TIGHT BINDING BOOK

Introductory Acoustics

BY

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FOURTH PRINTING

NEW YORK

D. VAN NOSTRAND COMPANY, INC.

250 FOURTH AVENUE

Special Edition, March 1932

Regular Edition Published, February 1933

Reprinted, February 1937, November 1940

June 1946

PRINTED IN THE U. S. A.

PREFACE

The accompanying text is an elementary treatise that undertakes to consider the most common phenomena in acoustics. The content assumes no previous preparation in physics, and utilizes very few mathematical expressions. The limitation in preparation of the student is met by the insertion in the text of the meaning of each technical term at the point where it is first employed.

The absence of mathematics places an increased responsibility upon language in presenting a clear analysis of all the phenomena. Thus at many points the writing is necessarily condensed and requires careful reading and re-reading. If the student will anticipate this type of effort he will have no serious difficulty. The text does not survey the field rapidly as most elementary texts do; it endeavors to study each topic with a thoroughness somewhat unexpected in a nonmathematical text. The student will secure an acquaintance that will not only serve as a background for any professional work involving acoustics, but also as valuable information that can be applied with success. As a matter of fact, the viewpoint of the book is utility in the broadest sense, including culture. The historical aspects of the subject are largely omitted to make room for the detailed explanations and analyses which are regarded as more important.

The number of students who need such a background and yet who cannot afford the time for mathematical studies is rapidly increasing. While these have been prominently in the author's mind, yet it is evident that the book can also be used in intermediate courses in physics. Moreover, the amount of acoustics in the usual elementary course in physics is so small that students with and without previous preparation in elementary physics can use this text in the same class. But it is preferable that the course be offered in the junior and senior years rather than in the first year of college.

Numerous demonstration experiments are suggested throughout. These and others which can be introduced by the instructor will prove invaluable.

The preparation of this text has extended over several years in connection with classes consisting for the most part of students specializing in music, speech and psychology. The attitude they have shown toward acquiring a clear understanding of acoustics and the pleasure they have derived from the nonmathematical analyses of phenomena have supplied the incentive for the revisions and final preparation of the manuscript.

I take pleasure in acknowledging my indebtedness to the students who have from time to time given excellent criticism and especially to my colleagues, Dr. P. G. Clapp, Director of Music, who prepared Section 14.6, and Dr. C. J. Lapp, who has critically examined the entire manuscript.

GEORGE WALTER STEWART

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To the Student who uses this Textbook:

This textbook represents many years of learning and experience on the part of the author. It does not treat of an ephemeral subject, but one which, since you are studying it in college, you must feel will have a use to you in your future life.

Unquestionably you will many times in later life wish to refer to specific details and facts about the subject which this book covers and which you may forget. How better could you find this information than in the textbook which you have studied from cover to cover?

Retain it for your reference library. You will use it many times in the future.

The Publishers.

CHAPTER I

SOUND WAVES

1.1. **Acoustics.** — Interest in acoustics has been increasing very rapidly during the past two decades. This has not occurred because of advances made in musical instruments or in our use of them. The instruments used today are largely the result of a gradual development over many centuries. They are not the result of scientific research, but of countless experiments in making sounds in every possible manner. Only with the advent of the telephone, radio loud speakers, the location of airplanes and guns in war, and, in general, all forms of acoustic reproduction, reception and transmission, has scientific research made invaluable contributions to development in acoustics. Today money expended annually in research in this one field probably exceeds half a million dollars. By the extension of scientific knowledge and its application many results have been achieved which would have been unattainable by a straightforward, experimental, trial and error method.

The applications have been made in a carefully reasoned manner. This has usually required the aid of mathematics not readily understood by the novice. The important fact for the student to bear in mind is that progress has depended generally not upon chance discoveries, but upon the most careful use of reasoning power. When phenomena occur that appear inexplicable, we lack either the necessary information concerning them, or the ability to reason correctly. For example, one might surmise that if speech could be heard and understood from a boat to shore, then speech should travel equally well in the reverse direction and should be heard just as easily from shore to boat. This surmise arises from many common experiences in calling to one another across open spaces or in a building, and also from what might be termed "common sense." But these do not suffice to give the correct answer, for the transmission is not the same in both directions. The wise way to proceed to an understanding of the transmission of sound from boat to shore is to study carefully what is actually involved in sound transmission and in its alteration in any manner. Not until these are clear in mind can the student consider with profit the transmission of sound in this case. This relatively simple problem is illustrative of the necessary procedure. When a problem is complicated, like the perfect reproduction of all sounds in a talking picture, the attitude of mind should be precisely the same. A careful understanding of all the phenomena involved, and logical reasoning based thereon, are essential. In the reasoning, mathematics is indispensable. The well-known processes that have been demonstrated in mathematics are capable not only of saving the human mind much complicated thought, but also of arriving at correct conclusions even in cases where the mind alone could not secure a solution. But, in the more simple situations in acoustics, the mind can follow the reasoning without the use of detailed mathematics. In such cases, it is all the more important that the phenomena involved be understood with *great clearness.* It is because of this fact that, in this text, emphasis will be placed upon clear understanding. The language must be concise and accurate and the reader because of this fact must follow no more rapidly than he can comprehend. The meagerness of the mathematics used is an advantage to the nonmathematical student, but this omission thrusts upon him a correspondingly greater care in understanding and reasoning. Every aspect considered in the early part of the text will be used repeatedly. Therefore, full comprehension must be obtained as one progresses. Acoustics is a particularly satisfying branch of physics because one can visualize the details of the phenomena which are concerned with matter and with vibrations therein. Both of these seem concrete and understandable.

A clear knowledge of the elements of acoustics is becoming increasingly important to any profession depending in any manner upon acoustics. Civilization is becoming acoustically conscious. It is studying the effects of noise. It is more critical of all acoustic effects whether in speech, music or sound transmission. The phonograph of yesterday will not be tolerated today. Auditoriums, music halls, and studio rooms must possess proper acoustic qualities. Acoustic effects satisfactory in the past will not be permitted in the future. Acoustics is an old subject, but with new responsibilities of everyday importance.

1.2. **Waves.** — The most fundamental concept to be grasped is the nature of sound itself. It is said to travel in waves. It actually consists of waves in matter. Our long familiarity with listening to all manner of sounds does not help us to understand what is meant by a sound wave.

Everyone has witnessed the movement of water waves and has recognized that they have a definite speed. A study of water waves discloses that the water itself moves neither horizontally forward with the velocity of the wave just mentioned, nor vertically upward and downward. Yet these are the two movements one recognizes visually as the most likely. It is found that any particular portion of the water itself has an approximately circular motion,* the plane of the circle being vertical and extending in the direction of motion of the wave. If this is the case, then if one says the wave has a definite velocity forward he does not refer to the water itself but rather to the physical shape of the surface of the water. This shape certainly moves with a horizontal velocity. To repeat, the term "wave" is used to refer to the physical shape and not to any portion of the water itself. But carry the ordinary use of the term "wave" a little further. The Weather Bureau announces that a "cold wave" is coming. Everyone understands that he may expect the thermometer to fall. It is a wave of low temperature and is said to be a wave because this physical condition has a velocity in a definite direction across the country. No one thinks that there is an actual movement of the same cold air from one point of the country to

* Reference is made to the case of waves that are not too great in magnitude and that are in deep water.

another. It is the movement of a physical condition. In a similar manner we may have a wave of atmospheric pressure.

It is noticed that in the above the word "wave" has acquired a definite technical meaning. When once clearly understood, no further difficulty is experienced. There are found numerous physical changes that have at any point a directional velocity. These alterations in physical relationships which are propagated through the medium concerned with a definite velocity are usually referred to as "waves."

1.3. **Properties of Waves.** — The water waves mentioned in the previous section are caused by the action of gravity, and are known as "gravity waves." They are quite different from what are termed "ripple waves." The latter are caused by a curious surface film of oriented molecules which acts very much like a very thin stretched membrane. If the ripple waves are very small the motion of the water is vertically upward and downward. Thus very small waves and the large wind waves on water are different in detail, for the active agency in the propagation of the one is the tension or pull in the surface film, and the other, the gravitational attraction of the earth. This section will not be concerned with this difference, but rather with the use of ripples as an illustration of the action of waves in certain respects. Figure 1.1 * is an instantaneous photograph taken of a series of such ripples on a water surface. They were produced by the continuous vibration of a thin wire projecting into the surface and vibrating perpendicular to it. This vibrator is located at the center of the concentric rings. The surface is brightly illuminated and the photograph shows the condition of the surface at one instant. There are two facts to be noticed. First, the waves form concentric circles. Evidently the different parts of any one wave have travelled equal radial distances in the same time interval. That is, the different parts of the wave have had the same speed of propagation at the same time. Is there any evidence

* The photographs shown in Figs. 1.1 to 1.6 are reproduced by the permission of the publishers of "Einführung in die Mechanik und Akustik" by R. W. Pohl, Julius Springer, Berlin, 1930.

the ripple waves seem more and more to have their origin in the hole itself.

The spreading of the waves in Fig. 1.2 may be described as bending around the edges of the hole. This would lead one to ex-

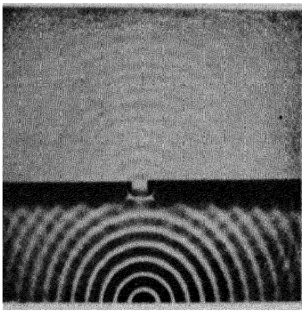


FIG. 1.3

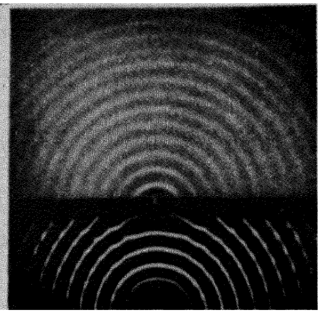


FIG. 1.4

pect the effect shown in Fig. 1.5. Here the bending prevents the obstacle from casting what might be termed a "sharp shadow."

What occurs at reflection is shown in Fig. 1.6. Here there seems to be a new set of waves issuing from the reflector. These

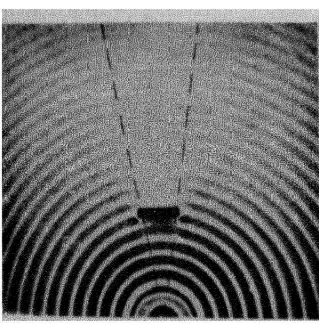


FIG. 1.5

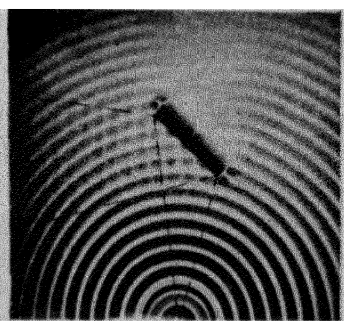


FIG. 1.6

are again circular, having a center as far behind the reflector as is the original source of the waves in front. A more definite understanding of the geometrical symmetry of the location of these two centers can be obtained from the discussion of reflection of acoustic waves in Section 2.1.

These ripple waves, with the properties shown, are fairly satisfactory as an analogy to sound waves. When one first becomes interested in the physics of acoustics he has need for something concrete, even though not strictly like sound waves. The properties of the ripple waves as described are similar to those of acoustic waves, but the nature of the waves themselves is entirely different. But this need not prevent one from gaining a better conception of acoustic waves by studying the properties of ripple waves. We see that the waves are not propagated in fairly straight * lines like light, and that they are not reflected as would be a tennis ball from a wall. Indeed, as shown by the action of the small hole in a wall, they spread out in all directions from a point on the wave. The properties of acoustic waves must be appreciated at the outset, though of course the reader cannot visualize in detail an acoustic wave. As he becomes more familiar with the properties of such waves there will be less need for analogies and visualization. The next section introduces the reader to the nature of acoustic waves by the consideration of waves travelling in but one direction. From the propagation in one direction will be developed the phenomena occurring when the observer is near the source, and the sound, travelling radially, has a different direction of propagation at different points.

1.4. A "Wave" in a Helix. — If one considers a helix of wire suspended as shown in the accompanying Fig. 1.7 he sees that it is possible to produce a wave. For if several of the turns at one end are pressed together and then the inside turn is released, the compressed part will at once expand, producing compression ahead. Thus a "wave of compression" will travel along the helix, having a velocity or speed of propagation † that depends upon the dimensions of the wire and helix and the physical nature of the material in the wire.

* Light does not travel in lines that are exactly straight, but nearly so. For our present purpose the straight line propagation of light will be assumed.

† "Velocity" is the distance passed over per unit of time. Technically it differs from "speed" in that the former includes the *direction* as well as the amount or magnitude.

1.5. Different Aspects of a Wave. — A wave of compression has just been described. What other physical alterations are there in this wave? First it is to be observed that a wave of compression is always accompanied by a wave of rarefaction.

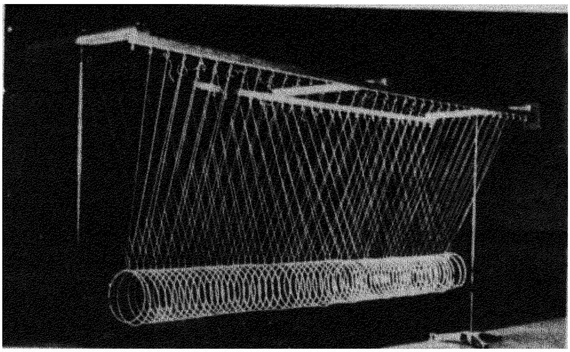


Fig. 1.7

For, consider a wave that has been established by giving the helix a sudden compression at the end and then restoring that end to its original position of rest. Suppose the compression to be travelling along the helix, both ends of which are in their original undisturbed positions. The actual length of the helix is unaltered, although there is a portion where the helix is compressed. If the total length is unaltered there must be a portion where the helix is elongated. So a wave of compression in the helix must be associated with a wave of rarefaction. These are indeed two aspects of the same wave. Again, one can observe in this experiment that the progress of the wave will cause any given turn to oscillate or vibrate to and fro during the passage of the wave at that point. This vibration is readily shown to the eye by tying a bit of string at the bottom of one of the turns of wire. Each and every point of the helix suffers a displacement * from its position at rest, and this displacement is first in one direction and then in another. We may say that a wave of displacement has travelled along the helix. In fact, it can be

* The word "displacement" is sufficiently defined by its use here. The amount or magnitude of the displacement is the actual distance from its position of rest.

shown * that a wave of displacement and a wave of compression not only are, but must be, coexistent.

The fourth aspect of the helix wave is somewhat more abstract. Velocity is the rate of movement in a given direction. It is the distance covered per unit of time. As a turn of the helix is displaced, it has a definite velocity in that direction. As the wave of displacement passes, the helix experiences a velocity (not constant or uniform) first in one direction and then in another. We could consider this aspect of the wave and call it a wave of impressed velocity. But one can see at a glance that the velocity referred to is *not the velocity of the* wave, but of a given portion of the helix. There is then a very definite distinction between the velocity of the wave described in Section 1.4 and a wave of velocity considered in this section. This is true of the helix and it is also true of a sound wave.

As above shown, there are four different aspects of this wave in a helix, compression, elongation, displacement and velocity. There is an analogy in the acoustic wave in a gas. A gas resists compression and will return to its former volume after compression (cf. automobile tire). So does the helix. A gas has inertia, that is, time is required to set it in motion. This is true of the helix. Indeed, these two qualities, which are called "elasticity" and "inertia," make possible the existence of a wave and its movement at a definite speed. It is sufficient for the present purpose to check up this thought with the helix. The wave of compression moves forward because the compression at one point exerts a force attempting to compress another point just in front. Without elasticity this would not occur. Moreover, if the helix were without inertia, any force would produce an effect instantly. This would result in an infinite velocity of the wave. This is, of course, not imaginable because we have never experienced anything without inertia and yet with ability to exert a force. A gas, having these two qualities, elasticity of compression and inertia, will transmit a wave of compression with a definite veloc-

* The phrase "it can be shown" will be frequently used. It refers to a possible demonstration but *not* one proposed for the student.

ity of propagation. Such a wave is called an acoustic or sound wave. For the purposes of this book, the wave will be so designated even if it cannot be detected by the human ear.

1.6. Gas as a Medium for Sound Waves. — That substance which transmits a sound wave is called a "medium" or means of transmission. In the case of the helix the medium is a continuous one, that is, the helix is a continuous piece of wire. Upon closer examination the wire is found to be made of fragments of crystals and in each of these fragments an orderly array of atoms, separated from each other, but nevertheless exerting forces upon one another. A force is required to compress or elongate such a crystal in any way. But for our present purpose we will not inquire as to what these individual atoms and crystals are doing. We will not go further than to appreciate the four aspects of the wave in the helix as previously described. In a similar manner we will neglect any consideration of the molecular constitution of a gas. The separation of these molecules is, on the average, very much greater than the diameter of one of them. To add to the detail, it should be stated that these molecules are moving to and fro in every direction, these motions corresponding to the heat the gas possesses. We are to be satisfied with the fact that the gas has elasticity and inertia, just as if it were continuous, and therefore it will act acoustically as an imagined continuous medium. No reference will need to be made to the motions of the individual molecules. Hereinafter when the phrase "a particle of the medium" is used it is understood that this does *not* refer to a molecule, but to a small portion, called a "particle," of the medium imagined to be continuous. This substitution of imagined continuity will be a great convenience.

1.7. Representation of a Sound Wave. — A sound wave may occur in a solid, a liquid or a gas. But in the last two, or fluids, the wave takes only the form of a wave of condensation and rarefaction, as already described. The discussion in this text is limited practically to that kind of a wave. In the case of most vibratory bodies producing aerial sound waves, such as a vibrating string or a vibrating air column in a wind musical instrument, the waves of compression and rarefaction produced in the air and reaching the ear follow one another in succession, but with the pressure * of the air at any point changing with time in an interesting manner. One may graph a series of such waves of changing pressures as in Fig. 1.8. Here the changing value of the pressure at a point *O* is represented by distances above and below the horizontal line, each positive and negative value being respectively an excess and a shortage of pressure when compared with the mean pressure. Figure 1.8 as drawn actually represents

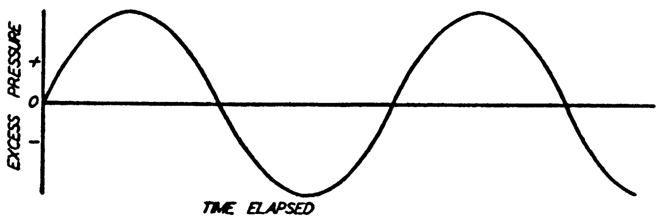


FIG. 1.8

a type of variation with time that has been found to be the simplest in every respect. This variation may be that of a quantity like pressure, but it also may be one of displacement from a mean position. It is, in fact, approximately the variation in displacement with time possessed by a vibrating pendulum. On first thought one would scarcely refer to the motion of a pendulum as simple, for its velocity changes constantly as it swings from its mean to its extreme positions. It might seem that a simpler motion would be one of uniform speed everywhere except at the ends of the arc where the pendulum could be stopped suddenly. But this is not the case, because two entirely different kinds of motion are assumed, one a uniform velocity and another an abruptly changing velocity. Being so different they could not be simultaneously described in a simple way. But with the pendulum the velocity changes progressively in a manner that permits a relatively simple specification. In its most condensed

* "Pressure" in a gas technically means the force per unit area which the gas would exert upon any containing wall.

form this statement is entirely mathematical. But a visual description can be obtained by observing certain actual motions. The three driving wheels on one side of a steam locomotive are connected and driven by a side or parallel rod which in turn is attached to the piston. At each wheel the side rod bears upon a crank pin which is made a part of the wheel itself. Assume that these wheels are being driven by the side rod, but that they are slipping on the track without any forward motion of the engine. Let the observer, who is standing alongside, fix his attention upon the pin on the wheel. It spins about in a circular motion. Suppose, however, that the observer were standing at a distance of perhaps fifty feet from the engine and yet alongside the track. Suppose also that although he is standing practically in the plane of the revolving driving wheel, he is yet able to see the moving end of the side rod or the crank pin itself. He will observe now not a circular motion of the pin but one upward and downward, with the entire movement appearing to occur in practically a straight line. In this apparent motion, the velocity of the pin will vary, being maximum at the center of its vertical path and zero at the extremities.

In the lecture room an experiment can be arranged as follows. A horizontal beam of light issues from a lantern and falls perpendicularly upon the screen. In its path is placed a bicycle wheel rotating at constant speed about a vertical axis. The shadow of the wheel rim or tire appears as a horizontal line. If a ball is fastened above the wheel and at the rim, its shadow remains on the screen during the rotation of the wheel. This shadow of the ball moves to and fro, apparently in a horizontal straight line. As will be surmised from an earlier statement this motion of the shadow has the same kind of varying velocity as that of a pendulum ball. The motion of the shadow is also a visual description of a pendulum's motion. Moreover, this description is simple, for it involves only a projection of a uniform motion in a circle, in itself a very simple kind of motion. It is not surprising to learn that when the motion of the shadow of the ball is described mathematically it proves to be very simple indeed. But it is now necessary to revise the conditions of the experiment to conform exactly with the mathematical statement mentioned. The light falling on the ball must be parallel and not at all divergent. Then the shadow has exactly the same dimensions as the wheel itself. With this alteration it can now be said that the motion of the ball's shadow is the kind of vibration or oscillation used throughout the subject of acoustics. It is called "simple harmonic motion," and is the simplest type known. Clearly one may similarly refer to a simple harmonic variation of pressure, or any other quantity, when its variation is like that of the displacement of the shadow of the ball from a mean position. Now it happens that the simplest vibration made by a tuning fork, by a piano string or by almost any vibrating mechanism, is precisely of the same character. A simple musical tone is a simple harmonic variation of pressure such as indicated in Fig. 1.8. The usual musical tone consists of a number of such simple tones. In the foregoing discussion the motion of a pendulum was selected because it is familiar. It must now be admitted that its motion is not strictly simple harmonic, as is that of the shadow of the ball moving in a circle, but approximates very closely to that condition. On the other hand, a simple pure tone does consist of a simple harmonic variation of pressure as stated. Moreover, the variations of displacement and particle velocity are also simple harmonic. This type of variation or vibration is the kind with which we are particularly concerned in acoustics. It is the building unit out of which we will construct or describe complex tones.

It has just been stated that the time variation of the excess pressure at a point in the medium can be represented graphically by a continuous curve. This is a wave of pressure. One could, however, represent the same kind of a wave by considering a row of little masses joined by an idealized weightless elastic cord as in line *A* of Fig. 1.9. Let there be a compressional wave along the cord similar to the one previously in the helix. Then line *B* will represent what is happening at a certain instant to the row of masses.

As the wave travels, for example, from left to right (just as in the helix), the masses suffer a *to and fro* displacement along this horizontal line. It is not possible, without confusion, to represent the displacements of all these masses by drawing lines in their actual directions, for these would all lie in the same straight line, i.e., the direction of the cord. But the displacements can be represented clearly by selecting a somewhat arbitrary method. If, from the mean positions of the particles, lines proportional but *perpendicular* to this actual displacement are drawn, a curve through the ends of these lines may be said to represent the displacements at that given instant. For example, the undisturbed

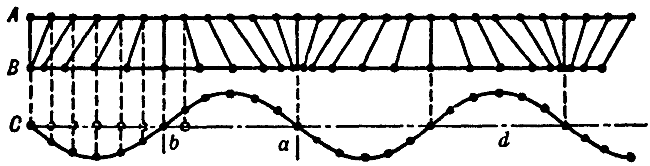


FIG. 1.9

A. An elastic cord with equidistant loading.
B. A compressional wave in the cord.
C. A graph showing displacements from mean positions of these loads.

position of the small masses is shown in *A*. But if a longitudinal wave is passing along this row of masses, then, *at a chosen instant*, the position of these same masses may be indicated by the drawing in *B*. If the actual *horizontal* displacement of a given mass, as shown by a comparison of *A* and *B*, is represented by a *vertical* line of the same length, perpendicular to the horizontal line in *C*, but drawn from the undisturbed position as in *A*, and if this representation is repeated for each particle, a curve drawn through the ends of these lines may be regarded as representing the displacements of the masses at the given instant. Displacements to the left and right have been represented by displacements down and up, respectively. This is an awkward method, because the graph does not truly represent the direction of a displacement. The graph in *C* nevertheless is said to represent the displacements of the row of masses at a given instant.* The masses

* The solid circles on curve *C* represent the pseudo-positions of the masses, the positions they would occupy had the displacement been up and down.

and the elastic cord can now be replaced in imagination by a gaseous medium, and Fig. 1.9 then gives a visualization of what is transpiring in a compressional wave in a gas. The graph of displacement is to be regarded as referring to the "particles" of the medium. Curve C in Fig. 1.9 is then a representation of a compressional or sound wave in a gas at any instant. If the wave can be considered as moving from the left to the right with a definite speed, one can prophesy just what will happen to a given particle at a certain time.

In curve C, the vertical distance above the horizontal line represents a displacement to the right, and the one below or downward, a displacement to the left. Having in mind the correspondence between the displacement to the *right* and the representation drawn *upward*, it is observed that the displacements in the neighborhood of the point "a" are directed toward that point from both sides. Consequently "a" is a point of maximum pressure at that instant. By similar reasoning "b" is observed to be the point of minimum pressure at that instant. There are two differences in the graphs in Figs. 1.8 and 1.9. Not only do they refer to variations of different quantities, but the former refers to the variation at a point *as time elapses*, while the latter is an instantaneous picture, so to speak, of the condition of a row of particles lying in the direction of the passage of the wave. There is a certain similarity between Fig. 1.9 and a gravity wave at the surface of water, though, in point of fact, the two are not of the same shape. The "crest" of a water wave is not the same shape as the "trough" and this is caused by the fact that the motion of the water particles is circular.

1.8. Velocity. — As already suggested by a statement in regard to the helix, the velocity of a wave depends upon the medium in which it is propagated. It is possible to prove that in a gas the velocity of a sound wave is

$$v = \sqrt{k\frac{p}{\rho}}, \qquad (1.1)$$

where p and ρ are the undisturbed pressure and density * under normal conditions, respectively, and k is a quantity depending upon what are called the specific heats † of a gas. While it is not proposed to examine the reasons for the form of this equation, we can with some satisfaction notice that the equation is in accord with the following considerations. A gas at high pressure recovers from a compression quickly just as a stiff spring under high pressure will, if released, return to its original position speedily. It appears reasonable that a quick recovery would result in a high velocity of the sound wave, and the formula (1.1) states that v is proportional to $\sqrt{p}$. But if one increases the mass of the spring, the recovery cannot be so rapid, for with the same force acting, the more massive the body the more slowly it can be set in motion. The formula states that the greater the density, pressure remaining constant, the less the velocity. This seems to be in accord with the variation just suggested.

A definition of "velocity of a sound wave" has been inferred. It is the speed with which the physical alteration is propagated in a definite direction. For example, if one represents a maximum displacement as at the point midway between b and a in Fig. 1.9, and if the wave is moving to the right, the velocity is the speed to the right which one must travel in order always to be at this point of maximum displacement.

It is necessary to refer to the velocity of sound in liquids and solids, for acoustic waves can be transmitted in any material. Formula (1.1) applies to gases only. In liquids it is customary to employ a slightly different form, as follows:

$$v = \sqrt{\frac{E}{\rho}}. \qquad (1.2)$$

Here E is a symbol representing what is technically called the volume elasticity of the medium. The reader will not be asked to become familiar with this technical definition, but merely with

* "Density" is the amount of the gas per ccm. Technically, it is measured in mass per ccm. or grams per ccm.

† The exact definitions of "specific heats" need not concern the reader. Obviously heat enters into the situation, for when a gas is compressed it is heated thereby and when expanded it is cooled.

the statement that volume elasticity is a measure of the force required to reduce the volume of the material by a fixed amount. The value E must be obtained for each liquid, so that there is no way in which one can compute the velocity in one liquid from the knowledge of the velocity in another. In gases, there is a slight variation in "k," but this variation is known to be caused by differences in the number of atoms in a molecule.

It is evident, however, that the nature of the vibrations in gases and liquids is the same. In both, molecules can change positions relative to one another freely, but in both there is an opposition to compression or expansion. In a solid, the molecules are close together and are fixed in relative positions. It will resist compression and expansion and there is possible the propagation of an acoustic wave in a given direction in the same manner as in a gas or liquid. There is also in a solid resisting

Table I

Substance	Temperature ° C.	Velocity in meters per second	Reference
Air	0°	331.41	Hebb (1919)
Air	0°	331.45	Foley
Air	0°	331.6	Reid (1930)
Carbon dioxide Atmospheric press.	0°	257 to 260	Various observers
Hydrogen Atmospheric press.	0°	1238 to 1269	" "
Oxygen Atmospheric press.	0°	315.2 to 317.2	" "
Water distilled	13.0°	1441	Dorsing (1908)
	19.0°	1461	"
	31.0°	1500	"
Sea-water	13.0°	1492.3	Eckhardt (1924)
Aluminum	—	5105.0	Masson (1858)
Copper	20°	3560.0	Wertheim (1849)
"	200°	2950.0	"
Cast steel	20°	4990.0	"
" "	200°	4790.0	"
Lead	15°–20°	1229.0	"
Hard rubber	—	150	Stefan (1872)
Vulcanized rubber	50°–70°	35	Exner (1874)
Brass	—	3617	Kundt (1868)

force to other kinds of motion such as a twist. Consequently, the transmission of sound in solids is more complicated than that in gases and liquids. In a wave of compression such as already discussed, the vibrations are in the *same direction* as the progress of the wave. Such vibrations are called technically "longitudinal." At a later point reference will be made to waves other than longitudinal that can be transmitted by a solid. Until then, any mention of sound transmission in solids will refer only to longitudinal waves.

The velocity of a sound wave is not wholly independent of the nature of the wave, as is evidenced by experiments with explosions wherein velocities have been found considerably in excess of the normal sound velocity. This is discussed in Section 1.9 but practically all sounds herein discussed travel with the same velocity, called the normal velocity. The accompanying Table I gives selected values.

1.9. A Variation of Velocity. — It will be noticed in (1.1) that we have $\sqrt{k\frac{p}{\rho}}$. If a gas is compressed so that p, the pressure, is doubled, it is found by experiment that ρ is doubled also. Hence the ratio of the two remains the same and consequently (1.1) states that there is no alteration in the velocity of sound. The same conclusion would be reached for an expansion of the gas. But this constancy of the ratio, with pressure changing, is true only if the temperature remains constant. For example, the temperature of a gas may be increased by heating, keeping the volume and hence the density constant and yet increasing the pressure. In fact, the ratio between p and ρ is determined by the temperature. Therefore the velocity of sound in a given gas depends only upon the temperature and not at all upon the values of pressure and density. It can be shown both experimentally and theoretically that the velocity is proportional to the square root of the absolute*

* Degrees on the absolute scale are very nearly the same size as on the Centigrade scale, but 0° C. is 273° absolute. Hence we generally add 273° to the reading of the Centigrade scale to get the absolute temperature. In the Centigrade scale water freezes at 0° and boils at 100°, these temperatures being written 0° C. and 100° C.

temperature. Hence at 0° C. for 1° C. rise in temperature the velocity must be $\sqrt{\frac{273 + 1}{273}}$ times the velocity at 0° C., and at t° C. it must be $\sqrt{\frac{273 + t}{273}}$ times the velocity at 0° C. or

$$v_t = v_0\sqrt{1 + \frac{t}{273}} = v_0\sqrt{(1 + .00366t)}, \qquad (1.3)$$

where v_0 is the velocity at 0° C. and v_t at t° C.

If a gas is under very great pressure, an exception must be made to the statements in the preceding paragraph. While it is true as stated that for a change of pressure under ordinary conditions the velocity changes but a negligible amount, yet at high pressure there is a marked change, especially at low temperature. This change is not only one of magnitude but is sometimes positive and sometimes negative. At − 103.5° C., the velocity with one hundred times the atmospheric pressure, or "100 atmospheres," is, according to Koch (1908), 293.2 meters per second, while with 150 atmospheres it is 346.9 and with 200 atmospheres it is 406.5 meters per second. Witkowski (1899) found that, at the same temperature, the velocity decreased from 260 to 245 meters per second when the pressure was increased from one to forty atmospheres.

The velocity in free air will change with humidity, because at the same pressure the presence of water vapor will alter the density. But this change is always less than one per cent if the atmosphere is saturated with moisture at ordinary temperatures.

Yet another exception will need to be made to the statement that the velocity of a wave in air depends only on the temperature. In the mathematical study of the passage of waves of condensation and rarefaction in a fluid, it has been proved that the velocity is independent of the magnitude of the displacements, but only if these are small. It might be anticipated, therefore, that waves of abnormally high velocities can be produced. This has been repeatedly accomplished by explosions. Even the waves near a large gun travel at a higher speed than the normal ones.

A review of earlier experiments is described by Professor A. L. Foley.* The speed of the waves produced by sparks has been studied by Foley and reported in the article just cited. He found that the speed close to the source depended upon the intensity and that in the case where he was able to obtain twice the normal velocity at a distance of 0.32 cm. from the source, the velocity had decreased to the normal value at 2 cm. from the source.

1.10. Frequency and Wave-Length. — The frequency of a sound vibration is usually the number of complete, or double, vibrations of the particles of the medium per second. The frequency is sometimes designated in "cycles." Yet some manufacturers mark on their tuning forks the number of single vibrations. The pitch of a musical sound is determined by the frequency. The higher the frequency the higher the pitch. The *wave-length* is the distance the sound travels during the time of one complete vibration. Thus the distance travelled in one second would be the length of one wave repeated that number of times which corresponds to the frequency. Hence the relationship between velocity, frequency and wave-length is as follows:

$$\text{Velocity} = \text{frequency} \times \text{wave-length}. \qquad (1.4)$$

In passing from one medium to another the frequency is constant, for adjacent particles at the boundary of the two media must vibrate together. But the velocity of the wave is not in general the same in the two media. If the frequency is constant but the velocity different, the wave-length must also be different.

1.11. Doppler's Principle. — Reference will now be made to a common phenomenon. If the source is in motion in the medium in a certain direction, then, though the frequency of the source remains unchanged, the wave-length measured in the medium will be altered. On the side of the source which is in the direction of motion the wave-length will become shorter and on the other side longer. To a stationary auditor, standing near

* Foley, *Physical Review*, 16, p. 449 (1920).

the path of motion, the frequency at the approach of the source will be greater than at the recession. This is a common experience with a train whistle or an automobile horn. It is not difficult to see that frequency heard will depend upon the auditor's velocity relative to the medium also. Illustrations of both aspects are left to the reader.

1.12. Velocity of the "Particle" of the Medium. — It is easy to confuse the velocity of sound with the velocity of a particle in the medium. The former is large, as has been shown in Table 1.1, but the latter is small. The reason is readily appreciated if one will notice that the maximum displacement in a sound wave is an exceedingly minute fraction of a millimeter. Even though the particle oscillates several thousand times a second, it can be shown that the velocity at its mean position is usually of the order of a small fraction of a millimeter per second. Thus these two velocities mentioned above differ not only in meaning but enormously in magnitude. The velocity of the wave is *not* the velocity of the particle.

QUESTIONS

1. Why is a "displacement" necessary in a "wave of condensation"?

2. Indicate in Fig. 1.9 the positions where the following conditions exist:
(*a*) The displacement is a maximum to the right, to the left.
(*b*) The displacement is approximately the same for neighboring particles.
(*c*) The displacements differ the most widely for neighboring particles.

3. Show that the maximum pressure and maximum displacement at a given point do not occur simultaneously in a sound wave.

4. An organ pipe changes its frequency with temperature because (as hereinafter shown) its length (assumed to change inappreciably with temperature) must remain one-fourth of the length of the wave. What will be the change in frequency if the room is heated from 0° C., where the frequency is 256 per second, to 20° C.?

5. Will the change in velocity of a sound wave *in air* caused by a change of temperature have any *direct* influence on the pitch of a piano or violin? Explain.

6. A water wave and a sound wave in the water travel horizontally to the right. What difference can you point out in the vibrations of the water in the two waves?

7. In what respect does the following expression seem misleading: "Sound travels in waves"?

8. Assume in Fig. 1.8 that the wave is travelling to the right. If you had an instantaneous picture (drawing) of the wave of pressure in a row of particles, how would it differ from the drawing of Fig. 1.8? Consider the drawing to represent the condition at the time indicated by 0 in Fig. 1.8.

9. What is the difference between the velocity of a sound wave and the velocity of a particle of the medium?

10. According to Eq. (1.1), what can produce a change in v if the gas concerned is open to the atmosphere?

11. Both the p and ρ of the atmosphere change from time to time. How would you determine the velocity at any one time if you knew the velocity for at least one condition of pressure, density and temperature?

(It is suggested that a review of Chapter I be made before proceeding with Chapter II since a clear understanding of these fundamental concepts is necessary for the comprehension of the remainder of the text.)

CHAPTER II

REFLECTION AND ABSORPTION IN AUDITORIUMS

2.1. Reflection at a Plane Surface. — Sound is not reflected like light, or like a tennis ball. It is a wave of pressure and of displacement. The purpose of this section is to describe what occurs as a result of reflection and that without the use of any analogy. Take a specific case. A source of sound is placed at *O*, Fig. 2.1, in front of a wall, represented in cross section by a line. It is desired to ascertain the effect of the wall upon the incident sound. At the wall at a point between *O* and *O′*, which is the same distance from the wall on the other side, the sound from *O* impinges upon the wall in a direction perpendicular thereto. At no other point along the wall does this perpendicularity exist. But a study of the reflection of a sound wave must include what occurs at all points of the wall. Obviously the situation is quite complicated and hence the physical action at the wall is perhaps too difficult to describe in detail. So the physicist seeks an indirect method, which, as will be found, permits him to describe the reflected wave without the necessity of detailing the action of the reflection over the entire wall. Let an imagined source, *O′*, like that at *O*, be placed on an extended line drawn from *O* perpendicular to the wall, the distance of *O* and *O′* from the wall being the same. By the method referred to, it will be shown that the sound coming from *O* will be reflected from the wall in such a manner as to produce a reflected wave precisely like that which would have come from a similar source, placed at *O′*, if the wall were absent. This is a remarkably simple description of a complicated physical action. It is interesting and instructive to follow the demonstration of its truth.

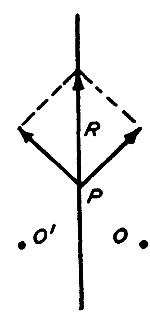


FIG. 2.1

Assume the sources O and O' as stated, but at first without the wall. The waves emitted by O and O' are spherical. Select any point P on the plane between O and O' and consider the sound wave arriving from O at the time it has a positive displacement. But if O' is a like source, then there is arriving simultaneously a wave from O' with a positive displacement of the same magnitude. Since a positive displacement is in the direction of the wave motion, the arrows as shown give the correct directions OP and $O'P$ of the displacements.

It is now necessary to consider how one can ascertain the resulting displacement. A well-known method is to complete the parallelogram with the two arrows representing the given displacements as sides and to regard the diagonal as the resultant R. This method, while apparently reasonable, is not easy to prove in a few words, and hence will be assumed.

Applying the foregoing to the two equal displacements at the point P, with the wall absent, the resulting displacement, R, is clearly in the plane which indicates the position of the wall when present. Since the resultant is the displacement which actually occurs, this means that there is a displacement in the plane mentioned but none perpendicular to this plane. But P is any point in the plane, and hence the condition of zero displacement just stated is true everywhere. This is true at the instant chosen; it will be true at any other instant since the two component displacements are always equal and always make equal angles with the plane. Inasmuch as the displacement and hence the motion perpendicular to the plane at any point are always zero, we can substitute a real motionless wall for the imaginary plane without modifying the resulting wave motion *on the right* of that plane. For with the wall present and assumed to be rigid, we can have no motion perpendicular to the plane. This is precisely the same condition stated above when the wall is absent and there are the two like sources O and O'. Thus with the source O, the rigid wall and the reflected wave, the resulting condition on the right is equivalent to the two sources O and O' without the wall. Hence the reflected wave from the wall is the same as if it originated at O', which is called the image of O. In Fig. 1.6 was shown the reflection of a series of ripple waves from a small plane reflector. Imagine this to be a part of an infinite plane as used in Fig. 2.1. Then it is easily seen in Fig. 1.6 that the phantom source of the reflected wave, or the image, is as far behind the plane containing the reflector as the source is in front, and that the two lie on a line perpendicular to that plane. The experiment in Fig. 1.6 thus illustrates the discussion in this section.

It is evident from the preceding that it is not difficult to obtain the result of reflection accurately, for the acoustician will in his computations treat the effect of the wall as that of an image at the point O' equally distant from the plane. But being able to compute the result is not the same as understanding precisely what occurs at the reflecting plane itself. If one desires to contemplate the act of reflection, he should avoid thinking of the displacements and consider rather a wave of pressure. Then it is easily appreciated that, as a variation of pressure is created at the wall, there will be a wave propagated therefrom.

2.2. Echo Reflecting from a Rough Surface. — The echo as commonly known is merely the sound from the "image" we have described. In the production of echoes, the best effects are found with plane surfaces of considerable dimensions. Nevertheless, a rough surface may be used, such as the edge of a grove of trees. But the indentations in the plane must be not large in comparison with the wave-length of the sound used, for then the argument which has been given in the preceding paragraphs, depending upon equality of phase at the surface, will not hold good. For exact equality of phase demands that P, which is any point in the plane, must be equidistant from O and O'. If the reflector is really not a plane, then there can be no single image O' that always is at this prescribed distance from any point P.

The chief reason for the failure of an echo with a small surface is virtually that the sound bends around it, leaving little to be reflected. This phenomenon is called diffraction and will meet our attention at a later point.

2.3. Reverberation. — If a source of sound is placed in a room all the walls reflect and consequently make the magnitude of the sound greater than if the walls were absent. The waves are reflected not once but again and again and it becomes impossible to continue to trace the waves originally sent out from the source. If a sustained sound is used, the resulting intensity * continues to increase and if there were no absorption of the sound energy, there would be no limit to the intensity of sound in the room. The absorption is quickly appreciated if the source is discontinued. Then it is observed that the intensity does not remain constant but decays gradually, evidencing an absorption of energy. In a large empty auditorium the residue of sound may continue for several seconds. The repeated reflection of sound in a room is called "reverberation" and the "time of reverberation" is the time required for the sound to become inaudible after the source is discontinued. It should be observed that reflection occurs from all the objects in the room as well as from the walls, ceiling and floor.

2.4. Absorption. — Usually by " absorption " of sound the physicist means the transfer of acoustic energy to heat energy. This is occasioned by the fact that if the adjacent portions of a gas are compelled to slip past one another heat is developed. This property of resistance to "slip" is termed "viscosity." There is viscosity in solids and in liquids as well. In considering the flow of fluids in a pipe it is customary to assume that the fluid at the wall does not move but that the viscosity of the fluid itself is the cause of the resistance to flow. It is internal friction. Obviously a method of producing sound absorption in a gas is to let the sound pass into a large number of small channels where the slippage already mentioned will occur. Thus, because of its physical construction, a heavy rug will produce absorption. Also, it is to be noted that the small fibres of the material will be caused to move slightly, though the displacement is a microscopic distance, and hence that there will be an additional absorption of

* "Intensity" refers not to loudness but to the amount of energy per unit volume in the sound wave.

energy in these vibrations on account of the viscosity of the material. These facts will receive attention in a later paragraph.

2.5. Reverberation in a Room. — The appreciation of the significance of the reverberation in a room and the methods of diminishing it to a desired degree marked the beginning of the scientific study of architectural acoustics. Professor Wallace C. Sabine in 1895 began the study of the relation between reverberation and the properties of the materials present in the room. He found in existence the widespread belief that the stringing of wires greatly assisted in reducing reverberation. Indeed, at that time, numerous auditoriums throughout the country were strung each with several miles of wire. From a later section devoted to resonance the reason for the inadequacy of wires can be ascertained. Dr. Sabine found very quickly that the time of decay of the sound to inaudibility was caused by absorption of all materials present. Reverberation is not wholly undesirable. Indeed, as previously shown, it augments the intensity. But if the sound of one syllable enunciated by a speaker lasts long enough, it will interfere with a clear understanding of the succeeding syllable. A similar undesirable confusion occurs with music.

The optimum (or most favorable) time of reverberation must be determined by the auditor. An interesting experiment performed by Dr. Sabine in regard to music rooms will illustrate the point. When the New England Conservatory of Music was completed, the piano rooms were found quite unsatisfactory. An appeal was made to Dr. Sabine, who agreed to undertake an investigation. An important part of the problem was to ascertain the opinion of musical experts in regard to the most desirable time of reverberation. A committee consisting of the director of the conservatory and four members of the faculty was asked to assist in the experiments. The opinion of the committee was ascertained by having each member pass judgment upon the effect of piano music in the various rooms, the amount of absorbing material present in each case being made variable by the insertion and removal of theater cushions. It was found that the committee agreed remarkably well. In fact, to express it in the terms of the unit used, they agreed as to the proper effect to within one theater cushion. All the rooms were tested, and at a later time Dr. Sabine actually measured the time of reverberation with the same amount of absorbing material, making due allowance for the number of persons originally present. The result for the optimum time of reverberation for a piano is 1.08 or practically 1.1 seconds. The rooms varied in size from one to three times. The furniture differed considerably. Yet the figures for the various rooms agreed to within the error produced by one cushion. One might conclude that the correct time of reverberation for small piano rooms is 1.1 seconds, and that a general statement for piano music would require further experiments in auditoriums of all sizes. But this conclusion must be modified for, in general, the preference for an optimum time depends upon the experience of the individual. Even orchestral directors differ as to the best time of reverberation. Perhaps 1.0 second for small auditoriums and 1.8 seconds for large auditoriums are approximately near the satisfactory figures.

If an effort is made to reduce the measurement of the time of reverberation to a basis whereby the time can be computed in advance of the construction of an auditorium, one must have a standard for a perfect absorber. If sound from within passes through an open window, very little is reflected and practically all absorbed, that is, never returns. It would be possible then to determine the absorbing quality of other materials in terms of an open window of the same area. If a piece of hair felt, for example, when placed against a plaster wall absorbs 50 per cent of the amount of sound energy absorbed by an equal area of open window, the "absorption coefficient" of hair felt is 0.50.

Assume a source of sound emitting acoustic energy at a fixed rate. Before any reflections take place, the sound reaching an auditor will have the same intensity as were the source and the auditor not surrounded by walls and other materials. But the waves strike the walls and other objects and these reflections multiply; the intensity continues to increase until the absorption occurs at the same rate as the emission. There are two factors involved in the continuation of the sound after the emission is discontinued. First, it is obvious that, since absorption occurs at each reflection, the rate at which the sound is absorbed will depend upon the number of reflections per second. Inasmuch as the velocity of sound is fixed, this then means that the rate depends upon the dimensions of the room, for the larger the room, the less the number of reflections per second. Hence the rate at which the sound is absorbed will decrease as the volume of the room is increased. Or the time of reverberation will increase with the volume of the room. Second, the rate of absorption will increase and the time of reverberation will decrease with the absorption coefficients of the room. When a careful study is made, it is found that the time of reverberation does depend upon the volume, V, and also upon what is termed the "absorbing power," hereafter referred to as "a," which is the sum of all products obtained by multiplying the area of each exposed material by its absorption coefficient. Dr. W. C. Sabine determined the time of reverberation experimentally with a certain source and found that it was,

$$t = \frac{0.164V}{a}, \tag{2.1}$$

all dimensions * being in meters.

His experiments showed that (2.1) was true in auditoriums usually met in practice. A few years after this first work of Dr. Sabine, a theoretical article by Dr. W. S. Franklin,† assuming that sound can reach any part of the room with ease and assuming a source similar to Sabine's, obtained the following:

$$t = \frac{0.162V}{a}. \tag{2.2}$$

The agreement of the experimental equation (2.1), with the the-

* If all dimensions are in feet, then $t = \frac{.050V}{a}$.

† *Physical Review*, 1903.

Table II

Material	Pitch: 128	256	512	1024	2048	4096
	Coefficient					
1. Acousti-Celotex, Type A perforated fiber board, 13/16″ thick, 441 holes per sq. ft., 3/16″ diameter, 1/2″ deep, plain side exposed		.20	.21	.19	.17	.24
2. Akoustolith Tile, 7/8″ thick, fine texture, cemented to clay tile	.06	.22	.28	.48	.50	.31
3. Balsam Wool, soft wood fiber, paper backing, scrim facing, 1″ thick, .254 pounds/sq. ft.	.10	.27	.50	.68	.56	.48
4. Standard Celotex, 7/16″ thick on 1″ furring		.16	.22	.20	.16	.15
5. Draperies, hung straight, in contact with wall, cotton fabric, 10 oz. per sq. yd.	.03	.04	.11	.17	.24	.35
6. The same, cotton fabric, 14 oz. per sq. yd.	.04	.07	.13	.22	.32	.35
7. The same, velour, 18 oz. per sq. yd.	.05	.12	.35	.45	.38	.36
8. The same as No. 7, hung 4″ from wall	.06	.27	.44	.50	.40	.35
9. The same as No. 7, hung 8″ from wall	.08	.29	.44	.50	.40	.35
10. Cotton Fabric, 14 oz./sq. yd., draped to 7/8 its area	.03	.12	.15	.27	.37	.42
11. The same as No. 10, draped to 3/4 area	.04	.23	.40	.57	.53	.40
12. The same as No. 10, draped to 1/2 area	.07	.31	.49	.81	.66	.54
13. Felt, Standard 1″, all hair	.09	.34	.55	.66	.52	.39
14. Felt, Asbestos-Akoustikos (hair and asbestos fiber), 1/2″ thick	.07	.14	.31	.51	.61	.43
15. The same 1″ thick	.11	.31	.59	.68	.58	.46
16. The same 1-1/2″ thick	.13	.41	.73	.73	.58	.46
17. The same 2″ thick	.21	.46	.79	.75	.58	.46
18. Flax-linum, semi-stiff flax fiber board, 1/2″ thick	.09	.15	.34	.57	.51	.47
19. Masonite, Standard 1/2″ board (pressed wood fiber), laid on 1″ furring, 18″ O.C.	.09	.30	.33	.32	.30	.37
20. 1″ Nashkote AAX, 1″ felt with cotton fabric cemented on surface, two coats, special paint	.11	.25	.34	.46	.48	.36
21. Plaster, gypsum on wood lath on wood studs, rough finish	.016	.032	.039	.050	.030	.028
22. The same with smooth finish ("lime putty")	.020	.022	.032	.039	.039	.028
23. Plaster, lime on wood lath on wood studs, rough finish	.027	.046	.060	.085	.043	.056
24. The same, smooth finish	.024	.027	.030	.037	.019	.034
25. Plaster, "Calacoustic," 1/2″ thick	.06	.10	.14	.15	.15	.20
26. Plaster, Sabinite, 1/2″ thick	.06	.16	.21	.29	.34	.37

oretical result (2.2), is noteworthy. The reason a definitely described source is necessary is that this time of reverberation could not be independent of the strength of the source, or the rate at which sound energy is emitted. It would really depend upon the sound intensity existing in the room at the instant of the discontinuance of the emission. A standard initial intensity must therefore be adopted and these two equations were obtained for cases where the initial intensity is one million times that just audible. (The nature of the ear is such that the intensity which is a million times that just audible does not appear very loud.) In considering the phenomenon of reverberation it should be noticed that the intensity of sound builds up to its maximum value in the reverse manner to its decay. The intensity increases until the rate at which it is absorbed equals the rate of emission.

2.6. Modern Absorbing Materials. — The significance of the researches of Professor W. C. Sabine in 1895 and of subsequent contributions from him and others were but slowly appreciated by architects and manufacturers. Prejudice was apparent and many insisted that the excellence of auditorium acoustics was really dependent upon the shape of the room. Given the proper ratios of the dimensions the results were claimed to be the same. Slowly and inevitably scientific facts, particularly the relation between volume and absorbing power, spread and today active interest in the subject is growing rapidly. Even Table II, or the list from which it is taken, quite inadequately represents the acoustic materials now available. The United States Bureau of Standards is actively engaged in the measurement of absorption coefficients and is making these values as well as general information on the subject available to the public.*

2.7. Absorption Coefficients. — Numerous absorption coefficients have been measured. A partial † list is shown in Table II.

* See circular of the Bureau of Standards No. 380, Jan. 4, 1930. Texts are Acoustics of Buildings by Watson, Architectural Acoustics by V. O. Knudsen, both John Wiley and Sons, 1930 and 1932 respectively, and Acoustics of Buildings by Davis and Kaye, Ball, London, 1927.

† These are taken from a list compiled by Dr. P. E. Sabine of the Riberbank Laboratories and published in Acoustics by Stewart and Lindsay, D. Van Nostrand.

Here the variation of the coefficients with frequency and with arrangement of material are indicated.

Formulas (2.1) and (2.2) assume that in determining V and a, the meter shall be used as the unit of length. Hence the computer must measure the volume in cubic meters and the areas of each kind of exposed surface in square meters. Each area must be multiplied by the corresponding coefficient of absorption and all these products added. If the foot is used as the unit, then the constant in (2.1) becomes .050, but the coefficients remain the same. An illustration in an actual case follows:

	Area in sq. ft.	Coef.*	Abs. power
Wood sheathing, including all wood surfaces...	9860	0.061	601
Plaster on lath..........................	1108	.033	366
Plaster on tile..........................	5480	.025	137
Glass....................................	613	.027	16.5
Iron.....................................	570	.029	17.8
Air vents................................	23.6	1.	23.6
Opera chairs, 922, absorbing power each, estimated................................		.176	162.2
			1,324.1

Volume, 165, 200 cu. ft., therefore

$$t = \frac{0.050 \times 165{,}200}{1324.1} = 6.24 \text{ sec.}$$

The time of reverberation in this auditorium was measured and found to be 6.26 seconds.

2.8. Absorption Coefficients and Frequency. — If the absorption occurs in the air pores or channels in the wall of a room, then the absorption coefficient should change with frequency, and the influence of viscosity can be shown to be greatest in waves of short wave-length.†

* These values were taken from measurements by Professor W. C. Sabine and were the only ones in existence at the time the computations were made.

† Rayleigh, Theory of Sound, Vol. II, Chapter XIX.

In Table II the variation of the absorption coefficient is not always that of an increase with frequency. One concludes that the effect of viscosity in the pores is not the only important effect. In addition there may be viscosity in the material itself. At any rate, the absorption coefficient can be determined only by experiment.

That painting a surface affects its quality is to be anticipated. The following * absorption Table III illustrates the effect of paint and of moisture.

Table III

Frequency	Absorption coefficients		
	1	2	3
64	.021		
128	.023	.0079	.0092
256	.026	.0084	.0097
512	.032	.0104	.0120
1024	.040	.0144	.0150
2048	.052	.0174	.0190
4096	.070	.025	.028

The column marked 1 contains values originally obtained by Dr. W. C. Sabine for an unpainted 18″ wall of hard brick set in mortar; column two is for a surface of gypsum plaster with a so-called "putty finish" taken about three months after placing on an 18″ brick wall and column three is for the same surface a year afterward. The data of the last two columns were obtained by Dr. P. E. Sabine. The change in the coefficient with time is doubtless caused by the evaporation of moisture.

It is an interesting fact that a slight film of water or of any substance of great density as compared with air, will prevent the transmission of sounds. This is because the sound striking such a surface experiences practically total reflection. Thus it occurs that a thin film of water or vaseline will more effectually prevent the transmission of sound than a surprising thickness of highly absorbing hair felt. An interesting experiment may be performed by the use of cotton in the ears. The great difference produced

* P. E. Sabine, *Physical Review*, 16, p. 514, 1920.

by the additional use of a thin film of vaseline at the opening can then be observed.

2.9. Other Effects in an Auditorium. — In the foregoing there has been discussed only reverberation. There are several other effects of importance, for example, resonance and the variation of intensity in various portions of the room. These will be mentioned in later paragraphs.

QUESTIONS

1. What is your reason for the statement on p. 24 that, inasmuch as there is no motion perpendicular to the plane, a wall can be substituted therefor?

2. In the discussion of echo, it was stated that the "deviation from a plane must not be large." Justify this statement in your own words.

3. If the reflecting plane were corrugated, but still made up of small plane surfaces, could you find an image by treating each surface separately?

4. In a given channel, where would the velocity of the air particles be the greatest, and where the least?

5. What is the relation between "reverberation" and "echo?"

6. Show, from a consideration of equation (2.1), that the time of reverberation must be less in an auditorium without sidewalls.

7. Why would the desirable time of reverberation in an auditorium depend upon the rapidity of speech?

8. Give one reason for the opinion that a fog does not have the same effect as a film of water in preventing transmission.

9. What objection would there be to a room in which all walls and objects are without absorption?

10. If a room has too much reverberation, what advice should be given to a speaker?

11. Show by assuming the walls large, that there is a multitude of images outside the walls of the room.

12. A piece of cheese cloth is known to stop the wind in a marked manner. Why is it not effective in stopping the passage of sound?

13. What phenomena in architectural acoustics have you observed that you can explain and that you cannot explain?

CHAPTER III

ACOUSTIC REFLECTORS

3.1. Nature of Interference. — In Chapter I we discussed the nature of sound waves of displacement, pressure and velocity. The easiest manner in which to conceive of interference is by regarding each wave as something that is propagated and that produces at every point in its path its own displacement, pressure and velocity. Its effect must be independent of every other wave. If two sound waves cross, then the air particles at the crossing must have values of displacement, pressure and velocity that are resultants, or combinations of the two waves. Thus the two displacements must be added, proper regard being given to their directions as well as to their magnitudes. If two waves of the same frequency are travelling in the *same direction* and if at a point their displacements reach positive maxima at the same instant the two vibrations are in the *same phase*. If one reaches its positive maximum displacement and the other its negative maximum displacement at the same instant, the waves are said to be *opposite in phase*. At the point where two displacements are equal and in the same direction, the resultant is twice either displacement. Where the two are equal and in opposite directions, the resultant is zero. In both cases the phenomenon is called "interference," for in both cases there is a combined effect which interferes with the occurrence of either displacement.

As an illustration of interference assume two tuning forks having almost the same frequency, one 255 and the other 256 vibrations per second. If the two are now held near the ear, the sound swells and diminishes to zero once each second, giving the phenomenon of "beats." This is explained in accord with the preceding paragraph, for if one has 255 vibrations in the same time as the other has 256 vibrations, then the latter gains one vibration in one second. Assume that at the beginning of a certain second the vibrations of the two forks are in the same phase. Then one-half a second later, they will be opposite in phase. At the end of the first second they will be again in the same phase. In the next second the same cycle will occur. Thus during each second there occurs complete agreement in phase and exact opposition in phase. In the former case displacements add and in the latter they subtract. If these displacements are equal in magnitude we have in the former case *four* * times the intensity of the sound from one fork and in the latter case no intensity at all. This effect gives us the phenomena of "beats" to which reference has been made.

3.2. Huyghens' Principle. — There are many interesting phenomena which are explicable only when one examines the interference of the sound waves. If sound travels with equal velocity in all directions from a simple source of sound, we say that a spherical wave results. But a sphere drawn about this source has a significant uniqueness. At every point on this sphere the vibration is simultaneously in the same phase, for since all points are equally distant from the origin, the displacements must be the same. To repeat, every point on this sphere is vibrating in the same phase. Such a surface is called a "wave front." Usually, in acoustics, the direction of propagation of a sound wave is perpendicular to the wave front; the exceptions of interest will be subsequently noted and the reason therefor given, but for the present purpose this direction of propagation will be assumed as correct. With this understanding, it is easy to designate a wave front and thus to determine the direction in which the wave is propagated.

What is termed *Huyghens' principle* is that *each point* on any wave front can be assumed to be the *source of a hemispherical wave* and a future wave front be thereby determined. This is illustrated effectively in Fig. 1.4. This principle is capable of a

* Technically, the intensity, frequency constant, is proportional to the square of the maximum displacement. See footnote to Section 2.3. This maximum value is called the "displacement amplitude" or frequently the "amplitude."

rigorous proof in acoustics, but here the principle will be assumed. Consider the spherical wave *a a* in Fig. 3.1 and select a number of points along this cross-section of the wave front. With each point as a center draw a semi-circle, each semi-circle having the same radius, equal to the distance traversed by the waves in some definite time. It is our task to ascertain the new wave front. Inspection shows that at the instant considered, at no other surface than *a′a′* can all the vibrations be in the same phase. For assume that the distance between *a a* and *a′a′* is "*n*" wave-lengths. Then draw any other curve you please from the upper point *a′* to the lower point *a′*. Remembering that only points an integral number of wave-lengths apart in the radial direction can be said to be in the same phase, one sees that *all* points on this newly drawn curve cannot possibly be in the same phase. Hence we cannot regard it as a wave front.

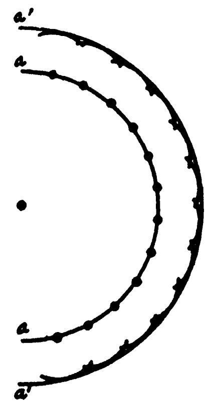


FIG. 3.1

3.3. A "Beam" of Sound.* — Assume we have a vibrating area, *ab*, in a plane wall as in Fig. 3.2. Assume that it is large in comparison with a wave-length of the frequency actually used. According to the previous paragraph we may now construct the later wave front *a′b′*. It is readily seen that the hemispherical waves from all points along *ab*, travelling in the directions *ac* or *bd*, would not have a surface in common, such as *a′b′*. In other words, the hemispherical wavelets are not in agreement as to phase in the directions *ac* or *bd*. This introduces interference and if *ab* is as long as many wave-lengths, it can be seen that there is destructive interference in these two directions. For, consider the direction more nearly along the wall *aA*. At a point *P* the displacement produced by the element of area at *a* will be equal and opposite to that produced by the element of area one-half

* This can be illustrated by means of a highly pitched whistle placed inside and at the closed end of a cylindrical tube. The open end may be considered approximately a wave front.

wave-length further away from P and between a and b. If ab is comparatively large, as has been assumed, there will be as many elements producing a displacement of one phase as there are elements producing a displacement of opposite phase. There is, then, approximate annulment at P. What is true of the waves in the direction aP is true of any direction other than aa' or bb'. Thus if ab is comparatively large, it will send out a "beam" of sound $a'b'$, similar to a beam of light sent out by a searchlight. Of course, the edge of this beam is not sharp, for ab is not infinitely large compared to a wave-length. This explanation of the condition which will produce a beam of sound demands careful study, for similar reasoning will subsequently occur.

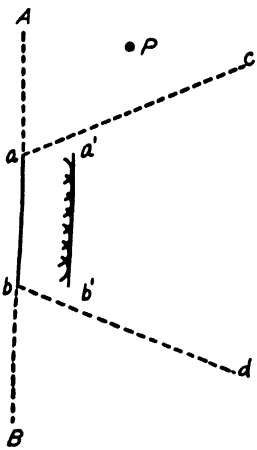


FIG. 3.2

If ab is now reduced in size, it is readily seen that the resulting wave front becomes more and more like a hemisphere. When ab is a point source, the wave is hemispherical.

As an application of the above reasoning, consider the directive property of a megaphone. It is known that the surface containing the large opening is a wave front. If the opening is large as in a large megaphone, the intensity produced is distinctly greatest along the axis of the megaphone. There is never a noticeably sharp beam of sound, and this is true because the wave-lengths used are not sufficiently small.

3.4. Acoustic Plane Reflector. — If reference is made to Figs. 1.2, 1.3 and 1.4, a series of ripples reflected from the wall will be noticed. This section is a brief study of the influence of the size of an area upon the reflection of acoustic waves therefrom. Consider Fig. 2.1. O is a source of sound in front of an infinite wall. Its reflected wave can be regarded as coming from O'. If, however, we are dealing with a reflector of ordinary size, or a part of the wall, the reflected wave cannot be the same as that coming from O', in the absence of the reflector. It must be of less intensity because it is of less area. According to Section 2.1 each area of the infinite wall will give a reflected wave which is just like the wave which would come from O' through this same area in the absence of the wall. Consider this hypothetical wave from the image O' in Fig. 3.3, passing through the circular area *ab*, which has replaced the small reflector of the same size. This wave will not remain in the conical volume indicated by the dotted lines. Fig. 1.2 has already illustrated an analogous action in the case of ripple waves. The section just preceding considers the divergence from a geometrical beam in the case of a plane wave front. Obviously this divergence will occur with a portion of a spherical wave as well. Consequently the wave front in *ab* will spread outside of the cone, unless the distance across *ab* is long compared to a wavelength. Moreover, the smaller *ab* the more the waves will diverge from it as from a point. Certain conclusions are now evident. *Not only will a small reflector reflect a small amount because of its size but a small reflector will scatter sound in all directions, thus further greatly reducing the sound reflected backward. This shows that, as the area of a small reflector is increased, there is a much more rapid increase in its effectiveness.* Yet another influence should be mentioned. The reflected wave will scatter not only in all directions on the same side as the reflector, but also around the reflector itself.

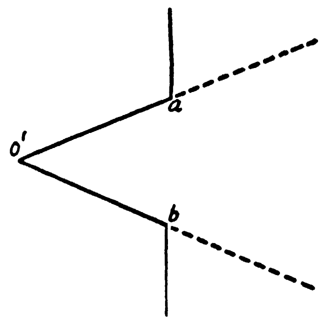


FIG. 3.3

3.5. Acoustic Parabolic Mirror. — It is shown in optics that light from a source located at a certain point within a parabolic * mirror called a "focus," will be reflected from the mirror as a

* The parabolic mirror has a concave shape similar to that of an automobile headlight. Its property of focussing is the important point and not its shape.

parallel beam. This property of a parabolic mirror is utilized in searchlights, the arc being placed at this focal point. Also light from a distant source such as the sun will be brought to this focus. During the war, acoustic parabolic reflectors were tried as sound detectors, since the waves from a distant source are approximately "parallel" and should be reflected to a focus. Huge mirrors 12 feet in diameter were constructed. But it was found that the concentration of sound was very poor. From our preceding explanations it is evident that sound will be reflected as will light only if the reflector is of dimensions very large in comparison with the wave-length. Moreover, in light, the smallness of the focus is caused by interference. The intensity is greatest where all the waves are in the same phase. The distances from likeness of phase to opposition in phase is of the same order of magnitude as a wave-length as will later be shown when discussing two waves travelling in opposite directions. Thus it occurs that the focus of an acoustic mirror is not sharp and may be regarded as of approximately the same diameter as a wave-length. If we have a plane wave of frequency 100 d.v. (complete or double vibrations per second) striking a 12 foot mirror, the focus would have approximately the diameter of 1100 ÷ 100, or 11 feet. This can scarcely be said to be concentration. Even with a note of high frequency, 1000 d.v., the diameter of the focus would be 1 foot. With a highly pitched whistle, a parabolic mirror and a sensitive flame, the concentration of sound may be demonstrated.

Small parabolic mirrors are sometimes used behind public speakers. They produce a noticeable difference but are not very effective.

3.6. Interference in Auditoriums. — Our discussion of interference of sound leads to an explanation of the well-known fact that the intensity of sound is not equal at all parts of an auditorium. In an enclosed space there are innumerable reflections and at any point the resultant intensity may be regarded as produced by many sources (images). But, if sound waves may interfere so that displacements rather than sound intensities must be added, the intensity of sound will not be the same throughout. Dr. W. C. Sabine was the first to explore the variation of inten-

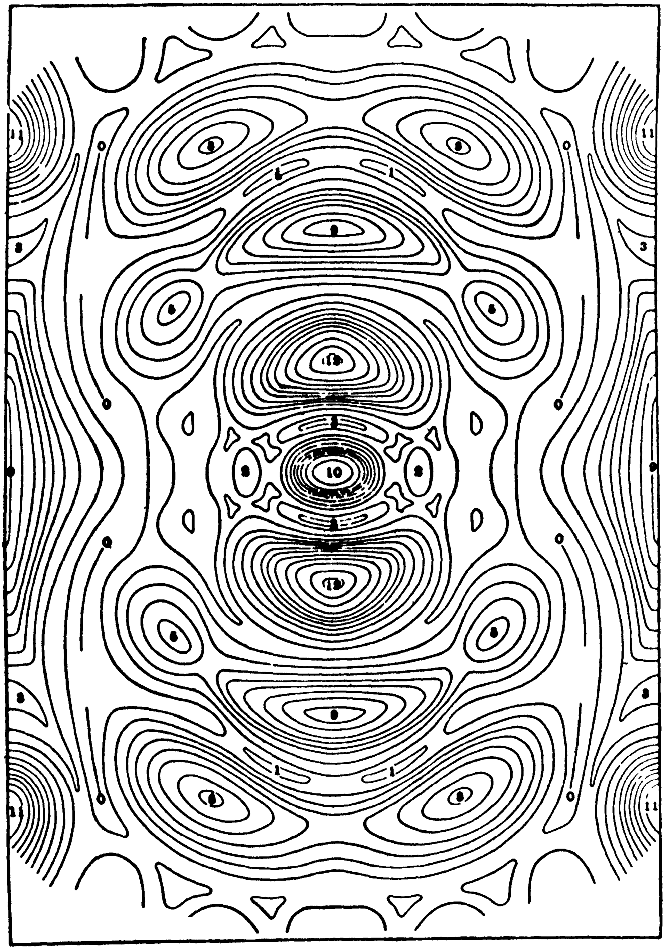


Fig. 3.4

sity throughout an auditorium. He has represented the variation in intensity in the manner that elevation of land is shown in topographical maps. Fig. 3.4 is taken from his work. In a previous chapter it was shown that the time of reverberation can be computed in advance with considerable accuracy. Acousticians have not yet learned a simple way to ascertain the presence of "bad" spots or areas of small intensities. Moreover, the location of such spots will vary with the frequency. It is possible, however, that a considerable area in an auditorium may be, in general, a poor place for an auditor.

3.7. Selective Property of Reflectors. — There is a curious property of reflectors to which this section will be devoted. Assume we have a vibrating disc, Fig. 3.5, and a point of observation at *O*. For the sake of simplicity the emission from only the right side of the disc will be considered. The effect at *O* will be due to the combined effects of all portions of the circular disc *AB*. But the sound from *A* will reach the point *O* later than the sound from *C*. Then the phase of the former will not be exactly the same as of the latter. If we increase the size of the disc, the intensity at *O* will increase unless the added area produces a displacement at *O* that nullifies a portion of the composite displacement already produced. Just at what size this will occur can be seen by the following discussion.

Suppose the circular area shown in cross section by *ACB* in Fig. 3.5 be divided up into concentric circles, in such a manner that the second circle adds an area that is exactly equal to the first one drawn about *C*, the third also adds an area equal to the first and so forth. Then each successive addition may be thought of as contributing an equal amount to the resulting amplitude at *O*. The whole area is vibrating in the same phase, but the above areas are at constantly increasing distances from *O*. Hence, the above successive contributions to a resultant amplitude must gradually decrease in phase. It can be shown that every additional area adds to the resulting amplitude until

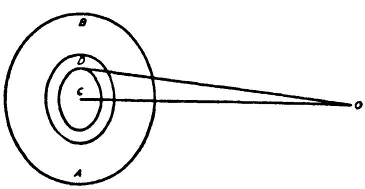


FIG. 3.5

the phase reached is opposite to that of the wave from C, when a decrease in the resultant begins. That the reader may have a better picture of this effect, the following discussion is given. If one adds two equal displacements as in Fig. 3.6, R, the diagonal, is the resultant. The resultant may be regarded as obtained by adding the two arrows, r_1 and r_2, end to end and connecting the terminus with the origin. This process may be continued with a third arrow. Suppose one adds a series of displacements as in Fig. 3.7, each equal in magnitude but differing in direction. The resultant is found by attaching the arrows end to end and drawing R_1, R_2, etc. It is noticed that the last displacement that makes a contribution to the length of R, ef, is opposite * to the initial displacement Oa. This method of adding displacements may be justified by a consideration of the nature of the variation of a displacement having the simple harmonic variation visualized in Section 1.7. There such a variation was described as corresponding to that of the shadow of the displacement of a ball on a screen, the ball itself moving in a circle in a plane containing the direction of the light. This simple harmonic variation of displacement is the one treated in this section and throughout the text. For, as stated in Section 1.7, a complex sound is made up of such units, and the effects produced by each unit may be added to determine the result

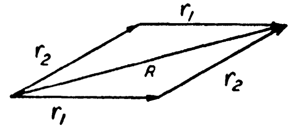


FIG. 3.6

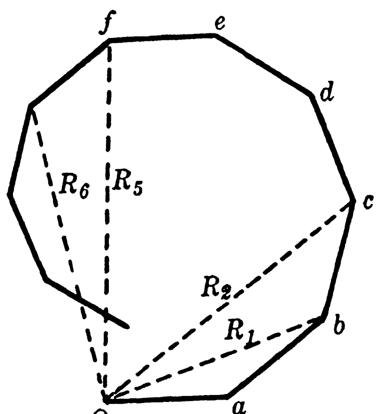


FIG. 3.7

* According to Fig. 3.7, the last one to make a contribution to R is de, but if the areas are made smaller and smaller, de more and more nearly approaches the condition of opposition in phase.

occurring with the complex sound. Consequently the treatment of reflection for a displacement having one frequency may be regarded as really of general application. Retaining the experiment with the ball in mind, consider the addition of two displacements differing in phase but represented on the same wheel. Assume the second displacement to have the same amplitude also indicated by the radius of the wheel. If the two displacements are opposite in phase, their corresponding radii are drawn in opposite directions. They have an angle between them of 180°. (It is customary to use such an angle as expressing numerically the difference of phase of the two displacements. Thus a difference in phase may be designated as 30°, 40° or 210°, as the case may be.) These two displacements, opposite in phase and equal in amplitude, will give zero if added. The addition can be made graphically by drawing the two radii in opposite directions. Assume that the difference in phase is 90° instead of 180°. The two balls are placed at the rim with an angle of 90° between the radii. The sum of the displacements at any instant can be found by adding the two values as found on the shadow on the screen. The resulting displacement evidently has a larger amplitude than the radius of the wheel. It can be shown by geometry that there is a radial line, drawn from the center of the wheel, which will have the value of this amplitude. Moreover, its shadow on the screen as the wheel rotates will always give the resulting displacement produced by the two components described. This radial line can be proved to be the diagonal of the parallelogram having the two indicated radii as sides. This, then, is the manner of securing a resulting amplitude, when two displacements are in the same straight line. The amplitudes are represented by arrows, the difference of phase by the included angle, and the resultant amplitude by the diagonal. This is precisely what was done in Fig. 3.6 and by an extension in Fig. 3.7. R is the resulting amplitude.

In Fig. 3.5 the displacements at O from the successive areas do not differ appreciably in amplitude or in direction if CO is long compared with CA. But they do differ in phase. If the angle between the direction of two arrows in Fig. 3.6 represents the phase difference and the equal lengths of the arrows, the amplitude, then *R* is the correct resultant amplitude. Figure 3.7 then states that, beginning at the center area of the vibrating disc in Fig. 3.5, the displacements at *O* produced by the successive areas conspire to increase the resulting amplitude at that point until opposition in phase is reached (180° in Fig. 3.7). Additional areas decrease this amplitude at *O*. A nullification will begin to occur when the difference in the paths *OC* and *OA* will cause the displacements from *C* and *A* to be opposite in phase. This critical difference in the paths is one-half a wave-length. When *AC* is increased beyond this critical distance the displacement at *O* will diminish. A more extended study shows that the resulting intensity at *O*, if the radius of the disc is gradually increased, will diminish from the above described maximum to a minimum, which is always greater than zero, will again increase and, in fact, will have a series of maxima and minima of less and less prominence, the intensity always remaining noticeably less than the maximum intensity occurring with the radius such that *AO–CO* is one-half a wave-length. This means that the action of the diaphragm is selective. For a given frequency there is a certain diameter which will give the maximum effect at *O*. This is specified by the condition that *AO–CO* is one-half wave-length.

If this interesting effect occurs with a vibrating diaphragm, it will of course occur with a reflecting surface as well, for the latter may be considered as sending out a wave. The following unpublished experiment was performed by the writer out of doors.

A circular screen *S* in Fig. 3.8 was placed about a cylindrical tube, *R*, from whose base a rubber tubing passed to the apparatus for measuring intensity. The tube *R* had the length which caused it to resonate with the frequency of the source placed at *O*. The observer compared the intensities in *R* and hence at *P* when the size of the screen was altered. Three annular rings, adding successive portions of the enlarged area of *S*, were constructed. The dimensions were adjusted so that the sum of the distances from *P* to the edge of the screen and from the edge to

O was as follows: (1) Without any of the rings this total distance was less than one-half wave-length greater than OP; (2) with one annular ring it was exactly one-half wave-length greater; and (3) with the second and third annular rings it was more than one-half wave-length greater than OP. The distance $OS + SP$ is

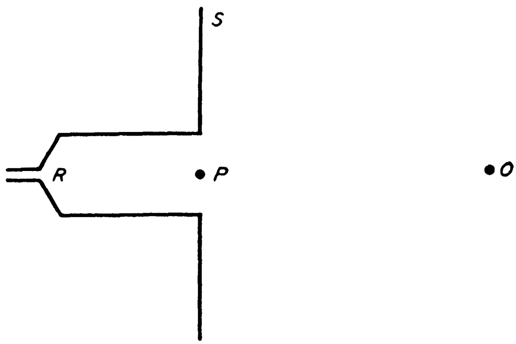


FIG. 3.8

regarded as a path of the sound wave, for the wave which is reflected at S travels therefrom in all directions. As would be expected from the preceding discussion, the greatest intensity observed in R was with one annular ring or with the difference in total distance one-half wave-length.

This selective property, namely, a maximum reflection for a given frequency, is, at present, more of a curiosity than a utility. Nevertheless, it sometimes occurs in echoes.

3.8. The Pinnae as Reflectors. — The reflecting power of the pinnae, or the auricles of the ear, is not of grave importance. As already pointed out both the incident sound and the reflected sound bend around small obstacles. In fact, the effect of an obstacle does not begin to be very marked until its size is comparable to a wave-length. Thus it is seldom that one listens to sounds of high enough frequency to permit the pinnae to be very effective. If one places a source of sound of 10,000 d.v. first in front and then behind the head, a marked difference in intensity can be noted. The wave-length is, in this case, less than the diameter of the pinnae. Similar remarks may be made in regard to the effect of cupping the hand at the ear. The reader may readily try experiments with the tick of a watch, which contains tones of high frequencies, and with ordinary sounds such as speech.

3.9. Acoustic Horns as Reflectors. — Contrary to the common view, a megaphone, used either to transmit or to receive, owes its advantage in increasing the sound intensity not to a reflection effect, such as would occur if the wall was silvered and light was used instead of sound, but rather to a resonating property which will be discussed in a later section. In fact, the discussion in this chapter shows that we cannot use the laws of light reflection unless all surfaces are very large in comparison with a wave-length. The directivity of a megaphone depends upon the area of the large opening, for this area is a wave front. Whether or not a good beam of sound is obtained may be determined by a discussion similar to that in Section 3.3.

QUESTIONS

1. If two waves are travelling in the same direction, under what condition are the two displacements at a given point in the same direction at all times?

2. If two waves are travelling in the same direction, under what condition can the displacement at one point be in phase, when simultaneously, at another point, they are opposite in phase?

3. In the case of a conical megaphone the text does not state whether the wave front at the opening is a plane or a section of a sphere. What would be the difference in effect if such two surfaces did not differ in position by more than one inch, for example?

4. In Fig. 2.1, assume the wall were made up of many small planes set in somewhat random orientations. Under what circumstances would this not materially affect the location or sharpness of the image, *O'*? Could this construction, if made of polished metal surfaces of an inch in diameter, prove a satisfactory mirror for light?

5. With a small reflector why does the amount reflected depend upon the wave-length?

6. When a train or automobile rushes by small objects such as tree trunks, telephone posts, bridge structures, are the sounds reflected to the passenger similar to those one would hear were he standing nearby on the ground, or what difference is noticed and why?

7. Can a blind man by hearing tell anything concerning the nature of the objects along the path upon which he is walking and why?

8. Assuming that clearness of enunciation depends upon the higher frequencies in the complex sounds, which will a small reflector improve the more, loudness or clearness of speech?

9. Why should the bad spots in an auditorium vary in position with the frequency?

10. One listening to a speaker talking through a megaphone will notice that there is a difference in the quality of voice caused by changing the direction of the megaphone relative to an auditor. State the difference that occurs and why.

11. If one talks through a megaphone rectangular in cross-section, in which directions from the axis of the megaphone will the sound spread the most easily?

12. Experiment with cupping the hands at the ears in order to improve hearing and report the nature of the sounds when distinct improvement was and was not made.

13. Is the presence of the head any acoustic advantage in increasing the intensity at the ear?

14. What does the experiment illustrated by Fig. 3.8 show as to whether or not sound is reflected like light?

CHAPTER IV

REFRACTION AND DIFFRACTION

4.1. Variations of Velocity in the Atmosphere. — Equation (1.1) shows the dependence of the velocity upon pressure, density and the ratio of the specific heats of a gas. In equation (1.3), however, we have an expression for the velocity in which neither the pressure nor the density of a gas occurs. In the atmosphere, which will here be assumed to have everywhere the same composition, there are changes in pressure, density and temperature. But equation (1.3) states that in discussing the velocity of sound in such a non-homogeneous medium *at rest* we need to consider only those variations in velocity of sound that are caused by variations in *temperature*. If there is a wind having a direction parallel to the earth's surface, for example, obviously the velocity of the sound wave is greater, relative to the earth, when the propagation of the sound is in the direction of the wind and less when the sound travels to the windward. In considering the propagation of sound in the atmosphere we have need, therefore, to take careful account of both the temperature and of the wind.

4.2. Effect of Temperature. — Equation (1.3) shows that the higher the temperature of a gas, the greater the velocity of sound therein. The influence of temperature may be studied by assuming the case of a sound wave passing from one stratum at a temperature t to another stratum at temperature t', $t' > t$, giving velocities v' and v. This is done in Fig. 4.1. Here the horizontal line represents the plane separating the two media. The initial plane wave front is AB and the final wave front after what is termed "refraction" is $A'B'$. The change of direction of propagation occasioned by a change in the nature of the medium which affects the velocity of sound is called "*refraction.*" * This term is also used in cases where there is a change in velocity even if the direction of propagation remains unaltered. Such would be the case if BB' were perpendicular to the boundary in Fig. 4.1. The position at $A'B'$ is obtained in the following manner. Since the sound will travel the distance AA' in the same time that it

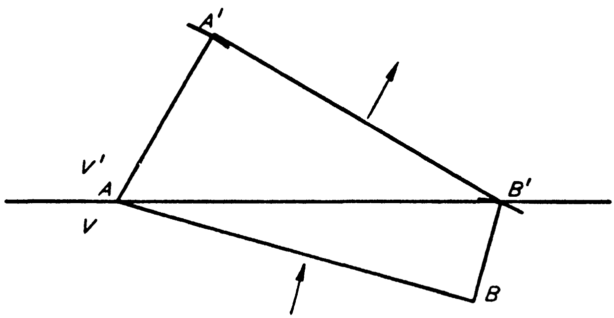


FIG. 4.1

will travel the distance BB', the wave front must contain the point B' and a point on the hemisphere about A with a radius AA'. If it is now assumed that the resulting wave front is plane, which a more extended discussion would justify, then $A'B'$ is the refracted wave front. It is thus seen that the direction of propagation of the wave in the second medium, AA', is perpendicular to the wave front, $A'B'$.

This effect of temperature produces an interesting result which is experienced usually in the early morning hours. If the night has been clear, the earth has been radiating heat rapidly and, in the absence of wind, the atmospheric layer near the ground may become cooler than that above. This is just the reverse of the usual daytime condition when the heat from the earth causes the lowest layer to have the highest temperature. When the layer adjacent to the earth has been cooled until its temperature is

* There will also be a reflected wave in the first medium, but it is not discussed at this point. The case of reflection (in gases) with perpendicular incidence is discussed in Section 5.4. In general all the energy will not pass from the first to the second medium.

lower than the stratum above, the temperature in this lowest region will increase gradually with elevation until a level is reached where the temperature begins to decrease. At further elevations the decrease continues indefinitely, so far as our present acoustic interest is concerned. Within the layer of temperature increasing with elevation, we have sound refracted in a manner similar to Fig. 4.1. In the discussion of this figure it will be assumed that the transition in temperature occurs suddenly. It can be proved that in the case of a gradual temperature variation we are justified in using the above conclusion qualitatively. It is therefore seen that the effect of this type of refraction is to bend the sound towards the earth. If the sound is propagated in a direction more nearly horizontal than shown in Fig. 4.1, it may reach a horizontal direction and then be refracted downward before reaching the upper limit of the stratum of temperature increasing with elevation. If so, the sound will be retained within the stratum very much as if transmitted between two parallel walls, one of them the earth. This accounts for the great distances at which sounds may be heard in the early morning hours. In a similar manner it can easily be shown that when the stratum is in its usual daytime condition the influence of temperature is to decrease the range.

Attention should be called to the fact that, in the foregoing paragraph, there has been described a case of what is essentially "total reflection" caused by the phenomenon of refraction. But one must not accept too literally the word "total." There will be sound diffused to some extent out through the layer, irrespective of the direction of propagation of the transmitted sound. The possibilities in Fig. 4.1 may be examined more closely. Suppose the temperature of the upper medium be gradually increased. At each increase AA' becomes longer while AB' and BB' will remain unchanged. Thus $A'B'$ will change its position and the direction of propagation AA' will become more nearly horizontal. As the temperature is further increased, causing v' to become greater, AA' will approach the length AB'. When AA' equals the length AB' the direction of AA' becomes horizontal. This condition means that the wave no longer enters the second medium but skims the surface between the two. If the temperature is now further increased the sound will be reflected at the boundary. The reader must not think such a reflection is impossible though we do not commonly have such large temperature differences as would be required for the angle of incidence used in Fig. 4.1. Instead of requiring the temperature of the upper medium to be raised, one may change the angle at which the direction of propagation of the wave meets the boundary. As this direction becomes more nearly parallel to the boundary the refracted wave also approaches coincidence with the boundary even more rapidly, then skims the surface and finally is totally reflected. So it appears that even a small difference in velocity in the two media may, if the grazing angle of approach is small enough, produce total reflection. This is further discussed in Section 5.11.

4.3. Effect of the Wind. — Assume that in Fig. 4.2 the line OO' is the cross-section of a plane which separates two regions of air, in the upper of which the medium is moving with a velocity

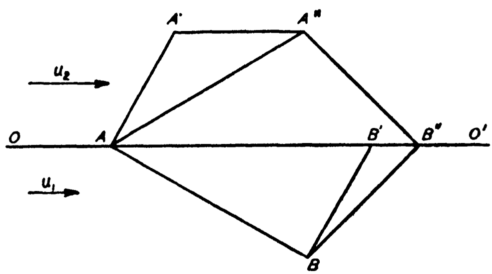


FIG. 4.2

relative to the earth of u_2 and in the lower with a velocity of u_1, u_2 being greater than u_1. Assume further that there is a plane sound wave represented by the cross-section of the wave front, AB, which is moving from the lower into the upper region. If the air everywhere were stationary, the wave front would move from AB to $A'B'$ in a certain time, say t. But simultaneously with this propagation each portion of the wave front will be carried forward by the motion of the medium in which it is located. At the expiration of the time, t, when the wave should have reached $A'B'$, it will actually be at $A''B''$, for the distances $A'A'' = u_2t$ and $B'B'' = u_1t$ are the distances which the respective media have transported their sound waves in the same time. Because of the fact that u_2 is not equal to u_1, $A''B''$ is not parallel to AB and we discover that the wave front has changed its angle with OO'. It is further easy to see that the direction of propagation from OO' is not perpendicular to $A''B''$, or the wave front, but is in the direction AA''. We can conclude that in a gaseous medium having convection currents, the direction of sound propagation is not, in general, perpendicular to the wave front.

But suppose an observer were moving with a velocity u_2, that is, with the upper medium. He would notice nothing unusual in the upper medium. The direction of propagation relative to him could be drawn normal to the wave front. Thus we see that this peculiarity, the movement of the wave front not perpendicular to itself, may be caused by the motion of the observer relative to the medium, and thus not be strictly an effect of the medium itself. But if the velocity of the medium varies from point to point in any direction, this peculiar movement of the wave front may exist and may be correctly attributable to the medium itself.

It is to be understood that the abrupt change of wind velocity with elevation, as here supposed, cannot really occur, and that refraction actually extends over a considerable depth in which the wind velocity varies gradually.

A practical point naturally arises as to the direction of propagation of sound as judged by an observer who judges wholly by hearing. As pointed out in a later chapter, the sound appears to come from a direction *perpendicular to the wave front.* As will appear, this is because equality of phase of vibration at the ears is the deciding factor.

4.4. Speaking in the Wind. — It is a simple step to the discussion of the influence of the wind upon the propagation of sound horizontally, and this is the case of out-of-doors speaking. In the diagram of Fig. 4.3 assume the source of sound to be at S and located in a wind having a velocity to the right that increases with elevation above the ground, and assume the wave which has proceeded from it to be AB. Proceeding as before, the lines AA', CC' and DD' are drawn as the distances that the sound wave would travel in the time t if the motion of the air were nil. But the medium has moved in this time, t, a distance $A'A''$ at the upper, $C'C''$ at the center and zero or a very small amount at the lower point. The considerable variation in wind velocity that is here represented really exaggerates the actual case, but, inasmuch as the wind velocity does increase with the vertical height, the assumption made for the variation will give us results that are qualitatively correct. It is seen that the wave has reached $A''D'$ in the time t, and is now a distorted spherical wave. Moreover, the direction of propagation from A is not AA' but AA''. In other words, the wave is bent or refracted toward the earth. The result of such refraction is the reduction of the spreading of the wave upward from the horizontal. The effect, therefore, is that the speaker's voice, or any other sound from S, is carried apparently along the earth. It is said his voice carries better in the direction of the wind.

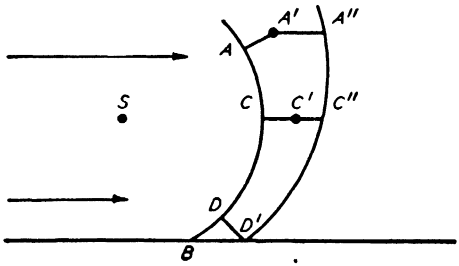


FIG. 4.3

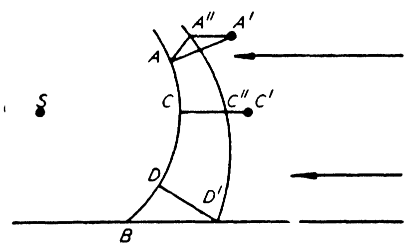


FIG. 4.4

In Fig. 4.4 is illustrated the effect when the sound travels against the wind. The reader can follow reasoning similar to the above without the necessity of its repetition here. The conclusion is that the sound wave is refracted upward, thus making it more difficult to be heard when speaking against the wind.

The wind may cause effects that at first thought seem curious. For example, two persons may be attempting to communicate in a high wind between shore and boat. They find that while the individual leeward can hear and understand, the other to windward can scarcely hear any sound whatever. In fact, sound will pass "more readily" in one direction than in the opposite. But if the observer to the leeward is elevated he will be able to make the observer to the windward hear much better, for now, so to speak, the sound may be refracted but will yet succeed in reaching the hearer. Such would be the case with sound travelling from the source in a somewhat downward direction. Elevated church bells can thus be heard more distinctly at points windward than if the bells are near the ground. The refraction toward the ground of the sound travelling leeward is not so serious a matter, for, assuming no obstacles, reflection will occur and the sound does not escape as in the case where the sound travels to windward upward. Of course, the presence of obstacles on the ground is an additional justification for the elevation of bells and whistles.

The previous discussion in regard to the scattering or diffusion of sound showed that in acoustics we rarely deal with a "beam" of sound. Therefore it is incorrect to suppose that sound transmitted to windward entirely leaves the ground. Likewise, it is impossible for all the sound transmitted to leeward to be refracted toward the ground. The effects as described are not complete.

4.5. **Silence Areas.** — In view of the influence of both wind and temperature upon sound refraction it is not surprising that conditions may obtain wherein the noise of an explosion may be distinctly heard in a distant place, and yet not heard at all at a nearer position. This phenomenon has been frequently observed. As an illustration on October 28, 1922, 2,000 pounds of explosives were fired in Holland. The noise was heard within a radius of from twenty to seventy kilometers, the difference in different directions being caused by the wind. No sound was heard between seventy and two hundred kilometers. But in a zone distant more than two hundred kilometers the sound of the explosion was again audible, in fact, up to a distance of nine hundred kilometers.

4.6. Refraction and Scattering of Airplane Noises. — In listening to airplanes in flight one observes several acoustic phenomena. The one most quickly noticed is that the sound from the airplane is of an uneven character. During experiments * in connection with airplane detection and location the observers noticed also that, with "poor listening" atmospheric conditions, the sound from the airplane at the greatest hearing distance was limited to the lowest frequencies in the emitted complex sound. These frequencies were for these particular airplanes approximately 90, 180, 270, etc., and the most prominent component was the one of lowest pitch. The sound from the same airplane heard at the greatest possible distance under excellent night conditions was distinctly different. The lowest frequencies just named were not noticeable and the sharp crackling sounds of the engine explosions with prominent components, probably of the order of 1,000, were most distinctly in evidence. The difference in the character of the sound in the two cases may be described as the cutting off of the higher frequencies in the former and of the lower frequencies in the latter. That there may be a more rapid decay of intensity of the higher frequencies is readily understood by a consideration of differences in wave-length. For the irregularities in the planity of the strata, for example, would be more effective in scattering, by reflection and refraction, the frequencies having the shortest wave-lengths. The reader can demonstrate this if he will make a drawing of a non-planar stratum and consider the refraction of waves in detail. In general, the irregularities of the atmosphere would effect more the shorter wave-lengths.

* See Stewart, *Phys. Rev.*, N.S., Vol. 14, No. 4, p. 376, 1919.

The apparently better transmission for the higher frequencies in the second instance is not to be explained by any influence of the medium but rather by the characteristics of audition. The sense of loudness for the different frequencies is not the same, whether the intensities * are measured in mechanical units or in terms of the least audible intensity. It is the latter unit of measurement that is of interest in the present case, for the nature of the sound heard at a great distance from the source depends upon audibility. A further explanation will be given in a discussion of audition.

The other experimental fact worthy of record is the rapidity with which the intensity falls off with distance in the atmosphere. If sound energy given off by the engine occurs at a constant rate, and if this energy spreads out in a spherical wave, then the amount of energy per unit volume or the intensity would vary inversely as the area of the expanding sphere. This means that the intensity varies inversely as the square of the distance from the source. In the experiments to which reference is here made, the observer used an instrument that amplified the sound intensity one hundred times. If the intensity of sound varied inversely as the square of the distance from the source, then the observer should hear an airplane at ten times the distance one could hear it with the unaided ear. But on fair days, cumulus clouds forming, airplanes at an elevation of one thousand yards could be heard only twice as far with the instrument as with the unaided ear. On days when the atmosphere was obviously more irregular, the decay of intensity was much more rapid. Under good night observing conditions with the airplane at an elevation of two thousand yards the maximum distance of hearing was increased to three times the distance possible with the unaided ear.

It thus appears that even when the atmosphere is favorable to sound transmission there is sufficient irregularity to cause a surprisingly rapid decay of sound from an elevated source.

4.7. Diffraction. — The word diffraction is used when a change of direction of propagation of sound is occasioned, not

* By "intensity" is meant the energy per unit volume.

by a difference in the medium itself but by the introduction of obstacles or reflecting surfaces, causing the sound to bend or to diffract around such objects. Diffraction has, as a matter of fact, previously been mentioned in Sections 1.3 (with ripples) and 3.4, but without being so designated. Sound will pass around the corner of a building or over a partition or out of a window. Everyone knows that the hearer need not see the source of the sound. Obviously, diffraction is a very common and also an important phenomenon. Without it we would be put to great inconvenience in the conduct of our daily affairs. Diffraction of sound must have played a significant role in self protection in the evolution of the race.

From our earlier discussion in 3.4 in regard to the dependence of the efficiency of reflectors upon the frequency of the sound concerned, it can readily be judged that the longer the wave-length the less the reflection and the greater the diffraction produced by an obstacle. It will be recognized also that our discussion of the reverberation in auditoriums assumed the free penetration of all frequencies to every corner and recess in the room. Fortunately the diffraction phenomenon is sufficiently pronounced to make this assumption approximately true.

4.8. Diffraction about the Head of a Speaker. — The diffraction about the head is an important consideration in two cases, speaking or singing and audition. In the former the source is at the head and in the latter it is removed at a distance. Results concerning diffraction about the head will now be discussed. The theoretical investigation, first begun by Lord Rayleigh and later continued by others,* assumes the problem to be the investigation of the diffraction of sound about a rigid sphere. This shape is necessary to make the mathematical solution possible. The term "rigid" means simply that the sphere is not set in vibration by the incident sound and hence that there is no absorption at any point on its surface. Let the circle in Fig. 4.5 repre-

* Stewart, *Physical Review*, 33, 1911, 467–479.
Hartley and Fry, *Bell System Technical Journal*, 1, 33, 1922.

sent the cross-section of the rigid sphere, the point A, the source of sound on the sphere, and P and P' positions of the observer, the former on the same radial line as A and the latter on a radial line making an angle θ with OP. The inquiry is now made as to the intensity of sound from A observed at any point P'. A glance at the problem shows that the solution is not easily obtained by physical reasoning or by utilizing any of the facts previously studied herein. The sound will spread out and will

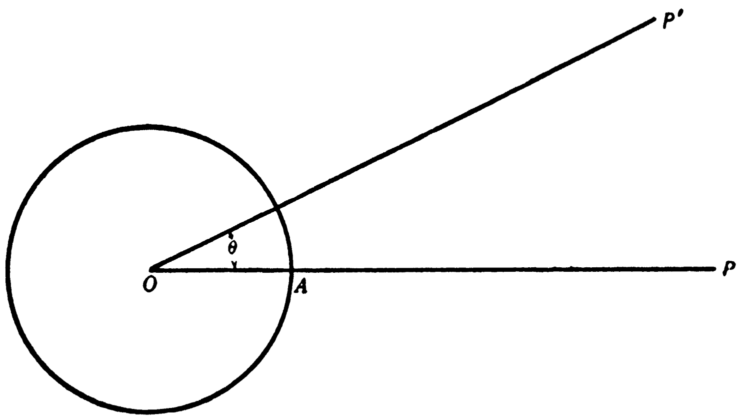


FIG. 4.5

pass in every direction from A around the sphere. Moreover the bending or diffraction will not cease when the sound has gone half way around the sphere. There is no reason why the wave should not continue to diffract around the sphere to the front. In fact, the effect at any point may be considered to be caused not only by the sound being diffracted around and close to the sphere repeatedly, but also by the entire wave front passing the sphere. The phenomenon seems hopelessly complicated. But the mathematical solution is relatively simple in principle. The method briefly stated is not without general interest. The equations describing the acoustic nature of the medium are set up. Then it is assumed that the correct solution must "satisfy" these equations and also the condition of no radial motion at all points of the sphere except at the source, *A*. The solution can then be obtained.

In Fig. 4.6 are shown curves representing the results computed from the theory and expressed in the form of the ratio of intensity at any point *P′* in Fig. 4.5 to that at the fixed point, *P*, the distance *OP* being equal to *OP′* and the angle θ lying between them. The sphere is assumed 60 cm. in circumference. Let us consider curve 2. Here *P′* is 19.1 meters from the center of the sphere. As *P′* moves about the sphere in a plane the intensity at *P′*

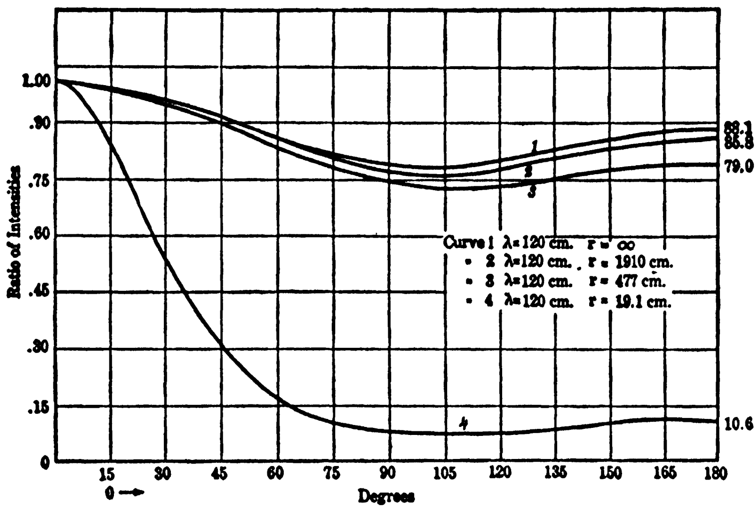


FIG. 4.6

changes so that according to the curve 2, at $\theta = 15°$, the ratio is .97, at 45° it is .91, at 105° it is .76 and at 180° it is .858. Curve 1 is for a very great distance from the sphere; curve 4, for a distance of 19.1 cm. These curves are for one frequency * only and show the variation of intensity experienced by the observer at the point *P′* as he travels in a circle about *O*. One of the interesting points is that the nearer the auditor to the speaker, the more advantageous is a position directly in front of the latter.

* In Fig. 4.6 occurs the Greek letter "λ," which represents "wave-length."

Another interesting point is that there is a maximum * of intensity immediately in front of the speaker and one also immediately behind.

That the change in diffraction with frequency is marked is shown by Fig. 4.7 wherein the range of three octaves is considered. The curves indicate that the higher the frequency, the more the advantage of the front position. It will subsequently be shown in Chapter IX that, in general, clearness of enunciation depends more upon the high than upon the low frequencies in the voice. The lower frequencies are more important in securing volume and the higher frequencies in securing clearness of speech. This fact should be given consideration in any use made of the results of the theoretical investigation.

It is true that the head is not a sphere 60 cm. in circumference as the above theory assumes, but it is safe to conclude that the results of the investigation are applicable as an approximation in any consideration of diffraction about the head.

4.9. **Diffraction about the Head of an Auditor.** — On account of an important theorem in acoustics called the "reciprocal" theorem, the conclusions of the foregoing theory can be transferred to the case of diffraction about the head of an auditor. In such a case the source is assumed at the point P' and the relative intensities at A are calculated for various values of θ. Figures 4.6 and 4.7 are correct for this case also. It can then be said that the sound shadow or variation in intensity, produced by the sphere at a distance OP' with a source on the sphere, is the same as the shadow at points on the sphere produced by the source at P'. The term "shadow" is permissible, for since in optics a shadow is a definite variation in light caused by an object, so in acoustics we may speak of the variation of sound intensity produced by an obstacle as a "sound shadow." From the above mentioned curves we may reach the following conclusions:

1. When listening to a distant sound of low frequency, say

* A "maximum" intensity occurs at a point if at all neighboring points the intensity is less. But here the graph refers only to a variation in the angle θ.

less than 200, there is but little to be gained by turning the head. (Here assume, for simplicity, that there is but one ear, but further examination shows the conclusion correct for binaural hearing.)

2. The closer the source the relatively more important becomes the position of the head.

3. The higher the frequency the relatively more important is the position of the head.

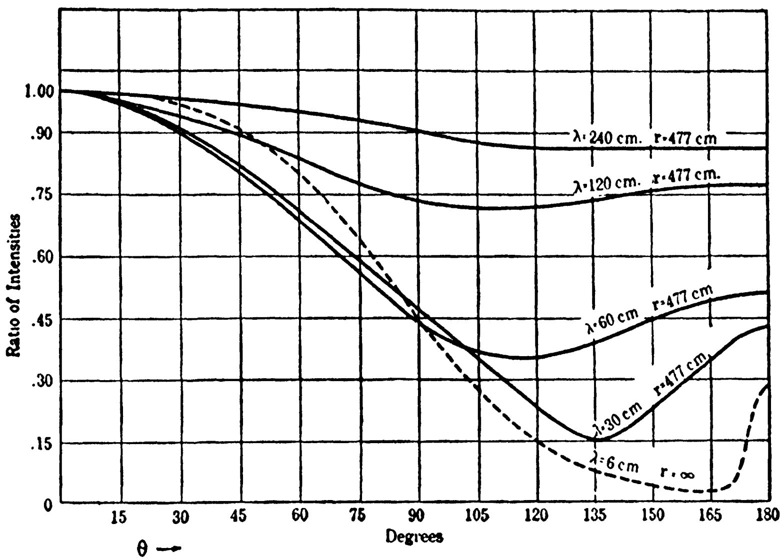


FIG. 4.7

4. The shadow decreases with distance of source from the obstacle.

5. With θ at 180°, A and P' are on opposite sides of the sphere. The closer the source at P', or the less OP', the greater is the shadow at A. This is indicative of the fact that an obstacle will cast a greater shadow the closer it is placed to the source.

That obstacles do cast acoustic shadows can be demonstrated by using the tick of a watch and passing it about the head while one ear is closed, also in the lecture room by a highly pitched whistle, a sensitive flame (Section 15.14) and a small obstacle.

4.10. Change of Quality by Diffraction. — From the above discussion it is evident that the music of a band in a city street will not have the same quality irrespective of the position of the observer. The higher frequencies are reflected more easily and are diffracted less easily than the lower frequencies. Consequently, because of the adjacent buildings, positions may be found where the high frequencies are relatively exaggerated or diminished. For the same reason, the quality of any music can be changed through the reflection from and diffraction about obstacles. The term "quality" is here used in a physical sense.

4.11. Principle of Least Time. — Sound does not always take the shortest geometric path. It is one of the possible deductions from Huyghens' principle, Section 3.2, that the time required for sound to pass from one point to another is a minimum. That is, sound will reach the ear from a source in the least time. Thus the shell wave of a swift projectile passing overhead will reach an observer on the ground from a certain point in the trajectory determined by this condition of least time. In fact, it can be shown that if u is the velocity of the projectile, if v is the velocity of sound, the line drawn to the trajectory from the observer makes with this path an angle θ such that the "cosine" of θ is equal to v/u. By "cosine" is meant the ratio of the adjacent side to the hypothenuse of a right angle triangle containing θ.

The more general statement of the principle described in this section is that the time is either a maximum or a minimum. But the latter is the usual case and consequently the phenomenon usually bears the title given to the section.

4.12. Passage of Aerial Waves about the Earth. — Krakatoa, a volcanic island between Java and Sumatra, was in violent eruption in 1883. On August 27th there was a culminating paroxysm and from this issued a wave of condensation which travelled out in all directions, passed about the earth and apparently again met at the antipodes to Krakatoa. It was then reflected, travelling back to the volcano, whence it returned in its original direction.

The wave was detected by the change in the barometric pressure. It was observed issuing four times from the region of the volcano and three times returning. The actual velocity was of the same order of magnitude as the velocity of a sound wave in air.

QUESTIONS

1. Draw two diagrams similar to Fig. 4.2, but assuming for one that u_1 is zero and for the other that u_2 is zero.

2. If the velocity of the wind, Fig. 4.2, varied not suddenly at OO' but constantly with elevation, would there be refraction and why?

3. Assume that the velocity of the wind relative to the earth increases with elevation; show by drawing what influence the wind will have upon the direction of propagation and the wave front, if a plane wave of sound with horizontal wave front is emitted at the surface of the earth.

4. By extending the discussion in the text prove more in detail the correctness of the statement that "the sound will be retained in this stratum very much as if transmitted between two parallel walls."

5. Show that, when the temperature of the air is highest near the earth, the influence of temperature is to decrease the distance at which sounds may be heard.

6. Show by drawing the possibility of refraction of sound from a source on the earth back to the earth again.

7. Explain why "irregularities in the planity of the strata would be more effective in scattering the frequencies having the shortest wave-lengths."

8. How does the quality of sound from an airplane change with its passage? (A report on actual observation desired.)

9. Does the refraction caused by wind or temperature vary with the frequency employed and why?

10. If the medium is moving with a uniform velocity relative to the earth, what effect would this have upon the shape of a spherical wave from a point source, as observed from a stationary point on the earth?

11. What is the justification for a reflector behind a speaker in the open air, as compared with indoors, and what effect would it have on the high and low frequencies?

12. Cite cases in your own experience where quality is modified by diffraction.

13. As you stand on the street in a busy city, in general how does the character of the noises from nearby sources compare with that from distant sources and why?

14. Draw a diagram illustrating the effect of the elevation of church bells and justify the statement concerning the transmission windward.

15. Is it possible to determine from Fig. 4.2 the magnitude of the exaggeration (in the drawing) of the wind effect?

CHAPTER V

PHASE CHANGE AT REFLECTION

5.1. **Phase Change.** — The terms "same phase" and "opposite phase" were used in Sections 3.1, 3.2 and 3.7. The former refers to two vibrations of the same frequency in which the positive maximum displacements occur simultaneously. But if one vibration is one-half of a period behind the other, they are said to be opposite in phase. But "phase," as in Section 3.7, is also used to indicate any difference in simultaneity whatever. The two vibrations are said to have a difference of phase or to be in different phases. Usually in acoustics only abrupt changes of phase occur and these are plus and minus half a complete vibration or a difference of phase of 180°.* In this chapter will be discussed "reflection with change of phase" and "reflection without change of phase." The former refers to reflection wherein there is a change of phase corresponding to half a period; and the latter to a reflection without change of phase.

5.2. **Reflection without Change of Phase.** — In Fig. 2.1, the waves from *O* and *O′* arriving at the point directly between these sources are in the *same phase*,† *yet the resulting displacement is zero.* By an extension of the reasoning involved, if instead of a wave from *O* we have a wave front whose plane is parallel to the wall, the resulting displacement everywhere at the wall surface would be zero. This reflection can be simulated by removing the wall and substituting a plane ‡ wave coming from the left that has (at the old position of the wall) the same phase as the

* The expression of phase in degrees is explained in Section 3.7.

† Note that, although the displacements are actually opposite one another, yet in each case the relation of the direction of the displacement to the direction of travel of its respective wave is the same. This relationship always determines the phase.

‡ A "plane wave" has a wave front that is a plane.

original wave. The substituted wave is equivalent to the wave reflected from the wall. Since the wave from the left has the same phase at the wall position as the incident wave from the right, the reflected wave must be considered as having that same phase also. Such a reflection is called a reflection *without* change of phase. This phenomenon can be illustrated by the helix of Fig. 1.7. If a block of wood is placed at the end of the helix, the arriving wave does not give a displacement at the end but the combination of reflected and incident waves gives zero displacement. But the condensation of the incident wave at the block occurs simultaneously with the condensation of the reflected wave, making a pressure variation at the reflecting surface. Thus, when the reflected wave is in the same phase as the incident wave, displacements are actually in opposite directions, but the two pressures are simultaneously positive or negative, meaning greater or less than normal pressure.

This reflection without change of phase apparently occurs at every boundary where the second medium may be said to have relatively infinite inertia, i.e., may be considered to be rigid or immovable. Thus such a boundary may be a wall of solid material or it may be a liquid such as a body of water. It is true that the inertia is not infinite and that there actually is a sound wave of small intensity passing into the second medium, but we are neglecting this wave in the claim that the displacement at the boundary is zero.

5.3. Reflection with Change of Phase. — The preceding section dealt with a reflection without change of phase, one in which the reflector was assumed to have infinite inertia. A case which is just the reverse will now be considered. Figure 5.1 represents an interface between a solid S and a gas G. If the sound wave comes from the right to the left, the solid has enormous inertia compared to the gas and the pressures in the gas are able to cause only very small vibrations in the solid. Neglecting these for the moment, it can be said that at the boundary we have total reflection, and this without change in phase as explained in the previous paragraph. Assume, however, that the wave is passing from left to right. It is then seen that the vibration in S is not impeded by G, because the gas has such a relatively small inertia and the vibration of S can occur approximately as if G were a vacuum. If G were really a vacuum, of course no sound energy could pass into it and there would be total reflection at the surface of S. With G a gas, some energy passes into it but only a small amount, for, as just stated, the vibration of S is approximately independent of the presence or absence of G. This can be appreciated by comparing the mass per unit volume in S with that in G. At the surface the solid and the gas execute the same vibrations, but the energy in a sound wave depends not only upon the amplitude of the vibrations but also upon the mass in vibration; the greater the mass, the greater is the energy in its vibration. Since the density of the solid, S, is at least several thousand times the density of the gas, and it cannot transmit to the gas a greater amplitude than the solid possesses, the energy of vibration in the latter must be very much greater than that in the gas. It is reasonable, then, to assert that but little energy is transmitted to the gas, and that the reflection may be regarded as approximately total.

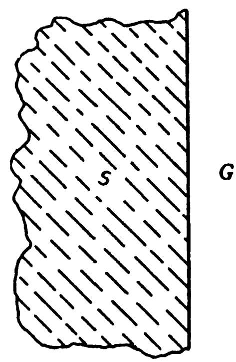


FIG. 5.1

The relatively small inertia of G produces another condition. It can be shown mathematically that the pressure amplitude in G for a wave of given intensity would be very much smaller than for a wave of the same intensity in S. One might surmise, then, that with this practically total reflection the pressure amplitude of the refracted wave in G would be relatively small, indeed, very much smaller than that occurring in S with the incident wave travelling to the right or the reflected wave travelling to the left. But the excess pressure in G is at all times equal to the excess pressure in S at the surface, or to the sum of the excess pressures of the two waves in S at that point. Thus, these two pressure waves in S are practically equal and opposite. Hence, there is a reflection with a change of phase of pressure also.

It is interesting to compare the displacements. Since the waves in S are travelling in opposite directions, an opposition in phase causes the displacement amplitude at the surface to be twice the displacement amplitude of either wave.

In the preceding section it seemed easier to determine the nature of the reflection by the consideration of displacements. In this section the pressure also is discussed. When a reflection is with change or without change of phase of the displacements, the same description must apply to pressures.

Reflection with change of phase may be illustrated by the helix with a free end. Here the vibration completes its swing and is reflected back a half period after its arrival.

5.4. Interesting Cases of Reflection in Gases. — It is not in general possible to determine the actual intensity of the reflected wave by a descriptive discussion. Hence a mathematical study must be made. The results of such a study in several cases will now be given.

1. If the two media concerned are gaseous, the ratio of the displacement amplitude of the reflected wave to that of the incident wave, which meets the boundary perpendicularly, is

$$\text{Ratio} = \frac{\sqrt{\rho_1} - \sqrt{\rho}}{\sqrt{\rho_1} + \sqrt{\rho}} \quad \text{or} \quad \frac{v - v_1}{v + v_1}, \tag{5.1}$$

where ρ is the density in grams per cu. cm. in the first medium and ρ_1 in the second medium. The corresponding velocities are v and v_1. In the case of a wave from hydrogen to air, $\rho = .00008837$ and $\rho_1 = .001276$. Computation by (5.1) shows that the amplitude of the reflected wave is about 0.58 of the amplitude of the incident wave. In other words, the reflected energy is about $(0.58)^2$ or (0.34) of the incident energy.*

2. If now we suppose the wave to be travelling in the opposite direction, that is, from air to hydrogen, the above fraction is negative, but of the same magnitude. This, the theory shows,

* It can be shown that the energy of a plane wave is proportional to the square of the displacement amplitude, frequency constant.

is to be interpreted to mean that now the displacements are opposite in sign, or that *this is reflection "with change of phase."* It is to be understood, then, that equation (5.1) will not only give the correct magnitude but will also state whether the reflection is with or without change of phase. From this equation we can conclude that if reflection occurs with the wave travelling from a *more dense to a less dense gas, the reflection is with change in phase,* but that if the propagation of the sound is from the rarer to the denser gas, the reflection is *without change of phase.*

While the preceding discussion is accurate, it is not descriptive. If one will consider Sections 5.2 and 5.3, he will observe that the conclusions therein would lead to the anticipation of the results just stated with gases. While one cannot extend the reasoning of those sections to gases of different density, yet they give a physical reason for the conclusions just stated concerning gases. There is, however, one fact that still lies hidden. A reflection is either with or without change of phase. The change must always be either 0° or 180°. There are no intermediate changes. It is to be observed, also, that in the broader statements of this Section the earlier limitation of equality of intensities of incident and reflected waves no longer holds.

3. If the difference in the two media is one of temperature, there is reflection, but computation shows it to be slight. For example, suppose the change in temperature to be from 18° C. to 98° C. Then the velocities can be calculated by using equation (1.3). If these two values of velocity be substituted in the latter part of equation (5.1), the ratio of the amplitude of the reflected wave to the incident wave is found to be .07 or 7 per cent. The reflection occurs with change of phase and the reflected energy is only $(.07)^2$ or .005 or one-half of one per cent of the incident energy. Assume this conclusion applied to the case of a large stream of hot air from a furnace entering a room. It is seen that the reflection at perpendicular incidence from such a column of hot air is very small indeed.

4. The effect of humidity may be considered by assuming sound passing from dry to saturated air. The latter is lighter

by about one part in two hundred twenty. It can be shown by (5.1) that the reflected intensity is only about one eight hundred thousandth part of the incident intensity.

5.5. The Image in Reflection without Change of Phase. — Referring to Fig. 2.1, the reader is reminded that the image O' is in phase with O. But reflection occurs without change of phase. This effect can produce a curious result. Imagine the source O in Fig. 2.1, with the image O' in phase, to be brought gradually closer to the wall, where reflection without change in phase occurs. Then O' must also approach the wall. When the distance between O and O' is very small compared to a wave length of the frequency considered, they will act like two equal and like sources indefinitely near each other. The displacement one produces at any distant point (assuming the reality of O' and the absence of the wall) is equal to and in the same phase as the displacement produced by the other. Hence the amplitude everywhere will be twice that produced by O alone. This means that if a source O is constant, it will, when placed very near a large reflecting wall, double the amplitude and quadruple the intensity everywhere on its side of the wall. The total amount of energy in the hemisphere may be compared with that in the sphere (for the wave is spherical) when the wall is absent. The total energy in the hemispherical wave is twice the total energy in the spherical wave. Therefore the source of sound placed near the wall is caused to emit sound energy at twice its previous rate. While this discussion is correct, there is an assumption involved which is not made clear; namely, that the source at O remains "constant." * The requirement of the constancy of the source limits the application of the conclusion concerning the doubling of the emitted energy, yet two points are made clear by the discussion. First, if the source is constant, the energy emitted is not always the same. Second, the surroundings may influence the amount of energy emitted from a source of sound, though the motion of the

* A "constant" point source is one which injects and removes a fixed volume of air. A telephone diaphragm is a constant source if its motion remains constant.

source, such as a vibrating reed or a telephone diaphragm, may remain the same.

5.6. The Image in Reflection with Change of Phase. — From the preceding section and by similar reasoning, one can readily see that if the two sources are in opposite phase each will annul the other's effect. But this is the case in reflection with change in phase. Thus when reflection occurs with change in phase and the source is near the surface the intensity of sound in the incident medium is greatly diminished by the reflection. This effect came strikingly to the attention of the physicists at work on submarine detection during the war. Since reflection in transmission from water to air is with change in phase, it would cause a diminution of intensity in the water in the case of any subaqueous source. The detector is of course placed in the water because even with this interference the intensity is much greater there. The velocity of sound in water is about 4.3 times that in air and hence the wave-lengths are correspondingly greater. The reader can readily show that the effect of interference just described is more pronounced at a given short distance of the source from the surface than would be the case if the velocity were that in air.

5.7. Reflection at a Change in Area of a Conduit. — We have discussed the fact that a change in the character of the medium produces a reflected wave. But a change in the restrictions of the medium may also produce a reflected wave. For example, in Fig. 5.2, assume a conduit in which sound is being transmitted from the left to the right, and that at P there is an abrupt change in area of the conduit. There are two conditions at the point P which are fulfilled. First, the excess pressure at the point P is simultaneously the same for both parts of the junction, and second, the total flow of gas (in the vibration) is the same in the two branches. Strange as it may seem, the fulfillment of these two conditions requires the reflection of a sound wave at the junction. For the moment this reflected wave will be assumed.

In the following section an explanation of its appearance will be given.

It can be shown that if S is the area of the larger tube in Fig. 5.2, and A that of the smaller one, the intensity of the re-

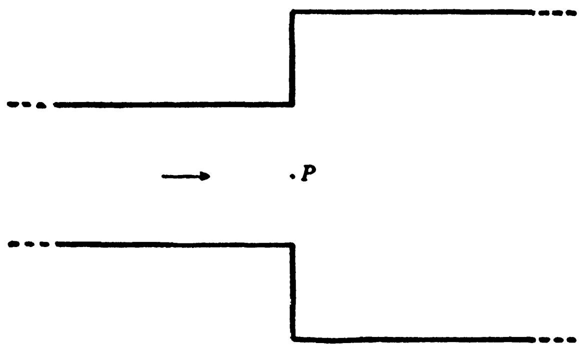


FIG. 5.2

flected wave divided by that of the incident one, or the percentage reflection, is correctly computed by the fraction,

$$\left(\frac{S - A}{S + A}\right)^2.$$

Thus, if the S is three times A, the amount reflected is 25 per cent. Let the same total alteration now be made in a series of many small steps instead of in one step as in Fig. 5.2. Then there will be a reflected wave at each small change in area. These reflected waves would not agree in phase. If there are many of them extending over a length long compared to a wave-length, then there would be destructive interference among them and the resultant reflected wave would be small. Thus it happens that if the change in area is accomplished very gradually, extending over many wave-lengths, the transmission is practically undiminished. As a practical procedure, if reflection is to be avoided, it is essential either to maintain the area of a conduit constant or to make the alteration gradually over a length of conduit containing a number of wave-lengths. In accomplishing the gradual alteration in area, it is not necessary that the inside surface of the conduit be without small abrupt changes in area. Thus a large number of small abrupt changes well scattered throughout a distance long compared with the wave length will eliminate much of the reflected flow of energy.

5.8. Cause of Reflection at a Junction. — One is entirely familiar with the reflection or rebounding of a tennis ball striking a wall. Yet it returns somewhat differently than an indoor baseball or a ball of putty. This is a problem in mechanics and must include such considerations as momentum and elasticity. It is mentioned here to show that, after all, unless one is a student of mechanics, he is not familiar with the details of even this simple phenomenon with the tennis ball. Hence, in considering acoustic reflection, the rebounding ball is of no assistance. Indeed, it is actually misleading for reflection in the case of acoustics is a very different matter. Here we are not dealing with an object, but with a wave of changing physical condition. It must be recognized that the cause of a reflection of a wave must rest in the acoustic conditions at the reflecting surface. For example, consider the reflection of a plane wave incident normally on a rigid wall. In Section 5.2 it was shown that a plane wave will be reflected, and specifically because of the condition required of the displacement at the wall. The resultant displacement at the wall, which is rigid, is zero at all times. But this is not the case in the incident wave, and consequently an incident wave only will not satisfy this condition of zero displacement. That condition can be satisfied by an additional reflected wave with the actual displacements of these two waves equal and opposite at the wall. The existence of the reflected wave is thus shown to be a necessity. It does not come from a vibrating surface, but simply arises from the conditions at the wall. Of course in considering reflection it is simpler to have in mind the approaching wave of pressure, which will produce a pressure at the wall, this pressure causing a return wave. But the important consideration at the moment is that the reflection occurs because the conditions there require an equal reflected wave.

With this explanation perhaps the reason for the reflection of a wave at the point *P* in the conduit shown in Fig. 5.2 may be understood. Let us first assume that there is one wave entering from the left and one passing out toward the right. Assume the conduit infinite in length so there is no return wave. Let it also be assumed for the purposes of trial that there is only the one wave present. It was stated in the previous section that one of the conditions fulfilled at *P* is that there is but one value of pressure. If, then, there is but one wave entering and passing through the junction, it must retain its pressure amplitude unchanged. But this cannot occur. For the wave at the right of *P* is similar to the incident wave at the left of *P* except that it has a larger wave front. The wave at the right of *P* must therefore contain not only as much energy * as the wave approaching from the left but also an additional amount corresponding to the increase in area. But the mere presence of a change of area could not increase the flow of energy. That would be a violation of a general principle called the "conservation of energy." Clearly the assumption of but one wave is incorrect and some way must be found to meet the condition of one pressure value at the point *P*. As another trial assume that there is a reflected wave travelling to the left from the junction. It is well to notice that one cannot assume a second additional wave travelling to the right of *P* for there would then be two waves travelling in the same direction and these will be united becoming one wave. And it is not possible to assume a third additional wave on the right of the junction, travelling to the left, for this was eliminated by assuming an infinite tube. But two waves in opposite directions on each side of its boundary complete the possible assumptions even without the assumption of an infinite tube on the right. Hence, in the present case, the only additional wave possible to

* It can be shown that the flow of energy per sq. cm. per sec. in a plane wave, and also a spherical one, varies with the square of the amplitude of the excess pressure.

assume is a wave beginning at the area containing the point P and passing to the left.

The problem is to determine if this assumed wave can meet the condition of pressure at the junction. Assume this reflected wave to occur and with change in phase. Then, as shown in Section 5.3, the pressures in the incident and reflected waves are opposite in sign. Since they are not of equal amplitudes there is a residue of pressure amplitude, and this pressure may be thought of as the source of the wave to the right. The assumed reflected wave can meet the condition of pressure at the junction. The excitation of the reflected wave and the fulfillment of the condition of pressure are two aspects of the same phenomenon. The fact that the wave has its origin without the vibration of a solid body need not disturb the student if he realizes that that is also the case when one whistles, when one plays the flute and when the wind howls about the corner. Space has been devoted to this discussion in order that the student may appreciate the essential difference between reflections in acoustics and reflections of a ball striking a wall.

5.9. Reflection at an Open End of a Pipe. — Consider the end of a pipe as in Fig. 5.3. At the point P we have the condition that the conduit opens out into the unconfined air where the pressure is normal. Obviously a wave passing to the right can-

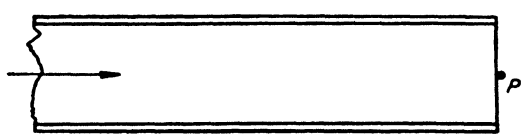


FIG. 5.3

not cause the entire region outside at the end of the pipe to experience as great changes in pressure as occurred in the wave within the tube. The reflected wave at P must then reduce the pressure at the opening, or it must furnish condensation for the rarefaction of the incident wave. This is reflection with change in phase, as we have previously seen. Another way of visualizing this result is as follows: at any point within the tube, displace a short length of the air column. The pressure ahead is thereby increased and resists the motion. If this experiment is tried at the open end, the displaced air spreads readily out into the atmosphere. The open end acts like the free end of a helix. As in that case, so here, the reflected wave increases the displacement and reduces the pressure amplitude. It is reflection with change in phase. One inherent difficulty in understanding why a reflection takes place at an open end of a pipe is that such a phenomenon seems at first thought not to be in accord with experience. A reflection from a wall is to be expected, but that from an open end is not so easy to anticipate. But, as will be shown in later pages, the action in such wind instruments as the cornet, trumpet, trombone and flute all depend upon the reflection of sound from an open end. This reflection builds up the intensity on the interior of these instruments and helps to make possible what is later described as "resonance." Such a reflection is common in experience.

There is considerable interest in the amount of reflection at the open end of a tube and this has been theoretically determined, but experimental data are lacking. If R is the radius of the tube and if the ratio of R to the wave-length λ is small, then the percentage of the wave travelling toward the open end dissipated into the open air is computed by the fraction.

$$\frac{8\pi^2 R^2}{\lambda^2}.$$

Thus if $\frac{R}{\lambda}$ is $\frac{1}{50}$, the amount dissipated is approximately 3 per cent. If $\frac{R}{\lambda}$ is $\frac{1}{20}$, the dissipation is approximately 19 per cent. A statement should be added to the effect that the theoretical investigation of the percentage dissipated assumes the open end of the tube to have a plane infinite flange. Although this is not in accord with Fig. 5.3 yet it is an approximation to the simple tube.

In Section 4.9 a certain reciprocal relation was mentioned. This relation, when applied to the case above, states that when a source of sound is placed in a tube with an open end P, the intensity at a given outside point has the same value as would be observed were the positions of source and point of observation interchanged. That is, if but little sound leaves the tube, but little would enter were the source on the exterior. In fact, a tube with a small opening does not supply a good means of egress of the sounds. It is likewise a poor receiver. These considerations explain why the receptive qualities of a tube need to be improved by flaring the open end.

5.10. Reflection at a Closed End of a Pipe. — At the closed end of a pipe the reflection occurs just as from a wall. It is reflection without change of phase.

5.11. Total Reflection at an Interface. — Assume that we have a sound wave of velocity v passing into a second medium and therein having a greater velocity v'. As in Fig. 4.1, the direction of the wave is changed and the angle measured between the perpendicular to the surface and the direction of the wave changes from θ to a larger angle θ', as in Fig. 5.4.* As explained in Section 4.2, if now one increases the "angle of incidence," θ, the "angle of refraction," θ', increases even more rapidly until finally the refracted wave just skims the surface, i.e., θ' is 90. If now θ be increased still further, there is no refracted wave, total reflection ensues, and we have only the reflected wave, which is not drawn. The minimum angle of incidence at which total reflection occurs

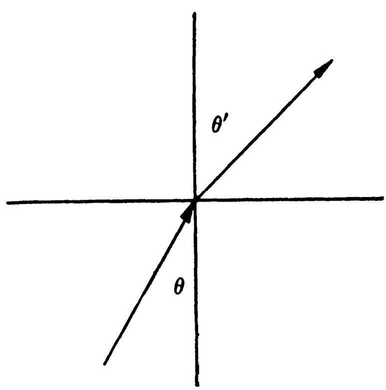


FIG. 5.4

* The reflected wave is not shown in Fig. 54.

is called the "critical angle." It is clear that the greater the difference in velocities of the sound wave the smaller the angle θ, at which total reflection begins. Thus, in passing from air into water, this angle has been computed to be approximately 13.5°. In the case of a column of hot air such as that described in Section 5.4, with the velocities having a ratio of 1.15, θ is approximately 60°. This phenomenon of total reflection can be reproduced in the laboratory and may often occur in experience. But if the refracted wave has a very small intensity as in the transmission from air to water, its presence or absence is not noted and the phenomenon of total reflection is not appreciated as such. Again, if the angle θ is large, the observer may think the sound is coming directly from the source and fail to recognize the fact of total reflection. Many reports have been made of apparent total reflection caused by unusual meteorological conditions. It should be observed that the refracted wave may cease to exist, but that the reflected wave is always present.

5.12. Absorption along a Conduit. — If a conduit has a rigid non-absorbent wall obviously the wave passes along it without being scattered. The wave front will remain plane as the wave travels along the tube. But it must not be thought that the wall of the tube plays no part in this transmission. Suppose it were made of felt. Then the wave of pressure travelling along the tube would cause motion of the air in the felt, producing absorption. Indeed, this is a method used in preventing the sound passing through a large ventilating flue. It is observed that although the wave may be said to travel along the tube there is nevertheless a divergence into the walls and a marked absorption.

QUESTIONS

1. If there are air currents in an auditorium, or if there are somewhat sharp variations in temperature, and if these conditions are not constant, what effect would be had on the sound intensity at any given point, assuming a constant sound source?

2. In a previous chapter was presented the phenomenon of absorption in the walls of a room. In the light of the present chapter,

assuming the velocity of sound in the walls to be much greater than in the air, what additional phenomenon would you note as occurring at the walls in a room?

3. Justify the statement, "That the effect of interference just described is more pronounced at a given short distance of the source from the surface than would be the case if the velocity were that in air."

4. In discussing the effect of change of area in a conduit the statement was made that "there would be destructive interference among them," etc. Show why this interference occurs.

5. What is the one condition depending on the physical property of the incident medium that determines the possibility of total reflection at an interface?

6. Which discussion is more general, that relating to Fig. (4.1) or that relating to Fig. (5.4)? Explain.

7. When like waves are travelling in opposite directions and their pressures at a point are always equal, what is the relation between the displacements? Justify the answer.

8. At a rigid boundary, the reflection occurs with or without change of phase of displacements?

9. When the phrase "with change of phase" has been used in the text, to what does it usually refer, pressure, velocity or displacement?

10. Can one hear a distant sound better if he listens at the surface of an "infinite" wall, and why? (Assume that the ear detects pressure and not displacement.)

CHAPTER VI

RESONANCE

6.1. General Phenomenon of Resonance. — The term resonance, when used in a broad sense, refers to the excitation of a vibration in a body by a wave from another sound source. The phenomenon appears most striking when the frequency of the initial wave equals the frequency of the natural vibration of the body caused to vibrate. For example, the air in an empty globular vessel may be made to speak loudly if its critical tone is sounded in the same room. There is an impression that the phenomenon of resonance provides a method of multiplying or amplifying the flow of energy *after* it has become sound energy. But this is incorrect. Resonance accomplishes, in general, two not altogether different results. When the resonator or the resonating body is at a distance from the original source such that the vibration of the latter can to no appreciable extent be affected by the sound wave from the former, then the total emission of sound from the original source is constant. In this instance the resonator does not increase the flow of sound energy or the flow of energy at the resonator. But there are cases to be discussed later where the resonator affects the source, for example, sounds from the vocal chords. In such a case there is a difference in the intensity of sound given off by the source when the resonating body is present. Hence one can say that resonance may increase the flow of energy which is becoming available as sound but it never can multiply the flow of sound energy already present. It may succeed in storing up energy over a period of time and thus make the final effect more powerful. This is true of a Helmholtz resonator described in Section 6.6. Or resonance may actually succeed in causing a source of sound such as a vibrating reed, a vibrating string, or the vibrating vocal chords to emit more sound energy than would be the case in the absence of resonance. In both cases the casual observer would claim that the amount of sound energy was increased.

As will be seen, resonance is not only of great practical importance but also of intrinsic interest. In order that the term "resonance" may be used with sufficient freedom, its meaning should be considered to embrace mechanical vibrations of all kinds.

6.2. Plane Stationary Waves. — There is a phenomenon, i.e., "stationary waves," described in all elementary texts in acoustics. The reader is reminded that no "wave" in the sense first used in this text could remain stationary. That the use of the word "stationary" is nevertheless appropriate will now be shown.

If two *plane* waves of the same amplitudes and frequencies are sent over the same path, they will combine, and if in the same phase they will give a wave of twice the amplitude of either. If they are in opposite phases they will completely annul one another. If, however, the waves are travelling in opposite directions over the same path, the resultant effect cannot be either of those just specified. What occurs is not easy to visualize for the waves are moving in opposite directions. One can understand the result most quickly by an analogy. If one fastens horizontally a long helical spring of small diameter at O, and grasps it at the end A, he can, by moving the hand up and down, send a series of transverse waves from A to O. The waves sent out will be reflected from O and, if the vibration at A is maintained, the resulting motion of the spring will be a combination of the two waves. By properly adjusting the frequency of motion at A, the resulting motion may be as in Fig. 6.1. The point A in the figure is represented at rest because when this exact frequency is secured, the motion of the hand is small in comparison with the motion of the spring. In (1) the spring vibrates between the position AaO and AbO. The reason an exact frequency must be used at A is that one must produce an interference such that A and O will remain at rest, and yet such that the other points along the spring will have motion. If now a more rapid and yet appropriate transverse vibration of A is tried, (2) or (3) will be secured.

The appearance of the vibration is similar to that in (1), save that instead of one segment we have two and three respectively. Now the resultant variations as shown in Fig. 6.1 could have been obtained graphically instead of by experiment. The graphical method would have been to superimpose two such drawings as in Fig. 1.9, causing one to move to the right and the other to the

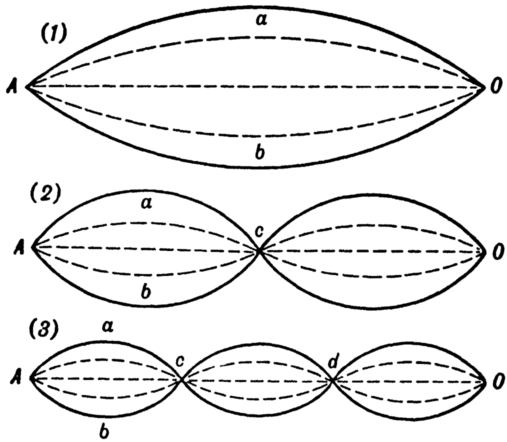


FIG. 6.1

left with the same speed, and then adding the displacements represented by the two graphs at different instants. The graphical method must of necessity give the same resultant at succeeding instants as occurs with the spring in the actual experiment or as shown in Fig. 6.1, for the graphical method is essentially like the experiment itself. In the uppermost drawing in Fig. 6.1, the curve *a* and the curve *b* of course represent the extreme positions of the spring which will occur at times differing by half of that required for a complete vibration. The dotted curves represent the positions of the spring at selected other intermediate instants.

It has just been stated that Fig. 6.1 could have been obtained by a graphical method as well as by experiment. Consider what steps would need to be taken to determine the resultant of two longitudinal waves of equal amplitude and frequency travelling in opposite directions. First, two curves similar to Fig. 1.9 would be prepared, the vertical distances now representing longitudinal displacements. Then these curves would be displaced in opposite directions and at each selected instant the two displacements everywhere added. The resulting curve for any instant would indicate the resultant longitudinal displacement at every point. But the preceding paragraph states that two such curves treated in that manner will give Fig. 6.1. Hence one will obtain similar final curves, but in one case having in mind transverse waves and in the other longitudinal waves. Fig. 6.1 may thus be taken to represent longitudinal waves as well as transverse waves. It will then be assumed that the above three figures, (1), (2) and (3), may in each case represent two longitudinal waves of like frequency travelling in opposite directions, the vertical distances being proportional to horizontal displacements.

It is now possible to discuss what is called a "stationary" sound wave in air. It is evident that at points A, c, d and O, there is approximately zero displacement. The medium is "stationary," at those points. But consider the direction of the displacements in the neighborhood of c and d. They are first toward these points and then away from them. Hence the medium at c and d experiences changes in pressure. The same is true of A and O. When A and d in (3) are points of condensation, c and O are rarefactions, and vice versa. These points of no motion are called "nodes." At the midpoint between A and c the medium has the greatest displacement. This midpoint is called a "loop." It has just been stated that when A is a condensation, d is also. The distance from A to c, or c to d, or d to O is one-half wave-length. Another interesting fact should not escape attention. In an ordinary progressive sound wave at any instant the phase differs from point to point along the wave. In a stationary wave the phase in one segment such as Ac is everywhere the same the displacements of the particles in this segment differing only in amplitude. But the phase in one segment is at the same instant opposite to the phase in the adjacent segment. At a loop, the adjacent particles have virtually the same displacements and hence there is no change in condensation or pressure.

6.3. Stationary Waves in a Cylindrical Pipe Closed at One End. — In the previous chapter it was shown that one can obtain reflection with or without change of phase. In the case of perpendicular incidence upon a wall where the reflected wave passes in a direction opposite to the incident wave, and where the amplitudes of the two waves are equal the conditions for stationary waves exist.

Consider a cylindrical pipe *AB*, Fig. 6.2, and assume that plane waves *OO′* from a distant source of sound enter this pipe and are reflected at *B*. If we assume that this reflected wave *B*

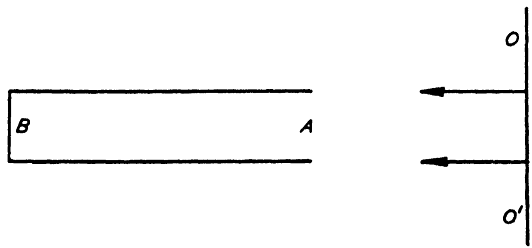


FIG. 6.2

to *A* does not suffer a reflection at *A*, but goes out into space, we will have in *AB* the condition of two like waves travelling in opposite direction or the condition for stationary waves. The reflection at *B* will be without change of phase, there being no displacement but variation of pressure only. *B* is therefore at a displacement node. Furthermore, in the distance from *B* to *A* there will be a node at each one-half wave-length, and between them, or at one-fourth, three-fourths, five-fourths, etc., wave-lengths, will appear a loop.

So far we have assumed no reflection at *A*. We now realize that if it is possible to have a stationary wave in a pipe closed at one end and open at the other, there may be a certain position for *A*, the open end of the pipe, which will not only permit but encourage the stationary wave to exist. The reflection at *A* of a wave travelling to the right, here called R_1, is with a change of phase, as shown in Section 5.9. R_1 and its reflection at *A* called R_2, would, if the latter were equal to the former, form a stationary wave with a loop at A. In fact they both conspire to this end. This suggests that we consider the idealized pipe in Fig. 6.2 cut off so that A will be at a displacement loop. Then from A to B is an odd number of quarter wave-lengths. Since the reflection at B is without change of phase, then from A to B and back to A would be equivalent to a path of twice this length and the wave R_1 when incident at A would be an odd number of half wave-lengths (or twice an odd number of quarter wave-lengths) ahead of the incoming wave at A. That is, it is out of phase with the incident wave. Then R_2, because the reflection is with change of phase, would be in phase with the incoming wave. Thus all three waves at the point A would have the necessary phase relations to form a displacement loop. The only other condition to be met for a stationary wave is that the combined amplitude of R_2 and the incoming wave would equal the amplitude of R_1. This requires an entering wave that has a flow of energy precisely equal to the difference between the corresponding values of R_1 and R_2. But this means that the flow of energy from the tube, or the energy of R_1 less that of R_2, is equal to the energy flowing into the tube. This is clearly the condition for steady operation, without any change in energy in the tube. With a steady source, this is of course a possible condition. Hence cutting off the pipe at one of the original loops places A at the correct point for a possible stationary wave. Assume that we have a source and cause a wave from it to enter a cylindrical tube of the length above described. The stationary state will not be attained at once, for a portion of the wave from B does not get out of the pipe but is trapped by reflection at A. Consequently the acoustic energy within the pipe will increase with time until the rate the energy is dissipated is just equal to the rate of influx.

By this process the energy per unit volume within the pipe region has been made much greater than if the pipe were absent. But this energy has not been created by the pipe, but rather stored over a period of time. It is to be observed that the agreement of the reflected wave at A with the incoming wave indicates that we are using the frequency which may be called a "natural" frequency of the pipe. It is the frequency with which the interior would oscillate after the volume of air is given a sudden blow. The phenomenon whereby we build up the intensity by using the natural frequency is called "resonance," though the term is not limited to such a case, as was stated at the beginning of this chapter.

One of the simplest ways of showing such a case of resonance is (see Fig. 6.4) by placing a vibrating tuning fork over a cylindrical jar in which the volume of air has been adjusted by the water content, so that its frequency is the same as that of the fork. The building up effect can be illustrated by varying the length of time the fork remains at the opening of the resonator. For short intervals, the longer the time, the greater the intensity.

6.4. Resonance.—In any case of resonance the greatest effect is obtained when the frequency of the stimulus is approximately equal to the natural frequency of the vibrating body. But we may also get an increased effect if the natural frequency is not so nearly matched. The apparatus shown in the accompanying drawing, Fig. 6.3, will illustrate this point. A disc, *D*, is suspended by a spring. If the disc is "offcenter," or if there is added to the disc along a radius a small weight *W*, then in each revolution of the disc there will be an impulse given the spring both upward and downward. The experiment is performed in the following manner. With its axis held stationary, the disc is given a spin. The axis is then released. At each revolution the disc gives the spring the impulses already mentioned, but these do not succeed in giving the spring a marked oscillation until, as the angular velocity decreases, the natural period of oscillation of the combined spring and suspended weight is approached. Then the vertical oscillation becomes evident, increasing until it is very vigorous at the agreement of the frequency of rotation, the natural one. Then the oscillation

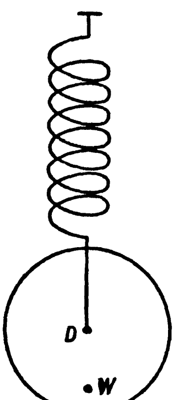


FIG. 6.3

decreases to a small value. This shows that while the maximum effect is obtained at a certain rotational frequency, the phenomenon of resonance is apparent at *adjacent frequencies also.*

6.5. Emission of Sound Increased by Resonance. — If an experiment is performed with a cylindrical tube and a vibrating tuning fork as in Fig. 6.4, it is found that when the air column has for its natural frequency that of the fork, there occurs the maximum emission of sound from the two. But since the cylinder cannot give out more or less energy than it receives, assuming no dissipation within, the emission of energy from the cylinder must be identical with the energy flowing into it from the fork. Hence we must conclude that when placed above the resonating cylinder the fork emits energy at a greater rate than when alone. This illustrates an important fact not usually understood, namely, that resonance *does not create energy* but may make possible *a greater emission from the source.* This increase in output can be explained by the phase relationship of the velocity and pressure at the source. An analogous case is setting a swing into vibration by pushing when at the midpoint of the arc or at the maximum velocity. The push and the velocity are in the same phase. It may be remarked that cases may arise in acoustics where the phase relationship is unfavorable rather than favorable. In such an event, the emission of sound is made less by the presence of the resonator near the source.

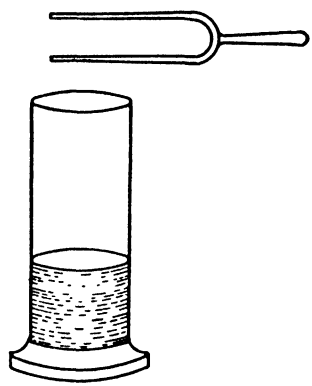


FIG. 6.4

That in the preceding experiment a fork has been caused to emit energy at a higher rate is based not only upon theory but also upon experiment. The literature records experiments by Koenig wherein a fork sounded about 90 seconds without a resonator and 10 seconds in the presence of one. Obviously the change that occurs depends upon the internal losses of energy in the fork and upon the dimensions of the resonator. Thus one should not expect the same relative change in all similar experiments.

The phenomenon of causing a vibrating source to give off energy more rapidly because of the presence of resonance is shown in practically all musical instruments including the vocal chords.

6.6. Resonance in a Volume Having an Orifice. — If, instead of a cylindrical pipe, we have a changing cross section, the theory becomes very difficult. In fact, the method then utilized is only an approximation. An illustration will be made of a volume containing an orifice as in Fig. 6.5. Such a volume and orifice is commonly called a "Helmholtz resonator." If this volume is set into vibration in its natural frequency, the maximum particle velocity (see Section 1.12) will occur at O. V is so large that the motion inside is slight. The volume thus acts like a cushion or a spring, the displacement at the orifice causing a compression (or rarefaction) throughout the interior. In the channel at the orifice, however, the motion is relatively violent and the mechanical forces arise not from condensation or rarefaction but from the rapid acceleration or rate of change of velocity of the mass of gas in the channel. In short, the predominant physical factor in the V region is *elasticity* and in the O region, *inertia.* We can therefore make this resonator analogous to a spring and a weight as shown in Fig. 6.5 (*b*). In mechanics it is shown that the frequency of vibration of such a spring is

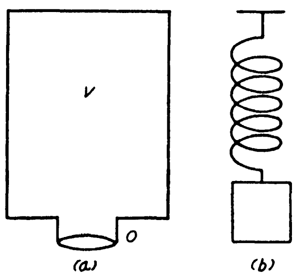


FIG. 6.5

$$n = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{k}{m}},$$

where m is the mass of the weight, k is the so-called "constant" or stiffness of the spring, n is the frequency and T is the period.

In acoustics an analogous equation is derived, and is found to be as follows:

$$n = \frac{1}{T} = \frac{a}{2\pi}\sqrt{\frac{c}{V}},$$

wherein a is the velocity of sound, c is called the "conductivity" of the orifice and V is the volume of the chamber. All distances are measured in cms. If the orifice is a circular hole in a thin wall, the value of c is two times the radius. To illustrate the formula, compare the case of a cylinder closed at one end (as in Fig. 6.4) and having a natural fundamental frequency of 200 cycles, with that of a Helmholtz resonator, Fig. 6.5. Let the cross sectional area of the cylinder be 10 cm.^{2} Its length would of necessity be one-fourth that of the wave-length of 200 cycles or $\frac{1}{4}$ (33200 ÷ 200), or 41.5 cm. This is the necessary height * of the cylinder resonating with a frequency of 200. If a similar cylinder, having a top closed, excepting a circular orifice 0.5 cm. in diameter, is used as a resonator, it is readily shown by the above formula that the height of this cylinder need be only 36 cm. in order to have 200 as a natural frequency.

If instead of a simple orifice for this Helmholtz resonator there is provided a neck 2 cm. long and 0.5 cm. in diameter, the natural frequency of 200 cycles will be obtained with a length of cylinder of only 4.9 cm. The formula for conductivity of such a neck is as follows:

$$c = \frac{\pi R^2}{L + \frac{\pi R}{2}},$$

where R is the radius and L the length of the neck. This is a more general value of the "c" which may be used in the formula for the frequency of a Helmholtz resonator.

The above example illustrates the possibility of securing low natural frequencies but with relatively small volumes of air. An illustration occurs also in the resonance of the human voice. A

* There is an "end correction" for every open pipe which is not considered in this example. It is discussed in Section 6.9.

deep bass note sung by a man would require a 10 foot open organ pipe to produce.

6.7. Resonance of the Voice. — In the case of the voice, the resonance cavities are found in the larynx, pharynx, mouth and nasal passages. To what extent the sphenoid sinuses and the right and left antrums may enter into the resonance of the voice is not known but is presumably small. The trachea below the vocal chords is not in a position to emit sound successfully and therefore its resonance is not of serious moment except as it may influence the emission of energy from the source. The phrase "throwing the voice" can scarcely refer to anything other than the control of quality by modifying the resonance through changes in the positions of the tongue and palate. This is the method used in producing different vowel sounds. The preceding formula assumes that the resonating cavity and the orifice are well defined. So long as this is true the shape of the volume is not of any significance. But in the mouth and pharynx cavities the volumes and orifices are less clearly differentiated, and consequently the shape of the cavities is a factor. The skill acquired by the individual in varying the resonance properties of the mouth, pharynx, nasal passages and larynx by variations in the two first named is indeed remarkable.

6.8. Resonance in Cylindrical Pipes. — In Section 6.3, it was shown that stationary waves may exist in a pipe and that there is a displacement loop at the open end and a displacement node at the closed end. Assume a pipe of length L open at both ends. The stationary waves formed in such a pipe must have a displacement loop at each end. The lowest frequency must have but one displacement node on the interior of the pipe. The successive higher frequencies must have two, three, four, etc., nodes. Thus it is simple to show that the lowest resonating or natural frequency is one having a wave-length $2L$, for a wave-length is the distance between alternate nodes or alternate loops. The next highest frequency has a wave-length of L, and the next $\frac{2}{3}L$.

The corresponding frequencies as multiples of the lowest or fundamental are 1, 2, 3, 4, etc. Thus the natural frequencies of an open pipe contain all integral multiples of the fundamental frequency.

The case of a pipe closed at one end is somewhat different. Here there is a node at the closed end and a loop at the other. Consequently the lowest frequency has a wave-length of $4L$, or four times the length of the pipe. The higher frequencies have wave-lengths of $\frac{4}{3}L$, $\frac{4}{5}L$, $\frac{4}{7}L$, etc. The natural frequencies of the pipe are thus, in terms of the fundamental, 1, 3, 5, 7, etc. The natural frequencies contain only odd integral multiples of the fundamental frequency. The difference in natural frequencies between an open pipe and a pipe closed at one end is of importance in wind instruments.

In the construction of the nodes and loops as in Fig. 6.1 (3), one must bear in mind that such graphs do not pictorially represent the longitudinal stationary waves in the pipe. The displacements are in the direction of the axis of the pipe, whereas the displacements in the graph are perpendicular thereto.

6.9. End Correction of an Open Pipe. — The relationship between the length of a pipe and the wave-length of a resonating frequency as stated in the foregoing section is not exactly correct. The pipe cannot be considered as ending, in an acoustical sense, *at* the opening. The reason is that the wave at this point cannot at once spread out into space. There is an additional equivalent length which is definitely related to the "conductivity" mentioned in Section 6.6. One of the latest determinations of the correct value for the additional length is that of Bate.* He states that the correction for the open end of an open-organ flue-pipe is 0.66 times the radius of the opening and that the value is independent of the frequency, at least over the octave used in the experiments. To the actual length of the pipe represented by L in Section 6.6 must be added a length equal to 0.66 times the radius of the opening.

* A. E. Bate, Philosophical Magazine 10, 65, 917, 1930.

6.10. Resonance in Conical Megaphones. — When the megaphone is used as a receiver, its resonating frequencies are not like a cylindrical pipe, closed at one end, but, strangely enough, they are the frequencies that would be obtained in a cylindrical pipe having the same length but open at both ends. The fundamental of the conical horn is therefore twice the fundamental of a cylindrical pipe of the same length closed at one end. The resonance of a conical horn differs very much from that of a cylindrical pipe. The latter will first be described. Fig. 6.6 shows the effect

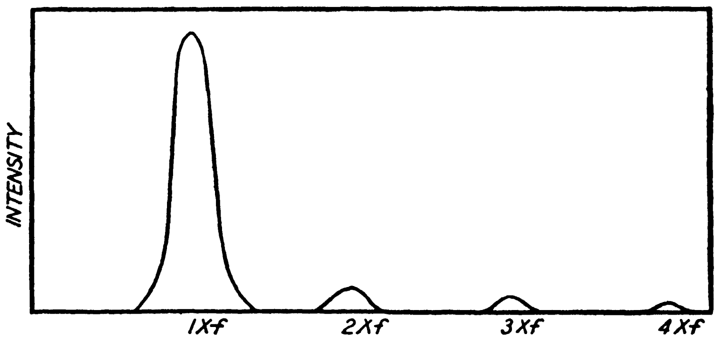


FIG. 6.6

of resonance in a cylindrical pipe open at both ends, the source of varying frequency being held near one end. The vertical height in the graph indicates the intensity in the pipe, assuming the source of varying frequency always to produce the same quantitative changes in condensation and rarefaction at the opening. The frequencies are indicated as being 1, 2, 3, or 4 times the fundamental frequency, "*f*."

It is to be observed that the intensity is reduced practically to zero, but more exactly to no amplification, in between each resonance frequency, and that the intensity at the successive natural frequencies, 2, 3, 4, $\times f$, decreases very rapidly. In point of fact, the intensities decrease much more rapidly than is here shown. The conical horn has a very different effect. Fig. 6.7 is a similar experimental graph by Stewart * for a conical horn.

* Stewart, *Physical Review*, XVI, Oct. 1920, p. 313.

(Actually, the experiments were performed with changing lengths, frequency constant, but one can show by theory that the two curves would be alike.) These figures show a striking difference between resonance in a pipe open at both ends and a conical tube closed at the vertex. With the latter, the intensity at resonance does not decrease rapidly with increasing frequencies but the maximum values are approximately equal. Moreover, the intensity does not become small at intermediate frequencies. Significantly, the minima increase with increasing frequencies. This

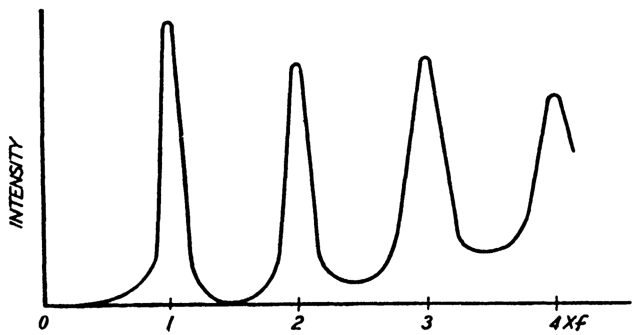


Fig. 6.7

is the secret of the fact that a conical horn acting as a receiver will amplify the emission of sound energy any frequency that is high compared with its fundamental. Thus a long conical horn will amplify all the tones existing in almost any sound and will therefore give a fairly faithful reproduction of the quality or timbre of the original sound. This is an important consideration in the construction of trumpet receivers and reproducers.

In considering the action of the conical horn as a transmitter, one should recall that a resonator can increase the emission of energy from a given source. Moreover, as indicated in Section 3.9, the area of the large opening is a wave front and consequently the horn can direct the sound or partially concentrate it in a favored direction. The conical horn is successful as a transmitter because of these two facts and because of the nature of its amplification as shown in Fig. 6.7.

In connection with the above, it should be emphasized that the megaphone does not ordinarily serve as a concentrator of sound but as a resonator. Of course if the frequency becomes very high the sound can be reflected somewhat as is light, but sounds usually encountered should not be considered as capable of concentration by a conical horn. The fallacy in the concentration idea can be observed experimentally as follows. If one holds a megaphone to the ear and listens to a sound having a frequency half-way between "1" and "0," Fig. 6.7, he will find that the intensity is practically the same with or without the megaphone.

6.11. Megaphones not Conical. — The phonograph or loud speaker horn is not conical but may be considered as made up of fulcra of horns of different angles and lengths. Hence it will have many resonating frequencies, but none are as marked as those occurring in the conical horn. A very common shape is that of the exponential horn. Here the diameter increases rapidly forming a flare. The harmonics can be computed. Mention will be made in Chapter XV of the use of horns in loud speakers.

6.12. Stationary Waves in General. — It must not be supposed that in all stationary waves the adjacent nodes are one-half wave-length apart. For example, in a conical horn closed at one end, the fundamental vibration has a node at the closed end and a loop at the open end, and, according to the discussion of stationary waves in this chapter, the distance between this node and loop should be one-quarter of a wave-length. Yet it has just been stated that the horn length is one-half of the wave-length of the fundamental frequency. The apparent contradiction is explained by an incorrect inference from the preceding discussion of stationary waves. The displacement nodes and loops were formed by two plane waves of equal frequency and amplitude travelling in opposite directions. Other more complicated cases of nodes and loops will be later discussed. No general statement can be made concerning the distance between a node and a loop. For such plane waves the distance between a displacement node and loop is one quarter of a wave-length. This is not the case in a conical horn for there the waves are spherical.

6.13. Resonance in Musical Instruments. — In stringed instruments the effort is made to get the energy from the strings to the atmosphere. On the violin this is done by the rocking of the bridge which sets the body of the violin in motion. Thus the surface exposed to the air is increased. Moreover, the air chamber within has a multitude of natural frequencies which in turn increase the emission of sound. This phenomenon of increase of emission by resonance has already been discussed. The function of the sounding board on the piano is similar to the body of the violin as has been described. The application of the two principles, (1) of increasing the area exposed and (2) of utilizing resonance, may be made in the explanation of the effect with any stringed instrument. A consideration of the possibility of resonance in the solid body of the violin or of the sounding board is omitted at this point but will be subsequently mentioned.

In wind instruments, the tone is produced through resonance. The quality of sound produced when a wind instrument voices any given fundamental depends upon the possible resonating frequencies, the presence of these in the source of sound, and the rate of emitted energy of each of these frequencies by the source. A difference in resonating frequencies and in the sources of the sound will merely be illustrated here, a more complete presentation being reserved for Chapter XV. The clarinet is a pipe practically closed at one end by a reed. Its resonating frequencies are therefore, 1, 3, 5, etc., times that of the fundamental. On the other hand, the oboe and the bassoon are conical tubes closed at the vertex by a reed. The resonating frequencies are 1, 2, 3, 4, 5 times that of the fundamental.

6.14. Resonance in Buildings. — There may be found resonance frequencies in any small room or recess. The organ builder is well aware of this difficulty and adjusts the intensity of his pipes to produce the correct proportion of intensity for the organ in its final position.

QUESTIONS

1. Draw a curve representing a stationary wave at the time of maximum displacement. Indicate the direction of the displacement, the points of condensation and rarefaction.
2. Describe the conditions existing at one-fourth of a complete vibration later than in the preceding question.
3. What is "stationary" in a so-called stationary wave?
4. What is peculiar about "phase" in a segment of a stationary wave?
5. What are three conditions necessary for a stationary wave such as discussed in this chapter?
6. In the three examples in Fig. 6.1 what are the relative frequencies?
7. Give a broad meaning of resonance.
8. What are the two possible fundamental accomplishments of resonance? Give illustrations of each.
9. From the discussion concerning conical horns, what would you anticipate concerning the effectiveness of a hearing trumpet 30 cm. long, and why?
10. The theory of resonators herein given does not discuss the nature of the material of the walls. Under what condition is the omission justified?
11. Why does it require time to build up the energy in a pipe closed at one end?
12. Why is it possible to build up energy in a pipe open at both ends?
13. Show that in the resonator, Fig. 6.5 (*a*), if the radius of the orifice is increased, the frequency is increased, and that if the length of the neck is increased, the frequency is decreased.
14. In the experiment illustrated in Fig. 6.4 what limits the intensity of sound produced? If the chamber were lined with absorbing material would the same intensity be produced?
15. Would it be possible for a man to play such a note on a musical instrument that a large steel bridge would be set into vibration and finally collapse?
16. Construct the graphs verifying the resonance wave-lengths of cylindrical pipes as given in Section 6.7. Compare the physical action with the graphs.
17. If one fills his lungs with hydrogen gas and then attempts to speak in the usual way, a marked effect is produced on the voice. Why?

CHAPTER VII

MUSICAL SOUNDS

7.1. Musical Tones. — It is not the purpose to enter here into either a physiological or a psychological discussion of the requirements for a tone that is "musical." It is a fact, however, that we demand for the most pleasing consonance a simple ratio between the frequencies of a complex musical sound. This is illustrated by the development of our musical instruments which in general give tones having the ratios of frequencies, 1 : 2 : 3 : 4 : 5, etc.

7.2. The Vibration of a String. — In a string we are usually concerned with transverse vibrations. It is true that longitudinal vibrations can be set up in a string or in any solid. A piano string, if rubbed so as to excite longitudinal vibrations, will give a very high tone. In all string instruments the transverse and not the longitudinal vibrations are utilized.* The longitudinal vibrations are displacements along the direction of wave-propagation and consequently possess condensations and rarefactions.† The velocity of such waves depends upon the force necessary to cause a given condensation and the density or mass per unit volume of the material. This dependence has been discussed in Chapter I. In the case of a steel wire the velocity is about 5,000 meters per second.

Suppose a stretched wire, fastened at *O*, Fig. 7.1, is by some means bent in the form shown and then released. The tension in the wire will cause a return to its original position. But it can be shown mathematically that the wire can retain the form in the

* The longitudinal vibration of a string is actually used in laboratory tests where a tone of high frequency is desired. The string may be rubbed by a sponge saturated with turpentine. The string is one-half wave-length long. From a knowledge of the wave-length and the velocity, the frequency can be computed.

† The transmission of waves in solids involves, in general, another type of vibration also.

same position given in Fig. 7.1 without any constraint whatever if the wire is moved to the left or right with a certain velocity v. In other words, a wave of this transverse displacement described will travel along the wire with a velocity v. From the very nature of the case one might expect the velocity in a perfectly flexible wire to depend only upon the tension of T, and the mass of the wire per unit length. That this is the case is stated in equation (7.1).

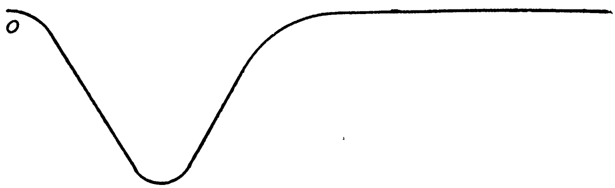


FIG. 7.1

If a transverse wave of displacement travels along the wire, it will be reflected at a fastened end and return in the opposite direction with the same velocity. The condition of stationary waves is fulfilled for there will then be two waves of equal amplitudes and frequency travelling in opposite directions. Since the wire is fixed in position at the ends, nodes will occur there. In Fig. 7.2 is what is called the fundamental or the lowest frequency

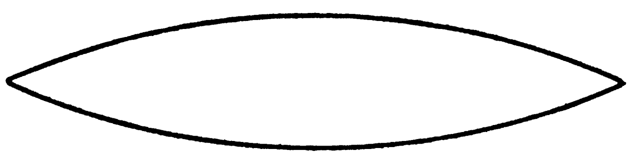


FIG. 7.2

of vibration. The vibration of next higher frequency with which the wire is capable of has one additional node as in Fig. 7.3. Successively higher frequencies that are possible have two, three, etc., additional nodes, respectively. The length of the wave in the wire is obtained in the same manner as is the wave-length in the stationary waves described in the Chapter VI. The distance between two adjacent nodes is one-half wave-length, but this is ***one-half of the wave-length in the wire.***

A string may vibrate simultaneously with the frequencies shown in Figs. 7.2 and 7.3. The lowest of these frequencies is called the "fundamental" and the others, respectively the first, second, etc., "overtones." Any one of these various frequencies is easily brought out by producing a corresponding node after the string has been bowed or plucked.

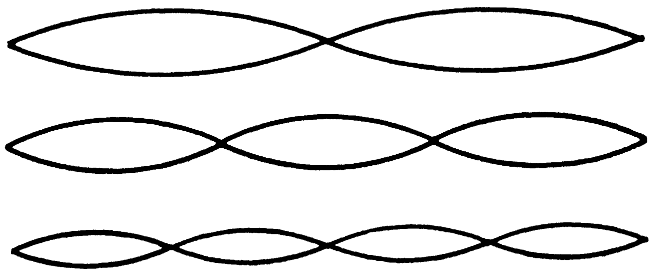


FIG. 7.3

It can be shown that the velocity of the transverse wave in the wire is

$$v = \sqrt{\frac{T}{m}}, \tag{7.1}$$

where T is the tension and m is the mass per unit length. In computations T is expressed in dynes * and m in grams per unit length. Let L be the length of the wire. If for the fundamental, L is equal to one-half of the wave-length, λ, then, since frequency is velocity divided by wave-length (see equation 1.4), the frequency, n, may be expressed as follows:

$$n = \frac{v}{\lambda} = \frac{1}{2L}\sqrt{\frac{T}{m}}.$$

This is the formula for the computation of the frequency of the fundamental of the string of any instrument.

7.3. Measurement of Relative Intensities of Fundamental and Overtones. — The nature of the aerial wave given off by a

* The weight of one gram is a force of about 980 dynes.

string is of interest. From mechanical considerations it is realized that the vibrations of the string are communicated to the body of the instrument, such as the violin, and subsequently to the air. A problem is to measure the aerial vibrations, obtaining the relative intensities of the components. This is by no means easy for there is no direct method. If the sound falls upon a diaphragm and the motion of the diaphragm be recorded on a smoked paper, a curve will be obtained. This curve can subsequently be studied by an expert and the nature of the component vibrations obtained. But the diaphragm, because of its own resonance characteristics, will distort the relative values of the amplitudes of the components in the sound wave and, unless a correction can be made for such errors, the relative amplitudes of the sound waves cannot be obtained. Fig. 7.4 * shows such a mechanically produced tracing on the top line and below the analysis of it. There are three component vibrations having relative frequencies of 1, 2, and 3. In this particular case, the fundamental has the greatest amplitude, but this is not always true, either in the case of the violin, which was used in the present case or of other instruments. Yet the fundamental determines what is called the "pitch" of a musical sound. It is a curious fact in audition that the fundamental is more prominent to the ear than is the same intensity of any overtone. The character of the sound from an organ pipe is shown in Figs. 7.5 and 7.6. The former shows the resultant vibration produced by an organ

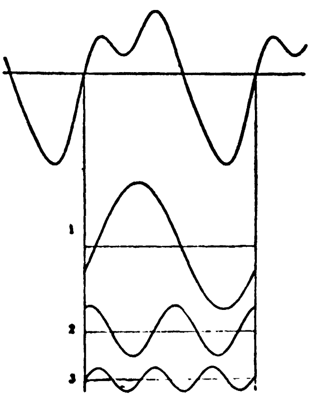


FIG. 7.4

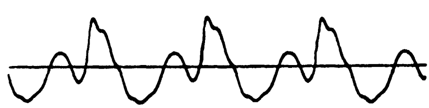


FIG. 7.5

* Taken from Miller's "The Science of Musical Sounds," Macmillan, 1916, as are also Figs. 7.5 to 7.12.

pipe and the latter the composition of it expressed in terms of the frequencies which have the ratios 1 : 2 : 3 : 4, etc., up to 12 times the fundamental.

7.4. Instrumental Quality. — The difference between the quality of a tone of an organ pipe and that of a violin must lie in the relative intensities of the fundamental and overtones. This indicates that the overtones, although difficult to recognize by ear as such, are compositely very prominent in a musical tone. As a matter of fact the ear is a very sensitive detector of overtones and the development of the various types of musical instruments with their ensemble in an orchestra has depended upon just this sensitiveness.

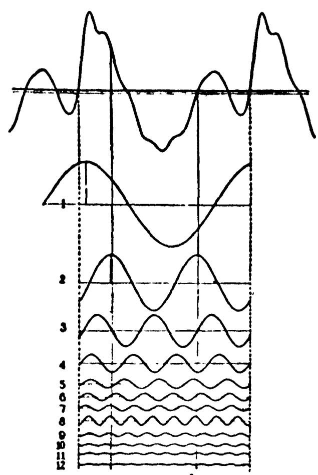


FIG. 7.6

7.5. Sounds from Various Instruments. — In Figs. 7.7 to 7.12 are presented charts showing the relative sound intensities of the components of various musical sounds. The vertical lines indicate the relative intensities. Along the horizontal the frequencies are distributed as on a piano, equal distances signifying an octave. These intensities have been secured from such curves as in Fig. 7.4 by making corrections for the resonating characteristics of the recording device and by transposing the facts concerning frequencies and displacement amplitudes into values of relative intensity. Relative sound intensities are always proportional to the square of the product of the frequency and the displacement amplitude. That is, if a tone of frequency 100 has an amplitude of 10 and a tone of frequency 500 has an amplitude of 2, the intensities would be equal. Hence, in obtaining relative intensities from amplitudes the frequencies must always be considered. Fig. 7.7 shows the relative intensities in the sound from a tuning fork, the voice, the flute, the violin and the French horn, when each is sounded on the fundamental tone. Fig. 7.8 shows an analysis of flute tones. The lowest line shows the average

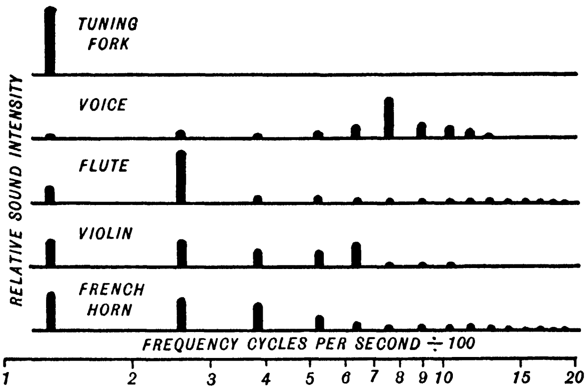


FIG. 7.7. Distribution of energy in sounds from various sources

composition of all the tones or the low register of the flute when played pianissimo. The second line from the bottom shows the middle register played *pianissimo*. When the lower register is

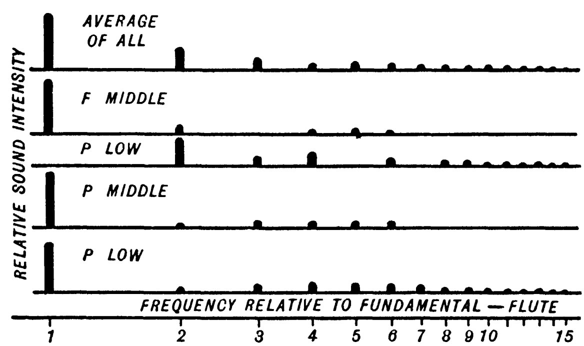


FIG. 7.8. Analyses of flute tones

played forte, the octave becomes the most prominent as shown in the third line. When the middle register is played *forte*, the fourth line results. Figs. 7.9, 7.10, 7.11, 7.12 and 7.13 show Professor Miller's results for the violin, the clarinet, the oboe, the

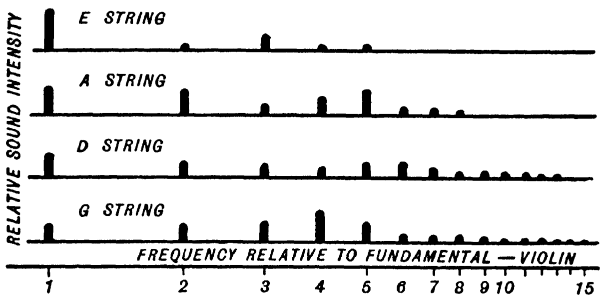


FIG. 7.9. Analyses of violin tones

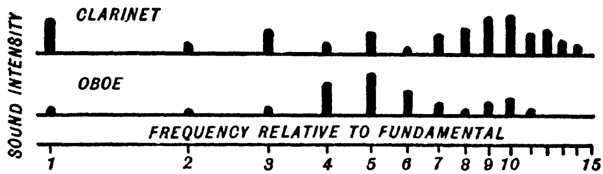


FIG. 7.10. Analyses of tones of the oboe and clarinet

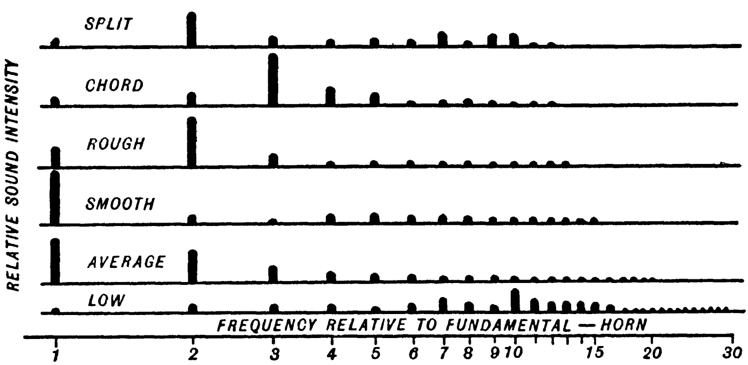


FIG. 7.11. Analyses of tones of the horn

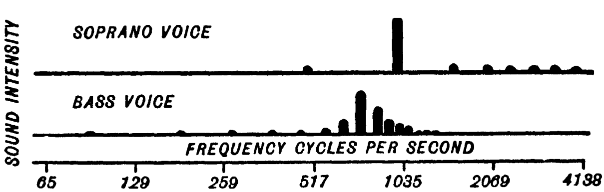


FIG. 7.12. Analyses of tones of bass and soprano voices

horn and bass and soprano voices. The variations in distribution are wide.

7.6. Recognition of Phase Differences of the Components. — Sound from a given instrument may vary in at least two respects, the relative intensities of the components and the relative phases of the components. It is a peculiar circumstance that the ear

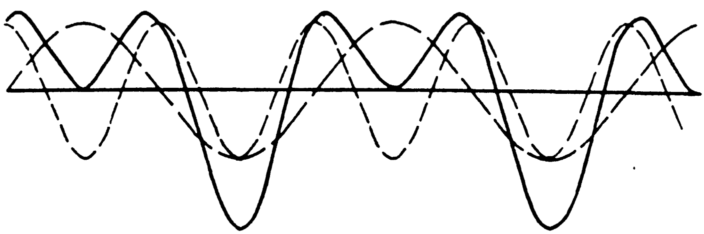


FIG. 7.13

recognizes the combined tone as the same irrespective of the phase relations among the components.* But the trace of the resultant vibration differs widely with changing phase relations of the components. The reader can readily see this from the accompanying drawings, Fig. 7.13 and Fig. 7.14. In Fig. 7.13 are drawn two

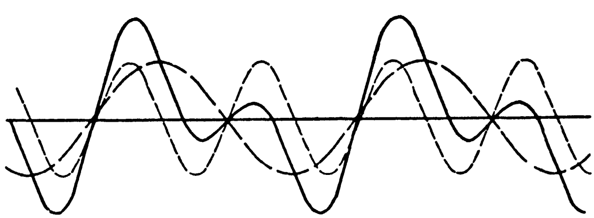


FIG. 7.14

dotted curves representing the displacements in two simple harmonic vibrations (of a diaphragm, let us say), one having twice the frequency of the other. It is a note and its octave. The only difference between Fig. 7.13 and Fig. 7.14 is that the components are shifted in phase relative to one another. In each

* Barton, "A Textbook of Sound," pp. 605–607.

case one-half the sum of the two curves, or the resultant amplitude, is obtained by drawing the mean. The mean value curves are very different in the two cases. Indeed an inexperienced person would claim that the component vibrations could not be the same. Yet these two resultant vibrations give the same auditory sensation. This circumstance, that two dissimilar vibrations give the same sensation, makes the appearance of the resulting vibration curve very deceiving.

QUESTIONS

1. Of what two types of vibration is a string capable? Which one is used in music?
2. Upon what factors does the velocity of a transverse wave in a perfectly flexible wire depend?
3. What frequencies are possible in a string in terms of the fundamental?
4. Why does the vibration of the string cause a vibration of the body of the violin?
5. Which component tone almost invariably determines the pitch of a sound?
6. Is there any difference in the perception of the fundamental and an overtone?
7. Can phase relations of components be recognized?
8. Upon what does the distinctive quality of sound of a musical instrument depend?
9. Assuming that the velocity of a longitudinal wave in a steel string is 5,000 meters per second, how long must a string fastened at both ends be in order to give a frequency of 1,000 per second?
10. What are the difficulties in detecting the components of a tone by eye inspection of the time trace of the vibration?
11. In at least what two respects is there an opportunity of affecting the quality of voice by training?
12. Why is the frequency of a string much higher with longitudinal than with transverse vibration?
13. What difference would be noticed if one was suddenly enabled to detect relative phase changes in components?
14. Why would the difference in quality of two violins depend upon the box, upon the nature of the string?
15. Would the quality of tone of any instrument alter during the entire time of its rendition? Answer by a consideration of one or two instruments.

CHAPTER VIII

THE NATURE OF VOWEL SOUNDS

8.1. The Nature of Speech Sounds. — It is well to observe the physical mechanism producing speech, a vibrator, resonance chambers and an egress for the acoustic waves. During speech these three are almost constantly changing. If one speaks a syllable slowly so that the vowel sound therein may be said to be sustained, then there is an approximation to a steady physical state of affairs at the middle of the time interval for the syllable. But the vowel is not usually defined as limited to this midpoint at the interval. Moreover the component sounds of a vowel would not be the same in magnitude and frequency with different speeds of enunciation of the syllable. This is because of two effects. The establishment of resonance requires time. Moreover if a resonant chamber receives an impulsive change of pressure it will give a rapidly decaying response in its own natural frequencies. For example, when the hands are clapped over the mouth of an empty jar, the sound heard is characteristic of the vessel itself. So it is easily realized that speech is a very complicated acoustic phenomenon, quite beyond the physicist's description at the present time. For that reason the discussion in this chapter is practically limited to the composition of the sustained vowel sounds and even here the results are not at all complete. In the first instance the results presented will be those obtained by Professor D. C. Miller,* whose work represents the beginning of modern physical research on vowel sounds.

8.2. The Vowels Used. — In an experimental test it is necessary to define the vowels tested. This was done by the following groups of words:

* Figs. 8.1 to 8.6 are taken with permission from Miller's "Science of Musical Sounds." Published by Macmillan, 1916.

*fa*ther,	f*a*r,	gu*a*rd,
r*a*w,	f*a*ll,	h*a*ul,
n*o*,	r*o*de,	g*o*al,
gl*oo*m,	m*oo*r,	gr*ou*p,
m*a*t,	*a*dd,	c*a*t,
p*e*t,	f*ea*ther,	bl*e*ss,
th*ey*,	b*ai*t,	h*a*te,
b*ee*,	p*i*que,	mach*i*ne.

In considering the results obtained by Professor Miller, one must bear in mind that they have been secured through careful observation and the devélopment of a technique in which all known corrections have been made. The intensities of the components in the first vowel as in *fa*ther are shown in Fig. 8.1.

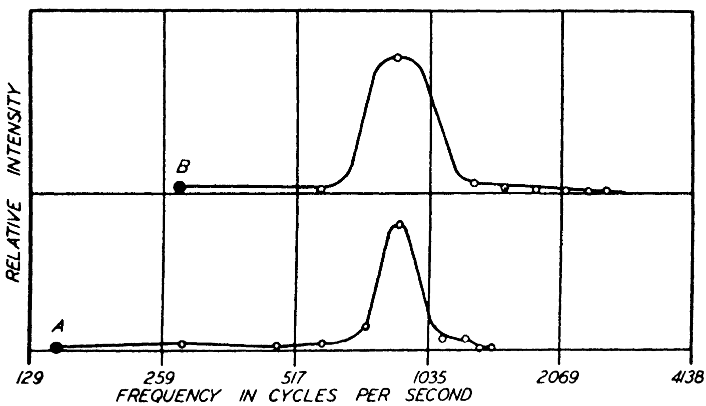


FIG. 8.1. Loudness of the several components of the vowel in *fa*ther intoned at two different pitches

Here the vowel is sounded on two different pitches and in each there is greater intensity in the region of 922 cycles. The frequencies for which the measurements are made are given by the circles, the largest one indicating the fundamental tone. The smooth curves are drawn merely to indicate the general magnitude of intensity in a frequency region. They do not indicate actual intensity for any selected frequency. Fig. 8.2 shows additional experimental values of the components involved in this same vowel. *B*, *C* and *D* are the results of intoning at three different pitches. The curve *A* deserves special attention, because it is a composite of twelve such graphs. Twelve different notes were used as fundamentals and thus a distribution of computed values

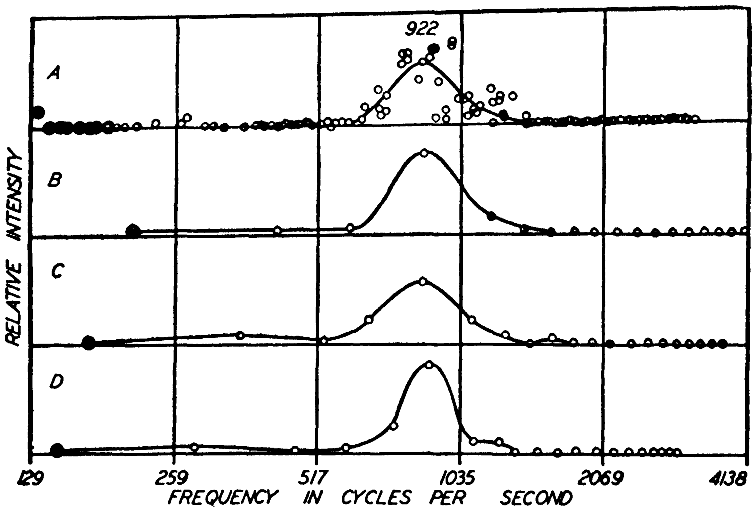


FIG. 8.2. Distribution of energy among the several partials of the vowel in f*a*ther, intoned at various pitches

were obtained. The reason that, with a given fundamental, the only frequencies indicated are integral multiples of the fundamental, is explained in Section 15.6. Briefly it may be stated that if one has a vibration that is periodic, i.e., recurs at fixed intervals of time, its component frequencies must * all be integral multiples of the fundamental whose period is this interval of time.

Hence in the analysis in any case only integral multiples of the frequencies are obtained. Returning to Fig. 8.2 it is easily seen that we have here a remarkable verification of the dependence primarily upon frequency regions. In Fig. 8.3 are shown the results with eight different voices sounding the vowel *a* in

* The proof is entirely mathematical.

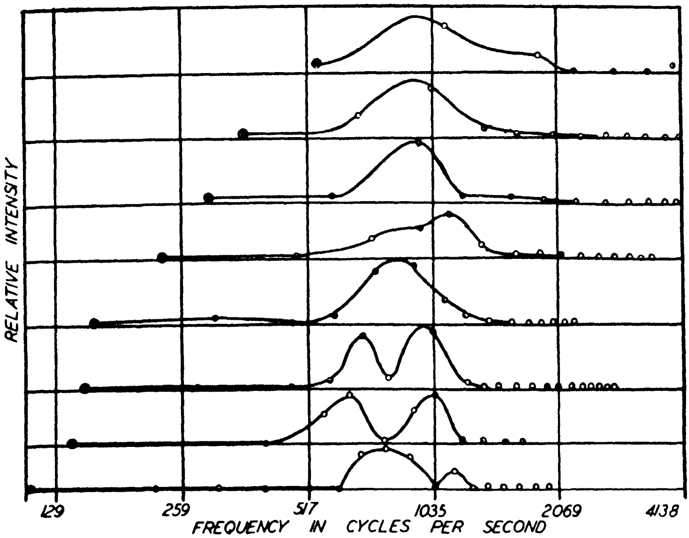


FIG. 8.3. Distribution of energy among the several partials of the vowel in *fa*ther as intoned by eight different voices

*fa*ther on eight different fundamental tones. Fig. 8.4 shows the results with voices of the vowel in b*ee*. Here the second resonance peak is drawn as a mean result of all eight voices. It

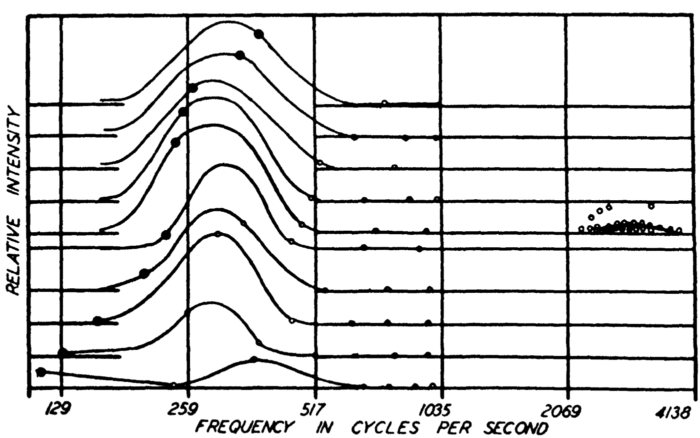


FIG. 8.4. Distribution of energy among the several partials of the vowel bee, as intoned by eight different voices

is evident that this is a vowel with two resonance regions; one about 300 and one about 3,000 cycles. This conclusion is verified by the use of an acoustic wave filter,* which will transmit the

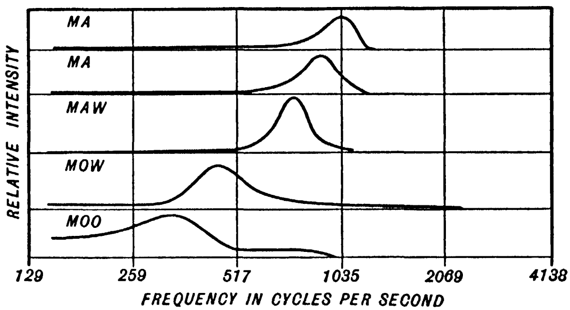


FIG. 8.5. Characteristic curves for the distribution of the energy in vowels of Class I, having a single region of resonance

lower and not the upper frequency region. If the vowel in b*ee* is spoken through such a filter, it becomes almost the same as the fourth vowel in the list or as in gl*oo*m. The reason therefor is

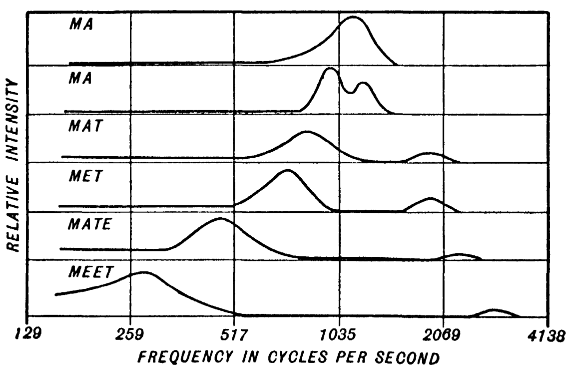


FIG. 8.6. Characteristic curves for the distribution of the energy in vowels of Class II, having two regions of resonance

readily seen in Fig. 8.5, wherein the latter vowel is shown to have a characteristic region in the neighborhood of 326 cycles or practically the same as the lower one of the two regions in Fig. 8.4.

Figs. 8.5 and 8.6 show two different types of vowels. In the

* Acoustics, Stewart and Lindsay.

former there is only one characteristic region and in the latter two characteristic regions. The vowel in m*a* is shown in both, for sometimes it shows two characteristic regions close together. This splendid group of curves shows with remarkable clearness the difference in the vowel characteristics.

8.3. Characteristic Regions Are Resonance Regions. — In the discussion of resonance in a previous chapter attention was called to the resonance of the mouth, pharynx and larynx, and to the fact that the presence of this resonance increases the energy given off by the vocal chords in corresponding frequencies. It is to be expected, therefore, that the characteristic vowel frequencies would be produced by the resonance chambers. That this is true can be verified by adjusting the mouth and lips for the sounding of a certain vowel and then by presenting before the lips a vibrating tuning fork having a frequency in the characteristic region of that vowel. The tuning fork will be strongly reenforced. The fact that the vowels can be whispered as well as spoken is also a verification of this point. Sir Richard Paget's * results show that there are always two simultaneous resonances, produced in two cavities separated by the tongue. Moreover he has by practice been able to demonstrate in the lecture room these two simultaneous regions, the source of sound being the clapping of hands in front of the mouth.

An elaborate investigation of the characteristic regions has been made by Crandall † of the Bell Laboratories. The results are shown in the accompanying Figs. 8.7 and 8.8. In both of these figures the relative amplitudes at the different frequencies are plotted after taking into consideration the sensitiveness of the ear (see Chapter X). The "ordinates," vertical distances, in any one graph then represent what might be termed the relative "effective amplitudes" with that vowel. The ordinates for one graph are not to be compared with those of another since it is not the intention to show such a relation between the vowel

* "Human Speech," Harcourt Brace & Co., London, 1930.

† *Bell Sys. Techn. Jl.*, IV, 4, p. 586 (1925).

sounds. The results of the investigations of Miller and Crandall are in general agreement. The differences in methods of measurements are probably accountable for the differences in con-

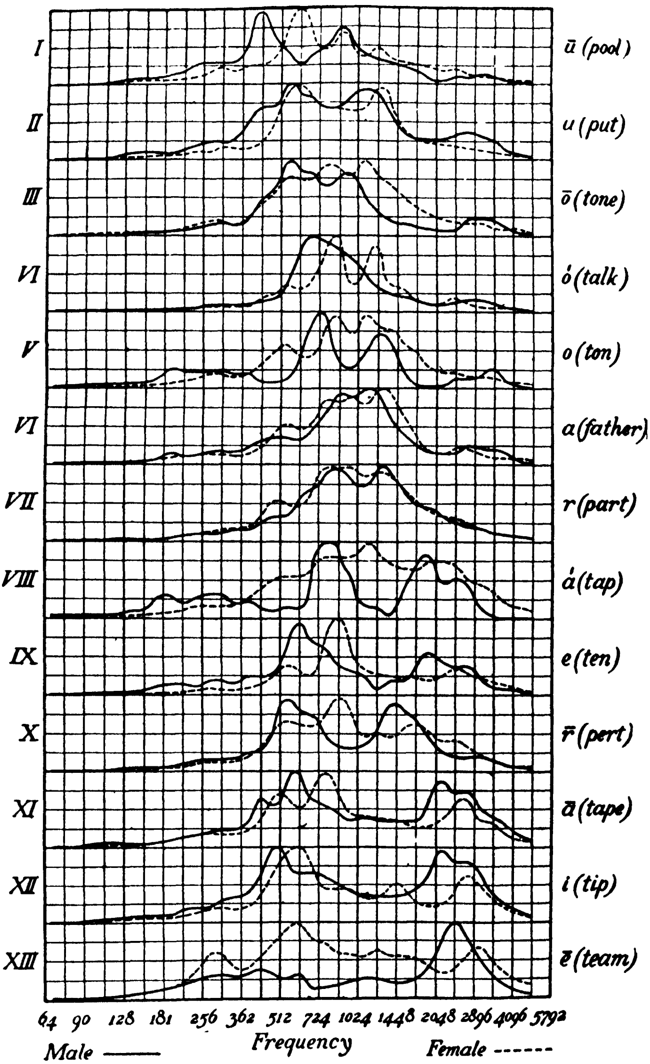


FIG. 8.7

clusions. Crandall's method of obtaining the amplitudes of the components involves the entire record of the vowel from start to finish. He finds the average characteristics throughout its duration. There is first a period of rapid growth of 0.04 sec., second a middle period of about 0.165 sec., with variations but with approximate constancy and third, a period of gradual decay lasting 0.09 sec. This is the approximate description of the duration

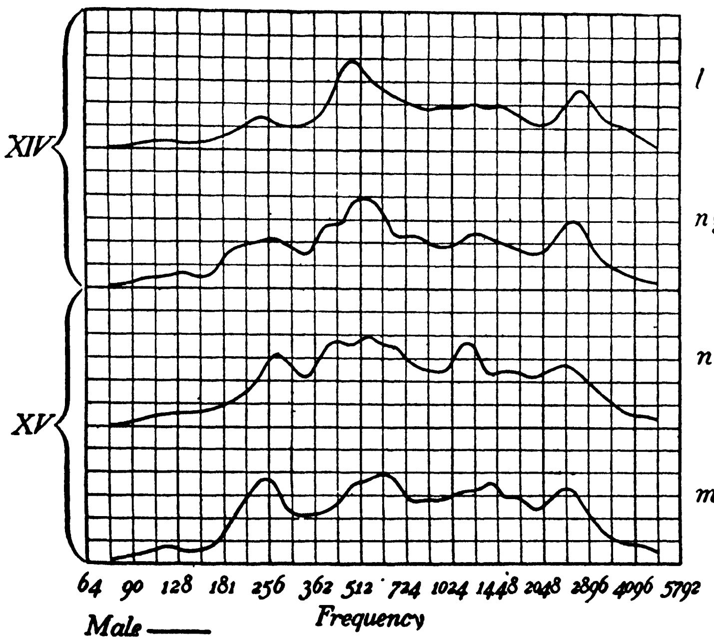


FIG. 8.8

of a vowel sound. There is a variation with the individual and with the vowel, but the general relative magnitudes are retained. Crandall's results leave us with not so clear a distinction as to the number of resonance regions possessed by a vowel. It cannot be stated that any given vowel has only one or two resonance regions. Indeed *any classification as to number of frequency regions* would seem to involve crude limitations not fully justified. The differences and similarities of male and female vowel sounds are also to be noted. As a matter of fact, "resonance region" is an omnibus term and a description using it cannot be regarded as sufficiently specific.

8.4. Clearness of Enunciation of Vowels. — All of the vowel sounds shown above have characteristic frequency regions that are in the upper half of the ordinary piano scale. Clear enunciation thus does not need the bass quality of voice. Indeed, clearness must depend upon the relative amounts of energy that go into these characteristic frequencies. If all the energy is in a characteristic frequency region (or in all the characteristic regions as the case might be) the maximum clearness will be secured. It has often been observed that a child's enunciation of vowels is more clear than a man's and presumably the foregoing is a reason therefor.

It is impossible to enunciate vowels clearly in singing unless the fundamental tone is lower than the characteristic frequency region of the vowel sounded. This is a well-known difficulty. Moreover, a great effort to sing the vowels clearly affects the quality of tone to an appreciable degree and prevents the artist from securing the best musical effect.

Since vowel characteristics depend upon resonance and since time is required in getting the maximum intensity, it follows that vowel sounds must be sustained for a short period of time in order to be the most intense for a given effort on the part of a speaker. The sustained vowel sound probably enters into speech in other respects as well. But it is certain that rapidly spoken vowels are detrimental to clearness.

In the discussion in this chapter the inference has been made that a steady-state vibration of the vocal chords can be secured. But practically this is not true. Indeed, some claim that the motion of the vocal chords is largely impulsive. One must expect that a detailed study of the mechanism will bring out many details at variance with the more simple picture in this chapter.

8.5. Variation in Vowel Sounds. — Professor Miller's records were made of *sustained* vowels only, for the reason that the records of the vowels while they were *being formed* did not contain a sufficient number of like vibrations to make measurement possible. The recent development of filters, both electric and acoustic, has made the variable character of the vowels demonstrable. The following experiment by Stewart,* briefly referred to in Section 8.2, shows the variable character of the vowel ē. The word ēat was spoken through an acoustic wave filter which prevented the transmission of all frequencies above 2,900. But the mouth was formed for the vowel before it was sounded. Hence it had from its beginning the same adjustment as for a sustained vowel. A listener heard the word practically as ōōt. The removal of frequencies above 2,900 transforms the ē into approximately ōō. But if the word mēēt was spoken through the filter, it was easily recognized for there was a distinct sound of ē. But if the vowel in mēēt was prolonged the ē quality disappeared. In other words, the ē heard consisted of frequencies below and near 2,900 which existed only temporarily while the vowel was being formed. The reason for the formation of lower frequencies can be understood by noticing the changes in the mouth during the utterance of the vowel. The mouth formed for an *m* must now be opened wider for an ē. Assuming that the interior of the mouth is correctly formed for an ē, the changing of the lips will increase the area of the orifice and raise the pitch of the resonance frequencies. The group of high frequencies should therefore alter during the formation of the vowel and in the direction demonstrated in the foregoing experiment. Inasmuch as the formation of vowels following consonants requires a change in the shape of the mouth, there is in general a change in the components of any vowel when preceded by a consonant. Similar considerations will apply to a vowel succeeded by a consonant. In the case cited above the consonant *t* shuts off the vowel so quickly that no noticeable change in the vowel is produced.

* Stewart, *Phys. Review*, 1923, Abstract of paper at Washington meeting of Am. Phys. Soc., April, 1923.

QUESTIONS

1. Is the clearness of vowel sounds dependent upon the speed with which they are voiced and why?

2. What reasons may be given for the differences in individual curves in Fig. 8.3?

3. Why are the vowels sounded universally alike?

4. In a previous chapter, the importance of the diameter of a sound reflector behind a speaker was emphasized. If the vowel sounds were the only ones of importance, what would you say about the minimum size of such a reflector?

5. Why may a vowel sounded by a bass and a soprano voice be recognized as the same?

6. Why does a highly pitched voice seem to carry relatively well in addressing a crowd?

7. Discuss the possibility of a "monotone" learning to speak.

8. According to the reasons given for vowel production, would it be possible to imitate speech by mechanical means only?

9. What are the causes of unavoidable variations of a vowel sound, considering the different parts of its duration? What additional variations can be introduced?

CHAPTER IX

CERTAIN PHYSICAL FACTORS IN SPEECH

9.1. Energy Distribution. — One of the inquiries arising in speech concerns the relative amount of energy in the various frequencies used by a speaker. The recent development of amplifying tubes which can multiply electric power has enabled Messrs. Crandall and MacKenzie to measure this frequency distribution of energy in speech. These measurements have been made by using a 50-syllable sentence of connected speech and also a list

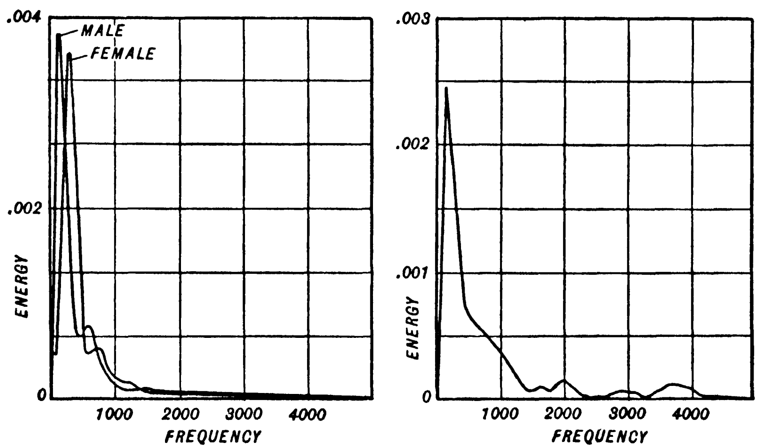


FIG. 9.1. Energy distribution

of 50-disconnected syllables. The results show that the energy distribution varied more with individuals than with the nature of the test material chosen. Fig. 9.1 * shows in the first plot two composite curves. The one marked "male" is the mean of four curves representing the results taken with men. The curve marked "female" is the mean of two curves taken with women.

* *Physical Review*, March 1922, p. 228.

In the second plot is drawn the mean of all six curves. The ordinates of the curve are arbitrary and need not concern us here excepting that they are proportional to flow of energy.

The test sentence was the first sentence of Lincoln's Gettysburg address but with the addition of two words "quite" and "nice" to bring the total number of syllables to fifty and to "improve the balance between the vowel sounds."

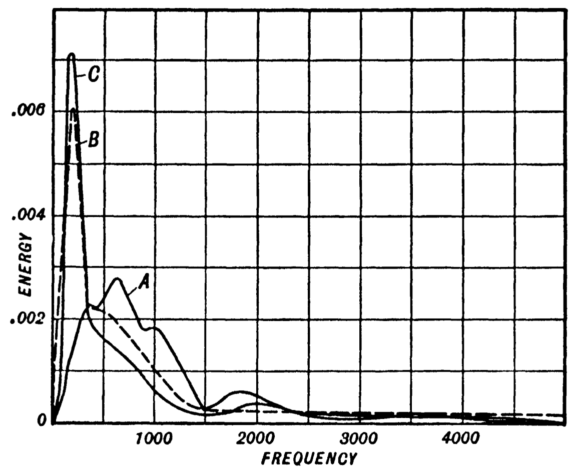


Fig. 9.2

The sentence was uttered slowly syllable by syllable, in order to give time for the instruments to be read for each syllable. This method was not wholly satisfactory and will be subsequently improved.

In Fig. 9.2 are shown certain energy distributions. *A* was obtained by assuming Professor Miller's results for vowels; *B* was obtained experimentally by disconnected speech sounds, and *C* by connected speech sounds.

The curves *A* and *C* do not agree. The difference should of course be contained in the consonants, but this explanation appears to the above workers as not sufficient. It is evident that the difference may be explained qualitatively by the variable character of the vowels and the incorrectness of assuming that the sustained vowels give the correct energy distribution.

These results, while giving the actual distribution of energy, do not give the relative importance of the different frequencies in clearness of speech, an important problem which is considered in the next section.

9.2. Useful Energy Distribution. — Dr. Harvey Fletcher has presented the first report * on the ability of the ear to interpret speech sounds under different conditions of loudness and of distortion caused by the elimination of groups of frequencies. This is a direct method of determining the useful energy distribution. Inasmuch as the actual pronunciation of a vowel occurs in sylla-

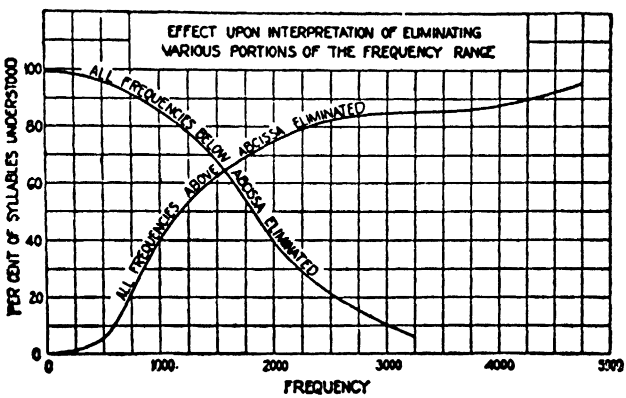


FIG. 9.3

bles, the tests were made with selected syllables. These were prepared in lists of 50, in which occurred a distribution of the three types, vowel-consonant, consonant-vowel, and consonant-vowel-consonant. In all 8,700 syllables were chosen. Fig. 9.3 shows the results of articulation tests in graphical form. The abscissae, or the horizontal distances, represent the *limit* of the frequencies used and the ordinates the percentage of correct judgments as to the sounds received. In the curve having its begin-

* *Bell System Technical Journal*, Vol. I, July 1922.

ning at 100%, for the frequency of 1,000 the articulation is 86%. The interpretation is that if all frequencies below 1,000 were eliminated, 86% of the syllables could be correctly understood. Again, using the same curve, it is found that 40% of the syllables were understood if all the frequencies less than 1,950 were eliminated. The interpretation of the other curve is similar but the elimination of frequencies is above the limit instead of below. On this second curve 40% of the syllables were understood if all the frequencies above 1,000 were eliminated. From the two graphs, the following additional conclusions may be derived: (1) A system which eliminates all frequencies above 3,000 has as low a value of articulation as one which eliminates all frequencies below 1,000 cycles per second. (2) At a frequency of 1,550, or in the third octave above middle *C*, the importance of the frequencies higher than this value is as great as that of the frequencies lower than this value, the limit of consideration being 5,000.

Several of the conclusions of Dr. Fletcher concerning the usefulness of speech sounds are as follows:

1. The short vowels, *u*, *o*, and *e* are seen to have important characteristics carried by frequencies below 1,000.

2. The fricative consonants *s*, *z*, and *th* are affected by elimination above 5,000.

3. The fricative consonants *s* and *z* are not affected by the elimination of frequencies below 1,500.

4. The sounds *th*, *f* and *v* are the most difficult to hear and are responsible for 50% of the mistakes of interpretation. The characteristics of these are carried principally by the very high frequencies.

One of the practical applications of the information gained concerning speech sounds is in long distance telephony. In designing these telephone message circuits it is important that a frequency range be selected within which the transmission shall be made as excellent as possible. At the present time in America the range selected * is between 250 cycles and 2,750 cycles, with the endeavor to make the transmission for these limits attain a

* Martin, *Bell Sys. Tech. Jl.*, IX, p. 483 (1930).

certain standard. The loss in transmission at 250 and 2,750 cycles is more than that 1,000 cycles, but by an amount that is not strikingly noticeable to the ear. The extension of the above stated range by a few hundred cycles evidently cannot, as shown in the foregoing figure, materially improve the understanding of telephone messages. But if, in the future, much more perfect station instruments are produced, more naturalness will be attained and largely because of the better reproduction of the fricative constants.

9.3. Speech Energy. — Crandall and MacKenzie,* using fifty syllables spoken in a normally modulated voice by six different speakers (four men and two women) give 125 ergs per second as the average acoustical output. Sabine † using a very different method obtains for certain vowels an output varying from 271 to 70 ergs per second. An acoustical output of 100 ergs per second would be 10 millionths of a watt or 10 microwatts. The non-technical reader can compare this value with the lamp at his study table, rated at 40 watts. The actual power output in speech is approximately two ten-millionths of that of a 40 watt lamp.

Interesting experiments have been performed to ascertain the relative amount of flow of energy or power occurring in conversational speech. Sacia and Beck ‡ have presented their results in the form shown in the accompanying Table IV.

The power during the existence of a given sound is averaged and this is called the "mean power." The "peak power" is the average of the maximum value for two speakers. As may be seen the vowels rank the highest, the semi-vowels next and the consonants the lowest.

* Crandall and MacKenzie, *Phys. Rev.*, 19, 1922, p. 221.
† P. E. Sabine, *Phys. Rev.*, 22, 1923, p. 303.
‡ *Bell System Tech. Jl.*, V (1926), p. 393.

Table IV

Speech Sound	Key	Relative Power, Arbitrary Units	
		A Mean Power Conversational Values for 16 Speakers	B Peak Power Normal Values for 2 Speakers
ò	talk	1870	688
a	top	1380	1430
ō	tone	875	630
ā	tape	808	632
e	ten	664	975
o	ton	616	688
ū	tool	532	344
ē	teem	484	402
r̄	err	384	—*
à	tap	366	2170
i	tip	346	688
n	no	84	78
m	me	74	185
sh	shot	73	192
ch	chat	58	87
s	sit	38	51
z	zip	29	52
j	jot	19	41
ng	ring	14	162
k	kit	14	10
l	let	13	218
t	tap	6	26
d	dot	3	7
f	for	3	6
v	vat	1	41
u	took	—*	688
zh	azure	—	63
dh	that	—	15
g	get	—	13
b	bat	—	11
p	pot	—	11
th	thin	—	1

* The dash indicates that observations were not available.

QUESTIONS

1. Show from a consideration of Figs. 9.1 and 9.2 that a man's clearness of enunciation of vowel is less than a woman's.

2. What other factors are involved in the production of superior clearness of speech?

3. What is a striking illustration of the fact that clearness of speech may depend upon very small sound intensities?

CHAPTER X

AUDIBILITY

10.1. Energy Required for Minimum Audibility. — There have been many observations made of the sensitivity of the ear and a summary of results and methods would be impracticable. The results of Wien,* Kranz † and Fletcher and Wegel ‡ are the most reliable, though they differ by much more than can be accounted for in the difference of the observers. The number of ears used in these investigations were, respectively, 3, 14, and 72. If the results are weighted by these numbers and the means taken, the best values of minimum audible pressure, expressed in dynes § per sq. cm., are found to be:

Frequency	64	128	256	512	1024	2048	4096
Threshold pressure	0.12	.021	.0039	.001	.00052	.00041	.00042

These values show that the threshold value of pressure (or the minimum audible pressure) is fairly constant above 1,000 vibrations per second and increases rapidly at lower frequencies. At an octave above middle *C* the pressure is approximately .001 dynes per sq. cm. This is a region of important frequencies and may be regarded as the most representative single value of the sensibility of the ear. From these values and a knowledge of the plane waves, it can be shown that the minimum rate of energy flow through a square centimeter in such a wave is of the order

* Wien, *Arch. f. ges. Physiol.*, 97, p. 1, 1903.

† Kranz, *Phys. Rev.*, 21, p. 573, 1923.

‡ Fletcher and Wegel, *Phys. Rev.*, 19, p. 553, 1922.

§ In expressing the dynes per sq. cm. of a pressure that varies periodically, positive, zero, negative, etc., one cannot give the average pressure for that would be zero. But since the intensity of the sound varies with the mean square of the pressure, it is customary to refer to the pressure of a sound wave as the square root of this mean square. This gives a value proportional to the square root of the intensity and proves convenient. The maximum pressure can be obtained by multiplying this value by $\sqrt{2}$.

4×10^{-16} (or 4 divided by the number ten raised to the sixteenth power) watts or 4×10^{-10} microwatts.*

10.2. Limits of Audibility. — The limits of audibility can be best expressed in the form of graphs. Fig. 10.1 is taken from the work of Fletcher and Wegel.† The full-line portion of the lower curve is a plot of the observations of minimum audibility mentioned in the foregoing paragraph. The ordinates represent the square root of the mean square of the pressure from the normal. The units at the right are explained in Section 10.6.

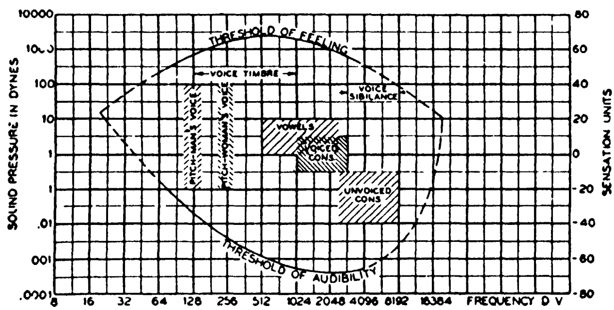


FIG. 10.1. Any sound that can be heard lies within the field outlined here. Areas covered by the most prominent speech sounds are indicated.

The upper curve represents experiments on 48 normal ears and is regarded as the "maximum" because at these pressures there is a sensation of feeling which limits the pressure used in any hearing device. This statement assumes that the sensation of feeling is approximately the same in abnormal ears, and hence it can be only an approximation, which varies with the nature of the abnormality. It is interesting to learn that the intensity for feeling is about equal to that required to excite the tactile nerves in the finger tips.

* The square root of the mean square of pressure in dynes can be shown to be equal to $\sqrt{20.5 \times E}$, where E is the flow of energy in ergs per sq. cm. per sec. and to $\sqrt{20.5 \times 10^7 \times E}$, where E is the flow of energy in watts per sq. cm., E being the flow of energy produced by a plane wave having this pressure.

† See Wegel, *Bell System Technical Journal*, Vol. I, No. 2, 1922. The form used in the figure is that suggested by J. C. Steinberg of the same laboratory.

The full-line curves have been extended as dotted curves and made to meet at approximate values of the frequency limits of audibility. The lower value, 20 vibrations, is really a high value selected because there is not an agreement as to the smallest frequency that can be recognized as a tone. The upper limit, here represented as 20,000, varies distinctly with age, being higher for the young and becoming lower with age. It has not been demonstrated whether this change is pathological or physiological. The cross hatched areas show the ranges of pressures and frequencies for the sounds indicated. The important frequencies in voice timbre and sibilance are also shown.

10.3. Deafness Defined in Dynamical Units. — These authors have found that in the region up to 4,000, persons of normal hearing require a pressure variation of approximately 1/1,000 dynes per sq. cm. for audibility. Persons called "slightly deaf" require a pressure of 1/10 dynes per sq. cm. A person requiring 1 dyne per sq. cm. can usually follow ordinary conversation. Those who require 10 dynes per sq. cm. need artificial aids to hearing.

In this connection it is also interesting to note that even if sounds could be amplified indefinitely, yet there is a limit to amplification because one cannot use an intensity great enough to cause pain. Hence only the deaf that have a range of pressure sensitivity between 1,000 and 1/1,000 dynes can be made to hear through artificial aids such as amplifying tubes and microphones. The above comments do not consider the totally deaf who by definition cannot be made to hear. Also, the conclusions as to painful effect have been obtained by a study of those not deaf and hence the numerical limits here fixed are open to question.*

10.4. Types of Deafness. — That deafness may have many causes has been well known to otologists but only in recent years have attempts been made to study types of deafness by actual

* Reger, of the Psychological Laboratory at the University of Iowa, reports that the congenitally totally deaf as well as those who have lost their hearing and vestibular reaction due to cerebral meningitis, experience the sensation of feeling at approximately the same sound pressure at various frequencies as do individuals with normal hearing. He believes that the sensation arises from the stimulation of pain receptors within the tympanic membrane.

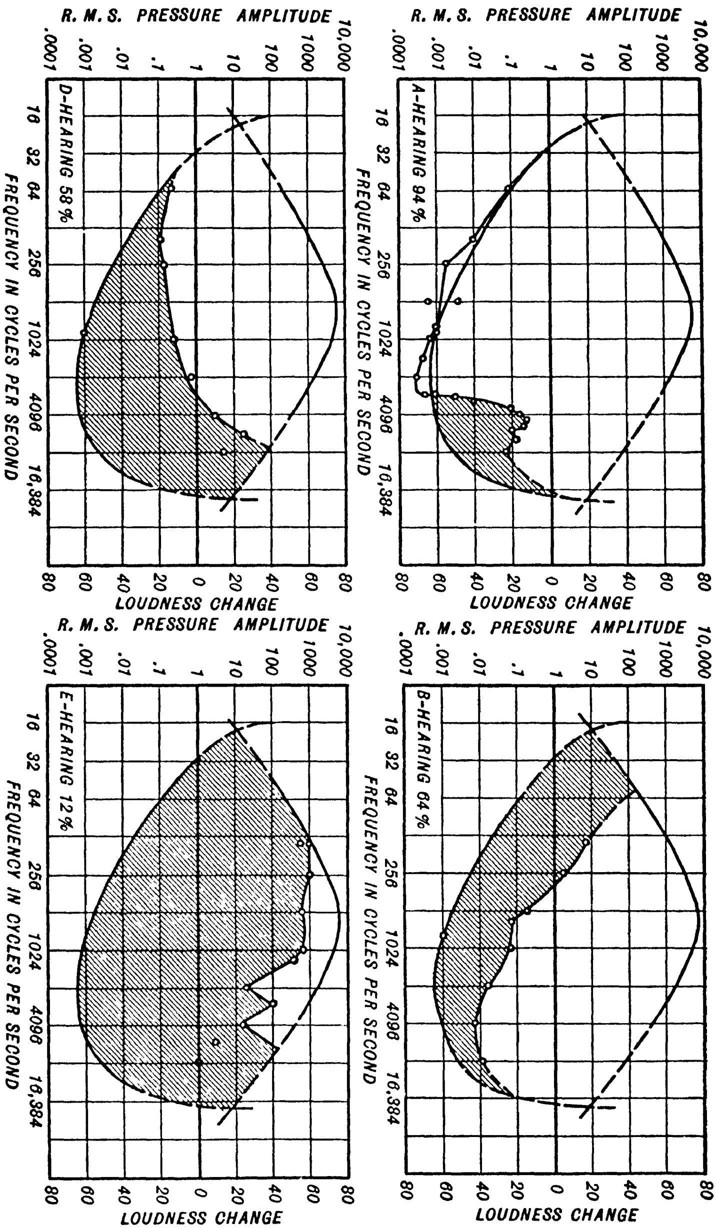


Fig. 10.2. Audiograms for typical cases of deafness

measurement of hearing. In Fig. 10.2 are represented * four curves of ear tests, plotted as in the original data. As will be readily seen the four cases vary widely. The ordinates on the right represent the relative loudness change as observed by a normal ear. The upper and lower curves in each case correspond to the similar curves in Fig. 10.1. A change of 10 in loudness means that there is an apparent change of 10 times the loudness.

10.5. Loudness. — One of the most interesting contributions to this field is that of Professor Wallace C. Sabine.† He used several different musical instruments, obtaining an equality of loudness for the seven frequencies shown in the accompanying Table V. In the second column of this table is shown an arbitrary number obtained for each frequency at equal loudness by dividing the actual intensity by the minimum intensity required for audibility of that frequency. This column therefore represents in each case the actual intensity in terms of the minimum audible intensity for that frequency. For a frequency of 64 the intensity is 0.7×10^6 times the minimum audible intensity for that frequency.

Table V

Frequency	Intensity Ratio
64	0.7×10^6
128	17. "
256	40. "
512	110. "
1024	133. "
2048	10.6 "
4096	.048 "

The application of these comparisons is perhaps not at once evident. Assume that one is listening to an airplane, flying first near by and then at a distance. Assume that when it is in the near position a number of frequencies are heard, apparently of

* These are from curves shown at Philadelphia by Dr. Harvey Fletcher, in 1923. See "Speech and Hearing," D. Van Nostrand Co., p. 200.
† Collected papers in "Acoustics," p. 129.

equal loudness. What will occur when the airplane recedes to a distance, assuming, for the moment, that the intensity of each frequency will decrease inversely as the square or any other power of the distance? The intensity of each frequency will be cut down to the same fraction of its original intensity. If this fraction is small enough some of the frequencies will be thereby cut down *below audible intensity* and *cannot be heard.* Hence the quality of sound from the airplane will change. Suppose the fraction is 1/100,000. Divide each number in the second column by 100,000 and it is seen at once the intensity of the 64 and the 4,096 frequencies have fallen below the intensity of minimum audibility in each case. If the fraction is 1/10,000,000, it is easy to see that the 128, 256 and 2,048 cannot be heard. As the fraction gets less and less, additional frequencies will disappear. The last frequencies to be heard will be in the region of 1,024. We have here, then, a clear explanation of the change of the quality of sound with distance only. As hereinbefore stated, diffraction and reflection are not discussed. They have independent influences in the variation of quality with distance. Steinberg* has shown that if a component tone has less than its own threshold pressure, it will not add to the loudness of the complex tone. Thus, in harmony with Sabine's results just stated, a complex sound can be heard no further than its most persistent component can be heard alone. It is obvious that if the entire energy available be expended in a tone of just one frequency, it can be heard further than the same energy distributed in any combination of frequencies whatever. This accounts for the apparently different carrying qualities of sounds from various sources. These considerations are, however, not the only ones applicable when other masking sounds are present.

10.6. Weber's Law — Fechner's Law — Sensation Units. — There is a general law in psychological literature that evidently is a law of the nervous system and is independent of the nature of peripheral organs. It may be stated as follows. The least

* Steinberg, *Physical Review*, 2, 26, p. 507 (1925).

detectable change in any stimulus is proportional to the intensity of that stimulus. This law of Weber has been verified in many directions, such as hearing, vision and feeling. In acoustics we are interested to know the relation between intensity and loudness. Although not an exact statement, a reasonably accurate guide has been found in assuming the sensation of loudness to be proportional to the logarithm * of the intensity in the stimulus.

This logarithmic law is usually known as Fechner's law and is expressed as follows:

$$S = c \log_{10} p + a.$$

Here S is the sensation or loudness, p is the pressure and a and c are constants for any one frequency. For frequencies varied from 100 to 4,000, c changes not more than 10%. MacKenzie † has performed experiments on the relative sensitivity of the ear at different levels of loudness and his results confirm the correctness of Fechner's law. He found that c is a constant. Now loudness should be measured in units so that twice the sensation of loudness would be indicated by twice as many units. It is observed in the preceding equation that, since the sensation of loudness varies with the logarithm of pressure, one cannot use for units of this sensation, units of pressure. A new unit is desirable and the literature is gradually adopting as a definition of *sensation units*, $S = 20 \log_{10} p$. The value of S is said to be in "decibels." The sensation level, "SL," of a tone is defined by

$$SL = 20 \log_{10} p/p_0,$$

where p_0 is the threshold pressure. The sensation level is really the number of sensation units in decibels that are required to reduce the tone to the threshold limit. A difference of one decibel

* The logarithm is a convenient measure of a number. The number 10 raised to the fourth power is 10,000. The logarithm of 10,000 is then 4, if the logarithm be written, $\log_{10} 10{,}000$. Also $\log_{10} 10 = 1$, $\log_{10} 100 = 2$, $\log_{10} 1000 = 3$. Likewise every number has a logarithm. It is noticed that the logarithm changes very slowly as compared to the number itself.

† MacKenzie, *Phys. Rev.*, 20, 1922, p. 331.

in a sensation level may be produced by approximately twice the minimum perceptible difference in intensity described in the next section. The range of speech sounds is usually from 40 to 60 decibels, an average whisper four feet away is 20 decibels, a rustle of leaves in a gentle breeze is 10 decibels, and very loud sound, almost painful, is about 100 decibels, in each case the number of decibels referring to the number required to reduce the tone to the threshold limit.

10.7. Minimum Perceptible Difference in Intensity. — The most comprehensive results of minimum perceptible difference have been presented by Knudsen * and may be expressed as follows:

SL	5	10	20	30	40	50	60	70	80	90	100
ΔE...........	0.4	0.23	0.14	0.12	0.11	0.106	0.102	0.10	0.10	0.10	0.10

Here SL is the sensational level or the number of units as above defined required to reduce the tone to the threshold, ΔE is the minimum audible increment in intensity, and E is the intensity. If Weber's law above stated were true, $\frac{\Delta E}{E}$ would be a constant. Wien in 1888 first showed that this could not be the case. Knudsen now shows that at the same loudness or sensation level, the ratio is practically independent of frequency, varying only 10% with frequencies of 100 to 3,200 cycles. Macdonald and Allen † have recently published data showing a similar variation of $\frac{\Delta E}{E}$ with E.

10.8. Audibility of a Tone Affected by a Second Tone: Masking Effect. — The experiments in this subject at the Bell Telephone Laboratories ‡ have been very extensive and have an im-

* Knudsen, *Phys. Rev.*, 21, p. 84, 1923.
† Macdonald and Allen, *Phil.*, May 9, p. 827, 1930.
‡ Wegel and Lane, *Phys. Rev.*, 23, p. 266, 1924.

portant bearing upon the theory of hearing. The phenomenon can be appreciated through a brief study of the results obtained by using a constantly sounding tone of 1,200 cycles and observing its masking effect upon other frequencies, the same ear being used. A definite arbitrary measure for "masking" must be given in order to express the results quantitatively. The investigators, Wegel and Lane, have selected the following: If without the masking tone, the threshold pressure of a given frequency is p_1, and with the masking tone it is p_2, then the measure of masking is $\frac{p_2}{p_1}$. In the accompanying Fig. 10.3 the three curves refer to

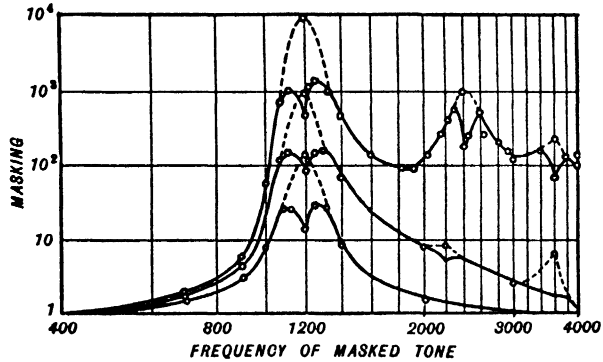


Fig. 10.3

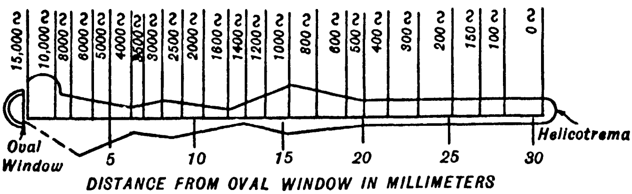
the results with three intensities of the tone having a frequency of 1,200; namely, 160, 1,000, and 10,000 times its minimum audible intensity. The ordinates are $\frac{p_2}{p_1}$ as already described. Several points are to be noted:

1. There is a decided masking which increases as the frequency of the masking tone is approached.

2. The masking effect increases with the intensity of the masking tone.

3. At higher intensities the masking occurs as if the additional tones of 2,400 and 3,600 were present. This indicates that those tones have been created by the ear itself, and, as will be explained at a later point, this is actually the case.

The authors just quoted have utilized the study of masking in a consideration of the frequency regions on the basilar membrane and their conclusions may be represented in the accompanying drawing, Fig. 10.4. As it is not the purpose of this text to discuss the anatomy of the ear, no explanation will be offered other than in the figure itself. Those who are not familiar with the ear are referred to the original article. The results in Fig.



Distance from oval window (Millimetres)

FIG. 10.4. Characteristic frequency regions on basilar membrane

10.4 should not be regarded as final but only as representing a stage in the progress of the establishment of a correct theory of hearing. It may be well to remark that there is at present no fully accepted theory of hearing. Several recent contributions to the discussion are contained in the account of a symposium of the American Acoustical Society in 1929.

10.9. Hearing in the Presence of Noise. — It has been repeatedly noticed that many people appear to hear better in the presence of a noise. The term used to describe this phenomenon is "paracusis." Knudsen and Jones * have removed much of the mystery in connection with the apparent better hearing of the "paracusic" in the presence of a noise. They find that, strictly speaking, the *acuity* of hearing is decreased by the presence of a noise for all persons of normal hearing and for all the partially deaf. Further, the individuals with impaired hearing of the perceptive † type do not hear as well in the presence of a noise.

* Knudsen and Jones, "The Laryngoscope," 36, p. 623, 1926.

† The terms "perceptive" and "conductive" are taken from the original paper. The former evidently refers to the usual means of hearing and the latter to the type where the vibrations reach the organ of hearing by bone conduction.

Paracusis is found only in the individual having hearing of a conductive type and usually one having a marked bilateral lesion, wherein the difficulty is the transmission through the bones of the ear. And in these cases the phenomenon occurs not because of increased acuity, but rather because of certain other advantages inherent in his impairments. For example, his partial deafness is chiefly in low tones, whereas his acuity is good in the region of most importance to speech, 1,000 to 2,000 cycles. In the presence of a low pitched noise, such an individual would enjoy a relative advantage in listening to conversation, when compared with the person having normal hearing. For the energy of noise is usually well below 512 cycles. Another advantage rests in the fact the lower tones are more important in the masking effect than the high ones. Hence, in the case of a noise covering a large range of frequencies, the elimination of the low tones by the individual mentioned would result in a noticeable relative advantage in the presence of such a noise.

10.10. Minimum Time for Tone Perception. — The data available on the minimum time for tone perception are discordant. Fletcher * states that at 128, 384, and 512 cycles the values of the time required for the perception of weak tones are those corresponding for 12.1, 24.1, and 29.6 cycles, respectively. These correspond to the time intervals, .0946, .0627, and .0579 seconds. For tones of medium strength, the time is noticeably reduced. But the phenomenon is more significant than appears upon the surface. The minimum time for tone perception may not have even the same meaning for different individuals. A tone is recognized as such only when it is identified as within a certain frequency range. With one individual this range may be larger than it is with another.†

10.11. Minimum Perceptible Difference in Frequency. — For frequencies between 500 and 4,000 the least perceptible difference in frequency is about 0.3 of one per cent of the frequency,

* "Speech and Hearing" (D. Van Nostrand Co.), p. 153.

† See Stewart, *Journal of the Acoustical Society of America*, II, 3, p. 325, 1931.

f, and is fairly constant. Calling this difference Δf, then this percentage is $\frac{\Delta f}{f} = 0.003$. At 260, 120, and 70, the values are 0.004, 0.006, and 0.009 respectively. These are reported by Knudsen.* The method was to sound one tone at a time. Just to what extent the least perceptible difference depends upon the length of time the tones are heard is not known. It is fairly safe to opine that this time duration is a factor which should be considered.

10.12. The Vibrato. — The interest of this paragraph is the audition of the vibrato rather than a report of present detailed knowledge concerning the vibrato itself. *Vibrato* and *tremulo* refer to effects which can be produced by the voice and by some musical instruments. The former appears to the ear to be a fluctuation of intensity of about 5.5 to 8.5 times per second. It occurs in the natural singing voice and Metfessel † states that the vibrato is a cycle of frequency variation with an average rate of seven cycles per second and an average extent of a musical half-tone. The ranges of variation of the number of alterations of frequency is from 5.5 to 8.5 per second. The range of the variation of the frequency of the tone is from a tenth to a whole tone. These results were obtained by tests made with the voices of over forty artists. A greater variation of frequency and intensity than found with artistic vibratos may be called a tremulo. Now one of the interesting points concerning the vibrato is that although the frequency alteration is well within the ability of the ear to detect under more favorable circumstances, yet here, with the rapid alteration of about 14 times per second, the ear cannot detect a change in frequency.‡ Evidently this is in part caused by the brevity of the time duration of the tones. Another curious result is that the change in intensity seems to be marked, indeed much more so than accountable by a direct intensity

* *Physical Review*, XXI, 1 (1923).

† M. Metfessel, Abstract, Acoustical Society of America, meeting December 13, 1930.

‡ Tiffin, Joseph, *Psychological Monographs*, 1931, 14.

effect. Whether or not this is explained in the nature of the ear response is not known.

The tremulo is similar to the vibrato except that the variation in frequency is large enough to be heard. Apparently it presents no new problem of the kind just described.

10.13. Loudness of Complex Sounds. — A study has been made of the absolute loudness of a complex sound and Steinberg * finds that the loudness can be expressed in the form of an equation the factors in which have been determined.

10.14. Combination Tones. — There are two chief combinational tones, called "summation" and "difference" tones. If two tones, having frequencies of n_1 and n_2, are sounded, the summation tone has a frequency $(n_1 + n_2)$, and the difference tone a frequency $(n_1 - n_2)$. If, on a piano, middle C or C_3 (see Section 14.4) and F_3 are struck vigorously, the tone F_1 can be heard. This is a *difference* tone. If the keys F_1 and C_2 are struck, the *summation* tone is that practically of A. The reader is referred to Barton † for a detailed description of the best method of bringing these tones into evidence.

10.15. Frequencies Introduced by Asymmetry. — If a vibrating body is displaced, it exerts a restoring force. It may be said that there is symmetry if the magnitude of this force is independent of the direction of the displacement and dependent only upon the amount. If there is asymmetry this is not the case, and the restoring force will change in *magnitude* if the displacement changes in sign only. It can be shown ‡ that in such a vibrator the fundamental will *always* be accompanied by all the overtones which are integral multiples of the fundamental. Also, mathematical analysis shows that if the restoring force is made proportional to the square of the displacement, and thus asymmetrical, the response will not consist merely of vibrations corre-

* Steinberg, *Phys. Rev.*, 25, 1925, p. 253.

† Barton, *loc. cit.*, articles 297 to 301.

‡ Barton, *loc. cit.* There are many asymmetrical vibrators. With this one, the restoring force depends upon both the amplitude and its square.

sponding in frequencies to those of the impressed forces. If this vibrator is set in motion by two forces having frequencies of p and q, there result vibrations * having the following frequencies: p, q, $(p + q)$, $(p - q)$, $2p$ and $2q$. But the amplitudes of the four additional tones are of peculiar interest, for they increase rapidly with increasing amplitudes of the primary vibrations p and q. The bearing of this conclusion of the assumed case upon the hearing of combinational tones is quickly understood. The drumskin of the ear is an asymmetrical vibrator and it is not surprising that, in case frequencies p and q are sounded with sufficient intensity, the summation tone $(p + q)$ and difference tone $(p - q)$ will be heard.

The condition of symmetry is not sufficient to cause a vibrator to give only that frequency which is impressed upon it. In addition, the restoring force must be proportional to the displacement. If the restoring force depends not only upon the first power, but also upon the cube of the displacement, then the vibrator, with an impressed force of frequency p will introduce the frequency $3p$.

10.16. The Ear an Asymmetrical Vibrator. — It has been previously stated that if two tones of different frequencies actuate an asymmetrical vibrator, there will be produced by the latter the summation and difference tones. This fact causes the ear sometimes to misjudge the frequencies present in a complex tone. Dr. Harvey Fletcher † has studied the criterion for determining the pitch of a musical tone. He found that if the frequencies, 100, 200, 300, etc., up to 1,000, each having the same pressure amplitude, were sounded together and then again with the 100 removed, no difference could be detected by the ear. Even if the elimination continued leaving only the 800, 900 and 1,000, the pitch seemed to correspond with 100 vibrations per second. In fact any three consecutive components gave this same pitch. If the components 200, 400, 600, 800 and 1,000 were used, the pitch corresponded to 200 cycles. Any consecutive three of these tones gave the tone of 200, but weakly.

* The statement as to frequencies represents an approximate solution.

† Fletcher, *Physical Review*, 23, 1924, p. 427.

One might conclude that the pitch of a musical tone is always determined by the common difference in the frequencies of the harmonics, but this is not correct. The combination 100, 300, 500, 700, 900, did not give a pitch of 200 but sounded more like a noise, but with the frequency 200 distinctly audible. The fact that components are multiples of a common difference seems to give the pitch with certainty.

These illustrations of the asymmetrical character of the ear lead to the conclusion that the quality of a complex sound as judged by the ear must depend not only upon the relative amplitudes of the components but also the actual amplitudes. For the asymmetrical character of the ear is brought into greater relative prominence by greater amplitudes of vibration. Hence if a complex sound is made fainter, but with the relative intensities of the components constant, the ear would notice a difference in quality. As the sound is made louder, the lower tones would become increasingly prominent. In this manner a loud-speaker which may give a faithful reproduction would, if sufficiently loud, appear to over-emphasize the lower tones. The asymmetrical nature of the ear introduces a difficult problem into the construction of acoustical instruments.

10.17. Use of Combinational Tones in the Organ. — The use of a combinational tone in a musical instrument actually occurs in the pipe organ. An open pipe 16 feet in length will give approximately 32 vibrations per second. But to give 16 vibrations per second a pipe 32 feet in length would be required. This is usually too long to install in the recess set apart for the organ. The tone is obtained by the combined use of two pipes of 16 ft. and $10\frac{2}{3}$ ft. acoustic length, giving 32 and 48 cycles. Thus there is obtained a combinational tone of 16 cycles. Apparently such practice is common.

But there is no reason why an organ pipe should be straight. The reflection at a bend in a pipe is caused by the fact that the successive wave fronts do not remain parallel and there is a certain amount of interference which depends upon wave-length.

The longer the wave-length, the less the interference and consequently the less the reflection at a bend in a pipe. Also, the larger the diameter of the pipe, the greater the reflection if the diameter of the pipe is small in comparison with the wave-length. This is shown in Fig. 12.4. It is possible, then, to have a pipe double back upon itself and thus reduce the length it occupies. This construction is now also utilized in organ building.

10.18. Pressure of Sound Waves. — Lord Rayleigh has shown that a plane wave of sound striking a wall perpendicularly will exert a constant excess of pressure determined by the formula,

$$p = 2E,$$

wherein p is the excess pressure and E is the energy per unit volume in the incident wave. This phenomenon has a bearing upon the preceding discussion, for the amplitude of the resulting waves when two frequencies of n_1 and n_2 are sounded fluctuates with a frequency $(n_1 - n_2)$. E in such a wave would fluctuate with this frequency and hence p also. Such a fluctuating pressure would cause any vibrator to respond with the difference frequency $(n_1 - n_2)$. But this pressure proves to be very small. Computation shows that $2E$ is about one millionth the value of the pressure of the incident wave, if the latter is one dyne. For less pressures in the incident wave the fraction is even less. Thus it is learned that although the difference tone $(n_1 - n_2)$ does have an objective existence in the air, yet the ear cannot hear it. It is usually referred to as a subjective tone. The hearing of the difference tone really depends upon the asymmetrical vibration of the ear.

10.19. Intermittent Tones. — If a frequency of n vibrations per second is interrupted u times per second, and if u is smaller than n, a frequency of u per second will be heard. If the amplitude of the n vibrations is varied u times per second, again the tone u is heard. F. A. Schultze * considers the case theoretically

* Schultze, *Annal d. Physik*, 26, 7, 1908, p. 217, and *Science Abstracts*, No. 1657, 1908.

and shows that there are objective tones present having the following frequencies: p, $(p - u)$, $(p + u)$, $(p - 2u)$ and $(p + 2u)$. From these may be formed the combinational tones u, $2u$, $3u$, etc., and $(2p + 2u)$, $(2p + 2u)$, etc. It is obvious that the frequency u may exist on account of the changes in mean pressure that occur with that frequency and on account of the asymmetry of the ear.

10.20. Intensity and Pitch of a Blend of Sounds. — Two tones of exactly the same frequency and the same phase at a point will give there an intensity four times that of one alone, for the amplitudes add and the intensity is proportional to the square of the amplitude. But suppose there are n such tones, then the intensity would be proportional to n^2. But only with extraordinary care could such a result be achieved. n violins playing in an orchestra must give phases at random at the auditor's ear, even if the frequencies are identical. The late Lord Rayleigh * has shown that in such a case the intensity is not proportional to n^2 but to n. It can also be shown that if there are a number of tones of nearly the same frequency and of approximately the same amplitudes, the resulting tone will be judged by the ear as having a pitch which is an approximate mean of the two extremes. The hum of a swarm of bees is an illustration.

QUESTIONS

1. What physical phenomenon may sometimes lead to an incorrect impression of the pitch of the lowest tone present?
2. What frequencies will an asymmetrical vibrator introduce when a pure tone is sounded?
3. In what case will a sound wave exert an excess of pressure?
4. What personal experiences can be explained by Sabine's work on loudness?
5. Explain why one tone of a frequency will "carry" farther than a complex tone of the same total acoustic flow of energy.
6. The frequency stated for least audible pressure, 2,048 cycles, as indicated in Fig. 10.1 is not in numerical agreement with the second column of Table V. Show that there is not necessarily a discrepancy.

* Rayleigh, *Scientific Papers*, Vol. 3, p. 52.

CHAPTER XI

BINAURAL EFFECTS

11.1. Binaural Intensity Effect. — There is no doubt but that difference of intensities at the ears is a factor, though small, in the auditor's location of a source of sound. It is the purpose of this section to state what sense of direction of the source is given by intensities at the two ears. Stewart and Hovda * discovered that there was a very precise mathematical relation between the *ratio of the intensities* at the ears and the apparent direction of the source. If one draws an imaginary plane midway between the ears and the perpendicular to the line joining them, then the apparent position of the source in a horizontal plane may be described by the angle its direction makes with this median plane. If this angle be denoted by θ and the intensities at the ear by I_1 and I_2, then the technical mathematical statement is

$$\theta \propto \log \frac{I_1}{I_2}. \tag{11.1}$$

This states that the angle, θ, between the median plane and the direction of the sound is proportional to the logarithm † of the ratio of the intensities at the ears. If I_1, the intensity at the right ear, is the greater, then θ is measured to the auditor's right of the median plane. If I_2 is the greater, the angle θ is to the left.

In obtaining the above results the apparatus was arranged so that the sound involved was a pure tone emitted by a tuning fork and there was *no difference of phase* at the ears.

From the data not here reproduced the following conclusions may be derived:

* Stewart and Hovda, *Psych. Review*, XXV, No. 3, 1918, p. 242.

† See reference in Section 10.6. In (11.1), the form, $\log_{10}$, could be used, but the equation would remain true if another number than 10 were used. So it is left without such a number indicated.

(1) Formula (11.1) was found correct for several observers and for frequencies of 256, 512, and 1,024.

(2) The binaural intensity effect does not account for a hearer's ability to locate a source of sound for to produce a certain apparent θ in the experiment, a much greater ratio of intensity is required than can exist in an actual case with the head casting an intensity shadow for the sound from a distant source.

Later experiments * were performed with 16 observers and with frequencies from 200 to 2,000. Four of the observers were tested at frequencies up to 4,000. These experiments were performed in such a manner that they did not show the accuracy of equation (11.1), but they did clearly indicate the frequencies at which the intensity effect did not exist at all. Some of the conclusions of these experiments are:

(1) With ten of the sixteen observers, the binaural intensity effect ceased to exist throughout one or more bands of frequencies. (When it ceased to exist, the source of sound appeared to remain directly in front of the observer, though the intensity ratio changed within wide limits.)

(2) These frequency bands all occurred above 800 cycles and were of different widths.

(3) In certain frequency regions there appeared to be two sources of sound, one stationary in front and one moving about with changes in intensity ratio.

Table VI

Observer	Lapse range	Observer	Lapse range
SC	none	EMB	none
ERK	1150–1250	AETF	850–1250
CEL	1450–3950	CKK	none
RH	none	HMH	all
GWS	850–1150	ML	1450–2100
EGR	none		2350–2940
ACR	1570–	IK	none
GRW	950–1850	CRB	850–1250
			2000–4000
		ES	1450–1850

* Stewart, *Phys. Rev.*, Vol. XV, No. 5, 1920, p. 432.

(4) There is a wide variation among individuals in regard to the above.

Table VI shows the results so far as the absence of the intensity effect is concerned.

11.2. Binaural Phase Effect. — By "binaural difference of phase effect" is meant the alteration of the angular displacement from the median plane of the apparent source of the fused sound when varying differences of phase of a given frequency are presented at the ears and the intensities are kept constant and equal. This "effect" has been known for a number of years; a review of the early literature is given in the *Physical Review*, IX, 1917, p. 502.

The experiments recorded in the article just cited show clearly that the angular displacement of the apparent source of the fused sound or "image" is strictly proportional to the phase difference at the ears, with, of course, the limiting provision that the linear relation is true only for a difference of phase, φ, less than 180°. (See discussion of phase angle in Section 3.7.) At $\varphi = 180°$ the image crosses from the maximum angular displacement on one side of the median plane to that on the other side. The experimental procedure was to ascertain this linear relation between θ, the angular displacement, and φ, the difference of phase, for a single frequency. If, as stated, θ is proportional to φ for any given frequency, then the interest centers upon the variation of $\frac{\varphi}{\theta}$ (which is constant at one frequency) with frequency. The accompanying three curves in Fig. 11.1 show the results obtained with three different observers. The circles and dots in the observations for the upper curve indicate two different observational methods. The observations for the other two curves are shown by squares and triangles.

The conclusions * from these curves are the following:

* The binaural phase effect is being actively studied by both psychologists and physicists, using both sustained and impulsive sounds. It is impractical to present here all of the important results. While all workers may not be wholly in accord with the discussion in the text, the author believes these data to be reliable and the impression conveyed essentially correct. The data have been verified by others.

(1) There is not a wide variation in individuals.

(2) If these straight lines went through the point "O," it could be stated (not here proved *) that the apparent position of the source indicated by θ is dependent only upon the difference in time of arrival of like phases at the ears. It can be claimed, therefore, that this conclusion as to time difference is approximately correct.

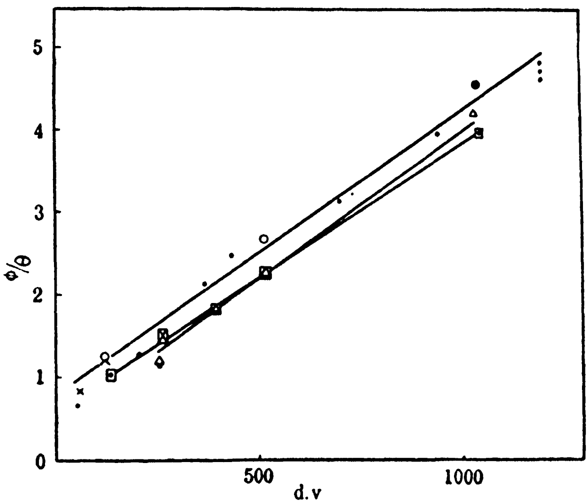


FIG. 11.1

(3) A consideration of the computed phase differences at the ears with the source of sound at any given θ shows that the above quantitative measurements fully account for the ability of the individual to locate the source of sound in the limited region discussed.

The second of the foregoing conclusions leads to interesting considerations. If a source is actually placed at an angle θ from the median plane, and if it emits several frequencies, all of these will have the same difference in time of arrival at the ears and hence, according to the second conclusion above, all would appear

* See "Acoustics," Stewart and Lindsay, p. 229.

to the auditor to come from the same direction. This is in accord with experience.

The third conclusion is very significant. It has been shown above that the binaural intensity effect cannot account for the ability to locate sounds. It is now shown that the binaural phase effect can do so in the limited region here discussed. There are limitations to this conclusion as will now appear.

Measurements of the frequency limit of the phase effect was made upon 16 observers. The values of frequency above which no phase effect existed are only approximate and are shown in the accompanying Table VII. Above each frequency limit there was no rotation whatever of the apparent source about the head with changing phase. Only frequencies less than 2,000 cycles were used.

Table VII

EMB	1360 d.v.
AETF	1335
CKK	1119
HMH	1474
ML	1767
IK	1249
ES	1392
CRB	1333
SC	1161
ERK	1146
CEL	1058
RK	1145
GWS	1248
EGR	1151
ACR	825
GRW	1393
	Mean 1260

There are two striking indications to be found in the table. The first is that the frequency limit is approximately the same for all individuals, and second, that there are exceptional wide variations from the mean value. The average deviation from the mean is 155 cycles. Omitting two observers, it is only 110 cycles. This constancy has a distinct bearing upon the conclusion that phase difference is the most important factor in localization up to 1,200 cycles.

Subsequent experiments * have shown that the phase effect is not limited to frequencies less than 1,200 cycles. In fact, with a few selected observers of considerable experience the binaural phase effect has been found to exist at frequencies of several thousand cycles. Nevertheless the frequencies below 1,200 cycles are evidently important in localizing ability. Other factors may become prominent at higher frequencies. This point is discussed in a later section of this chapter.

Psychologists are familiar with the influence of intensity-difference upon localization and this phenomenon is subject to generally accepted principles. But the recognition of a phase difference at the ears with the two intensities equal means, it would seem, a response to a different and more intrinsic feature of the stimulus. The suggestion that we have here to do with a response to the character of the stimulus will doubtless be regarded with skepticism, and in fact an attempt has been made by some to explain the "phase effect" in other terms. Fortunately it has been possible to get what seems to be conclusive evidence that any explanation in terms of *physical intensities at the ears* cannot be correct. This evidence is direct and readily understood. It has been shown above that with some individuals there are frequency regions or bands wherein the observers are not influenced in their localization by variations in the ratio of intensities at the ears, phase-difference remaining constant. With them, in this "lapse-region," the apparent source of the fused sound remains stationary in the median plane when the ratio of intensities is altered widely. But the significant fact is that with six † of sixteen observers *the "phase effect" is continuous in at least a portion of this lapse-region*. In short, the phase-phenomenon seems to be independent of the intensity displacement effect. The evidence may be found in the comparison of Tables VI and VII. For

* Halverson, *American Journal of Psychology*, 38, p. 97, 1927. There are also unpublished results by others.

† The reason for only six is that the lapse-regions were too high to be within the frequency limit of the phase effect. The phase effect was always entirely independent of the lapse-region.

example, note that G. W. S. and G. R. W. have the phase effect in a region where the intensity effect is entirely lacking.

11.3. Phase Effect with Complex Tones. — As shown above, the phase effect is effectively a difference in time of arrival at the ears, and hence the angular displacement is *independent of frequency*. This means that, so long as the phase difference of any overtone does not exceed 180°, all the tones will have the same angular displacement and hence there will be no confusion as to location. (A confusion in a horizontal semicircle only is being discussed.) This leads to the utilization of the phase effect.

11.4. Utilization of the Binaural Phase Effect. — The binaural difference of phase effect was utilized during the war for the location of submarines and of airplanes. Obviously, if attachments can be made to the ears which will virtually separate them further, then a small rotation of the apparatus will mean a larger difference in phase at the receivers than at the unaided ears. Thus a very high accuracy may be obtained. A few observers seem to locate the source of sound in the rear instead of the front, but this does not vitiate the method.

11.5. Complexity of Factors in Actual Localization. — One might conclude that, since the only physical factors in a pure tone of a given frequency are phase and intensity, and since we have all but eliminated intensity as a factor in localization below 1,200 cycles, the only important factor left is that of phase difference. But this cannot be true in the sense that the phase difference is produced merely by a single source and the diffraction about the head as a sphere. For there are always present reflecting surfaces which are extensive enough to produce images. This is especially true of frequencies higher than 1,000, having a wavelength less than 34 cm. Although the effect at the opening of the external meatus is still expressible in phase and intensity, yet, in contrast to the case of a simple source, we have, in general, the equivalent of several sources, with most of them on the same side of the median plane as the source. There would result from these reflections an apparently diffused source of sound instead of the original source only. Consequently the observer could distinguish between a location on the right and left side of the median plane. Thus, assuming that phase difference is the most important factor in localization, it by no means follows that the case is as simple or needs to be as simple as that of a source and a rigid sphere with the two ears located diametrically thereon. The complexity of conditions involving reflection gives the single factor, phase difference, a greater opportunity to secure accurate location than did the simple theoretical case. Doubtless there are other factors which enter into the localization of a pure tone. There are at least two additional ones in the case of a complex tone; they are the difference in quality at the two ears produced by diffraction, and the variation in the quality of a sound that depends upon the location of the source in the particular environment; an example of the latter is one's ability, in familiar surroundings in a home, to tell from which room a voice comes.

A suggestion has been made by Hartley and Fry * that the observer may have an appreciation of the distance of the source. Observers, however, agree that with pure tones no such appreciation exists.

It should be noted that the discussion in this chapter of localization of sound is limited to the apparent position of the source in a horizontal plane with θ not more than 90°. The whole problem of localization has therefore been merely touched upon.

11.6. Demonstration of Binaural Phase Effect. — If two tuning forks of almost the same frequency, producing a "beat" say every five to ten seconds, are held one to each ear, the hearer will observe the phase effect. The phantom source of sound, which seems to be a single source, moves continuously about and in front of the head in an approximately circular horizontal path, but does not complete the circle. Having reached a point almost directly opposite the ear it moves abruptly from one side to the other but continuously across in front of the observer.

* *Physical Review*, 13, 1919.

11.7. Binaural Beats. — There is an interesting phenomenon that may easily be observed. If two beating tuning forks are held one to each ear, the beats can be heard. In listening to two beating tones with one ear the combined intensity varies from a maximum to zero. With binaural beats the minimum intensity is distinctly not zero. Moreover, if one listens closely, he can hear two additional swells of intensity, one just before and one just after the minimum intensity. These additional or secondary maxima are present only if the beat period exceeds at least one second. The phenomena involved in binaural beats will, when fully investigated,* doubtless increase the understanding of audition.

QUESTIONS

1. Why cannot the phase effect be explained as an intensity effect?

2. Why cannot phase fully account for localization? Why cannot the phase and the intensity effect together fully account for it?

3. From the data in Fig. 11.1, compute the time difference that can be detected, assuming that one may notice the variation of θ from $0°$ if $\theta = 2°$.

* For a report on binaural beats with a record of new phenomena see Stewart, *Physical Review*, IX, No. 6, June 1917, p. 502, 509 and 514. The most interesting conclusion in these papers is, that the evidence points to the existence of a second and a different organ of hearing, the saccule. Recent experiments verify this conclusion. See also a later discussion by Lane, *Physical Review*, 26, p. 401, 1925, and by Stewart, *Journal of Acoustical Society of America*, April, 1930.

CHAPTER XII

ACOUSTIC TRANSMISSION

12.1. Transmission of Energy from One Medium to Another. — In previous chapters there have been discussed several cases of transmission where the medium remained the same and yet, because of the changes in the confinement of that medium, the energy transmitted was not 100 per cent. In Section 5.7 it was found that there was a reflection in a conduit at any sudden change in area, indeed, also when a conduit opened out into the unconfined atmosphere. In Section 6.9 the reflection at the open end of megaphones was mentioned. But the transmission of sound from a gas to a solid and from one solid to another involves new considerations. The physical factors entering the question of such transmission from one medium to another will now be described.

In the first chapter it was made evident that the transmission of a sound wave depends upon the elasticity of the medium and also upon the density. The former quality requires that the displacement return to zero value; it gives a return force which is essential in the process. The density indicates the existence of mass and hence of the requirement of time to produce a displacement. Both elasticity and density are requisite. Without either the wave would not be produced. It is therefore to be observed that in the value of the velocity of a sound wave in a fluid there are involved several physical factors. It may then not be surprising that in a plane wave, incident perpendicularly at a plane interface between two media, the percentage of energy in the transmitted wave depends upon the product of the density and the wave velocity in each medium. This product is called the "acoustic resistance," the second word being used also in electricity, but not in a closely analogous manner. Consider the plane interface, one medium being on the left and one on the right. Let the incident wave come from the left. Of the energy flowing to the right, part will be reflected at the interface and the remainder will be transmitted into the second medium. It is fortunate that the actual amount transmitted can be determined by the application of the following simple expression: *

$$\frac{\text{Transmitted flow of energy}}{\text{Incident flow of energy}} = \frac{4r}{(r+1)^2}.$$

Here r is an abbreviation for the ratio of the acoustic resistance in the second medium to that in the first.

It is also an important fact that this discussion can be extended to include solids if one limits the consideration to longitudinal waves. This would include the passage of sound from air to water and from water through a ship's hull.† It would be applicable to the transmission of sound from air to the ground and vice versa. But one must be warned that it is only applicable where the second medium does not act like a drum head or a diaphragm. In fact, in partitions and floors, diaphragm action is nearly the correct description, for it is found that mass is a very important factor. The actual sound entering a wall, treated as a medium as in this chapter, would be small indeed and this does not agree with the amount of transmission found in experience.

The following table gives the value of the acoustic resistance for several substances: ‡

Steel	40	$\times 10^5$
Cast iron	26	"
Brass	29	"
Lead	14	"
Water	1.5	"
Rubber	.029 –.066	"
Air	.0004	"

* For the derivation see Stewart and Lindsay, "Acoustics" (D. Van Nostrand), Chapter IV.

† Not precisely true, for the diaphragm action discussed in this section may not be disregarded.

‡ See Appendix I of Stewart and Lindsay, "Acoustics," p. 327.

As an example, if one applies the formula to the transmission from air to water, a very small fraction is found.

12.2. Transmission in Architectural Acoustics.* — From the foregoing one might correctly conclude that the structure of buildings to avoid transmission of sound from one room to another is not a very simple matter. Interesting experiments have been made with partitions in order to find the most economical construction that will give satisfactory acoustic insulation. The lighter the structure, such as in a home, the more difficult becomes the problem. For in a building requiring massive walls and floors, the inertia prevents diaphragm action. It is to be borne in mind that to reduce the sound transmission through a floor or partition there are chiefly two methods, absorption and rigid or massive construction. The absorption can be produced by the nature of the material itself or of its surface. This will prevent, through surface absorption, the transmission of sound, but the absorption on the interior is not so effective if the wall is light enough to vibrate as a diaphragm. Both mass and rigidity of a wall or floor will prevent transmission. An illustration of the use of absorption and rigidity is as follows. An ordinary wood floor in a home consists of the joists carrying two layers of flooring, the top one being the finished floor. Underneath the joists are the lath and the plaster. Not only will such a floor vibrate as a whole, but even the areas of flooring from one joint to another will have additional vibration. One way of producing a serious reduction of the transmission is to lay over the first flooring an absorbing material of perhaps an inch in thickness. Upon this, and separated by perhaps six inches, are laid 2 × 2's but without nailing. Then the finish floor is nailed thereto. Thus the finish flooring floats without any solid connection with the floor structure. The effect is very marked.

12.3. Machinery Noises. — Noises from machinery may be prevented by the removal of the cause, by absorption and by

* In this country there is an organization of physicists, acoustic engineers, architects, and construction engineers, the Acoustical Society of America, that is actively interested in all problems relating to architectural acoustics.

preventing the flow of energy from the machine to surrounding supports. Attention here will be devoted especially to the last method for the others are more obviously applicable. Assume that the desire is to prevent the 120 cycle hum of a motor from being conveyed to a support such as a table or floor. Everyone knows that a soft pad placed under the motor will be quite effective in preventing the transmission. This is not merely because of the absorption of the pad but also because of its elasticity. One can substitute a number of small springs and get a good effect also. This is because the springs cause a reflection of the energy, explained briefly as follows. The presence of the springs will allow the motor base to vibrate rather freely. There will then exist a condition much like that at the open end of a pipe, previously discussed. In other words the condition is one of a reflection. This will be particularly true if the springs are light and numerous, rather than heavy and few. The mathematical treatment gives a much better explanation,* but the chief point to be realized is that we are here dealing with a case of reflection of energy, rather than of absorption. Attention of engineers to the prevention of machinery noises is rapidly increasing.†

12.4. Case of Three Media. — Assume that we are dealing not with the transmission from medium one to medium two, but also from medium two to medium three, retaining perpendicular incidence of the sound and parallelism of the two planes separating the media. Then, if this second medium has a length such that resonance is obtained through the reflection at the two interfaces, there is a curious result. The transmission of energy from the first to the third medium is then of the same magnitude as would exist if the second medium were absent. Usually in order to ascertain the energy transmitted in the third medium it is necessary to use a complicated formula.

* Kimball, *Journal of the Acoustical Society*, II, 2, 297, 1930.

† See Slocum, "Noise and Vibration Engineering," D. Van Nostrand Company, 1931.

12.5. Constrictions and Expansion in Conduits. — (1) *Intensity Effect.* — It has been explained in a previous chapter that if the area in a tubular conduit is changed at any point, there is not 100% transmission, but a reflection. If there are two changes in area as illustrated in the accompanying Fig. 12.1, then there are

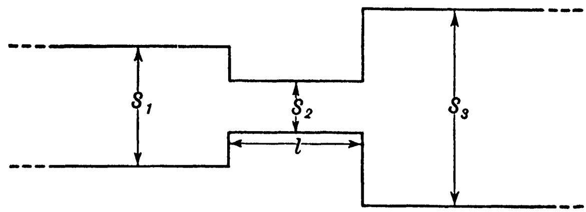


FIG. 12.1

reflections at both junctions where the areas are altered. It is assumed that the diameters of conduits 1, 2 and 3 are small in comparison with a wave-length. If the area of the first tube is S_1, of the second S_2, and of the third S_3, and if we put $m_1 = \frac{S_2}{S_1}$ and $m_2 = \frac{S_3}{S_2}$, then the following conclusions may be drawn:

(*a*) If the length of the tube area S_2 is very short compared with the wave-length, then the ratio of transmitted to incident energy depends only upon the first and third tubes and may be expressed by

$$\frac{4m_1m_2}{(m_1m_2 + 1)^2} \quad \text{or} \quad \frac{4S_1S_3}{(S_1 + S_3)^2}.$$

(*b*) If the tube S_2 has a length equal to one-half wave-length, then there is resonance set up in S_2 and the transmitted wave is the same as if S_2 were not present. This is also true if S_2's length is any integral number of half-wave-lengths.

(*c*) The ratio of transmitted to incident energy is known for any length of S_2.

The reader is now enabled to understand why it is difficult to diminish transmission in a conduit by a constriction which is very short in length. An example would be the insertion of a diaphragm across the tube with a small hole in it. According to item (*a*) just stated, since S_1 and S_3 would be equal, the entire incident energy would be transmitted. Of course viscosity and the assumption of plane waves in the theory would prevent the accuracy of this statement, but its truth is sufficiently approximate as a practical guide. If one attempts to pinch a rubber tube conduit and thus to reduce the intensity transmitted, practically no change in intensity will occur until the channel has been made very small. The ear canals may be nearly closed with impacted wax but no deafness will be noted until there is practically complete closure.

(2) *Phase Effect.* — At first thought it might seem that, although a constriction or expansion will change the intensity of flow of energy in a conduit, yet there would be no change of phase other than that which would ordinarily occur in the same length of tubing of constant diameter. But this is not the case, for there are repeated reflections at the ends of the constricted (or expanded) length and the transmitted wave is made up of not merely a portion of one wave incident at the S_3 end of the constriction, but of a large number. Its phase cannot be unmodified by this complexity. The theory for this change is known.*

12.6. The Stethoscope. — Consider an ideal stethoscope as shown in the accompanying figure. Medium 1 is a solid, liquid or any medium. The medium in 2 and 3 is air. The query arises as to the transmission of energy from medium 1 into the small tube. The theory is known † and the following conclusions may be drawn from it:

1. If the thickness of the lamina 2 is very small in comparison with a wave-length, the ratio of the transmitted energy to the energy incident in 1 is

$$\frac{4r_1m_2}{(r_1 + m_2)^2},$$

* See Stewart and Lindsay, Acoustics, p. 78.

† Brillié, *Le Génie Civil,* 75, 223, 1919.

where r_1 is the ratio of the acoustic resistances of medium 2 to medium 1 and m_2 is the ratio of the area of 3 to the area of 2.

2. If one considers the transmission from water to air, and considers the thickness of medium 2, he will find that this length can readily be adjusted so that the energy of sound in tube 3 is much greater than the energy which would pass from the given area of water to air. Moreover, the stethoscope tube permits all of the energy in tube 3 to flow into the ear, whereas without the stethoscope only a small part of the flow of energy from the given area of water would enter the ear. When the ear is pressed against medium 1, it is obvious that the ear becomes a stethoscope.

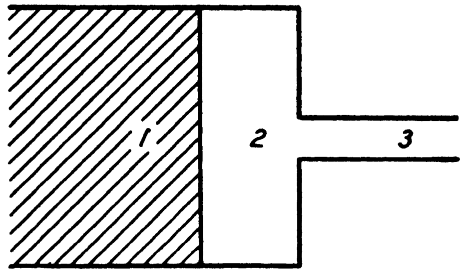


FIG. 12.2

12.7. Non-reflecting Conduit Junctions. — It is impossible to change the area of a conduit without introducing reflection. In quantitative measurements this is a serious consideration. In order to avoid reflection it is customary to connect the two different areas by means of a cone of very gradual slope.* The longer the cone, i.e., the less its slope, the less the reflections at the ends. Also, since the reflections at the ends of the cone introduce the possibility of resonance, all wave-lengths will not be transmitted equally well. Only at resonance will the ratio of transmitted to incident energy be unity. In many cases, the introduction of felt in the transmitting tubes will cut down resonance and prevent any material effect of reflected waves upon the source and thus will make possible satisfactory quantitative measurements of the relative intensities of different frequencies.

12.8. Velocity of Sound in Pipes. — There are two physical phenomena which enter into sound transmission in pipes but

* See also Section 5.7.

which are not important in the open air. The viscosity of a gas is made prominent in a pipe because of the presence of the stationary wall which necessitates slippage in the gas itself. Moreover, a gas is heated by compression and cooled by rarefaction. The exchange of heat between the wall and the gas is the second factor. The result of both of these factors is to diminish the sound intensity and also the sound velocity. It is found experimentally that the percentage decrease in the velocity of sound in a pipe is given by the following expression.

$$\frac{c}{2r\sqrt{\pi n}}$$

wherein r is the radius of the pipe in mms., n is the frequency and c a number which varies with the diameter and the material of the pipe. Schulze * in 1904 found experimentally that c varied from .0075 to 0.025.

12.9. Decay of Intensity in Pipes. — The decay of intensity of sound in transmission through pipes has been only slightly investigated. It is well-known that the percentage of decay per unit length should be the same throughout the pipe and should be dependent upon the frequency, the diameter and the material of the pipe. H. Brillie † presents the following data as having

Material	Diameter	Length
Rubber	5 mm.	165 cm.
Rubber	15 mm.	330 cm.
Brass	10 mm.	1575 cm.

been taken by Messrs. Clerget and Dessus in France. The lengths of pipe given are those required to reduce the sound intensity to 50% of its initial value. The frequency of the sound used in these experiments is not given by Brillié.

The accompanying Fig. 12.3 graphically exhibits the data ob-

* Schulze, *Ann. d. Physik*, 13, p. 1060 (1904).
† Brillié, *Le Génie Civil*, 75, p. 224, 1919.

tained with transmission through speaking tubes by Eckhardt, Chrisler and Evans.*

The loss in transmission depends importantly upon the diameter of the tube. Of course the transmission in a 20-foot tube

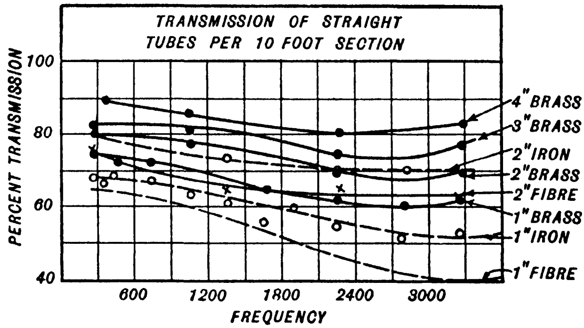


FIG. 12.3

would be the square of the values shown. Thus a 2-inch brass tube would transmit 80% of a tone of 300 cycles in a 10-foot tube, 64% in a 20-foot tube, and 51% in a 30-foot tube. Thus in long tubes the differences in Fig. 12.3 are accentuated. The

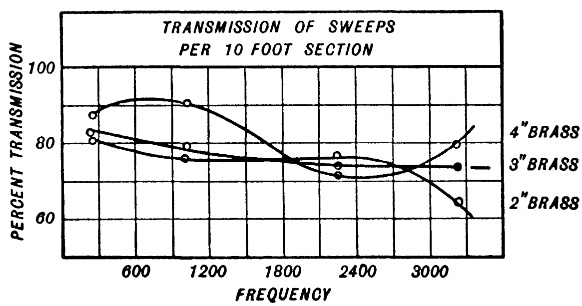


FIG. 12.4

loss in transmission also depends upon the material of the pipe. But it is not clear whether the difference is caused by friction or by absorption due to the lack of perfect rigidity in the pipe. The passage of sound around a bend in a pipe has already been

* Technological Paper of the Bureau of Standards No. 333, p. 163, Vol. 21, 1926–27.

mentioned in Section 10.18. The authors just mentioned have determined also the effect of 90° bends. The results are shown in Fig. 12.4. There is an unexplained peculiarity with high frequencies with a pipe of large diameter.

QUESTIONS

1. According to (2) of Section 12.6, how does the excess pressure in the tube containing medium 3 compare with the pressure which would be obtained if the tube containing 2 were extended indefinitely to the right and the former tube omitted?

2. Explain the apparent contradiction involved in the following: According to Section 12.1, the transmission of sound energy from medium 1 to medium 2 of Fig. 12.2 depends upon the ratio of the acoustic resistance in the second medium to that in the first. Then the attachment of the tube containing 3 could not increase the flow of energy from 1 to 2, and hence the energy computed as in 12.6 cannot be correct.

3. Compare Sections 12.7 and 5.7. For a wave-length long compared to the abrupt changes proposed in 5.7, there would be little difference in result between the long cone and the approximation to it by a succession of short cylindrical tubes connected by abrupt changes in area. From this fact what would you conclude concerning the reflection on the walls of the cone as well as at the ends?

4. Assume a conduit had walls that were thin and elastic so that they would be set in vibration by the sound wave. Can you venture a reason why such walls would modify the velocity of the sound wave in the tube?

CHAPTER XIII

SELECTIVE TRANSMISSION

13.1. Interference Tube of Herschel and Quincke.*— In transmitting sound through a tube or conduit, it is often desirable to eliminate certain frequency regions and this chapter describes the methods so far devised to produce such a selection. Consider the transmission of sound, from left to right, through the double tube shown in the accompanying Fig. 13.1.

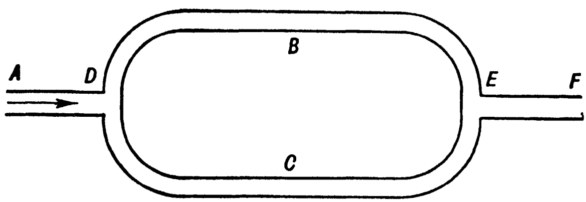


FIG. 13.1

As already noted in Chapter V, if the area of cross-section of tube *AD* is twice that of either *DBE* or *DCE*, then a wave passing from *A* to *D* will suffer no reflection at *D* but will divide equally and pass on to *E*. If *DBE* and *DCE* are alike in length and area, and if *EF* has the same diameter as *AD*, the combined wave at *E* will pass on through *EF* without reflection at *E* because there is no change in condition. But if *DBE* is, for a given frequency, one-half wave-length longer than *DCE*, then the two waves will meet at *E* out of phase. When the pressure of one is positive the other will be equal and negative, but the displacements will unite favorably for a *positive* displacement of one is the same actual direction as the *negative* displacement of the other. The pressures neutralize, but the displacements add. There can be no forward wave in *EF* for there is no pressure to produce it.

* See Rayleigh's "Theory of Sound," Vol. II, pp. 64, 65 and 210.

The energy cannot be destroyed by interference; the two waves proceed in their journeys around the loop through *C* and *B* and unite at *D*. Since the difference in paths is now zero, the pressures are in phase and the waves proceed from *D* to *A*. Thus, assuming the conditions specified, there will be no transmission of the energy through *EF*. If the sound entering *A* is not composed of a single frequency but is complex, that frequency for which the difference in path is one-half wave-length will be eliminated from transmission.

But the above explanation, although apparently satisfactory to physicists for almost a century (from 1833 to 1928 *), is in error in the inference that elimination will occur only for frequencies which are opposite in phase after passing over the paths *DBE* and *DCE*. The complete theory of the Herschel and Quincke tube shows that there are other frequencies, in fact, from two to three times as many, which will fail in transmission through *EF*. The secret of their appearance rests in the fact that, in general, the history of a sound wave leaving *D* is not simply passage from *D* to *E* by the two paths. One can see that the wave travelling by the path *DCE* will divide at *E* and part go out through *EF*, part through *B* back to *D* where it will again divide and part will be reflected back through *C* to *D*, etc. At these division points there will be in general a change of phase with the reflected waves. It is obvious that one cannot actually trace the paths of the waves in intensity and phase in the general case for they will not be limited by one circuit about the loop. Thus it becomes necessary to resort to mathematical methods of stating the conditions which must exist at *D* and *E*. When this is done and the equations solved, the additional eliminated frequencies are discovered. They are found to depend upon the sum of the lengths of the two branches rather than upon their difference. There is zero transmission when the sum of these lengths is an integral number of wave-lengths, provided that at the same time the difference of these lengths is not an integral number of wave-lengths. This is

* Stewart, *Phys. Rev.*, 31, 4, 696, 1928, or Stewart and Lindsay, "Acoustics," p. 90.

an interesting case of the advantage of mathematical methods.

Quincke has made use of a modification which is more simple in construction. The wave enters at *A*, Fig. 13.2, and either passes out at *F* or experiences what is equivalent to a reflection at *D*, for there is no opportunity for the energy to be disposed of otherwise. If the frequency corresponds to the natural period of vibration of the tube *DC*, then this tube will resonate. As noted in a previous chapter at resonance frequency the incoming and outgoing waves in such a closed tube agree in displacement at the open end, but have pressures that are approximately equal and opposite, forming a "loop" at the end. If this occurs, there is approximately no pressure to produce transmission out through *DF*. Thus the wave of this frequency is eliminated from transmission. Since the elimination is caused by resonance, the shape of the tube *DC* is inconsequential, if the elimination of this one frequency only is considered.

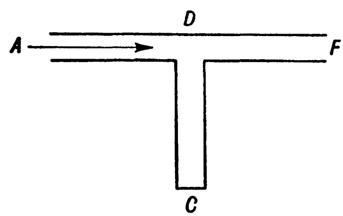


Fig. 13.2

But a more detailed description of the action in Fig. 13.2 will make the phenomenon clearer. Consider the number of possible waves involved. There is the flow of energy from *A* to *D*, which is wave one. There are the two waves from wave one, one entering *DC* described as two, and one passing on toward *F*, described as wave three. (For simplicity let us regard the reflection in passing into and out of *DC* as nil. We are not so much interested in an accuracy of treatment as in a further helpful description.) There is one wave from the tube *DC* which will pass partly to the left toward *A* and partly to the right toward *F*. Denote this wave in the tube by six, the one passing toward *A* by four, and the one toward *F* by five. We then see that there are two waves to the right, toward *F*, numbered three and five. As hereinbefore noted, time is required to build up resonance, and, if there is no viscosity, there is only the limiting case of vibration in *DC* where the energy escaping from the resonator is equal to that entering from the source. The details are as follows. Resonance in *DC* will build up until five is as great as three. But five is opposite in phase to three because it has traversed a half wave-length further. Wave five will not build up any further because at equality of three and five the total flow to the right becomes nil. Then the total flow in four must become equal to the flow in one. In other words, wave one is essentially reflected. This explanation gives a better insight into the phenomenon, though we have incorrectly neglected the reflections occurring in passing in and out

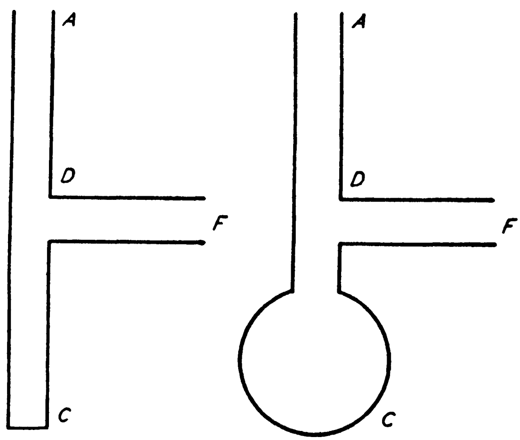


Fig. 13.3

of *DC* and the alteration of phases involved at this point. Even with these reflections we will never have any greater number of waves than those specified, though they arise from more causes than described. With resonance without viscosity one sees that wave three may equal wave five and wave four may equal wave one.

As illustrations of possible shapes, two arrangements are shown in Fig. 13.3. But, as shown below, the shape of this tube does determine the extent of the partial elimination of neighboring frequencies.

An investigation of the theory (see the following section) states that if it were not for viscosity, the elimination of the selected frequency would be complete and independent of the diameter of this side branch. In practice one branch will eliminate all but a fraction of the incident energy. Consequently more than one branch is sometimes essential to the production of the desired reduction in intensity. Krueger * has studied the use of such branch tubes and has concluded that the extent of the elimination depends upon the point of attachment of the side tube. He used a fork resonator and found that the elimination was the greatest when the point of attachment to the conduit was an integral number of half wave-lengths from the rear wall of the resonator and the least when the distance was an odd number of quarter wave-lengths. Undoubtedly these conclusions indicate

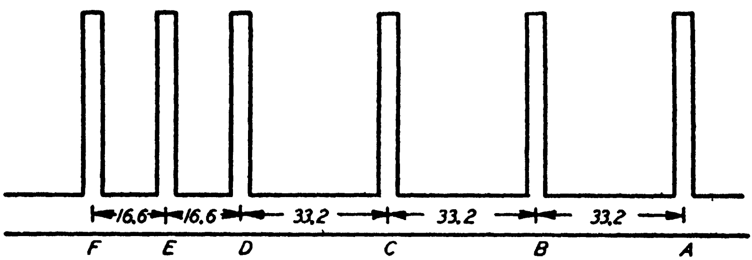


FIG. 13.4

the influence of the reflected wave on the source, but a discussion of the matter should be based upon additional experiments. It is not difficult, however, to understand that there is a desirable separation of side tubes when using more than one. As an illustration, attention is directed to Krueger's final design, shown in Fig. 13.4. It was constructed to eliminate a frequency of 256 cycles or an integral number of times this frequency.

Side tubes *A*, *C* and *F* will assist in eliminating 256 cycles per second if each is adjusted to have a length of one quarter of the wave length; *A*, *B*, *C*, *D* and *F*, 512 cycles if correspondingly adjusted; and similarly *A*, *B*, *C*, *D*, *E* and *F*, 1,024 cycles. The object of the spacing is to cause the reflected waves to be in agreement. Consider the fact that the incident and reflected waves

* Krueger, *Philos. Stud.*, 17, 1901, p. 223.

at each opening are opposite in the phase of the displacement. Assume a wave incident at the right in Fig. 13.4. If the distance *B* to *A* is one-half wave-length, then the phases of the waves reflected at the two points are opposite (for the incident waves are opposite in phase). But, since the reflected wave must travel from *B* to *A*, a half-wave-length, it will there be in the same phase with the reflected wave at *A*. Consequently there will be no interference in the reflected waves, a fundamental condition for the maximum elimination.

13.2. Theory of a Closed Tube as a Side Branch. — A theoretical investigation by Stewart * considers a single side tube, and only the incident and transmitted waves. The wave reflected at the opening of the side tube is assumed not to affect the source. This condition can be approximated by having a considerable amount of "damping," e.g., hair-felt, distributed along the conduit between the source and the branch tube. Also it is assumed that the transmitted wave is not reflected at the distant terminus of the tube. This ideal condition can be approximated by again inserting damping in the conduit between the side tube and the distant terminus.

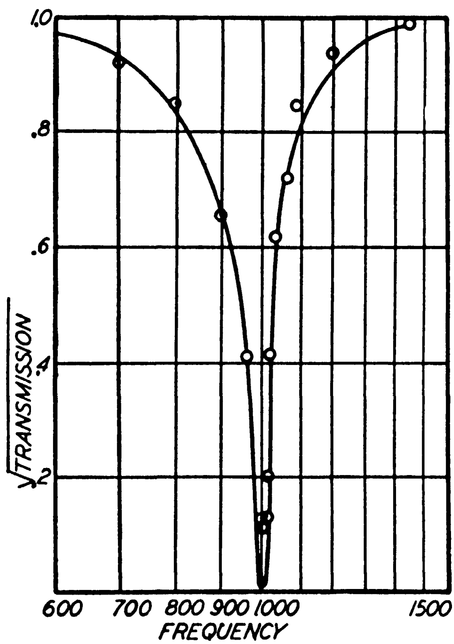


Fig. 13.5

Obviously the damping will greatly dissipate the original energy, but the arrangement will permit a comparison of the theory of

* "Acoustics," Stewart and Lindsay, p. 126.

the action and the experimental results. The comparison is found in Fig. 13.5. The full line curve gives the theoretical values * of the square root of the transmission.

13.3. Helmholtz Resonator as a Side Branch. — Here the theory and experiment bring rather unexpected results for the response of such a resonator in the open is "sharp," that is, frequencies other than the critical one produce very little vibration.

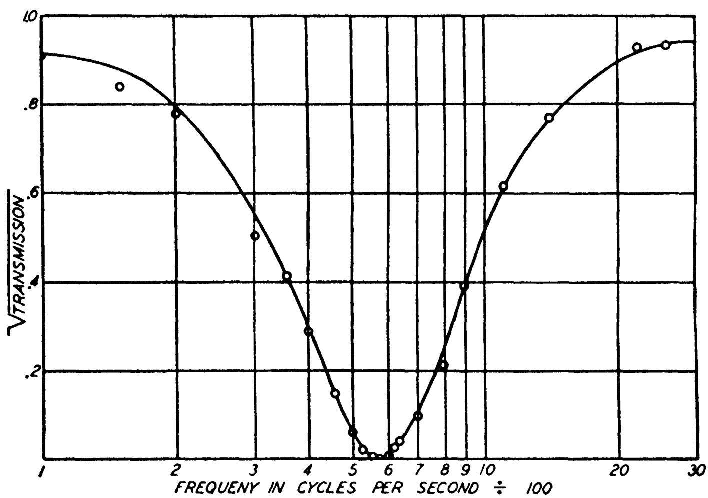


FIG. 13.6

* The theory shows that the ratio of the transmitted energy to the incident energy is

$$\left[1 + \frac{\sigma^2 \tan^2 kl^2}{4S^2}\right]^{-1},$$

where σ is the area of the branch tube, S the area of the conduit, k is 2π divided by the wave-length and l is the length of the side branch. At the critical frequency $\tan kl = \infty$, and the transmission is zero and independent of the areas of the tubes. The function "$\tan kl$" is a short expression used in trigonometry for indicating a certain value that depends upon the angle kl or $\frac{2\pi l}{\lambda}$, where λ is the wave-length. The formula is given here merely that the reader may see that a result complicated physically may have a simple mathematical expression.

The theoretical * and experimental results are shown in Fig. 13.6, the former by the graph and the latter by the circles.

It is to be observed that the transmission is affected very markedly over a wide range of frequencies. The Helmholtz resonator would not be efficient as a Quincke tube if only one frequency or a narrow range is to be eliminated.

13.4. Action of an Orifice. — In this connection the action of an orifice in a conduit is interesting. It might be supposed that the sound escapes from the orifice and thus diminishes transmission. But this is not the correct picture. Figure 13.7 shows a series of curves taken with orifices of four different diameters. The full line curve is the theory † and the designated points represent the square root of the measured values of the transmission. The ordinates on the right give the values in decibels. Curve 4 does not agree with experimental points. Curve 5 is the theoretical curve if c_0 be arbitrarily changed from its computed value 0.582 to 0.74.

A point of interest is that the orifice affects the low frequencies the most. However, this conclusion cannot be extended to indefinitely high frequencies. Two points which can be seen only through theoretical considerations are that the cause of the decreased transmission is more importantly the *reaction* of the mass in the orifice and that the viscosity of the orifice is relatively unimportant. The former point needs explanation. Of course there is a radiation from the hole outward and this energy is lost. But because of the action of the inertia of the mass of the gas in the hole, the reflected wave, similar to that discussed earlier in this chapter, is very much greater than the radiated wave and consequently is the chief factor in the resulting decrease of trans-

* The theory, Stewart and Lindsay, "Acoustics," p. 116, shows that the ratio of transmitted to incident energy is $\left\{1 + \left[4S^2\left(\frac{k}{c} - \frac{1}{kV}\right)\right]^{-2}\right\}^{-1}$. Here the symbols not defined in the foregoing are c the "conductivity" of the neck of the resonator and V its volume.

† The theory, Stewart and Lindsay, "Acoustics," p. 120, is too complicated for a brief footnote.

mission. However the relative importance of the radiated wave increases with frequency. The above interesting points can be applied to a musical instrument like a flute, but not without additional discussion, for here we have standing waves.

It has just been stated that the reaction of the mass is very important, indeed more so than the radiation from the orifice.

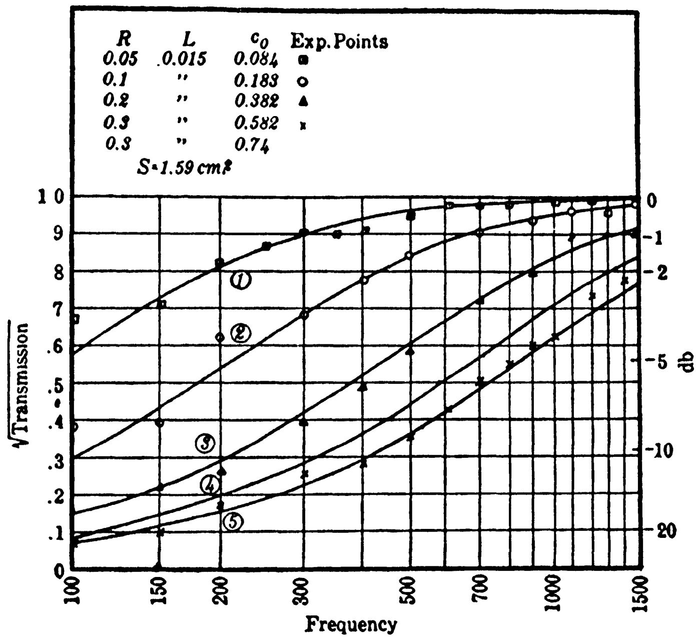


FIG. 13.7

Again the mathematical theory is the only adequate description, although one may perhaps wisely attempt a further discussion in language. In the early chapters the transmission of sound was seen to be possible because a medium possesses inertia (or mass) and elasticity (or the ability to return to the original position). In a tube or conduit having a constant cross-section there is no reflection backward. The reason is that these two qualities, inertia and elasticity, are the same throughout. If one can imagine the air losing its elasticity at a certain point, then there would be no pressure there and there would be reflection at that place approximately as at the open end of a pipe. In short, were the medium at any point along the tube to lose its elasticity and have inertia only, there would be reflection. Now an orifice acts very much like a medium having inertia only. Indeed, in the Helmholtz resonator, the orifice has already been assumed to have inertia only. This was because it led into a large volume, the elastic action of which was much more important. The experiment with the Helmholtz resonator illustrates that the orifice, although effectively so short, but opening out into space, may act as if it possesses inertia only. Thus the reflection may prove to be important and the transmission through it is small in comparison. Experimentally (and theoretically also) the radiation is not the important element. In fact, when one raises a key on the flute or clarinet it is not primarily for the purpose of allowing energy to escape, but rather to cause reflection with change of pressure phase. More concerning the action of such instruments will be given in Chapter XV. What is here stated is but one aspect in a very complex acoustic condition in such instruments.

13.5. Acoustic Wave Filters. — A very striking and effective means of eliminating specific ranges of frequencies has been found * in the acoustic wave filter. Its detailed description is beyond the scope of this presentation, but certain points of practical interest will be considered. The construction of three types of filters is shown in Fig. 13.8.

Type *A* consists of a series of Helmholtz resonators distributed at equal distances along a conduit, Type *B* of orifices similarly distributed, the side walls being extended to increase the inertia of the orifices, and Type *C* of the combination of the two preceding types. The characteristic curves of transmission are shown in Fig. 13.9, the letters corresponding to the types.

Type *A* is a low-frequency pass, Type *B* a high-frequency pass,

* See Stewart, *Physical Review*, 20, 1922, p. 528; 22, 1923, p. 502; 23, 1924, p. 520; and *Jl. of Opt. Soc.*, 9, 1924, p. 583; and Hall, *Phys. Rev.*, 23, 1924, p. 116; and Peacock, *Phys. Rev.*, 23, 1924, p. 525. Or see "Acoustics," Stewart and Lindsay, Chapter VII.

and Type *C* a single-band pass. The remarkable property of these filters is an almost total elimination of transmission in the attenuated frequency regions.

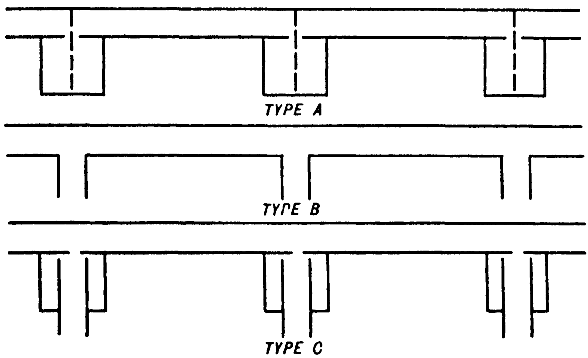


Fig. 13.8

There can be a vibration of a medium to and fro without any transmission of power. For example, consider a standing wave in a tube of infinite extension. Place walls across the tube at

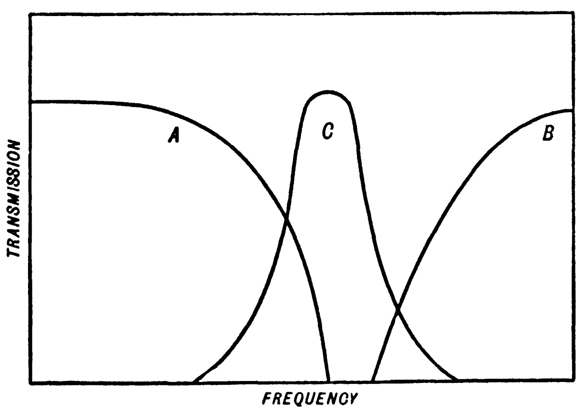


Fig. 13.9

two displacement nodes, not necessarily adjacent. The vibration would continue in this closed space indefinitely, were it not for the dissipation of energy caused by viscosity and absorption of the walls. A series of such closed spaces could be placed along the tube. One would then witness an oscillatory motion or vibration in each compartment, but no transmission of energy. This is not analogous to the wave filter but is introduced to show the possibility of no transmission of energy. Now theory shows that in the acoustic wave filter there is a region of frequencies where there is a vibratory motion in each "section" or space between dashed lines in Type *A* of Fig. 13.8, *without* a flow of energy from one to the other and *with* an amplitude of vibration constantly decreasing from section to section in the direction of the attempted transmission. This is the phenomenon in the non-pass region of frequencies.

If one wishes to set a pendulum in vibration by communicating energy throughout its vibration, the effort must be properly timed, that is, the applied force must have a certain phase relation to the movement. In a similar manner in the acoustic wave filter, it is possible to have a phase relation between pressure and particle velocity such that energy will be transmitted. Thus theory shows the possibility of a non-attenuated transmission region of frequencies in the acoustic wave filter.

From these two analogies the possibility of an acoustic wave filter having attenuated regions and non-attenuated region is suggested. Although this discussion does not consider the actual theory of the acoustic wave filter, enough has been said to indicate that the acoustic wave filter does not dissipate energy, but reflects it, thus refusing transmission.

13.6. Fictitious "Nodes."—The discussion in the preceding paragraphs makes this a convenient point to mention the absence of ideal displacement nodes in the stationary waves occurring in practice. No energy could be transmitted through such an ideal node. For, although here is a "loop" of pressure and ample variation from the mean pressure to do work, yet there is no motion of the medium and without movement a force cannot do work. In the case of standing waves in an open organ pipe, there is a flow of energy out of the open end. Thus there must be a flow from the mouth to this end. With a closed pipe, all the energy issues from the mouth, but even in this case the displacement nodes in the air column are not strictly stationary, for there is a loss of energy in the wave travelling along the tube. This loss occurs because the air next to the pipe remains stationary and a friction loss is introduced. Then, too, wave energy is communicated to the walls of the pipe. The wave travelling from the mouth to the end of the pipe thereby diminishes slightly in amplitude with distance of travel. The same sort of diminution occurs as the wave returns. Of course similar losses occur in the open pipe as well. From what has been stated, none of these cases can have ideal displacement nodes. This explanation of the existence of imperfect "nodes" must be modified by the statement that the viscosity loss must enter through the actual particle velocity of the resultant wave, rather than in the particle velocity which each wave, forward and back, would have if existing alone.

QUESTIONS

1. Show that for waves returning from *E* to *D* of Fig. 13.1, the displacements are favorable to transmission in *DA*, but unfavorable to transmission again in the branches, *DB* and *DC*, provided the difference in the lengths of the branches is one-half wave-length.

2. How would the introduction of viscosity affect the consideration of the divided tubes shown in Fig. 13.1?

3. What is the reason that the Quincke tube does not give zero transmission at the critical frequency?

4. Why does the presence of a mere mass of gas in the orifice prevent a serious amount of dissipation of energy out through the orifice?

5. In both Figs. 13.5 and 13.6 the effects of the resonators are not as sharp as would be expected from their response in the open. Can you suggest a possible reason?

6. Why might one expect that a "large" orifice would give a result not in conformity with theory as in Fig. 13.7?

CHAPTER XIV

MUSICAL SCALES

14.1. The Diatonic Scale.—There are a number of different musical scales employed in the world, but western countries use chiefly the one to be described.

Several reasons exist for expressing the relation between the frequencies of two tones not as a frequency difference but as a *frequency ratio.* It is a fact that to us the most pleasing combination of two tones is one which the frequency ratio is expressible by two integers neither of which is large. Thus what is called the octave has a frequency ratio of 2 : 1. The manner in which the range of frequencies between a note and its octave is divided determines the nature of the scale. Our scale consists of eight notes, the ratios of the frequencies to the first or "tonic" being as follows:

C	D	E	F	G	A	B	c′	d′	e′
1	9/8	5/4	4/3	3/2	5/3	15/8	2	9/4	5/2 etc.

It will be seen at once that the ratios of

$$C,\ E, \text{ and } G$$
$$F,\ A, \text{ and } c'$$
$$G,\ B, \text{ and } d'$$

are all 4 : 5 : 6, which is a major chord. The notes of *E*, *G*, *B* have the ratios 10 : 12 : 15, which is called a minor chord.

If the ratios of the frequencies of adjacent notes be now written we have

C		*D*		*E*		*F*		*G*		*A*		*B*		*c′*
	9/8		10/9		16/15		9/8		10/9		9/8		16/15	

An "interval" between two tones is the ratio of their frequencies. If the frequencies of three tones are represented by a, b, and c, then the three intervals are $\frac{a}{b}$, $\frac{b}{c}$ and $\frac{a}{c}$, the last being clearly the algebraic product and not the sum of the first two. Thus the interval *C* to *D* is 9/8 or 1.125, from *D* to *E*, 10/9 or 1.111, and the interval *C* to *E*, 1.125 × 1.111.

There are indicated above three distinct ratios or "*intervals*," 9/8, 10/9, and 16/15, or 1.125, 1.111, and 1.067. The first two are practically equal and approximately twice the third. If the first is called a whole tone, the latter is a semitone. If now additional semitones be inserted between *C* and *D*, *D* and *E*, *F* and *G*, *G* and *A*, and *A* and *B*, there are, as a result, 12 intervals in the octave. But with the above ratios these twelve intervals could not be all equal. Hence one could not pick out the tones that would produce the same scale if he began on *D* as a keynote. As an example, interval *D* to *E* is 10/9 whereas *C* to *D* is 9/8! Similar obstacles would arise throughout in attempting to have the same scale on *D* as a "tonic." One recourse would be to introduce a sufficient number of tones in the scale to make it possible to start on any note as the tonic. But this is not practical. Consider the impossibility of applying it to the piano forte. It is evident that a sacrifice of harmony must be made for convenience in execution. The modification in the tones necessary is called temperament. At least four suggestions as to temperament have been made, but only one has been retained. It is called the mean temperament.

14.2. Mean Temperament.—The mean temperament assumes twelve fixed intervals. If the scale is to be entirely independent of the tonic, then the twelve intervals must be exactly alike. In order for this to be true, each one of these intervals must be 1.059 instead of the 1.067 given above. The reason for this is that 1 × 1.059 × 1.059 (repeated, occurring twelve times) equals 2.0.* The intervals between *C* and the other notes of the scale

* This is the significance of what is meant by the twelfth root of 2.0. A semitone interval multiplied 12 times (which means 12 such intervals) equals 2.0 or the octave.

are shown for the "natural" and the "tempered" scales as follows:

	C	D	E	F	G	A	B	c′
Natural Scale......	1.000	1.125	1.250	1.333	1.500	1.667	1.875	2.000
Tempered Scale....	1.000	1.122	1.260	1.325	1.498	1.682	1.887	2.000

On the tempered scale *C*♯ would be 1.059 and would be the same as *D*♭. By placing the semitones, 1.059, in between *C* and *D*, *D* and *E*, *F* and *G*, *G* and *A*, *A* and *B*, we have twelve semitones in the octave and any one of them may be used as the tonic. In the actual tuning of a piano care is not taken to secure this equal temperament, but it is approximated. These introduced semitones would have the following intervals with the tonic:

C♯ and *D*♭	1.000 × 1.059
D♯ and *E*♭	1.122 × 1.059
F♯ and *G*♭	1.325 × 1.059
G♯ and *A*♭	1.498 × 1.059
A♯ and *B*♭	1.682 × 1.059

The difference between the natural and the tempered scale is slight and scarcely noticeable if the tones are played in succession. But in chords, it is said that the difference is very noticeable (see Section 14.6).

14.3. Frequency.—There is not universal agreement as to the frequency of a given note. The following can be found mentioned in various works in acoustics:

Standard Frequencies for *A*

French	435
Stuttgart	440
Concert Pitch	460
International	435
American Concert	461.6
Boston Symphony Orchestra	435

According to White * all piano manufacturers of the United

* White, *Science*, 72, No. 1864, p. 295 (1930).

States are using an *A* of 440 vibrations. With *A* 440, middle *C* becomes 261.6 on an equally tempered scale.

14.4. Nomenclature.—There are various ways of indicating the octave in which a given note is found. Two examples follow: C_{-1} to B_{-1}; C_0 to B_0; C_1 to B_1; C_2 to B_2; C_3 to B_3; etc., wherein "middle" *C* is C_3. This is used by D. C. Miller in the work so extensively quoted in this text.

C_{-2} to B_{-2}; C_{-1} to B_{-1}; *C* to *B*; *c* to *b*; c_1 to b_1; c_2 to b_2; c_3 to b_3; etc., wherein c_1 is middle *C*. This is used in Germany. The French use as *C*'s, *Ut*, *ut*, ut_1, ut_2, ut_3, etc., where ut_3 is middle *C*.

Table VIII

Musical Intervals

Interval Name	Note	Frequency Ratio				Millioctaves	
		Natural Scale		Tempered Scale		Natural Scale	Tempered Scale
Unison	C	1	1.000	1	1.000	0.	0.
Comma		81/80	1.013	1	1.000	17.92	0.
Semitone or diesis	C#	25/24	1.042	1 1/12	1.059	58.59	83.33
Limma		16/15	1.067	1 1/12	1.059	93.11	83.33
Minor second	Db	27/25	1.080	1 1/12	1.059	111.0	83.33
Minor tone		10/9	1.111	1 2/12	1.122	152.0	166.6
Major second	D	9/8	1.125	1 2/12	1.122	169.9	166.6
Augmented second	D#	75/64	1.172	1 3/12	1.189	228.8	250.0
Minor third	Eb	6/5	1.200	1 3/12	1.189	263.0	250.0
Major third	E	5/4	1.250	1 4/12	1.260	321.9	333.3
Diminished fourth	Fb	32/25	1.280	1 4/12	1.260	356.1	333.3
Augumented third	E#	125/96	1.302	1 5/12	1.335	380.7	416.5
Perfect fourth	F	4/3	1.333	1 5/12	1.335	414.8	416.5
Augumented fourth	F#	25/18	1.389	1 6/12	1.414	473.9	500.0
Diminished fifth	Cb	36/25	1.440	1 6/12	1.414	526.1	500.0
Perfect fifth	G	3/2	1.500	1 7/12	1.498	585.0	583.3
Augumented fifth	G#	25/16	1.562	1 8/12	1.587	644.0	666.6
Minor sixth	Ab	8/5	1.600	1 8/12	1.587	678.1	666.6
Major sixth	A	5/3	1.667	1 9/12	1.682	737.0	750.0
Augumented sixth	A#	125/72	1.736	1 10/12	1.782	795.8	833.3
Minor seventh	Bb	9/5	1.800	1 10/12	1.782	848.0	833.3
Major seventh	B	15/8	1.875	1 11/12	1.887	906.9	916.6
Diminished octave	Cb	48/25	1.920	1 11/12	1.887	941.1	916.6
Augumented seventh	B#	125/64	1.953	2	2.000	965.7	1000.
Octave	C′	2	2.000	2	2.000	1000.	1000.

14.5. Musical Intervals. — The accompanying table VIII gives the musical intervals in a comparative form. This table is taken from the preliminary report on acoustic terminology by the standardization committee of the Acoustical Society of America.

It is to be noted that in the last columns is evidence of the introduction into the literature of the term "millioctave," which is one thousandth of the interval of the octave.

14.6. Production of Music in the Natural Scale. — Singers and players of instruments whose pitch can be regulated by breath or touch, find the tempered scale less aesthetically satisfying than what they term the "pure" or "natural" scale, and consequently endeavor to perform in the latter. Certain choruses in the Roman and Greek churches do actually use or approximate the standards of pitch expounded by Pythagoras, and practice without instrumental accompaniment in order to form and maintain the habit of thinking in terms of this pitch standard. However, most performers who talk glibly of the "pure" scale actually increase major and augmented intervals and diminish minor and diminished intervals, as compared with the tempered scale, though, in the Pythagorean scale, the major third for instance, is not greater, but less, than the tempered interval; while these alterations are thus away from, not toward, the natural scale, they are aesthetically eminently justified, since in our contemporary music the contrast between major and minor, and the "tendency" of certain tones to progress to certain others because of our habit of thinking harmonically as well as melodically, are of relatively great psychological importance. To distinguish this practice of emphasizing "tendency" from a true attempt to approximate the scale of nature, I suggest that performers should speak of an "artistic" or "harmonic" scale, rather than make an inaccurate use of the terms "pure" scale and "natural" scale.*

QUESTIONS

1. If the semitone *B* to *C* is 1.067, why not take this as a standard?
2. What interval is the twelfth root of 2 and why?
3. If the interval of a whole tone is 1.122, what is the interval of a semitone and why?

* This paragraph has been kindly prepared by Doctor P. G. Clapp, Director and Professor of Music at the University of Iowa. To him the author's thanks are due.

CHAPTER XV

MUSICAL INSTRUMENTS, THE VOICE AND OTHER SOUND SOURCES

15.1. Development of Musical Instruments. — Musical instruments have been developed through experiment. Their use has preceded our full understanding of the physics of their operation. But with our increased knowledge of acoustics and of psychology, it is probable that the physicist will have greater influence in the development of musical instruments in the future than he has had in the past. That improvement can be made both by modifications of present instruments and the additions of others, there can be no doubt. The purpose of this chapter, in harmony with the remainder of the text, is not a detailed description of musical instruments, nor of the underlying theory. Its function is to emphasize physical principles so that the student may have his understanding increased and interest aroused.

At the present time there are excellent treatises * giving discussions that are more thorough than are here possible.

15.2. Production of Sound, General. — The resonance experiment with a fork placed over a jar, discussed in Chapter V, showed that the flow of energy from the fork was increased by the presence of the jar. The fork was closely coupled with the air column in the jar so that the latter could influence the former.

This is representative of the method of affecting the flow of energy from the source by resonance. Another method is to increase the vibrating surface exposed to the air. This may be illustrated by pressing a vibrating fork on a table top. The table becomes a sounding board, and can successfully convey to the air

* Barton, "Text-book on Sound," Macmillan and Co., 1922, gives both the theoretical and practical aspects. Richardson, "Acoustics of Orchestral Instruments and of the Organ," Oxford University Press, 1929, gives a non-mathematical and yet theoretical account.

more acoustic energy than could the fork directly. These are the two general methods used in musical instruments and in sound sources in general.

Returning to the fork placed at the opening of the resonating jar in Section 6.5, it is to be remembered that the fork and the air column each has its own frequency, and that the intensity of the acoustic output, although greatest when these two frequencies are alike, yet is noticeably large when the two frequencies are not exactly the same. An investigation of the situation shows that the resonating air column affects the frequency of the fork rather than vice versa. The nature of the change in frequency is curious. If the level of the water in the jar is raised, so that the natural frequency of the jar is gradually increased in the direction of equality to the natural frequency of the fork, the effect is opposite to what we would anticipate without resort to mathematical analysis. The effect is not to change the fork frequency in the direction of equality, but in the opposite sense. Then, with gradual increase in frequency of the air column, the fork frequency continues to shift slowly to higher frequencies. But when the natural frequency of the air column becomes equal to the natural frequency of the fork, there is a sudden jump in actual frequency of the fork back to its natural value with greatly enhanced intensity of resonance. The entire variation of frequency of the fork in a certain experiment performed as described was less than one-hundredth of one per cent. The effect is therefore not large. The experiment is cited to show that a vibrating air column can affect the frequency of such a rigid body as a tuning fork. This effect of one of the coupled vibrating systems upon the other, and the increase of flow of energy from the source at like frequencies, are the two most prominent effects to bear in mind. There are cases when the source is coupled to more than one vibrating system. This will be described at the appropriate place.

There is another feature that should be mentioned in stating a general view. One is concerned not only with the intensity of sound on the interior of an instrument, but with that on the exterior. In wind instruments with open orifices the transfer is very complicated.

15.3. Production of Sound by Strings. — Stringed instruments are actuated by impact, plucking and bowing. In the first two cases, the string is forced into a strained position and then allowed to vibrate freely. Since the string is capable of vibrating in any of its natural frequencies, which are integral multiples of the fundamental one, the relative amplitudes which the various components have will depend upon the original displaced position of the string. For example, assume that by a carefully constructed constraint it is easily possible to displace and hold the string throughout its entire length in that position corresponding to a displacement for the fundamental frequency alone. From this arbitrary position the string is suddenly released. Clearly it would then vibrate freely with only the fundamental frequency involved. Similarly, the string might be displaced to a position corresponding to the actual position of the string when possessing the frequencies of the fundamental and the first overtone. Then the subsequent vibrations would contain only these two frequencies.

When a string is plucked or is struck by a hammer, the displacement produced is much more complicated, and, moreover, the two cases would not be alike. In the case of plucking, the entire string is set free from practically a rest position. On the other hand, a blow of the hammer is very rapid and is actually completed before the string has had time everywhere to move into a displaced position. These two different starting conditions of the string will result in a difference of the values of the amplitudes of the component tones in each case.

For a similar reason, one can understand why the size and softness of a hammer would cause a difference in the quality of tone from a string. The hard hammer would give a sharper bend to the string. One associates sharp bends with short distances between nodes and hence with higher frequencies, and a careful study shows that such is really the case. The hard hammer accentuates high frequencies.

From the above discussion, it is rather clear that the quality of tone can be modified by the speed of plucking (which may not be a practical consideration), the nature of the hammer, the quickness of its blow, and the position along the string of either the plucking or the impact. But it happens that in the hammer instrument, the piano, the actuating key does not determine the quickness of the blow. After the hammer is in motion, the key no longer has any control. Give the hammer a certain energy or a certain velocity and it will always strike in the same manner. Thus the possibilities of "touch" on the piano are seriously limited, much more so than is commonly believed.

The quality of a bowed string can be modified at will much more readily. Bowing presents a complicated and an interesting phenomenon. The bow pulls the string to one side, the force arising from what is called "static friction." It is well known that such a force is greater than that existing when slippage once begins. So when the bow pulls the string to one side, the string finally slips under the bow. When the slip once begins it continues until the string has reached a displacement in the opposite direction. Then the string again follows the bow's motion and repeats its former action. Work is done by the bow on the string because the force acting in the direction of the bowing is very much greater in one half of the vibration than in the other half. From this description it is evident that the width of the bow, the place of bowing along the string, the pressure on the bow and the speed of bowing will all enter into the production of the tone quality. The possibility of "touch" in such an instrument is therefore relatively large.

15.4. Production of Sound by Reeds. — In the clarinet there is but one reed, and in the oboe and bassoon there are two. The reeds are set into motion by blowing. The velocity of the air blast lowers the pressure between the reed and its base, or between the two reeds, and the impact of the air behind also conspires to cause the closing of the reed or reeds. The pressure not being sufficient to keep the reed or reeds in a closed position, the original position is resumed and a continuous vibration ensues. In this vibration the reeds, acting alone, have a natural frequency, or rather a group of natural frequencies, with a quality of sound which is not very pleasing. But the reeds are usually, as in the clarinet and oboe, coupled with an air column. With such an air column and a thin reed, the former is the chief factor in determining the frequency of the coupled system. It is to be borne in mind that reeds are used in organ pipes as well as in orchestral instruments.

But there is another factor, the importance of which is not definitely known. The resonating cavity in the mouth of the blower is also coupled to the vibrator. It is taught by some authorities that the blowing of an instrument is made easier if one will put his mouth in the position of humming the note desired. Until quantitative data are obtained, a positive statement concerning the importance of this second coupling cannot be made.

What has been stated about the reeds would also apply to the use of the lips with brass instruments, except in the latter case there is a large opportunity for the alteration of the vibrator. The lips may be stretched more or less tightly and they may be thrust forward. The shape of the mouthpiece in the case of brass instruments and its influence on the quality deserves additional mention in a later section.

15.5. Production of Sound by an Air Blast. — The most common examples of the production of sound by an air blast are the blowing of the flute and of an organ pipe. If a sheet of air strikes a sharp edge a tone is produced which depends upon the shape of the edge, the velocity of the blast, and the distance from the blast opening to the edge. This tone is caused by the production of eddies first on one side of the sharp edge and then on the other. When such a source is coupled with a resonating air column, the frequency of the eddies or vortices is controlled thereby. The details are beyond the scope of this text, but enough has been said to remind the reader that here again one has two vibrating systems, with one controlling the other. The natural frequency of this "edge tone" is raised by an increase in air velocity and is lowered by an increase in distance of the edge from the air blast opening. The production of the edge tone can therefore be controlled. by the velocity of the air blast and the distance of the blast opening to the edge against which the blast impinges. This is shown in practice in the use of the flute and in the manufacture of organ pipes.

There is one application of the air-blast in which its importance is not quantitatively known. In most brass wind instruments the mouthpiece is cupped so that there is an edge at the opening of the tube of the instrument into the cup. The blast of air from the lips must impinge upon this edge. There is here a secondary source of sound produced. This may be the explanation, at least in part, of the effect of the shape of the mouthpiece upon the tone produced in such an instrument.

15.6. Harmonics and Overtones. — In dealing with the production of sound, one must have clearly in mind the possibility of regarding every sustained sound as composed of various frequencies, each of which is strictly a simple harmonic vibration. *Any vibration which repeats itself can be analyzed into its component simple harmonic frequencies.* In this analysis the frequencies of the components may include those whose relative frequencies are 1, 2, 3, 4, 5, 6, etc. But these frequencies may not all be present. The amplitudes of some of them may be zero. If the one represented above as "1" is present, all of the others present are in consonance with it, for each represents an integral number of octaves. Such overtones are called "harmonics." But if the fundamental, or lowest tone present, is represented by 2, then, while 4, 6, 8, etc., which are octaves of the first, may be called harmonics, yet 3, 5, 7, etc., are not octaves and to them is given the more general term "overtones." Thus the overtones may in this narrow sense be harmonic and they may not be. Also the overtones may be dissonant with the fundamental. For assume the lowest tone present is that represented by 8. Then overtone

9 would be dissonant with it. Every musical sound, then, has a fundamental, or the lowest tone, but the overtones need not be harmonics. This is true, for example, of most wind brass instruments. In them the tone represented in the above by "1" is not played.

15.7. Peculiarity of Action of Several Instruments. — In the violin, the vibration of the string is practically parallel to the body since the displacements are along the direction of the bow. This vibration causes a motion of the bridge in the same direction. Under the bridge, and more nearly under one foot of the bridge than the other, there is a supporting sound-post. About this side which is rather stable, the bridge rolls back and forth with the vibration of the string. There is thus conveyed to the body of the violin, and indeed to both the belly and back of the instrument, a vibration perpendicular to both. The sound from the instrument thus depends upon the natural frequencies of the violin itself and the air volume within. But the radiation of energy from the instrument is largely enhanced by the sounding-board effect, or the increase of exposure to the air of the vibrating surface. Thus the vibrating string is coupled with resonating bodies and also with an increased area which is forced to vibrate. That the natural frequencies of the violin are of importance is shown by the difference in instruments. H. Backhaus * has experimented with famous violins and has shown that one of their chief characteristics is emphasis upon the high overtones, or upon the actual number of overtones that have a measurable amplitude. A great deal may be said about the tone characteristics of violins and their construction, but this would take the reader too far afield. Attention may be called, however, to the fact that Norway spruce, which is frequently used in violin construction, has a great elasticity and small mass per unit volume. This gives a high velocity (15,000 feet per second) of sound and causes the vibration to spread rapidly over the violin. Of course the velocity across the grain is, unfortunately, much less.

* *Die Naturwissenschaften*, 18, 1929.

In the piano we find one bridge on the sounding board and one on the frame which carries the tension in the wires. The vibrations are conveyed to the sounding board by the former. The hammers are made to strike at one-seventh the length from one end. This location would help to prevent the formation of the seventh harmonic which is somewhat dissonant, but it is thought by Richardson (loc. cit.) that this is not the most important effect. In his opinion, if the string is struck at one-seventh from the end, its fundamental amplitude has the maximum value and consequently less of the energy can go into the high harmonics. Hence the excitation of natural vibrations of the sounding board are reduced to a minimum.

The clarinet is a cylindrical tube terminating in a bell. It is usually stated that the fundamental and harmonics have the relative frequencies of 1, 3, 5, 7, etc., as is the case with a cylinder closed at one end, for the mouthpiece acts as a closed end, though it is the source of sound. But it is possible that the mouthpiece may be made to act as an open end introducing noticeable even harmonics as well. There is a vent hole in the instrument near the beak which is opened in order to play the higher notes in its range. The function of this small hole is not very clear. It is claimed that it helps in the formation of an antinode near the beak end, and thus encourages the formation of high tones. But such an orifice would also be favorable to the transmission of the higher frequencies, as pointed out in Chapter XIII, and would thus have the tendency actually found in the use of the instrument.

The oboė has a conical tube, with a "closed" end at the reeds. Its natural frequencies include therefore all of the harmonic series.

Many brass instruments have a hyperbolical or an exponential shape. In these the tube widens out slowly at first and then very rapidly at the bell. The harmonics are the same as those of a conical tube. The bell on these instruments, as on the clarinet and oboe, enables the energy from the interior to escape more rapidly than otherwise. But the energy may escape from the opened holes also, as described in the following Section 15.8.

The organ pipe must be considered as open at the air-blast end. Thus the harmonics and fundamental have the variation of 1, 3, 5, 7, etc., for a pipe closed at the other end, and all the harmonics for an open end. In measuring the acoustic length, or the length referred to in Section 6.8, of an organ flue pipe, a correction must be made for the open end, and also for the mouth or air-blast end. The former correction has been mentioned in Section 6.9. The latter has recently been studied by Bate.* He found that the correction was proportional to the area of the pipe, inversely proportional to the area of the mouth, and independent of the frequency over the octave studied.

15.8. Emission of Sound from the Clarinet. — It is the purpose of this section to emphasize the very complicated action within an instrument such as a clarinet and the effect upon its emitted sound. A later section discusses the frequencies produced by opening the holes. But up to the present time no one has studied in detail the emission of sound from the instrument. Not only does the opening of the holes modify the resonance inside, but a pipe having open holes will inevitably have a filtering action inside, as discussed in the previous chapter. The sound in part issues from the opened holes as well as from the bell and with some notes the former is of high importance. In the absence of experimental facts, one can surmise that the quality of these two sounds would not be the same. The entire action, as one readily sees, is very complicated, for one has to deal at least with resonance, the filtering action of one or more orifices, waves travelling in both directions along the tube, radiation from the orifices and the affect of return waves upon the source.

15.9. Production of the Voice. — The vocal cords or ligaments require a blast of air from the lungs to pass through the slit between them.† They act in a manner similar to two non-rigid reeds. The frequencies are determined by tension, length and distribution of mass of the cords. The resonances of the larynx,

* Bate, *Philosophical Magazine*, 10, 65, p. 617, 1930.

† R. L. Wegel, *Jl. Acous. Soc. of Am.*, 1, 3, p. 1, 1930, discusses the "Theory of the Vibration of the Larynx." This paper is of interest particularly to physicists.

pharynx, mouth and nose control the harmonics that are emphasized and give the quality to the voice. As earlier stated, the differences in sustained vowels are caused by resonance. But it is also evident that resonance cannot fully control quality; for sustained tones, only the vibrations present in the vocal cords can exist in the tone produced. For the impulsive sounds which also proceed from the vocal cords, the quality is produced by the excitation of all the natural frequencies of the resonating chambers. This action is similar to the effect of sending a puff of air across the opening of a bottle. The natural frequencies die out rapidly. So, in speech and music both types of action are prominent.

15.10. Frequency of Pipes. — It has been well known that the frequency of a pipe, a flute for example, with one hole open, depends upon the resonating action of the column of air extending from the opened hole to the source of sound. In the case of the flute the source is the hole across which the player blows. If the frequencies are computed by assuming that the effect of the opened hole is to create a pipe with an open end at that point, then, since the blown end is also effectively an open end, the wave-length is twice the distance from the source to the open hole. This is only approximately true, and for the reason that the opened hole is not equivalent to an open end.

If two holes on the instrument are open, then the resonance frequency is modified. An examination of the theory * shows that now the frequency can be calculated by assuming the pipe to consist of two resonant systems consisting of a pipe from the source to the first hole, and a pipe from the first hole to the second. In fact, it is possible by making approximations in the theory to compute the frequency for the case of any number of open holes. Yet these approximations do not permit of manufacturing designs based solely on computations. Even now the location of holes depends upon experiment.

15.11. Aeolian Harp. — When wind blows over stretched wires sounds are produced which are not due to the vibrations of the

* Irons, *Phil. Mag.*, 10, p. 16 (1930).

wires but to the action of the air itself. When the air passes an obstacle, vortices or whirls may be established on the leeward side. It is the instability of these vortices that leads to oscillations and finally to sound production. The predominant tone has been found by recent observations * to have the following frequency,

$$n = 0.20\frac{v}{d},$$

wherein v is the velocity of the wind and d the diameter of the wire.

Of course if a natural frequency of the string corresponds to that given in the above formula, the tone is reinforced.

The noise of the wind at corners, in the trees, at all obstacles is caused by the instability of vortices and consequently the predominant term depends upon the shape of the obstacle and the velocity of the wind. The "howling" of the wind is the characteristic sliding of the tone in pitch which is precisely what would be caused by continuously varying wind velocities.

15.12. Singing Flames. — If a small flame is introduced into an open pipe, it may excite the natural vibration of the pipe. Its position must be between the open end and a node, or about half way to the mid-point of the pipe. When the frequency of the pipe is excited the height of the flame oscillates simultaneously with changing pressure, the pressure in the gas and in the air being in opposite phases. Heat is transferred to the air at each condensation.†

15.13. Singing Tubes. — Recently Professor C. T. Knipp has made an apparatus which exhibits the phenomenon in a convenient manner. A heated glass bulb is the source of energy and the tone is the natural vibration of the air in the chamber volume to which it is attached. One can tune this volume and thus secure a fairly constant source of sound.

* Richardson, *Physical Society Proceedings*, 36, 153, 1924, and 37, 178, 1925.
† See Barton, *loc. cit.*, article 265, et seq.

15.14. Sensitive Flames and Jets. — When a fluid jet issues from an orifice into quiet air, the pressure behind the jet may become so excessive as to cause an unsteady state in the jet. If, however, this pressure is not quite attained, a sound wave of certain frequencies may produce this unsteadiness. In the case of a flame, a flare results. A sensitive flame responds to sound waves because of the displacements of the air and not because of pressure changes. The action occurs near the orifice.

15.15. Tones from Membranes. — In acoustical apparatus membranes and diaphragms are in common use. Some of the best telephone transmitters for wireless broadcasting use a stretched membrane (or very thin flexible plate) and a vibrating plate (not stretched, but whose stiffness is caused by its thickness) is universally used in telephone receivers. To illustrate the nature of the natural vibrations of such pieces of apparatus, a brief description will be given of the modes of vibration of an ideal circular membrane. The assumptions are that the membrane is stretched uniformly in all directions with a certain tension that remains unaltered during the vibrations considered, and that the membrane is perfectly flexible and infinitely thin. A membrane is "perfectly flexible" if there is no force resisting bending. The membrane, under the assumptions, is fastened rigidly at its circular boundary. Its fundamental vibration is with the circumference as a node and a maximum displacement at the center of the circle. Inasmuch as our interest is in the ratios of the fundamental and overtones, it is satisfactory to represent all frequencies as multiples of the fundamental. Thus the frequency of the fundamental is regarded as unity. In Fig. 15.1 * are shown the nodal lines and the frequencies of the fundamental and eleven overtones. The first number under each figure is the frequency. The second and third numbers are the radii of the nodal circles as compared with the radius of the membrane. In each figure the nodal lines are those belonging to that overtone only. The chief points of interest are first, that the overtones are not mul-

* This figure is according to Rayleigh, "Sound," p. 206.

tiples of the fundamental and second, that the actual vibration of a membrane, when involving all these tones, is very complex.

The vibrations of plates are not the same as those of membranes because the stiffness in the former has a different origin;

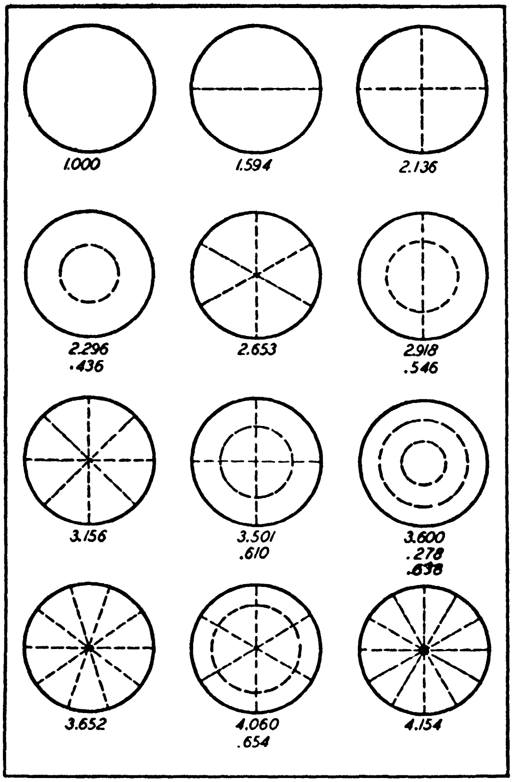


FIG. 15.1

namely, the rigidity of the plate and not the tension in it. Yet the general description of the nodal system as consisting of concentric circles and symmetrically distributed diameters remains correct. In the ordinary telephone transmitter diaphragm the fundamental has a frequency in the neighborhood of 800 cycles. A force having this frequency would set the diaphragm into a relatively large vibration. Reference to the discussion of the distribution of energy in speech will show that the maximum energy does not occur at 800. Experience has shown that this frequency gives the most satisfactory results when all the practical considerations of design are met.

Reference should be made to the sound of a drum. Its pitch is difficult to get, except with the kettle drum, where the pitch is modified at will by altering the tension. The quality of the sound of the drum is caused by the fact that its overtones are somewhat dissonant with each other.

15.16. Sound Waves in a Solid. — If a small rod is stretched or compressed longitudinally, there is an accompanying contraction or expansion laterally. There is therefore a change of shape of the material as well as a longitudinal dilation or compression. But if the solid is extended in all directions, this change of shape laterally is not free to occur and the velocity of the sound wave is not the same. The velocity of a longitudinal wave in an extended solid can be expressed by an equation similar to (1.2) of Chapter I, but E is then not a single constant but the sum of two. One of these elastic constants deals with volume elasticity and the other with shape elasticity.

But if instead of using a longitudinal force at the end of a rod, causing longitudinal waves, we had used a force twisting the rod first one way and then the other, we would have caused a wave of twist or a torsional wave in the rod. Thus it is possible to have in an extended solid a second kind of wave. Its velocity is expressed by an equation similar to that used for the longitudinal wave, E, depending upon the same two constants as before, but not in the same manner. The velocities of the two waves are not equal. Hence if we had spherical waves of both types starting from a point, one wave would travel faster than the other. In fact, the elastic constants are such that in most materials the velocity of the longitudinal wave is roughly twice that of the torsional wave.

Then it is possible also to produce transverse waves in a solid. Rest a thin horizontal bar on two edges placed approximately

0.224 of the length from each end. It will vibrate transversely with these edges as nodes, and the ends of the bar as loops. It will give a relatively pure tone because the natural frequencies of the rod are widely separated and, moreover, the fixed position of the nodes makes possible only a few of these. In fact, experiment and theory both show that the first overtone that would have a node at the same point has 13.3 times the fundamental frequency, or between three and four octaves above the fundamental. The position of this node is approximately at 0.226.

A tuning fork is a rod bent into a U shape, but with the addition of a mass of metal at the bend. Also the nodes of the bar approach each other as the rod is bent. The first overtone of a bar bent into a U shape is about six times the fundamental frequency. The wide separation of the fundamental from the first overtone and the relative faintness of latter are the reasons for the purity of the tone of the tuning fork.

15.17. Vibration of Bells.—A bell may be regarded as a vibrating cylinder. Consider Fig. 15.2 showing an exaggerated cross-section perpendicular to the axis of the cylinder. Assume that the bell experiences its fundamental vibration from one approximate ellipse to another with perpendicular axes. If the reader will draw the circle and the two ellipses he will see that the four "nodal" points on the circumference are not stationary. The points of intersection of one ellipse and circle are not the same as the crossing points of the other ellipse and circle. If there were stationary nodes we might assume that transverse (or torsional) waves were the only ones existing in the bell. But, under the circumstances, both longitudinal and transverse waves exist. It can be shown, as may be here surmised, that the overtones are not harmonics. (See Section 15.18.)

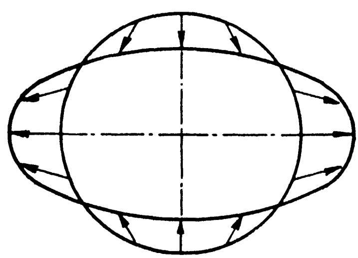


FIG. 15.2

The physical requisites of a good bell are that the material be homogeneous, relatively free from viscosity and have an elastic limit * that will not be exceeded in ordinary use. The fundamental tone of the bell is not prominent. In fact, it does not determine the pitch of the bell. The pitch of the strike note is an octave below the fifth partial or the fourth overtone, and in a bell not intrinsically tuned (see Section 15.18), does not correspond to any actual vibration of the bell. The cause of this judgment as to the pitch of a bell is not understood.†

15.18. Carillons and Chimes. — There seems to be some looseness of definition in the literature concerning the terms "carillon" and "chime." Both refer to a set of bells tuned to a musical scale, but in the carillon all the semitones appear. It is obvious that such bells must be carefully tuned to pitch. This is done by casting each bell for a slightly higher tone than desired and then by removing some of the casting by cutting and abrasion with the aid of machinery. Bell founders have for many years considered the seven desirable tones of a bell to have the following frequency ratios, 1 : 2 : 2.4 : 3 : 4 : 5 : 6. But this result cannot be secured except by the most skillful application of the art. Indeed, at the present time, a bell ranks with the best if only the fundamental and the first four overtones with the ratios stated are obtained. Even this requires machining in a manner that is regarded as a trade secret.

But bells with tuned overtones are unusual and occur only in the most expensive carillons. An illustration of what is considered a good bell will now be cited. Dr. A. T. Jones has carefully measured the frequencies of the largest bell of the Dorothea Carlile Chime at Smith College ‡ and has found that the first

* An elastic body when stretched will return to its original position when the stretching force is removed, provided a certain limiting tension is not exceeded. This limiting tension per unit area is called the elastic limit.

† See article by A. T. Jones, *Physical Review*, XVI, 4, 1920, p. 247, and *Journal Acoustical Society of America*, I, p. 373, 1930.

‡ A. T. Jones, *Jl. Acous. Soc. Am.*, I, p. 373, 1930.

seven partials* have the ratios 1.00 : 1.65 : 2.10 : 3.00 : 3.54 : 4.97 : 5.33, which are very different from the ratios given above for a bell with tuned overtones. It is surprising that a bell like the one described could be pleasing to the ear. But it must be remembered that harmonics of the seven tones are not present, and consequently the dissonance that would occur in sounding the same tones on a string instrument is not observed. The bells with tuned overtones are, however, a distinct improvement and will be increasingly used.

15.19. Acoustic Power Output. — Dr. P. E. Sabine has measured the acoustical power output of certain musical instruments. A violincello *in its fundamental*, when bowed strongly, varied from 100 microwatts (or 1,000 ergs per second) at 128 cycles to one microwatt at 650 cycles. A good violin gave a fairly uniform output of 60 microwatts for frequencies from 192 cycles to 1,300 cycles. Open diapason organ pipes gave an output of approximately 1,000 microwatts. All the values refer to the fundamental tone and not to the entire output.

15.20. Modern Loud Speakers. — This text has shown that with closely coupled systems one may get much more energy from the source of vibration with than without the condition of resonance. Moreover, it has been recognized that any mechanism having elasticity and inertia may be likened to a spring and a weight suspended therefrom. There is a natural period of vibration. Modern loud speakers frequently have a cone-shaped surface, actuated electrically at its vertex by the complex vibrations which eventually are to be given to the air. The vibration at the vertex cannot convey sufficient sound directly to the air. The cone is employed because of its large exposure to the air. It is a sounding board. But being light and far from rigid, the cone has natural frequencies of its own. Thus in actual use, any of the cone's natural frequencies occurring in the original sound at the transmitting station will be overemphasized by the loud

* The term "partial" is used to include both the fundamental (first partial) and the overtones.

speaker. Obviously the cone could be made of very rigid material, so rigid that its natural frequencies would be higher than any used at the transmitting station. But then it would be too massive to actuate. One of the difficulties is to obtain a cone light enough, large enough and yet without introducing a distortion of the complex sound. Another important feature of interest is the use of a plane, rim, or surface surrounding the base of the cone. It may be called a baffle plate. It has three functions. It separates the inside of the cone from the outside, thus preventing the slippage of air from the outside of the cone to the inside. Such a slippage could decrease the pressure caused by the vibration of the cone and hence the intensity of the sound wave. The baffle plate also reflects the sound. In Section 5.5 it is shown that by this reflection a greater amount of sound issues from the vibrator. The third function is that of shutting out the vibration from the other side of the cone. This is desirable for one can readily see that the vibrations on the two sides are opposite in pressure phase and will produce interference.

Horns, usually with diameters increasing much more rapidly than the length, are used in loud speakers in theaters and public address systems. They are coupled systems in which the horn and the moving diaphragm each has its own natural frequencies. The endeavor is made to reduce these resonance effects to a minimum. A practical discussion of horn design is given by Hanna.* He favors a horn of the "exponential" type, for theory points approximately to this shape. An exponential horn doubles its area at equal intervals along its length.

QUESTIONS

1. What mechanical conditions must be met if a musical instrument is to give great intensity of sound?

2. In what ways is the transfer of energy from the initial vibrating portion of a musical instrument to the atmosphere accomplished?

3. Under what conditions are two vibrating systems "coupled?" Give an illustration.

4. Give an illustration where three vibrating systems are coupled.

* Hanna, *Journal Acoustical Society of America*, II, 2, p. 150, 1930.

5. By what means (ideal and not practical) can a string be set into vibration of the fundamental alone?

6. In what way does the nature of the piano hammer affect the quality of tone?

7. While one holds a piano key down, what is the position of the hammer, of the damper?

8. Discuss the possibilities of the qualities of a tone in a brass instrument and the ease of blowing being altered by the adjustment of the mouth.

9. How do you know that the vibration communicated to the body of a violin is not one caused by the changes in tension of the string during its vibration?

10. What is the reason for using, in the violin, a wood having a high sound velocity for transverse displacements?

11. Physically how can tone quality be modified?

12. Upon what does the quality of the voice depend?

13. State the difference in action of the sustained and the impulsive sounds of the voice.

14. Explain the howling of the wind.

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