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PRINCIPLES OF PSYCHOLOGY

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PSYCHOLOGY: A New System
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PURPOSE AND EVOLUTION
THE IMMORTAL CARAVEL
UNE QUESTION DE REPRÉSENTATION GÉOMÉTRIQUE

PRINCIPLES
OF
PSYCHOLOGY

THE FOUNDATION WORK OF THE ALÉTHEIAN SYSTEM OF PHILOSOPHY

BY

ARTHUR LYNCH



LONDON
G. BELL AND SONS, LTD.
1923

PRINTED IN GREAT BRITAIN BY PURNELL AND SONS
PAULTON, SOMERSET, ENGLAND

PREFACE

Of the great tasks within the compass of man one of the most fascinating is surely that of tracing out the course of thought, so that this may be seen from the first glimmering of intelligence to the complex web of knowledge and reasoning that form our comprehension of the world.

Since all this is contained in our thought, the suggestion arises as to whether the thought be not capable of exhibiting its content lucidly in the modes of its experiences and processes of combination. The problem of discovering these processes and laying bare the entire mechanism might well appear so vast, and at the same time so tenuous and elusive, as to be impossible, unless indeed by favour of new methods of conducting the attack and the discovery of new forms of analysis. It is the object of the present work to show the investigations that furnish the solution.

Briefly the pith of the book is the elucidation of the manner in which all complexes of thought may be shown to be built up by syntheses of certain Fundamental Processes, which are in the psychic world the analogues of the chemical elements in the physical world. Once the central problem solved, the main position secured, I have shown a series of developments in which various great problems of psychology are dealt with in new illumination; and the work will have missed something of its purpose unless it be found to give to the reader the sense of standing at a central point of command from which he sees the development of the structural forms of thought, whether it be in the ordinary operations of the mind, or in the domain of the sciences, or in the applications wherein he may find the vast mechanical products of civilisation springing as corollaries from the delicate filaments of the thinker's mind.

All work of severe intellectual effort requires behind it a driving force of emotional interest, and this book owes its existence to

meditations that arose in ethical needs. The proper study of ethics involves as great a knowledge of the objective world as it is possible for us to acquire, and also the consideration of the relations of the individual to that world and to other beings. This implies a study of the sciences, biological, physical, and at length mental, and we are led then to establish the basis in the laws of thought, and to discover in what manner the rays of our Psychology interpenetrate all the other domains. Psychology is the matrix of all the sciences.

If I speak with enthusiasm, and if I desire to infuse especially into fresh, young, capable minds a sense of the fascination of these studies, it is not for the sake of that feeling of exaltation in itself, but chiefly to supply a motive strong enough to carry the student through the tedious passages of the patient explicative work involved. In the exposition I have especially sought for lucidity, and therefore I have not hesitated at occasional repetitions in order to make it clear that the course of argumentation begins from a deep base and proceeds by rigorous reasonings step by step to the conclusions reached. I have kept before me the example of the exact sciences, and I have insisted on the need of closely consecutive links in the exposition, otherwise we are once again in the region of shimmering but unfertile disquisitions; but, having sufficiently shown the method in the earlier part, I have in later developments taken longer steps in order to cover more rapidly an extended scope.

The pompous language which in philosophy is too often used to cover vague ideas has been avoided, and technical terms have been used as sparingly as possible, so that intelligent readers, not hitherto acquainted with Psychology, may begin at once with this volume; but even for the most ardent student the way will be found indicated to new developments.

What I have desired to demonstrate is that without the key of the Fundamental Processes no searching will lead to a veritable understanding of the subject; with this key the passage to the secrets of all problems is opened.

Two of these in especial are likely, I believe, to prove important as they have entered again and again into the question of the principles which have served to build the work. The first is that

of using this system as an instrument, of forming by its means athletes of intellect; the second, not disconnected, for the whole web of these thoughts will be found woven in an organic whole together, is that of eventually establishing the guiding lines of a logic of Research.

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NOTE:—The Table of Contents should be consulted during the reading, so that the consecutiveness of the whole argument may be kept in view. This is advisable because when the character of the reasoning has been rendered clear, larger steps in the onward progress are taken, though without sacrifice of cogency.

The general march of the exposition is as follows: After the Preface and the Introduction, Part I contains the enunciation of the central problem, and the indication of the terms of the solution by virtue of the Fundamental Processes. Part II enters into a demonstration of the manner in which we can obtain a schema (the meaning of which is explained), of the whole field of thought. This is co-extensive with the operations of mathematics. It is shown that by the adjunction of this schema to "sensile" experiences in the widest scope, the whole world of thought may be covered. Finally, we see completed the demonstration that the combinations of the Fundamental Processes serve to explain all modes of psychic acts.

Part III deals, in regard to these Fundamental Processes, with amplifications that clothe the skeleton forms with life, and relate them to the worlds of active thoughts that exist around us. Incidentally, some of the most remarkable problems that have baffled the mind of man for centuries are here illuminated with a new light.

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PRINCIPLES OF PSYCHOLOGY

INTRODUCTION

NOTE: In this chapter the germinal idea of the work is shown and the course of the reasoning indicated in skeleton form. In the first reading the full significance of this chapter may not be grasped. It will seem more vivid when the further exposition has been studied.

Psychology is the "Queen of the Sciences" in a sense more intimate and vital than that which induced Gauss to bestow the title on mathematics. It is not alone pre-eminent in its interest and fascination, but properly understood it controls all the other sciences; it forms the matrix from which they all proceed.

This will become clear in the course of the present work, though it is but an incidental result, for the main object of the book is to lay bare in ultimate analysis the nature of Reason and the foundation of Belief.

Preliminary Survey

To yield a formal assent to a demonstration is one thing; to comprehend a principle and feel in a more intuitive way the force of a proof is another. In this book, therefore, I desire to enter into closer relationship with the reader; I propose to show the *germinal idea* of the work, and to indicate the way in which I have been led to the modes of solution given.[1]

When early aspirations precipitated into more definite conceptions it was the notion of Guidance that first emerged; then with the rejection of lesser authority, the desire to seek in the conditions

[1] Clerk Maxwell reproached Ampère with having kicked down the ladder by which he had ascended to his proofs of the laws of the attraction and repulsion of electric currents. If then in an objective science it be advantageous to know the germinal idea, how much the more in questions of philosophy where the personal equation enters more into play

of the Universe itself, and of our world in particular, the ultimate sanctions of human conduct. That indeed is the problem of the *Data of Ethics.*

Then observing that the works that treated of Ethics were often based on assumptions that may not have been justified, and supported by unconvincing arguments, I asked myself: What then in complete analysis is the meaning of Reason, and what in the ultimate resort is the sanction of Belief?

In the meantime I had read the usual courses of philosophy, especially from Locke downward and had been deeply impressed by Herbert Spencer and pleased with Bain.

Then came Kant. Like a maiden to her bridegroom so went my mind to the *Kritik der Reinen Vernunft.* Never did I open a book with higher expectations. Never did I close a book with disappointment so blank. I had expected to find a mind of extraordinary subtlety and power surveying nature with comprehensive gaze, and winning out by the delicate instrument of thought new principles for the guidance of human life; I found a confused exposition, prepossessions, prejudices, the fumes of old theological disputes, the mists of mediæval metaphysics, assumptions in place of proofs, and no clear outcome in doctrines fertile of new visions and new developments. The Categories had not been obtained by any decisive analysis; they had been adapted from conventional books of logic.

Nevertheless, all this had a bearing on the present work. I began to see that a true Psychology might be worked out, not by metaphysical conceptions of the prerogative of Pure Reason, but by determined examination of the *processes* of normal reason pressed to the last resort of analysis. The Categories seemed to me both incomplete and redundant: the true method would be found in the establishment of processes of the mind which were both fundamental and inevitable.

From Kant to Herbert Spencer: I had acquired and retained a deep respect for the English philosopher, especially in regard to method, generalising power, and acuteness of philosophic vision; but I found that his Principles of Psychology left unanswered questions which appeared essential.[1]

In the meantime I had been applying my mind and exercising my analysis on various philosophical positions, such as that of Externality. The chapter on Externality in the present work is in the main part that which I had written as a young student in Melbourne.

[1] Since then, I have elsewhere submitted the foundation of Spencer's whole system, the principle of Evolution as enunciated by him, to a critical examination, and have given reasons for not admitting it as a scientific principle. *Cf. Ethics.*

Then, acting on a suggestion in the *Autobiography* of John Stuart Mill, where he said that at a certain stage of his *Logic*, he saw the advisability of taking a survey of the sciences, I resolved to look into this matter still more closely and to work through the chief subjects, in mechanical, biological, and mental sciences.

The advantage in so far as my Psychology was concerned was that it allowed me to see more clearly the fields of application of the system, and also that I was better able to estimate the various instruments of psychological research—those of Experimental Psychology, physiology, biology in the larger sense, studies of abnormal or pathological conditions of the mind, the more special indications of neurology.

* * * *

The Germinal Idea.

The study of different sciences carried on in some instances simultaneously suggested comparison of methods. Similarities in the mental processes became apparent, and especially when the processes themselves were generalised and freed as far as possible from the special concrete forms that distinguished one branch of science from another. At the same time I had cultivated a habit of analysis, consisting in the reduction of complex forms of thoughts into simpler factors, which again were subjected to a similar process.

The problem then became posed as to whether it were possible to carry on such processes of analysis until factors were obtained such as marked the limit of our powers of analysis.[1]

With these elements we could build again all the complexes from which they had been derived.

[1] Maudsley once reproached me with having resorted to "that old business of introspection". In his capacity of physiologist and alienist he had thrown out of perspective the importance of his special studies. And so with the professor of biology, the exponent of experimental psychology, the philosopher who devotes himself to some special study in the domain whether of Will, or Memory, or Attention, or Association; each magnifies his office and fails to see his own work in its proper relation to others. Not one of the instruments indicated should be neglected; but in the course of the exposition it will become clear that the physiologist who is deficient in aptitude for psychology produces absurdities in interpreting his own observation. To prove this it is only necessary to show how eminent physiologists reach opposite conclusions from the same facts. The mere bare facts from any of the subsidiary sciences carry us but a little distance in Psychology. In the interpretation there come into play new faculties, and when one enters the field which calls for clearness of vision, classification and co-ordination of results, the confirmed habit of generalisation in the scientific sense, and a combination of delicacy of perception with wide scope of comprehension; then Psychology is at work. The work of the despised faculty of introspection is not something idle or futile. There is no kind of effort in the whole range of thought that makes so trying a demand on the faculties. I have had the suggestion as if in concentrated thought I were setting up some extremely delicate and sensitive instrument which required a peculiar setting; and on occasions when this has been interrupted I have felt—so great was the strain—as if a shock had been given that sent quivering through all its structure a vast and delicate network of fibres.

Then came the further suggestions as to whether it were possible to obtain all these elements, and to demonstrate that by their combination all modes of thought could be represented up to the most complex expressions or recondite forms of which the human mind is capable. This problem and its solution, constitute the essential part of the present work.

Before indicating the suggestions that finally enabled me to grapple with this problem it may be of interest to refer to others of the numerous paths by which I was led to it.

Having practised various sports and gymnastics and having been struck by the efficiency gained by the right exercise of strength, I analysed as far as I could the movements involved in certain of these exercises.

It occurred to me, therefore, that the mind being an instrument more subtle and of wider range than the physique, it would be of enormous advantage to anatomise the mental constitution, so that its weak parts could be improved, and that the right combination of its functions would produce powers of the highest obtainable value. I thought it possible, to express the matter pithily, to produce a great thinking athlete and to devise generally a system of mental athleticism.[1]

Then again the analysis of the sciences themselves suggested to me that just as the chemical elements served to build up the immensely diverse objects of nature, so a comparatively few mental elements might serve to explain all mental syntheses.

The search for the complete meaning of Reason, and for the foundation of Belief, also led toward the same base.

I became then convinced that the problem of which I was in quest was that which would give me the key not only to the problems which had led to it but veritably to all problems in the mental world; that in fact this problem was the cardinal position, the central problem of Psychology.[2]

I became assured that if I could solve the questions which appeared before me I would have a sort of instrument of illumination which would reveal the structure of all complex forms and all obscure modes of thought in all fields.

* * * *

[1] This notion, worked out in accordance with a curriculum so chosen that the subjects taught would themselves supply the training, would constitute a science of education and provide an education on scientific lines.

[2] The formal statement of the general problem is given at p. 16.

The Methods.

I therefore resolved to hold this problem so persistently before my eyes that every thought in my mind would be, not remotely, related to it.

My studies in objective sciences offered me suggestions. In Engineering, in order to understand the functioning of a machine it is necessary to trace step by step the sequence of the operations, and the clue is not found till the finest wheelwork has been examined. Again, in biological work one finds a new world revealed by the microscope. I asked, therefore, whether in the mental world anything corresponded to this sort of minute study of function. This suggested the scrutiny of the most rudimentary mental processes. Hence I was led to the study of the axioms and postulates of geometry, the examination of the process of counting, and the patient observation of the forms of association most nearly automatic.

At the same time I paid particular attention to the examples of reasoning in which the steps are most closely and most inevitably linked, as in mathematics.

Side by side with this I considered a question to which many answers, all illusory, had been given, by celebrated thinkers, namely, as to how many objects the mind may hold in attention at one time. This study led me to the Concept of Unity.

All my studies led to the same foundation.

My analysis of mathematical operations indicated that the base of all the processes was counting.[1] Then I found myself led to the determination that at a given instant of time the mind could entertain only one subject of thought and its association with another, expressed in no matter what form of relation, this other appearing as the first receded.

All these problems were inter-related and the clue to one proved to be the clue to all, for having solved the question of the concept of Unity, and having found how this led me to the true meaning of

[1] A few words of explanation will make this clearer. When one is possessed of the meaning of counting, the process of addition is seen to follow from this, and another process is brought into evidence, that of symbolisation.

Subtraction is the inverse of addition. Here the Fundamental Process of what I have called Negation, of which the full significance will be found later, comes into relief. Multiplication is founded on addition just as addition is founded on counting. Division is the inverse of multiplication.

We continue this line of argument till we arrive at processes which at first seem to belong to another realm, such as those of the differential calculus and the integral calculus.

Integration is merely the summation of an infinite number of quantities exhibited under a general symbol. The only new processes that we will have met with are those involved in the use of Infinity as a mathematical form. In another domain of mathematics we have also the introduction of Imaginaries. These new terms I have studied in such a manner as to dispel their mystery and to bring them within the scope of the system. *Cf.* Part II. Chapter XII.

counting, I discovered the link associating in an organic manner two fields of study which had at first seemed separated.

The most difficult part of the solution of the central problem was that which was requisite to assure me that the Fundamental Processes, when finally determined, should be necessary and sufficient—to use the precise mathematic expression—to cover the entire scope of thought.

What was lacking in the system of Aristotle, and also of Kant, was a principle of division which should indicate whether or not the predicaments or the categories were exhaustive, that is to say, sufficient to the explanation of every act or combination of acts of thought.

My own principle may be now indicated: The analysis of the processes of mathematics shows that they all rest on counting, and that at each stage of development, throughout their whole extent and complexity, we can survey the manner of combinations of processes so as to be sure that no new mystery enters. Then by the study of the concept of the Unit we are assured that all our Immediate Presentations enter into the mind successively as units. We may make abstraction of the actual graphic form of such Presentations, so as to consider only the schema of successive units.[1]

The required demonstrations are found interpenetrating the whole body of the exposition here given, but particularly in the chapters on Counting and on the Concept of the Unit. The skeleton of the argument may be thus exhibited: The mind entertains, at one moment, a Unit, however presented. Counting is simply a process of proceeding from unit to unit with a reference at each unit to some series established, as, for example, by repetition, in the sequence of its associations. The actual Unit at any time, endowed in the graphic form or attributes derived from the senses[2] appears in consciousness as the Immediate Presentation at that

[1] As the conception of a schema, or "schematic forms", underlying actual modes of thought is frequently alluded to in the book a few words are here inserted to make the matter clear. The act of looking at a disc painted red is not the same as that of looking at a disc painted green. The difference consists in that of the sensations of the colours. Disregarding the colours we have a schema of the act. The schema in one case is similar to that of the other. The principle remains unaltered when we proceed to complexes of colours, and thence to any complexes of graphic forms.

If these explanations appear too simple then I beg to offer another by way of more developed illustration: There is a proposition in mathematics expressed as a differential equation, which is of the same form whether the subject studied be heat, or light, or electricity, or potential. The formulation of the proposition, representing in each case in symbolic form the sequence of the phenomena, constitutes a schema.

It may now be seen that all mental processes can be regarded as a sort of schema, or woof of the mental activities, on which the actual experiences are inset in their appropriate forms, as of sensations whether of colours or shapes, somewhat as a pattern is fitted into the underlying complex surface which supports it.

[2] Senses being understood with the widest meaning.

instant of attention. At each progressive step we may observe the manner of its Association with the following Immediate Presentation. Then, in conformity with our entire physical and mental constitution, we become aware, at a certain limit of discrimination, either of Agreement or difference. This reference implies Memory, and also Generalisation. The progress is determined by a process of Impulse. All the actual perceptions involved in these processes are accompanied by a Sense of Effort, and also at a certain level by an Hedonic Sense. All these processes are represented by a positive or a negative side, so that we must include the process of Negation. All are immersed in the conditions of our actual existence in Time and Space.

The Fundamental Processes may now be set forth as follows :—

(1)	Immediate Presentation.	(7)	Impulse.
(2)	The Conception of Unit.	(8)	Sense of Effort.
(3)	Association.	(9)	Hedonic Sense.
(4)	Memory.	(10)	Negation.
(5)	Conception of Agreement.	(11)	Conception of Space.
(6)	Generalisation.	(12)	Conception of Time.

I have spoken of the Unit endowed with its graphic or other sensile[1] forms. I show that our process of counting (which is itself not Fundamental, for it can be analysed) is possible because we are able to make abstractions of the sensile qualities, and to consider the Unit and the act of proceeding to the next Unit, in a schematic manner. The actual series of the Immediate Presentations may be taken as the combination of the schematic form with, added to it or interfused with it, the sensile attributes.

That being so, I seek for the sources of the sensile attributes on the one hand ; on the other I examine the full scope of the schematic forms apart from these attributes, and I find that scope co-extensive with that of the operations of mathematics.

If, therefore, I examine all the processes possible within the scope of the schematic forms, and all the processes involved in giving life and particularity to these schematic forms, then evidently the processes we so find to be necessary and sufficient will cover the whole world of thought, and that not merely actual but possible.

If then we submit the Fundamental Processes to such an examination that we become familiar with their nature, and if it be made evident that they represent the limits of our analysis, then, since we have shown that all modes or progressions of thought

[1] I use this word to indicate—derived from the operation of the senses. I take the whole range of the sources of entry into the psychic life as that of the scope of sensation.

are explained by their combination, it will be clear that we have solved the problem of the nature of Reason as well as that of the foundation of belief. This I call the central problem of normal Psychology.[1]

The discovery and the demonstration alike having hung together on the results of the forms of analysis that I have indicated, I am finally able to prove also that it was impossible to furnish the demonstration except, however the thinker may have been led to it, by traversing this route.

* * * *

Re-examination of Old Positions.

Having arrived at this point I had completed the essential part of my task. For a long time, however, I delayed the writing of my book because of uneasiness with regard to one in particular of the great philosophical positions I had essayed to bring within the scope of my system. This was Externality.[2]

The analysis which I had accomplished as a student remained valid, so far as it went, except that something seemed to be required for the final leap to the full recognition of objectivity.

What really was faulty was not so much a deficiency in the setting forth of the factors as the failure to throw these into their true perspective so that the crux of the problem and the method of dealing with it became perfectly clear.

I had not in the earlier analysis given enough weight to the effect of Disassociation together with Association, nor to those factors, whether of heredity or education, that tend to produce a sort of momentum, formed of innumerable subsidiary impulses, towards the acceptance of a mode of vision.

Then there was something which in the movement of rejection of Berkeley's paradox had not been properly brought to light. I found it in the fact that he had been led astray not by his idealism, but by the tacit assumption of forms of materialism, such as in

[1] It will also become clear in the course of the exposition that all forms of abnorma modes and progressions of thought—such as are observed in the study of lunacy—will be rendered more intelligible when approached in the manner I have indicated. It will also be made manifest that the claims are untenable of the specialists who assert that observations of such abnormalities constitute the whole or the bulk of the studies necessary for the comprehension of Psychology.

Once the central position is gained a clearer view will be afforded of various studies, auxiliary in this domain to Psychology, and the true perspective will be gained of the aids which various branches of biological science afford.

[2] This Introduction will here have a more vivid meaning when the entire exposition has been studied, and in regard to Externality particularly when the chapter on Externality has been read. After reaching the end of the book the student is invited to return to the Preface and Introduction.

the Middle Ages were associated with a high strained and impossible spirituality. This materialism consisted in the acceptance of localisation. The mind cannot be localised. All that we can say is that certain physical dispositions, particularly of the brain, are the correlatives of mental states. Where is our idea of Sirius? I answer, precisely where Sirius is, in our idea. This is not a verbal reply, in the style of the old schoolmen, to a paradox; it is the true and complete answer to the question.[1]

I was now in a position to begin the exposition of the system. I still delayed, however, until in the light of the Fundamental Processes, I had completed certain other analyses destined to show the fecundity of the principles. Of those questions the most important were: The Sense of Effort, The Ego, Will, Localisation of Functions, Fechner's Law, Relativity, Infinity, Memory, Dreams. Also I examined in association with psychological and physical facts the series of the Fundamental Processes.

* * * *

Localisation.

In dealing with the question of localisation of functions I offered proof, now condensed into a note, of the defectiveness of the theory of aphasia as related to localisation of speech in Broca's convolution. Demanding from physiology nothing but the assumption of the development of the brain on economic lines, I was able to show that the whole conception of localisation ordinarily entertained required re-casting.[2]

Memory.

In dealing with Memory, I employed methods of experimental psychology. I devised a series of experiments destined to throw a beam of light on the nature of Memory by giving the answer to questions so framed as to open hidden corners of the problem.

These experiments were extremely laborious, some of them extending over years. I have selected from my notes those which contain new points, and this part of the exposition will be found, I hope, not amongst the least interesting.

* * * *

[1] The point is, however, subtle, and it may not be completely grasped at first. If not, it will be clearer as the whole method and spirit of this work becomes familiar, and particularly after the chapter in Externality has been studied.

[2] This was the first time, I submit, that a result of this importance and of this definite character impinging on the domain of physiology had been shown as arising from psychological studies.

Applications.

In each one of these applications, the reader who has been familiar with these " positions " in other books will find that the treatment consists not in criticisms or improvements of old methods, but rather in approaching the matter from a fresh point of view and with new illumination.

There is no field of thought that the principles here set forth will not illumine, and that in a scope as wide as possible, from brain Localisations to Function theory in mathematics, from Relativity to Externality, from Fechner's Laws to Dreams.

In certain cases, as for instance, in dealing with Kant's Transcendantalism, or with Freud's Psycho-Analysis, I have indicated how essential arguments may be dealt with by way of corollary to the main results here obtained.[1]

If, for instance, I obtain a sure foundation, deeper than that of Kant, or Freud, or Bergson, and ascertained by methods that are demonstrable; and if from that foundation I proceed by rigorous arguments, consecutive and cogent, and arrive at conclusions inconsistent with any assumption of theirs; I say, even by way of corollary, that that assumption goes by the board.[2]

The Spirit of Psychology.

Psychology is often regarded as a cold and arid subject. I have heard a distinguished University professor scoff at the notion

[1] Again and again I must insist on the necessity for rigour. Without rigour the value of a step from one position to another is placed in doubt at least. The step in question may be entirely unjustified. In any case when we are dealing with probabilities, the chances of error are at each step multiplied by the odds against the probability concerned, and in a few steps the conclusions become reduced to so low a degree of certitude that the process of argumentation loses all value.

Such a principle is immediately accepted in mathematics. In the applications of mechanical science, as for instance in bridge-building, the conclusions are tested by the laws of nature, and the results of faulty reasoning redound in material loss.

But that is not the case in Psychology, or in any form of philosophy; therein lies the justification for the contempt into which philosophy, as generally taught, has fallen with keen scientifically trained minds. From the days of Aristotle downward generation after generation has been content with ill-defined terms and loose argument based on obscure conceptions.

Aristotle, certainly one of the greatest of minds of all time, in endeavouring to explain the phenomena of falling bodies spoke of the proper and improper motion of objects, and for some two thousand years philosophers wielding great authority were content with these terms.

The reasonings of Galileo and his experiments satisfied thinking persons, though not the authorities, of the fallacies of the old theories. I say, not the authorities, for philosophy had come to mean, not the research for knowledge, but a kind of language spoken among professors.

For generations Kant's Transcendentalism has been taught in *ex-cathedrâ* fashion; I have shown how to bring it to a crucial test, as I shall convince all who will sincerely and intelligently study the demonstrations here given.

[2] The usual answer to such argument is to cite the authority of these thinkers, the array of professors who promulgate their doctrines and the titles they bear, the volume of public opinion as shown in numerous editions published by orthodox houses. My reply is simply that such considerations have great weight in politics and business; in the realms of thought, none.

of any definite outcome of Psychology or Philosophy in general. He cited with approval a saying of Green of Oxford which seemed to indicate that even the truth or error of a thesis was of secondary consideration to the philosopher, and that the great gift of philosophy was that of a sort of Buddhistic contemplation, whether of states of mind or world-shaking religions.

Nothing is more foreign to my own conception. Philosophy is or should be, the means of revealing truth, and thence of obtaining instruments of thought by which we open up true and extended views of the universe in which we live.

My desire has been to find the steps by which from the basis of the physical universe we reach up to the intimations of guidance of the conduct of man. In regard to some of the successive links I can only, by evoking the sciences of physics, biology, and psychology, indicate main lines of guidance; but in the mental domain I here show the mechanism that serves to explain the most delicate movements of the mind, and through all, down to the physical foundation, I show a sort of organic chain of causative processes and effects.

The character of mind required for the study of philosophy, and especially of Psychology, is not common; for at times extraordinary patience is needed, and withal a sort of unresting, striving and wrestling quality, as of one determined to win secrets from the arcana of nature.

A bold quality of mind is requisite, that which is revealed in trenchant analysis, piercing to the foundations of a subject and displaying the structural lines of the science. With this there should be clearness both in vision and in expression, a rejection of ambiguity or of mystification, an impatience of obscurity.[1]

The mind of the true philosopher should retain a certain native innocence, a naiveté which sees things as they are without prepossessions or prejudices; yet it should be full of energy, and endued with a determination to press onward to definite outcome in results. In all this I find a recognition of the greatness of the work far beyond that of a tolerant or passive contemplation. The philosopher should deal with realities; he should open up new paths; he should point the way to the possession of new realms of human control.

These are the considerations that to my mind make Psychology supreme. Great is the man who discovers a remedy for some scourge of human life; but that discovery leaves untouched the vital question of man's destiny. Great is the discoverer of a new

[1] Gauss said, it is in searching the obscure corners usually passed over that discoveries are made.

world; but that gives to us only the greater physical scope of expansion. But the man who can win a secret from nature, the man who can endow the human mind with new powers, and show the means of refining and developing his old faculties—shall he be inferior?

Here then is immense stimulus to work, inspiration to thought. That impulse and that inspiration I have felt from my earliest years; through years of silence and wished-for seclusion and obscurity they have supplied the motive and sustained me amid arduous toils in the desire to advance the limits of the realm of thought and to

"Win new dominions of the passioned soul."

PART I

CHAPTER I

THE CENTRAL PROBLEM

ENUNCIATION: In this chapter is given the enunciation of the central problem. Reference is made to the difficulty of dealing with it, and then, in amplification of what has been said in the Introduction, the guiding lines of development are shown. A little repetition is here unavoidable but, unless the complete principle has been seized from the Introduction, this will be helpful.

* * *

A subsidiary problem in Psychology is that of finding an appropriate beginning to an exposition of highly general scope. If we take a suggestion from geometry, we observe that there the demonstration starts from a very simple basis and proceeds step by step by deduction in clearly consecutive arguments.

It is true to say rather, that is what an exposition in geometry appears to do. It will be verified by all who examine the books of Euclid searchingly that many assumptions are made on the way; that will be made clear also in the course of our arguments here. But the manner of treatment of Euclid, though impressive, is also remarkably artificial; it becomes practical only after the positions have been won and have been made familiar.

The solution of the Pythagorean problem, which is the main theme of the first book of Euclid, had been obtained before the systematic arrangement of Euclid was thought of; then an examination was made of certain subsidiary propositions, and of the ultimate assumptions implied in the demonstrations. Then following on this analysis the course became retraced in synthetic form; but this, though very orderly in form, is not so instructive as the method of the original analysis.

In more complicated forms of mathematics, or in subjects of physics, it is often advisable before dealing with an important proposition to enunciate and prove one or more lemmas, so that when the main problem which involves these is dealt with, the attention may not be distracted when the conclusions of the lemma are cited.

One objection felt by young students to Euclid's method is that they do not grasp the significance of the subject even after they have with some effort studied a series of propositions and recognised their truth. They feel that they are beating the air, and some who might have real aptitude for the subject if presented differently, become discouraged.

Any such reasons would weigh tenfold here. In order to present the matter in the most systematic form a number of propositions, abstruse, unfamiliar, and apparently irrelevant would have first to be demonstrated as a foundation for the subsequent propositions whose importance is more evident.

Such a mode of treatment will, I believe, be eventually adopted in regard to the system of Psychology here presented, and when the genius arises who will carry through such a work he will have accomplished more than he had thought of at the beginning of his undertaking; he will have laid the basis for a scientific structure of education. That time has not yet arrived, I think, for I found that one of the difficulties of the first work—"PSYCHOLOGY: A NEW SYSTEM"—was precisely that which constituted its value—its scope, its accuracy, and the originality of its methods.[1]

I propose, therefore, to adopt other methods, which are none the less completely scientific, and more in accord with the actual manner of research.

For example, if instead of demonstrating a subsidiary lemma, I say I will prove my proposition if you will accept my lemma, and if I undertake to demonstrate the lemma subsequently without any assumption derived from the main proposition, that method will be quite rigorous and the force of the total argument will not be diminished.

Having said so much, I will now enunciate the Central Problem, and I will posit the Fundamental Propositions of which I hold in reserve the proof that they are necessary and sufficient. My reason for this step is that I want to make the reader familiar with the terms and their employment, so that when concentrated attention is demanded in an argument the mind may not be disconcerted or fatigued by having to wrestle with such conceptions.

The Central Problem may be thus stated: Standing in the face of the whole world of mental activity, actual or possible, let the thinker proceed to analyse all complexes of thought with which he meets so as to exhibit them as composed of simpler forms, and let him reduce these forms again to greater simplicity. In the course of this investigation, having observed that the processes, or schemata, of thought in one domain resemble those of another—the variations being found in the differences of the objects referred to the schemata—and having observed that as a rule the processes consist in the combination of simpler processes, let him then

[1] The experience is often repeated in various forms. Grassmann's work in mathematics was unread in his lifetime because in order to present his theories he employed new symbols and unfamiliar terms. In another field Mendel's studies in biology were neglected because he opened up new fields of research. That now constitutes one of his glories.

pose the question—and this is the Central Problem—whether the analysis can be continued to a limit at which no further analysis is possible; whether then, examining with the closest scrutiny the mental processes that occur at that limit, he can show that these processes may be formulated, and that their nature may be defined with precision, and that it may be demonstrated that they are necessary to the simplest modes of thought, and that they are sufficient to explain the most complex mental operations; and that consequently by retracing the steps of the analysis and representing the course of the thoughts in synthetic form, it may be proved that by the combination of these Fundamental Processes the constitution of every possible synthesis in the domain of thought may be exhibited.

* * * *

In the introduction I have referred to this Central Problem, and I have indicated the modes of research which yielded the Fundamental Processes. The full story of the obtaining of these elemental forms would make demands too great upon the patience of the reader. I therefore now exhibit them, on the understanding that the exposition will justify them.

The Fundamental Processes are:

I. Immediate Presentation.
II. Conception of Unity.
III. Association.
IV. Memory.
V. Conception of Agreement.
VI. Generalisation.
VII. Impulse.
VIII. Sense of Effort.
IX. Hedonic Sense.
X. Negation.
XI. Conception of Time.
XII. Conception of Space.

These I have to show to be necessary and sufficient in all modes of thought.

The first impression that will occur to most readers is that the subject is so vast and complex that it offers no means of seizing it effectively, except perhaps in limited special studies.

What we must search for at the outset is some principle of division.[1]

[1] In a sphere which, compared to this universal realm, is extremely limited, namely, that of curves of the third degree, Newton made some remarkable observations but he had no means of ascertaining whether he had covered the whole field, and in fact he had not. "He lacked a principle of division," Euler said. *Cf.* also the reference to Aristotle and to Kant, page 5.

Means of Solution

What kind of principle of division seems advisable? How and where should we look for it?

The second of these questions seems at first to offer more grip than the first. What can be done in that regard?

Already we see the peculiar difficulties; we have not only to work out a line of thought, we have first to discover what our line of thought may be. We have no clue as yet, but we have to search for a clue and to find first what sort of a clue we may look for.

Certain considerations point the way: If, for example, we were to select any complex of thought at random, and reduce it by continuous analysis to its simplest forms we might arrive at some fundamental process. Then we might select another complex, and resume the search. In this way, if we had unlimited time and perfect skill we could complete immeasurable analyses, but even then we would not be sure of comprehending all cases.

Yet here we would find something that all these different essays in analysis possessed in common. We would have reduced each in turn to the simplest forms beyond which analysis is not possible.

The next suggestion then arises: Is it possible to help ourselves by examining, instead of complexes, the simplest forms of thought, in order to discover whether from that basis we may find common features, or ultimately processes which we can take to be universal?

Amongst the objects of our research, therefore, we would find precisely those which Euclid considers capable of being answered without examination at all. These are the axioms and postulates of geometry. Others assumed as equally simple are the processes of addition, or of counting.

These rudimentary forms I have examined with intense concentration of attention. I have, speaking metaphorically, put the microscope on the rudimentary forms of thought, in order to gain the understanding of the structure, and so having the secret of the minutest wheel-work and concatenation in the forms of thought I am able to trace out the building of the instrument, to attain the clearest idea of its functioning.

When we have carried out such an exploration, we will be able to pose the question: Having observed certain of these rudimentary forms and found as arising from this examination certain mental elements, or fundamental processes, and finding the same processes in examining other rudimentary forms, is it necessary to examine all examples of rudimentary forms?

To answer this question, we might pose another arising from it: How do these rudimentary forms differ? We may then find, as in fact we will, that the differences may in some cases, be resolved into different forms of sensation, or as I should say, for reasons that will clearly appear, Immediate Presentation; and when the characters of difference are removed, or abstraction of them is made in our minds, we shall arrive at a schema which in any one form is similar to that of another.

In the simplest cases this is evident, when we consider attentively what is implied in the term schema. If, for example, we look at a blue disc a different sensation arises from that which would occur if the disc had been coloured red, but making abstraction of the actual colour—and that act of abstraction is within our power—we find that what remains, the schema as I have called it, is similar in the two cases. It is evident, therefore, that if our mental activities were confined to successions and combinations of such colour sensations, we would find that, in all cases where the differences were only differences of the colours we would, when abstraction had been made of these colours, arrive at similar schemata.

What we have said of colour or colour sensations is equally true of other sensations, for we have but to adapt with any other sensation the reasoning already used. It is therefore, equally true when we are dealing with composites of sensations of different kinds affecting different senses.

* * * *

At this point we may for a time interrupt that line of argument in order to inquire: Are there any modes of receiving sensations, or any kind of analogous impressions, except by the senses as we know them? We have not yet investigated that matter, but here it may be said at once that if any new sense came into evidence we should not be compelled to revise our argument; we should simply add that new sense to our accepted list.[1]

* * * *

There is at least an act of attention. The instantaneous act of attention implies what I have called the Unit.

Now consider an actual Unit. That unit may be represented by a sensation, or by any Immediate Presentation whatever, but

[1] The sense of orientation exists to an appreciable extent in man; the organ of this sense is found in the system of semi-circular canals. In animals which are noted for their faculty of orientation, for instance homing pigeons, the semi-circular canals are enormously developed.

When the otoliths in the "auditory organs" (statocysts) of prawns and crayfish are replaced by magnetised iron filings, and a magnet is brought into proximity of these organs, the animal reacts to the stimulus and turns its ventral surface toward the magnet.

we may make abstraction of its sensational or graphic character, and fix our attention on the schema.

Now consider another Unit, in any kind of association with the first. Any kind of relation, even if it be merely of sequence, is what I here mean by Association; and a little consideration will make it evident that some Association under the conditions determined must exist.

In this, as in many of the arguments used in this chapter, the student may have the impression that when the statement is placed before him in its simplest form he is dealing with a truism. It will be at length evident that all demonstration when pursued to its elementary forms must be finally resolved to this consideration of truisms; but for the present I remark that if the argument is so simple as to appear a truism then we may accept it as true. From these simple arguments great truths will arise, and they will be the more readily accepted when we can trace them from their origins so closely.

Let us now proceed to a third Unit. There is an Association between the second Unit and the third. The nature of that relation depends on the character of the Immediate Presentations involved.

It may be inquired whether the Association between the second and the third has not been modified—as compared with what otherwise it would be—by the previous Association between the first and second.

When we come to study what I call Agreement, with which the question of discrimination is involved, we shall find that there are limits at which discrimination is not possible between two objects which objectively, or for other grounds than those of direct impression, we might consider as distinct.

Above these limits of discrimination, however, it is evident that the Association between the second and the third Unit just referred to might be modified by the Association between the first and the second. Nevertheless, in its own character it remains an Association.[1]

Pursuing our study of Units or Immediate Presentation, we find that the schema is developable according to a simple formula of numeration, for the schema makes abstraction of the character whether of the successive Presentations or of the Association between them.

[1] The following chapter deals with physiological data but very briefly and diagrammatically. It is inserted so as to make the arguments more graphic by reference to the underlying nervous organisms of our bodies, and what is said above will be clearer if we consider this physiological representation as an outline or plan by which to fix our meaning.

We arrive in this way at a conclusion of the highest importance, namely, that our mode of development as far as we have carried it out is co-extensive with the process of numeration.

After considering this process of numeration, discussed under the simplest processes of counting, we will find that we have here the key to the mathematical processes of addition and multiplication, and the inverse forms of subtraction and division, and that by aid of them we can see how the whole structure of mathematics is built up. Our schemata, it will be observed, are still co-extensive with mathematical forms.

In dealing with Immediate Presentations we have, for illustration, cited colour sensations. Certain spatial impressions, as seen in the immediate recognition of elementary forms or shapes, are as distinct as those of colour. In introducing them, however, we do not reach a sphere not covered co-extensively by mathematics; and when these spatial impressions are resolved, as they may be, into Immediate Presentations, we do not, in our schemata, get beyond the scope of enumeration. Hence, then, when we consider the structure and development of mathematics from the simplest forms upward, and observe what processes are called into activity in traversing that field; and when we further observe that any schematic form may be clothed with the characters of any sensations or impressions and their relations—all coming within the scope of Immediate Impressions and their Associations—and that by such are represented any actual experience, we may finally be sure of covering in this way the whole field of *direct experiences*.

We remark here, however, that the world of thought is made up of direct experiences and of the peculiar reproduction or recall of these which we cover by the term, Memory. Therefore we will examine and explain Memory. Then we will ask the question as to whether any other agency or influence can enter.

At length we will be convinced that the methods here set forth cover the *whole field of thought*, and it will remain for us to make sure of the demonstration of the necessity and the sufficiency of our formulated Fundamental Processes as tested in the examination of acts which in their combinations build up the whole scope of our thought.

* * * *

To sum up this chapter: We have formulated the central problem. We have found means of attacking it. We have shown how a certain number of the Fundamental Processes may be won.

CHAPTER II

PHYSIOLOGY AND PSYCHOLOGY

Certain data of Physiology and particularly the function of the brain will be frequently alluded to in course of the exposition; it is therefore advisable to insert here a note which will serve to fix our ideas with regard to psychic processes and their physical correlatives.

The value of a knowledge of physiology in regard to Psychology has become recognised more and more of recent years, and those who devote themselves to biological study are apt to exaggerate the importance of their science in its application to psychological problems.

The physiologist deals with a subject in which concrete facts arise in profusion, and where every fresh experiment, well-conceived and skilfully carried out, brings an increase of useful knowledge, and he contrasts unfavourably with these results the tentative strivings in psychology which have led to so little advance in two thousand years.

Nevertheless, the physiologist is wrong in his disparagement of the whole science of mental operations; his quarrel should rather be with the defective methods employed. The true method not only illuminates the field of Psychology, but also throws its light upon the questions which the physiologist has arrogated to his own domain. This statement will be made clear particularly in the references to the connection between aphasia and Broca's convolution.

What the physiologist really observes, as a series of fact, graphically displayed in the course of his researches, is small as compared to the superstructure of interpretation and theory that he raises upon it. When, however, he begins to interpret he lays down propositions which come immediately within the scope of Psychology, and if, as often happens, the physiologist is untrained in mental science, he may proceed to build theories which are unwarranted by his own observations and which fail in a searching criticism even on the ground of consistency with what he recognises as general principles.

It is possible to make these assertions confidently, for one has but to compare the works of eminent physiologists among themselves in order to be convinced how uncertain are their reasonings as soon as they leave the solid ground of fact.

Sir William Gowers, for example, held a theory, quite opposed to that of nearly all other physiologists, that the cerebellum exercised a control on the functions of the cerebrum. On the strength of this theory he established a system of diagnosis which in his hands proved satisfactory in dealing with diseases of the central nervous system.

Westphal, a German neurologist of no less authority, held the opposite view with regard to the relative functions of the cerebrum and cerebellum, but when Gowers gave the ground of his faith Westphal simply replied, " Perhaps ".

The physiologists base their claims to speak on physiological matters on some such reasoning as the following: " The brain is the organ of mind."[1] Therefore a knowledge of the constitution of the brain, its structure and functions, should give us knowledge of mental processes. The more we know of the functions of the brain the more we will know of psychological matters.[2]

All this argument must be taken with great circumspection. We have learned a great deal about the brain as one of the organs of the body, but relatively to all that we might possibly know of the brain our knowledge is small; with regard to functions our understanding is often extremely vague.

We are not able to observe the brain in activity in the same sense as we observe a machine in activity, and we are not able to dissect out the essential parts of the brain to a great degree of completeness.

We have several aids to the study of the brain, but every one of them presumes so much difficulty, while at the same time revealing a world full of interest, that each has been made into a definite science or rather separate discipline in the whole study of biological science.

One of the means of research, for example, consists in the study of development; another in that of degeneration of brain tissue. Such degeneration is often indicated by characteristic physical and mental reactions during the life of the patient. If the brain of the subject be examined after death it may be possible to find in the sections the area attacked by the degeneration. A correspondence is assumed between the observed physical and mental signs and symptoms and the state of the brain.

[1] It must be clearly impressed on the mind of the student that all we are able to assert is that certain physical conditions, interesting the brain in particular, are correlated to certain mental conditions; and that it may be assumed with high probability that any mental state implies a determinate physical correlative.

[2] Even at this point the inference that often seems tacitly drawn is not allowable: viz.: that the professor who speaks with the greatest knowledge of physiology up to date must be able to speak with the greatest authority on psychology up to date.

Comparative anatomy enables us to relate differences of structure or degree of development of brain to difference of function. Allied studies between individuals of the same species, and between normal and abnormal types, also assist us to form conclusions as to the relations of structure to function.

Clinical observations of persons suffering from brain lesions, and from lunacy, taken in conjunction with notes on the mental condition, form other means of study. These clinical observations may be supplemented by experiments either by way of stimulation of certain chosen areas, or by their elimination, with observation of the effect on the faculties.

Then there is the method which is perhaps the most obvious though one of the most arduous, that of tracing out step by step in successive fine layers of the brain the course of the nerve filaments of certain strands of nerves.[1]

The difficulties in the way of obtaining accurate and precise results by any of these methods are great. For example, before a thin slice of the brain can be studied in order to mark the position of the section corresponding to any strand of nerve filament, the brain must have undergone a preparation lasting about three weeks in order to harden it, and in this process the strands become warped. Then the section is submitted to staining processes which affect certain tissues differently from others, and so enable the nerve strands to be picked out.

These processes, even if they could be carried out to a degree of perfection which are still far beyond us, would only serve to give us the exterior aspects of the structure—their definition is entirely in the realm of topography and description of structure.

What passes within the nerve structure itself? Here we are at a stage where we can only guess, and the guesses of physiologists have hitherto been vague, and unsatisfactory though often mutually exclusive. It is evident that they cannot all be right; it is possible, however, that they are all wrong.

Even in regard to an elementary fact such as the rate of transmission of nerve impulse, there has been much discrepancy of opinion, and a good deal of erroneous teaching delivered with full weight of authority. Helmholtz found that the rate of transmission of nerve impulse varied from twenty-eight yards per second to thirty-two yards per second.

It must be noted that the experiments of Helmholtz were conducted mainly on the nerves of dead frogs, so that it was extraordinarily hazardous to apply the results immediately to the case of the living human body. Since Helmholtz's day many experiments

[1] First carried out by the German physiologist Schiff.

have been conducted with nerves of living persons, and the rate is now given in the text-books as 120 metres per second.[1]

Apropos of the variation given by Helmholtz of twenty-eight yards per second and thirty-two yards per second, Herbert Spencer makes the remark:[2] "Difference of constitution is doubtless the cause of this variation—a variation to which is due that individual peculiarity recognised by astronomers in what they call 'the personal equation'."

"This peculiarity affords yet another confirmation of the belief that a nervous discharge is a wave of isomeric transformation."[3]

Here on the basis of supposed facts, constituting false data, Herbert Spencer proceeds to build a theory concerning processes of nerve transmission of which we know little, except indeed that they must be exceedingly complex. His conclusion is completely without warrant.

The statement with regard to the personal equation here is absurd. Differences are no doubt produced by the personal equation, but the particular argument or illustration he adduces is valueless. The differences in question are merely due to the differences of dead frog nerve tissues. The differences due to the personal equation reach an enormously wider scope of variation than assumed by Spencer.

I dwell a little on these points in order to enforce the argument that so far from physiology being in any sense a sufficient

[1] Even these later experiments are subject to errors which have not been taken into account. What we would desire to measure is the rate of transmission of a sensory nerve impulse received in the ordinary way from the end-organ of the nerve, or of a motor nerve impulse carried to its termination in a nerve working under normal conditions. In the experiments we measure in a motor nerve the time of response to a stimulus, not received in the ordinary way, but delivered through the surrounding tissue as a shock to the nerve. Without reference to further grounds of criticism it is evident that the conditions in the experimental case and that of the ordinary reality are very different, and that moreover there are no data by which it can be estimated how far this difference may affect the results.

These determinations leave out of account also the enormous effect of training and practice in the rate of transmission of nerve impulse. Consider the act of catching a cricket ball, hit or thrown at great speed. The speed must at times exceed fifty yards a second. Yet when the ball is in view for a small fraction of a second, and while it is constantly changing its direction, it is caught with precision. In this case the impulse is carried first along the optic nerve to the brain. When it reaches the brain it sets up a long series of reactions necessary to bring about the proper co-ordination first of sensations and then of corresponding nerve impulses along the motor nerves. Considerations make it certain that the greater part of the time of the whole sequence of events is concerned in the operations of adjustment of which the brain is the seat. When the motor impulses are carried to the fingers, their degree and distribution are controlled and adjusted in accordance with tactile sensations which are carried along the nerves up the arms again; and yet the motor results—as including those of the hand—must operate in a very short space of time. We cannot measure the periods of time occupied, but all this points to the fact that the rate of 120 metres per second must often be greatly exceeded.

[2] *The Principles of Psychology*, Vol. I. Second edition (Williams and Norgate, 1878).

[3] By isomeric transformation Spencer means a transformation where "the components of each compound molecule have their relative positions altered."

preparation to enable us to discourse on Psychology, the works of some of the great physiologists are riddled with absurd errors when they broach psychological questions, and in this field even a thinker so keen and circumspect as Herbert Spencer produces arguments of which the premises are bad and the conclusions unjustified.[1]

All the methods employed in histology are coarse in comparison with the delicacy of the structure of nerves and of the complexity of the processes which take place during the transmission of impulses. The method of distinguishing nerve fibre from the surrounding tissues, neuroglia, for instance, by means of stains which have more affinity for nerve fibre tissue than for the supporting structure, is not sufficiently refined, though the best known at present. There are also various limitations of the miscroscope itself.

The difficulty of precisely relating the physical condition to the mental state is shown by the fact that it is rarely possible to detect the lesions in the central nervous system corresponding to well-marked hysteria, even though the disease may have produced excited physical displays and veritable storms in the world of ideas.

Physicians have been accustomed to make a distinction between diseases producing changes in the organism and "functional" diseases. Hysteria is called a functional disease. Then again a great many diseases used to be considered as having an "ideopathic" origin, that is to say, the diseases were supposed to take place without any definite grounds. With increase of knowledge the numbers of ideopathic diseases has become less, and unless we

[1] In Page 61 of Vol. I of *The Principles of Psychology* lately cited, Herbert Spencer gives a remarkable example of reasoning from an inadequate basis of knowledge to conclusions that present but a small connection with that basis:

"In brief, the hypothesis thus reached *a priori*, is that the cerebellum is an organ of doubly-conformed co-ordination in *space*; while the *cerebrum* is an organ of doubly-conformed co-ordination in *time*."

It would be easy to reply to Herbert Spencer here by quoting the words of great brain specialists as to the functions of the cerebellum and the cerebrum, and no matter which side were right the truth would be proved of what I have advanced about the danger of accepting the dicta of physiologists.

I prefer another method entirely, viz.: that founded on rigorous argument from the most fundamental base; and in this present work it will be made clear that there cannot be co-ordinations in space that are not also co-ordinations in time; and so the import of Herbert Spencer's conclusions vanishes.

In re-reading Spencer's *Principles of Psychology* after the lapse of many years, and remembering how deeply impressed I was formerly by a mind so comprehensive and so candid in outlook and expression, I was disconcerted to find with what carelessness he had sometimes relied on erroneous assumptions.

He was led astray by pre-conceived notions founded on his principle of evolution. I have shown elsewhere (*Purpose and Evolution*, and *Ethics*) that Spencer has not established here a principle of Nature, but that his enunciation of the evolutionary process amounts at best to a system of classification requiring great circumspection in its employment.

When, however, this doctrine of evolution fails, then the whole structure of Herbert Spencer's Synthetic Philosophy, in as far as it derives its sanction from a great central principle, falls to the ground.

are prepared to assume that effects can arise without causes, we may conclude that the only reason why hysteria is called a functional disease is that our methods of investigation are too coarse to enable us to distinguish the corresponding lesions, or morbid condition of the brain. Even in such grave disorders as epilepsy and lunacy it is often impossible to find any distinct central lesion or degeneration.[1]

We do not know what nerves correspond to the sensations of heat and cold and pain and various other less well-defined sensations. The study of localisation in the brain of various " centres ", such as of sight or of hearing, is still imperfect, and we shall find later that the meaning of localisation has been wrongly conceived.

Finally, opinions of great authority differ widely as to the functions of the cerebellum and the cerebrum.

In these circumstances hypotheses have a free run, partly because our knowledge is so vague that we cannot definitely overthrow even those theories that to the majority of physiologists appear absurd.[2]

It is evident that any special study of physiology does not offer good ground in which a professor may pose as an authority in psychology, but it does not follow that a knowledge of physiology is useless to the student of psychology.

If the main principles of psychology be once clearly grasped, then an extensive and intimate acquaintance not only with physiology but with the wider field of biology is of great value. This would remain true even if it were difficult to explain how any one fact or thesis of biology might help towards a clearer view of psychological principles.

For example, Locke speaks of the infant's mind as a *tabula rasa.* This was an assumption on his part, but we see more clearly why it is an assumption, and in how far that assumption warps his reasoning, if we are acquainted with the doctrines of development, evolution, and heredity.

Or again, in the wider field of biology it seems to me from the fragments that have come to us of the teachings of Empedocles, that the great Greek having expressed himself so clearly on the question of what is called " survival of the fittest " could not but have had a conception as clear as Darwin's of the main principles

[1] Recently an American physician has stated that in cases of epilepsy he has found effusion of blood under the pia-mater. This observation, if true, has escaped thousands of other investigators.

[2] When we consider that not even the most complete knowledge of the structure of the brain and the mode of working of nerves would give us any information as to sensations, it may be said without exaggeration that to study psychology simply by means of physiological data would be as hopeless as to survey the dome of St. Paul's from a balloon to know what the Dean and Chapter were saying.

of natural selection; yet little resulted by way of stimulation to psychology.

The merit of the English naturalist consisted not so much in the enunciation of the principle, as in the exemplication of it by means of hundreds of curious and interesting studies which not only served for illustrations or proofs but opened up new vistas of research and speculation. Each one of these in turn served to stimulate the mind to a fresh study of the question, and finally not merely to bring about the establishment of the theory of natural selection but to give to biology a new setting, a greater importance, and a fresh outlook.

And so at length the student of psychology will find that the deeper his acquaintance with physiology and biology, and more particularly that of the nervous system, the more will the various branches of his knowledge become united together into one whole, as of an organism, by ten thousand strands of association. He will continually find fresh stimulus to his psychological studies in this knowledge, and he will frequently employ the physical structure as a sort of schematic form to which to relate his thoughts.[1]

The tendency of this chapter, therefore, is not to discourage the study of biological science, but to warn the student that there lies before him in that direction a vast and intricate field of research, and that he cannot apply the results in psychology without having clearly seized the essentials in that subject. We shall see finally that all science may be regarded as extensions in special fields, either of research or of application, of the principles of psychology.[2]

It would be possible for me to incorporate into a work of psychology, which then, however, would comprise several volumes, all the biological knowledge, including development, heredity, physiology, and especially neurology, that might be thought to have a bearing on mental science. That, indeed, is what has been done in part by Wundt, who with true German thoroughness in his *Physiological Psychology* gives us as his first book an extensive treatise on the brain. He fails, however, to show the nexus between all this learning and the study of psychology. His *Physiological Psychology* is a Physiology added to a special Psychology.[3]

[1] I have heard a physiologist speak as if a study of the "kinetic drive" gave the key to psychology. This phrase simply sums up a series of processes by which the parts of the body may become activated in obedience to the will. Such phrases are never explanations. They are at best convenient terms by which to refer to composite forms. Similar remarks apply to the term "herd instinct." *Cf.* note.

[2] *Cf.* Note on aphasia, pp. 67 (note) and 368 (note); also p. 389.

[3] I have not neglected the aid obtainable in biological science and I have largely employed methods of Experimental Psychology; I only warn against undue claims of these methods.

Herbert Spencer in the first book of his *Principles of Psychology* also gives us a good deal of information which may be found in the text-books dealing with the nervous system.

The student, may be referred directly to such books. The section in Herbert Spencer's *Principles of Psychology* which deals with the structure of the nervous system is excellent but space does not here permit of the introduction even of a condensed account of the process of development there indicated. It must suffice to say that in the most elementary nervous reaction these constituents are required: An afferent nerve carrying an impulse; secondly, a portion of nervous structure forming a bond of connection between this afferent nerve and an efferent nerve; then the efferent nerve carrying a motor impulse.

This structure might be represented diagrammatically by two lines joined by a little circular dot. One line represents the afferent nerve, the other the efferent nerve, and the circular dot represents the centre which receives one impulse and transmits the other. The whole system may be called an arc. We may now suppose a second arc similar to the first. These arcs if kept entirely distinct would not lead to the development of a more complex system. But suppose now that in addition to the two centres, one for each arc, we lay down a third, and that we connect this third to each of the first two, and these two also between themselves, by lines.

If now we grasp this simple arrangement and if we proceed to extend the system, we are in possession of the principles of development which will at length produce, in schematic form, the most elaborate nervous system.

A striking manner of representing the complex development of the brain was given by Ferrier using the analogy of the telegraph service.

A small station is first set up, capable of receiving and transmitting messages. Then another station. These are connected to each other and they are both connected with a higher station. Similar higher stations are connected with each other and with others still higher, and so the process continues. The highest controls, it is generally believed, are to be found in the cerebrum, but we will not enter into this question now. I have simply desired at this stage to indicate the connection between psychology and physiology, and to suggest a diagrammatic form which will serve to fix the ideas in subsequent discussions.

We will now resume the march of the exposition, interrupted by this explanation, and at a later period we will consider once more the relation of the psychical processes to the physical side of our constitution.

CHAPTER III

THE FUNDAMENTAL PROCESSES

(1) *The Establishment of the Processes*

NOTE: In this chapter we refer to the sources of our impressions and ideas. We then deal with the Fundamental Processes separately so that they may be better defined. That is advisable so as to avoid digressions at a point where it is important to concentrate attention when we reach the critical chapters on the Unit and on Counting. We take notice here also of the manner in which the Fundamental Processes show mutual implications.

We have already received the suggestion to seek for the Fundmental Processes in the axioms; in the operations of counting; in the simplest examples of spatial relations; in the processes of mind which are at the base of the acts which preserve life.

From these we are led to consider the exercise of the senses, individually at first as far as possible, and then in various combinations. In these operations involving single senses we find a certain similarity of processes. Such of these as are unanalysable we are justified in adopting in our classification of Fundamental Processes. We may then ask whether the senses, as usually referred to, are the sole means of bringing to us knowledge of the external world; and we find that we must consider all sources for our mental activities, whether we apply to such sources the name of senses or not.[1]

At this point we may state a Fundamental division:

(A) External or objective factors.

(B) Internal or subjective factors.

Under the heading of External factors we consider the processes directly set in activity by the senses which bring us into contact with the external world. Under the heading of Internal factors we consider movements of the mind which do not depend directly on this stimulation.

We may then examine modes of combinations which build up our knowledge, and also we must make sure that in our review we fully consider the whole extent of the field of these combinations.

[1] A considerable amount of work has been done of late years in regard to this question. H. E. Beaunis in *Les Sensations internes* gave us an admirable exposition of the subject, and his book has been productive of new research. The names of Meumann, Becker, and E. Murray may be cited in this regard.

Then further we may examine particularly the mental operations involved in such subjective phenomena as memory, fancy, imagination, dreaming, invention, and the like, in order to obtain a clear view of the factors involved.

If, then, after adequate consideration of the senses, and all the sources of mental impressions besides the senses usually considered, we can show that the whole of our knowledge of the external world, and the whole of the new combinations of the various elements within our minds, may be built up by the use and combination of the mental processes we have called fundamental, we may be able to say that our classification of those Fundamental Processes is correct and sufficient. This accords with what has been previously said as to the scope of mathematics being co-extensive with the schemata of thought. For now we see that to cover all forms we have only to impress into these schemata the attributes derived from the Internal and External factors.

We find here a case in which elementary physiological facts aid us in research in matters where it is difficult to see that introspection alone would avail. Thus, if we consider the mechanism of the different senses, we find that in each the essential condition of sensation is the bringing of external impulses into contact with some specialised form of nerve-ending of which the supplying nerve is in uninterrrupted continuity with the central nervous system, which again must be efficient for its work. We are then led to consider other examples of nerve-endings.

Certain nerves terminate in muscle fibres, both striped and unstriped, found in various situations and including the gross motor muscles of the body, the heart, the muscles of the arteries, those of the outer parts of the walls and also of the mucous membrane of the intestines; there are nerves supplying the various mechanisms of secretion; there is the entire system of the sympathetic nerves with their inter-connection with the motor and sensory nerves; and there is the whole complex, with its extraordinarily intricate relations, of the central nervous system, including the spinal cord and the brain.

Through the nerve supply of the muscles we obtain in part that sense of energy [1] which we shall find to be one of the factors of our appreciation of Externality.[2] It would be fastidious at this moment to trace out in detail the sensations, impressions, informations, or demands brought by the nerves supplying internal

[1] This, which is now set forth on assumption, is considered especially in the chapter on the Sense of Effort.

[2] See chapter on Externality.

organs, and becoming revealed in consciousness, or, while working subconsciously, modifying the forms of consciousness.[1]

If now we examine the special senses in turn, we find, besides the peculiarities of each, certain Fundamental Processes of the mind necessary to this exercise.

For convenience I formulate them again.

(1)	Immediate Presentation.	(7)	Feeling of Effort.
(2)	Conception of Unit.	(8)	Impulse.
(3)	Association.	(9)	Hedonic Sense.
(4)	Memory.	(10)	Negation.
(5)	Conception of Agreement.	(11)	Conception of Time.
(6)	Generalisation.	(12)	Conception of Space.

A few explanations serving mainly for definition may be given at this stage, in order to make familiar the terms of subsequent fuller discussion:

(1) *Immediate Presentation*:—This may be taken as including Sensation, and indeed the schema of Sensation is coincident with it if the term be not restricted to the organs of sensation usually considered.

(2) *Conception of the Object as a Unit*:—The questions here involved are considered in the chapter on the Conception of the Unit and in the chapter on Counting.

(3) *Association*:—To this a wide range of application must be given. It is not merely a matter of Association, say, of one object seen with another object occupying the same field of vision, or again between two objects presented by different senses. Association is formed by myriad circumstances of the whole mental make-up of the moment—the sub-conscious forces in operation, the temperament and previous experiences of the individual, as well as by the accident of external association in the actual experience.

[1] We are now far removed from what is called the "Sensationalism" of Locke The mind of an infant is not a *tabula rasa*, but a magazine of stored-up forces, and though experience is necessary to its development, yet that experience conceived of as the chance impressions on a *tabula rasa* is extremely inadequate to explain a child's mental development. Even the experience itself is determined by multiple processes of instinct, choice, and by the actual scope of our senses and their limitations. Experience must be thought of not as an external something impinging on even a living apparatus; but as being a product formed by the mind from some external unknown. Consider the case of that lady, Helen Keller, who was born blind, deaf and dumb, though endowed with an organisation and mind, the heritage of a long line of ancestors, accustomed not only to the full exercise of the senses but also to mental operations of a high degree of complexity. Here it is evident that experience must be limited to the senses which Miss Keller possessed in activity; while the most vivid, the most stimulating of the senses have remained blank. Yet the intellectual development of Helen Keller has been extraordinary, and has evidently been subserved by factors which have come to her by heredity, and of which the possession was possible only through a physical form wrought out in previous generations in harmony with these special senses.

(4) *Memory*:—Memory is a Fundamental Process. For the purpose of exposition merely, let us assume something more than we have already considered, and let us speak of Idea as the image which the mind displays of a past experience, for instance, a particular sensation. Then Memory stands to Idea in a relation similar to that in which Immediate Presentation stands to Sensation. It may be that some will find a difficulty in distinguishing Immediate Presentation from Sensation, and the difficulty would be insuperable if there were but one sensation in experience; but since there are more, the very fact that we can think of them in some aspect common to all enables us to speak of Immediate Presentation without identifying that Fundamental Process with any one particular sensation. Similarly Memory and Idea are not identical. Memory is the Fundamental Process by which the Idea is made known as existing.

(5) *Conception of Agreement*:—This process involves the recognition of objects successively presented as being like, or in Agreement, with regard to calling up of a similar set of associations in similar relations.

We may find it indeed convenient here also to anticipate by referring to Discrimination, which is brought to light by the negation of Agreement, and which will frequently be referred to in place of Agreement, because the conditions to which attention is sought may be rendered thereby more evident.

When we say that Discrimination is brought to light by the negation of Agreement, we may indicate the process thus: The mind proceeds along a course marked by Agreement. For example, a child has been playing with a piece of red glass or ruby, and has noticed various associations about it. Then arises an interruption or negation of certain of these associations. The child has mistaken a live coal for the ruby. That negation calls attention to the differences. That is a process of Discrimination. The process is that of an Immediate Presentation of the difference, and this is joined in Association with those Presentations found in Agreement.

(6) *Generalisation*:—The process which becomes manifest at a crisis where Discrimination intervenes is Generalisation, with which will be considered Classification and Symbolisation.

This arises inevitably in connection with Agreement where a series, however long or short, of Presentations in Agreement is interrupted; for Generalisation is the recognition of common qualities, A, between two objects $(A+a)$ and $(A+d)$; while Discrimination is the recognition of the differences, a and d.

Generalisation as a fundamental movement really always proceeds from Impulse and Association, succeeded by Association in a Negative sense.[1] Thus, whereas on perception of $(A+a)$ and $(A+\acute{a})$ before the discrimination of a and $\acute{a}$, the mind's impulse proceeds to form for both the same associations, yet when Discrimination takes place, thus separating A from $A+a$ and $A+\acute{a}$, the recognition of A as distinct, and with distinct associations, constitutes Generalisation. It must be noticed that this is Generalisation in its simplest form.

Generalisation in its scientific use is not formed inevitably, but by exercise of a trained faculty; but this kind of generalisation bears the same relation to the Fundamental Process of Generalisation as the discrimination necessary to appreciate, say, a subtle legal point does to the elementary use of Discrimination in visual experiences.

I have associated with Generalisation the processes of classification and symbolisation. For long I hesitated as to whether any one of these terms should not be chosen as the representative with the other two subsidiary, or whether these, classification and symbolisation, should not be included among the Fundamental Processes. The fact that they arise from Generalisation in conjunction with other Processes is not decisive, because, as we shall soon observe, all the Fundamental Processes are inter-related.

Classification:—This is the conscious recognition of the combination of Discrimination and Generalisation. Thus:—

$$\frac{A}{A+a\,|\,A+\acute{a}}$$

Symbolisation:—This is a process involving all the previous factors. It may thus be ranked as a secondary movement of the mind, though considerations based on the study of Impulse may tend to make us regard the faculty of Symbolisation, not necessarily expressed in words, as inevitable if not absolutely fundamental. Thus in the system

$$\frac{A}{A+a\,|\,A+\acute{a}}$$

A, if conceived as a Unit, is the symbol either of $A+a$, or $A+\acute{a}$, in as far as the possession of A is alone considered.

[1] This remark indicates the implication, with which we will become familiar, of the Fundamental Processes, one with the others. We have not yet discussed Impulse and Negation, but nothing unallowable is here assumed.

The system may be possibly involved:

$$\frac{A}{\dfrac{A+a\,|\,A+\acute{a}}{A+a_{\text{I}}\,|\,A+a_{\text{II}}\,|\,A+\acute{a}_{\text{I}}\,|\,A+\acute{a}_{\text{II}}}}$$

and so indefinitely, and if A be elemental it becomes a symbol for any one of the systems in as far as A only is concerned.

Let us give an example, simply to make the meaning clearer, though it involves many complex factors far remote from the elementary cases we are considering. A soldier is instructed to fire on all enemies; the enemies are marked by a white cockade. The first enemy he encounters may be tall, the second short, another may have the uniform of an officer, another of a foot-soldier. Now with each of these variations there will be distinct associations, but these associations become disregarded in view of the main directive. Finally, we arrive at the recognition that the whole impulse of his attack is stimulated by the sight of a white cockade on a man; and this cockade is represented by A of the system given.

Symbolisation takes place inevitably at the moment Discrimination takes place. For if A be considered the whole object, the Impulse associated with A is summoned up on the Presentation of A. When A is found to be associated with other elements, and these are separated by Discrimination, A still remains as evoking the Impulse due to A; and the recognition of A in this relation is an act of Symbolisation.

(7) *Impulse*:—That is to say, some movement of the mind, proceeding from and linked with the Unit, or thing held in attention. This will be understood by the objective consideration of the nature of the mind: that the mind, or at least its physical substratum as a living thing, is a magazine of forces. The question of Impulse is of special interest.

(*a*) Impulse may be derived directly from the Unit in conjunction with something forming strong immediate associations. Impulse, it must be remarked, always involves association.

(*b*) Again, Impulse may arise with something not at all immediately associated with the Unit held in attention at that instant. In this instance the object is the starting point of associations stored up from past experiences, and these associations may be far stronger than such as are immediately and externally associated with the object. A rough analogy, which is simply used for the purpose of exposition, is that the object may have to the associations involved a relation suggesting that of the button of an

electric system to a mass of dynamite which is exploded when the button is pressed.

It is the Fundamental Process of Impulse which supplies the tentative efforts which we find continually necessary to mental progress in the course of reasoning. (See chapter on Reason.)

When we consider (*7b*) that the Impulse is not necessarily derived from immediate conjunctions with the object, we find the path leading us from too restricted limits of that "chain of Association" which is often popularly given as the representation of the course of mental processes.

Impulse must always be considered not only in relation to the object and its ordinary associations, but also in connection with that magazine of stored-up forces which finds in the object a point of instantaneous impact, while the new thought obtained and associated is the instantaneous resultant in the ever-changing form of the magazine of previously registered forces of impulse, ideas, and all that makes mind.

(8) *The Sense of Effort*:—This cannot be included among the senses, as usually understood, although it is always associated with them. It is not identical with what is known as the muscular sense, although its manifestations are more conspicuous in conjunction with that sense than with the others. It may become evident in the sense of hearing, as when one expects to hear a loud noise, and instead of that hears a faint noise. It is not identical with the actual sensation, for that may differ in quality and yet leave the Sense of Effort unchanged within the limits of Discrimination.

Impulse and the Sense of Effort are two physical phases of the magazine of stored energy which underlies, but may not except at moments become noticeable in, consciousness.

(9) *Hedonic Sense*:—(*a*) This consideration involves not only special nerves of pain,[1] or of physical dispositions in which in a negative sense the Hedonic effect is most apparent, but also pleasurable or displeasurable factors invariably discovered in all perceptions.

(*b*) In addition to an immediate Hedonic sense there may be found closely associated, in some elementary experiences at least, a more general emotional feeling. This has the same relation to the Hedonic sense as the Impulse of remote Associations connected

[1] The question of special nerves of pain cannot be regarded as entirely decided. The literature of the subject is now so abundant that for this reason I refrain from citations, and the opinions of great authorities are diverse.

with the mind in full working order has to the immediate Impulse of close invariable Association.[1]

(10) *Negation :*—It is necessary to include this amongst the Fundamental Processes, for it does not consist merely in the failure of these, but it introduces something of a distinct quality. It appertains to all, just as Presentation appertains to all sensations, or Memory appertains to all ideas. Thus the Negation of Association is Dis-Association; that of Agreement is Discrimination.

(11) *Conception of Time :*—This is essential to the perception of an object as a Unit, and to all Processes where there is a change of attention from one object to another. Objectively it is evident that it is necessary to Immediate Presentation; and subjectively, as a Presentation cannot be represented to our consciousness except by Association with some other Presentation. Time forms a factor even of elementary sensation.

(12) *Conception of Space :*—This is an invariable condition in regard to some at least of the objects of the senses.

* * * *

It will have been observed that it is impossible to refer to any one of the Fundamental Processes without reference to others. Nevertheless, they are distinct. An Immediate Presentation is not a process of Memory, but in all cases of Memory a former Immediate Presentation is implied, and also with it the associations that have been formed.

There is no especial difficulty in comprehending these relations. For example, we cannot think of a mother without implying a child; but the child is not the mother. We cannot think on one side of a penny without implying the other side. We cannot speak of a pentagon without implying five sides, all of which are self-subsisting. Or again, we have an analogy in the working of a steam engine, where every process implies other processes, which, however, are distinct.[2]

Let us revert to the question of Discrimination in order to make the explanations clearer. Thus a previous experience *A* tends to call up *B*. That is to say, *A*, as an Immediate Presentation, has

[1] An acute modern thinker, M. Guyau, finds a certain "aesthetic" quality in sensation. Bain attributes a pleasurable quality, or the reverse, to sensation. Wundt declared Gefuhlston (which is really included in the Hedonic Sense) a third characteristic of sensation, the others being quality and intensity. This was in the 4th Edition of his *Physiologische Psychologie*. Later in 1896 he speaks of "Lust und Unlust-Gefuhle", but he departs from the clearness of his original thought in confusing "Lust" and "Unlust" with affections. The matter is discussed by E. B. Titchener in the *American Journal of Psychology*, 1908: "The Tridimensional Theory of Feeling."

[2] Or, for those who dislike simple illustrations, in a still more complex way, in Abelian equations: each root of the equation is a function of all the others, but the roots are not identical.

had Association with B; so that when we find A either in Memory, or in reality again as an Immediate Presentation, then the image of B appears in consciousness; that is to say, Memory renews that Association. But as Memory is weaker in impression than the Immediate Presentation, only the chief features of B appear. But if in the reality A is now associated with $\acute{B}$, which has elements of Agreement with B, then the impressions of $\acute{B}$ override those of B; and where there is difference the consciousness of this is called Discrimination. Discrimination has implied Agreement, Memory, Association.

If it be disputed that Discrimination implies Agreement when we discriminate between a musical note and a billiard table, the answer is that we can discriminate in no closer sense than that in which Agreement exists. Here we have the succession of two different objects; but the fact that we consider them together as two objects already implies Agreement in some deep basis. That basis is discussed under the heading of Unit.

CHAPTER III—(*Continued*)

(2) *The Series is not Redundant*

NOTE: This chapter is not essential to those who have read so far and who will continue to read, without prepossessions, but it will be helpful to others in whose minds vague difficulties have occurred.

A criticism which has been suggested in regard to the Fundamental Processes is that the tabulation seems too detailed. There is no force in an objection of that kind, unless it can be shown in a particular instance that the series includes factors which should have no place there, or that it is in some respect redundant. I think it well, however, not to be content with refuting arguments, but rather to seek for some valid thought beneath the objection and to show its bearing upon the question.

The process of thought is often very rapid, and in thinking of ordinary affairs there appears to be a freedom, and even a simplicity inconsistent with the carrying on of a series of processes. Something of the sort may seem implied also in the position that the actual present thought is conceived as a Unit. Moreover, the new disturbing stimulus—sensation it may be—is appreciated as a Unit.

The expressions of such objections will help us to see the significance of these Fundamental Processes, and to demonstrate their necessity.

Suppose, that, in order to meet the underlying reserve we spoke of, we adopt for a moment the term Sensation with its popular meaning. Sensation embraces then Immediate Presentation, Unit, with something of Association, and Agreement. If now we give the term Memory a widened application, so as to include recollection with Association, Agreement, and also, though perhaps more vaguely, Generalisation with Classification and Symbolisation; and then if we form the popular synthesis of Will, including Feeling of Effort, Impulse, Hedonic Sense, with an underlying determinant from the other Fundamental Processes; while we speak of Time and Space, and all that conditions our experiences, as the environment; then finally we arrive at a means of speaking of the Processes of the mind, or of reasoning in general, as being a composite derived from Sensation, Memory, and Will, of a mind operating in a given Environment.

Now whether we accept this position or not, there is at least nothing in the notion that shocks us with a feeling of undue

complexity. We have been able thus to dispose of the criticism as to detail. The objection to such a position, however, is similar to the objection to Kant's formulation—the analysis is confused and incomplete, and the classification overlaps. When these objections become removed we are led again to the Fundamental Processes already set forth, and with that step we find that the original reserve in regard to these vanishes, while at the same time we have seen how it was possible for it to have risen.

But with a hint derived from these considerations we might form new modes of presenting. Avoiding the term Sensation, we might define a term to be used as lately as we had used Sensation, and call it Perception. Then we might form a new group of Memory, with its stored-up ideas, by virtue of Association and Classification, and the conjunction of all this with what has been termed Perception we might call Combination. Then the activity of the mind under this stimulus and interaction, in conjunction with Feeling of Effort, Impulse, Hedonic Sense, we might call adjustment. Then Reasoning might be defined in terms of Perception, Combination and Adjustment within certain universal conditions and special environment. There is nothing ostensibly absurd in that; it is clearer than most of the expositions found in books of Psychology or Philosophy, but the analysis is not basic.

Similar examples might be found by taking up almost at hazard a work of Psychology, and finding therein a treatment of reasoning.[1]

Suppose now that we take an illustration strikingly objective. Let the mind, with all its immense complexity of stored-up and inter-related ideas, be represented by a correspondingly complex mechanical system. Let a new external stimulus be represented

[1] Prof. Th. Ziehen in *Die Prinzipien und Methoden der Intelligenzprufung* speaks of faculties of Isolation, Complexion, and Generalisation.

Dr. Paul Kronthal, in *Nerven und Seele*, refers to the soul as "the sum of reflexes".

Prof. Störring, in a work which has been translated into English, *Mental Pathology in Relation to Normal Psychology*, speaks of "activity-feeling", which is a "condition of our guarding the spiritual self as within wide limits independent as against the material world."

G. Dwelshauvers forms a theory of synthesis at the base of mental life.

Freud, Jung, and their disciples, in psycho-analysis, build on Association, and by throwing this process out of its setting, fail not only to give a true account of the mechanism of thought, but contrive to write nonsensically even on Association.

Schopenhauer's famous work: *Die Welt als Wille und Vorstellung* suffers from similar incapacity of analysis.

A. Binet, in his work *La Psychologie du Raisonnement*, points out that Aristotle's formulation of the syllogism has injuriously affected the views of all subsequent thinkers—not merely Kant and J. S. Mill but even Herbert Spencer—in regard to the veritable processes of reasoning. Binet himself puts forward a "theory of substitution" but this is not at all convincing.

I refer to these particularly, for by comparison of any of them with the others we observe that in all the analysis is defective, and because each of these defective theories will find its veritable explanation in the course of this exposition.

by a projectile impinging on and entering into the system. Then let all the interactions of the mind, including such as we think of as associated with will, be represented by the interaction of the system. All this is considered as within a certain environment defined for the moment. The resultant of the forces of the system corresponds to the new activity of the mind. So that what we spoke of, though with insufficient accuracy, as Sensation, Memory, Will; or Reception, Combination, Adjustment; we can now describe in terms of Force, System, Resultant.

Once we clearly behold this graphic simile we see that, though not precise, it is in broad lines a correct representation. It might have been taken as a starting-point; and then the whole task of examination and exposition would have become the complete and precise definition of what corresponded to the mechanical processes.

Thus we are led to our Fundamental Processes with a clearer understanding why they seem complex, and why they are indissolubly united.

It is possible still further to simplify in appearance the process by which the mind proceeds in its course. Let me give first a curious objective example. I remember at one time seeing in Melbourne the following experiment: A small object was hidden in a warehouse in the city. A celebrated thought-reader [1] took as his starting-point the steps of the Town Hall. By means of a small light metallic chain one of his wrists was attached to the wrist of a man who knew where the object was concealed. The thought-reader, who was blindfolded, I believe, though that detail is really not essential, rapidly made his way to the place of concealment, and finally he discovered the object.

The explanation is that he had cultivated to a high degree the sense of touch, and the faculty of observation in such matters.[2] Whenever he showed a tendency to take a wrong direction, a slight indication, such as would be imperceptible to others, came by way of the chain, unconsciously tightened by the passive subject "concentrating" his attention. Now, just as in familiar games of children where indications are given by calling "hot" or "cold", it is sufficient to have at each point of progress affirmative or negative directions in order to arrive at the appointed goal.

[1] Mr. Stuart Cumberland.

[2] I have seen in various psychological publications excellent studies of questions of this sort. The conclusions correspond to what is here given. The names occur to me of J. E. Downer on "Muscle-reading", and B. Bourdon on the "Sensibilité tactile". A great number of physiologists and psychologists have written on a subject which has some connection with such experiments as that referred to above—tactile localisations. Names of authorities are Weber, Hering, Kottenkampf, Ulrich, Barth, Lewy, V. Henri, F. B. Dresslar, Pillsbury. Legerdemain, an allied subject, has been studied by Dessoir Külpe, Jastrow, Muensterberg, Pillsbury, and Triplett amongst others.

Reflecting on the whole circumstance it seemed to me that Reason might be represented by an operator who, at each turn receives the direction, " yes " or " no ", and so eventually reaches a desired conclusion.

Here then, in place of the apparent complexity of our Fundamental Processes, we have a very simple image. But let us examine this image to ascertain all that it implies. What is meant by saying " at each turn ? " That implies a crisis of experience. It implies Time and Space. It implies the experience. The simplest form of experience is that of Immediate Presentation of a sensation. " At each turn " implies also that the Fundamental Process of the concept of a Unit is involved, and moreover since there is movement this instantaneous Unit is linked or in some way related to a previous Unit. Here we are introduced to the Fundamental Processes of Memory and Association.

Then the directive is " yes " or " no ", at each turn ; it is possible that the directive may under other circumstance have been reversed. At this stage therefore we meet with the Fundamental Process of Negation.

But these directions are not given haphazard. The directive is given after comparison with some actual or implied chart of direction, or of something of which that forms the graphic picture. This comparison implies Agreement, and as the comparison implies also factors reproduced by Memory from the storehouse of the mind and represented by the help of symbolisation, we find here implied Generalisation.

In the fact of the operator's demand, either explicit for the directive, and in the state of expectancy and reception we have the Feeling of Effort ; and in the resolute setting out again, on receipt of the directive, we find the implication of Impulse. In the feeling, whether of satisfaction or disappointment, accompanying each of these acts we have the Hedonic Sense.

Accordingly from the consideration of this reduction to simplicity we are led again to the Fundamental Processes as already set forth.

It is easy to adapt this form of exposition to the image of a " chain of Association ". As a student, I once asked a German Professor[1] " if he could give me an analysis of Reason." He replied, " it is simply a ' chain of Association '." [2] This was not an analysis. But we have just observed in what way it might be made the basis of an analysis.

[1] Ebbinghaus.

[2] This substantially represents also the conclusion of John Stuart Mill's school.

CHAPTER IV

CERTAIN "POSITIONS" IN PSYCHOLOGY

NOTE: At this stage reference will be made to certain important positions usually set forth in works of Psychology, so that their relation to the series of the Fundamental Processes may be exhibited, and so that we may not have to discuss them when dealing with matters involving them.

Abstraction.

Abstraction may here be considered. Let us fix our notions, for example, by means of a sphere. The sphere has not only shape, but colour, mass, and a number of other properties associated with the material of which it is formed. But we may have another sphere of different material, and the associations are different. By Discrimination as before we arrived at a Generalisation; and in this way only is it possible, in that we still have before us a definite concrete something (A), from which a and $\acute{a}$ have been separated by Discrimination, but which possesses its own associations.

But Abstraction implies more than this. Abstraction implies dealing with some properties of an object, say a sphere, which is not conceived of as concretely existing. Generalisation in itself in its fundamental form is an inevitable, and partly unconscious, movement; but Abstraction implies a conscious recognition of the Process of Generalisation and also, not an inevitable, but a tentative exercise of separating certain associations and of conserving others.

Thus if in $A+a$ and $A+\acute{a}$ we disregard a and $\acute{a}$, we still have A as a concrete thing, say a sphere. Similarly in $B+b$ and $B+\acute{b}$, separating the incidental associations b and $\acute{b}$ we arrive at B, another sphere. Now we may consciously select and combine certain associations of A, and transfer them to B, and this with an active effort of disregarding associations arising from the difference between A and B; this is a process of abstraction.

Abstraction cannot get beyond the conditions formed by Association, and with it Dis-Association, Discrimination, and Generalisation, but it is not the same thing as the last step of inevitable Generalisation produced by Discrimination.

But since abstraction is so contained within the conditions of these operations we must seek an explanation of how it emerges

into something distinct. We shall find, as in further discussion on Discrimination (*cf.* pp. 50 and 160), that our mental operations are often determined, and, according to the conception we are compelled to form, even aided by the limitations of our faculties (*cf.* pp. 144 and 160). Thus if Discrimination were possible unto the absolutely distinct perception of the last intrinsic differences between things, and if the mind kept continually in activity all the associations formed by each of the elements, there would be no Generalisation, and there would be no Reason such as we now know it.

Now consider a line. A line is always a concrete thing—a piece of chalk drawn out in extreme tenuity, or a piece of graphite from a lead pencil, or the like. But apart from consideration of the material there is in drawing a line a certain form of movement given to the fingers, hand, and arm. It is also a fact that when the mind becomes accustomed to certain associations, these, being gradually formed with less and less effort, leave it free in its exercise to form new associations. So that the visual appearance of a line, associated moreover with a multitude of impressions involved in moving the ocular apparatus, become associated with the movements required in drawing it. Now these movements themselves are not very well defined in character, so that the movement in drawing one line does not leave in the mind an impression that enables it to be clearly discriminated from the movement involved in drawing a similar line. It is in the indefiniteness of such impressions, allowing the possibility of what is really a generalisation on a broad but vague basis, that we find the elements that in their combination give us abstraction.

Thus the notion of a straight line is gained by such acts as running the finger along the line, looking at it from one end to the other, or if it be represented as the edge of, say, a ruler, by turning the ruler round its edge as an axis and observing that the edge maintains its position. There are formed then in the mind associations with one straight object, and these are transferred to another object by virtue of a faculty of disregarding differences, that is to say by Dis-Association with similarities of processes not easily discriminated.

In the case of a sphere we have the combination of a series of associations, relatively complex, of resemblances. Thus, say, we have a hollow sphere; there is the recognition of a centre, the drawing of a line, perhaps represented by a rod, from the centre to any point in the sphere, and the observation that the rod remains of the same length while one extremity remains at the centre and the other is moved about on the periphery.

In all this, abstraction is greatly aided by the use of symbols, which become a strong bond for associations; but it must be remembered that symbols can only arise from the use, as has been indicated, of Association, Discrimination, Generalisation.

Thus, if we speak of a letter of the alphabet we must refer to some letter of the twenty-six. But our mental processes arrive at such a point of development that we can, by a conscious exercise of generalisation, form a phrase such as: A letter helps to form a word; and in this way we do not think of the particular form of any one letter. We have here an abstraction. But that employment of language depended necessarily on the generalisation, letter; then on the observation that, in a certain definite instance, a letter helped to form a word; then on the further observation that there was nothing particular in the circumstances of this letter, that is to say that Dis-Association might here operate in certain regards while the main associations necessary for the purpose in view were conserved; and it is mainly by virtue of symbolisation that we can make the transference of the associations we require while neglecting those we do require. Hence we arrive at an abstraction.[1]

An abstraction therefore consists in the associations of certain circumstances with a symbol, which symbol may be expressed by language, and which circumstances must be applied to certain concrete things, or the mental images corresponding, by virtue of conditions implied by the symbol, in disregard of other conditions not so implied.

* * *

The question has often been mooted as to whether the mental processes in the lower animals are similar to those of human beings. Many psychologists, especially psychologists of the more modern school, and including Herbert Spencer, believe that the processes are similar. The whole trend of the analysis of fundamental operations exhibited in this book would seem to point to like conclusions, but with this proviso, that the development of special

[1] Similar remarks would apply to the abstractions of mathematics, as for instance the formula of a curve involving the co-ordinates x and y. The formula has no meaning except in regard to definite particular instances of its application; that is to say, when definite values, in accordance with the formula, are ascribed to x and y. But the form of the symbols is the same, no matter what the particular definite values may be. In the simplest forms abstraction merges into what we have called Generalisation. The consideration of an ordinate x or y, or of any straight line, does not give us the simplest case. The meaning straight line involves a complex relation of elements capable of Immediate Presentation and combined by virtue of Fundamental Processes. This will be seen more clearly in the discussion of Externality. A Generalisation in regard to ordinates may be considered to refer to any ordinate, conceived of concretely, and in actual application to some definite ordinate. But any abstraction, as for instance of a straight line—length without breadth—not being representable concretely, really means applying a symbol for the processes of the mind themselves that operate on certain concrete things.

senses varies enormously in diverse animals, and such development affects the whole mental life.[1]

It may be questioned, therefore, whether an intelligent animal, such as a dog, could reproduce the process of abstraction. That would, no doubt, be impossible in regard to any high abstractions, for even with human beings a specialised form of language is required for such mental operations. But granting that a dog may exercise the Processes of Presentation, Discrimination, Association, and symbolisation, implying Generalisation, together with all the other processes herein involved,[2] then it seems possible to imagine a case where a rudimentary form of abstraction must necessarily follow. For example: If a dog be fed always at the same hour, and the hour be announced by ringing a bell, there then will be formed in the dog's mind strong associations connecting his mental impressions of the bell and the dinner. Now, suppose further that there are several bells, and that while their sounds, though resembling each other, are nevertheless distinct, then the dog would form the association of any one of these bells with his dinner. Suppose further that the dog did not see the bells, but that the movement of a latch, which he could see, always preceded the sound of any of the bells, there would thus be produced in the dog's mind the associations—latch, bell, dinner.

When these associations had become confirmed, we might arrange an instance where the dog was prevented from hearing, as, for instance, by having his ears plugged; then the movement of the latch would produce the associations of latch—bell—dinner, not quite so vividly as if he hear the bell really sounding, but still in the nature of most of our associations. The bell in this case would not be any special one of the bells. There would have already arisen in the dog's mind something similar to the generalisation: bell. The movement of the latch would have acquired the sense of a symbol of the whole associations. And then we should arrive at a fulfilment of the condition expressed in our consideration of abstraction in the case of human beings: An abstraction consists in the associations of certain circumstances with a symbol, which symbol may not necessarily be expressed by language, and which must be applied to certain concrete things, or the mental

[1] In this regard Hachet-Souplet's studies at the Institute of Zoological Psychology in Paris are interesting.

[2] Professor Lépinay, in a lecture delivered at the new Dogs' and Cats' Home near Reuil, Paris, expressed the belief that a simple kind of reasoning exists with such animals. The question has also been studied by Romanes, Lord Avebury, de Fouveau, de Courmelles, Fr. Houssay, E. Alix, Karl Groos, W. Wagner, C. J. Cornish, Fabre, and in an elaborate series of studies on monkeys conducted under the direction of Dr. Franz at the Government Hospital for the Insane, Washington. Most of the observers recognise in animals a manner of reason analagous to our own.

images corresponding, by virtue of conditions implied in the symbol and in disregard of other conditions not so implied.

In fact, proceeding from the associations of a symbol, the dog entertains the mental impression of a thing, which is not any one of the things in his immediate experience (the real bells), and which produces associations that are to be found in the case of each one of these things in his experience.

But the human mind in regard to the abstraction: sphere, moves similarly. It proceeds from a symbol: the word, sphere; forms mental impressions accordingly which are not necessarily identical with those arising from any particular sphere; then forms associations, that is to say, those of the properties of the sphere; recognising that these associations are to be found in the case of any one of the particular spheres considered.

Relativity.

The notion of Relativity[1] may here be referred to. Certainly it is true that all our knowledge is relative; that we cannot have an idea of the situation of a point in space absolutely; that we must consider it relatively to some other point. Our appreciations of sensations are relative to those that immediately precede them.

Further, in the more complex life it is certain that no two persons represent the external world alike, for there is much variation in the relative acuteness of senses, in association with other senses and with the complex impressions of effort.

Thus a person may hear the same objective sounds at one time faintly, and then, after some slight operation, say, on the ear, may hear them more loudly, and more distinctly, and more endowed with importance in regard to the impressions produced by other sensations and their combinations. He is then able to form a relative estimate of the previous impressions of the same objective sounds. But suppose that he had never experienced this change of hearing, then he would talk of these sounds as usual, apply the same names, associate them in the same way; but their standard in his mental life would be different.

But when we consider two persons, we should expect that the impressions brought by any of the senses would differ, even though relations persisted similar in any kind, but varied in the whole scale of importance in regard to other senses. That two persons are able to converse with close appreciation about colours,

[1] There is no special reference here to the use of the term Relativity in Einstein's theory. The discussion of that theory comes within the domain of physics, and it would be out of place here. Nevertheless in this Chapter and the later discussion of the axioms and on spatial relations, notions are introduced that underlie all theories of relativity.

pictures, sounds, music, does not imply that their sensations are not different, but simply that these sensations are sufficiently similar to allow of the same symbols, and that they possess similar relations to the complex of other sensations with their associations in the mind.

Thus, to give an example simply to make the meaning clearer: Two persons may each be looking at a map of a country, but the two maps may differ in their scale, in the relative size of the letters they apply to various towns, and also in the shade and vividness of the colours employed, though the colour schemes be similar. These two persons might converse about their maps without discovering that they differed in any particulars. Thus the principle of relativity runs through all our mental life.

Subjectively, the notion in its fundamental aspects is contained in that of Immediate Presentation. Relativity, apart from this, is an objective condition of our mental appreciations, but not anything subjectively recognised. We find such conditions, for example, in the sense of sight. If the undulatory theory be true, a necessary condition for the perception of a distinct object is that the vibrations of ether which it sets up should reach the eye, and that the undulations should be within the limits of a certain scale; if the emission theory, nowadays seriously reconsidered, be true, then minute particles from the body, within a certain scale of dimensions, reach the eye; but the sense has no direct information as to these conditions. Relativity is a condition of ideas, even when they are the simplest, but there does not seem anything in this position which, in regard to Fundamental Processes of the mind, would render inadequate the tabulation adopted.

CHAPTER V

THE FUNDAMENTAL PROCESSES CONSIDERED WITH REFERENCE TO THE DIVERSE SENSES

NOTE: We will now consider the Fundamental Processes in regard to the senses in turn, for their special characteristics will introduce us to new data This entire chapter could be skipped by those who desire only to see the skeleton structure of the exposition; the object of the chapter is to show how that skeleton becomes clothed with graphic forms, so that our Psychology becomes closely referred to our actual mental lives.

(1) *VISION*

In visual sensations it seems certain that particular unanalysable, Immediate Presentations are those of colour,[1] intensity of

[1] A curious and interesting study is this: By what process do we attain a sense of colour in general, so that although red things and green things are separate objects, we classify them under the idea of colour Even if we cannot solve such a problem the fact that it exists may convince us how profound, beneath even the processes of our conscious life, are the conditions which give rise to Agreement and Generalisation.

With regard to all that is involved in sensations due to coloured objects, Dr. F. W. Edridge-Green calls attention to distinctions between light perception and colour (*Cf.* Dr. Edridge-Green's *The Physiology of Vision* Series; and various articles in the *Proceedings of the Royal Society*).

Sir Ray Lankester, in one of his studies in *Science from an Easy Chair* indicates in what way vision has become developed from its rudimentary form, the perception of light.

Grant Allen in *The Colour Sense, Its Origin and Development*, puts forward the theory that the taste for bright colours has been derived by man from his frugivorous ancestors, who acquired it by exercise of their sense of vision upon bright coloured foodstuffs. One may argue quite convincingly on these lines, if all the uncertain and undetermined factors be conceded to operate in favour of the theory. But in an excellent little book on *Matter and Energy*, Professor F. Soddy points out that the greatest sensitiveness to colours corresponds to those parts of the spectrum where the light energy is highest. Upon this fact one might build a theory of development distinct from that of Grant Allen, and even more convincing.

These examples serve to show how dangerous it is to draw conclusions from arguments which are not entirely rigorous. Yet the reasoning both of Grant Allen and Soddy are models of cogent argument compared to those found in the famous works of Kant or Hegel, Schopenhauer or Bergson.

The general question of the course of Evolution has been illuminated, but certainly not finally answered, by the principles of Darwinism. With regard to the development of sight, Sir Ray Lankester has explained two quite diverse modes of development of the organ of vision which have terminated in similar results.

The development of vision in molluscs and vertebrates diverged long before the complete evolution of the eye, yet the process, though by adaptation of different structures, culminated in similar results. *Cf.* also Saint-Saens on the snail's eye, which though a rudimentary organ of vision seems to be used in feeling. (Note in *Revue Scientifique*, 1912).

Bergson, who writes interestingly when kept in hand by the discipline of positive science, has pointed out the concordance necessary of two lines of development in the eye, one by which the outer structure is prepared to allow the rays of light to pass, and the other by which a process from the central nervous system is adapted to receive the impressions conveyed.

This kind of co-ordination has been in operation not only in regard to what one may call the main plan of development in the case cited, but also in regard to successive modifications. Dr. Mott has dealt with the matter in an article in the *Archives of Neurology: The Progressive Evolution of the Structure and Functions of the Visual*

illumination, form, duration.[2] There are also included certain of the elements which form our appreciation of motion and of externality, which will be considered later.

We are fortunately enabled by physical science to assign to some of our fundamental perceptions certain objective correlatives, as, for example, that our sensations of colour depend on the frequency of the waves of light, red corresponding to 400,000,000,000,000 undulations per second, violet corresponding to 750,000,000,000,000 undulations per second.[3] These correlatives in the physical world permit us to make analyses and discriminations in an objective way far beyond the limit of direct Discrimination by our senses. Discrimination, though fundamental, is thus seen to be limited in application, and also as containing, beneath the limit of our Discrimination, a source of fallacy. Thus A and A' are two

Cortex in Mammalia. In *Felidae* Dr. Mott notes a specialisation of the fore limbs for prehension of the prey which would be less effective for the purpose without stereoscopic vision. In the case of the chameleon, the great Spanish histologist, Ramon y Cajal, has found that in the central area of the retina the cones become more delicate, and each cone is connected with a separate tripolar cell, and this again with a separate ganglion cell giving off an optic fibre. Dr. Wilfrid Harris has pointed out that this development is correlated with a special motor adaptation which is of service in seizing prey. M. Rochin-Duvigneaud, summarising the results of Ramon y Cajal's researches, says that Darwinism is inadequate to their explanation. Dr. Mott and Professor Sherrington, however, meet with no difficulty in solving the problem on Darwinian lines. The reason of these discrepancies is to be found, as has been already shown, in the lack of precision in application of the principle itself.

Professor A. C. Geddes, (1912) (now Sir Auckland Geddes) speaking of certain theories of bone formation advanced by Ranvier and Muller, and more recently by Retterer and Sir W. Macewen, said: "If this idea of the osteoblast ultimately be proved correct, it will be necessary for us to revise almost our whole conception of the course of organic evolution, of all histories and cell lineages."

Yet some one or other theory, we may be assured, will be accepted with equanimity by Evolutionists, for there has grown up in regard to this a manner of reconciling incompatibles which calls to mind the old theology.

The eyes have been developed in such a manner that the best conditions of vision correspond to the greatest energy of the rays from the sun. No doubt the principle of Natural Selection may operate here but what is it that has given the impulse to variations in that direction; and what is it that, with this mobility of variation, keeps our spectrum so well defined in its narrow range? These are questions to which no theory has yet given a satisfactory answer.

When various organisms submitted to the influences of the same environment develop each in its characteristic form, there is in operation, not merely a natural selection of favoured organisms by the environment, but also, and more markedly, a purposive selection by the organism from the environment of the favouring forces and material.

[2] Duration must be considered a necessary condition, and therefore, an inevitable presentation, of sensation. With regard to the use of the word Immediate, the difficulty has only to be mentioned in order to cause it to disappear. Immediate here does not imply a point of time without duration.

Bain gives massiveness as a factor, or attribute, of sensation. I think that in the case of sight this is compounded of intensity and extensity (form). The matter is made clearer by considering the objective conditions. The vibrations (adopting for a moment the wave theory) have frequency corresponding to colour, and amplitude corresponding to intensity. Any other psychic effect must be due to addition of vibrations, and hence introduces objectively, at least, extension. This objective manner of regarding will enable us to criticise the force of a remark by Edmund Hollands in the *American Journal of Psychology*, where, speaking of clearness, duration, and extension, he says Wundt refuses to class these with quality and intensity as attributes of sensation.

[3] These are the figures as usually given, but see note, page 52.

separate things, but if the difference be beneath the limit of our Discrimination we consider them as the same.

This point is of great importance in considering the foundations of belief, and the ultimate sanctions of reasoning. Our limitations enable us to build up a practical and intellectual life, served by the process of reasoning; but it must also be borne in mind that it is because of our limitations that such a mode of arriving at results is necessary.

The question of intensity of illumination as perceived by the visual sense leads us to that of the quantity of light as measured objectively, and here we find also that the means of measurement in this objective way are far more precise than those of direct visual appreciation.

The Hedonic sense is involved in the sensations of sight, though possibly less marked than in other senses. That it is so involved, however, is made evident by the fact that it is capable of being greatly developed, as by artists, and by the fact that most persons are pleased by certain combinations of colours. We may be satisfied also that this Hedonic sense exists when we consider great quantities of colours. For example, even in imagination we find the difference in this respect between a sky of soft azure hue and a sky of scarlet red.[1]

The tastes of artists, their appreciation of harmonies of colours, their delight in blending certain effects, cannot be cited as pure examples illustrating the Hedonic sense. Other considerations here intervene, such as the various associations involved in the word *métier* or profession, the sense of virtuosity in the handling of colours, the desire of experimenting for new effects, and the factitious associations therefrom arising; and also the cant of different schools of painting. The sense of sight perhaps gives us more pleasure than any other, but that again depends on a multitude of factors apart from the direct Hedonic sense.

The sense of sight is generally assumed to be the keenest in Discrimination within its range, but that range only extends a little beyond 400 millions of millions of vibrations a second for red to 750 millions of millions of vibrations per second for violet;

[1] Bain and Wundt are amongst those psychologists who find in sensation that attribute here definitely thrown into relief as the Hedonic sense (*Cf.* (note) p. 37).

Mr. E. Bullough gave an interesting study of this question in a paper: *Perception Problem in the Aesthetic Appreciation of Single Colours* (*British Journal of Psychology*). He enlarged the scope of the study in a communication to the British Psychological Society (1910): *The Aesthetic Appreciation of Simple Colour Combinations.*

Many experiments have been made in cases of insane persons showing that the influence on the mind of the colour of dwelling-rooms is appreciable. (*Cf. The Language of Colour*, M. Lukiesh, New York, 1918.) E. Haas (1923) confirms Broca and Sulzer in finding that blue causes fatigue the most quickly green, the least.

whereas the range of vibrations for some sounds perceptible to human senses is from 30 vibrations per second to 20,000 vibrations per second.[1] The medium of transmission differs—ether in the case of light, air in the case of sound—but what we are concerned with in sensation is not the medium itself, but the effect upon the organs of the nervous apparatus concerned in the sense.

(2) *HEARING*

It will be hardly necessary to discuss Hearing in terms of the fundamental operations in turn.

The faculty of perception has a very great range in this sense, as we have seen. The faculty of Discrimination within that range is not usually considered relatively so acute as in the case of sight. Thus, for example, while the whole range of vibrations which give rise to hearing extend from 30 to 20,000 vibrations a second, even the highly trained ear cannot well detect differences of 1/32 of a note; yet all the colours and their shades are contained within a scope of which the lower end represents more than one half the vibrations of the upper end.[2]

If we follow objective indications it would appear that pitch in a note corresponds to colour, since both depend on the rapidity of vibrations. Loudness would correspond to intensity of light. Timbre would correspond to shade of colour.

Association between the senses of sight and hearing is not merely that of actual experience, for there appears to be a deeper sort of

[1] More precise determinations give, for vibrations of the extreme visible rays, 395 millions of millions of vibrations, and 758 millions of millions of vibrations, per second. For hearing, the numbers given above are those of Chladni, but other *savants* show a more restricted scale. Biot takes the extremes as 32 and 8,192. Young gives 18,000 as the upper limit. This great range accounts for the fact that it is not possible to represent, for instance, a musical score by a spectrum of colours, although a certain relation between sounds and colours exists. This matter was discussed by Professor Sylvanus Thompson in a series of lectures on Sound at the Royal Institution (1911).

[2] Dr. Edridge-Green, who has carefully investigated the matter, finds that most normal-sighted persons make about eighteen monochromatic divisions in a bright spectrum; that is to say, there are found effectively eighteen colours. This result will be surprising to many, and Lord Rayleigh believed that his power of discrimination of colours was much finer. Edridge-Green, however, attributed the discrepancy to various disturbing conditions, as, for instance, the admixture of other light, in Lord Rayleigh's experiments. In his paper read before the Royal Society (1911) he refers to a former interesting examination by Helmholtz of Brewster's results in regard to three kinds of solar rays.

The discrimination of 1/32 of a note will also seem excessive, for many normal persons can hardly discriminate two succeeding notes. On the other hand J. Kerr Love, who has studied this subject, says that an untrained ear, or slightly trained ear, can detect 1/6 to 1/40 of a semi-tone, and a trained ear 1/64 to 1/80 of a semitone. According to George Gore, a practised ear can distinguish between 1,209 and 1,210 vibrations of the tuning-fork. These figures would indicate that discrimination in hearing is relatively sharper, as measured by objective tests, than in seeing; but the figures seem exaggerated for hearing. The explanation of discrepancies is partly to be found in the value of practice. It will be generally correct to say that for gross determinations the sense of sight is more constantly exercised; but that for the finer discrimination of experts the sense of hearing is more highly trained.

Association dependent on our physical constitution. Thus persons sensitive to impressions indicate certain associations between colours and the notes of various musical instruments, and there is a strong conformity in these indications when coming from different persons. A red colour is usually associated with the sound of a trumpet, a blue colour with that of a flute.

From objective considerations one might expect that a certain pitch of the note would rather correspond to red, and another to blue; but various other conditions enter into the matter. Thus the difference of timbre between two musical instruments is to some extent a question of pitch, for the timbre, as shown by the researches of Helmholtz, and more recently by the experiments of M. Devaux-Charbonnel, is determined by the value of the overtones, viz., in corresponding higher octaves, varying in various instruments. These overtones affect the shading of the note; and as the discrimination in respect to them is not clear in the sense of hearing, the differences corresponding to shading of notes become vaguely expressible in terms of difference of colour. Moreover, questions of the loudness of the notes are involved, and the fact that certain notes seem to set off the characteristics of an instrument better than others. Thus a loud, braying note on the trumpet would correspond to a red; a clear, soft, flute note of higher pitch to blue.[1]

That the intensity of light is associated with loudness of sound is recognised by most sensitive people, and particularly by those in certain conditions of nerve instability. Thus an idiot boy cited by Maudsley complained of the strong light of the full moon, describing it as "so loud."[2]

M. Hachet-Souplet, of the Paris Institute of Zoological Psychology, has in a series of observations and experiments furnished proof that certain animals have senses far more highly developed than human beings.[3] Thus, for example, the wolf hears sounds

[1] This corresponds to my finding, and is more usual, but there are great differences of appreciation with regard to these correspondences. The subject, under the title of colour audition, or synaesthesia, or pseudochromaesthesia, has been zealously studied of late, and considerable literature referring to it has already accumulated. Goethe referred to the matter. Hoffmann, a Swiss magistrate, studied it at the beginning of the last century. Since then Dr. Sachs, Wartmann, Fechner, I. Nuel, Von Kries and more recently Mary Whiton Calkins have contributed to our knowledge of the matter.

[2] The above remarks are put forward as suggestions of factors that are validly concerned in this matter, but the great secret probably resides rather in conditions of the development of the nervous system throughout the whole range of circumstances external and internal.

[3] Galton conducted a series of such experiments with an instrument devised by himself for increasing the pitch. At the London Zoological Gardens he found the animals of the cat-tribe to excel in hearing. *Cf.* O. Kalischer *Eine neue Horprufungsmethode bei Hunden* (*Sitz d Rge. A. C. d. Wiss*, 1907). W. T. Shephard: *On Sound Discrimination in Dogs.* (*American Journal of Psych:* 1919). *On Sound Discrimination by Cats. Journal of Animal Behaviour*, 1914).

that are imperceptible to human ears. Birds of prey have vision far more acute than men. Frogs have a certain sense so marvellously developed with regard to humidity that they are thus enabled to guide themselves towards distant places where water may be found. The recognition of such facts, together with those also of the greater importance to animals of the associations of their prodigiously developed senses, will help to explain many observations dismissed under the title of " Instinct " ; and to show how a continuous path may be found between the instinct and the highest reaches of the reason.[1]

The indications of Generalisation, classification, and symbolisation of sounds are not hard to discover. The notes of different instruments are not only distinct in themselves, but there are fine differences in the quality of the note sounded by musical instruments of the same kind but differing slightly in structure, even when the difference of structure is not perceptible to the eye. The note produced by a Stradivarius in constant use has a quality, which a musician would detect, superior to that which the Stradivarius would possess if rarely played upon. Yet though this difference may be appreciable, it is of another kind to that existing between instruments of different kinds. Hence we obtain classifications, following upon Generalisations and Discrimination.

Impulse may be well recognised in the case of musical sounds. If we take the case of a troop of cavalry accustomed to charge at the sound of some bar of warlike music, we should find that the first note would already produce its effect in causing the troop to prepare. A complete exposition of the aesthetic side of music would require not only to take such factors into account, but to relate them to questions of the mutual reactions of emotional and physical conditions.[2]

Memory, with regard to sounds, especially sounds in association, is in ordinary experience remarkably tenacious. A voice is remembered after an interval of years. But in this case the exercise of pure Memory of sounds is masked by the associations. This will be made apparent if we consider the difficulty one finds in

[1] *Cf. Animal Intelligence* by E. L. Thorndike.

[2] Famous philosophers, and among them notably Schopenhauer and Spencer, have sought in vain for an explanation of the effects of music. Professor Sarron more recently has believed that the great virtue of musical harmony is to produce a certain rest from distracting influences, but this explanation is not satisfactory. In *Art's Enigma*, Frederick Jameson discusses this matter interestingly. He finds the secret in " artistic imagination ", but he does not tell us very clearly what is artistic imagination.

None of the observations hitherto, and this remark applies to the whole system of Aesthetics of Croce, have struck deeply enough into the veritable nature of the mind's constitution, in its relation to the body, and the further relation to concordances of the external world. The indications of a true aesthetic in such matters will be found in the study of such suggestions as that offered above, and in the persistent meditation on the Hedonic principle in simple cases.

repeating the sounds of words of a foreign language. With regard to passages of music not only are the factors of Association generally strengthened, but other factors come into play, such as the response to rhythm and certain expectancies created by familiarity with common forms of music.[1]

The Sense of Effort will be observed if we take the instance of one expecting to hear a cannon fired, and if we compare this with the condition of expectancy when a note is about to be struck on a lute. The same Sense of Effort may also be observed in cases of accumulated efforts or repeated efforts. If a person listens to a simple sound repeated at short intervals it will be found that the sound drops occasionally from consciousness.[2]

A distinguished alienist, Dr. Hyslop, formerly of Bethlem Hospital, considers the mental fatigue caused by the incessant noises of London as a factor contributing to insanity. The appreciation of music also depends, amongst its multitudinous factors, on well-graduated and harmonised efforts of hearing.

The Hedonic Effect is evident in sounds, for not only are sounds immediately pleasant or unpleasant in themselves, but as music is formed by the combination of simple sounds there must necessarily be Hedonic effect in these sounds. The total effect of music cannot, however, be ascribed even to combinations of such pure elements. The truth of this may be perceived by reference to the effect of passages of poetry where the musical quality may be faulty, but where powerful feelings are stirred by the immediate images evoked, and by the deep associations they suggest. In music also there is an effect, which should be taken into account, of the immediate physical impacts of the undulations arriving in well-modulated harmonic vibrations (*Cf.* p. 236 and note).

Time is an invariable incident in the exercise of hearing. With regard to no other sense do we find, apart from the influence of other associations, such a clear appreciation of succession.

Space enters as a distinct element in every sensation of hearing.[3] We are so accustomed to associate our notions of space with the visual sense that we do not always remark the importance of the space relations in sounds. It is true that we recognise at once whether a familiar sound is near or far, but we are inclined to attribute our knowledge in this respect to past experience. Such

[1] *Cf.* H. F. Adams: *The Effect of Rhythm on Memory.* (*Psych. Rev.*, 1915).

[2] Here, however, a great many factors enter into the matter. I have observed in many trials that the sound does not fade away as the result of a gradual diminution. It would seem as if the ear had some sort of faculty of accommodation. Fernet, in the *Semaine Médicale* (1911), discusses the subject interestingly, and points out the importance of the action of the muscles of the middle ear.

[3] *Cf.* B. Bourdon: *Recherches sur les perceptions spatiales auditives, l'Année psychologique*, 1920.

experience is very helpful in enabling us to form just estimates of distance in connection with sounds, but similar remarks hold true with regard to the visual faculty.

It will be seen, in the analysis of Externality, that a great many factors combine with the Immediate Presentations of vision to give us our notion of space. It is evident in the case of hearing that the single unaided sense is inadequate to give us just notions of distance, and this consideration may help us to appreciate the analysis of Externality as presented mainly through vision. But it is also conversely true that a certain element of the notion of space is involved in hearing.

But sounds are not merely near or far. They have in the ordinary language "volume", that is to say, they convey the impression of volume in the external source of stimulus. Poets speak of the "thin" piping of a flute, of the "long, sequacious note" of the violin, of the roll of the organ, of the waving texture of the music of Wagner's *Valkyrie*. Keats, so often acute in introspection, speaks of "music that comes swooning over hollow ground". Human beings in civilised life generally find the sense of vision adequate for most of what they require to know of spatial relations, but savages and some of the lower animals have extensive fields of space relations also built up through the sense of hearing.

* * * *

(3) *TOUCH*

A detailed discussion of each sense in turn in its relation to the Fundamental Processes of the mind would take too much space for our present requirements. A suggestion, however, is hereby afforded for particular studies in this respect which will probably be found more and more interesting in proportion as they are further pursued.

In the tactile sense the attention may be called specially to the Hedonic principle or quality. By our sense of touch we are informed as to whether objects are hard, soft, solid, or liquid.

Some sensitive persons are greatly affected by peculiarities of touch. I knew a young man, a poet, who experienced peculiar feelings when he touched velvet. There is generally a direct pleasure in touching furs, and particularly in stroking the glossy fur of a well-groomed animal.

The sense of touch is not a single sense. It is the generic name for the various senses of touch in the different parts of the body, and these senses vary greatly in the fineness of their perceptions and in their quality.

A considerable number of experiments have been made in regard to the acuteness of the sense of touch, and it has been ascertained that the most sensitive parts of the body in this respect are the tip of the tongue and the tips of the fingers.[1] Thus though touch gives us information respecting space relations the information may be often defective, and in all cases it is seen to be not precise within the limit of the Discrimination of vision.

But in this respect touch differs only in degree from the other senses. From objective science we know that touch depends on the " end-organs "or tactile apparatus developed at the extremities of the terminal filaments into which the great nerve trunks ultimately branch. In some parts of the body these organs are more finely developed, and they are also closer together than in other parts.[2] But similar conditions hold with respect to eye-sight and hearing.

In the visual organ the sensitive portion is confined to the retina, which forms a relatively very small part of the human body, and the retina consists, as far as its sensitive elements are concerned, in an assemblage of organs, closely packed together, representing the terminal developments of nerve filaments, and consisting of a complex arrangement of " rods " and " cones " and nerve fibrillae.

The manner of operation of these rods and cones is not understood, but it is evident that some agitation of the organ is the usual preliminary to a visual sensation, and it is equally evident that the stimulus to this agitation is an impulse of " light ". What is the exact nature again of this stimulus we do not know; we are not even sure whether, as Newton supposed, and some modern philosophers have thought possible, the stimulus is derived from corpuscles emitted from the luminous body, or whether, as Huyghens and Thomas Young and Fresnel thought,[3] and most students of science since Young's days have believed, the stimulus is due to the vibrations of a pervading ether which is perturbed by the source of light.

[1] I refer the reader to text-books on Physiology, and particularly to the admirable work of Weber, *Tastsinn und Gemeingefuhl* (149 in Ostwald's series), which laid the foundations definitely in this branch of science But since Weber's time a great deal of work has been done in various countries, as by Hering, Funke, Volkmann, Vierodt, Kottenkampf, Ulrich, Barth, Lewy, Titchener, Dresslar, Pillsbury, and V. Henri. *Cf.* p. 41 (note). Goldscheider has devoted himself to this and kindred studies. J. Bernstein in the *Naturwissen. Rundschau* discusses a *New Theory of Sensation of Touch. Cf.* G. Störring: *Experimentelle Beitrage zur Lehre vom Gefuhl Arch. f. d. ges. Psych.*, 1905.

[2] *Cf*: *A Qualitative and Quantitative Study of Weber's Illusion* by Mabel E. Goudge (*Am. Journ. of Psych.*)

[3] Grimaldi and Hooke gave expression to this hypothesis before Huyghens. Thomas Young revived it in opposition to Newton's corpuscular theory. Fresnel conceived it more clearly in the manner now generally adopted, and with marvellous analytical skill gave it the modern mathematical form.

The retina is more closely packed with rods and cones and more highly developed in one region—the macula lutea—than nearer the periphery. It will be understood, therefore, when we affirm that there is not one sense of touch but innumerable senses distributed over the body, we do not in this way express an exceptional character. The sensation of vision depends on the co-operation of a number of separate organs each performing its own function. This is more strikingly shown when we consider both eyes. The impression made on the right eye is quite distinct from that made on the left eye by the same external object. The two impressions are not even fused together quite accurately, and it is indeed to this circumstance that we owe in part our notion of "solidity" or relief in the outside world.

When one eye squints it may form its image so entirely distinct from that of the other eye that no fusion occurs at all. Squinting may be caused by a great variety of causes but it is often due to the inefficient working of one of the muscles that direct the eye-ball, such as the external rectus or internal rectus. The reason of the formation of different images is in this case evident.

Accordingly when the whole scope of the visual faculty is considered, the muscular apparatus directing the eyeballs should be considered as part of the organ of sight. In the operations of sight with respect to determining our notion of Externality we find that such factors must be combined with many others before the ultimate impression is attained.

The sense of hearing likewise depends on the co-operation of a great number of complex parts, and even the end-organs—the organs of Corti—have varying degrees of development and concentration in that part of the apparatus which they occupy.

The fact that the sense of touch shows such independence of function of various parts, and such disparities of impression, is revealed to us by the sense of sight. We see that there are two needle-points resting on a part from which is conveyed the impressions as if from a single point. We have no sense which we can immediately direct upon the visual apparatus to give us similar information.

The sense of touch, though less acute in regard to certain aspects of the external world, has perhaps a stronger Hedonic effect. The very words "comfort" and "luxury" are associated more or less with this sense.

It also appears, and the remark is important, that as any sense takes less heed of the fine discrimination of its more sensitive apparatus, and becomes interested in the more massy general

conditions of agreeable contacts, the Hedonic effect is brought more into evidence.

(4) *SMELL*

The sense of smell has no doubt been enormously diminished in human beings during the course of their civilisation, and it differs widely in different individuals. Its Space relations are weaker than those of vision or hearing, and its acuteness within its range of Discrimination is generally considered less.[1] On the other hand its Associations-incidence and its Memory are often more tenacious.

Its Hedonic effect is also greatly marked. Wordsworth is said to have been very deficient in the sense of smell, but on one occasion he smelt fresh violets and remarked that it was like a glimpse of heaven.[2]

There is no doubt that the comparative deficiency of this sense is due in part to want of cultivation. It would be advisable to obtain by study and experiment a finely graduated scale of odours, and it would also be desirable to obtain a plan of classification of odours by their qualities. This would in turn afford the basis of an aesthetic use of the olfactory sense after the model of the development of the sense of hearing in music. The effect of music is only in part due to the immediate appreciation of the sounds by the ear; and as the Hedonic effect is originally even stronger in the sense of smell there seems to be no reason in nature why a distinct powerful art should not be built up on a gamut of odours.

The olfactory sense ought also to be investigated with a view to determining whether the appreciation of odours sums up its functions. The sense of pungency, as of various acids and of smelling salts, must be distinguished from that of odours. The sensations produced by the presence of noxious gases, the peculiar effects of chloroform or ether, even as affecting the olfactory sense

[1] Jacques Passy, whose studies in this sense were admirable, found it to be "the most sensitive, but on the other hand the least precise" of all. Individual variations are, however, enormous in regard to odours.

[2] Milton, in a fine passage of *Comus*, uses an expression derived from the sense of odours to reinforce the power of his description of music. *Cf.* p. 53 and (note); and p. 60 (note):

> "At last a soft and solemn-breathing sound
> Rose like a steam of rich distilled perfumes,
> And stole upon the air, that even Silence
> Was took ere she was 'ware, and wish'd she might
> Deny her nature, and be never more,
> Still to be so displaced. I was all ear,
> And took in strains that might create a soul
> Under the ribs of Death."

or some sense referred to a wider region, differ from those either of pungencies or of odours.[1]

(5) *TASTE*

The sense of taste is relatively not strongly developed with regard to Discrimination within its scope. On the other hand it is powerful in its effects of Memory and Association. Its Hedonic Effect is also relatively very strong, so much so in fact that in the perversion of the doctrines of Epicurus, such as we find in decadent modern societies, the importance of this sense has become exaggerated.

It is deserving of study, however, in a scientific way, and for this purpose it would be necessary to establish a scale of taste as accurately as possible. The quality of tastes differs widely, and here also accurate studies with nomenclature referring to good standards are called for.

The sense of taste will be found to be more complex than generally imagined. Alexander Bain calls attention to the distinction between tastes and relishes.[2] The discrimination can be carried much further, and it may be said that several senses are comprised under the generic term.

[1] Associated with Jacques Passy, as well as both before and after him, have been many *savants* who in the study of odours have given us a charming chapter of science. The principal names are Weber, Paulsen, Zwaardemaker, to whom we owe the invention of an olfactometer, Kayser, Funke, Aronsohn, Fischer, Petzoldt, Zuckerhandl, Patrizi, C. Henry, E. Morin. G. E. Muller has studied the Eindringlichkeit or insistency (to adopt Passy's term) of odours, by testing the minimum quantities necessary to excite sensation. Bailey and Nichols have worked in the same direction. Lombroso and Ottolenghi have tested this sense in criminals. Lehmann and Dr. Wm. Elder have studied memory in regard to smell. Eleanor A. M. Gamble has experimented on smell in regard to Weber's law. Exner and Lustig, working by aid of histological methods found all cells in the olfactory region to be sensitive. Classifications of odours have been given by Linnaeus, Geissler, E. Rimmel, and also, on the basis of chemical composition, by Berthelot.

For the work of Jacques Passy, the *Reliquiae* edited by his father may be consulted, as well as *L'Année Psychologique*, 1895–6. This may well form an introduction to the whole subject.

[2] Since Bain's time a considerable amount of research has been carried out with regard to the sense of taste by Ranvier, Engleman, Retzius, and Tanzi, among the histologists; and recently by Ponzo by experimental methods. Experiments have been carried out in Smith College, U.S.A., by A. H. Pierce on the effect on hearing of the stimulation of the sense of taste. Dr. P. Michelson has studied the sense of taste in the larynx. Bailey and Nichols have experimented on the delicacy of the sense. Lombroso and Ottolenghi continued their researches in the case of criminals with respect to this sense also. Camerer studied Weber's law in regard to taste. Keppler, J. Corin, and Karl Rittmeyer have devised various experiments. N. Cybulski and H. Beck found in a clinical case of extirpation of the tongue that taste persisted for sweet, bitter, and sour substances, but not for salt.

Cf. also *The Sense of Taste* by H. L. Hollingworth and A. T. Poffenberger (New York, 1917).

(6) *NEED OF ACCURATE STUDY OF THE SENSES*

Accurate study in regard to these special senses is still a need in the science of psychology. Most of the experiments in experimental psychology deal with the visual sense. One interesting fact, which lies at the threshold of the study, is that in visual sensations one-tenth of a second has been found to be the time required for the objective stimulus to make its full impression, and that one-tenth second is the period during which the impression persists.[1]

Similarly studies should be made in connection with all the other senses. Such data as would thereby be afforded would assist, in the establishment of a definite art of odour impressions.

The time during which the external stimulus must act seems to be longer in the senses in which Discrimination is less acute and the time of persistence of the impression is correspondingly longer. The Time element is thus less precisely marked. Correspondingly too we find the Space impressions becoming vaguer. The elements which are most concerned in intellectual development, Discrimination, Classification, are less marked in these senses. On the other hand, Memory and Association are strong with regard to them, and Impulse, Feeling of Effort, and the Hedonic Effect more remarkable than in the senses of finer discriminations.

This is well in accord with the necessities of nature. For the senses of the strongest Hedonic effects are those most fundamentally related to the necessities of life; and even in regard to comparisons in the individual senses themselves the strongest impressions have similar relations. Certainly the visual sense is of great importance in the preservation of life, although not vital; but a good deal of the development of its discriminative faculty is not so closely related, as in the indications of other senses, to the fundamentally physiological conditions of sustenance.

The five senses do not include all our sources of communication with the outside world. The distinction, for example, between tastes and relishes may lead us to inquire more closely in how far the other senses as usually accepted may have their functions subdivided, and also whether the different kinds of information they convey may not be subserved by different kinds of nerve-

[1] In Ribot's *Les Maladies de la Mémoire* some figures are given with regard to other senses, while those for light are taken higher. Wundt's results for reaction times have been generally accepted, but the celebrated Italian psychologist, Sergi, finds differently. E. Haas (1923) found that a luminous impression on the retina required 12 to 20 hundredths of a second to produce a distinct sensation. What is here suggested, however, is that the work should be carried out systematically on the basis of good classification.

endings. Already in this respect a difference has been established between the functions of the rods and cones.[1]

(7) *OTHER SENSES*

In addition to such possible subdivisions, we have other distinct senses, certain of which should hold equal rank with those of the five. The sensations of heat and cold in no wise resemble any of the other sensations. The seat of their end-organs no doubt is in the skin, but these sensations are not to be confounded with those of any kind of tactile impressions, and in fact they are subserved by a distinct sensory apparatus. The nerves conveying the impressions of cold are considered by some physiologists to be distinct even from those of heat.[2] Nerves of pain have been distinguished in recent years.

The examination of this part of the nervous system has advanced but little, but the general remarks regarding the increase of the Hedonic quality with the diminution of the acuteness of Discrimination will here also apply.

It was for long assumed that the organs of the ear were concerned in hearing alone, but the anatomical observations of the ear, and particularly the examination of the semi-circular canals, suggested other functions. The semi-circular canals are arranged approximately in three planes at right angles to each other. The analogy of the three planes to which Cartesian co-ordinates in geometry are referred would naturally suggest that the function of the semicircular canals has to do with the determination of position, and hence of the balance of the body, and of orientation, or estimations of direction.[3]

This supposition is strengthened by the fact that in homing pigeons the semicircular canals are enormously developed.[4] It is

[1] In the macula lutea (yellow spot), which is the most sensitive part of the eye, the proportion of cones to rods is greater than elsewhere. In the fovea there are no rods. The fine histological work of Ramon y Cajal, Ranvier, Schultze, Retzius, and others has revealed to us the complexity of these organs of vision, but hitherto not much has been ascertained precisely of their functions.

[2] Goldscheider, however, came to the conclusion that the difference of the sensations of hot and cold was not due to difference of nervous paths.

[3] Consider, for example, in a room the floor and two walls which meet in right angles. The position of any point in the room may be determined if we know its height from the floor and its perpendicular distances from the walls. On this principle Descartes founded his system of co-ordinates, as the distances which determine the point are called.

[4] A great number of experiments by Flourens and a host of subsequent scientists have been made to study the results of destroying the canals. The indications all tend to confirm the view that the semicircular canals are concerned in our appreciation of balance and orientation. The later experiments of Weill, Vincent, and Barre on the Voltaic Vertigo seem to confirm this theory of the functions of the semicircular canals. H. Held (*Untersuchungen über den feinern Bau des Ohrlabyrinthes der Wirbelthiere, Gesell. d. Wiss., Leipzig*, 1907) has well demonstrated the finer structure of the labyrinth.

not assumed that the semicircular canals are alone concerned in such judgments.[1] In man especially the appreciation of balance is derived from various nerve sensations, coming from the feet, for example, in walking, associated in experience with various positions of the body.

The muscular sense is distinct from the Sense of Effort noted in connection with the other senses, though that Sense of Effort appertains also to this sense.[2]

The muscular sense in regard to different muscles would present diversities analogous to those observed in regard to the sense of touch. Moreover, the fineness of Discrimination greatly depends on cultivation. Even in muscles which work automatically, and without giving any definite indication in consciousness, some derangement of function will often call attention to a definite muscular sense. Further, even when the exercise of the muscular faculty produces no definite effect in consciousness, yet the messages continually sent up to the central nervous system form part of those innumerable factors of the sub-conscious life which modify our conscious impressions.

The functions of the principal organs are carried on automatically but even in cases of perfectly normal working they are not without producing an effect in consciousness. The expansion and relaxation of the lungs, especially in deep breathing, are accompanied by well-recognised indications of the senses subserving the parts. Similarly the working of the normal stomach in full operation makes itself known.

All the organs give indications through the senses when difficulties arise through disarranged functions. The heart, which is a powerful hollow muscle acting as a pump, may in the case of highly sensitive people, give indications even of normal working. The heart labouring against obstruction makes its efforts known through the special sense affected. The arteries under undue tension also give indications of that condition through their nerves of supply.

It would lead too far to consider in turn each one of the natural processes and the indications of its normal and abnormal working; but from the suggestions offered it becomes apparent that the five senses are far from representing the whole of the impressions which arrive at the sensorium, which help to form the consciousness and to direct the current of the thoughts.[3]

[1] Hitzig's work, *Der Schwindel* (Vertigo), contains arguments in favour of a vertigo centre in the brain connected with the centre of equilibration in the medulla. He distinguishes between cerebral vertigo, ocular vertigo, and labyrinthine vertigo.

[2] *Cf.* the chapter on the Sense of Effort.

[3] H. E. Beaunis, in his work *Les Sensations internes* (1899) has laid broad foundations of systematic study. Amongst others who have worked at the subject are

The sense or feeling of hunger and that of thirst are quite distinct from any of those hitherto considered. That is also true of the senses connected with other deep processes of prime importance to the race. In these cases we find confirmed the inverse relation between the acuteness of Discrimination and the intensity of the Hedonic Effect.

To the cases of general hunger we may add those of local hunger and the sense of inanition or want of exercise of individual parts of the body.

Investigation is needed regarding other sources of information of the external world. For example, there appears to be a definite sense of humidity and dryness, analogous in human beings to that so enormously cultivated in the batrachians (*cf.* remark p. 54). It would be interesting to experiment so as to determine what parts of the body are most sensitive in this respect; what type of people, what ages, and what conditions of health best exemplify this function.

There is no doubt also a distinct sense of pressure, as, for example, atmospheric pressure. It would be well to inquire in how far any part of the auditory apparatus gives us indications in this regard. There would appear also to be a sense of vibrations as apart from those which come into consciousness as sounds.[2]

And there may be also a sense of rhythm in the change from one sound to another, for musical notes are not defined alone by

Kulpe, Hoffding, Wundt, Ladd, Kroener, Hermann, Schiff, Richet, Ebbinghaus, Titchener, Meumann, Sherrington. Elsie Murray (Am. J. of Psych., 1907) finds fault with the classification of Beaunis.

There are two sorts of receptive tissue in internal organs—free sensory endings and Vater-Pacini corpuscles. Poivier and Charpy found in the mucous membrane of the ileum nerve cells similar to those of tactile corpuscles.

[2] A case may be here cited, in which, however, it is difficult to exclude mystification: A Norwegian, Emil Knudsen, aged thirty-six (1910), claimed to possess a "sixth" sense. At the suggestion of Professor Friedenreich, a leading neurologist of Denmark, an experiment was tried. Knudsen was blindfolded, and placed at the wheel of a steamer in Copenhagen Harbour. He is said to have piloted the vessel through the intricate channel with perfect confidence.

Another experiment of a much more rigorous character was that of Professor Watson of Chicago, who destroyed the sense organs of the ordinary five senses of a rat The rat, nevertheless, found food which had been so placed as to make the test conclusive.

I once had a rare chance of studying the effect of vibrations in fishes on Coniston Lake. Looking over the side of a boat I saw a number of small fish perfectly motionless in the water.

The day was extraordinarily calm. Suddenly the fishes scuttled off rapidly in all directions. For a few moments I looked for the cause, then I observed a column of smoke and dust rising up about a mile away, where blasting operations had begun. Then after some seconds I heard the noise.

Of recent years there has been a good deal of writing on the subject of a "sixth sense", as by L. Truschel, *Das Problem des sogenannten sechsten Sinnes der Blinden*, and F. von Velden, *Der Sechstesinn* (*Fortschritte der Medizin*, 1906). V. Forti and B. Barrovechio in *Med Klin.* (1905) and Herzog in *Deutsch, Zeit f. Nervenk.* (1906) have discussed the Vibrationsgefuhl (Sense of Vibrations). Within recent years the question has been carefully studied in America, notably by F. B. Dresslar, whose paper on *The Pressure of the Drum* (*Am. J. Psych.*) may serve as an indication of the methods of experiment. Helen Keller, who, though blind and deaf, can enjoy and discriminate music, has said "every atom of my body is a vibroscope."

sounds or by the combinations of sounds that correspond to their chords.

There would appear also to be a distinct sense making us acquainted with the change of electrical conditions in our neighbourhood and consequently in ourselves, when such changes reach a certain importance.

And again there is probably a distinct sense of radiation, as may be observed in close dry atmospheres, and which is not to be confounded with any sense of light or heat.

A combination of indications such as those referred to, from the mention of humidity onwards, may account for the fairly clear premonitions some people have of the approach of storms.

CHAPTER VI

IMPRESSIONS FROM INTERNAL SOURCES

(1) *Normal Conditions*

But besides such sources of information the mind has to deal with subjective impressions. Let us consider[1] for a moment roughly the sequence of physical phenomena which precede a sensation of light or colour. We suppose first a luminous object. The luminous object communicates a stimulus, whose nature we do not know, which reaches the cornea, which traverse the anterior chamber of the eye, the crystalline lens, the vitreous body, and which finally acts upon the rods and cones of the retina in some manner likewise unknown to us.[2]

None of these passages or disturbances constitutes a sensation of light or colour. Nor can we say that the retina is the organ that causes sensation, for if the process be stopped at the point to which we have traced it, there will be no sensation. The retina is simply one of the essential parts of the optical apparatus. The crystalline lens, it will be noticed, is not an essential part, for it is removed in the operation for cataract, and its function in causing the rays of light to converge at the retina is more or less well carried on by the glass convex lens that the surgeon selects according to certain indications varying with the individual.

The retina, however, may be intact and yet from various causes no sensation of light or colour may occur. The excitation of the retina communicates in turn some form of stimulus to the nerves leading from the retina to the brain and which combine to form, for each retina, the corresponding optic nerve. Of the nature of this stimulus we really know nothing definite. We have only a choice of ingenious theories supported by celebrated names.

The course of the stimulus from the retina to the brain depends upon the part of the retina concerned. Most of the rays which fall on the inner side of the retina find their effects carried through

[1] Hachet-Souplet in an article entitled *Les Facultés mystérieuses des bêtes* refers to the high development of the sense of electricity in birds. Dr. Ph. Maréchal has published a work with the title *Supériorité des Animaux sur l'Homme.*

An observation by M. P. Delisle, an engineer on board the French war-vessel *Descartes*, in which he traces certain forms of nervous trouble to the use of Wireless Telegraphy may be found to have a bearing on this question; in view, however, of the direct effects of X-rays in producing dermatitis, and also deep-seated lesions it would be necessary to proceed here with great caution.

[2] The main parts only are here noticed. The cornea is itself formed of five layers, and in the retina eight layers are distinguished by histologists.

the optic chiasma, or crossing-place of the two optic nerves, to the opposite side of the brain. Along the optic track with which a vast number of junctions are formed to the different portions of the brain which it traverses, the stimulus is carried till it reaches what is called the centre of vision at the calcarine fissure in the occipital lobe of the brain. This centre has been located by various clinical observations, biological examinations, and experiments. What is the precise relation of a centre in regard to the stimulus or to the sensation we do not know. Nor is it even determined that this centre is the ultimate destination of the stimulus.[1]

Now at any part of the route which the stimulus traverses there is no sensation. The stimulus itself changes its character, and we are acquainted only with vague general indications regarding the conditions under which this stimulus operates.

It may be taken as established, for example, that the retina responds, in general, only to the excitation of which the source is a luminous object. But that excitation of the retina is not a sensation, nor is it an image of the object, though this term may be useful in ordinary language.

It is necessary to insist on these points, for much confusion has been caused by the loose use of language in this regard. A question has been frequently asked, for example, how is the image on the retina reversed that we do not see things upside down? It is true that as shown in elementary books on Physics, the stimulus proceeding from the lower part of the object affects a part of the retina which is above that affected by the stimulus from the upper part of the object. But that is in accordance with the whole manner in which we obtain information of the external world. To understand how it is that no confusion arises on this account is no more difficult than to understand how men can live and walk at the antipodes, and that in fact we ourselves are at the antipodes, if their standpoint be taken as the basis of comparison.

Now when we realize the fact that we may be considered as living at the antipodes, we find the answer more readily, viz., that we do not experience in any way the necessity of having to make corrections arising from this fact, that in fact the terms up and

[1] Sir William Gowers published in a pamphlet, *Subjective Visual Sensations*, some arguments in support of a theory given out by Ferrier that there is a higher visual centre in the region of the angular convolution. Monakow considers that these so-called centres are merely gates through which the nerve strands pass in closer approximation than elsewhere, and where the inter-communications by ganglia are more complete. A. W. Campbell's work, *Histological Studies in the Localization of the Cerebral Functions* (*Camb. Univ.* 1905), forms a good introduction to this subject. The whole field bristles with difficulties, and not less so, as we shall see, in cases where all recognised authorities had concluded that the question had been settled. The matter will be found discussed in a new light in the references to Broca's Lobe. (*Cf.* p. 368 (note).)

down are relative, and we call down what at the antipodes would be up; and all that we know in this way being similarly circumstanced, our view of the external world is consistent.

Even when by artificial means a correction is made, as for instance, when an engineer directs operations while looking through the telescope of his theodolite, which reverses the "image", it is remarkable how soon one becomes accustomed to disregard any difficulty that might arise on that account.

But with regard to the "image", the term "image" really presupposes a seeing eye, apart from the object, be it retina or screen, on which the "image" is supposed to lie. The luminous body does not of itself, and absolutely, send forth images. It sends forth perturbations, corpuscular or undulatory, which if brought into contact with a Crookes radiometer suitably arranged produce motion; which if brought into contact with a mixture of hydrogen and chlorine produce a chemical combination; which if brought into contact with a sensitised plate under suitable conditions produce a photograph; and which if brought into contact with the retina produce another kind of agitation to which the retina is specially adapted. But the disturbance of the retina is not a photograph, nor is it an "image"; for just as the photograph requires the combination, speaking broadly, of the luminous rays and the sensitised plate, so does the image require the combination of the luminous rays and the receiving systems which may be physically represented by the entire apparatus including the visual centre in the brain.[1]

If the previous exposition be clearly understood, then not only will all difficulties about seeing things upside down vanish, at least according to those terms; but also the usual explanations of physiologists and psychologists regarding them will be seen to be based on misapprehensions.

Further, many crude notions regarding the physical basis of thought will vanish also. The excitation of the retina is not a thought; it is not even an image. Yet the discovery of as much as we know of the curious mechanism of the retina has proved extraordinarily interesting and has marked an advance in our knowledge of vision. But if we could pursue our investigations so as to place on an equal level of certitude the existence and

[1] W. MacDougall (Brain, Vol. XXXIII, Pt. IV) has elaborately discussed the question of *The Relation Between Corresponding Points of the Retinae*. Newton supposed that there was a common centre in the brain for corresponding points, and he was followed by many authorities in this assumption. MacDougall points out a number of circumstances (as, for instance, rivalry of the images from the corresponding points) which are not consistent with that view. He concludes in favour of separate centres. Histologically, the matter is not determined. Psychologically, it will appear in the following exposition in a new light.

mechanism of the visual centres, and even the higher centres; or even if we could observe directly, or imaginatively, the final co ordinated movements of all the very molecules themselves of this intricate nervous apparatus, we should still be as far off as ever from the understanding of thought.

If one were to speak of thought as a function of the eyelids, the absurdity of the statement would be immediately evident. Yet, usually, in receiving the impressions from external luminous bodies the eyelids must be held apart, that is to say, in general language, must have a certain determined physical condition. But at what stage should we stop in tracing the process of stimulation from the external eye to the visual centre?

The nerve cells of a portion of the brain must undergo certain determined physical conditions before a sensation of light or colour arises in the mind; but here, so far as we know, or conceive, we have to deal with nothing more than a physical antecedent, just as the opening of the eyelids was a physical antecedent.

That the analysis may never be carried further it is impossible to say, for who can impose limits on the progress of the mind? But the questions become merged in those obscure depths where we seek to find the nexus between cause and effect, and where we have been unable to solve the far simpler problems in the tangible world about us.

But to speak of thought as a function of the brain, in any manner inconsistent with the foregoing exposition, is absurd. All that we can affirm is that a certain course of physical excitation is the antecedent to thought. We can say it is the cause of thought, if the word "cause" be invested with no other meaning than that which is involved in such a conception.

(2) *Morbid Subjective Impressions*

The foregoing discussion has been necessary in order to consider properly other sources of information to the mind, viz., those of subjective impressions.[1] The retina, we have noted, responds in general only to one kind of stimulus, viz., that of a limited section of the spectrum of the light rays. It is true that various experimenters—Helmholtz with electricity, and Purkinje with various other processes—have produced images probably due to the excitation of the retina, but such images are vague and not correspondent to the outside world.

[1] In this chapter the word "subjective" is used in a narrow sense. It refers to the impressions which are independent of any immediate stimulus from the external world.

The excitation of the nervous cells of the visual centre usually depends on that of the preceding excitation of the retina, but it appears that these cells may be excited independently, though whether in that case the stimulus is of a similar or different character we cannot tell.

The matter may be made more graphic thus: The excitation of the retina is representative of certain movements of the rods and cones, and the stimulus under various forms is carried at length to the visual centre, and in the course towards this the arrangements of simple nerve conduction have been replaced more and more by arrangements of inter-communication between nerves and cells, and between cells and cells. Now this web-like tissue suffers a peculiar perturbation corresponding to each kind of stimulus. That form of perturbation or excitation, the restrictions already noticed, we may speak of as the cause of sensation.

Now the nerve centre knows nothing directly of the retina. And moreover under various conditions of nervous instability it is possible that the excitations of the web-like structures of nerve ganglia, nerve cells, nerve filaments, may occur from conditions within the brain itself. Accordingly sensations are presented to the mind similar to those that arise from stimulation in the usual way.[1] These sensations may be combined in various ways; and so at length we may obtain subjective impressions of a complex and vivid character which make a total impression almost as strong as of the realities of the external world. In many instances they are mistaken for such realities; these are then a fruitful source of the stories of ghosts, apparitions, appearances of various kinds and of more or less distinctness.

It sometimes happens that these subjective impressions occur in the case of those whose minds are trained to observe and who have a good general knowledge of the causes of impressions objective and subjective. Accordingly a great deal of material for study has been collected, and many interesting facts about subjective impressions have been noticed.[2]

Subjective impressions are most commonly found in those of a highly sensitive temperament, either overwrought by excessive work, stress or deprivation, or afflicted with disease.

[1] Bain gives reasons for believing that the same central fibres are stimulated in subjective impressions as in those due to external stimulus. Intense subjective impressions of colour may even give rise to the complementary colour, as in the ordinary case.

[2] Brewster, Féré, and others have shown that certain hallucinations are deviated by the prism. This indicates their dependence on objective stimuli. Parinaud found that consecutive images—for instances, those of complementary colours—were not deviated by the prism.

Subjective impressions occur in well-marked forms in migraine and in epilepsy. In migraine the subjective impressions are often those of a kind of zigzag in colours, or "fortification spectrum", occupying a considerable part of the field of sight. The impression is definite, it lasts a considerable time, from a quarter to half an hour and it is followed by a headache for hours.

The subjective impressions in connection with epilepsy are much shorter in duration, lasting only a few seconds, but never elaborate, and they constitute part of the "aura" or premonitions of an epileptic fit.

Sir William Gowers mentions a case in which the sequence was the following: First the beating of the heart was felt; this impression seemed to ascend to the head, where it became transformed into sound. Then two lights appeared before the eyes and seemed to approach by jerks, synchronously with the pulsation. The lights then disappeared, and were replaced by the figure of an old woman in a red cloak, who offered something that had the smell of Tonquin beans. Then consciousness was lost.

This case is interesting for many reasons. It will be noted that the visual impression here is both elaborate and distinct. The first indication is the increased beating of the heart; then the fact that the pulse-beats mark the jerky progress of the approach of the lights indicates that nervous discharges which correspond to sensations and their combinations depend on the supply of blood. Moreover, as the disturbance becomes more severe a larger area of the nervous web is affected, as shown not only by the greater elaboration of the subjective images, but also by the fact that successively the areas corresponding to the centres for sound and the centres for smell are invaded and perturbed. Finally the disturbances become so great that the nervous discharges find outlet in the violent movements of an epileptic fit.

In this particular instance the impressions of sight dominated; but in other cases of subjective impressions, particularly in cases of delusions, the dominance of aural impressions is common.

Each sense may yield subjective impressions of a characteristic kind. It is not for our purpose necessary to work them out in detail, but attention may be called to the striking example of pains referred to a great toe in the case of a man who has had his leg amputated.

In all these cases we find subjective impressions arising spontaneously and even in spite of determined efforts of the will of the subject, finally dominating his consciousness. We find something analogous in the movements of Memory,[1] and in all

[1] *Cf.* the chapter on Memory.

manifestations of mental construction, invention, fancy, or imagination.

It may happen that the imaginative faculty is much more highly developed in an individual than the faculties which tend to criticism and good judgment. Also in such a subject the conditions may be found of overwork and stress, perhaps following an artificial stimulation. Again, highly intellectual men, from Julius Cæsar to the poet Alfred de Musset, and, it has been affirmed by some, Napoleon Bonaparte[1] have been subject to epileptic attacks. Hence has arisen a certain general impression summed up by Dryden in the lines:

> "Great wits to madness sure are near allied,
> And thin partitions do their bounds divide."

It would not be well, however, to push arguments of this kind too far or to draw hasty conclusions. We shall see that the very way of progress in thought is by a series of tentative combinations, afterwards tested by reference to the realities of the world. In men of high intellectual endowment and fine spirit the play of the constructive and inventive faculty is likely to be active, but there is no reason in the nature of things why the analytical faculties should be defective. If they were, the great wit would go only a small distance, even in achieving a reputation for ability.[2]

Moreover, the efforts of fancy and combination are within a certain measure controlled. They do not entirely tyrannise over the mind, as in the case of lunacy or epilepsy. Such diseases spring from material causes which are as likely to affect those of vulgar minds as the "great wits", though when a mind of great energy is so affected its aberrations become more remarkable.

Moreover in these matters of popular judgment the great wits are judged by a standard which is itself faulty, viz., the entire sanity and reasonableness of the average of mankind. All kinds of persistent errors in conduct or thought which do not stand the test of conformity with the great truths of the universe are in themselves elements of madness; and minds little cultivated are hardly more likely to form correct judgments than those of great reach and fine illumination.

[1] By Talleyrand, whose evidence would be decisive if it were reliable. Many French medical men class Napoleon with epileptics. Amongst other famous men subject to epilepsy may be mentioned Richelieu, Montesquieu, Pascal, Molière, Crébillon, Buffon, Chateaubriand, Flaubert, Daudet, Petrarch, Alfieri, Carducci, Lenau, Schiller, Swift, Dr. Johnson, Newton, Mozart, Handel, Paganini, Mahomet, Peter the Great, Charles V.

[2] The critical faculty is singularly difficient in such books as Nisbet's *Insanity of Genius*, and also in the work of Lombroso himself. The collection of facts has interest and value, but the deductions drawn from them are often absurd. A thesis recently read by Mr. Giraud at the University of Lyons, *Névroses à décharge* is no less interesting in regard to examples, and is couched in a more reasonable strain.

Nevertheless, the popular saying indicates the dangers which lie near in cases of great effort in mental operations. The man who flies is liable to fall, and the more audacious his flight the more striking will be his fall. But flying is not falling.

There must also be noted the aspect of fatigue and its consequences in faculties which become habitually over exercised. The failure of genius is sometimes a case of " trade maladies ".

We have now taken a review of the senses, and we have considered them in connection with the series of Fundamental Processes which have been posited; we have considered various sources of information, apart from the " five senses "; and we have noted in dealing briefly with subjective impressions that the physical organisation which subserves thought is to be considered otherwise than as something inert which comes into activity when touched by outside impressions: that to quote a simile used by Sir William Gowers, it must be viewed not as a piano but as an organ full of pent-up energy waiting to be released.

This manner of regarding the mind and its physical substratum will be confirmed in all that follows.

We will now consider more precisely some of the positions laid down, and particularly, such as demand for their full comprehension determined analysis and fine introspection.

PART II

CHAPTER I

CONCEPT OF UNIT

(1) *Leading Considerations*

The preceding chapters have cleared the ground to some extent; we now enter upon a discussion which deals with a cardinal point in the exposition. Let us recall the main lines of argument. It will be here shown that the mind can at any moment of attention conceive only one Immediate Presentation. Combining this result with the concept of Time and the process of Association we obtain the clue to the profound meaning of counting. By the use of processes already indicated we proceed to addition, subtraction, multiplication and division, and finally operations involving the notion of infinity. These together with spatial relations give us the control of the whole field of mathematics in the sense that we can now reduce all the operations to expressions in terms of our fundamental processes. But if we clothe the schema (*cf.* p. 6 (note), where this term is discussed) of any complex of processes with the appropriate appearances known through Immediate Presentation or Memory (with all the relations embraced by processes such as Association, Generalisation, Hedonic sense, Time, Space) then we shall have covered the scope of the whole domain of thought in mental exercise of any description. If then we examine the Fundamental Processes involved in the development of such schemata, and if we then examine the Fundamental Processes involved in clothing these schemata with appropriate forms or appearances, we shall effectively have covered the whole domain of thought or any psychic activities whatever.

If finally we prove that our Fundamental Processes are both necessary and sufficient in the sense in which these terms are used in mathematics, then we shall have demonstrated the main thesis of this work. There will remain the fields of application in which the fertility of the system will be observed. As these cannot be all dealt with, lines of development opening up veritable new branches of science will be indicated.

Finally it will be evident in this exposition that not only is this the true line of investigation for obtaining the solution of the central problem (*cf.* p. 16), but that no other method could give that solution. The reason of this assertion is found in the problem of proving that any "Categories" are first of all

fundamental and that they are necessary or sufficient; this last condition in turn involving the proof that they cover the whole field of thought.

* * * *

Before definitely entering the consideration of the Unit I would like in order to avoid possible misconceptions to refer to a theory of Monism put before me by a distinguished philosopher. He said that all the faculties of the human mind may be reduced to one, viz., Memory. He then entered into an explanation of the cosmos which in the last resort he declared consisted of nothing but vibrations. All was vibration. The combination of vibrations produced the world and what we know of it.[1]

Whether this be true or not, it is difficult for our minds to seize it, and it is still more difficult from that basis to develop a system of knowledge by which we can proceed to new and hitherto unknown results. The most fruitful products of human intelligence have been derived from minds in the operations of which each step has been founded on a solid basis of understanding, even though in the long line of the analysis the retrospect may give the impression of something beyond ordinary reason.[2]

The idea of Monism, that All is One may be tested by the series of the Fundamental Processes already set forth. It is evident that Immediate Presentation is not the same thing as Memory. Memory is called into play when the thing perceived awakens the image of another with which it becomes linked in Association. Similarly the recognition of a Unit cannot be an effort of Memory; nor Discrimination, though it depends on Memory; nor Generalisation with its corollaries, classification, symbolisation. The Impulse carrying the mind along with effort of Association is clearly not Memory, nor is the Feeling of Effort accompanying a conscious movement. The Hedonic sense is not Memory. Time and Space cannot be called Memory.

Therefore, if these represent real processes of the mind it cannot be denied on the ground of being involved in Memory that they are fundamental.

* * * *

[1] I have no wish to suggest that this intangible expression of Monism corresponds to the conception of all Monists. There is no lack of men of eminence in science amongst them. I cite at hazard Haeckel, Romanes, Binet, Carus, Dessoir, Morelli, and Mach, and the Biochemicals generally. Formerly Dubois-Reymond, Virchow, and Wundt might have been added, but they abandoned the position, and Haeckel scolded them soundly for doing so. On the other hand there is not one who has conducted his analysis to a profound base and exhibited a clear exposition of the principle of Monism.

[2] Newton's thoughts are less "lofty" in the sense of flight than close to earth always in the plainness and solidity of his reasoning. "The thoughts that wander through eternity" must first climb step by step; but all true thinking opens up a portion of the universe, and it is the contemplation of that result that inspires the imagination with the lofty and the wonderful.

The position gained in this chapter may be expressed: The mind attends to one thing at a time, and only one: the Unit.

I have used the phrase " attends to " in preference to any other because it is necessary to insist on the fact of attention. The mind may be influenced by a great number of objects at one time, and this manner of influencing may modify the concept of the Unit; but in the mind's attention at that moment there is only a Unit. As that Unit begins to disappear, another Unit arises and a relation or Association is formed.

The recognition of this position will be aided by clear introspection, but various suggestions help to make the matter evident; and even if the truth of the statement be not immediately conceded, it will be gained at length by persistently holding the question before the mental vision and by removing sources of confusion.

In the first place it must be understood that we may regard as a Unit something that is objectively formed of many separate parts and which on further consideration we recognise as composed of such parts. Take for example, a bundle consisting of a variety of flowers. That forms a multitudinous presentation to the visual sense. There is nothing to suggest a Unit in this. But if the whole be lifted up by the hand then the effort of attention associates that act, which in itself implies unity, with the total appearance. A symbol, such as the word " bouquet ", enables the mind to include this complexity under one label.

Now it may be argued that the mind from the first, and even after the application of the label, recognises these things as various, and that it separates the whole into its true units, the individual flowers. Here a close observation is necessary. When one perceives as a Unit something which is composed of many heterogeneous objects the first presentation as a Unit precedes the separation of the object into component parts, and when later the object is symbolised and again represented as a Unit, that Unit, apart from the succession of Time, is different from the original Unit, because the concept is modified by the associations.

The mind paying attention to each flower which it recognises in turn beholds in it a Unit. But remark that the impression of each flower in turn is capable of yielding by analysis an interminable number of separate presentations, to each of which in turn we could apply the term Unit, while if we were consistent in our objection we would refuse it to the individual flower.

On the other hand, the fact that we can indicate the first still greater complexity, the bouquet, by one symbol is proof that we can regard it as a Unit.

Even if the object were much more complex and diverse we could still think of it as a Unit. For example, we can designate by a term, implying the conception of Unit, each of the following complex things: A public meeting, a nation, an n^{th} differential coefficient, the analysis required to demonstrate it, a book, the world.

We may take then as established a part of the position set forth, viz., that anything whatever formed by whatever combination held in consciousness may be symbolised, and consequently regarded, as a Unit.

We must now proceed to the further consideration, that the act of attention on the Unit excludes, for the moment of that attention, the conception of another Unit.

The preceding discussion has cleared the way even for this position, for if the added object were simultaneously included in the same conception with the first, then the Unit would really be the complex of the two; and we should not recognise that complex as being composed of two until we had successively carried our attention from one Unit to the second.

Consider one of a pair of dumb-bells. We call it a Unit. But if we consider any two knobs separated by a little distance we call them two Units. That is because we have already made the separation by proceeding from one to the other. In the same way we may consider the two knobs of the dumb-bell as two Units. Or again we may consider the pair of dumb-bells as a Unit, when comparing with another pair of dumb-bells.

Presented in this way the position may be seen to resolve almost into a truism; but that is the characteristic of any truth seen clearly in all its relations. The main difficulty of accepting the position arises from the fact that the Unit present to the attention at a certain moment is usually a thing of great objective complexity and that the mind in regarding it is influenced by that complexity. But this is perfectly consistent.

I open my eyes, for instance, upon the room in which I sit. I see at once what may be resolved into a vast number of separate things—the walls, floor, carpet, chairs, a fire, pictures, a cat.

But we have already seen that no complexity prevents an object being regarded as a Unit. And though the impression immediately presented may be resolved into separate Units, it becomes clear on introspection, that that is only in consequence of successive acts of attention, each corresponding to a Unit.

It should be remarked again that the first Unit formed of the complex things is really determined in its character by the presence of these complex things. For instance, if the cat were absent from

the position it occupied the Unit presented would not be the same as the actual Unit.

We discover by subsequent analysis that the Unit is really of great complexity; but even at the moment of recognition of the Unit, its character, its Effort, its Impulse, the forms of its association, are really influenced by what we may subsequently recognise as its component parts.

The Unit is not present to a *tabula rasa* as the old notion of the Locke school would have it, but to a mind behind whose immediate activity is a vast fund of experience, of associations, the dynamical basis of Impulse, all magazined in certain particular forms, all ready to co-operate in new forms of associations and new impulses, and all influenced in that activity by the objects presented, even when these objects do not actually find expression in the consciousness.

If this consideration be well taken, and if the extreme activity of the passage of attention from one Unit to another be recognised, the difficulties will disappear of establishing the position set forth. The mind attends to one thing at a time—the Unit.

The study of the Unit is not closed by this preceding discussion. A great many questions become suggested, the analysis of which eventually leads towards the same conclusions, and which throw light not only on the Unit, but also on much of what follows.

We have seen that after accepting the flower as a Unit, we separate it, by directing the attention from one part to another part, into a great number of other units. Each of these in turn may be again separated into a number of Units.

The question may therefore be asked: To what will the ultimate analysis lead us? . . . . All these objects of mental phenomena have a chemical and physical base.[1] In the objective world the whole may be resolved into the things themselves, the elements, and the mechanical relations subsisting between their parts, considered atomically.

But the most persistent analysis and introspection will never lead us to the discovery of the so-called chemical elements, nor to the solution of the question now being attacked from many sides, as to whether these elements represent the final or original state of their matter.

Yet our analysis should lead us to some boundary which marks the limits of our conceptions, and the points so reached should represent elements of Immediate Presentation—Units.

Certain other questions may be put forward, which will on the one

[1] The works of Ernest Mach, original and profound, are interesting in regard to this question.

hand throw light on the nature of the problem we propose, and the solutions of which, on the other hand, will be found all consistent.

For example: We lift the bouquet with the right hand. By that act of a single movement we are helped at least in our contemplation of the bouquet as a Unit. Suppose now that we had lifted it with the left hand. We would arrive at a similar conclusion. But what is there in the nature of our consciousness that associates the movement of the right hand with that of the left so that we recognise this act as being of the same kind?

Or again we see a green leaf, then we see a red leaf. What is there in our consciousness that enables us to think in such a way that, though we are compelled to hold these things quite apart as to two Units impossible of coalescence (for red as a sensation can never be resolved into green), yet induces us to contain them under one head of classification? We reply that both are colours. But what is colour as distinct from the concrete realisations in red and green and the other distinct colours?

Or again if we proceed to sound: Why can one sound be linked in association with another distinct sound? And why can a number of distinct sounds, as for instance in a spoken word, be linked together to form a Unit?.... Here I am asking questions so as to indicate that the problem is far from being so superficial or simple as may at first have appeared.

The problem is more complex when we seek the ground of Association of very dissimilar things. For example, when I move my right hand this motion is habitually associated, let us say, with a certain sound. A certain expectancy of Association arises as I begin to move my right arm. Now if I were to move my left arm similarly, would the expectancy of this Association arise?

How is it that at length we can think of, as a Unit, "A song and dance"?

We may arrive at the answer to the problem thus: Consider the colour, red.

The actual sensation involves, as we have already noted, Sense of Effort and Impulse.

If immediately afterwards be presented a slightly different shade of red, there will be again a Sense of Effort and Impulse. Further, a process of Discrimination arises; and with two objects (the two shades of red) presented in consciousness, we obtain the whole scope of the Fundamental Processes established.

Now we have noted that as the discrimination becomes less, the more vital character of the movements becomes greater.

In these two colours then, while the shades of red are discriminated, the Sense of Effort and Impulse are so indistinguishably

in common that a basis is formed for the Association of the two.

We find in the whole course of the exposition that it is the fact of a limit to our faculties of Discrimination which affords a basis to our whole life of ratiocination.

In this elementary instance it enables us to form Association, thence Generalisation, thence Classification, and Symbolisation in regard to two shades of colour.

It is not necessary for this that the mind should have any clear perception of the analysis, or of the veritable underlying conditions or nature of this Association. The process is as natural and unconscious as that by which at night a black cat might be taken for a grey cat.

It must be noted that with exercise not only does the faculty of Discrimination become sharpened, but also the faculty of Generalisation, of finding essential resemblances in things apparently remote. Hence when we refer to a mind little developed we should expect first that colours of considerable difference are associated together by virtue of the inaccuracy of Discrimination. And also that when the difference is great the Generalisation may not extend so far as to include them in one class. It is only by experience then, I take it, that the relation of colour to colour is fully appreciated.[1]

By no amount of experience, or by the exercise of Generalisation, could a colour and a sound be referred to the same class in respect to their peculiar qualities. Yet they must have something in common in order to be counted in a series as Units. The Unit, therefore, must refer to something deeper than the external aspect of things. We find that common base in the act of attention itself, and attention is really associated indissolubly with the Sense of Effort in receiving an impression.

The process is: Effort or attention,[2] springing from a source

[1] Primitive people, in spite of their keen eyesight, have a limited discrimination of colour. The whole spectrum of the Greeks of the Iliad is comprised in remarkably few colours. It is generally supposed that women possess a more highly developed colour sense than men. In a recent number of the "University of Colorado Studies," Mr. V. A. C. Henmon shows that the experimental evidence in support of the theory is unsatisfactory and inconclusive. Experiments undertaken by Nichols showed that men were decidedly more sensitive in the recognition of red, yellow, and green, while women were more sensitive to blue. Tests on the discrimination of differences in colour show that women are in general superior to men in the discriminative sensibility, and as Mr. Henmon shows, particularly in their discriminative sensibility to blues and greens; for he, too, believes that men show a decidedly greater sensibility in discriminating reds. He finds that in schoolchildren between the ages of eleven and fifteen there is no significant difference in the perception in red, and that the variability and range are slightly greater in girls.

[2] This is not here intended as a definition of Attention as a position in Psychology. A note on Attention is given at a much later stage, but here the desire is to make the meaning plain with regard to the Unit.

fundamental enough to lie beneath Discrimination, and then the full impression received, the Immediate Presentation, having its own characteristic, whether it be of a colour or of a sound that determines that particular Unit.

When we speak of an "abstraction", as we have elsewhere noted (*cf.* p. 43 *et seq*), in the sense of considering numbers as apart from the circumstances of the things numbered, it is by virtue of the Fundamental Processes here described that we are enabled so to think.

In our active mental lives we are in all these cases greatly aided by the employment of symbols, which have become indeed so habitual to us that they mask our view of the progress itself. The symbol is taken as the Unit, and we carry on our thoughts without reference to all that it symbolises, until at any moment we arrest our attention more definitely and proceed to develop what the thing symbolised presents. And so it happens that anything that can be represented by a symbol can be taken as a Unit.

There is one more point to be made clear in this discussion. Although it is the case that the mind entertains but one thing at a time, yet in order that the procession of thought may be established, it is necessary that the mind should note also the passage from one Unit to another.

Here a reference may be made to the physical structure whose stimulation underlies the processes of thought.

The nerve fibres themselves seem to act only in the way of conducting stimuli. But when the stimulus reaches a ganglion in the brain, then what has been called an "explosion" takes place. The disturbance in the ganglion seems to be the immediate antecedent to a sensation or image. It is not meant that any explosion in any ganglion is followed by a phenomenon in consciousness, for there must be continually in the brain disturbances of the kind which, though modifying thought, do not reach the intensity necessary, or are not so situated as to "cause" ideation.

When the explosion in the ganglion takes place impulses are given out in various directions along nerve filaments to surrounding ganglia, which are disturbed according to the intensity of the stimuli, and according to the freedom of the paths along which the stimuli have passed, and according to the dominance of the stimuli in certain directions beyond others. Hence new ideas succeed the old ones.

Now graphically representing the Unit by means of a certain ganglion, the succeeding Unit would be represented by the next ganglion, the advent in consciousness of the second Unit closing the attention of the mind on the first. The pathway, which need

not be single between the ganglia, would represent the relations between these two Units.

This leads us to express more clearly what was suggested in a preceding paragraph. The mind has consciousness of a Unit, and also of the nature of the passage in thought from that Unit to the succeeding Unit, and the complete comprehension of that Unit closing the process in regard to the first and preparing the way to the passage to the next succeeding.[1]

The underlying mechanism must be thought as a multitudinous, highly complex, and well charged system of cells linked in an elaborate way, sensitive to impressions, and requiring strong inhibitory forces in the mechanism itself at every passage to prevent explosions. A stimulus not only causes explosion but produces something like a struggle for dominance of the currents along varied pathways.[2]

Those that fail to produce conscious impressions yet are not lost. Such a condition will be immediately understood by students accustomed to dynamical problems, where forces are spoken of as acting to their full extent, even though the result be equilibrium or a motion contrary to the direction of any force considered. This must be always kept in view in considering any Unit at any time. The Unit must be considered as the resultant of a vast system of profound forces, representing potential energy always on the point of becoming kinetic energy, yet in the main held in equilibrium.

When these forces by added stimulus burst beyond control the conscious thought arises rippling above the sea of the forces that have produced it. The Unit does not remain independent, and detached, but is modified by all that have preceded it, while in turn it must be considered with the whole system of these factors in determining the next.

The movement of attention at the Presentation of a new mental impression implies the cessation or modification of other activities, a suspension of pre-determined acts both of mind and body, so much so at times that a slight sensation may be the cause of considerable efforts of readjustment. Increased physical activity in any part implies in general a demand for an increased blood supply to that part, and consequently a diminution in other parts. Hence

[1] At any moment an Immediate Presentation is determined not only by the objective stimulus but by many conditions which may not be made apparent in consciousness. The Presentation immediately antecedent to a certain Presentation is one of the factors that determine the character of that Presentation.

[2] There is found here something like a struggle for existence of ideas, with a survival, if not of the "Fittest," at least of the most congenial to the environment and most favoured by the actual circumstances. All this is consistent with a phrase of Goethe: "*Dicht bei einander wohnen die Gedanken*" (closely dwell the thoughts, one with another).

in the movement of attention to any one sense other senses are indirectly involved, and these expenditures and modifications of energy are themselves obscurely presented in consciousness so that herein we find, at the limit of Discrimination, the base of Generalisation of which the immediate object of attention is the Unit.

At any time the impression made in consciousness by an active dynamic system in one part of the brain may be obliterated by explosions in another part of the brain arising from a powerful stimulus of a kind unconnected with that previously accounted for.

The foregoing arguments are the main part of what I have to offer at this stage in order to lead the mind of the reader to the concept of the Unit. The argument will be completed in the chapter on Counting. It will be also found determinate in the foot note which on account of its abstruse character I append here instead of placing it in the text.[1]

[1] At this point I will introduce a series of observations, progressively tending towards the positions we have gained, and which will serve to show how profoundly the principles which we have laid bare strike, and yet how simple is their application. The first is from Hermite, one of the great mathematicians who combined a philosophic spirit with his rare technical grasp of his instrument. He had found that his researches in elliptic functions had led to the study of propositions of Legendre and Gauss on the de-composition of numbers into three squares, and he remarked: "These illustrious mathematicians while pursuing at the cost of so much effort their profound researches into that part of the Higher Arithmetic, were progressing also though they knew it not, towards another *régime* of science, and giving a memorable example of that mysterious unity which manifests itself all times in analytical studies apparently the most remote."

Here, as we should expect, we have an example of the relation between schematic forms, which though intricate are formed by combinations of the Fundamental Processes.

The next carries further the suggestions that arise infallibly from our system. It is from the great work of Fourier: *Théorie analytique de la chaleur*: "They (the successors of Archimedes and Newton) have taught us that phenomena the most diverse are subject to a small amount of fundamental laws which become reproduced in all the acts of nature."

A further step which links these speculations with the principles we are now discussing I take from an article by Louis Couturat in the *Revue de Métaphysique et de Moral* (1898): "It is to Cournot after Poinsot that the merit is due of having brought clearly to light the importance of the idea of order as the foundation of mathematical science. They also distinguish two parts in Algebra, (1) A Universal arithmetic founded on the generalisation of elementary operations and on the correlative extension of the idea of numbers. (2) The science of order and of combinations; science formal and abstract, applicable to all sort of acts and operations. This distinction is just and profound, but it is too radical to allow two sciences quite heterogeneous to be confused under the same title."

Finally we have another citation which completes, in our sense, the notions of the last: "It required the genius of Galois to perceive the analogy between these two intellectual processes so diverse, and to discover the relations, fertile as unexpected, between the science of number and that of order."

These successive flashes of genius appear naturally as corollaries of the principles we have been considering, and so instead of using them as a basis of exposition we can treat them as illustrations by the way.

They afford here, however, a confirmation of the theory of the Concept of the Unit. The reasoning runs thus: We have already seen the framework of the development of mathematics, and we know that though the operations of mathematics are limitless, the kinds of operations are limited and amenable to classification. For example, we might have inumerable instances of addition, but we could classify them generally under the heading, addition.

We can, therefore, survey the field, and observe the crucial points where new operations come into play. In this way we have in part seen that there is no place where these operations are not covered by the Fundamental Processes.

We then direct our attention to the basic operations. We find that they are covered by the description of "science of order."

I do not know that I should be discouraged by a failure to attain this position at once. It is contrary to the pre-judgment of many, and these pre-judgments, even in the minds of those most ostensibly candid, are often insuperable.[1]

The conclusion was not one to which I had expected to be led when I first looked into the question, and it was only after many tentatives amid deep obscurities that its determination became revealed to me as consistent with reason and as inevitable.

Examination of Weber's Arguments

To throw light upon it from another source I will quote from the celebrated work of Weber, *Tastsinn und Gemeingefühl.* I do not wish to cite Weber for his authority, however great; for the virtue of this book is to seek new truths through an original course of thought, and moreover there are passages in Weber's book where I consider his analysis both incomplete and faulty.

Weber's style often suggests Locke's in its clearness, circumspection, and candour, but Weber had a great advantage over Locke in the discipline of his scientific training, his deep general knowledge, his bent towards quantitative expressions, and his acquaintance with the field of this research.

Weber, moreover, seems to have hesitated about accepting the Concept of the Unit, as here expressed, until moved to the position by the force of the reasons advanced. On p. 13 (vol. 149 of *Ostwald's Klassiker*) he says:

"I am able, while I count the ticking of a clock, also to see the form of a flame, and feel the form of an object which I hold in the hand, and it seems accordingly that one can at the same time entertain various sensations. Such an experiment, however, is not sufficient to prove that; for we may consider that in the intervals between

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That implies succession, counting, and hence, as the element of all, the Concept of Unit. There can be no element, in an arrangement of order, not conceivable as a Unit even though it be capable of analysis into other elements, which again are conceived as Units.

Here then we find a demonstration not only of the scope of the Fundamental Processes but also of their limitless fecundity of application.

[1] Sir Wm. Hamilton—the metaphysician, not the mathematician—thought that the mind could not entertain more than six presentations simultaneously. He advanced no good arguments, however. Speculations of this kind remind one of the philosophical feats of the Middle Ages in regard to the question of the number of angels who could stand on the point of a needle.

C. S. Myers in his text book of Experimental Psychology (1911) says that few are able to appreciate more than five different objects on an illuminated screen for a momentary time. In this case the form in which the objects appear constitute a Symbol of the number.

He also says that it is possible for most to detect the difference without counting, between seven and eight bells struck rapidly one after the other, but not between nine and ten. Here consistently with what has been said, the eight bells may be taken as a Unit. Note that they must be counted before or after the event to give assurance of the number.

The matter will be referred to again in the consideration of counting in its diverse aspects and (*cf.* p. 93, *et seq.*, esp. p. 95) an important demonstration will be given.

the tickings the attention was directed by the flame, and then to the form of the tactile object and that this changing of the attention may be so quick and so often repeated, that we receive the impression that, simultaneously and without interruption, the three sensations have been present in consciousness.

"How little time is necessary to receive impressions may be observed in the case of correctors of the Press, who read a sheet with considerable speed, and yet must look at each letter precisely in order to notice errors.

"In the case of vision I can show that the part of the retina with which we see clearly has the dimensions of 1/3 of a line only. We must therefore, move the eye from one part to another, so that every part of a large object may make its impression on this small sensitive spot of the retina. Nevertheless we believe that we take in at one view objects which could not possibly make their impression simultaneously on that spot of the retina. What we complete in successive efforts we believe we have accomplished at once; but the more precise observations of Bessel [1] seem to show, on the contrary, that it is impossible to entertain quite simultaneously a visual and an aural sensation.

"In observations with the transit instrument it falls to the astronomer's duty twice to estimate the distance of a star from the cross-thread which the star passes, and to determine how far the star was from the thread at the first pendulum beat of the clock before the star reached the thread, and how far on the other side of the thread at the second beat.

"It is brought to evidence that the observations of the most practised observers differ not inconsiderably because, as Bessel maintained, one first hears the pendulum beat and then sees the distance; another, on the contrary, first notices the distance of the star from the thread and then hears the beat.

"Bessel's assumption seems to me to be confirmed by the following observation which I have made and which appears to prove that one is not able to represent in their time-relations two aural sensations of which one affects the right ear and the other the left ear. When I hold two watches, which do not tick quite at the same rate, near one ear so that the ticking is heard by this ear and not by the other, then I am able to distinguish the periods when both tickings agree from the periods when the tickings of one fall in the interval between the tickings of the other, and I can make out a rhythm which becomes repeated. But if I hold one watch near to one ear, and the other near the other, I can observe certainly that one is ticking faster than the other, but I am not able to make out the repeated rhythm, and the tickings of the watches produce an impression quite different from that of the first case. On the same grounds one is prevented from hearing the beat of the heart and feeling the pulse at the same moment."

This passage from Weber is extraordinarily interesting on many grounds. It exhibits, for one thing, how objective science may be called in aid to deal with a question of psychology.[2] It also makes evident Weber's scrupulous care, and reluctance to form a decided opinion except on incontrovertible evidence. As a matter of fact I do not think this evidence in itself quite conclusive, but it reinforces all that has been said.

[1] Bessel, *Astronomische Beobachtungen*, VIII. Abteilung (Königsberg, 1823), Introduction.

[2] Observations, such as referred to with the transit instrument, also marked the first step in the history of Experimental Psychology. The Astronomer-Royal, Maskelyne, in the year 1795, noticed that the observations of his assistant differed from his own, and he dismissed the assistant. Subsequent examination revealed the cause of the discrepancy in what is now known as the "personal equation."

In a subsequent chapter Weber gives some interesting experiments which he undertook to find how many letters he could see at one moment with fixed eye when looking at a page of ordinary print, directing his gaze precisely at one letter. He thought that he could recognise three or at most four letters. This, however, is not inconsistent with what he has affirmed about the impossibility of entertaining two sensations simultaneously.

One may, for example, see the whole dome of the sky and all the stars as a Unit. Subsequently, and while not actually gazing at the stars, certain of them may be recalled and counted. So with the letters of a page. One may see, though not altogether clearly, the whole page as a Unit, but to ascertain how many letters one may recognise at a moment requires subsequent mental operations. Weber's observation, therefore, so far from being in conflict with the position here maintained, confirms what I have said about the Concept of Unit applying to an object that we may afterwards represent as complex.

There is another chapter of Weber (*cf.* p. 10. of *Ostwald's Klassiker*, 149) which, though not ostensibly dealing with this question, yet throws light upon it:

"We can bring two portions of the skin in contact with one another and thereby effect that one produces upon the other an impression by virtue of pressure, heat or cold. This is not possible with the other organs of sense. Thus we cannot, for instance, see into one eye with the other.

"The question now arises whether both impressions which affect simultaneously the organs in contact coalesce in one sensation in consciousness; or whether in that case we ourselves can determine, by domination and intentional direction of our attention, which of the impressions shall appear in consciousness, or what other circumstances bring about the result, that one or the other impression comes into consciousness.

"The experiments which I made proved that the impressions did not coalesce into one sensation. For example, if we bring a cold limb into contact with a warm limb, we do not feel a middle temperature, but in many circumstances cold, in others heat, and sometimes, alternately, cold and heat. When the sensations of hot and cold change quickly, we arrive at the representation that hot and cold objects lie near each other or behind each other, but we are not able to image the sensations hot and cold coalescing into one, somewhat as we represent a high note and a low note, while we consider their relation to a third."

Here again we have clearly shown the impossibility in Weber's mind of entertaining at the same instant two sensations even similar. It may be remarked that the two notes of music may coalesce into one, but in that case the sensation is only one. This is more clearly observable in the case of vision. If the stimuli which correspond to the colours, red, orange, yellow, green, blue, indigo, violet, be combined in a certain way, we have not a simultaneous presentation of the respective sensations. We have simply a sensation of white.

Another quotation from Weber, containing a passage wherein I do not find myself in accord with him, will make the whole position clearer. It occurs in the course of observations leading to the first of the quotations already given:

"In order that the representation (Vorstellung) of a sensation should become determined the attention must be directed upon the sensation to be represented, whereas the sensation alone also becomes determined when we direct our attention as earnestly as possible upon another object. Sensations which we have represented in the categories of Space, Time, and Number are more easily kept in Memory, while pure sensations, which have not been so represented, make no durable impression, and do not easily form associations. Every one knows by experience that many objects fall into his eye, while he sees only the few upon which he has directed his attention, and that, while he is absorbed by some occupation, many sounds come to his ear without his hearing them."

He goes on to say that he believes that even these impressions modify consciousness. All that has been discussed is in accord with that last opinion. It is in the opening remarks of Weber that I find a lingering false influence of Kant, from whom Weber, however, marks a decisive break. Weber speaks of the categories of Space, Time, and Number. But what has preceded in this work, and what will be shown in the chapter on Counting, will make clear that the conception of Number may be analysed, and that it may be shown to consist of a complexus, including Time, by the aid of the Fundamental Process of Association, and by those Processes which we have already discussed in dealing with Abstraction.

In regard to the representation of a sensation of which Weber speaks, it should be remarked that there is no word in English which precisely translates *Vorstellung*. It means not merely being aware of something, but perceiving it in its relations. It is a combination derived from Immediate Presentation and Association, where perhaps the scope of associations is wider than we have understood in regard to Association as a Fundamental Process, although capable of being covered by the repeated application of our Processes.

Now if, for the purpose of exposition, we keep in view the image of the mind as a system of active forces held, for the most past, in equilibrium—though that equilibrium be of moving parts, as in machinery—we will understand that everything that impinges on the senses, or indeed on the constitution of the body in any respect, and which consequently affects the physical apparatus correlated to consciousness, must have influence on the outcome of the Fundamental Processes.

We say, in the physical world, that the throwing of a stone alters the centre of gravity of the earth. That is true, though the effect

be inappreciable. When a disturbance reaches a certain degree of intensity the effect may be observable. And in like manner we can understand that the forces that play on our mental lives, and which really modify the results in consequence, may be so slight as to be virtually negligible or so intense as to dominate the whole physical activity.

At any instant that which is in consciousness is a Unit. It is not here meant, however, that the Unit is coincident with a sensation or that it is necessarily composed of factors from one sense. It is really determined by a vast complex of factors. I open my eyes and my ears to an aspect of the landscape with its multitudinous intimations of land, and sea and sky. That instantaneous aspect is a Unit.

I direct my attention more particularly to a feature of that landscape, a bright flower. But in that movement of attention there has been a change of Unit, a change involving a change, for example, of Association, of Impulse. The attention may now be "concentrated" on some one minute thing—as a coloured spot—in the flower. That implies a new Unit, with appropriate change in Effort, Impulse, Association.

Now reverting to the passage of Weber where he speaks of *Vorstellung*, it will be understood that something which, regarded objectively, may be present in the field of vision, and which, moreover, may possibly be recovered in Memory as a Unit, may in the original Presentation have been but a component of the Unit. The effects of the Process of Association depend on the intensity of the Presentation, and there are degrees of the strength of Association. But if the attention be diverted to a sensation, so as to obtain a Vorstellung, this means an intention of widening and strengthening the effects of Association, and of directing the new Impulse in particular Association with that sensation.[1]

We can make the sensation the object even of all the complicated operations of an extended argument; and we can thus by extending its associations give it greater durability in Memory, but in doing so we depart from a simple expression of Fundamental Processes, though the whole of the operation may at length, by analysis, be expressed in terms of these. And this is true, within a smaller scope, in regard to *Vorstellung*.

It may happen that the attention is being directed to a mere Presentation, and that the Sense of Effort and Impulse arise in Association with a new field of sensations and the like, by reason

[1] It is not implied that this is done consciously, but no difficulty is thus presented. We desire to walk, and we walk; and we thereby direct, although unconsciously, a complex muscular system with the nature of which no one, except possibly an anatomist, is acquainted.

of some sudden, violent stimulus, as for instance by a startling blaze of light, a thrilling screech, or by something of dominating importance appearing in the operations of our own bodies which had not hitherto been manifest in consciousness. The word "attention" therefore must in this case be employed in its involuntary aspect, and, therefore, with a meaning freer than seems usual.[1]

[1] If the above exposition be valid, then we shall find, as always happens upon the statement of a new and true principle, that the results reach far beyond the direct intention. For example, if the principles of the system of Copernicus were correct, he established not merely that the earth moved round the sun, but by the same breath he blew away the vast systems founded on false bases, and opened the way to new research. That is why M. Paul Painlevé calls Copernicus the founder of the modern world of science.

If the account here given of *Vorstellung*, and that which is subsequently given of Will, consistent with our Fundamental Processes, be true, then the whole system of Schopenhauer, "*Die Welt als Wille und Vorstellung*," must go by the board, in as far as it professes to be founded on an ultimate analysis, and be not merely a series of opinions, suggestions, aphorisms, criticisms, which may or may not be justified, though exhibited in a glowing, striking, and often captivating style.

And further, the inefficacy of the school of the doctrine of Association, or of the Chain of Association, is shown.

I am content to say, if the exposition here offered be valid, the answer must not be to take refuge in the illusory strongholds of great authorities—always the mark of inferiority of intellect—but to show definitely in what vital particular this analysis fails in what it professes to accomplish.

Let us strike all principles as with a Thor's hammer; let us probe into all positions as with Ithuriel's spear. If they be true, they gain in strength; if they be false, they are shattered; if they be in part faulty, then from that discovery arise new pathways.

We see (*cf.* pp. 144 (note) and 281; 386 (note)) that judged by the same unerring touchstone, the fabrics of Kant's Kritik and of Bergson's theories—in as far as these profess to be philosophies founded on the deepest basis of thought—are unscientific and invalid.

CHAPTER II

COUNTING

The discussion of the Unit has prepared the way for a close study of the nature of Counting.

We are so far familiar with the process of counting, in the ordinary conduct of our lives, that our clear view is apt to be confused by the ordinary methods which may seem to be necessarily associated with it.

A few questions that we meet with in the analysis of the position may therefore be set forth in order to disturb the mind from too facile acceptance of familiar things:

1. Is there any meaning in numbers apart from a series of concrete things?
2. In what way are we enabled to number unlike things?
3. If the answer to this question be the reference to the ordinary process of counting involving unlike things, might it not be remarked that these things are symbolised, that the symbols are like, that the first process has been the establishment of the series of the symbols?
4. Is there in things, as dissimilar as we please, a common element which enables us to number them in the same series?
5. Is it possible to count without the use of symbols?
6. What would be the operation of counting, if we had no language?
7. Could there be numbers if there were not things which occurred twice in experience; that is, if the march of phenomena was continuous but not repeating?
8. Do animals count?
9. If not, in what factors forming the operation are they deficient?
10. What are the faculties necessarily involved in counting?

In our everyday experience we do not find immediate answers to these questions, yet when asked they are seen to touch on matters of real importance.

We see an object, say a sheep. The operation of the mind in proceeding from one sheep to another is distinct; and if we consider the two sheep as similar so that we take no note of accidental circumstances, that is to say, if we proceed from a Unit to a like Unit, the processes involved are fundamental.

Counting of like things to the number two is possible without the use of symbols.

Also it appears that in this simplest form of counting we count not symbols, but things such as in our more developed arithmetic the symbols represent.

If now, without counting, we saw a succession of sheep pass, say about one hundred, and then a second series of about one hundred. It will be agreed that without counting symbols it would be impossible to say that the second series was equivalent in point of numbers, or repetitions of the objects, to the first. At what point then could the unaided faculty determine an equivalence of such repetitions? Would it be ten, or five, or three?

If we have three simple objects, such as three balls, placed side by side, thus . . . , we say at once that they are three. But even in this case we are liable to be deceived by appearances. The use of symbols has become so much like a second nature, and symbols may be formed by Association in so many ways, that we do not immediately recognise that in saying three, we may have performed no operation of counting at all, having simply taken the symbol . . . as familiarly and easily as if we had been told that the number of sheep was represented by the symbol three, which obviously does not imply that in the instance before us we have counted the objects.

A bank clerk counting a great number of coins may count them by fives, but this again is by the use of symbols. He arranges them thus . . . . . , or thus . . . . . , or even thus . . . . . , and each one of these symbols is almost as familiar as V, or 5. Hence this process of counting is analogous to that of counting little packets labelled 5.

Even when the clerk does not arrange the coins at all, it does not follow that he works without the aid of symbols. He might effect this in various ways. For example, if five sovereigns be weighed on good scales, the weight will be found so nearly equivalent to that of another series of five sovereigns that the possibility of mistake for four or for six sovereigns may be practically excluded. Considerable experience in simple weighing by hand would give delicacy of perception sufficient to exclude errors. The number of a pile of sovereigns might possibly be estimated in this way; but each five would really be determined by a symbol, which in this case would consist in the character of the muscular effort required.

Or again the length occupied by five sovereigns laid side by side is so constant as to exclude errors so gross as of four or six sovereigns occupying an equivalent length. With practise the eye

could estimate this length with sufficient accuracy as to avoid errors. If counting were performed by such an aid, then the symbol of five would be the notation of the length corresponding.

Or again the estimation of the fives might be due to an addition of three and two so rapid and automatic as to escape the attention of the operator himself if he were unaccustomed to mental analysis.[1] In this case again we would not have an instance of counting fives.

An ordinary person would find difficulty in estimating by fives in any case. An uneducated man finds a great difficulty in estimating by threes.

It seems to me that the pure faculty of recognising successions of units, apart from some form of symbol, does not carry beyond the recognition of two. In cases where we may be inclined to think otherwise, we must be careful to exclude such delicate estimation of effort as would imply a symbol analogous to that of weight, or of associations analogous to that of our ordinary use of symbols.[2]

It is well to consider the use of symbols in an unfamiliar manner.

Suppose we had a known number of cards placed along a path, and that we took a corner out of each card, and kept these corners in a heap. Now suppose that a number of dogs walked along a path, and that each time a dog passed a card we took a corner from the heap and placed it aside. Now at the end of the operation we might not have counted the cases of a dog having passed a card, but we would know that there were as many such passages as corners of cards, and therefore as many as the cards themselves, and therefore the known number above mentioned.

Further, if the operation were repeated we should know that at the end there were as many passages as on the first occasion. Now, further, if these corners were taken up always in the same order, and had each a characteristic shape, then when any particular corner were taken up, it would symbolise the corresponding stage, and serve to indicate the exact repetition with regard to number of the process. Yet this is not the ordinary process of counting.

Or again, supposing we become acquainted with a number of posts in series, the first painted red, the second orange, the third yellow, and so on to a violet post. But suppose that we had not counted the posts. Suppose also we had a horse tied to each of these posts. Now, still without counting we could observe that the last horse corresponded to the violet post, and we could at

[1] *Cf.* B. Bourdon, *Sur le temps nécessaire pour mesurer les nombres.* (*Rev., Phil.*, 1908).

[2] *Cf.* Note (2) p. 87.

any time reproduce the series, without mistake, simply by attaching a horse to each post in turn, until we arrived at the violet post. The violet post would then symbolise the full complement of horses.

If the series of the posts became so familiar that we could easily represent them in recollection, then if we saw a number of horses walking by we could, by referring each to a post, determine whether there were more or less than the usual number as symbolised by the violet post.

And what we say of the violet post is true of any of the other posts. And so here we should have a form not of counting, as ordinarily understood, but yet of estimation of numbers.

Similarly by making use of the sense of touch alone, in connection with any series we wished to determine, we would ascertain similarity and differences with respect to what in a familiar way we would speak as the numbers of any new series presented. It would be necessary that the series of touches should have distinct characteristics, and that they should be capable of reproduction always in the same order, and also of association with each Unit of the series.

Another form of estimation is when in listening to music, as for example of a marching tune, a step is taken at each beat of the music. Now all recollection of the peculiar character of each step may at a certain moment have passed away from the mind, and the recollection also of the characteristics of each beat of the tune; yet the whole series may be again produced without fault from the beginning, each bar of the music forming associations awakening recollection of the following, and the last beat would symbolise the number of steps.

It is only necessary to observe that the sequence of the bar is correct, and that a step takes place at each beat; then at the end we have the certitude that the steps have been reproduced, although in the meantime all intuitive sense may be lost of the early elements of the series.

The reference to a musical form of numerical estimation gains interest also from this peculiarity that in this case the passage from one Unit to the succeeding has a definitely marked and reproducible characteristic; whereas in turning the attention from one uncoloured post to another there may be nothing to seize the attention or facilitate recollection.

Other forms of estimation may be referred to, as for instance that of the Parliamentary orator who might divide his speech into parts, and look at one window, say, while dealing with one part, and a cornice while dealing with another, and at the Speaker's

chair for the peroration. These objects being always in the same place, no effort would be required in establishing the series.

A noted medical man[1] interested in history used to think of the centuries as associated with the rungs of a ladder which he climbed, each rung having some peculiarity.

Counting on the fingers is a very good way for those who are not sure of the numbers otherwise. Most of us, however, have been taught to use numbers which can be recollected at will. Here we have an artificial series, and one offering no special features for associations, but by constant repetition the series has become familiar and facile, and so it is generally more convenient than such as at first might seem easier of application.

If our life could be supposed to consist of a series of like sensations, counting would be simply repetition. With two sets of sensations, one being used to count the other, counting becomes repetition with association of the successive numbers of the series.

When we count three eggs, for example, and three twinges of gout, and three poems, what is there in common that we can apply the symbol three to each of the series?

That question has been answered in the main in discussing the Unit. The Sense of Effort itself in the Immediate Presentation of a new simple impression, which may be a symbol of complexity is the something common to these disparate things which enables us to apply the symbol of counting. It must be noted that the complexity, poem, has already been symbolised as a Unit by the word poem.

A further illustration may make this matter still more comprehensible. If we fire at a target, the operation of the firing is noted in itself. If then we fire at a tree, the similarity of the act of firing in the two instances is such that the differences of the object do not bring in an element of confusion such as would prevent the same term being employed to denote the act.

The act of repetition of the symbols of counting becomes so facile that each symbol is associated without difficulty at the rise into consciousness of the object counted. The various characteristics that appertain to the object are not noted. We count the acts of attention, or those operations of which the base is the Sense of Effort. We have been here led to a conclusion of especial interest.

It may be remarked by way of objection that in counting coins we may have occasion to examine the coin closely to detect a possible counterfeit; but in that case the examination is something apart from the process of counting.

[1] Sir William Gowers.

Thus, to revert to the previous illustration, a man may hesitate as to which target to choose, but when he has chosen his target the act of firing at one target is in itself similar to that of firing at another.

Counting essentially consists in the Presentation of a Unit, and the Association of another Unit with this, and a continuance in periods of Time, marked principally by the Sense of Effort, of such Processes of Presentation and Association, according to some recognition of Agreement in the successive units, and with the proviso that one of the series of Units has been established as reproducible.

Consequently the conditions under which counting is possible are those firstly under which the recognition of units is possible, and secondly the conditions under which a series of symbols may be established and linked by associations with any succession of units.

Accordingly, we have necessarily implied as a basis for counting all the Fundamental Processes which we have seen already to be involved in the Concept of the Unit; then we have the establishment of a series which implies a repetition of units, whether purposely or accidentally in experience, so that we have formed a strong Association between the successive numbers of the series; then we have the Association of each Unit of the counting series with the successive numbers of the counted series, the Association being with each only in reference to it as a Unit; the term series itself implying Agreement; the term succession implying Time; and finally all the Fundamental Processes.

By studying therefore the minute wheelwork of the mechanism of thought we have been here led to a formulation of Fundamental Processes. The study of mathematical processes in the greatest extension and development shows us that we need no other processes, the most complex thoughts having as their schemata the combinations of these Processes. We have also found the signification, in an elementary form, of the Sense of Effort. This has always been considered one of the most abtruse problems in psychology. A special chapter is reserved for it.

To complete what is ordinarily implied in the term counting, the associations must be applicable indifferently to any series, so that comparison may be thus established between any two counted series.

Further conditions are implied, but they lie beyond the bounds of mental analysis. They are the permanency of the constitution of the external world in as far as the existence of the series is concerned; the permanency of our own constitutions in regard to the response

to the stimuli corresponding to the series counted; the permanency of our constitutions in regard to the reproduction of a series by virtue of stimuli arising, not now from the external world, but by the succession of the series itself.[1]

The analysis of counting has therefore shown in the main lines: (i) Immediate Presentation; (ii) Succession, implying Memory and Association; (iii) Conception of a series in regard to mode of succession, especially as in musical notes, and to duration; this last marking also the concept of Time being always present even though requiring other aids to procure definition; (iv) Association with established series, of which one of the special examples is a known numerical series; this implies Memory, Association, and finally all that we have ascertained as implied in the Concept of Unity.

[1] The subject of counting will be resumed in the course of discussing addition and multiplication, for the study of these operations supply new considerations. I have not here entered into the discussion of the problem "do animals count?"—further than to pose the question in order to indicate the necessity for recognising that merely conventional forms of counting do not sum up the whole matter. The preceding exposition, however, makes it clear that animals may be capable of a rudimentary form of counting. Hachet-Souplet remarks: "Certain hymenopteres paralyse the larvae of other insects by stinging them, in order to place them in the nest where the young ones will be born, and the number so placed is always equivalent to the number of their young."

Some readers may object that this is not a case of counting but of instinct, but since an effective count is made we could call it counting by instinct. In this wise our own Fundamental Processes are also instinctive, that is to say, arising according to our physical and mental constitution.

Levi Leonard Conant has studied the faculty of counting in animals.

CHAPTER III

ADDITION AND SUBTRACTION

Addition

The study of counting has prepared the way for the proper study of addition.

In the first place we find that we add external objects, units, together; and that if we can speak of adding numbers, as abstractions, this only means that we take the numbers themselves as units, and indicate the result by a symbol. Thus if we take, say, a series of two peas, and then a series of three peas (. .) (. . .), we shall observe that consistently with what has already been discussed, we express each of the series as a Unit ; and when we add the first series to the second series the process of addition means that we now express these as one series, and then proceed to count the Units contained in that series. Thus $2 + 3 = 5$.

Here, however, by this experience we have done something more than merely count. The second series was associated with the symbols one, two, three. But now we have applied to it the symbols three, four, five; and we have assumed that the operation of counting in this case is equivalent to that of the previous case, that is to say, that we have added three units to two. Therefore we have really counted the symbols three, four, five, by means of the peas originally associated with the symbols one, two, three. If there is any difficulty in grasping this argument the question may be put: How do we know there are three in the series three, four, five? If we narrowly examine the manner by which we arrive at a correct answer the difficulty will disappear.

Thus by means of new associations we count the symbols themselves. It is more difficult to count the symbols directly.

It is true that in an ordinary way we do not count when we add; we simply say two and three are five; but it was necessary once to have counted in order to know what the sum would be. The formula of addition symbolises an operation and its results.

We may now examine whether anything further is implied in the formula: Three and two are five (. . . . .). In the first place we form the new series, that of three, and that of the two, each regarded as a Unit (though each containing units), since the problem is to add the first series to the second; with this difference the first three units are counted as before. In the second

series, which we have posited as consisting of two, we now count the symbols four and five.

There is a further step necessary in changing the order completely. Thus, suppose that for convenience of reference we name the objects *a,b,c,d,e*. Hitherto though adding different series we have preserved the order; but suppose now that we add *a,b,d,* to *c,e*. Here we call the series *a,b,d,* three, implying that for each object there is a corresponding symbol up to three. But to effect the operation it must have been brought about that the associations giving the sequence of the symbols one, two, three, have been made stronger in our minds than the associations of the original order of the objects. This last remark, apparently so simple, will be seen to be of importance in considering the ultimate sanctions of Reason.

The same observation is true in regard to the series *c,e,* which we recognise as of two. This series in its weakened Association serves to define the new associations of four and five, and to allow the application of these as symbols of two objects.

We may convince ourselves by remarking that we are still adding three and two, and to know that the symbols four, five, have been applied to a series of two, we must have counted *c,e,* as of two.

Hitherto in adding these series we have supposed them placed before our eyes. This fact has supplied to the operation certain assistance which we have assumed as a basis. That is to say, in changing the order we have assumed that in placing *d* after *a,b,* we observe in the actual experience that we have not created a Unit, but have simply changed its place in such a manner that it is easy to omit it, in the associations of sequence which it previously held, when we proceed to consider *c,e,* as the series of the objects that remain.

To bring the force of this remark clearly before the mind, we take an extreme case and suppose that a thousand objects, which we had seen mingled pell-mell, were divided into two unequal series which we had to count and add from memory. It would soon be discovered that we could not be sure of having counted each object once and only once.

In order to accomplish that we should require that they should be displayed in a definite order before the eyes. The actual experience thus affords the associations necessary in carrying out the operation. And when we are able to proceed to a similar operation without such ostensible aid, it is because the associations have been in our experience so often that we reproduce them in recollection.

Now when associations have been formed and frequently repeated so as to be very familiar, the associations cease to depend on sequence.[1] Thus *A,B*, is an association always repeated in experience in that sequence, but after familiarity it will be found in memory that not only does *A* recall *B*, but that *B* recalls *A*, and that the association of *A* and *B* is stronger than that of the notion of their ordinary succession. And if we have a series established, *A,B,C,D,E*, quite familiarly, it becomes easy to associate any three objects with *A,B,C*; and then two other objects with *A,B*; and finally to associate *D,E*, with *A,B*.

Now the symbols of counting have become so familiar that it is easy to recapitulate without regard to the continuance of their original succession. Accordingly when in recollection we add by counting two series taken from five units by selection in any order, we shall find in general that we have in the first place counted the objects in each of the two series, and that in forming the two series, in any manner we please, we form associations with the symbols of counting; that then we count the symbols themselves; and by virtue of the associations mentioned, we know that we have thus counted the objects so symbolised. The association between symbol and object need be strong enough only to indicate that each symbol does represent an object to be counted.

In the ordinary exercise of addition we make use of associations based on the operations already considered. We say two and three are five, four and five are nine, almost as familiarly as we repeat the numbers in counting. If we have to add, say, fifteen and twenty-three, fewer people can give the answer in the same direct way.

The Importance of Notation

In the manner of expressing fifteen, we meet with a great advance in arithmetical development. We have the numbers grouped in series of tens and numbers less than ten. This depends on a new operation of the process Generalisation, and classification and symbolisation. A series of ten is treated as a Unit, and our own enumeration tells us how many series of ten we have to deal with.

Thus in fifteen we have one series of ten, and five units besides. Thus fifteen is a compound way of expressing what otherwise we might express by $10+5$ or *XV*. Similarly twenty-three is a neat way of expressing two series of ten and three units in addition. $23 = 10 + 10 + 3 = XXIII$.

[1] Thus, in writing at great pressure, or in a condition of fatigue, it is not uncommon to displace a letter; terrible, for instance, may be written terrblie. The question will be considered in the chapter on Memory.

It is not only neater but more explicative than either of the last two enumerations, for it indicates at a glance, without the necessity of counting, how many series of ten, are added.

To add fifteen to twenty-three we add together the numbers expressing the number of series of ten. Thus adding one and two we find three as the number of series of ten. Then we add the series of the other units; five and three are eight. So that we have three series of ten and eight units in addition; and, according to the mode of expression adopted, this number is expressed by what we usually understand by thirty-eight. As expressed in familiar notion: $23 + 15 = 38$.

If we had required to add fifteen to twenty-nine we would have as before three series of ten and the sum besides of the numbers five and nine, that is to say fourteen. But fourteen consists of one series of ten and four units besides. So that we have in all three and one (that is to say, four) series of ten and four units besides. We express this as 44.

The great difficulties in the problem of counting, and accordingly in those of addition also, have now been overcome.

In our methods of enumeration we consider a series of ten series of ten as a Unit, which we call a hundred. And we express the number of hundreds taken by a symbol placed in the third position, beginning from the right and going to the left. 437 means four series of hundreds and three series of tens and seven units in addition. The clearness of this notation may be compared with that of the Romans, *CCCCXXXVII*.

These differences have proved, in the history of the world, to be of great practical importance.[1]

The Roman system of notation soon becomes unwieldy. In order, for example, that in this simple instance it should convey the same information as the notation 437 we must first count the number of *C's*, then we must count the number of *X's*, then to *V* we must add the result of counting the two extra units to obtain seven.

Now, in view of what has already been discussed regarding the Unit as the momentary fact of the mind, and also regarding the difficulty of remembering series which have not been associated in the mind by constant repetition, we shall readily conclude that the Roman method is not only more cumbrous in representation, but that it involves a far greater strain on the mind in each successive operation.

[1] C. F. Hill, who has written learnedly *On the early use of Arabic Numerals in Europe*, finds the first examples in MSS. of the Tenth century; Arabic numerals came into general use at the beginning of the thirteenth century.

Moreover, its development is not in accord in the best way with the principle that has suggested this notation. Thus in *XV* we have a single symbol representing five units, and we have a single symbol representing a series of ten. But in *XXXV* we have each of the series of ten expressed. That which was here lacking, in view of the establishment of a good system of notation, was the recognition that the series of ten might be considered a Unit, and that the numbers of such Units might be directly expressed by single symbol.

The Romans had, however, allowed an element of complexity to remain in that they did not use symbols sufficiently even for the numbers up to five; 3 is a better symbol than iii, for it is taken as a whole.

Acting on this principle we might now express the number of tens thus: $3xv$. If now we compare this again with $2xv$ and $4xv$, we see that by the operation of symbolification the numbers 2, 3, and 4, would express the numbers of the series of tens, even though the symbol x were faint or imperfect or finally absent; thus: $2v$, $3v$, $4v$. So that even by this means we might have been conducted to that Arabic notation 25, 35, 45, which in its simple beauty seems like an inspiration of genius.

The development of arithmetic has been made possible by an appropriate choice of notation. The development of algebra has also depended on this principle, that wherever a complex has to be frequently considered in an operation, it should be expressed by a simple symbol.[1]

The Commutative and Associative Properties

We have observed that in addition $2+3=5=3+2$. That is to say, the order of the number is indifferent. Now the operation

[1] The forms of the Differential Calculus are but extensions of algebra. Accordingly we should expect to find that the highest developments of mathematics would be found among the peoples who had employed the simplest and most scientific forms in their elementary expressions. History shows to us how profound and far-reaching the importance of such matters may be. The development of our mathematics made possible modern developments of the science of mechanics. Amongst the thousand products of this science we may cite the discoveries of astronomy, and particularly that of the law of Universal Gravitation; the principles on which big bridges and big ships are built; and so on in a thousand instances—discoveries whose consequences have determined the trend of our civilisation.

So true is it, that one might say that given two nations of equal material power at a certain moment, developing their civilisation apart from each other, the nation that had adopted the best form of mathematical notation would, *ceteris paribus*, inevitably lead the other.

The study of addition conducts us also in another way to questions of great importance in the development of the higher mathematics. It is by a minute examination of such problems as have been here presented that we may be best led to the understanding of the fundamental principles which are at the base of the system of Co-ordinates of Descartes, of Vector Analysis, Hamilton's Quaternions, Grassmann's "Ausdehnungslehre," Plucker's generalisations, which have in turn led Sophus Lie to a magnificent development of the powers of mathematical analysis.

would be similar with any two other numbers. Therefore by Generalisation we say, let a and b be symbols of any two numbers: Thus $a+b=b+a$.

This property is expressed technically by saying that in addition the numbers are commutative.

If now we add a third number whose symbol is c, we have $a+b+c=c+b+a$. It is also true that $a+b+c=(a+b)+c$, when in the second case a and b are added together and c is added to the result, for this is the way in which the addition $a+b+c$ has been effected. It is also true that $a+b+c=a+(b+c)$, where b and c are added together and the result added to a. This may be demonstrated by reference to the exposition of addition as based upon counting.

Accordingly we establish that in addition we may group together any numbers of the series and add the result of the addition to the remaining numbers. This property is expressed by saying that in addition the numbers are associative.

If then we had to deal with other objects, say geometrical qualities, in such a way that the processes were both commutative and associative, it would be suggested to employ the symbol of addition. It would be necessary to observe that nothing further was to be assumed in the use of this symbol, beyond the commutative and associative properties.

Now a vector is defined as a straight line having a certain length and a certain direction. Direction must in this instance be distinguished from situation. Thus if the vector be moved parallel to itself into any other situation it is not considered to have changed in its vectorial properties.

If to a vector $A\,B$ we apply a vector $B\,C$, so that the beginning of $B\,C$ coincides with the end of $A\,B$, then in the mathematics of vectors we say $A\,B+B\,C=A\,C$.

Here it is evident that the operation symbolised by $+$ is not similar to that which we have already considered as addition, but we will be consistent in its use if we observe that it is commutative and associative, and that it is not used otherwise than in accordance with these properties.

It is commutative as may be thus shown. Draw $C\,D$ equal in length and parallel to $A\,B$. Join the points $B\,D$. Now we know from geometry that $B\,D$ is equal in length to $A\,C$, and parallel to it. Therefore, by our definition of the vector we have $B\,D=A\,C$; and also $C\,D=A\,B$. And by our assumption regarding the symbol we have $B\,C+C\,D=B\,D$, therefore $B\,C+A\,B=B\,D$. Also, $B\,C+A\,B=A\,C$. Therefore $B\,C+A\,B=A\,B+B\,C$.

If now we add another vector $C\ E$, it is evident that $A\ B+B\ C+C\ E=A\ E$; and that $(A\ B+B\ C)+C\ E=A\ E$; and that $A\ B+(B\ C+C\ E)=A\ E$. It is easy to extend the demonstration to less simple cases.

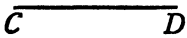
We have here a certain form of calculus at the basis of the science of vector analysis, which will be found capable of simplying many geometrical problems. Into these, however, we do not now enter.

We may now discuss a problem which arose before the development of vector analysis, and of which the terms appear to be simpler.

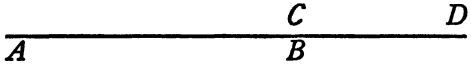
Consider a line $A\ B$,



then another $C\ D$,



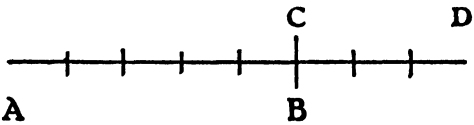
of which the direction is the same. For simplicity we may take the lines juxtaposed, so that the point C coincides with B.



Then we say $A\ B+C\ D=A\ D$. But although this process has become familiar to us, it really involved something beyond the forms of addition first discussed; for in that discussion we dealt only with the addition of series of units. Here we have the addition of two spaces, or lengths, which results in a length.

The Basis of Cartesian Co-ordinates

Suppose, however, that $A\ B$ were divided into a number of equal lengths, five, and that $C\ D$ were divided into a number of equal lengths, three, each equal to one of the divisions of $A\ B$.



Then we could consider each of these lengths as a unit. $A\ B$ would be taken as a series of five units, and $C\ D$ as a series of three units; $A\ D$ would be then a series of eight units. The principle could be extended to lines containing any number of units, if only the units were common measures of both lines. The principle would also apply to any number of lines in the same direction.

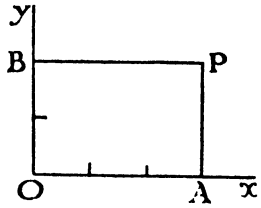
The suggestion would arise accordingly to apply the symbol + as in the simple case above.

Considerations of this kind apparently so obvious led Descartes to establish the system which gave an impulse to geometry which was productive of enormous development.

Descartes established his system of co-ordinates to define the position of a point in space: Let us consider a simple case of two co-ordinates, *Ox* and *Oy* at right angles.

Measure off on *Ox* three units to *O A*. Erect a perpendicular to *A x* at *A*: Measure off along *O y* two units to *O B*.

At *B* erect a perpendicular to *O y*, cutting the perpendicular to *O x* at the point *P*. *O A* is called the abscissa of *P*; *O B* the ordinate of *P*. Thus when we have given that the abscissa is 3,



and the ordinate 2, we define in this system the position of the point *P*.

The ancient Greeks, who were great geometers within the scope of their own conventions, had not taken the step of dividing the straight line into units of length to serve as measures. This fact alone proves that, although the notion is familiar to us, it required persistent and clear-sighted analysis on the part of Descartes to arrive at the principle involved, and to apprehend its importance. This also indicates that it is not trivial to endeavour to find, even in simple instances, the real meaning of operations. Without Descartes' co-ordinates we should not have had wireless telegraphy for there would have been lacking the mathematical apparatus that served to make, in the hands of Clerk Maxwell, an important link in the chain of discovery.

In the foregoing discussion, which has been offered to indicate the results that may flow from investigations of this sort, certain difficulties have been disregarded. For example, how do we know that we can find a Unit that will exactly measure two straight lines, so that they may be defined by the numbers which symbolise the repetitions of the Unit? It may be said that we need only take the Unit small enough; but this is an assumption. In mathematics we meet with incommensurable quantities, and with lines that cannot be measured by parts, however small, of other lines.[1]

[1] These matters are dealt with in the chapter on Infinity.

SUMMARY

The operations involved in Addition may be summarised as including: Firstly, all those involved in counting: then the operation of uniting one series with another so that they may be regarded as one series for the purpose of ascertaining by counting the number of objects in the series so regarded; the expression of such a result by symbols; the Generalisation of this process; the Association in Memory of the symbols of various series, taken in turn, with the symbols expressing the results of such combinations of the series; the recognition, and all that this involves, that the operation is "commutative", that is to say, that the order of the series is indifferent; the recognition that the operation is "associative"; operations involved in the classification of series in a selected system of notation; operations involved in the choice of methods, that is to say, in adding series of units to series of units, series of tens to series of tens, and so on; adaptation to the system of notation, as, for example, in "carrying over"; repetition producing familiarity so that all of the operations themselves becomes familiar by practice that they are carried on step by step without immediate reference to their meaning.[1]

(2) *Subtraction*

Subtraction is the "inverse" of addition; that is to say, if we have $2 + 3 = 5$, then if we consider first the series 5, and inquire what this series becomes if no account be taken of the series 2, we have the series 3. This is symbolised in the formula $5 - 2 = 3$.

Let us consider what is meant by taking no account of the series 2 in this process.

In the Immediate Presentation of what is externally an object there is found to be always associated a Sense of Effort and an Impulse. The Feeling of Effort determines the object as a Unit, the Presentation corresponding determines its particular character, and the Impulse carries the mind to an Association with an object which succeeds it in attention.

Accordingly we remark that in every succession of the kind the disappearance of the object from consciousness is as vital as its Presentation, although it is not so striking a phenomenon. It might possibly have been included in the series of Fundamental Processes, but that seemed an unnecessary refinement, for it is included in the processes Conception of the unit, Association, and Negation as applied to these.

We note also that in our active mental life, Association may produce a condition of expectancy in which the Sense of Effort becomes manifest without the object expected. For example, if one believed that it was midnight, and listened to a clock solemnly striking at regular intervals then after eleven had been counted the next stroke would be expected at the estimated time, and if it did not happen there would be felt a distinct shock of surprise.

[1] American psychologists have of recent years studied addition, but from other standpoints, and mainly in regard to failures, or speed tests, in the usual forms. L. D. Arnett and C. E. Brown have written interestingly in this regard on counting and addition in the *American Journal of Psychology*.

Now in applying a symbol, as in counting, there is also a Sense of Effort involved. Though not essentially the case, this is often associated with noticeable physical movements. We may observe that when school-children count the whole body is sometimes moved in unison. There is a suggestion of positing or casting the symbol as if it were a physical object. If then a symbol were used in a strongly associated series, and the object were not produced in experience, then there would be produced by the disappointment a sort of shock.

In such ways the mind becomes familiar with the Dis-association of a symbol and its object.

In our active life what is called the Will comes into play. We perform continually acts of addition with a certain purpose, and we continually perform acts which are similar in character, though represented by different objects, with that of positing a symbol upon its Units.

Thus the act of taking hold of an object is associated with that of dropping it. The act of seeking an object is associated with that of avoiding an object.

Accordingly we shall see that there is nothing in the nature of things why subtraction should not be performed independently of addition. We have the series, five, for example. In counting the objects of the series, five, we take note of the series, two, and we may here end our operation. But in so completing the count we do not necessarily note the remainder of the series as three. But now in order to know how many we have " subtracted ", we make account of the series left, and we find the number three.

Thus we need not have performed the process of addition, two and three are five. It was necessary to have performed the process of counting, however, if the operation were to have significance as subtraction. In this case the process of subtracting two from five is that of counting two, and then reversing the ordinary course of Association and renewing the count on the remaining Units we ascertain to be three. Subtraction therefore is in regard to counting co-ordinate with addition, not necessarily dependent on addition.

But in all our experience the rise of an object, simple or complex, in consciousness is more striking than its passing. And in regard to these processes the fact that addition is looked on as a positive process, and subtraction is the inverse, proves that addition is in experience the most important operation.

Subtraction, as the inverse of addition would be presented thus: We have become familiar with the operation of determining two and three are five. Two is considered as a whole, a Unit covering

two objects; similarly with three and with five. Moreover, with familiarity of these processes we not only neglect to observe all that they imply, but we also associate them with various operations of a more graphic character, which apart from certain differences contains something essentially similar.

Accordingly there would be associations of such addition with acts, such as placing a little stick in continuation with a longer stick and observing the combined length of the two sticks; or again adding a small weight to a greater weight and estimating the combined weight. Hence addition has Associations which make it comparable to positing or placing two things together.

Now we have already seen in the discussion on counting that when objects are familiarly associated together that association is stronger than the association of sequence.

Accordingly after placing two things together to make a whole we are prepared to comprehend the process of taking one of them from that whole and leaving the other.

It seems, therefore, probable that subtraction has risen from addition by conscious observation and by the exercise of will in seeking the " inverse " of addition.

CHAPTER IV

MULTIPLICATION AND DIVISION

(1) *Multiplication*

Multiplication is founded on addition, therefore eventually on counting. The process of multiplying three by two is that of taking a series of three; then another series of three so that we have two series of three; and then adding these together. The process is symbolised thus: $3 \times 2 = 6$.

In this simple instance it is equivalent to $3 \times 2 = 3 + 3 = 6$.

We might, in order to save the trouble of counting in expressing the number of times the series is taken, symbolise the process thus: $3\,(1) + 3\,(2) = 6$.

Pursuing the line of discussion regarding notation, we observe the great importance of simplicity in symbolisation, simplicity not only in the manner but also in the number of objects presented. In this way it is deemed inadvisable to distract the attention by anything beyond the essential denotation.

Thus instead of $3\,(1) + 3\,(2)$, we might be tempted to express the process thus 32 where the first symbol represents the number in the series to be multiplied, and the second represents the number of repetitions of that series.

But the expression 32 already means something different. To avoid confusion we separate the 3 and the 2 by a symbol thus: 3×2.

In algebra where numbers are symbolised by letters as a and b, we really do adopt the form, ab, for multiplication. No confusion here arises, for as the proposition concerning a and b is generally taken to apply to any numbers whatever, it is not necessary to divide them into a series of Units, and hundreds.

If now we multiply larger numbers, say 43 by 25, we introduce no new Fundamental Process, but we combine in the new forms some of the principles of counting, addition and notation, which we already have discerned. In multiplying 43 by 25, we multiply 43 first by 5; that is to say we obtain the result of the addition of the series of 43 repeated 5 times. Then we multiply 43 by 20, that is to say we obtain the result of the addition of 43 repeated 20 times. We have now the sum of 5 times 43 and 20 times 43. Now taking a series of 43 as a Unit, we have this: 5 of these Units and 20 of these Units, and we know from previous investigations,

that the addition gives us 25 of these Units; but (calling the Unit which formed the series 43, the original Unit) if we consider all units represented by taking the sum of 5 series of the original units 43, and add these to the units represented by taking the sum of 20 series of the original units 43; then it is not immediately evident that the total sum will be the same as if we added all the original units 43 together 25 times.

The Distributive Property

The determination of this matter in a general way brings into view the "distributive" character of multiplication. Let us begin with a single instance. We require to multiply by 3 the sum of 5 and 2. Translating this problem in the most direct way into the language of symbols, we have $(5+2)\times 3$.

We may at once take the sum $5+2=7$; then the problem becomes 7×3; that is to say the sum of 7 taken 3 times; or the addition of 3 series of 7 units; and this we know to be 21.

The question may be discussed thus;

. . . . . (1)
. . . . . (2)
. . . . . (3)

Arrange a series of 5 units in order; then add a series of 2 units; as in the first line above. Now consider this as a series of 7. We have introduced no new process. Mark this series by the symbol (1). Repeat the process, arranging the units in the manner shown. Mark the two following series of 7 by the symbols 2 and 3.

We have thus 3 series of 7; and it is evident that their sum is the sum of all the units shown above.

But from the manner in which the arrangement is made we see that we have also 3 series of 5, and 3 series of 2; and that if we first obtain the sum of the 3 series of 5, then the sum of the 3 series of 2, and then add the results together, the sum so obtained will be equal to that of all the units shown above. Therefore we shall have:

$$(5+2)\times 3=7\times 3=21$$

also

$$(5+2)\times 3=(5\times 3)+(2\times 3)=15+6=21.$$

The result here obtained, however, does not depend upon any peculiarity of the numbers 5, 2, or 3.

We therefore generalise the process; that is to say, as before discussed in connection with the application of symbols, we recognise the character of the operation in a manner independent of the differences of the symbols to which it is applied.

If therefore, instead of considering 5, we take the symbol a to represent any number; and similarly b to represent any number; and similarly c to represent any number, then:

$$(a+b) \times c = a \times c + b \times c;$$

or with greater simplicity of symbols in algebra

$$(a+b)c = ac + bc;$$

This formula expresses the "distributive" property in multiplication.

Commutative and Associative Properties

At this stage the "commutative" property may also be discussed; that is to say, taking a particular example, we ask, Is 3×7 the equivalent of 7×3?

This question may be solved by reference to the arrangements of Units (p. 112).

7×3 indicated taking 3 times the series of 7, and the result is the sum of all the units shown. Now we have already seen that the manner of arrangement of units is indifferent, except for purposes of convenience, in the process of counting, and therefore of addition.

We note accordingly that in each of the vertical rows we have a series of 3, and we see that by virtue of the first disposition of this arrangement we have 7 of these series.

The sum of the 7 series of 3 is the sum of all the units shown. Therefore finally $3 \times 7 = 7 \times 3$.

And we may generalise, as before, using algebraical symbols thus: $ba = ab$.

Another question in a similar order of ideas arises with regard to multiplication. Is it "associative"?

This question may be discussed also by means of a simple example. If we wish to multiply in order 2, 7, and 3, and if we express the problem symbolically thus: $2 \times 7 \times 3$; we may proceed in more ways than one.

If we multiply 7 by 3 we obtain 21; so that we might express the problem now as $2 \times 21 = 42$.

But we might have begun by multiplying 2 by 3, obtaining as the result 6. Then the problem would have been expressed $6 \times 7 = 42$.

But is this operation capable of generalisation? Consider again the arrangement of the objects (p. 112). Here we have a series representing the product 7×3, or 21. Consider now the first vertical row ⋮; if instead of each dot we now place two dots thus ⋮⋮, and similarly for all the other rows, we will have 21

series of two; that is, we will have $2 \times 21 = 42$. But in place of the first vertical row, whereas we had at first 3 dots ⋮ , we have now three series of 2, or 6, ⋮⋮ . And in the whole arrangement this series is taken seven times. So that we have for the whole number of units $6 \times 7 = 42$. Therefore $6 \times 7 = 2 \times 21$. Therefore remembering the way in which we obtained these results $(2 \times 3) \times 7 = 2 \times (3 \times 7)$, or generalising; with the symbols of algebra, we have $(ab)\, c = a\, (bc)$. This formula expresses the "associative" property of multiplication.

Origin of Multiplication

Do we multiply numbers or things? We propose to multiply 4 horses by 3 horses. We take a series of 4 horses. We consider this series as a Unit. We take another series of 3 horses; we associate the series of 4 horses with a Unit of the series of 3. We renew the operation but with different units; that is, we take another series of 4 horses and associate the series with another Unit of the series of 3. We renew this operation, associating another series of 4 horses with another Unit of the series of 3. The full operation is therefore: a process of taking, or positing, a certain Unit (in this case a series of 4 horses) in a manner of which the simplest example would be the positing of an object such as a stone; the repetition of this operation; the Association of each act of repetition with a Unit of a known series.

Now by the natural development according to which symbolisation arises, we find that the material characteristics of the series of 3 are indifferent. The result would be the same if the series consisted of 3 peas, or of 3 stones.

The essential is the repetition in order; and the positing of a particular series of 3 only serves to make the Association with each distinct, and to secure that the act of positing is not repeated beyond the number of that series. Now the repetition in order depends upon the element of Time, the conception of which in this way is a Fundamental Process. The fact that a series of any 3 units will serve for the Association required proves that these units are only the symbols to mark the successive efforts of the Time process. The symbols of counting are also nothing but such symbols, and they are really just as material, though defined by means of the sense of hearing, as the symbols previously used. The original process of multiplication is, however, that of multiplying—that is associating and counting—concrete things with concrete things.

When we desire now to "commutate"—as for example in showing that $3 \times 4 = 4 \times 3$—we would meet with confusion if

the units of the series of 4 were not of the same kind as the series of 3. Therefore the employment of the operation of commutation presupposes that, whatever be the series dealt with, the series of repetition is symbolised by the same kind of symbols; and as the symbols familiarly employed, those of the numbers in counting, are not such as arise in ordinary experience; and as the process of disassociation with the accidental characteristics of the first units demands repeated experiences; it appears that commutation can only have arisen after the habitual exercise of multiplication.[1]

We may now resume the discussion of the problem which we left at page 113. The digression has been necessary in order to throw light upon the difficulties which had occurred, but now the solution will become clearer.

[1] The questions discussed in this chapter, and those dealing with addition and counting, have proved in the history of science to be of great importance. It is true, for instance, that persons habitually multiply and arrive at correct results who have never occupied themselves with the analysis here exposed; but that merely proves that they may follow the indications taught by others A person may use a telephone by following a few clear instructions, but operations of that kind would give him a very imperfect notion of the mechanism employed, and would serve very inefficiently for the development of the science of electricity. Or a person may walk without having studied anatomy. But if the process of walking becomes defective, as by injury to any part of the mechanism, it often requires that the surgeon shall be well versed in anatomy in order to apply the proper remedy. The great mathematicians, notably Descartes, Gauss, Plucker, Hamilton, and Riemann, sought all their lives to enter more profoundly into the understanding of the nature of the assumptions involved in the simple operations of calculating.

Riemann's writings on the subject are very abstruse and highly technical, but they have been productive of brilliant results in the higher mathematics.

We have already noticed the value of Descartes' system of dividing the straight line into parts. For one thing, since it enables us to define a line by the number of units which measure it we are able to denote it by a symbol.

Thus if we have ——|——|—— straight line A B divided at C into two equal parts, of length b, and produced to D, and if the length of CD be a, then we prove in geometry that the rectangle contained by the whole line thus produced, AD, and the part of it produced, BD—that is to say, the rectangle AD, DB—together with the square on the half line bisected—that is to say, the square on CB—is equal to the square on the line made up of CB and BD—that is to say, the square on CD.

A C B D

The proof given by Euclid is somewhat long; but if we observe now that AD is equal to $a+b$; and BD is equal to $a-b$, and that $(a+b)\times(a-b)=a^2-b^2$, as is familiarly known in algebra; then we have a comparatively easy proof of the proposition. For now, $(a+b)\times(a-b)+b^2=a^2$; that is to say, re-translating the algebraical language, the rectangle AD, BD added to the square on CB is equal to the square on CD.

Considerations of this sort led Sir William Rowan Hamilton to form a calculus, which he called Quaternions, in which processes of multiplication take the place of geometrical transformations.

Hamilton, however, found it beyond his powers to devise a form of spatial multiplication which should have the property of commutation, although he was able to secure for his Quaternions the associative and distributive properties.

Thus if p, q, and r represent three quaternions, we would have $(p\ q)\ r=p\ (q\ r)$; and, $(p+q)\ r=pr+qr$, but not $pq=qp$.

This disadvantage has been severely felt in the Calculus of Quaternions, which, however has become developed into a remarkably interesting and powerful instrument for dealing with complex problems of geometry. It is worth noting, therefore, that the questions we have discussed are of great interest in themselves, apart from the purposes they serve of determining within their scope, the necessity and sufficiency of the Fundamental Processes we have set forth.

By the distributive property we have:

$$43 \times 25 = 43(20 + 5) = (43 \times 20) + (43 \times 5).$$

So that we may first multiply 43 by 5; then multiply 43 by 20; then add the results.

In multiplying 43 by 5 we will find it advantageous again to make use of the distributive property. Thus:

$$43 \times 5 = (40 + 3) \times 5 = (40 \times 5) + (3 \times 5).$$

Similarly:

$$43 \times 20 = (40 \times 20) + (3 \times 20).$$

Multiplying 40 by 5 we get 200. Multiplying 3 by 5 we get 15. Adding now according to the system of notation which we found so marvellously adapted to our needs, we obtain 215 as the result.

Multiplying 40 by 20 we get 800. Multiplying 3 by 20 we get 60. Adding these numbers together we obtain 860.

We must now add 860 to 215. We have thus five units, 7 tens and 10 hundreds, that is to say, 1 thousand. According to our system of notation we write 1,075.

This manner of obtaining the result will bring clearly to view the great practical value of what in the higher branches of mathematics is called "elegance" in the manner of solution, and of which an excellent example is the ordinary rule of multiplication:

```
 43
 25
---
215
 86
---
1,075
```

We multiply the units together, 15. If the product exceeds 9 we "carry on", that is to say, we note the number of tens—in this case 1. So that we set down 5 out ot the 15 and carry over 1.

We then multiply the number of tens, that is to say, 4, by 5, and we get 20 tens; and adding the 1 carried over we get 21 tens.

Carrying out the same principle with regard to the tens, when we get more than 9 tens, we have one or more hundreds, and we place the number representing the hundreds in the third place from the right. We have thus 1 ten and 2 hundreds. For convenience of addition simply we place the product of 43 by 20 in the line below the first product.

As 20 represents 2 tens, the product, 20 times 3, which is 60, contains tens. It is evident that such a product can never contain a series of units less than ten. Therefore there will be no number in the first place on the right in the second line. Therefore we always begin with the second place. It would be tedious to examine the process further in detail.

A few observations, however, will make clearer how it is that a process really involving so much ingenuity has become so familiar. The "rules" of multiplication are still simpler than we have

indicated in the last description. There is in the mind of the person who multiplies no thought of tens, of hundreds, or of systems by notation. He guides himself by a series of associations which by constant repetition have become "mechanical"; and the successive steps are also of this nature. Thus 5 threes are 15; 5 and carry 1; 5 fours are 20, and 1 are 21; and so on.

Referring now to the symmetrical arrangements of the objects on p. 112, it may be asked: What new element had been there introduced which enabled us to establish easily the distributive and the commutative properties of multiplication?

In the first place we see that there are seven in each of the three rows, by simply observing that the first row contained seven, and that each of the other rows contains a Unit associated directly with a Unit of the first row.

But in the exposition of counting we find that the simplest exercise is to associate one established series with another; and visual associations in this way are the most facile. We count the second row by using as counting symbols the units which we have counted in the first row.

When we commutate, we find the threes now arranged in series, the series being vertical. In each one of these observations there is a new effort involved in each Presentation: and new Processes are involved in each Association. But by constant exercise and repetition these processes become extremely rapid, and unless attention be called to them specially they make little impression on the mind.

After having established the series of threes, each series is now taken as a Unit, and as the first horizontal row was counted seven, so each of these units (series of three) is counted by being associated with the corresponding original Unit of the series of the horizontal row.

Now we have already seen in the discussion of counting how the mind establishes the principle that the order of counting is indifferent, provided that in both instances all the units are counted.

The visual aspect of the arrangement of the units enables us, what otherwise would require considerable efforts of Memory, to decide that in both forms of multiplication, 7×3 and 3×7, all the units are counted, and no Unit is counted more than once.

Moreover, Association is itself easier when objects are presented in some way, as for instance in straight lines, at regular intervals, which correspond to facilities of our movements, even to the slight, almost imperceptible, movements of vision, and which depend on innumerable experiences based on the physical condition also of the external world.

Considerations of this kind elaborated so as to apply to each one of the movements, associations, recollections, involved, will make clear why even for mental operations an orderly arrangement of objects is of great importance.

But another question arises: How does the mind arrive at the notion of this orderly arrangement? That is the result of a series of tentatives, combined with the associations and recollections of other orderly arrangements which have served useful purposes. By the nature of our lives our minds become storehouses of operations for securing orderly arrangements in the simple necessary occurrences of every day. If the arrangement required be more complex than these, the mind makes tentative efforts on similar principles, and when an orderly arrangement which serves its purpose is found in this way, it is retained, while the others are rejected.

Already, in the simple exercises of calculation which we have considered we have been acquainted with certain principles, the vital importance of which is seen throughout the whole range of mathematics. These principles might be referred to as good choice of symbols, good form of notation, orderly arrangements; symmetry where possible; knowledge of known forms, and operational "elegance"; tentative efforts, estimated according to these standards.[1]

Suppose that we proposed to define multiplication as some form of association different from that with which we are familiar; and suppose that we so selected our forms of association that we could speak of multiplying 3 pigs by 4 cows, meaning thereby that to each of the three pigs we adjoined in some way four cows. There is no difficulty in such a view of the process. But this system would not be commutative, for if we multiplied 4 cows by 3 pigs, then to each cow we should adjoin 3 pigs; and the result would not be the same as in the previous case.

The peculiarity of the association of ordinary multiplication is, then, that the series of the multiplicand takes the place of each unit of the multiplier, and so becomes in a manner substituted for that unit. The multiplier does not become reckoned in the count of the result. Thus, taking the first instance, we

[1] Mathematicians will recognise that even complex problems, such as that, for example, of the solution of elliptic integrals, may be referred to successively under the headings indicated above. The celebrated quarrel of the followers of Newton and of Leibnitz turned to some extent on a question of notation. That adopted on the continent was, for the development of the Differential Calculus, more convenient than that of Newton. It prevailed. The immense impulse given to mathematics by men of genius, such as Euler, Lagrange, Gauss, was therefore realised in forms that were a sort of foreign language to the Newtonians, and "English mathematicians stood still in reverence of their great countryman." More recently Newton's notation has been revived for special purposes.

would associate four cows to each of the three pigs; but we would not count the pigs, and we would count the cows only by referring them to a new series of counters, that is to say, associating them in a peculiar way with this new series.

Approached in this manner the ordinary process of multiplying seems less direct than that apparently fantastic one which we here first considered, and a similar remark applies also to the ordinary process of counting. It might appear, then, that a special effort of ingenuity had been required at a point of development where that ingenuity was of the greatest importance, and yet where that importance could not well have been foreseen.

Such a supposition, however, is not necessary. We find that by repetition the associations of any series may become very strong as compared with the associations of any one of the series with any other object. This is especially the case where the members of the series are mere symbols and the repetition very frequent. Hence a sequence is formed which at each step allows of any association with another object, and yet continues on in the regular succession of the sequence.

It must be noted that the association is that of symbol with symbol. For when we count a series of groups, for example, of four pigs, we consider each group as a unit, and the reference to the groups as a unit is only possible by symbolisation.

How the Mechanism of Counting is Involved

We may now introduce a question which is somewhat recondite: How is it possible to establish such a sequence of symbols as enables us to perform the operation of counting? The answer may be that by repetition we have established a series of associations, step by step, so that these associations have become stronger than any other. But I do not think that this answer reveals the whole truth of the matter.

Suppose that we have three vertical columns containing four objects each, say for simplicity, dots.



This arrangement may be viewed as three times four. But suppose now that we regarded these as four horizontal rows each containing three dots. This arrangement would represent four times three. How do we know then that four times three is equal to three times four? The readiest answer is to say that this conclusion is obvious. And if the grounds of this reply be sought it will be said that there has been no change in the objects. If it

were said, There is no change in the *number* of the objects, that would be assuming the truth of what we are seeking to establish. So that the further question arises; Is there any assumption in this that can be definitely separated as an assumption? This question compels us to look narrowly into the actual process of counting.

Before entering definitely into that matter, it may be well to point out that the problem is not idle. For if the dots represented voltaic cells, and these were in the first instance linked up in parallel circuit (that is to say, zinc to zinc, for example) in the vertical columns, and then these columns linked in series of three, we would get a different result from that obtained by linking the horizontal rows in parallel circuit and then linking these rows in series of four.

But it will be objected that in this case we have not merely left the cells in position, we have interposed an arrangement. But in counting, and in as far as the results of counting only are concerned, we interpose an arrangement. We must discover whether it affects the results; and if not, why not?

In counting the whole system we might begin with the first column, calling the first dot one, the second two, and so on. We would arrive at the number twelve. In the second place we might count the horizontal rows in order, calling the first dot one, as before, but the second dot of the row two, instead of, as before, the second dot of the column. Here then we have an essential condition, that the sequence of the counting symbols must be so strong that it is not disturbed by any variation of the associations.

The next step is to inquire how far this may lead us. Can we in fact suggest variations of the association likely to disturb the sequence?

The dots may really represent other objects. Let us take them to represent the symbols of counting themselves, and let us replace them by these symbols. Further let us place the symbols so that the first column begins with 2, and so reads, 2,3,4,5,

2	6	10
3	7	11
4	8	12
5	9	1

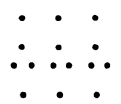
If then we count this column we refer the "one" of our counting to 2. If then we give attention to the fact that this object is really 2, the next association should be 3. But the next effort of the will causes us to return to the "two" of our counting, which will now apply to 3 of the column.

How the Conception of Unit is Envolved

Here is a case where the mind must proceed on one or other of these courses of association; it cannot proceed on both at the

same time. So that in closely considering counting we find not only examples of Association, but we discover experimentally a fact of Association, that repetition increases the strength of a series; and we are also brought to a clear view of the character of symbolisation (that is to say, Generalisation under another aspect) as a Fundamental Process; and finally we meet with a proof of the position set forth of the Conception of the Unit.[1]

If this last point is not quite clear another example may help to indicate it. Suppose that each item of the vertical row consisted either of an individual object or of a group, as for instance of men, and that the first consisted of one man but the third of two men then the counting would proceed normally to the third.



If the group of two men was taken as one object, the counting would still proceed normally. But if it were sought to recognise that the group was really formed of two, and that, without the intervention of a symbol, we desired to make the count under both suppositions at one and the same moment, we would find it impossible. Thus the validity of the argument with respect to the Conception of the Unit is established.

We discover that our counting is conditioned by this limitation of our faculties. But counting we will see to be the base of all mathematics, even to the highest reach, for all these are but adaptations of principles of combinations which can be reduced eventually to the type of simple addition and multiplication.

We have already found in certain instances (*cf.* p. 51), and subsequently we will see that this is general, that the whole process of Reason depends on limitations of Discrimination. The fact of experience that repetition strengthens the series of associations depends on conditions of organic growth. The establishment of a series depends on the constancy of physiological processes correlative to psychological processes. The study of counting therefore has brought us to the consideration of basic phenomena of thought, and shown us also the restrictions within which only we can speak of the certitude of Reason.

(2) *Division*

Division represents little difficulty after multiplication, and after the consideration of inverse processes already discussed (p. 109). For example we wish to divide 21 by 3. The problem

[1] *Cf.* note p. 86 and note p. 87.

may be expressed thus: How many series of 3 must we take in order that the total of the units thus presented be 21?

But our use of multiplication implies a knowledge and easy recollection of certain results. We know that seven threes are 21; that is to say, we know we must take 7 series of 3 to produce 21.

And this knowledge must be so familiar that (*cf.* Counting) the association of 7,3, and 21 in a certain relation is stronger than the associations essential to any form in which the proposition may be stated. That is to say, apart from any set enunciation, we know that 7 series of 3 produce 21, in such a sense that, varying the form of words, we must take 3, 7 times to produce 21, or that 21, divided by 3, gives 7.

But suppose we had to divide 23 by 3. We know that $3 \times 7 = 21$ and that $3 \times 8 = 24$. Thus, although we have no associations immediately between 3 and 23, yet the associations between 3 and 21, and 3 and 24, are so strong that they enable us to make the tentative efforts to show that 7 is too small a number and 8 too great.

Now the principle of notation indicates that in any category, as for instance, the tens, the greatest number of complete tens, up to 9, is taken, and the remaining units are placed separately; and similarly for the other categories. Consequently we would select here 7. Now we have two units remaining.

As we are not tracing out the actual development of mathematical forms, but inquiring what processes are involved in those we consider in order, we may assume some acquaintance with fractions, the most facile of employment of which are decimal fractions.

With these the Unit is divided into ten parts, and each of these being taken as the Unit under consideration, we find that the 2 remaining as above are reckoned as 20 tenths, or 20 decimal parts.

Dividing by 3 we obtain 6 and 2 over. We treat these two remaining decimal parts according to the same principle, expressing them now as hundredths; and we continue the process of division. The method of arranging the notation in the decimal system is admirable and simple. A dot indicates that the places to the right are occupied by tenth parts, hundredth parts, and so on.

In this particular instance we will observe also that as at each step we have a remainder 2, the process continues indefinitely.

We will have occasion in several places to refer to infinite series, and to inquire as to their significance. Meanwhile we may express the result of division of 23 by 3, thus $23 \div 3 = {}^{23}/_{3} = 7.666$ etc.

We may here leave the subject of division, for its discussion introduces no new Fundamental Processes, nor any derived processes which are new and different, except in application, from those already discussed.

CHAPTER V

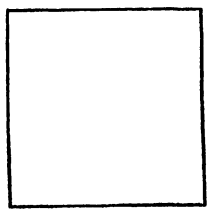
OPERATIONS WITH SPATIAL RELATIONS

NOTE:—We have now reached an important stage of the journey. We have acquired the result that the mind can entertain, at a given instant, only one Presentation. This gives us the Concept of the Unit, and we have demonstrated what is implied in that position. This has given us the key to Counting, and then to Addition and Multiplication,[1] *and, with a reservation on a postponed discussion on Infinity and Imaginaries, we have seen that the schematic forms of our thoughts, in as far as sequence is concerned, are co-extensive with the corresponding mathematical operations. And thus observing what Fundamental Processes are involved, we have obtained, within the limits hitherto discussed, the justification of the series we have given of these Processes. So far we have been mainly concerned with operations in Time.*

It now remains to examine whether a consideration of spatial relations introduces any new Processes. Already we may expect that what is new will be in the nature of new forms of Presentations, and that the operations in Time will apply equally in this field. We shall find this anticipation verified. We shall then have proved our schemata co-extensive with mathematical forms. Our next step will be the observation of the manner in which the schemata may be considered as marked by the actual Presentations—sensations and the like—of our experiences. Then having proved the central theme, there will remain the development of special studies such as Memory and Association and the re-examination, with our new instrument, of important positions, such as Externality, Will, Ego, Reason.

Problems involving spatial relations introduce questions that deserve separate consideration. If we take a square each side of which is equal, say, to one inch, and if then we say that the square is represented by 1×1, we are here performing a process which is not that of ordinary multiplication. The form, 1×1, as interpreted by the meanings we have already attached to these symbols, indicates taking a Unit once, that is to say, simply positing a Unit; and the Unit in this case is the side.

The operation of positing an area as a Unit arises from the Fundamental Processes involving Space. Certain aspects of the combinations of such processes are discussed in the chapter on Externality. In this place, however, we may call attention to the various impressions produced in the exercise of vision in regard to length from one point to another, with all the associations derived from corresponding movements of the ocular apparatus; and the impressions in regard to length in a direction perpendicular



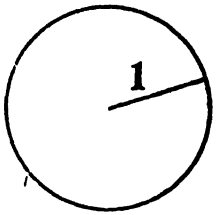
[1] In Jordan's valuable *Cours d' Analyse*, he says (Vol. I, 15). Arithmetic and Algebra are concerned with four fundamental operations: addition, subtraction, multiplication, division. One can conceive a fifth, consisting in replacing a variable quantity by its limit. It is the introduction of that new operation which characterises the Infinitesimal Calculus.

to the first. We find Space as an Immediate Presentation, but we also find associations, which we may group together as those of inclusion, with regard to the impressions of lengths just noted. Also in experience we may meet with many instanes of spaces being divided into parts symmetrically adjusted, as in seeing a number of regular small spaces filling a large space.

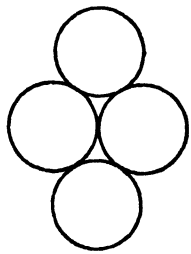
In this way we arrive at the operation of positing a certain defined area as a Unit area.

If the whole matter be considered in connection with that of the division of the line into units as already discussed (*cf.* pp. 106 *et seq.*), we see that in the adaptation of a unit area to the measurement of areas we have an advance similar to that which led from the old Greek treatment of homologies and homographies to Descartes' system with its facilities of transference to algebraical forms.

To bring clearly into view the convenient character of the square as the Unit of space we may inquire why the circle has not been so adapted. We might take a circle of unit radius as the unit of plane spatial measurement; [1] and the sphere of unit radius as the unit of measurement of space of three dimensions,[2] or what we call solid measure.

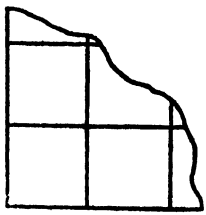


If, however, in tentative efforts we endeavour to comprise an area by means of a number of unit circles, we find a great difficulty in the fact that the circles, however small, cannot be made to fill up the area; and that the parts excluded from their areas are of a complicated character.



On the other hand the unit squares are exactly juxtaposed.

At the boundaries of an irregular figure we find that the squares do not become exactly applied. The smaller we take these squares the smaller will be the errors so occasioned. We may then suppose our unit sub-divided indefinitely, if only in the summation we take for one unit as many parts as those into which the unit was sub-divided. Thus if we had taken the square inch as a unit, we might divide this into 10,000 parts, and

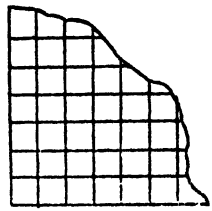


[1] It is a fact that in an advanced stage of mathematical development, Gauss in a remarkable treatise on the Theory of Surfaces, and Hamilton in his development of Quaternions, found great use in conventions of a similar nature.

[2] That is to say, according to the Cartesian system.

use such parts for applying to our space to be measured. We would get a closer application to the boundary than if we measured by the original unit.

Then in estimating the total area we could take 10,000 of these parts for the original unit, and estimate spaces left over by the number of parts.

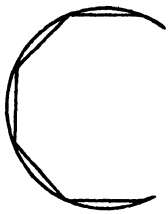


We have had occasion already to refer to a similar process in decimal fractions in the discussion of division. Considerations of this kind led to the formation of the Infinitesimal Calculus, or Differential Calculus, as it is usually called.

Reason of the Dominance of the Straight Line

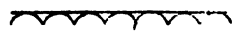
There is an interesting study in the inquiry as to the dominance of the straight line and the right angle in our simple geometrical processes. We measure the circumference of a circle, for example, by straight lines, which are arcs of the circle. It is evident that the smaller these arcs are taken the less is the difference of their total length from that of the circumference. Hence if we take the arcs small enough, and so make their number correspondingly great, we may have a length differing from that of the circumference by a deficiency as small as we please

Here again we have a suggestion which, given forth by the older geometers from the days of Archimedes, and developed later by Fermat and by Pascal, led, in accordance with notions arising from the problem of areas, to the Infinitesimal Calculus.



But it is possible to consider just as closely a straight line as made up of a great (unlimited or infinite) number of small arcs of circles.

There are reasons which might tend to make this method of estimation the more natural. Thus, our physical movements are mainly directed by the pulling of muscles on the bony framework. The movements of this framework take place at joints, or hinges. Hence the movements of parts are in the first place circular. Certain of the joints, it is true, permit slight sliding movements. Straight movements are, however, obtained mainly by the combinations in various suitable ways, taught by experience, of circular movements.



The eye, which is not attached to a joint, is moved by the combination of complicated movements, but the dominant character

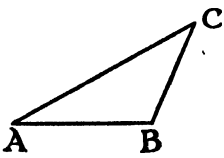
of the movements, even in this instance, is that of rolling around various axes.

In our contemplation of the external world we meet comparatively rarely with perfectly straight lines but very often with lines whose deviations are on all sides of a straight axis. Moreover, our mental habits are greatly influenced by the fact that the direction of propagation of light and of sound is in straight lines, and thus the impressions of our two most active and most discriminating senses are associated with straight lines. Then again, although our movements are compounded of circular movements round hinges, it is easier to draw a straight line than a circle; and, what is of greater importance, it is easier to note and correct the deviation from straightness than it is to correct the deviation from a perfect circle.

Hence the simplest of drawing instruments are straight edges. It is important too that, though in the human body there are comparatively few straight lines, yet here again the deviations are round well marked axes. If a man stand upright with his two arms extended and held sideways, then his body represents a structure round a vertical axis, his arms contain an axis at right angles to his body; his vision is on a line at right angles to both those axes. The direction of gravity is in the vertical axis. It is in fundamental conditions such as these that we must seek the origin of the choice of certain standards, bases, and units that we constantly apply without reflection.

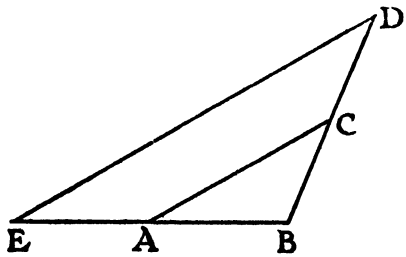
Multiplication Symbol in Square Measure

In order to show how purely conventional it is to apply the symbol of multiplication to a certain conjunction of the two sides of a square in order to obtain the area, we may adopt a convention inconsistent with this, yet by interpreting the symbol in a manner consistent with itself obtain under certain circumstances valid results.



Thus we may say $AB.\ BC$ = area ABC when the dot does not necessarily imply multiplication, and we then proceed to develop such an "algorithm" or system of calculation.

Call AB, a, BC, b; $a.b$ = Area ABC. Produce the sides BA to E, and BC to D, making $BE = 2a$, and BD $2b$. Then we would have $2a.\ 2b$ = Area EBD.

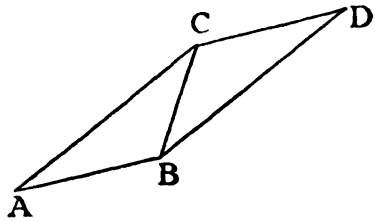


If in this formula we now assume that for the number 2 the process of multiplication is applicable, we have $4ab$ = area EBD. But $4ab = 4$ area ABC; so that the area of the triangle EBD is four times that of ABC. The result in this case is true; but there is an unjustifiable assumption in the use of the "algorithm" itself; for the area of a triangle cannot be expressed as dependent simply on two contiguous sides; it depends also on the included angle.

In the case we have considered the included angle remains the same for both triangles, hence a cause of error in the application of the algorithm disappears.

It is necessary, therefore, to examine any proposed algorithm not only to observe if its use be consistent in itself, but also as to whether it is justifiable in regard to the object to which it is applied.

Certain assumptions are made in regard to vectors (*cf.* p. 105) which, though justified, must be carefully noted. Thus if $AB + CB = AC$, and we wish to apply this formula, with the faculty of commutation, we have $BC + AB = AC$. But beginning with BC, we can in this system add AB, beginning at the end of BC, by assuming that CD, which is equal and parallel to AB, is its equivalent in regard to the formula. Then $AB + BC = BC + CD = BD$. That is to say, BD is the equivalent of the vector AC; that is to say, is parallel to it, and equal to it in length. Hence with this notation we arrive at a demonstration of the proposition that the lines which join the extremities, in order, of equal and parallel straight lines, are themselves equal and parallel; but in the posing of the algorithm itself this proposition is virtually assumed.



Similarly whereas Euclid lays down his axioms, such as that things that are equal to the same thing are equal to one another, etc., he assumes, in the statement of one of these axioms, what is less obvious, that all right angles are equal.

* * * *

It would be tedious to trace out in detail the various developments of arithmetical and algebraical calculations.

The result of the investigation is to furnish the proof that, at no stage are new methods introduced which are not continuations of the Fundamental Processes associated with actual objects of experience.

No really new calculus can arise, that is to say, a calculus which could offer a new faculty for solving problems, and which would be other than the continuation of those we have already discussed, and all that is involved in the multitudinous and complex forms of their applications, together with the necessary storing in memory of certain "known forms", and frequently repeated complex operations.

The Symbols of the Differential Calculus

The Infinitesimal Calculus introduces new symbols which give at first the appearance of a new order of ideas, but it is really algebra applied to conditions where we deal with the addition of "infinite" series, of which each term may be the product of a certain quantity and a quantity differing as little from zero as we please.[1]

Now in algebra we apply the symbol Σ which is simply a form of "S" to indicate a sum.

Thus $a_1b_1 + a_2b_2 + a_3b_3 + \text{etc.}$ might be expressed as Σab; that is to say, the sum of all the quantities, each term of which is of the kind of, or is generalised by, ab.

In the Integral Calculus we have the symbol $\int$, which is also a form of "S", for sum, and this indicates the summation of such infinite series, as are referred to above. $\int ydx$ means the summation of an infinite series, $y_1dx + y_2dx_2 + \text{etc.}$ of which ydx is the general term. It is understood that y is a "function" of x; that is to say, a quantity whose value depends on and changes with that of x; and dx means the "increment", or increase of x, or in other words, the difference (hence dx) between successive values of x, when this difference is considered to be as small as we may conceive.

But we have already considered the resources of algebra in respect of the Fundamental Processes. If then the Differential Calculus involve as new conceptions only those of indefinitely, or infinitely, large and indefinitely small numbers or quantities, we need only consider especially the processes involved in these conceptions (*cf.* Chapter on Infinity).

[1] Hermite, whose work in the higher mathematics is profound, called himself an "Algebraist."

CHAPTER VI

THE AXIOMS

NOTE:—The axioms of Euclid have generally been considered as the simplest basis upon which a series of reasonings may be built; and since Euclid starts with the statement of the axioms, the postulates, and the definitions to which he intends to make reference, it is assumed that the reasoning requires no further appeals to experience. We shall find, on examination, many grounds for regarding the whole matter in a different light.

Let us consider the axiom: Things that are equal to the same thing are equal to one another. The previous discussion of the concept of Unit has led us to the position of considering that things may never be equal one to another; that each depends eventually on the Immediate Presentation of Unit, these Presentations being separate and distinct things; and that combinations of these, depending on various operations in which Association plays a large part, are never the same for two objects. Even that two units presented in succession may be classified under one generalisation depends, as we have seen, on the limitation of our process of Discrimination.

If we confine our attention to the magnitude, for instance, of the area contained within the boundaries of two things, then we have already made an abstraction, which we have observed to be in such a case the result of the combination of several Fundamental Processes.

Before discussing the matter in this light it may be well to notice certain objections that may occur. For instance, there is ordinarily no hesitation in accepting the axiom as being both self-evident and basic, and moreover the theorems deduced by Euclid from such premises are found to be true.

In the first place the axiom assumes what is here the point contested, namely, that different things may be equal. But if the equality be extended to absolute identity then the axiom becomes reduced to a barren expression. If there be not identity, then since there are differences in the two things considered we must exercise Discrimination, and in paying attention to the features of resemblance we perform Association, and we make Generalisation, and thus finally abstraction.

That we habitually proceed in reasoning on assumptions of this kind and arrive at valid results is consistent with what has been said, for our assumptions and our results may be both true within

the scope of our consideration and within the limits of error of our discrimination.

Thus we may say of a number of eggs that they are all of the same size; they may not be absolutely of the same size; that would be practically impossible. But within the limits of error of our superficial observation and within the limits of precision imposed by the use to which to apply them, we may say the eggs are all of the same size.

Or again I bowl a cricket ball. It is hit over a fence, and after a search is found. I say it is the "same" ball. I may repeat this experience. But in the end it may be battered out of shape, and have lost some of its substance. It is not absolutely the same ball; and as the change has been gradual it is evident that absolutely it was not the same ball at any two successive trials. Yet no confusion arises in the ordinary use of the language.

But in studying, in regard to its processes and its products, such a delicate affair as the mind, so elusive in its operations, and yet capable of producing such complex and wonderful results, it is necessary to pierce as far as we can to the foundations and to examine the necessary parts in the finest movements of consecutive working. In this way alone can we hope to understand its mechanism and to indicate its development.

If an axiom were really elementary, any reasoning depending on the axiom should gain no force from the previous statement of the axiom. For the axiom acquires a definite meaning only when applied to a particular example. But it is easier to see the truth of the axiom in regard to the example, than first of all to express it in general terms, and then to particularise from the general expression.

Relativity of Measures

If any object be transported into another place it is not the same. A little discussion will make this clear, and it will be seen to be not a matter of futile metaphysical discussion, but finally of practical importance.

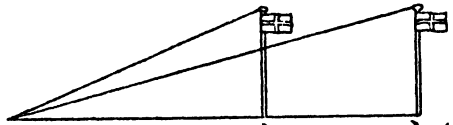
A body placed a metre higher than in its first position does not weigh the same as before. There was a time in the history of the world, and long after some of the most celebrated and elaborate systems of the Universe had been devised, when the proposition would have appeared absurd. To those to whom it might for some metaphysical reason appear not absurd, it would at least appear unimportant.

Yet the questioning of such possibilities led to various theories

of gravitation, to which at length the genius of Newton gave a clear and correct expression.

Suppose that we asserted that the shape of an object changed when the object is transported, what means would we have of disproving the statement, or of proving the contrary? To measure it would be beside the mark, because if the proposition were true the measure itself would change in the same proportion.[1]

The proposition that the magnitude of a body changes with its position seems really more nearly in accordance with actual experi-



ence than the contrary, for the farther a body is removed from the eye the less the angle which is subtends at the eye.

Moreover the conception of magnitude seems to depend more than that of weight on subjective impressions. Thus we appreciate the weight of a body by supporting it in the hand. But we also have a means of estimating its weight by observing its effects on a balance; and this experience is made manifest to us by different senses. Accordingly we gain a notion of objectivity in regard to weight.

This is less obvious in the conception of magnitude. Yet the complete conception of magnitude is formed by the combination of many elements of judgment brought from different senses (*cf.* Chapter on Externality).

The direct visual impression alone, apart from the impressions of the muscular sense in the movements of the eye, as in direction and accommodation, does not give the notion of the externality of the object perceived.

The idea of externality is derived from an elaborate system of co-operation with vision not only by the muscular senses of the ocular apparatus, but by the sense of Effort of the muscular system in approaching or receding from objects; by the sense of hearing, by the sense of smell, by the sense of touch, both in light touches and in impacts; all these combined in our experiences in all sorts of manners and degrees and repeated incessantly. Now the muscular sense is affected by the situation of the body, because the

[1] No philosopher regards a question as settled by appea to what is called "common sense" or ordinary experience. What operation of common sense could indicate that two lights could produce darkness? The deep investigations of Fresnel as to the veritable nature of light, however, enabled him to devise experiments showing the phenomena of interference.

weight of the parts moved by the action of the muscles is so affected.[1]

Therefore it is evidently not from direct impressions that we can know or deduce that objects do not change their magnitude with their situation.

If we look at the question in the light of the analysis of all the processes concerned in the conception of space, and of what we know of "objective" conditions, the postulate that a body, or a figure, remains unchanged when its situation is changed, so far from being self-evident, seems rather in danger of losing all meaning. For any material body changes by the operation of gravitation; the objective standards of measure change; the immediate subjective impressions change; they are liable to error. Moreover, it may be questioned whether we can think of a shape except in association with a material body. Already in the case of a straight line between two points, the association of the material forms seems predominant; but if in the terms of our proposition we suppose that two points may change relatively in position, how can we apply a fixed standard?

Can we even form a conception of a point fixed in space? If we could we would be able to speak of absolute motion of a point instead of merely relative motion. Clerk Maxwell, whose profound and perspicacious intellect was greatly concerned with this aspect of the question, concluded that the only motion we could think of was relative motion.[2] The appeal must here be made to one's faculties of introspection.

But if this be so, then it seems also that we cannot think of a point except in association with the presentation of material objects. For how otherwise can we define our point?

Abstraction in this case can get no further than the conception of external minute material things, with the faculty of throwing

[1] In Riemann's famous treatise on the Foundations of Geometry, which is one of the most abstruse discussions in the whole realm of philosophy, but which has nevertheless been fruitful of progress in mathematical science, the great analyst sends the plummet here and there profoundly. He treats as questionable assumptions the postulates, always implied in mathematical physics, of the persistence in regions infinitely great or small, of the most universally accepted laws. He is not sure that it is right always to assume the possibility of an absolutely plane figure, or of lines indefinitely parallel. The notion, however, will be found in the speculations of Gauss as to whether a geometry might be built up which ignored the twelfth axiom of Euclid, and it will be found in von Staudt's *Geometry of Position*, and afterwards in the speculations of Clifford. Bolyai and Lobatchewski followed Gauss in developing the Pan-Geometry. The posing of questions of the same kind as that discussed here has also given rise to the doctrine of Relativity; finally to Einstein's remarkable theories. The question is here approached from a different basis.

[2] It is true that as Clerk Maxwell points out (*cf. Matter and Motion*, articles cv and cvi, also articles xxix and xxx.) rotation may be absolute. Rotation is a case of relative motion where all the parts concerned obey certain conditions that determine in a certain particular manner their relative movement to any point, as for instance, to a point in the axis, or to the axis itself.

off from the mind the Impulse connected with the associations of any particular material thing. (*cf.* pp. 144 *et seq*).

If we look at the question in the light of our scientific knowledge, we find that we are constantly moving in space in movements compounded of our rotation around the earth's axis, of our movement in the earth's orbit, and of a number of perturbations; while again the whole solar system is moving in space. The idea of an absolute point or absolute motion vanishes. And to find a definition for points, relatively considered, we must resort to definite associations with external objects, though taking into account the faculty of abstraction already considered.

But adopting these conclusions, can we not form a notion of the permanancy of a line, or shape, independent of its situation?

We may study the question of weight in order to seek some guidance. Here the senses give illusory direct evidence, and no instrument that we can devise is delicate enough to detect slight differences of weight when a body is moved within a small space. But we have by a series of reasonings arrived at a theory of gravitation, expressed by a law, which has been found to give results always compatible with experience even with the most delicate measurements, and in regard to distant bodies, the sun, the moon, and the stars. We make an assumption that the law of gravity applies to all bodies within a scope which is very large and in all situations. Our reasoning on the matter has no certitude beyond this assumption.

We therefore seek in the problem of shape, with regard to space, some constant " objective " condition. We might find this in the permanency of the laws of light. Thus for example theoretically we might suppose the length of a line measured in a certain situation. We might suppose it transported into another situation. Then we might by using the formula of trigonometry so arrrange that we measure the length of the line in the second situation by means of angles and lines measured in the first situation; and therefore not involving the transport of the standards themselves.

But even this assumes certain permanent conditions with regard to light, and as by the researches of Faraday, Clerk Maxwell, Hertz and other physicists, we are led to believe in a close correspondence between light and electricity, at least in their objective condition, we begin to see at length under what limitations our " certitudes " are founded.

The object of the foregoing discussion has been to unsettle the mind a little in respect to the immediate acceptance of axioms, either as self-evident or fundamental. Regarded in the manner in which we now approach them they have quite a different aspect.

Instead of saying: Things that are equal to the same thing are equal to one another; we prefer to think in such terms as these: Suppose that in contemplating three objects we make abstraction except in regard to magnitude, we find in experience that with different objects we may reach the limit of our faculty of Discrimination in this respect, and that arising from the concepts of the three things, the variations of Impulse and of the Associations formed, and of the directives to the subsequent movements of the mind along a course of reasoning, may be taken as beneath the limit of Discrimination for any purpose concerned.

It is convenient to express all this in the familiar form of the axiom, and there will be no difficulty in its use, but a real philosophical gain will be found in the more precise conception.

It is evident, however, that the consideration of the axioms does not introduce any Fundamental Process except those we have already considered.

CHAPTER VII

PARADOXES IN MATHEMATICS

(I) *Problems of Infinity*

NOTE:—The study of infinity and of imaginaries may now be taken so as to make complete the review of the operations of mathematics in regard to the Fundamental Processes. On a first reading the study of this chapter, although it finds its logical place here, may be deferred until after the chapter on Association, so as to break the sequence of abstruse considerations.

The question of motion may first be considered. The discussion will be found to involve that of infinity.

We know of motion only by the change of position of something, *A*, which having been at one moment in position *a*, is discovered at another moment in a position, *á*. Thus we find in the consideration of motion conceptions of Time and Space associated with whose of the moving object in a peculiar way. The conception of motion therefore is not fundamental in the same way as those of Time and Space.

The old problem of Achilles and the tortoise is most instructive, for far from being worth only a smile it poses a problem which demands the subtlest delicacy of thought. The tortoise has, let us say, ten yards start, but Achilles runs ten times as fast. The old sophists asserted that it was impossible to think that Achilles could catch the tortoise, for when he reached the distance of ten yards the tortoise would be one yard ahead, and when Achilles reached that spot the tortoise would be one-tenth of a yard ahead; and so on *ad infinitum*

If one follows closely this kind of reasoning and keeps within the limits of a conception of motion formed as of a continuous progression, and if one must, as the sophists did, tacitly assume that this continuous progression can be described by means of small, even infinitesimal, intervals of Space and Time, having certain relations determined by the conditions under which the problem is offered for study; then there is no escape from the conclusion: Achilles cannot be thought of as overtaking the tortoise.

In reference to the conditions offered by the sophists, it must be remarked that as the interval between successive points becomes diminished, and the time diminished accordingly, the time is diminished at such a rate as precludes Achilles overtaking the tortoise within any interval considered. Thus suppose that

Achilles runs at the rate of ten yards a second and that the tortoise has a start of ten yards. We next consider the situation at the moment when Achilles has covered the ten yards interval. One second has elapsed, but this is too short a time to consider in view of the start the tortoise had. Similarly when Achilles has travelled another yard, we have to consider the relative positions after the lapse of one-tenth of a second from the previous moment. The interval of space between Achilles and the tortoise has been lessened but the time necessary to produce the next positions which we are asked to consider is again too short. And so on, as Achilles approaches the tortoise correspondingly smaller periods of Time are offered for our consideration. The solution of the problem is impossible on these terms.

But suppose motion to be effected in this way: An object reaches a position *a*, then is suddenly annihilated. It is restored to its entity in the position *á*. The fundamental conception of Space is here involved and also that of Time, but it is in their primordial aspects. *á* may be any distance we please from *a*, and the time required for the operation may be anything we please.

We have thus a new conception of the motion of a body from *a* to *á*, and if the limits of Space and Time involved in the operation be such that they do not conform to the notion of the infinite sub-division supposed by the sophists, then we should have disposed of the difficulty of Achilles and the tortoise.

Certainly we meet with a new difficulty, for we have evaded the conception of continuity, that is to say, continuity beyond even the limits of our infinitesimal concepts. But that is the crux of the whole question. Our minds cannot conceive of continuity in that sense.[1] The mental operation involved in the conception of motion is the same as if the objective process were, as described, a disappearance and reappearance of the object.

It is true that we do not commonly think that the object so disappears; but we are aided in our conceptions by the limitations of our visual organs. Thus we know for physical and physiological sciences that an impression lasts on the retina one-tenth of a second, and that the same period is necessary for a clear impression to be formed. So that if in verity the object did disappear for an extremely small part of a second and reappear, our visual organs could tell us nothing of that event. Thus in the familiar experiment of whirling a lighted stick in a circle we know that the lighted part does disappear from one position and reappear at another, but our direct visual sense would indicate that the lighted part occupied the whole circle and that it remained there continuously.

[1] *Cf.* p. 87 (note).

It is not here contended that the object actually ceases to exist from time to time, but that whatever be the condition of the objective reality, the conception of motion cannot be formed otherwise than as that of appearances at points at intervals from preceding points. Thus the answer to the sophist is that apart from the question of the decreasing time limit already discussed, he has no right to assume that motion is ultimately continuous; further, the mind is not capable of representing this hypothetical continuity by means of infinitesimal divisions. The sophists, in short, ask us at any moment to consider a finite image, whereas the division of time imposed by the conditions must be infinite and moreover they assume continuity which cannot be represented even by infinite division, in so far as what we call infinity, in this respect, is a veritable concept of the mind.

Let us look at the matter from another point of view. It has in recent years been determined that the old problem of squaring the circle is insoluble.[1] In other words circular motion cannot be represented by a series of small rectilinear motions. With this falls our conception of tangents, otherwise than by approximation. And thus the basis of our mathematics, so far as it is applied to these subjects, is tentative, that is to say, it marks the limit of our notion of things in our endeavour to conceive their ultimate reality.[2]

As the question of motion and that of infinity which arises from it are among the most obscure in the whole range of philosophy, it is well to seek light from any quarter that promises to offer a solution, and to re-examine even at the risk of repetition and insistence. Let us make the problem of Achilles and the tortoise simpler by taking it to assert that whether Achilles be pursuing the tortoise or not, it is impossible for him, setting out from point A, to arrive at a point B.

This is really the paradox of the old sophist who denied that there was such a thing as motion. A body must at each instant of time, he said, occupy the space where it is, but if it occupy the space where it is, it is at rest in that space.

All these paradoxes will be seen to depend on the meaning given to such words as instant, moment, point, infinity, or infinitely small.

[1] By Lindemann: *cf.* Klein's Lectures at the University of Evanstown; *cf.* also the *Oeuvres* of Hermite. The proof is valid only in so far as it assumes that the problem can be approached by the method of the infinitesimal calculus, and not by any new algorithm, if such were possible, free from difficulties that here arise.

[2] We seem thus to be brought face to face with Kant's Noumena, or things-in-themselves. But if our tentative efforts mark the limit of our powers of conception, it would follow likewise that we can never know the meaning of things-in-themselves.

Diogenes refuted the sophist[1] by getting up and walking. This was, it seems to me, a perfectly valid refutation, and not one that depended merely on superficial appearances. To make my meaning clearer, this may be contrasted with an apparent refutation of a statement. A man asserts, for example, that an object cannot be in two places at once. Another may whirl a lighted stick round his head and profess to show him the object in several places at once. Now what is true here is that there is a phenomenon occupying a certain space at a given moment, but it is not admissible to assume that that phenomenon is the lighted stick.

But in the case of motion, when motion may be defined as the change of situation of one body in relation to another, and where it is sought to deny fundamentally that there can be motion, it is sufficient in any one instance to exhibit a change of situation of two bodies.

The sophist is thus answered, for the appeal is made to tests as fundamental as those which he advances; but we may pursue the question farther and ask what was the origin of the sophist's paradox.

The Origin of the Paradox of Achilles and the Tortoise

Suppose then that we consider Achilles proceeding from *A* to *B*, and let us suppose the distance from *A* to *B* subdivided into intervals, *I*, *II*, *III*, etcetera.

Now if we consider Achilles traversing interval *I*, then since no one has contended that a body can be in two places at once, Achilles occupies an appreciable time in passing from the beginning to the end of interval *I*. Similarly with interval *II*; and so on; so that if his rate of progress were uniform and the spaces equal, then to arrive at the end of interval *XX* he would require *XX* times the period of time required to traverse interval *I*; and to arrive at the end of interval *n*, when *n* is any number however great, he would require *n* times the time spent on interval *I*.

But now the divisions of the entire distance *A* to *B* may be as small as we please, and even when Achilles had traversed *n* of these intervals, we might consider that his progress was contained within confines as small as we pleased. And this would also be the case when *n* was made as great as we pleased, greater than any assigned number; we have only to make the intervals of the original distance small enough in proportion. If then instead of merely employing an empiric dogma, as of the Common Sense school, we try to obtain a clear vision of Achilles's progress, forming

[1] This paradox appears to have been long in honour among the sophists from Parmenides to Zeno the Eleatic and his school, one of whom is said to have propounded it to Diogenes.

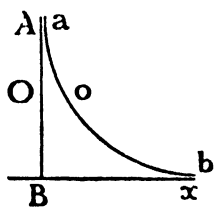
a conception of his appearing at each point, and so for any number of points greater than any given number, we fail.

The underlying fallacy of the paradox is that whereas in attempting to visualise the progress of Achilles we assume an appreciable measure of time employed, and this is brought into stronger relief by the fact that our mental operation occupies a considerable time—yet we assume the division of the distance of *A* to *B* as infinitesimal.

If we actually fall short of this and consider the division of the distance to be expressible finitely; and if we assume that Achilles actually traverses the first interval, then even though a lifetime may not suffice to consider him traversing each interval in succession, yet we become satisfied that the problem is soluble. And making the calculation that for a certain extremely small yet finite interval we would require a finite though extremely small period of time; we arrive at the common sense conclusion that in a certain finite, and not extremely small period of time, he will traverse the distance between *A* and *B*.

This mode of reasoning fails if the distance be infinitely divided. But in that case the assumption also fails that any appreciable and therefore finite period of time is occupied in traversing any one of the intervals. We are unable to form an appreciation of such an interval at all. This is the proper manner of answering the sophist.

Before asking, however, what is the meaning of infinity, or in what way we can form associations with this notion, it may be well to consider certain other cases.



Suppose we have a certain horizontal base line *Bx*; and that we have a straight line *AB* perpendicular to *Bx*, and that further we have a curve, *ab*, asymptotic to *Bx*, that is to say, which continuously approaches *Bx*, and which may be continued until it is less than any assigned distance from *Bx*, but which does not, at any finite distance, from the point *B*, meet *Bx*.

Suppose *A* and *a* to be at the same height above the base *Bx*, and suppose two points *O* and *o* to move, one along *AB*, the other along the asymptotic curve, in such a manner that they are always on the same level; that is to say, so that the height of *O* above the base is equal to the height of *o* above the base.

Now as *O* moves along *AB* it may be supposed to pass through all the points of the line *AB*. To each point occupied by *O* on *AB* will correspond to a point occupied by *o* in the curve.

Suppose first that *O* moves with uniform velocity, then as the

arm of the curve directed towards *b* becomes more and more near to the horizontal, the velocity of *o* will increase, and it can be thus increased above any assigned limit. But the point *o* is continually approaching the horizontal line *Bx* and yet it does not reach it; so that the point *o* may be approaching the line *Bx* continuously, and may be moving with a velocity as great as we please, and yet may not reach *Bx*.

On the other hand, if *o* moved with uniform velocity we would have the velocity of *O* continuously lessened, but *O* would nevertheless be continuously approaching *B*. In that case we would have a point *O* moving in the shortest path towards *B* and continuously approaching it, yet never arriving at that point.

But reverting to the case of uniform velocity of *B* along *AB*, we find that every point in *AB*, however close to *B*, will have a corresponding point in the curve. But eventually *O* arrives at *B*; where then is *o* to be found? It cannot be at any finite distance from the line *Bx*, for the point which it occupied would then correspond to a point marking the position of *O* on *AB*, distinct from *B*.

It will also appear that when *O* reaches *B*, *o* cannot be at a finite distance from the point *B*. In a region of Space therefore beyond any appreciable or conceivable distance from *B* the point *o* reaches the line *Bx*.

If we suppose Space, or in this case the line *AB*, to be indefinitely divisible, then it will be impossible to appreciate or conceive the point immediately preceding *B*, say $\grave{B}$, for if that were the case we could conceive the line $B\grave{B}$, and according to hypothesis we could divide that.

The Concept of Infinity

This shows that we cannot arrive at a notion either of infinity, or of an infinitely near point, by proceeding continuously from finity, however great we may make the finite.

But also we do not pose an insoluble paradox when we ask the question: Is it possible to conceive of infinity?—since indeed we actually employ the term. The question that is here appropriate is: What do we mean by infinity, or in what way do we arrive at the notion? It is evident from what has preceded that we arrive at the notion by a process of Disassociation followed by a process of Association, which is determined in part by former associations, uniting now new and suitable associations. The process of Impulse is here called into play. More popularly we should say that we arrive at the notion of infinity by a leap.

Let us consider another curious example cited by the Hon.

Bertrand Russell in his work on *The Principles of Mathematics.* It is that of Tristram Shandy, who took ten years to write the story of the first two days of his life. Supposing that he lived for ever, Mr. Russell says that at that rate he would eventually get all the story of his life written. That is the paradox that requires a little circumspection. On the one hand, it is evident that as his life proceeds he must leave the story of more and more years untold, and by suitably selecting the date we can make the number of years, which at that date will be unrecorded, greater than any assigned limit.

Certainly if we fix attention on any finite period of Tristram's life we shall find it possible to complete the story; but therein the character of the paradox comes to light—the substitution of a finite quantity however great for an indefinitely great or, as we say, infinite quantity.

In this case also we cannot arrive at infinity by continuity from finity. We have here again the series: Disassociation, Impulse, Association. If now we revert to the case of Achilles and the tortoise we say that the paradox is there based on the circumstance that we are asked to arrive at infinity by continuity through finity.[1]

The notion of degrees of infinity may be made clear by simple examples. If there be two curves such that the ordinates of one have always the same ratio to the ordinates of the other, then we may suppose the curves extended so that the ordinates become infinite, but the ratio will remain unaltered. Thus we may have $\frac{\infty}{\infty} = k$, where k is some finite quantity.

Similarly we may have $\frac{\infty}{3} = c$.

In mathematics indeed it is not possible to carry on operations by positing infinity absolutely or zero absolutely. We cannot multiply by 0 absolutely, except by removing the factor in question; multiplication has no sense in such a case; neither rightly has, of course, addition or subtraction. Hence no mathematical process has the usual sense in such a case.

But we may multiply, say, by a quantity which may then be understood to vary, and it may vary to infinity, or to zero. In this way we are informed of the manner in which it approaches infinity or zero, and we can deal intelligibly with symbols such as $\frac{\infty}{\infty}$, or $\infty \times o$.

[1] Considerations more difficult, but having a great practical outcome, arise in the study of mathematics, as for instance when integrating round a curve enclosing a point where the function becomes infinite. The problem seemed hopeless until Cauchy threw light on it and showed how in each case a correction of a certain residue was alone necessary. Herein was made apparent that there are degrees of infinity; and that the degree of infinity is determined by the manner at which the function approaches the value at the critical point.

It must be especially brought to mind that the use of the term " infinity " is merely a convention. It is only gradually and with difficulty that we entertain the notion of infinity. Therefore it is a matter of which the factors depend on experience. But of their synthetic result we can have no experience. No one has ever seen a line infinitely long, no one has ever lived for an infinite period.

Moreover, it does not follow that because we are able to conceive a complex, or synthesis, of known elements, or to form a symbol of such, that these have any existence in the objective world. We might talk of a million microbes and we might say that half a million would occupy half the space. And such a conclusion being consistent with regard to the numbers, we might substitute cows for microbes; but it might be impossible for the cows to exist in the space that sufficed for the microbes. Our processes of Association, Disassociation, Impulse, Generalisation, yielding symbolisation, do not imply the objective reality of the results.

The Assumption of Infinite Divisibility

Has this discussion any application to the larger question of infinity? It might well be asked: Have we any right to assume as a reality the indefinite, or infinite, extension of Space? That is a question that has seriously occupied the minds of the greatest and most fruitful of those thinkers whose study of mathematics has led them to deep philosophical speculations, notably Gauss, von Staudt, Riemann, and in England Clifford. Clifford propounded the notion of Space being curved.[1] Einstein has derived suggestions from these views.

The fact that such a position has been considered tenable is alone sufficient to disprove that the notion of indefinite extension of Space must be something both fundamental and inevitable. Riemann, moreover, in one of the profoundest studies in the history of philosophy, *The Foundations of Mathematics*, seems to question also the reality of the infinite divisibility of Space. Hertz in his *Prinzipien der Mechanik*, and later physicists show the importance of the question.

How does the question of infinite divisibility of Space look from the standpoint of objective sciences?

We must beware of preconceptions, or of such arguments as that of the " Sufficient Reason ".

From preconceptions arising from like sources I—and I suppose this is the case with most—was unable at one time to think otherwise than that Space, and hence solids occupying Space, must be

[1] *Cf.* Note, p. 132.

capable of infinite division. One says, if a thing have form we can suppose the form divided. And the old fallacy then intervenes of setting up the finite as symbol of the infinite.

But according to the atomic theory matter at least is not infinitely divisible. We come at length to the molecules which are now understood to be complex systems. If we divide one molecule we do not get homogeneous parts, we get resultants from the break-up of the system. If we divide the molecule of water we get parts of hydrogen and oxygen which in no wise resemble water.[1]

In another sphere, if we multiply or diminish the stimuli of the senses beyond certain limits we do not get increased or diminished sensations. We may get no sensation at all.

Here it may be objected that we cause the subject to intervene. That is so, but so does the subject intervene in the notion of infinity. That is built up from considerations which if analysed lead down to the base of Fundamental Processes, and these have meaning only relative to a subject.

But it may be urged: Let us come to the pith of the question of motion: Is it possible to conceive the passage of a point between A and B that does not imply the passage through all the points between A and B? I reply that it is impossible to conceive a point as defined at all, and that we can only employ the term as a convention by virtue of a synthesis of Processes resulting in abstraction.

Waiving, however, that discussion, I say that it should be shown by the propounder of the question that there are between A and B necessarily an infinite, or indefinitely great, number of points.

Finally I say that even if there were, I can conceive that a body might pass from A to B without passing through these points, for the transference might be that it ceases to exist on leaving A, and that it reappears at B. And that is not merely a metaphysical subtlety. I could demonstrate it by a gross, material experiment; for I could electrolyse a drop of water at A and synthesise the resultant gases again into water at B.[2]

Now the assumption is of metaphysical origin, that any portion of Space, as for instance a straight line, is built up through continuity of infinitesimally small parts. The onus of proving that

[1] The researches of physicists including Sir Ernest Rutherford, indicate that if we divide an atom we do not get a half atom, we get a nucleus and electrons, having properties unlike ordinary "matter."

[2] Here it may be objected that the elements pass from A to B. I could overcome that difficulty by a process which is indeed considered to be the veritable process in electrolysis, that of interchange of molecules with successive drops. It might then be said that the resultant drop at B would not be the same drop as that we first considered at A. To this I reply that if the drop were electrolysed at A and recompounded from its elements at B by another electric current, it would not be the same drop, and neither would it be the same drop finally if it were simply transported (*cf.* p. 130 *et seq.*)

should rest with the sophists, for it certainly has no claim to be considered as raising fundamentally or inevitably in Psychology. I do not suppose that the usual way for a drop of water to move from *A* to *B* is to vanish at *A* and to be reconstructed at *B*, but I do believe that the mental operation corresponding to our appreciation of that movement involves a break of continuity; or, expressing the matter in its fitting terms, there is found a certain process of Association in reference to the object and the place *A*; then there is Disassociation, and with new elements of Time and Space there is Process of Association again at *B*.

This explanation may give the impression of a certain imperfection in the working of the mind, but if we say simply limitation we will find nothing inconsistent with our rational conclusions. We have found already in dealing with the senses that limitation is the condition of their existence, and we have found that our faculty of reason depends on limitations, as for example of Discrimination. Were it not for these limitations we would not build our knowledge laboriously step by step but we would behold at once conclusions which Reason dimly gives us.[1]

Reason reconstructs as a man deprived of sight reconstructs a landscape which with that sense he would take in at a glance.

Does any problem still underlie all this? Yes. The mind is not satisfied with its own limitations. The problem that still underlies the analysis of infinity is not remote from that of "things-in-themselves".[2]

[1] We would "see as a god sees," to quote Keats in one of the marvellous flashes of his genius.

[2] I have known, for example, a psychologist to ask, "How do I know that the red of my sensation is like the red of the table-cover which gives rise to it?"

This question has an aspect of mere misconception. Looking at the matter with the guidance of physical science, we find that the sensation red is one result, and a remote one, of certain activities of matter in the table-cover.

If the undulatory theory of light be correct, there proceed from these activities regular disturbances of the ether which reach our bodies and create therein, and especially in the organs of sight, a series of other disturbances so intricate that we can only guess at their nature; and then comes the sensation red.

But, it may be repeated, the undulations of ether produce on a sensitized place a photograph. And in a phial containing hydrogen and chlorine they produce an "explosion," of which the result is a new substance, hydrochloric acid, formed by some combination of the elements. If we could imagine an animated photographic instrument it would inquire not what was really the colour of a table-cover but what was its actinic strength.

We are thus brought again to the question of Kant's Noumena, the things-in-themselves.

But in what way can we make known to ourselves things-in-themselves, except again by their results as they impinge on our organs of sense?

I do not mean to imply here that at any particular stage of knowledge one must cease investigation simply because one is confronted with a paradox; I show, moreover, in the chapter on Externality, that from the standpoint of Idealism, in its most illuminated aspect, it is possible to gain a clear notion of objectivity, and that the essential character of that objectivity is the containment of unknown phenomena and forces some of which may become revealed to our persistent searching. But any results obtained must be capable of enduring the closest scrutiny and they must be consistent with what we have ascertained to be the necessary conditions of our understanding.

Whenever we meet with a paradox[2] we may be assured that in the end some way out of that paradox will be found consistent with the nature of the Fundamental Processes of the mind.

Accordingly it will be found with regard to the positing of infinity, or of the infinitely small, as ordinarily understood, in spatial relations, that these concepts have meaning only within the scope of the Fundamental Processes.

(2) *Imaginaries*

The problem of imaginaries, so called, in mathematics really does not introduce anything new; that is to say, we can account for the use of this method by the reasonable application of principles already known.

The term Imaginary, and the symbol $\sqrt{-1}$, together with the remarkable results in facilitating the operations of the calculus in the hands of the great mathematicians, all have tended to give to this subject an air of mystery and of some sort of transcendental achievement.

The use of imaginaries has been the subject of many memoirs and I do not think that the theme has been yet exhausted.

It would lead too far from the present intention to discuss in detail the explanations that have been offered; suffice it to say that in mathematics the question resolves itself into a matter of convention in the designation of operations.

Briefly the paradox arises thus. The signs + and − in mathematics signify addition and subtraction. It is also found that if we multiply composed quantities, as for example $a-b$ by $c-d$, then we must give to the product of $-b$ and $-d$ the sign +, in order that the correct result may be obtained by the convenient rule of multiplying each term of the multiplier by each term of the multiplicand, and setting the terms so obtained in order with

Suppose, however, we were dealing with a problem much more circumstantial, such as that of algebraic equations. We might under certain conditions say that the problem was not completely soluble, because all the necessary data had not been made available to us. The character of the insolubility would be indicated. That is the method of the great mathematicians. But if a writer, ignorant of these limitations offers a solution in incomprehensible terms, he does not thereby show himself a superior geometer, but simply as an incapable analyst. And this is the manner of the Transcendentalists when they speak of things-in-themselves.

[2] At one time, when many suggestions arose that seemed to point to the earth being round, it was asserted, as a paradox, that this would involve the conception, if that were possible, of antipodes. But eventually not only did the conception of antipodes seem possible but that conception, leading to the discovery of universal gravitation, prepared the way for the enlargement of the whole scope of human understanding.

Again, when doubt was thrown on the theory that heat was a substance, the paradox arose that then it must be something imponderable, intangible, immaterial; and this seemed inconceivable. I remember that when as a boy I first encountered this notion I thought it most wonderful and entrancing, because it seemed to have come from a realm hitherto looked on as unattainable. Such a notion, however, was at length seen to correspond to well-ascertained principles of Nature, and here again the removal of a paradox was found prolific of important results.

the appropriate sign. Now if we square a quantity, whether its sign be + or −, as $(+a)\times(+a)$, or $(-a)\times(-a)$, we must accordingly have a result $+a^2$ in each case.

And as the symbol $\surd$, for example in $\surd b$, implies ascertaining the quantity of which the square is the quantity affected by the symbol, which is b in the case given, then it is evident that such an operation $\surd -b$ can have no meaning, according to our previous principles.

But in the form to which a reduction is permitted, $\surd -1$, we find great advantage in employing the symbol. Now it is not reasonable to say that we have made mathematics perform an impossible operation and yet obtain true results. The question must be posed thus: What interpretation can be given to the symbol $\surd -1$, so that the operation shall be consistent with the principles of mathematics? When the matter is so stated the paradox disappears.

Thus suppose we have a horizontal vector, or line of definite length and definite direction, considered as directing to the right hand. This we might call the positive direction, and mark by the sign +, and if the length were taken as the unit we might indicate the vector thus, 1; a similar vector, but oppositely directed, might be marked by the sign −1. Now we might interpret a vector of equal length but marked by the sign $\surd -1$, as a vector which had been obtained from the first vector by rotating it, the rotation being such that, if the operation be repeated, we obtain the vector marked by the sign −1. The condition would be fulfilled by a vertical vector.

This, which is the interpretation usually given in mathematical text-books, is not the only reasonable interpretation. The subject soon becomes technical, but those who are interested may consult the writings of Maurey, Argand, Cauchy, Hamilton, Bellavitis; and these will lead the way to an extensive consideration of the whole matter.[1]

What is here important to note is that we do not find any operation in mathematics that cannot be explained on the basis of those already taken into account and hence on the basis of our Fundamental Processes.

[1] I do not think that even in the writings of these celebrated mathematicians the last word has been said on the subject. On the contrary, it will be found necessary for the complete understanding of the problem to re-examine it in the light of the analysis of the operations of multiplication that has been already given. It will then be found that the usual explanations, as just indicated, and the suggestions of the authorities cited, refer only to special cases of the application of general principles, and that these special cases are not the most obvious, nor the most consistent, nor in all circumstances the most fruitful of results.

CHAPTER VIII

THE SCOPE OF THE FUNDAMENTAL PROCESSES

NOTE:—We have now reached a point where we can demonstrate that our system covers the field of thought.

(1) *Mode of Classifying Objective Sciences*

Let us search for some deep trenchant division in order to find a manner of survey of the range of objective science. It will have appeared to students who have pondered the question, and who have pursued their analysis persistently, that the tendency of sciences such as chemistry, electricity, heat, light, is more and more to seek explanations which finally resolve to matters of mechanics.

Newton in his *Principia* sought to base his exposition, as formerly Euclid in his geometry, on the fewest possible assumptions. The Science of mechanics consists in the study of material objects with regard to forces and motions. The application of this science is extended to the whole objective world considered in this respect. In the application appeal must be made to experience, but the experience is reduced to its most direct and simplest expression. The science is the *schema*, which with our experience supplying the concrete representation, applies to all the external world viewed with regard to forces and motions.

Now a deep division separates the animate from the inanimate world. How do things comport themselves in the animate world in especial?

The tendency of all modern sciences such as physiology or biology, is, again, to seek ultimate physical explanations as far as possible.

The science of physiology, for example, is compounded of chemistry, mechanics, and the complete study of anatomy with all that it should include; and, in addition, the action of low forms of life, either forming part of our bodies and controlled by an elaborate nervous system, or simply within our bodies.

Apart from the world of sensation, ideation, thought, which we have expressly placed beyond these spheres, most modern physiologists seek to find ultimately in vital action nothing more than what we might include in the most general sense amongst

mechanical causes.[1] Thus apart from phenomena purely mental we may adopt a conclusion expressed by Kirchoff, who declared: There is ultimately only one science: mechanics.

In considering this expression a certain allowance must be made for the aphorismal form. In this respect it is comparable to the philosophic epigram of Buffon: There is only one animal.

Kirchoff did not mean to imply that all science could be expounded from a basis of mechanics, for after all we have not yet reached that point with regard to certain sciences in which the truth of the aphorism is most apparent, chemistry, or even electricity.

Kirchoff simply asserted the mechanical action of the elements whose combinations give us the varied phenomena which constitute the field of the sciences. The limitations of the application of the principles of mechanics are simply the limitations of our knowledge and of our powers of mathematical expression.[2]

In regarding the whole field of biology, with its various subsidiary sciences, we behold many problems which are those of the adaptation of the organism to its environment, that is to say, the adaptation to physical conditions by virtue of the interplay of instincts, feelings, emotions, thoughts: accordingly we see all the factors included under the classification of mechanics and psychical phenomena.

But when we pursue our inquiries into the mysterious questions of embryology, of cell division and multiplication, into origins, who can say and offer proof that we have nothing here but mechanics?

[1] There is a school of physiological psychologists who speak of "vital action." Driesch is an eminent example. Bergson with his "évolution créatrice" and "élan vital" may also be included. Their defect is that they introduce vague though pleasing terms and give no demonstration of corresponding objective reality.

[2] I have met with suggested mathematical expositions of animate nature in quite unexpected quarters. Sir Ronald Ross in *The Prevention of Malaria*, 1911, writes "As a matter of fact, all epidemiology, concerned as it is with the variations of disease from time to time or from place to place, *must* be considered mathematically, however many variables are implicated, if it is to be considered scientifically at all."

Here however, we are far indeed from the direct application of such an aphorism as that of Kirchoff, which indicates something practically impossible at our present degree of mental development and in the present condition of our command of the Forces of Nature. Almost the only way in which it is at present feasible to apply mathematics to the study of diseases is by means of the science of statistics. Karl Pearson has distinguished himself in such researches. Other notable names are Farr, Brownlee, Goring, Greenwood.

A suggestion for the application of mathematics in a region still more unpromising is that of Professor Haret, of Bucharest. The title of his book, *Mécanique Sociale*, already implies something formidable. He deals with questions of sociology by means of applied methods of statics and dynamics, and we meet with terms such as "social space", "social force," "social rest," "social vector." A writer in *Nature* (G.U.Y.) has, however, pointed out that a system of Cartesian co-ordinates cannot be so adapted: "The three axes chosen do not represent qualities of the same kind or dimensions."

The theoretical *dictum* of Kirchoff must be conceived of as becoming realisable at a basis more profound.

The solution of such a question would not affect our present view. In studying even such a science as bacteriology we do not require to pierce to the ultimate nature of organisms. We deal with bacteria as substantial living things, and the difference of our study of bacteria and that of worms, for example, is due simply to the fact of the bacteria being so small that special means are necessary for observing their operations. Often we know them only by the results they produce, their appearance in " colonies ", their facility of cultivation on various media, their vitality at different temperatures, their gas-forming properties, or their capacity of living without oxygen; all of which properties may be determined without a single organism having been defined under the microscope.

Thus throughout the whole range of the organic sciences our knowledge is built up only on what we can know by experience, and of what we can apply from other sciences. It will be seen then that the study of these sciences can introduce no new Fundamental Process.

All the other objective sciences, that is to say, those dealing with inanimate nature, we may include under the classification: Mechanical. Here we have observation of material things, rearrangements for experiments, all that we may in short sum up as experience; and the application of mathematical science.[1]

[1] The laws of Newton are not fundamental. The concepts of force and of motion are both formed of complexes. Let us consider force Kepler spoke of a directive force in molecules as of a directing angel But Kepler did not further define his angel than to attribute to its directive powers certain effects which were real. We speak of that force of gravitation and we feel superior and satisfied. But all that we know of that force is by reference to its effects. Between Kepler's angel and Newton's force, there is nothing to choose but taste in nomenclature. Laplace is satisfied with the term force, and he gives a demonstration of the results. Keener minds than that of Laplace have not been satisfied; Lagrange is uneasy in the use of this term; Hertz has given us a work on mechanics which seeks to avoid the assumptions employed in the term. The whole question is interesting, but here I give this indication merely.

We now come to a very suggestive consideration: Suppose we are standing in a position symmetrically placed with regard to bodies moving in regular curves, we would be able to describe their movements simply and readily. Suppose that our position were shifted so that we could not perceive any regularity of movement, then we would find it hard to give a description of the appearances. Yet the actual phenomena would remain unaltered. This remark will prepare the way for the understanding of a keen observation of Henri Poincare: It is as true to say that the sun revolves round the earth as that the earth revolves round the sun. Thus relative motions remain unaltered by our point of view. We adopt the familiar form because then our calculus becomes simplified, and our statement of movements more easily intelligible. What we call " law " then has reference to our own understanding. We seek for orderly arrangements and simple statements embracing a great multitude of facts. Einstein has elaborated these notions. M. Painlevé has dealt with the question still more comprehensively in considering rotation.

Now as in the physical world the propagation of impulses may be represented by straight lines, and many phenomena are related to simple figures whose properties have been studied—circles, ellipses, etc.—all these facts aid us in the application of the results we have derived from principles of mathematics. The physical realities are adjoined to the schemata of representation of our mathematical forms. The laws are accordant with the modes of representation of which our minds are capable. These remarks will show the importance of the mathematical schemata we have referred to, for mathematics cover the extent of the physical world, as far as dealt with in our minds.

(2) *Conclusions from the Study of Mathematics*

We have now reached a point from which the ground covered may be surveyed and certain results indicated.

In regard to the inquiry into numerical mathematical operations, reserving the question of infinity, we showed in what way the system of mathematics depended on multiplication, addition, and more basically on counting in conjunction with the Fundamental Processes; we have now examined infinity and shown how its use in mathematics is explicable only on the basis of these Fundamental Processes; again all the mathematics of spatial relations are the combinations of mathematical numbers and the representations of space, which in ultimate analysis are also expressed in terms of the Fundamental Processes.

Hence the Fundamental Processes are necessary and sufficient to explain all the operations of mathematics whatsoever. But all things that may be considered in any way at all are capable of numeration,[1] and apart from their quantitative relations, they have only qualitative relations such as can be made known to us by Immediate Presentations. And all our knowledge of the combinations of such qualitative relations have been shown to depend on the Fundamental Processes.

Accordingly these Fundamental Processes are necessary and sufficient for the interpretation, so far as it is possible to our minds, of the External World.

But in a previous part of the exposition we made the division between what we may indicate as the External or Objective and the Internal or Subjective, provinces (*cf.* p. 30 and pp. 66 *et seq.*). We have also seen in what way the Subjective movements or affections of the mind can be analysed to show dependence on the Fundamental Processes.

Therefore, finally, the problem which we posed as the basis of this work is solved in these terms: That is to say, that the series of our Fundamental Processes is necessary and sufficient for dealing with the whole scope of what comes within human consciousness; or, again, that by the combinations of these Fundamental Processes in conjunction with actual experiences we build up the whole containment of our knowledge.

[1] This is proved in the chapter: Concept of the Unit.

CHAPTER IX

MUTUAL IMPLICATIONS OF THE FUNDAMENTAL PROCESSES

NOTE:—I insert here a little Chapter which did not appear in PSYCHOLOGY: A NEW SYSTEM, *because so many new conceptions, new methods, new conclusions, had been presented that I thought that this explanation, though it finally makes for complete comprehension, might be too abstruse. Now that the Fundamental Processes have become familiar this discussion will be helpful.*

In the preceding discussion we have seen the necessity and the sufficiency of the Fundamental Processes. Further it requires no great attention to be convinced that they are distinct; Immediate Presentation, for example, is not the same thing as Memory, nor Association synonymous with Generalisation.

Yet a suggestion that has arisen may leave unsettled the mind of the reader, that, though distinct, in these Fundamental Processes one seems to implicate the others. This points to something underlying their appearances, as if indeed they were but different facets of some incompletely revealed form.

* * * *

Immediate Presentation does not of itself imply the theory of the Unit, that is to say, does not immediately indicate that the mind can entertain only one object at a given moment. We have, however, shown that the mind can attend to only one object of thought at any moment, and this fact we designate conveniently by referring to the Fundamental Process included under the head of Unit.

Now, however, we see since this is an invariable circumstance of our mental life, the Immediate Presentation implies the Unit.

Hence we may reduce the number of our Fundamental Processes by placing the term Unit under that of Immediate Presentation.

Association is evidently not identical with Immediate Presentation or Unit, but also evidently the continuity of our psychic life would not be possible without Association. Association is therefore a condition of the functioning of our mental apparatus. It must arise inevitably and spontaneously.[1] The process of Agreement, or its converse, Discrimination, implies the comparison of an Immediate Presentation with a previous experience of an Immediate Presentation, and this involves Memory.

[1] It should always be borne in mind that all mental processes are inevitable and spontaneous when the conditions are prepared. This will indicate also that at the base of operations there is no distinction between reason and instinct.

Or again, if we first consider Memory, then finding on comparison of an Immediate Presentation with a past experience of Immediate Presentation, a similar functioning, or a certain difference, we are inevitably led to consider Agreement or Discrimination.

Now by the nature of things all Immediate Presentations are, when the matter is viewed objectively, seen to be different. Agreement then depends on the limitations of our faculties and Discrimination marks the limit. This is an important truth, and it must always be borne in mind when we consider the foundations of belief or the ultimate sanctions of Reason.

Agreement and Discrimination are conditioned by our own faculties and by the actual world we live in. They rise inevitably. We know them by experience.

The actual mode of Agreement and of Discrimination determines the character of the Generalisation that arises in any case. The extent of the Agreement is the extent of the genus in the classification; and the character of the Discrimination indicates the species. We find that whereas Generalisation is often described as marking the exercise of a highly developed mind, Generalisation is also rudimentary, conditioned, and inevitable.

Being inevitable and arising under Agreement we could arrange a list without it, while supplying an explanation. Yet the term is useful, and Generalisation is something definite itself. Generalisation implies classification immediately and classification is merely another form of expressing Generalisation; or if we consider a more formal classification we obtain this from Generalisation with the addition of symbolisation. The application of a symbol depends on Association; and if it be a symbol applied generally to a number of objects it depends on Association and Agreement. It is evident therefore that Generalisation, classification, and symbolisation, are different facets of the same process.

Impulse is the process which carries the mind from one Immediate Presentation to another. Looking at the matter narrowly, and applying a keen introspection, we will become convinced that Impulse and Association are not identical. Association depends on the characters of the Immediate Presentations or ideas, or other mental objects and then on the relation produced by the juxtaposition in thought by these mental objects.

Impulse is rather a process that deals with the movement along the underlying schema. We become convinced of its existence as a Fundamental Process when we consider all these matters objectively.[1]

[1] It need hardly be said that I reject no means by which I can obtain a clear vision of my subject. I use introspection to its utmost limits, but certainly I reject no hint or aid from any source. Hence also to avoid too technical a form of exposition I appeal to objective things.

The Sense of Effort offers, as has been already seen, one of the most difficult problems in the whole range of Psychology. For the moment, however, we need not consider the origin or the organs of that sense. It is certainly distinct from ordinary sensation, and that is a sufficient reason for placing it in another category. Even if the theory, unverified as it is, of William James should by any means, as for instance by physiological considerations, be found true, viz., that the Sense of Effort comes from the stimulus of afferent nerves which are ultimately influenced even in a discharge of motor impulses, then the position in regard to the distinction between sensation and Sense of Effort would remain untouched.[1]

But that being so, it is not only allowable, but also in accordance with rigorous method, to classify sensation and Sense of Effort under the general term, Immediate Presentation.

Negation is a form of Association, yet it is distinct from what is usually considered under Association, and it applies to all the processes. The recognition of this process as distinctively characteristic enables us to explain many anomalies. Yet it could be dispensed with in formal classification, if for instance, instead of Association we were to employ a special term—we might adopt or adapt the term Relation, and classify Association and Negation as co-ordinate forms under this head.

The Hedonic factor, usually omitted from consideration, or diminished in importance, is of particular interest. We will see how deeply strike the applications suggested by a clear conception of its rôle.

The Hedonic factor is evident in regard to some Immediate Presentations, as for instance, sensations, certain of these being more agreeable in given circumstances than others.

Departing from our own introspection and seeking some light from biology, we find in low forms of life that certain directive impulses depend on the stimuli produced by substances in the medium in which these organisms live.

This influence is explained by chemio-taxis, though the name means nothing more than a manner of recording the fact.

Whatever be the final explanation of chemio-taxis the directive impulse must evidently depend on something which it is possible to describe as an Hedonic factor.

The origin of the Hedonic factor is probably involved in that of life itself, for since the guidance of the individual organism

[1] Thus, in Memory it is probable on a great variety of grounds including physiological, and histological, considerations, that certain physical components such as nerve strands and ganglia in the brain are influenced in a manner comparable to that which produced the original impression, yet we are justified in speaking of Memory as a distinct Fundamental Process.

depends on this process, the Hedonic factor must be a life-preserving faculty or instinct which operates throughout the independent existence of the organism.

Now the human body is composed, in its ultimate element, mainly of cells each of which is really a microscopic organism. Everything that actually lives, as a component of the body, is such a cell. The principle of Hedonism therefore enters at as deep a level as that of these organisms, and it governs their activities, or reactions, therefore finally the response to all stimuli of the body.

If now we leave these considerations in turn and examine the question from an idealist standpoint, we find that the Hedonic factor is not a constant in all circumstances; it varies according to the external influences that impinge on our being, and also on the modifications produced in the body, or eventually on the mind, by the character of our experiences, as for instance by repetition of stimuli. To fix the ideas, we may say that if a sensation originally pleasurable be repeated frequently and monotonously then there may be a time when from pleasurable it becomes displeasurable. A sensation may at one time, and in a certain condition of bodily health, be agreeable, and at another time a similar sensation may be disagreeable. Or again, and this seems true of a great majority of our sensations, the sensation in itself may be apparently devoid of either agreeable or disagreeable qualities.

I say apparently, because very often, if we fix our attention, we will find that sensations which have seemed indifferent have really possessed a distinct hedonic quality.

Yet again, there are sensations in which this quality is not apparent. It seems incongruous to speak of an hedonic quality when the presence of this quality escapes observation. The question here requires a little delicate handling.

Every sensation that influences the mind makes itself appreciated not as a single mental experience but as one added to the countless series of sensations that have helped to determine the character of the mind, and at the same time, the character of the body itself. Its effect is comparable to that of a force which is adjoined to a given force, that given force being the resultant of myriads of preceding forces.

From the simple sensation of a mind receiving at the beginning of its experiences a sensation possessing a distinct hedonic quality we arrive at something of immense intricacy.

Here we touch on the question of the Sub-conscious, a point on which there are a great number of false and confused notions current.

The matter will be better discussed if we fix our attention on the physical correlatives of thought, and particularly on the network of nerves and ganglia which constitute the central nervous system. In this reference to the physical parts, as for instance, to the brain, we do not make any other implication than that of using this in a diagrammatic manner, although again it is true that though no physical process is anything but a precedent, or correlative of thought, yet, when the central nervous system is submitted to certain impulses that traverse its network in a certain manner, then if a mental experience follow, that mental experience will be inevitable and determined. In other words if the physical conditions could be exactly repeated, the mental experience would be exactly repeated.

It happens, however, that certain physical stimuli, even of the brain, do not produce any conscious state, and this is true even when the physical stimuli are intricate and of appreciable power. These stimuli, however, may modify this physical apparatus that underlies thought, and if, again, there be applied a new stimulus that does give rise to conscious impressions, these impressions will be different from what they would have been had the physical substrata not been modified in the manner indicated.

This simple explanation, if properly grasped, will suffice to guide us in dealing with the Sub-conscious.

Those who regard the matter with real acumen on the basis of the problem presented, but rather superficially in regard to the whole question, ridicule Sub-conscious thought.[1] Their objection is justified, but their objection should be directed to the misuse of terms.[2]

There is no Sub-conscious thought, but there is a physical apparatus in which may pass a series of modifications due to a variety of stimuli, and the modifications so produced will have influence on the subsequent experience of conscious self. Moreover, the stimuli here referred to may have their origin in precedent mental experiences of various kinds. The "Sub-conscious working" may therefore be a reality, and may indeed move along the lines of graphic representation of consecutive psychic processes, and may in the end rise to such a degree of energy that psychic experiences result.[3]

[1] As for instance the late Dr. Mercier in the *British Medical Journal.*

[2] The case is parallel to that of "centrifugal force." There is no centrifugal force though there are cases when the resultant of forces act in a direction away from the centre.

[3] Thus it may happen in remarkable cases that a thinker, as for instance a mathematician, may have the impression that a good part of his thinking has been done for him "sub-consciously." The famous electrician Nikola Tesla told me that he sometimes had the impression, as if he had started a kind of machine work in his brain, certain that in the end it would furnish the result sought.

Returning now to the question of the Hedonic Sense, it is evident that there may be stimuli which ordinarily would produce sensations affected with hedonic quality, but which do not produce that hedonic quality because they do not produce sensation.

It may happen that the hedonic quality of sensations of high intensity may be unperceived if there be an intense interest in some object which determines the course of the thought.[1] In that case the hedonic factor accompanies his thoughts nevertheless, and indeed helps to determine their direction.[2] Thus the hedonic factor may be present in pursuing the logical steps of an argument in geometry.

Therefore the argument that certain sensations which arise in the course of psychic experience may for some reason or other be not evidently accompanied by any hedonic quality does not itself destroy the position asserted, that the Hedonic Factor is one of the Fundamental Processes and that it is inevitably present in the act of psychic life.[3]

The two remaining Fundamental Processes are the Concepts of Time and Space.

All our experiences are conditioned in time. All the other Fundamental Processes would cease to have meaning if the Immediate Presentations did not occur in sequence. Indeed, if we consider the matter attentively, we will find that it is impossible to conceive of events happening except in a progressive series. Even when we reverse the order of events we are dealing with an imaginary series in progressive sequence.[4]

[1] This may be made clear by graphic example. A small boy who is generally all alive to the hedonic qualities involved in the sensations of eating a jam tart, may even with the tart in his mouth, forget its existence in the excitement of witnessing a close finish of a boat race.

[2] It must be ever borne in mind that though for illustration the term "sensation" is often used, yet our exposition does not depend on sensation; the Immediate Presentation is not necessarily a sensation in the ordinary sense, still less a sensation formed by any one single object ordinarily producing a sensation. The whole sky as it appears at one instant may form an Immediate Presentation.

[3] The question of the Hedonic factor has a great philosophical importance. Here indeed we come to another of the applications, of extraordinary interest, of the doctrines here exhibited. Kant in his *Kritik der Praktischen Vernunft* puts forward the famous doctrine of the "Categorical Imperative." When the Categorical Imperative is issued—Kant does not make it clear by whom, or by what—then the individual must obey, whatever be the protests of the whole feeling of his being.

Goethe in a witty epigram laughs at this doctrine as unnatural; so it is. Neither Kant nor Goethe has submitted the matter to a profound analysis. Goethe started from his fund of practical common sense. Kant, although he involved his reasoning in a cloud of ambiguities and assumptions, really starts from the prosaic conception of legal duties in an autocratic Prussian State. Even at the base at which our reasoning begins Nature has already decided that we must respond to the Hedonic Factor, for that in fact determines at each point of our psychic progress the direction the mind must take.

[4] In a Presidential Address to the Aristotelian Society, Dean Inge, speaking on the question of the time series being reversible, draws an argument from the representation at a kinema of a diving act, in which the film begins at the end of the real experience and finishes when that began. Here, however, we have no reversal of a time series;

Certainly I am not averse to the scrutiny of Time in order to ascertain whether anything more complex, more divisible into definite elements, can be discovered, but hitherto, even in learned and abstruse discussions, I have found nothing illuminating. Time seems, as far as our consciousness is concerned, elemental. A criticism may be offered that it is not so much a process as a condition; but I do not think that remark in itself is so decisive as it may at first seem.

It cannot be too often insisted, until the full meaning of the assertion is understood, that all the processes are conditions also, that is to say, that they are inevitable and spontaneous when the proper antecedents are present.

In the physical world, when a violin has a certain form and is composed of certain materials, with strings of certain length, section, weight, and quality, and when one of the strings is struck at a certain point the vibrations which will be given forth are not a matter of hazard or choice; they are conditioned by the structure of the Universe. Just so when the appropriate conditions are prepared, some sensation, for example, arises immediately; just so, Memory, and so on.

Certainly Time may be spoken of with regard to objective happenings, and therefore, so as to avoid perplexities, we may be content to consider Time simply under the heading of a condition.

Space is as elemental as Time. It is true that in order to form a conception of a greatly extended space we may add one space to another and the process involves Association and Time, but a certain conception of Space is fundamental; if it were not so we could not by successive acts of addition arrive at a greatly extended space.

It might be argued that as space has three dimensions we do not immediately arrive at a conception of space. That argument is not justified. We do not immediately arrive at a conception of all the relations of different elements of space; but it is also true that we do not immediately arrive at a conception of the complex relations that exist in a series of sensations.

Further, the statement that Space has three dimensions is entirely conventional. The origin of this expression must be sought not in a contemplation of Space as a concept of itself, but in the means adopted to define positions in space. If, for example, in an ordinary room we fix a small object and measure its distance

we have a progressive series artificially arranged so as to reverse the order of a previous progressive series. One of the simplest forms of such a case is where a small boy in counting the buttons of his coat might begin at the top on one occasion and proceed to the bottom, and subsequently begin at the bottom and proceed to the top. So stated there is neither paradox nor mystery in the matter. Neither is there if the series be made longer and the objects more complex.

from the floor, and from each of two walls, all meeting in a point, there we have data by which the position occupied by that object could at any time be re-discovered, even if the object were not there still. The height from the floor having been given, we could describe, or imagine, a plane parallel to the floor at that height. Any point in that plane would correspond to that indication, represented by the figure giving the height. That figure is called a co-ordinate.

Similarly, another plane parallel to one of the upright walls, would correspond to another co-ordinate; and this plane would cut the plane parallel to the floor, in a straight line. A plane parallel to the remaining wall would correspond to the third co-ordinate, and it would cut this line in a point. That point coincides with the position sought.

This is the principle of the Cartesian co-ordinates, which are so familiarly employed in geometry. There are three co-ordinates. These co-ordinates are employed in the definition of ellipsoids, for example, which occupy position in space, whereas ellipses on the other hand are plane figures and require only two co-ordinates for their definition. Space is therefore said to have three dimensions.

It may be objected that men had not waited for the invention of Cartesian co-ordinates to observe that Space had three dimensions. The answer is that in our own figures we are supplied with suggestions of modes of indicating position whose data differ from Cartesian co-ordinates only in not being precise. When we stand upright with arms extended horizontally, the direction of our bodies, that of our vision, and that of our arms, represent approximately three lines at right angles to each other. From that to the invention of Cartesian co-ordinates, though that invention came by a stroke of genius, the step is formed by the suggestion of rendering our appreciations exact; in other words, by measurement.

But a star-fish, supposing it could express itself well, would not speak of space having three dimensions. Its main lines of reference would not coincide with ours.

Or again, even mathematicians do not always speak of three dimensional space. The famous German geometer, Plücker, observed that there are certain arbitrary assumptions in the method of Descartes, and he devised a system by which spatial relations could be defined by four, or by five, co-ordinates. His system has been the basis of a great development of modern mathematics in the hands of Sophus Lie and Felix Klein.

Hence it is not otiose to deny that space has three dimensions; it is only when we come to estimate positions, shapes, or distances

by measurements that we form the conception of dimensions. The quest therefore for "the fourth dimension" has no meaning in reality except that of a search for a fourth co-ordinate. This is not only possible but actual in the Plückerian system, but if we are asked for the fourth dimensions in the Cartesian system we can only reply that it does not exist. As well might we ask; There being two ends of a finite straight line, where is the third end?

Certain philosophers of great authority have even suggested that Time is the fourth dimension. In view of what has been said this theory is absurd. The suggestion may have arisen from the fact that in certain equations in dynamics the position of a body is defined in terms of the Cartesian co-ordinates and in those of time—that is to say, the position varies with time; and, when a certain moment of time is given, then the position is defined by this and the Cartesian co-ordinates. In such an expression it would be correct to say that the term representing Time is a fourth parameter, that is to say, one of the four variables which serve to define the position of the body when particular values are assigned to each of the four; but that does not imply that time is "the fourth dimension" in the ordinary sense in which the expression is used.[1]

We have now reviewed the Fundamental Processes in the light of the previous exposition, and as a result of our examination, we may now arrange them thus:

Time, Space — *Universal Conditions*

Immediate Presentation + Impulse + Hedonism
— — *Conditions of Function*
including: Sensation
Sense of Effort

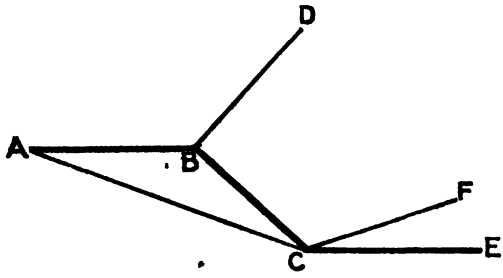
—
Implies Unit
Association + Memory
(Negation)

Producing Agreement (Discrimination) *Giving rise to* Generalisation, *Leading to* classification *and* symbolisation.

[1] Lagrange is the first I can find who spoke of time as a fourth dimension, but with him the matter is perfectly clear. He says (*Théorie des fonctions analytiques*, third Part, first Chapter): "Here the functions depend on time, which we will always indicate

Suppose that now we were to attempt to find some graphic memento, or eventually some diagram, which would seem to indicate these processes, we should assume that all is contained within the conditions of Time and Space; then making abstraction of the actual quality of the Immediate Presentation, whether of sensation or otherwise, we might in one diagram represent this Immediate Presentation by a material point, or dot *A*. This would imply Unit. The process of Impulse would be represented by a line drawn from this dot; and this line would be determined by another dot, *B*. The line of Impulse will represent the relation, or when the appropriate Hedonic factor is added, the Association between the Immediate Presentations.

Suppose from the second dot (*B*) two lines proceeded, one to a dot *C*, and one to a dot *D*. The Impulse carries the processes of thought to one or other. The choice is determined in the elementary case by the Hedonic factor, and, let us say, the movement is toward *C*. Note that it may have carried in a tentative way toward *D*, and then there may have been a return to *B*—this by virtue of the process of Negation—and then to *C*. When *C* is reached, and if *C* is like *A*, then may be revived by Memory the image of *A*, and if there be really Agreement the movement from *C* will resemble that from *A*. If there be Discrimination, the movement will not be quite the same, and the line representing Association will not be parallel to the first. We will now have a diagram a little more complicated, containing a line from *C* to *E*, and a fainter line representing Memory between *C* and *A*.



The process of Discrimination at *C* involves Generalisation, which we could indicate by a material process, or spoke, proceeding from the dot *C*.

by the letter '*t*'; and as the position of a point in space depends on three rectangular co-ordinates, *x*, *y*, *z*, these co-ordinates, in the problems of Mechanics, will be considered functions of '*t*.' Thus we may regard Mechanics as a geometry of four dimensions" . . . If instead of "dimensions" we say "parameters," or simply co-ordinates, there is no perplexity; but many philosophers, and amongst them particularly students of Einstein's theory of Relativity, have tortured their brains in order to imagine how time may be a "dimension" of a spatial character.

Space as a concept has no "dimensions," such as are necessarily expressed numerically. Dimensions refer to measurement, or definition of position, and the number of dimensions then depends on the system of co-ordinates adopted.

If now we continue this method of development, with all manner of return lines, indicating Memory, and increase the number of spokes proceeding from the dots, we would have at length an elaborate network, although built on a simple principle of development.

If now we refer to Chapter II, p. 29 we will see that what we here represent diagrammatically corresponds to the simple diagrammatic representation of the development of the brain. So that we have here a confirmation of the principle of the physical correlative of thought.

Once this is understood then every new part of the physiology of the brain adds to our grasp of the subject. We find a graphic representation, for example, in the manner in which the nervous current causes nervous "explosions" in the ganglia, the new impulse seeking outlet, some nerve paths being inhibited while others are left open; in the case of a strong explosion breaking down the barriers of inhibition; in the circumstance also of the nutrition of the nervous system by the blood. Hence again we observe the dependence of delicate functions of the brain on the condition of the entire bodily organism.[1]

[1] *The Whole Man Thinks; Cf. Human Documents.*

PART III

CHAPTER I

ASSOCIATION

NOTE:—We have now reached a point where, after obtaining in our schematic forms the webbing of thought, we may see in what manner are laid the forms and colours of these patterns which are our experiences of life. In other words we will find here and in the next four chapters some developments of the principles with which we have been concerned hitherto principally in regard to their elementary aspects.

Of the Fundamental Processes, Immediate Presentations needs no further elaboration; the Conception of the Unit has been fully dealt with in a special chapter, and in the subsequent investigation of mathematical forms. Agreement, or its inverse Discrimination, has been sufficiently exemplified; The Hedonic Sense has been made manifest as a rudimentary principle, and it finds an application in the reference to Kant's KRITIK. *The process of Negation needs no further study. The developed form of Generalisation will appear in the chapter on Reason and research. This leaves Association, with which the present chapter deals; Memory which is discussed in the light of testing experiments in the next Chapter (II); the Sense of Effort which is closely examined in a special Chapter (III); Time and Space which enter into the complete explanation of Externality (Chapter IV); and Impulse which gives the key to the study of Will, (Chapter V).*

Association has been abundantly discussed in the course of the development of Psychology, from the time of Hartley onward. There is no need now to travel familiar ground especially as Association has, in the preceding exposition, been not only incidentally dealt with, but referred to its proper relations among the Fundamental Processes of the mind.

There are, however, other positions involving Association which should be discussed, and these positions may be referred to as enlarging the scope of Association.

We have observed that Association is found not only between Presentations in immediate sequence, and between ideas that have arisen in immediate sequence, but also between ideas related in any way to each other and to the objects of the external world.

But there is found also Association between ideas and emotions and between different kinds of emotions; and between any of these and the greatest complexes that can be formed by combinations of all mental processes and feelings.

It would be well here to refer to the graphic representation provided by the structure of the nervous system itself (*cf.* Pt. I, Chap. II, p. 29 *et seq.*; Pt. II, Chap. IX, p. 160). We find that, correlated to the Association of idea to idea, we have stimulations producing disturbances along a nerve track from one neurone to another in the brain.

In active mental life there is a simultaneous stress along many related paths. In cases of mental excitement there are explosions

in brain cells and perturbations along paths that have no special relation to the immediate stimulation of external nature. In cases of profound emotion there is a wide area of disturbances throughout the nervous system.

We will have occasion to note the influence of the emotional factor on Reason. It is not only a factor in producing belief, but it influences the manner of expression of that belief. Shakespeare, who was a keen observer of human passions and also of their proper representation, has a fine passage where Cleopatra receives tidings of Antony (*cf.* Act II, sc. v) :

Messenger : Madam, he's married to Octavia.
Cleopatra : The most infectious pestilence upon thee !
(*Strikes him down*, etc.)

Here we find the Association of an emotion of anger with the source of the news that produced the anger. Observations of the kind have become proverbial, as in the advice not to be the bearer of bad tidings, and the notion, more valid than a mere superstition, that ill-luck attends the bearer.

Association of Mental with Physical States

Another passage of Shakespeare is more subtle, but not less dramatic. Othello, shocked by the tragedy of his own acts, works himself up to the point of suicide :

And say, besides, that in Aleppo once,
Where a malignant and a turban'd Turk
Beat a Venetian, and traduced the State,
I took by the throat the circumcised dog
And smote him—thus. (*Stabs himself*).
(Act V, sc. ii).

Here we have a reminiscence exciting a feeling of rage and desire for destruction, and this becomes powerful enough in Othello's despairing condition even to be turned against himself.

The Association here is contained not merely between a dominating emotional state and a certain notion, but also between that emotional state and the image of the act to which it led.

We generally find in cases of deep emotional disturbance abundance of gesticulation. This would be explained on the physical basis by saying that the stimulation along sensory nerve tracts in the brain produced disturbances in the areas of the motor nerve tracts corresponding.

Among people such as the French, where gesticulation even without great excitement is usual, habit has facilitated the passage of the disturbances from the sensory nerve tracts to the motor

nerve tracts to such a degree that it becomes difficult to express an idea without the accompaniment of motor displays. Frenchmen sometimes gesticulate when holding a conversation through a telephone. These actions are no doubt the natural accompaniment of speech but in Northern climates they are more repressed by habit.

In persons of unsound mind nerve instability, shown in incoherent ideas, finds a further expression in want of control of physical acts. A raving lunatic desires to do something, perhaps to kill.

Association of Thought with Temperament

The influence of Association is found in determining in a wider scope and over a greater length of time, the expression of a man's thoughts. Compare for example the works of Rabelais and Pascal. Both aimed at reformation in spiritual matters but their temperament and education were different.

With regard to Rabelais it is said that he was a profound moralist who cloaked his ideas in an extravagant manner in order that the essentials of his teaching would find easier acceptance. His gainsayers assert that he desired only to amuse or to outrage decency by the frivolity and ribaldry of his language and the licence of his descriptions. Both these positions seem overstrained. The works of Rabelais have an aspect of being a satire on the abuses of his time, but his mind was coarse and filled with licentious images and these he makes use of in the portrayal of his views.

The mind of Pascal was delicate and refined. His educational development was profound in certain directions. Ill-health with an ascetic *régime* provoked the hysterical tendency always underlying in his nature. Consequently, with the expression of a fine, luminous intellect we have in his writing the most poignant cries of distress in literature and the warping toward madness.

Now, if the style of Rabelais be purposely chosen to form a proper vehicle for preaching reforms, is it possible for us to imagine Pascal adopting such a manner of expression? We see in both the Association of temperament with intellectual acquirement.

In the case of Shelley we have a constitution not unlike that of Pascal except that the tendency both of his temperament and his education was to find expression rather in poetical flights than in ratiocination such as delighted Pascal. Shelley's mind, no less than Pascal's, was steeped in Christian influences, although his professed principles seemed absolutely opposed, at least in dogma, to the Christian religion. We find these tendencies expressed in

peculiar Association. Shelley's *Queen Mab* is ostensibly a revolt against religion, expressed even in terms which Christians consider blasphemous; but the essence of his whole discourse is that of appealing to the Christian spirit while condemning those whose example is in disaccord with Christian teaching.

Associations of Diverse Fields of Thought

The influence of Association is found in cases where a man with a strong bent toward one set of notions is also immersed in daily practice in another. Such was the origin of a book that had a large circulation some years ago: *Natural Law in the Spiritual World.* Drummond, the author, was a devout Christian and at the same time a lecturer on scientific subjects. Accordingly he desired to infuse the spirit of science, and even the examples drawn from his scientific work, into his religious views.

On the other side we find such cases as that of Huxley. For Huxley, being profoundly interested in ethical questions and being at the same time a brilliant student of biology, makes use of biology as the medium of expression of his opinions in ethics. It must not be supposed that Huxley found in biology alone the cogent proofs of all the conclusions he reached. His opinions depended on numerous other factors including other forms of his education, his early training, his temperament, and the more technical considerations of the field of ethics itself.

If, for example, a certain standard of conduct is found to prevail throughout the organic world, then its application to man may possibly be rejected on the simple observation that the new conditions introduced involve modifications of this standard. This is indeed the conclusion of Huxley himself in his later writings. Other biologists have refused to alter the continuity of their system. The decision depends partly on temperament and on the other factors noted.

Darwin's work on the *Origin of Species* caused great commotion, but this was in part due to conditions accidental to the history of science, namely, that it was declared by the foremost authorities of the Christian Church to be inconsistent with the Bible. The writings of Huxley were frequently a direct attack on the validity of Christian doctrines. Hence, as so many of his arguments were ostensibly derived from his biological work, there arose a notion among advanced thinkers that biology was in a peculiar sense the spiritual science, that salvation lay only in that way.

Biology was the science that attracted the enthusiasts of serious culture. But the great majority of these were disappointed. One does not find the data of ethics as the residuum of a chemical retort

nor in the dissection of an earthworm. The actual study of biology is a work of patience and it is only after a vast stock of knowledge has been patiently accumulated that any reliable lights on questions of ethics may be discerned. Even then, examples are abundant of men who are expert technicians in the realm of biology but whose opinion on other matters has no particular value.

It requires rather a deep contemplation of the principles and results of all related sciences to obtain, within the scope of these, some clear notion of the phenomena of the world; and the certainties met with in the way are rather those of the futility of weaker systems.

The influence of some great authority in impressing on a generation the importance of a science is itself an illustration of the widening of the scope of Association. From generation to generation we observe that there has been a succession of fashions in science according as some great discoveries have laid bare a portion of nature; and as no discovery of this kind is without a deep spiritual signification, it has happened that other sciences before biology have been considered especially as the spiritual sciences.

Mathematics had its great day in the time of Descartes and Pascal, and later reached its crowning glory with the discoveries of Newton.[1]

But even nowadays we find from time to time curious expressions of mathematics as the transcendental science. One of the most remarkable of these is that of Professor Keyser of Columbia University who, being a mathematician and a theologian endeavours to find, in pure mathematics, the assurances of the existence of the Deity. Here we have an instance of Association bridging over ideas that to most would have seemed extraordinarily remote.

Temperament in Philosophy

Mathematics in its own field is one of the most objective of sciences. There is not much ground for emotion in the fact that the angles at the base of an isosceles triangle are equal, yet even in pure mathematics we find scope for Association with temperament. Bernoulli said of an anonymous solution to a problem, which was really by Newton, that he knew the lion by his claws. In reading the exposition of Quaternions by Hamilton one is struck by the meticulous care of his manner. The ardent spirit of Abel is displayed in his attack of problems in bold generality and in his trenchant display of deep principles. This impression of Abel's style

[1] Dr. McCormack, an American mathematician, tells us of the days when fair dames discoursed of differential coefficients and the precession of the equinoxes, and when Euler's letters to the Princess Elizabeth had but one rival—those of Lord Chesterfield

has been rendered graphic, when it was said of the "good old geometer" Legendre that he could not keep pace with the young giant.

An illuminating remark on this subject occurs in a lecture on Riemann by Professor Klein of the University of Göttingen, in which he indicates how Riemann's profound conceptions had their origin in mathematical physics: "You see that even in the domain of mathematics individuality plays a great part."

In philosophical questions such as, for instance, the standpoint of ethics, we find at once a greater scope for temperament and a less strict control of results. Consequently we are not surprised to find in all ethical systems the imprint not only of the author's thoughts and experience but also of his temperament.

John Stuart Mill found this in his relations with Comte, for Mill with his liberty-loving character became shocked at length to discover that Comte, though a rebel to orthodoxy, would impose on the minds of men a tyranny founded on the principle of organisation of the systems he had renounced.

Herbert Spencer's theory of Evolution has no authority over certain parts of his writings as, for instance, his theory of the origin of baldness. It has not much more even on his views of Education or the relation of Man to the State. And although *The Data of Ethics* is the crowning of his work, there is no complete cogency of argument between the positions of the doctrine of Evolution and those of *The Data of Ethics*. We find here the influence of the author's surroundings and of his temperament. It represents the best side of the bourgeois ideal: peace, the development of trade, mutual help, justice.

Kant gives to ethical duty a sterner and more uncompromising character, representing an interpretation more akin to the Stoic philosophy, while Spencer's savours of Epicurus.

As these doctrines of conduct are inconsistent, it follows that they cannot both be right. Neither results from a study in objective style of all factors with a rigorous demonstration from accepted first principles. Such a demonstration would be extremely difficult, though I have reason to believe it, in the broad lines, to be possible.

We have seen how the principle of Association affects the total display of a man's personality and that among authors it colours even the expositions in objective realms.

Character Revealed even in Objective Drama

The converse problem may be posed therefore. Given writings of an ostensibly objective character, is it possible to find revealed in them evidence of the author's personality?

This is certainly the case. Consider, for example, the dramas of Shakespeare. Here we have a great deal of material on which to work. Many of the characters, moreover, are imaginary while those that are historical are liable to interpretation according to the dramatist's conception; and certainly Shakespeare has in some instances allowed himself the utmost latitude in this respect as, for example, in the character of Achilles in *Troilus and Cressida.*[1] But besides the manner of dealing with the plays themselves Shakespeare had free choice in the selection of subjects.

A method of analysis therefore, in order to discover the influence of Association of the character, temperament, origin, and acquirements of the author would in the first place set forth for consideration the actual choice of subjects, in their chronological order if possible, and the reasons of that choice.

Useful suggestions may be obtained by the study of the choice of other poets. Milton did not select *Paradise Lost* as the theme of his epic until "after much choosing", and in the course of the poem he gives the reason why he departed from the usual style of epic. The impress of the poet's individuality so strongly marks the poem that it may be considered in the light of a spiritual biography illustrated by the great imagery of the epic.

If we consider, in this aspect of Association, the dramas of Byron, of Shelley, or Sheridan, we find how intimately related to the personality in each case are *Manfred*, *The Cenci*, and *The School for Scandal*, respectively. Even in an author of most objective style the choice of subject gives indication of his character.

Then we consider the treatment of the subject. Here we have innumerable possibilities in the general atmosphere of the play, the selection of the persons of the drama, their relative importance in the whole perspective. Even in a subject of objective interest there are a thousand manners in which temperament may be shown.

Then there is the consideration of favourite words and phrases. With men dominated by some fixed purpose there is always to be found some characteristic expression such as the "Delenda est Carthago" of Cato, or the "Thorough" of Strafford, or "Le cléricalisme, voilà l'ennemi" of Gambetta.

But even where there is no dominating idea the temperament may be espied by the more frequent recurrence of certain words or phrases and these should be looked for in passages where they do not arise inevitably out of the situation.[2]

[1] This however is one of the disputed plays. But compare the delineation of character of Antony and Cleopatra with that of Ferrero. With the Italian historian we seem to move in realities; Shakespeare gives us a gorgeous fairy tale.

[2] If we study Shakespeare side by side with Calderon we find that the Spaniard has in no less degree the sense of dramatic movement. Calderon's mind was steeped

Another test would be that of the references to the dominating notions or purposes of others, or to the burning questions and fermenting ideas of the time in which the author lived. In Shakespeare's time particularly the question of religion was vital. England's position in the world's politics also had been in jeopardy and she had triumphed in brilliant style. In the realm of thought the genius of Galileo was enlightening the world; Gilbert was working in the approved methods of modern science; and Bacon " rang the bell that called the wits together ".

All these subjects of deep human interest must have thrown themselves with strong dynamic force into the attention of alert intellects of the time. Hence we obtain indications of character and intellect and temperament in the attitude toward these subjects, for in such matters even silence is significant. Shakespeare is silent on these questions.

Then we again obtain further indications in the dramas more or less ostensibly philosophic, such as *The Tempest*, or in those where reference is made to questions of philosophy or religion, as in *Hamlet*.

Then there is the consideration of the works taken as a whole, with the view of estimating intellectual energy according to the output extending over some years.

It would be out of place here to hazard opinions as to the results of such an analysis in the case of Shakespeare. The reference has been made to indicate in what way the principle of Association may be regarded. We have seen how in cases of known personalities it has influenced the expression of their thoughts; then it is reasonable to suppose that our line of criticism may be elaborated, having regard to this principle of Association; so that from the expression of thoughts we rise to an appreciation of the style of their author.

Association of Ideas and Faculties

The Association of Idea and Emotion has been worked out by Godfernaux in *le Sentiment et la Pensée*, a book remarkable alike for the depth of the thought and for the character of the investigations. Godfernaux finds emotion or sentiment as the force which produces what is called Association of Ideas. I will not enter into a criticism of these notions, partly because what is true in them

in religious thought. Note the frequency with which such words as cross and redemption occur in Calderon as compared with woman and love in Shakespeare.

On the lines indicated in the text I have studied Shakespeare's sonnets, with interesting results in regard to biographical notes. Sonnet 76 plays on the name of the Poet's patron, the Earl of Southampton. The line:

That every word doth almost tell my name

almost contains the name thus: HE'RY WROTHSELY.

will, I believe, be found expressed here, and partly because even this enlargement of the scope does not carry us far enough. We must embrace all that is contained in the aphorism: "The whole man thinks".

Thus it would be possible to elaborate a study of psychological politics based ultimately on physiological politics. A famous statesman in the decline of life, but full of vigour and in enjoyment of a commanding position, desires to signalise his career by some new great exploit. Here, in addition to all the objective considerations arising out of the condition of the nation, would necessarily be present the suggestion that the question to be tackled must be important and difficult, yet not such as to escape the compass of the years still allotted to his activity. Factors of this kind tell in the history of the world. They are to the great annals what the one last bumper is to a convivial gathering.

A patriot who is also a musician will in time of stress produce a national song. The greatest example is Rouget de Lisle. A great poet in disgrace, blind, undaunted, scornful, rebellious, will send forth a *Samson Agonistes*. And the philosophers have interfused their philosophy, especially their ethical conclusions, less with the essence of eternal verities than with the emanations of personal temperament.

The study of Fechner's Law is interesting in that respect. This law, which professes to indicate a numerical relation between objective stimulus and subjective impressions, is of general application; but a study of its origins reveals to us that it is compacted of the three dominant passions of Fechner's intellectual life—Psychology, Physics, and Mysticism. Many great physicists before Fechner had studied Psychology, and many even greater psychologists had studied problems of physics, but they had held these matters apart in their minds. Weber had indeed endeavoured to introduce into Psychology measurements as exact as possible, but compared with Fechner the difference was as that of looking at a sacred vestment with the judgment of a weaver or with the rapture of a believer. Fechner did not merely seek to measure phenomena; he desired to bridge the interval between two realms of speculation. Hence we discover in the end that he has performed that improbable feat, the application of the differential calculus to mental impressions, the testing of sensations by logarithms.

That Fechner should have been satisfied we can well imagine, but it is more difficult to understand how he could have persuaded independent thinkers to follow him. Certainly at the beginning many of the most eminent held aloof, but now Fechner's Law has

been established as among the precious mysteries of psychological faith.

Here we also obtain a clue. Whatever is wholly rational and explicable falls into the light of common day, and ceases to afford that which is to most of us a heritage of the ages, the thrill of wonder, of mingled resistance and attraction, of enjoyment and sacrifice, of strange imaginings, ecstasy and devotion, in the presence of superior things veiled in mystery or secluded from impious touch. Fechner's law becomes the palladium of an institution or the cabala of a faith.

Moreover, there enters into play the Associations derived from professional pride. Examples of these abound in profusion. I will select one that was revealed to me unexpectedly. I read an article in one of the medical journals on the thoroughly technical subject of the "Submucous Resection of the Nasal Septum". The whole matter was discussed in the style of a competent specialist, but the conclusion particularly interested me: "And now it can no longer be said that nasal practice has no major operation"!

Here at length the veil was lifted from the soul of the rhinologist. He had been doing good work; he had won that esteem always accorded to virtuosity in practice; he had the rich fields of anatomy, of comparative anatomy, of histology, of bacteriology, of experimental psychology, to roam in; but we find the secret gnawing of envy, jealousy of the exploits of enterectomy of the general surgeon, or of the operation on the Gasserian ganglion of the neurologist. All this was contained in that gem of professional pride thus unconsciously exhibited.

The psychological world, although a world of purer reason, has not entirely escaped such influence. It is found in pedantic language, in oppressive formalism of style, in that appreciation both of authority and obscurity which has made Hegelism the banner alike of Prussian autocracy and of intellectual socialism. It is found also in the creed of Fechner's Law. For hitherto the psychologist, especially the metaphysician, has felt the eye of the physicist on him, has felt even the unspoken reproach of barren studies in realms of reality. Now he can reply with Fechner's law. That is a great instrument, for few can understand it and it is worked with logarithms.

Moral Qualities in Scientific Work

Another aspect of Association in its wider operations is that of the atmosphere of the studies themselves; for it may shock some, contrary to the famous Horatian precept, to smile in the search

for truth or to laugh when some absurdity crops up.[1] This arises partly from the old traditions of religious strife, when truth itself was clad in harsh and forbidding aspects. It arises also from the belief that the man of science must necessarily be of a severe cast of mind.

Certainly a vast amount of technical work lies in his way and the influence of such toil is to conduce to seriousness; but the history of the world shows that the highest genius in such matters may consort with a temperament not always severe—Plato, Aristotle, Descartes, Fermat, d'Alembert, Ampère, Davy, Rumford, Galois, Clerk Maxwell, Huxley, and Clifford attest it.

We have already seen that Reason itself does not proceed by a concatenation merely formal of ascertained propositions. It will be evident that if what is advanced in this chapter be considered in all its import of life and interest, curiosity and attraction, a new notion will arise as to the conduct of the mind in research, and in the striking out of a life's work.

When we contemplate such a career as that of Darwin we discover him first a sportsman, then a naturalist, and gradually we observe the spirit of mere sport becoming less and less, and the interest of deeper questions steadily growing. Then we find a great and capable mind addressing itself to profound problems that lay within its scope, and of which the research was fraught with great importance to various spheres of human activity.

Here then we recognise as not loosely associated with the manner of thought, but interfused and providing for it the necessary motive force, the great moral qualities of patience, sincerity, judgment of values, and courage; and we see these in active play alike in dealing with questions that arise from time to time in course of studies, and in the planning and carrying to determinate issues the bold scheme of the life-work itself.[2]

[1] *Quamquam ridentem dicere verum quid vetat?—Horace.* There is also a passage of Phaedrus, usually rendered too stiffly, bearing on this. A free translation is: Let the mind play at times that it may return to work with vim.

[2] The question of Psycho-Analysis may be considered as a corollary to the study of Association. It has the same relation to psychology that any form of applied mechanics has to mechanics, and it is only one of unending applications of the theory. A discussion of the subject would ordinarily find no place in a work of pure psychology and I refer to it here, because the term psycho-analysis has become one of great import, and because all the expositions I have seen from those of Freud and Jung downward have been conspicuous for their lack of true scientific method.

In Freud's work there is no semblance of any investigation into the faculties of the mind, not a suggestion of the possibility of conducting analysis to the base from which we could determine and set forth the Fundamental Processes.

Freud has simply in a tentative fashion seized upon one of these Fundamenta Processes—Association—and proceeded to an exposition on the assumption that a study of Association was all that was required.

But if only one element of a complex be considered, and if effects due to the complex be ascribed to that element alone, there will be no satisfactory knowledge of the complex nor even of the element in question. If we know nothing of the mechanism of a clock

except that of the escapement, and if we ascribe what happens either normally or abnormally in the clock entirely to the escapement, then not only will our explanation of the happenings be worthless but we will talk nonsense of the escapement itself.

So it is with Freud's non-scientific conceptions of the mechanism of the mind. He has proceeded from a wrong assumption, and in his subsequent inferences he is as little bound by scientific method as in his haphazard selection of a starting-point.

Other philosophers have arbitrarily taken some one function, such as Will (Schopenhauer), or Attention (Nayrac), or Memory (Hering), as the essential fact of psychic life; it is only necessary to compare their assumptions with those of Freud to see that their solution is as well justified as his—and not more.

Freud, having fastened on Association and having failed to understand the true meaning of Association, selects next one of the so-called "instincts"—the sex instinct—as dominating the whole psychic life, even of children. Here again, apart from the grossly unscientific method of arbitrary assumptions, his main position and his inferences are alike absurd.

There are motives more fundamental, more imperious and more constant throughout all ages of life than that of sex; those made known by hunger and thirst and those concerned otherwise with preservation of life. Freud gives no hint of these; they are all submerged in sex. He ascribes overwhelming influence to sex in young children, although as a rule the manifestatation of sexual desire and impulse take place only at the period of full development of sexual organs.

He fails to take into account the motives formed of many involved factors which determine actions in a complex state of society, and he ignores those secondary motives of which Herbert Spencer speaks in scientific style, and which may become in certain cases more powerful than the original motives from which they take source.

On such a basis and with the employment of such methods it is not likely that anything of value can be built on Freud's psycho-analysis. Jung has elaborated the system with regard to the case of certain associated tests. In experimenting on the relative facility of associations and in measuring the time of reaction, he is using good scientific methods, but the central thesis, based on Freud's assumptions, vitiates the entire work.

I give one example, to show how grave may be the fallacies underlying investigations conducted with all the pomp of scientific terms. He tests the reaction-times of a female nurse. He gives the term: Male-nurse, and after a pause longer than usual in the tests, she replies: cupboard.

Jung expatiates on this result and displays the inevitable sex influence. The appropriate answer according to him was female-nurse; but as this arose first and yet was prevented from appearing on account of the inhibition due to sex feeling, the actual response was delayed and was of no particular significance.

Now it is evident that if the psycho-analyst be able to divine so much he has no need of psycho-analysis to inform himself of the condition of mind of the patient. But the whole comment is a tissue of unwarranted assumptions.

I had myself tested by that very word, male-nurse, and the response word was, cupboard. The steps by which I reached cupboard had no reference at all to sex. When the term male-nurse was given I could not at once think of anything in particular. This caused me to jog my thoughts urgently. Then male-nurse suggested a hospital ward, but too vaguely to enable me to respond with the word, ward. I had a vision, not of a complete ward, but only a part of a bed at the head of which stood the usual piece of furniture useful for containing utensils and surmounted by a frame for holding the card giving particulars about the patient. I could not think of the name of this convenience and so, as I had to give some word, I said, cupboard. It would be quite competent for Jung, if I were not present to contradict him, to give a fascinating account of the manner at which I had arrived at the cupboard by way of sex influence, or even remotely through the "Oedipus complex." The rub is that it would have been wrong.

There is another point in connection with the exposition of psycho-analysis which merits a remark. The psycho-analysts make great play with the unconscious mind or the sub-conscious mind. I have insisted even by repetition that the mind is influenced by a multitude of factors that do not make their own impress in consciousness, but that does not suggest to me to speak of an unconscious mind, except by a licence of speech pardonable enough when with an intelligent audience it is desired to shorten the tedium of explanations. But the psycho-analysts continually use this false expression, and nothing in their writings makes it clear that they have a true conception of the matter.

Their error is not an error of terms; it lies deeper. The roots are to be found in the old metaphysical notions of a constantly active, never intermitting, entity of mind. Ward in his definition of consciousness even speaks of this continuum. In mental operations there is no continuance. What is constant, or constant enough from one moment to another, to afford a sure reference, is the physical structure. The action of the mind is discontinuous; the conscious state arises when the physical correlative is stimulated according to certain requisite conditions. The sense of continuity is given by the exceeding brevity of the intervals, just as the air played on a violin or

SUMMARY OF ASSOCIATION

Association is a Fundamental Process. This refers to the simplest form of Association, as between Presentation and Presentation, and between Presentation and idea. But we may also find Association in cases where the objects are more complex, and where the understanding of the nature of an object involves a complexity of Fundamental Processes. Even in such cases the final act of Association is that of a Fundamental Process operating on Symbols.

Hence in using the word, now in a large scope, we find that the function of the mind is such that Associations are formed between Presentations in succession; between Presentations and ideas; between ideas and ideas; and hence by means of symbolisation between extensive systems of thought; between ideas and feelings; between ideas and complexes of ideas and emotions; between any of these incidents of the mental life.

Associations are determined by circumstances of the greatest complexity and involving many factors of the individual existence which do not come into consciousness. Thus Associations are determined in their course by temperament, education, and intention of life.

In the physical basis corresponding the course of nervous excitation may be determined not only by the mutual influence of contiguous cells, but also by connections established with remoter areas; and, from physical causes in the body, or as the result of stimulation from without, or from causes dependent of ideation or emotional disturbances, the course of nervous activity dominant at a given moment may be broken up, and new series of nervous excitations established.

Hence the notion of the mind moving along a "chain of association" is wrong if this refer to a pre-established chain; and if not, then the phrase loses meaning further than that ideas arise in sequence.

In reference to the application of the study of Association we find that since the resultant at each moment is determined by the combination of the subjective and objective factors, we have in the expression of that resultant a factor in determining the character of the subject

In the study, say, of Shakespeare by this aid we might inquire: What is the general temperament and atmosphere of inspiration of his works; or in a long series, if they can be determined chronologically, what are the sequences and the changing spirit of the atmosphere of these?

What are the evidences of education or of special training in the works?

In how far does the technical skill shown in reference to versification or stagecraft compare with that of contemporaries?

What is the character of the knowledge displayed? Is it strong in the science of the time, or in history as then known, or in the observation of character taken from life?

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organ may be continuous although it is due to a series of discontinuous notes, the effects of which merge with the precedent and the subsequent notes.

It may be asked why psycho-analysis has such a vogue if it be falsely founded. The history of medicine in general furnishes, alas, numerous examples where generations of scientists have talked of humours, or of phlogiston, or of causes of diseases which missed the essential nature of disease by ignoring bacteria; therefore no argument can be derived from that ground. If popular votes made a theory right or wrong we might find unexpected uses in the brains of legislators, but as a matter of fact nothing is more illusory than the value, in matters of philosophy or science at least, than a "volume of opinion."

Psycho-analysis has been founded on a base tentatively and wrongly selected, and by arguments which possess no cogency. That manner of founding beliefs has always been successful in a social sense and highly popular; I contend here simply that it is not scientific.

In all this, what conception arises of a development of mind ?

How is the bent of mind shown in oft-recurring words, in ideas brought into Associations not usual nor inevitable, in sportive moods, or in off-hand not seasonable expressions ?

What is the attitude toward great striking facts of the time either of politics or of religion ?

Is the cast of the mind analytical in a scientific sense ?

Where is admiration displayed ? Where fondness ? Where tenderness ? What is the character of passages that break forth most naturally or spontaneously, with enthusiasm, with warmth of language, with wealth of association, with an air of freedom and familiarity, or with imagery inspired, or surprise of happy touches ?

From all this might be built up, in works as extensive as Shakespeare's, especially with the help of direct biographical references, a well-defined and various character.

The Association of idea and emotion extends to the widest scope, so that we may say: The whole man thinks.

Every philosopher has interfused his temperament into his philosophy, in whatever objective terms this be expressed.

The study of Fechner's law indicates the influence not only of his studies of physics and psychology, but also of his bent toward mysticism.

Associations less definite, but persistent, affect the atmosphere in which either professional work or the discipline of a science is regarded.

Historical causes account in part for the austerity of the atmosphere of science.

Moral qualities are no less necessary than intellectual qualities for the carrying out of a great work of science.

CHAPTER II

MEMORY

NOTE:—Memory is another of the Fundamental Processes which, when studied in full development, relates our schematic considerations to rich fields of experience. The study of Memory has occupied the minds of men for thousands of years, and a great number of treatises, even extensive volumes, have been written on this subject. One writer has, however, copied another, and the expositions are generally descriptive rather than analytical.

Nearly all that I offer here results from my own observation and experiments. Some of these experiments were very laborious, and extended over long periods of time. I have selected out of a mass of material only such observations and experiments as determine definite points.

(1) *Experiments in Recollection*

Memory is a Fundamental Process of the mind. No mental life could be carried on without Memory, and on the other hand its elementary phenomena are as direct, spontaneous, and unanalysable in themselves as the Immediate Presentations of our experience.

But it is well to inquire into various circumstances on which Memory depends. We have already investigated conditions for the production of sensations; such as, for instance, in the broadest manner in the case of vision that the physical apparatus shall be in good order, and that an object be in the field of vision.

There is a Sense of Effort involved in the Process, yet that does not depend on the will. This must be clearly understood; it may require an exercise of will, for example, to open the eyelids, but when the conditions are obtained the Immediate Presentation follows. We are not aware of the mechanism by which the result is brought about; and we are not able, when we have fulfilled the conditions, to control the operation.

The conditions within our control are a part of the precedent physical conditions, but another and an essential part of these is beyond our control.

As Memory is a Fundamental Process, it will be readily understood that we cannot control the phenomena of Memory by an exercise of the will, except by endeavouring to alter the physical conditions and to influence Associations according to intention.

That the physical conditions are of importance becomes evident when we proceed to extreme cases. Thus, if a man be knocked

down by a violent blow on the head he may be altogether deprived of memory for a time. Again, if he be telling a story, and then suddenly discover that his coat is in flames, he will have the current of his recollections changed.

It is important to note that the appearances of what is called the "association of ideas" are frequently produced by physiological or pathological processes in the body, and this may be the case even when the sensations or feelings accompanying such processes make themselves but a feeble impression in the mind.

The exercise of the will may modify the conditions under which Memory works, but under given conditions it cannot influence the order of the ideas that arise in consciousness.

It is often most profitable to examine a structure in its breakdown, and to consider a process wherein it makes default. Therefore, in the study of Memory, I will begin with some observations of failure of Memory, and I will then proceed to the review of certain experiments in Memory, some of which extended over long periods of time. From all of these it will be possible to draw useful inferences. The observations and the experiments were all accurately described at the time, with dates and hours noted, and circumstances which seemed to have a bearing on the case mentioned. It does not seem necessary, however, here to reproduce these notes in the formal manner in which they were first set down.

Every Unit has its Recollection

I had occasion to recall the name of Fresnel, but, at the moment I required it, it "slipped my memory". I then resolved to study attentively the manner in which I should recover it.

The name had slipped my Memory because something accidental had arisen at the moment of recalling it which diverted my thoughts. This circumstance created a stronger bond of association with the previous chain of ideas than did the name itself.

Therefore every time I pursued the same route I found my train of association divided. Every successive change in associations not only implied a facility in the new route of ideas, but also an inhibition of other associations which I could yet "feel" as tending to assert themselves.

In such circumstances the mind cannot usually recover a name simply by persistent thinking along the old track. It is often necessary to allow a certain time to lapse, so that the association which has led us off the path, and which may not be a permanently strong one, may lose some of its force. Then the permanently strong association of the name with the facts to which it has reference finds easier play.

I have presented the matter graphically by an illustration (*cf.* Pt. I, Chap. II, p. 29, and Pt. II, Chap. IX, p. 160) which is at least true as regards the physical basis of the phenomena antecedent to thought. The elaborate network of the nerve-strands in the brain, with all their conducting lines, their stations of exchange, and their inter-communicating reticulations, forms a system which, though complex, is yet governed by a simple principle. This principle is reproduced in various forms in our human works. The service of an extensive telegraphic system, a very elaborate system of canalisation, the graphic representation of systems of classification in a large and varied field, all have analogies with the system of the nervous network of the brain. These analogies have their origin, in part, in the nature of things, in part also in the circumstances that our minds move according to that principle, so that everything that we create for reducing complexities to order is founded on Generalisation, classification, and all that these processes imply.

But in the brain this whole system is living. Its activities are affected by various forms of stimuli, by the supply of blood, by the quality of the blood, by the healthy development brought about by exercise, or by the disabilities produced by fatigue.

Again, from what we know from neurology as to the localisation of various "centres" in the brain, and from the fact that the mind holds in consciousness one Unit at a time, and that the progress of the mind is by a series of associations, one might suppose also that at any given moment only a comparatively small part of the brain is in activity, or at least in such activity as produces clear ideas in consciousness. This last point becomes evident when we consider the incoherence produced in uneducated persons by causes of excitement which disturb more areas than those normally stimulated, but which produce only ideas revolving round a few "fixed ideas".

This digression has been thought advisable in order to make clear the course of the exposition. It will be seen to imply nothing beyond what is accepted in neurological science. It should be especially noted, again, that however accurately we represent the physical basis of our minds, thought itself is something quite distinct, and unimaginable in terms of this physical basis, though the physical basis affords a good graphic representation for referring to movement of the mind.

This I had in view in making the study we are considering.

So as to avoid the influence of disturbing associations I thought quietly over various circumstances connected with Fresnel. I became satisfied that the name began with the letter "F".

The fact that the Memory of the initial letter often persists when nothing else is recalled is due to a variety of causes. The initial letter is made a capital letter, and thus it becomes doubly prominent as compared with the other letters. Moreover, being the first letter met with in the change of attention from ordinary words, as in reading, the associations formed round it are stronger by reason of the freshness of the attention. An extreme case of the like principle is that of the strength of the associations created by a joyful surprise. Moreover, the initial letter is often used, isolated, to indicate the person, and so a habit of mind is formed tending to impress attention on the initial letter.

Still endeavouring to recall the name, I formed in my mind a clear image of the bust of Fresnel which I had seen in one of the halls of the Institute at Paris; and I endeavoured to recall the name by associations with the notions I had formed in regard to the bust; the deeply thoughtful, absorbed look, the regular, classic features, the air of nobility of mind, the finely formed head, symmetrical and well-balanced, the well-arched dome, the large though not disproportionate forehead—all this recalled the man, but not the name. I then thought of his work; the rapidity of his progress in mathematics; his invention of reflectors for lighthouses; his intercourse with Thomas Young; that their work was on similar lines; and how certain results reached by Fresnel which filled him with a sense of triumph had already been attained by Young.

All of these circumstances brought a multitude of associations, but none of them produced the name. I then thought of him in connection with Foucault and Fizeau, but to no immediate purpose.

These efforts and observations I continued at intervals for three days. I then began associating the initial letter with various letters of the alphabet in order; and when I reached the letter "l", the name Fresnel came to my mind followed immediately by a series of new associations, prominent among them being that of Fresnel's rings, which in turn recalled the experiments of Newton, and various other matters in connection with the theory of light. Not only that, but I found that, by repeating the name Fresnel, the previous associations were made more vivid, and even the image of the bust became clearer.

I find this note appended to those set down at the time: There was also vague association with Napoleon, but I am quite unable, at the moment of writing, to add a single fact to this association or otherwise to define it.[1]

[1] Some years afterwards, while re-reading this passage, I remembered that the association with Napoleon was that I had once formed a list of men of genius with the clear-cut, classic features, and eager, emaciated countenance of Bonaparte, when young; this included Fresnel and Keats.

Now in this case the name Fresnel was not at all unfamiliar; but at the moment of desiring it I had a vague apprehension that I might not remember it. The slight emotional factor had an influence in inhibition. At the point I had reached then—using the graphic representation—the way was barred, and the movements of my thoughts were diverted in other directions.

In seeking other associations I was "trying back," seeking to get beyond a junction so that I could reach the name by paths that did not pass through the barred road. But in cases of the sort there seems to be often not merely a barred road but a veritable inhibition of associations with the name, as if the natural flow of associations were reversed.

I succeeded eventually because the inhibition was due to associations not deeply based, and the influence of these in diverting the current of associations diminished more rapidly with time than associations deeper and of longer date.

The fact that the letter "l" served to recall the name is interesting, for it seems to indicate that, in a name, each letter of the name forms its own associations. No doubt the endeavour to find the name during the three previous days had facilitated many paths close to that related to the name, so that some new slight stimulus was all that was required to complete the union. Thereupon the effect was comparable to that of an explosion of stimuli, from a centre corresponding to the name, out along many adjacent paths.

We may have occasion to conclude from a minute study of such cases that not only every letter of a name, but every element of a letter which we observe, and in short every unit clearly recognised, forms its own associations; that is the manner of its existence in the mental life.

* * * *

About the same period as the last observation I tried to remember the name of a great French neurologist, Duchenne, with whose writings I was to some extent acquainted. I had first met his name when studying the work of Sir William Gowers on the nervous system. Dr. Gowers spoke of the "elaborate hysterics" of the French women, and referred to Duchenne as an authority on that subject.

The chief association with that reference that remained in my mind was that of hysteria causing such a tension of the muscles in some cases that the patient was able to rest on the head and heels—*arc-boutée.*

I had other associations, for I had read a comparison of his work with that of Charcot, one of whose lectures at the Salpêtrière I had attended, and I had also read a biographical account of Duchenne in connection with a monument erected to his memory at Boulogne. I had been struck also by the Christian name, although it was not an unfamiliar one. Some years, however, had elapsed between these several associations.

Though unable to recollect the name, I was yet able to classify it to some extent. Thus I knew it was a typical French name, that it was not an unusual name, that it was of medium length. The name is really Guillaume Benjamin Amand. On reflection I see that the last is uncommon. I think that what was associated in my mind with the name was the designation "Duchenne of Boulogne", which the physiologist had adopted almost like a title.

It will be observed that these elements belong to broad lines of classification, and they are useful, therefore, in indicating how the mind forms its associations. The word Duquesne, on account of the peculiarity of the pronounciation in regard to the spelling, occurred to me frequently. I had been struck with it in glancing, not long before, at a book of Fenimore Cooper's.

In the case of Duchenne, however, I was unable to identify the initial letter, and I believed that the letters most likely to be found in the name were m and l. These facts I take from notes made at the time when I was still trying to recollect the name.

The degree of recollection was evidently less than that in the case of Fresnel, and no doubt the name was less familiar on the whole. The difficulty was partly due to the fact that, since having had occasion to note Duchenne's work, I had been engaged in other studies which had greatly occupied my mind. This produced a certain force of inhibition with regard to subjects less immediate.

It will be found, by experience, that if the mind be engrossed on certain matters of study, then subsequently find other absorbing studies, the recollection of the first will have become more difficult, with a sense of remoteness, than if there had been no severe studies intervening.

For a week I tried to recollect this name, finding new series of associations each day, but the inhibitive force was too strong. Afterwards I met with the name, accidentally, but even then the new associations provoked were not so vivid as I had expected. It would seem that, all along, I had associated what I had known of the man with his personality rather than with the name itself. This may easily happen with foreign names.

* * * *

We have now had an instance where the first letter was recalled with certainty, and an instance where there was doubt about the initial letter. I have notes of another where I believed the name began with a letter which was really not the initial letter. This was in the case of trying to recall the name Hesse, with whose work in the higher mathematics I had associated ideas of "elegance" of solutions. The name had escaped me, and in seeking, as in the previous examples, for associations I became convinced that the name began with F. At the same time, I knew that it was a short name, and that, though the man was a French mathematician, the name was German.

* * * *

Every Unit has its Association

The following is especially instructive. Late at night, while very tired after arduous work, I tried to recall the name of Berthelot, the great French chemist, a conversation with whom I remembered perfectly well. The name was obscured in my mind, but I resolved to recover it. After thinking for a considerable time I arrived at Berthollet. This satisfied me for a moment; but immediately afterwards I recollected that Berthollet was the name of another great chemist, and slowly, gradually, and by persistent thinking, I found the right name Berthelot.

During this process a name had often seemed dimly to suggest itself, but before I ascertained this name I knew it was that of the founder of the anthropometric system. It was really Bertillon.

I knew the name was like Berthelot and Berthollet, but it required a considerable amount of patient searching before I recognised the name clearly—Bertillon. In this case the name satisfied me immediately it was recovered, and its recollection brought in its train a number of associations.

This example indicates that each element of the name to be remembered has its own Memory, and that there are associations formed between the parts themselves, and also between the parts and external things independently associated.

* * * *

The unusual names are the most likely to fall out of memory. Here is a note regarding an uncommon, though not foreign name. I tried to recollect the Christian name, for I had the surname, of a young man whom I had met and whom I had heard sing—Dermot Sidford. Since I had seen him I had had my mind occupied with serious studies and much hard work.

*

I determined to remember the name by effort, and to observe the manner in which it would return. I felt sure the name would soon appear. That is a fact worthy of particular note. One has a distinct feeling in this respect which suggests the graphic representation of the name being held down by a network which is being distinctly agitated. Much might be explained by a scientific study of these presentiments—*Ahnung* of the Germans—extending from a single case like this to the vague but powerful impression which a scientist may have that he is on the verge of a discovery.

In seeking the name, suggestions came to me of Murdock, Mudford, and names having an assonance like them. I knew with certainty that these names contained some element of the real name, but that they lacked something characteristic. I then thought of Bernal and Bertrand, but on close attention I became convinced that these names arose from associations with that of Bernard Shaw. It will be noted that the names Shaw and Sidford have in common only one element, the sound S. This would suggest that, on the basis of our graphic representation, we should place the area for Si near that of Sh.

But as it is in the highest degree improbable that there could exist any organic provision for storing complex symbols of artificial construction, as not only our words but also the letters of our alphabet are; we are led to analyse to its elements all the combinations that go to form a letter; and we become convinced that each element of the letter, including not only its sense of sound, but its force, impact, anything of which we can form a distinct conception as a Unit, has its physical correlative.

It will be seen from this how marvellously complex are the foundations of ordinary speech.[1]

Returning to my desire to remember the name, and acting on a suggestion regarding the physical condition of brain activity, I determined an increased flow of blood to the brain by rubbing vigorously the back of my head, and applying my hands to my ears and bending down. The first effect of this was to obscure my quest by introducing too many associations connected with it.

Then after a time I recalled circumstances in which I had seen Dermot Sidford, had listened to him singing, and had heard his name pronounced. I repeated the name Sidford several times in quick succession and in an eager way, recalling all the circumstances as vividly as possible.

After a time there was a sense of the name arising. This was really curious. I gained no additional information at all with

[1] This observation may make clearer some of the remarkable phenomena found in aphasia, where certain elements of the alphabet are retained, while others are blotted out.

regard to the letters of the name. It was only just a sense of a veil rising and floating intangibly, and yet of expecting that this would uncover the secret. I knew it was near it, though in actual information I had gained nothing.

I tried again in the same way. Suddenly and with an air of surprise, as of a sudden dawning, the name was there in complete light—Dermot.

* * * *

Two cases, of which I have precise notes, introduce curious features of Association, which become easier to understand if we bear in mind the reticulation of the nerve paths in the brain which correspond on the physical side to the mental processes involved.

The first note, written on 12th May, 1917, reads thus:

In the year 1907 I glanced for a few moments at a book in two volumes in the library of St. Mary's Medical School: *Differential Diagnosis.*

Years afterwards from time to time I tried to remember the name of the author, and in 1917 I made a determined effort to that effect. My recollection was that the book was in one volume, but by two authors (it was really in two volumes by one author). I remembered the title well, and the general appearance and colour of the volume. The name also seemed to me to have something about it distinctly American. I thought of the names Ogden, Coolidge, Loomis, without being convinced, but with a partiality for Loomis. Now the name was Cabot, but about the period at which I had seen the book I had noted some researches in blood analysis of Loomis. In May 1917 I saw the book which had been bought, at my instance, by the British Museum, and I recognised the peculiar colour of the binding. This sight made the whole recollection firmer.

Reflecting on the whole matter, the suggestion of two names of authors arose from the fact that there were two volumes.

Cabot had struck me at the time as a name that recalled America; this book was by an American author, Cabot.

The original impression had been slight, and the experience had been interrupted as I was called away almost immediately after I had begun to read the book. The structure therefore of the recollection survived in its main parts, though smaller connecting parts had vanished or become diminished. Each part, moreover, as for instance the colour, the shape as well as the name, had its element of Memory distinct but associated.

In June 1917, I added a further note:

Finding that whereas the name I sought was Cabot, I had yet

thought of Loomis, I enquired how it was that Loomis came to my mind. Then I seemed to recollect that the name of Loomis had occurred to me in connection with another medical matter, viz.: the classification of leucocytes. I looked up a note-book where I had tabulated the classifications of various scientific men, and to my surprise I found amongst them the name of Cabot. The only name like Loomis was Lewis.

This proves that even though the precise recollection has been lost, yet the influence of the previous association persists. An image representing the matter might be formed by supposing that where there had been a little reticulation of conduits with all sorts of cross paths, some of them had been blocked so that certain communications once direct had become at length only possible by indirect paths leading through larger junctions.

* * * *

Associations Formed by Ideas Forgotten

The next note is dated 26th March, 1918, St. Ives, Cornwall:

Yesterday morning something recalled to my mind Dan Donnelly, the prize fighter, and I tried to remember the name of his opponent in a famous fight in Ireland—a name with which I was once familiar.

I recognised at once that it was not Tom Oliver, the suggestion of whose name recalled to me the passage in *Lavengro* describing the scene of the fight between Tom Oliver and Ned Painter.

I remembered the name of the first English opponent of Dan Donnelly—"Isle of Wight Hall"—but could not remember the name I wanted, although I recalled many circumstances connected with the fight. It seemed to me a short, familiar name, something like Bob Cross, although I knew it was not Cross. I remembered also the style of the man—that he was a more accomplished boxer than Donnelly, but smaller and not so tough.

While walking along near the sea, I saw on the beach at Carbis Bay that a soldier picked up an object and showed it to a well-dressed man in whose company he was. This man examined the object apparently with great attention. The suggestion then occurred to me of the mental steps which would be traced by a naturalist in the work of classifying a specimen. I then compared these in a general way in accordance with the spirit of this Psychology, with the steps which a grammarian would trace in determining the proper syntactical employment of a phrase. Then the notion became suggested to me with greater force than ever before that the great naturalist must not use these means merely as mechanical rules, but that his mind must be steeped in the

knowledge of the subject, that he must know ten thousand and one facts and their associations, and so that this knowledge is ever fresh in his mind.

I then proceeded to the recollection that Cuvier, the great French naturalist, was an example of this proficiency; that he knew each individual bone he had seen, not only by reason of its classification, but as an acquaintance.

Then I remembered that I had once observed that Cuvier, which now sounds a distinguished name, simply meant Cooper.

Then suddenly and without fresh effort I remembered that Cooper was the name I had been trying to remember. All this reflection occurred while I walked some eighty yards. On studying the matter I saw that a fact may be revived by a circumstance in association which may stimulate energy in various connecting relations without their being necessarily brought to memory. In other words, thinking of a network of relations we may be unable to proceed by routes (*a*) by direct Impulse, but by another route being reached (*b*) we may find that the Impulse for (*a*) has helped to make a connection from (*b*).

Another explanation, which as my experiments continue seems more important, is that although the name Cuvier suggested Cooper, yet precedently it was the name Cooper that suggested Cuvier, hence the meditations on the naturalist's method. The nerve filaments corresponding to the name Cooper, though not stimulated sufficiently to produce consciousness of that name, were yet stirred together with those surrounding and meeting them, so far as to have produced an activation of those corresponding to the name Cuvier.

Another study of a similar kind presents a peculiarity in the way in which the name returned. I had occasion at a later period than in the observation last mentioned to refer to the composer Mascagni. I had not used his name for years. I could not recollect it. After some reflection it seemed to be coming vaguely. It presented the letter M. I then thought of the opera "Cavalleria Rusticana"; of an occasion on which I had had an interesting conversation with the composer; of his appearance; of his voyage to the United States.

The name seemed at times as though ready to spring out all at once, but I could not quite attain it. I "concentrated" my thought, and I had the sense of applying energy with fervour. Then I made one or two tentative efforts. Then Magnardi seemed like it.

It was noticeable that before I recovered the name I knew that it was not longer than Magnardi, not so long as Magnardino.

Suddenly, quietly—and in a manner to suggest the explanation: not in the place at which I was looking—came the name Mascagni, naturally, freshly, familiarly, as though it had been disengaged from some encumbrance and were now floating easily.

* * * *

I have also notes made at the time of a case where there was a persistent error in regard to the first letter, but where, nevertheless, the letter wrongly taken played a prominent part in the name. I had occasion to recall the name of M. Doumer, the eminent French statesman, whom I had heard delivering an address in Paris, but of whom I had seen nothing for some years. The names that became successively suggested were those of Marieton, Moron, Merlou, Remon.

In the midst of other occupations I reverted from time to time during several days to my attempts to recover the name, but without seeming to make much progress. I had, however, acquired the certitude that the letters r and m were in the name. I inquired if I could find any others. The name of Drumont occurred to me, and with a sense that the real name seemed to be flashing near as though escaping from some encumbrance.

Then while walking in the afternoon I repeated as rapidly as possible, and in an agitated manner, several names such as Drumont, Remon, Durmer. Then suddenly, and with ease, I lighted on the name, Doumer. When the name was recalled there came with it a feeling of entire confidence, and many new associations were recalled.

* * * * *

Another case in which the Memory due to each letter becomes manifest I find recorded in my notes with dates. I desired to recall the name of a photographer in Paris, Branger, with whom I had once spoken. At first I could only form tentatives, Briaschi, Brigi, Vrass, but with the knowledge that these were incorrect. The name did not seem French. I knew it began with B, and that it consisted of one or two syllables.

I thought persistently on the name, but not till two days afterwards did I find a dim suggestion of "n" in the middle. The following day I made the tentatives Brianchi and Braschi, but recognised that they were wrong. On the next day I met the name Branger, accidentally, and became persuaded that it was the name, though there was lacking the bright flash of certain discovery. The explanation seems to be that I was under the impression, having in the first instance only heard the name pronounced, that it was

an Italian name like Branji. It will therefore be seen that the efforts of reproduction were consistent with the hypothesis that a distinct Memory appertained to each element, that is to say, not only to the letters, but to their forms of combination, and to associations so determined.

* * * *

One of the most instructive examples is that wherein the failure of Memory was partly produced artificially. I set down a note at the time: I was recalling the name of a firm, Swan & Edgar, being at the time fatigued with mental work. I would have had no difficulty in recalling the name, as it was familiar to me, except that I purposely followed for a moment another train of thought. On returning to find the name I could not recover it, the tendency being to pursue the new path artifically opened. I remembered that the first name began with an S, and the most frequently recurring tentative was Snow. Here it may be pointed out that this name contains the same number of letters as Swan. We have observed that each letter and element of a letter has its Memory. It would appear also that the Sense of Effort associated with the expression of the name has its Memory.

The tentatives in regard to the second name were less certain, Eglinton and Elbridge being among them. The first association Edg was the most vivid. It will be noticed that the "a" of Edgar does not appear in any of the tentatives, but the pronounciation of the name is really Ed-gr. Every element in the name, visual, aural, sense of Space, Sense of Effort, frequency of Association, vividness of Association, possibly on account of some Hedonic quality, all help the Memory.

* * * *

Sure Expectancy of Recall of Idea

The question in reference to each element having its own memory is so important that I add the notes of another case which had remarkable features. After a tiring day, involving much writing and ending in a long sitting in Parliament, I went to bed after midnight. My note taken at the time says: "Very Fatigued."

It occurred to me to try to recall the name of Chantemesse, the distinguished bacteriologist, whom I had known in Paris, and whose name, although not often seen in the meantime, was familiar to me. I had artificially interposed the name of another bacteriologist, Calmette. I tried in vain. My mind seemed dulled and lifeless. Calmette came forward every time. Here it may be

remarked that the French phrase, "*Ma langue se fourcha*" (My tongue forked) expresses well the physiological process, the stream of stimulus being diverted to a wrong path.

I thought of various circumstances in which I had seen or spoken to M. Chantemesse. This produced no effect. I thought of mutual friends, but again without effect. Then artificially I tried association of letters. I felt certain that C was the first letter. I was also certain that there was a "t"; and that the name in sound and style was somewhat similar to Calmette, and that I should find it lying under and lying longer than Calmette. This impression was remarkable. Suddenly, while trying combinations, there occurred to me the name of Chassemelle. There was a quivering as of under-growth, a stirring as if the whole would come to life. But the sense of fatigue came on again.

However, being convinced that the secret lay there I kept trying by repeating the name Chassemelle, but with searching here and there. I felt sure that the ending was of the form, ette, elle. Then suddenly the name came easily, and with a feeling of entire confidence—Chantemesse.

It may be observed that, though transposed, the letters had in the main been recalled, c, t,a,ss,e. This association with the name was stronger than that of the sequence of the letters themselves.

* * * *

Motor Tentatives in Thought

Here is another remarkable case, which permits of definite conclusions:

In Paris early in the month of November, I had occasion to remember the name of Félicien Champsaur (novelist), whom I had known, and whom I had met for the last time some three years previously. I failed to remember the name for the moment. I had other matters to attend to and forgot the incident. On the twenty-ninth of November in conversation with a member in the House of Commons, I referred to a French humourist, some of whose work I knew, but whose name escaped me at the moment. I had not seen him for many years, and had seldom occasion since to refer to him.

On succeeding days I tried several times to recover the name. I could not remember the first letter. Yet I seemed to have a decided notion of the "character" of the name, of its sound, of its being in some way characteristic of the man's writing.

I could only get so far as to eliminate certain letters as not being initial letters. The name seemed to me French of the French, not

usual, short, two or three syllables, and ending in some way with a smack. Grumbache, Gavroche, Eschelas, some affinity with D'Esparbès, an ending in uin, ain, an; sounds of g, r, nosch. I could get no further, try as I would, by recalling incidents relative to the writer, his appearance, voice, his plays, his style of humour, anecdotes. Suddenly one night after retiring to bed, I felt a little excitement and the name seemed to be on the point of coming.

I seemed to be sure of Car. Then nothing more.

Then next day while thinking desultorily of the name, suddenly and easily the other forgotten name—Félicien Champsaur—appeared. But that of the humourist remained as though imbedded in dough.

Here is a transcription of my notes.

11th December: Have been trying every day since at occasional intervals to recover the name. Now inclined to believe that it begins with G, but this is not certain. Though I have made no tangible progress towards recovery I feel at times that the name is on the point of appearing, quite familiarly. There is a suggestion of names like Gercache, Grulache, Kotzebru, Jastro, Guasclin, Guernan, Grumbache.

The name has certainly two syllables at least and, if three, not more than three.

17th December: Have been trying every day to recover the name. Find that names of others whose Christian name is George come forward. Georges D'Esparbès, George Stephens, George Dwelshauvers. But I am not sure that George is the Christian name in this case. Occasionally I have seemed on the point of touching the real name, and in those moments it seemed to be of three syllables. Have now a certainty that the name will be recovered, but cannot say what is the initial letter.

28th December: Every day since I have been trying to recover the name, I have tried in the morning when fresh, and at night when tired, but without avail. I have lost any certitude as to the initial letter. There was the suggestion of names like D'Esparbès but this I attributed to the name Georges, and I considered that this had been artificially introduced by a previous tentative which was like D'Esparbès, whose name is Georges. Then I resolved not to force the name, but to endeavour to "draw" it. I got no result but occasionally it seemed as if I was near to a revelation. I then visualised the man clearly, remembered a great number of circumstances connected with him. Then tonight while engaged in other matters I reverted to the subject again. I resolved not to play with old associations but in a sort of inviting way to consider the name familiar. I remembered that

Severine had once called "notre—" with the name. I had an idea of a name beginning with D; then Den or Dan, then with a termination, ellin, as Danevellin, or Danechellin. This seemed to promise as if there was a yeasty feeling. I then repeated the termination, ellin; then suddenly I was conscious of a name, dimly shadowed, floating, vibrating, as if printed in regular big letters in a sort of close thick medium. I saw the name was Georges Courteline. But even then there was not, as usual, the immediate recognition that that was the name sought. Then when I considered it again it seemed almost too familiar, too easy, as when we light upon some almost obvious solution of a problem that has been puzzling us. Then I saw clearly that it was the name, and I was astonished that I had not recognised it before. It was quite familiar. I felt as if a tension had been removed from my mind.

The study of this case seems interesting with the incessant suggestion of the George, even though that was not strong enough to produce the name. Also the suggestion laid aside of Car or Char. The D seems to have come by association with Georges, the name Georges D'Esparbès being very familiar to me. The difficulty of finding the association will, if studied, be found to have striking side-lights upon Freud's psycho-analysis. It will also be seen that the suggestions—forcing, inviting and so forth—really indicate that in thought there is in each Impulse a rudimentary motor tentative.

Finally this seems to confirm the theory that each element has its own memory-association.

* * * *

I do not intend to convey that the process of recovery is always, or usually, in accordance with those recently described. These examples have been chosen to show that each element of a complex, such as a word, has its own Memory. In most instances, however, it will be found that the associations between the parts of the word are so close that the whole word is recovered at once.

I find a note which illustrates this. Having laid aside certain studies involving great concentration I had occasion to recall the word "lumbrical". At first I had a vague impression fairly persistent, of a sequence of letters, m, l, b; but I could make no progress. I then recalled the form and the function of the lumbrical muscles. Then I recalled other associations. Suddenly the word came, "lumbricals", and with an unmistakable impression of energy.

This impression of energy is remarkable. Consideration of this phenomenon will give some light on certain of the thousand factors that go to form taste and style in poetry.

It may be noted also that in such cases there may be no Impulse towards any action as a consequence. The effect of recovery satisfies the intellectual and moral needs of the moment. This point should be remembered in the discussion of Will, for if Will could be analysed to mere relations of sequences and associations, there would not arise such a distinction of cases where Will is not marked from those where Will is well marked.

* * * *

Another illustration of the manner of return of former impressions I find thus recorded: I was thinking of persons known when I was a schoolboy in Australia, and whom I had not met since. Most of the names came readily. Some lagged, though I felt certain of remembering them. I tried to think of one whom I had seen as a young lady when I was a small boy—Ellen Moseley. I remembered the style, appearance, beauty, various circumstances. At length I became aware that the first name was Ellen. I believed that the second name began with M. I thought of Mooney, but I knew it was not Mooney, that it was not so familiar as Mooney, and yet that it was not altogether an unusual name. I went for a walk. Then suddenly with no marked Impulse the name was there all at once—Ellen Moseley. There was present a feeling of certainty and repose from the effort of searching.

* * * *

About the same period I wrote down the notes of another case which contains peculiar incidents. At half-past seven in the evening I tried to remember the name of a notable operatic manager, Hammerstein. I had met him once, but had not thought of the name for more than a year. The name of Heinemann, the publisher, came up before me, and I could not recover Hammerstein. I went to my study, and engaged in absorbing mental work, followed by writing, until half-past eleven. I then went to bed, not intending to think any more, but I could not throw off the prompting to find the name. Suddenly there was a glimmer which I felt sure contained the name. I felt sure also that very little was wanting to bring the name to my mind. When I say "glimmer", I am endeavouring as nearly as possible to indicate the actual impression in my mind. It seemed as if in a certain region there was a flash, long in form compared to breadth, and remotely suggestive of a flash of lightning along the horizon.

I am particular in regard to this description, for I believe that a study of such mental imagery may yet bring forth some curious and important facts. I then thought steadily on the name, but there were merely broken suggestions of H as the initial letter, with M to follow, and vague associations with the German character of the name.

I thought energetically, but felt that that was not the way to progress. I became convinced, however, that if I changed my position, as by turning over, I could recover the name.[1] I did not do so at once, but allowed my thoughts to subside till there was a general blank.

Then, on account of other causes, I gave an involuntary turn, and suddenly, as if in the midst of broken, semi-transparent hurdles, I found the name Hammerstein, not as presented vividly to the visual sense, but appearing as distinctly as a thing within my knowledge.

This manner of the name arising should be compared with what is said of the other cases. It may be noted also in connection with a remark I once heard made by a physiologist, an Irishman, whose ardent pursuit of science had not blunted the wit of his race. He said that it was sometimes best not to look directly at a problem, but to observe it sideways, for then the solution would occasionally come out " unbeknownst. "

* * * *

Consequential Effect of Idea Which is Itself Forgotten

Still another observation of a similar kind is noteworthy on account of a circumstance connected with the recovery of the name. I tried to remember, in conversation one evening, the name of a charming young actress I had seen in Paris some years before—Louise Bignon.

At first, although I remembered clearly the appearance of the actress—the colour of her hair, her manner, the circumstances under which I had seen her—yet the name was completely blank.

On the following morning, when my mind was fresh, I tried again and again to remember the name. These efforts continued at intervals for three days. Gradually a dim adumbration of the name seemed to arise—a suggestion of Marigny, but also with a certitude that that name represented only a few of the elements. I could not recall the initial letter.

[1] *Cf.* Mlle. Morand: *Le problème de l'attente.* (" Expectancy is an attitude "), *l'Année psychologique*, 1920.

On the following day I made but few efforts. Finally, about a week after the first attempt, I had occasion to look at one of the evening papers, *La Patrie*. I was not then thinking of the actress. Suddenly the name Bignon became quite clear. Immediately afterwards I recollected the whole name, Louise Bignon, and it seemed quite familiar.

A curious feature of this experience is that on reading *La Patrie* again, I observed the name Bignon in the news referring to a municipal councillor. The name is not a common one; I had not noticed it before in the paper. It is, however, though improbable, yet not impossible, that I had really seen the name in *La Patrie*, and that though it did not attract my attention it yet produced an impression which later and in another way made itself manifest in consciousness. But I think that I had not seen the name, and that its appearance was simply a coincidence.

The above was written about two years previously to the note which I now append, thinking it best, as I have no certitude in the matter, to allow both to stand:

On reconsidering the Bignon case I now incline to believe that the name must have caught my eye in *La Patrie*, otherwise the coincidence would be extraordinary. One may well ask, then, why did the name not immediately attract my attention? It will be observed, however, from the previous note, that my mind was intent on some other subject. Now it is possible for an impression to be received in consciousness, to be forgotten, and to become revived on some other occasion without the association of the first experience being vivid enough to produce again an effect in consciousness.

I will state certain incidents which appear to have a bearing on this problem. At one time, while a student in Melbourne preparing for an examination, I dreamt that a horse named Flying Dutchman won a race at a place near Melbourne called Kensington Park. I had not even known that such a horse existed, and I was too occupied at the time to take an interest in the races. I looked at a newspaper in the morning, and there I found that Flying Dutchman was entered for the race. I was surprised, but having no belief in any special interposition of Providence in such matters, I did not back the horse. As a point of fact, Flying Dutchman did not win.

Reflecting on the matter afterwards, I concluded that during the day preceding the dream I had read in a casual glance that Flying Dutchman was one of the horses engaged in the race. Other matters more important to myself then occupied my mind. During sleep, however, new associations arise, and in cases of great

fatigue those due to forced pressure may disappear. The name Flying Dutchman reasserted itself and with vividness possibly due to new associations produced in dreaming.

About the same period also I was walking along Collins Street, in Melbourne, absorbed in thought, when a friend of mine called me by name. He said that I walked on about twenty yards and then suddenly turned round. Here the circumstances form a sort of intermediate stage. I was thinking out a problem. My friend called my name. I heard him call, and the impression struggled with the tenor of my previous thoughts. During the walk of twenty yards I had reached a conclusion which gave me a resting-place, and then the call of the name entered with full effect.

Covering up of the Old Group of Associations

Recently I had another experience which shows how a body of associations may—to use a graphic representation—be covered over by later growths, and how they may slowly work their way to consciousness. I had looked into an old box. I found some old letters and verses of a young Australian poet, R————, unknown but veritable, who had been a friend of mine. These made a peculiar impression upon me. On the following day I was thinking for a long time over certain characteristics of Byron's poetry. In the early hours of the morning following I dreamt that while I was in company with R————, Byron came along and we had a conversation.

Still more recently I have record of an experience which seems to confirm my last interpretation of the Bignon case. At breakfast I read in the *Daily Chronicle* a review of a book of Dr. Poumiès de la Siboutie, *Recollections of a Parisian under Six Sovereigns, Two Revolutions, and a Republic.*

In this he tells a story of Horace Vernet, the painter, leaving Paris for Algeria on account of a slight put upon him by Napoleon III. In some way I formed the impression that Horace Vernet chose Algeria on account of the economy of living, and I read the paragraph again and again to account for that impression, but without result.

As I was putting the newspaper away my eye fell on a heading, "Where to Live Economically", which, it would seem, I had noticed previously to reading about Horace Vernet, and of which I had forgotten the exact associations.

Another curious little case may be added, as it allows one to observe with certainty the influence of associations which have not become realised in consciousness, but which have decisively affected the conscious life. I was looking over some typewritten

manuscript, reading quickly, when my eye fell on the name Romeford instead of Rumford: I struck out the e, and felt my attention called in a somewhat puzzled way to the first syllable. Here it may be mentioned that there is a place in Essex called Romford. The error did not make itself known to me as an error, and I passed on reading rapidly. In the next page, however, I met the name Rumford correctly spelt, and instantly, before I could express the matter deliberately, I turned to the first name and struck out the "o" and inserted "u." It appears to me that had I not seen the name for the second time I should have forgotten the error overlooked in the first name.

Meditating again on all these observations I think they throw light not only on the recollection of the name Bignon, but also on many subjects—such as premonition, telepathy—not within the immediate scope of this treatise.[1]

Since writing the above some years have elapsed during which I have made a great number of studies on this subject. Space does not permit to cite anything beyond the results. The overwhelming tendency of my observations is to show that in cases like that of Bignon here cited, the name had really been seen, but had very soon afterwards been forgotten, then subsequently recollected.

An experience that can readily be reproduced by others may be thus stated.

I go towards a room to get my watch. On the way there something happens, as for instance, a conversation, of some importance. Afterwards I continue my walk to the room, but on arrival do not know at once why I came. If, however, I return, in mind, to my first point of departure, and try to revive the circumstances then existing, the recollection of the watch occurs.

I conclude this part by repeating, with greater force, the remark in reference to premonition and telepathy.

(2) *The Manner of Failure in Memory*

On a certain occasion I had, in a letter, written a phrase from Shakespeare; but, as I had not referred to the passage for years, I misquoted it: "The labour we delight in lessens pain." Immediately I recognised that the word *lessens* was wrong.

I tried to think of the right word; sometimes it seemed ready to spring out, but again it vanished, leaving a dull impression of forgetfulness.

[1] *Cf.* F. Paulhan, *Histoire d'un Souvenir, Journal de Psych.*, 1907 E. Goblot, *Un Cas d'Association Latente.*

Then I tried the effect of stimulating neighbouring paths. I read various passages of Shakespeare, and I looked over a glossary. This seemed to stimulate recollection, but without producing the word. I knew, however, that it was an unusual word, conveying a sense of diminishing pain by some drastic process. Having other work to do, I decided to send my letter with the remark that I had not got hold of the exact quotation. I then put the letter aside, and gave my attention to other studies. Suddenly I remembered the exact quotation: "The labour we delight in *physics* pain."

In examining these cases we see that the most persistent associations are those connected with the import of the term. The strength of these associations corresponds with the manner in which such a term would arise in the mind of the author; for example, the term "lessens" not only fails to form the due associations in respect to its structure as a word, but it also diminishes the peculiar force and savour of the sentence.

It frequently arises that the sense of momentum, impact, and general effect of a phrase becomes shown forth to the mind while in the fervour of composition before the actual word corresponding is attained. It may happen that there is no word in the language available which gives the meaning desired, and yet fulfils other conditions of length and quantity. It is an accident of language that the word "physics" should exist in the form and with the expressiveness demanded by this phrase.

It is this felicity of phrasing that constitutes one of the charms of poetry, and that also marks the difficulty of composition. Those who have studied not only the final forms of the great poems, but also the erasures, the tentative efforts, the notes of some of the masterpieces, will observe that occasionally the word makes default. The rhythm, the force, the character of the phrase is already formed, and, if the accident of language be not propitious, a search is sometimes made in vain for a word already boded forth.

The famous line of Keats—

A thing of beauty is a joy for ever—

was originally expressed in a different form:

A thing of beauty is a constant joy.

Here the formal metre is better maintained than in the accepted reading. All the conditions of length, meaning, and quantity had been attained, but Keats, while not knowing how to replace the word, felt that it had not fully expressed the light, salient spontaneity of his thought. The associations thus formed were the deepest, and, by virtue of the principle of association which we will have occasion to discuss later in complex and in remote forms,

the phrase at length arrived that fulfilled his desire, and we have the immortal line:

A thing of beauty is a joy for ever.

By piercing deeply down into the nature of association and of the manner in which the Memory reproduces these associations, we gain a glimpse also of principles that should explain aesthetic appreciations. The true poet, in the force of his original quality, finds the "form" dependent on and adapted to his thoughts; and even if a word in this manner be a little forced from its usual meaning, it often gains strength and freshness; but an artist, by dint of mechanical working at the "form", at the instruments of expression themselves, cannot produce associations of strength beyond that with which the words have already been endowed by others.

It will be found on examination that all poetry, especially in long poems, consists of passages of "inspirations" followed by other passages which serve to hold the sense and to link the thought to the next inspirational movement. The "connective tissue" is less striking in form than the fine passages of poetry where the words often have the air of being inevitable and unchangeable.

If, then, long passages be learnt by heart, and then allowed to fall partly from recollection, it will be found that the Memory has unconsciously analysed the poem with respect to the poetry. The essential, the characteristic expression of a passage is retained, and the accidental is forgotten.

I have been able to observe this in regard to the poems of Milton, for example, learnt by heart perfectly and subsequently not referred to for long intervals.

In *L'Allegro* I remembered the opening:

Hence, loathéd Melancholy,
Of Cerberus, and blackest Midnight born
In Stygian cave forlorn.

Then my mind was blank until the new movement:

But come, thou Goddess fair and free,
In heaven yclept Euphrosyne.

Then followed, in perfect recollection, the whole passage down to—

Come, and trip it as you go,
On the light fantastic toe.

After this for a period I found no strong sequence, but various suggestions—

Nods, and becks and wreathéd smiles,

* * * *

Laughter holding both his sides.

Examining these circumstances, I found that my Memory had preserved the really characteristic inspirational passages. It seemed to me that the introduction to the poem was an afterthought, laboriously wrought by Milton with a sense of artistic effort; as, for instance, in regard to contrast.

Then followed the felicitous lines of the veritable introduction to the *Allegro* passages. Then followed a more or less artificial search for images of Mirth and Jollity.

Then I found that my Memory preserved, even more vividly than the form of the words, the "atmosphere" of the poem, the succession of delightful visions of English scenery, the spirit of cheerful meditation which is the characteristic of the poem.

Each of these scenes was marked by a phrase which formed its most apt reference:

Right against the eastern gate
Where the great sun begins his state.

The clouds in thousand liveries dight.

While the cock with lively din
Scatters the rear of darkness thin.

And every shepherd tells his tale
Under the hawthorn in the dale.

The upland hamlets will invite.

Mountains, on whose barren breast
The lab'ring clouds do often rest.

The Cynosure of neighb'ring eyes,
Rain influence, and judge the prize.

Then to the spicy nut brown ale.

She was pinched and pulled, she said,
And he by the Friar's lantern led.

And the busy hum of men.

If Jonson's learnéd sock be on,
Or sweetest Shakespeare, Fancy's child,
Warble his native wood-notes wild.

In certain instances there was no sense of the order being preserved. But all this represents fairly well, no doubt, how such a poem would be composed. There is no sense of consecutive "Argument", there is a brief recital, in fresh and glowing colours, of various scenes that at various times had impressed the poet's mind by a sense associated with the spirit of "allegro", which at the time, no doubt, also were represented in his mind by many of the phrases afterwards woven together.

It is usually easier to remember passages of rhymed poetry than passages of prose, because the similarity of the endings forms associations which recall the corresponding line directly, and also indirectly by virtue of the previous associations with that sound. The rhythm, the manner, and general atmosphere of the verse aid, and so may also peculiarities of expression forcible or felicitous.

The Law of Regression

I tested certain operations of Memory by first composing, and learning by heart, step by step, a prose story of over thirty thousand words, none of which was at that time written. I observed meantime the manner of the composition. First of all the general intention, then the selection of the principal characters, then the selection of the principal scenes, and the general course of the story; then the elaborations of the most striking or the most characteristic of these scenes; then the gradual appearance of the words, sometimes shown forth in striking phrases, at other times more connectedly; then the searching for a beginning, then the gradual weaving together of the words, led on from point to point by the course already marked out, and by certain passages already elaborated.

At first the progress was fairly rapid, but when half the work was completed the labour of retention and composition together produced a greater sense of effort. When the story was finished I found it possible to repeat it easily in three hours, and subsequently, after frequent repetition, in less time every day.

Some months elapsed before I wrote the story, and then I did not refer to it till three years afterwards. I then found that my recollection had become very uncertain with regard to the words. Examining the matter closely, I discovered that the gradual failure of the Memory traced out in reverse order the manner in which the story was composed. That is to say, the less striking passages disappeared first. This broke the continuity of the verbiage. There remained some of the more striking phrases first discovered. The surest factors were those first determined, the intention and general course of the narrative, the principal characters, the main scenes, the most characteristic expressions. The rest was perceived as through a fog.[1]

About the same period that I undertook this work I used to repeat every day a great number of lines mainly arranged in verses, observing day by day that the repetition became easier. The

[1] L. Dugas in *La Mémoire et l'Oubli* cites this passage in support of Ribot's "law of regression." He says "One could hardly imagine a verification more satisfactory, more instructive and more lucid, of the law."

whole series included extensively mnemonic systems for remembering facts in astronomy and physiology, in which numbers were represented by letters, thus enabling me to form phrases corresponding, for example, to food-tables; a few passages from Shakespeare; verses from Milton's poems, including a passage from *L'Allegro*, a passage from *Il Penseroso*, two passages from *Comus*, a passage from *Samson Agonistes*, a passage from *Paradise Regained*, and several passages from *Paradise Lost;* also eleven sonnets.

The effort of memorising, with slow recollection and hesitations, occupied me at first the greater part of the day. Subsequently, with continual repetition, I could repeat the whole series in half an hour, that is to say, at a far greater speed than I could have spoken the words.

With practice, I found that less and less effort was required. The image that was suggested to me was that of pulling a fine tape through my brain at a speed which corresponded to the necessities of my recollection, a speed which, though great, had no unpleasant feature. At length I was able to perform this operation so as to be sure, at least, that in no part was my recollection faulty, while listening to something else. Having arrived at this facility, I ceased the experiment.

Less than five years afterwards, not having referred to this series in the meantime, I re-examined the matter and found that my recollection was blank concerning a considerable part of it. The mnemonics had apparently left but little trace except the general principle of their formation, and the dim knowledge of the facts they represented. I had forgotten the poetry in so far that I could not repeat it in a consecutive manner, but I had a clear recollection of its general intention, its inspiration and atmosphere, the more striking phrases, and, particularly with regard to *L'Allegro* and *Paradise Lost*, clear mental pictures of what the passages had evoked.

I here offer some extracts from my notes referring to these experiments when at length, having in the meantime had my mind constantly occupied with other matters, I endeavoured to recall these passages:

> Of man's first disobedience, and the fruit
> Of that forbidden tree, whose mortal taste
> Brought death into the world
>
> * * * *
>
> Sing, heavenly Muse.

Here, in addition to the parts well remembered, I had a notion of the general sense of what I had missed; that is to say, I would

at once have recognised the words as being correct if I had read them again; and, if I had seen a version in which extraneous words occurred, these would certainly have attracted my attention and would have suggested rectifications.

In this instance, therefore, it will be noticed that I had retained what seemed to me the expression of the poetical movement. The parts that had fallen out had rather the air of afterthoughts, or literary amplifications.

The general "argument" had remained in my mind, but only in broad outline, and not always in sequence. On the other hand the spirit, the "atmosphere", of the poem was quite fresh in recollection; as also the clear sense of what I thought to be particular beauties, intentions, revelations of personal feeling, or warm inspirational passages; while many of the phrases were recollected as being associated with what I had noted of a peculiar tendency to paradoxical expression.

I found myself often lamentably unable to quote, even where I knew that I could "put my finger on" a quotation required, with the precise appreciation of its meaning. The inability to quote arose from having forgotten the sequence of words employed; in some cases of fairly clear images I did not seem to remember a single word.

Short phrases came to my mind which seemed to sum up the picture of a scene:

> Fretted gold.
>
> His looks and thoughts were always downward bent.
>
> Deep on his front engraven
> Deliberation sat, and public care—

which I associated with Strafford. And "yet he pleased", of Belial. Also there appeared readily a remarkable phrase attributed to Belial, in a manner which I thought little consistent with his character: "Those thoughts that wander through eternity."

Proceeding, and noting here and there: a wondrous image of the ship, hung in the skies, of which the full passage is:

> As when far off at sea a fleet descried
> Hangs in the clouds, by equinoctial winds
> Close sailing from Bengala, or the isles
> Of Ternate and Tidore, whence merchants bring
> Their spicy drugs; they on the trading flood
> Through the wide Ethiopian to the Cape
> Ply, stemming nightly towards the pole; so seemed
> Far off the flying Fiend.

I had remembered also a fine instance of *curiosa felicitas* of expression, although I had absolutely forgotten the words which might be thought to be the very essence of that example. The words are:

> A violent cross-wind from either coast
> Blows them transverse, ten thousand leagues awry,
> Into the devious air.

Though I had forgotten the words, I had a good impression of the very effect they produced in relation to the meaning intended to be conveyed.

Then, again, for a beautiful sense of the soft impress of flowery words; then the liquid melody of luted syllables:

> The pleasant dales of Sibma clad with vines.
> From Elealé to th' Asphaltic pool.

The splendid invocation commencing, "Hail, holy Light", was fairly well retained, and especially the sense of the salient swerve and movement of the first verses.

I had great difficulty—owing to lapses of words here and there—in recalling a passage with which I had been particularly familiar, because it seemed to me to be, if not the finest poetical flight, at least the most fervent, and the sole warm and unrestrained expression of the poet's inner feeling.

> But chief Thee, Sion, and the flowery brooks
> Beneath that wash thy hallowed feet . . .
> Nightly I visit.

That was as much as I remembered, though with the knowledge that this was only a note and that it did not represent the verse; and also with the recollection that in the passage there was a peculiar movement of rapidly swirling flow of verse laden with light and flushed with bright surprise.

The verse is:

> Yet not the more
> Cease I to wander where the Muses haunt
> Clear spring, or shady grove, or sunny hill,
> Smit with the love of sacred song; but chief
> Thee, Sion, and the flowery brooks beneath
> That wash thy hallowed feet and warbling flow,
> Nightly I visit.

Throughout the poem various phrases remained, each having some striking characteristics:

> All night the dreadless angel unpursued.
> Pomp of Graces.
> For contemplation formed, and valour, he;
> For sweetness she, and soft attractive grace.

The fairest of her daughters, Eve.
So easy it seemed once found which yet unfound
Most would have thought impossible.
Ramping the lion . . . and in his paws
Dandled the kid.

Then the fresco-like descriptions:

With clang despised the ground.
Onward they came in dance,
Pretty atheists.

In all of these examples it seems to me that the impressions retained, and the passages recollected, were those most nearly corresponding to the first dawning of the poem in the poet's mind; and then to the peculiar happy finding of rare and wonderful phrases, and of inspirational movements of verse.

In the *Paradise Regained* I remembered the whole course of the argument more clearly and consecutively than in the *Paradise Lost*, and I recognised the reason, precisely in this form of high philosophic discussion, that made Milton prefer it to *Paradise Lost*.

Of *Samson Agonistes* I remembered best the general spirit of the poem, the sense of its being an autobiographical relation in a more direct manner even than the other poems. Of individual passages I recalled the remarkable description of the coming of Dalila:

I wrote without reflection:

. . . What thing of sea or land
. . . With tackle trim
. . . And in her train
. . . Comes this way steering.
. . . 'Tis Dalila, thy wife!

I was well aware of the character of the gaps, and six days later I had a clearer recollection of the words.

The whole passage is:

But who is this, what thing of sea or land?
Female of sex it seems,
That so bedecked, ornate, and gay,
Comes this way sailing
Like a stately ship
Of Tarsus, bound for the isles
Of Javan or Gadire,
With all her bravery on, and tackle trim,
Sails filled, and streamers waving,
Courted by all the winds that hold them play,
An amber scent of odorous perfume
Her harbinger, a damsel train behind;
Some rich Philistian matron she may seem;
And now, at nearer view, no other certain
Than Dalila thy wife.

L'Allegro has been already discussed.

Of *Il Penseroso* I remember the impression of its being an afterthought to make a companion poem with *L'Allegro*. I remembered it mainly for special phrases.

Lycidas left me cold in respect of its qualities as a passionate expression of friendship. The recollections that remained were those of a supreme example of essay-writing according to a high classical model; and then turns of phrases, and touches of description.

> Once more, O ye laurels.
> . . . Forced fingers rude.
>
> He knew
> Himself to sing, and build the lofty rhyme.
> To sport with Amaryllis in the shade.
> Fame is the spur that the clear spirit doth raise.
>
> * * * *
>
> To scorn delights and live laborious days.
> To-morrow to fresh woods and pastures new.

Of *Comus* I remembered the atmosphere, a certain sense of noble poetical feeling overspread by artificial manner. Here and there fine passages; the movement, for example, of inspirational passages. Thus the actual words:

> Fragrant melody
> Rose like a steam—

took the place of the following verse:

> At last a soft and solemn-breathing sound
> Rose like a steam of rich distilled perfumes,
> And stole upon the air, that even Silence
> Was took ere she was 'ware, and wish'd she might
> Deny her nature, and be never more,
> Still to be so displaced.

Yet in all these, even when the gaps seemed formidable, my recollection was sufficiently sure that, on re-reading, I recognised that I had known before all the passages that rose above the ordinary level; and at any part of any of the poems I would have noticed had an error crept in of a manner of expression inconsistent with Milton's style.

With regard to Shakespeare similar remarks apply. I will only refer here to one passage which, forming part of the series already referred to, was at one period repeated so often as to pass through the mind with extraordinary facility. Less than five years afterwards I was only able to reproduce it thus:

What you do still betters what is done,
When you speak, sweet, I would have you do it
Ever;
Laugh, laugh so, . . .
But, so buy.

When you do dance I'd wish you a wave of the sea,
That you might dance ever,

Each your doing . . . crowns,
In each particular . . .
That all your acts are queens.

The correct passage is as follows:

What you do
Still betters what is done. When you speak, sweet,
I'd have you do it ever: when you sing,
I'd have you buy and sell so; so give alms,
Pray so; and, for the ordering of your affairs,
To sing them too: when you do dance, I wish you
A wave o' the sea, that you might ever do
Nothing but that; move still, still so,
And own no other function: each your doing,
So singular in each particular
Crowns what you are doing in the present deeds
That all your acts are queens.

It will be seen, in the comparison, that the memory was clear as to the general import of the passage. It was also clear with respect to the most gracefully turned of the phrases. The impression was vivid of a light and sparkling, somewhat fantastically turned, manner of compliment; and also, with regard to the phrases lost, of more artificial and less felicitious expression.

Of Byron's *Don Juan* once read with such keen attention that the whole sequence was vivid, several passages having been learnt by heart, I have a note over fifteen years after having read the poem. The opening is recalled thus:

I want a hero, an uncommon want,
Since every day brings forth a new one,
Till, after cloying the gazettes with cant,
The age discovers that he's not the true one.
. . . my old friend, Juan.

The verse really runs:

I want a hero: an uncommon want,
When every year and month sends forth a new one,
Till, after cloying the gazettes with cant,
The age discovers he is not the true one:
Of such as these I should not care to vaunt,
I'll therefore take our ancient friend, Don Juan—
We all have seen him in the pantomime,
Sent to the devil somewhat ere his prime.

From that point the recollection is lit up only by a series of passages and pictures, but with no determined sequence.

The most lively impression is retained of the spirit of the poem, its verve, its sparkling wit, its turn of humour, its *diablerie* of enjoyment even in shocking the prejudices of the bourgeois, the sense that it is the most characteristic of Byron's poems, that in which he spoke most truly for himself, and without regard for the frowns or the applause of others.

There was also a recollection of the manner of digression, like the variations round a central theme, and also that these digressions gave the poem its charm. Then, in regard to peculiar artistic effects, there was the impression of the growing passion of Adeline for Juan, first lightly touched on; then, after each digression, more deeply shaded in, with a sense of inevitableness; yet all without insistence.

Throughout the poem there were a great number of phrases which, for one reason or another, had struck my attention.

> Though young he was a Tartar,
> And not at all disposed to prove a martyr.
>
> Then some leaped overboard with dreadful yell,
> As eager to anticipate their grave,
>
> A wary, cool old sworder.
>
> Dusk as India, and as warm.
>
> He glanced like a personified Bolero.
>
> And seemed to say, Resist us if you can,
> Which makes a dandy while it spoils a man.
>
> Proud with the proud, yet courteously proud.
>
> Excel
> The serious Angles in the eloquence
> Of pantomime . . .
> As a poplar proportion'd or a pole,
> A handsome man, that human miracle,
> Yet there was something wanting on the whole
> Which women—certes, the sweet souls—call soul.

There was an understanding that this last verse was not correctly quoted, but no suggestion as to wherein it was faulty.

With this recollection of *Don Juan* I will compare that of a passage in which the closely continuous strain of deep thought and feeling, and the finely woven character of the verse aid remembrance:

> He who hath bent him o'er the dead
> Ere the first day of death is fled,
>
> * * * *

Before decay's effacing fingers
Have swept the lines where beauty lingers,
And marked the mild angelic air
The rapture of repose that's there.

Here my recollection stops. The lacunae are precisely those of the afterthoughts of artificial developments. There, as in most analogies, the associations between the lines and their application is strained.

Accordingly, although in some respects the recollection is more perfect than of passages in *Don Juan* formerly as well known, yet the associations are less varied, and there is less distinct impression of the place of these lines in the whole perspective of the poem.

The influence of beautifully woven verse, pleasant cadences, and good rhymes is well illustrated in a recollection of Longfellow's Prelude to the *Voices of the Night* learnt by heart and tested nearly twenty years afterwards.

Pleasant it was when woods were green
And winds were soft and low,
To lie amid some sylvan scene,
Where, the long drooping boughs between,
Shadows dark and sunlight sheen
Alternate come and go.

Here follows a lacuna. But throughout the poem passages are recalled with clear consecutiveness.

A contrast to this is Walt Whitman's. Here the original thought is striking for its amplitude, its profundity, its originality; so that the poem—such as *Starting from Paumanok*—brings with it a multitude of suggestions. Yet the form is faulty, although here and there vigorous expressions surge up which may astonish by their force, or even seduce by their beauty. Consequently, after having learnt many passages of Walt Whitman by heart, I find in my recollection ten years afterwards a clear idea of his intention, of his style, of his place in literature, of his value; but I can only repeat his words in disjointed phrases:

Starting from fish-shaped Paumanok,
A woman waits for me.
Tarrying and talked to by tongues aromatic.
Have the elder races halted?
Camerados!
I listened to the learned astronomer.

Of Shelley I had at one time learnt by heart a considerable number of passages.

Amongst these was the *Ode to the Skylark.* Fifteen years afterwards I remembered most distinctly the spirit of the poem; its form, its fine seizure, its sudden, brilliant flight, its "rain of melody". I also remembered that withal I thought certain of the verses rather far-fetched, and not in the spirit of inspiration of the rest of the poem. I remembered in sequential form thus:

Hail to thee, blithe Spirit,
Bird thou never wert,
That from Heaven or near it
Pourest thy full heart
In profuse strains of unpremeditated art.

* * * *

What thou art we know not;
What is most like thee?
From rainbow clouds there flow not
Drops so bright to see
As from thy presence showers a rain of melody.

Higher still and higher
From the earth thou springest;
Like a cloud of fire
The blue deep thou wingest,
And singing still dost soar, and soaring ever singest.

Then followed a verse which I remembered because, though beautiful in itself, it struck me as being a somewhat artificial fancy, a conceit, afterwards adjusted into the form of the poem:

Like a rose embowered
In its own green leaves,
By warm winds deflowered
Till the scent it gives
Makes faint with too much sweet these heavy wingéd thieves.

Then, finally, the touch of self-consciousness:

Teach me half the gladness
That thy brain must know;
Such harmonious madness
From my lips would flow
The world would listen then as I am listening now.

Of Keats I had learned a number of passages, including the opening of *Endymion.* Many years afterwards I found my recollection of the passage down to "Pouring into us from the Heaven's brink" intact. The reason of this was, partly, that though I had not re-read the passage, I had often reverted to it in my mind.

This explanation applies also to many facts and circumstances which we seem to remember because of their importance. As a

consequence of their importance we revert to them from time to time, and so continually freshen the recollection.

Referring to Keats' celebrated sonnet, *On first looking into Chapman's 'Homer'*, I find that I had at no time learnt it by heart; but I had at a certain period read it over frequently, and I had examined it part by part, line by line, and word by word, and by comparison also of successive versions. Then for three years I did not refer to it. I found my recollection thus:

> Much had I travelled in the realms of gold,
> And many goodly states and . . . seen;
> . . . been told
> That broad-browed Homer ruled as his demesne;
> Yet had I never breathed its deep serene
> Till . . . Chapman spake out loud and bold.
> Then felt I like some watcher in the skies
> When a new planet swims into his ken;
> Or like stout Cortez when, with eagle eyes,
> He stared at the Pacific . . . all his men
> . . . wild surmise—
> Silent on a peak in Darien.

The above fragment, imperfect and faulty, should now be compared with the original:

> Much have I travelled in the realms of gold,
> And many goodly states and kingdoms seen;
> Round many western islands have I been
> Which bards in fealty to Apollo hold.
> Oft of one wide expanse had I been told
> That deep-browed Homer ruled as his demesne,
> Yet did I never breathe its pure serene
> Till I heard Chapman speak out loud and bold;
> Then felt I like some watcher of the skies
> When a new planet swims into his ken;
> Or like stout Cortez when, with eagle eyes,
> He star'd at the Pacific—and all his men
> Look'd at each other with a wild surmise—
> Silent, upon a peak in Darien.

This sonnet, one of the most wonderful poems in all literature, came to the mind of Keats, accustomed as he was to the sonnet form, in a rare moment of inspiration bodying forth not merely its general atmosphere, but its salient images, and certain of its magical expressions, in associations of rapid succession already partly formed, but waiting the luminous flash which gave them life.

But in all such poems there is much work of mere craftsmanship, much adjustment so as to secure the necessary rhymes, and to give certain due proportion and sequence to thoughts, expressed under the conditions of a certain form. And here I find that the

parts which I thought to be the essentials, the inspirational effects, were those that my memory retained with the greatest certitude.

In foreign languages I found the discrepancy most pronounced between the recollection of what is common to all languages and what is peculiar to the language itself. Thus, at one time, being then quite familiar with German, I read Goethe's *Faust* carefully, and learnt by heart some of the passages.

Fifteen years afterwards I found my recollection of the general character of the poem, the delineation of the characters, the philosophic intention, quite clear; and this recollection went so far as to assure me of the qualities of the verse itself, the expression, the full-sounding phrases, the "tonicity" of Goethe's style. I had, moreover, a clear appreciation of various attempts at translation, including Shelley's, and I could tell in what way they fell short of the original. Yet of the actual words of *Faust* my recollection was very limited.

I remembered only two passages well; these by reason of frequent mental references in the meantime:

> Die Sonne tönt nach alter Weise
> . . . Sphaeren Wettgesang
> Und ihre vorgeschrieb'ne Reise
> Vollendet Sie mit Donnergang.

Then, in quite another vein:

> Es könnte kaum ein herziger' Narrchein sein
> Er liebte nur das Wandern all zu-viel
> Und fremde Weiber, und fremdes Wein,
> Und das verfluchte Wurfelspiel.

Of Schiller's tragedy *Wallenstein's Tod*, once read carefully, I found, fifteen years afterwards, recollection of the actual words very limited, only a few short phrases such as:

> Ich habe kein' Sohn mehr,

which I remembered from the intonation of an actor of the Herzog von Meiningen's Company which I had seen in the play at Antwerp; and the expression "Pflichtvergess'ne Kerls", which contained a point of irony not usual in German literature.

Of Camoëns' epic, *The Lusiad*, read in the original with the help of good translations, my acquaintance with Portuguese being very meagre, I had, three years later, a lively recollection of some of the characteristics, and a full appreciation of the justice of Humboldt's eulogy; yet, though I had committed passages to memory, only two lines remained, one being the famous:

> Por mares nunca de antes navigados,

Of Dante's *Divina Commedia*, read under similar circumstances, I remembered absolutely nothing verbal.

Of Virgil's *Aeneid*, read with care, I remembered, twenty years afterwards, not a word. I knew that "Arma virumque cano" occurred therein, but only by association with George Bernard Shaw's *Arms and the Man*.

Of the *Ode to Melpomene* of Horace, learnt by heart, I found that twenty years afterwards the general "argument" remained, and a few of the words, in disjointed style:

> Quem tu, Melpomene,
> Semel nascentem videris
> Illum non labor isthmius
> Clarabit pugilem . . .

But these I had frequently referred to mentally in the meantime.

Of Homer's *Iliad*, read with attention, I remembered, twenty years afterwards, only two words, although whole scenes of the poem were clear in my mind.

On the other hand, certain phrases of German which I had learnt from hearing when a child have not only remained clear, but in such a manner that I can remember a great variety of details referring to the circumstances under which I heard them uttered. In these cases attention was particularly devoted to the sound of the words themselves.

That the actual words should generally be apt to slip from recollection is easily understood. The words are, as we have seen, held in Memory by complex webs of association, and they are in themselves not generally important, but only the symbols by which we refer to the important things. In the first steps of learning a foreign language, however, the words themselves are not only of great importance, but also of a certain emotional interest; and thus many new associations are formed about them.

(3) *Special Forms of Aid to Recollection*
Recollection Aided by Reasoning

Amongst the most tenacious recollections are those of reasoning, and especially that form—geometrical reasoning—in which the actual objects of the reasoning are not of émotional interest, but the style of reasoning itself is very conspicuously displayed.

Having at a certain period been engaged in studies in the higher mathematics, I had occasion to revert to *Euclid*, which I had last read seven years before. I had entirely forgotten what the first proposition of the first book of *Euclid* referred to, and I was obliged to consult the book.

It will be observed that the first book of *Euclid* is mainly devoted to relations affecting triangles, right-angles, and parallel lines; and the first proposition, which is a problem, has no essential place in the theoretical considerations here involved. The series of the theorems begins with the fourth proposition, and from this I had a clear remembrance of the course of demonstrations throughout the book. This was not derived from the recollection of the forms or words used by any particular book, but simply by what I had observed of the lines of argument on which the whole chain of reasoning was conducted.

In the discussion of Reason it will be found that the base of the recollection also in the case of geometry is Association, the Memory here being specially tenacious because the associations are formed through operations in which the natural manner of our experience involves continual repetitions of the Fundamental Processes of the Mind.

* * * *

Recollection in Association Principally with Sight

In regard to Memory involving principally the sense of sight, with sense of direction, and various emotional associations, I select the following example: I inhabited a house in rue Cambon, in Paris, for a fortnight. I made no special note with regard to the house. Nearly two years afterwards, and under somewhat similar conditions, I again inhabited the house. On this occasion I noted my impressions on entering. I had forgotten the number of the house, and also the name, although the name seemed more familiar after I had recalled it than did the number. I remembered best the broadest characteristics, even some which were difficult to define because they were composed of many factors. That is to say, I remembered the general "atmosphere" of the house—for example, that it was a quiet house, well inhabited, not far from what might be called the centreof Paris, the Opéra, that the people were honest, the prices reasonable, and that one could live there with comfort and without molestation.

I remembered, also, that there was a post office close by on the opposite side of the street; that the street was busy with regard to wheel traffic, but quiet otherwise; that nearly opposite was a dressmaking house where young girls used to sit near the window working busily but occasionally talking and laughing; that horses fell in the street frequently, and that the drivers seemed rather incompetent and cruel.

Regarding a great number of details my recollection was confused and altogether blank.

I could remember the general situation of the street with regard to other streets, but the names of some of these other streets had disappeared. With regard to some streets I remembered their names, and I also had a notion of their being near, but they seemed to float vaguely in my mind. I had forgotten the way by which I should proceed to them.

I knew that the charges were not excessive, but I had forgotten the exact prices.

I could not remember exactly how the apartment was furnished, thought I had a clear impression that it was furnished in bourgeois taste, with nothing especially interesting either good or bad.

I could not remember how the apartment was heated, nor could I remember the places of things, pictures, furniture, or the like. I knew there were some pictures, and I remembered that there was a family resemblance in this style of decoration that one finds in apartments of the kind; but I could not recollect the subject, or appearance, or situation, of any picture.

I could not remember whether there was electric light in the apartment, or what was the style of the tables and chairs; I had forgotten whether I entered the apartment by a latch-key.

I remembered that the person in charge, who may have been the proprietor, was a dapper little Frenchman, intelligent, polite, and not neglectful of his business. I had a vague impression of his appearance, but I remembered him distinctly as soon as I saw him again, and this experience brought a number of other recollections in its train.

I remembered that his wife was a medium-sized, pleasant-faced young woman, but beyond that I could get no precise indication. When I saw her the recollection did not appear so clear as in the case of her husband, but I remembered why I thought her pleasant-looking, as she was healthy, well-balanced, and inclined to smile easily.

In this whole experience it will be seen that the recollection was perfectly good in what I had regarded as the essentials; that is to say, as to whether it was advisable to live in the house; and that the details became dimmer unless some special circumstance associated them with something more or less striking that had particularly demanded attention. Thus I remembered the horses falling as an unpleasant incident, while the work-room of neat-handed and intelligent-looking work-girls struck a pleasing note.

I noted similar experiences with regard to other houses, one especially in rue Chaptal which I had inhabited for years, and to which I returned after the lapse of some years. Here, though the whole scope of recollections was larger, yet the persistence or the

disappearance of the impressions conformed to the same general principles.

I remembered clearly the impressions that may be summed up as the "atmosphere" of the place. I also remembered a vast multitude of details which were associated with some impression of pleasure, pain, surprise, gaiety, sorrow.

But I had great difficulty in recollecting, for example, the place of the piano. I remembered at length that a young concert singer, afterwards famous in the world of music, Miss Amy Castles, had sung at the piano, and that the situation of the piano too close to the wall had tended to deprive the voice of some of its fine effects. Then I remembered a number of circumstances relating to the piano.

I have other notes regarding houses where a picture has formed the most dominant feature in the impression of recollection. In these cases there have been special circumstances which have served to weave a great number of strong associations with the picture. It may be admiration or the reverse, or the curiosity of puzzling out the meaning of an obscure symbolist picture, or it may be some strong impulsion towards action or inspiration; or it may be simply continual repetition due to the picture placed in such a position that it was frequently, and for long periods, in view.

In another case I had visited a house where I had noticed a picture, a pencil-sketch by Léandre. Years afterwards, owing to a change of residence, I saw the picture again in the hall of another house in entirely unfamiliar surroundings. I had forgotten the picture, but when I saw it again I recognised that I had been acquainted with it. There were two figures in the picture—one of a fine-looking man with a majestic beard, and the other of a thin man. I found that I had entirely forgotten the thin man; on the other hand, I not only recognised the majestic man, but I remembered why the picture had been impressed on my mind, viz., by the resemblance of the majestic man to a lunatic with whom I had some acquaintance.

Immediately afterwards the recollection came to me that the picture was by an artist whose style I knew. At that moment I observed the name Léandre, and that awaked vague associations which satisfied me, though the picture was not a characteristic example of that artist's work.

* * * *

(4) *The Déjà-Vu*

Here was a case, but fortunately explained in this particular example, of a mental phenomenon frequently observed. One sees

an object, or a composite of objects; an impression is produced in the mind: "I have seen that before, but I cannot recollect where."

This impression may be created in various ways. One may really have seen the thing in question before, as I had seen the picture by Léandre. My recollection of it was so faint that I should never have remembered where I had first seen it but for the fact that I met with it the second time in the house of a friend, and I knew that he had changed his domicile. Then there was a vague recollection of the artist's name, which, even before it was recovered was stirring a circle of associations formed round it.

The accident of seeing the name made my vague impressions definite.

But if I had not had these helps to a recollection complete enough to satisfy me it appears, nevertheless, that I would have known that I had at least seen the picture before. There were associations, not only with ideas independent of the composition of the picture such as I afterwards discovered, but with the parts of the picture itself. I expected, for example, to find with the expression of the eyes, and the form of the nose, the flowing beard of the majestic personage. But there is a recognition in the mind of the fulfilling of the expectancy of the associations. It is this that gives us the feeling of certitude in our recollections; but in the ordinary cases the recollection produces associations which, step by step, we can link on with some striking objective experience, and so again, step by step, to the actual external world which at the given moment surrounds us. It is precisely this recognition which distinguishes realities from dreams.

Now when the recollection is not complete we form associations then and there with the object and the aspect of the external world of the moment, and so we are not dreaming; but we cannot link the impression of fulfilment of a certain vague expectancy of associations also with the external world.

Similar impressions of the repetition of a former experience may be produced in a case where no former experience really corresponds. This is when the actual experience has certain elements of resemblance with some past experience; so that the impression of fulfilment of expected associations may arise confusedly, while at the same time there is no chain of associations that would link the experience to the actual external world.

We have already seen that a series of associations not yet made evident may nevertheless make some impressions of their existence. Now the fact that they are not evident indicates a want of definition in certain of the associations correlated to converging

paths. Thus it may happen that experiences, not really related to other associations so as eventually to form a consecutive chain, may nevertheless produce an impression of stirring vague formerly-known associations.

A theory has been put forward, from observation, that such a condition as the vague impression of a formerly-known experience really implies a state premonitory to epilepsy.

It is possible that this condition may occasionally precede epilepsy; but from the explanation given it will be evident that it is a state that might happen with any one. It is simply a condition brought about by faulty recollection of a complex, a portion of which is remembered particularly with regard to associations between the component parts.

Now this faulty recollection may be simply due to the original object not having been observed in such a way as to make much impression—at least in all its parts; or to lapse of time or to absorption of mental energy, or attention by other matters; or to inhibition due to associations arising along wrong tracks; or to fatigue which may be temporary. None of these conditions has any special connection with epilepsy.

On the other hand, if such a condition happened without any of the causes suggested it would indicate some inefficiency in the correlated part of the central nervous system. This might be due to a variety of causes, as for instance to defective nutriment following upon failing heart-action. Or it might be due to causes which, if progressive or cumulative, might lead to epilepsy.[1]

[1] In applying the principles that we have learnt we have been led to a natural explanation of the problem of imperfect reminiscence. To those interested in psychological questions, but who are not experts, it would probably be surprising to find how much literature has grown up round this problem of the *déjà-vu*, as the French call it. Pierre Janet has devoted his powers of analysis, as well as his great physiological learning, to the elucidation of the matter; and, amongst those who have in some way or other contributed to our knowledge, in this regard, may be mentioned Leroy, Emile Laurent, Méré, Kindberg, as well as many famous writers—Shelley, Lemaître, and Paul Bourget.

Lemaître calls attention to the profound feeling of surprise and of strange alarm on recognising as *déjà-vu* a place never before entered. Sensational examples of the sort do not disturb the explanations already given. A clue will be found when we study the case of Sir Walter Scott thinking he saw Byron. The observer is not in a normal condition, and the fancy plays a great part. The emotional effect is not produced directly by the *déjà-vu*, but by the associations involved in the things of fancy also. The *déjà-vu* is here only the fuse of the charge of dynamite.

Another aspect of the problem, that of "localisation," has been discussed by Vaschide in *l'Année psychologique*, 1896, and by various others. What Vaschide here means by localisation is localisation in the whole scheme of the observer's reminiscences. He finds that localisation is not always determined by Association, but is sometimes immediate and direct. The distinction will disappear, however, if we bear in mind that, by Association, it is not intended to imply a series of associations through a chain of ideas already formed.

Now consider what is meant when one asks, Have I seen that house before? We have observed something, either seen before or not; and a series of ideas is invoked which leads step by step to the present time. If the house itself had been known before, then the reminiscences would be associated with the present reality, with the multitude of its impressions, and with the liveliness of its discriminations, all in satisfying concord; but if the house prove to be different from that associated with the first impressions, then

(5) *Special Experiments to Determine Certain Questions Memory Relating to Physical Movements*

A study of Memory relating mainly to operations having a strong physical base is afforded by the following example:

While waiting for lunch in a restaurant in Bayswater I tried to make a knot called the "Tomfool's knot", which I had learned many years before on shipboard, and which I had since occasionally practised. But at least six years had elapsed since I had made the knot. I found myself unable either to make the knot or even to recollect how it looked when made.

Then there was a recollection that it had symmetrical appearance and that in the process of forming it the cord should be "thrown" in a certain style. I tried again and again. Then I laid the cord down. Nevertheless, although I had failed I had a strong impression that the obstacles to the recognition of the proper methods were being loosened.

Subsequently I took up the cord again; I made an effort which resulted in the cord coming right on one side. I tried twice again; then, in another trial, I found that I had accomplished the knot, but that it had become undone immediately. I did not know how I had made the knot, and I was unable to repeat the performance.

I again laid the cord down. Then, after an interval, I took it up again, and after two or three tentatives I made the knot, recognising also the position of the hands, and remembering, at the same time, that formerly I used to adopt that position as the basis of my manipulation.

we recognise, with a certain shock of surprise, that we are in a new dynamic relation to the series of ideas that has dominated the mind. We wake as from a dream. Dreaming is an extreme case of immersion in ideas where the check of present reality is not available.

We have, then, three distinct conditions, which may be indicated thus: normal memory, where the present reality is in accord with the series of ideas, really reminiscences; illusions of the *déjà-vu*, where the present reality is not in accord; dreams where the present reality does not appear as a test. A railway accident (1915) which caused fracture of two ribs (9th and 10th) and which deprived me for a short time of consciousness gave me an exceptional opportunity of studying the *déjà-vu*. My note on the following day reads: On getting out of the carriage in a dazed condition, I lay down for a few moments, then rose and looked around. I noted that I seemed to be realising something of which I had had previous experience, and of which the events had happened according to that experience. I did not know whether that previous experience seemed like a real experience, or like a dream, but I know it was very clear; I did not doubt it. After an interval of a day, I cannot find anything in my memory to warrant the notion of a previous experience. It is true that as soon as an accident appeared inevitable, I had placed my feet on the seat, acting on a remembered suggestion.

On reflection, I believe that the study of this case is facilitated if we form a diagrammatic picture of the strands of communication in the brain, and imagine the gradually returning tide of normal nervous energy. And so with all these cases.

Allied problems are dealt with by various authors, as, for example, Victor Egger, *Le Souvenir dans le Rêve*; Ed. Goblot, *Le Souvenir des Rêves*; Paul Tannery, *Sur la Mémoire dans le Rêve*; Bonatelli, *Il fenomeno della ricordanza illusoria*; Kräpelin, *Ueber Erringerungs-falschungen*. An article in Mind (1887) by F. H. Bradley, *Why do we remember Forwards and not Backwards?* may well be consulted.

Now in all this work I had made no effort whatever to revive the image of the knot, or to trace out logically the consequences of moving my hands in a certain manner and manipulating the cord. I was endeavouring to recover the operations which had become obscured, and with regard to them I made no effort to visualise them or recall them to consciousness in the form of any image.

During my former frequent experiments with the knot I had formed certain paths of associations which had, in relation to the knot, acquired facility of stimulus in a certain order of succession. With lapse of time, and having also learnt other knots, and having been accustomed to use them, I had brought about that the former paths had lost their facility. The tentative efforts finally resulted in finding the right paths, and, when once the right paths were restored, the facility rapidly increased.

Thus, although recollection had become dim, yet a considerable basis for reconstruction remained, for the experiment was not like that which would be observed in tying the knot for the first time, or in inventing it. It will be observed, too, that a considerable part of the work of reconstruction was subconscious, for I had no definite image of what I required. I knew that, with each failure, I had not got the knot, for the wrong knots did not awaken any new lines of associations, as I expected, and rightly so, that the proper knot would do.

Part of the process had formerly become automatic; that is to say, referring to physical correlatives, the proper lines of association had become facile, and incongruous lines inhibited. The wrong tentatives helped to inhibit wrong paths, and succeeding efforts made the discovery of the true path easier.

But when one considers the whole mechanism implied, the distribution of the nerves in the hand, the origins of these nerves in the brachial plexus, the rearrangements of the strands of the nerves in the complex network of the brain; and when one further, on this graphic model, works out the corresponding processes, and when the unconsciousness of much of the process is considered; the conclusion seems legitimate that, as far as physical correlatives are concerned, it is not only in the brain, but in all parts of the nervous system, that the physical base of Memory receives its impress from external stimulus, and has, in consequence, its paths facilitated so as to affect repetition and recollection.[1]

[1] L. Dugas cites this experiment above as decisive for motor memory without aid of images. B. Bourdon has written on Memory for complicated mechanical processes. Henle has taken a more general survey, *Ueber das Gedachtniss in den Sinnen*. John A. Bergström (*Am. Jour. of Psych.* 1893) describes *Experiments upon Physiological Memory by means of the Interference of Associations*. Vaschide has left behind an interesting book, *Essai sur la Psychologie de la Main* (1907).

Another curious incident which came within my observation may be added. The lunatic acquaintance already mentioned, who had been a sailor in his day, saw a sou'wester hat which he greatly admired. He requested to be allowed to try it on. In doing so he stopped suddenly in surprise, as though something in the recollection puzzled him. After a little reflection he remarked, "I remember that at that time I did not wear a beard."

The action of trying on the sou'wester awoke a series, not merely of recollections, but also of operations which did not all make an impression in consciousness. The series was checked owing to the new circumstances of the beard, and this circumstance threw the mind back with a shock to the previous recollection. Then came, distinctly enough evidently to allow the criticism to be made, the recollection that nothing formerly impeded the movements as performed, which, being traced in idea to the point of its arrest, drew attention to the beard as the cause of the check. In such a case it appears certain that there was not at first a clear image of the previous conditions, but simply a series of associations connecting the sight of the sou'wester with the desire of putting it on, which then awoke the series of automatic or partly unconscious processes.

In all our movements we are continually making use of a series of associations to which we have so trained ourselves that they have become automatic. The course of our heredity through countless generations has made certain of these paths so facile at each stage of our development that they serve, in part, to determine the subsequent development. Many movements along the paths have become so entirely automatic that they cause no clear and distinct impressions in consciousness, and even when any check happens in their regular course we cannot form a clear image of the condition of the process so obstructed.

We walk without knowing how we walk. Finally, we think without knowing how we think. Even when we desire to learn new combinations of physical processes, such as in bowling a cricket-ball, in playing a violin, or dancing a waltz, a great deal of the actual co-ordinations formed—nerves and muscles—are beyond the immediate reach of our consciousness. Our method of learning is not based on an analysis of the factors employed so as afterwards to direct them to desired combinations; it is rather by the persistent expression of the will forcing the physical organism to various tentatives of which those that are suitable are retained, and by incessant practice organically registered. The physical basis is thus formed of a series which perhaps only in incidents makes itself evident in consciousness.

It is generally very difficult to trace out in the organism the successive phases of such an education. One of the most interesting examples with which I am acquainted, where it has been possible to trace out and measure the successive physical modifications due to the effort to conform to a mental impression, is contained in certain studies of Dr. Marage presented to the Académie des Sciences (January 12, 1909) by M. d'Arsonval. Dr. Marage has studied the mechanism of singing by means of photographs representing the vibrations corresponding to the notes emitted, and also by observing the conditions of certain of the muscles involved at the moment of emission of these notes.

Singers have generally noted that, in ascending or descending the scale, a curious phenomenon makes itself known by symptoms arising in the larynx. At a certain moment they feel as if they were changing the vocal chords. They say they change from chest-register, or vice versa. Dr. Marage has shown that if this change is very marked the crico-thyroid muscle which is the chief agent in making the vocal chords tense, contracts brusquely. With highly trained singers, however, the contraction of the crico-thyroid muscle is gradual, and there is no marked change from the chest-voice to head-voice.

A considerable part of the education of a singer depends, accordingly, on the education of the muscles of the larynx, although the singer may be completely unacquainted with the mechanism of the larynx, or even with the extance of such muscles.[1]

Nevertheless, the correlatives of Memory become established in these muscles and their nerves, and following impulses derived from certain associations, the whole series is reproduced, while the result only as evidenced by the succession of notes is presented in consciousness.

Effects of Repetition

In most of the preceding observations a certain emotional factor enters, and this would have a different influence with different persons.

I have, however, a series of notes exactly recorded of various experiments which I made at different times with regard to Memory of subjects presenting no emotional features whatever. Certain of these experiments extended over years. That is to say, I

[1] Recently several men of science have turned their attention to the interesting question of the mechanism of the throat as a musical instrument: Dr. W. A. Aiken has published *The Voice, an Introduction to Practical Phonology*, in which he joins issue with Helmholtz in regard to the conditions of production of certain vowels; and Drs. E. J. Moure and A. Bouyer have written on *The Abuse of the Singing and Speaking Voice.*

experimented at one period, and then returned to the same series three or four years afterwards, and pursued a similar course of experiment, and then compared results.

The objects I chose to be memorised were the formulæ of integration as shown in books on the Integral Calculus. I had already been familiar with them, but, after having devoted much time to other studies, I discovered that I had forgotten them almost entirely.

The system I pursued was to commit to Memory a certain number of formulæ—ten on the first day. I repeated these on the following day, and learned ten more. I then repeated the first ten, and marked the errors of recollection. I counted these errors then learnt ten more; then, on the following day, repeated from the beginning. My notes say that, having learnt the first series of ten so that I could repeat them without mistake, I found that, on learning ten more, and then on the following day repeating the first ten, I failed to remember certain points in some of them; for example, whether a plus or minus sign was found in a certain place, whether $x-a$ or $a-x$ occurred in another place.

I then memorised these points, to which my attention had been thus specially drawn, by associating them with some circumstances of the integral. I then learnt ten more.

I discovered that, on the fourth day or fifth day, I sometimes failed to remember what had been perfectly familiar during the other days, and I observed that the faults of Memory generally occurred in those points to which my attention had not been specially called, and where I had not created associations by some direct attention. They were really the points in which the recollection by simple repetition had seemed to be the most facile. I also found that as I learnt still new formulæ the task of remembering the previous ones became more difficult, faults in Memory being found where previously the recollection had been quite sure.

All this is in accord with the observation regarding Memory, that recollection is more tenacious with regard to things associated with the more fundamental, and therefore more frequently exercised, processes of the mind.

The results indicate, too, that a great distinction must be made between two forms of mnemonic effort.

Mere Repetition Produces Small Permanent Effect

Compare, for example, the memorising of the name of a person well-known, but not having been present in the mind for a long time, and the memorising of words or formulæ having no special interest, but which have quite recently been committed to Memory

for a special purpose. If at the beginning we are unable to recollect the first name, it will at length by means of the revival of various associations come back to us. On the other hand, the recollection of indifferent things designedly committed to Memory, may be quite fresh for a day or two, but it soon fades away, and then cannot usually be restored.

If now we refer to the graphic representation of physical structure correlative to mental operations (*cf.* Pt. II, Chap. IX, p. 160) it would be found in accordance with results if we regarded the actual elementary work of Memory, of adhesiveness of impressions, as a constitutional faculty of each person, and that its value depends on a series of constitutional conditions.

Repetition, especially when the constitution is young and the organism developing, is a form of physical exercise of the brain which is finally represented in brain development and structure. Hence the Memory of the things which have been familiar to us, or the operations which have been habitual with us, in childhood produce a condition which is veritably organic.

But the Memory of things of recent date is correlated to excitation of nerve filaments which have lost the power of development, and to some extent of responsiveness to excitation. When the temporary excitation disappears, the Memory of the thing diminishes.

An application of these remarks may be made to what is called "cramming" for examinations. The Memory for things learnt in this way, without deep associations, is merely temporary, and in fact—still speaking in terms of the physical correlatives—it may be found that the over-excitation of a part of the brain may blur the connected paths between a series of cells affected, and so diminish the total Memory. This is a form of degeneration which is the inverse of that form of development which results in the organic associations of early memories.

An extreme case of this would be the obliteration of brain tracts, and their physical correlatives, by some form of brain disease or brain degeneration.

Similar results may happen even from diseases which are not originally brain diseases, such as in a form of thickening of the arteries, which deprives the parts affected of their efficient nourishment by the blood. Again, some forms of heart-weakness attract the attention of the physician by reason of the patient complaining of failing Memory. In this case the direct effect of malnutrition of the nerve substance of the brain is made evident.

We may now return to consider further observations in the experiments already mentioned. I found by renewed experiments

that repetition without distinct attention is less efficacious in fixing a formula than the special act of attention brought about by the correction of an error, and further that a formula repeated twenty times without an error may on the next occasion become vague or uncertain in some particulars.

If the formulæ be always repeated in a certain order they are easier to remember in that order than in any other order. If they are repeated about the same hour every day they are easier to remember then than at any other hour. Formulæ learnt at night, even when the brain is in good working order, are more difficult to recall next morning than formulæ learnt during the previous morning.

Effect of Lapse of Time

These experiments I resumed three years afterwards, not having in the meantime specially referred to these formulæ, though continuing to be occupied with various studies in mathematics. I found that of the first thirty formulæ I only remembered six distinctly, but also two others with some uncertainty. I had a faint recollection of some of the others, while some seemed entirely blank. I looked over them once.

A week later I tried the same series. I found that I remembered the first eight quite distinctly. As to the others, I retained only impressions of relative complexity of the formulæ.

I then studied the first twenty-five formulæ for twenty-five minutes. I found that though my recollection had been apparently blank with regard to some of the formulæ, yet that in relearning I had the sense of some of the formulæ being familiar. I recollected observations founded on their form, but which had not been strong enough to reproduce the whole formulæ.

I found, on trial, that altering the order of the formulæ constituted a difficulty. I continued the study for ten more minutes, thus making it thirty-five minutes in all; then tested recollection. I found the first error in No. 13. There were errors of particulars in four others; while in four cases I failed to remember the formulæ even badly.

Then after a short interval I studied the formulæ for seventeen minutes, and then tested the recollection. I found that the errors were fewer. The errors were not quite the same as before, nor in the same formulæ. Two of the previous blanks still remained blank.

Again after an interval of a week I resumed the study. I found that I remembered the first ten perfectly. There were slight errors in all the others, except in the case of two which remained blank.

One of them was a formula which had been blank on a previous occasion; but on the trial immediately preceding these now in question, it was perfectly remembered. The other blank had been blank in every trial.

I then studied the formulæ for thirty-five minutes, and again tested recollection. I found errors in one point in each of six formulæ out of twenty-five, while the formula which had always been blank remained so.

A week later I tested my recollection. I found Memory weak in the formulæ in which it had previously been weak, although the errors were not the same. The formula which had hitherto been always blank in recollection was not perfectly remembered.

I then tried, out of its turn, one which had hitherto been perfectly remembered, and found that my recollection was very defective.

This showed that distinct associations had been formed between the numbers representing the order and the result of the integration, this association being stronger than that of the form under the sign of the integration.

At the period during which these last experiments were made, I was engaged in other studies, and had a considerable amount of extraneous work on hand.

Three years previously, that is to say immediately after the study of the series first recorded of formulæ of the Integral Calculus, I had undertaken a much more fatiguing course. I committed to Memory the formulæ for elliptic functions which are amongst the most complicated, and at first sight most unattractive-looking, in the range of mathematics. At that period I was but little acquainted with the theory of the subject, and that made the learning more difficult, as I had fewer points for association. I selected this series in fact on account of its arduous nature and because it was, under the circumstances, an experiment in memorising a number of expressions conveying no meaning.

I had more leisure than during the experiments last mentioned, and as I had been employing myself with mathematical formulæ for some time previously, it would appear from the results that both these circumstances aided me in the experiment.

My plan was to learn seven formulæ, then repeat them; then to learn seven more; then repeat all from the beginning. I found that as I proceeded it was easier for me to add new formulæ, and to retain the recollection for short periods; but that as I repeated from the beginning the new formulæ rendered it more difficult to retain the formulæ already learnt.

When, however, I made an error, and then corrected it by observing some significant fact about the formulæ, the recollection

of these corrected formulæ became firmer than before. In three days I could repeat eighty formulæ of elliptic functions without error. At the same time my recollection was made firmer, by equivalent practice, in the case of the formulæ of integration previously learnt.

Adding a few formulæ day by day I had in three weeks learnt one hundred and sixty-seven formulæ which I repeated with but three errors. Immediately afterwards I repeated sixty-five formulæ of integration without error of any sort.

I found confirmed that even after many repetitions the weakest points were those which, even if at first learnt with great facility, were not associated in the mind with some facts of symmetry, or forms of remarkable sequence, or with some special observation.

I observed that the repetition was easier if it took place at a certain hour every day than if attempted at another time of day.

I also found that there becomes established a certain natural pace at which the repetition is the easiest, and that if that pace be forced blunders are apt to ensue.

The fatigue of recollection and repetition diminished with successive repetitions. It will be noticed also that though the absorption of energy and of emotional interest in one field of study will generally diminish the recollection of the facts of previous study, yet when the mind is greatly occupied with one field of study the learning of an extensive series of facts seems up to a certain point to be favourable to the recollection of another series of a somewhat similar character.

About three years later, having occupied myself with other studies during the interval, I returned to the examination of the formulæ of elliptic functions. I had acquired some knowledge of the theory, so that my mind was more receptive with regard to the formulæ. I found, however, that I had no recollection of the series that I had under other circumstances perfectly memorised. I recollected only the impression of the complexity and of the length of the formulæ.

After studying the formulæ for some time recollection began to appear, and now the recognition of symmetries, and sequences, seemed to be keener than previously.

Various Particular Associations in Aid of Memory

I have notes of another series of experiments where the task was to learn and remember the doses of drugs as given in a Pharmacopœia.

In this case there is found a slight stimulus in the fact that the series has a more evident meaning than in the case of the formulæ

of elliptic functions. Moreover, the actual separate facts to be committed to memory in regard to each drug were simply the lower and upper limit, generally in minims or grains. There were also aids to the fact that on each page the same dose would be found to apply to several drugs. Moreover, certain drugs, such as arsenious acid, would be expected to have smaller doses than others, such as acetic acid.

The task, therefore, was much less difficult than that of memorising and repeating without fault the intricate formulæ of elliptic functions, even though the total number of drugs was about four hundred.

I had learnt several pages of the drugs in succession until I could repeat them without error. I then tested my recollection in this way. I selected three pages, which for reference we may call the series "a" containing 16 items; "b", containing 11 items; "c" containing 20 items. Of these I had revised the list in "a" three hours previously, that in "b" one day previously, that in "c" two days previously. I have a note that the test took place at night when I was tired after a hard day's work. The result gave no error in 16 of series "a"; 1 error in 11 of series "b"; 11 errors in 20 of series "c".

Next morning, while fresh, I again tested the series which had been looked at, in order to verify the results, on the previous night. I found 1 error in 16 of series "a"; no error in 11 of series "b"; 10 errors in 20 of series "c".

I have noted, however, that in regard to series "c", although I did not seem ostensibly to have made much progress, yet there was a clearer notion based on relative quantities, although I was "hazy" about exact numbers. I noticed also a tendency to associate the dose with the position on the page, and the sequence of the drugs.

This remark indicates how in storing up in Memory associations apparently the most simple, and indifferent to emotional interest, we yet find by appropriate tests that associations are specially formed with regard to every circumstance of which we take cognisance in regard to the object.

I then tested three other series, which may be called: "d", containing 7 items; "e", containing 21 items; "f", containing 23 items; and which I had not referred to for three days, four days, and ten weeks respectively. In these the errors were: 7 in 7 for "d", 21 in 21 for "e"; 22 in 23 for "f".

But even here the previous work had not been entirely lost, for there was a recollection of relative quantities of different drugs, and also of the relative magnitude of the lower and upper limits.

That the previous study of the series had not been in vain became shown also by a test which I instituted between this series and a new series of a similar kind, which we may call " g ", containing 23 items.

The first operation began on the night of the last day mentioned when I was again tired. I tested series " f " and found 6 errors in 23. I then memorised the series for two minutes, and found 1 error in 23.

Then I took series " g ", and learnt the list for six minutes. I found 3 errors in 23. I studied the series " g " for another two minutes, and found no error in 23.

The following morning, when fresh, I tested the series " g ", and found $3\frac{1}{2}$ errors in 23; then, after one minute's revision, 1 error in 23. I then tested the series " f " and found $3\frac{1}{2}$ errors in 23; then after one minute's revision, no error in 23.

On the following day I again tested the series " f " and " g ". I found $\frac{1}{2}$ error in 23 for series " f "; and 3 errors in 23 for series " g ". I studied series " g " for one minute, and found no error in 23.

I then took another series, " h ", which I had not referred to for ten weeks. I found only vague recollections of relative strength, no precise number.

After studying for five minutes, I found 2 errors in 16; each only in one of the limits. I studied the series for one minute, and found 1 error, in one limit, in 16.

Every day I instituted some test of this nature, and I continued the experiments over three weeks.

My notes say that the influence of fatigue in diminishing recollection may be stated thus:

If we have two series, " a " and " b ", and the facility and correctness of repetition seem equal at a given time, but if that be because the series " b ", though committed to Memory with fewer repetitions, has been more recently learnt than the series " a "; then fatigue will have a greater effect in causing errors in series " b " than in series " a ".

I at length arrived at a point when I could repeat the doses of all the drugs without error. I then ceased to refer to them for eleven days. Then I tested my recollection again, having in the meantime increased the number of items in the series. I found in series " a " 1 error, at one limit, only, in 23; in series " b ", 6 errors in 20; in series " c ", $4\frac{1}{2}$ errors in 24.

In all the rate of recollection had diminished remarkably, and there was a sense of insecurity even in some cases where the right numbers were given. I found also that my recollection was most

certain in cases where I had especially noted corrections previously on account of errors. Also the recollection was perfect in a few cases where I had invented mnemonic aids.

I resumed these trials three years afterwards, beginning under conditions similar to those of the first test, viz., at night when tired.

I found 23 errors in 23 in series "a", and 23 errors in 23 in series "b".

I then studied series "a" for two minutes, and then found 10 errors in 23. I then studied series "b" for six minutes, and found 2½ errors in 23.

On the following morning, while fresh, I found 18 errors in 23 in series "a", and 12 errors in 23 in series "b". After two minutes' revision I found three errors in 23 in series "a". Then after six minutes' revision I found no error in 23 in series "b". I also observed in both cases that there was a clearer appreciation of values; that drugs having like doses were better associated; and that there was a more marked attention in regard to those doses which were exceptional; that many circumstances of similitude, which I had not noticed on the previous night, now became manifest.[1]

SUMMARY OF CONCLUSIONS WITH REGARD TO MEMORY

It may be well to exhibit in summary statements of all the observations which we have now gained respecting Memory.

Memory is not a special and separated faculty. Every idea presented in consciousness is reproduced by the Memory that appertains to itself.

When the conditions are established, Memory arises spontaneously or automatically.[2]

Every Unit, or every element of a complex, has its own Memory, and its own associations, some of which may be stronger than the associations formed by the element with the other elements of the complex.

Thus there is no such thing absolutely as good Memory or bad Memory; just as there is no such thing absolutely as muscular strength, considered apart from the strength of each muscle in turn. A man may have almost all his muscles well

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[1] Various writers have treated of analogous subjects. *Cf.* F. W. Edridge-Green, *Memory and its Cultivation*; Ed. Pick, *Memory and its Doctors*. Guicciardi and Cionini have written on the *Effect of Practice in Memory* in the *Riv. Sperim. di Fren*, and Fouillée on *La Survivance et la Sélection des Idées dans la Mémoire* in the *Revue des Deux Mondes*. At an earlier period H. Höffding has studied the effect of repetition, *Die psychologische Bedeutung der Wiederholung, Viertelj. f.w. Philosophie*, 1883.

[2] This observation, combined with a later discussion on Reason, will convince the student that the difference between Reason and Instinct does not depend on the nature of processes, but simply in the different character of form and quality of the Immediate Presentation, and on the restricted range of ideas in cases of Instinct. (*Cf.* note p. 46).

Here in a corollary we derive an application of great value, this being one instance of several. (*Cf.* Etienne Rabaud: *Document pour l'étude des rapports de l'intelligence et l'instinct, l' Année psychologique*, 1920. His conclusions derived from another series of arguments are in accord with these here given.

developed, and yet have some particular muscle weak; or a man may have most of his muscles comparatively weak, but have certain muscles strong. Similarly with Memory, one may have good Memory for certain ideas, but not for others.

Memory may be well considered in regard to its physical correlatives, the nervous structure of the brain.

Memory depends on the quality, the general good health, the nutrition, of this physical base. Memory becomes weakened with respect to certain ideas when the blood supply of the corresponding physical base becomes insufficient.

Generally speaking, any sources of degeneration of nerve substances tend to impair the Memory.

The pure plastic effort of retention can be "cultivated" only by developing the quality of the physical base.

Memory can be "cultivated" by various arts which form associations between the idea memorised and some series artificially well established or frequently met with in experience. Mnemonic aids are often useful in this respect.

Memory with regard to any series may be stimulated by repeating the recollection of a series whose physical correlatives are adjacent to the first.

Memory for any set of facts, or series of objects, is increased by moderate exercise of Memory with regard to facts, or objects, of a like character.

Mnemonic aids are most useful when they form direct and strong connection with frequently recurring and well-known series.

The best system in general is to classify well the objects to be remembered, to make associations corresponding to frequently repeated operations of the mind, such as in reasoning; to increase the lines of associations in all directions, in the manner expressed by "thoroughly understanding" a subject.

Memory depends on the emotional disturbances at the time of receiving the impression. Pleasant sentiments, surprise, or, though generally with less force, the opposite of these, aid the Memory.

Repetition is a powerful agent for securing remembrance. Repetition is especially valuable where matters of indifferent interest must be remembered for comparatively short periods of time.

The more recent the original impression, or the more recent the repetition of the exercise of recollection, the more facile and sure is the Memory.

If two series are held in Memory so as to be reproduced at a given time with equal facility; and if this is due in one series to many associations of old date, and in the other to constant repetition some little time previously; then after the lapse of a comparatively short interval, recollection of the second series may have disappeared, while that of the first remains but little altered.

Memory may be greatly diminished by fatigue, even to temporary extinction.

Memory may be entirely obliterated by some physical condition, such as brain disease; or even, at length, by disuse.

Memory may be greatly diminished with regard to a certain series of ideas, if, even without fatigue, the mind be much occupied with objects having no associations with the first.

Memory may be diminished by any condition contrary to certain favourable physical conditions, such as adequate but not excessive stimulation of the area physically correlated, and a general physical equilibrium not productive of incongruous associations.

Memory may be temporarily impeded through fatigue or unfavourable associations, and in that case the effort of forcing the Memory may have an effect contrary to that intended. It is then generally better to seek to restore recollection through other associations and without much effort.

The rapidity of recollection is generally greater in cases where the recollection is more sure.

Memory is aided by producing external conditions, as of time and place, similar to those that prevailed on the occasion of the original impressions.

The manner in which Memory gradually fails generally retraces in inverse order the acquisition of the knowledge of a subject.

The physical substratum correlated to Memory is, in part, composed of nervous structures whose activities are not directly represented in consciousness.[1]

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[1] One of the most interesting speculations of a profound thinker on Memory is that of Hering, who believes that Memory may reside in every cell of the body in a manner suggested by the analogy of an algebraical curve being contained implicitly in each of its elements. Hering seems to me to push this theory a little too far in the direction of definiteness; and also, while saying true things, to throw the importance of Memory out of the perspective: "So here we see that it is Memory to which we are beholden for almost all we are and all we know." I would refer the reader to Hering's own work, *Ueber das Gedachtniss als eine allgemeine Funktion der organischen Materie*, which will be found in Ostwald's *Klassiker der exakten Wissenschaften*. *Cf.* also Max Verworn, *Die Zelle als physiologische Grundlage des Gedachtniss, Zeitsch. f. allgem. Embriol.*, 1906.

In the preceding account of Memory I have relied as far as possible on my own observations and experiments, for beyond that lay a perilous sea of literature. It was from Ebbinghaus that I received the suggestions that led to some of the foregoing experiments. His book, *Ueber das Gedachtniss*, should be consulted (it has been translated); also that of Max Offner, *Das Gedachtniss*, the work of Ribot, *Les Maladies de la Mémoire*; of Paul Sollier, *Le Problème de la Mémoire*, based on the considerations of physiology and of pathological conditions. L. Dugas, *La Mémoire et l'Oubli* has already been alluded to. Bergson, *Matière et Mémoire*; which already contains the germs of his *Evolution Créatrice*; Adolf Lasson, *Das Gedachtniss* (1894); Ferri, *Le Malattie della Memora*; Ch. Richet, *La Mémoire élémentaire*; Biervliet, *La Mémoire*; an article by I. Madison Bentley in the *Am. J. of Psych.*, 1889, which refers extensively to previous treatises; and various works of Binet and Féré, such as Binet's *Les grands calculateurs et joueurs d'échecs*, which contains many useful observations; H. Piéron, *L'Evolution de la Mémoire*; and various passages from Taine, Hoffding, Meumann, and Maury.

With regard to special studies there should be mentioned: V. Henri, *La Fatigue intellectuelle*; W. Elder, *The Physical Basis of Memory*; various references in the works of Herbert Spencer, Maudsley, Wundt; Bourdon on the Influence of Age in Intermediate Memory; G. Muller, G. Schumann, Bigham, Kirkpatrick, and Smith on Retentivity; Beaunier, in a wider scope; W. Lewy on Memory in regard to tactile impressions, and to lengths of lines; Bernardini and Ferrari on Memory in regard to Music; Wolf in regard to Sounds; Lehmann in regard to Odours; and Arréat on Memory and Imagination.

The journals devoted to Psychology have also contained from time to time remarkable articles on special studies of Memory, as for example: *Un cas d'association latente*, by E. Goblot, *Revue philosophique*, 1909; *Histoire d'un Souvenir*, by F. Paulhan, *Journal de Psychologie*, 1904. Henri Pièron, *Recherches expérimentales sur les phénomènes de Mémoire, l'Année psychologique*; T. W. Harris, *On the Associative Power of Odours, Amer. Jour. of Psych.*, 1908; E. B. Titchener, *Affective Memory, Phil. Rev.*, 1895. Mentz and Kiesow have carried out various experiments in recollection somewhat similar in principle to those here offered. Of older writers, Aristotle and Roger Bacon have written interestingly on Memory.

These few indications will serve to point the way to the literature of the subject.

CHAPTER III

THE SENSE OF EFFORT

NOTE :—We have now reached a place where, after many tentative studies, we may consider to the best advantage the question of Sense of Effort.

In an exposition of such a subject as Psychology, so fundamental, and yet so intervolved with the whole web of experience, it is not possible to select each element, to define it separately, and to exhibit thence its combinations with others. And so it happens that although the term, Sense of Effort, has been so often used in the foregoing exposition, it is in regard to that exposition that we now proceed to consider the reasons that assure its existence.

I begin with references to the views of great authorities, especially of physiologists, for though I think little is to be gained directly by weighing authority against authority, the discussion will bring out several points. In the first place it is interesting to study the methods by which this problem has been attacked. Then it will be seen that great physiologists have often contented themselves with summary reasoning in the psychological domain, and therefore that, since they often contradict each other, the arguments of all must be submitted to some tests. Then even by virtue of the divergence of opinion the problem will be shown as of subtle character. Then in the next section a clearer view of pyschology will be demanded, and it will appear that so far from the data of physiology being sufficient for dealing with questions of psychology, a grasp of psychology is necessary to the interpretation of certain observations in physiology.

(1) *The Point of View of Physiology*

It should be observed that every outgoing impulse is not a motor impulse. In the simplest form of reflex we have an afferent or sensory nerve, a central connection, and an efferent or motor nerve; but in the complex structure of the brain a great proportion of the nervous lines of communication subserve the purpose of Association between ideas.

A new question now arises. Has the mind consciousness of these outgoing innervations, or of motor innervations of any kind?

The present exposition has been based on the supposition which in this chapter is justified, that there is a distinct Sense of Effort accompanying forms of nervous activity both afferent and efferent. It is not meant that every impulse along a nerve makes itself known. Even with regard to sensory nerves whose activities are well known to result in conscious impressions, it is not held that

every nervous impulse produces sensation, nor that we can distinguish by introspection the strands of nerves that have been affected. Impulses that do not produce sensation may nevertheless modify the phenomena of consciousness. An impulse too weak to produce conscious effects may, by continued repetition, at length produce such an effect.[1]

On the psychological side we have considered certain aspects of the matter in reference to Discrimination.

But while the position is well established with regard to sensory nerves, it is a matter of dispute in regard to efferent nerves. It would be impossible in the space of this work to deal *seriatim* with opinions on this subject coming even from high authorities. The very names of experts and their works would form an extensive bibliography; only a few salient observations here and there will be referred to.

Bain in *The Senses and the Intellect* says: "As the nerves supplied to muscles are principally motor nerves by which the muscular movements are stimulated from the brain and nerve centres, our safest assumption is that the sensibility accompanying muscular movements coincides with the outgoing system of nervous energy and does not, as in the case of pure sensation, result from influence passing inwards by ingoing or sensory nerves."

Ferrier enters objection to this hypothesis that it would make us conscious of the molecular processes of our brains. "If true," he says, "we might have been able to evolve from our consciousness a knowledge of the motor centres which have only recently been discovered after much experimental and clinical research."

It is well to bring forward this argument, for it indicates how misleading may be the views of an histologist or a clinician who gives interpretation in Psychology to the facts of physiology.

Ferrier's arguments could be applied to sensory nerves, and he might just as well argue that the stimulation of the nerves of hearing could not result in consciousness, for then we could evolve from our consciousness the localisation of the auditory centre.

William James is also opposed to the position as stated by Bain. He calls it a "pure encumbrance". This is a question-begging term; it simply means that it is a "pure encumbrance" to the psychology which he has built up. One is here again reminded of Sedgwick's argument that a certain boulder could not have been found in the place stated, for otherwise his big work would be based on error.

William James proposes to himself the question as to how the

[1] It was considerations of that kind that led Fechner to the study of the "threshold" of sensation, and Wundt to experiments in the same province to determine the "Reiz-höhe," or excitation-height,

notion arises of the feeling of voluntary movement. "I unhesitatingly answer: the aggregate of afferent feeling coming primarily from the contraction of the muscles, the stretching of the tendons, ligaments and skin, and the rubbing and pressing of joints; and secondarily, from the eye, the ear, the skin, nose, or palate, any or all of which may be indirectly affected by the movement as it takes place in another part of the body. The only idea of movement which we can possess is composed of images of these different afferent effects. By these differences are movements mentally distinguished from each other, and these differences are sufficient for all the discrimination we can possibly need to make when we intend one movement rather than another."

This quotation exhibits in short compass certain typical vices of argumentation. It really leaves the main problem untouched. No one has disputed the operation of the afferent nerves. But the question is as to whether that is sufficient to account for all we know of the movements of limbs. William James by the use of phrases such as "which we can possess", or "we can possibly need", is simply employing forensic arts of emphasis. And we are not advanced when he asserts: "I unhesitatingly answer". What we desire to know is not so much whether any particular exponent holds his opinion strongly, but simply what in Nature is the truth of this question.

Now Muensterberg cites Erb, the great neurologist, for cases of ataxia without anæsthesia, and anæsthesia without ataxia. I would not care to affirm that these instances alone are decisive against William James, for one can never be sure that the anæsthesia is complete in regard to all kinds of sensation; but such cases ought at least to prevent a philosopher from unhesitatingly answering in a contrary sense.

Weber cites from various observers, great surgeons and others, a number of experiments on the successive loss of impressions under anæsthetics, and since his time the question has been frequently studied. It would appear that more exact descriptions are required than that of the term "anæsthesia." And at each stage of the successive loss of particular sensations it would be interesting to study the effect on movements.

Weber, long before William James, had not only declared for the consciousness of efferent innervations, but he had given good reasons for it: "It is very interesting for the theory of the Sense of Touch and *Gemein-gefühl* (general sense, or general somatic feeling) that in addition to the organs of touch we possess a second class of organs, which afford us an impression of the magnitude of the pressure or the pull exercised upon our body; these are the

voluntary muscles with their nerves and the central nervous system. From the feeling of effort of these, when we lift a weight, and from the feeling of tension of these, when a weight drags upon relaxed limbs and so stretches the muscles, we form conclusions as to the magnitude of the weight working upon us or of the magnitude of the resistance which we must overcome in moving our limbs."

Weber's experiments show that by the feeling of muscular exertion which he has indicated, differences of weights may be detected when they are in the proportion of 39 to 40. The fineness of discrimination of the sense of touch on pressure was less, the most favourable showing perception of differences when the proportion was 29 to 30.

I do not wish to enforce Weber's authority. Further, I do not think that his experiments are conclusive in this matter, for those who deny the feeling of efferent innervation might say that the sense of touch or pressure should be supplemented by that from the joints, and so forth.[1]

Even in cases when a great display of neurological learning is shown it should be carefully investigated in how far this learning, even if all valid, really bears on the precise point. It will generally be found to have but a remote relation. An authority, say, has based his position on his own introspection, or has seized for some reason upon an opinion; he then discourses elaborately upon the topography of the brain; finally, he reasserts with conviction his previous opinion. But if the argument be examined closely we shall find that in the statement of the physiological results there is a psychological gloss not at all warranted by the actual objective facts revealed in the study of the brain. To take a gross example: Hitzig, who sought to locate the Will, never saw in the finest microscopical specimen any trace of the Will residing there, nor did Descartes before him ever behold the soul sitting on the pineal gland.

Ferrier is opposed to the theory of motor impressions. "In favour of the existence of a consciousness of effort, apart from afferent impressions conditioned by muscular contraction, various considerations have been offered, some of which are of little weight."

He gives in support of his own views arguments which, as far as I can judge, should rather be cited for the opposite theory:

[1] A. Goldscheider, whose work in this region is admirable, demonstrated sensory nerves in the joints. Huber and De Witt found sensory nerves in tendons and muscles (*A Contribution on Nerve Tendons in Neurotendinous Organs, Journ. of Comp. Neur.*, vol. xii.). Golgi had already, in 1890, written: *Sur nervi nei tendini dell' Uomo* (Mem. della R. Acad. dell. Sc. de Torno). Dr. Henry Head showed in 1908 (Brain, vol. 31) that the deep system of nerve fibres, supplying not only the muscles and joints but connective tissue, was sensory.

"The patient suffering from central motor lesion, after making futile attempts to carry out his ideally realised movements, not uncommonly bursts into tears at his failure. There is no defect in idealism, but only in the realisation of the movement."

In this instance the lower motor tracts are intact. What fails in the case is the passage to these lower tracts. Now judging from ordinary sensation, though the original stimulus from without is required, yet the activity of the brain suffices for the reproduction in the form of idea. It would seem to be sufficient also for the appreciation of a muscular movement which had been formerly realised. It is true that Ferrier says there is a central motor lesion, but he has left enough of the brain active to subserve his own theory, and therefore it is not asking anything additional that it should be sufficient also for the contrary theory. The limb is not moved, therefore the elements from which Ferrier could build up the feeling are not available, yet the Sense of Effort is there; for otherwise what is the meaning of "futile attempt"?

This Sense of Effort is doubtless not quite comparable to that of the Sense of Effort in the case of a normally executed movement; but neither is the voluntary reproduction of a visual sensation quite comparable to the sensation itself. I would add Ferrier's instances, though not Ferrier's interpretations, to the support of the sense of motor innervation.

Bastian says that the so-called motor centres are in reality sensory. Muensterberg agrees with this. Bastian refers to the "unfelt impressions which guide the motor activity of the brain by automatically bringing it into relation with the different degrees of contraction of all muscles that may be in a state of action."

Ferrier objects that movements are conducted even with absence of the cerebral hemispheres.

To this we may reply that certain movements of limited complexity are only so conducted. But in the whole course of this exposition we have noted the tendency of frequently repeated acts in association to become automatic, and we have seen that in the exercise of the Fundamental Processes when the conditions are established the mental act is so far automatic that it is inevitable. The more complex the condition of reasoning, the more extensively the higher regions of the brain become involved. And the hemispheres of the brain are necessary also to the carrying out of elaborate processes even of movements adapted to complex external conditions.

The consideration of such problems has given rise to the question whether muscles have sensibility, and even on this point there has been a wide divergence of opinion. Weber thought they had,

but still held, as we have seen, to the belief in the feeling of efferent innervation. Ferrier agrees with Weber in regard to the sensory nerves and muscles, and on this basis sees no necessity for motor-feeling. Duchenne on the purely physiological side finds sensory nerves in muscles. Wundt questions as to these sensory nerves, but finds an agreement with Weber on the second position. In his *Menschen und Thier-Seele* he says: "The strength of the sensation is dependent only on the strength of the mental impulse passing onwards through the centre, which acts on the innervation of the motor centres." Sherrington, and subsequently Rivers and Head have definitely settled the question by proving that the muscles are served by sensory fibres.

From a comparison of these deductions from physiology, which I have noted out of a great number equally incongruous, it is evident that in many instances physiological psychology is, as has been especially said in the case of Wundt, simply Physiology plus Psychology.

William James uses the argument that when the external rectus of one eye is moved the internal rectus of the other eye is moved correspondingly, and we are unable to discriminate the impressions. Certainly when the images derived from both eyes coincide we do not seem able to discriminate, but neither can we discriminate when tactual impressions of two fingers coincide. When, as in the case of squint, the two images do not coincide, then it is possible to discriminate with respect to the Sense of Effort involved, for the person may sometimes try by moving the squinting eye to make the images agree.

The fact, therefore, is not entirely as stated, and, moreover, if it were it would not sustain the position deduced from it. The question is simply one as to the limits of discrimination. Thus if two hard points be brought close enough together in any part of the body, the discrimination of the separate touches vanishes, and on certain areas of the arm or of the thigh the discrimination is so little precise that if the section of a hollow cylinder of two, or even three, inches in diameter be laid on the part, the impression produced will not differ from that produced by a solid cylinder. But no one would adapt the argument of William James and deny that sensation of touch existed.

Weber found that the tip of the tongue was fifty times more sensitive than the upper part of the thigh or of the arm. Correspondingly we have a series of degrees of discrimination on the motor side. We move fingers that are innervated from closely neighbouring sources, and we recognise the different feelings involved in the respective movements.

In order to settle this interesting question of motor-feeling, Bernhardt (*Archiv. f. Psychiatrie*, 1872) imagined a critical test, for he thought that muscular sense must depend on centripetal impressions if discrimination remained good when the muscles were made to contract artificially by means of electric stimulus. He concluded in favour of the motor-feeling.

Ferrier disputes these conclusions, and he says that if weight be estimated by the impression of cutaneous pressure alone, it requires a difference of one-third of the original weight to produce a distinctly perceptible difference of sensation. Here he is wrong on the point of fact according to Weber, who found the difference to be sometimes as small as one-thirtieth. But if Ferrier's figures were right Weber would draw from that fact the opposite conclusion, and I cannot but think that once again Weber would be right.

The question of the Sense of Effort is especially interesting because it has been bound up with that of Will. Here again the school that derives from Fechner and Weber have pursued the subject in the field of experiment with results which at least are wonderfully interesting in themselves—as, for example, Brieger's experiments in forms of muscular contraction, Toppel's experiments on the influence of amyl-nitrite on the activity of the Will, and those of Kräpelin in this and allied regions.

Schiff, who did great work in tracing out the strands of nerves in the brain, declared for motor localisation, but he thought that the movement followed what was psychically some form of representation derived from the periphery. In other words, his position in this respect was similar to that of William James. Schiff did not consider the motor district to be a veritable motor centre, for this he thought lay deeper in the development. He concluded also that the motor area was not artificially stimulable. Exner and Paneth performed experiments which, they believed, proved the contrary, but these experiments may have other interpretations.[1]

A. Bertrand, in *La Psychologie de l'Effort*, attributes to Bichat the first expression of the theory of "*sens musculaire*". The genius of Bichat shone resplendently not only in his actual accomplishments, but in his very intuitions (*cf.* p. 354 *et seq.*). In this particular regard, however, I would not care to attach high importance to

[1] One of the most original and interesting of the German thinkers in this realm is Ernest Mach, who in a work which has been translated into English, *Space and Geometry in the Light of Physiological, Psychological and Physical Inquiry*, has discussed the relations of the axis of the body to our notions of space. Mach, like Hering, has moreover not been content with saying that each organ of sense has its specified energy, he seeks for physiological and, ultimately, chemical and physical, conditions for our mental states. It is well to consult his work, *Die Analyse der Empfindungen und das Verhältniss des Physischen zum Psychischen*, published in Jena, 1906.

Bichat's authority; for whereas psychologists unacquainted with natural science have often been inclined to wander in illusory regions, those who are for ever contemplating limited physical conditions often fall into errors of materialism conceived in mere nudity. Bichat thought, for example, that it was possible to localise the emotions, and he made the heart the seat of the warm affections.

Bell, the great English physiologist, shared the opinion of Bichat as to the recognition of motor innervations; and in addition to the names of those, as Weber and Bernhardt, who sought for clinical cases, or attempted to devise experiments, which should isolate *Innervations-Gefühl*, we might cite Leyden, Helmholtz, Bernstein, Sternberg, Exner, and Beaunis.

One or two instances deserve to be noticed, if only to show how fallacious may be the interpretations of known phenomena. Helmholtz observed that if the external rectus, say, of the right eye, be paralysed, then if the left eye be closed, and the patient attempt to look to the right, the external objects will appear to move to the right. The right eye in fact remains immovable, but the sensations are similar to those formerly involved in the movement; and as the image falls upon the same part of the retina as before the attempted movement, the illusion is given that the external object has moved so as to bring about this correspondence.

Ferrier and William James offer the explanation that the left eye is moving owing to the innervation of the left internal rectus from the same centre as that of the right external rectus. The sensations that seem to afford recognition of the innervation are really derived from the movement of the left eye. It should be noted that if the left eye be opened the illusion does not take place.

Wundt considered the experiment to be in favour of *Innervations-Gefühl*. Certainly the mechanism seems simple according to that explanation, but no conclusion should be drawn from such an argument. There is more force in his remark that if afferent sensations alone give us the impression of motor activity, there should be a close parallelism between sensation and contraction. This is far from being verified in cases of paralysis, or in other conditions where a great amount of conscious effort seems necessary to produce small effects of movement.

Let us criticise the experiment from another standpoint. When in an ordinary way we have the experience of external objects moving to the right, we derive the impression from a great number of factors, such as habitually define our orientation to our surroundings. These factors are not of sight alone; they concern also the movements, depending partly upon tactile sensations, by

which we adjust our position. Yet the visual sensations, the associated movements of the eyes, form the most important of the factors referred to. If a muscular sense exists, then it is only a part, and a relatively small part, of these factors.

When the left eye, though prevented from seeing, is free to move, we introduce other factors of considerable importance, and the impression of the movement of the external objects is produced. We have therefore no clear proof of the existence of the muscular sense, since we can point to no experience dependent on such sense alone. When the eye is opened the external objects do not appear to move. This fact does not disprove the existence of the muscular sense, for even if it were active, its influence would be over-ridden by the relatively powerful factors which have been introduced corresponding to our ordinary means of estimating the position of the body with reference to external objects.

Hence the experiment gives us no conclusion as to the existence of the muscular sense.

Sternberg hit upon a very ingenious device. He remarked that if the phalanges of the index finger be flexed at the first (that is to say, the metacarpal-phalangeal) and middle joints, the other fingers of the hand be completely extended, and the thumb abducted, it will be impossible, on account of anatomical conditions, to flex the index at the third (or distal) joint. But if the eyes be closed and the attempt be made, one will have the illusion that the finger has been flexed.

This experiment is also inconclusive. In the first place, we must distinguish between an innervation of a motor nerve, and the motor effect which follows such innervation. We are endeavouring to obtain the recognition, by means of a special sensation, if such exist, of the actual innervation of, or release of nerve stimulus, into a nerve or strand of nerves.

The impression of the position of the parts as a consequence of motor action depends on sensations derived from various sources, partly visual, partly tactile.

Further, the occurrence of certain sensations tends, as we have seen in Helmholtz's experiments, to reproduce impressions of others ordinarily closely associated with them.

Again Sternberg's experiment is not so clear-cut as would appear from the statement above. There is some movement at the third joint, and the sensation does not give the impression of a complete movement. In the effort of movement there is a considerable innervation in associated parts and this is accompanied by contraction of muscles. Now, as Sternberg depends for his proof on something which really depends on sensation, his experiment

might be interpreted to show that the impression of muscular sense has no other ground than that of a confused impression of the other sensations. This conclusion, however, would not be justified either, for even did muscular sense exist, it could not be clearly perceived amidst the strong associated sensations in the case of other kinds. Hence once more we have no definite demonstration.

Weir Mitchell has written interestingly on this subject, but his cases do not appear to me decisive. One of the most notable experiments of all is that of Beaunis. Struck with the force of the argument that part of the impressions were formed by the articular nerves involved—an argument which Westphal appears first to have advanced—Beaunis sought for a part of the body whose movements might be studied, and where such an objection could not arise. The larynx appears to offer every advantage, for here the experimenter was able also to eliminate, by the use of cocaine, the sensations derived from the mucous membrane. Under these conditions he tested the accuracy of the notes of a tenor singer, and he found that the precision of pitch was hardly altered.

An examination of this case shows that the result might have been anticipated, quite apart from any assumption as to muscular sense.

It is necessary to remark again that we must distinguish between muscular sense, and the sensations which accompany the effects of innervation of the motor nerve.

When a person is singing he receives certain sensations derived from the movements involved; but the main sensations by which he adjusts his efforts are auditory. He hears whether his voice tends to be pitched too high or too low.

Now the question is not whether an adjustment of effort, that is to say, finally an adjustment of nervous stimulus in response to various sensations, is possible; it is proved that such adjustment is possible. The question is whether in the mechanism of such adjustment there is a sensation of a peculiar kind which indicates the appropriate release of the innervating energy.

In the experiment of Beaunis a comparatively unimportant series of the accompanying sensations became eliminated; the main sensations—auditory—remained. The experiment is interesting and suggestive, but not conclusive.

Leaving these great names, therefore, I will endeavour to make clear what it is really that inclines me to belief in the Sense of Effort as a Fundamental Process.

(2) *The Point of View of Psychology*

The Sense of Effort is not identical with the muscular sense, but one is involved with the other. Therefore we may consider the muscular sense with special attention.

In the first place the fact that various suppositions are made to account for mental phenomena, whether the explanation be right or wrong, affords grounds of belief in the existence, at least, of the phenomena. If one accounts for what others call the feeling of motor innervation by saying: No, that is not due to a consciousness of the excitation of motor nerves, but, in a complex and somewhat obscured way, to the excitation of various sensory nerves; then we have a something in dispute; the matter is resolved into a question of origins.

The experience of mistaking a bar of aluminium for a bar of lead may possibly be explained without the necessity of supposing a knowledge, or feeling, of the storing-up of energy. I do not mean to imply that such an explanation would be a good one, or the true one; but those who deny altogether the sense of motor innervation might argue: the whole process of storing up the nervous energy necessary for motor activity is unconscious. All the impressions that our senses convey to us respecting an object, together with all that Memory holds respecting that object, produce in the mind certain judgments respecting it.[1] When therefore the intention is formed of lifting an object, for example, a mechanism is set to work which prepares the necessary motor energy. But all this is unconscious. When the motor energy is discharged that act is also unconscious, and the feeling of surprise that may follow in a certain sense is formed of combinations of unexpected sensory effects, and the secondary effects also of judgment in the new situation (*cf.* pp. 73 (note) and 280).

Here it may be contended that there is a means of estimating the amount of the stored-up energy; but the means of estimation may not be by direct reference, it may be indirectly by reference to the sensory influences from the object and from various associations connected with it. Even if we think otherwise, how can we prove what we believe? How can we separate from all the complexity of our impressions some definite criterion as to the direct effect in consciousness of the nervous discharge along the motor nerves?

We see also that Weber's law does not help us. It is true that we can estimate so far as to say that one feeling of motor innervation seems to correspond to twice as much objective load as another.

[1] The term "judgment" is here used in a wide sense, and does not imply the formal expression of a judgment, or even its specific recognition as a judgment.

But the question is not thus settled. How do we know that this estimation is not really by means of combinations of sensory effects? Manual training, as for instance in the gradual refinement of the skill necessary to bowl a good ball, implies increasing accuracy of measuring the output of motor energy, and therefore of the amount stored up for the purpose of sudden release. But it does not seem possible to show definitely that the operation of storing up the energy may not be automatic and unconscious when the conditions have been determined by the consensus of all the sensory elements involved.

Of course similar difficulties are presented to the upholders of contrary views.

But in order to pose the problem better, suppose for a moment that we turn the tables on those who deny *Innervation-Gefühl* for motor nerves; that is to say, we declare the motor nerves are alone involved in the consciousness of innervation; explanations quite as feasible as those considered on the other side can be built up on this hypothesis. Thus, for example, a person is sitting in his room in profound quiet at night. Suddenly a bell rings. He is startled even before he clearly realises the sensation of sound in its specific quality. He might say: What has happened is that an impulse has reached the brain, and has then produced an explosion which finds outlet through various motor nerves, and the impression has arisen from the summation of the general muscular sensations.

I say that on the face of it the position thus indicated is quite as feasible as that of the contrary as expressed by William James.[1] This, however, only implies the unsatisfactory character of our investigations at that level. Let us therefore make a clean sweep of suppositions, and again apply ourselves to realities.

* * * *

For guidance of argument I say, anticipating proofs, that I believe that the Sense of Effort is manifest both in sensory and in motor innervations.

Such an experience as that just described happened to myself,

[1] This position is not put forward as a mere paradox. Factors from motor sources modify all our sensations. This observation has occurred to many who have studied attention, for though their attempt to build up a system by using complexes in place of elements is hopeless, yet many of their indications are of value when correctly interpreted. Ribot says of attention that its "constitutive elements" are motor. In a still wider application of the term attention, M. L. Billings and J. F. Shepard studied this aspect of the subject: *Change of Heart Rate with Attention* (*Psych. Rev.*, 1910). All this will be clear to those who have read patiently so far. The excitation of a sensory nerve is a necessary incident in sensation, but that stimulus impinges on a highly complex system, in the development of which have become implicated all manner of sensory and motor experiences and the combinations of these. If, beginning on the motor side, the matter were not correspondent with this, then the principle of development of the brain would incur a complication which seems unnecessary.

and I noted it carefully at the time. I believe that the peculiar Sense of Effort apart from the specific quality of the sensation was due to efferent innervation; the *quality* of the feeling seemed to bear that indication. I am not here placing too great a strain on the perception of quality, for it is by that mode alone that we distinguish any two sensations manifestly different. It is an application of our old instrument of introspection.

I notice, moreover, that when one touches an object which is too hot there is a feeling, a sensation, arising directly, and there is a feeling arising from the involuntary motor storm which is produced by our convulsive start. The feelings are capable of Discrimination.

It happened to me once when a young student to make experience in an intense degree of the Sense of Effort in a motor storm. I was walking one evening with a lady in Melbourne when, it appears, a drunken man suddenly surged up and lurched near her. In her terror she screamed and convulsively clutched my arm. I happened to have been absent-minded at the moment, and had not seen the man; so that the piercing shriek for help and the frantic clutch at my arm broke in on my senses all unprepared. I could not decide what to do, for I did not know what was happening. Yet in various directions motor impulses, tentative, uncertain, baffled and futile, were being sent out with intense force and rapidity. From the neighbourhood of the whole length of the vertebrae it seemed to me that a veritable escape of force proceeded in sudden disruptive clouds. So profound was the reverberation that there was with this a vague impression of form, —woolly clouds—and an association of colour—dark grey. I made a note of the occurrence soon afterwards. (*cf.* p. 53 and (note)).

The association of colour impressions could, I think, have resulted only from the communication of various currents to the nerve strands, or at least the cerebral portion, usually involved when such colour impressions normally occur. There was also, no doubt, an implication of various other sensory impressions such as those of touch in the regions most violently agitated by the motor efforts. Yet withal the main impressions could not have been formed, I believe, by any combination of sensory impressions however intense. Highly accentuated sensory impressions do not reach a pitch where they become confused with motor impressions; they seem to give way to sensations of pain.

The two examples that have been noted are typical. In both cases, the mind was unprepared. Sudden strong sensory stimulation was followed by motor impulses; but in the case of hearing the bell the sensory impressions seemed so far dominant that I

believed that it was possible to observe a distinct Sense of Effort associated with but not identical with the specific auditory sensation; in the other case, the motor elements were predominant, and quite apart from any direction given to the motor impulses, or from any guidance of sensory impressions. I believe I observed a distinct Sense of Effort in the motor discharge.

I have another curious note on this subject. On 26th August, 1906, an experience came to me which filled the conditions of an ideal experiment. The note says: I was lying in bed, fatigued, not asleep but not thinking. Suddenly I gave a start and heard an explosion as of a loud cracker in the street. Soon afterwards the explosion was repeated, and I noted the precise sequence of events. I gave a start, and then almost immediately, but yet perceptibly afterwards, I heard the noise.

I remarked at the time: It would seem that there is a faculty which prepares the organism for the amount of energy put forth in order to appreciate the sensation. That is to say, I had a feeling of Sense of Effort in direct association with the auditory sensation. In the first case the Sense of Effort seemed to be associated with the motor impulse. The explosion was again repeated. This time the noise was appreciably anterior to the start. The controlling mechanism had become adjusted by the help of the two previous experiences. This case prepares the way for other considerations where the Sense of Effort in association with afferent impulses appears.

Consider what is meant by intensity, especially in regard to those senses—sight and hearing—for which we can form a clear appreciation of the objective stimulus. Colour and pitch depend on wave length. Intensity and loudness depend on amplitude of vibration. The form of the wave may be determined by the superposition of simple wave structures. So that finally we see that the objective phenomena enable us to define the limits of variation of the subjective phenomena. When we have differences of colour the entire difference of subjective appreciation may be considered to be that of the sensations themselves in their specific character. But suppose we have two discs of the same colour, but one much brighter than the other, is the whole matter to be accounted for simply in terms of specific sensations?

We speak of one colour or shade being more lively than another. Does not that refer to a recognition of Sense of Effort purely on the sensory side? Or suppose that instead of a coloured disc we have a white disc; or a disc that gives an impression of lustre with as small an impression as possible of colour. Then if we increase the intensity of the effect, is there merely some change that can

be summed up by reference to specific sensation and to the exclusion of a Sense of Effort? It may be urged that this Sense is included in what we mean by the sensation; but even if it were recognised as something to be included the object of this particular argument would be gained.

For instance, I do not think we can conceive colour without form, or some manner of extensity, so that every visual sensation presents us with these factors; but though they are always associated we can readily make distinctions. We can vary the form and yet recognise that the colour is not altered.

It might be contended that if we had a square of red, and a circle of red of the same tint, and the same area, we have nevertheless two distinct sensations; and that is undoubtedly so; but we can form the Dis-association of the colour from the form in both cases; we can imagine the same square or circle of a different colour, as, for example, green, and we can form the Association of the two phenomena of red in a manner distinct from that of the green. Can we then separate intensity from that? Yes, I think we can. We can even say that the intensity of the illumination of a red disc is equivalent to that of a green disc.

The fact that we can speak of two shades of red, as being both red, or that we can speak of two discs of red and green as both being discs of different colours, implies something common in the midst of diversity. That something common corresponds to the feeling of innervation of the nervous tissue corresponding. That feeling, we may call, by definition, the Sense of Effort.

These arguments are strengthened by the consideration of fatigue. Given the same objective stimulus the impressions are livelier to a fresh, active mind than to a mind affected by fatigue. The fatigue, moreover, in regard to sensory impressions is of a different character from that which follows excessive motor action. If a man, as for instance in a berth of a steamer near the screw, be compelled to listen to a loud monotonous sound, he experiences a sense of weariness or fatigue, even though apart from this sound he be in repose. The character of this fatigue is different from that produced by wielding a sledge-hammer.

Again, if a man listen to a regularly repeated sound, as, for example, the ticking of a clock, he will find that now and then a beat seems to drop out. This may be in part ascribed to fatigue of the actual sense, even though we take into account the associated motor effects involved in directing the attention of the sound, and even though the actual mechanism for eliminating the sound be in part motor, viz., that of the apparatus regulating the tension of the drum.

The effect is quite different from that experienced when some motor apparatus, such as a co-ordinated set of muscles, rapidly repeats some tiring movements until there is a pause necessitated by the need of recuperation.

In fatigue it seems to me that a means of determination may be found. Consider for a moment a simple case. A man delivers a blow at a punching machine. He delivers it, let us say, with great force and accuracy and also with a certain joyous elasticity, which is a manifestation of the Hedonic principle. Suppose that he continues to punch till he is very tired. Then suppose him to rest for a time, and then to get ready to punch once more. Now the sensory impressions are not greatly altered, but there is a difference in the feeling of preparation of the necessary muscular effort, and a difference also in the feeling accompanying the delivery. Even here it might be said that, though the sensory nerves usually considered, those of sight particularly, are not greatly disturbed, yet the sensory nerves from the muscles are tired, and the feeling of fatigue is due to their changed condition.[1] The condition itself may be produced by excess of waste products in the nervous apparatus.

But since the fatigue affects the feeling of stored-up energy before these nerves are stimulated anew by the actual experience, the school that rejects *Innervations-Gefuhl* would require to suppose that the amount of energy is estimated by tentative communications along the motor nerves and thence, by reflex, along the sensory nerves. Those who maintain *Innervations-Gefühl* might say that such tentative communications took place with direct intimation in consciousness, or at least that the storing up of energy following sensory impressions was not unconscious.

Were the explanations of the sensory school susceptible of proof it would not seriously disturb, although it would modify the expression of, the positions set forth in this chapter. The only difference would be that the Sense of Effort on the motor side would be arrived at by a mechanism which might be contemplated as indirect, but which in its effects in consciousness gives no indication of that kind.

An argument which will be found helpful is the following: Bearing in mind, in regard to ordinary sensation, the dominant character of the central excitation; and considering also what has been said, in reference to localisation, that we should imagine not special centres or seats for powers and functions, but a wave of

[1] Here certain researches of physiology are of distinct value. Bowditch has established that sensory nerves may function for long periods without perceptible fatigue: *Note on the Nature of Nerve Force* (*Journ. of Phys.*, 1895) and *Nachweis der Unermudlichkeit der Saugethiernerven* (*Dubois Reymond's Archiv*, 1890).

impulses passing through the brain and affecting certain strands and regions more than others according to the nature of the ideas and activities involved;[1] we may now contemplate a case where motor activities occur as part of a sequence of associations.

For example, a man may fail to remember a figure of the lancers, but if he actually begin to dance them all the steps come back in turn. Or an actor may have forgotten a part which he used to play: but if he proceed to represent it on the stage he may remember it. Motor activities then have a part in the establishment of a stored-up system of ideas forming associations with new presentations. We have these as part of the antecedent factors in forming the new instantaneous resultant of mind. The resultant feeling is usually comprised in part of impressions from afferent nerves; but analogously, we have usually in sensation influences derived from motor sources. When there is a distinct movement of attention in regard to a sensation, then in the adjustment of all the bodily conditions favourable there is involved a complex of motor activities and also consequently of afferent and efferent impulses, even though there be no clear expression of these in consciousness. (*cf.* pp. 36 and 73 (note)).

* * * *

We have now reached a point where we can sum up the whole matter.

Ribot's remark that there is a certain motor element in all acts of attention is in accord with the whole conception of this chapter, and that particular fact may be verified by close observation.

Correspondingly it is clear that among the antecedents to the reproduction, in idea, of ordinary sensations we may find motor influences. Thus if an actor wished to recall a line he would be aided by throwing himself into the attitude suggested by the context. This so far proves nothing more than some kind of association that may be vague and indirect.

When we consider the case of a man estimating the force of a blow, or preparing to lift a bar, whether of lead or aluminium, and adjusting his effort accordingly, we find that such adjustment may be carried out with remarkable accuracy if there have been several previous trials. The adjustment is carried out not as a result of the actual sensations accompanying the movement—for the adjustment takes place beforehand—but the ideas of previous sensations are the determining factors. These ideas are correlated to cerebral activities. That is to say, the physical actions involved in the whole process of adjustment are central, and are

[1] This is von Monakow's view.

represented by the interplay between sensory cells, neurones or ganglia and motor cells.[1]

We can assert this on the same grounds as we assert that the physical correlative of any recollected idea is a disturbance of certain nervous elements in the brain.[2]

We have now greatly narrowed the issue. It may be that the actual disturbance of the motor ganglion may correspond to a special sensation, or it may be that, given the antecedent ordinary sensations (or rather in this case the corresponding idea) the operation of adjustment takes place automatically and without further note in consciousness.

Similarly with the sensation of the discharge of the prepared nerve forces; there may be a direct characteristic sensation, or there may be a sensation caused by the reaction in the sensory elements.

Certain it is, however, that a sensation corresponds to each of the physical states of priming and discharge. That sensation, even if it be formed of ordinary sensations, is in its composed quality quite distinctive.

We are aware that a kind of priming or loading of the nervous apparatus has taken place, and we know that there is a peculiar sense accompanying this process; and we are aware when the discharge takes place, and we know that the signal for that discharge does not consist in a massing of ordinary sensations; and we are aware when the discharge is complete, while at the same time we can distinguish, as apart from that feeling or sense, the usual tactile, visual or auditory sensations accompanying or succeeding the effort.

If now we form a picture of the essential parts of the central

[1] What change takes place in the nervous mechanism of the brain on stimulation through a sensory nerve is not known. In order to show how difficult it is to obtain a clear notion of the actual process I cite, though without accepting it, the pronouncement of one of those who have devoted time and research to this problem. Max Verworn (British Association, 1912) says:

"The molecular alterations brought about in the neurones by the stimulus during excitation consisted in a disturbance of the metabolic equilibrium by an increase of the processes of oxydative disintegration. Immediately following each excitation, however, metabolic equilibrium was again restored, and every trace of the excitation process seemed to be obliterated. Still, even though the process produced no molecular alteration, a secondary result remained, which consisted in an increase of the mass of the living cell. This only became clearly manifest when the excitation was of frequent recurrence, and was then expressed by functional hypertrophy. The presence of the latter could be demonstrated in the neurones as well as in every other living substance. It formed the cellular basis of memory by increasing the intensity of the discharge of impulse, and with this the specific action of the neurone. This recurring use led to a facilitation of the tracts (*Ausschleifen der Bahnen*), and thus to a greater intensity of the specific action of neurone chains resulting from the repeated demands."

[2] The argument of William James in regard to the "useless encumbrance" of supposing a motor sense is here turned against himself. The mechanism is simpler on that supposition. I do not press this argument, for to overthrow William James is not the same thing as to interpret Nature.

apparatus in the most elementary case, remembering the minuteness and the contiguity of the elements, sensory and motor, we see that the problem is hardly more complicated than that of the nature of the disturbance correlated to an ordinary sensation. We do not know the nature of that disturbance, except to surmise that considered in its molecular aspects it is extremely complex.

In ordinary sensation there is a reaction of the sensory upon motor cells; and in the muscular sense there is a reaction in the reverse direction.

The Sense of Effort we have seen to be not an attribute of the muscular sense alone, but to be associated with all kinds of sensation.

But even if all these considerations seem tentative and uncertain I now introduce a decisive argument. The previous discussion has prepared the way for its reception if only by compelling the mind to meditate closely on those subtle matters.

The question is linked with that of the concept of Unit. When from particular Units we proceed, by depriving them of their special attributes, to consider the most elemental form of unit, we find this in the act of attention itself, and in this act of attention we meet also the Sense of Effort in its elemental form.

Without this possibility of dealing with the act of attention as a unit, counting as we know it would be impossible, and then as will be evident from the previous discussion on this subject the whole system of our mathematics would be impossible.

Here then we see once more the elemental character of these Fundamental Processes, and we see how indissolubly one is linked to another.

If the reader fail to acquiesce at once in these arguments, the matter may not even then be disposed of. It will be understood that the question demands the most careful and delicate introspection, and that before forming a judgment it is well to have the matter continually in mind over a lengthened period, and to test it in as great variety of circumstances as possible.

In this way many features at first obscure will become clearer, though with the very clearness of view the difficulty of the problem will become more and more evident.[1]

[1] In the course of the discussion so many references have been given that there is no need now greatly to amplify these, especially as I have found the arguments of all unconvincing, and have relied on none for the demonstrations given. These derive from new points of view, new instruments, and new modes of attack. The following, however, will be found interesting: *The Sense of Effort: An Objective Study*, by Waller (Brain, 1891): also *Functional Attributes of the Cortex*, by Waller (Brain, 1892). Angelo Mosso, in various writings; Mott (*Journ. of Phys.*, 1894); Loeb, (*Pflüger's Archiv*, xlvi.) and a number of others have worked at the matter on the physiological side. Sternberg's work is of especial interest: *Zur Lehre von den Vorstellungen über die Lage unserer Glieder* (*Pflüger's Archiv*, xxxvii.). The great psychologists,

SUMMARY

Outgoing impulses are not all motor.

Great physiologists who have attempted to solve the question of the Sense of Effort have given contradictory theories. Many have shown an insufficient grasp of Psychology in the interpretation of observed facts in physiology.

Helmholtz, Sternberg, Beaunis, and others have instituted experiments with the object of isolating the Sense of Effort, but these experiments are not conclusive.

The solution must be sought in psychology aided by physiological data. Various experiences give assurance of Sense of Effort. The study of fatigue leads to the same conclusion.

The argument becomes narrowed down to a question of histology and of molecular action which is not more complex than that of ordinary sensation. In ordinary sensation there are motor effects; in motor innervation there are re-actions that tend to produce an impression of sensations.

The Conception of the Unit implies the Sense of Effort.

Wundt, Funke, Hermann, amongst many others, deal with the subject. In America E. C. Sandford and J. Dewey (*Phil. Rev.*, 1897) have expounded the matter. Amongst those who have favoured the Sense of Motor Innervation, Maine de Biran was one of the first to call attention to the significance of the question; in fact he bases a whole system of philosophy mainly on this position. See his correspondence with Ampère, where the great physicist shows himself an acute psychologist. Of late writers, in addition to those mentioned, Sully, Fouillée, Marillier, accept the principle; while to the list of formidable names of those opposed may be added E. Peillaube and I. P. Nuel.

CHAPTER IV

EXTERNALITY

NOTE:—The study of Externality is fascinating in itself, and partly because the lure of its conquest has led hosts of thinkers to philosophic discomfiture. The analysis of the position will also indicate in how subtle a manner the Fundamental Processes may operate, for the ideas here are amongst the most elusive. This analysis will develop notions inevitably associated with the Concepts of Space and Time, and affirm the sufficiency of the Fundamental Processes throughout the scope of the world of thought.

Scottish philosophers, Reid and others, formed the Common Sense school. They declared that the External world existed because they saw it existed; their common sense told them that it existed.

Berkeley declared that all this was explicable on the basis of Idealism; a complex of ideas gave us the impression of material things but ideas only existed.

We shall here find, not that these two theories are reconcilable, but that, if what is valid in the analysis of both be extended by still more persistent analysis, a basis will be discovered where all apparent inconsistencies become resolved.

* * * *

In the first place let us strip "Common sense" of some of its pretentiousness.

Bishop Butler, it will be remembered, when he endeavoured to get a universal standard of right and wrong, set up as final judge the "plain honest man". And Tolstoy has conferred on the same type pontifical qualities of infallibility in art. One may suspect that the plain honest man is in these cases rather a stalking-horse for a more powerful personality behind. But those of us who familiarly know the average man are by no means so impressed by the perspicacity of his intellectual vision or the lucidity of his expression.

By the fact that he is the average man too many streams of low origin have united to form his character. So far from it being the case that the plain honest man possesses unerring judgment, or that overwhelming majorities wield any valid authority in intellectual matters, we have found that at momentous turning points of the world's civilisation, the judgment of one man has deserved to outweigh that of all the rest of humanity; and that one has not been of the type of Reid's "common sense" or Butler's "plain honest man." Such a man was Empedocles

when he foreshadowed the theory of Natural Selection; such a man was Archimedes when he saw clearly the principles of specific gravity; such a man was Roger Bacon exploring the mysteries of chemistry; such a man was Copernicus beholding the verities of the solar system; such a man was Galileo when he let drop from the tower of Pisa the two pellets that announced in their fall the dawn of a new era.

The man of common sense scoffed at the notion of the earth revolving around the sun, for he said he saw that the sun revolved round the earth and his common sense sufficed.[1] The plain honest man ridiculed Darwin during a generation.

* * * *

Let us now examine Berkeley's position. We may begin with simple sensations. If we find that Externality be required to account for such phenomena, it will be necessary to the explanation of complex associations of these.

Berkeley must, of course, postulate mind, for if there were no mind there would be no sensation, and if there be no Externality for a developed mind there will be no Externality for a rudimentary mind.

We arrive at the consideration of a rudimentary mind.

There must also in the strict Berkeleyan sense be but one mind, for a mind cannot think without sensations; and if there be another mind that must have sensations also. These sensations must be different, for how can two minds have one and the same sensation?

If two minds existed, Externality would be implied; for what would be the meaning of saying that the actual sensation was not external to another whose experiences were different sensations?

If we consider a single rudimentary mind, we find that its sensations have characteristics of kind, intensity, duration. But there is nothing in this by which the mind can anticipate its sensations, or modify or check them. Even if we consider a developed mind we find that there are sensations which the mind cannot anticipate.

[1] It is as true that the sun revolves round the earth as that the earth revolves round the sun. Henri Poincaré, the famous French mathematician, expressed this clearly, and Einstein's theory of Relativity has been established by development from the principle involved. It should be made clear that when we speak of "law" we do not mean any sort of rule which we can impose on nature, but simply a mode of systematising and exhibiting in orderly form the course of phenomena. When we take the sun as revolving round the earth, the motions are none the less real, but they cannot be expressed in any simple form such as we require in a "law." Even Painlevé's subtle questions as to the effect of rotation are, I think, susceptible to answer in accordance with the present statement. The man of common sense is right when he says he sees the sun moving; that is to say, relatively to the earth. He is wrong when he scoffs at the conception of Copernicus which renders the whole matter more clear and gives to our intelligence new instruments of discovery.

On these lines, then, one would be reduced to the contemplation of a solitary mind, but without control of the phenomena produced within itself, or without any prescience of its own movements.

Such a position is not unthinkable, but at least one could hardly persuade another mind that it had no sensation and no existence beyond one's self. Descartes gave out the doctrine that animals were mere automata, and that a cry of pain, for example, was simply the creaking of the machine under certain circumstances. Berkeley does not appear to have believed anything of the kind concerning human beings, for he occupied himself with various schemes, not very practical as it turned out but at least practicable, for their benefit.

It would be competent, however, for a Berkeleyan to believe that his was the only mind in the world. But we will see that even on that basis he might arrive at a view of Externality which the common sense representative, except for the exclusion of his particular mind, might be led to adopt.

We might suppose a mind not endowed with sensations through either sight or hearing and yet attaining an extraordinary developed notion of the world. Helen Keller is a case in point. She has her other senses intact and greatly refined in exercise.

We can scarcely expect to find a person furnished with only the sense of touch; but we may imagine ourselves as on occasion refraining from the exercises of any of the other "five senses."

A sensation is now introduced. What we have learnt to speak of as intensity and duration could not be completely appreciated as such, but still they would afford their definite effect, in so much as that whatever the influence in the mind it would be different if these were different. If after the cessation of this sensation another were to follow of a different degree of intensity, the difference would be appreciated. In this way we might have a world consisting only of sensations of touch and varying in intensities.

Let us now add to the capabilities of our mind the muscular sense.[1] This brings to light clearly the Sense of Effort, which accompanies all sensation.

Now after the recurrence of the muscular sense, with certain duration, there may follow other sensations of touch. Imagine, in order that the exposition may be understood, the person stretching out an arm and touching something.

The muscular sense will be found on close attention to be not simple, but to have certain characteristics to the circular movement of the arm. Imagine these characteristics changed; effort

[1] Herein is assumed nothing as to the veritable nature of this sense or the origin of its manifestations.

with regard to the muscular sense being increased; the sensation of touch increasing in intensity. This would correspond to the familiar experience of the hand meeting with a firm object.

Here is a combination established in regard to what we will afterward consider in its complexity as the external world.

It must be noted that as far as we have gone we know of it only through changes of sensation of a definite character arising unexpectedly and therefore without control from ourselves; and that the changes of sensation are mainly of the nature of interruption of some of the factors of the muscular sense, intensification of others (as those muscles which aid in forcing against the resistance) and also intensification of the sense of touch and of the accompanying Sense of Effort or both.

We find also that except for this intensification the complex of experience does not change, and we find also that if a certain intensification be retained the whole process now persists in constancy. Looked at from the outside this means an even pushing against a firm object. The negative of this series would be followed by the sense of touch disappearing while the muscular sense derived from the negative movement continued. Let the whole operation be repeated till it is familiar.

We will introduce symbolisation to facilitate explanation, but we remark that this implies no new complexity; for though the symbolisation be not by words, yet the repetition of an experience invariably introduces symbolisation, even if it be by means of the initial steps of the experience symbolising the whole. We have already discussed this.

We will therefore adopt, in order to refer to the complex of our sensations, the symbol "External World". We must note particularly that it is not intended hereby to beg the question. We adopt the term for convenience of exposition, but we do not imply in its use anything further than we shall meet with as we proceed to develop our experiences. It will then remain to be seen whether the whole meaning that arises from our symbol, External World, contains anything less than, by appropriate analysis, the Common Sense philosopher would find in the term.

The symbol External World, by experience of the sort described, soon becomes diverse. To the complex we have already described therefore we apply the symbol "table", or more precisely, "top of table".

Consider now what happens when we come to the edge of the table. This would be presented thus: The first experience is renewed. The sensation of touch continues without intensification. Certain appreciated changes are given to the muscular sense;

then suddenly the sensation of touch disappears, but the muscular sense continues as before. On forming its negation the sensation of touch reappears.

The sensation of touch continues when the muscular sense undergoes another appreciated change. This would be known to us ordinarily as feeling over the side of the table.

A development of this method will convince us that, given the sense of touch and the muscular sense and the Sense of Effort, we form a conception of the External World with regard not only to touch but also to shape and size and also to resistance as far as this is expressed by the complex of the sensations of touch and the muscular sense.

So far we know nothing more than our combinations of sensations and their Associations in memory. These faculties we must always imply in mind. But we cannot say that nothing more can be known than these ideas which we have indicated. The table, for example, was met with unexpectedly, and in the External World we may at any time meet with some object unexpectedly, that is to say, in a manner to interrupt, independently of our control or desire, any sequence of ideas in association or any combination we may form.

We may now, for the purpose of exposition, abandon the tedious reference to first principles and employ ordinary terms, but without making assumptions unjustified by our method.

Let us suppose that the mind considered had not been endowed with the sensations of touch but with those of sight. With this sense alone there does not seem to be any suggestion of distance nor even of space in the full sense in which we ordinarily understand space. An appeal must here be made to introspection but to aid introspection the following experiment may be tried: First divest the mind as far as possible of ideas already formed. Then keeping the whole body still for a long time and without moving the eye or even winking gaze steadily on a quiet scene. It will be found that the impression of distance, of resistance, and of solidity become lost. This is especially the case if only the one eye be used. The sense of colour persists, that of space also remains, if this be interpreted to mean the impression of shape and of place, but neither of these terms must be considered as possessing all ideas usually associated with them, but rather a peculiar distinctive character as forming part of an assemblage of objects disposed in a certain way.

However, as we have already seen, the eye is adjusted for vision in various ways by different muscles; corresponding to the movements of these muscles there are definite sensations.

The action of the muscles of the iris, causing contraction or dilation of the pupil when the illumination is increased or decreased, is automatic, and probably there is no appeal in consciousness corresponding to this muscular action alone. In the case of accommodation for distance the action of the ciliary muscles is automatic, but if the act be energetic other muscles are really involved. Thus in general there is associated with the action of the accommodation a certain condition of the muscles directing the eyeball, and also of those controlling the eyelids.

It must be noted that the direction is never maintained by an absolutely rigid system but by constant balancing with appropriate innervation of the muscles and with the endeavour to reduce to a minimum the errors of direction oscillating round the true direction.

When change of accommodation for distance takes place the directing muscles move accordingly, so that associations are formed by this habit; and when, in an experimental case, we desire no change of direction, an effort of adjustment of direction, correcting the usual action, is necessary.

The combination in various ways, both of concurrence and succession, of the pure visual sensations first considered and those of locomotion, accommodation, and changes of illumination, gradually show the connection between the variations of the visual sense and locomotion, and so we add to our impressions and ideas the appreciations of what we afterwards know as solidity, and space.

It is necessary to use the phrase " afterwards know ", for there is nothing so far to tell us what effects of touch correspond to certain visual effects. We can only know this by experience.

We are fortunate in having been able to test the fact by actual experiments. The most famous of these is the case of the patient operated on by Cheselden for cataract.

Molyneux had asked Locke whether a blind man who knew a sphere and a cube by touch would be able to distinguish them correctly on receiving sight. Another philosopher expressed surprise that they should agree that he could not. It was ascertained that Cheselden's patient was unable to decide by sight alone which was the sphere and which the cube previously known by touch. He thought also that distant trees were resting on his eyes.

Although the result supported his opinion Locke went too far in pronouncing definitely, for who could tell before actual experiment what connection, having even a physiological base, might be discovered between the senses. But the wrongfulness of

dogmatising on the other side was clearly shown. Certainly symmetry of touch would generally be suggested by symmetry of vision.[1]

A distinct help toward a conciliation of the Idealist with the Common Sense Philosopher is found in the study of such an act as picking up a stone. The Idealist looks on the stone as made up of ideas in certain relations to other complexes of ideas. If the stone be taken up and removed then one set of complexes are held all together, while they change their relations to other sets.

It is found convenient therefore to indicate such a constant complex by a symbol, stone; and we find that the complex contains visual ideas, ideas of touch, ideas of resistance depending on the muscular sense, and more abstruse impressions, resolvable, however, into ideas which we speak of as weight.

If now we take an example of acts expressed in ordinary language we may show how they could be expressed also in the symbol of ideas we have found in our analysis.

Thus: I went into the garden and plucked a rose and smelt it. This would be expressed by relating the character and duration of the locomotive sensations in their successive phases, accompanied at each point by the other sensations corresponding to the term "walking in the garden"; then would follow the account of the more particular touch sensations with the more restricted locomotion of the arm and the fingers in grasping the rose, and the peculiar changes in the character of the combinations of those accompanied by the characteristic sight sensations of the plucking; then the whole series would be succeeded by the smell sensations, these being associated with certain locomotive and tactile sensations.

The difference between this elaboration of the successions and combinations of ideas and the ordinary form in which we tell of the experience is simply due to the development of symbolisation, applied to those combinations of ideas that have an intimate and persistent Association as well as frequent recurrence.

It must be noted that the sense of distance, as for instance of the rose tree, is produced by the visual sensations arising from the tree, by the muscular sensations, and by the sensation of changes of accommodation and illumination, in looking at the tree; then by the impression of the continuous series of objects from the tree to the place where the person is standing; then by the muscular

[1] A case has recently been related in *Harper's Magazine* by Dr. Edward Ayres where the patient, operated for cataract at the age of forty, recognised apples as round, and named the colours red and green at once correctly. A consideration of what has been previously said of Association between the senses would serve to explain this result.

sensations of locomotion in connection with the series of constantly changing efforts involved in so taking this series in order, that is to say, in getting close to the tree that the touch may be added to the others.

Is this all, however, that is contained in the meaning of the External World? We have already observed, in the case of the table, the factor of the unexpected.

Let us consider an object: Iron. The Idealist would say that this can be resolved into a bundle of associated ideas. This is true with reference to those attributes of which we have had experience, even of such remote relations as specific gravity. But this is not all that is contained in iron. The iron becomes electric and develops new and remarkable properties which no amount of the most skilful analysis or introspection employed in the "bundle" previously symbolised as iron could have indicated.

We could not predict these properties, nor, after their appearance, form any expectation as to their development or persistence. Nor could we by appealing to our previous knowledge of iron and sulphur as bundles of ideas predict the wonderful effect of their combination and the entirely new series of the bundle so formed.

In the next case we may consider another individual, and particularly in that individual what familiarly we speak of as mind. It will be remembered that hitherto in working out the problem from the Idealist position we have excluded other minds, but now though we use the word mind, we may consider it only in its phenomena. Thus Descartes, while believing that animals were automatic, might talk of them in ordinary terms.

But in dealing with mind we have a still more inefficient grasp of the "bundle" than in the previous cases and the unexpected becomes more marked.

The indications of other minds traverse a more extensive range than those of inanimate objects, and extend themselves beyond the scope of our calculations and yet from point to point reappear within the sphere of our knowledge and our anticipations.

In all this the character which is the most strongly objective is the unexpected: the complex of ideas that arise beyond the scope of anything that we have known or could elaborate by any kind of combination of ideas which had existed in our minds or which our minds could anticipate or in any way control.

Everything else may be expressed in ideas and their relations—that is to say, subjective states. This property of the unexpected we must except and make provision for by the "something, I know not what", of Locke. This phrase brought censure on Locke from other eminent philosophers but it seems admirably to

indicate the carefulness of his method. His analysis is not always final but within its scope is seldom wrong, while for delicate introspection and accurate record of mental experiences he has not often been equalled.

The "something, I know not what", is the something independent of previous mind, entirely dependent on experience and capable in its independence of being developed indefinitely in other forms.

But when once we admit anything of this character, which we may now consistently with our Idealism and without further assumption term, objectivity, we must admit it with respect to mind. When we admit another's mind the whole External World opens up precisely as the Common-Sense school, if it were sufficiently endowed with analysis and introspection, would demand.

In the previous discussion of mental operations we found that at every experience we had to take into account not only the actual objects of experience in their own relation, but also the nature of our previously acquired knowledge in all inter-relation, and the original character of the mind in which the knowledge had been stored; so that the whole formed a vast complex or resultant with which the actual experience in question produced a new resultant. Belief in this view was not merely a contemplation of certain Associations but an attitude of the mind charged with Impulses.

Now the Idealist, widening the scope of his mind, may say that even the unexpected in the External World cannot be set down as objective, in the sense of being independent of the mind. He may maintain that in these new experiences he is simply developing unexplored regions of his mind; and that the steps of a chemical experiment, for instance, are but the means, all expressible in ideas, by which he discovers new revelation in the ideas of his own mind.

This mode of argument cannot be gainsaid but there is no need in our process of reconciliation why it should be. Instead of attempting to batter it in breach we have only still further to widen and develop it. We have to give to the possibilities of our own mind the whole scope of the possibilities of the External World as we can ever know it; and even in regard to the entirely unexpected, unknown, uncontrolled, we have only to hold all this as a factor which may be associated in any form with the complex of our stored-up ideas, in determining that resultant of which we have spoken and the attitude of the mind toward the path of its next possible moment.

In this fullest development let us consider the phenomena of what for convenience we call another mind, although we are within the Idealist position of maintaining that this is part of our

mind. But to that other mind we grant the fullest measure of the unexpected and of the uncontrolled, and we grant also that the means of exploring that mind are those of receiving instruction by the hearing in connection with all the visual complexes by which we recognise a personality, or by visual sensations as in reading a book; and that the course of this exploration may lead to ratiocinations beyond anything we have hitherto attempted, or may expand our spiritual sense, or influence even our ideas of our own ideality, beyond our previous conceptions; and we further grant that in our review of the External World we must think of other minds possessing continuity beyond any space of time in which we contemplate immediate experience. We thus see finally that there are no circumstances of the most abstruse speculation, or any character of imagination, that we cannot express in ideas.

But it is quite consistent with the Idealist position to make use of symbols and to form all the combinations possible of the Fundamental Processes; and hence finally to speak of the External World in the easy language of every-day affairs. In all this, it must be noted, a distinction must be made between an Idealist and a Berkeleyan, for a consistent student of the subject might find himself in disaccord with many of Berkeley's conclusions.

But to what have we arrived in this way? Simply to the position of the Common Sense philosopher, if he also has pursued his common sense keenly and determinedly enough. He does not suppose that he can know phenomena of the External World, or of another mind, except by the means by which he can know anything; that is to say, he is dependent on his Presentations in their widest scope, these Presentations entering into the system of knowledge already acquired by a mind formed by its original character and the total of all its experience. Did he hold any other opinions he would be an Idealist, but of a wholly irresponsible kind.

It may be said that the Common Sense mind acknowledges the objectivity of the other mind, while the Idealist does not. But let us look at this very closely. The Common Sense mind must be restricted to this: That after all he cannot think with another mind, he can only think with his own mind, and that though he give to the terms "objectivity" and "other" mind the utmost independence, yet the expression of that independence is a mode of his own mind.

Let the Idealist on his part allow the exploration of his own mind to proceed to the limits of its capacity, and let his attitude toward the unexpected, or the uncontrolled, in that exploration be

the fullest expression of that possibility, then he will have no ideas different from those of Common Sense, for if any should be discerned he should widen still beyond them the region of the uncontrolled or as we may say, without straining the meaning, the region of the independent.

Yet withal there may linger a sense of discrepancy. It is a real difficulty and in removing it we shall find the last secret of reconciliation.

The Common Sense man says: "I see a distant tree. By no means can I persuade myself that it is only an idea in my mind. It is an external object."

The Idealist may reply that, as it is resolvable into ideas, the Common Sense man is wrong in regarding it as external.

But both in this regard fall into a tacit error of gross materialism. When the Common Sense man says that the tree is an external object, we may ask: "External to what?" That it is external to his body cannot present any difficulty for the Idealist could never suppose that it was in his body. That it is external to his brain is evident.

But he replies: "It is external to my mind".

The true answer of the Idealist is not to try to persuade him that his actual view of the object is wrong but to ask him, "Where is your mind?"

The brain is not the mind. Those who have advanced the idea of every mental act having its physical correlative have been accused of "materialism", but they are confronted with an assumption, even though a tacit assumption, of crude materialism both of the Berkeleyan and Common Sense Philosophers, that the mind is in the brain.

The veritable Idealist who seeks the reconciliation as there must be in all things, says: Of course the tree is external to me or you, as you behold us. But the tree is resolvable into ideas. The ideas are not in the brain. The ideas that make the tree known to me are where the tree is, as known to me. The tree, so far as I know it, forms those ideas, which are, so far as I know them, my ideas of the tree. That impression of Externality is the exact measure and account of the relation of those ideas, which constitute the tree, to the other ideas, which constitute the interval between the tree and any object, such as my physical self, to which it is external.

We have already advanced beyond the Berkeleyan position, but in departing here from Berkeley[1] it is with the feeling that the

[1] Byron said of Berkeley in amusing verse:

"Though his speculations are too airy
For the airiest human head."

sort of mystical paradox associated with him had arisen not from his subtlety but as a consequence of certain perverse misapprehensions of his own.

Berkeley, pursuing his own theories to practical conclusions, became opposed to the scientific spirit, as indeed have most of the idealists—Fichte, Carlyle, Emerson—though not so frankly as Berkeley.

They hoped to " evolve " out of their inner consciousness some particular revelation, as though the whole world were locked up in a human mind and could be expounded by diligent scrutiny. The extreme of this position has been ridiculed by Carlyle himself in reference to the Monks of Athos who gazed intently into their navels to behold the spiritual world revealed.

All this is not merely contrary to the Spirit of all that has made our civilisation great, but it is, again, the tacit acknowledgement of crude forms of " materialism ".

The Berkeleyan who says: " Why go to the outside world for instruction? "—is open to the rejoinder even from one he has instructed: There is no outside world—that is to say, apart from idea. What we are doing is to endeavour to find the clue to the sequence of our own mind. When we see a stone drop to the earth we are endeavouring, by active exploration and acquaintance with ideas, to trace the whole series of vast associated ideas between all the ideas of stone and of earth; but we shorten cumbrous language. Further we say that the process of evolving ideas is one of spiritual energy, and does not consist in the restriction of the mind to the little plot that has become in any manner already " evolved ". If the whole world be contained in the inner consciousness of the mind, we explore the mind in examining the world. But we must now pursue the matter to the final issue. The Common Sense philosopher does not seem quite satisfied that the Idealist has yielded enough to make their views concordant. He says in effect: Suppose that your mind becomes effaced; permanently, you may not admit as possible; but temporarily, you must acknowledge as often occurring. What then becomes of the External World?

The Berkeleyan may reply that it vanishes. But then it is within his experience that he meets it again, constant in many parts, changing in others, all beyond his expectancy or control, all independent so far as the Common Sense man can use the term. Who, then, thus uncontrolled, has recreated the External World?

The Berkeleyan may stubbornly refuse to admit anything beyond something unknown in his own mind; but we have already considered the nature of Belief (*cf.* pp. 44, 50 and 144) and we have

found that in the mind of man it has no immutable character, that in fact it is the resultant of myriad dynamic forces, some only of which appear in consciousness, and that it is expressed in a sort of dynamic representation of the mind toward associated new phenomena (*cf.* pp. 73 (note) and 160). This dynamic relation of Belief is the ground whereon we test the Idealist. He cannot hold or acquire the belief that his mind is all, and that there is nothing objective to it.

But "by shoeing one becomes a smith" and by analysing Common Sense has become analytic. He demands to know what the Idealist means by objective, suspecting here also some discrepancy. He has noted, for example, that a thing gives different effects as tested by different means. It has a certain appearance in vision. It has another relation in gravity, as may be tested by weighing it. And if light from it fall on a photographic plate we find a certain image.

Is there, according to the Idealist, an unknown something which in conjunction with the mind produces an idea; or is the idea itself the whole something?

If, while admitting objectivity, the Berkeleyan clings to his position of there being one mind, and that the idea itself is the full account of the object, then since he must consider what we call other minds, even though he consider them part of the development of his own mind, he would be forced into a sort of pantheism when the one mind has neither control nor knowledge nor expectancy of a great part of its domain. But also if the idea be all and that idea influence also the other minds which are part of his own, there would be a sort of pre-established harmony of a futile character.

If, however, with the acceptance of objectivity the Idealist acknowledges also the objectivity (that is to say, the independence) of other minds, but tries to save the position that the idea is all, then, as that idea arises simultaneously in other minds, we have here again a sort of pre-established harmony, but we have also another conception difficult to form, that one idea can be simultaneously in two or more minds.

But if the Idealist move his position so far as to say two like ideas are produced simultaneously in two minds, then we have added something to the idea, and that something is a cause, in so far as we are entitled to speak of a cause.

The Idealist, forced even on the basis of his own belief to abandon untenable positions, now reviews the whole field. He says: Yes, but even the postulate of an unknown cause is resolved into ideas. By idea I mean not merely sensation or the remembrance of

sensation; not only ideas in association, but also the process of Disassociation; not only the contemplation of ideas known but the positing of something as unknown; not only ideas in time and space within the actual contemplation, but in time before and after, and in space not yet explored. In this way I can admit other minds in the most complete independence, for the process of Disassociation is a process of my mind. I can also in this way consider my mind effaced and yet the processes continued in other minds.

But this enlargement of the scope of his theory really means that the Idealist has ceased to confine himself in ideas and has adopted the Fundamental Processes as being necessary and sufficient for the explanation of all mental phenomena. The Idealist, in other words, simply maintains that his mind can only think its own thoughts, and this everyone will grant. The Idealist therefore consents also to employ a form of language in accordance with the ways of Common Sense.

Common Sense, by persistent questioning of the Idealist, has been forced to find a deeper base for his own notions, until at last on the ground of the Fundamental Processes he is able to avoid the errors that have heretofore arisen from mere superficial appearances. In this way the reconciliation is complete.

The argument will be now briefly resumed, it being understood that the positions are set forth here only for convenience of reference and that they should be considered in the light of the whole previous exposition.

Externality is capable of analysis. It is the result of the combination of impressions brought from all senses and wrought into associations with previous knowledge stored in a mind prepared by evolution through countless generations.

The position of the Idealist school is opposed to that of the Common Sense school. The Idealist says that the External World is a bundle of ideas. The Common Sense philosopher says that by direct perception one sees that it is substantial and objective.

The reconciliation is to be found in conducting the analysis of both to routes which lead to deeper comprehension and clearer views.

The Idealist must drop too narrow an interpretation of ideas in this sense. He must for this reason drop even Berkeley himself.

Various crude positions of the Idealist must be abandoned on the way; such as that his mind only exists; that the ideas of his mind sum up an object; that the brain is the mind; that an idea can have location in the brain.

The idea is where the object is, in so far as that object is known in idea.

To this it may be answered: We recognise the star Sirius as part of the External World; is the mind in Sirius?

The Idealist might reply that the question should really have been expressed thus: Is Sirius in the mind?

Before answering that question I will take a more familiar case. I see the handle of a door. I desire to go out of the room and I intend to turn the handle. But as I sit I observe the handle. Is the handle in my mind?

In order again to answer that question we should have to retrace all the previous analysis. It becomes evident then that the question has been begged in the manner in which it has been proposed. I find that when I say, "I see the door handle," I have present certain sensations, and part of these are designated by a symbol. By the very nature of the movements of the mind we do not, when we use a symbol, bring to consciousness all that the symbol implies, but step by step all that it implies may become duly developed in the mind.

Now the sense of Externality of the handle is immediately present, for it is implied in the symbol, and the association with all the complex of the senses involved in my consideration of the handle. Part of this complex is the series of the muscular sense and the Sense of Effort accompanying. I rise, and step by step as I approach the handle I am really developing what has been implied in the symbol, though this again is rendered more correct, that is to say, more in accordance with nature, and more diversified and vivid by the actual new experience. Finally I touch the handle. I say that each step of that progress, in so far as I may have had consciousness of it, has been expressible as a mode of my mind; and at each step my mind has been where my conception of the reality was.

It is possible, however, to object that that is not the same as saying that the thing is where the mind is. Let us consider all that is implied in the objection. It will be better seen by taking another mind. That other mind may have conceptions respecting the handle that differ from mine. Factors of my conception may be entirely absent in another mind. Must there not be something constant in reality which may serve as a standard to show which is right?

To answer this question clearly we may go back to the considerations involved in a former question. "Is the red I see the same as the red another person sees? To decide, can we refer to the actual red of the tablecloth?"

We are brought to the conclusion here that the red of the tablecover has no existence independent of the mind; that red is the

result of the vibrations from the table-cover, and the operations of the visual apparatus. And as we have noticed previously we converse of red because our sensations, though not the same, are correspondent for our purpose. In the case of the two maps previously discussed the correspondence itself may be far from exact and yet conversation may be intelligible.

But the notion is ineradicable that there is something objective, even if of a different kind from our sensations. Yes, and in our minds that is expressed by the something independent, something that brings new development, and, in a certain circumstance, even a fund of new sensations, something in fact that now, freely, and with a full understanding of the symbol, we may call objective.

But the positing of the symbol, objective, with all that it implies, is itself of the mind. We have required here, no doubt, a certain Process of Disassociation, such as we have discussed in considering Infinity. But that Process of Disassociation is not apart from the mind.

So now with Sirius. That is at first a symbol, and the symbol is where my mind is. But the symbol may have become developed step by step, and among its implications is that of enormous distance not possible to be traversed by my body. But step by step as I know of these matters I am knowing them by Processes of my mind, ultimately by means of Fundamental Processes based on Presentation of real things.

But the Objector says: Sirius exists, quite apart from your mind; it would exist if your mind were extinguished. I reply that my knowledge of Sirius would be extinguished with the extinction of my mind. When you say it exists you posit another mind. But my extension of Idealism has enabled me to posit another mind and to speak of what that other mind knows. I hold that all that I can know of Sirius is all that Sirius is so far as I can know.

Here it is important to be sure that we are not dupes of a mere play of words such as stood for reasons in the disquisitions of the Middle Ages.

Therefore I will introduce not new arguments, for I hold that already the connection is definite, but new means of throwing light on the obscurities of the subject.

To a person freshly operated on for cataract Sirius appears as if resting on the eye; to a young child it appears as something within reach; to a Chaldean shepherd it appeared as a light in a dome; to a Chaldean astronomer as an object in an ocean-like stream over the earth; finally when astronomy demonstrates to us that Sirius is enormously distant, so far from making an appeal

to the Common Sense realism, our science conducts the reasoning on the finest and most delicate mechanism ever built by man in the world of demonstration, the apparatus of mathematics.

Our notion of that aspect of objectivity which arises from immense distances is really built up in all parts within the Idealist ambit. Further, those distances are incomprehensible in terms of blunt common sense; they are unimaginable by any direct intuitive effort and they must be appreciated only by reference to symbols.

In all the discussions we must bear in mind the enormous importance in the mind of symbols, and we want to give great weight, in the understanding of Externality, to the teachings of others, incessant in character from our childhood and depending not alone on the experience of those others but also on their heredity. The whole line of the previous argument must be enriched and supported at every turn by the accompaniment of the great influence of Heredity, personal Experience, Symbolisation, and Language generally; Education in its widest sense.

Now let us return to the question of the veritable meaning to us of Sirius. In an ordinary way, and especially in dealing with it in the mathematical calculations which have convinced us of the greatness of the distance from us, Sirius is simply an object symbolised.

As we endeavour to realise Sirius, then step by step we trace out, and again by symbols, some aspects of this object, and as we develop again the meaning of these symbols we see that finally we must come to a starting-point where we meet with the unanalysable, that is the Fundamental Processes, and we find that in the synthesis we will retrace steps such as we studied at the beginning of the chapter. At no time do we step out of Idealism, and at every point we can say nothing other than that our idea of the object is where the object is when the object is in our series of ideas. If it be distant then it is either a symbol or else the distance is brought home to us by the series of the ideas representing an immense complex of sensation and efforts between us and the object at that distance.

Then the objector cries: At last we have the real paradox. Is there not a difference between all that you can know of the world and all that the world is, as possible to be made known to you?

Here I recognise the veritable crux. We are face to face with a paradox again as deep, and as fundamental apparently to our powers, as that of Infinity. We solved that of Infinity by a Dissociation, or leap of the mind, with the formation of new Associations. We found that we had to stretch finity to infinity

if we desired to compare finity to infinity. Similarly here we must stretch the meaning of the term "independence", or objectivity, to the extent that the mind can only appreciate it by a process of Disassociation even though it recognise that process, and the subsequent process of Association, step by step, as processes of the Mind.

Having then kept the Idealist uncompromisingly to his own Idealism, having escaped the Serbonian bog of materialism into which Berkeley fell, and having pursued with the utmost energy his own well-begun but timidly abandoned analysis; having illuminated Common Sense and compelled it to sacrifice all that was not, in the clearest vision and with the aid of deeper analysis, consistent with fundamental Common Sense; we may declare that all that a mind can know of the world is known through the modes of that mind; that still the External World is real and external; that we reach, at the limit, something indistinct of separation, which we can bridge by a leap whose character we know as a mode of the mind; and that henceforward we speak of the External World in the familiar terms of Common Sense. And withal we must observe that the ultimate limit of division between illuminated Common Sense and determined Idealism is so indistinct that we cannot say definitely whether the leap spoken of is necessary or whether it differs at all from that discontinuity which is always in the mode of movement of the mind.[1]

[1] The whole argument in this chapter is so serried that a general summary cannot well be given.

CHAPTER V

EGO AND WILL

NOTE:—The question of the recognition of the Ego is so much involved with that of the Will that it will be found advisable to discuss them in close association.

I begin with a very simple example and I make use of this to exemplify the gradual education of a child to the conception of its own personality, the Ego. Then I discuss the general question, Will, and in the first place by citation of the views of great authorities, I show that these fail to offer a satisfactory account. Then I undertake an analysis based on suggestions arising from the consideration of the Fundamental Processes. In this way we arrive at a clearer notion of Will and of what is called Freedom of Will. Then we resume and complete the study of the Ego.

(1) *Synthetic Exposition of Will and Ego*

We may now select some simple illustrations by means of which to refer in more synthetic form to certain positions already discussed, and to observe arising in the discussion the forms of the Will and of the Ego.

A child sees a live coal.[1] He sees it at first in his mind; it is an Immediate Presentation altogether as a Unit. He stretches out his hand and touches the fire. He starts back in pain. This incident could be made the test of volumes of matter, for it opens up the whole ground of Psychology. Into the question of Locke's *tabula rasa* (*cf.* p. 27 and 32 (note)) or the doctrine of innate ideas, there is no necessity further to enter.

With respect to the movements of the child it must be noticed that many of his efforts result from no particular stimulation from without, but from abundant impulses within the brain. The child's brain is a magazine of energy which does not completely explode because to each impulse there is an inhibitive or controlling power. Sometimes the locked-up energy escapes control. We see this in an unhealthy form in St. Vitus's Dance and epilepsy, and in a healthy form in the lively play of young children.

[1] The psychology of children has been studied with particular attention of late years. In addition to well-known names such as those of Darwin, Preyer, Romanes A. F. Chamberlain, Wundt, Hoffding, Baldwin, Perez, and Storring, we find more recently Egger, Lindner, K.C. Moore, Oltuszewski, F. Tracy, Fr. Warner, Clara and William Stern. Amongst psychologists who have particularly directed their studies to the language of children may be mentioned Schultze, J. Dewey, E. Noble, E. A. Kirkpatrick, H. T. Lukens. The famous mathematician, Sophia Kowalewski, studied her own childhood; Perez has written on the first three years of childhood; Elizabeth S. Brown deals with the *Baby's Mind*; A. Kussmaul has published *Untersuchungen über das Seelenleben des Neugeborenen* (*new-born*) *Menschen*; H. Fehling has studied *Das Dasein vor der Geburt* (Being before Birth); and old Sir Thomas Browne gave a delightful philosophic conversation between twins who were waiting to be born! Treitel, A. E. Tait, and Dr. Eva McCall have written on Aphasia in children.

Superadded to this, however, are definite impulses due to pleasurable stimulation; the Hedonic Sense is in activity. The child would like to get the live coal, because its brightness is pleasing. He sprawls about, but he gets nearer. The sprawling is due to lack of co-ordination of the muscles, but in this sprawling he is helping his growth and educating his brain.

The difficulty of co-ordination may be seen in a man of advanced years attempting to learn an intricate dance. His case is more difficult, for he has to thresh out new brain connections, whereas the child has but to develop his natural growth.

This is in the movements of the child an illustration of instinct. If the live coal were pleasant to handle he would probably put it in his mouth. In judging the instincts of animals we must remember that the world of smell, as in a dog, is immensely more various and discriminated and more stimulating in its effects than with us, and that the propulsion of the muscles which follows thereupon is a matter of less strenuous exertion. A dog hunting on the trail is kept going by the persistence or the increase of the same kind of stimulation serving continually to produce the same kind of action. The action of a child sucking a lollypop is of the same class.

The same action of the child in sprawling towards the bright object introduces no extraordinary element; but from this experience we shall be able to talk of Reason. We will find that there is no line of separation between Instinct and Reason.[1]

(*Cf.* pp. 46 (note 2); 99 (note)).

Note on Instinct

The question of Instinct will be here briefly dealt with. For convenience of reference the matter may be thus expressed.

1. In the study of instinct importance should be given to the

[1] A symposium on instinct has appeared in the Journal of Psychology. An article by Wm. McDougall seems to me especially good. Lloyd Morgan and Dr. Myers express views in general accord with those set forth here. McDougall laments that Stout rejects human instinct. Wildon Carr follows Bergson, and with him I find complete disagreement. It will be generally found that those who have actually studied animals are the least inclined to make a barrier between their faculties and ours by virtue of instinct. In a study of protozoa (Hodge and Aikins, 1895) Hodge expressed the opinion that even unicells have some form of consciousness. Hering's theory of memory is bound in the supposition of psychic impression in single cells. Amongst others who have considered the psychic element in unicells, may be mentioned Metalnikow (*Archives de zoologie expérimentale*, Vol. xlix, p. 373); A. J. Stuart *On the Physics and Physiology of Protoplasmic Straming in Plants*; W. B. Hardy, *The Physical Basis of Life*, *Science Progress*, 1906; Gary N. C. Calkins, *Protozoa* (p. 301); G. Bohn, *La Naissance de l'Intelligence*; H. S. Jennings, *The Behaviour of Lower Organisms*; M. F. Washburn, *The Animal Mind*; A. Binet, *The Psychical Life of Micro-Organisms*; G. C. Crompton, *Paramœcia* (*Archiv. fur Protistenkunde*, Vol. xxvii); *cf.* also Prof. F. Darwin's *Presidential Address to the British Association*, 1908. One of the most valuable studies conforming to the conclusions here given is that of Etienne Rabaud: *Document pour l'étude des rapports de l'intelligence et de l'instinct*; *l'Année Psychologique*, 1920.

fact that certain animals have a great range in regard to some one sense or other, or possibly to several senses.

2. At the basis of our reason, that is to say, in the Fundamental Processes, there is an analogue of instinct, for, the conditions being present, the functions are produced automatically. This may be seen in considering each of the Fundamental Processes in turn.[1]

3. The whole scope of application of Association must be widened. There becomes ultimately involved the consideration of the entire sensory and motor conditions of our existence.

4. In considering the manner of corresponding associations in animals, regard must be shown to the different intensity of emotional states associated with certain objects and the different constitution of the associative motor conditions.[2]

5. Even before previous individual experience there has been developed throughout the existence of the species a ground-work that determines the subjective character of the experience.

6. On these general lines an exposition might be set forth that would show no solution of continuity of evolution from instinct to the highest grade of intelligence.[3]

* * * *

Resuming now our study of the child; the child touches the live coal, and starts back with a look of affright before he has even properly appreciated the sensation of pain. A great many questions which cannot all be discussed here have been opened up.

The expression of the emotions is of interest to us for it shows to us that the effects are not confined to a single strand of nerves.

[1] The experiments of Galton, Hachet-Souplet, L. Edinger, Ed. Claparède, and others are instructive in this regard.

C. Groos *On the Play of Animals* may well be consulted. In addition also to well-known standard works such as those of Lamarck, Darwin, Lord Avebury, Ch. Bonnet, and others already cited, we may note for curious interest such researches as that of W. Wagner, *L' Industrie des Araneides* and J. H. Fabre, *Moeurs des Insectes*, and some elaborate studies in American Universities on *The White Rat* (Small), *The Education of the Perch* (Triplett), *The Animal Mind* (Mary Floy Washburn), and allied questions. Note also, in regard to human capabilities, Arthur Carr, Hon. Sec. to the Blind Social Aid Society, says: "I know a man who though quite blind can tell the height of buildings through his highly developed sense of hearing." M. L. Truschel, has quite recently investigated this matter elaborately and has concluded that the perception is due to the refinement of the appreciation of sound. At the Institution of the Blind in Paris, M. Truschel concluded that the perception of solid objects by the blind is due to the fact that the objects reflect vibrations of air and so affect the auditory sense. To the objection that a deaf mute had been reported as possessing this faculty of perception he answers that persons deaf to speech and to music are often sensitive to very feeble noises.

[2] The human eye distinguishes, in normal vision, from red (400,000,000,000,000, vibrations per sec.) to violet (750,000,000,000,000, per sec.) and the human ear from thirty-two vibrations per sec. (even 16 vibrations per sec. have produced sensation) to 16,384 p.s. The wolf responds to 6 v.p.s., and the cat hears sounds corresponding to vibrations which make no effect on human ears.

[3] Professor Lépinay, whose lecture at the New Dogs' and Cats' Home near Reuil has been referred to, is of this opinion. Many other eminent scientific men have held the same view, but I do not wish to impress their mere authority.

The shock sends a storm of explosions throughout the brain and thence throughout the body. The whole action has been involuntary—the starting back, the shaking of the body, the straining of the muscles of the countenance, the catching for breath, the cry. Then follows a definite feeling of pain.

The child has made a very impressive experience. It is but a type of the world of stimulations of pleasures and pain by which our actions are influenced. Some of the more subtle of these, it is true, could never be analysed in terms so plain as in our example, but to such it stands in the same relation as instinct does to reason, or the rudimentary stage of an organism to the developed creature.

If we attempt to repeat the experiment, the child starts back before actual contact, and the whole process differs from the first in being more subdued and in losing altogether some of the minor features. Again, a number of interesting problems are opened up. We are face to face with Memory. Repetition of the process produces facility of conveying stimuli, but the ideas corresponding to the Presentations in Association produced by the original storm are weaker than the Presentations themselves. They give to Memory the features by which we know it.

The original experience brought vivid Presentations and Associations not only of the live coal, but the general appearance of surrounding objects. The ideas of these are subsequently less vivid, less persistent; and they are moreover over-swept by the Presentations of the immediate reality.

Let us consider a slightly different example. We see a lamp on a table on which are books, ornaments, or other objects that also attract the attention. The impressions we receive are more than the sum of the sensations, for these Immediate Presentations have not only definite and intricate relations between themselves, but also they are associated with previous experiences.

Now let us propose that we hold the same lamp at another time and at a different table. The Immediate Presentation of the lamp has a correlative in the same centre in the brain as before; and because the paths have already been opened up and so made more facile, those parts of the brain become awakened to activities which correspond to the image of the table that was associated with the lamp on the first occasion.

There is then the Immediate Presentation of the lamp repeated, and the image of the table, similar to but not less strong than, its former Presentation. But also there is the Immediate Presentation of the second table which at once overrides and submerges the image of the first, and continues to respond to all tests of reality, viz., Association of Presentations from all the senses, and

capacity for yielding new sensations. Thus though the increased facility of responding to stimuli like those producing a previous sensation is the essential of Memory, yet Memory is made known to us by the ideas in Association, and the distinct Memory of a specific object is rendered clear by certain associated ideas. It may happen that the recurrence of our experience may give a peculiar agitation to the mind, but which in its vagueness may not seem to us definitely to be of the kind to which we apply the term Memory. Thus we often say, " I am also sure that I have seen that face before, but where ? " (*Cf.* remarks on this subject in the Chapter on Memory, the *déjà-vu* pp. 218 *et seq* ; 220 (note)).

In such a case the facility of surrounding paths eventually leading to present realities has not been great enough to produce definite images in Association with those reproduced, but the stirring of these paths makes itself known in consciousness.

In other cases there may be a definite Memory, but it may not be possible to establish whether a certain experience thus revived in the mind was before or after certain other experiences. What there is needed for a complete recollection is not only the Association with definite images, but also a continued Association of such images with experiences that lead by recognised sequences of time up to the present reality. There may be all grades between this and an unconscious capacity for receiving stimulus more readily.

Suppose now that for the live coal in the experiment we introduce something closely resembling it, say a ruby. There is on the child's part a great repugnance to touching it, and the approach brings up a gathering and painful storm ; but when the finger is actually brought into contact with it there is a sudden collapse of the agitation and the entry of a new set of sensations. The shock of surprise is agreeable, and the activity of interest impresses the incident on the mind.[1]

[1] Such a shock of surprise contains the elements of one kind of humour. Another kind is formed in the collapse of an elaborate expectation of pleasure, but in such a way that any feeling of pain is small in comparison with the mind's capacity for recovery. The essential point in this also is not the disappointment of the expectation, but the recovery from the disappointment.

An example a little more complex than that of the child is where at an evening party a young man rises to sing and finds that some one has tied his coat tails to the chair. He turns round, sees what is the matter, extricates himself and joins in the laughter of the company. If he had been unable to rise on account of a paralytic stroke, it would not have seemed at all humourous. The humour does not arise until the pleasurable features have submerged the others.

If now we consider humourous recitation we shall find that the elements of humour are here similar. Take Hood's little piece, " We're putting a gaspipe down." A reciter works up the feeling of mystery and horror, then comes the homely fact regarding the gas pipe.

The pleasant surprise of a relief from the tension of overwrought feelings accounts for the success of judicial jokes. Remarks, not apparently very humourous, let fall from the judge often cause great merriment in court, except possibly to the parties against whom they are directed. For example, an Irishman condemned to death asked Lord Norbury for a " long day," meaning thereby a long interval between the sentence

Considering the child once more, we find that in the experience of a ruby which he mistakes for a live coal and towards which he places hi self in an attitude corresponding to that belief, we have the Process of Agreement. But when he touches the ruby, Discrimination arises by the Negation of the Agreement in certain particulars. What particulars are still contained within Agreement afford the grounds of a Generalisation which admits of a classification by taking into account also the particulars discovered in Discrimination. And we have also symbolisation, for that is the mode in which we obtain the terms " live coal " and " ruby." The child, though not acquainted with these names, symbolises nevertheless. Its symbols are formed by visual appearances perhaps in combination with the indications of other senses.

When our experience is repeated, and when certain sequences are observed to be invariable, the tendency is to symbolise a whole plexus of Presentations and ideas by means of the first clear indication—formed by Presentation and associated ideas—of the plexus following.

For example, in writing I dip my pen again and again into my ink-bottle. Temporarily I am placed in a fairly constant dynamic relation to all my surroundings, and if I require to fill my pen I find it sufficient to glance up and observe an indistinct black object—really my ink-bottle—and without thinking take a dip. If my attention be drawn to the matter, I find it possible to make an elaborate analysis.

and the execution. The judge replied: " Yes, to-morrow is the longest day of the year. You'll be hanged in the morning." Such sallies of wit amuse the court, but appeal only to a prisoner exceptionally gifted. It will be observed that the appreciation of humour depends not only on the form of the humour itself but on the attitude and temperament of the individual.

For example, if a young man who had his coat tails tied were a nervous and sensitive young man, his state of painful embarrassment might exclude any sense of humour. At the other end of the scale we have the story of the Irishman who kept laughing while he was being flogged. He explained that they had got hold of the wrong man.

For some kinds of humour a certain spirit of fortitude is necessary, for where painful incidents occur we have noted that the capacity for recovery must be elastic and extensive beyond that scope.

Hence it is that humour, and good humour, are often conspicuous in men of great courage and strength. The appreciation of humour is also influenced in individuals by the degree in which they enjoy laughter in itself. If in addition to this there is found considerable fortitude in bearing the pains of others, unusual incidents of humour may arise. Thus three Burmese were sitting on the bank of a river as a gun-boat went up. A cannon-ball from the gun-boat took off the head of the middle one. The other two laughed consumedly.

We may carry the principle even to the consideration of such a book as *Don Quixote*, where the humour is not merely in the incidents but pervasive throughout the whole conception of the story. Here we have the colouring of the mind of Cervantes himself who, an idealist, had found how often the ideal becomes defeated by its own extravagance; and, with a philosophic appreciation of things, " laughed his life's regret."

One might expect from what has been said that humour might also be found in animals. That is certainly the case. I have made some study of the matter in papers published elsewhere.

It seems then that the most subtle sports of humour might be typified by this example of the child, just as the processes of Reason may also be indicated from the child's experience.

All this is in accordance with what we have repeatedly observed, that ideas in Association have the facility of their Association increased by exercise. The facility of the paths of the physical correlative is likewise increased. The process involved is automatic and inevitable; it is Agreement implying in turn Generalisation, and is what we have called a Fundamental Process.

The child is not singular, therefore, in using symbols, although not yet acquainted with words. Such an observation should be borne in mind in studying animals, for though in order to express their meaning they use their own words, which are complex symbols produced by a combination of muscles acting on a sounding apparatus and producing their effect on hearing, they also use other symbols, some of which produce their effect on the visual sense, or on the tactile sense, and are formed by combinations of the movements of other sets of muscles.

Even in highly developed regions we find men working with symbols of their own. John Hunter, the surgeon, and Turner, the painter, are curious examples of men profoundly versed in their respective subjects, but deficient in the power of making their subjects clearly intelligible.

Turner, especially, when endeavouring to discourse on art, seemed to be incomprehensible. The explanation is that his study of art having been greatly original, he was not conversant with the phraseology of the schools, but was accustomed to think of certain things by signs and names of his own. Hence he was in nearly as difficult a position as if he were speaking in a foreign language with which he was little familiar.

The Sense of Effort is manifest in the energetic manner of appreciating sensations, while the Impulse is shown by the recoil of the child when lack of Discrimination had produced false routes of ideas in Association. The Hedonic principle is indicated in the character of the sensations. The succession of Time is also appreciated, and the forms of Space at each Presentation. The Unit varies at each moment of attention.

If the experiment with the child be repeated, there will be a conflict between the two Impulses. There will be a hesitation in action. With the interest awakened there will be a closer attention, and with the new experience there will be a certain Discrimination of the ruby as distinguished from the live coal. Here we have a complete example of reflection and of judgment.

Reflection is the operation of the mind, by which the diverse factors of a question are considered; and these factors are resolvable into the Fundamental Processes applied to the definite objects and ideas. Reflection prepares the way for belief. We

speak of judgment where Impulses are directed according to belief.

In attention we have a certain disposition of the body and we know that this involves the flow of blood to all the parts involved in this particular activity, so that they become stimulated to their effort. This physical correlative of attention is marked by appropriate sensations, although they do not all arise within the scope of the "five senses"; and Immediate Presentations, the Sense of Effort, and consciousness of the Unit involved, serve to make up attention.[1]

The sensations referred to are derived from the consciousness of the bodily changes produced by the flow of the blood and the redistribution of the nervous energy. They have their own Hedonic quality.[2]

But the experience of the child also makes known a new position; for the pleasurable desire, or the prompting to decision, having overcome the impressions of danger, we have an example of the exercise of Will. Let us examine with special attention this question of Will.

(2) *Will*

The difficulty of dealing with the problem of the Will has arisen from many sources. On the one hand, the Will played a great part in the fantastic and barren theological discussions of the Middle Ages which constitute symptoms of a kind of mental aberration. The minds of men had become directed to wrong routes, and it required the genius and sacrifice of a few great thinkers, from Roger Bacon to Galileo, to trace out by help of positive science some sure principles of direction in the otherwise hopeless maze.

Will had become elaborated into a sort of definite entity in order to secure that sense of responsibility which would warrant punishment for disobedience. Will was the arbiter, and a mistake of judgment, or at any rate, of decision, of this arbiter was fraught with serious consequences to the lord of that will.

[1] Attention which is here treated incidentally has been made the subject of long memoirs of celebrated philosophers, and has even formed the keystone of the system of some. If, however, the previous exposition be correct, it will serve to throw a new light even on the arguments derived from partial views. Jean Paul Nayrac has written a book on *Attention* with a preface by Ribot, whose writings are also affected by the prominence accorded to this position. Nayrac says: "It (Attention) should be placed in the highest scale of the psychical hierarchy." A book well worth consulting, not only in regard to this subject, but for the originality of its views and the boldness of their expression, is that of Heinrich: *Die moderne physiologische Psychologie in Deutschland, eine historischkritische Untersuchung mit besonderer Berücksichtigung des Problems der Aufmerksamkeit.* (Zurich).

[2] Herein we may find the rudiments of the aesthetic feelings of rhythm and harmony, and the various subtle likes and dislikes in music, poetry, painting, and sculpture.

But to give to such an ethical system a substance of justice it was necessary to reserve to that will a certain liberty of choice, hence the doctrine of the Freedom of the Will.

Side by side with that freedom, however, there was a notion corresponding to our own scientific principle of causality and order. To us moderns the full expression of this notion is found in the principle of determinism. To the devout but anxious and mystical minds of the thinkers of the Middle Ages, this notion derived from the contemplation of a Supreme Power, God, directing the Universe to certain ends. Giving therefore to God the attributes of omniscience and omnipotence it seemed impossible—to some at least, for anything absurd was possible to the majority—to escape the conclusion of predestination. How then was it possible to reconcile the doctrine of predestination with that of the Freedom of the Will?

Kant's Germinal Idea

That indeed might be said to be the germinal idea of Kant's *Critique of Pure Reason*, for the Kónigsberg philosopher had both the advantages and the disadvantages of deriving from Scottish ancestry saturated with mystic imaginings and psychic struggles, and of cultivating his intellect in a milieu of thinkers who were experts in the indefinite phrasings and cloudy speculations of the Middle Ages.

Kant mixes up the question of Will with that of reason in an inextricable entanglement; but his mind, capable of fine scientific work when kept within the true controlling grooves, perceived the necessity of an analysis of reason itself. This analysis was carried out in tentative and imperfect manner, as we may note in the discussion of the Categories examined in the light of the Fundamental Processes.

The difficulty of reconciling Freedom of the Will and Predestination gave Kant great concern. He does not face the problem in a scientific spirit. All through that vast *petitio principii* which constitutes his *Kritik*, he cries that he must save the Freedom of the Will.

He notes various paradoxes such as that of Freedom and Predestination; he calls them antinomies, and finding that they cannot be reconciled by reason, such as we know it, he imagines a world—the transcendental world—where neither reason nor will depends on the senses and where they work in harmony as between themselves and in due accord with predestination.

This transcendental world of Kant has dominated the minds of great authorities and the teachings of Universities for generations,

just as the doctrine of humours dominated the minds of great doctors who before Pasteur knew nothing of the real nature of disease, or as phlogiston was a sacred principle in the minds of the law-givers in chemistry before Lavoisier reduced it to non-existence. Amongst the results of this present work, but only as a corollary, will be found the annihilation of the Transcendental world.

I have begun with this reference because the influence of Kant, and hence of the Middle Ages, still infiltrates into the minds of most writers on Will.

Diverse Views of Great Thinkers

We will now briefly refer to others who have investigated the problem, if only to indicate how vast a literature has gathered round the question of Will, and how insecure is the reasoning of the great authorities.

James Mill analysed Will into what we may briefly speak of as something of this kind: the recognition of that feeling of pleasurable excitement which has in the conflict of feelings proved itself to be the strongest.

This analysis has been accepted by most of those influenced by the Utilitarian school of the Mills and Bentham, or by the Hedonistic school of Herbert Spencer.

Of course the term "pleasurable" must bear no restricted implication. Yet even so the analysis does not appear to me adequate.

Binet's *La Psychologie du Raisonnement* is a very interesting book, but I do not find it all conclusive; for had I done so I would have considered this present exposition, which really derives from the study of reason, as superfluous. [1]

Binet points out the baffling nature of the problem of reasoning.

[1] I do not know any French book in the psychological domain which is really original and basic (*grundlegend*, as the Germans say), though there are numerous interesting treatises. I do not say this in discredit of French intellect, which has produced in various spheres so much that is marked not only by the brilliancy of genius but also by the solidity of labour; but that makes it only more remarkable that no figure exists in France comparable in the realm of psychology to Locke or to Spencer. Condillac whom John Stuart Mill disparaged for fastening on the weaknesses of Locke, seems to me to have been worthy of a much more appreciative criticism. It was the constant endeavour of this great and sincere French thinker to analyse the nature of reasoning, and to direct his analysis to the elements of the process of thinking. The main problem of Condillac is not unlike that which has been here proposed; but I do not consider Condillac's analysis either fundamental enough, or lucid enough, or sufficiently free from involvement. Condillac's efforts are very remarkable for the period in which we find them.

When I consider, however, the work of a modern school of French writers, and contrast with the heavy obscurities of official German philosophy the clearness of the expression and the activity of these bright minds, it is there that I am inclined to look for the lead in future developments. Even where I cannot wholly accept the philosophy as in the case of Boutroux, I feel my spirit nevertheless invaded by the charm of style. But I cannot find that in any French thinker the purely technical work of analysis has been carried out to the last resort.

One old French school contented itself by saying that we reason by a faculty of reasoning; this is vaguely reminiscent of Molière's comedy. Others say that the problem is "irreducible." Taine does not offer a solution. Binet notes the failure to answer the question: What is reason? The analysis of Wundt, who reduces it to sensation, or that of James Mill, who is satisfied with association, are both too bare; they do not consider all the factors.

Binet is not content with describing mind merely as "that which reasons". And here it may be said that we do not find additional illumination from such definitions of mind as that offered in the *Analytique de l'Esprit humain et de la Vie* by Dr. J. A. Molinié: "A faculty of high differentiation as a superior polarisation of energy which it co-ordinates or unifies." Binet is satisfied with Herbert Spencer's exposition, and this he epitomises by the term "classification of relations." The French thinker himself formulates a "law of fusion", and he cites suggestions of Lotze, Wundt, and Helmholtz playing round the same notion.

But whether their positions could be established as true or not, what strikes us particularly is that the analysis has not descended to the elements so as to show how with these elements all processes may be built up synthetically. That has been the central problem of this book; if the analysis here presented were insufficient, or in some other way faulty, yet even the gainsayers would find that the true solution would be obtained by rectification of that analysis. Once that sea has been discovered, no other route can be thought of as alternative.

In the question of Will, Binet cites Ribot with approval: "The 'I will' gives evidence of a situation, but does not create it. Volition, which former psychologists have so often observed, analysed, noted, is not the cause of anything. Acts and movements which follow result directly from the tendencies, sentiments, images, and ideas which have arrived at co-ordinating themselves under the form of a choice. It is from that group that comes all its efficiency."

But the whole pith of the question is, What is the meaning of co-ordination under the form of choice?

Ribot introduces by a preface the work on *Attention* by Jean Paul Nayrac, in which the following passage occurs: "Attention is a general phenomenon which has a repercussion on all the psychical life and necessitates the co-operation of all the principal functions of the organic and mental life. It should be placed in the highest scale of the psychical; with the aid of effort and will it forms our faculty of mental adaptation."

William James, on the other hand, considers that effort and

attention are one and the same. In the course of the work Nayrac discusses whether the motor or the sensory side plays the more important part in attention, and there is found the usual divergence of opinion of celebrated physiologists and psychologists.

In Nayrac's passage I find terms used that are as difficult to define as attention itself. It seems to me that what I would mean by attention involves the Fundamental Processes particularly of Conception of Unit, Immediate Presentation, and the Sense of Effort; and if read in this light the character of Nayrac's passage is clearer, and the difficulties of the phsyiologists are capable of solution.

Some may have expected to find a factor of Will in attention, but I think that if the matter be closely regarded a distinction may be made. Thus, for example, we may by exercise of Will place ourselves in a situation to receive a sensation, but the sensation is not to be confused with the act of Will. The subject is, however, in part a matter of definition, though that again involves a question as to the degree of analysis. If Impulse, as a factor of Will, be thrown into relief, then we have a more determinate act of attention, and one involving an addendum to that previously considered. Similar remarks, in regard to diversity of application, would apply to attention as to Will. [1]

Ribot in his book, *La Volonté*, offers explanations which give to Will something more distinctive that would appear in the passage cited from that book by Binet: "Will, on the contrary, resolves into volitions of which each is a 'moment', a form instable of activity, a resultant varying according to causes that produce it."

This passage, if read in the light of all our present exposition, and expressed in accordance with the terms employed, is not unacceptable.

Further, Ribot disagrees with Maudsley and Lewes, and consequently with the whole school of James Mill, that Will can be analysed to an "excitation caused by ideas." He objects that this is simply a case of "laissez-faire." He says of Will: "But it is quite another thing. It is also a power of arrest, or, to speak the language of physiology, a power of inhibition."[2]

[1] So vast is the literature of Attention and so full of vitality—one of the latest works comes from Buenos Ayres: R. Senet, *Teoria de la Atencion*—that it seems hardly professional to dismiss it incidentally. Yet the fact that other syntheses—Memory Will, Judgment, or what not—have been taken as the keystones of systems simply indicates the necessity for analysis to the Fundamental Processes. In this view the matter becomes clarified.

[2] The mention of inhibition suggests the research by the physiologists of the "centre of inhibition," and we have the inevitable discrepancies based on the usual deficiency of analysis. Setschenow places the centre of the inhibition in the region of the optic thalamus and the corpora quadrigemina; Goltz and others in the cortex; Ferrier opines for the moderating centre in the frontal lobes. As well might we search, it seems to me, for the centre of Memory, or for the centre of delight.

In conclusion Ribot seems to weaken in his appreciation of Will. He does not accept the "laissez-faire" doctrine, but his own analysis is not definite. Previous to the passage cited by Binet we find · "Volition is a state of consciousness which results from co-ordination, more or less complex, of groups of states, conscient, subconscient, or purely physiological, which all united are translated into action or inhibition."

Of the great mass of literature on this subject a contribution which deserves to be specially noted is that of the great biologist Loeb. His analysis of Will agrees with that of the James Mill or William James schools. He says that the notions of the metaphysicians with regard to the Will are the outcome of an illusion due to the necessary incompleteness of self-observation. And further he remarks: "I think that we are justified in substituting the phrase 'activity of associative memory' for the term 'consciousness' used by the metaphysicians."

Here we find evidence of the impatience of a scientific worker in the presence of the absurdities of that mediæval philosophy which is still so carefully nurtured. But in ridding himself of a notion of an entity, Will, Loeb has been carried too far; for he gets rid of Will entirely.

The Only Valid Method of Analysis

It is significant also that all those who have wrestled with the question have involved it with that of reason even to the extent of confusion. Now, in setting forth the Fundamental Processes we have solved the main part of the problem of Reason, for its activities represented in schematic forms are formed from combinations of those Processes. Is there then any place in this analysis, or in subsequent synthesis, where we detect the operation of Will? In searching an answer our attention becomes directed particularly to Impulse.

Impulse is the process that directs at each moment the course of mental activity. In this Fundamental Process we gain a glimpse of the elementary activity of what we speak of as Will. Remember that so far we must think of all this expressed in rudimentary form, just as in the case of Memory we found it in rudimentary character related to every element of a complex thing.

At this stage we may return to the study of the mind engaged in simple activities.

We have seen in the case of the child, in one of the simplest examples we could find, that the Impulse corresponding to a certain path of ideas in Association could overcome, if not the impression of pain, at least the weaker impression of a possible danger.

In this case there does not seem to be clearly defined a balancing of possible pains and possible pleasures with the resolution of selecting a certain course of action accordingly. Or if confusion arise by what has been said about judgment, we may go back to the original incident of a child and the live coal.

Here we saw the child sprawling toward an object whose properties were for the most part unknown. Certainly the brightness acted as a pleasurable excitement, though another child might have the sensation of brightness as acutely but not a similar Impulse to action. The Impulse to action depends on the whole constitution of the child, and the brightness is not so much a cause which sets in motion the whole machinery as something which removes an inhibition or disturbs a temporary balance. To continue the metaphor of the machine, the stimulus removes the catch.

When the child touches the live coal he starts back. In this case there is no weighing pleasurable and painful emotions. The shock comes before the clear sensation of pain, and the storm overrides the weaker Impulse derived from desire.

The decision has been automatic, but it may be repeated that, when the conditions are determined, the Fundamental Processes are likewise automatic.

When we speak of any process being automatic, or subconscious, we do not thereby relegate it to a realm of hazard. The physical correlative of this has been formed by processes of evolution, by the selection of resultant forms which have been proved the best adapted to safety in experiences of which that of the child is a type.

When the child sees the ruby, the ideas and the Impulses produced by the brightness of the object are strong. Moreover, the repelling Impulses depend on Memory, and they are therefore weaker than the original, and Discrimination has operated on account of increased attention; consequently the Impulses tending to make the child touch the object begin to prevail. The Impulses are continually receiving fresh stimulation.

Here we see in an elementary form a factor that must always be taken into account in considering Will. The continual repetition of an incentive under circumstances of fair constancy will often cause a man not remarkable for will-power to accomplish a considerable undertaking.

When the child's new experience is completed it has added something to its stored-up knowledge which modifies the previous resultant. It has found that some present pain must occasionally be faced and overcome in order to obtain a future benefit; but

this is not registered simply as matter of profit and loss to the account of pleasures and pains.

There is an immediate satisfaction, or Hedonic principle, in the resisting of contrary Impulses, as in overcoming pain or putting forth new exertion.

This principle is stronger in some natures than in others, and it has a direct dependence on the vitality and energy of the person. It is one of the rudimentary factors of the great incentive of ambition, and is, as moreover ambition may be when not directed to selfish aggrandisement, one of the noble qualities which have their sanction deep in human experience and in the factors that have made human greatness.

For the same stimulus, pleasure and pain differ in individuals, and this difference also depends on the vitality and vigour of the individual.

Every subsequent experience of the child will not only give it fuller information of the world, and extend in its stored-up knowledge the associations between ideas and emotions of all sorts, but it will help to form habits according to the physical condition, the general character, and temperament of the individual. These habits are as important in regard to Will as they are in regard to Memory, or any other form of mental activity in which Association has part.

They indicate too that in the same sense in which Memory can be cultivated the Will can also be cultivated. If we extend the comparison we note that Memory has been considered as not a distinct faculty, but as pertaining in its peculiar quality to each idea. Will, according to the mode of representation we have arrived at, would be manifested as the resultant of a combination of Impulses, against contrary Impulses; and each of these Impulses has its own peculiar force.

But the case of Will may appear to differ from that of Memory in that we have to deal with a combination of Impulses, whereas in Memory, though the recollection depends on the force of a number of ideas in Association, yet the idea to be sought in Memory is single.

The difference is mainly due to use of popular language wherein Will is taken to imply some massy resistance; but in every conscious act, even the most simple, there is an expression of Will; whereas again in Memory there is a conflict of Associations, and the idea sought is involved in other Associations. Yet the distinction between Impulse and the reception of ideas, as between Will and Memory, is of prime importance.

The Study of Well Marked Examples

We have been led in the study of Memory, also, to find that though some individuals have generally strong Memories, and others weak Memories, yet it may be possible that the Memory is weak with respect to one set of ideas and strong in another region.

We find similar conditions in Will. Some individuals who are taken as types of strength of Will, because in the regions in which they exercise their faculties conspicuously their Will is strong, may yet be weak in Will in other directions hardly less important to their existence.[1]

Let us now consider a few cases which may cause difficulty. There are instances in which an individual may continue in the performance of acts which produce pain, even to the extent of being fatal, but which we do not consider as evidences of Will. For example, a maniac may dash his head against a wall, or an epileptic throw himself into the fire.

In such cases we may say that the mental derangement confuses the feelings of pleasure and pain. But no one considers that the maniac balances pleasures and pain and decides, on a wrong calculation, to dash his brains out. The maniac and the epileptic are carried on by impulses which they cannot control.

We have other examples in which the consciousness is much less disturbed, as in hysteria. An individual may have been brought

[1] General Grant, whose iron firmness crushed down the resistance of the South in the great Civil War of America, had before the war become a wastrel. Subsequently elected President of the United States, his career was such that I have heard him described by an American student of politics as a "weak man."

Some of the old prize-fighters displayed force of Will which Sparta or Rome never surpassed, but their Will was weak in other ways. Jem Ward was described as a lion in the ring, a child outside.

Bonaparte, who, at Arcola had again and again by the force of heroic example strengthened the wavering resolution of his soldiers, himself wavered on more than one occasion where his brother Lucien displayed tenacity of purpose. At the scene of the expulsion of the members of the Legislative Chamber on November 9, 1799, Bonaparte required the support of the will of the Abbé Siéyès, which in the circumstances was stronger than his.

The effect of habit of Will is illustrated by one of the delightful stories of Georges d'Esparbès in his *Guerre en Dentelles* (*War in Lace and Frills*). A French officer, during a period of war with Frederick the Great, disguised himself and acted as a spy. Having been taken prisoner, he was submitted to various ordeals, which he met with Stoic fortitude. Accidently he stumbled against a lady, and, forgetting his assumed character, he made a courtly bow and begged pardon in French.

As a contrast with Napoleon's failure to rally after Waterloo, a failure which had many causes, the example of Blucher after Ligny may be cited. Blucher was far from being the equal of Napoleon in intellectual power, but his general temperament formed a factor strong in Will, and familiarity with defeat had rendered the exercise of Will more definite and determined under such conditions. Hence followed the tactics which ensured victory for the Allies at Waterloo.

A man of indomitable Will, as an explorer, for example, might well show indecision if called upon to perform a surgical operation; while a surgeon, otherwise of weak character, might take charge of the proceedings in a style of masterly command.

In the case of the surgeon there has been a training which has defined the objects met with, which has again and again regulated Impulses along appropriate lines according to the ideas in Association, and which has at length formed a strong habit.

In this way we see a new meaning in the aphorism, Knowledge is power.

to a condition of hysteria by an inordinate pursuit of pleasure, taking the meaning of pleasure at a low standard. But in a certain state of body and mind the hysterical individual may persist in acts which give a great amount of pain. Or by remaining passive, as if paralysed, the hysterical individual may suffer grievously.

In neither of these cases do we hear of a strong Will. We say the state is due to a deficiency of Will. In the first case the dynamic forces are in a state of excitement, and the inhibitory power is weakened. In the second case the whole nervous energy is weakened, or its exercise obstructed in some manner.[1]

The fakirs of India, though apparently not men of great vitality or mental vigour, train themselves to resistance of feelings of pain to an extraordinary degree.

Having therefore seen that either by training, or some severe stress, it is possible to produce states of body and mind where the usual standards of pain and pleasure and their effects are greatly modified, we continue the consideration of Will.

In ordinary language strength of Will is not considered as coming into play in listening to sweet sounds, or in the soft enjoyment of a warm bath. We hardly speak of strength of Will in the active pursuit of some infatuating pleasure. We think of Will rather in connection with an arduous and long-continued struggle against difficulties. When the Roman youth, in spite of terrible protests of pain, holds forth his hand in the flames to be burnt, we speak definitely of Will.

The reply of those who explain these matters by the pleasurable impression being superior to the painful, is that even here there is the pleasure of conquering the tyrant, and of performing a service to one's country. But arguments of this kind really beg the question. For if everything that produces action be called pleasure, then it is evident that pleasure must always be the cause of the actions.

Moreover, a weaker individual might have stimulating feelings of high sentiments in equally powerful degree, and have as keen a pleasure in their realisation, and yet be conquered by the pain.

But if during years previously, and especially in the earliest years, a vast body of ideas in association be built up around even the pleasurable incidents of love of country and victory over

[1] *Cf.* pp. 73 (note) and 181.
An instance may be given where authoritative association produced physical results in hysteria. It was related to me by Jean Charcot, the celebrated Antarctic explorer, as having occurred in the practice of his no less famous father. Dr. Charcot had as patient a young woman who, though physically sound, as far as could be ascertained, believed that she was unable to rise from her bed. Her malady had been caused in part by excessive reading of religious books, and she had a particular respect for St. Peter. Dr. Charcot simulated St. Peter and, representing himself in this guise to the patient, commanded her to rise. She did so, and that was the starting-point of a cure.

tyrants, and if these associations be strengthened by symbols, and linked, not only to the early mental constitution, but also to the projects of a man's life, then all this will form within the mind a factor from which the effects of authoritative association will flow.

The immediate Hedonic principle must not of course be minimised. The Christian martyr whose body, smeared with honey, was stung to death by bees, smiled down on his human tormentors. He felt vividly that he was enduring but a painful passage to an eternity of bliss.

The Mahommedan who flung himself from a precipice at the bidding of his Sultan to testify his faith had within his mind the image of the waving green handkerchief, and the love-light of the houris in his eye.

The case would be different if they felt nothing but the " silence of infinite spaces," though even then martyrdom to a cause of negation is possible by reason of the strength of the other determinatives.

Indicative of the Factors at Work

The study of simple forms has helped us and the consideration of Impulse as supplying the characteristic element of Will has put us on the right track.

Let us now consider the complex system of the mind, and in order to fix our ideas let us refer to the brain if only as a diagrammatic form.

Bearing in mind always the image of the brain as a magazine of energies, co-ordinated and organised; and remembering also that all manner of impulses are proceeding along nerve strands, and that even if these do not all produce consciousness by their own activity nevertheless they modify the manner in which consciousness appears; then we have the image of each new course being determined as a resultant of a vast number of forces, some of which are derived from our entire constitution as moulded by the heredity of countless generations as well as modified by our own personal experiences. The totality of these factors is the whole system of the considerations on which we found a choice. The resultant implies a sort of integration of innumerable impulses, and the manifestation of them in consciousness is what we speak of as Will.

There is thus no entity Will, just as there is no entity Memory. Will is relative to each mental act, just as Memory is relative to each mental act. What in a general or popular way we speak of as Memory is the system of innumerable memories, and so the system of innumerable acts of will is what in a general way we speak of as Will.

The reason why the discussion of Will is so much involved in that of the Sense of Effort [1] is that where in ordinary appreciation Will is conspicuously exercised it is noted, not in regard to organs abundantly supplied with sensory nerves of high delicacy, but in the movements of massive muscles concerning the existence of whose sensory supply physiologists were long in doubt.

In that case the Sense of Effort would come into play.

Or again, Will may be exercised in a case where sensations result in such a degree of intensity as to be very painful, and yet where it may be necessary to endure the strain. Here again the Sense of Effort comes into play.

But the essential nature of Will will be observed to be something different from the sense of these stresses and strains, and once more we are led to the consideration of Impulse as giving us the characteristic element.[2]

[1] To prevent misconception it may be noted that the fact that Sense of Effort and Impulse are taken as Fundamental Processes does not imply that in all circumstances they can be appreciated as such. For instance, it is not implied that when a sensation of blue is noted the mind must have expressed consciousness also of a Sense of Effort. All that is meant is that when the mind has consciousness of a state which we have called a Sense of Effort, it is not possible to analyse this feeling into simpler elements, as Muensterberg proposes to do, but that the feeling of effort is fundamental. The appreciation of states of mind is largely a matter of subtlety of intellect and continued exercise of the faculties in that direction.

There is, however, in this regard nothing special in the cases of Sense of Effort or of Impulse. We find similar conditions in the case of sensation. Some persons have a much more delicate sense of differences of colours than others, and incessant practice may greatly develop the power of discrimination in these matters. Moreover, as Helmholtz remarked, we habitually fail to see objects presented to our vision which if we directed our attention to them would be strikingly visible. And in the rapid mental movements of ordinary life we fail to observe with much Discrimination, or to remember what we do see.

Now for the sake of clearness of exposition let us refer to the objective model. A stimulus affecting a sensory nerve may not produce a result in consciousness. If the stimulus be repeated, or its intensity be increased, we may reach a limit where an effect in consciousness follows. With some persons this may be reached much sooner than with others. Constant practice, attention, expectancy, freedom from distraction, all aid the Process of Discrimination.

When the sensation does arrive, it is fundamental. All that we mean is that we cannot analyse it, or compose it; and it is no answer to this to say that under certain objective conditions of a similar character no sensation was apparent.

Now keeping this model in mind, it will be found not reasonable to reject the Sense of Effort simply because anyone may say, "I have a sensation, but cannot detect a Sense of Effort."

There are occasions when those least sensitive to impressions, or those most recalcitrant to arguments of introspection, do experience something, even though confused in the mind, which contains a factor not further analysable, and which I have called a Sense of Effort.

Similarly with Impulse, it is no answer to say that any particular individual, who has not given full consideration to the matter, fails to find any such process. There are many positions, such as those of geometry, which are both true and fundamental, which require for their discovery a certain desire of knowledge and patience in its pursuit; and this is certainly true of Psychology.

Reverting now to the objective model, I say that just as stimulation of sensory nerve fibres is correlated to sensation, so we have at each instant a certain resultant of forces in the brain, and these serve to determine the new direction of outgoing nervous impulses. To this corresponds a certain mental phenomenon which I have called Impulse.

[2] What is called Action has been studied of late by many psychologists, including W. G. Smith, Judd, Alecksieff, Bergemann, and N. Ach. (*Cf.* Ach's *Ueber den Willensakt und das Temperament. Eine experimentale Untersuchung*, 1910). In another exposition.

Reverting now finally to the graphic image which we have already indicated, based on the structure of the brain, we find that what has helped to suggest Will as an entity is the tacit recognition that our course at any moment is dependent not only on the circumstances of that moment, or those most recently presented to us, but also on the indications, sometimes discordantly shewn, elusive but certainly forcible, of all that has contributed to our past experiences, our constitution and our heredity.

We recognise Will and we are able to speak of Will; but Will arises from the system of innumerable elemental wills, exercises of elemental Impulse.

We see then that the Will is not to be summed up in the balance of pleasures and pains; and especially, that we must not give a limited interpretation to pleasures and pains. In each case of the exercise of Will we must consider a resultant formed in the mind by a multitude of factors; the original constitution of the mind formed by myriad influences summed up in the term, heredity; the actual vitality and energy of the mind determined in part by physical health; the strength of its Impulses; the continual testing of these as against contrary Impulses in every act of life; the combinations of these; the appreciation of opposing Impulses formed in knowledge by experience; the immediate Hedonic principle in overcoming resistances; the habits formed and made organic; the associations formed with symbols that have powerful associations already woven round them; the stirring up a fund of authoritative association in this way; then the immediate Hedonic principle in the Sense of Effort itself, in the display of power against resistance; the refined pleasurable feelings of high ideals; even the potent promptings of vanity; then the force of dynamic Impulses which are beyond control. As the physical states are correlative to the mental, and in this sense may be said to exercise influence upon it, so the mental condition at any moment is a factor in determining the next resultant of mental progress. This adds in producing the appreciation of Will. Such are the factors that enter into the formation of Will.

In the first place I desire to free myself from the tyranny of the dogma of Freedom. I mean, not to create a world to suit my theology, but to interpret the world as it is, as far as I can by the illuminating rays of clear and honest thought. If there be Freedom, there is Freedom; if there be not Freedom, no appeals to

Ueber die Willenstätigkeit, und das Denken, Ach speaks of a fore, mid, and after period for the completion of a psychic effort. Here we are, as in the cases of Memory, Will, Attention, dealing with complexes. Analyse and translate into the terms here used all that is valid in these researches and they will be found to confirm the positions established in this treatise.

the high spirituality of that conception is worth considering. There is no greatness, no true spirituality, but truth.

Moreover, in regard to paradoxes we will find that as our minds are clarified and our comprehension extended, paradoxes tend to disappear. The grand order of nature, the wonderful precision and constancy of the few great principles we have discovered—the law of the conservation of energy, the law of gravitation, for example—incite us at least to examine paradoxes with the keenest scrutiny.

We find that our will depends on considerations, or motives, just as the movements of a body depend on the play of forces that act upon it.

These motives are all analysable into forms that depend ultimately on our constitutional make-up and the materials and energies of the circumambient world. All these systems cover the principle of determinism. In this universal scope of things the Will is determined.

Yet we all recognise the exercise of Will and we believe that we can, by exercise of Will, alter relatives in the Universe. Evidently then any analysis is false that seems to explain away this faculty. Will exists. Freedom of Will exists. The position here does not land us into a paradox.

Consider for a moment a pebble and the world. The mass of the world attracts the pebble. But placing ourselves from the standpoint of the pebble, the pebble attracts the world. The complete effect is expressed in the terms of an all-embracing law of gravitation.

Now in analysing Will we have at the same time been observing some of the functions which go to form the Ego. At every moment it is not the strength of immediate influences that determines our choice, but these taken in conjunction with that whole accumulated system which has resulted in the character of the Ego at that moment. But observing the whole matter in the widest scope we see that the factors that have served to the constitution of the Ego have been themselves determined.

The choice is noted in our minds as a peculiar activity of consciousness, and that is very wonderful. But so are sensations and memories wonderful activities, mysteries, linked also, if you please to use that term, to a spiritual side of things; but these activities are ultimately determined and, the circumstances being given, automatic or voluntary. The Will is determined but it is, on that account, none the less real.

It may be questioned whether there is not something more in the Will than is here set down, something that gives to the Will

a more distinctive ethical import. In what has been hitherto discussed we do not exclude such a supposition.

It must be remembered that the ultimate cause, and the ultimate sanction, of all our Processes, and therefore of all their combinations, escapes our analysis. When the conditions are established they arise spontaneously. Why? The solution of that problem contains the unknown factor which may bear the true ethical import.

Moreover, in the course of our evolution our whole constituton has been developed in such a way as to prepare certain conditions for these Impulses, and the course of our development has contained factors beyond our desire or our Will.

We are caught in a sort of Universal movement which moulds us and carries us on to certain destinies. Is there purpose in this, such as finds expression in whatever is true in our ethical conceptions?

The discussion of such subjects leads beyond the scope of this book.[1] I have here been content to set forth the analysis of factors which come clearly within the scope of what we may know in Psychology.

(3) *The Ego*

Let us return to the study of the child in our experiment. He sees his finger some distance away, and he observes that when the finger is brought in contact with the live coal, or the ruby, distinct sensations arise.

By continuing his experiences he observes that when the other parts which he recognises touch the object, sensations arise. He also observes with respect to the ruby the conditions that we have discussed with respect to the bouquet. (*Cf.* pp. 79 *et seq.*); that if a movement be applied to any part of it, the whole plexus moves together.

These conditions facilitate the Conception of Unit, as applied to the object.

Similar observations are formed with respect to the part of the child's body. He observes a continuity in all parts, and he observes that from any part may arise sensations.

He observes that when he turns his head he loses sight of the ruby, and that also when he places his hands before his eyes he no longer sees the ruby.

In this way by incessant experiences he explores his own body by the use of the same faculties as he has examined the live coal,

[1] In a brochure which was read originally before the Aristotelian Society: *Purpose and Evolution*, I have further considered these matters. *Cf.* also *Ethics*.

the ruby, the table. There is nothing transcendental to tell him that any part of his body belongs to himself. In point of fact he will never arrive at the complete knowledge of the parts of which his body consists. Most people know only superficially of what constitutes the body.

Similarly he finds by experience the relation between his sensations and feelings. We have already seen that nothing apart from experience tells him inevitably of the relations between sensations of touch and those of sight. But step by step the experiences are made under normal conditions, and these are facilitated by the whole constitution of the child. It may happen that certain of these experiences cannot be made by the child—as for example, if he be born deaf or blind, his world is limited accordingly.

But it may be thought that what the child, so limited, knows of himself is his whole Ego. That is not the case, for the obstruction to his senses may in some cases be removed, and the child discovers new faculties within himself. He may never discover the whole range of his sensations and feelings. Certainly he will never discover the full scope of their possible combinations.

If children be studied it will be found that their remarks often reveal the vagueness of their conceptions of their own personality. A child, for example, who was peevish on a certain occasion afterward explained that she had a pain in her temper.

A child suffering from ear-ache pitied her ear, as she might pity another child whose sufferings were vividly brought home to her. She cried: "Oh, my poor dear little ear!"

The process of education is helped by the instruction of others. She had been told that she had an ear, and an eye, and a finger, and a temper, and a bull-dog. She thought that she might have a pain in the temper as well as in her ear, she thought that it belonged to her somewhat as the bull-dog belonged to her.

A further illustration of the manner in which the conception of the Ego is formed may be found in this study: the same child told another still younger that she ought to be "good".

This one demanded: "Why?"

The elder child, puzzled for a moment, at length replied: "Because you have an immortal soul."

This conception had hitherto been unknown to the younger child, but she endeavoured to adjust it to what she had previously learned by experience and teaching of the Ego.[1]

[1] Great importance must be given to teachings of this sort heard at a time when the mind is most impressionable. This is the case whether the teaching be true or false, good or bad; whether a noble principle be inculcated or a foolish superstitition. Multiply these impressions not from one source only, but in the complex and various

The exposition here given of the growth of knowledge of the Ego is at variance with the assumption of Descartes as the basis of his system: *Cogito, ergo sum.* Both the Ego implied in the "sum" is complex and so is "cogito." Further if "cogito" have the meaning Descartes desires, the Ego is already implied in "cogito". Certainly the new-born child when it utters its first cry in contact with the world does not open the ball in these terms: *Cogito, ergo sum.*

Moreover, it would be merely begging the question to call the sensations it then experiences a conception of the Ego. It experiences the sensation at first in no definite relation with the other senses which are ready for activity. But the sensations take place in an organism prepared for development within certain lines. The process of that development has given the whole material for discussion in this volume.

(4) *Synthesis of Ego and Will*

As the child sprawls toward the ruby his movements are not altogether accidental. He is so constituted that the desire to reach the ruby tends to produce direction and co-ordination of movements necessary to the fulfilment of that desire.

It may happen that the Presentation of an object, or combination of objects, may produce co-ordination of movements that are not controlled, and not entirely represented in consciousness, and not even directed to the fulfilment of any desire.

For example, a dachshund of mine which had been taken from his parents when very young and which had never barked nor heard another dog bark, was playing with me when suddenly in the zest of the game he uttered a loud bark. He started back in surprise and alarm, but presently finding that no unpleasant consequences happened he repeated the bark with great enjoyment.

In this instance, though there may have been a Sense of Effort we could not speak of the effort itself as being due to Will, nor even to desire.

We will return to the consideration of desires, but there are certain aspects of Will now suggested that deserve to be discussed.

If any effect occurs frequently associated with efforts of our own, we are inclined to think of the effects as having been produced by these efforts.

This is probably the origin of our notions of cause and effect in which we have a tendency to see more than mere sequence, and

guise of general opinion, and we see how at an early age the influence of tradition and conformity is felt so deeply that few persons are able, by the sheer force of reasoning, to break through the bondage of conventional habits, or even of the authority of statements accepted without thought.

to which we ascribe some sort of anthropomorphic action. There may well be in cause and effect something deeper than mere sequence, even if invariable, but that has not been made known to us in consciousness; while to invest the term "force" with assumptions derived from Sense of Effort is simply a case of a confusion in the appreciation of the meanings.

But when our Sense of Effort is involved the tendency to think of cause and effect is so strong that we are apt to apply it even when the sequence is accidental. If, for example, I move my hand to a certain position at regular intervals, and a shadow appears on the wall, I regard the shadow as due to my hand. If I allow an interval to elapse and the shadow does not appear, my opinion is strengthened. And if the shadow reappears again when I move my hand to the former position, I shall have a strong belief that the shadow is dependent on my Will. But the effects may have been due by some rare coincidence to another cause.

Or again, an individual lying in bed on a quiet night hears a rustling sound. He thinks it may be due to his breathing; he stops breathing, and at the same time, to listen better, removes his ear possibly from the place at which it could well hear the sound. The sound ceases. He resumes his former position and recommences breathing. The sound reappears. He becomes convinced that it is due to his breathing and is therefore in some degree dependent on his Will. It may have been due to other causes producing sounds which were audible only in the original position of the ear.

We cannot tell except by experience how far any effects are due to our Will or what kind of phenomena are associated with the Sense of Effort.

The case of the dachshund proves this. But other indications are not lacking. The child finds that it moves its arms in correspondence with a certain effort; but if the arm be paralysed no visible effect would follow the effort.

Even when the characters of the phenomena are known it is only by experience that we can know what degree of external effect will be produced in accordance with a certain effort.

A great part of physical training consists in adapting the lessons of experience of the kind. For example, a boy bowls a ball and it falls wide of the mark. He observes in what way his effort has been too great and he tries again. Repeated trials with increasing accuracy at last form a strong bond of association between his conception of distance and the effort, or rather the co-ordinated efforts, required to bowl the ball correctly to "get the length."

Similarly a singer, training the voice with regard not only to

volume but also to pitch, can only be guided by experience, the fortunate tentatives being chosen for particular attention, so that a strong bond of Association is formed between the required sound and the various co-ordinated efforts of the different muscles required to produce the sound.[1]

If we now return to the child we find that among the various sprawling efforts not entirely controlled but nevertheless influenced by the desire to appropriate the ruby, some are better adapted than others. The representations of these become impressed on the child's mind in association with the efforts corresponding. The next efforts will be more consciously directed. This sort of education proceeds with every fresh experience.

In the first "spontaneous" movements of the child we do not speak of Will, as in the unexpected bark of the dachshund we do not speak of Will; but when the effort is consciously directed to a definite end, we speak of Will. The child finds pleasure not only in touching the ruby but also in the power of directing efforts with precision. This is the basis of the strength of secondary motives which may in the end override the primary motive for which they were formed.

We see this in the more developed state when a student, finding that he must keep in good health in order to work well, plays cricket. He may take such pleasure in his expertness in the game that he neglects his studies altogether. Or a surgeon after trying to kill his enemy in a duel may, if called upon, exercise a high degree of skill in saving his life.

* * * * *

We referred to the child's desire to touch the ruby. What is desire?

Desire is not merely the powerful representation of a pleasurable condition. The exercise of any faculty within due limits normally brings pleasure; the lack of exercise brings at length a vague sense not of pain but of unrest.

Again we have seen that the child's touch on the live coal produced a storm, which, though it resulted in appropriate directive efforts, in this case away from the source of sensation, yet extended far beyond the scope of these efforts.

Now the deepest desires of our nature have reference to the preservation of life, and in their most intense forms they do not attract us by a pleasurable representation, associated with purposive actions; they reverberate throughout the whole organism and the actions which they stimulate are often beyond the control of the Will.

[1] *Cf.* note 224.

Thus the thirsty seaman on the raft requires all his strength of Will to resist the temptation to drink the sea-water even though, if he give way, his actions may be appropriately directed. If it were sought to prevent him he might even show cunning and force in effecting his purpose. We could not withhold the word Will from such a case.

We have here two opposing forces; one prevails; a resultant of the whole organism is formed accordingly. There may be cases where a strong man hesitates as to which of two opposite courses to pursue. Having decided, he puts the whole strength of his nature into that which he has selected. Thus a captain may have made a *sortie* from a besieged town. He finds the enemy in strength before him. He retires, but finds his route to the town intercepted. He may hesitate whether to return or to force a passage through the ranks of the enemy he first encountered. In either case he may show strength of Will.

A man may hesitate whether to give way to the promptings of a personal ambition lower than the ideals for which he had hitherto fought, as Napoleon after Lodi, but in carrying out his designs he may in an extraordinary degree exercise Will.

Let us consider one or two examples where desire is less imperious than in vital matters but still strong.

A soldier on the battlefield has lost his way. He believes that he has come to the place where he left his horse but that the horse has disappeared. He thinks he will soon be taken prisoner. He has a desire for a horse but there is no strength in his desire, for he does not think the realisation possible. Suddenly in rounding a corner he finds that he was out of his bearings. He recognises his locality. He sees his horse and the desire to regain possession of it becomes intense. He exerts every energy to that end.

Here the desire is increased in proportion as Will is exercised in a definite way likely to lead to fulfilment of the desire. This will be found to be always the case.

We see in this example also that Will is strengthened by the exercise of purposive acts.

In the discussion on Externality we find that the object is recognised as external partly by means of the conception of the efforts necessary to reach it. It is this clear conception which helps to give it the impression of reality.

For example, a traveller in the desert has often been deceived by the mirage. He beholds again a smiling oasis of trees and grass and wells of water, but he feels inclined to turn away his head from the vision. But something attracts his attention. He recognises that this is not a mirage but a reality within his reach.

What was previously a brain-sick appreciation of a beautiful image changes now to an ardent desire, and the power of Will is called forth to its utmost scope.

John Alden believes that Priscilla is indifferent to him. Desire becomes a far-off dream. He learns that she loves him, a desire throbs through all his nerves and Will enables him to dare what he had not before thought possible.

Desire is, we find, related to Presentation associated with emotions of pleasure and with Will. In some forms its relation to pleasure is rather that of the uneasiness produced by the feeling of deprivation of something whose normal exercise gives pleasure. In this case the whole scope of the Impulses associated with it may be so great and their display so uncontrolled that Will is not conspicuous. Its most characteristic manifestation is when the pleasurable feelings are vividly represented and appear as realisable, while Will is exercised with energy.

It may be objected that all pleasurable feelings are associated with some desire of realisation, and that desire is therefore nothing more than the sentiment of pleasure; but though weak desires may in cases verging on indifference be merged in a mere sentiment of pleasure, and though even aesthetic pleasures may have a faint association with Will, yet the difference is in general marked. Thus we may speak of desire and pleasure as distinct, as we speak of animals and vegetables as distinct, though there are borderlands where one merges into the other.

* * * *

We have arrived, from the basis of the study of the child, at a conception of the Ego built up by complex factors; and we have found in analysis various conditions that contribute to the expression of Will. But these two conceptions must now be considered in more intimate Association.

The individual forms a representation of the Ego, which is not only associated with his previous experience, but with all manner of possibilities, many of them unrealisable, but all connected with the main directive opinions or ideals of his life.

At every experience also, such as we have observed, in the training of the Will, the Will is definitely associated with the Ego so represented.

Thus there grows up a body of co-ordinated ideas which form an authoritative association[1] derived not from the influences of others, but from the whole strength of the individual's own directives in life, from his organised character.

[1] The term "authoritative association" is better explained in the Chapter on Reason.

This then must always be considered in addition to the other factors already discussed, when a resultant is sought in regard to any condition with which the mind has to deal.

An example of the strength of this factor is found in the story of Alexander the Great and his friend, who in face of great difficulties said: "If I were Alexander, I would go back"; to which the conqueror replied: "So would I if I were Parmenion." [1]

We have in this synthetic exposition of the Ego noted how the child becomes acquainted with its sensations or Immediate Presentations, of any kind, as Units, the Association of these, how Memory perceives Agreements or exercises Discriminations, how a rudimentary faculty of Generalisation arises, and in what manner a Sense of Effort and Impulse appear, and that the activity of these faculties is accompanied by the Hedonic sense, and that with each Process there is a Negative process; and that all these faculties are exercised in Space and Time; that all the psychic manifestations may be formed by combinations of these Fundamental Processes.

We have seen in a brief illustration the kind of activities which finally make the child aware both of its mental faculties, and of its physical structure and qualities; we have also seen—this is shown more particularly in the study of Externality—how it acquires the notion of objectivity; we have seen the mystery of instinct in the rudimentary use of faculties; we have seen the manner of development which eventually produces our highest intellectual life, and we have seen complete continuity; we have noted the development also from the rudimentary sense of Hedonism into passionate desires or spiritual aspirations, and we have

[1] An example of this authoritative association arising from within and overriding a contrary expression of Will, occurs in the life of Napoleon when, seeing nothing before him but abdication and disgrace, he resolved to commit suicide. After he had swallowed the poison his conception of his whole character and career rose before him and he took the antidote.

An example of authoritative association arising from without and overriding a strongly formed Will is found in the meeting of Napoleon and Ney during the march to Paris after the evasion from Elba. Ney had sworn to bring Napoleon back to Paris in an iron cage, but the sight of the former leader and the tone of his words overcame this resolution. He returned to Paris with Napoleon to drive out the Bourbons.

Vanity, which is not altogether a bad quality, contains factors both of subjective and objective authoritative association.

Bonaparte's manifesto to his soldiers, "From yonder Pyramids forty generations look down on your actions," formed a strong incentive to Will. It was an example of authoritative association having a strong subjective appeal.

Nelson's signal at Trafalgar was based on authoritative association ostensibly more objective in character: "England expects that every man will do his duty."

Caesar's exhortation to his men at Pharsalia when he directed them to strike at the faces of Pompey's patricians was rather an address to the habit of legionaries inured to war and prone to victory.

Yet looked at closely there is not a great difference either in the manner of operation of these various exhortations or in the emotions which they stirred.

The soldiers of Bonaparte and the sailors of Nelson had equally with Caesar's troops formed the habit of tough fighting. A great part of the stimulus in each case was the desire to obtain their commander's approbation.

seen the growth of Will as the resolution of Impulses into a resultant; and we have noted how the observation of continuity in the physical parts, the sequence of mental operations, and the control of movements and mental faculties by Will, have co-operated till at length we have a conception of the Ego.

All this we have exhibited only in outline, and we have shown development by means of indications. This has been necessary on account of the limits of time and space, but with the aid of the principles elucidated we can easily satisfy ourselves that we have but to multiply the types of observation given in order to gain the complete impression, and to give full force not only to the effect of sensations, but to the great primordial feelings affecting preservation of life and continuance of life. That which gives the characteristic note to the whole summation of factors is Will, especially when with the Will is combined the consciousness of the next mental state in causal Association and the experience of directing and controlling forces and objects according to the Will. For that reason, among others, we have here discussed in close association the questions Will and Ego.

SUMMARY OF DISCUSSION OF EGO AND WILL

The Ego and Will may be considered by means of a study treated synthetically, namely, that of a child forming rudimentary experiences.

There is continuity between instinct and reason. All mental processes at the basis of reason are spontaneous, that is instinctive.

The child may make considerable advances in experience without having a conception of the Ego as a Unit. It is only by accumulated experiences that the child attains a conception of the Ego.

No one ever attains a full knowledge of the complex, afterward referred to as the Ego, either with regard to its physical basis or the possibilities of combinations of its faculties.

In the Middle Ages Will was considered as an arbiter, an entity. This conception pervades Kant's system. One of the main motives of his *Kritik* is the determination to defend the Freedom of the Will. We must start without any such predilections. James Mill in destruction of entity is carried too far. He explains Will away.

* * * * * *

No French work in Psychology has been sufficiently original and basic, though the French have supplied brilliant contributions of observation and suggestion. Condillac proposed problems similar to those here discussed but his analysis was not basic. A modern school of French psychologists is working in a scientific spirit.

Ribot and Binet consider that the declaration, "I will," indicates a state of affairs but adds nothing new.

Loeb, impatient of metaphysical doctrines of Will, is carried too far in the desire to reduce Will to ideation.

Loeb rides the theory of ions to death.

* * * * * *

The key to the position of Will is found in the study of Impulse.

We are aided in our investigation by discarding authorities and again studying the child.

The study of the Fundamental Processes indicates that we should attach great importance to Impulse. We find Impulse as rudimentary Will.

The factors that determine Will are various and complex (as indicated).

The Will is determined. Yet Will is real, and Freedom of Will exists within a certain scope. There is no paradox, for the factors of choice and influence are determined.

The child includes external things as part of Ego, and regards part of self as external.

The conception of the Ego is aided by the instruction of others; but in this way errors also arise.

The aphorism of Descartes, "*Cogito, ergo sum*," cannot be taken as the basis of a proper treatment of Psychology; it assumes what it desires to prove, and the position expressed is already of great complexity.

Will is not a distinct faculty having a certain relation to the totality of faculties which it governs.

As each idea has its own Memory, each Impulse has its own strength which is its element of Will.

The Will can be cultivated, but this must be understood with various restrictions comparable to those considered in discussing Memory.

Cause and Effect are regarded at first in an anthropomorphic manner and Will is associated with this view.

The Impulses arising in experience are the resultant of those experiences, the previous experiences, and the whole original constitution, with its physical basis, which has been produced by evolution through countless generations of experience.

The strength of Impulses which form Will depend on the general vigour of the constitution and the temperament of the individual.

There is always found Impulse toward the continuance or increase, within certain scope, of pleasurable conditions of the mind, sensations, or emotions.

There is always found Impulse toward overcoming resistance and enduring pain, and this is also associated with the Hedonic principle.

Desire may be considered as related to pleasure and effort.

The relation of desire to pleasure depends on vivid representation of pleasurable conditions, or vivid realisation of conditions of unrest produced by the lack of normal exercise of faculties.

The relation of desire to effort may be that it causes involuntary effort; but associated with this also is strong voluntary effort.

In connection with voluntary effort associated with desire, we speak of Will.

Will reaches high expression when purposively directed by strong desire within possibilities of realisation.

The exercise of faculties directed purposively by Will produces skill, and, when the combinations become elaborate, secondary motives are formed which may become stronger than a special operation of Will called into exercise on any occasion.

As the conception of the Ego becomes formed, the Will is associated with it, and it is this combination which gives peculiar characteristics to the Will. Will is not only referred to the Ego with all present experiences and with the record of experiences, but also by virtue of the representations of the Ego, particularly in regard to its dominating principles. This combination with certain qualities of temperament is a factor of pride. Hence arise powerful authoritative associations of subjective origin, tending to strengthen the Will in the direction involved.

Will is influenced, sometimes powerfully, by authoritative associations arising from others, in what may be called an objective way. The symbols by which these operate are obedience, and in some circumstances duty.

Combinations of various authoritative associations acting on certain qualities of temperament tend to form vanity, which is often a powerful factor in Will.

In cases where strong exercise of Will is demanded there are also factors of the vivid representation of pain following on failure, and the reward of success. These factors may be considered in various ways and they may be all factors in certain cases.

To these may be added unknown causes that lie beyond the bounds of our consciousness. For, as, when the conditions are given, all the Fundamental Processes follow spontaneously, and as these are all associated with the Hedonic principle, it may be that some purpose to which they are directed may be expressed also in some directive manner in the larger combinations of our Impulses, emotions, temperament, and that this expression may have its force in Will.

But here too we may at least propose questions as to some purposive arrangement which has produced these factors.

The Ego is gradually made clearer to our appreciation by a great variety and complexity of facotrs (as indicated). Of these, Will, which is itself determined by complex factors, gives the characteristic expression, especially when Will is recognised as directing thoughts or controlling movements. Hence Will and Ego have been studied in association.

PART IV

CHAPTER I

REASON

NOTE:—To those who have grasped the spirit of the preceding exposition, and to whom it has become habitual to perceive the lines of analysis ultimately leading to the Fundamental Processes, the question of Reason will have already been solved in regard at least to its schematic forms. The object of the chapter on Reason that follows is rather to invest the skeleton forms with muscle and blood taken from experience; I have endeavoured to appeal rather to the experiences of everyday life, with occasional reference to the domain of science.

In the first sections the Fundamental Processes will be discussed in turn; then as the discussion proceeds with growing complexity, reference to these will be less frequent and less direct, but it must be understood that that is because it will be within the powers of the student to trace out the steps that lead to these Fundamental Processes. The employment of this method ought to become so engrained a habit that at every stage of difficulty the mind beholds, with an ease that suggests intuition, the whole structural form of the complex, exhibited from the basis of the Fundamental Processes.

In the course of this discussion, moreover, the limitations of Reason and the meaning of Belief and the "sanctions" of Belief, or in other words the nature of any certitude of Belief, will be exhibited.

After that a chapter will be devoted to Questions of Research, then a Summary will be given expressing the results obtained in succinct form.

1. *Formal Examination of Operations of Reason*

The study of Reason is by no means facile; and the method of exposition here adopted is therefore that of examining simple cases by aid of the Fundamental Processes, and then proceeding to immerse the subject in wider and deeper relations until the whole scope becomes comprehensible. Let us begin with a tentative definition.

Reasoning is the series of processes by which we proceed from one set of facts, established in certain relations, to another set of facts established in certain other relations; or, as we express it in language less general but more easily understood: Given a certain true proposition, reasoning is the mode by which we ascertain that a certain other proposition dependent upon the first is also true.

We cannot at this stage insert any qualifying phrases, such as "by a rigorous line of deduction", or "by exceptionable proof", for the problem we have set before us is that of ascertaining in what deduction really consists, what are the laws of thought, and finally, what in this regard is the veritable meaning of the words "ascertain," or "proof."

The preceding exposition has already cleared the ground, as might have been expected from the fact that the investigation of the problem of reasoning itself led by successive steps of analysis to the establishing of the Fundamental Processes of Mind (*cf.* Introduction).

If therefore these Processes have been correctly exhibited, any mode of proceeding from the truth of one supposition to that of another must be compounded of these Fundamental Processes applied to the actual objects that occur in the experience. All these Fundamental Processes must be intact in a mind that reasons correctly. It may be well to consider the consequence of a failure of any of them.

Failure of Immediate Presentation must necessarily lead to error in so far as that process is concerned. The extreme cases are where there is no mental image at all evoked by an objective phenomenon. A man blind from birth cannot reason about the visual aspect of things.

He may certainly form ideas even about colours. Thus sensitive blind persons who have considerable culture assure us that they think of the colour red as of the sound of a trumpet (*cf.* p. 53 and (note)). A person not blind, but possessing a highly sensitive organism and a faculty of subtle introspection, might easily find this comparison intelligible. The colour red suggests ideas of a forcible stimulus, normally not of an unpleasant character. With that stimulus are connected various kinds of Impulse, and it is at that level that we form corresponding associations with the bright, lively stimulus of a trumpet.

All this, however, does not imply that a blind person can reason about visual phenomena in their own qualities, but in so far as such a person can reason there is no failure of a certain kind of Immediate Presentation, for the reasoning is to be found in the recognition of the kinds of Association produced in accord with that form of Presentation.

A person born deaf may learn to speak, but that again is from adopting as symbols, in place of our words, various combinations of positions and movements of the lips and other forms of expression.

It may happen that a brain capable of high development may be enclosed in a head provided with very inefficient outlook upon the external world. Such is the case of Miss Helen Keller, who though born blind and deaf, has continued by the extraordinary force of her genius to educate herself, and to interpret many phenomena for which one would have been inclined to think the special senses involved were necessary (*cf.* pp. 32 (note); 64 note)).

Her mind is deficient in the fund of experience brought by Immediate Presentation through sight or hearing, but there is no lack of such experience in regard to the ideas in which she actually reasons.[1]

It must even be borne in mind that our own normal senses are very limited in regard to the range of external phenomena which we know to extend beyond the spectrum, or the scales, of our senses. (*Cf.* p. 52 and notes 1, 2).

But at a point short of total deprivation there may be failure of Immediate Presentation. If a person looking at a red colour says that it is the same as green we set it down to a defect in his visual apparatus. He differs from the ordinary, therefore we conclude that he is, to that extent, deficient. It would not always be safe to carry out this doctrine quite rigorously, but if we found that the defect arose from a disease which otherwise showed deleterious consequences, we would be more justified.

Attention was called to colour-blindness by the case of the celebrated chemist, Dalton, who, though a Quaker and a man of modest demeanour, appeared at a meeting of the Royal Society in a gay suit of scarlet. He thought he was wearing drab. Dalton does not appear to have been otherwise a victim of disease, but the fact that the overwhelming majority of well-constituted mankind find red different from drab may be accepted as sufficient evidence that Dalton's vision was deficient.[2]

[1] Miss Helen Keller has published a volume of very remarkable poems, which are not without a "Sense" of form and colour. But then she also uses the expression "I see a young girl," and she writes of birds singing joyously. Evidently she uses the words figuratively, and she forms some representation of the deficient senses by means of others. Besides, it must be remembered that the part of her brain associated with the deficient sense organs is probably intact, and its spontaneous activities seek some kind of expression. *Cf.* J. Gensel, *Die Wahrheit über Helen Keller* (1909); and W. Stern, *Persönliche Eindrücke* (*Zeits. f. ang. Psychol.*, 1910). Stern describes her as appreciating, criticising, and enjoying music. To receive the impressions she placed her hands on the piano while the musician played. Miss Helen Keller is not unique as a blind and deaf person endowed with great intellectual power. Laura Bridgman was celebrated in this respect before her, and books and articles concerning her are numerous. E. C. Sandford has written interestingly on her attainments in the *American Journal of Psychology*. Henry H. Donaldson (*Am. Journ. of Psych.*, 1890), G. S. Hall, and M. S. Lamson have also studied her case.

A reference to statistics shows that in all countries the number of deaf and blind is considerable. In 1871 there were 111 in the British census. In the census of 1901 the total number of persons both blind and deaf, whether from birth or otherwise, was given as 470 in England and Wales, and 63 in Scotland. In the last census for which such returns are available (1911) the number for England and Wales was 432, for blind and deaf and dumb, 62.

[2] Some notes have been published by a physician, Dr. G. H. Taylor, of New South Wales, Australia, who has had much experience in testing for colour-blindness. He finds that there is in persons so deficient a lack also of animation in the countenance. Mr. E. Nettleship, whose experience is greater than that of most experts, is inclined to believe that there is a connection also between colour-blindness and abnormalities of hearing. Both these conclusions might have been anticipated from anatomical considerations referring to the corresponding nuclei in the brain. (*Cf.* also B. von Tschermak, *Lancet*, 1909). On the other hand, E. Edridge-Green, whose knowledge of colour vision is unrivalled, says, that colour-blind people are not deficient in animation and that the defect appears in a proportion above the average among musicians.

Now he might reason quite consistently with regard to his own observations, although his range of Discrimination would be limited in one particular; but his reasoning would be faulty in referring to facts of the external world, or to those dealing with this sensation, as expressed by other persons.

Practically, an engine-driver who sees a red danger signal and who, not distinguishing it from green, fails to stop his train, errs not in his process of ratiocination; though his reason is defective by virtue of the failure of Immediate Presentation.

Many disputes about questions of art have their origin in divergences of Immediate Presentation. Thus if a painter be short-sighted he will see a landscape in a different manner from that of a normal observer; and if he then paint in an impressionist style, he will see his picture also differently from the ordinary man. He will also range on his side those people whose eyesight is like his own; and so two "schools" may arise, each disparaging the other in various terms of contempt.[1]

The defect, or the divergence, of the vision from the normal or the ordinary may not be due to any thing so verifiable as short-sightedness. The case of Carrière, the painter, has been investigated. His portraits were noted for good general resemblances and for fine artistic qualities, but the tone of the colours was deficient, consisting mainly of whites and blacks, with a fulgurescent shimmer over all. His eyes were, however, found to be "normal".

Failures of Immediate Presentation are in the majority of cases so serious in regard to their consequences that we find them associated with lunacy. In these circumstances it is difficult to obtain very secure indications of what the nature of the failure may be. One case has come within my cognisance of temporary aberration of mind where the patient on recovery was able to give clear accounts of certain mental conditions during the period of the malady. On one occasion the patient thought that little seeds were really the hearts of friends. The seeds were of a shape that might suggest that of the heart, but all sense of proportion or scale seems to have been in abeyance, to say nothing of associations of place and function.

In general, hallucinations are not due to failure of Immediate Presentation, for it usually happens that there is no objective reality corresponding to the hallucination. For example: the commonest hallucinations are those of hearing. A person hears a voice frequently urging him to do something, possibly even to commit a crime. He may imagine that enemies are always telephoning

[1] Angelucci; *Sur les oeuvres des peintres daltoniens.* (*Ann. d'ocul.*, 1907).

to him, and in this way torturing him by insisting on his acting in some way contrary to his own inclinations. In these cases there is no fault of Immediate Presentation in as far as this is considered to be of objective origin, for the external stimulus does not exist.

Referring to the indications we have given of the physical correlatives, and relying upon the results of modern medicine, we should interpret the case thus: There is some source of irritation of the nerve substance due to some disease which has invaded the brain tissues, and certain nerve-cells become thus stimulated in a manner resembling an original stimulation which represented the physical correlatives of the ideas thus reproduced; that is to say, the nerve-cells corresponding to the words become excited.

It may happen that certain external objects become associated with hallucinations, but in such a case one must distinguish diverse factors. It may be that the Immediate Presentation has not been faulty except in regard to diminished Discrimination; but there may be, superadded to this, factors not derived from the external object.

This will seem clearer when we consider such a case as that recorded by Sir Walter Scott. He was sitting up late one night, when suddenly he thought he saw Lord Byron before him. The figure was very distinct, and the whole illusion was perfect. He subsequently observed that it was the armour of a knight of old, arranged as worn, that had served as a basis for the figure of Byron. On examining the circumstances we see that though the Immediate Presentation was not clear, yet, in as far as that was concerned, there was but an exaggeration of the effects of semi-darkness and distance in diminishing the powers of Discrimination. But to this dim figure were superadded combinations to which no objective reality corresponded. This also is but an exaggeration of what takes place under normal conditions.

Habitually we see only a part of what we believe we see of a person. If we are familiar with the details of the features and dress of an individual, then the sight of some one characteristic part may suffice to call up the image of the whole. This effect is due to the regular manner in which our minds work by Impulse and Association. It is by this means that we appreciate pictures which may only give an indication of the whole image that rises to our minds. A black-and-white sketch by Phil May, for example, if, as usual, struck in with characteristic skill and decision, causes a strong image to rise in the mind. Yet when we examine the actual objective realities we find how small a part these black lines form of a human figure.

This is true of all the senses. It may occasionally be tested for hearing by comparing the notes of the same speech by stenographers. Occasionally a phrase badly heard is rendered quite differently by two reporters.

A stenographer attentive to his own work will easily recognise that he really does not hear nearly the whole words, still less the whole phrases, which he reports *verbatim*. This fact becomes very evident when one tries to take a note in a foreign language which has not become quite familiar. A story of American Journalism takes note of this fact, for when the orator declared: "Amicus Plato, Amicus Socrates, sed major veritas", he was reported as saying: "'I may cuss Plato, I may cuss Socrates', said Major Veritas"!

There are certain conditions of the mind, provoked generally by nervous excitability due to want of rest or to over-stimulation or both, when the process of combination by Impulse and Association becomes exaggerated unduly; if at the same time the process of Discrimination be from any causes greatly diminished, then the conditions are favourable for such illusions as that of Scott (*cf.* p. 220 (note)).

Fortunately in Sir Walter's case the experience affected a mind of marked common sense as well as of high intellectual power, so that we have been thereby helped to understand the nature of such illusions. When, however, the conditions referred to previously affect minds not well balanced; and when, as for example in elusive moonlight, the power of Discrimination is in great part lost; and where terror, or previously wrought-up anticipations, still further inhibit Discrimination, and increase the excitement leading to Impulse and Association; then we have the familiar phenomena of ghosts.

Pascal, the great French mathematician, who was also a religious enthusiast, met in early manhood with an accident by which his carriage was precipitated over the bridge of Neuilly into the Seine. The whole circumstances made a deep impression on a mind and organism already highly strung and somewhat overwrought. Subsequently Pascal was subject to mental states comparable with that of Scott but showing certain inverse conditions.

He used to be seized with the notion of an abyss lying to his left side, and it required a special effort of his intelligence to make him know that this was but an illusion. Here the effect was to inhibit the proper view of external objects. There was an inhibition of Immediate Presentation, and superadded to that were positive images associated with the impression of an abyss.

Failure of Immediate Presentation may be due to a state of intoxication, as by alcohol, which is virtually like a temporary condition of lunacy. The eyes "see double", for example. In such a case there may be no great defect of actual vision in each eye, but the images are not overlaid and fused together. Each eye normally forms its own images, and there is indeed a certain energy required to produce the conditions of fusion. This is observed in the eyes of squinting children. If one of the muscles of the eyeball, such as the external rectus, be weak or its innervation be deficient, then the eye is not placed in the position required to secure good superposition of the images. A confusion of the images results. Thus the child acquires the habit of neglecting the image formed by one eye. This eye is left to some extent uncontrolled, and the squint is accentuated. With the intoxicated person there is an inability to adjust the necessary conditions, and the images appear distinct.

Such a state of affairs may be also produced by various forms of brain diseases, such as a tumour in the cerebellum. In this case the mind may be left quite lucid, and the patient may be able to measure his progress towards convalescence by the degree of proximity to which under given conditions he may be able to bring the presentations of the objects.

Hitherto we have discussed Reason by examining possible cases of its failure, and we have confined these cases to that of faultiness of Immediate Presentation. A complete discussion of such cases would take us far afield in the domain of medicine and particularly of psychiatry or mental aberration. But it has only been necessary for our purpose that we should indicate in what way, as referred to this Fundamental Process, the course of Reasoning may be misleading.

It would be tedious to enter exhaustively into a consideration of the failure of Reason with regard to each Fundamental Process in turn, for this again, as would be expected, must resolve into a discussion of the science of psychiatry. A few additional indications may therefore suffice.

Failure to recognise a Unit in mental operations seems inconsistent with any consecutive mental life whatever. There may, however, be a difficulty in dividing any object presented as a Unit into more particular units. The failure, however, in such a case would be due to limitations of Discrimination.

Total failure of Memory or of Association would be also inconsistent with any course of mental life. A similar remark is true with regard to the Sense of Effort and Impulse.

The normal course of Impulse may be defeated by some

condition, as in lunacy, in which an Impulse arising from morbid processes constantly makes itself evident. Thus a certain patient who had become mentally alienated through the shock of a sudden misfortune was unable to hold any kind of connective conversation; the mind always reverted to certain fixed ideas, diverting the ordinary course of associations. A conversation on the weather would be represented thus on her part: "It is a fine day, poor thing. The rain falls, poor thing. And the grass, and he was a lovely man. And the sun, too, poor thing".

It may, however, be true that in certain cases the Sense of Effort may be hidden or unnoticed; and certainly it is true that Discrimination with regard to this sense is capable of being greatly cultivated.

Without that recognition of sequence that gives the elementary notion of Time, a course of mental life would not be possible. There may, however, be errors of Reason with regard to Time by virtue of events which were really anterior to others being supposed to be posterior, and *vice versa*. We shall see that this cause of errors depends on faults of Memory and Association.

A failure to recognise Space as an Immediate Presentation would be inconsistent with a course of mental life. The notion of Space, however, such as we ordinarily possess, is built up by degrees and by diverse factors; consequently there is here considerable scope for error. This question has already been in part considered; it is dealt with more particularly in the discussion of Externality.[1]

The Hedonic quality of mental impressions has an influence on Reason because it has an influence on Association and Memory. A pleasurable surprise accompanying the Immediate Presentation is the surest means of making a strong Association and one that returns in Memory with a lively vivid style.

Repetition, which we have seen to be so powerful a factor in aiding Memory, acts partly by creating a sequence of images of which the facility of Association has itself introduced a peculiar Hedonic quality. This will be observed particularly when the sequence is interrupted unexpectedly, whereupon the attention

[1] Time and Space stand in a category, or categories, apart from that of the rest of the series of Fundamental Processes. I have included them simply because in their simplest appearance they mark the unanalysable.

Of recent years the whole question of Time and Space has been re-examined frequently. Neumann and various French writers, Fouillée, Guyau, Bergson, Roisel, Lechalas, derive more or less remotely from Kant. Charles Dunan is influenced greatly by the principles of Experimental Psychology. Tchermak finds a veritable Time-sense. Mach, profound as usual, and issuing here from Herbartian notions, concludes in a manner accordant with the analysis of Motion already given. Heinrich Czolbe finds all sensation extended, and he suggests that Time is the Fourth Dimension. Here possibly he is on the fringe of a great thought, though not in the manner in which he expresses it (*cf.* p. 159 (note)). In all this, however, I find nothing to disturb the present exposition.

is called to the failure of the sequence in a little shock of a disagreeable character. An exception may be found in that form of play which is immediately followed by a sense of abundant pleasant characteristics which thus gain greater zest. All play, such as, for instance, we may observe in the mutual teasing of a pair of kittens, is of the nature of mimicry of strife with the disagreeable factors diminished to harmlessness and the enjoyment of safety as well as of exercise thereby heightened.

The influence of Hedonic quality is often masked when a high development of intellect is considered, for then we have often a sense of following out strategic or tactical lines of conduct in which present conditions must be given importance only in regard to the general scheme and ultimate intention. Thus we may perhaps find a philosopher speaking of the "discipline of an uncongenial study".

Further, the importance of secondary motives, and the pleasurable exercise which their pursuit may involve, obscure at times the primary motives for which alone they were produced. Herbert Spencer has given so admirable an exposition of this aspect of the subject that it need not be further elaborated. What may be noted as new, however, is that the Hedonic principle which he has well discussed as a great controlling factor in Ethics, is here expressed as found in the Fundamental Processes themselves, and as therefore occurring at the base and exercising influence on the course of Reason itself.[1]

Reviewing now the Fundamental Processes we find that in considering the causes of error in Reason we may direct attention particularly to Discrimination, Generalisation (symbolisation and classification), Memory and Association. A total and immediate failure with regard to any of these Processes would be inconsistent with a course of mental life.

We shall see that the errors of Reason ordinarily met with may be made clear by discussion with regard to these Processes.

(2) *Forms of Reasoning Indicating the Amplitude of the Problem*

When we speak of Reason, or reasoning, ordinarily we may imply any number of operations which have certain features in common, corresponding to the general definition of Reason already given (*cf.* p. 307), but which may also present considerable differences.

[1] If this be borne in mind throughout the entire discussion whenever the Hedonic principle is involved, it will be seen that herein is the true sanction of that principle which Epicurus and Herbert Spencer sought in complicated forms, and consequently with certain erroneous implications.

Thus, there may be reasoning along a chain of positions already traced out, as for example in verifying the proof of a proposition of Euclid.

Or again, one is said to use Reason in the endeavour to find the solution of a proposition; that is to say, to find the intermediate positions between the conditions of a proposition and the recognition that a certain other condition, not immediately evident, is thereby necessarily determined as true.

Then again, there is exercise of Reason when, having ascertained certain conditions in nature, we endeavour to meet with other and as yet unknown conditions which are dependent on the first condition. This is called discovery or research. Or again, Reason is employed when we combine known things in a convenient way toward some useful end. This is called invention.

But apart from these formal exercises of Reason, it is equally true that Reason is employed in our ordinary observation of the objects and occurrences that come within our experience. Our minds do not proceed merely by set associations from one point of sequence to another; as, in that case, we should have but an assemblage of facts without meaning. But at every experience we recall past experience; and we compare objects and their associations; we use symbols, and apply them to new objects; and we note differences or resemblances between similar objects; we generalise and we classify.

These exercises of Reason, which are in part automatic, not only serve their own purpose but they give us the fund of experience to which we appeal in the more deliberate and definitely enunciated problems of Reason which we may set ourselves.

Reasoning in Regard to Set Propositions

An element of the Process of Reason may be thus expressed: If *a* is *b*, then *c* is *d*. It may happen, in some cases, that *c* is identical with *b*. But if we examine this simple formula in different fields of application we find that the Fundamental Processes involved are in different combinations.

In order to find an opening for discussion in detail let us consider the fourth proposition of the first book of Euclid.[1] If two triangles have two sides of the one equal to two sides of the other, each to each, and if the included angles be equal, then the bases of the triangles are equal, etc. Now if we had already proved, or if we assumed as true, that three independent "elements"—such as three sides or two sides and the included angle—determined

[1] This little discussion does not require any previous knowledge of geometry.

a triangle; and if we further assumed that a triangle did not change its dimensions by changing its situation, then the proposition would be one of those of which the truth is shown by the mere analysis of the terms of enunciation.

Or we might express the fact by saying that any one triangle of the kind is the generalisation of them all in the most exact sense, since, apart from the question of position, it becomes identical with each of the triangles possible.

But if we prove the proposition, as in Euclid, we see that it is still a proposition of which the truth is manifested by the analysis of the terms, if we make certain assumptions, as before, regarding the independence of situation of the magnitudes.

Thus, if we inquire what is the meaning of one side being equal to another we find that, unless we make further assumptions regarding the measurements which are not fundamental (*cf.* pp. 107 and 132), we can only realise the nature of one side being equal to another by superposing one on the other. This is the first part of Euclid's construction. Now this also applies to the remaining given sides. And this is the third part of Euclid's construction.

Again, if we realise the meaning of one angle being equal to another, apart from the notion of measurements, we find the suggestion of superposition of the triangles with the apex of one coinciding with the corresponding apex of the other. This is the second part of Euclid's construction.

Now suppose that we ask this question: What is there in this proposition that would prevent its truth being inevitably apparent from a consideration of the enunciation?

In the first place, we must know the meaning of the terms themselves; that is to say, we must know all that is implied or connoted in the symbols. This may appear obvious, but in less simple examples failure in this respect is a fertile cause of error. We are so accustomed to use words fluently that we do not always think of their "connotation" when we employ them.

Even in this case it requires a certain effort of consideration to make us know that, with the proviso mentioned, we can test the equality of the sides only by superposition. If this is fully understood that Impulse toward fulfilling that part of the construction must inevitably arise. With young scholars the question of connotation of the symbols is not merely confined to these few steps of analysis; the difficulty arises with regard to the whole style of the language employed in enunciation. These scholars find the same difficulty as ordinary people experience in reading the "jargon" of a legal document.

The absorption of energy and the confusion resulting prevents attention being given to the essential part of the enunciation (*cf.* pp. 44, 117 and p. 160), and thus the Impulse spoken of above, which is the element of all our tentative efforts in Reason, may be lacking. Further, young scholars have a difficulty in forming the notion of a triangle of Euclid. This difficulty, and the processes involved, have been discussed when considering abstraction (*cf.* pp. 43 *et seq.*) The difficulty regarding the changing situation of the applied triangle is one of those which escape most students owing to their want of analysis; it occurs only to the philosophic student. A young scholar, however, may have a difficulty in understanding the construction of superposition for quite another reason; for he has already had difficulty with abstraction, and now, to form a clear conception of superposition, he must reimpose some material aspect on his triangle.

Another difficulty may be that, even when he has made constructions separately of the sides, and the included angle, he may not have a notion so clear and unembarrassed that his mind associates their constructions in sequence, and then when combined, as a Unit.

There are all sorts of associations formed round the objects of our consciousness, and though to those who are familiar with geometrical constructions the associations required for our proof seem inevitable, yet it must be remembered that, to many minds, the associations lead them away from geometrical ideas altogether. Consequently there must be a check to certain forms of Impulse, and a solution implies guidance of the associations required.

This is true of the next simple step after the construction is formed; for, though it seems evident that if two points be given—the extreme points of the base—only one straight line can lie between, yet this is, in the ordinary demonstration, represented as being a case where the straight lines coincide; and the associations formed by the scholar with regard to these two points may not lead him to think at all of the straight line between them.

From the mere description of the steps of the process it will appear also that the question of Memory must be important; for, by the time the process of demonstration has neared the end, the mind, already tired by the effort of abstraction, and by the unfamiliar terms and associations, may have forgotten what has been already established; and that is more likely to happen by reason of the circumstantial language in which the demonstration is couched.

It will be seen from this example that a proposition almost

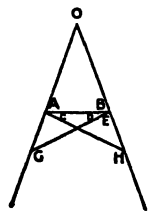
demonstrated by the meaning expressed in its terms may yet appear a stumbling-block to a mind little exercised in such notions. We have now, in a very simple case, advanced beyond the failure of Reason due to quite fundamental defects.

Reason may be at fault from failure of the Impulse producing the required Association, from a misuse of symbolisation, implying a failure of classification, and again from inefficiency of Memory.[1]

In this case the proposition *c* is *d* if *a* is *b*, resolves itself into establishing an equality of two things, where situation is not considered as affecting that equality. If now, to take a further step, we consider a proposition the truth of which is not implicitly contained in the manner of expression, we may select the ordinary demonstration of the fifth proposition of Euclid.

It is not necessary for our purpose to enter all the details of the proof; we may say that, the object being to prove that an angle *A* is equal to an angle *B*, we first prove that the sum of two angles, *A* and *C*, is equal to the sum of two angles *B* and *D*; and then we prove that the angle *C* is equal to the angle *D*; thus we conclude that the angle *A* is equal to the angle *B* (*cf.* p. 134).

This proposition presents so many difficulties to beginners that, from the olden days, it has been called the *pons asinorum* (the bridge of asses). The difficulties are of the nature mentioned in regard to estimation of numbers when too many objects are presented not clearly classified. (See pp. 34; 117 *et seq.*). There are also difficulties of Memory, for the proof that the sum of the angles *A* and *C* is equal to the sum of the angles *B* and *D* depends on the fourth proposition with which therefore the scholar should be familiar. Another application of the fourth proposition proves that the angle *C* is equal to the angle *D*. The whole demonstration is long, and here again Memory is taxed.



We know from what we have already discovered concerning Memory that these difficulties become less and less by repetition; but in the meantime the scholar may become discouraged especially if he have no apprehension of the importance of these propositions nor notion of whither they lead.

Another difficulty arises from the fact that, in this application of the fourth proposition, the sides of one triangle, *G O B*, lie upon and overlap those of the other, *H O A*. A distinct effort is

[1] A distinction is sometimes popularly drawn between intellect strong in Reason, and intellect strong in Memory. Differences of intellect do exist corresponding broadly to such distinctions; but Memory is an essential part of all reasoning, and some fine feats of Reason have been dependent on good Memory.

required to dissociate the triangles or to counteract associations which lead the mind away from the conception required.

The subtraction of the angle *C* from the angle which is the sum of *A* and *C* is not a process identical with that of substraction of numbers (*cf.* pp. 107 *et seq.*).

For the greater number, known ultimately by counting, must be recognised as made up of any lesser number and the number required to complete the total. Subtraction is the inverse of addition (*cf.* p. 109). But we prove that the whole angle, here presented by the sum of *A* and *C*, but really considered at first not with reference to these parts but as a whole, is equal to the corresponding angle represented by the sum of *B* and *D*.

To conceive all that is connoted here by equal we must conceive (*cf.* pp. 105, 106, 131) superposition. Similarly with regard to equality of the angles *C* and *D*.

The process by which we recognise that the residues, after removing the equivalent angles, are equal, is that of space-measurement which derives from experience involving certain Fundamental Processes in regard to the objects of that experience.[1]

In making use of the fourth proposition it may be observed that we might have again proved the fourth proposition in this particular instance. But the fourth proposition, though relatively simple, as we have seen, requires a series of operations for its full comprehension. It is simpler here, then, to say that the conditions of the fourth proposition are fulfilled, and that the consequences follow. Here is an example of symbolification. The examination as to the fulfilment of the conditions involves Discrimination, so that the same Fundamental Processes, though in different combinations, are employed as in the fourth proposition.

Assumptions and Sources of Error in Normal Reason

Hitherto we have considered propositions in which the sequence of reasoning has been already arranged. Before going further we may inquire what are ultimately the grounds, in such cases as those considered, for our declaring that our Reason is satisfied.

We have recognised that the Fundamental Processes, such as Immediate Presentation, or Time, are essential to any mental life. A narrow examination has convinced us that, in their elementary manifestations, a like statement is true of all.

[1] It is a useful exercise to examine the assumptions tacitly made throughout the range of geometry, and of mathematics generally. Such studies have helped to lay the foundation of this system of psychology. Duhamel has studied this matter interestingly.

But in a higher development of thought we speak of generalisation and classification, for example, as processes of value which may not be employed by ordinary minds, which nevertheless enable scientific men to carry on consecutive courses of mental life. We might be tempted to think that there was a distinction here between such processes and the others. For convenience of language and exposition we have seemed to recognise such a difference.

But, looking at the matter still more closely, we find that there are analogous variations, differences, degrees with regard to the other Fundamental Processes, each in its own quality.

Immediate Presentations differ greatly in certain cases (*cf.* pp. 32, 79 *et seq.*). The process of forming a Unit by new combinations may be thought to be just as truly a scientific faculty as generalisation and classification; if, for example, we are asked to consider as a Unit the amplitude of a Hertzian undulation, or the expansion of the second elliptical function as expressed by Jacobi.

We have seen that such Fundamental Processes as Immediate Presentations, Association, Memory (*cf.* pp. 99 (note); 223), are independent of our will in their fundamental exercise. We can intentionally produce certain conditions modifying the character of the Immediate Presentation, the Association, or the Memory; but when once the conditions of the external world and of our own constitutions are determined, the process follows. It is only necessary now to consider attentively each of the Fundamental Processes in turn to be convinced that a similar statement is true of all.

We feel then our Reason satisfied when all these Fundamental Processes are performed normally. That is to say, if we have to exercise Discrimination we know that, without check, we have exercised our Discrimination to the limit of its necessity in the case, or of its possibility as far as our conception of the case is concerned.

If we exercise Memory we feel assured that our Memory restores the idea of the original in a manner as lively and complete as we demand.

With regard to Association we find, for example, in shifting the position of a triangle, that certain associations are invariable, and it is these that we deal with. And a similar mode of observation applies to all the Fundamental Processes.

With regard to Association certain influences of experience become apparent (*cf.* p. 32, and p. 166 *et seq.*). But this is true also of all the other Fundamental Processes. We continually find, in experience, that our Discrimination is limited, that it may be

cultivated, that it is therefore modified by experience. Memory is modified by various conditions (*cf.* pp. 186, 191, 196).

The force of these observations will perhaps be better seen if we suppose that the normal exercise of any process is changed. For example, if by a fault, such as might be due to a defective Memory, we supposed that the fourth proposition proved that the angle formed by the sum of the angles *A* and *C* was equal to the angle formed by adding another angle, *E*, to *B* and *D*, then the conclusion would be faulty. In such a case we would know it to be faulty, for we had deliberately changed the indications of our Memory; but if the error had arisen unknowingly, then though it were error, we would have the same satisfaction of our Reason as when the right result was reached, and this satisfaction might remain until we found our results incongruous with some other result obtained also by a course which satisfied our Reason.

Or again, if we thought the conditions of the fourth proposition applied to a case where they did not really apply, then we would commit an error in symbolisation, an error perhaps finally referable to a fault of Memory or Discrimination; and the result would be wrong.

There is nothing mysterious in all this. Such errors are constantly committed. In attempting to prove a mathematical problem it is not unusual to be puzzled through some mistake, or to be convinced that the opposite of the true proposition has been proved. I have referred at this stage particularly to mathematical propositions because they present no great emotional associations, and the course of demonstration may be easily checked; but in all modes of Reason failure from any of the causes mentioned may arise.

What makes mathematics indeed so sure a science is the fact that, when the appeal is really made to experience it is experience of a kind that is absolutely fundamental to our mode of existence, experience constantly repeated; so that the road has been traversed, in its elementary parts at least, by a succession of minds including the keenest of all of humanity; and that finally its results can be checked more or less accurately in calculation. In the higher mathematics the check is less, but the minds themselves that traverse the route are necessarily superior. But in this most certain of all sciences we find Arago, for instance, doubting the validity of some of Abel's propositions in elliptic functions; we find Cauchy correcting and making more rigorous his own proofs in the integration of certain differential equations; and we find the whole characteristic system of Riemann, one of the most genially inspired of all, thrown in doubt.[1]

[1] In the course of reading I have met errors, acknowledged or demonstrated, of such brilliant intellects as Newton, Euler, Legendre, Jacobi, Abel. Mayer of Leipzig has written an illuminating essay on the *History of the Theory of Least Action*, in which the blunders of Maupertuis and a lapse of Euler are discussed.

The most inevitable kind of Association is that of cause and effect, yet we know that that means to our present intelligence no more than invariable sequence.

Thus it will be found, as in the cases we have already examined, that Reason is not only subject to errors, but that when these errors have been, as far as humanly possible, eliminated, then the certitudes of Reason extend no farther than those of a concordance between our minds—depending ultimately, as we have seen, on our physical constitutions—and the external universe. In this concordance we assume certain conditions of constancy. For example, if we look at a triangle, and then after diverting our attention again look at the triangle, we recognise that it is the same triangle. Underlying this process is the acceptance of a belief, though not expressed overtly or even considered, that our mental condition has remained unchanged, and that the objective reality, that is to say that part of the external world that in the instance comes within our purview, has remained unchanged.

But if we looked at a horse in this way we would say it was the same horse; and similarly if we looked at a stream of water. But as in these cases changes at length become obvious, we are led to the conclusion that the basis of our Reason depends also on the limitations of our Discrimination; and hence also in turn of all the Fundamental faculties (*cf.* pp. 44; 52 and (note)).

If we could " see as a god sees ", our Reason would be of another character; it would not be Reason, as we generally understand it, at all; it would be of the nature, so far as we can form a conception, of Intuition or the relations of Immediate Presentations. It is from our limitations that we have built up the marvellous processes of Reason which are the glory of our race.

* * * *

Mathematical examples are useful in the study of Reason because we are less liable to be led astray by accidental associations.

But keeping in view this danger we may take other examples. For instance, we read a story concerning the elder Pitt. Certain associations are formed around this symbol. We are acquainted with other stories concerning the Earl of Chatham. Now if we find that the symbols elder Pitt and Earl of Chatham apply to the same person, then we may have a proposition which is expressed in its very terms when we apply all the associations of the symbol elder Pitt to the Earl of Chatham.

This example may be compared with that of the fourth proposition, essential features of resemblance being noted and accidental differences disregarded.

Referring now to examples more important, we may cite the case of certain of Frauenhofer's lines found in the spectrum of the sun being identified with those of the spectrum of sodium. From this identity we proceed to apply the associations that have been gathered around sodium to part of the composition of the sun.

In this case there are really several assumptions. One is that these lines are due to sodium always; another is that in so far as we apply the same associations, sodium has the same properties in the sun as on earth.

Such assumptions are constantly being made in reasoning and though they are often sources of error, yet if they be true they lead to the establishment of certain laws of nature; and if they be untrue they nevertheless serve a purpose, for the criticism by which they are shown to be untrue reveals new truths.

Thus every generalisation bears an assumption and the progress of science has been thereby greatly aided.

When Newton enunciated his law of Universal Gravitation he certainly had not considered all the material particles of the universe. Yet the generalisation affords a basis on which we proceed to new discoveries, and the verification of these serves to confirm our belief in the law. [1]

But even when the generalisation seems devoid of the chance of error, as in proving a proposition for a given triangle, and so for all of that kind, the certitude is only great because the complex of associations is small; and the certitude is not greater than that of the assumptions we make regarding the invariability of the triangle in a varying situation (pp. 130, 134).

We may now observe how the principles we have investigated may apply to a complex case. The undulatory theory of light depends on the hypothesis of a universal ether which conveys the undulations. We cannot ascertain the presence of ether by any of our senses, we cannot weigh it, we cannot even very well conceive it. The suggestion arises from the study of other undulations —in water, in air; and so, for the phenomena of light, which

[1] The theories of the conservation of Energy and the conservation of Mass are really founded on assumptions which imply the truth of the theories. Of what avail is it to offer the most comprehensive and brilliant display of mathematics, if the whole process be conditioned by these assumptions, and the test of proof be only to show that the result is in concord with the assumptions? But it is said that the problem has been demonstrated. Landolt proved that in a reaction between silver nitrate solution and salt solution, producing silver chloride and sodium nitrate, there was no loss of weight. His methods were exquisitely fine, but they leave untouched the question: if a body escaped the law of gravity how could it be known by weighing? More widely, how could it be known at all? Moreover there is an assumption in applying universally a particular result. Finally, however, it does not follow that a principle is untrue because the arguments offered in its support are inadequate.

possess so many subtle associations, we imagine a medium correspondingly subtle.

Physical investigations have shown that if the ether be real it must possess qualities which at first we would not be inclined to ascribe to it. Kelvin for example thought it must be solid. Various difficulties that have grown up have suggested to great physicists that the hypothesis may be false. Even so, it has served to advance physical science.

Now the phenomena of light have been investigated on the assumption of the undulatory theory and a highly developed instrument of mathematics has been elaborated to express results and to aid in the search for new truths. Clerk Maxwell, in the course of such investigation, found many points of resemblance between the phenomena of light and of electricity. From the study of these arose the suggestion that the phenomena of electricity were due to undulations of the same ether. And finally we arrive at the theory that the phenomena of light, heat, electricity—all forms of radiant energy—are due to the undulations of ether.

Here is a case of generalisation based on an hypothesis which may not be true, but which has already produced marvellous results, even in practical domains.

Thus generalisation not only permitted the mathematical apparatus already evolved for light to be applied to electricity, but it also suggested to the genial mind of Hertz to attempt to reproduce, or to adapt, for electricity the experiments of reflection, of refraction, and of interference with which physicists were familiar in the case of light.

Among the consequences of Hertz's experiments may be noted that of wireless telegraphy, which therefore has one of its sources in Clerk Maxwell's brilliant and patient efforts of abstruse thought.

In the examples we have just cited the proposition if *a* is *b*, *c* is *d*, does not result in a mere identity, or equality. It is true that by the artifice of language we might express them as equalities, thus: If these Frauenhofer lines be the lines of sodium then the corresponding material in the sun is identical, as far as constitution is concerned, with the sodium of our planet.

But it is better to follow the natural process which arises in the mind itself, and to say that, if these lines be due to sodium, then we apply the symbol sodium to the material in the sun, and in applying this symbol we associate with the material so symbolised all the facts which we have hitherto ascertained regarding the sodium of our earth.

But even in the case where we say naturally if *a* is *b*, *c* is *d*, with the implication of equality, or (with certain abstractions) identity,

we may express the proposition also as that of associating with *d* the associations which we already formed about *c*. Moreover, a close study of the manner of movement of the mind, of the Fundamental Process therefore, will now convince us that that is the meaning ultimately implied in the copula, "is".

We have used the expression "equality", or (with certain abstractions) "identity" in order to lead to that conclusion; for we have already seen that there is no equality except that of identity (*cf.* p. 130); and when we speak of identity, with certain abstractions, we can only mean that there is a process of Disassociation (*cf.* pp. 44; 129 *et seq.*), and that there is a process of Association. The processes of Disassociation and then Association have already involved the whole series of Fundamental Processes.

Belief

The symbol, Belief, is used to indicate a state of mind in which these Processes are formed. No state of mind, it must be remembered, is merely passive (*cf.* pp. 73 (note); 181). At each experience, or with each proposition accepted, the mind places itself in a certain condition, and this condition is influenced by the physical correlatives; and these in turn are the resultant of innumerable forces and conditions which have come to us through countless ages of evolution, through our own individual development, modified by past experience brought to a state which has its expression in consciousness.

All this implies, not only by its factors but also by their resultant, a certain Hedonic quality involved in that state. Moreover, as we find not only the Impulse that produces in sequence some new Association formed in consciousness; but as, from a consideration of the physical correlative we know that there is a magazine of forces there endeavouring to send forth impulses in various directions; we have a state of expectancy created.

Now we have considered Disassociation as the inverse of Association (*cf.* pp. 44 *et seq.*); the same opposition exists in regard to Belief and disbelief.

Belief, accordingly, corresponds to that state of mind in which the natural Process of Association finds uninterrupted exercise, and in which the Hedonic quality is of the same nature as in Association, distinguished from Disassociation.

Belief then corresponds on the side of the physical correlatives to a dynamic disposition of forces having a resultant in a determined direction; and in consciousness to an Hedonic condition consonant with the fulfilment of expectancy in the movement along certain associations.

But as we have seen that Reason depends on the scope of all the Fundamental Processes as applied to definite experiences, and as these Fundamental Processes are all limited and fallible, it follows that Reason is fallible in so far as it may be supposed to offer demonstrations of the relation of things of the external world; but Belief is determined by these processes of which the failure cannot be made apparent, for otherwise we would simply revise the data within the limits of our Discrimination; consequently Belief gives no real certitude of a correspondence with the external world.

As a matter of fact we constantly meet with people who hold, as fundamental matters of belief, opinions or doctrines which, whether true or false, are inconsistent with the opinions or doctrines which other people hold as fundamental matters of belief.

Such opinions and doctrines cannot all be right. What then is the criterion? Simply this: As we have already seen in geometry, we must seek to rest our Reason on those experiences which are the most fundamental we can find, and to proceed from these by Reason rigorously examined step by step toward our new positions and conclusions; to submit these to every kind of test available in correspondence with the external world; and to modify them in accordance with indications so obtained; and, in order to discover the origin of any discrepancy, to submit again the whole process to our Reason (*cf.* pp. 307, 311; 322 (note)).

The discussion of such questions as that of Belief is difficult because it demands on the part of all a close introspection, and moreover it may happen that the words themselves do not convey the same meaning to one person as to another. Hence there has been repetition and a certain insistence, in the foregoing exposition, with the object of making the matter clearer, if possible, by holding the attention to it.

The discussion under a synthetic form (*cf.* also Chapter on Ego and Will, pp. 273 *et seq.*) will also serve to enable the mind to seize the explanations offered.

(3) *Synthetic Form of the Question of Reason*

Throughout the whole course of this exposition we have referred to the similarity of the processes in operation, for example, in different professions. This is easily understood when we consider that the basis to which analysis led us was that of the Fundamental Processes, by the combinations of which the whole of our operations are formed. And the whole of our knowledge is built up by the associations of actual objects of experience with these operations. Thus in quite different activities of life the formal schema

of the operations may be similar, but the actual concrete facts by which the schema is illustrated make the difference of knowledge.

Let us consider then how by a graphic example we may illustrate that movement of reasoning by which, proceeding from given conditions, we arrive at the demonstration of a certain other condition.

Thus we may express our example: We walk along a passage, not knowing from the beginning what route we may take, but having a sum of previous knowledge and guiding principles so that at each turn we may be able to decide what to do. We meet with a cross chalked on the first door. Associations already learnt indicate to us to take the first turn to the right. Had the mark been a circle it would have indicated a turn to the left. We proceed according to the directive, and meet a door on which we see printed: "Open the door and read the instructions left lying on a table in the hall". The instructions tell us to proceed to another place where we see a plan with an arrow pointing in the right direction. Arrived at a certain corner where two roads meet we consult a map in our pocket and we find ourselves directed to a house. There we seek a soldier looking out of a certain window. He advises us to take the road to the left. And so on we proceed to the end of the journey.

Now in a progress of this sort it is not necessary to remember the steps of the journey beyond the step actually dealt with and its path to the next. But step by step we move until finally we associate the end of the journey with the beginning.

Suppose now that instead of the line of reasoning being traced out in advance, our problem is to find it. For example, taking for convenience of reference an example already considered; what would be the course of an investigator of the fifth proposition of Euclid who, believing that the angles at the base of an isosceles triangle are equal, endeavoured to find the best demonstration of the fact?

Demonstrations of this sort are generally sought for analytically; that is to say, the mind proceeds from the conclusion to trace backward the steps by which it may be reached until in this way the starting point, namely, that of the given conditions, is arrived at.

Synthetic demonstrations are obtained by reversing this process. The close examination of the method of analysis is especially instructive. Thus in the fifth proposition of Euclid, if the angles at the base be equal then the angles formed by adding equal angles to these angles will be equal.

But how does such a suggestion arise? There is nothing inevitable in that suggestion, for it might as well have been suggested

that the angles formed by subtracting equal angles from the angles at the base will be equal, and it is indeed possible to form a demonstration in this way.

It becomes apparent on observation that we allow freedom to Impulse in forming various associations, and we select those that seem to lie in the way to the goal. There is no certainty in such suggestions and when the successive steps are very numerous it will be seen that the possible combination of associations upon association may be enormously large. Hence the difficulty of such problems.

With constant practice certain general principles, such as that of symmetry, the research of simplicity, suggestions from known forms, and previous demonstrations, come to our aid (*cf.* p. 216). Moreover just as we have seen in the case of Memory, so in all these processes the speed of operation is wonderfully increased by continual exercise.

In the fifth proposition the suggestion of angles formed by addition soon leads, since we are dealing with equality of angles, to the application of the fourth proposition, which demonstrates under certain conditions the equality of angles. The next step in this instance is to prepare these conditions. The rest of the demonstration is comparatively easy.

Here then we have an example of obtaining a solution by means of tentative efforts; that is to say, by giving free play to associations and testing those that seem to promise some real progress.

These associations are not necessarily confined to those that are habitual or soon apparent. It may happen, in searching for a solution, that the enunciation of the problem must be expressed in some form of deep generalisation before the appropriate suggestion arises.[1]

Analogies Between Forms of Reasoning in Diverse Fields

It is interesting to seek corresponding examples in quite different fields of thought, and this exercise will throw light on the developments that may be derived from the positions we have already

[1] The number of possible combinations to be exhausted may be enormously great—as for example in a field which has afforded the highest triumph to mathematical genius, namely, in the search for means of investigating elliptic integrals. Abel found that all possible combinations of ordinary methods seemed to be fruitless, and he also observed the fact, in his examination of the sources of these ordinary methods, that unless the properties of logarithms had been investigated it would have been impossible by those means to have obtained the integral of $\int \frac{dx}{x}$.

Hence he was led to study in what way one might obtain a clear notion of the logarithm by proceeding from the problem of integration of $\int \frac{dx}{x}$; and the methods so revealed were utilised in the analogous but more difficult problem.

established. For while the Fundamental Processes remain unchanged the actual objects to which they may be applied differ. Thus the processes of the mind in all combinations may be looked upon as a schema, and the character of the individual experience will determine the concrete details by which the schema becomes illustrated.

Hence we should expect, and we shall see this expectancy more and more clearly satisfied, that mental operations in different professions, or modes of employment having apparently little in common, may be shown to be similar.

The problem of finding a solution for a proposition considered as true might be compared to that of finding one's way from Hyde Park to the Tower of London. Various tentatives are made. Occasionally it is found necessary to abandon one route; occasionally the path taken seems to be making the journey longer; but, after a series of tentatives, keeping the object of the journey well in view, we succeed in accomplishing our purpose.

Suppose now we had already explored the route from Ludgate Circus; then if, in our tentative efforts, we had arrived at Ludgate Circus and recognised it, the difficulties of the problem would be at an end. But if we had not been perfectly familiar with Ludgate Circus, having observed its aspect only in one direction, then we might reach Ludgate Circus without recognising it.

An analogous case might arise in a mathematical problem where a formula, or a geometrical combination, might be unrecognised because met with in some unfamiliar form.

Such considerations indicate that to the mathematician good Memory is essential, and that especially the recollection of a considerable number of oft-recurring formulæ is very important. The advantage is also made clear of expressing formulæ and propositions in a general form so that they may be recognised even when associated with unfamiliar accidentals.

Another example might be drawn from the practice of a surgeon who has diagnosed appendicitis; but who, in his exploration, does not immediately find the appendix. If he recognises the cæcum, and traces it to its junction with the ileum, he is easily led to the discovery of the appendix.

We have often referred by preference to mathematical problems, because illustrations drawn from actual practice in various professions are always confused by complexes of other operations together with their associations; but the exercise is no less useful to find in them also analogies to processes met with elsewhere.

Thus with the warning against confusion we might compare with the problems already considered that of Napoleon in seeking

to free Toulon Harbour of the British vessels in the Harbour. His analysis of the situation led him to consider that a direct attack would be useless, but that if he secured the position called Little Gibraltar he would then be able in comparative safety to cannonade the ships in the harbour. And it so happened that when he seized this position the British ships departed.

At a later period of his career, and at the height of his ambition, he had conceived the project of subjecting the East and drawing therefrom immense resources of troops. In order to render his plans feasible it was necessary to secure the Fort of St. Jean d'Acre. Sir Sidney Smith defended it successfully. Napoleon's officers were astonished at his persistency in attempting to seize that *bicoque* (little hole of a place); but in military parlance such positions are the "keys" of the situation. Regarded from the purely intellectual side they are the intermediate steps in the way of the solution of a set problem.

The cases where solutions are obtained only by means of a great series of tentative efforts might be compared to the difficulty of finding one's way about a vast city in which at each corner a great number of roadways were found, some of which ended in blind alleys. This will also explain why the persistent exercise of trial and study helps toward the solution of problems. With each tentative some new area is explored and becomes known. We at length form associations not merely between one path and continuous paths, but also between distant points.

With practice and familiarity also, as we have already seen (*cf.* pp. 115; 204), the mind is rendered apt for further discrimination, and free to form more Associations. Memory is also aided by the greater facility of observations in regions that have hitherto absorbed energy too greatly and have prevented the exercise of Associations in other regions (*cf.* pp. 180 *et seq.*).

Thus for example, in exploring the town the mind may at first at each turn be confused by the multitude of objects presented. At length, in the manner indicated, comes a suggestion that a certain aspect of a building corresponds with what we have learnt of a certain landmark to which a definite symbol is attached. It may happen that by our previous knowledge the solution of the problem is known from that point.

Error may occur in the course of such a search for solution in various ways. Let us take an example from the problem of finding our way in a town. Thus, after an absence of some years from Paris, I find myself in a part of the town where I seek the rue de Londres. The rue de Londres begins near the Place de la Trinité, from which it ascends obliquely. To the east of the rue de Londres

another street, which begins at the Place, between it and the rue de Londres, also ascends obliquely but more nearly north and south. This Place is to the east of the Gare St. Lazare. I am in a place which has a similar disposition of streets, one ascending obliquely from a Place, while the second, between it and that Place, ascends more nearly north and south.

The fact is that these streets are to the west of the Gare St. Lazare; but from the general aspect of the streets, the suggestion comes to me that the street running more westerly is the rue de Londres. Now I have not identified this Place nor given it a name. The general aspect of the Place, considered as a Unit, has thus become itself the symbol with which I have associated the aspect of the rue de Londres and thence all its known associations.

I proceed to walk along this street and find it unfamiliar; that is to say, find that the associations presented in idea do not correspond with the Immediate Presentations afforded by the street.

I return to my starting point, thinking that I may have taken the wrong direction; that is to say, with the intention now of Disassociating erroneous associations which led me to the wrong street.

The symbol—that is to say, the general aspect of the Place—has Associations so strong that my Discrimination acting at the disadvantage of a Memory weakened by lapse of time, is not efficient enough to lead to the Disassociation of what are really the erroneous Associations. I again walk in a street I suppose to be the rue de Londres, and for a time I disregard the unfamiliar aspect; that is to say, I disassociate these associations. Then I see the name rue de Rocher. Here is a new factor which supplies associations with Impulses strong enough to disassociate the previous associations.

Proceeding along the line of these associations in a contrary direction, I arrive at the examination of the Place itself. Having no other outlet I am forced to pose the question: Why do I think this the Place corresponding to my former associations of a Place with the rue de Londres?

This condition of the mind, with the attention strongly held to the Place, with the broad general characteristics only known, and with an inhibitive or active disassociation of the previous associations, produces tentatives—that is to say, Impulses bearing new associations. Pursuing these, somewhat at hazard, but with the intention of arriving at a known point, I at length form associations of the other end of the rue de Londres with a situation on the east side of the Gare St. Lazare.

These Associations correspond to a good Memory, which brings a chain of recollections; that is to say, Associations in ideas, which

by their liveliness and their consistency with external objects, and ultimately with the Immediate Presentations, give me a feeling of distinct Hedonic quality, of certitude, or Belief. The Impulses become directives to action. I proceed to find the rue de Londres near the Place de la Trinité.

(4) *Sources of Error in Extended Reasoning: Errors Generally Considered as of False Premises but Capable of Analysis to Fundamental Processes*

Errors of the kind here investigated are formed in every field of Reason. The error might be expressed by saying that the premises are bad, though the reasoning may be good from that standpoint. But it must be understood that the premises from which the error flows may be found at any stage of reasoning, and that the error of the premises may be due to any of numerous causes.

What is called the error of false premises may be due to the failure of any of the Fundamental Processes.

The error of Immediate Presentation of mistaking a red light for a green may be considered as a case of false premises in regard to the reasoning proceeding from that point.

In fact, as our course of reasoning proceeds in ordinary life, every fresh step accomplished affords the premises for the succeeding steps, so that error of any kind in reasoning may result in false premises.

If we consider the star *a Centauri* of the pointers of the Southern Cross as a single star, and base a course of reasoning on that assumption inconsistent with its being a double star, we have a case of false premises due to wrong comprehension of the Unit. If we see a canvas daubed with paint, and fail to place ourselves so as to recognise the unity of the picture, we may in our reasoning commit an error of the same category, which we may describe as due to false premises. It will not be necessary to recapitulate every case in turn. The most frequent causes of error due to false premises are probably those which depend on wrong applications of a symbol, or wrong Associations with a symbol correctly applied; and these cases in turn may be due to want of Discrimination.

In the example cited of the error regarding the rue de Londres it might have happened that no incident followed which called my attention to the mistake. In that case the Associations of the symbol—namely, the general aspect of the Place—would be strengthened. If afterwards then I had learnt and had applied the verbal symbol Place de la Gare St. Lazare in question, then a strong association would be formed between this symbol and the rue de

Londres. If these associations were repeated frequently enough, and especially if they were learnt at an age when the mind was most impressionable and tenacious, then it might happen afterwards that, even if facts in contradiction with the original error were noticed, the tendency would be to explain them away by some other kind of false reasoning, rather then to revert to and examine the original source of error.

Thus for instance, I may cite as bearing on the subject—and it is always useful to trace such relations—a distinguished professor of engineering who used to describe the so-called " practical man " as the man who had perpetrated the same blunders so often that he could not do otherwise.

Taking an example from engineering: A practical man having had occasion frequently to strut up weak structures with extra pieces, proceeded on the assumption that this was always a good plan in case of doubt. He had to deal with a part of a structure consisting of a quadrilateral frame in iron with a diagonal piece. Now, as a triangle is fully determined by three sides, so the quadrilateral frame was fully determined by four sides and its diagonal. But the practical man inserted another diagonal crossing the other, and strained the framework almost to breaking before any weight at all was applied. The whole structure subsequently fell.

Another case was where a practical man having to construct a railway and a reservoir, hit upon the idea, brilliant in itself, of making the embankment of the railway serve for the bank of the reservoir. He was under the delusion that the pressure of water on an area of the bank depended, not on the depth of the area below the surface, but on the superficial area of the water in the reservoir. The reservoir was not large in this respect and accordingly the engineer thought it useless to fortify the bank by a puddle-wall. An appalling disaster duly happened.

Or again, a medical man, knowing that he must operate at once in certain cases of peritonitis, finds a patient presenting certain of the symptons, and performs an abdominal operation for what was really a case of acute glaucoma—a disease of the eyes—with intense suffering producing various symptoms, including those observed. Here the false premises have been due to inefficient diagnosis, that is to say, failure of Discrimination.

In many of these cases of false premises a distinct emotional factor plays an important part (see also pp. 166 *et seq*). For example, proceeding from the case just cited, our young surgeon, say, receives an admonition which causes him to take great interest in glaucoma, one of the symptoms of which is a hardness of the eyeball. Soon after he finds a patient with a tension of 3, as he

calls it. Now had he expressed himself in language less technical the association would be less strong; but he had learned that tension 3 indicates glaucoma. He decides to operate, but he discovers at length that the eye in question is a glass eye.

The Museum in Paris bought a scarabæus from the widow of an eminent Egyptologist. The scarabæus had an inscription which confirmed the recital of Herodotus about the celebrated circumnavigation of Africa organised by the Pharaoh Necho some six hundred years before the Christian era.

In this case also the emotional factor came into play. The inscription was so important and in every way so interesting that, in short, the desire to give free course to all the Associations it evoked prevented a critical examination of the validity of the claim made for the scarabæus. The symbol was erroneously applied through defective Discrimination. The premises were false. The scarabæus had in fact been manufactured in Montmartre.

A case celebrated in literary history is that of the letters of Pascal, which were sold to a distinguished mathematician and member of the Institute of France, M. Chasles, well-known for his work on unharmonic co-ordinates and for his researches into the history of the early development of mathematics. The purport of the letters was to prove that Pascal was really the inventor of the differential calculus. The delight of M. Chasles at this discovery blinded him to the lack of evidence of the authenticity of the letters, which were afterwards shown to be false. The argument which Chasles had drawn from them respecting the development of the differential calculus was thus erroneous, owing to false premises.[1]

Errors of Classification

For errors due to faulty classification the formal statement might be made thus: If *a,b,c,d,e,* be necessary conditions in the widest sense for the production of any phenomenon, and if we have frequent experience of *a,b,c,d,e,* producing *x,* and if *e,* for example, be a condition not readily observable in comparison with the others; then it may happen that already when we observe *a,b,c,d,* we anticipate *x*: and if at any time the series be really *a,b,c,d,f,* we may confuse *f* and *x,* and be not aware of our error until the non-appearance of *z* causes us to examine the conditions more closely. Here we have already had the series *a,b,c,d,f,* wrongly classified as that of *a,b,c,d,e,* and the due results of *a,b,c,d,e,* have been wrongly associated with *a,b,c,d,f.*

Popular errors are frequently due to this cause. Let us consider, for example, those of circumstantial evidence. The house of

[1] Alphonse Daudet used this story in his delectable novel, *l'Immortel.*

an artist, a painter, has been robbed. The wife of the artist says that she was bound and gagged by three men—one with a red beard, and another with a black beard—who were in company with a woman. Subsequently the police arrest a poor sculptor and his friend who live in the neighbourhood. The friend has a red beard. His description tallies with that given by the painter's wife. The sculptor has no beard; but it is proved that he used to wear a black beard previous to the robbery. The sculptor and his friend had set out for a long walking tour about the time of the robbery. Previously they had been in want of money. On the walking tour they had plenty of money. On the walking tour they were accompanied by a woman. Their habits were what is known as Bohemian. The public cried, "Down with the assassins!" But, as a matter of fact, the sculptor and his friend were a hundred miles away from the scene on the night of the robbery. The sculptor had removed his beard for reasons of artistic variety. The sudden access of means was due to the sale of his works. And so all the links of the circumstantial evidence fell one after the other. Moreover, the supposed robbery had never taken place.

In a case like this the value of the "chain of evidence" should be estimated in comparison with the value of other possibilities. Thus in a large city as Paris there might be hundreds of men with red beards and scores of artists who being pressed for money one day might well be in funds the next.

Yet at a certain moment the public were for the most part inclined to consider these men culprits. Such a movement of the mind is popularly called jumping at conclusions, and the fact that it is so designated, the designation acting as a warning, indicates how popular a form of error it is. It is associated often with what is called an impulsive character, and that also indicates its connection with errors of Association; for Associations in our typical case between a,b,c,d,x, are strong enough to give a dominant Impulse before they can be overridden by the Disassociation proper to f.

The fact that this error can be demonstrated either as a failure of Classification or a failure of Association, indicates once more this necessary connection between all the Fundamental Processes. (*cf.* p. 159).

It is convenient, however, to exhibit it as a failure of classification, because in this way attention is formally called to the necessary conditions; and the habit of ascertaining the existence of all the conditions becomes itself an association of influence in such reasoning.

For example, in any chemical experiment we must have certain conditions established before the experiment can be performed.

To take one of the simplest instances, we have oxygen and hydrogen mixed together in the proportion of one to two by bulk in a jar. Here already are essential conditions for the disappearance of these gases, as gases, and the production of water. But another condition is necessary, the passage of an electric spark. Then the whole of the conditions are satisfied and the expected phenomenon is shown.

Again, an amateur chemist reads in a book that if phosphorus be placed in contact with iodine spontaneous combustion will occur. He exhibits the experiment to a class and he finds that the expected result does not take place. By experience, however, he learns that the phosphorus, after being taken out of a jar containing water, should be dried carefully. This constitutes a condition which, though essential, is not so prominently brought to the attention as the other conditions.

We see here too the effect of training, for each experience of failure calls attention to the condition thus involved. Hence every man in his own trade or profession is relatively efficient in such reasoning, although possibly prone to error in other subjects. All adopt some form of Classification. Thus a lawyer in reading over a deed of conveyance examines all the necessary provisos. These are the conditions necessary to secure the validity or efficacy of the deed; that is to say, they are the conditions of classification of deeds with which certain associations, such as those of validity, are formed.

A hostess who casts her eye over her table to see that all the necessaries are present uses her faculties in a similar way. A policeman who tries to recognise a man from a description of physical characteristics considers one by one the elements of classification.

It is the mark of a well-trained mind to examine in all circumstances whether the necessary conditions are present before applying a symbol or expecting certain associations. This training gives the character of mind to which popularly the terms judgment, good balance, level-headedness, are applied.

All errors of the application of symbols may be expressed as errors of classification. For example, we call a whale a fish, and thence proceed to associate with the symbol, whale, all the qualities associated with the symbol, fish.

At length we discover certain incongruities, such as that the whole whale possesses a breathing apparatus unlike the analogous organs of the fish. We are then led, as a result of our examination, to define more closely the qualities according to which we would apply the name fish or whale. That is to say, we classify. The

application of the principles of this classification implies here a good deal of special knowledge. Sometimes the special knowledge may demand a considerable study of a branch of science, and even then the principles of division may be so difficult to express that the opinions of experts may differ. This may occur in the problem of placing low forms of life, such as sponges, in the animal or vegetable kingdoms.[1]

In certain instances the difficulty may be that even if the classification according to which we apply a symbol be well determined, we may be unable to ascertain whether a particular object possesses the properties indicated in the classification. Thus, if the tuberculosis of cattle were identical with the tuberculosis of human beings then it would be necessary to take stringent measures in regard to cattle living or dead, to minimise the possibilities of infection. But at one time there was a great conflict of opinion in regard to this problem. Most of those having some acquaintance with the subject were inclined to believe that the two kinds of tuberculosis were identical. The application of the same name already indicates this, and this symbolisation, again, has a consequence in making observers more liable to adopt associations in accordance with it. But Professor Koch and many other bacteriologists of the first rank held that the two diseases were not identical.[2]

Hence we have a very important practical problem which becomes reduced to the issue of determining the proper application of a symbol. But that determination involves a series of tests, intricate in themselves and demanding a mastery of technique and a power of observation so great that for years after the question had been mooted no solution which convinced scientific authorities has been obtained.[3]

[1] Sir Ray Lankester has given under the familiar title *Science from an Easy Chair*, two admirable chapters on species and the principles of classification of species, and he there treats of the difficulties of the "systematists" who make a special study of such principles of classification and their application to certain species. In Pleistocene times the muir-fowl or red grouse of Scotland and the rype or willow grouse of Norway, formed one species. Sir Ray Lankester poses the problem as to whether they should now be classified as of distinct species.

[2] An explanation of these discrepancies may be found in the Report of the Royal Commission on Tuberculosis (Human and Bovine), 1911. (1) The tubercle bacillus shows great variability. The general conclusion, according to Dr. A. Eastwood, is that with the exception of certain viruses of avian origin every variety of virus obtained from the lower animals has also been found in man.

[3] Cases of a somewhat similar kind may arise even in mathematics, but here the difficulty, though demanding a high degree of skill in the demonstration of properties necessary for classification, may become resolved with certainty. For instance, in the investigations in the field of integrations we arrive at the tabulation of certain kinds of elliptic integrals. Conditions may be set forth under which expressions differing from those of elliptic integrals may yet, by certain mathematical devices, be reduced to similar expressions. Thus we may seek to find the curve named Delaunay's Curve, which is traced by the focus of an ellipse which rolls without sliding on a straight line. In the terms of this problem there is no statement of elliptic integrals; but if with the knowledge of the properties of the ellipse we express the conditions of the problem in

In other cases it may be that the properties possessed by an object are not in doubt but that the question is: In what manner should those properties be estimated in order to decide the application of a symbol? For example, take the term Great Man. With this term we associate certain circumstances. After death the body of the Great Man is interred in Westminster Abbey; or in Paris in the Pantheon.

In Paris, the question was posed as to Zola being a Great Man worthy of such honour. His admirers urged the greatness of his literary career, and particularly his self-sacrificing endeavour to secure justice in the Dreyfus case. His detractors urged principally the grossness if not the immorality of his books. Finally the admirers of Zola carried the day, although not without violent protest on the part of his adversaries.

In England similar questions have been raised, though the consideration of the matter has taken place in a calmer atmosphere, and the less powerful representatives have been in a negligible minority. Thus, after the death of Herbert Spencer his admirers considered that he might be called a Great Man—on the grounds of his having possessed one of the noblest minds of all time, and that this mind was, with marvellous consistency and force, ever devoted to the cause of truth. His adversaries urged his want of conformity to certain opinions accepted by the majority. In this case the title of Great Man, with its necessary association of burial in Westminster Abbey, was denied.

In the case of Sir Henry Irving the adversaries urged that, even if Irving had been a great actor, no skill in a mimetic art should entitle any one to be called a Great Man. His admirers held that he was the representative of the histrionic profession in England, and in this case the title of Great Man was granted, with its association of burial in Westminster Abbey.

Emotional Influences in Reason

Emotional, or generally affective, conditions, play a part in influencing what at first sight might be taken to be cases of pure reasoning. (*Cf.* pp. 166 *et seq.*).

Let us consider an example used elsewhere for another purpose. A child has been playing with a ruby (*cf.* p. 273 *et seq.*). Then afterwards it mistakes a piece of live coal for the ruby, and this error leads to unpleasant consequences. Subsequently the child

mathematical symbols; and if we transform according to mathematical principles the expression so obtained, then we would arrive at length at a form which we see corresponds to that of an elliptic integral. In this case a high degree of skill is required to demonstrate that the conditions necessary exist for the classification in question; but when once that demonstration is accomplished no doubt remains.

sees the ruby again and proceeds to play with it. The mother of the child, mistaking the ruby for a live coal, calls out suddenly to the child to beware. Instantly the child draws back its hand.

In this case there has been no failure of Discrimination but a new series of associations of great dynamic Impulse has been set in motion by an external stimulus affecting another part of the brain.

In this case, as far as it is possible to judge in such matters, we find the physical correlatives in an actual conflict of physical forces, and the greater part of the whole process is carried out subconsciously.

With more experience and less faith in the maternal advice, or less inclination to obey the maternal command, the child might have relied on its own faculties, and in that case its first desire would have prevailed.

What we have here exhibited as the emotional factor in a simple form becomes developed in accordance with all the faculties which contribute to our reasoning in complex affairs. The emotional factor exists none the less when it has itself taken complex forms and when it is revealed in the predilections for one form of Belief or another in politics, in religion, or in the hopes and desires of personal ambitions. Popular observation has indicated these truths in the time-honoured *dictum* of convincing a man against his will.

Any strong desire to believe the consequences that flow from any fact will create in the mind a tendency to accept that fact. A judge has a prejudice against a prisoner whom he wishes to punish. He is content with scanty evidence of guilt. Or he has a strong predilection in favour of the prisoner and it is repugnant to him to inflict a penalty. In that case he requires proof amounting to metaphysical certainty. The story of the man who cried, "I see it, but I don't believe it," may not be apocryphal.

Faulty Association where Premises are Correct

The inverse instances, where premises are not false in themselves but the consequences attached to them are false may now be considered. The symbol is correctly applied but the Associations have not been found in experience or have been misapplied through want of proper Discrimination.

The Chinese for example, were able to predict eclipses. So far therefore their application of the symbol corresponding was justified. But they attributed to the eclipse consequences that were not necessarily connected with it.

This view of eclipses pervaded Europe and Milton refers to it in his *Paradise Lost:* "And with fear of change perplexes

monarchs." Nor were the false associations with regard to eclipses destroyed by any direct arguments but by the fact that, the Copernican System having been on various grounds well established, all that necessarily derived from it became accepted. Hence a rational view of the causation of eclipses prevailed and no associations derived from the false view were strong enough to counteract the true explanation.

The Chinese, believing that in the eclipse the moon was in danger of being eaten by a dragon, sought to frighten the dragon away by beating tom-toms. The fact that, after this performance, the moon remained intact served to convince them that their notion of the dragon was correct.

To express this formally, we would say, suppose that *a* is associated with *b*,*c*,*d*, by faulty reasoning, and that this reasoning is doubted. Then if afterward there be found veritably associations of the kind indicated between *b*,*c*,*d*, the tendency is to accept the reasoning, *a*,*b*,*c*,*d*.

In cases of this kind emotional factors very often exercise influence, as we shall see.

In the meantime it may be noticed in what way errors so arising are related to those arising from consistent reasoning on the basis of false premises.

For example, in searching for the rue de Londres (*cf.* p. 331), I met with certain experiences that formed a barrier to any further progress of my associations, and at length forced me to call in question my first assumption—that of the " premises "—erroneous on account of faulty symbolisation leading to faulty associations.

But if I had in my progress met with a number of experiences consistent with the notion that I had found rue de Londres, although really consistent also with a contrary notion, my Belief would have been strengthened that I had really found rue de Londres.

This observation is of considerable importance practically, for in matters of some complexity, and especially those involving vital interests or interests greatly exciting the emotions, it will seldom be found that the questions are argued reasonably between opponents, but that the issue to which the argument leads is often something quite irrelevant.

In the celebrated dialectical duel between Gladstone and Huxley relative to the morality of the affair of the Gadarene swine, it may be said, without here taking any side in the contest, that the discussion degenerated into a series of curious arguments on the authenticity of certain records or the weight of certain authorities. The two champions had their warm partisans who followed the

wrangle eagerly and who, as each of these inessential points seemed carried by one or the other combatant, felt their whole ethical system confirmed or menaced.

Or again, a politician argues that the whole principles and aims of the Russian Bolshevists are bad. An opponent challenges this view. From point to point the argument is conducted until it becomes a question whether Korniloff was really put to death by the Bolshevists who had discovered his treason to their cause. In this argument considerable knowledge of facts of Russian life and history is displayed and the discussion involves a good deal of mental energy. Finally it is declared that Korniloff was so put to death, and the anti-Bolshevist leaves with the assumption that he has carried his whole position; and though this may not be granted by his opponent, yet even he feels that a serious blow has been given to his ethical principles.

Or again, Gibbon in his *Decline and Fall of the Roman Empire*, propounds a theory of the downfall being in considerable part due to the establishment of the Prætorian Guards. This factor seems inadequate to those who have thought of the development of nations and consequently of the failures of these forces as accounting for their disintegration. But if the argument turn on the correctness or otherwise of Gibbon's account of the Council of Nice, then the investigation of such a subject requires considerable research and implies the absorption of a great deal of energy.

If a man then be led to agree with Gibbon in this quite extraneous matter, he will, while under the strain of his labours and with the condition of mind brought about by his concurrence, be inclined to accept his positions about the decline and fall of the Roman Empire.

Again, to consider another aspect of this question: If *b,c,d*, be associated with *a*, so that *d* might be consistent with the series *a,b,c*, which is well established; but also might be inconsistent without destroying the series *a,b,c*; then, if the series *a,b,c,d*, has been advanced and *d* has been disproved, the tendency is to hold that *a,b,c*, is not established.

For example, it is argued that the whole procedure with regard to the execution of Admiral Byng was correct and that he was properly executed for his timidity or want of enterprise in not engaging the French battleships near Minorca. The question of procedure is challenged. It is shown that politics affected the decision in the case, that the Cabinet desired a scape-goat. Then the tendency of the mind of the person so persuaded is to absolve Byng in the affair of allowing the French Ships to get away.

Or again, Lombroso, the celebrated criminologist, put forward

on insufficient grounds a generalisation to the effect that criminals display various physical signs, such as asymmetry; and conversely that persons displaying these signs are criminals. Already Lombroso's arguments have been demolished by various scientists, who have shown that several of the greatest men of the world and highest in that ethical scale, have displayed marked facial asymmetry; while others have again observed that asymmetry is normal and almost invariable owing to pre-natal conditions.[1] Prof. Debierre of Lille, after a minute examination anthropometrical, anatomical, and histological, of the skulls and the brains of four bandits, executed at Bethune in 1909, declared that there were no signs whatever in any of them of any physical degeneracy or anomaly. Thereupon the tendency of most who read these statements was to conclude that no connection existed between the physical conditions of brain or skull and moral development.

Again, the theory of Spontaneous Generation of life was held for centuries even by men of ability from von Helmont to Bastian. Microscopical investigation showing the great complexity of organisms formerly described as structureless and the discovery of "germs", served to discredit the theory. It seemed not only untrue but also absurd. Then Mr. Burke of Cambridge astonished the world for, in the course of experiments with radium on certain culture media, he obtained some results which he could not explain. Certain appearances were noted which seemed like living organisms, and these he found under conditions that would exclude germs.

Those who satisfied themselves as to the sterilised condition of the media jumped to the conclusion that Mr. Burke had produced life. Those of the general public who were interested reverted largely to the doctrine of Spontaneous Generation. Certain professors of special doctrines went further and declared that these results proved the whole of their teaching. A great creed of religion was declared to have been established by these experiments (*cf.* p. 339). Then came a reaction. Spontaneous Generation no longer held the field. Thinkers asserted that Mr. Burke had gained his reputation only by his ability to misinterpret phenomena, and that it was now clear that by no means could life be artifically produced. A certain reaction against the creed of religion followed.

Prof. Yves Delage announced the result of his researches in parthenogenesis. He had taken the greatest pains to exclude sources of error and he had with marvellous patience after incessant failures arranged conditions for the production of sea-urchins from

[1] Asymmetry is normal not merely with regard to the head but throughout the body. *Cf.* E. Gaupp, *Die normalen Asymetrien des menschlichen Korpers*, 1907; and Haemelinck, *l'Asymétrie du sens gustatif* (L.A.F., 1905).

eggs artificially fecundated. Moreover he had reared four of these urchins to the full-grown form.

The great reputation of Prof. Delage produced a readier acceptance for these results than could otherwise have been the case. This was such a shattering for preconceived opinions in the case of some and such a striking confirmation of the hopes of others, that once again far-off and even fantastic theories were considered as established.[1]

In the cases of this kind hitherto noticed the emotional element has considerable influence (*cf.* pp. 166; 167). Its effect is still greater to an individual when the question is one affecting either his interests or some theory deeply rooted in his prejudices or predilections. Darwin recounts that Sedgwick, one of his geological friends, tried to assure him, against all evidence, that a certain fossil was not found in a given locality, for otherwise the argument of his whole geology of the neighbourhood would be wrong. This incident made a deep impression on Darwin's mind. It showed him that the pursuit of truth was not merely a question of the ordinary exercise of mind, but that it demanded also qualities of courage and will, not less than those of intellect.

We have already noted the tenacity of impressions early created (*cf.* pp. 226; 334); and we have noted the force of constant repetition in influencing Memory and in facilitating associations (*cf.* pp. 224 *et seq.*). If then these influences be now considered in connection with the sources of erroneous reasoning we have been discussing, we shall be better able to account for the fact that two persons using the ordinary faculties of ratiocination but "brought up" under different *régimes* of religion or politics, may be unable to see any merit in each other's arguments though each be convinced of the infallibility of his own.

The counteractives which it is possible to apply are those which in most cases are inapplicable, namely, a discipline of looking upon truth as the sole great thing in these matters, a disregard of the questions of personal interest in anything but the truth, an absence of ill-founded prejudices or preconceived notions, a habit of searching to the base in the conduct of reason, a freedom of will toward new or unfamiliar ideas.

If we now revert to the consideration of Belief, we may find that our view of the matter has become clearer. The difficulty of a definition of Belief resides in this, that Belief itself, at least in its superficial aspect, is more easily understood than the terms by which we might seek to define it. We have, however, seen that

[1] Swedish girl students wrote to him in devoted admiration for having shown how future society could be constituted.

in the operation of Reason the mind proceeds along new experiences, applying the Fundamental Processes, but often erroneously, owing to the defect of any one of them. Errors are caused, for instance, by limited Discrimination, which may be due to imperfect Memory, producing faulty Symbolisation, which again brings with it a train of Associations inconsistent with the present reality. A certain experience may cause a closer attention to the sequence of experience. Discrimination may be made more precise. A more correct Symbolisation may be applied.

Errors Due to Momentum of Impulse

The study of errors having been introduced we may now formally consider what has been learned up to this point. In order to define our position by means of the Fundamental Processes, we may revert to the series *a,b,c,d,* where *a,b,* is a false link in the chain, and where the discussion has turned on *c,d,* with the result of establishing this link, and producing thereby a tendency of mind to accept the whole series. With persons little accustomed to connected series of arguments there is often a strong emotional feeling, be it respect or admiration or simple wonder, for logical processes in themselves, while the Discrimination is weak as to their correctness or falsity. In such a case the influence of mere authority may produce a Belief which after some repetition becomes strong, in regard to such a series as *a,b,c,d.*

Cases of the kind arise far more than one might suppose. A favourite author pleases our fancy, our prejudices, or he represents some special theory or emotional attraction that strongly appeals to us. We admit his whole philosophy accordingly. Thus a man may be immensely struck with Carlyle's great passages in *Sartor Resartus* on the Everlasting Nay and the Everlasting Yea, as holding up a vivid and powerfully drawn picture of states of mind within his own experience. His admiration leads him to accept Carlyle's whole philosophy in *Sartor Resartus* without having rightly comprehended it, or even without examining it. If subsequently he learns that Carlyle derived his philosophical ideas through Fichte and ultimately from Kant, he accepts their whole philosophy too, or at least so far as it coincides with what Carlyle produced.

Another aspect of this problem, though complicated with other matters, is that wherein young men of Byron's period, enthusiastic in their admiration, adopted his form of necktie. Or again we find it where the influence of Adam Smith's great work, *The Wealth of Nations,* was so marked among the more serious young students of the North that they adopted even his tricks of speech. Or again

a cultured feminine mind may resist George Bernard Shaw's Socialism until it finds in *Candida* a deep insight into the female heart shown in many subtle nuances; then under the influence of intense admiration, it accepts not only George Bernard Shaw's Socialism but possibly also his anti-vaccinationism.

With the minds more accustomed to lengthy ratiocination there becomes impressed at length a certain type, or model, of the chain of ratiocination itself. A form of the process of Reason becomes a Unit. In the process of analysis of a proposition, for example, the conditions are examined in which the solution of the problem will be accepted; any of these conditions not accepted may be examined in turn, and the conditions of its acceptance noted; and so on, until we arrive at a condition which is established.

A comparison may be made here with the methods of men of the law in dealing with an intricate case between two parties. The presentation of the case on one side is compared with that on the other; certain facts on which the parties are agreed are accepted; then those in which there is disagreement are examined. With regard to those which are essential the question resolves itself into the examination of two versions to discover which is correct or which is in accordance with the law. Thus the case is brought down to a definite issue or definite issues.

Now when such an issue is decided the mind reverts to the whole case in a mood of success or disappointment according as the issue is decided favourably or unfavourably.

Similarly, in the examination by way of analysis of a proposition, say of geometry. The proof is at length made to depend on a certain condition; and when this is established the mind reverts to the original proposition in a mood of acceptance or Belief.

Now in such a case as that of the series mentioned *a*,*b*,*c*,*d*, when the link between *a* and *b* is weak, the mind, absorbed by a long and intricate discussion which finally establishes *c*,*d*, has forgotten perhaps that the link *a*,*b*, was not properly established. We have already noted the influences on Memory of mental occupations which absorb attention and demand considerable mental energy. Hence in a failure of Memory of this kind is to be found one source of error.

But it may have been expressed originally that the link *a*, *b*, was true under certain conditions. The limitations may have escaped from Memory, either because of the subsequent absorption of energy or because in the first place their importance was not observed. This would be a failure of Discrimination.

But when after a discussion the link *c*,*d*, is established, an emotional condition arises which facilitates the acceptance of the

link *a,b*; then we have here—first a failure of Discrimination in confusing this problem with that of a known process of correct reasoning, and this failure of Discrimination is confirmed by false symbolisation; and this leads to the false association between the emotional condition, or intellectual satisfaction, and the condition of acceptance of the link *a,b*, or Belief respecting it and its consequences.[1]

We shall have no difficulty with the case when *a,b,c,d*, represents a series where *a,b*, forms a good link, but *c,d*, is determined as a bad link, and where in consequence although *c*, *d*, is not a necessary consequence of *a,b*, yet *a,c*, is rejected.

This case bears to the form of demonstration known as ***reductio ad absurdum*** a relation similar to that which the case previously considered has to the more direct analytic process. The causes of error become easily shown by this comparison.

A common form of error arises from the application of symbols of which the meaning is not fully determined, and where possibly the analysis necessary to express the meaning completely involves other symbols also not fully or clearly defined. Thus we have a symbol *A*, which covers certain properties *a,b,c,d*, etc.; where *d* is a symbol covering *z,y,x,w*; where *w* is a symbol which covers *m,n*, but which has been applied without ascertaining whether it covers *p* or *q*.

In such a case it may happen that the labour of a long analysis is itself sufficient to cause error to arise in regard to the proper application of the symbol. (*Cf.* pp. 317; 319). Moreover the analysis may lead to some difficulty which requires exceptional powers of Discrimination.

Now suppose the question to be one of practical import, such as those complex questions of changing the currents of the commerce of nations; the individual makes up his mind the more quickly in proportion as his analysis is limited and thus soon brought to an end. If a sufficient number agree with him, for similar reasons, the laws of the country are determined by what is called collective wisdom. But, as we have observed, there are possibilities of error even in the judgment of the multitude.

If a proposition be advanced that all fevers except those of the

[1] Even in the severest forms of reasoning and with those least likely to be influenced by any emotional considerations, errors of this kind may occur. For example, in treating of monodrome functions—that is to say, functions which return to the same value when the variable returns to the same value—we may prove that when the variable has left a point and subsequently returned to it after having moved along a contour enclosing no critical point, then the variable does return to its original valye.

We may afterward erroneously apply this conclusion so obtained, for we may fail to observe that the contour in this instance does enclose a critical point. The inconsistencies which result may cause us to search for the source of error, which may have been faulty Discrimination in the first instance, or simply failure of Memory.

nature of malarial fevers are due to microbes, we find a similar difficulty. In order to decide the question one way or another, we first seek to define the word, fever. But suppose we content ourselves with some general expression, involving rise of temperature and malaise. Then we have to define what is meant by malarial fever. In this case uncertainty might not arise. But suppose then that we included among fevers certain chronic diseases such as yaws, because they exhibit at one period rise of temperature and malaise; and if we discovered, in cases of yaws, a microbe of the form of spirochæte, then the problem would be resolved into determining this as the cause of the disease. In that case we might be without adequate means of determination. And so we should be unable to say whether we disagreed with the original proposition.

We take another example from subjects not at all technical. A lecturer advances the theory that high imagination is always found associated with low morality.

In this case it is necessary to define what is meant by the word imagination. Poets are said to be imaginative. Many persons might consider poetry to mean something expressible in rhythm or rhyme but without common sense. Or again, in the poetry of Moore what is called imagination would seem to be a certain constructive faculty which others would call fancy. Wordsworth seems to use the word Imagination in this sense. If Wordsworth's *Tintern Abbey* be taken as a poem of Imagination, then the symbol would indicate rather a faculty of reproduction, by the force of subtle associations, of deep-remembered feelings.

In the case of Keats the term imagination might be taken to mean a profound faculty of generalisation, as in scientific imagination, but applied to objects viewed in a poetic atmosphere and inspired by a soul exuberant in beautiful forms of association.

If finally the word were applied to the works of Burns then one aspect of the question would be the consideration also of the term morality. There are some who would condemn the poet formally and unsparingly on account of breaches of social observances. Others might endeavour to give the word morality a wider scope, embracing the whole aim, importance, and effect of a man's work; and they might point out the bold and manly character of Burns, his freedom from meanness, his ardent love of liberty, and the manner in which, giving expression in wonderful lyric form, or in racy pictures, to all that was genial in Scotland, he expressed in immortal tones the best feelings of the people and quickened the soul of his country to life.

But amid the difficulties of analysis and the uncertainty as to

the meaning of the symbols, and since there is conflict of judgment, it is evident that errors must arise.

(5) PROVISIONAL SUMMARY IN REGARD TO REASON AND BELIEF

What is the ultimate resort of operations of consecutive analysis? (*Cf.* p. 307). It is this: That we eventually reach a limit of the exercise of all our processes. We may hesitate with regard to the mind's progress in one direction or another—as depending on Discrimination, for instance. Certain Associations, however, are found to be invariable in our experience—such as, for example, simple immediate visible cases of cause and effect; the unchanged character of certain spatial relations in changed situations; the main recurrent phenomena of the order of the Universe. Now by no determination of physics, by no process of imagination, have we been able to indicate for cause and effect anything more certain than that of sequences invariable in our experiences and what we know of human experience. The constancy of geometrical figures and of certain phenomena of the Universe is a matter of experience, but only in regard to an experience limited in Discrimination and conditioned by limitations also of Time and Space. [1]

Nevertheless, certain sequences and certain constants have so prevailed in the Universe, as it has become known to human beings, that through countless generations their whole physical constitution and their mental processes have, through the action of various forces and processes, become developed so as to act in accordance with these phenomena. The first necessity for the individual has been the preservation of life; then there have been found various stimuli to exertion due to living in communities, and there have been corresponding adaptions; [2] then with increasing precision of powers and vast storehouses of Memory, and with a developed symbolisation, and all that it implies, there has been highly evolved that representation in idea which corresponds to experienced realities, and which enables the mind to form ideas, also, or notions, of its states and movements; then in referring these to the present realities, we form the operations which we include under the name Reason.

We have seen with regard to Memory (*cf.* p. 275) and Association, and it may be noted with regard to all the Fundamental Processes, that once certain conditions are posed the mind moves necessarily in a certain way.[3] This corresponds with all that we have hitherto discussed. But the mind has consciousness of certain of its processes and, since the processes extend throughout the scope of its consciousness, it has consciousness of some processes as being the removal, or the Negation of others. This also applies to those processes which, as fundamental as we know, arise in the necessary manner of activity of the mind. When the mind then, in experiences as fundamental as it knows, moves in accordance with the natural exercise of its Fundamental Processes, its condition at each step, corresponding to a certain Hedonic principle involved in the natural exercise of these Processes, is that of Belief. The condition involved in the conscious act of Negation in the natural exercise of these Processes is that of disbelief.

[1] At the Philosophical Congress at Bologna, 1911, Henri Poincaré discussed a question raised by Boutroux as to whether the laws of Nature were immutable. Poincaré in accord with Prof. Langevin, celebrated for his work in radio-activity, concluded that what were called laws might change, except possibly certain fundamental conditions of the molecules. All that is here written is in accord with these speculations, if it be well noted that they are philosophic and of extraordinarily remote application.

[2] Certain terms such as the "herd instinct" have become popular amongst men of science, who accept them as the full explanation of Psychology. I have not employed such terms, as I might do here, for they are composites, and they serve less to explain than to give cover for lack of explanation.

[3] When the conditions of their appearance are provided the Fundamental Processes may be called automatic or intuitive.

Belief is not a passive state. The mind is in continual progress. Consequently Belief has reference to the activity of the mind in meeting new experiences in accordance with the conditions of the mind already acquired.

If now we consider that graphic representation which is afforded by the physical substratum itself, the nervous system, we may rightly contemplate it as a complicated dynamic machine, developed in accordance with our world throughout countless ages; responding to various forces within a certain scope of the world; and becoming itself modified by that activity of response. And when we now consider that various of its activities, within certain scope, have correspondences in consciousness, and that its natural activities are accompanied by certain Hedonic conditions, often made more evident by the converse state; then we see that Belief corresponds to what we might in analogy call the resultant of all the previous forces acting on this dynamic machine; and we note that this resultant implies a certain phase of the dynamic machine in regard to new forces which make themselves manifest.

Now we have seen that the speed with which the mind may entertain ideas may be greatly increased by exercise; but in our progress through the world we meet with a multitude of impressions of all sorts in rapid succession, and even to those who have formed an analytic habit of mind, time is lacking to reduce our observations to any great degree of certitude. We apply symbols with little Discrimination; for this mode, though a source of error, is yet the only means by which we can continue our mental activities in the face of realities. Hence arise errors even in cases where the natural exercise of faculties has not been warped by prejudice or early false training.

But when the mind has been early trained wrongly, errors become inevitable which may produce errors on errors in their consequences. But error may be quite consistent with Belief; for example, a symbol may be applied to a certain individual or to a favourite theory, or to any nation, having associations not justified. This symbolisation may be applied in natural exercise, and Belief in its correctness may even cause a refusal to acknowledge the errors which become manifest as a consequence of its application (*cf.* pp. 326; 327; 340).

Belief in errors through a wrongly imposed Belief are in a high degree less likely to influence the mind in regard to those sequences of nature which are accepted as inevitable: But the whole course of reasoning from such a basis is so long that many causes of errors are by that circumstance introduced. This is shown in the consideration of mathematics which is yet the best example of such a course of reasoning.

That Belief does not necessarily correspond to the realities of the external world, that it may, in short, depend in great measure on factors peculiar to the individual, is shown by the cases of hesitancy or doubt in regard to propositions.

This is not necessarily produced by the failure of any of the Processes on which our Reason depends, for such conditions are quite consistent with Belief, even though erroneous Belief. But there are instances where our faculties of Discrimination are near their limit, and we cannot determine definitely whether we observe or do not observe either some condition of an external object or some state of relation between objects.

In the dusk all cats are grey, a French proverb says. And to the dusk they also apply the term "between dog and wolf". Here we have a condition of doubt about material things.

Failure of Memory may produce a hesitancy regarding Belief if something be in part remembered. The exact words of a conversation may be a matter of doubt. The guilt of a person accused has been known to turn on the question as to whether he mentioned a crime on a certain day when it had not been made public, or on the following day when it was possible for him to have read it in the newspapers.

The most perplexing form of doubt is that which follows after reading a long argument couched in highly general terms, or after working at a demonstration on which

certain assumptions that may be true are made. The mind is unable to decide whether the demonstration is true or false, and this condition bears with it, to some persons, a sense of intellectual humiliation.

Referring to the physical condition corresponding to these mental states, we find that Belief is represented by the manner in which a resultant is formed from our previous physical constitution, particularly of the brain, and the nerve stimulus produced by the presentation of new objects and their associations.

This Belief has a tendency to be expressed in some form of action or disposition toward action. A certain Hedonic condition corresponds to it. The condition of hesitancy implies a holding back, or inhibition, of the progress of the nerve stimuli. But this condition is not inactive either. Various tentatives are made which correspond to the flow of the nerve-impulses along certain strands. When a correspondence with desired conditions is not found this flow is checked and in succeeding tentatives it is directed along other strands or, in hesitancy, again inhibited. With impulsive or impatient people this physical condition is unpleasant. Hence the desire to arrive at a solution in general accordance with the temperament of the individual may override the desire to make certain that that solution is correct.

The individuals most impatient of deliberation are those who would not usually examine sufficiently the terms of a difficult classification (*cf.* pp. 338 *et seq.*), and those again who, on a stimulus from the external world, form associations beyond such as may be warranted by the conditions of the external world which supplied the stimulus (*cf.* pp. 343 *et seq.*).

In such a case decisions are not made according to a principle of true ratiocination but mainly on the basis of the individual character, or some dominant feature of an argument which has made an impression, or of direct personal interest, or by influences of environment, or by virtue of the last speech heard, or by following a rule, or by some multiplex prejudice or predilection, or by mere hazard. But in all this there is not the slightest guarantee that the result desired by the person so deciding will ever be realised.

The Processes of Reason are therefore usually hazardous; the sanctions of Belief are never complete certitudes.

CHAPTER II

(1) QUESTIONS OF RESEARCH

NOTE:—In this chapter certain broad general lines of guidance in study and research will be discussed by way of illustration of the principles already determined.[1]

Fields of Study

Hitherto we have dealt with cases of reasoning where a certain problem has been posed for solution; and we have considered the progress of the mind in tracing out solutions already obtained, and we have indicated the nature of tentative efforts tending to obtain such solutions.

We may now proceed to study processes of Reason where the mind sets out to discover new facts or new relations, as for instance, in a field of science.

A complete and satisfactory exposition of such processes would constitute a logic of Research; but it seems at present very difficult to formulate such a scheme. Nevertheless, in examining the matter in various fields, we find that in certain of these we may say beforehand that useful results may be obtained by proceeding according to ascertainable principles.

Thus if a naturalist set himself the task of observing the habits of the ornithorhynchus, which is an amphibian animal, he might be assured that for that reason alone he would make interesting discoveries. He might have been led to settle the question as to whether the ornithorhyncus is oviparous or viviparous; and that question was at a certain epoch of special moment to biological science.

The old anatomists who had begun to distrust Galen were sure of making discoveries of importance in their dissection of the human body, for the field itself was of great interest and the observations had hitherto been often erroneous or not sufficiently precise. Thus it happened that when van Graaf discovered the thoracic duct he paused long before he opened it with his knife, feeling that he was on the point of a discovery of prime importance.

[1] Many of the subjects here discussed could be expanded into volumes, but I have given nothing but brief indications. I have also sacrificed two elaborate studies of scientific methods which appeared in *Psychology: A New System*. The reason is mainly that interesting as these applications may be, I desire to avoid losing sight of the framework—the exposition of the Fundamental Processes—in the mass of subsidiary discussions.

Darwin in setting out on his Voyage of the *Beagle* had not formulated his problems but he knew that discoveries lay in his path. Even in meeting with coral reefs it was not till after some preliminary considerations and tentative researches that the problems presented themselves in more or less clear form.

The medical expert whose duty it is to examine a body in a case of suspected murder makes observations of many kinds before the crucial questions arise, as for example, at what distance a gun was fired that caused a wound in the head.

Or we may propose to examine Kant's *Kritik de Reinen Vernunft* to test the rigour of the reasoning; or the problem may be presented as to the genesis of the characteristic notions.

If one set forth on a study of the Scottish Philosophers of the eighteenth century one may be sure at least of finding material for a weighty book; but if a man throw off authority in the style of Descartes he may not know at first even where his thoughts will be led. Or if he search in the nature of things for the data of Ethics he may experience much uncertainty how to pose his problems. He may be shaken by still greater incertitudes if, regarding the scope of the sciences, he does not select any thesis for examination, but asks the questions from his place in the Universe: What is the problem that I shall attempt? How shall I know the source of my own impulses, my thoughts, my joys, my sorrows?

* * * *

The Personal Limitation

When we consider the manner of gaining experience of the external world by Immediate Presentation we recognise not only the limitations of our species, as being restricted to a scope of sensation, for example, which corresponds but to a small part of the scale of the external forces; but we also recognise in each individual that the scope of his possibilities is marked by his personal characteristics. But the sum total of knowledge and the operations of Reason are influenced likewise by the factors of heredity of the individual and the whole of his previous stored-up knowledge.

We have seen also the part played even in what are considered purely intellectual matters by the emotional factors. We have seen that not only in striking out a course of life but also in dealing with arguments that come to us in a purely intellectual appearance, moral qualities are of great importance, qualities not only of sincerity but also of courage.

* * * *

The Leap of Genius

Among those matters to be taken into account in regard to the question of Research are those also of the degree of complexity of the subject under consideration and the grade of development which the investigating mind has attained.

Even a theme of social science like that of Adam Smith's great work, *The Wealth of Nations*, though requiring fine faculties of analysis, is yet more circumscribed and determined than such studies as those relating to the influence on development of races of those geological upheavals and divisions of the earth's surface which Elie de Beaumont indicated in magnificent generalisations.

The studies of Robert Brown in Botany, collating a vast number of facts as a result of careful observation, are yet on a plane of more obvious tasks than those of Darwin on the Origin of Species.

The observations of diseases by the old physicians, though often acute, were certainly more superficial than the researches of Pasteur into the cause of anthrax, which he finally traced to a bacillus. The researches of Manson in diseases produced by filaria, Ross into the mode of transmission of malaria, and Laveran into yellow fever, dealt with problems having aspects still more elusive to direct observation, for they had to trace out the hidden mode of conveyance of the infection by mosquitoes.

The observations of the geology of a neighbourhood present problems more direct and tangible than that which Copernicus encountered when, after tracing and attempting to apply the complicated system of Ptolemy, his mind rose to the great conception of the solar system with the earth and other planets revolving round the sun.[1] The task which Kepler set himself, namely, that of ascertaining the orbits of the planets, though extraordinarily intricate and demanding a wonderful combination of the genius of insight and of patience for its solution, is yet, after all, more definite at least in its enunciation.

The problem of Universal Gravitation, again, was posed already before Newton, but it is the glory of the great author of the *Principia* to have divined the law and with masterly skill to have exhibited the demonstration. The nebular hypothesis of Kant, afterwards elaborated by Laplace, forms, even though the theory be not established, a rare leap of intellect beyond the realm of mere observation.

[1] This conception was familiar with Pythagoras, but the science of the Greeks had been lost amid the welter of the Middle Ages.

Importance of Classification, involving Generalisation

When the number of facts accumulated is large, it will be necessary to adopt some system of classification. We have already seen in what way this becomes necessary with regard to Memory.

A principle of classification must be sought that corresponds as far as possible to active, constant operations in the world, as for instance, those that we call cause and effect. This implies the greatest security of sequence, and hence the greatest faculty of associations. De Candolle's classification in Botany improved on that of Linnaeus by its corresponding more nearly to the vital functions of the organisms.

Darwin's theories of evolution, to which Herbert Spencer gave more precise expression, have throughout the vegetable world given new standards, because they express a profound conformity to the processes of nature. Herbert Spencer's theory of evolution may indeed be looked on simply as a formula of classification, though involving principles of division which are not always consistent nor clearly determined.

Generalisation is necessarily implied in classification. A generalisation may be a record of common features of a class exhibiting diversities in the individuals. Thus, for example, there are certain general features common to all fevers; such as malaise, high temperature, increased pulse rate. Each kind of fever has its special features.

Practically generalisation involves more than the recognition of community in a group that also shows divergencies. It involves assumptions and this, though a source of error, has also been a fruitful source of progress of human knowledge. Thus Newton, finding that a certain law of attraction explains the movements of the moon about the earth, propounds a law of Universal Gravitation. Now we have not tested this law in relatively many instances. The generalisation contains therefore an assumption which is possibly not universally true.

Again, the young German student, Schwann, in a few simple experiments laid the foundation for the development of the science of bacteriology and therefore the basis of the progress of modern medicine. Roughly stated his experiments were these: He placed some pieces of meat in jars which he sealed hermetically. No putrefaction took place. He then admitted the air. Putrefaction began.

Now the notion of microscopical organisms being found in the air was familiar to Schwann, as it was to other scientists before him.[1] Excluding other causes he therefore propounded the

[1] It had been formulated by the Italian scientist, Spallanzani.

theory that the putrefaction was due to access to the meat having been permitted to these microscopical organisms or "germs". But as putrefaction is only an example of organic change, and as Schwann's mind had been occupied with other instances of organic changes, especially fermentation, he gave to his theory the extraordinarily bold but happy generalisation that in all organic change the agency of such germs is necessary. Here then was a theory having a direct practical bearing on a thousand processes in the external world, physiological and pathological, in our bodies.

Pasteur's researches gave such an impulse to the study of fermentations and of diseases in the light of this theory that they overshadowed the original work of Schwann. The actual discovery of microbes in tissues affected by disease, such as cholera, gave strong support to Schwann's theory. The criteria which Koch laid down for ascertaining whether a disease was due to a certain microbe, and which in some cases were shown to be fulfilled, might be thought to have confirmed Schwann's theory.

Yet that theory involved great assumptions, and it seems in part unwarranted assumptions. For there was nothing in his experiments to exclude this possibility, that the organic changes did not absolutely require the presence of the germs, but of certain substances and ferments, for example, produced by the germs.

Meanwhile the assumption had been of incalculable value in developing a great field of science. It will be seen, moreover, that all operations of Reasoning have a character analogous to this mode of progress.

Again, if we generalise by saying that the atomic weights of elements are formed in regular gradations according to Mendelejeff's scale, it cannot be said for certain that we are right or wrong. Even wrong generalisations have been helpful in the cause of science. The discovery of their error renders our Discrimination more precise. Bold assumptions put forward as generalisations which have afterwards been found to be correct have been at times a stimulus and a guide to discovery. Mendelejeff's scale, for example, by pointing out that elements were still required to complete the gradations suggested experiments which led to Ramsay's discovery of argon and other elements.

It must always be borne in mind that Reason proceeds by means of a continual interaction between the new experiences coming to us from the external world and the whole of our previous knowledge stored up, marked by symbols, characterised by all manner of Association and Impulses.

Classification will be found accordingly to have influence on the order of our thought in more than one way. Classification is

necessary to avoid confusion in making reference, and also for the purpose of Memory. But classification will be found also to have a deep correspondence with external reality. The whole course of scientific discovery has been in a direction toward simplification by classification. We note a number of separate phenomena; subsequently we discover a certain "law" of Nature which explains the phenomena in such a way that we can predict their occurrence. Some of the most highly general laws, such as that of Universal Gravitation, have been found expressible in simple formulæ, but it must be understood that these formulæ refer to the sequences of phenomena, not to the essential character of the forces influencing them. We say that each particle in the universe attracts each other particle with a force which is great in proportion to its mass, but in proportion inversely to the square of its distance. But the "law" tells us nothing of the cause of the attraction. Indeed, many physicists find that herein we are presented with a notion inconceivable on the basis of our previous experience—the notion of force acting at a distance. Clearer views may be obtained by more closely investigating the meaning to be attached to the word "force" but this discussion lies beyond our present objects.[1]

It must be understood also that the term "law" means nothing further than a compact expression regarding phenomena as manifested to our experience. Consequently as our own minds, on account of their limitations in the immediate comprehension of ideas, move by classifications indicated by symbolisation, certain of these forms of groupings that we ascribe to the external world have meaning only with reference to the mode of our intelligence.

Nevertheless there appear to be in nature innumerable categories of things of which the developments are formed by variations on general plans. Thus in animal life we find, since there must be waste and consequent repair of material, some apparatus for conveying such material to and from the parts. This involves some sort of circulatory apparatus. The material thus conveyed not being found ready to hand must be prepared from various substances containing unnecessary adjuncts. Hence some kind of digestive apparatus is involved. The movements implied by these operations and other movements without which life seems incompatible, must be determined by stimuli from the outside world and within the organism itself. Hence we have the analogues of what in a more developed form we recognise as the muscular and nervous systems. But on this plan, which in the manner in which

[1] See the conclusion of Prof. Tait's article on Mechanics in the *Encyclopaedia Britannica*, IXth Ed.; also the writings of Clerk Maxwell, Poincaré, and Lord Kelvin and *cf.* Note p. 149.

we express it is relatively simple, endless varieties have been wrought.

And this example is offered only as an illustration of the fact that things are so ordered in the external world that generalisations and classifications, if well chosen, not only represent known correspondencies with the external world, but lead to discoveries of new correspondencies consistent with them. Consequently a habit of persistent generalisation and classification is of the utmost importance in Reason.

The whole course of science and the whole scope of Reason may be set forth as that of finding ultimately relations of Agreement. The final expressions of laws are not negative but positive statements. Even when Discrimination acts as in the way of Dissociation, we find at length a definite positive expression to mark the case so obtained. Thus in our course of life we meet with beans as edible food. Then subsequently we become acquainted with the Calabar bean. Experience proves that this contains poison. Closer attention is paid to its characteristics and from a study of these the Calabar bean is known and classified. At length we discover its advantages in medicine, and this association overrides that of its negative characteristics.

Certainly owing to our incomplete means of knowledge we must in our progress in the world of thought continually make use of safeguards, limitations, restrictions, but the tendency of these eventually is to guide us to positive associations having an outcome in exercise and use. But the bases of Agreement are often hidden beneath diverse appearances that in Immediate Presentation seem at first dominant.

To examine directly point by point all the discrepancies in the multitude of objects in the external world would be impossible even if only on account of the time involved; but following the scheme of a classification we find that if under each heading we have only two objects separated by some good principle of division, and under each of these two others, and so on, then with relatively very few applications of this principle of division we classify an immense number of objects.[1]

Thus the value of classification in our modes of Reason is of importance in the first place because diversities based on agreements prevail so largely in the world of our experience, and secondly because in this way of classification it is possible for us to define, and thence to form associations with, a vast number

[1] This is seen in the familiar game where anything in the world may be selected for guessing. Those who guess are allowed to ask—Is it in the organic world or inorganic? Is it an animal or a vegetable? Is it a mammal, and so on. After a few questions posed and answered any object selected from the world may be determined.

of objects of which otherwise the discrepancies would bewilder our minds.

What we call laws of nature are the expressions of certain of these most profound agreements.

In the consideration of errors of reasoning we were led especially to notice Discrimination, Generalisation, classification, symbolisation, Memory, Association, though remarking that these processes must imply the others.

Hence for the purpose of exposition we may lay special stress on these processes in regard to building up a scientific habit of thought.

Darwin, for example, took the faculty of generalisation to be the mark of a scientific mind. By generalisation we mean the discovery of resemblances amid accidental differences; and in dealing with complex matters these resemblances must be sought not on superficial grounds but on some vital principle, something such as the relation of cause and effect, that strikes deep into the constant order of the Universe. Classification in accordance with this form of generalisation enables a great complex to be held together, and progress is made as already noted, even in cases of wrong generalisation and classification.

Even such a quality as persistence, which might be thought to appertain rather to the emotional and moral side of men's nature, has become necessarily implied in the habit of deep generalisations, for the mind forms its associations around the things shown to have a common factor through our generalisation, and with persistent observation new problems, new discoveries of resemblance, of cause and effect, new generalisations are evolved. And since, as we have observed, Nature builds continually on similar plans, it is impossible to formulate any deep truth without afterward discovering applications which may not have appeared at first within the range of that speculation.

Great Thinkers Repeat their Methods in New Applications

Another reason why the habit of generalisation and classification is so important is that by virtue of the discipline of the mind involved it enables the thinker to wield his apparatus of analysis with mastery. Consequently if he makes one discovery he is almost certain to make other discoveries on account of what we have seen of the great diversities of things of Nature formed on similar plans.

Thus Newton had formed the basis of his life's work in reducing to simple expressions the principles of Mechanics. In the subsequent application of his system his mind was long occupied by

questions of attractions and, in accordance with his own third law of motion, of repulsions.

His discovery of the law of Gravitation had not only immersed his mind in a multitude of associations round the thesis of attraction, but had given him great skill in the handling of the mathematical apparatus involved in the calculations. When therefore Newton turned his attention to the study of light he sought to explain all the phenomena by means of attraction and repulsion of material corpuscles. And when Newton examined the problem of the tides it was always with the same notion of attraction and repulsion and with applications of the same mathematical apparatus.

It is interesting to compare with Newton's system that of Thomas Young. For Young, having adopted and expounded the undulatory theory of light, not only discovered some highly interesting facts regarding light itself, but also acquired a great facility in the use of the calculus corresponding. Thus also he formed associations with this theory in fields in which it would seem to others hardly applicable. Turning his attention to the theory of tides, he showed the influence not merely of the attractions actually subsisting at the moment of observation, but of the previous effect of attractions persisting in their influence on marine currents and undulations.

When electricity began to reveal its marvels Schelling cried out: "All is electricity! All is electricity!" At a still earlier epoch mathematics were thought to be the gate to all knowledge.[1] And at a later period when investigations of biology in the hands of great workers began to throw light on problems not immediately connected with that branch of study, it became assumed that biology held the key of all teaching.[2] So in this way we have seen in history a succession of veritable fashions in science.

The underlying reason has been that the objective realities corresponding cover wider spaces than those into which we have dipped the plummet of our research.

* * * *

Imagination in Science

The discussion of the scientific use of the imagination, on which Tyndall wrote a luminous chapter, has to do with a similar order of ideas. Imagination used in this sense is the discovery, through

[1] Mathematics, now regarded as a chilly and repellent subject, once formed the delight even of ladies of the Court. Sophie Charlotte in one of her letters speaks of Leibnitz talking to her of the Calculus, "as if any one could teach *me*, the wife of the Elector, anything about the infinitely Little!"

[2] Binet (*l'Année Psych.*, 1906) derided Haeckel for his display of biological learning in dealing with subjects only remotely connected with these specialised studies.

a mass of accidentals, of fundamental resemblances either of experience or construction.[1]

The work of Copernicus in dissociating from his mind all appearances due to the most striking Immediate Presentations, namely, those of the rising of the sun, in order to conceive the true solar system, is a triumph of imagination. In this case it is an imagination of construction, for erecting in his mind a symbolic representation of the solar system is as far as he knew it, it became easier to imagine the manner of movement.

The brilliant theory worked out by van t'Hoff and Sir J. J. Thomson and Sir E. Rutherford, of the atom being a minuscule system, comparable to the solar system, is another triumph of constructive imagination.

The notion of heat being a form of energy, although it is now familiar to us, has always seemed to me to be due to a subtle feat of imagination. Certainly we have, as in the case of boring of a cannon which drew the attention of Rumford to it, the association of mechanical work with heat; but it required preliminary deep thought on the essential nature of things to see the community in diverse manifestations of energy.

Yet men's minds had evidently been prepared, by a course of science fraught with propitious associations, for the acceptance of this doctrine, for there seems to be a doubt as to whom to ascribe its clear enunciation. Rumford seems to have been struck with it on the basis of independent thought and observation. Gay-Lussac had the same notion. So had Sir Humphry Davy.[2]

The theory of heat as a mode of energy was afterwards extended in scope until finally in the hands of investigators of wide knowledge, of whom Helmholtz was a type, we find the essentials of the theory included in that of the doctrine of the conservation of energy. In this case, however, although the knowledge, the skill of exposition, the command of scientific instruments of research, implied vast scientific capacity, yet the actual theme itself had become less recondite than before that leap of genius which enabled us to regard heat as a mode of energy. For having in this instance demonstrated that energy is not lost but only transformed, it

[1] L. Arréat has written on *Mémoire et Imagination*, 1895; and Pierre Boutroux on *l'Imagination et les Mathematiques*; and Adam (*cf. Rev. Phil.*, 1890) on *l'Imagination dans les Découvertes scientifiques*. George Gore's *History of Discovery* is full of interest from this standpoint, and G. Wilson's *Religio Chemici* fascinating.

[2] The theory seems to have been first set forth by Huyghens, but the works of Lavoisier, Rumford, Laplace, Carnot, Gay-Lussac, and Davy especially gave the modern impulse to the study, afterwards continued brilliantly by Mayer, Helmholtz, Clausius, Clerk Maxwell, Raoult and others.

Sir Humphry Davy represents indeed the faculty of scientific imagination in quite a special way, for in order to excite his imaginative faculties he used to attend Coleridge's lectures on poetry. We shall see a little later an explanation of such an aid to discovery.

becomes suggested without great difficulty to pursue that inquiry into every instance of apparent loss of energy.

The case is similar to that which arises when we have a certain number of objects included in one classification. We form some associations with one, such as the discovery of an unexpected peculiarity, then the natural Impulse connected with this association leads by further easy associations to examine each of the others in turn.

The exclamation of Buffon: "There is only one animal!" revealed a feat of imagination. For this in the epigrammatic French style expressed the theory that all animals have been developed by evolution from the primordial type. The whole work of Geoffrey St. Hilaire was conducted on this suggestion, and the theories he put forth in consequence, and the conflicts that ensued with the more orthodox Cuvier, fired in turn the imagination of the poet Goethe to new conceptions of the Universe.

Darwin's progress from the arguments of Malthus to the notion of the struggle for existence throughout all the organic world and thence to the doctrine of Natural Selection, was marked by efforts of imagination, beholding similarity in conditions having ostensibly great dissimilarity.

* * * *

Tentative Efforts

The mind in an operation of reasoning not being able inevitably to choose correct sequences, proceeds on a series of Impulses, forming associations, the relations of which to our previous knowledge are determined according to those principles of Reason which we have already discussed, those which are unsuitable being rejected.

When the chain of reasoning is already formed, and simply submitted to examination, then the advantage is that the Impulses are directed at once along the right path. In problems of a high degree of complexity we may, on the other hand, be carried by our Impulses producing associations along remote paths. We set forth propositions tentatively and in the course of their solution we ascertain whether they represent validly conditions that prevail in the external world. In certain cases these propositions may be capable of graphic representations. For example, Kepler devised a great number of models, destined to exhibit the course of the heavenly bodies, before he arrived at the correct representation.[1]

[1] Edison, before deciding in favour of the carbon filament for his electric light, had every corner of the globe "ransacked" by his agents. See an article by G. A. Lathrop, in *Harpers Magazine*.

Thus both in the successive steps of a chain of reasoning and in the larger processes of posing problems and searching tentatively for new paths leading to the discovery of new concordances in external nature, our progress is marked by a series of efforts of which the false or unsuitable results are rejected. Using a physical illustration, we might say we reason as we walk, by a series of fallings corrected.

The story of Sir Humphry Davy attending Coleridge's lectures on poetry now finds a clearer explanation. These lectures stimulated the general activity of his brain, causing associations to be formed between tracts not likely to be interlinked by severe studies in one domain. Among the multitude of suggestions passing through his mind, which he dealt with on the general principles of his science, Davy found some that called for investigation. His method of investigation was that of a series of bold guesses which he afterwards tried to verify. But this method is, as we have already gathered, not different, except in regard to the manner and temperament of the particular investigator, to that of all scientific research.

Faraday said that Davy by his example illustrated to him day by day what he should avoid in scientific research. But this criticism involved rather questions of the technique of the laboratory, cleanliness, neatness, accuracy, method, and system. Davy's mind was far more orderly than the arrangement of his crucibles, and his guesses were the result of profound thinking on the basis of generalisation and classification. Certainly his lack of care in regard to mechanical details was likely to produce errors and to cause him to overlook facts of importance. But the difference of his method from that of Faraday might be compared to that of a brilliant and dashing military leader who takes chances, from that of a slower, methodical and careful general who completes a carefully laid strategic plan point by point.

* * * *

"*Accidental*" *Discovery*

In a great many instances in the history of science, some accident has afforded the stimulus to the necessary flash of imagination. Thus Forbes, while pondering on the problem of the movement of glaciers, was nearly tripped up by an old woman pouring out beans from a bag in front of him. The forcible associations thus produced between the observance of the manner of sliding of the beans and the object of his study, gave him the necessary clue to movements of which he studied the applicability to the glaciers.

Again, d'Orbigny, while studying volcanoes and being puzzled by certain problems as to the vaporisation of the water in caverns at a high temperature, stepped into a cottage where an old woman was ironing linen. Being fatigued, he sat down near the fireside. Presently he observed that to test the heat of the iron the old woman spat on it, and that the saliva slid off before it had time to evaporate. This suggested to his mind an experiment by which it may be shown that with water in a crucible a sort of film of water nearest to the hot surface becomes evaporated, and the steam being a bad conductor of heat, retards the evaporation of the rest of the water.[1]

This explanation made clear various matters in connection with the character of the violence of eruptions in volcanoes.

In these accidental discoveries the mind has already been prepared, as Pasteur pointed out, to grasp the full significance of the phenomena presented by accident. For indeed we are constantly meeting with accidental phenomena which might reveal to us important secrets if we but knew how to interpret them. On the other hand, a mind completely engrossed in a subject and intent on examining it from every point of view, and at the same time observing the multitude of phenomena that comes within experience, is not unlikely to meet with such as throw illumination on the subject of study.

* * * *

Guiding Principles: Avoidance of Error

For guidance we depend to a great extent on acquired results. The time is past when a man, in the range of the physical sciences, at least, can reasonably proceed, like Descartes, taking nothing for granted. To persist in that course would mean that science could never progress. On the other hand, we know that a vast amount of work has been done by great and careful observers, and that their results may have been checked by the criticisms of hosts of minds, and that consequently we may accept the data so offered as the starting-points of new developments.

It is especially valuable to grasp thoroughly the leading principles of the sciences. For example, if a man propose to demonstrate a machine for perpetual motion, we may save the time necessary for examining this in detail. We know that this proposal runs counter to the basic principle of the conservation of energy. In

[1] This experiment is familiar at universities in the physical laboratories. I saw t performed in Berlin by Professor Kundt, the successor of Helmholtz, with a skill that bordered on prestidigitation. The room was darkened and he had projected on a screen the shadow of the drop of water floating above the surface of the heated iron. The iron was allowed to cool, the drop came in contact with the surface. This completed the circuit for an electric bell, which rang.

mathematics we may accept as insoluble such problems as squaring the circle.[1]

In each science the established principles become familiar and the mind, moving in the field of experience, or theory, or experiment, becomes so continually guided by these principles that to another not versed in the science it seems to act by a kind of instinct. Thus it has been said that even the guesses of Newton are profound.

A mathematician may cast his eye over a complicated calculation and declare at once that a mistake has been made. This may be because he observes on one side of an equation a certain symmetrical relation of symbols, and on the other side a corresponding symmetry preserved in every term but one. It must be noted, however, that in certain sciences, as in psychology, it is in the most fundamental phenomena that we must seek for guidance.

Particularly every case is deserving of the closest attention where in a law, ostensibly general, there is met with some instances of exception. Le Verrier and Adams independently established calculations destined to discover a planet whose influence would explain discrepancies between observed perturbations and the generality of the law of gravitation.[2]

The researches and careful estimations of Cavendish in regard to the constitution of the atmosphere were not satisfied by the assumption that oxygen, nitrogen, and carbonic acid gas were the sole constituents. The discrepancies were subsequently neglected, being ascribed to errors of estimation, or simply forgotten. It was not until after the researches of Ramsay that attention was again called to results set forth by Cavendish with entire correctness, though not with all the precision which improved methods rendered possible. Examples could be multiplied of valuable results obtained by the examination of the obscure corners left in previous explorations.

* * * *

Accuracy of Observation

In examining the origin of great discoveries and the course of the development of all that prepared the way and followed from them, we may be led to find some general principles of guidance. One of the chief requirements is that of accuracy in observations

[1] In fact Lindemann has shown that the problem is veritably insoluble by the use of finite series. (*Cf.* p. 137 (note)).

[2] The planet discovered by means of Le Verrier's indications is that which bears the name Neptune.

and the record of observations. We may profitably read at this day an account of an epidemic of mumps recorded by Hippocrates as raging amongst Greek soldiers of an expeditionary force, for Hippocrates was a very careful observer. The writings of the physicians of Louis XIV., on the other hand, are absurd, for they substitute theories for observation and their theories had no valid foundation.[1] Correct observation does not imply a mere colourless record, for the number of things capable of being recorded is infinite. Observations should be distinguished by discrimination, by the endeavour to seize the essential and to neglect the accidental.

Accuracy of observation must be insisted upon. This is not quite the same thing as the desire of truth, for inaccuracy may arise from lack of Discrimination, and this may occasionally be due to a want of training in systematic habits, particularly in the habit of classification.

Routh, the mathematician, having been asked for a characteristic phrase which would correspond to the important observations formed during his life replied, "Verify your references." This points to accuracy not only in observation but also in the application of symbols and in the acceptance of the associations implied.

Accuracy of observation and completeness of record have often been found of great importance in scientific discovery. They are in fact the guiding principle in the compilation of statistics and the study of statistics itself has grown into a useful and fascinating science.[2]

Again, an eminent neurologist, Sir William Gowers, described part of his work as scientific book-keeping. For example, he was accustomed to examine every epileptic patient with particular care in regard to all questions of heredity, accompanying diseases, such as those of gout, and heart disease, conditions of mental or physical stress, age, sex. After a great number of such cases had been examined the review of the records brought many curious points to light. This connection between epilepsy and gout conditions occurred too frequently to be due to accidental coincidences.

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[1] Recently in reading a report of Byron's last illness, with reference to the medical side, I discovered that "the inflammatory diathesis changed to the lymphatic." Thereupon the poet was bled again and again. Such a hotch-potch of nonsense as contented generations of scientific men would be laughable, except that it generally, as in Byron's case, conduced to the death of the patient.

[2] We learn, for instance, that in a great city the number of suicides is fairly constant year by year and that the maximum coincides rather with periods of depression in business than with climatic discomforts. Such an observation helps our understanding of curious problems of sociology. Remarkable also is the theory put forth, capable of being tested only by statistics, that crimes of impulse are more common in months of high temperature.

Concentrate, but Beware of Grooves

We have already seen how it is explained that constant exercise in a field of study helps to make clear problems that have seemed at first complicated and obscure. Certain aspects of the problem become familiar, the energy required in reviewing them is less, the mind is left freer for observations, and Memory is rendered more facile in operation.

By the fact of continual exercise along certain lines of thought the associations may lead to the mind being held within a certain field from which the suggestions of the solution cannot arise. Hence it has often happened in the history of research that the ideas leading to the solution have arisen at a time when the mind had ceased to be actively occupied with the problem. We have formed similar conditions with regard to Memory which represents, however, only one feature of the present study.

Poincaré, the brilliant author of *Nouvelles Méthodes de la Mécanique Céleste*, relates in an essay on *Invention in Mathematics*, that, having been greatly occupied with a problem that embodied new discoveries in the Higher Mathematics, he was in despair at a certain moment of ever finding a solution. Subsequently other duties called his attention away from the subject. One day when in the act of stepping into a coach he suddenly had a clear vision of the whole problem.

Here it was necessary for the mind to regain a condition of complete freedom before suggestions arose from other sources than those which had been persistently explored.

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Specialisation should not exclude Essential Parts

Thus the *dictum* of Newton that he had arrived at his discoveries by " always intending his mind ", must be read also in the light of such experiences. The aphorismal expression of Graham, " Follow game ", has a somewhat similar meaning. The question of " specialism " in science must also be examined in regard to this view of mental operations. The field of science still unexplored is so vast, and Nature is so diverse even in matters resting on similar principles, that persistent studies in limited fields of observation will almost invariably bring some new curious fact to light. But with the advance of knowledge we find that every science makes demands on every other science, and that the best type of specialist is one whose general knowledge is so vast and so precise that he can interpret the phenomena observed in his special province.

The studies of Dubois-Reymond, for example, on Animal Electricity involved a knowledge of physiology and of chemistry as well as that of the phenomena more exclusively related to electricity. The study of physiology involves that of chemistry, especially in regard to its later brilliant developments, and implies a clear comprehension of the principles of physics. The researches of Helmholtz with regard to theories of vision, which gave a concrete result in the invention of the opthalmoscope, involved a knowledge of physiology and mechanics. His researches on musical notes involved the application of a considerable range of the Higher Mathematics. The whole scope of his observations led him to throw new light on the field of psychology.[1]

Graham's researches in osmosis derived their importance mainly through their applications to various physiological processes of respiration, of digestion, and excretion. Graham studied the laws of the interchange between bodies, in a gaseous condition, for example, contained in chambers separated by a porous diaphragm.

[1] *BROCA'S CONVOLUTION*: Space prevents me giving more than an indication of the manner in which the question of Broca's Convolution might be best discussed in the light of psychology.

All that it is necessary to assume in the domain of physiology is that the development of the brain has progressed on the most efficient principle, which in this case is also the simplest, viz., that in which the web of connected nerve filaments and their junctions constitute the graphic image corresponding to the development of the psychic life, as in accord with the Fundamental Processes.

This being the case, let us now consider certain salient aspects of aphasia and their implications. The full meaning of the term, "word," involves an association of impressions from various sources of sensation, visual, aural, tactile, for example. The centres for these, we know from our physiology, are separate; therefore there must be strands of communication between them.

The word is spoken as well as heard. This involves a complex series of motor activities controlled by nerves serving different parts of the vocal apparatus, and therefore coming from different centres.

These again are related to the sensory system in a complex manner.

The individual word has no meaning unless it recalls appropriate images in association with such a continuous series of others as to lead to a definite form of thought. Here again we require corresponding strands of connection.

Next we reflect that since the brain, in order to function at all, must be regarded not as inert but as a storehouse of energy, then inhibition becomes as essential as stimulation; and the whole series of inhibitions required, standing in complex relation to each other and to the stimulation, imply also corresponding strands of connection.

We are now far away from the image of a "localisation" of function; we have rather the picture of the whole brain in readiness of function, and of a great number of nerve strands in activity of some form during the mental operations and their expression.

There are regions where the strands involved are more numerous than elsewhere. This appears to be the case at the third left convolution of the frontal lobe, and therefore aphasia may be produced by a lesion in that region. But if in other regions of the brain any of the essential strands of communication implied in speech be severed, then some degree of aphasia will result.

The mode of reasoning here offered in condensed form, is borne out by the researches of Marie, working at the Asylum at Bicêtre with cases, greater in number—more than a hundred—and more appropriate to the purpose than those of Broca himself.

If we proceed to the assumption that for certain senses—those for vision, hearing, smell, touch, taste, etcetera, as well as various motor activities—centres have been determined in various regions of the brain; and if we trace out the connections between these as corresponding to our psychological indications; then we have a conception of localisation of real value; and on the basis of this we find the possibility of building up a complete theory of aphasia.

Here again we have an instance where physiological problems lead to those of physics and chemistry.

The study of meteorology may be called a specialisation, but the proper comprehension of the subject involves an extensive acquaintance with the methods and results of various sciences. The questions that primarily present themselves may be said to be those of physics, such as the problems of the heating and expansion of the air in certain regions, the inflow of the colder air from other regions, the production of winds in consequence. But this problem soon becomes of extreme complexity owing to the influence of the earth's rotation in causing deviation of the direction of the winds; and owing to the various conditions of cooling on account of the distribution of continents, with ice-clad tracts, mountain ranges, wide expanses exposed to the sun; and the vast ocean areas, with their systems of warm and cool currents also. Here already we have added to the problems of physics considerations derived from physical geography and astronomy. The problems of astronomy involved are rendered complex by the influences produced by the movement of the earth in its orbit, its approaching and receding from the sun, its varying speeds in its orbit, its successive presentations of different areas to the greatest heat of the sun owing to the obliquity of the ecliptic with result to the equator.

These problems are again complicated by considerations of the effects of terrestrial and solar magnetism. This introduces us to a branch of astronomy usually treated as separate from that of gravitational astronomy, and demanding the co-operation of the work of specialists within its own field. In this way also we become introduced to a vast domain of terrestrial and solar chemistry, one of the aids of which is derived from spectrum analysis, which itself forms a special subject of absorbing interest.

The condition of the atmosphere is also influenced by the conditions of the central heat of the earth, and by disturbances of the earth's crust. Here we become introduced also to the very complex problems of geology.

As a tentative aid to our observations we seek to compare past results, to endeavour to form the means of demonstrating cycles of climatic disturbances. Then within a narrow scope the meteorologist keeps himself informed of the climatic conditions in various parts of the globe. Hence he is dependent also on a science of statistics.

It will therefore appear that for the proper understanding of special subjects, a vast amount of knowledge of other subjects is often not merely useful but essential.

Examples from Bacteriology

Let us consider in this respect one other subject, bacteriology. Bacteria being microscopical objects we must necessarily in our investigations be acquainted with the use of the microscope. This implies not only a knowledge of the principles of the microscope, but also a familiarity with various interesting points of technique, such as that of securing good illuminations, of illuminating from the side, or from above, and so studying the bacteria under various aspects; or by the use of the ultra-microscope, employing an exceedingly fine beam so as to bring to view objects otherwise invisible.

There is also a device by which the bacteria may be exhibited in movement. A drop of water containing living bacteria is deposited on a thin piece of glass called a cover slip. But as it would be impracticable to dip the end of the microscope containing the object-glass into this drop, the cover slip is inverted so that the drop now hangs down. But since it would again be impracticable for the cover slip now to touch the glass side on which it rests, it is separated from it by being allowed to rest on the ridge of some substance such as vaseline of a height slightly greater than that of the depth of the drop which is hanging downward from the cover slip. The technique is hence called that of the " hanging drop ".

The question of microscopy involves that of glass-making and this is so important that a distinguished German professor, Abbe, turned from his labours in physics to devote his attention to the manufacture of glass, and in the end laid the foundations of an important industry in Germany.

The question of increasing the power of the microscope has also demanded the attention of scientific men, and Gordon has devised a system which may be briefly described as superposing a microscope upon a microscope so as to multiply the effects; and this under certain circumstances is of the highest value. The question of microscopy is so vital to bacteriology that one of the most distinguished bacteriologists, Sir Almroth Wright, has written a substantial volume on the subject of microscopy alone.

The problem of the microscope in bacteriological work is complicated by that of staining materials. The bacteria cannot be studied in animal tissues without the use of stains by which they can be rendered distinct. The use of stains has also brought to light this fact, that some forms that might not be otherwise differentiated show different effects towards certain stains, some staining, for example, with an agent called Gram's stain, and others not staining at all with Gram's stain. Some of the most important

discoveries, such as those to which the name of Schaudinn is attached, in bacteriological work have been due to the selection of appropriate stains.

The whole question of the effects of stains on animal tissues and on bacteria forms itself an engrossing subject of study, and much work has been done towards its elucidation, particularly in Germany.

It is evident that the stains operate by reason of chemical reaction with the organic tissues, and by this means we are enabled to make such rough divisions as those of the parts having most affinity for acids or for bases. The whole question therefore involves a great range of chemistry; and also, for its interpretation, of physiology.

The observations of Koch respecting the possibilities of the culture of bacteria in certain substances, or culture media, gave rise to a branch of the science at which a host of scientific men have worked. The questions to be settled experimentally were those, for instance, of the best kind of media for different bacteria, the best conditions of temperature, and the limits both of heat and cold at which they were killed. Different behaviour under similar conditions suggested principles of division for further classification. The whole scope of the problem thus becomes that of the science of natural history of the bacteria, the conditions of their climate and habitat generally being varied experimentally.

Thus it becomes clear that if bacteriology be taken as a special study, mastery in this specialisation can be acquired only through the knowledge of a considerable number of associated sciences.

But if we now turn to the applications of bacteriology, we are led to believe that in medicine the problem of small-pox will almost certainly find its solution in some new discovery of bacteriology and that the study of cancer may possibly be advanced by similar means.

Subjects the most remote become in wider knowledge linked together. Studies of Natural History involve those of physiology and physiology involves electricity.

Examples from Mathematics

The science of electricity has been developed by means of an elaborate system of mathematics. The development of mathematics has often depended upon the considerations of questions of the finest analysis trenching on the domain of psychology. Thus the keenest insight of Abel or Riemann, the subtlest analysis of Huyghens or Fresnel, the recondite speculations of Pythagoras or Kepler, have in their developments resulted in fruits of practical value.

The question of specialisation is therefore never a question of the limitation of the subject, it is only to be considered in reference to the limitations of the student, his faculties, his quality of mind, his knowledge, his opportunities, his energy, his time.[1]

* * * *

Economy of Energy and of Time

In regard to diversity of knowledge there is a danger not so much from the incompatibility of studies, for the most remote experiences may finally become linked in a luminous chain of thought; but there is disability simply from the actual conditions of time and space under which we labour. Our lives are so very short in comparison with all we are capable of doing that all must renounce even the culture which would under more favourable conditions be necessary for the full appreciation of our own special science.

There is no incompatibility, except in point of space and time, between the labours of an astronomer and a bacteriologist; but few indeed would find it possible to acquire a mastery in both these sciences. Again, it is given to few to excel as a leader of armies and as admiral, though certainly the qualities of mind required are not dissimilar.

In olden times there was a frequent association between poetry and mathematics. Pythagoras studied problems of harmony, and to him is due the phrase, "the music of the spheres", which is again founded on a mathematical expression, namely, that of the order of the distances of the planets. Plato, whose thoughts are steeped in a poetic atmosphere, inscribed over the door of his school that none should enter who knew not geometry. In later days Leonardo da Vinci combined great mathematical and mechanical skill with a temperament ever inclined to research of poetical ideals. Pascal's mind was suffused with poetry which gives charm to his *Pensées* beyond their philosophical value. Even Kepler's speculations arose out of a kind of mystic Pythagorean poetry involving mathematical forms. In recent times such combinations have been rarer, principally no doubt owing to the exigencies of the study. Carlyle, however, had shown aptitude for mathematics, and Sylvester cultivated the graces of the Muses. The mind of Evariste Galois was of an ardent poetic type.

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[1] Even in special subjects, such as mathematics, we find that the results of one branch are often invoked in studies that apparently have little connection with it. Landen's transformation, for example, shows the applications of geometry to the problem of elliptic functions. Riemann was able to solve questions in the domain of the theory of numbers by reasonings which have passed through the route of elliptic functions, and the theory of functions in general.

Concentration Aided by Limitation

Yet withal, regarding the vastness of the survey now required in each of the great sciences, it is necessary severely to limit the range of mental activity. Bain has pointed out that it may be an advantage in regard to the prosecution of any given study to have distinct limitations of taste and aptitude for all that might prove of the nature of distraction.

Darwin, for instance, found as his interest in the scientific aspect of things increased, that his zest in sport diminished. Later he acknowledged that all his aesthetic tastes had departed except a liking for good prints. The only effect of music was to stimulate his thoughts. Newton saw in statues only " stone dolls ", while he considered poetry " a sort of ingenious nonsense ". And in the whole range of Shakespeare's work it would be difficult to find significant allusions to the profound scientific problems discussed in his day.

* * * *

In Complex Matters Seek Main Principles

The question of complexity of a problem may be separately considered.

Let us take a few examples. The steam engine has a very complicated appearance to anyone unacquainted with the principles by which it works. But if the principles be made clear, step by step, in such a manner that the student well understands what has been explained before proceeding to the next point, then the impression of complexity becomes less.

In this way it is best to explain the essentials first, and perhaps diagrammatically, though the use of diagrams should not be abused lest the student become again embarrased in observing the diversity of details with which he has been made familiar.[1] The movement of the piston in the cylinder owing to the expansion of the steam behind the piston is easily understood. The necessity of providing some means for the escape of the steam when the process is reversed is also easily suggested. The main problems of the difficulty are how to devise an arrangement of valves by which the entrance of the steam behind the piston at one end of the cylinder may be provided while, at the same time, communication between the other end of the cylinder and the external air (in the simplest type) may be allowed; and further, how the

[1] Edison avoided so far as possible the use of diagrammatic indications, preferring to make his studies on the actual system.

arrangement of valves and communications may be utilised in the reverse movement—namely, when the piston has traversed the length of the cylinder and must now be made to return.

Then there is the problem of converting an up-and-down movement, as of the piston, into a circular movement, as of a wheel. This transformation is effected by a crank, the working of which is easily understood on trial, although to those unfamiliar with machinery it is possible that many suggestions and tentative practical efforts would be necessary before the device of the crank became invented.

Once in possession of the main principle, and with a clear view of the most important parts of the machine, we find that the details present no great difficulty.

Though the best way of arriving at an understanding of the steam engine is that of first observing the essentials as described, yet it is possible that an individual always occupied about a steam engine and accustomed to the details, one by one, might arrive at length at such a comprehension of the working of the whole machine that the first impression of complexity would have been lost.

We see then that the impression of complexity is given by the consideration of a great number of objects associated in certain relations, particularly of dependence on other objects in such a way that the understanding of the nature of these objects is a condition necessary to a clear view of the relation of dependence. Hence in the understanding of a complex piece of machinery, as in the understanding of a number of diverse but associated objects in nature, we search by our analysis for some basis, which in one case is that of a generalisation affording a classification, and in the case of a machine is a principle of working, so that the associations that proceed from it are of that close kind which we express as of cause and effect. A reference to the discussion of the advantage of classification will make it clearer also why the work of Memory and of comprehension is easier when the machine is studied from its main principles onward.

We see that familiarity, or ease of reference, which results from repeated observations, diminishes the impression of complexity; for it leaves the mind free with all its energy to understand the step following those already considered.

The graphic representation, and particularly the study of the machine from an actual example, is of value in this respect, because the Memory is aided by the more vivid direct impression, and also by the multitude of associations that form about a concrete object observed in many relations.

The study of the steam engine also shows us that the part least difficult to understand is that of the principle of working. The ancients were familiar with the explosive and expansive effects of steam, but they did not know how to adapt this property to practical use, because the arts on which that use depended had not been sufficiently developed.

* * * *

Importance of Technique

This consideration reinforces what is said elsewhere in regard to the importance of a good knowledge of technique in matters where mechanical contrivances are necessary for the exploration of any domain of speculation. Watt, who invented the steam engine in a practical form not greatly different from the best steam engines of the present day, was an expert in mechanical contrivances. Thus the difficulty of complexity would not be present to his mind in the same degree as in the case of one who had only seized the main principle. Hence we learn with interest, but not with astonishment, that Watt prepared the main part of the work of invention during a quiet walk along the street.

Similar impressions of complexity occur to the mind of a lawyer who has to master a very long and involved case. It was Justice Hawkins, I believe, who declared that when he had looked at his brief for the Tichborne case he felt like a swimmer who had to swim across the Channel. In that case, to the impression of complexity was added the premonition of the physical endurance required.

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Find Development-Principle and Germinal Idea

The lawyer seeks for a guiding thread, or main line, or for the essentials of the case. These expressions refer to processes of analysis similar to those of obtaining the principles of classification or the principles of working of a machine. Considerations of the kind would eventually lead us, in regard to the works of permanent interest, to a discipline of study that might be called a logic of book-reading. In examining a great book, such as Spencer's *First Principles of Psychology*, or Bain's *Education as a Science*, it is not sufficient merely to proceed from position to position with the impression that the argument is justified. It is necessary to a good understanding so to analyse the book that, cutting away what has served its purpose of illustration, we see the structure of

the book in the form of a Classification, with a clear appreciation of the principles of division, and the relations between all the parts.

But that is not all that is advisible. We should search to find the germinal idea, the principle which gave rise to the book, and we should trace up the development from that basis. And in this way we should become so familiar with the essentials of the exposition that the details no longer have any air of complexity; so that the mind, being left free with its natural energy, may suggest new modes of exposition or illustration, or may anticipate arguments or make open the way to new developments.

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Complexity in Mathematics: Comparison with Daily Practical Problems

We must revert again to a consideration of mathematics, for it is there that we find the most striking examples of complexity.

In this science we have already noted the long consecutive arguments from a base easily comprehended, and we have noted that each step of that argument must be understood before the mind can profitably proceed on its course. We note also that as the development of the science proceeds there are frequent appeals to results already obtained. In such a work as Poincaré's *Nouvelles Méthodes de la Mécanique Céleste*, all the resources of graphic geometry, with its developments, are called into reference, as well as the resources of the analytical methods of the differential calculus, all the resources of that other development of algebra which makes the science of determinants, while in the determinants themselves, we have the expressions of special branches of study such as those of integral invariants.

Here is a high degree of complexity, and the impression of complexity is increased by the abstract nature of the conceptions and the recondite associations of the symbols. But in this complexity nothing new is introduced beyond what we have already considered. As we proceed with demonstrations following on demonstration we make demands on our Memory to secure that familiar ease of reference that leaves the mind with sufficient energy to grasp each succeeding demonstration and to behold it in all its relations. The familiarity should be of such a kind that, when one meets with an expression which is not of the form of a known result, but is capable of being reduced to it by means of known results, the mind ought to be lively in apprehension and suggestion.

Assuming that at any step the mind is familiar with acquired results, and that the Memory is good and active, then there is no

complexity in proceeding to the next result, for the terms of the demonstration should make it as plain as possible. Even in working over a new problem the method of analysis poses at each step a definite problem, which makes the matter as little complex as the mind finds convenient. Hence it is not uncommon for those accustomed to such abstruse speculations to work over and solve problems in the Higher Mathematics while walking about a street, or while engaged in other not absorbing occupations. Pascal solved the problem of the cycloid while tortured one night by the pains of neuralgia.

The impression of complexity is therefore entirely due to the sense of effort required to traverse the whole train of demonstrations, and hence in part to the work involved in retaining in Memory a considerable amount of acquired results which by reason of their abstract character have no strong associations. These conditions involve frequent repetitions or else the Memory becomes dim, and the effort of reproduction is notably increased. If in the end the mind becomes very familiar with the whole field involved, the actual feeling of effort becomes lost and with it the impression of complexity.[1]

To the complexity involved in the necessity of memorising a great number of formulæ and their relations may be added, in dealing with geometry, the complexity of constructing intricate figures in the mind. [2] In dealing with new problems it may be necessary to alter the construction tentatively from time to time. Hence it often happens that such problems make a greater demand

[1] Thus it requires a considerable mental effort to arrive at the idea of the multiplication of

$$\cos\frac{(2p+1)}{n}+i\sin\frac{(2p+1)}{n}$$ by itself.

Indeed the mind is unable intuitively to comprehend the process. But suppose that we have been familiar with the fact that the multiplication by itself of $\cos A+i\sin A$ gives as a result $\cos 2A+i\sin 2A$; and that we have already ascertained that in the process involved there is no reason why the angle $(2p+1)$ should not be symbolised by A ; then the result seems to offer no great complexity, for in the expression required we have only to double the original angle. Thus the proposition is that if

$$\cos\frac{(2p+1)}{n}+i\sin\frac{(2p+1)}{n}$$

be multiplied by itself the result is

$$\cos\frac{2(2p+1)}{n}+i\sin\frac{2(2p+1)}{n}$$

[2] The relation expressed is a truth ; the complexity has reference to our own minds and the means we adopt to arrive at this truth.

We might accordingly point out that if we were forced to depend on feelings of touch and the sense of muscular effort then we should find very complex a landscape, with all its relations which ordinarily we take in at a glance.

Or again if we say the principal ordinate of any point on a parabola is a mean proportional to its abscissa and the latus rectum, we express a truth that is so bound up in the nature of things that we cannot form a parabola but that, whether we actually draw any particular ordinate or not, the relation exists. There is no more complexity essential here than in the case of the falling of a stone. The complexity resides simply in our manner of arriving at the recognition of a truth that exists invariably.

on the attention than problems which depend on the transformation of formulæ.[1] We have seen in the case of the steam engine that the complexity was rendered less when the opportunity was offered of studying with the actual machine rather than by means of a representation in ideas.

This suggests in regard to the study of mathematics that a considerable advantage would be obtained by the actual tangible representation of all geometrical figures, and indeed all relations which can be graphically expressed.[2] Even complex geometrical constructions ought to be so exhibited; and this is especially useful when it is necessary to consider geometrical figures, under changing aspects. To each figure its formulæ should be conspicuously attached. Thus in the work of mental constructions we would be aided by Memory being made facile.

There is nothing unscientific in this method, for though in mathematics we deal with abstractions, yet these abstractions are founded on concrete objects. It must also appear reasonable that we may form as clear an idea of an isosceles triangle, or an ellipse, from a well-constructed representation in metal as from chalk lines on a blackboard.

Another illustration of complexity may be obtained from quite another field. Since, as we have seen, complexity refers to the difficulty of acquiring, without repeated experience, a knowledge of a great number of things and their relations; and in the second place to the difficulty of recalling our acquired knowledge with relations appropriate for comparison with new experiences; then the impressions, say of a countryman in a large city, would be those of complexity. And so we actually find it.

A problem of complexity in regard to movements in a large city might be of this character: Start from Clapham. Take the first train after noon that will bring you as near as possible to Charing Cross. There ask a policeman the way to the National Gallery. Take a guide-book and find your way to Turner's pictures representing Carthage. Study the colours by comparison with a plate of spectrum. Note the dominant colours. Then proceed to the British Museum. Look up in the catalogue a work bearing on Turner. Search in the index for reference to the picture. Compare your own note with the notions expressed by the book.

[1] Professor Klein pointed out a classification of mathematicians according to their bent towards graphic representations. At a recent meeting of the British Association a paper was read which expressed the opinion that some geometers were naturally inclined to the direct transformation of formulae, others to conceptions by means of graphic representations. Sir J. J. Thomson expressed concurrence.

[2] I have examined this question in *Une Question de Représentation Géométrique* (Paris).

The problem by reason of its complexity would be beyond the capacity of many; but to anyone already acquainted with London, and already versed sufficiently in the matters referred to, the suggestions of complexity would hardly arise.

Every cabman who finds his way about London and does his regular business deals with problems which from the base up are more complex than some of those of the Higher Mathematics; but incessant exercise from the earliest days of observation onward, combined with the advantage of dealing with visible, tangible things containing varied Associations, make the Memory more facile, and familiarity of experience removes the impression of complexity.

* * * *

Application to Discovery

If we now consider the whole question of discovery, we shall find no rules that lead inevitably to definite results, but we shall observe certain guiding principles that will be of service. Of these the most important is the desire of knowing the truth in preference to the defence of any kind of pre-conceived position.

We have already noted the progress of the mind in regard to a set problem. But as all discovery proceeds tentatively, the method is that of posing a problem which we seek to verify. It may be that the problem is of the nature of a somewhat recondite hypothesis, such as that of the undulatory theory of light, in the investigation of which many problems lie in the way.

The question of research then becomes resolved into that of setting forth hypotheses with the best judgment, or posing problems likely to lead to new discoveries.

We have had occasion also to consider what is meant in such cases by good judgment. It is the observation of concordances with what we have learnt to be the fundamental and well-established principles of a science; or the observation of cases of apparent, or, possibly, if the principles be not true, real discrepancies with these principles.

We should presuppose therefore in the investigator a knowledge of the directing principles of the subject investigated. The greater the acquired knowledge both of facts and principles, the better, other things being equal. But in some sciences the exhaustive study of all that has been investigated might make too great demands on the time and energy of the student. Thus erudition may be useless if it prove a bar to a certain liveliness of mind, freedom of association, and activity in setting forth and testing new problems.

With the necessary acquired knowledge there should be cultivated a habit of persistent analysis and that form of arranging and regarding facts, and seeking correspondencies in essentials, which we may speak of as Generalisation and classification. The progress of a scientific mind is marked by proceeding from the observation of particular cases to the search of the general principles involved; and from the general principle to the determination of other particular cases differing from those already observed; so that agreements between them may be thus exhibited and discrepancies at the same time explained.

A habit of classification will in general be found associated with accuracy of observation and record. A certain formal method is no doubt helpful in such matters, for many discoveries are due to the mere comparison of facts exhibited in statistics.

A natural energy expressed in activity of mind is in itself of great importance, for this implies, all things being equal, a greater rate of progress in the examination of problems, and it also implies a greater surety of Memory in so far as dependent on the quality of the mind itself.

The faculty of Imagination we have seen to be resolved into that of seeking resemblances and concordant associations whether of observation or construction; but the term would be more likely to be used where the fields thus associated might appear at first sight to have little connection with each other.

This implies that the mind must not be too narrowly absorbed in a certain field after that field has yielded all that it can give. The danger, however, is that the attention may become diffuse and that Memory and associations may be dispersed over too great an area.

Hence it may be a disadvantage to possess intellectual qualities or artistic tastes, even of a high order, but likely to lead to dissipations of energy; and conversely limitations in this respect may be a condition of greatness.

A deep interest in the matter of study is essential, for this implies the continual recurrence of the mind to the same field of exercise and associations with all that is otherwise observed in the course of other experiences. A mastery of the technique of the subject is very important. A man of science accustomed to the use of the instruments of research is able to test the suggestions which arise in his mind, and to learn something even from those that may not have seeemed productive. On the other hand, a person inefficient in the technique of a subject leaves these suggestions vague in his mind, and for that reason does not proceed.

In mathematics, skill and technique would be represented by

familiar acquaintance with formulæ. Efficiency in technique can be acquired only by practice. Hence the value to a scientific man of laboratory work. The constant daily service, the familiarity with methods, the faculty of testing new suggestions, are all matters of invaluable aids. The danger is that he may become limited to a certain routine; that he may be inclined to defend some preconceived thesis instead of searching always for the truth; or that he may be deficient in knowledge which, though really bearing on his subject, perhaps even being indispensable for the explanation of some of the phenomena observed, yet has lain outside a prescribed study of his speciality.

Or again, a good laboratory professor may be deficient in the speculative and imaginative intellect necessary to pierce through accidentals and see in remote things essential resemblances.

Certain it is that some of the greatest advances of science have been due to those whose study has been outside the schools of learning in these matters, and that frequently those who have least technical knowledge have thrown the most illuminating light on a subject.[1]

One reason is that their attention is not absorbed by a mass of details and that their minds pursue a course of analysis more persistently. Another reason not without its influence is this: That in science as in all other subjects of human activity, emulation is of great importance. Now one who is in constant touch with fellow workers is more apt to spend his energy on the points that arise in discussions and to obtain results which he knows how to set well in the light. A solitary thinker must strike down deep into general laws before he can obtain anything worth presenting to the world. The examples of Darwin, of Dalton, of Herbert Spencer, may be cited.

A methodical manner of work; great accuracy not only of observation, but also of record; a system of explanation well conceived and steadily worked through—these all seem factors of success in the way of discovery.

That which seems most important to all work of discovery is a great deal of preliminary work in the field of research. A mind persistently and actively occupying itself in a certain domain has, however, need occasionally of gaining ideas outside those found in a set routine.

Yet all that has been here written must be regarded in the light of guiding indications, not as rules. Some men, even of great

[1] Priestley affords an instance of a man accomplishing great work in science though but little versed in its learning. It is said that when making his discoveries in chemistry he could not have carried out a quantitative analysis.

general aptitudes, seem to have an irresistible bent towards certain fields of speculation. Thus we have the elder Herschel who, in the intervals of playing in a band, advanced the science of astronomy amazingly; Spinoza, whose whole passion in life was that of metaphysical speculation; Evariste Galois, who at the age of twenty-one had thrown illumination on the heights of mathemetics; Darwin, whose career is summed in the elaboration of his theories; Descartes, who hid in Holland the better to think in his solitude; Galileo, who faced the terrors of the Inquisition in his overwhelming desire to behold the truths of Nature.

CHAPTER III

GENERAL SUMMARY OF REASON

The whole of the preceding exposition may be regarded as an analysis of Reason Therefore Reason might be defined as the continued combination of the Fundamenta Processes of the mind so as to arrive at results in accordance with the conditions of the external world.

But such a definition requires to be expounded so that we may form a notion of the manner of such combinations. In view of the complexity of the matter that now arises we may have recourse to a graphic representation founded on the actual structure and constitution of the body and particularly of the nervous system.

Upon this system let a force impinge, and let it find a representation in consciousness, as, for example, a sensation. The sensation, if powerful enough, is accompanied by a Sense of Effort.

The tendency is for a disturbance of the system to find an outlet in motor activity. The form of activity is determined by a recognisable Impulse accompanied by a Sense of Effort.

This elementary scheme, however, is only to be regarded as an aid to exposition in an intricate subject, and we now proceed to consider it in more complex development.

Conceive then a system containing a vast number of forces mostly held in equilibrium. On this system impinge various forces of the external which form Immediate Presentations in consciousness. Even though capable of being expounded into a great complexity of separate factors, yet at each instant of Time the total of such impressions is summed up as a Unit. Its effect upon the system is continued after another Unit has made its impression. The internal forces of the system also, never in complete equilibrium, are producing various effects. At each instant of Time there is a resultant of all these. Certain impressions from without and from within may have no effect in consciousness. But a repetition of impressions, or a strengthening of impressions formerly too weak to produce an effect, may at length result in consciousness.

An effect in consciousness has not only its specific character as an Immediate Presentation, for example a sensation, but it may be accompanied by a feeling which is not identical with the specific character. It may be that this Sense of Effort is not perceptible, but by repetition, or by increase of force, it may become perceptible. In that case it is Fundamental. Like external forces acting on the system tend to produce like effects; accordingly the response of consciousness is affected by a recognition of Agreement. As, however, the recognition is made more manifest by an interruption of a movement, we find in that case the Negation of Agreement, and in general it is more convenient to speak of this process which we call Discrimination.

Though the mind recognises an Immediate Presentation as a Unit at each instant of Time, yet that Unit and a succeeding Unit are bound in an inevitable relation of which the mind has consciousness, and the term Association applies to all such relations. There are various factors on which the strength of such Associations depend. A strong factor is immediate succession, but a stronger factor may be derived from emotional accompaniments.

Sensations may be accompanied by an Hedonic sense. It may be that this is imperceptible until the sensation is frequently repeated; but, according to an analogy before used, it must be considered that with every sensation there is an Hedonic influence which, even if not perceptible, nevertheless affects the resultant in the mind. When the Hedonic sense is perceptible it is Fundamental.

A Unit once experienced may be revived by Memory, so as to have Associations, though less vivid than in the original experience, with other ideas; and Associations may take place between such ideas and Immediate Presentations. The fact of Agreement and of Association implies Generalisation as a Fundamental Process; and then inevitably arise also Symbolisation and Classification.

Considering now the resultant of the system at each instant of Time, we find its direction marked by Impulse accompanied by a Sense of Effort, particularly when the resultant finds expression in direct and powerful muscular exertion. But as, when the intellectual life becomes more and more developed, there is found an increasing extent and freedom of Association of ideas, so in the outgoing Impulse there is found an increasing extent of combination of Impulses, which have to the actual outcome in motor activity a relation analogous to that which ideas hold to actual experience. In this play of the mind each of the Fundamental Processes has its reverse, or Negation. The mutual implications of the Fundamental Processes are expressed at the end of Pt. II, Chapter IX.

Such is the general statement of the activity of the mind. If the statement appear somewhat colourless and barren, the explanation lies in its generality. This implies no vagueness or incompleteness; on the contrary. Just as a general theorem in mathematics may appear at first more vague and less interesting than some special example, until at length it is seen in operation covering that example, and is then beheld as far more fecund and more powerful in influence than any of its special cases; so the true importance of this analysis of Reason will be found by reading it in the light of all the examples that have hitherto been given.

The natural operation of the Fundamental Processes themselves, in relation to actual experience, give rise to a great number of secondary positions; and we thus build up what we know as Ego, Will, Abstraction, Symbolisation, and Generalisation and Classification in an extended sense; Fancy, Imagination, Speculation; Research and Discovery.

With regard to Research it has been shown that we proceed by throwing out hypotheses; and hypotheses are really only the complex analogies of what in the simplest form is the Association proceeding from Agreement, possibly an erroneous Association due to the defects of our Discrimination.

Now the question of verification or testing these hypotheses is not dissimilar to that of verification in any set and determined problem. The great special factor of research is therefore the setting forth of good hypotheses.

The criterion of a good hypothesis may be thought difficult to define, for we cannot know that it is good until we verify it. Yet by deep study of the world we find in it certain correspondences, and regularities, "laws", which assure us that the true explanation of phenomena will be eventually something more simple, more symmetrical, and more fecund of new suggestions and discoveries, than our previous view afforded.

Observance of these principles makes the differences between a good hypothesis and a mere futile guess. As the first hypothesis may not be right we see that to the reasoning, or discovering, mind there arises the advantage of imagination, energy, order, of familiarity with the subject only to be attained by immersing the mind in its facts and relations. We see too the necessity of piercing deep in regard to the subject, and also the advantage of knowing facts and relations from other realms that may serve to offer explanations of difficulties.

Hence we arrive at a true test of specialisation, which is not necessarily that of restriction to formal limits set forth by others, but rather an exploration determined by the veritable nature of the problems themselves in relation to the qualities and acquirements of the investigating mind.

All these matters must be considered in the study of Reason. Even in the ordinary course of life, and apart from scientific inquiries, we find nevertheless the occasion

for research, for making experiences, for the discovery of the unknown; a good reasoner is not thought to be one who merely tests the arguments of others to prove or disprove their intrinsic consistency.

In the scientific realm we find also the especial importance of the mastery of technique. This offers means of testing hypotheses which otherwise float in the mind as tentative and ill-defined. The actual testing offers new suggestions, and high technical skill will be found to have a valuable effect on energy, and even on the true use of the imagination.

Finally, all these principles will be seen to be vivified in actual experience, and it may be observed that every exercise of Reason from the closest criticism of argument to the most determined analysis of a problem, or from the casual, almost automatic, operations of the daily round of life to the subtle and airy imaginations of the thinker or the poet, are all expressible in terms of what have been here set forth as the Fundamental Processes.

If this be, however, an exposition of Reason, there still remains in the mind the desire to find the matter expressed in some succinct and tangible form. The problem becomes of extreme difficulty, for tangible here means popular, and that word implies a certain looseness of expression and at the same time a principle of easy applicability. Succinctness implies generality, but does not dispense with the need of exactitude. We must therefore attempt to hold within the limits of a definition some principle that will apply to the highest forms of scientific reasoning as well as to the most current modes of passage from one set of ideas to another. We must exclude mere aberrations, for otherwise the whole discussion becomes futile. But we cannot exclude false reasoning; for how is it possible in a mere definition to offer an infallible test of right or wrong ratiocination, and how can we remove from our purview so much that occupies our attention in debates of importance?

All these considerations show that the definition should be treated sympathetically, and that a certain latitude of language must be allowed. I will in the definition, therefore, speak of "facts", not necessarily implying that they are established truths, but taking this as a useful word more elastic than objects and more easily understood than any cumbrous phrase expressing relation of objects. The word "normal" will be used simply to exclude aberrations. It would not be well to refuse consideration to the errors, say, of the colour-blind. Such persons could reason quite intelligibly in their own company; and if it be objected that their conclusions would suffer from lack of Discrimination, the reply is that at a certain level we are all liable to that failure.

Therefore we say that a man may reason correctly, given defined conditions, or in regard to certain environment, and may reason incorrectly when the conditions or the environment vary.

Finally, therefore, with all these provisos we may sum up by stating: Reason is the mode of mental activity which consists in proceeding from fact to related fact in accordance with the normal exercise of the mind and in conformity with the conditions of the environment considered.

The Fundamental Processes Considered in Regard to a Dynamic Model

Throughout the preceding exposition, though the necessity of the Fundamental Processes has been shown in each successive incident of the progress of the mind, yet it will gradually have become apparent that they are not similar or co-ordinate, or, as we say in a manner to be understood, not all on the same footing.

It was well not to express this at the commencement, because we were introduced early to a series of new suggestions and to a manner of exposition not familiar.

Now, however, we may take a review, and, for the sake of that illumination necessary to progress in Psychology, to indicate at least certain features of distinction.

Time and Space, though inseparably bound up with such Immediate Presentations as Sensation, are yet something distinct. We gain the impression that they are conditions of the world in which we live. Certain philosophers have attempted to explain Time by means of Space. [1] But this is futile. It is more feasible to explain certain aspects of Space by means of the intervention of Time, as we have seen in the analysis of Externality. Yet, withal, in their simplest modes Time and Space are unanalysible.[2]

The Hedonic principle, again, may not appear essential to the mere mechanism of reasoning. Certainly we cannot exclude it, for it is part of our world; and, as I once heard a philosopher say [3], after a deep thought, in answer to a question of mine, "I believe in the necessity of the world". The Hedonic principle, while a necessary part of our mode of existence, may, however, be regarded as a boon of Nature; for it is the ultimate touchstone which shows us that a universal pessimist philosophy is false.

The Conception of the Unit we saw also to be an inevitable condition of the mode of action of our mind; it is a Fundamental Process. But it is not a Fundamental Process in the sense that it is recognised by way of Immediate Presentation, but rather it is a necessity, discovered in analysis, of the limitation of our powers. This principle then stands on a plane different from that of Sensation. It is a condition of action, a condition again not found at large in the structure of the objective world, but in the nature of our minds.

The Sense of Effort is essentially a part of mental movements correlated to the nervous activities involved in consciousness; therefore, as for instance when related to Sensation or to *Innervations-Gefuhl*, we may take it as included in the whole psychic state corresponding. Certainly it has been necessary, for the purpose of our analysis, clearly to define it as a Fundamental Process, and to exhibit it in this character subsequently as an element of synthesis in building up the course of thought. But now, having before our eyes a dynamic model of the working of the mind, we may consider the Sense of Effort as an incident, though an inevitable incident, of mental operations.

The recognition of Agreement might also, for the purpose of simplifying the dynamic model, be included under the heading of Association as forming, in conjunction with a mode of Memory, a certain form of Association.

Negation occupies also a position not co-ordinate with that of any one of the Fundamental Processes; for it applies to all, and in the dynamic model may be represented as the reversal of any action.

There is in this view now left to consider: Immediate Presentation, Memory, Association, Impulse, and Generalisation.

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[1] I take the following from J. M. Stewart's *Critical Exposition of Bergson's Philosophy*:

"The idea of number, even of abstract number, implies the simple intuition of a multiplicity of parts or units exactly similar to each other. They must all, at first, be grasped as side by side, and then, by a synthesising act of mind, added together. It is in space that this juxtaposition of units takes place. All explicit counting, presumably, whether it be numeral or ordinal, involves the holding of a number of exactly similar units in one moment, in a 'now,' before the mind—i.e., it involves the idea of space."

Herein is involved a failure of analysis of counting, on which is based an argument, which in vain I have tried to follow, of the relation of Time and Space; and from the standpoint of Bergsonians I would ask: Is it by such ratiocination, even if they themselves are content, that they can hope to bring the light to others? And do they appreciate the character of the movement of the mind by which a chasm of darkness is to be traversed by means of that psychic vehicle, 'now'? The true explanation, as given in this Chapter on counting, enables us to see what Bergson was groping at.

[2] Here at this fundamental level we get the true answer to certain metaphysical speculations of Einstein.

[3] Sir Ray Lankester.

With the first four of these we can form an image of a mental process corresponding to the simplest dynamic model but wherein a new force impinging on a system produces a resultant.

The Fundamental Process of Generalisation leads inevitably, as we have seen, to symbolisation and classification, and these faculties, playing on the whole of the diverse objective world, enable us to branch out from the simplest mental act (still using the dynamic model) to the vastness of that complexity which forms our complete psychic life.

We are now in a position to reply to two questions of quite opposite tendencies which may possibly have been posed in the form of objections. The first would dwell on the character of the total world of thought—its complexity, heterogeneity, obscurity, lack of definition, as well as the exceeding vastness of the range and the diversities within any limited scope of that range; how is it possible to express all this in terms of a few Fundamental Processes?

The other would indicate the expectancy of finding something in the ultimate mechanism simpler than the display of our Fundamental Processes; for how otherwise do we deal so familiarly with Reason, and how do we express its forms so as to be intelligible to minds of no great development?

It would not be reasonable to say that these two tendencies contradict each other, and that the truth lies in the middle; that is merely a forensic or judicial way of settling questions.

Both the positions indicated contain a basis of truth. We have already seen how the vast whole of thought falls within the operation of these Fundamental Processes, the impression of complexity being given by syntheses of these processes in conjunction with the diversity of objects; and we have already replied to those who on a confused view of simple forms might consider the list of Fundamental Processes excessive.

But now again, in this manner of holding up a dynamic model and of regarding the Fundamental Processes as not being all on the same footing in that regard, we arrive at a clear appreciation not only of the difficulties but of the explanation.

In other words, just as in the exposition of Externality we did not wish to find Common Sense abolished, but simply rendered more and more perspicacious; and as in the consideration of Will we still paid respect to the indefeasible demands of Common Sense; even while in both cases doing no violence to the true doctrine of Idealism, but illuminating it and rendering it both more consistent and more subtle; so in here dealing with Reason at large we may find a nexus between an Idealism imbued with the most exquisite delicacy of analysis and a Common Sense that sees essentials in the gross.

And in this view, and with all the provisoes and conditions already mentioned, we repeat the definition already given: Reason is the mode of mental activity which consists in proceeding from fact to related fact in accordance with the normal exercise of the mind and in conformity with the conditions of the environment considered.

The task of Psychology, and ultimately of all science, consists in illuminating this statement, in exemplifying it in diverse instances and in all manner of combinations, in rendering its terms precise and showing the relation between the objects of its reference. Those accustomed to the methods of mathematics would express the matter as: Showing all the modes of application of the general proposition; the Evolutionist might say: Developing the basic phase. The instrument of exposition is found in the system of the Fundamental Processes.

POSTSCRIPTUM

We are now in a position to deal with a question which might have been answered at the beginning, but only in a manner that would then appear somewhat tentative and vague.

That question is as to the advantage of this system, or to the use of Psychology in general.

To many, I believe, a sufficient reply will have been given already in the illumination that has been offered in the study of various problems that have beset and tantalised the minds of thinking men in all eras. The highest want of the human being is that of knowledge in the matters that point out to him the upward path of his efforts, and that afford him guidance in his spiritual life.

It was from such ethical considerations that I found myself led to the study of the mechanism of thought itself, and step by step I was brought to see that I could gain a clear view in these matters if I could discover the ultimate components, the elements, or, as I have here called them, the Fundamental Processes of the Mind.

Since by the combination of these all forms of mental activity are built up, it must be evident that a knowledge of these processes will be of use to the psychologist, as that of the physical elements to the chemist; and just as the world of material things becomes more tangible and more maniable to the chemist who has knowledge of its constituents, so also the operations of the mind are more intelligible to the thinker who beholds the modes of the synthesis.

But there are other uses of this system, for instance, as an instrument of training the mind. A knowledge of the anatomy of the body is useful in regard to physical education; a knowledge of the mechanism of intellectual operations is of value for mental training.

The application may not be immediate, but that does not impair the force of the argument, for the study of Applied Mechanics, for example, is not embraced within pure mathematics, yet acquaintance with a form of calculus conceived abstractedly is indispensable to the understanding of mechanical problems; so by analogy will a foundation in psychology be yet regarded as essential to the proper study of questions in the spheres of education and philosophy.

It is therefore well to accustom the mind to think in terms of the Fundamental Processes, so that automatically, and at length

without the feeling of strain, they become applied to all questions in the ordinary course of thought.

The great thinkers from Pythagoras onwards referred to in the course of the exposition have not been invoked in order to rest arguments on their authority; they are cited simply for purposes of illustration. Their achievements in regard to Psychology have been disparate and tentative; but in the light of the Fundamental Processes we perceive the principles that inter-penetrate all that work. By this system it is possible to find the alternate basis of correlation of all these efforts and results, and to cast a beam of illumination throughout the diverse fibres woven into that great organised body of thought.

The process may be likened to that by which a traveller in an unfamiliar part of a city traversed by an intricate maze of cross-streets guides himself by a plan showing reference to some well-known main thoroughfares. In regard to subjects of complexity the mind will, with practice, be enabled to behold the problem in its relation to the Fundamental Processes, and descending rapidly to that level will thence envisage the questions in their simpler aspects, and ultimately attain something approaching to an immediate " intuitive " view of the whole matter.

Let us observe on broad lines how this would apply to the study of the sciences. These may be conveniently divided into the physical, the biological, and the mental sciences; and they may be familiarly represented by the engineer, the physician, and the teacher.

A good deal of the engineer's work depends on mathematics, and mathematics deal in great part with the figures produced by combinations of straight lines and regular curves.

Again most of the occurrences in the physical world, the phenomena of attractions and repulsions exhibited in the domains of sound, light, electricity, magnetism, gravitation may be related to combinations of straight lines and regular curves, so that our mathematics serve in their interpretation. But in the research of the Fundamental Processes we have been led to the study of the foundation of mathematics, and thence to the examination of the character of new processes, the doctrine of limits, imaginaries, etc., that at various turning points have been introduced.

Here then we see from the ground up a flood of illumination making the whole domain clearer.

Turning to the biological sciences, we are led to attach great importance to classification, which is but another aspect of a mode of thought of which we find the rudiments in our Fundamental Process of Generalisation. The biological sciences are intertwined with the physical sciences, as for instance, in the employment of

instruments such as the microscope, and also in the reliance on chemistry.

A fair part of the work carried on from that foundation is that of observation and description and deduction, and here we come to a domain trenching on the mental sciences. We have already seen that principle in operation with regard to Broca's convolution and in the interpretation of various observations in physiology.

With regard to mental sciences it is not necessary here to elaborate, for the whole of the exposition has been immersed in that subject.

There are, however, special fields which seem specially to invite cultivation in the light of these principles. The study of Reason, for example, leads to a consideration of the forms of error. Here is a subject of which I have endeavoured to indicate the main lines; a development of the principles would result in a branch of science having a relation to formal Logic analogous to that of modern biology compared to the old infertile systems of classification.

Finally we reach a view of moral questions, founded on a consideration of the sciences which attempt to explain the Cosmos, as well as on those delicately-adjusted modes of the mind involved in human relationships; and as it was the endeavour to obtain light on these that led ultimately to the present exposition, reciprocally the value of a good basis in Psychology will be apparent in matters of Ethics.

Thus the assertion becomes justified that Psychology is the matrix of all other sciences, and that, properly understood, it affords an illuminated view throughout the range of the complex woof of thought.

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