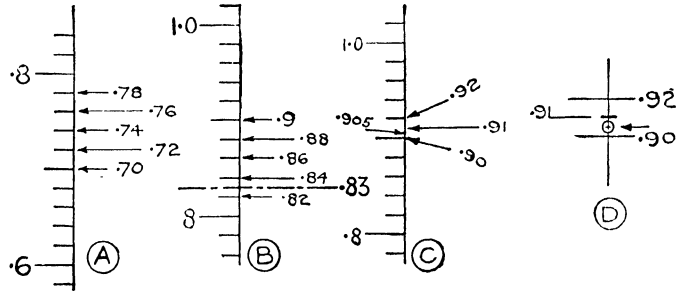


Fig. 124.—Estimation of Intermediate Readings.

line after the ·8 is ·82, and the next ·84. Hence ·83 is halfway between these, and a dot is made here on the 42 vertical line, as shown full size at B, Fig. 124. The remaining points are similar until the one H.P. 102, efficiency ·905, is reached. The position of ·905 is found as indicated at C, Fig. 124. From ·9 to ·92 is blank on the squared paper, and halfway between these points is ·91. Now ·905 is halfway between ·9 and ·91; hence ·905 lies one-quarter of a small square above ·9 as shown. When marking the actual dot the middle point between the ·9 and the ·92 lines should be estimated and marked; then the middle point between this and the ·9 line gives the point required, as shown twice full size at D, Fig. 124. Although this process is lengthy to describe it is quite easy to perform. A little practice is required in estimating the intermediate points, but if care be taken it is quite possible to estimate one-fourth or one-fifth of a $\frac{1}{10}$″ division.

Estimation of intermediate figures such as the above is only possible with a decimal scale. Thus if 1″ (*i. e.*, 10 squares) = ·2,

then one small square (*i. e.*, $\frac{1}{10}$") represents ·02 and a half of it ·01. Thus in this case any second decimal place can be accurately and quickly found. (With an odd scale such as $1\frac{1}{2}$", *i. e.*, 15 squares = ·2, one small square would be ·0133, and it would be practically impossible to find correctly any particular decimal figure that was not marked.)

When all the points have been plotted in pencil, their general appearance should be noticed. If they appear to lie on a fairly smooth curve, then the points may be marked in ink, but if any of them appear to be very much out of line with the others the plotting of these particular ones should be checked to see if a blunder has been made. When correct, *the points should be marked in ink.* This applies to all cases of tests and experiments. Only in a very few cases is it allowable to draw the graph without the points. The commonest method of marking the point finally is by a small fine dot and a ring round it, $\frac{1}{10}$" dia. or less, which may easily be put in by hand. When two or more curves appear on one sheet, and the points are liable to become mixed, it is desirable to use different methods of marking the points (three of which are shown in Fig. 135). Different curves on the same sheet can also be shown by differently coloured inks.

4. Drawing-in the Curve.—The curve should now be sketched in freehand with pencil. Sometimes all the points will be found to lie *exactly* on the curve as in Fig. 129 (top curve), where the values are calculated from a formula. But in plotting the results of experimental work the points seldom lie quite regularly on any curve. Instrument errors, errors in instrument reading and other circumstances, all affect the values to be plotted. The points will follow some *general direction*, and when sketching in the curve, it is this *general direction* which must be followed. The points must not be connected by a wavy line, but the curve sketched so as to lie *among* the points. As a general guide it may be taken that for every point on one side of the curve there should be one on the other side, and about the same distance from it. Where three or four points appear to be nicely in the path of the curve then, of course, the curve should be drawn right through them. In cases where the points do not follow an exact curve there may be a difference of opinion as to the best line to put among the points. Where answers are required to such examples they cannot be taken as being absolutely correct. Differences may be found in the third (and perhaps the second) significant figures. With a very few exceptions all plottings of natural or scientific observations will lie on a "smooth" curve, *i. e.*, one

without any sudden changes in direction. The most common exceptions are the temperature and the barometer readings.

For the final drawing of the curve some mechanical aid is desirable, such as an adjustable curve or, better still, a **French curve.** These are templates made usually in pear-wood and containing a large variety of curves; they can be purchased for a few pence. Several types may be seen in the instrument-maker's catalogue, but one which the author has found very useful is shown in Fig. 125. For graphs it is desirable that the curve should have a long portion, such as AB, only *very slightly* curved near the end. A portion of the French curve agreeing well with, say, half of the sketched curve should now be found, and the French curve moved about until its edge exactly " picks up " some portion of the sketched curve. Then carefully ink in this piece of the curve with a *fairly fine line,* using the *ruling pen.* Next move the French curve along and find another portion suitable for another piece of the curve and ink this in, taking care to make a *smooth* and *continuous* joint. Repeat until the curve is finished. Finally clean the pencilling off, and the finished curve will remain. With practice the French curve will be found quite easy to use.

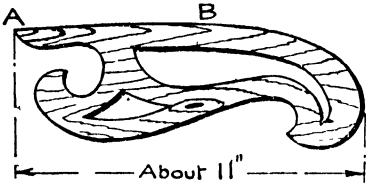


Fig. 125.—French Curve.

In all cases where any reading of values is required from the final curve, the line should be made fairly fine, but where only the general shape of the curve is required a fairly heavy line is advisable, as it stands out much better. Examples of this will be found in illustrations in technical books. For clearness the curves shown here are drawn thicker than is necessary.

5. Further Information Required on the Sheet.—This depends upon the style of work and the use to which the curve is to be put, but *in all cases the title of the curve should be written on the sheet.* In our case the title is " Efficiency Curve for 90 H.P. Electric Motor." For a drawing office job there would be some order number and a drawing number, with the name of the firm for whom the tests were made, the place of test and particulars of the type of motor, *e. g.,* " Shunt Wound, 500 volts at 400 revs. per min."

When two or more curves appear on one sheet, a *title must be placed on each curve,* as shown in Fig. 128, to distinguish between them, and to show which scale applies to each curve, for frequently

two vertical scales will be needed when two curves are drawn. In cases like this, the second curve is plotted *quite* separately from the first one. Any such information which applies to a particular curve should be neatly printed parallel to the curve.

Interpolation.—This is the process of finding from the curve values between those given in the table. Thus supposing it is necessary to know the efficiency of the motor mentioned on p. 347, at 50 H.P. and at 85 H.P., the required values are "interpolated" in the following way. Taking the 50 H.P., glance along the H.P. scale (Fig. 123) until this value is reached. Follow the 50 H.P. line upwards until the point is reached where the curve crosses this vertical line. Now glance horizontally to the left and read off this position on the efficiency scale. If the intersection comes upon one of the $\frac{1}{10}$" lines the reading is easy, but if it is *between* two $\frac{1}{10}$" lines, then the exact reading must be estimated as when setting out the points on p. 353 (see Fig. 124). In the case we are considering, the 50 H.P. line intersects the curve on the second $\frac{1}{10}$" line above ·8, the reading, therefore, being ·84. Thus we have interpolated the efficiency at 50 H.P. to be ·84.

In the second case, at 85 H.P., the intersection of the curve and the vertical line is about one-fourth of a small square below the ·9 line, *i. e.*, ·005 below the ·9. The reading is therefore ·895.

Example 354.—The following table gives the safe working load in lbs. allowable on a certain ball thrust bearing 2" dia., at various speeds, from 50 to 1000 revs. per min. Plot a curve of load upon a base of speed, and interpolate a suitable working load at speeds of 130 revs. per min., and 750 revs. per min.

Speed, r.p.m. . . .	50	100	200	300	400	600	800	1000
Safe load, lbs. . . .	1100	780	550	440	390	310	275	240

Refer to Fig. 126.

Scales.—*Base.* Max. speed = 1000. Taking 1" = 100 r.p.m., the base will require to be 10", which is suitable.

Vertical.—Max. load = 1100 lbs. Taking 1" = 200 lbs., this scale will require to be $5\frac{1}{2}$" long, which is suitable.

Plotting.—This should present no difficulty. The points will all be found to lie exactly on the curve.

Interpolation.—At 130 r.p.m. the curve reads 680 lbs.; at 750 r.p.m. the curve reads 280 lbs.

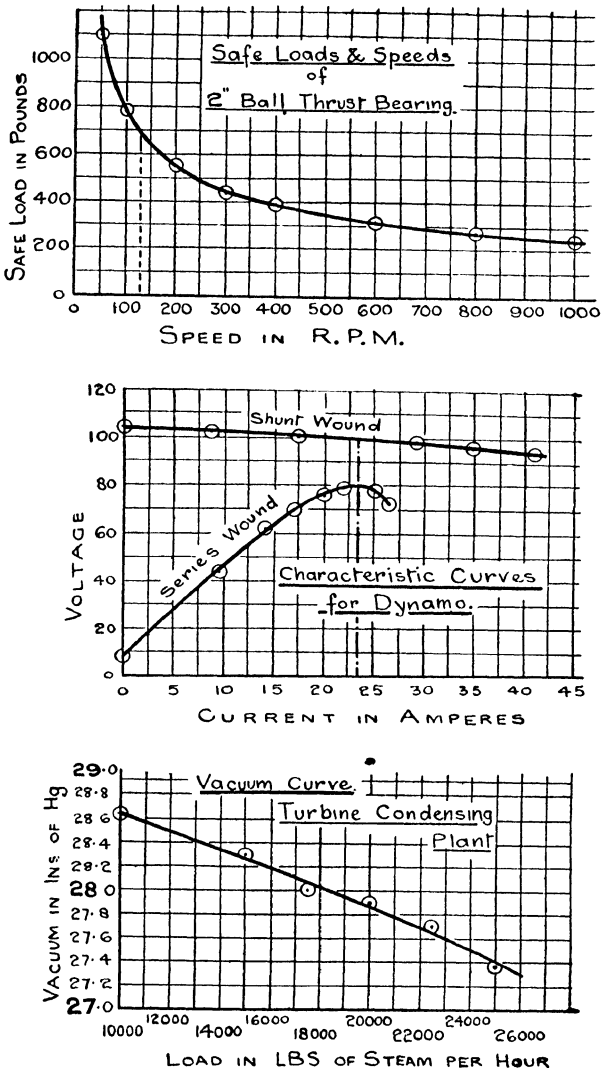


Fig. 126.—Curves to Examples 354, 355, and 356

Example 355.—An experimental dynamo could be arranged " series " or " shunt " wound. Tests were made with each kind of winding to determine the " characteristic " curves, measurements of voltage and current being taken. The results are given in the tables below. Plot the curves (voltage on base of current) for the two cases, upon the same sheet, using the same scales.

I. SERIES WOUND.

Current, amps. .	0	9·5	14·25	17	20	22	25·25	26·5
Voltage . . .	8	44	62	70	76	79	77·5	72·5

II. SHUNT WOUND.

Current, amps. .	0	8·75	17·5	29·25	34·75	41
Voltage . . .	105	103	101	99	96	94

Find from the curves the following information—

(*a*) The greatest voltage for the series machine.

(*b*) The current at this voltage for the series machine.

(*c*) The voltage of the shunt machine for the current in (*b*).

Refer to Fig. 126.

Scales.—Since both curves are to be plotted to the same scale and on the same sheet, the scales must be suitable for the largest values in either table.

Base.—The greatest current is 41 amps. (say 40) in the shunt-wound case. Taking 1″ = 5 amps., a base of 8″ is required.

Vertical.—The greatest voltage is 105, in the shunt-wound case. A suitable scale is 1″ = 20 volts, giving a height of 5″.

Plotting.—As the curves are quite separate, and the points of each very far apart, there is no need here to use two types of point. On the horizontal scale figures such as 29·25 can be exactly obtained as $\frac{1}{10}$″ = ·5 amps.; hence ·25 is shown by half of a small square.

On the vertical scale $\frac{1}{10}$″ = 2 volts, hence to show ·5 for the last two points of the series case, one-fourth of a small square must be estimated.

The Curves.—In the series case all the points seem to lie well on the curve. In the shunt case the curve passes slightly above the third point and slightly below the fourth, going right through the others. Remember that the two curves must be labelled " series " and " shunt."

Interpolation.—(*a*) On the series curve the highest point reached (*i.e.*, the greatest voltage) is 80 volts.

(*b*) The current at this max. point is 23½ amps.

(*c*) Following the vertical at 23½ amps. up to the shunt curve, the voltage is exactly 100.

Exercises 84. Cases Requiring the Origin.

[*Note.*—Answers can only be given where values have to be interpolated or equations found. A suitable scale for some of the exercises will be found in the answers.]

1. Some values of wind pressure upon a sloping roof are given in the following table. Plot a curve showing wind pressure on a base of angle, and read off the pressure for the following angles : (*a*) 38°, (*b*) 52°.

Angle of roof, degrees	0	10	20	30	40	45	50	55	60
Wind pressure, lbs. per sq. ft.	0	12	23	33	42	45	47·5	49·5	50·5

2. Plot a curve of temperature upon time from the following figures, which were obtained in a test on the heating of a small electric motor field-coil.

Time (mins.)	0	2	4	6	10	12	18	24	30	35	40
Temp. °C .	21·5	23·5	26	30	35	36·75	40	43	45	47	47

3. The following figures give the voltage drop across the carbon brushes of a direct-current motor, at varying currents. Plot a curve showing drop on a base of current. Read off the voltage drop for the following currents : (*a*) 20 amperes, (*b*) 40 amperes.

Drop, volts. . .	·35	·65	·88	1·3	1·5	1·75	1·8	1·85
Current, amps. .	4	9	13·5	21·5	27·5	37·5	42	47·5

4. The weights of a particular exhaust valve are given below, for various sizes. Plot a curve showing weight on a base of bore, and estimate the probable weight of the following sized valves : (*a*) 8″, (*b*) 16″, (*c*) 22″.

Bore of valve in ins. . .	6	10	12	15	18	20	24
Weight in lbs.	400	690	840	1300	2000	2300	3500

5. The following table gives the prices of certain expansion joints for steam-pipes. Plot a curve showing price on a base of diameter, and estimate the price of the following sizes : (*a*) $2\frac{1}{2}$″, (*b*) $3\frac{1}{2}$″, (*c*) 7″.

Diameter of pipe, ins. .	2	3	4	6	8	10
Price in shillings . . .	46	60	68	85	124	185

6. The following figures were obtained in tests on a dynamo arranged as shunt, series, or compound wound. Plot a chart showing volts on a base of current for each machine. using the same scale for each.

SERIES WOUND.

Voltage.	Current in Amperes.
5	0
16	19
25	35
36	61
45	85

SHUNT WOUND.

Voltage.	Current in Amperes.
83	0
80	19·5
77	34·5
75	46·5
70	71

COMPOUND.

Voltage.	Current in Amperes.
81	0
84	13
85	23·5
86	38
86	48

7. The following figures give the resistance to motion of an engine and train, in lbs. for every ton of total weight. Plot a curve showing train resistance on a base of speed. With the aid of the curve find the total resistance of a train whose complete weight is 285 tons and whose speed is 35 miles per hour.

Speed, miles per hr. .	0	3	5	10	12·5	15	17·5	20	25	30	40	50	70
Resistance, lbs. per ton	12	7·5	6·5	5·4	5·26	5·2	5·32	5·5	6	6·8	9	11·8	19·7

8. The following figures give some results of crushing tests of mixtures of sand and Portland cement. Plot a curve of strength on base of proportion. Interpolate: (*a*) the probable strength of a mixture of $4\frac{1}{2}$ to 1; (*b*) the proportion required to give a strength of 1500 lbs. per sq. in.

Proportion of sand to 1 cement	1	2	3	4	5	7	8	10
Crushing strength in lbs. per sq. in.	6600	6000	5000	2900	2050	1250	1050	650

9. The following figures give the least sizes of safety valve permitted by the B.O.T. requirements for steam boilers. Construct a chart showing area of valve on a base of boiler pressure. Find from the chart the area required for pressures of 50 lbs., 180 lbs., and 90 lbs. per sq. in.

Boiler pressure, lbs. per sq. ins.	15	30	45	60	80	100	120	140	160	200
Area of valve in sq. ins. per sq. ft. of grate area	1·25	·833	·625	·5	·394	·326	·277	·241	·214	·174

10. The breaking strength of tempered steel wire of various diameters is given in the following table, the diameters being given by their wire gauge number. Plot a curve of strength upon a base of gauge number.

Diameter, S.W.G.	0	2	4	6	8	9	12	16	20
Breaking strength in lbs.	18460	13400	9470	6490	4500	3650	1900	720	230

Find from the curve the sizes of wire with the following breaking strengths: (*a*) 15,000 lbs.; (*b*) 5000 lbs.; (*c*) 350 lbs. (*Note.*—As only whole numbers exist in the wire gauge the next *smaller* whole number must be given in the answer.)

Cases where the Origin is not Required.—In the examples shown up to the present the figures to be plotted have included values from 0 upwards, and each scale, therefore, when marked off, has been stated at 0, *i. e.*, we have plotted from the origin. In the majority of cases this is necessary or at least advisable, but it is sometimes advisable to omit the origin on the sheet, as seen in the example which follows. Here in the case of the load the figures vary from 10,000 to 25,000, *i. e.*, there are no figures from 0 to 10,000. Similarly the vacuum varies from 27·35 to 28·65, *i. e.*, there are no figures from 0 to 27. Now if this curve were plotted from the origin the result would be as shown in Fig. 127, the curve lying in a very small portion of the paper and leaving the majority of the sheet blank. In addition to this, the plotting of the second decimal place given in the vacuum figures would be practically impossible with the small scale which would have to be adopted. Therefore those portions of the scales in which there are no figures to plot are omitted; that portion of the sheet lying between the chain dotted lines is stretched out to fill our foolscap sheet. The load scale will then start at 10,000 and the vacuum scale at 27.

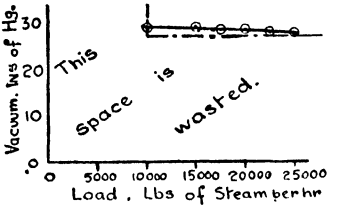


Fig. 127.

Example 356.—The following table gives the vacuum produced by the condensing plant of a steam-turbine installation. Plot a curve with the load as base.

Load in lbs. of steam per hour	10000	15000	17500	20000	22500	25000
Vacuum. Inches of mercury	28·65	28·3	28	27·9	27·65	27·35

The guarantee condition was 28″ at the full load of 18,000 lbs. Find from the curve the actual vacuum at 18,000 lbs.

Refer to Fig. 126.

Scales.—The origin will not be used on either scale.

Base Scale.—Start, say, at 10,000. Then the scale must accommodate from 10,000 to 25,000, *i.e.*, a distance of 15,000. A base of 15″ would be very suitable, giving 1″ = 1000 lbs., but is too large. The only convenient scale then is 1″ = 2000 lbs., giving a base of 7½″.

Vertical Scale.—Start, say, at 27 and go to 29, *i.e.*, a difference of 2. Taking 2½″ for 1, the total height will be 5″, which is suitable. The scale is then ½″ = ·2; and since ½″ is divided into five small squares, every small square is ·04, and the second decimal place (·01) can be plotted if a quarter small square be estimated.

Plotting.—The points follow a very flat curve, which might even be taken as straight, except that the general direction of the points suggests a curve; none of the points appears to be exactly on the curve.

Interpolation.—At 18,000 lbs. the curve reads exactly 28″, so that the guarantee condition is satisfied.

Example 357.—Experiments were made upon three different types of electric glow lamp to determine their resistance at various voltages. The results are given in the tables below. Plot upon the same sheet, and to the same scales, a curve of resistance with voltage as base for each lamp. State, from an inspection of the curves, how the resistance varies with increase of voltage, in each case.

I. Carbon Lamp.

Voltage .	75	80	84·5	90	93·5	96
Resistance in ohms	93·7	93	91·9	90	88·6	88

II. Tungsten Lamp.

Voltage .	70	75	80	85	90	95	98	100
Resistance in ohms	254	259	267	274	283	290	297	300

III. Tantalum Lamp.

Voltage .	78·5	84	90	94	96	100
Resistance in ohms	392	400	428	443	453	459

Refer to Fig. 128.

Scales.—The lowest voltage tabulated is 70 and the highest 100, *i.e.*, only 30 difference. The origin of the voltage scale, therefore, will not be shown. The lowest resistance is 88 and the highest 459, giving

a considerable difference. In this case, then, the origin may be shown. Thus, the resistance scale will start at 0, and the voltage scale at 70.

For this example it will be found advisable to use the foolscap sheet of squared paper with its *long side vertical*, as this arrangement enables better scales to be used.

Base Scale.—Requires from 70 to 100, *i.e.*, 30 difference. Taking 1″ to 5 volts, a base of 6″ is required, which is suitable for the short side of the paper.

Vertical Scale.—Required to take 460. Allowing 50 ohms per inch, a total height of 9″ is required and suits the long side of the paper.

Plotting.—In the case of the tantalum and tungsten lamps the plotting is not difficult. As 1″ = 50 ohms, every small square is 5 ohms, and to plot to 1 ohm, one-fifth of a small square must be estimated. For the carbon lamp the decimals will have to be neglected. As one-fifth of a small square is the smallest that can be reasonably estimated, then 1 ohm is the least that can be plotted.

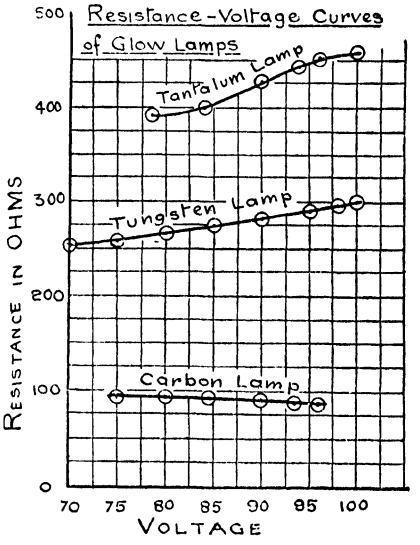


Fig. 128.—Curves to Example 357.

Therefore 93·7 must be called 94, and so on. Only one style of point need be used, as the curves are some distance apart.

Deduction.—In the tantalum and tungsten lamps the resistance evidently *increases* with increase of voltage. In the carbon lamps the resistance *decreases slightly* with an increase of voltage.

Exercises 85. Origin not Required.

1. Plot a curve of Indicated Horse Power on a base of speed from the following figures, the result of a trial of a Diesel-engined vessel. Interpolate the H.P. required at 10 knots.

Speed in knots .	7·7	9·0	9·65	10·8	11·7
I.H.P.	800	1200	1440	2200	2600

2. In an experiment to determine the effect of salt in water upon the boiling-point, the following figures were obtained. Plot a curve showing boiling-point upon a base of salt percentage. Estimate (*a*) the

boiling-point if there is 20% in solution; (*b*) the percentage of salt required to produce a boiling-point of 107° C.

Percentage of salt in solution	0	3·2	6·3	9·4	12·2	17·9	22·9	27·7	31·9
Boiling - point in °C.	100	100·5	101·2	102	102·5	104·5	106	107·8	108·8

3. The following figures give the horse power taken by the circulating and feed pumps of a small condensing steam-engine at various speeds. Plot, on a base of speed, a curve for each pump, using the same scales.

Speed. Revs. per min. .	40	45	50	55	60	65
H.P. of circulating pump	10·5	15·5	18	27	41·5	47
H.P. of feed-pump . .	4	5·5	6·5	7·5	9	15

4. The table below gives the specific resistance of the acid in accumulator batteries for different specific gravities. Plot a curve showing resistance on a base of specific gravity. Read off the specific gravity at which the resistance is least and also the resistance at that point.

Specific gravity .	1·04	1·06	1·09	1·1	1·15	1·2	1·25	1·3	1·35	1·4
Specific resistance	2·5	2	1·9	1·5	1·3	1·3	1·4	1·7	2·15	2·9

5. Friction tests on a certain bearing gave the following results, the speed being constant. Plot a curve with friction on a base of temperatures. Read off the friction when the temperature is 85°.

Temperature F. . .	60	70	80	90	100	110	120
Coefficient of friction	·008	·008	·0063	·0052	·0045	·0039	·0035

6. The following figures are from a turbo-electric plant, the load being constant while the vacuum varies. Plot a curve of steam consumption on a base of vacuum.

Vacuum in ins. of mercury	0	14	15·3	19·3	22	25·3	26·6
Steam consumption, lbs. per K.W. hour	34·3	31·2	29·4	27·4	25·6	24	23·2

Note.—As the measurement of large quantities of steam is troublesome, the points will appear rather irregular. Judgment only can settle the best curve through the points.)

7. The voltages obtained from an accumulator with varying strength of acids are given below. Plot a curve of voltage on a base of specific gravity, and interpolate the voltage when the specific gravity is 1·1.

Specific gravity of acid	1	1·006	1·015	1·025	1·038	1·08	1·15	1·2	1·25
Voltage . . .	1·5	1·7	1·8	1·85	1·88	1·94	2·0	2·03	2·07

8. The weight of 1 cu. ft. of water at various temperatures is given below. Plot a curve showing weight on a base of temperature. Interpolate the weight of 1 cu. ft. at temperatures of (*a*) 180°; (*b*) 320°.

Temperature F. . . .	32	39	46	80	100	120	140
Weight of 1 cu. ft., lbs.	62·42	62·425	62·42	62·23	62·02	61·71	61·38

160	200	220	260	300	400
61	60·08	59·58	58·46	57·26	53·63

9. The following table gives the safe currents that may be carried by the usual sizes of iron wires employed for starting resistances. Plot a curve of current on a base of diameter. What sized wires will be required to carry the following currents: (*a*) 10 amperes; (*b*) 25 amperes? (*Note.*—Take the next *smaller* whole number in each case.)

Diameter in S.W.G. .	12	14	16	18	20	22
Safe current in amps.	51	30	20	11·5	6·7	4·5

10. A certain water-supply problem was solved by plotting, the figures being as follows. Plot two curves to the same scales upon a base of *h*, (1) for *i*; (2) for I. Read off very carefully the value of *h* at which the curves cross each other.

h	80	90	100	110	120	130
i	·2343	·238	·242	·245	·2485	·252
I	·2826	·2706	·258	·2444	·2291	·2116

The Straight Line.—The name "curve" is applied to any graph, even though the points may lie upon an exact straight line. Probably none of the various curves met with in mathematics are more important than the straight line. In all cases where the relation or equation connecting two quantities is to be found from experimental results the straight line must be finally used.

In drawing a straight line through the plotted points a ruler or set-square may be laid down and a trial line drawn in pencil. It will be found easier, however, to space the line evenly among the points, by using a piece of cotton or thread. Stretch the cotton between thumb and forefinger of each hand. Laying this taut thread through the points, it can be seen at once how they are balanced on either side. When satisfactory it should be noticed if the thread passes through any two of the plotted points a fair distance apart, or any prominent intersection of the squares, and by such guides the line can be drawn.

Example 358.—The Fahrenheit and Centigrade temperatures are connected by the equation $F = \frac{9}{5}C + 32$. Calculate values of F for the following values of C: 0, 40, 80, 120, 150. Construct a "temperature conversion chart" to convert between Fahrenheit and Centigrade readings, and to read from 0° C. to 150° C.

The calculation is best carried out in tabular form thus, the values to be plotted being found in the first and last columns—

°C.	$\frac{9}{5}C + 32$.		°F.
0	$\frac{9}{5} \times 0 + 32$	$= 0 + 32$	32
40	$\frac{9 \times 40}{5} + 32$	$= 72 + 32$	104
80	$\frac{9 \times 80}{5} + 32$	$= 144 + 32$	176
120	$\frac{9 \times 120}{5} + 32$	$= 216 + 32$	248
150	$\frac{9 \times 150}{5} + 32$	$= 270 + 32$	302

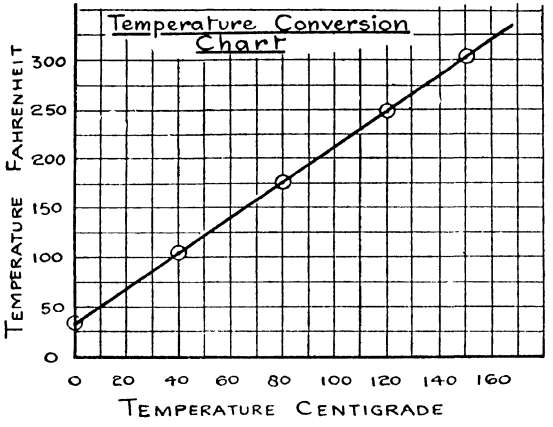
Refer to Fig. 129.

Scales.—Origin will be required.

Base.—Required to take 150. Allowing 1″ = 20°, a length of 7½″ is required, and is suitable.

Vertical.—Required to take 302, say 300. Allowing 1″ = 50°, a height of 6″ is required.

Plotting.—On the base scale the points are easily found. On the vertical scale, since 1″ = 50° every small square = 5°. Hence to plot to 1°, one-fifth of a small square must be estimated. All the points will be found to lie *exactly* on a straight line.



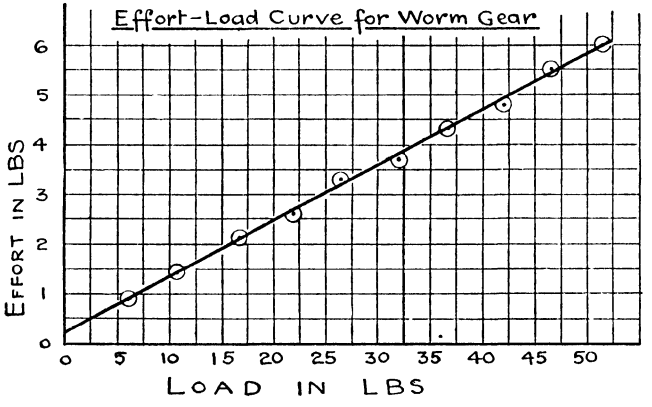


Fig. 129.—Curves to Examples 358 and 359.

Example 359.—The following table gives the corresponding values of effort and load in a small worm and worm-wheel lifting machine as determined by experiment. Plot a curve with load as base.

Load, lbs. .	6·0	10·7	16·7	22	26·5	32	36·7	42	46·5	51·5
Effort, lbs. .	·9	1·42	2·12	2·6	3·3	3·72	4·32	4·8	5·53	6·0

Refer to Fig. 129.

Scales.—The origin will be required.

Base.—Required to take 51·5, say 50. Taking 1″ = 5 lbs., a base of 10″ will be needed, and is suitable.

Vertical.—Required to take 6 lbs. Taking 1″ = 1 lb., a height of 6″ will be needed, and is suitable.

Plotting.—Since 1″ = 5 lbs. on the base scale every small square = ·5 lb., and the first decimal place can be plotted by estimating one-fifth of a small square.

On the vertical scale 1″ = 10 lbs. and each small square = 1 lb. As we cannot with accuracy estimate finer than one-fifth of a small square, one-fifth lb. (*i. e.*, ·2 lb.) is the smallest amount we can plot. The second decimal place must therefore be neglected.

The Curve.—It will be seen that the points generally follow a straight line. A trial with thread as described shows that the line will be suitably balanced if drawn through the two extreme points.

Exercises on plotting of straight lines will be found in Exercises 87, when finding laws.

The Plotting of Negative Values.—In the previous examples only positive values have been taken for each of the two quantities plotted, as required by the majority of curves met with. There are, however, certain cases where the values to be plotted are negative, or perhaps both positive and negative. These are accommodated as follows :—Instead of drawing our axes at the side and bottom of the sheet of paper, they are placed with the origin in the centre of the sheet and drawn right across, thus dividing the surface into four **quadrants** as shown in Fig. 130. Then on the vertical scale all + **values** are plotted from the origin **upwards,** and all − **values downwards.** On the horizontal scale all + **values** are plotted from the origin **to the right,** and all − **values to the left.**

Let us call the quantity plotted along the base x, and the quantity plotted vertically y. Then the four quadrants will take the various signs as follows :—

Right-hand top quadrant	x +	**and**	y +
Left-hand top	„	x −	„	y +
Left-hand bottom	„	x −	„	y −
Right-hand bottom	„	x +	„	y −

Taken in this order, *i. e.*, starting from the right-hand top quadrant, and going round " against the clock " the quadrants are numbered

for reference as 1st, 2nd, 3rd, and 4th (Fig. 130). The first quadrant where both quantities are positive is the one used up to the present. Evidently if our table of figures contains no negative values of any kind, only the right-hand top quadrant is needed, and is then arranged to fill the paper. The remaining three quadrants are more used in higher mathematics.

If necessary to specify any one point, it may be done by writing down the two co-ordinates with their signs, stating *the x or base value first.* Thus a point whose distance along the base is + 3, or 3 to the right, and whose vertical distance is − 2, or 2 below the origin, would be stated as (3, − 2). The following example is merely to illustrate the above.

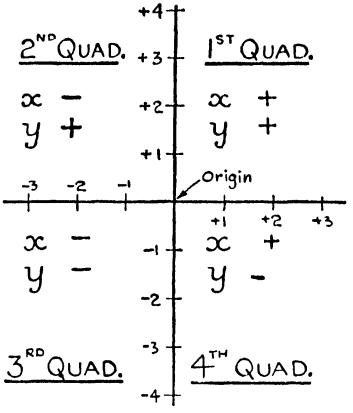


Fig. 130.—Complete Axes for Plotting.

Example 360.—Plot the following points and letter each one as stated: A (3, 4); B (− 2, 3·5); C (− 1·5, − 3); D (1·5, − 5·5), the figures being inches. As a check on the plotting, join up the points in the order given, and measure the four sides of the quadrilateral so formed, viz. the lines AB, BC, CD, DA. The sheet of paper is to be used with the long side vertical.

Refer to Fig. 131

The axes must first be drawn in the centre of the sheet.

Plotting.—*Point A.* Co-ordinates (3, 4). Plus sign understood. Therefore the point is in the 1st quadrant, 3 horizontally and 4 vertically.

Point B. Co-ordinates (− 2, 3·5). The horizontal distance is negative and the vertical distance positive. Thus the point lies in the 2nd quadrant, 2 horizontally to the left and 3·5 vertically upwards.

Point C. Co-ordinates (− 1·5, − 3). Since both are negative the point is in the 3rd quadrant, 1·5 to the left and 3 downwards.

Point D. Co-ordinates (1·5, − 5·5). The horizontal distance being positive and the vertical negative, the point is in the 4th quadrant 1·5 to the right and 5·5 downwards.

In Fig. 131 the lines to be measured as a check are shown by a

chain-dotted line, and are measured as follows : **AB = 5·02″, BC = 6·52″ CD = 3·9″, DA = 9·62″.**

The following example requires the axes to be in the centre of the sheet, since the figures to be plotted require equal space in all four quadrants :—

Example 361.—The following figures are taken from the results of an experiment on a sample of cast steel, to determine its curve of

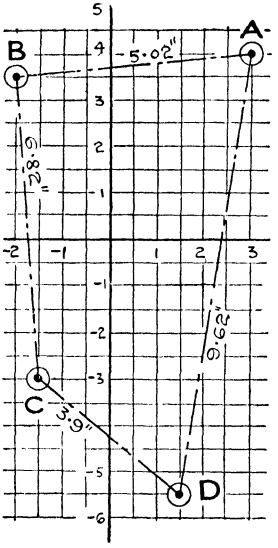


Fig. 131.—Curve to Example 360.

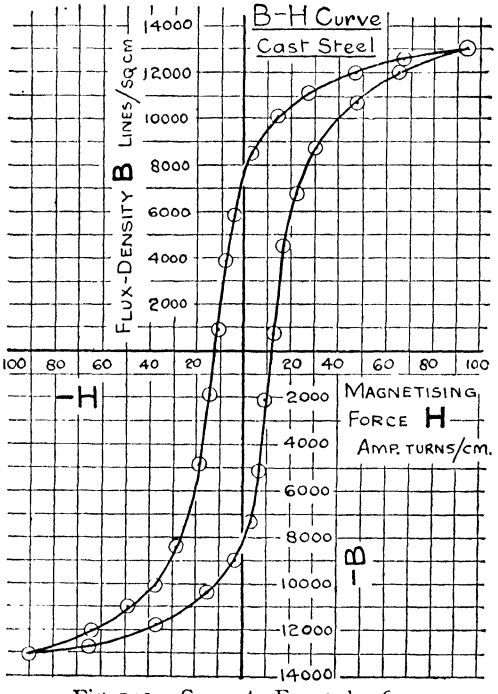


Fig. 132.—Curve to Example 361.

magnetisation (known as the " B-H " curve), which indicates by its shape, etc., the suitability of the material for magnet cores, etc., in electrical work. (*a*) Plot the curve, with the magnetising force H as base; (*b*) read off the values of B where the curve crosses the vertical axis, and, neglecting the minus sign, take the average of these two values (known as the residual magnetism); (*c*) read off the values of H, where the curve crosses the horizontal axis, and, neglecting the minus sign, take the average of these two values (known as the coercive force).

H. Magnetising force in ampere turns per cm.	94	67·5	47	27	14
B. Flux density in lines per sq. cm.	13050	12600	12000	11130	10100

H	3	−4	−7·5	−10·5	−14	−18·5	−28	−37·5
B	8520	5830	3900	960	−1920	−4840	−8400	−10100

H	−49	−64	−90	−65	−37	−15	−3
B	−11000	−12000	−13000	−12700	−11800	−10400	−9000

H	+3	+6·5	+9	+13	+17	+22	30	48	65
B	−7300	−5100	−2100	+775	+4550	6800	8700	10700	12000

Refer to Fig. 132

Scales.—*Base.* The greatest positive value is 94, and the greatest negative value − 90. Then the total value to be accommodated is 184, say 180. Allow 1″ = 20, then 9″ is required. This is rather wider than our foolscap sheet, but the scale is so convenient that it is advisable to use a larger sheet.

Vertical. The greatest positive value is 13,050, and the greatest negative one is 13,000. Then the total height required is about 26,000. Allow 1″ = 2000, then 13″ will be required. This is again longer than our sheet, but the scale is very convenient and very suitable for the larger sheet.

Plotting.—The first six points lie wholly in the first quadrant, both values being positive. The next three have base value −, and vertical still +, *i. e.*, they lie in the second quadrant. The following 11 points have both values negative, and thus lie in the third quadrant. The next three points have base values + and vertical values −, and therefore lie in the fourth quadrant. Finally, the remaining points lie in the first quadrant again, and complete the plotting. **The curve** lies practically through all the points, and the two branches should be joined to the extreme point at each end, making a " closed " curve.

Interpolation.—(*b*) The curve crosses the vertical axis in two places: at + 7600 and at − 8300.

Neglecting the — sign the average $= \frac{7600 + 8300}{2} = 7950$

$\therefore$ Residual magnetism = 7950 lines per sq. cm.

(*c*) The curve crosses the horizontal axis in two places : at + 12 and at — 11.

Neglecting the — sign the average $= \frac{12 + 11}{2} = 11{\cdot}5$

$\therefore$ Coercive Force = 11·5 amp. turns per cm.

Example 362.—The following values were calculated when solving a certain difficult equation—

Value of t .	362	362·5	363	363·5	364
Value of y ·	·0033	·0028	·0013	— ·0012	— ·0045

Plot the curve with the values of t as base, and read off accurately the value of t where the curve crosses the base line.

Refer to Fig. 133

Scales.—The origin will not be required on the *base*, as there are no figures up to 362, and the highest is only 364. Hence we start at, say, 361, which will keep the curve away from the vertical axis. Then from 361 to 364 is 3. Allowing 1″ = ·5, a base of 6″ is required, and is suitable.

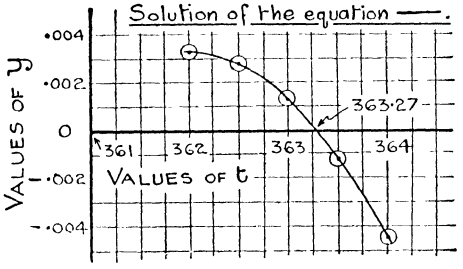


Fig. 133.—Curve to Example 362.

Vertically the largest + value is ·0033 and the largest — value is ·0045, giving a total depth of ·0078, say ·008. Allowing $\frac{1}{2}$″ = ·001, a total depth of 4″ is required. This is suitable, although by no means filling up our sheet. As only + values occur in the base scale, the second and third quadrants are not required. The vertical axis may then be placed towards the left-hand edge of the sheet, with the horizontal axis about halfway up as shown in Fig. 133.

Plotting.—As the scale is a very open one, the values can be plotted very exactly.

Interpolation.—The curve crosses the horizontal axis at 363·27.

Exercises 86. Plotting of Negative Values, etc.

Plot the following points, and as a check measure (in inches) the distances named. (Take a scale of $\frac{1}{2}'' = 1$ on both axes.)

1. A (5, − 1), B (− 1,5), C (2,1). Measure AB, BC, and CA.

2. A (− 4,3·5), B (− 4, − 3·5), C (·5, − 1), D (·5,1). Measure AC and BD.

3. In solving a difficult equation in connection with the steam engine the following figures were obtained. Plot y on a base of r, and find the value of r at which the curve cuts the horizontal axis.

Values of r .	5	6	7	8	9	10
Values of y .	·67	·45	·23	− ·03	− ·30	− ·59

4. The following figures were calculated when solving a difficult hydraulic equation by plotting. Plot the figures (with d as base) and read off the value of d at which the curve cuts the horizontal axis.

Values of d .	1·2	1·4	1·5	1·6	1·7
Values of y .	− 1·142	− ·39	·115	·705	1·385

5. In connection with the flow of water in pipes it was necessary to solve a certain equation by plotting. Plot the figures given below (with θ as base) and read off the value of θ where the curve cuts the horizontal axis.

Angle θ in degrees	298	303·5	309	315	321
Values of y . . .	2·173	·912	− ·29	− 1·435	−2·499

(*Note.*—The origin will not be required on the θ scale.)

6. The following figures are for a B-H curve of a sample of armature iron. Only the half curve above the horizontal axis is given. Plot the half curve and read off the intersection on the vertical axis.

B	0	1000	2500	5000	6000	7500	7000	6000	5000	3000	1000	0
H	1·55	1·7	2 0	2 6	3·1	4·0	2·3	65	− ·25	− 1·2	− 1·55	− 1·7

7. Plot the following figures with values of x as base. Interpolate the following points—

(*a*) The values of y where the curve cuts the vertical axis.
(*b*) The values of x where the curve cuts the horizontal axis.
(*c*) The smallest value of x.

Values of x	3	1·3	− ·4	− 1·4	− 1·7	− 1·77	− 1·5	− ·75	·3	2·25
Values of y	1·7	1·2	·5	− ·15	− ·5	− 1·25	− 1·67	− 2·2	− 2·73	− 3·4

8. Plot the following figures with x as base, and read off the three values of x where the curve cuts the horizontal axis.

Values of x .	-4	$-3{\cdot}5$	-3	$-2{\cdot}5$	-2	-1	$-{\cdot}4$	${\cdot}1$
Values of y .	-8	$5{\cdot}62$	14	18	18	10	$1{\cdot}94$	$-5{\cdot}5$

1	$1{\cdot}8$	$2{\cdot}3$	3	$3{\cdot}8$	$4{\cdot}1$
-18	$-25{\cdot}2$	$-26{\cdot}3$	-22	$-6{\cdot}1$	$3{\cdot}4$

9. Below are given some figures of the pressure at various temperatures of the two refrigerating agents in common use (sulphur dioxide, SO_2 and Ammonia, NH_3). Plot upon the same sheet and to the same scales, a curve showing pressure on a base of temperature for each agent.

Temp. F.		-40	-30	-20	-10	0	20	40	60	70
Press. lbs. per sq. in.	SO_2	$3{\cdot}1$	$4{\cdot}4$	6	8	$10{\cdot}4$	$17{\cdot}3$	$27{\cdot}3$	$41{\cdot}5$	50
	NH_3	$9{\cdot}2$	13	$17{\cdot}5$	23	$29{\cdot}5$	47	73	107	128

The Equation to a Straight Line.—In many cases after an experiment has been made, and the values plotted, it is necessary to find out some equation or formula connecting the two quantities plotted. This enables us to understand the relationship better, and reduces the result of the experiment to a simple statement which any one can use without having to obtain a graph. This equation is spoken of as the **law of the curve** or the **equation to the curve.** Only the case of the straight line will be taken here, but equations can be obtained to any curve.

Consider the straight line curve AD in Fig. 134. Draw a horizontal line Ad through the point A, where the curve cuts the vertical axis. Then any ordinate consists of two parts, a **constant piece** and a **variable piece.** Thus take any ordinate as bB. It is evidently equal to $bn + n$B.* Now it can be seen that the part bn is the same all along the curve, as the line Ad is parallel to the base. Let us call this the **1st constant.** The portion above the

* The expression bn here does not mean $b \times n$, but only the line from b to n in Fig. 134.

line A*d*, *e. g.*, *n*B, evidently changes with the position of the ordinate, *i. e.*, with the base value O*b*. Considering this variable piece only, it can be seen that the triangles such as AB*n* are always the same shape; in fact triangle AC*p* is only triangle AB*n* drawn to a bigger scale. These are known as similar triangles. Now, it is a principle in geometry that with triangles such as these "the ratio of the two perpendicular sides of each triangle is always the same." Thus if B*n* is half of A*n*, C*p* will be half of A*p*. (This the student should verify by measurement.) Thus the ratio "vertical side ÷ horizontal side" will be the same in any triangle, *e. g.*, AB*n*, AC*p*, etc. In triangle AB*n*, the vertical side is B*n* and the horizontal side A*n*, and in triangle AC*p*, the vertical side is C*p* and the horizontal side A*p*.

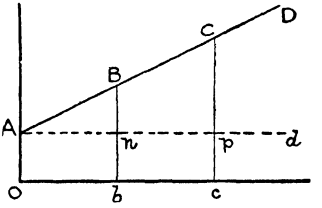


Fig. 134.

$$\text{Then } \frac{Bn}{An} = \frac{Cp}{Ap}, \text{ and so on for any other ordinates.}$$

Now the vertical side = variable piece of ordinate, and the horizontal side = base value. Then we have a ratio which is constant for our curve and which we can call the **2nd constant**, *i. e.*,—

$$\frac{\text{variable piece}}{\text{base value}} = \text{2nd constant}$$

$$\therefore \text{ variable piece} = \text{2nd constant} \times \text{base value}$$

Now, from some lines back,

$$\begin{aligned}\text{Any ordinate} &= \text{1st constant} + \text{variable piece} \\ &= \text{1st constant} + \text{2nd constant} \times \text{base value}\end{aligned}$$

To put this into a shorter and more mathematical form—

Let y represent any ordinate; and x the corresponding base value
Let a represent the 1st constant; and b the 2nd constant

Then writing these symbols in place of the words in our previous statement, we have

$$\mathbf{y = a + bx}$$

This is the form of the equation to *any straight line*. The constants a and b, of course, vary for different cases, but are constant for any one curve. The signs of a and b may be + or −, depending on the position of the line.

The following examples are intended to show how the values and signs of the constants a and b affect the line. When plotting

straight lines from equations such as the following there is no need to calculate and plot many points. It can be seen from Ex. 363 that such calculated points lie *exactly* on the line. To fix the position of a straight line only two points are required, the line being merely drawn through them with a straight-edge. But then any blunder in calculation would not be shown by the plotting. Thus it is necessary to *plot three points*, say two extreme ones and another approximately in the centre of the line. Then *all three should lie exactly on the straight line*, otherwise there is an error.

Example 363.—Plot on the same sheet the curves showing the three following equations, between the limits $x = -2$ and $x = +2$.

(*a*) $y = 1 + 1{\cdot}5x$; (*b*) $y = 1{\cdot}5x$; (*c*) $y = -2 + 1{\cdot}5x$.

Calculations:—

(*a*)

x	$1 + 1{\cdot}5x$	$= y$
-2	$1 + -3$	$= -2$
$+{\cdot}5$	$1 + {\cdot}75$	$= 1{\cdot}75$
$+2$	$1 + 3$	$= 4$

(*b*)

x	$1{\cdot}5x = y$
-2	-3
$+{\cdot}5$	${\cdot}75$
$+2$	3

(*c*)

x	$-2 + 1{\cdot}5x$	$= y$
-2	$-2 + -3$	-5
$+{\cdot}5$	$-2 + {\cdot}75$	$-1{\cdot}25$
$+2$	$-2 + 3$	1

The curves are shown plotted in Fig. 135.

The following points should be noted from the above plottings: **When the sign of the constant "a" is +, the line cuts the vertical axis above the origin.** See curve (*a*).

When the constant *a* is 0 (*i. e.*, when there is no constant *a*), the line passes through the origin. See curve (*b*).

When the sign of "a" is −, the line cuts the vertical axis below the origin. See curve (*c*).

Naturally the distance from the origin is given by the value of the constant *a*.

Example 364.—Plot on the same sheet the curves showing the following equations between the limits $x = -2$ and $x = +2$.

(*a*) $y = 2 + 3x$; (*b*) $y = 2 + {\cdot}5x$; (*c*) $y = 2 - 2x$.

Calculations:—

(*a*)

x	$3x + 2$	$= y$
-2	$-6 + 2$	-4
5	$1{\cdot}5 + 2$	$3{\cdot}5$
2	$6 + 2$	8

(*b*)

x	${\cdot}5x + 2$	$= y$
-2	$-1 + 2$	1
${\cdot}5$	${\cdot}25 + 2$	$2{\cdot}25$
2	$1 + 2$	3

(*c*)

x	$2 - 2x$	$= y$
-2	$2 - (-4)$	6
${\cdot}5$	$2 - 1$	1
2	$2 - 4$	-2

The curves are shown plotted in Fig. 136.

The following points should be noted from these plottings. When the sign of the constant b is $+$, the line slopes from the left-hand bottom corner to the right-hand top corner. This can be called a " positive " slope. When the sign is $-$, the line slopes from the left-hand top to the right-hand bottom corner. This can be called a " negative " slope.

The greater the value of the constant b, the greater is the amount of slope. Should the constant have for its value 0, then the slope

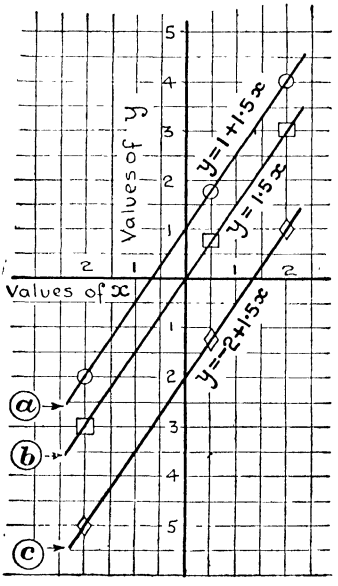


Fig. 135.—Curves to Example 363.

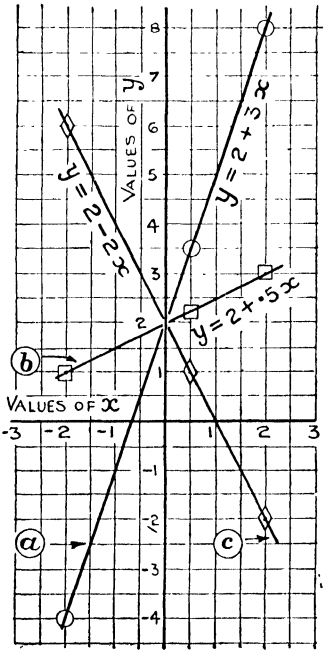


Fig. 136.—Curves to Example 364

is 0—*i. e.*, the line is horizontal—and its equation is then $y = a$, the a being the intercept on the y axis. The value of the constant b is called the **slope** of the line, and is the ratio " variable piece ÷ base value," as already stated. Thus, taking the ordinate at $+2$, on curve (a) in Fig. 136, the variable piece is 6, and the base value 2. Then, constant $b = \frac{6}{2} = 3$, which, of course, appears in the given equation.

To obtain the Equation to a Straight Line.—This easily follows from the previous section. In all cases the curve must be

plotted from the origin, if the law is to be found. If necessary, the line should be continued to cut the vertical axis.

The reading at which the line cuts the vertical axis is the value of the constant "a." The sign must, of course, be included when reading. Thus in Fig. 137 the line cuts the vertical axis at 1·57.

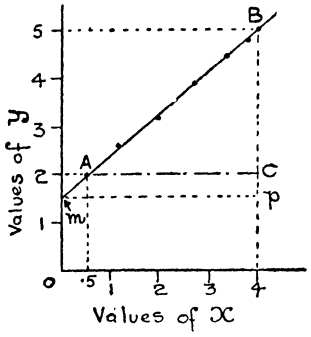


Fig. 137.

To find constant b glance along the curve near its extremities, and find a point somewhere near each end, where the co-ordinates are definite figures (*i. e.*, the reading does not have to be estimated). Thus in Fig. 137, at point A the line has the co-ordinates (·5, 2) and at B (4, 5). From these two points draw horizontal and vertical lines to cut, as shown, at the point C. Note the length of the vertical BC, on the vertical scale; in this case 3 (*i. e.*, 5 − 2): also the length of the horizontal line AC on the base scale; in this case 3·5 (*i. e.*, 4 − ·5).

Then, constant $b = \frac{\text{vertical}}{\text{horizontal}}$, and the sign is determined by the direction of the slope.

For the line in Fig. 137, $b = \frac{3}{3{\cdot}5} = {\cdot}857$, and the sign is + from the direction of the slope (see p. 377).
Then the equation is—

$$y = 1{\cdot}57 + {\cdot}857x$$

Any two points can be taken in estimating b, and it is therefore best to choose points of definite and easy co-ordinates, by which means it is often possible to see the value of b at a glance. The method of checking the equation is given on p. 379.

Obviously the whole variable piece of the ordinate at 4 (viz. Bp) and the whole base value mp (*i. e.*, 4) might have been taken; but as the point m seldom cuts the axis at some round figure, an estimation has to be made, which is longer and more liable to error. Since the triangles ABC and mBp are similar, the ratio "vertical side ÷ horizontal side" is the same in each case. Thus the vertical, Bp = 3·43, and the horizontal mp = 4, therefore the constant $b = \frac{3{\cdot}43}{4} = {\cdot}857$ as before.

Before plotting the straight line to determine the equation from any experiment, it must be decided which quantity is desired in

terms of the other, or in other words which is to be the "y" quantity, and which the "x." The quantity required in terms of the other is plotted vertically. Thus supposing figures were given of H.P. and steam used in an engineering test and it was desired to find the equation for *steam in terms of H.P.*, then steam would be plotted vertically (*i. e.*, the "y" quantity). In the examples given in this book it will be stated which quantity is required in terms of the other. In any case the equation can be transposed.

Example 365.—The following figures were obtained from a test on a small quadruple expansion steam engine.

B.H.P.	49	69	91	99	116	137	145
Steam used, lbs. per hr .	960	1190	1440	1550	1750	2000	2090

Determine the equation giving steam used in terms of the H.P., in the form $y = a + bx$. Check the equation by calculating from it the steam used per hour, when the B.H.P. is 36.

Refer to Fig. 138.

As steam is required in terms of B.H.P., plot steam vertically and B.H.P. horizontally. The scales and plotting should present no difficulty, and the points will be found to follow a straight line very well.

The Equation.—Let H = B.H.P. and W = lbs. of steam per hour.

Then, $W = a + bH$, where a and b are constants

Producing the line to cut the vertical axis, the intersection is at + 370, which is the value of a.

Two points have been selected near the ends of the line where the co-ordinates have definite values. These are: (124, 1840) and (20, 600). From these points vertical and horizontal lines have been drawn as shown.

The length of the vertical line is 1840 − 600 = 1240
The length of the horizontal line is 124 − 20 = 104

$$\text{Then constant } b = \frac{\text{vertical}}{\text{horizontal}} = \frac{1240}{104} = 11{\cdot}91$$

and the sign is +, since the slope is to the right and upwards. Then the equation required is

$$\mathbf{W = 370 + 11{\cdot}9H}$$

Check. when H = 36,

$$W = 370 + 11{\cdot}9H$$
$$= 370 + 11{\cdot}9 \times 36$$
$$= 370 + 428 = 798$$

For H = 36 the curve gives W = 800 barely, so our equation is reasonably correct.

Example 366.—The following table gives the diameters of the largest step on the cone pulley of some large lathes. Plot the curve, and assuming a straight line, find its equation, giving diameter of step in terms of height of centre.

Height of centres, ins. .	12	18	24	30	36	42	66
Dia. of largest step, ins. .	22	26	30	31	34	40	50

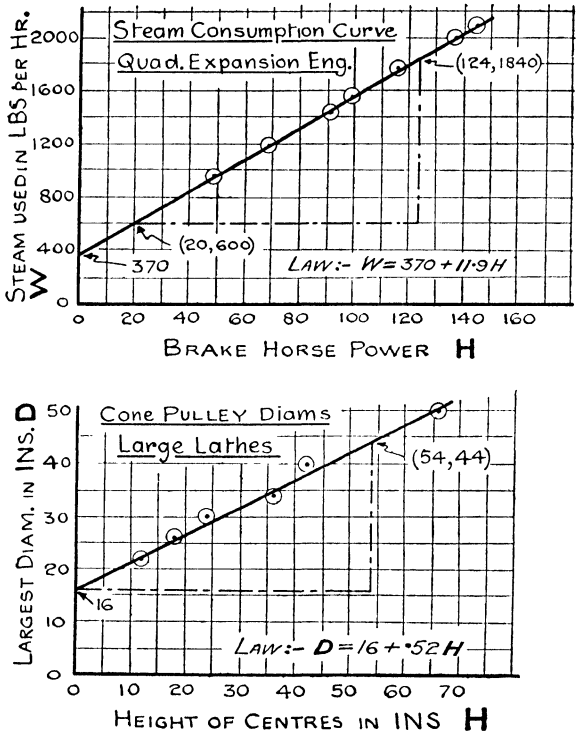


Fig. 138.—Curves to Examples 365 and 366.

Refer to Fig. 138.

To give diameter in terms of height of centre, diameter must be plotted vertically and height horizontally.

The scale and plotting are shown in Fig. 138. The points lie rather irregularly, but a straight line can be drawn to balance.

Equation.—Let H = height of centre in ins., and D = diameter of largest step in ins.

Then, $D = a + bH$, where a and b are constants

The line cuts the vertical axis at + 16, which is then the value of *a*.

This intersection is a good point to use for finding *b*; (54, 44) is taken at the other end. Then, the length of the vertical line is 44 − 16 = 28, and of the horizontal line 54 − 0 = 54.

$$\text{Then, constant } b = \frac{\text{vertical}}{\text{horizontal}} = \frac{28}{54} = \cdot 519, \text{ say, } \cdot 52$$

The slope being from left to right and upwards, the sign is +
Therefore the equation is,

$$D = 16 + \cdot 52H$$

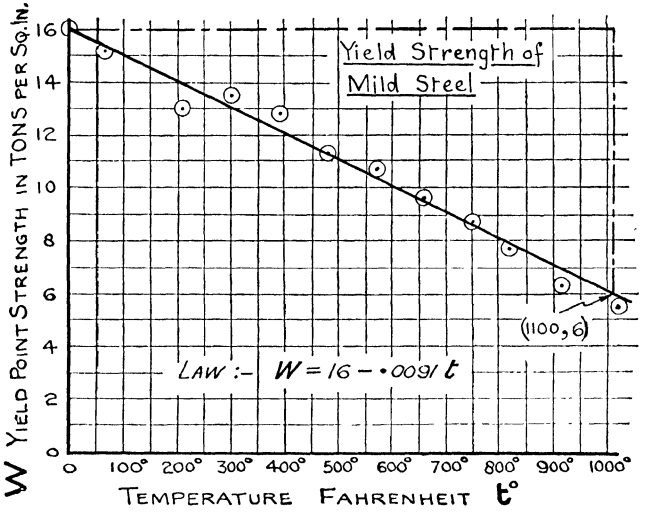


Fig. 139.—Curve to Example 367.

Example 367.—The following figures are the results of experiments on the yield point strength of mild steel at various temperatures. Plot the curve, and, assuming a straight line law, find the equation giving yield point strength in terms of the temperature.

Yield point strength, tons per sq. in.	16	15·2	13	13·5	12·8	11·3	10·7	9·6	8·7	7·7	6·3	5·5
Temperature °F . . .	0	66	210	300	390	480	570	60	750	840	930	1020

Refer to Fig. 139.

Strength is required in terms of temperature, therefore strength will be plotted vertically.

A larger sheet of paper is used to accommodate a better scale.

The points lie very irregularly, but the line shown is about the best that can be put in.

Equation.—Let W = yield point stress and t = temp. F.

Then, $W = a + bt$, where a and b are constants.

The line cuts the vertical axis at + 16, which is thus the value of the constant a.

This point is convenient for finding the slope; the other point taken is (1100, 6). Then, the horizontal distance = 1100, and the vertical distance = 16 − 6 = 10.

$$\text{Constant } b = \frac{\text{vertical}}{\text{horizontal}} = \frac{10}{1100} = \cdot00909, \text{ say, } \cdot0091.$$

In this case the slope is downwards from left to right, and therefore the constant b is negative.

Then the equation is—

$$W = 16 - \cdot0091t$$

Exercises 87. On Straight Lines and Finding Laws.

1. The following figures were taken when stretching a helical spring. Plot a curve of extension on a base of load, and find the equation giving extension (e) in terms of load (w).

Load, lbs. . . .	0	2	4	7	9	11
Extension, ins. .	0	·4	·8	1·4	1·8	2·2

2. The following table gives the prices of some geared pulley blocks. (a) Assuming a straight line law, find an equation giving the price (P) in terms of the capacity (W); (b) using the equation obtained, calculate the price of a block having a capacity of 5 tons; (c) read off the price of a 5-ton block from the curve.

Capacity in tons .	½	1	2	4	6	8
Price, £	4	5	7·25	10·5	15·25	18

3. The following figures give the voltages necessary to cause an electric spark between two needle-points in air, when d millimetres apart. Plot a curve and find the equation giving voltage (V) in terms of distance (d).

Distance apart in millimetres	6	10	14	18	22	26	30
Voltage	5000	7500	10000	12500	15000	17600	20100

4. In some experiments on the breaking strength of rolled copper at various temperatures the following results were obtained. Plot the

curve, and, assuming a straight line law, find the equation giving strength (f) in terms of temperature (t).

Temperature, F.	60	210	300	410	500	600
Breaking strength, tons per sq. in. .	17·8	17·4	16·4	16	15	14·2

5. Taking a number of lathes of somewhat similar designs, the width of the cone pulley steps is given in the following table. Plot the figures, and, assuming a straight line law, find the equation giving width (w) in terms of height of centres (H).

Height of centres. ins. .	6	8½	9	10½	12	15	18
Width of step, ins. . .	2½	3¼	3¾	3¾	4¼	5¼	6

6. The weights of some surface condensers are given in the following table, with the cooling surface. Plot a curve of weight on a base of cooling surface, and interpolate an approximate weight for condensers having (*a*) 5000 sq. ft.; (*b*) 2600 sq. ft. Find also an equation giving weight (W) in terms of square feet of cooling surface (S).

Cooling surface, sq. ft.	400	1000	2000	3000	4000	6000
Weight in lbs. . . .	3040	6310	11400	17100	21100	32700

7. The following figures were obtained in the tensile test of a piece of mild steel bar within the elastic limit. Plot a curve of load upon a base of extension, and find the "slope" of the line. *Note.*—The curve must pass through the origin.

Load in tons . . .	1·76	3·52	5·28	7·04
Extension in ins. . .	·0024	·0047	·0075	·0102

8. A test of a small oil engine gave the following results—

B.H.P.	1·0	2·1	3·0	4·2	4·7	5·3
Oil used per hr. in lbs. . .	1·07	2·16	2·85	3·91	4·40	4·90

Plot total oil used, to a base of horse power. Calculate also the oil used in lbs. per H.P. hour, and plot these figures upon the same base. Interpolate the total oil used and the oil used per H.P. hour at 5 H.P.

9. A test on the resistance of a metal filament lamp gave the following values—

Voltage in volts . .	30	40	50	60	70	80
Current in amps. . .	·2	·23	·24	·26	·27	·3

Calculate for each pair of values the resistance in ohms (*i. e.*, voltage ÷ current), and plot a curve showing resistance on a base of voltage. Find the equation giving resistance (R) in terms of voltage (V).

10. In a certain experiment the following figures were obtained. Find the equation giving L in terms of *l*.

Values of *l* . .	·6025	·81	1·017	1·243	1·6
Values of L . .	1·023	·933	·818	·746	·55

11. The following table gives the latent heat of ammonia at various temperatures. Find an equation giving latent heat (L) in terms of the temperature ° F. (*t*).

Temperature F. . .	— 30	— 10	20	40	80	90
Latent heat	589	573·5	550	534	502·3	494·2

12. The prices of some centrifugal pumps are given in the following table. Find an equation giving price (P) in terms of size (D).

Size of pump . .	6	7	8	9	10	12
Price, £	44	56	62	70	86	102

13. In connection with aircraft engine radiators the relation between the boiling point of water and the altitude is important. The following table gives the boiling point at various altitudes indicated by the ordinary altimeter.

Altitude in feet	0	5,000	10,000	15,000	20,000	25,000	30,000
Boiling Point degrees Centigrade . .	100	94·9	90·0	85·25	80·6	76·1	71·75

Find the law giving boiling point (T) in terms of altitude (H). Assume a straight line passing through the first point.

CHAPTER X

THE SLIDE RULE

[*The student should study this chapter with a slide rule at hand.*]

Introductory.—The slide rule is a mechanical aid to calculation; a "labour-saving" device by means of which many arithmetical calculations can be performed with very little expenditure of time and energy. Its principle is logarithmic, and although the slide rule may be used without any knowledge of logarithms, the student is advised to study the latter first, as he will then appreciate much better the possibilities of his instrument.

Some students appear very loath to learn the method of employing this most useful instrument, even when they possess one. This is partly due to the fact that at first their calculations with the slide rule are more troublesome and perhaps take longer than when worked in the ordinary manner. This difficulty must be expected in the early stages of using a strange machine and only practice will surmount it. The student should endeavour to use his slide rule upon *every possible occasion*, when its use will gradually become second nature to him. To employ the slide rule quickly and easily it is essential that he should be able to "approximate" with ease, and the material on p. 61 should first be thoroughly grasped.

Description.—The slide rule is made in three varieties, the straight, the circular, and the spiral. The straight type is the only one treated here, as it is by far the most common. The circular type is more easily carried in the pocket, but is hardly as accurate. The spiral form is very bulky, and very expensive, and is therefore beyond the reach of the average student. The straight type is made in various sizes, varying from about 5″ to 40″ long. For all general purposes the "10-inch" slide rule is by far the most suitable, and is the one most commonly met with. An example of the 10″ rule is shown in Fig. 140, half full size. It consists of a frame or stock F, grooved centrally to receive the slide S. Both these parts are of mahogany, and the surfaces carrying the various

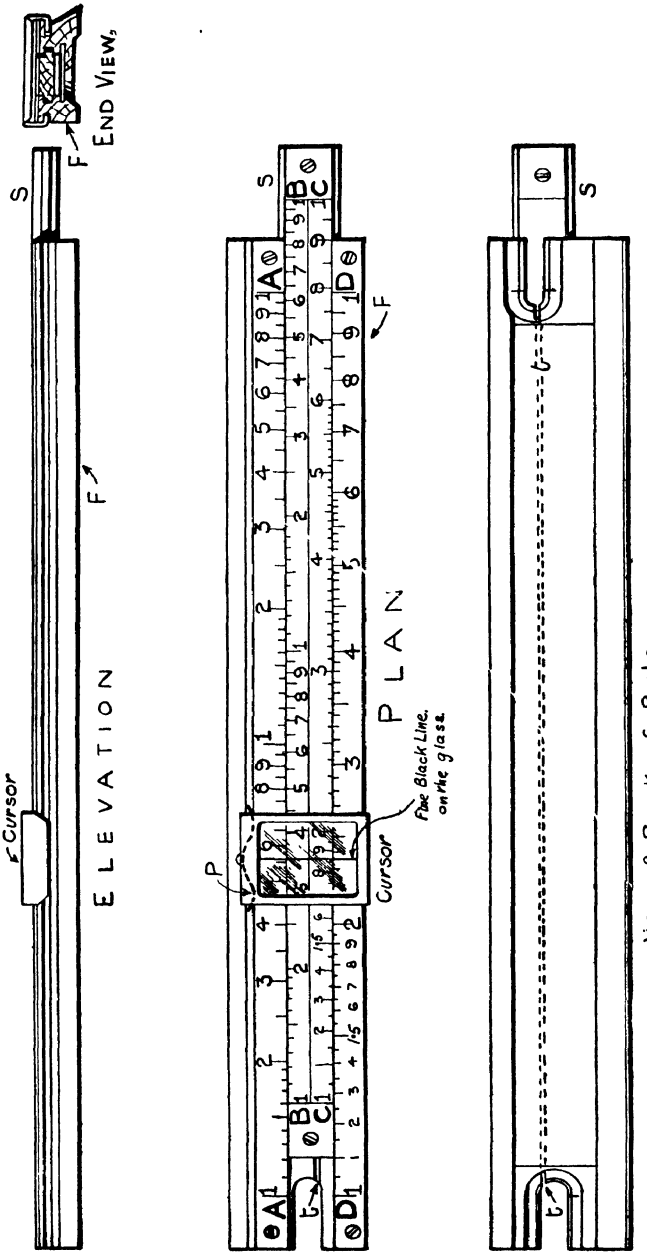


Fig. 140.—10″ Slide Rule, half-size.

scales are faced with white celluloid. The central portion of the stock is slotted lengthwise by the thin slit *t*, and a piece of thin sheet steel is placed at the back. This gives strength and elasticity, permitting the stock to be closed in, so as to grip the slide better, or opened out to give greater freedom to the slide. New rules are usually rather stiff in action, but with use they become more free, so it is unwise to open out the stock, as this will tend to looseness in later years. A stiff rule can be eased by the application of a little blacklead to the grooves and rubbing edges. When through use the slides become slack, the stock should be closed in, so that the slide cannot be shaken from any set position. This is done by removing the slide, and wrapping a handkerchief round the frame. Then holding it with both hands near the end, a steady grip should be applied, which brings the edges of the grooves slightly together. Then repeat further along, until the whole length is travelled. On replacing the slide it will be found appreciably tighter.

A light aluminium frame, called a **cursor** or runner, and carrying a piece of thin glass embraces the whole width of the rule, sliding in grooves upon the edges. A thin steel spring P, of the " carriage " type, in the top of the frame prevents any shake. A fine black line is engraved vertically in the centre of the glass, and passes across the 4 scales carried on the face of the rule. These scales are usually lettered at the ends as ABCD, starting at the top, and A and B are identical, A being on the stock and B on the slide. Also the scales C and D are identical, C being carried on the slide and D on the stock (see Fig. 140). Frequently these four are spoken of as two only, A and B being called the " top scale " and C and D the " bottom scale." These scales occupy a length of 10″ (the exact length is often 25 cms., but the rule is still known as a 10″ rule). In the later designs there is sufficient space beyond each end of the scales to permit of the cursor line being placed right over the end division.

Division of the Scales.—The " bottom scale " is divided into 9 spaces, the 10 lines starting from the extreme left being numbered 1, 2, 3.....8, 9, 1. These are the 1st significant figures of the numbers used. The lengths of the 9 spaces decrease steadily from left to right, and in consequence of this the subdivision is not uniform. From 1 to 2 the space is divided into 10 parts, and each of these into a further 10 parts. These also decrease steadily from left to right as seen at A, Fig. 141. The subdivision from 1 to 2 gives the 2nd significant figures (of numbers between 1 and 2),

and the smallest divisions are the 3rd significant figures. By estimation the 4th can be obtained, but it is not very reliable.

From 2 to 4 each main division is split into 10 parts, and each of these into 5 parts (see B, Fig. 141). The first 10 parts give the 2nd significant figure, while the 5 parts only give exactly the 3rd significant figures 2, 4, 6 and 8. By estimation the intermediate ones are obtained; an attempt at the 4th figure may be made, but is difficult. From 4 to the end each main division is also divided into 10 parts (see C, Fig. 141), but each of these divisions is only split into 2 parts. The 2nd significant figure is marked, and

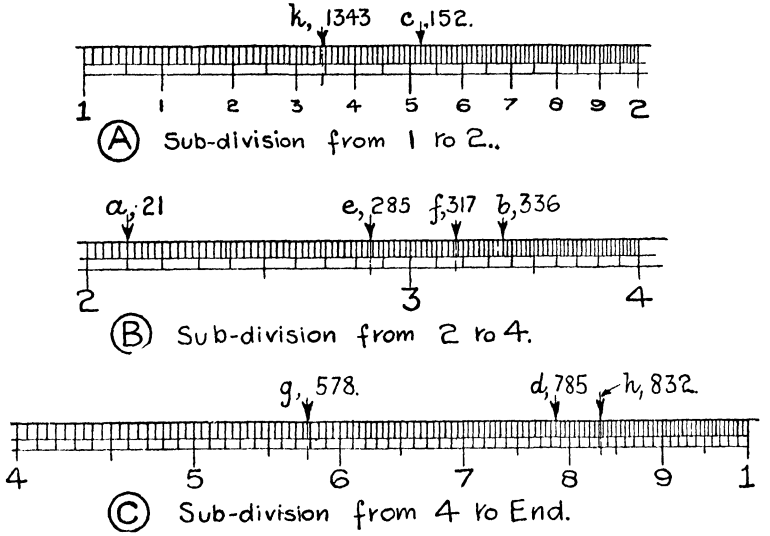


Fig. 141.—Subdivision of, and Readings on Scales C and D.

also the 5 in the 3rd significant figures. By estimating one-fifth of the smallest division the 3rd significant figure can be obtained, but no more. The accuracy of the 10″ slide rule is thus limited to the 3rd or 4th significant figure, which is quite sufficient, however, for ordinary engineering work.

The "top scale" consists of two halves, end on, and exactly the same, each half being similar to the "bottom scale." The subdivision is not as fine, since the lengths are not as long. This is dealt with further on p. 405 (see Fig. 149).

The back of the slide carries three scales, only one of which will be considered in this book.

Usually the edges of the body carry a scale of inches and tenths

on one edge and a scale of centimetres and millimetres on the other; these are very convenient for drawing, etc.; they are not shown in Fig. 140.

Many varieties of slide rules are made, with special scales for extracting various roots, areas of circles, resistances of electric conductors, etc., but unless one is doing a considerable amount of such special work, the extra scales do not warrant the expense.

Method of Reading.—As with logarithms, *only the significant figures are dealt with on the face of the slide rule*, and when finding any reading on the scales A to D only the significant figures of the number are noticed. The 1st significant figure is one of the main printed numbers 1 to 9. *Great care must be taken at first that the printed numbers from between* **1** *and* **2** *are not mistaken for these* **1st** *significant figures.* The 2nd significant figure is one of the next subdivision (which from 1 to 2 are numbered, but nowhere else). For ease of reading the fifth line of these subdivisions is rather longer than the rest. The 3rd significant figure may be actually marked (as between 1 and 2) or may have to be estimated. Accurate estimation can only be obtained by practice, preferably under careful supervision. The cursor is particularly valuable when estimating, as its line is fine and extends right across the scales. In all cases the beginner should be exact in his settings and readings; speed in using the slide rule is only attained by practice.

In Fig. 141 are shown various sample readings on the bottom scale, only the significant figures being given. In reading these values the digits should be read separately and not as a complete number; thus 21 is read as "two one," and not "twenty-one"; 336 as "three three six," etc. At *a* is shown 21; at *b* 336; at *c* 152; and at *d* 785, all of which require no estimation, the third figure being marked. At *e* is shown 285; the first two figures 28 are easily found. Then from 8 to 9 the subdivisions read 2, 4, 6, 8 and the 5 is halfway between the 4 and the 6. At *f* is shown 317, the last figure being halfway between the third and fourth subdivision lines. At *g* is shown 578; the 57 is easily found. The next short line is 575 and the following long one 58. Then, as 578 is 3 more than 575, three-fifths of the next division must be estimated. Similarly 832 is shown at *h*. An estimate of the fourth figure for numbers between 1 and 2 is given at *k*, which shows 1343.

Advice as to Holding the Rule.—Fineness of adjustment is essential, and the rule should be held so as to obtain this. It is usually convenient to hold the left hand end lightly in the left hand. Placing the right hand underneath the rule the cursor may

be moved with the forefinger and thumb on the top and bottom respectively. This is preferable to using the right hand on top, as no shadow obscures the cursor. The slide may be moved from its closed position by either forefinger. When once out it may be manipulated by finger and thumb. A very fine adjustment may be obtained by placing the thumb and forefinger holding the slide against the end of the frame as indicated in Fig. 142. A slight bending of the thumb and forefinger, while keeping them pressed against the end of the frame, will now produce a very fine motion of the slide. This will be found particularly useful when using a new rule, which is rather stiff. If the slide is manipulated without supporting the thumb and forefinger against the frame, it will continually be overshooting the mark.

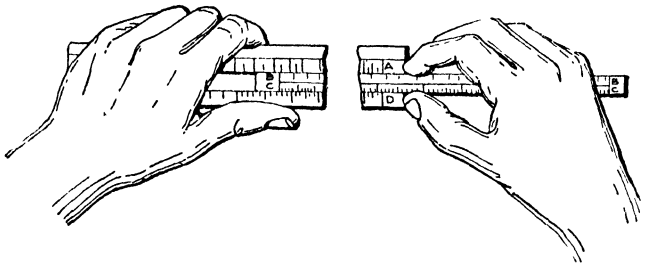


Fig. 142.—Holding Slide Rule to obtain Fine Adjustment.

Operations on the Slide Rule.—The rule can be used for multiplication, division, finding of various roots, finding logs and antilogs, and for other operations not treated here. It must be noted that *the slide rule cannot perform addition and subtraction.* By far the most common operation to be performed is that of combined multiplication and division.

All multiplication and division is to be performed on the bottom scale.

In every case the operation is started and finished on the frame or D scale. In the following descriptions the word " cursor " will frequently be used to denote the line on the cursor.

Division.—To divide one number (say, 7·5) by another (say, 3) proceed as follows :—

1. Place the cursor exactly over the number to be divided (in this case 7·5) on the frame or D scale.

2. Move the slide to bring the divisor (in this case 3) on the slide or C scale exactly under the cursor.

3. Shift the cursor to the end of the slide, setting it carefully over the extreme line.

4. Read under the cursor on the frame or D scale. This reading gives the significant figures in the quotient, in this case 25.

5. In approximating for a result settle the position of the decimal point and place it in the significant figures read.

The approximation in this case is very simple. Thus $\frac{7{\cdot}5}{3}$ is evidently a little more than 2; then our result is 2·5.

This order of operations must be strictly kept. At first the examples taken will be those not requiring any estimation of a reading. The setting of the rule is shown in the figures referred to for each case, the actual readings of the results being indicated.

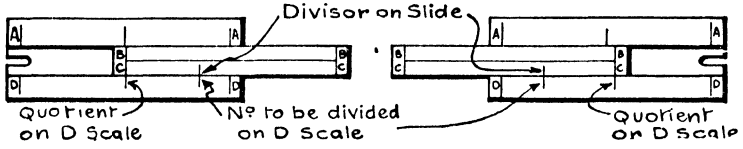


Fig. 143.—Division by Slide Rule.

The student should perform each step in the order given in the following examples, comparing his various settings with the diagrams. The method of carrying out the operation of division is shown in Fig. 143.

Example 368.—Work the following divisions with the slide rule.

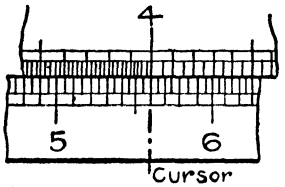
$$(a)\ \frac{56}{4};\quad (b)\ \frac{2{\cdot}3}{{\cdot}46};\quad (c)\ \frac{830}{3{\cdot}12};\quad (d)\ \frac{33{\cdot}2}{51{\cdot}5}$$

Refer to Fig. 144.

In the following examples the same letters are used in the figures as in the explanation.

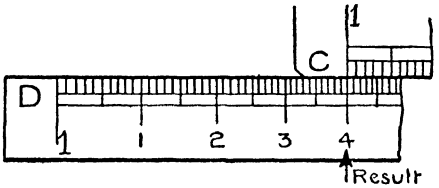
(*a*) **1.** Place cursor at 56 on D scale. **2.** Shift slide to right, placing 4 under the cursor. **3.** Shift cursor to left-hand end of slide. **4.** Read off 14 on D scale (the cursor will appear over a printed figure 4. Remember that this is 14, being between 1 and 2). Approximating, $\frac{56}{4}$ will be something over 12. Then our result is 14.

(*b*) **1.** Place cursor at 23 on D scale. **2.** Shift slide to left, placing 46 under the cursor. **3.** Shift cursor to right-hand end of slide. **4.** Read off 5 on D scale. Approximating, $\frac{2{\cdot}3}{{\cdot}46} = \frac{23}{4{\cdot}6}$ = about 5. So our result is 5.



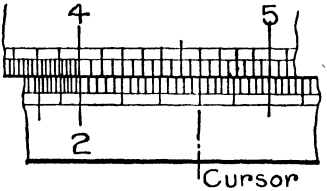
1. Set cursor at 56 on D
2. Set 4 on C under cursor

SETTING SLIDE



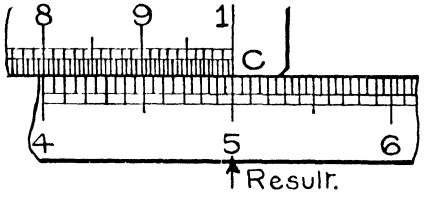
3. Under 1 on C, read 14 on D.

(a) READING RESULT



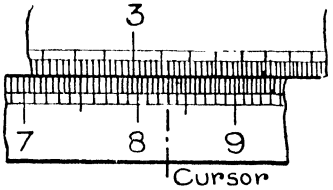
1. Set cursor at 23 on D
2. Set 46 on C under cursor

SETTING SLIDE



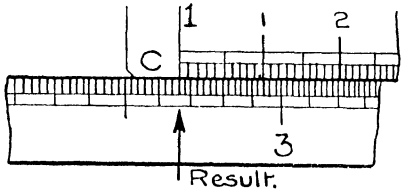
3. Under 1 on C, read 5 on D.

(b) READING RESULT



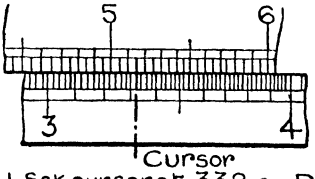
1. Set cursor at 83 on D
2. Set 312 on C under cursor

SETTING SLIDE



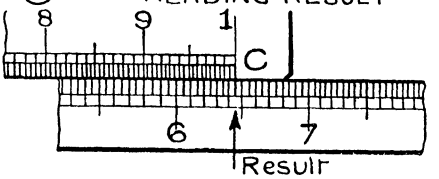
3. Under 1 on C, read 266 on D.

(c) READING RESULT



1. Set cursor at 332 on D
2. Set 515 on C under cursor

SETTING SLIDE



3. Under 1 on C, read 645 on D

(d) READING RESULT.

Fig. 144.—Settings for the Divisions of Example 368

(*c*) **1.** Place cursor at 83 on the D scale. **2.** Shift slide to right, placing 312 under the cursor. **3.** Shift cursor to left-hand end of slide. **4.** Read off 266 on D scale. Approximating, $\frac{830}{3{\cdot}12}$ = say, $\frac{800}{3}$, *i. e.*, something over 200. Then our result is 266.

(*d*) **1.** Place cursor at 332 on D scale. **2.** Shift slide to left, placing 515 under cursor. **3.** Shift cursor to right-hand end of slide. **4.** Read 645 under cursor on D scale. Approximating, $\frac{33{\cdot}2}{51{\cdot}5}$ = about $\frac{300}{500}$ or $\frac{3}{5}$, = ·6. Then our result must be ·645.

When the student can work exercises like the foregoing with accuracy, he may attempt those requiring estimation of readings such as the following :—

Example 369.—Perform the following divisions with the slide rule.

$$(a)\ \frac{11{\cdot}75}{5};\ (b)\ \frac{3{\cdot}45}{1{\cdot}2};\ (c)\ \frac{1000}{746};\ (d)\ \frac{608}{528}$$

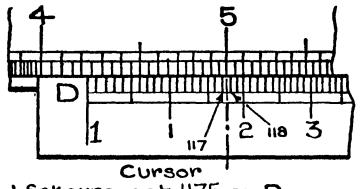
Refer to Fig. 145, which indicates concisely the operations.

(*a*) **1.** Place cursor halfway between the graduations denoting 117 and 118. **2.** Shift slide to left, placing 5 under the cursor. **3.** Shift cursor to right-hand end of slide. **4.** On the D scale the line will be found midway between the lines denoting 234 and 236. The reading is then 235. Approximating, $\frac{11{\cdot}75}{5}$ is evidently about 2; then our result is 2·35.

(*b*) **1.** Place cursor halfway between the graduations showing 344 and 346. **2.** Shift slide to right, placing 12 under the cursor. **3.** Shift cursor to left-hand end of slide. **4.** On the D scale the line will be found one-fourth of a small division to the left of the graduation 288 (see enlarged view in Fig. 145). The previous graduation reads 286. Then halfway is 287 and our reading is 2875. Approximating, $\frac{3{\cdot}45}{1{\cdot}2}$ is about $\frac{3}{1}$, *i. e.*, 3. Then our result must be 2·875.

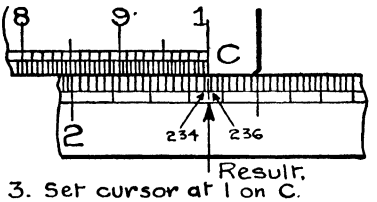
(*c*) **1.** Set cursor at either extreme end of the D scale, the right-hand for preference. **2.** Shift slide (to the right) placing 746 under the cursor. The graduation 745 is marked. Then one-fifth of the distance between 745 and 750 must be estimated (see enlarged view in Fig. 145). **3.** Shift cursor to (left) end of slide. **4.** Read 134 exactly. Approximating, $\frac{1000}{746}$ is evidently just over 1. Then our result is 1·34.

(*d*) **1.** Set cursor between the graduations of 605 and 610, estimating three-fifths of the smallest division to the right of 605, giving 608 (see enlarged view in Fig. 145). **2.** Shift slide to the right, placing 528 beneath the cursor. The graduations 525 and 530 are marked and three-fifths of the small division must be estimated beyond the 525.



1 Set cursor at 1175 on D.
2. Set 5 on C under cursor.

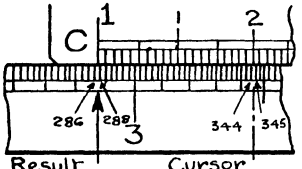
SETTING SLIDE



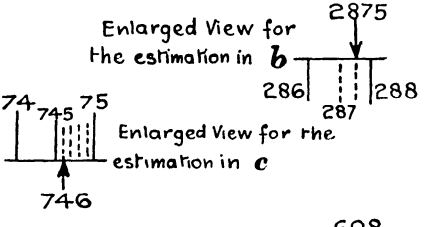
3. Set cursor at 1 on C.
4. Read 235 under cursor on D

READING RESULT

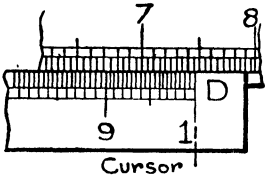
(a)



1. Set cursor at 345 on D.
2. Set 12 on C under cursor
3. Set cursor at 1 on C
4. Read 2875 under cursor on D

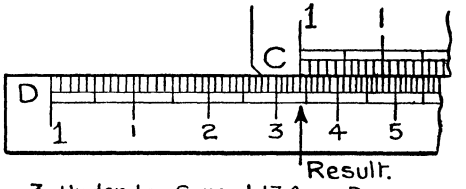


(b)



1. Set cursor at 1 on D.
2 Set 746 on C under cursor

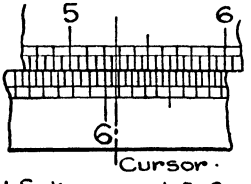
SETTING SLIDE



3. Under 1 on C, read 134 on D.

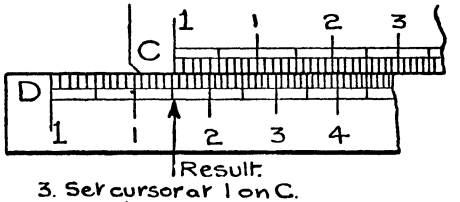
READING RESULT

(c)



1. Set cursor at 608 on D
2. Set 528 on C under cursor.

SETTING SLIDE



3. Set cursor at 1 on C.
4. Read 1151 under cursor on D.

READING RESULT

(d)

Fig. 145.—Settings for the Division of Example 369

3. Shift cursor to left-hand end of slide. **4.** The line will be found a shade to the right of graduation 1150, say 1151, this being our reading. Approximating, $\frac{608}{508}$ is just over 1. Then our result is 1·151.

The principle of the Slide Rule cannot be fully explained until the reader understands the theory of logarithms, but some idea of the principle may be given. It has been seen in Chap. VI, that to divide one number (X) by another number (Y) we subtract log Y from log X, and obtain the log of the result. Now the bottom scale of the slide rule is really a logarithm table. The distance of any graduation from the left-hand end represents the logarithm (decimal part only) of the reading at that graduation. Thus the distance from 1 to 2 represents the log of 2 (*i. e.*, ·301), the distance from 1 to 55 represents the decimal part of log 55 (*i. e.*, ·7404), and so on.

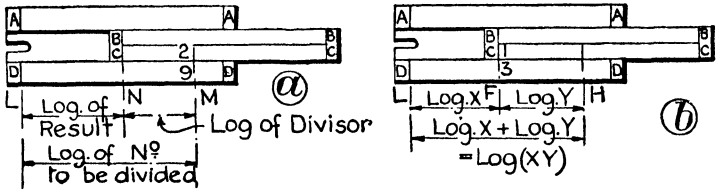


Fig. 146.—Illustrating Principle of Slide Rule.

Let us consider the case 9 ÷ 2, and refer to *a*, Fig. 146. The figure 9 is first found on the D scale; and distance LM is log 9. Now the slide is moved to the right to bring 2 on the C scale at point M; and distance NM is log 2. Then evidently distance LN = LM − NM = log 9 − log 2. Now at N on the D scale we read off 45 (read as four five). Then distance LN is log 45 and therefore 9 − log 2 = log 45; or since subtraction of logs means division of numbers $\frac{9}{2}$ = 45 (four five) as we know.

Exercises 88. On Division.

Perform the following divisions with the Slide Rule—

1. 24 ÷ 2.	**2.** 55 ÷ 25.	**3.** 6·8 ÷ ·81
4. ·392 ÷ ·67.	**5.** ·194 ÷ 6·3.	**6.** ·153 ÷ 15.
7. 96·5 ÷ ·109.	**8.** 2·28 ÷ ·0252.	**9.** ·384 ÷ 25·6.
10. 308 ÷ 725.	**11.** 223 ÷ 425.	**12.** 27·5 ÷ 1·1
13. ·083 ÷ 3·03.	**14.** 4475 ÷ 384.	**15.** 397 ÷ 84·5.
16. 5·28 ÷ 20.	**17.** 417 ÷ 182.	**18.** ·682 ÷ ·745.
19. 13·55 ÷ ·0209.	**20.** 1012 ÷ 9·17.	

Multiplication.—To multiply one number by another, for example, 3·5 by 2·2, proceed as follows :—

1. Place the cursor exactly over the number to be multiplied (in this case 3·5) on the frame or D scale.

2. Move the slide to the right (or left as may be required by operation 3) and place the end line exactly under the cursor.

3. Move the cursor until it is exactly over the multiplier (in this case 2·2) on the slide or C scale.

4. Read under the cursor, on the stock or D scale, the significant figures in the answer (in this case 77).

5. By approximating for a result fix the position of the decimal point, and place it in the significant figures read.

The approximation here is simple; 3·5 × 2·2 is about 3 × 2, *i. e.*, 6. Hence our answer must be 7·7.

The moving of the slide to the right or left in the second operation is necessary in order that the multiplier may appear within

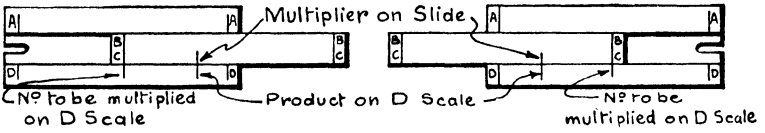


Fig. 147.—Multiplication by Slide Rule.

the limits of the stock or D scale. Thus taking 3·5 × 4, if the slide be moved to the right, its end being at 3·5 on the D scale, then when searching for the 4 on the slide or C scale, this will be found outside the stock altogether. Then the slide must be changed to the left-hand side, its right-hand end being opposite 3·5 on the D scale, when the 4 on the slide will be found opposite 14 on the D scale. When first using the instrument the student will be unable to say to which side the slide should be moved, but with practice he will recognise that a result is going to be smaller or greater than 10, and will move the slide naturally to the right or left respectively. The method is illustrated by Fig. 147.

Example 370.—Work the following multiplications on the slide rule : (*a*) 32 × 2·5; (*b*) ·785 × 4; (*c*) 1·34 × 48·5; (*d*) ·14 × 80.

When the student has worked Exercises 88, on Division, he should be able to follow this example without a figure showing details of setting.

(*a*) **1.** Set cursor to 32 on D scale. **2.** Move slide to *right* and place the left-hand end under the cursor. **3.** Move cursor to *right*

and place over 25 on the slide or C scale. **4.** Read off 8 on the D scale. Approximating, 32 × 2·5 will be something more than 64, hence our result is 80.

(*b*) **1.** Set cursor at 785 on the D scale. **2.** Move slide to *left* and place the right-hand end under the cursor. **3.** Move cursor to *left* and place over 4 on the slide or C scale. **4.** Read off 314 on the D scale. Approximating, ·785 × 4 will be about 3, hence our result is 3·14.

(*c*) **1.** Set cursor at 134 on D scale. **2.** Move slide to *right* and set left-hand end under the cursor. **3.** Move cursor to *right* and place over 485 on slide. **4.** Read off 65 on the D scale. Approximating, 1·34 × 48·5 will be rather more than 48·5. Hence our result is 65.

(*d*) **1.** Set cursor at 14 on the D scale. **2.** Move slide to *left* and place right-hand end under the cursor. **3.** Move cursor to left and place over 8 on slide or C scale. **4.** Read off 112 on D scale. (This may appear to be 12, but it should be remembered that the end of the D scale is 1, the next 1 to the right being 11.) Approximating, ·14 × 80 is rather more than one-tenth of 80, or 8. Hence our result is 11·2.

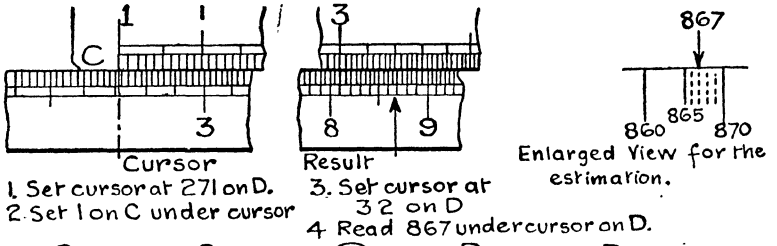
The above example requires only the graduations marked. The following will need estimation, and also introduces "continued multiplication," *i. e.*, the multiplication of three (or more) numbers.

Example 371.—Work the following multiplication on the slide rule: (*a*) 2·71 × 3·2; (*b*) 68·7 × 16·7 × ·366; (*c*) 5·33 × ·946; (*d*) ·349 × 20·3 × 8·35. Refer to Fig. 148, which indicates concisely the operations.

(*a*) **1.** Place cursor halfway between graduations 270 and 272, thus giving 271. **2.** Shift slide to the *right* and place left-hand end under the cursor. **3.** Move cursor to right and place over the graduation 32 on the slide or C scale. **4.** Read on the D scale; the cursor line appears to be a little less than halfway between 865 and 870; hence read as 8670 (see enlarged view in Fig. 148). Approximating, 2·71 × 3·2 is about 9; hence result is 8·670.

(*b*) **1.** Place cursor between graduations 685 and 690, estimating two-fifths of the small division beyond the 5, thus giving 687. **2.** Move slide to *left*, and place right-hand end under the cursor. **3.** Move cursor to left and place over the graduation 167 on the slide or C scale. [Cursor should now read a shade less than 115: in practice this result is not read; it is given here as a check.] **4.** Move slide to *right*, placing 1 under the cursor. **5.** Move cursor to right, placing it over 366 on slide. **6.** Read 42 on D scale under cursor. Approximating, 68·7 × 16·7 × ·366 is, say, 60 × 20 × ⅓ or 400. Hence result is 420.

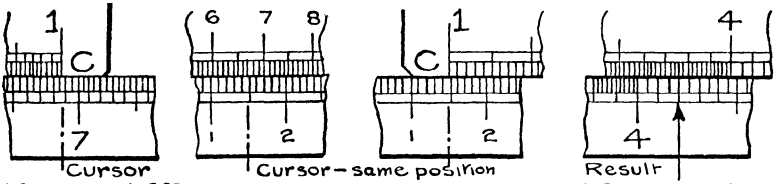
(*c*) **1.** Place cursor between graduations 530 and 535, estimating three-fifths of the small division beyond the 530, thus giving 533 (see enlarged view in Fig. 148). **2.** Move slide to the *left* and place the



1. Set cursor at 271 on D.
2. Set 1 on C under cursor

3. Set cursor at 32 on D
4. Read 867 under cursor on D.

SETTING SLIDE (a) READING RESULT



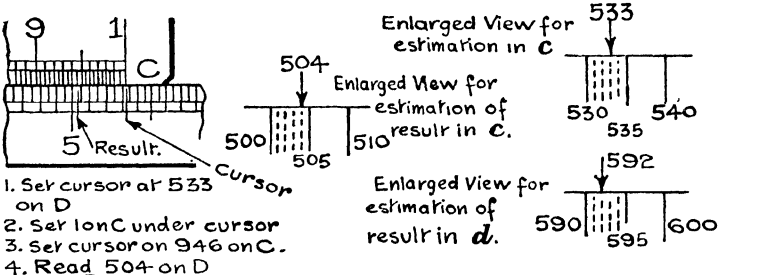
1. Set cursor at 687 on D
2. Set 1 on C under cursor

3. Set cursor at 167 on C

4. Set 1 on C under cursor

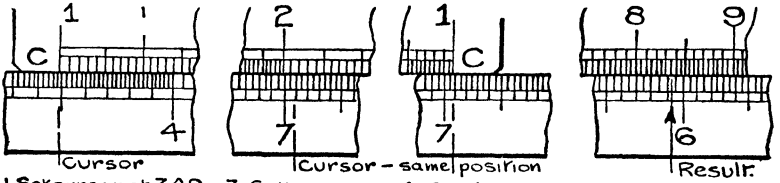
5 Set cursor at 366 on C.
6. Read 42 on D under cursor

1ST SETTING OF SLIDE (b) 2ND SETTING OF SLIDE



1. Set cursor at 533 on D
2. Set 1 on C under cursor
3. Set cursor on 946 on C.
4. Read 504 on D

(c)



1. Set cursor at 349 on D.
2. Set 1 on C under cursor.

3. Set cursor at 203 on C.

4. Set 1 on C under cursor

5. Set cursor at 835 on C.
6. Read 592 on D under cursor

1ST SETTING OF SLIDE (d) 2ND SETTING OF SLIDE.

Fig. 148.—Settings for the Multiplications of Example 371.

right-hand end under the cursor. **3.** Move cursor to the left and place it between graduations 945 and 950 on the C scale, estimating one fifth of the small division beyond the 945, thus finding 946. **4.** Read on the D scale; the line appears a shade to the left of graduation 505, say 504 (see enlarged view in Fig. 148). Approximating, $5{\cdot}33 \times {\cdot}946$ is practically $5{\cdot}33 \times 1$, so that our result must be 5·04.

(*d*) **1.** Place cursor halfway between graduations 348 and 350, thus giving 349. **2.** Move slide to the *right*, placing the left-hand end under the cursor. **3.** Move cursor to the right, placing it midway between graduations 202 and 204 on the C scale, thus giving 203. [Cursor should now read a shade less than 71, as a check.] **4.** Shift slide to *left*, placing the end under cursor. **5.** Shift cursor to left, placing it over 835 on C scale. **6.** Read 592 on D scale (see enlarged view in Fig. 148). Approximating, $\cdot 349 \times 20{\cdot}3 \times 8{\cdot}35$ is about $\frac{1}{3} \times 20 \times 8$ = about 50. Hence result is 59·2.

It is known from Chap. VI that the multiplication of two numbers by logarithms requires the addition of their logs. Now, when the slide is moved to the right and the left-hand end placed at (say) 3 on the D scale (see *b*, Fig. 146) the distance LF is log 3. When the cursor is moved to (say) 25 on the C scale, the distance FH is log 25 (decimal part only). Then evidently the total distance LH = LF + FH = log 3 + log 25. Now at H on the D scale (*i. e.*, counting from L) we read 75. Then distance LH = log 75. Therefore log 3 + log 25 = log 75; or since addition of logs means multiplication of numbers $3 \times 25 = 75$ as we know.

Exercises 89. On Multiplication.

Perform the following multiplications by slide rule—

1. $22 \times 3{\cdot}5$.	**2.** $\cdot 54 \times 4$.	**3.** $79 \times \cdot 26$.
4. $33{\cdot}8 \times 2{\cdot}1$.	**5.** $202 \times 4{\cdot}95$.	**6.** $\cdot 645 \times \cdot 49$.
7. $1{\cdot}99 \times 98$.	**8.** $6{\cdot}1 \times \cdot 141$.	**9.** $\cdot 0312 \times \cdot 173$.
10. $4{\cdot}25 \times 2{\cdot}66$.	**11.** $\cdot 205 \times 2{\cdot}78$.	**12.** $34{\cdot}9 \times 47$.
13. $2{\cdot}83 \times 38$.	**14.** $10{\cdot}25 \times 70$.	**15.** $\cdot 4375 \times \cdot 72$.
16. $50{\cdot}3 \times 3{\cdot}7$.	**17.** $\cdot 774 \times 119$.	**18.** $4{\cdot}08 \times 2{\cdot}29$.
19. $\cdot 866 \times 37{\cdot}3$.	**20.** $120{\cdot}5 \times 8{\cdot}775$.	**21.** $\cdot 285 \times 3{\cdot}12 \times 42$.
22. $32 \times 48 \times 86$.	**23.** $79{\cdot}2 \times \cdot 53 \times 1{\cdot}5$.	
24. $115 \times 1{\cdot}2 \times 5{\cdot}175$.	**25.** $345 \times \cdot 638 \times \cdot 205 \times 223$.	

Combined Multiplication and Division.—It is in calculations of this nature that the slide rule is particularly valuable. Consider an expression such as $\frac{5{\cdot}3 \times 33000 \times 12}{35 \times 144 \times 3{\cdot}14}$. Now by the aid of the previous section it would be possible to multiply up the top

line, giving 2,100,000. Similarly the bottom line can be multiplied up, giving 15,830. Then the division $\frac{2100000}{15830}$ could be performed as in the previous section but one, giving 132·5. But a quicker way of working, in which fewer settings of the slide are necessary, is to start from the first figure in the top line, divide by the first on the bottom, multiply by the second on the top, divide by the next on the bottom, and so on, zigzagging from top to bottom. It will be noticed from the detailed method about to be given for the above expression, that after the first setting on the D scale, the slide and cursor are moved alternately, and *all setting is done on the slide or C scale.* The final reading is, of course, taken on the D scale, and in this case comes under the end of the slide.

The intermediate results are *not read,* but to enable the student to check his setting at *each step* when learning, these intermediate results will be shown in square brackets to distinguish them from the necessary readings.

1. Set the cursor on the D scale, to the first number in the top line (in this case 53).

2. Set the slide with the first number in the bottom line under the cursor (in this case 35).

[Now the left-hand end of the slide should be opposite 1513 on the D scale.]

3. Move the cursor, placing it on the C scale over the second number in the top line (in this case 33).

[The cursor should read 499 on the D scale.]

4. Move the slide placing the second number in the bottom line (in this case 144) on the C scale under the cursor.

[The left-hand end should now read 347.]

5. Move the cursor to the next number in the top line reading on the slide or C scale.

Repeat this zigzag process until all the numbers have been used, when the answer is read off on the D scale, either *under the end of the slide* (see Ex. 375) or *under the cursor* (see Ex. 373). Finally approximate for the result, and fix the position of the decimal point.

Example 372.—Find the value of $\frac{13{\cdot}3 \times 4500 \times 19}{35 \times 1728 \times 4}$.

1. Set cursor at 133 on D scale. **2.** Move slide to left, placing 35 on C scale under cursor. [Right-hand end of slide should now read 38 on D scale.] **3.** Move cursor to 45 on slide or C scale. [Cursor should now read 171 on D scale.] **4.** Shift slide to right, placing 1728 on C scale,

under the cursor. [Right-hand end should now read 99 on D scale.] **5.** Move cursor, placing over 19 on C scale. [Cursor should now read 1877 on D scale.] **6.** Move slide to left, placing 4 under the cursor. **7.** Shift cursor to right-hand end of slide, and read off 47 on D scale.

Approximating, we have $\frac{1 \times \frac{1}{4} \times \frac{2}{3}}{3 \times \frac{2}{3} \times \frac{1}{4}}$ $\quad$ $\frac{11111}{1111}$

$= \frac{1}{3}$ or ·33 $\quad$ ∴ Approx. answer $= ·33 \times 10 = 3·3$

∴ Result = 4·7.

Example 373.—Find the value of $\frac{·214 \times 12500 \times 30}{13·6 \times 26}$.

1. Set cursor at 214 on D scale. **2.** Move slide to right, placing 136 under cursor. [Left end of slide should now read 1572.] **3.** Move cursor to left, placing it over 125 on C scale. [Cursor should now read 1967 on D scale.] **4.** Move slide to left, placing 26 on C scale under the cursor. [Right-hand end should now read 756 on D scale.] **5.** Move cursor to right, placing it at 3 on C scale. **6.** As this completes the expression, read the result under the present position of the cursor on the D scale, *i. e.*, 2268.

Approximating, for result $\frac{\frac{2}{3} \times 1 \times 3}{1 \times \frac{2}{3}}$ $\quad$ $\frac{11111}{111}$

$= 3$ $\quad$ ∴ Approx. answer $= 3 \times 100 = 300$

∴ Result = 226·8.

Sometimes when moving the cursor to find a certain number in the top line, it will not be found on that portion of the slide within the stock. Then an extra shift is necessary. The following example will illustrate :—

Example 374.—Evaluate $\frac{2·7 \times 23·6}{7 \times 5·93}$.

1. Set cursor at 27 on D scale. **2.** Move slide to left, placing 7 under the cursor. [Right-hand end should now read 386.] On moving cursor to pick up 236 on slide, this reading will be found beyond the left-hand end of the stock. Then proceed thus : **3.** Shift cursor to right-hand end of slide. **4.** Move slide from left to right, placing left-hand end under the cursor (which is at 386). **5.** Move cursor to right, placing it over 236 on slide. [The cursor should now read 91 on the D scale.] **6.** Move slide to left, bringing 593 under the cursor. **7.** Bring cursor to left-hand end of slide, and read off 1536 on D scale.

Approximating, we have $\frac{\frac{1}{3} \times \frac{2}{3}}{7 \times \frac{1}{6}}$ $\quad$ $\frac{1}{1}$

$= \frac{1}{7}$, say, ·14 $\quad$ ∴ Approx. answer $= ·14 \times 10 = 1·4$

∴ **Result = 1·536.**

When the bottom line contains one number more than the top line, then an extra shift has to be made, as shown in the following :—

Example 375.—Calculate $\frac{12{\cdot}3 \times 450}{25{\cdot}4 \times 3{\cdot}35 \times 14{\cdot}6}$

1. Set cursor at 123 on the D scale. **2.** Move slide to the left, placing 254 on the C scale, under the cursor. [Right hand end of slide should now read 484 on the D scale.] **3.** Move cursor to 45 on the slide. [Cursor should now be over 218 on the D scale.] **4.** Move slide to the right, placing 335 on the C scale under cursor. [Right-hand end of slide should now be opposite 65 on the D scale.]

This end position is the result of the working of the first four numbers, counting in our zigzag order; the value has now to be divided by 14·6. **5.** Move cursor to right-hand end of slide. **6.** Shift slide to the right, placing 146 on the C scale under the cursor. **7.** Move cursor to left-hand end of slide and read off 445 on the D scale.

Approximating, we have $\frac{1 \times 4}{4 \times 3 \times 1}$ (2)

$= \frac{2}{3} = \cdot 66$

$\therefore$ Result = 4·45.

$\frac{111}{11}$

$\therefore$ Approx. answer $= \cdot 66 \times 10 = 6{\cdot}6$

Example 376.—Evaluate the expression $\frac{5{\cdot}25 \times 8}{\cdot 785 \times \cdot 73 \times \cdot 00546}$

1. Set cursor at 525 on D scale. **2.** Move slide to left, placing 785 under cursor. [Right-hand end of slide should now be over 669 on D scale.] **3.** Shift cursor to 8 on slide. [Cursor should now be over 535 on D scale.] **4.** Move slide, placing 73 under cursor. [Right-hand end should now read 733 on D scale.] **5.** Move cursor to right-hand end of slide. **6.** Move slide to right, placing 546 under the cursor. **7.** Move cursor to left-hand end of slide, and read off 134 on D scale.

Approximating, we have $\frac{5 \times 8}{8 \times 8 \times 5}$

$= \frac{1}{8} = \cdot 125$

$\therefore$ Result = 13420.

11111

$\therefore$ Approx. answer $= \cdot 125 \times 100000$ $= 12500$

The student will find many examples for practice in the worked examples of Chaps. II, III, etc. These afford valuable practice, as there is a check on any errors.

Exercises 90. On Combined Multiplication and Division.

[In the answers to some of these exercises intermediate as well as final results are given to give a more ready check on the working.]

1. $1{\cdot}01 \times {\cdot}98 \div {\cdot}102$.
2. $11{\cdot}75 \times {\cdot}71 \div 81$
3. $\dfrac{35{\cdot}5 \times {\cdot}575}{32{\cdot}4}$
4. $\dfrac{4{\cdot}35 \times 25}{1{\cdot}7 \times 12{\cdot}3}$
5. $\dfrac{195 \times 3{\cdot}4}{13 \times 1{\cdot}2}$
6. $\dfrac{{\cdot}975 \times 10{\cdot}6}{2{\cdot}82 \times {\cdot}805}$
7. $\dfrac{20{\cdot}4 \times 6{\cdot}9 \times 2{\cdot}34}{25{\cdot}6 \times {\cdot}146 \times 76}$
8. $\dfrac{{\cdot}0735 \times 47 \times 109}{9{\cdot}55 \times 13{\cdot}6 \times 2{\cdot}96}$
9. $\dfrac{51{\cdot}75 \times 2{\cdot}52 \times 5}{15{\cdot}3 \times 7{\cdot}35}$
10. $\dfrac{{\cdot}325 \times 5{\cdot}2 \times 6{\cdot}6}{8{\cdot}4 \times 134}$
11. $\dfrac{7{\cdot}05 \times 7{\cdot}7}{23{\cdot}2}$
12. $\dfrac{114{\cdot}5 \times {\cdot}0183}{{\cdot}216}$
13. $\dfrac{{\cdot}303}{1{\cdot}775 \times 1{\cdot}16}$
14. $\dfrac{{\cdot}095}{{\cdot}243 \times 9{\cdot}6}$
15. $\dfrac{5{\cdot}38}{{\cdot}785 \times 14{\cdot}65}$
16. $\dfrac{{\cdot}239 \times 7{\cdot}45}{16{\cdot}55}$
17. $\dfrac{115}{{\cdot}00034 \times 7{\cdot}6 \times {\cdot}4375}$
18. $\dfrac{{\cdot}9375 \times 1{\cdot}7}{6{\cdot}06 \times {\cdot}753 \times 1{\cdot}27}$
19. $\dfrac{23{\cdot}8 \times {\cdot}94 \times 114}{2{\cdot}13 \times 17{\cdot}1}$
20. $\dfrac{11{\cdot}05 \times {\cdot}64}{1{\cdot}04 \times {\cdot}918}$

Ratios and Percentages. Special Case.—A ratio or percentage is easily worked on the slide rule as a division. When, however, several numbers have to be expressed as a fraction or percentage of some *one* number, the amount of time and labour necessary can be reduced by working the example as a multiplication instead of a division. For example, let it be required to express each of the numbers 256, 137, 373, 400, 65, and 78 as a decimal of 435. Now 256 as a decimal of 435 is $256 \div 435$, and similarly for the others, 435 being the denominator in *each case.* Then the ordinary operation of division would be as follows :—

Place cursor at each number in turn on D scale
Set slide with denominator 435 at this position of the cursor
Read under end of slide on D scale

Thus, *the slide has to be shifted each time.*

But $\dfrac{256}{435} = 256 \times \dfrac{1}{435}$ and similarly for each of the other numbers.

Thus we have, *decimal required* = *each number* $\times \dfrac{1}{435}$. Now $\dfrac{1}{435}$ is a constant quantity, and if the end of the rule be placed at this reading on the D scale, all the *multiplying* will be done without

shifting the slide, as the cursor will be placed at various positions on the slide or C scale and the result read on D. It is not necessary to know the value of $\frac{1}{435}$; the position of the end of the slide is found automatically if 435 on the C scale be placed at 1 or 10 on the D scale. After that only the cursor need be moved, but sometimes the slide may have to be shifted once.

The numbers to be expressed as fractions or percentages will generally be somewhat alike in size, and thus once the decimal point is fixed by approximating for one number, the others will follow naturally.

The following example will illustrate the method :—

Example 377.—Express each of the following numbers as a decimal of 435 : (*a*) 256; (*b*) 137; (*c*) 73; (*d*) 400; (*e*) 65; (*f*) 378.

Move slide to right, placing 435 on the C scale at the extreme end of the D scale. Then the left-hand end of slide gives the figures of $\frac{1}{435}$. Now numbers greater than 435 will be found beyond the end of the D scale, and cannot be taken on this setting, but will require the slide to project to the left. Then divide the numbers into two groups, one set being all below 435 and the remainder above. Indicate one set by, say, a tick.

Taking the numbers below 435 in turn, *i.e.*, *a*, *b*, *d*, and *f*. Then for (*a*) set the cursor at 256 on the C scale and read 588 on the D scale. The result is evidently about $\frac{1}{2}$. Then 256 is ·588 of 435.

For (*b*) move cursor to 137 on C scale and read 315 on D. Result is ·315.

For (*d*) set cursor at 4 on C scale and read 92 on D. Result is ·92.

For (*f*) set cursor at 378 on C and read 869 on D. Result is ·869.

(Note that the slide has not been moved the whole time, but it must now be moved to the left, with 435 on C scale at 1 on D scale.)

For (*c*) set cursor at 73 on C scale and read 1677 on D. Result is ·1677.

For (*e*) set cursor at 65 on C scale and read 1493 on D. Result is ·1493.

Exercises 91. On Percentages.

Work by slide rule, Exercises 18, Chap. II.

Square Root.—The actual operation upon the slide rule when extracting square root is very simple indeed, but difficulty may be experienced at first in fixing the position of the decimal point.

The slide is not used at all, only the extreme top or A scale, and the extreme bottom or D scale being employed. It will be seen in Fig. 149 that the top scale is divided into two halves or scales each exactly the same, and similar to the bottom scale, except that they are not so finely subdivided, since the space is more limited. The division and method of reading these scales will be readily understood from the diagram. The *left-hand half* of the top scale will be called the *first half*, and the *right-hand* the *second half*.

To extract the square root proceed as follows :—

1. Start from the decimal point and mark off the digits in pairs, to the left only if a mixed number, and to the right if wholly a decimal. This is similar to the pointing off, when finding a square root in the ordinary way, on p. 102.

2. If the extreme left-hand period contains one digit use the 1st half of the top scale ; if it contains two digits use the 2nd half. In the case of a number wholly a decimal, any noughts immediately after the decimal point are not considered as digits, as they are not significant figures.

3. Place the cursor on the A scale, at the number, in the particular half decided by (2) above, and read under the cursor on the D scale. This gives the significant figures in the square root.

4. Finally fix the decimal point in the result according to the following :—

(i) *When the number is a mixed number.* There will be as many figures in the whole number of the square root as there are periods in the number.

(ii) *When the number is a decimal.* There will be as many noughts immediately after the decimal point in the square root as there are *complete* periods of noughts in the number.

It is this setting of the decimal point which is found troublesome, and to better understand the above rules the following two examples should be worked through carefully: the first deals with the case when the number is a mixed one, and the second when the number is wholly a decimal. In any case a rough squaring of the root will give a check on the results. Should the student find that the graduations of the B and C scales on the slide confuse him, when reading from the A to the D scale, he should pull the slide out some distance so as to leave clear the region in which he is working.

Example 378.—Extract the square roots of the following numbers, with a slide rule : (*a*) 2250; (*b*) 259; (*c*) 44100; (*d*) 53·7; (*e*) 3·3.

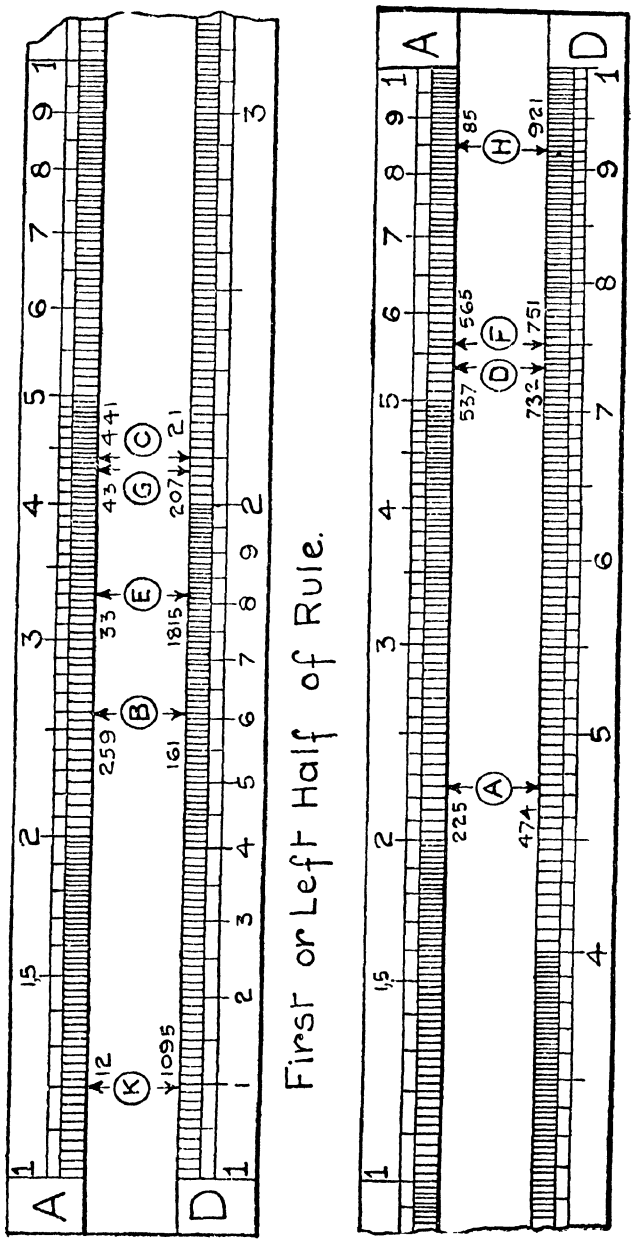


Fig. 149.—Subdivision of Scales A and B, and Settings for the Square Roots of Examples 378 and 379.

(*a*) Mark off in pairs from the decimal point thus : '22'50. The left-hand period contains two figures, and therefore the second half of the upper scale must be used. Place cursor over 225 on right of A scale and read 474 on D scale (see A, Fig. 149). As there are two periods in '22'50, there will be two figures in the whole number of our result. Thus required square root is 47·4.

(*b*) Mark off in pairs, giving 2'59. Left-hand period has only one figure, therefore first half of upper scale must be used. Place cursor over 259 on left of A scale, and read 161 on D scale (see B, Fig. 149). As there are two periods in 2'59, there will be two figures in the whole number of our result. Thus required square root is 16·1.

(*c*) Point off, giving 4'41'00, and as left-hand period has only one figure, use first half of upper scale. Under 441 on left of A scale we read 21 on D scale (see C, Fig. 149). As there are three periods in 4'41'00, then three figures must exist in the result. As two only exist, a nought must be added and the required square root is 210.

(*d*) Point off, giving '53·7. Left-hand period contains two figures, therefore second half of scale A is used. Under 537 on right of A scale we read 732 on D scale (see D, Fig. 149). There is one period in the whole number of '53·7, so there will be one figure in the whole number of the result. The required root is then 7·32.

(*e*) Point off, giving '3·3. Left-hand period contains only one figure, therefore first half of scale A is used. Under 33 on left of scale A, read 1815 on D scale (see E, Fig. 149). There is only one period in the whole number of '3·3, hence there will be one figure in the whole number of the result. Required square root is then 1·815.

Example 379.—Extract the square root of the following numbers, with the slide rule : (*a*) ·565; (*b*) ·043; (*c*) ·0085; (*d*) ·00012.

(*a*) Pointing off in pairs to the right from the decimal point gives ·56'5. Left-hand period contains two figures, hence second half of scale A is used. Place cursor over 565 on right of A scale, and read 751 on D scale (see F, Fig. 149). In the number ·56'5 there are no noughts following the decimal point. Hence there will be no noughts after the decimal point of the result. Thus the required square root will be ·751.

(*b*) Point off, giving ·04'3. Left-hand period contains one figure (the nought not being significant), hence first half of scale A is used. Under 43 on left of scale A read 207 on D scale (see G, Fig. 149). A period consists of *two* digits, so there are no complete periods of noughts in ·04'3. Thus there will be no noughts after the decimal point in the result. The required square root is then ·207.

(*c*) Point off, giving ·00'85. Left-hand period, with significant figures, contains two figures, hence second half of scale A is to be used. Under 85 on right of A scale read 921 on D scale (see H, Fig. 149).

There are two noughts after the point in the number ·0085 (*i. e.*, one complete period of noughts). Hence there will be one nought following the point in the result. The required root is therefore ·0921.

(*d*) Point off, giving ·00'01'2. The left-hand period, containing a significant figure, has one figure. Hence first half of scale A is used. Under 12 on left of scale A read 1095 (see K, Fig. 149). In the number 00'01'2 there is only one *complete* period of noughts following the decimal point, so there will be one nought following the point in the result. The required square root is therefore ·01095.

Exercises 92. On Square Root.

Find by slide rule the square roots of the following numbers, and say in each case which half of the top scale you would use—

1. 2600.	**2.** 44500.	**3.** 1·55.	**4.** 213.
5. 98·5.	**6.** 737.	**7.** ·0505.	**8.** ·0034.
9. 11·7.	**10.** ·307.		

Extract the following square roots by Slide Rule. In each case check your result by multiplication—

11. $\sqrt{250}$.	**12.** $\sqrt{7·3}$.	**13.** $\sqrt{·39}$.	**14.** $\sqrt{9200}$.
15. $\sqrt{106}$.	**16.** $\sqrt{5·25}$.	**17.** $\sqrt{·187}$.	**18.** $\sqrt{54·5}$.
19. $\sqrt{2·175}$.	**20.** $\sqrt{30·7}$.	**21.** $\sqrt{289}$.	**22.** $\sqrt{·833}$.

Squaring.—This being the reverse of extracting square root the operations are reversed, but are much less troublesome, as there is no need to consider which half of the top scale A, must be used. As before, only the A and D scales are needed, and the procedure is as follows :—

1. Place the cursor over the D scale, at the number to be squared.

2. Read on the A scale the significant figures in the result.

3. Fix the position of the decimal point by approximating for a result.

Example 380.—Find by aid of the slide rule the value of the following : (*a*) $2·37^2$; (*b*) $31·6^2$; (*c*) $66·3^2$.

(*a*) Place cursor at 237 on the bottom or D scale, and read 562 on the top or A scale. Now $2·37^2$ must be rather more than 2^2 or 4. Hence our result must be 5·62.

(*b*) Place cursor at 316 on the bottom scale and read 1 (*i. e.*, the central figure) on the top scale. Now $31·6^2$ must be rather more than 30^2, which is 900. Then the result must be 1000.

(*c*) Place cursor at 663 on the bottom scale and read 44 on top scale. Now $66·3^2$ must be more than 60^2, which is 3600. Hence the result must be 4400.

In some cases, when the result appears in the crowded portion of the top scale (as from about 7 to 10) it is difficult to estimate a third figure reasonably correctly. In these cases, if an accurate answer is desired, it is better to work the square as a multiplication example on the bottom scale, where finer estimation is possible. Thus taking $31{\cdot}6^2$, we multiply $31{\cdot}6 \times 31{\cdot}6$ on the C and D scales, reading off 999. The actual result to four significant figures being 998·6, this method gives a more accurate result than the former.

Exercises 93. On Squaring.

Square Examples 1 to 10 in Exercises 92.

Further practice may now be obtained by working more complicated exercises such as those in Exercises 64 and 65.

Reading Logarithms and Antilogarithms from the Rule.—Of the three scales on the back of the slide one is a scale of **equal parts**; it is generally the middle scale and is labelled L. This is used in conjunction with the A scale as a logarithm table, but the accuracy is limited to 3 decimal places. It must be noticed that the L scale reads from *right to left*, and is always used with the slide projecting to the *right*. The main divisions, which are numbered (the right hand end graduation is **O**), give the first decimal place in the logarithm, the next sub-divisions give the second decimal place, while the smallest graduations give every *even* third place, the odd figures in the third place being estimated as at the centres of the smallest sub-divisions.

To find a logarithm (decimal part),—*e. g.*, log 2·6.

1. Move the slide to the right, placing its left-hand end at the significant figures of the number, *i. e.*, at 26, on the A scale.

2. Turn the rule over (*not* end for end) and read the decimal of the log on the L scale opposite the appropriate small mark in the **U**-shaped gap at the right. The figures 415 should be found. Then the decimal part of log 2·6 is ·415, and the whole number must be chosen in the usual way. Then log 2·6 = 0·415.

To find an antilog. (significant figures)—*e.g.*, antilog 1·658. Reverse the order of operations in the previous paragraph.

1. Turn the rule over, moving the slide to the right, and set the *decimal* of the log., *i.e.*, 658, to the small mark in the gap.

2. Turn the rule back to its normal position, and read on the A scale under the left-hand end of the slide the figures 455, which are the significant figures in antilog 1·658. The decimal point must be placed in the usual way. Then antilog 1·658 = 45·5.

CHAPTER XI

TRIGONOMETRY

The Trigonometric Ratios.—In the Mensuration chapter the *right-angled triangle* is referred to as being of considerable importance —it is, in fact, the basis of a large branch of mathematics known as Trigonometry, which is of supreme importance in all branches of engineering and physical science.

At X, Fig. 150, are shown several right-angled triangles of various sizes, but all having the same acute angle A. The measured lengths of the various sides are shown. If now for every triangle, such as ABC, we calculate the value of the ratio—

$$\frac{\text{vertical side } (\textit{such as } BC)}{\text{inclined side } (\textit{such as } AB)}$$

we find the following results—

$$\text{In triangle ABC, } \frac{BC}{AB} = \frac{1{\cdot}44''}{1{\cdot}7''} = 0{\cdot}847$$

$$\text{In triangle ADE, } \frac{DE}{AD} = \frac{2{\cdot}08''}{2{\cdot}45''} = 0{\cdot}849$$

$$\text{In triangle AFG, } \frac{FG}{AF} = \frac{2{\cdot}88''}{3{\cdot}4''} = 0{\cdot}847$$

allowing for small errors in drawing and measurement, the value of this ratio is evidently *constant.*

If another set of right-angled triangles be taken, as at Y, having a different common acute angle, a similar result will be found; the ratio $\frac{\text{vertical side}}{\text{inclined side}}$ is 0·839 for all three triangles. This ratio has evidently a different value for different angles, but is always the same for the same angle whatever be the size of the triangle.

Again, the ratio $\frac{\text{horizontal side}}{\text{inclined side}}$ may be calculated; thus at

X in Fig. 150, $\frac{AC}{AB} = \frac{0{\cdot}9''}{1{\cdot}7''} = 0{\cdot}529$ and this ratio also will be found to be constant for all right-angled triangles having the same acute angle. These ratios are instances of what are called the **trigonometric ratios** of the angle at A.

Referring now to (Z) Fig. 150, the sides of the right-angled

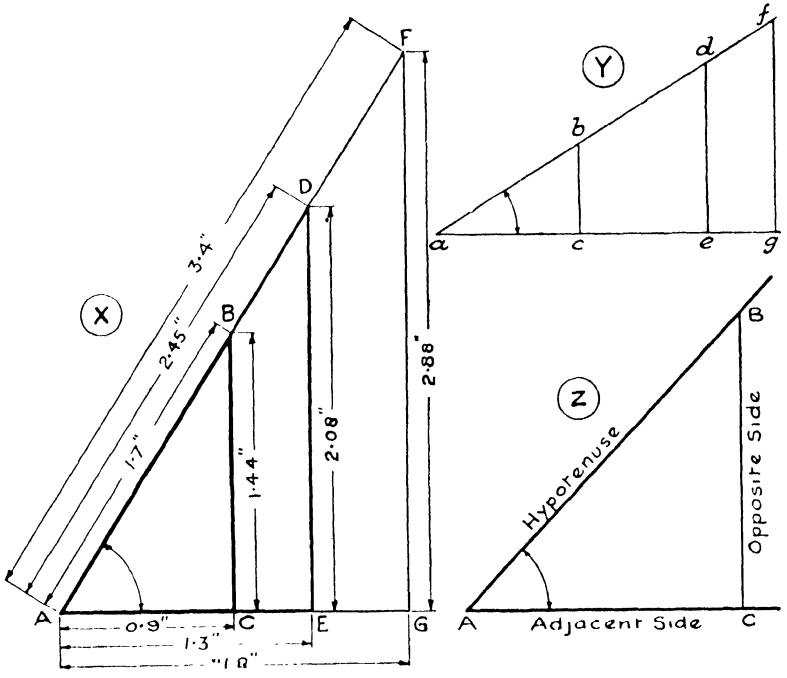


Fig. 150.—Right-Angled Triangles.

triangle are given more general names with reference to their positions regarding the angle A; thus,

The side BC, lying opposite to the angle A, is the **opposite side.**

The side AC, the shorter side enclosing the angle A, is the **adjacent side.**

The side AB, the longest side, is already known as the **hypotenuse.**

Of the various ratios that may be taken among the three sides, the three following are the most important :—

The ratio $\frac{\textbf{opposite side}}{\textbf{hypotenuse}} = \frac{BC}{AB}$ called the **sine.**

The ratio $\frac{\textbf{adjacent side}}{\textbf{hypotenuse}} = \frac{AC}{AB}$ called the **cosine.**

The ratio $\frac{\textbf{opposite side}}{\textbf{adjacent side}} = \frac{BC}{AC}$ called the **tangent.**

For convenience in writing in formulæ, etc., these names are abbreviated into the following forms :—

sin A for sine of the angle A.
cos A for cosine of the angle A.
tan A for tangent of the angle A.

The values of these ratios for all angles could evidently be obtained by careful drawing and measurement and subsequent division, a right-angled triangle being formed for every angle dealt with by drawing a line perpendicular to either side to cut the other side. Such a method would be liable to errors of drawing and measurement which for certain angles would be of serious proportions, and in all cases would be sufficiently large to limit the accuracy of the resulting quotients to a greater extent than would be permissible for many applications. It is possible, however, to calculate the values of the ratios, to any desired degree of accuracy, by the methods of advanced mathematics, and the values so obtained are recorded in various tables for all angles at intervals of every second. Such tables are naturally very large, and various abridged forms are obtainable. For many practical problems it is sufficient if the tables give the values of the ratios for every single degree of angle; in this case the tables can be arranged to occupy a single book page and are reproduced on p. 458.

Exercises 94. On the Trigonometric Ratios.

1. Make a sketch of a right-angled triangle ABC, with the right angle at B. Which trigonometric ratios of the angle A will be found by evaluating the following ratios :—

$$(a)\ \frac{BC}{AB} \qquad (b)\ \frac{AB}{AC} \qquad (c)\ \frac{BC}{AC}$$

and what ratios of angle C are represented by

$$(d)\ \frac{AB}{BC} \qquad (e)\ \frac{BC}{AC}.$$

2. A right-angled triangle PQR is shown fully dimensioned in Fig. 151. Calculate the values of the following ratios for the angles P and R :—

(*a*) tan P, (*b*) cos R, (*c*) sin P, (*d*) cos P, (*e*) tan R, (*f*) sin R.

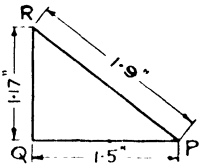


Fig. 151.

3. Draw very carefully angles of 35°, 68°, 20°, 47° and 73°, and from actual measurement calculate the values of the following trigonometric ratios :—

(*a*) sin 20°, (*b*) tan 35°, (*c*) tan 47°, (*d*) cos 68°, (*e*) sin 73°, (*f*) cos 35°.

Ratios of the Angles 30°, 45°, 60°.—These angles are of frequent occurrence and their ratios will therefore be considered before examining the complete table. The three angles are shown in

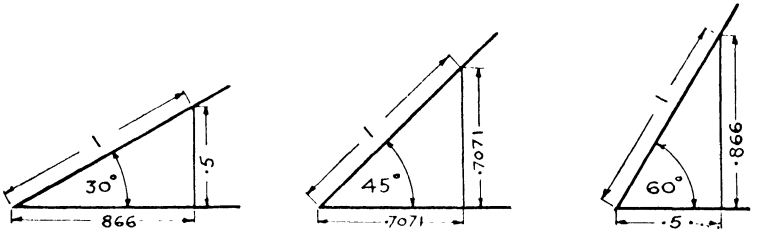


Fig. 152.—Trigonometric Ratios of Certain Important Angles.

Fig. 152, the hypotenuse of the right-angled triangle being taken as 1 in each case.

30°	45°	60°
$\sin 30° = \frac{·5}{1} = ·5$	$\sin 45° = \frac{·7071}{1} = ·7071$	$\sin 60° = \frac{·866}{1} = ·866$
$\cos 30° = \frac{·866}{1} = ·866$	$\cos 45° = \frac{·7071}{1} = ·7071$	$\cos 60° = \frac{·5}{1} = ·5$
$\tan 30° = \frac{·5}{·866} = ·5774$	$\tan 45° = \frac{·7071}{·7071} = 1$	$\tan 60° = \frac{·866}{·5} = 1·732$

It should be carefully noted that sin 60° is *not* twice sin 30°. In fact, no trigonometric ratio increases in the same proportion as the angle. This point is again referred to on p. 432.

Ratios of the Angles 0° and 90°.—At X, Fig. 153, is shown a very acute angle, viz. one of 5°, across which has been drawn a line BC, perpendicular to the side AC. The values of the ratios "sine," "cosine," and "tangent" for the angle A are expressed, of course, as—

$$\sin A = \frac{BC}{AB} \qquad \cos A = \frac{AC}{AB} \qquad \tan A = \frac{BC}{AC}$$

It should be noticed (*a*) that the length of BC is small compared with the length of AB, and also with the length of AC, and (*b*) that the length of AC is slightly less than, but nearly equal to, the length of AB. The values of the ratios sin A and tan A will then be small since the numerator is in both cases much smaller than the denominator ; actually the values are 0·0872 and 0·0875 respectively. In the case of the cosine, the numerator and denominator are very nearly equal, although the latter is the larger, so that the ratio is very nearly 1 ; actually it is 0·9962.

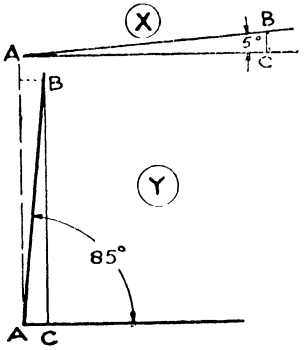


Fig. 153.

Now imagine that the angle A becomes steadily smaller owing to the line AB swinging about the point A towards the side AC. Then BC becomes still smaller, until when AB and AC are coincident, *i.e* the angle is zero, BC = 0. At the same time, the length of AC increases and gets nearer and nearer to the length of AB until, when the angle A is zero, AB and AC are equal in length and are, in fact, identical lines. With an angle of 0°, therefore, the length of the " opposite " side of the right-angled triangle is 0 and the length of the " adjacent " side is equal to the length of the " hypotenuse." The values of the three trigonometric ratios are then—

$$\textbf{sin } 0° = \frac{\text{opposite side}}{\text{hypotenuse}} = \frac{0}{\text{hypotenuse}} = \mathbf{0}$$

since the result of dividing nothing by any number is still nothing.

Similarly,

$$\textbf{tan } 0° = \frac{\text{opposite side}}{\text{adjacent side}} = \frac{0}{\text{adjacent side}} = \mathbf{0}$$

But,

$$\textbf{cos } 0° = \frac{\text{adjacent side}}{\text{hypotenuse}} = \mathbf{1},$$ since the top and bottom quantities are equal.

An angle of 85° is shown at Y, Fig. 153, lettered in the same way as the preceding angle of 5°. In this case it is the " adjacent "

side AC which is small, while the "opposite" side BC is nearly equal to the hypotenuse. The value of the cosine $\left(= \frac{\text{adjacent}}{\text{hypotenuse}}\right)$ is then small, being 0·0872, while the value of the sine $\left(= \frac{\text{opposite}}{\text{hypotenuse}}\right)$ is very nearly 1, being 0·9962. The value of the tangent is comparatively large, since it is the result of dividing a long length (opposite side) by a short length (the adjacent side); the actual value is 11·43.

If now this angle be increased until it ultimately becomes 90°, then the adjacent side BC diminishes to zero and the opposite side becomes equal to, and coincident with, the hypotenuse. The values of the three ratios then become—

$$\textbf{sin } 90° = \frac{\text{opposite side}}{\text{hypotenuse}} = \textbf{1,}$$ since the top and bottom quantities are equal.

$$\textbf{cos } 90° = \frac{\text{adjacent side}}{\text{hypotenuse}} = \frac{0}{\text{hypotenuse}} = \textbf{0.}$$

$$\tan 90° = \frac{\text{opposite side}}{\text{adjacent side}} = \frac{\text{opposite side}}{0}.$$

Now it is impossible to say what value is obtained by dividing any number by nothing. The result is larger than any number that we can think of and is termed **infinity**; it is usually denoted by the symbol ∞.

Then $\textbf{tan } 90° = \infty$.

Relation between Sine and Cosine of Complementary Angles.—A very important relation exists between the sines and cosines of any two angles which together total 90°. Such angles are termed "complementary angles" and each is said to be the "complement" of the other. Thus 30° is the complement of 60°, 20° is the complement of 70°, 45° is the complement of itself and so on, the sum of the two angles being 90° in every case. Thus the complement of any angle is the amount by which the angle differs from 90°.

Referring back to the right-angled triangle at (Z), Fig. 150, the values of the ratios sine and cosine of the angle A have already been expressed in terms of the sides AB, AC, BC. It is also possible to express the values of the same ratios for the angle B in terms of the same sides. The general names affixed to the sides in the figure will need changing over when referring to the angle B;

thus although AB will still be the hypotenuse, AC will become the " opposite " side and BC will be the " adjacent " side.

Then sin B will be given by $\frac{\text{(opposite)}}{\text{hypotenuse}} = \frac{AC}{AB}$

and cos B will be given by $\frac{\text{(adjacent)}}{\text{hypotenuse}} = \frac{BC}{AB}$.

But the ratio $\frac{AC}{AB}$ is already known as cos A.

and the ratio $\frac{BC}{AB}$ is already known as sin A.

Thus, in the triangle ABC of (Z), Fig. 150,

$$\sin A = \frac{BC}{AB} = \cos B.$$

and $$\cos A = \frac{AC}{AB} = \sin B.$$

Now the three angles of any triangle always total 180° (refer p. 264). Then, since the angle C is a right angle, *i.e.* = 90°, the remaining two angles A and B must together be equal to 90°, *i.e.* the angles A and B are " complements " of each other,

$$\therefore \quad B = 90° - A \quad \text{and} \quad A = 90° - B.$$

Then, substituting (90 — A) in place of B in the relations above,

$$\mathbf{\sin A = \cos (90 - A)}$$
$$\mathbf{\cos A = \sin (90 - A)}$$

or in general terms, the **sine and cosine of an angle are equal to the cosine and sine, respectively, of its complement.**

Thus $\sin 30° = \cos 60°$ $\qquad$ $\sin 20° = \cos 70°$
$\cos 30° = \sin 60°$ $\qquad$ $\sin 45° = \cos 45°$

and so on.

The last equation is easily seen to be true on referring to any 45° right-angled triangle, in which the " adjacent " and " opposite " sides are equal.

The Table of Trigonometric Ratios.—The abridged table giving the values, to 4 decimal places, of the various ratios for angles from 0° to 90° at intervals of one degree appears on p. 458. By making use of the relation discussed in the preceding section, it is possible to tabulate the values of *both* sine and cosine with only 91 values for the 91 angles involved, in place of the 182 values that would be necessary were the table arranged in a purely straight-

forward manner. The method of arrangement will be easily understood after a little use of the table.

The table contains 10 columns, of which the two outer columns contain the values of the angles in degrees, while the four central columns contain the values of the sines, cosines and tangents of the angles. From 0° to 45° the values of the angles appear in the extreme left-hand column and read downwards, while the angles from 45° to 90° appear in the extreme right-hand column and read upwards.

From **0° to 45°** the values of the sine, cosine and tangent are given in the columns so headed **at the top** of the table. Thus in the column headed "Sine" *at the top*, the figure ·5000 is found opposite the angle 30° in the extreme *left-hand* column; then sin 30° = 0·5. On the same horizontal line, in the columns headed "Cosine" and "Tangent" *at the top*, are found the values ·8660 and ·5774 respectively; then cos 30° = 0·866 and tan 30° = 0·5774.

From **45° to 90°** the values of the various ratios are given in the appropriate columns headed **at the foot** of the table. Thus the value of sin 75° is found by taking the value opposite 75 on the *extreme right*, but in the column headed "Sine" *at the foot* of the table, viz. ·9659.

Example 381.—Read the values of the following ratios from the table:—sin 81°, sin 10°, tan 55°, cos 37°, tan 5°, cos 63°.

Sin 81°.—This angle is over 45°, and therefore the right-hand column of angles will be referred to and the value taken from the column headed "Sine" at the foot of the table. Opposite 81° on the extreme right and above "Sine" at the foot is found the value ·9877. Then sin 81° = ·9877

Sin 10°.—Being smaller than 45°, the angle will be found in extreme left-hand column, and the column headed "Sine" at the top is used. Opposite 10 on the extreme left and under "Sine" at the top is found ·1736. Then sin 10° = 0·1736

Tan 55°.—Refer to right-hand column for angle and the heading "Tangent" at the foot. Opposite 55° and above "Tangent" is found the value . . . . . 1·4281

Cos 37°.—Use left-hand column of angles and top headings. Opposite 37° and under "Cosine" is found the value . . . ·7896

Tan 5°.—Use left-hand column and top heading. Opposite 5° and under top heading of "Tangent" find . . . . ·0875

Cos 63°.—Use right-hand column and bottom titles. Opposite 63° and above "Cosine," find . . . . . ·4540

Exercises 95. On reading Trigonometric Ratios from the Table.

Find, from the table on p. 458, the values of the following ratios :— (1) cos 15°, (2) tan 71°, (3) sin 18°, (4) sin 44°, (5) sin 46°, (6) tan 28°, (7) tan 89°, (8) cos 46°, (9) cos 77°, (10) cos 4°.

It is frequently necessary to determine the angle for which one of the trigonometric ratios has a known or given value, *i.e.* the table has to be used in the reverse order. It must be remembered that in each case two columns are devoted to the values of each of the ratios tabulated, one headed at the top and the other at the bottom of the table. Then, if the name of the ratio is at the *top* of the column in which the given value is found, the angle must be read from the extreme *left-hand* column; but if the name of the ratio is at the *foot* of the column, then the angle must be read from the extreme *right-hand* column.

In cases where the exact value given is not found in the table it will be sufficient for the purposes of this book if the nearest value is referred to.

Example 382.—Find from the table the angles whose sines have the following values :—

(*a*) ·2079; (*b*) ·9063; (*c*) ·7071; (*d*) ·6947.

(*a*) An examination of the two " Sine " columns shows that the value ·2079 occurs in the column whose title is at the *top*. The angle must then be read from the *left-hand* column and is . . 12°

(*b*) The sine ·9063 occurs in the column marked " Sine " at the *bottom*. The angle must then be read from the *right-hand* column and is . . . . . 65°

(*c*) The sine ·7071 is found in both columns of sines and 45° is read in both angle columns.

(*d*) The sine ·6947 is found in the column marked " Sine " at the *top*. The *left-hand* column of angles is then referred to and gives the value . . . . . 44°

Example 383.—Find the angles whose tangents are (*a*) 1·8807, (*b*) ·1051, and also (*c*) the angle whose cosine is ·5592.

(*a*) The tangent 1·8807 occurs in the column with title " Tangent " at the *foot*. The corresponding angle is then read from the *right-hand* column and is . . . . . 62°

(*b*) The tangent ·1051 is found in the column labelled " Tangent " at the *top*. The *left-hand* column is then referred to and gives the value . . . . . 6°

(*c*) The cosine ·5592 occurs in the column entitled "Cosine" at the *foot* of the table. The *right-hand* column of angles is then referred to and gives the value . . . . . 56°

Exercises 96. On reading the Trigonometric Table (*continued*).

Find the angles whose cosines are :—

(1) ·4384; (2) ·9703; (3) ·7771; (4) ·0349.

Find the angles whose tangents are :—

(5) ·1405; (6) 2·6051; (7) 1·0724; (8) ·6009.

Find the angles whose sines have the following values :—

(9) ·3090; (10) ·7547; (11) ·5150; (12) ·9511.

Use of the Trigonometric Ratios in Formulæ.—The various ratios are frequently met with in engineering formulæ, and in a great many such cases no more knowledge of trigonometry is required beyond the ability to read values from the tables, and once the numerical values have been substituted the expressions require only ordinary evaluation.

Certain points of writing must be emphasized, however, to ensure that the ordinary algebraic rules of expression are still complied with.

Thus, if a trigonometric ratio has to be raised to any power, the index must *not* be written at the right-hand top corner of the abbreviated expression for the ratio, unless brackets be used. Thus $\sin A^2$ does *not* represent the square of "the sine of angle A"; reading this expression in strict accordance with the rules of symbolic expression explained in Chapter III, it expresses the sine of the "square of angle A." The error is rectified if the expression be written as $(\sin A)^2$, but this form is never used, and the general practice is to write the index immediately after the abbreviated name of the ratio, thus $\sin^2 A$. This is read as "sine squared A" and indicates that the sine of angle A is to be squared. Emphasizing by a numerical example, suppose it necessary to evaluate the "square of sin 8°." Now sin 8° = ·1392, and we then write—

$$\sin^2 8 = (\cdot 1392)^2 = \cdot 0183.$$

If the form $\sin 8^2$ had been used, then, in strict accordance with principles, it would follow that—

$$\sin 8^2 = \sin 64 = \cdot 8988,$$

which is widely different from what was intended.

When the angle involved in the expression of a ratio is compound as, for example, a sum or difference of two angles, then the compound form must be enclosed in brackets. For example, the sine of the angle produced, say, by adding angles denoted by P and Q must be expressed as $\sin(P + Q)$; the form $\sin P + Q$ has not the meaning intended.

In cases where any ratio of an angle is to be multiplied by a factor, then the factor should always be written *in front* of the ratio name. Thus "twice the sine of angle A" should be written as $2 \sin A$ and *not* as $\sin A \times 2$; the latter form is ambiguous, as it may also mean "the sine of twice the angle A." If it is desired to express the latter idea the appropriate form of expression is $\sin 2A$. It has already been stated, and it will be further explained later, that no ratio increases in proportion to the angle, so that the expressions $2 \sin A$ and $\sin 2A$ indicate quite different values, as may easily be seen by giving any value to A, and evaluating the two expressions.

Expressions of the form $\cos \frac{A}{2}$ are also met with and should be written very carefully to avoid any possible confusion with the form $\frac{\cos A}{2}$ which means $\frac{1}{2} \cos A$.

Example 384.—The power developed, in watts, by an alternating electric current is given by the expression $AV \cos \varphi$, where A = current in amperes, V = voltage, and φ = the "angle of lag." If $A = 1{\cdot}05$, $V = 500$ and $\varphi = 32°$, find the power developed.

$$\text{Power} = AV \cos \varphi$$

Substituting given values—

$$= 1{\cdot}05 \times 500 \times \cos 32°$$
$$= 1{\cdot}05 \times 500 \times {\cdot}8480$$
$$= \underline{445{\cdot}2 \text{ watts.}}$$

From Table, $\cos 32° = {\cdot}8480$

Example 385.—The efficiency of a screw and nut, where θ is the angle of the thread and φ is the "angle of friction," is equal to $\frac{\tan \theta}{\tan(\theta + \varphi)}$. Calculate the efficiency for a case where $\theta = 5°$ and $\varphi = 8°$.

$$\text{Efficiency} = \frac{\tan \theta}{\tan(\theta + \varphi)}$$

Substituting given values—

$$= \frac{\tan 5}{\tan(5 + 8)} = \frac{\tan 5}{\tan 13}$$

Taking values from table—

$$= \frac{{\cdot}0875}{{\cdot}2309} = {\cdot}3789, \text{ say } \underline{{\cdot}379}.$$

Example 386.—When choosing a stock gear wheel cutter for milling spiral wheel teeth, the cutter is chosen by reference to the number of

teeth in an "equivalent spur wheel." The number of teeth in this "equivalent spur wheel" is $\frac{N}{\cos^3\alpha}$, where N = number of teeth required in spiral wheel and α = angle of the spiral teeth. If N = 12 and α = 45°, find the value of the given expression, taking the nearest whole number.

Number of teeth in "equivalent spur wheel"

$$= \frac{N}{\cos^3 \alpha}$$

Substituting given values— $$= \frac{12}{\cos^3 45} = \frac{12}{(\cdot 7071)^3}$$

$$= \frac{12}{\cdot 3535} = 33\cdot 94, \textit{ i.e. } \underline{34 \text{ teeth.}}$$

Example 387.—The following equation relates to the combination of stresses in a piece of material:—

$$\cos 2\theta = \frac{p - q}{p + q}.$$

If $p = 5\cdot 05$ and $q = 1\cdot 05$, find the value of the angle θ.

$$\cos 2\theta = \frac{p - q}{p + q}$$

Substituting— $$= \frac{5\cdot 05 - 1\cdot 05}{5\cdot 05 + 1\cdot 05} = \frac{4}{6\cdot 1} = \cdot 6558$$

$$\therefore \quad \cos 2\theta = \cdot 6558.$$

Referring to the table, the value ·6558 does not appear, but the nearest cosine value is ·6561, which is cos 49°,

$$\therefore \quad 2\theta = 49^\circ$$

$$\therefore \quad \theta = \frac{49}{2} = \underline{24\cdot 5^\circ}.$$

Exercises 97. On Evaluation involving Trigonometric Ratios.

1. When a ray of light is bent in passing through a piece of glass, the value of the expression $\frac{\sin i}{\sin r}$ is termed the "refractive index" of the glass, where i denotes the "angle of incidence" and r denotes the "angle of refraction." Calculate the value of the refractive index for the case where $i = 31^\circ$ and $r = 20^\circ$.

2. The expression $wh\frac{1 - \sin\varphi}{1 + \sin\varphi}$ relates to earth pressure on retaining walls. Find its value when $w = 125$, $h = 20$, and $\varphi = 22^\circ$.

3. The effective helix angle of an aeroplane propeller at various radii is given by the expression $\tan A = \frac{V}{2\pi nr}$ where A = helix

angle, V = translational velocity of the propeller, n = speed of rotation, r = radius. Determine the value of A when V = 190·8, n = 16·67, and r = 2·25.

4. The formula $\tan 2\theta = \frac{2t}{p - q}$ relates to " principal stresses." If p = 4·3, q = 3·63, and t = ·63, find the value of the angle θ.

5. The depth of a wall foundation is given by the expression

$$\frac{W}{Bw}\left(\frac{1 - \sin\varphi}{1 + \sin\varphi}\right)^2.$$

Find the value of the expression when W = 4500, B = 2, w = 125, and φ = 21°.

6. The velocity of the piston in a direct-acting engine is given by the expression $V\left(\sin\theta + \frac{\sin 2\theta}{2n}\right)$ where V = velocity of crank pin, θ = crank angle, and n = ratio of connecting rod length to crank length. If V = 7·9, θ = 38°, and n = 4·5, find the value of the expression.

7. When measuring distances by a tacheometer (a surveyor's instrument) the expression $Cs\cos^2\theta + k\cos\theta$ occurs. If C = 100, s = 1·18, k = 1·25, and θ = 28°, find the value of the expression.

8. In connection with Question 7, above, the expression $\frac{Cs}{2}\sin 2\theta + k\sin\theta$ also occurs. Using the same values as in Question 7, find the value of this expression.

9. The formula $\frac{P}{W} = \tan(\beta - \alpha)$ refers to an inclined plane. If P = 4·05, W = 58, and α = 7°, find the value of the angle β.

10. The angle θ of a conical plug is given by the expression $\tan\frac{\theta}{2} = \frac{D - d}{2l}$ where D and d are the large and small diameters respectively, and l is the axial length between them. Find the angle of a plug in which D = 2·5, d = ·95, and l = 2·25.

Problems involving Simple Trigonometry.—Problems frequently occur, particularly in drawing office and tool room, which require for their solution the use of the principles discussed in the earlier portions of this chapter. As with most examples of problem nature, it is impossible to give a rigid method by which such problems may be solved, and the process must be assimilated by working through as many illustrative examples as possible.

In all cases a sketch showing the conditions of the problem must be made, in which right-angled triangles should be sought for and, if necessary, introduced by the addition of suitable lines. A knowledge of geometry is of great assistance in this connection. From the various right-angled triangles then available, some

trigonometric ratio of some angle can usually be expressed in terms of one known side and one unknown side, whence the value of the latter may be calculated. A few illustrative examples are appended.

Example 388.—The height of a factory chimney stack is to be found. A theodolite (*i.e.* an angle-measuring instrument) is set up

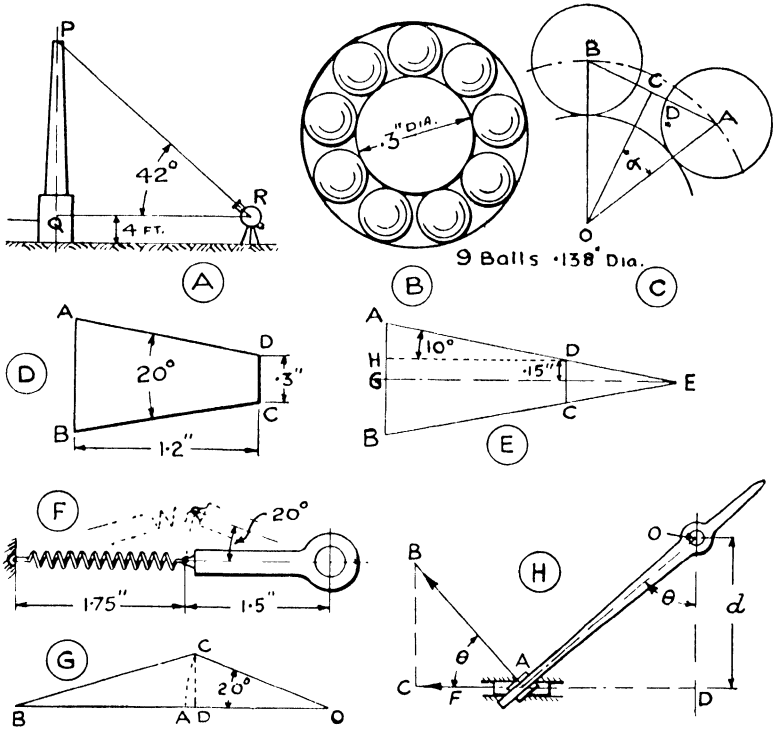


Fig. 154.—Diagrams for Examples 388–392.

at a point 200 ft. away from the base of the stack on level ground, and the top of the chimney is sighted, as indicated at (A), Fig. 154. The "angle of elevation" is found to be 42°. Calculate the height of the chimney if the theodolite stands 4 ft. high.

A right-angled triangle is evidently formed by the centre line of the stack (vertical), the line of sight, and the horizontal through the theodolite centre. In this triangle PQR, the side QR and the angle R are known, and the length of PQ is required.

Then since $$\frac{PQ}{QR} = \tan R$$

$$\frac{PQ}{200} = \tan 42° = \cdot 9004.$$

$$\therefore \quad PQ = \cdot 9004 \times 200 = 180\cdot 08', \text{ say } 180 \text{ ft.}$$

The height of the stack is evidently 4 ft. greater than this, since the point Q is 4 ft. above ground. Therefore the total height of the stack = 184 ft.

Example 389.—The proposed dimensions of a small ball bearing are given at (B), Fig. 154. Assuming the balls to be evenly spaced, calculate the clearance between two adjacent balls. (*Note.*—This clearance must be measured along the straight line joining the centres of the two adjacent balls.)

A sketch showing two adjacent balls to a larger scale is shown at (C), Fig. 154, in which the ball centres are lettered A and B respectively, and the centre of the inner race is lettered O.

Then on the assumption that the 9 balls are evenly spaced, the angle BOA is $\frac{1}{9}$ of a complete circle

$$i.e. \quad \text{angle BOA} = \frac{360°}{9} = 40°.$$

The triangle AOB is "isosceles," *i.e.* side OA = side OB. Then, if a line OC be drawn perpendicular to AB, this line OC will divide both the angle BOA and the side AB into equal parts.

$$\therefore \quad \text{angle COA} = \alpha \text{ (say)} = \frac{40°}{2} = 20°.$$

Now in the triangle AOC,

$$\frac{AC}{OA} = \sin \alpha,$$

and OA = radius of inner race + radius of ball

$$= \frac{\cdot 3''}{2} + \frac{\cdot 138''}{2} = \cdot 15'' + \cdot 069'' = \cdot 219''.$$

$$\therefore \quad \frac{AC}{\cdot 219} = \sin 20° = \cdot 3420$$

whence $$AC = \cdot 3420 \times \cdot 219'' = \cdot 0749''.$$

Now $$DC = AC - \text{ball radius}$$

$$= \cdot 0749 - \cdot 069 = \cdot 0059''$$

and DC is half the actual clearance,

$$\therefore \quad \text{Clearance} = 2 \times \cdot 0059 = \cdot 0118''$$

say $\cdot 012''$ to the nearest thousandth.

Example 390.—A gauge was required of the form and dimensions shown at (D), Fig. 154. The method of working was to first make a complete isosceles triangle AEB as shown at (E), Fig. 154, the sides

AB, AE, and BE, and also (as a check) the height, EG, being measured by micrometer. The top triangle EDG was then cut off until the required height of the remaining piece was obtained.

Calculate the dimensions AB, AE (which equals BE), and EG, to the nearest thousandth of an inch.

If the dotted line DH be drawn perpendicular to the base AB, then the triangle ADH is similar to the triangle AEG, and the angle ADH = 10°, *i.e.* half the apex angle, since the triangle AEB is symmetrical about the centre line EG.

Then, in triangle ADH

$$\frac{AH}{DH} = \tan 10°$$

$$\therefore \frac{AH}{1{\cdot}2} = {\cdot}1763. \qquad \therefore \quad AH = {\cdot}1763 \times 1{\cdot}2 = {\cdot}2116''.$$

Then the half width at the bottom,

$$= AG = AH + HG$$
$$= {\cdot}2116 + {\cdot}15 = {\cdot}3616''.$$

Then the whole width $\underline{AB} = {\cdot}3616 \times 2 = {\cdot}7232''$

say $\underline{{\cdot}723''}$.

Now in the triangle AEG,

$$\frac{AG}{AE} = \sin 10°$$

$$\therefore \frac{{\cdot}3616}{AE} = {\cdot}1736 \qquad \therefore \quad \underline{AE} = \frac{{\cdot}3616}{{\cdot}1736} = \underline{2{\cdot}083''.}$$

Also, in the triangle AEG,

$$\frac{AG}{EG} = \tan 10°$$

$$\therefore \frac{{\cdot}3616}{EG} = {\cdot}1763 \qquad \therefore \quad \underline{EG} = \frac{{\cdot}3616}{{\cdot}1763} = \underline{2{\cdot}051''.}$$

The required dimensions are then

$\underline{AB = {\cdot}723''}$ $\underline{AE = 2{\cdot}083''}$ $\underline{EG = 2{\cdot}051''}$

Example 391.—The movement of a rocking arm in a certain instrument is opposed by the extension of a helical spring, arranged as indicated at (F), Fig. 154. Determine the extension of the spring corresponding to a maximum rotation of the arm of 20°.

A skeleton diagram of the mechanism is shown at (G), Fig. 154, where OA is the initial position of the arm and OC is the deflected position, whilst AB is the unstretched, and BC the stretched, positions of the spring. The extension of the spring will be the difference between the lengths BC and AB.

The line CD is drawn perpendicular to the line BO. Then from the triangle OCD the lengths of CD and OD may be found.

Thus $\frac{CD}{OC} = \sin 20°$

$\therefore \quad \frac{CD}{1·5} = ·3420 \qquad \therefore \quad CD = ·342 \times 1·5 = ·513''.$

Also $\frac{OD}{OC} = \cos 20°$

$\therefore \quad \frac{OD}{1·5} = ·9397 \qquad \therefore \quad OD = ·9397 \times 1·5 = 1·409''.$

Now $OB = OA + AB$

$= 1·5 + 1·75 = 3·25''$

and $BD = OB - OD$

$= 3·25 - 1·409 = 1·841''.$

Now in the right-angled triangle BCD

$BC^2 = BD^2 + CD^2$ (refer p. 266)

$\therefore \quad BC^2 = 1·841^2 + ·513^2$

$= 3·388 + ·263 = 3·651$

$\therefore \quad BC = \sqrt{3·651} = 1·911$

$\therefore$ Extension of spring $= BC - AB$

$= 1·911 - 1·75 = \underline{·161''.}$

Example 392.—The arrangement of a ship's steering gear is shown at (H), Fig. 154. The turning moment on the tiller is the product of the force represented by AB and the arm OA. Find an expression for this turning moment in terms of the applied force F, represented by AC, the distance d and the angle θ.

From the information given

Turning moment = force AB × length OA . . . . . (1)

It is then necessary to find (*a*) value of AB in terms of F and θ from the triangle ABC, and (*b*) the value of OA from the triangle OAD.

Now in triangle ABC

$\frac{AC}{AB} = \cos\theta$

$\therefore \quad \frac{F}{AB} = \cos\theta \quad \text{and} \quad AB = \frac{F}{\cos\theta} \quad . \quad . \quad . \quad . \quad (2)$

And in triangle OAD,

$\frac{OD}{OA} = \cos\theta$

$\therefore \quad \frac{d}{OA} = \cos\theta \text{ and } OA = \frac{d}{\cos\theta} \quad . \quad . \quad . \quad . \quad (3)$

Then substituting in (1) the values found for AB and OA in (2) and (3) respectively,

Turning moment $= \frac{F}{\cos\theta} \times \frac{d}{\cos\theta} = \underline{\frac{Fd}{\cos^2\theta}}$

Exercises 98. On Simple Trigonometrical Problems.

[*Letters, thus* (*E*) *refer to Fig.* 155.]

1. The "gliding angle" of an aeroplane is given as "1 in 7·1," indicating that, when gliding, a horizontal distance of 7·1 units is travelled for a loss in height of 1 unit, as indicated at (A). What is the angle of the gliding path with the horizontal, *i.e.* angle θ?

2. The maximum gradient on the Pilatus mountain railway is given as 48%, *i.e.* a rise of 48 units for a distance of 100 units along the slope. What is the angle of slope to the horizontal, *i.e.* angle α at (B).

3. At (C), OE represents the effective radius of a steam engine eccentric. The "angle of advance" θ is arranged so that the projection of OE on the centre line, *i.e.* OP, equals "lap + lead." What must be the value of θ if the effective radius is $3\frac{1}{4}''$, the lead is to be $\frac{1}{8}''$ and the lap is $1\frac{3}{4}''$?

4. The Whitworth standard screw thread has an included angle between the side slopes of 55°, while the crest and root are so rounded that the sharp points are removed for a depth of $\frac{1}{6}$ of the pitch p, as shown at (D). Calculate, in terms of p, the value of the full depth d and also the actual depth D. (*Note.*—It will be sufficiently accurate if the value of any trigonometric ratio of $27\frac{1}{2}°$ be taken as half-way between the values for 27° and 28°.)

5. The governor mechanism of a centrifugal type tachometer is shown at (E). When running at the slowest and fastest speeds respectively, the governor runs "solid" in the positions shown at (F) and (G). The angles θ and φ for these positions are of importance with regard to the spring control. Determine the values of θ and φ from the dimensions given in the figure (E). The necessary right-angled triangles are indicated in figures (F) and (G).

6. The diagram at (H) indicates the skeleton arrangement of an Izod impact testing machine; OA is the initial position of a swinging hammer which, after breaking the specimen situated at C, rises to a position OB. The energy absorbed by the specimen is equal to W × distance DE, where W is the weight of the hammer. Find an expression for this energy in terms of the quantities W and L and the angles α and β.

7. A plug gauge is to be made of the form and dimensions shown at (J); the circular portion, if completed, passes through the corner A of the square ABCD. Calculate the diameter to which the plug must be ground before the shaded portions are cut away. [*Hint.*—Referring to the sketch at (K), the *radius* of the plug = OA = OE = EF − OF, and OF may be expressed in terms of OA by reference to the triangle OAF.]

8. A dovetail gap in a form gauge is to be tested by the fit of a turned disc as shown at (L). Calculate the necessary diameter of disc for the dimensions shown. [*Hint.*—The triangle essential for the solution is shown at (M) as OAB. Determine the value of r by reference to the angle OAB and the side AB.]

9. When measuring the thickness of spur wheel teeth by a gear tooth vernier caliper, as indicated at (N), the chordal thickness ADB and the height CD are measured. These dimensions differ from the

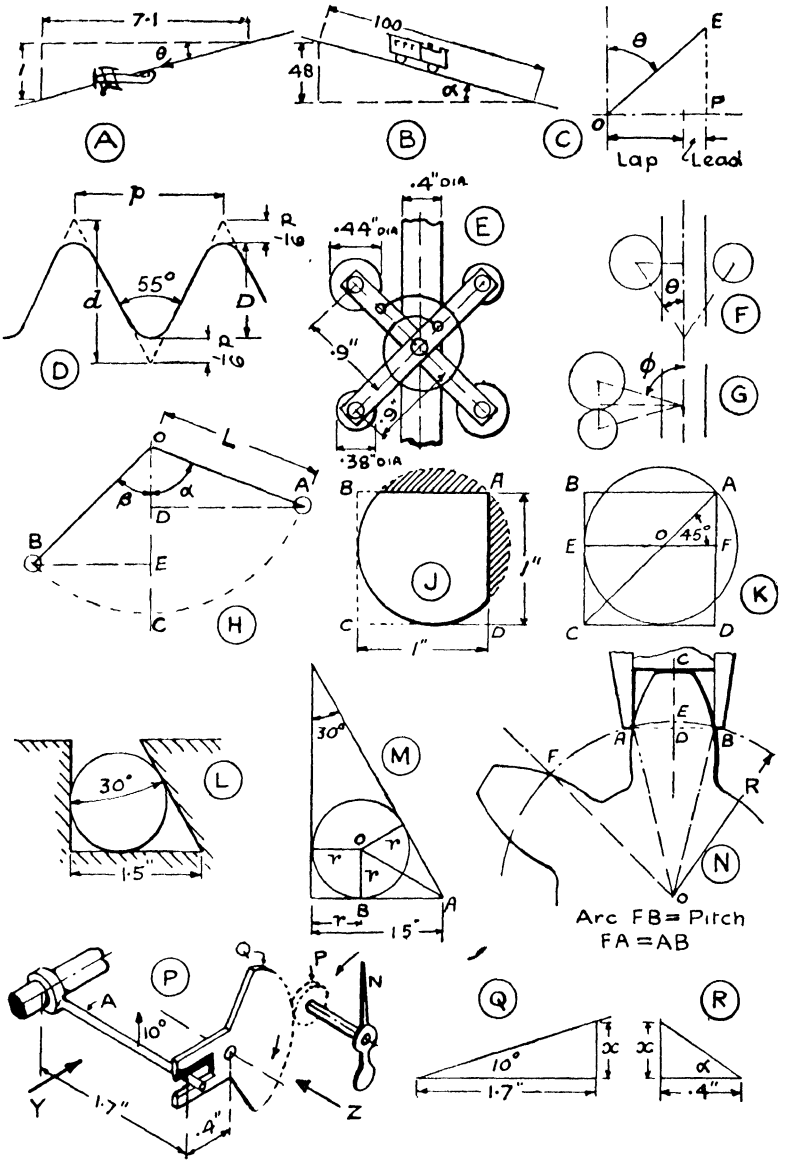


Fig. 155.—Diagrams for Exercises 98.

thickness and height determined from the circular pitch and need to be specially computed when the number of teeth in the gear is small and the pitch is coarse.

If R = pitch radius of wheel and N = number of teeth in the wheel (*i.e.* number of pitches), determine expressions for (*a*) the chordal thickness ADB, and (*b*) the additional height DE. [*Hint.*—Work from the triangle AOD.]

10. At (P) is shown the indicating portion of a certain gyroscopic instrument in which the tilting of an arm A is multiplied by the quadrant Q and the pinion P, the spindle of the latter carrying a pointer N. The essential dimensions of the multiplying gear being as shown, calculate the angle that the pointer will move if the arm A tilts 10° from the mean position shown; the ratio of the quadrant and pinion gear is 4 to 1. [*Hint.*—The skeleton diagram at (Q) is a view of the mechanism looking in the direction of the arrow Y; x is the vertical movement of the tilting arm in the plane of the quadrant Q. The diagram at (R) is a view looking along the arrow Z; the angle α turned through by the *quadrant* is determined by the value of x.]

The Reciprocal Trigonometric Ratios.—The ratios already considered, viz. sine, cosine and tangent, are by far the most important trigonometric ratios, but there are a few others which, although of lesser importance, are occasionally useful.

Referring to the right-angled triangle at (Z), Fig. 150, the three chief ratios of the angle A have been defined thus :—

$$\frac{\text{opposite side}}{\text{hypotenuse}} = \frac{BC}{AB} = \sin A; \quad \frac{\text{adjacent side}}{\text{hypotenuse}} = \frac{AC}{AB} = \cos A; \quad \frac{\text{opposite side}}{\text{adjacent side}} = \frac{BC}{AC} = \tan A.$$

Now if ratios be taken between the *same pairs of sides*, but in the *reverse order*, another three trigonometric ratios are obtained, which are named thus :—

$$\frac{\text{hypotenuse}}{\text{opposite}} = \frac{AB}{BC} = \textbf{Cosecant A}; \quad \frac{\text{hypotenuse}}{\text{adjacent}} = \frac{AB}{AC} = \textbf{Secant A};$$

$$\frac{\text{adjacent}}{\text{opposite}} = \frac{AC}{BC} = \textbf{Cotangent A}.$$

These names are abbreviated in a similar manner as in the case of the other ratios, thus :—

cosec A for cosecant of angle A.
sec A for secant of angle A.
cot A for cotangent of angle A.

It should be noted that each of the new ratios is derived from exactly the same two sides as those defining the more important ratios above, but the pairs of sides occur in the reverse order, *i.e.* the numerator and denominator are interchanged in every case.

Now, when a ratio between any two quantities is inverted,

the two forms are said to be " reciprocals " of each other (refer p. 79).

Therefore—

Sine and Cosecant are reciprocal ratios,
Cosine and Secant are reciprocal ratios,
Tangent and Cotangent are reciprocal ratios;

or, expressing these statements in equation form—

$$\text{cosec} = \frac{1}{\sin}; \quad \sec = \frac{1}{\cos}; \quad \cot = \frac{1}{\tan}$$

$$\sin = \frac{1}{\text{cosec}}; \quad \cos = \frac{1}{\sec}; \quad \tan = \frac{1}{\cot}.$$

The difference in the values of these ratios, for angles between 0° and 90° is that, whereas the sine and cosine are never greater than 1, the cosecant and secant are never less than 1. The cotangent is like the tangent in so far as its value varies from 0 to infinity.

It has already been shown that sine and cosine are inter-related for complementary angles,

i.e. $\sin\theta = \cos(90 - \theta)$ and $\cos\theta = \sin(90 - \theta)$.

Among the six ratios now known, similar relations can be shown to exist, viz.,

$\sec\theta = \text{cosec}(90 - \theta)$ and $\text{cosec}\,\theta = \sec(90 - \theta)$,
$\cot\theta = \tan(90 - \theta)$ and $\tan\theta = \cot(90 - \theta)$;

or in more general terms, each ratio of an angle has the same value as, and is identical with, the " co-ratio " of its complement.

Further points concerning the Table of Ratios.—A few other points relating to the table on p. 458 may now be examined.

It will be noted that the two central columns headed " Tangent " at one end are also headed " Co-tangent " at the other end. The reason for this is that the angles in the extreme outer columns on any horizontal line are complements of each other, and the tangent of an angle and the cotangent of its complement are equal. The arrangement of the tan and cot columns is thus similar to the arrangement of the sin and cos columns.

There are two other columns at each side of the table which have not yet been referred to. The " Radians " column merely gives the values of the angles in the extreme outer, or " Degrees," columns, expressed in terms of the " radian," a special unit, already referred to on p. 263, which is equal to 57·3°. This method of

measurement is of considerable importance in more advanced work, but need not be further considered at this stage.

The third and eighth columns headed "Chord" give the lengths of the chord of a circle [refer p. 250 and (N) Fig. 25] whose radius is 1 unit for various angles subtended at the centre. Referring to (C), Fig. 154, the line AB is a chord of the circle through the ball centres. Now the ratio of the length of this chord to the radius of its circle, *i.e.* $\frac{AB}{OA}$ in the figure, is always constant for the same angle subtended at the centre of the circle, and the "Chord" columns in the table give the values of this ratio for the various angles from 0° to 90°. The values in these columns are related to those in the sine columns; they are, in fact, the values of "twice the sine of half the angle."

Thus referring to (C), Fig. 154,

$$\text{the chord } AB = AC \times 2, \text{ and } AC = OA \sin \alpha.$$
$$\therefore \quad AB = 2 \times OA \sin \alpha.$$

Now the angle subtended by (*i.e.* opposite to) the chord AB is the angle AOB, which we may denote, say, by θ; then $\alpha = \frac{\theta}{2}$.

$$\therefore \quad AB = 2\, OA \sin \frac{\theta}{2}$$

$$\therefore \quad \frac{AB}{OA} = \text{"chord" ratio} = 2 \sin \frac{\theta}{2}$$

which is twice the sine of half the angle.

The chord columns are sometimes useful for setting out angles without using a protractor.

Important Relationships between the Ratios.—Much of the practical application of trigonometry is dependent upon a knowledge of certain very important relationships that exist between the various ratios other than those already referred to. It is not possible, nor desirable, to discuss these at this stage, but there are two very important instances which may be noted and remembered for use when the subject is further pursued.

Referring to the standard right-angled triangle at (Z), Fig. 150, it is known that

$$\sin A = \frac{BC}{AB} \qquad \cos A = \frac{AC}{AB} \qquad \tan A = \frac{BC}{AC}.$$

Now let the sine of angle A be divided by its cosine, when both ratios are expressed as above.

Then

$$\frac{\sin A}{\cos A} = \sin A \div \cos A = \frac{BC}{AB} \div \frac{AC}{AB}$$

inverting divisor and multiplying— $$= \frac{BC}{AB} \times \frac{AB}{AC}$$

and cancelling AB— $$= \frac{BC}{AC}$$

which relation is already known as the tangent of A.

$$\therefore \quad \frac{\sin A}{\cos A} = \tan A.$$

Again referring to the same right-angled triangle and the same three ratios, let the sine and cosine be each squared and their squares added together.

Then since $\sin A = \frac{BC}{AB}$ $\quad \therefore \quad \sin^2 A = \frac{BC^2}{AB^2}$

and $\cos A = \frac{AC}{AB}$ $\quad \therefore \quad \cos^2 A = \frac{AC^2}{AB^2}$

$$\therefore \quad \sin^2 A + \cos^2 A = \frac{BC^2}{AB^2} + \frac{AC^2}{AB^2}.$$

$$= \frac{BC^2 + AC^2}{AB^2}.$$

Now in any right-angled triangle the sum of the squares on the two sides enclosing the right angle is equal to the square on the hypotenuse (refer p. 266) so that $BC^2 + AC^2$ may be replaced by AB^2.

$$\therefore \quad \sin^2 A + \cos^2 A = \frac{BC^2 + AC^2}{AB^2} = \frac{AB^2}{AB^2} = 1.$$

$$\therefore \quad \sin^2 A + \cos^2 A = 1.$$

From this relation $\quad \sin^2 A = 1 - \cos^2 A$

so that $\quad \sin A = \sqrt{1 - \cos^2 A}$

also $\quad \cos^2 A = 1 - \sin^2 A$

so that $\quad \cos A = \sqrt{1 - \sin^2 A}.$

Graphs of the Important Ratios.—It has already been stated that no ratio increases in proportion to its angle; thus sin 60° is not equal to twice sin 30°, and so on. This general statement can easily be verified by reference to the tables, but a much better conception of the way in which the various ratios change as the angle is changed can be obtained by plotting a curve for each ratio to be examined.

If the values of the ratios sine, cosine and tangent be taken from the table for angles at intervals of, say, every 5° or 10°, and

these values be plotted vertically on squared paper with the corresponding angles plotted horizontally, then the resulting points will be found to lie on a perfectly smooth curve in every case. The shapes of these curves are characteristic and are reproduced to a small scale, for angles from 0° to 90°, in Fig. 156. None of the curves contains any straight portion, but the direction is everywhere different.

The following features should be noted :—

Sine Curve.—The curve is convex upwards, increasing from zero at 0° to 1 at 90°. The greatest slope is at 0°, and the slope steadily diminishes until at 90° the curve has become horizontal.

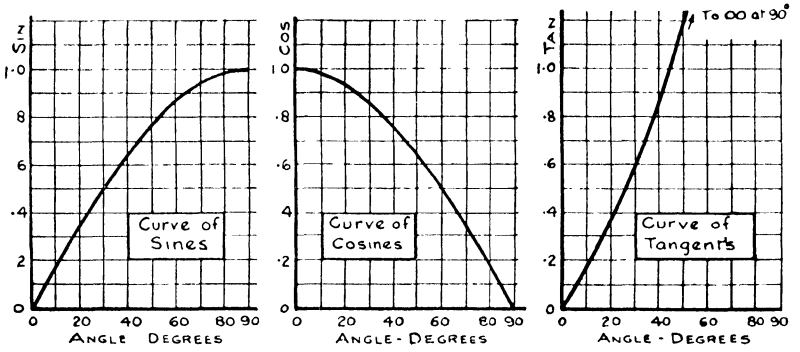


Fig. 156.—Sine, Cosine and Tangent Curves from 0° to 90°.

Cosine Curve.—The curve is convex upwards but decreases from 1 at 0° to zero at 90°, the general slope being downwards. The cosine curve is identical in shape with the sine curve but is reversed left for right.

Tangent Curve.—The curve is concave upwards and increases from zero at 0° to 1 at 45° after which it rises at a very rapid rate, becoming practically vertical in the neighbourhood of 90°. The actual value at 90° cannot be plotted as it is infinitely far up in the vertical direction.

In this book only angles between 0° and 90° have been considered, but if the subject be followed up then angles of greater than 90° must receive consideration. It will then be found that the various trigonometric ratios can possess negative as well as positive values, and that the sine and cosine curves become a series of ripples evenly spaced with regard to the horizontal axis. The study of this sine curve is of the greatest importance in engineering and all branches of science.

ANSWERS TO EXERCISES

Exercises 1

1. Proper fractions: $\frac{1}{8}$, $\frac{5}{6}$, $\frac{4}{5}$; Improper fractions: $\frac{161}{10}$, $\frac{22}{7}$
2. ,, ,, $\frac{7}{9}$, $\frac{31}{32}$; ,, ,, $\frac{41}{12}$, $\frac{13}{13}$, $\frac{33}{32}$
3. Improper fractions: $\frac{17}{16}$, $\frac{15}{15}$; Mixed numbers: $5\frac{2}{3}$, $30\frac{1}{4}$, $1\frac{1}{16}$
4. Proper fraction: $\frac{3}{17}$; Improper fractions: $\frac{28}{7}$, $\frac{9}{8}$; Mixed numbers: $6\frac{1}{4}$, $7\frac{10}{50}$
5. $\frac{3}{2}$, $\frac{9}{4}$, $\frac{11}{10}$, $\frac{10}{3}$, $\frac{17}{7}$
6. $\frac{46}{9}$, $\frac{23}{18}$, $\frac{31}{16}$, $\frac{25}{12}$, $\frac{30}{11}$
7. $\frac{21}{2}$, $\frac{61}{4}$, $\frac{91}{8}$, $\frac{39}{2}$, $\frac{225}{32}$
8. $\frac{162}{16}$, $\frac{222}{22}$, $\frac{71}{4}$, $\frac{171}{8}$, $\frac{94}{7}$
9. $\frac{73}{5}$, $\frac{101}{12}$, $\frac{275}{16}$, $\frac{1331}{32}$
10. $\frac{315}{16}$, $\frac{497}{12}$, $\frac{172}{11}$, $\frac{505}{16}$
11. $1\frac{1}{4}$, $3\frac{1}{2}$, $2\frac{2}{3}$, $1\frac{3}{7}$, $2\frac{2}{8}$
12. $1\frac{1}{16}$, $9\frac{1}{2}$, $6\frac{1}{4}$, $3\frac{1}{7}$, $9\frac{3}{8}$
13. $3\frac{1}{12}$, $9\frac{5}{8}$, $2\frac{5}{11}$, 1, 16
14. $1\frac{5}{16}$, $4\frac{1}{16}$, $4\frac{1}{8}$, $12\frac{1}{8}$, $9\frac{5}{12}$
15. 6, $3\frac{1}{18}$, $2\frac{6}{25}$, $11\frac{10}{11}$, $22\frac{1}{12}$
16. $11\frac{3}{16}$, $6\frac{13}{40}$, $3\frac{29}{128}$, $3\frac{10}{113}$

Exercises 2

1. $\frac{2}{3}$, $\frac{1}{3}$, $\frac{2}{3}$, $\frac{2}{8}$, $\frac{3}{8}$
2. $\frac{3}{4}$, $\frac{11}{12}$, $\frac{1}{4}$, $\frac{1}{1}$, $\frac{1}{2}$
3. $\frac{4}{5}$, $\frac{11}{16}$, $\frac{1}{3}$, $\frac{5}{8}$, $\frac{1}{5}$
4. $\frac{7}{80}$, $\frac{3}{4}$, $\frac{17}{18}$, $\frac{30}{37}$
5. $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{3}$, $\frac{1}{2}$, $\frac{19}{27}$
6. $\frac{39}{86}$, $\frac{109}{152}$, $\frac{8}{27}$, $\frac{4}{8}$
7. $2\frac{1}{4}$, $4\frac{2}{3}$, $1\frac{1}{2}$, $1\frac{3}{8}$, $1\frac{1}{8}$
8. $3\frac{1}{8}$, 5, $2\frac{1}{16}$, $6\frac{7}{8}$, $1\frac{6}{7}$
9. $5\frac{5}{12}$, $6\frac{1}{4}$, $10\frac{1}{8}$, $10\frac{2}{3}$, $1\frac{1}{20}$
10. $10\frac{1}{12}$, $4\frac{3}{7}$, $3\frac{3}{4}$, $1\frac{1}{12}$
11. $15\frac{1}{8}$, 88, $5\frac{8}{9}$, $3\frac{1}{8}$
12. 550, $1\frac{13}{38}$, $13\frac{1}{19}$

Exercises 3

1. (*a*) 1″; (*b*) $\frac{7}{8}$″; (*c*) $1\frac{1}{4}$″
2. (*a*) $\frac{5}{8}$″; (*b*) $1\frac{1}{16}$″; (*c*) $\frac{9}{16}$″
3. (*a*) $\frac{13}{32}$; (*b*) $\frac{15}{32}$
4. (*a*) $\frac{7}{32}$; (*b*) $\frac{21}{32}$
5. (*a*) $\frac{21}{32}$; (*b*) $\frac{13}{32}$
6. (*a*) $\frac{35}{64}$; (*b*) $\frac{47}{64}$
7. (*a*) $\frac{17}{64}$; (*b*) $\frac{61}{64}$
8. (*a*) $\frac{27}{32}$; (*b*) $\frac{31}{64}$
9. (*a*) $\frac{33}{64}$; (*b*) $\frac{19}{64}$
10. (*a*) $\frac{47}{64}$; (*b*) $\frac{51}{64}$
11. 24
12. 12
13. 72
14. 144
15. 60
16. 42
17. 144
18. 165
19. 300
20. 153
21. 102
22. 165
23. 750
24. 4200
25. 180
26. 306
27. $\frac{19}{20}$
28. $\frac{5}{6}$
29. $\frac{23}{30}$
30. $1\frac{9}{32}$
31. $\frac{71}{72}$
32. $13\frac{15}{16}$
33. $1\frac{37}{80}$
34. $7\frac{1}{16}$
35. $7\frac{9}{32}$
36. $\frac{13}{500}$
37. 3′–3″
38. $8\frac{1}{2}$″
39. 3′–$1\frac{1}{2}$″
40. $8\frac{3}{16}$″
41. $9\frac{7}{16}$″
42. $16\frac{7}{8}$″

Exercises 4

1. $\frac{1}{32}$″
2. $\frac{1}{64}$″
3. $\frac{1}{64}$″ larger
4. $\frac{1}{2}$
5. $\frac{13}{24}$
6. $\frac{5}{48}$
7. $\frac{1}{8}$
8. $1\frac{13}{72}$
9. $1\frac{37}{48}$
10. $\frac{13}{80}$
11. $2\frac{2}{45}$
12. $\frac{7}{8}$
13. $1\frac{1}{40}$
14. $2\frac{59}{70}$
15. $\frac{13}{28}$
16. $\frac{173}{176}$
17. $\frac{21}{40}$
18. $6\frac{11}{12}$
19. $\frac{55}{64}$
20. $\frac{53}{60}$
21. $3\frac{5}{16}$
22. $2\frac{57}{64}$
23. $\frac{9}{11}$
24. $\frac{33}{48}$
25. $\frac{1}{2}$
26. $2\frac{19}{60}$
27. $8\frac{33}{32}$
28. $2\frac{4}{5}$
29. $6\frac{15}{16}$
30. $1\frac{13}{20}$
31. $\frac{7}{32}$
32. $\frac{11}{40}$
33. $1\frac{9}{16}$
34. $3\frac{2}{3}$
35. $1\frac{1}{12}$
36. $11\frac{7}{8}$
37. $12\frac{5}{16}$
38. $\frac{9}{32}$
39. $1\frac{3}{10}$
40. 0
41. 2″
42. $20\frac{3}{8}$″
43. $3\frac{3}{4}$″
44. $7\frac{1}{8}$″
45. (*a*) $1\frac{23}{32}$″; (*b*) $\frac{5}{32}$″

Exercises 5

1. $2\frac{4}{5}$ **2.** $2\frac{3}{16}$ **3.** $\frac{5}{11}$ **4.** $3\frac{2}{5}$ **5.** $\frac{5}{3}$ **6.** $\frac{7}{15}$ **7.** $\frac{19}{38}$
8. $\frac{1}{6}$ **9.** $1\frac{5}{9}$ **10.** $\frac{23}{32}$ **11.** $4\frac{1}{8}$ **12.** 6 **13.** $9\frac{1}{2}$ **14.** $\frac{3}{100}$
15. $\frac{9}{40}$ **16.** $1\frac{1}{24}$ **17.** $3\frac{1}{8}$ **18.** 864 **19.** $2\frac{2}{3}$ **20.** $\frac{4}{385}$ **21.** $\frac{9}{160}$
22. $\frac{1}{12}''$ **23.** $\frac{1}{48}''$ **24.** $\frac{1}{12}''$ **25.** $\frac{1}{48}''$ **26.** $9\frac{1}{3}$ r.p.m. **27.** $\frac{1}{100}''$
28. (*a*) $\frac{1}{1000}''$; (*b*) $\frac{1}{10000}''$ **29.** 6080 ft. **30.** 12 miles. **31.** 1100 yards
32. 338 cu. yards

Exercises 6

1. 4 **2.** $2\frac{2}{3}$ **3.** $\frac{2}{5}$ **4.** 12 **5.** $\frac{4}{11}$ **6.** $\frac{3}{8}$ **7.** $2\frac{5}{8}$
8. 6 **9.** $\frac{1}{32}$ **10.** $\frac{5}{8}$ **11.** $3\frac{1}{12}''$ **12.** 100 **13.** 14″ **14.** $13\frac{1}{2}''$
15. 22″ **16.** 8″ **17.** 216 lbs. per inch **18.** 248 lbs. per inch
19. 256 lbs. per inch **20.** 624 lbs. per in. **21.** $\frac{1}{96}$ **22.** $\frac{1}{120}$
23. $\frac{1}{48}$ **24.** $\frac{1}{32}$ **25.** $\frac{1}{8}$ **26.** $\frac{3}{8}$ **27.** $\frac{1}{8}$ **28.** $\frac{1}{64}$ **29.** $\frac{1}{16}$
30. $\frac{1}{4}$ **31.** $\frac{1}{12}$ **32.** $\frac{3}{8}$ **33.** $\frac{1}{12}$ **34.** $\frac{3}{8}$ **35.** $\frac{1}{12}$ **36.** $\frac{1}{8}$

Exercises 7

1. $\frac{1}{16}$ **2.** $1\frac{1}{22}$ **3.** $3\frac{3}{4}$ **4.** $3\frac{3}{4}$ **5.** $16\frac{1}{2}$ **6.** $\frac{1}{12}$ **7.** $\frac{17}{27}$ **8.** $1\frac{1}{2}$
9. $1\frac{1}{13}$ **10.** $6\frac{4}{7}$ **11.** $\frac{3}{71}$ **12.** $1\frac{1}{16}$ **13.** $1\frac{17}{76}$ **14.** $\frac{3}{8}$ **15.** $\frac{5}{64}$ **16.** $\frac{47}{48}$
17. $\frac{15}{88}$ **18.** $6\frac{13}{48}$ **19.** 10 **20.** $2\frac{5}{12}$ **21.** $10\frac{31}{80}$ **22.** 8
23. 45 screws per bar; 39 bars. **24.** $50\frac{5}{8}''$, say 51″ **25.** $41\frac{1}{4}''$
26. 14 **27.** 138 **28.** $19\frac{3}{4}''$, say 20″ **29.** 163 ft.; 4 lengths
30. Cement 3 cwts.; coke breeze 12 cwts.
31. (*a*) Lead 90 lbs., tin 60 lbs.; (*b*) Tin 100 lbs., lead 25 lbs.; bismuth 25 lbs. **32.** Nitre 45 lbs.,; charcoal 9 lbs.; sulphur 6 lbs.
33. (*a*) Tin, 74 lbs. 11 ozs.; bismuth, 37 lbs. 5 ozs.
(*b*) Lead, 11 lbs. 3 ozs.; tin, 44 lbs. 13 ozs.; bismuth, 56 lbs.
(*c*) Lead, 35 lbs.; tin, 21 lbs.; bismuth, 56 lbs.
34. (*a*) Old sand $122\frac{2}{11}$, say 122 lbs.; new sand $81\frac{5}{11}$, say $81\frac{1}{2}$ lbs.; coal-dust $20\frac{4}{11}$, say $20\frac{1}{2}$ lbs.; (*b*) Old sand $790\frac{1}{11}$, say 791 lbs.; new sand $263\frac{9}{11}$, say 263 lbs.; coal-dust $65\frac{1}{11}$, say 66 lbs.
35. Lead $22\frac{1}{2}$ lbs., antimony 5 lbs., bismuth $2\frac{1}{2}$ lbs.
36. Cement $7\frac{1}{2}$ tons, sand $22\frac{1}{2}$ tons, aggregate 45 tons.
37. $31\frac{7}{8}''$; $50' - 9\frac{3}{4}''$, say 51 ft. **38.** $\frac{5}{12}$ **39.** $\frac{39}{37}$ **40.** $3\frac{43}{45}$ **41.** $7\frac{3}{10}$
42. $\frac{7}{18}$ **43.** 27

Exercises 8

1. (*a*) 55; (*b*) ·55; (*c*) 134; (*d*) ·0134; (*e*) 453600; (*f*) ·4536
2. (*a*) 1·118; (*b*) ·087; (*c*) ·00000066; (*d*) 350,000,000
3. (*a*) 35000 watts; (*b*) 7300 watts; (*c*) 80 watts
4. (*a*) 29·5 kw.; (*b*) ·231 kw.; (*c*) 305 kw.
5. (*a*) 270,000,000; (*b*) 500,000; (*c*) 300
6. (*a*) 4·26; (*b*) ·055

Exercises 9

1. (*a*) $\frac{13}{20}$; (*b*) $\frac{9}{20}$; (*c*) $\frac{17}{40}$
2. (*a*) $\frac{7}{250}$; (*b*) $\frac{7}{8}$; (*c*) $\frac{294}{625}$
3. (*a*) $2\frac{41}{200}$; (*b*) $1\frac{17}{80}$; (*c*) $\frac{61}{200}$
4. (*a*) $\frac{241}{250}$; (*b*) $3\frac{711}{5000}$; (*c*) $\frac{8}{3125}$
5. (*a*) $\frac{33}{64}$; (*b*) $\frac{25}{64}$

Exercises 10

1. 3937; 528; 2205; 3045; 365 **2.** 2985; 105; 807; 1083; 454
3. 66; 6065; 1118; 3209; 15708 **4.** 109; 87; 105; 6009; 2
5. 454; 30·5; ·434; 62·4; ·00112 **6.** 14400; 1390; 32·2; 601; 3750
7. ·081; ·28; 15; ·065; 5·4 **8.** 2·2; 610; ·016; 30,000,000; ·11
9. (*a*) 277·3 cu. ins.; (*b*) 277 cu. ins.
10. (*a*) 453·59 grammes; (*b*) 454 grammes
11. (*a*) ·14286; (*b*) ·1429; (*c*) ·14 **12.** (*a*) 2·7183; (*b*) 2·718; (*c*) 2·7

Exercises 11

1. 99·28 **2.** 263·81 **3.** 220·04375
4. 349·6 ft. **5.** 36 lbs. **6.** 67·05 chains
7. 11·23 **8.** Dowson 44·42; Mond 36; natural gas 95·75
9. 8·9 **10.** 29·95 **11.** (*a*) Brass 1·8; (*b*) steel 1·01
12. ·6888 grammes **13.** (*a*) 1163·7 grammes; (*b*) 51·9 grammes
14. (*a*) ·1888; (*b*) ·2878 **15.** ·0035″; ·0105″ **16.** 20·66 **17.** 10·995
18. Column R = 7·406, ,, F = 7·404 } Error = ·002 **19.** Error = ·002
20. 3·83 **21.** 6·23 tons **22.** 3

Exercises 12

1. 40·71, *i. e.*, 40·7 **2.** 404·2, *i. e.*, 404 **3.** ·7923, *i. e.*, ·792
4. 12·52075, *i. e.*, 12·5 **5.** 61·544, *i. e.*, 61·5 **6.** ·0004698, *i. e.*, ·00047
7. 48·24, *i. e.*, 48·2 **8.** 18·8819, *i. e.*, 18·9 **9.** ·09752, *i. e.*, ·0975
10. 21444·5, *i. e.*, 21400 **11.** 31·05 m.p.h., say 31 m.p.h.
12. Boat's speed = 12·1 m.p.h. ∴ boat is slightly faster
13. 25·95 **14.** 23·9 lbs. **15.** 97·5 **16.** 27·4 lbs.
17. 2713, say 2710 cu. ins. **18.** ·0783 in.
19. 515 lbs. per sq. in. **20.** 477·7, say 478 lbs. per sq. in.
21. 156 sq. ins. **22.** 28·16, say 28·2 ohms

Exercises 13

1. 43·9 **2.** 4·676 **3.** 18·19 **4.** 3096 **5.** ·08885
6. 24·31 **7.** ·475 **8.** ·00847 **9.** 2003. **10.** 282
11. (*a*) 11·2 atmospheres; (*b*) 6·12 atmospheres **12.** ·651; ·6405; ·62
13. 4·54 **14.** 2·205 lbs. **15.** 50·8 **16.** 1·34
17. 4·85 sq. ft. **18.** 22 lbs. per min. **19.** 42·2 lbs. per min.
20. 31·1 lbs. per min. **21.** ·0133 ohms. per ft. **22.** ·016 cu. ft.

Exercises 14

1. ·625 **2.** ·09375 **3.** ·5625 **4.** ·9048, say ·905
5. ·171875 exact, say ·172 **6.** ·152 **7.** ·45
8. ·555 **9.** ·417 **10.** 3·142 **11.** ·3 min.
12. ·567 min. **13.** ·183 min. **14.** ·9 min. **15.** ·217 min.

16. ·0416 ins. **17.** ·0556 ins. **18.** ·111 ins.
19. ·222 ins. **20.** ·286 ins. **21.** ·0909 ins.
22. ·0714 ins. **23.** ·0526 ins. **24.** ·0357 ins.
25. " Go in " end = 1·6246″ dia.; " Not go in " end = 1·6262″ dia.

Exercises 15

1. Yes **2.** Yes **3.** No; should be 31·5
4. No; should be ·155 **5.** No; should be ·0494 **6.** Yes
7. Yes **8.** Yes **9.** ·478 **10.** 64
11. 880 **12.** 71·4 **13.** 21·4 **14.** 272
15. 44000 **16.** 34700 **17.** ·0245 **18.** ·705

Exercises 16

1. 34 ft. **2.** 2·53 **3.** 2·55 **4.** ·199 amps.
5. (*a*) 6·63 ohms.; (*b*) 29·8 volts **6.** 20·3 **7.** 655 h.p.
8. ·619″, say $\frac{5}{8}$″ **9.** (*a*) 11520 watts; (*b*) 104·7, say 105 amps.
10. 24,600,000 lbs. per sq. in. **11.** 11,620,000 lbs. per sq. in.
12. 604 B.T.U. **13.** 9·16 **14.** 32·36 tons
15. 963 **16.** 49·75. **17.** 337
18. 8·59 tons **19.** 8·89 tons **20.** 21·3 tons
21. 544. **22.** 88 lbs. per sq. in. **23.** 15·05, say 15 lbs.
24. 53·6 lbs. per sq. in. **25.** 581 ft. tons **26.** 1600 lbs.

Exercises 17

1. 1·505 **2.** ·579 **3.** ·0278 ohms
4. ·03599″, say ·036″ **5.** ·396 **6.** 27·9 lbs. per. min.
7. 100·2, say 100 lbs. per sq. in. **8.** 110·45, say 110 volts
9. 221·1, say 221 amps. **10.** 592·5, say 593 cu. ft. per min.
11. ·462 mm. **12.** ·794.

Exercises 18

1. $1\frac{1}{4}$″ holes, 6*s.* per doz.; 1″ holes, 3*s.* 9*d.* per doz. **2.** £2 10*s.*
3. 29*s.* 3*d.* **4.** (*a*) £4 4*s.*; (*b*) £3 3*s.* **5.** 179 h.p. **6.** 40·8 lbs.
7. (*a*) ·477 cu. ft.; (*b*) 6·44 cu. ft. **8.** Max. 2958 lbs.; min. 2842 lbs.
9. 3·3 tons **10.** 2·5%. **11.** 14·1% **12.** 1·12%
13. (*a*) 317·5 galls. per min.; (*b*) 5·83% above
14. Copper 76·2%; tin 23·8% **15.** Copper 94·1%; tin 5·88%
16. Copper 65%; tin 5%; lead 30%
17. Copper sulphate 8·33%; sulphuric acid 8·33%; water 83·33%
18. Water 13·3%; carbon dioxide 17·8%; nitrogen 63%; oxygen ·9%
19. Carbon dioxide 10·4%; oxygen 12·25%; nitrogen 77·4%
20. 99·205% **21.** 98·136% **22.** 93·972% **23.** 99·377%
24. 204 and 196 r.p.m. **25.** 182·7 and 177·3 r.p.m.
26. 87·5 and 82·5 r.p.m. **27.** 20·6%
28. Water 97·6%; hypo 1·22%; sugar of lead 1·22%

Exercises 19

1. (*a*) 4·89 to 1; (*b*) 2·03 to 1 **2.** (*a*) 5 to 1; (*b*) 5·9 to 1
3. ·639 : 1 **4.** ·862 : 1 **5.** (*a*) 40·7 : 1; (*b*) 112·7 : 1
6. (*a*) 3·2 : 1; (*b*) 3·23 : 1 **7.** (*a*) 29·1 : 1; (*b*) 30·1 : 1
8. (*a*) ·829 : 1; (*b*) ·597 : 1 **9.** (*a*) ·529 : 1; (*b*) ·566 : 1
10. 1648 **11.** (*a*) 4·41 : 1; (*b*) 2·68 : 1 **12.** 6·32 : 2·38 : 1
13. 8·99 : 4 : 1·735 : 1 **14.** (*a*) 14·8 : 1; (*b*) 13·3 : 1 **15.** ·412
16. 12 : 1 **17.** 21 : 1 **18.** (*a*) 4; (*b*) 2·46
19. (*a*) 1·58; (*b*) 4·8; (*c*) 5·42; (*d*) 5·63; (*e*) 6·05; (*f*) 7·09

Exercises 20

1. 42 lbs. **2.** 4½ galls.
3. 50·2 sq. ins. **4.** 16·3 lbs.; increase = 1·8 lbs.
5. H.P. 15·94″, say 16″ dia.; I.P. 26·6″, say 26⅝″ dia.
6. H.P. 754·3 sq. in.; 1st I.P. 1660 sq. ins.; 2nd I.P. 3470 sq. ins.
7. 3520 sq. ins. **8.** 135 galls. **9.** 175 galls. **10.** 3·75 cwts.

Exercises 20a. *Refer p.* 453.

Exercises 21

1. 86·8 volts **2.** 13·28 grammes
3. (*a*) 104; (*b*) 1596 **4.** 1540 ft. per min.
5. 153,400 lbs. **6.** 710 **7.** 90 inch-lbs. **8.** 28·8 sq. ins.
9. 24·7 **10.** 211 tons-ft. **11.** 48 **12.** 2·55
13. 13·1 **14.** ·75″ **15.** ·632 **16.** 11·39
17. 28700 **18.** 29·7 H.P. **19.** 3·15 B.H.P. **20.** 6·47

Exercises 22

1. 1″ **2.** ·622″ **3.** 1185·5 **4.** 883
5. 1143·3 **6.** (*a*) 98·9 ft. per min.; (*b*) 21·6 ft. per min.
7. 6·52″ **8.** D = 1·492″, H = ·4264″, w = ·1447″, h = ·1421″
9. 1350 ft. per min. **10.** ·437 **11.** ·59 **12.** 28
13. 185° F. **14.** 38·4 amps. **15.** ·00833 **16.** 45·6
17. 4·67 **18.** 8·68 **19.** 3·41 tons
20. ·802″ **21.** 378·7 **22.** 5·04 H.P.
23. 2·33 **24.** 2·98 tons **25.** 1762 lbs. per hr

Exercises 23

1. 4 × 4 × 4; 10 × 10; 3 × 3 × 3 × 3; ·2 × ·2 × ·2; ·5 × ·5; 10 × 10 × 10 × 10 × 10

2. $r \times r$; $D \times D \times D \times D$; $a \times a \times a$; $h \times h \times h \times h \times h$

3. (i) 9; (ii) 6 **4.** (i) 64; (ii) 12 **5.** (i) 3; (ii) 2·25

6. (i) 81; (ii) 12 **7.** (i) 10; (ii) 32

8. (i) $c \times c \times r$; (ii) $c \times r \times r$; (iii) $c \times c \times r \times r$; (iv) $c \times r \times c \times r$; (v) Results of (iii) and (iv) are equal

9. (i) 2 × M × M; (ii) 3 × M × M; (iii) 2 × M × 2 × M; (iv) 3 × M × 3 × M

10. (i) $a \times a \times a \times r$; (ii) $a \times r \times r \times r$; (iii) $a \times a \times r \times r \times r$; (iv) $a \times a \times a \times r \times r$

11. (i) $3 \times n \times n \times p$; (ii) $3 \times n \times p \times p$; (iii) $3 \times n \times n \times p \times p$

12. (i) $3 \times n \times p \times n \times p$; (ii) $3 \times n \times 3 \times n \times p$; (iii) $3 \times n \times p \times 3 \times n \times p$

13. (i) 4·5; (ii) ·75; (iii) 2·25; (iv) 2·25

14. (i) 1; (ii) ·5; (iii) 4; (iv) 1

15. (i) 24; (ii) 54; (iii) 108; (iv) 72

16. (i) 43·2; (ii) ·36; (iii) 4·32; (iv) 4·32

17. (i) 4000; (ii) 1000; (iii) 30,000; (iv) ·0003

18. 4·91 sq. ins. **19.** 112,640 lbs. **20.** 72·13 **21.** 30,070
22. ·102 tons **23.** 3·39 H.P. **24.** 3·327 **25.** 2316 ft. lbs
26. ·438 H.P. **27.** 8·71 H.P. **28.** 33·8 **29.** 7·68 lbs.
30. 4·56 lbs. **31.** 1950 **32.** 887 watts **33.** 4·1 ft.

Exercises 24

1. 3·4 **2.** 15·5 **3.** 2·1 **4.** 29 **5.** 9·18 **6.** 10·5
7. 40·3 **8.** 109 **9.** ·78 **10.** ·27 **11.** ·095 **12.** 1·01
13. ·0031 **14.** 15·9 **15.** 49·9 **16.** 320 **17.** 2600 **18.** 4·76
19. 1·62 **20.** 50·6 **21.** 60·7 **22.** 27·6 **23.** 24·2 **24.** ·301
25. ·356 **26.** 7·58 nautical miles **27.** 74·6 ft. per sec.
28. 6·74 ins. **29.** 5·56 H.P. **30.** 12·4
31. 8·35, *i. e.*, $1\frac{9}{16}''$ **32.** 1·645″, *i. e.*, $1\frac{11}{16}''$ **33.** 15·9
34. 1·998 secs. **35.** 70·7 amps. **36.** ·917
37. 11·9 **38.** 2·12 ft. **39.** 1·25

Exercises 25

1. $\frac{l^2}{r^2}$ **2.** $\frac{d^2}{4}$ **3.** $\frac{D^3}{8}$ **4.** $\frac{D^4}{16}$ **5.** $\cdot 297h^3$ **6.** $\frac{x^2}{2}$ **7.** $\frac{w^2r^2}{4}$

8. $\frac{x^5}{243}$ **9.** $\frac{c^2}{9r^2}$ **10.** $\frac{T^2}{4\pi^2}$ **11.** $\frac{\pi^3d^3}{27}$ **12.** $2 \cdot 15V^2$ **13.** $\frac{\pi^2N^2}{900}$ **14.** $\frac{R^3}{3 \cdot 375}$

15. $\frac{r^2}{4\pi^2}$ **16.** $\frac{S_2^2}{S_1^2}$ **17.** $\frac{y^3}{L^3}$ **18.** $1 \cdot 69r^2n^2$ **19.** $\frac{w^{2/2}}{16}$ **20.** $\cdot 857\sqrt[3]{x}$.

Exercises 26

1. $\frac{54 \cdot 21}{\sqrt{H}}$ **2.** $\cdot 817\sqrt{RT}$ **3.** $\frac{d}{\cdot 866}\sqrt{\frac{P}{p}}$ or $1 \cdot 15d\sqrt{\frac{P}{p}}$

4. $\frac{D\sqrt{P}}{59 \cdot 2}$ **5.** $\cdot 346\sqrt{r}$ **6.** $8 \cdot 02\sqrt{h}$ **7.** $44 \cdot 3\sqrt{h}$

8. $\frac{4x}{\sqrt{y}}$ **9.** $\cdot 274D$ **10.** $224\sqrt{H}$ **11.** $12 \cdot 14\sqrt{p}$

12. $157\sqrt{\frac{M}{S}}$ **13.** $\cdot 0763\sqrt{p}$ **14.** $\cdot 707\sqrt{P} - 1$

Exercises 27

1. a^5 2. f^3 3. N^3 4. d^5 5. r^4 6. a^7
7. H^6 8. 10^9 9. 10^7 10. w^2r 11. px^2 12. b^5
13. D^6 14. R^4 15. 10^4 16. 10^6 17. f^3T 18. R^3E^4
19. π^2n^3 20. p^2d^5 21. x^5y^2v

Exercises 28

1. m 2. S^4 3. t 4. R 5. 10^4 6. 10^2a 7. an
8. πr 9. pd^3 10. rw 11. 10^6R 12. 10^6

Exercises 29

1. a^{12} 2. a^9 3. 10^6 4. N^8 5. D^{10} 6. R^{12}
7. 10^6 8. 10^{12} 9. R^9 10. m^{10} 11. r^4 12. c^3d^6
13. c^6d^3 14. c^6d^6 15. a^4p^6 16. a^3p^9 17. 10^{10} 18. $m^{12}p^4$

Exercises 30

1. $\frac{n^5}{2}$ 2. $2n^4$ 3. n^6 4. $\frac{n}{4}$ 5. 8 6. $\frac{16}{n^2}$
7. $\frac{8}{n^3}$ 8. $16n$ 9. (*a*) c^2R; (*b*) $\frac{E^2}{R}$ 10. $\frac{bh^3}{4}$ 11. $\frac{6}{25}B^3H$
12. $\cdot785d^2$ 13. $\frac{P^2v}{p}$ 14. $\frac{2}{3}b^3$ 15. $\frac{Hd}{12880}$ 16. $\frac{9pl^2}{128}$ 17. $\frac{pl^3}{24}$
18. $\frac{Q^2}{2K}$ 19. $\frac{\pi^2Ek^2}{l^2}$ 20. $\frac{ab}{4}$ 21. $\frac{Wk^2\omega^2}{64}$ 22. $\cdot5236D^3$ 23. $\frac{\pi d^3}{32}$
24. $2\cdot47Dd^2$ 25. $\frac{R}{4}$ 26. $\frac{D}{8}$ 27. $\frac{3}{5}B$ 28. $\frac{Wl^3}{48EI}$

Exercises 31

1. 8 2. 3 3. -15 4. -10 5. 17
6. 1·9 7. ·2 8. $-1\cdot6$ 9. $-1\cdot1$ 10. $-1\cdot55$
11. ·8730 12. $-\cdot1380$ 13. $-1\cdot0160$ 14. $-\cdot6700$ 15. 1·045
16. $-1\cdot695$ 17. ·954 18. ·95 19. $-2\cdot115$ 20. ·35
21. 2 22. 3 23. -6 24. -3 25. 0
26. 7 27. 1 28. -8 29. -3 30. -7
31. 0 32. $4x^2y$ 33. $-r^2$ 34. $4pm$ 35. $2a^2c$
36. $18wl$ 37. $8p^2$. 38. $-3d$ 39. $-5\cdot5n$. 40. $-8\cdot67w$

Exercises 32

1. 6 2. -11 3. -4 4. 11 5. -1 6. -18
7. -14 8. -1 9. 0 10. 2·2 11. $-3\cdot6$
12. 2·3 13. $-2\cdot7$ 14. $-2\cdot9$ 15. ·9 16. 6·72
17. $-3\cdot45$ 18. $-6\cdot91$ 19. $-1\cdot117$ 20. $-1\cdot7$ 21. 4·101
22. $-\cdot33$ 23. $-2\cdot002$ 24. ·845 25. $-7\cdot35$ 26. $7wl$
27. $-12rf$ 28. $25ad^2$ 29. $-2c^2p$ 30. $-5xy$ 31. $-1\cdot25xy$

Exercises 33

1. $-p^2-2pq-2q^2$ **2.** $4ar^2-2ar-4r^3$
3. $-w+2l-w^2l^2$ **4.** $4m^3-m^2-m$
5. a^3-b^3 **6.** p^3+q^3
7. $-2x^6-x^5y+5y^2-3x^3y^3-x^4$
8. $3ml-18+5l^2-4m^2$ **9.** $2-2r^2$ **10.** $t-4-12t^2$
11. $2x-\cdot55l$ **12.** $-\cdot25y^2-\cdot55y+7$ **13.** $7n^2+12$
14. $1\frac{1}{4}a+d$ **15.** $\frac{3}{4}c-\frac{1}{2}c^2$ **16.** $\frac{1}{16}\frac{1}{6}x^2+x-10$
17. $\frac{1}{4}d-3x+3x^2$ **18.** $\frac{5}{8}n$ **19.** $3bd^2-2b^2d-3b$ **20.** $9e-4f$

Exercises 34

1. 20 **2.** 20 **3.** -20 **4.** -20 **5.** $-2\cdot1$ **6.** $-5\cdot39$
7. $-3\cdot126$ **8.** $\cdot975$ **9.** -24 **10.** -24 **11.** 24 **12.** -16
13. 10 **14.** 6·75 **15.** -9 **16.** 4 **17.** -4 **18.** -4
19. 4 **20.** $-2\cdot5$ **21.** 1·875

Exercises 35

1. 4 **2.** -8 **3.** 16 **4.** -32 **5.** 64 **6.** -4 **7.** 8 **8.** -16
9. 32 **10.** -16 **11.** 64 **12.** -64 **13.** $64r^3$ **14.** $-64r^3$

Exercises 36

1. 711 **2.** 451 **3.** 436 **4.** 461 **5.** 283 **6.** 268 **7.** 273
8. 32 **9.** 4 **10.** 6·36 **11.** $-9\cdot44$°C **12.** $-17\cdot78$°C.
13. $-18\cdot89$° C. **14.** 0 **15.** 5° F. **16.** 32° F.
17. $-35\cdot5$° F. **18.** -1 **19.** 5 **20.** 3
21. $-1\cdot67$ **22.** $-\cdot737$ **23.** 175 **24.** 180·4
25. $-18\cdot95$ **26.** $-16\cdot96$ **27.** 6·44 **28.** 5·5
29. 4·23 **30.** 20·7 **31.** -2 **32.** 5·2
33. $-4\cdot71$ **34.** -10 **35.** -18 **36.** 60·25
37. 13·8 **38.** 28·25 **39.** 1218 **40.** -3
41. $-\cdot675$ **42.** 1·175 **43.** ·65 **44.** $-\cdot25$

Exercises 37

1. M **2.** C + M **3.** $x+3$ **4.** $5p-2$ **5.** 5C − 4
6. $22-3d$ **7.** $3w^2-w$ **8.** $1-a$ **9.** $4r-3r^2$ **10.** 7H + 8
11. mA **12.** $2p^3-2f$ **13.** $2v_2-v_1$ **14.** $3n-7$
15. $\cdot133-\cdot000279t$ **16.** 14500C + 62060H **17.** $99-\frac{99}{d}$
18. PR − W **19.** $\frac{2}{3}a+\frac{1}{3}b$ **20.** $\frac{Rl_1}{l_2}-R$ **21.** $\frac{Wx}{l}$
22. $\frac{x}{2}+\frac{h}{4}$ **23.** $\frac{C^2+1}{2}$ **24.** eC **25.** $\cdot997+\cdot0002t$
26. $561+\frac{100v}{s}$ **27.** $175\cdot6-\cdot27t$ **28.** $s+\cdot48p+950\cdot6$
29. $1082+\cdot3t$ **30.** $1\cdot64x-\cdot5x^2$ **31.** $1\cdot018+\cdot0000306t$
32. $\cdot7502-\cdot000504t$ **33.** 14500C + 62060H + 77570
34. $10\cdot76-\cdot01t+\cdot0048$T

Exercises 38

1. $a(1-e)$ **2.** $a(R-r)$ **3.** $c(A+ma)$ **4.** $M\left(1+\frac{x}{y}\right)$

5. $K\left(1-\frac{l}{L}\right)$ **6.** $d\left(\frac{n}{13}+7\right)$ **7.** $2M(l_1+l_2)$ **8.** $a(a+2b)$

9. $4h(2r-h)$ **10.** $w^2r\left(1-\frac{r}{l}\right)$ **11.** $x\left(1-\frac{x}{L}\right)$ **12.** $\frac{p}{4}\left(l_1^3+l_2^3\right)$

13. $l^2\left(5-\frac{4l}{x}\right)$ **14.** $bd\left(\frac{d^2}{12}+x^2\right)$ **15.** $\frac{n}{n-1}\left(PV-pv\right)$

16. $Wy\left(1-\frac{5x}{4l}+\frac{xy^2}{4l^3}\right)$ **17.** $ab(a+b)$ **18.** $p^2r(3p-2)$

19. $\frac{h}{A-a}\left(A+\sqrt{aA}\right)$ **20.** $lx^2\left(\frac{l}{2}-\frac{x}{3}\right)+\frac{x^4}{12}$ **21.** $(x+a)\left(l+\frac{l^2}{2}\right)$

22. $\frac{l^2x^2}{2}-\frac{x^3}{3}\left(l-\frac{x}{4}\right)$ **23.** $x(x+a)+c(x+a)=(x+a)(x+c)$

24. $\frac{ny}{b-1}$ **25.** $\frac{16}{105}HB^3$ **26.** $\frac{h}{2}\left(a+b\right)$

Exercises 39

1. $2x^2-5x-3$ **2.** x^4+3x^2-28 **3.** $2l^2+lx-x^2$

4. $2a^2-3ar-2r^2$ **5.** $2s^2+ps-3p^2$ **6.** $b^4-b^3-6b^2$

7. a^3+b^3 **8.** a^3-b^3 **9.** $4x^2+4xy+y^2$

10. $x^2-4xy+4y^2$ **11.** $9n^2-12np+4p^2$ **12.** $p^2-8np+16n^2$

13. $l^2+2pxl+p^2x^2$ **14.** $\frac{p^2}{4}+py+y^2$ **15.** $x^2-xl+\frac{l^2}{4}$

16. $\frac{a^2}{4}-\frac{ab}{3}+\frac{b^2}{9}$ **17.** $2{\cdot}25x^2-6x+4$

18. $9m^2+7{\cdot}2mp+1{\cdot}44p^2$ **19.** $\frac{x^2}{9}+\frac{2}{3}xl+l^2$

20. $4n^2-2n+{\cdot}25$ **21.** $4-l^2$ **22.** n^2-9 **23.** $\frac{l^2}{4}-x^2$

24. $r^2-\frac{x^2}{4}$ **25.** $x^2y^2-\frac{y^2}{x^2}$ **26.** $\frac{l^2}{4}-\frac{1}{x^2}$

27. $l^3-3lz^2-2z^3$ **28.** 396 **29.** 4475

30. 771 **31.** 855 **32.** 52 **33.** 82·875

34. $\frac{n}{n-1}\left(PV-pv\right)$ **35.** $\frac{h}{15}\left(8a+6c\right)$

Exercises 40

1. 3·3 **2.** 11·17 **3.** 21·33 **4.** ·946 **5.** 105·2 **6.** 4250

7. ·005378 **8.** ·2614 **9.** 27·61 **10.** 22·5 **11.** 1·624

12. 1·99, say 2 ft. **13.** 1·478″, *i. e.*, $1\frac{1}{2}$″ **14.** 2972 lbs. per sq. in.

15. 14·06 **16.** 183 lbs. per sq. in. **17.** 235 **18.** 42·75

Exercises 41

1. 30 **2.** 4·98 **3.** 13·2 **4.** 2·5 **5.** 109·8 **6.** 103·5

7. 22·05 **8.** 19·62 **9.** ·05775 **10.** 1·4 volts **11.** 172

12. 648 **13.** 2·47 **14.** 137·5 **15.** 1·437 **16.** 144

Exercises 42

1. 682 amps. **2.** 19·9 **3.** 9·69 **4.** 3·98″, say 4″ **5.** 106,700
6. 4 **7.** ·00671 **8.** 1250 ft. **9.** 152·4 **10.** 4·762
11. 13·33 **12.** 33·6 cu. ft. per sec. **13.** 16·27 **14.** 340
15. 64·4 **16.** ·0498 **17.** ·628 **18.** 628 **19.** 1·242 **20.** 9075

Exercises 43

1. 7 **2.** 81·3 **3.** 36·985 **4.** 961 **5.** 5050 **6.** 12·35
7. 6·7 **8.** 1·334 **9.** ·00015 **10.** 319·3 **11.** 2·686 **12.** 1·2
13. 6·1 **14.** — 6·85 **15.** 36·5 **16.** ·31 **17.** ·005 **18.** 1·18
19. 36 **20.** 125·71 **21.** ·54 **22.** 51·6 **23.** 939

Exercises 44

1. 30° C. **2.** ·45″ **3.** ·885 **4.** 1455 **5.** 1·5
6. 25 **7.** 20·8 **8.** 214 **9.** ·0346 sq. ins.
10. 61260 **11.** 8·33 **12.** 1·34″, say $1\frac{3}{8}$″ **13.** — 30210
14. 80° F. **15.** — 36·25° F. **16.** 59·25° F. **17.** 1·79
18. — 3·89° C. **19.** 31 pence **20.** 125 **21.** 405 **22.** — 41·5
23. 776 **24.** 114 **25.** 5 **26.** 7·5.

Exercises 45

1. 2 **2.** 5·91 **3.** 2·78 **4.** 1·105 **5.** — ·622 **6.** 91·4
7. — 2 **8.** ·75 **9.** 4 **10.** 5000 **11.** ·182
12. 3·29 **13.** 11·4 **14.** 10·15 **15.** 20·9 **16.** 2·84
17. 85 **18.** 14 **19.** — 3 **20.** 3 **21.** 4
22. 3393 **23.** ·0343 **24.** 41·7, *i. e.*, 42 **25.** 3·96 **26.** 1333

Exercises 46

1. 1·604 **2.** 79·1 **3.** 3·13 **4.** 161 **5.** 9·5° F. **6.** 19·33
7. 69 **8.** 3·78 **9.** 4·27 **10.** 1·36 **11.** ·00438 **12.** 1·296
13. 24·3 **14.** ·000158 **15.** 137 **16.** 1·318 **17.** 2·167 **18.** 52,840

Exercises 47

1. 7·5 **2.** ·6 **3.** 11 **4.** ·75 **5.** — 2 **6.** — 4
7. — 2 **8.** — 3 **9.** — ·286 **10.** ·4 **11.** — ·0909
12. 6 **13.** 5·04 **14.** 35·7 **15.** 25·25 **16.** 1·5

Exercises 48

1. 3·675 **2.** 1·12 **3.** 136 **4.** 1·475 **5.** 30,520
6. 11·5 **7.** ·664 **8.** 4·86 **9.** 9·375 **10.** 1·37, say $1\frac{3}{8}$″
11. 223 galls. per min. **12.** 3·39 **13.** 2348 **14.** 3
15. ·97 **16.** 5·57 **17.** 4·13″ **18.** 3·84″

Exercises 49

1. 6·25	**2.** ·458	**3.** ·956	**4.** 32·5	**5.** 39·3
6. ·000941	**7.** 5·56	**8.** 84·4	**9.** 1·675	**10.** 162
11. ·499	**12.** 5357	**13.** 6090	**14.** 1·5	**15.** 5·41

Exercises 49a. *Refer p.* 453.

Exercises 50

1. $\frac{F}{M}$	**2.** $\frac{w}{Ct}$	**3.** $\frac{pV}{v}$	**4.** $\frac{V}{\pi D}$	**5.** $\frac{R_2R_3}{R_1}$	**6.** $\frac{P}{\cdot 785d^2}$
7. $\frac{pr}{f\eta}$	**8.** $\frac{W}{Ep}$	**9.** $\frac{v^2}{2g}$	**10.** $\frac{d^2p}{\cdot 75D^2}$	**11.** $\frac{H}{w(T-t)}$	**12.** $\frac{L}{cpF}$
13. CR	**14.** $\frac{M}{f}$	**15.** cQ_1	**16.** $\frac{dW}{D}$	**17.** Vkr	**18.** $c(r-1)$

19. Pitch dia. × Real Pitch **20.** $\frac{1}{\cdot 00034N^2}$ or $\frac{2941}{N^2}$

Exercises 51

1. $\frac{PD}{13500}$	**2.** $\frac{1000K}{E}$	**3.** $\frac{746H}{E}$	**4.** $\frac{W\tau}{2\cdot 71}$	**5.** $\frac{12500t}{H}$
6. $\frac{33000H}{V}$	**7.** $\frac{6Z}{h^2}$	**8.** $\frac{12720H}{d^2n}$	**9.** $\frac{Wb}{ch^2}$	**10.** $\frac{Hl}{1\cdot 26}$
11. $\frac{Rt}{L}$	**12.** $\frac{\tau}{2\cdot 71V}$	**13.** $\frac{4\pi tK}{A}$	**14.** $\frac{fLv^2}{h}$	**15.** $\frac{d^2lN}{K}$
16. $\frac{966W}{H-h+h_1}$	**17.** $\frac{3t^2f}{2R^2}$	**18.** $\frac{Hd}{d-d_1}$	**19.** $\frac{C(R+nr)}{n}$	**20.** $\frac{3wl}{2bh^2}$

Exercises 52

1. P − 14·7	**2.** H − $\varkappa$L	**3.** T − 273	**4.** B − 4πI
5. $M_2 - mM$	**6.** $S - \frac{r}{n}$	**7.** $\frac{b}{e} - ad$	**8.** E − V
9. W − Cr	**10.** $u + at$	**11.** $\frac{V}{R} + \frac{V}{r}$	**12.** $\frac{Z}{h} - \frac{a}{6}$

Exercises 53

1. $\frac{L+12}{120}$	**2.** $\frac{W-7}{\cdot 362}$	**3.** $\frac{n-1\cdot 035}{\cdot 1}$	**4.** $\frac{K-48}{\cdot 45}$
5. $\frac{t-2\cdot 6}{1\cdot 54D}$	**6.** $\frac{4}{9}(F-32)$	**7.** $\frac{S-R}{r}$	**8.** $\frac{115-v}{1300}$
9. $\frac{566-L}{\cdot 8}$	**10.** $50x - 29\cdot 2$.	**11.** $\frac{D-K}{S}$	**12.** $\frac{v-u}{t}$
13. $\frac{mR}{n}$	**14.** $\frac{nr}{m}$	**15.** $\frac{B-H}{4\pi}$	**16.** $\frac{v^2-u^2}{2s}$
17. $\frac{l^2+1\cdot 8p^2}{6\cdot 5p}$	**18.** $150(s-1)$	**19.** $n(S-R)$	**20.** $\frac{13}{d}\left(D-7d\right.$
21. $r^2(p-A)$	**22.** $e(ad+c)$	**23.** $L(1-n)$	**24.** $cr+c$ or $c(r+1$

Exercises 54

1. $\frac{H}{ws} + t$ **2.** $\frac{P}{C} - mA$, **3.** $\frac{c^2}{h} + h$ **4.** $\frac{V}{333} - z$

5. $\frac{2{\cdot}5V}{V - 2}$ **6.** $\frac{c^2}{8h} + \frac{h}{2}$ **7.** $\frac{H}{T - t} - W$ **8.** $\frac{9C}{5d} - W$

9. $\frac{9}{5}C + 32$ **10.** $\frac{S}{600} - 1$ **11.** $\frac{PD}{20144} + \frac{1}{8}$ **12.** $\frac{H.P. \times 33000}{S} + T_2$

13. $\frac{R - R_0}{R_0 t}$ **14.** $\frac{H}{{\cdot}2d^2} - 1$ **15.** $\frac{f - p}{fc^2}$ **16.** $\frac{Wh}{6P} - 1$

17. $6\left(\frac{Z}{h} - A\right)$ **18.** $T - \frac{H}{W + w}$ **19.** ${\cdot}5053p$

Exercises 55

1. $\frac{n - a}{m - a}$ **2.** $\frac{2WH}{F - 2W}$ **3.** $\frac{CR}{E - Cr}$ **4.** $\frac{t}{1 - \eta}$

5. $\frac{N - kt}{1 - k}$ **6.** $\frac{WT - wL}{W + w}$ **7.** $\frac{13D}{n + 91}$ **8.** $\frac{G}{m - 1}$

9. $\frac{kd}{1 + k}$ **10.** $\frac{gH - T}{g - 1}$ **11.** $\frac{2S + Wb}{dh^2}$ **12.** $\frac{m(v - v_1)}{v_1}$

13. $\frac{em - n}{e - 1}$ **14.** $\frac{b(2h - 3g)}{3g - h}$ **15.** $\frac{b(R^2 - r^2)}{R^2r^2}$ **16.** $\frac{2}{3}h$

Exercises 56

1. $\sqrt{\frac{x}{R}}$ **2.** $\sqrt{Rx}$ **3.** $\sqrt{\frac{2E}{I}}$ **4.** $\frac{\sqrt{A}}{{\cdot}725}$ **5.** $3344\sqrt{\frac{P}{A}}$

6. $\frac{d}{{\cdot}866}\sqrt{\frac{p}{P}}$ or $1{\cdot}155d\sqrt{\frac{p}{P}}$ **7.** $t\sqrt{\frac{f}{p}}$ **8.** $\sqrt{\frac{Wl}{ab}}$ **9.** $2{\cdot}83\sqrt{\frac{Td}{w}}$

10. $\frac{{\cdot}9035}{\sqrt{H}}$ **11.** $a\sqrt{\frac{p}{f}}$ **12.** $\sqrt{\frac{bh}{S}}$ **13.** $\sqrt{\frac{EK}{I}}$

14. $\sqrt{2gJ(H_1 - H_2)}$ **15.** $\sqrt{2g(H - h)}$ **16.** $\sqrt{\frac{B}{A - q}}$

17. $2{\cdot}24\sqrt{\frac{H}{r + 1}}$ **18.** $9l - 1$ **19.** $\frac{t}{38{\cdot}7}\sqrt{\frac{5}{f} - 1}$ **20.** $y^2 + 1$

21. $\frac{v^2}{c^2i}$ **22.** $\frac{d^2}{1{\cdot}44}$ or ${\cdot}694d^2$ **23.** $f^2\mu b$ **24.** $\frac{B}{f^2b}$

25. $\frac{H^2}{{\cdot}16d^4}$ **26.** $\frac{\pi D^2}{4p^2}$ **27.** $d\sqrt{R}$ **28.** $\frac{3x^2}{a} - b$

29. $\frac{d_1f^2}{f_1^2}$ **30.** $\frac{4\pi^2W}{gt^2}$ **31.** $p \cdot \frac{D^2 + d^2}{D^2 - d^2}$ **32.** $f \cdot \frac{D^2 - d^2}{D^2 + d^2}$

33. $\frac{\lambda^2}{Lk^2}$ **34.** $\sqrt{b^2 - a^2}$ **35.** $f = 5{\cdot}55p$

Exercises 57

1. 1, 2, 0, 1, 1
2. 2, 4, 0, 1, 7
3. 1, 3, 5, 0, 0
4. $\bar{2}$, $\bar{3}$, $\bar{1}$, $\bar{3}$, $\bar{2}$
5. $\bar{1}$, $\bar{1}$, $\bar{6}$, $\bar{7}$, $\bar{1}$
6. 2, 3, $\bar{4}$, 0, $\bar{2}$

7. ·3096, ·8299, ·9809, ·0334, ·4900
8. ·7505, ·8388, ·0969, ·9031, ·0043
9. ·5999, ·9395, ·4362, ·3010, ·6021
10. ·3164, ·2919, ·8912, ·8006 **11.** ·6998, ·8483, ·5112, ·6056
12. ·9647, ·6430, ·6900, ·7481, ·4972
13. ·8012, ·1498, ·4453, ·5414, ·2915
14. ·6618, ·8362, ·9763, ·9972, ·0931
15. ·5598, ·1844, ·7901, ·9003, ·0038
16. ·2980, ·0124, ·0924, ·9763, ·0013 **17.** $3{\cdot}3502, \bar{3}{\cdot}0523, 0{\cdot}4972, 0{\cdot}0249$
18. $\bar{1}{\cdot}8949, \bar{1}{\cdot}7190, 2{\cdot}6567, 0{\cdot}2355$ **19.** $1{\cdot}4624, 5{\cdot}7634, \bar{2}{\cdot}9069, 0{\cdot}5228$
20. $2{\cdot}8910, 2{\cdot}6898, \bar{1}{\cdot}8495, 3{\cdot}4772, 3{\cdot}2375$
21. $3{\cdot}7226, 0{\cdot}4343, 0{\cdot}2385, \bar{3}{\cdot}699, 1{\cdot}4048$
22. $\bar{4}{\cdot}5328, 0{\cdot}1962, \bar{4}{\cdot}1440, 0{\cdot}4343, 1{\cdot}0026$
23. $0{\cdot}3623, 1{\cdot}7582, \bar{2}{\cdot}0249, 4{\cdot}7756, 3{\cdot}7839$
24. $0{\cdot}0004, \bar{3}{\cdot}7152, 2{\cdot}5912, \bar{1}{\cdot}1031, 1{\cdot}5076$

Exercises 58

1. 1758, 4560, 3162, 2000 **2.** 1012, 1001, 1687, 5093
3. 5616, 2505, 5379, 1408 **4.** 9974, 4720, 6390, 8910
5. 1862, 3111, 6929, 1000 **6.** 9118, 6246, 8015, 7960
7. 17·58, 456·0, 31620, 2000 **8.** 1·012, 1001, 1·687, 50·93
9. ·5616, ·02505, ·5379, 140·8 **10.** 9·974, ·0000472, ·639, 8910
11. 1·862, 3111, ·6929, ·01 **12.** ·0009118, 6·246, ·008015, 79·60
13. ·001022, 156, 20, 2·027 **14.** 21310, 22·89, ·2, 3·142
15. ·00001259, 2245, 3·941, 1,250,000
16. 5·722, ·1618, ·00000166, 30,000,000
17. 2240, 9·763, ·1005, 95·39, ·7854
18. 10, 1728, ·001293, 1·262, ·7501

Exercises 59

1. 78·12 **2.** 30·02 **3.** ·2496 **4.** ·4999 **5.** 3004

Exercises 60

1. 6·896 **2.** ·01227 **3.** 14890 **4.** ·005715 **5.** 1·397 **6.** 7·945

Exercises 61

1. log top line = 2·3200
log bottom line = 0·0239
Result = 197·7
2. log top line = 1·6444
log bottom line = $\bar{4}$·6885
Result = 90350
3. log top line = 6·5717
log bottom line = 4·5185
Result = 113·1
4. log top line = 1·9420
log bottom line = $\bar{4}$·9001
Result = 110,100
5. ·01171 **6.** ·00001535 **7.** 2·326
8. 2800 lbs. per sq. in. **9.** ·09228 sq. in. **10.** 1617 lbs.

Exercises 62

1. 6·822 **2.** 35·27 **3.** 7·709 **4.** 521·2
5. 277·1 **6.** 8·093 **7.** 6·122 tons

Exercises 63

1. 9·872 **2.** ·6169 **3.** 97·61 **4.** ·00005861
5. 775·6 **6.** 343·9 **7.** 2122

Exercises 64

1. 7·516 **2.** 14·76 **3.** ·4433 **4.** ·04062 **5.** 4·607 **6.** 104·3
7. 2·089 **8.** 2·823 **9.** 1·049 **10.** ·9895 **11.** ·2377
12. ·2235 **13.** ·144 **14.** ·02057 **15.** 2·158 **16.** 13·47 ins.
17. 5·343 sq. ft. **18.** 2·309 **19.** 3·773 **20.** 3·534 **21.** 9·668
22. 7·637 ft. **23.** 21·47 **24.** 4·024 **25.** 5459 amps.
26. 8·507 **27.** ·1748 in. **28.** 55·9 **29.** 8·299, say 8·3, H.P.
30 (*a*) 6·485, *i. e.*, 6 threads; (*b*) 8·698, *i. e.*, 9 threads.

Exercises 65

1. 3·142 **2.** 1·068 **3.** 1·646″, say $1\frac{3}{4}$″ **4.** ·4889
5. 9·21 ft. **6.** ·8549 **7.** 3·582 ft. **8.** ·001076
9. 21,200 cu. ft. per hr. **10.** 1·28 **11.** −35·141 **12.** 2·71
13. 1·31 **14.** 115·8 m.p.h. **15.** 8·692 **16.** 17,020 ft.

Exercises 66

1. 3855 yds. **2.** 1543 yds. **3.** 5751 ft. **4.** 4158 ins.
5. 334·2 ins. **6.** 210 ins. **7.** 66·5 ins. **8.** 11·896 ft.
9. ·601 yd. **10.** ·5274 mile **11.** 69′-8″ **12.** 183′-2″
13. 10″ **14.** 4′-4$\frac{1}{2}$″

Exercises 67

1. 13′-4$\frac{1}{4}$″ **2.** 15′-3$\frac{5}{8}$″ **3.** 10′-1″ **4.** 19′-2$\frac{1}{2}$″
5. 9′-5$\frac{1}{8}$″; 11$\frac{5}{16}$″ **6.** 9′-5″ **7.** 28 **8.** 44
9. 3·14″; 1·94″ **10.** 52 **11.** 7·64″ **12.** 24 ins.
13. 13·17″ dia.; ·524″ pitch **14.** 5650 ft. per min. **15.** 1528 r.p.m.
16. 864 ft. per min.; 137·5 r.p.m. **17.** 4′-7$\frac{3}{4}$″ **18.** 19′-5″
19. 7′-3$\frac{1}{2}$″ **20.** 4′-5$\frac{1}{4}$″ **21.** 11′-2″ **22.** 2′-7″
23. 5′-7″ **24.** 9′-2″ **25.** 55 ft. **26.** 12′-11$\frac{1}{2}$″
27. 11′-6$\frac{1}{4}$″ **28.** 15′-1$\frac{3}{8}$″ **29.** 34′-7″ **30.** 3′-3$\frac{3}{4}$″
31. 1500 ft. **32.** $\frac{\pi}{64}$″

Exercises 68

1. Simple 11·78″; more exact 12·96″ **2.** 39·27″; 39·68″
3. 34·54″; 34·6″ **4.** ·142″ **5.** 8·33″ **6.** 9″ **7.** 24
8. 5792 ft. **9.** 97 **10.** 116 **11.** 35 **12.** 1·78″

Exercises 69

1. 20′ too high **2.** Yes **3.** 45° 59′ **4.** 19° 21′
5. 17° 16′ 48″ **6.** 75° 3′ 36″ **7.** 47′ 6″ **8.** 25·25°
9. 30·314° **10.** 65·375° **11.** 11·0972° **12.** 49·23°

Exercises 70

1. 52′-2⅜″ **2.** 26′-11″ **3.** 13′-9″ **4.** 19′-5¼″
5. 23′-9½″ **6.** AB = 16′; AC = 27′-8½″ **7.** 13′-10¼″
8. 7′-1″ **9.** 5·96 ft. **10.** 10′-1⅛″ **11.** ·5001
12. ·684 **13.** 1½″ **14.** 34′-7″ **15.** 393′-10″
16. 16′-10¼″ **17.** 245′-10¼″

Exercises 71

1. 15′-0¾″ **2.** 37½″ **3.** 22′-5⅝″ **4.** 3′-8″
5. 4′-11½″ **6.** 2′-3⅝″

Exercises 72

1. 59·6 sq. ft. **2.** 111·75 sq. ft. **3.** 13,760 lbs.
4. 101¼ tons **5.** 1950 lbs. per sq. ft. **6.** 314·5 sq. ins.
7. 179 sq. ins. **8.** 338 sq. ins.; 34·1% **9.** 29·5 tons per sq. in.
10. ·412 sq. ins.; 36·8% **11.** 2·485 sq. ins. **12.** 45·1 sq. ins.
13. 13·1 amps. **14.** 8400 lbs. **15.** 33 sq. ins.
16. ·11 sq. ins.; ·001173 ohms. **17.** 60 sq. ins. **18.** 99 sq. ins.
19. 2·95 sq. ins. **20.** 132·6 sq. ft.; 16,570 lbs. **21.** 8·38 sq. ins.
22. $\frac{Wl^2}{8}$ **23.** 1080 sq. ins. **24.** 15 sq. ins.
25. 19·34 sq. chains **26.** 32·5 sq. ft. **27.** 1·92
28. 12·25 sq. ins. (both sides taken : edges neglected); 1·1 amps.
29. 8·435 sq. ins. **30.** 9·835 sq. ins. **31.** 2·183 sq. ins.
32. Embankment 389 sq. ft.; cutting 554 sq. ft.

Exercises 73

1. 254 sq. ft. **2.** ·00933 sq. in. **3.** 18·55 sq. ins.
4. Centre ·0579 sq. in.; Outer ·0427 sq. in.; Total ·1006 sq. in.
5. 12 sq. ins. **6.** 2486 lbs. **7.** 11·03 sq. ft. **8.** 158 sq. ft.
9. ·586 sq. in.; 58·6 tons **10.** ·697 sq. in.; 2·8 tons
11. 1372 lbs. **12.** 2·37 : 1 **13.** ·312 **14.** ·32 **15.** 1786 lbs. per sq. in.
16. 50·25 sq. ins.; Yes **17.** 8·44 sq. ins. **18.** 311 sq. ins.
19. 3 : 1 **20.** 5 **21.** 8¼″ **22.** ·048″ **23.** ·605″
24. 149·2 sq. ins.; 13¾″ dia. **25.** 10⅝″ **26.** ·182″, say $\frac{3}{16}$″
27. 2·83″, say 2⅞″ **28.** 3½″ **29.** 7·58″, say 7⅝″
30. H.P. = 2′-9″; I.P. = 4′-7½″

Exercises 74

1. 23 sq. ins. **2.** 44·1 sq. ins.; 154·5 tons
3. 1·23″, say 1¼″ dia. **4.** 4·1 sq. ft. **5.** 12·75 sq. ins.
6. 27·2 sq. ins. **7.** 19·1 sq. ins. **8.** 2¾″
9. L.P. = 176·7 sq. ins.; H.P. = 63·7 sq. ins.; L.P. : H.P. = 2·77:1
10. 154 lbs. per sq. in.

Exercises 75

1. 20·25 sq. ins. **2.** 2·33 sq. ft. **3.** ·417 sq. in.
4. 1·035 sq. ins. **5.** 2·46 sq. ft. **6.** 2·456 sq. ft.; ·163% low
7. 2·451 sq. ft.; ·366% low **8.** 1·48 sq. ft.
9. 1·42 sq. ft.; 4·05 % **10.** 4·63 sq. ft. **11.** 5·52 ft.; ·838
12. (*a*) 11·27 sq. ft.; (*b*) 8·61 ft.; (*c*) 1·31 **13.** 12·07 sq. ft.
14. 1371 sq. ft. **15.** 50·93 sq. ft. **16.** 60·9 sq. ins.; 2·155 times
17. 44 sq. ins.; 1·56 times **18.** 121·7 sq. ins. **19.** 86·8 sq. ins.
20. 127 sq. ins. **21.** 102 sq. ins. **22.** 7·06 sq. ft. **23.** 2170 lbs.

Exercises 76

1. 95·3 sq. ins. **2.** 46·8 sq. ft. **3.** 1124 sq. ft.
4. 245·5 sq. ins. **5.** 28·86 sq. ft. **6.** 26·1 inch tons
7. 1910 sq. chns. **8.** 7·38 sq. ins. **9.** ·96″; 48 lbs. per sq. in.
10. ·552″; 5·52 lbs. per sq. in. **11.** ·385″; 93·5 lbs. per sq. in.
12. ·329″; 98·7 lbs. per sq. in.

Exercises 77

1. 1736 cu. ins. **2.** 801 cu. ft. **3.** 33,700 gals. per hr.
4. 14·48 galls. per min. **5.** 28·35 cu. ft. per min.
6. 734 lbs., or 6 cwts. 2 qrs. 6 lbs.

Exercises 78

1. 371 cu. ft. **2.** 204·7 cwts., say 10 tons 5 cwts.
3. 25,450 galls. **4.** 390 galls. **5.** 31·5 cwts.
6. 162·5 lbs. **7.** 26 galls. **8.** 15 ft.
9. 90 lbs per yd. **10.** 3′-11″ **11.** 4′-9″
12. 206·3 cu. ins. **13.** 12 tons 14 cwts. **14.** 2·5 sq. ft.; 13⅜″

Exercises 79

1. 2,032,000 cu. ft. **2.** 49 cu. ft. **3.** 3·68 cu. ft.
4. 22 galls. **5.** 4·59 lbs. **6.** 4440 galls. per min.
7. ⅛″ **8.** 6·85″, say 7″ dia. **9.** 2·295′, say 2′-4″
10. 3200 cu. ft. per sec. **11.** 8250 cu. ft. **12.** 482·5 lbs.
13. 5490 galls. **14.** 6·75% of working volume; 6·32% of total volume
15. 825 cu. ft. **16.** ·927 lbs. per link; 33·35 lbs. per fathom
17. 13·75 lbs. **18.** 1180 lbs. **19.** 36·2 lbs. per ft.
20. 151·5 tons **21.** 10,240 galls. **22.** ·2275″, say ¼″

Exercises 80

1. 3·33 cu. yds.; 9450 lbs. **2.** 9130 lbs. **3.** 63,300 galls.
4. (*a*) 1·64 lbs.; (*b*) 2·18 lbs. **5.** 1·58 lbs. **6.** 2·97 lbs.
7. 2·29 galls. **8.** 12·86 galls. **9.** 31·6 lbs. **10.** 18·9 galls.
11. 104 lbs. **12.** 62 cu. ft.; 7440 lbs. **13.** 1·95 lbs.; 8*s.* 6*d.*
14. 230 lbs. **15.** 22·85 galls. **16.** ·475 cu. ft.; 57 lbs.
17. 404,000 lbs. **18.** 88·6 lbs. per yd.

Exercises 81

1. 2·84 lbs. **2.** 89·8 cu. ins.; 13·8, *i.e.*, 13 **3.** 2·045 cu. ft.
4. 86·7 galls. **5.** 58,200 galls. **6.** 80·5 cu. ft.
7. 37·8 lbs. **8.** 26·8 lbs. **9.** 12·8 lbs. **10.** 8 lbs.

Exercises 82

1. 104·2 cu. ins.; $\frac{1}{3}$ **2.** 8·54″ **3.** 99·6 cu. ins.; cone is one third of cylinder volume **4.** 615 cu. ins.; 7·82″, say 8″ **5.** 4·07″
6. 1 ton 14 cwts. **7.** 1918 cu. ft.; 47·95, say 48 tons
8. 13720 cu. ft.; 659,000 lbs. or 294 tons **9.** 417,000 galls.
10. 27·1 galls. **11.** 22·9 tons. **12.** 4,090,000 galls.

Exercises 83

1. 6653 sq. ft. **2.** 1440 sq. ft. **3.** 7480 sq. ft.; 6·95 lbs. per sq. ft.
4. 317 lbs. per sq. in. **5.** 402 units per sq. ft. **6.** 75·4 sq. ins.
7. $1\frac{1}{2}$″ **8.** $\frac{d}{4}$ **9.** 92·6 lbs. **10.** ·15 lb.
11. ·897 watt per sq. in. **12.** 440 sq. ins. **13.** 2·37″, say $2\frac{1}{2}$″
14. 134·6 sq. ft. **15.** 40·8 lbs. **16.** 4·39 ft., say 4′-4$\frac{3}{4}$″
17. 43·3 sq. ft. **18.** 30·3 sq. ft. **19.** 39·3 sq. ft.
20. 9530 lbs. **21.** ·568 watt per sq. in.; 2·41 cu. ins.
22. (*a*) 11·8 sq. millimetres; (*b*) 254. **23.** ·206 lb.

Exercises 83a. *Refer p.* 453

Exercises 84

1. Base 1″ = 10°; vertical 1″ = 10 lbs.; (*a*) 40·3 lbs. per sq. ft.; (*b*) 48·5 lbs. per sq. ft.
2. Base 1″ = 5 mins.; vertical 1″ = 10°.
3. Base 1″ = 5 amps.; vert. $\frac{1}{2}$″ = ·2 volt; (*a*) 1·22 volts; (*b*) 1·78 volts
4. (*a*) 525 lbs.; (*b*) 1500 lbs.; (*c*) 2920 lbs.
5. (*a*) 54*s.*; (*b*) 62*s.*; (*c*) 105*s.*
6. Base 1″ = 10 amps.; vertical 1″ = 20 volts
7. Base 1″ = 10 m.p.h.; vertical $\frac{1}{2}$″ = 2 lbs.; 2220 lbs.
8. (*a*) 2400 lbs. per sq. in.; (*b*) 6·3 to 1
9. (*a*) ·57 sq. in.; (*b*) ·195 sq. in.; (*c*) ·36 sq. in.
10. (*a*) 1·4, *i.e.*, No. 1 S.W.G.; (*b*) 7·4, *i.e.*, No. 7 S.W.G.; (*c*) 18·6, *i.e.*, No. 18 S.W.G.

Exercises 85

1. Base—from 7, 1″ = ·5 knot; vert.—from 0, ½″ = 200 H.P.; 1660 H.P.

2. Base—from 0, ½″ = 2%; vert.—from 100°, 1″ = 2°; (*a*) 105·1° C.; (*b*) 26%

4. Base—from 1, 1″ = ·05; vert.—from 1, ½″ = ·2; (*a*) 1·175 sp. gr.; (*b*) 1·28 resistance **5.** ·0057

7. Base—from 1, ½″ = ·02; vert.—from 1·4, 1″ = ·1; 1·96 volts

8. Base—from 0, ½″ = 20°; vert—from 53, 1″ = 2 lbs.; (*a*) 60·55 lbs.; (*b*) 56·6 lbs.

9. (*a*) 18·6, *i.e.*, No. 18 S.W.G.; (*b*) 14·9, *i.e.*, No. 14 S.W.G.

10. 109·5

Exercises 86

1. AB = 8·5, BC = 5, CA = 3·6. *Note.*—These measurements should be made on the squared paper and not with a rule, as the squares are often not exact to size.

2. AC = BD = 6·4 **3.** 7·9 **4.** 1·475 **5.** 308° **6.** 5350

7. (*a*) ·7 and − 2·6; (*b*) − 1·2; (*c*) − 1·84 **8.** − 3·7; − ·25; 4

Exercises 87

1. $e = ·2w$ **2.** (*a*) P = 3·1 + 1·94W; (*b*) £12 16*s.*; (*c*) £12 16*s.*

3. V = 1200 + 630*d* **4.** $f = 18·5 - ·0068\frac{4}{t}$ **5.** $w = ·65 + ·305H$

6. (*a*) 27,200 lbs.; (*b*) 14,600 lbs.; W = 900 + 5·22S **7.** 710

8. Total oil = 4·64 lbs.; oil per H.P. hour = ·93 lb.

9. R = 63 + 2·72V; the last point is disregarded; it is far from the general direction of the others and indicates a blunder when testing. **10.** L = 1·31 − ·471*l*

11. L = 565 − ·795*t*. *Note.*—On the vertical (latent heat) scale plot from 490, taking 1″ = 20. **12.** P = 9·56D − 13

13. T = 100 − ·000967H.

Exercises 88

1. 12	**2.** 2·2	**3.** 8·4	**4.** ·585
5. ·0308	**6.** ·0102	**7.** 885	**8.** 90·5
9. ·015	**10.** ·425	**11.** ·525	**12.** 25
13. ·0274	**14.** 11·65	**15.** 4·7	**16.** ·264
17. 2·29	**18.** ·916	**19.** 648	**20.** 110·4

Exercises 89

1. 77	**2.** 2·16	**3.** 18·2	**4.** 71	
5. 1000	**6.** ·316	**7.** 195	**8.** ·86	
9. ·0054	**10.** 11·3	**11.** ·57	**12.** 1640	
13. 107·5	**14.** 717·5	**15.** ·315	**16.** 186	
17. 92·1	**18.** 9·34	**19.** 32·3	**20.** 1058	
21. 37·35	**22.** 132,100	**23.** 63	**24.** 714	**25.** 10070

Exercises 90

1. 9·7	**2.** ·103	**3.** ·63	**4.** 5·2
5. 42·5	**6.** 4·56	**7.** 1·16	**8.** ·98
9. 5·8	**10.** ·00991	**11.** 2·34	**12.** 9·7
13. ·1471	**14.** ·0407	**15.** ·468	**16.** ·1076
17. 101,700	**18.** ·275	**19.** 70	**20.** 7·41

Exercises 91

See Answers to Exercises 18.

Exercises 92

1. 2nd : 51	**2.** 1st : 211	**3.** 1st : 1·245	**4.** 1st : 14·6
5. 2nd : 9·925	**6.** 1st : 27·15	**7.** 1st : ·225	**8.** 2nd : ·0583
9. 2nd : 3·42	**10.** 2nd : ·554	**11.** 15·81	**12.** 2·7
13. ·624	**14.** 95·9	**15.** 10·3	**16.** 2·292
17. ·432	**18.** 7·38	**19.** 1·475	**20.** 5·54
	21. 17	**22.** ·9125	

Exercises 93

1. 6,760,000	**2.** 1,981,000,000	**3.** 2·4	**4.** 45,400
5. 9700	**6.** 543,000		**7.** ·00255
8. ·00001156	**9.** 136·9		**10.** ·09425

Exercises 94

1. (a) tan A; (b) cos A; (c) sin A; (d) tan C; (e) cos C.
2. (a) ·78; (b) ·6158; (c) ·6158; (d) ·7894; (e) 1·282; (f) ·7894.
3. (a) ·342; (b) ·7002; (c) 1·0724; (d) ·3746; (e) ·9653; (f) ·8192.

Exercises 95

1. ·9659.	**2.** 2·9042.	**3.** ·3090.	**4.** ·6947.	**5.** ·7193.
6. ·5317.	**7.** 57·2900.	**8.** ·6947.	**9.** ·2250.	**10.** ·9976.

Exercises 96

1. 64°.	**2.** 14°.	**3.** 39°.	**4.** 88°.	**5.** 8°.	**6.** 69°.
7. 47°.	**8.** 31°.	**9.** 18°.	**10.** 49°.	**11.** 31°.	**12.** 72°.

Exercises 97

1. 1·505.	**2.** 1138.	**3.** 39°.	**4.** 31°.	**5.** 4·017.
6. 5·715.	**7.** 93·08.	**8.** 49·52.	**9.** 42°.	**10.** 38°.

Exercises 98

1. 8°. **2.** 29°. **3.** 35°. **4.** $d = \cdot 96p$; $D = \cdot 63p$.
5. $\theta = 28°$; $\varphi = 63°$. **6.** WL $(\cos\beta - \cos\alpha.)$ **7.** 1·171″.
8. 1·099″. **9.** (*a*) $ADB = 2R \sin \frac{90°}{N}$; (*b*) $DE = R\left(1 - \cos \frac{90°}{N}\right)$.
10. $\alpha = 37°$ approx.; angle turned by pointer = 148° approx.

Exercises 99

1. (A) 1·56″; (B) 3·92″; (C) 0·09″
2. (D) 2·731″; (E) 0·987″; (F) 1·069″; (G) ·028″; (H) 4·398″; (J) 0·01″; (K) 2·051″; (L) 3·549″; (M) 0·625″

Exercises 100

1. $\frac{1}{64}$ in. **2.** ·001″ **3.** ·002″ **4.** ·001″
5. 6 minutes **6.** 3 minutes **7.** 20 seconds
8. (A) $1\frac{3}{64}$ ins.; (B) $3\frac{5}{8} + \frac{7}{64} = 3\frac{47}{64}$ ins.; (C) $2\frac{1}{4} + \frac{4}{64} = 2\frac{5}{16}$ ins.
9. (D) $2\frac{3}{16} + 1\frac{1}{128} = 2\frac{25}{128}$ ins.; (E) $1\frac{5}{8} + 1\frac{5}{128} = 1\frac{85}{128}$ ins.; (F) $\frac{9}{16} + 1\frac{4}{128} = \frac{13}{32}$ ins.
10. (G) 29·98; (H) 17·416; (K) 26·134 **11.** (L) 4° 10′; (M) 18° 40′
12. (N) 23·3° = 23° 18′; (O) 5·8° = 5° 48′
13. (P) 10° 15′; (Q) 45° 39′ **14.** (R) 2° 18′; (S) 42° 43′

Exercises 101

1. (A) ·455″; (B) ·284″; (C) ·175″; (D) ·37″; (E) ·213″; (F) ·027″
2. (G) ·3692″, *i.e.* ·369″; (H) ·1875″; (J) ·5938″, *i.e.* ·594″

Exercises 102

(A) ·0813″; (B) ·3148″; (C) ·3512″

Exercises 20a

1. 5·734 **2.** 2828·55, say 2829
3. (*a*) 5·51 lb. per sq. ft.; (*b*) 16·54 lb. per H.P.
4. 3·762″ **5.** 9·23 % **6.** 6·4 to 1
7. (*a*) 10 lb. $5\frac{3}{4}$ oz.; (*b*) 3.62 % **8.** 27·74 lb.

Exercises 49a

1. ·553 **2.** ·000542 **3.** $\frac{1{\cdot}053}{10^7} \cdot \frac{W^2}{a^2}$ **4.** 175 **5.** 1·52

Exercises 83a

1. 4·71 sq. ins. **2.** 2727 lb. **3.** 208′ – 10″ **4.** 503 lb.
5. 38·3″, say $38\frac{1}{2}$″ **6.** (1) ·9995 sq. mm.; (2) 1000
7. 26 teeth; ·059″ **8.** 1·509″, say $1\frac{1}{2}$″ **9.** 8·06 lb.
10. ·708 gallon **11.** 88·4 lb.
12. No; calculated weight is practically 1 lb. **13.** 17·86 gallons.

Logarithms

	0	1	2	3	4	5	6	7	8	9	1 2 3	4 5 6	7 8 9
10	0000	0043	0086	0128	0170	0212	0253	0294	0334	0374	4 9 13 4 8 12	17 21 26 16 20 24	30 34 38 28 32 37
11	0414	0453	0492	0531	0569	0607	0645	0682	0719	0755	4 8 12 4 7 11	15 19 23 15 19 22	27 31 35 26 30 33
12	0792	0828	0864	0899	0934	0969	1004	1038	1072	1106	3 7 11 3 7 10	14 18 21 14 17 20	25 28 32 24 27 31
13	1139	1173	1206	1239	1271	1303	1335	1367	1399	1430	3 7 10 3 7 10	13 16 20 12 16 19	23 26 30 22 25 29
14	1461	1492	1523	1553	1584	1614	1644	1673	1703	1732	3 6 9 3 6 9	12 15 18 12 15 17	21 24 28 20 23 26
15	1761	1790	1818	1847	1875	1903	1931	1959	1987	2014	3 6 9 3 5 8	11 14 17 11 14 16	20 23 26 19 22 25
16	2041	2068	2095	2122	2148	2175	2201	2227	2253	2279	3 5 8 3 5 8	11 14 16 10 13 15	19 22 24 18 21 23
17	2304	2330	2355	2380	2405	2430	2455	2480	2504	2529	3 5 8 2 5 7	10 13 15 10 12 15	18 20 23 17 19 22
18	2553	2577	2601	2625	2648	2672	2695	2718	2742	2765	2 5 7 2 5 7	9 12 14 9 11 14	16 19 21 16 18 21
19	2788	2810	2833	2856	2878	2900	2923	2945	2967	2989	2 4 7 2 4 6	9 11 13 8 11 13	16 18 20 15 17 19
20	3010	3032	3054	3075	3096	3118	3139	3160	3181	3201	2 4 6	8 11 13	15 17 19
21	3222	3243	3263	3284	3304	3324	3345	3365	3385	3404	2 4 6	8 10 12	14 16 18
22	3424	3444	3464	3483	3502	3522	3541	3560	3579	3598	2 4 6	8 10 12	14 15 17
23	3617	3636	3655	3674	3692	3711	3729	3747	3766	3784	2 4 6	7 9 11	13 15 17
24	3802	3820	3838	3856	3874	3892	3909	3927	3945	3962	2 4 5	7 9 11	12 14 16
25	3979	3997	4014	4031	4048	4065	4082	4099	4116	4133	2 3 5	7 9 10	12 14 15
26	4150	4166	4183	4200	4216	4232	4249	4265	4281	4298	2 3 5	7 8 10	11 13 15
27	4314	4330	4346	4362	4378	4393	4409	4425	4440	4456	2 3 5	6 8 9	11 13 14
28	4472	4487	4502	4518	4533	4548	4564	4579	4594	4609	2 3 5	6 8 9	11 12 14
29	4624	4639	4654	4669	4683	4698	4713	4728	4742	4757	1 3 4	6 7 9	10 12 13
30	4771	4786	4800	4814	4829	4843	4857	4871	4886	4900	1 3 4	6 7 9	10 11 13
31	4914	4928	4942	4955	4969	4983	4997	5011	5024	5038	1 3 4	6 7 8	10 11 12
32	5051	5065	5079	5092	5105	5119	5132	5145	5159	5172	1 3 4	5 7 8	9 11 12
33	5185	5198	5211	5224	5237	5250	5263	5276	5289	5302	1 3 4	5 6 8	9 10 12
34	5315	5328	5340	5353	5366	5378	5391	5403	5416	5428	1 3 4	5 6 8	9 10 11
35	5441	5453	5465	5478	5490	5502	5514	5527	5539	5551	1 2 4	5 6 7	9 10 11
36	5563	5575	5587	5599	5611	5623	5635	5647	5658	5670	1 2 4	5 6 7	8 10 11
37	5682	5694	5705	5717	5729	5740	5752	5763	5775	5786	1 2 3	5 6 7	8 9 10
38	5798	5809	5821	5832	5843	5855	5866	5877	5888	5899	1 2 3	5 6 7	8 9 10
39	5911	5922	5933	5944	5955	5966	5977	5988	5999	6010	1 2 3	4 5 7	8 9 10
40	6021	6031	6042	6053	6064	6075	6085	6096	6107	6117	1 2 3	4 5 6	8 9 10
41	6128	6138	6149	6160	6170	6180	6191	6201	6212	6222	1 2 3	4 5 6	7 8 9
42	6232	6243	6253	6263	6274	6284	6294	6304	6314	6325	1 2 3	4 5 6	7 8 9
43	6335	6345	6355	6365	6375	6385	6395	6405	6415	6425	1 2 3	4 5 6	7 8 9
44	6435	6444	6454	6464	6474	6484	6493	6503	6513	6522	1 2 3	4 5 6	7 8 9
45	6532	6542	6551	6561	6571	6580	6590	6599	6609	6618	1 2 3	4 5 6	7 8 9
46	6628	6637	6646	6656	6665	6675	6684	6693	6702	6712	1 2 3	4 5 6	7 7 8
47	6721	6730	6739	6749	6758	6767	6776	6785	6794	6803	1 2 3	4 5 5	6 7 8
48	6812	6821	6830	6839	6848	6857	6866	6875	6884	6893	1 2 3	4 4 5	6 7 8
49	6902	6911	6920	6928	6937	6946	6955	6964	6972	6981	1 2 3	4 4 5	6 7 8
50	6990	6998	7007	7016	7024	7033	7042	7050	7059	7067	1 2 3	3 4 5	6 7 8

Logarithms

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
51	7076	7084	7093	7101	7110	7118	7126	7135	7143	7152	1	2	3	3	4	5	6	7	8
52	7160	7168	7177	7185	7193	7202	7210	7218	7226	7235	1	2	2	3	4	5	6	7	7
53	7243	7251	7259	7267	7275	7284	7292	7300	7308	7316	1	2	2	3	4	5	6	6	7
54	7324	7332	7340	7348	7356	7364	7372	7380	7388	7396	1	2	2	3	4	5	6	6	7
55	7404	7412	7419	7427	7435	7443	7451	7459	7466	7474	1	2	2	3	4	5	5	6	7
56	7482	7490	7497	7505	7513	7520	7528	7536	7543	7551	1	2	2	3	4	5	5	6	7
57	7559	7566	7574	7582	7589	7597	7604	7612	7619	7627	1	2	2	3	4	5	5	6	7
58	7634	7642	7649	7657	7664	7672	7679	7686	7694	7701	1	1	2	3	4	4	5	6	7
59	7709	7716	7723	7731	7738	7745	7752	7760	7767	7774	1	1	2	3	4	4	5	6	7
60	7782	7789	7796	7803	7810	7818	7825	7832	7839	7846	1	1	2	3	4	4	5	6	6
61	7853	7860	7868	7875	7882	7889	7896	7903	7910	7917	1	1	2	3	4	4	5	6	6
62	7924	7931	7938	7945	7952	7959	7966	7973	7980	7987	1	1	2	3	3	4	5	6	6
63	7993	8000	8007	8014	8021	8028	8035	8041	8048	8055	1	1	2	3	3	4	5	5	6
64	8062	8069	8075	8082	8089	8096	8102	8109	8116	8122	1	1	2	3	3	4	5	5	6
65	8129	8136	8142	8149	8156	8162	8169	8176	8182	8189	1	1	2	3	3	4	5	5	6
66	8195	8202	8209	8215	8222	8228	8235	8241	8248	8254	1	1	2	3	3	4	5	5	6
67	8261	8267	8274	8280	8287	8293	8299	8306	8312	8319	1	1	2	3	3	4	5	5	6
68	8325	8331	8338	8344	8351	8357	8363	8370	8376	8382	1	1	2	3	3	4	4	5	6
69	8388	8395	8401	8407	8414	8420	8426	8432	8439	8445	1	1	2	2	3	4	4	5	6
70	8451	8457	8463	8470	8476	8482	8488	8494	8500	8506	1	1	2	2	3	4	4	5	6
71	8513	8519	8525	8531	8537	8543	8549	8555	8561	8567	1	1	2	2	3	4	4	5	5
72	8573	8579	8585	8591	8597	8603	8609	8615	8621	8627	1	1	2	2	3	4	4	5	5
73	8633	8639	8645	8651	8657	8663	8669	8675	8681	8686	1	1	2	2	3	4	4	5	5
74	8692	8698	8704	8710	8716	8722	8727	8733	8739	8745	1	1	2	2	3	4	4	5	5
75	8751	8756	8762	8768	8774	8779	8785	8791	8797	8802	1	1	2	2	3	3	4	5	5
76	8808	8814	8820	8825	8831	8837	8842	8848	8854	8859	1	1	2	2	3	3	4	5	5
77	8865	8871	8876	8882	8887	8893	8899	8904	8910	8915	1	1	2	2	3	3	4	4	5
78	8921	8927	8932	8938	8943	8949	8954	8960	8965	8971	1	1	2	2	3	3	4	4	5
79	8976	8982	8987	8993	8998	9004	9009	9015	9020	9025	1	1	2	2	3	3	4	4	5
80	9031	9036	9042	9047	9053	9058	9063	9069	9074	9079	1	1	2	2	3	3	4	4	5
81	9085	9090	9096	9101	9106	9112	9117	9122	9128	9133	1	1	2	2	3	3	4	4	5
82	9138	9143	9149	9154	9159	9165	9170	9175	9180	9186	1	1	2	2	3	3	4	4	5
83	9191	9196	9201	9206	9212	9217	9222	9227	9232	9238	1	1	2	2	3	3	4	4	5
84	9243	9248	9253	9258	9263	9269	9274	9279	9284	9289	1	1	2	2	3	3	4	4	5
85	9294	9299	9304	9309	9315	9320	9325	9330	9335	9340	1	1	2	2	3	3	4	4	5
86	9345	9350	9355	9360	9365	9370	9375	9380	9385	9390	1	1	2	2	3	3	4	4	5
87	9395	9400	9405	9410	9415	9420	9425	9430	9435	9440	0	1	1	2	2	3	3	4	4
88	9445	9450	9455	9460	9465	9469	9474	9479	9484	9489	0	1	1	2	2	3	3	4	4
89	9494	9499	9504	9509	9513	9518	9523	9528	9533	9538	0	1	1	2	2	3	3	4	4
90	9542	9547	9552	9557	9562	9566	9571	9576	9581	9586	0	1	1	2	2	3	3	4	4
91	9590	9595	9600	9605	9609	9614	9619	9624	9628	9633	0	1	1	2	2	3	3	4	4
92	9638	9643	9647	9652	9657	9661	9666	9671	9675	9680	0	1	1	2	2	3	3	4	4
93	9685	9689	9694	9699	9703	9708	9713	9717	9722	9727	0	1	1	2	2	3	3	4	4
94	9731	9736	9741	9745	9750	9754	9759	9763	9768	9773	0	1	1	2	2	3	3	4	4
95	9777	9782	9786	9791	9795	9800	9805	9809	9814	9818	0	1	1	2	2	3	3	4	4
96	9823	9827	9832	9836	9841	9845	9850	9854	9859	9863	0	1	1	2	2	3	3	4	4
97	9868	9872	9877	9881	9886	9890	9894	9899	9903	9908	0	1	1	2	2	3	3	4	4
98	9912	9917	9921	9926	9930	9934	9939	9943	9948	9952	0	1	1	2	2	3	3	4	4
99	9956	9961	9965	9969	9974	9978	9983	9987	9991	9996	0	1	1	2	2	3	3	3	4

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	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
·00	1000	1002	1005	1007	1009	1012	1014	1016	1019	1021	0	0	1	1	1	1	2	2	2
·01	1023	1026	1028	1030	1033	1035	1038	1040	1042	1045	0	0	1	1	1	1	2	2	2
·02	1047	1050	1052	1054	1057	1059	1062	1064	1067	1069	0	0	1	1	1	1	2	2	2
·03	1072	1074	1076	1079	1081	1084	1086	1089	1091	1094	0	0	1	1	1	1	2	2	2
04	1096	1099	1102	1104	1107	1109	1112	1114	1117	1119	0	1	1	1	1	2	2	2	2
05	1122	1125	1127	1130	1132	1135	1138	1140	1143	1146	0	1	1	1	1	2	2	2	2
06	1148	1151	1153	1156	1159	1161	1164	1167	1169	1172	0	1	1	1	1	2	2	2	2
07	1175	1178	1180	1183	1186	1189	1191	1194	1197	1199	0	1	1	1	1	2	2	2	2
·08	1202	1205	1208	1211	1213	1216	1219	1222	1225	1227	0	1	1	1	1	2	2	2	3
09	1230	1233	1236	1239	1242	1245	1247	1250	1253	1256	0	1	1	1	1	2	2	2	3
·10	1259	1262	1265	1268	1271	1274	1276	1279	1282	1285	0	1	1	1	1	2	2	2	3
·11	1288	1291	1294	1297	1300	1303	1306	1309	1312	1315	0	1	1	1	2	2	2	2	3
·12	1318	1321	1324	1327	1330	1334	1337	1340	1343	1346	0	1	1	1	2	2	2	2	3
·13	1349	1352	1355	1358	1361	1365	1368	1371	1374	1377	0	1	1	1	2	2	2	3	3
·14	1380	1384	1387	1390	1393	1396	1400	1403	1406	1409	0	1	1	1	2	2	2	3	3
·15	1413	1416	1419	1422	1426	1429	1432	1435	1439	1442	0	1	1	1	2	2	2	3	3
·16	1445	1449	1452	1455	1459	1462	1466	1469	1472	1476	0	1	1	1	2	2	2	3	3
17	1479	1483	1486	1489	1493	1496	1500	1503	1507	1510	0	1	1	1	2	2	2	3	3
18	1514	1517	1521	1524	1528	1531	1535	1538	1542	1545	0	1	1	1	2	2	2	3	3
·19	1549	1552	1556	1560	1563	1567	1570	1574	1578	1581	0	1	1	1	2	2	3	3	3
·20	1585	1589	1592	1596	1600	1603	1607	1611	1614	1618	0	1	1	1	2	2	3	3	3
21	1622	1626	1629	1633	1637	1641	1644	1648	1652	1656	0	1	1	2	2	2	3	3	3
22	1660	1663	1667	1671	1675	1679	1683	1687	1690	1694	0	1	1	2	2	2	3	3	3
·23	1698	1702	1706	1710	1714	1718	1722	1726	1730	1734	0	1	1	2	2	2	3	3	4
·24	1738	1742	1746	1750	1754	1758	1762	1766	1770	1774	0	1	1	2	2	2	3	3	4
·25	1778	1782	1786	1791	1795	1799	1803	1807	1811	1816	0	1	1	2	2	2	3	3	4
·26	1820	1824	1828	1832	1837	1841	1845	1849	1854	1858	0	1	1	2	2	3	3	3	4
·27	1862	1866	1871	1875	1879	1884	1888	1892	1897	1901	0	1	1	2	2	3	3	3	4
·28	1905	1910	1914	1919	1923	1928	1932	1936	1941	1945	0	1	1	2	2	3	3	4	4
·29	1950	1954	1959	1963	1968	1972	1977	1982	1986	1991	0	1	1	2	2	3	3	4	4
·30	1995	2000	2004	2009	2014	2018	2023	2028	2032	2037	0	1	1	2	2	3	3	4	4
·31	2042	2046	2051	2056	2061	2065	2070	2075	2080	2084	0	1	1	2	2	3	3	4	4
·32	2089	2094	2099	2104	2109	2113	2118	2123	2128	2133	0	1	1	2	2	3	3	4	4
·33	2138	2143	2148	2153	2158	2163	2168	2173	2178	2183	0	1	1	2	2	3	3	4	4
34	2188	2193	2198	2203	2208	2213	2218	2223	2228	2234	1	1	2	2	3	3	4	4	5
·35	2239	2244	2249	2254	2259	2265	2270	2275	2280	2286	1	1	2	2	3	3	4	4	5
·36	2291	2296	2301	2307	2312	2317	2323	2328	2333	2339	1	1	2	2	3	3	4	4	5
·37	2344	2350	2355	2360	2366	2371	2377	2382	2388	2393	1	1	2	2	3	3	4	4	5
·38	2399	2404	2410	2415	2421	2427	2432	2438	2443	2449	1	1	2	2	3	3	4	4	5
·39	2455	2460	2466	2472	2477	2483	2489	2495	2500	2506	1	1	2	2	3	3	4	5	5
·40	2512	2518	2523	2529	2535	2541	2547	2553	2559	2564	1	1	2	2	3	4	4	5	5
41	2570	2576	2582	2588	2594	2600	2606	2612	2618	2624	1	1	2	2	3	4	4	5	5
·42	2630	2636	2642	2649	2655	2661	2667	2673	2679	2685	1	1	2	2	3	4	4	5	6
·43	2692	2698	2704	2710	2716	2723	2729	2735	2742	2748	1	1	2	3	3	4	4	5	6
·44	2754	2761	2767	2773	2780	2786	2793	2799	2805	2812	1	1	2	3	3	4	4	5	6
45	2818	2825	2831	2838	2844	2851	2858	2864	2871	2877	1	1	2	3	3	4	5	5	6
·46	2884	2891	2897	2904	2911	2917	2924	2931	2938	2944	1	1	2	3	3	4	5	5	6
·47	2951	2958	2965	2972	2979	2985	2992	2999	3006	3013	1	1	2	3	3	4	5	5	6
·48	3020	3027	3034	3041	3048	3055	3062	3069	3076	3083	1	1	2	3	4	4	5	6	6
·49	3090	3097	3105	3112	3119	3126	3133	3141	3148	3155	1	1	2	3	4	4	5	6	6

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	0	1	2	3	4	5	6	7	8	9	1 2 3	4 5 6	7 8 9
·50	3162	3170	3177	3184	3192	3199	3206	3214	3221	3228	1 1 2	3 4 4	5 6 7
·51	3236	3243	3251	3258	3266	3273	3281	3289	3296	3304	1 2 2	3 4 5	5 6 7
·52	3311	3319	3327	3334	3342	3350	3357	3365	3373	3381	1 2 2	3 4 5	5 6 7
·53	3388	3396	3404	3412	3420	3428	3436	3443	3451	3459	1 2 2	3 4 5	6 6 7
·54	3467	3475	3483	3491	3499	3508	3516	3524	3532	3540	1 2 2	3 4 5	6 6 7
·55	3548	3556	3565	3573	3581	3589	3597	3606	3614	3622	1 2 2	3 4 5	6 7 7
·56	3631	3639	3648	3656	3664	3673	3681	3690	3698	3707	1 2 3	3 4 5	6 7 8
·57	3715	3724	3733	3741	3750	3758	3767	3776	3784	3793	1 2 3	3 4 5	6 7 8
·58	3802	3811	3819	3828	3837	3846	3855	3864	3873	3882	1 2 3	4 4 5	6 7 8
·59	3890	3899	3908	3917	3926	3936	3945	3954	3963	3972	1 2 3	4 5 5	6 7 8
·60	3981	3990	3999	4009	4018	4027	4036	4046	4055	4064	1 2 3	4 5 6	6 7 8
·61	4074	4083	4093	4102	4111	4121	4130	4140	4150	4159	1 2 3	4 5 6	7 8 9
·62	4169	4178	4188	4198	4207	4217	4227	4236	4246	4256	1 2 3	4 5 6	7 8 9
·63	4266	4276	4285	4295	4305	4315	4325	4335	4345	4355	1 2 3	4 5 6	7 8 9
·64	4365	4375	4385	4395	4406	4416	4426	4436	4446	4457	1 2 3	4 5 6	7 8 9
·65	4467	4477	4487	4498	4508	4519	4529	4539	4550	4560	1 2 3	4 5 6	7 8 9
·66	4571	4581	4592	4603	4613	4624	4634	4645	4656	4667	1 2 3	4 5 6	7 9 10
·67	4677	4688	4699	4710	4721	4732	4742	4753	4764	4775	1 2 3	4 5 7	8 9 10
·68	4786	4797	4808	4819	4831	4842	4853	4864	4875	4887	1 2 3	4 6 7	8 9 10
69	4898	4909	4920	4932	4943	4955	4966	4977	4989	5000	1 2 3	5 6 7	8 9 10
·70	5012	5023	5035	5047	5058	5070	5082	5093	5105	5117	1 2 4	5 6 7	8 9 11
·71	5129	5140	5152	5164	5176	5188	5200	5212	5224	5236	1 2 4	5 6 7	8 10 11
·72	5248	5260	5272	5284	5297	5309	5321	5333	5346	5358	1 2 4	5 6 7	9 10 11
·73	5370	5383	5395	5408	5420	5433	5445	5458	5470	5483	1 3 4	5 6 8	9 10 11
·74	5495	5508	5521	5534	5546	5559	5572	5585	5598	5610	1 3 4	5 6 8	9 10 12
·75	5623	5636	5649	5662	5675	5689	5702	5715	5728	5741	1 3 4	5 7 8	9 10 12
·76	5754	5768	5781	5794	5808	5821	5834	5848	5861	5875	1 3 4	5 7 8	9 11 12
·77	5888	5902	5916	5929	5943	5957	5970	5984	5998	6012	1 3 4	5 7 8	10 11 12
·78	6026	6039	6053	6067	6081	6095	6109	6124	6138	6152	1 3 4	6 7 8	10 11 13
·79	6166	6180	6194	6209	6223	6237	6252	6266	6281	6295	1 3 4	6 7 9	10 11 13
·80	6310	6324	6339	6353	6368	6383	6397	6412	6427	6442	1 3 4	6 7 9	10 12 13
·81	6457	6471	6486	6501	6516	6531	6546	6561	6577	6592	2 3 5	6 8 9	11 12 14
·82	6607	6622	6637	6653	6668	6683	6699	6714	6730	6745	2 3 5	6 8 9	11 12 14
·83	6761	6776	6792	6808	6823	6839	6855	6871	6887	6902	2 3 5	6 8 9	11 13 14
·84	6918	6934	6950	6966	6982	6998	7015	7031	7047	7063	2 3 5	6 8 10	11 13 15
85	7079	7096	7112	7129	7145	7161	7178	7194	7211	7228	2 3 5	7 8 10	12 13 15
·86	7244	7261	7278	7295	7311	7328	7345	7362	7379	7396	2 3 5	7 8 10	12 13 15
·87	7413	7430	7447	7464	7482	7499	7516	7534	7551	7568	2 3 5	7 9 10	12 14 16
·88	7586	7603	7621	7638	7656	7674	7691	7709	7727	7745	2 4 5	7 9 11	12 14 16
·89	7762	7780	7798	7816	7834	7852	7870	7889	7907	7925	2 4 5	7 9 11	13 14 16
·90	7943	7962	7980	7998	8017	8035	8054	8072	8091	8110	2 4 6	7 9 11	13 15 17
·91	8128	8147	8166	8185	8204	8222	8241	8260	8279	8299	2 4 6	8 9 11	13 15 17
·92	8318	8337	8356	8375	8395	8414	8433	8453	8472	8492	2 4 6	8 10 12	14 15 17
·93	8511	8531	8551	8570	8590	8610	8630	8650	8670	8690	2 4 6	8 10 12	14 16 18
·94	8710	8730	8750	8770	8790	8810	8831	8851	8872	8892	2 4 6	8 10 12	14 16 18
·95	8913	8933	8954	8974	8995	9016	9036	9057	9078	9099	2 4 6	8 10 12	15 17 19
·96	9120	9141	9162	9183	9204	9226	9247	9268	9290	9311	2 4 6	8 11 13	15 17 19
·97	9333	9354	9376	9397	9419	9441	9462	9484	9506	9528	2 4 7	9 11 13	15 17 20
·98	9550	9572	9594	9616	9638	9661	9683	9705	9727	9750	2 4 7	9 11 13	16 18 20
·99	9772	9795	9817	9840	9863	9886	9908	9931	9954	9977	2 5 7	9 11 14	16 18 20

Trigonometric Ratios.

Angle.		Chord.	Sine.	Tangent.	Co-tangent.	Cosine.			
Degrees.	Radians.								
0°	0	0	0	0	∞	1	1·414	1·5708	90°
1	·0175	·017	·0175	·0175	57·2900	·9998	1·402	1·5533	89
2	·0349	·035	·0349	·0349	28·6363	·9994	1·389	1·5359	88
3	·0524	·052	·0523	·0524	19·0811	·9986	1·377	1·5184	87
4	·0698	·070	·0698	·0699	14·3007	·9976	1·364	1·5010	86
5	·0873	·087	·0872	·0875	11·4301	·9962	1·351	1·4835	85
6	·1047	·105	·1045	·1051	9·5144	·9945	1·338	1·4661	84
7	·1222	·122	·1219	·1228	8·1443	·9925	1·325	1·4486	83
8	·1396	·140	·1392	·1405	7·1154	·9903	1·312	1·4312	82
9	·1571	·157	·1564	·1584	6·3138	·9877	1·299	1·4137	81
10	·1745	·174	·1736	·1763	5·6713	·9848	1·286	1·3963	80
11	·1920	·192	·1908	·1944	5·1446	·9816	1·272	1·3788	79
12	·2094	·209	·2079	·2126	4·7046	·9781	1·259	1·3614	78
13	·2269	·226	·2250	·2309	4·3315	·9744	1·245	1 3439	77
14	·2443	·244	·2419	·2493	4·0108	·9703	1·231	1·3265	76
15	·2618	·261	·2588	·2679	3·7321	·9659	1·218	1·3090	75
16	·2793	·278	·2756	·2867	3·4874	·9613	1·204	1·2915	74
17	·2967	·296	·2924	·3057	3·2709	·9563	1·190	1·2741	73
18	·3142	·313	·3090	·3249	3 0777	·9511	1·176	1·2566	72
19	·3316	·330	·3256	·3443	2·9042	·9455	1·161	1·2392	71
20	·3491	·347	·3420	·3640	2·7475	·9397	1·147	1·2217	70
21	·3665	·364	·3584	·3839	2 6051	·9336	1·133	1·2043	69
22	·3840	·382	·3746	·4040	2 4751	·9272	1·118	1·1868	68
23	·4014	·399	·3907	·4245	2 3559	·9205	1·104	1·1694	67
24	·4189	·416	·4067	·4452	2·2460	·9135	1·089	1·1519	66
25	·4363	·433	·4226	·4663	2·1445	·9063	1·075	1·1345	65
26	·4538	·450	·4384	·4877	2 0503	·8988	1·060	1·1170	64
27	·4712	·467	·4540	·5095	1 9626	·8910	1·045	1·0996	63
28	·4887	·484	·4695	·5317	1 8807	·8829	1 030	1·0821	62
29	·5061	·501	·4818	·5543	1·8040	·8746	1·015	1·0647	61
30	·5236	·518	·5000	·5774	1·7321	·8660	1·000	1·0472	60
31	·5411	·534	·5150	·6009	1·6643	·8572	·985	1·0297	59
32	·5585	·551	·5299	·6249	1 6003	·8480	·970	1·0123	58
33	·5760	·568	·5446	·6494	1·5399	·8387	·954	·9948	57
34	·5934	·585	·5592	·6745	1·4826	·8290	·939	·9774	56
35	·6109	·601	·5736	·7002	1·4281	·8192	·923	·9599	55
36	·6283	·618	·5878	·7265	1·3764	·8090	·908	·9425	54
37	·6458	·635	·6018	·7536	1·3270	·7986	·892	·9250	53
38	·6632	·651	·6157	·7813	1·2799	·7880	·877	·9076	52
39	·6807	·668	·6293	·8098	1·2349	·7771	·861	·8901	51
40	·6981	·684	·6428	·8391	1·1918	·7660	·845	·8727	50
41	·7156	·700	·6561	·8693	1·1504	·7547	·829	·8552	49
42	·7330	·717	·6691	·9004	1·1106	·7431	·813	·8378	48
43	·7505	·733	·6820	·9325	1·0724	·7314	·797	·8203	47
44	·7679	·749	·6947	·9657	1·0355	·7193	·781	·8029	46
45°	·7854	·765	·7071	1·0000	1·0000	·7071	·765	·7854	45°
			Cosine	Co-tangent	Tangent	Sine	Chord	Radians	Degrees
								Angle	

APPENDIX

VERNIERS AND MICROMETERS

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Answers to Examples will be found on page 453.

APPENDIX

VERNIERS AND MICROMETERS

[This Appendix should preferably be studied with the various tools at hand.]

VERNIERS

Introductory.—Mention has been made on p. 263 of a special device being necessary to read fine angular measurements such as the minute and second. Similarly, when length measurements correct to $\frac{1}{100}$ inch, and finer, are required, a finely divided scale will be found very difficult, or even impossible, to use. The smallest division obtainable on a steel rule is $\frac{1}{100}$ inch, and the lines are then so close together that a magnifying glass becomes necessary to take a reading. Such fine measurements, both linear and angular, are rendered readily possible by the use of an openly divided scale of ordinary pattern, in conjunction with an auxiliary scale called a **vernier**. By suitable arrangement this auxiliary scale can be made to read to almost any desired degree of accuracy in a comparatively easy manner.

Vernier Calipers.—The most common engineering application of the vernier is in the **vernier caliper,** a tool used in workshop and drawing-office for the accurate measurement of moderate lengths. Such tools are made to read in various scales, and for purposes of explanation the simple type shown at (A), Fig. 157, will be taken, in which the main scale S is divided into inches and tenths, and the vernier scale V is arranged to read hundredths. Although not a common marking on vernier calipers, this provides the easiest introduction to the vernier principle.

The vernier scale V consists of 10 equal divisions, whose over-all length is that of 9 main scale divisions, as shown in the full-size view at (B), Fig. 157. These divisions are numbered 0 to 10, starting from that nearest the zero on the main scale. Each vernier division is then $\frac{1}{10}$ of 9 main scale divisions, *i. e.* $\frac{1}{10}$ of

·9″ = ·09″. With the tool set as shown at (A) and (B) the distance between the jaws is an exact number of tenths of an inch (actually 2·3″), and the two end lines on the vernier are in line (or "coincide") with lines on the main scale. The main scale division *a* being ·1″ and the vernier division *b* being ·09″, the distance *c* is evidently ·1 — ·09 = ·01″, *i. e.* one-hundredth of an inch. If now the moving jaw be moved to the right until line 1

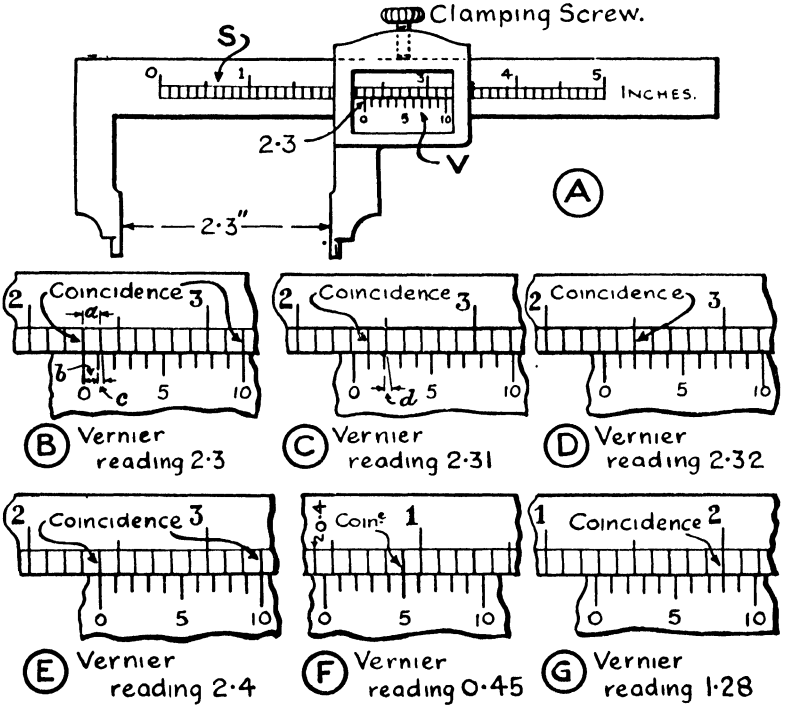


Fig. 157.—Simple Vernier Caliper, reading $\frac{1}{100}$ inch, with Sample Settings.

on the vernier coincides with the nearest line on the main scale, the jaws will have opened ·01″, and the new position of the vernier will be as at (C). Then the distance *d* is similarly ·1″ — ·09″ = ·01″, and if the moving jaw be again moved to the right until line 2 on the vernier coincides with its nearest main scale division, as at (D), a further opening of ·01″ will have taken place; the total distance between the jaws will then be 2·32″. This gradual movement of the moving jaw to the right can be continued, and the coincidence

of each successive line on the vernier will give the number of one-hundredths of an inch that the jaws are open beyond 2·3″. When line 10 on the vernier is in coincidence, a total movement of 10 one-hundredths or ·1″ will have been made from the first position, which is further indicated by line 0 on the vernier coinciding with 2·4″ on the main scale, as at (E). Then it will be seen that, at any particular setting, the number of the line on the vernier scale coinciding with any line on the main scale indicates the number of hundredths which the jaws are open beyond the number of exact tenths indicated by the zero of the vernier scale.

To take a complete reading then, proceed thus :—

1. Read the inches and complete tenths up to the zero mark on the vernier.

2. Note the line on the vernier which coincides with a line on the main scale.

3. Add the figure noted in (2) as hundredths to the reading taken in (1).

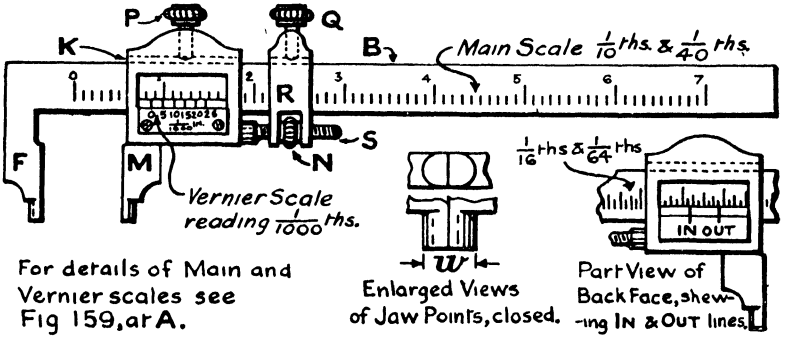


Fig. 158.—Vernier Caliper, reading $\frac{1}{1000}$ inch. (About half-size.)

Thus at (D), Fig. 157, the vernier zero cuts off 2·3 on the main scale, and coincidence occurs at 2 on the vernier scale. Then the full reading is 2·3″ + ·02″ = 2·32″.

With a very little practice this addition can be done mentally. As a further illustration, two other settings are shown at (F) and (G), Fig. 157, which should be easily understood. *Exercises on reading this vernier are given in Exercises* 94, *No.* 1.

The most widely used type of English-reading vernier caliper is shown in Fig. 158. The fixed jaw F is solid with the blade B, on which is marked the main scale of inches and tenths, each tenth being subdivided into 4 parts, *i. e. fortieths*. The moving jaw M carries the vernier scale, and can be clamped in any position

by the set-screw P and the key K. An auxiliary runner R, which can also be clamped by screw Q, carries a milled edged nut N, between lugs, which is threaded on the adjusting screw S. The ends of the jaws are shaped down and rounded so that inside diameters can be measured. The smallest hole which can be gauged is determined, of course, by the width, *w*, across the points when the jaws are shut. This is usually either ·25″ or ·3″, according to the size of the tool; this value, which may be found from the maker's catalogue, must be *added to the reading taken* when measuring inside diameters.

To use the caliper, both screws P and Q are slackened, when the moving jaw M and the runner R may be pushed, together, into nearly the required position. Screw Q is then tightened, securing runner R to the blade. The nut N is now rotated to advance or withdraw the moving jaw until the object to be measured is just held, without pressure, between the jaws. Screw P is then tightened to secure the moving jaw from accidental movement until the reading of the vernier is taken. This fine adjustment is essential when readings are to be made to ·001 inch, although not necessary when only ·01 is required.

The One-Thousandth Scale.—The main scale is divided into tenths, and each tenth is subdivided into four parts, so that the smallest main scale division is $\frac{1}{4}$ of ·1 = ·025 inch or $\frac{1}{40}$. The vernier scale has 25 divisions, whose total length is that of 24 main scale divisions, see (A), Fig. 159, and which are numbered at every five from 0 to 25. Each vernier division is then $\frac{1}{25}$ of 24 main scale divisions, *i. e.*—

$$\frac{1}{25} \text{ of } \frac{24}{40} = \frac{24}{1000} = \cdot 024,$$

which is ·001 inch shorter than the main scale division of ·025. The **smallest reading** is then **·001 inch** or one-thousandth, and is the amount added by successive coincidences of the vernier lines with the main scale. The complete reading of this vernier is then made as follows :—

1. Read the number of inches and complete tenths up to the vernier zero.

2. Read off the number of small main scale divisions between the last complete tenth and the vernier zero. In each tenth the small divisions, in order, are (1) **·025,** (2) **·050,** (3) **·075. Add the appropriate decimal to the reading in (1).**

3. Note the vernier graduation in coincidence with any line on the main scale and add this number of thousandths to the result of the addition in (2).

Sample readings are shown in Fig. 159, at (B), (C), and (D), the diagrams being twice full size.

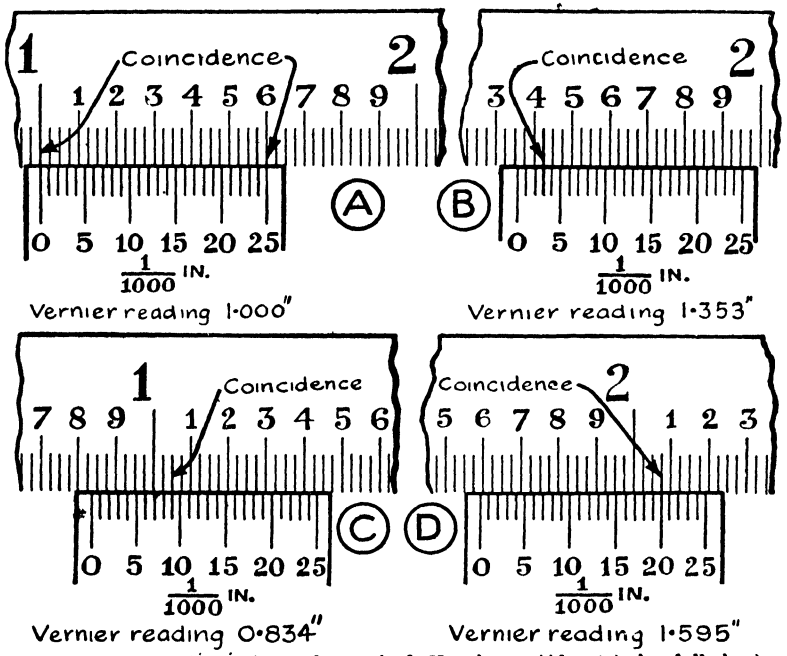


Fig. 159.—Sample Settings of $\frac{1}{1000}$ inch Vernier. (About twice full size.)

In Fig. 159:—

At (*B*).

The vernier zero cuts off 1·3 and *two* small main scale divisions, *i. e.* 1·3 + ·05 . . . . . = 1·35

Line 3 on the vernier coincides with a line on the main scale indicating an extra $\frac{3}{1000}$ . . . . . = ·003

Adding, the complete reading = 1·353

At (*C*).

Vernier zero cuts off 0·8 and *one* small main scale division, *i. e.* 0·8 + ·025 . . . . . = 0·825

Line 9 on the vernier coincides with a line on the main scale, indicating an extra $\frac{9}{1000}$ . . . . . = ·009

Adding, the complete reading = ·834

At (D).

Vernier zero cuts off 1·5 and *three* small divisions, *i. e.*

1 5 + ·075 . . . . . = 1·575

Coincidence on vernier is at 20, *i. e.* $\frac{20}{1000}$ . . . = ·020

∴ Complete reading = 1·595

Examination of the scales on an actual caliper will show that *very careful inspection is necessary* to distinguish the exact vernier line at which coincidence occurs. The lines immediately on either side of the coinciding line are only one-thousandth of an inch away from the nearest main scale graduations, *but in opposite directions.* This fact should be closely observed in the further sample settings of Fig. 160 (which are shown *full size*), as it assists considerably in identifying the coinciding line.

In Fig. 160 :—

At (A).

Vernier zero cuts off 2·2 and *three* small main scale divisions, *i. e.* 2·2 + ·075 . . . . . = 2·275

Coincidence on vernier is at 13, *i. e.* $\frac{13}{1000}$ . . . = ·013

∴ Complete reading = 2·288

At (B).

Vernier zero cuts off 4·5 and *two* small divisions, *i. e.*

4·5 + ·05 . . . . . = 4·55

Vernier coincidence is at 19, *i. e.* $\frac{19}{1000}$ . . . . = ·019

∴ Complete reading = 4·569

At (C).

Vernier zero cuts off *no tenths* and *one* small division, *i. e.*

0 + ·025 . . . . . = ·025

Vernier coincidence is at 11, *i. e.* $\frac{11}{1000}$ . . . . = ·011

∴ Complete reading = ·036

At (D).

Vernier zero cuts off ·3 and is very nearly in line with the first small division, but *not quite.* This is proved by coincidence being observed at 24 on vernier. Then vernier zero cuts off ·3 and *no* small main scale divisions, *i. e.* ·3 + ·000 = ·300

Vernier coincidence is at 24, *i. e.* $\frac{24}{1000}$ . . . . = ·024

∴ Complete reading = ·324

At (E).

Vernier zero cuts off 1·8 and appears to be in line with the *third* small division beyond this. It must not be assumed that this is actually the case unless *both* the vernier lines o and 25 are in coincidence. Inspection shows that both end lines of the vernier *are* in coincidence, and the reading is therefore 1·8 + ·075 . . . . . = 1·875

there being no thousandths to add.

At (F).

No tenths and no small divisions are cut off by the vernier zero, and coincidence is found at 21. The complete reading is then $\frac{21}{1000}$ . . . . . = ·021

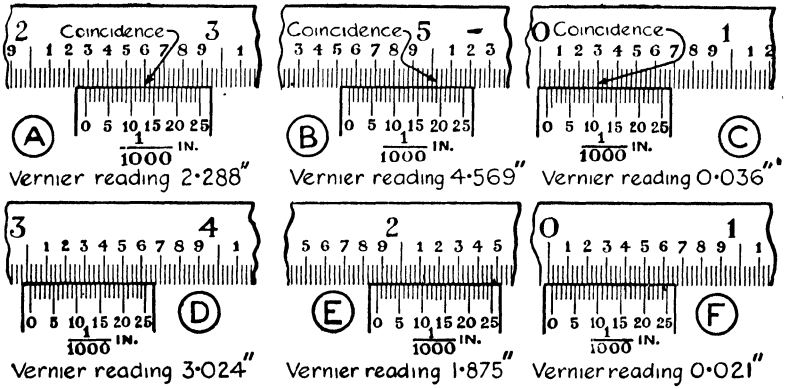


Fig. 160.—Sample Settings of $\frac{1}{1000}$ inch Vernier. (Full size.)

The examples at (D) and (E) above are important and should be carefully studied.

Whenever the vernier zero appears to coincide with the ·025, ·05 or ·075, the other end should be carefully examined, as a coincidence may possibly occur at 24. *It must not be assumed that there are no thousandths to add unless both end lines are in coincidence.*

After considerable practice in reading has been obtained the addition of the main scale reading and the vernier reading may be made mentally, but unless the user feels absolutely sure of his mental addition the separate readings should be written down.

Exercises 99. On Reading Verniers $\frac{1}{100}$ inch and $\frac{1}{1000}$ inch.

1. Read the settings of the $\frac{1}{100}$ inch vernier shown in Fig. 161, (A) to (C).

2. Read the settings of the $\frac{1}{1000}$ inch vernier caliper shown in Fig. 161, (D) to (M).

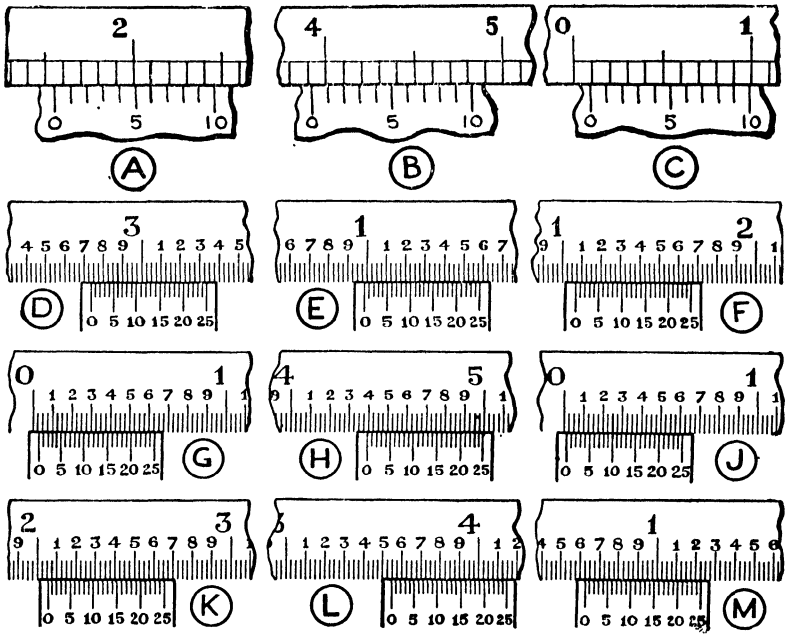


Fig. 161.—Exercises on reading Verniers. A to C, $\frac{1}{100}$ inch; D to M, $\frac{1}{1000}$ inch. (About full size.)

The Back of the Caliper.—The back of the blade is usually fully divided into inches and sixty-fourths. A vernier is not fitted to this scale, but measurements are made directly with the aid of two lines marked "*in*" and "*out*," as shown in Fig. 158. When the jaws are shut the line marked "*out*" will read 0 on the scale, while the line marked "*in*" will read off the distance *w* (see description of caliper on p. 463). The "*out*" line is used for *outside* measurements and the "*in*" line for *inside* measurements on the sixty-fourths scale, and no addition of the width *w* is then necessary.

Least Count.—To obtain the smallest reading possible with any given vernier from first principles, as done on pp. 460, 463,

is rather lengthy, but by means of a little algebra a very simple formula can be deduced for the **least count,** as the smallest possible reading is sometimes called.

Let v = length of 1 vernier division,
s = length of 1 main scale division,
and N = number of divisions on vernier.

With very few exceptions the N vernier divisions are made equal in length to *one less* their number of main scale divisions. Then (N − 1) main scale divisions will be the same length as N vernier divisions, *i. e.*—

$$Nv = (N - 1)s$$

$$\therefore \quad v = \frac{N - 1}{N} s \quad . \quad . \quad . \quad . \quad . \quad (1)$$

"Least Count" = scale division − vernier division

$= s - v$

$= s - \frac{N - 1}{N} s$, substituting from (1)

$= s \left(1 - \frac{N - 1}{N}\right)$, taking out the common factor (refer p. 147)

$= s \left(\frac{N - N + 1}{N}\right)$, combining the bracketed quantities

$= s \times \frac{1}{N}$ or $\frac{s}{N}$, since $N - N = 0$

i. e. the **least count** is the **length of the smallest main scale division divided by the number of divisions on the vernier,** provided that the N vernier divisions be made equal to N − 1 main scale divisions.

The least counts for the verniers already described will now be deduced from the formula :—

1. Main scale divided into tenths. Vernier with 10 divisions equal to 9 main scale divisions.

Then— $s = \frac{1}{10} = \cdot 1''$ $\quad N = 10$

$\therefore$ least count $= \frac{s}{N} = \frac{\cdot 1}{10} = \underline{\cdot 01''}$.

2. Main scale division $= \frac{1}{40} = \cdot 025''$. Vernier with 25 divisions equal to 24 main scale divisions.

Then— $s = \cdot 025''$ $\quad N = 25$

$\therefore$ least count $= \frac{s}{N} = \frac{\cdot 025}{25} = \underline{\cdot 001''}$.

3. As a further example we will take a marking, occasionally found on vernier calipers, in which the main scale is divided into sixteenths, and the vernier has 8 divisions equal to 7 main scale divisions.

Then— $s = \frac{1}{16}$ inch $\quad N = 8$

$$\therefore \text{ least count} = \frac{s}{N} = \frac{1}{16} \div 8 = \frac{1}{16} \times \frac{1}{8} = \underline{\frac{1}{128}} \text{ inch.}$$

Various other Vernier Scales.—Further examples of linear verniers of less importance will be found in Exercises 100.

As an example of an **angular vernier,** that usually found on a

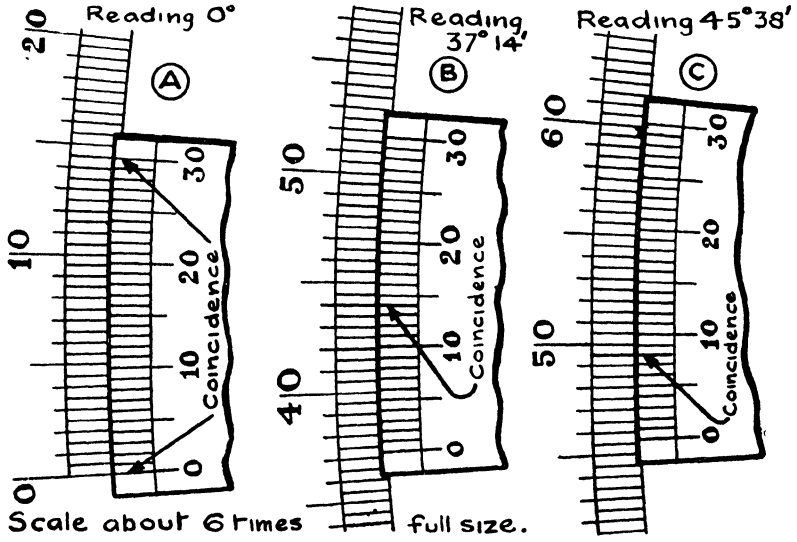


Fig. 162.—Sample Settings of Box Sextant Vernier, reading to 1 minute.

Box Sextant * will be taken. The instrument is arranged to be very light and portable; the scale is therefore of small radius, is finely divided on silver, and is read through a small magnifying glass carried on the instrument.

The main scale is divided in degrees and half-degrees, and the vernier has 30 divisions equal in total length to 29 small scale divisions, see (A), Fig. 162.

Then $s = \frac{1}{2}$ degree $= 30'$ (30 minutes) and $N = 30$,

$$\therefore \text{ least count} = \frac{s}{N} = \frac{30'}{30} = \underline{1 \text{ minute.}}$$

* A surveying instrument for measuring, in the field, the angle between two objects viewed from a third point.

The reading is carried out as follows :—

1. Read the degrees (and if one, the half-degree) cut off by the vernier zero.

2. Note the vernier coincidence.

3. Add the reading of (2), as minutes, to the reading of (1).

Two sample settings are shown in Fig. 162 at (B) and (C).

At (B).

Vernier zero is beyond 37° but has not reached the next half-degree . . . . . 37° 00′

Vernier coincidence is at 14 . . . . . 14′

Then complete reading = 37° 14′

At (C).

Vernier zero lies between 45½° and 46° . . . . 45° 30′

Vernier coincidence is at 8 . . . . . 8′

Then complete reading = 45° 38′

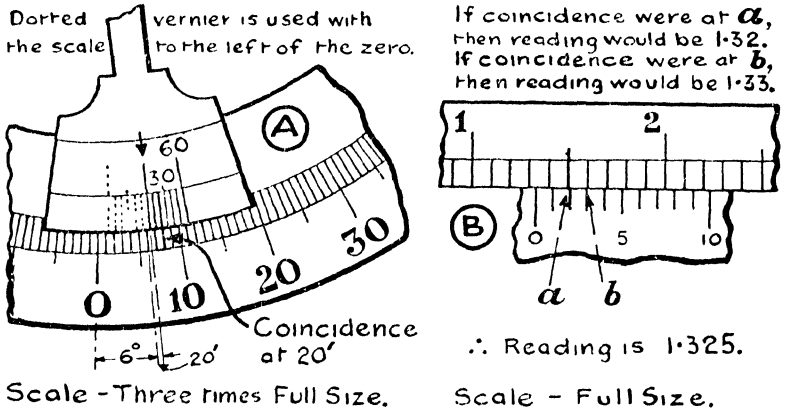


Fig. 163.—(A) Sample Setting of Abney Clinometer Vernier. (B) Reading $\frac{1}{10}$ inch Vernier with no coincidence.

With sufficient practice in reading, the addition may be done mentally and the result entered up at once.

The last example will be another angular vernier, that commonly found on the **Abney Clinometer.*** Here the main scale is divided to show single degrees, while the vernier has 6 divisions whose total length is equal to 5 main scale divisions.

* A surveying instrument for measuring angles in a vertical plane, such as the slope of the ground.

Then— $s = 1° = 60'$ and $N = 6$

$$\therefore \text{least count} = \frac{s}{N} = \frac{60'}{6} = \underline{10 \text{ minutes.}}$$

The reading of this vernier is very simple, the successive vernier coincidences indicating 10, 20, 30, 40, 50 minutes beyond the complete degrees. A sample setting is shown at (A), Fig. 163, and should be easily understood.

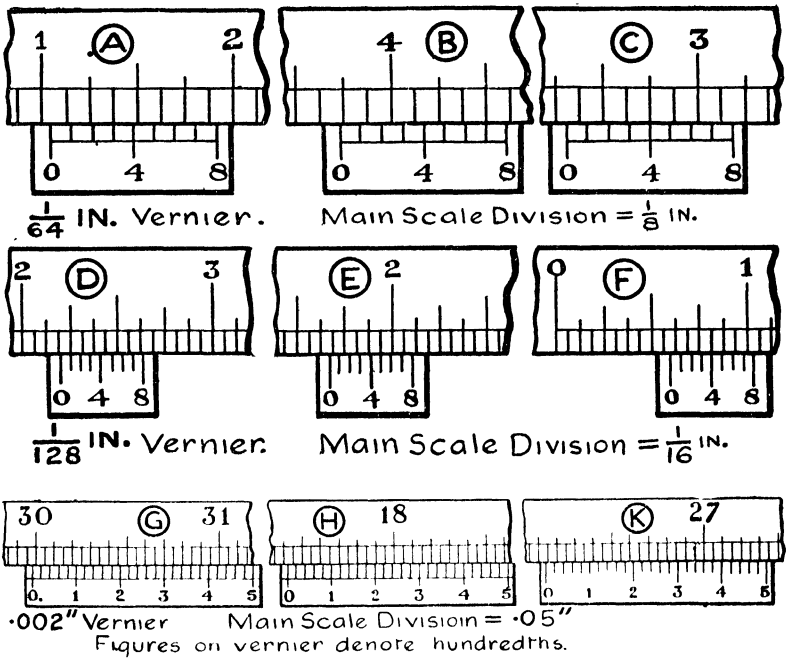


Fig. 164 (*part*).—Exercises on Reading Linear Verniers.

Lack of Coincidence.—With coarse reading verniers, such as the $\frac{1}{100}$ inch, it may happen that no coincidence can be detected, two vernier lines lying between two main scale lines, as at (B), Fig. 163. This indicates that the position is intermediate between the settings having coincidences at *a* and *b* respectively. It may usually be read as half-way between these two readings, *e. g.* if coincidence at *a* gave 1·32″ and coincidence at *b* gave 1·33″, then the actual reading could be taken as half-way between 1·32 and 1·33, *i. e.* 1·325″.

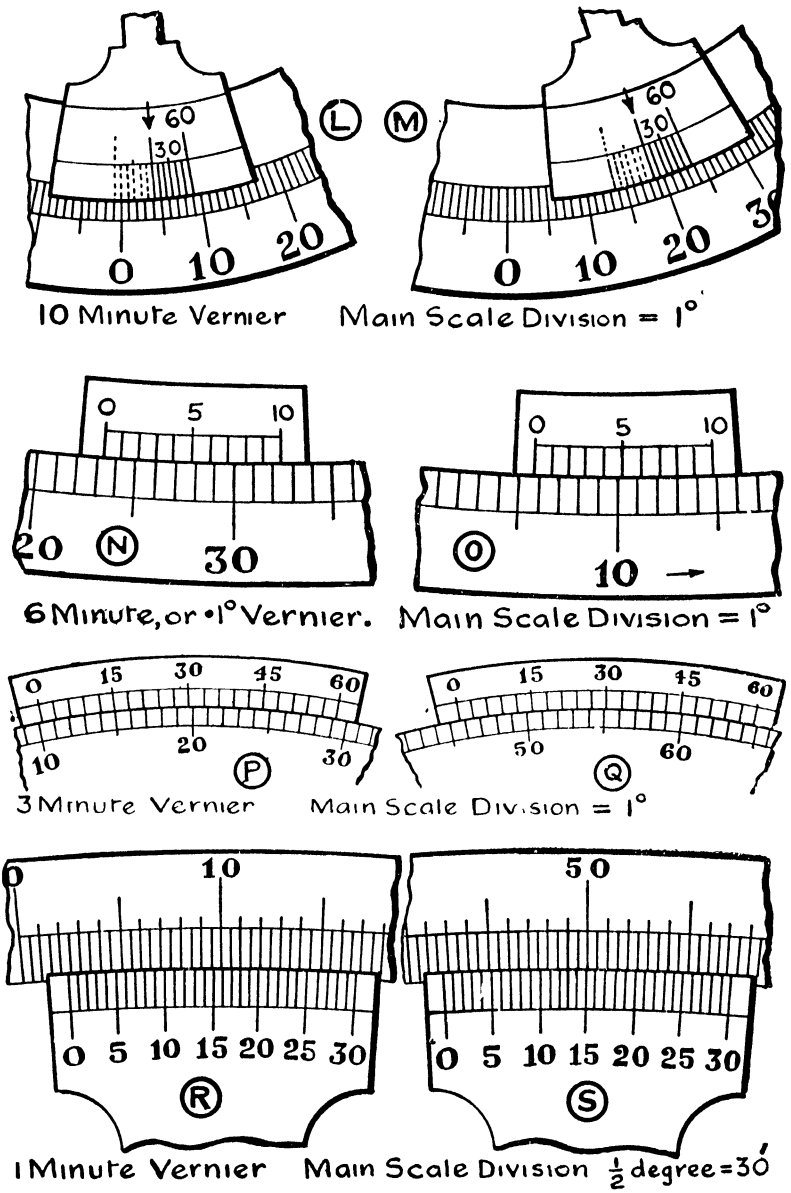


Fig. 164 (*contd.*)—Exercises on Reading Angular Verniers.

Exercises 100. On Various Verniers

Determine the " least count " of each of the following verniers (all actual examples) :—

1. Smallest main scale division = $\frac{1}{8}$ inch. Vernier has 8 divisions equal to 7 main scale divisions.

2. Main scale divided into $\frac{1}{25}$ inch, and 40 divisions on vernier = 39 divisions on main scale.

3. Main scale reads ·05". Vernier has 25 divisions equal to 24 scale divisions.

4. Smallest main scale division = ·02", and 19 main scale divisions equal 20 vernier divisions.

5. Main scale reads 1° and vernier has 10 divisions equal to 9 scale divisions. (Give result in minutes.)

6. Smallest main scale division = 1°, and 20 divisions on vernier = 19 divisions on main scale. (Give result in minutes.)

7. Smallest main scale division = $\frac{1}{8}$ degree. Vernier has 60 divisions equal to 59 main scale divisions. (Give result in seconds.)

Read the settings of the various verniers shown in Fig. 164:—

8. (A) to (C)	Least count	$\frac{1}{64}$	inch
9. (D) „ (F)	„ „	$\frac{1}{128}$	„
10. (G), (H) and (K)	„ „	·002	„
11. (L) and (M)	„ „	10	minutes
12. (N) „ (O)	„ „	6	„
13. (P) „ (Q)	„ „	3	„
14. (R) „ (S)	„ „	1	„

Micrometers.

Micrometer Wheel and Screw.—A method of fine measurement of considerable importance is that of the micrometer wheel

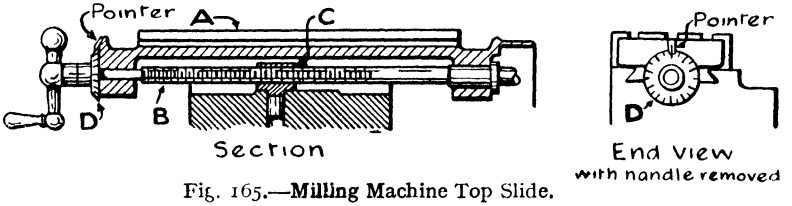


Fig. 165.—**Milling Machine Top Slide.**

and screw thread. It is employed on the better types of milling machine tables, slide rests of lathes, etc., where machining operations are required to a fine degree of accuracy, and it is also the basis of fine measuring machines.

Referring to Fig. 165, which shows a section and an end view of the top slide on a milling machine slide rest, A is the table supporting the work, B is the feed screw (usually of square thread form), C is the nut fixed to the remainder of the slide rest, while

D is a large diameter disc or wheel, graduated on its outer edge, and called the micrometer wheel, and which is rigidly attached to the screw. A pointer, or a line prominently marked on the end of the table, serves to indicate the reading on the disc.

Now the pitch (or better, "lead") of the screw is the distance that the table will advance for one complete revolution of the screw and disc. If the screw has 10 single threads per inch, then the pitch is ·1 inch, and for one complete revolution of the screw the table would advance (or withdraw) ·1 inch. If half a revolution be made, the advance would be $\frac{1}{2}$ of ·1 = ·05; if $\frac{1}{10}$ of a revolution be made, the movement would be $\frac{1}{10}$ of ·1 = ·01 inch, and so on. If the circumference of the micrometer disc be divided into 100 equal parts, then a rotation of the screw through *one* of these parts would move the table $\frac{1}{100}$ of ·1 = ·001 inch. The pitch of the screw and the number of divisions round the micrometer disc may be varied, but they are usually arranged so that the smallest advance registered is ·001 inch. Thus on a certain small tool-maker's lathe, the micrometer disc, which was about $\frac{1}{2}$-inch diameter, was divided into 50 parts; the pitch of the screw was $\frac{1}{20}$ inch, so that each small division was equivalent to $\frac{1}{50}$ of $\frac{1}{20}$ = $\frac{1}{1000}$ = ·001 inch.

Then by simply counting the number of graduations rotated past the pointer, any desired advance of the table, in thousandths of an inch, may be obtained. There is no indication, except counting, of the *complete* revolutions made, but usually this causes no difficulty.

The accuracy of the method is entirely dependent on the accuracy with which the screw is made, as regards pitch, and how the disc is divided. The latter can usually be done very exactly, while feed screws can be cut with a total error in pitch of not more than ·005 inch in 10 inches, which is quite suitable for ordinary manufacturing.

With this attachment to a milling machine table it is possible to graduate a scale, after working out the lengths of the divisions in thousandths of an inch.

In use it is essential that the pressure between the screw and its nut be kept in the same direction all the time, *i. e.* any back-lash must be eliminated. This is ensured by approaching all settings with the same direction of rotation. If the disc is overrun for any particular setting, then it must be rotated well back *to withdraw the table*, and then again rotated in the original direction, with greater care, until the desired position is attained.

The Micrometer Caliper.—The chief application of the foregoing principle is in the **Micrometer Caliper,** the most important workshop measuring tool. Fig. 166 shows the commonest form, which is adapted for outside or "male" dimensions, and which is spoken of as a *One-Inch Micrometer,* one inch being the largest dimension that it will measure.

The U-shaped body or bow A carries a fixed anvil B. The opposite side carries a hollow sleeve C, the end of which is tapped and split and on its outer surface is threaded with a coned thread. A nut D on this thread enables the sleeve to be adjusted to grip

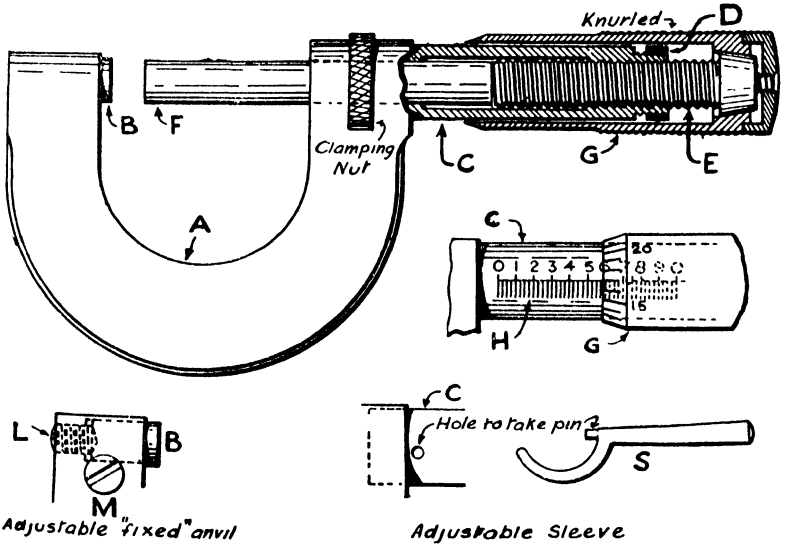


Fig. 166. —Micrometer Caliper.

the screw E with any desired degree of tightness, so that backlash may be eliminated. The screw E is the micrometer or gauge screw, and in English-reading instruments is invariably a single V-thread screw, right hand, of 40 threads per inch, *i. e.* pitch $= \frac{1}{40}$ inch $= \cdot 025$. Very great care is taken to make this screw accurate, and the total error in pitch on a 1″ length is usually less than ·0001″, *i. e.* one ten-thousandth. The inner end terminates in the anvil F, while the outer end is attached to the thimble G, which just passes over the sleeve C. The anvil faces should be ground and lapped quite flat and smooth, and should be perfectly parallel for all positions of the screw.

The sleeve C carries an axial scale H divided into tenths and fortieths, each tenth being numbered 0, 1, 2, etc., from left to right. One rotation of the thimble causes an axial movement of ·025″, *i. e.* the distance between two successive marks on the sleeve. This scale H registers the number of complete revolutions of the screw. The circumference of the thimble, on its bevelled edge, is divided into 25 equal parts, figured at every 5, as shown. A

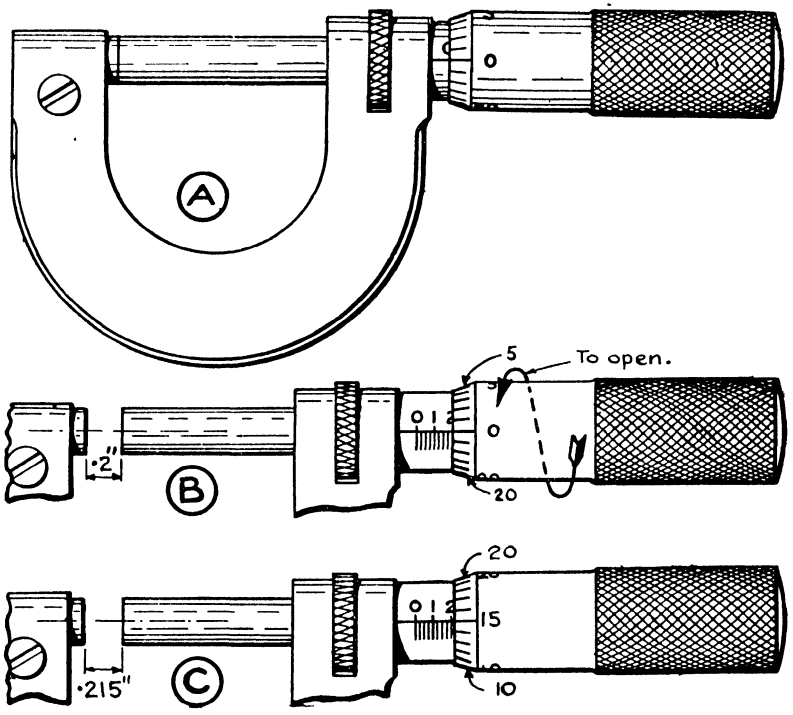


Fig. 167.—Sample Settings of Micrometer Caliper.

rotation of one of these parts, therefore, produces an axial movement of the anvil F of $\frac{1}{25}$ of ·025 = ·001 inch.

At (A), Fig. 167, the micrometer is shown with the anvils closed and reading 0. Referring to (B), Fig. 167, an opening of ·2″ between the anvils has been made, *i. e.* the thimble and screw have been rotated 8 times (·025 × 8 = ·2) from the closed position of (A), Fig. 167. If now a further rotation in the same direction be made until graduation 15 on the thimble coincides with the axial line

on the sleeve, as shown at (C), Fig. 167, an additional opening of 15 thousandths, *i. e.* ·015, will have been made. The distance between the anvils is now ·2″ + ·015″ = ·215″.

Outside micrometer calipers can also be obtained with larger bows to read up to 2″, 3″, 4″, etc., as desired, but in all sizes the travel of the gauge screw is invariably one inch. Then the distance between the anvils when the micrometer is reading zero is 1″ less than the "size" of the tool, *i. e.* is 1″, 2″, 3″, etc., respectively. This may be called the "*base*" of the caliper and *must be added* to any reading taken, since the thimble and scale only indicate the decimal part of the measurement.

Zero Adjustment.—When the anvils of a 1″ micrometer caliper are just closed, without pressure, after the faces have been wiped clean and dry, the edge of the thimble should be opposite the first line on the axial scale, marked 0, and the line 0 on the thimble scale should coincide with the axial line on the sleeve; as at (A), Fig. 167. An adjustment is provided to obtain this condition when the tool is first assembled, and to maintain it after wear and the subsequent truing-up of the anvil faces.

The mode of adjustment varies with the maker, but usually consists in *either* (1) making the fixed anvil B adjustable axially, *or* (2) making the sleeve C adjustable around its axis.

In method (1) the "fixed" anvil is a good push fit in the bow, and can be pushed out by a screw L; a locking screw M serves to lock the anvil when set. To adjust the tool, the locking screw M is slackened and the thimble set to read zero. The "fixed" anvil B is then set up to touch the anvil F by the adjusting screw L, and is finally locked in its position by screw M. *The adjustment should then be tested* by opening and closing the anvils, as it is possible to set the faces together with too much pressure by screw L.

In method (2) the sleeve C is secured to the bow A by friction only, and can be rotated by the special spanner S, provided with the tool. To adjust, the anvils are closed, without pressure, and the sleeve C is then rotated until the axial line of the scale H coincides with the 0 on the thimble. *The adjustment should then be tested* to check that the anvils were not closed with undue pressure.

With sizes larger than one-inch, a gauge is provided whose length is accurately made equal to the "base" of the tool. This is inserted between the anvils when setting to zero.

It is important, when using any micrometer for the first time,

to examine for any zero error and to adjust, if necessary. If the tool is in continual use then periodic examinations should be made.

To use the Caliper.—The thimble is rotated to open the anvils to pass over the work, and is then rotated in the reverse direction until the object is nipped, without pressure, between the anvils.

To take the reading :—

1. Read off the number of complete tenths exposed by the edge of the thimble.

Call these "large divisions."

2. Read off the last small division exposed beyond the last tenth.

These small divisions, in order, read—

(1) ·025, (2) ·050, (3) ·075. Call the particular decimal "small divisions."

3. Note the number of the graduation on the thimble which coincides with the axial line on the sleeve.

Call this the "thimble divisions" **and add, as thousandths, to the readings of (1) and (2).** The thimble reading will always be between ·000 and ·025.

Thus at (B), Fig. 167 :—

1. *Two* complete tenths are exposed;
∴ *Large divisions* = ·2
2. *No* small divisions are visible beyond the last tenth;
∴ *Small divisions* = ·000
3. Graduation 15 on thimble coincides with the axial line, *i. e.* $\frac{15}{1000}$; ∴ *Thimble divisions* = ·015

Adding, complete reading = ·215

The method of reading is closely allied to that for the one-thousandth vernier caliper. It is really simpler, as no vernier coincidence has to be detected. The micrometer caliper has advantages over the vernier type; it is easily capable of reading finer than one-thousandth, it is quicker and easier to handle, and, owing to its shape, is more sensitive for fine measurements. The vernier caliper, however, with its long thin jaws, can be used for measuring dimensions which are inaccessible to the micrometer.

Some further sample settings are shown in Fig. 168, (A) to (E).

At (*A*).

1. *Two* complete tenths are exposed by thimble.
   ∴ *Large divisions* = ·2
2. *Two* small divisions are exposed beyond the last tenth.
   ∴ *Small divisions* = ·050
3. Graduation 5 on thimble coincides with the axial line.
   ∴ *Thimble divisions* = ·005

∴ Complete reading = ·255

At (*B*).

Four complete tenths are exposed by thimble.
   ∴ *Large divisions* = ·4

2. *One* small division is exposed beyond the last tenth.
   ∴ *Small divisions* = ·025
3. Graduation 10 on thimble coincides with the axial line.
   ∴ *Thimble divisions* = ·010

∴ Complete reading = ·435

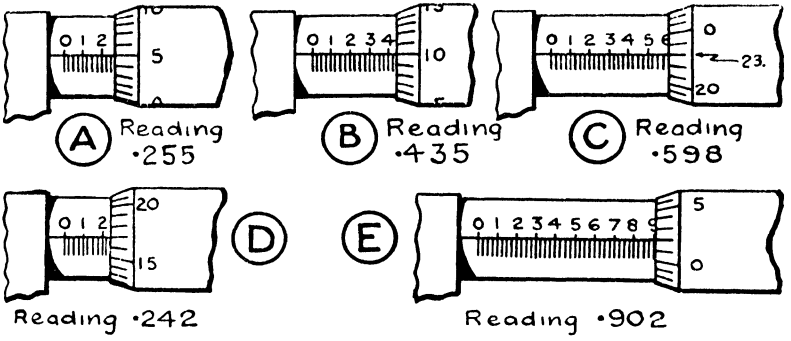


Fig. 168.—Sample Settings of Micrometer Caliper $\frac{1}{100}$ inch. (Full size.)

At (*C*).

1. *Five* complete tenths are exposed. ∴ *Large divisions* = ·5
2. *Three* small divisions are exposed in addition.
   ∴ *Small divisions* = ·075
3. Graduation 23 on thimble coincides with axial line.
   ∴ *Thimble divisions* = ·023

∴ Complete reading = ·598

At (*D*).

1. *Large divisions* . . . . . = ·2
2. *Small divisions* . . . . . = ·025
3. Coincidence at 17. ∴ *Thimble divisions* = ·017

∴ Complete reading = ·242

At (*E*).

1. *Large divisions* . . . . . = ·9
2. *Small divisions* . . . . . = ·000
3. Coincidence at 2. ∴ *Thimble divisions* = ·002

∴ Complete reading = ·902

In all the settings shown up to the present a graduation on the thimble has been in *exact* coincidence with the axial line on the sleeve. This will not always occur, and readings may have to be taken of such settings as appear in Fig. 169, where the axial line lies *between* two adjacent thimble graduations. In these cases the dimension does not consist of an exact number of thousandths, but contains a fraction of a thousandth.

For ordinary manufacturing purposes it is seldom necessary to read anything smaller than a thousandth. The extra fraction of a thousandth may, however, be estimated as *ten-thousandths*, or the reading may be stated to the *nearest* thousandth. The exact reading of ten-thousandths is treated later.

If the ten-thousandths are to be estimated, then the thimble reading is taken as the graduation immediately *before* the axial line on the sleeve; and the estimated tenths of a thimble division between this last graduation and the axial line are added to the reading in the *fourth* decimal place.

If the reading is required to the nearest thousandth, then the thimble graduation which is *nearer* to the axial line is read. Examples are shown in Fig. 169.

In Fig. 169:—

At (*A*).

1. *Large divisions* . . . . . = ·2
2. *Small divisions* . . . . . = ·000
3. Axial line lies between 17 and 18 on thimble.

∴ *Thimble divisions* = ·017

From 17 on the thimble to the axial line is estimated as ·7 of a thimble graduation . . . . . = ·0007

∴ Complete reading = ·2177

To the nearest thousandth this is ·218, as is shown by the graduation 18 being the nearer to the axial line.

At (B).

1. *Large divisions* . . . . . = ·7
2. *Small divisions* . . . . . = ·05
3. Axial line lies between thimble graduations 8 and 9.
   ∴ *Thimble divisions* = ·008

From 8 on thimble to axial line is about ·3 of a thimble division . . . . . = ·0003

∴ Complete reading = ·7583

To the nearest thousandth this is ·758, as is shown by graduation 8 being the nearer to the axial line.

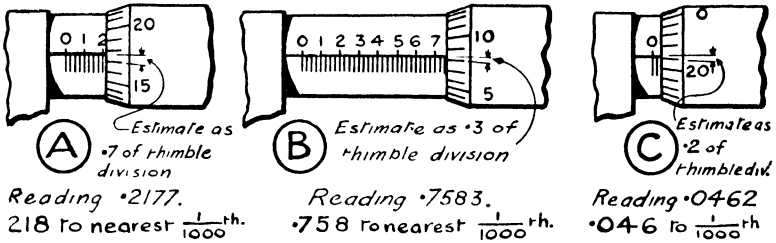


Fig. 169.—Estimating Ten-thousandths on Micrometer Caliper.

At (C).

1. *Large divisions* . . . . . = ·0
2. *Small divisions* . . . . . = ·025
3. *Thimble divisions* (axial line between 21 and 22) . . = ·021

The additional fraction of a thimble graduation is estimated as ·2 . . . . . = ·0002

∴ Complete reading = ·0462

To the nearest thousandth this is ·046.

When the axial line appears to be *exactly* half-way between two thimble graduations, the half-thousandth, ·0005, is read, and the dimension cannot be stated to a nearest thousandth.

The writing down of the component decimals of the reading should not be discarded until *very considerable practice* has been obtained. Even then mistakes of mental addition are possible, and it is safer to retain the writing down, reducing it, however, to two lines, the first containing the " large " and " small " divisions, and the second the thousandths and (if any) the ten-thousandths. This method is shown when dealing with the ten-thousandth micrometer.

Exercises 101. On Reading the Micrometer ($\frac{1}{1000}$ inch).

1. Read the settings of the micrometer caliper shown in Fig. 170, (A) to (F).

2. Read the settings shown in Fig. 170, (G) to (J), giving the readings including the *estimated* ten-thousandth and also to the nearest thousandth.

Various Micrometer Tools.—The measuring portion of the micrometer caliper, *i. e.* the sleeve, screw, and thimble, can be obtained separately, and is then known as a **micrometer head.** It can be attached to machines and tools where fine adjustment is

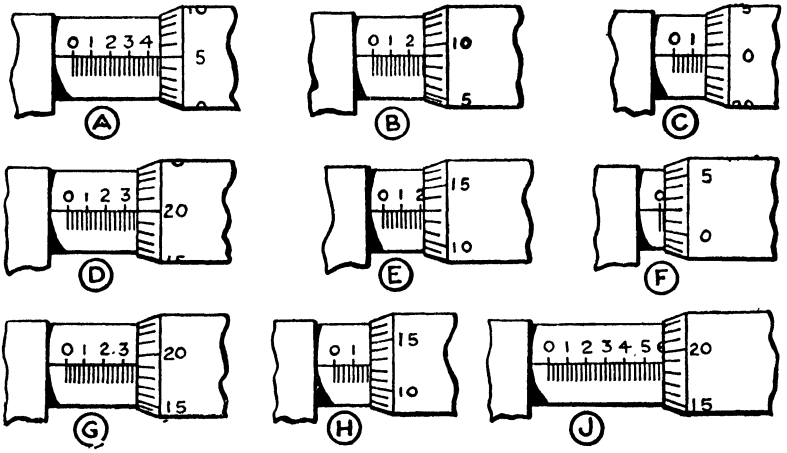


Fig. 170.—Exercises on Reading Micrometer, $\frac{1}{1000}$ inch.

required, being gripped by the portion *a* of sleeve. See (A), Fig. 171.

For measuring inside dimensions the **inside micrometer caliper,** shown at (B), Fig. 171, is employed. The measuring portion is the same as on the outside variety, and to enable a wide range of dimensions to be dealt with, extension rods of various lengths are supplied.

The depths of holes and recessed parts are accurately measured by the **micrometer depth gauge,** shown at (C), Fig. 171. The gauge screw has usually a total movement of half an inch, while the measuring rod is graduated by angular grooves in half-inches; by setting the rod to the nearest suitable groove, a variety of depths can be gauged.

There are many other applications of the micrometer which may be consulted at length in the manufacturers' catalogues.

The Ten-Thousandth Micrometer.—For fine work demanding measurement to a greater accuracy than one-thousandth of an inch, the micrometer caliper is fitted with a vernier scale by which ten-thousandths may be read.

Referring to Fig. 172 at (A), the sleeve is marked on the back with a vernier scale, the lines of which run the whole length of the sleeve, so that the vernier scale and the bevelled edge of the thimble are in contact for any position of the screw. The vernier has 10 divisions, numbered 0 to 10, as shown, their total length round the sleeve being equal to 9 divisions on the thimble. This

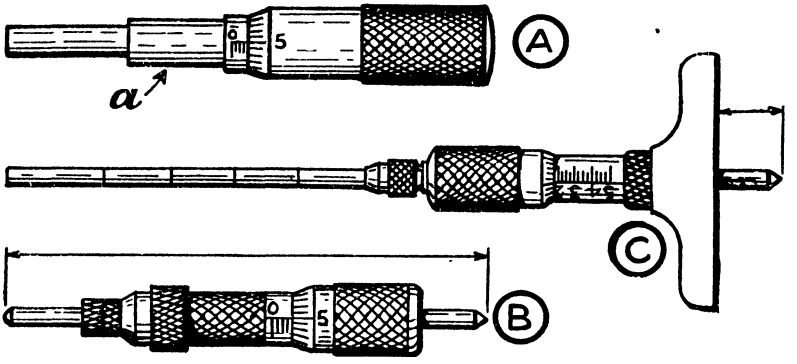


Fig. 171.—Various Micrometer Tools.

is an example of the simple vernier described on p. 460, and the least count is then $\frac{1}{10}$ of the amount indicated by one division on the thimble, *i. e.* $\frac{1}{10}$ of ·001 = ·0001 inch.

To read this caliper, proceed as described on p. 478, reading up to the thimble graduation *before* the axial line on the sleeve. This gives the number of complete thousandths in the dimension. **Then note which line on the vernier is in coincidence with a line on the thimble, and add this figure as ten-thousandths (4th decimal place) to the first reading.**

The caliper should be held with the axis vertical when reading the vernier, as otherwise the curvature of the vernier surface may cause a false coincidence to be read.

Sample settings are shown in Fig. 172 at (B) to (G).

In Fig. 172 :—

At (*B*).

1. Main scale reading, *i. e.* " large " and " small " divisions = 150
2. Axial line is between 17 and 18 on thimble . . = ·017
3. Vernier coincidence at 4 . . . . . = ·0004

∴ Complete reading = ·1674

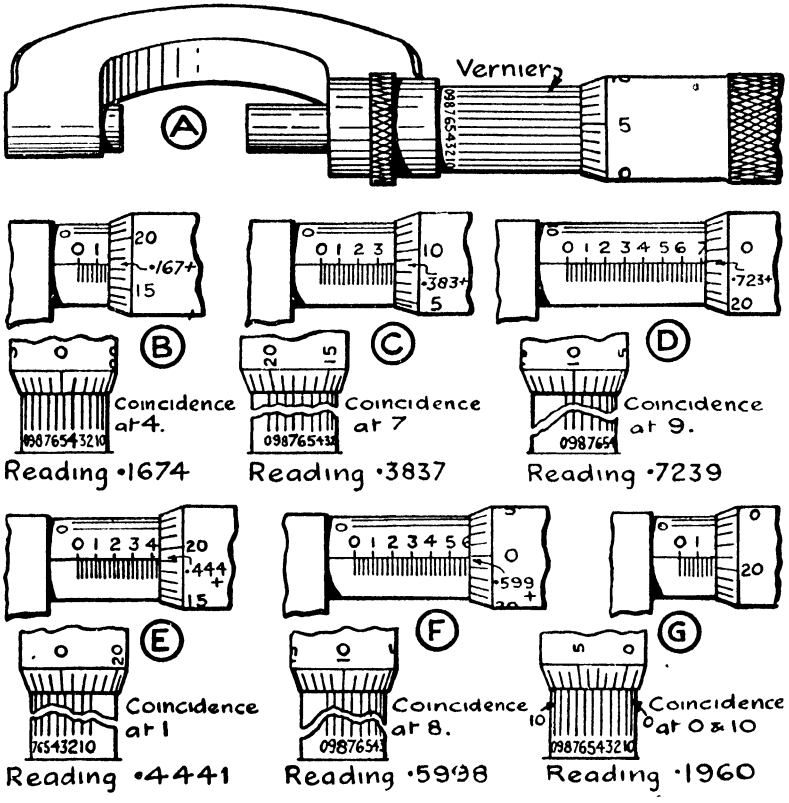


Fig. 172.—Sample Settings of Micrometer Caliper, reading to $\frac{1}{10000}$ inch.

At (*C*).

1. Main scale reading . . . . . = ·375
2. Thimble division before axial line = 8 . . . = ·008
3. Vernier coincidence at 7 . . . . . = ·0007

∴ Complete reading = ·3837

At (*D*).

1. Main scale reading . . . . . = ·700
2. Thimble reading . . . . . = ·023
3. Vernier reading . . . . . = ·0009

∴ Complete reading = ·7239

With further practice the thimble and vernier readings may be combined when read, the thimble reading consisting always of second and third places of decimals, and the vernier reading always the fourth place.

Should a thimble graduation appear to coincide with the axial line on the sleeve when reading to a ten-thousandth, the reading must be made with caution. The vernier should be carefully examined for a possible coincidence near the ends, and no reading should be taken as consisting of an exact number of thousandths unless *both* the 0 and the 10 lines of the vernier are in coincidence.

Examples are shown in Fig. 172 at (E) to (G).

In Fig. 172 :—

At (*E*).

1. Main scale reading . . . . . = ·425
2. Thimble graduation 19 appears very nearly in line with the axial line, but examination of the vernier shows a coincidence at 1. Then thimble and vernier = ·019 + ·0001 = ·0191

∴ Complete reading = ·4441

At (*F*).

1. Main scale reading . . . . . = ·575

but appears very nearly to be ·6, the line 0 on the thimble being just over the axial line.

2. The vernier reading is then ·024 and the vernier coincidence is seen to be at 8 . . . . . = ·0248

∴ Complete reading = ·5998

At (*G*).

1. Main scale reading . . . . . = ·175
2. Thimble reading is at 21 and vernier coincidence is at 0 and 10 . . . . . = ·0210

∴ Complete reading = ·1960

Exercises 102. On Reading the Micrometer ($\frac{1}{10000}$ inch).

In Fig. 173 are shown settings of a micrometer caliper, together with a view of the vernier in each case. Take the reading for each of these settings from (A) to (C).

When using a ten-thousandth micrometer certain practical points need careful attention. Under no circumstances may the screw be jammed on to the job with any pressure. A light, almost imperceptible, pressure is sufficient, which is judged by the "feel" and can only be gained by considerable practice.

It is frequently convenient to hold the bow of the instrument in one hand between the palm and the third and fourth fingers, the thimble being rotated by the thumb and first finger; the caliper should never be held very long, as the warmth of the hand is sufficient to cause an expansion of one ten-thousandth. It is a better plan, wherever possible, to hold the tool in a small bench stand, and to take the work to the instrument.

Lastly, a fine reading tool should never be used on rough

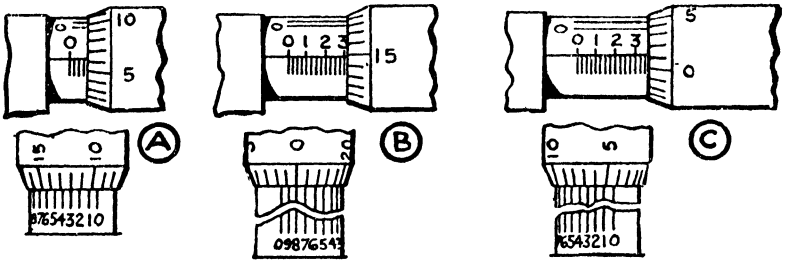


Fig. 173.—Exercises on reading Micrometer, $\frac{1}{10000}$ inch.

surfaces, or for any purpose where fine reading is not really necessary, as wear becomes of very considerable importance in such a tool.

Zero Setting of Ten-Thousandth Micrometer.—For reading to a ten-thousandth the micrometer cannot be said to read zero unless the vernier lines 0 and 10 are in coincidence, in addition to the thimble reading being 0. Zero must be read then, in *three* places, *i. e.* on the main scale, on the thimble and on the vernier. It is a rather tedious process to set a micrometer to zero *correct to one ten-thousandth*, and frequently, therefore, the tool is left reading one or two ten-thousandths *large* or *small*, the exact amount being subtracted or added, respectively, to the readings taken. The method is, of course, always open to the objection that the correction may be forgotten.

The Ratchet Stop.—The ratchet stop shown in Fig. 174 was introduced to eliminate any error due to gripping the job with varying degrees of pressure. It contains a ratchet and pawl, and

if the screw be always rotated by the small knurled end when closing on to the job, an approximately constant pressure is used on all occasions. If more than a certain pressure be applied the ratchet slips over the pawl without forcing the measuring screw round. Upon withdrawal, *i.e.* backward rotation, the ratchet engages positively with the pawl and cannot slip.

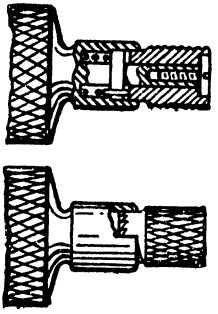


Fig. 174.—Ratchet-Stop for Micrometer Caliper.

The " feel " of the job between the anvils is, however, a more sensitive and better criterion for fine work. The ratchet, even when slipping, can exert an appreciable pressure, and by continued or sudden rotation can jam the anvils on to the job, thereby causing the thimble to overrun its true reading by one or two ten-thousandths. It is, however, of advantage for taking measurements rapidly, and also where more than one person use the same micrometer.

The use of verniers and micrometers graduated in metric units is treated in " Metric System for Engineers" (Chapman & Hall).

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