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DESCRIPTIVE
GEOMETRY

MANY TYPES OF SURFACES ARE REPRESENTED IN BUILDINGS FOR
CENTURY 21 EXPOSITION, SEATTLE, WASHINGTON.
(COURTESY CENTURY 21
EXPOSITION, 1962.)



HOLT, RINEHART AND WINSTON, New York

Descriptive Geometry

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PREFACE This book was written to serve as a practical textbook for beginning students of descriptive geometry. Therefore, emphasis has been placed on clear, concise, well-illustrated explanations of basic principles and methods. Comparison of different methods of solving problems is made easier by the authors' use of the same data for the explanation of each method. Great care has been taken to arrange the figures, so that it is seldom necessary to turn a page to refer to a figure while reading the corresponding text material. For convenient reference and review, a summary of important principles is provided at the end of most chapters. The notation system emphasizes the fact that only three types of views (plan, elevation, and inclined) are used, and it calls attention also to the type of view being drawn.

Problems based on the text material within each chapter are conveniently located at the end of that chapter. In addition, Chapter 10 contains miscellaneous problems, some of which are based on principles from several chapters. All problems can be solved on a standard 8½- by 11-inch sheet, with the 11-inch dimension placed parallel to the T square. Considerable time can be saved in setting up the problems by using Hoag's *Descriptive Geometry Problems* books, which contain most of these problems and many more.

The authors wish to express their appreciation to their many colleagues for their suggestions and encouragement. Special thanks are extended to Professor Herbert Boehmer of the Department of General Engineering for reviewing the manuscript; Professor Bates McKee of the Department of Geology for reviewing Chapter 9; Mrs. Miriam Jenks for preparing many of the illustrations and for proofreading; and the junior author's daughter, Harriet Hoag, for typing the manuscript.

CLARENCE E. DOUGLASS
ALBERT L. HOAG

Seattle, Washington
December, 1961

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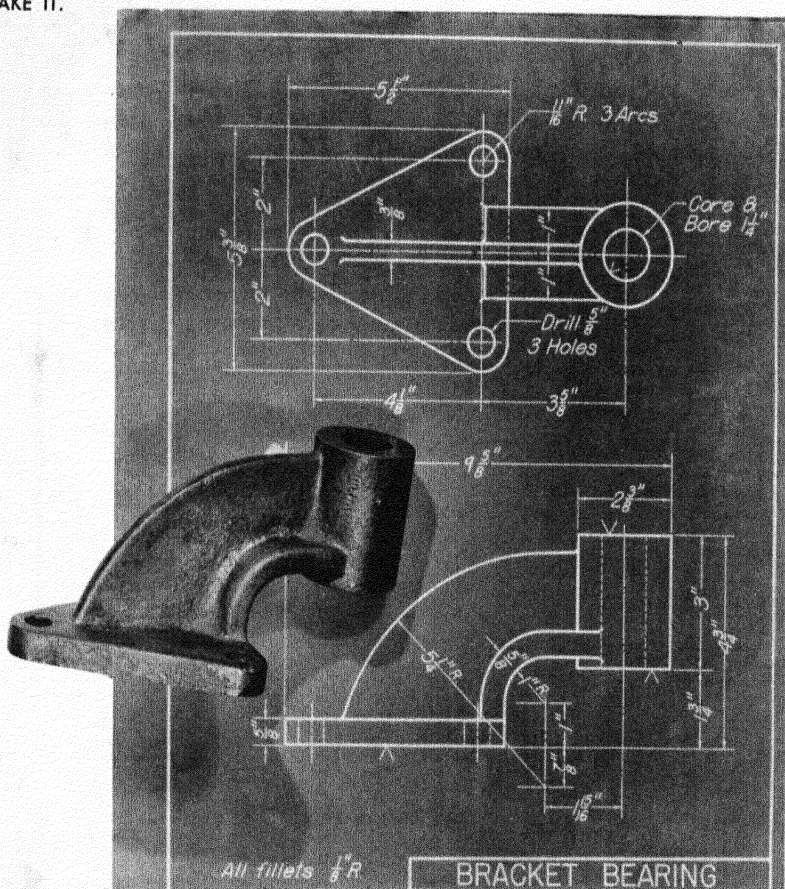
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CHAPTER 1

BRACKET BEARING AND BLUEPRINT
OF THE ORTHOGRAPHIC DRAWINGS USED
TO MAKE IT.



CHAPTER 1 Orthographic Projection

A Review of the important principles of orthographic projection and an introduction to nomenclature.

1.1 INTRODUCTION

The engineer deals with machines, structures, and space problems in three dimensions, but to represent them on paper, he must work in two dimensions only. To do this, he usually uses the *direct*, or *change-of-position*, method of orthographic projection to show the exact shapes and sizes of objects. This method is based on the assumption that the object remains in a fixed position and the observer changes his position to obtain different views of the object.

1.2 SURFACES AND LINES

Before discussing orthographic projection, the following definitions of surfaces and lines should be clearly understood:

A **plane surface** is one on which a straight line may be drawn in any direction.

A **single-curved surface** is a surface, such as a cylinder or cone, on which straight lines may be drawn in some, but not in all, directions.

A **double-curved surface** is a surface, such as a sphere, on which no straight line can be drawn.

A **level surface** is a surface that is parallel to the mean surface of the earth—for example, the surface of a still body of water. A level surface is a double-curved surface.

A **horizontal surface** is a plane surface that is tangent to a level surface.

A **vertical surface** is a plane surface that is at right angles to a horizontal surface.

An **inclined surface** is a plane surface that is neither horizontal nor vertical.

A **level line** is a line on a level surface.

A **horizontal line** is a line on a horizontal surface.

. **Note:** Since the divergence of a level line from a horizontal line is too small (0.02 ft in 1000 ft) to be shown on a drawing, the terms level and horizontal are often used interchangeably.

A **vertical line** (or **plumb line**) is a straight line that is perpendicular to a horizontal plane and if extended, would pass through the center of the earth. In other words, it is a line that is straight up and down.

1.3 TYPES OF VIEWS

In orthographic projection, the object is considered to be projected onto a transparent **image plane** that lies between the observer and the object. All **lines of sight** from the observer to the object are at right angles to the image plane for the particular view (Fig. 1.1).

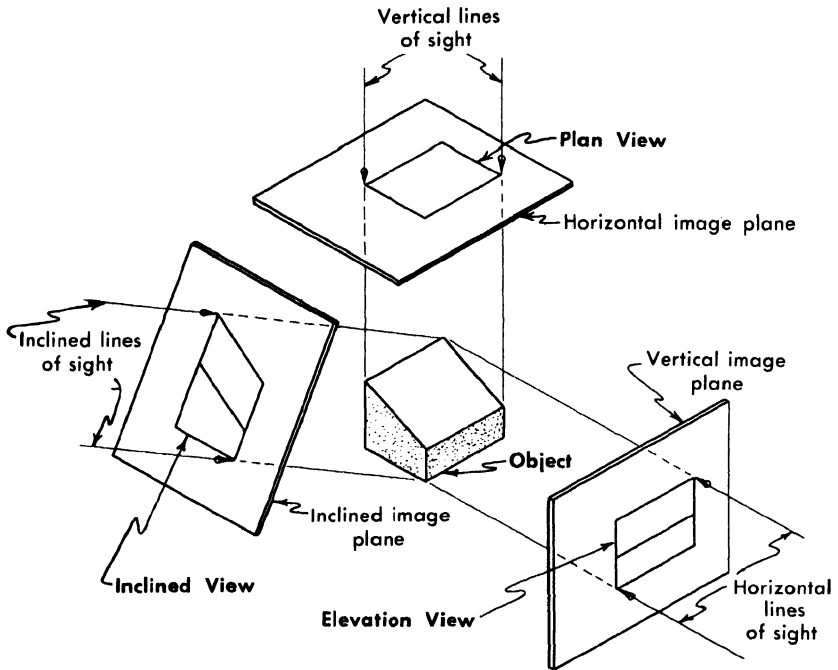


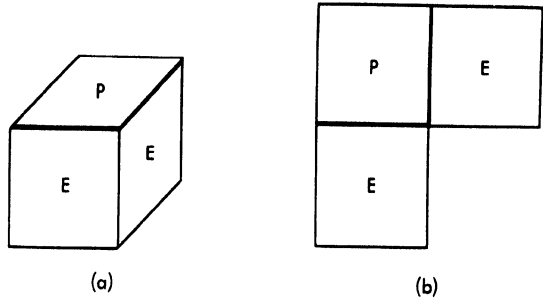
Figure 1.1 Lines of sight for orthographic views

When the observer is looking vertically downward, the image plane is horizontal and the view obtained is the **plan view**. Since the object remains in a fixed position, only one plan view is possible.

If the lines of sight are horizontal, the image plane is vertical, and the view obtained is an **elevation view**. An infinite number of elevation views of an object can be drawn.

If the lines of sight are inclined, that is, neither vertical nor horizontal, the image plane is inclined, and the view obtained is an **inclined view**. An infinite number of inclined views of an object can be drawn.

Figure 1·2 Rotation of image planes for views projected from plan views



1·4 ROTATION OF IMAGE PLANES

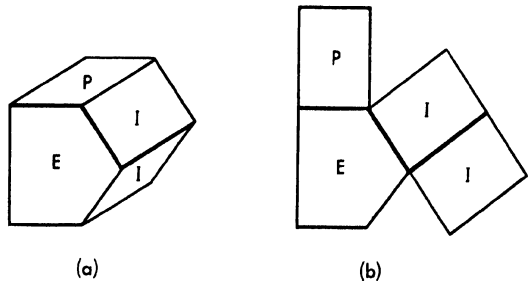
If plan, elevation, and inclined views are to be shown on a flat piece of paper, the image planes for all views must be rotated into the plane of the paper. Image planes may be rotated through 90 degrees with respect to each other when, and only when, they are at right angles to each other in space. Since there is only one possible plan view, for convenience we shall consider it in the plane of the paper. The elevation view is rotated upward about the line of intersection of its image plane with the image plane of the plan view (Fig. 1.2).

Inclined image planes obviously cannot be perpendicular to the image plane of the plan view. They may, however, be perpendicular to the image planes for either elevation or inclined views. This means that an inclined view may be rotated about the line of intersection of its image plane with the image plane of an elevation view or with the image plane of another inclined view.

Figure 1.3a illustrates the position in space of the image planes for the plan, one elevation, and two mutually perpendicular inclined views. These are brought into the same plane by rotation about the heavy lines, as shown in Figure 1.3b. These heavy lines are called rotation lines, for which the following symbol is used:

— — — — —

Figure 1·3 Rotation of image planes for views projected from elevation views



1.5 ROTATION LINE NOTATION

A foolproof system of referring to the views is necessary to prevent confusion when several views are to be drawn. The system used here is based on the type of view and the position of its image plane. Thus the plan view is the P view on the P image plane, elevation views are E views on E image planes, and inclined views are I views on I image planes. Since there is an infinite number of elevation views, the first elevation view drawn is called E1, the second E2, and so on. Similarly, the inclined views are designated as I1, I2, etc., in the order drawn.

Each rotation line is designated by the letters referring to the two views between which it lies. For example, $\text{---} \frac{P}{E1} \text{---}$ is the rotation line between the plan and the first elevation view drawn. Similarly, $\text{---} \frac{E2}{I3} \text{---}$ is the rotation line between the second elevation view and the third inclined view.

The reader may be familiar with the terms *front elevation*, *right side elevation*, and *auxiliary elevation* as used in standard drawing texts. In descriptive geometry, we deal with lines, planes, and solids for which terms such as *front* and *right side* have little or no meaning. As far as we are concerned, there are only three types of views: plan, elevation, and inclined views. Observe that the notation used here calls attention to the type of view being drawn. This method will help to prevent such mistakes as measuring slopes or bearings in the wrong views.

1.6 POINT NOTATION

If a letter is assigned to a specific point, the point may be identified in all views. Occasionally, in situations such as overlapping views, the view notation is used as a subscript to the letter indicating the point. For example, point *A* is designated A_P in the plan view, A_{E1} in the first elevation view, A_{I2} in the second inclined view, and so on.

1.7 VIEWS PROJECTED FROM THE PLAN VIEW

When one view is projected from another, the two image planes must be at right angles to each other in space. Thus, any view projected from the plan view is an elevation view.

Let us consider now an object in space and some of its views on their respective image planes.

As seen in Figure 1.4, the rotation line is the line of intersection between the image planes for the plan and elevation views. When we look at any elevation view, the rotation line represents the edge view of the image plane for the plan view. Since the point X is a distance d below the plan-view image plane, it must appear the distance d below the rotation line in all elevation views. This condition is true for every point on the object.

1.8 VIEWS PROJECTED FROM ELEVATION VIEWS

Views may be projected from any elevation view in the same manner as from the plan view.

In Figure 1.5a, the point Y is a distance s behind the image plane for view $E3$. When we look at the plan view, the image plane for view $E3$ appears as an edge and the distance s appears in its true length. When we look at the inclined views $I1$ and $I2$, the image plane for view $E3$ again appears as an edge, and the distance s is again in its true length.

In Figure 1.5b, the rotation line $\frac{P}{E3}$ represents the edge view of the image plane for view $E3$ when we look at the plan. The distance s therefore is in its true length in the plan view. Similarly, when we look at the inclined views $I1$ and $I2$, the rotation lines $\frac{E3}{I1}$ and $\frac{E3}{I2}$ represent the edge view of the image plane for view $E3$ and the distance s is true length. This condition will be true for the infinite number of views that may be projected from any elevation view.

Among the infinite number of views projected from an elevation view, it is possible to obtain only four views (plan, bottom, and two elevations) that are not inclined views. Let us then consider any view, except the plan, that is projected from an elevation view to be an inclined view. If additional elevation views are required, they should be projected from the plan view.

1.9 VIEWS PROJECTED FROM INCLINED VIEWS

Views may be projected from inclined views in the same manner as they are projected from plan and elevation views.

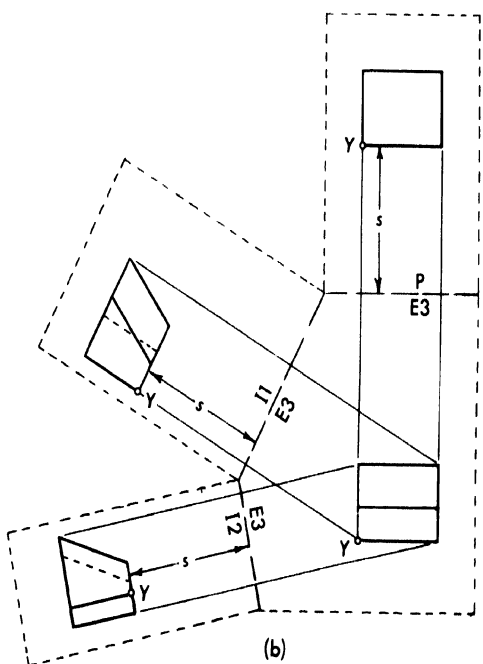
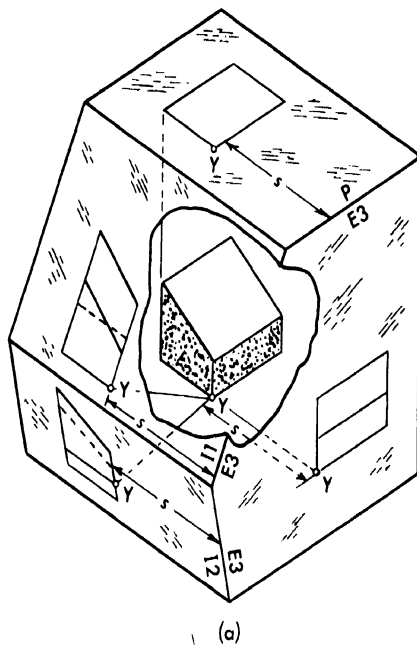


Figure 1-5 Views projected from an elevation view

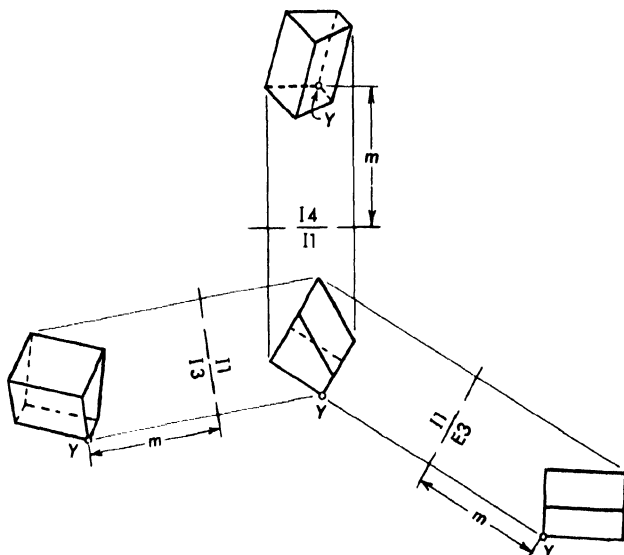


Figure 1-6 Views projected from an inclined view

Figure 1.6 shows an elevation view, E3, an inclined view, I1, and two more inclined views, I3 and I4, that are projected from view I1. Since the point Y is a distance m from the image plane of view I1, this distance must be measured in view E3 and used to establish point Y in views I3 and I4. Other points on the object are located in the new inclined views by similar measurements.

1-10 MEASUREMENTS FROM ROTATION LINES

. **Rule:** Any point on an object must appear the same distance from the rotation line in all views connected to a given view by projection lines.

This rule is illustrated in Figure 1.7. Point X is a distance d from the rotation lines in views E1, E2, and E3, each of which is connected to view P by projection lines. The distance from the rotation line to point Y is s in the three views connected to view E3, and m in the three views connected to view I1.

It is essential that this rule be thoroughly understood, since measurement from rotation lines is the key to drawing any new view.

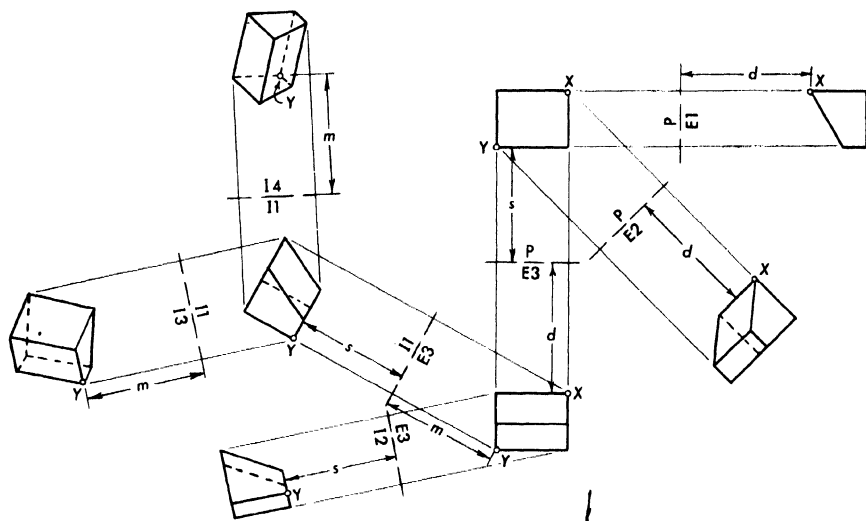


Figure 1-7 Measurements from rotation lines

1.11 VISIBILITY OF LINES AND SURFACES

To draw a complete view, it is necessary to know whether a line is visible or hidden. Any view of an object is bounded by visible lines, so visibility need be determined only for those lines within the outline of the view.

The views A and B in Figure 1.8 are any two connected views showing two triangular planes that intersect along the line ED . In view A, the lines CD and EF apparently intersect at M , although one

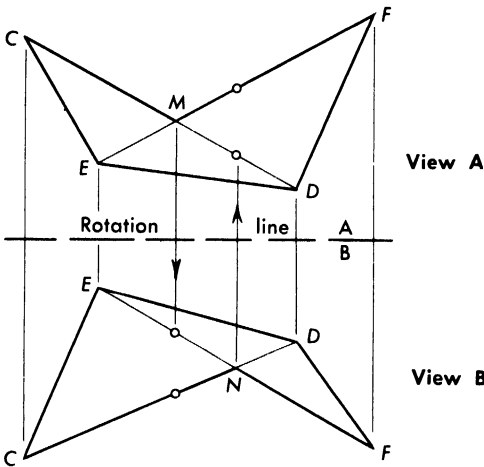


Figure 1.8 Visibility of lines

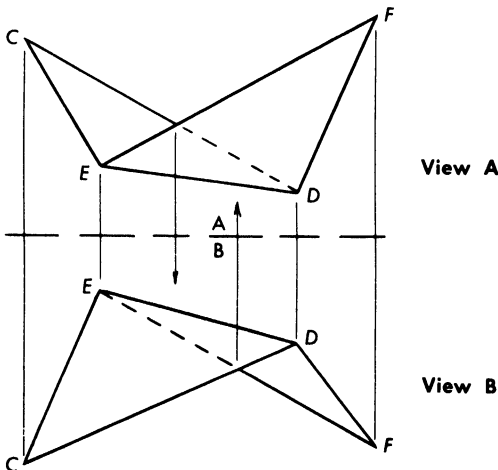


Figure 1.9 Visibility of surfaces

of the lines is closer to the observer than the other. A projection line drawn from *M* to view B intersects line *EF* first, showing that *EF* is closer to the observer than line *CD*. Therefore, line *EF* must be visible in view A, and part of line *CD* must be hidden. (See Fig. 1.9.)

In view B, the lines *CD* and *EF* appear to intersect at *N*, and a projection line drawn to view A shows that *CD* is closer to the image plane than *EF*. Therefore, *CD* will be visible in view B, and a portion of *EF* will be invisible.

When the procedure to determine visibility of lines is followed, the visibility of surfaces is also determined, as Figure 1.9 illustrates.

1.12 PROCEDURE FOR DRAWING VIEWS

For any drawing, sufficient data must always be given to allow the first two views to be constructed. Any other desired views are obtained by the following method:

First. Decide on the direction of the line of sight for the next view.

Second. Draw a rotation line at right angles to this line of sight.

Third. Draw a projection line perpendicular to the rotation line from each point on the given view.

Fourth. Select a surface that will be visible in the new view. If no surface is obviously visible, select the surface that is nearest the rotation line for the new view. (This surface must be visible.)

Fifth. Transfer the surface into the new view, placing each point on the proper projection line and measuring its distance from the rotation line according to the rule (Sec. 1.10). Connect the points in the same order in which they are connected in the given views.

· · · · · **Rule:** *If two points are connected by a line in space, they are connected by a line in all views.*

Sixth. Repeat the above procedure, selecting surfaces in order of increasing distance from the new rotation line. Any line is visible if it appears outside, and is invisible if it appears inside, a surface already drawn. If visibility is still in doubt, it may be determined by the procedure given in Section 1.11.

1-13 SUMMARY

1. Observer's *lines of sight* are at right angles to an image plane that is between the observer and the object.

2. For a *plan view*, the lines of sight are vertical (downward) and the image plane is horizontal.

3. *Elevation views* have horizontal lines of sight and vertical image planes.

4. For *inclined views*, both the lines of sight and the image plane are inclined.

5. *Surfaces* may be classified as *plane*, *single-curved*, or *double-curved*. They may also be designated as level, horizontal, vertical, or inclined surfaces.

6. Lines are either *straight* or *curved*, and may be designated as level, horizontal, or vertical.

7. A *rotation line* is the line of intersection of two mutually perpendicular image planes, about which one image plane is rotated to bring it into the plane of the other.

8. Every view projected from the plan view is an elevation view.

9. "Up" is toward the plan view in all elevation views.

10. *Inclined views* may be projected from elevation views and other inclined views, but never from the plan view.

11. *Measurement rule*: Any point on an object must appear the same distance from the rotation line in all views connected to a given view by projection lines.

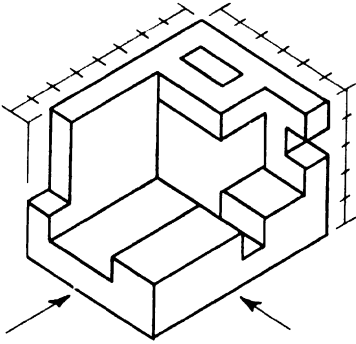
12. When two lines or planes apparently (but not actually) intersect, the line or plane nearest the observer is visible.

13. Each new view should be drawn *surface by surface*, and each surface *line by line*.

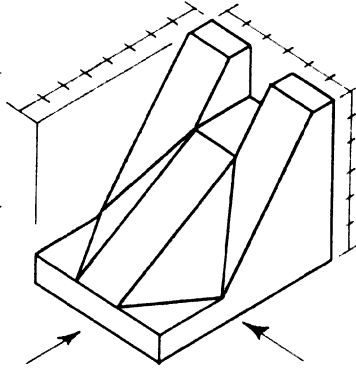
14. If two points are connected by a line in space, they are connected by a line in all views.

1-14 Problems

- 1-1. Draw a plan view and the two elevation views indicated by the arrows. Each space between the points on the marked line is equivalent to $\frac{1}{2}$ in. Scale: Full size.

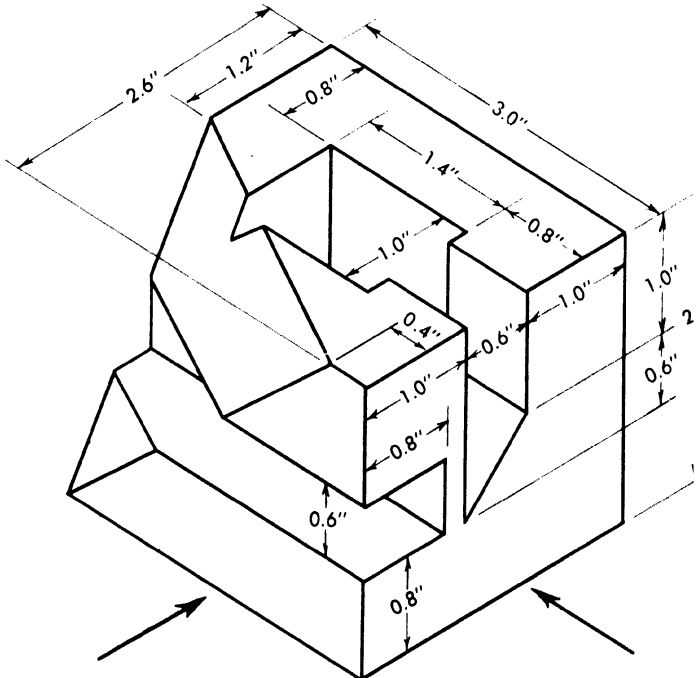


Problem 1-1

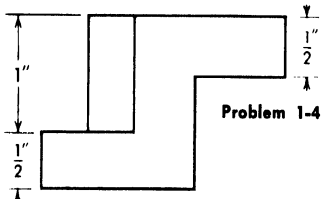
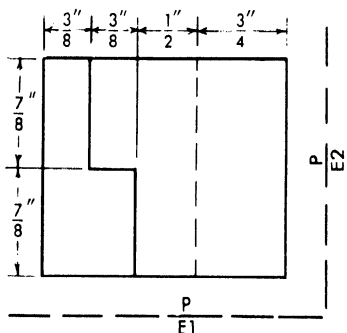


Problem 1-2

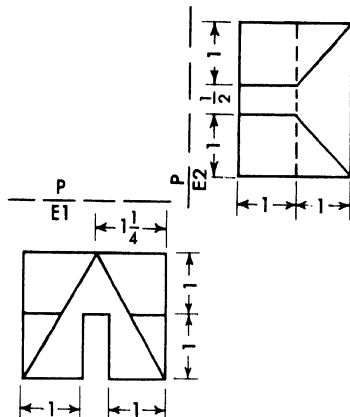
- 1-2. Draw a plan view and the two elevation views indicated by the arrows. One space equals $\frac{1}{2}$ in. Scale: Full size.
- 1-3. Draw a plan view and the two elevation views indicated by the arrows. *Note:* The hole in the center goes entirely through the object. Scale: Full size.



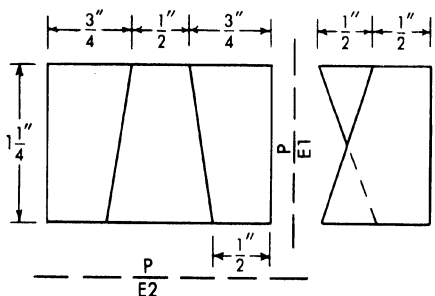
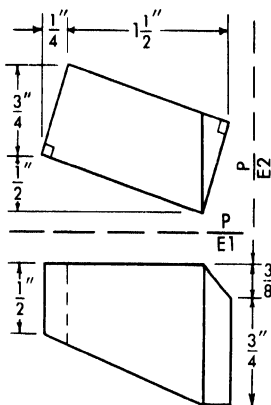
Problem 1-3



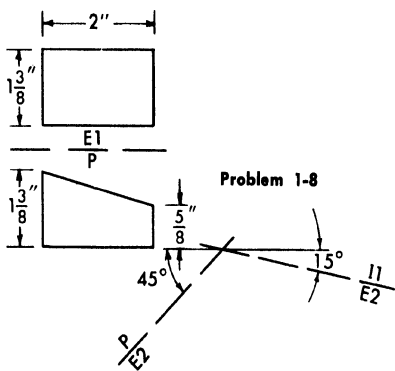
Problem 1-4



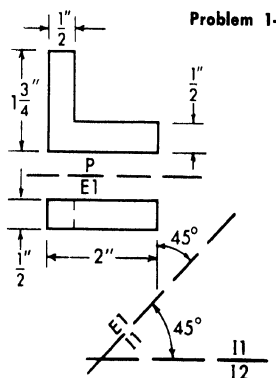
Problem 1-5



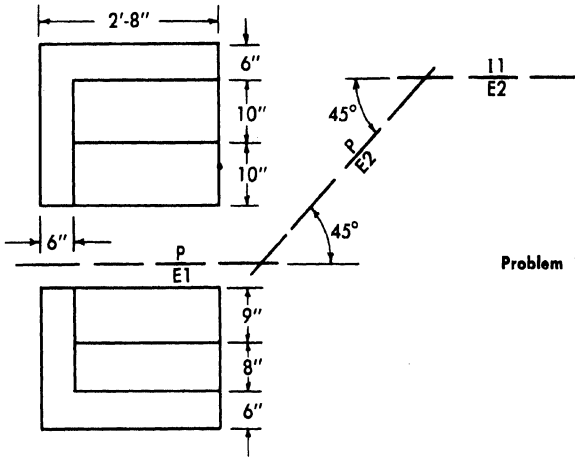
Problem 1-7



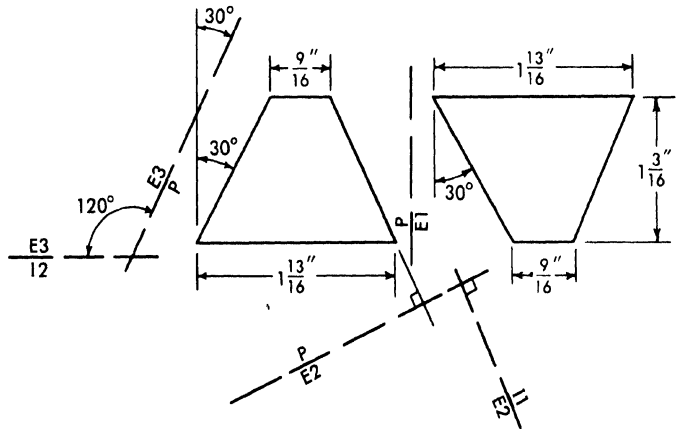
Problem 1-8



Problem 1-9



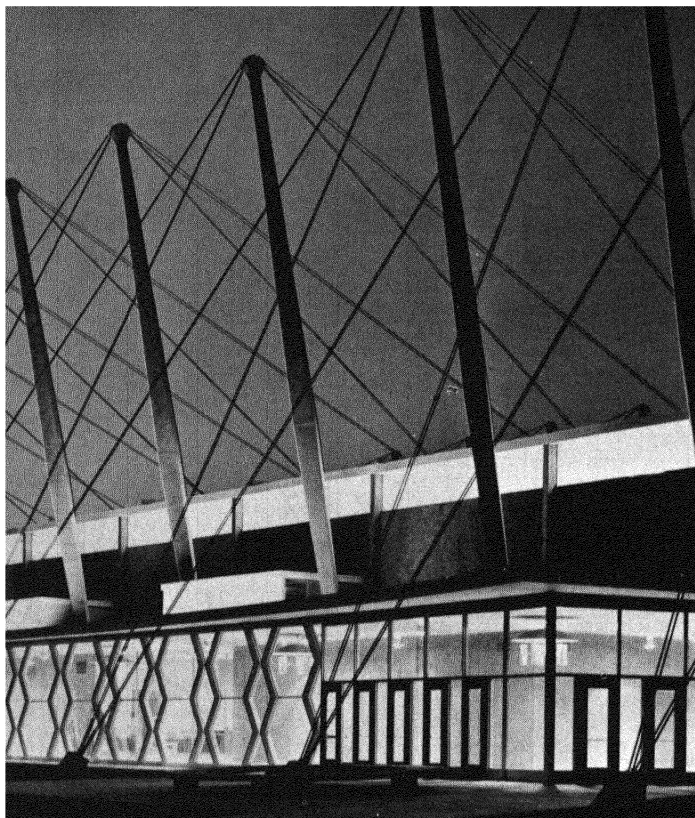
Problem 1-10



Problem 1-11

- 1-4. Draw view E2. Scale: Full size.
- 1-5. Draw the plan view. Scale: Full size.
- 1-6. Draw view E2. Scale: Double size.
- 1-7. Draw view E2. Scale: Double size.
- 1-8. Draw views E2 and I1. Scale: Full size.
- 1-9. Draw views I1 and I2. Scale: Full size.
- 1-10. Draw views E2 and I1. Scale: $\frac{3}{4}$ in. = 1 ft 0 in.
- 1-11. Draw views E2, E3, I1, and I2. Scale: Full size.

GYMNASIUM WITH SUSPENDED ROOF, CENTRAL WASHINGTON COLLEGE
OF EDUCATION, ELLENSBURG, WASH. (*Courtesy of architect, Ralph
H. Burkhard,
AIA, Seattle*)



CHAPTER 2 Lines and Planes

*The fundamental principles necessary for the solution
of problems involving lines and planes*

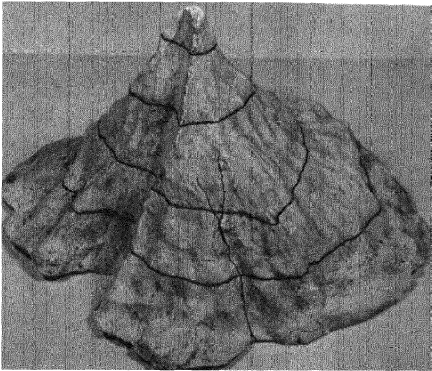
2.1 LINES

In this text, the term *line* means a straight line segment unless otherwise specified. When a line is designated by letters, the line segment has a definite length. For example, “line *AB*” means a straight line that extends from point *A* to point *B*.

. **Note:** *Any line may be extended indefinitely to aid in the solution of a problem.*

A **contour line** is a line, usually not straight, that represents points of equal elevation on the earth’s surface. Contour lines are therefore level lines, which may be considered as horizontal for the relatively short distances encountered in descriptive geometry. (See Sec. 1.2.)

Figure 2.1a shows a model on which the black lines represent contour lines. The view in Figure 2.1b, taken from directly above the model, shows the contour lines as they would appear on a map.



(a)

(b)

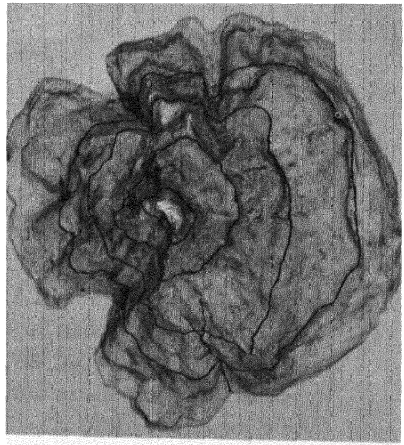


Figure 2.1 Model with contour lines

2.2 LOCATING POINTS ON LINES

If a point on a line is given in only one view, it may be located in another view by projection, as shown in Figure 2.2.

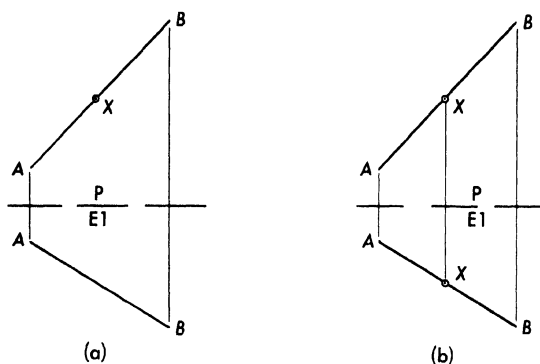


Figure 2.2 Locating a point on a line

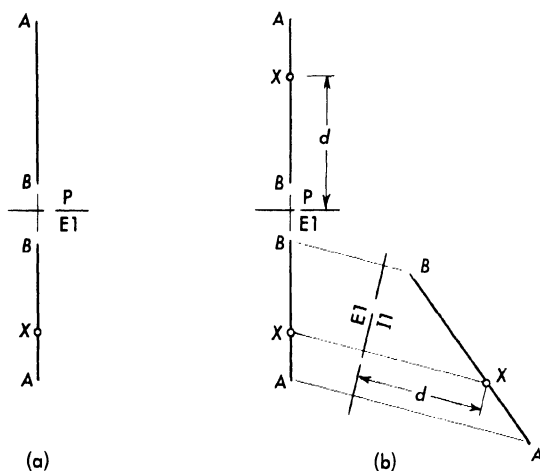


Figure 2.3 Locating a point on a line—special case

Where the line appears perpendicular to the rotation line between the two given views, the point may be located by drawing an additional view (Fig. 2.3). The point is first located in the new view by projection, and then in the given view by measuring from the rotation line. The point can also be located using only the two given views by constructing a plane containing line AB and a point not on line AB . (See Sec. 2.12.)

2.3 TRUE LENGTH OF LINE

The true length of a line will be seen only when the lines of sight are at right angles to the line. In other words, if a line is to appear true length in any view, it must be parallel to the image plane for that view. A line will appear shorter than true length in any view in which the lines of sight are not perpendicular to the line.

In Figure 2.4, the line *AB* appears true length in the plan view and the line *CD* appears true length in the elevation view, but the lines *KL* and *MN* do not appear true length in either view. In order to find the true length of *KL* or *MN*, a new view must be drawn.

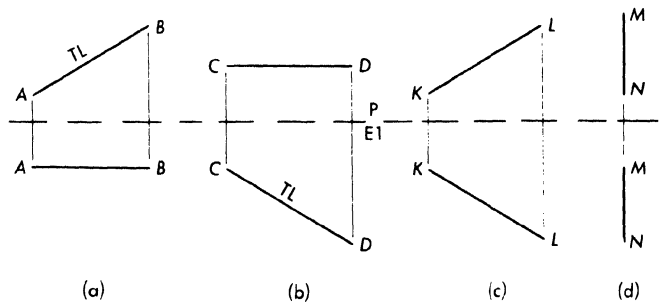


Figure 2.4 Conditions for true length of a line

As shown in Figure 2.5, the true length of the given line may be obtained in either an elevation view or an inclined view. In either case, a new rotation line is drawn parallel to the given line, and projection lines are drawn perpendicular to both the rotation line and the given line. Measurements for view E2 are obtained in view E1, for example, d ; those for view I1 are obtained in the plan, for example, s .

For future reference and to avoid confusion, it is advisable that the letters TL be placed on a line in all views that show its true length.

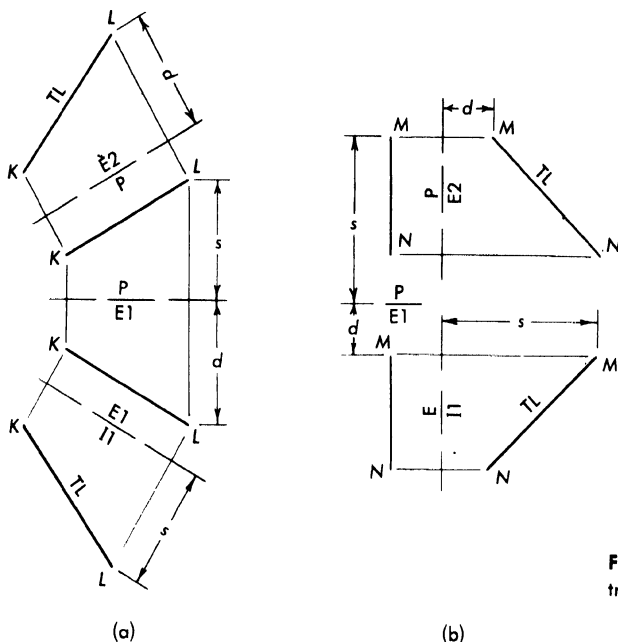


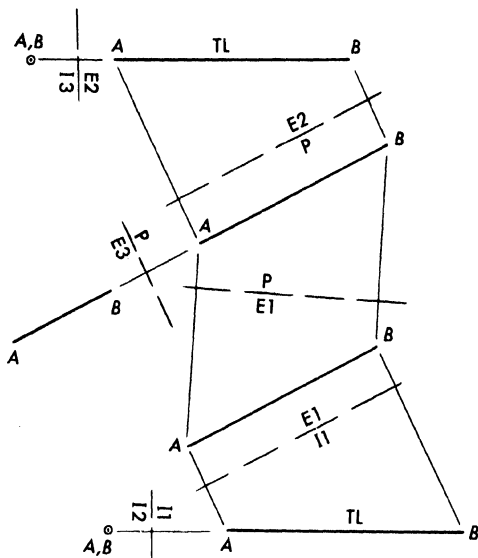
Figure 2.5 Finding the true length of a line

2.4 LINE AS A POINT

A line will appear as a point when the lines of sight are parallel to the line. In other words, the line must be perpendicular to the image plane for any view in which it appears as a point. This means that the image plane for the point view of a line must be perpendicular to the image plane for a true length view of the line.

. **Rule:** A line can never appear as a point in any view unless that view is projected from a true length view of the line.

Figure 2.6 Line as a point



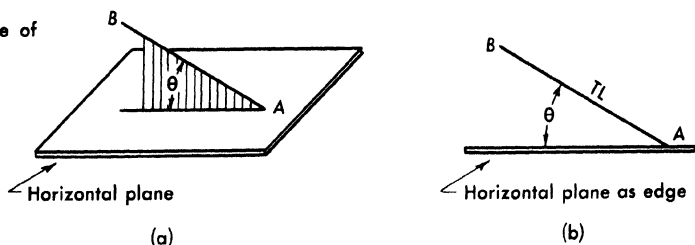
In Figure 2.6, consider that P and E1 views of the line *AB* are given. The true length of *AB* may be found in either view E2 or I1. A rotation line is then drawn perpendicular to the true length view of *AB*, and *AB* is projected into the new views (I2 or I3), where it appears as a point. Note that *AB* does not appear as a point in view E3 because this view is projected from a view in which *AB* does not appear true length. Remember, to get a line as a point, you must first find its true length.

2.5 SLOPE OF A LINE

The slope of a line may be defined as the angle the line makes with a horizontal plane.

As shown in Figure 2.7, the angle θ is the slope of the line *AB*. The true size of the angle θ is seen when the line is in its true length and the horizontal plane is an edge in the same view.

Figure 2.7 Slope of a line



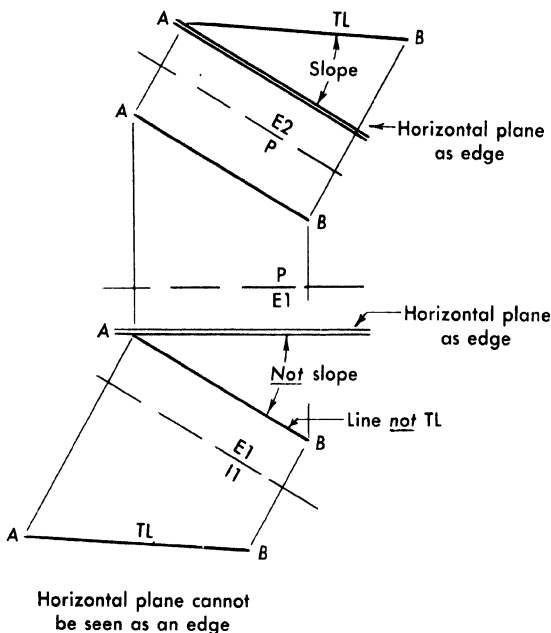


Figure 2.8 Finding the slope of a line

In Figure 2.8, the line AB is given in the views P and $E1$. So that the slope of AB can be seen, the new elevation view $E2$ is drawn to show the true length of AB . In this view, a horizontal plane appears as an edge and the slope angle is seen in its true size. Note that view $I1$ also shows the true length of AB , but that the slope cannot be seen because a horizontal plane cannot appear as an edge in an inclined view.

. **Rule:** The slope of a line can be seen only in an elevation view that shows the true length of the line.

Since B is lower than A , the line AB has a downward, or negative ($-$) slope. If the line had been designated as line BA , the slope would be upward, or positive ($+$).

Some other methods of designating slope are shown in Figure 2.9. Slope may be defined as the ratio of vertical rise to horizontal run, expressed as a ratio, a fraction, or a decimal (Fig. 2.9a). For streets, highways, etc., the slope is usually expressed as percent grade (Fig. 2.9b). Percent grade is defined as the number of units rise in 100 units run. In specifying the slope, or bevel, of a structural member, the longer leg of the slope triangle is always 12 (Fig. 2.9c). The inclination of piling, faces of retaining walls, bridge piers, etc., is often expressed as batter, the

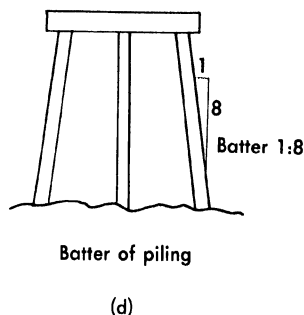
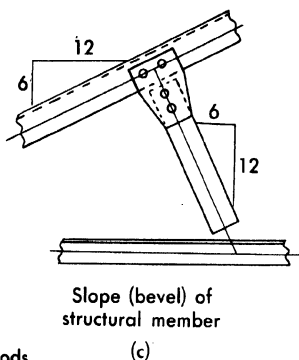
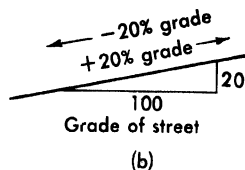
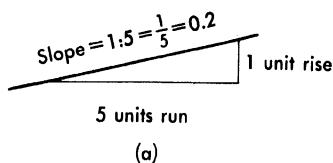
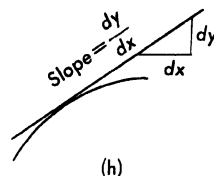
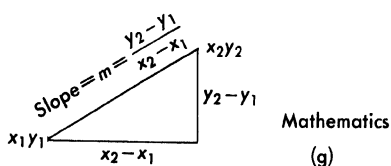
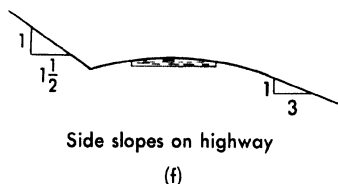
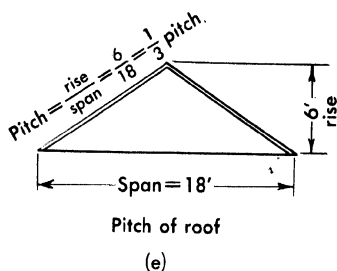


Figure 2-9 Methods of designating slope



horizontal leg of the triangle being always equal to unity (Fig. 2.9d). The pitch of a roof is expressed as the ratio of rise to span (Fig. 2.9e). Side slopes for highways and canals are expressed as a ratio of the number of units run to one unit of rise (Fig. 2.9f). In mathematics, the slope of a straight line is expressed as the ratio of rise to run—that is, tangent of slope angle (Fig. 2.9g); and the slope of a curve at any point ($\frac{dy}{dx}$) is the slope of the tangent to the curve at that point (Fig. 2.9h).

2.6 BEARING OF A LINE

The bearing of a line is the horizontal angle of 90 degrees or less measured *from* a meridian (north-south line) *to* the vertical plane containing the line. This angle may start from either the north end or the south end of the meridian and may be measured either toward the east or toward the west. Therefore, in giving a bearing, we must specify both the end of the meridian used and the direction of measurement of the angle.

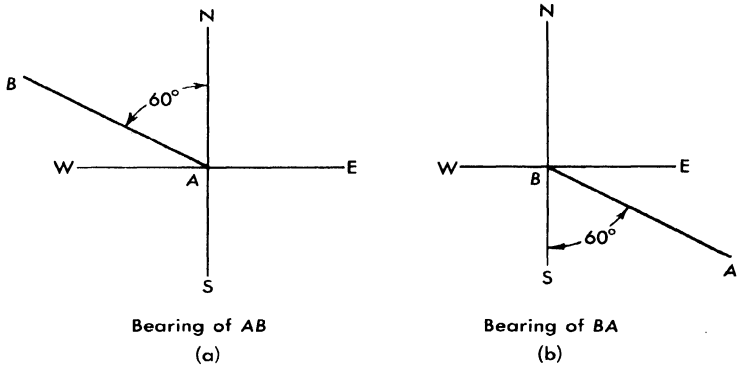


Figure 2.10 Bearing of a line

In Figure 2.10, the bearing of line *AB* is measured from the north end of a meridian, 60 degrees toward the west, and is written N 60° W. The bearing of *BA* is measured from the south end of a meridian, 60 degrees toward the east, and is written S 60° E. Since the bearing is measured *from* the meridian *to* the line, the angle is *always* preceded by N or S and followed by E or W. Note that the bearing is in no way affected by the slope of the line.

. **Rule:** *The bearing of a line must be measured in the plan view.*

2.7 TO LAY OUT A LINE OF GIVEN BEARING, LENGTH, AND SLOPE

GIVEN: The line AB with a bearing of $N 60^\circ E$, a true length of 500 ft, and a slope of $+35$ degrees.

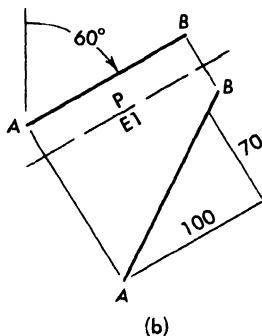
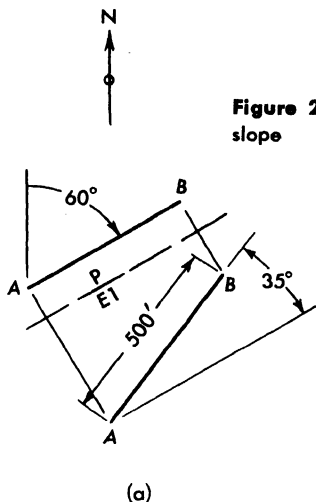


Figure 2.11 To lay out a line of given bearing, length, and slope

PROCEDURE: (Fig. 2.11a)

1. Locate A in the plan view.
2. Through A draw a line of indefinite length making an angle of 60 degrees toward the east from north.
3. Draw rotation line $\frac{P}{E1}$ parallel to this line.
4. Draw a projection line from A to locate A in view $E1$.
5. In view $E1$, draw a line of indefinite length through A at an angle of 35 degrees upward from horizontal.
6. On this line, locate B at a distance of 500 ft (to scale) from A .
7. Locate B in the plan view by projecting from view $E1$.

If the slope is given as a grade of $+70$ percent, follow the same procedure except for step 5. To lay out the $+70$ percent grade, measure 100 units horizontally using *any convenient scale*, and 70 units vertically using the *same scale* (Fig. 2.11b).

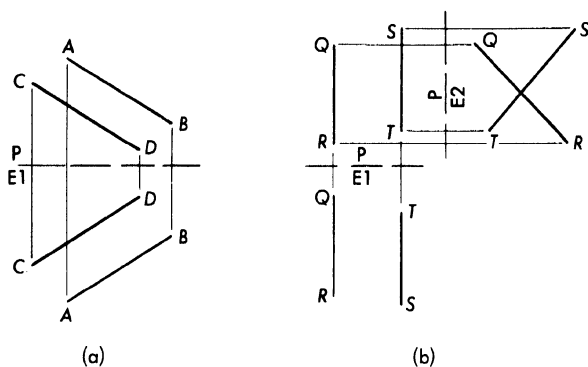


Figure 2.12 Parallel and nonparallel lines

2.8 PARALLEL LINES

Lines that are parallel in space will appear parallel in all orthographic views (except a view in which the lines apparently coincide) (Fig. 2.12a). Lines are parallel in space if they appear parallel in two adjacent views, *unless* they are at *right angles* to the *rotation line* between the two views (Fig. 2.12b). In this case, a third view will show whether or not the lines are parallel.

2.9 INTERSECTING AND NONINTERSECTING LINES

Lines intersect if they have a common point. If two lines apparently intersect in each of two adjacent views, they actually intersect only if the apparent points of intersection lie on the same projection line (Fig. 2.13).

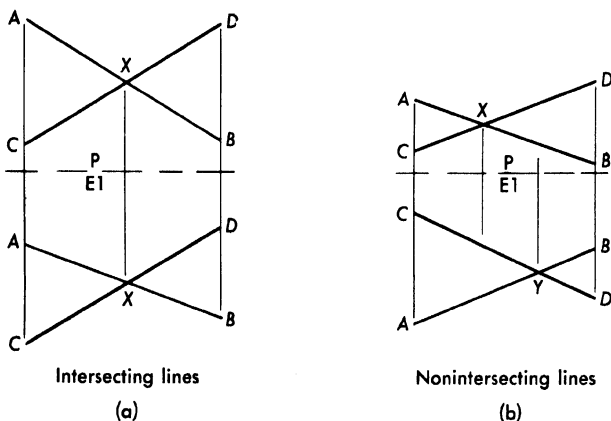


Figure 2.13 Intersecting and nonintersecting lines

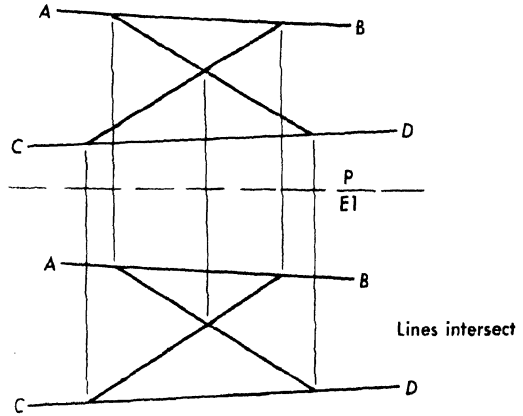


Figure 2.14 Method of determining intersection of lines

If two lines that are not parallel have no apparent point of intersection within the limits of the drawing, the method illustrated in Figure 2.14 will determine whether or not the lines intersect. The given lines intersect if any two lines drawn between the given lines have a common point.

2.10 RECTANGULAR COORDINATES

Points in space may be located by means of rectangular coordinates with three mutually perpendicular axes (Fig. 2.15).

Point *A* has coordinates of +2 in the *x* direction, +3 in the *y* direction, and +3 in the *z* direction, and point *B* has *x*, *y*, and *z* coordinates of -2, -2, and +4, respectively.

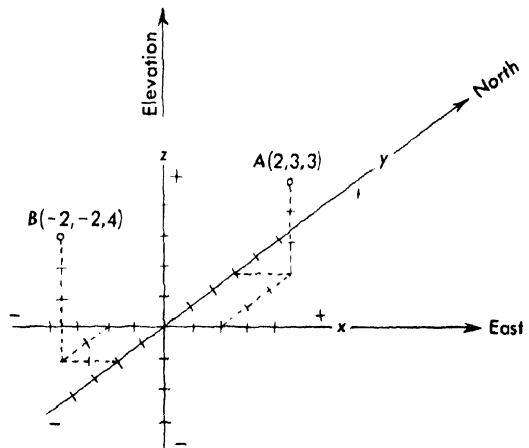


Figure 2.15 Rectangular coordinates

Note that the y axis runs north and south and the x axis runs east and west. This follows the conventional practice in surveying, especially with reference to State Plane Coordinate systems. The z axis is vertical and distances in the z direction denote differences in elevation.

A line can be designated either by its bearing, length, and slope, or by the x , y , and z coordinates of its ends.

2.11 PLANES

A plane is a surface such that any two points on the surface may be connected by a straight line that lies entirely on the surface. The positions of planes in space may be designated by the positions of two intersecting lines (Fig. 2.16a), two parallel lines (Fig. 2.16b), a point and a line (Fig. 2.16c), or three points not in a straight line (Fig. 2.16d).

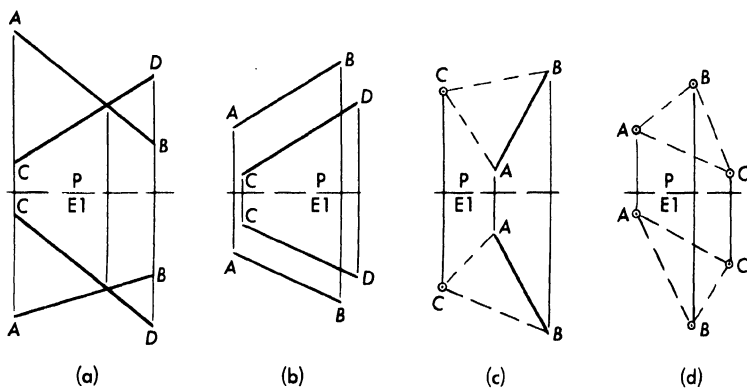
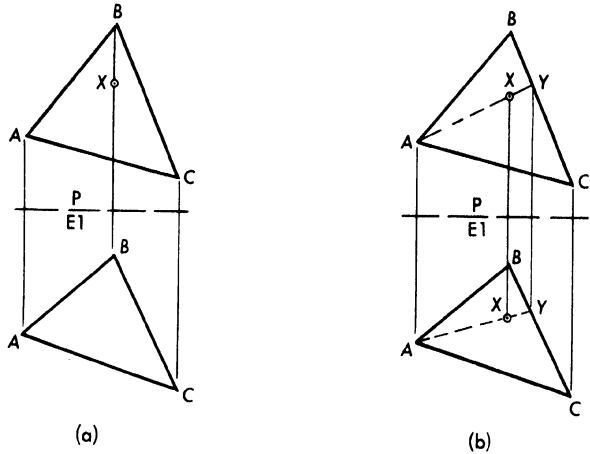


Figure 2.16 Designation of planes

Plane surfaces of definite objects, such as the top of a desk, have a particular shape and size, and are limited in extent. In solving problems, however, planes are considered to be unlimited in extent, and the plane, or any line on the plane, may be extended beyond the limits given for the plane. Also, any number of lines may be drawn on a plane to aid in the solution of the problem.

Figure 2-17 Locating a point on a plane

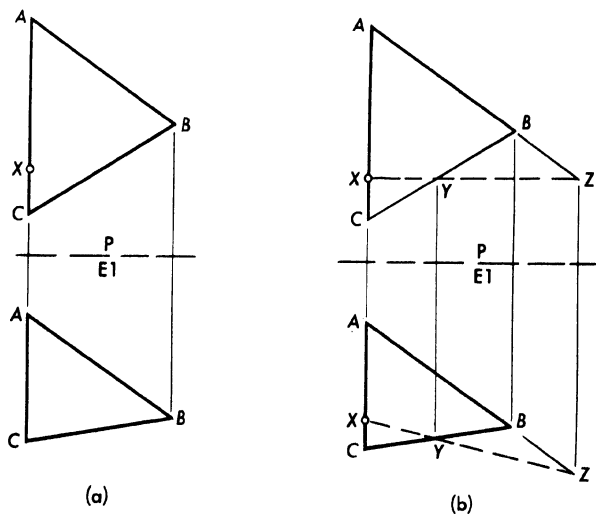


2-12 POINTS ON PLANES

Two views of a plane are required to show its position in space. However, the position of a *point on the plane* is fixed if it is shown in one view of the plane.

The point can be located in another view if a line is drawn on the plane through the point and is projected into the other view, as illustrated in Figures 2.17 and 2.18.

Figure 2-18 Locating a point on a plane—special case



In Figure 2.17a, the plan and elevation views of plane ABC are given, point X on the plane being shown only in the plan. In Figure 2.17b, line AY is drawn through X in the plan view. Point Y , being on line BC , is easily located in the elevation view by projection, and line AY is then drawn in this view. Since point X is on this line, it is located by projection from the plan. Although line AY was chosen for convenience in projection, almost any other line through X could have been used.

In Figure 2.18a, we are again given a plane ABC with a point X shown only in the plan. In this case, because point X is on line AC , which is perpendicular to the rotation line, it cannot be located in the elevation view by projection. One of the many ways of locating point X in the elevation view is illustrated in Figure 2.18b. Line AB is extended to point Z in both views, and line XZ is drawn in the plan. The point Y , at which XZ intersects BC , is projected into the elevation view, where a line is drawn from Z through Y to locate point X .

2-13 PLANE AS AN EDGE

If a plane is to appear as an edge (or a line) in any view, the lines of sight for that view must be parallel to the plane. As stated above (Sec. 2.4), a line appears as a point when the lines of sight are parallel to the line. Obviously, if the lines of sight are parallel to any line on a plane, the lines of sight are also parallel to the plane.

. **Rule:** *A plane must appear as an edge in any view in which a line on the plane appears as a point.*

Since the point view of a line must be projected from a view showing its true length, some line on the plane must appear in true length before an edge view of the plane can be drawn. In Figure 2.19, the P and $E1$ views of plane ABC are given. The horizontal line CX is drawn in view $E1$ and projected to the plan, where it appears in its true length. View $E2$ is then drawn, showing CX as a point and the plane ABC as an edge. Similarly, the edge view is found in an inclined view by drawing, in the plan view, the line AY parallel to the rotation line $\frac{P}{E1}$. Line AY then appears in its true length in view $E1$, and the inclined view $I1$ shows AY as a point and ABC as an edge.

Edge views could also be obtained in views with lines of sight in the directions indicated by the arrows. Note that all true-

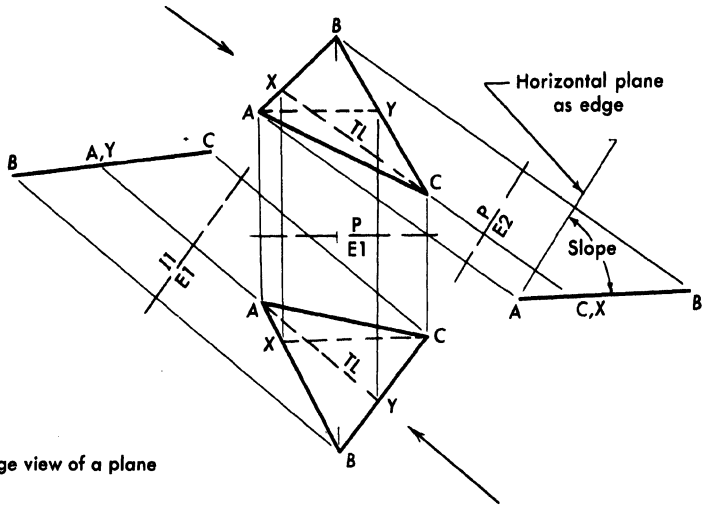


Figure 2-19 Edge view of a plane

length lines on a plane appear parallel in any view except the view that shows the true shape and size of the plane. (See Sec. 2.15.) Therefore, only two edge views of a plane can be projected from any view except a true-shape-and-size view. As will be seen later, all views projected from a true-shape-and-size view are edge views.

2.14 SLOPE OF A PLANE

The slope of a plane is the angle that the plane makes with a horizontal plane. This angle can be measured only if both the given plane and the horizontal plane are seen as edges. Since the horizontal plane appears as an edge only in an elevation view, the slope of a plane can be seen only in an elevation view that shows the plane as an edge. The slope of a plane can be expressed in the same manner as the slope of a line (Sec. 2.5).

In Figure 2.19, the slope of the plane ABC is shown in the elevation view $E2$, in which a horizontal plane appears as an edge. The slope cannot be seen in the inclined view which shows the plane as an edge, because a horizontal plane does not appear as an edge in an inclined view. For emphasis, we repeat, *slope* can be seen *only* in an *elevation* view.

2.15 TRUE SHAPE AND SIZE OF PLANE

A plane will be seen in its true shape and size in any view for which the lines of sight are perpendicular to the plane.

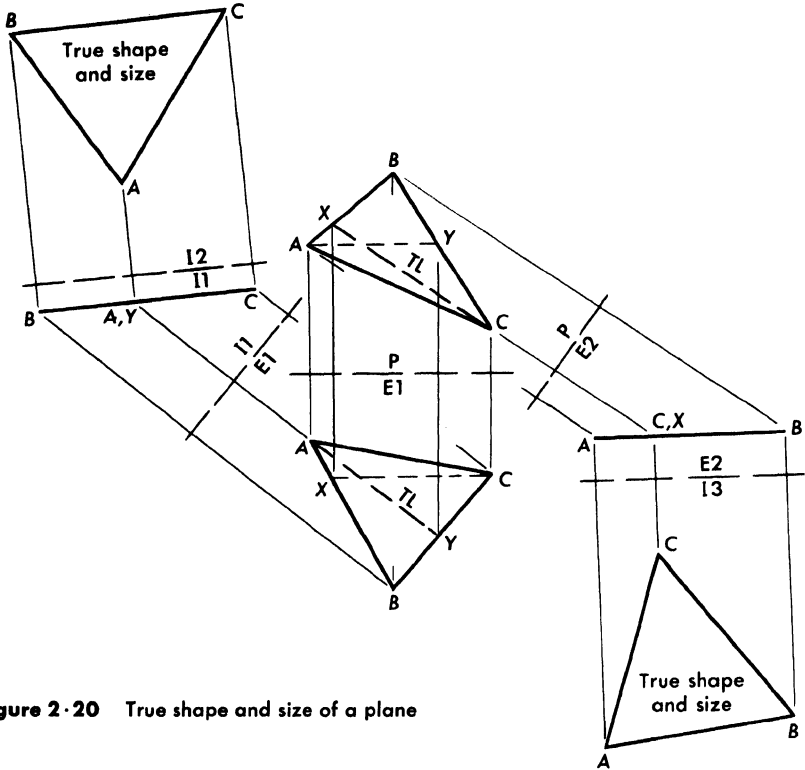


Figure 2.20 True shape and size of a plane

. **Rule:** The view showing the true shape and size of a plane must be projected from a view that shows the plane as an edge.

The method of obtaining the true shape and size of a plane is illustrated in Figure 2.20. The edge view of the plane is first obtained in either an elevation or inclined view by the method given in Section 2.13. A rotation line is then drawn parallel to the plane, and points are projected in the usual manner. The new view thus obtained shows the plane in its true shape and size. In this view, any line on the plane appears in its true length, and any angle on the plane appears in its true size.

2-16 PLANE FIGURES ON PLANES

In many problems, it is necessary to show a figure of a certain size and shape on a given plane. In general, the true shape and size of the plane should first be drawn. The required plane figure, in its true size and shape, is then drawn in this view in the position specified by the conditions of the problem. Any other required views of the plane figure are obtained by projection in the usual manner.

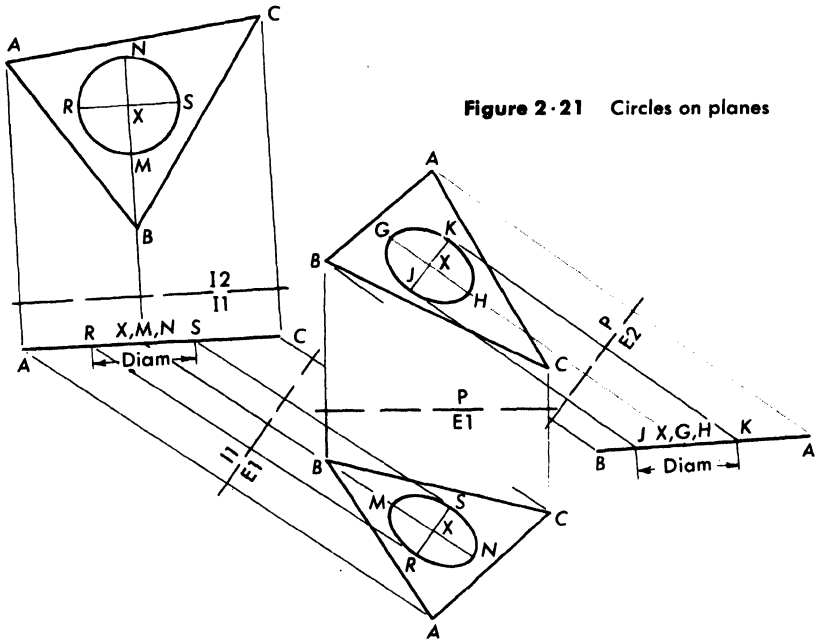


Figure 2-21 Circles on planes

Although the method described above can be used with circles, many points must be projected, making the process laborious and time-consuming. Because a circle appears as an ellipse in any view except the true-shape and edge views, the following special method is easier to use.

An infinite number of diameters may be drawn through the center of a circle. One of these diameters appears true length, and all the others appear foreshortened, in any view except the true-size view. The diameter that appears true length forms the major axis of an ellipse, and the one perpendicular to it appears shortest and forms the minor axis.

In Figure 2.21, a circle of a specified diameter is to be drawn so that its center is at given distances from AB and BC on the given plane ABC . The inclined view $I1$ is drawn to show the given plane

as an edge, and its true shape and size is then found in view I2. The center of the circle, X , is located in this view by construction, and in the other views by projection.

The circle can now be drawn true size in view I2. The diameter MN perpendicular to the rotation line appears as a point in view I1 and true length in view E1. The diameter RS , parallel to the rotation line, appears true length in view I1, and (foreshortened) parallel to the rotation line in view E1. In view E1, MN is the major axis and RS is the minor axis of an ellipse, which may be drawn by the trammel method (Sec. 2.17) or by means of an ellipse template (Sec. 2.18).

To draw the circle in the plan view, the major axis is drawn through point X parallel to any line on the plane that appears true length in this view. The minor axis is perpendicular to the major axis and appears true length in view E2, which shows the plane as an edge. The diameter is laid off in view E2 and projected into the plan to find the length of the minor axis.

. **Note:** The true-size view I2 could be eliminated if the center of the circle were already located.

2-17 ELLIPSE BY TRAMMEL METHOD

When the major and minor axes of an ellipse are known, the ellipse may be easily and quickly drawn by using a **trammel** (Fig. 2.22). Points A and B are located by laying off distances a and b , equal to the semiminor and semimajor axes respectively, along the straight edge of a card or piece of paper, from point O as shown. When the trammel is placed so that A is on the major axis and B is on the minor axis, the corner O of the trammel locates a point on the ellipse. By shifting the trammel along the axes, it is possible to locate sufficient points on which to draw the ellipse with a French curve.

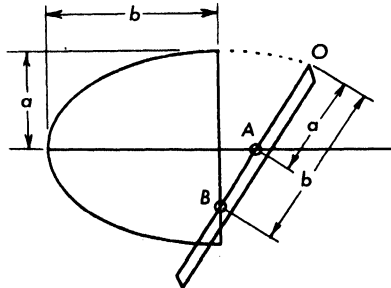


Figure 2-22 Ellipse by trammel method

2.18 ELLIPSE BY ELLIPSE TEMPLATE

Ellipse templates are sheets, usually of transparent plastic, with accurate elliptical holes in a number of different sizes. The exact shape of these elliptical holes is usually designated by the **ellipse angle**. This is the angle between the lines of sight and the edge view of the plane of a circle, as shown in Figure 2.23a. When the major and minor axes are known, the ellipse angle may be determined by construction, as shown in Figure 2.23b. Using the intersection of the axes as a center, X , an arc is drawn with a radius equal to the semimajor axis. Through the

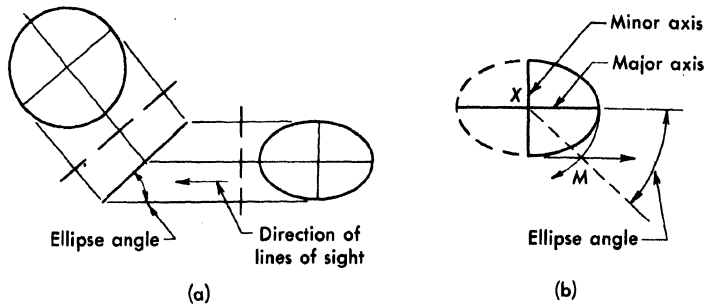


Figure 2.23 Ellipse angle

end of the minor axis, a line is drawn parallel to the major axis, intersecting the arc at M . The angle between the major axis and line XM is the ellipse angle.

Since ellipse templates are usually available with ellipse angles at intervals of 5 degrees, the ellipse is drawn using the ellipse template closest to the actual ellipse angle. For example, if the ellipse angle were found to be 41 degrees, the 40-degree ellipse template would be used.

2.19 SUMMARY

1. A *line* is a straight line segment unless otherwise specified.

2. A *contour line* is a line representing points of equal elevation on the earth's surface.

3. The *true length* of a line can be seen only if the lines of sight are perpendicular to the line.

4. A *point view* of a line must be projected from a true-length view of the line.

5. The *slope* of a line is the angle the line makes with a horizontal plane.

6. The slope of a line can be seen *only* when the line appears *true length* in an *elevation* view.

7. The *bearing* of a line is always measured in the plan view, *from* north or south *toward* east or west.

8. Lines which are parallel in space appear parallel in all orthographic views.

9. Intersecting lines contain a common point.

10. In this text, rectangular coordinates are arranged so that the positive direction of the *x* axis is toward the east; of the *y* axis, toward the north; and of the *z* axis, vertically upward.

11. The position of a plane in space may be defined by two intersecting lines, two parallel lines, a point and a line, or three points not in a straight line.

12. The edge view of a plane will be seen in any view in which any line on the plane appears as a point.

13. The slope of a plane can be seen *only* in an *elevation* view which shows the *plane as an edge*.

14. The view showing true shape and size of a plane must be projected from a view showing the plane as an edge.

15. Circles on planes (in all except true-shape views) can most easily be drawn by locating the major and minor axes.

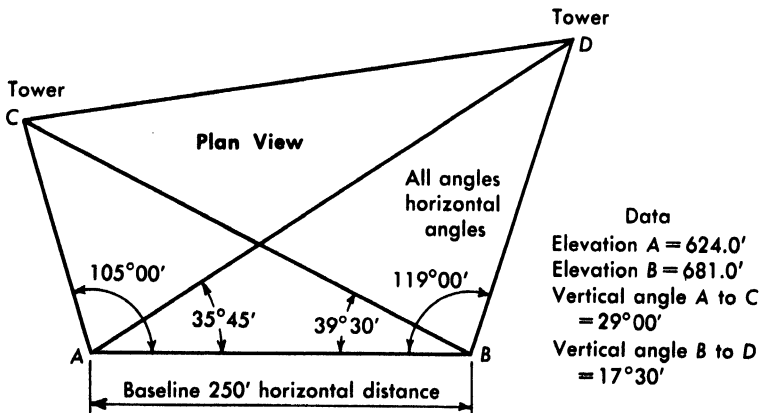
2.20 Problems

- 2-1. A pipe 450 ft long bears due south from A to B on a rising grade of 75 percent. Point C is 60 ft south and 150 ft west of A and is 50 ft lower than B . Point D is 80 ft south and 130 ft east of A and is 30 ft lower than B . From C a pipe bears due east until it intersects AB at E . From D a pipe bears $S 45^\circ W$ until it intersects AB at F . Find the true length and true slope of pipes CE and DF . Scale: 1 in. = 200 ft.
- 2-2. Points A and E on the surface of a mountain have the following coordinates:

Point	x	y	z
A	101,317.6	23,139.4	8,295.0
E	101,502.6	22,889.4	8,130.0

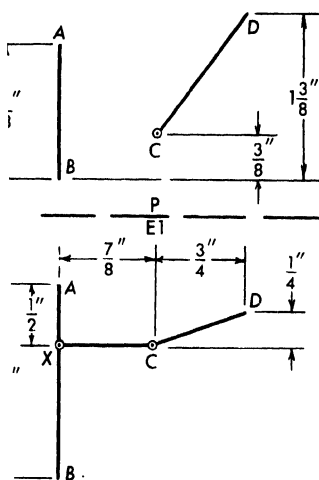
From A to B a tunnel with a true length of 150 ft bears $S 45^\circ W$ on a falling grade of 50 percent. From B to C , a vertical shaft extends downward 50 ft. A tunnel with a true length of 215 ft slopes up 15 degrees on a bearing of $S 15^\circ E$ from C to D . Determine the true length, bearing, and percent slope of a tunnel from the end of the cut at point D to the surface at point E . Scale: 1 in. = 100 ft.

- 2-3. The sketch and data shown were submitted to the office by a survey field party. Using a scale of 1 in. = 100 ft, determine the elevations of towers C and D and the airline distance between them.

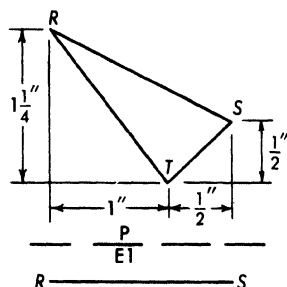


Problem 2-3

- 2-4.** The centerline of a sewer main runs $S 45^\circ E$ between points A and B . The true length of the sewer from A to B is 115 ft and B is 42 ft lower than A . A side sewer is to be constructed from a house at H to a point K , which is on the sewer line and is 75 ft (true length) from A . House H is 70 ft east and 20 ft north of A , and 15 ft higher than A . Find the bearing, true length, and percent grade of the side sewer. Scale: 1 in. = 50 ft.
- 2-5.** A service connection bearing $N 30^\circ W$ is to be constructed from the inlet of a tank to a water main. Point A on the water main is 145 ft north and 30 ft west of the inlet, and its elevation is 55 ft less than the elevation at the inlet. From A , the water main runs $S 40^\circ W$ on a grade of +27 percent. Find the true length and percent grade of the service connection, and the distance from A to the point where it strikes the water main. Scale: 1 in. = 50 ft.
- 2-6.** Using only the two given views, locate point X in the plan view. Scale: Full size.

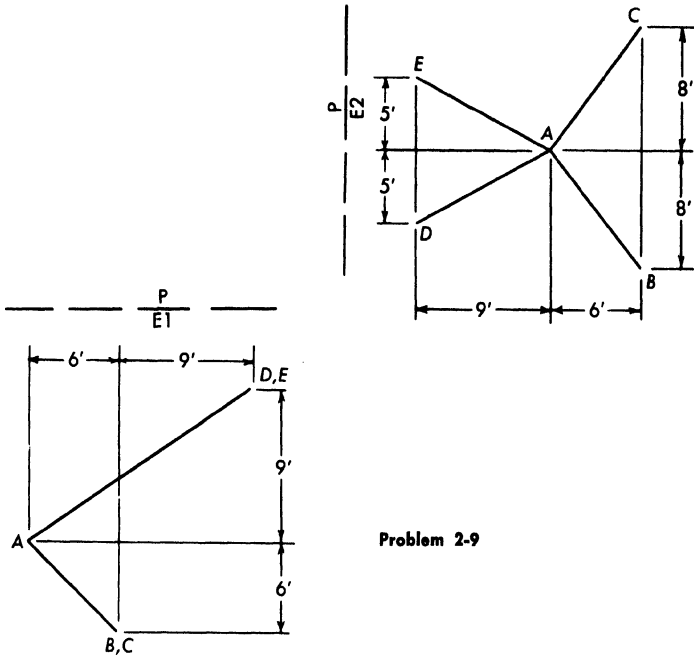


Problem 2-6

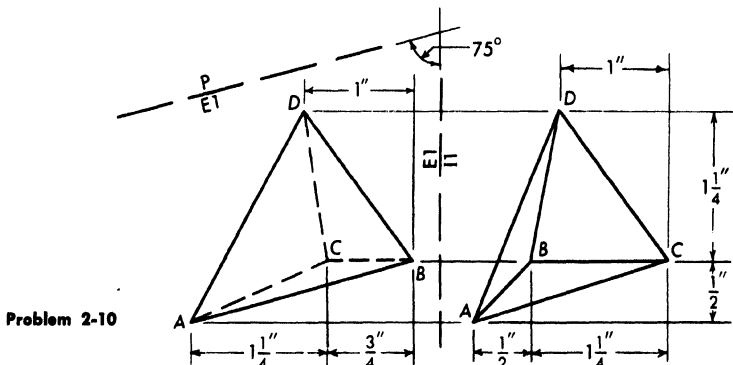


Problem 2-7

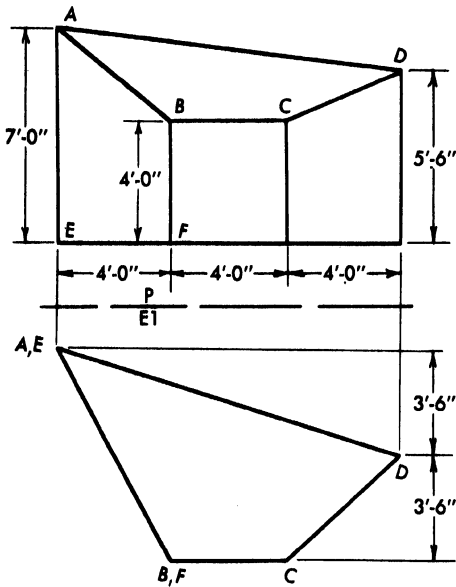
- 2-7.** The plane RST has a true slope of 60 degrees. Complete view $E1$, and find the true shape and size of the plane. Scale: Full size.
- 2-8.** Two tubes AB and BC are to be connected by a special fitting at B . Point B is 8 ft east of A and 5 ft north of C . The slopes of AB and BC are $+30^\circ$ and $+45^\circ$, respectively. Find the true size of the angle for the special fitting. Scale: $\frac{1}{4}$ in. = 1 ft 0 in.



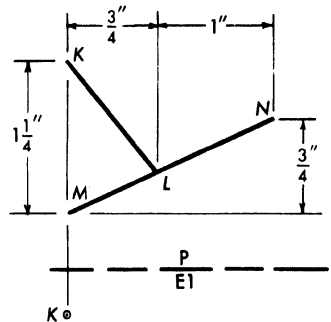
- 2-9.** Two elevation views of an A frame are given. Find and mark clearly (a) true size and true slope of plane BAC , (b) true slope of member AB , (c) true size and true slope of plane BAD , and (d) true length of member AC . Scale: $\frac{1}{8}$ in. = 1 ft 0 in.
- 2-10.** Draw the plan view and find (a) true length of line CD , (b) true slope of line AD , (c) true size of plane ABD , and (d) true slope of plane ACD . Give slopes in degrees and true lengths in inches. Scale: Full size.



- 2-11.** Plan and elevation views of a hopper are given. Find (a) true length of corner AB , (b) true slope of corner AD , (c) true slope of face $ABEF$, and (d) true size of face $ABCD$. Give slopes in degrees and true lengths in feet and inches. Scale: $\frac{1}{4}$ in. = 1 ft 0 in.



Problem 2-11

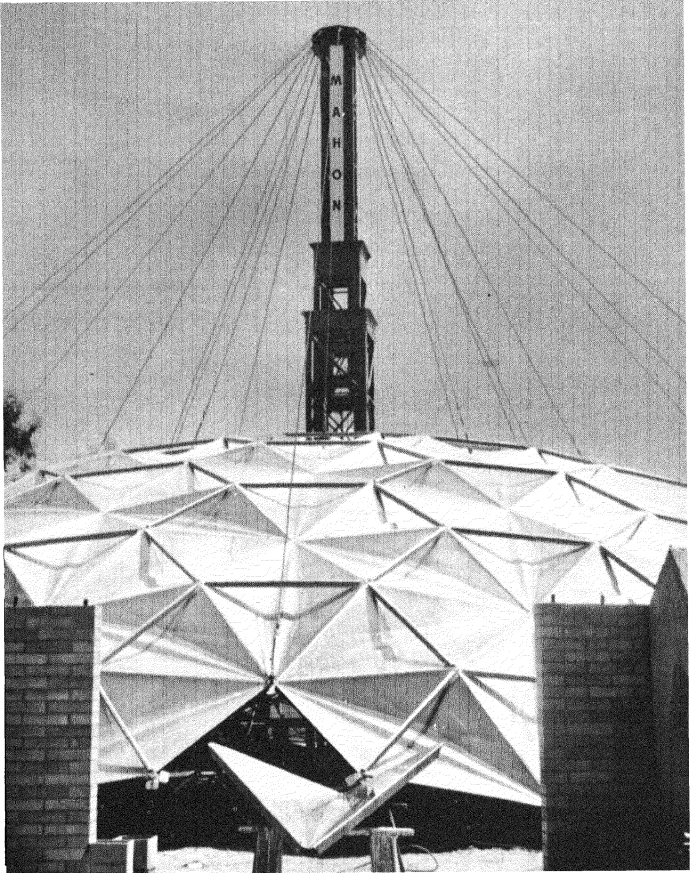


Problem 2-12

- 2-12.** Lines KL and MN in the plan view and point K in view $E1$ are given. Line KL is perpendicular to line MN , intersects it at L , and is $1\frac{1}{4}$ in. long. Show both lines in view $E1$. Scale: Full size.
- 2-13.** A shaft 2 ft 9 in. long, bearing $N 30^\circ E$, with a true slope of -45 degrees, is given. A 2-ft diameter disk is mounted with its center at the midpoint of the shaft. The plane of the disk is perpendicular to the shaft. (a) Draw the shaft in the plan view and in an elevation view looking due north. (b) Show the disk in these two views by locating the minor and major axes and using the trammel method. Scale: 1 in. = 1 ft 0 in.
- 2-14.** Use the same data and scale as for Problem 2-11. Show in the P and $E1$ views the largest circle that can be drawn on face $ABCD$.

CHAPTER 3

THE 210-FT DIAMETER KAISER ALUMINUM DOME BEING ERECTED FOR
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CHAPTER

3 Point, Line, and Plane Problems

*The principles and methods used to solve problems
involving various relationships among points,
lines, and planes*

3.1 IMPORTANT PRINCIPLES

The student must thoroughly understand the following principles regarding lines and planes if he wishes to analyze properly the methods for solving problems. (Figure 3.1 shows two 45-degree triangles, one horizontal and the other inclined. The circled letters in the plan view refer to the corresponding letters of the principles stated below.)

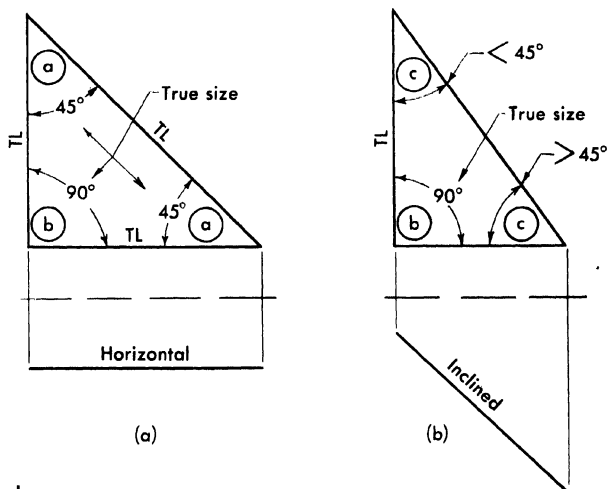


Figure 3.1 Projection of angles

(a) The true size of any angle (except a right angle) formed by two intersecting lines will be seen only in the view showing both lines in true length (Fig. 3.1a).

(b) When two lines intersect at right angles, the true size of the right angle will appear in any view showing either of the lines in true length (unless the other line appears as a point) (Fig. 3.1a, b).

(c) In an orthographic view, an angle may appear larger or smaller than its true size (Fig. 3.1b).

(d) Lines that are parallel in space must appear parallel in all orthographic views.

(e) Lines are parallel in space if they appear parallel in two adjacent views, *unless* they appear *perpendicular to the rotation line* between the two views.

(f) In any view except a true size view, all lines on a plane that appear in true length are parallel to each other.

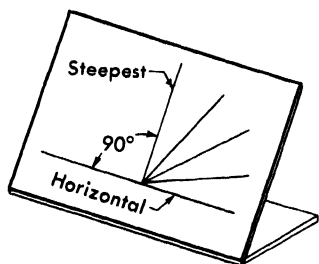


Figure 3.2 Slopes of lines on a plane

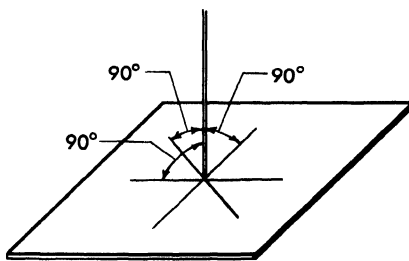


Figure 3.3 Line perpendicular to a plane

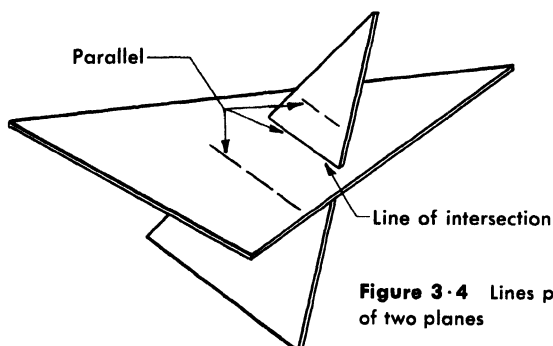


Figure 3.4 Lines parallel to the line of intersection of two planes

(g) Lines drawn on a plane may have slopes varying from zero to a maximum that is equal to the slope of the plane (Fig. 3.2).

(h) The steepest line on a plane has the same slope as the plane, and is perpendicular to a horizontal line on the plane (Fig. 3.2).

(i) Any two lines on a plane must either intersect or be parallel.

(j) A line that is perpendicular to a plane is perpendicular to all lines on the plane (Fig. 3.3).

(k) In any view, a line perpendicular to a plane appears at right angles to any line on the plane that shows in its true length in that view. (See (b) above.)

(l) A line is parallel to a plane if it is parallel to any line on the plane.

(m) The line of intersection of two planes is a straight line that is common to both planes (Fig. 3.4).

(n) Any two lines that are parallel to a third line are parallel to each other. Therefore, any two lines on different intersecting planes can be parallel only if they are parallel to the line of intersection of the two planes (Fig. 3.4).

(o) Two lines on different intersecting planes can intersect only on the line of intersection of the two planes.

3.2 SHORTEST CONNECTION FROM A POINT TO A LINE

The shortest connection from a point to a line is perpendicular to the line. Its location and true length may be found by either of the following methods:

Line Method

The shortest connection from a point to a line must appear perpendicular to the line in any view showing the true length of the line (Sec. 3.1b). The true length of the shortest connection must appear in a view showing the line as a point.

In Figure 3.5, the point C and the line AB are given in views P and $E1$. View $E2$ is drawn to show the true length of AB . The shortest connection, CX , is located in this view by drawing a line from C perpendicular to AB . The view $I1$ shows AB as a point and the true length (but not the slope) of the shortest connection, XC . The true slope is found in view $E3$, rotation line $\frac{E3}{P}$ being parallel to XC .

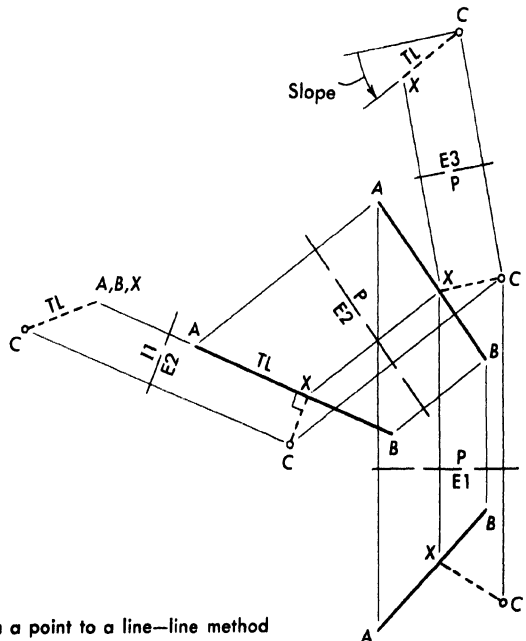


Figure 3.5 Shortest connection from a point to a line—line method

Plane Method

The point C and the line AB form a plane. The true shape view of this plane will show the true length of the shortest connection.

The lines AC and BC are drawn (Fig. 3.6) to form a triangular plane, which is seen as an edge in view $E2$. The position and true length of the shortest connection, CX , are found in the true-shape-and-size view, $I1$, by drawing a line from C perpendicular to AB . As in the line method, an additional view, $E3$, is required to find the slope.

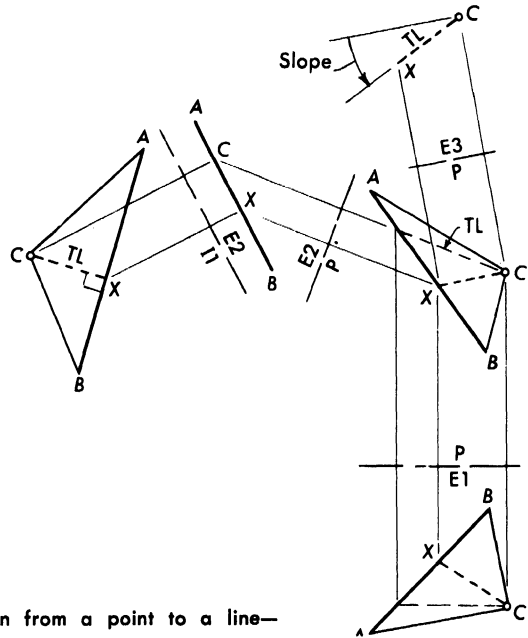


Figure 3-6 Shortest connection from a point to a line—plane method

3-3 STEEPEST CONNECTION FROM A POINT TO A LINE

The steepest connection from a point to a line is the steepest line that can be drawn on the plane formed by the point and the line.

Figure 3.7 shows plane ABC on which a horizontal line BX is drawn. Since the steepest line on a plane is perpendicular to any horizontal line on the plane (Sec. 3.1h), CY must make a right angle with BX in the plan view. The true length and slope of the steepest connection appears in the elevation view $E2$, showing the plane as an edge. Note that the slope is the same as the slope of the plane.

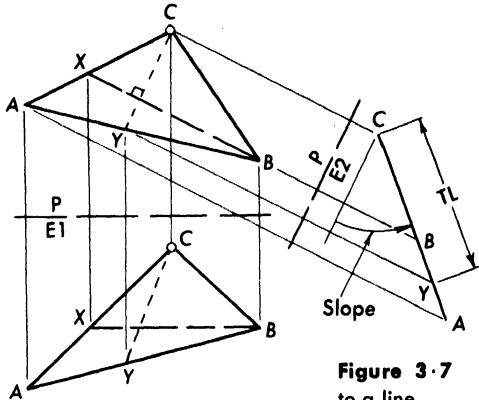


Figure 3.7 Steepest connection from a point to a line

3.4 SHORTEST DISTANCE BETWEEN TWO LINES

The shortest distance between any two lines must be measured perpendicular to both lines.

Intersecting Lines

Since intersecting lines have a common point, the shortest distance between them is zero.

Parallel Lines

There are two ways of finding the shortest distance; as follows:

- (1) Draw a view showing both lines as points.
the two lines.

Nonintersecting, Nonparallel Lines

The shortest distance may be found by either of the following methods:

Line Method. Since the shortest connection is perpendicular to both lines, it must appear true length in any view that shows either line as a point, and, in this same view, it must appear perpendicular to the other line (Sec. 3.1b).

In Figure 3.8, the lines AB and CD are given in views P and $E1$. View $E2$ is drawn to show CD in true length, and view $I1$ to show CD as a point. View $I1$, having lines of sight parallel to CD , will show the true length of any line that is perpendicular to CD . Since the shortest connection is perpendicular to both AB and CD , it is drawn at right angles to AB in view $I1$ to locate point S . Point S is then projected into view $E2$, where a line is drawn at 90 degrees to CD to locate point R . The shortest connection, RS , may then be projected into the given views, and its bearing can be measured in the plan. If the slope of this connection is required, a new elevation view ($E3$) must be drawn.

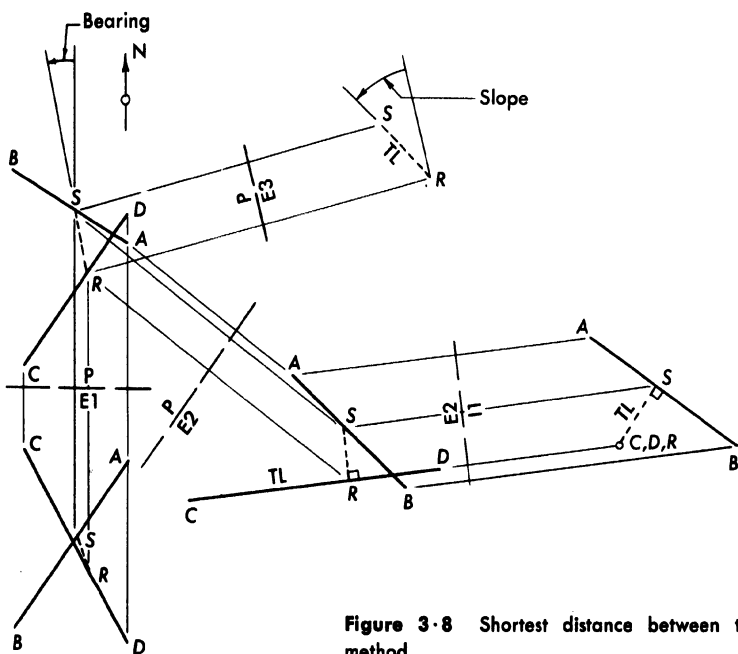


Figure 3.8 Shortest distance between two lines—line method

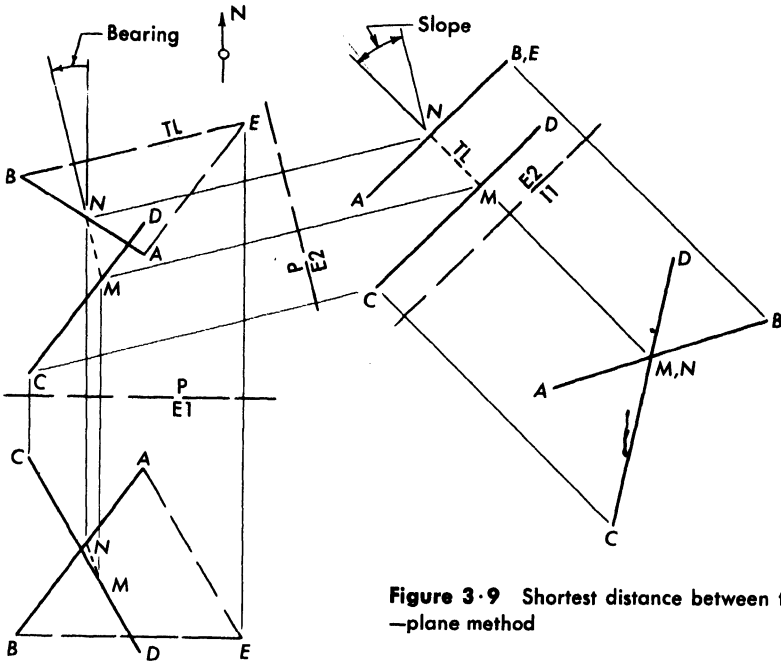


Figure 3.9 Shortest distance between two lines
—plane method

Plane Method. If a plane contains one line and is parallel to another line, the shortest connection between the lines, being perpendicular to both of them, must be perpendicular to the plane. Therefore any view showing this plane as an edge shows the true length of the shortest connection. If the edge view is an elevation view, it will also show the slope of the shortest connection.

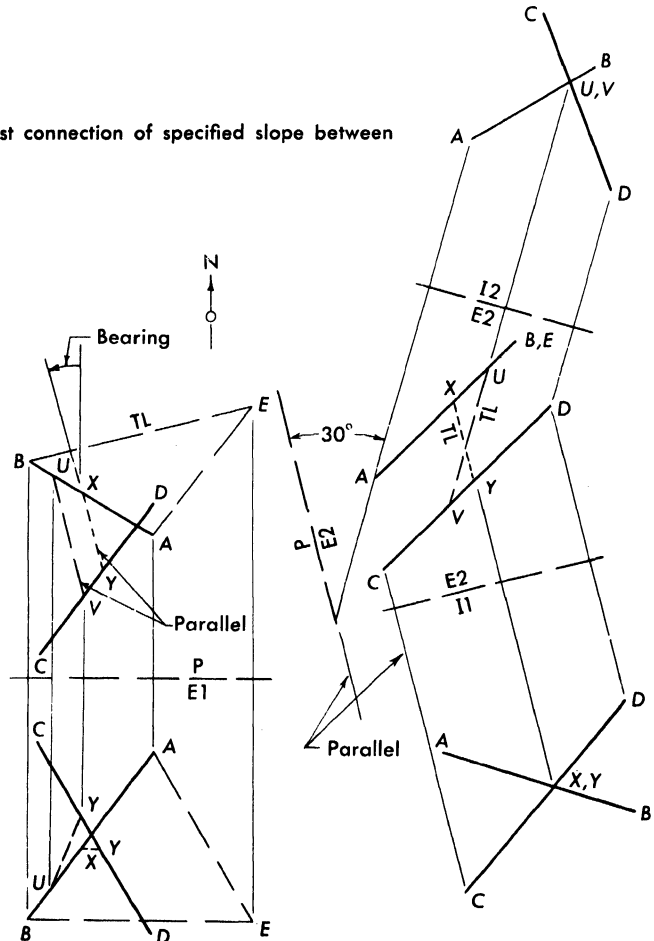
In Figure 3.9, the same lines AB and CD are given in views P and $E1$ as in the preceding example. First, a plane is constructed so that it contains one line and is parallel to the other. To do this, line AE is drawn parallel to CD in both given views, thus making AE and CD parallel in space (Sec. 2.8). The intersecting lines AB and AE form a plane. The line CD is parallel to this plane, since it is parallel to the line AE on the plane (Sec. 3.11). View $E2$ is drawn, showing the plane as an edge and the line CD parallel to the plane. The true length of the shortest connection can now be measured, since it is the perpendicular distance from CD to the plane. However, in order to determine the location of this shortest connection, an additional view must be drawn. View $I1$ is drawn with lines of sight perpendicular to the plane, thus showing both AB and CD in true length and the shortest connection as a point. The apparent

point of intersection of the lines AB and CD in this view locates the shortest connection, MN , which can then be projected back into the other views. Note that the slope of this connection can be seen in view $E2$, and that its bearing can be measured in the plan.

3.5 SHORTEST CONNECTION, OF SPECIFIED SLOPE, BETWEEN TWO LINES

Since the line method depends on perpendicularity of lines, it cannot be used to find the shortest connection of specified slope.

Figure 3.10 Shortest connection of specified slope between two lines



However, the shortest connection of any specified slope may be found by the plane method.

In Figure 3.10, the same lines AB and CD are given in views P and E1 as were given in Figures 3.8 and 3.9. By the method given in Section 3.4, the plane AEB is constructed parallel to CD . View E2 shows this plane as an edge, with line CD parallel to it.

Suppose a slope of zero is specified, thus requiring the location of the shortest horizontal connection. In view E2, any number of horizontal lines could be drawn, at different elevations, between the two given lines. Since the *apparent* length of all these horizontal connections is the same in this view, the one that appears in its true length must be the shortest (Sec. 2.3). We determine its position by drawing view I1, with horizontal lines of sight. In view I1 the shortest horizontal connection, XY , appears as a point at the apparent intersection of AB and CD . The connection XY can then be projected back into the other views.

Now, suppose that the shortest connection with an upward slope of 30 degrees from AB to CD is required. The above procedure is followed except that view I2 is drawn with lines of sight making an angle of 30 degrees with the $\frac{P}{E2}$ rotation line. The apparent point of intersection of AB and CD in view I2 locates this shortest 30-degree connection, UV , since it must appear true length in view E2. The shortest connection of any other specified slope may be found in a similar manner.

Note that UV and XY must be parallel in the plan view, because they are both true length in view E2. For the same reason, they are also parallel to the shortest connection, MN , shown in Figure 3.9. Thus it can be seen that all "shortest" connections between two given lines appear parallel in the plan. In other words, they all have the same bearing.

3.6 LINE PIERCING A PLANE

A straight line will intersect a plane at only one point, called the **piercing point**, which may be found by either of the following methods:

Edge-view Method

The point where a line pierces a plane is readily apparent in any view that shows the plane as an edge.

In Figure 3.11, the plane ABC and the line XY are given in views P and $E1$. View $E2$ is drawn to show the plane ABC as an edge. In this view, XY pierces the plane ABC at point P , which is then projected into the given views. The piercing point could have been found just as easily by projecting an inclined view from view $E1$ to show the plane as an edge.

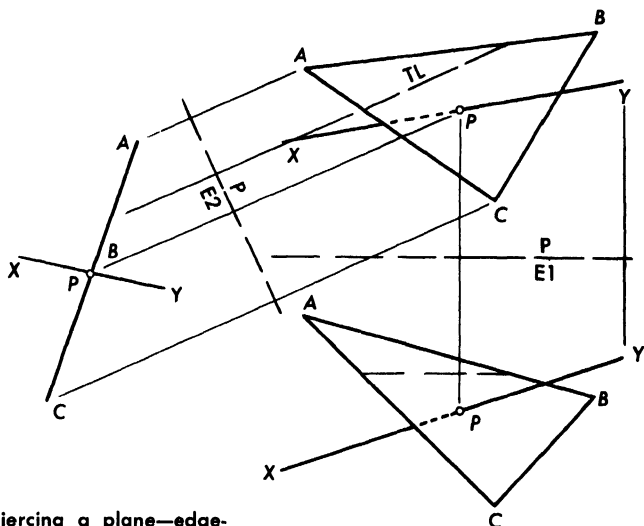


Figure 3.11 Line piercing a plane—edge-view method

Cutting-plane Method

If a line in one plane pierces another plane, it must do so on the line of intersection of the two planes.

Figure 3.12 illustrates the use of a vertical cutting plane to determine the piercing point P of line XY with plane ABC . The cutting plane contains XY and intersects the given plane along line FG . The intersection of FG with XY is the desired piercing point.

In Figure 3.13, the line XY and the plane ABC are given in the P and $E1$ views. A vertical plane that appears as an edge in the plan view is passed through XY and cuts AC at F and BC at G . Points F and G are two points on the line of intersection of the vertical cutting plane and the plane ABC . Points F and G are projected into view $E1$ and connected by a line that crosses XY at the piercing point P .

Next, a cutting plane that appears as an edge in view $E1$ is passed through line XY , cutting AC at H and AB at K . Points H

and K are projected into the plan and connected by a line that crosses XY at the piercing point P . The solution is then checked by projection between the piercing points in the two views.

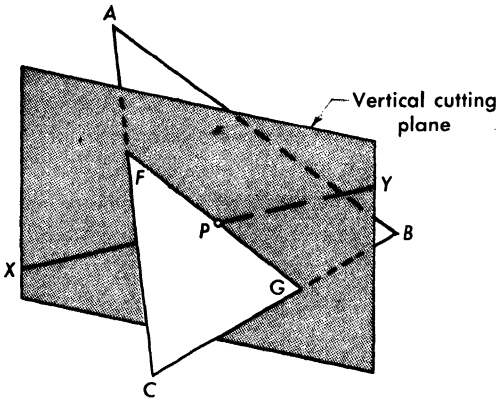


Figure 3-12 Use of a cutting plane in finding a piercing point

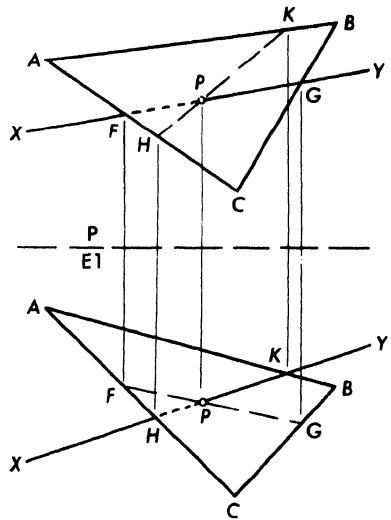


Figure 3-13 Line piercing a plane—cutting-plane method

3-7 SHORTEST CONNECTION FROM A POINT TO A PLANE

The shortest connection from a point to a plane is a line perpendicular to the plane, which may be found by either of the following methods:

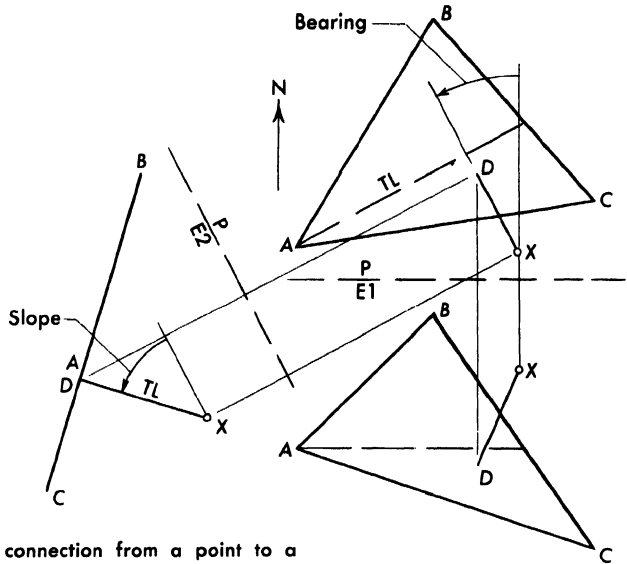


Figure 3-14 Shortest connection from a point to a plane—edge-view method

Edge-view Method

Any line that is perpendicular to a plane will appear at right angles to the plane in any view showing the plane as an edge. This view will show the true length of the perpendicular, or shortest, connection. If the edge view is obtained in an elevation view, it will also show the slope of the shortest connection.

In Figure 3.14, the plane ABC and the point X are given in the P and $E1$ views. View $E2$ shows the plane as an edge. In this view, line XD is drawn from X perpendicular to the plane, and its true length and slope can be measured. Since XD is true length in view $E2$, it must be parallel to the rotation line in the plan view. The piercing point D can now be located in the plan and $E1$ views by projection. The bearing is measured in the plan view.

Two-view Method

In any view, a line perpendicular to a plane appears at right angles to any line on the plane that is shown in its true length in that view (Sec. 3.1k).

Thus the *direction* of the shortest connection in any view is determined. The piercing point can then be determined by either of the methods described in Section 3.6.

In Figure 3.15, the plane ABC and the point X , given in the P and E1 views, are the same as in the preceding example. The horizontal line AY is drawn in view E1 and projected to the plan, where it appears in its true length. The shortest connection is drawn from X perpendicular to this line in the plan view. Similarly, the line CZ is drawn in the plan parallel to the rotation line so that it will appear true length in view E1. In this view, the shortest connection is drawn from X perpendicular to CZ . The *direction* of the shortest connection has now been determined in *both* views. The point D , where the connection pierces the plane, is then determined by the cutting-plane method (Sec. 3.6).

3-8 SHORTEST CONNECTION, OF SPECIFIED SLOPE, FROM A POINT TO A PLANE

The shortest connection of any specified slope from a given point to a given plane lies in a vertical plane that is perpendicular to the given plane and contains the given point. In the plan view, this vertical plane appears as an edge at right angles to any horizontal line on the given plane. An elevation view showing the given plane as an edge

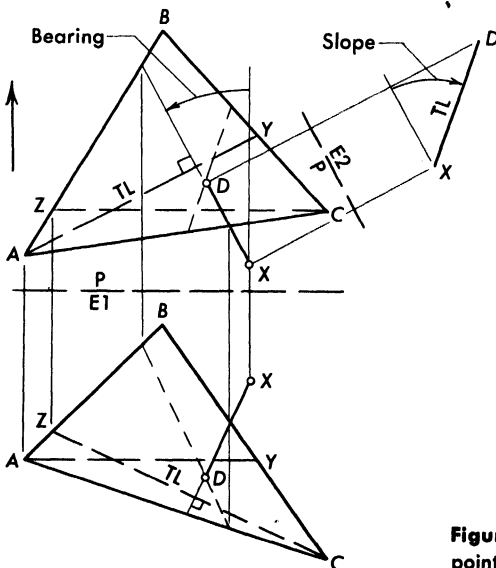


Figure 3-15 Shortest connection from a point to a plane—two-view method

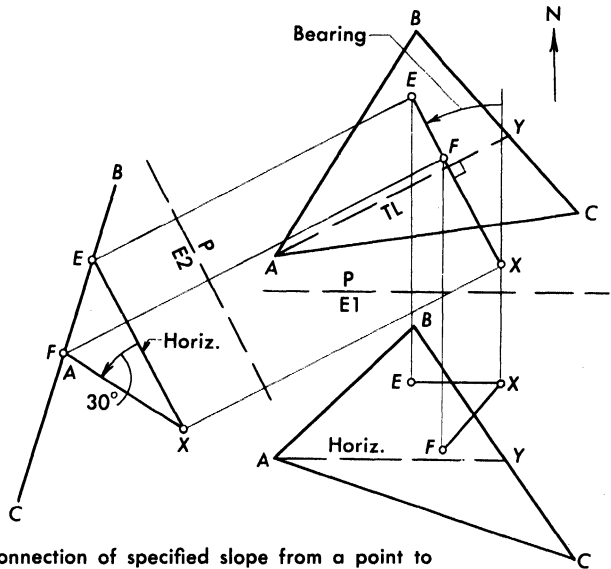


Figure 3.16 Shortest connection of specified slope from a point to a plane

shows this vertical plane in its true size. In this view, a line of any specified slope drawn from the given point to the given plane determines the piercing point.

In Figure 3.16, the same data is given as in Figures 3.14 and 3.15. View E2 is drawn showing plane ABC as an edge. In this view, the shortest *horizontal* connection, XE , and the shortest connection with a 30-degree slope, XF , are shown. In the plan view, both XE and XF appear at 90 degrees to the horizontal line AY on plane ABC . Both of these lines therefore have the same bearing as the shortest possible connection (Figs. 3.14, 3.15).

All shortest connections, whatever their slope, from a given point to a given plane must have the same bearing, since they all lie in the same vertical plane.

3.9 PROJECTION OF A LINE ON A PLANE

When projectors are drawn from a line perpendicular to a plane, the line connecting their piercing points forms the projection of the line on the plane. Thus any orthographic view of a line is the projection of that line on an image plane. Any line may be projected on a plane that is not perpendicular to the line of sight by applying the principles of Section 3.7.

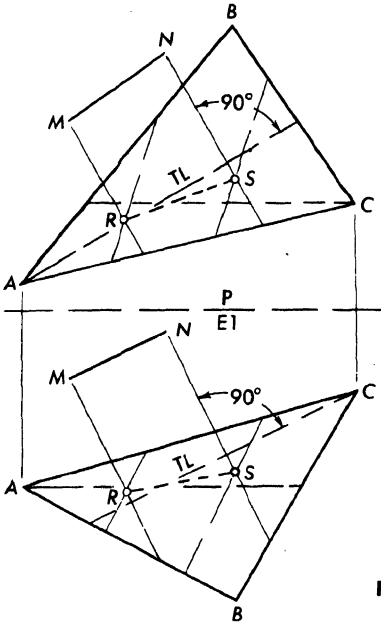


Figure 3.17 Projection of a line on a plane

In Figure 3.17, the plane ABC and the line MN are given in the P and $E1$ views. In each view, lines are drawn from M and N at right angles to true-length lines on the plane, and the piercing points, R and S , are found by the cutting-plane method. The line RS is the projection of the line MN on plane ABC . The projection of MN can also be found by an edge view of plane ABC , and perpendiculars from MN to the plane, as described in Section 3.7.

3.10 PLANE PERPENDICULAR TO A LINE

A plane that is perpendicular to a line will appear as an edge in any view showing the true length of the line. Therefore, a plane may be constructed perpendicular to a line by obtaining a true-length view of the line and drawing, at right angles to it, a line representing an edge view of the plane.

A plane may be drawn perpendicular to a line in two given views by applying the reverse of the principle used to draw a line perpendicular to a plane. In other words, when a plane is perpendicular to a given line, any line on the plane that appears true length in any view must be at right angles to the given line in that view.

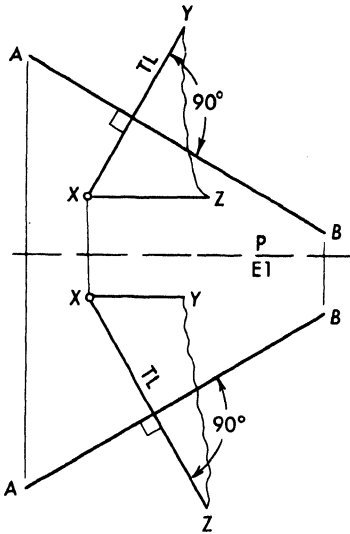


Figure 3-18 Plane perpendicular to a line

Figure 3.18 shows a plane containing the given point X and perpendicular to the given line AB . The line XY is drawn parallel to the rotation line in view $E1$ and perpendicular to AB in the plan view, where it appears true length. Similarly, the line XZ is drawn parallel to the rotation line in the plan view and perpendicular to AB in view $E1$. The two intersecting lines XY and XZ form a plane that is perpendicular to the line AB .

3-11 ANGLE A LINE MAKES WITH A PLANE

The angle a line makes with a plane is defined as the smallest angle between the line and the plane. Therefore it must be measured in a plane that is perpendicular to the given plane.

. **Rule:** *The angle a line makes with a plane can be measured only when the line appears in its true length and the plane appears as an edge in the same view.*

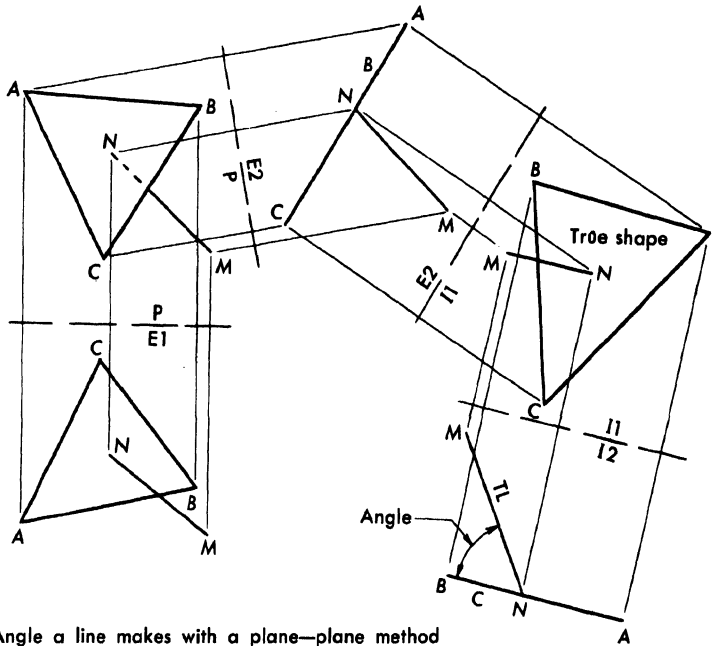


Figure 3-19 Angle a line makes with a plane—plane method

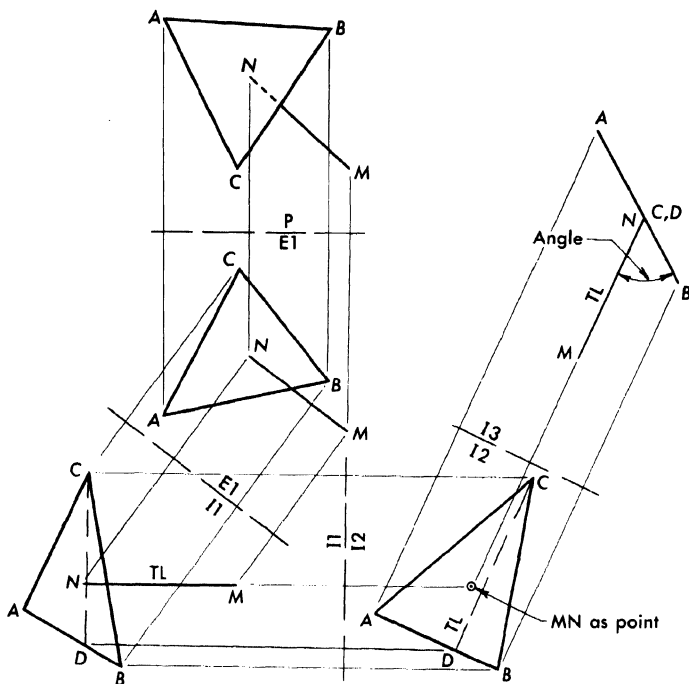
Plane Method

Although edge views of the plane can be obtained from any given view, it is unlikely that the line will appear in its true length in these views. However, all views projected from the true-shape-and-size view of a plane will show the plane as an edge, and two of these views will also show the true length of the line.

In Figure 3.19, the plane ABC and the line MN are given in the P and $E1$ views. View $E2$ shows ABC as an edge, and view $I1$ shows it in true shape, line MN being in both views. View $I2$ is then drawn with lines of sight perpendicular to MN . In this view, MN is true length, plane ABC is an edge, and the true size of the angle between MN and the plane can be measured.

Line Method

All views projected from the point view of a line will show the line in true length. Two of these views will also show the given plane as an edge.

Figure 3·20 Angle a line makes with a plane—line method

In Figure 3.20, the plane ABC and the line MN are given in the P and $E1$ views. View $I1$ is drawn to show MN in true length, and view $I2$ to show MN as a point. In view $I1$, line CD on the plane is drawn parallel to the rotation line $\frac{I1}{I2}$. Line CD therefore appears true length in view $I2$, and view $I3$ will then show CD as a point. In this view, the plane ABC is an edge, line MN is true length, and the angle between MN and ABC is the desired angle.

The true size of the angle a line makes with a plane can also be found by projecting the line onto the plane and then constructing the true size of the plane formed by the line and its projection on the plane.

3.12 LINE OF INTERSECTION OF TWO PLANES

The line of intersection of two planes is a line common to both planes. It may be found by any of the following three methods:

Edge-view Method

The fundamental procedure for finding the line of intersection of two planes is to draw a view showing one of the planes as an edge. In this view, the line of intersection coincides with the edge view, and its position on the other plane is clearly evident.

In Figure 3.21, the planes ABC and DEF are given in the P and E1 views. Both planes are projected into view E2, which shows plane DEF as an edge. In this view, the plane DEF crosses line BC at R and line AC at S . The line RS is the line of intersection between the two planes, and it is then projected into the given views. Note that this is the method used in locating piercing points to find the line of intersection between the cutting plane and the given plane (Sec. 3.6).

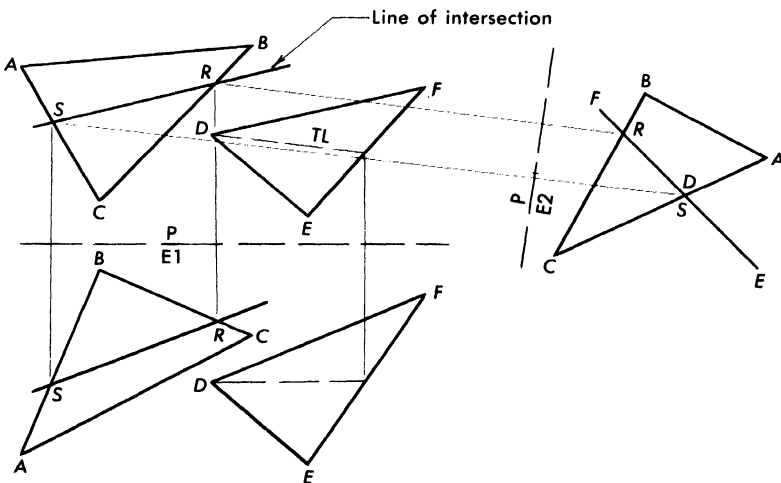


Figure 3.21 Intersection of planes—edge-view method

Piercing-point Method

An arbitrary line in one plane pierces another plane on the line of intersection of these two planes. Therefore any two such piercing points will determine the location of the line of intersection of the two planes.

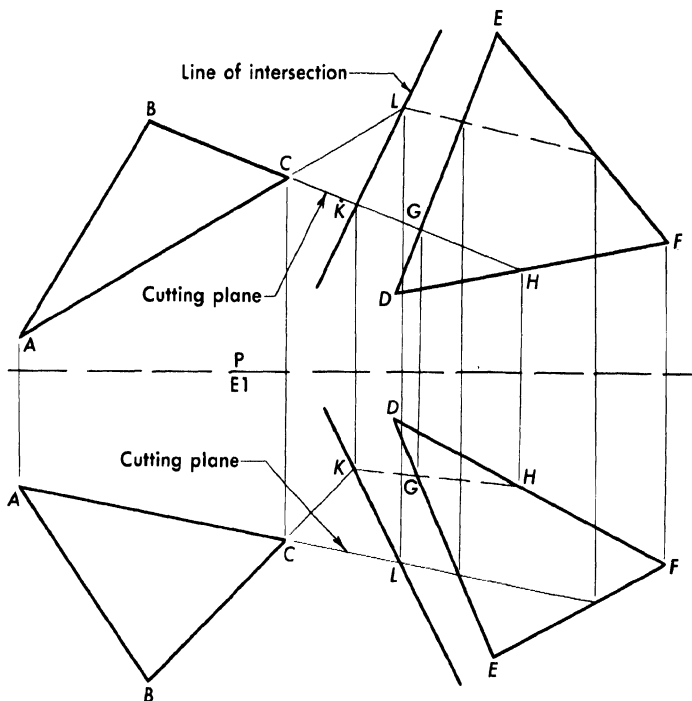


Figure 3-22 Intersection of planes—piercing-point method

In Figure 3.22, the planes ABC and DEF are given. A vertical cutting plane is passed through line BC in the plan view, cutting the line DE at G and the line DF at H . In view $E1$, the line GH intersects the extension of line BC at K , the point at which BC pierces plane DEF . Point K is then projected into the plan view. Similarly, a cutting plane appearing as an edge in view $E1$ is passed through line AC to locate the piercing point L . A line drawn through points K and L in both views represents the line of intersection. The piercing point of any other line on either plane could have been used with the same result.

Cutting-plane Method

When any two given planes are intersected by a third plane, the two lines of intersection thus formed must be either intersecting or parallel. If the lines intersect, they must meet on the line of intersection of the two given planes (Fig. 3.23a). If the lines are parallel, either they must be parallel to the line of intersection of the two given planes (Fig. 3.23b), or the two given planes must be parallel to each other.

Any two such cutting planes can be used to determine the line of intersection of two given planes.

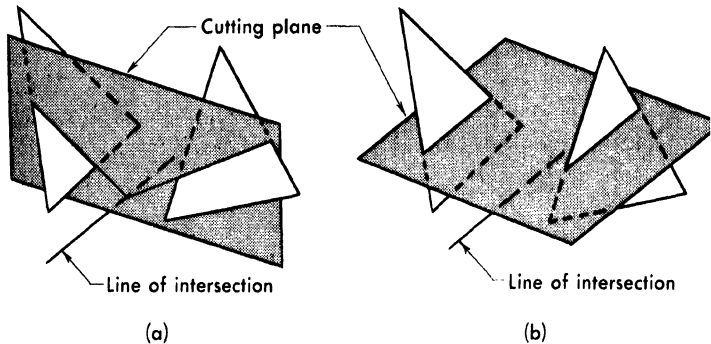


Figure 3-23 Use of cutting planes to find line of intersection

In Figure 3.24, the planes ABC and DEF are given. A vertical cutting plane ($CP1$) is drawn in the plan view so that it intersects plane ABC along the line GH and the plane DEF along the line JK . In view $E1$, the lines GH and JK intersect at the point R . Point R is then projected into the plan view, where it is on the edge view of the cutting plane. Similarly, a cutting plane ($CP2$) that appears as an edge in view $E1$ intersects ABC along line LM and DEF along line NF . In the plan view, lines LM and NF intersect at S , which is projected into view $E1$. Points R and S are both on the line of intersection of the given planes, since they are on each of the given planes (or in this case, the given planes extended). Although these two points are sufficient to locate the line of intersection, it is always advisable to check the line by finding a third point. All three points must lie in a straight line in both views.

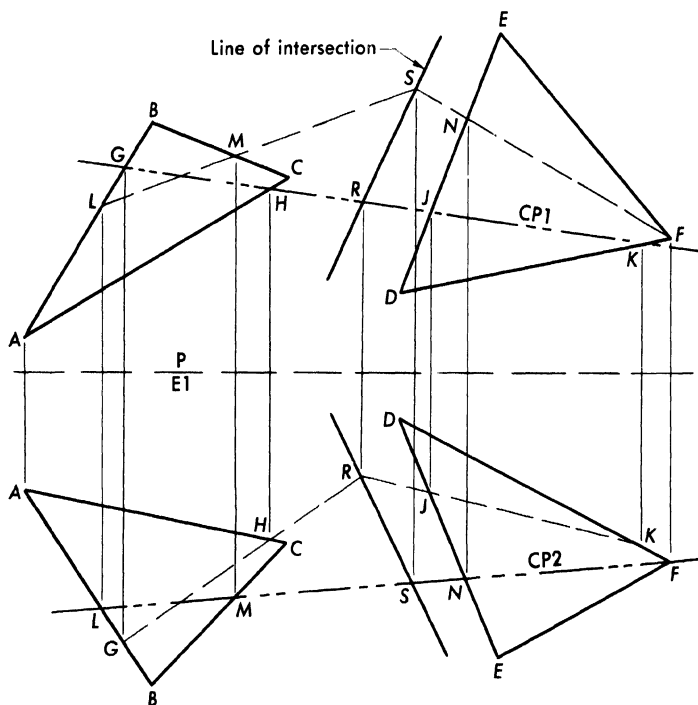


Figure 3-24 Intersection of planes—cutting-plane method

Additional cutting planes can be drawn in any position in either of the given views. If a cutting plane should cut lines (similar to GH and JK) that are parallel to each other, the cutting plane would be parallel to the line of intersection of the given planes. Although this would determine the *direction* of the line of intersection, another cutting plane, not parallel to this one, would be required to find its location. If the second cutting plane thus drawn again gave parallel lines, the two given planes would be parallel to each other.

Note that the edge-view method is actually used in all three methods. In the piercing-point and cutting-plane methods, the intersection of the cutting planes with the given planes was found by the edge-view method. Furthermore, in the cutting-plane method, the point R , for example, is the point at which GH pierces plane DEF and also the point at which JK pierces plane ABC . All three methods, then, are very closely related and are given separately only because one procedure may be easier to use in certain situations than either of the others.

3.13 ANGLE BETWEEN TWO PLANES

The true size of the angle between two intersecting planes can be seen only in a view which shows both planes as edges. There are two commonly used methods for finding this angle, often called the **dihedral angle**.

Line-of-intersection Method

Two intersecting planes will both appear as edges in a view that shows their line of intersection as a point.

In Figure 3.25, the planes ABC and BCD are given in the P and E1 views. View E2 is drawn showing the line of intersection BC in its true length. View I1 is then drawn to show BC as a point. In this view, both planes appear as edges and the dihedral angle is measured as shown. In cases where the line of intersection is not given, it will of course be necessary first to find the line of intersection, using any of the methods of Section 3.12.

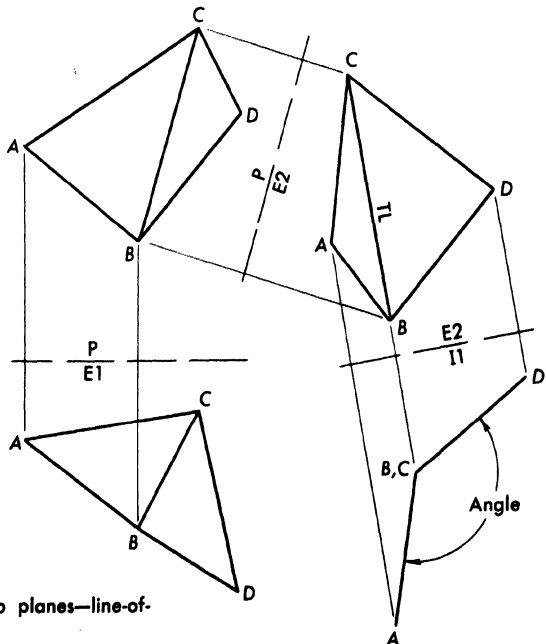


Figure 3.25 Angle between two planes—line-of-intersection method

True-size-of-plane Method

Any view projected from the true-shape-and-size view of a plane will show the plane as an edge. Also, an edge view of a plane may be projected from *any* view of the plane. Therefore, when a view shows one of the planes in true shape and size, another view may be projected from it to show both planes as edges.

In Figure 3.26, the planes *ABC* and *DEF* are given in the *P* and *E1* views. Since *DEF* appears as an edge in the plan view, view *E2* is drawn, showing *DEF* in true shape and size. A line is drawn from

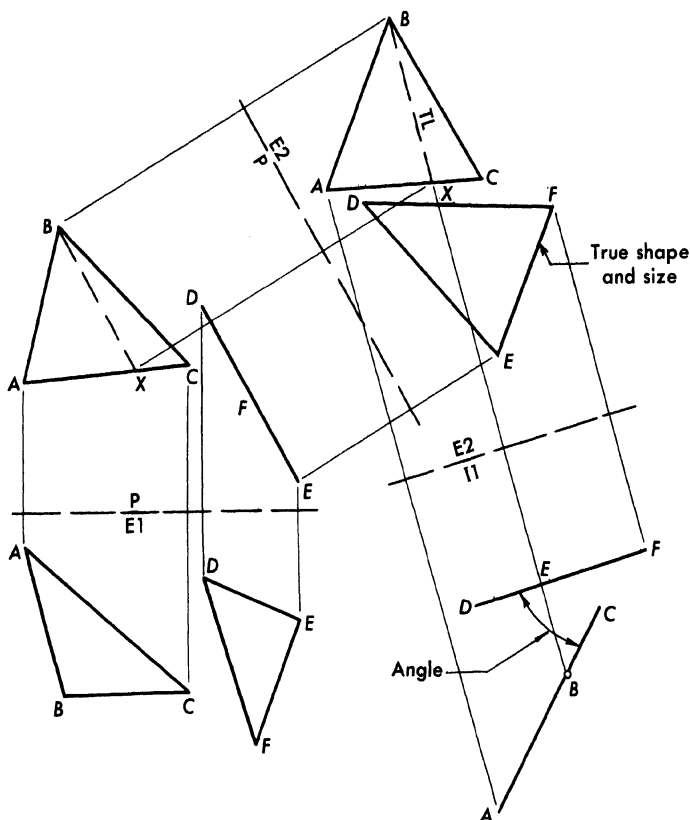


Figure 3-26 Angle between two planes—true-size-of-plane method

B to X in the plan view parallel to the rotation line $\frac{P}{E2}$, so that BX appears true length in view $E2$. View $I1$, drawn to show BX as a point, shows both planes as edges and the dihedral angle can be measured.

If plane DEF had not been given as an edge, an additional view showing either of the planes as an edge would have been required before the above procedure could be followed.

3-14 SUMMARY

1. Refer to Section 3.1 for review of important principles.

2. The shortest connection from a point to a line is perpendicular to the line. It may be found by either the *line method* or the *plane method*.

3. The steepest connection from a point to a line is a perpendicular to a horizontal line on the plane formed by the point and the line.

4. The shortest connection between two lines is perpendicular to both lines, and may be found by either the *line method* or the *plane method*.

5. The shortest connection of specified slope between two lines *must* be found by the *plane method*.

6. A line intersects a plane at a single piercing point that can be found either by the *edge-view method* or the *cutting-plane method*.

7. The shortest connection from a point to a plane is perpendicular to the plane and may be found by either the *edge-view method* or the *two-view method*.

8. In any view, a line perpendicular to a plane appears perpendicular to any line on the plane that appears true length in that view.

9. The shortest connection of specified slope from a point to a plane has the same bearing as the shortest connection, and is most easily found by the *edge-view method*.

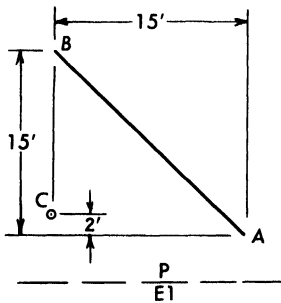
10. The angle a line makes with a plane is seen only when the plane appears as an edge and the line appears in true length in the same view.

11. The line of intersection of two planes is a straight line common to both planes. It may be found by the *edge-view method*, the *piercing-point method*, or the *cutting-plane method*.

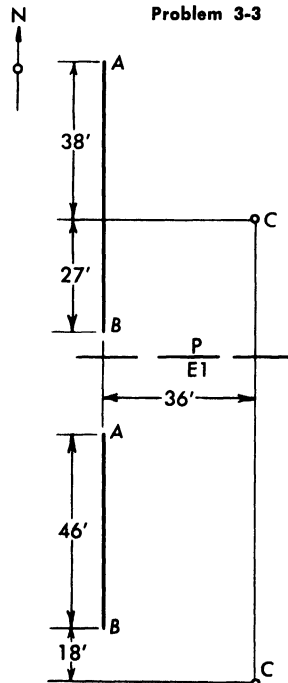
12. The angle between two planes, or *dihedral angle*, is seen in a view that shows both planes as edges. It may be found by the *line-of-intersection method* or the *true-size-of-plane method*.

3-15 Problems

- 3-1.** Show in the P and E1 views the shortest possible brace from the point C to the member AB, using the line method. Measure and record its bearing, true length, and slope. Scale: 1 in. = 10 ft.
- 3-2.** Solve Problem 3-1 using the plane method.
- 3-3.** (a) Find the bearing, true length, and slope of the shortest possible pipe to connect point C with the pipe AB. (b) Find the bearing, true length, and slope of the steepest pipe to connect C with AB. Scale: 1 in. = 30 ft.
- 3-4.** Two pipes AB and CD are given. Point A is 30 ft due north of D and at the same elevation. Both B and C are 6 ft north and 18 ft west of D. Point B is 15 ft higher than A and C is 9 ft lower than D. Find the bearing, true length, and slope of a pipe to connect AB and CD using 90-degree tees. Use the line method. Scale: 1 in. = 10 ft.



Problem 3-1



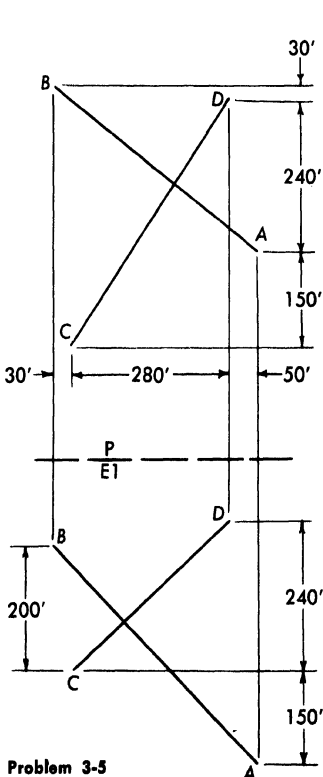
Problem 3-3

- 3-5.** Determine the bearing, true length, and percent grade of the shortest tunnel that could be constructed between the given shafts *AB* and *CD*.
Scale: 1 in. = 200 ft.
- 3-6.** Using the data given in Problem 3-5, determine the bearing and true length of the shortest horizontal tunnel between shafts *AB* and *CD*.
Scale: 1 in. = 200 ft.
- 3-7.** Two mine shafts, *AB* and *CD*, with the following coordinates, are given:

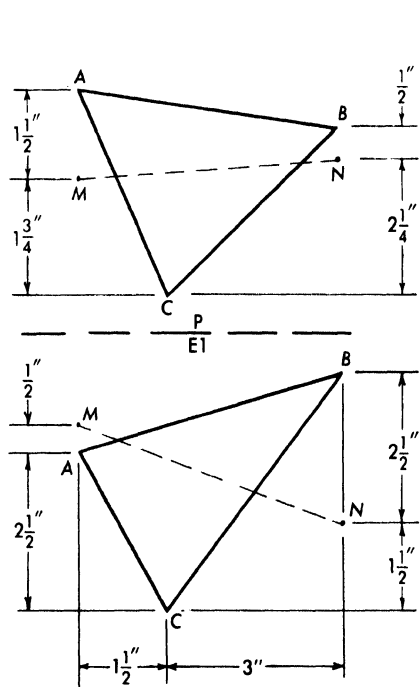
Point	<i>x</i>	<i>y</i>	<i>z</i>
<i>A</i>	840	1170	1130
<i>B</i>	400	690	700
<i>C</i>	900	470	650
<i>D</i>	400	750	1130

Find the coordinates at the ends of the shortest tunnel with a downward grade of 50 percent from *AB* to *CD*. Also, find the length and bearing of the tunnel. Scale: 1 in. = 300 ft.

- 3-8.** Find the point at which line *MN* pierces plane *ABC*, using the edge-view method. Scale: Half size.



Problem 3-5



Problem 3-8

3-9. Solve Problem 3-8 using the cutting-plane method.

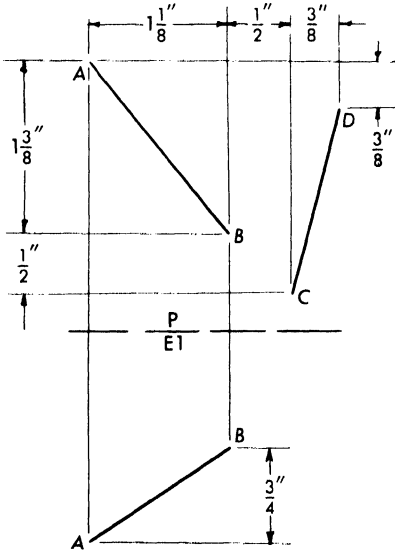
3-10. The plane ABC and a point X are given as follows: Point B is $2\frac{1}{4}$ in. south and 2 in. west of A , and C is $\frac{1}{2}$ in. south and $3\frac{3}{8}$ in. east of B . Point C is $1\frac{5}{8}$ in. lower than A and $1\frac{3}{8}$ in. higher than B . Point X is $1\frac{1}{8}$ in. east, $\frac{5}{8}$ in. south, and 2 in. above B . Find the bearing, true length, and slope of the shortest connection from X to the plane ABC . Scale: Full size.

3-11. Solve Problem 3-10 using only the plan view and an elevation view looking due north.

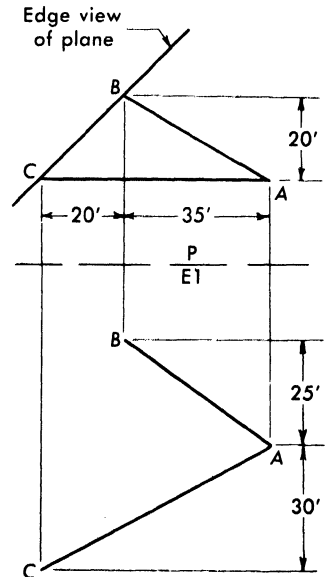
3-12. Using the data given in Problem 3-10, find the bearing and true length of the shortest horizontal connection from X to plane ABC . Scale: Full size.

3-13. Points B , C , and D lie in a plane that is perpendicular to line AB . Draw line CD in view E1, using only the two given views. Scale: Full size.

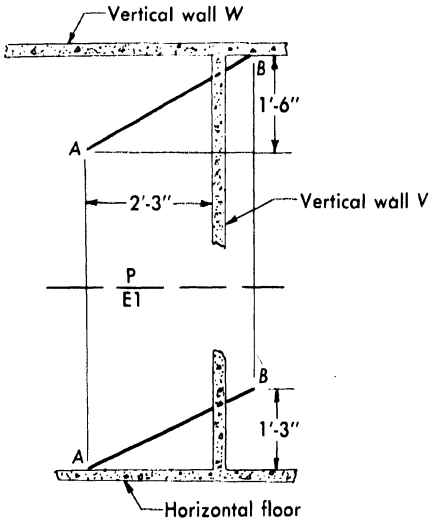
3-14. (a) Using the plane method, determine the true size of the angle between AB and the given plane. (b) Using the line method, determine the true size of the angle between AC and the given plane. Scale: 1 in. = 30 ft.



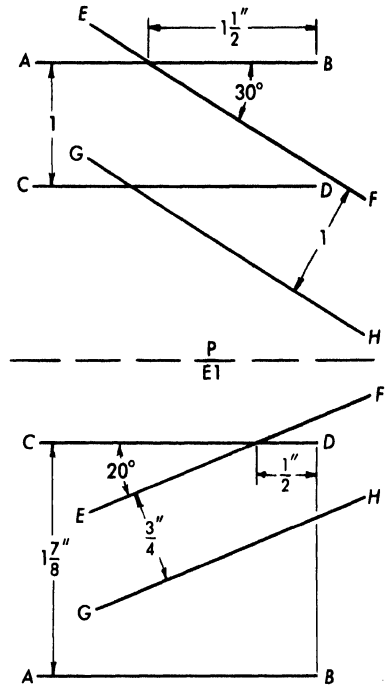
Problem 3-13



Problem 3-14



Problem 3-15

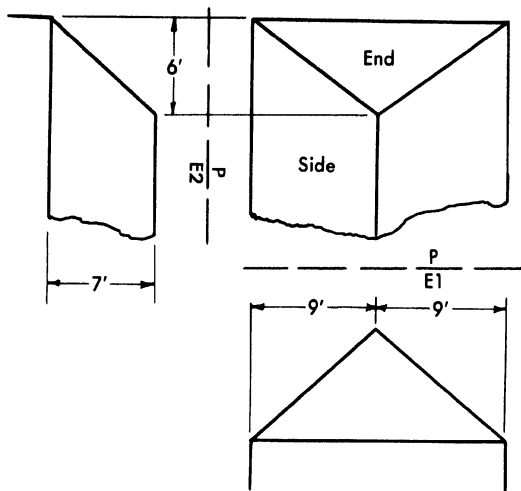


Problem 3-16

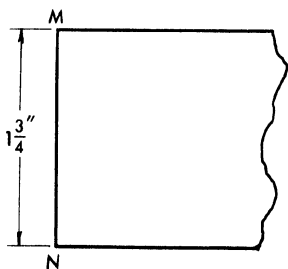
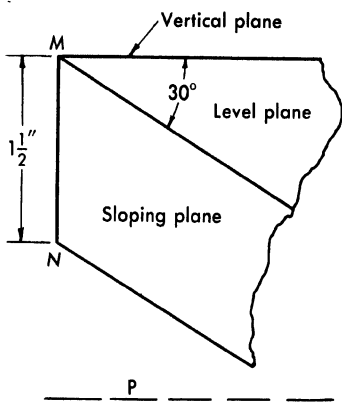
- 3-15.** Measure in degrees the angle that line AB makes with (a) the vertical wall W , (b) the vertical wall V , and (c) the horizontal floor. Scale: $\frac{1}{2}$ in. = 1 ft 0 in.
- 3-16.** Two planes, one designated by the parallel lines AB and CD and the other by the parallel lines EF and GH , are given. Find the line of intersection of the two planes by the edge-view method. Scale: Full size.
- 3-17.** Planes ABC and DEF with the following coordinates are given:

Point	x	y	z
A	0	11	8
B	15	18	24
C	23	0	3
D	33	9	13
E	45	21	17
F	56	5	0

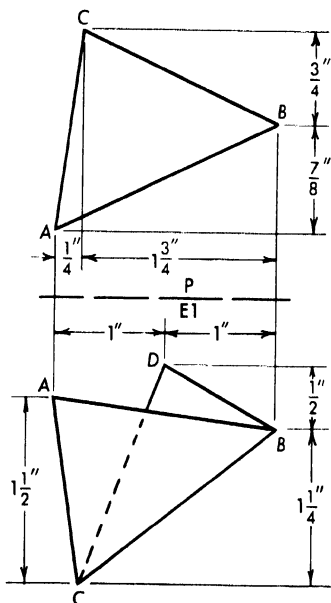
Find the line of intersection of the two planes using only the plan view and an elevation view looking due north. Scale: 1 in. = 10 ft.



Problem 3-18



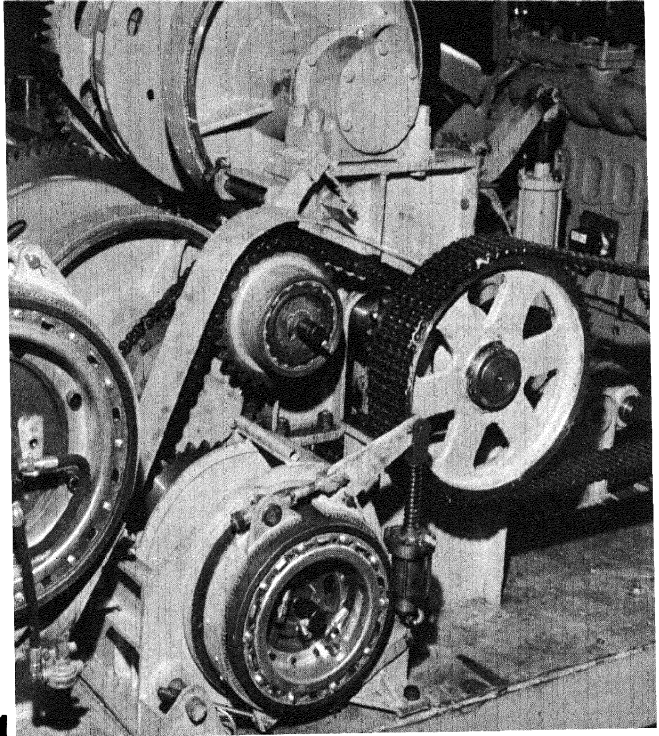
Problem 3-19



Problem 3-20

- 3-18.** Find the angle between the end and side planes of the roof. Scale: $\frac{1}{8}$ in. = 1 ft 0 in.
- 3-19.** Find the true size of the dihedral angle between the vertical plane and the sloping plane. Scale: Full size.
- 3-20.** The dihedral angle between planes *ABC* and *BCD* is 90° . Complete the plan view. Scale: Full size.

CHAIN DRIVE FOR POWER WINCH. (Courtesy Washington Iron Works, Seattle)



CHAPTER

4 Revolution

The basic principles of revolution, with emphasis placed on those procedures that provide particular advantages over other methods

4.1 INTRODUCTION

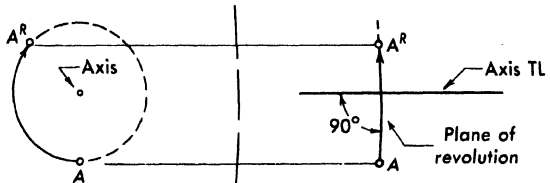
All the methods described in the preceding chapters are based on the *direct* or *change-of-position* method (Sec. 1.1). When this method is used, the observer's position is considered to be different for each view drawn, while the object remains in a fixed position. In the **revolution method**, the observer is considered to occupy a fixed position while the object is revolved or rotated into different positions to obtain different views. Most problems are best solved by the direct method, but certain problems involving moving parts of machines or structures can be more easily solved by revolution, or by a combination of the two methods. In general, revolution requires less space on paper, but it is apt to cause confusion, since views often overlap. Therefore most problems should be solved by the direct method, and the method of revolution should be used only when it has a definite advantage in a particular situation.

4.2 REVOLUTION OF A POINT

The following basic principles underlying the method of revolving a point must be fully understood before revolution can be used in the solution of problems. The student must have a clear picture of what is taking place in space when the revolution method is used.

(a) When a point is revolved in space, it must always move in a circular path about some straight-line axis. Therefore the path of the revolved point will appear as a circle in any view showing the axis of revolution as a point (Fig. 4.1).

Figure 4-1 Revolution of a point



(b) The circular path of the point being revolved must lie in a plane that is perpendicular to the axis of revolution. This plane will appear as an edge or straight line, at right angles to the axis, in any view showing the true length of the axis.

Figure 4.1 shows two views of a point that has been revolved from A to A^R about an axis in the direction indicated by the arrows. In the view that shows the axis as a point, the path of the point is a circle, while in the view showing the true length of the axis, the path of the point is a line at 90° to the axis. This line is the edge view of the plane of revolution. The usual designations of type of view have purposely been omitted, since the above statements are true for any two adjacent views.

Note that the circle of revolution of any point about an axis can be drawn only when the axis appears as a point. When using the method of revolution, the student should first be sure of the position of the axis of revolution in space, and then be sure it appears as a point, before performing the revolution. In a case where the true length of the axis is not shown in the given views, the point view of the axis is found by the change-of-position method before the point is revolved.

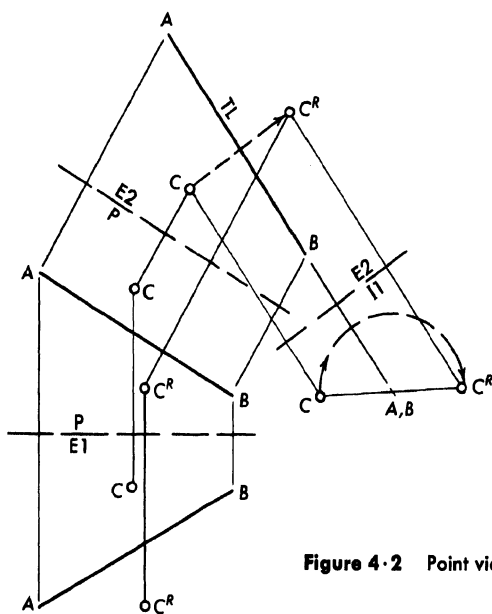


Figure 4.2 Point view of axis of revolution

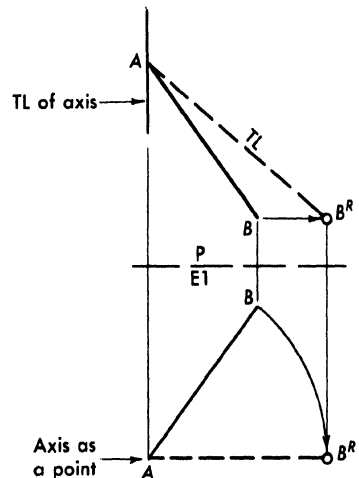
In Figure 4.2, the line AB and the point C are given in the P and $E1$ views. It is desired to revolve point C about AB through an angle of 180° . View $E2$ is drawn to show AB in its true length, and $I1$ to show AB as a point. In view $I1$, the circle of revolution is drawn using AB

as a center, to find the revolved position of C . In view E2, the axis AB is true length, and point C must move in a plane at right angles to AB , its position on this plane being found by projection from view I1. The revolved position of C can then be found in the given views by projection.

4.3 TRUE LENGTH AND SLOPE OF A LINE

The true length of a line may be found in any given view by revolving the line about an axis that is perpendicular to the lines of sight for that view.

Figure 4.3 True length of a line by revolution



In Figure 4.3, the line AB is given in the P and E1 views. To appear true length in the plan, AB must be revolved about a horizontal axis until it is parallel to the rotation line in view E1. Since all points on a line revolve through the same angle, it is necessary to revolve only one point on the line. In view E1, therefore, point B is revolved about a horizontal axis through A until it is at the same elevation as A . In the plan, point B moves parallel to the rotation line, or at right angles to the axis of revolution. The line connecting A and the revolved position of B in the plan view is the true length of the line AB . Any other horizontal axis through the line could have been used with the same results.

Note that the above procedure does not give the slope of the line AB , because the line does not appear true length in an elevation view. If the line is revolved about a *vertical* axis, the angle between the line and a horizontal plane remains constant. In other words, the slope of a line is not changed when the line is revolved about a vertical axis.

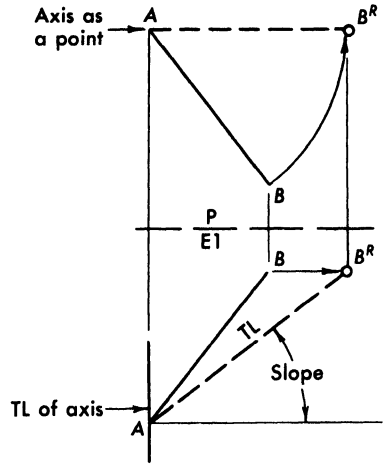


Figure 4.4 Slope and true length of a line by revolution

In Figure 4.4, the line AB is again given in the P and E1 views. The line is revolved in the plan view about a vertical axis through A until it is parallel to the rotation line. The revolved position of AB in the elevation view then shows both its true length and its slope.

4.4 A LINE WITH A GIVEN BEARING, LENGTH, AND SLOPE

GIVEN: The line AB with a bearing of $N 40^\circ E$, a length of 500 ft, and a slope of $+25$ degrees.

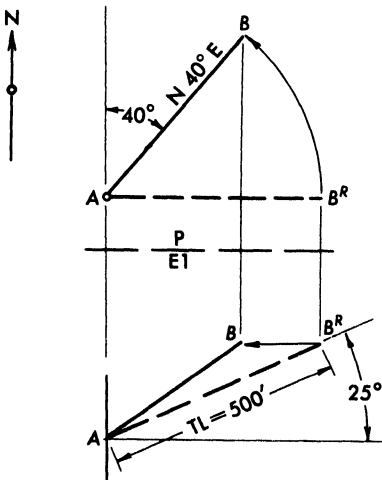


Figure 4.5 To lay out a line with given bearing, length, and slope

PROCEDURE:

1. Locate A in the P and E1 views.
2. In view E1, draw a line of indefinite length at an angle of 25 degrees upward from horizontal.
3. On this line, locate point B^R at a distance of 500 ft (to scale) from A .
4. Through A in the plan view, draw a line parallel to the rotation line and locate B^R by projection.
5. Using a vertical axis through A , revolve the line until it makes an angle of 40 degrees with north, thus locating B in the plan view.
6. When the line is revolved, point B moves parallel to the rotation line in view E1, and can therefore be located by projection.

Note that in this situation, the revolved position of the line was established first, and the line was then located by revolving it back to its true position.

4.5 TRUE SHAPE AND SIZE OF A PLANE

The true shape and size of a plane may be found by revolving the plane about an axis that lies on the plane. The revolution is always performed in a view that shows the plane as an edge, since the axis appears as a point in this view.

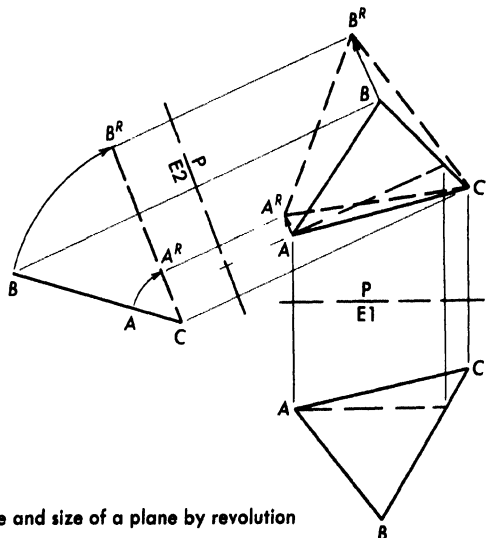


Figure 4-6 True shape and size of a plane by revolution

In Figure 4.6, plane ABC is given in the P and E1 views. View E2 is drawn showing the plane as an edge. The plane is revolved about an axis through C until it is parallel to the rotation line. The dashed lines, representing the revolved position of the plane in the plan view, show the true shape and size of the plane. Any other axis appearing as a point in view E2 could have been used to obtain the true size of the plane in the plan. The true shape and size could be obtained in view E1, or in any other desired view, by drawing the proper edge view.

4.6 ANGLE A LINE MAKES WITH A PLANE

The true size of the angle a line makes with a plane is seen only when the line appears in its true length in a view showing the plane as an edge.

. **Rule:** When the angle is to be found by revolution, the line must be revolved about an axis perpendicular to the plane, so that the angle is not changed as the line revolves.

In Figure 4.7, the plane ABC and the line MN are given in the P and E1 views. The plane is found as an edge in view E2 and an axis is then drawn perpendicular to it through N . Since the axis appears true length in this view, view I1 is drawn to show the axis as a point. In

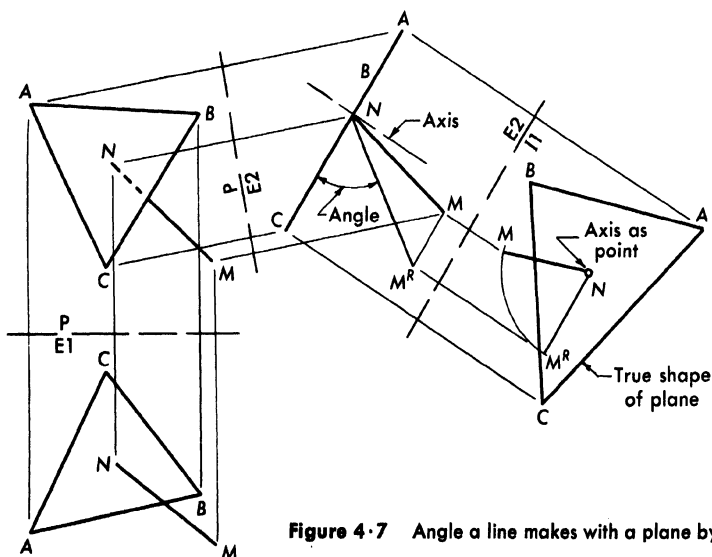


Figure 4.7 Angle a line makes with a plane by revolution

view I1, the line MN is revolved about the axis through N until it appears true length in view E2, where the angle between MN and the plane ABC can be measured. Note that point M remains the same perpendicular distance away from plane ABC during the revolution, and that the angle between MN and plane ABC therefore does not change.

Since view I1 shows the axis of revolution as a point, it also shows the plane ABC in true size, although the plane need not be drawn in this view. Notice the similarity between this method and the change-of-position method described in Section 3.11. Although the revolution method requires one less view, the total work involved is about the same in both methods.

When using revolution to find the angle between a line and a plane, remember that the axis of revolution must be perpendicular to the plane.

4.7 LIMITED REVOLUTION AND CLEARANCE

In many engineering situations, a revolving part of a machine or structure must be placed so as to clear other parts or surfaces by a specified distance, or else its rotation must be limited to an arc that allows proper clearance. Problems of this type are most easily solved by means of the principles of revolution, since the extreme point on any revolving part forms a circle at right angles to an axis of revolution. The line of intersection between the plane of revolution and any other surface determines the amount of clearance or interference between the part and the surface. If this line of intersection lies inside the circle, either the rotation must be limited to some maximum number of degrees, or the design must be changed to allow complete revolution.

In most problems of this type, the position of the axis of revolution is known. The true length and the point view of this axis of revolution should be found by the change-of-position method *before* revolution is applied.

4.8 SUMMARY

1. When a point is revolved, it follows a circular path in a plane perpendicular to the axis of revolution. The revolution is performed in a view showing the axis as a point.

2. The true length of a line in any view is found by revolving the line about an axis perpendicular to the lines of sight until it is parallel to the image plane for that view.

3. The slope of a line is found by revolving the line about a vertical axis until its true length is seen in an elevation view.

4. The true shape and size of a plane is found in any view by revolving the plane about an axis on the plane in an adjacent view that shows the plane as an edge.

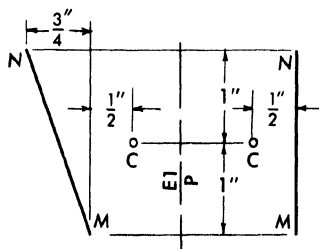
5. The angle a line makes with a plane is found by revolving the line about an axis perpendicular to the plane.

4.9 Problems

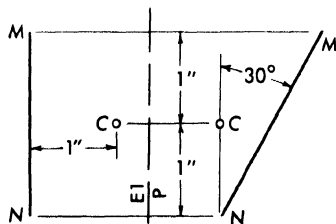
- 4-1. (a) Revolve point C through 180 degrees about the axis MN . Show the revolved position of C in the given views. (b) Revolve point C about the axis MN until it is $\frac{1}{2}$ in. higher than M . How many degrees did it revolve? Show the revolved position of C in the given views. Scale: Full size.
- 4-2. Find the least number of degrees through which C can be revolved about the axis MN to reach the same elevation as point M . If C is revolved through an additional 90 degrees, will it be higher or lower than M ? How much? Show both revolved positions of C in the given views. Scale: Full size.
- 4-3. Using the data of Problem 4-2, find the true length and slope of the lines MN and CM by revolution. Scale: Full size.
- 4-4. Find the true length and slope of members AC and BC by revolution. Scale: 1 in. = 6 ft.
- 4-5. Using only the plan view and an elevation view looking due north, lay out the following lines. Scale: 1 in. = 50 ft.

Line	Bearing	True Length	Slope
AB	N 30° E	150 ft	-45°
AC	N 45° W	200 ft	-50%

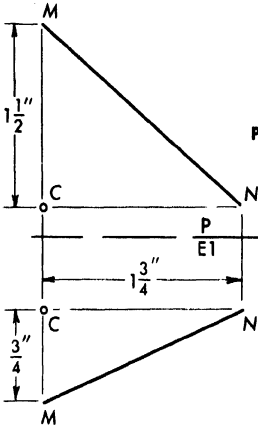
- 4-6. Find the true shape and size of plane ABC by revolution. Scale: 1 in. = 20 ft.



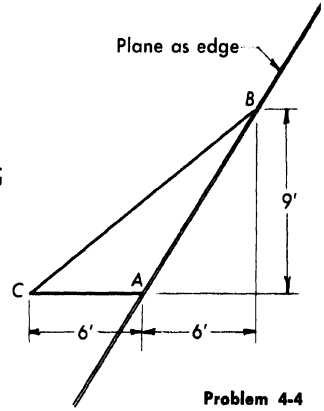
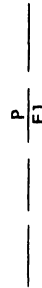
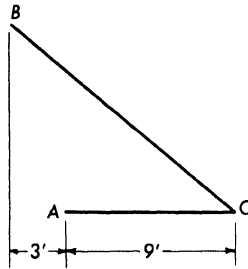
(a)



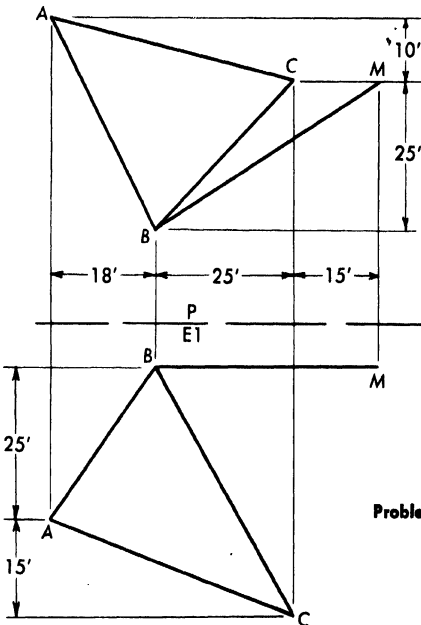
(b)



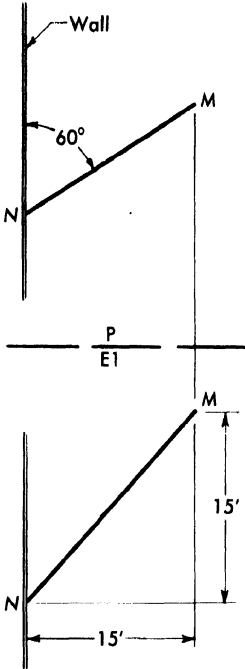
Problem 4-2



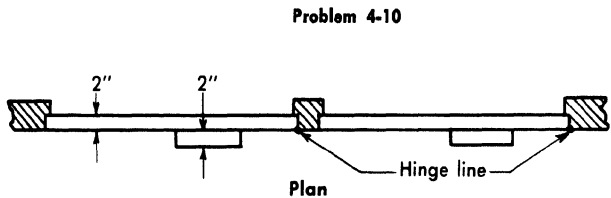
Problem 4-4



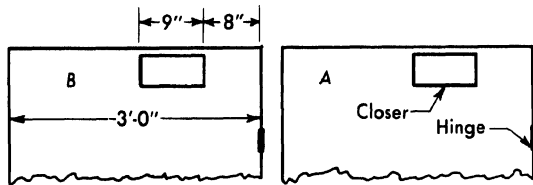
Problem 4-6



Problem 4-7



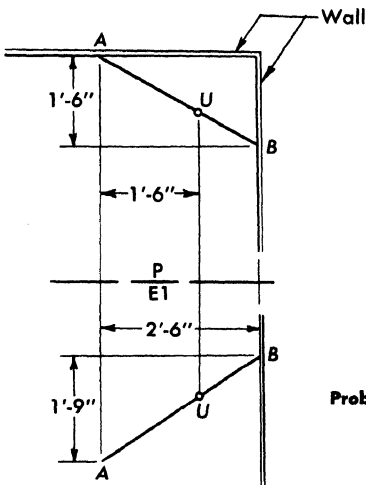
Problem 4-10



Partial Elevation

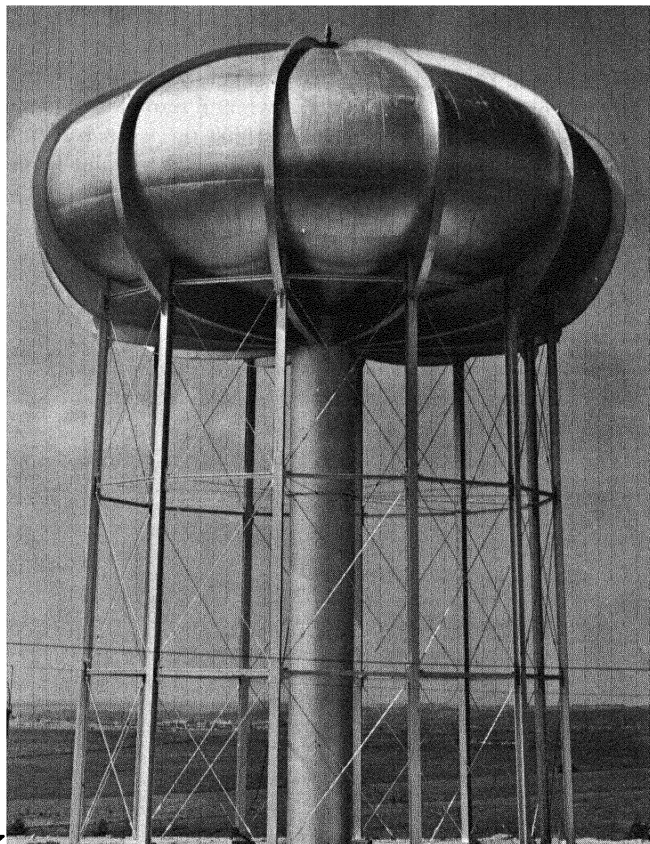
- 4-7. Find the true size of the angle between MN and the wall by revolution. Scale: 1 in. = 10 ft.
- 4-8. Using the data of Problem 4-6, find the true size of the angle between MB and the plane ABC by revolution. Scale: 1 in. = 20 ft.
- 4-9. Using the data of Problem 4-4, find the true size of the angles that AC and BC make with the given plane. Scale: 1 in. = 6 ft.
- 4-10. Two fire doors opening from a corridor into a stair well are shown in the plan and a partial elevation view. If door B is allowed to swing open through 110 degrees, will door A clear the closer on door B ? If not, how many degrees will door A swing before striking the closer? Scale: 1 in. = 1 ft 0 in. (Note: Only the plan view need be drawn.)

- 4-11. Use the same data as in Problem 4-10. Find the maximum number of degrees that door *B* can be allowed to swing open in order for door *A* to just clear the closer on door *B*. Scale: 1 in. = 1 ft 0 in. (Only the plan view need be drawn.)
- 4-12. Point *U* is a union connecting two sections of a pipe that passes through the walls at *A* and *B*. If an 18-in. wrench is used to remove the union, find the maximum number of degrees through which the wrench can be rotated. (Assume that the wrench moves in a plane perpendicular to the pipe.) Show the points at which the wrench handle strikes the walls in both given views. What length wrench could be used to allow a full 360-degree rotation? Scale: $\frac{1}{2}$ in. = 1 ft 0 in.



Problem 4-12

OBLOIDAL DESIGN WATER TANK—CAPACITY 1,000,000 GALLONS. (Courtesy
Pittsburgh-Des Moines Steel Company)



CHAPTER 5 Curved and Warped Surfaces

*The more common types of curved and warped surfaces
with emphasis on the principles used in solving problems
relating to the curved surfaces most commonly used—
cylinders and cones*

5-1 DEFINITIONS

The student should understand thoroughly the following definitions before proceeding to the solution of problems.

(1) A surface is generated by a moving line, called the **generatrix**, which may be either straight or curved.

(2) Any one of the infinite number of positions of the generatrix is called an **element** of the surface.

(3) A **directrix** is a straight or curved line that controls or directs the motion of the generatrix in forming a surface. A surface may have one or two directrices.

(4) In forming certain surfaces, the generatrix must make a constant angle with a given plane called a **plane director**.

(5) A **developable surface** is one that can be rolled out into a plane. In other words, it is a surface that can be cut from a plane and then rolled or bent into the desired shape without deforming the material.

(6) A **ruled surface** is any surface, such as a cylinder, that can be formed by a straight-line generatrix, or, in other words, a surface on which straight lines can be drawn.

(7) A **double-ruled surface** is a surface, such as a plane, on which two intersecting straight lines may be drawn through the same point.

(8) A **single-curved surface** is a surface, such as a cylinder or cone, that can be generated by a straight line moving along a curved directrix, so that any two adjacent straight-line elements are either parallel or intersecting. It is, therefore, a developable, ruled surface.

(9) A **double-curved surface** is a surface, such as a sphere, on which no straight lines can be drawn, generated by a curved line moving in a curved path. Double-curved surfaces cannot be developed, although in practice, approximate developments may be used to form surfaces that are nearly correct.

(10) A **warped surface** is a ruled surface that cannot be developed. Any two adjacent elements on a warped surface are non-intersecting and nonparallel.

(11) An **axis of a surface** is a straight line about which the surface is symmetrical. Some surfaces have more than one axis and some have none.

Every engineer should be familiar with the more common types of curved and warped surfaces. He should know how to develop any of the common surfaces that can be developed, and to recognize those surfaces that cannot be developed.

5.2 CYLINDERS

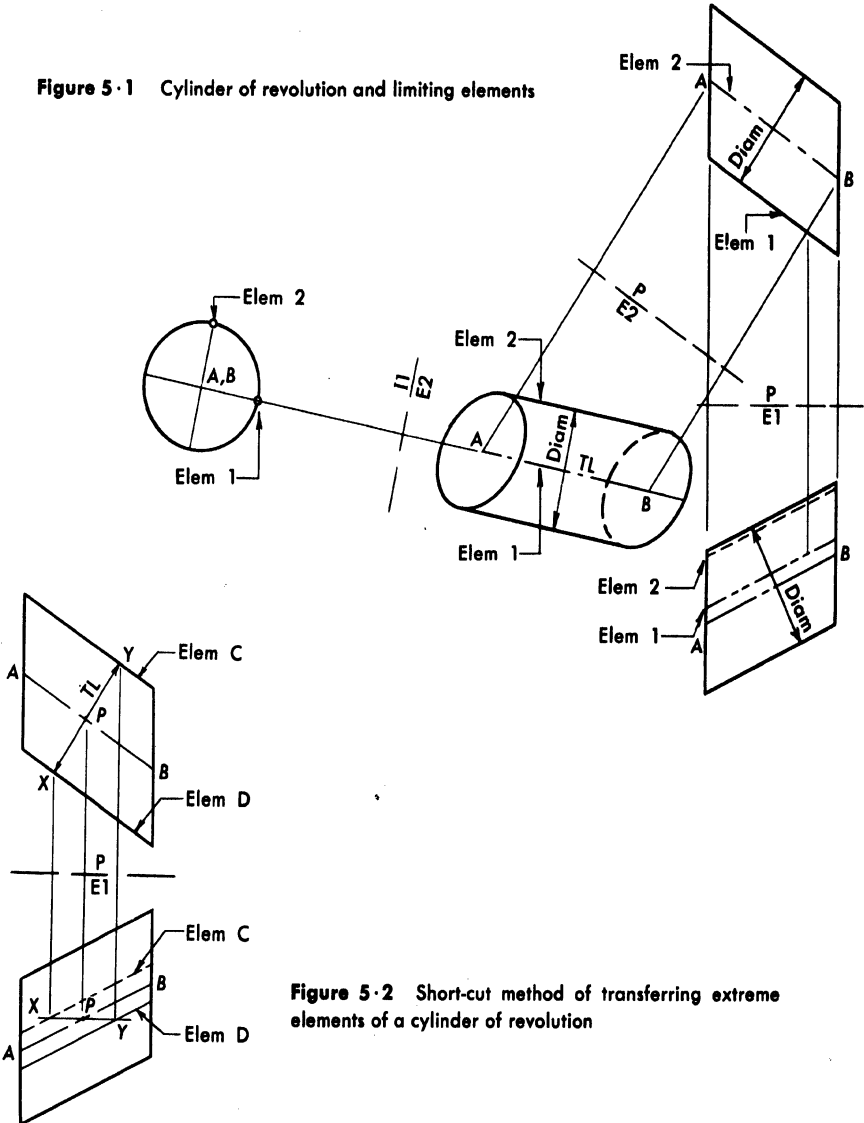
A **cylinder** is a surface formed by a straight-line generatrix that is perpendicular to a plane and follows a curved-line directrix in that plane. It can also be defined as a surface formed by a straight line that moves in a curved path while remaining parallel to a straight-line directrix. From the definitions, it can be seen that a cylinder is not necessarily round and does not even have to be a closed surface.

A plane that is perpendicular to the axis of a cylinder is perpendicular to all its elements, and cuts a **right section** of the cylinder. Cylinders are classified according to the shapes of their right sections. If the right section is a circle, the cylinder is called a **cylinder of revolution**, or a **right circular cylinder**. Other examples are the **elliptical cylinder** and the **parabolic cylinder**, where the right sections are an ellipse and a parabola, respectively.

Any view of a cylinder that shows the true shape and size of its right section also shows the axis and all elements as points, and is therefore an edge view of the cylinder.

In any view, the extreme outside elements are called **limiting elements** of the cylinder. The limiting elements are the only elements that need to be drawn in any view (except the edge view) to represent a cylinder. Since in practice a cylinder has a definite length, one or both bases are usually shown.

Figure 5.1 shows four views of a cylinder of revolution with axis AB and vertical bases. Notice that the shortest distance between extreme elements in every view is equal to the diameter. In view E2, the axis and all elements of the cylinder appear in true length. View I1 shows the axis and all elements as points, and therefore gives an edge view of the cylinder and the true shape and size of its right section. Note that the limiting element in any one view is not the limiting element in any of the other views. Element 1, which is the limiting element in the plan, appears directly in front of the axis AB in view E2, because it is the closest element to the rotation line in view I1. In view E1, element 1 appears higher than the axis AB , rather than directly in front of it, as can be verified by measuring from the rotation line. Element 2, which is a limiting element in view E2, appears directly above the axis AB in the plan view, since both must be the same distance from the rotation line in this view. (See view I1.) Element 2 is located in view E1 by measurement from the rotation line.

Figure 5-1 Cylinder of revolution and limiting elements**Figure 5-2** Short-cut method of transferring extreme elements of a cylinder of revolution

Since the diameter of a cylinder of revolution appears true length in every view, extreme elements in one view can be transferred to another as shown in Figure 5.2.

The plan and elevation views of a cylinder of revolution are given. To locate the limiting elements C and D in the elevation view, a diameter, XY, is drawn through any point P on the axis in the plan

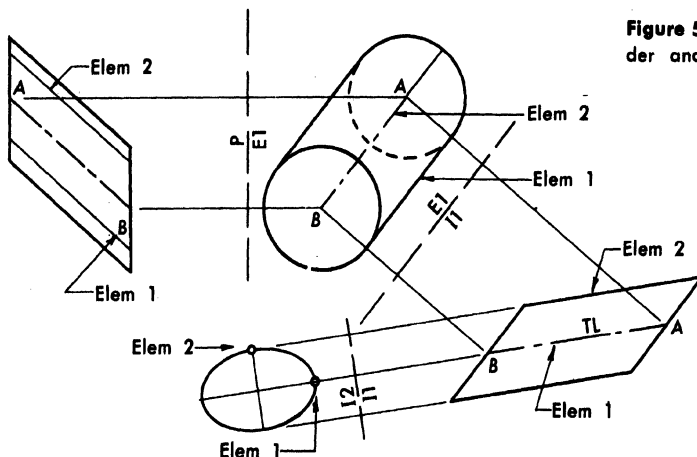


Figure 5.3 Elliptical cylinder and limiting elements

view. Because XY is true length in the plan, it must be drawn parallel to the rotation line in view $E1$. Elements C and D are drawn through X and Y , respectively, in view $E1$. The extreme elements of view $E1$ could be transferred to the plan by a similar procedure.

Four views of an elliptical cylinder are shown in Figure 5.3. The axis AB and all elements appear true length in view $I1$, while view $I2$ is an edge view of the cylinder showing the true shape and size of its right section. The extreme elements in one view lie along the same line as the axis in an adjacent view when, and only when, one of the views is a true-length view. Thus element 1 appears to coincide with the axis in view $I1$, but is at some distance from the axis in the plan view.

Since all elements of a cylinder are parallel to the axis, only one point on an element need be projected to transfer it from one view to another. The point projected is the point where the element passes through the base (or some other plane section).

5.3 LINE PIERCING A CYLINDER

Any line that passes through a cylinder must intersect at least one element on the cylinder. If the line is straight and the cylinder is a closed figure, the line must intersect two elements. The piercing points can be located by the following methods:

Edge-view Method

The points at which any line pierces a cylinder can be found by locating the line in a view showing the cylinder as an edge.

In Figure 5.4, the cylinder and the line MN are given in the plan and elevation views. The inclined view I1 shows the edge view of the cylinder, since view E1 shows its true length. Points on a number of elements are selected on the base in the plan view as shown. These points are projected to the base in view E1, and thence into view I1, where they are connected to form the edge view. In the inclined view, the line MN intersects the edge view at two points that determine the elements S and T . These elements are projected into view E2, where their intersections with line MN determine the required piercing points. These points may then be projected into the plan view. A check is obtained by projecting elements S and T into the plan view and again determining the piercing points.

Two-view Method

In Figure 5.4, any plane appearing as an edge in view I1 would be parallel to the elements of the cylinder. If this plane were drawn so that it contained the line MN , it would also contain the elements S and T . Applying this principle, it is possible to locate piercing points using only two views.

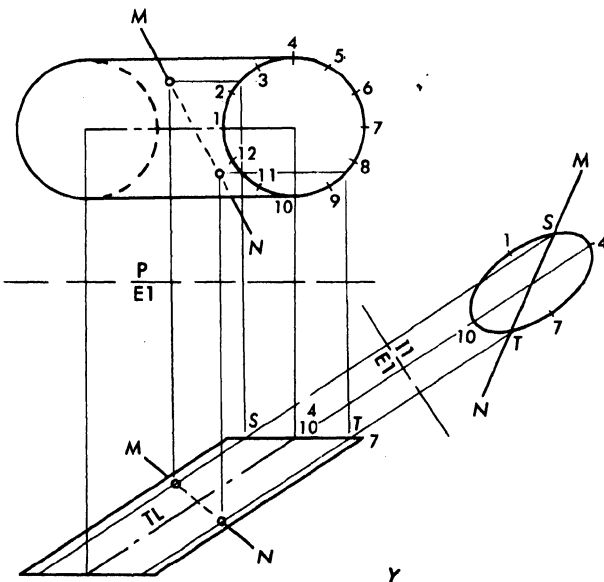
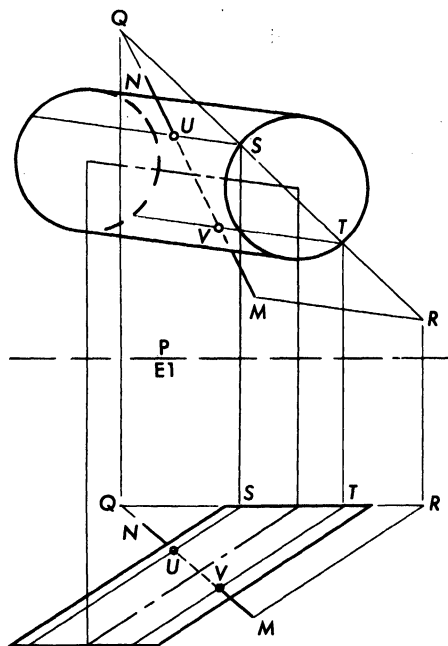


Figure 5.4 Line piercing a cylinder—edge-view method

In Figure 5.5, the piercing points of line MN with the given cylinder are desired. In each view, a line is drawn through M parallel to the elements of the cylinder. This line and MN form a plane containing MN that is parallel to all elements of the cylinder. This plane intersects the plane of the base along the line QR in view E1. In the plan, the points S and T , at which line QR crosses the base, are the ends of the two elements cut from the cylinder by the plane $MNQR$. Points S and T are projected into the elevation view, and the elements are drawn in both views to locate the desired piercing points, U and V . The location of these points can be checked by projection between the views.

Figure 5.5 Line piercing a cylinder—two-view method



The portion of the given line between piercing points is invisible and is shown by dashed lines. The portion between the piercing point and the limiting element is visible if the element through the piercing point is visible. Thus in the plan view, the given line is visible from N to U because element S is visible in this view, while the portion

between the limiting element and V is hidden because element T is hidden. Visibility in view $E1$ is similarly determined.

5.4 PLANE TANGENT TO A CYLINDER

A plane that is tangent to a cylinder contains one element of the cylinder, and intersects the plane of the base along a line which is tangent to the base.

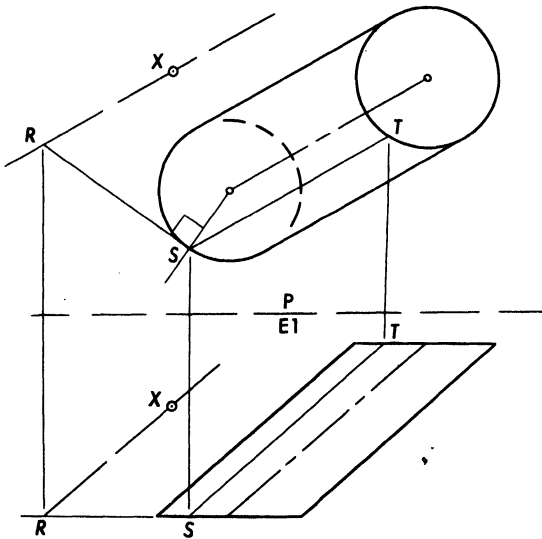


Figure 5.6 Plane tangent to a cylinder

In Figure 5.6, a plane tangent to the given cylinder is to be drawn through point X . A line drawn parallel to elements of the cylinder in both views is found to pierce the plane of the base at point R . From R , a line is drawn tangent to the base at S in the plan view, and S is projected into view $E1$. The exact point of tangency, S , is found in the plan view by drawing a radius of the circle at right angles to the tangent. The plane XRS thus formed is tangent to the cylinder along element ST . Note that one other plane can be drawn through X tangent to the cylinder.

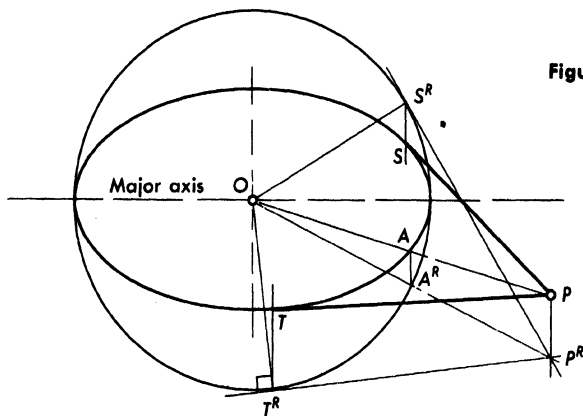


Figure 5-7 Tangents to an ellipse

5.5 TANGENTS TO AN ELLIPSE

When it is necessary to draw tangents to an ellipse, the exact points of tangency can be determined by the following method.

In Figure 5.7, it is desired to draw the tangents from point P to the ellipse and to determine the exact points of tangency. An ellipse can be considered as a view of a circle that is inclined with respect to the lines of sight. The diameter of this circle is equal to the major axis of the ellipse. We therefore revolve the ellipse about its major axis until it appears as a circle. A line drawn from O to P crosses the ellipse at point A , which moves to A^R as the ellipse is revolved. Point P is revolved about the same axis and therefore moves at right angles to the axis until it intersects line OA^R at P^R . Lines from P^R are drawn tangent to the circle at points S^R and T^R . The circle is then revolved back to its original position, and points S^R and T^R thus move to points S and T , which are the exact points of tangency of the tangents PS and PT .

5.6 PLANE SECTIONS OF CYLINDERS

In representing cylindrical shapes used in engineering, it is often necessary to draw views of cylinders that have been cut by some plane. The line of intersection of the plane and the cylinder is called a **plane section**. When the plane appears as an edge in a particular view, the plane section appears as a straight line. However, when the plane does not appear as an edge, the plane section appears as a curved line. This curved line can be located by plotting a sufficient number of

points at which elements of the cylinder pierce the plane. There are three different procedures for finding these piercing points.

First Method

In Figure 5.8, the plane $ABCD$ and a cylinder of revolution are given in the P and E1 views, and the plane section is to be drawn in the plan view.

View I1 is drawn to show the edge view of the plane and the true length of the cylinder. In view E1, the edge view of the cylinder is divided into equally spaced elements. These elements are projected into view I1 to find the points at which they pierce the plane $ABCD$. Each piercing point is then located in the plan view by measuring from the rotation line along the proper elements. For clarity in the sketch, only elements 2 and 10 are shown in the plan. After all points have been located in the plan, they are connected by a smooth curve. The visible portion of this curve is between points that lie on visible elements.

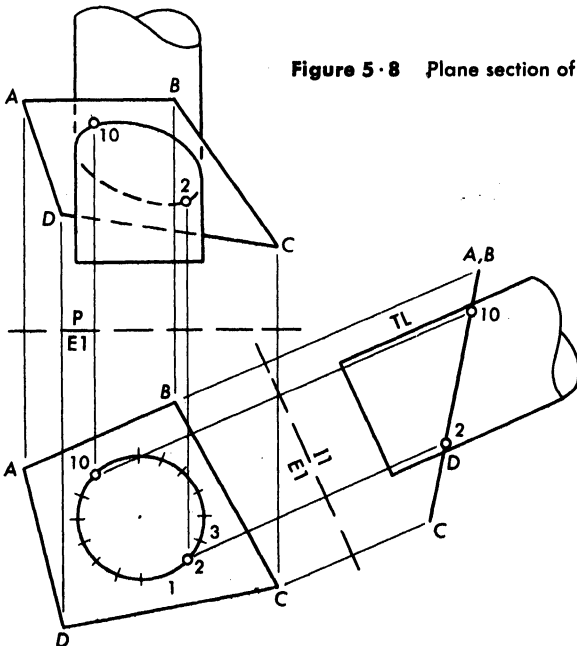


Figure 5.8 Plane section of a cylinder—I

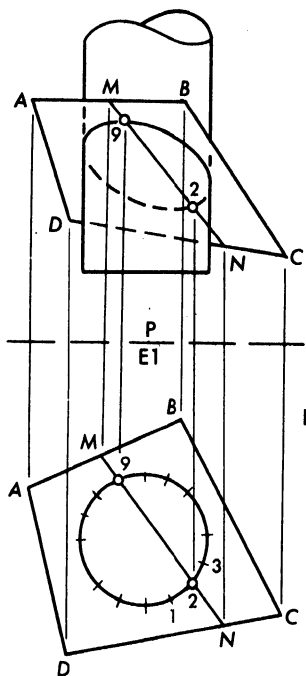


Figure 5-9 Plane section of a cylinder—II

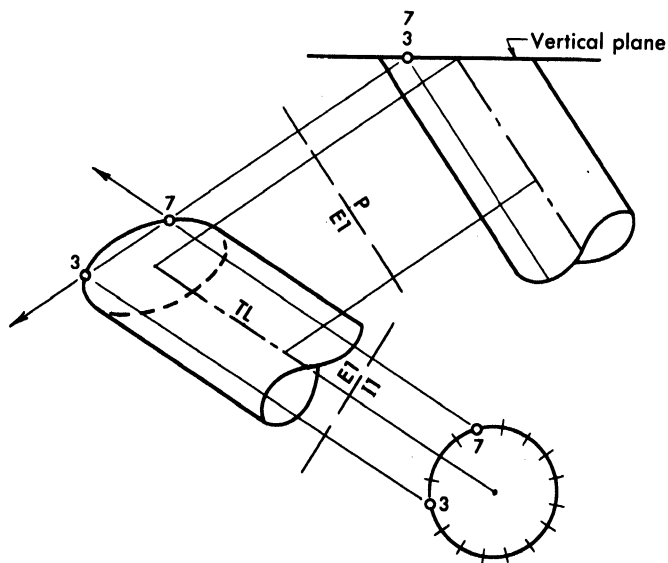


Figure 5-10 Plane section of a cylinder—III

Second Method

In Figure 5.9, the same information is given as in the preceding example, but the plane section is to be found by using only two views.

In view E1, where the cylinder appears as an edge, a cutting plane is passed through elements 2 and 9. This plane cuts across plane *ABCD* at points *M* and *N*, which are projected into the plan. Line *MN* in the plan view is the line of intersection between the cutting plane and plane *ABCD*. Since elements 2 and 9 both lie in this cutting plane, the points at which they intersect line *MN* are the piercing points. Other cutting planes are used to locate enough additional piercing points to allow the desired curve to be drawn.

Third Method

In Figure 5.10, a cylinder of revolution that is cut by a vertical plane is given in the P and E1 views, and the plane section is to be drawn in view E1.

The cylinder is true length in view E1, and view I1 is drawn to show the cylinder as an edge. In this view, a number of equally spaced elements are selected. These same elements are located in the plan view by measuring from the rotation line, and the points at which they pierce the given vertical plane are found. Projection lines drawn from these points in view P to view E1 intersect projection lines drawn from the corresponding elements in view I1, and thus determine points on the required plane section. The necessary lines for locating points 3 and 7 have been shown, the others being omitted for clarity. This procedure is called **cross projection**, because the two projection lines used to locate each point form a cross.

Note that although cylinders of revolution were used in each of the three preceding examples, the procedures work just as well with any other type of cylinder. In cases where the true length and/or edge view of the cylinder is not given, these views must, of course, be drawn before the solution is attempted.

In solving problems involving cylinders, the student should number elements in all views. This makes it easier to keep track of the elements and helps to eliminate mistakes in projection.

5-7 TRUE SHAPE AND SIZE OF PLANE SECTIONS OF CYLINDERS

The true shape and size of any plane section of a cylinder can be seen only in a view for which the lines of sight are perpendicular to the plane section.

In Figure 5.11, an elliptical cylinder cut by an inclined plane is given in the P and E1 views. A rotation line is drawn parallel to the edge view of the inclined plane to obtain a true shape view. A number of elements on the edge view of the cylinder are chosen in the plan view as shown. The extreme elements corresponding to those numbered 1, 3, 9, and 12 in Figure 5.11 should always be included. These elements are projected to view E1 to find the points at which they pierce the plane. The piercing points are then projected into view I1, where they are connected to give the true shape and size of the section.

If views of a cylinder corresponding to views E1 and I1 of Figure 5.11 were given, the true shape of the right section could be found by the same procedure, the elements being selected in view I1.

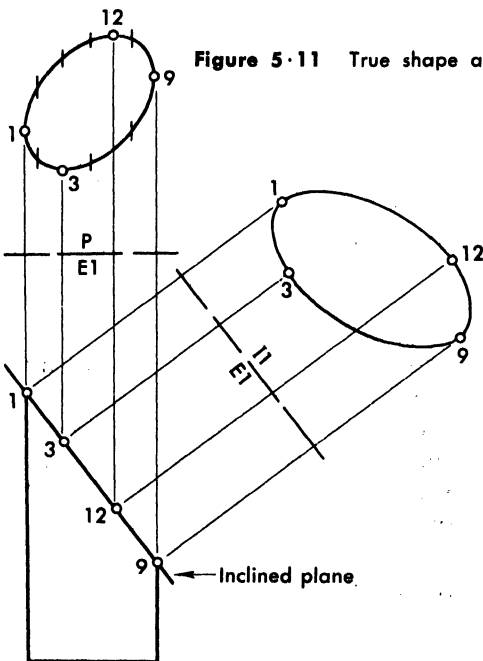


Figure 5.11 True shape and size of plane sections of cylinders

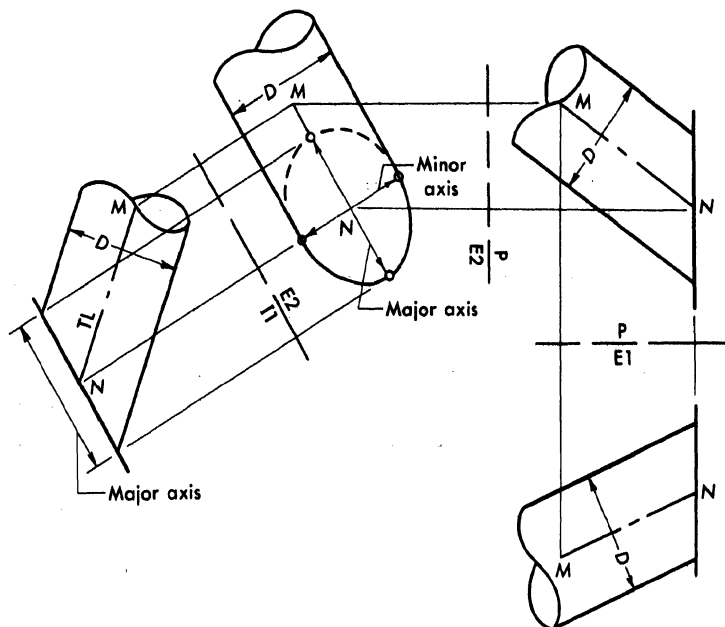


Figure 5.12 True shape and size of plane sections of cylinders of revolution

While the method above works for any cylinder, the following method is easier when the cylinder is a *cylinder of revolution* and its axis is given. Any plane section of a cylinder of revolution not perpendicular or parallel to the elements will be an ellipse with a minor axis equal to the diameter.

In Figure 5.12, a cylinder of revolution with axis MN is given in the P and $E1$ views. View $E2$ shows the true shape and size of the plane section. View $I1$ is drawn to obtain the true length of the axis MN . In this view, the plane section appears as an edge. The distance between the points at which the extreme elements of the cylinder strike this plane is the length of the major axis of the ellipse. The major axis is then projected into view $E2$, where it coincides with the axis of the cylinder. The minor axis is a diameter perpendicular to the elements. With these axes, the ellipse can be drawn by the trammel method (Sec. 2.17).

5.8 DEVELOPMENTS

When a surface is developed, it is laid out on a plane. The development shows the shape that could be cut from a flat surface, such as a sheet of metal, and then rolled or bent to form the desired shape. In metalwork, since pieces fabricated from sheet metal must be fastened together after rolling, an inch or two of extra metal is usually left on the edges to be joined. The joint may then be made by crimping, riveting, or lap welding. If the metal has sufficient thickness, the joint may be made by butt welding the material, in which case no extra allowance is necessary. The development is usually laid out so that the joint can be made along the shortest element, thus reducing the cost of fabrication. Since the development of any surface must be true size, it shows the true length of all lines and the true size of all angles on the surface. The development of a surface should always be marked to indicate whether the material is shown inside or outside up, to insure rolling or bending in the proper direction.

5.9 DEVELOPMENT OF A CYLINDER

The length of the development of a cylinder is equal to the perimeter of its right section. The perimeter will be seen in its true length in the view showing the cylinder as an edge. Since the cylinder must be seen as an edge, and since all elements appear true length in the development, *a true-length view of the cylinder is required.*

Figure 5.13 shows the development of a short piece of pipe (cylinder of revolution) between a level floor and a sloping wall. The position of the axis MN is given in the P and $E1$ views and the diameter D is known. Views $I1$ and $I2$ are drawn to show the true length and the point view of the axis. With the given diameter, the edge view of the cylinder is drawn in view $I2$. Equally spaced elements are located in the edge view and transferred into view $E1$ by measuring from the rotation line. Points on the ends of the cylinder in view $I1$ are located by cross projection from views $E1$ and $I2$, as shown for element 12. A stretch-out line is drawn at right angles to the true-length view of the elements. The elements are drawn perpendicular to this line at intervals equal to the spacing established in view $I2$. The ends of elements are found by projection from the true-length view $I1$, and connected to complete the development. If extra material is required for joining, it may be added

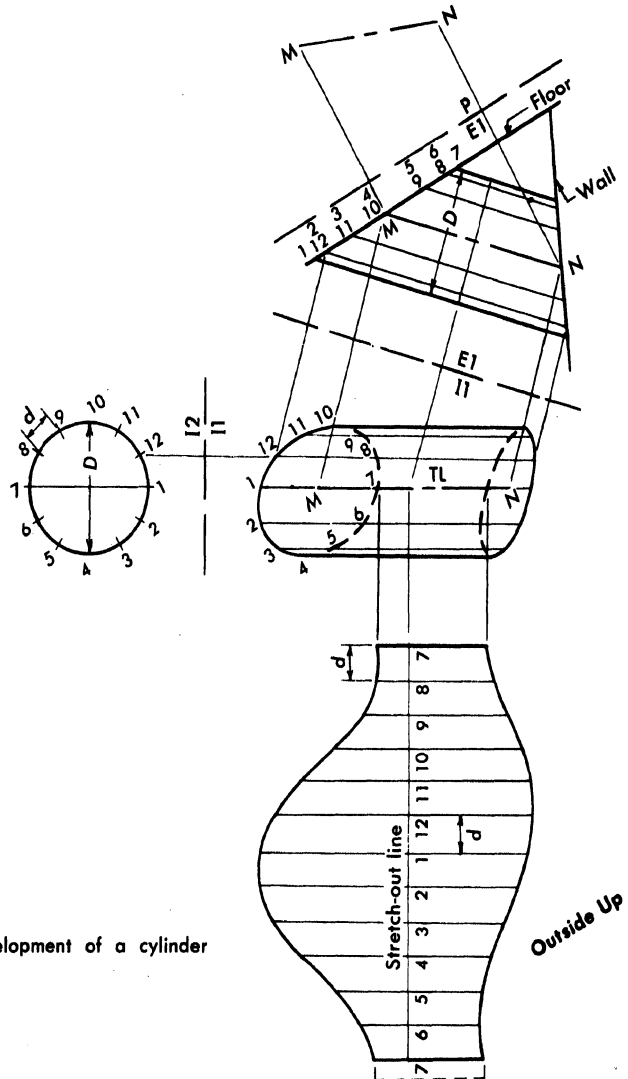


Figure 5-13 Development of a cylinder

as indicated by the dash lines. To prevent confusion, elements should be numbered in all views, including the development.

In this case, the development is purposely drawn to show the outside of the material up. This is determined by the position of two adjacent visible elements in the true-length view. For example, elements 1 and 2 have the same relative position in both the true-length view and the development.

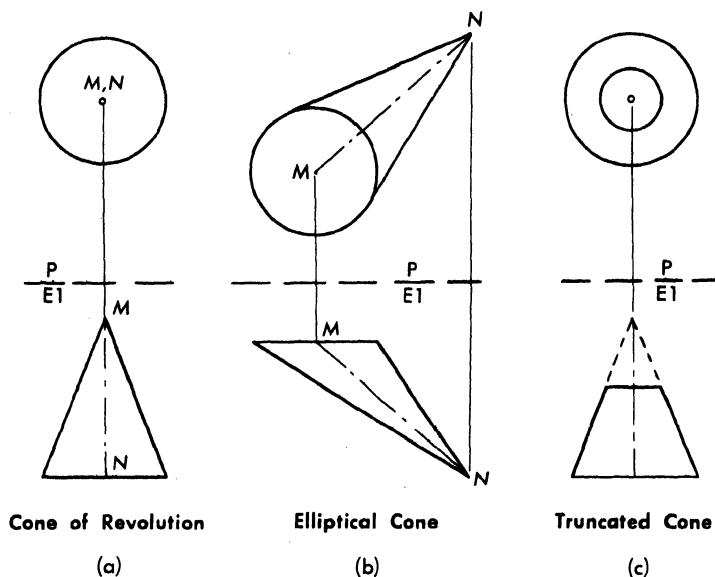


Figure 5.14 Cones

5.10 CONES

A **cone** is a surface formed by a straight line that passes through a fixed point called the **vertex** and follows a curved-line directrix. A cone may also be defined as the surface generated by a straight line moving in a curved path about a straight-line axis that it intersects at the vertex. Each of the infinite positions of the straight line generating the cone is an element of the cone.

If all elements make equal angles with the axis, a section perpendicular to the axis is a circle, and the cone is called a **cone of revolution** or a **right circular cone** (Fig. 5.14a). If the elements do not make equal angles with the axis, the right section may have many different shapes, the most common being the ellipse. If the right section is an ellipse, the cone is called an **elliptical cone** (Fig. 5.14b). When the vertex has been cut off, the cone is called a **truncated cone** (Fig. 5.14c).

If the generatrix of a cone extends through the vertex, it will generate two equivalent surfaces called **nappes** (see Fig. 5.16). In most cases, only one nappe of a cone is considered and it is usually limited by a plane section called the **base**. The vertex, the base, and two

extreme elements are usually shown to represent a cone, as indicated in Figure 5.14.

5.11 LINE PIERCING A CONE

Any given line that pierces a cone must intersect two straight-line elements of the cone. A plane that contains these two elements must also contain the given line and must pass through both the vertex and the base of the cone.

In Figure 5.15, the line AB and the cone of revolution with axis MN are given in the P and E1 views. In both views, a plane that contains both the vertex of the cone and the line is formed by drawing lines from M through A and B . In view E1, these lines strike the plane of the base at points Q and R , which are then projected into the plan view. In the plan view, the line QR crosses the base at points S and T , thus locating the two elements, MS and MT , which the plane cuts from the cone. Since both these elements and the line AB lie in the plane MQR , the points X and Y at which they intersect are the piercing points. Points S and T are projected into view E1, and elements MS and MT are drawn to locate the piercing points X and Y . The location of the piercing points can then be checked by projection between views.

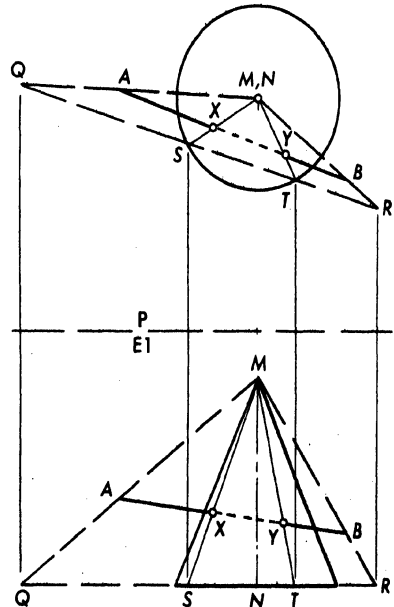


Figure 5.15 Line piercing a cone

5.12 CONIC SECTIONS

The curve cut from a cone of revolution by a plane that does not pass through the vertex is called a **conic section**. Any plane that passes through the vertex cuts straight-line elements from the cone.

Figure 5.16 shows both nappes of a cone of revolution cut by four different cutting planes.

The section cut by a plane is a *circle* if the plane, like A-A, is at right angles to the axis.

The section is an *ellipse* if the plane, like B-B, cuts all elements but is not perpendicular to the axis.

The section is a *parabola* if the plane, like C-C, is parallel to an element and cuts only one nappe of the cone.

The section is a *hyperbola* if the plane, like D-D, cuts both nappes of the cone.

The true shape of any section of a cone can be seen in the view that shows the true shape and size of the cutting plane. Sufficient points to draw the curve can be located by finding the piercing points of straight-line elements with the plane.

5.13 DEVELOPMENT OF CONE USING VERTEX

Cone of Revolution

If the base of a cone of revolution is a right section, the development will be a sector of a circle with a radius equal to the length of the elements. If a right section is not given, one should be constructed before the development is attempted.

The cone of revolution in Figure 5.17 is extended as shown by the dashed lines to form a right section, which is then divided into 12 equally spaced elements. To start the development, an arc is drawn with a radius X equal to the length of the elements of the extended cone. The circumference of the right section, divided into 12 equal spaces, is laid out along this arc, and the elements are drawn. Since the cone does not extend to this right section, its elements are revolved about the axis of the cone to determine their true lengths. The end of each element moves horizontally to the position of an extreme element, as shown for elements 4 and 10. Each of these true lengths is laid off on the corresponding element in the development. The points thus obtained are connected by a smooth curve, thus completing the development.

Figure 5-16 Conic sections

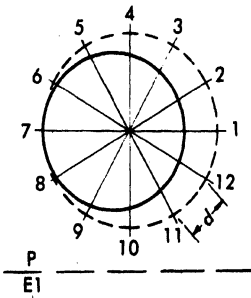
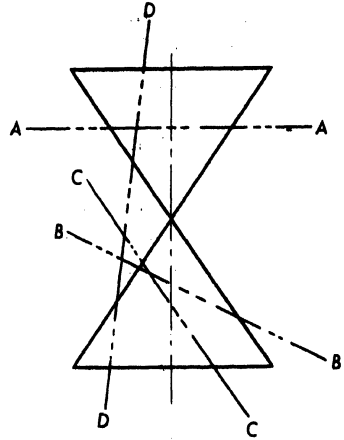
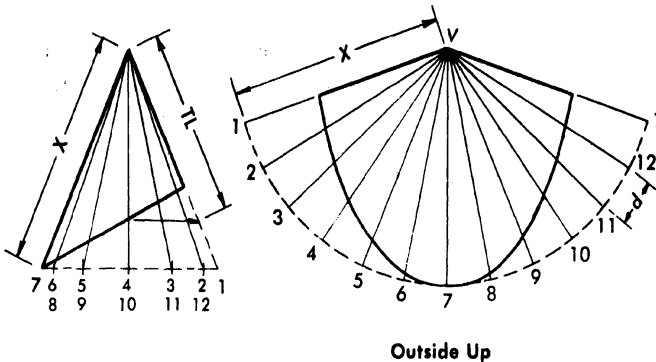


Figure 5-17 Development of a cone of revolution



Any Cone

When the given cone is not a cone of revolution, the elements are of different length, whether the base is a right section or not. This means that the preceding method cannot be used, because the base will not roll out as an arc of a circle. Therefore the cone is developed by the triangulation method, which assumes the surface to be divided by elements into a series of triangles. In the development, each triangle is constructed by using the true length of two adjacent elements, and the true distance

between the points where these elements strike the base. This method gives results that are not theoretically correct, since each of the triangles is drawn as a plane surface. However, since the error decreases as the number of elements is increased, any desired accuracy can always be obtained by using a sufficient number of elements.

In Figure 5.18, an elliptical cone with a circular base is given in the plan and elevation views. In the plan, points are established that divide the base into 16 equal spaces with a chord length equal to C . Elements are drawn from each of these points to the vertex, and their true lengths are found. To start the development, the true length of the shortest element, $V-7$, is laid out. Element $V-6$ is next located as shown, using the vertex and point 7 as centers on which to swing arcs with radii equal to the true lengths of $V-6$ and C , respectively. To locate element $V-5$, the radii used are the true lengths of $V-5$ and C , and the centers are at the vertex and point 6. This procedure is repeated until all elements have been drawn, after which the points are connected to complete the development, only half of which is shown. From the positions of any two adjacent *visible* elements in view $E1$, it is evident that the development of this cone is drawn with the inside of the material up. If the inside is to be up, the relative positions of the *visible* elements must be reversed between view $E1$ and the development. For example, element 2 is on the right of element 1 in view $E1$ and on the left of element 1 in the development.

If the given cone were truncated, the extreme elements would be extended to locate the vertex, and the above procedure followed, using the base farther from the vertex. The nearer base could then be drawn on the development by laying out the true lengths from the vertex to this base along the elements already located.

5.14 DEVELOPMENT OF CONE WITHOUT USING VERTEX

A truncated cone may be of such dimensions that the vertex cannot possibly be located on the drawing. The portion of the surface between any two adjacent elements is therefore assumed to be a trapezoid, which is then divided into two triangles by means of a diagonal. These triangles are then laid out consecutively to form the development.

In Figure 5.19, the vertex of the given truncated cone is not available within the limits of the drawing. A vertical plane is passed through the axis AB to locate element NO . Starting with points N and O , both bases are divided into the same number of equal parts to locate points that are then connected to form the elements of the cone. To

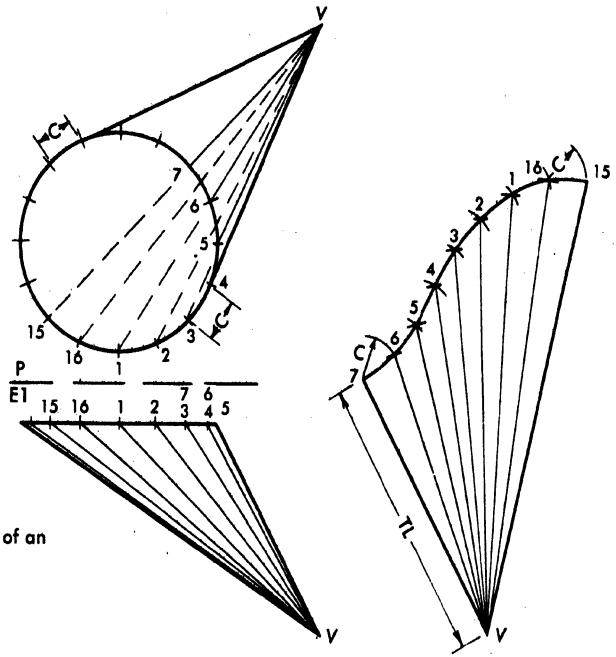


Figure 5-18 Development of an elliptical cone

Inside Up

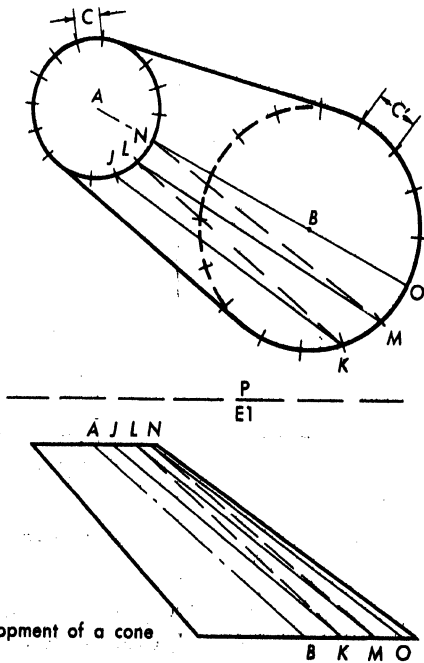
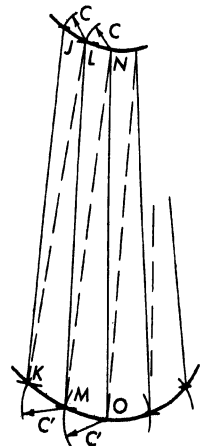


Figure 5-19 Development of a cone without using vertex



Outside Up

start the development, section $JKLM$ is divided into two triangles by the diagonal KL . Triangle JKL is laid out, using the true lengths of JK , KL , and chord distance C . Next, triangle KLM is laid out, using the true lengths of LM and chord distance C' . These steps are repeated for other sections to complete the development, of which only a portion is shown.

Although the method of revolution can be used to find true lengths of the elements, a confusing array of lines will result because each element has to be revolved about a different axis. In such cases, the true lengths are conveniently found by the following method.

5.15 TRUE LENGTH BY TRIANGULATION

The true length of any line can be found if the line is considered to be the hypotenuse of a right triangle whose base and altitude are equal to the horizontal and vertical projections of the line. The true length of the horizontal projection, called the **plan length**, can be measured in the plan view. The vertical projection is equal to the difference in elevation between the ends of the line; it can be measured in any elevation view.

Figure 5.20 True length by triangulation

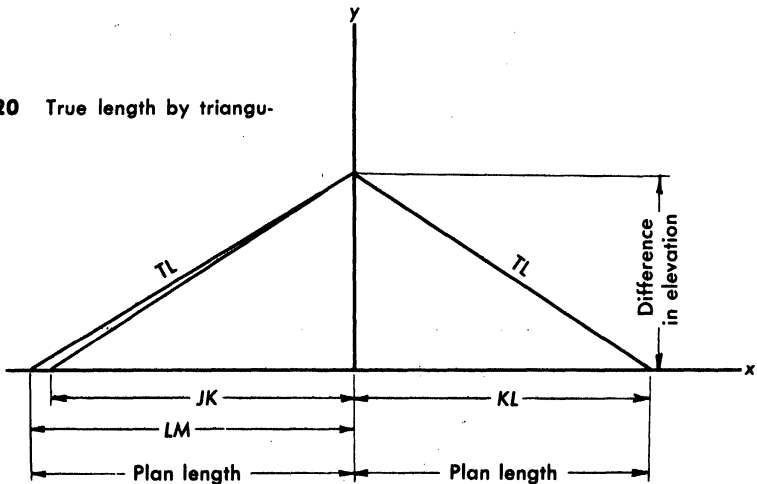
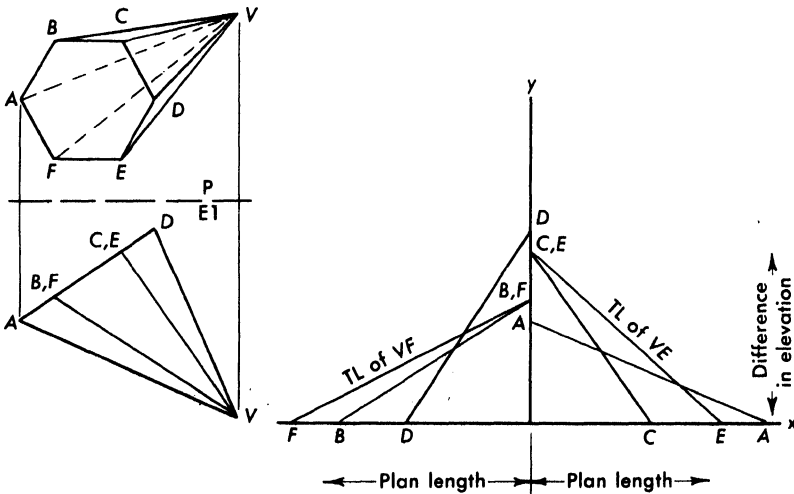


Figure 5.20 shows the procedure for determining the true lengths of lines JK , KL , and LM , used in the development of the truncated cone of Figure 5.19. The difference in elevation, which is the

same for all three lines, is laid out along the y axis, and the plan length of each line is laid out along the x axis. The hypotenuse of each right triangle thus formed gives the true length of one of the lines, as shown. The true lengths of other lines required for the development would be found in the same way.

Figure 5.21 Application of true length by triangulation



In Figure 5.21, the method is used to find the true lengths of corners of a pyramid, where the vertical projections are of different length. The plan length of each corner is laid out along the x axis, and the difference in elevation along the y axis. To keep track of the true lengths, each point is lettered as shown, so that AA is the true length of corner AV , etc.

5.16 PLANE TANGENT TO A CONE

Any plane that is tangent to a cone contains one, and only one, element of the cone. The tangent plane intersects the plane of the base along a line tangent to the base.

It is desired to draw a plane containing point *A* and tangent to the cone (Fig. 5.22). A line is drawn from the vertex through *A* in both views, giving the point *B* at which it pierces the plane of the base. In the plan, a line is drawn from *B* tangent to the base to establish point *C*, which is then projected into the elevation view. The plane *VBC* is the desired plane.

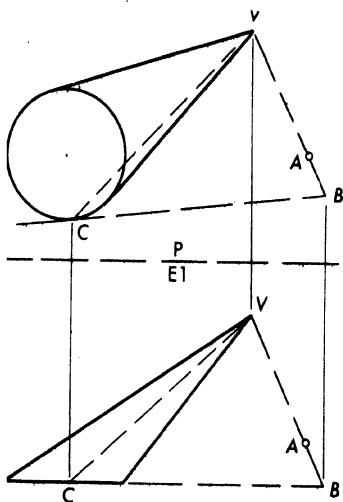


Figure 5.22 Plane tangent to a cone

Similar procedures can be followed to draw planes tangent to the cone that are parallel to lines not on the cone, or that contain points on the cone.

5.17 SPHERES

A sphere is a surface on which every point is at a fixed distance, the radius, from the center. A sphere may be generated by revolving a circle about any of its diameters. In every orthographic view, a sphere will appear as a circle whose radius is equal to the radius of the sphere.

The section cut from a sphere by a plane passing through its center is a circle with the same radius as the sphere, called a **great circle**. The section cut by a plane that does not pass through its center is a circle with a radius less than that of the sphere, called a **small circle**.

5.18 LINE PIERCING A SPHERE

The points at which a line pierces a sphere lie on a circle that is cut from the sphere by any plane containing the line. This circle may be either a great circle or a small circle. The piercing points are found by the following methods:

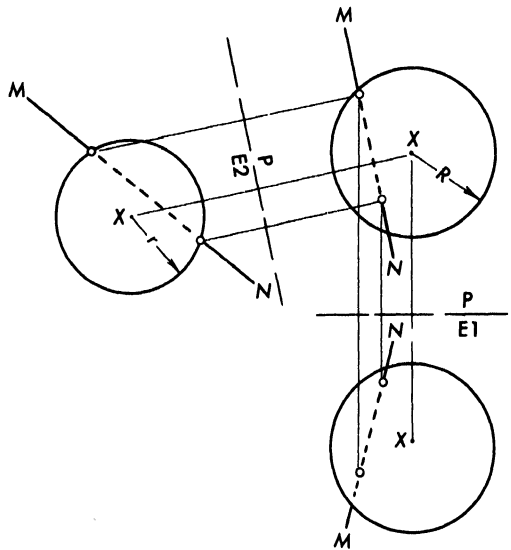


Figure 5.23 Line piercing a sphere—small-circle method

Small-circle Method

In Figure 5.23, the sphere with radius R and center X , and the line MN are given in the P and $E1$ views. A vertical plane passed through MN cuts a small circle, since it does not contain the center of the sphere. View $E2$ is drawn to show the true size of this small circle, with radius r . The points at which the line MN intersects this circle are the required piercing points, which are then projected back into the given views.

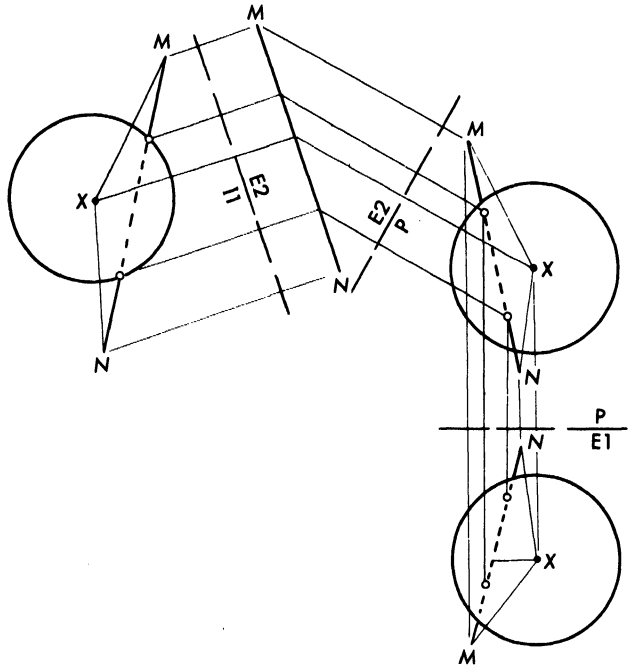


Figure 5-24 Line piercing a sphere—great-circle method

Great-circle Method

Figure 5.24 represents the same data given in the preceding example. A plane is drawn which contains MN and the center of the sphere, X , and therefore cuts a great circle from the sphere. This plane is first seen as an edge in view $E2$, and then true size in view $I1$, where the piercing points appear at the intersection of the great circle and the line MN . The piercing points are then located in the given views by projection.

5-19 APPROXIMATE DEVELOPMENT OF SPHERE

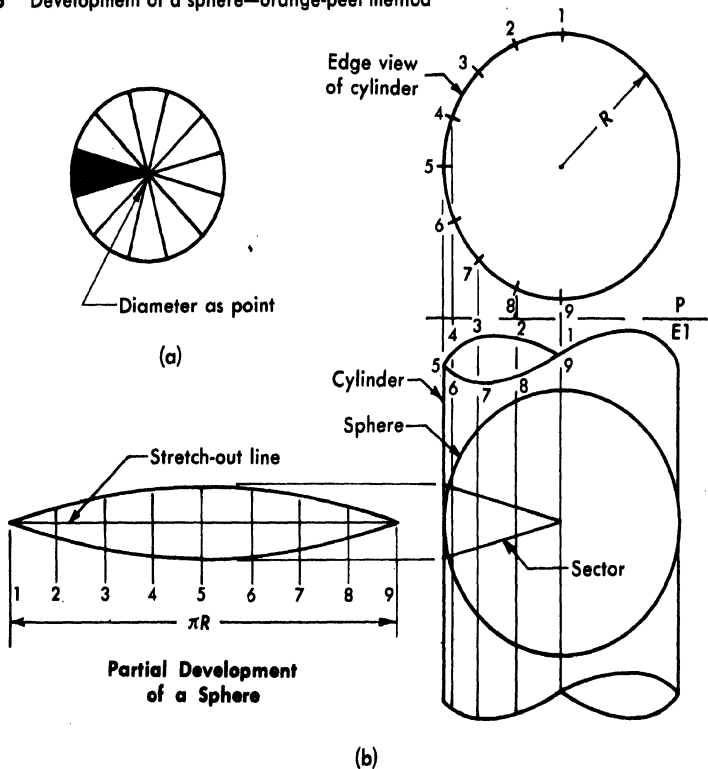
Since the sphere is a double-curved surface, it is not developable. However, a sphere may be approximately developed if it is divided into small sections, each of which is then considered as a portion of a cylinder or cone of revolution. Either of two methods may be used.

Orange-peel Method

The sphere may be divided into a number of sectors by planes passed through one diameter, as shown in Figure 5.25a.

The development of the shaded sector is shown in Figure 5.25b. The sector is assumed to be part of a cylinder, shown in the P and E1 views. A number of equally spaced elements are located on the edge view of the cylinder in the plan view and projected to the sector of the sphere in view E1. The same number of elements are established on a stretch-out line drawn at right angles to the cylinder. The lengths of the elements on the partial development are then located by projection. Because the cutting planes are equally spaced, all sectors will be of the same size and only one need be developed.

Figure 5-25 Development of a sphere—orange-peel method



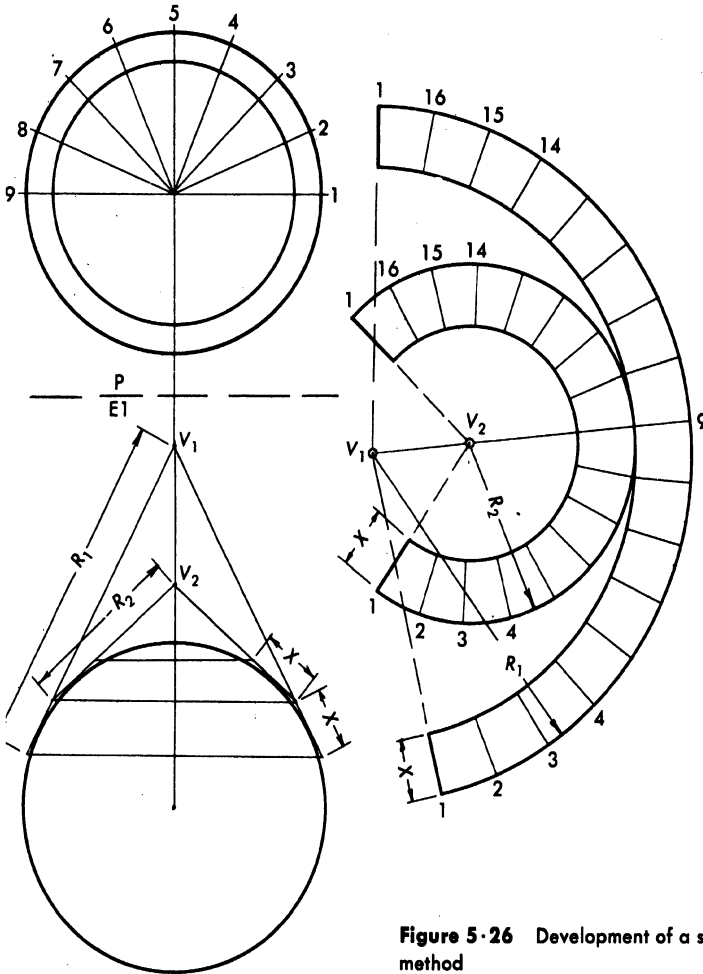


Figure 5.26 Development of a sphere—zone method

Polyconic, or Zone, Method

An approximate development can also be obtained by using parallel cutting planes to divide the sphere into segments and considering each as a portion of a cone of revolution.

The sphere in Figure 5.26 is divided into segments by horizontal cutting planes, and the approximate development of two of the segments or zones is shown. This consists of the development of two

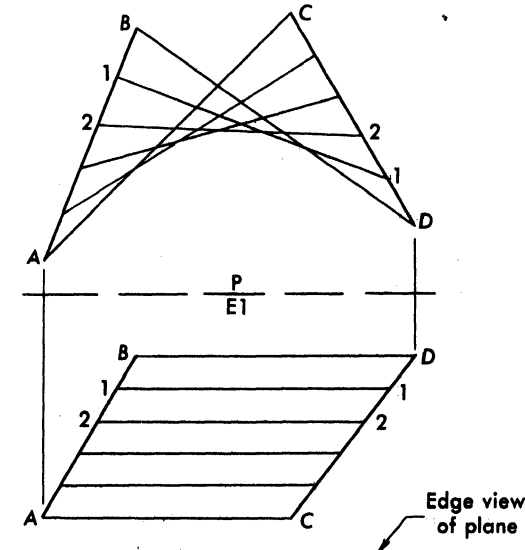
truncated cones with slanted heights equal to X , where the lengths of elements of the complete cones are equal to R_1 and R_2 respectively. Only half the sphere need be developed.

5.20 HYPERBOLIC PARABOLOID

The surface generated by a straight line that moves along two nonintersecting, nonparallel lines and at the same time remains parallel to a plane is called a **hyperbolic paraboloid** (Fig. 5.27). It has a straight-line generatrix, two straight-line directrices, and a plane director, and is a double-curved warped surface.

The hyperbolic paraboloid has been used for many years in such applications as the transition from a vertical to a sloping wall on the wing wall to a bridge abutment. In recent years, architects have increasingly designed roofs that are made up of a number of hyperbolic paraboloids. This surface gives a pleasing appearance and is very desirable for concrete construction, since the forms can be made with straight pieces of lumber.

Figure 5.27 Hyperbolic paraboloid



5.21 SUMMARY

1. A *cylinder* is a surface with parallel elements whose right section is a curved line.
2. A *cylinder of revolution* is a cylinder whose right section is a circle.
3. An *elliptical cylinder* is a cylinder whose right section is an ellipse.
4. A cylinder can be represented in any view except an edge view by showing only its two limiting elements.
5. The points at which a line pierces a cylinder are the points where the line intersects elements of the cylinder.
6. The shape of the *plane section* of a cylinder is determined by locating a series of points at which elements of the cylinder pierce the plane.
7. The true shape of a plane section of a cylinder of revolution not perpendicular to the elements is an ellipse with a minor axis equal to the diameter of the cylinder. The major axis of the ellipse is found in a view that shows the true length of the cylinder and the edge of the plane that cuts the section.
8. The *development of a surface* shows the shape of the surface as it would appear if laid out on a plane. The development should show the surface as if cut on the shortest element, and should always be marked to show whether the inside or the outside is up.
9. The development of a cylinder should always be made from a true-length view of the cylinder.
10. A *cone* is the surface formed by a straight line moving in a curved path about a straight-line axis which it intersects at the *vertex*. If all elements make equal angles with the axis, the cone is a *cone of revolution* and its right section is a circle.
11. The points at which a line pierces a cone are on elements of the cone and are found by means of a cutting plane that contains both the given line and the vertex of the cone.
12. *Conic sections* are curves cut by planes passing through a cone of revolution. The position of the cutting plane determines whether the section is a circle, an ellipse, a parabola, or a hyperbola.
13. The development of a cone requires the true length of all elements and is often made by triangulation.
14. A plane tangent to a cone contains one element of the cone and intersects the plane of the base along a line tangent to the base.
15. A *sphere* is a surface formed by revolving a circle about one of its diameters. The section cut from a sphere by a plane is a

great circle if the plane passes through the center of the sphere, and a *small circle* if it does not.

16. The points at which a line pierces a sphere can be found by either the great-circle method or the small-circle method.

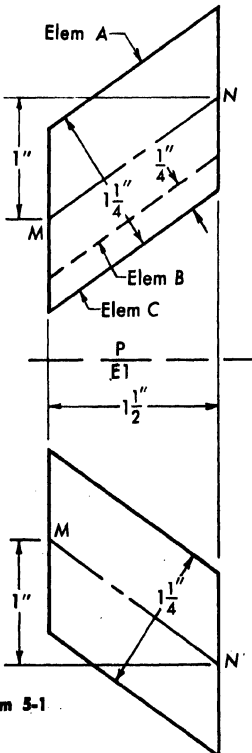
17. A sphere may be *approximately* developed if it is divided into small sections, each of which can then be developed as a portion of a cylinder or cone of revolution.

18. A *hyperbolic paraboloid* is a warped surface formed by a straight line moving along two nonintersecting, nonparallel lines while remaining parallel to a plane.

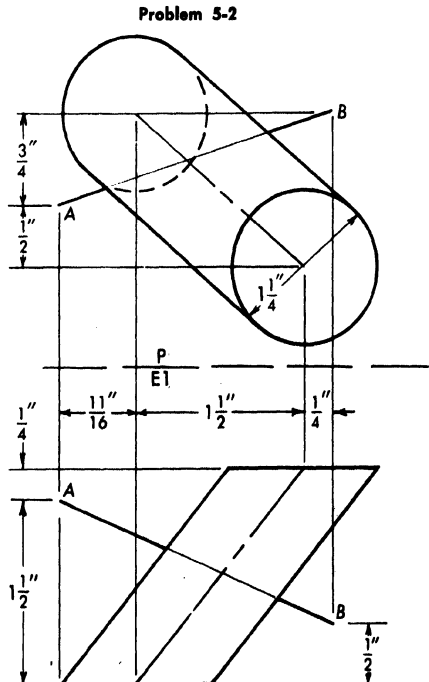
5-22 Problems

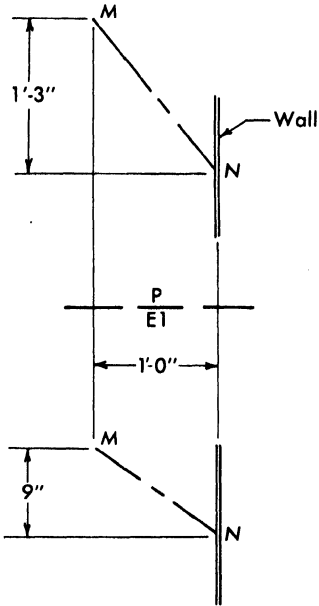
5-1. The given cylinder is a cylinder of revolution. Show elements *A*, *B*, and *C* in view E1. Scale: Full size.

5-2. Using only the given views, find where line *AB* pierces the cylinder. Show the piercing points in both views. Scale: Full size.

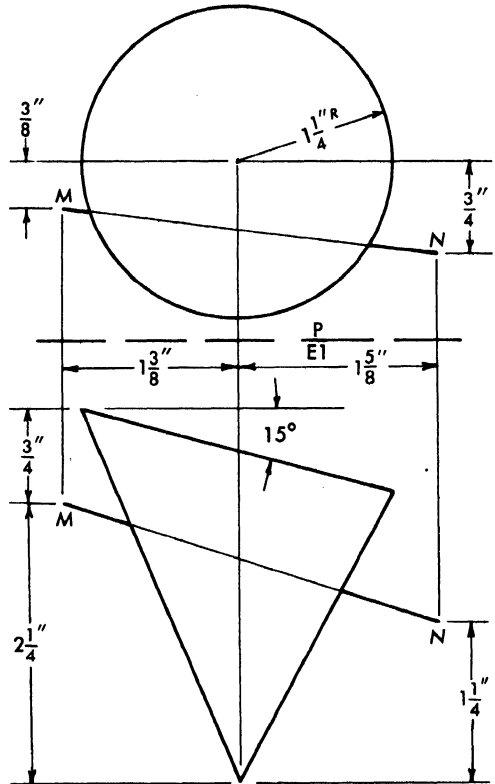


Problem 5-1





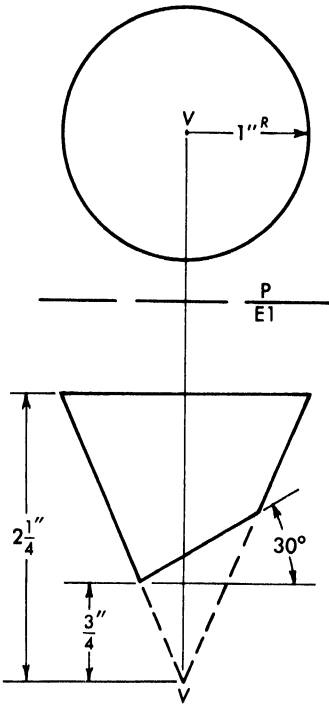
Problem 5-4



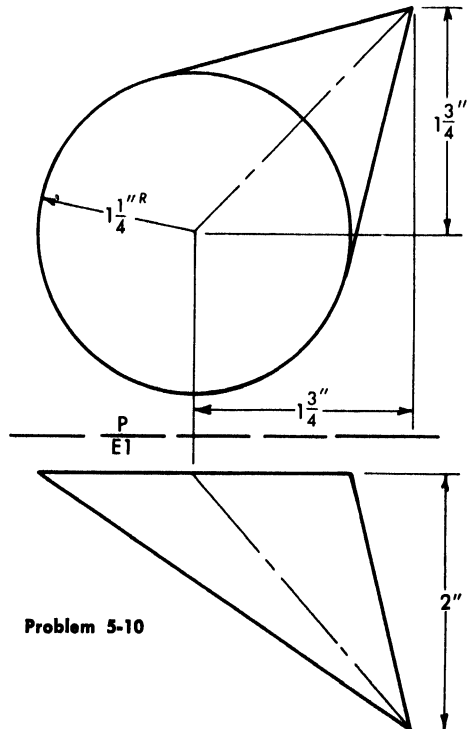
Problem 5-7

- 5-3. Use the data of Problem 5-2. Draw all possible planes that contain point *B* and are tangent to the cylinder. Show in both given views the elements at which these planes contact the cylinder. Scale: Full size.
- 5-4. Line *MN* is the axis of a cylinder of revolution 15 in. in diameter. Draw a view showing the true shape and size of the section of the cylinder cut by the wall. Scale: 1 in. = 1 ft 0 in.
- 5-5. Develop the cylinder of Problem 5-2. Show the inside of the cylinder up. Scale: Full size.
- 5-6. Use the data of Problem 5-4. Develop the portion of the cylinder between the wall and a right section through *M*. Show the inside of the cylinder up. Scale: 1 in. = 1 ft 0 in.
- 5-7. Find the points at which line *MN* pierces the given cone. Scale: Full size.

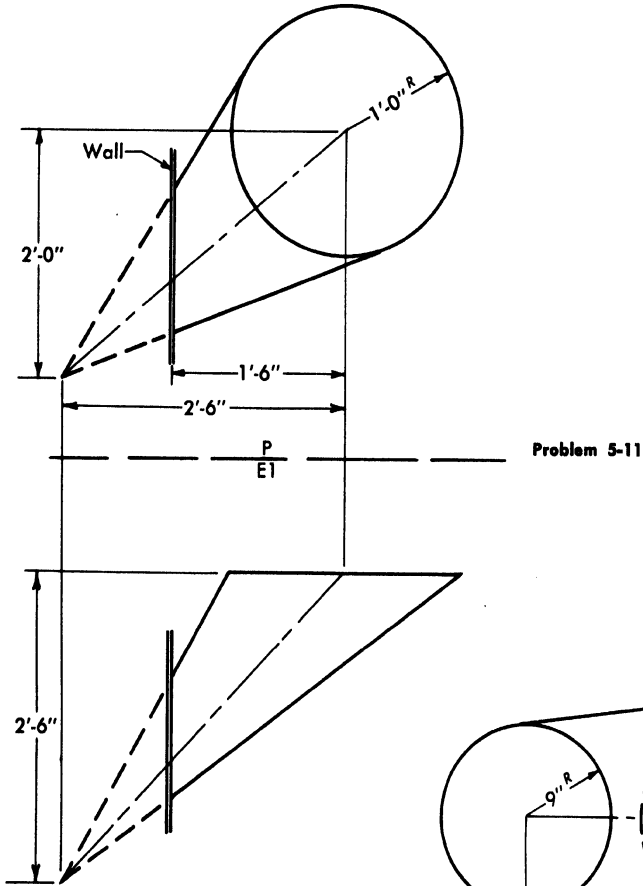
- 5-8.** Using the data of Problem 5-7, draw a plane that contains point *M* and is tangent to the given cone. Scale: Full size.
- 5-9.** Develop the given truncated cone of revolution. Scale: Full size.
- 5-10.** Develop the given elliptical cone. Scale: Full size.



Problem 5-9

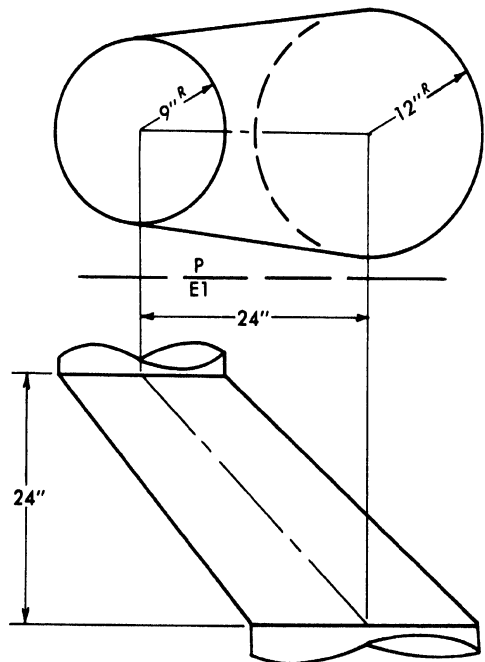


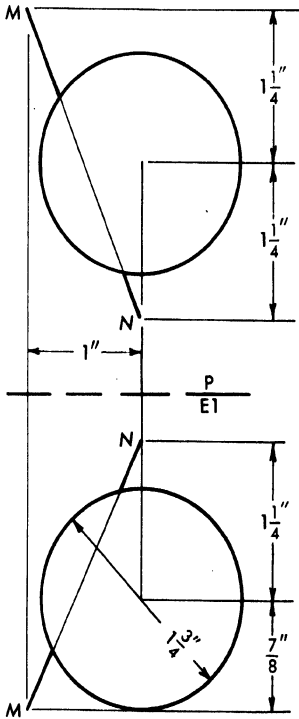
Problem 5-10



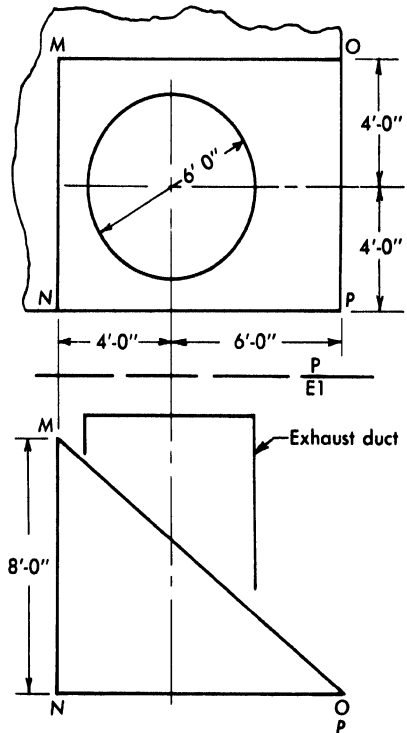
Problem 5-11

Problem 5-12





Problem 5-13

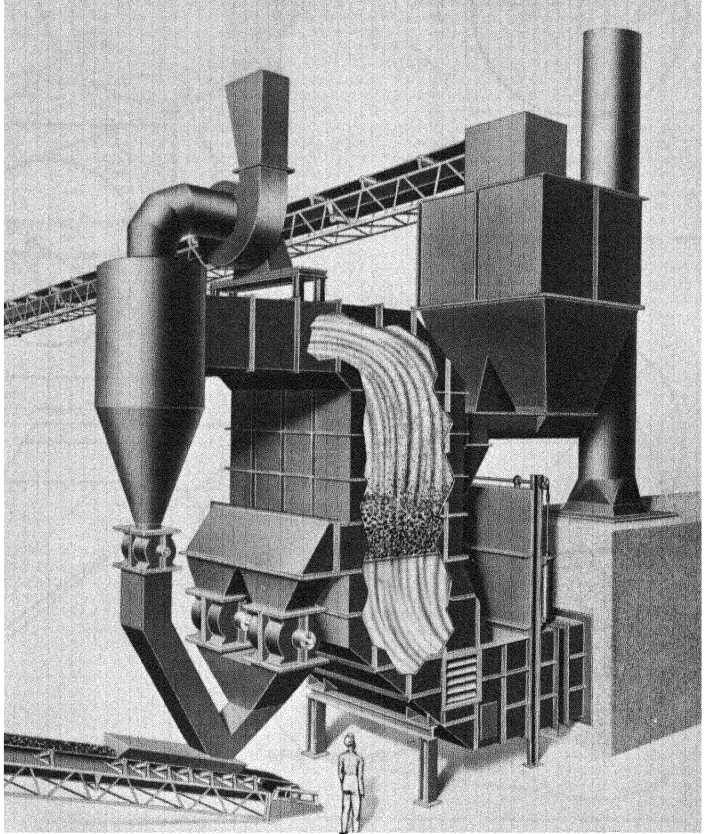


Problem 5-14

- 5-11. Draw a view showing the true size of the end of the conical hopper which fits against the wall. Draw a development of the hopper. Scale: 1 in. = 1 ft 0 in.
- 5-12. Develop the given offset reducer. Scale: 1 in. = 1 ft 0 in.
- 5-13. Find the points at which line MN pierces the given sphere, using the small-circle method. Check by the great-circle method. Scale: Full size.
- 5-14. Points M , N , O , and P are four points on a hyperbolic-paraboloid roof. An exhaust duct for a ventilation system extends through the roof. Show the line of intersection between the duct and the roof in the $E1$ view. Scale: $\frac{1}{4}$ in. = 1 ft 0 in.

FLUID-FLO DRYER HAS MANY INTERSECTING SURFACES.

(Courtesy Link-
Belt Company)



CHAPTER 6 Intersection of Surfaces

*The basic principles and methods for determining
the intersection of curved surfaces*

6.1 INTRODUCTION

Situations involving the intersection of surfaces occur in various types of construction. For example, in heating and ventilating work, ducts of many shapes and sizes must be fastened together. When any two or more pieces are to be joined, the line of intersection between the surfaces must be located accurately to insure a proper fit for the developed surfaces.

Many similar situations of more or less complexity exist in the various branches of industrial construction. Aircraft, space missiles, chemical retorts, pumping machinery, and pipelines are only a few examples.

Since there are many types of surfaces, there is obviously an extremely large number of possible problems involving intersections. This chapter deals primarily with the intersection of the more commonly used types of surfaces. If the basic principles are thoroughly understood, they can be extended to cover other types of intersections.

6.2 PLANE SURFACES

The intersection of any two plane surfaces can be found by the methods discussed in Section 3.12.

The intersection of a plane surface with any other type of surface may be found by using an edge view of the plane to locate the points at which elements of the other surface pierce the plane. These points can then be projected to any other view and connected to form the curved line of intersection. Another approach is to pass a series of cutting planes through both surfaces. Each of these will cut straight-line elements from the plane and either straight or curved elements from the other surface. The points at which these elements intersect are points on the required line of intersection. In this method, the cutting planes should be chosen so they will cut the simplest type of element from the non-planar surface.

The plane sections of cylinders, cones, and spheres discussed in Chapter 5 are examples of the intersections of planes with curved surfaces.

6.3 TWO CYLINDERS

The curved line of intersection of any two cylinders connects points that lie on elements of both cylinders. This line of intersection may be determined by either of the following methods:

Piercing-point Method

Straight-line elements of one cylinder will pierce the other cylinder on the line of intersection of the two cylinders. These piercing points can be found by either method of Section 5.3. This approach is preferred when the line of intersection is to be shown in the two given views and no development is required.

Cutting-plane Method

Using only the axes of the cylinders, a view is drawn to show the true length of both axes. Views that show each axis as a point are then obtained, and the edge view of each cylinder is drawn. In each edge view, cutting planes are drawn parallel to the axes of both cylinders. The line of intersection of the two cylinders is determined by the elements cut by these planes.

In Figure 6.1, the axes AB and CD of two cylinders are given in the P and E1 views. A line parallel to AB is drawn through D in both views, thus forming a plane which contains CD and is parallel to AB . The horizontal line CX is established and view E2 is drawn to show plane CDX as an edge. Since AB is parallel to plane CDX , view I1 shows the true length of both AB and CD . The point views of AB and CD are found in views I2 and I3, respectively, and the edge views of the cylinders are drawn. For purposes of illustration, one cylinder is shown as a cylinder of revolution and the other as an elliptical cylinder.

To find the line of intersection, a series of planes are passed through the cylinders in views I2 and I3. To make sure the same planes are used in both views, each plane is drawn parallel to, and the same distance from, the rotation line in views I2 and I3. The elements cut by these planes are projected into the true-length view, I1, where they intersect at points on the line of intersection of the cylinders. As shown, the elements cut by plane R intersect at points 1 and 2, and the elements cut by plane S intersect at points 3, 4, 5, and 6. The cutting planes used to

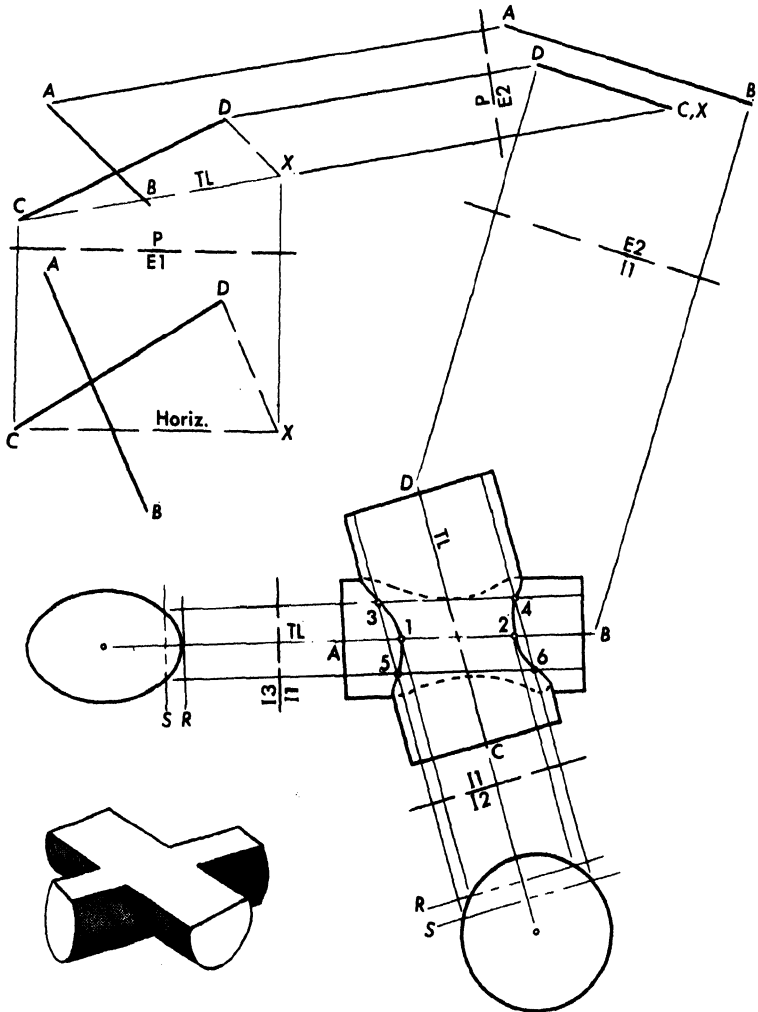


Figure 6-1 Intersection of two cylinders—cutting-plane method

locate the remaining points for the curve of intersection are omitted for clarity, but the entire curve of intersection is shown.

This method is preferred when the development of the cylinders is required, since the view in which the line of intersection is found shows the true length of the elements of both cylinders. The developments may then be projected directly from the true-length view.

6.4 TWO CONES

The line of intersection of two cones is found by locating a series of points at which elements of the two cones intersect. A plane can be passed through the vertexes of both cones so that it cuts straight-line elements from each cone (Fig. 6.2).

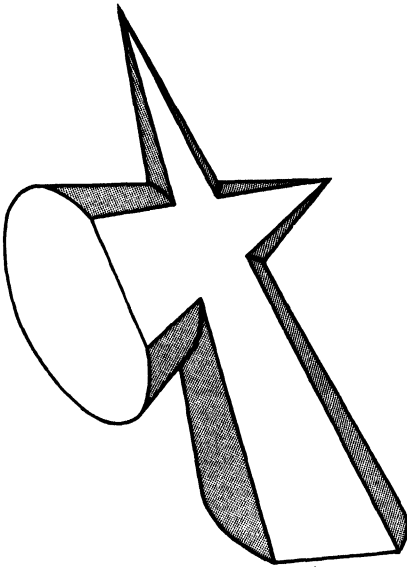


Figure 6.2 Plane cutting two cones

Since these elements lie on the same plane, they must intersect on the line of intersection of the two cones.

The various cases require use of different methods, as follows:

Bases in the Same Plane

Any cutting plane that passes through the vertexes of the two cones must contain the line connecting the two vertexes. This line can be extended to find the point at which it pierces the plane containing the bases of the two cones. The lines of intersection between the base plane and all cutting planes must pass through this piercing point. These lines of intersection can be used to locate elements on both cones.

In Figure 6.3, two cones are given in plan and elevation views, with bases in a horizontal plane. A line drawn through the vertexes M and N is found to pierce the base plane at point O . In the plan view, any line drawn from O through the bases represents the intersection of the base with a cutting plane that cuts elements from each cone. The line shown through limiting element Q establishes a plane that cuts two ele-

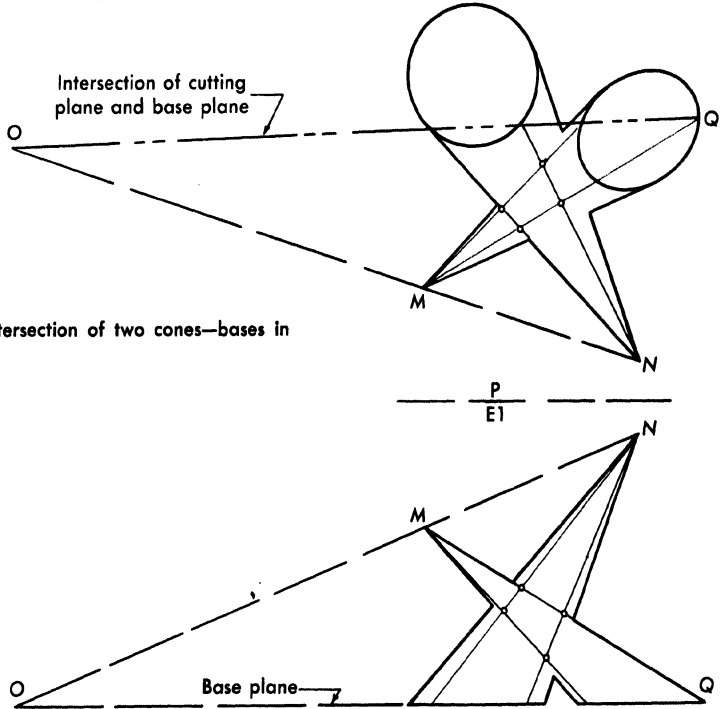


Figure 6.3 Intersection of two cones—bases in same plane

ments from each cone. The four points at which these elements intersect are four points on the line of intersection of the two cones. Although only four points are shown in Figure 6.3, sufficient points to locate the line of intersection would be found by using additional cutting planes. These cutting planes should include planes selected so as to locate points at which the curve of intersection is tangent to the extreme elements.

Bases in Parallel Planes

If the base planes of the two cones are parallel, one base plane may be extended to cut across the other cone, thus reducing the problem to the preceding situation.

Bases in Nonparallel Planes

As before, cutting planes are passed through the vertexes of both cones. When the bases are in different planes, the point at which the line of vertexes pierces each base plane must be found in order to draw the proper cutting planes.

In Figure 6.4, two cones with bases in nonparallel planes are shown in plan and elevation views. A line through the vertexes M and N pierces the horizontal base plane at O and the vertical plane at S . In the plan view, a line is drawn from O to any point T on the intersection of the horizontal and vertical base planes. The cutting plane that contains the vertexes and line OT intersects the vertical base plane along the line

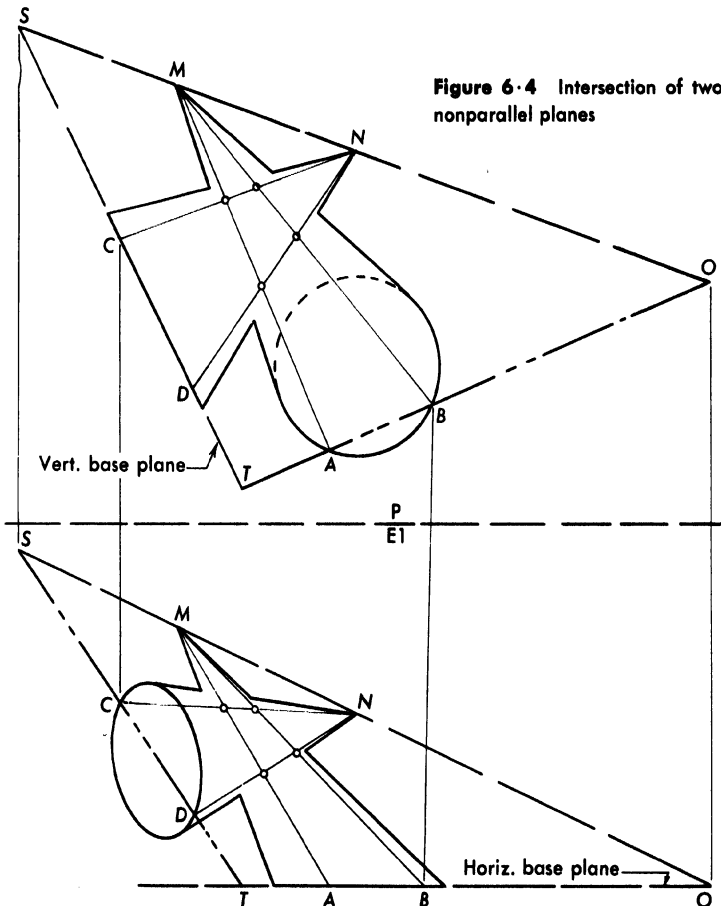


Figure 6.4 Intersection of two cones—bases in nonparallel planes

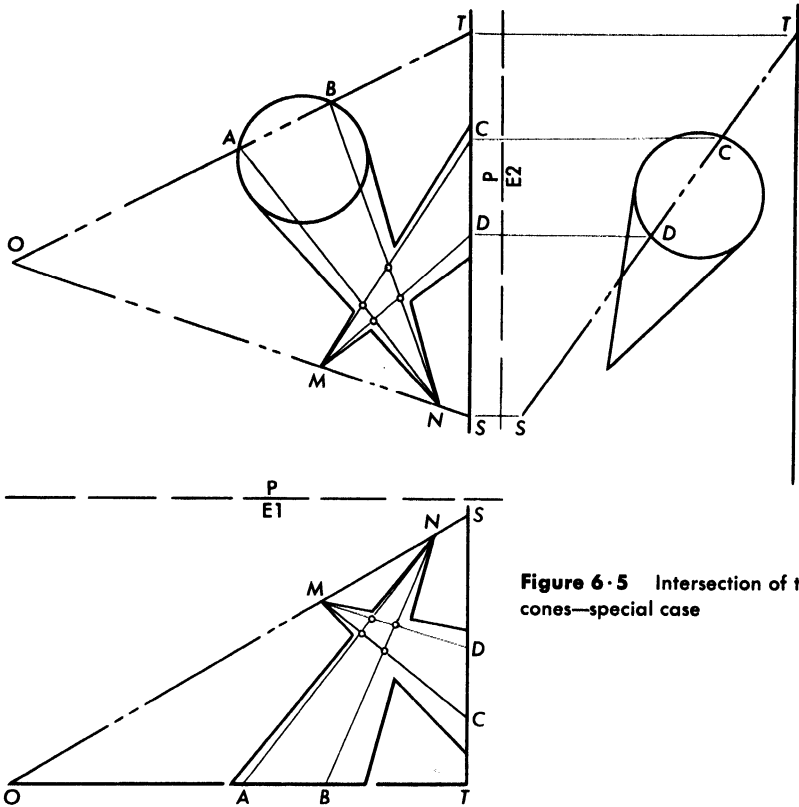


Figure 6-5 Intersection of two cones—special case

ST. The elements *AM* and *BM* cut from one cone are located by line *OT* in the plan view and projected to the elevation view. Similarly, elements *CN* and *DN* are located by line *ST* in the elevation view and projected to the plan. These elements intersect at points on the line of intersection. Additional cutting planes would be used to locate enough points to draw the line of intersection.

In the above example, the base plane of each of the cones is shown as an edge in one view. If the base of either cone does not appear as an edge in one of the given views, a plane appearing as an edge could be drawn in either view. This plane would then be used as a base plane to find the line of intersection.

Special Case, Bases in Nonparallel Planes

As shown in Figure 6.5, the base of one of the cones may be given so that it appears as an edge in both given views, *P* and *E1*.

The line of vertexes is found to pierce the horizontal base plane at O and the vertical base plane at S . When line OT is drawn on the horizontal base plane, the cutting plane thus constructed establishes line ST on the vertical base plane. To locate point T in the plan view, a third view is drawn with lines of sight in any convenient direction. In this third view, E2, only the base of cone M , the points S and T , and the horizontal base plane need be shown. Line ST is drawn in view E2, thus locating points C and D , which are projected back into the given views. The intersections of elements CM , DM , AN , and BN determine four points on the line of intersection of the cones.

This method could also be used when the base plane does not appear as an edge in both given views. For example, in Figure 6.4, if the vertical plane had been more nearly perpendicular to the rotation line, the base of the cone might have been so narrow in the elevation view that points C and D could not be located accurately by projection.

If, in Figure 6.5, both cones had been completely shown in view E2, the entire line of intersection could have been located in the P and E2 views. This would have reduced the problem to the same type as that shown in Figure 6.4.

6.5 CONE AND CYLINDER

The intersection of a cone and a cylinder is found by using cutting planes that cut straight-line elements from both surfaces. These cutting planes must contain the vertex of the cone and must be parallel to the axis of the cylinder. The points at which elements of the cone and cylinder intersect can be found most easily by drawing a view showing the cylinder as an edge. Since this view shows the point view of elements of the cylinder, the points at which the elements intersect the cone are readily apparent.

In Figure 6.6, the P and E1 views of a cone and cylinder are given. Since the axis of the cylinder appears true length in the plan view, view E2 is drawn to show the cylinder as an edge. In this view, a cutting plane that appears as an edge is drawn through the vertex of the cone as shown. This plane cuts elements VA and VB from the cone, and elements CD and EF from the cylinder. These elements are located by projection in the plan view, where they cross at points on the lines of intersection of the cone and cylinder. The cutting planes used to locate additional points are omitted for clarity, but the lines of intersection are shown.

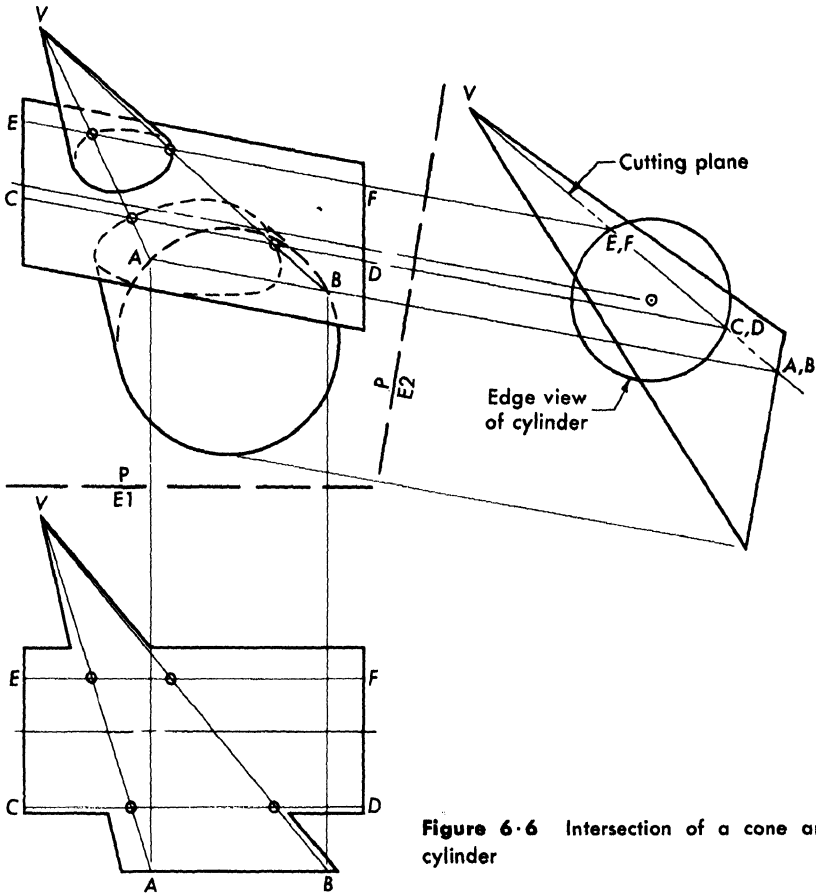


Figure 6.6 Intersection of a cone and a cylinder

When one base of the cylinder lies in the same plane as the base of the cone, the line of intersection can be located in the two given views. Cutting planes through the vertex of the cone and parallel to the axis of the cylinder are used to locate intersecting elements of the cone and cylinder. Since this procedure is exactly the same as the cutting-plane method for locating the points at which a line pierces a cone (Sec. 5.11) the solution is not shown.

6.6 SOME OTHER SURFACES

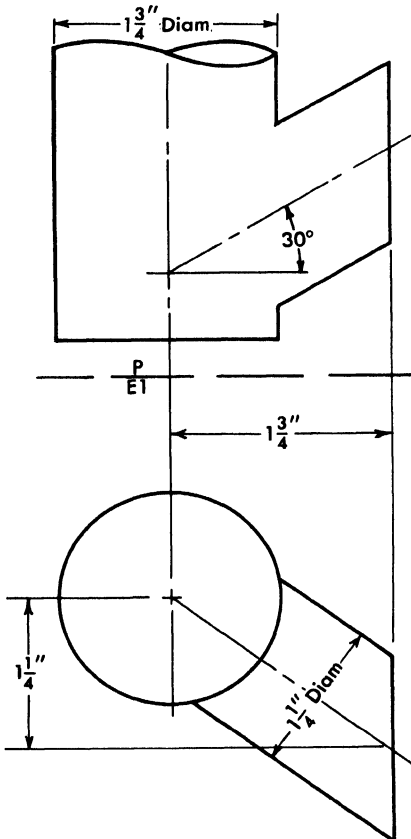
The methods described in this chapter for finding intersections of surfaces are sufficient for most purposes. On the basis of the

principles learned for these surfaces, the student can easily determine methods for finding intersections of other types of surfaces. Several examples of methods for other surfaces are as follows:

- (a) *Two prisms*: Same method as for two cylinders.
 - (b) *Two pyramids*:
 - (c) *Cone and pyramid*:
 - (d) *Cone and prism*:
 - (e) *Pyramid and prism*:
 - (f) *Pyramid and cylinder*:
- } Same method as for two cones.
- } Same method as for cone and cylinder.

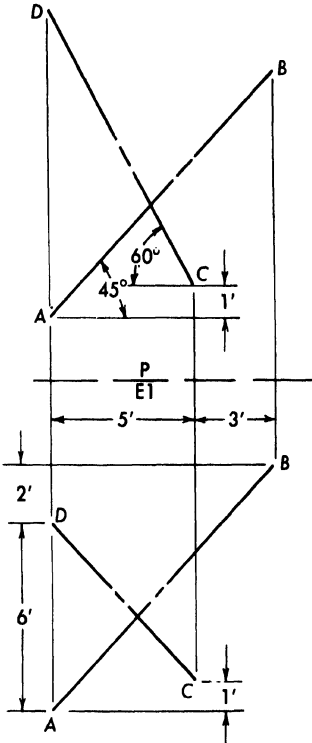
6.7 Problems

- 6-1.** Find the line of intersection of the two given cylinders and complete the plan view. Scale: Full size.

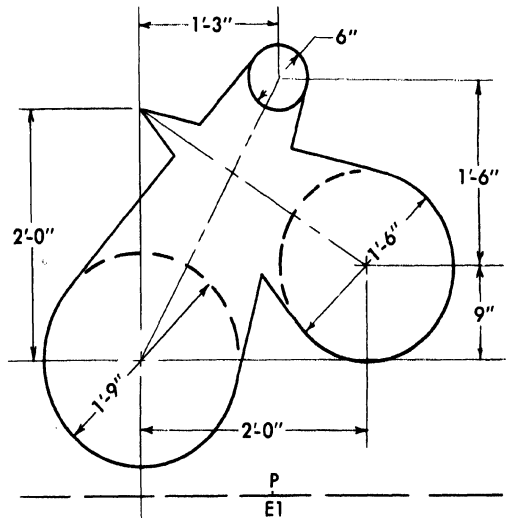


Problem 6-1

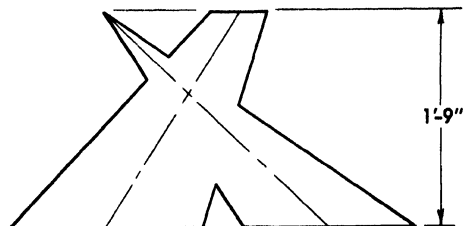
- 6-2.** The given lines AB and CD are the axes of two cylinders, with diameters of 6 ft and 5 ft, respectively. Draw a view showing the true length of both axes. In this view only, show the intersection of the two cylinders. Scale: 1 in. = 4 ft.
- 6-3.** Show the intersection of the two cones in both given views. Scale: 1 in. = 1 ft 0 in.



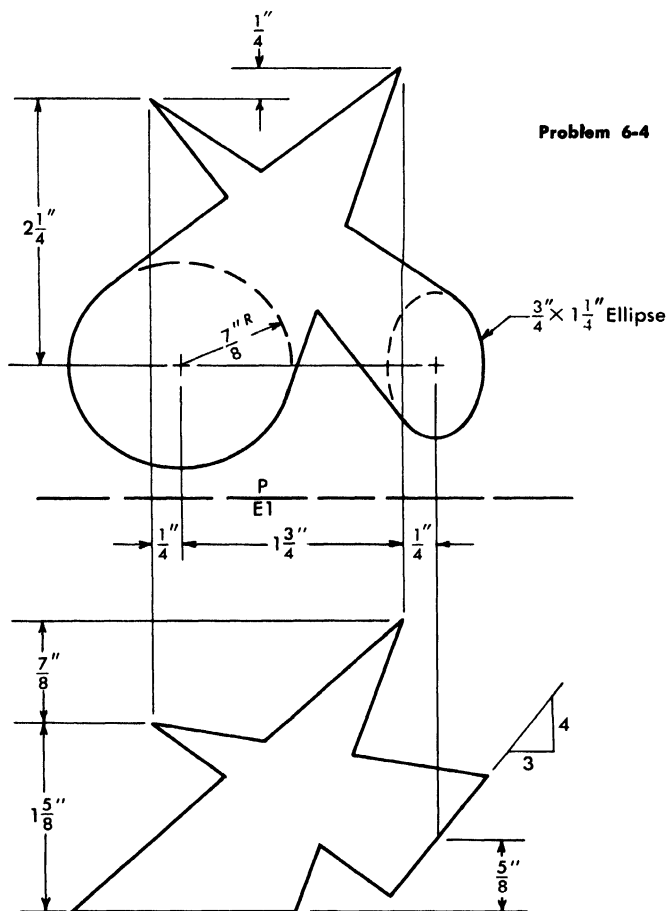
Problem 6-2



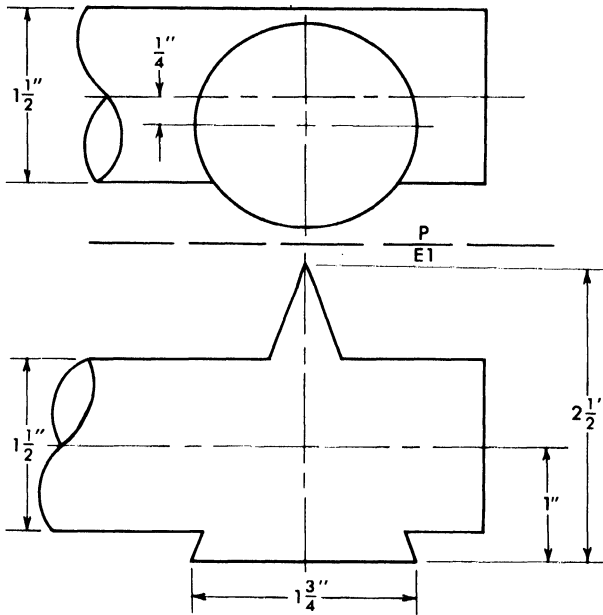
Problem 6-3



- 6-4. Show the intersection of the two cones in both given views. Scale: Full size.

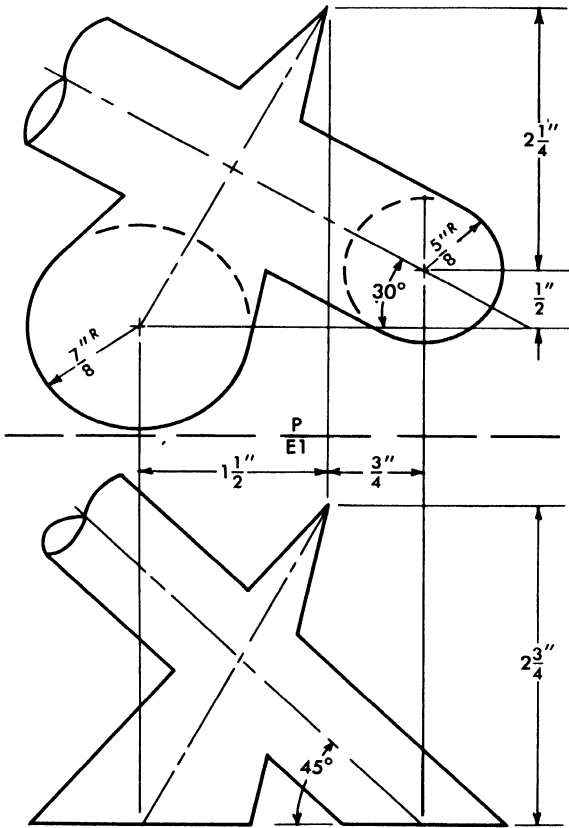


- 6-5. Find the line of intersection of the given cone and cylinder of revolution, and complete view E1. Scale: Full size.



Problem 6-5

- 6-6.** Find the line of intersection of the given elliptical cone and cylinder. Complete both given views. Scale: Full size.



Problem 6-6

CHAPTER 7

TWO PIPES CONNECTED BY THIRD PIPE USING 45-DEGREE ELBOWS.



CHAPTER 7 Locus of a Line

*Procedures for finding the locus of a line making
specified angles with given planes or lines*

7.1 INTRODUCTION

The **locus** of a point or line is the assemblage of all the possible positions of the point or line that satisfy a given set of geometric conditions. The locus of the point or line might also be defined as the line or surface formed when the point or line moves in a specified manner. For example, the locus of a point in a given plane at a certain distance from another point is a circle, and the locus of a straight line parallel to, and at a specified distance from, another straight line is a cylinder of revolution.

7.2 LOCUS OF A LINE MAKING GIVEN ANGLES WITH TWO GIVEN PLANES

The locus of a line that makes a specified angle with a given plane is a cone of revolution with its axis perpendicular to the plane. Each of the possible positions of the line is an element of the cone. If the line is to make a specified angle with some other plane, the locus of the line will again be a cone of revolution.

. **Note:** *The two cones must have the same vertex and elements of equal length, since both cones are formed by the same line.*

The elements common to both cones satisfy both given conditions, or, in other words, these elements are the only possible positions of the required line.

In Figure 7.1, a line 2 ft long is to be drawn in a southerly direction from *M* so that it makes an angle of 45 degrees with the given vertical plane and an angle of 30 degrees with the horizontal plane. In the plan view, cone 1 is constructed so that its extreme elements are 2 ft long and make an angle of 45 degrees with the given vertical plane. The base of this cone appears as an edge parallel to the vertical plane, and therefore appears as a circle in the elevation view E1. Similarly, cone 2 is constructed in the elevation view so that its extreme elements make an angle of 30 degrees with a horizontal plane. This cone also has elements 2 ft long. The base of cone 2 is horizontal and appears as a circle in the plan view.

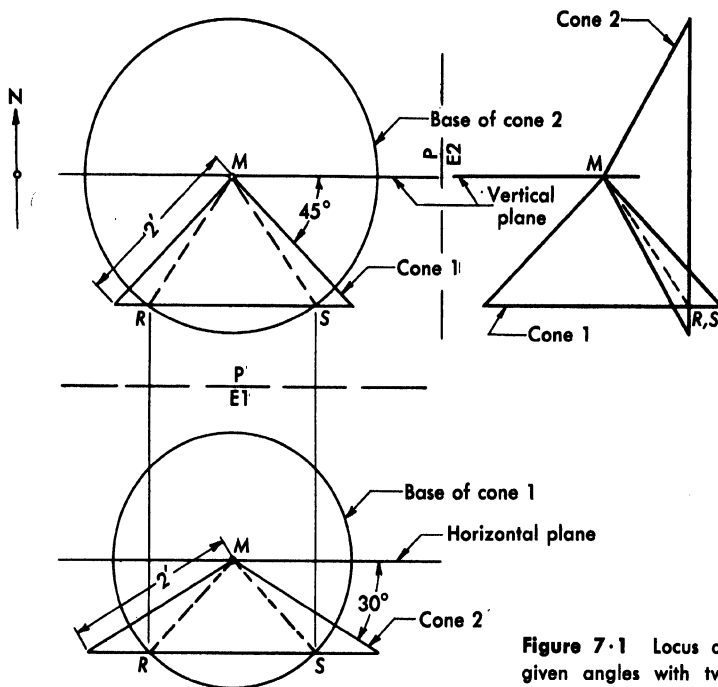


Figure 7.1 Locus of a line making given angles with two perpendicular planes

. **Note:** The diameter of each circle is equal to the distance across the base and not to the length of the elements.

Points R and S, at which each circle crosses the base of the other cone, are the points at which the two bases intersect. These points should check by projection between views. Lines MR and MS are elements of both cones and are therefore the two possible positions of lines that satisfy all the given conditions. If the conditions had specified that the line extend in a southeasterly direction, line MS would be the only possible solution.

The line could also be located by drawing view E2 showing the vertical plane as an edge. In this view, the bases of both cones appear as edges and their intersection is apparent.

When the two given planes are not perpendicular to each other, a view should be drawn to show both planes as edges. In Figure 7.2, the point M and the inclined plane ABC are given in the P and E1 views. A line is to be drawn from M so that it has a slope of 45

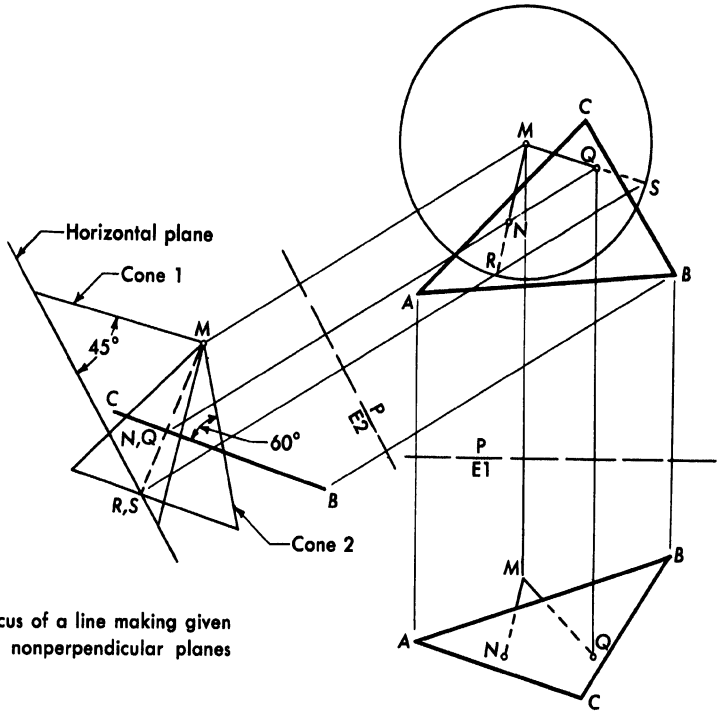


Figure 7.2 Locus of a line making given angles with two nonperpendicular planes

degrees and makes an angle of 60 degrees with plane ABC . View E2 is drawn to show both plane ABC and a horizontal plane as edges, and point M is located in this view. In view E2, the two cones are drawn with elements of any convenient length. Cone 1 has elements that make an angle of 45 degrees with horizontal, and cone 2 has elements that make an angle of 60 degrees with plane ABC . Since the bases of both cones appear as edges in this view, the points R and S at which they intersect are apparent. The base of cone 1 is drawn in the plan view and the points of intersection are projected to this circle. Lines MR and MS are the two possible lines that satisfy the given conditions. The points N and Q at which these lines pierce plane ABC are clearly seen in view E2, where ABC appears as an edge. Note that the elements of the cones can be any length as long as they are the same length on both cones. If, in Figure 7.2, the line had been required to make the 45-degree angle with some plane other than a horizontal plane, an additional view would have been necessary to show both given planes as edges in the same view.

7.3 LOCUS OF A LINE MAKING GIVEN ANGLES WITH TWO GIVEN LINES

The given lines may be intersecting, parallel, or non-intersecting and nonparallel.

Intersecting Lines

The two intersecting lines define a plane in which the required line must lie. To make a solution possible, the sum of the two given angles must be equal to the supplement of the angle between the two given lines.

Parallel Lines

The required line must lie in the plane defined by the parallel lines. A solution is possible only if the two given angles are equal (or supplementary) angles.

Nonintersecting, Nonparallel Lines

In any plane, a given line makes equal angles with all parallel lines. Therefore, any line that makes a given angle with one of the given lines will make the same angle with a parallel line through the other given line. This principle makes it possible to use two cones of revolution with the same vertex when the two given lines do not intersect.

In Figure 7.3, the lines AB and CD , given in the P and E1 views, are to be connected by a line that makes angles of 51 degrees with AB and 40 degrees with CD . The cones that show the possible positions of lines making these angles with the given lines can be drawn only in a view that shows both lines in their true length. Therefore the plane CDX is constructed parallel to AB and is found as an edge in view E2. View I1 is then drawn to show both AB and CD in true length. Point V is selected at *any* point on line AB , and a 51-degree cone is constructed with AB as an axis and V as a vertex. A line parallel to CD is drawn through V , and a 40-degree cone is constructed with this line as an axis and V as a vertex. View I2 is drawn to show AB as a point and the base of the 51-degree cone as a circle. The bases of the two cones are found to intersect at R and S , thus determining the elements VR and VS common to both cones. It can now be seen that neither VR nor VS actually inter-

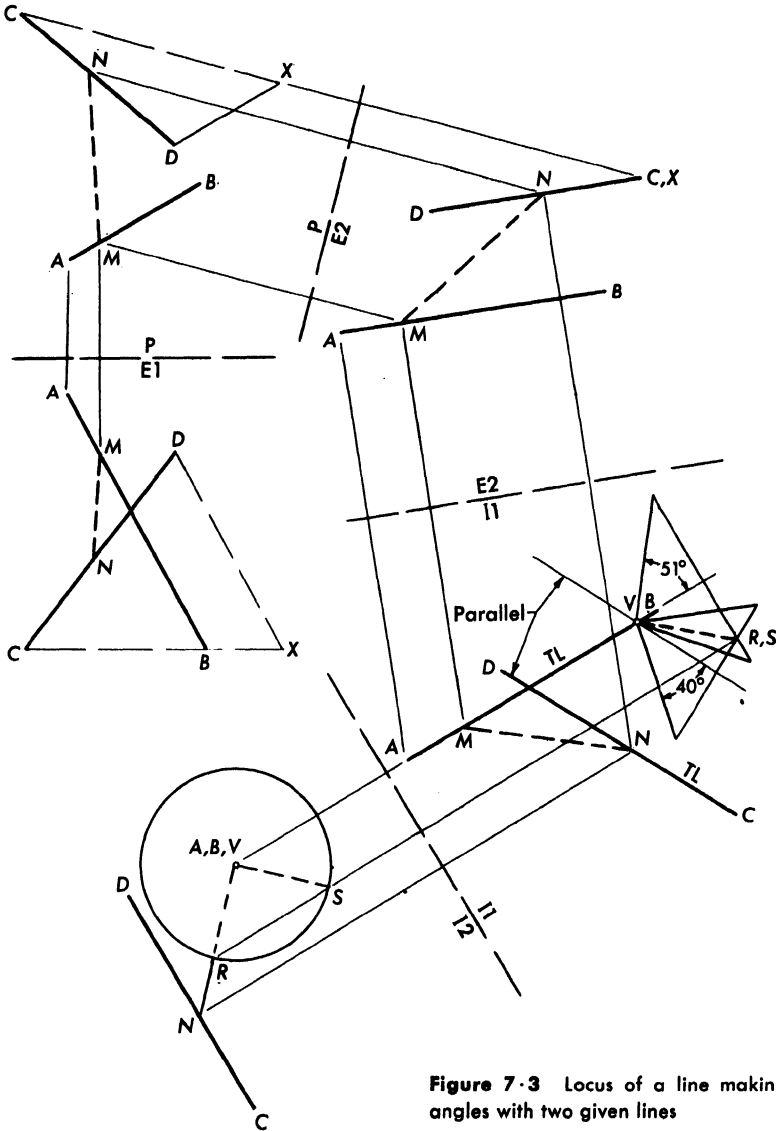


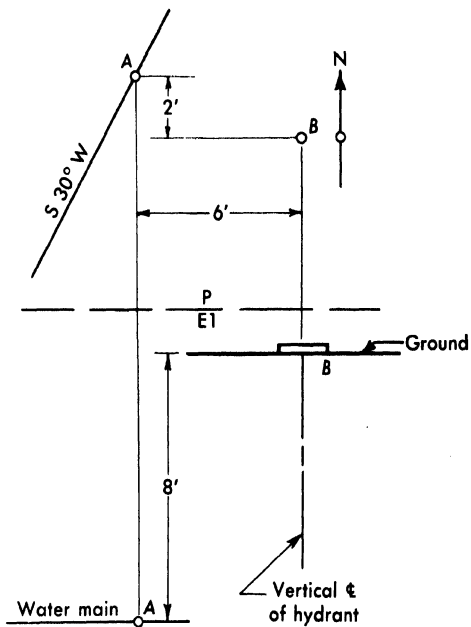
Figure 7-3 Locus of a line making given angles with two given lines

sects the line *CD*. This was to be expected, since it is unlikely that point *V* would be accidentally selected at the proper point. However, the *direction* of lines making the proper angles with *AB* and *CD* has been determined by the intersection of the cones. The correct *position* can now be determined by sliding the cones along *AB* until a common ele-

ment intersects CD . Therefore, a plane that appears as an edge in view I2 is passed through line VR , and CD is seen to pierce this plane at N . Point N is projected into view I1, and a line parallel to VR is drawn in this view, intersecting AB at M . This procedure is equivalent to sliding the cones along AB until the vertex is at M , although the cones are not actually drawn in this position. The line MN is the required line, which can now be located in the given views by projection. Note that if CD were extended, an alternate solution could be obtained by passing a plane through line VS in view I2.

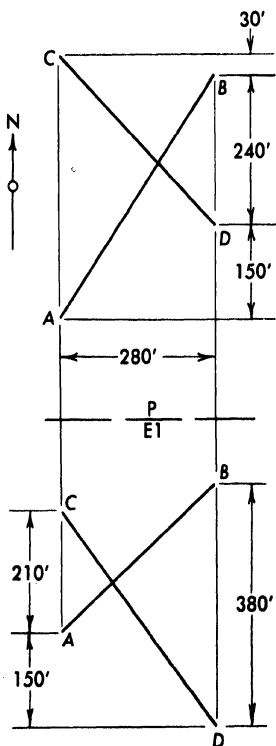
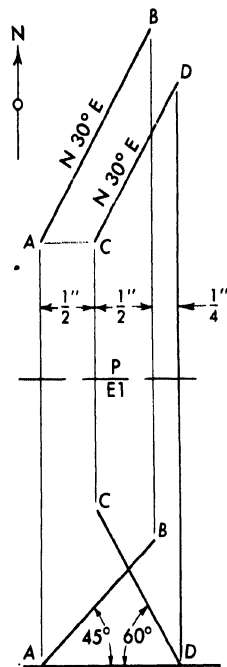
7.4 Problems

- 7-1. Show in plan and elevation views a line $2\frac{1}{2}$ in. long which slopes down 45 degrees and makes an angle of 30 degrees with a vertical plane parallel to the rotation line. Scale: Full size.
- 7-2. A vertical discharge pipe from a pump is to be connected to a horizontal pipe 10 ft above and 6 ft west of the pump outlet. The connecting pipe is to be fastened to the vertical pipe with a 45-degree elbow and to the horizontal pipe with a 60-degree elbow. How far above the pump outlet should the 45-degree elbow be placed? How far north of the pump outlet should the 60-degree elbow be placed? Scale: $\frac{1}{4}$ in. = 1 ft 0 in.
- 7-3. The hydrant is to be connected to the water main by a pipe using a 45-degree Y at the main and a 60-degree elbow on the vertical center line of the hydrant. What will be the bearing of the connecting pipe? How far will the elbow be below the ground? How far from A will the Y be placed? Scale: 1 in. = 4 ft.
- 7-4. Find the bearing, true length, and slope of a line that makes angles of 45 degrees with each of the given lines. Scale: Full size.
- 7-5. Find the bearing, true length, and slope of a connecting pipe that would make an angle of 30 degrees with pipe AB and 45 degrees with pipe CD . Scale: 1 in. = 200 ft.



Problem 7-3

Problem 7-4



Problem 7-5

LARGE TRUCK BEING LOADED ON BARGE. (Courtesy Hal Porter Photos,
Seattle)



CHAPTER 8 Vectors

The principles and methods used for the graphical solution of both coplanar and noncoplanar vector problems

8.1 INTRODUCTION

Many common quantities can be specified by a single number of units of a particular dimension, such as a temperature of 70°F or a pressure of 30 lb per sq in. These quantities are called **scalars**, or **scalar quantities**, and can be defined as quantities that have magnitude but not direction.

Certain quantities, such as force and velocity, can be completely specified only if a direction and a magnitude are given. These quantities are represented graphically by an arrow to indicate direction, the magnitude being shown by the length of the arrow. Further, these quantities can be combined with each other by the parallelogram law, as shown in Figure 8.1. All quantities that have magnitude and direction, and that can be combined (or added) by the parallelogram law are called **vectors**, or **vector quantities**.

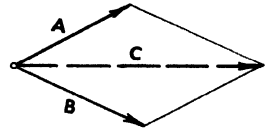


Figure 8.1 Parallelogram law

In Figure 8.1, the two given vectors **A** and **B** intersect as shown. A parallelogram is constructed on **A** and **B**, the diagonal **C** being equal to the **resultant**, or vector sum, of **A** and **B**. It can be shown experimentally that the effect of vector **C** is exactly the same as the combined effect of vectors **A** and **B**. Thus, vectors **A** and **B** could be replaced by the vector **C** without any change in the total effect.

8.2 CLASSIFICATION OF VECTOR SYSTEMS

A vector system is any group of two or more vectors. These systems can be divided into those in which all vectors lie in the same plane, called **coplanar** vector systems, and those in which the vectors lie in different planes, called **noncoplanar** vector systems. Vector systems may also be classified by the relative positions of the vectors within the system. Thus, a **concurrent** vector system is one in which all vectors intersect at a common point; a **nonconcurrent** vector system is one in which the vectors do not intersect at a common point; and **parallel** vector systems are those in which all vectors are parallel. Actually, the parallel systems are a special case of nonconcurrent systems, but they are considered

separately for purposes of classification. Since each of the last three types of systems can exist within each of the first two classes, six different classifications of systems are possible. These are: (1) *Coplanar, concurrent*, (2) *Coplanar, nonconcurrent*, (3) *Coplanar, parallel*, (4) *Noncoplanar, concurrent*, (5) *Noncoplanar, nonconcurrent*, and (6) *Noncoplanar, parallel* vector systems.

8.3 COPLANAR, CONCURRENT VECTOR SYSTEMS

The resultant of a system of coplanar, concurrent vectors is a single vector that is equivalent to the original system. The resultant must have exactly the same effect as the original system, and must

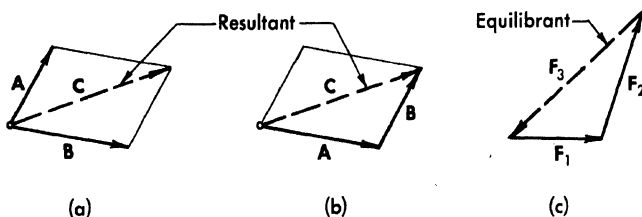


Figure 8.2 Resultant and equilibrant of two concurrent vectors

act through the point of concurrency (the point at which the vectors intersect). As previously stated, the resultant of two vectors may be found by the parallelogram law (Fig. 8.2a). The resultant of two vectors may also be found by a corollary of the parallelogram law called the triangle rule. As shown in Figure 8.2b, the vectors are connected so that the tip of one vector is connected to the tail of the other, the resultant being drawn from the tail of the first vector to the tip of the second. When the vectors represent forces, the force necessary to balance the system—that is, put the system in equilibrium—is often required. This **equilibrant** is exactly equal in magnitude to the resultant but is in the opposite direction, as shown in Figure 8.2c.

When the original system consists of more than two vectors, as in Figure 8.3a, the resultant may be found by a vector polygon, formed by applying an extension of the triangle rule. The vectors are connected one after the other in the same tip-to-tail manner, as shown in Figure 8.3b. Again, the resultant is found by connecting the tail of

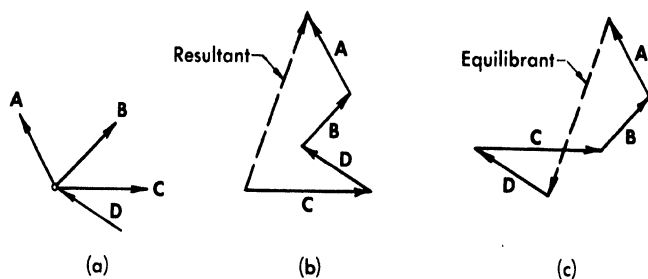


Figure 8.3 Resultant and equilibrant by vector polygon

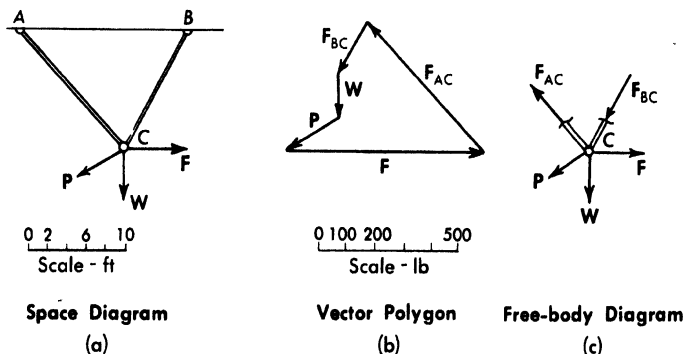
the first vector to the tip of the last vector. If the equilibrant is desired, the polygon is closed by drawing the closing side from the tip of the last vector to the tail of the first, as shown in Figure 8.3c.

The vectors may be combined in any order in the vector polygon without changing the value or direction of the resultant or equilibrant. Also, vectors may cross each other in the vector polygon, as in Figure 8.3c, without changing the result.

8.4 SOLUTION OF COPLANAR, CONCURRENT FORCE SYSTEMS

In Figure 8.4a, a simple structure consisting of two members connected at C is supporting several loads in the same plane. Before members AC and BC can be designed, the magnitude of the axial force imposed on each of them by the given loads must be determined.

Figure 8.4 Solution of coplanar, concurrent vector systems



For design purposes, it is also necessary to know whether this force applies a compressive or a tensile stress to the member. A member is in **compression** if the applied forces tend to shorten it and in **tension** if the forces tend to lengthen it.

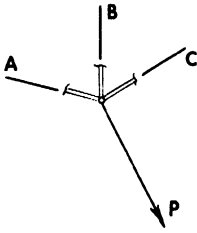
To solve this problem, the space diagram (Fig. 8.4a) is drawn to scale to establish the correct directions of the members and applied loads. Note that the length of the load vectors is not important as long as they are drawn in the proper directions.

Since the structure is in equilibrium, the known loads and the stresses in the members must form a closed vector polygon in which the vectors are connected in tip-to-tail order. This vector polygon is therefore drawn to scale, as shown in Figure 8.4b. First, the known force vectors $\mathbf{W}$, $\mathbf{P}$, and $\mathbf{F}$ are connected tip-to-tail, each vector being drawn parallel to the corresponding force on the space diagram. The vector diagram is then completed by drawing one line through the tip of vector $\mathbf{F}$ parallel to member AC and a second through the tail of vector $\mathbf{W}$ parallel to member BC . The point at which these lines intersect determines the length of vectors $\mathbf{F}_{AC}$ and $\mathbf{F}_{BC}$, which represent the stresses in members AC and BC . Arrowheads can now be placed on $\mathbf{F}_{AC}$ and $\mathbf{F}_{BC}$, and their values can be scaled. Comparing the vector diagram with the space diagram, it is seen that vector $\mathbf{F}_{AC}$ acts away from the joint at C , thereby indicating that member AC is in tension. Similarly, member BC is in compression because vector $\mathbf{F}_{BC}$ acts toward the joint.

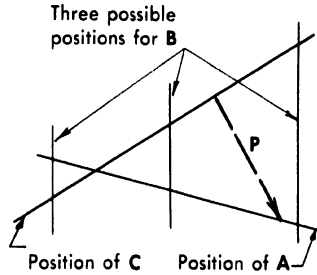
The student may find that he can determine tension and compression more easily by drawing a free-body diagram, as shown in Figure 8.4c. In the free-body diagram, the members AC and BC are considered to have been cut off, the portions not drawn being replaced by the forces $\mathbf{F}_{AC}$ and $\mathbf{F}_{BC}$. These forces have exactly the same effect as did the original members; that is, they keep the joint in equilibrium. On the free-body diagram, the member AC is seen to be in tension, since the arrow representing $\mathbf{F}_{AC}$ points away from the joint and therefore indicates that the force is tending to lengthen member AC . Similarly, the member BC is in compression because the arrow representing $\mathbf{F}_{BC}$ points toward the joint, indicating that the force tends to shorten member BC . It should be noted that the directions on the free-body diagram can be approximate, since they are used only to determine tension and compression.

The solution of coplanar vector systems in equilibrium is limited to cases with not more than two unknowns.

The free-body diagram given in Figure 8.5a shows a joint acted upon by a known load $\mathbf{P}$ and three unknown forces $\mathbf{A}$, $\mathbf{B}$, and



Free-body Diagram
(a)



Attempted Vector Polygon
(b)

Figure 8-5 Attempted solution for three unknowns

C. When we attempt to draw a vector diagram by plotting **P**, **A**, and **C** as shown in Figure 8.5b, we find that **B** can be plotted in many different positions, three of which are shown. Since it is impossible to tell where to plot vector **B**, the vector diagram cannot be completed and no solution is possible.

8-5 COPLANAR RELATIVE MOTION

The relative motion of one object with respect to another is equal to the vector difference of their two velocities. The vector difference of any two vectors is found by reversing the direction of one vector and adding it to the second, using the parallelogram law. The resultant velocity is then considered to be the velocity of the second object while the first remains stationary, as in the following example.

Two airplanes in level flight at an elevation of 10,000 ft are at the positions shown by points *A* and *B* in Figure 8.6. Airplane *A* is flying N 45° E at a ground speed of 400 mi per hr, and airplane *B* is flying due west at a ground speed of 300 mi per hr. It is desired to find the distance between the two airplanes when they are closest to each other, and the distance each airplane travels to reach this position.

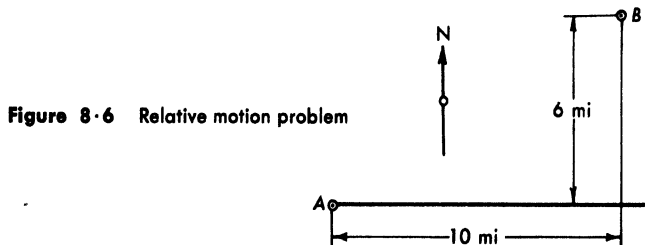


Figure 8-6 Relative motion problem

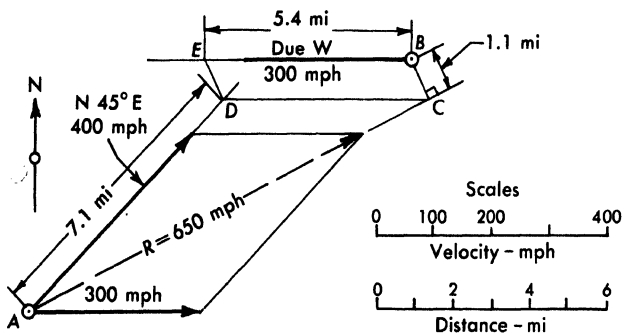


Figure 8-7 Solution of relative motion problem

In Figure 8.7, the positions of airplanes *A* and *B* are plotted, using the distance scale. Next, using the velocity scale, the velocity vector of airplane *B* is reversed and added to the velocity vector of airplane *A*, giving a resultant velocity of 650 mi per hr. Airplane *B* is then temporarily considered to remain stationary while *A* travels in the direction shown by this resultant. The perpendicular distance from *B* to point *C* on the line of the resultant shows that the clearance between the two airplanes is 1.1 mi. A line is then drawn from *C* parallel to the direction of motion of *B* until it intersects the actual direction of flight of *A* at point *D*. A line from *D* drawn parallel to *BC* intersects the direction of flight of airplane *B* at point *E*. Points *D* and *E* represent the positions of the two airplanes when they are closest to each other. The distances from *A* to *D* (7.1 mi) and from *B* to *E* (5.4 mi) are then scaled to find the distance that each airplane traveled to reach this position.

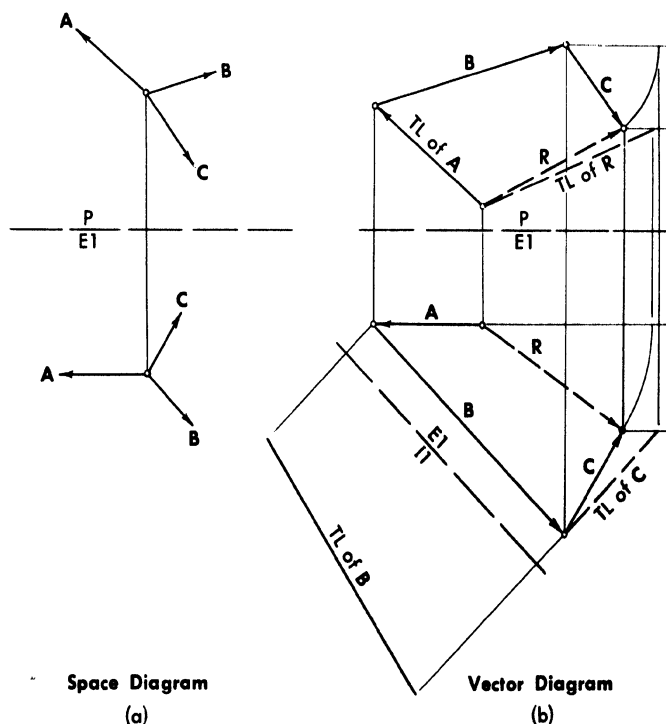
8-6 NONCOPLANAR, CONCURRENT VECTOR SYSTEMS

The principles used in solving noncoplanar, concurrent vector systems are exactly the same as in coplanar systems, except that a third dimension is involved. This means that two orthographic views are required to show the positions of the vectors in space, and, similarly, to show the shape of the corresponding vector diagram.

The *P* and *E1* views of a space diagram representing the positions in space of vectors *A*, *B*, and *C* are shown in Figure 8.8a. The resultant of these three vectors is found by vector addition, as shown in the vector diagram, Figure 8.8b. In the plan view of the vector diagram,

each vector is drawn parallel to the corresponding vector in the plan view of the space diagram, and in the E1 view, each vector is drawn parallel to the corresponding vector in the E1 view of the space diagram. Since each vector must be drawn to scale in the vector diagram, its length must be scaled in a view that shows the true length of the vector. Vector **A** can be scaled in the plan, where it appears true length, and its length in view E1 is then found by projection. Since vectors **B** and **C** do not appear true length in either view, their true lengths must be found either by drawing another view, as shown for vector **B**, or by using revolution, as indicated for vector **C**. Revolution is preferred because it requires less space. The resultant **R** is now drawn from the tail of vector **A** to the tip of vector **C** in both views, and its true length is found either by revolution (as shown), or by an additional view. As before, an equilibrant can be found by reversing the direction of the resultant vector. Also, the vectors can be combined in any order without changing the result.

Figure 8-8 Resultant of noncoplanar, concurrent vector systems



8.7 SOLUTION OF NONCOPLANAR, CONCURRENT FORCE SYSTEMS

While the same principles are used in solving noncoplanar, concurrent force systems as in coplanar, concurrent systems, the fact that a third dimension is involved means that two orthographic views of the force vector diagram are required. Furthermore, since there are usually three unknowns, the two views of the vector diagram must be carefully chosen. If all three unknowns appear in different directions in a view, the corresponding view of the vector diagram is in the same form as that shown in Figure 8.5. If both views are in this form, neither view of the vector diagram can be completed. The views, therefore, must be chosen so that the unknowns appear in only two different directions in one of the views of the vector diagram. This can be accomplished by drawing either a view that shows the edge view of a plane containing two of the unknowns, or a view showing one of the unknowns as a point.

A three-dimensional frame supporting a 2000-lb load, and the corresponding vector diagram, are shown pictorially in Figure 8.9. The values shown for the vectors can be found by either the edge-view method or the point-view method.

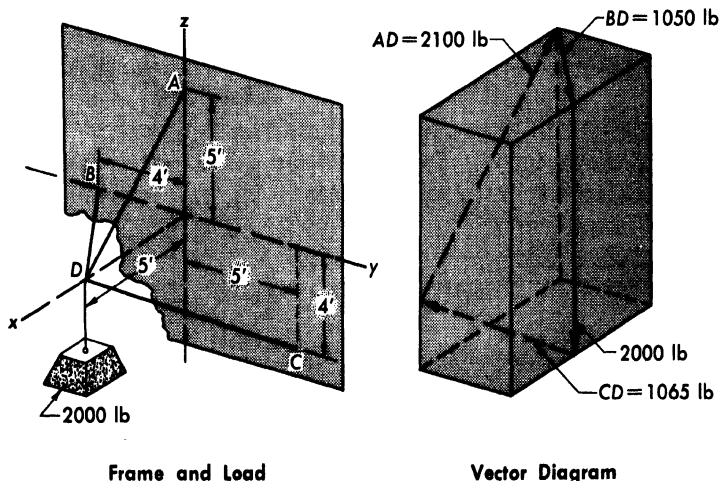


Figure 8.9 Noncoplanar, concurrent force system

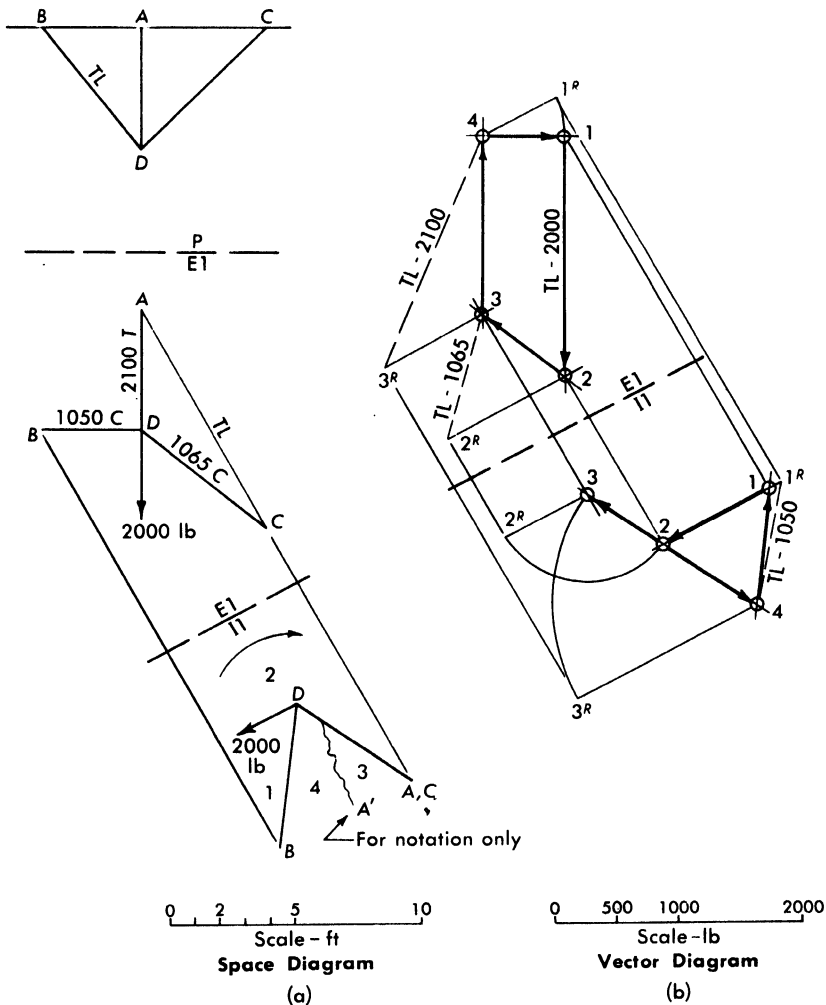


Figure 8.10 Solution of a noncoplanar concurrent force system—edge-view method

Edge-view Method

The P and E1 views of the frame are first drawn to scale as shown in Figure 8.10a, the load being represented by an arrow.

The plane containing members *AD* and *CD* is seen as an edge in view I1, where the unknowns therefore appear in only two directions.

. **Note:** In drawing the vector diagram, it is absolutely essential that, in both views, (1) the vectors be kept in the same sequence and (2) the two vectors in the plane that appears as an edge be kept consecutive (that is, adjacent to each other).

To insure this, Bow's notation is applied in the view showing the plane as an edge (view I1). A number is placed in each space between members or between members and the load. To allow a number to be placed between members AD and CD , member AD is considered to be moved to the position ($A'D$) shown by the wavy line. This position is for purposes of notation only, and does not change the actual direction of member AD . The curved arrow is drawn to indicate the order in which the vectors are to be connected in the vector diagram. The direction can be either clockwise (as shown) or counterclockwise, but once the choice is made, the vectors *must* be laid out in that order in *both* views.

The values of the unknown forces are determined by drawing the E1 and I1 views of the vector diagram, as shown in Figure 8.10b. A rotation line is established parallel to the $\frac{E1}{I1}$ rotation line of the space diagram so that directions for the vectors can be established parallel to the corresponding members of the space diagram.

The step-by-step development of the vector diagram is illustrated in Figure 8.11.

First, as shown in Figure 8.11a, a line is drawn, in each view of the vector diagram, parallel to the known load in the corresponding view of the space diagram (Fig. 8.10a). The known load is laid out to scale in view E1, where it appears true length, and is then projected into view I1. In view I1 of Figure 8.10a, where Bow's notation was applied, the load is between numbers 1 and 2 as we read around the joint in the chosen clockwise direction. Thus, the direction of the vector is established (by Bow's notation) as extending from number 1 to number 2 in the direction indicated by the arrowhead. Therefore, the number 1 is placed at the tail of the vector and the number 2 at its tip in both views of the vector diagram.

Next, as shown in Figure 8.11b, a line parallel to member CD is drawn through point 2 in each view. Point 3 must lie somewhere along this line because member CD is between numbers 2 and 3 of Bow's notation. Since the length of vector 2-3 is unknown, the exact position of point 3 must be established by locating its intersection with

vector 3-4. Clearly, since point 3 is not known, vector 3-4 cannot be drawn until point 4 has been located.

In view I1, point 4 must lie along the line drawn through point 2, because member AD , which appears between numbers 3 and 4, coincides with member CD in view I1 of the space diagram. Also, point 4 is on vector 4-1, which is shown by Bow's notation to be parallel to member BD . Therefore, as shown in Figure 8.11c, a line is drawn through point 1 parallel to member BD in both views. Point 4 is found in view I1 at the intersection of this line with the line previously drawn through point 2, and is then located in the plan view by projection.

Next, as shown in Figure 8.11d, a line is drawn parallel to member AD , through point 4 in view E1, to intersect the line previously drawn through point 2, thus locating point 3. Point 3 is then found in view I1 by projection. The vector diagram is completed by adding arrowheads as shown, since the vectors must be connected tip-to-tail in the vector polygon.

Note that each of the points is marked by the intersection of fine lines for accuracy in scaling the values. To preserve this accuracy, each point is "protected" by a small circle, and the arrowheads are drawn so that they touch the circles, rather than the points. It is strongly recommended that this procedure be followed in solving vector problems.

The true length of each vector is found either by revolution as shown in Figure 8.10 or by drawing an additional view, and the value is scaled. For easy reference to the answers, the values are placed along the members in one view of the space diagram.

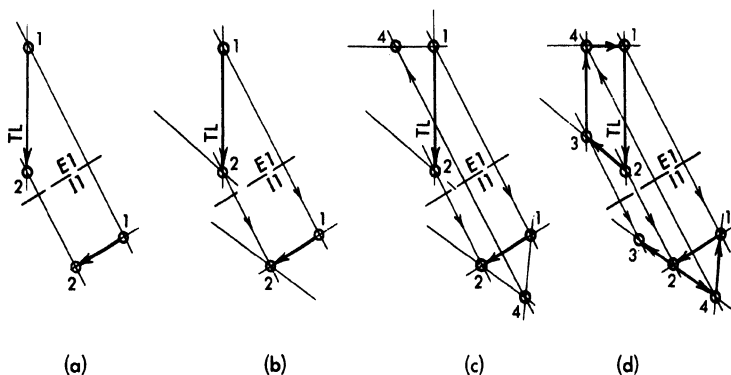
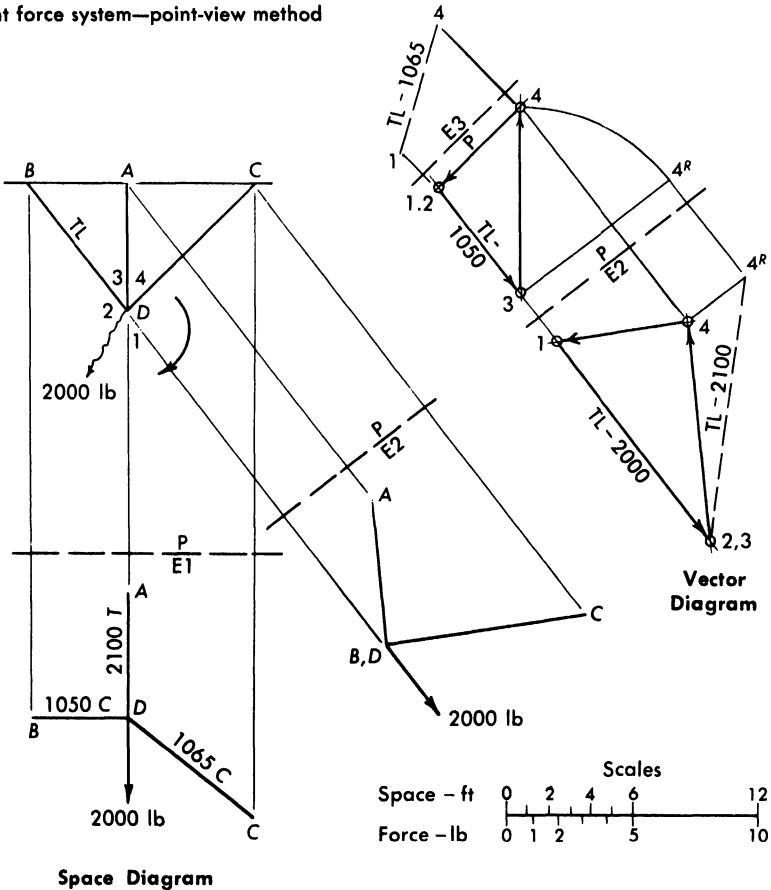


Figure 8.11 Development of vector diagram

The final step is to determine whether the stress in each member is tension or compression by comparing directions in the vector and space diagrams (Fig. 8.10). For example, the stress in member CD is shown by the vector from 2 to 3. In either view, the arrow on vector 2-3 is seen to point along CD toward the joint at D , thus indicating that CD is in *compression*. Similarly, the stress in AD is *tension*, since vector 3-4 points away from the joint, and the stress in BD is *compression* because vector 4-1 points toward the joint. The letters T for tension and C for compression are placed after the values of the stress on the space diagram, thus completing the solution.

Figure 8.12 Solution of a noncoplanar concurrent force system—point-view method



Point-view Method

The P and E1 views of the space diagram are drawn (Fig. 8.12) using the same data as in the preceding example. Since BD is in its true length in the plan, view E2 is drawn to show BD as a point. The unknowns therefore appear in only two directions in view E2.

Bow's notation is applied in the plan view, where the 2000-lb load is moved to one side for purposes of notation. The curved arrow is drawn to show that the vectors will be connected in clockwise sequence in the vector diagram.

The rotation line $\frac{P}{E2}$ for the vector diagram is drawn parallel to the corresponding rotation line for the space diagram. Vector 1-2, representing the known load, is laid out to scale in view E2, where it appears in true length. Vector 2-3 appears as a point in view E2, and perpendicular to the rotation line in the plan. In view E2, lines are drawn through point 3 parallel to AD and through point 1 parallel to CD , thus locating point 4 by intersection. A line parallel to CD is then drawn through point 1 in the plan view, and point 4 is located by projection from view E2. Next, point 3 is located by drawing a line parallel to AD through point 4 in the plan view, and the vector diagram is completed by adding arrowheads as shown. The value of the stress in BD is found by scaling vector 2-3 in the plan, where it appears true length. The true lengths of the other vectors are found either by revolution (as shown for vector 3-4) or by additional views (as shown for vector 4-1). Finally, tension or compression is determined as in the preceding example, and the stresses are recorded on one view of the space diagram.

8.8 OTHER VECTOR PROBLEMS

The principles discussed in this chapter can be used to solve many other types of vector problems. Since the only essential difference is in the names of the vectors, no additional examples are shown.

8.9 SUMMARY

1. *Scalars*, or scalar quantities, have magnitude only.
2. *Vectors*, or vector quantities, have both magnitude and direction, and can be added (or subtracted) by the parallelogram law.

3. *Coplanar vectors* are vectors that lie in the same plane.

4. *Noncoplanar vectors* are vectors that do not lie in the same plane.

5. *Concurrent vectors* are vectors that intersect at a common point.

6. *Nonconcurrent vectors* are vectors that do not intersect at a common point.

7. *Parallel vectors* are vectors that are parallel to each other.

8. The *resultant* of a system of vectors is a single vector that has exactly the same effect as the original system. It is usually found by drawing a vector polygon.

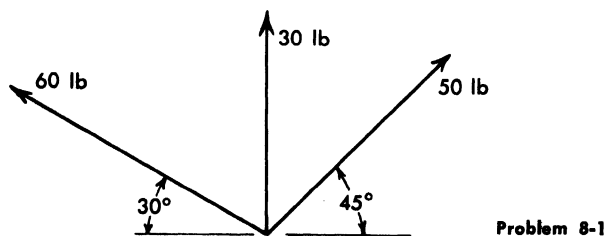
9. The *equilibrant* of a system of forces is the single force necessary to put the system of forces in equilibrium. It is equal in magnitude to the resultant, but is in the opposite direction.

10. The relative motion of two objects is equal to the vector difference of their velocities.

11. Vector problems are solved by drawing the necessary views of the vector diagram and then finding the true length of each vector. For noncoplanar, concurrent vector problems, the most common methods are to draw views that show either (a) the point view of one unknown, or (b) the edge view of a plane containing two unknowns.

8-10 Problems

8-1. Find the resultant of the three given forces. Measure its magnitude and the angle that it makes with the 30-lb force. Scale: 1 in. = 20 lb.

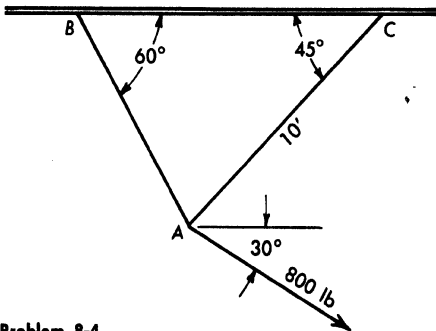


Problem 8-1

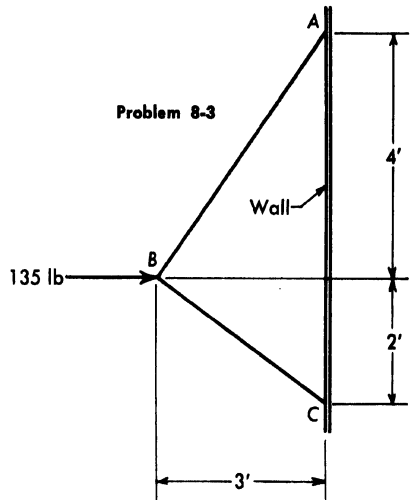
8-2. A boat is following a compass course of due E at 15 knots across a current which flows N 30° E at 6 knots. (a) What is the resultant speed

and true course of the boat? (b) What compass course should be followed for the boat to pass a buoy 8 nautical miles east and 5 nautical miles north of its present position? Velocity scale: 1 in. = 3 knots. Distance scale: 1 in. = 2 nautical miles.

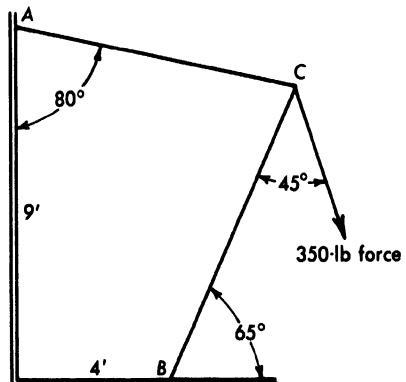
- 8-3.** Find the stresses in members AB and BC of the coplanar frame shown. Space scale: 1 in. = 2 ft. Force scale: 1 in. = 40 lb.
- 8-4.** Find the stresses in members AB and AC of the coplanar structure shown. Space scale: 1 in. = 4 ft. Force scale: 1 in. = 200 lb.
- 8-5.** Find the stresses in members AC and BC of the coplanar structure shown. Space scale: 1 in. = 3 ft. Force scale: 1 in. = 100 lb.



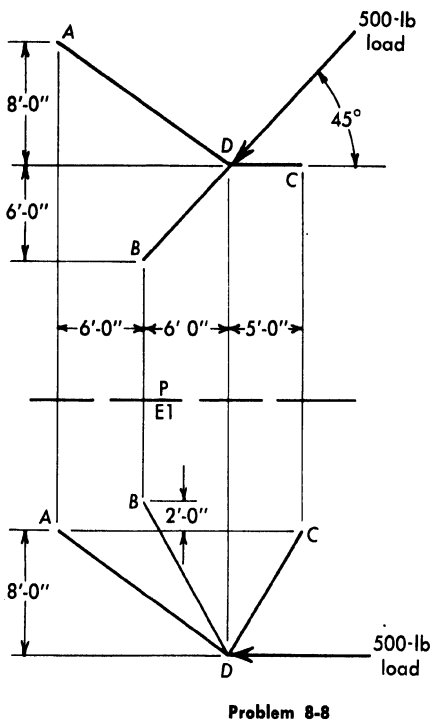
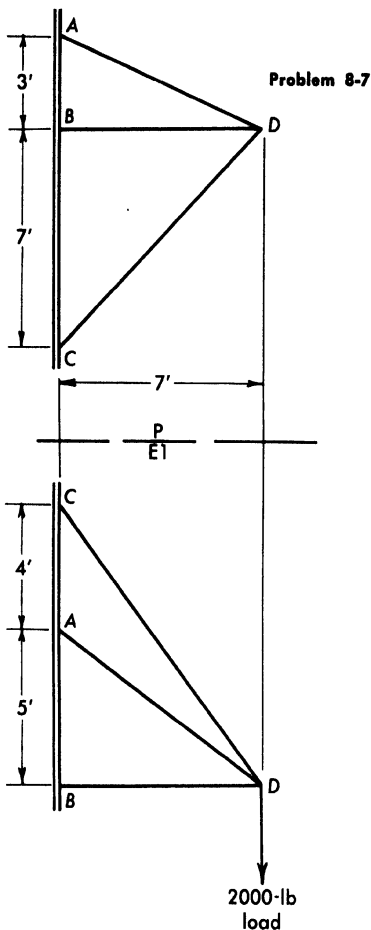
Problem 8-4



Problem 8-3

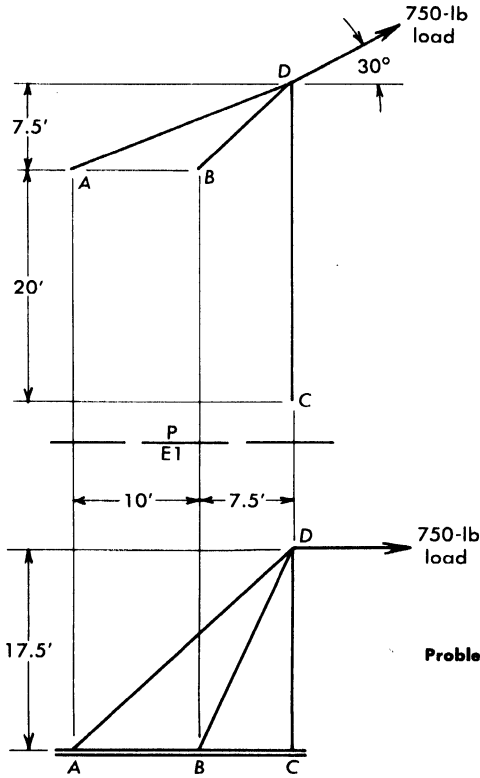


Problem 8-5



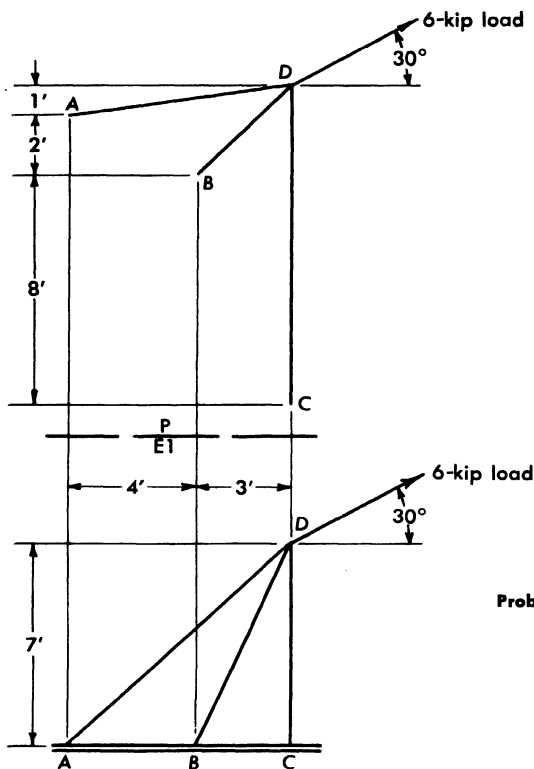
- 8-6.** Ship *A* is 4 nautical miles south and 6 nautical miles west of ship *B*. Ship *A* is traveling at a speed of 12 knots on a true course of $N 30^\circ E$, and ship *B* is traveling at a speed of 9 knots on a true course of $S 75^\circ W$.
 (a) How far apart are the two ships at their closest position? (b) When they are at this position, what is the bearing from ship *A* to ship *B*? (c) To reach this position, what distance was covered by ship *A*? by ship *B*? (d) How long did it take ship *A*? ship *B*? Distance scale: 1 in. = 1 nautical mile. Velocity scale: 1 in. = 4 knots.
- 8-7.** Find the stress in each member of the structure shown. Space scale: 1 in. = 4 ft. Force scale: 1 in. = 1000 lb.
- 8-8.** Find the stress in each member of the structure shown. Space scale: $\frac{1}{8}$ in. = 1 ft 0 in. Force scale: 1 in. = 200 lb.

- 8-9. Find the stress in each member of the tripod shown. Space scale: 1 in. = 10 ft. Force scale: 1 in. = 500 lb.



Problem 8-9

- 8-10.** Find the stress in each member of the tripod shown. Space scale: 1 in. = 4 ft. Force scale: 1 in. = 3 kips.

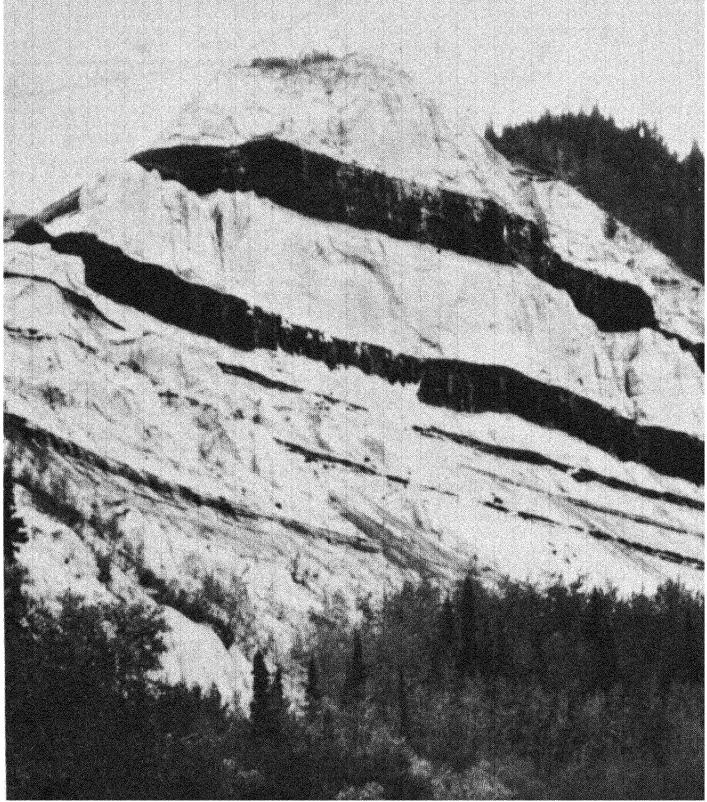


Problem 8-10

CHAPTER 9

STRATA OF COAL OCCURRING IN SANDSTONE, SUNTRANNA, ALASKA.

(Courtesy M. R.
Geer, Seattle)



CHAPTER 9 Geology, Mining, and Topography Problems

The basic principles and methods used in solving problems such as dip, strike, and thickness of rock strata, net slip and plunge of faults, and limits of cuts and fills in earthwork

9.1 CONTOURS AND TOPOGRAPHIC MAPS

A contour is a line that represents points of equal elevation on the earth's surface (Sec. 2.1). A map which shows ground relief by means of contour lines is called a **topographic map**. All contour lines are continuous and form closed figures, although the figure may not close within the limits of the map.

A topographic map of a small area is shown in Figure 9.1. The scale given is the horizontal scale, and the contour interval is the difference in elevation between successive contours. Every fifth contour is drawn heavier, and its elevation is given. Note that the sharpest curves in the contour lines occur at the bottom of a valley and the crest of a ridge, the convex side of the curve being toward higher ground in the valleys and toward lower ground on the ridge lines. The character of a slope is indicated by the spacing of contour lines: close together for steep slopes, far apart for gentle slopes, and evenly spaced for uniform slopes.

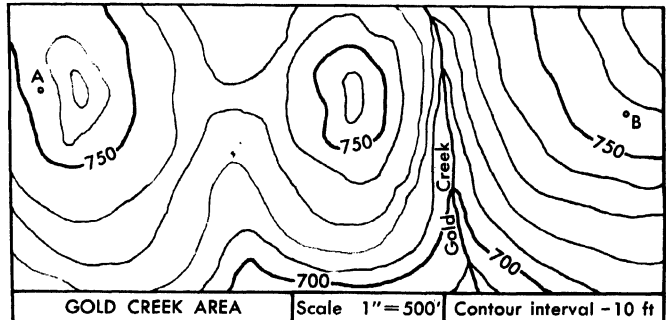


Figure 9.1 Topographic map

9.2 PROFILES

A **profile** is a vertical section of the surface of the ground along some fixed line. It is usually obtained in practice by plotting elevations that have been determined in the field. However, the profile of any line can be easily constructed from a topographic map by plotting the contour elevations, horizontal distances between contours being scaled from the map. The vertical scale of the profile is usually made greater than the horizontal scale to make the irregularities of the surface more apparent.

The profile of the ground along line *AB* (Fig. 9.1) is shown in Figure 9.2, using a vertical exaggeration of ten—that is, a vertical scale equal to ten times the horizontal scale. Comparison of the topographic map and the profile illustrates that the steepness of the slope is shown by the spacing of the contour lines.

It should be noted that nothing is shown by the profile that cannot be determined by a study of the contour lines. On the contrary, the contour lines show the shape of the ground better than a series of profiles would. Profiles are drawn for determining such matters as the best placement of towers for electrical transmission lines, rather than for determining the shape of the ground.

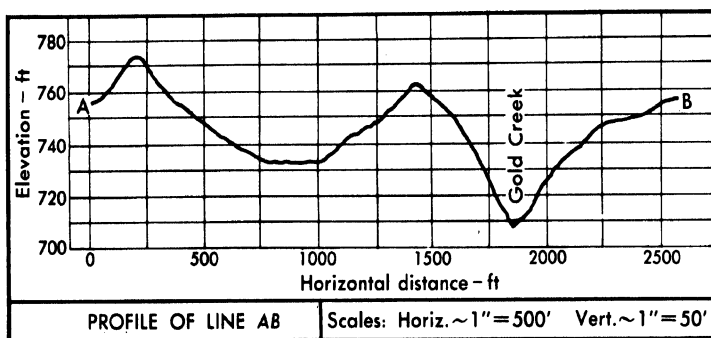


Figure 9.2 Profile

9.3 CUT AND FILL PROBLEMS

In many types of construction, such as highway work, quantities of earth must be moved to establish the proper gradient or elevation. This usually involves excavating material from the higher areas and depositing it in the lower areas. The material removed, the process of removing it, and the area from which it is removed are all referred to as **excavation** or **cut**. Similarly, the material, process, and area deposited above the original surface are called **embankment** or **fill**. The boundaries of the cut and fill areas often must be established on a map in advance of construction to allow computation of the quantities of earthwork. These boundaries are established by locating the proposed contours on the map, marking the points where the proposed contours

intersect the original ground contours, and then connecting the marked points.

The 80-by-120-ft rectangular area in Figure 9.3 is to be graded level to allow construction of a tennis court. The finished grade elevation is to be 335 ft and the slopes are to be $1\frac{1}{2}$ to 1 in cuts and 2 to 1 on fills.

Contours on the cut and fill slopes will be parallel to each other and to the boundaries of the rectangle. The horizontal spacing between 5-ft contours will be $7\frac{1}{2}$ ft ($1\frac{1}{2}$ times 5) in cuts and 10 ft (2 times 5) on fills.

Since there will be neither cut nor fill at the 335-ft contours, the intersection of these contours with the rectangle determine the points of change between cut and fill slopes. Proposed contours, shown

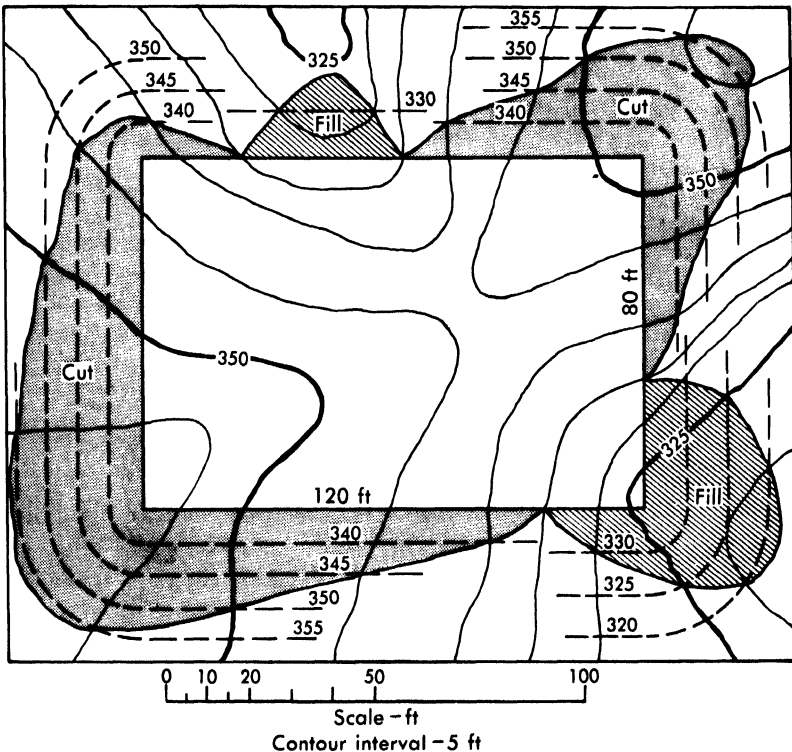


Figure 9.3 Cut and fill boundaries

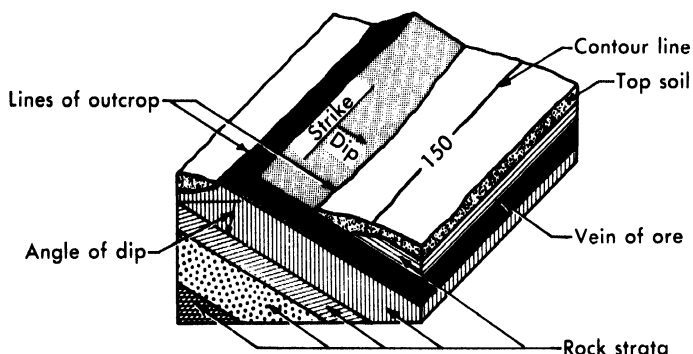
by dashed lines, are drawn at horizontal intervals of 10 ft from the rectangle across the contours below 335 ft. The points of intersection of proposed and original contours having the same elevation are then connected by smooth lines. These lines represent the boundaries of the fill slopes, shown cross-hatched in Figure 9.3. Similarly, proposed contours at horizontal intervals of $7\frac{1}{2}$ ft are used to determine the boundaries of the cut slopes, shown shaded in Figure 9.3.

Sufficient information can now be taken from the drawing to determine the volume of cut and fill. Note that all material within the rectangle above 335 ft will be excavated, as well as that between the rectangle and the cut slope. It is evident from Figure 9.3 that the volume of cut will be considerably greater than the volume of fill. These could be made more nearly equal if the finished grade were set at an elevation higher than 335 ft.

9.4 STRATIFIED ROCK AND ORE

Material deposited by sedimentation at the bottom of ancient bodies of water has been compacted and cemented to form sedimentary rock. While originally deposited in nearly horizontal layers or **strata**, this rock may have been tilted, folded, and fractured by forces acting on the earth's crust in subsequent eras. As a result, rock strata are often inclined and of relatively limited area. Within these areas, however, the strata may be considered to have uniform thickness between parallel plane surfaces, called **bedding planes**. Intrusions of metal-bearing rock

Figure 9.4 Block diagram showing strata and outcrop



(ore) into cracks or fissures of strata are called **veins**, the upper and lower surfaces of which are called the **hanging wall** and the **footwall**, respectively.

Any portion of a bedrock that is exposed above the surface of the ground (or only lightly covered) is called an **outcrop**, and is bounded by outcrop lines formed by the intersection of the stratum with the surface. Figure 9.4 is a block diagram showing several strata of rock, most of which have been buried by soil through weathering, leaving an outcrop of the more resistant vein of ore.

9.5 STRIKE AND DIP

The position of a stratum or vein in space is usually given by its strike and dip, as indicated on the exposed hanging wall of the vein in Figure 9.4. **Strike** is the bearing of a horizontal line on either surface of a stratum or vein, and **dip** is the angle at which the stratum or vein is inclined *downward* from horizontal. The angle of dip is measured in a vertical plane at right angles to the strike.

The *direction* of dip must be specified as well as the angle. Since only two directions of dip are possible when the strike is stated, only the general direction of the dip need be indicated. For example, the designation N 35° E, 30° SE means a strike of N 35° E and a dip of 30 degrees in the southeasterly direction. The amount of exposed surface of a stratum may be insufficient to allow the strike and dip to be measured directly, or there may be no outcrop at all. In many instances, it is necessary to locate subsurface points on the stratum or vein by drilling test holes or **boreholes**. The methods previously described for solving plane problems can be used to determine the strike and dip of any stratum, once three or more points not in a straight line have been located on the same surface of the stratum. If the thickness and lines of outcrop are to be found, at least one point on the other surface of the stratum is also required.

9.6 STRIKE, DIP, THICKNESS, AND LINES OF OUTCROP

Small portions of outcrop on the hanging wall of a vein of ore have been discovered at points *A* (Elev. 834) and *B* (Elev. 892),

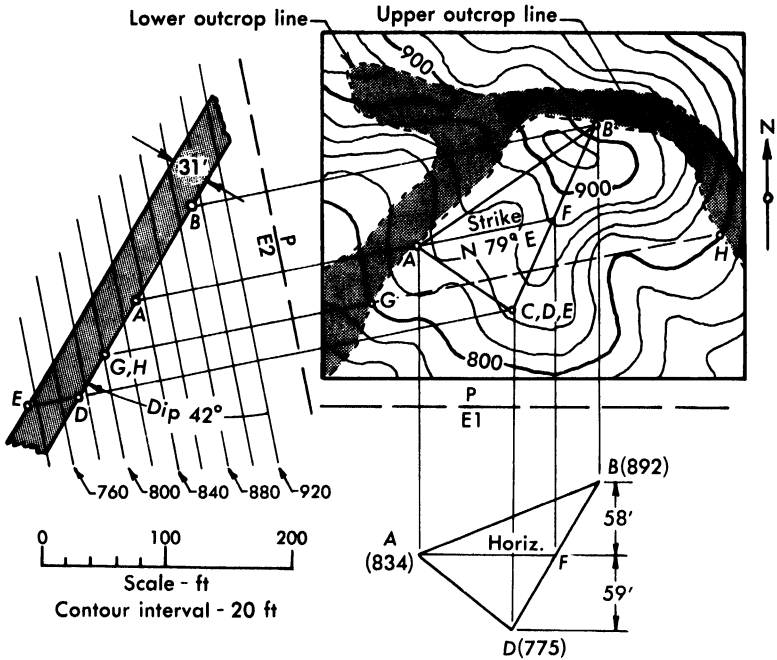


Figure 9.5 Strike, dip, thickness, and outcrop

shown on the topographic map in Figure 9.5. Since no other points of outcrop were noted, additional points on the vein were located by means of a vertical borehole drilled from a point C on the surface at an elevation of 843 ft. This borehole struck the hanging wall of the vein at a depth of 68 ft (point D) and the footwall at a depth of 111 ft (point E).

Points A, B, and D are plotted in view E1, thus giving two views of the triangle ABD on the hanging wall of the vein. The horizontal line AF is drawn in view E1 and projected into the plan, where its bearing of N 79° E is the strike of the vein. View E2 is then drawn to show the hanging wall as an edge, and the dip is seen to be 42 degrees in a southeasterly direction. Point E is now plotted in view E2 at a depth of 43 ft (111 minus 68) below point D, and the footwall is drawn parallel to the hanging wall. The thickness of the vein is found to be 31 ft by measuring the perpendicular distance between the two walls.

Contour lines on either surface of a stratum or vein are called **structure contours**. Intersections of a structure contour with a ground contour of the same elevation determine points on the line of outcrop, since these points are on both the vein and the ground surface.

In view E2, parallel lines are drawn at the contour elevations to locate the structure contours. Note that the structure contours appear as points in this view, and are therefore strike lines. Each structure contour on the hanging wall is projected into the plan to find its intersection with the corresponding ground contour, as illustrated by points *G* and *H* at the 800-ft level. After all intersections have been located, they are connected to form the upper line of outcrop. Similarly, structure contours on the footwall are used to locate the lower line of outcrop. Any additional points of outcrop would lie within the shaded area between the outcrop lines.

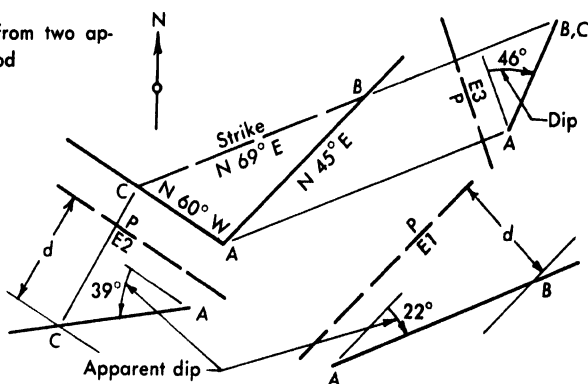
9.7 STRIKE AND DIP FROM TWO APPARENT DIPS

The angle of inclination of a line on a stratum measured in any vertical plane not at right angles to the strike is called the **apparent dip**. The apparent dip is always less than the true dip, becoming zero if measured in the strike direction. The strike and dip can be determined from any two apparent dips that are not measured on parallel vertical planes. Either of two methods may be used.

Conventional Method

In Figure 9.6, two apparent dips are given, one of 22 degrees (NE) along a line bearing N 45° E, and the second of 39 degrees (NW) along a line bearing N 60° W.

Figure 9-6 Strike and dip from two apparent dips—conventional method



The two lines of apparent dip are assumed to intersect at point *A*, since any two nonparallel lines on a plane must intersect at some point. View E1 is drawn to show the true length of the line bearing N 45° E. The apparent dip (slope of the line) is then laid off and a point *B* is arbitrarily selected. View E2 is drawn to show the true length and slope of the second line of apparent dip, and point *C* is established at the same distance, *d*, below the rotation line as point *B*. Points *B* and *C* therefore lie on the same structure contour, or strike line. Points *B* and *C* are located in the plan view by projection, and the strike is found to be N 69° E. Next, line *BC* is seen as a point in view E3, where the true angle of dip is found to be 46 degrees in a northwesterly direction.

Geologist's Method

Figure 9.7 gives the solution of the same problem by the compressed method often used by geologists.

Careful examination of Figure 9.7 shows that this method is the same as the conventional method with the following exceptions:

- (1) The image plane for the plan view is passed through point *A*.
- (2) The rotation lines $\frac{P}{E1}$ and $\frac{P}{E2}$ coincide with the lines *AB* and *AC* in the plan view.
- (3) Points *B* and *C* are designated as *B'* and *C'* in the elevation views.

This method requires a little less construction than the conventional method, and also takes up less space on the paper.

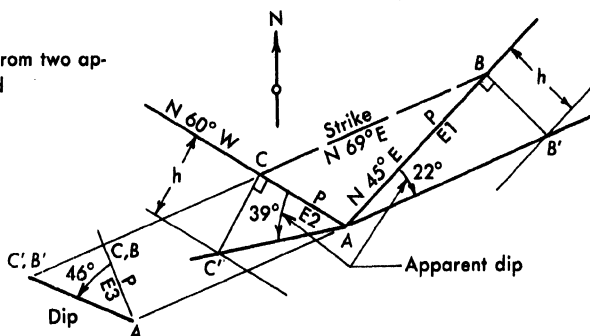
9.8 DIP FROM STRIKE AND APPARENT DIP

The true angle of dip can be determined when the strike and one apparent dip at some known angle to the strike are known. Since the strike is a line with an apparent dip of zero, this is merely a special case of the preceding problem, and the solution is therefore not shown.

9.9 BEARING, PLUNGE, AND RAKE OF SUBSURFACE LINES

It is frequently necessary to describe the position in space of a subsurface line; for example, the line of intersection of two strata. Such positions are usually designated by giving the bearing and the

Figure 9.7 Strike and dip from two apparent dips—geologist's method



plunge (dip) of the line. The **rake** of the line with respect to one of the strata may also be required. The rake of a given line with respect to a stratum is the angle between the given line and a horizontal line on the stratum, measured in the plane of the stratum.

Figure 9.8 shows a stratum of limestone striking $N 60^\circ W$ and dipping $45^\circ SW$ that crops out at point A at an elevation of 740 ft. Point B at an elevation of 600 ft is 800 ft east of A on a vein of ore striking $N 45^\circ E$ and dipping $51^\circ NW$. The bearing and plunge of the line of intersection and its rake on the vein are required.

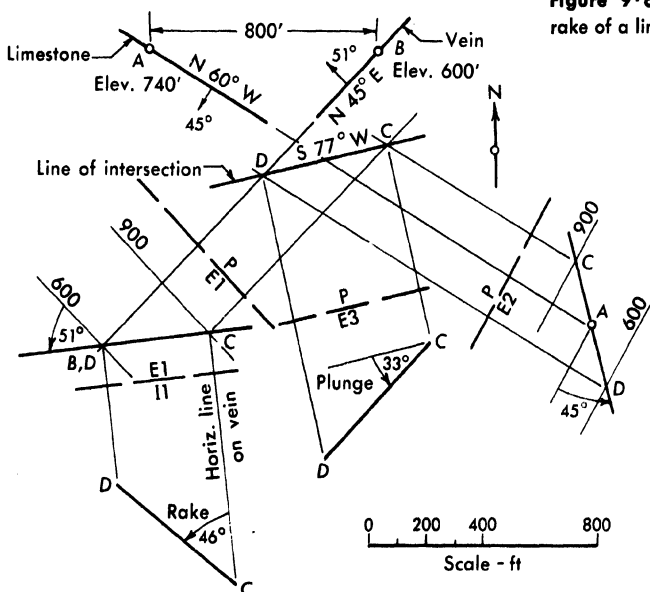


Figure 9-8 Bearing, plunge, and rake of a line

Views E1 and E2 are drawn, showing the vein and the stratum as edges. Structure contours at any two elevations, such as 900 and 600 ft, are established in these views and projected to the plan view to locate points *C* and *D* on the line of intersection, which bears $S\ 77^{\circ}\ W$. View E3 is drawn to show *CD* in true length, and the plunge is found to be $33^{\circ}\ (SW)$. To find the rake on the vein, a true-shape-and-size view of the vein, I1, is drawn. In this view, the true size of the rake angle between *CD* and a horizontal line on the plane is found to be $46^{\circ}\ (SW)$.

9-10 FAULTS

Fractures or breaks in rock strata accompanied by a displacement of one portion with respect to the other are called **faults**. A plane surface along which the movement has occurred is known as a **fault plane**. The location of fault planes is specified by strike and dip in the same manner as the location of strata and veins. The rubbing action during faulting often leaves relatively smooth, striated surfaces called **slickensides** on the fault plane. The striations (grooves or scratches) on the slickensides are parallel to the direction of the motion during faulting.

The movement of the surfaces along the fault may be translation along straight lines, rotation, or combined translation and rotation. When the motion is translation, it is called **strike slip** if parallel to the strike of the fault, **dip slip** if parallel to the dip of the fault, and **oblique slip** if parallel to neither. Note that oblique slip has both dip-slip and strike-slip components.

Any nonrotational fault displacement can be specified by its bearing, **net slip** (true length) and plunge. The horizontal distance along the fault between strike lines of the original and displaced strata, known as **strike separation**, may also be given. Methods of solving fault problems are shown in the following examples. Although oblique-slip faults are used as illustrations, the methods can be used equally well with either strike-slip or dip-slip faults.

9-11 BEARING, NET SLIP, AND PLUNGE FROM DISPLACEMENT OF SINGLE VEIN

A vein which strikes $N\ 60^{\circ}\ E$ and dips $45^{\circ}\ SE$ is interrupted by a fault striking $N\ 10^{\circ}\ W$ and dipping $60^{\circ}\ NE$. The strike separation is 220 ft east side north. The rake of the striations on the slickensides is $53^{\circ}\ NE$. The bearing, net slip, and plunge of the displacement of the vein along the fault are desired.

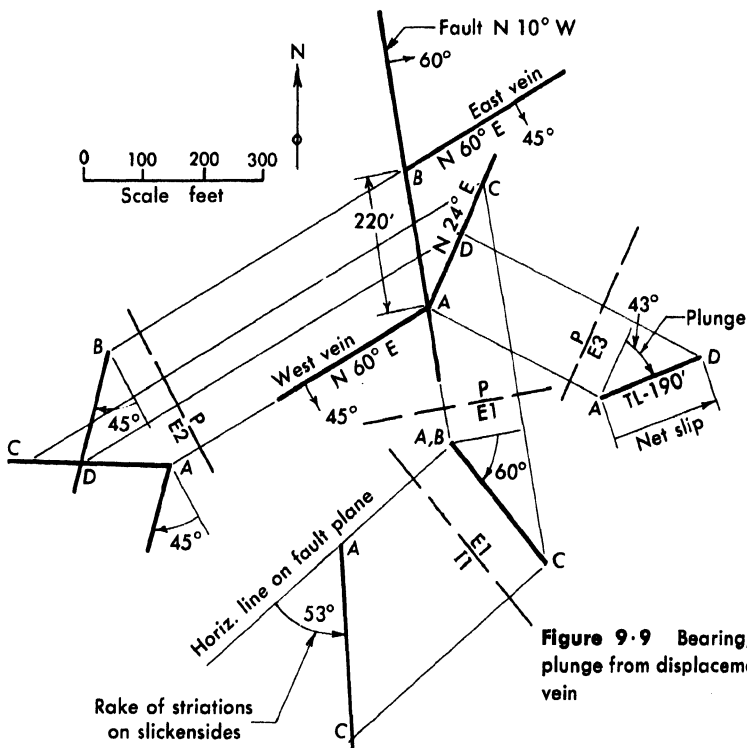


Figure 9-9 Bearing, net slip, and plunge from displacement of single vein

The east and west veins are plotted along the fault in the plan view. The fault plane is seen as an edge in view E1 and in true shape and size in view I1. The rake of the striations on the slickensides is laid out in view I1 and an arbitrary point C is selected. Point C is projected into the plan view, where the bearing of the fault displacement (along line AC) is found to be N 24° E.

Next, view E2 is drawn to show the edge views of both the east and west veins. Point C is projected into view E2, where the line AC is seen to intersect the east vein at D. Point D is located in the plan view by projection, and the true-length view E3 is drawn to find the net slip (190 ft) and the plunge (43°) of the displacement.

9-12 BEARING, NET SLIP, AND PLUNGE FROM DISPLACEMENT OF TWO VEINS

A fault which strikes N 30° E and dips 50° SE causes strike separations of 150 ft east side north for vein 1 (N 40° W, 35° NE)

and 270 ft east side south for vein 2 ($N 80^{\circ} E, 40^{\circ} SE$). On the east side of the fault, vein 2 is 360 ft (horizontal distance) northeasterly from vein 1. The bearing, net slip, and plunge of the fault are required.

The east and west positions of the two veins are plotted along the fault in the plan view. Structure contours in views E1, E2, and E3 are drawn to find the lines of intersection of each of the veins with the fault plane in the plan view. The lines of intersection (BF and CH) of the east veins with the fault plane are found to intersect at point J . Similarly, for the west veins, the lines AE and DG intersect at K .

Before faulting occurred, the points J and K coincided. Therefore, the line between J and K , bearing $S 82^{\circ} E$, is the displacement. These points are projected into view E2, where they lie on the edge view

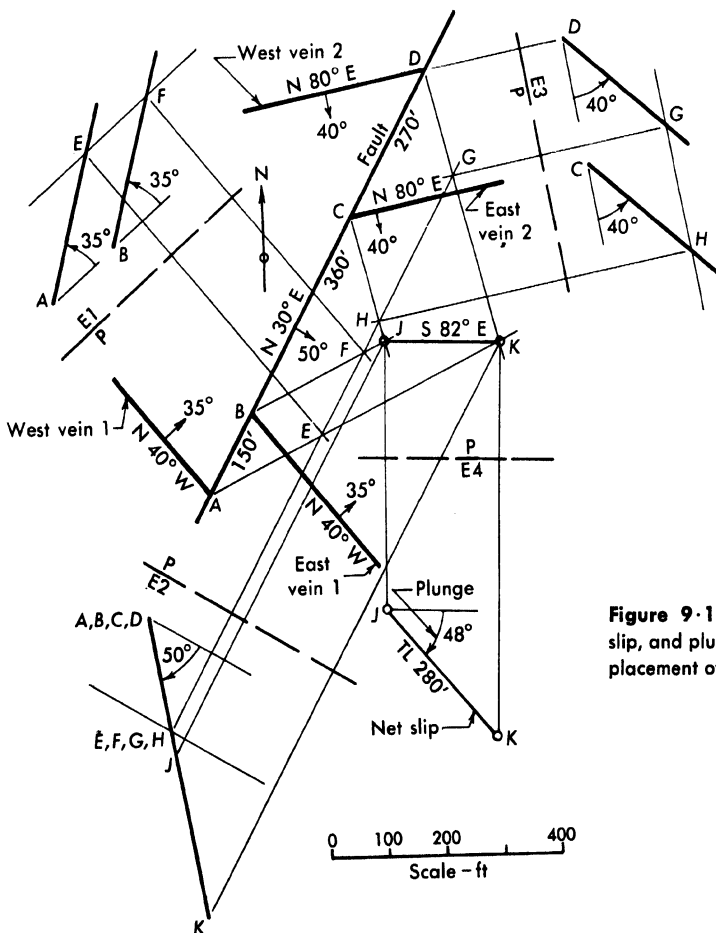
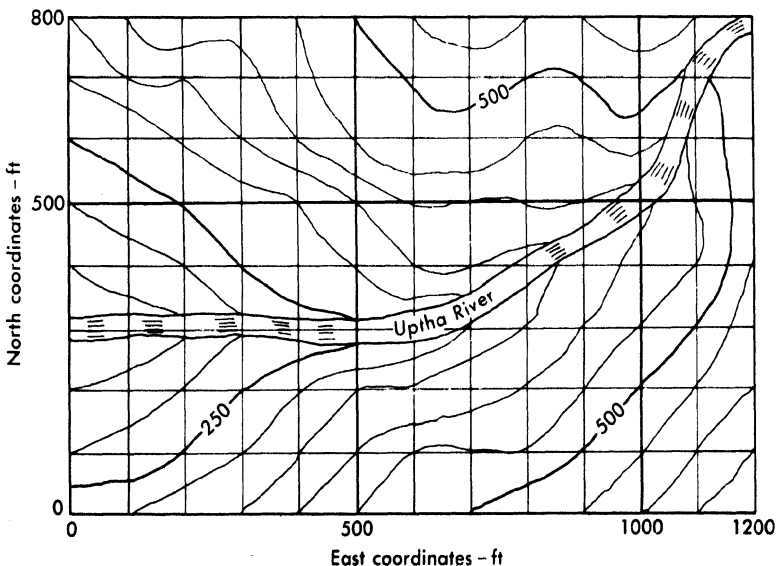


Figure 9-10 Bearing, net slip, and plunge from displacement of two veins

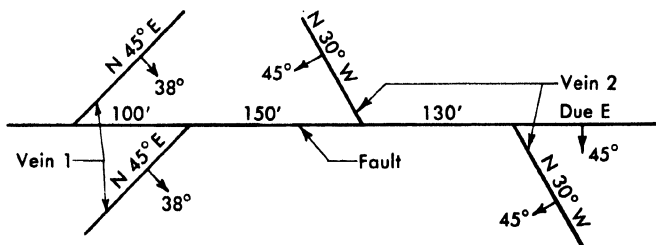
of the fault plane. Next, view E4 is drawn to find the net slip (280 ft) and plunge (48°).

9.13 Problems

- 9-1. An earth-fill dam is to be built across Uptha River. The centerline of the dam is to be a 700-ft radius circular curve, with the center of the circle at coordinates of East 0, North 300. Draw the profile of the ground along the centerline of the dam, starting at the north edge of the given map and extending to the south edge. Horizontal scale: 1 in. = 200 ft. Vertical scale: 1 in. = 100 ft.
- 9-2. The elevation of the top of the dam in Problem 9-1 is to be 450 ft. Both upstream and downstream faces are to have a slope of 2 to 1. Find the fill boundaries of both the upstream and downstream faces of the dam. Scale: 1 in. = 200 ft.
- 9-3. A point of outcrop on the upper surface of a stratum of rock is found at an elevation of 550 ft at E 600, N 800. A borehole at E 900, N 700 strikes this stratum at an elevation of 450 ft. A second borehole at E 800, N 500 strikes the upper surface of the stratum at an elevation of 300 ft and the lower surface at an elevation of 250 ft. (a) Find the strike, dip, and thickness of the stratum. (b) Draw the lines of outcrop on the map. Scale: 1 in. = 200 ft.



- 9-4. Points *A*, *B*, and *C* are three points on the hanging wall of a vein of ore and *D* is a point on the surface of the ground. Point *B* is 150 ft south and 150 ft east of *A*, *C* is 100 ft north and 325 ft east of *A*, and *D* is 100 ft due west of *C*. The elevations of the points are: *A*, 780 ft; *B*, 855 ft; *C*, 630 ft; and *D*, 830 ft. The vertical distance from *C* to the footwall is 80 ft. Find (a) the strike, dip, and thickness of the vein; (b) the apparent dip of the vein along a line bearing N 30° W; and (c) the length, bearing, and slope of the shortest shaft from *D* to the vein. Scale: 1 in. = 100 ft.
- 9-5. The apparent dip of a stratum is 40° (NE) on a cliff that trends N 60° E and 15° (SE) on the bank of a stream that flows N 75° W. Find the strike and dip of the stratum.
- 9-6. A stratum of rock has a strike of N 60° W. The apparent dip of the stratum along a line bearing due W is 30° (W). Find the dip of the stratum.
- 9-7. Point *A* is on vein 1 (N 40° W, 37° NE) at an elevation of 1700 ft. Point *B* on vein 2 (N 65° E, 50° NW) is 550 ft east and 150 ft south of *A*, and is at the same elevation. Find the bearing and plunge of the line of intersection and its rake on vein 1. Scale: 1 in. = 300 ft.
- 9-8. The line of intersection of a stratum of shale and a stratum of sandstone bears S 15° E and plunges 32° (SE). The sandstone stratum strikes N 45° E and the shale stratum strikes N 55° W. Find (a) the dips of the two strata and (b) the rake of the line of intersection on the sandstone stratum.
- 9-9. A stratum of limestone (N 45° W, 60° NE) has a strike separation of 300 ft south side east on a fault which strikes N 75° E and dips 45° SE. The rake of the slickenside striations on the fault plane is 60° SW. Find the bearing, net slip, and plunge of the fault displacement. Scale: 1 in. = 200 ft.
- 9-10. A fault strikes N 30° W and dips 46° NE. On the northerly side of the fault a vein strikes due E and dips 60° S. The fault displacement has a bearing of N 45° E, a plunge of 45°, and a net slip of 60 ft. Locate the vein on the southerly side of the fault and find the strike separation. Scale: 1 in. = 50 ft.
- 9-11. Two veins have strike separations along a fault as shown. Find the bearing, plunge, and net slip of the displacement and its rake on the fault plane. Scale: 1 in. = 100 ft.



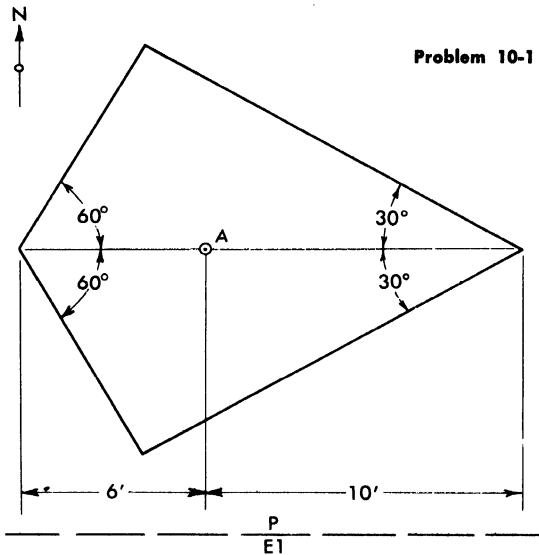
Problem 9-11

CHAPTER 10

CHAPTER 10 Miscellaneous
Problems

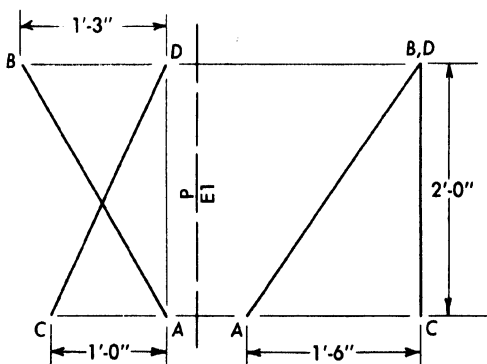
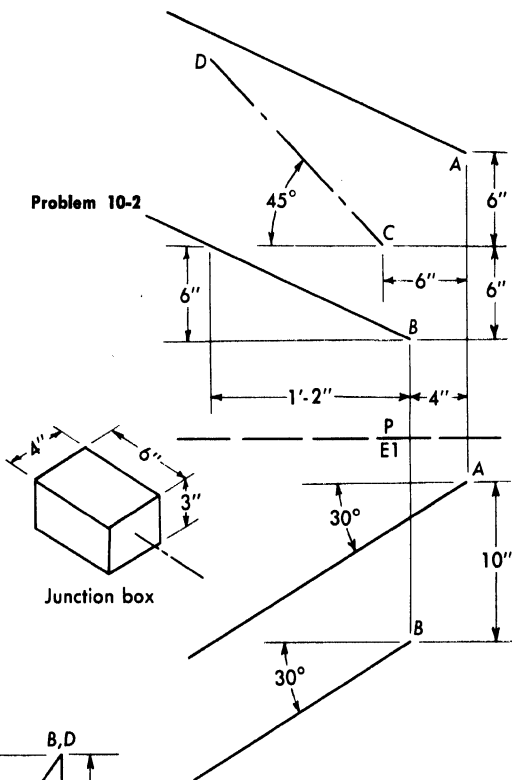
Many of the problems in this chapter require the use of methods from more than one of the preceding chapters for their solution. The student should read each problem carefully and thoroughly and select the appropriate method or methods before attempting the solution.

- 10-1.** The waterline of a concrete pier is shown in the plan view. The top of the pier is level and is 9 ft above water. All sides have a batter of 1 horizontal to 3 vertical. It was necessary to imbed an inclined marine

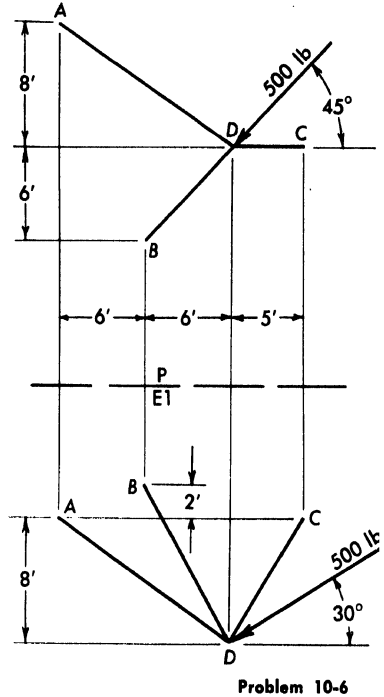
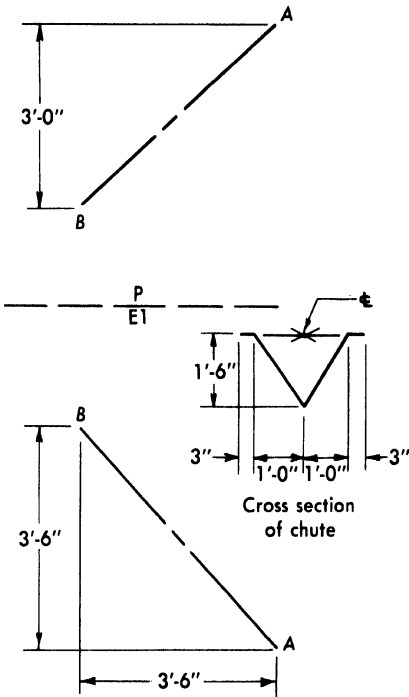


cable conduit in the concrete. This conduit is 5 ft above water at point A, which is its highest point. From this point, one end runs due north and slopes down at a 50 percent grade, and the other end runs S 30° W on a 30-degree downward slope. (a) Draw P and E1 views of the part of the pier that shows above water. (b) Show in all views only the imbedded part of the conduit. Scale: $\frac{1}{4}$ in. = 1 ft 0 in.

- 10-2.** A plane is determined by two parallel lines from A and B . The center-line of a conduit CD , which is parallel to the given plane, is located in the plan view only. The junction box shown is to be placed anywhere along the conduit so there is a 2-in. clearance between the box and the plane. The largest face of the box is to be parallel to the given plane. Show the box in position in both given views. Scale: $1\frac{1}{2}$ in. = 1 ft 0 in.
- 10-3.** Locate the smallest circular saw that can be mounted on the axis AB so that it is just touching the center line CD of a 2-by-10-in. plank.

**Problem 10-3**

Problem 10-5

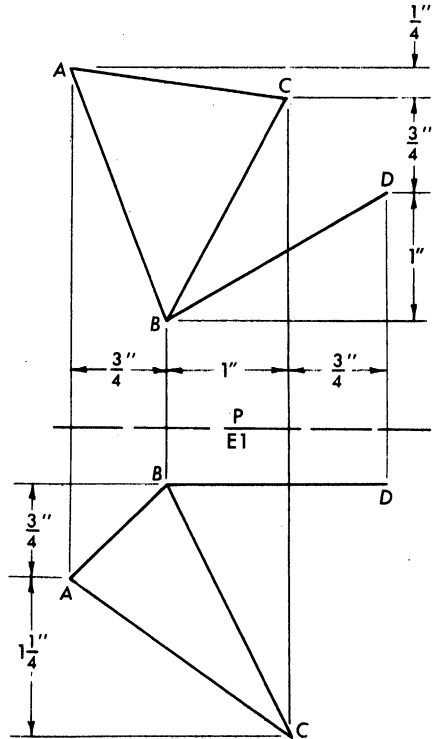
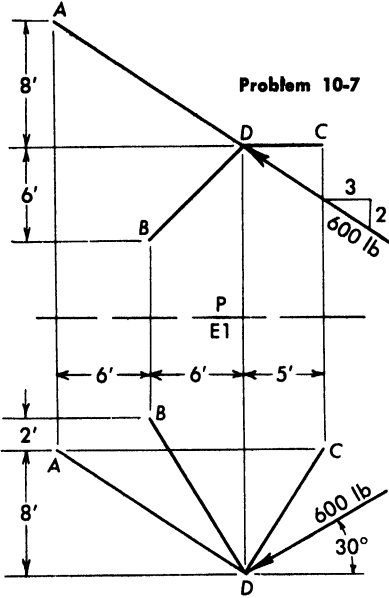


Show the center of the saw in the P and $E1$ views. Find (a) the distance from C to the point where the saw cuts the center line CD , (b) the diameter of the saw, and (c) the true size of the angle between the saw cut and the center line CD . Show the saw in the given views. Scale: $1\text{ in.} = 1\text{ ft } 0\text{ in.}$

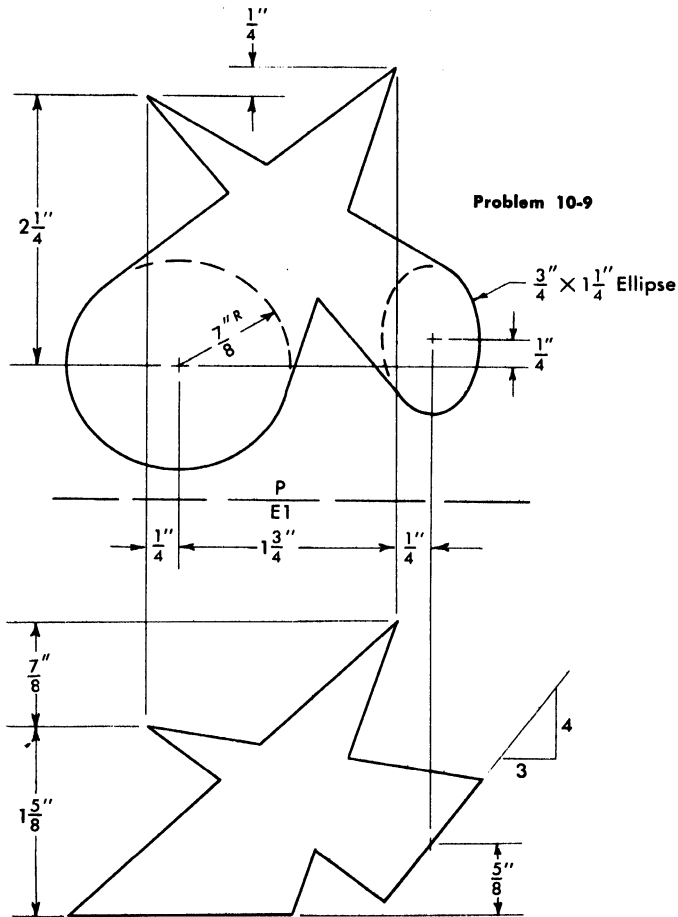
- 10-4.** A triangular plane ABC is given as follows: Point B is 90 ft north and 65 ft west of A and 100 ft above A . Point C is 30 ft north and 130 ft west of A and 15 ft below B . Point D , 60 ft above and 65 ft west of A , is on plane ABC . Using the minimum number of views, draw a line 40 ft long, perpendicular to the plane at D and entirely above the plane. Scale: $1\text{ in.} = 30\text{ ft}$.
- 10-5.** Line AB is the centerline of a chute with a cross section as shown. The chute is to be placed so that a plane through the centerline and the vertex of the V is vertical. The ends of the chute are to be cut off square at A and B . Show the entire chute in the given views. Scale: $\frac{1}{2}\text{ in.} = 1\text{ ft } 0\text{ in.}$
- 10-6.** Find the stress in each member of the given structure. Space scale: $\frac{1}{8}\text{ in.} = 1\text{ ft } 0\text{ in.}$ Force scale: $1\text{ in.} = 200\text{ lb}$.

10-7. Find the stress in each member of the given structure. Space scale: $\frac{1}{8}$ in. = 1 ft 0 in. Force scale: 1 in. = 200 lb.

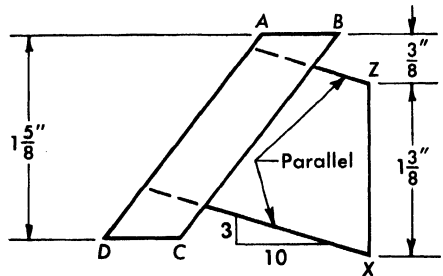
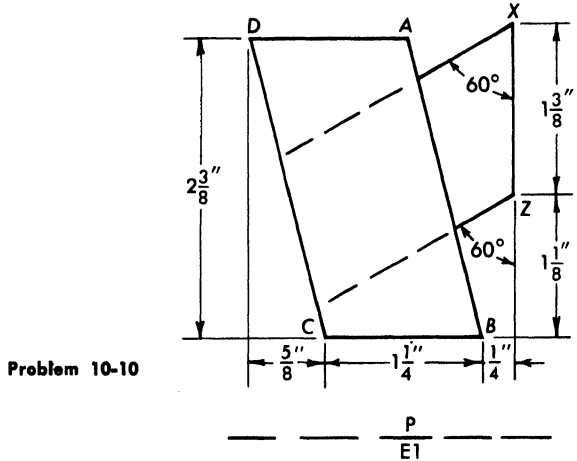
10-8. Find the true angle between line BD and the plane ABC . Use the line method. Scale: Full size.



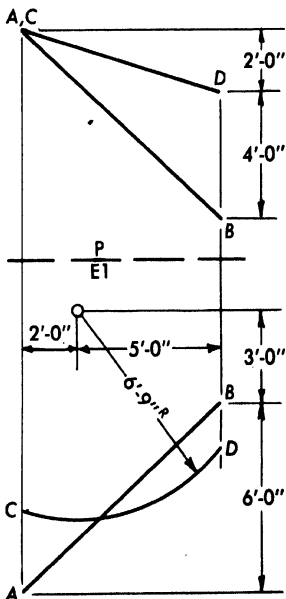
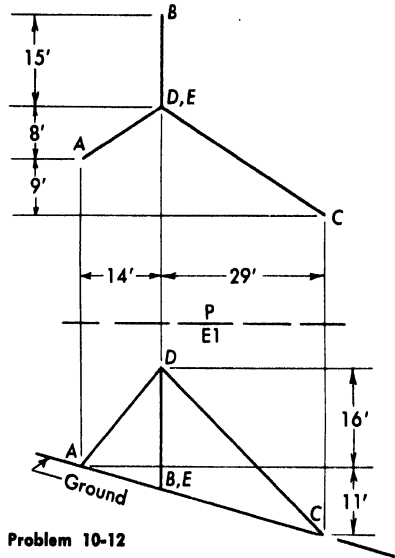
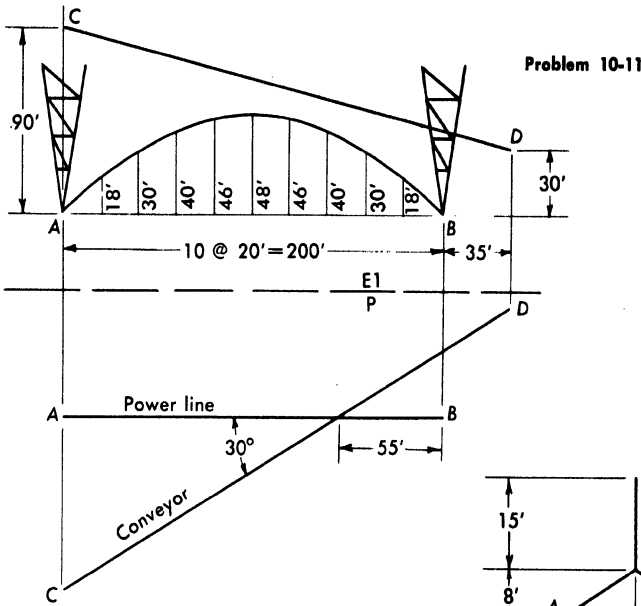
- 10-9. Show the entire line of intersection of the two cones in both given views. Scale: Full size.



- 10-10.** The plate extending from XZ is to be bent, or flanged, so that it can be fastened to plate $ABCD$ (a parallelogram). The flange is to be $\frac{1}{4}$ in. wide, and the bend radius is to be neglected. Determine the number of degrees (less than 90 degrees) through which the plate must be bent from the flat to make the flange fit. Show the flange in the plan view. Scale: Full size.

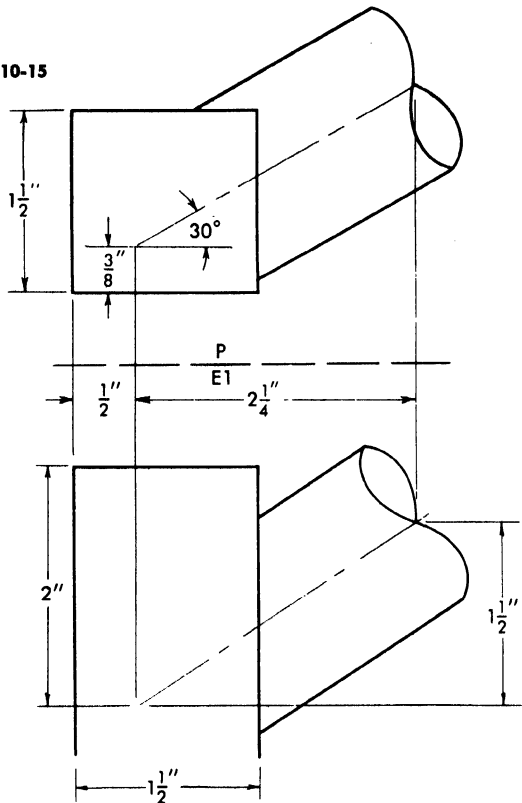


- 10-11.** A power line is supported by towers at A and B . Investigate the clearances between the power line and the proposed belt conveyor CD (a) when the power line hangs in its normal position, and (b) when the power line is blown 45 degrees out of plumb during a severe wind-storm. Scale: 1 in. = 60 ft.
- 10-12.** The vertical mast DE is held in position by guy wires to anchors A , B , and C . Determine the horizontal distance that B and C must be moved so that all three guy wires make equal angles with the mast. Do not move A or change the bearings of the guy wires. Scale: 1 in. = 20 ft.
- 10-13.** Find the bearing, true length, and slope of the shortest possible connection between tubes AB and CD . Scale: $\frac{1}{4}$ in. = 1 ft 0 in.



- 10-14.** An airplane at an altitude of 21,000 ft is flying $N 75^\circ E$ at a ground speed of 450 mi per hr and is descending at a rate of 9000 ft per minute. A second airplane is 5 mi east and 3 mi south of the first at an altitude of 9400 ft. The second airplane is flying $N 45^\circ E$ at a ground speed of 360 mi per hr and is climbing at 3000 ft per min. How far apart will the two airplanes be at their closest position? How far does each plane travel, and how long does it take, to reach this position? Scales: Distance, 1 in. = 2 mi; velocity, 1 in. = 4 mi per min.
- 10-15.** A $1\frac{1}{2}$ -inch diameter cylinder of revolution intersects the rectangular prism as shown in the plan view. (a) Complete view E1. (b) Develop a portion of the cylinder so it will fit against the prism. Scale: Full size.

Problem 10-15



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