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THE PROGRESS OF ATOMIC THEORY.

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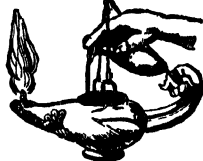
ALBERT C. CREHORE, Ph.D.

LONDON.

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PREFACE.

THIS volume contains an account of the Author's conception of the structure of atoms based upon the classical electrodynamics. The models of the atoms shown in figs. 2 and 2 A were constructed before the recent experimental results of Rutherford and those who have since been using his methods were known to the Author. It will stimulate the reader's interest in the work to know of the subsequent experimental confirmation of many features of these models.

The atomic theory here presented differs radically from other existing theories and embraces phenomena which other theories do not include. The first of these is Newton's law of gravitation and his law of equal action and reaction. Another is the demonstration that the force concerned in chemical reactions, the so-called chemical affinity, is chiefly an electrostatic phenomenon. The ordinary law of electrostatic action common to all forms of the classical electrodynamics is sufficient, when used in connection with these models, to account for this phenomenon. It is not entirely an electrostatic effect but chiefly so, for the directions of the axes of rotation of the fundamental charges are controlled by forces due to their motion.

Perhaps the greatest departure from all other atomic theories is the absence of the so-called valence electrons, the whole atom being neutral and self-contained within a space of the order of 10^{-12} cm., or in the larger atoms perhaps 10^{-11} cm.

across. There has already been noticed in the literature on this subject a tendency to reduce the number of electrons formerly supposed to exist in rings at a considerable distance from the centre of the atom, as, for example, in the mercury atom, and it is the Author's opinion that this reduction will continue until there are no electrons standing alone at a distance from other electrons.

It must occur to everyone familiar with this subject that the fundamental difficulty in building an atomic theory upon any form of the classical dynamics is Planck's quantum theory. It is believed that this issue is squarely met, and that the quantum theory will be established eventually as a consequence of the classical electrodynamics when considered in the light of the correct form of an atom. The statements made in this work as to how this may possibly be may not go all the way, but it is believed that the suggestions will prove fruitful.

The conclusion is reached that all atomic phenomena are intimately bound up with the energy content and properties of the elementary fundamental bodies. The energy changes in chemical reactions are traced to this same source, the energy content of these fundamental charges, the atoms in their free state not having exactly the same energy content, that is, not being exactly the same atoms, as when they enter into a molecular structure.

More chapters might be added to cover certain phenomena not touched upon, such as the photoelectric effect, the secondary beta-rays generated by X-rays, etc., on which the theory sheds much light, but it seems unwise to discuss these topics at the present time until the matters here presented have proved their value.

In the main only the results of the mathematical investigations by means of electrodynamics are given in this book, but,

even with the omission of the processes of their derivation, the introduction of some equations is required as being a more concise way to tell the story. The discussion and interpretation of these results will be of more interest to the general reader. For more details the reader may refer to the Author's series of original papers in the Philosophical Magazine beginning in October 1921.

ALBERT C. CREHORE.

Cleveland, Ohio, U.S.A.,

August, 1922.

More than three years have passed since the above preface was written. An interruption of the investigations occurred, but they were resumed two years ago. The delay seems fortunate on the whole because it is now possible to add considerable material which supports the form of atomic model with which the work is concerned by comparisons with experimental data. The new material is contained in Chapters IV, V, VI, and XII. The discovery has been made that there exist atomic constants characteristic of each atom and closely related to the properties of the negative electron in connection with the manner in which they are attached to the atoms. The physical meaning of these constants is clearly understood in terms of the atomic models, and the universal distance between the centres of nearest atoms in solids of the order of 10^{-8} cm. is shown to be directly due to the effects of these constants.

It has been possible to add the chapter on crystal structure because the expression for the electrostatic force between electrons in general positions in space has now been obtained.

This was restricted to the special case of parallel axes until quite recently, which prevented any study of crystal structure in terms of the models. The forces between two hydrogen or two carbon atoms in general positions have been obtained, and their application to crystal structure in Chapter VI shows a complete agreement with experimental results so far as known. All of this new material is independent of any special form of electrodynamics.

ALBERT C. CREHORE.

October, 1925.

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THE PROGRESS OF ATOMIC THEORY.

CHAPTER I.

INTRODUCTION.

THE word "atom," derived from the Greek, literally meaning not cut-able or, better, indivisible, was originally chosen to designate these bodies because it was thought that they represented the ultimate particles of matter always the same. To-day this is known to be untrue, and these once ultimate particles are observed to be changing, going to pieces. Radioactive substances such as uranium, thorium, and particularly radium make it quite certain that their atoms are automatically breaking into pieces. They project from themselves two different kinds of bodies, the one a negatively charged electron, and the other what has been termed an alpha-particle. The negative electrons are particles always having the same amount of negative electricity from whatever source they are derived, while the alpha-particle always carries a charge of positive electricity of twice the amount of the charge on the negative electron.

The relative masses or the weights of all atoms from the lightest, hydrogen, to the heaviest, uranium, are included between the numbers 1 and 238, a very moderate range of variation considering that the mass of the negative electron on this same scale must be expressed by the fraction $1/1846$. This body is, therefore, in a class by itself and is not regarded as a new atom, though so far it is one of the very few bodies we know that most satisfies the original meaning of the word a-tom.

The alpha-particle, on the other hand, has a mass of four on the atomic scale, and is not in a class with the negative electron. It is known to be what is left of a helium atom after it has lost two of its negative electrons.

These automatic radioactive bodies, radium and all its by-products, continue on their independent way breaking to pieces at a fixed rate, different for different substances but fixed for the same substance, some taking a long time to reduce to half their value, thousands of years, and others a short time, days, hours, and minutes. Nothing that man has yet been able to do to such bodies has had any effect in accelerating or in retarding these rates of decay characteristic of each substance.

However, very recently Sir Ernest Rutherford has succeeded in inducing some of those atoms that do not automatically disintegrate to fly to pieces or explode, as it were. For this purpose he has used as projectiles the alpha-particles that are automatically ejected from some product of radium. He has succeeded in breaking up several of the lighter elements in this way, and has obtained hydrogen from a nitrogen atom and also alpha-particles from it.

From what has been said it is evident that the former idea of the atoms can no longer be entertained, but this has been known for so long a time that every one with even a slight interest in science has read something about the words "atomic structure," which in itself implies that the atoms of the elements are built up of parts. All of the different theories that have been proposed regarding atomic structure make use of the negatively charged electron as one constituent and of a positive charge or charges as a second constituent, but these are the fundamental things generally supposed to form the atoms, the bricks, so to speak, that compose the structure.

What are these bricks? How large are they, and what are some of their properties? How many of them are to be found in any neutral atom? Where are they located with respect to each other, and what is the state of their relative

motion or rest? Do any of them exist in contact with others, or are they separated by distances considerable compared with their own size?

• The several different theories of atomic structure may be classified in accordance with the answers they give to these questions. The fundamental assumptions of a theory play their part in shaping these answers. To make certain assumptions and then reason from them by means of certain principles of electromagnetism to certain results seems to be the only available mode of procedure. If the results compare favourably with experimental observations the assumptions and the deductions therefrom are strengthened or justified, but, if not, they must be changed until all discord is made to disappear. Let us first consider these bricks, so to speak, that compose the atoms.

The Negative Electron.

Fortunately, the approximate size of the negative electron has recently been determined experimentally by Sir Ernest Rutherford, and this will effectually put an end to the many different opinions that have been entertained in regard to it in the past. All those who have theories of atomic structure will have to make their electrons of this size. It is a body about 3.5×10^{-13} cm. across. Its shape, whether spherical or not, was not determined by Rutherford, but good evidence has been obtained since that it is not a spherical body, though what its shape is has not been experimentally determined.

The negative electron used in the Author's atomic models, with which this volume is particularly concerned, was made in accordance with electrodynamics in so far as this offered a manageable solution of the problem at a time before the experimental results of Rutherford, above mentioned or to be mentioned below in connection with the atom models, were known to the Author. This statement seems important in view of the very close agreement between the theoretical and experimental results described here.

Let us try to imagine, if possible, a spherical surface within which there is a quantity of electrical charge, say E , uniformly distributed throughout the volume, and let us also suppose that each small element of the charge throughout the whole volume is repelled by every other element with a force that is directly proportional to the product of the charges of the two elements in question and inversely as the square of the distance between the elements. The result of all these forces added together upon any element near the surface of the sphere must evidently be a large force directed along a radial line away from the centre of the sphere, and if there is nothing to prevent its moving outward along this line, such forces would cause the whole sphere to expand without limit. While the volume within the spherical surface is supposed to be filled with the electrical charge, the volume or space outside of this surface is supposed to contain nothing unless it be the universal medium or ether. It is not necessary to raise this old question of the existence of an ether here, for it will have to be left in much the same state as it now stands.

For the moment the source of the pressure on the surface of this imaginary sphere to prevent the electrical charge from expanding without limit need give us no concern. To compress the sphere from a larger radius a_1 down to a smaller radius a_2 must require that work be done against these mutual forces of repulsion we have supposed to exist. The amount of the work that must be done to contract this sphere from the one radius to the other has been calculated and is to be found in standard text-books on the subject. Assuming that the density remains uniform as the contraction occurs, the resulting work may be shown to be equal to

$$W = \frac{3E^2}{5k} \left(\frac{1}{a_2} - \frac{1}{a_1} \right) \dots \dots \dots (1)$$

against these forces. Here k is the dielectric constant of a vacuum or the space outside the sphere, E is the charge within it, a_1 and a_2 are the initial and final radii of the sphere. If

the contraction began from a very large radius the second term of the equation would be vanishingly small in comparison with the first term, so that the total work to contract such a sphere from a very large radius down to a small radius, a , would be simply

$$W = \frac{3E^2}{5ak} \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot (2)$$

According to this the work to be done is inversely proportional to the radius. This may be thought of as potential energy possessed by the sphere, for, if it were allowed to expand indefinitely, it would be capable of performing this amount of work. The expression might, therefore, be regarded as the energy content of the charge. And if ordinary mass is connected with the energy content of a charge, it seems reasonable to think of the most massive body as that having the smallest dimensions—that is, for a given charge.

This conception is in exact agreement with another result also found in the text-books on electrodynamics. The mass of such a solid sphere as described, as given by the Lorentz mass formula when the sphere has no translational motion, is

$$m = \frac{4}{5ak} \left(\frac{E}{c} \right)^2, \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot (3)$$

where m is its mass, a its radius, E its charge, c the velocity of light, and k the dielectric constant of a vacuum.

It is now to be noticed that $E^2/5ak$ is a factor common to both equations (2) and (3). Hence it may be eliminated between them so that the energy, W , may be expressed in terms of the mass, m , and the velocity of light only, thus

$$W = \frac{3}{4} mc^2. \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot (4)$$

This shows that the mass, m , and the energy, W , are proportional to each other, since the velocity of light is constant, and there is justification in saying that the more massive bodies are those that have the greater energy content, and in direct proportion to the mass.

Now Einstein and others have come to the conclusion, on independent grounds, that the total energy content of a piece of matter is just mc^2 and not some fraction of it like $3/4$ or $1/2$. Giving weight to this conclusion it becomes difficult to believe that the whole energy content of the charge is correctly represented by the formula (4). There must be an additional amount of energy not accounted for by this formula equal to $mc^2/4$. Before considering this point, however, let us stop to see whether the size of the sphere as determined by the formula (3) agrees even in an approximate manner with the experimental result of Rutherford above mentioned as being 3.5×10^{-13} cm. across the negative electron, for the error in the energy as determined by (4) is not so very great.

The only quantity that has not been experimentally measured in equation (3) is the radius of the sphere, a . Using the most reliable determinations of the three quantities E , c , and m , and taking $k=1$, the resulting value of the radius, which may now be designated by r , is

$$r = 2.244 \times 10^{-13} \text{ cm.}$$

or

$$2r = 4.488 \times 10^{-13} \text{ cm.} \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

This figure is so close to the diameter 3.5×10^{-13} cm. found by Rutherford experimentally as to leave little doubt that there is much of truth in the reasoning by which the theoretical value has been obtained. If it were far wrong the value would not have the same order of magnitude, 10^{-13} cm.

Let us now return to consider the discrepancy between the energy derived in (4) for the static electron and the energy mc^2 , which probably represents the true total energy. If such a sphere as supposed should accidentally be set into rotation in some way, electrodynamics shows that new forces due to the rotation would be brought into existence, and the question should be considered whether these forces would tend to stop the rotation and thus restore the charge to the original static state, or whether they show that the static state is one of

instability. By this it is meant that the least initial motion of rotation either to the right or to the left might conceivably set up such forces that the motion would increase instead of stop up to a certain equilibrium speed such that all forces attain their perfect equilibrium at some critical speed. If this be the case such a thing as a static charge becomes a pure fiction, and something very essential was left out from the original hypothesis above stated.

It is admitted that this problem has not yet been solved in a rigid manner by the use of electrodynamics, but there are good reasons for thinking that a rotating electron is in perfect harmony with this doctrine. There are strong reasons that have led the Author to make the hypothesis of a rotating electron. A purely static atom is untenable from the standpoint of electrodynamics, and unless one is willing to make hypotheses diametrically opposed to any form of electrodynamics, the only kind of motion of an electrical charge that can be permitted in a neutral atom when not radiating energy must be such motion that no energy is radiated. A spherical charge in rotation theoretically radiates no energy. It may be thought of as made up of elementary continuous rings of charge, each of which by itself radiates no energy.

On the other hand, a point charge describing a closed orbit of any kind does radiate energy according to any of the forms of electrodynamics. Motion in a straight line at a uniform velocity only is excepted, but a closed orbit could not be formed by such motion. Hence, in order to adhere strictly to electrodynamics, all forms of orbits have been abolished from the models of atoms in their normal and neutral state in the Author's models to be described. The only motion in these atoms when at the absolute zero of temperature is supposed to be the rotation of the individual electrical charges upon axes.

But the rotation on an axis must cause the bounding surface hitherto supposed spherical to change its shape. The shape of the electron has been deduced in the following manner, and the fact that the experimental results already

obtained bear out these conclusions derived before the experimental results were known is sufficient justification for giving an account of the matter here.

It has been stated above that the static spherical charge would have a tendency to expand, and the work required to contract the sphere against these forces was given in equation (1). The force itself was not given, but it is just as well known as the work, and the pressure per square centimetre perpendicular to the surface is given as follows:—

$$P = \frac{3E^2}{8\pi a^4 k} \dots \dots \dots (6)$$

Mathematically, a pressure upon a surface in general is resolvable into a tension along the surface. If the surface is not spherical and has two radii of curvature at any point in question, say r and r' , the relation between the tension, T , and the pressure, P , above given, is as follows:—

$$T = P \left(\frac{1}{r} + \frac{1}{r'} \right) \dots \dots \dots (7)$$

In the case of the sphere the two radii r and r' are each equal to a , and the formula becomes

$$T = aP/2 \dots \dots \dots (8)$$

The pressure given by (6) may, therefore, be equally well expressed as a tension along the surface by combining (6) and (8) as follows,

$$T = \frac{3E^2}{16\pi a^3 k} \dots \dots \dots (9)$$

Let it now be assumed that the volume of the charge—that is, the volume of the sphere—does not change when the rotation begins and that the same surface tension also remains that has just been computed for the sphere. The shape of the surface will change because of the rotation, but the sphere has the minimum surface possible for its volume. Any change in its shape must increase the area of its surface, provided its volume is not to change. If, therefore, its area due to the rotation is increased over that of the sphere by an amount, say A , then

there must have been a certain amount of work done against the surface tension equal to AT in order to make the change in shape, provided the surface tension has remained the same. To find this work required it is only necessary to multiply (9) by the area, A , giving

$$W = AT = \frac{3AE^2}{16\pi a^3 k} \cdot \cdot \cdot \cdot \cdot \cdot (10)$$

Now it has been pointed out above that the total energy content of the static sphere was deficient by the amount of the difference between mc^2 and that given in (4)—namely, $3mc^2/4$. This is a deficiency of $mc^2/4$, and, if an amount of work equal to (10), to change the shape from the sphere into some other shape, will account for the whole of this deficiency, then we may equate (10) to $mc^2/4$, giving

$$\frac{3AE^2}{16\pi a^3 k} = \frac{1}{4} mc^2 \cdot \cdot \cdot \cdot \cdot \cdot (11)$$

But, according to (3) above, we have

$$\frac{1}{4} mc^2 = \frac{E^2}{5ak} \cdot \cdot \cdot \cdot \cdot \cdot (12)$$

Hence by (11) and (12)

$$\frac{3AE^2}{16\pi a^3 k} = \frac{E^2}{5ak}, \cdot \cdot \cdot \cdot \cdot \cdot (13)$$

whence the area, A , by which the surface of the sphere must be increased, comes out

$$A = 16\pi a^2/15 \cdot \cdot \cdot \cdot \cdot \cdot (14)$$

That is to say, the shape of the new form caused by the rotation ought to be such that its area is a little more than one great circle of the sphere greater than that of the sphere itself. The area of the sphere, being four great circles, $4\pi a^2$, the area of the new surface ought to be $76\pi a^2/15$.

The most natural shape to assume for such a rotating body, lacking any rigid proof of it by means of electrodynamics, is that of the oblate spheroid, the same geometrical figure as that assumed by the Earth or a rotating raindrop. Immediately this assumption is made the foregoing reasoning about the

area makes it possible to determine the ellipse which represents its meridian section. Its eccentricity comes out

$$e=0.945. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (15)$$

The ratio of the semi-major to the semi-minor axis is

$$a/b=3.058.$$

Assuming that the original sphere has a radius given in (5) above and that the volume of this ellipsoid is equal to that of the original sphere, the semi-major axis, a , of the ellipse comes out

$$a=3.257 \times 10^{-13} \text{ cm.}$$

and the semi-minor axis

$$b=1.065 \times 10^{-13} \text{ cm.} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (16)$$

This means that the smallest diameter is about 2.13×10^{-13} cm. When all orientations to be encountered during a bombardment of a gas by alpha-particles are allowed for, the electron as experimentally measured must be a proper kind of mean or average between these two extreme diameters. This average may easily be closely equal to the experimental diameter found by Rutherford, 3.5×10^{-13} cm., for the experimental value lies between the greatest and the least theoretical diameters, and in about the right proportion.

The Positive Electrons.

As to the positive charges that constitute the other kind of bricks that compose the structure of atoms, there is not the same clear experimental evidence as there is in the case of the negative electrons just described. As was stated above, the alpha-particle, which is automatically shot out from radioactive substances, carries a positive charge of twice that on the negative electron. The hydrogen ion carries a positive charge equal to that of the negative electron.

There is some evidence which goes to show that the alpha-particle is not a single body of positive charge, but that it is composed of three bodies at least—two negative electrons and a positive charge. The positive charge must be four units if

this is the case, since the total charge of the combined individuals is two, and $4 - 2 = 2$. There has been much speculation as to how this charge of four units is made up, whether it is one body or is composed of several together. It has been the view of Rutherford and others that this positive charge of the alpha-particle is made up of four single units, each equal to the charge of the negative electron. And at the same time it is their view that this unit charge is equal to that of the hydrogen ion, according to which the hydrogen ion represents nothing but a single unit of positive charge.

There is very little evidence to support these opinions. If they represented the truth it might well be expected that an alpha-particle would under some circumstances break up into hydrogen atoms or ions. This has never been experimentally accomplished. If it were it would settle the matter, but there are reasons, which will be given in a later section, that point to the hydrogen ion as being itself a complex structure analogous to the alpha-particle. In the Author's atomic models the hydrogen ion is made up of at least two individual bodies, one negative electron and a positive charge of two units, so that the total charge is one positive unit, since $2 - 1 = 1$. To make a neutral hydrogen atom requires that the ion shall acquire one more negative electron. The normal hydrogen atom thus consists of a positive charge of two units and two negative electrons. It, therefore, differs from the alpha-particle only in the value of the positive charge.

It is still an open question whether or not the positive charge of either the hydrogen atom or of the alpha-particle is a single body, or whether it is composed of units equal and opposite to the negative electron. In the Author's models they are taken to be single individual units, of charges two and four respectively. It is not expected, according to this, that a hydrogen atom will ever be formed from alpha-particles. According to these conceptions a positive charge is never encountered alone detached from all negative electrons. At least one negative electron accompanies every positive charge.

On this view, it is not surprising that there have been no experimental measurements as yet of the positive charge when alone. The best that can be done to estimate its size is, then, to use the same mass formula of Lorentz which, was so successfully employed in the case of the negative electron, equation (3) above, from which it immediately appears that the diameter of the positive bodies must be very much smaller than that of the negative electron. For the radius of the sphere is inversely as the mass, and the mass of the hydrogen ion is 1846 times greater than that of the negative electron. This conception connects the mass of the whole atom chiefly with the positive charge where it belongs, and the small size is very natural, because it requires much more work to compress the charge within the smaller volume, as pointed out above.

Using the known mass of the hydrogen atom as the m in equation (3) above, let the radii of the charges of the positive spheres be computed on the three assumptions that the charge is e , $2e$, and $3e$, so as not to assign as yet any definite charge to the strictly positive portion of the hydrogen atom. The radii come out:

<i>Charge.</i>	<i>Radii.</i>
e	1.215×10^{-16} cm.
$2e$	$4 \times 1.215 \times 10^{-16}$ cm. = 4.86×10^{-16} cm.
$3e$	$9 \times 1.215 \times 10^{-16}$ cm. = 10.935×10^{-16} cm. (17)

The same reasoning that was used above for the negative electron applies with equal force to the positive charge, in order to make its energy content equal to mc^2 instead of $3mc^2/4$, which it would be if assumed to be a static sphere. The result is that the eccentricity of the ellipse representing a meridian section is, as before, $\epsilon=0.945$, and the ratio of the major to the minor axes, as before, $a/b=3.058$. If the above spheres are converted into ellipsoids each having this same eccentricity and maintaining the same volume as the respective spheres above, we have three ellipsoids of the following

dimensions, as representing the possible positive charges, e , $2e$, and $3e$:

<i>Charge.</i>	<i>Semi-major axis.</i>	<i>Semi-minor axis.</i>
e	1.763×10^{-16} cm.	0.577×10^{-16} cm.
$2e$	7.054×10^{-16} cm.	2.307×10^{-16} cm.
$3e$	15.867×10^{-16} cm.	5.193×10^{-16} cm. (18)

It must be understood that only one of the three sets of dimensions corresponding to the charges e , $2e$, and $3e$, can be correct, depending upon which of the three charges is correct for the hydrogen atom. The matter has been presented in this manner, because there is a way to pick out the approximate size of the positive charge of the hydrogen atom, which is discussed in a subsequent chapter, better than the charge of it. The size comes out very close to the second row of figures in the above table, thus giving the preference to $2e$ for the positive charge of a hydrogen atom which has been used in the model. This, of course, assigns two negative electrons to the normal atom to neutralize this positive charge.

According to all the figures above given, the dimensions of the positive charge are expressed by numbers of the order of 10^{-16} cm., whereas the dimensions of the negative electron are expressed by 10^{-13} cm. In any model of an atom, therefore, which attempts to represent these charges to scale, the positive charges would scarcely be visible when the negative electrons have a convenient size. The difficulty is of the same kind as one meets in attempting to represent the solar system to scale in a model. If the Sun is made of a convenient size the Earth is scarcely visible. Moreover, if the Earth's orbit is represented to the same scale as the Earth itself, the Earth, again, is scarcely visible when the orbit has any convenient dimensions. The negative electron is by far the largest single individual body of charge in size that enters into the structure of an atom or that has any existence. Hence, the principal things that one would observe in atoms, or models of them, must be the negative electrons. If these are represented as of moderate size, the positive charges would be represented apparently by points

or very small bodies, the location of which becomes the important thing.

Atomic Distances.

The distances between the centres of the nearest atoms in crystals, and in solids generally, is known to be of the order of 10^{-8} cm. roughly by means of measurements, now familiar to all those having a slight acquaintance with experimental physics, making use of X-rays. Indeed, the order of magnitude of these distances is, perhaps, more firmly established than anything else in the atomic realm. Accordingly, it would require an enormous number of negative electrons of the dimensions above given to fill the volume allotted to one atomic unit in a crystal, if they were so close to each other as to be in contact. It would, for example, require between 10,000 and 100,000 of them to bridge the distance 10^{-8} cm. if they touched each other and were in one straight line only, to say nothing of filling the volume, for 10^{-8} cm. is 100,000 times greater than 10^{-13} cm.

All theories of the structure of atoms agree that the number of electrons per atom is comparatively small—nothing like one million or more. Granting this, it follows that there must exist immense spaces where there are no charged bodies between the atoms or between the electrons composing an atom or both. To represent this immense distance, 10^{-8} cm. (for all distances are relative), in a model where the negative electron has a convenient size would be, as in the example cited, like trying to represent the Earth's orbit in a model where the Earth has some convenient size.

Atomic Structure.

These statements make it clear that the task before the investigator is to find, in this immense space allotted to an atom, where the negative electrons and the positive charges belonging to it are located, and, if they are not in any one particular place in the normal condition of the atom, but are

in motion, to discover the character of the motion. But, after all this has been done, one may ask what good has been accomplished. Why may they not be located anywhere we please, and why is one plan of distribution any better than any other? The answer must be that there is little good unless it serves a useful purpose enabling one to understand better the phenomena of observation. Experimental observation does not reveal directly the locations of the fundamental charges within the atoms. It is only by a process of reason or thought that their positions may be deduced.

The word "deduced" implies that some fundamental underlying principle must have been used in reasoning back from the experimental observations to find something as to the state of the electrons. And, conversely, the use of the same principle, if broad enough, should enable one to predict phenomena that ought to be observable, if the true distribution and motion of the electrons were known. But, if we possessed no governing principle, not to mention no knowledge of the distribution of the electrons in atoms, the futility and uselessness of any theory of atomic structure is evident. And yet physicists think there is no more important subject in science than that of atomic structure. It has been made clear that there can be no success unless there be established a governing fundamental principle upon which the whole structure rests.

It is the Author's belief that the classical electrodynamics will prove to be the only principle that will be required, and the models of atoms described in the next chapter are based upon this thought. No other theories of atomic structure that have been proposed are based unreservedly upon this fundamental idea. They all depart from electrodynamics in their fundamental assumptions. The chief reason for this is that no way has ever been found to reconcile Planck's quantum theory with the classical electrodynamics. The quantum theory has now become an established fact, to which every form of atomic model must conform. In the past there has been an attempt to make use of two independent principles at once,

the classical electrodynamics and the quantum theory, without clearly realizing that the one may include the other. The classical dynamics is a broad doctrine and the quantum theory partakes more of the nature of a fact than a doctrine, which should require some explanation in terms of electrodynamics.

The reason for making the statements in the preceding paragraph is to make it clear that there is no intention of evading the real issue, and to show that the vital point at issue is understood. Anyone who succeeds in establishing an atomic theory upon electrodynamics is bound to connect the quantum theory in some way with electrodynamics. A suggestion as to what this connection may be is given in a subsequent chapter, but no claim is made for it that it represents the ultimate solution of this great question. It seems to the Author, however, that these suggestions have opened profitable channels for investigation, and that something closely akin to them represents the truth of the matter.

Proceeding, then, with the understanding that nothing shall be used in the atomic models which is in direct violation of the principles of the classical electrodynamics, this is already a great step in helping to eliminate certain things that must not go into the atomic model. But one must be willing to follow this lead without fear of the consequences, even though it may lead at times to conclusions that make it necessary to abandon preconceived ideas on the subject.

At the very beginning is encountered a demand of electrodynamics that presents the greatest difficulty to those who have been trained in a different way of thinking. Point charges in accelerated motion must radiate energy out into space. Electrons moving in orbits where the radius of curvature is large compared with the size of the electron are essentially point-charges, and any atom that contains electrons moving in such orbits with accelerated motion must radiate energy continuously. The atoms in all solid substances at moderate temperatures do not, as a matter of fact, radiate energy out into space at any such rate as they would be obliged

to if every electron in every atom were sending out energy at the rate electrodynamics demands. Even though it were assumed that some of this energy is reabsorbed by surrounding atoms in the interior of the solid, yet those atoms on the surface would have to lose energy at a greater rate than observation can justify. Recognizing this fundamental difficulty, all of the present theories of atomic structure make the pure assumption that no energy is radiated by the accelerated motion that they have adopted for the normal steady states of their electrons in the atomic models. This assumption is tantamount to saying that the classical electrodynamics is set aside as being inapplicable to the electrons in atoms.

This procedure has introduced great confusion of thought. The authors of these theories have recourse to electrodynamics for other matters in connection with their electrons, but they insist that in the matter of radiation it must not be used. In following electrodynamics without exception, one is forced to the conclusion that in the normal state of the atoms the electrons do not move in any kind of orbits, but remain with their centres stationary relative to each other. The only motion that exists at the absolute zero of temperature is the internal motion of the charge within the positive and the negative electrons. This conception has led to an electron as above described which is in close agreement with experimental observation. There are other experimental results, to be mentioned in the next chapter, which help to prove that the form of atomic models arrived at by following the lead of electrodynamics is in good agreement with the observed facts.

CHAPTER II.

ATOMIC MODELS.

IF a negative electron and a positive charge equal to twice that on the electron are now considered by themselves in the light of electrodynamics, each of these being such bodies as are partially described in the first chapter, there exists an attractive force between them, which, if unopposed, will draw the two charges together at all distances, and without limit until they come into actual contact with each other. The only forces that might prevent this result are those which may be generated by the motion of the charges, either translational motion as a whole, or internal motion such as described. It may not be necessary to repeat here the reason why these two bodies cannot continue to describe orbits about each other, but this result is so contrary to the prevalent thought of many about these matters that it must be stressed. If such orbits continue in a permanent state there must, according to any form of electrodynamics, be a continuous radiation of energy out into space, but there does not exist any such amount of radiated energy as demanded theoretically by electrodynamics.

There is an additional reason against the existence of such orbits on the ground of electrodynamics that may now be cited, even though it may show that electrodynamics itself is seemingly vulnerable, and may apparently weaken not only this but the preceding argument. It seems best to mention this matter now, however, in order to make it clear at the beginning that weaknesses are admitted to exist in the classical electrodynamics itself. It is a doctrine that has had a gradual historical development since the first mighty impulse given it by Maxwell, and it is far from the writer's thought to imply that it has reached its ultimate perfection. Neither can the

claim be made that it has not reached its perfection. There are two forms of the classical electrodynamics that may be mentioned: the one due to H. A. Lorentz, which has received more attention than any other form; and the other, more recent, due to Megh Nad Saha, which has not yet been investigated with thoroughness.

When applied to a so-called ring of two electrons circulating about a positive nucleus of charge $2e$ as a centre in the same orbit, one at each end of a common diameter, and in the same direction, both of these forms of electrodynamics show that the second electron exerts a force upon the first. When this force is resolved in the direction of motion along the tangent line to the orbit, the result obtained from the two different forms of electrodynamics is radically different. The Lorentz form makes this force due to the second electron in the direction of motion of the first electron. Without a retarding force from some other source, the effect of this force would be to make the electrons go faster and faster in their orbit. The Saha form makes this force due to the second electron in exactly the opposite direction—that is, opposite to the direction of motion. The effect of this force would be eventually to stop the orbital revolution of the electrons.

Had it not been for the early result obtained by the use of the Lorentz theory, which goes to show that rings of electrons may automatically keep themselves in revolution, it is extremely doubtful whether rings of electrons would ever have been suggested to form any part of any atomic model. But, even with this support, this theory encountered a formidable difficulty, because at the same time it demanded that there be a radiation of energy out into space from these single moving point charges or electrons. The only possible source of such energy was within the electrons themselves. To admit of this possibility, however, implied that there must be a continuous drain of energy for an indefinite time upon each electron. The absurdity of this is so great that no importance has ever been attached to it, though the suggestion

has been made in all seriousness. If there were any real support for rings of electrons, even by the Lorentz form of theory, the problem should have had a definite solution, so that the speeds of the rings could be exactly determined, and this would be one of the results that would help to prove the correctness of the assumption. If they approach a certain limiting speed this value should have been determinable, whereas it is impossible to compute it.

For this reason and others to be given in subsequent chapters, the Saha form of electrodynamics is preferred by the Author to that of Lorentz. This form of theory is consistent in demanding that electrons, which are started in orbits such as described, will gradually come to rest. No orbits are permissible without radiating energy. This latter statement is also true of the Lorentz theory, and of all forms of electrodynamics worth consideration.

The best test of any of the forms of electrodynamics is in comparing the results obtained by following the theory with the results of observation, for "by their fruits ye shall know them." So many of the results obtained by the use of Saha's electrodynamics agree with the results of observation that the Author has given the preference to this form of the theory rather than to that of Lorentz. This does not mean that the Saha electrodynamics is regarded as the ultimate form of its expression, but it may be. At least it may be said that nothing has yet appeared among experimental observations that is positively at variance with it. Until something does appear that positively contradicts its predictions, it seems most desirable to adhere to it in all strictness so far as this is possible, and to extend the field of its application. For it must be remembered that this theory has already pointed to a very different conception of an atom, and that very little work of investigation has been done with these atoms, and the correct form of atom is just as essential to success as the form of electrodynamics applied to it.

Having considered the translational orbital motion it

remains to consider the internal motion to see whether electrodynamics gives forces sufficient to overcome the electrostatic attraction between the two charges in question. It may be shown that the speed of the internal motion of rotation would have to approach the velocity of light in order that the repulsive force due to the motion shall be great enough to counterbalance the electrostatic force at any distance. On the one hand, there is reason to think that a rotational velocity approaching that of light does not exist within the charged bodies in their steady states, and, on the other hand, there is reason, which will be discussed in a subsequent chapter, to think that the actual velocity is comparatively slow.

The conclusion has now been reached that there are no forces revealed by electrodynamics due to an orbital, or to an internal motion that can prevent the positive and negative charges from coming together until there is contact between them, as stated in the opening sentence of this chapter. There is very good evidence, obtained experimentally, that the fundamental charges do not interpenetrate each other, but maintain their independent individuality at all times. Good evidence is afforded by considering the law of scattering of the alpha-particles when they are projected through hydrogen gas. Down to a certain very critical distance between the hydrogen atom and the alpha-particle, the law of scattering, which holds at greater distances, is followed; but at this critical distance there is a sudden departure from the law. At the same distance the hydrogen atoms are driven forward in the same direction, and with a much greater velocity than that of the alpha-particle, just as if the hydrogen atom had been struck a blow by the alpha-particle, and had taken up the momentum of it.

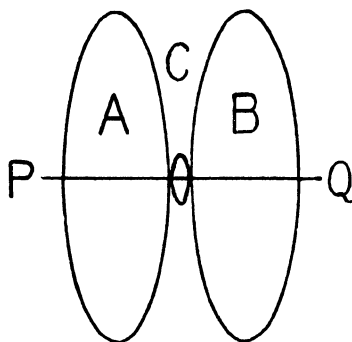
Until more is known about the nature of the positive and the negative electricity within the fundamental charged bodies it is safe to assume because of this experimental observation that they are such that they will not interpenetrate each other, or mix with one another, and that they maintain a definite

surface or boundary between the space within, where the charge is, and the space without, where there is none. The model atoms to be described are based upon this understanding.

The Hydrogen Atom.

The neutral hydrogen atom, according to this, must be as represented in the diagram (fig. 1), which is a meridian section

FIG. 1.



through the three bodies, the central positive charge, C, of $2e$ and the two electrons, one on either pole of the positive ellipsoid. They all have one common axis, PQ, and the centres of the charges are as near together as possible when arranged in this way. If the rotation of the charges in the negative electrons, A and B, is clockwise when viewed from Q, that of the positive charge, C, must be counter-clockwise, so that the magnetic poles will alternate, plus minus, plus minus, plus minus, along the axis.

The Alpha-Particle.

In a similar manner the alpha-particle may be represented by the same fig. 1, if the central positive charge, C, equal to $2e$ in hydrogen, is replaced by a charge of $4e$ for the alpha-particle. In order to show the positive charge in the figure

at all, it has been greatly enlarged compared with the dimensions of the negative electron, as before explained. If these bodies should be directly viewed by eyes of some kind adapted to see them, the negative electrons would be the principal objects visible, and the neutral hydrogen atom would appear very much like the alpha-particle.

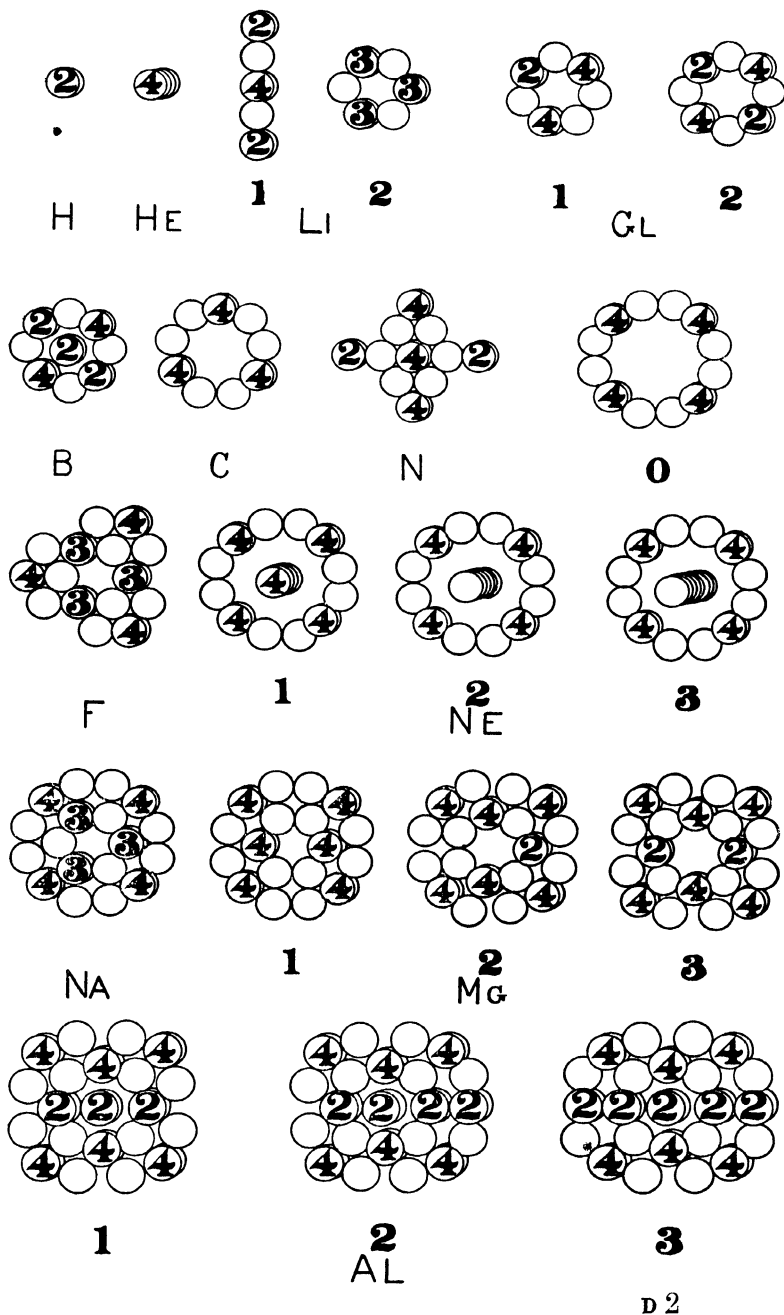
Consequently the distance through the alpha-particle along the polar axis must be twice the diameter of the negative electron, for the dimensions of the positive charge may be neglected. By equation (16) of Chapter I this distance is 4.26×10^{-13} cm. The longest dimension of the alpha-particle, a compound structure consisting of the three bodies, would be a diagonal line about 7.11×10^{-13} cm. in length, and it would not behave like a spherical body.

Quite recently Chadwick and Bieler, working along the same line as Rutherford's experimental investigations, have concluded from experiments they have made with alpha-particles that they are not spherical bodies, but are bodies having an apparent greatest length of 8×10^{-13} cm., and least length of about 4×10^{-13} cm., the ratio of the two lengths being roughly estimated as two to one. This result, obtained subsequently to the construction of the models, comes so close to the theoretical values above given, that it affords a very remarkable confirmation of the ideas that have led to the construction of these models. It even seems probable that these experimentalists will admit that the figures above given (7.11×10^{-13} cm. and 4.26×10^{-13} cm.) will fit their observations just as well as the approximate figures at which they have arrived, for they may easily fall within the probable error of their observations not stated.

Description of the Atomic Models.

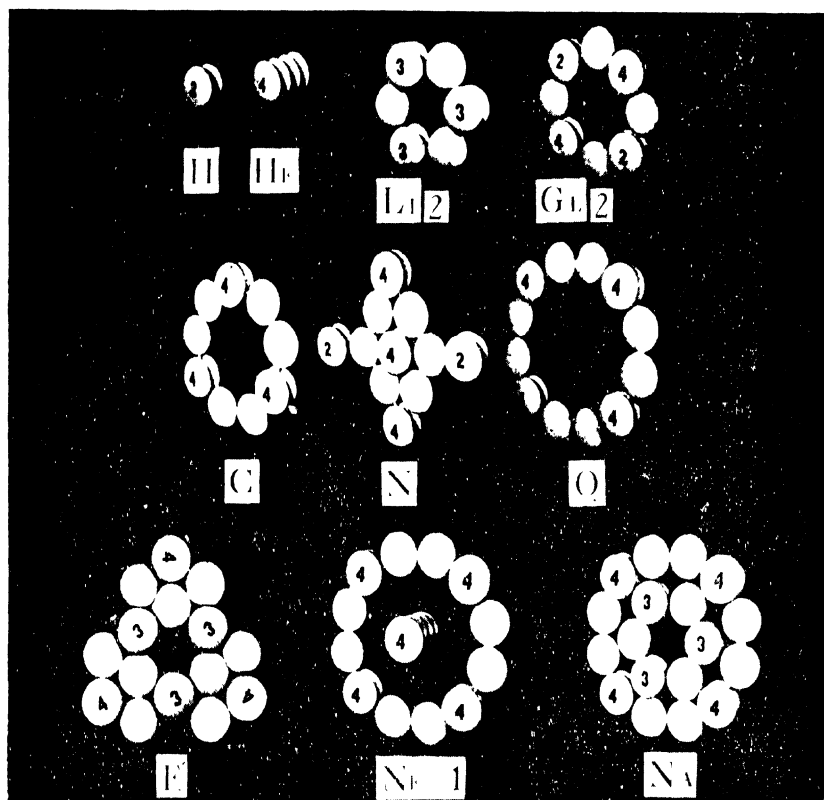
The model atoms in the accompanying fig. 2 are composed for the greater part of neutral hydrogen atoms, and alpha-particles bound together by the requisite number of single

FIG. 2.



negative electrons. But use is made also of another fundamental body of positive charge of $3e$, which is the only integral charge between $2e$ and $4e$. The reason for its use is that all of the atoms, and their isotopes, cannot be constructed in a symmetrical form at least by the use of hydrogen atoms

FIG. 2A.



and alpha-particles alone. As yet, however, there is no experimental evidence for the existence of this intermediate charge of $3e$, but at the same time there is no evidence of any kind against it. Let us first describe the atoms, in order of their atomic number, which are composed entirely of hydrogen atoms, alpha-particles, and negative electrons.

At H in fig. 2 is a drawing of a model of the neutral hydrogen atom described in detail in the preceding paragraphs. A photographic reproduction is shown in fig. 2A. The number 2 upon it designates the value of the charge of the positive central body, too small to be seen in the model.

Helium. Atomic Number 2.

Helium (He in figs. 2 and 2A) is shown as a pile of four negative electrons in contact with each other. The central pair of electrons, however, has a positive charge of $4e$ as one individual body between them, and forms an alpha-particle to all appearances just like the hydrogen atom. If the two outside electrons are removed from this helium atom the alpha-particle remains. The attraction between the positive charge of $4e$ and the adjacent electrons is so great, that it is difficult to dislodge them, and, as a matter of observation, the alpha-particle preserves its integrity with great tenacity. The two outside electrons, however, may be more easily dislodged, because the electrostatic force holding them is evidently much less than that holding the inside pair when they are alone.

With the neutral hydrogen atom, on the other hand, it is difficult to remove the last electron compared with the ease of removing the first one, and thus far experimental observations give no reason to suppose that the last electron is ever removed.

Since this model was constructed, Millikan has studied the ionization of helium atoms experimentally, and has announced that the single ionization of helium atoms occurs with much greater frequency than the double ionization when the atoms of the gas are bombarded with alpha-particles under the same conditions. The ratio of the frequency of occurrence of the single ionization to the double was determined to be about six to one. The ratio is quite understandable by the use of this model, although it would be difficult to predict the precise numerical ratio. In reasoning from his experimental evidence

in this investigation, Millikan was led to abandon the original form of the Bohr model of the helium atom as being untenable. In this Bohr atom the two negative electrons are supposed to revolve in the same circular orbit, one at each end of the same diameter, around a central point charge of $2e$.

The mass of the hydrogen atom, which is located almost entirely in the positive charge of $2e$, may be taken as 1.008 on the scale where oxygen is 16, and to this positive charge is to be assigned the mass 1.008, wherever it occurs in the more complex atoms. Similarly, the mass of the helium atom, which is located almost entirely in the positive charge of $4e$, has a value of 4.00 on the same scale, and is to be assigned this value wherever it occurs in the more complex atoms.

It should, perhaps, be stated that the form here assigned to the helium atom will be shown, in a subsequent chapter, to be such that it will neither unite with itself to form a compound molecule consisting of several helium atoms united together, nor will it unite with any of the other atoms shown in fig. 2. This result is in entire agreement with the well-known facts that helium is monatomic and that it refuses to form compounds with any of the other elements.

The total number of electrons in helium is four, a number equal to the atomic weight. The ratio of the number of electrons in helium to those in hydrogen is 2, which is equal to the atomic number.

Glucinum or Beryllium. Atomic Number 4.

This element has an atomic weight according to the latest international table of 9.1. Its weight is given to only two significant figures, and is more uncertain or inaccurate than that of any other element, if the accuracy may be judged by the method of changing the last figure given by unity, when the error would be more than 1 per cent. The fractional value given—namely, .1, indicates the probable existence of isotopes, that is to say, more than one form of the atom having the same atomic number, and consequently the same name; for

experimental observations have made it clear that the masses of the individual atoms are represented by whole numbers, or near-whole-numbers, on the scale of oxygen (16). The mass may vary by a few thousandths, or possibly by a few hundredths from a whole number, but a variation of as much as .1, at least in the lighter elements, has not been observed.

For example, hydrogen varies by .008 from unity, and nitrogen is .01 more than 14. Phosphorus is .04 more than 31, and sulphur is .06 more than 32. There are no isotopes of these elements known, and these numbers represent their true masses; but in a case like chlorine, where the atomic weight is 35.46, there are known to be isotopes. So in the case of glucinum the fraction .1 indicates the existence of isotopes, but, so far as the writer is aware, but one form of this atom has been definitely measured, and this has been found by G. P. Thomson to have a mass of 9. He states that if an isotope of mass 10 exists it has not been established by his investigation.

The isotope of glucinum of mass 9.008 is shown in fig. 2 as Gl-1. The mass is obtained by adding together the masses of the positive charges, there being two alpha-particles ($2 \times 4 = 8$), and one hydrogen atom (1.008), making a total of 9.008. To distinguish between masses of 9.000 and 9.008 requires considerable accuracy in the experimental measurements, and it is only very recently that the technique has been perfected to such an extent as to establish the fact that these small fractional values have any existence. There are in this model four single electrons which form the necessary connecting links to hold the whole structure together in the form of a ring.

A possible isotope of glucinum of mass 10.016 is shown at Gl-2 (figs. 2 and 2A). This is like the first isotope, only differing from it in having one additional hydrogen atom inserted in the ring. The number of the single connecting electrons is four, as before, and this is also the atomic number of glucinum, the same for each of the isotopes.

If these four negative electrons were removed to a distance, or if they are supposed to be held in some way at a distance from the nucleus of the atom as some have supposed, then the charge on this central nucleus would be equal to the atomic number 4. This charge would then be made up of two alpha-particles of two units each, the hydrogen charge being zero. It has been the general opinion that the charge on the nucleus, so-called, of an atom is equal to the atomic number. According to these models, however, the whole atom is practically the nucleus, and there are not supposed to be any outstanding electrons at considerable distance from the nucleus. All the atoms are thus normally neutral. The atomic number is equal to the positive charge that would remain if the connecting electrons were removed to a distance, or, which comes to the same thing, the atomic number is equal to the number of these connecting electrons.

One might be led to expect from the two isotopes, Gl-1 and Gl-2, that there ought to exist still another form obtained by removing the hydrogen atom from Gl-1, thus forming a hexagon ring containing but the two alpha-particles, which would have mass 8. In doing this the two alpha-particles must come nearer to each other, and their repulsion for each other, each having an effective charge of plus 2, is greater than the binding effect of the single electrons. If one arranges a hexagon of point charges to obtain a first approximation to the result, having two charges of plus $2e$ each on the opposite ends of one diameter, and single negative charges of e each at the four remaining positions, and computes the electrostatic forces between the two groups of charges, including one alpha-particle with its two adjacent electrons on the one hand, and the other alpha-particle with its two electrons on the other hand, resolving these forces along a line parallel to the diameter through the two alpha-particles, it will be found that the repulsive forces between the groups exceeds the attractive forces, and the ring will not be stable. There has been no atom discovered having

a mass of 8, but there exist atoms with multiples of this figure, oxygen, 16; magnesium, 24.

In studying the two isotopes of Gl shown in fig. 2, the question arises why the atom of mass 9 should apparently be the one of most frequent occurrence, for the weight of 9.1 obtained by the chemist is much nearer to 9 than to 10. Granting that the form Gl-1 is a stable structure, it is not difficult to see that the form Gl-2 may not be as stable, for the hydrogen atoms, being neutral, form a line of weakness in the sense that the atom may be more easily disrupted or broken up by bombarding alpha-particles. This conclusion seems to be borne out in the cases of boron and aluminium described below.

In the writer's opinion there is a question whether this atom shown as Gl-2 has not been mistaken for one of the isotopes of boron, for an isotope said to be boron has been reported by Aston having a mass 10. It does not seem impossible that this atom should be found as an impurity in boron, because it resembles it in a striking manner, the difference being that boron is the same atom having one additional hydrogen atom at its exact centre. This opinion has not been formed because it is considered that different atoms with the same mass do not exist, but rather because two symmetrical atoms of mass 10 seem improbable.

Boron. Atomic Number 5.

The atom of boron is shown at B (fig. 2), and is, as above stated, formed by placing one neutral hydrogen atom at the exact centre of Gl-2. The atom has a very symmetrical form, but there are three neutral hydrogen atoms in it all lying on the same diameter of the ring. One might expect that it would on this account be comparatively easy to break it up by bombarding with alpha-particles. Rutherford has succeeded in breaking it up in this way, and has obtained long-range hydrogen atoms from it.

The mass of this atom is the sum of two alpha-particles ($2 \times 4 = 8$), and three hydrogen atoms ($3 \times 1.008 = 3.024$), making a total of 11.024. Aston has found that the principal boron isotope has a mass 11, and there is some doubt whether his measurements at that time would enable him to be sure whether the mass is 11.00 or is 11.024.

The atomic number of this form of atom is 5, for there are four connecting electrons in the outside ring, and one must be counted for the hydrogen atom at the centre. Just as hydrogen itself has atomic number 1, so 1 must be counted when hydrogen is found alone at the exact centre of a more complex atom. Similarly, when an alpha-particle occupies the exact centre alone 1 must be counted to obtain the correct atomic number. An example is to be found in the case of nitrogen below. If, however, a whole helium atom occupies the centre of a more complex atom, two must be counted to obtain the correct atomic number, just as 2 is the atomic number of helium. An example of this is found in the case of neon described below.

Up to the time of the last table of atomic weights issued by the international committee, the atomic weight of boron was given as 11.0. The latest table gives the weight as 10.9, a change of nearly one per cent. This revision has been made subsequent to the work of Aston in finding isotopes. It goes to confirm the opinion above expressed that there may still be a question as to whether pure boron has been used in finding these weights.

Carbon. Atomic Number 6.

In figs. 2 and 2A carbon is represented at C as a ring, having a very symmetrical form with three alpha-particles marked by the numbers 4, forming an equilateral triangle and six connecting electrons, two for each alpha-particle. The mass is thus exactly 12, agreeing with the chemist's weight of 12.00. The atomic number is that of carbon (6), there being six connecting electrons. It is not supposed that these

six connecting electrons all lie in the exact equatorial plane as shown in the model, because of the repulsion between the single electrons that are adjacent to each other, but they are supposed to be slightly out of the equatorial plane, the one above and the next below it alternately all the way around the ring.

A word of caution is offered here concerning this and all of the other forms of models, for a chemist at first sight cannot see how such a ring form of the atom can possibly represent carbon. In his thought the carbon atom must have a form that will harmonize it with the tetrahedral quality exhibited by carbon having a valency of four. Methane, for example, is supposed to consist of one carbon atom at the centre surrounded by four hydrogen atoms forming a regular tetrahedron. The difficulty probably arises in the conjecture that the shape of the atom itself must have some evident relation to the positions occupied by the atoms that unite with it. When attention is directed to the fact that all of the atoms here described are comparatively small points of the order of 10^{-12} cm. in diameter, instead of the large structures that are probably in the minds of those to whom this form of atom presents some difficulty, this changes the situation considerably. To make such objections really valid one must be able to show that these atoms cannot possibly behave as carbon atoms do, and this means that one must be able to compute on the basis of some definite principle just what they will and will not do.

Some progress has already been made in this direction, although the subject has not been investigated with the thoroughness that is hoped for it. Nothing has been found, however, which contradicts experimental observations, and much has been found that throws light upon the nature of the forces concerned in the process of forming molecules with such atoms as these. Some account of this is given in subsequent chapters.

The total number of negative electrons in the carbon atom

is 12, equal to the atomic weight, but the ratio of the number of electrons in it to the number in the hydrogen atom is $12/2=6$, equal to the atomic number.

Nitrogen. Atomic Number 7.

The nitrogen atom is shown at N in figs. 2 and 2A, having one alpha-particle at its exact centre. This is surrounded by a hexagon of single connecting electrons, and outside of these are two other alpha-particles diametrically opposite. On a diameter at right angles to these are two neutral hydrogen atoms. The total mass is thus the sum of three alpha-particles and two hydrogen atoms, making $3 \times 4 = 12$, and $2 \times 1.008 = 2.016$, or a total of 14.016.

No isotopes have been found for the nitrogen atom, and the weight given in the international table probably represents the true weight with great accuracy, for the nitrogen atom has been investigated with perhaps as much care as any other atom. When the substance is obtainable as a stable gas at ordinary temperatures and pressures a greater accuracy may be expected. The weight given in the table is 14.01, and agrees with the theoretical value 14.016 very closely.

The atomic number of nitrogen (7) is obtained from the model by counting the six single connecting electrons in the hexagon, which, it should be stated, are supposed to be staggered alternately above and below the equatorial plane just as those of carbon are, and adding 1 for the single alpha-particle in the exact centre. The total number of negative electrons in the atom is 16, which is two more than the atomic weight. The ratio of the total number of electrons to those in hydrogen is 8, one more than the atomic number.

Bearing in mind that this model was constructed at a time before the results obtained by Rutherford experimentally were known, the following points of confirmation of the model may have a greater interest.

He has obtained from pure nitrogen in the form of a gas both hydrogen atoms and alpha-particles in breaking up the

atom by the method of bombarding with alpha-particles from one of the products of radium. Moreover, he has succeeded in determining the distance from the centre of the nitrogen atom to the position normally occupied by the hydrogen atoms in the nitrogen, and finds it to be just two diameters of the negative electron. This is exactly the position assigned to the hydrogen atoms in the model of N in figs. 2 and 2A. This is considered to be a remarkable confirmation of the model. It may be regarded also as justification, not only for this atom, but for the general scheme of the others in the use of the plane form of assemblage of the hydrogen atoms and the alpha-particles to form rings and plane-figures rather than building them up in three dimensions in space.

An inspection of the model shows that the two hydrogen atoms ought to be more easily detached or dislodged than any of the bodies in it, because they are neutral atoms and are only held in place by the excess force upon the positive charge of the hydrogen over and above that upon the two electrons, the former being situated on the equatorial plane, whereas the latter are above and below it. If these hydrogen atoms were knocked off from the atom, probably carrying with them some of the electrons, there must be left a residue or remainder of the nitrogen atom, a kind of unstable structure which probably has but a brief life as such.

Rutherford also obtained a record of these residues, and in reasoning about them concluded that they had a mass of three on the scale of oxygen sixteen. To obtain this result, however, he had to know the charge upon the residue, which was estimated to be a positive charge of $2e$, the same as that on one alpha-particle. This result would mean that a new particle had been discovered having a mass of only 3, which falls between the mass of hydrogen (1.008) and the next (helium 4.00), and on this account assumes great importance.

It may be stated now that Rutherford also succeeded in breaking up the oxygen atom and examined the residue from

it, finding it to be of precisely the same mass, 3, as the residue from the nitrogen atom. This result has been very difficult to accept for many reasons, for it is difficult to see how a mass of 3 can enter into both the nitrogen and the oxygen atoms. It does not fit in with these models at all, and, moreover, has not been confirmed by the later experimental results announced from time to time. That is to say, the particle of mass 3 has not been found elsewhere since this early result of Rutherford. A possible explanation of his results has been offered by the writer in the light of these models, some account of which is given under the next atom, oxygen.

Oxygen. Atomic Number 8.

In figs. 2 and 2A oxygen is represented at O as a symmetrical ring, being an extension of the carbon ring above described to include four alpha-particles in the form of a square instead of three in the form of an equilateral triangle. The mass is thus exactly 16, which agrees with the value in the international table, there being no isotopes of oxygen. The atomic number is 8, equal to the number of connecting electrons. The total number of electrons is 16, this being the same number as contained in the nitrogen atom. The ratio of the number of electrons to those in the hydrogen atom is also 8, equal to the atomic number.

No hydrogen atoms or long-range particles have been obtained by the breaking up of the carbon or the oxygen atoms, as they have been from the breaking up of boron and of nitrogen. Although the former is a negative experimental result, yet it is in perfect agreement with the forms of these models. As above stated, Rutherford has succeeded in breaking up the oxygen ring, and in examining the residue of the atom found the same particle he had obtained in the breaking up of nitrogen. He deduced from theoretical reasoning that the mass of this particle is 3. There are three unknown quantities to be determined, the mass, velocity, and charge of the unknown particle. Two simultaneous equations were

available connecting these quantities, so that, if one of the unknown quantities could be determined by some other independent means, then the two remaining quantities might be found.

There seemed to be some independent evidence that the charge on the unknown particle was two units, the same charge as on the alpha-particle, and Rutherford used this value of the charge to determine the mass and the velocity by means of the two simultaneous equations referred to. The mass came out 3.1, a result that is near enough to a whole number 3 to make it likely that the one-tenth may be accounted for as an error.

Thinking it to be barely possible that there is evidence that the charge on the unknown particle may have been something different from two units, the author has tried in these same simultaneous equations the next higher integral values of charge, $3e$, $4e$, and $5e$. With $3e$ and $4e$ substituted in these equations the mass comes out larger than 3, but is not near to any integer. With $5e$, however, the mass comes out 12.1, which is just as close to an integer as the 3.1 when $2e$ is used, and, in fact, the result is a little closer because the ratio of 121/120 is a little better than 31/30. It makes the mass four times greater than that obtained by Rutherford, but it is in perfect agreement with the models of nitrogen and oxygen, for, when the two hydrogen atoms are removed from the nitrogen atom, the mass of the residue is just three alpha-particles, that is 12. Also, when one alpha-particle is removed from the oxygen atom three are left, making the mass of the residue 12.

The above explanation has been offered as a suggestion only, because it harmonizes the observations with these models, and because it does not appear that the charge on the unknown particle was determined in a direct manner. It is, of course, possible to repeat this experiment for the purpose of determining the charge directly, and, if it is shown to be $5e$, the answer is complete.

Neon. Atomic Number 10.

The atomic weight of neon given in the international table is 20.2, which indicates the existence of isotopes. Two isotopes have, in fact, been measured by Aston, the one with mass 20 and the other with mass 22, there being also a possible trace of one with mass 21. The isotope of mass 20 is shown at Ne-1, figs. 2 and 2 A, consisting of the oxygen ring at the centre of which is a single helium atom. The total mass is thus five alpha-particles giving 20 exactly.

The atomic number of neon is 10, and this may be obtained from the model by counting the eight connecting electrons in the outside ring and adding 2 for the helium atom at the exact centre, as above mentioned. Not only does the sum of the atomic weights of helium and oxygen give that of this isotope of neon, but also the sum of the atomic numbers gives the atomic number of neon. The total number of electrons is 20, equal to the atomic weight, and the ratio of the total number of electrons to those in hydrogen is 10, equal to the atomic number.

An isotope of neon of mass 22.016 is shown at Ne-3, fig. 2, consisting of the same atom Ne-1 to which is added two hydrogen atoms on the axis at the centre, one on either pole of the helium atom, thus making a pile of three atoms at the centre, one helium atom in between two hydrogen atoms. This does not change the atomic number, which still remains 10, because only two of the electrons, those on the outside, are removable from the central pile just as they are from the single helium atom.

A third isotope with mass 21.008 shown at Ne-2, fig. 2, might possibly exist formed by adding only one hydrogen atom instead of two to the central helium atom in Ne-1. This would, however, be unsymmetrical, and it does not seem likely to persist long. The atomic number would still be 10 as before.

In a subsequent chapter it is shown that a helium atom

will not unite to form molecules either with itself or with other kinds of atoms. The presence of the helium atom at the centre of the neon atom for the same reason will prevent neon from uniting with itself or with any other atom. This accounts for the observed fact that neon is mon-atomic and that it does not form compounds with other elements.

Magnesium. Atomic Number 12.

The atomic weight of magnesium in the international table is 24.32, indicating the existence of isotopes. Three isotopes have been reported found having masses 24, 25, and 26. The isotope of 24 is shown as Mg-1, fig. 2. Although the atom of mass 8 described above under glucinum* has no independent existence, yet when surrounded by the oxygen ring it becomes stable. So the isotope of 24 consists of a ring with two alpha-particles and four connecting electrons within a greater ring of four alpha-particles and eight connecting electrons. The mass of this atom is $6 \times 4 = 24$, and the atomic number is $4 + 8 = 12$, which is correct for magnesium. The total number of negative electrons is 24, equal to the atomic weight, and the ratio of the total number of electrons to those in hydrogen is 12, equal to the atomic number.

An isotope of mass 25.008, Mg-2, fig. 2, is formed by inserting one hydrogen atom in the inside ring, thus making this ring just like Gl-1. This does not change the number of connecting electrons, and consequently does not change the atomic number, which is still 12.

An isotope of mass 26.016, Mg-3, fig. 2, is formed by inserting two hydrogen atoms in the inner ring, thus making this ring like Gl-2. This does not change the atomic number, which remains 12. The total number of electrons in this latter form is 28 and the mass number 26, the former being two more. The ratio of the total number of electrons to those in hydrogen is 14, two more than the atomic number.

No particles having a great range have been obtained from magnesium by the method of bombarding with alpha-

* See page 40.

particles. This negative result is in agreement with the form of the principal isotope Mg-1, which contains no hydrogen. This may have been the only kind of magnesium that was used by Rutherford in his experiments.

Aluminium. Atomic Number 13.

The atomic weight of aluminium given in the international table is 27.1. No isotopes have as yet been reported so far as the writer is aware, but the atomic weight indicates their existence. The principal isotope is shown at Al-1, fig. 2, and consists of a boron atom surrounded by the oxygen ring. The atomic weight is thus the sum of 11.024 for boron and 16 for oxygen, making 27.024 for this isotope of Al. The atomic number of aluminium is 13, the sum of 5 for boron and 8 for oxygen. The number of electrons in this form of Al is 14 for the boron part plus 16 for the oxygen part, giving a total of 30.

In an analogous manner with magnesium two other isotopes may be formed by the addition of one or two hydrogen atoms to the outside ring. The form Al-2 has a mass of 28.032, and the form Al-3 a mass of 29.040. According to the official weight the first isotope is of the most frequent occurrence.

The same diameter of weakness through the three hydrogen atoms is found in this atom, as was pointed out in boron above. The extra load placed upon them by the presence of the outside ring ought to make it easier to break this atom into two halves than it is to disrupt the boron atom.

The aluminium atom is perhaps the most noteworthy of them all, because Rutherford has obtained from it the greatest range particles in air, presumably hydrogen, obtained from any element, fully ninety centimetres. Moreover, these particles or hydrogen ions are shot out at random in every direction, without being entirely governed by the original direction of projection of the bombarding alpha-particles

from the radium. The case of aluminium is so conspicuous by comparison as to leave no uncertainty that some of the internal energy of the atom is being liberated, and that the high velocity of projection is produced at the expense of this internal energy, because it is greater than the energy of the alpha-particle projected from the radium.

In this connection it may be stated that these models make it clear that there are several apparent sources of energy, or different manifestations of the same source, which are eventually identified with the energy content of the fundamental electrical charges themselves, and are of increasingly different orders of magnitude as the ultimate source is approached. First, there is the gravitational field at great distances giving the lowest order of energy: then, as molecular range is approached, there is the next greater order of energy, the chemical energies of combustion and dissociation: next, going closer still within the structure of the atom itself, is encountered a third order of energy much greater than either of the others, which has just been pointed out in connection with aluminium as the most conspicuous example. This might be called the interatomic energy, because it is concerned with pulling the structure of the atom to pieces, or in building it up from its fundamental electrons. The fourth grade of energy is by far the greatest of them all, and may be considered to be the source of them all, the inter-electronic energy discussed in the first chapter, which is equal to mc^2 , where m is the mass of the body and c the velocity of light.

In studying any of the first three classes of energy intelligently it will be necessary to trace their effects back to the original source eventually. These models have been useful in stressing this point, and some progress has been made in this direction, as will be shown in a later chapter.

The aluminium atom is the last that we shall attempt to describe in detail, that is to say, the atom having the highest atomic number. Several atoms of lower atomic number have

been passed over in the foregoing description as stated, because they cannot very readily be made up by the use of hydrogen atoms and alpha-particles alone. These atoms will now be taken up individually.

Lithium. Atomic Number 3.

The atomic weight of lithium in the international table is 6.94. Two isotopes have been measured by Aston and others, the one having a mass of 6 and the other of 7. The principal one is the latter, which is close to the official weight of 6.94.

The isotope of mass 6.016 is shown at Li-1, fig. 2, and this is composed of two hydrogen atoms connected each by one negative single electron to one alpha-particle at its exact centre. The atomic number of lithium is 3, which may be obtained from the model by counting the two single connecting electrons and by adding one for the alpha-particle at the exact centre, as was done in the case of nitrogen above. The total number of electrons is eight, the ratio to the atomic weight being $8/6=4/3$. The ratio of the total number of electrons to those in hydrogen is 4, one more than the atomic number.

The isotope of mass 7 introduces for the first time the new hypothetical positive charge of three units, to which is to be assigned a mass of $7/3=2\frac{1}{3}$, intermediate between 1.008 for the positive charge of $2e$ in hydrogen and 4.00 for the positive charge of $4e$ in helium. This isotope of lithium is shown at Li-2, figs. 2 and 2A, consisting of three tau-particles connected together by three connecting single electrons in the form of a hexagon ring. The neutral hydrogen atom and the alpha-particle have names, and since they differ from each other only in the amount of the positive charge, it is convenient to have a name for the new hypothetical particle, for it is just like the others, except that it has a charge of $3e$ instead of $2e$ or $4e$ at its centre. The name tau-particle is

suggested because tau is the first letter in the Greek word for three.

The atomic number of this isotope is 3 because there are three connecting electrons. The total number of electrons is 9, which is two more than the atomic weight. The ratio of the total electrons to those in hydrogen is 4.5, being 1.5 more than the atomic number. The total mass is $3 \times 7/3 = 7$.

The mass of the tau-particle has, of course, not yet been measured, and has been estimated approximately by assuming that the masses of the charges of $2e$, $3e$, and $4e$ are roughly proportional to the squares of the charges. Thus, a charge of plus e , if it has any independent existence, would have a mass equal to a quarter of 1.008 or 0.252, and a charge of $3e$ nine times 0.252 or 2.268, also a charge of $4e$ sixteen times 0.252, or 4.032. This result would come about by a strict application of the Lorentz mass formula (3) of Chapter I on the assumption that all other quantities remain fixed except the mass and the charge. If the radius of the sphere changes but slightly, only slight variations would be introduced into the mass. Such a slight variation actually exists in the mass of the charge $4e$ which has been measured, and a slight variation from 2.261 may be supposed for the charge of $3e$. If it is made equal to 2.333, or 0.065 more than 2.268, it meets the requirements for this isotope of lithium and also of fluorine and sodium to be described.

If one attempts to make up a symmetrical atom having a mass 7 and atomic number 3 to represent this isotope of lithium on the principle that the total mass is equal to the sum of its parts, and that each part has the same mass as when it is isolated, or very approximately the same, this cannot readily be done by the use of hydrogen atoms and alpha-particles alone. If any alpha-particles are present there can be but one, for two would exceed the mass seven. If there is one, then there must be three hydrogen atoms. All attempts to arrange one alpha-particle and three hydrogen atoms in a symmetrical form and in accord with

the scheme of building-up in one plane fail for the atomic number 3.

If no alpha-particle is present, then there would be seven hydrogen atoms, and it is difficult to see how these can be bound together. There would be no connecting electrons to fulfil the rule for the atomic number, and the mass would be 7.056, an amount in excess of seven sufficient to be detected. Another reason against this possibility is, that no long-range particles have been obtained * from lithium, and if it were all hydrogen there would be sure to be such long-range particles found. This statement also applies to the form previously mentioned, consisting of one alpha-particle and three hydrogen atoms. This line of reasoning has led to the adoption of the alternative possibility of there being a charge of $3e$ besides $2e$ and $4e$, for this fits in well with all requirements. It is recognized that this matter will soon be decided by experimental evidence, though this has not as yet been obtained one way or the other, in favour or against.

Fluorine. Atomic Number 9.

The atomic weight of fluorine is 19.0 as given in the international table, and, since no isotopes have been found or are suspected, this probably represents the true weight. Nineteen is the sum of 12 and 7, that is the sum of the masses of the carbon and the lithium atoms. Likewise, the atomic number 9 is the sum of 6 for carbon and 3 for lithium. The fluorine atom built in this manner around the lithium atom of mass 7 is shown at F, figs. 2 and 2A. It forms a perfectly symmetrical figure, having three alpha-particles in the outside ring and three tau-particles in the inside ring. The total number of electrons is the sum of 9 for lithium and 12 for carbon, making 21, two more than the atomic weight.

Rutherford has obtained long-range particles from the fluorine atom, but he does not state that all of these long-range

* In 1922, when these sections were written.

particles are hydrogen ions, though this had been suspected at one time to be the case. It is clear that the ionized tau-particle, that is the tau-particle that has lost one electron and retained one, will behave very much like the hydrogen ion, and it will require accurate measurement to distinguish between them. The only practical way to distinguish between them is by varying the magnetic and electric field strengths along their path. By this means the values of e/m , the charge divided by the mass, may be compared. For the hydrogen ion this may be represented for comparison by the number $1/1.008=0.992$, and for the ionized tau-particle by the number $2/2.333=6/7=0.86$ approximately, and these two numbers differ by only about 13 per cent., so that some care must be exercised to distinguish them.

It is believed that the fluorine atom would be more easily disrupted than the simple lithium ring because it is weighted on the outside by the three alpha-particles. The energy imparted to these tau-particles would, it seems, be greater in the case of fluorine than in the case of lithium, for the same reason that the particles from aluminium have a much greater range than those from boron. It is quite possible that Rutherford did break up the lithium ring without being aware of it, because he has only studied those particles whose range in air exceeds about 28 or 29 cm., the range obtained by the use of hydrogen-gas. If these tau-particles from lithium have a less range than this they would have escaped notice.

Sodium. Atomic Number 11.

The atomic weight of sodium given in the international table is 23.00, and, since no isotopes have been discovered or suspected, this represents with great accuracy the true weight. This is the sum of 7 for lithium and 16 for oxygen. Moreover, the atomic number, 11, is also the sum of 3 for lithium and 8 for oxygen. The sodium atom is represented at Na, in figs. 2 and 2 A, as being the lithium ring inside of the oxygen

ring. The total number of electrons is the sum of 9 and 16, making 25, two more than the atomic weight.

Rutherford has also obtained long-range particles from this atom, and they are thought to be ionized tau-particles as in the case of fluorine. This question may be decided experimentally by making sure that the particles are or are not hydrogen, but this has not yet been accomplished.

All of the atoms from hydrogen to aluminium have been discussed in detail above, including possible and known isotopes. This may be extended through the whole periodic table of the elements, but it had best be deferred until there is more experimental evidence bearing upon some of the questions already raised.

CHAPTER III.

GRAVITATION.

HAVING before us concrete pictures of several of the simpler atoms, let us in this and the following chapters state some of the conclusions reached by applying the classical electrodynamics to these atoms. The first problem is evidently to find the effect which one of the individual electrical charges when isolated by itself must have upon a similar charge situated at a distance from it. For this purpose let us select the positive unit $2e$, which forms the positive part of the hydrogen atom, and find its effect upon another at a distance from it.

On the supposition that this positive charge has the shape of an oblate spheroid in revolution, we are confronted at once with a problem that has never been solved by means of any of the classical theories. Even though the charge were supposed not to be in revolution, and yet to have the shape of the spheroid, even this simplified form of the problem involving purely electrostatic forces has never been solved for two bodies of this shape situated in their most general positions with respect to each other.

It has been necessary to resort to an approximation to the form of this body on account of the difficulties of the mathematical analysis. The first and most obvious approximation that would occur to anyone is to substitute a circle in revolution for the oblate spheroid. The charge on the circle is supposed to be uniformly distributed and the total charge upon it equal to the total charge in the oblate spheroid. This is the actual approximation which has been made in order to obtain the following results, for it has been possible to find the complete solution for two such circles placed in their most

general positions with respect to each other. It seems worth while giving a complete statement of the result that follows from the use of the Saha form of electrodynamics, but this is most easily expressed by an equation containing a number of different quantities that will now be considered.

The ring upon which the force is to be expressed is denoted by the subscript 1, all the quantities pertaining to it having this subscript, while the other ring which is supposed to be the cause of the force, so to speak, for of course both are concerned in the cause, is denoted by the subscript 2. Let the first ring be located with its centre at the origin of co-ordinates, x , y , and z , and its plane in the x , y plane, its axis of rotation being coincident with the z -axis. Let the centre of the second ring be located at any general point in space having the co-ordinates, x , y and z , and the direction cosines, X , Y , and Z , these being equal respectively to x/r , y/r , and z/r , where r is the distance between the centres of the two rings, which distance is supposed to remain fixed and constant.

Not only is the distance, r , supposed to be constant, but each of the co-ordinates, x , y , and z , is supposed to be fixed so that the relative centres of the two rings do not change. The planes of the two rings are also supposed to remain fixed, and the only motion with which we are concerned is the motion of the charges around their respective circles. The velocity of this charge divided by the velocity of light is denoted by the quantity β , the symbol β_1 being the value for the first ring and β_2 for the second. The charge upon the first ring is E_1 and upon the second E_2 . The angle, α , denotes the fixed angle between the two axes of rotation of the rings.

Imagine a perpendicular line drawn from the origin, that is from the centre of the first ring, to the axis of revolution of the second ring produced, which may be denoted by p . The quantity k denotes the dielectric constant of a vacuum taken as unity on the electrostatic system of units. With this understanding of the quantities concerned the whole force

upon the first ring resolved in the direction of the line joining the centres of the rings is found to be

$$F_r = \frac{E_1 E_2 (1 - \beta_2^2)}{kr^2 (1 - \beta_1^2)^{1/2}} \left[-1 - \frac{1}{2} \beta_2^2 \left(\frac{p}{r} \right)^2 - \frac{3}{8} \beta_2^4 \left(\frac{p}{r} \right)^4 \dots \right]. \quad (1)$$

When this expression is positive the force upon the first ring is towards the second ring and consequently attractive. When negative it is repulsive.

To render this expression useful when applied to the parts of atoms which may be oriented in any way with respect to each other, it seems required to find an average value of the force taking into account all possible orientations of the axes of the two rings. If a collection of atoms, as in hydrogen gas for example, is contemplated, it seems probable that the axes are just as liable to be turned in one direction as in any other direction. So an average of the force expressed by (1) has been taken on the probable supposition that all directions are equally probable.

Keeping the distance between the rings, r , constant the only quantity that changes in the expression for the force is p , the perpendicular distance from the centre of the first ring to the axis of the second ring produced. It may be proved that the average value of $(p/r)^2$ on this supposition is equal to $2/3$, and that the average value of $(p/r)^4$ is $22/45$. With these numbers substituted in (1) the force upon the first ring due to the second on the average is

$$\bar{F}_r = \frac{E_1 E_2 (1 - \beta_2^2)}{kr^2 (1 - \beta_1^2)^{1/2}} \left[-1 - \frac{1}{3} \beta_2^2 - \frac{11}{60} \beta_2^4 \dots \right]. \quad (2)$$

This result includes all orders of β up to β^5 and merely the inverse square of the distance. There are other terms which enter into the higher orders of the distance, r , but these all become vanishingly small when the distance is great as compared with the very small radius of the rings, because the terms are inverse powers of r . When the quantity $(1 - \beta_1^2)^{-1/2}$ in the coefficient of (2) is expanded into infinite series, and this

series is then multiplied by the quantity $(1-\beta_2^2)$ also in the coefficient, the result is

$$(1-\beta_1^2)^{-1/2} = 1 + \frac{1}{2}\beta_1^2 - \beta_1^4 + \frac{3}{8}\beta_1^6 - \frac{1}{2}\beta_1^8\beta_2^2 \dots, \quad (3)$$

which includes all terms in the fourth power of β . Multiplying this series by the last series in (2) gives the following as an equivalent expression for the force in (2),

$$F_r = \frac{E_1 E_2}{kr^2} \left[-1 + \frac{2}{3}\beta_2^2 - \frac{1}{2}\beta_1^2 + \frac{3}{20}\beta_2^4 + \frac{1}{3}\beta_1^2\beta_2^2 - \frac{3}{8}\beta_1^4 \dots \right], \quad (4)$$

which includes all terms to the order of β^4 .

It is of interest to compare this result obtained from the Saha form of electrodynamics with that obtained from the Lorentz form, although the steps of the process in finding the latter result are omitted. Instead of (4) the Lorentz theory applied to the same two rings of charge gives

$$F_r = -\frac{E_1 E_2}{kr^2} \left[1 + \frac{2}{3}\beta_2^4 + \frac{4}{5}\beta_2^6 + \frac{6}{7}\beta_2^8 + \dots \right]. \quad (5)$$

The difference between the two results is very marked in that there are no terms in the Lorentz expression involving the product of the two speeds. In fact, β_1 does not appear at all in equation (5). This fact in itself makes it difficult to accept the latter result. The success of the result in (4) in explaining gravitation is due, as we shall show, directly to this product term, $\frac{1}{3}\beta_1^2\beta_2^2$, which has its origin in the quantity $(1-\beta_1^2)^{-1/2}$ in the coefficient of (2). Such a term as this in the coefficient is not found in the expression derived from the Lorentz electrodynamics.

When any equation of the type of (5) is applied to the structure of a neutral atom containing as much negative as positive charge, it will appear that the whole force between them comes out exactly zero at great distances, whereas the force between such atoms at a great distance obtained by means of (4) is not zero but a small residual force that is always one of attraction, thus agreeing with the well-known fact of observation.

Let us take the example of two hydrogen atoms at a great distance from each other and apply equation (4) to them. In its ordinary condition each atom has three charges, a positive charge of $2e$ and two single negative electrons of $-e$ each. Considering the forces between the pairs of individuals, one in each atom, independently, and then adding them together there must be nine pairs of charges to include. But equation (4) is applicable to any pair provided the proper values of the charges and the speeds are inserted.

In writing down the force upon a negative electron due to a positive charge of $2e$ the value of E_1 is $-e$ and of E_2 is $+2e$. β_1 is the velocity of the charge in the ring representing the negative electron divided by the velocity of light, and β_2 a similar quantity for the positive charge. In the following we shall assume that the velocity of the charge in the negative electron is very small and negligible as compared with that in the positive charge, but its value will be obtained later. For the present let us put $\beta_1=0$ and let β_2 denote the proper value for the positive charge of $2e$. Then we have the average force upon a negative electron due to the charge $2e$ at the distance of r as follows, using (4),

$$F_r = \frac{(-e)(2e)}{kr^2} \left[-1 + \frac{2}{3}\beta_2^2 + \frac{3}{20}\beta_2^4 \dots \right]. \quad (6)$$

In a similar manner the same equation (4) may be used to find the force upon a positive charge of $2e$ due to a single negative electron by making $E_1=2e$ and $E_2=-e$; also the β_2 in the equation now becomes zero and the β_1 is numerically equivalent to what we have called β_2 above, as denoting the proper value for the positive charge. With these substitutions the average force of an electron upon the positive charge becomes

$$F_r = \frac{(2e)(-e)}{kr^2} \left[-1 - \frac{1}{2}\beta_1^2 - \frac{3}{8}\beta_1^4 \dots \right]. \quad (7)$$

The force of a positive charge of $2e$ upon a similar charge of $2e$ is obtained by making E_1 and E_2 equal to $2e$ and β_1 and

β_2 equal to each other and equal to β_2 . The average force of the positive charge upon the positive charge is then

$$\bar{F}_r = \frac{(2e)(2e)}{kr^2} \left[-1 + \frac{2}{3}\beta_2^2 - \frac{1}{2}\beta_1^2 + \frac{3}{20}\beta_2^4 + \frac{1}{3}\beta_1^2\beta_2^2 - \frac{3}{8}\beta_1^4 \dots \right] \quad (8)$$

The force of one negative electron upon another is obtained by making $E_1 = -e = E_2$, and by putting all the β -terms equal to zero, giving merely the electrostatic term

$$\bar{F}_r = \frac{(-e)(-e)}{kr^2} [-1]. \quad \dots \quad (9)$$

To find the whole force upon the first atom due to the second taking into account the nine pairs of forces, it is necessary to add together two times equation (6), two times equation (7), making four pairs, once (8) making five pairs, and four times (9) making nine pairs. The four equations thus obtained are

$$2 \times \bar{F}_r = \frac{4e^2}{kr^2} \left[1 - \frac{2}{3}\beta_2^2 - \frac{3}{20}\beta_2^4 \dots \right], \quad \dots \quad (10)$$

$$2 \times \bar{F}_r = \frac{4e^2}{kr^2} \left[1 + \frac{1}{2}\beta_1^2 + \frac{3}{8}\beta_1^4 \dots \right], \quad \dots \quad (11)$$

$$\bar{F}_r = \frac{4e^2}{kr^2} \left[-1 + \frac{2}{3}\beta_2^2 - \frac{1}{2}\beta_1^2 + \frac{3}{20}\beta_2^4 + \frac{1}{3}\beta_1^2\beta_2^2 - \frac{3}{8}\beta_1^4 \dots \right], \quad (12)$$

$$4 \times \bar{F}_r = \frac{4e^2}{kr^2} [-1]. \quad \dots \quad (13)$$

In taking the sum of these equations not only do all the electrostatic terms, the first terms within the brackets, cancel but also the terms in the even powers of β cancel, leaving only the product-term $\beta_1^2 \beta_2^2 / 3$ within the brace of (12), which is the expression for the force upon the positive charge of the one atom due to the positive charge of the other atom. The sum of (10) to (13) inclusive is

$$\bar{F}_r = \frac{4e^2}{kr^2} \frac{1}{3} \beta_1^2 \beta_2^2. \quad \dots \quad (14)$$

This may also be expressed

$$\overline{F}_r = \frac{E_1 E_2}{k r^2} \frac{1}{3} \beta_1^2 \beta_2^2 = \frac{1}{3k} (E_1 \beta_1^2) (E_2 \beta_2^2) r^{-2}. \quad (15)$$

H-atom on
H-atom

If a similar process of adding up the forces obtained by the use of any equation of the type of (5) derived from the Lorentz theory were carried out, the above example must make it evident that no product-terms would appear in the equation corresponding to (12), and that the β -terms in the equations corresponding to (10) and (11) would exactly cancel all terms in (12) so as to give the resulting force of zero.

Again it is interesting that the product-term is the only one remaining in (15) derived from the Saha form of theory because by interchanging the β_1 and the β_2 no difference in the force results. That is to say, the force upon the one atom is equal and opposite to that upon the other, and Newton's law of equal action and reaction between bodies is strictly true. Moreover, the sign of this product-term is positive, indicating that the force is always attractive. If the matter had no connection with the phenomenon of gravitation it would have been just as likely that this term would come out negative denoting a repulsion as positive.

If equation (4) is applied to a more complex atom consisting of several positive charges instead of to a single one, or to two bodies containing many such atoms, the resulting attractive force that would be obtained may be stated by means of the very simple equation, which we shall not take the space to derive here, namely,

$$\overline{F}_r = \frac{1}{3k} \Sigma (E_1 \beta_1^2) \Sigma (E_2 \beta_2^2) r^{-2}, \quad (16)$$

where the summations indicated must be extended to cover each positive charge in each of the two atoms, or in each of the two bodies.

Newton's law of gravitational attraction between two

masses, M_1 and M_2 , at a great distance from each other as compared with their dimensions, may be written

$$F = k' M_1 M_2 r^{-2} \quad . \quad . \quad . \quad . \quad . \quad (17)$$

where k' denotes the so-called gravitational constant, which has been experimentally determined to be 666×10^{-10} when the masses and the distance are expressed in grams and centimetres, r being the distance between the centres of gravity of the two bodies. If the mass of the body is written

$$M = \frac{1}{\sqrt{3kk'}} \sum (E\beta^2), \quad . \quad . \quad . \quad . \quad . \quad (18)$$

the summation being extended to every positive charge within the body, the subscript 1 being used for the first and 2 for the second body, then a substitution of these values in (17) gives equation (16). The equation (16) obtained from the Saha form of electrodynamics may then be regarded as the equivalent of Newton's law of gravitation to the effect that the attractive force is proportional to the product of the masses and inversely proportional to the square of the distance. Each individual positive charge in the bodies thus contributes its proportional quota of attraction. If there is but one positive charge of $2e$, as in the hydrogen atom, the mass of this charge is simply

$$M = \frac{1}{\sqrt{3kk'}} (2e\beta^2). \quad . \quad . \quad . \quad . \quad . \quad (19)$$

This furnishes another expression for the mass of the same charge $2e$ obtained before by the Lorentz mass formula (3) of Chapter I, namely,

$$M = \frac{4}{5ak} \left(\frac{4e^2}{c^2} \right), \quad . \quad . \quad . \quad . \quad . \quad (20)$$

which gives the mass in terms of the charge and the radius of the equivalent sphere instead of in terms of the charge and the velocity of rotation as in (19). It is natural that there should exist two such relations if a definite speed is always associated with a definite mass.

There is but one unknown quantity in equation (19),

namely β , hence it may be determined numerically for the positive charge of the hydrogen atom. It may be more evident how this comes about by equating the forces of attraction expressed by (14) for two hydrogen atoms, in which both β_1 and β_2 are equal to each other, say β_H , derived from electrodynamics, and that expressed by Newton's law in (17), when $M_1 = M_2 = M_H$, giving

$$\frac{4e^2}{3kr^2} \beta_H^4 = k' m_H^2 r^{-2}, \quad (21)$$

whence

$$\beta_H^4 = 3kk' m_H^2 / 4e^2 \quad \text{and} \quad \beta_H^2 = \frac{m_H}{2e} \sqrt{3kk'} = 0.778 \times 10^{-18} \quad . (22)$$

and

$$\beta_H = 0.882 \times 10^{-9}. \quad (23)$$

The actual velocity of the charge in the ring, which has been assumed as the equivalent of the oblate spheroid in rotation, is then

$$v = \beta_H c = 26.5 \text{ cm./sec.} \quad (24)$$

This result tells us nothing about the radius of the circle in which the charge revolves but merely its speed. Before the radius can be obtained a knowledge of the frequency of revolution is required from some other source. In the hydrogen atom, according to the model of Bohr, the frequency of revolution of the single electron in an orbit is $2K$, twice the Rydberg constant. This constant is accurately obtained by measurements of the spectrum of hydrogen. Atomic models are designed to conform to the observed facts of spectrum analysis and to throw some light upon these facts. In changing over from one form of model to another it is natural to retain the same frequency of revolution to accord with the spectrum, and we shall now assume that the frequency of revolution of the charge in the equivalent ring of the positive charge of $2e$ is equal to $2K = 2 \times 3.29 \times 10^{15}$.

The velocity of a point moving with uniform motion around a circle is equal to

$$v = 2\pi an, \quad (25)$$

where a is the radius and n the frequency. With a frequency of $2K$ this becomes

$$v = 4\pi aK, \quad \dots \dots \dots (26)$$

or the radius is

$$a = \frac{v}{4\pi K} \dots \dots \dots (27)$$

Using the value of v obtained from (24) and of K given above, the radius of the ring equivalent to the positive charge in a hydrogen atom is

$$a = 6.4 \times 10^{-16} \text{ cm.} \dots \dots \dots (28)$$

It is remarkable that the numerical value of the radius obtained in this way by the use of the law of gravitation and electrodynamics should come out anywhere near to the order of magnitude of the semi-axes of the ellipsoids as computed by an entirely different method in (18) of Chapter I. But the coincidence is closer than a mere agreement in the order of magnitude, as we shall see, for it enables us to pick out which of the three charges, e , $2e$, or $3e$ best represents the hydrogen atom. In Chapter I the dimensions of three ellipsoids were computed and given in (18) assuming that hydrogen might possibly have a positive charge of e , $2e$, or $3e$. This computation depended primarily upon the Lorentz mass formula, an entirely different basis from that used above.

If now, instead of assuming that the charge of the positive body in hydrogen is $2e$, as has been done in (21) above, it is assumed to be e , $2e$, and then $3e$ in succession, three corresponding values of the velocity and radii would have resulted as follows:—

<i>Charge.</i>	$\beta_H = v/c.$	<i>Radius.</i>
e	1.248×10^{-9}	$9.06 \times 10^{-16} \text{ cm.}$
$2e$	0.882 „	6.4 „ „
$3e$	0.7205 „	5.23 „ „ (29)

Upon comparing the values of the radii in (29) with the semi-major axes of the ellipsoids in (18) of Chapter I it is seen that, if the charge of the positive body in hydrogen is e , the radius of the equivalent ring, which is supposed to be

a kind of average value for the ellipsoid, comes out many times greater than the semi-major axis of the ellipsoid. This result is absurd on the face of it, for an average value must be less than the maximum. The equivalent ring must of necessity lie within the body of the spheroid.

In comparing the radii for a charge of $2e$ the semi-major axis is 7.054×10^{-16} cm. and the radius of the equivalent ring is 6.4×10^{-16} cm., which is about right to represent a possible average. Thus the charge, e , is ruled out and the decision falls upon $2e$ as best representing the charge in hydrogen. The decision really lies between these two charges rather than the charge of $3e$, which need not be seriously considered for other reasons. However, the radius of the equivalent ring for the charge $3e$ is only about one-third of the semi-major axis of the ellipsoid, a value that seems entirely too small to represent a proper average, and so the charge of $3e$ does not fit the case for hydrogen.

The principal features of the law of gravitation, including the magnitude of the force in agreement with the known value of the gravitational constant, have been accounted for in a satisfactory manner by means of electrodynamics applied to the models described in Chapter II. It may be said that the models embrace gravitation as their first and chief characteristic. The same cannot be said for any other form of atomic model that has been proposed. With this as a beginning there is good reason to expect that the models will not be found wanting in aiding the understanding of many other phenomena, provided the way to apply electrodynamics to the atoms can be found and is manageable.

It is possible that some of the other features of the law of gravitation may receive an adequate explanation along the lines of the theory above traced. Reference is made to the several phenomena which astronomers have observed that do not receive a ready explanation in terms of the very simple statement of the law as given by Newton. There exists a difference between the observed position of the Moon and

the computed position which requires correction from time to time. The difficulty of these computations is so great even when use is made of the simple law only that one hesitates to say that the simple law, when applied in a rigid manner without making any approximations, is not entirely adequate to account for the discrepancies between the observations and the approximate calculations. But it may be inadequate.

The one conspicuous example where astronomers seem most certain that the simple law is inadequate to account for the discrepancy between the observations and computations, is in the case of the planet Mercury. All such discrepancies are of the second order of smallness, for the simple law takes care of the principal features of the motion with remarkable approximation. The orbit of Mercury is an approximate ellipse with the Sun in one focus of it. The major axis of this ellipse does not remain in a fixed position but revolves very slowly because of the influence of the other planets in the solar system upon it. The rate of this revolution has been both observed and computed from the simple law of Newton. The two rates do not exactly agree.

The terms in the series involving the higher inverse powers of the distance, r , have not been computed in obtaining the law of gravitation as expressed by (16) above, but they exist, and this equation is an approximation to the total force, but a very close approximation. It is not impossible, however, that these higher terms may have an appreciable effect of the second order of magnitude in the motions of the astronomical bodies. If so, one must expect that it will make its appearance first in those cases where the distance is least, such as the Moon's motion and the motion of Mercury. In advance of an actual demonstration, that is, in advance of finding the complete series expressing the gravitational force and applying it to the complicated question of these astronomical discrepancies, of course there is no way of knowing that the observations can be accounted for in the way suggested. But the whole subject is new, and it seems well worth while to watch for all

possible chances to rectify existing discrepancies, for the decision whether or not to undertake a work of considerable magnitude is usually made because of the importance attached to the theory and to the end in view.

There is one other feature that the gravitational equation (1) accounts for according to gravitational observations without requiring any greater approximation, which may be briefly mentioned. It occurred to experimentalists some time ago that possibly the gravitational action of a single atom is not the same in all directions, but has a symmetry about some axis. The theory above given has confirmed this conjecture and shows that it makes a difference when two atoms only are concerned how they are oriented with respect to each other.

Crystals being solid bodies in which the atoms are marshalled in a systematic manner, it was thought possible that two crystals might show a different attraction for each other depending upon how they are oriented. This was put to an experimental test by Mackenzie, who found that there was no variation in the attraction between certain crystals no matter how they were oriented with respect to each other to within the accuracy of about one part in 500, that is, to within the error of measurement of his apparatus.

Since, therefore, the above theory states that there should exist a variation so far as a single pair of atoms is concerned, it has seemed necessary to reconcile this with the observation that has been made with crystals. In order to do this something has to be known concerning the directions of the axes of the atoms in crystals. Nothing, however, is positively known concerning these. None of the many observations made by means of the X-rays on the structure of crystals has revealed them. If there exist definite directions for any one of the elementary groups of atoms of which the crystal is composed, it is most probable that any such groups will be just like all the other similar groups as to the directions of the axes as well as to locations. What is true of one group will, therefore, be true of the whole crystal in respect to the

gravitational force. If there can be found arbitrarily any possible directions for the axes of the atoms in one of these groups, by which the computed force, according to equation (1), comes out the same for all orientations of the group, then it is reasonable to assume that this may be the actual arrangement, and the more especially so if it is also shown for other reasons that this is the only arrangement possible.

Such a result has been obtained, and it also agrees with the directions of the axes in cubic or isometric crystals predicted theoretically some years before this gravitational theory was suspected. It makes all the atoms in the crystal divisible into four equal groups, each group having all axes parallel to one of the diagonals of a cube, or to the medial lines of a regular tetrahedron, which comes to the same thing. It is only necessary to treat of one group of four similar atoms having their axes parallel to these four medial lines respectively by equation (1) in order to obtain the character of the whole effect upon the crystal. It has been shown that it is immaterial how such a group is oriented; the total force upon the four atoms of a group is the same in every direction.

This result is due to the fact that the quantity, p in (1), is the only one which varies when changing the orientation. It is the perpendicular line from an external point upon one of the four directions representing the axis of one atom of the group. In getting the sum of the forces for the group of four such atoms it is necessary to add together the squares of the four perpendiculars represented by p upon each of the four different atoms. This sum has been proved to be constant independent of the orientation of the group. The statement thus made is equivalent to a geometrical theorem, and the truth or falsity of it was unknown at the time the gravitational equation suggested it. To add a new theorem to geometry, and especially from such a source as gravitation, is especially significant.

CHAPTER IV.

MOLECULES :

WITH SYMMETRICAL ATOMS.

THE distance between the centres of the nearest atoms in crystals is in round numbers found experimentally to be of the order of 10^{-8} cm. rather than 10^{-7} , 10^{-9} , or 10^{-10} cm. It may be as small as 5×10^{-8} cm., or as large as 4×10^{-8} cm. in special cases perhaps, but the minimum distance always falls within these rather narrow limits. This was briefly referred to under the heading "atomic distances" in the introductory chapter, but it is a fact that should be emphasized because it is the key to the proper understanding of the structure and behaviour of atoms, and is of the utmost importance. Since the variation of the minimum distance between nearest atoms in crystals, and no doubt in all solids, is so small, it may be called the universal distance of the order of 10^{-8} cm. Considering the very small size of the negative electron itself, which is now experimentally established, the experimental and theoretical dimensions agreeing very satisfactorily, and remembering that the negative electron is the largest fundamental body of charge in point of size that enters into the structure of an atom, for the negative electron occupies a far greater volume than any positive charge, taking all these things into consideration, it is not at all evident why 10^{-8} cm., instead of 10^{-7} or 10^{-9} cm., should have been this universal distance. And indeed it has not been evident why there should exist such a universal distance of any value whatever, for there is plenty of room for distances much smaller or greater than 10^{-8} .

From the fact of the existence of this universal distance it seems to follow, as a matter of necessity, that there must exist

a causal connection between the constitution or structure of the atoms themselves and this distance, because one would expect a much wider departure from 10^{-8} cm. than is found experimentally if there exists no controlling principle within each atom. If, however, it can be shown that this universal distance is closely related to the properties of negative electrons and to the manner in which they are attached to the atoms so as to be controlled by them, as is shown in the pages that follow, then it is not surprising but very natural to expect a universal distance between nearest atomic centres because the negative electron forms an integral part of each and every one of these atoms. The main distance itself between the atoms will then be determined by the negative electron, and the very small variations from this main distance will depend upon the particular grouping of these electrons in the different kinds of atoms.

In this chapter, as in the last, we shall deal with the steady states of atoms when they are neither radiating nor absorbing energy in the form of high frequency waves, and examine the property of the atomic models described in Chapter II conferred by electrodynamics, that enables them to unite together to form chemical compounds. The subject as thus stated is of immense proportions, and only a modest beginning has been made, but this beginning is of importance, because no other models to which electrodynamics has been applied without reservation have revealed the character of the forces concerned in the so-called chemical affinity. There exist hundreds of thousands of compounds, and it will eventually tax the highest types of mathematical analysis yet developed, as well as the patience of the investigator, to compute them all, but the fortunate thing is that there already exists a vast store of experimental evidence accumulated by the chemist, and at each step the theoretical results may be compared with and tested by these observations.

In advance of the reasons given later in this chapter, it may be stated that the forces concerned in holding the atoms

apart at this universal distance of the order of 10^{-8} cm. are chiefly electrostatic forces, the forces due to the rotation of the fundamental charges merely serving to determine the direction of the axis of each atom. The two things that are chiefly responsible for the universal distance are the shape of the electron, it being non-spherical, and the existence of two atomic constants which have been discovered in connection with these atomic models. Each of these constants is directly related to the properties of the negative electron, so that it is really the presence of negative electrons in all atoms which is responsible for the existence of the universal distance above referred to. One of these constants is connected with the energy required to compress the figure of the electron so as to shorten the minor axis of it and consequently increase its surface area. The other constant is concerned with the force required to displace the positive charge, C of fig. 1, slightly to one side, that is, slightly off from the line joining the two electrons. These matters will be fully described in due course, but in order to present the ideas so that they may be intelligible it seems best to begin at the beginning and develop the subject gradually.

Quite apart from all thoughts as to the actuality of things, the mathematical aspects of the problems suggested by the atomic models with which we are now concerned have many features of great beauty and sufficient merit to maintain the interest, to say nothing of the new field that lies open for atomic physics. How to give readers an understanding of the subject with the minimum of effort on their part has received some study. The historical order of development would be the easiest way, but an objection is that the results become clouded with a mass of detail. It is necessary, however, to pass through many successive steps to attain the final result to be described, and if these steps were omitted there is a question whether the presentation would have much meaning or be understood.

At the risk of repeating some of the figures derived in the

opening chapter, it is best to keep in mind at the outset the properties of the free fundamental bodies, the negative electron, and the positive charge of $2e$, for these will be used in determining their behaviour in the atoms and molecules. The dimensions of the negative electron supposed to be an oblate spheroid with charge uniformly distributed throughout its volume are

$$\text{semi-major axis : } a = 3.257 \times 10^{-13} \text{ cm.} \quad . \quad . \quad . \quad . \quad (1)$$

$$\text{semi-minor axis : } b = 1.065 \quad , \quad , \quad . \quad . \quad . \quad . \quad (2)$$

$$\text{eccentricity : } \epsilon = 0.945. \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

It is equivalent in volume to a sphere of radius

$$r = 2.244 \times 10^{-13} \text{ cm.} \quad . \quad . \quad . \quad . \quad . \quad (4)$$

The total energy content of the negative electron is equivalent to its mass ($m_0 = 0.898 \times 10^{-27}$ grams) multiplied by the square of the velocity of light ($c^2 = (3 \times 10^{10})^2$). Hence the energy, E_0 , becomes

$$E_0 = 8.08 \times 10^{-7} \text{ erg.} \quad . \quad . \quad . \quad . \quad . \quad (5)$$

Similarly, the positive charge of the hydrogen atom is supposed to be an oblate spheroid with uniformly distributed charge having the same eccentricity as the negative electron, (3) above. But its charge is $+2e$ instead of $-e$, and its dimensions are

$$\text{semi-major axis : } a = 7.054 \times 10^{-16} \text{ cm.} \quad . \quad . \quad . \quad . \quad (6)$$

$$\text{semi-minor axis : } b = 2.307 \quad , \quad , \quad . \quad . \quad . \quad . \quad (7)$$

It is equivalent in volume to a sphere of radius

$$r = 4.86 \times 10^{-16} \text{ cm.} \quad . \quad . \quad . \quad . \quad . \quad (8)$$

The total energy content of this positive charge is also given by the product of its mass (1.66×10^{-24} grams) by the square of the velocity of light, namely

$$E = 1.49 \times 10^{-3} \text{ erg.} \quad . \quad . \quad . \quad . \quad . \quad (9)$$

For comparison with these energies in (5) and (9), it is important to keep in mind that the energy required to

separate the two atoms of hydrogen forming one hydrogen molecule to a great distance from each other is

$$E = 5.8 \times 10^{-12} \text{ erg.} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

This value has been determined experimentally by Langmuir, and the order of magnitude of most of the energies of the heats of formation and the heats of dissociation in many chemical reactions is well represented by this figure. The energy content of the negative electron is more than 100,000 times, and of the positive charge more than 250,000,000 times this value. These facts have a very important bearing upon the whole behaviour of atoms.

It seems best to give at the outset the reasoning that shows the important part played by the very large energy content of the electron in the formation of atoms and molecules. The oblate spheroid given by (1) to (4) is supposed to be that of a free electron which is in stable dynamic equilibrium. Two such electrons will be attracted by the positive charge ($+2e$) of a hydrogen atom at all distances and will approach until they come into contact * with it, the three bodies aligning themselves on a common axis as in fig. 1, which represents a meridian section of a hydrogen atom.

The pressure of each electron upon the positive charge due to the resultant attraction of the other two bodies must cause a slight flattening of the original figure of the free electron. But either to flatten or to expand the figure of the free electron, which is in dynamic equilibrium, requires that work be done, and, if the amount of this change of shape were known, the energy may be computed, and *vice versa*, if the work done were known, the change in shape may be computed. Now the energy so involved represents the difference between the energy contents of the free bodies and these same bodies as they exist in the atom, that is to say, it is the energy of ionization of the free hydrogen atom.

But there are two energies of ionization for an atom which

* Consult page 34.

carries two electrons, the first when only one electron is detached and the second when both are detached. The first of these is the only one that has ever been measured for hydrogen, and this is of little service to us. The second energy must be far greater than the one which has been measured. In fact, it is so large that the last electron never leaves the positive charge, at least in laboratory experiments so far conducted, so that it cannot, of course, be measured. And so we have never heard of a hydrogen ion with two positive charges. The maximum charge it has been possible to give it in the laboratory is one positive charge.

It seems profitable to picture the change that takes place in the atom during the removal of one electron. Were it not for the fact that much of the energy of the electron on the opposite side is transferred over to the one electron during the process of removal of the one, it would not be easy to remove even one, for in the symmetrical position in the free atom, of course, each electron shares alike and represents one half of the very large energy of the double ionization. But when one electron begins to move away from the centre of the atom, its minor axis expands because of the release of the pressure, and thereby it gives up some of the energy that was expended upon it during the formation of the atom. At the same time the minor axis of the opposite electron must contract slightly because of the increased pressure on that side due to the removal of the first electron, and thus it is apparent that there occurs a transfer of some of the energy across from the one to the other electron.

The comparatively small difference between the gain of energy of the one that remains and the loss of energy of the one that departs represents the rather small energy of the first ionization of the hydrogen atom. This is known experimentally, but is not of much importance for this problem. The more important energy, which is not known experimentally and probably will not be measured for some time to come, is the second or total ionization energy.

It seems important to attack the problem from the standpoint of energy, and we shall now consider the problem of finding a relation between the energy absorbed or emitted by a negative electron because of a definite change in its semi-minor axis. When the hydrogen atom is formed as in fig. 1 because each electron is attracted at all distances until they both come into actual contact with the positive charge, then the attraction, which still continues, results in a deformation of the shape of the electron. Since this change is extremely small for a comparatively large energy change, it may be assumed that each electron remains an oblate spheroid as before. The result of the pressure between the electron and the positive charge is to flatten the spheroid slightly, thus increasing its eccentricity and decreasing its semi-minor axis.

The Change in the Energy Content of the Electron for a given Change in its Semi-minor Axis.

It is assumed as before that the volume of the spheroid, which has a uniform distribution of charge throughout, remains constant and fixed, equal to that of a sphere of radius, r , given by (4) above.

This volume is

$$V = \frac{4}{3} \pi r^3. \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

The volume of an oblate spheroid of polar semi-axis, b , and equatorial radius, a_0 , is

$$V = \frac{4}{3} \pi a_0^2 b. \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

Equating these volumes gives an immediate relation between a_0 and b , namely,

$$a_0^2 b = r^3 \quad \text{or} \quad \frac{b^2}{a_0^2} = \frac{b^3}{r^3}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

But in an ellipse the relation between the axes and the eccentricity is

$$\frac{b^2}{a_0^2} = 1 - e^2, \quad . \quad . \quad . \quad . \quad . \quad . \quad (14)$$

whence, equating (13) and (14), we have

$$\frac{b^3}{r^3} = 1 - \epsilon^2. \quad (15)$$

The area of an oblate spheroid, S , in terms of its semi-axes and eccentricity is

$$S = 2\pi a_0^2 + \frac{\pi b^2}{\epsilon} \log \frac{1+\epsilon}{1-\epsilon}. \quad (16)$$

By means of the above relations this area is expressible in terms of the eccentricity and a constant alone, for it is to be remembered that the radius of the equivalent sphere, r , is constant, as

$$S = 2\pi r^2 \left[(1-\epsilon^2)^{-1/3} + \frac{1}{2\epsilon} (1-\epsilon^2)^{2/3} \log \frac{1+\epsilon}{1-\epsilon} \right]. \quad . . . (17)$$

From this expression we desire to find the change in the area of the spheroid that corresponds to a given change in the semi-minor axis, and since this change is extremely small, the differential of S may be employed. Hence, differentiating (17) with respect to ϵ gives

$$\begin{aligned} \frac{dS}{d\epsilon} = 2\pi r^2 \left\{ \frac{2}{3} (1-\epsilon^2)^{-1/3} + \frac{1}{\epsilon} (1-\epsilon^2)^{-1/3} \right. \\ \left. - \left[\frac{2}{3} (1-\epsilon^2)^{-1/3} + \frac{1}{2\epsilon^2} (1-\epsilon^2)^{2/3} \right] \log \frac{1+\epsilon}{1-\epsilon} \right\}. \quad . . . (18) \end{aligned}$$

By means of the equation (15) this may also be expressed equally well in terms of r , b , and ϵ as follows,

$$\frac{dS}{d\epsilon} = 2\pi r^2 \left\{ \frac{2}{3} \left(\frac{r}{b} \right)^4 + \frac{1}{\epsilon} \left(\frac{r}{b} \right) - \left[\frac{2}{3} \left(\frac{r}{b} \right) + \frac{1}{2\epsilon^2} \left(\frac{b}{r} \right)^2 \right] \log \frac{1+\epsilon}{1-\epsilon} \right\}. \quad . . . (19)$$

Since we desire the rate of change of the area with respect to b rather than with respect to ϵ , this may be obtained by multiplying (19) by $d\epsilon/db$, and an expression for this may be found by differentiating (15) above, giving

$$\frac{d\epsilon}{db} = -\frac{3}{2\epsilon r} (1-\epsilon^2)^{2/3} = -\frac{3b^2}{2r^3\epsilon}. \quad (20)$$

Hence, denoting the quantities within the brace of (19) by $f(b)$ for brevity, and multiplying (19) by (20), we obtain

$$\frac{dS}{db} = \frac{dS}{d\epsilon} \times \frac{d\epsilon}{db} = -\frac{3\pi b^2}{r\epsilon} f(b). \quad (21)$$

When the change in the area is small as now contemplated, we may say that dS becomes $\Delta S = S_2 - S$ and that db becomes $\Delta b = b_2 - b$, whence the change in the area in terms of the change in the semi-minor axis is now determined to be

$$S_2 - S = -\frac{3\pi b^2}{r\epsilon} (b_2 - b) f(b). \quad (22)$$

In order to find the energy change which this change in the area represents, use may be made of the expression (9) of Chapter I denoting the surface tension over the entire surface of the electron. If this is multiplied by the change in area the work done against the tension is obtained. This tension is

$$T = \frac{3}{16} \frac{e^2}{\pi r^2 k}. \quad (23)$$

Hence to increase its surface by the amount $S_2 - S$ and to flatten the electron thus reducing its minor axis, requires an amount of energy

$$\Delta E = T \cdot \Delta S = T(S_2 - S), \quad (24)$$

and this becomes by the use of (22)

$$\Delta E = -\frac{9}{16} \frac{b^2 e^2}{r^3 \epsilon k} (b_2 - b) f(b), \quad (25)$$

which is the expression that was to be found. Taking b in round numbers as 10^{-13} cm. and r as given by (4), the eccentricity obtained from (15) is

$$\epsilon = 0.9547. \quad (26)$$

Hence, after computing the value of $f(b)$, which is the brace in (19), and multiplying it by the coefficient as given in (25), we obtain numerically a value for the energy as follows

$$\Delta E = -\frac{e^2}{k} (307 \times 10^{26}) (b_2 - b). \quad (27)$$

And, again, putting in the numerical value of the charge on the electron as $e=4.763 \times 10^{-10}$ E.S.U., the energy of (27) becomes

$$\Delta E = -6.964 \times 10^6 (b_2 - b). \quad (28)$$

That is to say, if the decrease in the semi-minor axis of the electron in centimetres is multiplied by nearly seven million the energy is obtained in ergs. To illustrate the very small contraction required in the minor axis for a comparatively large energy change, consider the formation of the hydrogen atom. In its free state the value of b given by (2) above is 1.065×10^{-13} cm. If this is contracted down to exactly 10^{-13} cm. when the hydrogen atom is formed, it would mean that

$$b_2 - b = (1 - 1.065) \times 10^{-13} = -0.065 \times 10^{-13} \text{ cm.} \quad (29)$$

The corresponding energy change according to (28) becomes

$$\Delta E = 0.4527 \times 10^{-7} \text{ erg.} \quad (30)$$

And according to the energy content of the negative electron given in (5) this is the $1/17.85$ part of the total. This would make the energy of double ionization of the free hydrogen atom double this value because in the free atom the two electrons are supposedly symmetrical. The numerical value thus obtained for the energy of the double ionization is probably much too great, and one reason for giving it in this way is to show that the contraction of the semi-minor axis of the electron is probably less than from 1.065 down to 1×10^{-13} cm. as arbitrarily assumed. However, since the experimental value of this is unknown, and since the minor axis must be less than it is in the free electron, it may be assumed, without affecting any of the results that follow, that $b=10^{-13}$ cm. in round numbers, and that the eccentricity is 0.9547 in the free hydrogen atom as computed in (26).

The Hydrogen Molecule.

Let us first consider in a preliminary way the process of the formation of the hydrogen molecule by the uniting together of two hydrogen atoms of the kind above described. The first striking result of an application of the equations expressing the electrostatic forces which are given below is, that the second atom exerts a comparatively large force in one direction upon the two negative electrons of the first atom and a correspondingly large force upon the positive charge of the first atom in the opposite direction. The effect of these larger forces is necessarily to change the pressure between the electrons and the positive charge as well as to displace very slightly the location of the positive charge to one side of the centre line of the two electrons. These larger forces, in other words, have the effect of slightly deforming the atom itself, thereby producing an internal strain so that it is no longer a symmetrical atom as it exists in the free state. We really have to deal with a slightly different atom when it forms a part of a molecule, as well as during the process of formation of the molecule, than when free.

It required time and experience to realize that these deformations are of any practical importance, since they are evidently very small effects. The reason why they become important, when examined in terms of the equations expressing the electrostatic forces between the fundamental bodies, is because the asymmetry of the deformed atom introduces new terms in the lower orders of the distance between centres of the two atoms of the molecule, denoted by r . When it is understood that these forces are necessarily expressed by an infinite series in terms of r^{-2} , r^{-3} , r^{-4} , etc., and that without some special condition imposed upon the coefficients of the terms the series is extremely convergent so that its first term represents practically the whole force, it appears that for perfectly symmetrical atoms the whole force of the one upon the other obeys the inverse sixth power of the distance law.

All of the coefficients of the odd powers of r vanish because of the symmetry, and the inverse second and fourth powers vanish in summing up the nine forces on the individual charges taken in pairs, one from each group of three charges in each of the two atoms. This matter will be proved in detail later in this chapter.

With a deformed atom, however, not only do earlier terms appear of lower orders in r , but the odd powers are present as well. Because of the very small amount of deformation the inverse fourth and fifth power terms compare as of equal magnitude with the important inverse sixth power term, and they are therefore not to be neglected. It is the simultaneous co-operation of these three terms of the force series which produces an equilibrium position for the two atoms at a

FIG. 3.



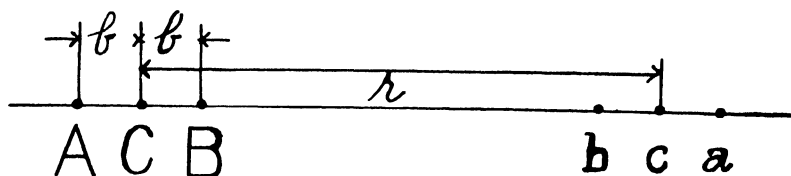
Two coaxial hydrogen atoms.

distance of the order of 10^{-8} cm., for a single term never could produce this result. The distance between the atoms turns out to be immediately dependent upon the atomic constants above referred to, which makes it depend directly upon the properties of the electron, common to all atoms. This matter may be understood so much more readily when the various steps of the process are clearly understood that we shall proceed at once with the systematic development.

Let us begin with the simplest case and imagine two hydrogen atoms according to the model placed so that they have a common axis as in fig. 3. The one atom is denoted by ABC and the other by abc , C and c being the positive charges of $2e$ each, the distance between C and c being denoted by r . We are confronted at once by the difficulty that no solution

has ever been obtained for the electrostatic force upon a solid oblate spheroid of charge due to another similar charge, assuming the inverse square law as between the infinitesimal elements of the charges. It is necessary to make some kind of an approximation to the shape of the charged bodies in terms of things for which the solution has been obtained. A very rough approximation may be obtained by making each of the bodies spheres, but it will be shown that this approximation is not sufficiently near to be trustworthy. The result when spheres are substituted for the oblate spheroids is of some interest for purposes of comparison with the results obtained by a closer approximation given below and seems worth some consideration on this account.

FIG. 4.



Representing the centres of the charges in two hydrogen atoms coaxial with each other. C and c are positive charges of $2e$ each, and A, B, a, and b are the centres of the negative electrons.

It is well known that uniformly charged spheres behave electrostatically towards each other just as if all of their charge were transferred to the centres of the spheres. This does not have reference to charged conductors within which the charge has an opportunity to flow from part to part. Even with them, however, their behaviour is such that all of the charge may be considered concentrated at one point, but this point is not the exact centre of the sphere and is farther and farther from it as the distance between the spheres becomes smaller and smaller compared with the radii of the spheres.

Let us therefore substitute fig. 4 for fig. 3, in which each charge is represented as concentrated at a single point. The method of computation of the forces is so straightforward and

simple that it may be of interest to follow it in detail. There are nine separate forces to be computed, since there are nine different pairs of charges when but a single charge is taken from each group, but some of these pairs give the same result as others, so there are less than nine expressions to find.

The force of the positive charge, c , upon the positive charge, C , at the distance r is

$$F_{c \text{ on } C} = \frac{e^2}{k} (-4)r^{-2}. \quad . \quad . \quad . \quad . \quad . \quad (31)$$

The negative sign indicates that the force is repulsive. The force of a on C at the distance $r+b$ is attractive, one charge being positive and the other negative, giving

$$F_{a \text{ on } C} = \frac{e^2}{k} (2)(r+b)^{-2}. \quad . \quad . \quad . \quad . \quad . \quad (32)$$

The force of b on C at the distance $r-b$ is also attractive, giving

$$F_{b \text{ on } C} = \frac{e^2}{k} (2)(r-b)^{-2}. \quad . \quad . \quad . \quad . \quad . \quad (33)$$

If these three forces are added together we obtain the force of the whole atom, abc , upon the positive charge, C , of the first atom. To facilitate the addition it is best to develop each expression in infinite series. Expanding $(r+b)^{-2}$ by the binomial theorem gives

$$\begin{aligned} (r+b)^{-2} = r^{-2} - 2br^{-3} + 3b^2r^{-4} - 4b^3r^{-5} + 5b^4r^{-6} - 6b^5r^{-7} \\ + 7b^6r^{-8} - 8b^7r^{-9} + 9b^8r^{-10} \dots \end{aligned} \quad (34)$$

The expansion of $(r-b)^{-2}$ gives exactly the same series of terms except that all the signs are positive instead of alternating as in (34). When these series are used in (32) and (33) and both these forces are added the first term is $\frac{e^2}{k}(4)r^{-2}$. If the force (31) of the positive charge on the positive charge is now added to the other two forces, the r^{-2} term cancels. And, again, when the series (34) is added to

another with all signs positive, all of the odd power terms cancel. This is because the distance $\overline{ca}$ of fig. 4 is assumed to be exactly equal to $\overline{cb}$, the atom being symmetrical. If these distances were slightly different the odd power terms would not cancel. The sum of (31), (32), and (33) gives the force upon the positive charge, C, of the first atom due to the whole of the second atom, and is

$$F_{H_2 \text{ on } C} = \frac{e^2}{k} (12b^2r^{-4} + 20b^4r^{-6} + 28b^6r^{-8} + 36b^8r^{-10} \dots). \quad (35)$$

This force being positive shows that the second atom produces an attraction upon the positive charge of the first atom. Neglecting the numerical coefficients, which are very small as compared with the literal parts, it is seen that the ratio of each term to the next succeeding term is not only constant but assumes a very large value, for we have

$$\frac{b^2r^{-4}}{b^4r^{-6}} = \frac{b^4r^{-6}}{b^6r^{-8}} = \frac{b^6r^{-8}}{b^8r^{-10}} \dots = b^{-2}r^2, \quad \dots \quad (36)$$

and when numerical values are substituted, b being of the order of 10^{-13} cm. and r of the order 10^{-8} cm., it is seen that this ratio is of the order of 10^{10} . Hence the first term of the series is all that is effective, and the law of variation of the force is as the inverse fourth power of the distance. The magnitude of the force is $\frac{e^2}{k}(12)10^6$ dynes, the equivalent of the force between two electrons at a distance of 2.89×10^{-4} cm. from each other.

Let us next compute the force exerted by the atom, abc , upon the single electron, A, of the first atom. The force of c upon A at the distance $r+b$ is evidently the same as that of a upon C at the same distance, given above by (32). The force of a upon A at the distance $r+2b$ is

$$F_{a \text{ on } A} = \frac{e^2}{k} (-1)(r+2b)^{-2}. \quad \dots \quad (37)$$

When expanded into series this is equivalent to

$$F_{a \text{ on } A} = \frac{e^2}{k} (-1)(r^{-2} - 4br^{-3} + 12b^2r^{-4} - 32b^3r^{-5} + 80b^4r^{-6} - 192b^5r^{-7} + 448b^6r^{-8} - 1024b^7r^{-9} + 2304b^8r^{-10} \dots). \quad (38)$$

Lastly the force of b upon A at the distance, r , is merely the inverse square term, namely

$$F_{b \text{ on } A} = \frac{e^2}{k} (-1)r^{-2}. \quad \dots \quad (39)$$

Hence the sum of (32), (38), and (39) gives the whole force of the atom, abc , upon the single electron, A , of the first atom, giving

$$F_{H_2 \text{ on } A} = \frac{e^2}{k} (-6b^2r^{-4} + 24b^3r^{-5} - 70b^4r^{-6} + 180b^5r^{-7} - 434b^6r^{-8} + 1008b^7r^{-9} - 2286b^8r^{-10} \dots). \quad (40)$$

The odd powers of r appear in this expression because the single electron, A , unbalanced by the other electron, is not a symmetrical structure around the centre, C . The force upon the remaining electron, B , due to the whole atom, abc , may be found in a similar manner. The force of c on B is evidently the same as that of b on C given by (33). The force of b on B is evidently the same as (38) provided the signs in the series do not alternate but are all positive. And the force of a on B at the distance, r , is the same as (39). Hence by adding (33), (39), and (38) with signs of all terms the same we obtain precisely the same series of terms as that given in (40), except that all signs are negative instead of alternating negative, positive, and so on. Therefore when the forces upon the two electrons, A and B , due to the atom, abc , are added together all the odd power terms cancel and the even terms have coefficients double those of (40), giving

$$F_{H_2 \text{ on } A \text{ plus } B} = \frac{e^2}{k} (-12b^2r^{-4} - 140b^4r^{-6} - 868b^6r^{-8} - 4572b^8r^{-10} \dots). \quad (41)$$

The first term of the series as before gives practically the whole force, and the odd power terms have vanished because

of the symmetry. Comparing with (35) it appears that the force upon the positive charge is equal and opposite to that upon the two electrons together, each force being of the order of $\frac{e^2}{k}(12)10^6$ dynes.

The force of the whole atom, abc , upon the atom, ABC , is found by adding together (35) and (41), giving

$$F_{\substack{abc \text{ on} \\ ABC}} = \frac{e^2}{k} (-120b^4r^{-6} - 840b^6r^{-8} - 4536b^8r^{-10} \dots). \quad (42)$$

Here the force is repulsive and the first term represents practically the whole force. Taking as before b of the order of 10^{-13} cm. and r of the order 10^{-8} cm., $b^4r^{-6} = 10^{-4}$, and the force given by (42) becomes $\frac{e^2}{k}(-0.012)$ dynes, a force about equal to the repulsion between two electrons at a distance of 9 cm. from each other. This force is 10^9 times smaller than the force upon the positive charge (35) or upon the two electrons together (41). It also shows that the force is repulsive at all distances, the sign of every term being negative, and hence there is no possibility of reducing it to zero and attaining an equilibrium at any distance for these two coaxial atoms.

Let it now be assumed that the two atoms are not situated in the coaxial position as represented in fig. 4, but that they occupy a general position having parallel axes as represented in fig. 5, the co-ordinates of the centre of the second atom, abc or H_2 , namely c , being (x, z) , the z -axis being the polar and the x -axis the equatorial axis. The method of computation is the same as that just carried out and will not be given in detail here. The result, however, must evidently be expressed in terms of the position occupied by the two atoms. For convenience all forces are resolved into two components, the one along the x - and the other along the z -axis. Denoting the direction cosines x/r by X and z/r by Z , it may be shown

without much difficulty that instead of (35) the force upon the positive charge, C, of the first atom at the origin is given by the two components,

$$F_{x \text{ on } C} = \frac{e^2}{k} X [-6b^2r^{-4}f_{x,4}(Z) + \frac{1}{2}b^4r^{-6}f'_{x,6}(Z) \dots], \quad . \quad . \quad (43)$$

$$F_z = \frac{e^2}{k} Z [-6b^2r^{-4}f_{z,4}(Z) + \frac{1}{2}b^4r^{-6}f'_{z,6}(Z) \dots], \quad . \quad . \quad (44)$$

where the functions of Z are given by

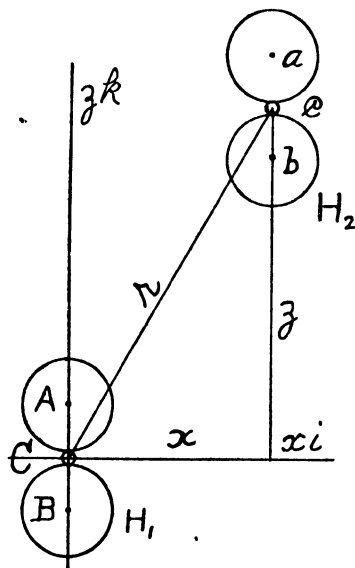
$$f_{x,4}(Z) = 1 - 5Z^2, \quad . \quad . \quad . \quad (45)$$

$$f_{x,6}(Z) = 1 - 14Z^2 + 21Z^4, \quad . \quad . \quad . \quad (46)$$

$$f_{z,4}(Z) = 3 - 5Z^2, \quad . \quad . \quad . \quad (47)$$

$$f_{z,6}(Z) = 15 - 70Z^2 + 63Z^4. \quad . \quad . \quad . \quad (48)$$

FIG. 5.



Hydrogen molecule assuming spherical electrons.

The series, as now developed, includes the r^{-6} term only because it may be shown that this is as far as it will be required on account of its very rapid convergence. In a

similar manner the process above outlined may be carried out to find the force upon the two electrons, A and B together, due to the second atom, and instead of (41) above for the coaxial position we have for the general position

$$F_{x, \text{H}_2 \text{ on } A+B} = \frac{e^2}{k} X [6b^2 r^{-4} f_{x,4}(Z) - \frac{1}{2} b^4 r^{-6} f'_{x,6}(Z) \dots], \quad (49)$$

$$F_z = \frac{e^2}{k} Z [6b^2 r^{-4} f_{z,4}(Z) - \frac{3}{2} b^4 r^{-6} f'_{z,6}(Z) \dots], \quad (50)$$

where the functions of Z are given by (45) to (48).

To find the force upon the first atom due to the second take the sum of (43) and (49) for the x -component, and of (44) and (50) for the z -component, giving

$$F_{x, \text{H}_2 \text{ on H}_1} = \frac{e^2}{k} X [-45b^4 r^{-6} f_{x,6}(Z) \dots], \quad (51)$$

$$F_z = \frac{e^2}{k} Z [-15b^4 r^{-6} f'_{z,6}(Z) \dots]. \quad (52)$$

These more general equations reduce immediately to those previously given for the coaxial position of the two atoms when $X=0$ and $Z=1$. The x -components (43), (49), and (51) vanish because X is a factor, and (44) becomes equal to (35), (50) to (41), and (52) to (42).

It is not profitable to pursue this simplified type of atom any further, because these equations show that there can be no stable equilibrium position where the two atoms will stand in equilibrium at any distance. Subsequent terms in the higher powers of r are of no value, and no values of the two variables Z and r will reduce both the x - and z -components, that is the total force, to zero as it must be at the position of stable equilibrium.

At first thought, without giving the matter full consideration, one might have imagined that no equilibrium for a system of point charges of this kind should be expected because of the general theorem of electrostatics due to Earnshaw. The theorem, however, can have no bearing on this problem, for

the reason that another condition has been imposed, since the two axes of the atoms have been assumed to be parallel to each other in all positions. This result is brought about by the rotation of each fundamental charge upon its own axis, which gives rise to an electromagnetic force that is not included in Ernshaw's purely electrostatic theorem. It is this rotation that keeps the two axes parallel, but after that we are merely concerned with the electrostatic forces, and these are all that need be computed.

Circle-and-Point Electrons in a Symmetrical Atom.

The next form of atom subjected to computation approaches the form of fig. 1, representing oblate spheroids much closer than the above assumption of simple spheres. This new computation now to be described still assumes, however, a perfectly symmetrical atom, and makes no allowance for any internal strain. On this account, in the light of subsequent experience, it could not be expected to be entirely successful and satisfactory. Because the solution of the electrostatic problem of finding the force exerted upon one oblate spheroid such as above described due to a second one, on the assumption of the inverse square law as between the infinitesimal elements of charge, because this problem has never been solved, it has been necessary to approach the condition of the model of fig. 1 by the use of solutions which are available.

In this closer approximation to the spheroid of the electron the sphere inscribed within the spheroid, having a radius, b , equal to the semi-minor axis of the spheroid, is imagined to be cut out, and the electron thus divided into two different parts. This leaves a ring of charge thickest at the equator and tapering in thickness toward the poles. The total charge of the electron is thus divided mentally into the sum of two, a sphere and a ring. The sphere may be represented for mathematical computation by a charged point at its centre, and the ring may be approximated by considering it as the

equivalent of a circle of the proper radius, whose centre coincides with the centre of the spheroid and whose plane is the equatorial plane. The oblate-spheroid electron is thus replaced for mathematical requirements by a circle-and-point electron. The approximation is close, however, and the eccentricity of the spheroid is represented in the circle-and-point electron. A change in the eccentricity of the spheroid changes the size of the inscribed sphere, and because the charges are proportional to their volumes, some of the charge changes over from the point to the circle and *vice versa* in the circle-and-point electron.

Denoting by ρ the ratio of the charge on the circle to the whole charge of the electron, e , it is easy to show that ρ is equal to the square of the eccentricity of the spheroid, as follows:— Let E_1 denote the charge on the inscribed sphere and E_2 that on the ring or circle, then evidently

$$E_1 + E_2 = e. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (53)$$

And by the definition of ρ above evidently the charge on the point is

$$E_1 = (1 - \rho)e, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (54)$$

and the charge on the circle is

$$E_2 = \rho e. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (55)$$

Because of the assumption of a uniform density of charge in the electron, the charges on the sphere and ring are proportional to their volumes; whence it follows that

$$\frac{\text{Volume ellipsoid}}{\text{Volume inscribed sphere}} = \frac{e}{E_1} = \frac{e}{e - E_2} = \frac{1}{1 - \rho} = \frac{\frac{4}{3}\pi a_0^2 b}{\frac{4}{3}\pi b^3} = \frac{a_0^2}{b^2}, \quad (56)$$

where a_0 and b are the semi-axes of the ellipsoid.

Solving (56) for ρ we have

$$\rho = \frac{a_0^2 - b^2}{a_0^2} = \epsilon^2, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (57)$$

where ϵ is the eccentricity.

This ratio, ρ , like ϵ , is always unity or less, and as the form of the spheroid changes from a sphere to a very flat

spheroid more and more charge shifts over from the point to the circle. For the sphere both ρ and ϵ are zero, and $E_2=e$, while $E_1=0$. As ρ approaches unity, E_1 approaches zero, and E_2 approaches e .

The electrostatic force of one spheroid upon another is thus simplified to depend upon the force of one circle upon another, and fortunately this latter is obtainable*. The resulting series expressing the force of one circle upon another and the force of one circle-and-point electron upon another derived from it are too long to be printed here, and the reader is referred to the original sources given in the references. It will be necessary to pass over the processes of computation and content ourselves with pointing out some of the results obtained with this form of electron. In the first place, the law of force between two circle-and-point electrons is no longer the simple inverse square law as it was between the two spheres considered in the last example, and not only so but the force has a different expression according to whether the two electrons are in the coaxial position or are in the equatorial position, or are located at some intermediate position. Then, too, the electrons may not have exactly the same eccentricity, which is the case when they constitute parts of atoms, so that different values of ρ and of the radii of the circles denoted by a have to be employed. The subscript (1) refers to the values belonging to the first electron and (2) to those of the second. It will be of interest to give the expressions for the forces upon a first electron due to a second in the two important positions, the coaxial or polar position and also the equatorial position, the axes of the two electrons being parallel to each other in every case. The force upon the first electron in the coaxial position is given by the series of terms,

$$F_z = \frac{e^2}{k} \left[-r^{-3} + \frac{3}{2}(\rho_2 a_2^2 + \rho_1 a_1^2)r^{-4} - \frac{15}{8}(\frac{1}{2}\rho_2 \rho_1 a_2^2 a_1^2 + \rho_2 a_2^4 + \rho_1 a_1^4)r^{-6} \dots \right]. \quad (58)$$

* A. C. Crehore, Phil. Mag. p. 69, July 1913; p. 901, May 1922; p. 854, May 1925.

And the force upon the first electron due to a second electron located in the equatorial plane of the first electron is given by

$$F_x = \frac{e^2}{k} \left[-r^{-2} - \frac{3}{4}(\rho_2 a_2^2 + \rho_1 a_1^2) r^{-4} - \frac{45}{84}(4\rho_2 \rho_1 a_2^2 a_1^2 + \rho_2 a_2^4 + \rho_1 a_1^4) r^{-6} \dots \right]. \quad (59)$$

When both of these electrons have the same eccentricity, so that $\rho_1 = \rho_2$ and $a_1 = a_2$, (58) becomes

$$F_x = \frac{e^2}{k} \left[-r^{-2} + 3\rho a^2 r^{-4} - \frac{15}{4}(2\rho + 1)\rho a^4 r^{-6} \dots \right], \quad . \quad . \quad (60)$$

and (59) becomes

$$F_x = \frac{e^2}{k} \left[-r^{-2} - \frac{3}{2}\rho a^2 r^{-4} - \frac{45}{32}(2\rho + 1)\rho a^4 r^{-6} \dots \right]. \quad . \quad . \quad (61)$$

These series show that the force of repulsion between the two electrons in the equatorial case is slightly greater than when the electrons are spheres, since the first term represents the repulsion between spheres, and the other terms, all being negative, are added to this force according to (61). It is apparent, however, that the amount of change in the force at distances of the order of 10^{-8} cm. must be very small, because ρ is equal to the square of the eccentricity of the spheroid, of the order of unity, and a must be only slightly greater than 10^{-13} cm. Taking ρa^2 as of the order of 10^{-26} and r of the order of 10^{-8} , the terms within the brackets of (60) and (61) become respectively 10^{16} , 10^6 , 10^{-4} , etc., decreasing by 10^{10} from term to term. At this distance, therefore, the force is but slightly increased in the equatorial case expressed by (61) and is decreased in the polar case expressed by (60) by twice the amount, the r^{-6} and following terms being quite negligible.

As the distance between the electrons is decreased from 10^{-8} cm. the higher order terms become greater and greater, but the second term of the series does not become as great as the first term until the distance is reduced to the order of 10^{-13} cm. At such a small distance the r^{-6} term and following

terms are each as large as the first term, the series becoming non-convergent and of course useless. It cannot be applied when the distance becomes less than the value where the convergence is slow, but it is not designed for use under these circumstances. The circle-and-point approximation to the form of an oblate spheroid ceases to be approximate when the distance between the electrons compares with their own dimensions. It might seem at first that at distances of the order of 10^{-8} cm. these very small terms might be omitted, but this is an error, because these terms are all that is left in certain cases after the inverse square term has been cancelled by the presence of other charges.

Let us again pass over the process of computation, which follows the general plan given in full above in the case of spherical electrons, except that rapidly convergent infinite series must be used instead of the more simple inverse square law to express the forces between any pair of elementary charges, and write down the result as follows:—

$$F_x = \frac{e^2}{H_2 \text{ on } C} X \left[-6(b^2 - \frac{1}{2}\rho a^2)r^{-4}f_{x,4}(Z) \right. \\ \left. + \frac{1}{4}(2b^4 - 6b^2\rho a^2 + \frac{3}{4}\rho a^4)r^{-6}f_{x,6}(Z) \dots \right], \quad . \quad . \quad (62)$$

$$F_z = \frac{e^2}{k} Z \left[-6(b^2 - \frac{1}{2}\rho a^2)r^{-4}f_{z,4}(Z) \right. \\ \left. + \frac{5}{4}(2b^4 - 6b^2\rho a^2 + \frac{3}{4}\rho a^4)r^{-6}f_{z,6}(Z) \dots \right], \quad . \quad . \quad (63)$$

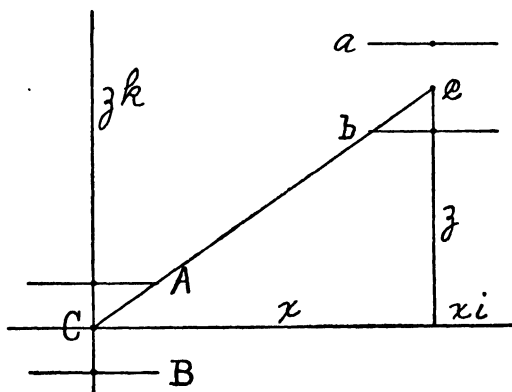
where the functions of Z are as given by (45) to (48).

A diagram representing the hydrogen molecule with circle-and-point electrons to which these equations apply is shown in fig. 6.

These equations are general in so far that the form of the electron may be changed at pleasure by assigning different values to ρ , b , and a , which changes the eccentricity of the ellipsoid without altering its volume or its charge. If, for example, ρ is put equal to zero, it is equivalent to saying that the electrons are spheres, and making this substitution for ρ in

(62) it becomes identical with (43) above computed for the case of spherical electrons. Similarly (63) reduces to (44). So the result in these last equations is more general, including the former as a special case. Just as before, however, the series begins with the inverse fourth power of the distance term and only the even power terms are present. This is because the atom is still assumed to be symmetrical and undeformed. The value of the force now, however, is completely altered, since it depends upon the relative values of b^2 and $\frac{1}{2}\rho a^2$. If these quantities were equal to each other the r^{-1} term would vanish entirely, or, if the latter part is

FIG. 6



Hydrogen molecule with symmetrical atoms using circle-and-point electrons.

larger than b^2 , the sign of the r^{-1} term is reversed. Let us denote the binomial which occurs in the coefficient of the r^{-1} term by P so that we have

$$P = b^2 - \frac{1}{2}\rho a^2. \quad . \quad . \quad . \quad . \quad . \quad . \quad (64)$$

Derivation of the Numerical Value of P.

The numerical value of P may be derived as follows. For the purposes of approximation to the oblate spheroid by the substitution of the circle-and-point electron in its stead, it may be assumed that the ring is best represented by a circle whose

radius is equal to the radius of gyration of the ring, and it will be recalled that the ring is that part of the spheroid that remains after cutting out an inscribed sphere. The moment of inertia of the oblate spheroid about its minor axis is, assuming a uniform density, d , throughout its volume,

$$I_{0,s} = (\frac{4}{3}\pi a_0^2 b d)(\cdot 4 a_0^2), \quad (65)$$

the first factor in parenthesis being the mass and the second the square of the radius of gyration of the spheroid. The moment of inertia of the inscribed sphere is

$$I_s = (\frac{4}{3}\pi b^3 d)(\cdot 4 b^2). \quad (66)$$

The moment of inertia of the ring excluding the inscribed sphere is the difference between (65) and (66), namely

$$I_R = I_{0,s} - I_s = \frac{4}{3}\pi b d (a_0^2 - b^2) a^2 = \frac{4}{3}\pi b d (a_0^4 - b^4)(\cdot 4), \quad . (67)$$

where a represents the radius of gyration of the ring, a_0 the semi-major and b the semi-minor axis of the spheroid. Hence

$$a^2 = \cdot 4(a_0^2 + b^2), \quad (68)$$

where a is the radius to be found that is used in the circle-and-point electron. Now according to (14) above we have the relation between the eccentricity and the semi-axes as follows,

$$a_0^2 = \frac{b^2}{1 - \epsilon^2}, \quad (69)$$

and according to (26) above the eccentricity of the electron in the atom is $\epsilon = 0.9547$, or

$$1 - \epsilon^2 = 1 - 0.91145 = 0.08855. \quad (70)$$

Whence by (69) we find

$$a_0^2 = 11.3 \times 10^{-26} \text{ cm.}^2 \quad (71)$$

And by (68) we have

$$a^2 = \cdot 4(11.3 + 1) \times 10^{-26} = 4.92 \times 10^{-26}, \quad . . . (72)$$

or

$$a = 2.218 \times 10^{-13} \quad \text{and} \quad a_0 = 3.36 \times 10^{-13} \text{ cm.} \quad . . (73)$$

The value of P may now be found by the use of this value of a , for by (57) we have $\rho = \epsilon^2$, and hence

$$\rho a^2 = 0.91145 \times 4.92 \times 10^{-52} = 4.484 \times 10^{-52} \text{ cm.}^2 \quad . . (74)$$

And, since $b^2 = 10^{-26}$ cm. it follows that

$$P = b^2 - \frac{1}{2}pa^2 = (1 - 2.242) \times 10^{-26} = -1.242 \times 10^{-26}, \quad (75)$$

which was to be found. Accordingly the value of P is negative, the second term of the binomial exceeding the first in value. For the purpose of computation in subsequent sections the value of P used is the round number minus one and a quarter times ten to the minus twenty-sixth power.

*The Force upon the Two Electrons and upon
the Whole Atom.*

After this brief digression in deriving the numerical value of P let us return to a consideration of the forces. The force upon the positive charge of the first atom due to the second atom was given by (62), (63). In a similar manner the force upon the two electrons, A and B together, of the first atom due to the second atom is given by

$$\begin{aligned} F_x &= \frac{e^2}{k} X [6Pr^{-4} f_{x,4}(Z) \\ &\quad - \frac{1}{4}(14b^4 - 18b^2pa^2 + 3p^2a^4 + \frac{3}{4}pa^4)r^{-6} f_{x,6}(Z) \dots], \quad (76) \\ F_z &= \frac{e^2}{k} Z [6Pr^{-4} f_{z,4}(Z) \\ &\quad - \frac{5}{4}(14b^4 + 18b^2pa^2 + 3p^2a^4 + \frac{3}{4}pa^4)r^{-6} f_{z,6}(Z) \dots], \quad (77) \end{aligned}$$

where the functions of Z are given by (45) to (48).

Since we now know that the value of P which occurs in the coefficient of the r^{-4} term is not extremely small but is of the order of b^2 , it follows that at distances between the atoms of the order of 10^{-8} cm. the r^{-4} term is approximately 10^{10} times greater than the r^{-6} term just as it was in the previous example with spherical electrons given by (43) and (44) above. The force upon the positive charge of the first atom or upon the two electrons due to the second atom, therefore, obeys the inverse fourth power law very approximately. But it is to be noticed also that the r^{-4} terms of (62) and (63) are exactly equal and opposite respectively to the r^{-4} terms of (76) and (77), and, therefore, in taking their sum in order to obtain the force of the second atom upon the first atom, these terms cancel and leave the series beginning with the r^{-6} term precisely as

was the case with spherical electrons. Adding (62) and (76) for the x -component of the force gives

$$F_x = \frac{e^2}{k} X[-45P^2 r^{-6} f'_{x,6}(Z) \dots], \quad \dots \quad (78)$$

and adding (63) and (77) for the z -component gives

$$F_z = \frac{e^2}{k} Z[-15P^2 r^{-6} f'_{z,6}(Z) \dots], \quad \dots \quad (79)$$

which may be compared with (51) and (52) for the case of spherical electrons. The only difference between the two expressions is in the substitution of P^2 for b^4 in the coefficients, and since P^2 is always positive the sign of the force is not changed but only its magnitude in the ratio of about 1.56 to 1. When ρ is made zero the latter equations reduce to the former as before. And, just as before, the next term of the series is the r^{-8} term, that is about 10^{10} times smaller than the r^{-6} term and is of no consequence. It is not profitable to pursue this symmetrical type of atom any further for the same reasons that were given in the case of the use of spherical electrons. There can be no position of stable equilibrium where the two atoms will stand in equilibrium at any distance. No values of the two variables, Z and r , will reduce both the x - and the z -components, that is the total force, to zero as it must be at the position of stable equilibrium.

The part played by the comparatively large forces expressed by the r^{-1} terms in (62) and (63) cannot be neglected because the energy changes that take place as the molecule is being formed are necessarily connected with these larger forces, but these are manifested only within the atom itself producing an internal strain. The internal stress deforms the atoms themselves very slightly so that they are no longer the perfectly symmetrical bodies that have been assumed up to this point of the investigation. When these strains are taken into account the problem becomes much enlarged in scope and more complex, but if this is the truth of the matter the situation must be faced.

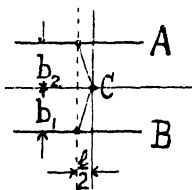
CHAPTER V.

MOLECULES (*continued*):

WITH UNSYMMETRICAL AND DEFORMED ATOMS.

THE effect of the x -component of the force of the second atom upon the positive charge and upon the two electrons of the first atom of the hydrogen molecule is to pull the one to the right and the other to the left, thus very slightly displacing the positive charge off from the line joining the centres of the two electrons, as illustrated in fig. 7. The sum of these displacements in the two atoms of a molecule may be denoted by l , while $l/2$ denotes that in the single atom of fig. 7. The amount of displacement in one atom will be shown to be less

FIG. 7.



Hydrogen atom with circle-and-point electrons illustrating the x -displacement.

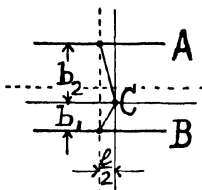
than the radius of the positive charge itself, and in order to show it in the figure it has been greatly magnified. Circle-and-point electrons are illustrated, but it should be kept in mind that the electrons are oblate spheroids in contact with the positive charge.

The effect of the z -component of the force is to increase the pressure between one electron, say B, and the positive charge, and to reduce the pressure between the other electron, A, and

the positive charge. This increases the area of the surface of the one electron, B, and reduces slightly its semi-minor axis, but not necessarily by the same amount as that of A is increased, since the force of the second atom upon A differs slightly from that upon B because of their different distances.

Fig. 8 represents the effect of the z -component of the force upon the first atom together with that of the x -component. The vertical distance from A to C has increased from b in fig. 6 to b_2 in fig. 8, while that from B to C has diminished from b to b_1 . It is of course conceivable that the minor axis of A increases by just the same amount as that of B decreases, in which case the vertical distance from A to B.

FIG. 8.

Hydrogen atom illustrating both the x - and z -displacements.

would remain unchanged. But when the axis of A increases, this electron performs work giving up some of its energy content, and similarly, when the axis of B decreases, work must be done upon it to increase its surface area, and it consequently absorbs energy. If the amounts of the increase of the axis of A and the decrease of that of B were the same, then all the energy given up by A is reabsorbed by B, and there occurs no total change in the energy content of the atom. As long, therefore, as the vertical distance from the centre of A to that of B remains the same as it was in the free atom, so long does the energy of the whole atom remain unchanged. But if the distance AB increases over and above its value in the free atom, then work has been done by the atoms in forming the molecules at the expense of some of the energy content of the electrons as they existed in the free

atoms. Since it is true that work must be done by outside influences to separate the molecule of hydrogen into free atoms and thus restore the two electrons to the condition they have in free atoms, it may be concluded that the distance AB must be very slightly greater when the atom forms part of a molecule than when the atom is free. This also means that the over-all axial distance from the top surface of A to the bottom surface of B in fig. 1 must be slightly greater in the molecule than in the free atom.

The Energy of Dissociation of the Hydrogen Molecule.

Let it be supposed, therefore, that the semi-minor axis of the electron, A, increases from b to b_2 and that of B decreases from b to b_1 , so that the change in that of A is $b_2 - b$ and in that of B, $b_1 - b$, then according to the result derived in Chapter IV, equation (28), the change in their respective energy contents is

$$\text{for A,} \quad \Delta E_A = -6.964 \times 10^6 (b_2 - b), \quad . \quad . \quad . \quad . \quad . \quad (1)$$

$$\text{and for B,} \quad \Delta E_B = -6.964 \times 10^6 (b_1 - b). \quad . \quad . \quad . \quad . \quad . \quad (2)$$

The sum of these energy changes represents the work done by the atom in forming the molecule, and is

$$\Delta E = \Delta E_A + \Delta E_B = -6.964 \times 10^6 [(b_2 + b_1) - 2b], \quad . \quad . \quad (3)$$

which must be negative as representing a loss of energy content on the whole. Now $2(b_2 + b_1)$ is the polar diameter of the atom when it is in the molecule, and $4b$ its diameter when the atom is free, so that it is evident the new diameter must be greater than the free diameter. This neglects the very small diameter of the positive charge of the order of 10^{-16} cm. which cancels, however, in subtracting.

The new semi-minor axis of A may then be expressed as

$$b_2 = b + a + h, \quad . \quad . \quad . \quad . \quad . \quad (4)$$

and of B,

$$b_1 = b - a, \quad . \quad . \quad . \quad . \quad . \quad (5)$$

where a now represents the major part of the change in the semi-minor axis as the molecule is formed from free atoms,

and h represents the very small excess of change of electron A over that of B. From this we find that

$$b_2 - b_1 = 2a + h \quad \text{and} \quad b_2 + b_1 = 2b + h, \quad (6)$$

or

$$h = (b_2 + b_1) - 2b. \quad (7)$$

Hence by (3) and (7) h is not only proportional to the whole work done in forming the molecule, but it also represents half the difference between the new and old polar diameters of the atom.

The experimental value of the energy of dissociation of the hydrogen molecule as determined by Langmuir, namely 5.8×10^{-12} erg per molecule, may now be employed to determine the very small length, h . Since the energy per atom is one half of the energy per molecule, we have by means of (3)

$$\Delta E = -\frac{5.8}{2} 10^{-12} = \Delta E_A + \Delta E_B = -6.964 \times 10^6 h, \quad . . (8)$$

whence the value of h becomes known numerically as

$$h = \frac{5.8}{2 \times 6.964} 10^{-18} = 0.4164 \times 10^{-18} \text{ cm.} \quad (9)$$

This is its value at the point of equilibrium of the two atoms when in the molecule, and it is about one two-hundred-and-forty-thousandth part of the semi-minor axis of the electron itself. The principal change in the semi-minor axis is, however, greater than h , that is to say the a of (4) and (5) is considerably larger than h . The value of a will be determined later in this chapter.

The Forces acting upon the Individual Bodies of the First Atom of the Deformed Molecule.

The complication that arises when allowance is made for the internal stresses within the atom itself as the molecule is formed, is due to the circumstance that there is no longer a fixed atom that may be moved about from place to place as there was in the cases considered in Chapter IV, but the atom itself now changes with each new position. Instead of

having but two variables to manage, Z and r as before, there are now five variables, namely, Z , r , l , b_2 , and b_1 , all of which change in value with the relative positions of the atoms.

Although the complete expressions for these forces have been derived without omitting any terms, they are too long to be given here, and the reader is referred to the original source* for the full equations. However, when the correct numerical values are introduced, these longer expressions take a rather simple form because many of the terms are negligibly small, and the shorter expressions may be given here. Denoting by $l/2$ the small displacement of the positive charge, shown in figs. 7 and 8, and by D the difference between the semi-minor axes of electron A, and electron B, namely $D=b_2-b_1$, the force upon the positive charge, C, of the first atom due to the second atom of the hydrogen molecule is as follows, recalling the definition of P in (64) of Chapter IV:

$$F_{\text{H}_2 \text{ on C}} = \frac{e^2}{k} \{ -2[(2-3Z^2)l-3XZD]r^{-3} - 6(1-5Z^2)XP r^{-4} \dots \}, \quad (10)$$

$$F_z = \frac{e^2}{k} \{ -2[3XZl+(1-3Z^2)D]r^{-3} - 6(3-5Z^2)ZP r^{-4} \dots \}. \quad (11)$$

These may now be compared with the corresponding forces, (62) and (63) of Chapter IV, for the case of symmetrical atoms. It is to be observed that they reduce immediately to the r^{-4} terms of the latter respectively when the displacements l and D are put equal to zero. And the r^{-6} term is not required here any more than it was there, because it is about 10^{10} times smaller than the r^{-4} term. The noticeable change that has taken place is the appearance of an r^{-3} term in (10) and (11) which did not occur in (62) and (63), but these contain l and D as factors, and consequently vanish when they are vanishingly small. These terms cannot be neglected, however, because very small values of l or D will make this r^{-3} term of equal importance with the r^{-4} term simply because the order of the term as to r is lower. The r^{-3} term as here given is complete, no parts of it having

* Phil. Mag. May 1925, p. 864.

been omitted, but in the full expression there are other parts of the r^{-4} term containing both l and D which have been omitted because they are all very small as compared with the principal part that has been retained.

The Values of the Displacements, "l" and "D."

Now, as before stated, the force (10) upon the positive charge including the equal and opposite force upon the two electrons, A and B, is the cause of the small displacement, $l/2$, in the equatorial direction. And, similarly, the force (11) in the polar direction is the cause of the change in pressure between electrons and positive charge, that is to say, is the cause of the small difference in the semi-minor axes, D . For very small displacements such as those contemplated it is legitimate to assume that the displacement is proportional to the force producing it. This is the same as saying that the force F_x in (10) may be set equal to the displacement, $l/2$, multiplied by some constant, say $\frac{e^2}{k} \rho_x$, which is strictly characteristic of this form of atom. Hence we may write

$$F_{x \text{ on } C} = \frac{e^2}{k} \rho_x \frac{l}{2} \quad \dots \dots \dots (12)$$

And, similarly, for the polar component of the force F_z in (11), we may write

$$F_{z \text{ on } C} = -\frac{e^2}{k} \rho_z D, \quad \dots \dots \dots (13)$$

where $\frac{e^2}{k} \rho_z$ is also a constant strictly characteristic of this form of atom, but differs from the first constant. The negative sign is required in (13) because of the definition of the quantity D as equal to $b_2 - b_1$. Had D been defined as equal to $b_1 - b_2$ the negative sign would not have been required. This is made evident by a reference to fig. 8, which represents the first atom of the molecule, H_1 , or by fig. 9, which shows both the atoms, H_2 and H_1 , of the molecule. If the z -component of the force upon C due to the second atom is positive, it acts in the upward direction in fig. 9 and must

this is not yet known. It is also evident that the approximate form of the electron used above, namely, the circle-and-point electron, will not answer even approximately for such a purpose where the charges are so close as to be in contact with each other. The displacement contemplated is so small that the dimensions of the positive charge itself are large in comparison with it, so that the positive charge may no longer be considered as a point charge for this purpose.

On the other hand, the constant, $\frac{e^2}{k\rho_z}$, defined by equation (13) can be determined approximately from the change in the energy content of the electrons because of the change in their surface area which has been discussed in Chapter IV, and a numerical value of this important constant will be derived in a subsequent section of this chapter.

Let us now return to the derivation of the values of the displacements, l and D . By equating the two expressions for F_x in (10) and (12) the quantity $\frac{e^2}{k}$ cancels giving the equation

$$-2(2-3Z^2)lr^{-3}+6XZlDr^{-3}-6(1-5Z^2)XP r^{-4}=\rho_z\frac{l}{2}, \quad (14)$$

which may easily be solved for the displacement, l , in terms of D , r , Z , X and constants as follows :

$$l=\frac{3XZlDr^{-3}-3(1-5Z^2)XP r^{-4}}{(2-3Z^2)r^{-3}+\rho_z/4} \dots \dots (15)$$

And again, by equating the two expressions for F_z in (11) and (13) the quantity $\frac{e^2}{k}$ cancels giving

$$-6XZlr^{-3}-2(1-3Z^2)Dr^{-3}-6(3-5Z^2)ZPr^{-4}=-\rho_z D. \quad (16)$$

Solving this for l gives a second independent expression for this variable as follows :

$$l=\frac{\frac{\rho_z}{2}-(1-3Z^2)r^{-3}}{3XZr^{-3}}D-\frac{3-5Z^2}{X}Pr^{-1}. \quad \dots \dots (17)$$

The latter expression for l involves the constant ρ_z , while the former (15) involves ρ_r . Upon equating the two expressions

Therefore, adopting for the present the ratio $p=1/2$, the value of D in (18) reduces to

$$D = -3ZPr^{-4} \frac{(3-Z^2)r^{-3} + \frac{1}{4}(3-5Z^2)\rho_z}{2r^{-6} - \frac{1}{4}\rho_z r^{-3} - \frac{1}{16}\rho_z^2} \dots \dots \dots (20)$$

From this it appears that D must pass through all values from $-\infty$ to $+\infty$ according to the relative positions of the two atoms. D is evidently equal to zero when the numerator of (20) is equal to zero. The numerator is independent of the ratio, p , whence it follows that for any value of this ratio $D=0$ when

$$r^{-3} = -\frac{1}{4}\rho_z \frac{3-5Z^2}{3-Z^2} \dots \dots \dots (21)$$

This equation is plotted in polar co-ordinates in terms of Z and r in fig. 10, in the curve marked " $D=0$." When $Z=1$ and the two atoms are then in the coaxial position, we have

$$r^{-3} = \frac{1}{4}\rho_z \dots \dots \dots (22)$$

the curve having two infinite branches, the radius vector approaching infinity as Z^2 approaches a value equal to $\cdot 6$, corresponding to an angle of latitude of about $50^\circ 44'$.

And, again, when the denominator of (20) is zero, then $D=\infty$, and we have the equation

$$2r^{-6} - \frac{1}{4}\rho_z r^{-3} = \frac{1}{16}\rho_z^2, \dots \dots \dots (23)$$

which happens to be independent of the direction cosine, Z , merely because of the particular assumption of $p=1/2$ in (18). Hence D has the same infinite value at one fixed radius, r , for all angles or positions of the two atoms, r being the root of this equation.

Solving for r^{-3} the two roots are

$$r^{-3} = \frac{1}{4}\rho_z \text{ or } r^{-3} = -\frac{1}{8}\rho_z \dots \dots \dots (24)$$

The negative root has no physical significance, and the positive root is the same value as that above given in (22). The circle which this represents is shown in fig. 10 and marked " $D=\infty$." The curves for $D=0$ and $D=\infty$ meet on the axis where $Z=1$. This is a critical point where both the numerator and

denominator of D in (20) have become zero. The value of D when $Z=1$ may be evaluated by putting $Z=1$ in (20), giving

$$D = -3Pr^{-4} \frac{2r^{-3} - \frac{1}{2}\rho_x}{2r^{-6} - \frac{1}{4}\rho_x r^{-3} - \frac{1}{16}\rho_x^2} \quad \dots \quad (25)$$

Now when $r^{-3} = \frac{1}{4}\rho_x$ both numerator and denominator are zero, but the numerator is an exact factor of the denominator, and when the vanishing factor is cancelled, we have

$$D = -3Pr^{-4} \frac{1}{r^{-3} + \frac{1}{8}\rho_x} \quad \dots \quad (26)$$

whence the value of D becomes

$$D = \frac{2}{(4)^{1/3}} (-P\rho_x^{1/3}) = 1.26(-P\rho_x^{1/3}) \quad \dots \quad (27)$$

It is, of course, evident that infinite values of the small displacements, l or D, are manifestly impossible in the atom model. The meaning of the value of D here contemplated is simply the value that it must have in any position to satisfy the conditions of equilibrium between the two atoms of the molecule, and since infinite values are impossible, the meaning is that equilibrium is also impossible anywhere on the circle of radius such that $r^{-3} = \frac{1}{4}\rho_x$, or of radius

$$r = (4)^{1/3} \rho_x^{-1/3} = 1.587401 \rho_x^{-1/3} \quad \dots \quad (28)$$

It will be seen from equations (15) and (17) that l also becomes infinite when D is infinite, so that the same circle answers for the infinite values of both l and D.

In tracing the value of D along any radius vector such as that represented in fig. 10, it appears that the two curves where $D=0$ and $D=\infty$ are the loci of points where D changes sign from positive to negative or *vice versa*. Along the portion of the radius vector from the origin that lies between the two curves, and marked D (-), its value is everywhere negative, changing to positive at all points outside the curve where $D=0$, and to positive again for points within the circle where $D=\infty$.

Let us next examine the expressions for the force of the second atom upon the whole of the first atom, for these two equations, the x - and z -components of the force, supply the

remaining expressions to complete the four independent simultaneous equations required to find the four variables Z , r , l , and D .

*The Simplified Equations of the Force of the
Second Atom upon the First.*

The final test of the equilibrium positions of the two atoms of the molecule must be that the complete equations expressing the force of the second atom upon the first are satisfied. Although these expressions have been obtained in full without omitting any terms up to and including the r^{-6} term, we are obliged to omit the complete expressions here and, as before, give the simplified form only. It is to be recalled that the corresponding result for symmetrical atoms was given in full in (78) and (79) of Chapter IV, with which the following expressions may be compared. In that case the first term of the series is the r^{-6} term, but now the series begins with the r^{-4} term, and all terms after that are present including the odd power terms which do not appear with symmetrical atoms. In making the simplification by omitting certain terms because of their negligible numerical values no parts have been omitted from the r^{-4} term, and only those parts containing the higher powers of l and D have been omitted in the r^{-5} term. It is found that either l or D is a factor of all parts in both the r^{-4} and r^{-5} terms, so that when l and D are each equal to zero the series reduces immediately to the expressions in (78) and (79). While there are, of course, parts of the r^{-6} term that contain both l and D , yet they are very small numerically as compared with the important term which has been retained. The simplified equations are then as follows:

$$F_{x \text{ on } H_1} = \frac{e^2}{k} \{ [-3(2-5Z^2)Xl^2 + 6(4-5Z^2)ZlD + 3(1-5Z^2)XD^2]r^{-4} \\ + [-6(4-35Z^2-35Z^4)l + 30(3-7Z^2)XZD]Pr^{-5} \\ - 45(1-14Z^2+21Z^4)XP^2r^{-6} \}, \quad (29)$$

$$F_z = \frac{e^2}{k} \{ [-3(4-5Z^2)Zl^2 - 6(1-5Z^2)XlD + 3(3-5Z^2)ZD^2]r^{-4} \\ + [-30(3-7Z^2)XZl - 6(3-30Z^2+35Z^4)D]Pr^{-5} \\ - 15(15-70Z^2+63Z^4)ZP^2r^{-6} \}, \quad (30)$$

In the last section there was given at least a hint that the values of l and of D , as in (27), come out in terms of the quantity $(-P\rho_z^{1/3})$ with a small multiplier that depends upon the position of the second atom, and that the distance between the atoms, r in (28), comes out in terms of $\rho_z^{-1/3}$ with a small multiplier that also depends upon the position of the second atom. As a matter of fact, the solution of the four simultaneous equations must express each variable in terms of this fundamental constant. Consequently, by a change of notation, the constant may be omitted during the process of solution and restored when the solution is obtained. It is, therefore, found convenient to substitute the new variables, l' , D' , and u for the old variables l , D , and r respectively, as follows:

$$l = l'(-P\rho_z^{1/3}), \quad . \quad . \quad . \quad . \quad . \quad (31)$$

$$D = D'(-P\rho_z^{1/3}), \quad . \quad . \quad . \quad . \quad . \quad (32)$$

$$r = u\rho_z^{-1/3}. \quad . \quad . \quad . \quad . \quad . \quad (33)$$

This substitution is equivalent to changing the scale by which l , D , and r are measured, so that l' , D' , and u may be regarded as the displacements and the distance between the atoms, respectively, in terms of a new unit. The four simultaneous equations required to be solved are (15), (17), (29), and (30). Upon substituting the new variables of (31) to (33) these equations become respectively the following, in which the value of the ratio, p in (19), is taken as equal to $1/2$.

$$l' = \frac{3XZu^{-3}D' + 3(1-5Z^2)Xu^{-4}}{(2-3Z^2)u^{-3} + \frac{1}{4}}, \quad . \quad . \quad . \quad . \quad . \quad (34)$$

$$l' = \frac{\frac{1}{4} - (1-3Z^2)u^{-3}}{3XZu^{-3}} D' + \frac{3-5Z^2}{X} u^{-1}, \quad . \quad . \quad . \quad . \quad . \quad (35)$$

$$\begin{aligned} F_x = \frac{e^2}{k} \{ & [-3(2-5Z^2)Xl'^2 + 6(4-5Z^2)Zl'D' + 3(1-5Z^2)XD'^2]u^{-4} \\ & + [6(4-35Z^2+35Z^4)l' - 30(3-7Z^2)XZD']u^{-5} \\ & - 45(1-14Z^2+21Z^4)Xu^{-6} \dots \} P^2\rho_z^2, \quad (36) \end{aligned}$$

$$\begin{aligned} F_x = \frac{e^2}{k} \{ & [-3(4-5Z^2)Zl'^2 - 6(1-5Z^2)Xl'D' + 3(3-5Z^2)ZD'^2]u^{-4} \\ & + [30(3-7Z^2)XZl' + 6(3-30Z^2+35Z^4)D']u^{-5} \\ & - 15(15-70Z^2+63Z^4)Zu^{-6} \dots \} P^2\rho_z^2. \quad (37) \end{aligned}$$

It is evidently not practicable to solve these equations in a direct manner by elimination of the variables, the solution must be effected by a process of approximation to the roots which will next be described.

The Process of Computation of the Equilibrium Position.

In the position of equilibrium the total force upon each atom must be equal to zero, so that each of the components of the force in (36) and (37) must be equated to zero. Upon so doing the constant factor $\frac{e^2}{k} P^2 \rho_x^2$ disappears and the quantities within each brace are equated to zero. The process followed is to compute and plot two curves, which are shown in fig. 10, the one marked " $F_x=0$ " and the other " $F_z=0$," each point of which gives the position of the second atom when the respective forces are zero. The point of intersection of the two curves thus obtained gives a position where the total force upon each atom due to the other is zero. It then remains to be found whether the position is one of stability or of unstability for small displacements away from this position, for in both cases the force at the point in question is zero.

It may be stated in advance of the detailed account of the process that a point has been found at H_2 in fig. 10 where the two curves intersect which is one of stability for the two atoms, and the approximate numerical values of the four variables at this point are as follows :

$$\begin{array}{ll} Z^2 = 0.8065 & \text{and } u = 1.30, \\ l' = 3.066 & \text{and } D' = 2.935. \quad . \quad . \quad . \quad (38) \end{array}$$

This value of Z corresponds to an angle of latitude of approximately $63^\circ 50'$ as shown in the figure. Restoring the values of l , D , and r in centimetres according to (31) to (33), we have

$$l = 3.066(-P\rho_x^{1/3}); \quad D = 2.935(-P\rho_x^{1/3}); \quad r = 1.30\rho_x^{-1/3}. \quad . \quad (39)$$

Or, since in this example it was assumed that $p = 1/2$ and $\rho_z = \frac{1}{2}\rho_x$, the above values may equally well be expressed in terms of ρ_x instead of ρ_z , as follows:

$$l = 2^{1/3}l'(-P\rho_z^{1/3}) = 3.863(-P\rho_x^{1/3}), \quad . \quad . \quad . \quad (40)$$

$$D = 2^{1/3}D'(-P\rho_z^{1/3}) = 3.698(-P\rho_x^{1/3}), \quad . \quad . \quad . \quad (41)$$

$$r = \frac{u}{2^{1/3}}\rho_z^{-1/3} = 1.032\rho_x^{-1/3}. \quad . \quad . \quad . \quad . \quad (42)$$

From this it is apparent that the distance between the two atoms of the molecule is equal to a small multiplier, which depends upon the position of the second atom, times the reciprocal of the cube root of the constants ρ_x or ρ_z . These constants are entirely independent of the position of the second atom, but are chiefly dependent upon the property of the electron itself and the manner in which it is attached to the atom. The absolute distance in centimetres cannot be determined with exactitude, of course, until both constants have been computed numerically from the theory, but it has now been proved that the main part of the distance depends for its numerical value upon the constants, and that the small variations from this main part come from the small variations in the function, u , which differs for the different kinds of atoms.

The numerical value of ρ_z , however, which is derived in a subsequent section of this chapter, is

$$\rho_z = 30.7 \times 10^{24}, \text{ or } \rho_z^{1/3} = 3.13 \times 10^8 \text{ and } \rho_z^{-1/3} = .32 \times 10^{-8}. \quad . \quad (43)$$

Upon substituting this in (42) the absolute distance between the two atoms of the hydrogen molecule is obtained in centimetres as approximately

$$r = \frac{1}{3} \times 10^{-8} \text{ cm.} \quad . \quad . \quad . \quad . \quad (43A)$$

under the assumption made for this particular solution that $\rho_z = \frac{1}{2}\rho_x$.

The distance between the two atoms in the hydrogen molecule is not known from experimental determinations with

any such approach to precision as that obtainable in the measurement of crystals. Such distances cannot be measured experimentally as if one were measuring with a ruler the distance between two points on a sheet of paper. When concerned with a gaseous molecule, which as a whole is always in a state of rapid translational motion, these measurements are quite indirect. Certain theories of atomic structure are necessarily assumed, that may or may not be correct, and the resulting distance, which is said to be determined experimentally but is in reality only partially so, must differ from the true distance in proportion to the error introduced by such assumptions.

Perhaps as good an estimate as any of the distance between the two atoms of the hydrogen molecule has been made by E. C. Kemble*. He begins with the assumption of a so-called dumb-bell form of molecule. This means that his molecule may be likened to two equal masses connected together by an imaginary massless rod, something like a dumb-bell. Such a structure may whirl around the central point, the centre of gravity half-way between the two masses, in any one plane as well as another with an equal degree of freedom. After adopting this molecular conception use is then made of the kinetic theory of gases combined with the quantum theory, by means of which a value of the moment of inertia for the molecule is deduced. It comes out 2.0×10^{-41} gm. cm.², and from this, together with the known mass of each atom, it may be deduced that the distance between the two atoms of the molecule is about $\frac{1}{2} \times 10^{-8}$ cm.

The value above determined in (43A), namely $\frac{1}{3} \times 10^{-8}$ cm., is in very good agreement with Kemble's result. At least the order of magnitude 10^{-8} cm., which has been referred to above as the universal distance between nearest atoms, is well established by either result. By means of this atomic model it has been definitely connected with a universal cause, the

* Phys. Rev. Feb. 1918, p. 156.

existence of the universal constants, ρ_x and ρ_z , the meaning of each of which is clearly understood.

It should be pointed out that the model of the hydrogen molecule described in the foregoing pages differs quite markedly from a so-called dumb-bell form. The angle of latitude determined by Z given in (38) and corresponding to about $\lambda=63^\circ 50'$ remains approximately fixed as the molecule moves about in its translational motion in the gas. The two atoms are free to revolve about an axis through their common centre of gravity like a conical pendulum around one circle of latitude only, but not at any other latitude. In other words, this molecule does not have the same number of degrees of freedom as the dumb-bell form: it is more restricted. This restriction will undoubtedly necessitate some revision of the result obtained by the use of the dumb-bell molecule above mentioned, and there has as yet been no attempt to carry this out.

On the other hand, no reliance can be placed on the coefficient $1/3$ in (43), but only on the general order of magnitude as being 10^{-8} cm., for it was obtained after assuming a ratio for the two constants ρ_x and ρ_z . Until ρ_x can be found in an *a priori* manner from the electrostatic forces an exact result is not to be expected. In fig. 10 only a small portion of each curve, $F_x=0$ and $F_z=0$, is charted between the values of Z^2 ranging from about $\cdot 7$ to $\cdot 86$, and the whole quadrant has not been covered. This is because the charting of these curves requires an unusual amount of labour. To obtain a single point on one of these curves, as shown below, requires the construction of an auxiliary curve for the purpose of approximation to a root of a higher degree equation. So when the plotting of a set of curves for a succession of assumed values of the ratio of the constants, p , is contemplated the task becomes almost prohibitive. However, it may be stated that, with an assumed ratio of $p=1/4$ instead of $p=1/2$ as in the above example, no position of equilibrium has been found, although most of the quadrant has been covered. At this writing it is

too soon to give any results with a different ratio, but it is entirely possible that the result may come out $r = \frac{1}{2} \times 10^{-8}$ cm. instead of $\frac{1}{3} \times 10^{-8}$ cm. If, for example, with a ratio of $p=1$ the equilibrium distance, u , should happen to be 1.55 instead of 1.3 as before, which is quite possible, then the distance, r , comes out $\frac{1}{2} \times 10^{-8}$ cm.

Numerical Values near the Position of Equilibrium.

It is of some interest to know the magnitude of the forces near to the position of equilibrium and to study how the various terms of the equations above given change with the changing position of the second atom. Let us now consider in some detail the method employed to solve the four equations (34) to (37). First assume a definite radius vector corresponding to some fixed value of Z , say such that $Z^2 = .8$. and work along this particular radius until the points are found where the forces given by (36) and (37) vanish. When $Z^2 = .8$, the four equations (34) to (37) become numerically, respectively

$$l' = \frac{1.2u^{-3}D' - 4.02489u^{-4}}{-0.4u^{-3} + 0.25}, \quad . \quad . \quad . \quad . \quad (44)$$

$$l' = \frac{0.25 + 1.4u^{-3}}{1.2u^{-3}} \quad D' = \frac{u^{-1}}{0.44721}, \quad . \quad . \quad . \quad . \quad (45)$$

$$\begin{aligned} F_x = \frac{e^2}{Z^2} \Big|_{\substack{\text{H}_2 \text{ on H}_1 \\ Z^2 = .8}} \{ & [2.6833l'^2 - 4.02489D'^2]u^{-4} \\ & + [-9.6l' + 31.2D']u^{-5} \\ & - 65.2032u^{-6} \} P^2 \rho_x^2, \quad . \quad . \quad . \quad (46) \end{aligned}$$

$$\begin{aligned} F_z = \frac{e^2}{k} \{ & [8.04978l'D' - 2.6833D'^2]u^{-4} \\ & + [-31.2l' + 8.4D']u^{-5} \\ & + 9.1232u^{-6} \} P^2 \rho_z^2. \quad . \quad . \quad . \quad (47) \end{aligned}$$

Some arbitrary value of u may now be selected, say $u = 1.3$, whence both l' and D' may be found by means of (44) and (45), giving

$$l' = 3.09 \quad \text{and} \quad D' = 2.961. \quad . \quad . \quad . \quad . \quad (48)$$

With these numerical values of l' and D' substituted in (46) and (47) we have numerically

$$F_z = \frac{e^2}{k} \left\{ \begin{array}{ccc} r^{-4} \text{ term} & r^{-5} \text{ term} & r^{-6} \text{ term} \end{array} \right\} P^2 \rho_r^2 = \frac{e^2}{k} (0.102) P^2 \rho_r^2, \quad (49)$$

$H_2 \text{ on } H_1$
 $Z^2 = 8$
 $u = 1.3$

$$F_z = \frac{e^2}{k} \{17.55 - 19.27 + 1.982\} P^2 \rho_r^2 = \frac{e^2}{k} (0.262) P^2 \rho_r^2. \quad (50)$$

According to this result each component of the force is very small as compared with the individual terms that are added, showing that the position selected is very close to the place where the forces vanish. The z -component of the force being positive indicates that the distance, u , should be reduced to make this component vanish, because this means an upward force on the first atom at the origin and a downward force on the second atom tending to reduce the distance. Trying another value of u less than 1.3, say $u=1.29$, gives for l' and D' by (44) and (45)

$$l' = 3.023 \quad \text{and} \quad D' = 2.946. \quad (51)$$

With these values of l' , D' , and u substituted in (36) and (37) we obtain for the forces

$$F_x = \frac{e^2}{k} \left\{ \begin{array}{ccc} r^{-4} \text{ term} & r^{-5} \text{ term} & r^{-6} \text{ term} \end{array} \right\} P^2 \rho_r^2 = \frac{e^2}{k} (-0.283) P^2 \rho_r^2, \quad (52)$$

$Z^2 = 8.2$
 $u = 1.29$

$$F_z = \frac{e^2}{k} \{17.47 - 19.52 + 1.982\} P^2 \rho_r^2 = \frac{e^2}{k} (-0.068) P^2 \rho_r^2. \quad (53)$$

According to this both components of the force have changed sign somewhere between the values $u=1.3$ and $u=1.29$, and by interpolation between these values it is seen that

$$F_x = 0 \quad \text{when} \quad u = 1.2972$$

and

$$F_z = 0 \quad \text{when} \quad u = 1.2920. \quad (54)$$

Thus far only a single point on each $F_x=0$ $F_z=0$ curve has been found, and the value of Z in the general equations may next be changed so as to work along a different radius vector. In effect, the process above outlined is the

plotting of a curve of the variation of these forces along one radius vector and then approximating to the roots of the equations, that is finding the values of u where the forces change sign. In practice many more points must be computed along the radius vector than the two which have been given, for these were purposely selected near to the position of the roots merely to illustrate the method.

When Z^2 is set equal to $\cdot 82$ instead of $\cdot 8$ and the process above given is again carried out, it is found that

$$F_x = 0 \text{ when } u = 1.3065$$

and

$$F_z = 0 \text{ when } u = 1.317. \quad . \quad . \quad . \quad . \quad . \quad (55)$$

Comparing the two results at $Z^2 = \cdot 8$ and $Z^2 = \cdot 82$ shows that the $F_z = 0$ curve lies outside of the $F_x = 0$ curve at $Z^2 = \cdot 82$ but inside of it at $Z^2 = \cdot 8$, and at some point between these values of Z the two curves must therefore intersect. By interpolation the intersection is found to be at $Z^2 = 0.8065$ and $u = 1.30$. The four simultaneous values of r , D , Z , and u corresponding to this point have already been given in (38) above and need not be repeated. It is represented in fig. 10 at the intersection of the $F_x = 0$ and the $F_z = 0$ curves.

It may now be shown that the point so determined represents a stable position for the atoms rather than an unstable one by giving attention to the directions of the forces on either side of the curve of zero force. When the radius vector is greater than the value corresponding to the $F_x = 0$ curve, the x -component of the force of the atom H_2 on H_1 is positive and of H_1 on H_2 , negative, as indicated by the horizontal arrows directed toward the curve in the figure. When the radius vector is smaller than those corresponding to this curve the force changes sign and the arrows are again directed toward the curve. Any displacement of the atom to the right or left of this curve sets up forces that tend to bring the atom back to the curve, showing stability for the x -component of the force.

And, similarly, small displacements up or down from the curve where $F_z=0$ tend to restore the atom to this curve as indicated by the vertical arrows. All forces surrounding the point of equilibrium are thus directed towards this point, showing that the position is one of stability.

Determination of the Atomic Constant, ρ_z .

The definition of the constant ρ_z is contained in the assumption given by equation (13) above, namely,

$$F_z = -\frac{e^2}{k} \rho_z D, \quad \text{H}_\tau \text{ on C} \quad (56)$$

and the constant $\frac{e^2}{k} \rho_z$ may be said to denote this force when

D is equal to one centimetre—that is, the force per unit of change in D assuming proportionality. It was shown in equation (27) of Chapter IV that the work done by the electron A in forming the molecule is

$$-\frac{e^2}{k} (30.7 \times 10^{24}) (b_2 - b) = -6.964 \times 10^6 (b_2 - b), \quad (57)$$

and that the work done upon the electron B during the same interval is

$$-\frac{e^2}{k} (30.7 \times 10^{24}) (b_1 - b). \quad (58)$$

The sum of these two energies, the first being negative and the second positive, was given in (3) above as representing the work done by A over and above that done upon B, which is equivalent to one half of the total energy of dissociation of the hydrogen molecule. On the other hand, the difference between these energies is

$$-\frac{e^2}{k} (30.7 \times 10^{24}) (b_2 - b_1) = -\frac{e^2}{k} (30.7 \times 10^{24}) D. \quad (59)$$

And, since each of the above energies is large as compared with their numerical difference, the last expression is very approximately double the energy change for each electron

disregarding the sign. One half of this, then, represents the energy given up by A and received by B. Hence the energy change to decrease the semi-minor axis of B by the amount a or to increase that of A by the same amount (see (4) and (5)), is

$$\frac{1}{2} \frac{e^2}{k} (30.7 \times 10^{24}) D. \quad . \quad . \quad . \quad . \quad . \quad (60)$$

But for a unit distance, or for D equal to unity, the energy and the force required to make this change in the energy are numerically equal to each other. And, since the force upon the positive charge in (56) is approximately double the force upon each electron numerically, it follows that the constant, ρ_z , is merely the numeric that occurs in (59), and we have

$$\rho_z = 30.7 \times 10^{24}, \quad . \quad . \quad . \quad . \quad . \quad (61)$$

which was to be found.

*Numerical Values of the Properties of the Atom at
the Position of Stable Equilibrium.*

The distance between the two hydrogen atoms forming the molecule has been discussed above. It is given by equation (42) and in centimetres by (43 A). The small displacements l and D are given by (40) and (41). To find these in centimetres the numerical value of P obtained in (75) of Chapter IV and of ρ_z in (61) above may now be substituted in (40) and (41), giving

$$l = 3.066 \times 1.25 \times 10^{-26} \times 3.945 \times 10^8 = 15.11 \times 10^{-18} \text{ cm.}, \quad (62)$$

$$D = 2.935 \times 1.25 \times 10^{-26} \times 3.945 \times 10^8 = 14.47 \times 10^{-18} \text{ cm.} \quad (63)$$

Hence the displacement of the positive charge C off from the centre line of the two electrons, A and B of fig. 1, is equal to one half of 15.11×10^{-18} cm. or 7.56×10^{-18} cm. Bearing in mind that the semi-major axis of the positive charge C is 7.054×10^{-16} cm. in the free state according to (6) of Chapter IV, it appears that this displacement is only a small

fraction of the semi-major axis of the positive charge, approximately one hundredth of it. This fact is sufficient to show that it would be futile to attempt to derive the constant ρ_x by assuming that the positive charge may be regarded as a point charge, for only a minute portion of the charge crosses the centre line joining the two electrons. The major portion of it still remains on either side after the displacement, and yet this very small displacement, together with the displacement D, is sufficient to so alter the electrostatic forces between the two atoms as to produce an equilibrium position where none existed before.

The numerical value of $D = b_2 - b_1$ is given by (63) above, and the equivalent expression for it by (6) of Chapter IV. Hence we have

$$D = b_2 - b_1 = 2a + h = 14.47 \times 10^{-18} \text{ cm.} \quad . \quad . \quad . \quad (64)$$

The numerical value of h obtained in (9) of Chapter IV may now be used to find the principal part of the change in the semi-minor axis of the electron A and the whole change in that of B denoted by a in (4) and (5) of Chapter IV. We have

$$2a = (14.47 - 0.4164) 10^{-18} = 14.054 \times 10^{-18} \text{ cm.,} \quad . \quad . \quad . \quad (65)$$

whence

$$a = 7.027 \times 10^{-18} \text{ cm.} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (66)$$

This represents the contraction in the semi-minor axis of the electron B, and the expansion of that of A is greater than this by the amount h , so that the new semi-axes, which were assumed to be exactly 10^{-13} cm. in the free state of the atom, become after the molecule is formed

$$\text{for A,} \quad b_2 = 1.000,074,43 \times 10^{-13} \text{ cm.} \quad . \quad . \quad . \quad (67)$$

$$\text{for B,} \quad b_1 = 0.999,929,73 \quad , \quad , \quad . \quad . \quad . \quad (68)$$

In making these changes in the axes while the molecule

is being formed the electron A gave up an amount of energy, according to (1) of Chapter IV,

$$\Delta E_A = -6.964 \times 10^6 (l_2 - l) = -51.83 \times 10^{-12} \text{ erg, . . . (69)}$$

while B absorbed an amount of energy

$$\Delta E_B = -6.964 \times 10^6 (l_1 - l) = +48.93 \times 10^{-12} \text{ erg. . . (70)}$$

The sum of this negative and positive energy represents the part of the original energy content of the two electrons that is used in forming the molecule, which is

$$\begin{aligned} \Delta E_A + \Delta E_B &= -6.964 \times 10^6 [(l_2 + l_1) - 2l] \\ &= -6.964 \times 10^6 h = -2.9 \times 10^{-12} \text{ erg. . . (71)} \end{aligned}$$

This is just half the energy of dissociation of the molecule, namely 5.8×10^{-12} erg as determined experimentally, and the same value is obtained by adding (69) and (70) and multiplying the result by two. According to this nearly seventeen times as much energy is passed across from the electron A to B as is used in forming the molecule, that is in binding the second atom to the system. The actual change in the minor axis of either electron is minutely small as compared with the minor axis itself, the minor axis being about 14,000 times greater. It has been shown conclusively, however, that on the basis of the electrostatic forces alone this very small asymmetry of the two electrons is responsible for the formation of the molecule.

The Stress within the Atom.

The force upon the positive charge C in the direction of the equatorial plane is given by the x -component of the force in (12) above, which now becomes numerically

$$F_{x \text{ on } C} = \frac{e^2}{k} \rho_1 \frac{l}{2} = \frac{e^2}{k} (464) 10^6 \text{ dynes. . . . (72)}$$

This is a comparatively large force, being the equivalent of the repulsion between two electrons at a distance of only about 4.6×10^{-5} cm. from each other. On the other hand, for

comparison the order of magnitude of the forces upon the whole atom near to their position of equilibrium, by (36) and (37), is

$$\frac{e^2}{k} P^2 \rho_x^2 = \frac{e^2}{k} (.590) \text{ dynes, ... (73)}$$

which is nearly 10^6 times smaller than the forces producing the internal strain. It follows naturally that the energy transformations must follow in the wake of these larger forces and manifest themselves as deformations of the atom.

*The Coaxial Position of the Two Atoms
of the Molecule.*

Considerable time and effort have been given to an investigation of the coaxial position of the two atoms of the hydrogen molecule, because it seemed natural to expect that an equilibrium might be found in this situation; but this was an erroneous conjecture, and since it has been possible to prove that there can be no equilibrium for the coaxial position, it seems worth giving the proof here. In the coaxial position we have $Z=1$ and $X=0$, so that the first of the four fundamental equations (14) above reduces to

$$2lr^{-3} = \rho_x \frac{l}{2}, \quad \text{or} \quad r^{-3} = \frac{1}{4} \rho_x, \quad (74)$$

from which l cancels, leaving the same result as given by (22) or (24), which must hold true for any and every value of l . That is to say l cannot be determined from (14) or (74).

And, again, putting $Z=1$ and $X=0$ in (36) and equating the force to zero gives

$$-6l'D'u^{-4} + 24l'u^{-5} = 0, \quad (75)$$

whence

$$D' = 4u^{-1}. \quad (76)$$

Now when Z is made equal to unity and X equal to zero in (16) we find

$$4Dr^{-3} + 12Pr^{-4} = -\rho_x D = -p\rho_x D. \quad (77)$$

This may be converted over into terms of D' and u by the use of (32) and (33), giving, after solving for D' ,

$$D' = \frac{12r^{-4}\rho_x^{2/3}}{4r^{-3} + p\rho_x} \dots \dots \dots (78)$$

At the position of equilibrium it has been shown above that it is true that $r^{-3} = \rho_x/4$ for any position of equilibrium on the axis, and upon substituting this in (78) we find

$$D' = \frac{3r^{-1}\rho_x^{-1/3}}{1+p} = \frac{3}{1+p} u^{-1} \dots \dots \dots (79)$$

Thus two different expressions for D' have been found in (76) and (79), the latter containing the ratio p , to which as yet no numerical value has been assigned. Upon equating these values of D' and solving for p , it appears that

$$p = -1/4, \dots \dots \dots (80)$$

and must, therefore, have a negative value, which is impossible because ρ_x and ρ_z are essentially positive constants and p denotes their ratio. It has been proved by this that no possible value of p can be assigned which will permit of satisfying all of the conditions for equilibrium in the case of two coaxial hydrogen atoms.

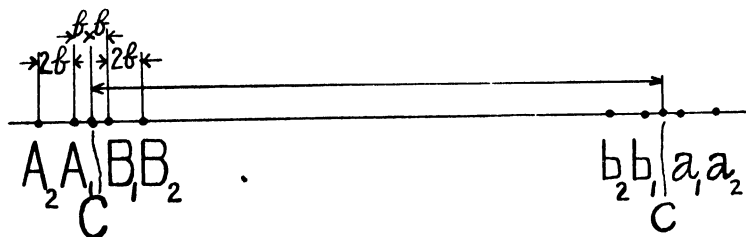
The Helium Atom.

Up to this point we have dealt almost exclusively with the atom of hydrogen and have traced in some detail the formation of a molecule by the combining of two hydrogen atoms. It is now proposed to give some consideration to the atom of helium constructed according to the model in figs. 2 and 2A, and treated in a manner similar to that of hydrogen with the object of ascertaining whether or not two such atoms ought to unite to form a molecule of helium. As a matter of fact, it is well known that helium is monatomic and the atom of helium will neither unite with another helium atom nor with any other form of atom to form a molecule. So, if some reason appears in connection with this model of the helium

atom that renders it quite different from that of hydrogen showing that it ought not to form a molecule by the combination of two or more atoms, then this form of model receives a very strong support from the experimental observations.

Let us place two helium atoms, such as represented in fig. 2 or 2A, in the coaxial position at some distance from each other and compute the force exerted by one upon the other, using the circle-and-point approximation to the form of the oblate-spheroid electron as was done for hydrogen in Chapter IV. The centre of the positive charge of $4e$ in the first atom is represented at C (fig. 11), the centre of the second atom being at c at a distance r . The centres of the electrons in the pile

FIG. 11.



Representing the centres of the charges in two helium atoms coaxial with each other.

of four negative electrons belonging to one atom are shown at $A_2 A_1 B_1 B_2$ and of the other at $a_2 a_1 b_1 b_2$. The resulting force upon the positive charge C, due to the whole of the second atom $a_1 a_2 c b_1 b_2$ developed in series of the inverse powers of the distance r , comes out as follows :

$$F = \frac{e^2}{k} [(240b^2 - 24a^2)r^{-4} + (3280b^4 - 1200a^2b^2 + 30a^4)r^{-6} \dots]. \quad (81)$$

He atom on C

In a similar manner, the total electrostatic force of the second atom upon the four negative electrons together is found to be

$$F = \frac{e^2}{k} [(-240b^2 + 12a^2)r^{-4} + (-15280b^4 + 3600a^2b^2 - 90a^4 - 15a^4)r^{-6} \dots]. \quad (82)$$

He atom on four elects.

It is interesting to compare these with the corresponding equations obtained with hydrogen atoms in Chapter IV. The equation corresponding to (81) is given by (63) of Chapter IV after putting $Z=1$. And the equation corresponding to (82) is (77) of that chapter after putting $Z=1$ for the coaxial case. In the case of hydrogen the r^{-4} term of (63) is exactly equal and opposite to the r^{-4} term of (77), but this is not true of the r^{-4} terms in (81) and (82) above. Now since the force upon the whole of the first atom due to the second is evidently the sum of (63) and (77) in the hydrogen case, or of (81) and (82) in the case of helium, therefore the r^{-4} term cancels in taking the sum in the case of hydrogen, leaving the resulting series to begin with the r^{-6} term. This was given as equation (79) of Chapter IV, in which Z is to be made equal to unity for the coaxial case. In contrast with this the r^{-4} term does not cancel in taking the sum of (81) and (82) above, but gives a resulting series beginning with the r^{-4} term as follows:

$$F_{\text{He atom on He atom}} = \frac{e^2}{k} [-12\rho a^2 r^{-4} + (-12000b^4 + 2400\rho a^2 b^2 - 90\rho^2 a^4 + 15\rho a^4)r^{-6} \dots]. \quad (83)$$

For reasons similar to those given in Chapter IV, equation (36), it can be shown that the r^{-4} term is about 10^{10} times larger than the r^{-6} term, and that this is in turn the same number of times larger than the r^{-8} term and so on, the series being very rapidly convergent, so that the first term represents the sum of the series very closely. Although this computation has only been carried out as regards the coaxial position, it is quite evident from the experience with the hydrogen atom that the complete force in the general position of the two atoms will come out equal to the r^{-4} term of (83) multiplied by some function of Z as yet undetermined, but similar in kind to the functions given by (45) and (47) of Chapter IV.

And, just as in the case of hydrogen, it may be concluded that there can be no position of stable equilibrium for the two atoms of the molecule until the small deformations of the

atom are taken into the account, that is, until the case of unsymmetrical atoms is considered. When this is done for the helium atom the result differs very radically from the case of hydrogen, for the reason that the force series begins with the r^{-4} term instead of the r^{-6} term as above pointed out in the example with symmetrical atoms. The deformed hydrogen atoms caused terms of lower order beginning with the r^{-4} and including the r^{-5} term to appear in the expression for the force of the second atom upon the first, thus introducing two additional terms as given in (29) and (30) of Chapter V. It is the simultaneous co-operation of these three terms which makes the position of stable equilibrium possible. But, now, in the case of helium it is only possible that one term of lower order can appear, namely the r^{-3} term, for all the r^{-2} terms cancel. This gives at most but two terms in the series instead of three. In the case of hydrogen the r^{-3} terms cancel when equations (10) and (11) of Chapter V, expressing the force upon the positive charge, are added to the force upon the two electrons. If this were also true of the deformed helium atom then there would be no additional terms of lower order to appear, and (83) after including the proper function of Z would represent the whole force. This shows no possible position for stable equilibrium so as to form a molecule. Even though the deformation were such as to introduce a possible r^{-3} term, which does not appear in the case of hydrogen, even then we should not expect any equilibrium position, because there are but two terms present.

This result explains why the atom of helium is monatomic if it is constructed like the model in fig. 2 or 2A. In searching further for the peculiar reason why such a result in the case of helium should come out so different from the result in the case of hydrogen, it may be stated that it is because the helium atom has four electrons in a pile instead of two in a pile as in hydrogen. Such a result would not have been obtained if the two outside electrons of the helium atom had been placed differently, say, in a second pile of two on the same equatorial

plane with the alpha-particle constituting its centre. This is not suggested as a possible arrangement for helium but as an illustration of an arrangement that would give a different result. Moreover, whenever such a pile of four electrons occurs in any atom, such, for example, as the neon atom Ne of fig. 2, the same reasoning above given in the case of helium will cause the expression for the force to begin with the r^{-4} term and prevent the atom from uniting with another to form compounds.

The only two atoms in figs. 2 and 2A which have this pile of four electrons are helium and neon, the inert gases, and it has been shown that neither of them will combine with other atoms like themselves. It remains to be shown that they will not unite with any other atoms of the type of those shown in fig. 2, and for this purpose we shall next consider the combination of a hydrogen and a helium atom as being typical of the others. This result with helium offers a reason why a plane distribution of the alpha-particles and hydrogen atoms, as, for example, in the model of the nitrogen atom, has been adopted for the other atoms, and in a sense it offers some justification for the arrangements illustrated in fig. 2.

Attention should also be called to the fact that the coefficient of the r^{-4} term of (83) is in terms of ρa^2 instead of b^2 , which means that, if the shape of the negative electrons was spherical instead of non-spherical, ρ would be zero and the r^{-4} term would vanish, and with it would vanish the reason that prevents helium from forming compounds. Although it may seem to have a rather indirect bearing upon the shape of the electron, yet such a finding must be regarded as still another indication of the existence of a non-spherical shape for the electron.

*A Helium and a Hydrogen Atom will not combine
to form a Molecule.*

Carrying out a computation of the electrostatic forces for the case of two coaxial atoms similar to the case represented

in fig. 11, except that one of the helium atoms is replaced by a hydrogen atom, gives for the force upon the whole hydrogen atom,

$$F_{\text{He on H}} = \frac{e^2}{k} (-6\rho a^2)r^{-4} \dots \dots \dots (84)$$

the series beginning with the r^{-4} term as was the case in (83). This force is just half as great as that given by (83), and the same reasoning that applied to that equation, given in full in the last section, also applies to this case and need not be repeated. From this it appears that both the atoms of the molecule need not consist of a pile of four electrons as in the helium atom, but that it is sufficient that one such atom only be present in order to prevent the formation of compounds with this atom. This also means that neon will not form compounds with hydrogen.

Since writing the following Chapter VI on crystal structure more than a year has been devoted to the investigation of diamond and graphite, during which period the fundamental force-expressions have been verified, and the resulting forms somewhat simplified. It is much to be desired that the process of derivation be published in detail, but this has not been possible in the present work. The resulting forces are necessarily set down here as required in their application to crystals, but without proof.

One purpose of this note is to call attention to an amendment in certain parts of the following Chapter VI. It has seemed best to take care of the matter by the use of a note not only because the chapter was already in type, but because the principal conclusions are not affected. The manner in which the forces have been derived demands that the positive direction of the y -axis, where just two atoms are concerned, shall be along the vector $k' \times k$ instead of in the opposite direction $k \times k'$. Here k and k' represent unit vectors along the direction of the axes of the first and second atoms respectively. The inadvertent use of the wrong one reverses the direction of the y -axis, and consequently also of the x -axis. Hence the signs of the direction-cosines X and Y only require to be reversed in Table I, p. 147, and this in turn changes the signs of $\Sigma(f_i p^{-3})$ in Tables I, II, and III. This does not affect the general conclusions that hydrogen or carbon atoms of this type can form neither a simple face-centred, nor a body-centred cubic lattice, and that carbon satisfies the conditions for diamond.

As a simplification of equation (11), p. 151, it has been proved that f_6 in (15) reduces to f_4 in (13) and that f_6 in (18), p. 152, reduces to $-(1/2)f_7$ in (16).

August, 1926.

CHAPTER VI.

THE FORCES OF CHEMICAL AFFINITY. A STUDY OF CRYSTAL STRUCTURE IN THE LIGHT OF THE ATOMIC MODELS.

IN the two preceding chapters on Molecules the treatment was limited to the special case of parallel axes as between two atoms. At the time that investigation was made the expression for the electrostatic force exerted by one circle of charge upon another in general positions in space had not been obtained; it was restricted to the special case of parallel planes or axes. Until the expression for this electrostatic force is found for the general positions of the two circles, which enables one to write down the force upon a first circle-and-point electron due to a second such electron, it is quite apparent that no progress could be made in the study of crystal structure, because it is certain that the axes of all atoms, which means of all electrons too, are not parallel to each other in such bodies.

This general expression for the force has now been obtained, but its derivation and length are such that it is not practicable to give it here. By its means, however, the electrostatic forces upon one hydrogen atom at the origin of co-ordinates due to a second hydrogen atom, as well as that upon a first carbon atom due to a second carbon atom in general positions have been obtained. The carbon atom is shown in figs. 2 and 2A at C, and a more detailed projection drawing of it is given below in fig. 18. As yet only the hydrogen and carbon atoms have been thus computed because the process involved is somewhat tedious, that of summing up the forces due to the individual charges situated in slightly different positions within each

atom. As a consequence the treatment is limited in this chapter to a study of hydrogen and carbon only, although certain generalizations for other atoms may be made. The reason for this limitation will be apparent when it is realized that, in order to effect the summation of the forces, say, between two carbon atoms, there are 225 expressions to be summed. This number is the square of 15, which is the number of individual charges in the carbon atom as shown in the figures referred to. There are 12 negative electrons and 3 positive bodies of charge, and the number of pairs taking one at a time from each atom is evidently the square of 15. Fortunately the perfect symmetry of the carbon ring as seen in the model has simplified some of the expressions and facilitated the process of addition. We shall give the resulting expressions for hydrogen and carbon later in this chapter in connection with their applications to crystal structure.

There are many forms of space-lattice in which different substances are known to crystallize, but there are four in particular that will be considered here both because of their common occurrence and of their greater simplicity. These are (1) the simple cubic lattice, (2) the so-called face-centred cubic lattice, (3) the body-centred cubic lattice, and (4) two interpenetrating face-centred lattices such as that of the diamond.

The process of the X-ray analysis of a crystal by means of the spectrum lines recorded on the photographic plate enables one to deduce in many cases the exact locations of the centres of each of the atoms in the crystal. The system of points thus located form what is called a space-lattice. But this is the extent of the information obtainable from such an analysis. It indicates the form of the lattice and tells the kind of atom situated at each point of it, and what is most important, determines all the dimensions in centimetres with a surprising degree of accuracy. In fact the determination experimentally of certain forms of crystals may be regarded as among the most reliable and accurate in the atomic realm.

The force exerted by one atom upon another, however, cannot be computed from the force equations by a knowledge of the kind of atoms and the locations of their centres alone. Every atom possesses a definite axis, and to alter the angle between any two axes not only changes the force of the one upon the other, but it changes it in a marked degree, to such an extent that the force may be made zero, strongly attractive or strongly repulsive merely by changing the relative orientations of their axes without altering the positions of their centres at all.

Before any application of the force equations can be made to a space-lattice of atoms, therefore, something more is required than is given by an X-ray analysis of a crystal. The relative directions of the axes of all atoms must be determined, a matter not revealed by the experimental data.

If two atoms only are now considered having their centres located in fixed positions, the atoms being free to turn their axes in any directions, the force equations show that they will always tend to turn their axes until they become parallel with each other. The electromagnetic forces connected with the individual charges rotating upon their axes are responsible for this effect. The effect of these forces is to produce only a torque and not a translational effect, which vanishes when the two axes are brought into parallelism. This torque is a function of the angle between their axes, increasing at first with a departure from parallelism up to a maximum and then decreasing again to zero as the axes again become parallel but turned in opposite directions. The stable condition for slight displacements is when the axes have the same direction, the moments of the force then tending to restore them to parallelism, while the unstable condition is when the axes have opposite directions.

Due to this circumstance alone the directions of the axes of all the atoms in any one of the four space-lattices above mentioned may be determined without making any use of the expressions for the electrostatic force between atoms. The

result is that there exist in such lattices but four different directions for the axes of all atoms, namely parallel to the four diagonal lines of a cube, or, which comes to the same thing, to the four medial lines of a regular tetrahedron. The whole lattice is thereby divided up into four equal groups of atoms, there being the same number having axes parallel to one of the medial lines of the tetrahedron as to any other.

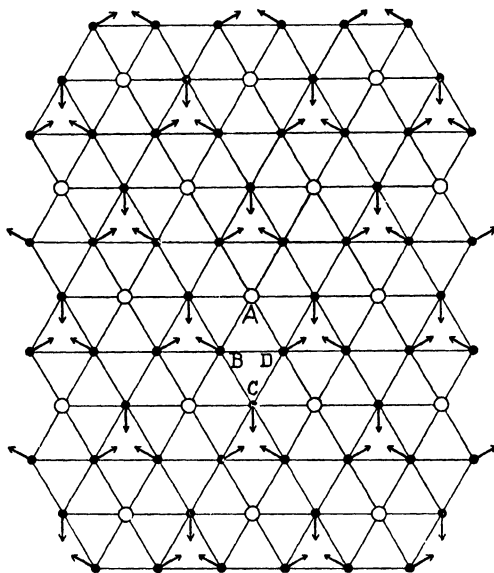
This matter may be more readily understood by considering a definite form of lattice, and we shall begin by a study of the simple cubic lattice. Such substances as sodium chloride (NaCl), potassium chloride (KCl), potassium bromide (KBr), and potassium iodide (KI), have been proved by the X-ray analysis to form simple cubic lattices. It is to be noted that there are at least two different kinds of atoms in each of these substances. Moreover, no simple cubic crystal has ever been discovered, as far as the Author is aware, composed entirely of one kind of atom, one element. There must exist some reason for this connected with the character of the forces acting between two atoms of the same kind. In the cases of hydrogen and carbon the force-equations given below reveal the reason, and the process of their derivation makes it quite probable that the same result will be found in this respect for all other atoms, so that no single element ought to crystallize as a simple cubic lattice on the basis of these atomic models.

In the simple cubic lattice, of which we may take sodium chloride, common rock salt, as an example, it has been found that the atoms of sodium and of chlorine occur alternately along the three directions parallel to the three edges of the cube. From this it follows that in any one plane perpendicular to any diagonal of the cube, a so-called (111) plane, all the atoms are of the same kind, either sodium or chlorine as the case may be, all of the even planes containing but one kind of atom, while the odd planes alternating with the even planes contain only atoms of the other kind.

Because each atom exerts a torque upon every other atom in the crystal tending to turn its axis into parallelism with

itself, as above mentioned, it may be shown that the condition for equilibrium of these turning moments of force demands that the axes of the atoms in any and every one of these (111) planes shall be as represented in fig. 12. There are but four different directions for the axes of atoms in this plane figure, namely, parallel to the four medial lines of a regular tetrahedron erected upon any one of the equilateral triangles

FIG. 12.

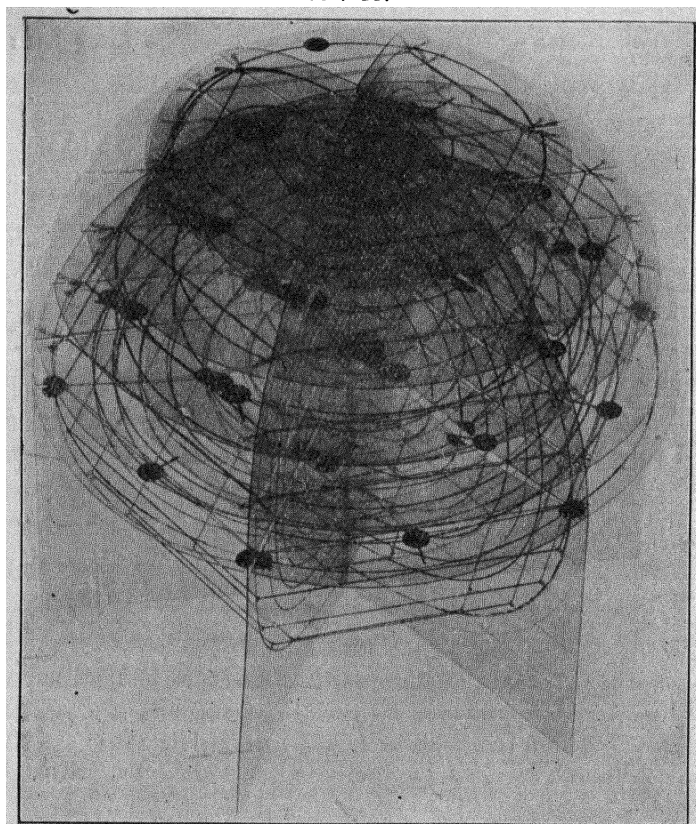


Arrangement of the centres of atoms and the directions of their axes in any (111) plane in a crystal of the cubic or isometric system. The axis of atom A is upward perpendicular to the paper, while those of B, C, or D are downward away from the centre of the tetrahedron having BCD as a base.

as a base. These four directions are exhibited by the four atoms A, B, C, and D. The axis of A is denoted by a small circle meaning that the atom's axis is directed upward perpendicular to the plane of the paper. All atoms shown with such circles have axes perpendicular to the paper. The axis of the atom at B is shown in projection by an arrow

directed outward from B away from the centre of the equilateral triangle BCD. This axis is not supposed to be in the plane of the paper but is shown in projection being in reality directed outward and downward from B away from the centre of the tetrahedron upon BCD as a base. And, similarly, the

FIG. 13.



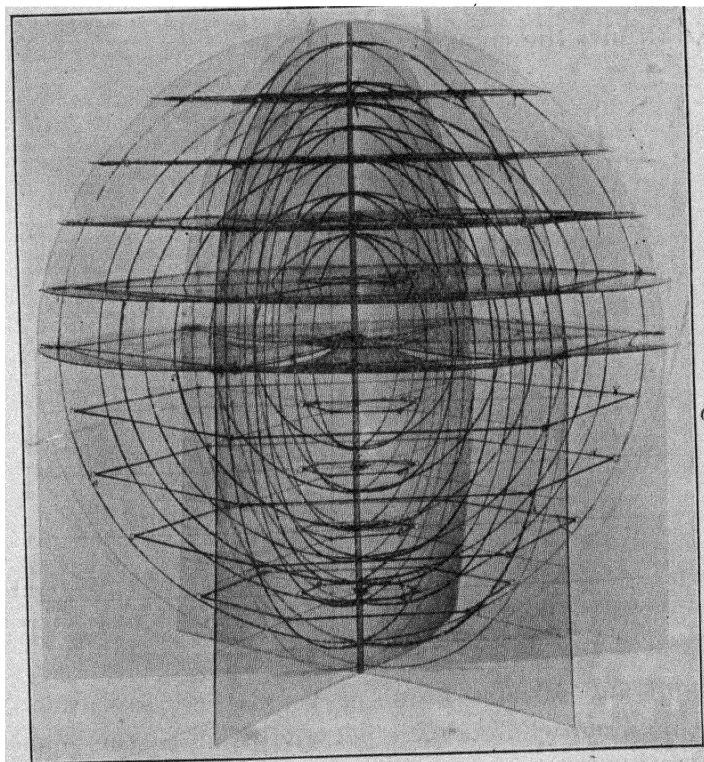
Celluloid model of the simple cubic lattice taken from a point which shows the elementary cubes.

axes of C and D are directed away from the centre of this same tetrahedron. The plane is thus made up of atoms in four groups with equal numbers parallel respectively to the axes of A, B, C, and D. And because every (111) plane in

the cubic lattice is exactly similar to that shown in fig. 12 the whole lattice is thus divided into four equal groups of atoms.

To form a cubic lattice of such planes as that of fig. 12, however, each atom in the plane next above any given plane comes directly over the centre of an equilateral triangle in the given plane instead of directly over the points or vertices of these triangles. The intervals between the planes are uniform,

FIG. 14.

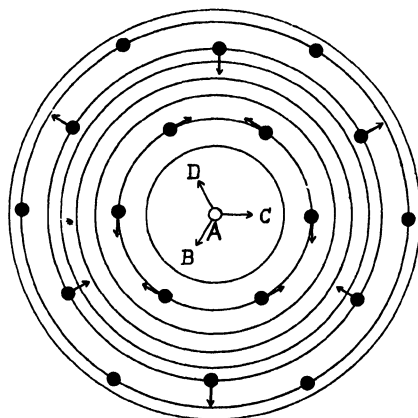


A second view of the same model showing the equally spaced (111) planes perpendicular to the cube diagonal.

each being equal to one-third the diagonal of an elementary cube, and it is not until one passes over three such intervals, a distance equal to the cube diagonal, that a plane occurs having its atoms directly over each atom of the original plane, each axis being parallel to the corresponding one.

To represent a simple cubic lattice and exhibit the directions of the axes of all the atoms requires more than is included in the models of such a lattice as at present constructed. A photographic reproduction of a model of the cubic lattice is shown in two different views in figs. 13 and 14. It is made of sheets of transparent celluloid on which black dots are drawn to indicate the atoms at the points of the lattice. Six vertical sheets set at 60° apart support the parallel (1 1 1) planes, which are equidistant, their separation being $1/3$ the cube diagonal or $\frac{1}{3}\sqrt{3}$ times the cube edge.

FIG. 15.

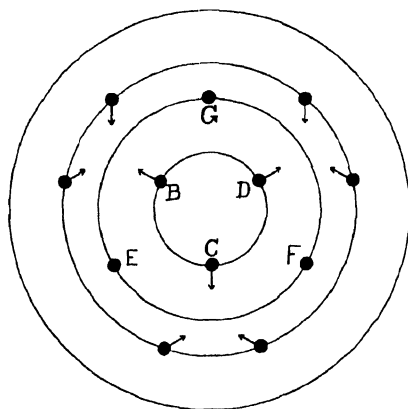


The central or zero plane of the model of figs. 13 and 14. This figure will also answer for the zero plane of any lattice of the cubic or isometric system.

About any selected atom of the lattice, say atom A of fig. 12, an atom whose axis is perpendicular to the plane, the neighbouring atoms are all arranged in several concentric spheres of increasing radii. Seven of these spheres are contained in the model, their intersections with the six vertical planes and with the zero plane through the selected atom being clearly shown by the seven concentric circles drawn in these respective planes. The horizontal (1 1 1) plane through the selected atom at the centre is called plane No. 0, the next

plane above it plane No. 1, the next No. 2, and so on. The top plane in the model is No. 4 containing atoms belonging to the sixth and seventh spheres only. The fifth sphere, counting from the centre outwards, does not intersect this plane. Planes below the zero plane are numbered -1 , -2 , etc., but these have been omitted from the model because each negative plane is an exact counterpart of the corresponding positive plane, being symmetrical with respect to the centre of the sphere, the directions of the axes at corresponding points being parallel to each other.

FIG. 16.



Plane No. 1 of the model of the cubic lattice, figs. 13 and 14. This figure will also serve for Plane $+2$ of the face-centred cubic lattice, or for Plane -1 of the diamond lattice.

Planes Nos. 0, $+1$, and $+2$ of the model are shown in figs. 15, 16, and 17 respectively. The seven concentric circles of fig. 15 represent the intersections of the seven spheres of the model with the zero plane. The outside circles represent the limits of the celluloid plates respectively.

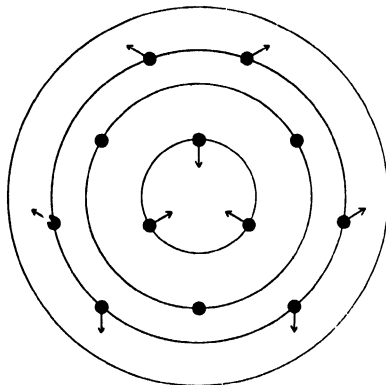
The atoms in this plane, however, do not occur in every one of these spheres but only in spheres 2, 6, and 7, not counting the one selected atom at the centre. In plane No. 1, fig. 16, the atoms occur only in spheres Nos. 1, 3, and 5 in the

model, while in plane No. 2 of fig. 17 they occur only in spheres Nos. 2, 4, and 6.

Planes -6 , -3 , 0 , 3 , 6 and so on in the cubic lattice are exact duplicates of plane No. 0, each atom occurring directly over its corresponding atom and having parallel axes. Similarly planes Nos. -5 , -2 , 1 , 4 , 7 and so on are duplicates of plane No. 1, while planes Nos. -4 , -1 , 2 , 5 , 8 and so on, are duplicates of plane No. 2.

Let us now study the effect of the plane of atoms shown by fig. 16, plane $+1$, upon the selected atom at the centre of

FIG. 17.



Plane No. $+2$ of the model of the cubic lattice, figs. 13 and 14. This figure will also serve for Plane $+1$ of the face-centred cubic lattice, or for Plane $+2$ of the diamond lattice.

the spheres and at the centre of fig. 15, atom A, in so far as the moments of the forces are concerned. The atom B of plane No. 1, fig. 16, exerts a torque upon A of fig. 15 tending to turn it into parallelism with itself. The torque takes place about an axis perpendicular both to the axis of B and to that of A. Such a perpendicular drawn from A lies wholly in the zero plane, fig. 15, the line AB. The amount of the torque may therefore be represented by the line AB, its length being drawn to a suitable scale. In a similar manner the turning

moment upon A due to the atom C of fig. 16 may be represented by the line AC in the plane of fig. 15. This line must have the same length as AB because the atoms B and C are necessarily of the same kind, all atoms in any one plane being alike, and because their distances from A are the same and also the angles between their axes. The angle between the lines AB and AC is 120° because of the equilateral triangle formation of the atoms B, C, and D of plane No. 1.

And again the line AD of fig. 15 represents the torque of D upon A, being equal to each of the other two torques. The sum of the three vector torques, AB, AC, and AD in one plane at an angle of 120° with each other gives a geometrical sum equal to zero. Hence the three atoms B, C, D of plane No. 1 have no turning moment upon the selected atom A. The next circle of atoms in plane No. 1, namely E, F, and G, have axes parallel to that of A and consequently they have no turning moment upon A. They do produce a condition of stability for turning moments of force to hold the axis of A where it stands, however, in case of small displacements that arise from any cause. This matter was referred to above.

The third circle of atoms in plane No. 1 may easily be shown to exert a zero torque upon the atom A by the same reasoning used for the first circle, the atoms B, C, and D. And so for each and every circle of atoms in plane No. 1 there is no turning moment of force upon atom A in the zero plane, while all those atoms having parallel axes produce great stability. What is true of plane No. 1 may by similar reasoning be shown to hold true for every other plane of atoms in the cubic lattice. Hence the condition for zero turning moments of force upon any selected atom in the cubic lattice is satisfied by arranging each (1 1 1) plane of atoms with the directions of their axes as shown in fig. 12.

But it might be objected that we have selected a particular atom A of fig. 12 or 15 whose axis is perpendicular to the plane instead of one that makes an angle with it, and this does not fairly represent *any* atom. The answer to this is that, had

we selected one of the other atoms in plane zero, a different set of (1 1 1) planes, of which there are four sets, would have been chosen, the set that is perpendicular to the axis of the selected atom. However, a study of the lattice shows that with the lattice as above described with A as centre, the symmetry is perfect in respect to B, C, or D as a centre, it being possible to turn the same model in four different ways so as to make the axis of either A, B, C, or D vertical, when the (1 1 1) planes will be found arranged exactly like fig. 12 in the four different ways. It is believed that there is no other possible arrangement of the directions of the axes in a (1 1 1) plane than that of fig. 12 that can produce this result of four-fold symmetry and yet reduce the moments of the forces upon each atom to zero.

It may be concluded, therefore, that the directions of the axes of all the atoms in a cubic lattice have now been determined by making use of the principle of turning moments of force alone without any reference to the translational electrostatic forces that are next to be studied. The form of the crystal must be like that of the model shown in the photographic reproductions figs. 13 and 14.

The Electrostatic Forces.

There are two conditions that must be satisfied by the electrostatic forces acting upon any selected atom in a crystal in general, both of which require investigation before the problem becomes determinate. By far the largest forces concerned act upon the individual charges in any given atom, upon the positive charges in one direction and upon the negative charges in the exact opposite direction. The force of the whole crystal of atoms upon the positive charge of any atom may be regarded as the cause of the deformation of the atom, which has been discussed in Chapter V. It is met by the equal reaction of the negative electrons. The component of this force upon the positive charge along the direction of

the axis of the atom, the z -component with which alone we shall be concerned in studying these crystals, has the effect of slightly flattening the figure of the electrons on one side of the hydrogen atoms, or of the alpha-particles in the carbon atoms, and of slightly expanding those on the opposite side.

As pointed out in Chapter V, the reactive force upon the positive charge accompanied by a change of shape of the upper and lower electrons may be taken as proportional to the displacement, D , and the actual force (axial component) upon the positive charge of either a hydrogen atom or of an alpha-particle may be set equal to the expression given by (13) of Chapter V, namely,

$$F_z = -\frac{e^2}{k} \rho_z D = -\frac{e^2}{k} \rho_z (b_2 - b_1) = -30.7 \times 10^{24} \frac{e^2}{k} D, \quad . \quad . \quad (1)$$

where b_2 and b_1 refer respectively to the semi-minor axes of the negative electrons on the positive and negative sides of the positive charge, the positive or negative z -axis.

The first condition to be satisfied is, therefore, to set the sum of the forces upon the positive charge of a hydrogen atom in the axial direction due to the whole lattice of atoms in the crystal equal to equation (1). Or in case of a carbon atom containing three positive bodies in three alpha-particles the force is three times equation (1). If the nature of the force due to the lattice as shown by the expressions obtained for this force is such that it cannot be equated to (1), the inference is that such an atom cannot form such a space-lattice as that chosen.

The second condition is that the total force of the whole lattice of atoms, the crystal, upon the whole of each and every atom within it shall be set equal to zero for equilibrium.

It will now be shown that neither hydrogen nor carbon can form a simple cubic lattice or crystal because neither of the two conditions mentioned above will satisfy the expressions for the forces between two hydrogen or two carbon atoms. Although this might be regarded as a kind of negative result,

inasmuch as it does not tell the kind of crystal that either hydrogen or carbon may form, yet it is certainly an experimental result that neither of these elements alone do crystallize as a cubic lattice, and it seems important to point out the reason why. Every instance where there is agreement with an experimental result establishes more confidence in the expressions for the forces derived from the atomic models of Chapter II.

The component of the electrostatic force upon the positive charge of a first hydrogen atom at the origin of co-ordinates resolved along the z -axis, the axis of the first atom, due to a second hydrogen atom in any general position, x, y, z, α , has been derived from the atomic model of hydrogen to be as follows.:

$$F_{z \text{ on } +} = \frac{e^2}{k} \{ 2f_1 D_B r^{-3} + 6f_2 (b^2 - \frac{1}{2}\rho a^2) r^{-4} \dots \}, \quad . \quad . \quad . \quad (2)$$

where f_1 and f_2 are different functions of X, Y, Z , the direction cosines, and of α , the angle that the axis of the second atom makes with that of the first. These functions are

$$f_1 = (1 - 3Z^2) \cos \alpha + 3XZ \sin \alpha, \quad . \quad . \quad . \quad . \quad (3)$$

$$f_2 = -Z + Z(5Z^2 - 2) \cos^2 \alpha + 5X^2 Z \sin^2 \alpha \\ + 2X(1 - 5Z^2) \sin \alpha \cos \alpha, \quad . \quad . \quad . \quad . \quad (4)$$

The quantity D_B denotes the displacement previously discussed, equal to $b_2 - b_1$ for the second atom, the subscripts A and B referring hereafter to quantities pertaining to the first or second atoms. The quantity $(b^2 - \frac{1}{2}\rho a^2)$ is substantially a constant, the quantity P of equation (64) of Chapter IV, its numerical value being given in (75).

The force is expressed in the form of a series with increasing inverse powers of r , and it may be proved that the r^{-5} and following terms may be neglected as negligible in comparison with the r^{-3} and r^{-4} terms above given, the series being very rapidly convergent.

It has been pointed out above that the simple cubic lattice

is symmetrical with respect to any selected atom as a centre both as regards the location of the centres of the atoms and with regard to the direction of the atom's axes. This means that for any one pair of such symmetrical atoms the values of the direction cosines X , Y , and Z are each reversed in sign for the second atom of the pair, while the angle α remains unchanged. Therefore, in summing the forces expressed by (2) for any one such pair of atoms upon the positive charge at the origin, the r^{-4} term gives a sum equal to zero. This is because f_2 in (4) contains the direction cosines in odd degrees only and therefore gives equal and opposite values for the given pair of atoms. On the other hand, the r^{-3} term containing f_1 in (3) does not vanish because f_1 is an even-degree function. Hence the r^{-4} term will vanish for the whole cubic lattice, because it is wholly made up of such symmetrical pairs of atoms. And after writing

$$r = pl, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

where l is the length of the edge of the elementary cube of the lattice, it is evident that the sum of the forces due to all the atoms of the lattice upon the positive charge of any selected atom may be written, dropping the r^{-4} term, as follows:

$$\Sigma F_x = 2 \frac{e^2}{k} D l^{-3} \Sigma f_1 p^{-3}, \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

where the summation indicated is extended to every atom. The quantity D has been set outside the sign of summation because it must have the same value for each atom in the cubic lattice, since each atom is related to the whole in exactly the same manner as every other. The quantity p may be considered as the distance of the second from the selected atom in terms of a new unit, the edge of the elementary cube, and it is to be noted that the whole force upon the positive charge is made to depend upon the inverse *third* power of the distance, p . This implies that many more of the outlying atoms will have an appreciable effect upon the

selected atom than would be the case if the force followed a higher inverse power of the distance. The force upon the *whole atom*, however, does follow a higher inverse power, as we shall presently see.

To find the force expressed in (6) it is best to proceed in a systematic manner to find the sum of $f_1 p^{-3}$, which depends only upon the geometry of the cubic lattice as described. For this purpose it is convenient to consider the atoms in concentric spheres around the selected atom of successive increasing radii, because the value of p is the same for all the atoms in any one sphere. The necessary geometrical description of the simple cubic lattice for this purpose is given in Table I to include eight concentric spheres. The first column of the Table gives the number of the sphere counting from the centre outward. The second the number of the (111) plane in which the atoms concerned lie. The third the number of the circle in this plane counting from the centre outwards. The fourth the value of $\cos a$, the fifth the number of atoms in this circle, the sixth the radius of the sphere. The seventh, eighth, and ninth columns give the values of the direction cosines, X, Y, and Z. Where the double sign occurs and the cosines are given for both positive and negative planes together, the upper sign is to be taken for the atoms in the positive plane and the lower for those in the negative plane. The tenth column gives the sum of f_1 for all atoms in the respective planes, the eleventh the sum of $f_1 p^{-3}$ for each sphere, and the twelfth the total $\Sigma f_1 p^{-3}$ including all spheres preceding.

There are several points of interest to be noted concerning the cubic lattice.

1. The points of the lattice, at each of which but one atom is centred, in any one sphere about the selected atom all lie either in the even or in the odd planes, but never in both even and odd.

2. The angle, a , is the same for each atom in any one sphere, $\cos a$ being either $-\frac{1}{3}$ or $+1$ for any given sphere.

TABLE I.
Cubic Lattice.

Sphere.	Plane.	Circle.	$\cos \alpha$	No. atoms circle.	$\rho = \frac{r}{l}$.	X.	Y.	Z.	Σf_1 plane.	$\Sigma f_1 \rho^{-3}$ sphere.	$\Sigma f_1 \rho^{-3}$ total.
1	± 1	1	$-\frac{1}{3}$	3	1	$\frac{1}{3}\sqrt{6}$	0	$\frac{1}{3}\sqrt{3}$	+4	+8	+8
2	0	1	$-\frac{1}{3}$	6	$\sqrt{2}=1.41421$	0	± 1	0	0	-2.42	
"	± 2	1	$-\frac{1}{3}$	3	"	$\pm \frac{1}{3}\sqrt{3}$	0	$\pm \frac{1}{3}\sqrt{6}$	-4	-2.82842	+5.17158
3	± 1	2	+1	3	$\sqrt{3}=1.73205$	$\frac{2}{3}\sqrt{2}$	0	$\pm \frac{1}{3}$	+2		
"	± 3	0	+1	1	"	0	0	± 1	-2	0	"
4	± 2	2	+1	3	2	$\frac{1}{3}\sqrt{6}$	0	$\pm \frac{1}{3}\sqrt{3}$	0	0	"
5	+1	3	$-\frac{1}{3}$	6	$\sqrt{5}=2.23607$	$-\frac{2\sqrt{30}}{15}$	$\pm \frac{\sqrt{10}}{5}$	$+\frac{\sqrt{15}}{15}$	$-\frac{16}{5}$		
"	-1	3	"	6	"	+ "	"	- "	$-\frac{16}{5}$		
"	+3	1	"	6	"	0	$\pm \frac{\sqrt{10}}{5}$	$+\frac{\sqrt{15}}{5}$	0	$-\frac{32}{125}\sqrt{5}$	
"	-3	1	"	6	"	0	"	- "	0	$=-57243$	+4.59915
6	0	2	"	6	$\sqrt{6}=2.44949$	± 1	0	0	0		
"	± 2	3	"	6	"	$\pm \frac{2}{3}$	$\pm \frac{\sqrt{3}}{3}$	$\pm \frac{\sqrt{2}}{3}$	$+\frac{16}{3}$	$+\frac{4}{9}\sqrt{6}$	
"	± 4	1	"	3	"	$\pm \frac{1}{3}$	0	$\pm \frac{2\sqrt{2}}{3}$	$+\frac{8}{3}$	$=+1.08866$	+5.68781
7	0	3	+1	6	$2\sqrt{2}=2.82842$	1	0	0	+6		
"	± 4	2	"	3	"	$\frac{\sqrt{3}}{3}$	"	$\pm \frac{\sqrt{6}}{3}$	-3	0	"
8	± 1	4	$-\frac{1}{3}$	6	3	$\pm \frac{\sqrt{6}}{9}$	$\pm \frac{2\sqrt{2}}{3}$	$\pm \frac{\sqrt{3}}{9}$	$+\frac{8}{9}$		
"	+3	2	"	6	"	$\pm \frac{\sqrt{6}}{3}$	0	$+\frac{\sqrt{3}}{3}$	0		
"	-3	2	"	6	"	$\mp "$	0	- "	0	$-\frac{8}{81}$	
"	± 5	1	"	3	"	$\mp \frac{\sqrt{6}}{9}$	0	$\pm \frac{5\sqrt{2}}{9}$	$-\frac{20}{9}$	$=-98765$	+4.70016

3. The sum of the quantity $(1-3Z^2)$ taken for all the atoms in each and every sphere is equal to zero. Consequently this sum is zero for the whole cubic lattice.

4. In those spheres where $\cos \alpha = +1$, the axes of all such atoms being parallel to that of the selected atom, $\sin \alpha = 0$ and, therefore, the second term of equation (3) vanishes for all such spheres. Since the first term of (3) also vanishes because of the preceding statement in 3, it follows that f_1 vanishes for all spheres in the lattice where $\cos \alpha = 1$.

These statements make it possible to simplify the expression for $\Sigma f_1 p^{-3}$ and set it equal to

$$\Sigma f_1 p^{-3} = 2 \sqrt{2} \Sigma (XZ p^{-3}), \quad . \quad . \quad . \quad . \quad . \quad (7)$$

since $\sin \alpha = \frac{2}{3} \sqrt{2}$ in every case and the first term of f_1 in (3) containing $(1-3Z^2)$ vanishes for each sphere.

Including eight spheres Table I gives the numerical value of $\Sigma f_1 p^{-3}$ as $+4.700$. This includes 122 of the nearest surrounding atoms up to a distance of $r=3l$, three times the cube edge. Its value, however, has been computed to include 30 spheres embracing 882 atoms to a distance of $r=\sqrt{35}l=5.9161l$, but these spheres have not been included in Table I on account of the space required. In Table II the total $\Sigma f_1 p^{-3}$ is given up to 30 spheres, and the final value of this geometrical quantity for an infinite cubic lattice may be taken as not far from

$$\Sigma f_1 p^{-3} = +4.47. \quad . \quad . \quad . \quad . \quad . \quad (8)$$

By this Table it is seen that atoms as far away as six times the cube edge still have a small but appreciable effect upon the selected atom.

Let us now return to equation (6) and write down the force upon the positive charge of one hydrogen atom on the assumption that the whole cubic lattice contains only hydrogen, using the numerical value in (8).

We have

$$F_z = \frac{e^2}{k} (8.94) l^{-3}. \quad . \quad . \quad . \quad . \quad . \quad (9)$$

But it has been pointed out above that this force must also be equal to equation (1) arising from the equal and

TABLE II.
Cubic Lattice.

Sphere No.	Total No. Atoms.	$\Sigma f_i p^{-1}$.
1	6	8.00000
2	18	5.17153
3	26	"
4	32	"
5	56	4.59915
6	80	5.68781
7	92	"
8	122	4.70016
9	146	4.19419
10	170	"
11	178	"
12	202	4.56180
13	250	4.47453
14	256	"
15	304	4.38054
16	340	4.62497
17	364	"
18	388	"
19	436	4.33995
20	460	4.26947
21	484	"
22	514	4.34371
23	586	4.42726
24	618	"
25	690	4.57564
26	738	4.49125
27	750	"
28	786	4.52192
29	834	4.47444
30	882	"

opposite restoring force exerted by the negative electrons. Upon equating to that expression the unknown quantity, D ,

cancels as well as the factor e^2/k , giving a numerical value for l , the edge of the cube of the lattice, namely,

$$l^{-3} = -\frac{30.7}{8.94} \times 10^{24}, \text{ and } l = -0.66 \times 10^{-8} \text{ cm.} \quad (10)$$

This result is evidently impossible because of the negative value for l , which is essentially a positive quantity, although its magnitude comes out of the right order. Hence the first condition above referred to is not satisfied by the hydrogen atom, and this supplies the proof that hydrogen cannot crystallize as a simple cubic lattice. In order to satisfy the first condition the geometrical sum of $f_1 p^{-3}$ for the lattice must have a negative sign instead of positive as found in (8). The reason for this should perhaps be repeated here. If both D and $\Sigma f_1 p^{-3}$ are positive, then the force upon the positive charge of the selected atom by (6) is positive, that is along the positive z -axis. Such a force will compress the electron on the positive side and enlarge that on the negative side of the positive charge. Since b_2 is by definition the semi-minor axis of that on the positive side, and b_1 of that on the negative side, and since $D = b_2 - b_1$ by definition, a positive force must produce a negative value for D . But, if D is negative so also must $\Sigma f_1 p^{-3}$ be negative to make the force in (6) positive. On the other hand, if the force (6) were negative, then D must be positive, and if D is positive then $\Sigma f_1 p^{-3}$ must be negative as before. Therefore, to satisfy the requirements $\Sigma f_1 p^{-3}$ must be negative for the lattice. But it has been proved that this sum is positive for the cubic lattice, and hence this lattice will not be possible for hydrogen*.

Before leaving this subject it ought to be added that the force in (6), said to be the z -component of the force, really represents the whole force upon the positive charge. The symmetry of the cubic lattice makes the sum of the x - and y -components of this force vanish individually, thus leaving the z -component as the total force, which, therefore, acts in the direction of the axis of the atom.

* See note page 130.

The Second Condition.

While it may seem superfluous to consider hydrogen further in connection with a cubic lattice after having proved that the first condition is not met, yet it seems best to show that the second condition is not satisfied either, because the same reasoning will be found to apply to carbon also. The second condition is that the total force upon each and every atom shall reduce to zero for equilibrium.

It requires more space to write down the force upon the whole of the first hydrogen atom due to a second atom than was required in (2), (3), and (4) above for the force upon the positive charge because it involves higher order terms in r^{-4} , r^{-5} , and r^{-6} and seven functions of X, Y, and Z instead of two as before. However, it has been proved that the z -component of the total electrostatic force upon the first hydrogen atom at the origin due to a second hydrogen atom in the position (x, y, z) is as follows:

$$\begin{aligned} F_z = \frac{3e^2}{k} \left\{ f_3 D_A D_B r^{-4} \right. \\ + \left[f_4 b^2 D_A + f_5 \left(b^2 - \frac{1}{2} \rho a^2 \right) D_B - f_6 \left(\frac{\rho a^2}{2} \right) D_A \right] r^{-5} \\ \left. + 5 \left[f_7 b^2 \left(b^2 - \frac{1}{2} \rho a^2 \right) + f_8 \rho a^2 b^2 - \frac{1}{2} f_9 \rho^2 a^4 \right] r^{-6} \dots \right\}, \quad (11) \end{aligned}$$

where the seven functions of the direction cosines, X, Y, and Z, and the angle, α , are as follows:

$$* f_3 = 5Z^2\xi - 3Z \cos \alpha + X \sin \alpha, \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

$$\begin{aligned} f_4 = & -(3 - 30Z^2 + 35Z^4) \cos^2 \alpha \\ & - 10XZ(3 - 7Z^2) \sin \alpha \cos \alpha \\ & + [5(X^2 + Z^2) - 35X^2Z^2 - 1] \sin^2 \alpha, \quad . \quad . \quad . \quad . \quad (13) \end{aligned}$$

$$\begin{aligned} f_5 = & (3 - 30Z^2 + 35Z^4) \cos \alpha \\ & + 5XZ(3 - 7Z^2) \sin \alpha, \quad . \quad . \quad . \quad . \quad . \quad . \quad (14) \end{aligned}$$

$$\begin{aligned} f_6 = & 2 - 10Z^2 - 5Y^2 + 35Z^2(\xi^2 + Y^2) - 10Z\xi \sin \alpha \\ & - 20XZ \sin \alpha \cos \alpha \\ & + 2(2 - 15Z^2) \sin^2 \alpha - 5X^2 \cos^2 \alpha, \quad . \quad . \quad . \quad . \quad (15) \end{aligned}$$

* See note page 130.

$$f_7 = -63Z^3\zeta^2 + Z(21X^2 + 7Z^2 - 3)\sin^2\alpha \\ + Z(70Z^2 - 15)\cos^2\alpha + 6X(1 - 14Z^2)\sin\alpha\cos\alpha, \quad (16)$$

$$f_8 = -(1 - 7\zeta^2)(Z + \xi\sin\alpha) \\ + \zeta\cos\alpha(5 - 7\zeta^2) \\ + \frac{7}{2}Z(1 - 9\zeta^2)(\xi^2 + Y^2), \quad . \quad . \quad . \quad . \quad . \quad (17)$$

$$f_9 = 3Z - 7Z(X^2 + 4Y^2 + \xi^2) + \frac{63}{2}Z(\xi^2 + Y^2)(X^2 + Y^2) \\ + 2\xi[1 - \frac{7}{2}(X^2 + Y^2)]\sin\alpha \\ + (Z\cos\alpha + 2X\sin\alpha - 14XZ\xi)\cos\alpha, \quad . \quad . \quad . \quad (18)$$

$$\text{where} \quad \zeta = Z\cos\alpha - X\sin\alpha, \quad . \quad . \quad . \quad . \quad (19)$$

$$\text{and} \quad \xi = Z\sin\alpha + X\cos\alpha, \quad . \quad . \quad . \quad . \quad (20)$$

These seven functions show that the coefficients of the r^{-4} and r^{-6} terms of the force-equation in taking the sum of (11) for a cubic lattice vanish, because the functions f_3 in the r^{-4} term and f_7 , f_8 , and f_9 in the r^{-6} term are odd-degree functions in X , Y , and Z . All these odd-degree functions sum to zero, but the even-degree functions that occur in the coefficient of the r^{-5} term do not sum to zero. The total force is then expressed by the r^{-5} term alone, the r^{-7} and following terms being quite negligible.

Now the coefficient of the r^{-5} term is not zero and the force upon the selected atom, therefore, cannot be equated to zero for equilibrium so as to fulfil the second necessary condition. Even though the coefficient of the r^{-5} term were possibly zero, yet there would exist no stability for an equilibrium. The condition of stability demands that there be a balance between the three terms, r^{-4} , r^{-5} , and r^{-6} , so that either an increase or a decrease in the length of the elementary cube edge, l , will be opposed by the forces acting.

It has now not only been proved that the second condition is not satisfied for hydrogen in a cubic lattice, but the proof given applies with equal force to any lattice in which each atom is a centre of symmetry. It follows that hydrogen

cannot crystallize as either a face-centred or a body-centred cubic lattice. Of course it is not yet known experimentally by X-ray analysis just how hydrogen does crystallize, but the subject has been presented in this manner because it will presently be shown that precisely the same reasoning can be applied to the carbon atom, and it is known that pure carbon does not form any one of the three lattices above mentioned.

The Face-centred Cubic Lattice.

By adding one atom to the centre of each square face of the simple cubic lattice above described, still retaining all atoms at the corners of the cubes as in the cubic lattice, a so-called face-centred cubic lattice is obtained. All of the points thus added will be seen to be situated in one of the same (111) planes as they existed before. The addition of these face-centred points merely increases the number of atoms in each (111) plane, which it will be observed are still arranged in equilateral triangles, but the side of each triangle is only one-half as long as it is in the cubic lattice. The above statement applies to the location of the centres of the atoms only, but the directions of the axes are changed. The axes of each (111) plane, however, are just as represented in fig. 12. No complete model of this lattice is shown here, but one may easily be constructed by the use of fig. 15 for the zero plane, fig. 17 for plane +1, and fig. 16 for plane +2, the centres of the three figures being placed vertically one above the other. Plane +3 is an exact repetition of plane 0, plane +4 of plane +1, and so on.

A study of this face-centred lattice shows that the complete data for it may be obtained from that of the simple cubic lattice above described. This is also one of the reasons for having computed the cubic lattice as far as thirty concentric spheres, because the same computation answers for the face-centred lattice merely by the process of omitting certain of the spheres from the cubic lattice. There are several points that should be noted.

1. The (111) planes perpendicular to the cube diagonals are equally spaced at a distance of one-third the elementary cube diagonal, namely $\frac{1}{3}\sqrt{3}l$.

2. The edge of the elementary equilateral triangle is one-half of that in the simple cubic lattice. In the latter it is $\sqrt{2}l$ and in the face-centred lattice $\frac{1}{2}\sqrt{2}l$.

3. A face-centred cubic lattice may be formed from a simple cubic lattice merely by omitting all of the odd (111) planes from the cubic lattice. All dimensions of the remaining lattice must then be divided by a factor of 2, because the side of the triangle of the cubic is twice that of the face-centred lattice. Consequently the same computations made above for the cubic may now be used for the face-centred lattice, provided $r/2$ is substituted for r . In the r^{-3} term of the force-equation this change makes r^{-3} and p^{-3} eight times its former value.

4. By the use of Table I and its extension to include a larger number of spheres, not given on account of space, it is possible to find $\Sigma f_1 p^{-3}$ for the face-centred lattice by picking out the sums obtained for the even planes only, omitting those of the odd planes, and then multiplying the result by 8 for the reason given in 3 above. The omission of the odd planes of the cubic lattice is seen by Table I to mean the complete omission of certain spheres, because an odd plane never occurs in the same sphere with an even plane and *vice versa*. Hence a face-centred lattice may be made from a cubic lattice by retaining the spheres of the latter numbered 2, 4, 6, 7, 9, 11, 13, 14, 16, 18, 20, 21, 23, 26, 27, and 29 just as they are, and renumbering these spheres consecutively from 1 to 16 as is done in Table III for a face-centred lattice. The radius of each sphere, p , is half as great in the face-centred as in the cubic lattice. These 16 concentric spheres comprise 428 atoms.

The resulting value of $\Sigma f_1 p^{-3}$ for the face-centred lattice is -17.66 , a negative quantity instead of a positive one as found for the cubic lattice*. In order to satisfy what was

* See note page 130.

termed the first condition above mentioned this sum should be negative. Upon substitution in the force-expression (6) and equating to (1) as before, we find

$$F_z = \frac{e^2}{k} (35.32Dl^{-3}) = -30.7 \times 10^{21}D, \quad \text{on H}^+ \quad (21)$$

whence

$$l^{-3} = 0.87 \times 10^{21} \text{ and } l = 1.05 \times 10^{-8} \text{ cm.} \quad (22)$$

This would be the value of the cube edge of the face-centred lattice for hydrogen if it could form such a lattice,

TABLE III.

Face-centred Cubic Lattice.

Sphere No.	Corresponding Sphere No. Cubic Lattice.	Total Atoms.	$\Sigma f_i p^{-3}$ Sphere.	$\Sigma f_i p^{-3}$ Total.	$\cos \alpha$.
1	2	12	-2.82842	-2.82842	-1/3
2	4	18	0	"	+1
3	6	42	+1.08866	-1.73976	-1/3
4	7	54	0	"	+1
5	9	78	-0.50597	-2.24573	-1/3
6	11	86	0	"	+1
7	13	134	-0.08727	-2.33300	-1/3
8	14	140	0	"	+1
9	16	176	+0.24443	-2.08357	-1/3
10	18	200	0	"	+1
11	20	224	-0.07048	-2.15905	-1/3
12	21	248	0	"	+1
13	23	320	+0.08355	-2.07550	-1/3
14	26	368	-0.08439	-2.15989	"
15	27	380	0	"	+1
16	29	428	-0.04747	-2.20736	-1/3

Radius sphere 16 = 2.9155 l . Total $\Sigma f_i p^{-3} = -8 \times 2.20736 = -17.66$.

but as shown above the second condition for zero force upon the whole atom cannot be satisfied for any lattice symmetrical with respect to a centre, and so hydrogen cannot form such a lattice. There are certain elements that are known by the X-ray analysis to form face-centred lattices, such as aluminium,

copper, lead and others, but there are none that form a simple cubic lattice. The simple cubic lattice may be considered as inadmissible for any element because $\Sigma f_i p^{-3}$ is positive instead of negative. The face-centred lattice is admissible so far as this sum is concerned, but is inadmissible in those cases where the functions of X, Y, and Z are of an odd degree only in the r^{-4} and r^{-6} terms of the expression for the force upon the whole atom. In those cases where these r^{-4} and r^{-6} terms do not cancel out in taking the sum, as seems probable in the case of aluminium for example, then both conditions may be satisfied by the force-equations as applied to a face-centred lattice and so crystals may be formed. As we shall now show, however, this second condition is not satisfied for the carbon atom. In a later section some further consideration of hydrogen will be given, but it seems best to study the carbon atom next.

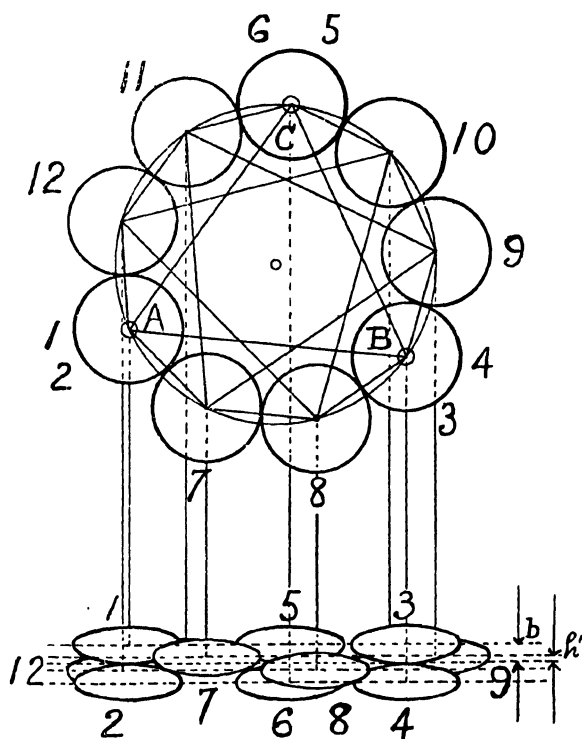
Carbon.

The model of the carbon atom that will be used for the purpose of computing the electrostatic force of a second such atom upon a first is shown at C in figs. 2 and 2A of Chapter II. A detailed drawing of it is given below in fig. 18 in two projections, a plan and elevation. The three positive charges, A, B, and C, in the three alpha-particles of the carbon atom form a perfect equilateral triangle in the equatorial plane of the atom. There are twelve negative electrons, six of them, 1-6, belonging to the alpha-particles, and six, 7-12, serving as the connecting electrons to bind the structure together. There are thus formed four perfect equilateral triangles of negative electrons, the upper ones, 1, 3, 5, in the alpha-particles forming the first, the lower ones, 2, 4, 6, a second, the upper three connecting electrons, 7, 9, 11, a third, and the lower three, 8, 10, 12, a fourth triangle.

The connecting electrons are supposed to be staggered alternately up and down all around the ring: that is to say, they are not in the exact equatorial plane, but located at about

half the distance from it that the electrons in the alpha-particles have. The reason for this separation from the equator is the mutual repulsion of the adjacent connecting electrons. This is counterbalanced, however, by the strong attraction they have for the alpha-particles, each having an effective charge of $+2e$.

FIG. 18.



Detailed drawing of the carbon atom in plan and elevation. See also figs. 2 and 2 A at C for the carbon atomic model.

In the free carbon atom the common distance of all electrons in the alpha-particles from the equator is the same, equal to b , the semi-minor axis of the negative electron very approximately. Similarly the connecting electrons have the same distance from the equator, the one as any other, say

equal to h' , which differs from b and is thought to be about half the value of b .

When a carbon atom forms part of a molecule instead of being free, the upper distances, b and h' , change to b_2 and h'_2 , while the lower distances become $-b_1$ and $-h'_1$, each of which differs from b or from h' by a very small amount. These differences, however, $D=b_2-b_1$ and $H=h'_2-h'_1$, which are zero in the free carbon atom, become appreciable when a molecule is formed. In fact a finite value for D and for H is demanded in order that the molecule be formed, their values being proportional to the force upon the positive charge caused by the other atoms of the molecule or crystal.

We shall now give the force upon the three positive charges of a first carbon atom at the origin of co-ordinates due to a second carbon atom in the position x, y, z , as derived from the atomic model for carbon by summing the electrostatic forces in the manner above explained. The first atom at the origin has its axis along the z -axis, while the axis of the second atom makes an angle α with the z -axis. The direction of the y -axis in the equatorial plane of the first atom is perpendicular to the axes of both atoms. We have

$$F_z = \frac{36e^2}{k} \left\{ f_1(D+H)_B r^{-3} + 3f_2[(b^2+h'^2)-\rho a^2] r^{-4} \dots \right\}, \quad (23)$$

$\begin{matrix} \text{C}_2 \text{ on} \\ 3 \text{ positive} \\ \text{charges of} \\ \text{first atom.} \end{matrix}$

where $(D+H)_B$ refers to the D and H of the second carbon atom, and not the atom at the origin upon which the force on the positive charge is expressed. Both f'_1 and f'_2 are the same functions before obtained in (3) and (4) in the case of hydrogen. The corresponding force in the case of hydrogen was given above in (2), and if h' in (23) were made equal to b and H to D , then it is seen that this force for carbon becomes exactly 36 times that for hydrogen in (2), 36 being the square of the atomic number, 6, for carbon. But h' is not and cannot be equal to b nor can H be equal to D in the carbon atom, and it is just here that the character of the expression for carbon differs from that of hydrogen.

Without as yet making any use of (23) we shall give the *z*-component of the force upon the whole of the first carbon atom due to the second carbon atom as follows :

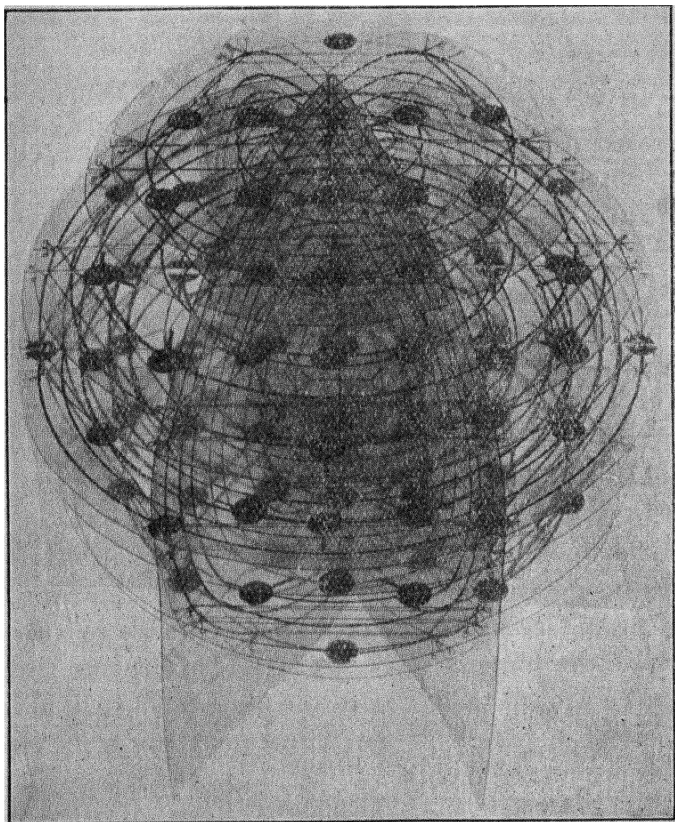
$$\begin{aligned}
 F_z = \frac{27e^2}{k} & - \{f_3(D+H)_A(D+H)_B r^{-4} \\
 & + [f_4(D+H)_A(b^2+h'^2) + f_5(D+H)_B[(b^2+h'^2) - \rho a^2]] \\
 & - [f_6(D+H)_A \rho a^2] r^{-5} \\
 & + 5[f_7[(b^2+h'^2) - \rho a^2](b^2+h'^2) \\
 & + 2f_8 \rho a^2(b^2+h'^2) - 2f_9 \rho^2 a^4] r^{-6} \dots\}. \quad (24)
 \end{aligned}$$

The subscript A refers here to the (D+H) of the atom at the origin, while B refers to the second atom. The functions f_3 to f_9 are given above by (12) to (18), obtained by the use of hydrogen. As pointed out in connection with (23) so here too it is seen that, if h' were equal to b and H to D, the force in (24) for carbon would be exactly 36 times (11) for hydrogen. If this were true the same conclusions would apply to carbon as were found above for hydrogen; but it is not true that $h'=b$, and it is this fact which gives rise to important differences in the behaviour of hydrogen and of carbon.

In commenting upon this carbon force-equation it should perhaps be pointed out that the radius of the ring of the carbon atom (see fig. 18) makes no appearance in this *z*-component of the force in (23) or (24). This seems surprising, but may be proved to be due to the regular equilateral triangle arrangement of the charges in the atom, there being five such triangles as above mentioned. The *x*- and *y*-components of the force are not required for the study of these crystals because it is easy to show that they sum up to zero due to the symmetrical arrangement of the atoms in the lattice. In these crystals the total force takes the direction of the *z*-axis, that is the axis of the atom upon which the force is expressed, and the *z*-component is, therefore, the total force. The axis of the atom, therefore, sets itself in the direction of the total force upon it.

Now that it has been shown that the functions f_3-f_9 are the same for carbon as for hydrogen, it is apparent that for any lattice which is symmetrical with respect to a centre, the r^{-4} and r^{-6} terms of (24) must vanish in taking the sum for each symmetrical pair of atoms as before, that is vanish

FIG. 19.

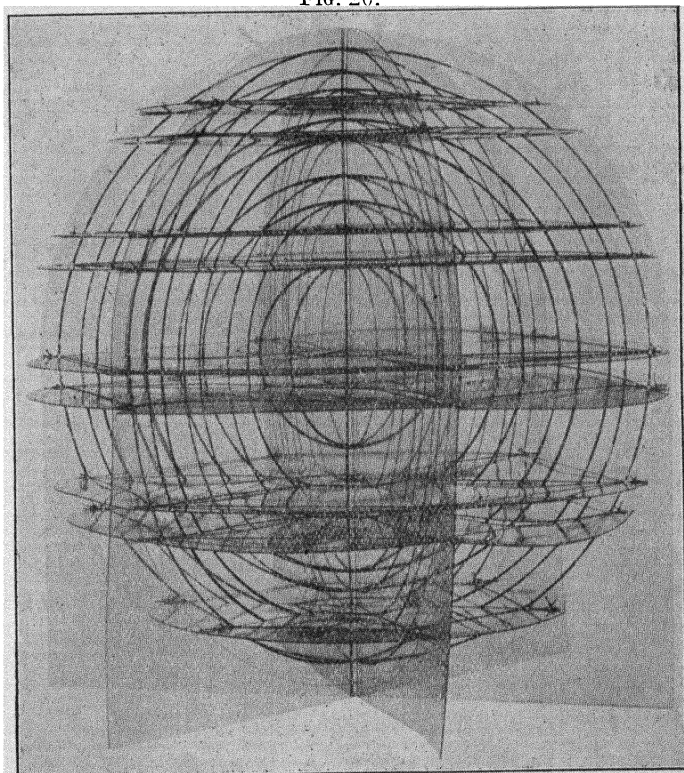


Celluloid model of the diamond lattice taken from a point which shows the elementary cubes.

for the whole lattice. The coefficient of the r^{-5} term does not vanish because it contains even-degree functions, and consequently the sum of the forces upon the selected atom cannot be equated to zero for equilibrium. Just as for hydrogen it may be concluded that carbon will form neither a simple cubic,

face-centred cubic nor body-centred cubic lattice. This conclusion is in exact accord with the facts, for it is known that carbon does not so crystallize, but this atomic model has supplied the reason why it does not. The question that naturally presses for consideration is how may carbon crystallize in so far as these force-equations are concerned. The diamond is one of the forms of pure carbon and its structure is well

FIG. 20.



A second view of the same model showing the spacing of the (111) planes, perpendicular to the cube diagonal.

known from the X-ray analysis. In the first place it is seen to conform immediately to one of the conditions found to be demanded by the above equations for carbon, namely that it shall not be a lattice that is symmetrical with respect to a centre. We will now pass to a study of the diamond lattice.

The Diamond Lattice.

The diamond lattice is made up of two face-centred lattices together, each exactly like the face-centred lattice above described. These two displaced lattices composing the diamond lattice are exactly alike, but the one is moved or displaced as a whole with respect to the other along the direction of a

TABLE IV.

Diamond Lattice.

Plane No.	z/l .	Fig. 15.	Fig. 16.	Fig. 17.
-8	$-16\left(\frac{\sqrt{3}}{12}\right)$		"	
-7	-13 "		"	
-6	-12 "	"		
-5	-9 "	"		
-4	-8 "			"
-3	-5 "			"
-2	-4 "		"	
-1	-1 "		"	
0	0	"		
+1	+3 "	"		
+2	+4 "			"
+3	+7 "			"
+4	+8 "		"	
+5	+11 "		"	
+6	+12 "	"		
+7	+15 "	"		
+8	+16 "			"

diagonal of the cube by a distance equal to $3/4$ ths the altitude of the tetrahedron erected upon one of the elementary equilateral triangles. A model of the diamond lattice made with sheets of transparent celluloid is shown in two views in figs. 19 and 20. There are nine of the (111) planes shown

in the model, which embrace 98 atoms in 8 concentric spheres surrounding the selected atom. Because the lattice is made up of these two identical face-centred lattices, at a

TABLE V.
Diamond Lattice.

Sphere No. Diamond.	Sphere Radius, $\rho = r/7$.	Total $\Sigma f_i \rho^{-3}$.	Sphere No. Cubic lattice.
1	$\sqrt{3}/4 = 0.43301$	65.6896	2
2	$\sqrt{2}/2 = 0.70711$	-88.3170	
3	$\sqrt{11}/4 = 0.82915$	-75.5589	
4	1 = 1.00000	„	
5	$\sqrt{19}/4 = 1.08973$	-77.5112	6
6	$\sqrt{6}/2 = 1.22475$	-68.8018	
7	$3\sqrt{3}/4 = 1.29964$	-73.7084	
8	$\sqrt{2} = 1.41421$	„	
9	$\sqrt{35}/4 = 1.47903$	-69.4165	9
10	$\sqrt{10}/2 = 1.58115$	-73.4643	
11	$\sqrt{43}/4 = 1.63935$	-71.6907	
12	$\sqrt{3} = 1.73205$	„	
13	$\sqrt{51}/4 = 1.78535$	-72.9035	13
14	$\sqrt{14}/2 = 1.8709$	-73.6017	
15	$\sqrt{59}/4 = 1.92028$	-74.2527	
16	2 = 2.00000	„	
17	$\sqrt{67}/4 = 2.04635$	-75.6740	16
18	$3\sqrt{2}/2 = 2.12132$	-73.7186	
19	$5\sqrt{3}/4 = 2.16506$	-72.6886	
20	$\sqrt{5} = 2.23607$	„	
21	$\sqrt{83}/4 = 2.2776$	-72.1216	20
22	$\sqrt{22}/2 = 2.34520$	-72.6855	
23	$\sqrt{91}/4 = 2.3849$	-72.1410	
24	$\sqrt{6} = 2.44949$	„	

distance of $(\sqrt{3}/4)l$ apart (the wider interval between (111) planes in the model) adjacent planes separated by a wide interval are identical, namely, plane 0 is like plane +1,

plane + 2 like + 3, and so on. The scheme for this lattice is exhibited in Table IV, in which the first column gives the number of the (111) plane, the second the distance of the plane from the zero plane, namely the value of z , in terms of the elementary cube edge. The last three columns indicate the arrangements of the axes of the atoms as being like either fig. 15, the zero plane, fig. 16, plane -1, or fig. 17, plane + 2.

Table V gives the radii of 24 concentric spheres in the diamond lattice embracing 488 atoms. The last column gives the number of the sphere of the simple cubic lattice of Table I which is identical in all respects with the corresponding sphere of the diamond lattice, provided only the linear dimensions of the cubic lattice are each divided by a factor of 2 as above explained. The third column of Table V gives the important total $\Sigma f_1 p^{-3}$, that has been computed from the complete table of the direction cosines for these spheres, which is too long to be given in full here. It is apparent from this that the first sphere gives the largest sum, but that for a very accurate value for an infinite lattice probably more than 24 spheres should be computed. The resulting value for the 488 atoms nearest to the selected atom in the diamond is

$$\Sigma f_1 p^{-3} = -72.141. \quad . \quad . \quad . \quad . \quad . \quad (25)$$

And because the diamond lattice is not symmetrical with respect to a centre, as those above described were, the r^{-4} term of equation (23) does not sum to zero but gives a definite value that must be computed. The value of $f_2 p^{-4}$ according to (4) has been summed for the first six spheres to include 70 atoms in the diamond lattice by the use of the same table of direction cosines referred to, giving as the result

$$\Sigma f_2 p^{-4} = -55.55. \quad . \quad . \quad . \quad . \quad . \quad (26)$$

The force upon the positive charge of the selected carbon atom according to (23) therefore becomes

$$\begin{aligned} \text{on 3} \\ \text{+ charges} \end{aligned} \quad F_z = \frac{36e^2}{k} \{ -72.14(D+H)l^{-3} + 3(-55.55)[(b^2+h^2)-\rho a^2]l^{-4} \dots \}. \quad . \quad . \quad (27)$$

The force upon just one positive charge of one of the alpha-particles of the carbon atom is, therefore, one-third of the above force, and this may be equated to equation (1) above as before explained. Hence the first equation of condition that must be satisfied if carbon is to form the diamond lattice is

$$-865.68(D+H)l^{-3}-1999.8[(l^2+h'^2)-\rho a^2]l^{-4}=-30.7 \times 10^{24}D. \quad (28)$$

The Second Condition.

It will be recalled that the second condition is that the sum of the forces of the lattice upon any selected atom of it shall be zero for equilibrium. If the whole lattice were constructed of hydrogen atoms the condition would be that the sum of the forces expressed by (11) shall vanish, but if constructed of carbon atoms the sum of the forces expressed by (24) must vanish. It will be profitable here to consider both the hydrogen and the carbon atoms together because these equations involve the same functions of X , Y , and Z , in order to bring out the difference between the behaviour of the two kinds of atoms on the basis of these expressions for the forces. It will also be recalled that the sums of the functions f_3p^{-4} , f_5p^{-5} and so on are purely geometrical quantities depending entirely upon the character of the lattice investigated and not at all upon the kind of atom supposed to be situated at the points of it. Hence these sums are the same for either the hydrogen or the carbon atoms, and have been computed to be as follows:

$$\Sigma f_3p^{-4}=+89.218 \quad . \quad . \quad . \quad . \quad . \quad (29)$$

$$\Sigma (f_4+f_5)p^{-5}=-1022.60 \quad . \quad . \quad . \quad . \quad . \quad (30)$$

$$\Sigma (f_5+f_6)p^{-5}=-1015.58 \quad . \quad . \quad . \quad . \quad . \quad (31)$$

$$\Sigma f_7p^{-6}=-2217.50 \quad . \quad . \quad . \quad . \quad . \quad (32)$$

$$\Sigma (-f_7+2f_8)p^{-6}=+1917.83 \quad . \quad . \quad . \quad . \quad . \quad (33)$$

$$\Sigma f_9p^{-6}=+775.37 \quad . \quad . \quad . \quad . \quad . \quad (34)$$

Upon substituting these sums in (11) for hydrogen, the

force of the whole diamond lattice upon one hydrogen atom, if hydrogen could form such a space-lattice, would be

$$F_z = \frac{3e^2}{k} \left\{ +89 \cdot 218 D^2 l^{-4} - 1022 \cdot 6 l^2 D l^{-5} \right. \\ \text{Diamond} \quad \quad \quad + 507 \cdot 79 \rho a^2 D l^{-5} - 11087 \cdot 48 b^4 l^{-6} \\ \text{Lattice} \quad \quad \quad + 4794 \cdot 58 \rho a^2 b^2 l^{-6} - 1938 \cdot 44 \rho^2 a^4 l^{-6} \} = 0. \quad (35) \\ \text{on H.}$$

The force of the whole diamond lattice upon one carbon atom by the use of (24) is

$$F_z = \frac{27e^2}{k} \left\{ +89 \cdot 218 (D+H)^2 l^{-4} - 1022 \cdot 6 (b^2 + h^2) (D+H) l^{-5} \right. \\ \text{Diamond} \quad \quad \quad + 1015 \cdot 58 \rho a^2 (D+H) l^{-5} - 11087 \cdot 48 (b^2 + h^2)^2 l^{-6} \\ \text{Lattice} \quad \quad \quad + 9589 \cdot 15 \rho a^2 (b^2 + h^2) l^{-6} - 7753 \cdot 74 \rho^2 a^4 l^{-6} \} = 0. \quad (36) \\ \text{on C.}$$

The expression for the force in (35) proves immediately that the hydrogen atom does not fulfil the necessary condition to enable it to form a crystal in the same form as that of the diamond lattice, and so hydrogen is easily disposed of in connection with this lattice. For, after putting in the known numerical values .

$$b^2 = 10^{-26} \text{ and } \rho a^2 = 4 \cdot 484 \times 10^{-26}, \quad . \quad . \quad . \quad (37)$$

multiplying each term by l^6 , and dividing by the coefficient of $D^2 l^2$, we obtain from (35) the quadratic expression in terms of Dl as follows:

$$D^2 l^2 + 14 \cdot 05 \times 10^{-26} D l + 95 \cdot 0 \times 10^{-52} = 0, \quad . \quad . \quad . \quad (38)$$

$$\text{whence} \quad D l = (-7 \cdot 025 \pm \sqrt{-45 \cdot 65}) 10^{-26}, \quad . \quad . \quad . \quad (39)$$

an imaginary quantity. Both D and l must have real values to satisfy the requirements for the lattice. It should perhaps be stated that the value of D must be the same for each and every atom in the diamond lattice, because each atom is related geometrically to the whole lattice in the same way as any other atom.

Let us next consider the equations (28) and (36), the two conditions that must be satisfied with carbon in the diamond lattice. Instead of only two unknown quantities, D and l ,

as in the case of hydrogen there are now four, D , l , h' , and H , and two equations are not sufficient to determine four unknown quantities. It requires further information, but this may be obtained from the model of the carbon atom itself. It is apparent from the model that h' is substantially a fixed quantity in every carbon atom everywhere in the same manner that b is fixed by the property of the negative electron. It must be a certain definite fraction of b , and was estimated in working out the model to be not far from one-half of b . Let us call the ratio of h' to b the new quantity, s , which must be less than unity, probably not far from $1/2$. Now when a crystal or a molecule is formed and the electrons in the alpha-particles are deformed, a study of the very slight movement of the connecting electrons shows that the value of $H = h_2' - h_1'$ will be the same fraction of $D = b_2 - b_1$ that h' is of b , namely s . It is necessary that the connecting electrons maintain the same positions relative to the adjacent electrons in the alpha-particles because of the internal forces within each atom, so that when one electron in an alpha-particle is slightly raised or lowered as the case may be the connecting electrons will be raised or lowered proportionally. Of course this is not considered to be a rigid proof of this question, but lacking this it is a most reasonable supposition to make, and it leads to a definite solution as follows.

Let us now substitute in (28) and (36)

$$h' = sb \text{ and } H = sD. \quad . \quad . \quad . \quad . \quad . \quad (40)$$

This reduces the variables from four to three, namely D , l , and s . But the experimental value of l for the diamond is

$$l = 3.56 \times 10^{-8} \text{ cm.}, \quad . \quad . \quad . \quad . \quad . \quad (41)$$

and instead of treating this as an unknown quantity the values of D and s may be found by the use of the experimental value. The solution is obtained easily by approximation trying different values of s until the same value for D is obtained from both (28) and (36). When $s = 0.42$, it is found from (28)

$$D = -11.92 \times 10^{-18} \text{ cm.}, \quad . \quad . \quad . \quad . \quad . \quad (42)$$

and from (36) that

$$D^2 + 7.4296 \times 10^{-18} D = 52.88 \times 10^{-36}, \dots (43)$$

from which

$$D = -11.88 \times 10^{-18} \text{ cm.}, \text{ or } +4.45 \times 10^{-18} \text{ cm.} \dots (44)$$

The negative values of D in (42) and (44) are very closely the same, and a value of s very slightly in excess of 0.42 will make the two values of D identical. The positive value has no significance. It is to be noted, incidentally, that this value for D is not far different from that found for the hydrogen molecule, namely 14.47×10^{-18} cm. in (63) of Chapter V. The negative value of D shows that the force upon the positive charge of the atom is positive according to (1) and (27), meaning that it acts in an upward direction away from the base and towards the apex of the tetrahedron constituting the first surrounding sphere of atoms. Hence the electron on the side of this apex is the one that is compressed, while that on the side towards the base is the one that is expanded.

Both conditions are, therefore, perfectly satisfied by the form of the diamond lattice for carbon, and it may be concluded on theoretical grounds alone that the carbon atom will permit of the forming of such a crystal. The exact experimental determination of the cube edge of the diamond has, however, made it possible to locate with the same precision the positions of the connecting electrons in the carbon atom itself, that is the important distance $h' = 0.42 b = 0.42 \times 10^{-13}$ cm. This distance must be substantially the same wherever carbon is found and will prove to be of service when it is possible to work out the details of any other crystals or molecules containing carbon. The distance agrees very closely with that previously estimated from the model of the carbon atom.

Concluding Remarks on Molecules.

As long as the expressions for the forces are restricted to the atoms of hydrogen and carbon it is quite apparent that

the field of their application is limited in so far as any comparison may be made between experimental and theoretical results. This is because the X-ray analysis does not as yet reveal the locations of hydrogen atoms. Their mass is too small. The form in which hydrogen crystallizes is still unknown. The only crystals containing pure carbon only are the diamond and graphite. Considerable theoretical work has been done upon graphite in connection with the force-expressions for carbon, but there is not as yet a complete agreement among those who have studied it with the X-rays, and it is even possible that there is more than one form of graphite. Using the best experimental information* obtainable, however, it appears that graphite is constructed of equally spaced planes of hexagons instead of equilateral triangles. If one imagines that the planes in the diamond lattice, shown in figs. 19 and 20 above, which are separated by a narrow interval, equal to $(\sqrt{3}/12)l$, are brought into coincidence in one plane, this will give just such a hexagon formation in one plane as intended. There appears to be still a question as to whether these planes are in exact coincidence or are separated by a very small interval. Assuming for the purpose of discussion, however, that they are thus brought into coincidence, it may be seen by means of the model that each atom is no longer geometrically related to the whole crystal just as every other atom is, as is the case with the diamond lattice. Taking into account the directions of the axes as well, it appears that there are at least four different sorts of carbon atoms in such a crystal, that is to say each of these four different carbon atoms is related in a different manner to the whole crystal. The meaning of this is that each of these four carbon atoms experiences a different force upon its positive charge, so that there must be four different values of the displacement, D , to consider. This is equivalent to saying that there are really four different kinds of atoms in the crystal, although each is still a carbon atom. The problem of the computations is very

* J. D. Bernal, Roy. Soc. Proc. 106. pp. 749-773, Dec. 1, 1924.

much more complex under such conditions, and until all doubt is removed as to the exact construction of the graphite lattice by X-ray analysis, it does not seem worth the labour to treat it theoretically. It is of much interest, however, to ascertain whether the same value for h' for the carbon atom would not be obtained from it as from the diamond.

There exist, of course, the hydrocarbon compounds in great numbers, but here again the X-ray analysis does not reveal the locations of the hydrogen atoms. Some progress* has been made, however, in locating the positions of the carbon atoms in crystals of the chain hydrocarbon compounds, but as yet the precise positions of even the carbon atoms is clouded in uncertainty.

If the precise form of even one molecule containing either hydrogen or carbon or both were known from experimental data, merely as to the locations of the centres of their atoms, it would be of very great assistance in checking theoretical results. At the present time it may be said that the diamond is the one such molecule that is known with precision, and this satisfies the theoretical requirements completely.

In concluding these three chapters on Molecules, considering crystals as a special form of molecule, it may be pointed out that one of the important results of the investigation makes the stable equilibrium positions of the atoms depend upon a balance between the inverse fourth, fifth, and sixth powers of the distance between atoms. It should, therefore, result from experiments in the compression and expansion of solids in general that the apparent law of force between their atoms obeys some power intermediate between -4 and -6 , probably approximating to the inverse fifth power of the distance. This result is in close agreement with the experimental evidence as far as it is known.

* A. Müller and W. B. Saville, Chem. Soc. Journ. 127. pp. 590-603, March 1925.

CHAPTER VII.

AN ELECTROMAGNETIC OSCILLATOR.

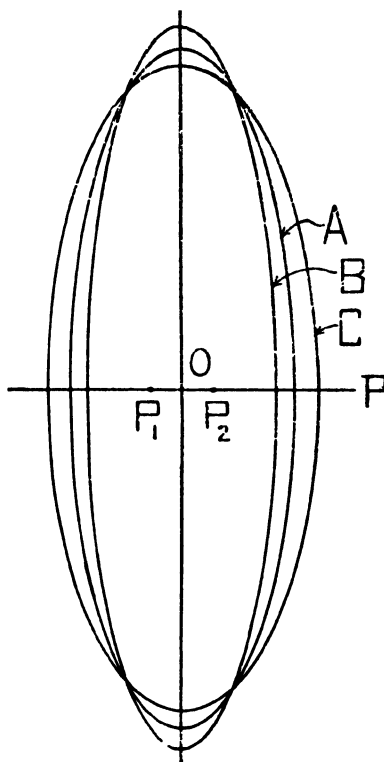
THE phrase "An Electromagnetic Oscillator" may be misleading to some, for those oscillators which have been treated in the literature of the subject have consisted of two charges of opposite sign, a positive and a negative, whereas in this chapter we shall consider an oscillator the two elements of which have charges of the same sign and equal values. Moreover, the results here given are derived from the Saha form of electrodynamics, which in itself gives the matter new interest.

A word of explanation as to why such a problem has been suggested, and what its application to atoms may be, seems called for at the beginning in order that the reader may see the practical nature of it rather than think of it as a purely abstract academical question. There is good reason to believe that the elementary charges, the negative electron, and the several positive elementary charges above described are at times struck a blow by the impact of other similar charges, and that when this happens the whole elementary charge is set into vibrations in a manner characteristic of itself, just as a bell is set into vibration by the blow of a hammer.

In the conception of these elementary charges which has already been formed they really behave much like solid bodies, and the analogy with the vibrations of a bell may be closer than formerly expected. According to this the vibration is not simple, and is intimately connected with the shape of the body of the elementary charge. One conceivable form of vibration might be as illustrated in fig. 21, in which the form of the oblate spheroid changes its shape from the normal

represented by the ellipse A to a more eccentric ellipse, B, and a less eccentric one, C, as extreme variations. This fundamental style of vibration might not take place at all on the other hand, and the ellipsoid might break up into several

FIG. 21.



Illustrating the change of shape when executing high-frequency vibrations natural to the positive charge. The effect of this vibration is taken as equivalent to two point charges, P_1 and P_2 , vibrating from and toward each other on the axis OP to a first approximation.

different segments, each having its own vibration at a higher frequency than the fundamental.

However this may be, we do not hope even to treat of the complex vibration of the fundamental, or simplest kind, with

the present limited power of analysis. In considering fig. 21, however, it is seen that the centre of the charge on one side of the equatorial plane represented by the point P_1 must move nearer to and farther from the equatorial plane during a cycle of the vibration. The centres of the half-charges on either side of this equatorial plane, P_1 and P_2 , will then move toward and from each other alternately during a complete cycle, and to a first rough approximation the behaviour of the whole vibrating charge would be simulated by substituting for the ellipsoid two point charges harmonically vibrating from and toward each other along the line of the axis.

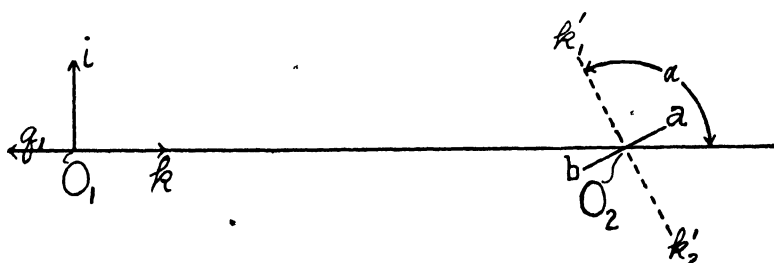
Notwithstanding the possibility that the actual vibration of the elementary charge may not take this fundamental form at all, nevertheless it seems most desirable to obtain an expression by means of electrodynamics for the force that an oscillator consisting of two point charges vibrating harmonically in the manner described must have upon an electron at a distance from it, for any more complex form of vibration might be approximated by multiplying the number of such oscillators, each having different periods and charges.

It will be shown that the force exerted upon the distant electron due to this vibration alone may easily be so great that it exceeds the electrostatic force between the charges. Under these circumstances, while the vibration lasts, a positive and negative charge at a distance might repel each other instead of attract, and an entirely new expression for the law of force between them exist. If such a law is found, it must of necessity be a function not only of the distance between the two charges but a function of the time as well, for the vibration will not persist indefinitely, and at a later time the two charges may be at precisely the same distance apart, and yet the force between them be quite different from the force when they occupied these same positions before.

In order to present the results clearly, let us represent by fig. 22 a diagram showing the instantaneous positions of the centres of two charges, O_1 being a negative electron of

charge $-e$ and O_2 a positive charge equal to one-half the positive charge in a hydrogen atom, $+e$, and let it be supposed that the positive charge is in vibration along the line ab in the plane of the paper having an harmonic motion about the centre O_2 , between the extreme limits of a and b . The negative charge is not supposed to have any vibration, but to be in motion in a straight line away from the centre O_2 , with a velocity $q_1 = \beta_1 c$, represented by the arrow in the figure. If we suppose three coordinate axes, i , j , and k , to have their origin at O_1 , the k -axis being toward the point O_2 , and the i -axis at right angles to it upward in the plane of the paper,

FIG. 22.



Illustrating an electron at O_1 moving away from O_2 with velocity q_1 , and a positive point charge vibrating with harmonic motion along ab about O_2 as centre.

then the j -axis will be perpendicular to the plane of the paper towards the observer.

By an application of the Saha form of the ponderomotive force between point charges, it has been proved that the average force upon the electron at O_1 , due to the vibrating charge at O_2 , the derivation being omitted here, is

$$\begin{aligned}
 \bar{F} = & \frac{(-e)(+e)(1-\beta_2^2)}{k^2(1-\beta_1^2)^{1/2}} \left\{ \left[\beta_2^2(1-\beta_1) \sin \alpha \cos \alpha \right. \right. \\
 & + \frac{3}{2} \beta_2^4(1-\beta_1) \sin^3 \alpha \cos \alpha \dots \left. \right] i \\
 & + \left[-1 - \frac{1}{2} \beta_2^2 \sin^2 \alpha - \frac{3}{8} \beta_2^4 \sin^4 \alpha \dots \right] k \left. \right\} . \quad . \quad . \quad . \quad (1)
 \end{aligned}$$

The quantity $\beta_2 c$ is the maximum velocity of the harmonically vibrating charge at O_2 , and the angle α is that represented in the figure. When this angle is zero the vibration of the point is at right angles to the line joining the centres of the charges, corresponding to the case when the electron at O_1 is in the equatorial plane of the vibrating positive charge, and when the angle is $\pi/2$ the electron at O_1 is on the pole of the positive charge.

The harmonically vibrating charge at O_2 above described represents but one-half of the positive charge of the hydrogen atom. If a similar expression is derived for the other half, vibrating with an harmonic motion of the same amplitude and frequency but in a different phase, since they move from and towards each other as above described, the result for the second half is represented by a similar equation in which the i -component differs in sign. The result is obtained by reversing the sign of the cosine but not that of the sine of the angle α , as follows:

$$\begin{aligned} F = \frac{(-e_1)(+e_2)(1-\beta_2^2)}{kr^2(1-\beta_1^2)^{1/2}} \left\{ \left[-\beta_2^2(1-\beta_1) \sin \alpha \cos \alpha \right. \right. \\ \left. \left. - \frac{3}{2}\beta_2^4(-\beta_1 \sin^3 \alpha \cos \alpha \dots) \right] i \right. \\ \left. + \left[-1 - \frac{1}{2}\beta_2^2 \sin^2 \alpha - \frac{3}{8}\beta_2^4 \sin^4 \alpha \dots \right] k \right\}. \quad (2) \end{aligned}$$

The total effect upon the electron at O_1 is obtained by adding together (1) and (2), giving

$$\bar{F} = \frac{2e^2(1-\beta_2^2)}{kr^2(1-\beta_1^2)^{1/2}} \left[1 + \frac{1}{2}\beta_2^2 \sin^2 \alpha + \frac{3}{8}\beta_2^4 \sin^4 \alpha + \dots \right] k, \quad (3)$$

which may also be written

$$F = \frac{e^2}{kr^2(1-\beta_1^2)^{1/2}} \left[\frac{2(1-\beta_2^2)}{(1-\beta_2^2 \sin^2 \alpha)^{1/2}} \right] k, \quad (4)$$

since the series in the last bracket of (3) is the equivalent of the development of the quantity $(1-\beta_2^2 \sin^2 \alpha)^{-1/2}$, as may be seen by expanding by the binomial theorem.

It is to be remarked that the i -component of the force, which appears in both (1) and (2), has cancelled out in taking the sum. And, since there was no j -component of the force, the total force acts along the line joining the centres of the two charges. The electrostatic force, of course, acts in this direction, but we now see that the forces due to the motion also have this same direction. The result in (3) or (4) may be taken to represent the force upon the distant electron due to the positive part of a hydrogen atom, when it has been set into vibration in the manner supposed.

If there were present with this positive charge at the point O_2 , a negative electron not supposed to be in vibration, the combination of the positive charge with it would constitute what we have shown to be the so-called hydrogen ion. And, since we have supposed that the positive charge never loses the last electron, and has no independent existence by itself as these bodies are met with experimentally, it seems fitting to add to the force in (4) the force that this additional negative electron* has upon the distant electron at O_1 . According to the Saha electrodynamics this force is

$$F = -\frac{e^2}{k r^2 (1 - \beta_1^2)^{1/2}} k, \quad \dots \dots (5)$$

where as before $\beta_1 c$ represents the velocity of the point charge at O_1 . The sum of (4) and (5) is

$$F = \frac{e^2}{k r^2 (1 - \beta_1^2)^{1/2}} \left[-1 + \frac{2(1 - \beta_2^2)}{(1 - \beta_2^2 \sin^2 \alpha)^{1/2}} \right] k, \quad \dots (6)$$

which may, therefore, represent the force upon a single negative electron near to a hydrogen ion when its positive charge is in vibration as from the effects of a blow as above described.

Of course, the analogy between this hypothetical case employing point charges as the equivalent of the oblate spheroid must not be pressed too far, but it is believed that this approximate result will assist in understanding the process

by which an electron may be driven out from its normal position in the hydrogen atom to a maximum distance and back again in a straight line, thus by its accelerated motion radiating energy out into space away from the atom. On this account these equations seem worth examining in some detail.

Let us begin by studying how the force upon the electron varies merely with a change in the angle α , all other quantities remaining fixed and constant. Considering, first, equation (4) as representing the force due to the positive charge alone in vibration, and supposing F and a to be the only variables, it is seen that the equation is the polar equation of an ellipse referred to the centre as origin, the force being the radius vector and the angle α being the angle between the radius vector and the minor axis of the ellipse.

Starting with the ordinary equation of the ellipse referred to rectangular co-ordinates, x along the direction of the major axis, and y along that of the minor axis, a and b being the semi-major and semi-minor axes respectively, the common form of the equation of the ellipse is

$$y^2 = \frac{b^2}{a^2}(a^2 - x^2), \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

the eccentricity being expressed by

$$\frac{b^2}{a^2} = 1 - \epsilon^2. \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

Putting $x = r \sin \alpha$ and $y = r \cos \alpha$, where α is the angle between the radius vector and the minor axis, gives

$$r^2 \cos^2 \alpha = r^2(1 - \sin^2 \alpha) = \frac{b^2}{a^2}(a^2 - r^2 \sin^2 \alpha). \quad . \quad . \quad . \quad (9)$$

Whence

$$r^2 \left[1 - \left(1 - \frac{b^2}{a^2} \right) \sin^2 \alpha \right] = b^2. \quad . \quad . \quad . \quad . \quad (10)$$

or

$$r^2(1 - \epsilon^2 \sin^2 \alpha) = b^2,$$

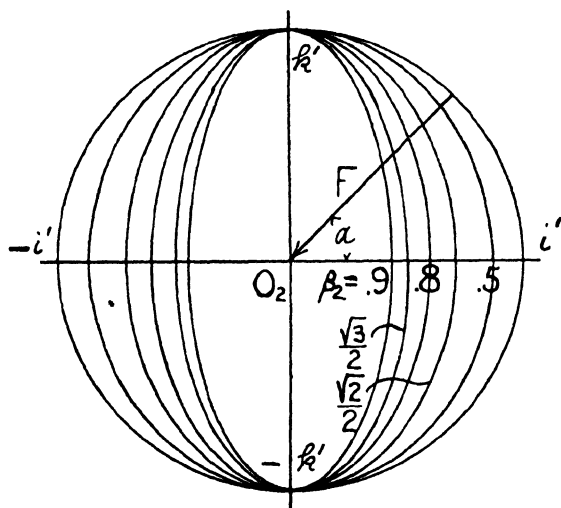
and

$$r = \frac{b}{(1 - \epsilon^2 \sin^2 \alpha)^{1/2}}, \quad \dots \dots \dots (11)$$

the equation of the ellipse in polar co-ordinates referred to the centre.

Comparing with (4) above, it is evident that it is the

FIG. 23.



The radius vector from the curve to the origin O_2 represents the whole attractive force upon the electron at a fixed distance, r , due to a pair of positive point charges at O_2 vibrating in the direction $O_2 k'$. Each ellipse shows the change in the force with the change in the angle α (see fig. 22), other quantities being constant. The six ellipses correspond to six different values of β_2 shown.

equation of an ellipse in polar co-ordinates in which F is the radius vector, and the minor axis b is represented by the constant

$$b = \frac{2e^2(1 - \beta_2^2)}{kr^2(1 - \beta_1^2)^{1/2}}, \quad \dots \dots \dots (12)$$

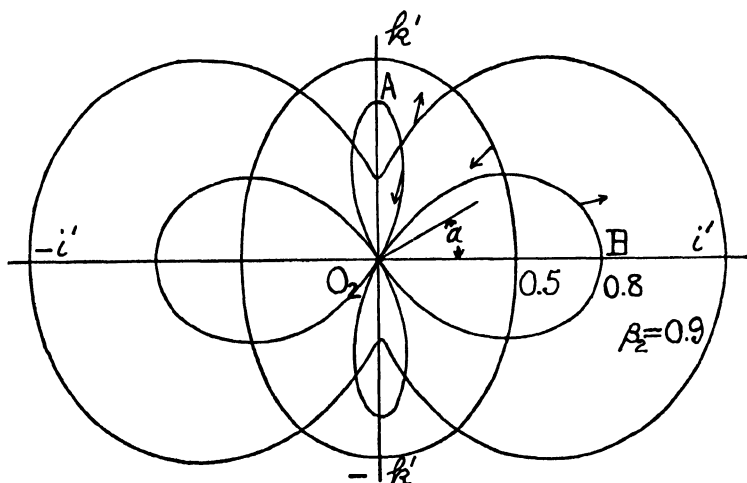
the eccentricity being equal to β_2 .

A system of ellipses is represented in fig. 23, corresponding to eccentricities, that is to values of β_2 , 0, 0.5, $\sqrt{2}/2=0.707$, 0.8, $\sqrt{3}/2=0.866$ and 0.9 respectively. They have all been given the same semi-major axes equal to the radius of the circle when $\beta_2=0$, namely

$$\frac{2e^2}{kr^2(1-\beta_1^2)^{1/2}}.$$

This means that they each have a different scale, because

FIG. 24.



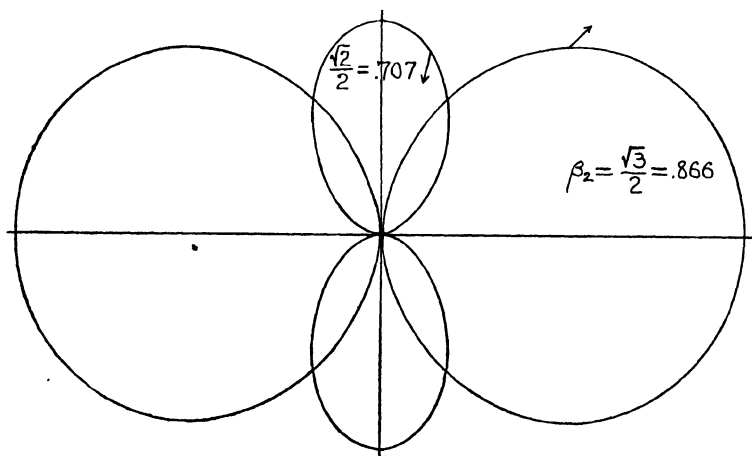
The radius vector represents the whole force upon an electron at a fixed distance, r , due to a pair of positive point charges at O_2 vibrating in the direction O_2k' together with one negative electron, the combination representing the hydrogen ion. Each curve shows the variation in the force with a change in the angle α (see fig. 22), other quantities being constant. When $\beta_2=0.5$ the force is attractive at all angles. At $\beta_2=0.8$ the force is attractive in the loop A and repulsive in the loop B. When $\beta_2=0.9$ the force is repulsive at all angles.

the major axes would not be the same for a fixed value of β_1 . The important feature, however, is the shape of the curve, and the scale may easily be changed by supposing a change in the value of β_1 for each curve. It may be concluded from the

above reasoning that it makes considerable difference in the force upon the distant electron how the positive charge is oriented, and the more difference the greater the velocity of the vibrating points represented by β_2 .

Passing now to a consideration of the force upon the electron due to the hydrogen ion as represented by equation (6), it becomes evident that these curves may be drawn by using the ellipses in fig. 23, and subtracting a constant distance as representing the first term of (6) from the radius vector of

FIG. 25.



Representing the same as fig. 24, but for two critical values of β_2 , namely $\frac{1}{2}\sqrt{2}$ and $\frac{1}{2}\sqrt{3}$.

each ellipse. This procedure gives rise to curves of a more complicated form shown in figs. 24 and 25. Those in fig. 24 correspond to the values of β_2 , 0.5, 0.8, and 0.9, the curve for $\beta_2=0$ being a circle of half the radius of the circle in fig. 23 not shown here. In fig. 25 are shown the two critical cases corresponding to the values of β_2 , $\sqrt{2}/2$ and $\sqrt{3}/2$ mentioned below.

When $\beta_2=0.5$ the force according to fig. 24 is attractive at all angles, α , but is greatest when the pole of the positive

charge points towards the distant electron, and $\alpha = \pi/2$. When $\beta_2 = 0.8$ the force is still an attraction along the polar direction following the loop curve O_2A , but is a repulsion in the equatorial area within the loop curve O_2B . In this case there exists a critical angle, say α_0 , at which the whole force vanishes. This angle may be obtained by setting the quantity in the bracket of (6) equal to zero, giving

$$\frac{1 - \beta_2^2}{(1 - \beta_2^2 \sin^2 \alpha_0)^{1/2}} = \frac{1}{2}, \quad \dots \dots \dots (13)$$

or

$$\sin^2 \alpha_0 = 8 - 4\beta_2^2 - \frac{3}{\beta_2^2} \dots \dots \dots (14)$$

When $\beta_2 = 0.8$, then $\sin^2 \alpha_0 = 0.7525$, $\sin \alpha_0 = 0.8675$ approx., and $\alpha_0 = 60^\circ 10'$, which corresponds to the curve in the figure.

It appears from (13) that the curves will have two branches or loops, the one representing attractive forces nearer the pole, and the other repulsive forces nearer the equator for a certain limited range of β_2 , such that $\sin \alpha_0$ in (14) is real, this range being between the limits $\sqrt{2}/2$ and $\sqrt{3}/2$. The former limit is obtained by making $\alpha_0 = 0$ in (13), and the latter by making it $\pi/2$. The special forms of the curves for these critical values are shown in fig. 25.

As the value of β_2 begins to exceed $\sqrt{3}/2 = 0.866$, the force becomes repulsive at all angles. This is illustrated by the curve for $\beta_2 = 0.9$ showing a repulsive force even at the pole, though it is there a minimum. As before the scale of these different curves is not given as being unimportant, since the scale may be changed by assuming a different value for $\beta_1 c$, the velocity of the distant electron. The shape of the curves is the chief thing required for present purposes.

In order to make the action of the system as represented by equation (6) better understood, let us consider several hypothetical cases. Suppose, for example, that the distance between the ion and the electron and the angle α remain

fixed, and that the maximum velocity of the vibration of the positive charge of the ion begins at a value close to the velocity of light, and gradually and continuously decreases in value. At first the force upon the electron is repulsive, and then becomes less and less repulsive. When β_2 falls to 0.866 the radius vector at the angle α_0 , along which the force is zero, starts at the pole $\pi/2$, and swings around from the pole to the equator as α_0 continues to decrease. When it passes the particular fixed angle α , that the ion is now supposed to occupy, the force upon the electron is zero, and at this value of β_2 changes over from a repulsion to an attraction, remaining an attraction for all smaller values.

Again, let us suppose that the speed of vibration does not vary, but that the distance, r , increases, because of an initial velocity of projection of the electron away from the ion. And for illustration assume that β_2 is such that the angle, α_0 , exactly coincides with the position the axis now happens to occupy. The force upon the electron is, therefore, zero; and will remain zero unless β_2 changes in value, which is contrary to the hypothesis. Hence the electron will continue its outward course at a uniform velocity along the radial line indefinitely, and finally become separated from the ion. In so doing no energy is lost through radiation from the electron because of its uniform motion, for according to electrodynamics a charge moving with a uniform velocity in a straight line loses no energy by radiation. And, because no energy is lost in this way, the value of β_2 need not decrease perceptibly.

Let it next be supposed that all other quantities remain constant except the angle α , and that it begins at $\pi/2$ and gradually passes through all angles down to 0° . If the value of β_2 is such that there exists a real angle, α_0 , at which the force is zero, then the force upon the electron will be a repulsion at first and gradually reduce to zero, reversing to an attraction at the critical angle, α_0 , and remaining an attraction after that. Again, it may be imagined that there is some cause in operation which makes the direction of the axis of

the ion oscillate backwards and forwards instead of gradually decreasing as just supposed. If the oscillation happens to be between limits that include the critical angle α_0 , then the force upon the electron will also oscillate between repulsion and attraction at every crossing of the critical angle.

As a matter of fact, no one of the quantities just considered as constant probably is constant, and it will be necessary to imagine a case where r is changing and β_2 decreasing, as well as the pole of the ion oscillating, all at the same time. This suggests a complicated problem that need not be discussed here, but it may be stated that there is reason to expect that there is a cause in operation tending to make the actual angle occupied by the axis approach the critical angle, where the force is zero, as rapidly as possible, and in so doing it must oscillate about this position because of the moment of momentum of the ion. A further discussion of this subject is given in Chapter VIII.

CHAPTER VIII.

THE PROCESS OF LIGHT-RADIATION.

WHEN negative electrons are projected through a gas, as by the so-called cathode rays, they cause the gas to emit light under certain conditions. When the light is analysed by the spectroscope, it is seen to consist of fine spectral lines of varying degrees of brightness. These lines each correspond to a definite frequency of vibration, and it has been found that atoms emit lines that are characteristic of themselves. The art of spectrum analysis owes its existence to this fact, that each atom exhibits sets of lines characteristic of itself and different from the sets of lines produced by other atoms, no matter how complex the compound may be of which the particular atom to be studied forms a part. When this particular set of lines is observed in the spectrum obtained from any compound substance that belongs to some known atom, then it is certain that this atom is to be found in the compound, and hence compounds may be analysed in this way without the necessity of obtaining them in a chemically pure form. This also makes it clear that this set of lines must have its origin in the atom itself rather than in the molecule to which the atom may happen to belong. Other lines may make their appearance because of the molecules, but this does not prevent the set of lines associated with the atom itself from being produced.

What does the electron projected through the gas do to the atoms of it to cause it to emit even one such characteristic line in a spectrum? If this question can be answered satisfactorily by means of electrodynamics applied to the special forms of the atoms above discussed, there will be reason to

think that the production of groups of lines may receive some explanation, but in this chapter the attention will be confined to a single line of the spectrum in the endeavour to understand what must be the nature of the phenomenon. For this the simplest case is again selected, the passage of the cathode rays through hydrogen gas.

We shall trace the effect that one of these negative electrons passing near to an atom of this kind, but not colliding with it, will have upon the atom merely because of the mutual dynamical action between the two bodies. It is not necessary to assume a collision to account for the action; and, moreover, collisions will not account for it because of their infrequent occurrence.

The total effect may be studied by considering independently the effects that the passing electron exerts upon the three individual charges making up the hydrogen atom by means of the general expression for the ponderomotive force due to Saha, that a second point charge in motion, the passing electron, exerts upon the first point charge, the separate individuals of the hydrogen atom. This force is expressed as follows:—

$$\mathbf{F} = \frac{E_1 E_2 (1 - \beta_2^2)}{k(RA_2)^3 (1 - \beta_1^2)^{1/2}} \left\{ \left(1 - \frac{1}{c^2} \mathbf{q}_1 \cdot \mathbf{q}_2 \right) \mathbf{R} - \frac{R A_1}{c} \mathbf{q}_2 \right\}, \quad (1)$$

where the subscripts 1 and 2 refer to the first and second point charges respectively. The letters A_1 and A_2 are abbreviations for the expression

$$1 - \frac{\mathbf{q} \cdot \mathbf{R}}{cR}$$

for the first and second points respectively. The vector, $\mathbf{q}$, denotes the velocity of the charge, numerically equal to βc . The vector, $\mathbf{R}$, is the instantaneous distance from the second to the first charge. c is the velocity of light.

For clearness of presentation let us at first neglect the fact that each individual body of charge is supposed to be a

rotating body, and treat each simply as a static charge. Considering the positive charge of the atom $2e = E_1$ as a body at rest upon which we have to express the force due to the passing negative electron $-e$, we have

$$q_1=0, \quad \beta_1=0, \quad A_1=1, \quad \text{and} \quad E_1 E_2 = -2e^2.$$

Hence (1) reduces to

$$\mathbf{F} = -\frac{2e^2(1-\beta_2^2)}{k(RA_2)^3} \left\{ \mathbf{R} - \frac{\mathbf{R}}{c} \mathbf{q}_2 \right\} \dots \dots \dots (2)$$

An estimate of the probable value of the order of magnitude of $\mathbf{q}_2$, the velocity of the passing electron, may be obtained from the experimental observation that light-radiation does not begin to manifest itself in hydrogen gas until the velocity of the passing electron corresponds to about 11 volts, that is to say a velocity of about 2×10^8 cm./sec., or $\beta_2 = (2/3)10^{-2}$, less than the one hundredth part of the velocity of light. At this small velocity the part of the force expressed by the second term of (2) has less than one hundredth of the value of the part expressed by the first term, and may for a first approximation be neglected. If, therefore, we also put

$$q_2=0, \quad \beta_2=0, \quad \text{and} \quad A_2=1, \quad (2) \text{ reduces to}$$

$$\mathbf{F} = -\frac{2e^2}{kR^3} \mathbf{R}, \quad \dots \dots \dots (3)$$

which is merely the electrostatic force, this being an attraction, since by definition the vector $\mathbf{R}$ has its positive direction from the second to the first point charge.

In a similar manner the force upon each individual negative electron of the atom becomes

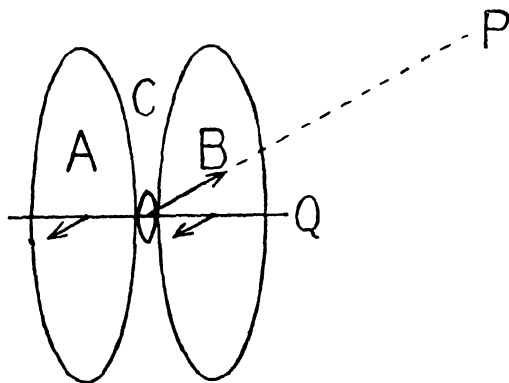
$$\mathbf{F} = \frac{e^2}{kR^3} \mathbf{R}, \quad \dots \dots \dots (4)$$

a repulsion of half the value of (3) being in the opposite direction.

These forces are represented in fig. 26 by vectors drawn from the centre of each charge of the atom parallel to $\mathbf{R}$, that

is, to the line from the distant-passing electron P to the atom. The sum of these forces is zero so far as any translational effect upon the atom as a whole is concerned, considering the three R's to be approximately the same, but there is produced an internal effect within the atom itself. In general, unless P happens to lie exactly in the equatorial plane of the atom, there is a component of each force which acts in the direction of the axis. The one pair of charges, B and C, is held by this component more firmly together, while the other pair, A and C, is less firmly held and tends to separate. The chief effect

FIG. 26.



Illustrating the forces upon the three parts of the atom due to the passing electron, P, electrostatic terms only.

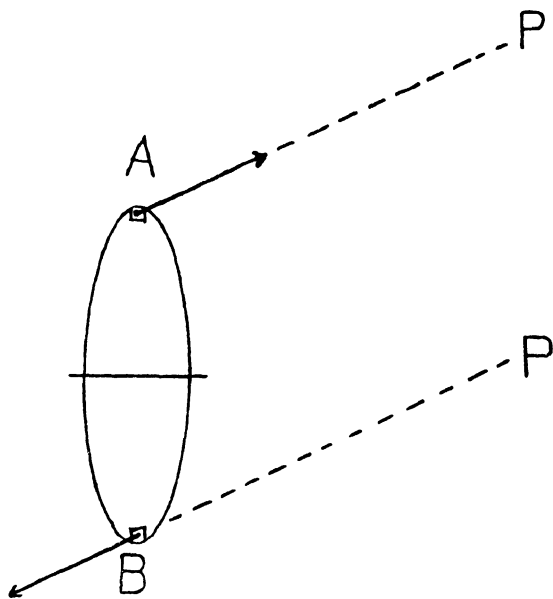
of this component is, therefore, to increase the momentary pressure between B and C and to decrease that between A and C, thus rendering A more readily detachable from the atom than B when other effects arise from any cause.

The other component of the forces resolved in the equatorial plane produces a couple tending to displace the two electrons A and B in the same direction off from the axis of C, but these electrostatic forces are not great enough to cause much displacement, and cannot be until the passing electron is near enough to be in contact with the atom. This near approach

is not necessary to account for radiation from the atom, as will be shown.

The rotation upon their axes of the charges A, B, and C having a non-spherical shape is the key to the behaviour of this atom. Let us next consider the effect that the passing electron P must have upon the positive charge C as a

FIG. 27.



Illustrating the couple acting upon a pair of elements A and B of the positive charge due to the passing electron P originating in the term $(1/c^2)(q_1, q_2)R$, which tends to rotate the axis.

rotating charge. In fig. 27 the ellipse represents a meridian section of the positive charge, and P the distant passing electron. Apply the formula (1) to find the force upon an element dE_1 of the positive charge in its upper side at A due to the passing electron of charge $-e_2$ as point charges. For small velocities of P of the order of, say, the 1/100th the

velocity of light the quantities A_1 and A_2 in the general formula are approximately unity, and (1) becomes

$$\mathbf{R} = -\frac{e_2 d \mathbf{E}_1}{k R^3} \left\{ \mathbf{R} - \frac{R}{c} \mathbf{q}_2 - \frac{\mathbf{q}_1 \cdot \mathbf{q}_2}{c^2} \mathbf{R} \right\}. \quad (5)$$

The first term within the brace, $\mathbf{R}$, is the electrostatic force discussed above acting along the line $\mathbf{R}$. The term $-(R/c)\mathbf{q}_2$ is small by comparison when β_2 is less than $1/100$, and has a direction the same as $\mathbf{q}_2$, the velocity of P . Since the velocity, $\mathbf{q}_1$, of the first point A does not enter this expression, the force due to this term is substantially the same upon every element of the positive charge.

This term, therefore, has the same kind of effect upon the positive charge as has the electrostatic force, only much smaller and in a different direction, so that the first two terms of (5) may be classed together and dismissed from further consideration, as merely producing a slight change in pressure between the electrons A and B upon C of the model fig. 26.

The remaining term of (5) becomes the important one, and the effective force due to this term upon the element at A , fig. 27, due to P to produce a rotation of the axis of the positive charge becomes

$$\mathbf{F} = -\frac{e_2 d \mathbf{E}_1}{k R^3} \left\{ -\frac{1}{c^2} (\mathbf{q}_1 \cdot \mathbf{q}_2) \mathbf{R} \right\}, \quad (6)$$

which is represented in the figure by the vector from A toward P , having the direction of $-\mathbf{R}$. When the element, A , is moving either from or towards the observer perpendicular to the paper, then the motion of the element, B , diametrically opposite is in exactly the opposite direction. For this element the cosine of the angle represented by the dot between $\mathbf{q}_1$ and $\mathbf{q}_2$ has the opposite sign, and the force upon the element, B , has the opposite direction along $\mathbf{R}$, represented by the equal vector from B away from P .

This pair of forces upon the two elements forms a couple having a tendency to turn the plane of rotation of the positive charge unless P happens to lie exactly in the plane of the

equator as a special case. The total effect of this term in the force equation, (6), upon the positive charge at any moment is, of course, the integral of these elementary forces throughout its volume. Without undertaking to integrate it, however, the chief conclusion is that there is no doubt that the total effect of the motion of P is to produce in the positive charge a tendency to turn its axis of rotation.

There exists at the same time a similar effect of the passing electron upon each negative electron A and B in the atom; but it may be shown that the angular acceleration produced in these electrons is small in comparison with that produced in the positive charge, and, therefore, for the present purpose it is negligible.

To compare their orders of magnitude consider a pair of elements of charge as before, and take the position of P on the axis so that the vector, **R**, is perpendicular to the plane of the orbit and the effect is a maximum. The moment of the couple for two opposite elements, A and B of fig. 27, of the positive charge is $2aF$, where a is the radius or distance from the element to the axis and F the force upon one element as expressed by (6). Setting this equal to the product of its moment of inertia I by the angular acceleration of its axis, α , we find the angular acceleration in terms of the element of mass, dm , to be

$$\alpha = \frac{2aF}{I} = \frac{2F}{adm} \cdot \cdot \cdot \cdot \cdot \cdot (7)$$

Next suppose the negative electron to be divided up into the same number of elements as the positive charge, and select a pair of elements similarly situated in the oblate spheroid to those chosen in the positive charge. The elementary positive charge will then be twice as great as the elementary negative charge because the whole positive charge is twice the negative electron. A similar expression may now be written for the angular acceleration of the negative electron's elements, namely,

$$\alpha' = \frac{2F'}{a'dm'} \cdot \cdot \cdot \cdot \cdot \cdot (8)$$

According to the mass formula the mass of a charge supposed to be spherical is inversely as its radius, the charge remaining the same. And, since each charge has been divided into the same number of elements, the masses of these elements are inversely as the radii of the said spheres, and again the radii of the spheres are directly as the radii of the two similarly placed elements.

Hence, for equal charges it is true that

$$a/a' = dm'/dm \quad \text{or} \quad adm = a'dm'. \quad . \quad . \quad . \quad . \quad . \quad (9)$$

But for a positive charge of double the negative, since the radii vary as the squares of the charges, we have

$$adm = 4a'dm'. \quad . \quad . \quad . \quad . \quad . \quad (10)$$

Hence the ratio of their angular accelerations by (7) and (8) is

$$\frac{\alpha}{\alpha'} = \frac{2F}{adm} \times \frac{a'dm}{2F'} = \frac{1}{4} \frac{F}{F'}. \quad . \quad . \quad . \quad . \quad . \quad (11)$$

But, by (6)

$$\frac{F}{F'} = \frac{dE_1}{dE_1'} \frac{q_1}{q_1'} = 2 \frac{q_1}{q_1'} = 2 \frac{\beta_1}{\beta_1'}. \quad . \quad . \quad . \quad . \quad . \quad (12)$$

Hence

$$\frac{\alpha}{\alpha'} = \frac{1}{2} \frac{\beta_1}{\beta_1'}. \quad . \quad . \quad . \quad . \quad . \quad (13)$$

But the gravitational equation and the expression for the mass given in (18) of Chapter III shows that the value of β for a ring equivalent to the positive charge of $2e$ is about 31.4 times the value of a corresponding ring equivalent to a negative electron. Hence the value of β for corresponding elements may be assumed to have this approximate ratio, and we have $a=15.7a'$, showing that the angular acceleration produced by the passing electron in the two elements of the positive charge is about sixteen times greater than that produced in the corresponding elements of the negative electron. The total integral effect for all elements in each

of these two bodies may be assumed to have approximately this same ratio, and hence the effect upon the negative electrons as above stated may be neglected and attention centred upon the positive charge.

*The Effect of the Rotation of the Axis of the
Positive Charge.*

While it is true that this whole process cannot be worked out in exact detail because of the limitations of analysis, yet we are able to obtain a glimpse of what probably happens in a general way. If preferred, the processes now to be described may be regarded as assumptions, but with the understanding that they are clearly within the possibility of being substantiated by electrodynamics.

Tracing the process as the passing electron has various velocities from zero up, it is clear that for a slow velocity the axis of C (fig. 26) may be turned slightly out of its normal position, the body of the positive charge rolling over smoothly on the surface of the electrons, A and B, without losing contact with them. Some energy must be taken from the passing electron if any alteration of this kind takes place in the atom. But, whatever disturbance takes place upon the approach of the electron, P, the normal condition is restored again as it departs, and all of the energy received is again returned to the electron, so that it passes the atom leaving with as much energy as it had upon approach.

As the velocity of the passing electron becomes greater and greater the positive charge will be turned with more and more violence, and when the impulse is sufficient, it may turn with sufficient rapidity to leave contact with that electron, say A, whose pressure upon C is the least. This electron then returns and comes into actual collision with the positive charge, thereby setting up vibrations within its body that are characteristic of itself.

The case may be likened to rolling an elliptical wheel along a horizontal table. If the velocity of the wheel exceeds a

definite critical value, the wheel will leave contact with the table, and upon its return bump against it thus setting up sound waves. Some of the translational energy of the wheel is then transformed into sound waves and is radiated out into space. So some of the translational energy of the passing electron is transformed in character into the high frequency vibrations of the positive charge, and the energy thus transformed or received by the atom is not returnable to the passing electron because it has been thus transformed. This transformation and consequent reception of the energy by the atom is, therefore, only possible when the velocity of the passing electron exceeds a certain critical value, which prediction is in close accord with the known facts.

And further, just as is the case with the wheel set into vibration so as to emit waves of sound and thus radiate its energy into space to be reflected, refracted, and absorbed by all immediately surrounding objects, so the waves emitted by the vibrating positive charge are reflected from the objects surrounding it. The objects, however, immediately and practically completely surrounding it are the two negative electrons, A and B of fig. 26, and these effectually confine this received energy within the limits of the atom itself, none of it being radiated out into space because of these barriers.

Whatever the details of this complicated process may be, at a certain critical energy, which means a critical velocity on the part of the passing electron, the positive charge may be set into vibration say like that assumed in fig. 21 of Chapter VII. The normal figure of the rotating positive charge is tentatively represented in section by the ellipse A. The vibration may be supposed to change its shape between the extreme limits of the curves B and C. Since the volume is supposed to remain constant, the surface area changes, and there occurs an interchange of energy or an oscillation between the potential energy and the rotational energy, the potential energy being associated with the surface pressure, or the surface tension if preferred. The whole energy content of the positive charge thus takes part in this vibration.

The actual vibration of the positive charge is probably far more complicated than that represented in fig. 21, and it is possible that the fundamental vibration there represented does not take place at all. It must be true, however, that the value of the natural free frequency of the positive ellipsoid regarded as a quasi-solid body far transcends any that has ever been measured*, for it has the free period of the smallest known body.

It is not material to the arguments that follow what the precise value of this frequency is, provided it is very great, that is to say, much greater than the highest frequencies observed anywhere, and probably having a wave-length comparable with the dimensions of the negative electrons themselves, which are, of course, many times greater in diameter than the positive body now referred to. When this is the case the energy of vibration may be prevented from escaping from the atom, for it has been shown† that bodies having dimensions close to the order of magnitude of the received waves become great absorbers of this radiation. The present case differs from this, however, in that the word absorbed implies a transformation in the form of the energy whereas the phenomenon above contemplated is more analogous to total reflection, the whole energy being confined within the region between the two negative electrons of the atom and without any transformation in character.

The treatment of an electrodynamic oscillator was undertaken in Chapter VII in order that the results obtained may now be used, for in effect when the electron whose pressure is the least bumped into the positive charge, it became an oscillator, and new forces were immediately brought into play which were not present before. In applying the results of the last chapter to the atom, which now becomes an ion and an electron in close proximity, since the surfaces of the electron

* See Note at the end of Chapter IX.

† L. Page, *Astrophys. Journ.* lii. p. 65, Sept. 1920.

and the positive charge become separated in the manner described, the first thing to notice is that the total force upon the electron still acts along the radial line from the ion, but that its magnitude differs from the electrostatic force. Let it be assumed, for example, that the maximum velocity for the oscillating positive charge exceeds 0.9 of the velocity of light. The force upon the electron is then repulsive in every direction, but has its maximum value along the equatorial direction, as shown by the curve for $\beta_2=0.9$ in fig. 24, Chapter VII.

It is also to be noticed that the force upon the electron depends upon its own speed on account of the factor $(1-\beta_1^2)^{1/2}$ in the denominator of (6) Chapter VII, and that when this speed approaches the velocity of light the force approaches a very large value. But it is well known that the mass of the electron also depends upon the speed, becoming greater with increasing speed according to the formula

$$m=m_0(1-\beta^2)^{-1/2}, \quad \dots \quad (14).$$

which is precisely the same form of function as governs the force in (6) referred to. Hence it would not be strange if the velocity of projection of the electron were very great, even, approaching the velocity of light, for the force upon it increases just as fast as its mass increases.

From considerations that are given below it is probable that the initial velocity is very close to the velocity of light, but that it rapidly decreases. If the reactive force due to the radiation of energy from the electron as it moves outward from the ion is neglected for the present and the whole force upon it set equal to the mass times its acceleration, $\frac{d^2r}{dt^2}$, we have

$$\frac{d^2r}{dt^2} = \frac{F}{m} = F(1-\beta_1^2)^{1/2}/m_0. \quad \dots \quad (15)$$

Putting in the value of F from (6) of Chapter VII, the acceleration becomes

$$\frac{d^2r}{dt^2} = \frac{e^2}{kr^2m_0} \left[-1 + \frac{2(1-\beta_2^2)}{(1-\beta_2^2 \sin^2 \alpha)^{1/2}} \right]. \quad \dots \quad (16)$$

The chief thing that we know about this acceleration is that it takes place in one straight radial line because the whole force in (6) is radial. As to the value of the acceleration along this line the expression does not afford much light, because there are in it three quantities which vary both with the time and with the distance, namely, r , a , and β_2 . It does not seem possible without further help to make valid assumptions as to these variables as functions of the time and distance.

Some light upon the question is afforded by a consideration of the energy relations. Regarding the hydrogen ion at O_2 (fig. 22) as a single physical system, let us apply to it the ordinary rules of dynamics. Since the entropy of any material system left to itself is known to tend towards a maximum, this may be interpreted here to mean that the ion strives to be rid of its energy at a maximum rate. Now the kinetic energy of translational motion of the electron at O_1 is a part of the energy originally possessed by the positive charge of the ion. If the ion succeeds in driving this electron free completely away from itself carrying with it this kinetic energy, then its own energy will be reduced at a far greater rate than it would be if it depended merely upon the energy lost by radiation.

For, as will be shown later, the kinetic energy at some points of its path is much greater than the total energy radiated by the whole motion. To render this entropy a maximum, therefore, the ion strives to get rid of an electron. As shown above, there exists a certain critical angle a_0 , which reduces the whole force to zero, and, if the ion could immediately turn itself to this angle, the force would remain zero and the translational velocity of the electron would then become constant because it would retain its original velocity of projection. No energy would be radiated out into space by this uniform velocity and zero acceleration, for the rate of radiation is zero for uniform motion in a straight line, but the condition for maximum entropy would be fulfilled.

The legitimate inference from this reasoning is that the

forces governing the direction of the axis of the positive charge strive to make it assume this critical angle, say α_0 , as soon as possible. But in so doing the mass of the ion, or its angular momentum, requires that a certain amount of time be consumed in this operation, causing the axis first to be carried beyond the critical angle and then to be reversed in its motion, thus oscillating about the angle, α_0 , with a damped motion. The damping must be present because the value of β_2 is continually decreasing due to loss of energy by radiation from the electron into space.

This gives rise to a periodical force the maximum value of which must, according to fig. 24, be greater the greater the maximum departure from the critical angle, α_0 , during a complete period. But the amplitude of this angular oscillation must be a function of the frequency of it, and it seems most logical to assume that the greater the frequency the smaller the amplitude of the motion. This may mean that the maximum force at the extreme angular deflection varies on this account alone as the inverse of the frequency.

The force or the acceleration according to (16) is also inversely as the square of the distance, r , but r itself will be a periodic function of the time, and it is shown below that the maximum values of r vary inversely as the frequency. These two causes operating together, therefore, make the maximum values of the periodic force vary directly as the frequency of oscillation of the pole of the positive charge. This matter is stressed because it is unusual to find a force in mechanical systems whose maximum values vary as the frequency, but the above model points to this condition.

With the above understanding it seems entirely justifiable to assume that the acceleration expressed by (16) may be written as a simple periodic function of the time as follows,

$$\frac{d^2r}{dt^2} = k_1 \omega e^{-\omega t} \cos \omega t, \quad (17)$$

where ω is the angular velocity of the pole of the positive

charge corresponding to a frequency, ν , such that $\omega = 2\pi\nu$. The presence of the factor ω in the coefficient makes the maximum values of the force, as above stated, directly proportional to the frequency, ν , k_1 being a constant to be determined. The presence of the damping factor, $e^{-\omega t}$, has been accounted for by the loss of energy due to the radiation from the electron moving with accelerated motion.

Granting that this is the typical form that the acceleration expressed by (16) would take, were it possible to deduce it as a function of the time alone, the velocity may be found by a first integration with respect to the time, giving

$$\frac{dr}{dt} = \frac{1}{2} k_1 e^{-\omega t} (\sin \omega t - \cos \omega t) + k_2, \quad (18)$$

k_2 being the constant of integration. When the time is infinite it may be assumed that the electron has returned to resume its original position in the atom, and that $dr/dt = 0$. Hence $k_2 = 0$, and no constant is required. A second integration gives the distance, r , as follows,

$$r = -\frac{1}{2} \frac{k_1}{\omega} e^{-\omega t} \sin \omega t + k_3, \quad (19)$$

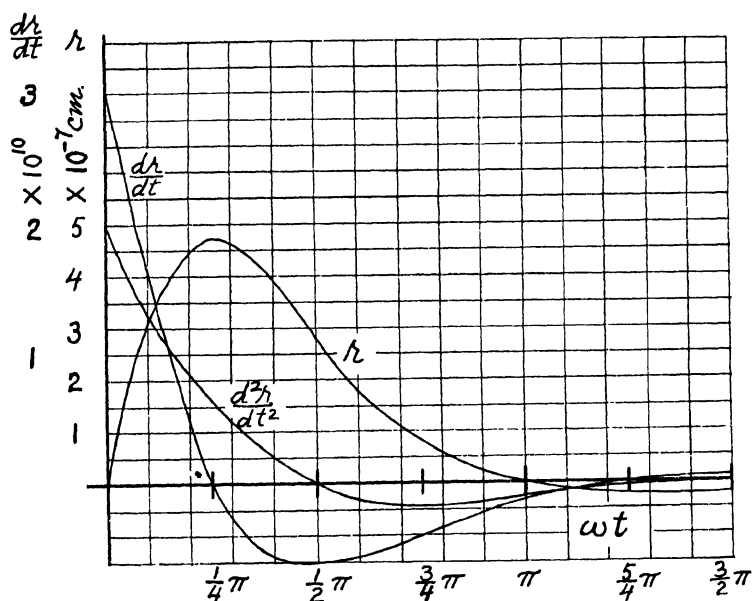
k_3 being a second constant of integration. This is also negligible because when the time is infinite, $r = k_3$, and the distance from the centre of the atom to the normal centre of the electron in the atom is of the order of 10^{-13} cm. and quite negligible.

This expression for r shows that the maximum values of it vary inversely as the frequency other things being equal, which confirms the statement made above in regard to the distance r that occurs as r^{-2} in the expression for the force.

Three curves marked $\frac{d^2r}{dt^2}$, $\frac{dr}{dt}$ and r in fig. 28 represent the equations (17), (18), and (19) for certain assumed numerical values of the constants. $k_1/2$ according to (18) is equal to the

initial velocity of projection of the electron from the atom. The frequency ν is assumed for these curves to be equal to the Rydberg constant, 3.29×10^{15} as representing the maximum

FIG. 23.



Equations (17), $\frac{dr}{dt}$, (18), $\frac{d^2r}{dt^2}$, and (19), r , plotted as ordinates against the angle ωt , proportional to the time, as abscissæ. The assumed frequency is equal to the Rydberg constant, and the initial velocity approximately the velocity of light. r denotes the distance of the ejected electron from the ion sending it forth.

frequency in the spectrum of hydrogen. The angles ωt are the abscissæ in each of the three curves.

The maximum distance of the electron from the ion is reached when $\omega t = \pi/4$, one eighth of a period, at which time the velocity is zero and reverses direction. Putting $\omega t = \pi/4$ in (19), and giving the initial velocity, $k_1/2$, its maximum possible value, approximately the velocity of light, c , the

maximum distance the electron is driven from the ion is given by

$$r_{\max.} = \frac{c}{\omega} e^{-\pi} \sin \frac{\pi}{4} = \frac{c}{\omega} \times 0.456 \times 0.707 = 0.322 \frac{c}{\omega}. \quad (20)$$

Since $\omega = 2\pi\nu$, and $\lambda\nu = c$, we have

$$r_{\max.} = 0.0513\lambda, \quad . \quad . \quad . \quad . \quad . \quad . \quad (21)$$

where λ is the wave-length of the radiated waves.

According to the type of motion expressed by (17), (18), and (19), the velocity of the electron decreases rapidly as it moves away from the ion in a straight line from a value near to the velocity of light down to zero in one eighth of a period of the oscillation of the pole of the positive charge. At this point it reverses its direction, at a distance about one twentieth of the wave-length from the ion, and begins to move toward the ion again with an accelerated motion. In this direction it passes a point where the velocity is a maximum after one quarter of a period, which represents the greatest velocity it has on the return path to the ion. The actual velocity at this point is about 0.323×10^{10} cm./sec. assuming that the initial velocity is close to that of light. The mass of the electron at this slower velocity is then not far from its normal mass at zero velocity, and its kinetic energy of motion is, therefore, about 47×10^{-10} erg.

The motion of the electron thus described is one of great acceleration, and according to all forms of electrodynamics the electron so moving must radiate out some energy into space. Moreover, this radiation will have a periodic character at one definite frequency, that contained in the equation of motion (19). It is not difficult to show that an electron so moving will exert a ponderomotive force upon each individual elementary charge of an atom at a great distance from it, which is periodic in character and of the same period as that of the original motion. This is almost axiomatic that the same period is transmitted through without change and will make

itself felt upon distant atoms upon which the radiation falls, say the retina of the eye or a photographic plate.

At present we shall not try to picture what permanent change this radiation effects in the silver salts used in the photographic plate, but may rest content with a picture that covers the process of bringing this radiation up to the receiver from the original passing electron which caused the disturbance.

In this chapter the process has been outlined with a view to making the probable operation of the mechanism of radiation of one characteristic line of the spectrum understood. It is clear that the frequency of this line must be a characteristic of the atom itself, for after the energy was received from the passing electron and transformed in character into very high frequency waves, it is only the properties of the fundamental bodies belonging to the atom in question that can impress a definite frequency upon the subsequent and secondary action described. In the next chapter will be considered the amount of the energy radiated by the electron in making its complete excursion out and back from the ion.

CHAPTER IX.

THE TRIGGER EFFECT AND PLANCK'S QUANTUM.

WHEN a point charge is set in motion all the forms of classical electrodynamics state that energy is radiated out from it in all directions. The one exception is for uniform motion in a straight line when the radiated energy falls to zero. The flow of energy at each point of the field surrounding the point takes the direction of the Poynting vector, and, in order to obtain an expression for the total energy radiated as well as the rate of radiation, the surface integral of the Poynting vector over the surface of a sphere surrounding the moving point was obtained by Liénard in 1898, using the Lorentz form of electrodynamics.

An electron is not strictly a point charge, but it is equivalent to one provided the radius of curvature of the path along which its centre moves is large compared with the linear dimensions of the electron. The rate of the radiation of energy from the moving point charge as found by Liénard is as follows:

$$\dot{W} = \frac{2}{3} ce^2 \left[\frac{\beta^4}{\rho^2(1-\beta^2)^2} + \frac{1}{(1-\beta^2)^3} \left(\frac{d^2r}{dt^2} \right)^2 \right], \quad \dots (1)$$

where ρ is the radius of curvature of the path, $\frac{d^2r}{dt^2}$ the acceleration along the path and βc the velocity of the point, c being the velocity of light. This formula has been checked by others, notably by G. A. Schott in his book 'Electromagnetic Radiation,' who uses the same form of electrodynamics, and the presumption is that the expression has been correctly derived from the Lorentz form of theory.

The Poynting vector in the Saha form of electrodynamics

has a different form from that on the Lorentz theory, but, so far as the Author is aware, the rate of radiation from a point charge found by integrating this Saha-Poynting vector over the surface of a sphere has not been published by anyone. The result may differ from that above given or it may possibly be the same, though the chances are that it differs because of the difference in the Poynting vector. This matter is mentioned because we have followed the Saha form of dynamics heretofore, but now must use equation (1) or wait until the Saha expression is available. The expression (1) may be right in the main, but it seems that it must be defective or incomplete for the following reasons.

If E denotes the total energy radiated, the rate of change of this energy with the time, dE/dt , may be denoted by $\dot{R}$, and, if F denotes the force upon the electron due to the reaction of the radiation along the element of the path, dr , then the work done by this reactive force in the distance, dr , is Fdr , and the rate of working is $Fdr/dt = Fv = F\beta c$. If (1) is divided by the velocity, v , of the electron, we shall, therefore, obtain the reactive force due to the radiation when due regard is paid to the sign of the force, that is, whether it is the force of reaction we wish or a force equal and opposite to it.

In his 'Electromagnetic Radiation' Schott has worked out examples of radiated energy in circular motion (p. 110), simple harmonic motion in a straight line (p. 122), and elliptical motion (p. 126), and shows that his results are obtainable from (1) above. Let us consider the case of simple harmonic motion in the light of equation (1). At the middle point of the path the velocity is a maximum and the acceleration zero. Hence at this point R is zero and no energy is radiated as in uniform motion. And by dividing by v the reactive force is also zero as it should be for no radiation.

However, as the point approaches the end of its path its velocity approaches zero and its acceleration a maximum. Hence in dividing a finite quantity by zero or a very small velocity, just before the end of the path is reached the reactive

force approaches infinity. This does not mean that the total energy radiated per cycle must be infinite. It comes out finite, although the electron must pass through a point where the reactive force upon it becomes exceedingly large. That there ever exists so great a force of reaction anywhere is physically improbable. No such difficulty arises with the circular motion, and one can scarcely believe that there is any more reason why simple harmonic motion should present any more difficulty than circular motion. The indication is that something is lacking or incomplete about the formula, and this is borne out by the other instances mentioned in the preceding chapters, where the Lorentz form of dynamics is not satisfactory.

This is a disconcerting kind of an introduction to give just before proceeding to use formula (1) to obtain some sort of a useful result from it. Even so the reader will certainly prefer to have one's true opinion of the formula. It is not wholly wrong, and the results obtained from it are suggestive rather than final. It is quite likely that a similar result will be obtained from the Saha formula, when it is available, in all the essential particulars, and some interest may attach to the use of (1) in throwing some light upon a very obscure phenomenon.

For straight line motion of the electron out and back from the ion described in the last chapter, the radius of curvature of the path, ρ , is infinite, and the formula (1) reduces to

$$\dot{r} = \frac{2e^2}{3kc^3} \frac{\left(\frac{d^2r}{dt^2}\right)^2}{(1-\beta^2)^3} \cdot \cdot \cdot \cdot \cdot \cdot (2)$$

The dielectric constant k is inserted here in order to keep the dimensions of both members of the equation the same. It should also have been written in (1), but that stands as quoted from the original author.

Assuming that the motion of the electron at O_1 , fig. 22, is completely described by the equations (17), (18), and (19).

of Chapter VIII and illustrated by fig. 28, the total energy radiated during one whole excursion may be obtained from (2) by integration with respect to the time between the limits zero and infinity. Writing for β , $\frac{1}{c} \frac{dr}{dt}$, we have the total energy

$$E = \int_0^\infty \dot{K} dt = \frac{2e^2}{3kc^3} \int_0^\infty \left[1 - \frac{1}{c^2} \left(\frac{dr}{dt} \right)^2 \right]^{-3} \left(\frac{d^2r}{dt^2} \right)^2 dt, \quad \dots (3)$$

or expanding into infinite series, this becomes

$$E = \frac{2e^2}{3kc^3} \int_0^\infty \left[1 + \frac{3}{c^2} \left(\frac{dr}{dt} \right)^2 + \frac{6}{c^4} \left(\frac{dr}{dt} \right)^4 + \frac{10}{c^6} \left(\frac{dr}{dt} \right)^6 + \dots \right] \left(\frac{d^2r}{dt^2} \right)^2 dt. \quad (4)$$

Using the values of the acceleration and velocity given in (17) and (18) above, and integrating each term separately, gives finally the total energy radiated in one excursion of the electron out and back :

$$\dot{E} = \frac{\pi e^2 k_1^2}{2kc^3} \left[1 + \frac{23}{90} \left(\frac{k_1}{c} \right)^2 + \dots \right] \nu. \quad \dots (5)$$

The next term of this series will be k_1^4/c^4 with a different numerical coefficient, and so on, the powers of k_1/c increasing by 2 from term to term. But k_1 is constant by hypothesis according to (17), and there exist, therefore, no variables in the right member of (5) except the frequency of the radiation, ν . Hence the whole energy may be written

$$E = h\nu, \quad \dots (6)$$

which is Planck's quantum expression, if h may denote the coefficient of ν in (5). That is to say, exactly one quantum of energy is radiated away from the atom by one excursion of one electron out and back. Moreover, the atomic model affords a concrete picture of just what it is that possesses this frequency, ν .

The numerical value of Planck's constant, h , cannot be deduced from (5) because the value of the constant, k_1 , is unknown, but the known value of h may be used to give some information about k_1 . According to (18) it is seen that $k_1/2$ represents the initial velocity of ejection of the electron from the atom. The series in the parenthesis of (5) may have any value from unity to infinity according to the value of k_1 . It is evident by (2) that the radiated energy approaches an infinite value as the initial velocity of the electron approaches that of light.

For all initial velocities less than that of light the series is finite, but less and less convergent as the velocity of light is approached. Suppose that the sum of the series within the parenthesis of (5) is denoted by S . In order that the coefficient of ν may be numerically equal to Planck's constant, h , the value of S must be 138.2 . This indicates immediately that a large number of terms of the series is required, that it is not very rapidly converging, and that the initial velocity $k_1/2$ must be very close to that of light. Its exact value might be evaluated by the use of this information, but the process is laborious and not worth while. When $k_1=2c$ approximately, the constant, h , is then given by

$$h=2\pi e^2 S / ck. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

The Trigger Effect.

It was stated above that the maximum kinetic energy of the electron on its return journey is about 47×10^{-10} erg. The greatest frequency in the hydrogen spectrum is equal to the Rydberg constant $K=3.29 \times 10^{15}$. Multiplying this by $h=6.56 \times 10^{-27}$ gives 0.2154×10^{-10} erg as the maximum quantum of energy radiated by hydrogen. Assuming that this represents the total energy radiated during one excursion of the electron out and back, it is seen that the kinetic energy

at one point of the path on the return journey is two hundred and twenty times greater than the total energy radiated, and this is smaller than the kinetic energy at other points on the outward journey.

The importance of this conception, if true, is so great that some of the more fundamental points of the process given in the last chapter may bear restating. An initial velocity closely approaching that of light may be produced in an electron which is shot out from the ion in a straight line and then returns again to the ion. The energy thus lost by radiation from the electron is no greater than that received from the passing electron from which the energy was originally derived. Because the form of this energy received from the passing electron is entirely transformed in character into high frequency vibrations of the positive charge, the whole internal energy of the positive charge is used to give the ejected electron its large kinetic energy of translational motion, but this is all returned again to the atom as the electron comes back.

The positive charge undoubtedly contains a store of energy, $mc^2 = 1.49 \times 10^{-3}$ erg, which is very great compared with any of the other energies above mentioned. The kinetic energy of the electron cited is of the order of 10^{-9} erg, a million times smaller, the maximum quantum of energy above computed of the order of 10^{-11} , and the energy of dissociation of the two atoms of a molecule of the order of 10^{-12} erg. Hence it becomes possible for a very much smaller amount of energy to release or to bring into action a much larger amount of energy, namely some of that stored in the positive charge. This may be called the trigger effect.

Although such an effect has been mentioned in the literature of the subject as a remote possibility, its probability from the standpoint of dynamics has never been pointed out. To admit of its existence removes some otherwise insurmountable difficulties connected with the photoelectric effect and the secondary β -rays generated by X-rays. These two phenomena

in particular have seemed to demand the artificial conception of "light quanta," which of itself introduces difficulties of an order far more formidable than to admit of a true trigger effect.

*The Maximum Distances to which an Electron
is Projected from Hydrogen Atoms.*

According to the result in (21) of Chapter VIII the maximum distance to which the electron is projected from the ion during the characteristic radiation from hydrogen gas is equal to the 0.0513 part of the wave-length of the radiated waves. The shortest wave-length characteristic of the hydrogen spectrum is 911.8×10^{-8} cm., corresponding to the frequency $\nu = K = 3.28880 \times 10^{15}$, the Rydberg constant. Hence in this case the distance given by (21) is 4.677×10^{-7} cm.

In order to compare this distance with the average distances between the molecules of the hydrogen gas, let it be supposed that the gas is at 0° C. and one millimetre of mercury pressure. The approximate distance between the molecules is then on the average about 65×10^{-7} cm. This is about fourteen times greater than the maximum distance that the electron goes from the atom as above computed. On the average, therefore, the ejected electrons will not come within the sphere of influence of the next molecule of the gas, and will experience no disturbance or interference from this cause.

As the frequency of the characteristic radiation is reduced and the wave-length increased to correspond with other lines of the spectrum, the maximum distance according to (21) becomes greater in direct proportion. The wave-length must be increased fourteen times, therefore, in order that the electrons may go on the average as far as the next molecule of the gas under the conditions of temperature and pressure named. This would make the wave-length about $12,760 \times 10^{-8}$ cm., which is not far from the second line of the Paschen

series in the infra-red. Hence, practically the whole of the hydrogen spectrum so far as it has been observed may be emitted before an electron travels so far as the next adjacent molecule on the average, and no interference will thus be experienced.

It has just come to our notice that Prof. R. A. Millikan, in a paper entitled "High Frequency Rays of Cosmic Origin" before the National Academy of Science at Madison, Wisconsin, has announced a new discovery of high-frequency waves of much higher value than those hitherto known. These waves seem to be just those with which Chapters VII to X are concerned, and it has been anticipated that they would eventually be detected experimentally, though it could not be anticipated when this would be accomplished. The waves that Prof. Millikan announces apparently have all the ear-marks of the waves treated of in these chapters. As explained in these pages they are free to radiate out into space during the times when an electron is being detached, but not at other times, because in the normal states the presence of the two electrons in the hydrogen atom or in the alpha-particle prevents this radiation from escaping.

November, 1925.*

CHAPTER X.

THE PROCESS OF X-RADIATION.

IN examining in Chapter VIII the process of light-radiation from a gas a stream of cathode rays, that is negative electrons, was supposed to be passed through it, and the action of one electron upon a single hydrogen atom was considered, which being one of the atoms of a gas is free to orient itself in any manner.

In order to generate X-rays the usual method is to cause the stream of cathode rays to impinge upon a solid substance, usually a metal plate such as tungsten, rubidium, or platinum, in an exhausted bulb. The point where the cathode rays strike the plate becomes a source of X-rays, which are emitted in all directions from the point.

As thus stated it is evident that the two phenomena of light and X-radiation are distinct in this, that the atoms of the gas are free, while the atoms of the metal, being a part of a solid body, are not free to be oriented simply by the mutual forces between the passing electron and the atom only.

In view of certain recent experimental results some may think that this distinction, as between a gas and a solid, does not cause the difference between light- and X-rays, but there is no experimental evidence as yet which establishes the fact that the X-ray spectrum has ever been produced from the atoms while in the state of a gas, when bombarded by a stream of electrons. Until very recently there has existed a wide gap between the lowest X-ray frequencies and the highest light-frequencies measured, the X-ray frequencies lying in a region from one to ten thousand times higher than light-frequencies formerly measured.

Recent experiments of Millikan may occur to the reader as

an answer to the above statement that the X-ray spectrum has not yet been produced by the atoms in a state of gas. In these experiments use was made of an electric arc, its two electrodes being separated in the neighbourhood of one tenth of a millimetre from which light-frequencies of an unusually high order were obtained, bridging the gap spoken of between light- and X-ray frequencies as they had previously been measured.

No precautions were taken in these experiments to prevent the light from the metal electrodes themselves from passing through the slit to the photographic plate, and indeed this would have been difficult of accomplishment considering the small distance between the two electrodes of the arc. It may be assumed to be possible, therefore, in the absence of other evidence, that X-rays may have been generated by the electric discharge at the surface of the metal. The only evidence, however, that X-rays were obtained in these experiments seems to be that the light-spectrum comes to an abrupt end at a definite high frequency limit, which fits closely with the quantum relation. There is no reason why such a limit should not also be present in light-radiation. The only satisfactory evidence that the characteristic X-radiation is being obtained must be the actual production of the X-ray emission spectrum which consists of several series, the K, L, etc., groups, each consisting of several distinct lines. No such spectrum has ever been produced from a gas so far as the writer is aware.

To anticipate the results to be described it seems probable that the source of X-rays is fundamentally different from light-rays. Light-rays in accordance with the atom models are intimately connected with the high-frequency vibrations of the positive charge as described in Chapters VIII and IX, while X-rays are intimately connected with the high-frequency vibrations of the negative electron itself. There is some experimental support for this view. At first thought one might expect that all X-ray spectra would be exactly alike if they are due to the same body, the negative electron. But they are actually very much alike, the same lines appearing in the

spectrum of each element only being a little displaced with increasing atomic number from element to element. No such marked similarity exists between the light-spectra of these same elements. They are very markedly different the one from the other.

There are certain reasons why the X-ray spectra of successive elements, while being almost identical, yet should be shifted slightly from element to element, even though they are generated in every case by the same body, the negative electron. If the negative electron is projected from the atom with a velocity at all approaching the velocity of light, as was supposed to be the case in light-spectra discussed in the preceding chapters, then its mass is appreciably different, and to all intents it is not the same electron, but differs in its properties. The same is never true of the positive charges, because they do not have velocities approaching that of light. This different mass must affect all of its motions alike, which makes it clear that the spectra will all have the same general character but be shifted somewhat from element to element.

In a recent experimental investigation by Zecher* in measuring X-ray intensities over the surface of a hemisphere, he makes the remark that it is in harmony with his observations to conceive of the source of the X-rays as the vibrations of rapidly moving particles, which is in perfect agreement with the above statements.

According to these views the way to produce the X-ray spectrum of hydrogen is to obtain hydrogen in the solid form, and then bombard it with cathode rays just as is done with other substances, but to do this presents considerable experimental difficulty. Until it is done, however, it cannot be stated that the X-ray spectrum of hydrogen will prove to be the same as the light-spectrum, and indeed it may be expected to be different with more reason.

Heretofore in the theoretical discussions in the preceding

* Ann. Physik, lxiii. p. 28 (1920).

chapters the hydrogen atom has been used almost exclusively on account of its simpler construction, but now in considering solid substances it seems advisable to picture one of the more complex atoms. The structure of the atoms is only given in fig. 2 as far as aluminium, and, since it is possible to generate X-rays by bombarding an aluminium plate, let us use the aluminium atom for illustration.

An aluminium crystal has its elementary cube edge 4.07×10^{-8} cm., which is an immense distance as compared with the longest dimension of the aluminium atom, say Al-1 (fig. 2). This distance is between five and six major diameters of the negative electron, that is about $6 \times 6.51 \times 10^{-13}$ cm. = 3.9×10^{-12} cm., about ten thousand times smaller than the edge of the cube lattice.

The forces of all the other atoms in the metal acting upon the one atom, however, determine not only its position in the crystal but also the position of the axis of the atom. In the latter the rotation of the positive elements on their respective axes is by far the most potent factor in directing the axis of the whole atom, because the absolute values of the speeds of rotation are so much greater than those of the electrons.

When one of the electrons of the cathode rays, which we have called the passing electron in discussing the phenomenon in Chapter VIII, passes close to one of these aluminium atoms, close, that is, as compared with the space lattice distances, it will have an effect similar to that described with the hydrogen atom upon each positive charge and each electron tending to twist or turn the plane of its axis of rotation. But the whole atom is held by the other members of the surrounding metal, so that it must require considerably more torque on the part of the passing electron before one of the charges can be turned with sufficient violence to cause it to lose contact with one of the electrons. This means more energy and greater velocity on the part of the passing electron before the critical velocity is reached. When the critical velocity is passed, however, it is most probable that there is some one electron in the

atom most favourably situated so that it will be the first to respond. Let us suppose that this is the central electron in the atom A1-1 (fig. 2) in order to make the illustration concrete.

It will then bump against the positive charge setting both itself and the positive charge into high frequency vibrations similar to that illustrated in fig. 21. The plane of the atom as a whole is not affected by this action, however. To rotate the plane as a whole would require more time involving the whole moment of momentum of the atom.

The negative electron may then be assumed to be projected from the atom in a straight line because of the new forces brought into existence due to these high frequency vibrations. The oscillations of the axis of the positive charge, however, which does not leave the atom must be quickly stopped because of the influence of the other positive charges in the same atom, and the force upon the now distant electron will not be oscillatory because of a positive charge in the atom.

It is oscillatory, however, because of the turning of its own axis, and, as was shown in Chapter IX, it will endeavour to orient its axis so as to come into the angle of zero force, and will oscillate about this angle with a damped motion because of the loss of energy through radiation. It may be assumed, therefore, that the motion of the electron away from the ion may be expressed by the same equations, (17), (18), and (19) of Chapter VIII as used before.

The same general conclusions may, therefore, be applied, that the total energy radiated in one excursion out and back is equal to one quantum, $h\nu$. Since the frequency of the X-rays is higher than that of the light before considered, so also is the quantum of energy proportionally greater, and we have just seen why it must require more energy on the part of the passing electron to produce the effect.

The second general conclusion is that the electron goes out to a maximum distance from the ion expressed by (21) of Chapter VIII, namely, the 0.0513 part of the wave-length. Since the wave-lengths are less for the X-rays so also are the

maximum distances to which the electron goes in direct proportion.

This affords a qualitative test of the above ideas, because, if the distances so computed were very great as compared with the distances to the nearest atoms in the metal, there should occur obstacles to the motion that must result in disturbing the path of the ejected electrons, and especially so if adjacent atoms chance to be sending out radiating electrons simultaneously towards each other.

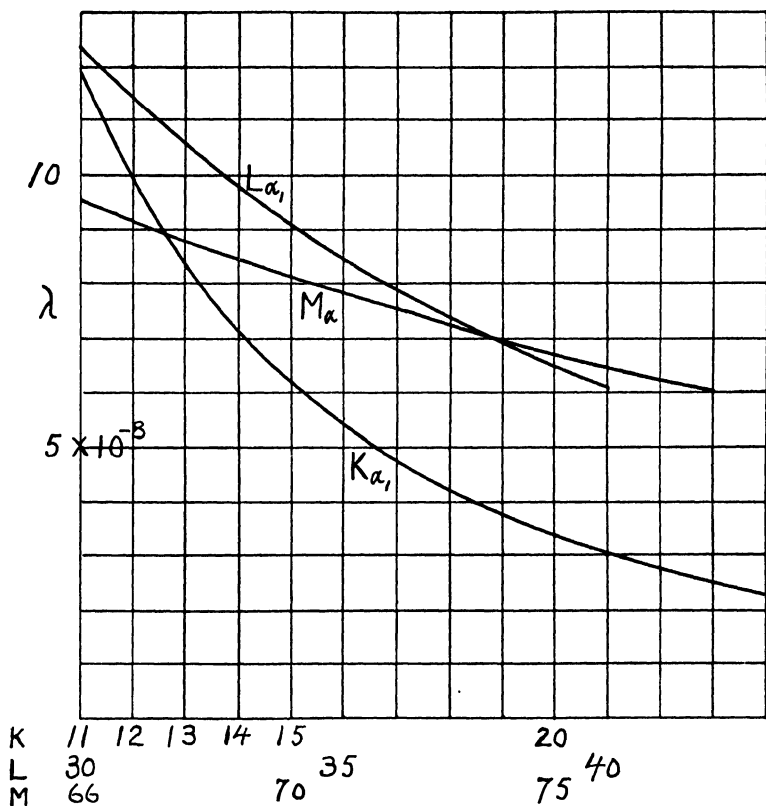
The interest centres, therefore, in the longest X-ray wave-lengths that have been observed. These are charted in fig. 29, from data taken from a paper by Duane. The element of lowest atomic number for which the emission X-ray spectrum has been partially obtained is sodium, atomic number 11, and the longest emission wave for it is 11.88×10^{-8} cm., the K_{α_1} line. The curve marked K_{α_1} gives these wave-lengths in the K-series for increasing atomic numbers. They go on decreasing until $N=30$ (zinc), when the L-series first makes its appearance, the longest wave-length for zinc being 12.346×10^{-8} cm., about the same as the longest wave-length in the K-series for sodium.

These are shown by the curve marked L_{α_1} and decrease in the same manner, though not so rapidly from element to element as the K-series. The L-series gives the longest waves recorded until we come to dysprosium ($N=66$), when the M-series is first observed, the longest wave-length being 9.509×10^{-8} cm., shown in the curve marked M_{α} . The waves of this series decrease from element to element but at a rate still slower than for the L-series.

If these wave-lengths are each multiplied by the same factor, 0.0513, the results give the maximum distances to which the electrons go from the ion in each case. In the K-series for Na, Mg, and Al these distances are respectively 0.61, 0.506, and 0.427 times 10^{-8} cm. In the case of Al the elementary cube edge, as above stated, is known to be

4.07×10^{-8} cm., and the side of the elementary equilateral triangle of certain planes in the lattice is 2.88×10^{-8} cm. Consequently in Al the electron that goes out the farthest covers but 14.8 per cent. of the distance to the nearest neighbour.

FIG. 29.



Observed X-ray wave-lengths as ordinates plotted against the atomic numbers as abscissæ, in the K, L, and M series.

As the atomic number increases these maximum distances decrease until for copper ($N=29$) the longest observed wave-length is the K_{α_2} line, 1.543×10^{-8} cm., corresponding to a maximum distance of 0.0729×10^{-8} cm. The cube edge of the

copper lattice is 3.61×10^{-8} cm. and the elementary equilateral triangle 2.55×10^{-8} cm. Consequently the maximum distance is but 3.1 per cent. of the distance between the nearest neighbours of the lattice.

The L-series begins with the next number ($N=30$) zinc, as far as yet observed, and the maximum distance now jumps up to 0.663×10^{-8} cm., which is very close to the value for sodium in the K-series. These distances decrease again with increasing atomic number until the M-series begins at dysprosium ($N=66$), when the maximum distance again jumps up to 0.488×10^{-8} cm.

The space lattice distances have not been determined for dysprosium so far as the writer is aware, but the lattice distances do not depend intimately upon the atomic number. The actual distances to which the electrons go, therefore, enable one to estimate roughly the proportional distance to the nearest neighbour with sufficient approximation.

The greatest distance computed from any observed wavelength does not exceed about one quarter of the distance to the next neighbour in the lattice, that for zinc in the L-series being the greatest of any line recorded. The maximum distance falls off most rapidly for the K-series from Na to Mg and Al. When the actual distances approach, say, one quarter of the distance between adjacent atoms, which may both be sending out electrons simultaneously, it may be inferred that their mutual influence upon each other will produce some disturbance, and it may be anticipated that in elements of still lower atomic number than 11 this disturbance rapidly becomes more pronounced and the spectrum possibly more complicated, other lines making their appearance, there being more general radiation, and a departure from the uniform characteristics exhibited by the heavier elements.

If the remark made by Moseley in his original paper concerning the elements sodium and magnesium, whose spectra he did not give, is to be trusted to the same extent as the rest.

of his work now acknowledged universally, there is an indication that this effect has already set in with sodium and magnesium. He says: "The very light elements give several other fainter lines, which have not yet been fully investigated, while the results for Mg and Na are quite complicated, and apparently depart from the simple relations which connect the spectra of the other elements."

This theory thus attaches considerable interest to the experimental determination of the characteristic emission for the elements of lower atomic number than have yet been measured. It is recognized that the chief difficulty in making such measurements is the fact that the wave-lengths are so long that there are no crystals available that can resolve the lines. It requires a grating with a greater distance between rulings.

The theory above outlined is also in harmony with the experimental results of Bragg and others as to the scattering of the electrons of the original cathode stream in their passage through sheets of metal. It is observed that there is no increase in the number of electrons in the original stream, but merely their direction of motion is changed and their velocities somewhat reduced, never increased by their passage through the sheet. On this point W. H. Bragg* says: "All these secondary radiations may well be electrons of the original cathode stream diverted by the atoms upon which they have fallen." According to the present theory the atoms of the metal are not ionized by losing electrons permanently, because the electron merely goes out to a certain maximum distance and then returns again to its original place within the atom.

* 'Studies in Radioactivity,' p. 80.

CHAPTER XI.

LINE-SPECTRA.

IN the preceding chapters we have considered the mechanism of the process by which one characteristic line of the light- and of the X-ray spectra may be produced by such atoms as described in Chapter II. The radiation is first traceable back to the character of the straight-line motion of the ejected electron out and back, and second, in the case of light to the oscillations of the pole of the positive part of the atoms, and in the case of X-rays to the oscillation of the pole of the negative electron itself, each body being under the influence of the other body.

The origin of the characteristic frequencies is thus traceable ultimately to the little-known properties of these fundamental bodies, the positive and negative electricity. It should be emphasized that each of these ultimate units of charge is a rotating body possessing an axis. In general, such a system of bodies cannot be disturbed without producing a couple tending to alter the directions of the axes of rotation. In plain words, the conclusion has been reached that even the simplest atom, hydrogen, is no simpler in its behaviour than a system of three gyroscopic bodies would be, and no doubt not as simple because of the semi-liquid or solid character of the charges, which are subject to deformation. It is very evident that, granting the atoms are bodies built up as suggested in fig. 2, the problem of unravelling a complete explanation of line-spectra in terms of the models is one of the most recondite questions possible. One might wish that this were not so, and possibly hopes have been entertained that a new

conception of an atomic model would offer an easy road toward the understanding of spectrum lines. The fact that this is not so and that the problem appears even more complex than formerly is no argument at all against the form of these models. More labour and time of investigation on the part of experimentalists has been devoted to line-spectra than possibly to any other one subject in physics, and the meagre results to show for it in correlating these results theoretically really demonstrates the great complexity of the problem.

The greatest strides in this direction have been made by an application of the so-called quantum theory to the subject, which, as regards spectra, has now become identified with the Bohr theory. It has not been possible or advisable to devote much space in this volume to any discussion of this important theory, which has certainly put an entering wedge into this most obscure subject. A condensed statement of the Bohr theory as it is now understood may be given as follows, or rather the two hypotheses which underlie the theory. There are recognized to be n different states of an atom, and the difference between the energies of any two states may be expressed by the frequency condition $\epsilon_m - \epsilon_n = h\nu$. The Bohr theory began by having a definite reference to a special form of atomic model, but the two great generalizations which have emerged from it and now form the real basis of Bohr's theory are really detached from any form of model. In other words, any form of model which admits of these two conditions may represent the truth just as well as the model originally proposed by Bohr. The valuable results already obtained by the use of this generalized Bohr theory need not, therefore, be abandoned because they may have been construed to apply to a definite model. If another model admits of the two general hypotheses this is sufficient to justify an adaptation of the theory to it.

The most that can be expected at present in explaining the existence of series spectra by means of the models above described, is to show that the classical dynamical theory seems

adequate to cope with these questions when sufficient data are available and the power of analysis is sufficiently developed. It is gratifying under these circumstances to be able to show with some probability that the ordinary equation of series spectra in the case of hydrogen is derivable by means of the classical theory from just such an atomic model as described. This was first pointed out by E. T. Whittaker in 1911, in connection with another conception of the source of the series spectra. His reasoning is very general, however, and the suggestion assumes a renewed interest because it is equally applicable to this form of model. He postulated that there must exist a gyroscopic connection between the two separate atoms of a hydrogen molecule, and derived the equation of the Balmer series from this idea by the use of the classical dynamics. This explanation of the series spectra, however, attributed it to the molecule rather than to the atom to which it is now admitted to belong. The same treatment may, however, be applied with equal force to the gyroscopic connection that has been pointed out in preceding chapters between the ejected electron and the remainder of the atom from which it came.

His demonstration is short and seems worth giving in full. If u and v are some scalar functions of both time and position of two elements, the one in the electron and the other in the ion, such that the potential energy of the element of the electron is represented by

$$\frac{1}{2}au^2 + \frac{1}{2}b\left(\frac{\partial u}{\partial \theta}\right)^2, \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

and the potential energy of the element of the ion by

$$\frac{1}{2}av^2 + \frac{1}{2}b\left(\frac{\partial v}{\partial \theta}\right)^2, \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

where a and b are constants and θ is a position angle referred to some fixed point as an origin; then, if it is legitimate to

assume that the gyroscopic connection between the two bodies may give rise to a term in the kinetic potential of the type

$$\frac{\partial u}{\partial \theta} \cdot \frac{\partial^2 v}{\partial t \partial \theta}, \quad \dots \quad (3)$$

the equations of motion take the form

$$au - b \frac{\partial^2 u}{\partial \theta^2} - \frac{\partial^3 v}{\partial t \partial \theta^2} = 0 \quad \dots \quad (4)$$

and

$$av - b \frac{\partial^2 v}{\partial \theta^2} + \frac{\partial^3 u}{\partial t \partial \theta^2} = 0. \quad \dots \quad (5)$$

If, now, the functions u and v are assumed to be simple periodic functions both of time and of position, an assumption which is justified by the models, namely

$$u = A \sin (nt + m\theta), \quad \dots \quad (6)$$

$$v = B \cos (nt + m\theta), \quad \dots \quad (7)$$

where m denotes any integer and n is the angular velocity, the solution of (4) and (5) gives

$$\left. \begin{aligned} (a + m^2 b)A &= m^2 n B \\ (a + m^2 b)B &= m^2 n A \end{aligned} \right\} \quad \dots \quad (8)$$

From these two equations A and B may be eliminated, giving

$$\pm n = b + a/m^2. \quad \dots \quad (9)$$

Since m is an integer this equation has the form of the hydrogen series spectra usually written

$$\nu = A - \frac{N}{r^2}. \quad \dots \quad (10)$$

This example has been cited to show that it is not beyond the power of the classical dynamics to show that such a system of rotating charges as the model hydrogen atom contains, may give rise to the peculiar series that is of very frequent occurrence in the spectra of the elements.

Assuming that this demonstration is applicable to the atoms shown in fig. 2, it is seen that there exists a very large number of states of the atom corresponding to the integer, m , 1, 2, 3, etc. This is one of the two fundamental hypotheses underlying the Bohr theory, which, therefore, applies with equal reason to this form of the atoms.

CHAPTER XII.

AVAILABLE AND UNAVAILABLE INTERATOMIC ENERGY.

IN the preceding chapters a fairly comprehensive range of phenomena has been studied with the help of the new atomic models, though it is by no means an all-inclusive one. It embraces phenomena that fall under two broad classifications: first, "the steady states," when the atoms neither radiate nor absorb energy, as in Chapters III to VI, which treat of the subjects of gravitation and of the equilibrium positions of the atoms in molecules; and, second, "the active states," when there occur both radiation and absorption of energy, as in the remaining chapters that touch upon both light- and X-radiation. In reviewing these chapters, the thing that perhaps makes the greatest impression is the conspicuous part played by the very great energy content of the electrons, great, that is, in comparison with the small amounts of energy that they give up or receive back again in any of their activities. There was no success in forming a molecule with two hydrogen atoms as long as the atoms were assumed to be symmetrical and the electrons all exactly alike. The individual electrons temporarily give up or receive back several times as much energy as is used in forming the molecule. And, similarly, in the process of light- or of X-radiation the electrons temporarily give up and receive back again several times as much energy as the quantum of radiated energy. These great reservoirs of energy are tapped to a certain extent in every operation or change of state. But the amount of the energy they thus give up or receive back again is extremely small in comparison with their total energy contents.

It has been recognized for some time that there exist great stores of energy in all ordinary masses of matter, great in

comparison with the very small amount of energy that man has yet been able to extract and make available for his practical needs in supplying power for commerce and the arts. This belief, that the energy is there and that such a very minute portion of it has as yet been turned to commercial use, is the inspiration impelling scientists and inventors throughout the world to try to increase the small proportion of the whole which is now available energy.

It is our belief that the energy content of any piece of matter may be expressed as the product of its mass by the square of the velocity of light. Consequently, for one gram of matter of whatever kind the energy content is equal to the square of the velocity of light, that is, approximately 9×10^{20} ergs. This is an immense amount of energy equivalent to that required to supply one kilowatt continuously day and night for a period of no less than 2852 years' time. The heat produced by the complete combustion of one gram of the very best grade of coal is about 8500 small calories, equivalent to 3.554×10^{11} ergs. This is more than two and a half billion times smaller than the energy, 9×10^{20} ergs, that expresses the total energy content of the gram of coal or of any other substance. To state it in other words, the complete combustion of the gram of coal will supply energy at the rate of one kilowatt for only about thirty-five and a half seconds of time instead of for a period of 2852 years' duration.

One reason for mentioning the matter here is that there evidently exists a rather general misunderstanding on the part of inventors and many others as to what goal of attainment may reasonably be expected. This misunderstanding may be traced to a misuse of the words "interatomic energy." To be explicit there should be a clear distinction drawn between the three phrases, intermolecular energy, interatomic energy, and interelectronic energy. In one sense the first of these might be interpreted to include both of the others, because the atom is within the molecule and the electron within the atom, but this is not the common and preferred usage. When one

speaks of intermolecular energy he has in mind those energies required to cause the atoms to change partners so to speak as between different molecules, represented by the heats of combustion and heats of dissociation, of the order, say, of 10^{-12} erg per atom. This process takes place without altering in an appreciable degree the atoms themselves; they continue to retain the same number of positive charges as they move about from place to place whether forming a part of one molecule or of another.

And in a similar manner the phrase "interatomic energy" might be interpreted to include interelectronic energy as well, because the electron is within the atom, but this should not be the common and preferred usage. When one speaks of interatomic energy he should have in mind those energies required to cause the electrons to change partners, so to speak, as between different atoms represented by such energies as those of the ionization of atoms, that is to say, the energy to pull one or more electrons off from an atom or to cause them to unite with an atom that has already lost electrons, energies between ten and one hundred, or perhaps a thousand, times 10^{-12} erg per atom. The energy to pull one electron off from the hydrogen atom, for example, is about 35×10^{-12} erg. To pull the second electron off certainly requires several times this amount. Just as in the former example of intermolecular energy where the atoms were undisturbed, so here in the example of interatomic energy the individual charges or electrons both positive and negative pass through the change without alteration in an appreciable degree.

Now intermolecular energy is not only available energy, but it is also that which has already been in practical use in the combustion of coal and other fuels. On the other hand, interatomic energy has not been obtained as yet in such a way that any practical use has been made of it in commerce and the arts, but it, too, must be regarded as available energy in the sense now employed. Some of it, on a very small scale to be sure, has already been of great use in physical and chemical

research, and no one is bold enough to try to set any limits for future attainment in extending the proportion for commercial use. As already pointed out, the probability is that this energy per atom may be as much as one hundred or more times the energies of combustion or of the dissociation of molecules, but it certainly will never amount to a hundred thousand times as much, at least until the electrons themselves are made to give up their entire energy contents and dissipate themselves. For this there has as yet been no possible suggestion of encouragement on either the experimental or the theoretical side. That is to say, the interelectronic energy is by far the greatest of the three classes of energies above mentioned, and this, as a whole, may for the present be considered as unavailable energy in distinction to the first two classes that are both available.

The confusion that has arisen is primarily due to a misuse of the words interatomic energy. It is often erroneously used to include the unavailable interelectronic part of the energy which is millions of times greater per atom than intermolecular energy. This is misleading, because it includes both the available and the unavailable parts, and may in certain quarters foster an unwarranted expectation of realizing this large store as available energy for practical commercial use. When our technique has advanced to such a stage that interatomic energy employed in the sense above mentioned is realised for practical commercial uses, then at the most we may expect to multiply the present sources of energy per atom by a factor of the order of 100 or 1000 perhaps, but not by 1,000,000 or even 100,000, which is sometimes implied by a thoughtless use of the words interatomic energy.

